Inflation and Effective Shift Symmetries

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Abstract

Cosmic inflation not only sets the initial conditions for the evolution of the universe but also provides the origin of structure formation. It is hence both from a theoretical and observational point of view a highly successful paradigm. Precision measurements of the cosmic microwave background constrain inflation to be effectively driven by a single scalar field whose potential maintains an approximate and continuous shift symmetry $V \sim const.$ sufficiently far from its minimum. In effective field theory, such a shift symmetry amounts to tuning an essentially infinite number of coefficients of all higher dimensional operators involved. Here, we study different realisations of shift-symmetric inflaton potentials to examine if the amount of fine tuning can be reduced. We begin by considering a UV example and find that underlying parameters do not evade tuning as the intrinsic suppressions do not suffice. Continuing to study non-canonical dynamics, we formulate a condition on the non-canonical kinetic term equivalent to the potential shift symmetry and provide expressions for universal corrections and phenomenological imprints resulting from a broken shift symmetry. Studying modified gravity, we derive all order expressions for broken shift symmetries that allow for observationally viable inflation to occur. Finally, we study scalar field dynamics non-minimally coupled to gravity. After developing an understanding of the phenomenological implications of different types of shift symmetry breaking, we propose a mechanism that realises a sufficient intermediate shift symmetry for inflation to occur by essentially only tuning one parameter. This parameter sets the spectral index as well as the normalisation of the cosmic microwave background temperature spectrum and is found to satisfy both observational constraints while at the same time pushing all higher order terms sufficiently far away in field space.

Zusammenfasung

Kosmische Inflation bestimmt nicht nur die Anfangsbedingungen des Universums sondern liefert auch den Ursprung der Strukturbildung. Sie ist sowohl aus theoretischer als auch experimenteller Sicht ein erfolgreiches Paradigma. Präzisionsmessungen der kosmischen Mikrowellenstrahlung deuten darauf hin, dass Inflation durch ein einziges Skalarfeld verursacht wird, dessen Potential eine kontinuierliche Symmetrie $V \sim konst.$ besitzt, die hinreichend weit vom Minimum entfernt ist. In effektiver Feldtheorie bedeutet solch eine Symmetrie das Einstellen aller Koeffizienten der höher dimensionalen Operatoren, die zur Wirkung beitragen. In dieser Arbeit geht es um verschiedene Realisierungen dieser Symmetrie hinsichtlich einer möglichen Reduzierung der einzustellenden Parameter. Zuerst untersuchen wir ein UV Beispiel, in dem die beitragenden Parameter nach wie vor eingestellt werden müssen, da die aus dem UV Bereich sich ableitenden Unterdrückungen nicht ausreichen. Darauf folgend untersuchen wir nicht-kanonische Dynamiken. Wir formulieren eine Entsprechung zur genannten Symmetrie in nicht-kanonischer Sprache und leiten universelle Korrekturen und phänomenologische Effekte her, die aus einer Symmetriebrechung resultieren. In modifizierter Gravitation können wir die Koeffizienten aller Terme höherer Ordnung bestimmen, sodass Inflation im Einklang mit den Beobachtungen stattfinden kann. Schliesslich untersuchen wir Felddynamiken nicht-minimal gekoppelter Felder. Nachdem wir verschiedene Arten der Symmetriebrechung untersucht haben, entwickeln wir einen Mechanismus, der ein hinreichendes Inflationspotential realisiert, wofür effektiv nur ein Parameter eingestellt werden muss. Dieser Parameter bestimmt sowohl den spektralen Index als auch die Normalisierung des Temperaturspektrums der Hintergrundstrahlung. Ist er in Übereinstimmung mit den Messungen eingestellt, schützt er das Inflationspotential vor allen Korrekturtermen höherer Ordnung.

This thesis is based on the following publications and preprints:

- B. J. Broy, D. Roest, and A. Westphal "Power Spectrum of Inflationary Attractors", Phys. Rev. **D91**, 023514 (2015), 1408.5904
- B. J. Broy, F. G. Pedro, and A. Westphal "Disentangling the $f(R)$ - Duality", JCAP03 (2015) 029, 1411.6010
- B. J. Broy, M. Galante, D. Roest, and A. Westphal "Pole Inflation - Shift Symmetry and Universal Corrections", JHEP12 (2015) 149, 1507.02277
- B. J. Broy, D. Ciupke, F. G. Pedro, and A. Westphal "Starobinsky-Type Inflation from α' - corrections", JCAP01 (2016) 001, 1509.00024
- B. J. Broy, D. Coone, and D. Roest "Plateau Inflation from Random Non-Minimal Coupling", JCAP06 (2016) 036, 1604.05326

Contents

Introduction

The cosmological concordance model (ΛCDM) successfully describes the universe as we observe it today.[1] Among its main predictions is a cosmic microwave background (CMB) radiation originating from a primordial plasma that became transparent due to cooling caused by the expansion of space. Often coined the 'echo' of the big bang, this observation is a crucial probe of primordial dynamics. CMB experiments [4–7] beautifully confirm the predictions of the ΛCDM concordance model. Moreover, not only do they probe the predicted dynamics but also the initial conditions. In principle, initial conditions are not expected to be part of a physical theory, the theory should merely predict the correct dynamics given a certain choice of boundaries. However, if the dynamics observed are such that they may only result from extremely tuned input parameters, the initial conditions seem to require a physical mechanism on their own in order to explain the observed dynamics.

CMB experiments measure the universe to be spatially flat and isotropic with anisotropies in the CMB temperature spectrum of the order of $\Delta T/T \sim \mathcal{O}(10^{-5})$. Spatial flatness is not an attractor of ΛCDM cosmology.[2] Furthermore, the CMB comprises roughly 10^5 regions that have never been in causal contact according to ΛCDM dynamics, yet they display only the aforementioned small anisotropies. An unsatisfying choice is simply to accept the high degree of fine tuning of the initial conditions required to match observations. A more intriguing route to take is the study of the inflationary paradigm. In its simplest version, a single scalar field φ is postulated to play the role of a dynamical cosmological constant in the early universe [8–12]. This is achieved by having a potential such that the

[1] For a comprehensive review, see e.g. [1–3].

[2] We will explain this in more detail by means of expression (1.14).

field φ is dominated by its potential rather than kinetic energy. The field then has an equation of state of a cosmological constant. This causes the universe to undergo accelerated expansion which not only drives the curvature small but also ensures that all patches of the CMB sky have been in causal contact. Eventually, the inflaton φ settles in its minimum and the effective dynamical cosmological constant is hence switched off.

The CMB is a rich resource for inflationary cosmology as it not only provides the incentive to study the inflationary mechanism in the first place but also constraints inflationary models. Measurements imply that inflation is driven by a potential displaying an approximate shift symmetry $\varphi \to \varphi + \varphi_0$, i.e. $V(\varphi) \sim const.$ for some regime of φ. Interestingly, the same dynamics can be achieved when adding a hidden scalar degree of freedom in the form of a term quadratic in the Ricci scalar to the Einstein-Hilbert action.[3] While a shift-symmetric potential requires higher order terms to be sufficiently suppressed, the same holds for higher order curvature scalar terms. There are hints in the CMB data that the temperature power spectrum is slightly suppressed at large angular scales corresponding to earlier inflationary dynamics. This could signal the onset of a shift-symmetry breaking at larger fields. It is the realisation of a shift-symmetric inflaton potential and the possible breaking at larger fields that is the topic of this work.

In the first chapter after the introduction, i.e. chapter two, we study an explicit UV example. Compactified extra dimensions enter the four-dimensional (4D) effective theory as scalar fields. At first, these moduli are free fields and there hence exist numerous flat directions in field space. Employing a combination of perturbative and non-perturbative effects, we generate a potential that stabilises the overall compactification volume while at the same time allowing for a lifted but sufficiently flat direction for a combination of moduli serving as the effective inflaton upon canonical normalisation. The canonical potential can be tailored to maintain an approximate shift symmetry $V \sim const.$ for intermediate fields provided the parameters involved are appropriately tuned. As the potential terms come with different powers of volume suppression, there exists a restoration of the no-scale

[3] It is noteworthy that it was the formalism of modified gravity in which a first model of inflation was formulated [9]. Moreover, this was also the model to which cosmological perturbation theory was first applied in order to link the temperature fluctuations of the CMB to quantum fluctuations of the inflaton field [13].

property in the decompactification regime, i.e. at parametrically large volume. It is this feature that descends into the effective Lagrangian at lower volume in form of intrinsic suppressions of potentially dangerous terms. However, this mechanism is not sufficient to realise a shift symmetry for a sufficing field range. Thus, the necessity to tune parameters cannot be evaded. Allowing for tuning, we find a mini landscape of viable slow-roll potentials where inflation can occur by rolling down each side of a prolonged hilltop potential.

Since UV examples usually yield non-canonical kinetic terms as an intermediate result, we turn to an extensive study thereof in chapter three. We recall how a potential shift symmetry can be realised if the kinetic function has a pole and the non-canonical field is placed in vicinity of it. Given pole and potential minimum do not coincide, the Lagrangian has a plateau potential in canonical fields. We continue to study universal corrections to the pole structure and derive leading order corrections to the inflationary observables. Driven by the observational hint for power suppression at large angular scales, we construct a toy model where moving the kinetic pole to the complex domain naturally realises a shift-symmetry breaking at larger canonical fields. The complex pole proves to be a realisation of the universal corrections obtained before and induces an inflection point in the potential that in principle may blueshift the spectral index n_s and suppress power at low multipoles. We then outline how a perturbed pole structure could be obtained from UV scenarios.

Turning to a rival paradigm in chapter four, we focus on modified gravity or shortly $f(R)$. The main objective is to understand whether or not shift symmetry breaking that is hinted at by power loss at low-ℓ, can be realised as an extension of e.g. R^2 inflation. We begin with the study of non-integer powers of the Ricci scalar and find how logarithmic $f(R)$ theories translate to chaotic large field inflationary scenarios in the Einstein frame. Breaking an Einstein frame shift symmetry with rising exponentials, we obtain closed form and approximate $f(R)$ duals that do not display the leading order quadratic term any longer. Closed form expressions of the function $f(R)$ that break the shift symmetry at larger fields are to leading order R^n with $n < 2$ and have the quadratic term only survive as part of their series expansion. We further demonstrate that the correspondence between $f(R)$ and Einstein frame only allows for exact dual formulations of the dynamics provided the shift-symmetry breaking does not involve higher powers in the rising exponentials. Otherwise, the duality can only be cast in different regimes but not over the whole domain of the scalar degree of freedom. At last, we match modified gravity to specific UV models and find that regions of the models can only be matched separately. We modify the compactification volumes involded and conclude that whether or not a Kähler modulus scenario allowing for an exact dual $f(R)$ description can be constructed, remains to be seen.

In the fifth chapter, we study the dynamics of a scalar field non-minimally coupled to gravity. The strategy employed is allowing for higher order terms in the Jordan frame potential and investigating their effect when transformed to the Einstein frame. First, we consider toy models and focus on the phenomenology of different powers of higher order Jordan frame terms. While higher powers in principle can induce a larger suppression, their placement in the effective Lagrangian has to be tuned for the effect to be visible within the observable range of e-folds while at the same time not violating the observational bounds on the spectral index at horizon crossing. Nevertheless, lower powers have universal imprint in the CMB and studying power suppression may be understood as a tool of EFT spectroscopy.

In a second part, we generalise our ansatz to arbitrary series in the non-minimal coupling as well as the potential and find that if the field is stabilised and the non-minimal coupling is an arbitrary series expansion starting with some linear term, an Einstein frame effective shift symmetry will be realised regardless of any infinite number of higher order terms. The precise length of the inflationary plateau is governed by the non-minimal coupling strength. An analytic analysis as well as a numerical scan both pinpoint the non-minimal coupling strength to a value that at the same time sets the normalisation of the CMB temperature spectrum in accordance with PLANCK. This hence proves to be an effective one parameter ansatz satisfying all observational bounds while being robust against all higher order corrections.

In conclusion, this thesis presents different approaches to realise the effective shift symmetry in the inflaton potential hinted at by observations. While often tuning is involved, we also present a universal mechanism by invoking a non-minimal coupling. The results presented in this work have previously been published in [14–18]. While this work emphasises the fact that observations strongly favour shift symmetric inflaton potentials, a recent astronomical measurement [19] claims to observe the Hubble parameter of today H_0 to be higher than the one obtained from CMB experiments. The situation remains unresolved at time of writing, but provided H_0 were indeed larger, the spectral index n_s of the CMB temperature spectrum would be blueshifted, thus deviating from vanilla plateau inflationary predictions.[4] However, as we will argue in the conclusion, a mere blueshifting of n_s can most naturally be accommodated for within the scenarios presented, if not to say would be more natural than the current n_s measurement. We will discuss this in more depth in the final chapter of this work.

Conventions: Unless stated otherwise, we will work in units of $c\,,\hbar\,,8\pi G = 1$. The metric signature is taken to be $(+,-,-,-)$. Greek indices run from 0 to 3 whereas Roman ones run from 1 to 3. Furthermore, use of the Einstein summation convention is implied.

[4]The central panel of Figure 3 of [20] demonstrates how the value of the spectral index n_s depends on a combination of the baryon density and the reduced Hubble parameter. The spectral index increases for larger H_0.

Chapter 1

Inflation

1.1 Foundations

This chapter aims to provide the foundations and notation on which the subsequent work is built. We will begin with a review of the bare essentials of space and time and continue to describe the shortcomings of conventional hot big bang cosmology.[5] This serves as the motivation to introduce the inflationary paradigm.[6] We will review the physics driving inflation and outline extensions of the conventionally used formalism. To connect theory with observation, we will introduce the cosmic microwave background (CMB) as a powerful probe of primordial physics. The study of higher order effects within the CMB will prove to be a useful tool in examining pre-inflationary dynamics, or more generically, physics beyond the horizon.

Spacetime geometry

The universe has a length scale above which it is statistically homogeneous and isotropic. This is known as the cosmological principle and may be seen as the modern form of the Copernican principle. While local departures from homogeneity and isotropy may be identified [22, 23], the cosmological principle remains unchallenged due to its statistical nature. In fact, if we assume not to inherit any

[5]For a more comprehensive review, see e.g. [21].

[6]The foundational works of the inflationary paradigm are [9, 12].

special position in the universe, or more generically that all observers are equal, it suffices to postulate isotropy about three non-colinear points to prove that the universe must also be homogeneous.[7] Mathematically, isotropy and homogeneity respectively imply that the spatial line element may only be rescaled by a time dependent function and that the 3-space has to be maximally symmetric.[8] Thus the metric suitable to describe the universe is the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 d\boldsymbol{x}^2 \,, \tag{1.1}$$

where t denotes coordinate time labelling a spacelike hypersurface, $a(t)$ is the scale factor and $\boldsymbol{x}$ are comoving coordinates of the maximally symmetric 3-space. Considering an observer with four-velocity u^μ and the energy-momentum tensor of a perfect fluid

$$T^{\mu\nu} = (\rho + P)\, u^\mu u^\nu - P g^{\mu\nu} \,, \tag{1.2}$$

in the above spacetime with ρ and P being density and pressure respectively and requiring conservation of energy-momentum $\nabla_\mu T^{\mu\nu} = 0$, one obtains the continuity equation

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \,. \tag{1.3}$$

Furthermore, combining ansatz (1.1) with the Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} \,, \tag{1.4}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R the Ricci scalar and $T_{\mu\nu}$ the energy-momentum tensor of a perfect fluid, one obtains the Friedmann equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2} \,, \qquad H^2 + \dot{H} = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + P) \,. \tag{1.5}$$

[7]Consider three non-colinear points P, Q and R. Consider lines through P but not Q. Any inhomogeneity along the lines becomes an anisotropy when viewed from Q (contradiction). So space must be homogeneous except for the line PQ. But now view from R. This implies homogeneity from isotropy about three non-colinear points.

[8]The above mentioned scale of homogeneity decreases for earlier times. The Friedmann-Robertson-Walker metric may hence be applied to small patches when considering dynamics very close to the initial singularity.

Here, k denotes whether the universe is open ($k = -1$), flat ($k = 0$) or closed ($k = +1$). The continuity equation (1.3) and the Friedmann equations (1.5) are sufficient to describe the dynamics of the universe given the equation of state of the dominant matter content. For non-relativistic matter (MD), radiation (RD) and a constant vacuum energy density (Λ), the scale factor evolution for $k = 0$ can be summarised as:

	$P(\rho)$	$\rho(a)$	$a(t)$
RD	$\rho/3$	a^{-4}	$t^{1/2}$
MD	0	a^{-3}	$t^{2/3}$
Λ	$-\rho$	a^0	$e^{\Lambda t}$

We now turn to the causal structure of spacetime. Introducing conformal time $d\tau = dt/a(t)$ and only considering propagation along a radial direction $d\boldsymbol{x}^2 = dr^2 = (1 - kr^2)d\chi^2$, metric (1.1) reads

$$ds^2 = a^2(\tau)\left(d\tau^2 - d\chi^2\right) . \tag{1.6}$$

For a light ray, i.e. a null geodesic $ds^2 = 0$, we get the solution

$$\chi(\tau) = \tau + \text{constant} . \tag{1.7}$$

The comoving and physical distance distance a light ray travels in an interval of coordinate time hence are

$$\Delta\chi = \int_{t_1}^{t_2} \frac{dt}{a(t)} , \qquad \Delta\chi_{phys} = a(t_2) \int_{t_1}^{t_2} \frac{dt}{a(t)} . \tag{1.8}$$

The behaviour of the above integrals determines whether or not two observers can be in causal contact. Provided there exists an initial singularity which may serve as the lower limit of integration, the expressions (1.8) yield the maximum width of the past light cone (particle horizon) and hence the maximum distance of past causal interaction. Likewise, sending the upper limit of integration to infinity gives the range of possible future interactions (event horizon).

Initial conditions

Observations indicate that the CMB is nearly isotropic with fluctuations in the temperature of $\Delta T/T \sim \mathcal{O}(10^{-5})$ [5, 7, 24]. However, given the conventional hot big bang paradigm, there are roughly 10^5 regions in the CMB sky we observe today, that do not have overlapping past light cones, i.e. they have never been in causal contact. This means that there are 10^5 causally disconnected regions that nevertheless display nearly the same properties. While in principle it does not constitute a failure of a theory if it does not predict its own initial conditions, it remains highly unsatisfying to accept such a high degree of fine tuning. This is called the horizon problem.

In technical terms, the expression for comoving distance of (1.8) evaluated at CMB decoupling t_{dec}

$$\Delta\chi(t_{dec}) = \int_{t_1}^{t_{dec}} \frac{dt}{a(t)} \tag{1.9}$$

ought to allow for some $t_1 \to t_i$ such that all events of CMB photon emission share a common causal past. In other words, to solve the horizon problem we want the above integral to potentially diverge when t_1 is sent back as far as physically possible.[9] Consider recasting the above as

$$\Delta\chi(t_{dec}) = \int_{t_1}^{t_{dec}} \frac{dt}{a(t)} = \int_{a_1}^{a_{dec}} \frac{1}{\dot{a}\, a} da\,. \tag{1.10}$$

The advantage of the above substitution is that we can now consider the universal lower bound $a_1 \to 0$ instead of differentiating between the two cases for the coordinate time. The integral (1.10) diverges for $a_1 \to 0$ when

$$\frac{d}{da}\left(\frac{1}{\dot{a}}\right) < 0\,. \tag{1.11}$$

From this, we find

$$\frac{d}{da}\left(\frac{1}{\dot{a}}\right) = \frac{d}{dt}\left(\frac{1}{\dot{a}}\right)\frac{dt}{da} < 0\,, \tag{1.12}$$

which, as $\dot{a} > 0$ for an expanding spacetime, readily implies the requirement of an

[9] In case of an initial singularity, integral (1.9) ought to diverge for $t_1 \to t_i = 0$, otherwise for $t_1 \to -\infty$.

accelerating inflationary phase

$$\ddot{a} > 0\,. \tag{1.13}$$

Note that we derived this from the simple requirement of having a diverging particle horizon (1.9) at time of CMB decoupling t_{dec}. Any scale factor satisfying (1.13) can hence solve the horizon problem.

A second motivation to study inflation is the flatness problem. Consider introducing a density parameter $\rho_{crit} = 3H^2$, where $H = \dot{a}/a$. Rescaling the density of the matter component in (1.5) with the inverse of ρ_{crit} as $\Omega \equiv \rho/\rho_{crit}$ simplifies the first Friedmann equation to

$$\Omega - 1 = \frac{k}{\dot{a}^2}\,. \tag{1.14}$$

Observations [7, 24] indicate that $\Omega - 1 < \mathcal{O}(10^{-3})$. Considering that during matter and radiation dominated periods $\ddot{a} < 0$, Ω ought to diverge as the universe evolves. Current observations do imply that $\Omega - 1 < \mathcal{O}(10^{-16})$ during big bang nucleosynthesis [25]. The question hence arises why the universe was so extremely flat to begin with. Obviously, we stated earlier that $k = -1, 0, +1$ and hence could simply have been $k = 0$ from the start. Thus, the flatness problem seems to require less fine tuning than the horizon problem. Nevertheless, considering $\ddot{a} > 0$ also drives the right side of (1.14) to zero. Hence, spatial flatness is an attractor during inflation.

Furthermore, there are arguments that some UV theories predict e.g. magnetic monopoles [3] which so far have not been observed. Inflation is then invoked to drastically reduce the density of such defects that way explaining the lack of observation. More intriguingly, we will later see that inflation provides a mechanism of structure formation [13]. While this was never thought of as a problem before, the realisation that inflation seeds the primordial density perturbation makes inflation not only theoretically appealing but also highly predictive.

Overall, an inflationary phase with $\ddot{a} > 0$ may be invoked to set the initial conditions for the evolution of the universe as we observe it today. Also, it can be deduced [2, 3] that the inflationary phase has to last for roughly $60 - 70$ e-folds in order to provide the necessary initial conditions for the evolution of the universe, meaning the scale factor a has to grow by a factor of $e^{60} - e^{70}$.

Physics of inflation

We now seek a physical mechanism responsible for the inflationary phase $\ddot{a} > 0$. We begin by introducing two new parameters which have to satisfy given conditions for inflation to be realised. Rewriting (1.12) as

$$\frac{d}{da}\left(\frac{1}{\dot{a}}\right) = -\frac{1}{a}(1-\epsilon) < 0 \,, \quad \text{where} \quad \epsilon = -\frac{\dot{H}}{H^2} \,, \tag{1.15}$$

we find inflation to occur for $\epsilon < 1$. Inflation has to last for a sufficient amount of e-folds in order to set the initial conditions observed, we hence seek a measure of the change of ϵ and write the fractional change of ϵ per scale factor growth as

$$\eta \equiv \frac{d\ln\epsilon}{d\ln a} = \frac{\dot{\epsilon}}{\epsilon H} \,, \tag{1.16}$$

where the last equality follows from the definition $H = \dot{a}/a$ and hence $H dt = d\ln a$. Thus $\eta < 1$ ensures $\epsilon < 1$ for a sufficient amount of e-folds. Combining the second Friedmann of (1.5) equation with (1.15), one obtains

$$\epsilon = \frac{3}{2}\left(1 + \frac{P}{\rho}\right) < 1 \,, \quad \text{and therefore} \quad \omega \equiv \frac{P}{\rho} < -\frac{1}{3} \,. \tag{1.17}$$

Thus the physical mechanism responsible for the inflationary phase must violate the strong energy condition, i.e. has to have negative pressure. A prime example realising $\omega < -1/3$ is a cosmological constant Λ. Adding a constant term $\Lambda\, g_{\mu\nu}$ to the Einstein equations (1.4) without matter yields an energy-momentum tensor from which the equation of state $P_\Lambda = -\rho_\Lambda$ can readily be read off. This hence satisfies the constraint on ω. However by definition, a cosmological constant is non-dynamical, implying a never ending inflationary phase which is clearly at odds with observations. We hence seek a dynamical mechanism mimicking the behaviour of a cosmological constant for intermediate times.

Consider a scalar field φ minimally coupled to gravity with Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{2}\left(\partial\varphi\right)^2 - V(\varphi) \,, \tag{1.18}$$

where we assume the background spacetime to be of Friedmann-Robertson-Walker type.[10] From the expression for the energy-momentum tensor, one obtains

$$\omega_\varphi \equiv \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)}\,, \tag{1.19}$$

which satisfies $\omega_\varphi \sim -1$ when the potential dominates over the kinetic term and hence mimics a cosmological constant. Substituting for ρ in the first Friedmann equation and deriving the equation of motion for φ from Lagrangian (1.18), one has

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]\,, \qquad \ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0\,, \tag{1.20}$$

from which we obtain $dH/d\varphi = -\dot{\varphi}/2$. Inserting this into expression (1.15) and using the result to evaluate (1.16), we arrive at

$$\epsilon = \frac{1}{2}\left(\frac{\dot{\varphi}}{H}\right)^2\,, \qquad \eta = -\left(\frac{\ddot{\varphi}}{H\dot{\varphi}} + \frac{1}{H}\frac{d\ln H}{d\varphi}\right)\,. \tag{1.21}$$

To realise $\omega_\varphi \sim -1$ we now impose $\dot{\varphi}^2 \ll V(\varphi)$ and $\ddot{\varphi} \ll H\dot{\varphi}$. This simplifies expressions (1.20) and has the parameters (1.21) evaluate to

$$\epsilon \approx \frac{1}{2}\left(\frac{d\ln V(\varphi)}{d\varphi}\right)^2 \equiv \epsilon_V\,, \qquad \eta \approx \frac{1}{V(\varphi)}\frac{d^2V(\varphi)}{d\varphi^2} \equiv \eta_V\,, \tag{1.22}$$

which defines the potential slow-roll parameters. For successful inflation, one requires $\epsilon_V, \eta_V \ll 1$. Hence a scalar field has to maintain a potential satisfying these conditions to be a possible inflaton candidate. Furthermore, we can quickly deduce from the first expression of (1.20) that during slow roll, the scale factor is $a(t) \propto e^{Ht}$ which self-consistently has $\ddot{a} > 0$. The duration of inflation is quantified in terms of the number of e-folds

$$N_e(\varphi) \equiv \ln\frac{a_f}{a_i} = \int_{t_i}^{t_f} H dt \approx \int_{\varphi_i}^{\varphi_f} \frac{1}{\sqrt{2\epsilon_V}} d\varphi\,, \tag{1.23}$$

as $Hdt = d\ln a$. In the CMB, the observable window of inflation comprises the last 60 to 40 e-folds before the end of inflation, we henceforth write $N_{CMB} \sim 60$.

[10] This makes the field φ only dependent on time.

Leaving the Einstein frame

One may also consider and study a scalar field ϕ non-minimally coupled to gravity. An exemplary Lagrangian is

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \frac{1}{2}\Omega(\phi)R_J - \frac{1}{2}\left(\partial\phi\right)^2 - V_J(\phi)\,, \tag{1.24}$$

where the subscript J denotes that we refer to the above Lagrangian as being formulated in the Jordan frame as opposed to the minimally coupled case, which is called the Einstein frame. The metric can be rescaled with the non-minimally coupling or frame function

$$g^E_{\mu\nu} = \Omega(\phi)g^J_{\mu\nu}\,. \tag{1.25}$$

In order to reduce to ordinary General Relativity (GR) at low energies, one has $\Omega(\phi) > 0\,,\ \forall\phi$.[11] Lagrangian (5.1) then becomes

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{1}{2}R_E - \frac{1}{2}\left[\frac{1}{\Omega} + \frac{3}{2}\left(\frac{\partial\ln\Omega}{\partial\phi}\right)^2\right]\left(\partial\phi\right)^2 - \frac{V_J}{\Omega^2}\,. \tag{1.26}$$

If the frame function Ω is sufficiently large such that the first term of the kinetic function in the above is suppressed, canonical normalization yields

$$\Omega(\varphi) = e^{\kappa\varphi}\,, \tag{1.27}$$

where $\kappa = \sqrt{2/3}$. Provided $\Omega(\phi)$ is invertible, the potential V_J/Ω^2 can then be recast in terms of the Einstein frame field φ and a conventional analysis of e.g. the slow-roll parameters (1.22) can be done. Likewise, one may also consider scenarios where a scalar degree of freedom is hidden within an extension of the Einstein-Hilbert term R of the Lagrangian. This will be introduced in section 4.1.

[11] Note that $\Omega(\langle\phi\rangle) = c$, c = constant with $c > 0$ and $c \neq 1$ implies a rescaling of the Planck mass.

1.2 Phenomenology

After having introduced a physical mechanism able to drive inflation, we now quickly review how to connect theory with observations. While we have introduced the inflaton φ as being only dependent on time, it features quantum fluctuations

$$\varphi \to \hat{\varphi}(\tau, \boldsymbol{x}) = \bar{\varphi}(\tau) + \delta\hat{\varphi}(\tau, \boldsymbol{x}) \,, \tag{1.28}$$

where we have split the field φ into a time-dependent (given in conformal time $d\tau = dt/a$) background value and quantum fluctuations that depend on both space and time. The quantum fluctuations do not alter any of the results presented previously but eventually induce the curvature perturbation $\zeta = z^{-1}a\delta\varphi$ with $z = 2a^2\epsilon$ serving as the seed of all structure we observe in the universe today.[12]

Deriving the equation of motion for a classical $\delta\varphi$ in an FRW background is lengthy. Thus we will merely outline the steps involved.[13] Ansatz (1.28) perturbs the energy-momentum tensor (1.2) which then can be related to the Einstein equations (1.4). This yields relations between a perturbed FRW metric and the background value $\bar{\varphi}$ as well as the perturbation $\delta\varphi$. Using these relations, one may insert the perturbed FRW metric into the Klein-Gordon equation for the inflaton φ to eventually obtain an equation of motion for the rescaled variable $f = a\delta\varphi$. In Fourier space, the result reads

$$f_k'' + \left(k^2 - \frac{y''}{y}\right) f_k = 0 \,, \tag{1.29}$$

where k now labels a Fourier mode and should not be confused with the curvature parameter of (1.5), primes denote derivatives with respect to conformal time, and we have further introduced $y = (a\,\partial_t\bar{\varphi})/H$. This is the Mukhanov-Sasaki equation and resembles a simple harmonic oscillator with time-dependent mass. In de Sitter

[12]Note that while we have introduced $\delta\hat{\varphi}$ as a quantum fluctuation, the curvature perturbation ζ is given as a classical object. This is because the fluctuations undergo a quantum to classical transition, see e.g. [26].

