An algorithm to derive wind speed and direction as well as ocean wave directional spectra from HF radar backscatter measurements based on neural network
Als Dissertation angenommen vom Department Geowissenschaften der Universität Hamburg

Auf Grund der Gutachten von Prof. Dr. Detlef Stammer

und Dr. Klaus-Werner Gurgel

Hamburg, den ____________

Prof. Dr. Jürgen Oßenbrügge

Leiter des Departments Geowissenschaften
Abstract

The technology of land-based High Frequency (HF, 3–30 MHz) radar has the unique capability of continuously monitoring ocean surface parameters up to 200 km off the coast. The HF radar system developed at the University of Hamburg can provide reliable surface current and wave observations. Wind direction measurement is also possible, however, wind speed measurement is still a problem. In the coastal area with a complex topography, the atmospheric and oceanic conditions vary spatially and temporally. For example, the thermal contrast between the land and the ocean produces the daily changing land-sea breeze, and mountains at the coast affect the wind speed and direction significantly. All these make the mesoscale weather systems and associated surface winds in the coastal region complicated. HF radar can solve this problem due to its high resolution (300 m - 1500 m) and it can be operated in real-time and at all weather conditions.

A large amount of ocean data is nowadays collected by remote sensing methods using electromagnetic waves scattered from the rough sea surface. Various techniques for solving inversion problems have been proposed over the last few decades. Among these, Artificial Neural Network (ANN) is ideally suited for applications where the relationship of input and output is either unknown or too complex to be described analytically. In this work, the basic idea is to use the input-output pairs generated by the radar data and in-situ measurements to train the network. This study therefore addresses the issue using a neural network to tackle the complexity and non-linearity of both radar remote sensing and the wind-wave relationship.

In order to investigate how wind acts on the sea surface in a controlled environment, the HIPOCAS (HIndcast of dynamic Processes of the Ocean and Coastal AreaS) wave model data is analyzed to get a better understanding of the relationship between the wind and waves. As a result, new methods are proposed for wind inversion from HF radar backscatter. In this dissertation, the wind inversion from HF radar remote sensing is verified by two experiments: the Fedje experiment in Norway and the Ligurian Sea experiment in Italy. The radar operates at a frequency of 27.68 MHz during the Fedje experiment, providing shorter radar working range but higher range resolution. During the Ligurian Sea experiment, the radar operates at 12-13 MHz, covering a range up to 120 kilometers. In-situ wind and wave measurements are used to train the neural network. This dissertation presents the wind wave and HF radar scattering theory as well as the wind inversion using neural networks and conventional approaches.
Acknowledgments

I would like to express my gratitude to Dr. Klaus-Werner Gurgel for guiding me throughout this study. I am grateful to him for introducing me to the study of ocean surface dynamics and wind inversion techniques. It has been my great pleasure to work with him, and I always obtain more help than I expected. His kindness, generosity, patience and invaluable ideas help me out of difficulties from my first day here.

I would like to thank my advisor, Prof. Dr. Detlef Stammer, for offering me the opportunity to finish this dissertation in the remote sensing group at the Center for Marine and Atmospheric Sciences. I want to thank him for his continuous assistance, guidance and suggestions for my dissertation.

Thanks are also given to Dr. Heniz Günther at the Helmholz-Zentrum Geesthacht (HZG, former GKSS), for providing me the HIPOCAS WAM model data. The model result provided invaluable assistance for developing some new algorithms in this dissertation.

Many thanks to my colleagues and friends, especially to Thomas Schlick, for his help of processing the Radar RAW data, sharing ideas and programming skills with me. Andreas, Frauke and Meike also help me during my daily life. Thanks are also given to Jian Su, Cui Chen, Chao Li and many other friends who have helped me during my stay in Hamburg.

Thanks are also delivered to Prof. George Voulgaris from University of South Carolina, for his invaluable suggestions to my presentation and dissertation. We had a good time in Hamburg.

This work was funded by a scholarship from the China Scholarship Council (CSC) of People’s Republic of China under the contract number 2007U13032.

Finally, I am very grateful for my family and friends who have given me constant support.
Contents

1 Introduction ................................................................. 1
  1.1 State of Research ................................................. 2
  1.2 Scientific Goals ................................................... 3
  1.3 Scope and Outline ............................................... 4

2 Wind Wave Theory and Wave Models ................................. 5
  2.1 Wind waves at the sea surface .................................. 5
    2.1.1 Wave basic definition .................................... 5
    2.1.2 Ocean wave spectrum ..................................... 6
  2.2 HIPOCAS WAM data analysis .................................... 9
    2.2.1 HIPOCAS WAM mode introduction ......................... 9
    2.2.2 Spatial and directional analysis ......................... 9
    2.2.3 Temporal and frequency analysis ....................... 11
  2.3 Wind direction and wave directional distribution .......... 14
    2.3.1 Half-cosine 2s-power type spreading function .......... 14
    2.3.2 Hyperbolic secant-squared type spreading function .... 15
  2.4 Wind speed inversion from wave spectrum .................... 16
    2.4.1 Dimensionless parameters ............................... 17
    2.4.2 SMB curves for wind speed inversion .................. 17

3 HF Radar Remote Sensing and Wind Inversion .................... 21
  3.1 Introduction to HF radar remote sensing .................... 21
    3.1.1 WERA system ........................................... 23
    3.1.2 Physical scattering model and radar cross section .... 24
  3.2 Wind direction and radar backscatter echoes ................ 28
  3.3 Wind direction determination with two radars ............... 32
    3.3.1 Least Square Minimum (LSM) method ................... 33
    3.3.2 Multi-beam method using one radar site ............... 34
    3.3.3 Pattern fitting with a varying spreading parameter ... 35
  3.4 Wind speed and radar backscatter echoes .................... 40
    3.4.1 Wind speed inversion from HF echoes .................. 42
    3.4.2 Wind speed inversion from the first-order peaks ....... 42
3.4.3 Wind speed inversion from the second-order sidebands .......................... 44
3.5 Summary ........................................................................................................ 48

4 Neural Network and Approaches of Wind Inversion ........................................... 49
4.1 Neural network and remote sensing ............................................................... 49
4.2 Principle of artificial neural network ............................................................. 50
  4.2.1 Artificial neuron models and transfer functions ....................................... 50
  4.2.2 Neural network structures ........................................................................ 51
  4.2.3 Introduction to back-propagation network .................................................. 53
4.3 Neural network design .................................................................................... 59
  4.3.1 Layers and number of neurons ................................................................. 59
  4.3.2 Training, validation and test data ............................................................. 60
  4.3.3 Dependent variables selection for neural network .................................... 61
4.4 Methodology of wind inversion from waves and radar remote sensing .......... 63
  4.4.1 Wind inversion from waves at certain frequencies ..................................... 64
  4.4.2 Method of wind inversion from radar first-order backscatter .................... 65
  4.4.3 Wind inversion from wave spectra ......................................................... 67
  4.4.4 Method of wind inversion from radar second-order effects ......................... 69
  4.4.5 Method of directional wave spectra inversion from radar second-order
        backscatter .................................................................................................. 69
4.5 Extension of wind measurements to the other locations within radar
        coverage ........................................................................................................ 71
4.6 Summary ......................................................................................................... 75

5 Radar Experiments and Results of Inversion ....................................................... 77
5.1 Radar experiments and in-situ measurements ............................................... 77
  5.1.1 Fedje experiment ...................................................................................... 77
  5.1.2 Ligurian Sea experiment .......................................................................... 79
  5.1.3 Wind and resonant waves ....................................................................... 81
5.2 Wind inversion from radar first-order peaks using new pattern fitting method 83
  5.2.1 Wind direction inversion during the Fedje experiment ............................... 83
  5.2.2 Wind direction inversion during the Ligurian experiment .......................... 86
5.3 Wind inversion from second-order sidebands using conventional methods 88
  5.3.1 SNR of the second-order sidebands ......................................................... 89
  5.3.2 Wind speed inversion from radar second-order spectra .............................. 90
5.4 Wind inversion from radar first-order peaks using neural networks ............ 92
  5.4.1 Wind inversion during the Fedje experiment ............................................. 93
  5.4.2 Wind inversion during the Ligurian Sea experiment ................................ 97
  5.4.3 Extension the wind measurements to the other locations within radar
        coverage using neural network ................................................................. 98
5.5 Wind speed inversion from HF radar second-order backscatter using neural network

5.5.1 Wind speed inversion from second-order sidebands during the Fedje experiment

5.5.2 Wind speed inversion from second-order sidebands during the Ligurian Sea experiment

5.5.3 Discussion of the wind speed inversion at the other locations within radar coverage using the second-order sidebands and NN

5.6 Wave inversion from radar second-order backscatter using neural network

5.7 Summary

6 Conclusions and Outlook

6.1 Conclusions

6.2 Outlook

A Wind Direction and Power Ratio of Radar First-order Peaks

A.1 Half-cosine 2s-power spreading function

A.2 Hyperbolic secant squared spreading function

Bibliography
## List of Figures

1. Test points in the North Sea .................................................. 10
2. Wind and Wave spectrum (HIPOCAS Data) ................................. 10
3. Feather vector wind at location C (bold arrow is at the time in b) and wind map 11
4. Directional wave patterns at certain wave frequencies (Note that the scales of the radial axis are different) .................................................. 11
5. Wind speed and direction varies with time ................................ 12
6. Wave growth with time at an increasing wind speed ..................... 12
7. Integrated wave energy at certain frequencies versus wind speed .......... 13
8. Examples of directional spreading function .................................. 15
9. Directional spreading function for different $\beta$ values .................. 16
10. Dimensionless wave height as a function of dimensionless fetch (Holthuijsen, 2008) 18

3.1 Sketch of HF surface wave radar(©IFM, University of Hamburg) .................. 21
3.2 HF Radar backscatter Doppler spectrum ....................................... 22
3.3 Principle of beam forming (a) and photo of receiving antenna array (b) .......... 23
3.4 Antenna directional beam patterns ($d = 0.45\lambda$, provided by Gurgel) ........ 24
3.5 Curves of received power against ranges at different operating radar frequencies: a - 7.5, b - 15 and c - 30 MHz, the transmitted power is 250 W, the solid curves are at a smooth sea state and the dashed curves are at a high sea state. (Shearman,1983) 25
3.6 Range-Doppler spectra at Fedje site during the Fedje experiment ............ 26
3.7 Range-Doppler spectra at Palmaria site during the Ligurian Sea experiment .... 26
3.8 Illustration of the second-order interaction process (Lipa 1986) ............... 28
3.9 Schematics of wind directions (red arrow), wave directional patterns and radio beam direction ................................................................. 29
3.10 Half-cosine 2s-power spreading function is a periodic function, while the hyperbolic secant squared function is a non-periodic function ......................... 30
3.11 Comparison of ratio as a function of wind direction (radio beam $\phi = 0^\circ$) ........ 31
3.12 Wind direction derived from the ratio of approaching and receding wave components ($\phi = 0^\circ$) ................................................................. 32
3.13 Diagram of wind wave pattern and radio beam direction .................... 33
3.14 Conventional methods for wind direction determination ................... 34
3.15 Radio beams during the Fedje experiment ..................................... 36
3.16 Direction curves and cross point for half-cosine 2s-power function ............ 37
3.17 Wind direction $\theta$ as a function of $\beta$ and given values of $R_1$ and $R_2$ ........ 38
3.18 Wind direction as a function of spreading parameter $\beta$ and given value of $R_1$ ($R_1 < 1$) and $R_2$ ($R_2 > 1$) .................................................. 39
3.19 Doppler spectra at different wind speed (simulated Doppler spectra at 25 MHz) .. 40
3.20 Simulated Doppler spectrum at different operating frequencies .................. 41
3.21 An example of the SNR of first-order peaks and second-order spectra during the Fedje and Ligurian Sea experiment .......................................................... 41
3.22 Wind wave pattern and wave energy along radio beam ............................ 43
3.23 Simulated second-order spectra (a) for different wind directions. Radar frequency: 30 MHz, Wind speed: 22 knots, directional spreading factor: $s^* = 2$; (b) for different wave spreading parameter at radar frequency: 30 MHz, wind speed: 22 knots, wind/radio angle (figures from Lipa 1977) .................................................. 45

4.1 Structure of artificial neuron model (from Duch 1999) .......................... 50
4.2 Example of multi-layer neural network (from Demuth 2009) ................. 52
4.3 Sketch of supervised learning (from Demuth 2009) .......................... 53
4.4 Extended network for the computation of the error function (from Rojas 1996) .. 55
4.5 Mean square error of the ANN during the training (from Demuth 2009) ...... 58
4.6 Network performance and training state (net.trainParam.max_fail = 20) in wind data inversion from radar first-order backscatter during the Fedje experiment (details of the network configuration are given in Section 5.4) .................. 62
4.7 Sketch of data set selection for neural network ...................................... 62
4.8 Sketch of network application for wind inversion at the same grid point .... 64
4.9 Wave directional patterns at frequency of 0.5476Hz .......................... 65
4.10 Sketch of neural network for inversion wind speed from Bragg waves using WAM model data ................................................................. 65
4.11 Scatter plot of wind speed inversion from waves at certain frequencies (HIPOCAS WAM data) ................................................................. 66
4.12 Derivation of wind data from the radar first-order backscatter ............... 66
4.13 Sketch of wind speed inversion from wave power density and direction .... 67
4.14 Wind speed inversion from the wave spectrum measured by wave buoy .... 68
4.15 Wave data at Location E in 2004 .................................................. 68
4.16 Wind speed inversion from wave power density spectra and direction (WAM model data) ................................................................. 69
4.17 Wind speed inversion from the second-order sidebands ....................... 70
4.18 Wave power spectrum and direction inversion from the second-order sidebands using neural network .......................................................... 70
4.19 Schematic of wind directions (red arrow), wave directional patterns at three grid points and radio beam directions ($\phi_1, \phi_2, \phi_3$) ................................. 71
4.20 Schematic of range circles of two radar sites for azimuth compensation factor ... 73
4.21 Sketch of network application for wind inversion within radar coverage ..... 74

5.1 Radar coverage during the Fedje experiment 2000 .................................. 78
5.2 Waverider measurements during the Fedje experiment .......................... 79
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3</td>
<td>Wind speed and direction measured by the anemometer at Fedje</td>
<td>79</td>
</tr>
<tr>
<td>5.4</td>
<td>Interface of radar data analysis during the Ligurian Sea experiment</td>
<td>80</td>
</tr>
<tr>
<td>5.5</td>
<td>Wave measurements by waverider during the Ligurian sea experiment</td>
<td>80</td>
</tr>
<tr>
<td>5.6</td>
<td>Wind speed and direction measured by the anemometer at the Ligurian Sea</td>
<td>81</td>
</tr>
<tr>
<td>5.7</td>
<td>Statistics of wind speed measurements by the anemometer</td>
<td>81</td>
</tr>
<tr>
<td>5.8</td>
<td>Wave energy at certain frequencies (two Bragg wave frequencies corresponding to the two radar frequencies) vs. wind speed (anemometer measurement) during the two experiments</td>
<td>82</td>
</tr>
<tr>
<td>5.9</td>
<td>Mean wave direction at Bragg frequencies (waverider) vs. wind direction (anemometer) during two experiments</td>
<td>83</td>
</tr>
<tr>
<td>5.10</td>
<td>Comparison of wind direction measured by radar using pattern fitting method and LSM method during the Fedje experiment</td>
<td>84</td>
</tr>
<tr>
<td>5.11</td>
<td>Spreading parameter $\beta$ vs. wind speed using the pattern fitting method ($\text{sech}^2(\beta \cdot \theta)$) during the Fedje experiment</td>
<td>85</td>
</tr>
<tr>
<td>5.12</td>
<td>Wind direction map using pattern fitting method during the Fedje experiment</td>
<td>85</td>
</tr>
<tr>
<td>5.13</td>
<td>Scatter plots of wind (mean wave) direction during the Ligurian Sea experiment</td>
<td>86</td>
</tr>
<tr>
<td>5.14</td>
<td>Spreading parameter $\beta$ vs. wind speed using the pattern fitting method ($\text{sech}^2(\beta \cdot \theta)$) during the Ligurian Sea experiment</td>
<td>87</td>
</tr>
<tr>
<td>5.15</td>
<td>Wind direction map derived from the first-order backscatter using pattern fitting method at the Ligurian sea (radar measurement: wind direction at the buoy location is $331^\circ$)</td>
<td>88</td>
</tr>
<tr>
<td>5.16</td>
<td>SNR of the second-order left sideband around the negative Bragg peak during the Fedje and Ligurian Sea experiment</td>
<td>89</td>
</tr>
<tr>
<td>5.17</td>
<td>Scatter plots of wind speed derived from radar second-order sidebands using the SMB method and anemometer measurements during the two experiments</td>
<td>90</td>
</tr>
<tr>
<td>5.18</td>
<td>Wind speed (from second-order sidebands using SMB method) and direction (from the first-order peaks) map during the Fedje experiment (radar measurement at buoy location: wind speed = 9.03 m/s, direction = $331^\circ$)</td>
<td>91</td>
</tr>
<tr>
<td>5.19</td>
<td>Wind speed (from second-order sidebands using SMB method) and direction (from the first-order peaks) map during the Ligurian experiment (radar measurement at buoy location: wind speed = 6.08 m/s, direction = $119^\circ$)</td>
<td>91</td>
</tr>
<tr>
<td>5.20</td>
<td>Structure of data set for wind inversion from radar first-order backscatter</td>
<td>93</td>
</tr>
<tr>
<td>5.21</td>
<td>Wind direction and speed derived from radar first-order backscatter using neural network and the anemometer wind measurement during the Fedje experiment</td>
<td>94</td>
</tr>
<tr>
<td>5.22</td>
<td>Comparison of wind speed derived from the first-order backscatter using neural network and the anemometer measurement during the Fedje experiment</td>
<td>94</td>
</tr>
<tr>
<td>5.23</td>
<td>Correlation coefficients of training, validation and testing data for wind speed inversion from radar first-order peaks during the Fedje experiment</td>
<td>95</td>
</tr>
<tr>
<td>5.24</td>
<td>Records for manual testing and the network training</td>
<td>96</td>
</tr>
<tr>
<td>5.25</td>
<td>Scatter plots of the wind speed, (a) is the result of network training, (b) is the result using the trained network for the new data set.</td>
<td>96</td>
</tr>
</tbody>
</table>
List of Figures

5.26 Scatter plots of wind direction and speed derived from radar first-order backscatter (neural network output) and in-situ wind at the Ligurian Sea ........................................... 97
5.27 Sketch of wind speed inversion from the direction spreading information of Bragg waves .......................................................................................................................... 98
5.28 Scatter plots of the wind speed derived for spreading parameter using neural network and the wind speed measured by anemometer during the Fedje and Ligurian Sea experiments ................................................................. 99
5.29 Wind map (direction and speed) derived from first-order peaks using neural network at the Norwegian Sea (at buoy location: wind speed = 7.05 m/s, direction = 331°) ......................................... 101
5.30 Wind map (direction and speed) derived from first-order peaks using neural network at the Ligurian Sea (at buoy location: wind speed = 5.1 m/s, direction = 119°) ........................................... 101
5.31 Structure of data set for wind speed inversion from radar second-order sidebands ................................................................. 102
5.32 Comparison of the wind speed inversion from radar second-order backscatter using neural network during the Fedje experiment ........................................................................ 103
5.33 Comparison of the wind speed inversion from radar second-order backscatter using neural network during the Ligurian Sea experiment .......................................................... 104
5.34 Structure of data set for directional wave spectra inversion from radar second-order sidebands ....................................................................................................................... 105
5.35 Comparison of wave power spectrum given by the in-situ buoy measurement and neural network during the Fedje experiment ............................................................................. 106
5.36 Comparison of wave direction given by the in-situ buoy measurement and neural network during the Fedje experiment .......................................................................................... 107
5.37 An example of wave comparison for the network output and buoy measurement ................................................................. 107

A.1 No cross point for curve \( \theta_1^+ \) and \( \theta_2^+ \) .......................................................................................................................... 117
A.2 One cross point for curve \( \theta_1^+ \) and \( \theta_2^+ \) .......................................................................................................................... 117
A.3 Two cross points for curve \( \theta_1^+ \) and \( \theta_2^+ \) .......................................................................................................................... 117
A.4 No cross point for curve \( \theta_1^- \) and \( \theta_2^- \) .......................................................................................................................... 119
A.5 One cross point for curve \( \theta_1^- \) and \( \theta_2^- \) .......................................................................................................................... 119
A.6 Two cross points for curve \( \theta_1^- \) and \( \theta_2^- \) .......................................................................................................................... 119
A.7 Threshold for \( R_2 \) having a cross point of \( \theta_1^+ \) and \( \theta_2^+ \) \((R_2 = R_{2,\text{min}})\), the cross point is \((\beta_{1,\text{min}}, \phi_1)\) .......................................................................................................................... 123
A.8 Threshold for \( R_2 \) having a cross point of \( \theta_1^+ \) and \( \theta_2^+ \) \((R_2 > R_{2,\text{min}})\) .......................................................................................................................... 123
A.9 Threshold for \( R_2 \) having a cross point of \( \theta_1^+ \) and \( \theta_2^+ \) \((R_2 < R_{2,\text{min}})\) .......................................................................................................................... 123
## List of Tables

2.1 Beaufort wind force and sea state scale .............................................. 6
3.1 Radar operating frequencies and Bragg wave properties .......................... 22
3.2 Wind direction $\theta$ and ratio $R$ ............................................................ 36
3.3 The start point of direction curve $(\beta_{i,\min}, \theta_{i,\beta_{\min}})$ and the power ratio of the first-order peaks $R_1$ and $R_2$ .......................... 39
5.1 Example of wave parameters provided by waverider at the Norwegian Sea .......... 78
5.2 Comparison of the RMS Error of wind direction related to wind speeds using the pattern fitting method and the conventional LSM method during the Fedje experiment 84
5.3 Comparison of the RMS Error of wind direction related to wind speeds for the radar and the in-situ meteorological buoy as well as the mean wave direction measurements during the Ligurian Sea experiment .......................... 87
5.4 Specification of neural network for wind inversion from the first-order backscatter 93
5.5 Specification of neural network for wind inversion from the spreading information of Bragg waves .......................... 99
5.6 Specification of neural network for wind speed inversion from the second-order backscatter (at the buoy location) .......................... 103
5.7 RMS error for the wave power density and direction inversion during the Fedje experiment .......................... 108
5.8 RMS error analysis for wind speed inversion using different methods .......................... 109
A.1 $R_1, R_2$ and number of the cross points for $\theta_1^+$ and $\theta_2^+$ .......................... 116
A.2 $R_1, R_2$ and number of the cross points for $\theta_1^-$ and $\theta_2^-$ .......................... 118
A.3 The possible cross points analysis for half cosine squared function .......................... 120
A.4 The possible cross points analysis for hyperbolic secant squared function .......................... 124
Chapter 1

Introduction

As the largest source of momentum at the ocean surface, wind affects the full range of ocean motion – from individual surface waves to complete current systems. Winds at sea surface modulate the coupling between atmosphere and oceans, which establishes and maintains both global and regional climates, and more importantly, the heat and gas exchange at the air-sea interface. The wind exerts a force or stress on the ocean surface, which produces not just ocean waves but also injects momentum into surface layer of the ocean. As we know the wind changes in strength and direction from place to place, which causes a spatially variable Ekman transport and ocean surface wave field. Wind observations have proven their significant impact on the forecasting of fast developing and severe weather as well as the global wind driven current circulation. As a consequence, winds at the sea surface are one of the most important sources data for the oceanographic research, and wind observations can also be implemented for data assimilation within models.

Wind at sea has been measured for centuries. Recently, the National Oceanic and Atmospheric Administration (NOAA) has collected and digitalized millions of observations going back over a century. The bulk of wind data over the ocean is provided by ships. These data nevertheless suffer from various sources of errors related to different anemometer heights, effects of ship movement and other boundary layer processes. Surveying vessels and buoy measurements can only provide point measurements and they can not be carried out in severe weather conditions. In the last decades, more and more remote sensing techniques have been implemented for wind measurements, which are based on actively illuminating the sea surface with electromagnetic energy and detecting the corresponding reflection. The sensors may be installed at the coast, on oil platforms or moving platforms (aircrafts or satellites). Spaceborne scatterometer can cover a large area of measurements, but with a coarse spatial resolution of 25~50 km [1], and it can not provide real-time measurements. More globally distributed wind data are inferred from cloud motions recorded by the geostationary satellites, but these are not provided with a uniform spatial density and need to be corrected from cloud level to the earth surface [2]. A shore-based HF radar can cover a large area (up to 200 km offshore), at a high resolution (300 m - 1500 m) and it can be
operated in real-time at all weather conditions. It is especially useful for the ocean current and waves as well as wind observations in the coastal areas.

The variability and change in atmospheric, oceanic and topographic conditions in the coastal areas complicate mesoscale weather systems and associated surface winds. Measurements of the mesoscale structure of the wind field in coastal areas are required to improve our understanding of the processes. Besides these, the wind observations could be exploited for economic use. For example, over the last decade, the deployment of offshore wind farms has received significant attention. The evaluation of wind fields is required to predict the energy capture and machine power generation levels, and the real-time monitoring of the wind field is also important for the maintenance of the turbines, for example, the turbines must be adjusted to adapt the wind conditions for generating the power efficiently.

1.1 State of Research

The basic physics of backscattering of electromagnetic waves from a rough sea surface was discovered by Crombie [3] in 1955. He found that the frequency shifts (Bragg peaks) in the Doppler spectrum corresponded uniquely with the ocean waves, which have the wavelength exactly half the radio wavelength, hence the mechanism was explained as “Bragg scattering”. The Doppler spectrum of the backscattered radar signal is characterized by two strong peaks which are caused by the Bragg-resonant scattering from the ocean surface. These peaks are surrounded by a continuum due to the second-order scattering.

Because there is no HF electromagnetic waves reflection from the movement of the atmosphere, the wind is measured indirectly, from the ocean wave parameters. For the wind direction measurement from HF radar backscatter, it is assumed that the wind direction is identical to the mean direction of the short waves, which are sensitive to the changes of the local wind. Extraction of the wind direction from the first-order peaks has been discussed for decades. In 1972, Long and Trizna [4] suggested using the amplitude of the two first-order peaks to determine the wind direction. Harlen [5] used an empirical approach finding a simple linear relationship between the power ratio of the first-order peaks and wind direction. Stewart [6] suggested using a cardioid model to describe the directional distribution of wind-waves. Several models are currently available for extracting wind direction from HF radar backscatter. All require the power ratio of the two first-order peaks, combined with an assumed directional distribution function for the Bragg resonant waves. Although details of the techniques differ, the principle is now well established. The main uncertainty lies in the dependency of the assumed wave directional distributions, which is also related to the prior knowledge of wind speed [7]. Most of researchers simplify the calculation by setting the directional spreading parameter to be a constant value. In this work, the author proposes a new method for extracting the wind direction as well as the directional spreading value of Bragg resonant waves.

Up to now, published work in this area presents solutions to estimate wind speed from
the ocean wave power spectra, which is derived from the HF radar second-order backscatter. In 1971, Hasselmann [8] first suggested that the amplitude of the second-order sidebands ought to be proportional to the non-directional wave power spectrum. From the radar-deduced wave parameters, Dexter et al. [7] proposed a method to invert wind speed using the dependency of significant wave height and dominate wave frequency on wind speed and fetch [9]. The basic principle is that, in a purely wind-driven sea, \emph{i.e.}, where the swell is negligible, the development of the wave energy is a function of wind speed, fetch and duration. The presence of swell leads to an overestimation of wind speed since it both increases the wave height and mean period [10]. However, the measurement of the wave spectrum requires a good Signal-to-Noise Ratio (SNR) of the second-order spectrum [11]. In case of low wind conditions, especially when the radar works at a lower frequency, the SNR of the second-order spectrum might be quite low [12]. Sometimes the second-order spectrum even can not be distinguished from radar background noise. The first-order peaks present the dominating feature in the radar spectrum and the power of positive and negative first-order peaks is proportional to the strength of the approaching and receding Bragg resonant waves. In this work, the amplitude and the directional spreading of Bragg waves are exploited for wind speed inversion using neural networks.

Estimating high quality geophysical parameters from remote sensing measurements is a very important issue in geosciences. To solve such inversion problems, the number of neural network applications increased steadily in the last decade. The rapid uptake of neural approaches in remote sensing is mainly due to their widely demonstrated ability of learning complex patterns, taking into account any non-linear complex relationship between the explicative and the dependent variables [13]. Although some neural network methods have been tried in HF radar signal processing [14, 15] and some other applications [16, 17, 18], few works about the wind inversion using neural networks have been reported.

### 1.2 Scientific Goals

The aim of this work is to derive wind speed and direction from HF radar backscatter. Based on different mechanisms of the radar first-order and second-order scattering as well as the experimental conditions, this dissertation focuses on the following questions:

1. The conventional methods for the wind speed inversion are based on the radar second-order backscatter. Could the first-order backscatter also be used for the inversion?

2. The power of the first-order peaks represents the energy scattered by the Bragg resonant waves along the radio beam. How could wind speed be inverted from the amplitude and directional information of the Bragg waves?

3. The neural network is trained using the radar data and in-situ observations at the location of in-situ data collection. How could the wind speed measurement be extended
to the other locations within radar coverage?

4. If the Bragg resonant waves come to a state of saturation, the wind speed has to be inverted from radar second-order backscatter. Could the neural network also be used for this inversion?

With these goals, the study of wind inversion addresses the issue using the neural network to tackle both the complexity of the radar remote sensing and wind-wave relationship.

1.3 Scope and Outline

This dissertation focuses on several aspects: wave model data analysis, HF radar remote sensing and wind inversion from HF radar backscatter using neural networks. The content of this dissertation is therefore arranged as follows:

Chapter 2 introduces the wind wave theory and principle of wind inversion from waves. In order to investigate how the wind acts on the sea surface, HIPOCAS wave model data is analyzed to get a better understanding of ocean surface waves. Based on the model results, new methods are proposed for deriving wind speed from HF radar backscatter.

Chapter 3 takes a close look at the high frequency radio backscatter from the rough sea surface and the theory of wind inversion from radar backscatter. When the radar operates at different frequencies, the signatures of the radar Doppler spectra are also different. For the wind direction inversion, the author proposes a new pattern fitting method, which gives a unique solution for wind direction as well as the spreading parameter of the Bragg resonant waves. The spreading parameter describes the directional distribution of resonant waves, which could also be used for the wind speed inversion.

