

Dissertation

Robust Control and Persistence in New Keynesian Monetary Policy Analysis

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Prof. Dr. Michael Funke
Institut für Wirtschaftssysteme, Wirtschafts- und Theoriegeschichte
Arbeitsbereich Makroökonomik und Quantitative Wirtschaftspolitik

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Erstgutachter: Prof. Dr. Michael Funke

Zweitgutachter: Prof. Dr. Thomas Straubhaar

Vorsitzender des Prüfungsausschusses: Prof. Dr. Christian Scheer

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1 Introduction

The relationship between inflation, real economic activity, and monetary policy has been of major economic interest since the 1950s. Conditions under which monetary policy is neutral, in the sense that it influences prices without affecting real economic variables, are rarely fulfilled in any economy. Thus monetary neutrality is at best only guaranteed in the long run, and the monetary authority is constantly confronted with the temptation to influence, not just inflation, but also real economic variables in the short run. Increasing the interest rate too much to dampen inflationary pressures could cause a recession with high costs in terms of foregone output and rising unemployment. Therefore central bankers have to base their decisions on a broad investigation of current economic conditions and expectations about the future. At least since Keynes (1936) no economist would deny that uncertainty should be one major issue entering this decision process. Expectations are forecasts and thus always uncertain, but also the study of current conditions is shrouded by uncertainty about real data. In addition, when it comes to concluding policy makers have to assume specific relationships between the interest rate and the economic variables their decision will influence. It seems that the role of a central banker involves a lot of worrying.

Academic research has yielded a vast amount of models differing in assumptions about economic relationships in order to derive helpful suggestions for the conduct of monetary policy. However, none of these models will ever map reality perfectly and every model can only be an approximation. Thus policy makers who base their decision on academic research face the problem not to know which model to trust. Researchers also dispute about the "real" relationship. The major controversy in the recent debate about monetary policy models is about the (un)importance of expectations in the dynamics of inflation and output. Empirical studies lead to different results that raise more questions than they answer. Thus, being a researcher seems to involve a lot of worrying, too.

This thesis tries to contribute some helpful insights to policy makers and researchers and analyzes uncertainty in models that differ in their degree of persistence in inflation and output dynamics. We do so by employing recently developed robust control techniques, which capture the fear of private agents and the central banker against model misspecification by adding a restricted worst case shock process to the model equations. This reflects the worries of households, firms and the monetary authority, who all know that models are only approximations of reality. The worst case process can be interpreted as a malevolent player, trying to distort the economy as strong as possible. Thus he hits the model where his disturbance has the most damaging consequences and adequately reflects the concern of a policy maker, who fears the worst possible outcome. A robust decision is one that bears this concern in mind and works well in the worst possible scenario.

Robust control techniques, developed by Hansen and Sargent (2007) among others, gain more and more popularity in analyzing monetary policy under uncertainty.

While the first generation of robust control applications focusses on backward-looking models, more recent studies investigate the influence of a preference for robustness against model misspecification in forward-looking economies. For both model specifications, academics suggest reacting more aggressively on deviations from equilibrium, but so far no study considers the connection between uncertainty and persistence. Thus the analysis in this thesis ties up to the most recent work in monetary policy by filling this important research gap between purely forward- and purely backward-looking models. To do so, we build a hybrid New Keynesian framework, which has become the state-of-the-art model for monetary policy analysis during the last two decades. The model is based on a huge microstructure, including modern macroeconomic techniques like utility maximizing households and firms, and Calvo-type price stickiness. Furthermore, by assuming persistence in consumption habits of households and partly backward-looking price-setting of firms, a small-scale model including persistence in output and inflation dynamics is derived, where the persistence depends on structural parameters of the private sector. Thus the model is resistant to the criticisms of conventional aggregate demand-aggregate supply models, that often lack of microeconomic foundations. Moreover, the analysis does not investigate changes in the persistence of inflation and output without explaining where they come from.

Due to the increased openness of many countries and the foundation of the ECB, international interdependence between economies has undeniably increased during the last years, and research on open economies and two-country models receives more and more popularity. In fact a new strand of promising literature has been established, including nominal rigidities and imperfect competition, that maps real trade relations, international interdependence and financial markets very well. This strand proves to be very helpful, when we are talking about exchange rates and trade balances. More important, the inclusion of a utility maximizing framework allows one to derive welfare conclusions that are useful for policy makers. So far, the analysis of uncertainty in these models have not received the appropriate amount of attention. Since increased interdependence could increase uncertainty, as central bankers additionally have to worry about future exchange rates and possible feedbacks from foreign monetary policy, we believe that analyzing a two-country model under Hansen-Sargent robustness is straightforward. Thus the single-country economy is extended to a two-country setting to investigate the influence of a preference for robustness of private agents and central bankers on optimal monetary policy. In order to give helpful conclusions with respect to the persistence of the economy, we use the same assumption on consumption habits and partly backward-looking price-setting, used in the single-country analysis.

The remainder of this thesis is organized as follows. Section 2 gives a brief introduction to the art of monetary policy and the corresponding types of uncertainty. In section 3 we discuss the New Keynesian perspective on monetary policy and differences to previous approaches are worked out. Subsequently, section 4 summarizes the Hansen-Sargent methodology. The following section 5 surveys studies on un-

certainty in monetary policy models, before section 6 analyzes the consequences of Hansen-Sargent robustness in a hybrid closed New Keynesian framework. Section 7 describes the historical evolution of open economy and two-country models from the Mundell-Fleming model of the early 1970s to the latest two-country models, while in section 8 the influence of a fear about model misspecification in a hybrid two-country New Keynesian economy is investigated. The final section concludes.

2 Monetary policy and uncertainty

As already stressed in the introduction, uncertainty dictates that monetary policy is a combination of art and science. Thus the following section presents some stylized facts about monetary policy, derived from academic research. Subsequently, 2.2 classifies different types of uncertainty faced by a monetary authority.

2.1 The science of monetary policy

When it comes to investigating monetary policy, academic researchers often assume a central banker setting the interest rate in reaction to inflation and deviations of actual output from its natural level (the output gap) to guarantee economic stability. The most famous policy rule of this type is that referred to as the Taylor-rule,

$$i_t = 1.5\pi_t + 0.5y_t, \quad (1)$$

where i_t , π_t and y_t represent the short-term interest rate, inflation and the output gap at time t , respectively.¹ Taylor (1993) shows that his hypothetical policy rule is in fact representative, as it approximates the Federal Reserve policy throughout the 1990s quite well. Since this seminal paper many economists derive similar policy rules using economic models, most often by introducing a quadratic social loss function, which conventionally includes inflation and the output gap. Optimal monetary policy is derived by minimizing this loss function with respect to the assumed transition equations of the economy.

There is some controversy concerning the weights in the policy maker's objective, as a trade-off between reducing the variability of inflation and the variability of the output gap appears.² Taylor (1998a) refers to reduced inflation stability as the opportunity costs of improved output stability (and vice versa) if monetary policy lies on an efficiency curve.³ He also believes that these opportunity costs

¹Throughout the whole thesis we assume a short-term interest rate as the policy instrument, and do not participate on the discussion of the optimal instrument choice. Furthermore, we assume all variables to be measured in percentage deviations from mean.

²For empirical evidence on the output/inflation trade-off see Owyong (1996).

³Such a curve follows the logic of a production possibilities curve, but here output and inflation stability make up the axes. See Taylor (1998a), p. 38, or Clarida, Galí, and Gertler (1999), p. 1672 f., for more details.

are increasing, so that the additional costs of reducing output fluctuations by one unit are greater, the more reduction that has taken place previously.⁴ This trade-off arises from the different impacts of shocks on inflation and output and was originally emphasized by Taylor (1979a). An upward demand shock for example causes output to rise, bringing about a positive output gap, if prior to this shock actual output had been equal to its potential level. However, the same demand shock causes inflation to rise. A central banker has two possibilities to react to these circumstances. The first is a very sharp response in order to reduce the inflationary pressure, but this policy also slows down economic growth and could lead to a recession. The second possibility is to react with caution and therefore dampen the pressure on inflation to a lesser extent, but also have a smaller effect on output and avoid the danger of a recession. The first response increases inflation stability with the costs of a higher output instability, while for the second response the opposite is true. Concerning a supply shock the problem becomes even greater, since inflation is shifted upwards, while the decreasing real money balance leads to a negative output gap, so that every increase in the interest rate to reduce inflation implies that the output deviates further from its potential level. Seeing that such shocks occur continually, it becomes clear why there must be this trade-off. Thus a policy on the efficiency curve has to take a position on the importance of inflation stability versus output stability.⁵

In addition to the controversy about the relative weight of output stabilization to inflation stabilization, there are also other variables that could enter into the objective function of an authority. There are good reasons to include a term representing a stabilization of the interest rate itself (i_t^2) - often referred to as *interest rate stabilization* - or the change in the interest rate $(i_t - i_{t-1})^2$ - often referred to as *interest rate smoothing*.⁶ With regard to the variability of the interest rate there are two main motives for including it. First, in a regime with a low average interest rate there is no possibility to overreact to deflationary shocks because nominal interest rates are bounded by zero, so that interest rate reductions *cannot* be so substantial. A rule that possibly results in a negative interest rate is not useful, and including the interest rate in the objective can prevent such a scenario. More importantly, volatile nominal interest rates always imply distortions (as resources are wasted on unnecessary efforts) that are related to the size of interest rate *fluctuations*. As a result policy makers have an incentive to stabilize the interest rate.

Woodford (2003) argues that an authority should also be concerned about the variability of the *change* in the interest rate, since adding lagged endogenous variables makes policy history-dependent. Without this history-dependence "(...) it

⁴There is also some empirical evidence that these opportunity costs increase sharply (see Taylor (1992)).

⁵As with every production possibilities curve there is the chance to shift out the curve with the help of improved technologies. In this context this could be achieved by improved financial market data (see Ball (1994)).

⁶Note that deriving a Taylor-type rule from an objective, including a preference for interest rate smoothing, would lead to a rule which also depends on the lagged interest rate.

is not possible to arrange for expectations to respond to shocks in a desired way unless subsequent policy is affected by those past shocks in the way that one would like the private sector to anticipate" (Woodford (2003), p. 863).⁷ In this he follows Goodfriend (1991) who suggests that output and prices respond to *longer-term* interest rates, so that a policy maker should affect these longer-term rates which are determined by the expectations of the future short rates. As a consequence the goal of stabilization could only be reached by making the private sector *believe* that the future path of short rates has changed. In his view, maintaining interest rates at a high level for some periods longer than necessary or implementing a sequence of small interest rate changes can achieve this job and represents a moderate adjustment of short-term rates which has a great effect on long-term rates. Thus significant effects upon aggregate demand are not necessarily accompanied by volatile short-term interest rate changes, and a central banker has a rationale to smooth the interest rate.⁸

2.2 Model uncertainty

If we are to talk about the different types of uncertainty faced by a central bank, we have to distinguish between at least four types: data uncertainty, shock uncertainty, policy uncertainty, and model uncertainty.⁹ The focus of this thesis is on the latter, which can be further divided into at least four different types. The resulting classification scheme is shown in figure 1.

A central banker faces *data uncertainty* due to the fact that he is dealing with incomplete information about the current state of the economy. The output gap, for example, is only available with a time lag of weeks, sometimes even months. Thus, monetary policy is based on estimations, which are often deficient, or based on old data. In addition, data of different countries are often difficult to compare. This is particularly problematic for new monetary unions like the euro area, as a monetary policy for the whole union is required, but evaluating the consequences for the various members of that union is highly complicated. Furthermore a policy maker must deal with at least some unobservable data such as equilibrium exchange rates and interest rates or the output gap. Different methods of measuring these unobservable data can lead to substantially diverging results. That is why estimates of non observable data are often revised considerably, even after a time period of several years.

⁷We will come back to the discussion about the management of expectations in section 3.3.

⁸See Goodfriend (1991) and Goodhart (1997) for a more detailed discussion about interest rate smoothing and stabilization. More recently, researchers developed the concept of inflation forecast targeting, which provide forecasts of inflation to be the best intermediate target of a central banker. As such an intermediate target is not included through the whole thesis, this concept is not discussed here. See Svensson (1997) for details.

⁹See Feldstein (2003) and Walsh (2004c) for more detailed surveys about different types of uncertainty.

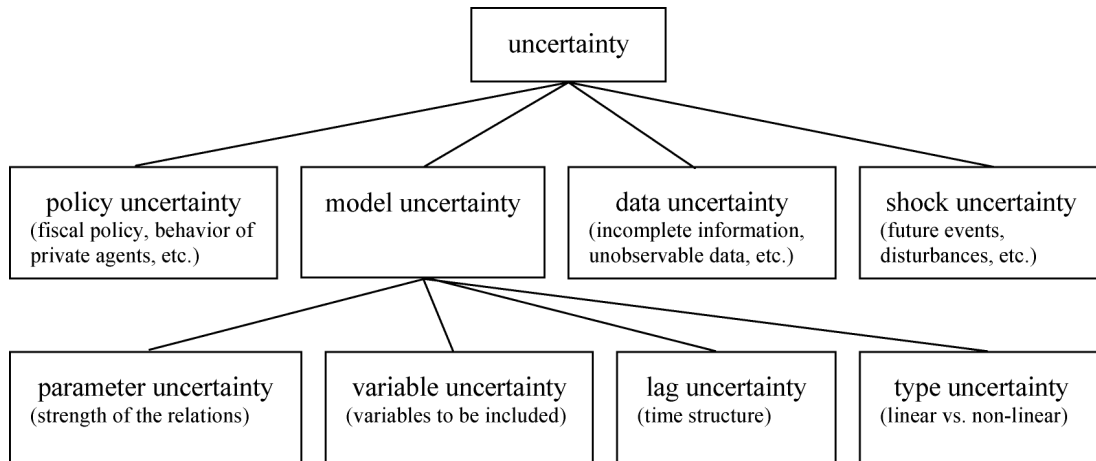


Figure 1: Classifying different types of uncertainty

A central banker also faces *policy uncertainty*, that is uncertainty about future fiscal policy and the future behavior of private agents. Both affect the impact of monetary policy. Of course he also does not know the time, the size and the persistence of future events and disturbances (*shock uncertainty*), as most shocks appear to be accidental.

Model uncertainty is related to the limited knowledge about economic structures, processes and the interaction of economic variables. Models are just approximations of reality, while the "true model" is unknown. For this reason they risk emphasizing only specific relations, while ignoring other relevant issues and thus are no substitute for a judgement based on all relevant information. An authority that bases its decisions on a model, has to deal with uncertainty about the strength of the relations (*parameters*), the type of relations (*linear vs. non-linear*), the time structure (*lags*), and the relevance of different *variables*. As a decision based on too few variables could lead to a wrong judgement of the economic climate, choosing the correct variables is essential.

As a result of uncertainty, parameters are often based on estimations, which are dependent on the data. Thus data uncertainty can enforce model uncertainty, and data and model uncertainty have a stronger link than it appears to be at a first glance. Of course, estimations are at their most dubious when the data is hardly comparable, as it is the case with new monetary unions. Different databases make empirical work complicated. In addition, the time period for estimation can change the outcomes, especially in the case of structural breaks. After a structural break estimated relations are no longer useful for future policies. This is an even more pertinent point in new monetary unions, as the foundation of a union is a structural break in itself. Furthermore shocks in oil prices or the introduction of a new technology can alter the political-institutional framework.

Structural breaks raise some further questions in connection with parameter uncertainty. If the effectiveness of a policy after such a change in the structure is higher than before, the question is: does the change merely improve the effectiveness or

does the strategy actually cause the change. Of course, the magnitude of a response to a policy action depends on the policy action itself.¹⁰ Consider an example where the output gap tomorrow depends on today's output gap and a short term interest rate:

$$y_{t+1} = \alpha y_t + \beta i_t \quad (2)$$

Who can be sure that a change in the monetary strategy (represented by a change in i) does not change α ? Maybe the true relationship is in fact a nonlinear one and $\alpha = \alpha(i_t)$. Because of these unknown effects political interventions can raise the uncertainty about the impacts of monetary policies.¹¹ This leads many economists to conclude that more defensive policy rules should be employed.

Uncertainty regarding the time structure is reflected in unknown lags for the evolution of the model's variables and in uncertain time lags for monetary policy to affect economic conditions. These long and variable time lags are often mentioned as the main reason against a fine adjustment of the economic situation, as interventions can impact the economy in a procyclical way if they are effective only after some time, when the actual economic situation is completely different from that at the time of implementation.¹²

In addition, if aggregate demand is the sum of *aggregate demand without active policy* plus *aggregate demand with current and previous active policy*, then the variance of demand is the sum of the variances with and without political interventions plus *twice the covariance of both scenarios*. Bearing this in mind, active policy is only stabilizing if the covariance is negative with an absolute value twice as great as the variability due to the intervention.¹³ For this reason it is also important to account for the costs and the possibilities of correcting or undoing an active policy. Since changes in the political-institutional framework are extremely persistent, only well considered strategies should be implemented, and much caution is required with respect to uncertainty.¹⁴ On the whole, the time lags in the effectiveness of monetary policy and the related risk of implementing procyclical actions, that maybe irreversible for a long time, are normally interpreted in favor of policies of less magnitude.

However, more recent studies argue that a less than appropriate reaction could lead to a similar procyclical result. In contrast to the work advocating to more defensive policy rules, they argue that uncertainty about the quality of a model should

¹⁰This is related to the Lucas-Critique, which argues that estimations of econometric models can not be used for the conduct of monetary policy, as estimated model parameters depend on the policy and vice versa. See Lucas (1976) for an analytical derivation.

¹¹See Brainard (1967), Bundesbank (2004) and the discussion in section 5.

¹²This argument was mentioned first by Friedman (1961).

¹³See Friedman (1953) and Feldstein (2003).

¹⁴In terms of credibility and transparency a permanently changing policy would also not be helpful for the "*management of expectations*"(see Svensson (2004), p. 2).

make policy rules more aggressive, for example, because of the underestimation of shock persistence.¹⁵ In general the implications of uncertainty derived in the literature are diverse. The most consistent conclusion is that a central bank should not react mechanically on decision rules based on a model theoretic analysis, but should account for all relevant information. Contrary, the question of whether policy rules should be more aggressive or more defensive in order to take uncertainty into account is a more controversial one, even if the focus is restricted to the field of *model* uncertainty. Nonetheless it should still be clear that a central bank has to react to undesirable inflation, because *"uncertainty is not an excuse for allowing inflation to go back to the bad old days"* (Feldstein (2003), p. 4).

3 The New Keynesian Perspective

In the following we describe the evolution of monetary policy analysis and the key differences of the New Keynesian perspective to previous approaches for the investigation of monetary policy.¹⁶

3.1 A new workhorse model in monetary policy analysis

The historical background of today's monetary models goes back to the first IS-LM models of Hicks (1937) and Modigliani (1944). Some years later, the addition of a Phillips (1958)-curve enables this framework to study inflationary effects. However, the "stagflation" during the 1970s made economists doubt about the theoretical framework, and the Rational Expectations (RE) revolution exposed that the standard IS-LM framework is inconsistent with optimizing behavior of households and firms.¹⁷ In the RE augmented IS-LM model monetary policy was unable to influence real variables systematically since every attempt to raise output by a surprise inflation is condemned to failure as under the assumption of model consistent expectations the private sector anticipates this policy. Only unanticipated shocks not economic policy are able to cause deviations from the natural level of output. This feature of the often called New Classical Macroeconomics is well-known as the policy ineffectiveness proposition (PIP).¹⁸

Since most economist do not believe the PIP in this strong manner, New Keynesians like Fischer (1977) show that under longer-term contracts or sticky wages the

¹⁵ A detailed discussion of studies concerning model uncertainty in monetary policy will be given in section 5.

¹⁶ This section is heavily inspired by Goodfriend and King (1997), King (2000) and the seminal paper of Clarida, Galí, and Gertler (2001).

¹⁷ The breakdown of the Phillips curve was predicted and analytically shown by Phelps (1967) and Lucas (1980).

¹⁸ For applications of policy analysis in the RE IS-LM model see Lucas (1972), Lucas (1973) and Sargent and Wallace (1975).

PIP fails to hold.¹⁹ Moreover, a stream of empirical studies during the late 1980s, finds evidence, that monetary policy in fact *does* influence real variables in the short-run.²⁰ Encouraged by these studies, economists built a vast array of models, including fixed prices and/or wages for at least one period.²¹ However, all modelling strategies are heavily criticized for their lack of microfoundations and modern dynamic macroeconomics, including utility optimizing agents and profit maximizing firms. Therefore the RE IS-LM model has been improved by combining the Keynesian assumption of rigid prices with the Dynamic Stochastic General Equilibrium (DSGE) techniques of the Real Business Cycle (RBC) literature (hence *New Keynesian*).²² In opposition to the view of the New Classical Macroeconomists, the sticky price framework allows monetary policy to have real effects. Furthermore, the New Keynesian model class contains the RBC-type models as a border case, when prices are assumed to be completely flexible.²³

3.2 The importance of sticky prices

The most often cited reason for monetary policy to be non neutral are sticky prices. The argumentation is very simple: If there is a time gap between the policy action of a central bank and the reaction of the private sector, the monetary authority has a *moving* advantage to exploit. Imagine, for example, a temporary positive demand shock. Under complete flexibility, prices would adjust immediately and consumption and output would not react at all. Thus there would be no reaction of real variables, whereas under sticky prices a temporary effect on output appears, due to the sluggish adjustment of firms, even under RE.²⁴ In this case, a rise in the interest rate would dampen the shock-effect by reducing demand, and speed up the return of inflation to its steady state. Due to the lower deviation of consumption

¹⁹See Buiter (1980) for a discussion of other reasons for a failure of PIP. Buiter already mentions the importance of price stickiness in this context.

²⁰See for example Romer and Romer (1989), Galí (1992), Bernanke and Blinder (1992), Christiano, Eichenbaum, and Evans (1996), Christiano, Eichenbaum, and Evans (1999) and Bernanke, Gertler, and Watson (1997).

²¹The most famous examples are Taylor (1979b) and Taylor (1980).

²²In fact in New Keynesian models price rigidity is no assumption any more, but follows from optimizing agents, that face costs when it comes to adjust prices. We will come back to this in the next subsection.

²³The literature on these models is not distinct, when it comes to categorize models that combine DSGE techniques with rigid prices. Goodfriend and King (1997) for example denote this model class as New Neoclassical Synthesis. Due to this disagreement in scientific literature, we simply call this model class New Keynesian, as this seems to be used by most academics. Early examples of New Keynesian models are Rotemberg and Woodford (1995), Rotemberg and Woodford (1997), Yun (1996), Goodfriend and King (1997) and McCallum and Nelson (1999b).

²⁴It is possible not to raise prices immediately but raise output due to the assumption of monopolistic competition. Goods prices include a mark-up over real wages, so that production could temporarily be increased at the costs of profits per good without an increase in prices.

from equilibrium, firms do not have to raise prices as much as they would have to do without the countercyclical policy, and the deviations of inflation and output from equilibrium are smaller. Since the monetary authority recognizes the disturbance before it becomes optimal for the firms to reset prices, it has a moving advantage.

This modeling framework seems to be the superior approach for at least two reasons: Firstly, assuming sticky prices gives a more realistic picture of an economy. Since price adjustments are costly and firms are uncertain about future developments, they do not adjust prices continuously.²⁵ Secondly, incorporating sticky prices reproduces the output persistence that is revealed by the data. Taylor (1979b) and Taylor (1980) show that even when prices are set only two periods in advance, monetary surprises already have long-lasting effects on output, depending on the elasticity of labor supply with respect to the real wage.²⁶ Nelson (1998) is critical of the fact that the two-period sluggish price adjustment only accounts for output persistence, but not for the observed persistence in inflation. U.S. inflation rates seem to be highly serially correlated, even when monetary policy shocks are serially uncorrelated, an empirical pattern that is not captured by Taylor's models.

The standard framework used today is the Calvo (1983) sticky price setting, which implies more persistent inflation dynamics, even when monetary policy shocks are serially uncorrelated. Furthermore, this setting is based on a profit-maximizing representative firm, setting prices infrequently, depending on randomly occurring adjustment opportunities. If ω is the probability for a randomly selected firm to adjust prices, the average fixation of prices is $\frac{1}{1-\omega}$. Thus, the smaller the probability to adjust, the longer prices are fixed, and the less sensitive is inflation to movements in output. In equilibrium inflation depends on expected future inflation and current marginal costs, since firms maximize the discounted value of current and future profits, which in turn depends on current real marginal costs and future prices.²⁷

In open economy or two-country models, respectively, there are several additional reasons for incorporating sticky prices. In the seminal paper of Dornbusch (1976), for example, the exchange rate overshoots its equilibrium value in response to an unanticipated monetary expansion due to different reaction times between goods and finance markets, and because of expectations. Furthermore, Obstfeld and Rogoff (1996) argue that the strong volatility in exchange rates relative to that observed in prices is another rationale for assuming that prices are sticky. This seems to be especially the case in floating exchange rates regimes with open financial markets.²⁸ Rigid prices imply that nominal and real exchange rates behave in a similar manner

²⁵For the macroeconomic consequences of menu costs see for example Akerlof and Yellen (1985), Mankiw (1985), Ball and Romer (1991) and Romer (2001).

²⁶However, Chari, Kehoe, and McGrattan (2000) and Ascari (2000) show that for Taylor's two period price or wage stickiness, respectively, a very high labor supply elasticity is required to obtain responses on a monetary shock that matches U.S. data.

²⁷A detailed explanation of the Calvo price setting framework will be given in 6.1.2.

²⁸Evidence on this can be found in Mussa (1986), Baxter and Stockman (1989) and Flood and Rose (1995).

in the short-run, and reproduce the empirical regularities. The stickier prices are, the more volatile are nominal exchange rates, and the greater the co-movement with real rates.²⁹

3.3 The importance of expectations

Apart from the assumption of sticky prices, there is another very important innovation in New Keynesian models, namely the inclusion of expectations. In previous investigations inflation and output dynamics are determined by past values of inflation and the output gap, whereas the New Keynesian Phillips Curve (NKPC) and the forward-looking IS curve depend on the expected values. This is justified by assuming the existence of dynamically optimizing agents and firms. Households maximize their utility by smoothing consumption over time. Hence, households' consumption decision today depends on the expected future consumption, as does the output gap. Additionally, firms choose their optimal price by maximizing the discounted sum of future profits, so that price setting today depends on expected future prices.

The forward-looking behavior of households and firms has important consequences for the dynamics of an economy. Consider for example a simple forward-looking IS curve, following directly from the optimal consumption decision of optimizing households:³⁰

$$y_t = -\varphi(i_t - E_t\pi_{t+1}) + E_ty_{t+1}, \quad (3)$$

where the negative effect of the real interest rate reflects the intertemporal substitution of consumption. Iterating (3) forward gives

$$y_t = -\varphi E_t \sum_{i=0}^{\infty} (i_{t+i} - \pi_{t+1+i}), \quad (4)$$

which illustrates how beliefs about the evolution of the central bank and about future inflation influence current production. If a policy maker signals his intention to keep inflation at a low level, he can reduce current inflation at a lower cost in terms of output loss. And the same arguments are valid for the NKPC

$$\pi_t = \lambda y_t + \beta E_t \pi_{t+1}, \quad (5)$$

which can be derived from the optimization procedure of forward-looking firms (thus β represents the discount factor). Forward iteration leads to

$$\pi_t = \lambda E_t \sum_{i=0}^{\infty} \beta^i y_{t+i}, \quad (6)$$

²⁹Microeconomic empirical evidence on sticky prices can be found in Carlton (1986), Blinder (1991) and Kashyap (1995).

³⁰The derivation of the following equations will be shown in 6.1.

which illustrates the dependence of inflation from the current and the expected future output gap. Firms maximize the sum of their discounted future profits by equating the optimal nominal relative price with the expected discounted value of future nominal marginal costs. Since equilibrium marginal costs equal the real wage, which in turn depends on consumption and employment,³¹ there exists a stable relationship between marginal costs and the output gap, so that the expected future output gap enters the equation for current inflation. Again, the belief about future monetary policy is of great importance as it affects the expectations of future production. As shown in (4) a credible central banker reduces inflation, with lower costs associated with output reduction, which due to (6) reduces current inflation further. To summarize the arguments of this section: To the extent that wage and price setting depends on the expected future inflation, it also depends on the monetary course of the central bank. Therefore managing expectations is crucial for monetary policy.

3.4 The importance of endogenous persistence

In addition to the persistence in prices there is also empirical evidence on the persistence in inflation dynamics, in the sense that inflation today depends also on past inflation rates.³² Actually, it is a stylized fact that inflation is a non-stationary variable, which implies persistent dynamics. To the extent that this is the case, decisions affecting today's inflation also affect future values of inflation. This should be taken into account. In previous monetary policy models this dependence of inflation from past values is justified with the assumption of adaptive expectations, whereas the hybrid version of the NKPC is most often derived under the assumption of partly backward-looking price setters due to costly collection of information about future developments.³³

Clarida, Galí, and Gertler (1999) show that with endogenous persistence in inflation, the equilibrium path of monetary policy influences the speed of convergence of inflation to its target, where the speed is decreasing in the degree to which inflation is persistent. Furthermore, it makes the trade-off between inflation and output less favorable, as expectations play a minor role, so that monetary policy should react more aggressively on fluctuations in inflation and output.

A new strand of research emphasizes also the persistence in output, which can be microeconomically justified by a habit formation in consumption. Fuhrer (2000) argues that real variables as well as inflation display a gradual response to monetary policy, with a peak after one year. While these humped-shaped pattern is illustrated in the empirical work of Christiano, Eichenbaum, and Evans (1996) and

³¹In fact the equilibrium real wage equals the marginal rate of substitution between leisure and consumption, as we will show in 6.1.

³²Empirical results are quite diverse. See for example Fuhrer (1997), Rudebusch (2002), Galí, Gertler, and Lopez-Salido (2001), Sbordone (2002) and Roberts (2005).

³³See 6.1.2 for details.

Leeper, Sims, and Tao (1996), standard models imply a jump in these variables for the first period. He develops a model in which consumer's utility depends on current consumption relative to past consumption and finds evidence that the existence of a habit formation in consumption can not be rejected, using U.S. data from 1966:1 to 1995:4. Following Abel (1990) and Carroll, Overland, and Weil (2000) he introduces multiplicative habit persistence in consumption functions, and shows that these improve the dynamics of all variables. Batini, Justiniano, Levine, and Pearlman (2005) shows evidence for the Europe and the U.S. in a comparable model assuming that today's consumption minus a part of yesterday's consumption enters household's utility function. This modelling strategy seems to be superior, as it leads to the same conclusions, but is analytically more tractable. Assuming persistence in consumption decisions leads to a hybrid version of the IS curve. Under these circumstances, the optimal interest rate reaction depends on lagged values of the output gap as well.³⁴

3.5 Policy analysis in a New Keynesian Framework

As has been already mentioned in 3.3 the introduction of expectations has important consequences on monetary policy due to credibility issues. Therefore it has become standard to distinguish between policies under *commitment* and *discretion*. Furthermore, in light of the popularity and empirical evidence on simple rules of the Taylor type it is pertinent to evaluate the use of these rules.³⁵

3.5.1 Discretion

Under discretion a central bank minimizes the expected loss in each period separately, so the policy maker faces a single-period optimization problem. From a technical viewpoint the authority finds the optimal interest rate in a sequential decision process. Firstly, he chooses the optimal output gap, maximizing the objective function. In a second step he chooses the implied nominal interest rate, conditional on the optimal output gap value. Assuming a central banker, who tries to minimize the weighted sum of the variance of the output gap and inflation ($\alpha y_t^2 + \pi_t^2$) subject to (5), taking expectations as given, leads to an optimality condition of $y_t = -\frac{\lambda}{\alpha} \pi_t$ in the first stage.³⁶ In the second stage the authority sets the interest rate by using the desired value for y_t in (3) and solving for i_t . According to this, a policy maker

³⁴This is only the case in single-country models. In a two-country analysis it is analytically impossible to derive a stable relationship between domestic consumption and domestic output, since domestic output depends on foreign consumption as well. Therefore we use a hybrid Euler equation for the evolution of consumption for our two-country analysis in 8.1.

³⁵See Walsh (2003) for a more detailed discussion.

³⁶Note that under discretion this is equal to minimizing $\alpha y_t^2 + \pi_t^2 + E_t \left\{ \sum_{i=1}^{\infty} \beta^i (\alpha y_{t+i}^2 + \pi_{t+i}^2) \right\}$, when expectations are taken as given. See Clarida, Galí, and Gertler (1999) for details. Since all variables are measured in percentage deviations from mean, the variances are simply the squared values.

should reduce demand by raising the interest rate, whenever inflation is above target. Without credible commitment, private sector expectations are taken as given (exogenously), and do not enter the optimal reaction function. Thus future inflation and output are not affected by today's decision, the central bank is unable to manage the expectations of the private sector, and equilibrium inflation is independent from lagged variables.

3.5.2 Commitment

There are several problems that arise under a discretionary policy. One is the well-known inflation bias, first illustrated by Kydland and Prescott (1977) and Barro and Gordon (1983). Due to the temptation to raise output above its natural level, inflation stays above target under RE, without any gain in output. The classic inflationary bias under discretion can be illustrated by augmenting the objective of the central bank:

$$\alpha (y_t - k)^2 + \pi_t^2, \quad (7)$$

where $k > 0$ is the target for the output gap, representing the tension to raise output above potential. Solving this for y_t gives $y_t^{ib} = -\frac{\lambda}{\alpha}\pi_t^{ib} + k$, which can be used in (5) and (3) to get the equilibrium values of the output gap and inflation: $y_t^{ib} = y_t$ and $\pi_t^{ib} = \pi_t + \frac{\alpha}{\lambda}k$. This shows that inflation will be systematically above target. The private sector anticipates the incentive of the policy maker to raise output above potential by lowering the real wage due to a surprise inflation, and thus equilibrium inflation rises to the point, where the authority is no longer tempted to raise output.

In contrast to the case of discretion, a central bank acting under commitment chooses the optimal paths for future inflation and the output gap and promises *credibly* to implement a monetary policy that leads to exactly these paths. Thus, under commitment a monetary authority would simply set $k = 0$.³⁷ Economists such as McCallum and Nelson (1999b) and Blinder (1997) argue that a central banker will recognize the longer term costs of a discretionary policy and therefore bind himself to a commitment strategy.³⁸ However, even a policy maker who does not resist the temptation of raising output above potential still experiences credibility gains from using the optimal policy under commitment, since credibility improves the inflation/output trade-off, as demonstrated in 3.3. For example, under a lack of credibility, disinflating the economy might be more problematic, because the private sector will not believe that the authority fights inflation.

³⁷Another possibility to reduce the inflationary bias is to appoint a conservative central banker, who assigns higher relative costs to inflation than society. With shrinking α , the inflationary bias vanishes, see Rogoff (1985) for details. Some authors also argue that fiscal policy could eliminate the inflation bias by offsetting the distortions using tax subsidies. See for example Dixit and Lambertini (2003).

³⁸Furthermore, in a repeated prisoner's dilemma game the public might adopt a tit-for-tat strategy, so that the policy maker gets punished if he misbehaves. Assuming a dynamically optimizing policy maker, this decreases the temptation to raise output above potential. See Barro and Gordon (1983) and Backus and Driffill (1985).

Contrary to a discretionary policy, future values of inflation and the output gap are affected by monetary policy to the extent that the authority influences private sector expectations, and thus equilibrium inflation depends on the lagged output gap. By solving for the optimal policy in a class of simple rules, Clarida, Galí, and Gertler (1999) show that a policy maker under commitment should raise the interest rate more aggressively in response to a rise in expected inflation relative to the discretionary case. However, this does not necessarily imply that nominal interest rates are in fact higher, since under a credible central bank, expectations perhaps do not rise by such an amount. Furthermore, they show that the optimal policy response changes from a level rule to a difference rule of the form $y_{t+i} - y_{t+i-1} = -\frac{\lambda}{\alpha}\pi_{t+i}$.

Because of the different forms of building expectations, the dynamics of the output gap and inflation also differ. To see this, imagine a positive supply shock increases inflation: A central bank acting under the optimal precommitment policy chooses a path for the output gap that stays output below its potential value for some periods to lower private sector expectations of future inflation, as this improves the policy trade off. Therefore inflation undershoots its equilibrium level before it returns to zero. Contrary to this, a discretionary policy leads to an effect often referred to as *stabilization bias*: Inflation and the output gap return to their steady state values one period after a one-period shock has occurred. This can only be achieved at the costs of a very high interest rate and stronger deviations from equilibrium than under commitment, since expectations are unaffected. An authority acting under discretion is unable to improve expectations, since future inflation and output are unaffected by today's actions, and adjusts the interest rate to offset the impact of the shock immediately.

The major problem with the optimal policy of *precommitment* is that it leads to dynamic inconsistency as the central bank's optimal commitment policy changes as soon as the next period appears, because the private sector has already built expectations. Technically speaking, the lagged Lagrangian multiplier in the first order condition for inflation at the time of optimization is equal to zero. As this is the case for every period, the first order conditions change with the period the central bank optimizes. A possible way to escape dynamic inconsistency is the *timeless perspective approach* by Woodford (1999), who simply sets the lagged Lagrangian multiplier in the time t first order condition for inflation different to zero, so that the conditions are the same for all periods. This can be interpreted as a policy having been chosen in the distant past, so that the current values of inflation and the output gap are chosen from an earlier perspective.

3.5.3 Simple rules

Intense debate is ongoing as to whether monetary policy should be adhered to as a strict rule. The fundamental reasons why the rule-advocates feel that there is a rationale for using a strict rule are: the problem of time inconsistency, the influence

on expectations of the private sector (anticipation of current actions upon equilibrium determination at previous dates), as well as the increased credibility; which improves the policy-performance; the reduction in uncertainty as the market participants can forecast policy with such a rule (and related to this, the simplicity in explaining policy to the public), and finally the rise in accountability, which should discipline policy makers.³⁹ On the other hand those who advocate discretion "(...) would not play in a basketball team whose coach had given the players a fixed set of rules at the beginning of the season: on the third possession of the second quarter, the off-guard should dribble twice, get to the baseline and shoot from the corner." (Solow (1998), p.92).⁴⁰

The truth probably lies somewhere in between these two positions: Taylor (1998b) argues for a mixture of a pure rule and a pure discretion scenario as he believes that a mixture is superior to pure discretion as it reduces uncertainty (improves predictability) and at least retains some advantages cited by the discretion-advocates. He also believes that there is a clear need to deviate from a rule in specific circumstances; for example the 1987 stock market crash; making a hybrid scenario also more appropriate than following a rule mechanically. This has become the most widely-held view among economists. There is also a wide field of empirical studies which suggest that simple policy rules approximate the behavior of central banks quite well, and make the research on simple rules also interesting from a practical viewpoint.

In a New Keynesian framework, studying simple rules is of immense interest since a monetary authority who credibly sticks to a rule, reduces uncertainty about the path of monetary policy. Once a policy maker has committed to a rule, private sector expectations about the future course of the central bank can simply be calculated by using this rule. Since it is difficult to achieve the necessary amount of credibility to implement the optimal precommitment policy, a simple rule might be one way to achieve at least enough credibility to reach a solution close to the optimal solution. In addition, a simple rule avoids the time inconsistency problem of commitment. Recapitulating, "(...) understanding the qualitative differences between outcomes under discretion versus rules can provide lessons for the institutional design of monetary policy." (Clarida, Galí, and Gertler (1999), p. 1671)

3.5.4 Objectives

In order to derive monetary policy rules, most models assume a heuristic quadratic loss function that depends on the output gap, deviations of inflation from target, the interest rate and changes in the interest rate. Since in a New Keynesian framework all variables are measured in percentage deviations from equilibrium, there is no

³⁹See Taylor (1998a) for a discussion.

⁴⁰Notice that the discretionary behavior meant in this context is different from the discretionary policy that we recently discussed, since the responses meant here are also based on economic principles, see Poole (1998) for a discussion.

loss of generality to set target inflation to zero in the policy objective. Furthermore, the underlying DSGE framework of New Keynesian models allows one to draw up a policy objective that represents the approximation of the utility function of the representative household. With the assumption of a continuum of differentiated goods normed to an Interval $[0, 1]$, Woodford (2002) derives a loss function as log-linear approximation to the utility function that depends on the variance of the output gap and inflation.⁴¹ As this loss function is derived from the general equilibrium framework with monopolistic competition on goods markets, one has to assume that there is no gap between the actual steady-state level and the steady-state efficient level of output, which is the one without distortions due to monopolistic competition. Distortions would imply that a central bank acting under discretion would produce an average *inflation bias*. As the average inflation in the major industrialized countries was low during the 1990s, Woodford believes that the assumption of no gap seems to be plausible.

However, there are several short-comings with this approach. Clarida, Galí, and Gertler (1999), for example, mention that the representative agent approach could be a highly misleading guide to welfare analysis. Since some groups suffer more than others in a recession and incomplete insurance and credit markets avoid many of those people from borrowing money in a recession. Therefore the hypothetical representative agent might not be an accurate measure of cyclical welfare fluctuations.⁴² Furthermore, central banker's opinions about the optimal objective of central banks - in particular the optimal weights in the objective function - are quite diverse. And even if they are derived from the underlying utility maximizing framework, the parameter of the household's utility function are unknown. Therefore a rather pragmatic approach is used by assuming different scenarios with different weights in the loss function of the authority and investigate consequences of model uncertainty.⁴³ Moreover, we believe that assuming no monopolistic distortions in equilibrium seems to be inappropriate, since the whole microfoundations of our models are based on monopolistic competition.⁴⁴

⁴¹Steinsson (2003) shows that in a model with forward- and backward-looking price adjusters also the squared change in inflation appears.

⁴²See Kirman (1992) and Hartley (1996) for critical assessments of the representative agent approach.

⁴³Due to technical reason, we could only analyze the case of a pure inflation stabilization in our two-country setting. See 8.1.5 for details.

⁴⁴In addition, the assumption of monopolistic competition has very interesting implications for monetary policy, since an unexpected rise in the money supply, for example, can correct the monopolistic distortion. Since product prices include a mark-up over marginal costs, a rise in demand due to an increase in real money balances can be served, even when real wages have to rise in order to recruit more workers. Thus monetary policy can shift equilibrium to its efficient level and thus may have real effects even in the long-run. We will come back to this when we are to discuss the two-country model.

3.6 Key insights from New Keynesian policy analysis

There are some insights from New Keynesian models that have become stylized facts in modern monetary policy analysis. The first one is that a central banker should adopt an inflation targeting strategy and adjust for the optimal inflation rate gradually. The reason for this is that to the extent that a supply shock is present, there exists a trade-off between output and inflation variability. In the absence of a cost-push shock, inflation depends only on current and future demand and the central bank sets the interest rate so that $y_t = 0$. As a consequence inflation *and* output hit their target values. This implies that one should completely offset demand shocks, but accommodate supply shocks. Extreme inflation targeting, in the sense that a monetary authority should set the interest rate to hit the inflation target immediately, represents the optimal strategy only in the absence of cost-push shocks or when there is no concern for stabilizing the output gap, respectively. A demand shock raises inflation and output in the same direction, so that an increasing interest rate pushes both variables back to equilibrium, and there exists no trade-off.

Moreover, the nominal interest rate should be adjusted more than one-for-one with expected future inflation in order to alter the real interest rate, manage aggregate demand, and neutralize deviations of expected inflation from target. This is necessary since output is demand-determined in the short-run. Every reaction parameter smaller than one would lead to an unstable equilibrium, in the sense that the implied change in the real interest rate as reaction to rising inflation expectations is not strong enough to stop expectations to increase. Clarida, Galí, and Gertler (2000) find evidence that this more than one-for-one rule was in fact violated in the pre-Volcker era of 1960-1979, whereas for the Volcker-Greenspan era this rule seemed to be followed.

Another conclusion from a New Keynesian analysis is that central banks should not react to shocks on potential output. A permanent rise in productivity raises potential output. However, output demand also rises by exactly the same amount due to the increase in permanent income, and completely offsets the demand gap. Nevertheless, the most important insight from a New Keynesian analysis is the one we already discussed in the preceding sections: An authority, who sticks himself credibly to a commitment strategy eliminates the inflation bias and improves the output/inflation trade-off.

4 Hansen-Sargent robustness

Since the investigation of uncertainty is essential in monetary policy analysis, there are many different techniques to study the consequences of uncertain model structures. The most popular ones are Bayesian methodologies, which all have in common that the expected loss of a policy maker is minimized, given a prior probability of uncertain parts of the underlying model. In contrast, robust control techniques do not need any information about probabilities in advance. So far, most applications of

Hansen-Sargent robust control techniques are restricted to quadratic problems with linear transition equations. However, in chapter 17 of Hansen and Sargent (2007) the methodology is extended to allow for more general functional forms. As this thesis concentrates on linear-quadratic problems we do not discuss this extension.

The following sections give a brief summary of the Hansen and Sargent robust control techniques, built to reflect the decision maker's fear about model misspecification. In 4.1 the main idea is sketched. Subsequently, technical details for solving robust (and non-robust) RE programs are given in 4.2. 4.3 critically assesses the potential of this approach, and finally 4.4 gives robust control applications apart from the analysis of monetary policy.⁴⁵

4.1 Methodology

Doubts about models have existed for as long as models themselves have. This tradition is one normative reason for studying robust decision rules. Another, positive one, is a phenomenon, referred to as the "equity premium puzzle": The value of prices to be assigned to macroeconomic risks seem to be underestimated in RE models. This result follows from the fact that under RE agents could trust the model because subjective and objective probability distributions (i.e. their models) coincide. Each agent's model is an equilibrium outcome, not to be specified by the model builder. So the decision maker has not to worry about model misspecification, because RE remove agents' personal model as element of the model.⁴⁶ Hansen, Sargent, and Tallorini (1999) show that agent's caution in responding to fears about model misspecification can raise the theoretical value of the risk premium.

This is reminiscent of the well known Lucas critique, which shows that under the assumption of RE, decision rules are a functional of the serial correlation of shocks.⁴⁷ He argues that the way to build expectations changes so that estimated relations and simulations of models of the past are no longer useful for forecasting under a new environment. While RE econometrics achieve parameter identification by exploiting the structure of the function, mapping shock serial correlation properties to decision rules, robust control theorists alter the mapping by treating the decision maker's model as an approximation and seek one rule to use for a set of models that might govern the data.⁴⁸ Following Muth (1961) they put econometricians and the agent being modelled on the same footing: If econometricians doubt a model specification, the agents in the model should doubt this specification too.

⁴⁵ An overview of robust control applications in monetary policy is given in 5.4.

⁴⁶ See Hansen and Sargent (2007) for a discussion.

⁴⁷ See Lucas (1976).

⁴⁸ See Hansen and Sargent (1991).

4.1.1 The core idea

The core of the idea is to treat the decision maker's model as an approximation of the true model. Let x be a vector of state variables and let the true data follow a markov-process with a transition density $f(x^*|x)$.⁴⁹ Moreover, let the approximating model be described by a transition density $f_\alpha(x^*|x)$ ($\alpha \in A$, where A denotes a compact set of parameter values). If there exists no α such that $f_\alpha = f$ the model is considered to be misspecified. The maximum likelihood estimator $\hat{\alpha}_0$, often used by econometricians, fulfills

$$\text{plim } \hat{\alpha}_0 = \arg \min_{\alpha \in A} I(\alpha, f), \quad (8)$$

where $I(\alpha, f)$ denotes the *relative entropy* of f and f_α respectively.⁵⁰ Relative entropy measures the "expected distance" between f and f_α and is defined as the expected value of the logarithm of the likelihood ratio evaluated with respect to the true conditional density $f(x^*|x)$:

$$I(\alpha, f)(x) = \int \log \left(\frac{f(x^*|x)}{f_\alpha(x^*|x)} \right) f(x^*|x) dx^* \geq 0. \quad (9)$$

If the model is misspecified I is strictly positive while $I = 0$ if the decision maker's model is the true data generating process. While the maximum likelihood estimator tries to minimize entropy, robust control inverts this approach by taking f_{α_0} as given, and building a set of possible data generating processes around this model, so that the true model is one model in this set. As the decision maker does not know the true model, he is interested in decision rules that are robust against model misspecification and work well in a set of models, which is restricted by $I \leq \delta$.

4.1.2 The distorted shock process

To describe the distorted shock process that reflects the preference for robustness, first some key insights of standard control theory are reviewed, by considering the following problem, which from now on will be referred to as the approximating model:

$$x^* = Ax + Bu + C\tilde{\varepsilon}^*, \quad (10)$$

$$z = Hx + Ju, \quad (11)$$

where (10) and (11) represent the transition and the target function, and x , u and z denote vectors of state variables, control variables and target variables, respectively, and the vector of shock processes $\{\tilde{\varepsilon}_{t+1} : t = 1, 2, \dots\}$ is a conditionally homoskedastic Gaussian martingale differences process that satisfies $E_t(\tilde{\varepsilon}_{t+1}) = 0$ and $E_t(\tilde{\varepsilon}'_{t+1}\tilde{\varepsilon}_{t+1}) = I$.

⁴⁹* denotes next periode values.

⁵⁰See A.1.1 for a definition of the limes in probability (plim).

The decision maker uses a (one-period) loss function $r(x, u) = z^2$ to express his preferences in an objective function that he wants to maximize by choosing a sequence of control vectors $\{u_t\}$:

$$\min_{\{u\}} E_0 \sum_{t=0}^{\infty} \beta^t r(x, u) = \min_{\{u\}} \sum_{t=0}^{\infty} \beta^t E_0 (z^2). \quad (12)$$

His aim is to get a decision rule of the form $u = -Fx$ that minimizes his objective. Under the assumption that the value function of this problem is the solution of the fixed-point problem $V(x) = x'Px$, dynamic programming reduces the problem to the following Bellman equation:

$$x'Px = \min_{\{u\}} E_t \{z'z + \beta x'^* P x^*\} \quad (13)$$

subject to (10) and (11). The following certainty equivalence principle results directly from the linear quadratic nature of the problem:⁵¹

Lemma 1: Certainty Equivalence Principle

In a linear quadratic control problem of the form (10) - (12) the matrix P and the decision rule F are independent from the volatility matrix C .

The Certainty Equivalence Principle makes robust control theorists doubt standard control theory. Because of their experience with misspecified models they believe that this theory does not capture doubts about the model's specification in an adequate way. A robust control theorist particularly fears about misspecified dynamics, which influence the impulse response functions of the target vector. He is unsatisfied with using normally identically and independently distributed (i.i.d.) shocks with zero mean, as he believes a robust solution needs a shock process that feeds back on the state variable.

Therefore an additional vector process $\{\omega_t\}$ is introduced that depends in a possibly nonlinear way on the history of the state variables. This is similar to the work of Gilboa and Schmeidler (1989), who implement a malevolent second player to reflect concerns about model misspecification. Here this malevolent player tries to maximize the objective by choosing a feedback rule

$$\omega_t = g_t(x_t, x_{t-1}, \dots), \quad (14)$$

where $\{g_t\}$ is a sequence of functions which are measurable (in a stochastic sense) with respect to $x^t = [x_t, \dots, x_0]$. To represent dynamic misspecification, we surround the approximating model with a set of models of the form

$$x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \omega_{t+1}), \quad (15)$$

where the vector of shock processes $\{\varepsilon_{t+1}; t = 1, 2, \dots\}$ is another conditionally homoskedastic Gaussian martingale differences process that satisfies $E_t(\varepsilon_{t+1}) = 0$ and

⁵¹See A.1.2 for a proof.

$E_t(\varepsilon'_{t+1}\varepsilon_{t+1}) = I$. Equation (15) and (10) coincide, when $\tilde{\varepsilon}_{t+1}$ is normally distributed, conditioned on x^t , with mean ω_{t+1} , and identity covariance matrix.

As the decision maker believes that his model is not that bad, and the second player could distort the system in a way that leads to completely nonsense solutions, distortions need to be restricted. For this reason remember relative entropy

$$I(f_a)(x) = \int \log \left(\frac{f_d(x^*|x)}{f_a(x^*|x)} \right) f_d(x^*|x) dx^* \quad (16)$$

where f_a and f_d denote the transition densities of the approximating and the distorted model, respectively. Following Bayesian analysis these densities are equal to the corresponding likelihood functions. So the relative entropy could be interpreted as the expectation of the log likelihood ratio conditioned on the distorted model:

$$I(f_a)(x) = \int \underbrace{\log \left(\frac{f_d(x^*|x)}{f_a(x^*|x)} \right)}_{\text{value of the log likelihood ratio}} \times \underbrace{f_d(x^*|x)}_{\text{probability under the distorted model}} dx^* \quad (17)$$

The transition densities of the models are

$$f_a(x^*|x) \sim N(Ax + Bu, CC'), \quad (18)$$

$$f_d(x^*|x) \sim N(Ax + Bu + C\omega^*, CC'). \quad (19)$$

Because ω_{t+1} is measurable with respect to x^t , the log likelihood functions of x_{t+1} conditioned under the distorted and the approximating model are

$$\log L^d = -\log \sqrt{2\pi} - 0.5\varepsilon'_{t+1}\varepsilon_{t+1}, \quad (20)$$

$$\log L^a = -\log \sqrt{2\pi} - 0.5(\varepsilon_{t+1} + \omega_{t+1})'(\varepsilon_{t+1} + \omega_{t+1}). \quad (21)$$

A combination of (20) and (21) gives the log likelihood ratio with respect to the distorted model

$$\log \left(\frac{f_d(x^*|x)}{f_a(x^*|x)} \right) = \log L^d - \log L^a = 0.5\omega'_{t+1}\omega_{t+1} + \omega'_{t+1}\varepsilon_{t+1}. \quad (22)$$

Averaging over ε_{t+1} , evaluated when the data are generated by the distorted model gives the expectation of the log likelihood function conditioned on the distorted model. This is what in (17) is defined as relative entropy⁵²

$$I(f_a) = 0.5\omega'_{t+1}\omega_{t+1}. \quad (23)$$

To get a measure of the size of model misspecification for dynamic models relative entropy is extended to an intertemporal measure of the form

$$R(\omega) = 2E_0 \sum_{t=0}^{\infty} \beta^{t+1} I(f_a) = E_0 \sum_{t=0}^{\infty} \beta^{t+1} \omega'_{t+1}\omega_{t+1} \leq \delta_0, \quad (24)$$

where expectation conditioned on x_0 is evaluated under the distorted model.

⁵²The term $\omega'_{t+1}\varepsilon_{t+1}$ vanishes when we build the expectation over all x^* -values.

4.1.3 The distorted model

Under the assumption that the value function is of the form $V(x) = x'Px$ the distorted model can also be displayed by a Bellman equation:

$$x'Px = \min_{\{u\}} \max_{\{\omega^*\}} E_t \{z'z - \theta\beta\omega^{*'}\omega^* + \beta x^{*'}Px^*\} \quad (25)$$

subject to (15), where $\theta > 0$ represents the decision maker's preference for robustness, that falls as θ rises, so that the problem is equal to its non robust version when θ reaches infinity, since for $\theta = \infty$ (25) implies that the optimal solution for $\{\omega^*\}$ is zero. The second player embodies pessimism and the solution of (25) incorporates now a worst case ω as a function of x and u . This allows for misspecification in a more adequate way, since the corresponding decision rule $u = -Fx$ depends on C for all finite values of θ (as we will see soon). Thus certainty equivalence is broken.⁵³

It follows from the properties of ε_{t+1} that $E[\varepsilon_{t+1} + \omega_{t+1}|x^t] = \omega_{t+1}$, so that ω_{t+1} could be interpreted as the mean of the new shock process $\{\varepsilon_{t+1} + \omega_{t+1}\}$ conditioned on the information set up to time t . The ω 's can also be interpreted as multiple prior probabilities in the transition function at time t .⁵⁴ Their size is directly penalized through θ . This θ is equivalent to the Lagrangian multiplier on an intertemporal misspecification constraint in a min-max problem of the following type:

$$\min_{\{u_t\}} \max_{\{\omega_{t+1}\}} E_0 \sum_{t=0}^{\infty} z'z \quad (26)$$

$$\text{subject to } x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \omega_{t+1})$$

$$\text{and } E_0 \sum_{t=0}^{\infty} \beta^{t+1} \omega'_{t+1} \omega_{t+1} \leq \delta_0.$$

Since the value function of (25) is monotonous and concave in δ_0 there is bijective negative function from δ_0 to θ , so that θ limits the size of model misspecification introduced by the distorted shock process.

In what follows the relationship between the distorted and the true model is clarified. Let the true model be

$$x_{t+1} = A^*x_t + B^*u_t + C\varepsilon_{t+1}. \quad (27)$$

Then the distorted model

$$x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \omega_{t+1}) \quad (28)$$

and the true model coincide, if we can solve the equation $C\omega_{t+1} = (A^* + A)x_t + (B^* + B)u_t$ for some ω_{t+1} . Thus the volatility matrix C limits the form of the

⁵³See Hansen and Sargent (2000).

⁵⁴To make this clear take the urn example of Ellsberg (1961); see Gilboa and Schmeidler (1989), p. 141 f.

misspecification and the restricted worst case model is equal to the true one with different matrices. Because of the very different possibilities of interpretation (non-linearities, time-varying parameters, etc.) this kind of robustness is referred to as *unstructured* robustness. The solution of the inner minimization of (25) leads to the following formula of the worst case error ω_{t+1} as function of x_t and u_t :⁵⁵

$$\omega_{t+1} = \frac{1}{\theta} \left(I - \frac{1}{\theta} C' P C \right)^{-1} (C' P (A x_t + B u_t)), \quad (29)$$

which implies that

$$A^* = A + \frac{1}{\theta} \left(I - \frac{1}{\theta} C' P C \right)^{-1} C' P A \quad (30)$$

$$B^* = B + \frac{1}{\theta} \left(I - \frac{1}{\theta} C' P C \right)^{-1} C' P B. \quad (31)$$

Depending on A , B , θ and P a context specific and distorted model arises, which provides more robustness than the conventional one. As θ goes to infinity (29) - (31) illustrate, that the worst case process disappears and the matrices of the approximating and the worst case model coincide.

4.1.4 Detection error probabilities

As robust decision rules depend crucially on the choice of θ , we need a methodology to find a value for θ , which corresponds to a plausible range of uncertainty. Using detection error probabilities links the choice of the worst case model with Bayesian statistical detection theory. Detection error probabilities depend on the log-likelihood ratios of the approximated and the worst case model, that we already introduced in 4.1.2.

The log likelihood function under the approximating model (10), given that this model is the data generating process, is

$$\log L^{aa} = -\frac{1}{T} \sum_{t=0}^{T-1} \left\{ \log \sqrt{2\pi} + 0.5 \tilde{\varepsilon}'_{t+1} \tilde{\varepsilon}_{t+1} \right\}, \quad (32)$$

and the log likelihood function under the distorted model (15), given the same data generating process, is

$$\log L^{ad} = -\frac{1}{T} \sum_{t=0}^{T-1} \left\{ \log \sqrt{2\pi} + 0.5 (\tilde{\varepsilon}_{t+1} - \omega_{t+1}^a)' (\tilde{\varepsilon}_{t+1} - \omega_{t+1}^a) \right\}, \quad (33)$$

where $\{\omega_{t+1}^a\}$ is the worst case process when the data generating process is the approximating model. Therefore the log likelihood ratio under the assumption that the approximating model is the data generating process is given by

$$r_a \equiv L^{aa} - L^{ad} = \frac{1}{T} \sum_{t=0}^{T-1} \{0.5 \omega_{t+1}^{a'} \omega_{t+1}^a - \omega_{t+1}^{a'} \tilde{\varepsilon}_{t+1}\}. \quad (34)$$

⁵⁵See A.1.3 for details.

Now, suppose the distorted model is the data generating process. The log likelihood function for the distorted model is then given by

$$\log L^{dd} = -\frac{1}{T} \sum_{t=0}^{T-1} \left\{ \log \sqrt{2\pi} + 0.5 \varepsilon'_{t+1} \varepsilon_{t+1} \right\}. \quad (35)$$

Defining $\{\omega_{t+1}^d\}$ as the worst case process when the distorted model generates the data, the log likelihood function for the approximating model, assuming that the distorted model is the data generating process, is

$$\log L^{da} = -\frac{1}{T} \sum_{t=0}^{T-1} \left\{ \log \sqrt{2\pi} + 0.5 (\varepsilon_{t+1} + \omega_{t+1}^d)' (\varepsilon_{t+1} + \omega_{t+1}^d) \right\}. \quad (36)$$

In analogy to (34) the log likelihood ratio under the assumption that the distorted model is the data generating process is given by

$$r_d \equiv L^{dd} - L^{da} = \frac{1}{T} \sum_{t=0}^{T-1} \left\{ 0.5 \omega_{t+1}^{d'} \omega_{t+1}^d - \omega_{t+1}^{d'} \varepsilon_{t+1} \right\}. \quad (37)$$

To derive a value for θ based on statistical detection theory, the log likelihood ratios for the approximating and the distorted model are computed for a large number of simulations. With equal prior weights the Bayesian detection error probability is defined as

$$p(\theta) = \frac{1}{2} (p_a + p_d),$$

where $p_i = \text{freq}(r_i \leq 0)$, $i = a, d$, represents the frequency of simulations with a log likelihood ratio r_i smaller or equal to zero. If $r_i \leq 0$, an econometrician would reject model i , although we generated the data with this model. Thus p_i can be interpreted as the probability to reject model i wrongly. And $p(\theta)$ is the equally weighted mean of of this probability for both models. When θ rises, both models become more similar to each other, and the probability to reject a model wrongly increases. Furthermore, when θ reaches infinity, $p(\theta) = 50\%$ and both models have the same probability to be rejected wrongly and thus coincide. Hansen and Sargent (2000) suggest to set $p(\theta)$ at a plausible value and then invert $p(\theta)$ to find a plausible value for the robustness parameter. This parameter is the negative inverse of the risk-sensitivity parameter σ of Whittle (1990) and Jacobson (1973): $\sigma = -\theta^{-1}$. Hansen and Sargent among others⁵⁶ advise to use a value for the detection error probability around 10% in a sample of size 150. Note that θ increases as the sample size does, so that θ goes to infinity and uncertainty disappears when the number of samples does.⁵⁷

⁵⁶See for example Giordani and Söderlind (2004).

⁵⁷See Hansen, Sargent, and Wang (2002) or Anderson, Hansen, and Sargent (2003) for more details about computing detection error probabilities.

4.1.5 Interpretation

As uncertainty is a wide field, this section provides interpretations of the presented methodology to show which kind of uncertainty a Hansen-Sargent robust control theorist is concerned about.

Following the seminal work of Knight (1921) a Hansen-Sargent robust planner models the risk between Knightian risk and Knightian uncertainty (measurable versus unmeasurable uncertainty). Unlike a Bayesian planner, who would only be concerned about calculated risk, this kind of robust planner faces unorganized uncertainty and cannot express his beliefs in probabilistic statements (he is unable to build a prior probability). So he takes the least favorable prior, given a restricted set of priors. Therefore robust control theory covers a wide range of misspecified dynamics:

- Wrong parameters (ω_{t+1} is a linear function of x_t),
- autocorrelated errors (ω_{t+1} is a linear function of x^t) and
- nonlinearities (ω_{t+1} is a nonlinear function of x_t)

A Hansen-Sargent robust planner handles all these types of misspecification by specifying only one parameter θ . This is an advantage because it makes the analysis very easy, but it also implies that it is not possible to investigate specific uncertainty because Hansen-Sargent robustness only allows to analyze all types of uncertainty at once.

The methodology introduces an imaginary evil agent, who reflects the cautionary behavior of the policy maker. This evil agent tries to wreak the largest possible havoc, given his budget constraint (24). So he hits the model at the point where the variance of the forecast error is the largest. Obviously this reflects the concern of the policy maker, who fears misspecification most, where forecast errors are the largest. Furthermore, a comparison of the dynamics of the approximating and the worst case model allows *identifying* the type of misspecification the policy maker fears most, when the approximating model uses the same robust policy without an evil agent. Giordani and Söderlind (2004) argue, that for models including expectations, Hansen-Sargent robustness is also a good way of interpreting deviations from the RE hypothesis of the term structure.

4.2 Solving robust RE programs

After the preceding sections illustrated the motivation behind Hansen-Sargent robustness, this section briefly sketches the solution algorithms needed to solve robust (and non-robust) linear quadratic problems. As the inclusion of forward-looking variables implies different solution techniques under commitment, discretion and simple rules, we also explain the differences related to these concepts.⁵⁸

⁵⁸For a more detailed description see A.2.

Henceforth, the approximating model is characterized by the following transition equations

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B u_t + C \varepsilon_{t+1}, \quad (38)$$

where x_{1t} is a $n_1 \times 1$ vector of predetermined (backward-looking) variables with x_{10} given, x_{2t} is a $n_2 \times 1$ vector of nonpredetermined (forward-looking) variables, u_t is a $k \times 1$ vector of control variables (policy instruments), and ε_{t+1} is a vector of $n_1 \times 1$ vector of innovations (linear combinations of Gaussian i.i.d. shock processes with zero mean and constant variance) to x_{1t} with covariance matrix Σ .⁵⁹ Furthermore, the matrices A , B and C are filled with the appropriate structural model parameters.

4.2.1 Commitment

Given the loss function of the authority

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + 2x_t' U u_t + u_t' R u_t) \right], \quad (39)$$

where R , Q and U are matrices filled with preferences of the policy maker, $x_t = (x_{1t}', x_{2t}')'$ and x_{10} is given, the planner's reaction function $u_t = -F x_t$ can be derived by minimizing (39) subject to (38), using the standard algorithm provided by Söderlind (1999).

A preference for robustness expands the problem to a max-min problem of the form (25), which can be transformed into a standard RE program:

$$\min_{\{u_t\}} \max_{\{\omega_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + 2x_t' U^r u_t^r + u_t^{r'} R^r u_t^r) \right] \quad (40)$$

$$\text{subject to } x_{t+1} = (A - B^r F^r) x_t + C \varepsilon_{t+1},$$

where $U^r = \begin{pmatrix} U & \mathbf{0}_n \end{pmatrix}$, $R^r = \begin{pmatrix} R & \mathbf{0}_{k \times n} \\ \mathbf{0}_{n \times k} & -\theta \mathbf{I}_n \end{pmatrix}$, $B^r = \begin{pmatrix} B & C \end{pmatrix}$, $F^r = \begin{pmatrix} F_r \\ F_\omega \end{pmatrix}$, $u_t^r = (u_t', \omega_{t+1}')'$, ω_{t+1} is a $n \times 1$ vector of shock processes, and x_0 is given. After this transformation the robust rule is given by $u_t = -F_r x_t$, and as the first order conditions for minimization and maximization are the same, the control law can be derived by the same standard RE algorithms. Thus, the easiest way to solve robust and non-robust solutions is simply to solve (40) subject to (38) for $p(\theta) = 10\%$ (robust) and $\theta = \infty$ (standard).

Under discretion and simple rules solution algorithms for (39) and (40) need to be modified. The ideas behind these modifications are illustrated in the following sections, and are due to Giordani and Söderlind (2004), and based on Backus and Driffil (1986). A more detailed technical summary is given in the appendix.

⁵⁹ As we build expectations of the vector of forward-looking variables x_{2t} the last n_2 entries of ε_t are empty.

4.2.2 Discretion

This section assumes a central bank that is unable to commit credibly to the optimal interest rate path. In what follows changes in the solution algorithm related to this concept are presented. Thus we first summarize the sequential decision process, assumed for deriving a standard RE solution under discretion, provided by Backus and Driffil (1986):

1. The private sector builds its expectations for x_{2t+1} : $E_t x_{2t+1} = K_{t+1} E_t x_{1t+1}$. As the authority moves after the private sector, the matrix K_{t+1} , mapping the backward looking variables to the forward looking ones, includes a guess of the central bank's policy function.
2. The planner is now able to observe K_{t+1} and chooses a policy function $u_t = -F x_t$ that minimizes (39) subject to (38), but also subject to $E_t x_{2t+1} = K_{t+1} E_t x_{1t+1}$.
3. The RE Equilibrium is found when the private sector's K_{t+1} coincides with its mathematical expectation. The policy function is consistent with private sector's expectations in the sense, that the F implied by K_{t+1} also solves the problem given K_{t+1} . Therefore K_{t+1} and the control law F are constant in equilibrium.

