Essays on Efficiency in Bargaining

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Introduction

Negotiation is a ubiquitous phenomenon of social life. It usually takes place when parties' interests are to some extent *common* but at the same time also *conflicting*. These situations emerge when parties depend on each other in the realisation of a project, but diverge in their interest regarding the exact settlement. For such conflict of interest, negotiation is a process to help parties decide how to divide the fruits of cooperation. The agreement, in the case one is found, necessarily implies a compromise located somewhere between the coordinates of parties' particular positions. The prospect of such an agreement creates an incentive for parties to meet at the negotiating table and, equally important, their incentive not to leave it. The set of agreements for which this mutual incentive exists delimits the space of feasible outcomes. Ex ante, this is the space of *common* interest. After the identification of this space, *conflict* arises from the parties' interest to find an agreement close to their particular position, i.e. to minimize their part of the compromise. Now, while negotiation is mostly used in a broader sense, *bargaining* usually refers to the 'tug war' within the (bargaining) space of feasible outcomes. This is what Schelling (1956, p. 281) calls "the 'distributional' aspect of bargaining: the situations in which more for one means less for the other. When the business is finally sold to the one interested buyer, what price does it go for? When two dynamite trucks meet on a road wide enough for one, who backs up?"

To influence the bargaining outcome in their interest, besides haggling and deceiving, parties make use of threats. These threats are projections of scenarios with outcomes disfavored by their opponent. The more to the opponent's disfavour and the more credible such a projection, the more it weakens the opponent's position. But what makes a threat credible? One possibility is that a party binds itself to the execution of the threat. For example, to deliberately crash into the other truck in case he does not back up is certainly not a very credible threat for a driver. In contrast, this strategy becomes credible if it is commonly known that the driver dismantled his brakes. In absence of such a mechanism to bind oneself, when does one party believe another party that it will stick to its strategy in the future? Quite simply, if the party believes that it is the best choice, i.e. rational, for the other party to stick to it. Hence, to be credible, the execution of the strategy has to be rational or at least appear to be rational. For a strategy to be rational, its expected benefits have to exceed its costs. For example, is it rational to always go on strike if a company does not agree to the latest wage claim? In turn, is it rational for the company not to agree and suffer strikes if they arise?

Indeed, sometimes negotiations appear to be a rather wasteful way to implement an agreement, e.g. collective bargaining, armed conflicts or international trade conflicts. Inefficient or delayed agreements and costs from upholding threat scenarios may shrink parties payoffs in such negotiations. In these cases a different outcome would imply a pareto-improvement, i.e. would leave both parties better off. This is what Schelling (1956) calls the ’efficiency’ aspect of bargaining. However, the arising inefficiency in such scenarios seems puzzling as it is in no party’s interest. Obviously, either the bargaining parties do not behave rational, or individual rationality does not necessarily lead to collective rationality. In game theory, to assume the former is rather inconvenient as by throwing away “the assumption (even in cases where this is a realistic assumption) that the players will act rationally and will also expect each other to act rationally” the very uninformative statement results that “every player should always do whatever seems best to him” (Harsanyi, 1982, p. 121 & p. 123). In contrast, the assumption that parties act rational and that they have formidable powers of calculation is stringent. On the other hand, it enables us to study the effect of different procedures and frameworks. Kennan and Wilson (1990, p. 249) argue as follows:

> “Calculation includes selection of one’s own strategy and, less credibly, anticipation of the other’s strategy. This assumption of equilibrium — the parties’ strategies are optimal responses, each to the other — is a hallmark of game theory. Rather than a normative axiom or an approximation of empirical fact, which is ambiguous at best, it is a prerequisite here for identification of fundamental barriers to quick resolution of negotiations.”

Explanations for inefficient outcomes in non-cooperative bargaining are plenty.[1] Some of them rely on the fact that negotiations encompass multiple issues. In the corresponding models, inefficient outcomes arise because parties’ ability to make mutu-

[1] Note that in cooperative games, parties are able to make binding contracts while in non-cooperative bargaining they are not.

ally beneficial inter-issue trade-offs is restricted.[2] Parties cannot bind themselves (by contract) to the division of future issues such that inefficient outcomes for today's issues result. For example, would you trust your mean (but rational) brother and entirely concede him his favoured chocolate pie today, hoping for a larger share of your favoured strawberry cake tomorrow? More often, though, inefficient outcomes are explained by assuming that players face either *fundamental* or *strategic* uncertainty.[3] In the former case, at least one party lacks information about the *fundamental* facts such that it may be unable to calculate his or others parties best answer in a given situation. As a result, parties can only estimate the equilibrium and delay may arise if they fail to do so correctly. If uncertainty is *strategic*, players are fully informed about the fundamentals, but face multiple equilibria. In this case, multiple strategy combinations are mutually best answers and two sources of inefficiency arise. First, parties do not know on which equilibrium to coordinate and miscoordination may cause delay. Second, in the presence of multiple equilibria in bargaining, we typically find equilibria which support inefficient outcomes.[4] In these equilibria, parties remain obstinate, because they are afraid of being pulled over the barrel if they make a first concession.

It seems to be the case that certain bargaining frameworks, i.e. the conditions under which bargaining takes place, give rise to inefficient outcomes. Naturally, we are interested in frameworks which are likely to produce efficient outcomes. Kennan and Wilson (1990, p. 249) bring up a useful analogy: "[...] battles among animals competing for mates or prey select winners; the costs are injuries or energy consumption. In some species, however, [...] ritualized battles reduce costs." In this sense, the research on the bargaining framework is the search for the right 'ritual'.

In this thesis, I analyse two different aspects which may influence the efficiency of the bargaining process. One aspect is the bargaining agenda. If multiple issues are at stake, the agenda specifies the order in which bargaining takes place, e.g. whether offers may encompass multiple issues or just one, whether the sequence of issues is exogenously determined. The other aspect concerns parties' option to call in a third-party for assistance. In real-life, to avoid extreme inefficiencies, negotiations are often accompanied by mediators, and ongoing disagreements are resolved by arbitration.

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[2] Compare for example Busch and Horstmann (1997); Flamini (2007); In and Serrano (2004); Inderst (2000); Lang and Rosenthal (2001); Fershtman (1990); Weinberger (2000) and Weinberger (2000).

[3] Compare for example Slantchev (2003); Rubinstein (1985); Haller and Holden (1990); Fernandez and Glazer (1991); Admati and Perry (1987) or Cramton (1992).

[4] Compare also Avery and Zemsky (1994) and Manzini and Mariotti (2001).

The thesis consists of three articles all written with no co-author. Every single article is fully self-contained and can be read on its own. The first paper, 'One Thing at a Time: Efficient Agendas in Multi-Issue Bargaining', has been submitted to *Games and Economic Behaviour* and is invited for resubmission after revision. The revision is still in work. The second paper and third paper, 'The Agenda in Multi-Issue Bargaining with Punishment' and 'Arbitration and Mediation in a Bargaining Model with Punishment' have been completed only recently and not been submitted so far.

'One Thing at a Time: Efficient Agendas in Multi-Issue Bargaining' analyses sequential agendas in multi-issue bargaining. On the basis of Rubinstein's (1982) alternating offer model, this paper analyses a bargaining situation with two issues which are both essential to an overall agreement. In this sense, it continues the work of Fershtman (1990) and Weinberger (2000). Two main results emerge. First, the paper weakens Fershtman's result that a sequential agenda leads to inefficient outcomes. In his approach, this occurs when players have opposing preferences and the first issue on the agenda is the one less preferred by the first-mover. I show that this result strongly relies on the structure of moves which is assumed. By following the alternating offer structure more literally, the results are contrasting: players nearly always prefer the agenda such that their preferred issue is negotiated first. In this case the outcome is always efficient. The second result shows that by adopting an ex ante perspective – a perspective prior to the selection of the first-mover – the sequential agenda may be beneficial for both players. By assuming that players are uncertain whether they are first or second-mover, their incentives fundamentally change. The preferred agenda of a player who is aware that he moves first is the one in which he can *capture* the maximal value, and this is always the simultaneous agenda. From the ex ante perspective, his interest in *creating* value increases and this is facilitated by the sequential agenda.

The second paper, 'The Agenda in Multi-Issue Bargaining with Punishment', analyses the effect of the agenda when players have the ability to execute punishments. It builds upon the models of Haller and Holden (1990), Fernandez and Glazer (1991) and Avery and Zemsky (1994). In their models, the ability to strike enlarges players' strategy space and triggers a multiplicity of equilibria, including such of immediate agreement and others which allow for delay and strikes. These inefficient equilibria have both players upholding their punishment threat for a number of periods before they agree on a compromise. Both players find this strategy favorable to a deviation, as this leads to quickly implemented but even worse outcomes for the deviator. In this approach I extend this single-issue punishment bargaining model to a multi-issue setting and analyze the influence of the agenda. The comparison of the equilibria shows that the choice of the agenda influences the degree of inefficiency. A restriction to single-issue offers can have positive or negative effects on the range of inefficient equilibria and on the potential level of inefficiency of these equilibria. As long as no player is too powerful, the results suggest issues to be negotiated one at a time and the issue of lower importance first. In contrast, if one player is sufficiently powerful the simultaneous agenda is favorable in terms of efficiency.

In the last paper of this thesis, I Analise the influence of arbitration and mediation on the outcome in the bargaining model with punishment. In particular, I investigate whether the presence of such third-party may influence the potential level of inefficiency in equilibrium. So far, the approaches which analyse arbitration in the context of non-cooperative bargaining either apply the standard Rubinstein game or a bargaining game of concessions.[5] In the absence of a third-party, the equilibrium concept in these models never allows for frictions such that two points of criticism arise.[6] First, it seems that the need for a third-party in these models is questionable. Second, these approaches basically find "that arbitration is essentially a source of inefficiency" (Compte and Jehiel,1995, p. 34).[7] However, in 'Arbitration and Mediation in a Bargaining Model with Punishment', I apply a bargaining model with punishments which allows for inefficient equilibria. In this bargaining model, I implement the arbitration scheme of Manzini and Mariotti (2001) and show that the influence of arbitration on efficiency crucially depends on the proposed arbitration outcome. For sufficiently balanced arbitration outcomes, the presence of an arbitrator erases all inefficient equilibria, while for rather inappropriate proposals the possible level of inefficiency increases. Mediation, by contrast, never increases the level of inefficiency. Instead, it generally reduces the possible level of inefficiency in equilibrium or prevents it completely if mediation is a rational option. This is always the case if players are sufficiently patient.

[5]Manzini and Mariotti (2001) and McKenna and Sadanand (1995) analyse and compare arbitration schemes by applying Rubinstein's procedure. Adamuz and Ponsatí (2009) and Compte and Jehiel (1995) work within a model of bargaining by concession.

[6]Note that in the former model no inefficient equilibrium exists in the absence of a third-party. In the latter, some rather artificial delay of one period may cause inefficiency.

[7]Compare also Adamuz and Ponsatí; Manzini and Mariotti and McKenna and Sadanand (1995).

Chapter 1

One Thing at a Time: Efficient Agendas in Multi-Issue Bargaining

Abstract: This paper analyses sequential agendas in multi-issue bargaining. A bilateral two-issue alternating offer model with complete information and players with opposing ranking of the issues is studied. The model assumes issues to be linked such that the implementation of any result requires all issues to be settled. The analysis shows that if offers are restricted to single-issues, then the agenda which emerges endogenously leads only exceptionally to inefficient outcomes, i.e. if bargaining frictions are sufficiently low and preferences are sufficiently similar. Moreover, by adopting an ex ante perspective, it is shown that full exploitation of inter-issue tradeoffs can be guaranteed only if issues are discussed one at a time. As a consequence, it is primarily the sequential agenda which generates ex ante efficient outcomes.

1.1 Introduction

Negotiation is a process that helps parties decide how to divide the fruits of cooperation. Often it is the case that multiple issues are at stake: political parties negotiating over a coalition agreement, two companies bargaining over the exact conditions of their joint-venture, or a labour union and a company trying to find an agreement on wages and working hours.

Multi-issue bargaining models frequently assume that the disagreement on one issue does not threaten the possible benefit from other issues so that the result from bargaining over one issue is unaffected by the other (In and Serrano, 2003, 2004; Inderst, 2000). Issues are not linked and can be *implemented separately*. On the other hand, there might be several issues which are linked in such way that they are essential for cooperation. Here the overall agreement depends on the successful settlement of these linked issues. Consequently, these issues can only be *implemented jointly*. For example, assume the success of the union-employer negotiation requires that both issues, wages and working hours are settled. In this case the agreed wage can not be implemented until an agreement on the working hours is achieved (and vice versa).

On the basis of Rubinstein's (1982) alternating offer model, this paper analyses a bargaining situation with two issues which are both essential to an overall agreement. In this sense, it continues the work of Fershtman (1990) and Weinberger (2000). However, in contrast to the assumption of separate implementation, joint implementation generally allows players to make compromises. In Raiffa's (1982, p. 142) words: "the art of compromise centers on the willingness to give up something in order to get something else in return". Because each issue might threaten a player's benefit from several issues, negotiations over linked issues allows the opponent to give up something without the fear that he will not get something in return. Hence, joint implementation allows players to focus on *one thing at a time* without losing the ability to make mutual beneficial compromises between the issues, i.e. *inter-issue tradeoffs*. These tradeoffs are impossible when issues are implemented separately and are negotiated issue-by-issue.

A main task of this paper is to analyze the efficiency of different bargaining agendas. By *bargaining agenda* I mean the sequence in which the issues are discussed. The agenda is *simultaneous* if all issues at stake are negotiated in a bundle, it is *sequential* if negotiation takes place issue-by-issue. Moreover, the agenda is *exogenous* if the sequence of issues is prescribed while it is *endogenous* if not.

It is well-known that if offers are unrestricted, players chose to make bundle-offers over all issues and the simultaneous agenda emerges (Fershtman, 1990; In and Serrano, 2003; Inderst, 2000). From Fershtman (1990) we further learn that a restriction to single-issue offers may lead to inefficient outcomes; in his approach this occurs when players have opposing preferences and the first issue on the agenda is the one less preferred by the first-mover. In Fershtman's setting the first-mover prefers this inefficient sequence to the sequence which sets his preferred issue first such that the endogenous sequential agenda generally leads to an inefficient outcome. In this paper I show that this result strongly relies on the structure of moves which is assumed. By following the alternating offer structure more literally the results are contrasting: players nearly always prefer the agenda such that their preferred issue is negotiated first.[1] In this case the outcome is always efficient. In my approach the inefficient sequence emerges endogenously only when players are relatively patient and their preferences are relatively similar.

By adopting an ex ante perspective – a perspective prior to the selection of the first-mover – I propose the notion of ex ante efficiency as in Chen (2006). Assuming that players are uncertain about the sequence of moves, i.e. whether they are first or second-mover, their incentives fundamentally change. The preferred agenda of a player who is aware that he moves first is the one in which he can *capture* the maximal value, and this is always the simultaneous agenda. But as uncertainty increases, his interest in *creating* value increases. As a result we can ask the following questions:

> From an ex ante perspective: (a) Which agenda maximizes a player's expected payoff? (b) Which agenda maximizes efficiency? (c) Does an agenda exist which maximizes both players expected payoffs simultaneously?

We will see that a player's payoff in equilibrium is the higher the more disparate players' preferences for the issues are. This results from the fact that with increasing differences in the preferences the possibility to make inter-issue tradeoffs increases. It will further become clear that players can exploit the tradeoffs to a higher extent in the sequential agenda. As as result, I will show that the sequential agenda is always ex ante beneficial for both players if preferences are sufficiently different or first mover probabilities are sufficiently similar. To get this straight, this result shows us that a

[1] In Fershtman (1990) the proposer on each of the issues is selected randomly. I assume a random selection of the proposer only once for the first issue to be negotiated. Thereafter the offer structure is strictly alternating.

sequential agenda may expand the ex ante reachable bargaining space and thereby increase the ex ante efficiency.

Classifying the literature on multi-issue bargaining, we find that the assumption made on the implementation of issues is of major importance: it influences the structure and the result of the game most profoundly. This explains why the results derived with separate implementation are far more general.[2] Fershtman (1990) assumes joint implementation and shows (in addition to the inefficiency result mentioned above), that the agenda does not influence efficiency but does influence the distribution of payoffs if players' preferences are identical. In a similar framework Weinberger (2000) shows that inefficient equilibria result when partial acceptance of bundle offers is allowed. Consequently, she finds that package bargaining tends to improve efficiency.[3]

This paper is organized as follows. In section 2, I will present a formal model and discuss equilibria for the simultaneous and sequential agenda. In section 3, I will introduce the ex ante perspective and derive the ex ante efficient agenda. In Section 4, I conclude with a discussion of the results. Proofs are provided in an appendix.

1.2 Multi-Issue Bargaining with Joint Implementation

1.2.1 The Model

Consider a game of complete information in which two players, A and B, bargain over drafting a contract for the arrangement of two issues or *pies*, X and Y. Each issue is assumed to be of size 1, desirable and infinitely divisible. Players' evaluations of the issues differ, precisely, their evaluation is exactly the reverse.

[2] The early works of Bac and Raff (1996) and Busch and Horstmann (1997) discuss the influence of unilateral incomplete information on the agenda setting. Working in a complete information setting, Busch and Horstmann analyse the influence of different types of bargaining frictions (1997b) and of different agendas (1999). Working with arbitrary sets of issues, In and Serrano (2003) generalise the result of Inderst (2000) and show for a larger class of utility functions that simultaneous offers form the unique s.p.e.. In and Serrano (2004) analyse the endogenous agenda when offers are restricted to single-issues. Their model suffers from multiple equilibria when bargaining frictions become small. Chen (2006) tackles this problem and shows that a unique s.p.e. agenda exists if the alternating offer structure is interpreted slightly different. Flamini (2007) finds a way to overcome the multiple equilibria by introducing a small unit of time in between the bargaining stages.

[3] Lang and Rosenthal (2001) show under the assumption of unrestricted offers, that there are s.p.e. in which players offer only on subsets instead of on the whole bundle of issues. This result is surprising but it depends strongly on the nonconcavity assumption. Applying the Nash solution, Horstmann et al. (2005) and Harstad (2001) compare joint with separate implementation in a two-issue bargaining game. Both find that only joint implementation generally allows for inter-issue tradeoffs and is therefore preferable.

I adopt the Rubinstein (1982) alternating offers bargaining model. Specifically, the procedure is as follows: In the first period a first mover is selected randomly and announces an offer to his opponent. Generally, this offer may contain either one proposal for the division of one issue or a menu of two proposals, each for a division of one issue. I will consider these cases separately and refer to the former as *sequential agenda* and to the latter as *simultaneous agenda*. An offer can be accepted or rejected only as a whole; partial acceptance is not possible. If an offer on a set of issues is accepted by the recipient, this set is assumed to be settled. Renegotiation of settled issues is excluded. No matter whether the recipient accepts or rejects the offer he will be the next proposer if an unsettled issue remains. *Joint implementation* is assumed. Hence, issues are linked such that players obtain any utility only if an agreement on both issues is found. This implies that a disagreement on one issue threatens the entire negotiation. The game starts in period zero. Only a rejection of an offer causes a bargaining friction and leads to a delay of one time period t. Time is assumed to be valuable such that each player prefers an early settlement. The game has no predetermined number of rounds and the outcome of (permanent) disagreement is zero for both players. The game ends when both issues are settled.

Given an agreement on both issues after period t, the partition is such that player A receives share x of issue X and share y of Y. Players' utilities are

$$u_A = (\alpha \cdot x + (1-\alpha)y)\,\delta^t, \qquad u_B = ((1-\alpha)(1-x) + \alpha(1-y))\,\delta^t$$

where δ is the constant rate of time preference and is equal for both players, with $0 \leq \delta < 1$. It is further assumed that $1/2 < \alpha < 1$, i.e. player A evaluates a share of X and B a share of Y higher than an equal share of the alternative issue.

1.2.2 The Simultaneous Agenda Equilibrium

Given a simultaneous agenda, the offer of the first mover contains a proposal on how to divide each of the two issues. The second-mover may accept or reject the offer only as a whole. In this case there is a unique subgame perfect equilibrium (s.p.e.) which always gives the first mover all of his preferred issue and, depending on α and δ, some share of his less preferred issue. Independently of the sequence of moves, the utility in equilibrium is $\frac{\alpha}{\alpha+\delta-\alpha\delta}$ for the first and $\frac{\alpha\delta}{\alpha+\delta-\alpha\delta}$ for the second-mover respectively. The proof is provided in Appendix 1.1. It comes as no surprise that in equilibrium the second-mover's utility takes the value of the discounted first-mover utility as this result coincides with Rubinstein (1982) for symmetric players. Now, Figure 1.1 illustrates the bargaining outcomes of the first mover $i \in \{A, B\}$ and the second mover $j \in \{A, B\}$ with $j \neq i$ depending on α and δ. The continuous rays illustrate the bargaining frontier depending on α and the dashed rays illustrate the distribution depending on δ. The intersection points depict the bargaining outcome for the respective values of α and δ.

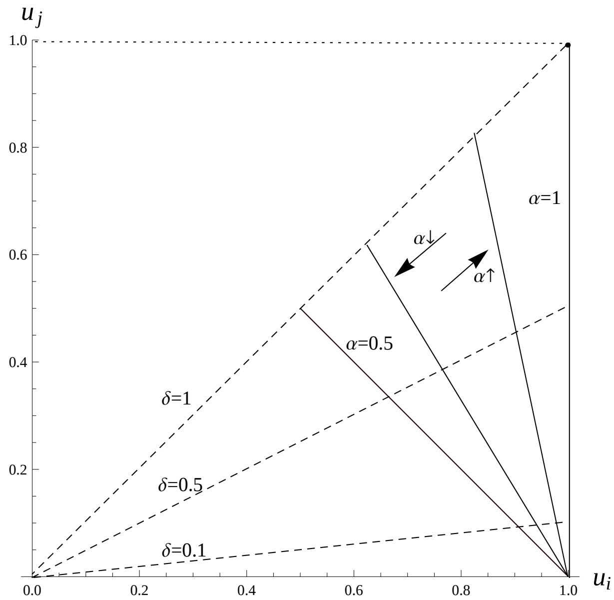

Figure 1.1: First (u_i) and second-mover (u_j) payoffs given a simultaneous agenda

As can be expected, decreasing δ increases the first-mover's payoff (u_i) while it decreases the second-mover's payoff (u_j) independent to α. If patience reaches zero $(\delta = 0)$ the game naturally converges to the ultimatum game; and if $\delta \to 1$ then it converges to the symmetric Nash solution. Note that for $\alpha = 0.5$, $u_i + u_j = 1$. For all $\delta > 0$, both payoffs increase in α. Hence, a higher discrepancy in the evaluation of the issues expands the bargaining frontier. This is due to the fact that opposing evaluations allow players to make *explicit inter-issue tradeoffs*. Figure 1.1 shows that the inter-issue tradeoffs diminish when bargaining frictions increase and vanish if $\delta \to 0$.

1.2.3 The Sequential Agenda Equilibrium

Unlike in the equilibrium of the simultaneous agenda, the determination of the equilibrium for the sequential agenda requires consideration of two cases: players who are restricted to single-issue offers either offer firstly on their preferred issue (*Case I*) or may prefer to offer on their less preferred issue (*Case II*). It will be shown that for every bargaining situation (i.e. every combination of α and δ) for both players one of these strategies is strictly preferable to the other.[4] Further, for all bargaining situations the preferred strategy is represented by a unique s.p.e. in which both players accept their one-issue offers immediately.