Chapter 4 presents the principle of neural networks and the wind inversion methods using neural networks. The details of the network design are discussed. The wind inversion from waves are based on two possible procedures: (1) the amplitude and directional response of waves at a certain frequency to the changes of the local wind; (2) the wind speed inversion from the wave spectra. The WAM model and in-situ buoy data are investigated for the wind inversion, which leads to the ideas for the wind inversion from HF radar backscatter.

In Chapter 5, the wind inversion from HF radar backscatter is verified by two experiments: the Fedje experiment from February to April 2000 in Norway and the Ligurian Sea experiment from April to September 2009 in Italy. The radar operates at a frequency of 27.68 MHz and 12-13 MHz respectively. Based on different scattering mechanisms and experimental conditions, several neural networks are implemented for the wind inversion. Details of the experiments and data analysis are presented.

Finally, Chapter 6 summarizes the principal findings and presents potential ideas which could be investigated in the future.
Chapter 2

Wind Wave Theory and Wave Models

In HF radar remote sensing, the wind conditions at the sea surface are inverted from the signatures of the ocean surface waves. Wind generated waves are the most impressive phenomena found at the sea surface, ranging from capillary waves to storm surges. Since water moves easily because of its “fluid” nature, flat seas seldom occur and undisturbed water surface is rarely found at sea. The rough sea surface is a superposition of waves with different wavelengths and directions.

2.1 Wind waves at the sea surface

Ocean surface waves may be summarized as the interaction of different forces. First of all, there must be some kind of generating forces, in form of pressure or stress from the atmosphere (especially through the winds), which provides perturbations at the surface. When the water surface is no longer flat, restoring forces bring back the surface to its equilibrium state. The characteristics of the waves depend on the controlling forces and the waves can be classified by their periods. The most common waves (gravity waves) have a period between 1 s and 30 s are generated by the wind and restored by gravity. Wind-generated gravity waves are almost always present at sea. The description of the wind effects at the sea surface which has been used by mariners for observing the various intensities of the wind. The criteria are the results of long experience and represent individually distinguishable steps on a specific scale. The Beaufort scale was recommended for international use in 1874. Many studies have been made to determine wind speeds equivalent to the steps of the Beaufort scale and probable wave height [19], details are given in Table 2.1.

2.1.1 Wave basic definition

The basic mathematical representation of an ocean wave is given by the sinusoidal curve:

\[ \eta(x, t) = a \sin(kx - \omega t) \]  

(2.1)
Chapter 2. Wind Wave Theory and Wave Models

<table>
<thead>
<tr>
<th>Beaufort wind scale</th>
<th>Mean Wind Speed</th>
<th>Limit Wind Speed</th>
<th>Probable wave height</th>
<th>sea states</th>
<th>Sea descriptive terms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Knots</td>
<td>m/s</td>
<td>Knots</td>
<td>m/s</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>&lt;1</td>
<td>0.0-0.2</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1-3</td>
<td>0.3-1.5</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4-6</td>
<td>1.6-3.3</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>7-10</td>
<td>3.4-5.4</td>
<td>0.6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>11-16</td>
<td>5.5-7.9</td>
<td>1.0</td>
<td>3-4</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
<td>17-21</td>
<td>8.0-10.7</td>
<td>2.0</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>22-27</td>
<td>10.8-13.8</td>
<td>3.0</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>28-33</td>
<td>13.9-17.1</td>
<td>4.0</td>
<td>5-6</td>
</tr>
<tr>
<td>8</td>
<td>37</td>
<td>34-40</td>
<td>17.2-20.7</td>
<td>5.5</td>
<td>6-7</td>
</tr>
<tr>
<td>9</td>
<td>44</td>
<td>41-47</td>
<td>20.8-24.4</td>
<td>7.0</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>52</td>
<td>48-55</td>
<td>24.5-28.4</td>
<td>9.0</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>60</td>
<td>56-63</td>
<td>28.5-32.6</td>
<td>11.5</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>64+</td>
<td>32.7+</td>
<td>14+</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 2.1: Beaufort wind force and sea state scale

where \( k = 2\pi/\lambda \) is the wave number and \( \lambda \) is the wavelength, \( \omega \) is the angular frequency and \( a \) is the wave amplitude. Equation 2.1 contains both time \((t)\) and space \((x)\) coordinates. For all types of truly periodic progressive waves, one can write:

\[
\lambda = c_p T
\]  
(2.2)

where \( c_p \) is the wave phase speed, from above we know that \( c_p = \omega/k \). The variation of wave speed with wavelength is called dispersion and the relation is given [20]:

\[
c_p = \frac{\omega}{k} = \pm \sqrt{\frac{g}{k}} \tanh(kD)
\]

(2.3)

where \( g \) is gravitational acceleration and \( D \) is the water depth.

2.1.2 Ocean wave spectrum

Ocean surface waves can be expressed by a superposition of linear waves which are called fundamental waves. Nonlinear processes are important for the interactions between fundamental waves.

2.1.2.1 Definition of wave spectrum

The three-dimensional (frequency and wave number vectors) wave spectrum for a stationary and homogeneous wave field is defined by Y. Hisaki [21, 22] as follows:

\[
X(\omega, k) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C(r, t) \exp[-i(k \cdot r - \omega t)] dt dr
\]

(2.4)
where \( C(r, t) = \langle \eta(r_0, t_0) \eta(r_0 + r, t_0 + t) \rangle \) is the covariance of the sea surface displacement, \( r \) is the spatial separation vector, \( t \) is the time separation and \( \langle \cdot \rangle \) denotes an ensemble averaging. \( k \) is the vector wave number \( (k = (k \cos \theta, k \sin \theta)) \), \( \theta \) is the wave propagating direction. The wave spectra can be defined in the reduced forms as follows [21]:

- Wave number vector spectrum

\[
S(k) = 2 \int_0^{+\infty} X(\omega, k) d\omega \tag{2.5}
\]

- Directional angular frequency spectrum

\[
G(\omega, \theta) = 2 \int_0^{+\infty} X(\omega, k) kd\theta \tag{2.6}
\]

- Angular frequency spectrum

\[
\Psi(\omega) = \int_{-\infty}^{+\infty} G(\omega, \theta) d\theta \tag{2.7}
\]

The spectra are expanded by perturbation series:

\[
X = X_1 + X_2 +, ..., \\
S = S_1 + S_2 +, ..., \\
G = G_1 + G_2 +, ..., \\
\Psi = \Psi_1 + \Psi_2 +, ..., \tag{2.8}
\]

Subscript 1 denotes spectra composed of fundamental waves, subscript 2 denotes spectra composed of bound waves, which are the product of non-linear combination of two fundamental waves. In deep water, the first-order wave spectrum is expressed as:

\[
X_1(\omega, k) = \frac{1}{2} \sum_{m=\pm 1} S_1(mk) \delta(\omega - m(gk)^{1/2}) \tag{2.9}
\]

where \( \delta \) is the Dirac-delta function, \( m \) represents the positive and negative wave spectra in frequency domain. The second-order wave spectrum is expressed as:

\[
X_2(\omega, k) = \frac{1}{2} \sum_{m_1=\pm 1} \sum_{m_2=\pm 1} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Gamma^2 H S_1(m_1, k_1) S_1(m_2, k_2) \cdot \delta(\omega - m_1(gk_1)^{1/2} - m_2(gk_2)^{1/2}) dp dq \tag{2.10}
\]

Here the fundamental waves, whose wave number vectors are \( k_1 \) and \( k_2 \), are coupled non-linearly, satisfying the relation \( k = k_1 + k_2 \), where \( k_1 = k/2 + \kappa \) and \( k_2 = k/2 - \kappa \) for \( \kappa = (p, q) \), here, the spatial vector \( p \) is defined to lie along the generated waves \( k \), and
Chapter 2. Wind Wave Theory and Wave Models

$q$ is perpendicular to vector $p$. $m_i = \pm 1 \ (i = 1, 2)$ gives four solutions for the wave-wave coupling. In Equation 2.10, the coupling coefficient $\Gamma_H$ is written as follows:

$$\Gamma_H = \frac{1}{2} \left[ k_1 + k_2 + \frac{(k_1k_2 - k_1 \cdot k_2)}{m_1m_2(k_1k_2)^{1/2}} \left( gk + \omega^2 \right) \right] (2.11)$$

### 2.1.2.2 Wave spectrum examples

A wave spectrum is commonly used for modeling the sea state. Models enable the spectrum to be expressed as some functional forms, usually in terms of frequency. Here, some widely used wave spectrum models are introduced.

- **Pierson-Moskowitz (P-M) wave spectrum**

  The Pierson-Moskowitz spectrum $\Psi(\omega)$ is often used for a fully developed sea, which means that the wind blows steadily for a long time over a large area, the waves would come into equilibrium with the wind. To obtain a spectrum of fully developed sea, they used measurements of waves made by accelerometers in the north Atlantic [23, 24].

  $$\Psi(\omega) = \frac{\alpha_{PM}^2}{\omega^5} \exp[-\beta_{PM}(\frac{\omega_0}{\omega})^4] (2.12)$$

  where $\alpha_{PM} = 8.1 \times 10^{-3}$, $\beta_{PM} = 0.74$, $\omega_0 = g/U_{19.5}$ and $U_{19.5}$ is the wind speed at a height of 19.5 m above the sea surface, which is the height of anemometers on the weather ships used in their experiments.

- **JONSWAP wave spectrum**

  After analyzing data collected during the JOint North Sea WAve Project (JONSWAP), Hasselmann et al. [25] found, that the wave spectrum is never fully developed. It continues to develop through non-linear wave-wave interaction even for a very long time and distance. They therefore proposed the spectrum $\Psi(\omega)$:

  $$\Psi(\omega) = \frac{\alpha^2}{\omega^5} \exp[-\frac{5}{4}(\frac{\omega_0}{\omega})^4] r \ (\gamma = 3.3) \quad (2.13)$$

  $$r = \exp[-\frac{(\omega - \omega_p)^2}{2\sigma_J^2\omega_p^2}]$$

  Wave data collected during the JONSWAP experiment are used to determine $\alpha$, $\omega_p$ and $\sigma_J$ in Equation 2.13:

  $$\alpha = 0.076(U_{10}^2/g)^{0.22} \quad (2.14)$$

  $$\omega_p = 22[g^2/(U_{10}F)]^{1/3} \quad (2.15)$$
\[ \sigma_J = \begin{cases} 
0.07 & \omega \leq \omega_p \\
0.09 & \omega > \omega_p 
\end{cases} \quad (2.16) \]

2.2 HIPOCAS WAM data analysis

In order to investigate how wind acts at sea surface in a controlled environment, the HIPOCAS wave model data is analyzed to get a better understanding of the relationship between the wind and ocean surface waves. HIPOCAS is a project to obtain a 40-year hindcast of wind, wave, sea-level and current climatology for European waters and coastal seas for the application in coastal and environmental processes. Circulation models are used in the North Sea and some other regions of the north Atlantic ocean. The data is processed with the horizontal resolution of 10 km and the temporal resolution of 3 hours.

2.2.1 HIPOCAS WAM model introduction

In wave modeling, theoretical and observational knowledge on ocean surface waves are combined and expressed mathematically. The wave spectrum is the most common way of describing the wave condition at a certain location. Its evolution in time and space is often calculated using the wave energy balance equation, expressed by

\[ \frac{\partial E}{\partial t} + \nabla \cdot (c_g E) = S_{\text{Total}} = S_{\text{in}} + S_{\text{nl}} + S_{\text{ds}} \quad (2.17) \]

The evolution of the spectrum depends mainly on three source functions, wind input \((S_{\text{in}})\), nonlinear interaction \((S_{\text{nl}})\) and dissipation \((S_{\text{ds}})\). Details of the WAM model are given by G.J.Komen [2].

The wave models used in HIPOCAS project are based on WAM with nested grids in order to produce high resolution data. In the North Sea, this model takes into account tidal currents that influence the waves. Several points are analyzed as shown in Figure 2.1. Each point contains the wind data (speed and direction) and the directional wave spectra. The wave frequency range \((0.0418-0.5476 \text{ Hz})\) is divided into 28 bins, the wave direction is divided into 24 bins (every 15 degrees), so the total number of bins is \(28 \times 24 = 672\).

2.2.2 Spatial and directional analysis

In order to demonstrate the presence of swell and residual waves, an example of wave spectra and wind data (speed and direction) is shown in Figure 2.2. The higher frequency part are wind waves, which are forced by the local wind. In this example, the waves at lower frequencies are definitely not generated by the local wind if their directions are with a large deviation to wind direction. They might be swell traveling from far away or residual waves. Figure 2.3a shows the wind records in previous days, the north-east wind and north
wind have blown for nearly two days. After the wind changes its direction to south-west, the energy of the longer waves still remains.

From the wind map demonstrated in Figure 2.3b, the wind at location B, E, F, G blows from north-west and the swell travels from the E, F, G to the region C. The 2-D wave spectra present the information of wave directions at different frequencies.

<table>
<thead>
<tr>
<th>Wind Direction</th>
<th>Wind Speed</th>
<th>Wave Direction</th>
<th>Wave Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>228.9°</td>
<td>6.24 m/s</td>
<td>92.2°</td>
<td>1.1166 m</td>
</tr>
<tr>
<td>fp = 0.23225</td>
<td>E(fp) = 1.9574 m² · s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.2: Wind and Wave spectrum (HIPOCAS Data)

Figure 2.4 presents directional wave patterns at four frequencies. At the higher wave frequencies, the directional spreading patterns are smoother, while at the lower frequencies, the patterns are more or less disturbed by other effects, such as swell or residual waves (if the wind turns its direction). The waves travel with a certain directional pattern which is closely related to its frequency and surface wind conditions.
2.2. HIPOCAS WAM data analysis

Figure 2.3: Feather vector wind at location C (bold arrow is at the time in b) and wind map

Figure 2.4: Directional wave patterns at certain wave frequencies (Note that the scales of the radial axis are different)

2.2.3 Temporal and frequency analysis

In order to analyze the time response of waves to the changes of wind speed, some data is chosen from the wind record. As shown in Figure 2.5, at the beginning the wind is weak
(less than 1 m/s). As time goes on, the speed increases and the wind almost blows from a constant direction after the wind speed exceeds a certain value. The wind speed and direction are shown in Figure 2.5.

![Wind Speed Record](image1)

(a) Wind speed

![Wind Direction Record](image2)

(b) Wind direction

Figure 2.5: wind speed and direction varies with time

The wave growth with time in terms of wavelength is shown in Figure 2.6. Figure 2.6a shows the wave growth curve of short waves (e.g., \( f = 0.5476 \) Hz). The short wave increases quickly and gets saturated at a low wind speed. Regarding the long wave as shown in Figure 2.6b (e.g., \( f = 0.1745 \) Hz), these take more time to get saturated. So the short waves are more sensitive to change of the wind, which could be used to invert the local wind conditions (especially wind direction).

![Wave growth with time (short wave)](image3)

(a) At 0.5476 Hz (\( \lambda = 5.21 \) m)

![Wave growth with time (long wave)](image4)

(b) At 0.1745 Hz (\( \lambda = 51.28 \) m)

Figure 2.6: Wave growth with time at an increasing wind speed

In Section 2.1.2, in order to describe wave spectrum from different perspectives, several forms of wave spectrum are given. Here, the frequency spectrum \( \Psi(f) \) in WAM model is

\[
\Psi(f) = \sum_{n=1}^{N} \mathcal{G}(f, \theta_n) \quad (\Delta \theta = \frac{\pi}{12}, N = 24)
\]

(2.18)
Where $G(f, \theta)$ is the wave directional spectra, the integrated wave power $\Psi(f_i)$ at wave frequency $f_i$ is calculated by Equation 2.18. The data for a full year at location $E$ is analyzed, the growing curves of wave power versus wind speed are given in Figure 2.7. Several wave frequencies are selected that coincide to Bragg frequencies often used in radar measurements (Table 3.1). The wave energy increases due to the wind speed, the longer waves need higher wind speed to get saturated. In Figure 2.7a, the saturation of waves even can not be seen due to few wave records in extreme wind conditions, while the saturation of short waves at a certain wind speed can be seen in Figure 2.7b and 2.7c. A linear regression method is used to calculate the increasing rate ($k_i$) for the wave energy before the turning point of saturation.

![Wave Energy vs Wind Speed](image)

(a) $k_1 = 0.0995$

(b) $k_2 = 0.048$

(c) $k_3 = 0.0061$

(d) Wave energy vs. wind speed

Figure 2.7: Integrated wave energy at certain frequencies versus wind speed

The wave measurement at a certain frequency can be conducted by HF radar Bragg scattering, and wind speed can be estimated from the wave power before the waves get saturated (the directional spreading pattern at certain frequencies is also related to wind, which will be discussed later).
2.3 Wind direction and wave directional distribution

The directional wave spectrum \( G(f, \theta) \) describes how the wave energy is distributed over the ranges of the frequency \( f \) and the angle \( \theta \). It is expressed as the product of the frequency spectrum \( \Psi(f) \) and the directional spreading function \( G(f, \theta) \):

\[
G(f, \theta) = \Psi(f) \cdot G(f, \theta)
\]  \hspace{1cm} (2.19)

The directional spreading function indicates how a given energy density at each frequency is spread over the directional angle and thus it is made dimensionless as follows:

\[
\int_{-\pi}^{\pi} G(f, \theta) d\theta = 1
\]  \hspace{1cm} (2.20)

There have been several proposals for the directional spreading functions. Here, two widely used: half-cosine 2s-power and Hyperbolic secant-squared are discussed.

2.3.1 Half-cosine 2s-power type spreading function

The earliest directional spreading function is the cosine-squared type which was used by Pierson, Neumann and James [26] in their spectral wave forecasting method. The function is later evolved into the half-cosine 2s-power type by Loguet-Higgins and H. Mitsuyasu et al. [27, 28]. The function is given:

\[
G(f, \theta) = A \cdot \cos^{2s} \left( \frac{\theta}{2} \right)
\]  \hspace{1cm} (2.21)

\( A \) is a normalizing factor satisfying Equation 2.20, so we have:

\[
A = \frac{1}{\pi} 2^{2s-1} \frac{\Gamma^2(s + 1)}{\Gamma(2s + 1)}
\]  \hspace{1cm} (2.22)

where \( \Gamma \) denotes the Gamma function. For example, by setting \( s = 4 \), \( A \) turns out to be 0.5821 and the directional spreading function is depicted in Figure 2.8.

Mitsuyasu et al. [28] presented a reasonable comprehensive set of estimates for \( s \) using measurements obtained from a cloverleaf buoy. The directional spreading function has the features that the parameter \( s \) represents the degree of directional energy concentration. The directional spreading of wave energy is narrowest around the spectral peak energy, the original proposal of Mitsuyasu relates the spreading parameter \( s \) to the wind speed. Goda and Suzuki [29] have given the original equation into the following form by introducing the peak value of \( s \), denoted as \( S_{\text{max}} \):

\[
s = \begin{cases} 
(f/f_p)^5 \cdot S_{\text{max}} & : f \leq f_p, \\
(f/f_p)^{-2.5} \cdot S_{\text{max}} & : f \geq f_p.
\end{cases}
\]  \hspace{1cm} (2.23)

The degree of directional spreading of wave energy greatly affects the extent of wave
2.3. Wind direction and wave directional distribution

refraction and diffraction. Thus, the estimation of the value of the parameter $S_{\text{max}}$ is important. The observation by Mitsuyasu et al. showed that the peak value increases as the parameter $2\pi f_p U_{10}/g$, which represents the state of wind wave growth. They introduced the relation [30]

$$S_{\text{max}} = 11.5(2\pi f_p U_{10}/g)^{-2.5}$$  \hspace{1cm} (2.24)

Based on the argument that the primarily non-linear processes determine the wave spectrum. Hasselmann et al. [31] suggested that $s$ depends mainly on $f/f_p$ for $f \geq f_p$. As a referee [32] has pointed out, the spreading parameter $s$ might be expected to depend both on $U_{10}/c_p$ and $f/f_p$ even if the directional distribution is mainly governed by nonlinear transfer, where $c_p$ is the phase speed at peak frequency $f_p$. The peak enhancement parameter $\gamma$ and the “constant” $\alpha$ for the JONSWAP spectrum [25] depend on $U_{10}/c_p$. Nevertheless, the general tendency should be as follows: If the spectral shape is entirely governed by input from the wind, the relation $s = s(U_{10}/c_p; f/f_p)$ should degenerate into $s = s(U_{10}/c_p)$; If, on the other hand, the spectral shape is governed by non-linear interactions, we should expect $s$ to depend mainly on $f/f_p$ with a slight dependency on $U_{10}/c_p$. Hasselmann presented the spreading parameter as follows:

$$s = \begin{cases} 9.77(f/f_p)^{-2.33-1.45(U_{10}/c_p)-1.17} & \text{if } f \geq f_p \\ 6.97(f/f_p)^{4.06} & \text{if } f < f_p \end{cases}$$ \hspace{1cm} (2.25)

2.3.2 Hyperbolic secant-squared type spreading function

In 1985, M.A.Donelan et al. [32] proposed hyperbolic secant function based on the observations of wind and water surface elevation by 14 wave staffs in Lake Ontario and a
large laboratory tank. The directional spectrum of wind-generated waves in deep water is determined using a modification of Barber’s method [33]. The angular spreading is given:

\[ G(f, \theta) = 0.5\beta \text{sech}^2(\beta \cdot \theta) \]  

where \( \beta \) is the spreading parameter. The three-dimensional evolution of freely propagating, second-order Stokes gravity wave groups [34, 35] indicates that an envelope soliton group propagating around the main wave direction is described by a hyperbolic secant \( \text{sech}^2(\beta \theta) \). The width of the spectral spread is determined by the parameter \( \beta \). Several examples of the spreading functions are given in Figure 2.9:

\[ G(\theta) = 0.5 \beta \text{sech}^2(\beta \cdot \theta) \]

(a) Cartesian coordinate  
(b) Polar coordinate

Figure 2.9: Directional spreading function for different \( \beta \) values

M.L.Banner [36] presented the parameter \( \beta \) based on the wave frequency spectral techniques with an improved angular resolution pitch-and-roll buoy. Using an extension of the wave gauge array technique to higher wave numbers, it shows a continuous increase in spreading beyond \( f/f_p > 2.56 \) with a spreading cutoff at much shorter scales, consistent with the broad directional distribution observed by Banner et al. [37] at higher \( f/f_p \). On this basis, the spreading distribution is given:

\[ \beta = \begin{cases} 2.28(f/f_p)^{-0.65} & \text{for } 0.97 < f/f_p < 2.56 \\ 10^{-0.4+0.8393\exp[-0.567\ln(f/f_p)]} & \text{for } f/f_p > 2.56 \end{cases} \]  

(2.27)

### 2.4 Wind speed inversion from wave spectrum

In an attempt to describe the relationship between wave and wind, a number of empirical/theoretical equations have been formulated in the past thirty years. In a purely wind-driven sea, the state of development of the wave spectra is a function of wind speed \( (U_{10}) \), fetch \( (F) \) and duration \( (T) \). In particular, such gross sea parameters as total wave energy
2.4. Wind speed inversion from wave spectrum

$E$ (or significant wave height $H_S$) and peak wave frequency $f_p$ are functions of both $F$ and $U_{10}$. Thus, for a given measurement of $E$, a variety of solutions of $F$ and $U_{10}$ are possible. To simplify this further, since the two parameters ($F$ and $U_{10}$) are unknown (the duration $T$ could be involved in $F$ as given in Equation 2.29), it is possible to derive these quantities from simultaneous measurements of such two parameters with some appropriate theoretical/empirical relationships.

2.4.1 Dimensionless parameters

The wave power spectrum $S(k)$ at a wind-generated sea is a function ($\mathbb{F}$) of surface wind speed $U_{10}$, fetch $F$, duration $T$ and gravitational acceleration $g$:

$$S(k) = \mathbb{F}(g, U_{10}, F, T) \tag{2.28}$$

In practical applications, the four parameters are often reduced to three by expressing the duration $t$ in terms of an equivalent fetch $F_{eq}$. Considering that at time $t$, the waves have traveled a distance $c_g t$ along wind direction since the wind started to blow ($c_g$ is the group velocity of the wave component). So $F_{eq}$ could be given with an integrand [38]:

$$F_{eq} = \int_0^T c_{g, \text{peak}}(t) dt \tag{2.29}$$

where $c_{g,\text{peak}}$ is the group velocity of the evolving peak frequency. With Equation 2.29, the number of parameters can be reduced from four ($F, T, U_{10}, g$) to three ($F, U_{10}, g$), which can be combined into one dimensionless parameter, the dimensionless fetch $F^*$:

$$F^* = \frac{g F}{U_{10}^2} \tag{2.30}$$

Besides the dimensionless fetch, there are also many other dimensionless parameters for wave growth which have been derived from large data sets. These formulas make no attempt to separate the physical processes involved. They simplify computation if the variables are all made dimensionless [39].

- Peak frequency $f_p^* = U_{10} f_p / g$
- Duration $T^* = g T / U_{10}$
- Height $H_S^* = g H_S / U_{10}^2$

2.4.2 SMB curves for wind speed inversion

By far, the most widely used is the equation first developed by Sverdrup and Munk [40, 41] and later modified by Bretschneider [42], the so-called SMB curves [9, 43]. Despite some criticism of their lack of a proper theoretical basis and simplistic description of the sea
surface [9], they continue to find wide application in both coastal engineering and wave forecasting [44, 45]. Most researchers have fitted the following function to their observations [38]:

\[ H^* = H^*_\infty \tanh[k_1(F^*)^M] \]  

(2.31)

where \( k_1 \) and \( M \) are constant values, \( H^*_\infty \) is the dimensionless \( H^* \) for \( F^* = \infty \). Pierson and Moskowitz [24] analyzed the observations of such fully developed waves in the North Atlantic. Since the fetch is not relevant in such cases, the significant wave height and period depend only on the local wind speed. This implies that, under these fully developed conditions, the dimensionless significant wave height and period are universal constants, and they gave the value of \( H^*_\infty = 0.24 \). The dimensionless wave height \( H^* \) is depicted in Figure 2.10.

![Figure 2.10: Dimensionless wave height as a function of dimensionless fetch (Holthuijsen, 2008)](image)

Dexter [7] gave in the principal relationship of significant wave heights \( H_S \), fetch \( F \) and wind speed \( U_{10} \):

\[ \frac{gH_S}{U_{10}^2} = 0.26 \cdot \tanh[\frac{1}{10^2}(\frac{gF}{U_{10}^2})^{1/2}] \]  

(2.32)

In JONSWAP project, Hasselmann presented the relationship between dimensionless fetch \( F^* \) and dimensionless spectral peak frequency \( f_p^* \) [30]:

\[ f_p^* = 3.5(F^*)^{-1/3} \]  

(2.33)

where \( f_p^* = f_p \cdot U_{10}/g \) and \( F^* = F \cdot g/U_{10}^2 \). Thus in terms of wave period \( T_p \) at the spectral peak, Equation 2.33 becomes

\[ \frac{T_p}{U_{10}} = \frac{1}{3.5g} \left( \frac{gF}{U_{10}^2} \right)^{1/3} \]  

(2.34)
Equation 2.32 and 2.34 are sufficient to allow a unique evaluation of $U_{10}$ in terms of the radar measured $H_S$ and $T_p$. An algebraic manipulation is given:

$$\frac{gH_S}{U_{10}^2} = 0.26 \tanh\left(\frac{T_p}{U_{10}}^{3/2} \frac{(3.5g)^{3/2}}{10^2}\right) \tag{2.35}$$

In HF radar remote sensing, the extraction of wave parameters (significant wave height $H_S$ and the wave peak frequency $f_p$) from radar second-order backscatter is presented in Section 3.4.
Chapter 3

HF Radar Remote Sensing and Wind Inversion

3.1 Introduction to HF radar remote sensing

Sea echoes at HF have been observed by radars since World War II. In 1955, Crombie [3] found that the discrete frequency shifts (the first-order peaks) above and below the carrier frequency observed in the Doppler spectrum corresponded uniquely with the ocean waves, which have the wavelength exactly half the radio wavelength (grazing incidence) moving towards or away from the radar site. These waves are called “Bragg waves”. Wait [46] analytically verified Crombie’s observation by examining the reflection of electromagnetic waves from a gently rippled surface. Barrick and Peake [47] confirmed the effect of the resonant phenomena by examining the scatter from slightly rough sea surface. The boundary conditions proposed by Rice [48] involved a perturbation approach to examine the problem of radio wave scattering from rough surfaces. Based on Rice’s theory, Barrick [49] derived a model for the first-order cross section of the ocean surface that is consistent with Crombie’s observations. Ward [50] suggested that the continuum surrounding the first-order peaks is due to higher order interactions.

![Figure 3.1: Sketch of HF surface wave radar](https://example.com/sketch)

The high frequency radio band covers frequencies between 3 and 30 MHz with wavelengths of 100-10 m, which is in the same order as the wavelengths of ocean waves, so
the Bragg scattering theory is applicable. The signal therefore interacts with the ocean surface waves and the back-scatter echo contains a wealth of information about the sea state. Figure 3.1 gives the sketch of HF radar remote sensing, in which, the wavelength of electromagnetic wave is $\lambda_r = 10 \text{ m}$, so the wavelength of the resonant ocean wave is $\lambda_w = 5 \text{ m}$.

The roughness of the ocean surface is the combination of waves of different wavelengths and directions. When the radar works at different frequencies, the corresponding Bragg wave frequency (wavelength) varies as well. The Bragg wave frequency is given as follows:

$$f_B = \frac{c_{\text{Bragg}}}{\lambda_w} = \sqrt{\frac{g}{2\pi\lambda_w}} = \sqrt{\frac{gF_r}{2\pi c}} \quad (3.1)$$

where $c_{\text{Bragg}}$ is the phase velocity of Bragg waves, $\lambda_w$ is the wavelength of Bragg waves and $F_r$ is the radar frequency. Table 3.1 gives a list of often used radar frequencies and the corresponding Bragg wavelengths and frequencies. The property of wave growth rate and directional spreading is quite different due to their wavelengths. This relationship can be used to derive wind information.