[13]While the original result is presented in [13], an instructive derivation can be found in [2] or [21]. Issues of gauge fixing that we have neglected in our outline are thoroughly treated in the given references.

space, $y''/y \to a''/a = 2/\tau^2$ and the general solution to the above is

$$f_k(\tau) = A_k \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + B_k \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right) , \tag{1.30}$$

where we impose normalisation $|A_k|^2 - |B_k|^2 = 1$. Recalling that the inflaton perturbation is introduced as being a quantum object $\delta\hat{\varphi}$, one may quantise (1.29) and impose the Minkowski vacuum on (1.30) at the infinite past

$$\lim_{\tau \to -\infty} f_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau} . \tag{1.31}$$

This is called the Bunch-Davies vacuum.[14] We hence set $A_k = 1, B_k = 0$ and thus fix the mode function of the rescaled inflaton fluctuation $\hat{f} = a\,\delta\hat{\varphi}$ as

$$f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) . \tag{1.32}$$

To extract an observable quantity, we calculate the two-point function for the field $\delta\hat{\varphi} = a^{-1}\hat{f}$ to be

$$\frac{k^3}{2\pi^2} \langle 0|\delta\hat{\varphi}_k^\dagger \delta\hat{\varphi}_{k'}|0\rangle \to \left(\frac{H}{2\pi}\right)^2 \quad \text{for} \quad k\tau \to 0 . \tag{1.33}$$

This result shows that the power spectrum of the inflaton fluctuations freezes once they have crossed the (event) horizon. In eternal de Sitter, i.e. without inflation ending, the spectrum would remain exactly scale invariant (i.e. independent of the Fourier mode k). The inflaton fluctuation $\delta\varphi$ seeds a curvature perturbation $\zeta = z^{-1}a\delta\varphi$. It is this curvature perturbation whose power spectrum we observe today in form of the temperature spectrum of the CMB. Similarly, inflaton fluctuations also cause tensor fluctuations. The explicit form of these spectra given by

$$\Delta_s^2(k) = \frac{1}{8\pi^2} \frac{H^2}{\epsilon}\bigg|_{k=aH} , \qquad \Delta_t^2(k) = \frac{2}{\pi^2} H^2\bigg|_{k=aH} . \tag{1.34}$$

[14] While often found in literature, the Bunch-Davies vacuum must not be imposed at $k\tau \to -\infty$ but only at $\tau \to -\infty$. Given an inflationary phase which does not extend to the infinite past, this immediately poses the question whether or not the Bunch-Davies vacuum may consistently be imposed in such a scenario. As of now, a true consensus has not been established.

These spectra are evaluated for each scale at horizon crossing $k = aH$. We may immediately infer that the ratio of tensor and scalar fluctuations is $r \equiv \Delta_t^2/\Delta_s^2 = 16\,\epsilon$. As inflation is not eternal de Sitter but has slight time dependence, the time of horizon crossing for each k is different. Thus it is the time dependence of the background that induces a k dependence to the power spectrum of the curvature perturbation and the tensor fluctuations.[15] The scale dependence of $\Delta_s^2(k)$ can be quantified in terms of the spectral index n_s which is defined as

$$n_s - 1 = \frac{d \ln \Delta_s^2}{d \ln k}\,. \tag{1.35}$$

When demanding the slow-roll conditions $\epsilon_V, \eta_V \ll 1$ to hold, the quoted results of this section can be recast and summarised as

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2} \frac{V}{\epsilon_V}\bigg|_{k=aH}\,, \quad n_s = 1 + 2\eta_V - 6\epsilon_V\,, \quad r = 16\,\epsilon_V\,. \tag{1.36}$$

The above three parameters are the powerful observables of inflationary cosmology. If all three were measured, one could reconstruct the inflationary potential in the vicinity of φ_{CMB} as scale, first and second derivative of the potential were known. The latest results from the Planck collaboration (PLANCK) [7, 20, 27] are

$$\ln\left(10^{10}\Delta_s^2\right) = 3.094 \pm 0.034\,, \quad n_s = 0.968 \pm 0.006\,, \quad \text{and} \quad r < 0.1\,, \tag{1.37}$$

which - as the last measurement only yields an upper bound - leaves room for speculation about the inflationary potential. Recalling that $n_s = 1 + 2\eta_V - 6\epsilon_V$ and $r = 16\epsilon_V$, we note that the above quoted results imply a hierarchy between the two slow-roll parameters $|\eta_v|$ and $|\epsilon_V|$. Concretely,

$$|\epsilon_V| \sim \mathcal{O}(10^{-3})\,, \qquad \text{and hence} \qquad |\eta_V| \sim \mathcal{O}(10^{-2})\,. \tag{1.38}$$

It is this hierarchy that motivates us to consider a certain type of inflaton potential as we will outline in section 1.3.

[15] In eternal de Sitter spacetime, the power spectrum of the curvature perturbation diverges as $z \sim \epsilon$ vanishes. This resembles the fact that the curvature perturbation can only be defined for a slowly time varying background.

While the results (1.37) may be used to sufficiently reconstruct an inflationary potential, there are higher order effects in the CMB temperature spectrum which can indicate possible features of the inflationary evolution beyond the horizon. Namely, the running $\alpha_s = dn_s/d\ln k$ of n_s can hint at a sufficiently fast change of the slow-roll parameters implying a certain behaviour of the inflationary potential. The observable effect is the following: At large angular scales, the temperature power spectrum of the primordial curvature perturbation is given by [21]

$$l(l+1)C_l^{TT} \propto \Delta_s^2(k) \propto \left(\frac{k}{k_*}\right)^{n_s-1} , \tag{1.39}$$

where k_* is a pivot scale which we take to have left the (event) horizon 55 e-folds before the end of inflation. When the spectral index increases with decreasing k, the temperature power becomes suppressed at low multipoles. Indeed, observations [7, 20, 27, 28] measure a percent level power loss of 3-5% at scales $\ell \lesssim 40$ as compared to a spectrum with $n_s = 0.968$ and no running. This had already been noted in the first CMB measurement [5]. Cosmic variance [29] limits any measurement of the c_ℓ to $\Delta c_\ell \sim (2\ell+1)^{-1/2}$. At low-$\ell$, the Planck temperature data already reaches this limit. At smaller scales, Δc_ℓ is not yet reached experimentally everywhere, and adding future data may still lead to slight variations of the value of n_s. Adding future polarization data will provide additional independent data at low-ℓ. Moreover, future large-scale structure surveys and 21-cm tomography may provide even more modes at low-ℓ due to an increased sample volume compared to the CMB alone [30]. Thus the significance of the observed power loss may still change considerably in the future. We now review how to study power loss at low-ℓ numerically [30–35]. To obtain power loss within the first observable e-folds, we require n_s to fall sufficiently fast. We thus parametrise the scalar field equation in terms of the number of e-folds $N_e = \ln(a/a_{end})$ with $a = a_{end}e^{Ht}$, where the precise value of a_{end} depends on the details of reheating and shall not concern us any further.[16] One has

$$\frac{\partial^2\varphi}{\partial N_e^2} + \frac{1}{2}\left[6 - \left(\frac{\partial\varphi}{\partial N_e}\right)^2\right]\left(\frac{\partial\varphi}{\partial N_e} + \frac{\partial \ln V}{\partial\varphi}\right) = 0\,. \tag{1.40}$$

In slow roll, $\partial^2_{N_e}\varphi \ll \partial_{N_e}\varphi$ and thus $\partial_{N_e}\varphi \approx -\partial_\varphi \ln V$. Hence (1.40) may be solved numerically to give $\varphi(N_e)$. We can then evaluate the slow-roll parameters ϵ_V and η_V on the numerical solution to investigate whether n_s falls off sufficiently fast. At last, to identify N_e with the wave number k, we recall that a mode k exits the horizon when $k = a_k H_k$, where H_k denotes the inverse event horizon during inflation and a_k is the size of the scale factor at horizon exit. Thus

$$k = a_k H_k = a_{end} e^{N_e} H_k\,, \tag{1.41}$$

where $N_e < 0$. Rearranging, we find

$$N_e(k) = \log\left(\frac{k}{a_0 H_0}\right) - \log\left(a_{end}\frac{H_k}{H_0}\right)\,, \tag{1.42}$$

in terms of Hubble parameter of today. The second term on the right hand side is $\sim$ 62, the exact value again depending on the details of reheating and the inflationary energy scale. From expression (1.42) we find that the scale $k_* = 0.05$ Mpc^{-1} left the horizon at $N_e \sim -55$. Having a relation between wave number k and number of e-folds N_e, we may investigate (1.39) with n_s being dependent on k through N_e. To obtain the percentage of suppression $\%(N_e)$, we can then compare $\Delta_s^2(k)$ at the onset of observable e-folds to a spectrum with no running of n_s.

If the running of the spectral index n_s is also tractable analytically, the above procedure does not have to be invoked. Recalling $n_s - 1 = d\ln\Delta_s^2/d\ln k$, it then suffices to study the expression

$$\left.\frac{\delta\Delta_s^2(\delta n_s)}{\Delta_s^2}\right|_{N_e+\Delta N_e}^{N_e} = \int\limits_{N_e+\Delta N_e}^{N_e} \delta n_s\left(N_e\right)\,, \tag{1.43}$$

[16] We approximate inflationary spacetime as de Sitter space, thus $a \propto e^{Ht}$ with H being the Hubble parameter during inflation. N_e is taken negative throughout inflation and becomes zero at the end of inflation. As a shorthand, $n_s(62)$ means n_s at $N_e = -62$.

provided the change δn_s of the spectral index can be cast in terms of the number of e-folds N_e. We will make use of this in section 3.4.

1.3 Shift symmetry primer

Having outlined the fundamentals of the inflationary paradigm and observational manifestations thereof, we know turn to classify which kind of inflaton potentials are currently favoured from an experimental point of view. We begin by recalling hierarchy (1.38), namely $|\eta_V|$ being roughly one order of magnitude larger than $|\epsilon_V|$. A possible ansatz naturally satisfying this hierarchy is assuming the potential to approximate a constant value at large fields as

$$V(\varphi) \sim 1 - e^{-\kappa\varphi} + \ldots \to \text{const.} \quad \text{for} \quad \varphi \to \text{ large}\,, \tag{1.44}$$

where the dots denote subleading terms in the region $V \to$ const. It can be shown that for such a potential, the slow roll parameters scale as

$$\eta_V = -\frac{1}{N_e} + \ldots\,, \qquad \text{and} \qquad \epsilon_V = \frac{1}{2\,\kappa^2\,N_e^2} + \ldots\,, \tag{1.45}$$

which readily satisfies the observational hierarchy constraint for $N_e \sim N_{CMB}$. We hence conclude that a field with a potential of plateau type (1.44) is a natural inflaton candidate. In other words, the potential energy driving inflation has to have, at least for intermediate fields, an approximate and continuous shift symmetry $\varphi \to \varphi + \varphi_0$. The inflaton potential not only has to mimic the equation of state of a cosmological constant, but also the form of the potential energy, despite a weak breaking to ensure a graceful exit of inflation.

Considering an arbitrary and minimally coupled scalar field

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(\varphi)\,, \tag{1.46}$$

we recall that integrating out heavy fields or generally radiative contributions lead us to generically expect an infinite series of higher order corrections to $V_0(\varphi)$, containing in particular pieces of the form

$$\Delta V = V_0(\varphi) \sum_{n\geq 1} c_n \frac{\varphi^n}{M_{Pl}^n} \,. \tag{1.47}$$

In the spirit of Wilsonian effective field theory (EFT) we assume $c_n = \mathcal{O}(1)\ \forall\ n$. Then corrections to the η_V-parameter will generically be expected as

$$\Delta\eta_V|_{\varphi=\Delta\varphi} = \sum_{n\geq 2} c_n n(n-1)\frac{\Delta\varphi^{n-2}}{M_{Pl}^{n-2}} \gtrsim 1 \tag{1.48}$$

and are readily greater than unity as soon as $\Delta\varphi \gtrsim 1\, M_{Pl}$, i.e. when the field is trans-Planckian. We thus generically expect terms spoiling a controlled inflationary regime and inflation mostly never acquires a sufficient amount of e-folds to solve the horizon and flatness problems. An inflationary model that successfully addresses the problems inflation needs to solve while at the same time being in accordance with latest observations, hence must first address the above described η_V-problem. Controlling such trans-Planckian field excursions effectively requires

$$c_n \lesssim \eta_0 \frac{1}{n^2}\frac{M_{Pl}^{n-2}}{\Delta\varphi^{n-2}} \lesssim 1 \tag{1.49}$$

to be put in either by hand or through some mechanism from the UV. The potential needs to be tuned flat if no protective UV symmetry is at hand.

While the η_V-problem as such is not defined in the framework of modified gravity, the same shift symmetry requirement is evident in the structure of $f(R)$-gravity versions producing inflation models close to the $R+R^2$ Starobinsky model. If one allows for arbitrary powers of the Ricci scalar in the Einstein-Hilbert action

$$f(R) = R + \frac{c_2}{M_{Pl}^2}R^2 + \sum_{n\geq 3}\frac{c_n}{M_{Pl}^{2n-2}}R^n \tag{1.50}$$

with $c_2 \gg 1$, there are stringent limits on higher order terms from solar system requirements, i.e. weak field limits of GR and constraints on 5^{th} force measurements. Combining those limits with the $f(R)$ properties for a successful inflationary phase, i.e. the existence of an enhanced R^2 term, one may effectively cast the shift symmetry structure as

$$\frac{c_n}{c_2^{n-1}} \ll \frac{1}{c_2^{n-1}} \ll 1 \quad \forall\, n \geq 3\,. \tag{1.51}$$

Then the exponential approach to a shift-symmetric plateau potential

$$V(\varphi) \sim 1 - e^{-\sqrt{2/3}\varphi} + \ldots \tag{1.52}$$

for the associated canonically normalised scalar φ dominates the resulting scalar potential at least for intermediate fields relevant during horizon exit of CMB scales. Condition equation (1.51) again marks the pattern of an effective weakly broken shift symmetry.

Furthermore, a breaking of the shift symmetry at large field values corresponding to the onset of observable e-folds can induce an inflection point in the inflationary potential. Inflection points in an approximately shift-symmetric potential mostly maintain $d^2V/d\varphi^2 > dV/d\varphi$ but $d^2V/d\varphi$ tends to be large. Hence the spectral index is very sensitive to inflection points. This can manifest itself in a large running α_s of the spectral index and thus lead to the aforementioned power-suppression at low angular multipoles. Again, as we naturally expect the shift symmetry to be broken for some field values, the effect of power loss can be readily accommodated for provided the breaking appears in the vicinity of N_{CMB}.

It is the topic and task of this thesis to study different realisations of effective shift-symmetries in order to realise inflation. We will begin by embedding inflationary dynamics within a UV framework. Here, the shift symmetry derives mainly from tuning the parameters involved. In a second approach, we consider non-canonical inflation. We derive conditions on the pole structure of the kinetic function and describe the shift-symmetry breaking in a universal way. Turning to modified gravity, we give $f(R)$ duals corresponding to intermediate Einstein frame shift-symmetries and are thus able to determine the coefficients in an all order expansion of the function $f(R)$. At last, we will provide a minimalist and universal mechanism to realise effective shift-symmetries. While the other approaches always invoke some amount of tuning, this mechanism effectively only depends on one parameter for which two independent observational indications exist.

Chapter 2

UV example

In this chapter, we describe the realisation of an effective shift symmetry $V \sim$ *const*. of the inflaton to drive inflation within a UV framework [17]. As we will consider a part of the UV theory where no inherent shift-symmetries exits, constructing observationally viable inflaton potentials will amount to balancing and tuning the parameters and coefficients of the higher dimensional operators involved. We will be working in the Large Volume Scenario (LVS) of IIB Calabi-Yau flux compactifications [36–39]. Depending on the specific geometry under consideration, the F-Term scalar potential for the Kähler moduli generated by non-perturbative effects leaves flat directions for certain combinations of moduli. It is these flat directions in field space that are of interest for inflationary model building. Different perturbative effects may be used to lift the flat directions and hence to generate potentials capable of driving observationally viable slow-roll inflation. We will be employing a combination of string loop effects and recently computed higher derivative α'-corrections. Inflation is then driven by a Kähler modulus whose inflationary potential arises from the latter correction, while we use the inclusion of string-loop effects only to ensure the existence of a graceful exit and Minkowski minimum. The effective shift symmetry required and thus control over higher corrections relies in part on tuning underlying microscopic parameters by hand, and in part on intrinsic suppressions. The intrinsic part of control arises as a leftover from an approximate effective shift symmetry at parametrically large volume. Precisely, the potential recovers its no-scale property at infinite volume.

The rest of the chapter is structured as follows; we start with a review of the bare essentials of the Large Volume Scenario. We continue to quote the perturbative corrections we will employ to generate the inflaton potential in section 2.2 and describe the resulting inflationary dynamics in section 2.3. Following a discussion of theoretical parameter bounds, we turn to the extraction of inflationary observables in section 2.4 and continue to significantly constrain the parameter space by considering higher order observables.

2.1 Large Volume Scenario in a nutshell

We begin with a short review of the Large Volume Scenario (LVS) [36–39]. While this was originally studied with moduli stabilisation in mind [36], it was quickly discovered [39–41] that, for certain geometries, flat directions within a compact field space remained; or in other words that not all moduli involved we readily stabilised. It is these flat directions that are of interest in order to realise slow-roll inflation.

The results we will primarily base our construction on are the large volume limit of the low-energy 4D $\mathcal{N} = 1$ - effective action of type IIB Calabi-Yau-orientifold compactifications with background fluxes [42,43] including the leading order $(\alpha')^3$-corrections to the bulk fields [42] and non-perturbative corrections from gaugino condensation on wrapped D7-branes or from Euclidean wrapped D3-branes. The background fluxes stabilise the dilaton and complex structure moduli so that, once having replaced these fields with their respective minima in the effective Lagrangian, the theory is specified by the Kähler and Superpotential

$$K = -2\log\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) \qquad \text{and} \qquad W = W_0 + \sum_i A_i e^{-a_i\tau_i}\,, \tag{2.1}$$

where $\hat{\xi}$ parametrises the leading order α'-corrections [42], W_0 is the Gukof-Vafa-Witten superpotential and A_i, a_i can be seen as constants depending on the specific mechanism generating the non-perturbative contributions. The volume modulus

$\mathcal{V}$ describes the total volume of the compactification geometry and is given as

$$\mathcal{V} = \frac{1}{6} k_{ijk} t^i t^j t^k \,, \tag{2.2}$$

with k_{ijk} denoting the triple intersection numbers and t^i the two-cycle volumes. The variables of 4D $\mathcal{N} = 1$ theories however are the four-cycle volumes which descend from the real parts of the Kähler coordinates as

$$\frac{1}{2}\left(T_i + \bar{T}_i\right) \equiv \tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} k_{ijk} t^j t^k \,, \tag{2.3}$$

where the last equality relates two- and four-cycle volumes.

In the original work [36], the compactification volume was taken to be controlled by a single four-cycle. When a blow-up cycle is added for which non-perturbative effects are assumed to exist, the resulting F-term scalar potential features a minimum for the volume modulus at exponentially large volume. While such a scenario suffices in terms of moduli stabilisation, no flat directions of phenomenological interest remain in this minimal set-up. Kähler modulus inflation [40] went beyond the minimal set-up to include an additional blow-up four-cycle. One blow-up then served to stabilise the volume while a sub-leading dependence of the F-term scalar potential on the remaining blow up was used to effectively lift the leading order flat direction of the additional blow-up. This realised a potential viable for slow-roll inflation. Yet string-loop effects quickly introduced the η-problem to that set-up.

We will consider a compactification geometry for which the volume is stabilised at exponentially large values and a flat direction exists which can be lifted in a controlled way. Adding a blow-up cycle to the $K3$-fibered threefold $\mathbb{CP}^4[1,1,2,2,6]$ which has $h^{1,1} = 2$, the volume in terms of the four-cycles is of the form

$$\mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_2 t_3^2 = \alpha\left(\sqrt{\tau_1}\tau_2 - \gamma \tau_3^{3/2}\right) \quad \text{with} \quad \lambda_i = \text{const.}\,, \tag{2.4}$$

where τ_1 is associated with the volume of the $K3$-fibre, τ_2 controls the overall volume, τ_3 denotes the blow-up and we identify $\alpha = 1/(2\sqrt{\lambda_1}), \gamma = \frac{2}{3}\sqrt{\lambda_1/(3\lambda_2)}$. Large volume implies that $\tau_1, \tau_2 \gg \tau_3$. This hierarchy shuts off all non-perturbative contributions to the superpotential except those for the blow-up τ_3; one has

$$W \to W = W_0 + A_3 e^{-a_3\tau_3}\,. \tag{2.5}$$

Thus the resulting F-term scalar potential has the form

$$V^{LVS}(\mathcal{V},\tau_3) = g_s\left(\frac{8a_3^2A_3^2}{3\alpha\gamma}\frac{\sqrt{\tau_3}}{\mathcal{V}}\mathrm{e}^{-2a_3\tau_3} - 4W_0a_3A_3\frac{\tau_3}{\mathcal{V}^2}\mathrm{e}^{-a_3\tau_3} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3}\right) \tag{2.6}$$

This does not depend on τ_1 and τ_2 but only on the particular combination of the two controlling the overall volume. V^{LVS} has minima to stabilise

$$\langle\tau_3\rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma}\right)^{2/3}\,,\quad \langle\mathcal{V}\rangle = \frac{3\alpha\gamma}{4a_3A_3}W_0\sqrt{\langle\tau_3\rangle}\mathrm{e}^{a_3\langle\tau_3\rangle}\,, \tag{2.7}$$

which demonstrates that the volume is driven towards an exponentially large value. Note how this construction manages to fix two of the three degrees of freedom involved; τ_3 is stabilised directly and a combination of τ_1, τ_2 is fixed through the volume $\mathcal{V}$. The remaining scalar degree of freedom may hence play the role of the inflaton. Note that τ_1 is massless at tree level. Therefore, we expect corrections that lift the flat direction to induce terms which are subleading with respect to V^{LVS}, i.e. we expect τ_1 to remain the lightest modulus and hence to be a prime inflaton candidate. In what follows, we will lift the flat direction of τ_1 with new α'-corrections and string loop-effects.

2.2 Perturbative corrections and inflation

We now turn our attention to the perturbative corrections we will employ to lift the previously flat directions in field space.

2.2.1 Higher-derivative corrections

First, we will consider a recently computed α'-correction [44]. This correction was derived by matching higher derivative terms in $\mathcal{N}=1$ superspace to Kaluza-Klein reduced α'-corrections from ten dimensions (10D). While overall, there are additional two- and four-derivative terms in the effective action, we will only focus on the contribution to the F-term scalar potential, which reads

$$V_{(1)} = -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \Pi_i t^i \,, \tag{2.8}$$

where $\hat{\lambda} = \lambda(\alpha')^3 g_s^{-3/2}$ with λ being an undetermined combinatorial constant. The Π_i are integer numbers encoding geometric information and always have the same sign.

In a first step, we consider the inflationary dynamics of the LVS F-term scalar potential with the additional α'-correction, i.e.

$$V(\tau_1) = V^{LVS} + V_{(1)} \,. \tag{2.9}$$

For the geometry (2.4), the correction reads

$$V_{(1)} \simeq -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right) \,, \tag{2.10}$$

where we have omitted τ_3 dependent terms; this is because correction (2.8) is $1/\mathcal{V}$ suppressed with respect to (2.6), we thus do not expect the stabilisation of τ_3 to be significantly altered. Moreover, while there may be subleading corrections to the vev of τ_3, the blow-up modulus will certainly remain stabilised and hence will not be dynamical during inflation. Thus, we write the potential as

$$V(\tau_1) = V^{LVS}(\langle\tau_3\rangle, \langle\mathcal{V}\rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle\mathcal{V}\rangle^4} \left(\Pi_1 \frac{\langle\mathcal{V}\rangle}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right) \,. \tag{2.11}$$

Before we turn to the features of the above potential, let us consider canonical normalisation of the particular combination of moduli that will take the role of the inflaton in our subsequent discussion. Recalling that the complexified Kähler moduli are $T_i = \tau_i + i b_i$, the kinetic part of the Lagrangian is

$$\mathcal{L}_{kin} = K_{i\bar{j}} \partial_\mu T_i \partial^\mu \bar{T}_j = \frac{1}{4} \frac{\partial^2 K}{\partial\tau_i \partial\tau_j} \left(\partial_\mu \tau_i \partial^\mu \tau_j + \ldots \right) \,. \tag{2.12}$$

Hence upon replacing $\tau_2 \to \tau_2(\tau_1, \tau_3, \mathcal{V})$ via the expression for the compactification

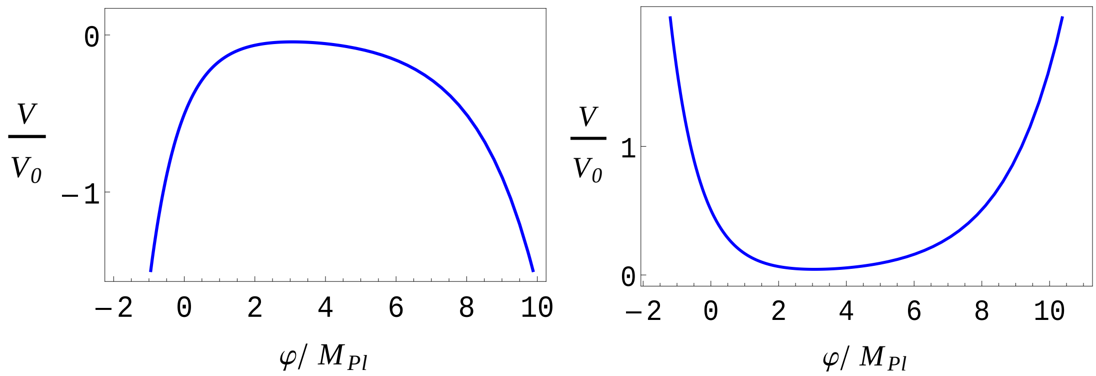

Figure 2.1: **Left:** *Exemplary potential* (2.15) *where* $\hat{\lambda}$ *and* Π_i *share the same sign. While this is observationally viable plateau inflation, the potential is unbounded from below.* **Right:** *When* λ, Π_i *have opposite sign, the resulting potential can drive inflation and ensures a graceful exit and hence the start of reheating. However, the inflationary observables for this potential are excluded by observations as demonstrated by expression* (2.18)*.*

geometry, the kinetic terms are

$$\mathcal{L}_{kin} \supset -\frac{3}{8\tau_1^2}(\partial_\mu \tau_1 \partial^\mu \tau_1) + \frac{1}{2\tau_1 \mathcal{V}}(\partial_\mu \tau_1 \partial^\mu \mathcal{V}) - \frac{1}{2\mathcal{V}^2}(\partial_\mu \mathcal{V} \partial^\mu \mathcal{V}) + \dots \quad . \tag{2.13}$$

The canonical inflaton is then defined as

$$\tau_1 = e^{\kappa\varphi} \qquad \text{and hence} \qquad \varphi = \kappa^{-1} \log \tau_1 \, , \tag{2.14}$$

with $\kappa = 2/\sqrt{3}$. Potential (2.11) then becomes

$$V(\varphi) = V^{LVS}(\langle\tau_3\rangle, \langle\mathcal{V}\rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle\mathcal{V}\rangle^4} \left(\Pi_1 \langle\mathcal{V}\rangle e^{-2/\sqrt{3}\,\varphi} + \Pi_2 \lambda_1^{-1/2} e^{\varphi/\sqrt{3}} \right) \, . \tag{2.15}$$

If $\hat{\lambda}$ and both Π_i have the same sign, the above potential features a global maximum and is unbounded from below. While such a Hilltop potential can drive observationally viable slow-roll inflation, it remains highly unsatisfying that the inflaton stays eventually unstabilised after inflation. If $\hat{\lambda}$ and both Π_i have the opposite sign, the potential manages to stabilise the inflaton due to the minimum as well as to provide an inflationary phase. In this case however, inflation is effectively driven by a single rising exponential

$$V(\varphi) \sim e^{\nu\varphi}\,, \tag{2.16}$$

for which it is readily verified that

$$n_s = 1 - \nu^2 \qquad \text{and} \qquad r = 8\,\nu^2\,. \tag{2.17}$$

For potential (2.15), one has $n_s = 2/3, r = 8/3$ which is clearly incompatible with PLANCK [20]. Moreover, eliminating the coefficient ν from the above set of equations, the resulting line

$$n_s = 1 - \frac{r}{8} \tag{2.18}$$

in an (n_s, r) plot is never inside the bounds set by PLANCK, i.e. no coefficient ν in the exponent of (2.16) can remedy this situation. Figure 2.1 depicts the explained scenarios.