Based on this algorithm, Giordani and Söderlind (2004) propose to use the Hansen-Sargent robustness concept by assuming that an evil agent tries to maximize the planner's loss at the time (and only at this time!) the planner chooses his policy. As a policy maker under discretion reoptimizes at every period, given the private sector expectations, the evil agent should also be allowed to do so. Every time the authority chooses a policy rule he faces uncertainty and wants to design a robust rule. Solving the robust problem given by (40) subject to (38) thus implies the following additions to the preceding steps:

1. K_{t+1} incorporates now also a guess of the policy function of the evil agent ($\omega_{t+1} = -F_\omega x_t$). This represents the assumption that private agents and the planner share the same concern for robustness.
2. The evil agent designs a policy function $\omega_{t+1} = -F_\omega x_t$ at the same time as the planner does. He faces the same constraints as the planner, but also his own constraint given by $E_t \sum_{t=0}^{\infty} \beta^{t+1} \omega'_{t+1} \omega_{t+1} \leq \delta$, and implemented by the robustness parameter θ .⁶⁰
3. In equilibrium also the evil agent's policy rule (in particular F_ω) has to be constant and consistent with private sector's expectations (in particular K_{t+1}).

⁶⁰See 4.1.3 for the relation between δ and θ . Note that as δ is time-independent the size of deviations from the reference model is constant.

Once the treatment of the evil agent described above is accepted, the robust and the non-robust versions of the problem can both be solved, using the solution algorithm for discretionary RE problems of Backus and Driffil (1986) described above.⁶¹ The equilibrium dynamics are given by

$$x_{1t+1} = Mx_{1t} + C\varepsilon_{t+1}, \quad (41)$$

$$\text{and } \begin{bmatrix} x_{2t} \\ u_t \\ \omega_{t+1} \end{bmatrix} = \begin{bmatrix} N \\ -F_r \\ -F_\omega \end{bmatrix} x_{1t}, \quad (42)$$

where M is a matrix specified in the appendix. The only difference between the worst case and the approximating model in the dynamics of the backward looking variables is due to this matrix M . Therefore the differences between these matrices help to find out the misspecifications feared most by the policy maker.

4.2.3 Simple rules

As already discussed in 3.5.3 and 2.1, simple rules become more and more attractive for several reasons, for example the improved credibility and the reduction of uncertainty. A side effect of assuming a planner, who commits to a given simple rule, is that this allows to analyze the effect of private sector deviations from the RE hypotheses. As soon as one assumes that the planner commits to a rule, he is no longer involved in any decision, and the differences between the RE and the robust solution must be due to private sector expectations. The solution concept is here exemplarily illustrated for the robust problem. Changes regarding the non-robust problem are straightforward and only affect the matrices.

After implementing a simple rule the problem faces an additional constraint: the policy function of the authority:

$$\max_{\{\omega_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + 2x_t' U^* u_t^* + u_t' R^* u_t) \right] \quad (43)$$

$$\begin{aligned} \text{subject to } x_{t+1} &= (A - B^* F^*) x_t + C\varepsilon_{t+1}, \\ \text{and } u_t &= -F_s x_t, \end{aligned}$$

where $F^* = \begin{pmatrix} F_s \\ F_\omega \end{pmatrix}$, $U^* = \begin{pmatrix} U & \mathbf{0} \end{pmatrix}$, $R^* = \begin{pmatrix} R & \mathbf{0}_{\mathbf{k} \times \mathbf{n}} \\ \mathbf{0}_{\mathbf{n} \times \mathbf{k}} & -\theta \mathbf{I}_{\mathbf{n}} \end{pmatrix}$, $B^* = \begin{pmatrix} B & C \end{pmatrix}$, and $u_t^* = \begin{pmatrix} u_t' & \omega_{t+1}' \end{pmatrix}'$.

As solving this problem would lead to infinite loss for every degree of robustness, another constraint is needed for deriving a plausible solution to this problem. Without allowing the authority to use a robust rule the evil agent could, for example,

⁶¹Like in the commitment case, we solve (40) subject to (38) for $p(\theta) = 10\%$ (robust) and $\theta = \infty$ (standard).

choose an exponentially increasing or decreasing series $\{\omega_{t+1}\}$ - the policy maker would be defenceless. This seems to be no plausible way for modelling fear about misspecification. The technical reason for this problem is due to the specification of the evil agent's policy function. As long as the evil agent is allowed to use any rule reacting to private sector expectations, he would strategically exploit these forward looking expectations to derive infinite loss. The solution is to constrain the evil agent's policy function so that it depends only on the predetermined variables of the model: $\omega_{t+1} = -F_\omega x_{1t}$. Now the misspecifications are given exogenously and do not depend on the formation of expectations.⁶²

Putting both new constraints together the vector of controls is modified to $F^* = \begin{bmatrix} F_s \\ F_\omega \mathbf{0}_{\mathbf{n}_1 \times \mathbf{n}_2} \end{bmatrix}$ and the solution to the problem is given by

$$u_t^* = - \begin{bmatrix} F_s \\ F_\omega \mathbf{0}_{\mathbf{n}_1 \times \mathbf{n}_2} \end{bmatrix} x_t. \quad (44)$$

4.3 Criticism

With increasing popularity of Hansen-Sargent robust policies, more and more economists have started to criticize the methodology. For example, Svensson (2002b) argues that robust policies depend crucial on the choice of θ , as in linear quadratic problems the solution will always be on the boundary of the set of possible models. In his opinion a robust planner could also be a fool, taking too much care about implausible catastrophes, just by setting the value for θ too low. It is truly right that this is a weakness of the methodology, but with the use of detection error probabilities the choice of θ is not that arbitrary any more. Rather it is based on the idea to take a set of models, that are not easy to distinguish by available data. As the policy maker does not know the true data generating process, he chooses a probability of making the wrong choice between the two models on the basis of in-sample fit, for a sample of given size.⁶³ Contrary to this, Svensson's way of restricting the set of possible parameter values appears to be more arbitrary.

Furthermore Svensson puts forward the argument that the boundary solution is important, but all other solutions (that might be only a small epsilon away) are not considered. This is of course true, as this is exactly the robust control philosophy: minimize the worst case loss. So one *has* to consider the boundary solution. Sims (2001) raises concerns over some other issues regarding a minimax approach to model uncertainty. In his opinion minimax expected utility theory is merely a way of constructing a prior someone has to consider when assessing actual prior believes. He argues that it can be used at best as a shortcut when there is not enough time to construct a prior in a more appropriate way. For example, he believes that modelling a robust private sector and a robust central bank is not

⁶²See Giordani and Söderlind (2004) for a more detailed discussion.

⁶³See also Giordani and Söderlind (2004).

appropriate, as this implies that central banks have the same sub-rational behavior than private agents. He suggests modelling uncertainty only in the private sector, whereas a Hansen-Sargent robust planner believes that the private sector and the central bank should have the same model, the same loss function and the same degree of fear about misspecification.⁶⁴ In short, his argument is that it would be better to form prior distributions in a reasonably way, and that the methodology presented here is only concerned about a very general and unspecific type of uncertainty. As already mentioned above, this can be an advantage as well as a disadvantage: If a policy maker is not able to express his beliefs, this approach seems plausible. And of course, the results of this approach should be compared to results of more direct approaches (when possible).

Sims also argues that most studies using Hansen-Sargent robustness deal with uncertainty about relatively unimportant sources of model uncertainty. In his view the uncertainty about actual values of coefficients in log-linear approximations is not the most important lack of knowledge. Rather, the question whether there is a long-run trade-off between inflation and output, and the uncertainty about the danger of deflationary or inflationary spirals, are much more important. As these aspects of uncertainty are not included in Hansen-Sargent robust control applications, he proposes not to label this as "model uncertainty" when one puts strong assumptions on some important sources of uncertainty. He is definitely right when he says that most studies using such an approach include a lot of critical assumptions. However, (a) these assumptions normally stem from the underlying model and are not related to the methodology, and (b) by keeping these assumptions in mind, one gains insights into how uncertainty changes results, given these assumptions. Different models lead to different results under uncertainty and should of course be compared to each other. As long as a policy maker knows the weak points, this methodology helps him in making a decision. With these criticisms in mind, Hansen-Sargent robustness remains "(...) a tool for assessing uncertainty, not a replacement (...)" (Sims (2001), p. 53).

4.4 Applications apart from the analysis of monetary policy

Before we discuss the consequences of a Hansen-Sargent robust central banker, we believe that it is straightforward to show results from robust control applications in other fields of economic interest. For example, Hansen and Sargent (1999) use their methodology in a forward-looking model of a monopoly (acting as a Stackelberg leader) facing a competitive fringe in an industry producing a single nonstorable homogenous good. When robustness is only assumed in one sector, pessimistic forecasts about demand push firms' output down and the robust sector produces less, while the other produces more. However, when both sectors share the same

⁶⁴This becomes somewhat different when we analyze simple rules, as uncertainty about the policy disappears, when assuming the policymaker to use a predictable rule. See 3.5.3 for details.

concern for robustness the monopolist produces enough more (as reaction to the cautious production of the other sector) to overcompensate his own uncertainty, and drives the price below the value of the non-robust case.

A number of papers analyzes market prices of risk under Hansen-Sargent robustness.⁶⁵ These studies provide a solution to the "equity premium puzzle" as robustness leads to increasing risk premia. Hansen, Sargent, and Tallerini (1999) develop a robust decision and pricing theory that incorporate a market price for *Knightian model uncertainty*. Therefore a multiplicative adjustment to a stochastic discount factor, reflecting a preference for robustness, is introduced. As a result the premium for model uncertainty increases the conventional market price of risk. Hansen, Sargent, and Wang (2002) extend this analysis by assuming that the state of the economy is partly unobservable, so that planner and agent have to use a Kalman filter to estimate the unobservables. Unfortunately, comparing the resulting market price of risk with the one derived from a fully observable state does not suggest that the introduction of unobservable parts of the economy raises the risk premium.

Cagetti, Hansen, Sargent, and Williams (2002) show how a Hansen-Sargent concern for model misspecification changes prices and quantities in a stochastic non-linear growth model under uncertain growth. Investors make inferences about the growth rate of the technology (modeled as a hidden Markov model) by observing movements in the technology level without the possibility to distinguish between infrequent large shocks and continuous small shocks hitting the growth rate. Preferences of investors are modeled using a penalty approach that penalizes departures from a reference or approximating model. As a reaction to a concern for robustness investors are more cautious and robustness enlarges the risk premium. This motive can be offset by making investors discount the future more.

5 Uncertainty in closed economy models

"(...) uncertainty is not just a pervasive feature of the monetary policy landscape; it's the defining characteristic of that landscape."

(Alan Greenspan in his Remarks at the Meeting of the American Economic Association, 2004)

Following Alan Greenspan, an assessment of uncertainty is essential for the analysis of monetary policy. Since uncertainty is not just "the defining characteristic of the monetary policy landscape", but also a wide research field, and as this study concentrates on model uncertainty, this section only presents a brief survey and integration into the "modeling model uncertainty"-literature. Seeing as doubts about models have existed for as long as models themselves have, this is already a vast research area.

⁶⁵See Anderson, Hansen, and Sargent (2000), Anderson, Hansen, and Sargent (2003), Hansen, Sargent, and Tallerini (1999), Chen and Epstein (2002), Hansen, Sargent, and Wang (2002), and Cagetti, Hansen, Sargent, and Williams (2002).

The two most influential economists have already been mentioned above: Friedman (1953) and Friedman (1961), who stress the uncertainty of the time structure in economic models, and Brainard (1967), who argues that political interventions can give rise to uncertainty about the impacts of monetary policy. Both conclude that policy actions should be dealt with cautiously and not in a heavy handed manner. Nowadays a vast amount of complicated and advanced methodologies have been developed and implemented to deal with the impact of different types of uncertainty in macroeconomic models and their conclusions are diverse. One that can be drawn from all these results is that various ways of modeling different types of uncertainty can lead to quite different results.⁶⁶

Most studies concerning model uncertainty use Bayesian approaches or robust control techniques, that provide a robustness analysis with respect to a single reference model. However, some more recent studies evaluate monetary policy rules across different models, often by mixing Bayesian with robust control approaches. Figure 2 classifies the bulk of this literature according to the approach, the model type (forward-looking vs. backward-looking) and the kind of model uncertainty (general vs. parametric uncertainty). As a detailed discussion of every study is clearly impossible the following survey is limited to the main characteristics of the most important ones.

5.1 Bayesian techniques

Studies using Bayesian techniques are all based on the principle of minimizing the expected loss, given a prior probability of an uncertain part of the model. After Brainard's seminal work (based on multiplicatively added error terms), which leads to "Brainard's Conservatism Principle", other investigations also propose that a more aggressive policy rule can be an appropriate reaction to uncertain parameters. The analysis of Chow (1975), for example, show that one can say almost nothing even qualitatively about the changes in the presence of uncertainty about several parameters, whereas Craine (1979) and Söderström (2002) both derive the result that uncertainty in inflation dynamics can lead to more aggressive policy rules. Craine uses a simple dynamic random coefficient model to demonstrate that if uncertainty about policy impact is dominant, this results in a fixed money growth rate policy, while the dominance of uncertainty over transition dynamics leads to very aggressive countercyclical policy rules. Söderström models uncertainty by assuming coefficients that are subject to multiplicative random shocks and suggests that in the case of uncertainty about the persistence of inflation, it may be optimal to respond more ag-

⁶⁶In contrast, studies of data uncertainty most often lead to clearly less aggressive rules: Aoki (2003) and Rudebusch (2001) for example suggests reacting more defensively to first publications of data as they are often revised. However, Wieland (1998) shows that monetary strategies under an uncertain natural unemployment rate can also motivate an element of experimentation, so that in the case of very high uncertainty and inflation close to target, the authority responds more aggressively.

Applications modeling model uncertainty		
	general uncertainty	parameter uncertainty
Bayesian approaches	forward	Kimura/Kurozumi (2003), Kurozumi (2003) Levin/Williams (2003a), Walsh (2004c)
	backward	Craine (1979), Onatski/Williams (2003)
robust control	forward	Giannoni (2001, 2002), Kara (2002), Levin/Williams (2003a)
	backward	Sargent (1999), Stock (1999) Tetlow/von zur Muehlen (2001), Onatski (2000a,b), Onatski/Stock (2002)
evaluation across models	Taylor (1999), Coenen (2007), Christiano/Gust (1999), Levin/Wieland/Williams (1999, 2003), Levin/Williams (2003b), Angeloni/Coenen/Smets (2003), Brock/Durlauf/West (2003, 2007)	

Table 1: Uncertainty in closed economy models

gressively to shocks to reduce uncertainty about future inflation dynamics, whereas uncertainty about other parameters makes policy rules more defensive. When the impact of lagged inflation is uncertain, the variance of future inflation increases if a central bank fails to stabilize current inflation and a more aggressive policy reaction reduces the impact of the coefficient uncertainty on the unconditional variance of inflation.

The studies of Levin and Williams (2003a), Kurozumi (2003), Walsh (2004c), and Kimura and Kurozumi (2007) use micro-founded models with a micro-founded loss function and include uncertainty even in the social loss function. These studies also conclude that uncertainty can lead to more aggressive rules. Kimura and Kurozumi show in a hybrid New Keynesian framework that, particularly when uncertainty about inflation dynamics increases, the central bank should place more weight on price stability and respond to shocks more aggressively. Aggressive policy rules as a response to uncertain output dynamics are justified by the positive correlation between the policy multiplier and the transmission of natural interest rate shocks as well as by the effect of loss function uncertainty. Furthermore a more aggressive policy response with a highly inert interest rate policy reduces Bayesian risk. Levin and Williams use Bayesian and robust control methods to show that multiplicative uncertainty about the elasticity of inflation with respect to output leads to more

aggressive rules when effects on the objective function are taken into account. The analysis of Walsh studies the costs of using a simple rule in face of uncertainty about the degree of inflation inertia and price stickiness. He suggests that the robust implicit instrument rules derived by Giannoni and Woodford (2003b) and Giannoni and Woodford (2003a) are in fact robust to uncertainty about inflation inertia, but not robust to a misspecification in the degree of nominal price rigidity. Furthermore he shows that optimal first difference rules (proposed by some studies using an evaluation across different models) are robust to uncertainty about inflation inertia. For most combinations of actual and perceived structural inflation persistence, however, losses of using such rules instead of the optimal targeting rule exceed the costs arising from parameter misspecification.

So far all the aforementioned studies have been based on model theoretic analysis. An empirical approach is taken by Onatski and Williams (2003), who use a Bayesian model error modeling (MEM)-approach based on Ljung (1999). The idea is to estimate a reference model, take the reference model's errors, and try to fit them with a general set of explanatory variables. Afterwards a set of uncertainty models is built around the reference model, which is consistent with all regressions for the errors, not rejected by formal statistical procedures. For this purpose they use frequency domain methods, as well as Bayesian posterior distributions. This allows them to analyze the performance of different Taylor-type rules for different types of uncertainty, which they characterize in the following way: model uncertainty (uncertainty about the parameters and the model specifications), shock uncertainty (uncertainty about the serial correlation properties of the noise-processes), and data uncertainty (uncertainty about the data generating process). The results suggest that real-time data uncertainty is less dangerous as model uncertainty, and that the effects of a pure shock uncertainty are relatively mild. They also evaluate different robust monetary policy rules, related to different types of model uncertainty, by calculating an upper bound on the worst possible losses under these different uncertainty sources. These evaluations lead one to conclude that the most dangerous part of model uncertainty involves the effect of the real interest rate on the output gap (the slope of the IS-curve), while the least dangerous part is - contrary to the recent ECB studies we will discuss shortly - the dynamics of inflation. In the field of model uncertainty all resulting policy rules are more aggressive than the rules ignoring uncertainty.

5.2 Evaluation across different models

Another branch of literature focusses on the performance of policy rules across different models to analyze the robustness of policy rules. Some of them achieve this by using both Bayesian and robust control techniques, others by employing a decision theoretic approach.⁶⁷ The contribution of this strand of literature to the under-

⁶⁷As our approach uses Bayesian detection error probabilities as well as an investigation of different reference models - differing in their degree of persistence - it shares an overlap with these approaches. We will come back to this issue in 6.3.

standing of policy rules in different models is important, but these papers are, in fact, unable to compute one robust rule that performs well across all models. Most of these studies suggest using a rule which includes a weight on the lagged interest rate.

Following the arguments of McCallum (1988) and McCallum (1999) the studies of Taylor (1999), Christiano and Gust (1999), Levin and Williams (2003b), Levin, Wieland, and Williams (1999) and Levin, Wieland, and Williams (2003) analyze the performance of optimal rules for one model when the true model is described by a different model. Levin, Wieland and Williams conclude that first difference rules (those with a parameter on the lagged interest rate of one) perform well in a wide range of models including current and lagged inflation and show that forecast based rules provide no substantial gains in stabilizing the performance of the economy compared with purely outcome based ones. Furthermore longer horizon rules perform badly and lead to multiple equilibria because of self-fulfilling expectations that are not related to fundamentals in five models that differ in output and price dynamics. In a similar analysis Levin and Williams (2003b) suggest that rules with a moderate parameter on lagged inflation are good when an authority wants to stabilize inflation *and* output. However, there is no simple rule that performs well across models for a regime solely targeting inflation. They also show that rules derived by robust control methods might perform very poorly in models different to the reference model. In all of these studies non-nested models represent competing perspectives about controversial issues like expectation formation or inflation persistence. Average loss functions are minimized across different models, which from a Bayesian perspective is equivalent to assuming a policy maker with flat prior beliefs about the extent to which each model provides an accurate description. As such a policy maker is unable to express probability statements about the different models he simply puts an equal weight on each model.

The ECB studies the behavior of monetary strategies under uncertainty in inflation and output gap persistence in a number of papers, which suggest that it pays for a central bank to overestimate the inflation persistence. Angeloni, Coenen, and Smets (2003), for example, show in a dynamic stochastic general equilibrium model that uncertainty about inflation persistence is a greater problem than uncertainty about output persistence. They use an estimated Smets and Wouters (2003) euro area model and conclude that it is better to proceed under the assumption of a relatively high degree of inflation persistence, as the costs of assuming a lower degree of inflation persistence than the one prevailing in reality are greater than the costs of making the opposite mistake. Coenen (2007) confirms this result in two small-scale models using parameter estimates by Coenen and Wieland (2005). He also concludes that policy makers, under uncertainty about the degree of inflation persistence, are well-advised to design rules under the assumption of a high degree of inflation persistence. His investigation follows partly Levin, Wieland, and Williams (2003), who evaluate policy rules across different models, but as Coenen only assumes uncertainty in the inflation persistence, his analysis is restricted to

two model *specifications* of one single reference model differing in the underlying staggered contracts specification.

More recently, Brock, Durlauf, and West (2003) and Brock, Durlauf, and West (2007) develop a decision theoretic approach by employing Bayesian model averaging. In contrast to the three studies mentioned above, they do not put equal weights on the different models but use posterior model probabilities that are derived by the relative likelihoods of each model in an empirical analysis. The applications within a class of hybrid and a class of backward-looking models suggest that the standard Taylor rule performs quite well, but that rules with a weight on the lagged interest rate perform better.

5.3 Some other approaches

Recently some completely new approaches providing robustness in economic models have been developed, which can neither be categorized in one of the previous methodology classes nor in the class of robust control. For example, Giannoni and Woodford (2003a) and Giannoni and Woodford (2003b) provide a general method for deriving an optimal monetary policy rule in the case of a dynamic linear rational-expectations model and a quadratic objective function. Their rule results in a determinate equilibrium in which the responses to shocks are optimal. Furthermore their "robust implicit instrument rules" are (i) independent of the specification of the stochastic disturbances, (ii) can be justified from a "timeless perspective",⁶⁸ and (iii) can be represented by a generalized Taylor rule. A first application concludes that optimal policy rules require history-dependence and do not place nearly as much weight to future inflation and output projections as the current practice of central banks using inflation-forecast targeting might suggest. Walsh (2004a) and Walsh (2004b) show that these rules lead to exactly the same rules as the robust control policies in a wide class of models, while the implied macroeconomic behavior differs because of different assumptions about the formation of expectations.

An even more recent developed methodology for the investigation of monetary policy rules under uncertainty has been provided by Svensson and Williams (2005). They extend the standard linear-quadratic framework to a Markov-jump linear-quadratic system, where model uncertainty takes the form of different modes (or regimes) for random transition matrices following a Markov process with constant transition probability. These modes correspond to a finite number of random coefficient values that are independent of the conventional additive innovations. This allows one to approximate very different kinds of uncertainty, as for example simple i.i.d. model deviations, serially correlated model deviations, estimable regime-switching models, and so forth. They present examples of this method based on two empirical models of the US economy, a backward- and a forward-looking one. They estimate three-mode regime-switching models using Bayesian methods to locate the

⁶⁸Commitment to such a rule does not imply time-inconsistent policy.

maximum of the posterior distributions, and compare the results to the constant-coefficient versions. Their analysis suggests that model uncertainty clearly leads to a change in the nature of monetary policy, and that the median instrument-rate response under uncertainty is less aggressive. Unfortunately, they have not provided a reasonable economic interpretation of the three regimes at the time of writing.

5.4 Robust control approaches

We now turn to studies using methods similar to those employed here.⁶⁹ Robust control techniques are based on minimizing the worst case loss for a given set of models around a single reference model. Studies using these techniques can be classified according to the modelling of unstructured uncertainty (Sargent (1999), Hansen and Sargent (1999)), structured non-parametric uncertainty (Stock (1999), Onatski (2000a), Onatski (2000b), Tetlow and Von Zur Muehlen (2001), Onatski and Stock (2002), Leitimo and Söderström (2004), Kilponen (2003), Kilponen (2004), Kilponen and Leitimo (2006)), and parametric uncertainty (Giannoni (2001), Giannoni (2002), Kara (2002), Levin and Williams (2003a)). Unstructured uncertainty is modelled by adding a shock term to the model equations to represent model misspecifications. These misspecifications are limited by the statistical distance between the approximating and the distorted model.⁷⁰ Structured non-parametric approaches modify this technique by imposing structures on the set of possible models and compute robust policy rules for the maximal size of uncertainty that still leads to a determinate equilibrium and a stable model. Studies modelling parametric uncertainty impose bounds on parameter values and compute rules that minimize the maximum loss for all models with parameters inside this specified range. Nearly all studies using one of these techniques conclude that more aggressiveness is the appropriate reaction to model uncertainty.

Early studies employing Hansen-Sargent robustness concepts are based on small backward-looking models. For example Sargent (1999) investigates the open economy model of Ball (1998), whereas Stock (1999), and Onatski and Stock (2002) analyze the closed economy of Rudebusch and Svensson (1998). The unstructured approach of Sargent suggests using more aggressive rules for a plausible parametrization of the Ball-model. The more aggressive reaction is appropriate because shocks become larger and serially correlated as uncertainty arises, so that a higher aggressiveness is needed to dampen the effects these shocks. The analysis of Stock, and Onatski and Stock impose different structured model sets for mixed model and shock uncertainty, as well as for pure model uncertainty. The key findings of their studies are that (a) nearly all uncertainty sets lead to more aggressiveness and (b) the more structure is placed on the perturbation sets, the less aggressive are the

⁶⁹We will discuss the results from the most important studies in more detail when we compare them with ours in 6.3.

⁷⁰Details of this statistical distance have already been discussed in 4.1.4.

rules. However, they also conclude that uncertainty in the lag structure of the model can lead to a more defensive reaction to inflation but a more aggressive one with respect to the output-gap.

The last finding is confirmed by Giannoni (2001) and Giannoni (2002) who use robust control to derive policy rules for the case of uncertainty of the parameters in a standard New Keynesian model. His robust rules also include in general a stronger response of the interest rate to fluctuations in inflation and the output gap. In Giannoni (2001) it is also shown that uncertainty may amplify the degree of super-inertia in optimal rules (a coefficient on the lagged interest rate greater than one). An interesting equivalence result concerning expectation modelling is obtained by Kara (2002), who shows that under parameter uncertainty robust control methods lead to exactly the same rules, no matter if both the policy maker and the private sector fear misspecification, or only the private sector is modelled in a robust fashion.⁷¹ His results suggest that uncertainty about the slope of the Phillips curve leads to less aggressive responses, whereas imperfect knowledge about the shock persistence results in more aggressive rules.

The studies closest to our are those of Leitemo and Söderström (2004), Kilponen (2003) and Kilponen (2004), who also employ robust control techniques to forward looking New Keynesian frameworks.⁷² Kilponen (2003) argues that the higher aggressiveness in robust policy rules calls for a more inflation-averse policy maker. In the application of Kilponen (2004) the partial information model of Svensson and Woodford (2003) is chosen and only private expectations are modeled robust. His results suggest that robust private agents should overestimate the persistence of exogenous shocks (shock uncertainty), but underestimate the policy response to the output gap (policy uncertainty). The latter leads, together with imperfect measurements, to larger and more persistent responses of private consumption to government expenditure shocks (precautionary savings). More recently, Kilponen and Leitemo (2006) argue that using the Friedman rule of increasing money supply can reduce the costs of robustness.

Leitemo and Söderström (2004) analyze a small open economy version of the standard New Keynesian framework under structured uncertainty, namely uncertainty for each of the three model equations separately. Depending on the type of shock reduced form solutions of the robust optimal policy rules can be more aggressive or more defensive in an open economy, whereas the results for a closed version of their model suggest reacting more aggressively, regardless of the uncertainty source. Unfortunately robustness can have serious consequences when more likely scenarios

⁷¹Notice that dynamics differ due to different private sector behavior.

⁷²The modification of the solution to a standard RE program to one that proposes robustness for the case of an authority acting under commitment is originally due to Hansen and Sargent (1999) and Hansen and Sargent (2003). More recently, Kasa (2002) builds an extension to a frequency domain approach, and Giordani and Söderlind (2004) extend the solution principle to an authority acting under discretion, and also provide a method for the evaluation of simple policy rules. See 4.2 for details.

than the worst case appear to be the true specification: Fear about misspecification in the Phillips curve or the exchange rate equation leads to an inefficiently high output variability, and robustness against misspecification in the output equation leads to a higher inflation variability.

Contrary to all previous studies analyzing Hansen-Sargent robustness discussed above, we use a *hybrid* New Keynesian closed economy to investigate how *different degrees of backward-looking behavior* in the NKPC and the expectational IS-curve change robust and non-robust results under the assumption of an unstructured uncertainty set. We choose the unstructured approach to receive the most general results possible when investigating the connection between robustness and different degrees of persistence. Furthermore we build our model on a microeconomic foundation that leads to persistence in inflation and output gap dynamics, so that differences in optimal policy strategies can be attributed to the preferences of households and firms. This illustrates the influence of private sector behavior on optimal policy. Since optimal commitment solutions are very hard to achieve, and discretionary solutions often lead to undesirable results, we also investigate the performance of simple rules in a hybrid model under uncertainty, and calculate losses and impulse-response functions for different generalized Taylor-type rules.

6 Robustness in a hybrid New Keynesian model

This section presents a hybrid version of a New Keynesian framework, including all important features discussed above. We build on a DSGE model, based on optimizing behavior of forward-looking households and firms, combined with Calvo-type price stickiness. In order to capture the inflation and output persistence suggested by most empirical studies,⁷³ we allow a fraction of the price adjusting firms to use the recent history of the aggregate price index to set their prices, and assume a habit formation in consumption. The following section presents the benchmark model, before 6.2 calibrates it and shows robust and non-robust solutions in form of losses and impulse response functions. Finally, 6.3 concludes and provides a comparison with the existing literature.

6.1 The benchmark model

The New Keynesian framework is based on the assumptions of competitive goods markets (drawn from Dixit and Stiglitz (1977)) and sticky prices (drawn from Calvo (1983)). For a detailed model derivation see appendix A.3.

⁷³See for example Roberts (2005), Fuhrer (1997), Sbordone (2002), Galí, Gertler, and Lopez-Salido (2001), Rudebusch (2002) and Altissimo, Ehrmann, and Smets (2006).

6.1.1 Households

Households supply labor (L_t), purchase consumption goods (C_t), and hold money (M_t) and bonds (B_t). A representative household maximizes the expected sum of its discounted utility with respect to its preferences:

$$\max_{\{C_t, M_t, N_t, B_t\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - H_{C,T})^{1-\sigma}}{1-\sigma} + \frac{\chi_1}{1-b} \left(\frac{M_t}{P_t} \right)^{1-b} - \chi_2 \frac{N_{t+i}^{1+\eta}}{1+\eta} \right], \quad (45)$$

where β represents the discount factor, b represent the risk aversion parameter (the inverse of the intertemporal elasticities of substitution) of real money holding, and $H_{C,T} = h_C C_{t-1}$ represents the habit formation in consumption. The higher h_C , the more the households try to consume at least as much as in the last period. Therefore h_C is a measure of persistence in the habit formation. If $h_C = 0$, σ represent the risk aversion parameter of consumption, for values different from zero σ is merely an index of the curvature of the utility function.⁷⁴ The parameters χ_1 and χ_2 represent weights, defining household preferences.

Firms are normalized to a continuum of measure one and produce differentiated products (c_j). Under the assumption of monopolistic competition the composite consumption good is given by

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}}, \quad \zeta > 1, \quad (46)$$

where ζ represents the price elasticity of demand.⁷⁵

Households choose the cost minimizing combination of individual goods to achieve any given level \bar{C} of the composite consumption good:

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj \quad (47)$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}} = \bar{C}_t, \quad (48)$$

where p_j represents the price of the individual good c_j . The first order condition for good j implies

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\zeta} C_t. \quad (49)$$

where $P_t \equiv \left[\int_0^1 p_{jt}^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}}$ represents the aggregated price index, defined as a generalized weighted mean of all prices. The price elasticity of demand for every good

⁷⁴For convenience we refer to σ simply as the risk aversion parameter of consumption or the inverse of the intertemporal elasticity of substitution.

⁷⁵The composite consumption good is in fact the generalized mean of a continuous function $x(j) = c_{jt}$ (representing the individual good of firm j) weighted by the function $h(x) = x^{\frac{\zeta-1}{\zeta}}$.

c_j is given by the same ζ . With increasing ζ , goods become closer substitutes, and firms loose market power. As a consequence, a change in the price of any individual good has greater influence on demand due to substitution.

Given the cost minimizing combination of individual goods for a given level of the composite consumption good, households now choose the utility maximizing levels of consumption, labor supply, and real money and bond holdings. The decision problem is given by (45) subject to household's budget constraint (in real terms):

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t, \quad (50)$$

which simply says that the sum of consumption, and real money and bond holdings in period t must equal the sum of the real wage bill (W_t represents the nominal wage) and real firm profits (Π_t) in period t , and real money and bond holdings (including interest payments $(1 + i_{t-1})$) from period $t - 1$.⁷⁶ The first order conditions of this problem are given by (50) and the following three optimality conditions

$$(C_t - h_C C_{t-1})^{-\sigma} = \beta (1 + i_t) E_t \left[\left(\frac{P_t}{P_{t+1}} \right) (C_{t+1} - h_C C_t)^{-\sigma} \right], \quad (51)$$

$$\chi_1 \left(\frac{M_t}{P_t} \right)^{-b} = (C_t - h_C C_{t-1})^\sigma \frac{i_t}{1 + i_t}, \quad (52)$$

$$\frac{\chi_2 N_t^\eta}{(C_t - h_C C_{t-1})^{-\sigma}} = \frac{W_t}{P_t}, \quad (53)$$

which represent the Euler condition for the intertemporal allocation of consumption, the intertemporal optimality condition setting the marginal rate of substitution between money and habit adjusted consumption equal to the opportunity costs of holding money (the interest rate), and the intertemporal optimality condition setting the marginal rate of substitution between leisure and habit adjusted consumption equal to the opportunity costs of "holding" leisure (the real wage).

The Euler equation implies a flat consumption path over time (consumption smoothing). Due to (51) habit adjusted consumption in period t is chosen such that it equals the discounted, and inflation, interest earnings, and habit adjusted consumption of period $t + 1$. The second condition represents the money market equilibrium, with money demand depending directly on consumption rather than income. According to this the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. By holding money the representative agent loses the opportunity to eliminate the costs of inflation (by earning interest payments), or the opportunity to receive utility from immediate consumption, respectively. The third equation can be interpreted as a labor-leisure trade-off equation, requiring that in equilibrium the marginal utility of a higher revenue from producing one extra unit of output equals the marginal disutility of the work effort needed.

⁷⁶As the prices for the individual goods determine the aggregate price index, the composite consumption good is already measured in real terms.

6.1.2 Firms

The profit maximizing firms face three constraints: The production function, the demand curve and the price adjustment mechanism. Production is assumed to follow a constant returns to scale production function which solely depends on labor input, $c_{jt} = Z_t N_{jt}$, $E(Z_t) = 1$, where Z_t is a stochastic zero mean aggregate productivity disturbance. Every firm j first chooses the cost minimizing labor demand subject to the production function:⁷⁷

$$\min_{\{N_{jt}\}} \left(\frac{W_t}{P_t} \right) N_{jt} + \varphi_t (c_{jt} - Z_t N_{jt}). \quad (54)$$

It follows directly that φ_t is equal to the firm's real marginal production costs. The first order conditions implies that real marginal costs equal the real wage divided by the marginal product of labor:

$$\varphi_t = \frac{W_t}{Z_t P_t}. \quad (55)$$

Regarding the price adjustment we assume a variant of Calvo's price stickiness due to Galí and Gertler (1999). Like in the original paper of Calvo (1983) a randomly selected fraction of firms $(1 - \omega)$ adjusts their price while the remaining fraction of firms ω does not adjust. Thus ω is a measure of the degree of nominal rigidity and can be interpreted as the probability of a firm not to adjust in period t . In addition we assume that a fraction of $(1 - \tau)$ firms behave in a forward-looking way (just like in Calvo's model) and the remaining fraction τ use the recent history of the aggregate price index when they set prices. According to this, τ is a measure of the degree of *backwardness* in price setting.

First consider the forward-looking fraction of firms. Those firms that adjust their price at time t maximize the expected discounted value of current and future profits. Profits at some future date $t + s$ are affected by the price decision at time t if (and only if) the firm does not receive another opportunity to adjust between t and $t + s$. As the probability for the event "no adjustment" is ω , the probability for "no adjustment between t and $t + s$ " is ω^s and the maximization problem is given by⁷⁸

$$\max_{\{p_{jt}\}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right], \quad (56)$$

where (since all variables are measured in real times) $\Delta_{i,t+i}$ represents the real discount factor, which is determined by (51) to be

$$\Delta_{i,t+i} = E_t \frac{\left(\frac{P_{t+i}}{P_t} \right)}{(1 + i_t)^i} = \beta^i \left(\frac{C_{t+i} - h_C C_{t+i-1}}{C_t - h_C C_{t-1}} \right)^{-\sigma}. \quad (57)$$

⁷⁷Since all firms face same constraints, we assume that equilibrium wages for all firms are the same.

⁷⁸ $P(\text{"no adjustment in } s \text{ periods"}) = P(\text{"no adjustment in one period"})^s$.

All firms are identical in the sense that they use the same technology and face the same demand curve, but differ in current prices due to different adjustment dates. Firms that adjust their price at date t all chose the same optimal price p_t^{fl} (the superscript fl simply clarifies that it is the optimal price of forward-looking firms), determined by the first order condition of (56):

$$\left(\frac{p_t^{fl}}{P_t}\right) = \mu \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i} (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\zeta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i} (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\zeta-1}}, \quad (58)$$

where we used (57) and $\mu = \frac{\zeta}{\zeta-1}$.⁷⁹ The aggregate price level is an average of the price charged by the adjusters and the price charged by the nonadjusters.

As the adjusting firms are randomly selected, the average price of the nonadjusters equals the average price (of all firms) at time $t-1$. Defining \bar{p}_t^n as an index for the prices newly set in period t , the law of large numbers determines the aggregate price level to be

$$P_t = \left[\omega P_{t-1}^{1-\theta} + (1-\omega) (\bar{p}_t^n)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (59)$$

The index for newly set prices is defined as the weighted average of the prices of the forward-looking adjusters and the prices of the backward-looking adjusters:

$$\bar{p}_t^n = (1-\tau) p_t^{fl} + \tau p_t^{bl}, \quad (60)$$

where p_t^{bl} represents the optimal price of a backward-looking firm adjusting the price at time t . We assume that the p_t^{bl} evolves according to the following equation:

$$p_t^{bl} = \bar{p}_{t-1}^n + \pi_{t-1}, \quad (61)$$

$\pi_t \equiv \hat{p}_t - \hat{p}_{t-1}$ represents inflation, defined as the difference between the actual percentage deviation of the price level from its steady state value (\hat{p}_t) and its corresponding lagged percentage deviation (\hat{p}_{t-1}). A backward-looking firm that adjusts at time t simply corrects the average price of last period's price adjustment for inflation. For this correction it uses last period's inflation to forecast future inflation. These assumptions ensure that the evolution of prices (i) converges to optimal behavior (as long as inflation is stationary), and (ii) implicitly incorporates future information since \bar{p}_{t-1}^n is partly determined by forward-looking price setters.⁸⁰

6.1.3 Linear approximations

In the following we derive the key equations of the New Keynesian model by approximating the preceding equations in percentage deviations from flexible price

⁷⁹Note, that for the derivation of (58) the fact is used, that as long as firms do not adjust $p_{jt} = p_{jt+i}$.

⁸⁰See Galí and Gertler (1999) for a discussion.

equilibrium.⁸¹ This has the advantage of "easier-to-handle" additive representations and allows us to assume an equilibrium with zero values for inflation, output and the interest rate *relative* to their steady states, without concerning *absolute* steady state values, and without any loss of generality. Henceforth a variable \hat{v} denotes the percentage deviation of a variable V around its steady state and the superscript f denotes the flexible-price steady state. The Euler equation implies an equilibrium steady state interest rate $\bar{i} = \frac{1-\beta}{\beta}$.

The New Keynesian Phillips Curve In the special case when all firms are able to adjust ($\omega = 0$) (58) is reduced to the standard result of monopolistic competition: every firm sets its price equal to a markup $\mu > 0$ over nominal marginal costs $P_t \varphi_t$:

$$\left(\frac{p_t^{fl}}{P_t} \right) = \mu \varphi_t, \quad (62)$$

When prices are flexible, all firms are able to adjust and all prices have to equal the average price ($p_t^* = P_t$), and thus (62) gives $\mu^{-1} = \varphi_t$. Together with (55) and the labor-leisure trade-off equation (53) this leads to

$$\frac{Z_t}{\mu} = \frac{\chi_2 N_t^\eta}{(C_t - h_C C_{t-1})^{-\sigma}}. \quad (63)$$

Approximating (63) and the production function, under the market clearing assumption that output equals consumption in equilibrium, allows us to derive an expression for percentage deviations of output from flexible price equilibrium:

$$\hat{x}_t^f = \frac{1}{\gamma} \left[(1 + \eta) \hat{z}_t + \sigma \varkappa \hat{x}_{t-1}^f \right], \quad (64)$$

where $\gamma \equiv \eta + \frac{\sigma}{1-h_C}$, $\varkappa \equiv \frac{h_C}{1-h_C}$, and x denotes output.

Approximating (58) around a zero average inflation steady state gives

$$\hat{p}_t^{fl} = (1 - \omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}), \quad (65)$$

which can be interpreted as follows: The optimal nominal relative price of the forward looking price setters equals the expected discounted value of future nominal marginal costs. Combining (65) with approximations of (59), (60) and (61) leads to an expression for aggregate inflation,

$$\pi_t = (1 - \tau) (1 - \omega) (1 - \beta\omega) \phi \hat{\varphi}_t + \phi (\omega\beta E_t \pi_{t+1} + \tau \pi_{t-1}), \quad (66)$$

where $\phi = (\omega + \tau [1 - \omega(1 - \beta)])^{-1}$. Equation (66) represents the supply side of the economy and is often referred to as the *hybrid* version of the NKPC. In the

⁸¹Recall that equilibria of flexible and sticky price models are the same and only dynamics differ, since flexible prices imply an immediate adjustment. Thus, we can use the assumption of flexible prices, when we are to approximate around equilibrium. See also 3.1 and 3.2.

border case of $\tau = 1$ the NKPC differs from (66) since (65) disappears when there are no forward-looking price setters. If all prices are adjusted in a backward-looking manner the influence of marginal costs and expected inflation vanishes and (66) is reduced to $\pi_t = \pi_{t-1}$. As all price adjustments only correct for the last period's inflation rate, price-changes are always of the same size. Therefore, no one takes marginal costs into account when it comes to set the prices. This follows directly from (60) and (61) by setting $\tau = 1 : \bar{p}_t^* = \bar{p}_{t-1}^* + \pi_{t-1}$.

Following (66) the measure of the degree of backwardness in price setting τ also represents a measure of the degree of persistence in inflation dynamics. For an increasing τ the influence of the lagged inflation rate rises, whereas for $\tau = 0$ we derive a purely forward-looking variant of the NKPC. A rise in the degree of price stickiness ω or in the discount factor β increases the relative importance of inflation expectations, simply since price adjustments become more rare, or the future is discounted less, respectively.

In contrast to the conventional Phillips Curve (66) depends on real marginal costs instead of the output gap. However, real marginal costs can be related to an output gap measure, namely the gap between actual output and flexible-price equilibrium output: $y_t \equiv \hat{x}_t - \hat{x}_t^f$. The difference between this concept to the conventional one is the following: The conventional output gap is output relative to its natural level, which depends solely on productivity disturbances, and is often derived under the assumption of wage rigidity, whereas the output gap used here depends on the utility function of labor supplying households. Thus, it reflects changes in equilibrium employment after a On the contrary, the output gap measure used here represents the difference between output relative to its steady state level under price rigidity ($\omega > 0$) and output relative to its flexible price steady state value ($\omega = 0$) and reflects only *deviations* from steady state, adjusted for particular movements in equilibrium.⁸²

With the help of (55), (62), (53), the production function and $\hat{x}_t = \hat{c}_t$, real marginal costs can be approximated by⁸³

$$\hat{\varphi}_t = \gamma y_t - \kappa \sigma y_{t-1} \quad (67)$$

Marginal costs are related to the real wage due to cost minimization of the firms, as illustrated in (55). As the real wage is in turn related to the labor supply decision of the households via the labor-leisure trade-off equation (53), labor supply

⁸²For convenience we refer to y_t simply as the output gap. See Walsh (2003) for a more detailed discussion of the two gap-concepts.

⁸³Note that the following approximation can only be derived under very restrictive assumptions. First of all, in a single-country model consumption always equals output in equilibrium. For the two-country analysis we have to drop this assumption. More important, the production function solely depends on labor. Thus we deny any influences of capital accumulation. This is a standard assumption for short-run analysis in monetary policy models (see Walsh (2003) for a discussion). In a model with variable capital the relationship between output and marginal costs would no longer be proportionate. However, simulations suggest that the relation remains close to proportionate and justify this assumption. See Galí and Gertler (1999) for a discussion.

and consumption decisions depend on the real wage *and* influence marginal costs. Since consumption equals production in equilibrium, and production solely depends on labor, marginal costs and output have the equilibrium relationship shown by (67). Output is only able to rise, when households work more. Just as households suffer opportunity costs in terms of foregone leisure, firms have to pay more to recruit enough workers, until the marginal rate of substitution between leisure and habit adjusted consumption equals the opportunity costs of "holding" leisure. Thus, real wages and marginal costs rise. In addition to the conventional New Keynesian Framework, our hybrid model also includes lagged output, as household's consumption decisions include habit formation, and the relationship between marginal costs and the consumption-leisure decision exhibits this persistence as well. Workers alter their consumption behavior slowly, and thus demand changes slowly, and the feedback on marginal costs also includes a lag.

Substituting (67) into (66) leads to

$$\pi_t = \kappa (\gamma y_t - \sigma \chi y_{t-1}) + \phi (\omega \beta E_t \pi_{t+1} + \tau \pi_{t-1}), \quad (68)$$

where $\kappa = (1 - \tau)(1 - \omega)(1 - \beta\omega)\phi$. The parameter κ can be interpreted as follows: It represents the impact of real marginal costs on inflation and depends on the structural parameters β , τ and ω . When the discount factor β rises a firm gives more weight to the future and therefore κ declines, because current inflation is less sensitive to *current*, but more sensitive to future marginal costs. An increase in nominal price rigidity also reduces the impact of current marginal costs, simply because the firms are able to adjust more frequently. The same is true for the degree of backwardness in price setting: When the fraction of backward-looking firms rises, the forward-looking behavior becomes less important and, as a consequence, the impact of current marginal costs decreases, since firms adjust prices according to "the rule of thumb" given by (61).

An increase in the fraction of backward-looking price-setters has several implication on the dynamics of the NKPC. The first concerns the impact of marginal costs on inflation, represented by κ . As the central bank controls inflation by influencing demand via the real interest rate, and marginal costs depend on output via (67), this is an important channel for the transmission of monetary policy. As we have just discussed, κ falls when τ rises:

$$\frac{\partial \kappa}{\partial \tau} = -(1 - \omega)(1 - \beta\omega) < 0. \quad (69)$$

Thus, the effectiveness of monetary policy declines with a rising fraction of backward-looking price-setters. The more firms take last period's inflation to correct for expected future inflation, and do not set prices equal to the expected discounted value of future nominal marginal costs, the less effective is the demand management of the monetary authority. In the extreme case $\tau = 1$, $\pi_t = \pi_{t-1}$, monetary policy would have no effect, and every temporary change in inflation would be permanent.

Another effect is the impact on the relative importance of expected and lagged inflation:

$$\frac{\partial \omega \beta \phi}{\partial \tau} = -\frac{\omega \beta [1 - \omega (1 - \beta)]}{(\omega + \tau [1 - \omega (1 - \beta)])^2} < 0 \quad (70)$$

and

$$\frac{\partial \tau \phi}{\partial \tau} = \frac{\omega}{(\omega + \tau [1 - \omega (1 - \beta)])^2} > 0. \quad (71)$$

According to (70) and (71) expectations lose relative importance in inflation dynamics and deviations of inflation from equilibrium are more persistent. Thus an increase in the fraction of backward looking price-setters calls for stronger interest rate reactions to control for inflationary pressures when the policymaker acts under commitment, since the possibility to influence expectations declines. Furthermore, the persistence in inflation dynamics tends to strengthen the return to steady state when inflation is out of equilibrium.

According to (68) an increase in h_c has two effects. Contrary to the persistence in price-setting behavior, the influence of output on marginal costs increases with h_c :

$$\frac{\partial \gamma}{\partial h_C} = \frac{\sigma}{(1 - h_C)^2} > 0 \quad (72)$$

This effect is due to the influence of consumption habits on the optimal labor-leisure decision. The habit adjusted inverse of the intertemporal elasticity of substitution of consumption ($\frac{\sigma}{1-h_C}$) rises when last period's consumption becomes more important for the decision about the optimal amount of labor supply. Recall that a rise in output can only be achieved by recruiting more worker, and, as a consequence, by paying higher wages to increase the opportunity costs of holding leisure. This increase in real wages has to be the higher, the more persistent the consumption habits of the households are, since households base their decision partly on last period's consumption and change their labor supply slowly. If the consumption behavior is persistent, any change in real wages in order to change labor supply needs to be stronger. Thus, the influence of a change in consumption (and thus production in equilibrium) on real wages and marginal costs increases also. Contrary to the effect described by (69) an increase in consumption habits improves the influence of monetary policy, as the transmission channel in the NKPC via marginal costs becomes more important.

As a second effect of a rise in h_c also lagged consumption comes into play:

$$\frac{\partial \sigma \varkappa}{\partial h_C} = \frac{\sigma}{(1 - h_C)^2} > 0. \quad (73)$$

The higher consumption habits are, the more the *change* in consumption becomes the relevant variable for deciding between consumption and leisure.⁸⁴ A temporary rise in C_t would provoke the households to claim for higher real wages, as they

⁸⁴To clarify, see that in the extreme case of $h_c = 1$ (53) is given by $\chi N_t^\eta \Delta C_t^\sigma = \frac{W_t}{P_t}$.

would otherwise be willing to substitute work with leisure and abandon on some consumption. Independent from the habit formation, any fall in consumption afterwards makes households more willing to work and tends to lower real wages again, but consumption habits imply a slower adjustment as the dynamics back to equilibrium exhibit persistence as well. Therefore, the influence of deviations of C_t from equilibrium on real wages, and thus marginal costs, become more persistent.

The hybrid IS curve Equation (68) is only one half of the key equations of a New Keynesian model. The second half is a linearized version of (51)⁸⁵

$$y_t = \left(\frac{1}{1 + h_C} \right) E_t y_{t+1} + \left(\frac{h_C}{1 + h_C} \right) y_{t-1} - \sigma^{-1} \left(\frac{1 - h_C}{1 + h_C} \right) (i_t - E_t \pi_{t+1}). \quad (74)$$

Equation (74) represents the demand side of the economy and is often referred to as the *expectational forward-looking IS curve*. From (74) follows that h_C , the measure of the persistence in the habit formation, is also a measure of the degree of persistence in the output gap dynamics. Moreover, for $h_C = 0$ (74) is purely forward-looking.

Concerning the dynamics of (74) an increase in h_C rises the relative importance of y_{t-1} with respect to y_t :

$$\frac{\partial \left(\frac{1}{1+h_C} \right)}{\partial h_C} = -\frac{1}{(1+h_C)^2} < 0, \quad \frac{\partial \left(\frac{h_C}{1+h_C} \right)}{\partial h_C} = \frac{1}{(1+h_C)^2} > 0. \quad (75)$$

This is due to the household's optimal allocation of consumption over time. As habit adjusted consumption in period t is chosen such that it equals the discounted, and inflation, interest earnings, and habit adjusted consumption of period $t + 1$, an increase in consumption habits implies that every change in consumption becomes more persistent. Households base their decision partly on last period's consumption and do not prior smooth consumption over time, but rather habit adjusted consumption. This has no influence on equilibrium outcomes, since consumption does not change in the steady state, but any change of consumption, and thus output in equilibrium, becomes more persistent.

A second effect concerning the effectiveness of monetary policy is the impact on the influence of the real interest rate the decision of the households:

$$\frac{\partial \left(\sigma^{-1} \left(\frac{1-h_C}{1+h_C} \right) \right)}{\partial h_C} = -\frac{2\sigma^{-1}}{(1+h_C)^2} < 0. \quad (76)$$

The habit adjusted inverse of the intertemporal elasticity of substitution of consumption ($\frac{\sigma}{1-h_C}$) increases and the opportunity costs of consumption (the real interest rate) lose importance. Thus a higher h_C calls for stronger interest rate reactions in order to manage demand and control inflation. Thus, consumption habits have two opposing effects. On the one hand, consumption habits increase the effectiveness of

⁸⁵For convenience we use i_t instead of \hat{i}_t .

monetary policy by rising the influence of demand on marginal costs, as shown in (69). On the other, they reduce the impact of changes in the interest rate, since the opportunity costs of consumption become less important. In addition, the influence of the real interest rate falls faster with rising h_c :

$$\frac{\partial^2 \left(\sigma^{-1} \left(\frac{1-h_c}{1+h_c} \right) \right)}{\partial h_c} = \frac{4\sigma^{-1}}{(1+h_c)^3} > 0. \quad (77)$$

6.1.4 State space representation

The model so far ignores any disturbances in the inflation or output gap dynamics other than the productivity disturbance. Walsh (2003), chapter 5.4, for example, shows how including a taste shock in the utility function leads to a more general disturbance term in the expectational IS-Equation. Clarida, Galí, and Gertler (2001) derive a New Keynesian Phillips curve with inflation shocks by adding a stochastic wage markup that capture deviations between the marginal rate of substitution between leisure and consumption and the real wage. For convenience and generality we simply add two demand (e_{yt}) and supply ($e_{\pi t}$) shocks to study the dynamics of the model. These are assumed to follow an AR(1)-process with orthogonal Gaussian zero mean innovations:

$$y_t = \left(\frac{1}{1+h_C} \right) (E_t y_{t+1} + h_C y_{t-1}) - \sigma^{-1} \left(\frac{1-h_C}{1+h_C} \right) (i_t - E_t \pi_{t+1}) + e_{yt}, \quad (78)$$

$$\pi_t = \kappa (\gamma y_t - \sigma \pi_{t-1}) + \phi (\omega \beta E_t \pi_{t+1} + \tau \pi_{t-1}) + e_{\pi t}, \quad (79)$$

$$e_{y_t} = \rho_y e_{y_{t-1}} + \xi_{y_t}, \quad \xi_{y_t} \sim N(0, \sigma_y), \quad (80)$$

$$e_{\pi_t} = \rho_\pi e_{\pi_{t-1}} + \xi_{\pi_t}, \quad \xi_{\pi_t} \sim N(0, \sigma_\pi). \quad (81)$$

Combining these model equations in the appropriate state-space form leads to

$$A_0 \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B_1 i_t + C_1 \varepsilon_{t+1}, \quad (82)$$

where A_0 , A_1 , B_1 and C_1 are matrices filled with appropriate structural parameters, $x_{1t} = (e_{y_t}, e_{\pi_t}, y_{t-1}, \pi_{t-1}, i_{t-1})'$ is the vector of predetermined (backward looking) variables with x_{10} given, $x_{2t} = (y_t, \pi_t)'$ is the vector of forward looking (jump) variables, and $\{\varepsilon_{t+1} : t = 1, 2, \dots\}$ is a conditionally homoskedastic Gaussian martingale differences process, which satisfies $E_t(\varepsilon_{t+1}) = 0$ and $E_t(\varepsilon'_{t+1} \varepsilon_{t+1}) = I$.⁸⁶ Solving this for the state variables leads to the conventional representation of a transition equation:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B i_t + C \varepsilon_{t+1}, \quad (83)$$

where $A = A_0^{-1} A_1$, $B = A_0^{-1} B_1$ and $C = A_0^{-1} C_1$. Since central banker's opinions about their optimal objective are diverse, we assume a very general quadratic loss

⁸⁶To allow for interest rate smoothing and/or stabilization the lagged interest rate is included into the vector of state variables.

function including the possibility for interest rate smoothing and/or stabilization. The corresponding minimization problem to (39) is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \right), \quad (84)$$

where λ_y , λ_i and $\lambda_{\Delta i}$ represent the weights regarding the policymaker's preferences for output gap and interest rate stabilization, and interest rate smoothing, respectively. The planner's reaction function $i_t = -Fx_t$ can be derived by minimizing (84) subject to (83), where x_{10} is given. For deriving a robust solution the problem is extended to a min-max standard *RE* problem of the form (40). The following section shows the calibration and the results under commitment, discretion and simple rules for the closed economy hybrid model.⁸⁷

6.2 Calibration and results

parameter		scenario					
h_c	degree of persistence in habit formation	0	0.4	0	0.4	0.8	
τ	degree of backwardness in price setting	0	0	0.4	0.4	0.8	
ω	degree of nominal price rigidity	0.8598					
σ	inv. of consumption elasticity	0.1571					
η	inv. of labor elasticity	0.824					
β	discount factor (quarterly periods)	0.99					
ρ_y, ρ_π	AR parameter of shocks	0.35					
σ_y, σ_π	standard deviation of shocks	1					

Table 2: Calibration, single-country economy

The values for the model parameters for five different scenarios are summarized in Table 2. All structural parameters are chosen so that the parameters in the NKPC and the IS Curve for the purely forward-looking version of the model coincide with the empirical results of Rotemberg and Woodford (1997), who estimate a complete purely forward-looking New Keynesian Framework, including the loss and the reaction function of the monetary authority, which provides a good description of the macroeconomic behavior in the United States between 1979 and 1995. For reasons of comparability we take the baseline value for the inverse of labor elasticity of Giannoni (2002), who studies robust optimal Taylor rules in the Rotemberg and Woodford (1997)-model. We focus on varying only h_c and τ to investigate the influence of backward-looking behavior in the private sector separately. Since the information about the real degree of persistence in consumption habits and the degree of backward-looking firms is rare, we simply experiment with the values to

⁸⁷See 4.2 and A.2 for details of the solution algorithms.

0.4 and 0.8 to study the qualitative influence of rising persistence. Thus, the results should be interpreted carefully, especially for the case of $\tau = h_c = 0.8$, which seems to be unrealistic high with respect to the corresponding κ -value, representing the influence of marginal costs on inflation, and should thus be interpreted as an experiment, done to clarify the qualitative effects of a change in persistence.

The following sections show losses and impulse response functions under commitment and discretion for different weights in the objective of the policymaker, inter alia the estimated objective of Rotemberg and Woodford (1997), which is given by $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0.0483, 0.2364, 0)$. Losses are computed for the approximating model using the optimal RE rule (l_a^{RE}) and the worst case model using the robust rule (l_w). Additionally, we show losses for an authority and a private sector with unfounded fear against misspecification, using the robust rule although the approximating model is the true one (l_a^r), and there is no imaginary evil agent. A comparison of l_a^{RE} and l_r illustrates how losses under uncertainty rise, whereas the difference between l_a and l_a^r shows the disturbance caused by the suboptimal robust rule. Robust solutions are computed for robustness parameters θ that correspond to detection error probabilities near 10% and near 35% in a sample of 150, using Monte Carlo simulations.

For the following technical reason it is not possible to compute the solution for an authority, using the approximating rule in the distorted model. In equilibrium the dynamics of a solution to (39) are given by

$$\begin{bmatrix} x_{1t+1} \\ \rho_{2t+1} \end{bmatrix} = M \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix} + \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{\mathbf{n}_2 \times 1} \end{bmatrix}, \quad (85)$$

and

$$\begin{bmatrix} x_{2t} \\ u_t \\ \rho_{1t} \end{bmatrix} = N \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix}, \quad (86)$$

where M and N are matrices, specified in the appendix A.2, ρ_{it} are Lagrangian multipliers, and $\rho_{20} = 0$.⁸⁸ The matrix M , which determines the dynamics of the backward-looking variables, depends on the control variable u_t , and thus on the reaction functions of the central bank and the evil agent. Therefore, we can study the dynamics of an authority who uses the robust policy in the approximating model (simply by canceling the evil agent's reaction function), but we are not able to simulate the approximating rule in the worst case model, as assuming the approximating rule would alter the matrix M , and thus the dynamics of (85) are not those of the worst case model any more. Note that when we talk about the robust policy in the approximating model, this model uses the same policy function and expectation formation as the worst case model, but simply sets the evil agent (ω_{t+1}) to zero.

⁸⁸For the discretionary solution and the solution using simple rules, the Lagrangian multipliers are zero for all t and the matrices are slightly different, but the following arguments remain valid nevertheless. See appendix A.2 for details

Therefore the approximating model uses not only the robust policy, but also robust expectations. This can be interpreted as assuming that central bank and private sector share the same reference model, the same loss function, and the same degree of robustness.

6.2.1 Commitment

Table 3 gives results for different weights in the objective. Recall that for the non-robust versions $p(\theta) = 50\%$, what corresponds to $\theta = \infty$.⁸⁹ Unfortunately, we are only able to find a determinate equilibrium under uncertainty for $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0, 0.3, 0.3)$ and $(0, 0.6, 0)$, as for objectives including a preference for output gap stabilization the problems show an infinite number of stable solutions. Under indeterminacy every expected inflation can be justified.⁹⁰

$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0, 0.3, 0)$			$(0.0483, 0.2364, 0)$			$(0.1, 0.5, 0)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	45	48,55	50, ∞	219	ind	ind	239	ind	ind
$h_c = 0.4, \tau = 0$	47	45,49	43, ∞	220	ind	ind	241	ind	ind
$\tau = 0.4, h_c = 0$	153	166,272	183, ∞	831	ind	ind	977	ind	ind
$h_c = \tau = 0.4$	138	148,174	160, ∞	834	ind	ind	982	ind	ind
$h_c = \tau = 0.8$	540	599,4158	689, ∞	6151	ind	ind	8299	ind	ind
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.5, 0, 0)$			$(0, 0.6, 0)$			$(0.5, 0, 0.3)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	249	ind	ind	52	54,125	59, ∞	250	ind	ind
$h_c = 0.4, \tau = 0$	249	ind	ind	49	51,66	54, ∞	250	ind	ind
$\tau = 0.4, h_c = 0$	920	ind	ind	176	187,8618	212, ∞	879	ind	ind
$h_c = \tau = 0.4$	920	ind	ind	157	166,578	183, ∞	876	ind	ind
$h_c = \tau = 0.8$	8265	ind	ind	629	691,1117	811, ∞	4880	ind	ind

For some scenarios the steady state of the model is indeterminate (ind).

Table 3: Losses under commitment, single-country economy

First of all, the non-robust results suggest that a preference for stabilizing output seems to complicate the optimal reaction of a central banker and raises losses. This is a plausible result, since a supply shock brings the monetary authority to raise the interest rate to dampen inflationary pressures, but also causes a fall in output and thus raises losses, when the aim of the central bank is also to stabilize the

⁸⁹When both models have the same probability to be rejected wrongly they coincide. See 4.1.4 for details.

⁹⁰For a discussion of indeterminacy see Clarida, Galí, and Gertler (1999), Batini and Pearlman (2002) and Batini, Levine, and Pearlman (2004).

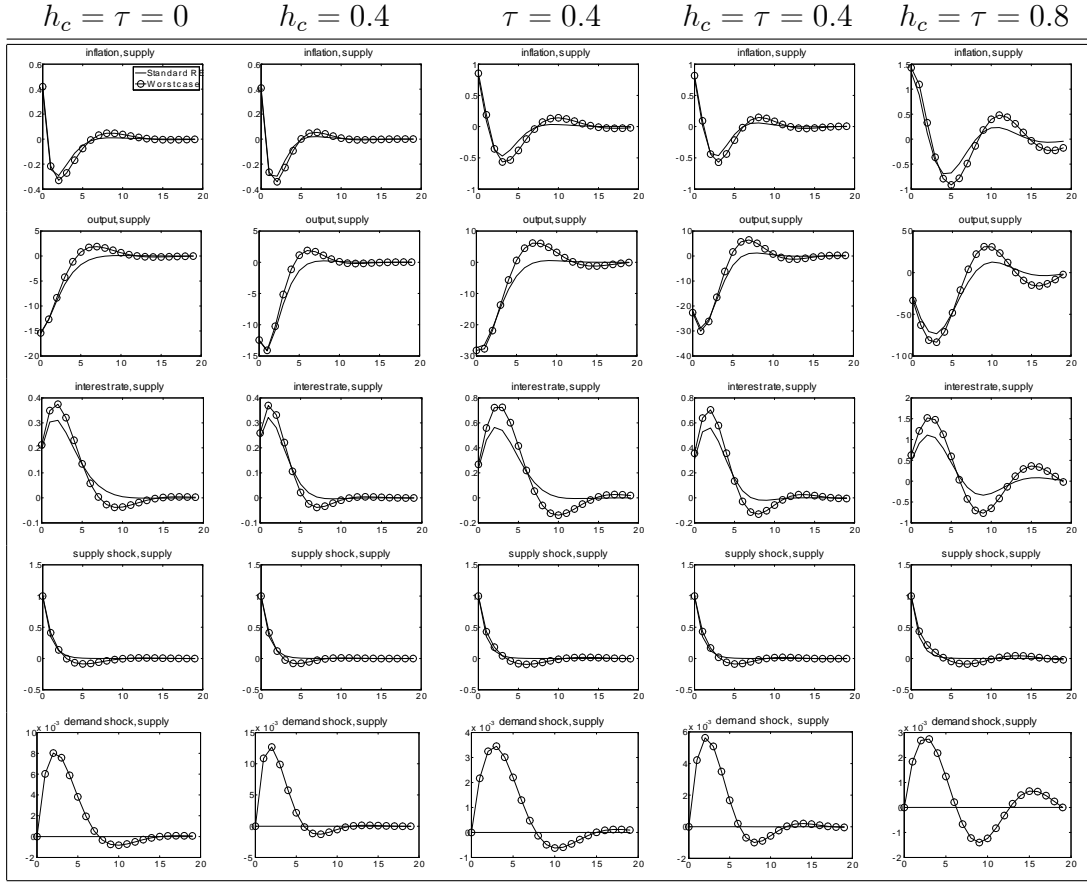
output gap. Furthermore, persistence in inflation dynamics increases losses substantially, whereas persistence in the output gap does not change outcomes considerably or even decreases losses. This is due to the effects described in 6.1.3. Whereas the effects of increasing consumption habits on the effectiveness of monetary policy are twofold (see (72) and (76)), an increase in the fraction of backward-looking price-setters clearly deteriorates the transmission of monetary policy. With increasing persistence in price-setting behavior monetary policy loses effectiveness in two important channels: (i) expectations are less important, so that influencing expectations is less effective (this argument is also valid for an increase in consumption habits), and (ii) deviations from flexible price output play a lower role in the Phillips curve, as marginal costs are of less importance in price setting (here consumption habits imply the opposite).

Regarding the robust solutions, table 3 clearly shows that losses under uncertainty increase. Furthermore, the results suggest that under commitment the model is very sensitive with respect to uncertainty. Using the robust rule in the approximating model does not stabilize the problem for $p(\theta) = 10\%$ in the cases where we could determine one single steady state. For $p(\theta) = 35\%$, corresponding to a lower degree of uncertainty, the losses in the case of an unfounded fear against misspecification already rise substantially, whereas using the robust rule in the worst case scenario keeps the evil agent under control, and implies that losses are near the optimal *RE* solution even for $p(\theta) = 10\%$. Thus, a central banker acting under commitment is confronted with a very difficult decision, since he is faced with two options: The first is to react on uncertainty and use a robust policy, risking an immense increase in losses, when his fear is unfounded. The second is simply to ignore uncertainty, even if a Hansen-Sargent robust policy provides a very good advice, when the worst case scenario actually arrives.

If we compare the influences of uncertainty with those of persistence in price-setting or consumption habits, respectively, Table 2 suggests that already a small degree of inflation persistence increases losses more than the evil agent could, when the policymaker uses a robust monetary strategy. This indicates that the choice of the degree of backward-looking price-setting is more important for the conduct of monetary policy than an appropriate degree of general model uncertainty.