<table>
<thead>
<tr>
<th>Radar frequency (MHz)</th>
<th>“Bragg” wave frequency (Hz)</th>
<th>“Bragg” wave length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.228</td>
<td>30</td>
</tr>
<tr>
<td>7.5</td>
<td>0.279</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>0.3534</td>
<td>12.5</td>
</tr>
<tr>
<td>27.68</td>
<td>0.5368</td>
<td>5.419</td>
</tr>
</tbody>
</table>

Table 3.1: Radar operating frequencies and Bragg wave properties

Figure 3.2 shows a typical HF radar Doppler spectrum, which contains dominant peaks due to first-order (Bragg) scattering and a structured continuum due to higher-order scatter (mainly second-order spectrum).
3.1.1 WERA system

Researchers of the University of Hamburg started working on HF radars in 1980. WERA (WEllen RAdar) has been developed within the European project - Surface Current And Wave Variability Experiment (SCAWVEX). Information on the WERA system design is presented by Gurgel [51]. One advantage of the system is the ability to use different configurations of receive antennas. With a linear array, information about the sea state can be obtained via second-order spectra bands [52]. Another advantage is the flexibility in range resolution between 0.3 km and 3 km by using Frequency-Modulated Continuous Wave (FMCW), which can simply be achieved by reconfiguring the bandwidth of the chirp. In addition, this technique avoids the blind range in front of the radar because there is no transmit to receive switching involved. The transmit antenna array is designed to make sure that the null produced in the antenna pattern points towards the receive antennas to reduce the energy transmitted on the direct path from the transmit to the receive antenna.

Beam forming is a signal processing technique used in sensor arrays for directional signal transmission or reception. Information from different antennas is combined in such a way that the expected pattern of radiation is preferentially observed. The advantage of beam forming is that the beams can be steered to achieve a particular area coverage which may be located around a buoy (which is mainly implemented in this dissertation). Beam forming generally increases the antenna gain which also increases the signal-to-noise ratio of the echoes received. Each antenna element in the array has its own receiver and A/D converter and beams are formed by digital processing all the outputs. The system is flexible and beams can be recalculated for further processing, different weight (window) functions can be applied to control the antenna side lobes [53].

![Diagram of Beam Forming](image)

Figure 3.3: Principle of beam forming (a) and photo of receiving antenna array (b)

As described in Figure 3.3a, a wave front from direction $\theta$ arrives at antenna 1 first. Then after traveling an additional path distance $\Delta l$, it arrives at antenna 2, and we have $\Delta l = d \sin \theta$. The path difference results in a phase difference $\Delta \phi$ between the signals from the two antennas:
\[ \Delta \varphi = 2\pi \frac{\Delta l}{\lambda_r} = 2\pi d \sin \theta / \lambda_r \quad (3.2) \]

Figure 3.3a shows a simple example of 2-element beam forming. When the number of elements increases, the side lobes become smaller and the central beam becomes narrower. During the WERA experiments in Norway and Italy, a 16-element antenna is used. One photo of receive antenna array is shown in Figure 3.3b.

The grid is defined within radar coverage, for example, during the Fedje experiment, a \( 60 \times 50 \) grid is defined. According to the longitude and latitude of the grid points, the distance from the grid point to each antenna is calculated in WGS84 coordinates [54], and the distance between each adjacent antenna element is less than half of radio wavelength \( (d < \lambda/2) \). So the phase difference \( \Delta \varphi_i \) \( (i \) is the number of antenna) is determined. The size of cell is determined by the azimuth resolution \( \Delta \phi \) (beam-width at -3dB) and the range resolution \( \Delta R \). Two typical antenna directional patterns pointing to \( 0^\circ \) and \( 45^\circ \) are presented in Figure 3.4.

![Antenna directional beam patterns](image)

Figure 3.4: Antenna directional beam patterns \((d = 0.45\lambda, \text{provided by Gurgel})\)

### 3.1.2 Physical scattering model and radar cross section

The earliest approach to the problem of the scattering of electromagnetic waves from the rough surface is “perturbation”, initiated by Lord Rayleigh in 1896 [55], and implemented by Rice [48]. Following Rice, many investigators, including Wait [56], Barrick [49, 57, 58, 59]
3.1. Introduction to HF radar remote sensing

and later J. Walsh [60] and E. Gill et al. [61, 62] gave the contribution to the scattering theory. The basic requirements in the application of the perturbation method are that (1) the surface profile variations are small compared to the radio wavelength; (2) the surface slopes are much smaller than unity; (3) the impedance of the surface medium is small in terms of the free space wave impedance. The three requirements must be satisfied for the applications of scattering theory in HF radar remote sensing.

3.1.2.1 HF radio propagating on conductive ocean surface

For the radio wave propagation at sea surface from a vertically polarized transmit antenna, the high conductivity of the sea water results in a low attenuation. The performance of the radar detecting and characteristics of propagating channel as well as “target” (ocean waves) determine the signature of radar echoes. The sea surface has a scattering capability described by the backscatter coefficient $\sigma_0$, the echoing area per unit area, which is usually expressed in dB. The expression for the received power (mono-static condition) is:

$$P_R = \frac{P_T G_T G_R \lambda_r^2 F^4(d)}{4\pi^3 d^9} \cdot \sigma_0 \Delta d \Delta \phi \quad (3.3)$$

Here $P_T$, $P_R$ are transmitted and received power, $\lambda_r$ is the radio wavelength, $G_T$ and $G_R$ are the gain factors of transmit and receive antennas relative to isotropic, $F(d)$ is the Norton field attenuation factor over sea, $d$ is the detecting range and $\Delta d$, $\Delta \phi$ are the range and angular extent of spatial resolution cell. Figure 3.5 shows the results of received power against ranges at different radar frequencies. The figure also indicates the additional attenuation due to the high sea state, the solid curves are the received power at a smooth sea, while the dashed curves show the extra two-way loss for sea state 6 on Douglas sea scale [63]. The increase in the attenuation is due to the roughness at sea surface and the range (two way loss). For example, for curve $a$, the offset between solid and dashed line are larger at 200 km than that at 100 km.

Figure 3.5: Curves of received power against ranges at different operating radar frequencies: a - 7.5, b - 15 and c - 30 MHz, the transmitted power is 250 W, the solid curves are at a smooth sea state and the dashed curves are at a high sea state. (Shearman,1983)
In the use of HF radar remote sensing, the weaker second-order scattering mechanism is used to deduce the wave height and direction. Normally, the range coverage of second-order spectrum is nearly half of the first-order Bragg peaks. Of course, it also depends on the radar frequency, sea state and the noise level outside the radar. Figure 3.6 and Figure 3.7 present the example of range Doppler spectra during the Fedje experiment ($F_r = 27.68$ MHz) and Ligurian Sea experiment ($F_r = 12.254$ MHz) in nearly same sea state (wind speed) respectively. Doppler spectra at certain distances are also given. From the two range-Doppler spectra, it is obvious that the propagating distance at a lower frequency is much farther than that at a higher frequency. Besides the attenuation of the first-order peak power along the distance, the signature of the power attenuation of the second-order continuum is also different. Details will be discussed in Section 3.2.

Figure 3.6: Range-Doppler spectra at Fedje site during the Fedje experiment

Figure 3.7: Range-Doppler spectra at Palmaria site during the Ligurian Sea experiment
3.1.2.2 Radar cross section and backscatter modeling

When radiowaves are backscattered from the sea surface, the Doppler spectral density is proportional to the radar cross section per unit frequency $\sigma(\omega_D)$, where $\omega_D$ is the Doppler angular frequency. The Bragg scattering at a grazing incidence is caused by the wave component whose wave number vector is $\pm 2k_0$ (radio wave number vector $k_0$). The radar cross section $\sigma(\omega_D)$ is proportional to the energy of the wave component whose frequency is $\omega_D$ and the wave number vector is $2k_0$. That is

$$\sigma(\omega_D) \propto X(\omega_D, 2k_0) \quad (3.4)$$

In addition to the second-order scattering by the contribution of hydrodynamic effect, there is a double Bragg scattering. A pair of fundamental waves whose wave number vectors sum to $\pm 2k_0$ causes double Bragg scattering. Barrick [49] presented the first-order and the second-order Cross Section (CS) in the following form:

$$\sigma_{\text{CS,1}}(\omega_D) = 2^6 \pi k_0^4 \sum_{m=\pm 1} S(-2mk_0)\delta(\omega_D - m\omega_B) \quad (3.5)$$

$$\sigma_{\text{CS,2}}(\omega_D) = 2^6 \pi k_0^4 \sum_{m_1=\pm 1} \sum_{m_2=\pm 1} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\Gamma_E - i\Gamma_H|^2 S(m_1k_1)S(m_2k_2) \cdot \delta(\omega_D - m_1\sqrt{gk_1} - m_2\sqrt{gk_2})dpdq \quad (3.6)$$

where $\omega_B = \sqrt{2gk_0}$ is the Bragg angular frequency, $S(\cdot)$ is the directional wave number spectrum and $(p, d)$ is the Cartesian coordinate as shown in Figure 3.8. The electromagnetic coupling coefficient $\Gamma_E$ represents double Bragg scattering and it is expressed as:

$$\Gamma_E = \frac{1}{2} \left[ \frac{(k_1 \cdot k_0)(k_2 \cdot k_0)/k_0^2 - 2k_1k_2}{(k_1 \cdot k_2)^{1/2} - k_0\Delta} \right] \quad (3.7)$$

where $\Delta$ is a normalized surface impedance. When the radar works at HF band, the impedance is approximate to $\Delta = 0.011 - i(0.012)$ [64]. In Equation 3.6, the spatial wave number $p$ is defined to lie along the radio beam, with $q$ perpendicular to $p$. In the second-order scattering process, a first set of waves of wave number $k_1$ interacts with the incident radar wave to produce a scattered wave $\overrightarrow{k_m}$. A second interaction with waves of wave vector $k_2$ takes the incident intermediate wave and scatters it back toward the radar site. The scattering wave vector $k_1$ and $k_2$ are defined by

$$k_1 = (p - k_0, q) \quad k_2 = (-(p + k_0), -q) \quad (3.8)$$

as illustrated in Figure 3.8, and hence they obey the constraint:

$$k_1 + k_2 = -2k_0 \quad (3.9)$$
As introduced above, $S(k, \theta)$ is the directional wave number spectrum. In Lipa’s scattering model, Pierson-Moskowitz (P-M) non-directional spectrum and a cardioid directional spreading function are used. We set $s = 1$, so

$$S(k, \theta) = \frac{4}{3\pi} \cdot 0.005 \exp(-0.74(k_c/k)^2) \cdot \cos^2\left(\frac{\phi - \theta}{2}\right)$$

P-M spectrum describes the characteristic falloff of saturated waves above a cutoff region defined by wave number $k_c$, which is related to wind speed, $U_{10} = \sqrt{g/k_c}$, and $\phi$ is the radio beam direction, $\theta$ is the mean wind-wave direction. HF radar backscatter modeling is based on the first-order and second-order backscatter cross section combining with the given wave spectrum $S(k, \theta)$. The first-order peak can be easily calculated, while the second-order backscatter is based on the non-linear integral equation (Equation 3.6), obeying the constraint given by Equation 3.9 and $k_1 \cdot k_2 = 0$ for electromagnetic coupling component.

### 3.2 Wind direction and radar backscatter echoes

The ocean surface wind determines the directional spreading distribution of ocean waves. Once the radio beam direction $\phi$ is given, the wind (speed and direction) at selected grid point and the directional spreading pattern of the Bragg waves determine the strength of receding and approaching wave components to the radar site. To derive information about wind field by observing sea-wave spectrum, the spectral power density of Bragg waves is proportional to the scattering cross section [65]:

$$R = \sigma_1(f_B)/\sigma_1(-f_B) = S(-2k_0)/S(2k_0)$$

Figure 3.9 shows the sketch of the radar map and wind direction: three examples of wind direction and wave directional spreading patterns are illustrated, the red cardioid patterns represent the directional distribution of Bragg waves. The line A-O-B is the radio beam passing through the directional pattern, line O-B represents the approaching wave component, which generates the first-order (Bragg) backscatter on the positive side of the Doppler spectrum, while line O-A represents the wave component receding from the radar.
3.2. Wind direction and radar backscatter echoes

site, which generates the corresponding Bragg peak appears on the negative side of the spectrum. The length of O-A and O-B represents the relative strength of the two wave components. As shown in the figure, if the wind changes its direction, the strength of wave components O-A and O-B changes as well.

Figure 3.9: Schematics of wind directions (red arrow), wave directional patterns and radio beam direction

Assuming $\phi$ is the azimuth of the radio beam and $\theta$ is the azimuth of the wind vector, the power ratio of Bragg peaks is related to wind direction $\theta$ (at the fixed grid point, $\phi$ is a constant value):

$$R(\theta, \phi) = \frac{\sigma_1(f_B)}{\sigma_1(-f_B)} = \frac{G(\pi + \phi - \theta)}{G(\phi - \theta)}$$

(3.12)

In order to simplify the calculation, $\phi$ is set to $0^\circ$, so

$$R(\theta) = \frac{\sigma_1(f_B)}{\sigma_1(-f_B)} = \frac{G(\pi - \theta)}{G(-\theta)}$$

(3.13)

If the half-cosine $2s$-power spreading function $G(\theta) = A \cdot \cos^{2s}(\theta/2)$ is used, we have

$$R(\theta) = \frac{G(\pi - \theta)}{G(-\theta)} = \frac{\cos^{2s}(\pi - \theta/2)}{\cos^{2s}(\theta/2)} = \tan^{2s}(\theta/2)$$

(3.14)

Equation 3.14 is depicted in Figure 3.11a and the spreading parameter $s$ is defined to be $s_1 = 1$, $s_2 = 2$ and $s_3 = 4$ for comparison. Actually, while $\theta = \pi$ or $\theta = -\pi$, the ratio $R \to +\infty$, and while $\theta = 0^\circ$, the ratio $R \to -\infty$. So in the figure, the $\theta$ is defined close to
±π and 0 but not equal to.

If the azimuth of the radio beam φ is considered, we have the wind direction

$$\theta = \phi \pm 2 \arctan(R^{1/2s})$$  \hspace{1cm} (3.15)

M.A. Donelan [32] suggested using the hyperbolic secant squared function $G(\theta) = 0.5 \cdot \beta \text{sech}^2(\beta \cdot \theta)$. Wyatt implemented it in HF radar experiment with the conclusion of having a better fit than half-cosine 2s-power type spreading [66]. The difference between these two functions is that: For the half-cosine 2s-power function, when $\theta = \pi$ and $\theta = -\pi$, the spreading value $G(\theta) = 0$, as shown in Figure 2.8. For the hyperbolic secant squared function, when $\theta = \pi$ and $\theta = -\pi$, the spreading value $G(\theta) > 0$, as shown in Figure 2.9. That is to say, the hyperbolic secant function allows some energy to propagate opposite to the wind direction. In HF radar measurements, if the wind blows along the radio beam direction, the measured approaching wave component (which is proportional to positive first-order peak power) is never equal to zero, and even in the buoy measurement accomplished by M.A. Donelan, there is some minor wave components propagating against the wind direction.

Figure 3.10: Half-cosine 2s-power spreading function is a periodic function, while the hyperbolic secant squared function is a non-periodic function

Another important difference is, that the half cosine 2s-power function is periodic function, but the hyperbolic secant function is not. Figure 3.10 gives the half-cosine type and hyperbolic type at the angle of $\theta \in [-2\pi, 2\pi]$. For the hyperbolic secant function, if the angle $\theta$ exceeds the range of $[-\pi, \pi]$, it still gives some value of the ratio because of the non-periodicity, but this makes no sense for the wind direction inversion.

If the hyperbolic secant function is used for calculating the ratio of approaching and...
receding wave components, the direction range should be separated into two cases:

\[
R(\theta) = \frac{G(-\pi - \theta)}{G(-\theta)} = \left[\frac{e^{\beta(-\theta)} + e^{-\beta(-\theta)}}{e^{\beta(-\pi - \theta)} + e^{-\beta(-\pi - \theta)}}\right]^2 \quad (\pi \leq \theta \leq 0) \tag{3.16}
\]

\[
R(\theta) = \frac{G(\pi - \theta)}{G(-\theta)} = \left[\frac{e^{\beta(-\theta)} + e^{-\beta(-\theta)}}{e^{\beta(\pi - \theta)} + e^{-\beta(\pi - \theta)}}\right]^2 \quad (0 \leq \theta \leq \pi) \tag{3.17}
\]

Figure 3.11b demonstrates the ratio \( R \) and the wind direction \( \theta \), in which, the spreading parameter \( \beta \) is set to be: \( \beta = (0.6, 0.8, 1.0, 1.4) \) for comparison. Inversely, the wind direction \( \theta \) can be derived from the ratio \( R \) (for \( \phi = 0^\circ \)):

\[
\theta = \frac{1}{2\beta} \ln \left| \frac{1 - R^{1/2} \cdot e^{-\beta \pi}}{R^{1/2} \cdot e^{-\beta \pi} - 1} \right| \quad (-\pi \leq \theta < 0) \tag{3.18}
\]

\[
\theta = \frac{1}{2\beta} \ln \left| \frac{1 - R^{1/2} \cdot e^{\beta \pi}}{R^{1/2} \cdot e^{\beta \pi} - 1} \right| \quad (0 \leq \theta < \pi) \tag{3.19}
\]

and we have

\[
\frac{1}{2\beta} \ln \left| \frac{1 - R^{1/2} \cdot e^{-\beta \pi}}{R^{1/2} \cdot e^{-\beta \pi} - 1} \right| = -\frac{1}{2\beta} \ln \left| \frac{1 - R^{1/2} \cdot e^{\beta \pi}}{R^{1/2} \cdot e^{\beta \pi} - 1} \right| \tag{3.20}
\]

In this work, piecewise processing is implemented to get the whole direction range \( \theta \in [-\pi, \pi] \), and to simplify the calculation, only Equation 3.18 is used. For \( \theta \in [0, \pi] \), based on Equation 3.20, it can be easily calculated.

![Figure 3.11](image-url)

(a) Ratio vs. wind direction (half-cosine type)  
(b) Ratio vs. wind direction (hyperbolic type)

Figure 3.11: Comparison of ratio as a function of wind direction (radio beam \( \phi = 0^\circ \))
beam direction. In this example, if we set $\beta = 0.8$, the lower limit value is $R_1 = 0.0251$, which makes

$$\pm \frac{1}{2\beta} \ln \left| \frac{1 - R_1^{1/2} \cdot e^{-\beta \cdot \pi}}{R_1^{1/2} e^{\beta \cdot \pi} - 1} \right| = 0 \quad (3.21)$$

The upper limit of the ratio is the value making $\theta = 180^\circ$ or $\theta = -180^\circ$. In this example, the upper limit value of $R_2$ is 39.81. The value of lower and upper limit of $R$ also depends on the spreading parameter $\beta$. Figure 3.12b shows the curves with different spreading parameters $\beta_1, \beta_2, \beta_3$, and the corresponding lower limit ratio $R(\beta_1)$, $R(\beta_2)$ and $R(\beta_3)$ are also given in the figure. Discussion of a varying $\beta$ for extracting wind direction is discussed in Section 3.3.3.2.

![Figure 3.12: Wind direction derived from the ratio of approaching and receding wave components ($\phi = 0^\circ$)](a) $\beta = 0.8$

(b) $\beta_1 = 0.6, \beta_2 = 0.8, \beta_3 = 1.0$

3.3 Wind direction determination with two radars

The principle of wind direction inversion from the radar first-order backscatter has been introduced above. The uncertainty of the wind direction is mainly due to the ambiguity and the directional spreading pattern as well as the spreading parameter. In Equation 3.15, there are two possible wind directions matching the ratio of approaching and receding wave components, the “±” sign is due to the mathematical ambiguity which can not be solved using a single beam azimuth. Figure 3.13 shows the diagram of the wind direction ambiguity.

In order to remove the wind direction ambiguity, a variety of methods have been attempted, including: (1) general wind circulation information on the maps of air pressure; (2) Least Square Minimum method (LSM) to get the best fit to wind-wave pattern [67]; (3) one radar switching radio beam direction [65, 68]. Besides the methods above, in this dissertation, a new pattern fitting method with varying spreading parameter is proposed.
3.3. Wind direction determination with two radars

The use of air-pressure maps is somehow subjective and maps are often of insufficient quality [69]. In most cases, geometric one or two radar mapping methods are used to determine the unique wind direction. Details are introduced in the section.

3.3.1 Least Square Minimum (LSM) method

Gurgel proposed a unique solution for the direction determination by means of Least Square Minimum (LSM) principle [67]:

\[
LSM(\theta_i) = [R_1 - \frac{G(\phi_1 - \theta_i + \pi)}{G(\phi_1 - \theta_i)}]^2 + [R_2 - \frac{G(\phi_2 - \theta_i + \pi)}{G(\phi_2 - \theta_i)}]^2
\]  

(3.22)

where \( R_1 \) and \( R_2 \) are the power ratios of Bragg peaks at two radar sites. \( \theta_i \) is the variable, which is the direction of wind-wave. In the estimation, \( \theta_i \in [0 \sim 360^\circ] \), i.e., the wind-wave directional pattern rotates a complete circle. The wind direction \( \theta'_i \) could be decided by giving the minimum value of Equation 3.22. Figure 3.14a shows an example of estimation of wind direction using LSM method, in which, the radio beam direction \( \phi_1 = 215.5^\circ, \phi_2 = 305.5^\circ \) and the ratio \( R_1 = 2, R_2 = 0.8 \). In the figure, the wind-wave direction is decided by the minimum of the variance sum (indicated by one red circle). In [67], the function \( \cos^{2s}(\theta/2) \) is suggested with the spreading parameter \( s = 1 \) or \( s = 2 \). Hyperbolic secant function could also be implemented with a predefined \( \beta \) value, a value of \( \beta = 0.8 \) is suggested based on the range of Bragg peak power ratio. All these direction patterns assume a fixed spreading parameter. In reality, the spreading of wind-wave pattern varies with wind speed and wave age. Although the method can find the minimum error for the wind direction estimation, in some cases (e.g., \( R_1 \approx R_2 \)), there might be two minima.

Figure 3.13: Diagram of wind wave pattern and radio beam direction
3.3.2 Multi-beam method using one radar site

The ambiguity removal by switching radio beam direction relies on the assumption of an approximately uniform wind direction over the area of ocean being observed. This method is proposed by M.Heron [65] and later discussed by Huang [69]. In their work, the ambiguity is solved by a spatial analysis using the ability of the radar system to steer the beam to different azimuths and sample the ocean over a short period of time (about 30 min). As shown in Figure 3.14b, within the area of interest, the surface wind directions are assumed to be uniform for stable sea state. This means that there is a continuity in the wind direction between closely spaced radio scatter points and the sum of the differences of the real wind directions in neighboring cells ought to be zero or near to zero. In M.Heron’s work [65], he gave the idea that, if the radio beams $\phi_A$ and $\phi_B$ are steered to two cells at the surface from one radar station, the radio beam directions are determined. The power ratio $R$ can be calculated from the radar Doppler spectrum, so the wind direction is the function of wind-wave spreading parameter $s$ ($\cos^2 s(\theta/2)$ is used), so $s$ can be predefined $s \in [0, 10]$, and wind direction $\theta$ can be given as a function of $s$ by assuming that the wind direction at these cells are identical. As a result, one value of $s$ can be determined. But it is complicated to calculate and there might be more than one solution for $s$. In this case, a third beam direction is needed. In Huang’s work [69], he states that: the wind direction at these cells doesn’t need to be identical, it can be determined by the minimizing the difference at these cells. In the Figure 3.14b, three cells $A$, $B$ and $C$ are in three neighboring radio beam directions, there are two possible wind directions on each cell. There are labeled as $\theta_{AWi}$, $\theta_{BWi}$ and $\theta_{CWi}$ (i=1,2). The sum of the differences of the wind direction in cell $B$ from its two neighboring cells is defined by

$$\Delta \theta_{ijk} = |\theta_{BWi} - \theta_{AWj}| + |\theta_{BWi} - \theta_{CWk}| \quad (i,j,k = 1,2) \tag{3.23}$$

The value of $\theta_{BWi}$ that minimizes $\Delta \theta_{ijk}$ is chosen as the true wind direction on cell $B$. 

![Figure 3.14: Conventional methods for wind direction determination](Image)
3.3. Wind direction determination with two radars

In this manner, the wind direction on cell C can also be found by using cells B and D (D doesn’t appear in the figure) and for subsequent cells. This technique is applied sequentially to the whole field of view of the radar starting with nearest cells. In Huang’s calculation, a choice of \( s = 1 \) is used for the spreading parameter throughout the experiment [69].

3.3.3 Pattern fitting with a varying spreading parameter

Various techniques have been introduced for extracting wind direction from HF radar backscatter. All require essentially the power ratio of the two first-order peaks, combined with an assumed directional distribution functional form of the resonant ocean waves. Although the details of the techniques differ, the principle is now well established. The main uncertainty lies in the dependency of the assumed wave directional distribution, implying a dependency of measured wind direction on the prior knowledge of wind speed [7]. In this dissertation, the author proposes a novel method for removing the wind direction ambiguity.

The wind direction can be written as:

\[
\theta_i = \phi_i \pm \Theta(R_i) \quad (i = 1, 2)
\]  \hspace{1cm} (3.24)

where the subscript 1, 2 represent the two beams starting from the two radar sites to the grid point, identifying a small patch at the sea surface. As illustrated within radar coverage in Figure 3.15, the red star is the location where the wave buoy is deployed. \( \Theta(R) \) is the angle between radio beam direction and wind direction. In this example, \( \phi_1 = 215.5^\circ, \phi_2 = 305.5^\circ \).

3.3.3.1 Half-cosine \( 2s \) – power spreading function

If \( \cos^2 s(\theta/2) \) is used, Equation 3.24 can be rewritten as

\[
\theta^\pm_i = \phi_i \pm 2 \arctan(R_i^{1/2s}) \quad \text{(for } i = 1 \text{ and } 2) \hspace{1cm} (3.25)
\]

In Figure 3.15, the site Fedje locates at a higher latitude and both sites face to the west, so we have \( \phi_2 > \phi_1 \) and \( 0 < \phi_1 < \phi_2 < \pi \). We take the radar beam direction \( \phi_1 \) as an example and define \( R = [0.2, 0.8, 1, 2, 8] \), the wind direction curves \( \theta^\pm_1 \) are illustrated Figure 3.16a.

Regarding the two pairs of wind direction curves calculated from two radar sites, curves \( \theta^\pm_1 \) and \( \theta^\pm_2 \) are a function of \( s \) value and the ratios are set to \( R_1 = 0.1 \) and \( R_2 = 4 \) in this example. The results are given in Figure 3.16b. The wind direction curves \( \theta^\pm_2 \) start at direction \( \theta^\pm_{2,s\rightarrow 0} = \phi_2 + \pi \) and \( \theta^\pm_{2,s\rightarrow 0} = \phi_2 - \pi \), the curves \( \theta^\pm_1 \) start at \( \phi_1 \). All the values at the start points \( \theta^\pm_{i,s\rightarrow 0} \) are determined by the ratio \( R \). Details are given in Table 3.2

The discussion of the possible number of cross points for these four curves (\( \theta^\pm_1 \) and \( \theta^\pm_2 \)) are given in Appendix A.1. In some cases, they give more than one cross point, which
also brings some ambiguity to wind direction inversion. In the next section, the hyperbolic secant-squared function is discussed.

### 3.3.3.2 Hyperbolic secant squared spreading function

As introduced in Equation 3.24, if the hyperbolic secant-squared function is used, the wind direction can be written as:

\[
\theta_i^\pm = \phi_i \pm \frac{1}{2\beta} \ln \left| \frac{1 - R_i^{1/2} e^{-\beta \pi}}{R_i^{1/2} e^{\beta \pi} - 1} \right|
\]  

(3.26)

As shown in Figure 3.10, the sech²(\(\beta \cdot \theta\)) is not a periodic function. We must define that the angle between radio beam direction and wind direction \(\theta - \phi\) is in the range of \([-\pi, \pi]\). Figure 3.11b gives the range of ratios when the \(\beta\) value is known, and if the ratio \(R\) is known, the spreading value \(\beta\) has a lower limit value \(\beta_{\text{min}}\). For example, if \(R_i < 1\), in order to cover all the direction range \([-\pi, \pi]\), we need to make sure that: when \(\theta = 0^\circ\)
3.3. Wind direction determination with two radars

(Wind-wave direction as a function of spreading parameter $s$

\[
\theta = \phi - 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 0.2
\]

\[
\theta = \phi + 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 0.8
\]

\[
\theta = \phi - 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 0.8
\]

\[
\theta = \phi + 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 2
\]

\[
\theta = \phi - 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 2
\]

\[
\theta = \phi + 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 8
\]

\[
\theta = \phi - 2 \arctan \left( R^{1/2} s \right)
\]

\[
R = 8
\]

\[
\theta = \phi + 2 \arctan \left( R^{1/2} s \right)
\]

\[
\theta = \phi - 180^\circ
\]

\[
\theta = \phi + 180^\circ
\]

(a) Wind direction vs. spreading value $s$ ($\cos^{2s}(\theta/2)$)  
(b) Cross point and two pairs of direction curves

Figure 3.16: Direction curves and cross point for half-cosine 2s-power function

(here, $\phi$ is set to $0^\circ$), the spreading parameter $\beta$ has a lower limit, which makes:

\[
\frac{\text{sech}^2(\beta_{i,\text{min}} \cdot \pi)}{\text{sech}^2(\beta_{i,\text{min}} \cdot 0)} = R_i
\]

(3.27)

Hence

\[
\beta_{i,\text{min}}^\pm = \frac{1}{\pi} \ln \left[ \left( \frac{1}{R_i} \right)^{1/2} \pm \left( \frac{1}{R_i} - 1 \right)^{1/2} \right]
\]

(3.28)

If $R_i > 1$, from Figure 3.11b, we also need to make sure the direction covers range $(-\pi, \pi)$, so when $\theta = \pi$ (or $-\pi$)

\[
\frac{\text{sech}^2(\beta_{i,\text{min}} \cdot 0)}{\text{sech}^2(\beta_{i,\text{min}} \cdot \pi)} = R_i
\]

(3.29)

Hence

\[
\beta_{i,\text{min}}^\pm = \frac{1}{\pi} \ln \left[ R_i^{1/2} \pm (R_i - 1)^{1/2} \right]
\]

(3.30)

Considering that

\[
R^{1/2} - (R - 1)^{1/2} = \frac{1}{R^{1/2} + (R - 1)^{1/2}} < 1 \quad \text{for} \quad (R > 1)
\]

(3.31)

Therefore $\beta_{i,\text{min}}^- < 0$, we just consider the condition of $\beta > 0$, so

\[
\beta_{i,\text{min}} = \frac{1}{\pi} \ln \left[ \left( \frac{1}{R_i} \right)^{1/2} + \left( \frac{1}{R_i} - 1 \right)^{1/2} \right] \quad (R_i < 1)
\]

(3.32)

\[
\beta_{i,\text{min}} = \frac{1}{\pi} \ln \left[ R_i^{1/2} + (R_i - 1)^{1/2} \right] \quad (R_i > 1)
\]

(3.33)
Chapter 3. HF Radar Remote Sensing and Wind Inversion

Figure 3.17a shows the results of $\theta_1^\pm$, without the lower limit of $\beta_{\text{min}}$, that is to say, with the given value of ratio $R_i$, not only the direction in the range $[-\pi, \pi]$ can be satisfied, but also the angle exceeding the range of $[-\pi, \pi]$. Due to the non-periodicity of $\text{sech}^2(\beta \cdot \theta)$, the lower-limit of the spreading parameter $\beta$ is computed according to Equation 3.32 and 3.33. The direction curves are given in Figure 3.17b.