2.2.2 String-loop effects

We now turn to a second class of perturbative corrections, namely string-loop effects [45]. These perturbative corrections arise from the exchange of closed strings carrying Kaluza-Klein momentum and winding strings between stacks of branes. While explicit results are lacking, their general form is conjectured to be

$$\delta K^{KK}_{(g_s)} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{C_i^{KK}(a_{ij}t^j)}{\mathcal{V}} \qquad \text{and} \qquad \delta K^{W}_{(g_s)} \sim \sum_{i=1}^{h^{1,1}} \frac{C_i^{W}(a_{ij}t^j)^{-1}}{\mathcal{V}}\,, \tag{2.19}$$

where the first term denotes the contribution from the exchange of closed strings between D3 and D7 branes and the latter the exchange of winding strings between D7 branes. While a_{ij} are combinatorial constants, C_i^{KK}, C_i^W are functions of the complex structure moduli. It was shown in [46] that without fine-tuning, one expects

$$C_i^{KK} \simeq C_i^W \simeq \mathcal{O}\left(\frac{1}{128\pi^4}\right)\,, \tag{2.20}$$

which we will assume for the following discussion. The string-loop corrections to the scalar potential for the compactification geometry (2.4) are [39]

$$\delta V_{(g_s)} \simeq \frac{g_s|W_0|^2}{\mathcal{V}^2}\left(g_s^2\frac{(C_1^{KK})^2}{\tau_1^2}+2g_s^2(\alpha C_2^{KK})^2\frac{\tau_1}{\mathcal{V}^2}\right) . \tag{2.21}$$

Note that we omitted the contribution to the blow-up τ_3 as we take τ_3 to be fixed during inflation by the leading order LVS potential. Furthermore, we purposefully did not include corrections from winding strings. This is the first occurrence of an explicit tuning; as a winding mode contribution to the four-cycle τ_1 comes with the same g_s and $\mathcal{V}$ dependence as correction (2.10), we have to

- either *tune* the coefficient C_1^W small by making assumptions about the stabilisation of the complex structure moduli,
- or simply to postulate a brane configuration where the D7 branes only wrap those four-cycles associated with τ_2, τ_3 and where therefore winding-mode contributions are absent,

for the winding-mode not to spoil the inflationary dynamics driven by (2.10). To ease the subsequent analysis, we opt for the latter case.

The string-loop corrections in (2.21) are suppressed by additional powers of $g_s^{5/2}$ with respect to $V_{(1)}$ in (2.10). The typical size of the topological numbers and the constant λ was inferred in [44] to be $\Pi_i \sim \mathcal{O}(10\ldots100)$ and $|\lambda| \sim \zeta(3)/(16\pi^3)$. Combining this with the estimate (2.20) we find that

$$|C_1^{KK}|^2 \sim |C_2^{KK}|^2 \ll |\lambda||\Pi_i| . \tag{2.22}$$

Thus, for moderately small $g_s \lesssim 10^{-1}$ and $W_0 \gtrsim 1$ the string-loop corrections are suppressed with regard to $V_{(1)}$ for some domain of τ_1. However, for sufficiently small as well as large τ_1 the contributions $\delta V^{KK}_{(g_s),1}$ and $\delta V^{KK}_{(g_s),2}$ will become important and eventually dominate over the terms in $V_{(1)}$. Since $\delta V^{KK}_{(g_s),1}$ and $\delta V^{KK}_{(g_s),2}$ are strictly positive, the potential will thus globally be bounded from below. In the intermediate τ_1 regime the string-loop effects will remain subleading compared to $V_{(1)}$. Hence, we now do not need to worry about instabilities of the potential and the $V_{(1)}$-correction may be invoked to drive inflation in some intermediate regime of the modulus τ_1.

The resulting scalar potential for the modulus τ_1 hence reads

$$V(\tau_1) = V^{LVS} + V_{(1)} + \delta V_{(gs)} + \delta_{up} \,, \tag{2.23}$$

where we have included an uplifting term $\delta_{up} = \epsilon/\mathcal{V}^p$ with $p = 1 \ldots 3$, which may be induced by numerous mechanisms (see e.g. pp. 134 in [47]). Upon canonical normalisation and having absorbed the uplift term into the τ_1-independent V^{LVS}, the full inflationary potential reads

$$V(\varphi) = V^{LVS}_{\delta_{up}} + V_0 \left(-\, \mathcal{C}_1 e^{-2/\sqrt{3}\varphi} - \mathcal{C}_2 e^{\varphi/\sqrt{3}} + \mathcal{C}_1^{loop} e^{-4/\sqrt{3}\varphi} + \mathcal{C}_2^{loop} e^{2\sqrt{3}\varphi} \right) \,, \tag{2.24}$$

where we have defined

$$V_0 = g_s^2 \frac{|W_0|^4}{\mathcal{V}^4} \,, \quad \mathcal{C}_1 = \hat{\lambda}\, \Pi_1 \mathcal{V} > 0 \,, \quad \mathcal{C}_2 = \hat{\lambda}\, \Pi_2 \lambda_1^{-1/2} > 0 \,,$$

$$\mathcal{C}_1^{loop} = \frac{\mathcal{V}^2}{|W_0|^2} g_s (C_1^{KK})^2 > 0 \,, \quad \mathcal{C}_2^{loop} = \frac{2 g_s}{|W_0|^2} (\alpha C_2^{KK})^2 > 0 \,. \tag{2.25}$$

While the $\mathcal{C}_i^{loop}$ are always positive, we have now explicitly chosen scenarios where $\hat{\lambda}$ and the Π_i have the same sign to fix also the $\mathcal{C}_i > 0$; the rationale behind this is that if the $\mathcal{C}_i$ were allowed to be negative, then inflation will effectively be driven by potential (2.16), regardless of the inclusion of further perturbative corrections. As we have explicitly shown that inflation driven by a positive exponential is not in agreement with observations, we are hence - by experiment - directed only to consider scenarios in which $\hat{\lambda}\, \Pi_i > 0, \forall i$.

Realising observationally viable inflation within the Kähler moduli sector by invoking known perturbative corrections thus means balancing the exponentials of (2.24), i.e. tuning the parameters in the definition of the coefficients (2.25). Provided these can be set to suitable values, configurations of falling and rising exponentials may then provide the approximate and continuous shift symmetry $V \sim$ *const.* in the inflaton potential to drive inflation compatible with PLANCK.

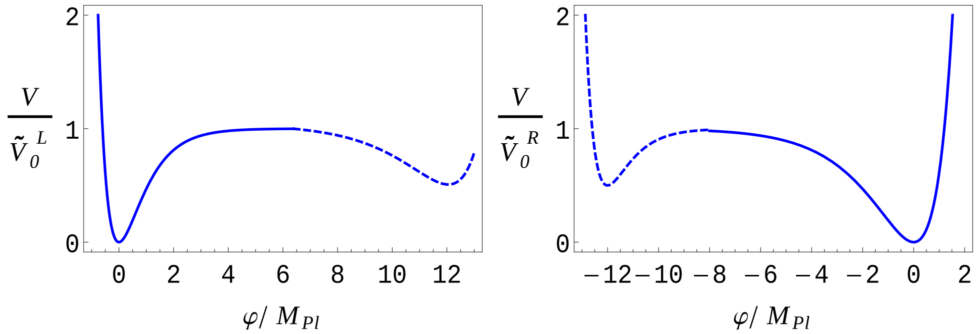

Figure 2.2: **Left:** *Potential* (2.24)*, where the parameters have been chosen such that the inflaton rolls to the left. The thick line is effectively captured by expression* (2.40)*. The dashed line is the second falling exponential. Both falling exponentials are eventually stabilised by string loops.* **Right:***The thick blue line resembles potential* (2.32)*. Now, inflation occurs for growing fibre.*

2.3 Inflationary dynamics

As observations require to choose

$$\mathcal{C}_1 = \hat{\lambda}\,\Pi_1\mathcal{V} > 0\,, \quad \mathcal{C}_2 = \hat{\lambda}\,\Pi_2\lambda_1^{-1/2} > 0\,, \tag{2.26}$$

for inflation not to be driven by a single rising exponential (2.16), we will eventually have two minima towards which the inflaton may roll. The relevant terms of the potential during inflation are

$$V_{inf} \sim -\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2\sqrt{\tau_1} \tag{2.27}$$

in non-canonical fields. Without the inclusion of string-loops, each term would quickly drive the potential to large negative values. Hence, as depicted in Figure 2.2 the set-up allows for two minima, one of which has to be tuned Minkowskian through the uplift. We hence seek only to drive inflation with one of the terms in potential (2.27), namely the one which in combination with string-loops and uplift has the inflaton settle in the Minkowski minimum. Hence, we have a mini-landscape of inflation being possible by rolling to the left or to the right. Thus, we want the observable ~ 60 e-folds of inflation to occur when the terms in V_{inf}

are not of the same order. They are of equal order of magnitude for

$$\tau_1^c \sim \left(\frac{\mathcal{C}_1}{\mathcal{C}_2}\right)^{2/3} . \tag{2.28}$$

Now, depending on whether inflation is occurring by the inflaton rolling to the left or right as depicted in Figure 2.2, we have to ensure that the plateau appears around values $\tau_1^c > \tau_1$ or $\tau_1^c < \tau_1$. In order to have the potential sufficiently flat, there are bounds on the $\mathcal{C}_i$ and $\mathcal{C}_i^{loop}$ coming from the running of the spectral index, which have to be obeyed when considering viable models of inflating to the right or the left. These are subject of subsection 2.4.2 and will be found in expression (2.56) and (2.60) respectively.

In what follows, we will in a first step work in non-canonical fields to establish the bounds on the parameter space and turn to canonical fields in a second step in order to confront the parameters with constraints from observations.

2.3.1 Inflation to the right

We first consider the inflaton rolling to the right, i.e. we have inflation driven by the $\mathcal{C}_2$-term and the stabilisation ensured by the $\mathcal{C}_2^{loop}$-term. The inflaton has to be initially placed on the right side of the stationary point τ_1^c and to the left of the minimum. The leading terms in the potential hence are

$$V_{inf}^R \sim V_0 \left(-\mathcal{C}_2\sqrt{\tau_1} + \mathcal{C}_2^{loop}\tau_1\right) , \quad \tau_1 > \tau_1^c . \tag{2.29}$$

Uplifting the above by its value at the minimum, we obtain

$$V_{inf}^R = \tilde{V}_0^R \left(1 - \frac{\beta_R}{2}\sqrt{\tau_1}\right)^2 , \quad \beta_R \equiv \frac{4\,\mathcal{C}_2^{loop}}{\mathcal{C}_2} = 2\,\frac{g_s^{5/2}(C_2^{KK})^2}{\lambda\,|W_0|^2\Pi_2} , \quad \tilde{V}_0^R = \frac{V_0}{\beta_R} . \tag{2.30}$$

For V_{inf}^R to exhibit an approximate shift symmetry $V_{inf}^R \sim const.$, we require $\beta_R\sqrt{\tau_1} \ll 1 \leftrightarrow \tau_1 \ll \beta_R^{-2}$, i.e. we need $\beta_R \ll 1$ to have a plateau at $\tau_1 \gg 1$. Recalling expressions (2.25), $\beta_R \ll 1$ may may be satisfied and hence ensures an inflationary plateau. Further, as we want the minimum to be on the right side of

τ_1^c in (2.28), we require

$$\tau_1^{min} > \tau_1^c \quad \Rightarrow \quad \mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{\mathcal{C}_1}\right)^{1/3}, \tag{2.31}$$

which can easily be satisfied using (2.22). Turning to canonical variables via (2.14) and shifting the canonical scalar φ by the vacuum expectation value $\varphi \to \varphi - 2\,\kappa^{-1}\log(\beta_R/2)$, we arrive at the effective inflaton potential

$$V_{inf}^R(\varphi) = \tilde{V}_0^R \left(1 - e^{\frac{\kappa}{2}\varphi}\right)^2, \tag{2.32}$$

where we recall that $\kappa = 2/\sqrt{3}$. Having canonically normalised, we now confront our set-up with observational bounds (1.33) arising from the normalisation of the scalar density perturbations [20, 48]

$$\frac{1}{8\pi^2}\frac{H^2}{\epsilon_V}\bigg|_* \sim 2.2 \times 10^{-9}, \quad \text{and thus} \quad \left(\frac{1}{8\pi}\frac{V_{inf}}{\epsilon_V}\right)^{1/4}\Bigg|_* = 0.027\, M_{Pl}\,, \tag{2.33}$$

where the star denotes evaluation at horizon exit which we assume to occur 55 e-folds before the end of inflation. Thus, when inflating to the right, we have

$$\tilde{V}_0^R = 8\pi\, 0.027^4\, M_{Pl}^4\, \epsilon_V\big|_* = 5.7 \times 10^{-9}\,, \tag{2.34}$$

where we have set $M_P = 1$ in the last equality. This hence sets

$$\tilde{V}_0^R \equiv \frac{g_s^2 |W_0|^4}{\mathcal{V}^4}\frac{\mathcal{C}_2^2}{4\,\mathcal{C}_2^{loop}} \sim \lambda^2\,\frac{|W_0|^6}{\mathcal{V}^4} g_s^{-2} (C_2^{KK})^{-2} \overset{!}{=} 5.7 \times 10^{-9}\,, \tag{2.35}$$

which is in accordance with natural choices for g_s, $\mathcal{V}$ and W_0. We thus summarise the demands and resulting bounds for a viable inflationary regime as follows:

demand	*resulting bound*
plateau at $\tau_1 \gtrsim 1$	$\beta_R \ll 1$
$\tau_1^{min} > \tau_1^c$	$\mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{\lvert\mathcal{C}_1\rvert}\right)^{1/3}$
COBE	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4} g_s^{-2} (C_2^{KK})^{-2} \sim 5 \times 10^{-9}$

If these requirements are met, inflation is fully captured and described by potential (2.32). Note that this scenario corresponds to an increasing fibre at fixed compactification volume.

2.3.2 Inflation to the left

Given the perturbative corrections induce a mini landscape where the inflaton may roll in two different directions, we now study the inflationary dynamics for the field rolling towards small values. The terms responsible for the inflationary plateau and minimum are

$$V_{inf}^{L} = V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} \right) . \tag{2.36}$$

To be able to safely neglect non-perturbative correction for the fibre modulus we require that

$$\tau_1^{min} \gtrsim 1 \ , \tag{2.37}$$

thus the minimum of potential (2.36) has to be at values sufficiently greater than unity. Again Minkowski uplifting the potential by its value at the minimum, one has

$$V_{inf}^{L} = \tilde{V}_0^{L} \left(1 - \frac{\beta_L}{2\tau_1} \right)^2 , \quad \beta_L = \frac{4\,\mathcal{C}_1^{loop}}{\mathcal{C}_1} = 4\frac{\mathcal{V} g_s^{5/2} (C_1^{KK})^2}{|W_0|^2 \lambda \, \Pi_1} , \quad \tilde{V}_0^{L} = \frac{V_0}{\beta_L} . \tag{2.38}$$

We find $\tau_1^{min} = \beta_L/2$. hence we require $\beta_L \gtrsim 1$ to keep control over the theory. Recalling (2.25) and given reasonable choices for $\mathcal{V}, g_s$ and W_0 we find $\beta_L \gtrsim 1$ to be readily satisfied. Also, as inflation now occurs for the field rolling to the left, we seek

$$\tau_1^{min} < \tau_1^{c} \quad \Rightarrow \quad \mathcal{C}_1^{loop} < \frac{\mathcal{C}_1^{5/3}}{2\,\mathcal{C}_2^{2/3}} , \tag{2.39}$$

which is easily fulfilled using (2.22) and $\mathcal{V} \gg 1$. Canonical normalisation yields

$$V_{inf}^{L}(\varphi) = \tilde{V}_0^{L} \left(1 - e^{-\kappa\varphi} \right)^2 , \tag{2.40}$$

where we have shifted the field φ to $\varphi \to \varphi + \kappa^{-1} \log(\beta_L/2)$ with $\kappa = 2/\sqrt{3}$. Confronting the set-up with the normalisation of the curvature perturbations (2.33), we find the condition

$$\tilde{V}_0^L = 8\pi\, 0.027^4\, M_{Pl}^4\, \epsilon_V\Big|_{N_e=55} = 1.5 \cdot 10^{-9}\,, \tag{2.41}$$

or equivalently

$$\tilde{V}_0^L \equiv \frac{g_s^2|W_0|^4}{\mathcal{V}^4}\frac{\mathcal{C}_1^2}{4\,\mathcal{C}_1^{loop}} \sim \lambda^2\,\frac{|W_0|^4}{\mathcal{V}^4}|W_0|^2 g_s^{-2}(C_1^{KK})^{-2} \stackrel{!}{=} 1.5\cdot 10^{-9}\,, \tag{2.42}$$

which may be satisfied given reasonable choices of the involved parameters. We thus summarise the demands and resulting bounds for a viable inflationary regime as follows:

demand	*resulting bound*
minimum at $\tau_1 \gtrsim 1$	$\beta_L \sim g_s^{5/2}\,\mathcal{V}(C_1^{KK})^2 \gtrsim 1$
$\tau_1^{min} < \tau_1^c$	$\mathcal{C}_1^{loop} < \frac{1}{2}\left(\frac{2}{\mathcal{C}_2}\right)^{2/3}\mathcal{C}_1^{5/3}$
COBE	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4}g_s^{-2}(C_1^{KK})^{-2} \sim 10^{-9}$

Last but not least, in both cases - inflating for increasing or shrinking fibre - the inflaton has to remain the lightest scalar in the effective theory. We have already pointed out in the discussion about LVS (section 2.1) that the leading order flat direction for τ_1, i.e. τ_1 being massless at tree level, should in principle lead to a natural mass hierarchy in which, when lifted by perturbative corrections, τ_1 remains the lightest scalar.

In technical terms, fluxes and the stabilisation at large volume yield masses for the complex structure moduli, the axio-dilaton and the blow-up τ_3 respectively as

$$m_{cs}^2 \sim m_S^2 \sim m_{\tau_3}^2 \sim g_s\frac{|W_0|^2}{\mathcal{V}^2}\,. \tag{2.43}$$

The volume modulus is $1/\mathcal{V}$ suppressed with respect to the above masses. In order for the volume modulus not to be as light as the inflaton, we hence require

$$V_{inf}^{L/R} \sim H^2 \ll m_{\mathcal{V}}^2 \sim g_s\frac{|W_0|^2}{\mathcal{V}^3}\,. \tag{2.44}$$

For inflating with increasing and decreasing fibre respectively, the above translates to

$$\frac{W_0^2}{\mathcal{V}}\lambda\Pi_2 \ll \sqrt{g_s}\beta_R\,, \quad \text{and} \quad W_0^2\lambda\Pi_1 \ll \sqrt{g_s}\,, \tag{2.45}$$

where it is noteworthy that already $\beta_R < 1/20$ in the inflationary regime.

2.4 Inflationary observables

Having outlined the set-up for inflating with increasing and decreasing fibre as well as having given bounds on the parameter space, some from theoretical concerns and some from observations, we now study the phenomenology of the models presented. After giving the leading order results, we will loosen our requirement that only one term of potential (2.27) be responsible for the inflationary phase and will therefrom derive further constraints on the parameter space.

2.4.1 Leading order results

Recall the effective inflationary potentials (2.32) and (2.40)

$$V_{inf}^R(\varphi) = \tilde{V}_0^R\left(1 - e^{\frac{\kappa}{2}\varphi}\right)^2\,, \quad V_{inf}^L(\varphi) = \tilde{V}_0^L\left(1 - e^{-\kappa\varphi}\right)^2\,. \tag{2.46}$$

The predictions for a potential of type

$$V_{inf} = V_0\left(1 - e^{\pm\nu\varphi}\right)^2 \tag{2.47}$$

for the spectral index n_s and the tensor to scalar ratio r can be cast in terms of N_e via expressions (1.45) and are approximated by

$$n_s \sim 1 - \frac{2}{N_e} + \dots\,, \qquad r \sim \frac{1}{\nu^2}\frac{8}{N_e^2} + \dots\,,$$

respectively, where N_e denotes the number of e-folds before the end of inflation and we have omitted sub-leading terms in $1/N_e$ for conciseness. The coefficient of the $\mathcal{O}(1/N_e^3)$ term can be obtained analytically (see e.g. [49]) but we omit its tedious and lengthy form here. Its numerical value differs between our two model classes of 'inflation to the left' and 'inflation to the right', which explains the split in the values for r in Table 2.1, where the first order phenomenological fingerprint is shown. Note how the predictions of inflating to the right are in line with [39].

	$n_s(50)$	$n_s(60)$	$r(50)$	$r(60)$
right	0.960	0.967	0.0077	0.0055
left	0.960	0.967	0.0024	0.0016

Table 2.1: Numerical values for the spectral index n_s and the tensor-to-scalar ratio r calculated at 50 and 60 e-folds before the end of inflation.

When inflating to the left however, the tensor to scalar ratio decreases whereas the value of the spectral index remains universal. This is because for 'inflation to the right' the exponential term forming the inflationary plateau is the square-root of the analogous term when inflating to the left.

2.4.2 Higher order analysis

We now reconsider the inflationary part of the potential, but do not require the inflaton to be placed sufficiently far away from the stationary point (2.28) such that only one of the terms of (2.27) dominates the inflationary dynamics. Now, we allow for both terms to contribute.

Considering inflation corresponding to increasing fibre, the inflationary potential with the $\mathcal{C}_1$-term included reads

$$V_{inf}^{R} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2\sqrt{\tau_1} + \mathcal{C}_2^{loop}\tau_1 \right) . \tag{2.48}$$

The inflaton inflates to the right while the $\mathcal{C}_1$-term may break the plateau at smaller τ_1 and induce a local maximum. We consider the $\mathcal{C}_1$-term arising from $V_{(1)}$ to be of importance while we have still omitted the string loop induced $\mathcal{C}_1^{loop}$-term. The reasoning is that first the string loop term is τ_1^{-1} suppressed with regard to the higher derivative term. Second, for typical values of C_1^{KK} as given in (2.20), the C_1^{loop}-term will be additionally suppressed. At last, while the string loop term scales with g_s, the higher derivative term is $g_s^{-3/2}$ enhanced. The canonically normalised and uplifted potential hence receives a falling correction at large e-foldings, i.e.

$$V^R_{inf}(\varphi) \sim \tilde{V}^R_0 \left(1 - 2e^{\frac{\kappa}{2}\varphi} - \varepsilon^2 e^{-\kappa\varphi}\right), \quad \varepsilon^2 = \frac{\mathcal{C}_1}{4\,\mathcal{C}_2}\beta^3_R\,, \tag{2.49}$$

where we have already expanded the inflationary potential and have omitted the string loop induced term for notational ease.[17] From the above we may already infer that $\varepsilon^2 \ll 1$ for the inflationary plateau not to be spoiled. The slow-roll parameters (1.22) then receive ε^2 dependent corrections of the form

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\left(-\kappa\, e^{\frac{\kappa}{2}\varphi} + \kappa\,\varepsilon^2 e^{-\kappa\varphi}\right)\,, \tag{2.50}$$

$$\eta_V = \frac{V''}{V} = -\frac{1}{2}\kappa^2 e^{\frac{\kappa}{2}\varphi} - \kappa^2\varepsilon^2 e^{-\kappa\varphi}\,, \tag{2.51}$$

where we have taken the potential to be slowly varying during inflation, i.e. $V^R_{inf} \sim$ *const*. Recalling (1.23) $dN_e = (2\epsilon_V)^{-1/2} d\varphi$, i.e.

$$N_e \sim 2\kappa^{-2} e^{-\frac{\kappa}{2}\varphi} + \mathcal{O}(\varepsilon^2)\,, \tag{2.52}$$

we hence arrive at the expression for the spectral index n_s including higher order corrections

$$n_s = 1 - \frac{2}{N_e} - 3\,\varepsilon^2\kappa^4 N_e + \frac{\varepsilon^2\kappa^6}{2} N_e^2 + \dots \quad . \tag{2.53}$$

The above suggests that there is a further phenomenological fingerprint in the form of running of the spectral index. Considering the next-to-leading correction to the spectral index

$$\delta n_s = -3\,\varepsilon^2\kappa^4 N_e + \frac{\varepsilon^2\kappa^6}{2} N_e^2\,, \tag{2.54}$$

and requiring $\delta n_s \lesssim 0.008$ for $N_e = 55$, which is the 2-σ range for the n_s measurement from Planck, we find an upper bound on ε^2 to be

$$\varepsilon^2 = \frac{|\mathcal{C}_1|}{4\,\mathcal{C}_2}\beta^3_R \lesssim 2.4 \times 10^{-6}\,. \tag{2.55}$$

[17] During inflation, the term ensuring the existence of the minimum is negligible.

This is in agreement with earlier works employing exponential corrections to the inflationary plateau [14, 30–33, 35, 50, 51].[18] The bound on ε^2 also restricts

$$\frac{|\mathcal{C}_1|}{4\,\mathcal{C}_2}\beta_R^3 \sim \lambda^{-3}\,\mathcal{V}\,g_s^{15/2}\,\Pi_1\,\Pi_2^{-4}(C_2^{KK})^6 \lesssim 2.4\times 10^{-6} \tag{2.56}$$

and hence gives a constraint that has to be fulfilled in the first place when considering observationally viable inflation to the right.[19]

Considering the scenario where inflation occurs for the inflaton rolling to the left, we now start with the potential

$$V_{inf}^L \sim V_0\left(-\frac{\mathcal{C}_1}{\tau_1}+\frac{\mathcal{C}_1^{loop}}{\tau_1^2}+\mathcal{C}_2\sqrt{\tau_1}\right)\,. \tag{2.57}$$

The $\mathcal{C}_2$-term destroys the plateau. Again, we have omitted the string loop induced term as it is suppressed with regard to the higher derivative $\mathcal{C}_2$ term by similar reasoning as was employed when justifying the omission of the $\mathcal{C}_1^{loop}$-term when studying inflation to the right. The canonically normalised and uplifted potential hence receives a falling correction at large e-foldings, i.e.

$$V_{inf}^L(\varphi) \sim \tilde{V}_0^L\left(1-2e^{-\kappa\varphi}-\varepsilon^2 e^{\frac{\kappa}{2}\varphi}\right)\,,\quad \varepsilon^2=\frac{\mathcal{C}_2}{\sqrt{2}\,\mathcal{C}_1}\beta_L^{3/2}\,, \tag{2.58}$$

where we have again expanded the inflationary potential and have omitted the string loop induced term as it plays no role on the inflationary plateau. Similarly to the rolling to the right case above, we can derive n_s and obtain

$$n_s = 1-\frac{2}{N_e}-\frac{3\sqrt{2}\varepsilon^2\kappa}{\sqrt{N_e}}+\frac{\varepsilon^2\kappa^3\sqrt{N_e}}{\sqrt{2}}-\frac{3}{2}\varepsilon^4\kappa^4 N_e+\ldots\quad . \tag{2.59}$$

Considering the 2-σ bounds by PLANCK, i.e. requiring $\delta n_s \lesssim 0.008$ at $N_e = 55$,

[18] Note that potential (2.49) may not account for power loss at low-ℓ in the CMB temperature spectrum as the correction comes with a minus sign and hence induces a local maximum and not an inflection point as would be required for power suppression.

[19] For $\mathcal{V}\sim 10^3$ and $\Pi_1\sim\Pi_2$ the bound requires $\beta_R \lesssim 10^{-3}$, thus placing a stronger constraint on β_R than the minimal required length of the plateau.

we obtain the upper bound

$$\varepsilon^2 \sim \lambda^{-3/2}\,\mathcal{V}^{-1}\left(g_s^{5/2}\mathcal{V}\right)^{3/2}\,\Pi_2\,\Pi_1^{-5/2}(C_1^{KK})^{3/2} \lesssim 10^{-3}\,. \tag{2.60}$$

Given the bounds on ε^2 induced by the data-compatible range of δn_s, we also find that the contribution of the δn_s to the magnitude of running $dn_s/d\ln k = -dn_s/dN_e$ is typically $\lesssim \mathcal{O}(10^{-4})$ and hence at least an order of magnitude smaller than the contribution to the running from $n_s^{(0)} - 1 = -2/N_e$ which is about $dn_s^{(0)}/d\ln k \sim -10^{-3}$. Finally, let us provide one example of parameter choices for a model of inflation with shrinking fibre τ_1 in Table 2.2.

	W_0	g_s	$\mathcal{V}$	τ_1^{min}	Π_1	Π_2	C_1^{KK}	C_2^{KK}	n_s
$\mathcal{L}_1$	2	0.3	460	3	100	1	0.163	0.0288	0.966

Table 2.2: Example of compactification parameters and inflationary observables for inflation to the left ($\mathcal{L}_1$).

In this chapter, we have presented a way to realise an effective shift symmetry by tuning and balancing different perturbative corrections against each other. While a part of the effective shift symmetry derives from an intrinsic one at parametrically large volume, this could not serve to protect the inflaton potential against higher orders in the parameter regime required to drive observationally viable slow-roll inflation Hence balancing and tuning terms had to be invoked.

