Appendix C

Derivation of Critical Sharpe Ratio

Proposition 1 *Suppose index returns come from a population with mean, μ, and standard deviation, σ, and:*

$$\sqrt{260} * \frac{\mu}{\sigma} = 0.32 \tag{C.1}$$

If $\{X_1, ..., X_N\}$ is a random sample drawn from this population, then the following approximation holds for large N: $P\left(\sqrt{260}\frac{\overline{X}}{S} > 0.32 + \frac{1.645}{\sqrt{\frac{N}{260}}}\right) \cong 0.05$

Proof. *One version of the central limit theorem states that:*[1]

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{N}}} \xrightarrow{D} N(0,1)$$

$$\Rightarrow \sqrt{260}\left(\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{N}}}\right) \xrightarrow{D} N(0,260) \tag{C.2}$$

$$\Rightarrow \sqrt{N}\left(\sqrt{260}\frac{\overline{X}}{\sigma} - \sqrt{260}\frac{\mu}{\sigma}\right) \xrightarrow{D} N(0,260) \tag{C.3}$$

$$\Rightarrow \sqrt{N}\left(\sqrt{260}\frac{\overline{X}}{\sigma} - 0.32\right) \xrightarrow{D} N(0,260) \tag{C.4}$$

$$\Rightarrow \sqrt{N}\left(\sqrt{260}\frac{\overline{X}}{\sigma}\right) \xrightarrow{D} N(0.32,260) \tag{C.5}$$

Now, one of the Slutzky Theorems states that for sequences of random variables U_n and

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[1] *See, e.g., Goldberger (1991), p.99.*

V_n, if $U_n \xrightarrow{P} c$ and V_n has a limiting distribution, then the limiting distribution of (U_nV_n) is the same as that of cV_n.[2] *If we let*

$$V_n = \sqrt{N}\left(\sqrt{260}\frac{\overline{X}}{\sigma}\right) \tag{C.6}$$

and

$$U_n = \frac{\sigma}{S}, \tag{C.7}$$

then from (C.5) and because $U_n \xrightarrow{P} 1$ it follows that

$$U_nV_n =\xrightarrow{D} N(0.32, 260). \tag{C.8}$$

$$\therefore \sqrt{N}\left(\sqrt{260}\frac{\overline{X}}{S}\right) \xrightarrow{D} N(0.32, 260) \tag{C.9}$$

$$\Rightarrow \left(\sqrt{260}\frac{\overline{X}}{S}\right) \xrightarrow{A} N(0.32, \frac{260}{N}) \tag{C.10}$$

$$\Rightarrow P\left(\sqrt{260}\frac{\bar{X}}{S} > 0.32 + \frac{1.645}{\sqrt{\frac{N}{260}}}\right) \cong 0.05 \tag{C.11}$$

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[2] *See, e.g., Goldberger (1991), p.102.*