The cross point of two pairs of curves ($\theta_1^\pm$ and $\theta_2^\pm$) defines the value of $\beta$ and the wind direction $\theta$. As indicated in Figure 3.17b, $\beta = 0.5732$ and wind direction $\theta = 188.3^\circ$.

When the ratio $R_i \geq 1$, the wind direction at $\beta_{i,\text{min}}$ is given by:

$$
\theta_{i,\beta_{i,\text{min}}} = \frac{1}{2\beta_{i,\text{min}}} \ln \left| \frac{1 - R_i^{1/2} e^{-\beta_{i,\text{min}} \cdot \pi}}{R_i^{1/2} e^{\beta_{i,\text{min}} \cdot \pi} - 1} \right| \quad (\phi_i = 0)
$$

$$
= \begin{cases} 
-\pi & (R_i > 1) \\
0 & (R_i < 1) \\
-\pi/2 & (R_i = 1)
\end{cases}
$$

(3.34)

Figure 3.18 shows the wind direction derived from the ratio $R_1$ and $R_2$ ($R_1 < 1$, $R_2 > 1$), and it gives the cross point ($\beta = 0.9788$, $\theta = 142.6^\circ$).

The pattern fitting method is tested using the hyperbolic secant function and half-cosine 2s-power function. For the hyperbolic secant function, the curves $\theta_1^\pm$ and $\theta_2^\pm$ start from the spreading parameter value $\beta_{1,\text{min}}$ and $\beta_{2,\text{min}}$ respectively, which are also related to the ratio $R_1$ and $R_2$. Table 3.3 gives the values of $\beta_{1,\text{min}}, \beta_{2,\text{min}}$ and wind-wave direction at the start point $\theta_{i,\beta_{i,\text{min}}}$. 

38
3.3. Wind direction determination with two radars

**Figure 3.18:** Wind direction as a function of spreading parameter $\beta$ and given value of $R_1$ ($R_1 < 1$) and $R_2$ ($R_2 > 1$)

<table>
<thead>
<tr>
<th>Ratio $R_1$</th>
<th>Ratio $R_2$</th>
<th>Spreading Parameter $\beta_{i,\text{min}}$</th>
<th>Wind Direction $\theta_{i,\beta_{\text{min}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 &gt; 1$</td>
<td>$R_2 &gt; 1$</td>
<td>$\beta_{1,\text{min}} \geq \beta_{2,\text{min}}$ ($R_1 \geq R_2$)</td>
<td>$\theta_1^\pm = \phi_1 \pm \pi$, $\theta_2^\pm = \phi_2 \pm \pi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{1,\text{min}} &lt; \beta_{2,\text{min}}$ ($R_1 &lt; R_2$)</td>
<td></td>
</tr>
<tr>
<td>$R_1 &lt; 1$</td>
<td>$R_2 &gt; 1$</td>
<td>$\beta_{1,\text{min}} \geq \beta_{2,\text{min}}$ ($R_1^{-1} \geq R_2$)</td>
<td>$\theta_1 = \phi_1$, $\theta_2^\pm = \phi_2 \pm \pi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{1,\text{min}} &lt; \beta_{2,\text{min}}$ ($R_1^{-1} &lt; R_2$)</td>
<td></td>
</tr>
<tr>
<td>$R_1 &gt; 1$</td>
<td>$R_2 &lt; 1$</td>
<td>$\beta_{1,\text{min}} \geq \beta_{2,\text{min}}$ ($R_1 \geq R_2^{-1}$)</td>
<td>$\theta_1^\pm = \phi_1 \pm \pi$, $\theta_2 = \phi_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{1,\text{min}} &lt; \beta_{2,\text{min}}$ ($R_1 &lt; R_2^{-1}$)</td>
<td></td>
</tr>
<tr>
<td>$R_1 &lt; 1$</td>
<td>$R_2 &lt; 1$</td>
<td>$\beta_{1,\text{min}} \leq \beta_{2,\text{min}}$ ($R_1 \geq R_2$)</td>
<td>$\theta_1 = \phi_1$, $\theta_2 = \phi_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_{1,\text{min}} &gt; \beta_{2,\text{min}}$ ($R_1 &lt; R_2$)</td>
<td></td>
</tr>
<tr>
<td>$R_1 = 1$</td>
<td>$R_2 = 1$</td>
<td>$\beta_{1,\text{min}} = \beta_{2,\text{min}} = 0$</td>
<td>$\theta_1^\pm \equiv \phi_1 \pm \pi/2$, $\theta_2^\pm \equiv \phi_2 \pm \pi/2$</td>
</tr>
</tbody>
</table>

Table 3.3: The start point of direction curve $(\beta_{i,\text{min}}, \theta_{i,\beta_{\text{min}}})$ and the power ratio of the first-order peaks $R_1$ and $R_2$

Another important difference is: for hyperbolic secant squared function, the direction curves of $\theta_1^+$ and $\theta_2^-$ give only one cross point by limiting the range of spreading parameter $\beta_i \geq \beta_{i,\text{min}}$, which makes the cross point $(\beta_0, \theta_0)$ unique. For example, one direction curve $\theta_2^-$ could only have one cross point with curve $\theta_1^+$ or with the curve $\theta_1^-$, because the start point of direction curve $\beta_{1,\text{min}}$ is not fixed at the value of zero. $\beta_{i,\text{min}}$ also changes with the ratio $R_i$. Proof is given in Appendix A.2. So with the hyperbolic secant-squared function, the wind direction could be determined as well as the spreading parameter $\beta$. 

39
3.4 Wind speed and radar backscatter echoes

The traditional methods for deriving wind speed from radar echoes are based on the second-order spectrum. The second-order continuum is normalized by the power of first-order peaks to cancel unknown factors, such as path loss and system gains [8]. Normally, the SNR of the second-order sidebands is nearly $20 \sim 40$ dB (depends on wind conditions and radar operating frequency) below the first-order peaks. Once the distance and azimuth of radio beam are determined, the SNR of second-order spectrum depends on the ocean surface roughness at the patch ($\Delta d, \Delta \phi$) of interest. In Lipa’s backscatter numerical model [70], P-M wave spectrum is implemented to give the simulated wave information from the predefined wind speed. For example, we set the wind speed $U_1 = 4 \text{ m/s}$ and $U_2 = 12 \text{ m/s}$ respectively, radar operating frequency is 25 MHz, the SNR is set to 50 dB (Ratio of stronger first-order peak to noise level). In these examples, at different wind speeds, the simulated Doppler spectra are demonstrated in Figure 3.19.

The signature of radar second-order spectra is also related to radar frequencies. Barrick gives a basic relationship between the normalized second-order spectra and radio wave number [12]:

$$\frac{\sigma_2(\omega_D)}{\sigma_1(\omega_D)} \propto k_0^2 \quad (3.35)$$

where $\omega_D$ is the Doppler angular frequency, $k_0$ is the radio wave number, $\sigma_1(\omega_D)$ gives the power of the first-order peaks, $\sigma_2(\omega_D)$ is the second-order continuum around the first-order peaks. In Equation 3.35, the second-order continuum $\sigma_2(\omega_D)$ is divided by the adjacent first-order peak power. So at the same wind speed, the ratio of second-order continuum to the first-order peak is proportional to the square of radio wave number $k_0$. Figure 3.20 illustrates the simulated Doppler spectra at the radar frequencies of 13 MHz and 25 MHz. The higher operating frequency gives a higher ratio of second-order continuum to the first-
3.4. Wind speed and radar backscatter echoes

order peaks.

![Simulated Doppler Spectra](image)

(a) $U = 12$ m/s, $f_{\text{radar}} = 13$ MHz

(b) $U = 12$ m/s, $f_{\text{radar}} = 25$ MHz

Figure 3.20: Simulated Doppler spectrum at different operating frequencies

Two examples of HF radar range-Doppler spectra are shown in Figure 3.6 and 3.7. In order to demonstrate the SNR of both first and second-order backscatter varies with the radar range, Figure 3.21a gives the SNR of the first-order peak and the second-order spectra during the Fedje experiment (27.68 MHz), and in the figure a line of 5 dB is also depicted, which gives the SNR threshold for extracting wave spectrum from the radar spectrum. As seen in this example, the range coverage of the second-order spectrum is only half that of first-order peak (see the cross points of power curves and threshold). But when radar is operated at a lower frequency (Lugrian Sea experiment, 12.254 MHz), as shown in Figure 3.21b, the second-order spectra only covers nearly one-third of the distance covered by the first-order peak. So at the same sea state, the SNR of the back-scattered second-order spectra are lower when the radar operates at a lower frequency, which brings some difficulties for extracting waves from the second-order spectrum.

![SNR of first and second-order spectrum as a function of distance](image)

(a) SNR decreases with distance, Fedje

(b) SNR decreases with distance, Palmaria

Figure 3.21: An example of the SNR of first-order peaks and second-order spectra during the Fedje and Ligurian Sea experiment

41
3.4.1 Wind speed inversion from HF echoes

In principle, the first-order peaks could be used to determine the ocean wave power spectrum if the radar frequency could be varied at will. Unfortunately this can seldom be carried out due to the practical limitations on radar design and strong radio interference. Hasselmann [8] first suggested that the second-order Doppler sidebands ought to be proportional to the non-directional wave height spectrum. The wave spectrum can be estimated from the Doppler spectrum by solving a nonlinear integral equation, which relates the second-order sidebands to the wave spectrum [71]. Dexter et al. [7] proposed a method using the dependency of significant wave height and dominant wave frequency to estimate a wind speed, which employs the Sverdrup, Munk and Bretschneider [43] (SMB) curves. The algorithm overestimates wind speed because of the presence of swell. To develop this idea further, it is necessary to partition the full two dimensional spectrum [10]. However, the extraction of the wave spectrum requires a good signal-to-noise ratio of the second-order spectrum [11]. In case of low wind conditions, especially when the radar works at a lower operating frequency, the SNR of second-order spectrum is quite low [12]. The first-order backscatter gives the dominant feature in Doppler spectrum and its strength is proportional to the heights of the corresponding Bragg waves [46, 72]. Some researchers (Y. Hisaki [11, 73], L.Wyatt [74]) use Bragg waves backscatter to derive short-wave directional distribution. However, the method of deriving wind speed from first-order peaks hasn’t been applied until recently.

3.4.2 Wind speed inversion from the first-order peaks

The first-order backscatter energy is proportional to that of the Bragg waves along the radio beam. The wind direction and the shape of wind-wave (Bragg wave) directional pattern determine the ratio of the first-order peak power. Both wave height and directional spreading pattern of Bragg wave could be used to invert wind speed at sea surface, the wind speed can be expressed as follow:

\[ U_{10} = F[\sigma_1(f_B), \sigma_1(-f_B), G(f_B, \theta), \phi] \]  

(3.36)

where \( F(\cdot) \) is the function for inverting wind speed from the signature of first-order peaks, which is a function of the positive and negative Bragg peak power \( \sigma_1(\pm f_B) \), the directional spreading pattern of Bragg waves \( G(f_B, \theta) \) and the radio beam direction \( \phi \). If the radar measurement is fixed to one certain grid point at the sea surface, the radio beam direction is a constant value, which is an independent variable in Equation 3.36.

As shown in Figure 3.9, the radio beam goes through an area cell (rectangle in the radio beam) with a direction range \( (\phi_1' \sim \phi_2') \) and we define the averaged beam direction \( \phi_0' = (\phi_1' + \phi_2')/2 \). In Figure 3.22a, the radio beam boundary line \( \phi_1' \) and \( \phi_2' \) can be parallely
shifted to the center of polar coordinate. In this example, we assume that the wind blows from west and the spreading function is a half-cosine 2s-power spreading function with \( s = 1 \). From the sector covered by the direction range \( \phi \in (\phi_1', \phi_2') \) and the opposite component \( \phi \in (\phi_1' + \pi, \phi_2' + \pi) \), the wave components in these two opposite sectors give the positive and negative Bragg backscatter.

![Polar coordinate and Cartesian coordinate](image)

**Figure 3.22:** Wind wave pattern and wave energy along radio beam

Because of the narrow radio beam width (normally, \( \phi_2' - \phi_1' \leq 15^o \)), the curve \( G(\theta) \) at \( \theta \in [\phi_1', \phi_2'] \) could be in proximity to a straight line. So the approximation is given as follows:

\[
\int_{\phi_1'}^{\phi_2'} G(\phi - \theta) d\phi \approx G(\phi_0' - \theta) \cdot (\phi_2' - \phi_1')
\]  \( \text{(3.37)} \)

Equation 3.13 can be written as:

\[
R(\theta, \phi) = \frac{\sigma_1(f_B)}{\sigma_1(-f_B)} = \frac{\int_{\phi_1'}^{\phi_2'} G(\pi + \phi - \theta) d\phi}{\int_{\phi_1'}^{\phi_2'} G(\phi - \theta) d\phi} = \frac{G(\pi + \phi_0' - \theta)}{G(\phi_0' - \theta)}
\]  \( \text{(3.38)} \)

where \( \phi_0' = (\phi_1' + \phi_2')/2 \). The integral in Equation 3.38 could be demonstrated in Figure 3.22b, in which, the shadow regions give the integral value. According to Equation 3.13 and the discussion above, the amplitude of Bragg peaks in the Doppler spectrum can be given:

\[
\begin{cases}
\sigma_1(-f_B) = \kappa \cdot E(f_B) \cdot \int_{\phi_1'}^{\phi_2'} G(\phi - \theta) d\phi \\
\sigma_1(f_B) = \kappa \cdot E(f_B) \cdot \int_{\phi_1'}^{\phi_2'} G(\pi + \phi - \theta) d\phi
\end{cases}
\]  \( \text{(3.39)} \)

where \( \kappa \) is a constant value based on Equation 3.11, \( E(f_B) \) is the wave energy integrated over direction at Bragg frequency.

In the example above, the directional spreading function is \( \cos^{2s}(\theta/2) \) with a given value of \( s = 1 \). But in HF radar remote sensing, the directional pattern might not be as regular
as given in the figure, and the spreading parameter is also related to the wind speed. For the inversion of wind speed from the first-order peaks, both the wave energy at Bragg frequency and the directional spreading of Bragg waves are used. This method is valid before the saturation of Bragg waves (including the wave height and directional spreading). The saturation of directional spreading parameter and the method of using neural network to help the wind speed inversion will be discussed in Chapter 4.

### 3.4.3 Wind speed inversion from the second-order sidebands

As discussed above, the wind speed might be derived from the radar first-order backscatter, which is a new approach proposed in the dissertation. In case of strong wind conditions, the Bragg waves are saturated and the SNR of second-order sidebands is sufficient for deriving wind speed. Many researchers have investigated the methods for extracting wind speed from ocean wave parameters such as significant wave height $H_S$ and peak frequency of wave spectra $f_p$. All these methods are based on the second-order backscatter effect.

#### 3.4.3.1 Theoretical wave inversion method

Barrick [49, 72] showed that the second-order continuum of energy in HF radar backscatter is produced by the combination of hydrodynamic non-linearity and a double scattering mechanism. He derived an inversion technique for obtaining the wave height non-directional spectrum [12, 75]. Approximations are used in the derivation and the technique is acceptable for $k_0 h_s > 0.2$, where $k_0$ is the radar wave number and $h_s$ is the $rms$ wave height. He employed one of the stronger second-order Doppler sidebands and divides it by a parameterless, dimensionless weighting function $w(\nu)$ and then divides this result by the adjacent first-order spectral energy. The non-directional wave height spectrum $S_t(\omega)$

$$S_t(\omega_B | \nu - 1 |) = \frac{4\sigma_2(\omega_B \nu)/w(\nu)}{k_0^2 \int_{-\infty}^{+\infty} \sigma_1(\omega_d) d\omega_d} \tag{3.40}$$

where $\nu$ is the normalized frequency, $\nu = \omega_d/\omega_B$, $w(\nu)$ is the weighting function of the Doppler shift scaled by the Bragg frequency $\omega_B$. The coupling coefficients $\Gamma_H$ and $\Gamma_E$ are for the sea wave height and electromagnetic scatter respectively. Barrick [12] removed the coupling factors from the integral as a constant value, and the $rms$ wave height is also given [75]:

$$h_s^2 = \frac{2 \int_{-\infty}^{+\infty} \sigma_2(\omega_B \nu)/w(\nu) d\omega_d}{k_0^2 \int_{-\infty}^{+\infty} \sigma_1(\omega_d) d\omega_d} \tag{3.41}$$

The weight is assumed to be invariant with the directional wave spectrum $(0.5 < \nu < 1.5)$. Barrick pointed out that the error associated with this assumption is dominated by the angle between the wind direction and that of radar beam [76].
3.4. Wind speed and radar backscatter echoes

The significant wave height $H_S$ can be calculated by

$$H_S = 4 \times h_*$$

After Barrick proposed the method for extracting the non-directional wave spectrum. Lipa [77, 78] showed that directional information can be derived from the pair of second-order sidebands, she expressed the ocean wave spectrum as:

$$S(\vec{k}) = F(k)G(\theta)$$

The spectrum term in Equation 3.6 becomes a quartic function:

$$S(\vec{k}_1)S(\vec{k}_2) = F(k_1)G(\theta_1)F(k_2)G(\theta_2)$$

in which, $F(k)$ is the Pierson-Moskowitz model for the amplitude spectrum, and $G(\theta)$ is the spreading function (half-cosine type is used)$^1$

![Normalized Doppler Spectrum](image1.png)

(a) Wind/radio direction: (1) 0° (2) 60° (3) 90°

![Normalized Doppler Spectrum](image2.png)

(b) Spreading: $2s^* = 8$; $2s^* = 4$; $2s^* = 2$

Figure 3.23: Simulated second-order spectra (a) for different wind directions. Radar frequency: 30 MHz, Wind speed: 22 knots, directional spreading factor: $s^* = 2$; (b) for different wave spreading parameter at radar frequency: 30 MHz, wind speed: 22 knots, wind/radio angle (figures from Lipa 1977)

In Lipa’s method, the amplitude spectrum and the directional factor are separately estimated using two regions of the spectrum, region 1 is for $|\nu - 1| > 0.4$, where the two scattering waves are saturated, the integrated waves could be represented by PM spectra, Equation 3.44 can be written as:

$$S(\vec{k}_1)S(\vec{k}_2) \propto G(\theta_1)G(\theta_2)/(k_1^4k_2^4)$$

This is linearized by the substitution of a trial function for $G(\theta_2)$. The integral equation

$^1$In Lipa’s paper, the spreading parameter $s$ is used instead of $2s$, so the $s$ value here is the half value of Lipa’s original value (for example, in Figure 3.23a the $s$ value in Lipa is $s = 4$)
Chapter 3. HF Radar Remote Sensing and Wind Inversion

is converted using numerical quadrature to a matrix equation which solved for a new estimation of $G$. This process is repeated until convergence occurs. At the completion of this step, the directional factor and the wind-wave direction are known. Second, treating the frequency spectrum in the region $|\nu - 1| \leq 0.4$ and substituting the parameters derived for $G$ and the saturated amplitude spectrum for $F(k_2)$, as $k_2$ always corresponds to a saturated waves, reducing Equation 3.44 to a linear function of $F$:

$$S(\vec{k}_1)S(\vec{k}_2) = \left[ G(\theta_1)G(\theta_2)/k_4^2 \right] F(k_1)$$ \hspace{1cm} (3.46)

With the assumption of separability, it is assumed that all ocean wavelengths have the same directional characteristics. The nature of the equation allows the directional characteristics and the long-wave amplitude spectrum to be calculated separately. She based the numerical technique on a regularization method developed by Phillips [79] and Towmey [80]. The approximations made are reasonable for wind-driven seas, but it would not be applicable if there is a well component propagating at a finite angle to the wind direction [81].

There are other significant approaches to the problem of inverting the HF ocean backscatter spectra to determine parameters of the directional ocean wave spectrum. For example, a model-fitting technique to extract wave directional spectrum was presented by L.Wyatt [82], which is an extension of Barrick and Lipa’s method [83] and extended the wave frequency range that can be measured using radar frequencies in the lower HF band. Results of the application to a wide variety of simulated radar spectra are presented in [82], which gave some of the weakness as well as its strengths. The method predicted accurate estimates of the long wave amplitude spectrum as long as wave components are propagating at an angle which is not perpendicular to the beam, but there is an associated over-prediction in amplitude. Except L.Wyatt’s method, Howell and Walsh [84, 85], Hisaki [71] gave solutions for directional analysis but they lack the speed or robustness of one-parameter (significant wave height) analysis based on Barrick’s approach.

3.4.3.2 Regression method

WERA group [67, 86] proposed an empirical method using the regression method during EuroROSE project in 2000. The regression parameters are adjusted by in-situ buoy measurements. With HF radar backscatter spectra measured at the same position by two distinct sites, it is possible to determine the spectral amplitudes and the mean wave direction. After selecting the stronger first-order Bragg peak and normalizing the power of the associated second-order sidebands by this first-order Bragg peak power, it is assumed that the measured radar spectra $S_k$ depends on the wave height spectrum measured by the wave buoy $H_k$ by
3.4. Wind speed and radar backscatter echoes

\[
\begin{align*}
\alpha_f S_{mk} &= H_k G(\phi_r - \theta_k) \\
\alpha_f S_{pk} &= H_k G(\phi_r - \theta_k + \pi)
\end{align*}
\]  

(3.47)

where the indices \( m \) and \( p \) of \( S \) refer to minus (negative) and plus (positive) Doppler shift relative to the first-order Bragg peak respectively, \( \alpha_f \) is the regression coefficients at radar frequency \( f \), \( k \) counts the spectral frequencies, \( e.g., \ k = 1, 2, ..., 21 \ (0.05 \sim 0.25 \ Hz \ with \ frequency \ step \ 0.01 \ Hz) \), \( G(\cdot) \) is an angular spreading function and \( \theta_k \) is the wave direction at frequency \( k \) measured by the wave buoy and \( \phi_r \) is the radio beam direction. During the experiment, the buoy can give the wave measurement at the frequency range of \( f \in [0.025 \sim 0.58] \ Hz \), but the radar measurement only gives an range of \( f \in [0.05 \sim 0.25] \ Hz \), so the wave spectrum is preprocessed to the range which is same as the radar derived wave spectrum.

The wave direction at different frequencies can be estimated from the ratio of second-order sidebands. Lipa has given a basic relationship for the wind direction and the ratio of second-order sidebands. In her model, the fully-developed wave model (P-M spectra) and half cosine 2s-power directional model are implemented. Only the mean direction is derived from the second-order amplitude ratio. Lipa [81] expressed that the long waves may be derived from the second-order sea echo spectral peaks. This reversal results in a closer examination of models of the radar Doppler spectrum produced by these waves. In measured radar spectrum, the ratio of second-order spectrum is not a constant value, because the shape of one side second-order spectrum is not a replica of other side.

With HF radar backscatter spectra measured at the same position by two distinct sites, it is possible to determine the spectral amplitudes and the mean wave direction with a resolution of \( e.g. \ 0.01Hz \) between \( 0.05 \sim 0.25Hz \). K.W.Gurgel gives the regression coefficient \( \alpha_{f_0} \) as follow \( (f_0 = 27.68 \ MHz) \) from the analysis of Fedje experiment:

\[
\alpha_{f_0} = \begin{cases} 
0 & 0 < f \leq 0.0125 \\
23.75 + 500 \times (f - 0.06) & 0.0125 < f \leq 0.0825 \\
35 & 0.0825 < f \leq 0.09 \quad \text{for } f_0 = 27.68MHz \\
31.25 - 375 \times (f - 0.1) & 0.09 < f \leq 0.16 \\
10.625 - 21.875 \times \sqrt{f - 0.16} & f > 0.16
\end{cases}
\]  

(3.48)

While the radar operates at other frequencies, according to Equation 3.35, the normalized second-order sidebands are proportional to radio wave number \( k_0^2 \) and radar frequency \( (f_0) \), so the regression coefficients can be used for other radar frequency if they are multiplied by a factor

\[
\alpha_f = (27.68 \times 10^6/f_0)^2 \alpha_{f_0}
\]  

(3.49)
The total wave energy is given by

\[ <\xi^2> = \int_0^\infty S_k(f) df = \sum_{k=1}^{21} S_k(f) \Delta f \]  \hspace{1cm} (3.50)

where \( S_k(f) \) is the final wave spectrum derived from radar backscatter using regression method and the significant wave height is given:

\[ H_S = 4 <\xi^2>^{1/2} = 4 \left( \sum_{k=1}^{21} S_k(f) \Delta f \right)^{1/2} \]  \hspace{1cm} (3.51)

The peak frequency of the spectrum could be given by making \( S_k(f_p) = \text{max} \). With the SMB method (Equation 2.35), the wind speed could be derived from the radar second-order spectrum.

### 3.5 Summary

In this chapter, the principle of the HF radar remote sensing and wind direction as well as wind speed inversion from HF radar backscatter are presented.

When radar operates at different frequencies, the signature of backscatter spectra is different as well. At a higher radio frequency, the radar range is shorter due to the higher attenuation at the conductive ocean surface. Besides that, the roughness of the sea surface brings some additional attenuation. In the radar Doppler spectrum, the SNR of the second-order sidebands is quite lower than that of the first-order peaks. At the same wind condition, the SNR of the second-order sidebands strongly depends on the radar frequency; at a lower radar frequency, the SNR of second-order sidebands is lower, which brings some difficulties for the wind speed inversion from radar second-order backscatter. But the first-order backscatter gives a much higher SNR and covers a much larger area within radar coverage than the second-order backscatter method.

From HF radar first-order backscatter, a new pattern fitting method is proposed for the wind direction inversion as well as the directional spreading parameter (\( \beta \)) of Bragg resonant waves. The method gives a unique solution for the wind direction. As introduced in Chapter 2, at a certain wave frequency, the wave directional spreading parameter (\( s \) or \( \beta \)) is closely related to the local wind conditions, so the spreading value \( \beta \) could be implemented for the wind speed inversion.
Chapter 4

Neural Network and Approaches of Wind Inversion

4.1 Neural network and remote sensing

An Artificial Neural Network (ANN) can be defined as a highly connected array of elementary processors called neurons. The use of artificial neural networks for remote sensing has been motivated by the realization that the human brain is very efficient at processing vast quantities of data from a variety of different sources. Artificial neural networks do not approach the complexity of the brain. There are, however, two key similarities between biological and artificial neural networks. First, the building blocks of both networks are simple computational devices that are highly interconnected. Second, the connections between neurons determine the function of the network [87]. A network can be trained to perform a particular function by adjusting the values of the connections (weights) between the elements. Research in the field of neural networks has been attracting increasing attention in recent years [88, 89, 90]. ANN representations are capable of developing functional relationships from discrete values of input-output quantities obtained from experimental results or complex computational approaches. The generalization property makes it possible to train a network on a representative set of input-output examples and get good results for new input without further training the network.

In order to estimate ocean wave parameters from wind speed and direction at ocean surface, various wave models have been developed in the last half century. Wind at sea surface acts as the main input energy source for wave growth. Besides the wind speed, other parameters such as fetch, duration and even the temperature stability (temperature difference between air and sea) make the wind inversion from waves complicated. In HF radar remote sensing, the complexity of radar equation also makes it difficult to derive wave and wind information [76]. ANNs are ideally suited for applications where input to output relationship is either unknown or too complex to be described analytically. They simply require a sufficiently variable data set consisting of measurements of the input parameters and
Chapter 4. Neural Network and Approaches of Wind Inversion

the corresponding output parameters. ANN can be trained to learn the mapping process between the input and output. The basic idea is to use the input-output pairs generated by the radar data and in-situ measurements to train the neural network. In this dissertation, a back-propagation (BP) learning method is used for training the network. This algorithm uses the gradient descent algorithm to get the best estimates of the interconnected weights: the weights are adjusted after each iteration. The iteration stops when a minimum of the difference between the desired and the actual output is reached. After training, the networks can automatically process the radar data and derive the wind data. This study therefore addresses the issue using neural network to tackle the complexity and non-linearity of the ocean waves scattering measurement by radar and the wind-wave relationship.

4.2 Principle of artificial neural network

An artificial neural network is a mathematical or computational model which consists of an interconnected group of artificial neurons and processes information using a connectionism approach. The neurons and the structure of neural network are introduced as follows:

4.2.1 Artificial neuron models and transfer functions

The fundamental processing element of a neural network is the neuron. An elementary neuron with \( n \) inputs is shown below: each input is weighted with an appropriate \( w_{ij} \) (\( i \) is the neuron node number in layer \( j \)). The sum of the weighted inputs and the threshold \( \theta_j \) (or bias \( b_j = -\theta_j \)) forms the input to the transfer function \( f(x) \). Neurons can use any differentiable transfer function to generate their output. The neuron model can be written as follow:

\[
o_j = f_j\left(\sum_{i=1}^{n} w_{ij} x_j - \theta_j\right) = f_j(W_j X_j - \theta_j) \tag{4.1}
\]

where \( W_j = [w_{1j}, w_{2j}, ..., w_{nj}] \), \( X_j = [x_{1j}, x_{2j}, ..., x_{nj}]' \) and \( f_j \) is the transfer function in the layer \( j \). The structure of a neuron model is illustrated in Figure 4.1.