Chapter 3

The non-canonical point of view

Embedding inflation in some UV theory usually yields a non-canonical kinetic term in the effective Lagrangian in an intermediate step (as an example, recall the kinetic Lagrangian (2.13) of chapter 2). Conventionally, inflationary dynamics are studied once the kinetic term is canonically normalised. However, obtaining a non-canonical kinetic term as an intermediate step suggests to analyse the kinetic function directly and hence to - independently of the specifics of the potential - use the non-canonical kinetic term as a short cut to the inflationary dynamics. In this non-canonical language, recent work established a reformulation of plateau-type inflaton potentials in terms of a certain pole structure of the kinetic function [52].

In this chapter we enhance these ideas by establishing an extended duality between a kinetic function with a certain pole structure and shift symmetry of the Einstein frame canonically normalised inflaton potential [16]. Moreover, we study the breaking of the shift symmetry at large fields. Since non-canonical kinetic terms are a generic consequence of compactifications of higher-dimensional models such as string theory, this may provide a new avenue of constructing this set of phenomenologically promising models from more fundamental embeddings. One of the main aims of this chapter is to provide the analogue formulation of this shift symmetry for non-canonical models of inflation, to which we turn next.

The rest of the chapter is structured as follows. First, we recall the formulation of inflation where the inflationary dynamics' complexity has been shifted in parts to the kinetic term rather than the potential. Given a suppression hierarchy for poles of increasing order, we continue to describe corrections to the aforementioned formalism and derive leading order corrections to the inflationary observables. Assuming the corrections to follow the pattern of shift symmetry in an EFT sense, we then study an infinite tower of corrections and demonstrate that the leading order corrections coincide with the structure obtained before. After outlining phenomenological fingerprints deriving from the corrections, we attempt to embed the previous considerations into some UV theory. Following a generic argument as to what the coarse structure of the UV candidate Kähler potential might be, we give perturbative and exact examples which reproduce the kinetic functions under study in this work. We then turn our attention to String Theory and argue that the necessary terms may be obtained. Specifically, we recall that the more general form of string loop corrections to the volume moduli Kähler potential in string compactifications spoil the log-structure of the Kähler potential, and we hence expect them to break the shift symmetry at large field ranges. We conclude by discussing our results, and point out that our steepening corrections generically produce a moderate loss of CMB power at large angular scales for which we give an analytical estimate.

3.1 Pole inflation

We begin with a quick recapitulation of the formalism presented in [52] and provide extensions in order to translate the results presented to canonically normalised fields.

3.1.1 Laurent expansion

If the kinetic term may be cast as a Laurent series and given reasonable assumptions about the potential, one can study and understand inflationary dynamics mostly in terms of the leading order pole of the Laurent expansion and its residue, as we will now recall. Consider an Einstein frame Lagrangian of the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{2}K_E(\rho)\left(\partial\rho\right)^2 - V_E\left(\rho\right) \ , \quad K_E(\rho) = \left(\frac{a_p}{\rho^p} + \dots\right) \ , \tag{3.1}$$

where we assume the kinetic function $K_E(\rho)$ to be given by a Laurent series with a pole of order p at $\rho = \rho_0 = 0$ (without loss of generality) plus sub-leading terms, which are higher-order in ρ (not higher order in ρ^{-1}) and are thus irrelevant close to the pole.[20] In principle, higher order terms in ρ^{-1}, i.e. higher orders in the pole, are of increasing importance when $\rho \to 0$. We will neglect those terms for now to ease our analysis of the first pole and give a condition which has to be satisfied in order to do so in (3.13).

The location of the pole corresponds to a fixed point of the inflationary trajectory, which is therefore characterised almost completely by this point. Upon canonical normalisation, the fixed point translates into a nearly shift-symmetric plateau in the potential. As the inflationary behaviour will be determined by the trajectory of the non-canonical field in the vicinity of the pole, one may approximate $V_E(\rho)$ to be

$$V_E = V_0(1 + c\rho + \ldots)\,, \tag{3.2}$$

where we may leave the coefficient c unspecified. Our results will not depend on any choice of c as we will later demonstrate. All higher order terms may also have arbitrary coefficients as they will be sub-dominant close to the pole $\rho_0 = 0$.

The crucial assumption in Lagrangian (3.1) is that kinetic pole and potential minimum do not coincide.[21] In other words, scenario (3.1) and (3.2) may also be recast - by means of a field redefinition $\rho \to \rho + \rho_0$ - to read

$$K_E(\rho) = \frac{1}{(\rho - \rho_0)^p} + \ldots\,, \quad V_E(\rho) = V_0(\rho^n + \ldots)\,. \tag{3.3}$$

with $n \geq 1$. Going back to $\rho \to \rho - \rho_0$, the above will become $V_E \sim 1 + c\rho \ldots$ with $c = -n/\rho_0$ in the vicinity of the pole ρ_0 and hence results in the same inflationary dynamics as scenario (3.1) and (3.2). For $n = 2$, this argument may also be understood in the following sense; if the specifics of a scalar field potential are unknown, as long as the scalar field is stabilised at ρ_{min} and the kinetic function has a pole $\rho_0 \neq \rho_{min}$, inflation compatible with PLANCK will be realised.

[20] We can move any pole ρ_0 to $\rho_0 \to \rho_0 = 0$ by means of a field redefinition $\rho \to \rho - \rho_0$ without changing the dynamics of the system.

[21] If the potential also had a minimum at $\rho_{min} = \rho_0 = 0$, expansion (3.2) would obviously not be suitable. This issue was not mentioned explicitly in the original work. We will revisit this when turning to perturbations of the pole structure in section 3.2.

For simplicity, we now assume the pole to be located at $\rho_0 = 0$ and the potential hence to be given by (3.2). For this non-canonical set-up, the slow-roll parameters (1.22) are

$$\epsilon_V = \frac{1}{2K_E(\rho)} \left(\frac{1}{V_E(\rho)} \frac{\partial V_E(\rho)}{\partial \rho} \right)^2 , \quad \text{and}$$

$$\eta_V = \frac{1}{K_E(\rho)\, V(\rho)} \left[\frac{\partial^2 V(\rho)}{\partial \rho^2} - \frac{\partial V_E(\rho)}{\partial \rho} \frac{1}{2K_E(\rho)} \frac{\partial K_E(\rho)}{\partial \rho} \right] , \tag{3.4}$$

where φ is the canonically normalised inflaton and $K_E(\rho)$ is the kinetic function of Lagrangian (3.1). An explicit calculation then yields

$$\epsilon_V = \frac{1}{2a_p} \rho^p , \quad \eta_V = -\frac{p}{2a_p} \rho^{p-1} . \tag{3.5}$$

The number of e-folds N_e is obtained as

$$N_e = \int \frac{1}{\sqrt{2\,\epsilon_V}} d\varphi = \int K_E(\rho) V_E(\rho) \left(\frac{\partial V_E(\rho)}{\partial \rho} \right)^{-1} d\rho . \tag{3.6}$$

Sufficiently close to the pole at $\rho = 0$, i.e. at large N_e, the number of e-folds hence evaluates to

$$N_e = \frac{a_p}{(p-1)\rho^{p-1}} , \quad \text{and thus} \quad \rho = \left(\frac{a_p}{(p-1)N_e} \right)^{\frac{1}{p-1}} . \tag{3.7}$$

Since we assume $p > 1$, indeed the number of e-folds increases as the field ρ approaches the pole. At lowest order in $1/N_e$, the inflationary predictions for this model are therefore given by

$$n_s = 1 - \frac{p}{p-1} \frac{1}{N_e} , \qquad r = \frac{8 a_p^{\frac{1}{p-1}}}{(p-1)^{\frac{p}{p-1}}} \frac{1}{N_e^{\frac{p}{p-1}}} , \tag{3.8}$$

where a_p is the leading coefficient of the Laurent expansion as in (3.1). The above derivation is indeed independent of the linear coefficient c of (3.2).

Putting all of this together, we observe that the presence of a fixed point of the kinetic function that does not coincide with the minimum of the potential translates to an effective shift symmetry of the canonically normalised inflaton at large field values, provided that all higher-order poles in the kinetic function beyond the leading-order pole defining the fixed point have successively suppressed coefficients as in equation (3.13) below. This provides us with a new handle on finding regimes where inflaton potentials show an effective shift symmetry via analysing the local structure of the non-canonical kinetic function. In other words, vastly enhancing the kinetic term such that it becomes dominant with regard to the potential - e.g. with a pole in the kinetic function as above - enters the canonical normalisation such that, given reasonable assumptions about the specific potential, the canonically normalised field will slowly roll down its effective potential which will be of plateau type.

The case $p = 2$ is special for a number of reasons. First of all, this gives rise to a value of the spectral index that agrees exceedingly well with PLANCK. Secondly, from a theoretical perspective, large classes of models with different interactions actually give rise to nearly identical predictions (3.8) with $p = 2$. In what follows, we will therefore focus on corrections to $p = 2$ poles.

To finish the discussion, we now turn to canonical variables. We begin by demonstrating that the coefficient c may indeed be kept arbitrary when studying the inflationary dynamics also in canonical fields. Recalling potential $V_E = V_0(1 + c\rho \ldots)$ and the pole $\rho_0 = 0$, we observe that for

$$\begin{aligned} &c < 0\,, \quad \rho > 0 \quad \text{and thus} \quad N_e \to \infty \quad \text{for} \quad \rho \to 0_+\,, \\ &c > 0\,, \quad \rho < 0 \quad \text{and thus} \quad N_e \to \infty \quad \text{for} \quad \rho \to 0_-\,, \end{aligned} \tag{3.9}$$

where inflation occurs for decreasing ($c < 0$) or increasing ($c > 0$) field ρ. For the exemplary case $p = 2$, canonical normalisation introduces

$$\rho \propto e^{\pm\varphi/\sqrt{a_p}} \quad \text{and} \quad \rho \propto -e^{\pm\varphi/\sqrt{a_p}} \tag{3.10}$$

for $c < 0$ and $c > 0$ respectively.[22] The sign of the exponent is arbitrary and

[22] For $p = 2$, canonical normalisation $\partial\rho/\rho = \partial\varphi$ evaluates to $\log|\rho| = \varphi + \varphi_0$. For $\rho > 0$, this is solved by the first term of (3.10) while for $\rho < 0$, it is solved by the second. The constant of integration φ_0 may be tuned to absorb any value $|c|$.

simply reflects whether inflation occurs for increasing (plus sign) or decreasing (minus sign) canonical field φ. Moreover, any value $|c|$ can be absorbed by means of a field redefinition, i.e. choosing a suitable integration constant φ_0. We hence see that also the potential in canonically normalised fields is independent of the linear coefficient c and universally reads

$$V_0(\varphi) = \begin{cases} V_0 \left(1 - A\,\varphi^{\frac{2}{2-p}}\right) & , \quad p \neq 2 \\ \\ V_0 \left(1 - e^{-\frac{\varphi}{\sqrt{a_p}}}\right) & , \quad p = 2 \end{cases} \tag{3.11}$$

where $A = \left(\frac{2-p}{2\sqrt{a_p}}\right)^{\frac{2}{2-p}}$. This shows the plateau at $\rho \to 0$ occurring for $\varphi \to \infty$ if $p \geq 2$ and for $\varphi \to 0$ otherwise. The higher powers in ρ of $V_0(\rho)$ beyond the linear term are irrelevant due to the fact that the pole structure has inflation taking place for $\rho \to 0$. Higher powers in the Laurent expansion of $K_E(\rho)$

$$K_E(\rho) = \frac{a_p}{\rho^p} + \sum_{q>p} \frac{a_q}{\rho^q} \tag{3.12}$$

will perturb $V_0(\rho) \to V(\rho) = V_0(\rho) + \Delta V(\rho)$. Therefore, an extended plateau in the potential equation (3.11) requires us to restrict to the regime where the following condition holds

$$\frac{a_q}{\rho^q} \ll \frac{a_p}{\rho^p} \;\; \forall \;\; q > p\,. \tag{3.13}$$

Similar to the suppression of higher-dimension operators in some scalar potential, there is a priori no reason why condition (3.13) should hold. We hence propose condition (3.13) as a statement dual to the requirement to suppress higher-dimension operators in the canonical picture and will give a toy model realisation of (3.13) in section 3.2 and specifically via expression (3.32). This suppression pattern of the residues of the Laurent expansion dictated by the approximate shift symmetry on the plateau forms a complete analogue of the two known requirement to suppress higher order terms in the canonical formulation.

3.1.2 Universal corrections

Above, we discussed how a real pole corresponding to a fixed point in field space translates to an approximately shift-symmetric plateau upon canonical normalisation. We now perturb the duality between fixed points and shift symmetry. Consider a higher-order pole with small coefficient a_q (i.e. imposing (3.13)):

$$K_E(\rho) = \frac{a_q}{\rho^q} + \frac{a_p}{\rho^p} + \dots\,, \tag{3.14}$$

while the scalar potential is still given by the Taylor expansion, and the dots represent less singular terms in ρ. This gives rise to the relation

$$N_e = \frac{a_q}{(q-1)\rho^{q-1}} + \frac{a_p}{(p-1)\rho^{p-1}} \tag{3.15}$$

for the field ρ close to the pole. However, to invert this relation, one has to assume that the perturbation is small with respect to the original term:

$$\frac{a_q}{\rho^q} \ll \frac{a_p}{\rho^p}\,. \tag{3.16}$$

We thus rediscover condition (3.13). As an expansion, we then obtain the solution

$$\rho = \rho_0 + \delta\rho\,, \quad \delta\rho = \frac{a_q}{a_p(q-1)}\rho_0^{p-q+1}\,, \tag{3.17}$$

where the subscript zero refers to the unperturbed respective result of the previous section. The corrections to the slow-roll parameters become

$$\delta\epsilon = -\frac{(q-p-1)}{2(q-1)}\frac{a_q}{a_p^2}\rho_0^{2p-q}\,, \quad \delta\eta = -\frac{(q-p)(q-p-1)}{2(q-1)}\frac{a_q}{a_p^2}\rho_0^{2p-q-1}\,, \tag{3.18}$$

at lowest order in a_q. We therefore obtain

$$\delta n_s = -\frac{a_q}{a_p^{\frac{q-1}{p-1}}}\frac{(q-p)(q-p-1)}{(q-1)(p-1)^{\frac{q-1}{p-1}-2}}N_e^{\frac{q-1}{p-1}-2}\,,$$

$$\delta r = -\frac{8a_q}{a_p^{\frac{q-2}{q-1}}}\frac{(q-p-1)}{(q-1)(p-1)^{\frac{2p-q}{p-1}}}N_e^{\frac{q-2p}{p-1}}\,. \tag{3.19}$$

These are universal corrections deriving from a perturbation of the shift symmetry at large field values of the canonically normalised inflaton field.

Motivated by observational evidence, we now restrict ourselves to the case $p = 2$. Other cases are qualitatively identical. For $p = 2$ the expressions for n_s and r reduce to

$$n_s = 1 - \frac{2}{N_e} - \frac{a_q}{{a_p}^{q-1}} \frac{(q-2)(q-3)}{(q-1)} N_e^{q-3} \,, \quad r = \frac{8a_p}{N_e^2} - \frac{8a_q}{{a_p}^{\frac{q-2}{q-1}}} \frac{(q-3)}{(q-1)} N_e^{q-4} \,. \tag{3.20}$$

These are the universal corrections to the cosmological attractor predictions provided the corrections respect the effective shift symmetry structure of equation (3.13). They should be understood as a double expansion, both in $1/N_e$ as well as in $a_q N_e^{q-2}$. The latter requirement follows from the approximation to obtain $\rho(N_e)$ (and is given by $a_q N^{\frac{q-p}{p-1}}$ in general). Here we have assumed that a_p is of order one. In terms of these expansion parameters, the correction term to the spectral index is bilinear in both, while the correction to the tensor-to-scalar ratio is an order in $1/N_e$ higher. Corrections bilinear in N_e will become of increasing importance for larger N_e. This will nicely be illustrated in the next subsection when transforming to canonical fields.

3.1.3 Canonical formulation

We will now turn to a description of the corrections to the plateau potential arising from the least suppressed residue a_q in the Laurent expansion. Starting from the perturbed Laurent expansion with two poles (3.14), we find the relation $\rho(\varphi)$ for the canonically normalised field φ for $p \neq 2$ to leading order in a_q to be

$$\rho(\varphi) = A\, \varphi^{\frac{2}{2-p}} + \frac{a_q}{2\sqrt{a_p}} A^{\frac{p-2(q-2)}{2}} \varphi^{\frac{2}{2-p}(p-q+1)} \,. \tag{3.21}$$

For the special case $p = 2$ the relation becomes exponential and we get

$$\rho(\varphi) = e^{-\frac{\varphi}{\sqrt{a_p}}} + \tfrac{1}{4} a_q e^{(q-3)\frac{\varphi}{\sqrt{a_p}}} \quad , \quad q > p = 2 \,. \tag{3.22}$$

We obtain the resulting canonical scalar potential $V(\varphi) = V_0(\rho(\varphi))$ to $\mathcal{O}(a_q)$ by plugging expressions (3.21) and (3.22) into the original potential (3.2)

$$V(\varphi) = \begin{cases} V_0\left(1 - A\,\varphi^{\frac{2}{2-p}} - a_q\,B\,\varphi^{\frac{2}{2-p}(p-q+1)}\right), \; p \neq 2 \\ \\ V_0\left(1 - e^{-\frac{\varphi}{\sqrt{a_p}}} - \frac{1}{4}a_q e^{(q-3)\frac{\varphi}{\sqrt{a_p}}}\right), \; p = 2 \quad . \end{cases} \tag{3.23}$$

Here, the correction coefficient B for $p \neq 2$ has the form $B = \frac{1}{2\sqrt{a_p}}A^{\frac{p-2(q-2)}{2}}$. Consequently, for $a_q < 0$ the plateau potential universally acquires a rising correction, increasing for $\varphi \to \infty$ for $p \geq 2$, and for $p < 2$ rising towards $\varphi = 0$. We see that if all microscopic parameters a_p, p, q take $\mathcal{O}(1)$ values, then $A, B = \mathcal{O}(1)$ as well, and their precise values are irrelevant for the general arguments given here. The only relevant quantities are:

- p which determines the leading functional form of the plateau potential,
- $a_q \ll 1$ which controls the magnitude of the correction, and
- the difference $q - p$ which controls the functional dependence of the first correction in the scalar potential.

This structure of the scalar potential allows for two observations. First, the case

$$q = p + 1 \tag{3.24}$$

leads to a rather curious observation. Namely, the corrections to the scalar potential are constant. For this reason, they only serve to redefine the constants in the original form (3.11):

$$\tilde{A} = \frac{A}{1 - a_q B}\,, \; \tilde{V}_0 = V_0\,(1 - a_q B) \quad , \quad p \neq 2 \tag{3.25}$$

$$\tilde{V}_0 = V_0\,(1 - \tfrac{1}{4}a_q) \quad , \quad p = 2\,. \tag{3.26}$$

This can be understood as follows: We look again at the kinetic function, writing

$$K_E(\rho) = \frac{a_p}{\rho^p} + \frac{a_{p+1}}{\rho^{p+1}} + \sum_{q > p+1} \frac{a_q}{\rho^q}\,. \tag{3.27}$$

Now perform a field redefinition $\rho \to \rho + \varepsilon$, and insert this into K_E. We get

$$K_E(\rho + \varepsilon) = \frac{a_p}{(\rho+\varepsilon)^p} + \frac{a_{p+1}}{(\rho+\varepsilon)^{p+1}} + \ldots = \frac{a_p}{\rho^p} - \frac{p a_p \varepsilon}{\rho^{p+1}} + \frac{a_{p+1}}{\rho^{p+1}} + \ldots \tag{3.28}$$

Hence, by adjusting the field definition to $\varepsilon = a_{p+1}/(p a_p)$ we can *always* absorb the pole of order $p+1$, but not other poles of higher-order at the same time. This is the reason why a pole at order $p+1$ does not contribute at that order to the scalar potential. Beyond leading order, the field redefinition generates contributions to poles at order $p+2$ and higher. Consequently, we expect a pole in K_E at order $p+1$ to contribute at higher sub-leading orders to the scalar potential. At the next order, i.e. for

$$q = p + 2 \tag{3.29}$$

the correction scales as the inverse of the leading plateau potential term. The cases $p = 1, 2, 3$ form illustrative examples for this situation

$$V(\varphi) = \begin{cases} V_0 \left(1 - A\,\varphi^2 - a_q\,\frac{B}{\varphi^2}\right) \quad , \quad p = 1 \\[2ex] V_0 \left(1 - e^{-\frac{\varphi}{\sqrt{a_p}}} - \frac{1}{4} a_q e^{\frac{\varphi}{\sqrt{a_p}}}\right) \quad , \quad p = 2 \\[2ex] V_0 \left(1 - \frac{A}{\varphi^2} - a_q\,B\,\varphi^2\right) \quad , \quad p = 3 \quad . \end{cases} \tag{3.30}$$

3.2 Complex poles

The above analysis only considers a single correction. In order to further link the above discussion with our argument in section 1.3, we will now consider an infinite tower of corrections to the leading pole of a kinetic function and hence readily demonstrate that in order not to spoil the inflationary dynamics, a hierarchy between the corrections reminiscent of the EFT argument has to arise. To that end, consider higher powers in the Laurent expansion of $K_E(\rho)$

$$K_E(\rho) = \frac{a_p}{\rho_p} + \sum_{q>p} \frac{a_q}{\rho^q}\,. \tag{3.31}$$

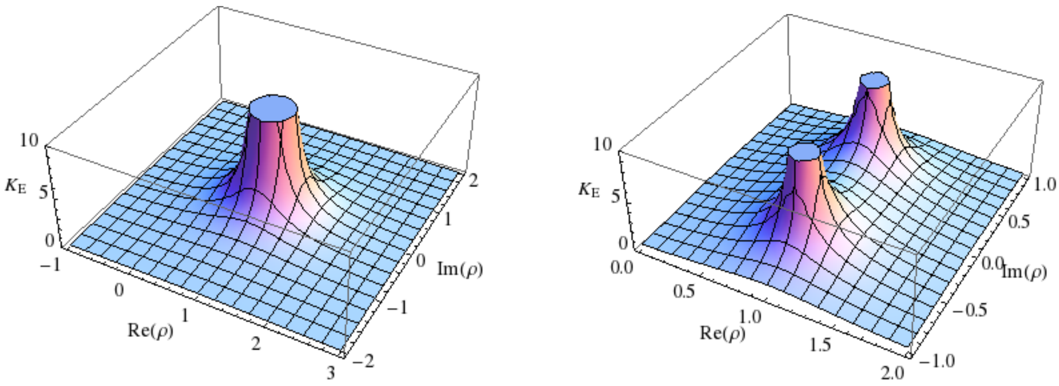

Figure 3.1: *Pole structure of K_E.* **Left:** *A pole of order $p = 2$, localised in the real part of ρ.* **Right:** *The perturbed case, showing the split of the original pole in two complex poles of order one. The non-canonical inflaton may now move over the hilltop along the real line which corresponds to shift symmetry breaking in canonical fields.*

As an example, we will assume the above to arise from a toy model of the closed form

$$K_E(\rho) = \frac{a_p}{\rho^2 + \varepsilon^2}\,, \tag{3.32}$$

where $\varepsilon^2 \ll 1$. The perturbation ε^2 affects the pole structure, moving the pole at $\rho_0 = 0$ from the real to the complex plane at $\rho_0 \to \pm i\varepsilon$, as shown in Figure 3.1. It is important to note that the function $K_E(\rho)$ does not become complex itself at any point, it merely contains a complex pole.

The inflationary predictions from the presence of a complex pole follow at lowest order from the universal corrections that we derived earlier: expanding the complex pole

$$K_E = \frac{a_p}{\rho^2} - \frac{a_p\varepsilon^2}{\rho^4} + \frac{a_p\varepsilon^4}{\rho^6} + \dots\,, \tag{3.33}$$

it is clear that at lowest order in ε^2 the form of the kinetic function, and hence the inflationary predictions, is exactly that of the perturbed Laurent expansion considered previously with $p = 2$ and $q = 4$. As a consequence, the inflationary predictions are given by (3.20) with $q = 4$ and $a_q = -a_p\varepsilon^2$. Note that the latter always corresponds to a blue-shifting of the spectral index at large N_e. Further note how (3.33) realises (3.31) with $a_q \ll a_p \, \forall \, q > p$, i.e. satisfies (3.13). This suppression pattern of the residues of the Laurent expansion dictated by the approximate shift symmetry on the plateau is in complete analogy to the known earlier cases given in section 1.3.

If the above kinetic term only has a complex pole (and no sub-leading corrections), the transition to a canonical inflaton field φ can be done exactly and reads

$$\rho = e^{-\varphi/\sqrt{a_p}} - \tfrac{1}{4}\varepsilon^2 e^{\varphi/\sqrt{a_p}} \, . \tag{3.34}$$

The scalar potential around the would-be pole reads

$$V_E = V_0 (1 - e^{-\varphi/\sqrt{a_p}} + \tfrac{1}{4}\varepsilon^2 e^{\varphi/\sqrt{a_p}} + \ldots) \, . \tag{3.35}$$

Again, the nearly shift-symmetric plateau of the canonically normalised inflaton is broken at large field values, the exact value depending on the perturbation ε^2 of the kinetic pole structure.

Note how the above argument assumed $\rho_0 = 0$ and the minimum of the potential to be at $\rho_{min} \neq \rho_0$. As argued in section 3.1.1, the same dynamics may also be found when perturbing a pole $\rho_0 \neq 0$ and only considering the potential in the vicinity of its minimum, as we will now show. Consider the kinetic function and potential (for the exemplary case $p = 2$)

$$K_E(\rho) = \frac{a_p}{(\rho - \rho_0)^2 + \varepsilon^2} \, , \quad V(\rho) = V_0(\rho^2 + \ldots) \, , \tag{3.36}$$

where again the argument readily extends to scenarios $V \sim \rho^n$ with $n \geq 1$. Canonical normalisation can be done exactly and yields

$$\rho = \frac{1}{2} \left(2\rho_0 - e^{-\varphi/\sqrt{a_p}} + \varepsilon^2 e^{\varphi/\sqrt{a_p}} \right) \, . \tag{3.37}$$

Upon field transformations, the leading order scalar field inflationary potential in the vicinity of ρ_0 is precisely given by (3.35).

3.3 Towards a UV embedding

Within the framework of non-canonical inflation, the complexity of the inflationary dynamics has been shifted to the kinetic function. Poles in the kinetic function then translate to nearly shift-symmetric potentials and complex poles or higher order terms in $1/\rho$ break the shift symmetry at large fields. It is therefore an important question whether one can embed kinetic functions with the aforementioned structure into a UV theory. We start with a general observation.

3.3.1 Kähler potentials

Consider a toy potential of the form $K = \log f$ where the function f in the argument of the logarithm is a real function, e.g. an arbitrary polynomial, of $\Phi + \bar{\Phi}$ or $\Phi\bar{\Phi}$. Now assume that f has a real zero of order n at e.g. $\Phi_0 = 0$. Close to the pole, the function can then be approximated as $f = (\Phi + \bar{\Phi})^n + \ldots$. The corresponding Kähler metric takes the following form

$$K_{\Phi\bar{\Phi}} = \frac{f_\Phi f_{\bar{\Phi}} - f_{\Phi\bar{\Phi}} f}{f^2} = \frac{n}{(\Phi + \bar{\Phi})^2} + \ldots . \tag{3.38}$$

Upon identifying $\Phi = \bar{\Phi} = \rho$ and $n = 2a_p$ this becomes the previously considered Laurent expansion. The order of the pole is therefore independent of the order of the zero in the argument of the logarithm; instead, the order of the zero determines the residue of the pole, which always has order two. Changes to the location of the zero of f and to its order do not affect the resulting pole structure of order two. Note how - provided f has a zero - the denominator f^2 of $K_{\Phi\bar{\Phi}}$ in expression (3.38) always factorises on the real line. However, the key feature of structures such as (3.31) and (3.32) was precisely the absence of a real pole. Hence by construction of a logarithmic Kähler potential, one cannot obtain a Kähler metric where the denominator does not factorise which however would be required to have a perturbed and complex pole structure of the kinetic function.

Turning to the type of corrections corresponding to complex poles

$$K_{\Phi\bar{\Phi}} = \frac{n}{(\Phi + \bar{\Phi})^2 + \varepsilon^2} = \frac{n}{(\Phi + \bar{\Phi})^2} - \frac{n\varepsilon^2}{(\Phi + \bar{\Phi})^4} + \ldots , \tag{3.39}$$

that become relevant at large field values, we first stress that the denominator does not factorise on the real but only on the complex plane. Hence in order to generate such Kähler potentials, we must resort to a different structure than the one described above. A prototypical example would be $f = (\Phi + \bar{\Phi})^2 + \varepsilon^2$. Indeed this will induce additional terms in the Kähler metric that correspond to higher-order poles, similar to (3.39):

$$K_{\Phi\bar{\Phi}} = \frac{n((\Phi + \bar{\Phi})^2 - \varepsilon^2)}{((\Phi + \bar{\Phi})^2 + \varepsilon^2)^2} = \frac{n}{(\Phi + \bar{\Phi})^2} - \frac{3n\varepsilon^2}{(\Phi + \bar{\Phi})^4} + \ldots . \tag{3.40}$$

In order to obtain exactly the Kähler metric (3.39), we note that it can actually be integrated to yield

$$K = \frac{\Phi + \bar{\Phi}}{\varepsilon} \tan^{-1}\left(\frac{\Phi + \bar{\Phi}}{\varepsilon}\right) - \frac{1}{2} \ln\left(\varepsilon^2 + (\Phi + \bar{\Phi})^2\right) . \tag{3.41}$$

Expanding the Kähler potential at small ε, we find

$$K = -n \log(\Phi + \bar{\Phi}) - \frac{n\varepsilon^2}{6(\Phi + \bar{\Phi})^2} + \ldots . \tag{3.42}$$

The leading term outside the logarithm corresponds to the pole of order four (necessarily with opposite sign, to counter the pole of order two) that is the first to become relevant at large field values, i.e. at large N_e. As we have argued, this gives rise to a universal signature in terms of the spectral index and tensor-to-scalar ratio. We thus conclude that one has to resort to corrections *outside* of the logarithm in order to realise shift symmetry breaking at large fields without invoking the construction of specific features in the inflaton potential. These terms outside of the logarithm are expected to cancel any additional terms that may arise when constructing the argument of the logarithm without any real zero. As a further example, consider a more general expansion of the form (3.42) with arbitrary corrections and order:

$$K = -n \log(\Phi + \bar{\Phi}) + \frac{n'}{(\Phi + \bar{\Phi})^{q-2}} + \ldots . \tag{3.43}$$

In a way, terms outside of the logarithm may be thought of as a way to reintroduce the η-problem, but in a controlled way such that this only occurs at large fields. As we have argued, this gives rise to a universal signature in terms of the spectral index and tensor-to-scalar ratio at leading order in $a_q = 2^{-q}(q-1)(q-2)n'$.