Case I: Assume a sequential agenda such that the players are restricted to make offers on one issue at a time. Next, assume the agenda is set exogenously such that players offer solely on their preferred issue. In this case we can state:

Proposition 1.1. (sequential exogenous agenda) *For all δ and α, there is a unique s.p.e. in which holds: each player receives his higher evaluated issue entirely, the bargaining solution is always on the efficiency frontier and the first (second) mover payoff is always smaller (higher) compared to the simultaneous structure.*

Proof (Sketch).[5] The second-mover will accept to give away all of his less preferred issue because he foresees that in return he can demand and receive all of his preferred issue. As the issues are only implementable if both issues are settled – meaning that an disagreement on an unsettled issue threatens players' benefit from the settled issue – the second-mover is secured against exploitation. Note that each issue is negotiated in a separate stage, this means that stage *II* starts after the first issue is settled.[6] Now, let $(x^s_A, 1 - x^s_A)$ be the division of X player A proposes in stage $s \in \{I, II\}$ and let $(y^s_B, 1 - y^s_A)$ be the division of Y player B proposes in stage $s \in \{I, II\}$. Assume that player A is first mover and proposes a division $(x^I_A, 1 - x^I_A)$ on his preferred issue X in stage I. After rejection or acceptance of his offer, it is up to B to offer A a share of Y. Note that the stage *II* agreement depends on the stage *I* division as a decision to reject would also discount the partitions of the stage *I* solution. That is, the stage *II* equilibrium is a reaction function of the stage *I* solution. Assume that A's proposal in

[4]Note that the payoff of the first and second mover is generally independent of the type of player (A or B), as both types are symmetric except that their evaluation of the issues is exactly reverse. Hence, for any given δ and α each player who is about to offer faces the same situation. So for example, if it is optimal for one player to offer on his preferred issue first, then the same holds for the other player. This property substantially simplifies the analysis as for any proposer's offer, the corresponding reservation value of the responder can be calculated (rather easily). Without this assumption, a calculation of the sequential endogenous agenda equilibrium analysis is far more difficult or may even be impossible to carry out as reservation values have to be calculated for any possibly evolving sequence of responses. Here only two cases have to be considered: either both players prefer to offer on their preferred or on their less preferred issue first.

[5]For more details see Appendix 1.2.

[6]Note further that stage *II* begins immediately after the settlement of the first issue. So, for example, if the first offer of stage *I* is accepted right away, no friction occurs and the negotiation of stage *II* continuous in t=0.

stage I was accepted such that now B's stage II offer $(y_B^{II}, 1-y_B^{II})$ has to fulfill:

$$\alpha x_A^I + (1-\alpha)y_B^{II} \;\geq\; \delta\left[\alpha x_A^I + (1-\alpha)y_A^{II}\right],$$

while A's counteroffer after a rejection would be restricted by:

$$(1-\alpha)(1-x_A^I) + \alpha(1-y_A^{II}) \;\geq\; \delta\left[(1-\alpha)(1-x_A^I) + \alpha(1-y_B^{II})\right].$$

Solving these conditions for y_B^{II} makes consideration of different cases necessary. This results from the fact that X and Y are bounded by 0 and 1. Calculating A's possible payoff r_A from both issues as a function of his share x^I from his stage I offer, we obtain:

$$r_A(x^I) = \begin{cases} \frac{\delta - x^I\delta - \alpha\delta + 2x^I\alpha\delta}{\alpha+\alpha\delta} & , for\, x^I \geq k \wedge x^I < l \\ \alpha x^I & , for\, x^I \geq k \wedge x^I \geq l \\ \delta(1-\alpha+\alpha x^I) & , for\, x^I < k \wedge x^I < m \\ \alpha x^I & , for\, x^I < k \wedge x^I \geq m \end{cases}$$

with

$$k = \frac{(1-\alpha)(1-\alpha-\alpha\delta)}{1-\alpha(2-\alpha-\alpha\delta)},\; l = \frac{\delta-\alpha\delta}{\alpha^2(1+\delta)-2\alpha\delta+\delta},\; m = \frac{\delta-\alpha\delta}{\alpha-\alpha\delta}$$

while $r_A(x^I)$ is increasing in x^I over all cases and for all $x^I \in [0,1]$. Note that for the case that B rejects, his possible payoff would be $\delta r_B(y^I)$ which is decreasing in y^I for all $y^I \in [0,1]$. The calculation of the borders k, l and m and the reaction function r_A is provided in Appendix 1.2. Given these payoff functions, we can see that both players will try to maximize their overall payoff by maximizing their share of their preferred issue. For A's stage I offer on X the following conditions must hold:

$$(1-\alpha)(1-x_A^I) + \alpha(1-y_B^{II}) \geqq \delta\left[\alpha(1-y_B^I) + (1-\alpha)(1-x_A^{II})\right]$$

$$(1-\alpha)y_B^I + \alpha x_A^{II} \geqq \delta[\alpha x_A^I + (1-\alpha)y_B^{II}].$$

If we check for the proposers payoff maximizing offer ($x_A^I = 1$, $y_B^I = 0$), we find that the stage II reactions for all $\alpha \in (0.5, 1)$ and all $\delta \in [0,1)$ are $y_B^{II}(x^I = 1) = 0$ and $x_A^{II}(y^I = 0) = 1$ (see Appendix 1.2). Inserting these reactions in the above stage I conditions, we have:

$$(x_A^I =)\, 1 \leq \frac{1-\alpha\delta}{1-\alpha} \quad and \quad (y_B^I =)\, 0 \geq \frac{\alpha\delta-\alpha}{1-\alpha}.$$

Both conditions hold for all $\alpha \in (0.5, 1)$ and all $\delta \in [0, 1)$. According to this result, the first-mover (player A) will demand and receive all his preferred issue. Then B as first-mover in stage *II* will take advantage of A's impatience and can successfully claim the other issue entirely. The first (second) mover payoff is thus of value α and is always lower (higher) than his corresponding payoff $\frac{\alpha}{\alpha+\delta-\alpha\delta}$ $(\frac{\alpha\delta}{\alpha+\delta-\alpha\delta})$ in the simultaneous agenda game. For the case that players are infinitely patient ($\delta \to 1$), the result of the game converges to the simultaneous agenda equilibrium. It is obvious that this result is the *unique* s.p.e. of the given sequential bargaining game and is on the corresponding efficiency frontier. ■

The above result shows that players' payoffs in equilibrium are independent of the common discount factor but are sensitive to changes in the evaluation of the issues. As before, a higher discrepancy in the evaluations allows for higher tradeoffs. But in contrast to the simultaneous offer game, where inter-issue tradeoffs are an explicit part of the proposal, the sequential agenda forces the players to consider possible tradeoffs in an implicit way. Note, that two properties are crucial for the existence of *implicit inter-issue tradeoffs*. Firstly, the sequential agenda reduces the bargaining power of the first proposer. Secondly, joint implementation allows the responder to forego the first issue without any worry, as this very sacrifice increases the impatience of his opponent in stage *II* and thereby enlarges his share of the second issue.

Case II. As before, the agenda is sequential such that players are restricted to offer on one issue at a time. But now assume that in contrast to *Case I*, the agenda is endogenous. This leads us to the following question: when is it optimal for the proposer *not* to offer on his preferred issue first? The following proposition gives the answers.

Proposition 1.2. (sequential endogenous agenda) *If players are sufficiently patient* ($\delta \gtrapprox 0.82$) *and preferences are sufficiently similar* ($\alpha \lessapprox 0.65$), *there are combinations of* α *and* δ *for which the unique s.p.e. of the sequential endogenous agenda game has players offering on their less preferred issue first. In this case a first-mover advantage exists, the result is never on the efficient bargaining frontier and the first mover's payoff is always smaller compared to the simultaneous agenda.*

Proof. Appendix 1.3 [7]

The results contained in proposition 1.2 are illustrated in Figures 1.2, 1.3 and 1.4 and discussed in the following. The dark region θ in Figure 1.2 illustrates the combi-

[7] A sketch of this proof is omitted here as it basically follows the proof of proposition 1.1.

nations of α and δ for which in the unique s.p.e. the first mover starts offering on his less preferred issue as his payoff ($\underline{u}_1^*$) by doing so exceeds his payoff from offering on his preferred issue first (recall: in *Case I* $\bar{u}_1^* = \bar{u}_2^* = \alpha$).[8]

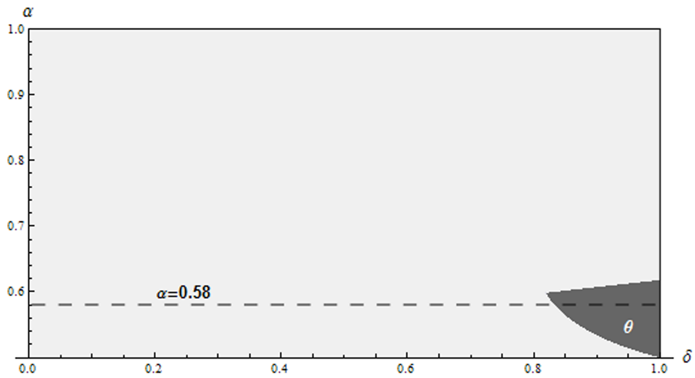

Figure 1.2: Region θ - values of α and δ for which the first-mover prefers to offer on his less preferred issue

It is obvious that this inefficient agenda emerges only exceptionally; specifically if preferences are sufficiently similar and bargaining frictions are sufficiently small. This stands in contrast to Fershtman (1990) where a sequential endogenous agenda always leads to inefficient outcomes.[9] Figure 1.3 plots the equilibrium payoffs of the first and second-mover for $\alpha = 0.58$, while Figure 1.4 shows the division of both issues in the s.p.e. for $\alpha = 0.58$ and A being first-mover. [10]

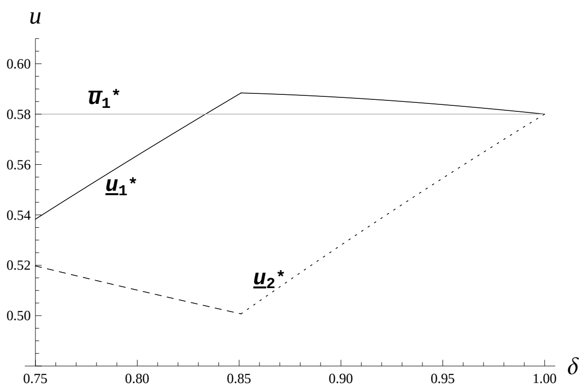

Figure 1.3: First and second mover payoffs for $\alpha = 0.58$

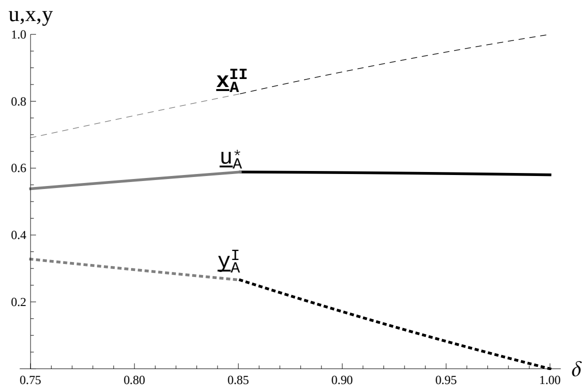

Figure 1.4: Division of issues in stage I and II for $\alpha = 0.58$

[8] Region θ is defined by the following two conditions (4.12) and (4.15). The derivation of these conditions are provided in Appendix 1.3

[9] In Fershtman (1990) players are uncertain about the sequence of moves (if they are proposer or responder) for the bargaining procedure on the second issue while they bargain on the first. Hence, players are ex ante uncertain about the division of the second issue and expect some average. As a result the first mover advantage for the first issue is comparably smaller such that each player prefers to bargain firstly on the issue he prefers less.

[10] The value $\alpha = 0.58$ has been chosen solely for explanatory reasons. For any other value in θ the picture is similar but less illustrative.

From Figure 1.3 we learn that the payoff ($\underline{u}_1^*$) of the first-mover for $\delta > 0.84$ is higher than the second-mover's payoff ($\underline{u}_2^*$) and higher than $\overline{u}_1^* = \alpha$. In addition, we can see in Figure 1.3 that the summed payoffs of both players ($\underline{u}_1^* + \underline{u}_2^*$) is smaller than the summed payoff when players offer on their more important issue first ($\overline{u}_1^* + \overline{u}_2^* = 2\alpha$).[11] Furthermore, the plot for $\alpha = 0.58$ in Figure 1.4 shows us that for $\delta < 1$ each of the players receives at least a little share of both issues. This means that each player receives a share of an issue which is more important to his rival. Consequently the initial bargaining result is not on the efficiency frontier as renegotiation could lead to a higher payoff for both players. All observations derived from Figure 1.3 and 1.4 hold for the entire region θ.

The assumption of joint implementation implies that player's impatience concerns both, the shares of the settled issue as well as the (expected) shares of the issue to be settled. Therefore common sense might tell us that the proposer of the last stage can exploit the common impatience at best. However, this line of thought is deceptive. Actually it is the proposer of the first stage who can set the course such that his opponent acts in his favor. The following example helps us to elucidate this point.

Example 1. Assume A moves first, $\delta = 0.9$ and $\alpha = 0.58$ as in Figure 1.4. The game is in θ, i.e. it is optimal for A to offer on his less preferred issue Y. In equilibrium player B accepts a share of $1 - y_A^I = 0.829$ of Y in stage I (see the lower dashed line in Figure 1.4). Given his large share of his higher evaluated issue, B is impatient regarding an implementation and cannot avoid to give away $x_B^{II} = 0.888$, an even larger share of X in stage II (see the upper dashed line). Adding up we have $\underline{u}_A^* = 0.586\,(> \alpha)$ and $\underline{u}_B^* = 0.528$. However, note that neither in stage I nor in stage II B can escape from this equilibrium path. A rejection in stage I is excluded by the fact that $\underline{u}_B^* = \delta \underline{u}_A^*$, such that any deviation of B in stage I would not increase his payoff. Moreover, once having followed this path to stage II, B finds himself in a relatively powerless position: his minor interest in stage II issue combined with his impatience regarding an implementation of his large share from stage I would force him to accept any potential counter-offer from his opponent A – as a consequence his own offer has to set A at least indifferent.

[11]Note the following: If the game is θ, then for $\underline{u}_1^*$ and $\underline{u}_2^*$ the following holds: the sum of both player's payoffs is always smaller, the first mover's payoff is always smaller and the second mover's payoff may be larger or smaller compared to the respective payoffs from the simultaneous agenda.

1.3 The Ex Ante Efficient Agenda

Given a restriction to single-issue offers, we have seen that in most cases players offer on their preferred issue first. The simultaneous agenda gives strictly higher utility to the first mover than any of the sequential agendas. Hence, if offers are unrestricted the simultaneous agenda would emerge. This is also the standard result in literature.[12] However, do these results still hold if we take an perspective ex ante to the choice of the first-mover?[13] Note that even though the result of the simultaneous agenda is always on the efficiency frontier, this efficiency is only relative with respect to the agenda. Put simply, the joint payoff under the sequential agenda is always higher than under the simultaneous agenda as long as the game is not in θ. Or in other words, the restriction to endogenous single-issue offers expands the efficiency frontier and yields the *ex ante efficient* allocation. The following example illustrates this.

Example 2. Assume that players are completely impatient ($\delta = 0$) and have a strong preference for one issue ($\alpha = 0.83$). If offers are unrestricted, the first-mover would always chose to offer on the bundle and receive both issues entirely: the payoff is 1 for the first and 0 for the second-mover. Now assume that players are uncertain who will be the first-mover; i.e. nature's choice falls on each player with probability one half. In the unrestricted case, the expected utility of each player is 0.5. In contrast, if offers are restricted to single issues and the game is not in θ, both players expected payoff is $\alpha = 0.83$. In this example, an exogenous restriction of the bargaining agenda to single-issue-offers meets the ex ante interest of both players even though it does not meet the interest of the first-mover once he is selected. To generalize this result it can now be stated:

Proposition 1.3. *If $p \in [0, 1]$ is the probability for player A to be the first-mover, then for all games not in θ the sequential agenda yields*

(i) for all $p \in [0, 1]$ higher joint payoffs

(ii) for each of both players a higher ex ante expected payoff if $p \in (1 - \alpha, \alpha)$

in comparison to the simultaneous agenda.

[12] Compare for example In and Serrano (2003); Inderst (2000) and Fershtman (1990).

[13] A somehow constitutional perspective in which players - under a veil of ignorance - discuss about the agenda.

Proof.

(i) Outside of θ both players' payoff under the sequential structure is α, hence players' joint payoff is 2α. For the simultaneous structure the first and second-mover payoffs are $\frac{\alpha}{\alpha+\delta-\alpha\delta}$ and $\frac{\alpha\delta}{\alpha+\delta-\alpha\delta}$, respectively. The joint payoff in the simultaneous structure is $\frac{\alpha+\alpha\delta}{\alpha+\delta-\alpha\delta}$ and always lower than 2α.

(ii) Notice that A's ex ante expected utility in the simultaneous case is $Eu_A = p\frac{\alpha}{\alpha+\delta-\alpha\delta} + (1-p)\frac{\alpha\delta}{\alpha+\delta-\alpha\delta}$ (for B: $Eu_B = (1-p)\frac{\alpha}{\alpha+\delta-\alpha\delta} + p\frac{\alpha\delta}{\alpha+\delta-\alpha\delta}$) which is always smaller than α as long $p < \alpha$ (for B: $(1-p) < \alpha$). ■

Note that part (ii) of the proposition answers questions a) and c) from the introduction: the sequential agenda maximises each player's expected payoff as long as neither of them has a probability higher α to be the first mover. Part (i) answers question b): the sequential structure maximises ex-ante efficiency.

Figure 1.5 illustrates players' expected utility from bargaining with a simultaneous agenda as subject to δ and p for $\alpha = 0.8$. As in Figure 1.1 the bargaining frontier expands with higher α and has the extreme points (0,1), (α, α) and (1,0). Players' expected payoff is depicted by the points of intersection of the dashed lines, reflecting a certain level of p (player A's probability to move first) and the continuous lines, reflecting a certain discount factor δ. For p=1, A is always the first mover and his expected payoff is linearly decreasing with increasing delta (between $Eu_A = 1$ for $\delta = 0$ and $Eu_A = 0.8$ for $\delta \to 1$). For p=0, player A's expected payoff is linearly increasing with delta (between $Eu_A = 0$ for $\delta = 0$ and $Eu_A = 0.8$ for $\delta \to 1$). Now, for p=0.8 it holds that $Eu_A = 0.8$ (=alpha) independent to the discount factor while for all p>0.8 it holds that $Eu_A > 0.8$ and for all p<0.8 it holds that $Eu_A < 0.8$. For example, given $\delta = p = 0.5$ (intersection point E) both players expect a payoff $Eu_A = Eu_B = 0.\bar{6}$.

From section 1.2.2 we know that α expands the bargaining frontier and both players' payoffs increase ceteris paribus if α increases. If δ approaches 1, both issues will be split by maximizing the overall utility and a player's utility equals his expected utility α (in this case 0.8) independently of p. This naturally coincides with the cooperative division of the Nash Bargaining Solution. However, the more impatient the players are, the more asymmetric is the bargaining power and the more unequal players' payoffs. Moreover, the bargaining space shrinks in δ, because players' ability to make inter issue tradeoffs diminishes. Hence, if ex ante uncertainty comes into play the expected utility of each player also decreases in δ ceteris paribus. Figure 1.5

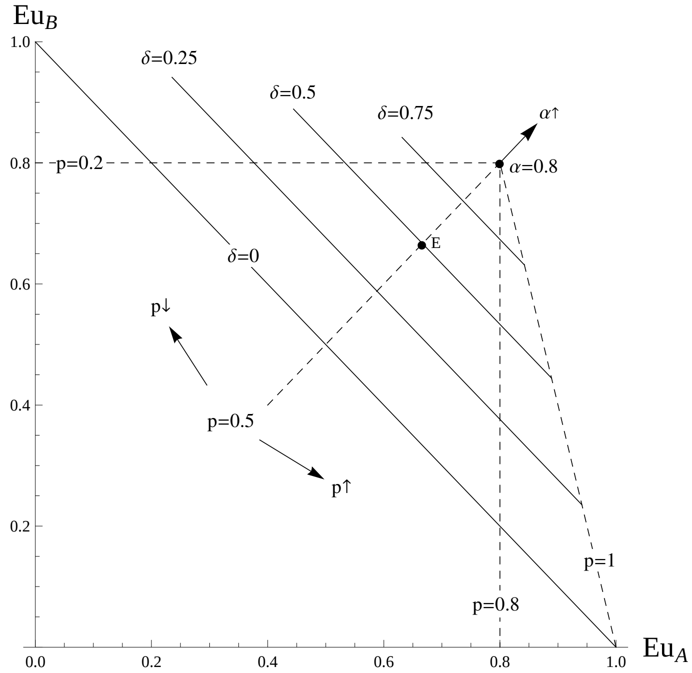

Figure 1.5: Simultaneous agenda expected payoffs subject to p and δ for $\alpha = 0.8$

now shows us that a player's expected utility, independent of δ, exceeds the value of α only if the first-mover probability p (for B (1-p)) exceeds α. Thus, we can conclude that the presence of frictions in multi-issue bargaining a simultaneous (unrestricted) agenda imposes not only a high ex ante risk on each player, but may also lower the bargaining frontier and thereby reduce player's ex ante expected utility.

1.4 Conclusion

This paper explored the importance of the bargaining agenda from two perspectives. Firstly, does a certain agenda create an efficient outcome? Secondly, by applying the ex ante perspective, how efficient is this outcome compared to those from other agendas? The insights are new and provide novel arguments for the sequential agenda: negotiating one issue at a time only seldom leads to inefficiency and it is almost always ex-ante efficient. Admittedly, the results are anything but general – however, the essential results are likely to prevail in less restricted settings.

By adopting an ex ante perspective it became obvious that a sequential agenda allows for *implicit inter-issue tradeoffs* if joint implementation is assumed. We learned that in this case players fully exploit the inter-issue tradeoffs independently of the bargaining frictions present. In contrast, we saw that the simultaneous agenda only allows for full exploitation of tradeoffs if no frictions are present and that tradeoffs decrease when frictions rise. Similarly, for separate implementation we know from Chen (2006) that the sequential agenda only allows for full exploitation of tradeoffs if bargaining frictions are maximal and that tradeoffs decrease if frictions decrease. We can thus conclude that linking issues (joint implementation) and restricting offers to single-issues (sequential agenda) makes inter-issue tradeoffs generally achievable to a higher extent and thus expands the bargaining frontier. Under these settings, the outcome of our non-cooperative bargaining game equals the (ex-ante) efficient outcome of the cooperative (Nash-) Solution. This insight is a strong result itself and a valid argument against a rejection of the sequential structure.[14]

Nevertheless we have also seen that a sequential structure may cause inefficiency. If the first mover offers on his less preferred issue first, then both players receive a share of the issue preferred by their opponent. Like Fershtman (1990) we find that for this agenda the result is clearly not on the bargaining frontier. But in contrast to Fershtman, under our assumptions this inefficient sequential agenda is a rather exceptional result: it emerges endogenously only if players are sufficiently patient and their preferences are not too opposing. For most bargaining situations the emerging endogenous agenda has players offering on their preferred issue first and the result is efficient. This result clearly weakens the inefficiency result of Fershtman (1990) and is a second argument in favor of bargaining *one thing at a time*.

We have further seen that the restriction of the agenda has distributional effects. Payoffs in the sequential structure are nearly always independent of the sequence of moves which removes risk. Ex post, the restriction to single-issue offers is always unpleasant for the first-mover and throughout beneficial for the second-mover. Ex ante, the effect is always beneficial for both if the achievable inter-issue tradeoffs are comparably higher than any player's first-mover advantage. From this perspective, the simultaneous structure leads the proposer to exploit his relative advantage and to *capture value* while the sequential structure leads both players to exploit possible inter-issue tradeoffs and thus to *create value.*

[14]Compare for example Fershtman, 1990; Weinberger, 2000; In and Serrano, 2004 and Raiffa, 1982.

Chapter 2

The Agenda in Multi-Issue Bargaining with Punishment

Abstract: This paper investigates the effect of the agenda in alternating-offer bargaining on multiple issues when players have the ability to execute a punishment in every period of rejection. This setting reflects labor conflicts or warfare negotiations. The results show that the choice of the agenda has influence on the range of inefficient equilibria, which are usually present in such models. Whenever the first mover is not too powerful, a sequential agenda which targets the unimportant issue first reduces inefficiency. The simultaneous agenda, i.e. players make bundle offers, does so in the opposite case.