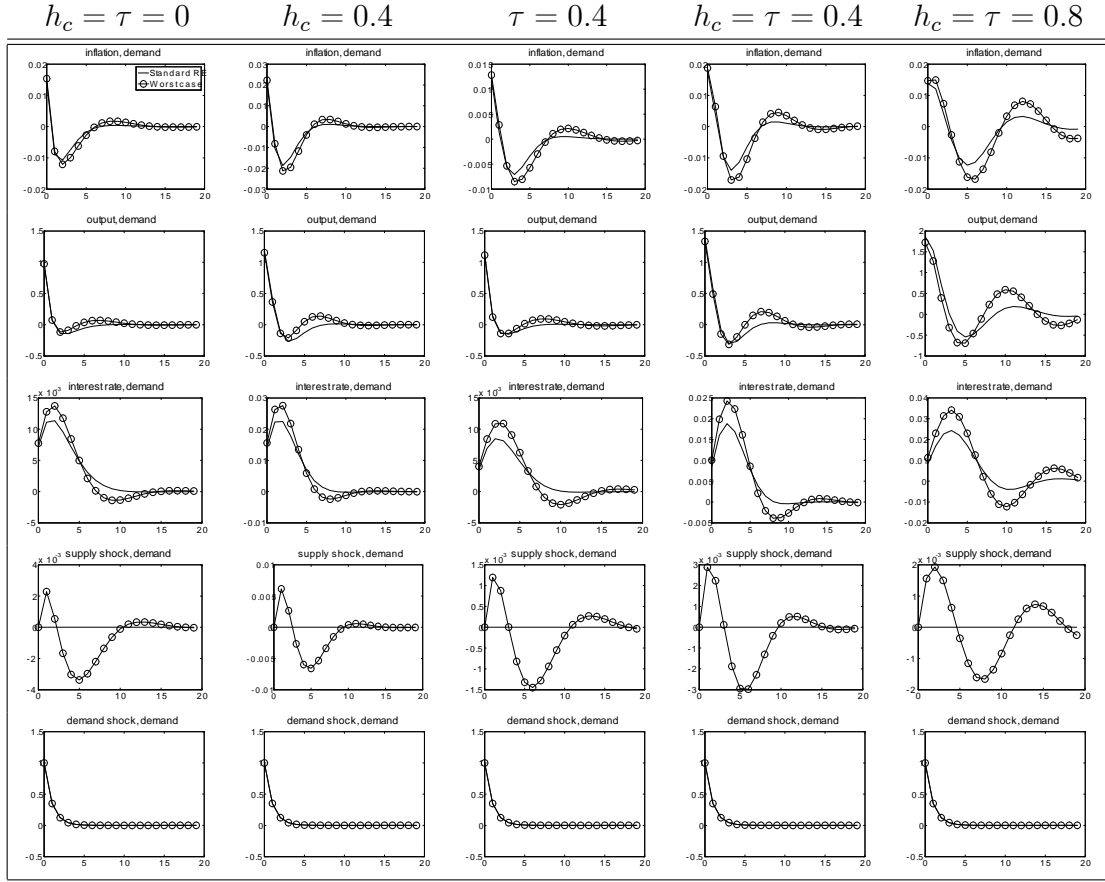
Figures 2 and 3 illustrate the dynamics of the model for $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0, 0.3, 0)$ after supply and demand innovations of size one for the standard *RE* solution ($\theta = \infty$) and the robust policy for $p(\theta) = 10\%$. Impulse responses show that uncertainty justifies a stronger equilibrium interest rate for all scenarios and objectives. Note that the evil agent starts distorting in the second period, so that the contemporaneous response of the approximating and the worst case model are the same.

In the case of a supply shock the impulse is a rise in inflation, which provokes the central banker to increase the nominal interest rate strong enough to cause a positive real interest rate in order to reduce demand, and thus inflationary pressures. The fall in the output gap causes a period of disinflation immediately, and inflation undershoots its equilibrium value for some periods. This is the reaction we already

Figure 2: IRFs, commitment, $p(\theta) = 0.1$, supply innovation

discussed in 3.5.2. Under commitment the monetary authority chooses a path for the output gap that keeps output below its potential value for some periods to lower private sector expectations of future inflation and improve the policy trade-off. Thus, inflation undershoots its equilibrium level before it returns to zero. Additionally, the interest rate is adjusted gradually under commitment, because of its preference for interest rate stabilization, and due to the influence on expectations. The gradual response exploits private sector expectations as it signals that the central bank is willing to fight inflation. As we will see in 6.2.2, for the discretionary solution this gradual response vanishes even for an objective including a preference for interest rate stabilization.

When comparing the optimal reactions of the purely forward-looking model with those including persistence, the graphs confirm the results of Table 2. Introducing persistence in consumption habits seems not to change the degree of deviations considerably, whereas persistence in price-setting clearly does. For $h_c = 0.4$ reactions simply include a more cyclical adjustment of the output gap, without altering any other reaction in a noteworthy manner, except the interest rate. The introduction of both consumption habits and backward-looking price-setting leads to stronger reactions due to the loss in effectiveness of monetary policy. Thus, the interest rate responses in Figure 2 increase from the left to the right column. Furthermore,

Figure 3: IRFs, commitment $p(\theta) = 0.1$, demand innovation

the case of a small persistence in price-setting behavior ($\tau = 0.4$) already confirms that deviations become much stronger for all variables. In the extreme case of $h_c = \tau = 0.8$ persistence leads to cyclical adjustment paths of inflation and the output gap, which seem to amplify each other, and are transmitted to the interest rate as well.

Concerning the robust solutions all graphs justify stronger interest rate reactions when the policymaker fears model misspecification. Moreover, deviations from equilibrium are stronger and more persistent for all variables. The imaginary evil agent, reflecting the policymaker's fear about misspecification, wreaks the most possible havoc, given his budget constraint, by causing stronger and more persistent shock impacts. As this represents the worst possible scenario for the monetary authority, it is plausible to react to uncertainty with a more aggressive interest rate policy in order to protect against this worst case. Since the effectiveness of monetary policy decreases with rising persistence, this effect becomes even stronger when we introduce consumption habits and backward-looking price-setting.

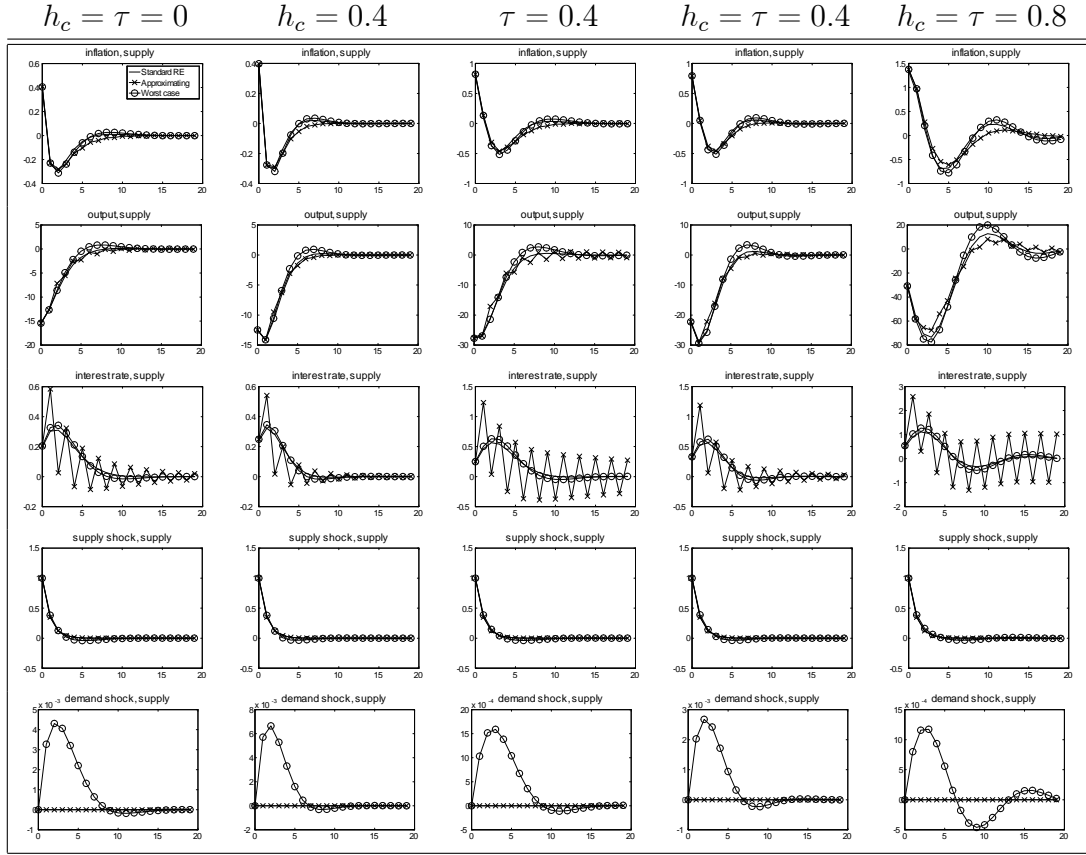
The last two rows in Figure 2 clarify the influence of the evil agent and the model misspecification that the policymaker fears most. In order to distort the model as severely as possible, the malevolent player strengthens the downturn in the supply shock which implies an undershooting of equilibrium. At the same time he

generates a demand shock in the opposite direction to enlarge reactions of inflation and output, complicates the decision of the central banker, and introduces a cyclical component. This cyclical component becomes bigger when persistence increases, as the evil agent exploits the dynamics of inflation and the output gap. The reactions of supply and demand shocks represent the greatest possible deterioration the model can exhibit, given the restriction of the distortion, and therefore reflect the concern of a policymaker who fears model misspecification most where the disturbance is the greatest.

Comparing the influence of uncertainty with the influence of persistence, the graphs confirm the result from Table 3, and also suggest that the degree of consumption habits and backward-looking price-setting seems to influence outcomes more than an appropriate degree of uncertainty. The responses also suggest that persistence justifies even stronger interest rate reactions than uncertainty. Thus, uncertainty about the right degree of persistence (the right reference model) seems to be more important than general model uncertainty, represented by the imaginary evil agent.

In Figure 3 the responses after a demand innovation are illustrated. The primary impulse is an increase in consumption, leading to an increase in inflation as well. The policymaker reacts with an increase in the interest rate to dampen inflationary pressures by reducing demand via a positive real interest rate. Similar to the supply shock, an authority under commitment keeps output under potential for some periods to improve expectations, and output and inflation undershoot their equilibrium values before they return to zero.

In contrast to the supply innovation, an increase in persistence in price-setting does not imply a more aggressive policy. Since the transmission of marginal costs on inflation decreases with τ the increase in inflation is smaller when more firms set prices in a backward-looking manner and do not care about today's and future marginal costs. This clearly decreases the effectiveness of the central banker, but as the primary effect on inflation becomes smaller, it calls for a more defensive policy reaction. Due to the same arguments, the opposite is true for consumption habits. Since an increase in consumption habits increases the primary transmission on inflation and deteriorates the effectiveness of monetary policy (as the opportunity costs of consumption are of less importance in household decisions) the optimal policy has to be more aggressive. In addition, as the importance of expectations falls, which tends to complicate optimal reactions for a central bank under commitment, inflation and consumption dynamics become more persistent, and for high values of h_c and τ persistence leads to cyclical adjustments of inflation and the output gap which are transmitted to the interest rate as well. Nevertheless, the reactions after a demand innovation for $h_c = \tau = 0.8$ are still of less strength than the reactions after a supply shock for $h_c = \tau = 0$. This indicates that reactions after demand disturbances should in general be more defensive than after supply innovations of

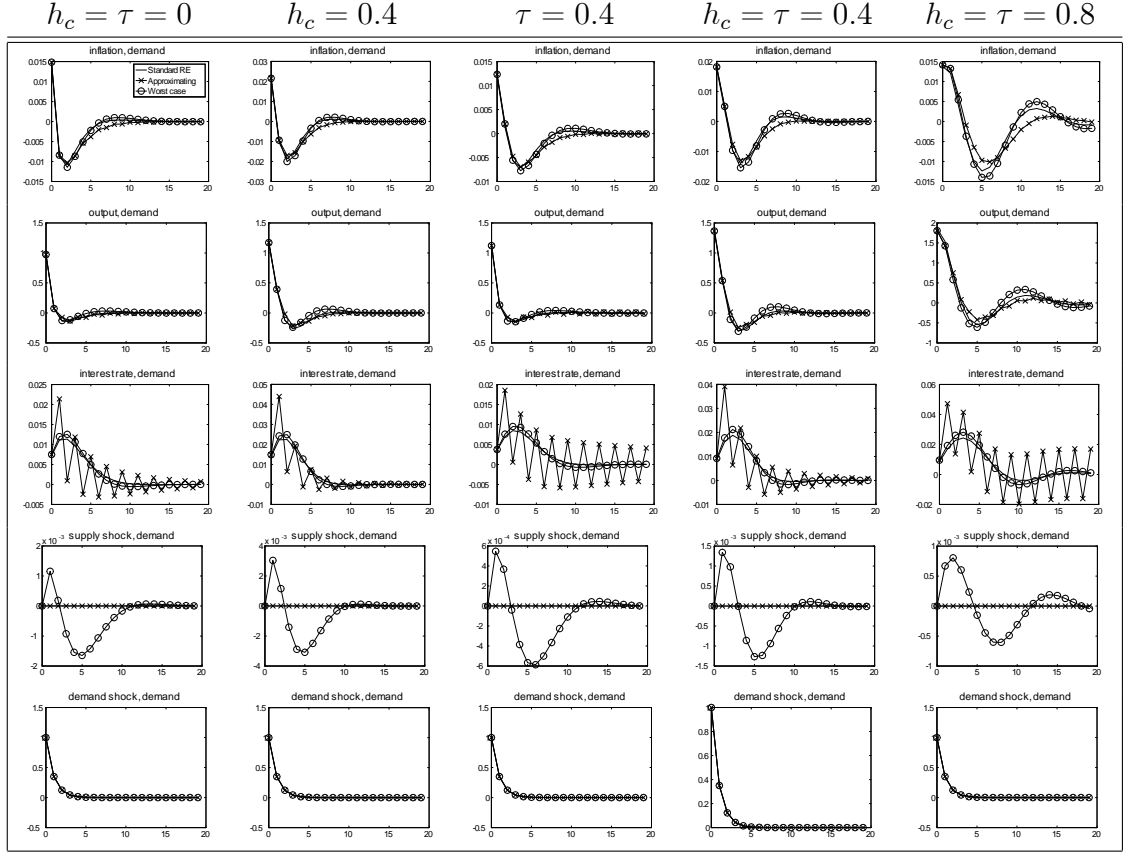
Figure 4: IRFs, commitment, $p(\theta) = 0.35$, supply innovation

the same size.⁹¹

The robust solutions validate the hypothesis that a robust policymaker should react more aggressively. The greatest fear of the policymaker is represented by a cyclical supply shock that contradicts the response of the policymaker and complicates the decision of the monetary authority. The evil agent causes stronger and more persistent reactions of all variables, and the central banker is well advised to implement a more aggressive interest rate policy to deal with this problem. In addition, the cyclical behavior of the induced supply shock leads to cyclical adjustment paths even for $h_c = \tau = 0$. As in the case of a supply innovation, the last two rows of Figure 3 can be interpreted as the greatest possible havoc an imaginary evil agent can do, and thus reflect the concerns about model misspecification.

Analogous to the result of the supply innovation, when comparing the influence of uncertainty with the influence of persistence, the degree of persistence in consumption habits and price-setting behavior seem to have a greater impact than an appropriate degree of model uncertainty, given a reference model with a particular degree of persistence.

⁹¹Note that the effect of a demand innovation is less damaging than a supply innovation of same size, as the primary impulse of risen demand can be damped directly in the IS curve by increasing the real interest rate, so that the effect on inflation is much smaller.

Figure 5: IRFs, commitment, $p(\theta) = 0.35$, demand innovation

Figures 4 and 5 illustrate responses on supply and demand shocks under commitment for the case of $p(\theta) = 35\%$, and the dynamics of an unfounded fear against misspecification by showing the response of the robust rule in the approximating model. Recall that the *RE* and the approximating solution share the same dynamics for the predetermined variables (the same approximating model), but the latter uses robust expectations *and* a robust policy rule. Central bank and private sector share the same reference model, the same loss function, and the same degree of robustness.

The reactions of the standard *RE* solution and the robust solution in the worst case model are the same as before, except that robust reactions are of less strength due to the lower degree of uncertainty. However, the reactions of the robust solution in the approximating model show a highly undesirable cyclical adjustment of the interest rate and the output gap, which seems to become even stronger and more persistent when the fraction of backward-looking price-setters increases, whereas an increase in consumption habits tends to decrease the variability. The last result is due to the increase in the transmission of output deviations on inflation. As the monetary policy becomes more effective, when the influence of consumption on real wages and marginal costs increases, the robust policy does not need to be as aggressive as for an increase in backward-looking price-setting. Therefore, the robust policy does not lead to that strong changes in the interest rate when the evil agent

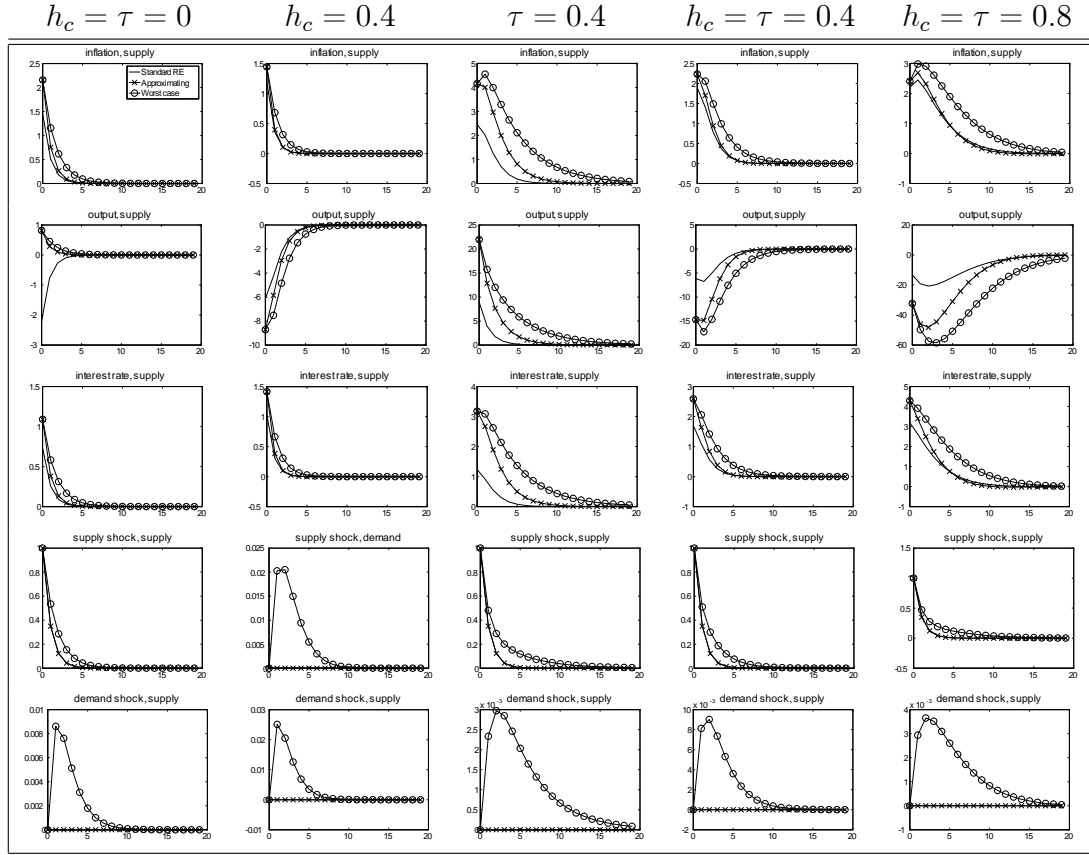
is absent. All graphs confirm the results from table 3 and underline the difficulty to choose an optimal interest rate reaction under commitment, since the case of unfounded fear of the central banker and the private sector leads to disastrous results, even for a small amount of uncertainty ($p(\theta) = 35\%$). With respect to uncertainty, these results suggest that a monetary policy under commitment seems to be on the razor's edge.

6.2.2 Discretion

In Table 4 losses are given for a central banker, who reoptimizes every period. As the policymaker is unable to influence expectations, losses rise compared to those of the commitment case for all scenarios and objectives. Similar to Table 3 consumption habits do not rise losses substantially, and even imply a decline for $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0, 0.3, 0)$ and $(0, 0.6, 0)$. In contrast, a rise in the fraction of backward-looking price adjusters leads to an immense increase in losses. This is due to the same effects on the effectiveness of monetary policy as described above.

$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0, 0.3, 0)$			$(0.0483, 0.2364, 0)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	255	305,317	491,566	270	315,327	410,452
$h_c = 0.4, \tau = 0$	190	209,216	260,290	271	316,328	400,437
$\tau = 0.4, h_c = 0$	1302	2045,2095	5541,5471	1107	1495,1520	2596,2715
$h_c = \tau = 0.4$	758	879,889	1154,1196	1114	1510,1536	2508,2598
$h_c = \tau = 0.8$	2827	3151,3100	3880,3653	16187	26819,26300	42490,27327
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.1, 0.5, 0)$			$(0.5, 0, 0)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	282	333,348	504,586	263	304,315	447,508
$h_c = 0.4, \tau = 0$	283	335,350	523,612	263	305,316	448,509
$\tau = 0.4, h_c = 0$	1266	1778,1819	2890,3007	985	1321,1337	2079,2110
$h_c = \tau = 0.4$	1271	1756,1795	4856,5590	985	1322,1338	1998,2008
$h_c = \tau = 0.8$	24229	28158,25912	42714,32509	10830	24370,23520	42173,29152
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0, 0.6, 0)$			$(0.5, 0, 0.3)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	294	364,381	544,621	268	301,315	404,474
$h_c = 0.4, \tau = 0$	252	293,306	428,499	268	302,315	404,474
$\tau = 0.4, h_c = 0$	2215	5727,5876	12358,10028	924	1137,1161	1976,2139
$h_c = \tau = 0.4$	1360	1720,1759	2608,2810	921	1134,1158	1986,2144
$h_c = \tau = 0.8$	5529	6127,6018	15437,10961	5307	8912,7437	12158,8598

Table 4: Losses under discretion, single-country economy

Figure 6: IRFs, discretion, $p(\theta) = 0.1$, supply innovation

However, it seems to be easier for a discretionary central banker to handle uncertainty, since robust losses are near the optimal *RE* solution, and using the robust rule in the approximating model still stabilizes the economy. In fact, losses under unfounded fear are near the robust solution in the worst case model. Thus, under discretion a policymaker is well advised to use Hansen-Sargent robust policies. Additionally, the problem of indeterminacy vanishes under discretion.

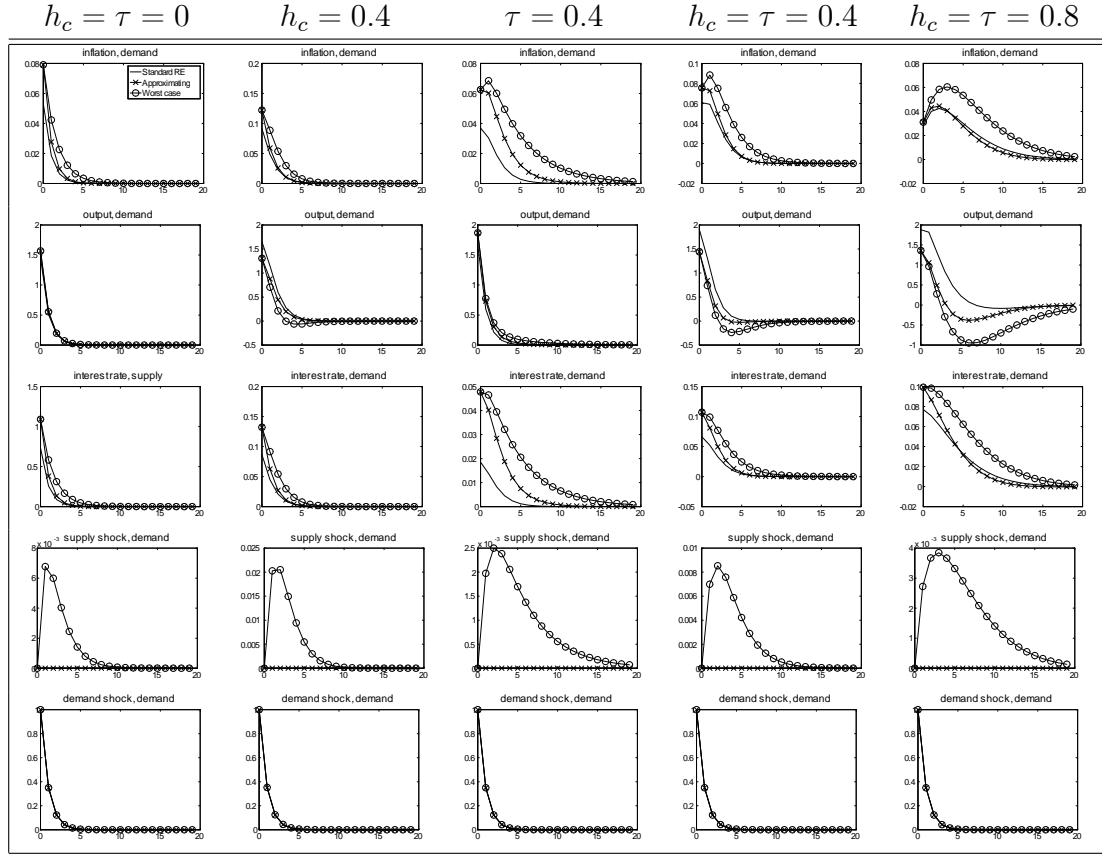
Figures 6 and 7 show the impulse responses of the standard *RE* solution ($\theta = \infty$), as well as for the robust solution in the worst case scenario, and the robust solution in the approximating model for $p(\theta) = 10\%$. All graphs show that also a discretionary policymaker is well-advised to react to uncertainty with a more aggressive policy.

Furthermore, the dynamics of the model under discretion clearly show the stabilization bias due to the inability to manage expectations, which we already discussed in 3.5.2. A discretionary policymaker adjusts the interest rate to offset shock impacts immediately, which can only be achieved at very high costs in terms of strong deviations from equilibrium of inflation, the output gap and the interest rate. Furthermore, the interest rate is not adjusted gradually, but is risen very sharply at the beginning, even though the authority has a preference for interest rate stabilization. For a discretionary policymaker a gradual interest rate adjustment brings no advantages, as he could not improve private sector expectations. Thus, also the cyclical responses of the commitment case for $h_c = \tau = 0.4$ vanishes.

The reactions with respect to a supply innovation show similar dynamics as for the commitment solution for all scenarios, except for the case of $\tau = 0.4$. Here the primary increase in inflation is accompanied by an increase in output, as the interest rate has not risen sharply enough to generate a positive real interest rate. Since for $\tau = 0.4$, but $h_c = 0$, the transmission from marginal costs to inflation is very weak, this seems to be a plausible result. As the real interest rate becomes positive, output and inflation slowly return to their steady states. Similar to the observations in the case of commitment, the introduction of both, consumption habits and backward-looking price-setting, leads to a more aggressive monetary policy. Moreover, the responses of inflation increase with τ and decrease with h_c , due to the influence on the transmission of demand on inflation.

The robust reactions in the worst case model illustrate again that uncertainty leads to stronger and more persistent shock impacts and justifies stronger and more persistent interest rate reactions. In addition, for the dynamics of the robust rule in the approximating model, the graphs confirm the suggestion from Table 3: Under discretion the case of unfounded fear lies not far away from the worst case solution. As the initial impact is the same, since both models share the same dynamics of the backward-looking variables, the reactions illustrate that an unfounded fear would lead to stronger and more persistent reactions of the interest rate compared to the *RE* solution, but they also illustrate how in the absence of an evil agent all variables react with less strength than in the robust solution under the worst case. For the policymaker under commitment, the ability to manage expectations makes decisions under uncertainty more complicated, since in the case of unfounded fear the private sector builds robust expectations about shock impacts that are too strong for the approximating model, and thus lead to the undesirable result of a highly variable interest rate and output gap. This becomes especially clear by taking a look at the impulse responses of supply and demand shocks, which clearly show that the cyclical dynamics vanish under discretion, as monetary policy does not keep output under potential for some periods to improve expectations, and thus the malevolent player is unable to exploit the undershooting of output and inflation by generating a cyclical demand disturbance to contradict monetary policy. As under commitment neither the policymaker nor the evil agent are able to influence expectations, the authority earns higher losses than in the commitment case, but also less disturbances due to model misspecification. In the absence of undershooting inflation and output, the greatest fear about model misspecification of the monetary authority is no longer represented by a cyclical demand shock. Again, this illustrates that achieving a robust commitment solution is highly improbable and in fact a monetary policy on the razor's edge.

The dynamics of the model after a demand innovation are given in Figure 7. They show similar primary reactions as in the commitment case, except for the size. Like in the commitment solution, rising persistence in price-setting does not imply more aggressive policies, whereas rising consumption habits do, due to the same effects on the marginal costs transmission channel mentioned above. Furthermore, all scenar-

Figure 7: IRFs, discretion, $p(\theta) = 0.1$, demand innovation

ios and objectives illustrate that robust policies are more aggressive and persistent. However, using the robust policy in the approximating model does not lead to undesirable reactions of the interest rate and the output gap under discretion, since robust expectations are unaffected and thus private sector and central banker do not overestimate the shocks in such a strong way, that the implied response of the interest rate, inflation, and the output gap are not to handle for the approximating model. Similar to the reaction after a supply innovation, the discretionary policymaker does not keep output under potential to improve expectations any more, and thus his fear about model misspecification is not described by cyclical disturbances, but by a positive supply shock that contradicts his aim to stabilize inflation by increasing the real interest rate, and forces the policymaker to react more aggressively. However, with respect to the comparison of uncertainty and persistence both Figures validate again that the degree of persistence in consumption habits and price-setting seems to be more important than the general model uncertainty, described by Hansen-Sargent robustness.

6.2.3 Simple rules

Since Taylor (1993) shows that his hypothetical policy rule approximates the behavior of the Federal Reserve during the 1990s quite well, simple rules have received

(g_π, g_y, g_i)	(8.294, 0.5, 0)			(2.217, 0.5, 0)		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	1215	1325,1402	1568,2063	335	372,408	ind
$h_c = 0.4, \tau = 0$	1160	1271,1351	1518,2048	332	376,426	ind
$\tau = 0.4, h_c = 0$	4239	4609,4917	5553,8097	1201	1355,1578	ind
$h_c = \tau = 0.4$	4099	4514,4921	5390,7852	1201	1390,1748	ind
$h_c = \tau = 0.8$	21819	25641,29381	30577,56971	8214	ind	ind
(g_π, g_y, g_i)	(1.5, 0.5, 0)			(4.5, 1.5, 0)		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	288	ind	ind	312	348,383	ind
$h_c = 0.4, \tau = 0$	288	ind	ind	317	356,401	ind
$\tau = 0.4, h_c = 0$	1082	ind	ind	1165	1327,1577	ind
$h_c = \tau = 0.4$	1086	ind	ind	1181	1376,1764	ind
$h_c = \tau = 0.8$	9217	ind	ind	8378	ind	ind
(g_π, g_y, g_i)	(8.294, 0.5, 0.5)			(4.5, 1.5, 1.5)		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	1228	1342,1419	1602,2067	314	350,397	ind
$h_c = 0.4, \tau = 0$	1192	1297,1368	1552,2029	320	360,412	ind
$\tau = 0.4, h_c = 0$	4410	4791,5084	5937,8520	1189	1378,1733	ind
$h_c = \tau = 0.4$	4351	4794,5172	5821,8248	1217	1415,1820	ind
$h_c = \tau = 0.8$	23716	27638,31606	33586,63911	8357	ind	ind

For some scenarios the steady state of the model is indeterminate (ind).

Table 5: Losses using simple rules, single-country economy

more and more popularity among academic researchers as well as policymaker. The pros and cons of a rule-solution are already summarized in 3.5.3. No matter whose arguments are more convincing, studying simple rules in the context of Hansen-Sargent robust policies represents an interesting deviation of the private sector from the *RE* hypothesis. As soon as one assumes that a policymaker commits to a rule, he is no longer involved in any decision, and all differences between the optimal *RE* and the robust solution must be due to private sector expectations.

Table 5 shows losses for six different simple rules of the form $i_t = g_\pi \pi_t + g_y y_t + g_i i_{t-1}$ for the policy objective estimated by Rotemberg and Woodford (1997): $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0.0483, 0.2364, 0)$. Among other hypothetical policy rules, we also investigate the consequences of uncertainty for the famous Taylor-rule, $(g_\pi, g_y, g_i) = (1.5, 0.5, 0)$, and the optimal rules of Giannoni (2002), who studies Hansen-Sargent robustness in a forward-looking variant of our model under the same parameterization. For the case of parameter uncertainty and a shock process on the equilibrium interest rate,

he derives an approximating, $(2.217, 0.5, 0)$, and a robust Taylor rule, $(8.294, 0.5, 0)$.

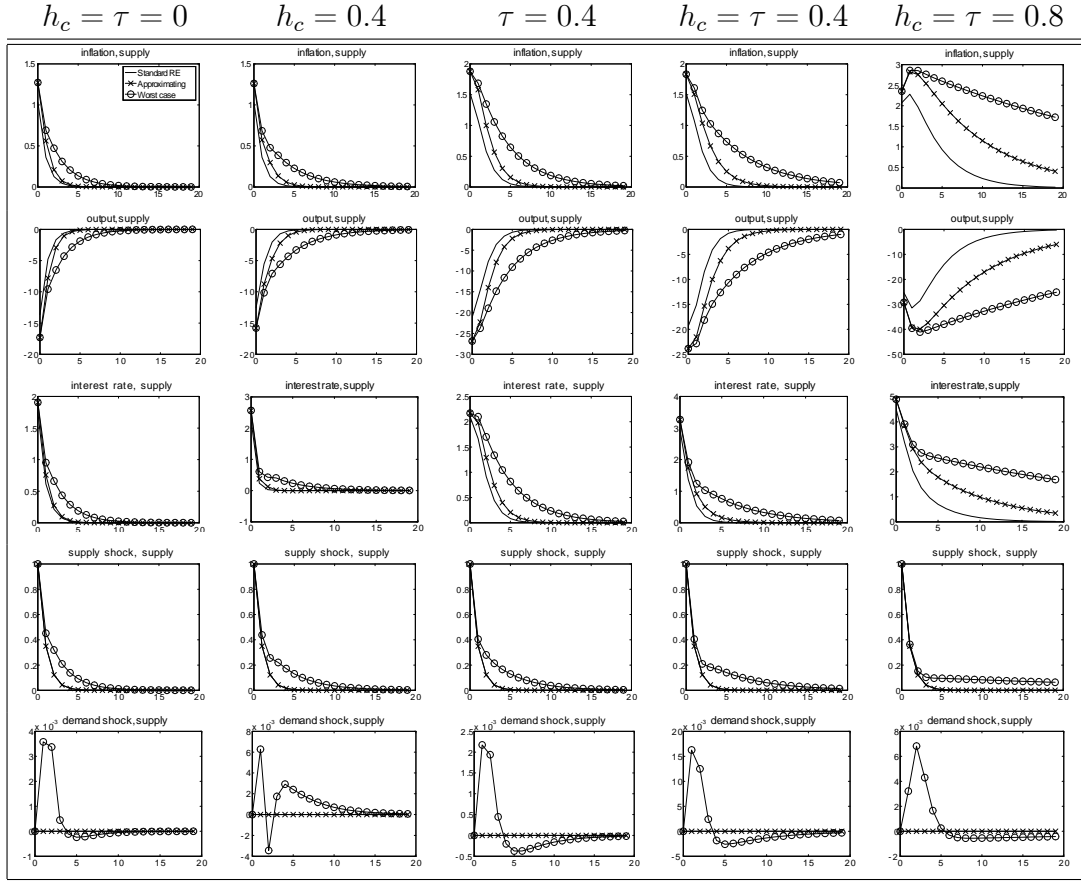
We compute losses for the optimal *RE* solution (l_a), the robust solution under the worst case scenario (l_r , private expectations are robust), and for the approximating model, using robust expectations (l_a^r). Recall that for the last case, the dynamics of the forward looking variables and the interest rate are the same and given by (85) and (44). The only difference to the robust solution under the worst case scenario is that we cancel ω_{t+1} and F_ω . Thus, the private sector builds (unfounded) robust expectations, even though the evil agent is absent.

There are several similarities between the results under simple rules, and that for a policymaker, who commits credibly to the optimal interest rate trajectory. Similar to the commitment case, under uncertainty the problem becomes indeterminate for some objectives. Furthermore, the table provides that the degree of backward-looking price setters seems to cause higher problems than the degree of consumption habits, and that these measures of persistence can be more important than the policymaker's fear about a general model misspecification. The results also suggest that the authority has to decide between the two options, that we already described for the commitment case: He can implement a very aggressive policy rule that handles uncertainty quite well, but leads to very high losses in the case of no uncertainty, or he ignores uncertainty and uses a more defensive rule (like the Taylor-rule), which leads to lower losses if the approximating model is the true one, but could imply disastrous results under uncertainty. For example, the optimal robust rule of Giannoni (2002) implies only very small increases in losses under uncertainty, but already implies very high losses for the case of no uncertainty.

When we are to compare the results under simple rules with those under discretion, respectively, for the case of $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0.0483, 0.2364, 0)$, there is no distinct, scenario-independent regularity. For the case of a purely forward-looking model the Taylor-rule, which is the one leading to the lowest losses for the case without uncertainty, performs worse than the optimal discretionary policy. Contrary to this, the introduction of a small persistence in consumption habits or price-setting behavior, respectively, yields the polar opposite of this outcome. For $h_c = \tau = 0.8$ the simple Taylor rule implies much smaller losses than the optimal discretionary solution, and is near the optimal, but highly improbable solution under commitment. For the highly backward-looking scenario this result stays valid for all rules, except for the highly aggressive ones, $(8.294, 0.5, 0)$ and $(8.294, 0.5, 0.5)$. This suggests that using a moderately more aggressive simple policy rule, like $(4.5, 1.5, 0)$, might be good guidance for a monetary policymaker.

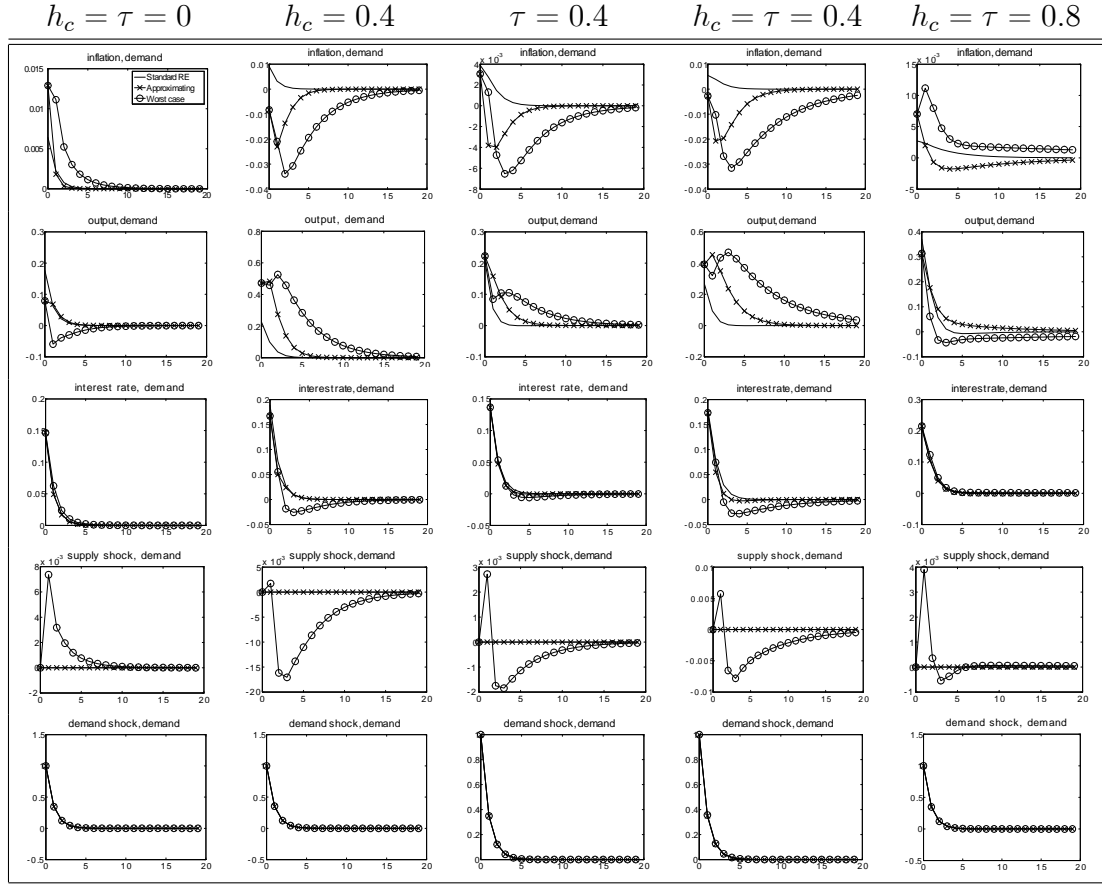
In Figures 8 and 9 the dynamics of the model for the optimal robust rule of Giannoni (2002), $(8.294, 0.5, 0)$ are given. Again we show impulse responses for the standard *RE* solution ($\theta = \infty$), as well as for the robust solution in the worst case scenario, and the robust solution in the approximating model for $p(\theta) = 10\%$.

The dynamics after a supply innovation in Figure 8 are very similar to the impulse responses under discretion. The central bank raises the interest rate to generate a positive real interest rate in order to dampen inflationary pressures. This causes

Figure 8: IRFs, simple rules, $p(\theta) = 0.1$, supply innovation

a downturn in output which brings inflation back to equilibrium. With falling inflation, the interest rate is reduced very fast, the real interest rate declines, and all variables return to their equilibrium. As in the case of commitment or discretion, respectively, the degree of persistence in price-setting behavior causes stronger deviations of inflation from equilibrium, whereas a higher degree of consumption habits causes the opposite. Furthermore, the loss in effectiveness calls for stronger responses with respect to both types of persistence. Due to the fact that we assume no reaction parameter on the lagged interest rate, the interest rate is not adjusted gradually, but is risen sharply at the beginning of the shock.

Regarding the robust solutions the graphs confirm our previous results from the discretionary case as well as the results from Table 5. The central banker reacts on uncertainty with stronger and more persistent interest rate responses as he fears stronger and more persistent shock impacts. In particular, for the extreme case of $h_c = \tau = 0.8$, the dynamics show very strong and long-lasting deviations under uncertainty, suggesting that for a very high persistence uncertainty becomes more difficult to handle, as deviations of inflation and output from equilibrium become more persistent. Furthermore, it seems that unfounded robust expectations in a model without evil agent do not lead to such catastrophic outcomes as shown in the commitment case. This suggests that using a simple rule might be the silver

Figure 9: IRFs, simple rules, $p(\theta) = 0.1$, demand innovation

bullet that can handle an appropriate amount of uncertainty and, on the other hand, does not cause too much trouble when the fear about model misspecification is unfounded. Of course, one should keep in mind that the rule of Figure 8 is very aggressive and leads to very high losses compared to a more defensive rules for the case of no uncertainty. In addition, the graphs illustrate again that the choice of the right degree of persistence in price-setting behavior for the reference model can be more important than an appropriate amount of general model uncertainty.

For the case of a demand shock impulse responses are shown in Figure 9 and once again confirm all of our previous results. First of all, an increase in persistence in price-setting does not imply a more aggressive policy, whereas a higher degree of persistence in consumption habits does. Furthermore, reactions under uncertainty are more aggressive, and unfounded robust expectations of the private sector (without evil agent) lead to outcomes near the optimal worst case reaction. Concerning the comparison of uncertainty and persistence, the graphs also suggest, that the choice of the right reference model - the choice of h_c and τ - can be more problematic than a general model uncertainty.

6.3 Discussion and comparison with related literature

This section summarizes the results of the preceding sections, compares them with the existing literature, and draws conclusions for a single-country economy. First of all, persistence in price-setting decreases the transmission of monetary policy from the IS curve to the NKPC, whereas persistence in consumption habits does the opposite. Thus the reactions on rising degrees of persistence after demand and supply innovations differ. After a demand innovation, an increase in h_c calls for higher interest rates, since the transmission of the primary impulse (a rise in output) becomes stronger, and the interest rate loses effectiveness, whereas the opposite is true for an increase in τ , as the primary impulse is transmitted less. Considering a supply innovation, the interest rate response becomes more aggressive when both types of persistence increase, as the primary impulse is already a rise in inflation. Here, the negative impact on the effectiveness of monetary policy due to consumption habits in the IS curve dominate the increase in the transmission of output gap deviations on inflation. Furthermore, an increase in the fraction of backward-looking price adjusters leads to higher interest rates due to the loss in transmission in the NKPC. However, the losses for all scenarios, objectives, and monetary strategies show that an increase in persistence of price-setting behavior causes much more damage than an increase in consumption habits. Impulse responses show that this is due to the fact that demand innovations are less damaging, since the rise in output can directly be dampened via the real interest rate in the IS curve, and thus the transmission on inflation is already much smaller. Leitemo and Söderström (2004) bring forward a similar argument, as they show that in the case of uncertainty solely about the output gap equation, the higher interest rate response of the central bank directly offsets any misspecification - a result we will discuss in more detail later.

The analysis of model uncertainty by employing robust control techniques clearly provides for the use of more aggressive policy rules in the case of an unstructured model uncertainty. However, the results also suggest that when the central bank uses the robust policy and the private sector builds robust expectations in the approximating model, which is the most likely outcome, this can lead to disastrous results and infinite losses, especially in the case of commitment. Therefore, with respect to uncertainty, a commitment solution seems to be a monetary policy on the razor's edge. As the commitment to optimal interest rate dynamics is unlikely, since a monetary authority needs to be highly credible (see for example Walsh (2003) and Clarida, Galí, and Gertler (1999) for a discussion), the results concerning uncertainty suggest that optimal commitment solutions should be seen as a theoretical benchmark for optimal monetary policy that one can not, and should not, implement in reality. Nevertheless, the dynamics under commitment, as well as the comparison with the discretionary solutions and the solutions using simple rules are interesting for academics and practitioners.

Similar to the results under commitment, the analysis of discretion and simple rules also shows that using robust policies in the approximating model can lead

to undesirably high losses, suggesting that the monetary authority has to decide whether he wants to reflect his concerns about misspecification and use a more aggressive policy that works well in the worst case, but might lead to a very strong increase in losses when the more likely approximating model is the true one, or ignores his concerns and uses a standard policy which handles the approximating model very well, but could lead to infinite losses under uncertainty. A robust policymaker should bear in mind that a good performance in the worst case can only be achieved at the cost of average performance. Thus, the silver bullet could be to use a moderately more aggressive policy. As using simple rules implies losses near the optimal commitment solution, a moderately more aggressive Taylor rule could work as a benchmark and reduce the uncertainty of the private sector.

Comparison with the robust control literature Our results are confirmed by a vast amount of studies on uncertainty in monetary policy. We will only discuss the most important ones, beginning with robust control approaches similar to the ones that we employ. As we already discussed in 5.4, early studies using robust control techniques investigate the consequences of a preference for Hansen-Sargent robustness in backward-looking economies, like the closed economy of Rudebusch and Svensson (1998) or the open economy of Ball (1998). Since our analysis is focussed on a forward-looking closed New Keynesian economy, we skip these studies and turn to more recent investigations of model uncertainty, that are a better comparison with ours. However, all studies which employ an unstructured approach in a backward-looking model confirm that uncertainty leads to more aggressive policies.⁹²

Robust control applications to purely forward-looking closed economies can be found in Giannoni (2001), Giannoni (2002), Kara (2002), Giordani and Söderlind (2004), Kilponen (2004), Kilponen and Leitemo (2006) and Leitemo and Söderström (2004). The studies of Giannoni both concentrate on simple Taylor rules and a structured approach of parameter uncertainty and also conclude that policy rules under uncertainty have to be more aggressive for all parameters. Contrary to this, Kara suggests that uncertainty about the slope of the NKPC can lead to more defensive reactions, whereas uncertainty about shock persistence leads to more aggressive rules. His study differs from those of Giannoni as he derives optimal linear robust targeting rules in the spirit of Svensson (2002a) and Svensson (2003), rather than instrument rules. Assuming these rules, the policymaker commits credibly to a rule of the form $y_t = -\psi\pi_t$. He argues that, as parameter uncertainty affects the short-run trade-off between output and inflation by rendering inflation stabilization more costly, this requires the policymaker to act as if the instrument is less effective. However, his results for the case of uncertainty about shock persistence, comparable to ours, confirm that also targeting rules should be more aggressive.

⁹²In fact, the structured approaches of Stock (1999) and Onatski and Stock (2002) only lead to a more defensive reaction with respect to the output gap when there is uncertainty about the lag structure of the model.

Giordani and Söderlind (2004) summarize the solution algorithms for a policy-maker under commitment, discretion and under the usage of simple rules, and give a brief example for each strategy for one particular parameter set. Independent from the monetary strategy, all examples show that, due to the fear about stronger and more persistent shocks, robust interest rate reactions are more aggressive.

The work of Kilponen combines model uncertainty and data uncertainty by combining robust control with imperfect information approaches.⁹³ He follows Kasa (2001), who extends the prediction formula for rational expectations of Hansen and Sargent (1981) to a rule used by a robust forecaster. Robustness leads to overestimation of shocks and revisions that are more often than under standard forecasting. In Kilponen (2004) he develops a method to solve robust decision problems when the policy is uncertain and some state variables are unobservable. For this reason he uses the partial information model of Svensson and Woodford (2003), and investigates robust Taylor rules in a closed New Keynesian economy including government expenditures, under a policy objective with no preference for interest rate stabilization nor smoothing. To measure uncertainty he uses the sum of standard errors of the worst case shocks, instead of error detection probabilities. The results concerning an expenditure shock are similar to ours, as inflation reacts stronger on shocks under uncertainty and robust consumption implies precautionary savings. More interestingly, as agents are prepared for deviations of exogenous expenditure from equilibrium, the robust steady states imply that output is below and inflation above their equilibrium in the approximating model. Thus, nominal shocks have real impacts. For the experiment of policy uncertainty - a shock process on the Taylor rule to model that a policymaker might deviate from the rule under specific circumstances - agents underestimate policy responses to output, but overestimate responses to inflation. In a recently published working paper, Kilponen and Leitemo (2006) argue that this might be a rationale to re-establish the Friedman rule of increasing the money stock, as the price paid for robustness is smaller. Although we do not study the effects of an increasing money stock under uncertainty, we concur with these authors that committing to a rule decreases uncertainty and thus the costs of robustness.

Leitemo and Söderström (2004) investigate a forward-looking open New Keynesian economy, including a real interest rate parity condition, under commitment. They follow the modelling approaches of Clarida, Galí, and Gertler (2002) and Galí and Monacelli (2005), but assume a time-varying premium on foreign bond holding. The model converges to our closed version when the degree of openness goes to zero. Due to the simple structure of the model - shocks are not modelled by an AR(1)-process, and the policy objective does not include a preference for interest rate stabilization nor smoothing - it can be solved analytically. Contrary to our

⁹³In a previous paper Kilponen (2003) confirms the result, that a robust policymaker should react more aggressively, and argues that with larger model misspecifications the central banker should be more inflation-averse.

analysis, robustness is not modelled for the economy as a whole, but for every equation separately, as the authors want to reflect the fact that policymakers are more confident in some relationships than in others.⁹⁴

Similar to our results, the solutions show that in a closed economy the interest rate reacts with more aggression as the central banker fears that inflation and output are more volatile. Furthermore, the authors also argue that the robustness against uncertainty in inflation dynamics leads to undesirably high output variability when the robust policy is used in the more likely approximating model instead of the worst case scenario.⁹⁵ However, this only illustrates the trade-off between inflation and output variability faced by the authority. Due to their analytical approach, they could not derive any measure of uncertainty comparable to the detection error probabilities used here. Beyond controversy, there is of course an amount of uncertainty that would lead to a catastrophe, but the more interesting question is whether assuming this amount of uncertainty is reasonable. Our simulations suggest that only for the case of commitment and a reasonable amount of uncertainty (reasonable due to the detection error probabilities near 10%) we see an undesirably high variability in the interest rate and the output gap that can not be found in the dynamics for the cases of discretion and simple rules.

Comparison with other methodologies With respect to Bayesian analysis, we validate the results of previous robust control approaches, and also overcome Brainard's conservatism principle. Furthermore, we can confirm the results of more recent Bayesian studies, like Söderström (2002), who shows that in the case of increasing uncertainty about inflation persistence, optimal policy in a backward-looking model becomes more aggressive, since the central bank wants to reduce the probability for inflation to move away from its target.

Studies using techniques that evaluate monetary policy in different reference models are closely related to ours in the sense that we also use the robust policy in the approximating model, and investigate the performance of simple rules in models, differing in the degree of persistence, which can also be interpreted as different reference models. A paper comparable to ours is Angeloni, Coenen, and Smets (2003). They explore the influence of uncertainty about nominal and real persistence in an estimated hybrid DSGE model for the Euro area. The model is an extended version of a standard New Keynesian economy, including partially backward-looking

⁹⁴Technically speaking, those policymakers are able to express their beliefs (at least partially) in probability statements. Critics would argue that this assumption is inappropriate. See also the discussion in 4.3.

⁹⁵For the open economy, uncertainty in output dynamics does also lead to undesirably high inflation variability in the approximating model. As the authors do not add general demand and supply shocks, but shocks that were derived from structural equations, there is no uncertainty for the IS curve for technical reasons. The central banker offsets specification error in the IS curve directly by adjusting the real interest rate. Since its objective includes no preference for interest rate smoothing this does not affect its loss function.

wages and prices, consumption habits, and adjustments costs in investment. The investigation builds up on Coenen (2007), who focusses on inflation persistence and concludes that it pays for a policymaker to overestimate persistence, since losses increase when making the opposite mistake. Angeloni, Coenen, and Smets (2003) analyze both inflation and output persistence and confirm Coenen's result. Furthermore, optimized simple rules are quite robust against persistence in the output gap, illustrating that output persistence seems to be a rather small problem. This supports our results regarding persistence in price setting and consumption habits. For rising inflation persistence, the optimal reaction parameter on inflation increases, but the parameter on the output gap decreases, while for investment persistence the opposite is the case, so that a general statement about the aggressiveness of policy rules is not possible. In contrast to our finding, the analysis shows that a commitment policy is more robust with respect to inflation persistence than a simple rule, and confirms the findings of Amato and Lauenbach (2003), who show that the optimal commitment policy derived in a forward-looking model works reasonably well also in a backward-looking version. However, keeping in mind that we derive robust rules in the case of general model uncertainty and use them in a model without evil agent to evaluate the performance, our results do not contradict those of Angeloni, Coenen, and Smets (2003). Optimal rules under commitment derived in a forward-looking model might work well in a backward-looking economy, but commitment under uncertainty causes problems in the approximating model nevertheless, since private sector and central bank expectations of shock impacts are too strong under a preference for robustness. Thus, our analysis suggests that the costs of assuming that inflation can be controlled through expectations can be very high with respect to uncertainty, whereas Angeloni, Coenen, and Smets (2003) argue that a commitment policy is quite robust with respect to inflation persistence. However, we clearly differ in conclusions, since our investigation questions the gains from commitment under uncertainty.

The study of Levin and Williams (2003b) use a diagnostic approach, where non-nested models represent competing perspectives about controversial issues like expectation formation or inflation persistence. They compare optimal rules under three different models and show that rules that are derived in order to work well in the neighborhood of one model might perform very badly in one of the others. The model variants cover a standard forward-looking New Keynesian economy, the purely backward-looking model of Rudebusch and Svensson (1998), and the model of Fuhrer (2000), including persistence in output and inflation. They show that rules derived by robust control methods perform very poorly in a different model. Similar to our analysis this provides that the right reference model - in particular the right degree of persistence - seems to be of immense importance, as the choice of persistence in the reference model can have a stronger influence than the fear about misspecification.

Conclusions for a closed single-country economy After summarizing our own results and those of the most important literature, there are a few robust conclusions to mention. First of all, due to the overestimation of shock persistence, robust private agents and policymaker imply more aggressive policies. However, a lot of academic work suggests that robust rules might perform very badly in models different from the reference model. We support this view by showing that the influence of persistence on losses and dynamics can be stronger than the influence of Hansen-Sargent robustness, at least with respect to the price-setting behavior. Furthermore, we conclude that even using the robust rule in the same model without an evil agent can cause problems, especially for a policymaker under commitment. Thus, Sims (2001) is right in his belief that robust control can only be one methodology among many others. A robust policymaker should bear in mind that robustness against the worst case implies higher costs when a more likely model appears to be the true one.

Furthermore, our results contribute to the discussion about whether optimal policy should be built under commitment, the argument that the costs of assuming that inflation can be controlled through expectations can be very high with respect to uncertainty, since robust expectations overestimate shocks in a manner that is difficult to handle in the approximating model. This questions the already very difficult to achieve commitment solution to be the best advice for the monetary authority. Moreover, the results under discretion and simple rules give little support for using a moderately more aggressive Taylor rule as a benchmark. In this we support the view of Taylor (1998b) who also argues that a mixture of a pure rule and a pure discretion scenario is superior to pure discretion as it reduces uncertainty (improves predictability), and includes the necessary possibility of deviating from such a rule under unforeseeable events.

7 The return of two-country models

In the following sections we sketch the road to a new workhorse model in international macroeconomics. Parallel to the development of a single-country economy including nominal rigidities and imperfect competition, open economy and two-country models with these features have been derived. Due to the increased openness of many economies this field of research received more and more attention over the last three decades. Furthermore, the foundation of the European Central Bank forced academic researchers to extend previous models, and include realistic treatments of trade relations, financial markets, and policy interdependence, in order to give helpful suggestions. The two countries in these models can be interpreted as two economies with an own monetary policy, which are big enough to have an influence on each other, like the U.S. and the EU. Since international interdependence might make monetary policy even more complicated, as a central banker now faces several additional constraints, like his influence on exchange rates or the influence

of foreign monetary policy on the domestic country, analyzing uncertainty remains essential. Thus 7.2 summarizes the few studies that already incorporate an analysis of uncertainty in one of the recently developed models. That these studies are quite rare calls for further investigations.

7.1 New Open Macroeconomics

This section gives an overview of the evolution of open economy and two-country models including nominal rigidities and imperfect competition.⁹⁶ Table 6 categorizes the most important studies according to characteristics that are explained in the following.

Dornbusch (1976) was the first, who implemented consistent expectations, a sluggish price adjustment and perfect capital mobility in an open economy IS-LM extension of Mundell (1962) and Fleming (1962). In his model the exchange rate overshoots its equilibrium value after an unexpected monetary expansion due to a slow adjustment of goods markets relative to asset markets. The overshooting result is a consequence of perfect capital mobility, since assuming that domestic and foreign assets are perfect substitutes ensures that the expected net yields have to be equal. An unexpected domestic monetary expansion leads to a fall in the domestic interest rate, which triggers a capital outflow and a depreciation in the exchange rate. However, since uncovered interest rate parity holds, agents expect an appreciation proportional to the interest rate differential and the exchange rate has to stay above equilibrium until this differential disappears. The rise in domestic aggregate demand (due to relatively lower domestic prices) leads to an increase in domestic prices and a fall in real balances, so that the domestic interest rate rises to keep the money market in equilibrium. This is accompanied by capital movements, implying the expected appreciation, until the exchange rate value is consistent with purchasing power parity (PPP) and domestic and foreign interest rates are equal.

This analysis demonstrates that the exchange rate is a key transmission channel for international monetary policy. Since a depreciation lowers the relative price of domestic goods, aggregate demand shifts from foreign to domestic goods. This is often referred to as the expenditure-switching effect and is controversially debated in the context of the stabilizing role of flexible exchange rates, as we will see soon. In addition, Dornbusch impressively demonstrates that for a short-run analysis, expectations about future dynamics seem to be more important than the current interaction of markets.

However, there are a number of shortcomings in the Mundell-Fleming-Dornbusch model. The most important one is the lack of microfoundations for the agent's intertemporal choice, which makes researchers unable to study welfare implications. In contrast, an analysis based on a dynamic utility maximization of households

⁹⁶The following subsection is heavily influenced by the more detailed surveys of Lane (2001b) and Sarno (2001). For surveys about closed economy models see Dixon and Rankin (1994) and Goodfriend and King (1997).

	Local-currency pricing	Producer-currency pricing	Preferences and nontradables	Complete financial markets	International policy interdependence	Small Open economies	Small open New Keynesian models	Two-Country New Keynesian Models
Basu (1995)			X					
Batini et al. (2005)		X	X		X			X
Batini/Levine/Pearlman (2004)		X	X		X			X
Batini/Pearlman (2002)		X	X		X			X
Benigno (2002)		X	X		X			
Benigno/Benigno (2001)					X			X
Bergin/Feenstra (1998)			X					
Bergin/Feenstra (2001)			X					
Betts/Devereux (2000)	X		X		X			
Betts/Devereux (2001)			X		X			
Chari/Kehoe/McGrattan (1997)	X		X	X				
Chari/Kehoe/McGrattan (2000)	X		X	X				
Clarida/Gali/Gertler (2001)		X		X			X	
Clarida/Gali/Gertler (2002)		X		X	X			X
Corsetti/Pesenti (2001)		X	X	X	X			
Corsetti/Pesenti (2002)		X	X	X	X			
Corsetti/Pesenti (2005)		X	X	X	X			
Corsetti/Pesenti/Roubini/Tille (1999)				X	X			
Devereux/Engel (2003)	X	X		X				
Engel (1999)	X		X					
Gali/Monacelli (2005)		X		X			X	
Hau (2000)			X					
Lane (1997)			X			X		
Lane (2001a)			X			X		
McCallum/Nelson (1999a)		X					X	
McCallum/Nelson (2000)		X					X	
Obstfeld (2001)		X	X					
Obstfeld/Rogoff (1995)			X			X		
Obstfeld/Rogoff (2000)		X	X					
Obstfeld/Rogoff (2002)		X	X					
Pappa (2004)		X	X		X			X
Senay (1998)	X			X				
Sutherland (1996)				X				
Svensson/Van Wijnbergen (1989)			X					
Tille (2001)					X			
Warnock (1998)			X					

Table 6: Classifying different two-country models

allows for an analysis of welfare, current accounts, and budget deficits. Furthermore, a monopolistic competition framework leads to a decentralized equilibrium output lower than social optimum (the marginal utility from producing an additional output unit is greater than the costs in terms of forgone leisure), so that an active monetary policy could have positive real effects in the long run by raising output to its efficient level - a possibility that is excluded in the Dornbusch paper. Moreover, monopolistic competition justifies the determination of output via aggregate demand in the short-run: When there is a price markup on marginal costs, it is still profitable to increase output, even when prices are fixed.

For these reasons many economists search for a new workhorse model in open economy macroeconomics superior to the hitherto existing reference model. The seminal Redux paper of Obstfeld and Rogoff (1995) has established imperfect com-

petition and nominal rigidities in open economies and therefore became a benchmark. However, it is fair to say, that the heavily influential forerunner model by Svensson and Van Wijnbergen (1989) had already included many of the microfoundations adopted in the more recent literature.⁹⁷ As understanding this model is essential for understanding the numerous modifications that have followed, as well as for understanding our two-country setting, we first briefly review the main features and outcomes of the Redux model, before we turn to extensions and more recent developments.⁹⁸

7.1.1 Exchange rate dynamics redux

Obstfeld and Rogoff combine the Keynesian assumption of rigid prices from the Dornbusch model (in the short-run output is demand-determined) with the advantages of modern intertemporal economics. They develop a two-country setting with a world population, normalized to a continuum of measure one, where a fraction of $(0, n)$ live in the home and the remaining $[n, 1)$ live in the foreign country. All agents are consumer-manufacturers that produce differentiated products. Each agent chooses her dynamically utility maximizing bundle of consumption goods, real money balances and labour supply with respect to a sequence of budget constraints including transfers from the government. Assuming zero government consumption, these transfers have to be of exactly the same size as the money creation. Furthermore, all individuals share identical preferences and trade is assumed to have no barriers, so that the law of one price (LOOP) holds for each good, implying that PPP holds and the consumption-based real exchange rate is unity. Since prices are set one period in advance, short-run and long-run effects differ.

The paper reexamines the Dornbusch experiment of an unanticipated and permanent increase in the domestic money supply, leading to a decreasing nominal domestic interest rate, implying an increase in domestic consumption and output, as well as an exchange rate depreciation due to financial arbitrage (like in the Dornbusch model).⁹⁹ The falling world real interest rate (in the proportion to the increase

⁹⁷However, Svensson and Van Wijnbergen (1989) model home and foreign output as stochastic exogenous endowments, whereas the subsequent literature uses an endogenized production side of the economy.

⁹⁸Corsetti and Pesenti (2001) mention that all models including nominal rigidities legitimise sticky prices by the assumption of fixed underlying menu costs, although a shock, large enough to raise marginal costs above prices, would lead to an *immediate* change in prices. Therefore all studies should be interpreted as analysing the effects of shocks of relevant size.

⁹⁹Note, that the model assumptions exclude an exchange rate overshooting. As both countries face the same real interest rate, domestic and foreign consumption grow with identical rates. A permanent increase in the money stock implies that short-run and long-run changes in domestic real balances relative to its foreign counterpart are identical, so that the permanent increase in the nominal exchange rate equals exactly its initial jump. However, a model extension introducing nontradables allows for different real interest rates and is able to capture the conventional Dornbusch overshooting phenomena.

in the world money supply) and the depreciation impact on the terms of trade brings about foreign consumption to rise, whereas the impact on foreign output is ambiguous because of two opposed effects: on the one hand, foreign prices become relatively more expensive, which has a negative impact on foreign output due to expenditure-switching, on the other hand, increased world consumption results in rising foreign production.

Independent from the question for the dominating effect, the domestic current account moves into surplus, since in the short-run the currency depreciation shifts world demand toward domestic goods and raises domestic income above consumption. Since domestic residents want to smooth consumption over time, they partly save their extra income. As the short-run surplus implies a permanent improvement in net foreign assets, the new equilibrium includes a permanent domestic trade deficit, since domestic (foreign) consumption remains permanently above domestic (below foreign) output to the extent that the home country receives a positive net investment income inflow. However, in the long-run domestic agents shift from work to leisure due to their increased wealth. This reduces domestic output and generates a permanent improvement in the home country's terms of trade. In equilibrium the current account is in balance again, but money is not neutral in the long-run.

Studying the welfare effects demonstrates that a permanent unanticipated increase in the domestic money supply is able to raise output to the level consistent with perfect competition. Astonishingly, welfare rises in both countries by the same amount.¹⁰⁰ The temporary increase in relative domestic production does not change relative utility of domestic agents, as the extra revenue is exactly cancelled out by the increased work effort. From a welfare perspective the effects of switching expenditures from foreign goods to those produced in the home country are only of second-order, since optimizing agents had initially set the marginal utility of an extra revenue equal to the marginal disutility of extra work. The same is true for current-account imbalances, as the initial equilibrium does not account for marginal gains from reallocating consumption and leisure over time periods. The only first-order effect is the increase in aggregate demand, which corrects the initial world output distortion due to monopolistic competition. This illustrates the major benefit from using microfounded models: evaluating the net welfare effects on the basis of the agent's utility function prevents the rash conclusion that a higher relative production would imply higher relative welfare.

7.1.2 Basic extensions

Even if the Redux model changes the view on international monetary policy considerably, the analysis has some obvious shortcomings. The most evident one is the oversimplified price setting behavior. As prices are simply preset one period in advance, all variables reach their new steady states after one period. One possibility to escape the large and discrete jumps in the price level, which is incompatible with

¹⁰⁰Except for a minor gain due to a relative permanent increase in real balances.

empirical observations, is to use the Calvo (1983) staggering price adjustment mechanism, where the opportunity to adjust prices arrives stochastically to every firm, which leads to a smooth price level adjustment. Kollmann (2001) compares predetermined price and wage setting to the Calvo-type price stickiness and concludes that the latter captures the observed serial correlation of nominal and real exchange rates and the gradual adjustment in the price level, while differences between the model implied output correlations with other macroeconomic variables and empirical observations are greater in the Calvo setting.¹⁰¹ Another basic modification is Hau (2000), who considers flexible prices but sticky nominal wages. By assuming a constant elasticity of demand, he shows that monopolistic firms set prices as a constant markup over the wage, so that sticky wages in fact imply sticky prices.