3.3.2 Comments on matching to string theory

Which of these structures can be obtained in string theory settings? If we look at the peculiar behaviour of non-canonical inflation with $p = 2$ and $q = 3$, we discover by comparison a relation to a well known fact of the Kähler geometry argued for 1-loop corrections in string theory to the volume moduli Kähler potential K in supergravity [37, 45, 53–55]. Namely, for $p = 2$ we can think of $K_E(\rho)$ as arising from a logarithmic Kähler potential for a chiral modulus field χ

$$K_0 = -2a_p \ln(\chi + \bar{\chi}) \quad , \quad \chi + \bar{\chi} = 2\rho \,, \tag{3.44}$$

where we get $K_E(\rho) = \partial_\chi \partial_{\bar{\chi}} K \equiv K_{\bar{\chi}\chi}$. A string loop correction to K is the generically argued [37, 45, 53–55] to change K with a quantity

$$\delta K = -\frac{2^q}{(q-2)(q-1)} \frac{a_q}{(\chi + \bar{\chi})^{q-2}} \quad , \quad q = 3, 4 \,. \tag{3.45}$$

Here, we have chosen the prefactor of the loop correction such that the induced term in $K_{\bar{\chi}\chi}$ matches the form eq (3.14). Hence, according to [37, 45, 53–55] the corrections form degree $-(q-2)$ polynomials in K. From the general analysis in [37, 45] we know that for constant superpotential W_0 the leading-order supergravity scalar potential for such a modulus χ induced by the above Kähler potential correction scales like

$$\delta V \sim (2-q)(3-q)\delta K \,. \tag{3.46}$$

Again, we see that for $q = 3$ the leading correction to the potential vanishes. In this context of string loop corrections in type IIB compactifications this phenomenon was named "extended no-scale structure" in [37, 45, 53, 54] as the above leading correction to the no-scale potential of the Kähler moduli (which have $a_{p=2} = 3/2$) was observed there to vanish (and hence "extend" no-scale) for all loop corrections to K which scale with power $q = 3 = p + 1$ in the resulting Kähler metric $K_{\bar{\chi}\chi}$.

Our analysis of the scalar potential above shows that for models with pole-dominated kinetic terms this extended no-scale structure holds for kinetic functions with an arbitrary leading pole of order $p > 0$ even if $p \neq 2$. Moreover, it has a natural explanation as a shift redefinition of the modulus. We can now take a look a the leading-order structure of both the string 1-loop and the leading $\mathcal{O}(\alpha'^3)$-corrections to the type IIB volume moduli Kähler potential

$$\begin{aligned} K &= -2\ln(\mathcal{V}+\xi/2) - \frac{C}{T+\bar{T}} - \frac{D}{(T+\bar{T})^2}\,, \\ &= -2\ln\mathcal{V} - \frac{\xi}{(T+\bar{T})^{3/2}} - \frac{C}{T+\bar{T}} - \frac{D}{(T+\bar{T})^2}\,, \end{aligned} \tag{3.47}$$

with $\mathcal{V} \sim (T+\bar{T})^{3/2}$ and at lowest order in ξ. In such a simple situation of a single Kähler modulus the above inflationary regime would correspond to working close to $T = 0$ where the α'-corrections are out of control. However, the simple toy example serves us here to point out that a comparison with string theory as a possible UV completion fixes concrete numbers for the possible values for p and q. Namely, from the single modulus toy example we get $p = 2$ and $q = 3, 7/2, 4$ of which the $q = 3$ contribution drops out of the scalar potential at leading order as discussed above. Moreover, matching to a string example would allow us also to compute the compute c and a_q in terms of the microscopic parameters ξ, C, D. As C, D are g_s-suppressed in the string coupling compared to the tree level terms and ξ, this may allow also for an understanding of the smallness of a_q in terms of small g_s. It remains to be seen, whether an embedding of this structure in a concrete controlled string theory setting (either away from small volume regimes, or in a better-controlled singular regime) is possible.

3.4 Phenomenology and discussion

The topic of this chapter was non-canonical inflation. We have recalled how a leading pole in the Laurent expansion of the kinetic function translates into a nearly shift-symmetric plateau in the effective scalar potential of the canonically normalised inflaton field, i.e. a fixed point of the cosmological evolution. This is a generic feature and does not depend on the order of the pole.

Subsequently, we have investigated higher-order poles as perturbations of the Laurent expansion of the kinetic term. The fixed point hence vanishes which results in the approximate shift symmetry of the inflaton potential to be broken at large fields. Given a hierarchical suppression of higher order poles, we have outlined the leading corrections to the inflationary predictions in terms of the number of e-folds and the perturbation of the pole structure, and found that such corrections induce terms with positive powers of N_e in the spectral index n_s, which therefore rise to prominence at sufficiently large-N_e (i.e. at large field values). Moreover, we have provided an explanation of the irrelevance of the first higher-order pole and have argued this to be an alternative way to view the extended no-scale structure in string theory: the effect of a pole one order higher than the leading one can be absorbed in a redefinition of the field. We can use our results for n_s to analytically estimate the power-loss at large angular scales resulting from the blue-shifting of the spectral index. Recalling (1.43), we get

$$\left.\frac{\delta\Delta_s^2(\delta n_s)}{\Delta_s^2}\right|_{N_e+\Delta N_e}^{N_e} = \frac{a_q}{a_p^{\frac{q-1}{p-1}}}\frac{(q-p)(q-p-1)}{(q-1)(p-1)^{\frac{q-2p+1}{p-1}}}N_e^{\frac{q-2p+1}{p-1}}\Delta N_e + \mathcal{O}(\Delta N_e^2)\ . \quad (3.48)$$

For the particular case of exponential potentials arising from $p = 2$ and $q = 4$, setting $a_2 = 1$ and $a_4 = -\varepsilon^2$ we obtain

$$\left.\frac{\delta\Delta_s^2(\delta n_s)}{\Delta_s^2}\right|_{N_e+\Delta N_e}^{N_e} = -\frac{2}{3}\varepsilon^2 N_e\Delta N_e + \mathcal{O}(\Delta N_e^2)\ . \quad (3.49)$$

Using $N_e = 60$ we see that a bound $\varepsilon^2 \lesssim 2\times 10^{-4}$ limits the shift of the spectral index to $\delta n_s \lesssim 0.008$ which is the 2-σ range for the n_s measurement from PLANCK. By plugging in these numbers and the range of e-folds $\Delta N_e \simeq 5$ over which power-loss occurs we find the power loss for this case to be

$$\left.\frac{\delta\Delta_s^2(\delta n_s)}{\Delta_s^2}\right|_{N_e+\Delta N_e}^{N_e} = -\frac{2}{3}\varepsilon^2 N_e\Delta N_e \simeq -0.04\,, \quad (3.50)$$

which is about 4%. This is in qualitative agreement with previous studies employing exponentially rising corrections [14, 30–35, 50, 51]. For order-one values of a_p, p and q, one obtains similar results.

Finally, we have discussed the possible UV embedding of non-canonical inflation. While Kähler potentials of logarithm type are bread and butter in string theory compactifications, loop corrections can induce higher-order terms in the Kähler potential. These would generically result in a shift symmetry breaking at large field displacements. We leave a concrete embedding of these terms into a reliable string theory set-up for future investigation. In particular, the properties of complex structure moduli space close to a conifold point may provide a viable path to embedding our structure into string theory, while working with volume moduli close to zero volume (if one took the above toy comparison literally) is clearly a badly controlled regime.

While elegant, rephrasing the formulation of inflationary dynamics in terms of a non-trivial kinetic function does not reduce the severity of the need to fine tune the scenario. Having to impose condition (3.13) is one of our central findings. Further, one formulation may not be understood as more fundamental than the other. However, it recasts the context in which the fine tuning has to occur such that new insights may be possible. Concretely, since a non-canonical kinetic term arises an intermediate step in any UV derived 4D effective theory, it is instructive and may even provide a short cut to know which types of non-canonical kinetic terms affect possible inflationary dynamics in what way.

Chapter 4

Shift symmetry and $f(R)$

Having established how current observations strongly favour models of inflation that exhibit an approximate and continuous shift symmetry when formulated in canonical fields and moreover providing two distinct exemplary frameworks of realisation above, we now turn our attention to a seemingly rival paradigm to describe early universe dynamics, namely modified gravity or hereafter simply $f(R)$.

In fact, one of the earliest models of inflation [9], for which also cosmological perturbation theory was first fully worked out, was presented in the language of a modified Einstein-Hilbert Lagrangian. As the name suggests, $f(R)$ theories replace the Einstein-Hilbert term in the Lagrangian with some function of the Ricci curvature scalar. While at first this seems to provide a mechanism to change spacetime dynamics without relying on some matter content, introducing an arbitrary function of the Ricci scalar in fact adds a hidden scalar degree of freedom to the theory. Hence, two possible avenues to analyse the resulting dynamics may be taken: One may either vary the action to derive and then solve the modified Einstein equations or attempt to conformally transform the theory to a formulation with the conventional Einstein-Hilbert term and the additional scalar degree of freedom made explicit.

We are precisely interested in the second route. In what follows, we will investigate the duality between $f(R)$ and scalar field theories [15]. More precisely, we study generic exponential plateau-like potentials to understand whether an exact

$f(R)$-formulation may still be obtained when the asymptotic shift symmetry of the potential is broken for larger field values. Thus, we will identify the properties any $f(R)$ theory has to maintain in order to drive a sufficient amount of slow-roll inflation complying with observations.

While doing so, we further find a lean and instructive way to obtain a function $f(R)$ describing $m^2\varphi^2$-inflation which breaks the Einstein frame shift symmetry with a monomial. We demonstrate how potentials with exponentials $\exp(-\gamma\kappa\varphi)$ with $0 < \gamma < 2$ (i.e. effectively rescaling the exponents $\kappa \to \kappa'$ of the exponential) induce corrections of type $R^{2-\gamma}$ or equivalently (at leading order if $\gamma \ll 1$) $\log R$ corrections to the corresponding $f(R)$-dual. This is in line with the observations made in [56], and then [57–61] that adding a term $R^{2-\epsilon} \supset R^2 \log R$ can enhance the tensor mode signal significantly over the pure $R + R^2$ case. We show that these models can provide for chaotic inflationary dynamics within the observable range of e-folds. Hence, they yield $f(R)$ duals to chaotic inflation models from logarithmically broken scale invariance in the Jordan frame discussed in [62, 63]. Additionally, we relate the function $f(R)$ corresponding to chaotic inflation to a more general Jordan frame set-up. We continue to consider $f(R)$-duals of two given UV examples, both from supergravity and string theory and link the considerations of this chapter to the UV examples given previously. Finally we apply our models with rising exponential terms to some of the aspects of the suppression of CMB power at large angular scales. We find that the corrections to the scalar field potential required to have a strong suppression effect on large-angle CMB power no longer have an exact but only an asymptotic $f(R)$-dual.

4.1 $f(R) \to R^2$ – Shift symmetry at large fields

We start this section with a brief review of the essentials of $f(R)$-theory (we refer the reader looking for a more thorough review to e.g. [64]). Consider a modified Einstein-Hilbert Lagrangian of the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2} f(R) \,, \tag{4.1}$$

with the Ricci scalar and tensor given by $R = g^{\mu\nu}R_{\mu\nu}$ and $R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu}$ respectively. Performing a conformal transformation on the metric $g_{\mu\nu}$ with conformal factor Ω

$$\tilde{g}_{\mu\nu} = \Omega\ g_{\mu\nu}, \quad \sqrt{-\tilde{g}} = \Omega^2\ \sqrt{-g}\,, \tag{4.2}$$

one can show that the Ricci scalar transforms as

$$R = \Omega\left(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right), \tag{4.3}$$

with $\omega \equiv (1/2)\ln\Omega$ and $\tilde{\Box} = \tilde{\nabla}^\mu\tilde{\nabla}_\mu$. The behaviour of the Ricci scalar under this Weyl transformation makes the additional degree of freedom, ω, of equation (4.1) apparent. Let us now rewrite the Jordan frame Lagrangian (4.1) as

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}f'R - U\,, \tag{4.4}$$

where the prime denotes differentiation with respect to R and we have introduced $U = (1/2)(f'R - f)$. Substituting (4.3) into (4.4) and applying (4.2) yields

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{1}{2}\Omega^{-1}f'\left(\tilde{R} - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right) - \frac{U}{\Omega^2}\,, \tag{4.5}$$

where we have omitted the $\tilde{\Box}\omega$ term as its contribution to the action vanishes due to Gauss' theorem. Defining $\varphi \equiv \sqrt{3/2}\ln\Omega$ normalises the kinetic term.[23] Choosing the conformal factor $\Omega = f'$ for $f' > 0$ brings the action to the Einstein frame where the Lagrangian takes the form

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{\tilde{R}}{2} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\,, \tag{4.6}$$

with the potential for the scalar degree of freedom given by

$$V(\varphi) = \frac{f'R - f}{2f'^2}\,. \tag{4.7}$$

[23] We could also have chosen $\varphi = -\sqrt{3/2}\ln\Omega$. This essentially mirrors the resulting potential with $V(\varphi) \to V(-\varphi)$.

The relation between the Jordan frame scalar curvature and the canonically normalised scalar field, $R(\varphi)$, can be found by inverting $\varphi = \sqrt{3/2}\ln f'$ for a given $f(R)$. Thus an $f(R)$-theory may be recast in terms of gravity with a scalar field, provided f' is invertible to allow for a relation between the Ricci scalar and the canonically normalised field φ. The frame function f' has to satisfy the same constraints as non-minimal coupling function (1.27) given previously.

Examples

In this chapter we study the mapping between the Jordan frame $f(R)$ and the Einstein frame scalar potential $V(\varphi)$. We first demonstrate how to obtain a potential $V(\varphi)$ from a given $f(R)$ and vice-versa for the well known Starobinsky model [65]

$$f(R) = R + \alpha R^2 \,. \tag{4.8}$$

Choosing $f' = e^{\kappa\varphi}$, we find $R(\varphi) = (2\alpha)^{-1}(e^{\kappa\varphi} - 1)$. Evaluating the expression for $V(\varphi)$ of (4.6) then yields

$$V(\varphi) = V_0 \left(1 - e^{-\kappa\varphi}\right)^2 , \tag{4.9}$$

where $\kappa = \sqrt{2/3}$ and $V_0 = 1/8\alpha$. This is the well know Starobinsky potential that provides a model for cosmic inflation along the plateau of the potential, where $V \sim V_0$.

Let us consider a small deformation of the theory (4.9) and contemplate a potential of the form

$$V(\varphi) = V_0 \left(1 - C_0 e^{-\frac{\kappa}{2}\varphi} + C_1 e^{-2\kappa\varphi}\right) , \tag{4.10}$$

with $C_0 - C_1 = 1$ to ensure $V(0) = 0$. The difference to potential (4.9) is that the second exponential is no longer the square of the first. Even though this does not significantly affect the inflationary phenomenology, the nature of the duality changes as we will show in the following. Identifying $f' = e^{\kappa\varphi}$, we recall the relation between Einstein frame $V(\varphi)$ and Jordan frame $f(R)$, equation (4.7), and thus consider the differential equation

$$\frac{f'R - f}{2f'^2} = V_0 \left(1 - \frac{C_0}{\sqrt{f'}} + \frac{C_1}{f'^2}\right). \tag{4.11}$$

By solving this equation one may find the $f(R)$ theory that is dual to the potential of equation (4.10). Multiplying the above with $2f'^2$ and differentiating with respect to either R, f, or f' gives

$$f' - \frac{3}{4}C_0\sqrt{f'} - \frac{1}{4V_0}R = 0\,, \tag{4.12}$$

provided $f'' \neq 0$. This can be solved as

$$f' = 2\left(\frac{3}{8}C_0\right)^2 + \frac{1}{4V_0}R \pm \frac{3}{4}C_0\sqrt{\left(\frac{3}{8}C_0\right)^2 + \frac{1}{4V_0}R}\,. \tag{4.13}$$

Considering (4.11) as a boundary condition[24], we can then integrate the above to find the corresponding $f(R)$-theory to be

$$f(R) = \frac{R}{2} + \frac{8V_0}{3}\left(\frac{1}{4} + \frac{R}{4V_0}\right)^{3/2} + \frac{R^2}{8V_0} - \frac{V_0}{3}\,, \tag{4.14}$$

where we have chosen $C_0 = 4/3$ and $C_1 = 1/3$ such that $V(0) = 0$ and the positive sign in (4.13) so that $f(R) \geq 0$ and $f(0) = 0$. Note that the expression (4.14) is $\sim R^2$ in the large R regime but has a modified behaviour for intermediate R. This comes as no surprise since potential (4.10) also has an approximate shift symmetry for large field values but, as opposed to (4.9), does not display the quadratic relation among the exponentials of the potential, which in turn influences the behaviour of the theory at intermediate R and φ.

In principle, any potential $V = V_0(1 - 2e^{-\frac{\kappa}{n}\varphi} + e^{-2\kappa\varphi})$ with $n > 1$, which hence breaks the square relation of the two exponentials, has an approximate dual $f(R) \sim R^2$ for large R. Similarly, breaking the square relation with the second exponential such that $V = V_0(1 - 2e^{-\kappa\varphi} + e^{-n\kappa\varphi})$ with $n > 2$ also has $f(R) \sim R^2$ as

[24]The integration constant is chosen such that the resulting function $f(R)$ satisfies (4.11) when inserted. Thus the information which has been lost upon differentiating (4.11) is regained.

the large R dual when the corresponding differential equation may not be solved explicitly any more.[25] Both results are expected as the corresponding potentials maintain an approximate shift symmetry at large field values. This can further be demonstrated by simply considering $f(R) = \alpha R^2$ which reduces to a pure cosmological constant $\Lambda = (8\alpha)^{-1}$ in the Einstein frame.

4.2 Logarithmic corrections to $f(R)$

4.2.1 Changing the coefficient κ

The line of argument given in the previous section requires the potential to contain exponentials with a coefficient $\kappa = \sqrt{2/3}$ in the exponent.[26] However, in the same spirit that led us to deviate from the square relation between the exponentials in V, we may now consider potential (4.9) with a change of the coefficient of the exponent, i.e. with $\kappa \to \kappa' \neq \kappa$:

$$V(\varphi) = V_0 \left(1 - 2e^{-\kappa'\varphi} + e^{-2\kappa'\varphi}\right), \tag{4.15}$$

and try to understand to what extent this deformation of the original model alters the gravitational $f(R)$ description. To apply the previous method, we have to recast potential (4.15) with coefficient κ' in terms of $\kappa = \sqrt{2/3}$. When we choose e.g. $\kappa' = 2/\sqrt{3} = \sqrt{2}\,\kappa$, we write (4.15) in terms of κ as

$$V(\varphi) = V_0 \left(1 - 2e^{-\sqrt{2}\kappa\varphi} + e^{-2\sqrt{2}\kappa\varphi}\right). \tag{4.16}$$

Rather than attempting to analytically solve a differential equation with irrational powers in f', we now integrate implicitly to find

$$f(R) = \frac{R^2}{8V_0} + \alpha \int f'^{1-\sqrt{2}} dR - \beta \int f'^{1-2\sqrt{2}} dR\,, \tag{4.17}$$

[25] We would like to refer the reader to appendix A.1 for a concise demonstration of these statements.

[26] Applying the above procedure with $f' = e^{\kappa'\varphi}$, where $\kappa' \neq \sqrt{2/3}$, introduces a non-canonical kinetic term in (4.6).

where α, β depend on V_0 and the φ - coefficients of the exponentials in (4.15). Considering the inflationary regime, i.e. imposing the regime of large R, we approximate $f' \sim R$ and write

$$f(R) = \frac{1}{8V_0}R^2 + c\ R^{2-\sqrt{2}} + \dots \tag{4.18}$$

where c is a rescaled α due to the integration. Hence a given inflaton potential with a rescaled exponent such as

$$V(\varphi) = V_0\left(1 - e^{-\gamma\sqrt{\frac{2}{3}}\varphi}\right)^2, \tag{4.19}$$

where $0 < \gamma < 2$, has an approximate $f(R)$ dual given by

$$f(R) = \frac{1}{8V_0}R^2 + cR^{2-\gamma} + \dots \tag{4.20}$$

up to sub-leading terms during slow-roll inflation (see appendix A.2 for an explicit argument).

4.2.2 Chaotic inflation from $f(R)$-theory

Observable inflation in the Starobinsky model proceeds in the transition region between the flat plateau and the minimum of the potential, ending at $\varphi \sim 1$ M_{Pl}. Observationally, it yields a spectral index compatible with observations and very low primordial tensor fluctuations. The phenomenologist may ask if there is a deformation of the standard Starobinsky model that significantly changes the observational signatures, making them more in line with those of chaotic inflation and if so what is its corresponding gravitational dual.

Note that potential (4.19) may well approximate chaotic inflation for intermediate field values provided $\gamma \ll 1$. To see this, consider the series expansion of (4.19) around the point $\varphi = 0$

$$V(\varphi) \approx \kappa^2\gamma^2\varphi^2 - \kappa^3\gamma^3\varphi^3 + \mathcal{O}(\gamma^4\varphi^4)\,. \tag{4.21}$$

Hence for $\gamma \ll 1$, inflation occurs in the concave region of the potential, i.e. before

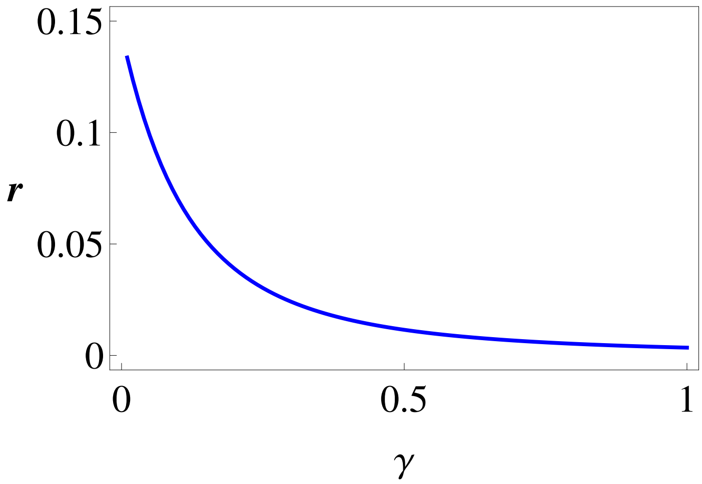

Figure 4.1: *The plot depicts the observable tensor to scalar ratio r for potential* (4.19) *or equivalently its dual* (4.20) *as a function of the parameter γ in the range* $0 < \gamma < 1$*. For sufficiently small γ, the value of r converges to the prediction of $m^2\varphi^2$ inflation. The tensor to scalar ratio has been calculated 55 e-folds before the end of inflation.*

the nearly shift-symmetric plateau. Thus (4.20) can be an $f(R)$ dual for a potential that has a chaotic regime before approaching the shift-symmetric plateau. Hence adding a term of type $R^{2-\gamma}$ with $\gamma \ll 1$ to a Starobinsky $f(R)$ can induce a chaotic regime and hence change the predictions for the spectral index n_s and tensor-to-scalar ratio r to those of φ^m models, depending on the specific value of γ as depicted in Figure 4.1. This confirms the findings of [56, 59, 61, 66] but without omitting the R^2-term in the first place. The R^2-induced plateau can simply be pushed beyond the last 60 e-foldings of inflation and hence does not influence the inflationary predictions, yet it still determines the behaviour at large field values or e-foldings corresponding to super-horizon scales.

Now consider $\gamma \ll 1$ and expand (4.20) as

$$\begin{aligned} f(R) &\approx R^2 + R^{2-\gamma} + \dots \\ &\approx R^2 \left[1 + e^{-\gamma \ln R}\right] + \dots \\ &\approx R^2 \left[1 + \sum_{n=0}^{\infty} \frac{(-\gamma \ln R)^n}{n!}\right] + \dots \\ &\approx R^2 - \gamma R^2 \ln R + \frac{\gamma^2}{2} R^2 \ln^2 R - \mathcal{O}(\gamma^3 \ln^3 R) + \dots \end{aligned} \tag{4.22}$$

The first two terms of (4.22) resemble the ansatz of [57]. Revisiting Starobinsky's original idea, it was investigated whether or not logarithmic corrections to R^2-inflation change the inflationary observables, specifically whether a larger amount of tensor modes is produced. It was found that even though a first-order logarithmic correction lifts the plateau, a significant enhancement of tensor modes does not occur.

We hence realise that although higher order terms seem to be sub-leading at first sight, they change the inflationary dynamics drastically. Whereas a linear logarithmic correction only causes a slight deviation in the inflationary observables, higher-order terms can indeed make the inflationary dynamics chaotic within the accessible range of inflationary e-folds. Thus if one assumes higher-order terms to exist, they have to be considered. This confirms the findings of [60], where parametric methods are used to obtain an $f(R)$-theory corresponding to $m^2\varphi^2$-inflation and it is found that a linear logarithmic correction is not sufficient but a squared term is necessary.

Moreover, hierarchy (1.38) tells us that the above scenario is not realised in nature, as it would violate the bound $r < 0.1$ (the value for the spectral index is similar). We hence conclude that corrections to the Starobinsky model ought not to come as an expansion in logarithms. Corrections not at odds with observations will be considered in section 4.3.

4.2.3 Another Jordan frame

We have seen in the previous sections of this chapter how to obtain a function $f(R)$ describing the dynamics of chaotic inflation. However, one may also ask how a corresponding scalar field theory being non-minimally coupled to gravity looks like, hoping to learn something more general regarding the mapping between some Jordan and the Einstein frame.

One can always go from the Starobinsky frame to the Einstein frame and then try to find another Jordan frame again. We now propose how to go from the Starobinsky frame to a specific Jordan frame directly, namely the Jordan frame with a non-minimal coupling function

$$\mathit{\Omega}(\phi) = 1 + \xi\phi^2\,. \tag{4.23}$$

For the remainder of this section we will use $\tilde{R}$ to denote the Starobinsky frame curvature scalar, while R denotes scalar curvature in the new Jordan frame. We start by first recalling a slightly rewritten Lagrangian (4.5), i.e. the Weyl-transformed Lagrangian of some $f(\tilde{R})$-theory,

$$\frac{\mathcal{L}}{\sqrt{-g}} = \Omega^{-1} f' \left[\frac{R}{2} - \frac{3}{4}\left(\partial \log \Omega\right)^2\right] - \frac{U^2}{\Omega^2}\,, \tag{4.24}$$

where we have rescaled the metric with the function Ω. For (4.24) to simply reduce to the Einstein frame, we would just choose $\Omega = f'$. However, now we seek a Jordan frame given by (4.23), hence in (4.24) we require

$$\Omega^{-1} f' \equiv 1 + \xi\phi^2. \tag{4.25}$$

We then have to normalise the kinetic term accordingly in order to obtain a relation between Ω and ϕ. We thus write

$$\frac{3}{4}\left(1 + \xi\phi^2\right)\left(\partial \log \Omega\right)^2 \equiv \frac{1}{2}\left(\partial\phi\right)^2\,, \tag{4.26}$$

which, in the regime of strong coupling, i.e. $\xi \gg 1$, may be solved to yield

$$\Omega(\phi) = \phi^{\Delta}, \tag{4.27}$$

where $\Delta = \kappa/\sqrt{\xi}$ with $\kappa = \sqrt{2/3}$. Thus for strong coupling, $\xi \gg 1 \Leftrightarrow \Delta \ll 1$. Combining the above with (4.25) gives

$$f' = \phi^{\Delta}\left(1 + \xi\phi^2\right) . \tag{4.28}$$

The resulting Jordan frame potential can then be written as

$$V_J(\phi) = \frac{\phi^{-2\Delta}}{2}\left(f'\tilde{R} - f\right) , \tag{4.29}$$

where $\tilde{R}$ denotes the Starobinsky frame curvature scalar. Then $\tilde{R} = \tilde{R}(\phi)$ as well as $f(\tilde{R}(\phi))$ can be obtained from specifying a certain $f(\tilde{R})$ and obtaining the relation between $\tilde{R}$ and ϕ from (4.28). The resulting Jordan frame Lagrangian is then given as

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = \left(1 + \xi\phi^2\right)\frac{R}{2} - \frac{1}{2}\left(\partial\phi\right)^2 - \frac{\phi^{-2\Delta}}{2}\left[f'(\tilde{R}(\phi))\tilde{R}(\phi) - f(\tilde{R}(\phi))\right] . \tag{4.30}$$

In case of the Starobinsky model $\tilde{R} + \alpha\tilde{R}^2$, one indeed recovers a non-minimally coupled scalar field theory with a ϕ^4 potential. One finds

$$\tilde{R}(\phi) \sim \phi^{2+\Delta} , \tag{4.31}$$

and therefore

$$V_J(\phi) \sim \frac{\phi^{-2\Delta}}{2}\tilde{R}(\phi)^2 \to \phi^4 , \tag{4.32}$$

which is the leading-order potential of the well-known non-minimally coupled ϕ^4-inflation model [67, 68].