2.1 Introduction

Early in 2007 the German train driver union (GDL) and the German railway civil servants (GDBA) canceled their joint labor contract with the Deutsche Bahn (DB), the by far biggest German railroad company. GDL followed up by starting a tariff conflict that lasted until the beginning of 2008 and was accompanied by heavy strikes causing immense costs for all parties, the GDL, the DB and rail travelers. The claim of the GDL contained two main issues, firstly a significant higher wage (in the beginning around 30%) and secondly an independent tariff contract. It was widely perceived that both parties' interest in the latter issue was comparably higher but their positions regarding its exact structuring seemed hardly reconcilable. However, even though negotiations took place in camera, it was obvious that parties bargained over both issues simultaneously, i.e. negotiations were not limited to one issue at a time. This paper addresses the question if this conflict could have been settled less costly by setting the bargaining agenda differently. More generally, it analyzes if the bargaining agenda has an influence on the level of inefficiency in multi-issue bargaining settings in which parties have a punishment ability, e.g. calling strikes or arranging lock-outs. Indeed, the analysis will show that the length and intensity of the GDL strike might have been reduced under a different agenda. To develop this answer, this paper draws on two strands of research that build upon Rubinstein's (1982) alternating offer model, models of wage-bargaining and multi-issue bargaining models.

One challenge of the models on wage-bargaining is to explain the existence of strikes and thereby to overcome the so called Hicks Paradox: "The main obstacle is that if one has a theory which predicts when a strike will occur and what the outcome will be, the parties can agree to this outcome in advance, and so avoid the costs of a strike. If they do this, the theory ceases to hold." (Kennan, 1987, p. 1091). This paradox can be avoided by models in which players face some kind of uncertainty. Uncertainty, whether about the game being played, i.e. by assuming incomplete information, or about the equilibrium being played in settings of multiple equilibria. In the early papers the assumption of one- or two-sided incomplete information was essential for the explanation of strikes. In these models strike and delay is generally used by parties as signaling device.[1] Later Haller and Holden (1990) and Fernan-

[1]See for instance Admati and Perry (1987); Chatterjee and Samuelson (1987); Cramton (1984, 1992); Fudenberg et al. (1985); Grossman and Perry (1986); Hart (1989); Rubinstein (1985); Sobel and Takahashi (1983). In Goerke and Holler (1999) the strike threat of the union becomes credible if ballots provide sufficient commitment.

dez and Glazer (1991) demonstrated that the assumption of incomplete information is dispensable. In their models the ability to strike enlarges players strategy space and triggers a multiplicity of equilibria, including such of immediate agreement and others which allow for delay and strikes.[2] Avery and Zemsky (1994, p. 155) conclude that in these models "delay results from the existence of multiple equilibria with immediate agreements. Players are kept from making acceptable offers by the threat of getting their worse possible equilibrium payoff."

Multi-issue bargaining models, by contrast, mostly focus on the study of the bargaining agenda, i.e. the order in which issues are put on the table. For various multi-issue bargaining settings it has been shown that the agenda itself can be a source for inefficiency if offers have to be made piecewise.[3] In these models the simultaneous agenda (offers target all issues at once) is generally preferable in terms of efficiency. Exceptions to these results have been contributed by Chen (2006) and Tiedemann (2009). Both show that by adopting an ex ante perspective to the bargaining game, a better exploitation of inter-issue trade-offs and therefore a higher efficiency can be achieved by the sequential agenda, i.e. by offering on one issue at a time.

This paper extends a single-issue punishment bargaining model to a multi-issue setting and analyzes the influence of the agenda. More concretely, it is investigated if the range of inefficient equilibria and the level of inefficiency of these equilibria is sensitive to the agenda setting. For this purpose a bilateral, two-issue bargaining model with complete information is set up. In this model I assume that the two issues can only be implemented jointly, i.e. no benefit becomes available before an agreement is found on all issues. This reflects settings in which multiple issues are essential for the overall agreement, as for example the train driver labor conflict is only resolved if an agreement is found for all issues.

The analysis of the extended model shows that the structure of the single-issue equilibria equally applies in the multi-issue case. As before, multiple efficient and inefficient subgame perfect equilibria emerge under both agendas though differences between the two agenda outcomes can be observed. The comparison of the equilibria then shows that the choice of the agenda influences the degree of inefficiency. A restriction to single-issue offers can have positive or negative effects on the range of inefficient equilibria and on the potential level of inefficiency of these equilibria. As long as no player is too powerful, the results suggest issues to be negotiated one at a

[2]Slantchev (2003) uses a similar model to show that equilibria with costly hostilities can arise in warfare negotiations between completely informed states.

[3]See for example Fershtman (1990); In and Serrano (2003, 2004); Inderst (2000) or Inderst (2000).

time and the issue of lower importance first. In contrast, if one player is sufficiently powerful the simultaneous agenda is favorable in terms of efficiency.

The paper is structured as follows. In section 2 the single-issue model is introduced and the analysis identifies multiple subgame perfect equilibria. In section 3 the model is extended to two-issues. The equilibrium analysis is provided for the sequential and the simultaneous agenda. En suite the structure and scope of the inefficient equilibria is compared under both agendas. Section 4 concludes.

2.2 Punishment Ability and Multiple Equilibria

2.2.1 The Model

Assume a Rubinstein bargaining game of the discounting type where, in addition, players have the option to punish their opponent. This means that after the rejection of an offer, each of the players independently and simultaneously decides to carry out a costly action which inflicts costs on the opponent. Each player is free to carry out his threat once in any period following a rejection. This ability enlarges players' strategy spaces such that a rejection can entail different levels of costs, depending on whether a punishment follows or not. As a consequence a multiplicity of equilibria arises in this bargaining game. A similar model (single issue and only one punisher) was provided by Avery and Zemsky (1994).

The formal setup is as follows. Two completely and perfectly informed players, A and B, bargain on the division of an infinitely divisible pie X of size 1. Players make offers in alternating order about the division of X, x_{ij} is the share player $i \in \{A, B\}$ offers to $j \in \{A, B\}$. Offers can be accepted (a) or rejected (r). The acceptance of a proposed division leads to its immediate implementation. Player A is assumed to make offers in period t=0 and all even periods. B offers in any odd period. The passing of a period causes discounting of players' payoffs by δ_A and δ_B. The game continues as long as no agreement is found, players' payoffs for perpetual disagreement is zero. So far it equals the Rubinstein setting. Now, assume that the rejection of an offer compulsorily leads to the punishment game Γ, which takes place in the same period. Let Γ be some normal form game where both players independently and simultaneously choose between two actions, *punish* or *not punish*. A punishment by player i inflicts costs $c_{ij} > 0$ to $j \neq i$ and $c_{ii} > 0$ to i himself. Figure 2.1 illustrates the game.

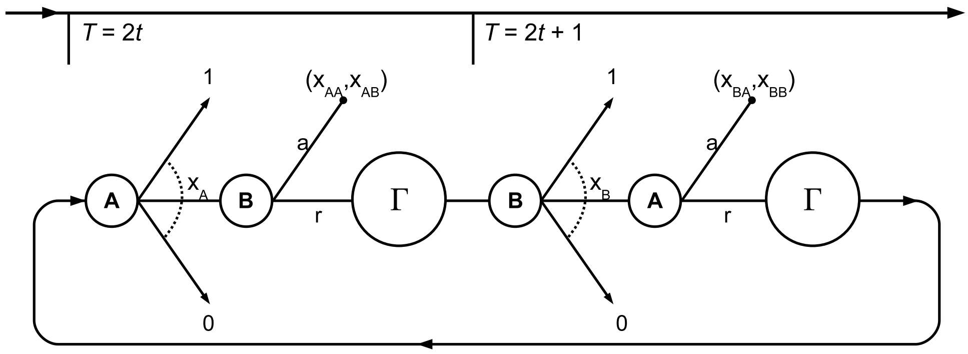

Figure 2.1: Two periods of the bargaining procedure

An agreement established in period T which gives the share x to player i yields i the following payoff:

$$u_i = \delta_i^T x - \sum_{t=0}^{T} \delta_i^t c_{i,t} \,.$$

The cost for player i in period t are as follows, $c_{i,t} = 0$ if no player punishes, $c_{i,t} = c_{ii}$ $\left(c_{i,t} = c_{ji}\right)$ if only *i*(*j*) punishes, and $c_{i,t} = c_{ii} + c_{ji}$ if both punish.

2.2.2 The Effect of Punishment

For the following assume that the decision to punish is set exogenously (the choices in Γ are given). Given this assumption we can easily compare the outcomes of players' punishment strategies. We find that for each player a best and a worst combination of (rational) punishment strategies exists. Each of these strategy profiles has the same structure: player j never punishes and $i \neq j$ applies a disrupted punishment strategy, i.e. he punishes always after his own offer is rejected. I will refer to this profile as R^i, i.e. the regime of player i. If we assume that players follow R^i, player i receives his best and j his worst payoff. Let x_{ij}^k denote the share player i offers j in the regime of player k.

Now assume that R^i is played. With his disrupted punishment strategy, i has a strategical advantage; for j the costs of his opportunity 'rejection' increase such that in equilibrium he will accept a lower share. Making use of Shaked and Sutton's (1984) method it can be shown that for subgames in which i proposes, the unique subgame perfect equilibrium (s.p.e.) partition is:

$$\left(x_{ii}^i, x_{ij}^i\right) = \left(\frac{1 - \delta_j + c_{ij}}{1 - \delta_i \delta_j}, \frac{\delta_j - \delta_i \delta_j - c_{ij}}{1 - \delta_i \delta_j}\right). \tag{2.1}$$

Note that this regime yields i the largest share he can achieve by any punishment strategy.[4] Analogously, if regime R^j is played, the unique s.p.e. partition in subgames in which i offers, is:

$$\left(x_{ii}^{j}, x_{ij}^{j}\right) = \left(\frac{1-\delta_j-\delta_j c_{ji}}{1-\delta_i\delta_j}, \frac{\delta_j-\delta_i\delta_j+\delta_j c_{ji}}{1-\delta_i\delta_j}\right). \tag{2.2}$$

None of these s.p.e. support delay or punishment.[5] In contrast, if we assume that no player ever punishes the game converges to the standard Rubinstein game, denoted R, and the unique partition supported in equilibrium is:

$$\left(x_{ii}^{R}, x_{ij}^{R}\right) = \left(\frac{1-\delta_j}{1-\delta_i\delta_j}, \frac{\delta_j-\delta_i\delta_j}{1-\delta_i\delta_j}\right).$$

2.2.3 Two Landmark Equilibria

Now there is to check if the execution of the punishment (and the investment of c_{ii}) is indeed rational for player i. To implement his regime R^i player i has to execute his punishment if his offer gets rejected. In this case he can expect at best a payoff of x_{ji}^{i} in the next period. If i forgoes to punish instead the Rubinstein outcome results and yields him x_{ji}^{R} in the next period. Thus, the punishment is rational for player i if the following holds:

$$-c_{ii}+\delta_i x_{ji}^{i} \geq \delta_i x_{ji}^{R}. \tag{2.3}$$

In Appendix 2.1. it is argued that (2.3) is a weak constraint. I summarize by stating the following:

Proposition 2.1. *If punishment is rational for i, then R^i constitutes a s.p.e. which supports the partition $\left(x_{ii}^{i}, x_{ij}^{i}\right)$.*

Proof. To show that R^i is a s.p.e. which supports the partition $\left(x_{ii}^{i}, x_{ij}^{i}\right)$ if (2.3) holds, I show that no player has an incentive to deviate. In R^i players strategies are the following:

[4] The s.p.e. which yields i an even higher payoff will be neglected as it requires irrational punishments by j. If both players punish always after j rejects, i would receive a higher share than by punishing alone. The decision to punish after his own rejection is therefore irrational for j.

[5] Note that these results are in accordance to the single punishing player model in Avery and Zemsky (1994).

- Player i: offer x^i_{ij}, accept all $x \geq x^i_{ji}$, reject all $x < x^i_{ji}$ and punish always after j rejects.
- Player j: offer x^i_{ji}, accept all $x \geq x^i_{ij}$, reject all $x < x^i_{ij}$ and never punish.

If player i fails to punish after j has rejected an offer, player j's counteroffer is x^R_{ji} and players follow R.

Given this profile, a deviation is never rational for j as he has to bear the costs of punishment without expecting a larger share in the end. Because (2.3) holds, it is preferable for i to invest the punishment costs of c_{ii} in order to implement his regime R^i and receive x^i_{ji} in the next period than to forgo the punishment and continue with R and receive x^R_{ji} in the next period. ■

2.2.4 Efficient Equilibria

The two extreme equilibria of R^A and R^B allow for a (kind of) Folk theorem construction with an infinite number of s.p.e. including equilibria with immediate and efficient (no punishment) agreement as well as inefficient ones with delay and punishment. First, to show the existence of the efficient equilibria I state the following.

Proposition 2.2. *If the rationality constraint* (2.3) *holds for both players, then every*

$$\bar{x} \in \left[x^B_{AA}, x^A_{AA}\right], \tag{2.4}$$

can be supported as s.p.e., in which A demands $\bar{x}$ for himself in t=0 and B immediately accepts.

Proof. To prove that these actions are supported as s.p.e. we make use of the two landmark equilibria which emerge in R^A and R^B. The structure of the equilibrium strategies is as follows: Each player proposes $(\bar{x}, 1-\bar{x})$ and accepts the partition $(\bar{x}, 1-\bar{x})$. If player i deviates from the proposition or the acceptance of $(\bar{x}, 1-\bar{x})$, player j starts his disruptive punishment, i.e. the strategies immediately switch to R^j.

Concretely, assume that A's offer in period t=0 is rejected by B. For A's offer $\bar{x} \in \left[x^B_{AA}, x^A_{AA}\right]$, B's rejection constitutes a deviation from the equilibrium strategy, the game switches immediately to R^A, this means A punishes (thereby inflicting c_{AB} to B and c_{AA} to himself) and players follow R^A in t+1 where B receives x^A_{BB}.[6] Conversely, if

[6]At this point the question may arise whether A's punishment after the rejection of B in t=0 is a best response, as it inflicts costs c_{AA} to A. To understand the necessity of A's punishment, note that in

A deviates by making any other offer than $\bar{x}$, players follow R^B in t+1 and A receives x^B_{BA}.[7] Comparing the payoffs in t=0, it is optimal for A to demand $\bar{x}$ right away, because any other offer yields him $\delta_A x^B_{BA} < x^B_{AA} \leq \bar{x}$. It is optimal for B to accept the proposal of $\bar{x}$, because $1 - \bar{x} \geq x^A_{AB} = \delta_B x^A_{BB} - c_{AB}$. ■

2.2.5 Inefficient Equilibria

The structure of the efficient equilibria relies on the threat that any deviation from the equilibrium path causes a switch to the opponent's regime. The same threat allows for the construction of inefficient equilibria including outcomes with delay and punishment. In such inefficient equilibrium each player follows his own regime, i.e. he makes extreme offers and punishes the opponent for a certain period of time before a compromise is mutually accepted. In this structure each player has the choice between this wasteful path with delay and punishment and a deviation from it, i.e. the acceptance of the opponent's regime. The following proposition states that the wasteful path is indeed an equilibrium.

Proposition 2.3. *If the rationality constraint* (2.3) *holds for both players and $\hat{x}$ is such that*

$$\hat{x} \in \left[\left(x^B_{AA} + \frac{1-\delta_A^N}{1-\delta_A} c_A \right) \delta_A^{-N},\ 1 - \left(x^A_{AB} + \frac{1-\delta_B^N}{1-\delta_B} c_B \right) \delta_B^{-N} \right], \tag{2.5}$$

then there is a s.p.e. in which there is no agreement and punishment for a positive integer number of N periods followed by agreement on the partition $(\hat{x}, 1-\hat{x})$ in period N+1. The punishment can have the following structure (with c_A and c_B defined analogously):

a) A and B punish in every period.

b) A or B punishes in every period.

c) A and B punish disruptively after their own offer is rejected.

Proof. I restrain the analysis to show (a) as the structure of the proof is identical in all cases. The equilibrium strategies are the following as long as no player has deviated:

case of a deviation by B, the strategies impose a (immediate) switch to R^A even before the punishment game Γ in t=0 starts . Hence, if A fails to punish after B's deviation, then players follow the Rubinstein procedure R which yields A the payoff $\delta_A x^B_{BA}$ instead of $\delta_A x^A_{AA} - c_{AA}$ if he punishes. Given (2.3) holds, the latter is higher and A's punishment is rational.

[7] Remember that in subgames where A offers, a share for A outside of $[x^B_{AA}, x^A_{AA}]$ cannot be supported as s.p.e. because A can never be forced below x^B_{AA}, nor can A demand more than x^A_{AA}, as B cannot be forced below $x^A_{AB} = 1 - x^A_{AA}$.

- Player A: offer x^A_{AB}, reject all $x < x^A_{BA}$, accept all $x \geq x^A_{BA}$, and punish for N periods in every period, then demand $\hat{x}$, reject all $x < \hat{x}$ and accept all $x \geq \hat{x}$, punish disruptively in periods where B rejects.

- Player B: offer x^B_{BA}, reject all $x > x^B_{AB}$, accept all $x \leq x^B_{AB}$, and punish for N periods in every period, then offer $\hat{x}$, reject all $x > \hat{x}$ and accept all $x \leq \hat{x}$, punish disruptively in periods where A rejects.

If player i deviates from above strategies, e.g. i offers an early compromise or refuses to agree on $\hat{x}$ in period $N+1$, player j immediately switches to his strategy of regime R^j. From Proposition 2.1 we know that in this case i's equilibrium payoff is x^j_{ii} as first and x^j_{ji} second mover. In contrast, if both players follow the suggested strategies, then the equilibrium payoffs are:

$$u_A = \delta_A^N \hat{x} - \frac{1-\delta_A^N}{1-\delta_A}\left(c_{AA} + c_{BA}\right) \qquad u_B = \delta_B^N\left(1-\hat{x}\right) - \frac{1-\delta_B^N}{1-\delta_B}\left(c_{BB} + c_{AB}\right).$$

Given B follows his equilibrium strategy, by any deviation A cannot enforce a payoff higher than x^B_{AA}, i.e. his payoff if he immediately complies on regime R^B. It is thus optimal for A not to deviate as long as $\hat{x} \geq [x^B_{AA} + \frac{1-\delta_A^N}{1-\delta_A}(c_{AA} + c_{BA})]\delta_A^{-N}$ which is the lower bound of $\hat{x}$. By any deviation B cannot enforce a payoff higher than x^A_{AB}, which is his payoff if he immediately accepts A's first equilibrium offer. Therefore, it is optimal for B not to deviate from the equilibrium strategy if $\hat{x} \leq 1 - [x^A_{AB} + \frac{1-\delta_B^N}{1-\delta_B}(c_{BB} + c_{AB})]\delta_B^{-N}$, the upper bound of $\hat{x}$. ■

For Proposition 2.3 to be of any value it remains to be shown that the lower and upper bound indeed open a range for $\hat{x}$. For small discount factors, such range is not generally given (compare equation (2.5)). I will therefore give a positive example of case (a). Figure 2.2 plots the boundaries of equation (2.5), case (a) for the indicated values as a function of N. For values of $\hat{x}$ around 0.6, $\hat{x}$ lies in between the boundaries for any $N \leq 10$. Hence, a delay with both sided punishment for $N \leq 10$ periods followed by agreement on x close to 0.6 is preferable for each player to an immediate agreement on the opponents regime and can thus be supported as s.p.e. Figure 2.3 compares, somehow more intuitively, players' payoffs from an immediate agreement on one of the extreme regimes to the payoffs of an agreement on (some arbitrarily chosen) $\hat{x} = 0.5$ after N periods of delay and punishment. Player A's (B's) payoff is depicted on the left (right) hand scale. The continuous horizontal lower (upper) line plots the payoff of A (B) given they have immediately agreed in the first period to their unfavored regime R^B (R^A). An upward (downward) directed vertical line at any point in time N indicates the payoff of A (B) after N periods of punishment followed by agreement on $\hat{x}$. The vertical dashed line at N illustrates the level of inefficiency, which is the lost payoff induced by the discounting and the costs of punishment.

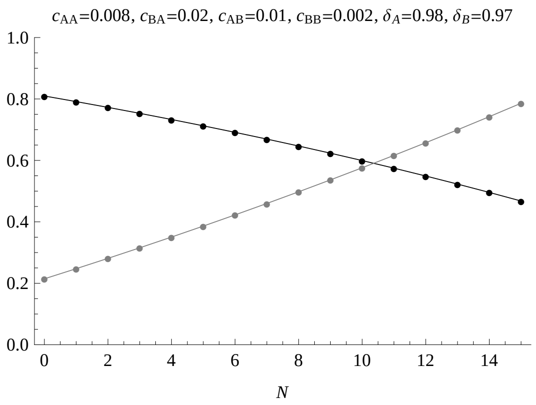

Figure 2.2: Range of inefficient equilibria

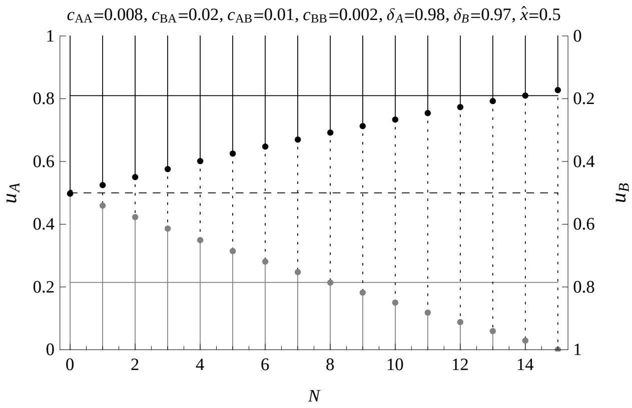

Figure 2.3: Social costs of punishment equilibria

2.3 Punishment Ability and the Bargaining Agenda

2.3.1 A Two-Issue Model and the Agenda

In real-life negotiations there is often more than one issue at stake. Or, to put it differently, negotiations can be split in multiple sub-issues. In the following I will apply the above bargaining structure to a multi-issue setting and compare the outcomes under two different bargaining agendas, the *simultaneous* agenda and the *sequential* agenda. More precisely, in this setting player A and player B bargain over two issues Y and Z, each of size 1. Let y_{ij} (z_{ij}) be the share player $i \in \{A, B\}$ offers to $j \in \{A, B\}$ of issue $Y(Z)$. Both players evaluate their share of Y with ρ and their share of Z with $1 - \rho$. For simplicity it is now assumed that both players' costs for punishing the opponent are zero, i.e. $c_{AA} = c_{BB} = 0$, and further that both players are equally patient, i.e. $\delta_A = \delta_B = \delta$. [8]

By referring to a simultaneous agenda, I assume that players make offers which contain a proposition for the division of issue Y and Z, often referred to as bundle

[8]The focus of the analysis lies on the question whether the agenda has influence on the bargaining result in general and on the possible degree of inefficiency in particular. This will be positively answered, a more general approach may qualify but not negate this. These assumptions surely cause a loss of generality, but do not affect the results in regard to our question. Instead, they allow for more clarity by making further case differentiations unnecessary. Moreover, opposing or completely uncorrelated evaluations of the issues would yield more general results but appear to be not tractable.

or menu offer. The responder can only reject or accept this offer as a whole. For the sequential agenda, we assume that players have to find an agreement on one issue in stage *I* before they continue bargaining on the other in stage *II*. If an agreement is found on one issue, I will refer to this issue as *settled.* In this approach the bargaining agenda as well as the sequence in which issues are negotiated is assumed to be exogenously set.[9]

The sequential agenda requires further assumptions. I assume renegotiation to be excluded, i.e. a settled issue is taboo for further haggling. This assumption simplifies the analysis but is not a necessary condition as it will later turn out that players have no incentive to renegotiate anyway. As before, I assume that only the rejection of an offer causes a friction and thus a delay in bargaining, i.e. if the first offer in both stages is accepted immediately there is no delay. This assumption surely needs justification. It should capture the idea that time delay in bargaining is caused by frictions in the procedure. To my understanding these frictions are more likely to be caused by rejection (and the inevitable reevaluation of the situation) itself rather than by the submission of a new offer.[10]

Now, if player j accepts i's offer on the first issue at some point in time, then two more questions need clarification. Firstly, which player makes the first offer on the second issue? Secondly, are players allowed to benefit from their share of the settled issue right away or do they have to wait until all issues are settled? Regarding the first, I will assume that the player who accepted the offer of his opponent, in this case j, makes the first offer on the next issue. With this assumption we literally follow the alternating offer structure, acceptance/rejection - new offer.[11] Regarding the second question, I assume implementation to be joined, i.e. benefits cannot be derived before both (all) issues are settled. As a consequence, disagreement on one of the issues leads to failure of the entire negotiation. This reflects situations in which several issues are crucial for a contract or an overall agreement.