7.1.3 Pricing-to-market

As the assumption of the LOOP in the original Redux model cannot be empirically confirmed, some authors introduce international market segmentation into the model to reflect international deviations in tradable prices, which these authors believe to be responsible for a large proportion of the observed real exchange rate fluctuations.¹⁰² In most of these approaches at least some firms are able to charge different prices for the same good in different countries. An European exporter facing a dollar depreciation would rather lower the export's euro price and reduce the profit margin to defend the U.S. market share than raise the export's dollar price. Since prices are now sticky in the currency of the buyer, these approaches are often referred to as pricing-to-market with *local-currency-pricing*.

Local-currency-pricing Betts and Devereux (2000) show that under the assumption of a low consumption elasticity of money demand the possibility of a short-run exchange rate overshooting rises with the fraction of firms able to set different prices. Since a change in the value of the currency has only a very limited impact on relative prices and consumption, expenditure-switching plays only a minor role. Therefore the size of an exchange rate movement required to satisfy the monetary equilibrium is enlarged, as only an immense depreciation affects the relative price strong enough to shift aggregate world demand to domestic goods.¹⁰³

Implications of local-currency-pricing are much in line with the empirical obser-

¹⁰¹Note, that the persistence in prices for a staggered price setting depends on (i) the sensitivity of prices to costs, and (ii) the sensitivity of costs to output. In a model where prices are a constant mark-up over marginal costs and marginal costs are increasing in the level of output, a firm will raise its price as soon as possible. Only the introduction of a price elasticity which is increasing in the price level (for example by introducing convex demand schedules) leads to a smooth adjustment in prices. See Chari, Kehoe, and McGrattan (2000) for details.

¹⁰²See Engel (1999), Rogoff (1996) and Devereux (1997) for empirical evidence.

¹⁰³Like in the Dornbusch model, a falling interest rate to clear the money market in reaction to a monetary expansion can only be achieved if the exchange rate is expected to appreciate.

vation, that exchange rates are more volatile than relative prices.¹⁰⁴ When all prices are preset in the domestic currency, exchange rate movements change profit margins, but do not affect the consumption based price index (CPI), so that these models capture the strong correlation between nominal and real CPI exchange rates. Moreover, the reduction of the co-movement of consumption and the rise in the co-movement of output, since an increase in domestic demand now raises the demand for imports at the fixed relative price of imports in terms of domestic currency, mirrors empirical regularities of international business cycles.¹⁰⁵

As under perfect pricing to market the expenditure-switching effects completely vanishes, the current account remains in balance.¹⁰⁶ However, evaluating welfare displays asymmetric gains across countries. Since export prices are fixed in terms of the foreign currency, a depreciation raises the "price" of exports in domestic currency, while import prices are unaffected. This possibility of an improvement in the domestic terms of trade introduces a beggar-thy-neighbor effect by adversely affecting the foreign terms of trade.

A critical assessment The assumption of perfect pricing to market in local currency, however, has attracted strong criticism. Obstfeld (2002), for example, argues that the assumption of a complete exchange rate pass-through to *import competing goods prices* (and therefore the overall CPI) reestablishes the exchange rate pessimism of the early postwar period and is based on a misinterpretation of the empirical record. The empirical evidence on stabilizing adjustment effects of exchange rates remains strong. In a world of sticky *foreign* currency prices, the steep depreciation after the launch of the euro would have led to immediately rising import prices, but the implied output inflation failed to appear.

He argues that one should assume a rapid pass-through to *import prices*, but a slow pass-through to the CPI, since the major component in production, wage costs, are nominally sticky and very sluggish. Therefore output prices adjust slowly, whereas exporters have to maintain their domestic-currency prices to maintain profit margins. In contrast to static empirical studies,¹⁰⁷ which argue that among OECD countries exporters lower (on average) their own-currency price by around 5 % in response to a depreciation of 10% (in the same year), implying a median pass-through to *manufacturing import prices* (CPI pass-through) of 50 percent, dynamic studies suggest a complete long-run pass-through of exchange rate changes to *import prices* (import price pass-through).

¹⁰⁴See also Chari, Kehoe, and McGrattan (2000) and Devereux and Engel (2003) for recent applications of local-currency-pricing.

¹⁰⁵See Backus, Kehoe, and Kydland (1992).

¹⁰⁶Not even a distinction between tradables and nontradables changes this result since due to the highly variable nominal exchange rate international tradable prices move much in line with international nontradable prices. See Engel (1999).

¹⁰⁷See for example Goldberg and Knetter (1997).

Obstfeld criticizes the missing distinction between consumer prices of imported goods and producer prices domestic retailers pay to manufacturers. One of his key arguments is that import prices paid at the point of entry to a country display a very different behavior than CPI prices of imported goods. On their way to the consumer, imports incorporate a substantial nontradable marketing input and pass through imperfectly competitive retailing networks. Furthermore, import prices measured in the currency of origin are closely linked to nominal wages, so that a depreciation will tend to worsen the terms of trade by lowering relative wages.¹⁰⁸ As wages move more slowly than international prices the pass-through to import prices should be more rapid than to the general CPI and a nominal depreciation causes the observed real depreciation.

Moreover he is of the opinion that it may rather be firms as opposed to consumers who decide to switch expenditures. When import prices at the point of entry change, importing firms might switch to domestic goods, so that the critical relative price for the switching effect is the real exchange rate measured with respect to relative unit labor costs. Rangan and Lawrence (1999) find striking evidence that intrafirm trade responds very strongly to exchange rate changes by showing correlations between the dollar's real depreciation and U.S. content levels in sales made by majority-owned foreign affiliates of U.S. multinationals from 1985 to 1989, indicating substantial expenditure-switching. Obstfeld himself strengthens this evidence by showing strong correlations between relative export prices and nominal exchange rates, and strong co-movements of relative export prices with the bilateral exchange rate between Canada and the U.S. for a wide range of products and a sample period from 1993 to 2001. Moreover, the ratio of the Canadian import prices to the industrial prices of competing goods shows for the 1990s that a nominal effective depreciation of the Canadian dollar led to an increase in relative import prices.

Obstfeld (2001) develops a model including retail firms which are able to switch between imported and domestic inputs. Their relative price depends on the relative wages. Prices for the final goods and wages are preset, arbitrage between national markets is prohibitively expensive. The model displays strong expenditure-switching, but nevertheless captures the high empirical correlation between nominal and real CPI exchange rates. However, the sticky final goods prices are irrelevant with respect to the allocative role of flexible exchange rates.

Producer-currency-pricing Since all foregoing considerations lead to the conclusion that prices appear to be sticky in the producer's currency, Obstfeld and Rogoff (2000) and Obstfeld and Rogoff (2002) develop a model where exporters set prices in their own currency. The authors further justify this assumption by the direct evidence on invoicing, which is largely inconsistent with local-currency pric-

¹⁰⁸For empirical evidence see Obstfeld and Rogoff (2000) and Obstfeld (2001).

ing.¹⁰⁹ Additionally they assume rigid nominal labour markets, since they believe that sticky prices due to sticky wages are more likely to explain the observed persistent macroeconomic fluctuations than the invoicing in local currency, which largely applies to contracts of only 90 days. To capture deviations from the LOOP Obstfeld and Rogoff assume a fraction of nontradables in each country.¹¹⁰ Since it is not possible to offset the monopolistic distortions in labor and product markets and raise welfare above its flexible-wage level, the optimal reaction on a productivity shock would be to achieve the flexible-wage allocation associated with the realized values of the shock - a constraint-efficient solution, since it is subject to monopoly distortions.

Corsetti and Pesenti (2005) develop a model where exporters can choose whether they would like to invoice in their own currency or the foreign one. The degree of pass-through of exchange rates to export prices is captured by a parameter η , where $\eta = 0$ corresponds to local-currency-pricing, whereas for $\eta = 1$ the model converges to a dynamic version of Obstfeld and Rogoff (2000) without nontradables. For $\eta = 1$ optimal monetary policy would replicate the flexible-price equilibrium, whereas for $\eta = 0$ this equilibrium cannot be maintained and the optimal policy should reduce export markups by aiming at a fixed exchange rate since this strategy would stabilize the exporters' revenues in their own currency. For $0 < \eta < 1$ a trade-off arises and exchange rate movements are muted. In an extension of this model by Corsetti and Pesenti (2002) producers are allowed to choose η so as to maximize expected utility. If the policy maker targets the flexible-price equilibrium it is optimal for firms to practice producer-currency-pricing, whereas they are indifferent to the choice of η when monetary policy aims at fixing exchange rates. However, since welfare is higher for all countries under a floating-rate regime, cooperative policy authorities would choose an optimal floating strategy, and producers would practice producer-currency-pricing.

7.1.4 Preferences and nontradables

In addition to conventional pricing to market approaches, a number of authors introduce translog preferences, generating variable mark-ups over marginal costs, by assuming that the expenditure share for each good is inversely related to its relative price.¹¹¹ As firms are now reluctant to raise prices, a monetary shock has persistent effects on the real exchange rate and leads to persistent deviations from the LOOP. Furthermore, the slower price adjustment also leads to a larger accumulation of net foreign assets.

¹⁰⁹Thygesen (1995), for example shows that imports are mostly invoiced in foreign currencies, whereas exports tend to be invoiced in the exporter's currency.

¹¹⁰Obstfeld (2001) argues that this is a much less extreme assumption to explain the close association between deviations from the law of one price and exchange rates than the complete CPI pass-through of, for example, Devereux and Engel (2003).

¹¹¹See, for example, Bergin and Feenstra (1998), Bergin and Feenstra (2001) and Basu (1995).

The ideas of assuming only a limited degree of substitutability between home and foreign goods or asymmetric preferences, first used by Svensson and Van Wijnbergen (1989), is picked up again by some more recent studies. Corsetti and Pesenti (2001), for example, analyze a model with a unit elasticity of substitution between home and foreign goods. Income shares for home and foreign agents are constant, since an increase in domestic output by x percent would automatically be accompanied by a decrease in prices by x percent. In addition, these offsetting terms of trade movements imply a zero equilibrium current account, so that shocks have no real long-run effect any more. In this framework, an unanticipated exchange rate depreciation can be a "beggar thyself"- or a "beggar thy neighbor"-policy, since the positive output effect of a monetary expansion could be more than offset by the negative terms of trade effect. In contrast to the Redux model, it is therefore not optimal to raise output to its competitive level by a monetary expansion. Furthermore the asymmetric welfare effect raises the potential for conflicts and calls for policy coordination.¹¹²

Chari, Kehoe, and McGrattan (1997) assume an elasticity of substitution of home and foreign goods less than the elasticity of substitution between different varieties of home and foreign goods and include a home bias in consumption. Reacting to the criticism concerning the separable treating of consumption and leisure in the Redux model,¹¹³ they further consider preferences which imply a unit elasticity of substitution between consumption and leisure, so that a monetary shock also raises relative domestic consumption and compensates for the extra work effort. As this mitigates the impact of monetary shocks, the observed volatility in real exchange rates can only be generated by an elastic labor supply and a high intertemporal elasticity of substitution in consumption. In addition, they believe that adding capital is important even for a short-run analysis, since a monetary shock causes an investment boom by reducing the interest rate and may bring about a current account deficit in contrast to the surplus predicted by the Redux model. Concerning the long-run effect, the investment boom would lead to an increase in current relative labor supply, so that the shock persistence diminishes.¹¹⁴

Hau (2000) introduces a fraction of non-traded goods, leading to an increase in the size of the initial exchange rate response to a monetary shock. Since non-traded prices are tied down by sticky nominal wages, a larger exchange rate movement is required after a given change in the aggregate price level. However, this effect is dampened, since the demand expansion is biased towards home goods, so that relative domestic consumption increases. This consumption differential is strengthened by the relatively low domestic real interest rate.

¹¹²In addition, the assumption of asymmetric preferences allows one to derive policies as functions of structural parameters and the policy stance of the other country.

¹¹³The assumption of separability of consumption and leisure is incompatible with a balanced growth path: as a country grows richer, labor supply continually declines to zero.

¹¹⁴See also Betts and Devereux (2000).

Warnock (1998) includes asymmetric preferences across goods by assuming higher utility gains from domestic goods. The results are intuitive: A domestic monetary expansion increases relative domestic demand and raises domestic welfare more than foreign welfare. Furthermore, the accumulation of net foreign assets affects wealth not only by the labor supply channel, but also by influencing the composition of demand. The real exchange rate now depends on the terms of trade and nominal exchange rate overshooting occurs.

7.1.5 Including complete financial markets

As Obstfeld and Rogoff believe that one should not analyze imperfections and rigidities in goods markets and on the other hand, assume complete international capital markets, they only allow for riskless international bonds trading in the Redux model. However, some authors of a different opinion have included complete financial markets. Chari, Kehoe, and McGrattan (1997), for example, compare monetary shocks in a model featuring complete financial markets to a model in which only a fraction of bonds are traded. As they choose a pricing-to-market approach, the differences are small since foreign consumption is changed only slightly by the domestic monetary expansion. The increase in domestic imports is financed by an increase in the domestic currency value of export earnings, because of fixed foreign currency prices, but depreciating exchange rates. For log-separable preferences the current account stays always in balance and there are no persistent effects at all.¹¹⁵

Sutherland (1996) includes trading frictions by assuming convex adjustment costs for the purchase of foreign bonds, but perfect goods markets. This allows the domestic interest rate to deviate from the foreign one. The increase in relative domestic consumption is higher than in the Redux model, implying a smaller exchange rate depreciation, less rising domestic output, and exchange rate undershooting. Since consumption and output dynamics are more closely linked to each other, the current account imbalances are smaller. Furthermore, a perfect financial market seems to lower (increase) the volatility of most macroeconomic variables when shocks originate from real demand or supply (the money market). In addition, Sutherland concludes that impacts due to trading frictions rise with the degree of price inertia due to the slower adjustment of output accompanied by the higher willingness to smooth consumption via financial markets. Senay (1998) extends this model by including goods market segmentation and pricing-to-market behavior, and shows that complete financial markets reduce the output response to monetary shocks only if goods markets are segmented, whereas under integrated goods markets the output response increases.

¹¹⁵This result is confirmed by Betts and Devereux (2001).

7.1.6 Including international policy interdependence

In the Corsetti and Persenti model introduced above, as well as in the original Redux model, monetary policies have positive spillover effects, in the sense that a domestic expansion raises foreign welfare as well. However, they are strategically independent in the sense that a domestic expansion does not affect the optimal policy of the foreign country. This follows directly from the assumption of an intertemporal elasticity of substitution of home and foreign goods equal to the intratemporal elasticity. If the *intertemporal* elasticity is larger (smaller) than the *intra*temporal elasticity, the policies are strategic substitutes (complements), and the foreign central bank responds with a monetary contraction (expansion) when the domestic country increases its money supply, as the expansion raises (decreases) foreign output and leads to higher (lower) costs in terms of foregone leisure, so that its optimal for the central bank to react with a reduction (expansion) in the money supply. In the Corsetti and Pesenti model a country acting unilaterally takes the negative terms of trade effect into account, it would choose too tight (loose) a policy, so that the optimal output level could only be achieved by policy coordination.

Benigno (2002) analyses two countries of different size to show that under non-cooperation a contractionary bias arises since each country does not internalize the gain of the other one. The optimal cooperative solution leaves the terms of trade unchanged and leads to a joint expansion to push both countries output levels to their competitive solutions. Since the larger country (if it is big enough) has lower terms of trade in the competitive case, it is possible that such a policy is unsustainable, as this country is now worse off. Focussing on those policies that make both countries better off illustrates that under this restriction only the smaller country reaches its competitive equilibrium, whereas in the other one the steady state retains some monopoly power.

Tille (2001) shows that a monetary expansion can be a “beggar thyself” policy since the negative terms of trade effect dominates the domestic output expansion, when domestic and foreign goods are poor substitutes. However, for close substitutes the “beggar thy neighbor” result applies. In a multi-country setting, Corsetti, Pesenti, Roubini, and Tille (1999) assume two peripheral countries A and B, selling similar goods to a country C, which exports these goods to the periphery. An expansion in country A could be a “beggar thy neighbor policy” with respect to country B, as the demand in country C switches to goods from country A due to the devaluation, but a “beggar thyself” policy with respect to country C through the negative terms of trade effect.

Betts and Devereux (2000) investigate international policy coordination when the LOOP fails to hold and illustrate in a pricing to market approach, how a domestic monetary expansion reduces foreign welfare by decreasing its terms of trade. In contrast to studies assuming that the LOOP holds, policy spillovers under full pricing-to-markets are negative and coordination implies a slower rate of monetary expansion.

7.1.7 Small open economies

As the two-country models described so far include highly complex assumptions, that may be not of relevance for a small open economy, Obstfeld and Rogoff themselves sketch a small open economy model in the appendix of their Redux paper, where domestic currency prices in the traded sector are equal to the world price times the exchange rate. Furthermore, they assume log separable preferences for traded and nontraded goods, international asset markets, which provide a riskless real bond denominated in units of the tradable good, and a discount rate equal to the world interest rate. The first and the last assumptions imply that the optimal path for tradables consumption is perfectly flat, so that the current account – in contrast to their two-country model – always remains in balance. However, they illustrate that for a consumption elasticity of money demand less than one, the monetary equilibrium requires a decline in the short-run nominal interest rate, so that the Dornbusch (1976) overshooting result can be revised in this model. Lane (1997) analyses discretionary monetary policy in this model to show that a more open economy has a lower equilibrium inflation rate. The reason is that a more open economy gains less from a surprise inflation, as the output gain, stemming from a monetary expansion, diminishes when the nontraded sector decreases.

An alternative specification is analyzed by Lane (2001a), who assumes a CRRA utility function which solely depends on a consumption index, which is a CES aggregate of traded and nontraded goods. The response of the current account to a monetary shock depends on the interplay between the inter- and the intratemporal elasticities of substitution between traded and nontraded goods. When the willingness to substitute consumption across periods is smaller (greater) than the willingness to substitute between traded and nontraded goods, the rise in nontraded output and consumption, due to the monetary expansion, decreases (increases) traded consumption and leads to a current account surplus (deficit).

7.1.8 Small open New Keynesian models

Over the last years New Keynesian models, formerly focussed on closed economies, have been extended to small open economies. Model equations are built by a log-linear approximation of equilibrium (first order) conditions from a dynamic stochastic general equilibrium framework with optimizing agents, nominal rigidities and imperfect competition. The main advantage of these models is high tractability, since for most of the models, equilibrium dynamics are simple representations of domestic inflation and the output gap. However, the most important difference of nearly all models of the new open economics type described so far is that monetary policy is not modelled by an exogenously given stochastic process, but endogenously by assuming a monetary policy instrument like the interest rate.¹¹⁶ These models assume a quadratic loss function to derive optimal monetary policy rules by minimizing this

¹¹⁶One exception of the models discussed above is Obstfeld and Rogoff (2000).

function.

The study of Galí and Monacelli (2005) differs from most other small open economy models in modelling one economy among a continuum of infinitesimal small economies, aggregated as rest of the world. By assuming identical preferences, technologies and market structures, as well as complete financial markets, a two-equation dynamic system, consisting of the New Keynesian Phillips Curve and the expectational dynamic IS Curve, emerges which is closed by a monetary policy rule. Moreover, they assume a complete exchange rate pass-through to prices of imported goods. The authors derive a loss-function by a second-order approximation to the consumer's utility function under the assumption of a log-utility and a unit elasticity of substitution between domestic and foreign goods and show that optimal policy should fully stabilize the domestic price level.

They investigate the welfare implications of three different monetary policy rules: one rule where the interest rate responds to domestic goods price inflation, one responding to CPI inflation, and one pegging the effective nominal exchange rate. The optimal policy can simultaneously stabilize inflation and the output gap, but leads to a greater volatility in the exchange rate and the terms of trade than the two Taylor-type rules and/or the exchange rate peg. The simple rules show an excess smoothness in the nominal exchange rate, which combined with the assumption of price inertia precludes a fast enough adjustment of prices in response to a productivity shock, so that there is a significant deviation from the optimal solution. Ranking the rules by their welfare implications demonstrates that for a broad range of parameter configurations the domestic (goods) inflation-based Taylor-rule dominates the CPI-based one, which in turn dominates the exchange rate pegging. Furthermore, they conclude that volatility in the terms of trade is adversely related to volatility in inflation and the output gap, and therefore also to welfare.

Clarida, Galí, and Gertler (2001) develop a model similar to that in Galí and Monacelli (2005), Obstfeld and Rogoff (2000) and Svensson (2000), but add nominal labor market rigidities that (re)introduce a short-run trade-off between inflation and output. They show that under certain conditions the problem is isomorphic to the closed economy counterpart, so that the qualitative results are the same as those given in the closed economy version of Clarida, Galí, and Gertler (1999). However, quantitatively it is the degree of openness that determines optimal monetary policy. Their analysis includes an expenditure-switching effect when the elasticity of substitution between domestic and foreign goods is large enough, and what they propose seems to be empirically reasonable. Like Galí and Monacelli they find that to the extent that there is a complete exchange rate pass-through, the central bank should target domestic producer price inflation.

McCallum and Nelson (1999a) and McCallum and Nelson (2000) promote an alternative approach by modelling imports not as finished consumer goods, but as inputs in domestic production, leading to simple analytics and correlations between the exchange rate and inflation, which are more realistic, compared to conventional approaches. Model simulations replicate the experience of some small OECD countries

during the East Asian crisis of 1997-1998 quite well. For example, Australia, Canada and New Zealand experienced double-digit depreciations of their trade weighted exchange rates during this period. The latter two countries tightened their monetary policy in the hope of meeting their inflation target, whereas Australia did not. Ex post, Australia's policy seemed to be the most appropriate, since it was consistent with strong economic growth and ensured that inflation was on target. By the simple experiment of adding a shock to the uncovered interest parity, the authors illustrate that the implied depreciation of the exchange rate causes an output boom in their model. Since they assume a policy rule responding to expected nominal income growth, the optimal interest rate declines, since the temporary output boom implies an expected lower change in future output. By treating imports as producer inputs, total consumer price inflation does not depend on the real exchange rate or the terms of trade any more, so that optimal policy should account only for shock-implications on inflation in terms of its effect on the output gap. This provides a little support for practicing pure inflation-targeting, unless depreciations are associated with large increases in the output gap.

7.1.9 New Keynesian two-country models

After the extension of the New Keynesian literature to the analysis of open economy models, the most recent studies assume two-country settings. Benigno and Benigno (2001), for example, derive the conditions under which price stability (defined as a stable GDP-index) can implement the flexible-price allocation as a Nash equilibrium in a model with imperfect competition and price stickiness. In this setting a stable equilibrium is only possible if both countries maintain a certain positive degree of monopolistic competition, so that the monetary policy makers have no incentive to surprise price setters ex post. Since volatility is exactly the same as under flexible prices, there is no need for the policy maker to stabilize the exchange rate. The intuition behind the association of stable prices with monopolistic distortions is simply that otherwise policy makers would have the incentive to deflate. Since in a competitive equilibrium real wages are equated to the marginal rate of substitution between consumption and labor this strategy would reduce marginal utility of labor and the marginal disutility of production by the same amount. However, the implied appreciation of the terms of trade, following such a policy, would decrease the disutility further.

A two-country version the small open economy of Clarida, Galí, and Gertler (2001) is given in Clarida, Galí, and Gertler (2002). They show that their isomorphism result is preserved in a non-cooperative equilibrium and optimal rules are qualitatively the same than in the closed economy case. However, since there are potentially gains from coordinating policy rules, the isomorphism result breaks down. As the domestic marginal costs of production and domestic potential output depend on the terms of trade, a coordination of central banks could improve welfare by taking this into account. Furthermore, the authors show that this can be

achieved by using a Taylor-type rule, which also responds to foreign inflation.

Batini and Pearlman (2002) and Batini, Levine, and Pearlman (2004) study inflation-forecast-based rules in a modification of the Clarida et al. framework including a habit formation in persistence and inflation indexing. This leads to persistence in the New Keynesian Phillips curve and the expectational IS curve that capture empirical regularities in the euro area and the U.S.. In addition they assume a home bias in consumption, so that long-run deviations from the consumption-based PPP are possible. This helps replicating the observed movements in the real euro/dollar rate. By analyzing the uniqueness and stability conditions for an equilibrium under inflation-forecast-based rules,¹¹⁷ they find out that for any feedback parameter on inflation indeterminacy occurs when the horizon parameter is too low. Furthermore, the problem of indeterminacy seems to rise in an open economy, and this result is compounded when central banks respond to expected consumer rather than producer price inflation. An extension of this model is estimated for the euro area and the U.S. in Batini, Justiniano, Levine, and Pearlman (2005) using Bayesian techniques. In addition, this study includes an imperfect exchange rate pass-through and wage stickiness by assuming a staggered wage setting. They compute optimal inflation-forecast based rules for the cooperative and the non-cooperative case based on the estimated parameters to provide an empirical assessment of coordination gains, which shows that gains are fairly low in terms of average loss. However, they find a significant increase in the aggressiveness of the European monetary policy rule at the expense of the U.S.

Pappa (2004) comes to similar conclusions in a theoretical approach. He investigates implications for macroeconomic stability and welfare in three international policy regimes: cooperation, no cooperation and monetary union. He shows that a non-cooperative equilibrium may be suboptimal because of beggar-thyself and beggar-thy-neighbor effects, while a monetary union maybe suboptimal due to the sluggish price adjustment. He argues that for deriving significant coordination gains between the ECB and the Fed, one had to assume a high degree of trade link and unrealistically high values for the international elasticity of substitution and the risk aversion parameter. However, concerning the UK, he finds the co-ordination gains to be significant.

7.2 Analyzing uncertainty in open economy models

To date only a very few studies include an analysis of uncertainty in a two-country model. Most of them have focussed on *monetary* uncertainty by assuming a money stock that follows a stochastic process and analyze the effect of an unexpected rise in the money supply leading to a risk-premium into output prices.¹¹⁸ As our

¹¹⁷A monetary rule leads to indeterminacy when its reaction parameter on inflation is not high enough, so that real interest rates fall and enforce demand to rise. Under these circumstances any exogenous expected inflation can be confirmed.

¹¹⁸See Lane (2001a), p. 33 ff. for a brief overview.

uncertainty analysis focuses on *model* uncertainty and whether a monetary policy should be more aggressive under uncertainty, the only analysis comparable to ours is Leitemo and Söderström (2005). They use the Clarida, Galí, and Gertler (2002) model to derive optimal policy rules with the robust control techniques developed by Hansen and Sargent (2007) and Giordani and Söderlind (2004), which are also employed in this paper. The simplicity of the model enables the authors to derive analytical solutions under uncertainty for a monetary authority acting under discretion. Furthermore, they generalize the robust control framework to allow a policy maker to have different robustness preferences for different model equations. This is an advantage when the policy maker has a prior belief regarding which equations he should trust more than others. In contrast to a closed economy version of the model, where policy rules under uncertainty should be more aggressive in general, the open economy robust policy reaction can be either more aggressive or more cautious, depending on the type of disturbance and the shock.

The model presented in this paper is very much in the spirit of the most recent developments of New Keynesian two-country models like Batini, Levine, and Pearlman (2004) and Clarida, Galí, and Gertler (2002). Instead of assuming a complete set of Arrow-Debreu securities (like in Clarida et al.), we assume one riskless internationally traded nominal bond, so that the uncovered interest parity should equalize net yields (like in Batini). Similar to Batini et al. we introduce persistence in inflation and consumption. However, instead of inflation indexing we use a fraction of backward-looking price setters for this reason, as we believe that this leads to similar outcomes, but is more straightforward to solve. In keeping with Batini et al. we assume intermediate goods producers, practicing producer-currency-pricing, and a home bias in consumption due to nontradables, but allow for different country sizes. This setting captures the observed long-run deviations from the CPI based LOOP. In contrast to most two-country studies we do not investigate possible gains from cooperation between central banks, but concentrate on gains stemming from the optimal commitment policy of a social planner compared to the solution under discretion, for three different assumptions on the degree of backwardness in expectations.¹¹⁹ Therefore we study the optimal interest rate reactions by generating impulse responses to general supply, demand and exchange rate shocks, and evaluate welfare implications. Furthermore we investigate the consequences of Hansen-Sargent robust modelling.

8 Robustness in a hybrid two-country New Keynesian model

After the preceding sections' discussion of the most recent work in monetary macroeconomics, this section derives a two-country New Keynesian economy and analyzes

¹¹⁹Clarida, Galí, and Gertler (2002) investigate welfare gains from cooperation and illustrate how the solution of an optimal planner can be implemented.

the influence of a preference for robustness against model misspecification on optimal monetary policy. In 8.1 the benchmark model is introduced, 8.2 calibrates the two-country economy and studies the consequences of uncertainty, and a discussion of the results and a comparison with the related literature is given in 8.2.3. For a detailed model derivation see appendix A.4.

8.1 The benchmark model

The model is based on a two country DSGE framework. The microfoundations feature a home and a foreign country which share same preferences and technologies, but differ in their size. The size of the foreign country is γ , so that $(1 - \gamma)$ is the size of the home country. Foreign country variables are denoted with the superscript $*$, foreign price indices are measured in the foreign currency. Households and firms face the conventional New Keynesian optimization problems based on a competitive goods market (drawn from Dixit and Stiglitz (1977)) and sticky price set-up (drawn from Calvo (1983)). In a two-stage production, final goods producers use the output of intermediate goods producers as input. The final goods producers are assumed to produce competitively using a CES technology consisting of a continuum of nontraded intermediate goods. Furthermore, we assume that prices are sticky in the producer's currency and believe that the uncovered interest rate parity should equalize net yields from riskless internationally tradable bonds across countries. Under these assumptions the LOOP applies to the PPIs, since exchange rate movements are followed by price changes of exporters in the buyer country's currency (complete exchange rate pass-through to *import* prices). Furthermore, the assumption of a home bias due to nontradables allows for additional deviations from the CPI-based LOOP. While the producer-currency-pricing implies a strong expenditure-switching, the home bias dampens this effect. The home bias can be due to nontradables or trading frictions, which imply that a switch to another country is impossible. Contrary to most previous studies we assume a fraction of backward-looking price setters and a habit formation in consumption. In the following we focus on the microfoundations relevant for the domestic country for as long as foreign counterparts are derived in exactly the same fashion. For convenience all variables used in this section are summarized in table 7.

8.1.1 Households

Let C_t (C_t^*) be the domestic (foreign) aggregate Cobb-Douglas consumption index, where $C_{H,t}$ ($C_{H,t}^*$) and $C_{F,t}$ ($C_{F,t}^*$) denote domestic (foreign) consumption of home and foreign goods, respectively:

$$C_t \equiv C_{H,t}^{\gamma(1-\varpi_H)} C_{F,t}^{\gamma\varpi_H}, \quad (87)$$

$$C_t^* \equiv C_{H,t}^{*\gamma(1-\varpi_F)} C_{F,t}^{*\gamma\varpi_F}, \quad (88)$$

$$\varpi_H = \frac{\gamma\varpi}{\gamma\varpi + (1-\gamma)(1-\varpi)}, \quad \varpi_F = \frac{\gamma(1-\varpi)}{\gamma(1-\varpi) + (1-\gamma)\varpi},$$

$C_{H,t} (C_{F,t})$: consumption index for domestic (foreign) products
$c_{H,jt} (c_{F,jt})$: demand for domestic (foreign) final good j
$P_{H,t} (P_{F,t})$: domestic (foreign) producer price index (PPI)
$P_{H,jt} (P_{F,jt})$: price of domestic (foreign) intermediate good j
$p_{H,jt} (p_{F,jt})$: price of domestic (foreign) final good j
p_t^{fl}	: optimal price of forward-looking price setters
p_t^{bl}	: optimal price of backward-looking price setters
\bar{p}_t^n	: price index for newly set prices
$\pi_{H,t} (\pi_{F,t})$: domestic (foreign) producer price inflation
C_t	: composite consumption index
P_t	: domestic (foreign) consumption price index (CPI)
π_t	: consumer price inflation
$H_{C,t}$: habit formation in consumption
X_t	: aggregate output
X_{jt}	: output of intermediate goods producer j
M_t	: money holding
B_t	: bond holding
N_t	: labor supply
S_t	: terms of trade $\left(\frac{P_{F,t}}{P_{H,t}} \right)$
W_t	: nominal wage
Π_t	: real firm profits
Q_t	: nominal exchange rate
$RE R_t$: real exchange rate
TB_t	: trade balance
i_t	: interest rate
φ_t	: real marginal costs
$\Delta_{i,t+i}$: expected real discount factor

Table 7: Model variables, two-country economy

where $\varpi \in [0, \frac{1}{2}]$ represents the home bias in consumption and measures the degree of openness. For $\varpi = 0$ both countries live in complete autarky and buy only domestic products, whereas $\varpi = \frac{1}{2}$ corresponds to a complete integration scenario without any trading frictions, so that the relative consumption decisions only depend on the relative size of the countries.¹²⁰ Thus ϖ_H and ϖ_F represent the home bias adjusted country sizes. Note that without a home bias (87) and (88) are reduced to the indices in Clarida, Galí, and Gertler (2002), whereas for equally-sized countries they are exactly the same as in Batini, Levine, and Pearlman (2004).

Producers are normalized to a continuum of measure one and produce differenti-

¹²⁰The home bias can be justified by trading frictions or nontradables and is analytically more tractable than introducing a fraction of nontradables consumption C_N as in Obstfeld and Rogoff (1995).

ated products. Under the assumption of monopolistic competition, the consumption indices of domestic and foreign goods are given by

$$C_{H,t} = \left[\int_0^1 c_{H,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}}, \quad C_{F,t} = \left[\int_0^1 c_{F,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}}, \quad \zeta > 1, \quad (89)$$

where $c_{H,jt}$ ($c_{F,jt}$) represents the domestic demand for domestic (foreign) individual good j and ζ represents the price elasticity of demand.

The first order conditions of the household's cost minimizing problem lead to the following demand equations:

$$c_{H,jt} = \left(\frac{p_{H,jt}}{(1 - \varpi_H) \psi_t} \right)^{-\zeta} C_{H,t}^{-(\zeta-1)} C_t^\zeta, \quad (90)$$

and

$$c_{F,jt} = \left(\frac{p_{F,jt}}{\varpi_H \psi_t} \right)^{-\zeta} C_{F,t}^{-(\zeta-1)} C_t^\zeta, \quad (91)$$

where ψ_t is the Lagrangian multiplier on the condition to achieve any given level \bar{C} of the composite consumption good. By combining (90) and (91) with the definition of the domestic aggregate consumption index, the household's optimization problem gives an expression for the domestic and foreign CPI:

$$P_t \equiv k_H^{-1} P_{H,t}^{(1-\varpi_H)} P_{F,t}^{\varpi_H}, \quad (92)$$

$$P_t^* \equiv k_F^{-1} P_{H,t}^{*(1-\varpi_F)} P_{F,t}^{*\varpi_F}, \quad (93)$$

where $k_H = \varpi_H^{\varpi_H} (1 - \varpi_H)^{(1-\varpi_H)}$ and $k_F = \varpi_F^{\varpi_F} (1 - \varpi_F)^{(1-\varpi_F)}$, and domestic currency PPIs of home and foreign goods are defined as generalized means of all corresponding prices ($p_{H,jt}, p_{F,jt}$):

$$P_{H,t} \equiv \left[\int_0^1 p_{H,jt}^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}}, \quad P_{F,t} \equiv \left[\int_0^1 p_{F,jt}^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}}. \quad (94)$$

Defining the terms of trade S_t as the foreign PPI divided by the domestic PPI (both measured in the domestic currency), the domestic and foreign CPIs are given by

$$P_t = k_H^{-1} P_{H,t} S_t^{\varpi_H}, \quad P_t^* = k_F^{-1} P_{H,t}^* S_t^{\varpi_F}, \quad (95)$$

where we use the assumption of producer-currency-pricing, which implies $P_{H,t} = Q_t P_{H,t}^*$, $P_{F,t} = Q_t P_{F,t}^*$.¹²¹ Given the PPIs for domestic and foreign goods, domestic and foreign households optimal intratemporal decisions are given by

$$P_{H,t} C_{H,t} = (1 - \varpi_H) P_t C_t, \quad P_{F,t} C_{F,t} = \varpi_H P_t C_t \quad (96)$$

¹²¹Note that without a home bias in consumption ($\omega = \frac{1}{2}$) home and foreign price indices are equal to those in Clarida, Galí, and Gertler (2002), since all consumers share the same preferences and therefore consume the same proportion of domestic and foreign goods.

and

$$P_{H,t}^* C_{H,t}^* = (1 - \varpi_F) P_t^* C_t^*, P_{F,t}^* C_{F,t}^* = \varpi_F P_t^* C_t^*. \quad (97)$$

Eliminating the Lagrangian multiplier in (90) and (91) leads to the demand for domestic and foreign individual goods:

$$c_{H,jt} = \left(\frac{p_{H,jt}}{(1 - \varpi_H) P_t} \right)^{-\zeta} C_{H,t}^{-(\zeta-1)} C_t^\zeta \quad (98)$$

and

$$c_{F,jt} = \left(\frac{p_{F,jt}}{\varpi_H P_t} \right)^{-\zeta} C_{F,t}^{-(\zeta-1)} C_t^\zeta. \quad (99)$$

These equations show that the price elasticities of demand for domestic and foreign products are the same: $-\frac{\partial c_{H,jt}}{\partial p_{H,jt}} \frac{p_{H,jt}}{c_{H,jt}} = -\frac{\partial c_{F,jt}}{\partial p_{F,jt}} \frac{p_{F,jt}}{c_{F,jt}} = \zeta$.¹²² A change in the price for an individual foreign good j leads to a change in the demand for good j in exactly the same manner as a change in the price for an individual domestic good k leads to an adjustment in the demand for good k .

Defining the real exchange rate as $RER_t = \frac{Q_t P_t^*}{P_t}$, there is a link to the terms of trade given by

$$RER_t = \frac{Q_t P_t^*}{P_t} = \frac{k_H}{k_F} S_t^{(\varpi_F - \varpi_H)}. \quad (100)$$

Since $\varpi_F - \varpi_H = \frac{1-2\varpi}{[\gamma\varpi+(1-\gamma)(1-\varpi)][\gamma(1-\varpi)+(1-\gamma)\varpi]} \geq 0$, an appreciation of the real exchange rate (a fall in RER_t) is accompanied by an improvement in the terms of trade (a fall in S_t represents an increase in the relative value of domestic exports to foreign imports, *ceteris paribus*), as, assuming constant quantities, the real term relationship between both countries improves from a domestic perspective. For equally sized countries the real exchange rate relationship is reduced to $S_t^{(1-2\varpi)}$ and is exactly the same as in Batini, Levine, and Pearlman (2004). Furthermore, under the assumption of no home bias (so that the home bias adjusted country sizes are reduced to $\varpi_F = \varpi_H = \gamma$) (100) implies a real exchange rate of value one as in Clarida, Galí, and Gertler (2002), and represents the LOOP.

Given the cost minimizing combinations of individual goods for a given level of the CPI, a representative household supplies labor (N_t), purchases domestic and foreign consumption goods ($C_{H,t}, C_{F,t}$), and holds money (M_t) and domestic and foreign bonds (B_t, B_t^*) to maximize¹²³

¹²²This result follows directly from the assumed aggregate consumption index of the Cobb-Douglas type. As preferences across countries are the same, this price elasticity also applies to a foreign country facing domestic and foreign goods and prices.

¹²³Note that for reasons of convenience we use the standard assumption that domestic citizens do not hold foreign money, but pay foreign products with domestic money (see for example Batini and Pearlman (2002) and Batini, Levine, and Pearlman (2004)). Including a term M_t^* in (101) and (102) would only complicate the following analysis without altering the log-linear representation of equilibrium relationships. In fact the money market equilibrium would become slightly more complicated without any further consequences.

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - H_{C,t+i})^{1-\sigma}}{1-\sigma} + \frac{\chi_1}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi_2 \frac{N_{t+i}^{1+\eta}}{1+\eta} \right], \quad (101)$$

where β represents the discount factor, and b and η represent the risk aversion parameters (the inverses of the intertemporal elasticities of substitution) of real money holding and labor supply, respectively, and the term $H_{C,t} = h_C C_{t-1}$ captures the habit formation in consumption. The weights χ_1 and χ_2 are individual preferences of the representative household. In addition, households sequence of budget constraints (in real terms) is given by

$$\begin{aligned} & C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \frac{Q_t B_t^*}{P_t} \\ &= \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + (1 + i_{t-1}^*) \frac{Q_t B_{t-1}^*}{P_t} + \Pi_t, \end{aligned} \quad (102)$$

and simply says that the sum of consumption,¹²⁴ and real money and bond holdings in period t must equal the sum of the real wage bill (W_t represents the nominal wage), the real firm profits (Π_t) in period t , and real money and domestic and foreign bond holdings (including interest payments) from period $t - 1$.

Similar to the single-country model, the standard first order conditions of (101) are given by (102) and

$$(C_t - h_C C_{t-1})^{-\sigma} = \delta (1 + i_t) E_t \left[\left(\frac{P_t}{P_{t+1}} \right) (C_{t+1} - h_C C_t)^{-\sigma} \right], \quad (103)$$

$$\chi_1 \left(\frac{M_t}{P_t} \right)^{-b} = (C_t - h_C C_{t-1})^{\sigma} \frac{i_t}{1 + i_t}, \quad (104)$$

$$\chi_2 N_t^{\eta} (C_t - h_C C_{t-1})^{\sigma} = \frac{W_t}{P_t}, \quad (105)$$

which represent the Euler condition for the intertemporal allocation of habit adjusted consumption, the money market equilibrium, and the labor-leisure trade-off equation.

Households accumulate assets in form of either domestic or foreign bonds. The assumption of internationally frictionless tradability implies that financial arbitrage should equalize the expected net yields on bonds, so that the uncovered interest parity holds:¹²⁵

$$1 + i_t = \frac{E_t Q_{t+1}}{Q_t} (1 + i_t^*). \quad (106)$$

¹²⁴As the prices for domestic and foreign individual goods determine the aggregate price index, the CPI good is already measured in real terms.

¹²⁵In agreement with Obstfeld and Rogoff (1995) we do not include a complete asset market in an analysis with nominal rigidity in the goods markets. See Clarida, Galí, and Gertler (2002), Corsetti and Pesenti (2001) and Benigno and Benigno (2001) for a complete asset market equilibrium in a simple asset market.

8.1.2 Firms

We assume a two-stage production process: The output of a continuum of *intermediate goods* producers is used as input by *final goods* producers who produce output according to the following CES technology:¹²⁶

$$X_t = \left(\int_0^1 X_{jt}^{\frac{(\varepsilon-1)}{\varepsilon}} dj \right)^{\frac{\varepsilon}{(\varepsilon-1)}}, \quad (107)$$

where X_t denotes aggregate output, X_{jt} is the input produced by intermediate goods firm j (both expressed in per capita terms) and ε represents the price elasticity of demand. The profit maximization problem of the final goods producers is given by

$$\max_{\{X_{jt}\}} P_{H,t} X_t - P_{H,jt} X_{jt}$$

and leads to the following set of demand equations

$$X_{j,t} = \left(\frac{P_{H,jt}}{P_{H,t}} \right)^{-\varepsilon} X_t, \quad (108)$$

where $P_{H,jt}$ is the price of a domestic individual intermediate good j .

Note that for the single-country analysis we assume a single final good producer without modeling intermediates, as it would only complicate the derivation without altering the equilibrium relationships. For a two-country economy the assumption that final goods producer use only domestic intermediate goods is crucial for solving the optimization problem of domestic producers, as this allows to express the demand for intermediate goods as a function of solely domestic variables ($P_{H,jt}$, $P_{H,t}$ and X_t). An approximation of the first order condition from the Calvo price-setting would include foreign PPI without this assumption and complicate the equilibrium relationship substantially. This implies that the only transmission channel from one country to the other is the demand for final goods. However, this assumption is standard in monetary policy analysis (see for example Batini and Pearlman (2002) and Batini, Levine, and Pearlman (2004)) and can be justified, since for the analysis of short-run fluctuations it should not change outcomes considerably. Furthermore, for large economies like the U.S. and the EU trading of intermediates should remain to the greatest extent inside their own country. In addition, McCallum and Nelson (1999a) and McCallum and Nelson (2000) argue that this modeling strategy leads to highly realistic correlations between the exchange rate and inflation.

Similar to the single-country analysis, intermediate goods production is assumed to follow a constant returns to scale production function, solely depending on labor input, $X_{jt} = Z_t N_{jt}$, $E(Z_t) = 1$, where Z_t is a stochastic zero mean aggregate productivity disturbance. Every firm j chooses the cost minimizing labor demand,

¹²⁶We assume that intermediates are nontradable, so that producers use only intermediates of their own country.

subject to the production function,

$$\min_{\{N_{jt}\}} \left(\frac{W_t}{P_{H,t}} \right) N_{jt} + \varphi_t (X_{jt} - Z_t N_{jt}), \quad (109)$$

what directly implies that φ_t is equal to the firm's real marginal production costs. Furthermore, the first order conditions give

$$\varphi_t = \frac{W_t}{P_{H,t} Z_t} = \frac{W_t S_t^{\varpi_H}}{k_H P_t Z_t}, \quad (110)$$

so that real marginal costs equal the producer price adjusted real wage divided by the the marginal product of labor. According to (110) a rise in domestic producer prices is accompanied by a fall in domestic real marginal costs.

Price adjustment is assumed to follow the same variant of Calvo's price stickiness due to Galí and Gertler (1999) as in 6.1.2. A randomly selected fraction of firms $(1 - \omega)$ adjusts prices while the remaining fraction of firms ω does not adjust. In addition, a fraction of $(1 - \tau)$ firms behaves in a forward-looking way and the remaining fraction τ uses the recent history of the aggregate price index when they set prices. Thus τ is a measure of the degree of backward-looking price-setting.

For the forward-looking fraction of firms the optimization problem has the same form as (56) in 6.1.2,

$$\max_{\{P_{H,jt}\}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\frac{P_{H,jt}}{P_{H,t+i}} X_{jt+i} - \varphi_{t+i} X_{jt+i} \right], \quad (111)$$

and leads to

$$\left(\frac{p_{H,t}^{fl}}{P_{H,t}} \right) = \mu^* \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \varphi_{t+i} \frac{P_{t+i}}{P_{H,t}}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \frac{P_{t+i}}{P_{H,t+i}}}, \quad (112)$$

where $\mu^* = \left(\frac{\varepsilon}{\varepsilon-1} \right)$ and differs from the parameter μ , defined in 6.1.2, as it does not depend on the price elasticity of demand for consumption goods but for intermediates used by the final goods producer.

Defining the domestic index for the prices newly set in period t ($\bar{p}_{H,t}^n$) as a weighted average of the forward- and backward-looking prices, and assuming the same rule of thumb as in 6.1.2 for the backward-looking price-setters ($p_{H,t}^{bl}$), equations similar to (59) - (61) can be derived:

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1 - \omega) (\bar{p}_{H,t}^n)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (113)$$

$$\bar{p}_{H,t}^n = (1 - \tau) p_{H,t}^{fl} + \tau p_{H,t}^{bl}, \quad (114)$$

$$p_{H,t}^{bl} = \bar{p}_{H,t-1}^n + \pi_{H,t-1}, \quad (115)$$

where $\pi_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1}$ represents the domestic producer price inflation.

8.1.3 Equilibrium output and trade balance

In equilibrium goods markets, money markets and the bond market all clear. Equating supply and demand for domestic and foreign goods, and using (96) and (97) gives $(1 - \gamma) X_t = (1 - \gamma) C_{H,t} + \gamma C_{H,t}^*$ and $\gamma X_t^* = (1 - \gamma) C_{F,t} + \gamma C_{F,t}^*$ and determines domestic and foreign output to be¹²⁷

$$X_t = \frac{P_t}{P_{H,t}} \left[(1 - \varpi_H) C_t + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) RER_t C_t^* \right], \quad (116)$$

and

$$X_t^* = \frac{P_t}{P_{F,t}} \left[\frac{1 - \gamma}{\gamma} \varpi_H C_t + \varpi_F RER_t C_t^* \right], \quad (117)$$

where we use (100). These equations show that a fall in the terms of trade leads to a rise in foreign output, and a drop in domestic, since relative prices change in favor of the foreign country.

Combining the Euler equations (103) of both countries with the uncovered interest rate parity (106) gives

$$E_t \left[\left(\frac{Q_{t+1} P_{t+1}^*}{P_{t+1}} \right) \left(\frac{C_{t+1} - h_C C_t}{C_{t+1}^* - h_C C_t^*} \right)^{-\sigma} \right] = Q_t \frac{P_t^*}{P_t} \left(\frac{C_t - h_C C_{t-1}}{C_t^* - h_C C_{t-1}^*} \right)^{-\sigma}. \quad (118)$$

If a variable is defined as $z_t \equiv \frac{Q_t P_t^*}{P_t} \left(\frac{C_t - h_C C_{t-1}}{C_t^* - h_C C_{t-1}^*} \right)^{-\sigma}$, then (118) gives $E_t z_{t+1} = z_t$. Linearizing z_t around a symmetric steady state $z = 1$, then implies $z_t = 1$ in any stable equilibrium, so that¹²⁸

$$\left(\frac{C_t - h_C C_{t-1}}{C_t^* - h_C C_{t-1}^*} \right)^{\sigma} = \frac{Q_t P_t^*}{P_t} = RER_t \quad (119)$$

According to (119) the real exchange rate balances the consumption differential across countries.

Due to the consumption habits and the home bias in consumption the model includes net foreign asset dynamics, which can be illustrated by the trade balance, defined as the value of exports minus imports denominated in the domestic currency $TB_t = \gamma P_{H,t} C_{H,t}^* - (1 - \gamma) P_{F,t} C_{F,t}$. Using (119) gives:

$$TB_t = P_t (\gamma (1 - \varpi_F) RER_t C_t^* - (1 - \gamma) \varpi_H C_t) \quad (120)$$

Without a home bias in consumption ($\varpi_H = 0, \varpi_F = 1$) $RER_t = 1$ and trade is balanced as in Clarida, Galí, and Gertler (2002). This also true for $\sigma = 1$ and without a habit formation in consumption ($h_c = 0$), since (119) is reduced to $RER_t C_t^* = C_t$ under these conditions, and $\varpi_H = \varpi = 1 - \varpi_F$.

¹²⁷Recall that all variables are measured in per-capita terms.

¹²⁸Without loss of generality we have to choose a particular value for z . See Batini, Levine, and Pearlman (2004) for details.

8.1.4 Linear approximations around a baseline equilibrium

As the model can be solved for an infinite number of steady states, we have to restrict the initial equilibrium. Without loss of generality we assume a baseline symmetric steady state with equal and constant consumption and a real exchange rate of value one, and therefore a balanced trade. Domestic and foreign Euler equations imply an equilibrium steady state interest rate $\bar{i} = \frac{1-\beta}{\beta}$. In the following we derive the key equations of the New Keynesian model by approximating preceding equations in percentage deviations from flexible price equilibrium. Similar to the approximation for the single-country a variable \hat{v} denotes the percentage deviation of a variable V around its steady state and the superscript f denotes the flexible-price steady state.

The hybrid New Keynesian Phillips Curve Since (111) has the same form as in 6.1.2 optimization leads to the familiar condition, equating the optimal price of a forward looking price setter ($\hat{p}_{H,t}^{fl}$) to the expected discounted value of future nominal marginal costs ($E_t(\hat{\varphi}_{t+i} + \hat{p}_{H,t+i})$):

$$\hat{p}_{H,t}^{fl} = (1 - \omega\delta) \sum_{i=0}^{\infty} \omega^i \delta^i E_t(\hat{\varphi}_{t+i} + \hat{p}_{H,t+i}). \quad (121)$$

Together with an approximation of (112) and (113) around a zero average inflation steady state using (114) and (115) this gives an expression for the aggregate domestic producer price inflation,

$$\pi_{H,t} = \kappa \hat{\varphi}_t + \omega\beta\phi E_t \pi_{H,t+1} + \tau\phi \pi_{H,t-1}, \quad (122)$$

where $\kappa = (1 - \tau)(1 - \omega)(1 - \delta\omega)\phi$ and $\phi = (\omega + \tau[1 - \omega(1 - \delta)])^{-1}$. Equation (122) is a hybrid version of the New Keynesian Phillips curve, similar to the one derived in 6.1.3. However, as for a two-country economy producer and consumer prices differ, the relevant price index is substituted. For κ the same arguments with respect to the impact of real marginal costs on inflation are valid as for the single-country NKPC.

Deriving a relationship between marginal costs and output for a two-country setting is somewhat more complicated than for a single-country economy, since domestic output is no longer equal to domestic consumption. Thus the influence of consumption and output on marginal costs represented by (105) and (110) leads to

$$\hat{\varphi}_t = \eta y_t + \frac{\sigma}{1 - h_c} (\hat{c}_t - h_C \hat{c}_{t-1}) + \varpi_H \hat{s}_t, \quad (123)$$

where we used $X_t = Z_t N_t$ and $y_t \equiv \hat{x}_t - \hat{x}_t^f$ is the output gap, that was already defined in 6.1.3 as the difference between the percentage deviations of output from its natural level and its flexible price equilibrium level. Since output does not equal consumption it is not possible to substitute \hat{c}_t as in the single-country analysis. Thus the effects of output and consumption enter (123) separately: η represents the influence of the work effort needed to produce output and $\frac{\sigma}{1-h_c}$ represents the impact of habit

adjusted consumption in the labor-leisure trade-off equation (105). An increase in output or consumption, respectively, raises the marginal rate of substitution between leisure and consumption, and the opportunity costs of holding leisure has to increase as well. Moreover, the terms of trade enter (123) due to the influence on real marginal costs and the real wage. An increase in \hat{s}_t represents a fall in $P_{H,t}$, and thus an increase in the PPI adjusted real marginal costs, or an increase in $P_{F,t}$, and thus a fall in the CPI adjusted real wages, provoking worker's to call for higher nominal wages, or both. A fall in $P_{H,t}$ has the opposite effect on real wages, but as relevant price indices for firm and household decisions differ, the decreasing effect on the real wage due to a fall in domestic prices can only partly compensate for the increase of the firms real marginal costs (the PPI adjusted real wage).

Implementing this in (122) leads to

$$\begin{aligned} \pi_{H,t} = & \kappa \left(\eta y_t + \frac{\sigma}{1 - h_c} (\hat{c}_t - h_C \hat{c}_{t-1}) + \varpi_H \hat{s}_t \right) \\ & + \phi (\omega \beta E_t \pi_{H,t+1} + \tau \pi_{H,t-1}). \end{aligned} \quad (124)$$

This New Keynesian Phillips Curve depends not only on output and inflation, but also on relative prices and consumption. Notice that, as domestic output depends also on foreign consumption, there is a link of the two-countries additional to the terms of trade relationship.

The foreign counterparts of (123) and (124) are given by

$$\hat{\varphi}_t^* = \eta y_t^* + \frac{\sigma}{1 - h_c} (\hat{c}_t^* - h_C \hat{c}_{t-1}^*) - (1 - \varpi_F) \hat{s}_t \quad (125)$$

and

$$\begin{aligned} \pi_{F,t}^* = & \kappa \left(\eta y_t^* + \frac{\sigma}{1 - h_c} (\hat{c}_t^* - h_C \hat{c}_{t-1}^*) - (1 - \varpi_F) \hat{s}_t \right) \\ & + \phi (\omega \beta E_t \pi_{F,t+1}^* + \tau \pi_{F,t-1}^*). \end{aligned} \quad (126)$$

The hybrid Euler equation After presenting the supply side equations for the model, the demand side is derived by linearizing the Euler equation (103):

$$\hat{c}_t = \left(\frac{1}{1 + h_C} \right) (E_t \hat{c}_{t+1} + h_C \hat{c}_{t-1}) - \sigma^{-1} \left(\frac{1 - h_c}{1 + h_C} \right) (\hat{i}_t - E_t \pi_{t+1}). \quad (127)$$

For the hybrid Euler equation the same implications follow as for the hybrid IS equation of 6.1.2. The measure of the persistence in the consumption habits h_c , is also a measure of the degree of persistence in consumption dynamics, and for $h_C = 0$ we derive a purely forward-looking variant of (127). Moreover, h_c has the same influence on the effectiveness of the real interest rate, as an increase in consumption habits decreases the habit adjusted inverse of the intertemporal elasticity of substitution $\left(\frac{\sigma}{1 - h_c} \right)$.

Domestic and foreign output paths The paths of domestic and foreign output are determined by linearizations of (116) and (117) together with the equilibrium relationship between the real exchange rate and the terms of trade (100):

$$y_t = \alpha_{H,c} \hat{c}_t + \alpha_{H,c^*} \hat{c}_t^* + \alpha_{H,s} \hat{s}_t, \quad (128)$$

$$y_t^* = \alpha_{F,c} \hat{c}_t + \alpha_{F,c^*} \hat{c}_t^* - \alpha_{F,s} \hat{s}_t, \quad (129)$$

where

$$\begin{aligned} \alpha_1 &= (1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F), \alpha_2 = \frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F, \\ \alpha_{H,c} &= \frac{1 - \varpi_H}{\alpha_1}, \alpha_{H,c^*} = \frac{\frac{\gamma}{1 - \gamma} (1 - \varpi_F)}{\alpha_1}, \alpha_{H,s} = \frac{(1 - \varpi_H) \varpi_H + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) \varpi_F}{\alpha_1}, \\ \alpha_{F,c} &= \frac{\frac{1 - \gamma}{\gamma} \varpi_H}{\alpha_2}, \alpha_{F,c^*} = \frac{\varpi_F}{\alpha_2}, \alpha_{F,s} = \frac{\frac{1 - \gamma}{\gamma} \varpi_H (1 - \varpi_H) + \varpi_F (1 - \varpi_F)}{\alpha_2} \end{aligned}$$

For equally sized countries ($\gamma = \frac{1}{2}$) $\alpha_{H,s} = \alpha_{F,s} = 2\varpi(1 - \varpi)$ and (128) and (129) are exactly the same as in Batini, Levine, and Pearlman (2004). Furthermore, since $\alpha_{H,s} > 0$ and $\alpha_{F,s} > 0$, a rise in the price for domestic goods relative to those produced in the foreign country (a fall in the terms of trade) triggers the households to switch their expenditures from domestic to foreign products. As a consequence domestic output falls, whereas foreign output rises. As $\frac{\partial \varpi_H}{\partial \varpi} \geq 0$ and $\frac{\partial \varpi_F}{\partial \varpi} \leq 0$ a stronger home bias (a smaller value for ϖ) implies smaller expenditure switching (smaller values for $\alpha_{H,s}$ and $\alpha_{F,s}$). The more households prefer goods which are produced in their own country, the less willing they are to switch to products from the other country.

There are two interesting special cases. The first is the case of a full home bias ($\varpi = 0$, $\varpi_H = 0$, $\varpi_F = 1$). Since both countries live in complete autarky under this scenario, $y_t = c_t$ and $y_t^* = c_t^*$. Without any trading, the expenditure switching effect vanishes and the terms of trade are irrelevant for the determination of the output paths. The second one is the case of no home bias ($\varpi = \frac{1}{2}$, $\varpi_H = \gamma = \varpi_F$), which corresponds to full openness. Since we assume a baseline scenario with constant and equal consumption, (128) and (129) are reduced to

$$y_t = \hat{c}_t + \gamma \hat{s}_t, \quad (130)$$

and

$$y_t^* = \hat{c}_t^* - (1 - \gamma) \hat{s}_t, \quad (131)$$

and incorporate exactly the same country-size adjusted expenditure switching effect as in Clarida, Galí, and Gertler (2002).

Equilibrium terms of trade Since we assume equal and constant consumption, and a real exchange rate of value one, using (96) and (97) to solve (116) and (117) for S_t gives

$$S_t = \left(\frac{\frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F}{(1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F)} \right) \frac{X_t}{X_t^*} = \left(\frac{\gamma(1 - \varpi) + (1 - \gamma)\varpi}{(1 - \gamma)(1 - \varpi) + \gamma\varpi} \right) \frac{X_t}{X_t^*}. \quad (132)$$

According to (132), equilibrium terms of trade depend positively on the output differential $\frac{X_t}{X_t^*}$, indicating the plausible inverse relationship between prices and output of the two countries. When the equilibrium ratio of foreign to domestic prices falls (increases) the equilibrium ratio of foreign to domestic output increases (falls). Notice that without home bias ($\varpi = \frac{1}{2}$, $\varpi_H = 0$ and $\varpi_F = 1$) this equation is reduced to $S_t = \frac{X_t}{X_t^*}$ like in Clarida, Galí, and Gertler (2002).¹²⁹ Furthermore, the same is true for $\gamma = \frac{1}{2}$ (implying $\varpi_H = \varpi$ and $\varpi_F = 1 - \varpi$) since for an equal home bias and equally sized countries the effect is the same. Only if countries differ in their size, a home bias, which is assumed to be equal in both countries, improves (132) in favor of the bigger country. Consider for example the case where the domestic is bigger than the foreign country ($\gamma < \frac{1}{2}$). Then the coefficient in front of $\frac{X_t}{X_t^*}$ is less than one, and every increase in the equilibrium terms of trade (a fall in $\bar{P}_{H,t}$ or a rise in $P_{F,t}$) is accompanied by a stronger increase in $\frac{X_t}{X_t^*}$ than under equally sized countries. As the demand for products of the bigger country is greater, the strength of the inverse relationship between equilibrium prices and output increases from the perspective of the bigger country. Approximating (132) gives

$$\hat{s}_t = y_t - y_t^*. \quad (133)$$

Complete model dynamics The model dynamics are finally completed by linearizations of the terms of trade, the real exchange rate, the trade balance, and the uncovered interest rate parity.¹³⁰

$$\begin{aligned} y_t &= \alpha_{H,c} c_t + \alpha_{H,c^*} c_t^* + \alpha_{H,s} s_t, \\ y_t^* &= \alpha_{F,c} \hat{c}_t + \alpha_{F,c^*} c_t^* - \alpha_{F,s} s_t, \\ c_t &= \left(\frac{1}{1+h_C} \right) (E_t c_{t+1} + h_C c_{t-1}) - \sigma^{-1} \left(\frac{1-h_c}{1+h_C} \right) (i_t - E_t \pi_{t+1}) + e_{c_t}, \\ c_t^* &= \left(\frac{1}{1+h_C} \right) (E_t c_{t+1}^* + h_C c_{t-1}^*) - \sigma^{-1} \left(\frac{1-h_c}{1+h_C} \right) (i_t^* - E_t \pi_{t+1}^*) + e_{c_t^*}, \\ \pi_{H,t} &= \kappa \left(\eta y_t + \frac{\sigma}{1-h_c} (c_t - h_C c_{t-1}) + \varpi_H s_t \right) \\ &\quad + \phi (\omega \beta E_t \pi_{H,t+1} + \tau \pi_{H,t-1}) + e_{\pi_{H,t}}, \\ \pi_{F,t}^* &= \kappa \left(\eta y_t^* + \frac{\sigma}{1-h_c} (c_t^* - h_C c_{t-1}^*) - (1 - \varpi_F) s_t \right) \\ &\quad + \phi (\omega \beta E_t \pi_{F,t+1}^* + \tau \pi_{F,t-1}^*) + e_{\pi_{F,t}^*}, \\ q_t &= E_t q_{t+1} + (i_t^* - i_t) + e_{q_t}, \\ s_t &= s_{t-1} + \pi_{F,t}^* + \Delta q_t - \pi_{H,t}, \end{aligned}$$

¹²⁹Recall that output is measured in per-capita terms.

¹³⁰For convenience we use ν instead of $\hat{\nu}$ for all variables.

$$\begin{aligned}
\pi_t &= (1 - \varpi_H) \pi_{H,t} + \varpi_H (\pi_{F,t}^* + \Delta q_t), \\
\pi_t^* &= (1 - \varpi_F) (\pi_{H,t} - \Delta q_t) + \varpi_F \pi_{F,t}^*, \\
rer_t &= (\varpi_F - \varpi_H) s_t, \\
tb_t &= tb_{t-1} + \pi_t + \frac{\gamma (1 - \varpi_F)}{\gamma (1 - \varpi_F) - (1 - \gamma) \varpi_H} (\Delta rer_t + \Delta c_t^*) \\
&\quad - \frac{(1 - \gamma) \varpi_H}{\gamma (1 - \varpi_F) - (1 - \gamma) \varpi_H} \Delta c_t,
\end{aligned}$$

where we add four general autocorrelated and orthogonal demand and supply shocks, and an exchange rate shock, which are assumed to be serially uncorrelated,

$$\begin{aligned}
e_{ct} &= \rho_c e_{ct-1} + \xi_{ct}, \quad \xi_{ct} \sim N(0, \sigma_c), \\
e_{\pi_{Ht}} &= \rho_{\pi_H} e_{\pi_{Ht-1}} + \xi_{\pi_{Ht}}, \quad \xi_{\pi_{Ht}} \sim N(0, \sigma_{\pi_H}), \\
e_{c_t^*} &= \rho_{c^*} e_{c_{t-1}^*} + \xi_{c_t^*}, \quad \xi_{c_t^*} \sim N(0, \sigma_{c^*}), \\
e_{\pi_{Ft}^*} &= \rho_{\pi_F^*} e_{\pi_{Ft-1}^*} + \xi_{\pi_{Ft}^*}, \quad \xi_{\pi_{Ft}^*} \sim N(0, \sigma_{\pi_F}), \\
e_{qt} &= \rho_{e_{qt}} e_{qt-1} + \varepsilon_{e_{qt}}, \quad \varepsilon_{e_{qt}} \sim N(0, \sigma_{e_{qt}}),
\end{aligned}$$

and coefficients are given by

$$\begin{aligned}
\kappa &= (1 - \tau) (1 - \omega) (1 - \beta \omega) \phi, \quad \phi = [\omega + \tau (1 - \omega (1 - \beta))]^{-1}, \\
\alpha_1 &= (1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F), \quad \alpha_2 = \frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F, \\
\alpha_{H,c} &= \frac{1 - \varpi_H}{\alpha_1}, \quad \alpha_{H,c^*} = \frac{\frac{\gamma}{1 - \gamma} (1 - \varpi_F)}{\alpha_1}, \quad \alpha_{H,s} = \frac{(1 - \varpi_H) \varpi_H + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) \varpi_F}{\alpha_1}, \\
\alpha_{F,c} &= \frac{\frac{1 - \gamma}{\gamma} \varpi_H}{\alpha_2}, \quad \alpha_{F,c^*} = \frac{\varpi_F}{\alpha_2}, \quad \alpha_{F,s} = \frac{\frac{1 - \gamma}{\gamma} \varpi_H (1 - \varpi_H) + \varpi_F (1 - \varpi_F)}{\alpha_2}, \\
\varpi_H &= \frac{\gamma \varpi}{\gamma \varpi + (1 - \gamma) (1 - \varpi)}, \quad \varpi_F = \frac{\gamma (1 - \varpi)}{\gamma (1 - \varpi) + (1 - \gamma) \varpi}.
\end{aligned}$$

8.1.5 State space representation

Assuming a social planner whose aim is to minimize the variance of world inflation $\pi_t^W = (1 - \gamma) \pi_t + \gamma \pi_t^*$ and who has a preference for a stabilization of world output $Y_t^W = Y_t^{(1-\gamma)} (Y_t^*)^\gamma$, and stabilization and smoothing of the world interest rate $i_t^W = (1 - \gamma) i_t + \gamma i_t^*$, the world loss function is given by¹³¹

$$L^W = E_t \sum_{i=0}^{\infty} \beta^i \left((\pi_t^W)^2 + \lambda_c (c_t^W)^2 + \lambda_i (i_t^W)^2 + \lambda_{\Delta i} (i_t^W - i_{t-1}^W)^2 \right), \quad (134)$$

¹³¹World inflation can be derived by approximating world prices $P_t^W = P_t^{(1-\gamma)} + (P_t^*)^\gamma$. The world interest rate follows from combining both Euler equations to world consumption. Recall that the interest rate is measured in percentage deviations from equilibrium.

where we use the fact that world output equals world demand in equilibrium, $C_t^W = C_t^{(1-\gamma)} (C_t^*)^\gamma$, and where the λ_i 's represent the corresponding weights, defining the preferences of the countries or the social planner, respectively. As our focus is on the influence of uncertainty and persistence we do not investigate possible gains from coordination.¹³² However, solving the world loss function might also be interpreted as cooperation of both countries, where the country size is used as a measure of bargaining power. Notice that this loss function implicitly includes the exchange rate as $\pi_t = (1 - \varpi_H) \pi_{H,t} + \varpi_H (\pi_{F,t}^* + \Delta q_t)$ and $\pi_t^* = (1 - \varpi_F) (\pi_{H,t} - \Delta q_t) + \varpi_F \pi_{F,t}^*$.

