ESSAYS IN THE THEORY OF VOTING POWER

by

Frank Steffen

Abstract

The measurement of voting power plays a useful role in the investigation of structural properties of collective decision-making rules which can be modelled as a simple (voting) game. Such rules can be found in legislative bodies, committees, and a variety of organizations. Measures of voting power have an established history in game and social choice theory, going back more than half a century. In the early 1980s the field has gained a reputation that it has become a somewhat exhausted mine. This may have been true since the early 1990s, but such a view has not kept up with more recent work in the field. The last decade has seen a resurgence of research into this field, with many new discoveries about the properties of classical power measures as well as new developments in probabilistic techniques and new areas of applications. This thesis includes contributions to all these aspects.

The central aim of the introductory chapter, chapter 1, is to discuss the meaning of the term *'voting power'*. This is essential to understand for which purposes measures of voting power are applicable. The debates in the literature indicate that more attention on this issue is required especially in order to help those who are not familiar to this area of research, but either seek to criticise it or just to apply its concepts and methods in an unreflective manner.

The remaining thesis consists of two parts. Part I concerns theoretical aspects of the theory of voting power while the part II deals with applications of the theory of voting power to political and organizational questions. The chapters of both parts are laced together by their common focus on questions of a prioricity and local monotonicity and by the analysis and application of Straffin's probabilistic partial homogeneity approach to the measurement of power.

Chapter 2 concerns a discussion of a prioricity properties of measures of voting power, in particular, questioning the position taken by Felsenthal and Machover. The analysis in this paper is: (i) There is little ground to support Felsenthal and Machover's position that the Penrose/Banzhaf measure, derived from an assumption that each player behaves independently under Straffin's approach, is the only pure a priori measure or is 'more' a priori than the Shapley-Shubik index, which results from Straffin's approach if it is assumed that all players behave as clones according to the so-called 'homogeneity assumption'. (ii) That, in contrast to Straffin's statement that partial homogeneity assumptions are by their nature ad hoc, a partial homogeneity framework could also be a priori if the additional information which is used has an a priori 'character'.

In chapters 3 and 4 the a prioricity discussion is examined in more detail devoting a separate section of each chapter to this issue. While chapter 3 contains a more detailed discussion of the question whether one can distinguish between Straffin's independence

and homogeneity assumption behind a Rawlsian 'veil of ignorance', chapter 4 elaborates when a measure based on a partial homogeneity structure fulfils the conditions to be aprioristic.

However, the main focus of chapters 3 and 4 are on different issues. Chapter 3 deals with the occurrence of abstentions in simple voting games. This is a very young and as yet under-developed part of the theory of voting power. A first approach to dealing with abstentions was made by Felsenthal and Machover. They proposed a ternary voting game (TVG). Chapter 3 provides an alternative way to model abstention by using an abstention voting game (AVG). The basic difference is that a TVG treats 'yes, 'no' and 'abstain' as simultaneous choices, while under an AVG setup voting is conceptualised sequentially: a player first chooses whether to vote at all, and then, if he or she has decided to vote, between casting a 'yes' or a 'no' vote. Both approaches can be conceptually justified. We can distinguish between two different forms of abstention: *abstention by default* and *active abstention*. By the former is meant the act of not showing-up to vote; by the latter is meant the case a player declaring 'I abstain'. While the TVG model can be regarded as assimilating all abstentions to those of the active kind, the AVG model, can be regarded doing the same for all abstentions that occur by default.

The focus of chapter 4 is on the ongoing and fundamental debate in the literature on voting power about what constitutes a 'reasonable' measure of *a priori* voting power. A central topic in this debate is whether or not a reasonable measure of voting power should fulfil *local monotonicity* (LM). This is a postulate which says that in weighted voting games, i.e. simple voting games characterized by a vector of voting weights attached to each player and a quota, if a player i has at least as much weight as a player j, then player i should have at least as much power as player j. While the Shapley-Shubik index and the Penrose/Banzhaf or Coleman measures are locally monotonic, the Deegan-Packel and the Public-Good measures are not. Some authors have argued that the violation of LM is 'pathological' and, thus, measures of voting power that exhibit such behaviour are unreasonable, while other authors, have argued that the violation of LM is a simple social fact of power and, therefore, LM cannot be used to determine the reasonableness of a measure of voting power. So far the debate has ignored the violation of LM by another set of measures derived from Straffin's partial homogeneity approach. By examining violations of LM in this context we show that the different sides to this debate are in a sense 'both wrong'. We argue that LM is a special case of a more general montonicity condition that relates 'resources' to 'power'; in LM the resources are but the voting weights. However, given that it is not clear that a priori voting power is based on, and only on, the vector of voting weights and the decision rule, it turns out that a violation of LM can be 'reasonable'. This, however, does not imply that power is not monotonic in resources per se.