Figure 4.1: Structure of artificial neuron model (from Duch 1999)
4.2. Principle of artificial neural network

The input of the transfer function is the sum of the weighted inputs and the bias. Transfer functions should provide maximum flexibility of their contours with a small number of adaptive parameters. Large networks with simple neurons may have the same power as a small network with more complex neurons. The choice of transfer functions may influence the complexity and performance of neural networks [91]. Here, several commonly used transfer functions are introduced:

- Step function
  \[ f(x) = \begin{cases} 
  1 & x \geq 0 \\
  0 & x < 0 
  \end{cases} \]  (4.2)

- Piecewise linear function
  \[ f(x) = \begin{cases} 
  0 & x \leq x_1 \\
  ax + b & x_1 < x \leq x_2 (a, b \text{ are constant values}) \\
  1 & x > x_2 
  \end{cases} \]  (4.3)

- Logistic sigmoid function
  \[ f(x) = \frac{1}{1 + e^{-cx}} (c \text{ is a constant value}) \]  (4.4)

- Tangential sigmoid function
  \[ f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]  (4.5)

4.2.2 Neural network structures

A network may have several layers, each layer has a weight matrix \( W_j \), a threshold vector \( \theta_j \) (or bias vector \(-b_j\)) and an output vector \( o_j \). To distinguish between the weight matrices, output vectors, etc., for each of these layers, the number of the layer is appended as a superscript to the variable of interest. For example, a three layer neural network\(^1\) is illustrated in Figure 4.2.

In the multi-layer network, one of the output \( o^i_j \) can be written as

\[ o^i_j = f_2(W_2 \cdot f_1(W_1 \cdot X + \theta_1) + \theta_2^i) \]  (4.6)

Multi-layer networks are quite powerful, for instance, a network of three layers, where the second layer uses sigmoid function and the third layer uses linear function, it can be trained to approximate any function arbitrarily well.

\(^1\)Some authors refer to the inputs as not a layer, this work does not use that designation.
Figure 4.2: Example of multi-layer neural network (from Demuth 2009)

The structure of artificial neural network can be separated into two basic types: **feed-forward** and **feedback** network. Feed-forward ANNs allow signal to travel one way only: from input to output, there is no feedback (loops) *i.e.*, the output of any layer doesn’t affect the same layer. It tends to be straight forward networks that associate inputs with outputs. Each input neuron is connected to all neurons in the hidden layer and each hidden neuron is connected to all neurons in the output layer. Feedback networks can have signals traveling in both directions by introducing loops and allowing connections between input and output neurons and between hidden and other hidden neurons [92].

The neuron model and the architecture of a neural network describe how a network transforms its input to output, this transformation can be viewed as a computation. Each processing element is interconnected to others following a specific interconnect scheme. The feed-forward neural network is the most popular and widely used model in many applications. The earliest neural network is a single-layer perceptron network. The multi-layer perceptron is a modification of the standard linear perceptron in that it uses three or more layers of neurons with nonlinear transfer functions. It uses a variety of learning techniques, the most popular is back-propagation. Here, the output values are compared with the correct answer to compute the value of some predefined error-function. By various techniques, the error is then fed back through the network. Using this information, the algorithm adjusts the weights of each connection in order to reduce the value of error function by some small amount. After repeating this process for a sufficiently large number of training cycles, the network will usually converge to some state where the error is small enough. In this case, the network has learned a certain target function. To adjust weights properly, one applies a general method for non-linear optimization that is called gradient descent. From this derivation of the error function, the network weights are calculated.
4.2. Principle of artificial neural network

and the weights are then changed so that the error decreases. For this reason, the back-propagation can only be applied on networks with differentiable transfer function.

4.2.3 Introduction to back-propagation network

Error Back Propagation (BP) neural network is the most widely used neural network, which is used by different research communities in different contexts. It was proposed in 1969 [93] and rediscovered in 1985 [94]. The aim of the technique is to train the network so that the response to a given set of inputs corresponds as closely as possible to a desired output. There are two distinct steps in the back-propagation algorithm. The first step is calculating the transformations of both the hidden layer(s) and the output layer units with respect to the summarized weighted input variables in the network model called forward propagation. It comes up with a predicted value and checks to see how well it compares to the observed value by calculating the error. The second step is evaluating the derivatives based on the error function with respect to network weights. Therefore, the algorithm then evaluates the derivative of the error function by back-propagation the error terms backwards through the network by performing the descent algorithm with corresponding adjustments made to the weight estimates. This process continues until a best fit is achieved, that is, when the vector of errors are all zero or when any one of the convergence criterion values are met.

4.2.3.1 Supervised learning

BP neural network is a supervised learning method and it requires a teacher who knows the outputs for any given input. The sketch of supervised learning is given in Figure 4.3.

![Figure 4.3: Sketch of supervised learning (from Demuth 2009)](image)

4.2.3.2 Differentiable transfer functions

BP requires that the transfer function used by artificial neurons is differentiable. The error propagates backwards from output nodes to inner nodes. So technically speaking, back-propagation is used to calculate the gradient of the error of the network with respect to the network’s modifiable weights. This gradient is then used in a simple stochastic gradient descent algorithm to find weights that minimize the error. Back-propagation usually allows quick convergence on satisfactory local minima of error in the kind of networks to which
it is suited. Since this method requires computation of the gradient of the error function at each iteration step, we must guarantee the continuity and differentiability of the error function. Obviously sigmoid functions are suitable for the transfer function.

1. Logistic sigmoid \((\mathbb{R} \rightarrow (0,1))\)– also called \textit{logsig}, defined by the Equation 4.4, the function \textit{logsig} generates output between 0 and 1 as the neuron’s net input goes from negative to positive infinity. The constant \(c\) can be selected arbitrarily. The shape of the sigmoid changes according to the value of \(c\). Higher values of \(c\) bring the shape of the sigmoid closer to that of step function. Here, \(c\) is set to 1, which is often used in BP network.

2. Tangential sigmoid \((\mathbb{R} \rightarrow (-1,1))\) – also called \textit{tansig}, this is derived from the \textit{hyperbolic tangent}, defined by Equation 4.5. It has advantages over the \textit{logsig} of being able to deal directly with negative numbers. The function \textit{tansig} generates output between -1 and 1 as the neuron’s net input goes from negative to positive infinity.

3. Linear function – occasionally, the linear transfer function \textit{purelin} is also used in BP neural network. If the last layer of a multilayer network has sigmoid neurons, then the outputs of the network are limited to a small range. If linear output neuron is implemented, the network outputs can be any value.

In back-propagation, it is important to be able to calculate the derivatives of any transfer functions used. Each of the transfer functions above, \textit{logsig}, \textit{tansig} and \textit{purelin}, can calculate its own derivative. The three transfer functions described here are the most commonly used transfer functions for back-propagation, but other differentiable transfer functions can also be created and used if desired.

### 4.2.3.3 Error back-propagation and weight updating

Consider a feed-forward network with \(n\) input and \(m\) output units. It can consist of any number of hidden units and can exhibit any desired feed-forward connection pattern. We are also given a training set \(\{(x_1, t_1), ..., (x_p, t_p)\}\) consisting of \(p\) pairs of \(n\) and \(m\) dimensional vectors, which are called the input and output patterns. Let the primitive functions at each node of the network be continuous and differentiable. The weights are randomly selected. When the input pattern \(x_i\) from the training set is presented to this network, it produces an output \(o_{i,j}\) different in general from the target \(t_{i,j}\). What we want is to make \(o_{i,j}\) and \(t_{i,j}\) identical for \(i = 1, 2, ..., p, j = 1, 2, ..., m\), so we have:

\[
E_{i,j} = \frac{1}{2}(o_{i,j} - t_{i,j})^2
\]

The back-propagation algorithm is to find a local minimum of the error function, the network is initialized with randomly chosen weights. The gradient of the error function is computed and used to correct the initial weights. The first step is the minimization process.
4.2. Principle of artificial neural network

of extending the network, so that it computes the error function automatically. Figure 4.4
shows the sketch of the structure. Every one of the $j$ output units of the network is
connected to a node which evaluates the function $\frac{1}{2}(o_{ij} - t_{ij})^2$, where $o_{ij}$ and $t_{ij}$ denote the
$j$-th component of the output vector $o_i$ and of the target $t_i$.

Figure 4.4: Extended network for the computation of the error function (from Rojas 1996)

We now have a network capable of calculating the total error for a given training error
for a given training set. The weight in the network is the only parameter that can be
modified to make the quadratic error $E$ as low as possible. Because $E$ is calculated by the
extended network exclusively through composition of the node functions, it is a continuous
and differentiable function of the $\ell$ weights $w_1, w_2, ..., w_\ell$ in the network. We can thus
minimize the $E$ by using an iterative process of gradient descent, for which we need to
calculate the gradient

$$\nabla E = \left( \frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, ..., \frac{\partial E}{\partial w_\ell} \right)$$

Each weight is updated using the increment

$$\Delta w_i = -\gamma \frac{\partial E}{\partial w_i} \text{ for } i = 1, 2, ..., \ell$$

where $\gamma$ represents a learning constant, \textit{i.e.}, a proportionality parameter which defines
the step length of each iteration in the negative gradient direction, the minus ($-$) sign
indicates a down-hill direction towards a minimum. Now, by the steepest descent (gradient)
procedure, we have that

$$\omega_{ij}(k + 1) = \omega_{ij}(k) + \Delta \omega_{ij}(k)$$

The correction step is needed to transform the back-propagation algorithm into a learn-
ing method for neural networks. After choosing the first weights of the network randomly,
the back-propagation algorithm is used to compute the necessary corrections. The algo-
rithm can be decomposed in the following four steps [95]:

1. Feed-forward computation
2. Calculation of output error
3. Error back-propagation

4. Weight updates

These four steps are conducted as a loop, and the algorithm will be stopped when the value of the error function has become sufficiently small (a predefined small value).

4.2.3.4 Learning and training algorithms

Learning and training are fundamental for all neural networks. Training is an external process. It is the procedure by which the network learns. Learning is the result that takes place internal to the network. It is the process by which a neural network modifies its weights in response to external inputs. Weights are changed when the output(s) are not what is expected [96]. Training can take place in three distinct ways: supervised, reinforcement and unsupervised. In the supervised training, the network is provided with an input stimulus pattern along with the corresponding desired output pattern. The learning law for such a network typically computes an error, that is, how far from the desired output network’s actual output really is. This error is then used to modify the weights on the interconnections.

At present, researchers on NN mostly focus on how to get an efficient learning algorithm and optimize the architecture. Traditional BP learning algorithm adopts a gradient descent algorithm, which converges slowly and tends to trap into local minima. These defects lead to weak learning abilities. To solve a nonlinear discrete problem, choosing a suitable training algorithm is critical to enhance the training speed and accuracy of the results [97, 98]. Many methods have been proposed subsequently and they mainly focus on the following two ways: (1) Heuristic algorithm, including adding momentum terms, adopting adaptive learning rate and spring-back algorithm; (2) Numerical optimization algorithms, including Newton algorithm, conjugate gradient algorithm and Levenberg-Marquardt algorithm. Among these methods, the Newton algorithm processes a second-order convergence property. But for its high computation cost, the Newton algorithm often fails in applications. As one of the improved Newton algorithms, the Levenberg-Marquardt (L-M) is the most widely used algorithm which can maintain high convergence rate and good practicability. It outperforms simple gradient descent and other conjugate gradient methods in a wide variety of problems. L-M [99, 100] method provides a stepwise weight modification formula that can be incorporated into a network training. The basic idea is to reduce the gradient to zero which can be envisioned as making jumps directly toward the closest minimum on the error surface. The weight modification formula is

\[
\Delta w = -(H_d + \gamma I)^{-1} g
\]  

(4.11)

where \(H_d\) is the diagonal approximation of the Hessian matrix with second-order error derivations, \(g\) is the gradient vector, and \(\gamma\) is the regularization parameter, \(I\) is the unit matrix. The diagonal elements of the Hessian can also be written in terms of the Jacobian.
4.2. Principle of artificial neural network

matrix entries of the derivatives of the output with respect to the weights:

\[
[H_d]_{ij} = \sum_{n=1}^{N} [J]_{i,n} [J]_{j,n}
\]  

(4.12)

where \([J]_{i,n} = \partial P(x_n)/\partial w_i\). Having this correspondence is useful for rewriting of the training algorithm in a more useful format. Details of Hessian and Jacobian matrix are given in [101].

The Levenberg-Marquardt training rule is expressed using the elements of the Jacobian is given alternatively by matrix equations:

\[
\Delta w = -(J^T J + \gamma I)^{-1} g
\]  

(4.13)

where \(g\) and \(J\) can be obtained by back-propagation. In order to gain efficiency the diagonal approximation of the Hessian is adopted, which can be easily computed when back-propagating the error, and to avoid numerical computation instabilities it includes a regularization factor. When the regularization parameter goes to infinity, i.e., \(\gamma \to \infty\), Equation 4.13 approaches the generalized delta rule for gradient descent learning. The L-M method has an advantage over these methods as it is less sensitive to the ill-posedness of the Hessian matrix due to the use of regularization.

Use of the L-M algorithm is recommended for training neural network in batch mode. Alternatively, it can be used in incremental mode with the formula [102]:

\[
\Delta w_n = -\frac{1}{[J]_{i,n}} [J]_{j,n} + \gamma I) g_n
\]  

(4.14)

where \(i\) and \(j\) are the weight vector indices.

4.2.3.5 Mean square error and flat spot

Once the network weights and biases are initialized, the network is ready for training. The performance function for the feed-forward networks is MSE, which is the error calculated by determining the Mean Square Error (MSE) between the network outputs \(o_1, o_2, ..., o_N\) and the targets \(t_1, t_2, ... t_N\) (\(N\) is the number of output note):

\[
MSE = \frac{1}{N} \sum_{j=1}^{N} (o_j - t_j)^2
\]  

(4.15)

During the network training, the value of the MSE decreases, as shown in Figure 4.5, which is a typical MSE curve in the network training. The weight updating is done by using the first derivation of the error index function with respect to the weights, because the transfer functions are non-linear in general, the phenomenon of “flat spots” may appear [103]. The derivative of the error index function with respect to the weights approaches
to zero, which makes the weight update nearly equal to zero. This phenomenon is called “fake saturation” or “flat spot” [104]. The flat phenomenon is formed by the total input of neurons getting into the saturation region of neuron transfer functions. During the training, if a unit in a multilayer network receives a weighted signal with a large magnitude, it outputs a value close to one of the saturation levels of its transfer function, while the corresponding target value is substantially different from that of the saturated unit, the unit is incorrectly saturated. When this happens, the step of the weight updating will be very small, even though the error is relatively large, and it will take an excessively long time for such incorrectly saturated units to reverse their states [105]. The method to avoid the “flat spot” should be considered by decreasing the total input of neurons and the appropriate choice of transfer functions. The total input of neuron transfer functions is the weighted sum of neuron outputs in the front layer. The bigger the number of total input of neurons is, the more possible the neurons go into the saturation state. Therefore, the neuron outputs and the amplitude of weights in the front layer must be limited. The investigations of transfer functions for eliminating the “flat spot” in multi-layer feed-forward networks are discussed in [106, 107]. The neurons in the output layer using the linear function and the neurons in the hidden layer using the sigmoidal function can be used to approximate any non-linear function. Obviously, if the linear transfer functions are used in output layer, the neurons have no saturation state. Therefore there is no “flat spot” in the output layer and the convergence speed of BP algorithm can be increased. Of course, some other improvements have been developed to solve the “flat-spot” and speed up the BP convergence [108].

![Figure 4.5: Mean square error of the ANN during the training (from Demuth 2009)](image-url)

### 4.2.3.6 Generalization capability

From a biological perspective, generalization is very important for our creation of models of the world. Think of generalization according to the following example. If you just memorize some specific facts about the world instead of trying to extract some simple
essential regularity underlying these facts, then you would be in trouble when dealing with
the novel situations where none of the specifics appear [109].

The generalization ability of neural networks is also considered as an important perfor-
manace criterion. In the BP network, not only a training set is needed, but also a testing
net is necessary for evaluating the performance of the network working on the new patterns
that have not been used in the network training process. Generalization is measured by
the ability of a trained network to generate the correct output for a new randomly chosen
input drawn from the same probability density governing the training set. Several factors
affect the capabilities of the neural network to generalize, that is, the ability of the neural
network to interpolate and extrapolate to data that it has not seen before. These include:

(1) **Number of nodes and architecture.** If a large number of simple processing elements
are used, the mathematical structure can be made very flexible and the neural network can
be used for a wide range of applications. This may be not necessary for all applications.
For example, very simple topologies using a small number of data points have been inves-
tigated. In general terms, the larger the number of nodes in hidden layer(s), the better
the neural network is able to represent the training data, but at the expense of the ability
to generalize; (2) **Size of training set.** The data set used must be representative of the
entire distribution of values likely to be associated with a particular class. If the extent of
the distribution of the data in feature space is not covered adequately the network may fail
to classify new data accurately. A consequence of this is that large quantities of data are
often required for training and researchers are often concerned with finding the minimum
size of data set necessary [110]; (3) **Training time.** The time taken for training also affects
the generalizing capabilities of the network. The longer a network is trained on a specific
data set, the more accurately it will be able to classify those data, but at the expense of
the ability to classify previously unseen data. In particular, it is possible to over-train a
network so that it is able to memorize the training data, but it is not able to generalize when
it is applied to different data [111]. Researchers have been making an effort to promote
the generalization ability and presented several methods, for example, early stopping [112],
regularization [113], fuzzification of input vector [114], neural network ensembles [115, 116],
etc.

### 4.3 Neural network design

In order to select an appropriate neural network configuration to perform wind inversion
from HF radar echoes, many major factors need to be considered.

#### 4.3.1 Layers and number of neurons

In this work, a multi-layer structure is used for wind inversion. The number of layers can
be set to three (with one hidden layer) or four (with two hidden layers) or more, which
depends on the complexity of the inversion. The number of nodes in the input and output layers corresponds to the number of input arguments and desired outputs, which are easy to determine. The number of nodes in hidden layer(s) affect both the network accuracy and the time required for the training. If the number of nodes in the hidden layers is too small, the internal structure of the data even can not be identified and therefore produces low accuracies, and if the number is too large, it is likely to over-fit the training data and make the computation too complicated. The selection of node numbers in the hidden layer(s) can be determined by the designer’s tests and experiences. Some rules could be used as a reference [117]:

- \( n_1 = \sqrt{n + m + a} \), where \( n_1 \) is the number of nodes in the hidden layer, \( n \) and \( m \) are the numbers of the node in the input and output layer respectively, \( a \) is a constant ranged from 1 to 10.

- \( n_1 = \log_2 n \), where \( n \) is the number of input nodes.

### 4.3.2 Training, validation and test data

In the application of a BP neural network, initially, each weight \( w_i \) is set to some arbitrary small random value. The process then goes through an iteration using the back-propagation convergence technique with the training data set. Depending on the nature of the problem, the neural network may be designed to approximate a function describing the training data, or may learn relationships between input and output data within the training set. Training sets can be significant in size with several thousand training examples. After each iteration, the learning algorithm continues to adjust the network weight coefficients. The goal of the training is that, after training the network to some stage, the internal neural network parameters are developed to satisfy the designed requirements. Optimization procedures are used to evaluate the derivatives of the error function with respect to the weight estimates. But, none of the convergence algorithms guarantees a global minimum. The reason is that the nonlinear error surfaces might consist of a number of minima. Adding more input layer units and hidden layer units increases the propability of occurance of the multi-minima. The validation data set is used to prevent over-fitting in monitoring the error in the iterative process. The process uses the training data set to drive the gradient-descent grid search and then uses the validation data set to produce the smallest squared error in minimizing the objective function. Therefore, the process continues until a minimum error is reached in the validation data set. The split sample procedure works best when there is enough data allocated to the validation data set. Conversely, over-fitting can occur with a small sample size of the training data set. Fitting a model to an enormous amount of data eliminates over-fitting. There are no general rules for the precise allocation of the original data or the input data set in the neural network. The allocation schema used in partitioning each one of the data sets depends on both the amount of the available cases from the input data set

60
4.3. Neural network design

and the noise level in the underlying data set.

In the Matlab neural network toolbox, early stopping is the default method for improving generalization, which is automatically provided for all of the supervised networks. In this technique, the available data is divided into three subsets: the training data set, the validation set and the test set. The training data set is used for computing the gradient and updating the network weights and biases. The error on the validation set is monitored during the training process. The validation error normally decreases during the initial phase of training, as does the training set error. However, when the network begins to over-fit the data, the error on the validation set typically begins to rise. When the validation error increases for a specified number of iterations, the training is stopped, and the weights and biases at the minimum of the validation error are returned. The test set error is not used during the training, but it is used to compare different models (architectures) [118]. If the error in the test set reaches a minimum at a significantly different iteration number than the validation set error, this might indicate a poor division of the data set. Matlab also provides four functions for dividing data into training, validation and test sets. They are dividerand (Random Data Division), divideblock (Block Data Division), divideint (Interleave Data Division) and divideind (Index Data Division). For example, the Random Data Division divides the input data randomly, 60% of the samples are assigned to the training set, 20% to the validation set and 20% to the test set. The other functions are introduced in Matlab neural network documentation [119].

An example of validation performance and the training state are given in Figure 4.6, which is one result of wind speed inversion from radar first-order backscatter during the Fedje experiment. The network input and output data set as well as the network configuration will be introduced in Section 5.4. During the training, the “max fail” number is set to 20, which means the training stops when the validation error continuously increases for 20 iterations starting from the best validation performance, which is the minimum of the MSE. In this example, the training stops at the 48th iteration, that means, the 28th iteration gives the best validation performance, as shown in Figure 4.6a. The training state is illustrated in Figure 4.6b, in which there is local minimum starting from 17th epoch, but after several training epochs, the network jumps out of the local minimum. Besides the validation check state, the intermediate parameter’s gradient value and Marquardt adjustment parameter (mu) are also given [119].

4.3.3 Dependent variables selection for neural network

During the training, the selection of data set from radar echoes is very important. In principle, only the wind or wave dependent variables should be used for the training, although the noise in the radar Doppler spectrum is the independent variable, the noise could not be suppressed easily and it strongly affects the performance of the neural network, especially when the SNR of the input data set is low. Figure 4.7 shows a sketch of data selection for
Chapter 4. Neural Network and Approaches of Wind Inversion

Figure 4.6: Network performance and training state (net.trainParam.max_fail = 20) in wind data inversion from radar first-order backscatter during the Fedje experiment (details of the network configuration are given in Section 5.4)

Another important issue that needs to be considered is, that for different applications of the neural network, the selection of “dependent” variables for the input data is also different. For example, the power of the first-order peaks depends on the directional distribution of Bragg waves and the radio beam direction as well as the radar range. For the wind inversion from radar first-order backscatter at the buoy location, the radio beam direction and the radar range are constant values, which are not used as the input data set for the network. But for the wind speed inversion at the other locations (also from radar first-order backscatter), the radio beam direction and radar range are not constant values any more. So they must be taken into consideration for the wind speed map inversion.

Figure 4.7: Sketch of data set selection for neural network


4.4 Methodology of wind inversion from waves and radar remote sensing

For many years, in order to meet the growing requirements for wave information, oceanographers have been developing models to predict ocean waves from measurement of wind data. In all these models, wind speed is an important input parameter, which determines the growth and decay of waves. Besides the wind speed, there are some other parameters such as wind fetch, duration and intermediate parameters (wave age, friction velocity, etc.). All these contribute to the waves and make the wind inversion very complicated. In HF radar remote sensing, the motion of atmosphere could not give any reflection of electromagnetic waves. The only way for deriving wind is from the signature of waves, which can also be derived from the radar backscatter echoes. During the experiments, an in-situ buoy and an anemometer give the wave and wind measurements at the sea surface (anemometer is installed at the lighthouse at Fedje). So the in-situ measurements could be used as the target data for the network training and give the solution of wind at sea surface. The Bragg scattering gives the signature of ocean waves at a certain frequency \( f_B \). As we have intensively discussed, the integrated Bragg wave energy and the directional spreading parameter are closely related the local wind conditions. The second-order backscatter gives the ocean wave spectra measurement at a wider frequency range \( f \in [0.05 \sim 0.25] \text{ Hz} \). So from both first-order and second-order backscatter, the wind speed could be inverted. Besides the wind speed, with the help of directional waverider buoy and neural network, the directional wave spectra could also be inverted from the radar backscatter.

At the buoy location, the in-situ wind and wave measurements provide the target data for training the neural network. After a successful training, the network can process the radar data automatically and invert wind data independently. For example, the meteorological buoy is deployed at the grid point A for several days and moved to the other locations within radar coverage B or C, etc. If at the location A, the buoy has already acquired sufficient variables for the training, so after the buoy is moved away, the radar could still invert the wind data using the trained the network. The same process is also conducted at the grid point B and C, etc. Or during the experiment, if there are several meteorological buoys deployed at the sea surface and measure the wind simultaneously, using the in-situ measured wind data, the networks \( \text{net}_A, \text{net}_B \) and \( \text{net}_C \) could be trained. When the buoys are moved away, these networks could be used to calculate the wind direction and speed at these gird points. Finally, the wind map could be depicted from the discrete measuring points at the sea surface. The sketch of network application is given in Figure 4.8.

Because the first-order peak power and second-order sidebands strongly depend on the radio beam direction, especially the power ratio of first-order peaks is directly related to the angle between the radio beam direction and wind-wave direction. So when calculating the wind speed within radar coverage, the power of the first-order peaks or the second-order spectra can not be used to give the wind measurements at the other locations within
radar coverage except the buoy location. In this work, some beam direction and radar range independent parameters are tried in the neural network for the wind field. Before introducing that, the wind derivation at the buoy location is discussed as follows:

### 4.4.1 Wind inversion from waves at certain frequencies

Both directional spreading pattern and wave power density of Bragg resonant waves give the information on ocean surface wind. A radar could measure the resonant waves along the direction of the radio beam, including the approaching and receding wave components. In the HIPOCAS WAM model data, the complete wave directional patterns are given at different wind conditions. Before inverting the wind speed from HF radar backscatter, the HIPOCAS WAM model data is analyzed to find the relationship between Bragg waves and ocean surface wind. In WAM data, we could not give the exact same value of Bragg frequency due to the discrete wave frequency steps, but the waves of adjacent frequencies are analyzed.

As introduced in Section 2.3, the spreading parameter of wave directional pattern depends on the wind speed. The WAM model data gives the two-dimensional wave directional spectra as well as the wave directional pattern at certain frequencies. In Chapter 2, some of the WAM data is selected as shown in Figure 2.5. In this example, the wind speed increases almost linearly and the wind direction becomes stable. The waves at the Bragg frequency of 0.54764 Hz are taken as an example and the wave directional patterns at three different wind speeds are illustrated in Figure 4.9.

In the WAM model data, there are 24 direction bins. The power density at each direction bin is $S(\theta_i)$, where $\theta_i = \frac{i\pi}{12}, i = 1, 2, \ldots, 24$. The power density $S(\theta_i)$ is a function of wind speed and direction. If the parametric methods are used for the wind inversion from the directional pattern and the power at each direction bin, 24 equations are needed and it is difficult to invert wind speed from such equations. Here, a neural network method is implemented to invert the wind speed from $S(\theta_1), S(\theta_2), \ldots, S(\theta_{24})$.

The wave directional spreading pattern and the power density at each direction bin are used as the input data, the wind speed of WAM data is used for the target data, the sketch
4.4. Methodology of wind inversion from waves and radar remote sensing

is given in Figure 4.10.

In the network, the WAM model data at location E in the whole year of 2004 is analyzed and the number of input neurons is 24, which is the number of direction bins. The scatter plots of wind speed of WAM data and the neural network output are shown in Figure 4.11. Two wave frequencies are selected based on the two corresponding radar frequencies used in the experiments dissertation. Comparing with the integrated wave results in Figure 2.7, the saturation of waves is not obvious, although the tendency of the saturation could also be observed in Figure 4.11a. The reason is that the directional spreading pattern of resonant waves is involved in the wind speed inversion, which might give a wider range of possible wind speed inversion than that just using the integrated wave power density.

4.4.2 Method of wind inversion from radar first-order backscatter

Two radars are used during the radar experiments. The sketch of two radio beams and wind-wave pattern is given in Figure 4.12. The shadow and dark regions indicate the wave components which give the Bragg scattering to the radar sites respectively. HF radar measurement could only give the measurements of four wave components on this Bragg resonant wave directional distribution, and each beam sector is with a directional range of
Chapter 4. Neural Network and Approaches of Wind Inversion

Figure 4.11: Scatter plot of wind speed inversion from waves at certain frequencies (HIPOCAS WAM data)

\[ \phi_w. \] Although there is no complete wave directional pattern used for the inversion, with the help of neural network, wind speed and direction could also be inverted from radar first-order backscatter.

Figure 4.12: Derivation of wind data from the radar first-order backscatter

The first-order peaks are the dominant feature in the radar Doppler spectrum and the SNR (Signal to Noise Ratio) is much higher. \( \sigma_1(f_B), \sigma_1(-f_B) \) are the positive and negative first-order peak power. Once one grid point is selected within radar coverage, the radio beam directions are determined as well as the beam width \( \phi_w \). As shown in Figure 4.12, the two pairs of first-order peak power \( \sigma_1(f_B), \sigma_1(-f_B) \) (at radar site 1) and \( \sigma_i' (f_B), \sigma_i' (-f_B) \) (at radar site 2) are used as the input data set and the anemometer wind speed and direction are used as the target data for training the neural network. The wind direction inversion has been proved to be reliable with the pattern fitting method (hyperbolic secant function) proposed in this dissertation, but still the neural network method is also tested. Details of
the inversion and results are presented in Section 5.4.

4.4.3 Wind inversion from wave spectra

In Chapter 2, the method of wind speed inversion from the wave parameters such as significant wave height $H_s$ and wave peak frequency $f_p$ is introduced, but in case of the presence of swell or residual wave components, the wind speed will be overestimated. But if there is a large deviation for the wave direction of the long waves and the wind waves, the swell or residual wave components might be suppressed. The Bragg resonant waves are located at the tail of the wave power spectrum, which are sensitive to the variability of wind direction, so the wind direction inversion is only based on the first-order method. Here, only the wind speed is inverted from the wave spectra. As shown in Figure 4.13, the input of the network is the wave frequency spectrum and direction. The waverider buoy and the WAM model data could give both wave direction and frequency spectrum. Regarding to the wind speed inversion from the HF radar backscatter, the normalized pairs of second-order sidebands are used as the input data set, which gives the information of wave amplitude and direction.