All equations can be combined in the appropriate state-space form,

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B u_t + C \varepsilon_{t+1}, \quad (135)$$

where $x_{1t} = (e_{c_t}, e_{c_t^*}, e_{\pi_{H,t}}, e_{\pi_{F,t}}, e_q, i_{t-1}, i_{t-1}^*, c_{t-1}, c_{t-1}^*, \pi_{H,t-1}, \pi_{F,t-1}^*, q_{t-1}, s_{t-1})'$ is the vector of predetermined variables (x_{10} given), $x_{2t} = (c_t, c_t^*, \pi_{H,t}, \pi_{F,t}^*, q_t)'$ is the vector of forward-looking variables, $u_t = (i_t, i_t^*)'$ is a vector of control variables, and $\{\varepsilon_{t+1} = (\varepsilon_{c_{t+1}}, \varepsilon_{c_{t+1}^*}, \varepsilon_{\pi_{H,t+1}}, \varepsilon_{\pi_{F,t+1}}) : t = 1, 2, \dots\}$ is a conditionally homoskedastic Gaussian martingale differences process, which satisfies $E_t(\varepsilon_{t+1}) = 0$ and $E_t(\varepsilon_{t+1}' \varepsilon_{t+1}) = I$. A , B and C represent appropriate matrices, filled with the corresponding structural parameters. The optimal reaction function $i_t = -F x_t$ can now be derived by minimizing

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + 2x_t' U u_t + u_t' R u_t) \right], \quad (136)$$

subject to (135), where Q , R and U are appropriate matrices, filled with the corresponding preferences of the two countries.

8.2 Calibration and results

The calibration of the two-country model is given in Table 8. For reasons of comparability, the same values are taken from Giannoni (2002) as for the single-country economy. For the parameter representing the home bias in consumption we choose a value of 0.2, which refers to a median home-bias (recall that $\varpi = 0.5$ represents full integration). Thus the two countries could be interpreted as two large currency unions which have a slightly greater amount of within-trade than trade with each other. This seems to be a reasonable value if we are talking about currency unions like the U.S. and Europe.

The following sections show robust and non-robust losses and impulse responses for an authority under commitment or discretion, respectively. Unfortunately we were unable to find a Taylor-rule, where the social planner sets both interest rates according to world inflation, output, and the world interest rate, that leads to one

¹³²See Batini, Justiniano, Levine, and Pearlman (2005) for an empirical or Pappa (2004) for a recent theoretical investigation of possible gains from cooperation.

parameter		scenario				
h_c	degree of persistence in habit formation	0	0.4	0	0.4	0.8
τ	degree of backwardness in price setting	0	0	0.4	0.4	0.8
ω	degree of nominal price rigidity	0.8598				
σ	inv. of consumption elasticity	0.1571				
η	inv. of labor elasticity	0.824				
γ	size of the foreign country	0.7				
ϖ	degree of integration	0.2				
β	discount factor (quarterly periods)	0.99				
ρ_i	AR parameter of shocks	0.35				
σ_i	standard deviation of shocks	1				

Table 8: Calibration, two-country economy

single equilibrium. Due to the model's complexity every simple rule leads to indeterminacy, and any expected inflation can be justified. This result is clearly no argument in favor of using a simple rule. As the results for the discretionary solution are much more straightforward to interpret, the next section starts with this case.

8.2.1 Discretion

The results of the model for a discretionary policy maker are given in Table 9 and confirm all results of the single-country analysis under discretion.¹³³ Persistence in consumption habits seems to have no significant impact, whereas persistence in price-setting behavior clearly leads to an immense increase in losses due to the lowered effectiveness of monetary policy. Furthermore, a discretionary policy maker deals with uncertainty quite easily, as long as private sector behavior is not too persistent, since losses of the robust strategy in the worst case model are close to the optimal solutions even for $p(\theta) = 10\%$. In addition, a central banker seems to be no fool when using Hansen-Sargent robust rules, as the losses of an unfounded fear against misspecification do also not increase substantially, except for the extremely unrealistic case of $h_c = \tau = 0.8$. A discretionary central banker reoptimizes every period and thus he can react on unforeseeable events. Table 9 suggests that this makes the discretionary solution quite robust against model misspecification.

In addition, Table 9 validates the finding that the choice of the reference model is more important than a general model uncertainty given this particular model. Introducing a small degree of persistence in price-setting behavior increases losses to greater extent than a worst case scenario with $p(\theta) = 10\%$.

¹³³For the commitment solution we have to assume for a minimum value of 0.02 for the preference on output-gap stabilization as otherwise algorithms fail. The generalized Schur-decomposition needed to solve the model leads to a matrix Z which is close to singular. For reasons of comparability a minimum value of 0.02 is chosen also for the discretionary case, which does not change the outcomes considerably.

$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.02, 0.3, 0)$			$(0.0483, 0.2364, 0)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	164	193,200	266,291	163	192,199	241,272
$h_c = 0.4, \tau = 0$	166	196,203	235,259	165	194,201	238,268
$\tau = 0.4, h_c = 0$	716	999,1019	2594,2640	680	964,983	1151,1018
$h_c = \tau = 0.4$	747	963,969	2332,2379	698	987,1006	1964,2032
$h_c = \tau = 0.8$	13413	15950,13313	52401,23307	15159	35255,32795	59768,34226
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.1, 0.5, 0)$			$(0.5, 0, 0)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	171	203,212	241,266	159	185,192	230,258
$h_c = 0.4, \tau = 0$	173	206,216	245,275	159	185,192	255,291
$\tau = 0.4, h_c = 0$	779	1098,1124	1841,1747	604	839,850	1580,1611
$h_c = \tau = 0.4$	798	1123,1148	2184,2096	605	839,850	1597,1643
$h_c = \tau = 0.8$	33309	66101,65900	325130,373380	6648	13218,10923	67901,20866
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.02, 0.6, 0)$			$(0.5, 0, 0.3)$		
$p(\theta)$	50%	35%	10%	50%	35%	10%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	176	210,219	347,416	161	183,191	244,283
$h_c = 0.4, \tau = 0$	178	214,224	313,324	160	182,189	243,280
$\tau = 0.4, h_c = 0$	893	1283,1315	2419,2509	563	697,710	1100,1170
$h_c = \tau = 0.4$	932	1472,1514	2837,2649	556	690,702	1201,1267
$h_c = \tau = 0.8$	21015	76859,52846	97749,148360	3405	5662,5510	24129,14985

Table 9: Losses under discretion, two-country economy

Scenario 1 ($h_c = \tau = 0$) The impulse responses for the purely forward-looking variant of the model are given in Figures 10 - 14 for a discretionary policy maker and a robustness parameter θ , which corresponds to a detection error probability $p(\theta) \approx 10\%$.

After a domestic supply innovation, both central banker raise their interest rates to dampen the pressure on world inflation. As the domestic interest rate is risen more sharply, foreign households buy more domestic bonds and, as a consequence, the domestic currency appreciates and q falls. The rise in the domestic interest rate implies a positive domestic real rate which causes domestic consumption to fall. In contrast, the foreign interest rate is not risen sharply enough to generate a positive real rate, and thus foreign consumption increases. Due to the home bias in consumption, the consumption dynamics are reflected by the output reactions, strengthened by the fall in the terms of trade. The rise in foreign consumption and output raises the foreign producer price inflation and thus world inflation. The dynamics of output and consumption indices illustrate that domestic households

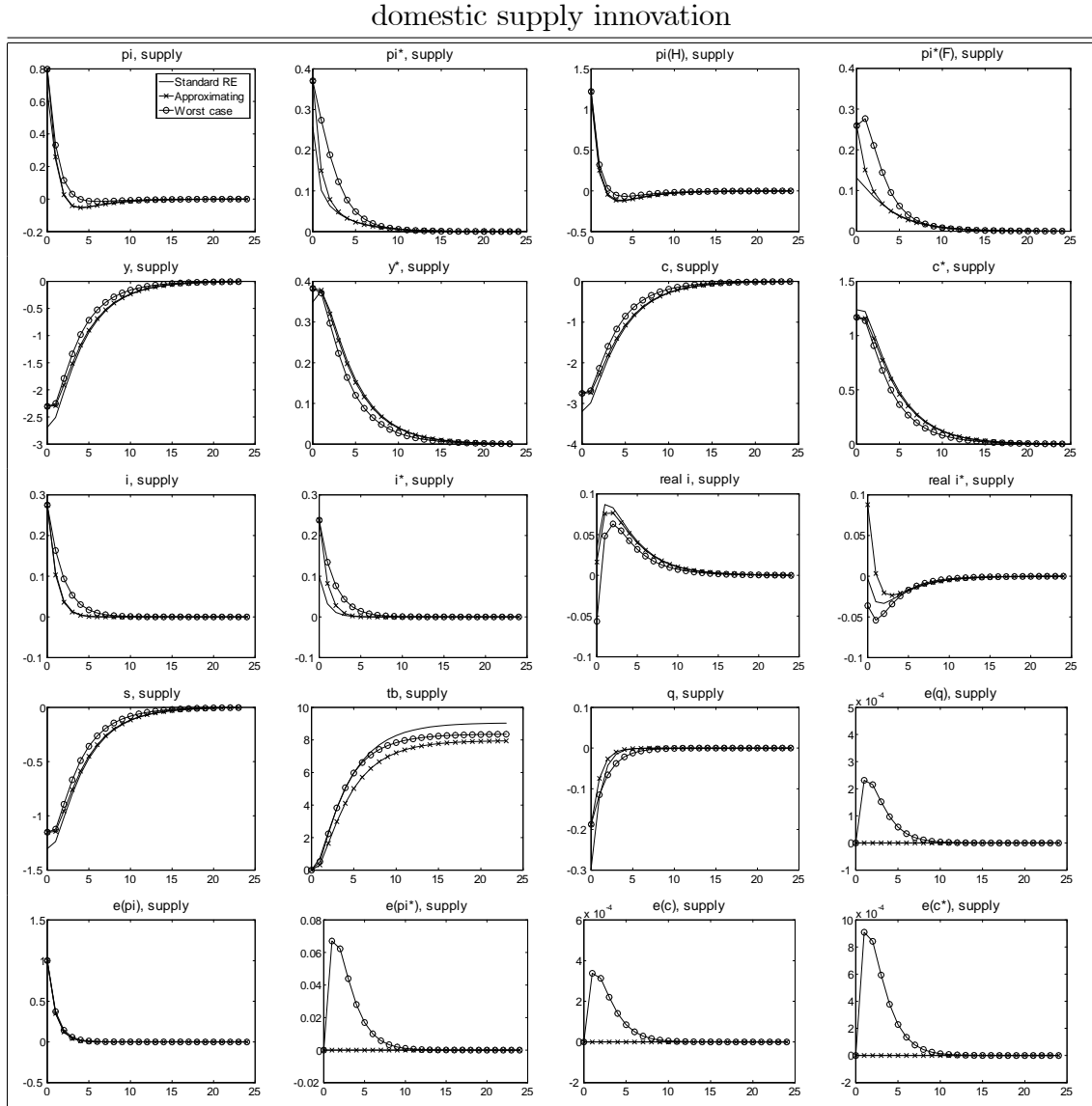


Figure 10: IRFs, discretion, domestic supply innovation, scenario 1

produce more than they consume, indicating that the relative value of domestic exports to domestic imports increases and the trade balance becomes positive. The fall in domestic demand is partly compensated by the foreign country and overweighs effects from expenditure switching due to the relative change in prices.

According to Figure 10 a robust social planner would not raise the domestic interest rate more aggressively, but raises the foreign one. This seems to be plausible, since the greatest fear of the policy maker is represented by a positive foreign supply shock, which would raise foreign and thus world inflation. As foreign inflation can be controlled best by a stronger increase in the foreign interest rate, a robust foreign central banker increases the interest rate more aggressively. As a consequence, the exchange rate appreciates less and the reaction of the terms of trade, as well as the impact of the terms of trade on output indices, decreases. In addition the imaginary evil agent, representing the policy maker's fear about misspecification, causes

small domestic and foreign demand shocks and an exchange rate shock in order to raise world inflation. As a result foreign producer price inflation and domestic and foreign consumer price inflation rates increase more strongly and more persistently. The stronger increase in π_F^* further dampens the fall in the terms of trade and domestic producer price inflation increases a little more strongly because of the higher domestic output. The higher domestic inflation rate makes the domestic real interest rate negative in the first period which dampens the fall in consumption. Due to the strong rise in foreign producer price inflation the foreign real rate becomes even more negative under uncertainty, although the nominal rate is risen more sharply. The effects on consumption are reflected by the output gaps as well. As the evil agent generates slightly positive demand shocks, the differences between consumption and production decrease and the trade balance response is smaller in the worst case scenario.

Furthermore Figure 10 validates the suggestions from Table 9 that an unfounded use of robust expectations and a robust policy does not change the outcomes substantially. The (robust) expected rise in foreign producer price inflation results, in fact, in an increase in inflation, but the strong foreign interest rate reaction corrects this effect very quickly. All other responses are straightforward. In the end the increase in the trade balance is smaller as the exchange rate returns faster to equilibrium, and thus the deviation of the terms of trade are smaller. This implies smaller deviations of domestic and foreign output, and thus smaller output-consumption differentials, since foreign demand for domestic products decreases. The exchange rate response follows directly from the faster fall in the foreign interest rate, when the evil agent is absent. Thus a preference for robustness of central bankers and private sectors has an impact on equilibrium trade balance, independent from the actual appearance of the evil agent.

The responses after a foreign supply innovation are illustrated in Figure 11. Similar to the domestic supply shock, both interest rates are risen. However, even in the case of a foreign shock, the domestic rate increases more, since the optimal social planner wants the exchange rate to fall in order to dampen deviations of the terms of trade from equilibrium. As the origin of the innovation lies in the bigger country it pays to counteract the terms of trade response, whereas for the domestic shock it does not as the increase in π_H has a smaller impact on world inflation. The dynamics of all inflation rates, as well as those of both output gaps and consumption indices mirror those from the domestic innovation, except that they are adjusted for the hump-shaped terms of trade response. In addition, as the foreign interest rate is risen to a lesser extent in order to decrease the exchange rate the foreign real rate falls slightly, enforcing the return of consumption to equilibrium, whereas the domestic real rate becomes positive. However, as domestic (foreign) households expect an increase (fall) in output, consumption expectations are positive (negative) and cause a boom (fall) in domestic (foreign) consumption nevertheless. Since the fall in foreign demand is partly compensated by domestic households, foreign workers produce more than they consume and one would expect the trade balance to fall. On

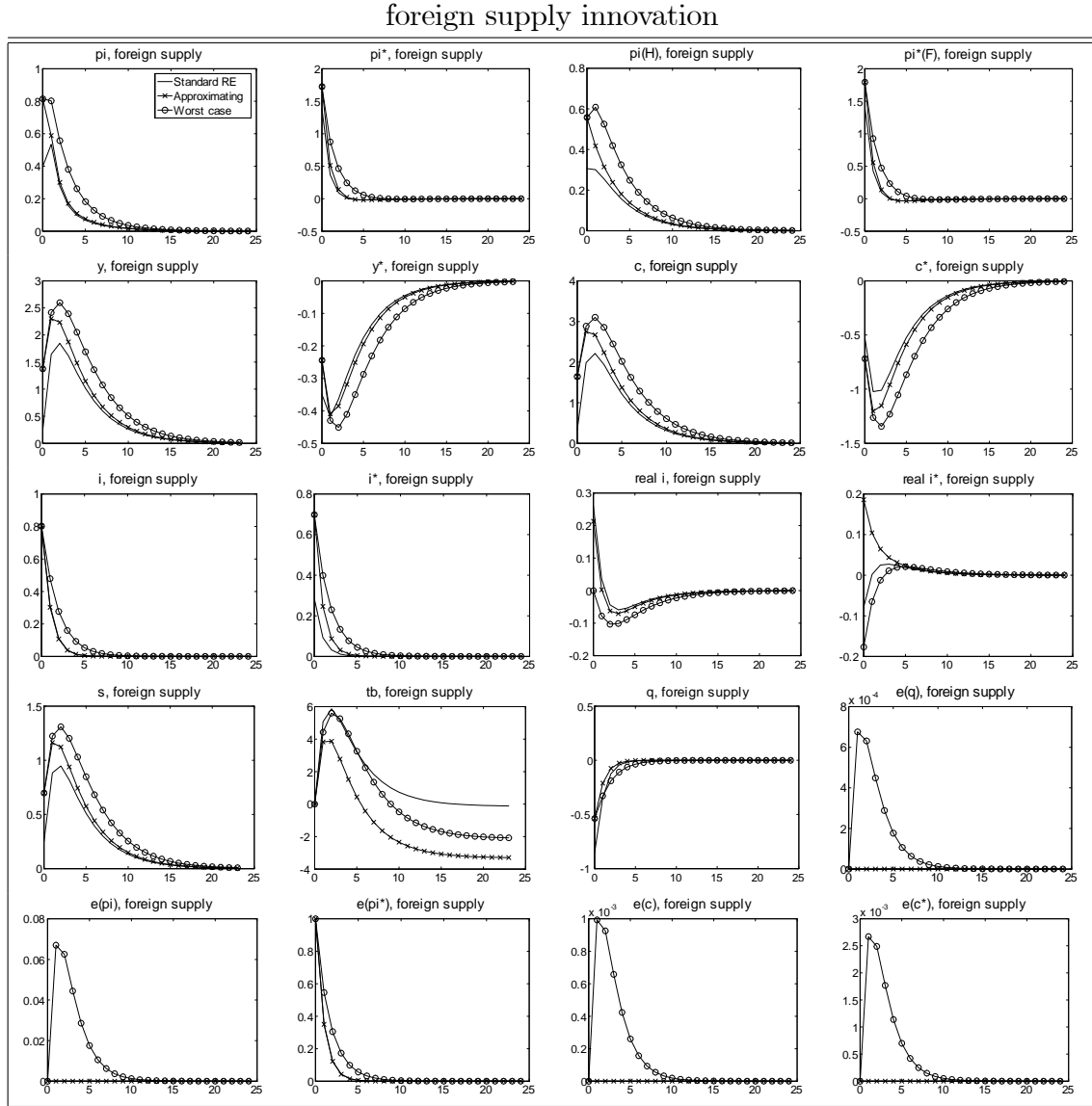


Figure 11: IRFs, discretion, foreign supply innovation, scenario 1

the other hand, expenditure switching implies an increase in domestic exports. As the innovation origin lies in the bigger country this effect initially overweighs simply since foreign households are of a greater quantity, and thus the country-size adjusted expenditure switching is greater. The relative value of domestic exports increases first, but with decreasing terms of trade this effect vanishes and the dynamics show the expected fall. In the long run equilibrium trade is more or less balanced.

For the robust dynamics in the worst case the reactions illustrate the same changes as was the case for the domestic innovation. The interest rate of the bigger country is risen more aggressively in order to dampen the stronger responses of domestic and foreign supply shocks due to the distortion of the evil agent. Due to the greater effect on world inflation the malevolent player increases the foreign supply shock, which could be controlled best by rising the foreign interest rate. Similar to the domestic case he further brings about an increase in both demand shocks, in order

to increase inflation, and increases the exchange rate shock to counteract on the policy maker's intention to decrease the distortions in the terms of trade. As a consequence all inflation rates rise more aggressively and the domestic and foreign real interest rates fall. Now the domestic real rate also becomes negative initially, and domestic consumption increases much stronger, leading to a further increase in domestic inflation as well. However, as the real rate becomes positive, consumption and inflation decrease again. Furthermore, the very strong increase in domestic consumption partly compensates the foreign downturn and outweighs the expenditure switching effect implied by the terms of trade dynamics. Thus the trade balance turns into negative territory.

Moreover, a policy maker using the robust rule in a model without evil agent seems to behave wisely as the first impact of the robust expectations results in reactions of all other state variables equal to the worst case, but the more aggressive foreign interest rate response makes their dynamics converge very quickly to the optimal *RE* response. As the expected inflation rates are smaller in the absence of the evil agent, both real interest rates become positive in the beginning. Equilibrium trade balance becomes more negative due to the much smaller terms of trade response.

The dynamics following domestic and foreign demand innovations are shown in Figures 12 and 13 and confirm the conclusion from the single-country analysis regarding the relative distortion of demand to supply innovations. The deviations of all variables are much smaller than for both supply innovations, since these would imply opposing reactions of consumption and inflation, which clearly complicates the optimal reaction of the policy maker. A domestic demand innovation leads to a rise in the domestic producer price inflation and increases world inflation. Thus both central bankers react by increasing the interest rate, where the interest rate of the foreign country is risen more aggressively in order to depreciate the exchange rate and dampen the fall in the terms of trade, as this tends to increase foreign producer price inflation due to its effect on marginal costs.¹³⁴ As a consequence deviations of foreign producer and consumer price inflation turn positive first when the terms of trade reach their peak. Both real rates become positive and foreign consumption decreases. The responses of domestic and foreign consumption are reflected by both output gaps. Since part of the domestic demand is for foreign products, domestic consumption lies above production and the trade balance falls.

For the foreign demand innovation the dynamics of all inflation and consumption indices mirror those of the domestic one. However, similar to the response after a domestic innovation the foreign interest rate increases more strongly, resulting in an increasing exchange rate. This strengthens the effect on the terms of trade

¹³⁴Similar to the case of a foreign supply innovation, the interest rate of the country where the shock does not originate increases the interest rate stronger to influence exchange rate dynamics. However, note that for a domestic demand shock, the reaction of foreign PPI is a consequence of the interest rate response, whereas for the foreign supply innovation the increase in PPI is the impulse.

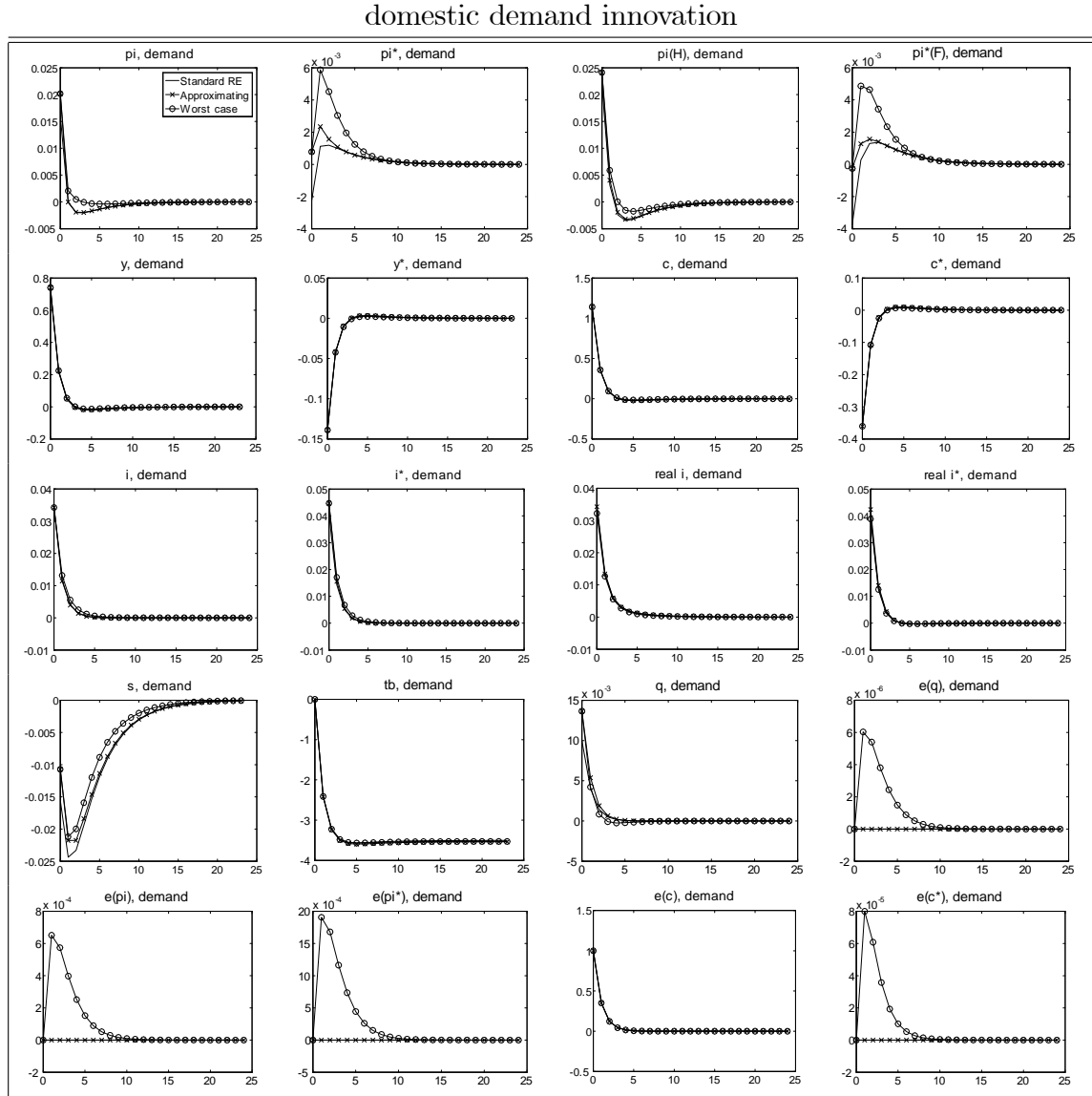


Figure 12: IRFs, discretion, domestic demand innovation, scenario 1

and dampens the inflationary pressure on foreign producer price inflation. As a proportion of the demand of foreign households falls on domestic goods, the increase in foreign consumption is greater than the increase in production and the trade balance becomes positive from a domestic perspective.

Regarding the worst case dynamics Figures 12 and 13 suggest that robust agents do not fear strong misspecifications when the economy is hit by demand innovations. For both innovations the misspecification feared most is represented by very small increases in all shocks in order to increase world inflation. The robust central bankers show no reaction, but interest rate dynamics become slightly more persistent due to the stronger responses of inflation rates. In the case of unfounded fear robust expectations overestimate inflation compared to the *RE* solution, but even for the same interest rate response as in the *RE* solution all variables converge very quickly to the optimal solution. Since the reactions of consumption and output are virtually

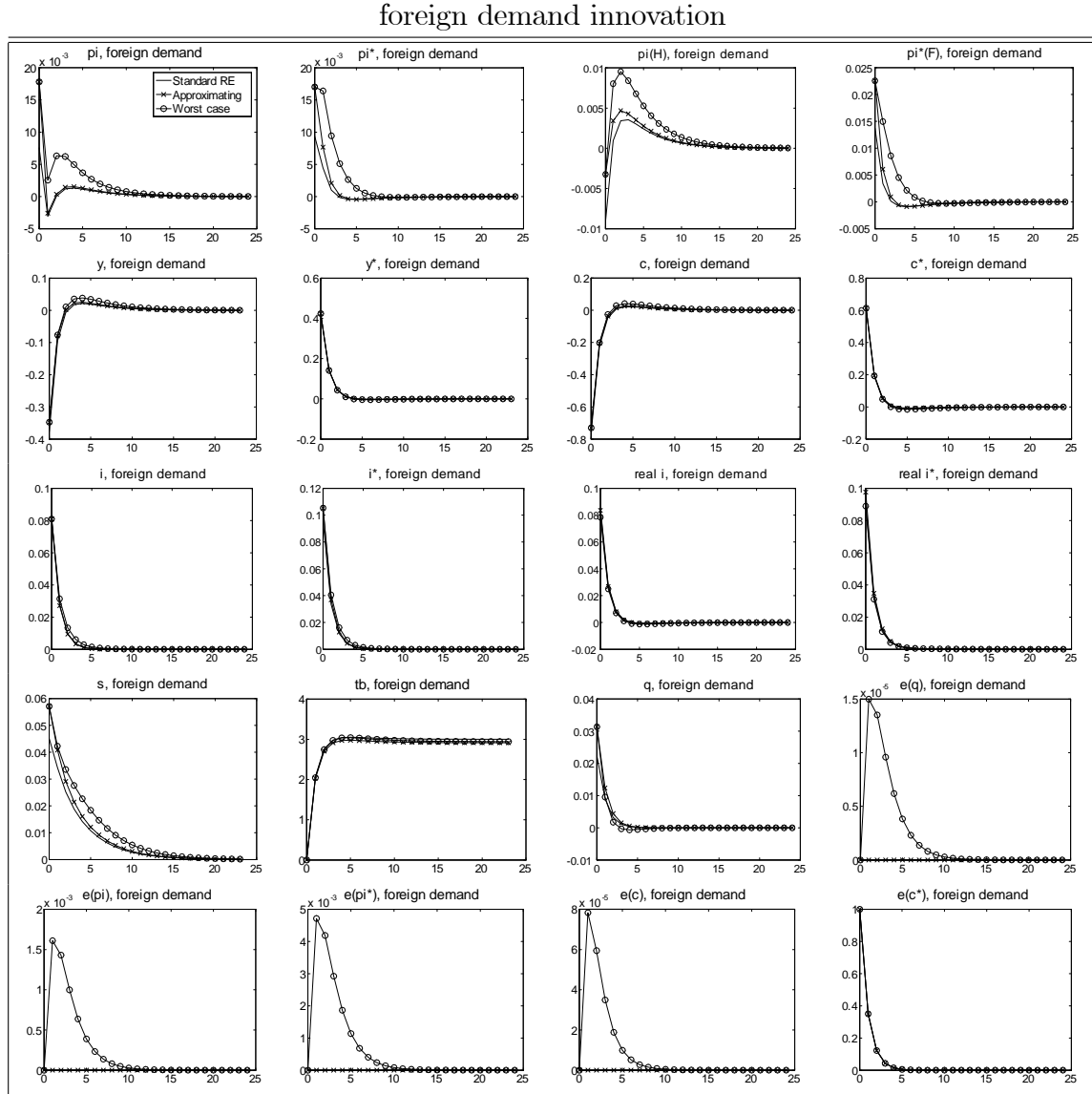


Figure 13: IRFs, discretion, foreign demand innovation, scenario 1

unaffected in both countries trade balance dynamics do also not change considerably.

Impulse responses after an exchange rate innovation are given in figure 14. Surprisingly, the innovation does not lead to an increase in the exchange rate, as it is completely offset by the interest rate differential generated due to a sharp increase in the domestic and a fall in the foreign interest rate. Inflation expectations imply an immediate fall in domestic and a rise in foreign inflation indices, leading to an increase in the terms of trade. The real interest rates reflect the reactions of their nominal counterparts, and as a consequence domestic consumption and output falls, whereas the corresponding foreign indices rise. As the fall in domestic output is partly compensated by foreign demand, the trade balance becomes positive.

The robust dynamics lead to the conclusion that the authority's fear of misspecification does not lead to any change in the optimal interest rate responses after an exchange rate innovation. In fact only the exchange rate shows an identifiable

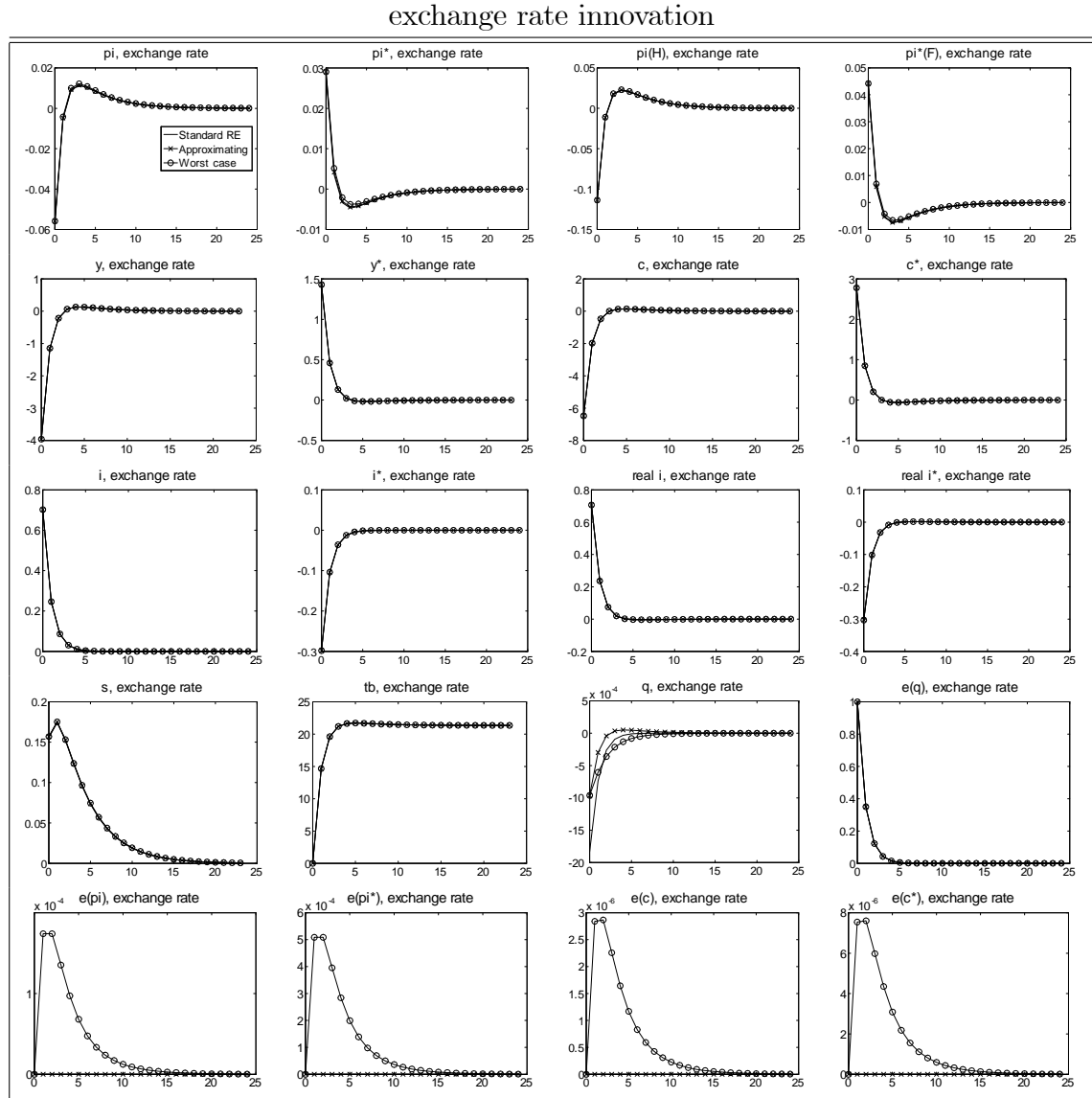


Figure 14: IRFs, discretion, exchange rate innovation, scenario 1

change, which might be due to slightly more persistent reactions of the interest rates which are so small that they can not be seen in Figure 14.

Summarizing the effects in the purely forward-looking variant of the model, the impulse responses suggest stronger interest rate reactions only after supply innovations, and only for the bigger country, whereas in the case of demand and exchange rate innovations optimal interest rate responses are unaffected. This suggests that a robust policy maker fears model misspecification most in the case of supply disturbances. Furthermore the robust responses in the absence of the evil agent lead one to conclude that using a Hansen-Sargent robust strategy is an appropriate way to reflect fears about model misspecification and do not imply strong deviations when the more likely approximating model describes the real world.

Scenario 2 ($h_c = 0.4, \tau = 0$) The dynamics of the model variant including a small persistence in consumption habits are illustrated by the Figures 15-19. The impulse responses confirm the result from Table 9; as for the *RE* solutions all dynamics are very similar to those of the first scenario. The only difference is a small persistence in consumption and output dynamics that leads to cyclical and more persistent responses that are partly transmitted to other variables as well. Introducing a small persistence in consumption habits does not lead to strong deviations from the purely forward-looking variant.

The dynamics following a supply innovation are shown in Figure 15. Compared to the first scenario the initial increase in both interest rates is quite the same. However domestic consumption reacts less due to consumption habits, which imply a hump-shaped response. This reaction is reflected by domestic output dynamics. The smaller reaction of domestic consumption is due to the increase in transmission of consumption on inflation because of its effect on marginal costs. Similar to the single-country analysis, every increase in consumption and/or output requires an increase in the opportunity costs of leisure, and thus domestic producers have to pay higher wages in order to recruit enough workers. This also seems to be a rationale not to raise the interest rates stronger. As the interest rate differential causes a stronger appreciation of the exchange rate, the terms of trade react slightly stronger, but foreign output does not change considerably compared to scenario 1 due to the consumption dynamics. According to Figure 15, consumption habits seem to have an important impact on the trade balance dynamics, and first lead to a small slump that is not present in the first variant of the model. Under persistence in consumption habits, the fall in domestic demand is smaller and indicates that domestic households switch their expenditures to foreign products to maintain their consumption level although the relative domestic product prices increase. This effect raises the relative value of imports to exports and tends to make the trade balance negative. As the terms of trade return to equilibrium, the trade balance turns positive.

Regarding the robust responses the results differ substantially from the first scenario, as both interest rates are risen much more aggressively. The greatest fear of the monetary authorities is represented by an even stronger increase in the domestic supply shock, and positive reactions of the foreign supply shock and both demand shocks. All of these distortions imply stronger reactions of overall inflation and thus both central bankers react with more aggressiveness. Robust inflation expectations increase the initial impact on domestic producer price inflation and thus the terms of trade passivate to a greater extent. As a consequence, domestic output increases and foreign output falls stronger. As the interest rate differential is higher than for the first scenario, the implied appreciation of the exchange rate strengthens this effect. Moreover both central bankers fear a negative exchange rate shock, which amplifies the exchange rate appreciation. Concerning international trade, uncertainty leads to an increase in trade balance from a domestic perspective. The expenditure switching outweighs in the beginning and the trade balance becomes

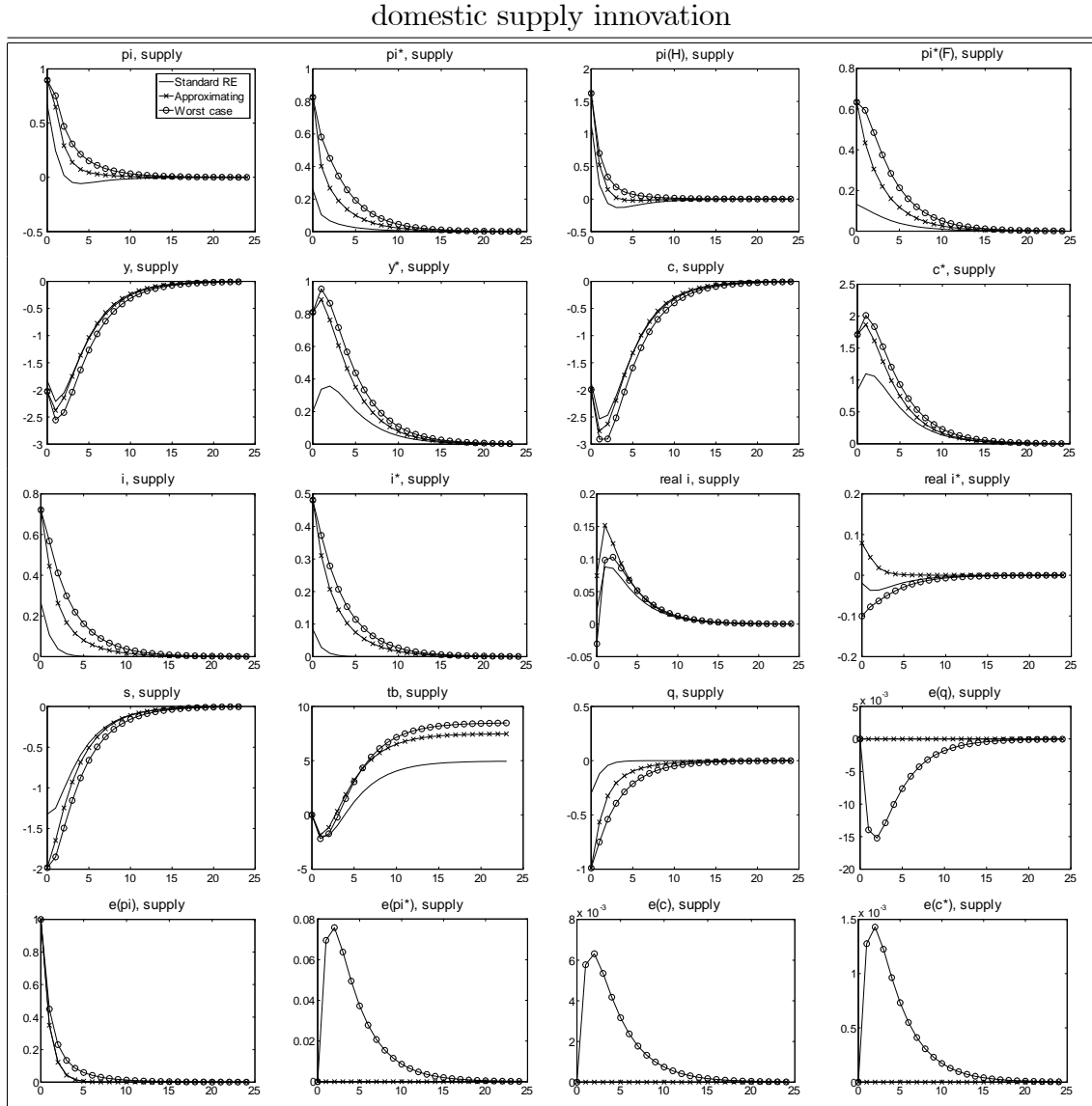


Figure 15: IRFs, discretion, supply innovation, scenario 2

negative, but as the terms of trade return to equilibrium, the immense increase in foreign demand is partly served by domestic firms and activates the trade balance. The robust solution without distortion confirms previous suggestions and illustrates that a robust policy maker is not a fool when the approximating model appears to be the true one. The reactions of all variables lie between the *RE* and the worst case solution and validate the hypothesis that robust control is a plausible way to model uncertainty in the discretionary case.

For a foreign supply innovation (Figure 16) persistence in consumption habits also does not lead to noteworthy changes in either interest rate compared to the purely forward-looking model. In fact, persistence decreases the reactions of both consumption and thus output indices, and seems to ease monetary policy. And the same is true for the robust dynamics. Similar to the first scenario only the foreign interest rate is risen more sharply in order to decrease the exchange rate

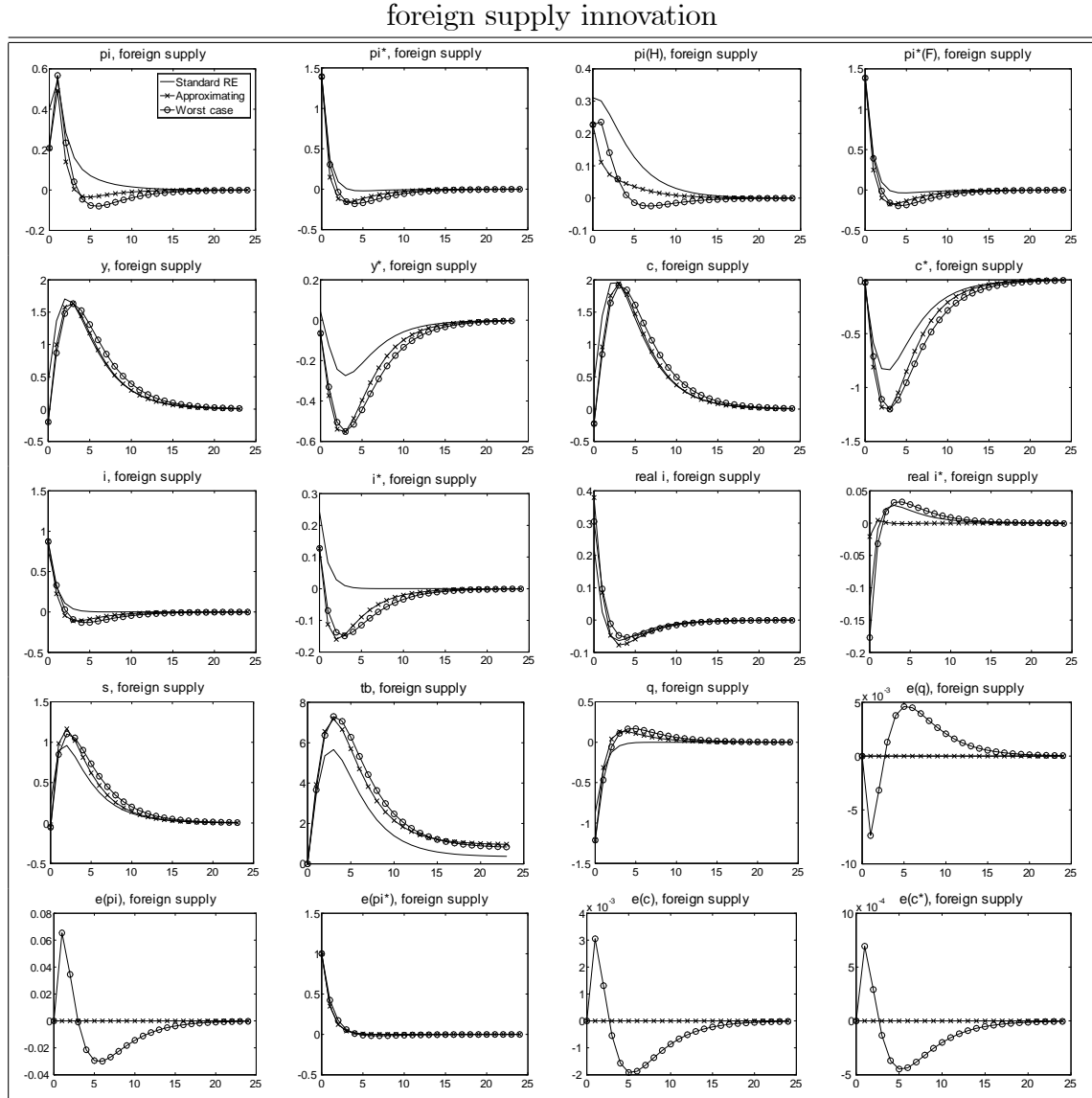


Figure 16: IRFs, discretion, foreign supply innovation, scenario 2

further and counteract the terms of trade evolution. Changes in the worst case dynamics are only present for shocks and the trade balance. The assumption of persistence in consumption habits implies a cyclical response of e_{c_t} , $e_{c_t^*}$, $e_{\pi_{Ht}}$ and e_q in order to exploit consumption dynamics and complicate the central banker's decision. In addition, the more aggressive foreign interest rate response causes a much stronger and more persistent slump in foreign consumption, decreasing the demand for domestic products as well. Thus under uncertainty the trade balance does not become negative (like in the first scenario), and the stronger reaction of relative prices increases the trade balance in the worst case as well as for the robust policy without evil agent. Similar to the domestic innovation foreign households react with stronger expenditure switching in order to keep last period's consumption level. Moreover, the responses for an unfounded fear against model misspecification again indicate that a robust policy maker is well-advised to use Hansen-Sargent

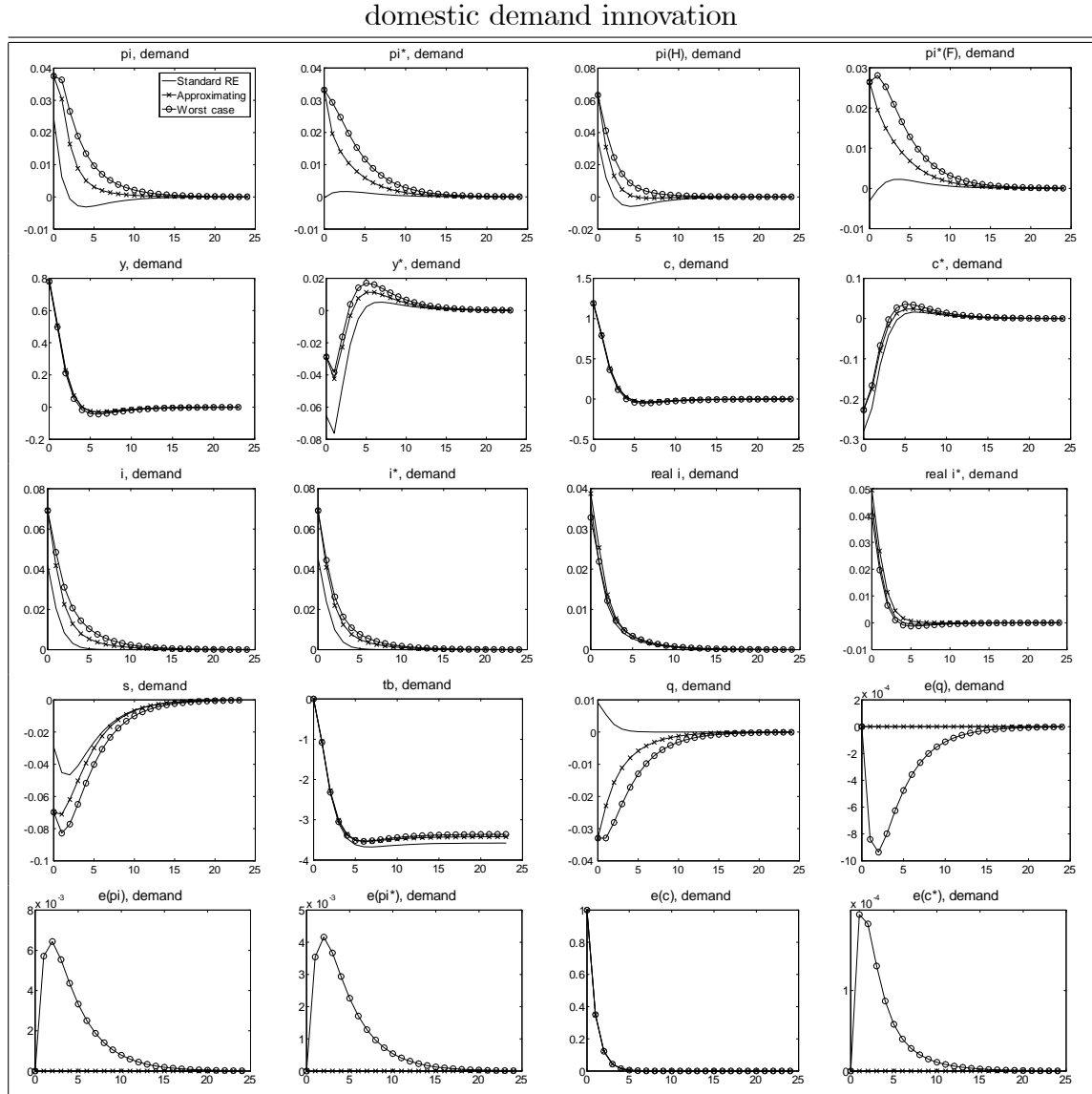


Figure 17: IRFs, discretion, domestic demand innovation, scenario 2

rules.

Impulse responses following demand innovations are given in Figures 17 and 18. Similar to the previous figures the dynamics are quite the same as in scenario 1. The reaction on a domestic demand innovation is a slightly more aggressive increase of the domestic interest rate compared to the purely forward-looking model, in order to react on the slightly more persistent shock impact, and thus the exchange rate depreciates less. Furthermore, domestic variables react stronger, whereas foreign variables do the opposite. The stronger deviation of domestic variables is due to the more persistent impulse, whereas the smaller deviations of foreign variables is due to the smaller reaction of the exchange rate, which strengthens the fall in the terms of trade and dampens the downturn in foreign output and producer price inflation. The smaller fall in y^* increases expected foreign consumption and deviations of c^* are also smaller. All inflation indices reflect the dynamics of domestic and foreign

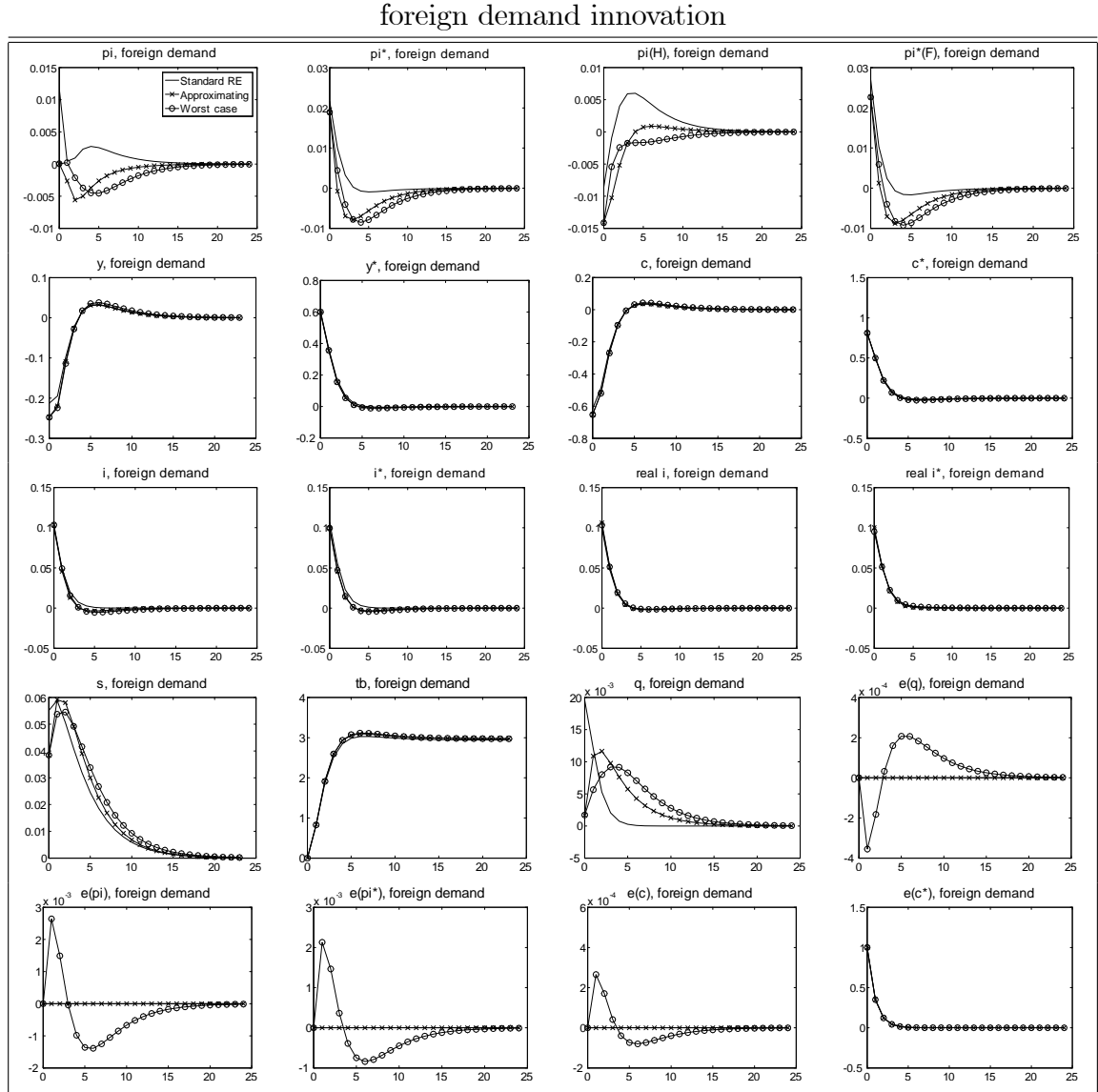


Figure 18: IRFs, discretion, foreign demand innovation, scenario 2

demand.

Whereas a preference for robustness in scenario 1 seems to have no impact on monetary policy at all, it calls for stronger interest rate reactions of both countries in the second scenario. The robust policy maker fears that the domestic demand shock is accompanied by a foreign one, and domestic and foreign supply shocks, which all tend to increase overall inflation. This overestimation of shock impacts justifies stronger interest rate reactions. Furthermore the malevolent player generates a negative exchange rate shock, implying stronger terms of trade deviations, followed by smaller reactions of foreign output and consumption. The increase in overall demand brings all inflation rates above the *RE* solution. As the increase in foreign demand leads to an increase of the value of domestic exports, the trade balance reacts less under uncertainty. The dynamics of an unfounded fear against model misspecification are straightforward and illustrate again that for a robust policy maker acting

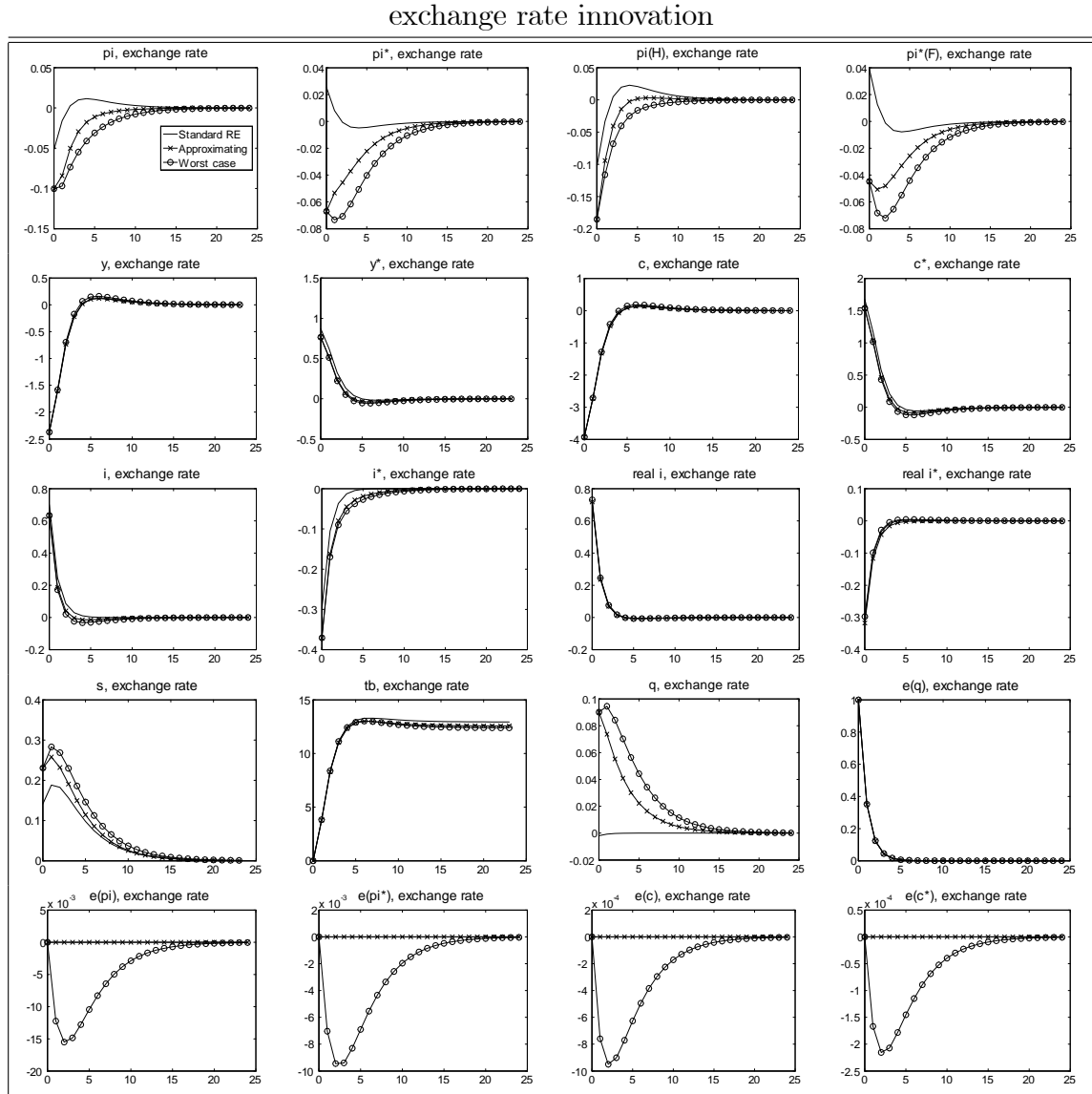


Figure 19: IRFs, discretion, exchange rate innovation, scenario 2

under discretion robust control seems to be an appropriate methodology that helps finding the optimal policy under uncertain circumstances.

The dynamics in the second scenario for a foreign demand innovation, shown in Figure 18, illustrate again that consumption habits do not have a very strong impact on the economy. The domestic interest rate is risen more aggressively in order to generate a smaller depreciation in the exchange rate. This dampens the stronger and more persistent increase in the terms of trade due to the stronger and more persistent increase in foreign producer price inflation. Comparable to the domestic shock, this implies stronger reactions of foreign, but smaller responses of domestic variables.

The worst case dynamics lead to the same conclusion than for scenario 1, since robust interest rate responses do not substantially differ from those of the *RE* solution. Introducing persistence in consumption dynamics leads the policy maker to

fear cyclical distortions, represented by the dynamics of the shocks in the worst case model. As these cyclical responses imply a more aggressive fall in all inflation rates, this leads to undershooting for some indices, the dynamics of the interest rates are even more defensive under uncertainty. However the changes due to a preference for robustness are so small that the reactions for the *RE* and the worst case solution, as well as for the robust strategy in the approximating model, are very hard to distinguish from each other.

In Figure 19 impulse responses after an exchange rate innovation are given. Again the influence of consumption habits on optimal interest rate policy is very small. The domestic interest rate is risen slightly less, and thus the exchange rate appreciates less. As consumption dynamics exhibit a persistent component, consumption and output of both countries react less and so do all inflation rates. This indicates that consumption habits clearly decrease losses after an exchange rate shock. As all variables deviate less from equilibrium also the trade balance is activated to a smaller extent. When households are assumed to change their consumption behavior more slowly, they do react less on interest rate changes (this is the loss in monetary policy effectiveness due to the fall in the habit adjusted substitution elasticity $\frac{\sigma}{1-h_c}$), and thus the interest rate differential, which is created in order to offset the exchange rate shock, leads to less deviations from equilibrium. As a consequence, trade also deviates less from the balanced baseline scenario.

Contrary to the previous scenario, assuming a preference for robustness in a model including habit consumption has an impact on optimal monetary policy. As the central bankers fear negative demand and supply shocks that tends to disinflate the economy, the foreign interest rate is decreased more aggressively to react to the fall of the foreign producer price inflation. Moreover, since the influence of interest rates on household's consumption decisions is smaller, consumption reacts less to a more aggressive monetary policy. As the robust agents expect the exchange rate to depreciate, it does so slightly, and strengthens the impact on the terms of trade. Foreign consumption, and thus foreign demand for domestic products, increases less and the trade balance reacts to a lesser extent. However, the robust solutions in the worst case as well as in the approximating model clearly show that a Hansen-Sargent robust policy maker does quite a good job and is close to the optimal *RE* solution.

Scenario 3 ($h_c = 0, \tau = 0.4$) Figures 20-24 show the dynamics of the two-country model for the third scenario. The dynamics for all innovations are very similar to those of the first scenario, but differ in size. Furthermore, the graphs confirm the results from Table 9 and from the single-country analysis. Introducing persistence in price-setting behavior has a strong impact on losses. Moreover, the additional loss due to this persistence is greater than the one following from a general model uncertainty.

Impulse responses for a supply innovation show stronger increases in both interest rates. The stabilizing effects of the transmission from output, consumption and terms of trade deviations decrease when more prices are adjusted in a backward-

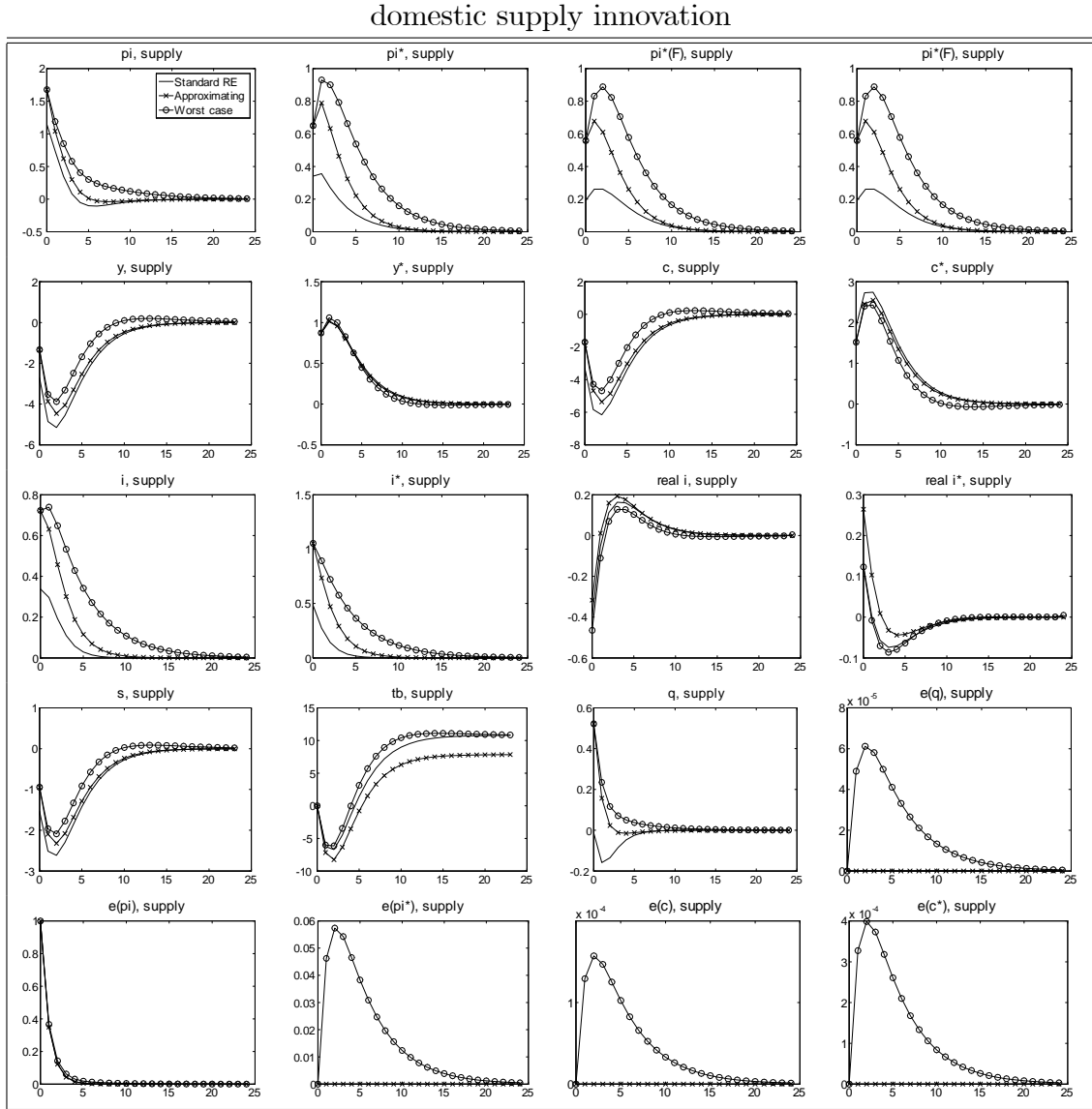


Figure 20: IRFs, discretion, domestic supply innovation, scenario 3

looking manner, due the smaller influence of marginal costs. The interest rates have to be risen stronger in order to decrease demand to a greater extent, and decrease inflationary pressures. As a consequence all variables deviate stronger and some exhibit a small cyclical component, that reflects the inflation persistence. This cyclical adjustment is also present in the trade balance dynamics as the hump-shaped patterns in consumption dynamics (implied by hump-shaped real interest rate responses) lead to an increase in the value of imports as domestic households switch expenditures to foreign products. However, as the terms of trade return to equilibrium this effect vanishes and the trade balance activates.