Let us now investigate, whether we can directly find a corresponding non-minimally coupled scalar field theory that is dual to an inflationary period driven by a term $\tilde{R}^{2-\gamma}$ with $\gamma \ll 1$. To that end, consider

$$f'(\tilde{R}) = (2-\gamma)\tilde{R}^{1-\gamma} \tag{4.33}$$

as the derivative of the function $f(\tilde{R})$ for the range of observable e-folds.

The potential (4.29) then is

$$V_J(\phi) = \frac{\phi^{-2\Delta}}{2}\left[\tilde{R}(\phi)^{2-\gamma} - \gamma\tilde{R}(\phi)^{2-\gamma}\right], \tag{4.34}$$

where the second term in the brackets is sub-leading as $\gamma \ll 1$. We further have from (4.28) at large ξ

$$f' \sim \phi^{2+\Delta} \tag{4.35}$$

and thus

$$\tilde{R}(\phi) \sim \phi^{(2+\Delta)/(1-\gamma)}\,. \tag{4.36}$$

Inserting the above back into (4.34) gives, upon expansion of the exponent

$$V_J(\phi) \sim \phi^4\phi^{2\gamma+\mathcal{O}(\gamma^2)}\,. \tag{4.37}$$

This can then be rewritten as

$$V_J(\phi) \sim \phi^4\left[1 + c_1\gamma\log\phi + c_2\gamma^2\log^2\phi + \mathcal{O}(\gamma^3)\right], \tag{4.38}$$

and hence may be understood as a quantum corrected non-minimally coupled model of ϕ^4 inflation[27].

This demonstrates how logarithmic corrections to some function $f(R)$ can be directly related to logarithmic corrections to the potential in another Jordan frame. Hence our results here close the $f(R) \to V_J \to V_E \to f(R)$ circle started by the analysis in [63] where such logarithmic corrections to a ϕ^4 Jordan frame potential were found to produce Einstein frame quadratic large-field inflation for arbitrary frame function $\Omega(\phi)$.[28]

[27] These results may easily be extended to models in which the non-minimal coupling scales as $\Omega(\phi) \sim \sqrt{V(\phi)}$ such as in [69], given the frame function is to leading order of power law type.

[28] This can nicely be demonstrated by considering a Jordan frame Lagrangian $\mathcal{L}_J = \sqrt{-g}\left[\Omega(\phi)R/2 - 1/2\,(\partial\phi)^2 - V_J\right]$ with $V_J \propto [\Omega(\phi)\log\Omega(\phi)]^2$. This is an immediate general-

4.3 Exponential shift symmetry breaking

4.3.1 Rising exponentials

Returning to Starobinsky-esque potentials, we now want to determine functions $f(R)$ corresponding to a class of potentials which break the shift symmetry of the inflationary plateau by a rising exponential at large field values. Hence consider exemplary potential

$$V(\varphi) = V_0 \left(C_3 - C_0 e^{-\frac{\kappa}{2}\varphi} + C_1 e^{-2\kappa\varphi} + C_2 e^{\kappa\varphi} \right) , \tag{4.39}$$

where $C_2 \ll C_0, C_1, C_3$ and the sum over all C_i is unity. We recall the expression for $V(\varphi)$ in terms of $f(R)$ and its derivative, equation (4.7), and obtain the differential equation

$$f'^2 + \frac{2C_3}{3C_2} f' - \frac{C_0}{2C_2} \sqrt{f'} - \frac{1}{6V_0 C_2} R = 0 \,. \tag{4.40}$$

As in the previous section one would like to solve this equation to determine the exact $f(R)$ dual to the potential of equation (4.39), however (4.40) is not easily solvable. We hence learn that adding a rising exponential to the potential (4.10) prevents us from finding a dual $f(R)$ description for the entire field range. Nevertheless asymptotic results are still within reach. Focusing just on the rising exponential, we may now want to find the approximate $f(R)$-dual for potentials that are generically

$$V(\varphi) \sim V_0 \; e^{n\kappa\varphi} \,, \tag{4.41}$$

at large field values, with $n \geq 1$. Recalling (4.7), we write

$$V_0 f'^n = \frac{f'R - f}{2f'^2} \,, \tag{4.42}$$

which we rearrange as

$$2V_0 f'^{n+2} - f'R + f = 0. \tag{4.43}$$

ization of the observation made in [63]. In the Einstein frame, this theory has a purely quadratic potential for a canonically normalised inflaton.

Differentiating with respect to R and solving for f' then yields

$$f' = \frac{1}{2(n+2)V_0} R^{1/(n+1)} . \tag{4.44}$$

Integrating, we find the leading order solution

$$f(R) \sim R^{(n+2)/(n+1)} . \tag{4.45}$$

Even though the full analytic expression cannot always be found, it is possible to give an approximate form, where the resulting $f(R)$ is always R^n with $n < 2$ to leading order. Considering a correction with very large n, we see that the resulting $f(R)$ corresponds to a theory where the exponent of the non-linear term may effectively be understood as a perturbation around unity. Similarly, we saw above that a term $R^{2-\gamma}$ with $\gamma \ll 1$ produces a chaotic regime, with the specific inflationary signatures depending on the tuning of γ. This can be understood, as has already been pointed out in [70–73], as a correction of an R^2 term around its exponent. Thus we see that small deviations from integer powers in a polynomial function $f(R)$ have a major effect on the resulting potential inflationary observables.

4.3.2 Maintaining a plateau

That a R^n term with $n > 2$ cannot steepen the corresponding potential could also have been foreseen from the following argument. Consider a function $f(R)$

$$f(R) = \sum_{n=1}^{N} a_n R^n , \tag{4.46}$$

where all $a_n > 0$. In the limit of large R, $f(R) \to a_N R^N$ and hence $f' \sim R^{N-1}$. We now seek the behaviour of the potential for very large φ and thus very large R. Considering expression (4.7) for $V(\varphi)$, we obtain

$$V(\varphi(R)) \to R^{2-N}. \tag{4.47}$$

Hence it appears that only $N = 2$ produces a plateau in the potential at large φ, any power $N < 2$ curves the potential upwards whereas powers of $N > 2$ have it asymptote the axis. This is in line with [74].

4.3.3 Finite order corrections

We now consider whether there is a way to steepen the potential by considering finite higher order corrections to an $R+R^2$ theory, where the coefficients of higher order terms may be negative. As we established above, terms with positive coefficients and power $n > 2$ curve the potential downwards towards the axis. Thus consider a function $f(R)$ such that

$$f(R) = R + \alpha R^2 + \beta R^3, \tag{4.48}$$

but with $\beta < 0$. We then find

$$\varphi = \sqrt{3/2}\ln\left(1 + 2\alpha R - 3|\beta|R^2\right)\,, \tag{4.49}$$

or equivalently

$$R(\varphi) = \frac{\alpha}{3|\beta|} \pm \sqrt{\frac{1}{9|\beta|^2} + \frac{1}{3|\beta|} - e^{\kappa\varphi}}\,. \tag{4.50}$$

Thus the field φ is only defined as long as $f' > 0$, in other words, the field space is limited.[29] Equivalently, the Ricci scalar R becomes complex when f' changes sign. For $f' \to 0$, we find that $\varphi \to -\infty$. This demonstrates that the spacetime region where $f' < 0$ is disconnected from the $f' > 0$ region as it is pushed infinitely far away in field space. Further, it is known (see e.g. [64]) that ghosts appear once $f' < 0$. However, a negative coefficient β will introduce some steepening over a finite range.

We further note that higher powers in R with negative coefficients do not increase the steepening of the potential but simply shorten the field range over

[29]This modification has already been investigated in [75]. Importantly, it was argued that a universe with $f' = 0$ for some R consists of two causally disconnected regions, one in which $f' < 0$ and one with $f' > 0$. Hence a universe with $f' < 0$ can never evolve into Minkowski space.

which the field φ is defined, as depicted in the first plot of Figure 2. Again, higher powers are only tractable as long as (4.49) remains analytically soluble for R.

4.3.4 A full $f(R)$ toy model

Having established result (4.45), we may now however ask if there exists a function $f(R)$ that is not dominated by a quadratic term at large R, but - in its series expansion - has a quadratic term that dominates in an intermediate regime.

In the Einstein frame language this corresponds to having a scalar field potential that has a plateau separating the minimum at the origin from a growing exponential at large field values (see the right plot of Figure 4.2). Consider

$$V(\varphi) = V_0 \left(1 - 2e^{-\kappa\varphi} + e^{-2\kappa\varphi} + \delta e^{n\kappa\varphi}\right) - \delta\, V_0 \,, \tag{4.51}$$

with $\delta \ll 1$ and $n = 1$. The subtraction of $\delta\, V_0$ ensures $V(0) = 0$. Following the usual method we find the differential equation

$$f'^2 + \frac{2}{3\,\delta}(1-\delta)f' - \frac{2}{3\,\delta} - \frac{R}{6\,\delta\,V_0} = 0 \,, \tag{4.52}$$

from which we obtain an equation for R, namely

$$R = 6V_0\delta f'^2 + 4V_0(1-\delta)f' - 4V_0 \,. \tag{4.53}$$

Substituting $f' = e^{\kappa\varphi}$ then yields $R(\varphi)$ for the potential of equation (4.51). The solution to (4.52) reads

$$f(R) = \frac{\delta - 1}{3\delta}R + 4\delta V_0 \left[\frac{(1-\delta)^2}{9\delta^2} + \frac{2}{3\delta} + \frac{R}{6\delta V_0}\right]^{3/2} + K \,, \tag{4.54}$$

where K is a constant of integration determined by a boundary condition as in (4.11). This constitutes one of the main results of this chapter. Explicitly, the integration constant K has to be chosen such that the function $f(R)$ satisfies the boundary condition (4.11)

$$\frac{f' \cdot R - f}{2f'^2} = V_0 \left(1 - 2e^{-\kappa\varphi} + e^{-2\kappa\varphi} + \delta e^{n\kappa\varphi}\right) - \delta \cdot V_0 \,, \tag{4.55}$$

where again $e^{\kappa\varphi} = f'$. Satisfying (4.11) also ensures that $f(0) = 0$ and hence a Taylor expansion of (4.54) recovers the Starobinsky model to leading order. Thus adding a rising exponential to the Starobinsky potential (4.9) yields a dual $f(R)$ description (4.54) that is not dominated by a quadratic term at large R. Note that the above holds for $n = 1$ in the rising exponential. A higher n increases the leading power in (4.52) and induces logarithmic and inverse trigonometric terms in the $f(R)$ description. Analytic solutions of (4.52) in terms of f' only exist up to some n. We hence focus on the case $n = 1$. Further note that for the above, we find from (4.53)

$$R|_{\varphi=0} = 2\delta V_0 \,, \tag{4.56}$$

which is $\ll \mathcal{O}(10^{-10})$ in Planck units. Thus there is a lower bound on R when $\varphi = 0$. We also find that

$$f(R|_{\varphi=0}) = 2\delta V_0 \,, \tag{4.57}$$

and, as expected, $f(0) = 0$. Hence when the field φ has reached its minimum at $V(0) = 0$, the corresponding $f(R)$-description displays a cosmological constant type term. This is also indicated by the fact that $f'|_{R=0} < 1$ but $f'|_{R=2\delta V_0} = 1$. Importantly, switching off the perturbation by sending $\delta \to 0$ restores $R|_{\varphi=0} = 0$ and thus the Starobinsky model. This can be demonstrated by Taylor expanding (4.54) to find the coefficient of the linear and the quadratic term in R to approach the value of the coefficients of the Starobinsky model when $\delta \to 0$.

Let us stress that (4.54) is an $f(R)$ theory with an infinite number of terms in its series expansion where the term $c_2 R^2$ induces the inflationary plateau. The infinite number of higher power terms does not curve the potential down but sums to an contribution that is to leading order $R^{3/2}$ and hence steepens the potential. In an EFT sense, (4.54) gives the full theory and determines the coefficients of every power.

4.4 Matching $f(R)$ to the UV

We now turn our attention to examples of inflationary potentials with intermediate shift symmetry that can be embedded into a candidate UV theory and study, whether a dual $f(R)$ formulation may be obtained.

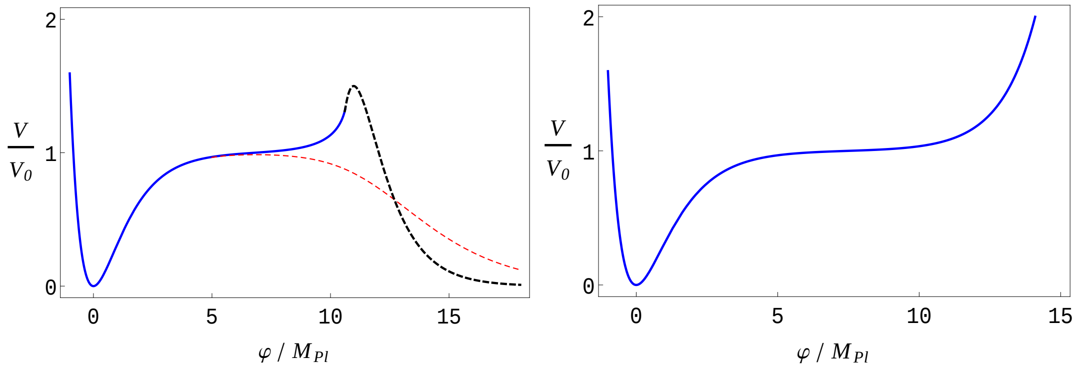

Figure 4.2: **Left:** *The blue line depicts the potential dual to* (4.48). *The black dashed line is the real continuation of the potential once R has become complex. The red dashed line displays the potential dual to a finite $f(R)$-theory, where the highest order term is $\sim R^3$ and has a positive coefficient.* **Right:** *The Einstein frame potential dual to the $f(R)$-theory* (4.54). *Contrary to* (4.48), *the field range is not limited, but the potential is lifted infinitely after the c_2R^2-term induced and intermediate plateau. In both cases, we have normalised the potential such that $V_0 \sim 10^{-10}$ in Planck units.*

4.4.1 No-scale supergravity

Let us start by considering the scenario of [76], where the inflaton superfield is described by a Wess-Zumino model. The potential for the real part of the inflaton superfield reads

$$V(\varphi) = \mu^2 \sinh^2\left(\frac{\varphi}{\sqrt{6}}\right)\left[\cosh\left(\frac{\varphi}{\sqrt{6}}\right) - \frac{3\lambda}{\mu}\sinh\left(\frac{\varphi}{\sqrt{6}}\right)\right]^2, \tag{4.58}$$

with φ driving inflation and μ, λ being parameters of the model. By expanding the hyperbolic, functions the potential (4.58) can be written in terms of exponentials as

$$V(\varphi) = C_0 e^{2\kappa\varphi} + C_1 e^{\kappa\varphi} + C_2 e^{-\kappa\varphi} + C_3 e^{-2\kappa\varphi} + C_4\,, \tag{4.59}$$

where the C_i are dependent on μ, λ, the sum over all C_i is zero and $\kappa = \sqrt{2/3}$. Judiciously choosing the coefficients of the quadratic and cubic terms in the Wess-Zumino superpotential such that $\lambda/\mu = 1/3$, the coefficients C_0, C_1 vanish and the above reduces to

$$V(\varphi) = \mu^2 e^{-\sqrt{2/3}\varphi} \sinh^2\left(\frac{\varphi}{\sqrt{6}}\right), \tag{4.60}$$

which is the Starobinsky potential and hence has $R+\alpha R^2$ as an exact dual formulation. Contrary to (4.51), the above (4.59) does not allow for a simple analytic solution for a corresponding function $f(R)$ in the general case $\lambda/\mu \neq 1/3$. However, just considering the leading term at large field values gives the differential equation

$$C_0 f'^2 = \frac{f'R - f}{2f'}\,, \tag{4.61}$$

which is easily solved to yield

$$f(R) \sim R^{4/3}\,. \tag{4.62}$$

This is an example for (4.45) and shows that even though the explicit function $f(R)$ is hard to find over the entire range of R, it must asymptote $R^{4/3}$ for large values.

Let us now investigate the limit of (4.58) when the ratio λ/μ is perturbed around the value of $1/3$ such that

$$\lambda/\mu \to \frac{1}{3} - \delta\,, \tag{4.63}$$

where δ is infinitesimally small. From (4.58) we can infer the coefficients in (4.59) to be

$$C_0 = \frac{1}{4}\left[\frac{1}{4} - \frac{3\lambda}{2\mu} + \left(\frac{3\lambda}{2\mu}\right)^2\right], \quad C_1 = \frac{1}{4}\left[3\frac{\lambda}{\mu} - \left(3\frac{\lambda}{\mu}\right)^2\right], \tag{4.64}$$

where the other C_i are not of interest for reasons to become clear in a moment. Plugging (4.63) into the above, we find that[30]

$$C_0/C_1 \sim \delta\,. \tag{4.65}$$

In other words, if λ/μ is perturbed with an infinitesimally small δ, the squared rising exponential in (4.59) is drastically suppressed with respect to the C_1-term. Thus one can well approximate (4.59) by a scalar potential such as (4.51) for

[30] For $\lambda/\mu = 1/3$, we have $C_0, C_1 = 0$. It is the fact that $C_0 \sim \delta^2$ and $C_1 \sim \delta$, in other words, C_0 approaches zero faster than C_1 which has (4.65) scale as δ even though $C_1(1/3) = 0$.

which the corresponding $f(R)$-dual (4.54) is exactly known. This scenario [76] is not only a supergravity realisation of the vanilla R^2-inflation model, but also maintains the duality for an infinitesimal perturbation of the parameters of the model as in (4.63).

4.4.2 Fibre inflation

We now return to the LVS [36–39]. As a representative thereof, we will consider inflation solely being driven by string-loop effects [39]. Thus, we will now include the winding term of (2.19) but omit the higher-derivative correction (2.8). The canonically normalised inflaton potential then takes the form

$$V(\varphi) = V_0' \left(\mathcal{C}_0 e^{\kappa'\varphi} - \mathcal{C}_1 e^{-\kappa'\varphi/2} + \mathcal{C}_2 e^{-2\kappa'\varphi} + \mathcal{C}_{up} \right), \tag{4.66}$$

with $\kappa' = 2/\sqrt{3}$. This potential maintains an approximately shift-symmetric plateau before a rising exponential starts to dominate. As the first negative exponential is the fourth root of the second term and the coefficient in the exponent is larger than the Starobinsky κ, this model features an enhanced gravitational wave signal compared to the Starobinsky model, without reaching a tensor signal of comparable order of magnitude to that of chaotic inflation. In this section we are interested in investigating whether the fibre inflation potential has an approximate $f(R)$-description. As before it is useful to recast (4.66) in terms of $\kappa = \sqrt{2/3}$. In the low φ regime, where the rising exponential is negligible, one obtains

$$V(\varphi) = V_0 \left(1 - \mathcal{C}_1 e^{-\frac{\kappa}{\sqrt{2}}\varphi} + \mathcal{C}_2 e^{-2\sqrt{2}\kappa\varphi} \right). \tag{4.67}$$

We thus have

$$f(R) = \frac{1}{8V_0} R^2 + \alpha \int f'^{1-1/\sqrt{2}} dR - \beta \int f'^{1-2\sqrt{2}} dR, \tag{4.68}$$

which, upon enforcing the large R regime[31], yields

$$f(R) = \frac{1}{8V_0} R^2 + \alpha' R^{2-1/\sqrt{2}} + \beta' R^{2-2\sqrt{2}} + \dots, \tag{4.69}$$

[31] Here, we require large R, yet still sufficiently small such the rising exponential has no effect.

where α', β' are rescaled coefficients due to the integration. We hence find an approximate $f(R)$-dual for the inflationary regime of the fibre inflation potential. Considering the regime in which the rising exponential dominates, we have

$$V(\varphi) \sim \mathcal{C}_0 e^{\sqrt{2}\kappa\varphi} \,. \tag{4.70}$$

Using equation (4.7) we obtain

$$f'^{1+\sqrt{2}} \sim R \,, \qquad \text{and therefore} \qquad f(R) \sim R^{\sqrt{2}} \,. \tag{4.71}$$

This is in accordance with our previous findings and demonstrates that the leading order term in an $f(R)$-theory corresponding to a rising exponential has to be of type R^n with $1 < n < 2$.

4.4.3 Changing the compactification

One can ask whether it is possible to modify the above set-up such that the coefficients in the exponents are the ones required to have integer powers of f' in the differential equation one has to solve to find a corresponding $f(R)$ theory. We would then effectively have a situation such as (4.40) which though not fully soluble, has separate analytic solutions for both the regime in which the rising exponential dominates as well as the inflationary regime. These two solutions would be exact and the point of matching could in principle be determined. To answer that question, we have to take a closer look at the string construction of fibre inflation and see how it may be modified.

The coefficient in the exponent of the exponential comes from the canonical normalisation of the kinetic term of the fibre modulus that drives inflation.[32] The kinetic term itself is derived from the volume of the compactification which, in the case of Fibre inflation, is given as

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1}\tau_2 - \gamma \tau_3^{3/2} \right) . \tag{4.72}$$

[32]We are excluding the possibility of manipulating the form of the string loop generated Kähler potential. Though this is possible in principle we consider it to be less well motivated.

This choice leads to $\kappa' = 2/\sqrt{3}$, corresponding to the following relation between the fibre modulus τ_1 and the canonically normalised field φ: $\tau_1 = e^{\kappa\varphi}$. Equation (4.72) when combined with the conjectured form of the Kähler potential generated by string loops on Calabi-Yau manifolds [37, 38] determines the potential (4.66) for the lightest Kähler modulus, τ_1. As we have seen above this does not allow us to find an exact $f(R)$ formulation for the model. One may however consider a slightly modified volume form, where the would be inflaton is still called τ_1 but now corresponds to the volume of the base manifold rather that of the fibre as in the original setting. This amounts to considering

$$\mathcal{V} = \alpha \left(\sqrt{\tau_2}\tau_1 - \gamma\tau_3^{3/2} \right) , \tag{4.73}$$

which yields $\kappa' = 1/\sqrt{3}$ and so once again one ends up with irrational powers of f' and is therefore unable to solve the associated differential equation. Alternatively one may compactify on a torus, such that

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1\tau_2\tau_3} - \gamma\tau_3^{3/2} \right) , \tag{4.74}$$

yielding $\kappa' = 1$. This exhausts the set of more obvious choices for $\mathcal{V}$ which has to be a polynomial of degree $3/2$ in the four-cycle volumes τ_i. Whether or not Fibre inflation or a variation can be modified in such a way as to maintain the Starobinsky $\kappa = \sqrt{2/3}$ is inconclusive as of now.

4.5 Phenomenology and discussion

So far we have considered theoretical implications for a dual $f(R)$-description once a rising exponential has been added to the potential $V(\varphi)$. We found that we either have to give up the dominating quadratic term, or that we have to limit the field range of φ. We now turn our attention to a phenomenological fingerprint of a steepened potential $V(\varphi)$.

As discussed in various contexts [30–34, 50, 51], a steepening of the inflaton potential in the vicinity of the point of 55 e-foldings can suppress power in the CMB temperature spectrum at large angular scales, i.e. low-ℓ. We now employ the numerical method reviewed in section 1.2. Considering potential (4.51) or equivalently its $f(R)$ dual (4.54), we find $n_s(55) = 0.970$ and $n_s(62) = 0.975$ and the power suppression is 1.7% to 3.2% where we have chosen $\delta \sim \mathcal{O}(10^{-4})$ to allow for a sufficient amount of e-folds and a red n_s. Considering scenario (4.48), we choose $\alpha \sim 10^9$, $|\beta| \sim 2 \cdot 10^{14}$ to satisfy the boundary conditions for the number of e-folds and a red n_s. The spectral index takes similar values as for (4.51) and the power suppression is 1.8% to 3.3%. Lowering the amount of observable e-folds to $N_e^{obs} \sim 50$ by assuming intermediate reheating temperatures and adjusting $|\beta| \to 10^{15}$ yields $n_s(45) = 0.978$ and $n_s(50) = 0.985$ and thus gives a suppression of about 5%.

In this chapter, we investigated the duality between scalar field theories being coupled minimally and non-minimally to gravity and $f(R)$-Lagrangians. We showed that any potential with an infinitely long plateau may be recast as an $f(R)$-theory which is $\sim R^2$ to leading order. We further demonstrated how to obtain an expression for an $f(R)$-Lagrangian driving chaotic inflation within the accessible range of inflationary e-folds. Weyl-rescaling from the $f(R)$-frame to another Jordan frame, we found a general expression relating an arbitrary $f(R)$-theory to a scalar field theory non-minimally coupled to gravity and consequently established that a series of logarithmic corrections in any Jordan frame leads to chaotic inflationary dynamics as becomes apparent when analysing the resulting Einstein frame inflaton potential. Having noted the different dynamics of the inflationary spacetime regarding the cut-off of the series of logarithmic corrections, we learn that higher order terms of the Ricci scalar are of crucial importance and may not be neglected if they exist. Turning to modifications of plateau-like potentials at higher field values, we gave the example of potential (4.10) which looses its closed form dual $f(R)$ description once a rising correction is added. We showed that any scalar potential which is dominated by rising exponentials at higher field values at least allows for an $f(R)$-dual that is $\sim R^n$ with $1 < n < 2$ to leading order. The Starobinsky potential (4.9) maintains a closed form dual formulation (4.54) when a rising exponential is considered (4.51). The important implication however is that the leading R^2-term is removed from the $f(R)$-dual, as expected. The resulting $f(R)$ theory is an infinite series which sums to an expression which is to leading order $R^{3/2}$. Furthermore, the correction may not come with arbitrary powers of exponential functions as (4.52) has to remain soluble to find an analytic expression for the function $f(R)$. Thus the form of correction to the Starobinsky potential is strongly limited if the duality shall be given by explicit expressions obtainable in closed form by known methods over the entire range. Only considering the limit of large field values, approximate $f(R)$ duals may easily be found explicitly. It is important to note that the inability to find closed form $f(R)$ duals does not exclude the existence of a dual description in terms of a theory of modified gravity. It merely demonstrates that the dual description is out of our reach with current methods.

Concluding, the inflationary behaviour and predictions can be substantially changed by considering slight corrections to the first two powers in a polynomial $f(R)$-theory. Investigating only finite order corrections, we pay the price of either having to place the inflaton on the right side of a hilltop or having a finite field range for the inflaton φ, depending on the sign of the coefficient of the higher power. Therefore, corrections to the Einstein frame potential with rising exponentials - provided an $f(R)$ dual may be found - remove the leading R^2-term whereas higher powers in R with negative coefficients limit the field range. Both are interesting theoretical consequences. Considering concrete UV examples, we find that scenario (4.58) can maintain an $f(R)$-dual (4.54) given a certain choice of model parameters. The string inflation scenario (4.66) does not allow for an exact $f(R)$-dual. Modifications of the string theory set-up, i.e. considering different Calabi-Yau compactifications, did not prove to be successful at a first attempt. Phenomenologically, rising exponentials coming to dominate the potential at higher field values induce some running of the spectral index n_s. However, considering (4.54) the running is small and the consequent effect of power suppression at low-ℓ is up to $\sim 3\%$. Hence if the observational significance of power suppression increases and the effect is found to be $\gtrsim 3\%$, one requires corrections to an Einstein frame potential of higher order than those that can be provided by an exact dual $f(R)$ description.

Chapter 5

Non-minimally coupled inflation

Until now, we have studied how to realise an effective shift symmetry first within UV theories, then by requiring features of a non-canonical kinetic term and at last within the framework of $f(R)$. Already in the previous chapter, we introduced, as a small detour, a non-minimally coupled Jordan frame field in order to demonstrate how logarithmic corrections in generic Jordan frames give rise to chaotic, i.e. leading order monomial inflationary potentials. In this chapter, we revisit the Jordan frame but with the specific goal in mind to realise an Einstein frame shift symmetry. Generically, the shift symmetries under consideration will always be intermediate and eventually be broken by higher order terms. Thus in a first section, we will in detail study the phenomenology of the breaking at large fields in the light of possible power suppression in the CMB temperature spectrum at low-ℓ [14]. The strength of power suppression will be linked to the order of the term breaking the inflationary plateau. With no assumptions about the UV, this can be seen as EFT spectroscopy. In a second section, we turn our focus towards a minimal realisation of observationally viable inflation with a non-minimally coupled scalar field and identify properties which are in excellent agreement with observational data [18]. This hence will describe a lean and robust mechanism to realise a shift symmetry in a minimal way.