![Figure 4.13: Sketch of wind speed inversion from wave power density and direction](image)

4.4.3.1 Wind speed inversion from wave buoy measurements

During the Fedje and Ligurian Sea experiments, the waverider buoy measures the wave power density and direction. The wind data is collected by an anemometer. Both the wave power density and direction are used as the input for the network, the anemometer wind speed is used as the target data. After the training, the results of wind speed inversion from wave buoy during the two experiments are given in Figure 4.14. Here, the scatter plots of wind speed during the Fedje and Ligurian Sea experiments are presented respectively. At the Norwegian Sea, the result of the network gives a better result than that at the Ligurian Sea because the wind speed is relatively high. The statistics of the wind conditions is presented in Section 5.1.3.1.
4.4.3.2 Wind speed inversion from WAM model data

The wave power density spectra and wave direction of the HIPOCAS WAM data are shown in Figure 4.15, which are used as the input data set. The wind speed of WAM data is the target data set. After the network training, the scatter plot of network result and wind speed in WAM data is shown in Figure 4.16. The wave direction in Figure 4.15b gives the information of wave direction at each frequency bin, which helps the network to identify the possible swell or residual wave components and suppress them if the wave direction at lower wave frequencies has a great deviation to that at higher frequencies.
4.4 Methodology of wind inversion from waves and radar remote sensing

4.4.4 Method of wind inversion from radar second-order effects

In Barrick’s second-order backscatter equation, both the non-linear wave-wave interaction and the double scattering contribute to the second-order sidebands. The two dimensional integral equations make the calculation complicated. Here, a new inversion method is used with the help of neural network. The wind speed measured by anemometer is used as the target data. In this inversion, the input data set is different from that used for wind inversion from the first-order backscatter. At one radar site, the pair of sidebands of higher signal-to-noise ratio is used as the input. The offset frequency range of the second-order sidebands to the adjacent Bragg peak is from 0.05 to 0.25 Hz with a step frequency of 0.01 Hz (21 frequency points), so each sideband contains 21 spectra points. One radar site gives an input matrix of $21 \times 2$, the two radar sites give an input matrix of $21 \times 4$. The pair of second-order spectra at one site is written as $\sigma_2(f_{Di}), \sigma_2(-f_{Di}), (i = 1, 2, ..., 21)$, at the other site, they are written as $\sigma'_2(f_{Di}), \sigma'_2(-f_{Di}), (i = 1, 2, ..., 21)$. The sketch of the wind speed inversion is illustrated in Figure 4.17, at Fedje site, the pair of second-order sidebands (VM(1) and VP(1)) around the negative Bragg peak are with higher signal-to-noise ratio, while at Lyngoy site, the pair of the second-order sidebands (VM(2) and VP(2)) around the positive Bragg peak are with higher signal-to-noise ratio.

4.4.5 Method of directional wave spectra inversion from radar second-order backscatter

The second-order sidebands surrounding the first-order peaks give the information of directional wave spectra, the regression method has been successfully implemented for the wave height inversion during the EuroROSE project [67]. But the wave direction inversion is also based on the assumed spreading function $(\cos^2(\theta/2))$. However, as the spreading pattern
at lower wave frequencies is not regular, the mathematical description of the pattern might be very complicated. Here, a neural network is used to invert directional wave spectra from the second-order sidebands. The sketch of directional wave spectra inversion from the second-order sidebands is illustrated in Figure 4.18. The target data is the wave power density spectrum and wave direction given by the waverider buoy. During the experiment, the buoy can give the wave measurement at the frequency range of \( f \in [0.025 \sim 0.58] \) Hz, but the radar measurement only gives a frequency range of \( f \in [0.05 \sim 0.25] \) Hz, so the wave spectrum is preprocessed to the range which is same as the radar second-order sidebands. The output of the network contains the wave power density spectrum and wave direction. Details of the inversion will be discussed in Section 5.6.
4.5 Extension of wind measurements to the other locations within radar coverage

The wind or wave inversion methods introduced earlier were designed for radar measurements at the grid point where the buoy is deployed. The radio beam directions from the radar sites to the buoy and the radar range are constant values. But regarding to the other locations within radar coverage, the radio beam directions are definitely changed, and the radio beam-width also changes slightly, which affects the backscattered first-order peak power as introduced in Section 3.4.2 (two radio beams steering at the direction of 0° and 45° are presented in Figure 3.4). During the radar experiment, even if the wind speed and direction are constant on the radar coverage, the power of the first-order peaks strongly depends on the angle between wind direction and radio beam direction \((\phi - \theta)\), which will change if the radio beam is steered to some the other locations within radar coverage. An example of radio beams pointing to three grid points within radar coverage is given in Figure 4.19. On the map, the wind direction at the grid point 1,2 and 3 are identical \((\theta_0)\). \(\phi_i\) \((i = 1, 2, 3)\) is the radio beam direction from the radar site to the grid point 1,2 and 3. We assume that the point 2 is the location where the buoy is deployed and the beam direction is \(\phi_2 = \phi_0\), and the other two direction beam directions are \(\phi_1 = \phi_0 + \Delta \phi\) \(\phi_3 = \phi_0 - \Delta \phi\). 

![Figure 4.19: Schematic of wind directions (red arrow), wave directional patterns at three grid points and radio beam directions (\(\phi_1, \phi_2, \phi_3\))](image)

When training the neural network at the buoy location, the radio beam direction is an independent argument for the wind inversion. But if we apply the trained network to the other locations within radar coverage, the power of the backscattered first-order peaks depends on the radio beam direction and the radar range. So some compensation factors need to be used to “normalize” the radar echoes at other locations to the buoy
location (azimuth and range compensation factors). For example, when the wind at the sea surface is homogeneous, the Bragg wave heights are nearly at the same level. As shown in Figure 4.19, although these three grid points are with the same distance off the radar site, the backscattered first-order peak power is redistributed by the angle between the radio beam and wind direction ($\phi - \theta$), and the power ratio of the first-order peaks changes with the radio beam direction as well. In Figure 4.19, at each grid point, the wave component $OB$ and $OA$ give the positive and negative first-order peak power respectively, and not only the power ratio of the first-order peaks is different, but also the total power of the positive and negative peaks (the length of line $AB$ at the grid points are different). But in case of the homogeneous wind field, the integrated Bragg wave energy $E(f_B)$ (integrated Bragg wave energy over all directions) at these grid points is assumed to be constant.

In this work, the hyperbolic secant spreading function $0.5\beta \sech^2(\beta \cdot \theta)$ is used, as given in Equation 3.37 and Figure 3.22. The wave component $OB$ at grid point 2 is given as an example:

$$ S(f_{B2}) = E(f_B) \cdot \int_{\phi_0 - \Delta \phi_2/2 + \pi}^{\phi_0 + \Delta \phi_2/2 + \pi} 0.5\beta_2 \sech^2[\beta_2 \cdot (\pi + \phi - \theta_2)] d\phi $$

$$ \approx 0.5\beta_2 E(f_B) \cdot \sech^2[\beta_2 \cdot (\pi + \phi_0 - \theta_2)] \cdot \Delta \phi_2 \quad (4.16) $$

where $\beta_2$ and $\theta_2$ are the values of the spreading parameter and wind direction derived by the pattern fitting method at the grid point 2. $\Delta \phi_2$ is the beam-width of the radio beam pointing to the grid point 2.

At grid point 1, the wave component $OB S(f_{B1})$ is:

$$ S(f_{B1}) = E(f_B) \cdot \int_{\phi_0 + \Delta \phi - \Delta \phi_1/2 + \pi}^{\phi_0 + \Delta \phi + \Delta \phi_1/2 + \pi} 0.5\beta_1 \sech^2[\beta_1 \cdot (\pi + \phi - \theta_1)] d\phi $$

$$ \approx 0.5\beta_1 E(f_B) \cdot \sech^2[\beta_1 \cdot (\pi + \phi_0 - \theta_1)] \cdot \Delta \phi_1 \quad (4.17) $$

where $\beta_1$ and $\theta_1$ are the values of the spreading parameter and wind direction derived by the pattern fitting method at the grid point 1, $\Delta \phi_1$ is the beam-width of the radio beam pointing to the point 1.

As has been introduced in Equation 3.39, the positive first-order peak power $\sigma_1(f_{B2})$ at the grid point 2 is:

$$ \sigma_1(f_{B2}) = \kappa \cdot S(f_{B2}) = \kappa \cdot 0.5\beta_2 E(f_B) \cdot \sech^2[\beta_2 \cdot (\pi + \phi_0 - \theta_2)] \cdot \Delta \phi_2 \quad (4.18) $$

So we have:

$$ E(f_B) = \frac{\sigma_1(f_{B2})}{\kappa \cdot 0.5 \cdot \beta_2 \sech^2[\beta_2 \cdot (\pi + \phi_0 - \theta_2)] \cdot \Delta \phi_2} \quad (4.19) $$

Substituting $E(f_B)$ to the positive first-order peak power $\sigma_1(f_{B1})$:
4.5. Extension of wind measurements to the other locations within radar coverage

\[ \sigma_1(f_{B1}) = \kappa \cdot S(f_{B1}) = \frac{\beta_1 \text{sech}^2[\beta_1 \cdot (\pi + \phi_0 + \Delta \phi - \theta_1)]}{\beta_2 \text{sech}^2[\beta_2 \cdot (\pi + \phi_0 - \theta_2)]} \cdot \frac{\Delta \phi_1}{\Delta \phi_2} \cdot \sigma_1(f_{B2}) \]  

(4.20)

In order to generalize the application of the neural network trained at the buoy location to the other locations on the map, for example, to the grid point 1, the azimuth compensation factor for the positive first-order peak power is:

\[ \Delta \sigma_1(f_B, \Delta \phi) = \frac{\sigma_1(f_{B2})}{\sigma_1(f_{B1})} = \frac{\beta_2 \text{sech}^2[\beta_2 \cdot (\pi + \phi_0 - \theta_2)]}{\beta_1 \text{sech}^2[\beta_1 \cdot (\pi + \phi_0 + \Delta \phi - \theta_1)]} \cdot \frac{\Delta \phi_2}{\Delta \phi_1} \]  

(4.21)

As shown in Figure 4.20, for the grid points on the range circle 1, the radio beam azimuth compensation factor is used to normalize the first-order peak power to the grid point where the buoy is deployed. But this is only valid for the grid points with the same distance to the radar site (same path loss and attenuation). During the radar experiments, two radars are placed at the coast. The azimuth compensation factor could not be used in the case of two radars. For example, at the grid points on the circle 1, the power of the first-order peaks at the site 1 can be normalized to that at the buoy location by the azimuth compensation factor. But for the radar site 2, the azimuth compensation factor is invalid, because the grid points on the circle 1 are not with the same distance to the site 2 (except for the other cross point).

Figure 4.20: Schematic of range circles of two radar sites for azimuth compensation factor

Besides the variance of radio beam direction, the compensation of the radar range needs to be considered. The attenuation of the radio wave propagating at the sea surface also plays an important part in the radar first-order peak power. The propagation of electromagnetic waves at a conductive sea surface has been briefly introduced in Section 3.1.2.1.
In Figure 3.5, the received power decreases with the radar range, and the roughness of the sea surface also increases some additional attenuation. This effect has been explained by Shearmann [63]. At a high wind speed, the roughness of sea surface increases, which increases some more path loss and decreases the radar range. That means, if there is a target at the sea surface, e.g., a ship, the backscattered echoes are of a lower power (not significant) due to the increased dual path loss between the radar site and the target. But if the target is the Bragg resonant wave, the wave height increases with the wind speed. So there are two factors acting on the backscattered first-order peak power due to the wind speed: (1) the increasing wind speed increases the resonant wave height; (2) the increasing roughness also brings additional attenuation due to the path loss, which decreases the power level of the first-order peaks.

The power of the echoes scattered by the resonant waves are expressed by the negative and positive first-order peaks, which depends on the angle between radio beam and wind direction. So only with the empirical attenuation curves, the range compensation factor could not easily to be calculated for the positive and negative Bragg peaks. All these factors make it difficult to extend the wind speed measurement to the other locations using the network trained at the buoy location.

The neural network can track all these variation of Bragg peak power and wind speed at the buoy location with sufficient large of samples, but at the other locations within radar coverage, we have no in-situ measurements to train the network. So we have to find some parameters which are independent of the azimuth of the radio beams and radar range. In this dissertation, the spreading parameter $\beta$, which is derived by the pattern fitting method, is used to calculate wind speed at the other locations. During the experiment, the pair of spreading parameter and wind direction ($\beta, \theta'$) are derived at the buoy location, which are used as the input data, and the in-situ measurements of wind speed and direction ($U, \theta$) are used as the target data. The network is trained along the time series. With the extension of the trained network to the other locations, the wind map could be calculated. The sketch of wind inversion within the radar coverage using neural network is given in Figure 4.21 and details are discussed in Section 5.4.3.

![Figure 4.21: Sketch of network application for wind inversion within radar coverage](image-url)
4.6 Summary

This chapter presents the neural network design and the methodology of wind inversion from HF radar backscatter.

In the wind inversion, the in-situ measurements are achieved by the anemometers on the meteorological buoy or the lighthouse, which are used as the target data for the network training. The wind could be inverted from the signatures of waves at a certain frequency (amplitude and directional distribution) and the wave frequency spectra, which are related to the radar first-order and second-order backscatter mechanisms respectively. In this chapter, in order to verify the neural network methods for the wind inversion, the WAM model data and the in-situ wave measurements are used as the input data and the corresponding wind records are used as the target data. Several examples are given to illustrate the functional performance of the networks. For example, for the wind speed inversion from waves at a certain frequency, the WAM model data provides the complete information of the wave amplitude and the directional distribution. Although the radar backscatter could not give the exact same data structure as the WAM data or the wave buoy data, with two radar sites at different locations, the two pairs of first-order backscatter peaks are used as the input in the network, and power of the four first-order peaks is the echo scattered by the four Bragg wave components along the two radio beam directions. Besides the methodology of wind inversion from radar first-order backscatter, the methods of wind speed and directional wave spectra inversion from radar second-order backscatter are introduced.

The wind speed inversion at the other locations within radar coverage using neural network is discussed. In the application of neural networks, the selection of the input data set depends on the networks and the experimental conditions. For example, the power of radar first-order peaks is related to the wind direction, radio beam direction and the radar range. But at the buoy location, the radio beam direction and radar range are constant values, so these variables are not used in the inversion. While at the other locations within radar coverage, the radio beam directions and radar range should be considered. In order to normalize the first-order peaks from the other locations to the buoy location, the calculation of the azimuth and range compensation factors is discussed. But they are too complicated and can not be applied. So the idea of using the radio beam direction and radar range independent variable is proposed. In this work, for the calculation of the wind speed at the other locations, the directional spreading parameter of the Bragg resonant waves is used, which is derived by the new pattern fitting method. The spreading value ($\beta$) is independent of radio beam direction and radar range and it is only related to the wind conditions at the sea surface.
Chapter 5

Radar Experiments and Results of Inversion

5.1 Radar experiments and in-situ measurements

Data from two HF (WERA) radar experiments are used for wind speed and direction inversion. One experiment was the Fedje experiment within the EuroROSE (European Radar Ocean SEnsing) project, which was carried out in Norway (Fedje and Lyngoy) from February to April in 2000. The radar operating frequency was 27.68 MHz. The other experiment was carried out at the Ligurian Sea coast. The radar sites were located at Palmaria and Rossore in Italy and the experiment took place from May to September 2009 and the radar operating frequency was 12~13 MHz.

5.1.1 Fedje experiment

EuroROSE was an EU-funded project which demonstrated the possible advantage in operational oceanography by combining area covering radar measurements with fine-scale numerical models using advanced data assimilation techniques. The project was organized in close cooperation with Vessel Traffic Service (VTS) operators in Fedje, Norway. Within the EuroROSE Fedje experiment, surface current and wave measurements by HF radar were obtained from two sites (Fedje and Lyngoy) at the Norwegian coast. The Fedje experiment took place from February 8 to April 3, 2000. Radar frequency was 27.68 MHz and the coverage area was approximately $40 \times 40$ km$^2$. The measuring period was 9 minutes and in order to avoid interference, the two sites were operated successively and repeated every 20 minutes. Both WERA systems were configured to use a 16-element linear receive antenna array. Azimuthal resolution was achieved by beam-forming techniques, allowing access to the second-order sidebands in the Doppler spectrum. Each radar covered a sector of 120° as shown in Figure 5.1. The circles show the ranges of 25 km and 40 km respectively, the grid on the map is the cells with a size of $1.2 \times 1.2$ km.
Chapter 5. Radar Experiments and Results of Inversion

Figure 5.1: Radar coverage during the Fedje experiment 2000

<table>
<thead>
<tr>
<th>Time:</th>
<th>2000.03.16 14:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height $H_S$ (m)</td>
<td>1.96</td>
</tr>
<tr>
<td>Peak period $T_p$ (s)</td>
<td>10.53</td>
</tr>
<tr>
<td>total mean wave direction (°)</td>
<td>64.98</td>
</tr>
<tr>
<td>total directional spread (°)</td>
<td>42.52</td>
</tr>
<tr>
<td>Wave frequency spectra (0.025 ∼ 0.58Hz)</td>
<td>$1 \times 64$</td>
</tr>
<tr>
<td>Mean wave direction (direction at each frequency bin)</td>
<td>$1 \times 64$</td>
</tr>
</tbody>
</table>

Table 5.1: Example of wave parameters provided by waverider at the Norwegian Sea

In-situ wave measurements were provided by a waverider buoy, which was deployed nearly 8 km offshore as shown in Figure 5.1. The buoy acquired wave data every half hour. The buoy grid location within radar coverage is shown as a star. The in-situ buoy measurement is used to verify the wave inversion from radar data. One example of wave measurements is given in Table 5.1. The wave power density spectrum and mean wave direction at each frequency bin are shown in Figure 5.2.

Unfortunately, there was no in-situ wind measurements at the sea surface, but there was an anemometer installed on a lighthouse, located close to the Fedje radar site. Figure 5.3 shows the observed wind speed and direction at the lighthouse.
5.1. Radar experiments and in-situ measurements

(a) Wave spectral density  
(b) Mean wave direction

Figure 5.2: waverider measurements during the Fedje experiment

5.1.2 Ligurian Sea experiment

The Ligurian Sea experiment was used to investigate HF radar wind inversion in Italy from April to September 2009. The radars were placed at Palmaria and Rossore within a distance of 50 km from each other. The interface of radar data processing is shown in Figure 5.4. A meteorological buoy and a waverider buoy were deployed between radar sites about 30 km offshore ([43.876N, 9.873E]). The meteorological buoy provided in-situ wind direction and speed measurements at sea surface and the waverider buoy provided in-situ...
wave power density and wave direction measurements. As shown in Figure 5.4, the two buoys are deployed closer to the Palmaria radar site. The maximum range of the radar coverage can reach 120 km, which is much farther than that during the Fedje experiment due to a lower radar frequency.

![Figure 5.4: Interface of radar data analysis during the Ligurian Sea experiment](image)

During the Ligurian Sea experiment, the waverider buoy collected wave data for a period of 30 days on a half an hour interval, as shown in Figure 5.5. Compared to the wave conditions during the Fedje experiment, the wave power density and the peak frequencies of the spectra are quite different. The wave amplitude during Ligurian Sea experiment is relatively lower and the peak frequencies are relatively higher. The meteorological buoy provided wind speed and direction information for 68 days, acquiring wind data every 10 minutes. The time series of the wind speed and direction are shown in Figure 5.6.

![Figure 5.5: Wave measurements by waverider during the Ligurian sea experiment](image)

(a) Wave power density spectra (waverider)  
(b) Mean wave direction (waverider)
5.1.3 Wind and resonant waves

During the two experiments, the radar operating frequencies were different as well as the corresponding wavelength of Bragg resonant waves, so the responses of the Bragg resonant waves to the wind acting on the sea surface are also different.

5.1.3.1 Statistics of the wind speed during two experiments

The statistics of the wind speed are shown in Figure 5.7. The wind speed at the Norwegian Sea is higher than that at the Ligurian Sea: 67.4% of the wind records are higher than 5 m/s. During the Ligurian Sea experiment, only 18.9% of the wind records exceed 5 m/s.

![Figure 5.6: Wind speed and direction measured by the anemometer at the Ligurian Sea](image)

![Figure 5.7: Statistics of wind speed measurements by the anemometer](image)
5.1.3.2 Integrated Bragg resonant wave energy vs. wind speed

The waverider buoy measures the integrated wave power spectra over all directions, so the wave power at a certain frequency can be extracted from it. Considering the two radar frequencies (12 ~ 13 MHz and 27.68 MHz), the corresponding Bragg wave frequencies are given. In the wave spectrum measured by the waverider, the wave frequencies which are close to the Bragg wave frequencies are selected for analysis. Figure 5.8 shows the power of Bragg waves increases with the wind speed at two frequencies. Comparing these results with the WAM data (Figure 2.7), the scatter plots are less clustered, because the number of samples is not as many as that in the WAM data, and during the Ligurian sea experiment, the wind speed is quite low.

![Wave power density at 0.54Hz Vs. Wind speed (Fedje 2000)](image)

(a) $f_{\text{Bragg}} = 0.54\text{Hz} \ (f_{\text{radar}} = 28.01\text{MHz, Fedje})$

![Wave power density at 0.36Hz Vs. Wind speed (Ligurian, 2009)](image)

(b) $f_{\text{Bragg}} = 0.36\text{Hz} \ (f_{\text{radar}} = 12.5\text{MHz, Ligurian Sea})$

Figure 5.8: Wave energy at certain frequencies (two Bragg wave frequencies corresponding to the two radar frequencies) vs. wind speed (anemometer measurement) during the two experiments

5.1.3.3 Wind direction and mean wave direction at Bragg frequencies

The waverider gives the mean wave direction measurement at each frequency bin. Regarding the two radar frequencies, the comparison of the mean wave direction at corresponding Bragg frequencies and the anemometer measured wind direction are presented in Figure 5.9. As seen in these figures, the wind direction and the mean direction of Bragg resonant waves are not always identical, especially when radar operates at a lower frequency (longer Bragg resonant waves). Besides that, the wind speed is also an important factor. During the Fedje experiment, the wind speed is high and the mean wave direction agrees with the wind direction well, but during the Ligurian Sea experiment, the wind speed is low and the wind direction measurement is unreliable.
5.2 Wind inversion from radar first-order peaks using new pattern fitting method

Several conventional methods for wind direction inversion have been described in Chapter 3. Because the wind direction inversion from one radar is ambiguous, two radars are used during the Fedje and Ligurian Sea experiments. In this work, the new pattern fitting method and the hyperbolic secant function \( 0.5 \beta \text{sech}^2(\beta \cdot \theta) \) are used for the wind direction inversion, which give a unique solution for wind direction \( \theta \) and the directional spreading parameter \( \beta \) of the Bragg resonant waves.

5.2.1 Wind direction inversion during the Fedje experiment

During the Fedje experiment, the scatter plot of wind direction is shown in Figure 5.10a\(^1\). Besides the wind direction measurement given by the pattern fitting method, the Least Square Minimum (LSM) method is also tested with the directional function \( \cos^2(\theta/2) \) (\( s = 1 \)). The result is given in Figure 5.10b. The pattern fitting method improves the direction measurement (not significantly), because the spreading parameter is not predefined to a constant value during the wind direction inversion. In reality, the spreading parameter varies with the wind conditions at the sea surface. Many researchers simplify the spreading parameter (\( s \) or \( \beta \)) to a constant value, because the spreading parameter is difficult to calculate. In the forward method (\( i.e. \), wave model), the wind speed is a prior known

---

\(^1\)Here, “RMSE” is the RMS (Root Mean Square) Error of wind direction measurement (\( U > 3 \) m/s, a criteria for wind direction measurement). During the RMS error calculation, if the angular difference between radar measurement and the anemometer data is larger than 180°, the system will automatically reprocess the data and make sure the difference is less than 180° (by adding 360° to the smaller value).
parameter, which determines the wave peak frequency \( f_p \) and wave age \( c_p/U_{10} \), which are used to calculate the spreading. But in the wind inversion from the remote sensing data, the wind speed is unknown. So in this work, the spreading parameter \( \beta \) has been determined using the pattern fitting method.

![Scatter plot of wind direction (RMSE = 23.2°, U>3m/s)](image1)
(a) Pattern fitting method

![Scatter plot of wind direction (RMSE = 26.6°, U>3m/s)](image2)
(b) LSM method (data provided by Gurgel)

Figure 5.10: Comparison of wind direction measured by radar using pattern fitting method and LSM method during the Fedje experiment

The error of the wind direction measurement also depends on the wind speed and the radar-derived wind direction is the mean direction of Bragg resonant waves. At a high wind speed, the mean wave direction agrees well with the wind direction. At a low wind speed, especially when the wind speed is near to zero, the wind direction measurement is meaningless. The RMS error of the wind direction measurements using the pattern fitting and LSM method is presented in Table 5.2, which illustrates the error analysis for the two methods at different wind conditions.

<table>
<thead>
<tr>
<th>Comparison of inversion methods</th>
<th>Wind speed (m/s)</th>
<th>Different wind speed range (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( U &gt; 3 )</td>
</tr>
<tr>
<td>Pattern fitting method</td>
<td>23.2</td>
<td>72.5</td>
</tr>
<tr>
<td>LSM method</td>
<td>26.6</td>
<td>75.9</td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of the RMS Error of wind direction related to wind speeds using the pattern fitting method and the conventional LSM method during the Fedje experiment

In this dissertation, the pattern fitting method is proposed not only to invert the wind direction, but also provides the directional spreading parameter of the Bragg resonant waves. The spreading parameter increases with the wind speed, as shown in Figure 5.11. It indicates that the resonant waves are more directive at a higher wind speed (the value \( \beta \) is larger).
5.2. Wind inversion from radar first-order peaks using new pattern fitting method

Figure 5.11: Spreading parameter $\beta$ vs. wind speed using the pattern fitting method ($\text{sech}^2(\beta \cdot \theta)$) during the Fedje experiment

Regarding to the other locations within radar coverage, the pattern fitting method is also used and the lower limit for SNR of the first-order peak is set to 3 dB. One example of wind direction map during the Fedje experiment is shown in Figure 5.12.

Figure 5.12: Wind direction map using pattern fitting method during the Fedje experiment (radar measurement: wind direction at the buoy location is 331°)
5.2.2 Wind direction inversion during the Ligurian experiment

During the Ligurian Sea experiment, the radar frequency is \(12 \sim 13\) MHz, the wavelength of Bragg waves is 12 m (when the radar operates at 12.5 MHz) and the Bragg wave frequency is 0.3607 Hz. The wavelength of the Bragg waves is nearly twice of that during the Fedje experiment. For longer Bragg waves, the directional spreading pattern might not be as regular as that of the short waves, and the wave response (time and amplitude) to the changes of the wind is different as well. The scatter plot of wind direction measurements using the pattern fitting method is shown in Figure 5.13a\(^2\).

![Scatter Plot of Wind Direction](image)

(a) Radar vs. in-situ wind measurement

![Scatter Plot of Mean Wave Direction](image)

(b) Radar vs. in-situ mean wave direction

Figure 5.13: Scatter plots of wind (mean wave) direction during the Ligurian Sea experiment

From the in-situ wind and wave direction measurements (Figure 5.9), we know that the wind direction and the mean wave direction of the Bragg waves does not agree well. So another comparison between the radar measurement and the mean wave direction by the waverider is shown in Figure 5.13b. From which, it is clear that the radar measurement has a better agreement to the mean wave direction given by the waverider buoy, that is because the radar could not measure the wind direction directly, so the mean wave direction at Bragg frequency is used as an approximation for the wind direction.

The RMS error analysis is presented in Table 5.3. As seen in the table, the radar measured wind direction is in a better agreement with the mean wave direction at the Bragg frequency and at a higher wind speed, the wind direction measurement is more reliable. The RMS error of the wind direction is larger than that during the Fedje experiment. The reason is: In the wind direction inversion from HF radar backscatter, there is a hypothesis that the wind direction is identical to the short wave direction. As shown in Figure 5.9, the mean

\(^2\)Although the radar and meteorological buoy measure the wind every 10 minutes, the waverider measure the mean wave direction every 30 minutes. In order to consistent with the waverider buoy measurements, for the wind direction measurement by the radar and meteorological buoy, the results are averaged to one observation every 30 minutes.
wave direction at the frequency of 0.54 Hz is in a good agreement with the wind direction, however, at a lower Bragg frequency (0.36 Hz), it gives a larger deviation. Besides that, the wind speed is low at the Ligurian Sea, which makes the wind direction measurement not so reliable. So the result of wind direction during the Ligurian Sea experiment is more scattered. Another possible reason is, that the wave directional spreading pattern at a lower frequency is not as regular as that of the waves at a higher frequency. The wind direction inversion using the hyperbolic secant function is based on the mathematical function. The function is in a good agreement with the directional wave pattern at higher frequencies, but at the relatively lower wave frequencies, the directional spreading pattern might be disturbed by the possible swell or residual wave components due to the difference in the wave direction.

<table>
<thead>
<tr>
<th>Comparison of measurements</th>
<th>wind speed (m/s)</th>
<th>Different wind speed range (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar – wind direction (meteo buoy)</td>
<td>U &gt; 3</td>
<td>0 &lt; U ≤ 3</td>
</tr>
<tr>
<td>Radar – wave direction ( (f_B, \text{waverider}) )</td>
<td>57.2</td>
<td>80.3</td>
</tr>
</tbody>
</table>

Table 5.3: Comparison of the RMS Error of wind direction related to wind speeds for the radar and the in-situ meteorological buoy as well as the mean wave direction measurements during the Ligurian Sea experiment

The spreading parameter \( \beta \) vs. the wind speed is presented in Figure 5.14. Compared to the spreading parameter increases with the wind speed during the Fedje experiment \( (f_B = 0.54\text{Hz, Figure 5.11}) \), the increasing rate of \( \beta \) during the Ligurian Sea experiment \( (f_B = 0.36 \text{ Hz}) \) is higher. This is due to the difference between the two Bragg frequencies. At a lower wave frequency, the increasing rate is higher.