Regarding the robust solutions, the graphs show that the policy maker fears the same worst case scenario than before, represented by the increase in domestic and foreign demand and supply shocks, and a positive exchange rate shock. However, due to the smaller effectiveness in monetary policy and the higher persistence in inflation

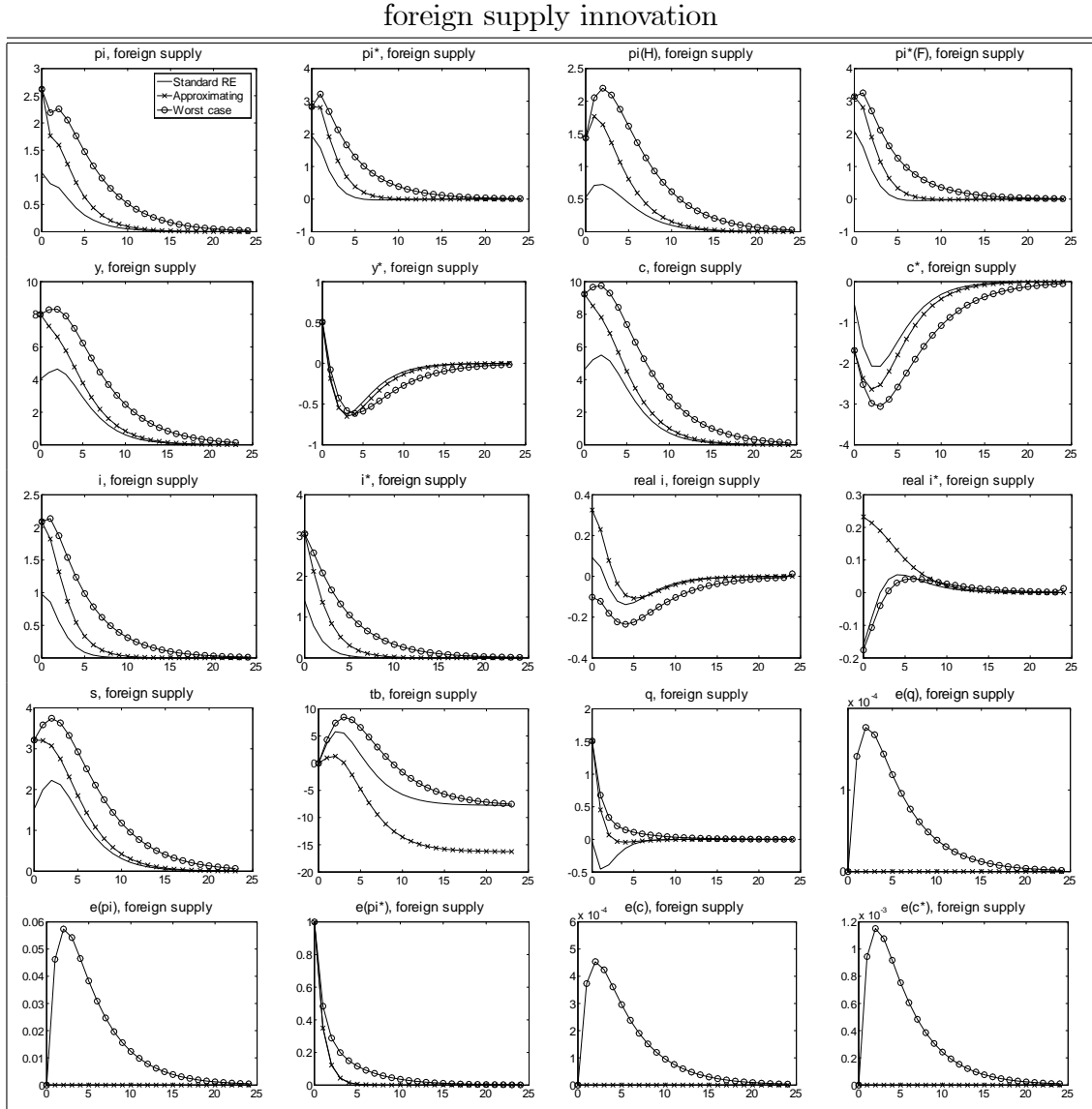


Figure 21: IRFs, discretion, foreign supply innovation, scenario 3

dynamics, the malevolent player leads to much stronger deviations in the third scenario, calling for a much more aggressive monetary policy reaction. Moreover, even for this stronger reaction the consumption and output indices react to a lesser extent, and thus the deviations of the inflation rates from equilibrium are much more aggressive. With respect to the influence of persistence in price-setting behavior this confirms the result of Table 9 and the single-country analysis, and illustrates that the amount of backward-looking price adjusters has a stronger influence on optimal policy than a general model uncertainty, given a particular value for this degree. However, given the reference model the graphs also confirm that a Hansen-Sargent robust policy represents an appropriate strategy to react on uncertainty for a discretionary policy maker.

Reactions after a foreign supply innovation are illustrated in Figure 21. Monetary policy in both countries becomes more aggressive due to the loss in monetary

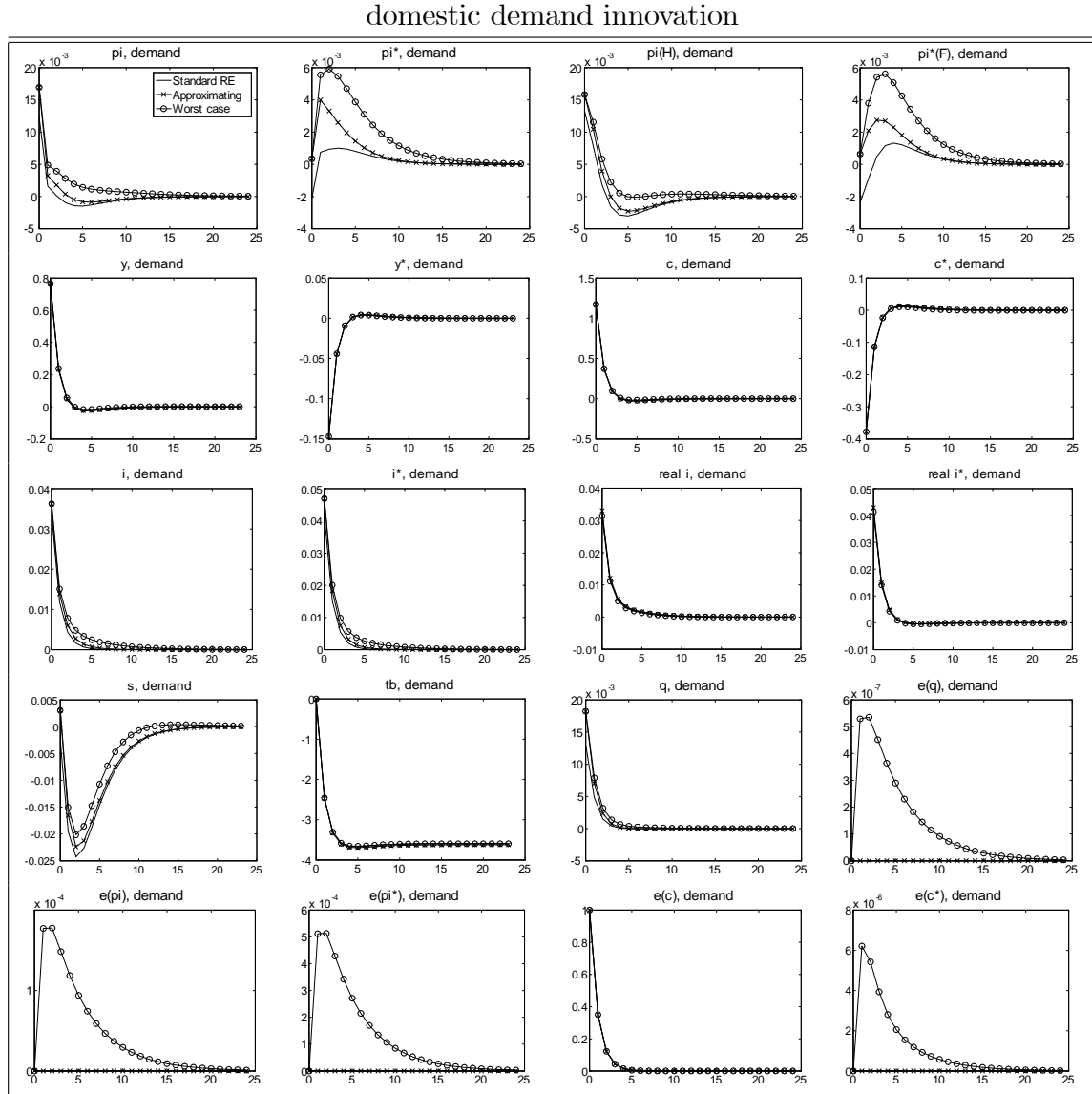


Figure 22: IRFs, discretion, domestic demand innovation, scenario 3

effectiveness, when prices are set in a more backward-looking manner. As managing demand via the real interest rate is less effective in reducing inflation, all inflation rates increase and thus the domestic real rate also decreases first. This causes domestic consumption to increase more strongly, which further increases domestic producer price inflation. Not until the real rate turns positive, can inflationary pressure be reduced. Furthermore, the increase in overall consumption leads to an initial increase in foreign output. As the stronger fall in foreign consumption leads foreign households to buy more domestic products, the trade balance falls to a greater extent than in the first scenario.

The robust dynamics suggest that more aggressive interest rate reactions are an appropriate reaction to uncertainty when the price-setting behavior includes persistence. The fear of the policy maker is represented by the same reactions of e_{c_t} , $e_{c_t^*}$, $e_{\pi_{H_t}}$, $e_{\pi_{F_t}^*}$ and e_q as in the first scenario, but the stronger persistence in in-

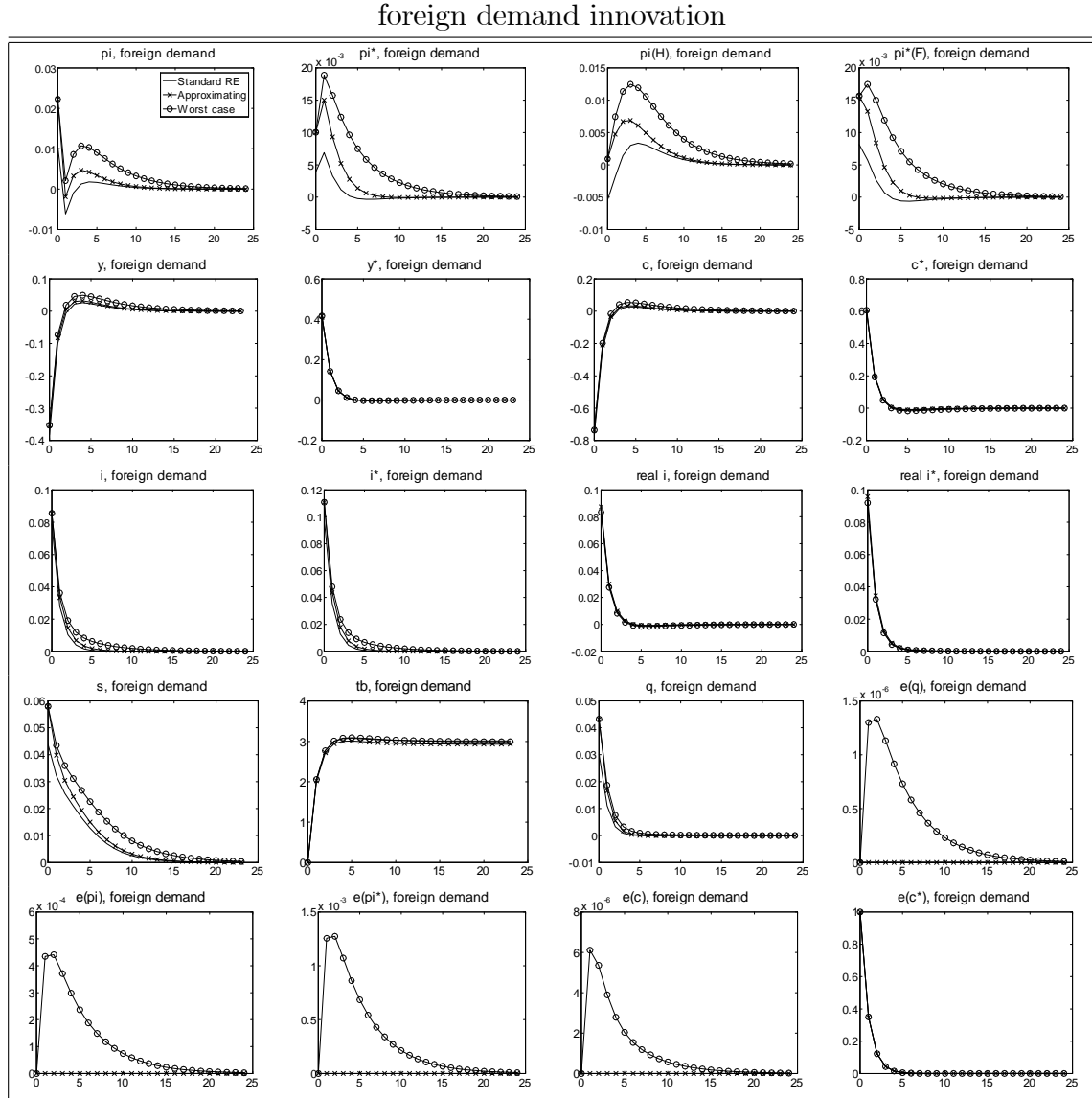


Figure 23: IRFs, discretion, foreign demand innovation, scenario 3

flation dynamics and the loss in policy effectiveness implies a much more aggressive robust solution. However, even this much stronger reaction could not prevent all other variables to deviate much stronger and persistent in the worst case. The positive exchange rate shock implies a depreciation which strengthens the response of the terms of trade, and expenditure switching effects increase the trade balance first. Robust expectations and monetary policy in the approximating model validate our previous results and are close to the optimal *RE* solution. Furthermore, comparing the dynamics with those of the first scenario confirms that the choice of the degree of persistence in price-setting behavior causes stronger deviations from equilibrium than a general fear about misspecification and calls for stronger interest rate responses - especially under uncertain conditions.

Concerning the responses after a domestic demand innovation (Figure 22) the graphs seem to replicate the purely forward-looking results. The reactions of all

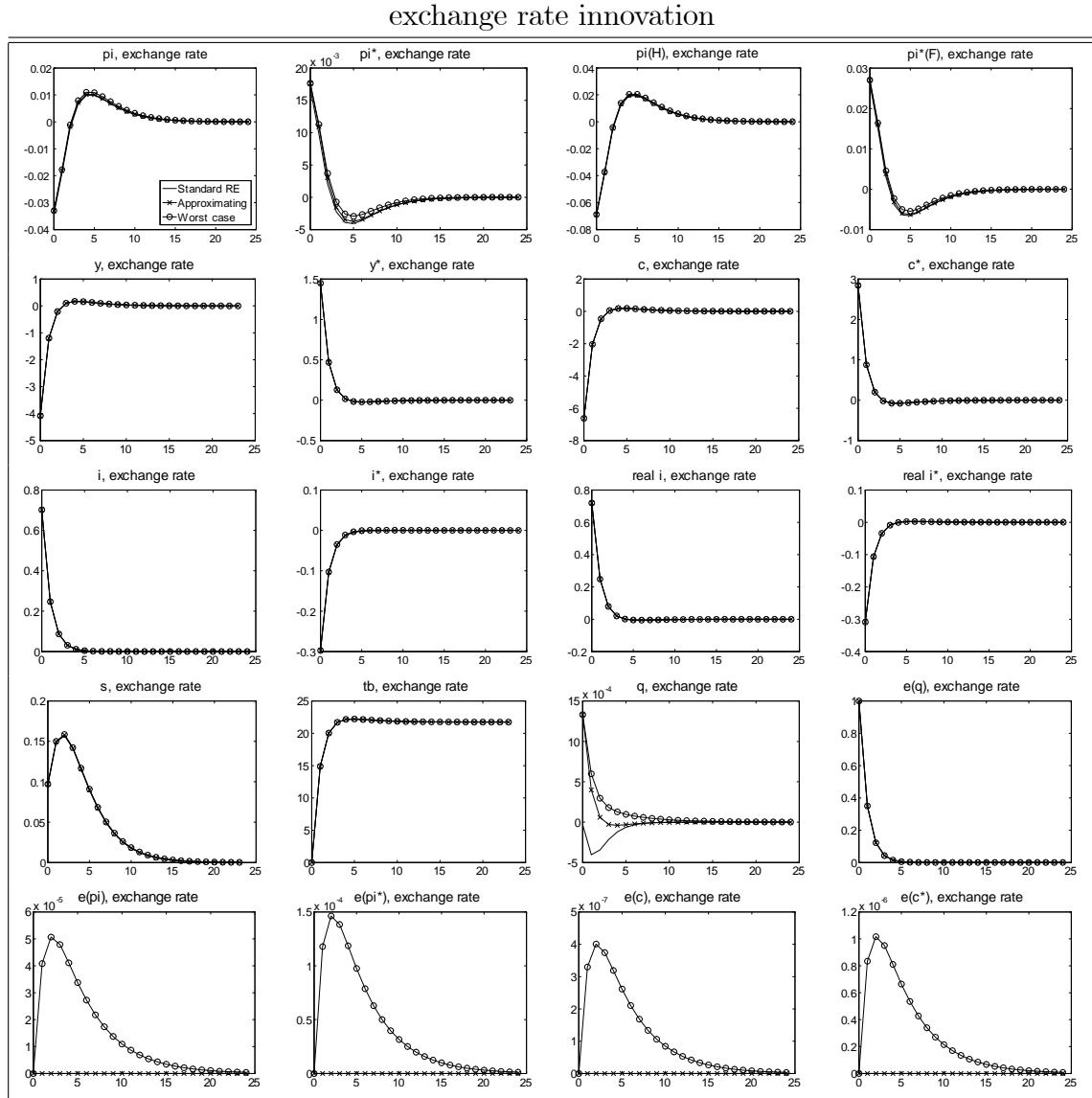


Figure 24: IRFs, discretion, exchange rate innovation, scenario 3

variables are only slightly smaller, since the initial impulse is less transmitted in the third scenario. Optimal interest rates seem to react a bit more aggressively, but changes are negligible. And the same is true for both robust solutions, leading one to conclude that introducing persistence in price-setting seems not to change optimal dynamics after a domestic demand innovation considerably. Robust interest rate responses do not change, but are slightly more persistent due to persistence in inflation dynamics.

For the foreign demand innovation, illustrated in Figure 23, conclusions are the same as for the domestic one. Introducing partially backward-looking price adjusters does not have a noteworthy impact on the model dynamics after a foreign demand shock. Due to the smaller transmission of the initial impulse on inflation, all variables deviate less, and the policy reactions are quite the same as for scenario 1. Furthermore, the robust responses do not change considerably either, and again

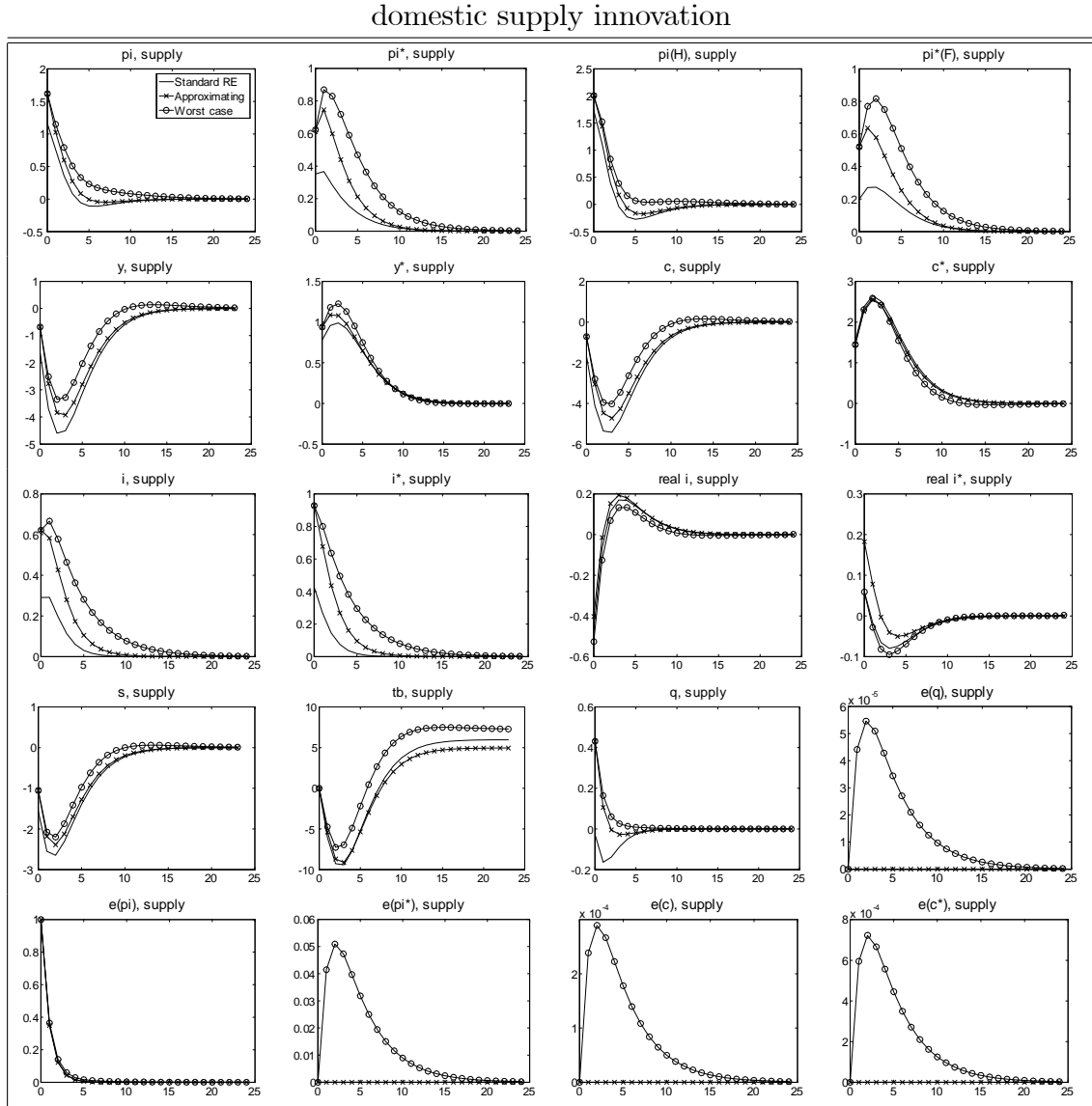


Figure 25: IRFs, discretion, domestic supply innovation, scenario 4

confirm the previous results.

Figure 24 gives the dynamics of the model following an exchange rate innovation, which provide similar conclusion as those of both demand disturbances. The responses in the third scenario are very close to those of the first one. In fact, the responses of both interest rates, and consumption and output indices seem to replicate the first scenario nearly exactly, but as the transmission on the NKPCs decreases, all inflation rates react to a lesser extent. Thus consumption habits seem to decrease distortions from an exchange rate innovation. Furthermore, the robust reactions indicate that an exchange rate innovation does not lead to any noteworthy changes in dynamics, which suggests that an exchange rate innovation is quite easy to handle for the discretionary policy maker.

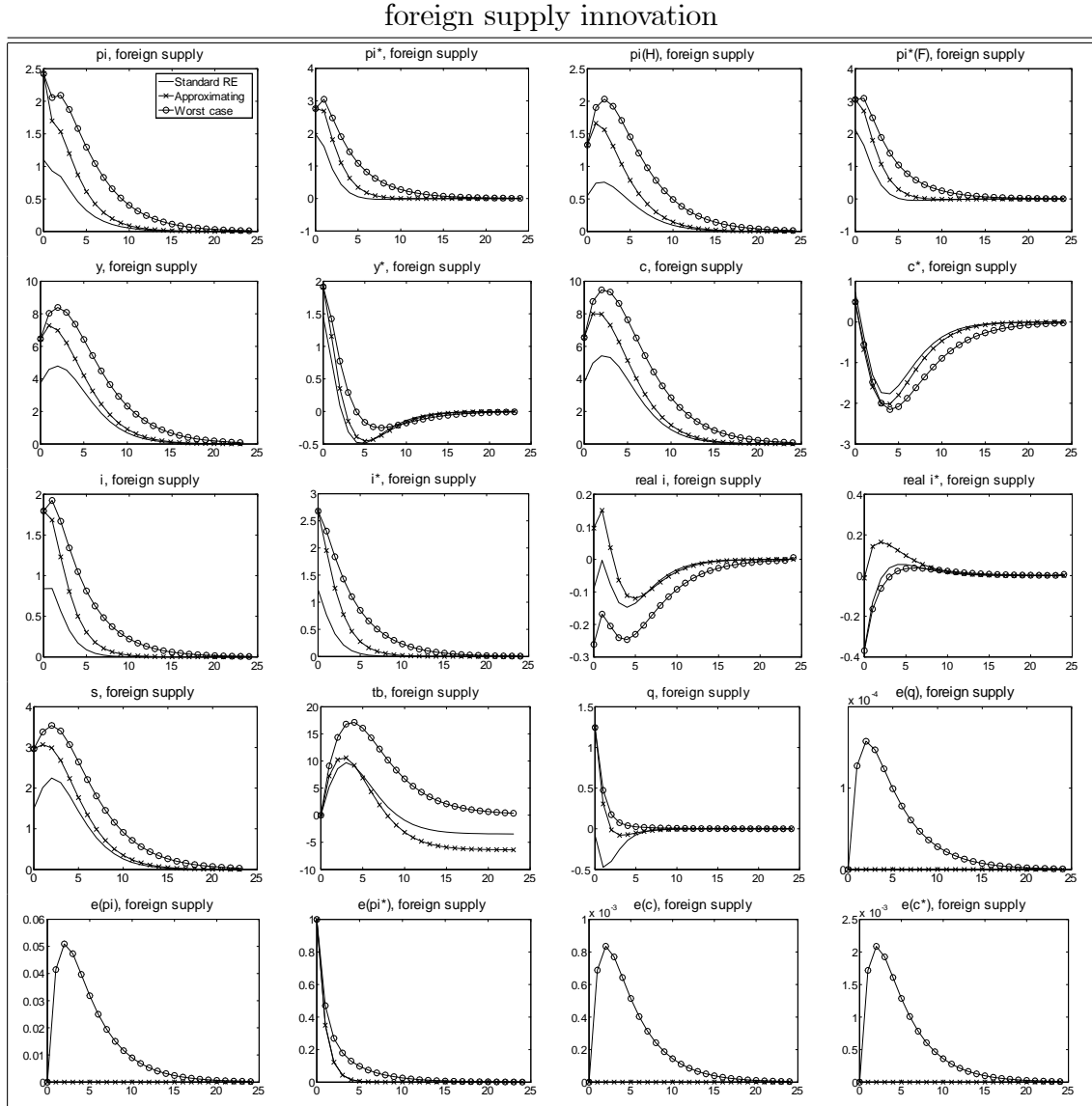


Figure 26: IRFs, discretion, foreign supply innovation, scenario 4

Scenario 4 ($h_c = \tau = 0.4$) Dynamics for the two-country model including both, persistence in consumption habits and in price-setting behavior, are illustrated in the Figures 25-29 and are very similar to those of the third scenario. This validates our previous suggestions concerning the influence of consumption habits. Whereas introducing persistence in price-setting behavior leads to stronger deviations and increasing losses, the effect of persistence in consumption habits is rather small and negligible.

For a domestic supply innovation robust and *RE* solutions are shown in Figure 25 and completely replicate the results from Figure 20, except for a slightly more cyclical adjustment path due to the additional persistence in consumption dynamics. The interest rates are risen less due to the increase in the transmission from demand to inflation, which implies that smaller deviations of consumption are necessary in order to influence inflation. All other dynamics are straightforward and follow the

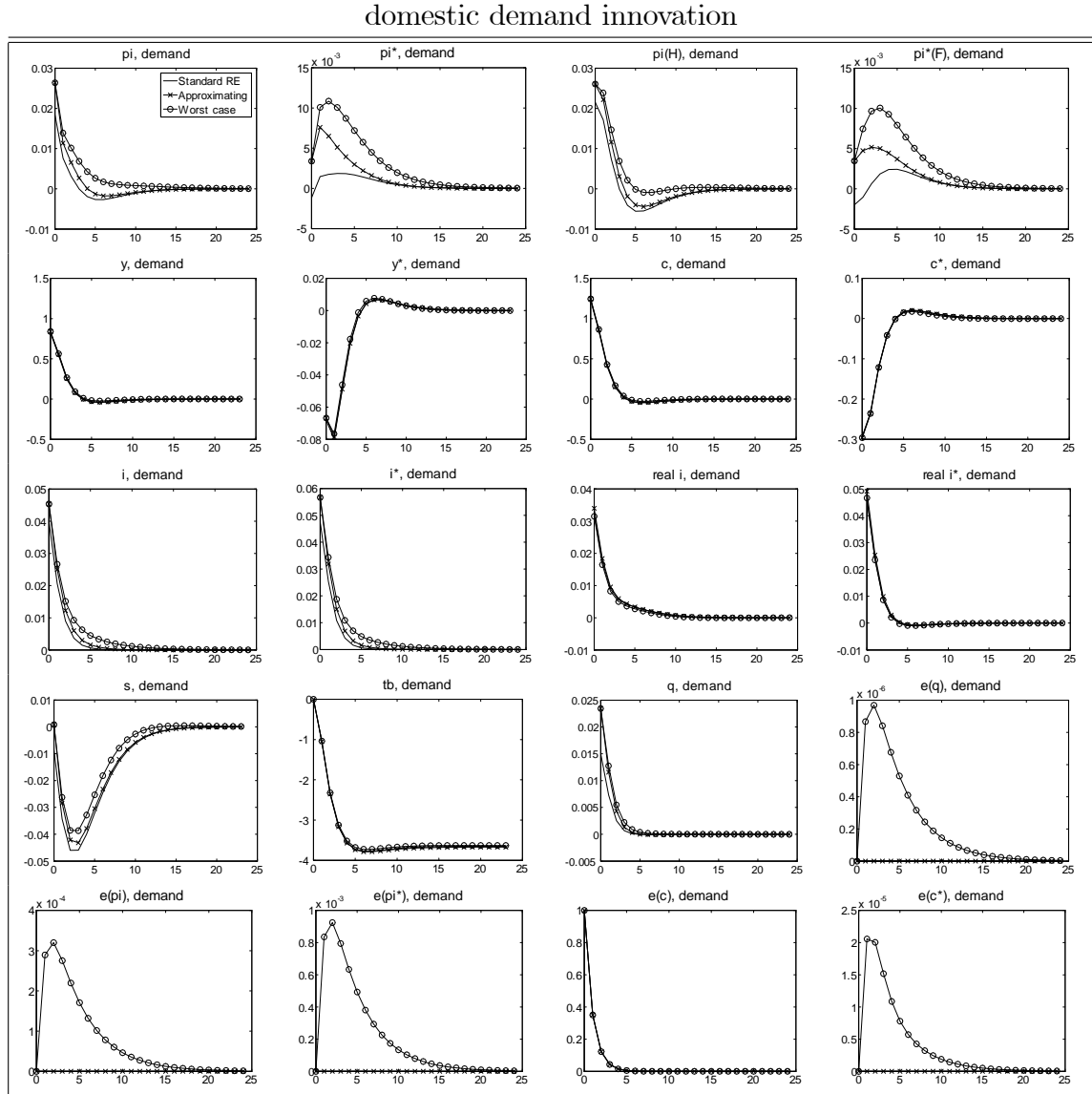


Figure 27: IRFs, discretion, domestic demand innovation, scenario 4

same arguments discussed for the previous scenario.

The dynamics after a foreign supply innovation, given in Figure 26, provide the same indications. The additional introduction of persistence in consumption habits has only a very small influence, and most of the graphs completely replicate the third scenario. Furthermore, the increase in transmission from consumption to inflation calls for more defensive interest rate reactions, and consumption and output indices exhibit a small amount of cyclical adjustment. However, as this implies that world consumption lies above its value for the third scenario, the foreign output gap initially increases more. Altogether, the responses of output and consumption are of less strength and reflected by the trade balance dynamics. The worst case solution as well as the robust strategy in the approximating model give same conclusions as those in scenario 3.

Demand innovation dynamics are illustrated in the Figures 27 and 28. Again the

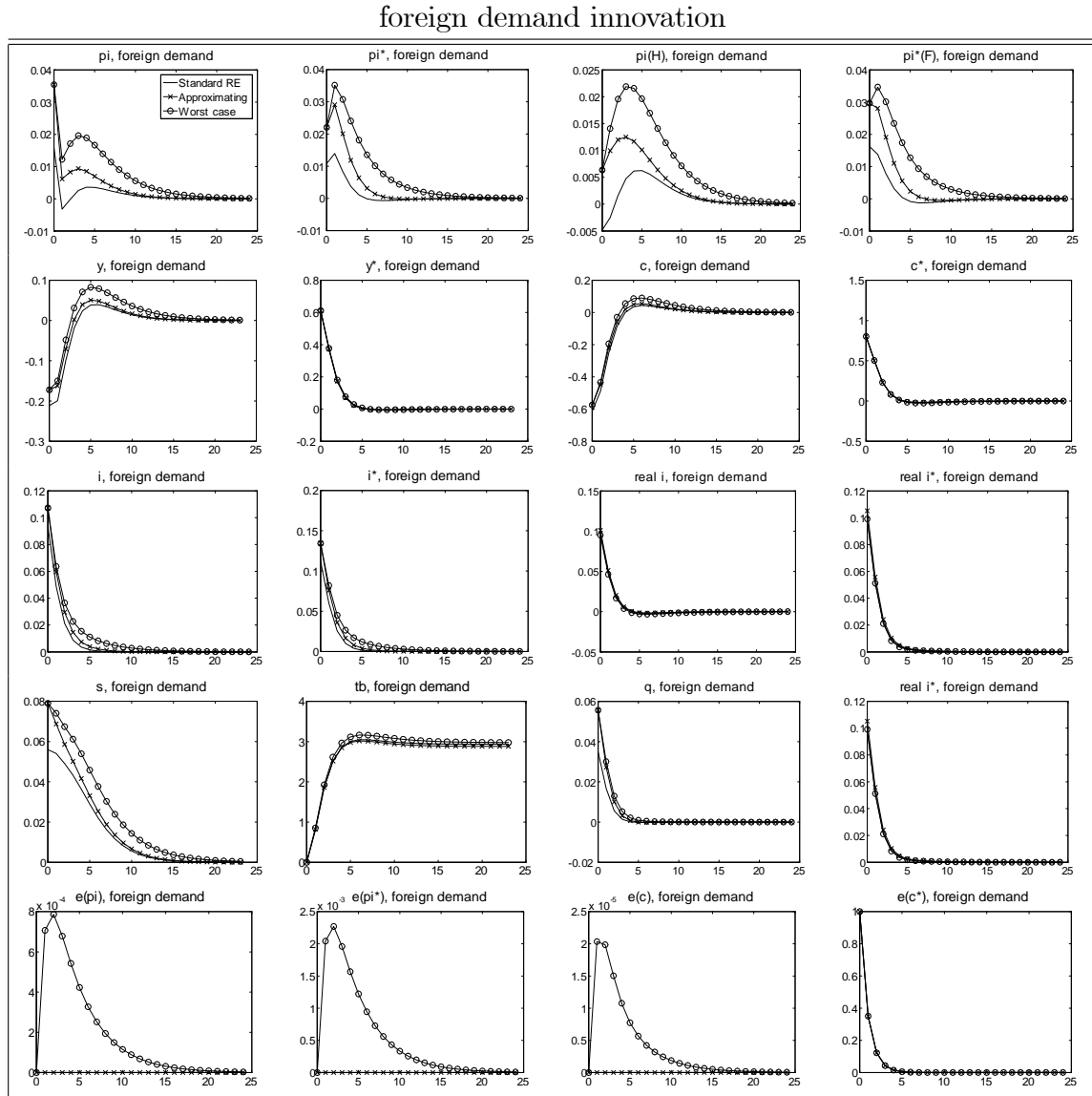


Figure 28: IRFs, discretion, foreign demand innovation, scenario 4

graphs are very similar to the previous scenario and suggest that demand shocks are much easier to handle. However, the stronger transmission from demand to inflation calls for more aggressive interest rate reactions, as in the case of demand shocks the initial impulse is already the increase in consumption, which leads to a stronger increase in inflation when consumption habits become more persistent. All other reactions are straightforward and illustrate the same dynamics than without consumption habits.

Additionally, the robust responses illustrate anew that a Hansen-Sargent robust policy maker does not change his interest rate reaction under uncertainty in the case of a demand innovation. The stronger and more persistent deviations of all variables due to the influence of the evil agent, that represents the fear of the robust policy maker, makes interest rate reactions become more persistent as well, but the initial increase is exactly the same. Besides, this also leads one to conclude that building

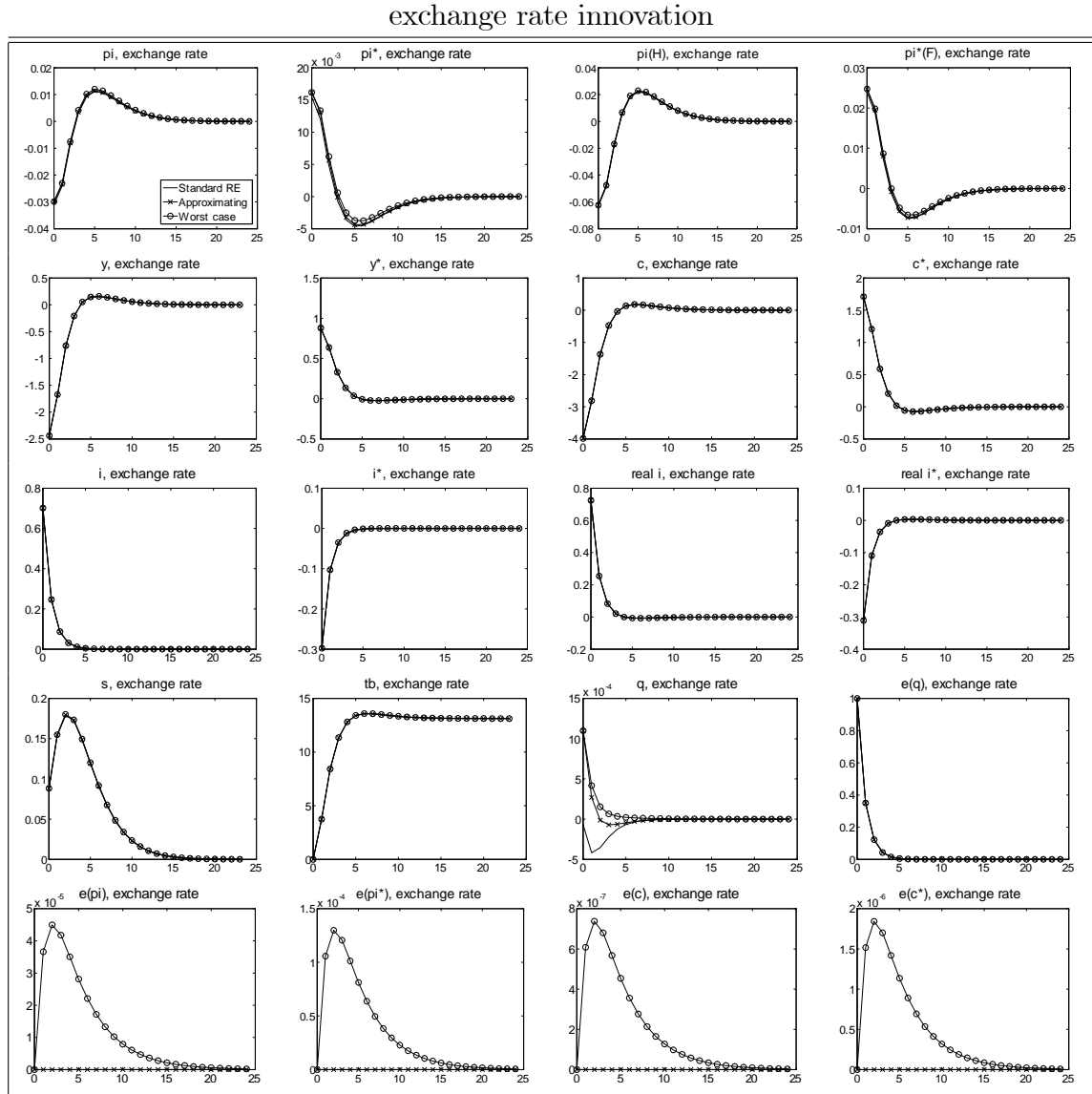


Figure 29: IRFs, discretion, exchange rate innovation, scenario 4

robust expectations and using a robust monetary policy in the absence of an evil agent implies dynamics close to the *RE* solution.

Concerning an exchange rate innovation in the fourth scenario, the corresponding graphs in Figure 29 suggest what already followed from the second scenario. Introducing consumption habits decreases the influence of the interest rate on household decisions, and thus the interest rate responses in order to offset the exchange rate shock imply smaller deviations of consumption from equilibrium and thus smaller output and inflation reactions as well. Moreover the robust solutions illustrate again that a preference for robustness against a general model misspecification does not imply any noteworthy changes in the responses to an exchange rate innovation.

Summarizing the results for the discretionary policy maker The results for the discretionary solution of the two-country economy are similar to those for the

single-country analysis and can be summarized as follows. First of all, persistence in price-setting behavior implies substantially stronger deviations, whereas the introduction of consumption habits seems to be negligible. Moreover, the results suggest that the discretionary solution is quite robust against model misspecification. As a discretionary policy maker reoptimizes every period, he is able to manage a general model uncertainty quite well. In addition the previous sections illustrate that the fear of a robust policy maker about misspecification does not lead to noteworthy changes in the case of demand or exchange rate innovations, as these can be easily offset by corresponding interest rate reactions which directly impact consumption or the exchange rate, respectively. In contrast, supply innovations can only be offset by reducing demand. In this case, robust central bankers react with more aggressive monetary policy rules to respond on an overestimation of shock impacts. Although the robust solution works well in the approximating model, impulse responses and losses suggest that the choice of the degree of persistence - the choice of the right reference model - has a much stronger impact than general model uncertainty given a particular value for this degree.

8.2.2 Commitment

The results for the policy maker acting under commitment are given in Table 10. When we compare these with those of the discretionary solution, Table 10 illustrates the gains from a commitment strategy, since losses for all scenarios and objectives are below the corresponding values of the previous section. Moreover, and similar to the single-country analysis, the commitment solution is very sensitive with respect to uncertainty. In order to derive any results we have to choose detection error probabilities of $p(\theta) = 45\%$ and $p(\theta) = 15\%$, as for most objectives and scenarios the model exhibits indeterminacy, even for very small amounts of uncertainty. In addition, the solutions for the very small amount of uncertainty that corresponds to $p(\theta) = 45\%$ already suggest that uncertainty implies an increase in losses. Furthermore, the result for $(\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0.02, 0.3, 0)$ and $p(\theta) = 15\%$ in the model with very high persistence leads to instability in the case of robust expectations and monetary policy but no evil agent, indicating that the indeterminacy of the other solutions might be due to instability of the robust rules in the approximating models. This supports the conclusion we derived in the single-country analysis that credibly committing to the optimal interest rate path is particularly problematic when the true model of the economy is unknown.

According to the introduction of persistence the results confirm all previous suggestions, and provide that consumption habits have only a negligible impact on losses, whereas persistent price-setting behavior implies a substantial increase. A comparison with the losses for $p(\theta) = 45\%$ also suggests that the price-setting behavior of firms has a greater impact than uncertainty modelled by robust control techniques.

$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.02, 0.3, 0)$			$(0.0483, 0.2364, 0)$		
$p(\theta)$	50%	45%	15%	50%	45%	15%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	115	121,117	ind	129	134,131	ind
$h_c = 0.4, \tau = 0$	115	120,117	ind	129	134,131	ind
$\tau = 0.4, h_c = 0$	431	474,454	ind	497	564,546	ind
$h_c = \tau = 0.4$	428	470,451	ind	494	558,540	ind
$h_c = \tau = 0.8$	2822	3086,3736	5251, ∞	3535	5153,3090	ind
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.1, 0.5, 0)$			$(0.5, 0, 0)$		
$p(\theta)$	50%	45%	15%	50%	45%	15%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	142	ind	ind	146	ind	ind
$h_c = 0.4, \tau = 0$	142	ind	ind	146	ind	ind
$\tau = 0.4, h_c = 0$	584	ind	ind	551	ind	ind
$h_c = \tau = 0.4$	582	ind	ind	551	ind	ind
$h_c = \tau = 0.8$	4665	7235,7231	ind	4812	ind	ind
$(\lambda_y, \lambda_i, \lambda_{\Delta i})$	$(0.02, 0.6, 0)$			$(0.5, 0, 0.3)$		
$p(\theta)$	50%	45%	15%	50%	45%	15%
scenario	l_a^{RE}	l_r, l_a^r	l_r, l_a^r	l_a^{RE}	l_r, l_a^r	l_r, l_a^r
$h_c = \tau = 0$	116	122,118	ind	142	ind	ind
$h_c = 0.4, \tau = 0$	116	121,118	ind	141	ind	ind
$\tau = 0.4, h_c = 0$	443	489,469	ind	484	ind	ind
$h_c = \tau = 0.4$	437	481,463	ind	477	ind	ind
$h_c = \tau = 0.8$	2996	3756,3660	5607,5295	2194	ind	ind

For some scenarios the steady state of the model is indeterminate (ind).

Table 10: Losses under commitment, two-country economy

scenario 1 ($h_c = \tau = 0$) The following sections show impulse response functions for the first 4 scenarios of the model and $p(\theta) = 45\%$. The dynamics of most state variable are very similar to those of the discretionary solution. However, the reactions are of a much smaller size and the interest rates differ substantially.

Impulse responses for a domestic supply innovation are given in Figure 30. Due to the possibility to improve private sector expectations, deviations of all variables are much smaller. The impulse brings about a rise in domestic producer price inflation and in order to dampen world inflation the domestic central banker raises his interest rate. Contrary to the discretionary policy maker, the foreign authority slightly decreases his interest rate under commitment. As a consequence the exchange rate appreciates and negatively impacts the terms of trade. The increase in the domestic interest rate leads to a fall in consumption despite the negative real rate. This is due to negative expected consumption, implied by the fall in domestic output. As the fall in overall demand causes both output gaps to fall, the same is true for foreign

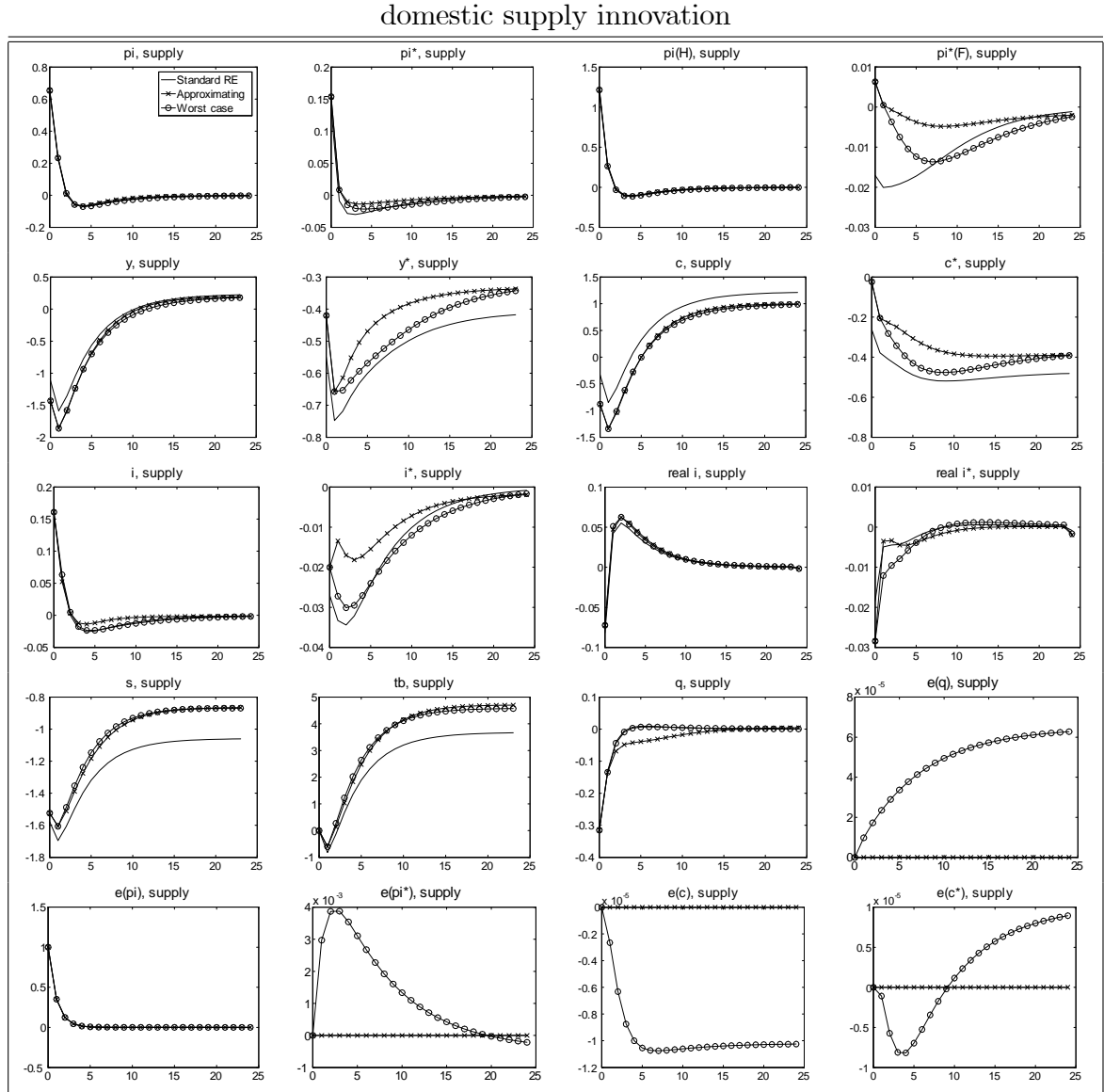


Figure 30: IRFs, commitment, domestic supply innovation, scenario 1

consumption. The fall in domestic (foreign) output is strengthened (dampened) by the fall in the terms of trade. In contrast with the discretionary case, the downturn in foreign output and consumption, as well as the fall in the terms of trade, imply a decreasing foreign producer price inflation.

The graphs further suggest that credibly committing to the optimal interest rate response has little impact on the equilibrium values of the individual countries of the economy. Thus managing expectations seems to influence the contribution of real variables across countries. The rise in domestic consumption expectations, partly due to the negative real interest rate, is so strong that domestic consumption overshoots its previous steady state due to self-fulfilling expectations. As the real rate becomes positive expectations are dampened and consumption remains in a new equilibrium. According to Figure 30 the opposite is true for foreign consumption.

The downturn in expectations is too strong with respect to the only slightly negative foreign real rate and consumption is stabilized on a lower equilibrium. In the new steady state domestic households partially compensate for the lower foreign demand, but due to the home bias output responses reflect the consumption dynamics. Domestic (foreign) output dynamics are dampened (strengthens) by the terms of trade, which converge to a lower steady state, but this effect is too weak to completely offset the increase in domestic demand. A social planner under commitment seems to suffer lower losses when stabilizing world output and inflation by improving expectations and allowing consumption and output to deviate from their previous equilibrium values.¹³⁵ This is possible under commitment as the monetary policy rule also depends on the forward-looking variables. By exploiting expectations, the social planner chooses the least distorting dynamics for the world variables, although the distribution of world output and consumption across countries changes. The trade balance turns negative first due to expenditure switching of households, but the higher domestic production in equilibrium raises the export values and the trade balance is positive in the new equilibrium.

With respect to the robust solutions, the same is true for the evil agent than for the policy maker. By exploiting expectations, the imaginary evil agent finds a rule that leads to a permanent influence on equilibrium values. Under commitment the robust policy maker is aware of that and fears such distortions most. His fear of model misspecification is represented by an additional foreign supply shock that undershoots its equilibrium value in the long run. Moreover, he fears a negative and persistent domestic demand shock, as well as a cyclical foreign demand shock that remains in a positive steady state, and a permanent positive exchange rate shock. Due to permanency, the worst case scenario includes long run values that differ from the *RE* equilibrium. The increase in $e(\pi^*)$ leads to rising foreign producer price inflation expectations, implying in fact an increase in foreign producer price inflation. Since this leads to a stronger fall in the foreign real interest rate, foreign consumption and thus output decrease less, although the foreign interest rate falls to a lesser extent in response to the increase in foreign producer price inflation. The domestic interest rate reaction does not change, but the slightly stronger response of domestic inflation brings about a small increase in the real interest rate which is accompanied by a stronger fall in domestic consumption, implying a lower equilibrium as in the *RE* solution. These reactions are reflected in both output dynamics as well. The reaction of the trade balance indicates that the smaller increase in domestic consumption hits the demand for foreign products more and decreases the value of imports. This might be due to the stronger increase in foreign prices, implying expenditure switching to domestic goods. The possibility of finding a reaction func-

¹³⁵Notice that this can not be interpreted as an increase (fall) in domestic (foreign) welfare. Domestic and foreign households both choose their optimal labor supply by equating the marginal utility from a higher revenue of producing one extra unit with the marginal disutility from the needed work effort. Thus, if lower consumption is accompanied by a corresponding increase in leisure, utility is unaffected.

tion that leads to permanent impacts clearly shows that uncertainty causes much stronger distortions under commitment. Recall that the worst case solutions are derived for a very low amount of uncertainty corresponding to $p(\theta) = 45\%$. Figure 30 suggests that more appropriate amounts of uncertainty might lead to very strong permanent shock impacts that might be the reason for indeterminacy of most objectives and scenarios.

When using the robust rule and building robust expectations in the approximating model, the graphs provide that robust expectations lead to the same increase in expected foreign producer price inflation than in the worst case. However, the smaller downturn in the foreign interest rate gets this increase under control very fast. As the expected foreign supply shock does not occur, the foreign interest rate returns much faster to equilibrium. As first reactions of the real rates are similar to those of the worst case scenario, both consumption expectations behave similar, and unfounded fear implies consumption and output dynamics that are comparable to those of the worst case. Thus fear of misspecification has an impact on equilibrium values of the individual countries under commitment, independent from the actual appearance of the malevolent player. To fear the distortions and reflect this fear in forming expectations suffices to imply new steady states.

With respect to a foreign supply innovation impulse responses are given in Figure 31. Again the dynamics are similar to the discretionary solution, but differ in size. Furthermore, improving expectations implies less aggressive interest rate dynamics. Similar to the domestic innovation the domestic interest rate is risen but the foreign one decreases slightly. This is comparable to the reaction under a discretionary policy maker who chooses to raise the domestic interest rate stronger in order to cause an exchange rate appreciation and counteract on the terms of trade response. In the commitment solution the fall in i_t^* causes an appreciation strong enough to overcompensate the increase in foreign producer price inflation, and implies a fall in the terms of trade. Consumption expectations mirror the domestic shock and imply a fall in foreign and an increase in domestic consumption indices, which imply corresponding output dynamics. The rise in domestic consumption implies an increase in the demand for foreign products and the trade balance becomes negative.

Regarding the equilibrium values Figure 31 illustrates that the policy maker under commitment stabilizes world output by decreasing foreign and increasing domestic consumption and output, independent from the origin of a supply shock. In addition the robust solution shows that the fear of the social planner is also represented by similar distortions as before. Thus the responses of all variables in the worst case follow the same arguments as before and lead to lower domestic, but stronger foreign, consumption and output dynamics. The robust solution in the worst case illustrates again that a fear of model misspecification has an impact on equilibrium values, also in the absence of the evil agent.

The dynamics after a domestic demand innovation, given in Figure 32, are very similar to the discretionary counterparts, where again the size of distortions is smaller due to the improvements in expectations. The increase in domestic con-

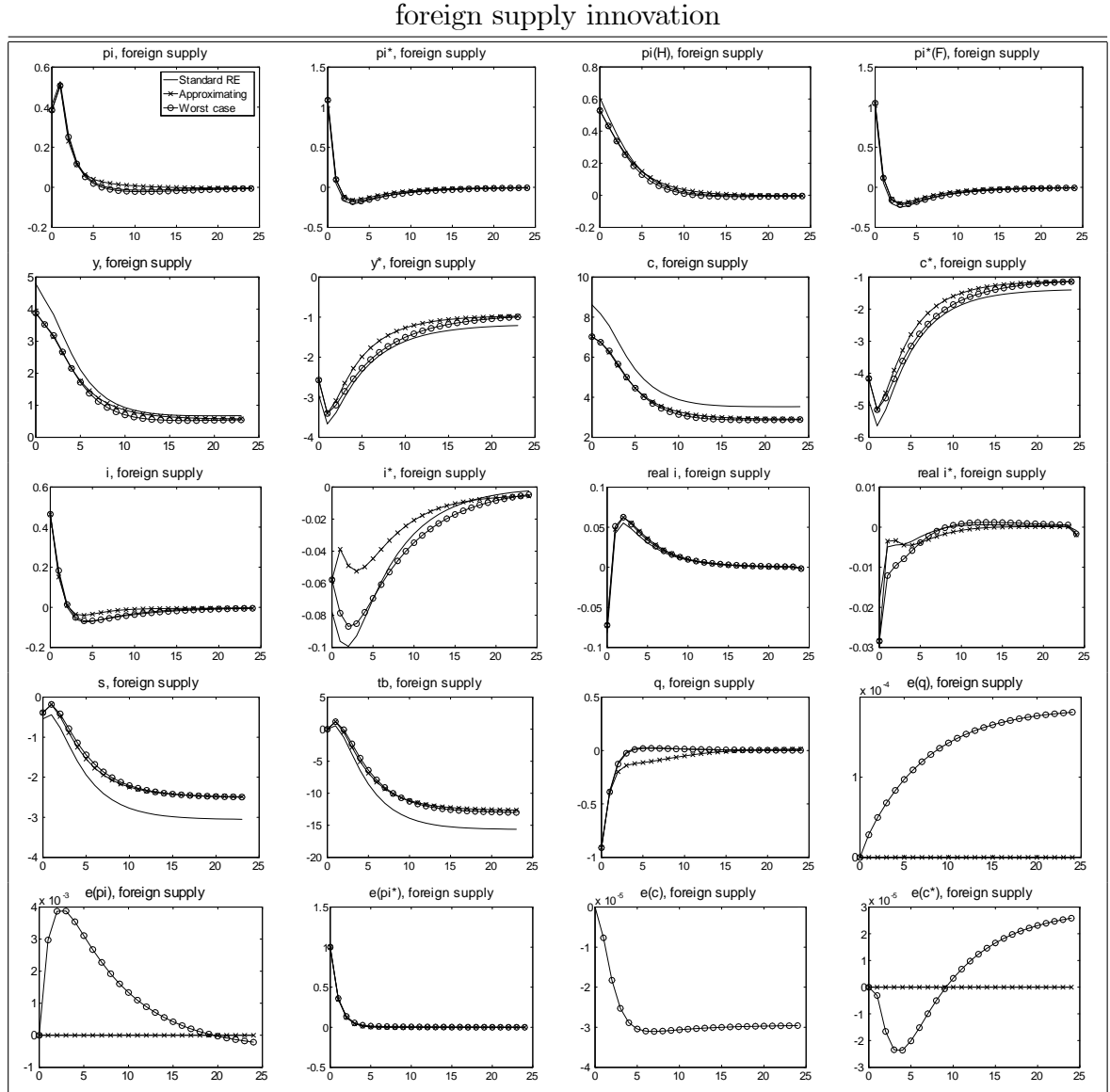


Figure 31: IRFs, commitment, foreign supply innovation, scenario 1

sumption implies a demand overhang, and both interest rates are risen in order to dampen world inflation. Similar to the discretionary dynamics, the foreign interest rate is increased more strongly and causes the exchange rate to depreciate, what counteracts the fall in the terms of trade. As the increase in domestic demand partially falls on foreign products, the trade balance becomes negative.

However, an important difference from the discretionary solution is that under commitment a domestic demand innovation also implies new equilibria for domestic and foreign consumption and output indices. The new equilibrium of domestic (foreign) consumption and output lies above (below) the previous steady state. Furthermore, uncertainty has long run influences, shown by the permanent reaction of the exchange rate shock. By exploiting the expectations of the agents, the evil agent is able to find a strategy that has a permanent impact on the economy. This

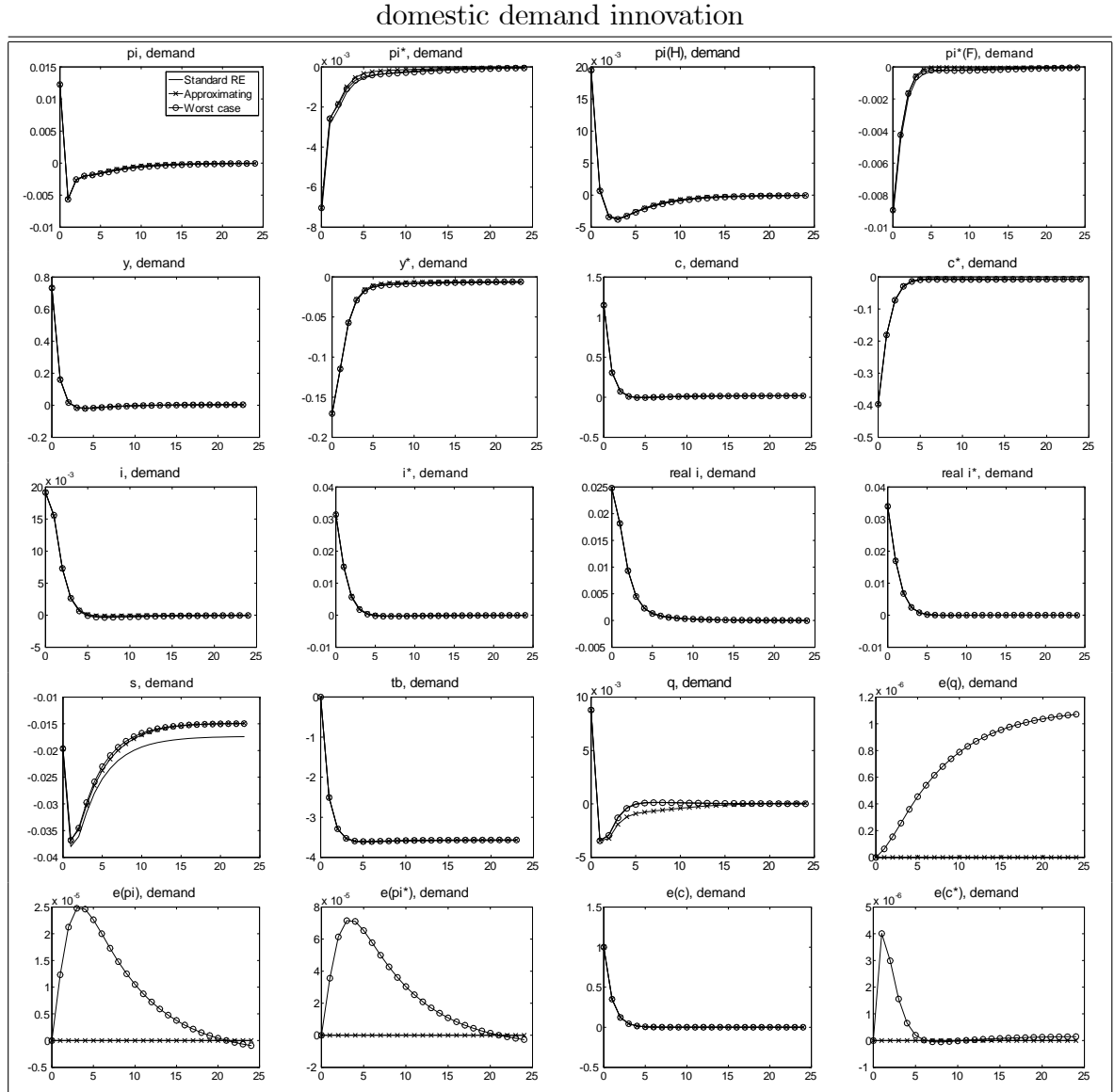


Figure 32: IRFs, commitment, domestic demand innovation, scenario 1

indicates again that for higher values of uncertainty the evil agent might cause disturbances that a policy maker acting under commitment is unable to deal with. However, comparable to the results under discretion, the policy maker's fear about misspecification is relatively small with respect to a domestic demand innovation. Thus, optimal interest rate responses are unaffected and the influence of uncertainty on all other variables is too small to be present in the graphs. The same is true for the robust solutions in the absence of the malevolent player.

The dynamics implied by a foreign demand innovation (Figure 33) follow exactly the same argumentation than for the discretionary solution, except that the size of reactions is smaller and there is an influence on equilibrium values of consumption and output. Similar to the domestic demand innovation and both supply innovations domestic (foreign) equilibrium consumption and output are slightly above (below)

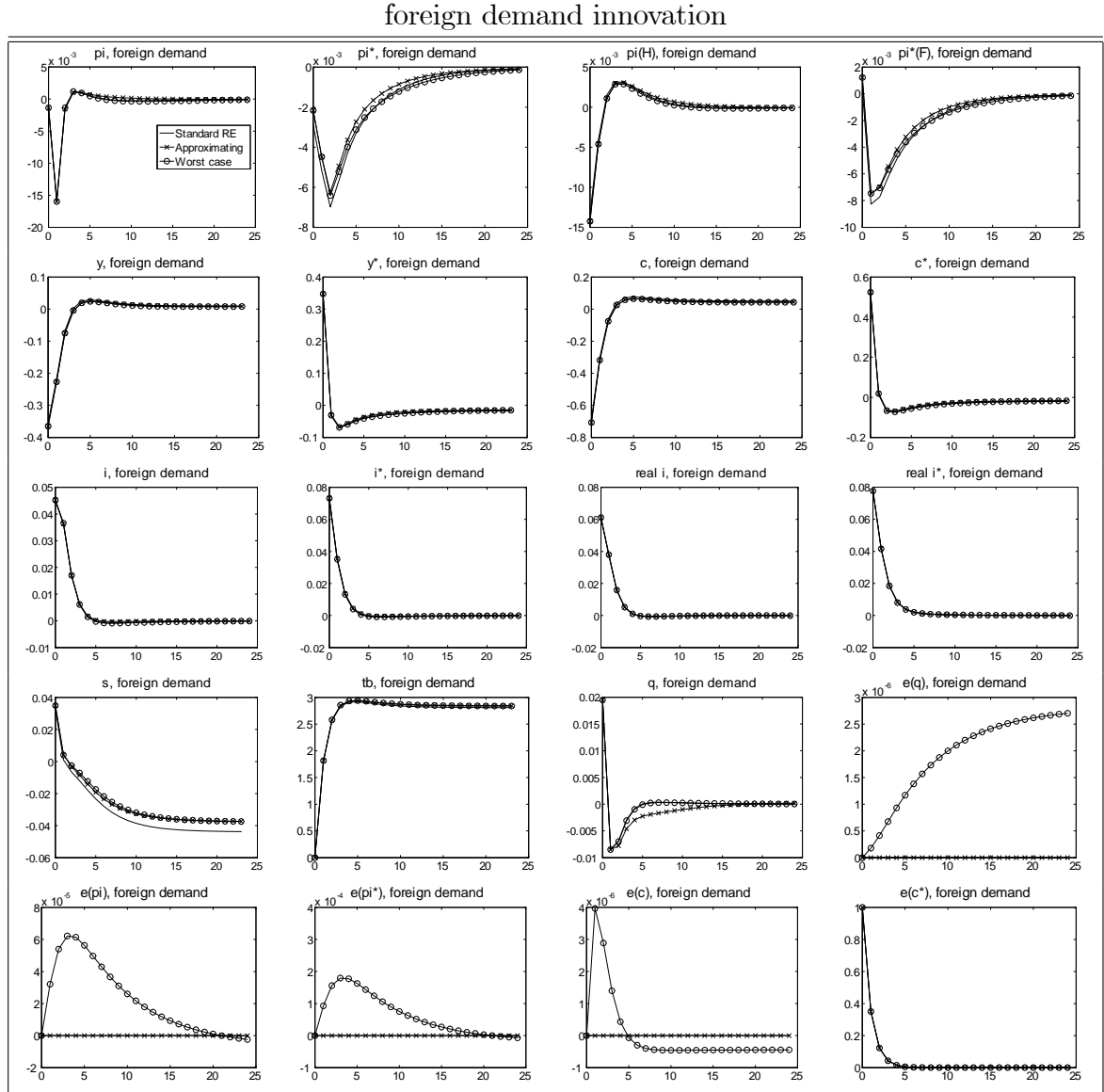


Figure 33: IRFs, commitment, foreign demand innovation, scenario 1

their previous steady states. Similar to all other innovations discussed so far it seems to be optimal for the social planner to bring the domestic interest rate back to equilibrium so fast that domestic consumption expectations, as well as actual consumption, overshoots its previous equilibrium.