5.1 Coupling the inflaton to gravity

We begin with a short review which also serves as our motivation by recalling a simple model of non-minimally coupled inflation. Making the underlying mechanism apparent, Higgs inflation [67, 68] can be generalised to arbitrary potentials. This is called the Universal Attractor (UA) [69]. The details are as follows: Consider the Jordan frame Lagrangian of a scalar field ϕ

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \frac{1}{2}\Omega(\phi)R_J - \frac{1}{2}\left(\partial\phi\right)^2 - V_J(\phi)\,, \tag{5.1}$$

with

$$\Omega(\phi) = 1 + \xi f(\phi), \quad V_J(\phi) = \lambda f(\phi)^2\,. \tag{5.2}$$

The metric can be conformally transformed by rescaling it with the non-minimally coupling function

$$g^E_{\mu\nu} = \Omega(\phi) g^J_{\mu\nu}\,, \tag{5.3}$$

where the superscripts denote Einstein and Jordan frame respectively. As stated in the beginning, to reduce to ordinary GR at low energies, one has $\Omega(\phi) > 0\,, \forall\phi$. Lagrangian (5.1) then becomes

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{1}{2}R_E - \frac{1}{2}\left[\frac{1}{\Omega} + \frac{3}{2}\left(\frac{\partial \ln\Omega}{\partial\phi}\right)^2\right]\left(\partial\phi\right)^2 - \frac{V_J}{\Omega^2}\,. \tag{5.4}$$

If the non-minimal coupling strength ξ is sufficiently large and provided the following relations are satisfied

$$\Omega \ll \frac{3}{2}\Omega'^2\,, \quad N_{CMB} \sim \frac{3}{4}(\Omega - 1)\,, \tag{5.5}$$

where N_{CMB} denotes the number of e-folds at horizon exit of scales now observable through the CMB, canonical normalization

$$\left(\partial\varphi\right)^2 = \frac{3}{2}\left(\partial \ln\Omega\right)^2 \tag{5.6}$$

yields

$$\Omega(\varphi) = e^{\kappa\varphi}\,, \tag{5.7}$$

where $\kappa = \sqrt{2/3}$.[33] The potential becomes

$$V_E = \frac{\lambda}{\xi^2}\left(1 - e^{-\kappa\varphi}\right)^2\,. \tag{5.8}$$

This is conformally dual to R^2-inflation [9,77]. The ratio $(\lambda/\xi)^2$ sets the amplitude of the CMB power spectrum.[34] Inflationary predictions are give by the formulae

$$n_s = 1 - \frac{2}{N_e} + \frac{3}{2}\frac{\log(N_e)}{N_e^2} + \ldots\,, \quad r = \frac{12}{N_e^2} - 18\frac{\log(N_e)}{N_e^3} + \ldots\,, \tag{5.9}$$

where N_e denotes the number of e-folds before the end of inflation and we have included subleading corrections from [49] to the well known leading order result. Note that also with the first subleading terms, these expressions only provide an approximation of the exact result which may easily be obtained numerically.

5.2 EFT spectroscopy

In this section, we will consider simple toy corrections to the UA relation (5.2). The focus lies on the non-minimal coupling strength ξ and the steepening of the potential induced by the corrections. As these corrections - depending on their position in field space - can leave a clear observable fingerprint in the the CMB temperature spectrum, we can infer which type of correction may be linked to the observed loss of temperature power at large angular scales, or low-ℓ.

5.2.1 Corrections

We will introduce corrections in the Jordan frame and will analyse the effect of such corrections on the Einstein frame potential and the attractor behaviour. To

[33] For simple monomial potentials $V_J \sim \phi^n$ with $2 \leq n \lesssim \mathcal{O}(10)$, a non-minimal coupling $\xi \sim \mathcal{O}(1)$ is sufficient for relations (5.5) to hold.

[34] The amplitude of the CMB temperature fluctuations (1.36) is given as $A_s = (24\pi^2)^{-1}V_E/\epsilon$, where we have chosen the notation of [7] instead of [21]. For the measured value of $A_s \sim \mathcal{O}(10^{-9})$, one readily obtains $\xi \sim 10^5\sqrt{\lambda}$.

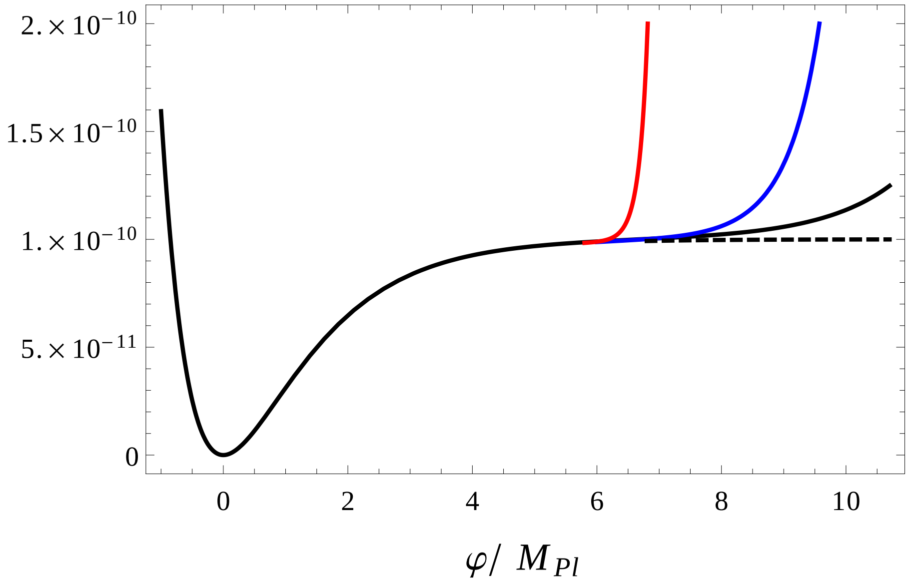

Figure 5.1: *Potential* (5.12) *with* $n = 1, 2, 8$ *(black, blue, red from right to left). Higher* n *require lower* ξ *if one seeks just* 55 *flat e-folds. The dashed line depicts the potential without corrections.*

this end, we may consider either a correction in ϕ with the cost of having to specify an invertible $f(\phi)$ in order to obtain a relation $\phi(\varphi)$, or - to allow for arbitrary $f(\phi)$ - simply parametrise a correction in terms of $f(\phi)$. Whereas the former leaves the coefficients of the series parametrizing the corrections unspecified, the latter maintains the beauty of the universal attractor, as any function $f(\phi)$ is allowed (examples are chaotic, natural and induced inflation, see [69,78]; moreover, we have verified for the chaotic case that corrections in ϕ or $f(\phi)$ yield similar findings). Hence we consider the latter case and replace the attractor relation (5.2) with

$$V(\phi) = \lambda^2 h(f(\phi))^2 \,, \tag{5.10}$$

where λ^2 remains a free parameter. The deviation of $h(f)$ from a linear function encapsulates the correction to the attractor relation. To develop a first intuition for such corrections, we consider a toy model with a single additional term. Taking

$$h(f) = f(1 + c_n f^n) \,, \tag{5.11}$$

and choosing $c_n \sim \mathcal{O}(1)$ in Planck units, one obtains

$$V_E = V_0 \left[1 + \mathcal{O}(1)\, \xi^{-n} \left(e^{\sqrt{\frac{2}{3}}\varphi} - 1\right)^n\right]^2 , \tag{5.12}$$

with V_0 being the unperturbed potential as in (5.8). Expanding the correction to leading order shows that its main contribution to the potential comes from a term $(\Omega/\xi)^n$. Hence the potential starts to deviate significantly from its plateau when the ratio Ω/ξ is greater than unity (as illustrated in Figure 5.1). The point at which the deviation occurs is set through requiring the inflaton φ to traverse a certain distance in field space *on the plateau* and enters the ratio through Ω. In other words, a minimal length of a nearly flat plateau, or equivalently a required number of flat e-folds, translates into a lower bound of the coupling ξ that is independent of n for larger n. For lower values of n, $(\Omega/\xi)^n$ starts to contribute earlier than for higher n, hence ξ increases in order to ensure $n_s(55) < 0.980$. Thus any correction of order n affects the attractor around the same point in field space for a value of ξ that is set as to allow for at least $|N_e|$ flat e-folds. Corrections of higher power steepen the potential in a sharper way and thus the running of n_s increases. Hence we find a larger running of n_s to come from dominating higher-order terms in the correction.

In order to quantify the above considerations, we have calculated the percentage of power suppression $\%(N_e)$ of $\Delta_s^2(k)$ at the onset of observable e-folds for exemplary values of n (see Table 5.1) with the procedure presented in section 1.2. In all cases, we have tuned ξ such that $n_s(55) = 0.970$; this is slightly higher than the universal value (5.9) and hence signals the onset of the pre-Starobinsky phase of the scalar potential. Repeating this analysis for a redder or bluer $n_s(55)$ somewhat increases or decreases the non-minimal coupling ξ respectively.

5.2.2 Examples

Understanding (5.10) not as a full UV theory but as an effective description, we consider not just a monomial correction but a series

$$h(f) = \sum_{n=1}^{N} c_n f^n , \tag{5.13}$$

n	%(60)	%(62)	ξ	$n_s(62)$	N_{total}
1	1.9	3.4	$\mathcal{O}(10^4)$	0.975	272
2	2.0	3.8	$\mathcal{O}(10^3)$	0.976	173
8	3.7	7.7	$\mathcal{O}(10^2)$	0.981	90

Table 5.1: *The effect of single higher-order corrections to the attractor relation. The suppression increases with n. Variation of ξ compensates for the varying sharpness of the steepening. For larger n, $\xi \to \mathcal{O}(10^2)$ and N_{total} approaches ~ 62.*

where we again naturally assume all $c_n \sim \mathcal{O}(1)$. We find that lower order terms dominate the steepening of the potential in the vicinity of the 55 e-folds point and hence the running of n_s is weak regardless of any higher power terms in the series. Assuming a natural variation $\Delta c_n \sim \mathcal{O}(1)$ of the coefficients and thereby suppressing the first three terms of the series leads to an exemplary power suppression of about 2.5% to 4.8%, given a cut-off $N = 20$ and a coupling $\xi \sim \mathcal{O}(10^3)$ as to allow for at least 55 flat e-folds with $n_s(55) = 0.970$. In this scenario, $N_{total} = 115$.

Thus understanding power suppression as a tool of effective field theory spectroscopy, we argue that a higher suppression indicates a cancellation or suppression mechanism of lower-order terms in the correction. To maintain numerical control, we now seek to impose natural summation schemes in the expansion such that higher c_n are effectively suppressed. We take

$$h(f) = f \sum_{n=0}^{N} \frac{c_n}{n!} f^n \,, \tag{5.14}$$

which may be understood as requiring that higher order terms are of decreasing importance. Without a cut-off and taking all $c_n = 1$, the above yields

$$V_E = V_0 e^{\frac{2}{\xi}(\Omega - 1)} \,. \tag{5.15}$$

Again the requirement of a minimum number of flat e-folds translates to a lower bound on the coupling ξ. Allowing for at least 55 flat e-folds and requiring $n_s(55) = 0.970$ induces a running of n_s such that the power suppression is about 2.0% to 3.6%. More importantly, the above translates into a value of the coupling $\xi \sim \mathcal{O}(10^5)$, which is, having a natural $\lambda^2 \sim \mathcal{O}(1)$, the required value to fit the normalisation of the power spectrum (see e.g. [79]). Hence we find the normalisation of the power spectrum as well as the level of power suppression to be linked to the parameter ξ, which in turn is set by the amount of e-folds we require before any significant deviation from the nearly flat plateau occurs. Here, $N_{total} = 264$. Truncating (5.14) after the first 10 terms yields the same results, hence we conclude that higher order terms are phenomenologically negligible. In fact, provided the first few c_n are of order one and given some cutoff, higher-order coefficients may be completely arbitrary.

To study more exemplary corrections, consider a $\mathbb{Z}_2$ symmetry, i.e. we only invoke even terms in the correction. Applying this to (5.14) yields $h = f \cosh(f)$ and

$$V_E = V_0 \cosh^2 \left[\xi^{-1}(\Omega - 1)\right] . \tag{5.16}$$

In this case, tuning ξ such that $n_s(55) = 0.970$ gives a suppression of about 2.2% to 4.0%, where $\xi \sim \mathcal{O}(10^3)$. Considering a natural variation $\Delta c_n \sim \mathcal{O}(1)$ such that the first few lower order terms are suppressed and mimicking this by omitting the first two terms in the series expansion of the hyperbolic cosine, we find the suppression level to be increased to 3.1% to 6.1% where $\xi \sim \mathcal{O}(10^2)$. Hence scenarios with stronger suppression due to omitted lower order terms in the correction yield a sufficient amount of flat inflationary e-folds already for $\xi < \mathcal{O}(10^5)$.

Finally, we vary our ansatz for (5.10) and consider the Jordan frame potential as a power series in $f(\phi)$, i.e.

$$h(f) = \sum_{n=1}^{N} \frac{c_n}{n!} f^n . \tag{5.17}$$

The example of $h = e^f - 1$ gives a suppression of up to 3.0% and $n_s(55) = 0.970$ for $\xi \sim \mathcal{O}(10^4)$. Restricting to only odd terms we have $h = \sinh(f)$ and find $n_s(55) = 0.970$ with a suppression of up to 4.4% for $\xi \sim \mathcal{O}(10^3)$. Considering the first five terms of a sum as in (5.17) without suppressing coefficients gives a power loss of up to 2.9% and $n_s(55) = 0.970$ for $\xi \sim \mathcal{O}(10^4)$.

Remarkably, in all examples considered in this section, the condition of 55 flat e-foldings translates into a range for the non-minimal coupling of the order 10^3 up to 10^5, depending on the specific correction. The upper end of this range leads to a power spectrum amplitude in concordance with the measured value. In contrast, the lower end of this range would have a larger amplitude. However, these values were obtained by requiring exactly 55 flat e-foldings and no more; tuning ξ to the observed value around 10^5 would simply lead to a longer inflationary plateau in these cases, and hence more flat e-foldings. Thus, the requirement of *at least* 55 flat e-foldings is remarkably consistent with the observed amplitude of the power spectrum, for a range of different examples: none of these examples require a non-minimal coupling of the order 10^6 or higher to have a sufficiently long inflationary plateau for 55 e-foldings.

These results also apply to Higgs inflation [67, 68]. This model has a non-minimal coupling set by $f = \phi^2$ and a scalar potential that takes the form, including corrections of type (5.14),

$$V_J = \lambda^2 \left[\phi^4 + c_6\phi^6 + c_8\phi^8 + \mathcal{O}(\phi^{10})\right] . \tag{5.18}$$

Taking both λ^2 and the coefficients c_i of order one, there is a relation between the power normalisation and N_e. As shown in the previous section, both point towards a large non-minimal coupling (up to 10^5). Claims that such a large coupling leads to a cutoff scale M_p/ξ [80, 81] that is problematically close to or lower than the inflationary scale have been addressed in various ways [78, 82–84]. The vacuum stability regarding higher-order terms was discussed in [85]. A different, logarithmic correction was considered in the context of Higgs inflation at the critical point [86,87]. While they also rely on this correction to perturb the inflationary plateau, their aim and results are different: the non-minimal coupling is chosen such that the correction affects the entire observable period of the inflationary regime. Accordingly, they require a lower ξ and end up with predictions different from (5.9).

5.2.3 Implications for eternal inflation

If quantum fluctuations of the inflation field $\delta\phi_Q = H/(2\pi)$ dominate over the classical variation $\delta\phi_C = \dot{\phi}/H$, on average half of the fluctuations drive the field upwards its potential. Thus if a potential supports a regime where $H^2/(2\pi\dot{\phi}) > 1$, inflation globally never ends [88]. It can be shown that potentials have to support at least 1000 e-folds of inflation for the above scenario to be realised [89]. As demonstrated, natural corrections to the Universal Attractor yield models, where generically $N_{total} < 300$, hence quantum fluctuations will always remain sub-dominant to the classical evolution. Thus if no high energy effects restore the flatness of the potential at large φ, slow-roll eternal inflation appears disfavoured in this scenario. This is similar to the landscape [90], where N_{total} is generally not much larger than the minimal amount of e-fold

In this section, we studied the effects of corrections to a universal class of inflation models. Remarkably, these provide a link between the observed normalisation of the power spectrum and the number of flat e-foldings: both the height and length of the inflationary plateau are determined by the non-minimal coupling parameter ξ, which is required to be around or below 10^5. We stress that, given either some cut-off or suppression mechanism, this single parameter determines the spectral index and amplitude of the power spectrum, the tensor-to-scalar ratio as well as the number of flat e-foldings. Moreover, for a range of corrections we predict a power loss of a few percent at low-ℓ in the temperature power spectrum of the CMB.

5.3 Generic plateau inflation

Above, we have considered toy model corrections to a generic class of inflation models. We have given a detailed account of the low-ℓ CMB phenomenology and emphasized how height and length of the inflationary plateau are linked by the non-minimal coupling strength ξ. Furthermore, we found that in order to induce the power loss observed, any correction ought to be dominated by lower order terms in the vicinity of the field range corresponding to the scales being probed by the CMB.

Now, we seek to turn this argument into a more general form such that we obtain a mechanism which, when combined with a minimal set of assumptions, automatically provides observationally viable slow-roll inflation. We will find that we only need the Jordan frame field to have a minimum and the non-minimal coupling function to have series expansion; for $\xi > O(N_e^2)$, an effective Einstein frame shift-symmetry appears and for the value $\xi \sim \mathcal{O}(10^4)$, the predictions generically

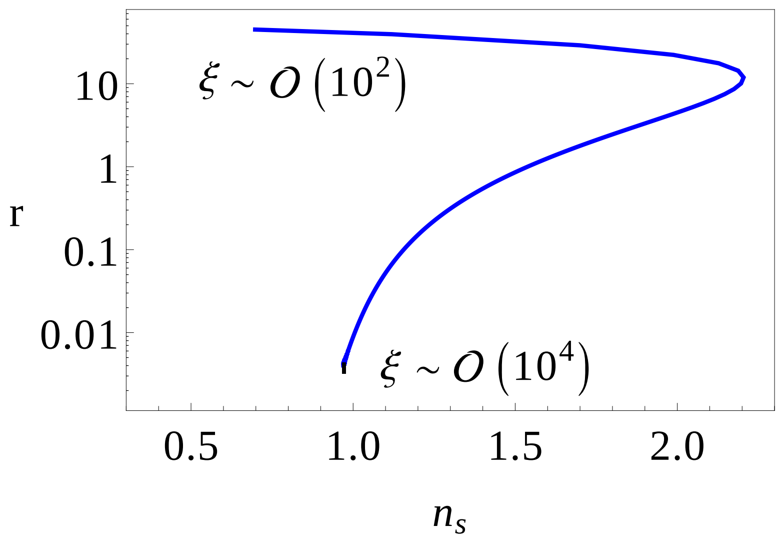

Figure 5.2: *Depiction of the (n_s, r) predictions of a gradually increased non-minimal coupling ξ for $V_J = e^\phi - 1 - \phi$ and $\Omega(\phi) = 1 + \xi\phi$. While different potentials may display a different path of convergence, $\xi \sim \mathcal{O}(10^4)$ is sufficient for the Starobinsky point to be reached. The point, marked as black, remains the attractor for $\xi \to \infty$.*

converge towards the ones measured by PLANCK. We hence provide a minimal model which effectively depends on one parameter only and not just predicts the measured observables but also automatically protects the required shift symmetry from any correction. Figure 5.2 is an exemplary depiction of our ansatz.

5.3.1 Analytic predictions

We now demonstrate how coupling a scalar field non-minimally to gravity may realize an approximately shift-symmetric Einstein frame potential employing a minimal set of assumptions. The aim of this section is to explicitly show the robustness of the inflationary potential from an arbitrary number of higher order terms.

To that end, consider the non-minimal coupling or frame function as well as the potential to be given by arbitrary series with the only requirement that the Jordan frame potential and the square of the frame function share the order of their first zero for $\phi \geq 0$.[35] In other words, we require the Jordan frame potential to have a minimum and the frame function to contain a term linear in the Jordan frame field ϕ. We thus write the setup in full generality as

$$\Omega(\phi) = 1 + \xi \sum_{n=1} a_n \phi^n, \quad V_J(\phi) = \lambda \sum_{m=2} b_m \phi^m, \tag{5.19}$$

where we have kept the factor λ to be consistent with the original work but will assume it to take a natural value of $\lesssim \mathcal{O}(1)$. We have further omitted to specify the cut-off of either series as it will not play a role in the subsequent analysis. In principle, both series may contain an infinite number of terms. The assumption that the Jordan frame field ϕ is stabilised translates into the requirement $b_2 > 0$. We further, as already stated, take $a_1 > 0$, i.e. assume the non-minimal coupling to be approximated by a polynomial series expansion around the minimum of the potential $\phi = 0$. This also implies that the canonical Einstein frame inflaton φ and Jordan frame field ϕ decrease correspondingly, i.e. $d\varphi/d\phi > 0$. This is necessary for the canonical field φ not to have a runaway direction in the potential which might prevent the inflaton from gracefully exiting slow roll.

In the Jordan frame, higher order terms are sub-leading if the Jordan frame field ϕ remains sub-Planckian. From this, two questions arise. First, is it possible to generate a sufficient amount of e-folds within one Planck distance in the Jordan frame field. Secondly, how does this argument carry over to the Einstein frame and the non-minimal coupling strength ξ.

For set-up (5.19), and for now assuming to be in the regime $\phi < 1$, the expression for the number of e-folds of (5.5) obtains corrections as

$$N_e \sim \frac{3}{4}\Omega - \frac{b_3\,\Omega}{8\,b_2 a_1}\left(\frac{\Omega^2}{\xi}\right) + \mathcal{O}^{(2)}\left(\frac{\Omega^2}{\xi}\right) = \Omega\left[\frac{3}{4} - \frac{b_3}{8\,b_2 a_1}\left(\frac{\Omega^2}{\xi}\right) + \dots\right], \tag{5.20}$$

which may be understood as an expansion in Ω^2/ξ. Given that we seek a mechanism yielding inflationary dynamics compatible with PLANCK, we require the corrections to the leading order term of the above to be sub-dominant. This is the case for $\Omega^2 << \xi$, and hence leads to a self-consistent expansion. Setting

[35] We are only considering zeroes and not poles in this set-up.

$N_e \equiv N_{CMB} \sim \mathcal{O}(60)$, where N_{CMB} denotes the number of e-folds at horizon exit of scales now observable through the CMB, we thus find that the lower bound on the non-minimal coupling strength for generating a sufficient amount of inflation within $\Delta\phi < 1$ is

$$\xi > \mathcal{O}(N_e^2)\,. \tag{5.21}$$

The following discussion hence assumes this lower bound. This result is crucial as it demonstrates that the predictions of the general ansatz (5.19) begin to converge towards the universal predictions (5.9) when the non-minimal coupling strength is of the order of the amount of e-folds minimally required.

In the regime $\phi < 1$ the first zeros in both series of (5.19) are leading. We hence infer that non-canonical field and frame function may be related as

$$\phi \sim \frac{1}{a_1 \xi}\left(\Omega - 1\right)\,. \tag{5.22}$$

It is readily verified from ansatz (5.19) that the Einstein frame potential then becomes

$$V_E = \frac{\lambda}{a_1^2 \xi^2}\left(1 - \frac{1}{\Omega}\right)^2 \left[b_2 + \sum_{k=1} b_{k+2}\left(\frac{\Omega - 1}{a_1 \xi}\right)^k\right]\,. \tag{5.23}$$

First we note that as long as the summand is less than unity, the series of all corrections converges. We conclude that making ξ sufficiently large can, regardless of an *infinite* tower of higher order corrections with order one coefficients, induce a Starobinsky-like inflationary plateau over a finite field range. This is the Einstein frame manifestation of the fact that during inflation, $\phi < 1$ and hence all higher order terms in the Jordan frame potential are sub-leading.

In other words, the effect of higher order terms can simply be pushed far away in field space by sufficiently enlarging the non-minimal coupling strength ξ. Thus the inflationary dynamics are independent of whether or not the tower of higher order corrections is truncated at some order. Hence we see that, given inflation occurs for the non-canonical field $\phi < 1$ which can be ensured via having $\xi \gtrsim \mathcal{O}(N_e^2)$, the set-up is independent of the truncation of the potential, and the non-minimal coupling strength ξ therefore protects a finite plateau. Another way to look at this is the following: The expression for the displacement of the non-canonical

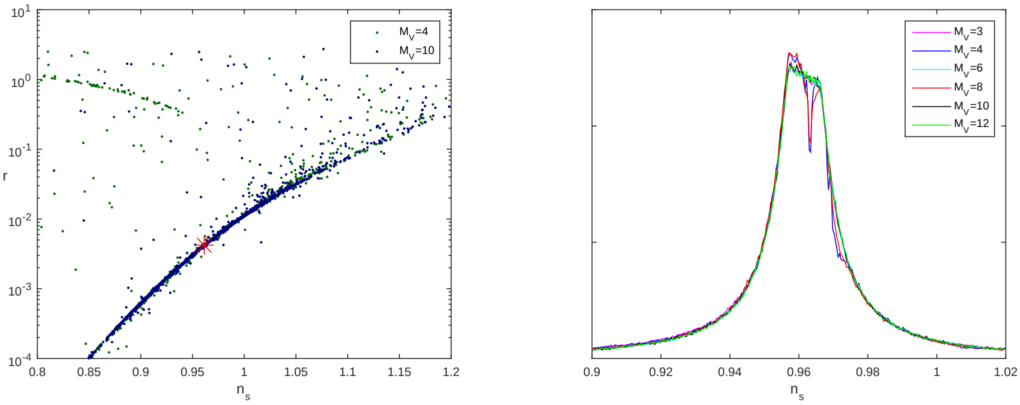

Figure 5.3: **Left:** *The plot shows a scatter plot with* $\xi = 10^4$, $M_\Omega = 1$ *and* $M_V = 10$ *containing* 5000 *trajectories. The red dot represents the Starobinsky point* $n_s(50) = 0.962, r = 0.004$. *The green line in the upper left displays chaotic signatures. The slope of the line in the vicinity of the Starobinsky point is, as predicted,* $\delta r/\delta n_s \sim 0.1$. **Right:** *An* n_s *density plot, on a linear scale, for different values of* M_V. *The plot peaks around the Starobinsky value.*

field during inflation reads to leading order

$$\Delta\phi \sim \frac{1}{\xi}\Delta\Omega\,, \tag{5.24}$$

where $\Delta\Omega$ denotes the change of the frame function between horizon exit of CMB scales and the end of inflation and is typically $\Delta\Omega \sim \mathcal{O}(60)$. One immediately understands that an increase in ξ can force $\Delta\phi$ sub-Planckian as $\Delta\Omega$ is fixed through N_{CMB}.

To obtain a value for ξ that ensures the corrections to be sufficiently far away from the minimum of the inflaton potential and to have inflation matching observations by PLANCK, it is most useful to study the inflationary observables and their dependence on the infinite tower of higher order terms. To leading order, the expressions for the inflationary observables n_s and r of (5.19) and thus (5.23) are

$$\begin{aligned} n_s &= 1 - \frac{2}{N_e} + 8\,\kappa^6 \frac{b_3}{b_2}\left(\frac{N_e}{a_1\xi}\right) + \mathcal{O}^{(2)}\left(\frac{1}{N_e}, \frac{N_e}{a_1\xi}\right), \\ r &= \frac{12}{N_e^2} + 32\,\kappa^4 \frac{b_3}{b_2}\left(\frac{1}{a_1\xi}\right) + 32\,\kappa^8\left(\frac{b_3}{b_2}\right)^2\left(\frac{N_e}{a_1\xi}\right)^2 \\ &\quad + \mathcal{O}^{(3)}\left(\frac{1}{N_e}, \frac{N_e}{a_1\xi}\right), \end{aligned} \tag{5.25}$$

where again $\kappa = \sqrt{2/3}$. Expressions (5.25) are expansions in $1/N_e$ and $N_e/(a_1\xi)$. For the spectral index n_s, the leading order terms are the linear contributions of the $1/N_e$ and the $N_e/(a_1\xi)$ expansions. For the tensor to scalar ratio r, the leading order terms are the quadratic and bilinear expressions of both expansions. Subleading terms stem from cross terms and higher orders in $1/(a_1\xi)$ and $N_e/(a_1\xi)$ and are denoted by $\mathcal{O}^{(n)}$. Note that we have omitted the subleading corrections of [49], i.e. higher order terms in $1/N_e$, for clarity. For n_s and r to be dominated respectively by the linear and quadratic term in $1/N_e$, i.e. for prolonging the Einstein frame potential's intermediate plateau, we quickly identify that

$$\xi > \mathcal{O}\left(N_e^2\right), \tag{5.26}$$

provided $a_1 \sim \mathcal{O}(1)$. This hence marks the onset of a convergence of the inflationary predictions towards the values measured.

Considering that we eventually seek to study models with random a_n, b_m, expressions (5.25) predict a range of n_s, r pairs where however the slope of a scatter plot r vs. n_s ought to be independent of the random draws as the leading order corrections, i.e. the next to leading order terms, both come with the same a_1, b_2, b_3 dependence. Thus consider the ratio of the next to leading order terms

$$\frac{\delta r}{\delta n_s} = \frac{6}{N_e} \sim \mathcal{O}\left(0.1\right) \tag{5.27}$$

for $N_e \sim N_{CMB}$. Note that this implies a scatter plot of (n_s, r) pairs to demonstrate a $1/N_e$ scaling in the slope at the Starobinsky point, i.e. scatter plots for different values of N_e will show a different slope. Generally, this predicts that in the vicinity of the Starobinsky point, there will be signatures to the bottom left and top right, roughly aligned with a slope of $\mathcal{O}(0.1)$ for $N_e \sim N_{CMB}$. In other words, given (5.25), we see that when b_3 is positive the effect of steepening corrections is to increase the spectral index and the tensor to scalar ratio. Thus if coefficients are arbitrary and a large ξ not automatically protects the plateau fully against higher order terms, we expect signatures to appear in the n_s, r plot that are higher than and to the right of the Starobinsky point. This corresponds to an upward curve in the potential plateau, roughly indicating the onset of a monomial dominated chaotic phase. Allowing for the first higher order coefficient b_3 to be negative, potential (5.23) may obtain a hilltop feature. This means that the first order correction of (5.25) now comes with a minus sign. We hence expect signatures to appear below and to the left of the Starobinsky point. The left panels of Figures 5.3 and 5.4 nicely depict this behaviour. Note that a_1 and b_2 have to be positive to guaranty the positivity of the frame function and the potential around the minimum.