Figure 5.14: Spreading parameter \( \beta \) vs. wind speed using the pattern fitting method \( (\text{sech}^2(\beta \cdot \theta)) \) during the Ligurian Sea experiment
Chapter 5. Radar Experiments and Results of Inversion

One example of the wind direction map during the Ligurian sea experiment is presented in Figure 5.15, the lower limit for SNR of the first-order peak is 3 dB. Each radar covers a sector of 120°. For Palmaria site, the coverage is \( \phi \in [150° \sim 270°] \), for Rossore site, the coverage is \( \phi \in [222.4° \sim 342.4°] \). The wind direction is calculated in the overlapped covering area. At the buoy location, the wind direction derived from radar first-order peaks using the pattern fitting method is 119°, and at this moment, the in-situ wind direction measurement given by the meteorological buoy is 128°.

Figure 5.15: Wind direction map derived from the first-order backscatter using pattern fitting method at the Ligurian sea (radar measurement: wind direction at the buoy location is 119°)

5.3 Wind speed inversion from radar second-order using the conventional method

Besides the radar first-order peaks, the second-order sidebands also give the information on the ocean wave spectra. From the radar-deduced wave spectra, the wind speed could be derived from the wave parameters (i.e., significant wave height and wave peak frequency) using the SMB method. During the Fedje experiment in the EuroROSE (European Radar Ocean SEnsing) project, Gurgel proposed a regression method to calculate the significant wave height from the second-order sidebands.
5.3.1 SNR of the second-order sidebands

As introduced before, the SNR of the second-order sidebands depends on the wind conditions and the radar frequency. The radar works at a higher frequency during the Fedje experiment and the wind speed is high. As shown in Figure 5.16, we take the SNR of the second-order sideband on the left side of negative Bragg peak during the two experiments as an example. During the Fedje experiment, most of the second-order sidebands are with a SNR higher than 5 dB. But during the Ligurian Sea experiment, the radar operates at a lower frequency and the wind speed during the experiment is quite low. So the SNR of second-order sidebands is also low. In Figure 5.16c and d, the SNR of the second-order sidebands is much lower (note that the illustrated examples are the SNR of one sideband in the two pairs of sidebands). So during the Ligurian Sea experiment, only the second-order sidebands with the SNR above 5 dB are used for the wind and wave inversion.

Figure 5.16: SNR of the second-order left sideband around the negative Bragg peak during the Fedje and Ligurian Sea experiment
5.3.2 Wind speed inversion from radar second-order spectra

If the wind speed is inverted from the wave parameters (significant wave height and wave peak frequency), which are derived from radar second-order sidebands, it will be overestimated in case of the presence of possible swell or residual wave components. Figure 5.17 presents the scatter plots of the wind speed between radar data and anemometer measurement during the Fedje and Ligurian Sea experiment respectively. The regression coefficient is derived based on the wave height measurement during the EuroROSE project, but the measured waves are not pure wind-waves, so the results don’t agree with the anemometer measurement very well.

![Scatter plot of wind speed derived from radar second-order sidebands](image)

(a) Wind speed at the Norwegian Sea

(b) Wind speed at the Ligurian Sea

Figure 5.17: Scatter plots of wind speed derived from radar second-order sidebands using the SMB method and anemometer measurements during the two experiments

The wind speed measurement based on the second-order method covers an area which is much smaller than that given by the first-order method. The wind (speed and direction) map during the Fedje and Ligurian Sea experiments are shown in Figure 5.18 and 5.19 respectively. The wind direction is derived using the pattern fitting method (hyperbolic secant squared function). The wind speed is derived from the second-order sidebands using the conventional method (regression method and SMB method). As shown in the figures, the wind speed coverage based on the second-order method is smaller than the wind direction map, and the wind speed during the two experiments is overestimated. For example, during the Fedje experiment in Figure 5.18, at the buoy location, the inverted wind speed is 9.03 m/s, while the wind speed measured by the anemometer is 7.2 m/s. During the Ligurian Sea experiment in Figure 5.19, at the buoy location, the wave speed derived from the radar data is 6.08 m/s, while the wind speed measured by the meteorological buoy is 4.9 m/s.
5.3. Wind inversion from second-order sidebands using conventional methods

Figure 5.18: Wind speed (from second-order sidebands using SMB method) and direction (from the first-order peaks) map during the Fedje experiment (radar measurement at buoy location: wind speed = 9.03 m/s, direction = 331°)

Figure 5.19: Wind speed (from second-order sidebands using SMB method) and direction (from the first-order peaks) map during the Ligurian experiment (radar measurement at buoy location: wind speed = 6.08 m/s, direction = 119°)
5.4 Wind inversion from radar first-order peaks using neural networks

Although the wind-wave directional spreading functions such as half-cosine 2s-power and hyperbolic secant squared function have been widely used for wind direction inversion, the investigation of wave spreading function is still under development. Some further researches are conducted to find a better description of directional distribution of ocean waves. For example, in finite-depth water or fetch-limited condition, the wave directional pattern is even more complicated [120]. The conventional methods based on half-cosine type or hyperbolic secant type could give a good agreement when the radar operates at a higher frequency (shorter Bragg resonant waves). But if the radar works at a lower frequency and the corresponding Bragg waves are with a longer wavelength and the directional spreading pattern is not as regular as that of the shorter waves. So in this case, the irregular directional spreading pattern brings some uncertainty for the wind inversion.

The neural network doesn’t need to know the functional form of the directional spreading pattern. With the pairs of the input and target data, after the training, the network will automatically generate a “function” for wind inversion. The sketch of wind inversion from radar first-order peaks has been introduced in Figure 5.12. During the two experiments, the radar data at the buoy location is intensively investigated. At this point, the radio beam directions $\phi_1$ and $\phi_2$ are constant values, which are the independent variables in the wind inversion, so they are not used as the input data. Only the two pairs of the first-order peak power (two radar sites) are used as the input data. Wind speed and direction measured by an anemometer are used as the target data set. The wind direction is the angular degree ($0^\circ \sim 360^\circ$), so in the network, the sinusoidal component (y-component) and cosine component (x-component) are used instead of one single value of the angular degree, so the three elements $(U, \cos \theta, \sin \theta)$ are used as target. The structure of data set for wind inversion is shown in Figure 5.20, in which, $\sigma_1(\pm f_B)$ and $\sigma'_1(\pm f_B)$ are the two pairs of the first-order peak power at two radar sites. The network output data is the wind speed $U'$ and the direction $(\cos \theta', \sin \theta')$. Finally, the wind direction can be calculated from the output of neural network:

$$\theta' = \text{atan2}(\sin \theta', \cos \theta')$$

The specification of a three-layer neural network is given in Table 5.4. Error back-propagation principle is implemented in the network. The number of input and output neurons depends on the variables of the system, and the number of neurons in hidden layer is based on some rules as given in Section 4.3.1. Some network parameter definitions are also given in the table.
5.4. Wind inversion from radar first-order peaks using neural networks

Figure 5.20: Structure of data set for wind inversion from radar first-order backscatter

<table>
<thead>
<tr>
<th>Network type</th>
<th>BP network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hidden layers</td>
<td>1</td>
</tr>
<tr>
<td>Number of Neuron of each layer</td>
<td>4-12-3</td>
</tr>
<tr>
<td>Net.trainParam.epoch (Maximum number of epochs to train)</td>
<td>1000</td>
</tr>
<tr>
<td>Net.trainParam.goal (Performance goal)</td>
<td>5e-3</td>
</tr>
<tr>
<td>Net.trainParam.lr (Learning rate)</td>
<td>0.1</td>
</tr>
<tr>
<td>Net.trainParam.max_fail (maximum failure number for validation)</td>
<td>20</td>
</tr>
<tr>
<td>Net.trainParam.mu (Marquardt adjustment parameter)</td>
<td>0.05</td>
</tr>
<tr>
<td>transfer function</td>
<td>tansig (2nd), purelin (3rd)</td>
</tr>
<tr>
<td>train algorithm</td>
<td>Levenberg-Marquardt</td>
</tr>
</tbody>
</table>

Table 5.4: Specification of neural network for wind inversion from the first-order backscatter

5.4.1 Wind inversion during the Fedje experiment

After the training, validation and testing, the scatter plots of the wind direction and speed during the Fedje experiment are shown in Figure 5.21. Compared to the pattern fitting method and LSM method given in Figure 5.10, the neural network gives a better result. Besides the wind direction, the wind speed could also be inverted from the radar first-order backscatter, the result is presented in Figure 5.21b. In WAM model data analysis, the integrated wave energy increases with the wind speed at certain wave frequencies, which gives a one-dimensional description of wave growth. In Section 4.4.2, not only the integrated wave energy, but also the wave directional spreading pattern is considered, the two-dimensional information of the Bragg waves are used to invert wind speed, which contains more information and makes the wind speed inversion more accurate. Besides that, the threshold for Bragg wave saturation is higher if both amplitude and direction information are used for the wind speed inversion. For example, the wind speed inverted from the two-dimensional WAM data (Figure 4.11a) and radar first-order peaks (Figure 5.21b) shows a higher saturation value than if only the integrated Bragg wave energy is used (Figure 2.7c), which means, it will give a wider range for wind speed inversion if both Bragg wave amplitude
and directional spreading information are used. In Figure 5.21b, at a higher wind speed, more results of wind speed are located below the line $y = x$. That means if the wind speed is increasing beyond the saturation limit, the Bragg waves will come to a state of saturation (including integrated wave energy and the wave directional spreading pattern). In this case, the wind speed could no longer be inverted from radar first-order backscatter.

![Scatter plot of wind direction](image1)

![Scatter plot of wind speed](image2)

Figure 5.21: Wind direction and speed derived from radar first-order backscatter using neural network and the anemometer wind measurement during the Fedje experiment

![Comparison of wind speed](image3)

Figure 5.22: Comparison of wind speed derived from the first-order backscatter using neural network and the anemometer measurement during the Fedje experiment

The comparison of wind speed is demonstrated in Figure 5.22. The wind speed derived from radar first-order peaks using neural network can cover the low wind speed measurement. For example, from the data record 800 to 900, the radar measurements agree with the anemometer measurements very well, which is an advantage of the method using first-order peak power, because the Bragg resonant waves are sensitive to the change of the
wind speed. But for wind speed inversion from radar second-order sidebands, there is a disadvantage, because the sea surface could not calm down immediately when the wind becomes weak, the residual waves still remain. As a result, the wind inverted from the waves (significant wave height and peak wave frequency) is overestimated.

The network randomly divides the input and target vectors into three sets: 60% are used for training; 20% are used to validate that the network is generalizing and to stop training before over-fitting; The last 20% are used as a completely independent test of network generalization. The correlation coefficients of the training, validation and testing are given in Figure 5.23 as well as the correlation coefficient of the total data set. The illustrated example in Section 4.3.2 is the result of the network used in this application (Figure 4.6). The network output tracks the targets very well for training, validation and testing, the CC-value (Correlation Coefficient) is higher than 0.85 for the total data set.

![Figure 5.23: Correlation coefficients of training, validation and testing data for wind speed inversion from radar first-order peaks during the Fedje experiment](image)

During the training of the network, the network automatically verifies its performances. For the application of the network, the trained network is used independently (only the new input data is used). So the capability of the generalization is an important issue. In order to verify the trained network, we manually select some data for testing. In the anemometer measurement, we have 929 valid wind records. The first 200 wind records are selected for manual testing and the last 729 data for the network training, validation and testing. The 729 data wind records consist of high and low wind speeds, which are sufficient and variable.
for the network training. Figure 5.24 shows the time series of the wind record.

![Figure 5.24: Records for manual testing and the network training](image)

The scatter plot of the network output and the anemometer measurement for the 729 samples is given in Figure 5.25a, which gives a correlation coefficient of 0.8484. After the training, the network net\(_1\) is generated and saved. We use the first 200 samples as the input data for the network net\(_1\), they are the new input data. The scatter plot of the network output and anemometer measurement for the manual selected testing data is given in Figure 5.25b, the correlation coefficient value is 0.8366, which is acceptable for wind speed inversion.

![Figure 5.25: Scatter plots of the wind speed, (a) is the result of network training, (b) is the result using the trained network for the new data set](image)

Figure 5.25: Scatter plots of the wind speed, (a) is the result of network training, (b) is the result using the trained network for the new data set.
5.4.2 Wind inversion during the Ligurian Sea experiment

The wind speed during the Ligurian Sea experiment is quite low. More than 85% of wind records are lower than 5 m/s. The results of wind direction and speed inversion during the Ligurian Sea experiment are shown in Figure 5.26\(^3\). Although the RMSE of the wind direction inversion is 49.8\(^\circ\), it gives a better result than the conventional method (Figure 5.13a). In the wind direction inversion, neither the pattern fitting method nor the neural network method could give a result as good as that during the Fedje experiment. This is because the wind speed is quite low, which makes the wind direction measurement unreliable. Another reason is that the directional distribution of the Bragg resonant waves is irregular due to the longer wavelength (double of that during the Fedje experiment). The neural network can tackle the complexity of the Bragg wave directional distribution, but if the directional distribution is very irregular (or unstable) due to the influence of the swell or other effects, the network could not find the inversion “function” easily. If the relationship of the nonlinear mapping is stable, although it is complicated, the neural network still can accomplish the inversion successfully. But if the relationship is unstable (the swell or residual wave direction is uncertain), the result of wind direction at the Ligurian Sea is not as good as that at the Norwegian Sea.

The wind speed is also derived from radar first-order backscatter based on the neural network method. The scatter plot of wind speed measurement is presented in Figure 5.26b. From which, it is obvious to find that, most of wind records are clustered at the low wind speed region, and the result using neural network is much better than that using SMB method given in Figure 5.17b.

![Scatter plots of wind direction and speed derived from radar first-order backscatter (neural network output) and in-situ wind at the Ligurian Sea](image)

Figure 5.26: Scatter plots of wind direction and speed derived from radar first-order backscatter (neural network output) and in-situ wind at the Ligurian Sea

\(^3\)In order to compare to the conventional method in Figure 5.13a, the result of wind direction is averaged into one observation every 30 minutes. The wind speed measurement for radar and meteorological buoy is every 10 minutes, so this result presents the original value of the wind speed (every 10 minutes).
5.4.3 Extension the wind measurements to the other locations within radar coverage using neural network

The conventional solution for wind speed inversion is based on the second-order method, but the SNR of the second-order sidebands is much lower than that of the first-order peaks. Especially when the radar operates at lower frequencies or the wind at sea surface is quite low, the wind speed measurement by the first-order method could give a larger coverage within radar coverage than the second-order method.

The power of radar first-order (positive and negative) peaks is related to the resonant wave height, the roughness of the sea surface and the angle between radio beam and wind direction. As introduced in Section 3.1.2.1, the roughness of the sea surface (due to surface wind speed) brings some additional attenuation for the dual pass loss (radiation and reflection). Although the neural network can track all these variations of Bragg peak power and wind speed at the buoy location with sufficient large of samples, for the other locations on radar map, we have no in-situ measurements to train the network. The radar first-order peak power is related to the angle between the radio beam and the wind direction. In these cases, the spreading parameter $\beta$ of the Bragg resonant waves is used to calculate wind speed within radar coverage. The spreading value $\beta$ vs. wind speed during two experiments are presented in Figure 5.11 and 5.14. Both indicate that the spreading parameter $\beta$ at the corresponding Bragg frequencies increases with the wind speed before the saturation. Besides the spreading parameter $\beta$, the mean direction of Bragg waves derived by the pattern fitting method is also used, because it is also important for the directional pattern of the resonant waves. For example, in Section 4.5, the wind direction at the three grid points are used for calculating the azimuth compensation factors. The directional distribution of the Bragg resonant waves is described by the spreading value and the wind direction.

$\beta \cos \theta' \sin \theta$

Figure 5.27: Sketch of wind speed inversion from the direction spreading information of Bragg waves
In the network training, the Bragg wave directional spreading parameter and its mean wave direction at the buoy location are used as the input data set \((\beta, \sin \theta', \cos \theta')\). The wind measured by the anemometer is used as the target data \((U, \sin \theta, \cos \theta)\). The network is trained along the time series. After training, the trained network is extended to the other locations. The spreading \(\beta\) and wind direction \(\theta'\) at other grids points are used as the new input data for the trained neural network. After the processing, the wind speed map is presented. This is called the “spatial extension” for the application of the neural network.

The specification of a three-layer neural network is given in Table 5.5. Error back-propagation principle is implemented in the network. The number of input and output neurons depends on the variables of the system.

<table>
<thead>
<tr>
<th>Network type</th>
<th>BP network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hidden layers</td>
<td>1</td>
</tr>
<tr>
<td>Number of Neuron of each layer</td>
<td>3-10-3</td>
</tr>
<tr>
<td>Net.trainParam.epoch</td>
<td>1000</td>
</tr>
<tr>
<td>Net.trainParam.goal</td>
<td>5e-3</td>
</tr>
<tr>
<td>Net.trainParam.lr</td>
<td>0.1</td>
</tr>
<tr>
<td>Net.trainParam.max_fail</td>
<td>20</td>
</tr>
<tr>
<td>Net.trainParam.mu</td>
<td>0.05</td>
</tr>
<tr>
<td>transfer function</td>
<td>tansig (2nd), purelin (3rd)</td>
</tr>
<tr>
<td>train algorithm</td>
<td>Levenberg-Marquardt</td>
</tr>
</tbody>
</table>

Table 5.5: Specification of neural network for wind inversion from the spreading information of Bragg waves

Figure 5.28: Scatter plots of the wind speed derived for spreading parameter using neural network and the wind speed measured by anemometer during the Fedje and Ligurian Sea experiments

(a) Fedje experiment \((f_B = 0.54764 \text{ Hz})\)  
(b) Ligurian Sea experiment \((f_B = 0.3764 \text{ Hz})\)
Figure 5.28 shows the scatter plots of the wind speed inverted from the spreading parameter and mean wave direction at the Bragg wave frequency. In Figure 5.28a, there is a tendency of the saturation, which means that at the Bragg wave frequency, when the wind speed increases up to some level, the directional spreading pattern of the Bragg waves comes to a state of saturation. In this case, the wind speed could no longer be inverted from the directional spreading of Bragg waves. Unlike the wind speed inversion from the first-order peak power presented in Figure 5.21b, for the other locations within radar coverage, only the wave directional information including spreading and mean wave direction at Bragg frequency are used for the wind speed inversion instead of both the Bragg wave amplitude and the directional spreading information, the directional spreading information contains less information than both wave amplitude and directional spreading for wind speed inversion. So the threshold for the saturation of Bragg wave directional spreading might be lower. For example, during the development of Bragg waves, the response of directional spreading at a certain frequency might be different from that of wave amplitude. The saturation of Bragg waves for wind speed inversion from their directional spreading information appears in Figure 5.28a.

During the Ligurian Sea experiment, the Bragg wave frequency is lower and the waves need higher wind speed to get saturated, but the wind speed during the experiment is quite low. Therefore, the threshold for the saturation is higher than that during the Fedje experiment, and due to few high wind records, the tendency of saturation is not obvious in Figure 5.28b. If the wind speed exceeds the threshold of the saturation, the wind speed could no longer be derived from the directional spreading information of Bragg waves. So this method is only valid before the saturation of directional spreading of Bragg waves.

An example of a wind map (speed and direction) during the Fedje experiment is shown in Figure 5.29. Because the wind speed is derived from radar first-order backscatter using the neural network method, it gives a larger coverage than that derived from the radar second-order backscatter. The wind speed measurement covers an area as large as the wind direction map (the threshold for the SNR of the first-order peaks is 3 dB). The wind speed and direction map during the Ligurian sea experiment is shown in Figure 5.30. In both figures, the anemometer measured wind speed and direction are given as well. For example, during the Fedje experiment, the wind speed derived from radar data at the wave buoy location is 7.05 m/s and the anemometer measured wind speed is 7.2 m/s. During the Ligurian Sea experiment, the wind speed derived from radar data at the meteorological buoy is 5.1 m/s and the anemometer measured wind speed is 4.9 m/s. Because at the Bragg wave frequency, the resonant waves are not easily affected by the possible swell and residual wave components, the results of the wind speed are much better than that calculated from the radar second-order sidebands using the conventional method (as shown in Figure 5.18 and 5.19).
5.4. Wind inversion from radar first-order peaks using neural networks

Figure 5.29: Wind map (direction and speed) derived from first-order peaks using neural network at the Norwegian Sea (at buoy location: wind speed = 7.05 m/s, direction = 331°)

Figure 5.30: Wind map (direction and speed) derived from first-order peaks using neural network at the Ligurian Sea (at buoy location: wind speed = 5.1 m/s, direction = 119°)
5.5 Wind speed inversion from HF radar second-order backscatter using neural network

The wind speed inversion from the radar second-order spectra using the conventional method has been presented in Section 5.3. The wave inversion from the second-order spectrum contains the possible swell or residual wave components, which overestimates the wind speed measurement. Here, a neural network method is introduced, which has the advantage to tackle the issue of the complex non-linear mapping and suppress the possible swell or residual wave components if the wave directions at lower frequencies have a large deviation to that at higher frequencies. The network is trained at the buoy location and two pairs of the second-order sidebands (four sidebands from two radar sites, the pair of sidebands with higher SNR is selected) are used as the input. The target data is the wind speed $U$ given by the anemometer. All data sets are shown in Figure 5.31, in which, $\sigma_i(\pm f_{Bi})$ and $\sigma'_i(\pm f_{Bi})$ ($i = 1, 2, \ldots, 21$) are the selected pair of second-order sidebands of the two radar sites respectively. The sketch of wind speed inversion from the second-order sidebands has been introduced in Figure 4.17 and the specification of the network is given in Table 5.6.

![Neural Network Diagram](image)

Figure 5.31: Structure of data set for wind speed inversion from radar second-order sidebands

5.5.1 Wind speed inversion from second-order sidebands during the Fedje experiment

During the Fedje experiment, the results of the wind speed inversion are shown in Figure 5.32. As presented in the wind speed comparison (Figure 5.32b), the wind speed derived from the second-order radar backscatter could not track the weak wind (data record 590~610). That is because at the low wind conditions, the wind speed decreases almost
5.5. Wind speed inversion from HF radar second-order backscatter using neural network

<table>
<thead>
<tr>
<th>Network type</th>
<th>BP network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hidden layers</td>
<td>2</td>
</tr>
<tr>
<td>Number of Neuron of each layer</td>
<td>21-16-12-1</td>
</tr>
<tr>
<td>Net.trainParam.epoch</td>
<td>1000</td>
</tr>
<tr>
<td>Net.trainParam.goal</td>
<td>5e-3</td>
</tr>
<tr>
<td>Net.trainParam.lr</td>
<td>0.1</td>
</tr>
<tr>
<td>Net.trainParam.max_fail</td>
<td>20</td>
</tr>
<tr>
<td>Net.trainParam.mu</td>
<td>0.05</td>
</tr>
<tr>
<td>transfer function</td>
<td>tansig(2nd), tansig (3rd), purelin (4th)</td>
</tr>
<tr>
<td>train algorithm</td>
<td>Levenberg-Marquardt</td>
</tr>
</tbody>
</table>

Table 5.6: Specification of neural network for wind speed inversion from the second-order backscatter (at the buoy location)

to zero, but the wave energy still remains, which overestimates the wind speed. But if the wind speed is inverted from the first-order backscatter, which can track the weak wind conditions, that is because the short waves (Bragg resonant waves) are sensitive to the change of the local wind conditions.

Figure 5.32: Comparison of the wind speed inversion from radar second-order backscatter using neural network during the Fedje experiment

5.5.2 Wind speed inversion from second-order sidebands during the Ligurian Sea experiment

The SNR of radar second-order sidebands depends on the radar frequency and wind conditions. During the Ligurian Sea experiment, the wind speed is quite low as well as the radar working frequency, so the SNR of the second-order sidebands is also low. In this case, for the wind speed inversion from the second-order sidebands, only the second-order sidebands with a SNR higher than 5 dB are used, and the comparison of wind speed given by the
Chapter 5. Radar Experiments and Results of Inversion

radar and anemometer measurements are shown in Figure 5.33.

Figure 5.33: Comparison of the wind speed inversion from radar second-order backscatter using neural network during the Ligurian Sea experiment

Because the wind speed is inverted from the radar second-order sidebands, at the low wind conditions, the wind speed is overestimated. As shown in Figure 5.33b, when wind speed is lower than 2 m/s, the wind speed derived from radar second-order sidebands is relatively higher than the in-situ wind data. At high wind speeds, the method could present a good agreement.

5.5.3 Discussion of the wind speed inversion at the other locations within radar coverage using the second-order sidebands and NN

The extension of wind speed measurement from the radar first-order backscatter using a neural network has been successfully implemented to cover the whole radar map (cf. 5.4.3). But the extension of wind speed measurement to the other locations within radar coverage from the second-order sidebands is much more difficult, which is not applied in this work.

When second-order sidebands are used for wind speed inversion, they are normalized by the adjacent first-order peak power. The power ratio of the second-order sideband pair around the first-order peaks depends on the angle between the radio beam and wave directions. The azimuth compensation factor for the first-order peaks has been introduced in Section 4.5, in which the spreading parameter $\beta$ and mean direction of the Bragg resonant waves are used. For the waves at higher frequencies, the directional spreading pattern is more regular, the result of the directional spreading parameter of the Bragg resonant waves is calculated based on the hyperbolic secant function. This method is available on the premise that the directional spreading pattern can be approximately described using the
5.6. Wave inversion from radar second-order backscatter using neural network

The methodology of directional wave spectra inversion has been introduced in Section 4.4.5. In the inversion, the two pairs of second-order sidebands (two radars) are used as the input data set, the wave power density spectra and wave direction given by waverider buoy are used as the target data. The in-situ buoy measures the wave spectra at the frequency of \( f \in [0.025 \sim 0.58] \) Hz, but the radar second-order sideband only gives an range of \( f \in [0.05 \sim 0.25] \) Hz with the frequency step 0.01 Hz, so the in-situ wave measurements are processed to be consistent with the radar second-order sidebands (wave power spectra: \( S(f_n) \) and wave direction \( \theta_n(n = 1, 2, ..., 21) \)). The direction \( \theta_n \) is represented by \( \sin \theta_n \) and \( \cos \theta_n \). The input data is a matrix of \( 21 \times 4 \) and the target data is a matrix of \( 21 \times 3 \). All data sets are shown in Figure 5.34.

![Figure 5.34: Structure of data set for directional wave spectra inversion from radar second-order sidebands](image)

During the Fedje experiment, the SNR of the radar second-order sidebands is high,
and during the Ligurian Sea experiment, there is some strong radio interference located at the frequency range of the second-order sidebands. Some methods have been tried for the suppression of the interference, but still, the shape of the second-order spectra is still somehow polluted by the interference. So we just take the Fedje data as the example. The wave power density of the target data (buoy data) and the output of the neural network are given in Figure 5.35. As shown in the figure, some of the buoy measured wave spectra are with a higher amplitude than the neural network output. Barrick’s first-order and second-order scattering theory is available on the premise of three requirements as introduced in Section 3.1.2. During a storm within the Fedje experiment, the wave height was not small in terms of the radio wavelength and the surface slope was not small any more, so the preconditions of the scattering theory have been vivated in this case. Besides that, the second-order sidebands might increase to a value higher than the first-order peaks and make it difficult to normalize the second-order sidebands. As a result, the amplitude of second-order sidebands is underestimated in storm conditions. The comparison of wave direction is presented in Figure 5.36. At the lower frequencies, there are some sudden changes in the wave direction, which may due to the plotting in degree.

![Wave power density spectrum (in-situ buoy at Fedje, 2001)](image1)

![Wave power density spectrum (Neural network output at Fedje, 2001)](image2)

(a) In-situ wave power spectra measurement (b) Wave power spectra given by neural network

Figure 5.35: Comparison of wave power spectrum given by the in-situ buoy measurement and neural network during the Fedje experiment

The training performance could not be easily expressed in these three-dimensional data sets, so some example is given for the comparison. As seen in Figure 5.37, one buoy-measured spectrum and the output of the neural network are given. We can find that both the wave power spectrum and wave direction agree well with the in-situ measurements.

The RMS error for wave power density and wave direction is presented in Table 5.7. In the wave inversion, significant wave height $H_S$ is normally used to describe the strength of wave power. So the RMS error of the significant wave height is also given. Considering the wave power spectra shape, the wave power dominates around the wave peak frequencies. At the tail of the wave power spectra, the value of the power density is quite smaller than that at the peak of the spectra. So the frequency range is separated into three frequency
5.6. Wave inversion from radar second-order backscatter using neural network

(a) In-situ wave direction measurement
(b) Wave direction given by the neural network

Figure 5.36: Comparison of wave direction given by the in-situ buoy measurement and neural network during the Fedje experiment

(a) Comparison of wave power
(b) Comparison of wave direction

Figure 5.37: An example of wave comparison for the network output and buoy measurement

ranges: 0.08-0.14 Hz is the frequency range where the peaks of spectra are normally located; 0.15-0.20 Hz is the transition frequency range for the short waves growing into long waves; 0.21-0.25 Hz is the tail of the spectra. Besides the absolute value of the RMS error, the relative RMS error is also presented, which is the averaged error of the wave power density (or wave height) divided by the corresponding averaged wave power density (or wave height). At the frequency range 0.15-0.20 Hz, the RMS error of the wave power density and the wave direction is smaller than the RMS error at the other two frequency ranges, because at the lower frequency range, the wave power density is underestimated due to the strong wind conditions and the limitation of the first-order and second-order scattering theory. At the higher frequency range, the spectra amplitude is low, which might be smeared into the background noise. The wave direction is also presented in the table. In the frequency range 0.15-0.20 Hz, the direction measurement is relatively more accurate.
Table 5.7: RMS error for the wave power density and direction inversion during the Fedje experiment

<table>
<thead>
<tr>
<th>Frequency range (Hz)</th>
<th>wave height Hs (m)</th>
<th>power density (m² · s⁻¹)</th>
<th>wave direction (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08-0.25²</td>
<td>0.52</td>
<td>13.2%</td>
<td>0.93</td>
</tr>
<tr>
<td>0.08-0.14</td>
<td></td>
<td>2.01</td>
<td>59.9%</td>
</tr>
<tr>
<td>0.15-0.20</td>
<td></td>
<td>0.44</td>
<td>48.1%</td>
</tr>
<tr>
<td>0.21-0.25</td>
<td></td>
<td>0.21</td>
<td>61.2%</td>
</tr>
</tbody>
</table>

1. the percentage is the relative RMS error of the wave power density spectra 
2. at the lower boundary of the frequency range, the power density might be influenced by the broadening first-order peaks in the Doppler backscatter spectra, so the starting frequency is 0.08 Hz in the calculation

5.7 Summary

This chapter presents the two radar experiments and the results of wind and wave inversion. The wind direction inversion using the new pattern fitting method is presented and it gives a better result than traditional methods, and at a higher radar frequency (Fedje experiment), the result of wind direction is in better agreement with the in-situ measurement.