The issue of LM and its violation is also central to chapter 5. It deals with the violation of LM in voting weights by Public-Good measures which most prominent measure is the Public-Good Index. The underpinning argument of the Public-Good measures is the existence of a decision-making situation that includes an incentive structure such that only those winning subset (coalitions) of players ought to form which contain no *excess-player*. The chapter introduces two constrained versions of LM: (i) *player-constrained* LM by restricting the number of non-dummy players in a game and (ii) *partial* LM by

applying specific constrains on voting weights. It is shown the Public-Good measures fulfil *partial* LM for every *proper* weighted voting game, i.e. for WVG in which two disjoint subsets are never winning at the same time, and *player-constrained* LM for every weighted voting game with a simple majority rule and up to *four* non-dummy players. Chapter 5 concludes with a discussion that points out that whether a specific measure of voting power is appropriate depends on the properties of the model of collective decision-making which one wants to analyze, and not necessarily on some intuitive notions of monotonicity.

Part II of this thesis contains two applied chapters, which make use of parts of the results provided in the previous chapters. While chapter 6 is an application of the theory of voting power to an actual decision-making situation, chapter 7 deals with what one may call a 'theoretical application', i.e. the application of the voting power to answer questions in another theoretical area of research. In the case of chapter 7 this is the study decision-making situations and the nature of power in hierarchical organizations.

Chapter 6 contains an analysis of the voting rules for the National Assembly for Wales, which was established in 1999 for the first time. The rule for electing members to the National Assembly for Wales is the *Additional Member System* (AMS), i.e. not the otherwise usual *first-past-the-post system* for Westminster Parliament. The AMS gives each voter two votes, to be cast at the Assembly Constituency level, and at the bigger Assembly Electoral Region level. One third of the members to the assembly are elected by a form of proportional representation, where party support is calculated by aggregating the two votes. The voters are allowed to cast the second vote in favour of a different party than the one they earlier voted for, at the Assembly Constituency level. It is shown that this additional degree of freedom can frustrate the objective of obtaining better correspondence between party support and the number of seats. Also, the effects of this additional degree of freedom on the voting power of the parties on the Assembly Electoral Region level are shown using Straffin's partial homogeneity approach. Based on this analysis, a different system of proportional representation and a method of equating the distribution of voting power and the seat distribution are proposed. The result of the study of the voting rules for the National Assembly for Wales turns out to be being that the switch from the *first-past-the-post system* to the AMS for electing the assembly can frustrate voters and implies the possibility that some parties in the assembly will be rendered powerless. However, they may at least give some parties the chance of being involved in the business of government.

Chapter 7 deals with the nature of a priori voting power in hierarchical organizations. It is shown that every *'restricted' game with a permission structure*, which is a simple game where the winning subsets are additionally restricted by a *permission structure* can be represented as a *compound game*. Furthermore, it is pointed out that the existing research in voting power in hierarchical structures is necessary, but not sufficient to understand the nature of a priori voting power in hierarchical organizations, because it does not take into account: (i) that players who participate in a decision-making in hierarchical organizations in general have a *damatis personae*, which we model via Straffin's partial homogeneity approach applied as an aprioristic framework, and (ii) that the top of a hierarchical organizations can have a board-structure. Taking both aspects into account

we not only come out with violation of LM which one would expect based on the results of chapter 4. Moreover, there are some further counterintuitive results, i.e. the violation of known monotonicity properties of power in hierarchical organizations such as (*weak*) *structural montonicity* and *dis-* and *conjunctive fairness*. (*Weak*) *structural montonicity* more or less says that a player in a hierarchy who dominates another player should have at least as much voting power as the dominated player; *dis-* and *conjunctive fairness* roughly stipulate that the deletion of a hierarchical relation between two players under *disjunctive fairness* should change their voting power and that of the superiors of the dominating player by the same amount and in the same direction, while under *conjunctive fairness* the voting power of the dominated player and his superiors should be changed by the same amount and in the same direction. Moreover, it is illustrated that dropping a player belonging to an intermediate hierarchical level, does not necessarily imply that his voting power is transferred downwards to the lower hierarchical levels which has an important implications to two related management concepts which are known as *empowerment* and *lean management*. Both are based on the idea that (i) by removing intermediate layers or parts of layers of a hierarchy power can be transferred downwards to employees on the lower levels and that (ii) such a change will lead to increased motivation due to employees having more of a say in the organization's destiny and thus, increased responsiveness and productivity gains for the organization. But as indicated above (i) is not necessarily true if we remove layers or parts of layers. The practical implications of this perspective is that when we come to look at the performance of organizations, it is necessary to abstract from the particular personalities that are involved. The success or failure of an organization may not be so much a matter of its 'leadership' and 'management style' – its 'corporate culture' – but of the interaction of *incentives* and *decision-making rules*.