With respect to the robust dynamics, the graphs give rise to the same congestions than for the domestic innovation, illustrating that the policy maker's fear against misspecification in response to a foreign demand shock seems to be rather small. However, the shock responses in the worst case scenario also indicate that due to the possibility to exploit expectations the malevolent player finds a rule that has a permanent impact on equilibrium values.

Figure 34 shows the model dynamics after an exchange rate innovation and replicates the result under discretion nearly perfect. The only difference is that under

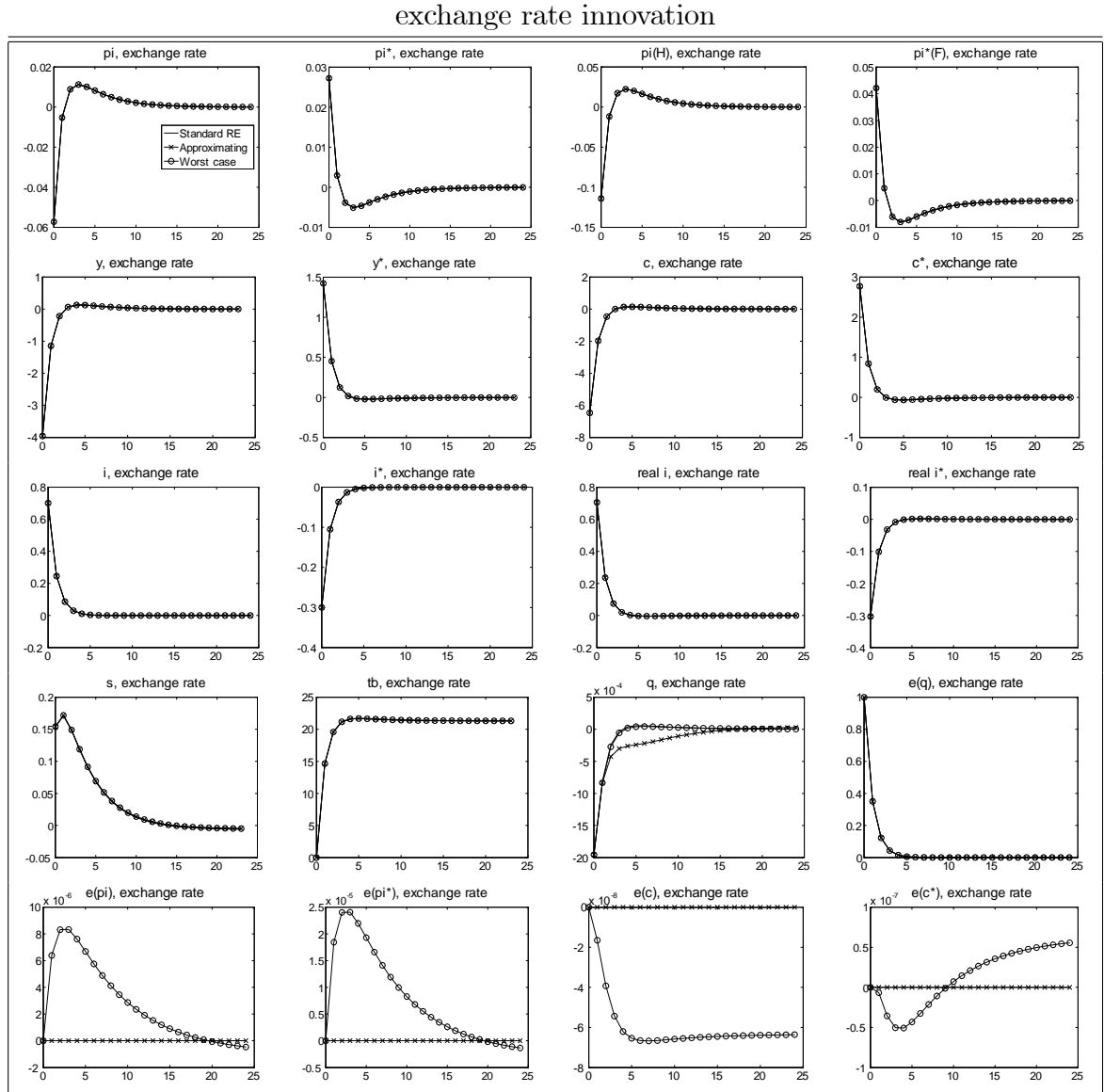


Figure 34: IRFs, commitment, exchange rate innovation, scenario 1

commitment the interest rate differential implies a stronger appreciation of the exchange rate due to improved expectations. All other responses are quite similar to the discretionary case and thus straightforward to interpret. Regarding the robust solution Figure 34 again shows that an exchange rate innovation does not lead to a substantial change in reactions, validating the hypothesis that a fear about misspecification is rather small with respect to a disturbance in the exchange rate. Moreover, the graphs confirm the results from the previous innovations and show that under commitment a robust policy maker's fear is best represented by permanent distortions.

Scenario 2 ($h_c = 0.4, \tau = 0$) The dynamics for the second scenario are illustrated in Figures 35-39 and confirm all previous results regarding persistence in

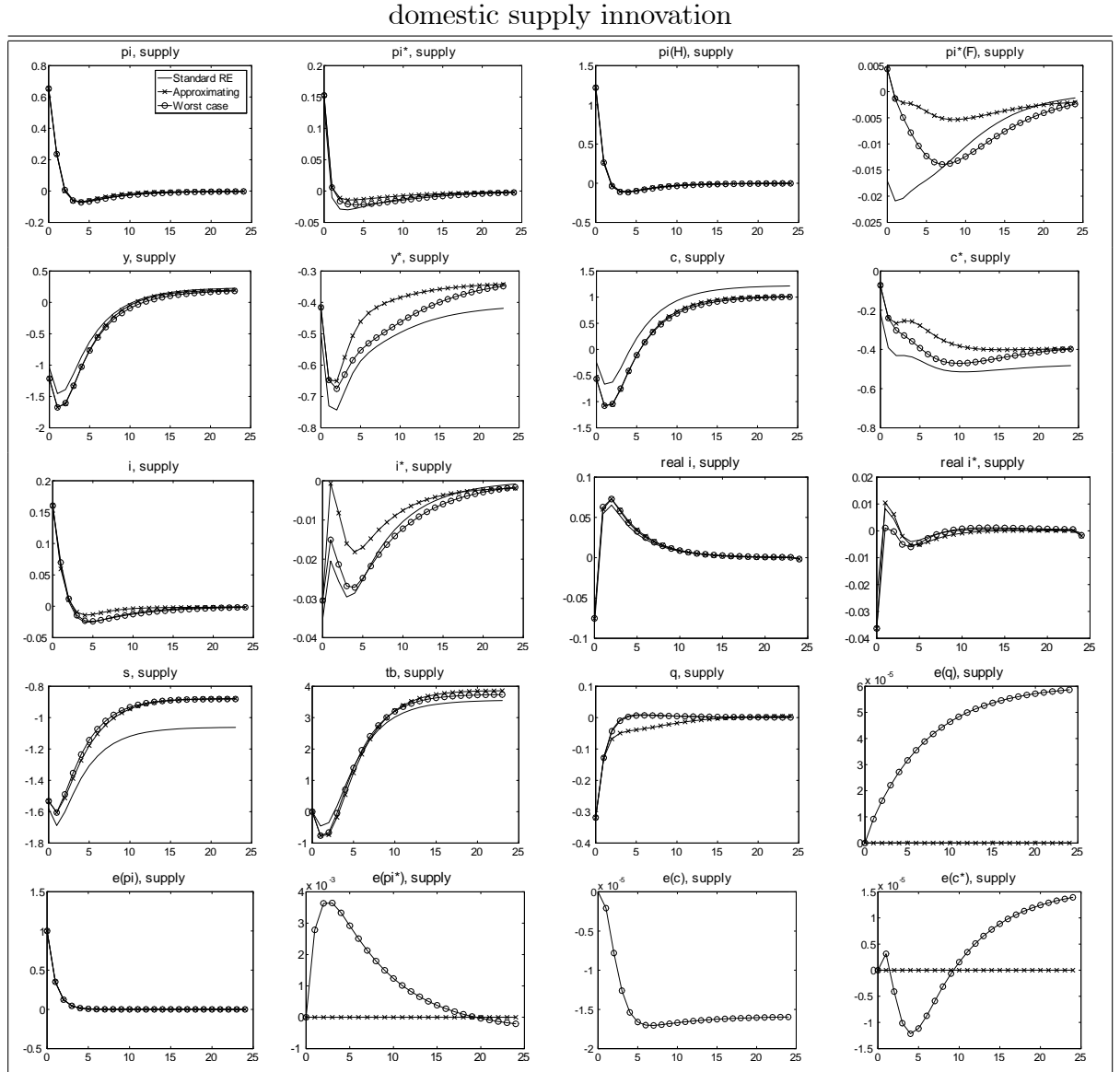


Figure 35: IRFs, commitment, domestic supply innovation, scenario 2

consumption habits. The dynamics are exactly the same as for the first scenario, except for a very small cyclical behavior in consumption indices which is transmitted to both output gaps as well.

When we compare the graphs of the domestic supply innovation with the purely forward-looking scenario, the only difference is a slightly more aggressive fall in the foreign interest rate due to the loss in effectiveness of the monetary policy instrument. However this difference is so small that all other variables completely replicate the corresponding dynamics from the first scenario. Only the consumption and output indices exhibit a slightly more cyclical adjustment due to the persistence in optimal household decisions. Concerning the robust solutions, the only difference to the previous scenario is the stronger reaction of both consumption shocks and a small cyclical component in $e(c^*)$. In an economy including consumption habits

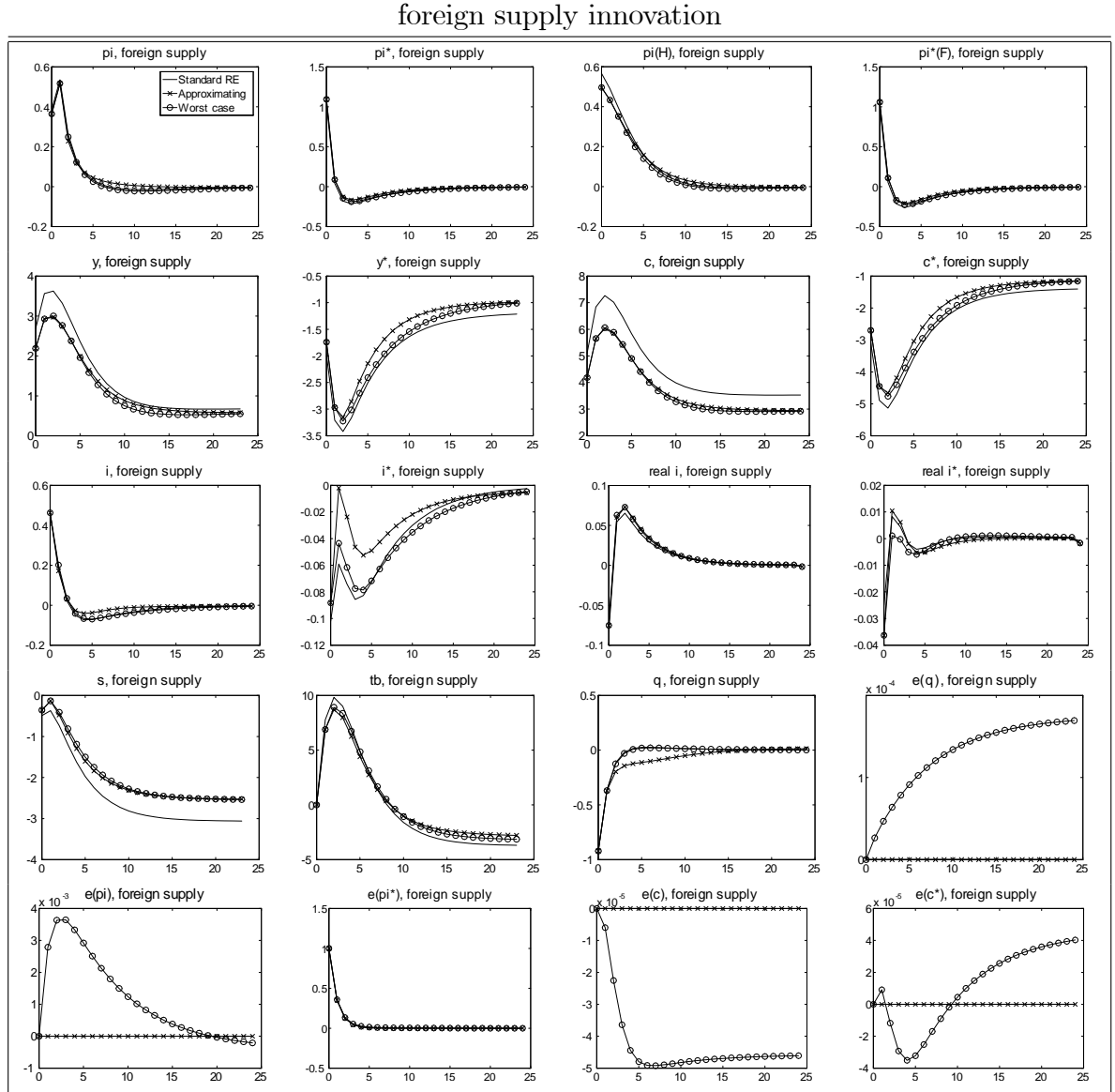


Figure 36: IRFs, commitment, foreign supply innovation, scenario 2

demand shocks have a stronger impact as the transmission on inflation is greater and the monetary authority has to react more aggressively due to the loss in policy effectiveness. Thus the imaginary evil agent generates a foreign demand shock.

In Figure 36 the dynamics after a foreign supply innovation are given. Again the only noteworthy difference from the previous scenario is a slightly more aggressive fall in the foreign interest rate due to the loss in policy effectiveness when the habit adjusted intertemporal substitution elasticity ($\frac{\sigma}{1-h_c}$) decreases. As a consequence, the real foreign interest rate falls more aggressively and dampens the downturn in foreign consumption. On the other hand, domestic consumption increases due to the smaller impact of the real interest rate on consumption decisions. Both consumption dynamics exhibit a small cyclical adjustment path which is reflected by both output gaps as well. Similar to the responses after a domestic innovation the demand shock

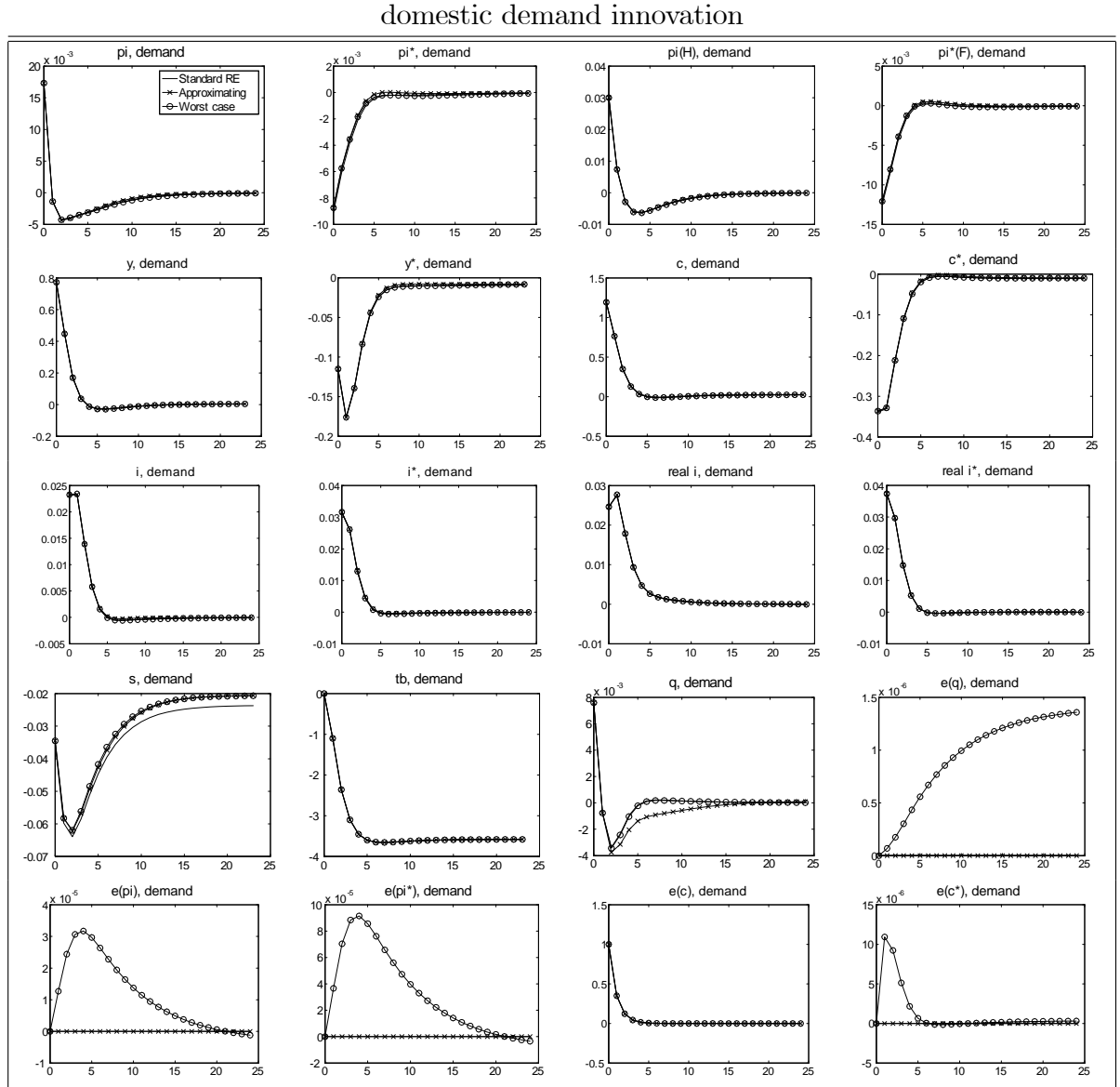


Figure 37: IRFs, commitment, domestic demand innovation, scenario 2

dynamics in the worst case are stronger compared to those of scenario 1. In addition the foreign demand shock dynamics illustrate a small cyclical component.

The impulse responses following demand innovations in Figure 37 and 38 suggest once more that demand disturbances are easier to manage for the central bank, since interest rate reactions and deviations of all other variables from equilibrium are much smaller than for the supply innovations. Additionally the graphs illustrate again that the policy maker's fear about misspecification is smaller in the case of a demand disturbance. The robust dynamics are so close to the *RE* solution that it is difficult to see any differences.

Compared to the first scenario, the domestic interest rate is risen more aggressively in order to manage demand after a domestic innovation. All other responses are quite the same as before. Because of the increased transmission from consump-

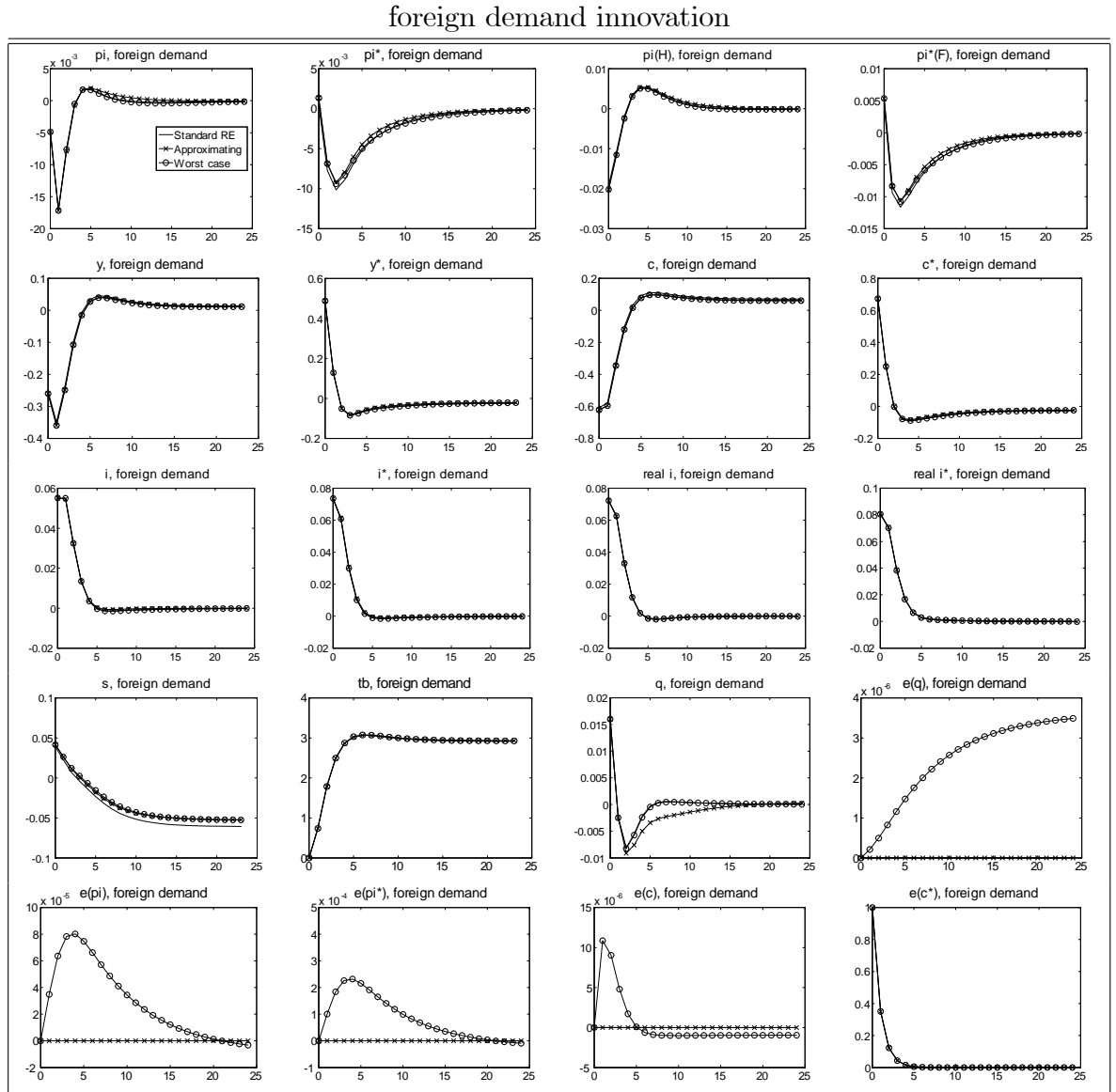


Figure 38: IRFs, commitment, foreign demand innovation, scenario 2

tion to inflation due to the stronger impact on marginal costs, all inflation indices react to a greater extent. Furthermore, the persistence in consumption dynamics imply a small cyclical component in foreign consumption and demand. The robust responses confirm that the evil agent does not generate strong disturbances in response to a demand innovation, but exhibits a stronger increase in the foreign consumption shock, as managing demand requires stronger interest rate reactions due to the loss in policy effectiveness.

The foreign demand innovation validates the results from the domestic one, as the dynamics are nearly the same as before. The domestic interest rate is increased a bit stronger to dampen the increase in overall demand and domestic consumption reacts less and cyclical due to consumption habits. As the optimal response of the foreign interest rate is unaffected, consumption habits increase the primary impact

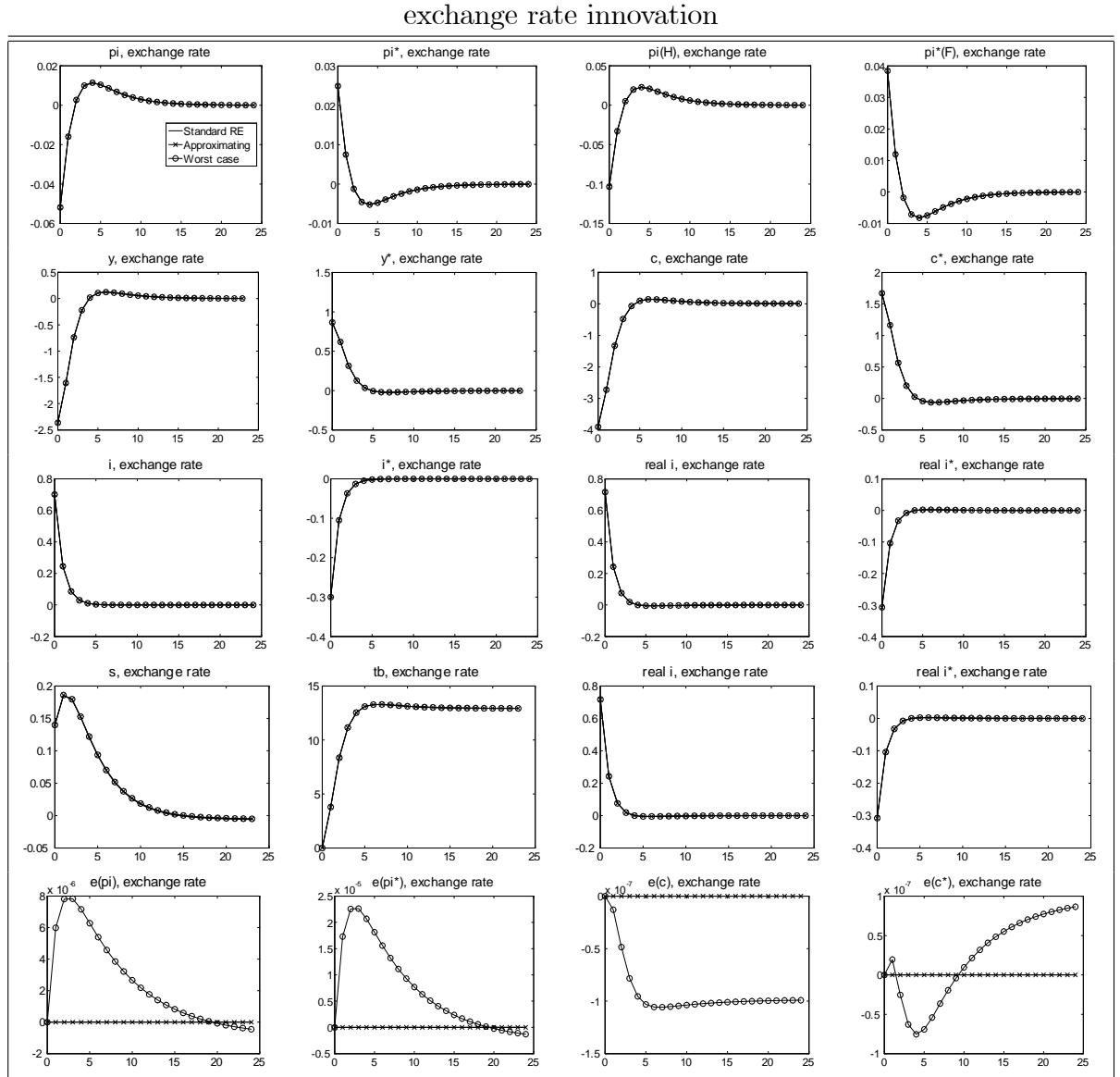


Figure 39: IRFs, commitment, exchange rate innovation, scenario 2

on foreign consumption with corresponding implications for foreign inflation rates. With respect to the robust solutions, the dynamics also exhibit a stronger reaction of the domestic consumption shock to imply a more aggressive interest rate reaction.

The reactions after an exchange rate innovation in Figure 39 are straightforward to interpret as they completely replicate the dynamics of the first scenario. Similar to the discretionary solution, the only difference is that consumption habits decrease the impacts of the interest rates on consumption, so that the deviations from equilibrium are smaller for all variables. This confirms that persistence in consumption habits decreases losses in response to an exchange rate shock. The robust solution seems not to change any response to a noteworthy manner, confirming that the policy maker's fear about the wrong model is rather small when we are to talk about an exchange rate innovation.

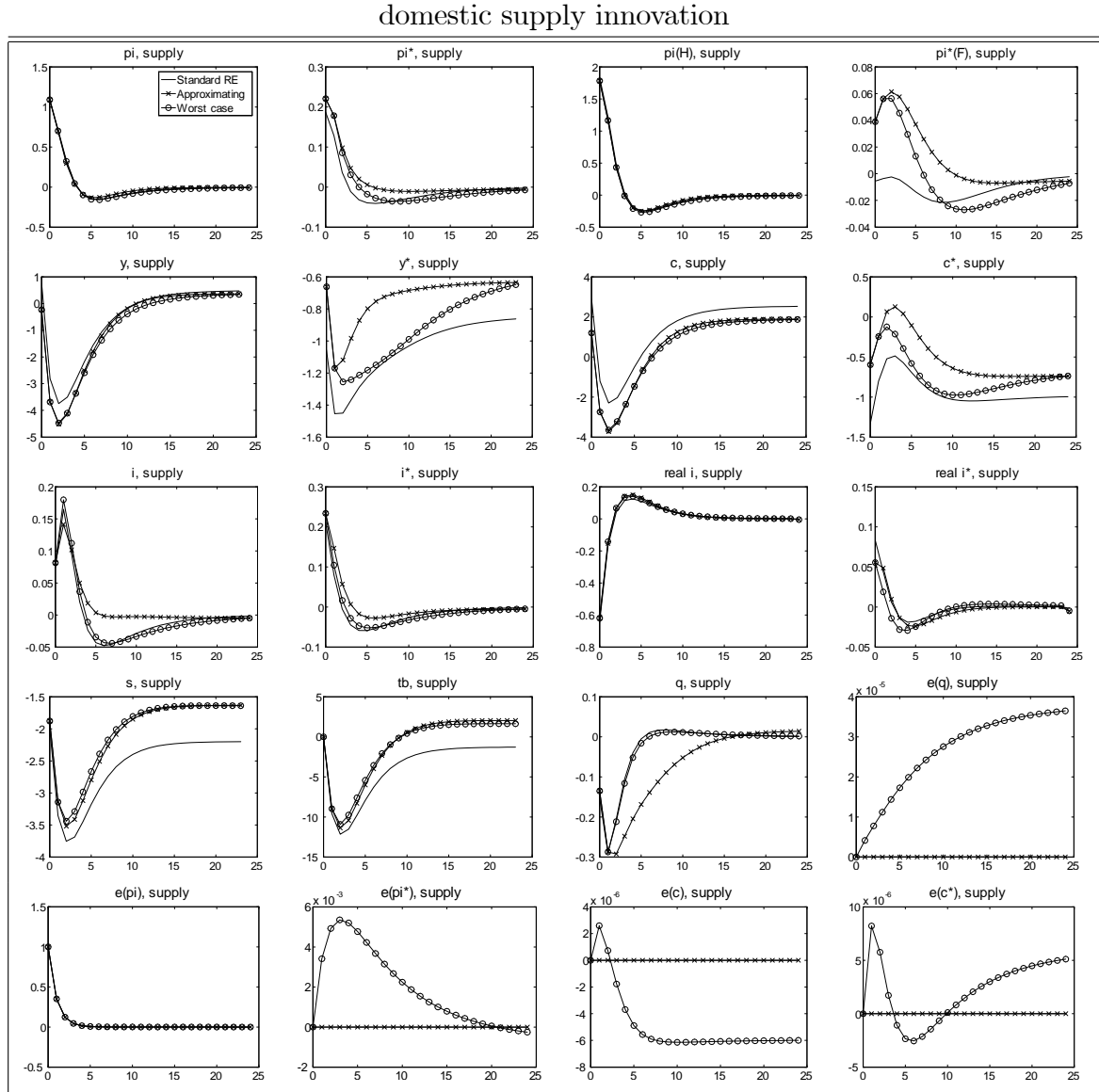


Figure 40: IRFs, commitment, domestic supply innovation, scenario 3

Scenario 3 ($h_c = 0, \tau = 0.4$) Impulse response functions for the model variant including a fraction of backward-looking price-setters are illustrated in Figures 40-44. The graphs give rise to the same conclusions we derived for the discretionary policy maker. As the transmission from output and consumption on inflation via the marginal costs channel deteriorates, the monetary authorities have to increase the interest rate stronger, since a stronger fall in demand is needed in order to decrease inflation. This implies an immense increase in losses for a supply shock where the impulse is already a rise in inflation, but eases monetary policy in the case of a demand shock since the transmission on inflation rates is smaller. Furthermore, the results support our previous finding that the choice of the degree of persistence seems to be more important than a general model uncertainty, modeled by a malevolent second player.

The dynamics for the case of a domestic supply innovation are similar to the purely forward-looking economy for all state variables. However, due to higher inflation expectations under persistent price-setting the initial increase in domestic producer price inflation is already stronger than for scenario 1. Both interest rates are risen more aggressively, where the domestic interest rate exhibits a slow increase and does not reach its peak until the next period. The dynamics of the exchange rate also reflect this kinked interest rate response. As the domestic real interest rate turns negative first, domestic consumption increases initially but suffers a deep slump as the real rate starts rising. The domestic interest rate is risen sharp enough to cause a positive real rate from the beginning and brings about foreign consumption to fall much stronger than under the first model variant. Due to the stronger increase in domestic producer price inflation and both consumption indices, all other variables also deviate more strongly and exhibit a cyclical adjustment path due to inflation persistence. The strong increase (downturn) in domestic (foreign) demand leads to a rise (fall) in the value of exports (imports) and the trade balance becomes negative.

The results in the worst case model are straightforward to interpret. The persistence in inflation dynamics increases the policy maker's fear about supply innovations represented by a stronger increase in the foreign supply shock in expense of the demand shocks. Moreover the partly backward-looking price-setting makes the imaginary evil agent generate cyclical patterns in domestic and foreign demand innovations. The reactions of both interest rates become more aggressive under the worst case even for $p(\theta) = 45\%$. All other dynamics in the worst case are similar to those of the first scenario. Using the robust policy in the approximating model illustrates that due to unfounded overestimation of shocks the foreign producer price inflation expectations increase the actual producer price inflation and thus overall inflation. However, the more aggressive interest rate response of a robust policy maker corrects this distortions very fast for the assumed small amount of uncertainty. Similar to the first scenario, a preference for robustness has an influence on equilibrium outcomes, independent from the actual appearance of the worst case. Moreover, when we compare the influence of introducing a small persistence in price-setting with the impact of uncertainty the graphs clearly provide that the first leads to much more distortions than the evil agent does.

In Figure 41 the dynamics in response to a foreign supply innovation are illustrated. Like the domestic innovation already the initial increase in foreign producer price inflation is stronger due to higher inflation expectations in this scenario. Moreover the smaller transmission from consumption and output to inflation brings about stronger and more persistent disturbances compared to the first scenario. Both interest rates increase more aggressively and the domestic interest rate response again exhibits a slow increase, implying a small cyclical behavior. As all responses are stronger and more persistent, the state variables deviate to a greater extent from their previous steady state. The robust solutions confirm all results from the domestic innovation and imply more aggressive interest rate reactions in order to manage the rise in the domestic supply shock, representing the policy maker's fear about

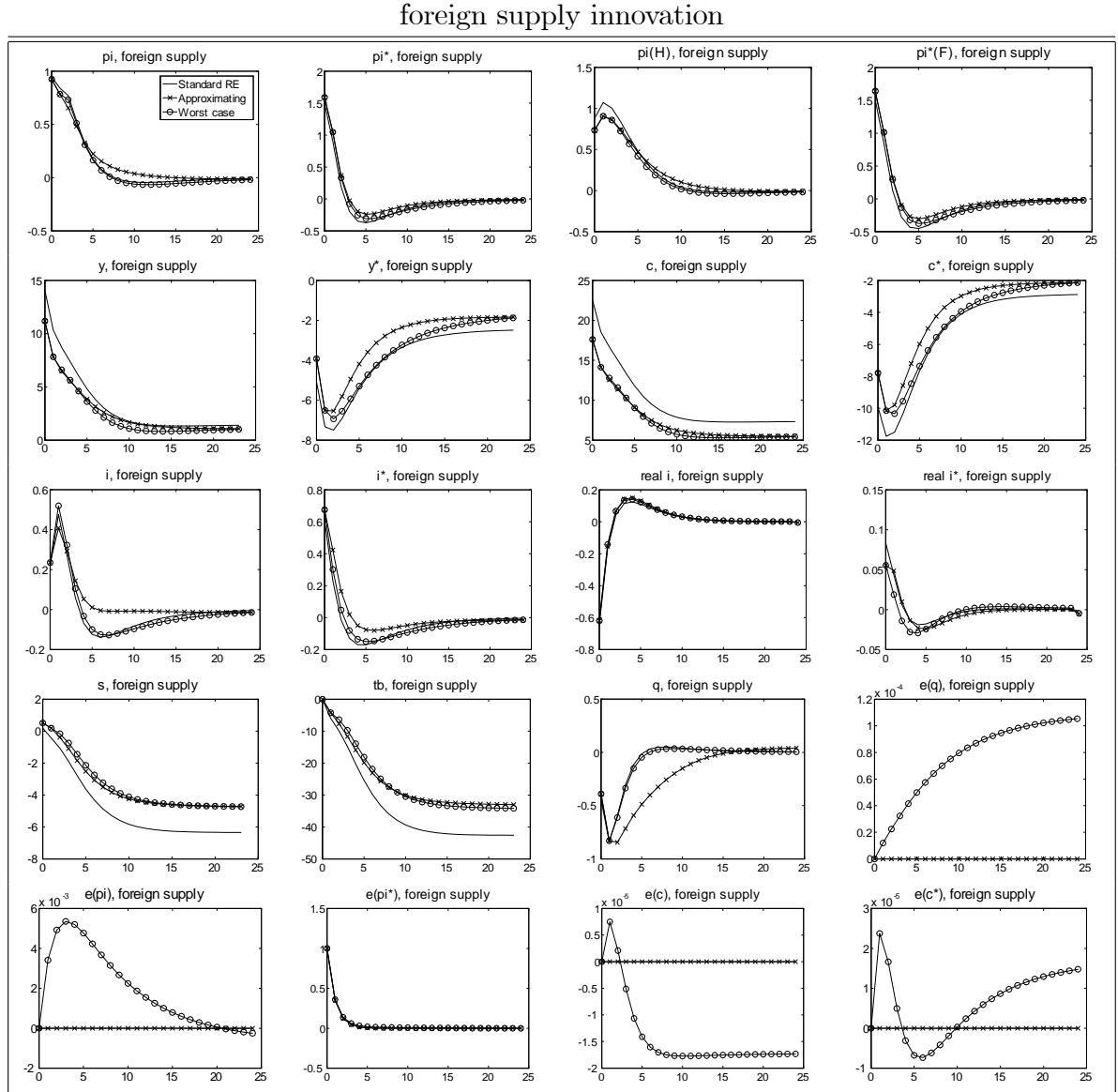


Figure 41: IRFs, commitment, foreign supply innovation, scenario 3

model misspecification. In the case of an unfounded fear, robust expectations lead to slightly stronger disturbances than in the *RE* solution that can be dealt with quite easily for $p(\theta) = 45\%$. Compared to the purely forward-looking model, the impulse responses also indicate that a fraction of backward-looking price adjusters has a stronger impact on the dynamics of the economy than a preference for robustness against a general type of model uncertainty.

The Figures 42 and 43 show the dynamics for the model variant including persistent price setting behavior after demand innovations. Comparable to the discretionary solution the interest rate responses seem not to react to the introduction of a backward-looking fraction of price adjusters. In addition, since the transmission from consumption and output on inflation is smaller, all inflation rates deviate less from equilibrium, illustrating again that, in the case of demand innovations, persis-

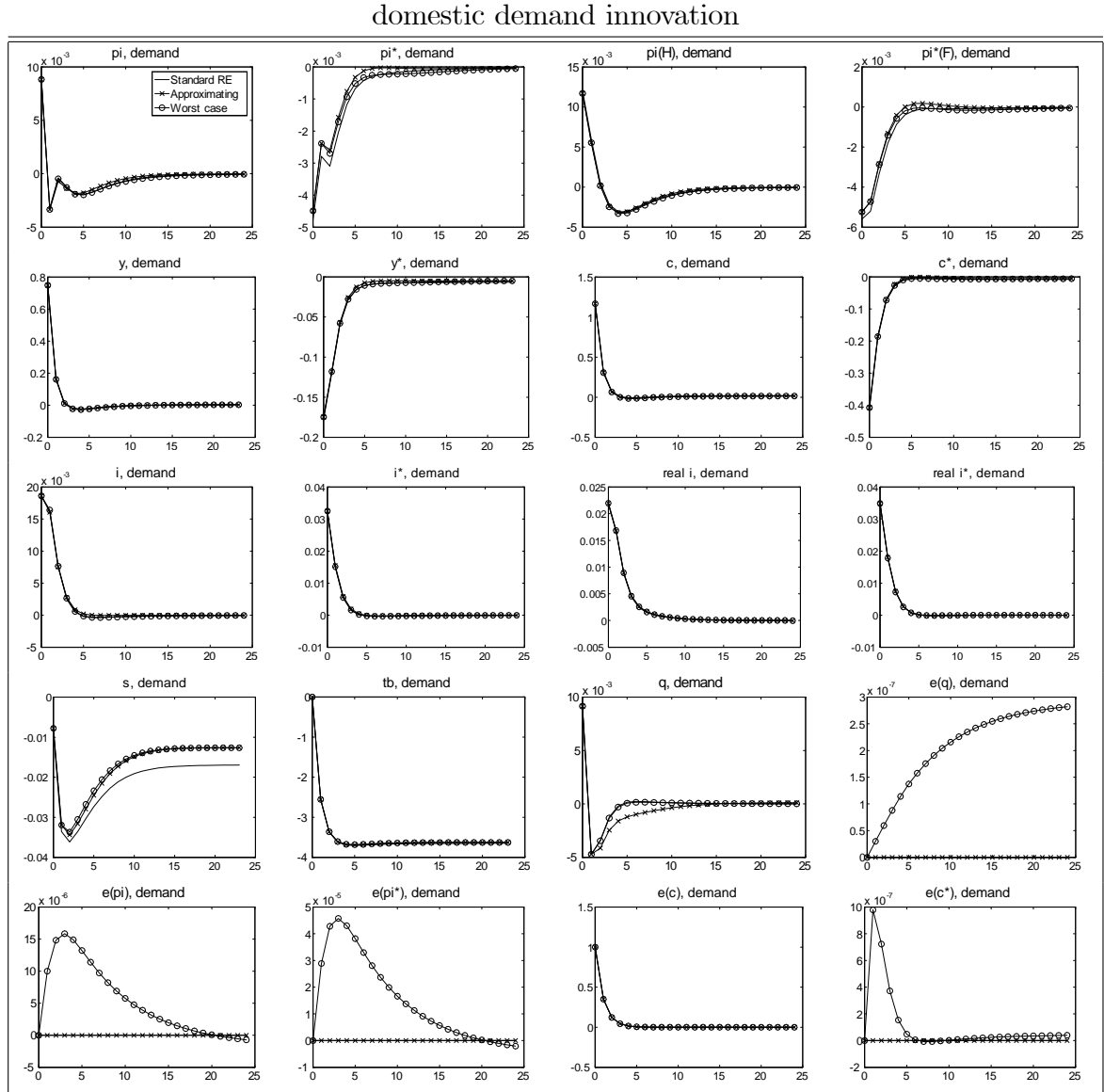


Figure 42: IRFs, commitment, domestic demand innovation, scenario 3

tence in prices represents an advantage for the monetary authority. With respect to uncertainty, the graphs also confirm that the model misspecification the policy maker fears most is rather negligible in response to a demand shock. Furthermore the positive influence of backward-looking firms in the case of demand shocks is greater than the influence of the evil agent.

Dynamics of the model following an exchange rate innovation are shown in Figure 44. All previous suggestions are validated also for the commitment solution under persistence in price-setting. The interest rate responses do not differ to a noteworthy extent compared to the purely forward-looking case, and completely offset the exchange rate innovation. Furthermore, the lower influence of consumption on inflation leads to lower deviations from equilibrium for all variables. Since the impulse is to completely offset the innovation, all deviations are due to the interest

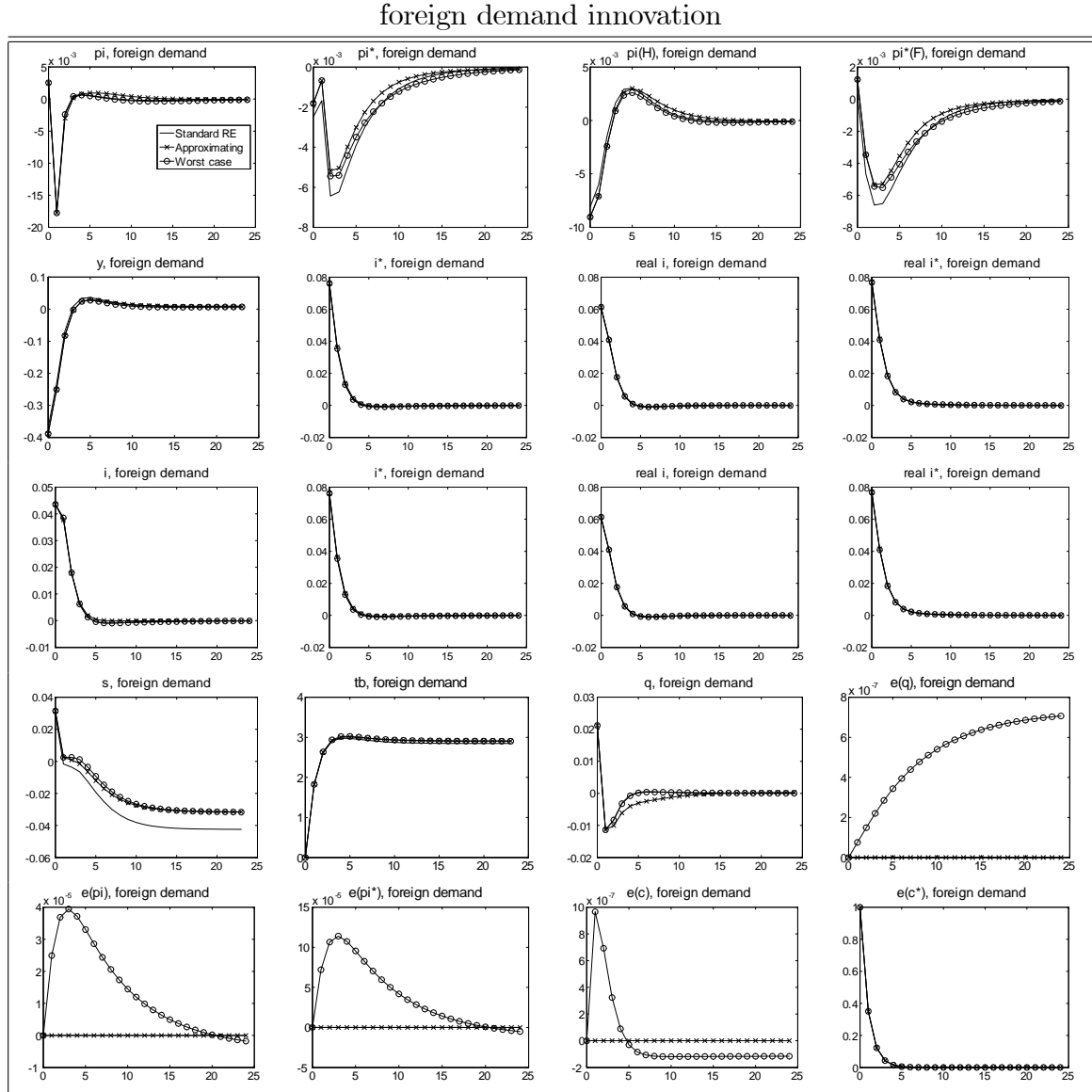


Figure 43: IRFs, commitment, foreign demand innovation, scenario 3

rate responses. As these only influence consumption decisions that are transmitted to a lesser extent due to the lower importance of marginal costs, all implied deviations are smaller. In addition, the robust solution shows no noteworthy difference and confirms that the policy maker fears only small distortions due to model misspecification in the case of an exchange rate shock. Although the interest rates do not substantially differ from those of the first scenario, a comparison leads one to conclude that the influence of persistence on the dynamics of all other variables is greater than the influence of uncertainty.

Scenario 4 ($h_c = \tau = 0.4$) Figures 45-49 illustrate the model dynamics for the fourth scenario. In accordance to all the previous suggestions concerning the influence of consumption habits, the additional assumption of persistence in consump-

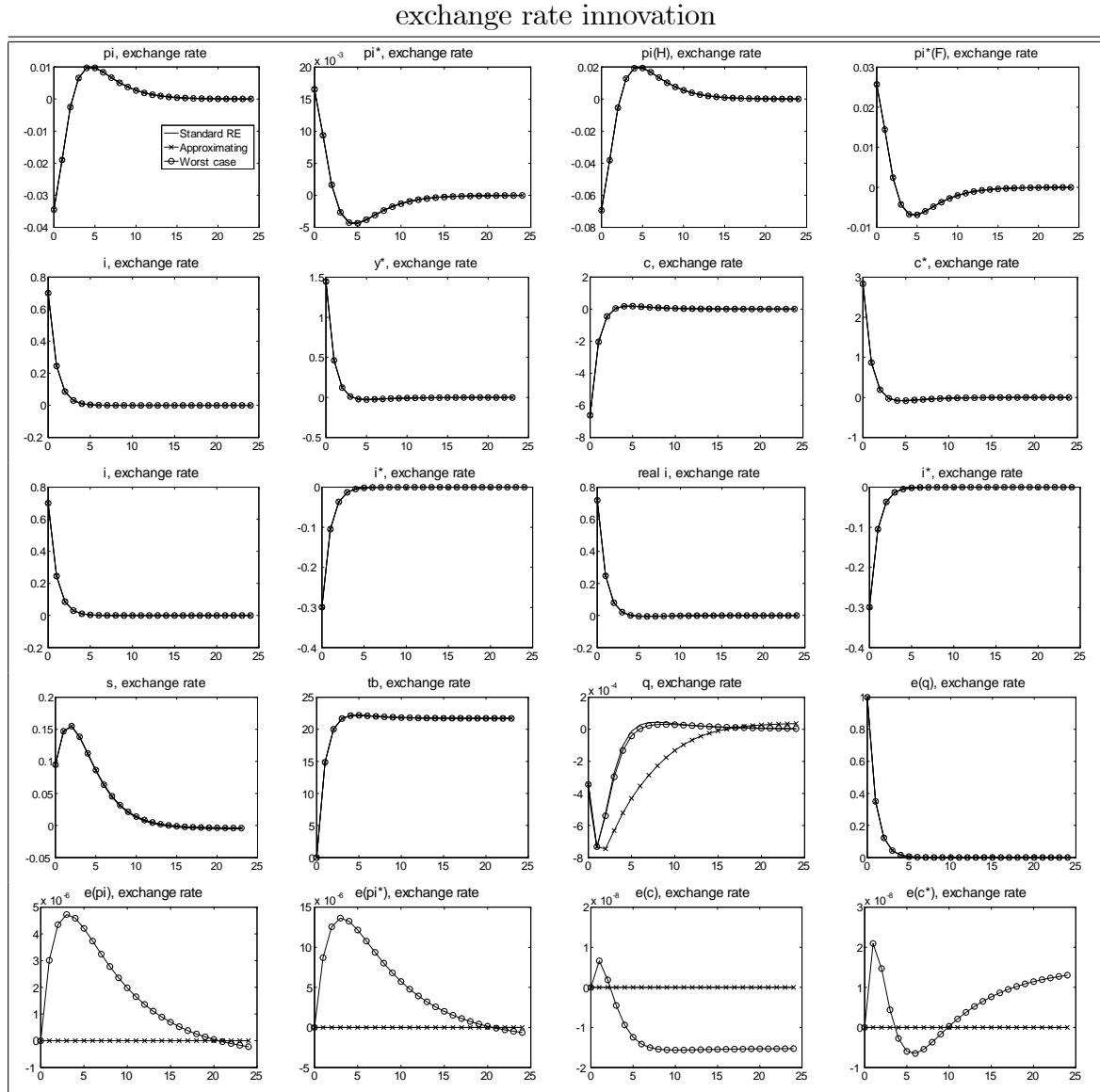


Figure 44: IRFs, commitment, exchange rate innovation, scenario 3

tion dynamics does not lead to strong differences compared to the previous scenario. However, even these minor changes are greater than the influence of a preference for robustness, which refers to $p(\theta) = 45\%$.

The dynamics following a domestic supply innovation are given in Figure 45. Similar to the single-country analysis and the investigation of the discretionary policy maker, the increase in the transmission from consumption and output on inflation serves to be helpful in the case of a supply shock, and, as a consequence, the interest rate reactions are slightly smaller. Furthermore, the smaller responses of the monetary authorities implies smaller consumption reactions as well, and the persistence in consumption dynamics imply a small cyclical behavior in the adjustment of foreign consumption. Since equilibrium values of consumption indices are closer to the previous steady state, domestic demand for foreign products decreases (and vice

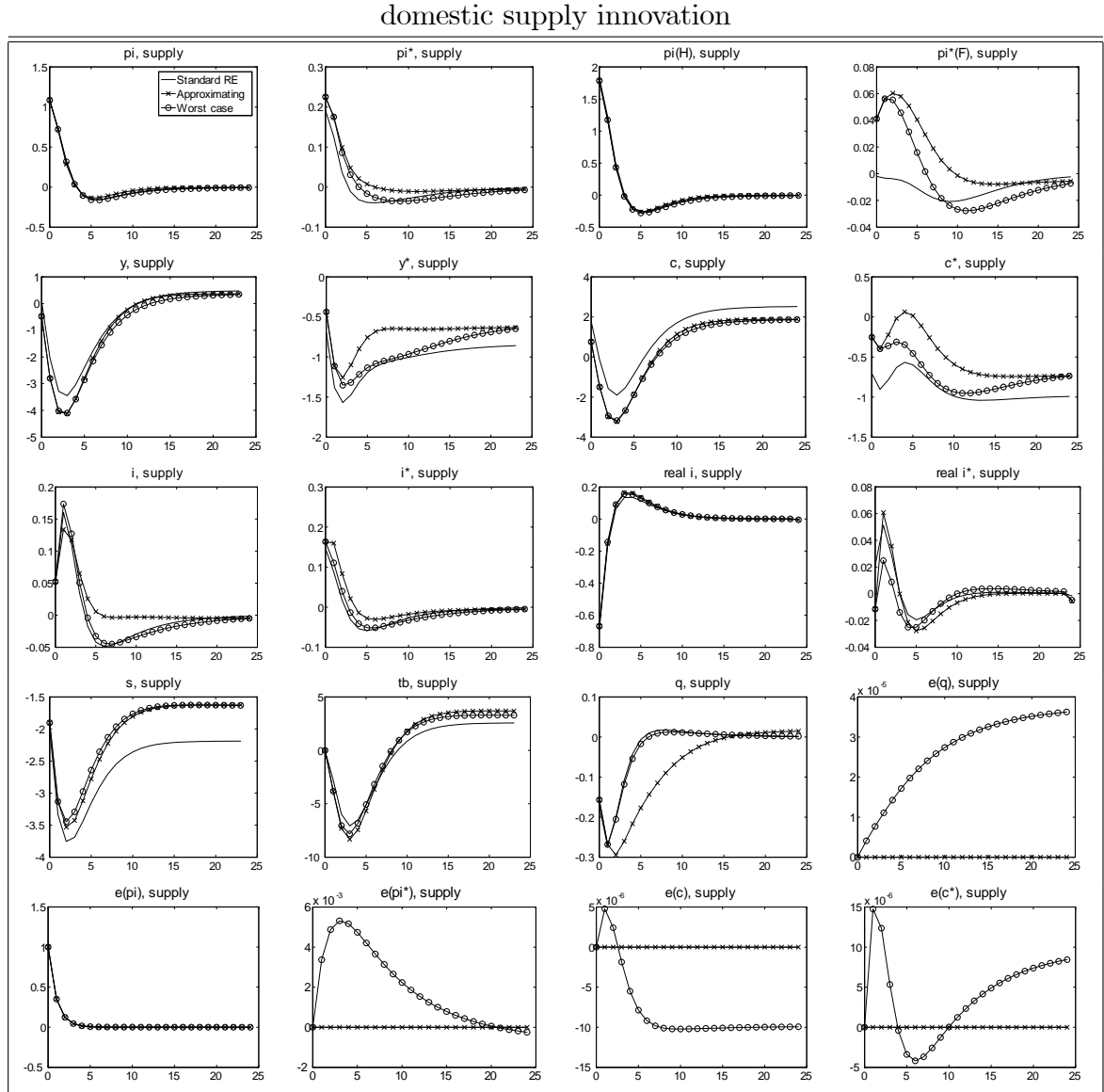


Figure 45: IRFs, commitment, domestic supply innovation, scenario 4

versa) and the trade balance activates. All other responses are straightforward to interpret. Concerning the robust solutions, the only difference to the previous section is the stronger distortions in demand shocks, due to the fact that consumption habits require more aggressive interest rate reactions in order to manage demand, as monetary policy loses effectiveness. In order to reflect concerns about a possible model misspecification the foreign interest rate is increased more aggressively.

A foreign supply innovation implies dynamics given in Figure 46. The graphs exhibit the expected changes due to the introduction of consumption habits, suggested by previous scenarios. Similar to the domestic innovation the interest reactions are a bit smaller, since the transmission from demand to producer price inflation increases, and stabilizing world inflation thus requires a smaller fall in demand. As a consequence, domestic and foreign consumption deviates less from equilibrium and

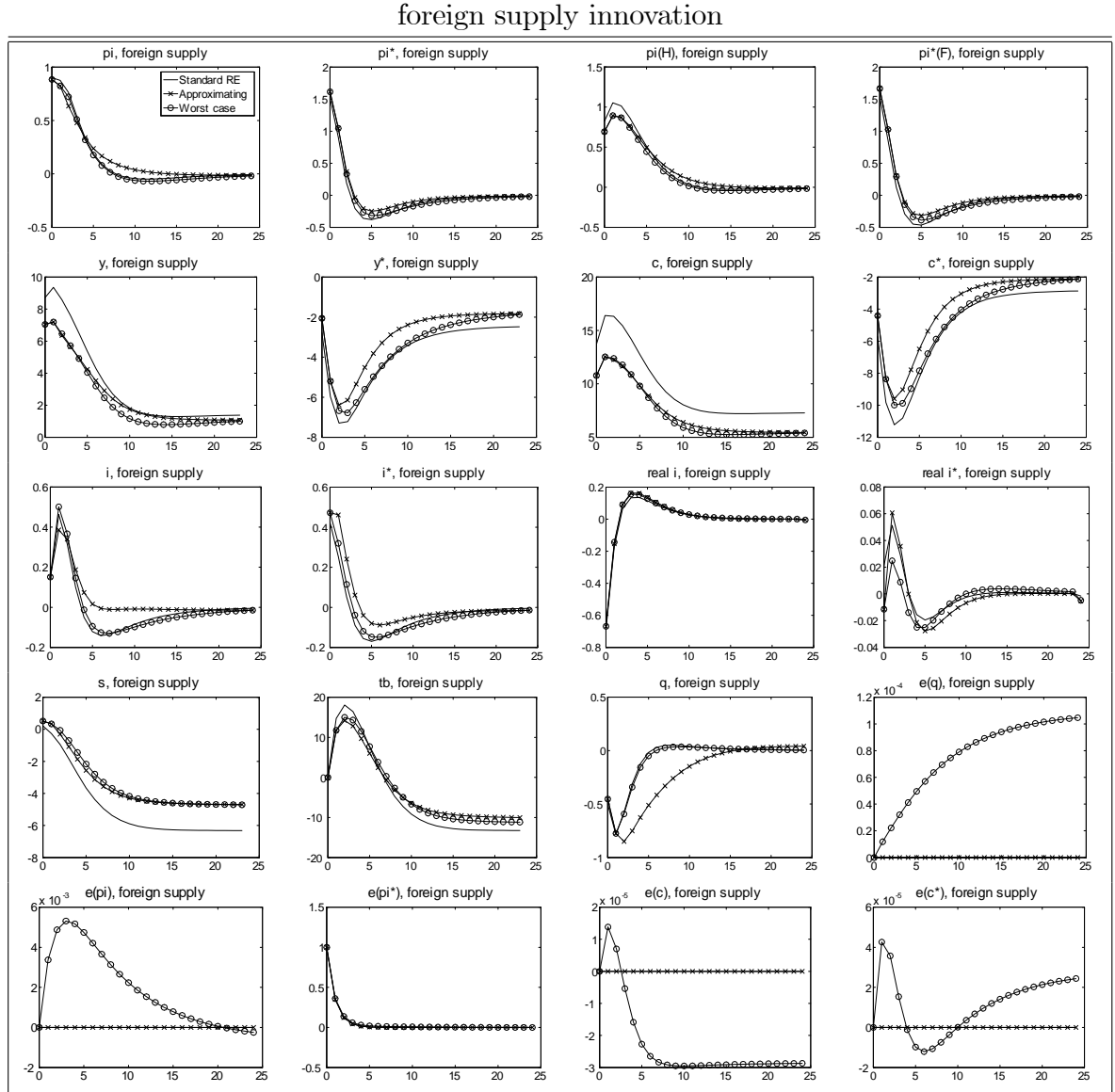


Figure 46: IRFs, commitment, foreign supply innovation, scenario 4

the trade balance activates first, as the smaller domestic equilibrium consumption implies a downturn in domestic demand for foreign products. Concerning the robust dynamics, conclusions are the same as before and confirm that in response to a foreign supply shock, the foreign interest rate is risen more strongly in order to fight the greater shock impacts, feared by private sector and the central bankers. All other dynamics follow the same logic as for the domestic innovation.

The resulting graphs in response to a domestic demand innovation are given in Figure 47. Contrary to the supply shock, but in accordance with the previous suggestions, the interest rate reactions are stronger than in the last scenario. As the impulse is an increase in demand, and as the transmission on inflation is stronger, both central bankers raise their interest rates more aggressively in order to manage demand and dampen inflationary pressures. This is even enforced by the loss in pol-

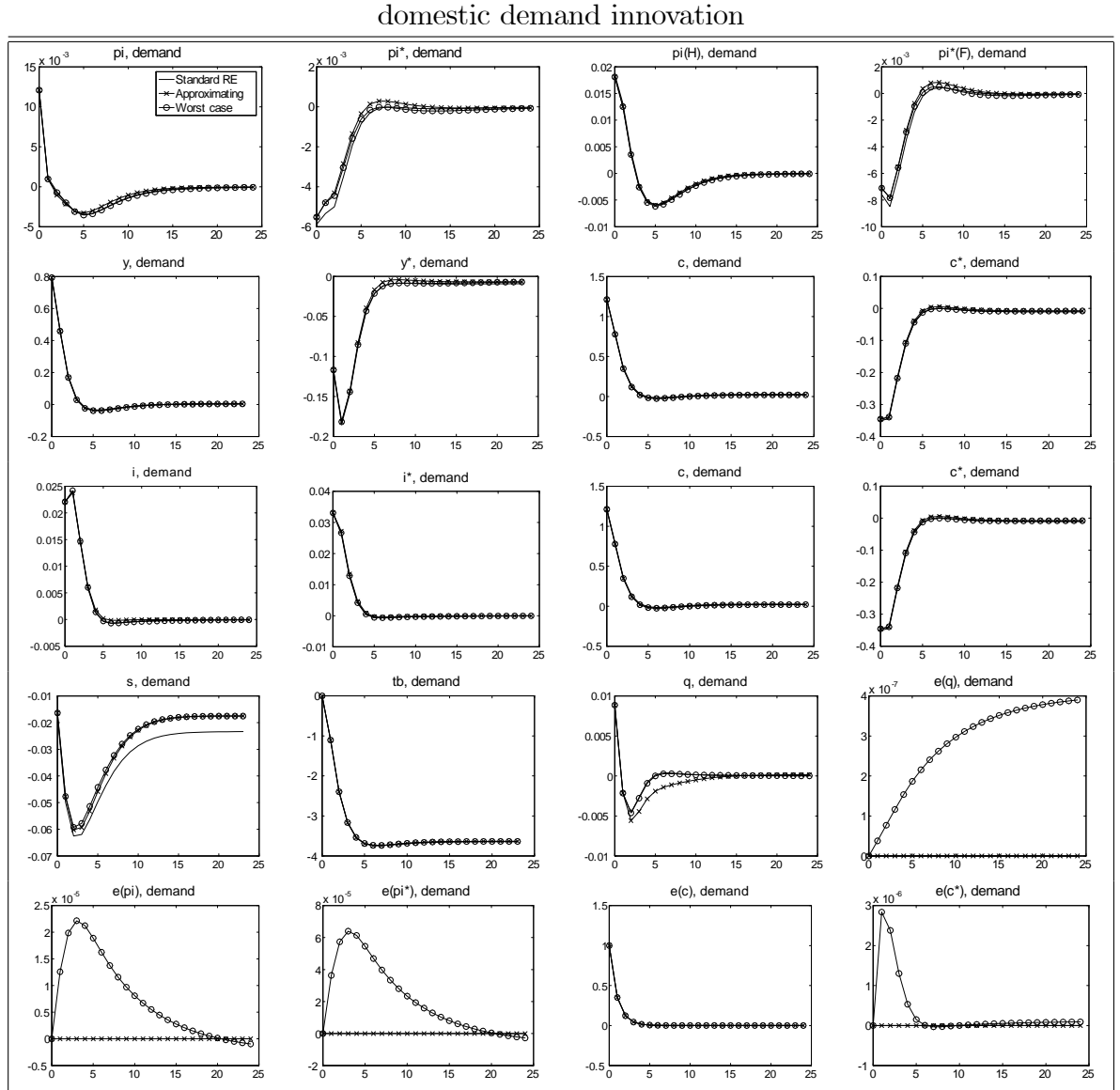


Figure 47: IRFs, commitment, domestic demand innovation, scenario 4

icy effectiveness when household decisions include persistent behavior. The stronger transmission increases the deviations of all variables and confirms that persistence in consumption habits increases losses following a demand innovation. The robust dynamics lead to the same conclusions derived for the supply shocks as the fear about misspecification under persistent consumption behavior is represented by stronger demand shocks, since managing demand becomes more complicated. However, in contrast to the supply shocks, the graphs show that interest rates under uncertainty are unaffected, confirming that the policy maker's fear about misspecification is rather small in response to a demand shock.

Analyzing the dynamics in response to a foreign demand innovation, shown in Figure 48, gives rise to similar conclusion than for the domestic one. Due to the stronger transmission from demand to inflation, all inflation rates react to a greater

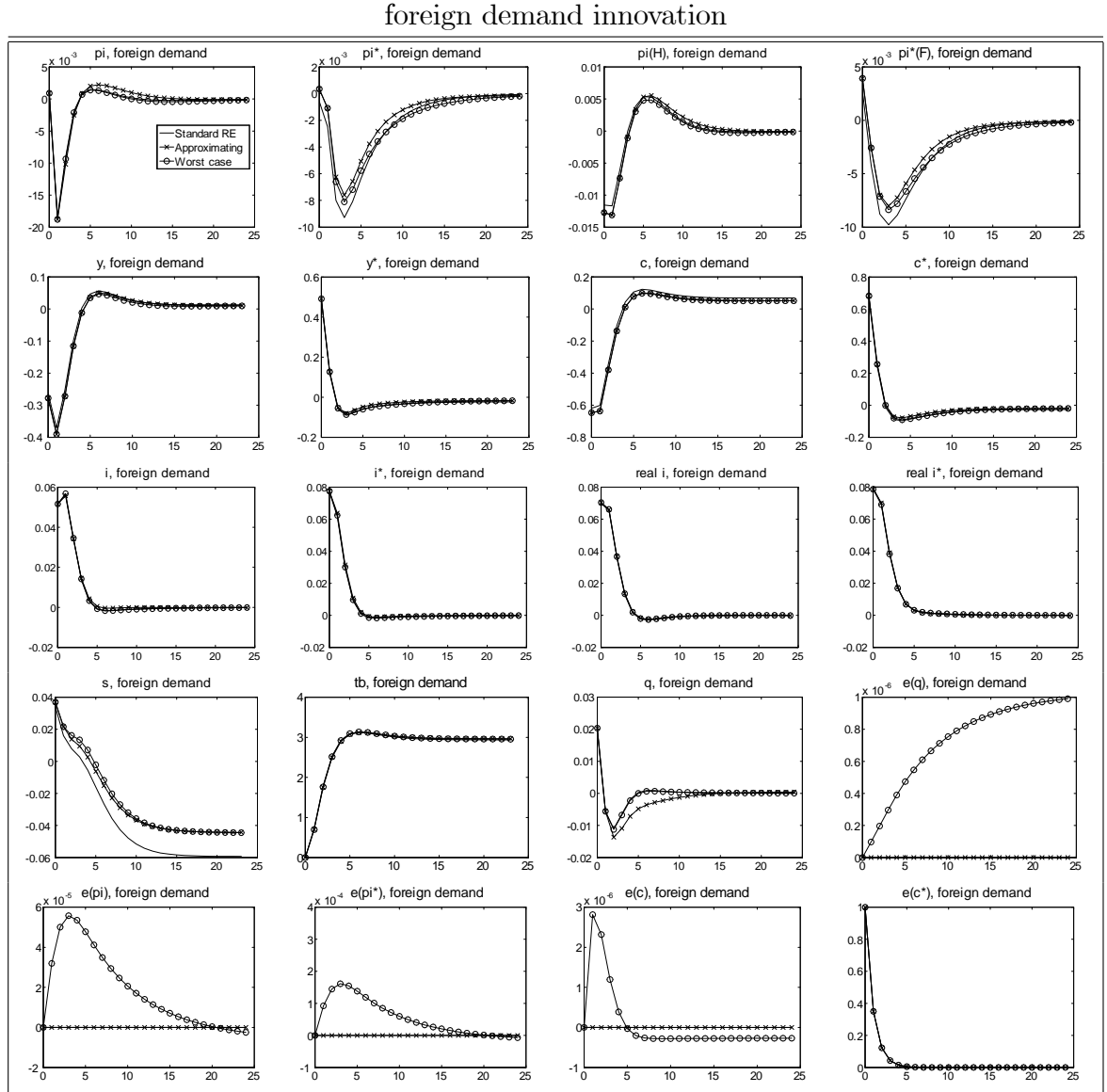


Figure 48: IRFs, commitment, foreign demand innovation, scenario 4

extent, and thus the interest rates are risen more aggressively. All other dynamics follow the same arguments as before and validate the hypothesis that the interest rate responses under uncertainty do not change in the case of demand shocks, illustrating again that the policy maker's fear about misspecification after a demand shock is rather negligible.