For higher order terms not to spoil the value of n_s observed by PLANCK, i.e. for the observables to enter the Planck contours, we consider the 2-σ bound by PLANCK of $\delta n_s < 0.008$ at $N_e = 55$ and find, given $a_1, b_2, b_3 \sim \mathcal{O}(1)$,

$$\xi \gtrsim \mathcal{O}\left(10^4\right) . \tag{5.28}$$

This hence sets, given order one coefficients, a lower bound on the non-minimal coupling strength ξ to realize observationally viable slow-roll inflation.

A few comments are in order. Not only does a fixed value of ξ prevent *all* higher order terms from becoming important before very roughly

$$\varphi \sim \kappa^{-1} \log\left(a_1 \xi\right) \tag{5.29}$$

but also is the value of ξ obtained from the requirement of matching the observed spectral index n_s similar to the value needed to match COBE normalization (provided $\lambda \lesssim \mathcal{O}(1)$). Thus two independent observational indications - in technical terms the spectral index n_s and the amplitude A_s - hint towards an otherwise ad hoc value of the theory's parameter. The length and the height of the inflationary plateau are correctly set by the single parameter ξ. The rare case that coefficients a_n, b_m are randomly drawn such that $b_i \gg b_2, b_3$ for $i > 2$ and hence that higher order terms evade the ξ induced flattening will be discussed in appendix B.

5.3.2 Numerical study

The above argument considered all coefficients to be around unity. However, to make the statement about the robustness of the inflationary plateau for given values of the non-minimal coupling stronger, we now introduce random coefficients in order to study whether or not our previous findings hold.[36] We will hence study ansatz (5.19) with coefficients randomly chosen (drawn) from some interval.

Our setup is Lagrangian (5.19), where we now draw $a_n \in [-1/n!, 1/n!], \forall n > 1$ and $b_m \in [-1/m!, 1/m!], \forall, m > 2$. We invoke a factorial suppression of the coefficients to maintain numerical control over large series of corrections and will later consider non-suppressed coefficients in appendix B. We perform a Monte Carlo analysis based on the procedure of [91–94]. We work with a non-canonical kinetic term in the Einstein frame. The slow-roll parameters (1.22) hence become

$$\epsilon_V = \frac{1}{2\,K}\left(\frac{1}{V_J}\frac{\partial V_J}{\partial \phi} - \frac{2}{\Omega}\frac{\partial \Omega}{\partial \phi}\right)^2 , \tag{5.30}$$

$$\eta_V = \frac{\Omega^2}{K\,V_J}\left[\frac{\partial^2}{\partial \phi^2}\left(\frac{V_J}{\Omega^2}\right) - \frac{1}{2\,K}\frac{\partial K}{\partial \phi}\frac{\partial}{\partial \phi}\left(\frac{V_J}{\Omega^2}\right)\right] , \tag{5.31}$$

in terms of the non-canonical kinetic function

$$K = \frac{1}{\Omega} + \frac{3}{2}\left(\frac{\partial \ln \Omega}{\partial \phi}\right)^2 . \tag{5.32}$$

Here, φ is the Einstein frame canonical inflaton and ϕ is the non-canonical Jordan frame field. The number of e-folds then follows as

$$N_e = \int \frac{1}{\sqrt{2\epsilon_V}}\, d\varphi = \int \frac{\sqrt{K}}{\sqrt{2\epsilon_V}}\, d\phi \, . \tag{5.33}$$

[36]We still impose a_1 , $b_2 > 0$ for the same reasons as given in the previous section.

With the above expressions, we proceed as described in the following way: Iterating 10^6 times, we draw parameters a_n and b_m of Lagrangian (5.19) being distributed uniformly.[37] For each draw the slow-roll parameters and the number of e-folds are calculated in order to check whether or not the resulting Lagrangian yields slow-roll inflation at all. We classify the resulting trajectories according to their late time behaviour and - provided the effective potential maintains at least 50 e-folds - calculate the inflationary observables n_s and r. Three late-time behaviours can occur: If ϵ_V approaches 1 for decreasing non-canonical field ϕ and eventually becomes unity, inflation gracefully ends. Following the terminology of [92], we hereafter refer to this as a *non-trivial* ending. In case the potential behaves as just described but does not feature a sufficient amount of e-folds, we call the scenario *insuf.* Unphysical trajectories occur when there appears a zero in the frame function or when the Jordan frame potential becomes negative. Those are labelled Ω, V*-negative.* Finally, a very small fraction of the models does not include an inflation phase at all, but this fraction is negligibly small for the values of ξ discussed in this work. In what follows, we will focus on the non-trivial trajectories.

As computation resources are limited, it is crucial to understand if Lagrangian (5.19) yields dynamics that prove to be independent of the truncation of either series. Fortunately, at the ξ values we are considering, it is computationally possible to include a sufficient number of terms in both the non-minimal coupling and the scalar potential to render the results truncation independent. This is illustrated in the right panel of Figure 5.4. In what follows, we will consider the specific case of $M_V = 10$ and $M_\Omega = 5$, but none of our results depend on these specific numbers.
The outcome of the numerical simulations is as follows; the scatter plot of Figure 5.4 depicts the (n_s, r) pairs for two different choices of the non-minimal coupling strength ξ, where the red dot denotes the Starobinsky point. Clearly, there are hilltop and plateau signatures visible.[38] Two observations are noteworthy: First, the predictions clearly converge towards a pronounced line when increasing ξ by two orders of magnitude. Secondly, the slope in the vicinity of the Starobinsky point is precisely captured by expression (5.27). As the finite point size blurs information about the true spectrum of (n_s, r) pairs, we now turn to density plots. This

[37]We constrain $a_1, b_2 > 0$.
[38]Chaotic signatures will be subject of appendix B.

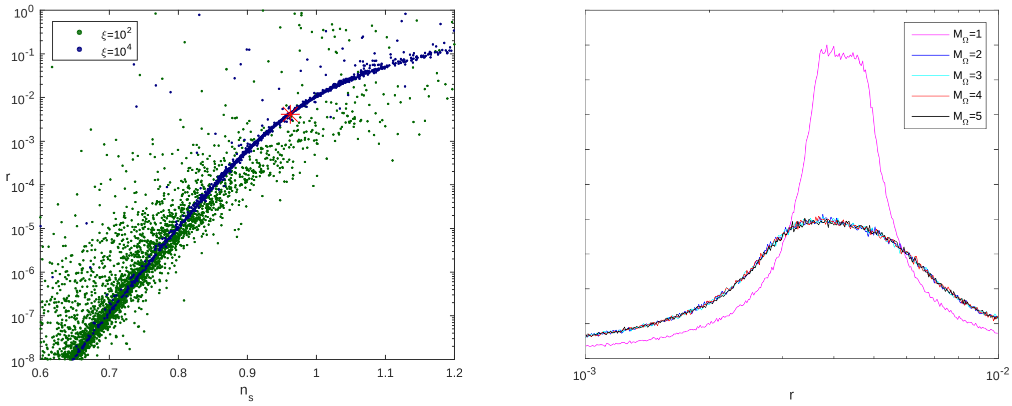

Figure 5.4: **Left:** *Scatter plot of* 5000 *trajectories with* $M_\Omega = 5$ *and* $M_V = 10$ *for* $\xi = 10^2$ *in green and* $\xi = 10^4$ *in blue. The red star represents the Starobinsky point* $n_s \approx 0.962, r \approx 0.004$ *at* $N_e = 50$. **Right:** *An* r *density plot on a linear scale for different values of* M_Ω *with* $M_V = 10$ *and* $\xi = 10^4$*. For* $M_\Omega > 2$ *the system is truncation independent.*

means binning the data in small bins of either δn_s or δr and counting the number of points in each bin. The resulting curve is a rough measure of the probability distribution of the variable, due to the number of points over which is sampled is large.[39] To normalise we calculated the number of points in a bin and divided by the total number of points. This in principle depends on the chosen binsize; however, our conclusions are not binsize dependent. Density plots for n_s and r are shown in Figure 5.5. For $\xi = 10^2$, no peak around the Starobinsky point is visible. When $\xi = 10^4$ a peak clearly has emerged and this peak sharpens when ξ increases, just as the analysis in section 5.3.1 demonstrated. This centering around the Starobinsky point is a continuous process, starting from around $\xi \approx N_e^2$. At last, we want to study the occurrence of different scenarios outlined before, i.e. we seek to count how many of the 10^6 random draws actually feature a sufficient amount of observationally viable slow-roll inflation. To probe this we plot the percentage of the number of outcomes in Figure 5.6. The probability that a model ends non-trivially indeed increases when ξ increases, and the number of models

[39] Note that that the leading order corrections to the inflationary observables (5.25) depend on ratios of now random coefficients. Strictly speaking, the moments of ratio distributions are not defined, we thus refrain from projecting true statistical meaning onto the density plots.

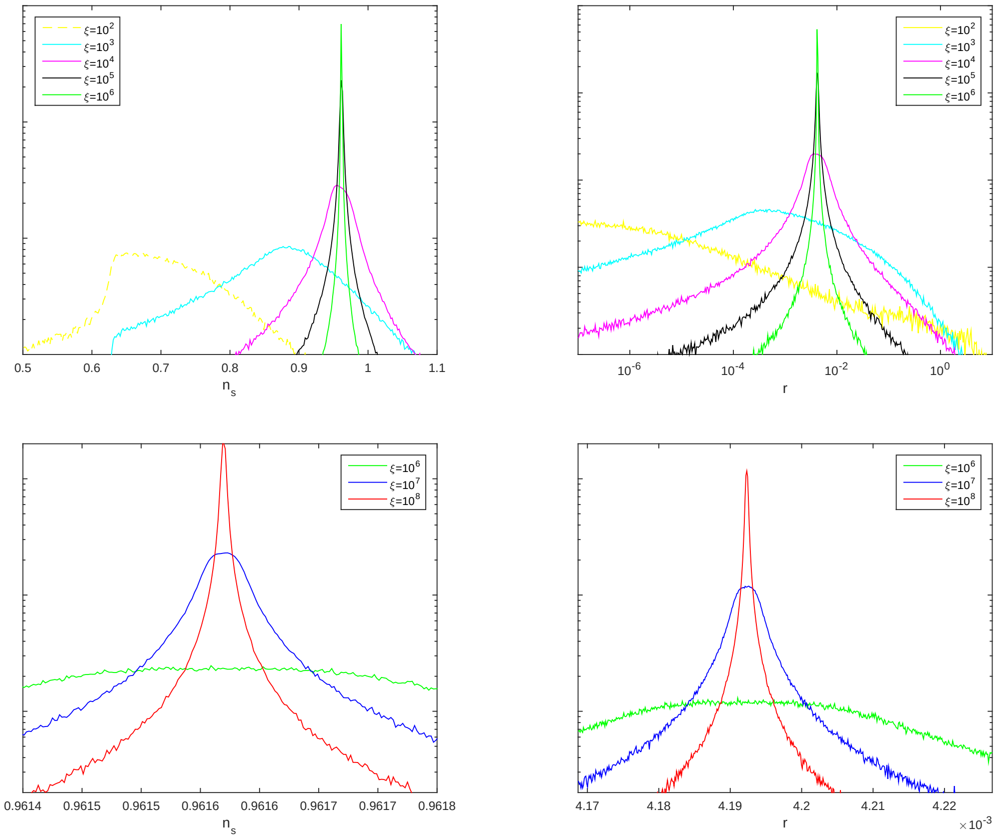

Figure 5.5: *Density profiles (on a log-scale) for different values of ξ for n_s (left) and r (right). The bottom frames zoom in on the Starobinsky peak.*

with insufficient e-folds to account for the observations (*insuf*) and the number of models with negative potential and/or frame function during inflation (*Vneg*) decrease. Figure 5.6 demonstrates a maximal increase in observationally viable models once $\xi \sim \mathcal{N}_e^2 \sim 10^3$ for $N_e = 50$. This is in line with our predictions. For $\xi \sim \mathcal{O}(10^4)$, roughly nine in ten draws feature Starobinsky type inflation. We hence conclude that the lower bound $\xi \gtrsim 10^4$ appears first from CMB normalization arguments and our toy model analysis in subsection 5.3.1 and follows to be a special value also in the numerical study.

We revisited non-minimally coupled inflation models in the spirit of [67–69]. Our interest was whether there exists a value of the non-minimally coupling

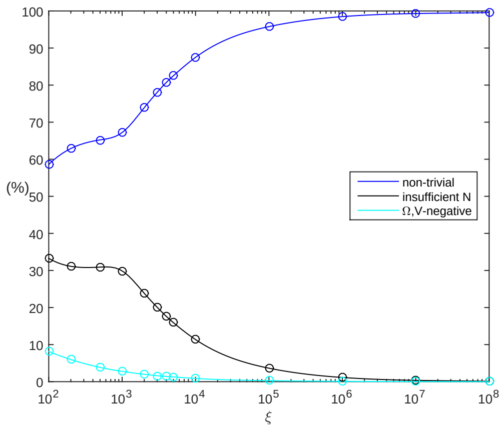

Figure 5.6: *The occurrence of different late-time behaviours as a function of ξ. The circles denote actual data points, the lines are only to guide the eye.*

strength that is preferred not only by matching COBE normalisation. We recalled how the non-minimal coupling ξ may be used to induce an effective shift-symmetry which is protected against a possibly infinite tower of higher order corrections. The size of the non-minimal coupling determines the field range of this Einstein frame shift-symmetry. We continued to parametrise non-minimal coupling functions and potentials as arbitrary series with a minimal set of assumptions. Drawing the series coefficients randomly, we examined the resulting Einstein frame potentials to find out whether observationally viable slow-roll inflation occurs. We found that with increasing non-minimal coupling ξ, the number of Starobinsky-like inflation trajectories increases. Remarkably, $\xi \sim \mathcal{O}(10^4)$ is the value when the number of Starobinsky-like trajectories increases the fastest. Thus a non-minimal coupling ξ can induce a shift-symmetry protected against all higher order terms (i.e. length of an inflationary plateau) while also matching COBE normalization (i.e. height of the inflationary plateau). An analysis with all coefficients $a_n, b_m \sim \mathcal{O}(1)$ as well as an analysis with all coefficients random both point towards a preferred value of $\xi \sim \mathcal{O}(10^4)$. This result may also be obtained when choosing the random interval $[-1, 1]$, i.e. without factorial suppression. Appendix B considers such a scenario. Following the argument of CMB normalisation, this chapter provides a further way to pinpoint the non-minimal coupling strength ξ and describes a minimal mechanism realising observationally viable slow-roll inflation.

Chapter 6

Conclusion and Outlook

CMB measurements find a hierarchy between the slow-roll parameters ϵ_V and η_V. The latter is roughly one order of magnitude larger than the former. Potentials maintaining a shift symmetry $V \sim const.$ naturally satisfy this constraint. However, from the point of effective field theory, an approximate shift symmetry amounts to suppressing, i.e. tuning the coefficients of all higher dimensional operators. Higher order coefficients in an EFT expansion may be specified if the UV is known. Otherwise, the suppression could be understood as ad hoc. This work considered different approaches to realise an approximate shift symmetry of the inflaton potential in order to investigate whether or not the amount of tuning can be reduced or the need to tune avoided at all. We discussed different scenarios and found that tuning cannot be entirely avoided in most cases. In the UV, parameters have to be carefully balanced. Corrections to non-canonical dynamics must also be under control. Considering modified gravity, a suppression pattern akin to that of potential suppression surfaced. Nevertheless we were able to provide $f(R)$ toy models explicitly specifying an infinite number of higher order coefficients. Finally, we provided a minimal mechanism to realise an effective shift symmetry by means of a non-minimal coupling. This mechanism can be made robust against an infinite number of higher order terms via the coupling strength. Having the coupling strength satisfy CMB temperature spectrum normalisation, observationally viable slow-roll inflation is realised as all corrections can be pushed sufficiently far away in field space.

Recently, the claim surfaced that the value of the Hubble parameter today is roughly eight percent larger than the one inferred from CMB measurements [19].[40] It is not the first time that CMB data has been critiqued for being at odds with astronomical measurements [95]. A single measurement [19] leaves the situation inconclusive for now. However, it has to be noted that a larger H_0 blueshifts the spectral index n_s. If H_0 indeed turns out to be larger, would this imply that inflation is driven by potentials adhering a paradigm other than that of an approximate shift symmetry? Quite the opposite: It can readily be shown that allowing for symmetry breaking at larger fields, n_s is blueshifted while r increases only slowly. Considering a potential with a ε suppressed rising exponential breaking the plateau at larger fields (i.e. $\varepsilon \ll 1$) and recalling expressions (5.25), we immediately see that n_s obtains corrections $\delta n_s \sim \mathcal{O}(\varepsilon\, N_e)$ while r is corrected as $\delta r \sim \mathcal{O}(\varepsilon^2 N_e^2)$. Corrections to an approximate shift symmetry are natural and expected. A larger spectral index n_s would merely imply that the shift-symmetry breaking occurs for slightly lower φ but would still favour shift-symmetric inflaton potentials discussed in this thesis.

As a final remark let us note the following: The most simple attempt to realise a quantum theory of gravity, quantising the Einstein equations (1.4), fails due to the non-renormalisability of the resulting theory (see e.g. [96]). However, when linking the quantum fluctuations to metric perturbations in order to induce the primordial density perturbation, one implicitly assumes that gravity can safely be quantised on the perturbative level. Measuring tensor modes in the CMB hence would be the first experimental handle on perturbative quantum gravity. The absence of such a measurement has been the main motivation to study potentials that only induce a small tensor signal, i.e. shift-symmetric potentials. Nevertheless, a minimalist interpretation of the non-detection could in principle question whether or not the quantisation of tensor modes is realised in nature in the first place. However, as it was already argued in the original work on gravitational waves [97], if gravitational waves exist, they ought to be quantised similar to the electromagnetic case as otherwise, particle orbits would be unstable. We thus believe the perturbative approach to the quantisation of gravity to hold in the inflationary scenario and the study of shift-symmetric potentials therefore to be well motivated.

[40] I would like to thank David Ciupke for making me aware of this result.

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Appendix A

More on f(R)

In this appendix, we will explicitly derive some of the claims made in chapter 4. The following sections further serve to demonstrate the way first order differential equations of rank 2 have been solved in this work.

A.1 The $f(R)$ dual for $V \sim V_0$

Recall the potentials $V = V_0(1 - 2e^{-\frac{\kappa}{n}\varphi} + e^{-2\kappa\varphi})$ with $n > 1$ and $V = V_0(1 - 2e^{-\kappa\varphi} + e^{-n\kappa\varphi})$ with $n > 2$. Both potentials have the exponentials departing from their square relation characteristic of the R^2 dual Starobinsky potential. The aim of this appendix is to prove the claim made in section 4.1, namely that regardless of the specific values chosen for n, both potentials will always admit at least an approximate $f(R)$ dual which is to leading order R^2. Essentially, one may argue that the potential mimics a cosmological constant for large field values and hence all one has to do is finding the $f(R)$ dual to general relativity with a free scalar and a cosmological constant. To that extent, consider that both potentials display a shift symmetry in the inflationary region, i.e. one may well approximate both of the above as

$$V(\varphi) \sim V_0 \tag{A.1}$$

during inflation. We recall equation (4.7) and hence write

$$V_0 = \frac{f'\,R - f}{2f'^2}\,, \tag{A.2}$$

which, upon rearranging, may be recast as

$$2V_0 f'^2 - f'\,R + f = 0. \tag{A.3}$$

Differentiating the above with respect to R gives, for $f'' \neq 0$,

$$4V_0 f' - R = 0\,, \tag{A.4}$$

which may then simply be integrated to yield

$$f(R) = \frac{1}{8V_0} R^2\,. \tag{A.5}$$

The integration constant has been set to zero by considering the boundary condition (A.2). We thus see that any potential which approximates a cosmological constant, i.e. $V \sim V_0$, may be recast in terms of a leading order R^2 $f(R)$ formulation. The same argument also applies vice-versa, i.e. the scale invariant theory $f(R) = \alpha R^2$ may readily be recast in terms of an Einstein-Hilbert Lagrangian with a cosmological constant $\Lambda = (8\alpha)^{-1}$.

When considering e.g. the full potential $V(\varphi) = V_0(1 - 2e^{-\frac{\kappa}{n}\varphi} + e^{-2\kappa\varphi})$ with $n > 1$, one has, according to expression (4.7),

$$\frac{f'R - f}{2f'^2} = V_0\left(1 - 2f'^{-\frac{1}{n}} + f'^{-2}\right)\,, \tag{A.6}$$

where we have identified $f' = e^{\kappa\varphi}$. Rearranging and differentiating with respect to R gives

$$f' = \frac{R}{4V_0} + \left(2 - \frac{1}{n}\right) f'^{1-\frac{1}{n}}\,. \tag{A.7}$$

For large values of R, we hence approximate $f' \sim \mathcal{O}(R)$. Therefore, we insert $f' \sim R$ into (A.7) to obtain

$$f(R) = \frac{R^2}{8V_0} + R^{2-\frac{1}{n}} + \dots\,, \tag{A.8}$$

where the dots denote sub-leading terms during inflation and indicate that the above was obtained iteratively. We thus find that the leading order behaviour is indeed R^2. Applying the same procedure to the potential $V = V_0(1-2e^{-\kappa\varphi}+e^{-n\kappa\varphi})$ with $n > 2$ yields as a first iterative step

$$f(R) = \frac{R^2}{8V_0} + \frac{1}{2}R^{2-n} + \dots\,, \tag{A.9}$$

where again the dots denote sub-leading terms. For the above, one might wonder whether or not the function $f(R)$ becomes singular for small R. However, expression (A.9) was explicitly obtained with a method relying on limiting the range of validity of the solution to the large R regime. Thus it is found that the enhanced R^2 term dominates, as was foreseen from expressions (A.1) to (A.5).

A.2 An explicit derivation

Recall the potential

$$V(\varphi) = V_0\left(1 - e^{-\gamma\kappa\varphi}\right)^2, \tag{A.10}$$

with $\kappa = \sqrt{2/3}$ and $0 < \gamma < 2$. By considering (4.7), we write

$$V_0\left(1 - 2e^{-\gamma\kappa\varphi} + e^{-2\gamma\kappa\varphi}\right) = \frac{f'\,R - f}{2f'^2}\,, \tag{A.11}$$

which, upon identifying $f' = e^{\kappa\varphi}$ and multiplying by $2f'^2$, can be rewritten as

$$2V_0\left(f'^2 - 2f'^{2-\gamma} + f'^{2-2\gamma}\right) + f - f'\,R = 0\,. \tag{A.12}$$

Differentiating with respect to R yields

$$2V_0\left[2f' - 2(2-\gamma)f'^{1-\gamma} + (2-2\gamma)f'^{1-2\gamma}\right] - R = 0\,, \tag{A.13}$$

where, as always, we are taking $f'' \neq 0$. Having reduced the rank of the differential equation, we may now rearrange terms and integrate to write

$$f(R) = \frac{1}{8V_0}R^2 + (2-\gamma)\int f'^{1-\gamma}dR - (1-\gamma)\int f'^{1-2\gamma}\,dR\,. \tag{A.14}$$

This establishes that for large R, $f'(R)$ may be approximated as being of order $\sim R$. We hence state the approximate $f(R)$ dual to potential (A.10) as

$$f(R) = \frac{1}{8V_0}R^2 + R^{2-\gamma} - \frac{1}{2}R^{2-2\gamma} + \dots \tag{A.15}$$

up to sub-leading terms where the above solution may be understood as the first result of an iterative approach. Connecting to the expressions (4.68) in the text, we thus identify

$$\alpha = 2 - \gamma\,, \quad \beta = \gamma - 1\,, \quad c = 1\,. \tag{A.16}$$

Appendix B

Evading ξ

In this appendix, we briefly summarise the phenomenology of non-minimally coupled models, where a random draw of coefficients evades the otherwise generic ξ-induced flattening. The presented analysis in section 5.3 has demonstrated that given $a_1, b_2, b_3 \sim \mathcal{O}(1)$ and $\xi > \mathcal{O}(N_e^2)$, inflation occurs with a leading order Starobinsky (or Hilltop) signature and a value of $\xi \gtrsim \mathcal{O}(10^4)$ can serve to push *all* higher order corrections sufficiently far away in field space to arrive at an observationally viable model. We hence find an inflationary regime independent of the truncation of either series in (5.19).

However, due to the randomness of the coefficients a_n, b_m, it could in principle happen that terms $b_m\phi^m, m > 2$ in the potential evade the ξ-induced flattening and influence the inflationary dynamics. Changing our set-up to $a_n, b_m \in [-1, 1]$, we now examine whether or not the set-up remains truncation independent when the coefficients are drawn such that terms $b_m\phi^m$ for $m > 2$ are important, i.e. greater than unity, during inflation; in other words, the Jordan frame field ϕ is trans-Planckian to maintain the required amount of e-folds. Having the coefficients a_n, b_m resemble a factorial suppression pattern, the non-canonical field has to be $\phi \gtrsim \mathcal{O}(M)$ during inflation (M is the order of the frame function's truncation) for higher order terms to be non-negligible. Simply taking $a_n, b_m \in [-1, 1]$, the non-canonical field has to be $\phi \gtrsim \mathcal{O}(1)$ during inflation to feel the effect of higher order terms. In what follows, we study the case $a_n, b_m \in [-1, 1]$ and $\phi \gtrsim \mathcal{O}(1)$ but the argument readily extends to the scenario $a_n, b_m \in [-1/n!, 1/n!]$ and $\phi \gtrsim \mathcal{O}(M)$.

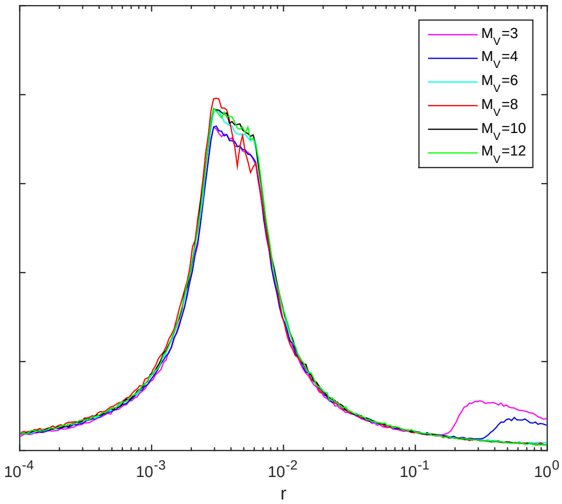

Figure B.1: *Density profile for r with $\xi = 10^4$, $M_\Omega = 1$ and with coefficients b_m that are not factorially suppressed.*

Consider

$$\Omega(\phi) = 1 + \xi \sum_{n=1}^{M} a_n \phi^n, \quad V_J(\phi) = \lambda \sum_{m=2}^{2M+\Delta} b_m \phi^m, \tag{B.1}$$

where Δ is a positive integer and hence parametrizes how much the highest order term of the Jordan frame potential departs from a square relation with the highest order term in the non-minimal coupling function Ω. When $\phi > 1$, we obtain the effective potential

$$V_E \sim \frac{\lambda}{a_M^2 \xi^2} \left[b_{2M} + \sum_{k=1}^{\Delta} b_{2M+k} \left(\frac{\Omega}{a_M \xi} \right)^{\frac{k}{M}} \right]. \tag{B.2}$$

If the potential departs from the square relation between potential and frame function at highest order, the Einstein frame potential in principle feels this effect. While also this effect can be made negligible by tuning Δ or simply pushing it away in field space by enlarging ξ, it could as such play an important role when the coefficients b_m are drawn such that terms of the order $> 2M$ become dominant in the inflationary region of the Einstein frame potential.

As coefficients $b_{m>2M}$ may have either sign, the effect of these higher order terms on the inflationary dynamics can either be to curve the potential upwards and hence increase the number of chaotic signatures in the n_s, r plot or to induce a hilltop and thus to enlarge the number of signatures with redder n_s and very small r. We conjecture that a large Δ will increase the number of hilltop signatures while chaotic signatures may only be visible when $\Delta \sim \mathcal{O}(1)$ and M is not too large. This is because a large Δ will allow for an interplay of coefficients $b_{m>2M}$ with possibly different signs such that hilltops occur whereas if there exists just one or two higher order terms, a positive highest order coefficient could be sufficient to steepen the potential before lower order terms will have induced a hilltop. The phenomenology of this analysis is depicted in figure B.1. This shows how chaotic signatures are only visible for $\Delta \sim \mathcal{O}(1)$.

We thus find that once sufficiently large $\xi \gtrsim \mathcal{O}(N_e^2)$ drives the non-canonical field displacement sub-Planckian, the form of the higher order coefficients is mostly irrelevant for the inflationary predictions.

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Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Hamburg, den 10. Mai 2016 Unterschrift