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[9] The endogenous agenda is of minor relevance for this topic. However, in this approach both players would always prefer to start offering on their higher evaluated issue. See Tiedemann (2009) for an approach in which players have opposing preferences regarding the issues and, at least in some cases, the first mover prefers to offer first on the issue of lower value to him.

[10] In the standard Rubinstein bargaining procedure, each counteroffer generally implies a precedent rejection such that there is no need to ask what actually causes the delay. Note that in accordance to my interpretation, the submission of the very first offer in Rubinstein bargaining does not lead to discounting either.

[11] Note that other assumptions are possible. Fershtman (1990) for instance assumes that after settlement of one issue, each players expects by one half to be first mover on the next issue.

Now, if an agreement on both issues is found in period T and y and z are player i's share of issue Y and Z, then i's payoff is:

$$u_i = \delta^T \left[\rho y + (1-\rho) z\right] - \sum_{t=0}^{T} \delta^t c_{i,t}$$

with $c_{i,t} = c_{ji}$ if j punishes in t and $c_{i,t} = 0$ otherwise.

2.3.2 The Effect of Punishment and the Agenda

For now, again we assume the punishment decision to be exogenous. Let us first consider bargaining to take place in a simultaneous agenda. It can be shown that Proposition 2.2 and 2.3 (by simply replacing x with $[\rho y + (1-\rho) z]$) hold without further modification for the simultaneous agenda of this multi issue setting. That is, for all regimes R, R^A and R^B (and all pairs of s.p.e. strategy profiles) players' payoffs in the simultaneous agenda game equal their payoffs in the single issue case.[12]

For the sequential agenda game the solution is not straightforward. The outcomes are derived in the following. Without loss of generality it is assumed that bargaining takes place in the YZ-agenda, i.e. players first have to settle issue Y before they bargain for Z.[13] In Lemma 2.1 I will argue, that in no regime it exists delay in equilibrium, i.e. the offer of the first mover on Y in stage I and the offer of his opponent on Z in stage II will be immediately accepted. For simplicity the sum of players' evaluated shares $(\rho y + (1-\rho) z)$ is denoted as $\tilde{x}$.

Lemma 2.1. *If the bargaining agenda is sequential and the punishment decision is exogenous, then there is a unique s.p.e. in which there is immediate acceptance of the first offer in both stages. In the YZ-agenda, for subgames in which i offers, the following payoffs are supported in equilibrium of regime* R^j :

$$\left(\tilde{x}_{ii}^j, \tilde{x}_{ij}^j\right) = \begin{cases} \left(min\left\{1-\delta x_{ij}^j, \rho\right\}, max\left\{\delta x_{ij}^j, 1-\rho\right\}\right), & for\, \rho \geq x_{ii}^j \wedge \rho \leq x_{ij}^j \\ \left(min\left\{1-\delta\rho, \rho\right\}, max\left\{\delta\rho, 1-\rho\right\}\right), & for\, \rho \geq x_{ii}^j \wedge x_{jj}^j > \rho > x_{ij}^j \\ \left(x_{ii}^j, x_{ij}^j\right), & for\, \rho \geq x_{ii}^j \wedge \rho \geq x_{jj}^j \\ \left(\rho, 1-\rho\right), & for\, x_{ii}^j > \rho > x_{ji}^j \\ \left(x_{ji}^j, x_{jj}^j\right), & for\, \rho \leq x_{ji}^j \end{cases},$$

[12]As this argument is straightforward a proof is omitted. Note, that this result depends on two assumptions, (i) that players' evaluation of the issues is identical and (ii) that the issues lack any complementarities. Under these assumptions, the statement also holds for any arbitrary set of issues.

[13]The ZY-agenda which sets Z before Y is implicitly included in this case. As players' evaluation of Y (Z) is ρ (1-ρ) the ZY outcomes for any ρ equal the YZ outcome for $1-\rho$.

and in regimes R^i and R, respectively:

$$\left(\tilde{x}_{ii}^{k},\, \tilde{x}_{ij}^{k}\right) = \begin{cases} \left(x_{ii}^{k},\, x_{ij}^{k}\right) & for\, \rho \geq x_{ii}^{k} \\ (\rho, 1-\rho)\,, & for\, x_{ii}^{k} > \rho > x_{ji}^{k} \\ \left(x_{ji}^{k},\, x_{jj}^{k}\right) & for\, \rho \leq x_{ji}^{k} \end{cases} \quad , \, with\, k \in \{i, R\}\,.$$

Proof. Appendix 2.2.

Obviously, the sequence of issues in such setting matters independent to the regime in play. It is clearly advantageous for a player to have the first offer on the issue of high value. The relative advantage of having the first offer on one issue is limited by the value of this issue. For example, assume that issue Y has a low value for the players (i.e. ρ is small) and A moves first in the YZ-agenda. In this case the relative advantage of A is limited to ρ as he cannot ask for more than the entire issue. In turn, B accepts this offer because after that he can exert the advantage of having the first offer on the important issue Z in stage II. In this setting, A would prefer either to move second or the ZY-agenda to be in force. This result is independent to the regime in play. However, if one player is first mover in the regime of his opponent, e.g. A moves first in R^B, under certain conditions his equilibrium payoff is higher than his first mover payoff in the simultaneous agenda $\left(x_{AA}^{B}\right)$.[14] In this case A takes advantage of B's relatively small reservation value which in turn results from the fact that B cannot fully exert his first mover advantage in case he rejects.

2.3.3 The Equilibria and the Agenda

The derivation of the existence of multiple equilibria follows as before. Again, for endogenous punishment decisions we need to check if players' decisions to punish are rational. Punishment is rational in both stages of the YZ-agenda for player $j \in \{A, B\}$ after his offer is rejected, iff

$$\delta \tilde{x}_{ij}^{j} \geq \delta \tilde{x}_{ij}^{R}\,. \tag{2.6}$$

In Appendix 2.3. it is shown that this constraint generally holds for $c_{ji} \geq \frac{1+\delta^2-2\delta}{\delta}$.[15] Now, the set of efficient and inefficient equilibria can be constructed as before.

[14]This is the case when $\rho \geq x_{AA}^{B}$ and $\rho < x_{BB}^{B}$. Here, A receives $min\,\{1-\delta\rho, \rho\}$ or $min\,\{1-\delta x_{AB}^{B}, \rho\}$ which is under respective conditions never smaller than x_{AA}^{B}.

[15]The weakness of this constraint relies on the assumption that players' costs for executing a punishment are zero. For most evaluations ρ the constraint allows for $c_{jj} > 0$ (compare Appendix 2.3).

Lemma 2.2. *In the sequential agenda game, if* (2.6) *holds, then Proposition 2.2 with* (2.4) *replaced by*

$$\bar{x} \in \left[\tilde{x}^{B}_{AA}, \tilde{x}^{A}_{AA}\right],$$

and Proposition 2.3 with (2.5) *replaced by*

$$\hat{x} \in \left[\left(\tilde{x}^{B}_{AA} + \frac{1-\delta^{N}}{1-\delta}c_A\right)\delta^{-N},\ 1 - \left(\tilde{x}^{A}_{AB} + \frac{1-\delta^{N}}{1-\delta}c_B\right)\delta^{-N}\right] \tag{2.7}$$

equally apply.

Proof. The structure of the proof is analogue to those of Proposition 2.2 and 2.3.

2.3.4 Inefficiency and the Agenda

We are now able to compare the range of possible equilibria under the simultaneous and the sequential agenda. In particular we are interested in the potential range of inefficient equilibria under these agendas, more specific in the maximum of potential periods of punishment. The analysis so far demonstrated that players' equilibrium payoffs in the sequential agenda game may differ from those in the simultaneous one. For a sufficiently low evaluation of the first issue (ρ is small) the first mover advantage is reduced or reversed (it becomes a disadvantage). This has influence on the range of inefficient equilibria and the following can be stated:

Proposition 2.4. *Assume that the rationality constraints* (2.3) *and* (2.6) *hold, and A is the first mover in the YZ-agenda, then*

(i) *for* $x^{A}_{BA} \geq \rho \geq x^{B}_{AA}$ *the maximum number of periods with punishment in equilibrium is smaller for the sequential than for the simultaneous agenda.*

(ii) *for* $\rho \leq x^{B}_{BA} \wedge 1 \leq x^{A}_{BA}$ *the maximum number of periods with punishment in equilibrium is smaller for the simultaneous than for the sequential agenda.*

Both, (i) and (ii) equally apply if B moves first and the ZY-agenda is played.

Proof. Appendix 2.4.

Figure 2.4 and 2.5 illustrate examples for case (i) and (ii). In both Figures the descending functions illustrate the upper and the ascending functions the lower boundaries of (2.5), marked by points and of (2.7), marked by triangles. Note that for both examples the rationality constraints (2.3) and (2.6) hold. Now, Figure 2.4 plots an example of (i) in which the s.p.e. concept supports at the maximum one period of delay and punishment in the sequential game but two periods in the simultaneous game. Here, A's advantage of being first mover is limited by the sequential agenda to the size of the first issue Y. For $\rho \leq x^A_{BA}$, player A has a first mover disadvantage in R^A and his payoff $\left(\tilde{x}^A_{AA} = x^A_{BA}\right)$ is smaller than his payoff in the simultaneous agenda $\left(x^A_{AA}\right)$. Thus, the upper bound in the sequential agenda lies below the upper bound in the simultaneous agenda. The lower bound depends on players' payoffs in regime R^B and it holds for the given values $\left(x^B_{BB} > \rho \geq x^B_{AA}\right)$ that A's payoff in the sequential agenda is higher than in the simultaneous agenda. Therefore, the lower bound in the sequential agenda lies above the lower bound in simultaneous bargaining.

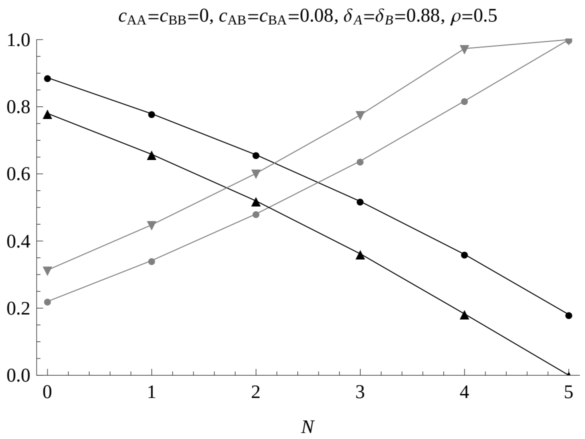

Figure 2.4: The sequential agenda reduces maximal punishment periods

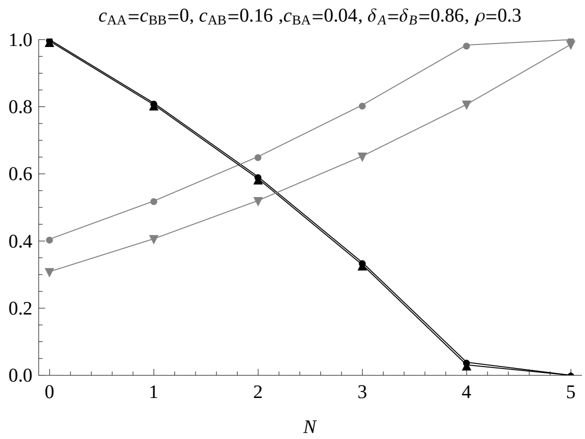

Figure 2.5: The sequential agenda increases maximal punishment periods

Figure 2.5 illustrates case (ii). In this example A's bargaining position is strong as his punishment is relatively costly for B. Here, the equilibrium supports maximal two periods of punishment in the sequential agenda game, while it supports only one in the simultaneous. For the given values, the upper bound is equal in both agendas as A's payoff as second mover in both agendas is 1, as $x^A_{BA} \geq 1$ and $\tilde{x}^A_{BA} = x^A_{AA} > 1$. The lower bound in the sequential agenda lies below the lower bound of the simultaneous agenda, as $\rho \leq x^B_{BA}$ such that $\tilde{x}^B_{AA} = x^B_{BA}$ which is smaller x^B_{AA}.

2.4 Conclusion

The ability to punish the opponent for rejecting ones offer enlarges players' strategy space in alternating offer bargaining. As a result different strategy profiles are mutually best answers in each period. Thus, each of them constitutes a subgame perfect equilibrium. This multiplicity includes equilibria which allow for a quick compromise without delay as well as equilibria with punishment and delayed agreement. Each particular equilibrium implies certain punishment strategy for each player.

Along with the assumption that the negotiation concerns multiple issues the question of the bargaining agenda emerges. We learned that under the given assumptions, the simultaneous and the sequential agenda supports solely immediate agreements in equilibrium if the punishment decision is exogenous. As one may expect, players' payoffs in the equilibrium of the simultaneous agenda game are identical to those of the single issue case. The calculation of the sequential agenda equilibria showed that for sufficiently high evaluations of the second issue, the first mover's advantage is reduced, vanishes completely or becomes a disadvantage.

For endogenous punishment decisions a multiplicity of efficient and inefficient outcomes can be supported in equilibrium, identical in structure to the single issue case. The comparison of the equilibrium outcomes under the two agendas then showed an influence on the potential level of inefficiency of the bargaining game. Depending on the evaluation of the issues, the sequence of the agenda and the distribution of bargaining power, the choice of the sequential agenda may have a positive or negative effect on the inefficiency. Compared to the simultaneous agenda, the sequential agenda reduces the potential level of inefficiency if the lower evaluated issue is set first but may increase the potential level of inefficiency if the first mover is sufficiently powerful. A more precise statement regarding this influence would be desirable but is prevented by the multitude of influencing variables.

Caution should be used by the interpretation of these results. They may surely not serve as policy recommendation for tariff conflicts or warfare negotiations in general. Instead, the insight is twofold. On one hand the influence of the agenda choice is limited and by far not substantial or decisive for arising inefficiencies in such negotiations. On the other hand, in the absence of solid reasons for a certain agenda, the following rules of thumb may tip the balance: If one player is relatively powerful the simultaneous agenda is preferable. If this is not the case, the sequential agenda is preferable and players should bargain on the less important issue(s) first. For the tariff conflict between the DB and the GDL the latter may have been the case. In these negotiations the decision to bargain one issue at a time and by setting the less important issue first (which was presumably the issue 'wage' as the issue 'independent tariff contract' was heavily disputed) might have reduced the bargaining power of the first mover (which was presumably the GDL as they made the first demand after they canceled the contract) and thereby decreased the potential level of inefficiency.

The model is kept quite general with regard to the punishing power of the players but quite restricted in the assumptions about the sequential agenda game. In particular the assumptions made with regard to the implementation procedure and to the successive proposer are somewhat arbitrary. Here different assumptions could be justified and would certainly lead to different results. This paper does not claim to solve the agenda problem in general rather it opens the discussion about the relevance of the agenda in bargaining settings with punishment ability. Future work focusing on the agenda problem in settings closer to those of Haller and Holden (1990) and Fernandez and Glazer (1991) might reveal interesting results.

Chapter 3

Arbitration and Mediation in a Bargaining Model with Punishment

Abstract: Instead of implementing efficient states, we observe that negotiations often end with inefficient outcomes. Outcomes which may be even worse if parties can punish their opponent, e.g. with strikes in collective bargaining. On the one hand we find that alternating offer bargaining models with punishment are frequently applied to abstract these negotiations, as they may explain inefficient outcomes. On the other hand, real-life negotiations are often accompanied by a third-party, an arbitrator or mediator, to avoid extreme inefficiencies. Therefore, it is surprising that the influence of third-parties in these models has not been analysed yet. This paper aims to make a first step to fill this gap. The results show that both arbitration and mediation can have a positive effect on negotiations by reducing the potential level of inefficiency in equilibrium, though arbitration may also increase inefficiency.

3.1 Introduction

In negotiations parties make use of threats to support their position. These threats are quite generally projections of credible scenarios which entail an outcome disfavored by their opponent. The more credible such projection, the more it weakens the opponents' position. A threat might be to terminate the negotiation, e.g. if one party can resort to an outside option, or the delay of agreement if a party is comparably patient. In various settings like collective bargaining or trade disputes parties have the additional ability to execute some kind of punishment, e.g. strikes or trade embargoes. Punishment inflicts costs on the opponent, but it usually also requires an investment by the punisher. By making this investment, a party can enforce the credibility of its position. Such scenarios are likely to result in highly inefficient outcomes as costs arise from delay *and* punishment. To avoid extreme inefficiency, often negotiations are accompanied by mediators or, alternatively, arbitration is applied to resolve ongoing disagreements. This paper aims to analyse the influence of mediation and arbitration on negotiations in which parties have the option to punish their opponent. Concretely, this paper aims to shed light on the following questions:

- Does arbitration or mediation influence the outcome with regard to its efficiency?
- Under which conditions do parties tend to accept the intervention of an arbitrator or mediator?
- Which conditions make it more likely for arbitration or mediation to reduce inefficiency?

To target these questions I set up a alternating offer bargaining model with complete information in which both players have a punishment option. This model is leaned on Avery and Zemsky (1994) and was already presented in Tiedemann (2010).[1] It has the property of creating multiple equilibria, including inefficient ones with delay and punishment. Generally these inefficient equilibria have both players upholding their punishment threat for a number of periods before they agree on a compromise. Both players find this strategy favorable to a deviation, e.g. to offer an early compromise, as this leads to quickly implemented, but even worse outcomes for the deviator.

[1] Bargaining models with punishment which create multiple equilibria go back to the analysis of Fernandez and Glazer (1991) and Haller and Holden (1990).

Therefore, this model is adequate to reflect real-life negotiations in which players remain obstinate, because they are afraid of being pulled over the barrel if they make a first concession. However, the influence of a third-party in such models has not been approached so far. To analyse if a third-party can help players out of their quandary, i.e. eliminate or mitigate the inefficient equilibria, I examine two different scenarios. In the first, players can decide on the intervention of an arbitrator. In the second, players can call in a mediator.

Generally, arbitration refers to interventions which terminate the bargaining process by making a binding decision on the topic, or, as Hunter (1977) puts it: "Arbitration is a *substitute* for continued bargaining and leaves the parties with no control over the final outcome".[2] Beside this, arbitration mechanisms can vary substantially in their design. Under *unilateral* arbitration a single player's call is sufficient to bring in the arbitrator while in *consensus* arbitration the opponent's approval is necessary as well. If the arbitrator bases his settlement on 'fairness' considerations independent to the previous bargaining process we speak of *conventional* arbitration. In contrast, under *final offer* arbitration the arbitrator bases his decision on the concessions and offers made by the players.[3] In this paper, I apply the arbitration scheme by Manzini and Mariotti (2001) in which the arbitrator intervenes only on *consensus* and then implements a *conventional* outcome which is commonly known by both players at all time. Note that the arbitrator has no particular interest in the outcome of the bargaining game.

While the economic literature on arbitration is substantial,[4] only few non-cooperative bargaining models analyse the influence of arbitration on the outcome or compare different arbitration systems. Manzini and Mariotti (2001) and McKenna and Sadanand (1995) analyse the influence of arbitration within the alternating offer model by Rubinstein (1982).[5] Among other results both find that the presence of an arbitrator induces inefficient equilibria in a model which has a unique efficient equilibrium otherwise. From this we learn that "arbitration may generate inefficient delays" (Manzini and Mariotti, 2001, p. 194), which reverses, at least to my understanding,

[2] Compare also Ashenfelter (1987); Goldberg et al. (2003) and Raiffa (1982). An exception is Compte and Jehiel (1995) who consider the agreement proposed by the arbitrator as non-binding.

[3] Or, as Adamuz and Ponsatí (2009) denote it, arbitrators may act on *principle* or act *pragmatic*.

[4] Compare for example Ashenfelter (1987); Farber and Bazerman (1986); Gibbons (1988); Goltsman et al. (2009).

[5] McKenna and Sadanand (1995) assume arbitration to be conventional, i.e. the arbitration outcome is independent to the history of offers and, further, that arbitration starts in some predetermined or exogenously given period.

the main motivation of arbitration.[6] In contrast to Rubinstein's model, the bargaining model with punishment used as reference in this paper quite generally creates inefficient equilibria. As a result I find that the influence of arbitration on efficiency crucially depends on the proposed arbitration outcome. For sufficiently *balanced* arbitration outcomes the presence of an arbitrator erases all inefficient equilibria, while for rather *inappropriate* proposals the possible level of inefficiency increases. If players are rather impatient, their favoured equilibria may be extreme and far apart. In these settings the range of balanced outcomes is large and arbitration is likely to reduce inefficiency. If players are rather patient, then the risk of inappropriate arbitration is high and successful arbitration can only be guaranteed if the arbitrator is sufficiently informed.

Upfront, modelling mediation is an altogether difficult task for two reasons. The process of mediation encompasses various aspects which influence the result. Moreover, the process of mediation is not well specified and can be of different types. Like arbitration, mediation is traditionally considered as a form of dispute or conflict resolution with the difference that it leaves the bargaining process intact. This means that, in contrast to an arbitrator, a mediator has no power to impose an outcome on the players. Instead, a mediator is likely to, or at least intends to increase cooperation by structuring the negotiation and by "acting as a neutral discussion leader, helping to set the agenda, [...] smoothing out interpersonal conflicts, [...] and preparing neutral minutes" (Raiffa,1982, p. 108). Generally speaking, there are at least three distinct aspects of the mediation process which influence the disputing parties willingness to come to an agreement. First, the mediation technique of the mediator, i.e. his ability to improve the dialogue. Second, the informational aspect, i.e. the exchange of information should "help the parties to understand each other's views" (Goldberg et al., 2003, p.111) but it also bears the risk for them to disclose valuable information. Third, the structure of the mediation process, i.e. the rules the parties (voluntarily or already have contractually agreed to) accept in mediation. To get a handle on the issue, most technical approaches focus on single aspects of mediation, often on the informational aspect.[7]

[6] In similar approaches Compte and Jehiel (1995) and Adamuz and Ponsatí (2009) analyse the influence of arbitration in a model of bargaining by concessions. In both approaches arbitration can increase inefficiency such that Compte and Jehiel (1995, p. 34) state: "it may seem that arbitration is essentially a source of inefficiency".

[7] Jarque et al. (2003) and Copic and Ponsatí (2008) analyse the influence of mediation in an incomplete information, double auction model. Here, the mediator filters extreme bids and only transmits compatible ones. As an result the level of inefficiency decreases in this model. Goltsman et al. (2009) compare mediation and arbitration to negotiation in Crawford and Sobel's (1982) model of strategic

The mediation model in this paper focuses on the legal and contractual rules of mediation services and agreements applied in particular in collective bargaining. Not least due to the assumption of complete information in the underlying bargaining game, the aspect of information disclosure and exchange is not taken into account in this paper. The model offers an abstract representation of services offered by various private and public (mediation) institutions. For example, it resembles the mediation service offered by the US National Mediation Board and by the United States Arbitration and Mediation offices.[8] In addition, the German collective bargaining system offers various mediation (or conciliation) agreements (Schlichtungsvereinbarungen) which differ in detail but generally share the framework captured by my model.[9] I assume mediation to take place as soon as one party requires it, e.g. by proclaiming failure of the negotiation. This means, either party has, beside the acceptance or the rejection of the offer, the third alternative to call in a mediator. The mediation process starts immediately on this unilateral call and takes an entire period of time. For this period, both players have to follow the rules of mediation: proposals and offers are not binding and peace is obligated, i.e. punishment cannot be executed.[10]

The results which I derive in this paper propose that mediation never increases the level of inefficiency. Instead it generally reduces the possible level of inefficiency in equilibrium or prevents it completely if the game is *mediatable*. This is always the case when players are sufficiently patient and their opponents' threat of punishment is sufficiently strong. In such settings players try to avoid the costly punishment and prefer to call in the mediator even though mediation is time-consuming.

The rest of the paper is structured as follows. In section 2 the basic bargaining model with punishment is introduced and the 'level of inefficiency' is defined as measure of welfare. In section 3 the arbitration scheme is implemented in the bargaining game. The influence of arbitration on the equilibria and on the level of inefficiency is analysed. Section 4 repeats this analysis for mediation and section 5 concludes.

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information transmission. Here, mediators and arbitrators filter information or add noise.