The dynamics for the case of an exchange rate innovation in scenario 4 are given in Figure 49. Once again, the graphs confirm the results from previous sections. Including persistence in consumption dynamics leads to less disturbances after an exchange rate innovation as the primary impulse is completely offset by the interest rate responses and all deviations are due to the implied consumption responses. As these react less, all deviations are smaller, although the transmission on inflation increases. With respect to the robust dynamics there are no noteworthy differences

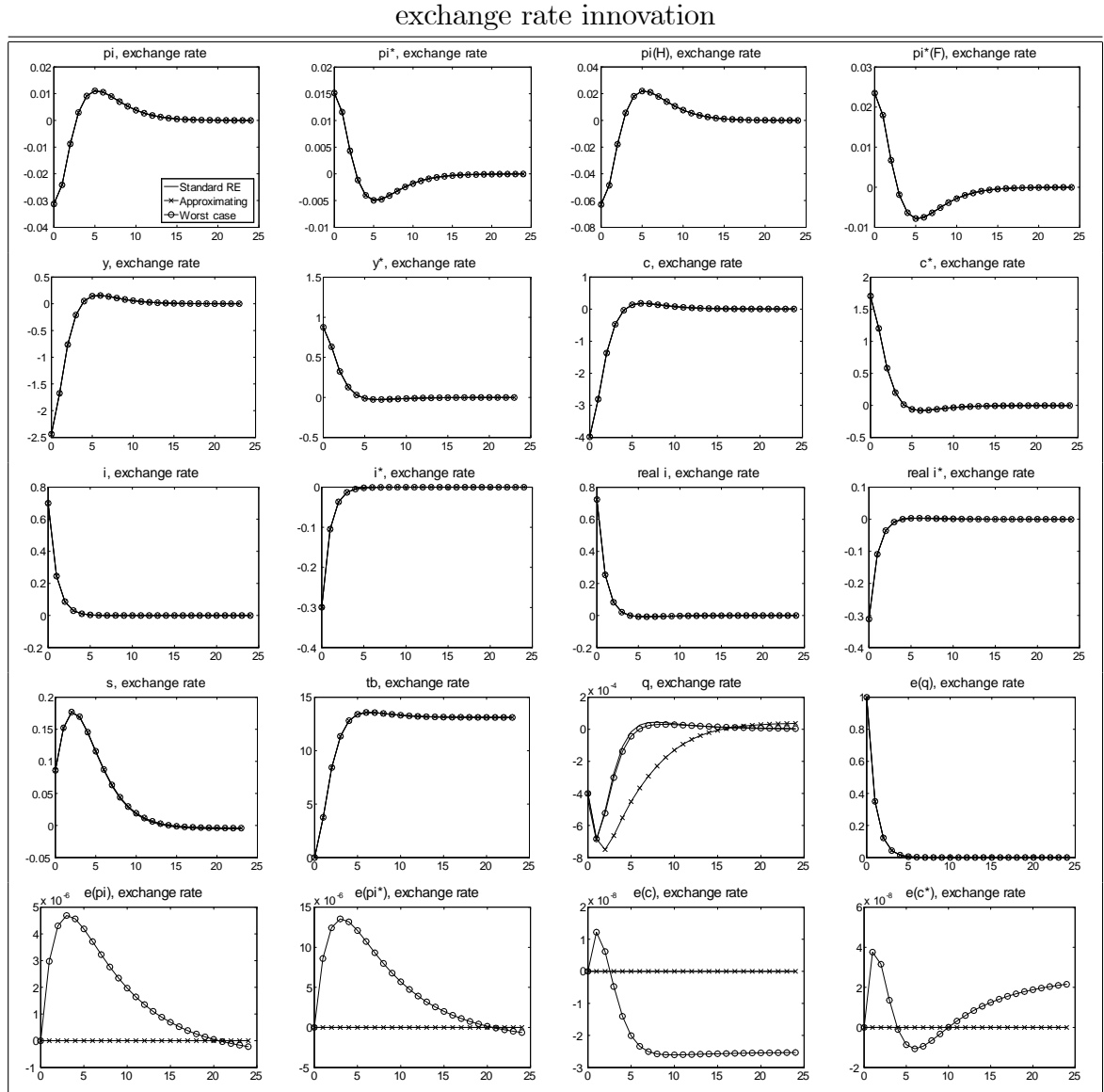


Figure 49: IRFs, commitment, exchange rate innovation, scenario 4

to the previous scenario.

Summarizing the results for a policy maker acting under commitment

Taken as a whole, studying the behavior a policy maker under commitment confirms most findings from the discretionary analysis. However it also provides interesting differences from the previous section. First of all, binding oneself to a commitment allows a policy maker to influence the equilibrium distribution of consumption and output across countries. As this influence is not present in the discretionary solution, it must be due to the improvement of private sector expectations. For both, supply and demand innovations, the domestic interest rate drops so fast after the initial increase in order to dampen inflation, that domestic expectations overshoot. In the new steady state expectations and actual values coincide and are above their

old equilibrium. As domestic (foreign) consumption and output are slightly above (below) their old equilibrium values, the new terms of trade relationship differs from the one shown in (132), and derived under the assumption of equal and constant consumption. The deviations of the new equilibriums from the old steady state terms of trade are negatively related to the output differentials. Since domestic households consume more than before, and due to the home bias, reflecting the preferences for own products of both countries, the demand for domestic products increases, although the relative prices have changed in favor of the foreign country.

Apart from this result, commitment implies less aggressive interest rate responses and smaller losses compared to the discretionary case, due to the influence on expectations. However, the gains from commitment seem to be difficult to realize. The analysis of the worst case solutions illustrate that by exploiting expectations the malevolent second player finds a strategy that results in a long run deterioration of the economy. In connection with Table 10, which shows that for most objectives and scenarios uncertainty leads to indeterminacy, this supports the suggestion from the single-country analysis, that committing to the optimal interest rate path can lead to instability and infinite losses.¹³⁶ Thus also the two-country investigation questions whether a commitment strategy is merely an academic concept that serves as an unachievable benchmark for theoretical analysis, but also has several practical deficits. As long as economists disagree about the "true" reference model, a central banker is unable to achieve enough credibility to implement the optimal commitment solution, simply because he does not know which model the credibility should be tied to. Moreover the analysis illustrates that the choice of the reference model has a stronger impact on outcomes as the general uncertainty, modelled by a worst case shock process, where we have to keep in mind that $p(\theta) = 45\%$. In addition, the dynamics of the model exhibit some highly unrealistic patterns, which further question the practicability of commitment solutions. For the purely forward-looking model the foreign interest rate is slightly decreasing in optimal response to a domestic and/or foreign supply innovation. Even if interpreted as no reaction, this can not be brought in accordance with reality. In this respect, the results support Clarida, Galí, and Gertler (1999) among others, who believe that a commitment strategy is hardly achievable and remains a theoretical concept discussed by academic researchers.

Conclusions for the two-country economy Introducing persistence in consumption habits and price-setting has similar implications for the conduct of monetary policy as in the single-country analysis. Since consumption habits increase the impact of output and consumption on producer price inflation through the marginal costs channel, demand management needs to be less aggressive in order to dampen inflationary pressures. However, as interest rates have to be increased stronger in order to influence household's consumption decisions, the reactions in response to a demand innovation are slightly more aggressive in order to manage demand, whereas

¹³⁶ Recall that for indeterminate solutions any expected inflation can be justified.

reactions are quite the same for a supply innovation. When prices are partly set in a backward-looking manner, the additional introduction of consumption habits leads to an even smaller increase. When the impulse is a rise in inflation, the impact of consumption habits is twofold, as less aggressive deviations of demand are needed to bring back inflation to steady state, but more aggressive interest rate responses are required to influence consumption. Thus the overall influence of consumption habits on average loss are negligible, whereas the impact of persistence in price-setting behavior implies strong increases in losses as with the lowered importance of marginal costs for price decisions, the stabilizing influence of demand vanishes.

With respect to exchange rate innovations the results for both solutions, discretion and commitment, suggest that it is optimal for the policy maker to completely offset the shock immediately by opposing interest rate reactions. Furthermore, and comparable to the demand innovations, the resulting consumption disturbances cause less damage than the supply innovations, as demand can be directly managed by manipulating the real interest rate. As introducing consumption habits decreases the influence of the interest rate on consumption, deviations are smaller when household behavior exhibits persistence. Persistence in price-setting also results in smaller deviations due to the smaller transmission from the consumption disturbances to inflation. Therefore persistence in both consumption and price dynamics decreases the losses suffered by central bankers in response to an exchange rate innovation.

The results concerning a preference for robustness reflect the fact that demand and exchange rate innovations cause less damaging disturbances. The fear of the policy maker about model misspecification is rather small for these perturbations and the influence on optimal interest rates is negligible. In contrast, the imaginary evil agent has a strong impact in the case of supply innovations, illustrating that the policy maker's fear is greater because of these disturbances. The robust solutions confirm the suggestions of the single-country analysis and a vast amount of other studies using robust control techniques for a supply innovation and provide that monetary policy should react more aggressively under uncertain circumstances.

Analyzing the robust strategies in the approximating model gives little support for the conclusions of the single-country analysis, as the commitment solution seems to be very sensitive with respect to uncertainty, and using the robust rule in the approximating model might lead to instability. Moreover the results clearly confirm that the choice of the right reference model is more crucial than uncertainty around a particular model specification. A robust policy maker has to bear this in mind and react slightly more aggressive under uncertainty, but much more aggressive when prices are partly set in a backward-looking manner, especially in response to a supply disturbance.

8.2.3 Discussion and comparison with related literature

Comparison with the single-country analysis When we are to compare the results from the two-country analysis with the single-country investigation, most

conclusions are the same. However, contrary to section 6 the two-country analysis suggests that dynamics after demand innovations are quite robust with respect to uncertainty. In a two-country economy the imaginary evil agent causes only very small disturbances in case of demand innovations, but distorts the model quite heavily in the case of supply innovations. This is plausible since supply innovations lead to opposing reactions of inflation and consumption. As this is also true for the single-country, the results suggest that introducing international trade does even strengthen the relative importance of supply to demand innovations. And the same is true for exchange rate innovations, which are completely offset at the beginning and show no noteworthy difference with respect to robustness. Altogether this suggests that the policy maker fears misspecification most in the case of supply shocks, and can be interpreted as fearing misspecification in the NKPCs more than in the Euler equations of the households. This confirms some results from the most recent literature that we will discuss soon. In addition we find little support for the argument that international trade can increase the uncertainty, as the results under commitment suggest that stabilizing the two-country model with a preference for robustness is more difficult.

Furthermore, in the two-country economy the optimal commitment solution implies persistent deviations of output and consumption from equilibrium that are not present in the single-country case. This is plausible as for one closed single economy output and consumption need always to be equal, whereas for a two-country model multiple combinations of c , c^* , y and y^* can lead to world market clearing $Y_t^W = C_t^W$. Moreover, different combinations of these variables can lead to the same utility. When the additional domestic consumption is greater than the additional work effort needed to produce the additional output, marginal utility of consumption might again equal the marginal disutility of producing. As long as the additional consumption compensates for the additional work effort, households are willing to abandon leisure. For the foreign country the opposite is true, when the fall in consumption is compensated by a fall in work effort. As foreign households hold more leisure, they are willing to abandon consumption.

Comparison with related literature First of all the two-country analysis leads to conclusions similar to those of the robust control literature on single-countries, since the policy maker's fear about misspecification justifies more aggressive interest rate reactions. Furthermore, the investigation suggests that the policy maker's fear about misspecification in the NKPC is greater than for the other equations. This confirms the results of Angeloni, Coenen, and Smets (2003), who show in the estimated Smets and Wouters (2003) euro area model that uncertain inflation dynamics are a greater problem than uncertain output dynamics. The strong relative influence of persistence in price-setting on dynamics and losses of our two-country model is a further result in favor of this conclusion. In contrast, the work of Onatski and Williams (2003) draws the conclusion that the most dangerous uncertainty is that associated with the slope of the IS curve. However, this analysis

uses the backward-looking closed-economy of Rudebusch and Svensson (1998), and is thus hardly comparable to the hybrid euro area model of Smets and Wouters (2003), or the hybrid two-country economy of our investigation. In addition, the work of Leitemo and Söderström (2004) illustrates that the central banker in a New Keynesian economy offsets any misspecification simply by manipulating the interest rate. Our results are quite similar, since deviations in response to demand innovations are rather small and the impact of uncertainty is negligible.

Concerning the persistent influence of uncertainty on equilibrium outcomes a comparable result is found by Kilponen (2004). This paper uses robust Taylor rules in a closed New Keynesian economy including government expenditures, and shows that robust consumption implies precautionary savings, that can lead to self-fulfilling expectations. Thus, for using the robust rule in the approximating model equilibrium inflation is above and output below their previous values. Similar to our analysis, nominal shocks have an impact on equilibrium real variables. However, we have no influence on equilibrium inflation rates, but on the distribution of output and consumption across countries.

Related to this, a vast amount of studies which evaluate optimal rules across different models (see for example Levin, Wieland, and Williams (2003)) conclude that many rules lead to multiple equilibria due to self-fulfilling expectations that are not related to fundamentals. This is comparable to our result for commitment under uncertainty and confirms that a credible commitment is a dangerous strategy when the true model is unknown. Furthermore, another comparable finding from this strand of literature is that there is no simple rule that performs well across all models, since for the two-country economy we are not even able to find a rule that leads to determinacy in this single model. Additionally, and in accordance with the literature on the evaluation of strategies across different models, also the two-country model confirms that the choice of the right reference model seems to be of more importance than uncertainty around a single reference model.

With respect to robust control analysis, there is no study so far which investigates the influence of a preference for robustness in a two-country framework. The study closest to ours is Leitemo and Söderström (2004), who analyze a small open economy under commitment. Similar to the results of our single-country economy, they show that robustness against uncertainty in inflation dynamics for a closed economy version leads to undesirably high output dynamics and vice versa, when the robust rule is used in the approximating model. For the open economy, uncertainty about inflation dynamics also results in highly undesirable volatility in the exchange rate for the approximating model. The model of Leitemo and Söderström (2004) is purely forward-looking and, due to simplicity, analytically solved. Thus, the study illustrates how the trade-off between output and inflation stability affects the macroeconomic volatility without evaluating the results for a plausible amount of uncertainty. This suggests that for a plausible robustness parameter the dynamics of the model under commitment might be as volatile as those derived in the single-country economy. However, as commitment leads to indeterminacy even

for very small amounts of uncertainty, we cannot validate these outcomes directly, but find some evidence for increasing losses when the robust policy is used in the approximating model, as commitment leads to indeterminacy for most objectives and plausible values of uncertainty, and in one case even to infinite losses.¹³⁷

As mentioned several times, the two-country model presented here is based on the work of Clarida, Galí, and Gertler (2002), Batini and Pearlman (2002), Batini, Levine, and Pearlman (2004) and Batini, Justiniano, Levine, and Pearlman (2005). By assuming no home bias in consumption, the purely forward-looking variant of our model converges to the one in Clarida, Galí and Gertler, who show that potential gains from coordination can be implemented by a Taylor type rule that also responds to foreign inflation. The results from our analysis show that it seems to be very difficult to find such a rule that leads to determinacy. The studies of Batini and Pearlman (2002) and Batini, Levine, and Pearlman (2004) investigate countries of similar size, including a home bias, and draw similar conclusions, as they illustrate that the problem of indeterminacy for inflation-forecast-based rules increases in open economies. Our findings are in line with this result, as it seems to be much easier to determine a simple Taylor rule to stabilize the closed single-country economy.

9 Conclusions

To summarize all aforementioned results, arguments and suggestions to one single robust conclusion is akin to squaring a circle. The most robust result of this study is that a monetary authority should at best have visionary powers. However, there are some conclusions in robust New Keynesian monetary policy analysis, which are summarized in the following.

Firstly, all results confirm the vast amount of the most recent robust control literature and illustrate that due to the overestimation of shock impacts, a robust policy maker reacts more aggressively. Furthermore, the investigation provides the plausible suggestion that supply disturbances imply stronger perturbations in the economy than demand shocks, since influencing demand can be done directly via the real interest rate. In addition, the study illustrates that in a two-country economy this leads the central banker to fear model misspecification most in the case of supply innovations. Thus, for the case of supply disturbances the robust policy maker clearly reacts more aggressive, whereas the responses to demand and exchange rate perturbations do not change considerably under uncertainty.

With respect to the impact of persistence in price-setting and consumption habits, the model dynamics demonstrate that one has to distinguish between demand, sup-

¹³⁷Concerning the aggressiveness of monetary policy, the results of the study are not comparable to ours as the authors investigate the influence on closed form coefficients and thus the optimal interest rate rule only reacts to shocks. In contrast, and following the work of Giordani and Söderlind (2004) and Hansen and Sargent (2007), we write the model in state-space form and assume shocks to be predetermined variables. The evil agent is allowed to respond to all state variables, including lagged shocks as well.

ply and exchange rate innovations. An increase in consumption habits has a twofold impact, as the transmission from demand to inflation increases, but the habit adjusted intertemporal substitution elasticity $\left(\frac{\sigma}{1-h_c}\right)$ falls too. Thus a demand disturbance is transmitted less to inflation and interest rates react less, whereas for an increase in inflation, strong responses are needed in order to cause a fall in demand, strong enough to bring inflation back to steady state. The losses of both models suggest that average performance of the economy seems to be unaffected by persistence in consumption dynamics. For persistence in price-setting the effect is unambiguous and implies a smaller transmission from demand to inflation, since the influence of consumption and output via marginal costs decreases when more firms set prices according to a backward-looking rule. Thus the responses after a demand disturbance are smaller as inflation increases to a lesser extent, but a supply innovation calls for stronger interest rate responses in order to dampen inflationary pressures. Concerning losses, persistence in price-setting behavior implies a strong increase, since the loss in effectiveness through the marginal costs channel clearly deteriorates the policy instrument of the central banker without any opposing effect, as in the case of persistence in consumption habits. This also illustrates the importance of the marginal costs channel for the effectiveness of monetary policy.

When we compare the influence of model uncertainty with the influence of persistence, the findings suggest that introducing persistence in price dynamics is more damaging than an realistic amount of general model uncertainty. This is in line with the most recent research, evaluating the performance of policy rules across different models. These studies also conclude that a policy rule that works well in one model might at the same time lead to a catastrophe in a different model. Thus the choice of the right reference model seems to be of more importance than robustness against misspecification in one single reference model.

Most importantly, the study presents a vast array of arguments in favor of not implementing the credible commitment solution, although the losses under commitment are substantially lower. Since under uncertain conditions the credible binding to commitment can lead to very undesirable outcomes, relying on the improvement of expectations seems to be a costly strategy, when the true relationships in the economy are uncertain. What is more, the dynamics of the two-country economy under commitment are quite unrealistic. In contrast, the discretionary policy maker is able to react to uncertainty quite easily. Reoptimizing every period seems to be a superior strategy when the structure of the economy is uncertain, as a discretionary policy maker is able to react to unforeseeable events.

Using a simple rule could reduce the uncertainty of the private sector as to the monetary strategy of the central banker. However, the single-country analysis suggests that it might be difficult to find a rule that leads to a determinate equilibrium in the case of uncertainty, and suggests that policy makers should deviate from such a rule to react on unexpected incidents. This is again confirmed by studies which evaluate across different reference models, as one robust conclusion of all these stud-

ies is that there is no simple rule that works well across models that, for example, differ in assumptions on expectations formation. Furthermore, for the two-country economy no simple rule can be found which leads to determinacy, even for the standard RE solution. As under indeterminacy any expected inflation rate could be justified, this investigation suggests that also using a simple rule should at best be a benchmark for monetary policy analysis, that might reduce uncertainty about the monetary strategy of a central bank, but should only be one measure for optimal monetary policy among many others. In this respect we are in line with Clarida, Galí, and Gertler (1999) and Walsh (2003) among others who believe that the optimal commitment strategy is a theoretical concept for researchers, whereas real monetary policy is (and should be) conducted in a discretionary manner. Thus we support the view of Taylor (1999) and most other economists that in order to react on unforeseeable events, and due to changing economic structures, it is necessary to break any binding. Since the policy maker does not even know on which model to base his credible behavior, it is clear that commitment in the case of uncertainty is only a spurious commitment. Academics and practitioners should bear this in mind when contemplating monetary policy.

A Mathematical appendix

A.1 Hansen-Sargent Robustness

A.1.1 Limes in probability

Definition A.1¹³⁸ Denote an estimator of a parameter θ based on a sample of size T by $\hat{\theta}_T$. Then $\hat{\theta}_T$ converges in probability to the true value θ if

$$\lim_{T \rightarrow \infty} P \left(\left| \hat{\theta}_T - \theta \right| < \varepsilon \right) = 1, \forall \varepsilon > 0$$

$$\Leftrightarrow \text{plim } \hat{\theta}_T = \theta.$$

A.1.2 Certainty equivalence principle

Under the assumption that $V(x) = x'Px$ (where P is assumed to be a positive semidefinite symmetric matrix) the problem becomes (after expectations erased ε_{t+1} and the transition function eliminated next period's state)

$$x'Px = \max_{\{u_t\}} \left\{ u'Ru + x'Qx + \beta (Ax + Bu)'P(Ax + Bu) \right\}. \quad (137)$$

The first order conditions of (137) are (note that R is a positive definite symmetric matrix and Q is a positive semidefinite symmetric matrix)

$$(R + R')u + \beta B'PA\tilde{x} + \beta (x'A'PB) + \beta (B'PB + (B'PB)')u = 0$$

¹³⁸See Judge, Hill, Griffith, Lütkepohl, and Lee (1982) for details.

$$\Longleftrightarrow u = -(R + \beta B'PB)^{-1} \beta B'PAx$$

Substituting u into the right side of (137) and rearranging gives the following Riccati equation:

$$P = Q + \beta A'PA - \beta A'PB(R + \beta B'PB)^{-1} \beta B'PA.$$

To proof that P and F are independent of C it is sufficient to show that P is independent of C , because F could only depend on C through P . The independence of P follows directly from the Riccati equation \square ¹³⁹

A.1.3 Derivation of the worst case error ω_{t+1}^*

After expectations erased ε_{t+1} the inner maximization problem is

$$\max_{\{\omega_{t+1}\}} \left\{ \begin{array}{c} z_t' z_t - \theta \beta \omega_{t+1}' \omega_{t+1} \\ + \beta ((Ax_t + Bu_t + C\omega_{t+1})' P (Ax_t + Bu_t + C\omega_{t+1})) \end{array} \right\},$$

what assuming a symmetric matrix P leads to the following first order conditions:

$$\begin{aligned} \beta \left\{ \begin{array}{c} (x_t' APC)' + (u_t' BPC)' + C' P Ax_t + C' P Bu_t \\ + [C' PC + (C' PC)'] \omega_{t+1}^* \end{array} \right\} - 2\theta \beta \omega_{t+1}^* &\stackrel{!}{=} 0 \\ \Longleftrightarrow \theta \omega_{t+1}^* - C' PC \omega_{t+1}^* &= C' P (Ax_t + Bu_t) \\ \Longleftrightarrow \omega_{t+1}^* &= \frac{1}{\theta} \left(I - \frac{1}{\theta} C' PC \right)^{-1} C' P (Ax_t + Bu_t) \quad \square \end{aligned}$$

A.2 Solution methods to RE programs

The solution methods described here are drawn from Söderlind (1999) and Giordani and Söderlind (2004), and exemplarily shown for a model including forward-looking expectations. The corresponding problem without forward-looking variables can be solved by canceling out these variables (which is equivalent to setting n_2 equal to zero). Therefore the transition equation of the economy is given by

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + C\varepsilon_{t+1}, \quad (138)$$

where x_{1t} is a $n_1 \times 1$ vector of predetermined (backward-looking) variables with x_{10} given, x_{2t} is a $n_2 \times 1$ vector of nonpredetermined (forward-looking) variables, u_t is a $k \times 1$ vector of control variables (policy instruments), and ε_{t+1} is a vector of $n_1 \times 1$ vector of innovations (linear combinations of Gaussian i.i.d. shock processes with zero mean and constant variance) to x_{1t} with covariance matrix Σ .¹⁴⁰ The matrices

¹³⁹See Ljungquist and Sargent (2000) for a discussion of certainty equivalence in the stochastic optimal linear regulator.

¹⁴⁰As we build expectations of the vector of forward-looking variables x_{2t} the last n_2 entries of ε_t are empty.

A and B are filled with structural and constant model parameters. Combining x_{1t} and x_{2t} in one single vector $x_t = (x'_{1t}, x'_{2t})'$ leads to the following loss function of the policy maker:

$$J_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t (x'_t Q x_t + 2x'_t U u_t + u'_t R u_t) \right], \quad (139)$$

where β represents the discount factor and the matrices Q and R are (without loss of generality) assumed to be symmetric, and Q , R and U are constant. For convenience we rearrange (138) to get

$$\begin{aligned} Ax_t + Bu_t + C\varepsilon_{t+1} - \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} &= 0 \\ \iff Ax_t + Bu_t + \begin{bmatrix} C\varepsilon_{t+1} \\ x_{2t+1} - E_t x_{2t+1} \end{bmatrix} - x_{t+1} &= 0. \end{aligned} \quad (140)$$

A.2.1 Solution to a standard RE program under commitment

Under the assumption of a policy maker that is able to commit to a constant policy rule, the corresponding Lagrangian to (138) and (140) is given by

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [x'_t Q x_t + 2x'_t U u_t + u'_t R u_t + 2\rho'_{t+1} (Ax_t + Bu_t + \xi_{t+1} - x_{t+1})], \quad (141)$$

where $\xi_{t+1} = \begin{bmatrix} C\varepsilon_{t+1} \\ x_{2t+1} - E_t x_{2t+1} \end{bmatrix}$, and x_{10} is given.

The first-order conditions of this problem are given by

$$\begin{aligned} \{\rho_{t+1}\} : Ax_t + Bu_t + \xi_{t+1} - x_{t+1} &= 0, \\ \{x_t\} : \beta Q x_t + \beta U u_t + \beta A' E_t \rho_{t+1} + \rho_t &= 0, \\ \{u_t\} : U' x_t + R u_t + B' \rho_{t+1} &= 0, \end{aligned}$$

and can be written in matrix form:

$$\begin{aligned} & \begin{bmatrix} I_n & \mathbf{0}_{n \times k} & \mathbf{0}_{n \times n} \\ \mathbf{0}_{n \times n} & \mathbf{0}_{n \times k} & \beta A' \\ \mathbf{0}_{k \times n} & \mathbf{0}_{k \times k} & -B' \end{bmatrix} \begin{bmatrix} x_{t+1} \\ u_{t+1} \\ E_t \rho_{t+1} \end{bmatrix} \\ &= \begin{bmatrix} A & B & \mathbf{0}_{n \times n} \\ -\beta Q & -\beta U & I_n \\ U' & R & \mathbf{0}_{k \times n} \end{bmatrix} \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0}_{n \times 1} \\ \mathbf{0}_{k \times 1} \end{bmatrix}. \end{aligned} \quad (142)$$

Expanding $\rho_t = (\rho'_{1t}, \rho'_{2t})'$, taking expectations of (142) conditional on the information set in t , and reordering the rows by placing ρ_{2t} after x_{1t} leads to

$$GE_t \begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = D \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}, \quad (143)$$

where $k_t = (x'_{1t}, \rho'_{2t})'$ and $\lambda_t = (x'_{2t}, u'_t, \rho'_{1t})'$. This reordering allows to exploit the initial conditions for k_t . As the initial state vector, x_{10} , is given, the forward-looking variables can be chosen freely in the initial period, so that their shadow prices, ρ_{20} , must be zero.¹⁴¹

As the matrix G in (143) is typically singular, we have to use a *generalized* Schur decomposition to find a stable solution to the problem. We decide to use the algorithm provided by Klein (2000) which is well known for its quick solutions: Following a generalized Schur decomposition, for given square matrices G and D there have to exist square complex matrices S, T, Y and Z such that

$$G = YSZ^H \text{ and } D = YTZ^H, \quad (144)$$

where Z^H denotes the transpose of the complex conjugate of Z . The matrices Y and Z are unitary ($Y^HY = Z^HZ = I$), and S and T are upper triangular.¹⁴² Using (144) in (143) leads to (after premultiplying with Y^H)

$$SZ^H E_t \begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = TZ^H \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix}. \quad (145)$$

This decomposition can now be reordered so that the block corresponding to the stable generalized eigenvalues (the i th diagonal element of T divided by its corresponding element in S) comes first.

After defining χ_t as the stable and ψ_t as the unstable eigenvalues we can use

$$\begin{bmatrix} \chi_t \\ \psi_t \end{bmatrix} = Z^H \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} \quad (146)$$

in (145) to derive (after partitioning S and T)

$$\begin{bmatrix} S_{\chi\chi} & S_{\chi\psi} \\ \mathbf{0} & S_{\psi\psi} \end{bmatrix} E_t \begin{bmatrix} \chi_{t+1} \\ \psi_{t+1} \end{bmatrix} = \begin{bmatrix} T_{\chi\chi} & T_{\chi\psi} \\ \mathbf{0} & T_{\psi\psi} \end{bmatrix} \begin{bmatrix} \chi_t \\ \psi_t \end{bmatrix}. \quad (147)$$

This equation shows that ψ_0 has to be zero as otherwise ψ_t would diverge (because the lower right blocks contain the unstable roots). Furthermore (147) shows that when ψ_0 is equal to zero ψ_t has to be zero for all t and therefore¹⁴³

$$E_t \chi_{t+1} = S_{\chi\chi}^{-1} T_{\chi\chi} \chi_t. \quad (148)$$

It follows from (146) by premultiplying with Z and partitioning the result that

$$\begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} Z_{k\chi} & Z_{k\psi} \\ Z_{\lambda\chi} & Z_{\lambda\psi} \end{bmatrix} \begin{bmatrix} \chi_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} Z_{k\chi} \\ Z_{\lambda\chi} \end{bmatrix} \chi_t. \quad (149)$$

¹⁴¹See Currie and Levine (1993) for details.

¹⁴²See Golub and Loan (1989).

¹⁴³Notice that $S_{\chi\chi}$ is invertible as the determinant of triangular matrix equals the product of its diagonal elements, which are non-zero because of the reordering of the eigenvalues.

As we know that $k_0 = (x_{10}, \mathbf{0}_{n_2 \times 1})$ (149) allows us to solve for χ_0 .¹⁴⁴

$$\chi_0 = Z_{k\chi}^{-1} k_0 \quad (150)$$

From (138) follows that $x_{t+1} - E_t x_{t+1} = C\varepsilon_{t+1}$, and Backus and Driffil (1986) show that $\rho_{2t+1} - E_t \rho_{2t+1} = \mathbf{0}_{n_2 \times 1}$. By combining these results we get

$$Z_{k\chi} (\chi_{t+1} - E_t \chi_{t+1}) = k_{t+1} - E_t k_{t+1} = \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}, \quad (151)$$

where $Z_{k\chi} \chi_{t+1} = k_{t+1}$ from (149) is used in the first part of the equation. Solving this for χ_{t+1} leads to

$$\chi_{t+1} = Z_{k\chi}^{-1} \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix} + E_t \chi_{t+1} = Z_{k\chi}^{-1} \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix} + S_{\chi\chi}^{-1} T_{\chi\chi} \chi_t, \quad (152)$$

where we used (148) in the second step. The equations (148) and (152) describe the dynamics of the model. Solving (149) for χ_t and using this in (152) allows us to express the dynamics in terms of the original variables:

$$\begin{bmatrix} k_{t+1} \\ \lambda_{t+1} \end{bmatrix} = \begin{bmatrix} Z_{k\chi} \\ Z_{\lambda\chi} \end{bmatrix} S_{\chi\chi}^{-1} T_{\chi\chi} \begin{bmatrix} Z_{k\chi} \\ Z_{\lambda\chi} \end{bmatrix}^{-1} \begin{bmatrix} k_t \\ \lambda_t \end{bmatrix} + \begin{bmatrix} Z_{k\chi} \\ Z_{\lambda\chi} \end{bmatrix} Z_{k\chi}^{-1} \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}. \quad (153)$$

By using the definition of k_t and λ_t we finally derive

$$\begin{bmatrix} x_{1t+1} \\ \rho_{2t+1} \end{bmatrix} = M \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix} + \begin{bmatrix} C\varepsilon_{t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}, \text{ where } M = Z_{k\chi} S_{\chi\chi}^{-1} T_{\chi\chi} Z_{k\chi}^{-1}, \quad (154)$$

and

$$\begin{bmatrix} x_{2t} \\ u_t \\ \rho_{1t} \end{bmatrix} = N \begin{bmatrix} x_{1t} \\ \rho_{2t} \end{bmatrix}, \text{ where } N = Z_{\lambda\chi} Z_{\lambda\chi}^{-1}, x_{10} \text{ given, and } \rho_{20} = 0. \quad (155)$$

A.2.2 Solution to RE programs under discretion

Under discretion a policy maker reoptimizes every period given the process private expectations are built with. These expectations have to be consistent with actual policy. In this linear-quadratic model, the solution in $t+1$ gives the following quadratic value function, $x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1}$, and a linear relation between the forward-looking and the state variables, $x_{2t+1} = K_{t+1} x_{1t+1}$, that is used by the private sector to form expectations. Therefore the optimization problem is expanded to

$$x'_{1t} V_t x_{1t} + v_t = \min_{\{u_t\}} [x'_t Q x_t + 2x'_t U u_t + u'_t R u_t + \beta E_t (x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1})] \quad (156)$$

¹⁴⁴Blanchard and Kahn (1980) show that a necessary condition for $Z_{k\chi}$ to be invertible is that in (143) the number of predetermined variables (rows in $Z_{k\chi}$) equals the number of stable roots (columns in $Z_{k\chi}$).

subject to

$$E_t x_{2t+1} = K_{t+1} E_t x_{1t+1},$$

and (138), x_{1t} given. We can rewrite this problem by partitioning the matrices and combining the two restrictions:

$$x'_{1t} V_t x_{1t} + v_t = \min_{\{u_t\}} \left[x'_t \tilde{Q}_t x_t + 2x'_t \tilde{U}_t u_t + u'_t \tilde{R}_t u_t + \beta E_t (x'_{1t+1} V_{t+1} x_{1t+1} + v_{t+1}) \right] \quad (157)$$

subject to

$$x_{1t+1} = \tilde{A}_t x_{1t} + \tilde{B}_t u_t + \varepsilon_{t+1}$$

and x_{1t} given, where¹⁴⁵

$$\begin{aligned} D_t &= (A_{22} - K_{t+1} A_{12})^{-1} (K_{t+1} A_{11} - A_{21}) \\ G_t &= (A_{22} - K_{t+1} A_{12})^{-1} (K_{t+1} \tilde{B}_1 - \tilde{B}_2) \\ \tilde{A}_t &= A_{11} + A_{12} D_t \\ \tilde{B}_t &= B_1 + A_{12} G_t \\ \tilde{Q}_t &= Q_{11} + Q_{12} D_t + D'_t Q_{21} + D'_t Q_{22} D_t \\ \tilde{U}_t &= Q_{12} G_t + D'_t Q_{22} G_t + U_1 + D'_t U_2 \\ \tilde{R}_t &= R^* + G'_t Q_{22} G_t + G'_t U_2 + U'_2 G_t. \end{aligned} \quad (158)$$

The first order condition of (157) with respect to u_t is

$$u_t = -F_{1t} x_{1t}, \quad F_{1t} = \left(\tilde{R}_t + \beta \tilde{B}'_t V_{t+1} \tilde{B}_t \right)^{-1} \left(\tilde{U}'_t + \beta \tilde{B}'_t V_{t+1} \tilde{A}_t \right). \quad (159)$$

Together with (157) this leads to

$$x_{2t} = K_t x_{1t}, \text{ with } K_t = D_t - G_t F_{1t}, \quad (160)$$

and

$$V_t = \tilde{Q}_t - \tilde{U}_t F_{1t} - F'_{1t} \tilde{U}'_t + F'_{1t} \tilde{R}_t F_{1t} + \beta \left(\tilde{A}_t - \tilde{B}_t F_{1t} \right)' V_{t+1} \left(\tilde{A}_t - \tilde{B}_t F_{1t} \right). \quad (161)$$

The equations (158) - (161) allow to iterate ("backwards in time") until convergence.¹⁴⁶ it should be started with a symmetric positive-definite V_{t+1} and some K_{t+1} . In those cases where F_{1t} and K_t converge, the dynamics following from (138) are given by

$$x_{1t+1} = M x_{1t} + C \varepsilon_{t+1}, \text{ where } M = A_{11} + A_{12} K - B_1^* F_1,$$

$$\begin{bmatrix} x_{2t} \\ u_t \end{bmatrix} = N x_{1t}, \text{ where } N = \begin{bmatrix} K \\ -F_1 \end{bmatrix}$$

Note that for a backward-looking model, the discretionary and the commitment solution coincide.

¹⁴⁵From these Definitions follows that $x_{2t} = D_t x_{1t} + G_t u_t$.

¹⁴⁶This is the algorithm first provided by Oudiz and Sachs (1985) as well as Backus and Driffil (1986).

A.2.3 Solution to RE programs using simple rules

The problem using a given simple rule is given by:

$$\max_{\{\omega_{t+1}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t (x'_t Q x_t + 2x'_t U^* u_t^* + u_t^{*'} R^* u_t^*) \right] \quad (162)$$

$$\text{subject to } x_{t+1} = (A - B^* F^*) x_t + C \varepsilon_{t+1}, \quad (163)$$

$$\text{and } u_t = -F_s x_t, \quad (164)$$

where $F^* = \begin{pmatrix} F_s \\ F_\omega \mathbf{0}_{n_1 \times n_2} \end{pmatrix}$, $U^* = \begin{pmatrix} U & \mathbf{0}_{n \times 2} \end{pmatrix}$, $R^* = \begin{pmatrix} R & \mathbf{0}_{1 \times 2} \\ \mathbf{0}_{2 \times 1} & -\theta I_2 \end{pmatrix}$,
 $B^* = \begin{pmatrix} B & C \end{pmatrix}$, and $u_t^* = \begin{pmatrix} u_t \\ \omega_{t+1} \end{pmatrix}$.

As described in section 4.2.3 we have to restrict the evil agent's policy function so that it depends solely on the predetermined variables, $\omega_{t+1} = -F_\omega x_{1t}$, whereas the rule of the policy maker can still depend on all state variables, $u_t = -F_s x_t$.¹⁴⁷ Putting this together into $u_t^* = -F^* x_t$ and substituting this in (163) leads (after taking conditional expectations) to

$$E_t x_{t+1} = (A - B^* F^*) x_t.$$

As this expression has the same form as (143) the same solution method can be used. The only differences are that $G = I$ and $D = A - B^* F^*$. Therefore S and Y are equal to I and Z , respectively, like in the original "ungeneralized" Schur decomposition. Provided that $Z_{k\chi}$ is invertible (F implies a unique equilibrium) the solution is the same as in (154) and (155), except that u_t , ρ_{1t+1} , and ρ_{2t+1} are now empty vectors:

$$x_{1t+1} = M x_{1t} + C \varepsilon_{t+1}, \text{ where } M = Z_{k\chi} T_{\chi\chi} Z_{k\chi}^{-1}, \quad (165)$$

and

$$x_{2t} = N x_{1t}, \text{ where } N = Z_{\lambda\chi} Z_{\lambda\chi}^{-1}, \text{ and } x_{10} \text{ given.} \quad (166)$$

The loss function for given F_s and F_ω is then given by

$$J_0 = x'_{10} V x_{10} + \text{trace} (V C_5 C'_5) \frac{\beta}{1 - \beta},$$

where the matrix C_5 consists of the first 5 rows of C the matrix V is the fixed point in the (backwards) iteration on

$$V_s = P' \begin{bmatrix} Q & U^* \\ U^{*'} & R^* \end{bmatrix} P + \beta M' V_{s+1} M, \text{ where } P = \begin{bmatrix} I_n \\ -F_s \\ -F_\omega \end{bmatrix} \begin{bmatrix} I_{n_1} \\ N \end{bmatrix}.$$

¹⁴⁷Note that in this paper we only investigate *simple* linear rules that depend on lagged values of inflation, the output gap, and the interest rate. The more general solution for a simple rule depending on all state variables is shown here because the algorithm is the same.

The evil agent maximizes this loss function by choosing the optimal elements in F_ω , subject to the restriction that decision rule should give a unique equilibrium and that x_{10} is given. This rule must typically found by a non-linear optimization algorithm and depends on C_5 and the initial state vector x_{10} .¹⁴⁸

A.3 The New Keynesian single-country framework

A.3.1 Household optimization

Households choose the cost minimizing combination of individual goods to achieve any given level \bar{C} of the composite consumption good. The Lagrangian of this problem is given by

$$\mathcal{L} = \int_0^1 p_{jt} c_{jt} dj - \psi_t \left[\left[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta}{\zeta-1}} - \bar{C}_t \right] \quad (167)$$

and the first order conditions for good j imply

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_j} &= p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{1}{\zeta-1}} c_{jt}^{-\frac{1}{\zeta}} \stackrel{!}{=} 0. \\ \Leftrightarrow &\left(\left[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{1}{\zeta-1}} c_{jt}^{-\frac{1}{\zeta}} \right)^{-\zeta} = \left(\frac{p_{jt}}{\psi_t} \right)^{-\zeta} \\ \Leftrightarrow &c_{jt} = \left(\frac{p_{jt}}{\psi_t} \right)^{-\zeta} C_t. \end{aligned} \quad (168)$$

With the help of the definition of the composite consumption good, an expression for the Lagrangian multiplier can be derived, $\psi_t = \left[\int_0^1 p_{jt}^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}} \equiv P_t$, which is equal to the aggregated price index (defined as a generalized weighted mean of all prices). Eliminating the Lagrangian multiplier from (168) gives

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\zeta} C_t. \quad (169)$$

A.3.2 Approximation around the steady state

In this section we use the following notations: (i) variables written in small case letters express the logarithm of its upper case versions, $x_t = \ln(X_t)$, and (ii) variables written in small case letters with a hat express the percentage deviation of its upper case versions from its steady state (X_{ss}):

$$\hat{x}_t = \frac{X_t - X_{ss}}{X_{ss}} \approx \ln \left(1 + \frac{X_t - X_{ss}}{X_{ss}} \right) = \ln X_t - \ln X_{ss} = x_t - x_{ss}. \quad (170)$$

¹⁴⁸For all algorithms used in this paper matlab codes written by Giordani and Söderlind (2004) have been modified. For a more detailed description see Currie and Levine (1993).

Equilibrium output First we approximate (63) by a second-order Taylor approximation: $\hat{z}_t = \eta \hat{n}_t^f + \frac{\sigma}{1-h_C} \left(\hat{c}_t^f - h_C \hat{c}_{t-1}^f \right)$ or

$$\hat{n}_t^f = \frac{\hat{z}_t - \frac{\sigma}{1-h_C} \left(\hat{c}_t^f - h_C \hat{c}_{t-1}^f \right)}{\eta}. \quad (171)$$

Rewriting the production function as $\hat{c}_t^f = \hat{z}_t + \hat{n}_t^f$ and assuming that output equals production in equilibrium, $\hat{c}_t^f = \hat{x}_t^f$, finally allows us to derive an expression for the flexible price equilibrium output, expressed in percentage deviations from its steady state:

$$\hat{x}_t^f = \frac{1}{\gamma} \left[(1 + \eta) \hat{z}_t + \sigma \varkappa \hat{x}_{t-1}^f \right], \quad (172)$$

where $\gamma = \eta + \frac{\sigma}{1-h_C}$ and $\varkappa = \frac{h_C}{1-h_C}$.

The New Keynesian Phillips curve To derive an expression for the deviations of inflation around its steady state we first have to define inflation as the difference between the actual percentage deviation of prices from flexible price equilibrium and last period's deviation: $\pi_t = \hat{p}_t - \hat{p}_{t-1}$. Given this definition we can, without loss of generality, assume that $\pi_t = 0$ in equilibrium, since prices do not deviate from their steady states. Let $Q_t = \frac{\bar{p}_t^p}{P_t}$ be the average relative price chosen by adjusting firms and divide (59) by $P_t^{1-\theta}$ to get

$$1 = \omega \left(\frac{P_{t-1}}{P_t} \right)^{1-\theta} + (1 - \omega) Q_t^{1-\theta}.$$

Now we approximate this equation by a second-order Taylor approximation around the steady state values of q_t , $q = 0$ (as under flexible prices *all* firms adjust to the *same price*, so that $\bar{p}_t^* = P_t$), and of p_t and p_{t-1} , which both are equal to the same p (as prices do not change in a zero inflation steady state):

$$\hat{q}_t = \left(\frac{\omega}{1 - \omega} \right) \pi_t, \quad (173)$$

where $\hat{q}_t = q_t - q = q_t$.

To approximate (58) we rewrite the equation as

$$\left[E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i} (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\zeta-1} \right] \left(\frac{p_t^{fl}}{P_t} \right) \quad (174)$$

$$= \mu E_t \sum_{i=0}^{\infty} \omega^i C_{t+i} (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\zeta}. \quad (175)$$

Defining the average relative price chosen by the forward-looking firms as $Q_t^{fl} = \frac{p_t^{fl}}{P_t}$ leads to

$$\left[E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\zeta-1} \right] Q_t^{fl} \quad (176)$$

$$= \mu \left[E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\zeta} \right]. \quad (177)$$

A second-order Taylor approximation around the steady state values c , p and q^{fl} gives

$$\begin{aligned} \left(\frac{1}{1-\omega\beta}\right)\hat{q}_t^{fl} &= \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i} - \hat{p}_t) \\ &= \sum_{i=0}^{\infty} [\omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i})] - \left(\frac{1}{1-\omega\beta}\right)\hat{p}_t \\ \Leftrightarrow \hat{q}_t^{fl} + \hat{p}_t &= (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}), \end{aligned} \quad (178)$$

where we heavily used $q^{fl} = 0$ (as $\mu\varphi = Q^{fl} = 1$ when prices are flexible and all firms adjust to the same price) and the fact that p_{t-1} and p_t have the same equilibrium value p . This equation can be interpreted as follows: The optimal nominal relative price of the forward looking price setters ($\hat{p}_t^{fl} = \hat{q}_t^{fl} + \hat{p}_t$) equals the expected discounted value of the future nominal marginal costs. As the right hand side of (178) can be written as

$$\begin{aligned} &(1-\omega\beta)(\hat{\varphi}_t + \hat{p}_t) + (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+1+i} + E_t \hat{p}_{t+1+i}) \\ &= (1-\omega\beta)(\hat{\varphi}_t + \hat{p}_t) + \omega\beta (E_t \hat{q}_{t+1}^{fl} + E_t \hat{p}_{t+1}) \end{aligned} \quad (179)$$

we can rewrite the equation as

$$\hat{q}_t^{fl} = (1-\omega\beta) \hat{\varphi}_t + \omega\beta (E_t \hat{q}_{t+1}^{fl} + E_t \pi_{t+1}) \quad (180)$$

Using $\hat{q}_t = \hat{p}_t^n - \hat{p}_t$ and $\hat{p}_t^n = (1-\tau)\hat{p}_t^{fl} + \tau\hat{p}_t^{bl}$, because of (60), allows us to express a relationship between the (average) relative price chosen by all new price setters, \hat{q}_t , and the relative price chosen by all forward looking new price setters, \hat{q}_t^{fl} , expressed in percentage deviations from equilibrium:

$$\begin{aligned} \hat{q}_t &= (1-\tau)\hat{p}_t^{fl} + \tau(\hat{p}_{t-1}^n + \pi_{t-1}) - \hat{p}_t \\ &= (1-\tau)\hat{q}_t^{fl} + \tau\hat{q}_{t-1} + \tau\pi_{t-1} - \tau\pi_t. \end{aligned} \quad (181)$$

Using (173) in (181) and solving for \hat{q}_t^{fl} gives

$$\begin{aligned} \left(\frac{\omega}{1-\omega}\right)\pi_t &= (1-\tau)\hat{q}_t^{fl} + \tau\left(\frac{\omega}{1-\omega}\right)\pi_{t-1} + \tau\pi_{t-1} - \tau\pi_t \\ \Leftrightarrow \hat{q}_t^{fl} &= \left(\frac{(1-\omega)\tau + \omega}{(1-\tau)(1-\omega)}\right)\pi_t - \left(\frac{\tau}{(1-\tau)(1-\omega)}\right)\pi_{t-1}. \end{aligned} \quad (182)$$

Together with (180) this finally leads to the *New Keynesian Phillips curve*:

$$\pi_t = \kappa\hat{\varphi}_t + \omega\beta\phi E_t \pi_{t+1} + \tau\phi\pi_{t-1},$$

where

$$\phi \equiv (\omega + \tau [1 - \omega (1 - \delta)])^{-1},$$

and

$$\kappa \equiv (1 - \tau)(1 - \omega)(1 - \beta\omega)\phi.$$

To derive a New Keynesian Phillips curve that depends on an output measure we first approximate (55) around the steady state values φ , w , p and z :

$$\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t \quad (183)$$

An approximation of (53) gives:

$$\Leftrightarrow \hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \frac{\sigma}{1 - h_C} (\hat{c}_t - h_C \hat{c}_{t-1}). \quad (184)$$

Using this and an approximation of the production function, $\hat{x}_t = \hat{z}_t + \hat{n}_t$, in (183) under the assumption that output equals consumption leads to

$$\begin{aligned} \hat{\varphi}_t &= \eta \hat{n}_t + \sigma (1 - h_C)^{-1} (\hat{x}_t - h_C \hat{x}_{t-1}) - \hat{z}_t \\ &= (\eta + \sigma (1 - h_C)^{-1}) \hat{x}_t - (1 + \eta) \hat{z}_t - \sigma \frac{h_C}{1 - h_C} \hat{x}_{t-1} \\ &= \gamma \left[\hat{x}_t - \frac{1 + \eta}{\gamma} \hat{z}_t - \frac{\sigma \varkappa}{\gamma} \hat{x}_{t-1} \right] \\ &= \gamma \left[\hat{x}_t - \hat{x}_t^f - \frac{\sigma \varkappa}{\gamma} (\hat{x}_{t-1} - \hat{x}_{t-1}^f) \right] \\ &= \gamma y_t - \sigma \varkappa y_{t-1} \end{aligned} \quad (185)$$

where $y_t \equiv \hat{x}_t - \hat{x}_t^f$, and we used (172). Using this in (199) leads to

$$\pi_t = \kappa (\gamma y_t - \sigma \varkappa y_{t-1}) + \omega \beta \phi E_t \pi_{t+1} + \tau \phi \pi_{t-1}. \quad (186)$$

If all firms use (115) for price adjustments, $\hat{q}_t = \hat{q}_{t-1} + \pi_{t-1} - \pi_t$, and inflation is the same for all periods.

The forward-looking IS-curve The IS curve is simply derived by an approximation of the Euler equation (51) around the steady state values:

$$\hat{x}_t = \left(\frac{1}{1 + h_C} \right) E_t \hat{x}_{t+1} + \left(\frac{h_C}{1 + h_C} \right) \hat{x}_{t-1} - \sigma^{-1} \left(\frac{1 - h_C}{1 + h_C} \right) (\hat{i}_t - E_t \pi_{t+1}),$$

where we used $\hat{c}_t = \hat{x}_t$. Subtracting $\hat{x}_t^f + \zeta E_t \hat{x}_{t+1}^f + \zeta h_C \hat{x}_{t-1}^f$ finally gives

$$y_t = \left(\frac{1}{1 + h_C} \right) E_t y_{t+1} + \left(\frac{h_C}{1 + h_C} \right) y_{t-1} - \sigma^{-1} \left(\frac{1 - h_C}{1 + h_C} \right) (\hat{i}_t - E_t \pi_{t+1}) + u_t,$$

where $u_t \equiv \left(\frac{1}{1 + h_C} \right) E_t \hat{x}_{t+1}^f - \left(\frac{h_C}{1 + h_C} \right) \hat{x}_{t-1}^f - \hat{x}_t^f$ only represents exogenous productivity disturbances as it solely depends on the utility function of the households supplying labor (\hat{x}_t^f represents deviations from equilibrium, when prices are perfectly flexible).

A.4 The New Keynesian two-country framework

A.4.1 Household optimization

Households choose the cost minimizing combination of individual goods to achieve any given level \bar{C}_t of the composite consumption goods:

$$\min_{\{c_{H,jt}, c_{F,jt}\}} \int_0^1 (p_{H,jt} c_{H,jt} + p_{F,jt} c_{F,jt}) dj \quad (187)$$

subject to

$$C_{H,t}^{1-\varpi_H} C_{F,t}^{\varpi_H} = \bar{C}_t, \quad (188)$$

where p_j represents the price of the individual good c_j . The Lagrangian of this problem is given by

$$\mathcal{L} = \int_0^1 (p_{H,jt} c_{H,jt} + p_{F,jt} c_{F,jt}) dj - \psi_t \left[\left[\int_0^1 c_{H,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{(1-\varpi_H)\zeta}{\zeta-1}} \left[\int_0^1 c_{F,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\varpi_H\zeta}{\zeta-1}} - \bar{C}_t \right]$$

and the first order conditions for the domestic and foreign good j imply

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{H,jt}} &= p_{H,jt} - (1 - \varpi_H) \psi_t C_{F,t}^{\varpi_H} \left[\int_0^1 c_{H,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta(1-\varpi_H)-(\zeta-1)}{\zeta-1}} c_{H,jt}^{-\frac{1}{\zeta}} \stackrel{!}{=} 0 \\ \Leftrightarrow c_{H,jt} &= \left(\frac{p_{H,jt}}{(1 - \varpi_H) \psi_t} \right)^{-\zeta} C_{H,t}^{\zeta(1-\varpi_H)-(\zeta-1)} C_{F,t}^{\zeta\varpi_H}, \end{aligned} \quad (189)$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{F,jt}} &= p_{F,jt} - \varpi_H \psi_t C_{H,t}^{1-\varpi_H} \left[\int_0^1 c_{F,jt}^{\frac{\zeta-1}{\zeta}} dj \right]^{\frac{\zeta\varpi_H-(\zeta-1)}{\zeta-1}} c_{F,jt}^{-\frac{1}{\zeta}} = 0 \\ \Leftrightarrow c_{F,jt} &= \left(\frac{p_{F,jt}}{\varpi_H \psi_t} \right)^{-\zeta} C_{H,t}^{\zeta(1-\varpi_H)} C_{F,t}^{\zeta\varpi_H-(\zeta-1)}. \end{aligned} \quad (190)$$

Combining (189) and (190) with the definition of the domestic consumption index, an expression for the Lagrangian multiplier can be derived, which equals the domestic consumption price index (CPI):

$$\begin{aligned} C_t &= C_{H,t}^{1-\varpi_H} C_{F,t}^{\varpi_H} \\ &= \psi_t^\zeta \varpi_H^{\zeta\varpi_H} (1 - \varpi_H)^{\zeta(1-\varpi_H)} C_{H,t}^{\zeta(1-\varpi_H)} C_{F,t}^{\zeta\varpi_H} P_{H,t}^{-\zeta(1-\varpi_H)} P_{F,t}^{-\zeta\varpi_H} \\ \Leftrightarrow \psi_t &= k_H^{-1} P_{H,t}^{(1-\varpi_H)} P_{F,t}^{\varpi_H} \equiv P_t, \end{aligned}$$

where $k_H = \varpi_H^{\varpi_H} (1 - \varpi_H)^{(1-\varpi_H)}$. The corresponding foreign household optimization follows the same operations and leads to $P_t^* \equiv k_F^{-1} P_{H,t}^{*(1-\varpi_F)} P_{F,t}^{*\varpi_F}$, where $k_F = \varpi_F^{\varpi_F} (1 - \varpi_F)^{(1-\varpi_F)}$.

A.4.2 Approximation around the steady state

Linearization is done around a baseline symmetric steady state with equal and constant consumption. We assume a real exchange rate of value one and therefore a balanced trade. Throughout this section we use the same notations as in A.3.2.

The New Keynesian Phillips curve Following the same logic as for the single-country the definition of the average relative price chosen by adjusting firms is given by $Q_{H,t} = \frac{\bar{p}_{H,t}^n}{P_{H,t}}$, and similar computations than before give

$$\hat{q}_{H,t} = \left(\frac{\omega}{1 - \omega} \right) \pi_{H,t}. \quad (191)$$

As all firms that adjust their price at date t chose the same optimal price $p_{H,t}^{fl}$, the first order condition of (111) subject to (108) implies¹⁴⁹

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} X_{t+i} \left[(1 - \varepsilon) \frac{p_{H,t}^{fl}}{P_{H,t+i}} + \varepsilon \varphi_{t+i} \right] = 0, \quad (192)$$

what using (57) leads to

$$\left(\frac{p_{H,t}^{fl}}{P_{H,t}} \right) = \mu^* \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \varphi_{t+i} \frac{P_{t+i}}{P_{H,t}}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \frac{P_{t+i}}{P_{H,t+i}}}, \quad (193)$$

where we used $\mu^* = \frac{\varepsilon}{\varepsilon-1}$. To approximate (193) we rewrite the equation as

$$\begin{aligned} & \left(\frac{p_{H,t}^{fl}}{P_{H,t}} \right) E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \frac{P_{t+i}}{P_{H,t+i}} \\ &= \mu^* E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C_{t+i} - h_C C_{t+i-1})^{-\sigma} X_{t+i} \varphi_{t+i} \frac{P_{t+i}}{P_{H,t}}, \end{aligned} \quad (194)$$

Defining the average relative price chosen by the forward-looking firms as $Q_{H,t}^{fl} = \frac{p_{H,t}^{fl}}{P_{H,t}}$ approximating and rearranging gives

$$\hat{q}_{H,t}^{fl} + \hat{p}_{H,t} = (1 - \omega\beta) \sum_{i=0}^{\infty} \omega^i \delta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{H,t+i}) \quad (195)$$

Similar to the optimal price decision of the single-country, the optimal nominal relative price of the forward looking price setters ($\hat{p}_{H,t}^{fl} = \hat{q}_{H,t}^{fl} + \hat{p}_{H,t}$) equals the expected discounted value of the future nominal marginal costs. Rearranging (195) gives

$$\hat{q}_{H,t}^{fl} = (1 - \omega\beta) \hat{\varphi}_t + \omega\beta \left(E_t \hat{q}_{H,t+1}^{fl} + E_t \pi_{H,t+1} \right) \quad (196)$$

¹⁴⁹ As long as a firm does not adjust their price $P_{H,jt} = P_{H,jt+i} = p_{H,t}^{fl}$.

Using $\widehat{q}_{H,t} = \widehat{\bar{p}}_{H,t}^n - \widehat{p}_{H,t}$, $\widehat{\bar{p}}_{H,t}^n = (1 - \tau)\widehat{p}_{H,t}^{fl} + \tau\widehat{p}_{H,t}^{bl}$, because of (114), and (115) allows to express a relationship between the (average) relative price chosen by all new price setters, $\widehat{q}_{H,t}$, and the relative price chosen by all forward looking new price setters, $\widehat{q}_{H,t}^{fl}$, expressed in percentage deviations from equilibrium:

$$\widehat{q}_{H,t} = (1 - \tau)\widehat{q}_{H,t}^{fl} + \tau\widehat{q}_{H,t-1} + \tau\pi_{H,t-1} - \tau\pi_{H,t}. \quad (197)$$

Using (191) and solving for $\widehat{q}_{H,t}^{fl}$ gives

$$\widehat{q}_{H,t}^{fl} = \left(\frac{(1 - \omega)\tau + \omega}{(1 - \tau)(1 - \omega)} \right) \pi_{H,t} - \left(\frac{\tau}{(1 - \tau)(1 - \omega)} \right) \pi_{H,t-1}. \quad (198)$$

Together with (196) this finally leads to the *New Keynesian Phillips curve*:

$$\pi_{H,t} = \kappa\widehat{\varphi}_t + \phi(\omega\beta E_t\pi_{H,t+1} + \tau\pi_{H,t-1}), \quad (199)$$

where

$$\phi \equiv (\omega + \tau[1 - \omega(1 - \beta)])^{-1},$$

and

$$\kappa \equiv (1 - \tau)(1 - \omega)(1 - \beta\omega)\phi.$$

If all firms are backward-looking ($\tau = 1$), the inflation dynamics given by (197) are reduced to $\widehat{q}_{H,t} = \widehat{q}_{H,t-1} + \pi_{H,t-1} - \pi_{H,t}$ and inflation is the same for all periods.

In order to derive a relationship between real marginal costs and an output gap measure as in the single-country economy, we approximate (110) to derive

$$\widehat{\varphi}_t = \widehat{w}_t - \widehat{p}_t - \varpi_H\widehat{s}_t - \widehat{z}_t. \quad (200)$$

Next, we use an approximation of (105) with an approximation of the production function $\widehat{x}_t = \widehat{z}_t + \widehat{n}_t$

$$\widehat{\varphi}_t = \eta y_t + \frac{\sigma}{1 - h_C}(\widehat{c}_t - h_C\widehat{c}_{t-1}) - \varpi_H\widehat{s}_t + v_t, \quad (201)$$

where $v_t = -(1 + \eta)\widehat{z}_t + \eta\widehat{x}_t^f$ solely depends on exogenous productivity disturbances. Using this in (199) finally gives

$$\begin{aligned} \pi_{H,t} = & \kappa \left(\eta y_t + \frac{\sigma}{1 - h_C}(\widehat{c}_t - h_C\widehat{c}_{t-1}) - \varpi_H\widehat{s}_t \right) \\ & + \phi(\omega\beta E_t\pi_{H,t+1} + \tau\pi_{H,t-1}) \end{aligned} \quad (202)$$

Domestic and foreign output paths For approximating we rewrite (116) and (117) in logarithms

$$e^{x_t} = (1 - \varpi_H)e^{p_t - p_{H,t} + c_t} + \frac{\gamma}{1 - \gamma}(1 - \varpi_F)e^{p_t - p_{H,t} + r_{er,t} + c_t^*}, \quad (203)$$

and

$$e^{x_t^*} = \frac{1 - \gamma}{\gamma}\varpi_H e^{p_t - p_{F,t} + c_t} + \varpi_F e^{r_{er,t} + p_t - p_{F,t} + c_t^*}. \quad (204)$$

As we assume a baseline scenario with constant and equal consumption, and (119) implies an equilibrium value for the real exchange rate of $RE R = 1$, the steady state relationships can be expressed by

$$e^x = \left[(1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) \right] e^{p - p_H + c} \quad (205)$$

and

$$e^{x_t^*} = e^{p - p_F + c} \left[\frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F \right]. \quad (206)$$

Thus approximating (203) gives

$$\begin{aligned} e^x (1 + \hat{x}_t) &= (1 - \varpi_H) e^{p - p_H + c} (1 + \hat{p}_t - \hat{p}_{H,t} + \hat{c}_t) \\ &\quad + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) e^{p - p_H + r_{er} + c^*} (1 + \widehat{rer}_t + \hat{p}_t - \hat{p}_{H,t} + \hat{c}_t^*) \\ &= e^{p - p_H + c} \left[\frac{(1 - \varpi_H) (\hat{p}_t - \hat{p}_{H,t} + \hat{c}_t)}{1 - \gamma} + \frac{\gamma(1 - \varpi_F)}{1 - \gamma} (\widehat{rer}_t + \hat{p}_t - \hat{p}_{H,t} + \hat{c}_t^*) \right] + \alpha_1 e^{p - p_H + c}, \end{aligned} \quad (207)$$

where $\alpha_1 = (1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F)$. Using an approximation of (100), (205) and $\hat{p}_t - \hat{p}_{H,t} = \varpi_H \hat{s}_t$ finally leads to

$$y_t = \frac{(1 - \varpi_H) (\hat{c}_t + \varpi_H \hat{s}_t) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) (\hat{c}_t^* + \varpi_F \hat{s}_t)}{\alpha_1} + o_t, \quad (208)$$

where $o_t = -\hat{x}_t^f$ solely depend on exogenous productivity disturbances. Following the same steps for (204) leads to

$$\begin{aligned} e^{x^*} (1 + \hat{x}_t^*) &= e^{p - p_F + c} \frac{1 - \gamma}{\gamma} \varpi_H (1 + \hat{p}_t - \hat{p}_{F,t} + \hat{c}_t) \\ &\quad + e^{r_{er} + p - p_F + c^*} \varpi_F (1 + \widehat{rer}_t + \hat{p}_t - \hat{p}_{F,t} + \hat{c}_t^*) \\ &= e^{p - p_F + c} \left[\frac{\frac{1 - \gamma}{\gamma} \varpi_H (\hat{p}_t - \hat{p}_{F,t} + \hat{c}_t)}{1 - \gamma} + \varpi_F (\widehat{rer}_t + \hat{p}_t - \hat{p}_{F,t} + \hat{c}_t^*) \right] + \alpha_2 e^{p - p_F + c}, \end{aligned} \quad (209)$$

where $\alpha_2 = \frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F$. Using (100), (205) and $\hat{p}_t - \hat{p}_{F,t} = -(1 - \varpi_H) \hat{s}_t$ finally gives

$$y_t^* = \frac{\frac{1 - \gamma}{\gamma} \varpi_H (\hat{c}_t - (1 - \varpi_H) \hat{s}_t) + \varpi_F (\hat{c}_t^* - (1 - \varpi_F) \hat{s}_t)}{\alpha_2} + o_t^*, \quad (210)$$

where $o_t^* = -\hat{x}_t^{f*}$ again solely depends on exogenous productivity disturbances.

Equilibrium terms of trade Assuming a baseline scenario with constant and equal consumption and a real exchange rate of value one, simplifies (116) and (117):

$$X_t = \frac{P_t}{P_{H,t}} \left[(1 - \varpi_H) + \frac{\gamma}{1 - \gamma} (1 - \varpi_F) \right] C_t, \quad (211)$$

and

$$X_t^* = \frac{P_t}{P_{F,t}} \left[\frac{1 - \gamma}{\gamma} \varpi_H + \varpi_F \right] C_t. \quad (212)$$

Dividing the first by through the second equation leads to

$$\frac{X_t}{X_t^*} = \left(\frac{(1 - \varpi_H) + \frac{\gamma}{1-\gamma} (1 - \varpi_F)}{\frac{1-\gamma}{\gamma} \varpi_H + \varpi_F} \right) S_t. \quad (213)$$

Solving for S_t directly gives (132).

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