[8]As for example formulated in the Railway Labor Act on the website www.nmb.gov or in the "mediation clause" on www.usam.com.

[9]Most *Schlichtungsvereinbarungen* usually involve peace obligation (*Friedenspflicht*), delay and the intervention of a mediator (*Schlichtungsstelle*) after a unilateral proclamation of negotiation failure (compare e.g. Franz, 2006).

[10]To gather the parties in the 'mediation room' and "help the parties to understand each other's views, [...] encourage flexibility, shift the focus from the past to other" (Goldberg et al., 2003, p. 111) is surely a time consuming process and requires ceasefire.

3.2 A Bargaining Model with Punishment

3.2.1 The Basic Model

Assume a Rubinstein bargaining game of the discounting type where, in addition, players have the option to punish their opponent. This means that after the rejection of an offer, each of the players independently and simultaneously decides to carry out an action which inflicts costs on the opponent but is also costly for the punisher. Each player is free to carry out his punishment once in every period in which an offer was rejected. This ability enlarges players' strategy spaces and as a consequence a multiplicity of equilibria arises in this bargaining game. Similar models were provided by Avery and Zemsky (1994), where only one player has the punishment option, and Tiedemann (2010).

The formal setup is as follows. Two completely and perfectly informed players, A and B, bargain on the division of an infinitely divisible pie X of size 1. Players make offers in alternating order about the division of X, x_{ij} is the share player $i \in \{A, B\}$ offers to $j \in \{A, B\}$. Offers can be accepted (a) or rejected (r). The acceptance of a proposed division leads to its immediate implementation. Player A is assumed to make offers in period t=0 and every even period. Player B offers in any odd period. The passing of a period causes discounting of players' payoffs by δ. The game continues as long as no agreement is found and players' payoffs for perpetual disagreement is zero. So far it equals the Rubinstein setting. Now, assume that the rejection of an offer compulsorily leads to the punishment game Γ which takes place in the same period. Let Γ be some normal form game where both players independently and simultaneously choose between two actions, *punish* or *not punish*. A punishment by player i inflicts costs c_{ij} to j and c_{ii} to i himself. Figure 3.1 illustrates the procedure.

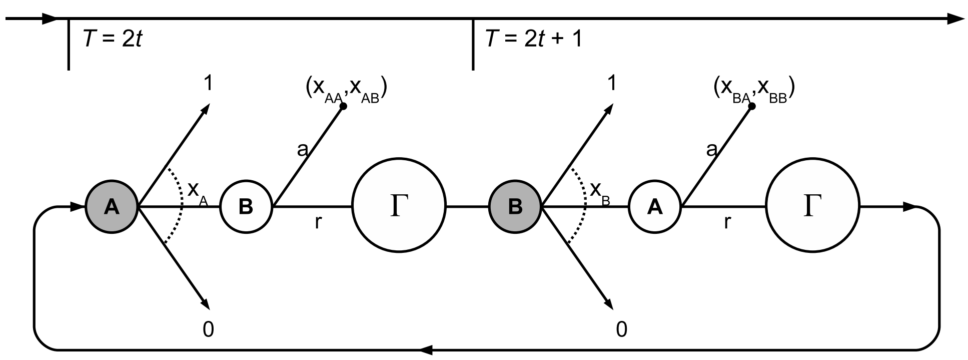

Figure 3.1: Two periods of the bargaining procedure

An agreement established in period T which gives player i the share x yields i the following payoff:

$$u_i = \delta^T x - \sum_{t=0}^{T} \delta^t c_{i,t} \,.$$

The cost for player i in period t are as follows, $c_{i,t} = 0$ if no player punishes, $c_{i,t} = c_{ii}$ $\left(c_{i,t} = c_{ji}\right)$ if only i(j) punishes, and $c_{i,t} = c_{ii} + c_{ji}$ if both punish in t.

3.2.2 The Effect of Punishment

We find that for each player a best and a worst combination of (rational) punishment strategies exists. Each of these strategy profiles has the same structure: player j never punishes and his opponent $i \neq j$ applies a disrupted punishment strategy, i.e. he always punishes after his own offer is rejected. I will refer to this profile as the regime of player i, in short R^i. With his disrupted punishment strategy, i has a strategical advantage; for j the costs of his opportunity 'rejection' increase such that in equilibrium he will accept a lower share. Note that there is no punishment profile which yields player i a higher or player j a smaller equilibrium payoff than the regime R^i.

Now assume that R^i or R^j is played and that i is proposer. Further, let x_{ij}^k denote the share player i offers j in the regime of player $k \in \{A, B\}$. Then by standard arguments, it follows from the stationary structure of the game that the unique s.p.e. partition, which i offers and j immediately accepts, is:

$$\left(x_{ii}^i, x_{ij}^i\right) = \left(\frac{1-\delta+c_{ij}}{1-\delta^2}, \frac{\delta-\delta^2-c_{ij}}{1-\delta^2}\right); \quad \left(x_{ii}^j, x_{ij}^j\right) = \left(\frac{1-\delta-\delta c_{ji}}{1-\delta^2}, \frac{\delta-\delta^2+\delta c_{ji}}{1-\delta^2}\right).$$

Note that regime R^i yields i the largest share he can achieve by any punishment strategy.[11] In contrast, if we assume that no player ever punishes the game converges to the standard Rubinstein game, denoted R, and the unique partition supported in equilibrium is:

$$\left(x_{ii}^R, x_{ij}^R\right) = \left(\frac{1-\delta}{1-\delta^2}, \frac{\delta-\delta^2}{1-\delta^2}\right).$$

Next, I check if the execution of the punishment (and the investment of c_{ii}) is indeed rational for player i. To implement his regime R^i, player i has to execute his punishment if his offer gets rejected. In this case he can expect at best a payoff of x_{ji}^i in

[11] The s.p.e. which yields i an even higher payoff will be neglected as it requires irrational punishments by j. If both players punish always after j rejects, i would receive a higher share than by punishing alone. The decision to punish after his own rejection is therefore irrational for j.

the next period. If i forgoes to punish, the Rubinstein outcome, which results instead, yields him x_{ji}^R in the next period. Thus, the execution of the punishment is rational for player i, denoted as p-rational for i or, in short, as p^i, if the following holds:[12]

$$-c_{ii} + \delta x_{ji}^i \geq \delta x_{ji}^R. \tag{3.1}$$

I summarize by stating the following:

Proposition 3.1. *If punishment is p-rational for i, then R^i constitutes a s.p.e. which supports the partition $\left(x_{ii}^i, x_{ij}^i\right)$.*

Proof. Appendix 3.2.

The two extreme equilibria of R^A and R^B allow for a (kind of) Folk theorem construction with an infinite number of s.p.e. including equilibria with immediate and efficient (no punishment) agreement as well as inefficient ones with delay and punishment. To show the existence of the efficient equilibria I state the following.

Proposition 3.2. *If punishment is p-rational for both players and iff $\overline{x}$ is such that*

$$\bar{x} \in \left[x_{AA}^B, x_{AA}^A\right],$$

then there is a s.p.e. in which A demands $\bar{x}$ in t=0 and B immediately accepts.

Proof. Appendix 3.3.

The efficient equilibria are enforced by i's threat that any deviation of j is punished with a switch to regime R^i. The same threat ensures the existence of the inefficient equilibria. Here players prefer to suffer punishment and delay in order to receive the share $\hat{x}$ instead to end up with the extreme unfavorable regime of their opponent.

Proposition 3.3. *If the punishment is p-rational for both players and $\hat{x}$ is such that*

$$\hat{x}_A \in \left[\left(x_{AA}^B + \frac{1-\delta^N}{1-\delta}\left(c_{AA}+c_{BA}\right)\right)\delta^{-N}, 1-\left(x_{AB}^A + \frac{1-\delta^N}{1-\delta}\left(c_{AB}+c_{BB}\right)\right)\delta^{-N}\right], \tag{3.2}$$

then there is a s.p.e. in which for a positive integer number of N periods, no agreement is found and both players punish in every period followed by agreement on the partition $(\hat{x}_A, \hat{x}_B)$ in period N+1.

Proof. Appendix 3.4.

[12] In Appendix 3.1 it is argued that (3.1) is a weak constraint.

3.2.3 The Level of Inefficiency

As a measure for the potential welfare loss of the bargaining outcome I now define the *maximum level of inefficiency* in equilibrium. For this purpose, I analyse the nature of the equilibria in respect to the loss induced by discounting and punishment. For illustration, Figure 3.2 compares each player's payoff from an immediate agreement on his opponent's regime to his payoff from an agreement on $\hat{x} = 0.6$ after N periods of delay and punishment. Player A's payoff is depicted on the left and B's on the right hand scale. The continuous horizontal lower (upper) line plots the payoff of A (B) given they have deviated and right away agreed to their unfavored regime R^B $\left(R^A\right)$. A upward (downward) directed vertical line at time N indicates the payoff of A (B) after N periods of punishment followed by agreement on $\hat{x}$. The vertical dashed line at N illustrates the *level of inefficiency*, i.e. the payoff which is lost for both players due to discounting and the costs of punishment compared to an immediate agreement without delay, e.g. on $\overline{x}$. The *maximum level of inefficiency* which is possible in equilibrium is the distance between the two horizontal lines which is given by the conflict range (3.2) for N=0, i.e. the difference between a player's payoff from an immediate agreement on R^A or on R^B.

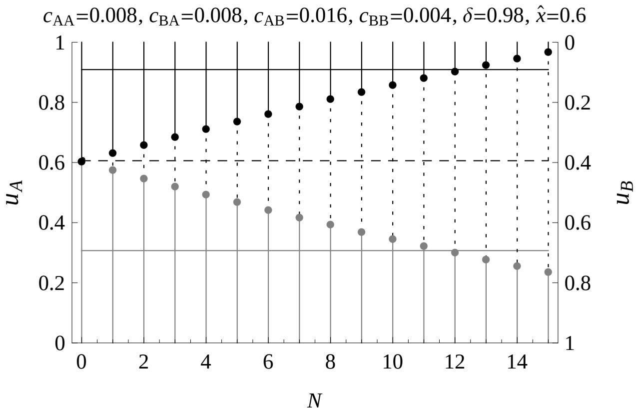

Figure 3.2: Level of inefficiency in equilibrium

3.3 Arbitration

3.3.1 The Arbitration Model

In this section I integrate arbitration in the bargaining model with punishment as described in the last section. Therefore I adopt the arbitration scheme by Manzini and Mariotti (2001) who, by contrast, apply it to a standard Rubinstein Game. In their scheme arbitration relies on the following assumptions. Arbitration terminates the negotiation by implementing a binding outcome. Each party may propose the intervention of the arbitrator but both parties have to agree before arbitration takes place. The arbitration process can be costly but does not inflict delay on the bargaining process.[13] The outcome implemented by the arbitrator is common knowledge.

Concretely, the model is as follows. After receiving an offer of player i, besides accepting (a) and rejecting (r), j now has the third alternative to call for the intervention of an arbitrator (s). As before, the punishment game follows in case player j rejects. If, by contrast, j chooses to call the arbitrator, then player i can agree (y) or disagree (n) to arbitration. In the case i disagrees he continues by proposing a division of X. If player i agrees to the proposed intervention, the arbitrator implements the partition (s_A, s_B) with $s_A + s_B = 1$. The intervention of the arbitrator is costly, equally split on both players.[14] Precisely, arbitration reduces each player's payoff by $\varepsilon \leq min\left[s_A, s_B\right]$. Figure 3.3 illustrates the procedure.[15]

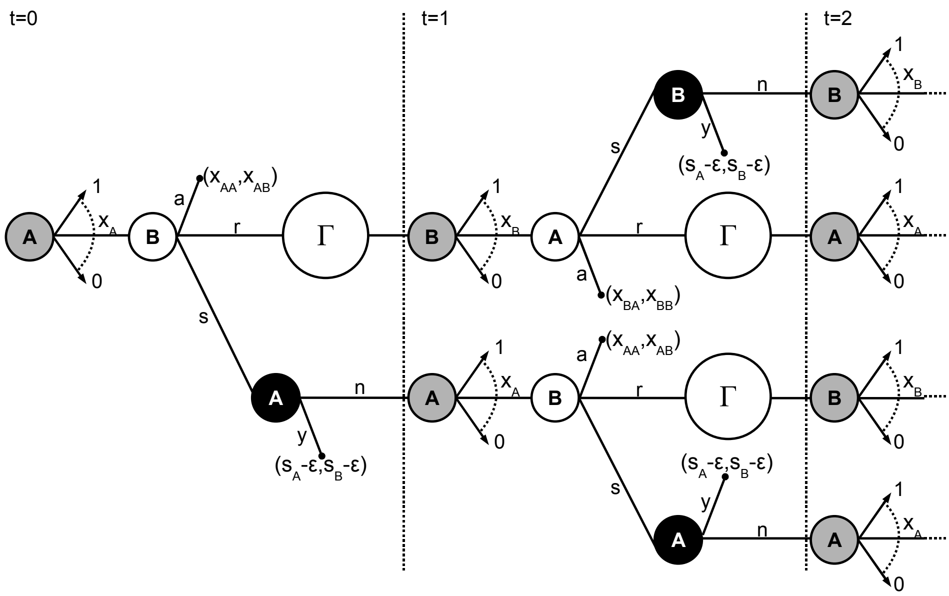

Figure 3.3: Arbitration in a bargaining model with punishment

[13]The structure of the model allows the responder to call in the arbitrator immediately and without delay in case of disagreement. This assumption needs justification as the arbitration process needs time in real life. Like Manzini and Mariotti (2001), I justify this assumption as follows. The discount factor summarises all transaction costs associated with bargaining, and arbitration is an institution which reduces some of them. Parties rely on arbitration because it appears more efficient to them than carrying on with bargaining. Consequently, arbitration has a different cost structure which is captured by the costs of arbitration.

[14]I assume this for notational simplicity. The quality of all results remains if different arbitration costs for the players are assumed.

[15]Decision nodes in which players offer are depicted in Grey, in which they respond to an offer by choosing (a), (r) or (s) are white and those where they agree (y) or disagree (n) are black.

3.3.2 Equilibria with Arbitration

In the bargaining game with punishment and arbitration there is a s.p.e. in which the agreement is *almost* determined by the partition which would be implemented by arbitration itself.

Proposition 3.4. For all $(s_A, s_B) \in [0,1] \times [0,1], \forall (c_{ii}, c_{ij}) \in [0,1] \times [0,1], \forall \delta \in (0,1),$ $\exists \varepsilon_\delta$ *such that,* $\forall \varepsilon \leq \varepsilon_\delta$*, there exists a s.p.e. in which agreement is reached immediately on the partition* $(s_A + \varepsilon, s_B - \varepsilon)$.

Proof. The following strategy for each player $i \in \{A, B\}$ supports $(s_A + \varepsilon, s_B - \varepsilon)$ as a s.p.e. partition:

(i) offer j the share $s_j - \varepsilon$;

(ii) reject any $x < s_i - \varepsilon$ accept all $x \geq s_i - \varepsilon$;

(iii) always accept arbitration when player j proposes it;

(iv) always propose arbitration when rejecting an offer.

I denote this strategy profile R^S. To prove subgame perfectness of R^S it is to show that no player can profit from unilateral deviation. The stationary structure of the game allows us to reduce the analysis to two successive periods. Assume player $j \neq i$ follows the above strategies, for i the optimality of part (i) is straightforward: Any offer of less than $(s_j - \varepsilon)$ will not be accepted by j according to (ii) and arbitration follows according to (iv). Checking for optimality of part (ii): Player i's rejection of any offer smaller than $(s_i - \varepsilon)$ is optimal as according to (iii), j would accept arbitration in turn, which then yields i exactly $(s_i - \varepsilon)$. Further it is optimal for i to accept any offer larger $(s_i - \varepsilon)$ as he cannot receive a higher payoff by arbitration or by counteroffer in the next period. The former yields i exactly $(s_i - \varepsilon)$ and according to (i) the latter yields him at maximum $\delta (s_i + \varepsilon)$. For

$$s_i - \varepsilon \geq \delta (s_i + \varepsilon)$$

to hold for all $\varepsilon \leq \varepsilon_\delta$, we define

$$\varepsilon_\delta = \frac{(1 - \delta)\, min\, [s_A, s_B]}{(1 + \delta)}. \tag{3.3}$$

For part (iii) to be optimal, the payoff from arbitration $(s_i - \varepsilon)$ has to be at least as high as the payoff from not accepting arbitration and making a counteroffer $\delta\,(s_i + \varepsilon)$, this is assured by (3.3). For part (iv) the same inequality applies. ■

In the above equilibrium there is a first mover advantage as by immediate agreement players avoid the costs of arbitration: the second mover accepts his share minus his costs from arbitration, i.e. his reservation value. The smaller the arbitration costs, the more precisely the arbitrated partition is implemented. In the absence of arbitration costs, it follows that the exact arbitration outcome is supported in equilibrium:

Corollary 3.1. *If* $\varepsilon = 0$, *then* $\forall\,(s_A, s_B) \in [0,1] \times [0,1]\,, \forall\,(c_{ii}, c_{ij}) \in [0,1] \times [0,1]\,,$ $\forall \delta \in (0,1)\,,$ *there exists a s.p.e. with immediate agreement on the partition* $(s_A, s_B)\,.$

The structure of players strategies in R^S is similar to those in R^A and R^B. If one player follows the strategy of R^S, hence he always calls the arbitrator if he rejects an offer, then his opponents best answer is to comply in R^S. The result implies that even for extremely unbalanced arbitration outcomes, the equilibrium of R^S supports the immediate agreement of a partition arbitrarily close to the arbitration outcome. This includes equilibria which support a partition where player i receives nothing, even if he is relatively patient (i.e. if δ is high). The driving force behind this result is the 'commitment effect' created by the arbitration proposal. In a standard Rubinstein game player j's commitment to always reject offers which yield him less than 1 is not credible as i can easily call the bluff by offering j the share $(\delta + \mu)$. This offer will always be accepted by j as by rejection he cannot receive more. In the arbitration model, j can avoid the costs of delay by calling the arbitrator for low or no costs and can thus credibly commit to not accept $\delta + \mu$.

3.3.3 Arbitration and the Level of Inefficiency

In the following I show that arbitration may have disparate effects on the level of inefficiency in equilibrium. Depending on the outcome proposed in arbitration, the mere presence of an arbitrator either increases the potential level of inefficiency, leaves it unchanged or resolves it entirely. For simplicity and clarity I assume in the following that arbitration is costless. However, similar results can be derived with costly arbitration.

First, I show that an inappropriate arbitration proposal increase the conflict potential by enlarging the scope of inefficient equilibria. The arbitration outcome is defined as inappropriate if it concedes a player i a share larger than the share i receives at best in the game otherwise. Hence, the proposed arbitration outcome is *appropriate* if

$$s_i \leq x^i_{ii} \quad \forall i$$

and *inappropriate* otherwise.[16]

Proposition 3.5. *If punishment is p-rational for both players, $\varepsilon = 0$, and the arbitrated outcome is inappropriate then*

(i) *there is a s.p.e. in which players do not agree for a positive integer number of N periods before they agree in N+1 on the partition $(\tilde{x}_A, \tilde{x}_B)$ with*

$$\tilde{x}_A \in \begin{cases} \left[x^B_{AA}\delta^{-N},\ 1 - s_B\delta^{-N}\right], & if\ s_A > x^A_{AA} \\ \left[s_A\delta^{-N},\ 1 - x^A_{AB}\delta^{-N}\right], & if\ s_B > x^B_{BB} \end{cases}$$

(ii) *the maximum level of inefficiency in equilibrium is higher in the presence of an arbitrator.*

Proof. Appendix 3.5.

Proposition 3.5 shows that the presence of an arbitrator in the bargaining process bears the risk of increasing the inefficiency of the bargaining outcome if the arbitration proposal is not *appropriate*. Next, proposition 3.6 shows that if the arbitration partition is *appropriate* and *balanced* then this partition is uniquely supported in equilibrium. In this case the presence of an arbitrator causes any inefficient equilibria to vanish. I call the arbitrated outcome *balanced* if no player has reason to disagree with arbitration. This is true if each player prefers his share from arbitration to what he will receive at best in the next period if he disagrees to arbitration and continues bargaining, i.e. if

$$s_i \geq \delta x^i_{ii} \qquad \forall i.$$

Note that any outcome which is *balanced* is also *appropriate* for both players.[17]

[16]Note that it is never true that $s_i > x^i_{ii}$ holds for both players. If $s_i > x^i_{ii}$, then $s_j < x^i_{ij}$ as $s_i + s_j = x^i_{ii} + x^i_{ij} = 1$. Further, as $x^i_{ij} < x^i_{jj} \leq x^j_{jj}$ it holds that $s_j < x^j_{jj}$ if $s_i > x^i_{ii}$.

[17]Because for both players $i \in \{A, B\}$ it holds that $\delta x^i_{ii} = x^i_{ji} \geq x^j_{ji}$ it follows from the *balance* condition that $s_i \geq x^j_{ji}$ holds for both players which tantamount to the *appropriate* condition.

Proposition 3.6. *If the arbitrated outcome is balanced, then the unique payoff pair supported in equilibrium is* (s_A, s_B) *.*

Proof. I will argue that no player has an incentive to deviate from the proposition of (s_A, s_B) and its immediate acceptance in the first period. First, player B has no incentive not to accept A's offer of s_B as neither by rejection and counteroffer (because $s_B \geq \delta x_{BB}^B$) nor by calling the arbitrator he can increase his payoff. Second, player A has no incentive to offer less than s_B because by offering $s_B - \mu$, B has an incentive to call the arbitrator (because $s_B \geq s_B - \mu$) and A has no incentive not to accept arbitration as $s_A \geq \delta x_{AA}^A$. ■

From proposition 3.5 and 3.6 we learn that arbitration may influence the potential level of inefficiency in equilibrium. If the arbitration outcome is *inappropriate*, in the sense that one player's share in arbitration is larger than his maximum share without arbitration, then the maximum inefficiency in equilibrium increases. In contrast, given that arbitration is *balanced*, no player has an incentive to disagree to arbitration. Here the arbitration outcome is the unique partition supported in equilibrium. In this equilibrium players immediately agree such that no equilibrium with inefficient outcome remains. The range of balanced proposals is the larger the less patient players are and the more costs their punishment inflicts on their opponent.[18]

3.4 Mediation

3.4.1 The Mediation Model

In contrast to an arbitrator, a mediator neither makes binding proposals nor implements an outcome. Instead his task is to moderate and mediate the negotiation and soothe parties' tempers in order to make them ponder on their strategy. Therefore, I assume that the intervention of the mediator causes the immediate obligation of peace and the suspension of the negotiation for one period.[19] The mediator intervenes on

[18]Note that, if punishment is not p-rational for both players, then the game without arbitration uniquely supports the standard Rubinstein partition in equilibrium. Hence, in this simple setting no inefficient equilibria exist without an arbitrator, but the presence of an arbitrator can create such. If players are sufficiently patient the arbitration outcome is easily inappropriate and creates inefficient equilibria. Manzini and Mariotti (2001) obtain this result in the standard Rubinstein setting.

[19]During this period the mediator may propose an outcome, work with players on feasible agreements or simply pause. For the results to hold, it is simply relevant that no binding proposal can be made in this period.

request of one player without awaiting the other player's agreement. This ensures that a player who is confronted with an aggressively negotiating opponent can unilaterally involve the mediator.

On the basis of the bargaining game with punishment, I assume now, that a player who receives an offer in period t has the additional option to call a mediator. Mediation is assumed to be costless.[20] Concretely, if player i resorts to the mediator in period t, his intervention inhibits the punishment game Γ in t and obliges both players to suspend negotiation for one period (t+1) before bargaining continues in t+2 with an offer of i. Figure 3.4 illustrates the game. [21]

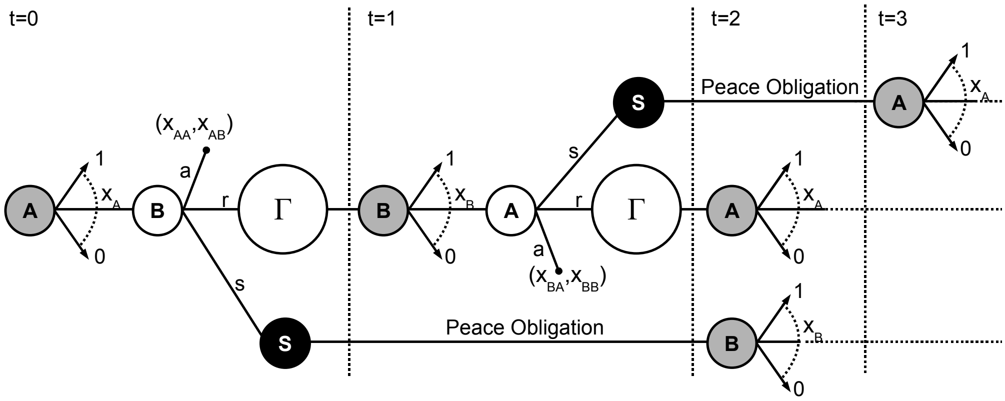

Figure 3.4: Mediation in a bargaining model with punishment

3.4.2 Equilibria with Mediation

First, I show that mediation can inhibit the extreme regimes R^A and R^B to be subgame perfect. If player j is sufficiently patient he will prefer to call the mediator over rejecting his opponent's offer and being punished, simply because his costs from doing so are lower. In this case R^i cannot be supported as equilibrium. Furthermore, as j's costs of not accepting an offer have decreased, i's maximum payoff in equilibrium is smaller.

To verify this, assume for the moment that player j either accepts an offer or calls for mediation but never rejects. Further assume that i either accepts or rejects but never calls for mediation and that no player punishes if i rejects. I call this strategy profile the j-mediation regime R^{Mj}. Given players follow R^{Mj}, the unique equilibrium

[20] This assumption simplifies the analysis. All results can also be derived with costly mediation.

[21] Again, decision nodes in which players offer are depicted in Grey, in which they respond to an offer by choosing (a), (r) or (s) are white and the mediation stage (S) is depicted black.

partitions in subgames in which i or j offers can be derived by standard arguments:

$$\left(x_{ii}^{Mj}, x_{ij}^{Mj}\right) = \left(\frac{1-\delta^2}{1-\delta^3}, \frac{\delta^2-\delta^3}{1-\delta^3}\right); \qquad \left(x_{ji}^{Mj}, x_{jj}^{Mj}\right) = \left(\frac{\delta-\delta^3}{1-\delta^3}, \frac{1-\delta}{1-\delta^3}\right).$$

In the following I derive necessary conditions to show subgame perfectness of R^{MA} and R^{MB}. If and only if R^i is an equilibrium, i.e. punishment is p-rational for i, and

$$c_{ij} > \frac{\delta - 2\delta^2 + \delta^3}{1-\delta^3},$$

then mediation is a rational option for player j and I denote the game as *mediatable* for player j, short M^j. Given M^j, then player j's share in regime R^{Mj} is larger than his share in R^i, i.e. $x_{ij}^{Mj} > x_{ij}^i$. Hence, the threat of i's punishment (c_{ij}) is sufficiently strong such that j prefers to accept the costs of delay caused by mediation (put differently, j is sufficiently patient).[22] Further, if the game is mediatable for j and

$$c_{ii} \leq \frac{\delta^3}{(1+\delta)\left(1+\delta+\delta^2\right)},$$

then it holds that $-c_{ii} + \delta x_{ji}^{Mj} \geq \delta x_{ji}^R$ and I denote punishment as m-rational for i or short p^{Mi}. In this case the execution of punishment is rational for player i, even though i cannot enforce R^i and expects the mediation regime R^{Mj} to emerge.

Now, we can show that for games which are mediatable for player j, first, the punishment regime of his opponent R^i is not longer subgame perfect and, second, depending on p^{Mi} either the payoff from the mediation regimes R^{Mj} or the standard Rubinstein share is the smallest payoff for j supported in equilibrium.

Proposition 3.7. *If the game is mediatable for*

(i) *player j and m-rational for i, then R^{Mj} is a s.p.e. which supports the partition* $\left(x_{ii}^{Mj}, x_{ij}^{Mj}\right)$.

(ii) *player j then regime R^i is never subgame perfect.*

(iii) *both players and <u>not</u> m-rational for both players, then $\left(x_{AA}^R, x_{AB}^R\right)$ is uniquely supported as s.p.e.*

Proof. Appendix 3.6.

[22] Note that mediation is *never* a rational option for i if p^j does not hold, i.e. if R^j is no equilibrium. Further, note that M^j equally implies that a rejection of x_{ij}^{Mj} is never a best answer for j as his payoff in the next period less his costs from i's punishment is smaller $\left(\delta x_{jj}^{Mj} - c_{ij} < x_{ij}^{Mj}\right)$.

From Proposition 3.7 we learn that in settings in which mediation is a rational option for player $i \in \{A, B\}$, the mediation regime R^{Mj} is subgame perfect if punishment is *m*-rational for *j*. As *i*'s choice to call the mediator is precedent to the punishment game and because M^j holds, *i* can prevent *j*'s disrupted punishment strategy such that R^j ceases to be subgame perfect. If *m*-rationality does not hold for *j*, R^{Mj} is no equilibrium and the Rubinstein share is the largest share *j* can enforce. Here, the threat of *i*'s punishment is never a rational option. First, because *j* can prevent regime R^i and second, because *i*'s punishment threat is not credible for mediatable games, i.e. punishment is not *m*-rational for *i*.

3.4.3 Mediation and the Level of Inefficiency

Making use of the defined equilibria we are now able to compare the potential level of inefficiency in the game with and without mediation. If mediation is a rational option, then the presence of a mediator leads to a decrease of the maximum level of inefficiency in equilibrium or prevents inefficient equilibria completely. For simplicity I restrict the analysis to a game which is mediatable for both players. The analogue analysis follows for games which are mediatable for one player.

Proposition 3.8. *If the game is mediatable for both players*

(i) *and m-rational for at least one player, then there is a s.p.e. with no agreement for a positive integer and odd number of N periods and agreement in N+1 on the partition* $(\check{x}_A, \check{x}_B)$ *with*

$$\check{x}_A \in \begin{cases} \left[x_{AA}^{MA}\delta^{-N},\ 1 - x_{AB}^{MB}\delta^{-N}\right] & for\ p^{MA} \wedge p^{MB} \\ \left[x_{AA}^{R}\delta^{-N},\ 1 - x_{AB}^{MB}\delta^{-N}\right] & for\ p^{MA} \wedge \neg p^{MB} \\ \left[x_{AA}^{MA}\delta^{-N},\ 1 - x_{AB}^{R}\delta^{-N}\right] & for\ \neg p^{MA} \wedge p^{MB} \end{cases}$$

(ii) *the maximum level of inefficiency decreases or vanishes in the presence of a mediator.*

Proof: Appendix 3.7.

The results show that the presence of a mediator influences the set of equilibria. For a sufficient patient player who faces a sufficient strong threat, i.e. his costs from being punished are high, the call for mediation becomes the best answer. In settings where punishment is p-rational the absence of a external party allows for extreme regimes (R^A and R^B) - in mediatable games the presence of a mediator generally eliminates these extreme equilibria. Concretely, for mediatable games two scenarios are possible: if the costs of punishing are sufficiently small, i.e. punishment is m-rational, then the more moderate mediation regimes R^{Mi} replace the punishment regimes R^i and the maximum level of inefficiency decreases. In contrast, if m-rationality is not given for any player, punishment is never a rational option, such that the Rubinstein partition is uniquely supported in equilibrium.

3.5 Discussion

The aim of this paper is to analyse the influence of third-party intervention in bargaining with punishment. In the absence of a third-party the basic bargaining model with punishment quite generally creates multiple equilibria including inefficient ones with punishment and delay. I show that the presence of a third-party in this model influences the set of equilibria in general and the set of inefficient ones in particular. As a measure for the influence of the third-party the maximum level of inefficiency in equilibrium is introduced. The choice of this measure has two reasons. First, it indicates the possible worst case scenario in equilibrium from a social welfare perspective. It is therefore a valid criterion for agenda setters and may serve as reference point for arbitration- and mediation-boards and as policy recommendation. Second, we lack other measurable criteria, e.g. to learn about the probability of inefficient outcomes is surely desirable, however, a probability measure seems impossible in the context of this model.

In the arbitration model the mere presence of the arbitrator suffices to drive the results. This is puzzling as, independent of players' bargaining power, extremely unbalanced partitions can be supported in equilibrium. In this model the arbitrator is relatively powerful, as independent from his proposed partition this proposal becomes an equilibrium outcome. If his proposal is inappropriate, i.e. it lies outside of the range of equilibria which exists in his absence, it creates a new equilibrium and alters the maximum level of inefficiency. In contrast, if the arbitration proposal lies inside this range and is sufficiently balanced, it turns to be the only partition supported in equilibrium and erases all inefficient equilibria. This result highlights the importance of the particularly chosen arbitration outcome, at least if we assume that arbitration aims to diminish inefficiency. From this point of view, arbitration is recommendable only if sufficient information about the bargaining situation are available to the arbitrator.

In contrast to an arbitrator, in my mediation model the mediator cannot actively drive the bargaining outcome to a certain direction. His intervention is rather passive, it slows down negotiations and offers parties a break to reflect their strategies. Patient and conflict averse players prefer the intervention of a mediator to the opponents punishment if their costs from being punished are sufficiently large. By preventing punishment, mediation soothes negotiations and reduces the maximum level of inefficiency. The remaining equilibria then solely depend on players' impatience, the particular costs of punishment become irrelevant. Under certain conditions mediation resolves the game from any inefficient equilibria.

By comparing the conditions for successful third-party-intervention, players' impatience turns out to be the decisive factor. For settings in which players are relatively impatient mediation is no rational option and has no influence on the set of equilibria at all. For such settings the arbitrator has a relatively wide range to successful chose a balanced arbitration outcome which prevents inefficient equilibria. In contrast, if players are rather patient and punishment threats are small, the arbitrator needs rather precise information about players' bargaining power to make a balanced proposal. In this case the risk of inappropriate arbitration proposals is high which in turn would increase the potential level of inefficiency. For these settings a mediator is more likely to reduce inefficiency as such games tend to be mediatable.

The provided analysis relies on the rather strong assumption that players have common knowledge of the arbitration proposal. Clearly, a relaxation of this assumption is indicated for further research. However, in contrast to other non-cooperative approaches, the results of this paper can explain for the motivation to resort to third-party assistance in negotiations. Thereby, the conditions under which arbitrators or mediators are likely to be asked for assistance, as well as the outcomes they implement, seem easily to go along with intuition.

Chapter 4

Appendices

Appendix 1.1 (simultaneous agenda)

Assume A is the first-mover. His offer maximizes his utility $u_A = \alpha x + (1-\alpha)y$ under the condition that the second-mover B accepts. As players have equal discount factors and following Rubinstein (1982), B always accepts if $u_B \geq \delta u_A$. Therefore, in the unique s.p.e. the following has to hold $u_B = (1-\alpha)(1-x) + \alpha(1-y) = [\alpha x + (1-\alpha)y]\delta = \delta u_A$.
By rearranging we obtain:

$$x = \frac{1 - y(\alpha(1-\delta)+\delta)}{1-\alpha(1-\alpha)}\,.$$

Note that x, which is A's share of issue X exceeds one if $y < k$ with $k = \frac{\alpha(1-\delta)}{\alpha+\delta-\alpha\delta}$, . Remember that X is bounded by 0 and 1, therefore for all $y < k$ the share x is set to 1. We can express A's payoff depending on y, α and δ :

$$u_A(y,\alpha,\delta) = \begin{cases} \alpha\frac{1-y(\alpha(1-\delta)+\delta)}{1-\alpha(1-\alpha)} + (1-\alpha)y, & if\, y > k \\ \alpha + (1-\alpha)y & if\, y \leq k \end{cases}.$$

As u_A is strictly increasing in y for all $y < k$ and strictly decreasing in y for all $y > k$, the s.p.e. is unique and has A proposing a partition in which B receives nothing of issue X and 1-k of Y. B accepts this offer. The payoff of A and B reflect the general utility of the first (u_1) and second-mover (u_2) in the unique s.p.e. of the simultaneous offer game:

$$u_A = u_1 = \alpha + (1-\alpha)\frac{\alpha(1-\delta)}{\alpha+\delta-\alpha\delta} = \frac{\alpha}{\alpha+\delta-\alpha\delta}$$

$$u_B = u_2 = \alpha(1-\frac{\alpha(1-\delta)}{\alpha+\delta-\alpha\delta}) = \frac{\alpha\delta}{\alpha+\delta-\alpha\delta}$$

Appendix 1.2 **(Proposition 1.1 - sequential exogenous agenda)**

Proof of Proposition 1.1.[1] The structure of the proof is as follows. We assume that A is proposer in stage I and that B immediately accepts his offer how to divide issue X. In the first step (i) we derive the B's stage II s.p.e. offer how to divide Y which is depending on stage I division of X. Using the stage II s.p.e. we can now derive A's payoff as a reaction function depending on his share of issue X , which is step (ii). We find that A's payoff is strictly increasing in his share of X and we will show in the last step (iii) that in the s.p.e. B accepts A's stage I proposal to keep the entire issue X for himself while B in turn receives all of his preferred issue Y in stage II.

(i) Assume that B accepted A's proposal $(x_A^I, 1-x_A^I)$ how to divide issue X in stage I. The stage II offer of B on Y $(y_B^{II}, 1-y_B^{II})$ will be accepted by A if the following holds:

$$\alpha x_A^I + (1-\alpha)y_B^{II} \;\geq\; \delta\left[\alpha x_A^I + (1-\alpha)y_A^{II}\right],$$

solving for y_B^{II}, the share B offers A of issue Y:

$$y_B^{II} \geq \frac{-\alpha x(1-\delta)}{1-\alpha} + \delta y_A^{II}\,. \tag{4.1}$$

If (4.1) does not hold and A rejects, his counteroffer finds B's acceptance if the following holds:

$$\delta[(1-\alpha)(1-x_A^I) + \alpha(1-y_A^{II})] \;\geq\; \delta^2\left[(1-\alpha)(1-x_A^I) + \alpha(1-y_B^{II})\right]\,.$$

Solving for y_A^{II}, we obtain the share A would offer himself:

$$y_A^{II} \leq \frac{(1-x_A^I(1-\alpha))(1-\delta)}{\alpha} + \delta y_B^{II}. \tag{4.2}$$

[1]This proof completes the sketch in the main text. Further explication and intuition are provided in the main text.

When solving for the stage *II* offers, one should keep in mind that both issues are assumed to be of size 1, such that offered partitions are thus naturally restricted by 0 and 1. By inserting equation (4.1) in (4.2) we find that y_A^{II}, the partition Player *A* would demand for himself in the case of a counteroffer, is never below 0 but exceeds 1 always when

$$y_B^{II} > \frac{-1+\alpha+\delta+x_A^I(1-\alpha-\delta+\alpha\delta)}{\alpha\delta}, \tag{4.3}$$

such that y_A^{II} will be set equal 1 if (4.3) holds. Inserting (4.2) in (4.1) we obtain the minimal share which *B* has to offer *A* in stage *II*, this is *B*'s s.p.e. offer in stage *II*:

$$y_B^{II} = \frac{-x_A^I\alpha(1-\delta)}{1-\alpha} + \delta \tag{4.4}$$

if (4.3) holds and $0 \leq y_B^{II} \leq 1$, while it is

$$\begin{aligned} y_B^{II} &= \frac{-x_A^I\alpha(1-\delta)}{1-\alpha} + \delta\left(\frac{(1-x_A^I(1-\alpha))(1-\delta)}{\alpha} + \delta y_B^{II}\right) \\ &= \frac{\alpha\delta-\delta+x_A^I(\alpha^2+\delta-2\alpha\delta+\alpha^2\delta)}{-(1-\alpha)\alpha(1+\delta)} \end{aligned} \tag{4.5}$$

if (4.3) does not hold and $0 \leq y_B^{II} \leq 1$, otherwise we have

$$y_B^{II} = 1 \text{ if } y_B^{II} > 1 \text{ and } y_B^{II} = 0 \text{ if } y_B^{II} < 0\,.$$

Note that y_B^{II} (*A*'s share of *Y* in the stage *II* s.p.e. of *B's* offer) is continuous and monotonically decreasing in x^I over all above cases. Therefore we can now state that (4.3) only holds as long as $x^I < \frac{(1-\alpha)(1-\alpha-\alpha\delta)}{1-\alpha(2-\alpha-\alpha\delta)} = k$. Checking for the natural boundaries (1 and 0) of y_B^{II} we find the following: if $x^I \geq k$ it holds that y_B^{II} is defined according to (4.5) and is never larger than 1 but is always smaller than 0 if $x^I \geq \frac{\delta(1-\alpha)}{\alpha^2(1+\delta)-\delta(2\alpha-1)} = l$. Further, if $x^I < k$, the s.p.e. offer y_B^{II} is now defined according to (4.4) and is again never larger than 1 but is smaller than 0 for all $x^I \geq \frac{\delta(1-\alpha)}{\alpha(1-\delta)} = m$.

(ii) Given *B*'s optimal offer in stage *II* we can now calculate *A*'s expected payoff as a reaction function $r_A(x^I)$ depending on x^I and the bargaining situation (the values of α and δ):

$$r_A(x^I) = \begin{cases} \frac{\delta(1+2x^I\alpha - x^I - \alpha)}{\alpha+\alpha\delta} & , for\, x^I \geq k \wedge x^I < l \\ \alpha x^I & , for\, x^I \geq k \wedge x^I \geq l \\ \delta(1-\alpha+\alpha x^I) & , for\, x^I < k \wedge x^I < m \\ \alpha x^I & , for\, x^I < k \wedge x^I \geq m \end{cases} .$$

As described in the main text, r_A is increasing in x^I for all $\alpha \in (0.5, 1)$ and all $\delta \in [0, 1)$. In stage I, the share of X which would maximize A's payoff is thus $x^I = 1$.

(iii) We now check the stage I conditions for A's proposal to find B's acceptance. As we know, B accepts the proposal only if he cannot expect greater payoff by rejecting:

$$(1-\alpha)(1-x_A^I) + \alpha(1-y_B^{II}) \geqq \delta\left[\alpha(1-y_B^I) + (1-\alpha)(1-x_A^{II})\right]$$

$$(1-\alpha)y_B^I + \alpha x_A^{II} \geqq \delta[\alpha x_A^I + (1-\alpha)y_B^{II}] .$$

Now, assume that A's stage I partition is $x_A^I = 1$. It is then easy to show that for all α and δ the share A receives in the s.p.e. of stage II is always $y_B^{II} = 0$: therefore note that firstly, if $k \leq 1$, then according to (4.5) the following has to hold: $\frac{\alpha(\alpha+\alpha\delta-\delta)}{-(1-\alpha)\alpha(1+\delta)} \leq 0$. This is generally true as the denominator is always negative while the nominator is positive for all α and δ in the defined range. Secondly, $k > 1$ never comes into play as k is never larger than 1. Inserting the values $x_A^I = 1$ and $y_B^{II} = 0$ in the above stage I conditions, we have:

$$(x_A^I =)\, 1 \leq \frac{1-\alpha\delta}{1-\alpha} \quad and \quad (y_B^I =)\, 0 \geq \frac{-\alpha(1-\delta)}{1-\alpha} .$$

Both conditions hold for all $\alpha \in (0.5, 1] \wedge \delta \in [0, 1)$. Hence, in the unique s.p.e. the first mover (A) will demand all of his preferred issue (X) in stage I. The second mover (B) will accept this proposal and forego any share of his less preferred issue in stage I. As a consequence he can expect to receive his preferred issue Y entirely in stage II. Note that B would not reject A's proposal in stage I as he would then face the same situation A faced before. As this holds for all α and δ, under this sequential exogenous agenda each player receives his preferred issue entirely and each player's payoff is his evaluation of this issue: α. ■

Appendix 1.3 **(Proposition 1.2 - sequential endogenous agenda)**

Proof of Proposition 1.2. From Proposition 1.1 we know that the first and the second mover's payoff is α if players offer on their preferred issue. We need to show that for all α and δ in the region θ there is an unique s.p.e. in which the payoff of first mover is higher than α if he offers on his less preferred issue first.[2] Further we show that in this s.p.e. players' joint payoff is strictly lower than 2α and therefore not on the bargaining frontier.

The proof is structured as follows. We assume that A is proposer in stage I and that B immediately accepts his offer how to divide issue Y. Remember that issue X is preferred by A while Y is preferred by B. In the first step (i) we derive the B's stage II s.p.e. offer how to divide X which is depending on stage I division of Y. Using the stage II s.p.e. we can now derive A's payoff as a reaction function depending on his share of issue Y , which is step (ii). We derive conditions on α, δ and A's stage I offer y_A^Isuch that his payoff exceeds α. In the next step (iii) we define the conditions under which B would accept such offer and show that for all combinations of α and δ within the region θ all conditions hold. In the last step (iv) we calculate the s.p.e. offers and payoffs and conclude the proof.

(i) Assume player A is the first mover. In stage I, A proposes a division $(y_A^I, 1-y_A^I)$ of issue Y. Again, the proof is by backward induction. Given B accepts the stage I proposal, then in stage II, B himself proposes a division $(x_B^{II}, 1-x_B^{II})$ of issue X. Player A will accept this offer only if:

$$\alpha x_B^{II} + (1-\alpha) y_A^I \geq \delta[\alpha x_A^{II} + (1-\alpha) y_A^I]\,.$$

Solving for x_B^{II}, the minimal share B has to offer A of issue X is:

$$x_B^{II} \geq \delta x_A^{II} - \frac{(1-\alpha)(1-\delta) y_A^I}{\alpha}\,. \tag{4.6}$$

Given a rejection, A's counteroffer would find acceptance if the following holds:

$$\delta[(1-\alpha)(1-x_A^{II}) + \alpha(1-y_A^I)] \;\geq\; \delta^2 \left[(1-\alpha)(1-x_B^{II}) + \alpha(1-y_A^I)\right]\,.$$

[2] The two conditions (4.12) and (4.15) which define θ are derived in this Appendix and plotted in Figure 1.2 and 4.1.

Solving for x_A^{II}, we obtain the share A would offer *himself*:

$$x_A^{II} \leq \delta x_B^{II} + \frac{1-\delta-\alpha(1-\delta)y_A^I}{1-\alpha}. \tag{4.7}$$

Inserting (4.6) into (4.7), it results that x_A^{II} is never smaller 0 but is equal or larger 1 if the following is true:

$$x_B^{II} \geq \frac{-\alpha+\delta+\alpha(1-\delta)y_A^I}{(1-\alpha)\delta}. \tag{4.8}$$

We can now calculate the condition for B's stage II s.p.e. offer By inserting (4.7) into (4.6) we can now calculate the minimal share which B has to offer A in stage II, which is B's s.p.e. offer in stage II:

$$x_B^{II} = \frac{\alpha\delta-(1-\alpha(2-\alpha-\alpha\delta))y_A^I}{(1-\alpha)\alpha(1+\delta)} \tag{4.9}$$

if (4.8) does not hold and $0 \leq x_B^{II} \leq 1$. The condition is

$$x_B^{II} = \delta - \frac{(1-\alpha)(1-\delta)y_A^I}{\alpha} \tag{4.10}$$

if (4.8) does hold and $0 \leq x_B^{II} \leq 1$. Otherwise for $x_B^{II} < 0$, set $x_B^{II} = 0$ and if $x_B^{II} > 1$, set $x_B^{II} = 1$.

Note that we now have B's stage II equilibrium offer as a function depending on α, δ and the solution of stage I $(y^I, 1-y^I)$. Note further that following (4.9) and (4.10), A's share of X is monotonically decreasing in y^I. Therefore it can be stated that (4.8) always holds if $y^I \leq \frac{\alpha(\alpha-(1-\alpha)\delta)}{\alpha^2+(1-\alpha)^2\delta} = \hat{k}$. Checking for the natural boundaries (0 and 1) of x_B^{II}, we obtain the following: for all partitions $y^I > k$ of stage I, it holds that x_B^{II} is defined as in (4.9) and is (for the relevant parameters) never greater than 1 but always smaller than 0 if $y^I > \frac{\alpha\delta}{1-\alpha(2-\alpha-\alpha\delta)} = \hat{l}$. For all partitions $y^I \leq k$ of stage I, x_B^{II} is defined as in (4.10) and is never greater than 1 but always smaller 0 if $y^I > \frac{\alpha\delta}{(1-\alpha)(1-\delta)} = \hat{m}$.

(ii) Given B's optimal offer in stage II we can now calculate A's expected payoff as a reaction function $r_A(y^I)$ depending on the stage division of issue Y and the values of α and δ:

$$r_A(y^I) = \alpha x_B^{II} + (1-\alpha)y^I = \begin{cases} \frac{(\alpha - y^I(2\alpha-1))\delta}{(1-\alpha)(1+\delta)} & , for\, y^I > \hat{k} \wedge y^I \leq \hat{l} \\ (1-\alpha)y^I & , for\, y^I > \hat{k} \wedge y^I > \hat{l} \\ \alpha\delta + (\delta - \alpha\delta)y^I & , for\, y^I \leq \hat{k} \wedge y^I \leq \hat{m} \\ (1-\alpha)y^I & , for\, y^I \leq \hat{k} \wedge y^I > \hat{m} \end{cases}$$

Note that $r_A(y^I)$ is a continuous function over all possible cases. Now, as $\alpha \in (0.5, 1] \wedge y^I \in [0, 1]$ we find that the following holds: $(1-\alpha)y^I < \alpha$. Consequently, player A will not offer in these regions as his payoff is smaller than α, the payoff he expects by offering on his preferred issue first. Further, if both $y^I > \hat{k}$ and $y^I \leq \hat{l}$ hold, it is easy to see that A's payoff is monotonically decreasing in y^I and we can calculate that B's payoff is also decreasing in $y^I : (1-\alpha)(1-x_B^{II}) + \alpha(1-y^I)$, with x_B^{II} defined as in (4.9) we have:

$$r_B(y^I) = \frac{\alpha - y^I(2\alpha - 1)}{\alpha(1+\delta)}\, , for\, y^I > \hat{k} \wedge y^I \leq \hat{l}\, .$$

Consequently A would not propose y^I in this region as smaller y^I increase the payoff of both players. In contrast, if $y^I \leq \hat{k} \wedge y^I \leq \hat{m}$ we have $r_A(y^I)$ increasing and $r_B(y^I)$ decreasing in y^I. Moreover, $r_A(y^I) = \alpha\delta + \delta(1-\alpha)y^I$ is higher α always when $y^I > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}$. Note that $y^I \leq \hat{m}$ always holds if $y^I \leq \hat{k} \wedge y^I > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}$ hold. Further note that a share y^I which fulfills:

$$y^I \leq \hat{k} \;\wedge y^I > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}\, , \tag{4.11}$$

only exist if α and δ are such that the following holds:

$$\frac{\alpha(1-\delta)}{(1-\alpha)\delta} < \hat{k} = \frac{\alpha(\alpha - (1-\alpha)\delta)}{\alpha^2 + (1-\alpha)^2\delta}\, ,$$

which in turn holds only if:

$$\delta > \frac{\alpha^2}{3\alpha - 1 - \alpha^2}\, . \tag{4.12}$$

Hence, only if (4.12) applies, then there can exist a first stage offer y^I which gives the first mover a payoff greater than α if he offers on his less preferred issue first. Consequently, for A's payoff to be greater than α, his stage I offer has to fulfill (4.11), be no greater than 1 nor smaller 0 *and* find the acceptance of B.

(iii) Player B will accept A's stage I offer only if a rejection does not yield him a greater payoff. As we assume an endogenous sequential agenda, this statement has to hold no matter if B's subsequent offer after a rejection would be on his preferred issue (Y) or his less preferred issue (X). Here the fact that the players' evaluation is exactly reverse helps us to put ourselves in the shoes of player B: we can deduce that player B faces the same situation as player A did before. Hence, what has been issue X for player A means issue Y for player B. Assume a bargaining situation in which (4.12) holds and A's offer fulfills (4.11), A expects a payoff greater α. Now, given player B rejects he would also offer on his less preferred issue (X) and expect the discounted payoff $\delta r_B(x^I) = \delta r_A(y^I) > \delta\alpha$. Consequently, if (4.12) holds, B will accept A's stage I offer if the following condition holds:

$$r_A(y^I) \geq \delta r_B(x^I) = \delta r_A(y^I). \tag{4.13}$$

By reformulating we obtain:

$$y^I \leq \frac{\alpha(1-\delta)(1+\alpha\delta)}{2\alpha - 1 + (1-\alpha)^2\delta + (1-\alpha)\alpha\delta^2} = \hat{n}. \tag{4.14}$$

From (4.11), $y^I > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}$ has to hold, thus

$$\frac{\alpha(1-\delta)(1+\alpha\delta)}{2\alpha - 1 + (1-\alpha)^2\delta + (1-\alpha)\alpha\delta^2} > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}.$$

This results in the condition

$$\delta > \frac{2\alpha - 1}{\alpha(1-\alpha)}. \tag{4.15}$$

A share y^I fulfilling (4.11) and (4.14) consequently exists if, and only if, α and δ fulfill (4.12) and (4.15). Note that in this case the values of $\hat{n}$, $\hat{k}$ and $\frac{\alpha(1-\delta)}{(1-\alpha)\delta}$ are always in the interval of 0 and 1. Figure 4.1 plots the boundaries of conditions (4.12), (4.15) and (4.16). The region above (4.12) and (4.15) constitutes θ. In θ the first mover (here player A) expects a higher payoff from offering on his less preferred issue (here Y) first. Note that Figure 1.2 plots the inverse function.

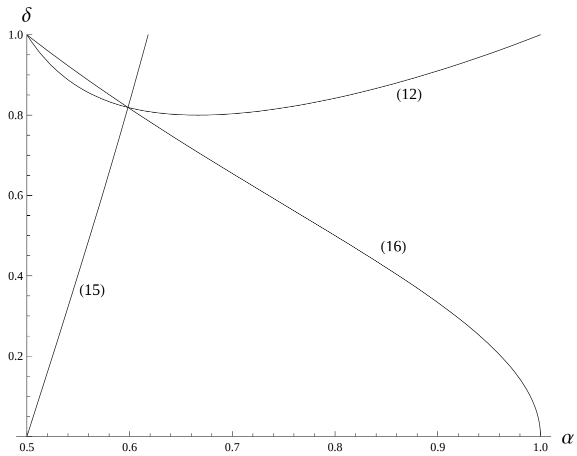

Figure 4.1: Plot of condition (4.12), (4.15) and (4.16)

(iv) Having defined region θ, we need to calculate the exact s.p.e. stage I offer (y_A^I) and the corresponding payoffs. As we know that A's payoff is monotonically increasing in y^I we know A will chose the highest y^I fulfilling the three conditions $y^I \leq \hat{k}$, $y^I > \frac{\alpha(1-\delta)}{(1-\alpha)\delta}$, $y^I \leq \hat{n}$ of (4.11) and (4.14). Further we find that in θ, i.e. if (4.12) and (4.15) hold, the share $y^I = min(\hat{k}, \hat{n})$ is always greater $\frac{\alpha(1-\delta)}{(1-\alpha)\delta}$. Therefore, $y^I = min(\hat{k}, \hat{n})$ is the unique s.p.e. offer of A, as no other offer in stage I (neither on X or Y) yields A a higher final payoff. Regarding $y^I = min(\hat{k}, \hat{n})$ it holds that $\hat{k}$ is smaller $\hat{n}$ if

$$\delta < \frac{\sqrt{1-\alpha}}{\sqrt{a}}\,. \tag{4.16}$$

Hence, A will demand the share $\hat{k}$ for himself if (4.16) holds and $\hat{n}$ otherwise. Condition (4.16) is also depicted in Figure 4.1. For all α and δ above the continuous but below the dotted line, A's equilibrium offer is $y^I = \hat{k}$; while if α and δ are above the dotted and the dashed line, A's s.p.e. offer is $y^I = \hat{n}$. Note that these two areas constitute region θ.

Now, we are able to calculate the s.p.e. payoffs for A and B in θ :

Case (a): If (4.16) holds, A offers $y^I = \hat{k}$, B accepts and offers $x_B^{II} = \delta - \frac{(1-\alpha)(1-\delta)\hat{k}}{\alpha}$ (see (4.10)) to A in stage II. A in turn accepts such that we obtain the payoffs:

$$u_A = \alpha(\delta - \frac{(1-\alpha)(1-\delta)\hat{k}}{\alpha}) + (1-\alpha)\hat{k} = \frac{\alpha^2\delta}{\alpha^2 + (1-\alpha)^2\delta} \text{ and}$$

$$u_B = (1-\alpha)(1-\delta + \frac{(1-\alpha)(1-\delta)\hat{k}}{\alpha}) + \alpha(1-\hat{k}) = -\frac{-(1-\alpha)\alpha}{\alpha^2 + (1-\alpha)^2\delta}\,.$$

For α and δ in θ, A's payoff is always greater than the value α and the payoff of B is always smaller α.

Case (b): If (4.16) does not hold, A offers $y^I = \hat{n}$, B accepts and offers $x_B^{II} = \delta - \frac{(-1+\alpha)(-1+\delta)\hat{n}}{\alpha}$ to A in stage II. Again, A in turn accepts and in this case the payoffs are:

$$u_A = \alpha\delta + (\delta - \alpha\delta)\hat{n} = -\frac{\alpha^2\delta}{1 - 2\alpha + (1-\alpha)^2\delta - (1-\alpha)\alpha\delta^2} \text{ and}$$

$$u_B = -\frac{\alpha^2\delta^2}{1 - 2\alpha + (1-\alpha)^2\delta - (1-\alpha)\alpha\delta^2}.$$

Again, we find that u_A is always greater α and u_B is always smaller α. [3] ■

Appendix 2.1 (rationality constraint - single issue game)

I will argue that player i's rationality constraint is a weak one by examining the following two cases:

(i) *If the costs of a punishment are equal for both players, i.e. if $c_{ii} = c_{ij} = c_{ji} = c_{jj}$, then i's constraint reduces to:*

$$\delta_i^2 + \delta_i\delta_j \geq 1.$$

In this case the punishment costs players inflict to themselves and their opponent are equal. The constraint holds if the punishing player is sufficiently patient.

(ii) *If players have equal discount factors, i.e. $\delta_i = \delta_j = \delta$, the rationality constraint of Player i simplifies to:*

$$\delta^2 c_{ij} \geq (1-\delta^2)c_{ii}.$$

In this case of equal discount factors, the constraint reduces to a comparison of the punishment costs induced to the individual players. If punishment does not leave the opponent completely unaffected, there is always a level of patience which allows the constraint to hold. When players are sufficiently patient, it is still rational to punish even if the punishing player himself suffers most from it. If the assumptions of both corollaries apply the constraint simplifies to $\delta \geq \sqrt{1/2}$. [4]

[3] Figure 1.3 illustrates these payoffs for $\alpha = 0.58$ where the increasing part of the first mover payoff reflects Case (a) and the decreasing part reflects Case (b).

[4] Note, that Bolt's (1995) restriction of Fernandez and Glazer's (1991) result does not apply here. In our model there exists no (stream of) payoff in the absence of an agreement such that Bolt's 'no concession' strategy is never rational.

Appendix 2.2 **(Lemma 2.1 - sequential agenda with punishment)**

I restrain the proof of Lemma 2.1 to show R^j, the proof of both other regimes is analogue. Remember that R^j is exogenously given, i.e. j always punishes after i rejects and i never punishes. The proof consists of 2 steps. First, I define the equilibrium strategies for stage II (issue Z) and argue why no player can improve by deviation. Second, I show subgame perfectness for the five different subcases in stage I.

Stage II: Assume that in stage I an agreement was found on issue Y which gives the share y_i to i and y_j to j. Remember that, first, a settled issue remains settled and cannot be reopened, second, the punishment decision is assumed to be exogenous, third, a delay in stage II concerns both issues, i.e. it diminishes players payoff from their shares of issue Y and Z. Now, players' equilibrium strategies are the following:
Player i:

- reject all $z_{ji} < \frac{x^j_{ji}-\rho y_i}{1-\rho}$ and accept all $z_{ji} \geq \frac{x^j_{ji}-\rho y_i}{1-\rho}$.
- demand $z_{ii} = min\left\{\frac{x^j_{ii}-\rho y_i}{1-\rho}, 1\right\}$, never punish.

Player j:

- reject all $z_{ij} < \frac{x^j_{ij}-\rho y_j}{1-\rho}$ and accept all $z_{ij} \geq \frac{x^j_{ij}-\rho y_j}{1-\rho}$.
- demand $z_{jj} = min\left\{\frac{x^j_{jj}-\rho y_j}{1-\rho}, 1\right\}$, always punish after i rejects.

Now, if $1-\delta \leq \frac{x^j_{ii}-\rho y_i}{1-\rho} \leq 1$, then i demands $z_{ii} = \frac{x^j_{ii}-\rho y_i}{1-\rho}$ which, if accepted, yields him an payoff (from issue Y and Z) of: $(1-\rho)\,\frac{x^j_{ii}-\rho y_i}{1-\rho} + \rho y_i = x^j_{ii}$. Further i rejects all shares z_{ji} which yield him a payoff less than x^j_{ji} and accept all which yield him more than x^j_{ji}. Analogue, if $1-\delta+c_{ji} < \frac{x^j_{jj}-\rho y_j}{1-\rho} < 1$ player j demands a share $z_{jj} = \frac{x^j_{jj}-\rho y_j}{1-\rho}$ which yields him a payoff of x^j_{jj} and rejects all offers which yield him less than x^j_{ij}. If $i[j]$ offers, then (z_{ii}, z_{ij}) $[(z_{ji}, z_{jj})]$ is the unique partition for Z which solves the following two equations

$$(1-\rho)\,z_{ij} + \rho y_j \geq \delta\left[(1-\rho)\,z_{jj} + \rho y_j\right]$$

$$(1-\rho)\,z_{ji} + \rho y_i \geq \delta\left[(1-\rho)\,z_{ii} + \rho y_i\right] - c_{ji}$$

and is therefore subgame perfect. No player has reason to deviate from above strategies and immediate agreement is reached. This holds independent of the partition (y_i, y_j) agreed on in stage I.

Stage I: Given the results of stage II, I show that for stage I subgames in which i offers, there is a unique s.p.e. for each case (every ρ) in which j immediately accepts i's offer.

(i) If $\rho \leq x^j_{ji}$, then player i offers some share $0 < y_{ij} < 1$ and j immediately accepts. Note, that for $\rho \leq x^j_{ji}$ it generally holds that $1 - \delta + c_{ji} \leq \frac{x^j_{jj} - \rho y_j}{1-\rho}$, further, $\rho \leq x^j_{ji}$ implies $1 - \rho > x^j_{jj}$ and it holds that $\frac{x^j_{jj} - \rho y_j}{1-\rho} < 1$. This means, for all $y_j \in [0,1]$ player j proposes in stage II the partition $(z_{ji}, z_{jj}) = \left(\frac{x^j_{ij} - \rho y_i}{1-\rho}, \frac{x^j_{jj} - \rho y_j}{1-\rho}\right)$, i immediately accepts and players' overall payoffs are $\left(x^j_{ji}, x^j_{jj}\right)$. Hence, by no deviation i can improve his payoff. Now, I argue why player j can not improve on x^j_{jj} by deviating. Note, that $\rho < x^j_{ji}$ equally implies $1 - \rho \geq x^j_{ii}$ which in turn implies $\rho < x^j_{ij}$ and it holds that for all shares $0 < y_{ji} < 1$ player j offers i in stage I the following payoffs result $\left(x^j_{ii}, x^j_{ij}\right)$ in the next period. If j rejects i's offer in stage I, then by any counteroffer y_{ji} his payoff is $\delta x^j_{ij} = x^j_{jj}$.

(ii) If $x^j_{ii} > \rho > x^j_{ji}$, then i demands the entire issue Y for himself and j immediately accepts. Note, that $\rho > x^j_{ji}$ implies $1 - \rho \leq x^j_{jj}$ which in turn implies for $y_j = 0$ that $\frac{x^j_{jj} - \rho y_j}{1-\rho} \geq 1$ such that in stage II, j would demand the entire issue Z, i.e. $z_{jj} = 1$. As $\rho > x^j_{ji}$ implies for $y_i = 1$ that $\frac{x^j_{ji} - \rho y_i}{1-\rho} < 0$ such that i accepts j's offer $z_{ji} = 0$ and players payoffs are $(u_i, u_j) = (\rho, 1 - \rho)$. Player i cannot improve by demanding less of Y in stage I as in stage II he receives the share $z_{ji} = min\left\{\frac{x^j_{ji} - \rho y_i}{1-\rho}, 1\right\}$ such that his payoff is $u_i = \rho y_i + (1-\rho) min\left\{\frac{x^j_{ji} - \rho y_i}{1-\rho}, 1\right\}$ which is increasing in y_i. Hence, i's best option is to claim the entire issue Y. Player j can neither improve by deviation. Note, that $\rho < x^j_{ii}$ implies that $\rho < x^j_{jj}$ which means that by rejection and a counteroffer he either receives ρ or x^j_{ij} as payoff in the next period. As $\rho < x^j_{ii}$ equally implies $1 - \rho \geq x^j_{ij}$, first, it holds that $1 - \rho > \delta x^j_{ij}$, and, second, by $x^j_{ij} = \delta x^j_{jj}$ and by $\rho < x^j_{jj}$ it holds that $1 - \rho > \delta\rho$. Therefore, j would diminish his payoff by deviating. Player i cannot exploit this fact by offering j exactly his reservation value as i's offer is limited by the size of Y.

(iii) If $\rho \geq x^j_{ii} \wedge \rho \geq x^j_{jj}$, then i demands $y_{ii} = \frac{x^j_{ii}}{\rho}$ and j accepts immediately. Note, that for $\rho \geq x^j_{ii} \wedge \rho \geq x^j_{jj}$ all first stage I agreements for which holds $y_i \geq \frac{x^j_{ii}}{\rho}$ (which implies $y_j \leq \frac{x^j_{ii}}{\rho}$) lead to a stage II equilibrium in which j receives the entire issue Z, as $\frac{x^j_{jj}-\rho y_j}{1-\rho} \geq 1$ (equally for $y_i \leq \frac{x^j_{jj}}{\rho}$ holds that $\frac{x^j_{ii}-\rho y_i}{1-\rho} \geq 1$). Hence, for $y_{ii} \geq \frac{x^j_{ii}}{\rho}$ and $y_{jj} \geq \frac{x^j_{jj}}{\rho}$ the problem reduces to:

$$\rho y^j_{ij} + (1-\rho) \geq \delta \rho y^j_{jj}$$

$$\rho y^j_{ji} + (1-\rho) \geq \delta \rho y^j_{ii} - c_{ji}$$

If i offers first, then the unique Y-partition which fulfills both equations is $(y_{ii}, y_{ij}) = \left(\frac{x^j_{ii}}{\rho}, 1 - \frac{x^j_{ii}}{\rho}\right)$. Players' equilibrium payoffs in this case are $(u_i, u_j) = \left(\rho \frac{x^j_{ii}}{\rho}, \rho\left(1 - \frac{x^j_{ii}}{\rho}\right) + 1 - \rho\right) = \left(x^j_{ii}, x^j_{ij}\right)$.

(iv) If $\rho \geq x^j_{ii} \wedge x^j_{jj} > \rho > x^j_{ij}$, then i demands $y^j_{ii} = min\left\{1 - \delta + \frac{1-\rho}{\rho}, 1\right\}$ and j accepts immediately his share $y^j_{ij} = max\left\{\delta - \frac{1-\rho}{\rho}, 0\right\}$. Note, that for $x^j_{jj} > \rho > x^j_{ij}$ and for $y_j \leq max\left\{\delta + \frac{1-\rho}{\rho}, 0\right\}$ player j's equilibrium share in stage II is 1, as under these conditions it generally holds that $\frac{x^j_{jj}-\rho y_j}{1-\rho} \geq 1$. In this case players' payoffs are $(u_i, u_j) = (min\{1-\delta\rho, \rho\}, max\{\delta\rho, 1-\rho\})$. I show that no player can gain by deviation from this strategy. Player i cannot gain by demanding less in stage I as his payoff

$$u_i = \rho y_i + (1-\rho)\, max\left\{\frac{x^j_{ji} - \rho y_i}{1-\rho}, 0\right\}$$

is increasing in y_i. If j deviates and rejects i's offer his payoff is ρ in the next period, as $x^j_{jj} > \rho > x^j_{ij}$.[5] Therefore i's offer for j has to fulfill only the following condition:

$$\rho y_{ij} + (1-\rho) \geq \delta\rho.$$

In equilibrium j accepts $y_{ij} = \delta - \frac{1-\rho}{\rho}$ and i receives the share $y_{ii} = 1 - \delta + \frac{1-\rho}{\rho}$. Note that $1 - \delta + \frac{1-\rho}{\rho} > 1$ for $\rho < \frac{1}{1+\delta}$.

[5]The proof is analogue to case (ii) above where $x^j_{ii} > \rho > x^j_{ji}$.

(v) If $\rho \geq x^j_{ii} \wedge \rho \leq x^j_{ij}$, then i demands $y^j_{ii} = min\left\{\frac{1-\delta x^j_{ij}}{\rho}, 1\right\}$ and j immediately accepts his share $y^j_{ij} = max\left\{\frac{\delta x^j_{ij}-(1-\rho)}{\rho}, 0\right\}$. Note, that for $\rho \leq x^j_{ij}$ it holds for all shares $y_j \leq max\left\{\frac{\delta x^j_{ij}-(1-\rho)}{\rho}, 0\right\}$ which j receives in stage I that in the equilibrium of stage II player j receives the entire issue Z, as under these conditions it generally holds that $\frac{x^j_{jj}-\rho y_j}{1-\rho} \geq 1$. In this case players' payoffs are $(u_i, u_j) = \left(min\left\{1-\delta x^j_{ij}, \rho\right\}, max\left\{\delta x^j_{ij}, 1-\rho\right\}\right)$. Player i's payoff in this case is never smaller than his payoff in case (iii) which is x^j_{ii}, as it is generally true that $1-\delta x^j_{ij} > x^j_{ii}$ and by assumption it holds that $\rho \geq x^j_{ii}$. Now, I show that no player can gain by deviation from this strategy. Player i cannot gain by demanding less in stage I as his payoff

$$u_i = \rho y_i + (1-\rho)\, max\left\{\frac{x^j_{ji} - \rho y_i}{1-\rho}, 0\right\}$$

is increasing in y_i. If j deviates and rejects i's offer his payoff is x^j_{ij} in the next period, as $x^j_{ij} > \rho$. [6] Therefore i's offer for j has to fulfill only the following condition:

$$\rho y_{ij} + (1-\rho) \geq \delta x^j_{ij}.$$

In equilibrium j accepts $y_{ij} = \frac{\delta x^j_{ij}-(1-\rho)}{\rho}$ and i receives the share $y_{ii} = \frac{1-\delta x^j_{ij}}{\rho}$. Note that $\frac{1-\delta x^j_{ij}}{\rho} > 1$ for $\rho < 1-\delta x^j_{ij} = \frac{1-2\delta^2+\delta^3-\delta^2 c_{ji}}{1-\delta^2}$. ■

Remark: In regime R^i with i as first mover the result given in the main text has only three subcases. Consequently the same holds for R^j with j as first mover. This may appear odd to the reader, as for $\rho \leq x^j_{ij}$ (the least case) i reservation value may differ depending on the exact ρ. Remember that in R^j, i's reservation value is i's payoff in the next period minus his cost from j's punishment. For $\rho \leq x^j_{ij} \wedge \rho \leq x^j_{ji}$ player i's reservation value is $\delta x^j_{ji} - c_{ji}$, for $\rho \leq x^j_{ij} \wedge x^j_{ii} > \rho > x^j_{ji}$ it is $\delta\rho - c_{ji}$ and for $\rho \leq x^j_{ij} \wedge \rho \geq x^j_{ii}$ it is $\delta \cdot min\left\{1-\delta x^j_{ij}, \rho\right\} - c_{ji}$. Under the respective conditions all

[6] The proof is analogue to case (i) above where $\rho \leq x^j_{ji}$.

three values are smaller than i's payoff in this case $\left(x_{ii}^{j}\right)$ such that no further subcases emerge here. As in case (i) above, the bargaining power of the second mover does not result from his reservation value, but from the fact that the first issue is relatively small such that the first mover cannot fully exert his advantage.

Appendix 2.3 **(rationality constraint - sequential agenda game)**

I show that $\delta\tilde{x}_{ij}^{j} \geq \delta\tilde{x}_{ij}^{R}$ (equation (2.6)) holds for all $\rho \in [0,1]$ if $c_{ji} \geq max\left\{\frac{1}{\delta} - 2 + \delta^3, 0\right\}$.

- For $\rho \leq x_{ji}^{j} \wedge \rho \leq x_{ji}^{R}$ it results that $\tilde{x}_{ij}^{j} = x_{jj}^{j}$ and $\tilde{x}_{ij}^{R} = x_{jj}^{R}$. For all $c_{ji} \geq 0$ it holds that $x_{jj}^{j} \geq x_{jj}^{R}$.
- If $x_{ji}^{j} \leq x_{ji}^{R}$, then it is possible that $\rho > x_{ji}^{j} \wedge \rho \leq x_{ji}^{R}$ such that $\tilde{x}_{ij}^{j} = 1 - \rho$ and $\tilde{x}_{ij}^{R} = x_{jj}^{R}$. For $\rho \leq x_{ji}^{R}$ it holds that $1 - \rho \geq x_{jj}^{R}$ which in turn implies that in this case $\tilde{x}_{ij}^{j} = 1 - \rho \geq x_{jj}^{R} = \tilde{x}_{ij}^{R}$.
- For $x_{ii}^{j} > \rho > x_{ji}^{R}$ it results that $\tilde{x}_{ij}^{j} = \tilde{x}_{ij}^{R} = 1 - \rho$.[7]
- If $x_{ii}^{j} \leq x_{ii}^{R}$, then it is possible that $\rho \geq x_{ii}^{j} \wedge \rho < x_{ii}^{R}$ such that $\tilde{x}_{ij}^{R} = 1 - \rho$ and j's share in regime R^j is either $\tilde{x}_{ij}^{j} = x_{ij}^{j}$ (if $\rho \geq x_{jj}^{j}$), $\tilde{x}_{ij}^{j} = max\{\delta\rho, 1 - \rho\}$ (if $x_{jj}^{j} > \rho > x_{ij}^{j}$) or $\tilde{x}_{ij}^{j} = max\left\{\delta x_{ij}^{j}, 1 - \rho\right\}$ (if $\rho \leq x_{ij}^{j}$). For $\rho \geq x_{ii}^{j}$ it holds that $1 - \rho \leq x_{ij}^{j}$ such that $\tilde{x}_{ij}^{R} = 1 - \rho$ is in no case larger than $\tilde{x}_{ij}^{j}$.
- For $\rho \geq x_{ii}^{j} \wedge \rho \geq x_{ii}^{R}$ it results that $\tilde{x}_{ij}^{R} = x_{ij}^{R}$ and j's share in regime R^j is either $\tilde{x}_{ij}^{j} = x_{ij}^{j}$ (if $\rho \geq x_{jj}^{j}$), $\tilde{x}_{ij}^{j} = max\{\delta\rho, 1 - \rho\}$ (if $x_{jj}^{j} > \rho > x_{ij}^{j}$) or $\tilde{x}_{ij}^{j} = max\left\{\delta x_{ij}^{j}, 1 - \rho\right\}$ (if $\rho \leq x_{ij}^{j}$). First, for $\rho \geq x_{jj}^{j}$ it holds that $\tilde{x}_{ij}^{j} = x_{ij}^{j} \geq x_{ij}^{R} = \tilde{x}_{ij}^{R}$. Second, for $x_{jj}^{j} > \rho > x_{ij}^{j}$ it holds that $\tilde{x}_{ij}^{j}$ is never smaller than $\delta\rho$ which is in this case by assumption $(\rho > x_{ij}^{j})$ larger $\delta x_{ij}^{j} = \delta\frac{\delta - \delta^2 + \delta c_{ji}}{1 - \delta^2}$. As for all $c_{ji} \geq \frac{1}{\delta} - 2 + \delta$ it holds that $\delta\frac{\delta - \delta^2 + \delta c_{ji}}{1 - \delta^2} > \frac{\delta - \delta^2}{1 - \delta^2} = x_{ij}^{R}$ it follows that $\tilde{x}_{ij}^{j} > \tilde{x}_{ij}^{R}$. Third, for $\rho \leq x_{ij}^{j}$ it holds that $\tilde{x}_{ij}^{j}$ is never smaller than δx_{ij}^{j} and the argument follows as before.

Appendix 2.4 **(Proposition 2.4 - inefficiency comparison)**

The proof compares the range of inefficient equilibria under the sequential agenda given by (2.7) and the range of inefficient equilibria under the simultaneous agenda

[7] Note that in this case the rationality constraint holds only if $c_{ii} = 0$.

given by (2.5). In case (i), first, given (2.3) and (2.6) hold, for $x^B_{AA} \leq \rho$ the lower bound in the sequential agenda game never lies below the lower bound in the simultaneous agenda game, because $\tilde{x}^B_{AA} \geq x^B_{AA}$. Second, if $\rho \leq x^A_{BA}$, then for all N the upper bound in the sequential agenda game lies below the upper bound in the simultaneous agenda game, because $\tilde{x}^A_{AB} = x^A_{BB} > x^A_{AB}$. An example of this case is plotted in Figure 2.4.

In case (ii), given (2.3) and (2.6) hold, for $1 \leq x^A_{BA}$ the upper bound is identical in both agendas, because in this case $\rho \leq 1 \leq x^A_{BA}$ such that $\tilde{x}^A_{AA} = x^A_{BA}$ and further $x^A_{AA} > x^A_{BA}$ such that for $x \in [0,1]$ it results that $\tilde{x}^A_{AA} = x^A_{BA} = 1$ and $x^A_{AA} = 1$ which implies $\tilde{x}^A_{AB} = x^A_{AB} = 0$. If $\rho \leq x^B_{BA}$, then the lower bound in the sequential agenda lies always below the lower bound in the simultaneous agenda game, as $\tilde{x}^B_{AA} = x^B_{BA} < x^B_{AA}$. This case is illustrated by Figure 2.5.

Note, that for all ρ not part of these cases, the maximal periods of punishment can vary slightly in both directions between the two agenda games. ■

Appendix 3.1 (rationality constraint)

I will argue that p^i is a weak constraint by examining the following two cases:

(i) *The p-rationality constraint can be reformulated as*

$$\delta^2 c_{ij} \geq (1-\delta^2) c_{ii}.$$

It is now easy to see that for sufficiently high δ the constraint holds and punishment is rational for player i even if i himself suffers most from his punishment.

(ii) *If the costs of a punishment are equal for both players, i.e. if $c_{ii} = c_{ij}$, then i's constraint reduces to:*

$$\delta \geq \sqrt{0.5} \approx 0.7.$$

In this case the punishment costs players inflict to themselves and their opponent are equal. The constraint holds if the punishing player is sufficiently patient.

Appendix 3.2 (Proposition 3.1 - s.p.e. of regime R^i)

To show that R^i is a s.p.e. which supports the partition $\left(x^i_{ii}, x^i_{ij}\right)$ if p^i holds, I show that no player has an incentive to deviate. In R^i players strategies are the following:

- Player i: offer x^i_{ij}, accept all $x \geq x^i_{ji}$, reject all $x < x^i_{ji}$ and punish always after j rejects.
- Player j: offer x^i_{ji}, accept all $x \geq x^i_{ij}$, reject all $x < x^i_{ij}$ and never punish.

If player i fails to punish after j has rejected an offer, player j's counteroffer is x^R_{ji} and players follow R.

Given this profile, a deviation is never rational for j as he has to bear the costs of punishment without expecting a larger share in the end (and thus a smaller payoff). Confronted with j's rejection of his offer, if and only if punishment is p-rational for i, then it is preferable for him to invest the punishment costs of c_{ii} in order to implement his regime R^i and receive x^i_{ji} in the next period instead to forgo the punishment and continue with R and receive x^R_{ji} in the next period. ■

Appendix 3.3 (Proposition 3.2 - efficient equilibria)

As a first step (i) I will argue why in subgames where player A offers, no partition outside of $\left[(x^B_{AA}, x^B_{AB}), (x^A_{AA}, x^A_{AB})\right]$ can be supported as subgame perfect equilibrium. In the second step (ii), I show that every partition in $\left[(x^B_{AA}, x^B_{AB}), (x^A_{AA}, x^A_{AB})\right]$ can be supported as s.p.e. in subgames in which A offers.

(i) In our model, δ is the common discount factor. In addition, let c^j_i be all costs inflicted to player $i \in \{A, B\}$ always after $j \in \{A, B\}$ rejected an offer. Now, given the stationary structure of the game and given X_{ij} $\left(\underline{x}_{ij}\right)$ is the supremum (infimum) of payoffs supported in s.p.e. which player j can receive in subgames where i offers, then by standard arguments the following conditions for a s.p.e. result:

$$\begin{array}{l} X_{AA} \leq 1 - \delta \underline{x}_{BB} + c^B_B \\ X_{BB} \leq 1 - \delta \underline{x}_{AA} + c^A_A \\ \underline{x}_{AA} \geq 1 - \delta X_{BB} + c^B_B \\ \underline{x}_{BB} \geq 1 - \delta X_{AA} + c^A_A \end{array},$$

which have the unique solution for subgames in which A offers:

$$(x_{AA}, x_{AB}) = \left(\frac{1 - \delta + c_B^B - \delta c_A^A}{1 - \delta^2}, \frac{\delta - \delta^2 - c_B^B + \delta c_A^A}{1 - \delta^2} \right).$$

It is obvious that A's (B's) share is increasing in the costs inflicted on his opponent B (A) in periods after B (A) rejects and decreasing in the costs inflicted to A (B) himself in periods after he, A (B), rejects. It follows that player i's share in s.p.e. is maximised if both players punish after j rejects $\left(c_j^j = c_{ij} + c_{jj}\right)$ and no player punishes after i rejects $\left(c_i^i = 0\right)$. Note that in this case player j's share in the s.p.e is greater if we assume that he always abstains from executing punishment after rejecting i's offer. Hence, j prefers the s.p.e. outcome in which he does never punish after his rejection over the outcome in which he always punishes after his rejection. Given the stationary structure of the game, player j's execution of punishment after his own rejection can therefore not be an optimal choice in any period.

Now, given the punishment choices are endogenous, we know that player j never punishes after his own rejection. Thus, the disrupted punishment strategy of i $\left(R^i\right)$ is *the* punishment strategy which yields i the greatest share (x_{ii}^i if i offers and x_{ji}^i if j offers). In the proof of proposition 3.2. it is shown that R^i constitutes a s.p.e. if the punishment is p-rational. We can conclude that for subgames in which i offers, regime R^A is the s.p.e. strategy profile which yields A the greatest payoff $\left(x_{iA}^A\right)$ and R^B the profile which yields him the smallest payoff $\left(x_{iA}^B\right)$.

(ii) The structure of the equilibrium strategies is as follows: Each player proposes $(\bar{x}, 1 - \bar{x})$ and accepts the partition $(\bar{x}, 1 - \bar{x})$. If player i deviates from the proposition or the acceptance of $(\bar{x}, 1 - \bar{x})$, player j starts his disruptive punishment, the strategies immediately switch to R^j.

Concretely, assume that *A's* offer in period t=0 is rejected by B. For *A's* offer $\bar{x} \in \left[x_{AA}^B, x_{AA}^A\right]$, B's rejection constitutes a deviation from the equilibrium strategy, the game switches immediately to R^A, this means A would punish (thereby inflicting c_{AB} to B and c_{AA} to himself) and players follow R^A in t+1 where B receives x_{BB}^A.[8] Conversely, if A deviates by making any other

[8]At this point the question may arise whether A's punishment after the rejection of B in t=0 is a

offer than $\bar{x}$, players follow R^B in t+1 and A receives x^B_{BA}.[9] Comparing the payoffs in t=0, it is optimal for A to demand $\bar{x}$ right away, because any other offer yields him $\delta x^B_{BA} < x^B_{AA} \leq \bar{x}$. It is optimal for B to accept the proposal of $\bar{x}$, because $1 - \bar{x} \geq x^A_{AB} = \delta x^A_{BB} - c_{AB}$. ■

Appendix 3.4 (Proposition 3.3 - inefficient equilibria)

I show that no player has an incentive to deviate from his equilibrium strategies which is the following for both players $i \in \{A, B\}$ as long as no player has deviated:

- offer x^i_{ij}, reject all $x < x^i_{ji}$, accept all $x \geq x^i_{ji}$, and punish for N periods in every period, then demand $\hat{x}_i$, reject all $x < \hat{x}_i$ and accept all $x \geq \hat{x}_i$, punish disruptively in periods where j rejects.

If player i deviates from above strategies, e.g. i offers an early compromise or refuses to agree on $\hat{x}_i$ in period $N+1$, player j immediately switches to his strategy of regime R^j. From Proposition 3.1 we know that in this case i's equilibrium payoff as first (second) mover is $x^j_{ii}\left(x^j_{ji}\right)$. Checking for subgame perfection of the strategies we find that if both players follow the suggested strategies, then the equilibrium payoffs are:

$$u_i = \delta^N \hat{x}_i - \frac{1-\delta^N}{1-\delta}\left(c_{ii} + c_{ji}\right).$$

Given the second mover B follows his equilibrium strategy, by any deviation A cannot enforce a payoff higher than x^B_{AA}, i.e. his payoff if he immediately complies on regime R^B. It is thus optimal for A not to deviate as long as $\hat{x}_A \geq [x^B_{AA} + \frac{1-\delta^N}{1-\delta}(c_{AA} + c_{BA})]\delta^{-N}$ which is the lower bound of $\hat{x}$. By any deviation B cannot enforce a payoff higher than x^A_{AB}, which is his payoff if he immediately accepts A's first equilibrium offer. Therefore, it is optimal for B not to deviate from the equilibrium strategy if $\hat{x}_A \leq 1 - [x^A_{AB} + \frac{1-\delta^N}{1-\delta}(c_{BB} + c_{AB})]\delta^{-N}$, the upper bound of $\hat{x}$. ■

best response, as it inflicts costs c_{AA} to A. To understand the necessity of A's punishment, note that in case of a deviation by B, the strategies impose a (immediate) switch to R^A even before the punishment game Γ in t=0 starts . Hence, if A fails to punish after B's deviation, then players follow the Rubinstein procedure R which yields A the payoff $\delta_A x^A_{BA}$ instead of $\delta_A x^A_{AA} - c_{AA}$ if he punishes. Given (3.1) holds, the latter is higher and A's punishment is rational.

[9] Remember that in subgames where A offers, a share for A outside of $\left[x^B_{AA}, x^A_{AA}\right]$ cannot be supported as s.p.e.

Appendix 3.5 (Proposition 3.5 - inefficiency with arbitration)

To prove (i) I show that no player can improve by deviating from his equilibrium strategy. I restrain the analysis to the case $s_A > x^A_{AA}$. Here the equilibrium strategies are the following as long as no player has deviated:

- Player A: offer s_B and never punish if B rejects, accept all $x \geq s_A$, not accept all $x < s_A$ and always call for arbitration for N periods, then demand $\tilde{x}_A$, accept all $x \geq \tilde{x}_A$, reject all $x < \tilde{x}_A$ and call for arbitration.

- Player B: offer x^B_{BA}, accept all $x \geq x^B_{AB}$, reject all $x < x^B_{AB}$ and never punish for N periods, then demand $\tilde{x}_B$, accept all $x \geq \tilde{x}_B$, reject all $x < \tilde{x}_B$ and always punish after A rejects.

If player A deviates from his equilibrium strategy, player B immediately switches to his strategy of regime R^B. From Proposition 3.1 we know that in this case A's equilibrium payoff as first (second) mover is x^B_{AA} $\left(x^B_{BA}\right)$. If B deviates, players switch to their equilibrium strategies of R^S. From Corollary 3.1 we know that this yields B a payoff of s_B.

In contrast, if both players follow the suggested strategies, then the equilibrium payoffs are:

$$u_i = \delta^N \tilde{x}_i.$$

By deviation A cannot enforce a payoff higher than x^B_{AA}, i.e. his payoff if he immediately complies on regime R^B. It is thus optimal for A not to deviate as long as $\tilde{x}_A \geq x^B_{AA}\delta^{-N}$ which is the lower bound of $\tilde{x}_A$. By deviating, B's best payoff is s_B, which is his payoff if he immediately accepts A's first equilibrium offer. Therefore, it is optimal for B not to deviate from the equilibrium strategy if $\tilde{x}_A \leq 1 - s_B\delta^{-N}$, the upper bound of $\tilde{x}_A$.

To proof (ii) there is to show that the maximum loss due to punishment and delay is larger with arbitration than without. Or, put differently, in the presence of an arbitrator, the minimum sum of players' payoffs supported in equilibrium is comparably smaller. As shown in (i), in the game with arbitration the minimal payoff of player $A(B)$ in equilibrium is x^B_{AA} (s_B) if $s_A > x^A_{AA}$ and s_A $\left(x^A_{AB}\right)$ if $s_B > x^B_{BB}$. In the game without arbitration the minimal payoff of $A(B)$ in equilibrium is x^B_{AA} $\left(x^A_{AB}\right)$ as shown in proposition 3.3. Note, that $s_B < x^A_{AB}$ if $s_A > x^A_{AA}$ as $s_A + s_B = x^A_{AA} + x^A_{AB} = 1$ and equally $s_A < x^B_{AA}$ if $s_B > x^B_{BB} \geq x^B_{AB}$ as $s_A + s_B = x^B_{AA} + x^B_{AB} = 1$. It is now straightforward that $x^B_{AA} + s_B < x^B_{AA} + x^A_{AB}$ and $s_A + x^A_{AB} < x^B_{AA} + x^A_{AB}$. ■

Appendix 3.6 (Proposition 3.7 - equilibria with mediation)

To prove (i) I show that no player has an incentive to deviate from his strategy in R^{Mj}. The equilibrium strategies are the following as long as no player has deviated:

- Player i: Offer x_{ij}^{Mj}, accept all $x \geq x_{ji}^{Mj}$, reject all $x < x_{ji}^{Mj}$.
- Player j: Offer x_{ji}^{Mj}, accept all $x \geq x_{ij}^{Mj}$, not accept any $x < x_{ij}^{Mj}$ and call the mediator.

If player j deviates by rejecting without calling the mediator, then i punishes and players continue with R^{Mj}. If player i fails to punish after j rejects, players switch to the standard Rubinstein game R.

First, it is straightforward that player i cannot improve by unilateral deviation. Second, I argue why player j cannot improve by rejecting and not calling the mediator. If j rejects an offer $x \geq x_{ij}^{Mj}$ he will bear the costs of i's punishment (c_{ij}) and players follow R^{Mj} in the next period (where j receives x_{jj}^{Mj}). If the game is mediatable for j $\left(M^j\right)$ then it holds that $-c_{ij} + \delta x_{jj}^{Mj} < x_{ij}^{Mj}$ and j has no incentive to deviate. Third, i's punishment after j's deviation is rational if i's payoff from punishing and following R^{Mj} is higher than from not punishing and R, i.e. $-c_{ii} + \delta x_{ji}^{Mj} \geq \delta x_{ji}^{R}$, which is guaranteed by m-rationality for i.

The proof of (ii) can be derived from two facts. First, player j has an incentive to deviate from his strategy in R^i. Second, j can prevent R^i. Given that the game is mediatable for j and that i offers, then by definition it holds that j's payoff from R^{Mj} is higher than his payoff from R^i. Concretely, if j deviates from R^i by referring to the mediator and playing R^{Mj}, his share is $\delta^2 x_{jj}^{Mj} = x_{ij}^{Mj}$ which is by definition of M^j larger than x_{ij}^{i}, his share in R^i. As j's decision to call the mediator is precedent to the punishment game Γ, j can prevent i from punishing. It follows immediately that R^i is not subgame perfect. Note, that in this case i's (j's) payoff in R^{Mj} $\left(R^{Mi}\right)$ is the highest payoff for i (j) which can be supported in equilibrium.

The intuition behind the proof of (iii) is as follows. First, no player can enforce his punishment regime. Second, as punishment is not m-rational no player can enforce the mediation regime. Third, by standard arguments it follows straightforward that the unique partition supported in equilibrium is $\left(x_{ii}^{R}, x_{ij}^{R}\right)$. Concretely, I prove (iii) by showing that no player $i \in \{A, B\}$ has reason to deviate from his equilibrium strategy, which is as follows as long as opponent j has not punished in any period before:

- offer x_{ij}^{R}, accept all $x \geq x_{ji}^{R}$, reject any $x < x_{ji}^{R}$ and not punish.

If j punishes in any period t, in period t+1 player i switches immediately to his mediation regime R^{MI} (offer x_{ij}^{Mi}, accept all $x \geq x_{ji}^{Mi}$, not accept any $x < x_{ji}^{Mi}$ and call for mediation).

I argue why it is never rational for j to punish. First, assume that j has punished in some period t and i switches to regime R^{Mi} and offers j the share x_{ij}^{Mi} in t+1. In (i) and (ii) of this proof it is shown that this is the largest share j can enforce. Second, j's decision in t is rational if his payoff from investing c_{jj} and receiving x_{ij}^{Mi} in t+1 is not smaller than his payoff from not punishing and receiving x_{ij}^{R} in t+1, i.e. $-c_{jj} + \delta x_{ij}^{Mi} \geq \delta x_{ij}^{R}$, which is the definition of m-rationality. Hence, if m-rationality does not hold for any player, no player has reason to punish. Now, it is straightforward that $\left(x_{AA}^{R}, x_{AB}^{R}\right)$ is uniquely supported in equilibrium (remember that A moves first). ■

Appendix 3.7 (Proposition 3.8 - inefficiency with mediation)

To prove (i) I show that no player can improve by deviating from his equilibrium strategy. I restrain the analysis to the case where p^{MA} and p^{MB} hold, i.e. punishment is m-rational for both players. The equilibrium strategy for each player $i \in \{A, B\}$ are the following as long as no player has deviated:

- Player i: offer x_{ij}^{Mj}, accept all $x \geq x_{ji}^{Mj}$, not accept all $x < x_{ji}^{Mj}$ and always call the mediator until period N+1, from then on demand $\check{x}_i$, accept all $x \geq \check{x}_i$, not accept all $x < \check{x}_i$ and call the mediator.

If player i deviates from his equilibrium strategy, for example by rejecting without calling the mediator, players immediately switch to regime R^{Mi}.

If both players follow the suggested strategies, then the equilibrium payoff for both players $i \in \{A, B\}$ is $u_i = \delta^N \check{x}_i$. By any deviation A cannot enforce a payoff higher than x_{AA}^{MA}, i.e. his payoff if he immediately complies on regime R^{MA}. It is thus optimal for A not to deviate as long as $\check{x}_A \geq x_{AA}^{MA}\delta^{-N}$ which is the lower bound of $\check{x}_A$. By deviating player B's payoff is x_{AB}^{MB}, which is his payoff if he immediately accepts A's first equilibrium offer. Therefore, it is optimal for B not to deviate from the equilibrium strategy if $\check{x}_B \leq x_{AB}^{MB}\delta^{-N}$ which is tantamount to $\check{x}_A \leq 1 - x_{AB}^{MB}\delta^{-N}$, the upper bound of $\check{x}_A$.

To prove (ii), I compare the maximum inefficiency supported in equilibrium in the game with mediation and in the game without. More concretely, I compare the minimum sum of players' payoffs in equilibrium. As shown in (i), player A's minimum payoff supported in equilibrium is x_{AA}^{MA} if p^{MB} and x_{AA}^{R} if $\neg p^{MB}$, and B's is x_{AB}^{MB} if p^{MA} and x_{AB}^{R} if $\neg p^{MA}$. As shown in proposition 3.3, in the game without mediator the minimum payoff supported in equilibrium is x_{AA}^{B} for A and x_{AB}^{A} for B. Remember that $x_{AA}^{B} < x_{AA}^{MA} < x_{AA}^{R}$ and that $x_{AB}^{A} < x_{AB}^{MB} < x_{AB}^{R}$. It is now straightforward that the minimum sum of players' payoffs in equilibrium is greater with mediation, i.e. the maximum inefficiency is smaller. Further, if $\neg p^{MA} \wedge \neg p^{MB}$ then there are no inefficient equilibria in the presence of an mediator. ■

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