For the wind speed inversion from radar backscatter, based on the different mechanisms of the first-order and second-order scattering as well as the experimental conditions, three neural networks are implemented for the inversion:

- **The network using radar first-order backscatter as the input.** At the buoy location, the power of the two pairs of first-order peaks (two radars) is used as the input data and the in-situ wind data is used as the target data. The input data represents the four wave components along the two radio beams, which give the information of the wave directional distribution and strength of Bragg resonant waves. The results prove that the neural network is suited for the wind inversion from radar first-order backscatter. After training, if the buoy is moved away, the network could continue to process the radar first-order peaks and calculate the wind speed and direction. This is called “time extension” for the application of the network.

- **The network using the spreading parameter of resonant waves as the input.** The power of the first-order peaks is radio beam direction and radar range dependent. If we want to invert the wind speed within radar coverage, the training data set at the buoy location must be radio beam and radar range independent. In this case, the Bragg wave directional information (spreading parameter and mean wave direction) derived by the pattern fitting method and the in-situ wind measurements are used for the training. With the network trained at the buoy location, the wind speed map is covered by extending the network to the other locations. This is called “spatial extension” for the application of the network. Besides that, the wind speed map covers the same area as the wind direction map, because both are derived from radar
first-order backscatter. So the wind speed map using neural network covers a larger area than that derived from the second-order method using the conventional method.

- **The network using the second-order backscatter as the input.** In the former two neural networks, the signature of the Bragg resonant waves is used for the wind speed inversion, which is valid before the Bragg waves get saturated. If the Bragg waves are saturated, the wind speed could not be derived from the first-order peaks any more. In this case, the wind speed has to be inverted from the second-order backscatter. The two pairs of second-order sidebands at two radar sites are used as the input data, and the wind speed given by the meteorological buoy is used as the target data. After training, the network can process the second-order sidebands and derive the wind speed at the buoy location. This is also called “time extension” for the neural network.

In this chapter, several neural networks and conventional methods are used for the wind speed inversion. In order to compare the performance of these methods, the error analysis of the wind speed at different wind conditions during the two experiment are presented in Table 5.8.

<table>
<thead>
<tr>
<th>Experimental conditions</th>
<th>Wind speed $U$ (m/s)</th>
<th>Inversion methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar experiments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fedje Sea (27.68MHz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U &gt; 3$</td>
<td>1.85</td>
<td>2.45</td>
</tr>
<tr>
<td>$0.1 &lt; U \leq 3$</td>
<td>1.98</td>
<td>2.19</td>
</tr>
<tr>
<td>$3 &lt; U \leq 10$</td>
<td>1.70</td>
<td>2.06</td>
</tr>
<tr>
<td>$U &gt; 10$</td>
<td>2.27</td>
<td>3.34</td>
</tr>
<tr>
<td>Ligurian Sea (12-13MHz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U &gt; 3$</td>
<td>1.36</td>
<td>1.47</td>
</tr>
<tr>
<td>$0.1 &lt; U \leq 3$</td>
<td>1.13</td>
<td>1.29</td>
</tr>
<tr>
<td>$3 &lt; U \leq 10$</td>
<td>1.34</td>
<td>1.46</td>
</tr>
<tr>
<td>$U &gt; 10$</td>
<td>2.13</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Table 5.8: RMS error analysis for wind speed inversion using different methods

- NN method1: wind speed inversion from radar first-order peak power using neural network
- NN method2: wind speed inversion from the directional spreading information of the Bragg waves using neural network
- NN method3: wind speed inversion from radar second-order sidebands using neural network
- Conventional method: wind speed inversion from radar second-order sidebands using regression and SMB method
- For wind speed inversion from radar second-order backscatter during the Ligurian Sea experiment, only the spectra with the SNR higher than 5 dB are used for the inversion.
- There is an exception for wind speed higher than 10 m/s during the Ligurian Sea experiment, because there is only few high wind speed record, 1.67% of record is higher than 10 m/s, so in this case, it is not statistically significant

As shown in the table, the network using the power of the first-order peaks (NN method1) presents more accurate results than that using the directional spreading parameter (NN method2), because both the amplitude and directional distribution of Bragg resonant waves are used for the wind speed inversion. But the NN method1 is only valid for the wind speed measurement at the buoy location, while the NN method2 is designed for
the wind speed inversion within radar coverage. It covers the same area as the wind direction field map. In case of the saturation of the Bragg resonant waves, the wind speed can be inverted from the second-order backscatter. The neural network method (NN method 3) also provides better results than the conventional method.

Besides the wind inversion from the radar backscatter, the inversion of the directional wave spectra from radar second-order backscatter is also given. The second-order sidebands with higher SNR are used as the input data set and the waverider buoy measured wave spectra are used as the target data. It gives a better result than the conventional method. The network has the advantage to tackle the irregularity of the wave directional distribution for the wave direction inversion and the complexity of two-dimensional non-linear integral equations for the wave power spectra inversion.
Chapter 6

Conclusions and Outlook

6.1 Conclusions

Estimating high quality ocean surface wind in coastal areas is an important issue for scientific research and offshore resources investigation such as planning of offshore wind farms. A land-based HF radar provides the unique capability to continuously monitor coastal ocean surface dynamics at ranges up to 200 km. In HF radar remote sensing, the traditional method for wind speed inversion is based on second-order backscatter measurements, but the complexity of the nonlinear radar second-order integral equations makes it difficult to derive waves as well as wind speed, and the SNR of the second-order sidebands is quite low. In this dissertation, the HIPOCAS WAM data is analyzed to get a better understanding of the wind acting on the ocean surface. As a result, new methods are proposed for wind speed inversion from HF radar backscatter.

Referring to the goals in the introduction, the work accomplished in this dissertation describes several new ideas for wind inversion from HF radar backscatter:

1. In former works, the Bragg resonant waves, which give the radar first-order backscatter, are considered to be saturated due to the short wavelength. However, the HIPOCAS WAM data shows that the resonant waves are not always saturated, especially when the radar operates at lower frequencies (longer resonant waves). In this case, the amplitude and directional spreading of the resonant waves are related to the local wind speed, so the wind speed can be inverted from HF radar first-order backscatter.

2. In the radar backscatter Doppler spectrum, the power of the positive and negative first-order peaks is proportional to the strength of the approaching and receding Bragg resonant waves along the radio beam. To handle the uncertainty of both the wave directional distribution at Bragg frequency and the complexity of the wind-wave relationship, neural network methods are used for the wind inversion. The power of the two pairs of the first-order peaks are used as the input data, and the in-situ wind
measurements are used as the target data for the network training. The network doesn’t need to know the functional form of the wave directional spreading. After the training, the network automatically generates a “function” for the nonlinear mapping between radar echoes and wind data.

3. A new pattern fitting method is proposed that gives a unique solution for both wind direction and directional spreading parameter $\beta$ of Bragg resonant waves. The spreading value varies with the wind speed, which gives a new possibility for the wind speed inversion. The network trained at the buoy location using the power of the first-order peaks normally can not be extended to the other locations within radar coverage, because the first-order peak power is radio beam direction and radar range dependent. But in this work, another neural network method using the directional spreading information is presented. The spreading value of Bragg waves is independent of the radio beam direction and the radar range. The network is trained using the spreading parameter and the in-situ wind measurement at the buoy location. After training, the trained network is extended to the other locations and the spreading parameter $\beta$ derived by the pattern fitting method is used as the new input for the wind speed field inversion.

4. If the Bragg resonant waves come to a state of saturation, the wind speed could not be inverted from the radar first-order backscatter any more. In this case, the second-order backscatter has to be used. Although the conventional method for the wind speed is also based on the radar second-order backscatter, it might overestimate the wind speed in case of the presence of swell or residual waves. In this work, the neural network method is investigated for the wind speed inversion from radar second-order backscatter. Two pairs of second-order sidebands are used as the input data and the in-situ wind speed measurement are used as the target data. After training, this method gives a better result than the conventional method.

In the wind inversion from radar echoes, the wind speed derived from radar first-order backscatter can cover the low and moderate wind speed conditions. Especially when the radar works at a low frequency (long Bragg resonant waves), the first-order method can be used over an extended wind speed range. At the buoy location, the network trained using the first-order peak power gives a more accurate wind speed measurement, because both the amplitude and the directional distribution of the Bragg resonant waves are used for the inversion. But for the wind speed inversion at the other locations within radar coverage, only the directional spreading information can be used, so the result is not as accurate as that using both amplitude and directional spreading information. With the help of the neural network, the wind speed measurement is also reliable. The wind speed inversion from radar first-order peaks also has some limitations, in case of the saturation of the Bragg resonant waves, the wind speed could not be inverted from the first-order
backscatter any more, the second-order sidebands have to be used and the neural network method is also implemented for the inversion.

The RMS Error analysis is presented in Table 5.8 comparing the performance of the wind speed inversion using different methods. The neural network methods improve the wind speed measurement, for wind speeds higher than 3 m/s, with the RMS error being reduced by a factor of 1.5 to 2. Besides these, the wind speed field inverted from radar first-order backscatter covers a much larger area than that inverted from radar second-order backscatter, because the SNR of the first-order peaks is much higher than that of the second-order sidebands.

For the wind direction measurement from HF radar backscatter, the accuracy strongly depends on the radar frequency. When the radar works at a higher frequency, the wavelength of the Bragg resonant waves is shorter, which leads to a better sensitivity for the wind direction. Because the direction derived from the radar first-order backscatter is the mean direction of the Bragg resonant waves, it is better to use a higher radar frequency for the wind direction measurements.

6.2 Outlook

The amplitude of the radar first-order peaks depends on the radar range and the roughness of the sea surface as well as the radar radio beam direction. In this work, the azimuth compensation factor is calculated to normalize the first-order peaks at the other locations to the buoy location (on the range circle where the buoy is located), but still, there are some limitations. The range compensation factor is difficult to describe, not only the electromagnetic wave attenuation at the sea surface, but also the roughness (sea state) has to be considered. Besides that, the echoes of the resonant waves are split into the positive and negative first-order peaks due to the Bragg wave directional distribution and radio beam direction. But still, some statistical or empirical method could be tried for the calculation of the compensation factors, or if we have several in-situ buoys deployed at the sea surface, more target data could be used for the inversion. With all these future investigations, we may find a way to use the first-order peak power for wind speed inversion at all the grid points within radar coverage.

Concerning the relationship between the amplitude of the normalized second-order spectra and the radar frequency (Equation 3.35), during the Fedje experiment, the wind speed is high and the radar frequency is also high. During this high sea state, the second-order sidebands might increase to a value higher than the first-order peaks, which brings some trouble for the identification and normalization of the second-order spectra. In contrast, the wind speed at the Ligurian Sea is quite low and the radar frequency is also low. During the weak wind conditions, the second-order sidebands sometimes can not be distinguished from the background noise. As a consequence, it is better to measure the higher wind speed with a lower radar frequency and the lower wind speed with a higher radar frequency.
Appendix A

Wind Direction and Power Ratio of Radar First-order Peaks

A.1 Half-cosine $2s$-power spreading function

In the wind direction from HF radar first-order backscatter, in order to remove the ambiguity of the mathematical solution “±” and uncertainty of spreading parameter $s$, two radar sites are used and the wind direction can be derived from each site:

$$\theta_i = \phi_i \pm 2 \arctan(R_i^{1/2s}) \quad (i = 1, 2) \quad (A.1)$$

where $\phi_i$ is the radio beam direction, $R_i$ is the power ratio of the Bragg peaks, $s$ is the spreading parameter and $\theta_i$ is the wind direction measured by each site. Here, the author defines that $\theta_i^+ = \phi_i + 2 \arctan(R_i^{1/2s})$, $\theta_i^- = \phi_i - 2 \arctan(R_i^{1/2s})$.

The wind direction $\theta$ and the directional spreading parameter $s$ at the sea surface is unique. That is to say, for all these four direction curves ($\theta_i^\pm$, $i = 1, 2$), there should be one cross point, or if they might have more than one cross points, but only one of these is true. The radar sites are located on the coast facing to the west, and the author defines that the radar site at higher latitude is site 1, the other one is site 2, so we have $\phi_1 < \phi_2$.

Considering different values of the $R_1$ and $R_2$, firstly, we take $R_1 < 1$ and $R_2 < 1$ as an example. As introduced in Section 3.3.3, if $R < 1$, so

$$\theta_{1,s\rightarrow0} = \phi_1$$
$$\theta_{1,s\rightarrow\infty} = \phi_i + \pi/2$$
$$\theta_{1,s\rightarrow\infty} = \phi_i - \pi/2$$

Note that $0 < \phi_2 - \phi_1 < \pi$, therefore, $\phi_2 - \pi/2 < \phi_1 + \pi/2$. So there is one cross point for the curves $\theta_2^-$ and $\theta_1^-$. For direction curves $\theta_2^+$ and $\theta_1^+$:

$$\theta_2^+ - \theta_1^- = \phi_2 + 2 \arctan(R_2^{1/2s}) - (\phi_1 - 2 \arctan(R_1^{1/2s})) \quad (A.3)$$
$$\phi_2 - \phi_1 < \theta_2^+ - \theta_1^- < \phi_2 - \phi_1 + \pi \quad (A.4)$$
Because $0 < \phi_2 - \phi_1 < \pi$, hence, the direction curve $\theta_2^{\pm}$ and $\theta_1^{\pm}$ don’t have a cross point.

With respect to the curves $\theta_1^{\pm}$ and $\theta_2^{\pm}$, suppose there might be cross points satisfying the following equality:

$$\theta_2^{\pm} - \theta_1^{\pm} = (\phi_2 - \phi_1) + (2 \arctan(R_2^{1/2s}) - 2 \arctan(R_1^{1/2s})) = 0$$  \hspace{1cm} (A.5)  

$$\tan\left(\frac{\phi_2 - \phi_1}{2}\right) = \frac{R_1^{1/2s} - R_2^{1/2s}}{1 + R_1^{1/2s} R_2^{1/2s}}$$  \hspace{1cm} (A.6)  

Due to $0 < R_1 < 1, 0 < R_2 < 1$ and $0 < \phi_2 - \phi_1 < \pi$, hence

$$0 < \frac{R_1^{1/2s} - R_2^{1/2s}}{1 + R_1^{1/2s} R_2^{1/2s}} < 1$$  \hspace{1cm} (A.7)  

In case of $\pi/2 \leq (\phi_2 - \phi_1) < \pi$, $\tan\left(\frac{\phi_2 - \phi_1}{2}\right) \geq 1$. Therefore, only when $0 < \phi_2 - \phi_1 < \pi/2$, there might exist cross point. From Equation A.6:

$$0 < R_1 = \left[\frac{-\tan\left(\frac{\phi_2 - \phi_1}{2}\right) + R_2^{1/2s}}{1 - R_1^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}\right]^{2s} < 1$$  \hspace{1cm} (A.8)  

$$0 < R_2 = \left[\frac{-\tan\left(\frac{\phi_2 - \phi_1}{2}\right) + R_1^{1/2s}}{1 - R_1^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}\right]^{2s} < 1$$  \hspace{1cm} (A.9)  

so we have the range of parameter $s$:

$$\frac{\ln R_1}{2 \ln\left(\frac{\phi_2 - \phi_1}{2}\right)} < s < \frac{\ln R_2}{2 \ln\left[1 - \tan\left(\frac{\phi_2 - \phi_1}{2}\right)\right] - \ln\left[1 + \tan\left(\frac{\phi_2 - \phi_1}{2}\right)\right]}$$  \hspace{1cm} (A.10)  

The variables $(\phi_2 - \phi_1)$ and $R_2$ can be determined during the experiment, $s$ is uncertain variable. So we define that:

$$R'_1 = [\tan\left(\frac{\phi_2 - \phi_1}{2}\right) + R_2^{1/2s}]^{2s}$$  \hspace{1cm} (A.11)  

in which, $s$ is a value in the range of Equation A.10. In the examples below, let $R_2 = 0.2$, $\phi_1 = 205^\circ$, $\phi_2 = 250^\circ$, with Equation A.11, the $R'_1$ is depicted in Figure A.1a. The minimum value of $R'_1$ is 0.647. So the measured $R_1$ is used to compare with the threshold $(\text{Min} R'_1)$, three cases are discussed in Table A.1:

<table>
<thead>
<tr>
<th>Number of cross points for $\theta_1^{\pm}$ and $\theta_2^{\pm}$ (K is the number)</th>
<th>( R_1, R_2 ) and $\text{Min}(R'_1)$</th>
<th>Note</th>
<th>K</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>$R_1 &lt; \text{Min}(R'_1)$</td>
<td>Figure A.1a</td>
<td>0</td>
</tr>
<tr>
<td>0.647</td>
<td>0.2</td>
<td>$R_1 = \text{Min}(R'_1)$</td>
<td>Figure A.2a</td>
<td>1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>$R_1 &gt; \text{Min}(R'_1)$</td>
<td>Figure A.3a</td>
<td>2</td>
</tr>
</tbody>
</table>

Table A.1: $R_1, R_2$ and number of the cross points for $\theta_1^{\pm}$ and $\theta_2^{\pm}$
A.1. Half-cosine 2s-power spreading function

Figure A.1: No cross point for curve $\theta_1^+$ and $\theta_2^-$.

Figure A.2: One cross point for curve $\theta_1^+$ and $\theta_2^+$.

Figure A.3: Two cross points for curve $\theta_1^+$ and $\theta_2^+$. 

$\theta_{2} = \phi_{2} - 2 \text{atan}(R_{2}^{1/2s})$, $R_{2} = 0.2$

$\theta_{2} = \phi_{2} + 2 \text{atan}(R_{2}^{1/2s})$, $R_{2} = 0.2$

$\theta_{1} = \phi_{1} + 2 \text{atan}(R_{1}^{1/2s})$, $R_{1} = 0.8$

$\theta_{1} = \phi_{1} - 2 \text{atan}(R_{1}^{1/2s})$, $R_{1} = 0.4$

$R_{1'} = F(R_{2}, s, (\phi_{2} - \phi_{1}))$

$\text{Min}(R_{1'})$

$S$ value at $\text{Min}(R_{1'})$

$R_{1} = 0.647$

$R_{2} = 0.2$

$R_{1} = 0.8$

$R_{2} = 0.2$

$R_{1} = 0.4$

$R_{2} = 0.2$
Appendix A. Wind Direction and Power Ratio of Radar First-order Peaks

Regarding to the curves $\theta_1^-$ and $\theta_2^-$, we have

$$\theta_1^- - \theta_2^- = \phi_1 - 2 \arctan(R_1^{1/2s}) = \phi_2 - 2 \arctan(R_2^{1/2s})$$  \hspace{1cm} (A.12)

$$\tan\left(\frac{\phi_2 - \phi_1}{2}\right) = \frac{R_2^{1/2s} - R_1^{1/2s}}{1 + R_1^{1/2s} \cdot R_2^{1/2s}}$$  \hspace{1cm} (A.13)

Note that $\tan\left(\frac{\phi_2 - \phi_1}{2}\right) > 0$ and $1 + R_1^{1/2s} \cdot R_2^{1/2s} > 0$. So $R_1 < R_2$ and also $\frac{R_2^{1/2s} - R_1^{1/2s}}{1+R_1^{1/2s} \cdot R_2^{1/2s}} < 1$. Therefore, only when $0 < \phi_2 - \phi_1 < \frac{\pi}{2}$, they might exist cross point.

$$R_1^{1/2s} = \frac{R_2^{1/2s} - \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}{1 + R_2^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}$$  \hspace{1cm} (A.14)

Since $0 < R_1^{1/2s} < 1$ and the denominator $1 + R_2^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right) > 1$, in order to satisfy Equation A.14, from Equation A.12, we also have

$$0 < R_2^{1/2s} = \frac{R_1^{1/2s} + \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}{1 - R_1^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)} < 1$$  \hspace{1cm} (A.15)

$$0 < R_1^{1/2s} = \frac{R_2^{1/2s} - \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}{1 + R_2^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)} < 1$$  \hspace{1cm} (A.16)

So we have the range of parameter $s$:

$$\frac{\ln R_2}{2 \ln\left(\frac{\phi_2 - \phi_1}{2}\right)} < s < \frac{\ln R_1}{2(\ln[1 - \tan\left(\frac{\phi_2 - \phi_1}{2}\right)] - \ln[1 + \tan\left(\frac{\phi_2 - \phi_1}{2}\right)])}$$  \hspace{1cm} (A.17)

The variables $(\phi_2 - \phi_1)$ and $R_1$ can be determined in the experiment, $s$ is a variable, we define:

$$R'_2 = F[\phi_2 - \phi_1, R_1, s] = \left[\frac{\tan\left(\frac{\phi_2 - \phi_1}{2}\right) + R_1^{1/2s}}{1 - R_1^{1/2s} \cdot \tan\left(\frac{\phi_2 - \phi_1}{2}\right)}\right]^{2s}$$  \hspace{1cm} (A.18)

In which, $s$ is a value in the range of Equation A.17. In the example below, let $R_2 = 0.2$, $\phi_1 = 205^\circ$, $\phi_2 = 250^\circ$. With Equation A.18, the $R'_2$ is depicted in Figure A.4a, the minimum value of $R'_2$ is 0.647. So the measured $R_2$ is used to compare with the threshold (Min$R'_2$), three cases are discussed in Table A.2:

<table>
<thead>
<tr>
<th>Number of cross points for $\theta_1^-$ and $\theta_2^-$ (K is the number)</th>
<th>$R_1$, $R_2$ and Min($R'_2$)</th>
<th>$R'_2$: Equation A.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>$R_2$</td>
<td>Min($R'_2$) = 0.647</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>$R_2 &lt; \text{Min}(R'_2)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.647</td>
<td>$R_2 = \text{Min}(R'_2)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.7</td>
<td>$R_2 &gt; \text{Min}(R'_2)$</td>
</tr>
</tbody>
</table>

Table A.2: $R_1$, $R_2$ and number of the cross points for $\theta_1^-$ and $\theta_2^-$
(a) $R_2 < \text{Min}(R'_2)$

(b) No cross point for curve $\theta_1^-$ and $\theta_2^-$

Figure A.4: No cross point for curve $\theta_1^-$ and $\theta_2^-$.

(a) $R_2 = \text{Min}(R'_2)$

(b) One cross point for curve $\theta_1^-$ and $\theta_2^-$

Figure A.5: One cross point for curve $\theta_1^-$ and $\theta_2^-$.

(a) $R_2 > \text{Min}(R'_2)$

(b) Two cross points for curve $\theta_1^-$ and $\theta_2^-$

Figure A.6: Two cross points for curve $\theta_1^-$ and $\theta_2^-$. 

A.1. Half-cosine 2s-power spreading function
The other cases are given in Table A.3, as shown in the table, in some cases, the pattern fitting methods using the half-cosine 2s-power function presents more than one cross point (solution) for the wind direction and the spreading parameter of the Bragg resonant waves.

<table>
<thead>
<tr>
<th>$R_1, R_2$</th>
<th>curves having cross points</th>
<th>*condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 &lt; 1, R_2 &lt; 1$</td>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$\phi_1 = \frac{\pi}{2} - \theta_1, \theta_2$</td>
</tr>
<tr>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$R_1 &gt; 1, R_2 &lt; 1$</td>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$\phi_1 = \frac{\pi}{2} - \theta_1, \theta_2$</td>
</tr>
<tr>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$R_1 &lt; 1, R_2 &gt; 1$</td>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$\phi_1 = \frac{\pi}{2} - \theta_1, \theta_2$</td>
</tr>
<tr>
<td>$0 &lt; \phi_2 - \phi_1 &lt; \pi/2$</td>
<td>$R_1 &lt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
<tr>
<td>$\pi/2 &lt; \phi_2 - \phi_1 &lt; \pi$</td>
<td>$R_1 &gt; R_2$</td>
<td>$(\theta_1^+, \theta_2^-); (\theta_1^-, \theta_2^+)$</td>
</tr>
</tbody>
</table>

* Only when $R_1, R_2$ and $(\phi_2 - \phi_1)$ meet these agreements, there could be cross point for the curves with the star “*”

Table A.3: The possible cross points analysis for half cosine squared function

### A.2 Hyperbolic secant squared function

In Section A.1, the half-cosine function has been analyzed to find the possible cross points for determining wind direction and spreading parameter. Here, hyperbolic secant squared function is also investigated to find whether the solution of wind direction $\theta$ and spreading parameter $\beta$ is unique or not.

We also take $R_i < 1$ as an example, and the radar beam direction is defined $0 < \phi_2 - \phi_1 < \pi$, on the premise of $\pi/2 < \phi_2 - \phi_1 < \pi$, the curve $\theta_1^+, \theta_2^- = \theta_1 + \pi/2 < \phi_2$, likewise, there is no cross point for $\theta_1^+$ and $\theta_2^-$ either. If $0 < \phi_2 - \phi_1 < \pi/2$, the curves $\theta_1^+$ and $\theta_2^-$ might exist cross point, the analysis is given as follow:

$$
\phi_1 + \frac{1}{2\beta} \ln \left| 1 - \frac{R_1^{1/2} \exp(-\beta \cdot \pi)}{R_1^{1/2} \exp(\beta \cdot \pi) - 1} \right| = \phi_2 + \frac{1}{2\beta} \ln \left| 1 - \frac{R_2^{1/2} \exp(-\beta \cdot \pi)}{R_2^{1/2} \exp(\beta \cdot \pi) - 1} \right|
$$

(A.19)

$$
\exp[2\beta(\phi_2 - \phi_1)] = \frac{[1 - \exp(\beta \pi)R_2^{1/2}][1 - \exp(-\beta \pi)R_1^{1/2}]}{[1 - \exp(\beta \pi)R_1^{1/2}][1 - \exp(-\beta \pi)R_2^{1/2}]} \geq \max(\beta_{1,\min}, \beta_{2,\min})
$$

(A.20)
A.2. Hyperbolic secant squared spreading function

\[\phi_2 - \phi_1 \geq 0 \implies \exp[2\beta(\phi_2 - \phi_1)] > 1\]

if \(R_1 \geq R_2\)

\[
\frac{[1 - \exp(\beta\pi)R_2^{1/2}][1 - \exp(-\beta\pi)R_1^{1/2}]}{[1 - \exp(\beta\pi)R_1^{1/2}][1 - \exp(-\beta\pi)R_2^{1/2}]} \leq 1 \quad (\beta \geq \beta_{2,\text{min}})
\] (A.21)

So there is no cross point when \(R_1 \geq R_2\), and there might be cross point when \(R_1 < R_2\), according to Table 3.3, we know that \(\beta_{1,\text{min}} \geq \beta_{2,\text{min}}\), so, \(\max(\beta_{1,\text{min}}, \beta_{2,\text{min}}) = \beta_{1,\text{min}}\). Equation A.20 is complicated to find the solution, therefore, the monotonicity principle is analyzed as follow:

We define that:

\[
F_1(\beta) = \exp[2\beta(\phi_2 - \phi_1)]
\] (A.22)

\[
F_2(\beta) = \frac{[1 - \exp(\beta\pi)R_2^{1/2}][1 - \exp(-\beta\pi)R_1^{1/2}]}{[1 - \exp(\beta\pi)R_1^{1/2}][1 - \exp(-\beta\pi)R_2^{1/2}]}
\] (A.23)

\[
\frac{\partial F_1}{\partial \beta} = \exp[2\beta(\phi_2 - \phi_1)] \cdot 2(\phi_2 - \phi_1) > 0
\] (A.24)

\[
\frac{\partial F_2}{\partial \beta} = \left[\pi R_1^{1/2} \exp(-\beta\pi) - \pi R_2^{1/2} \exp(\beta\pi)\right]\left[1 - \exp(-\beta\pi)R_1^{1/2} - \exp(\beta\pi)R_1^{1/2} + (R_1R_2)^{1/2}\right] \frac{[1 - \exp(\beta\pi)R_1^{1/2}]^2[1 - \exp(-\beta\pi)R_2^{1/2}]^2}{[1 - \exp(\beta\pi)R_2^{1/2}]^2[1 - \exp(-\beta\pi)R_1^{1/2}]^2} - \left[\pi R_2^{1/2} \exp(-\beta\pi) - \pi R_1^{1/2} \exp(\beta\pi)\right]\left[1 - \exp(-\beta\pi)R_2^{1/2} - \exp(\beta\pi)R_1^{1/2} + (R_1R_2)^{1/2}\right] \frac{[1 - \exp(\beta\pi)R_2^{1/2}]^2[1 - \exp(-\beta\pi)R_1^{1/2}]^2}{[1 - \exp(\beta\pi)R_1^{1/2}]^2[1 - \exp(-\beta\pi)R_2^{1/2}]^2}
\]

\[
\therefore [1 - \exp(\beta\pi)R_1^{1/2}]^2[1 - \exp(-\beta\pi)R_2^{1/2}]^2 > 0 \text{ and } R_1 < R_2
\]

\[
\therefore \frac{\pi(R_1^{1/2} - R_2^{1/2})}{[1 - \exp(\beta\pi)R_1^{1/2}]^2[1 - \exp(-\beta\pi)R_2^{1/2}]^2} < 0
\] (A.26)

and

\[
[\exp(-\beta\pi) + \exp(\beta\pi)] > 2 \quad (\text{Here} \beta \neq 0)
\] (A.27)

\[
(1 + R_1^{1/2}R_2^{1/2})[\exp(-\beta\pi) + \exp(\beta\pi)] - 2(R_1^{1/2} + R_2^{1/2}) > 2[(1 + R_1^{1/2}R_2^{1/2}) - R_1^{1/2} - R_2^{1/2}]
\]

\[= 2[(1 - R_1^{1/2})(1 - R_2^{1/2})] > 0
\] (A.28)

Therefore

\[
\frac{\partial F_2}{\partial \beta} = \frac{\pi(R_1^{1/2} - R_2^{1/2})[(1 + R_1^{1/2}R_2^{1/2})(\exp(-\beta\pi) + \exp(\beta\pi)) - 2(R_1^{1/2} + R_2^{1/2})]}{[1 - \exp(\beta\pi)R_1^{1/2}]^2[1 - \exp(-\beta\pi)R_2^{1/2}]^2} < 0
\] (A.29)

Note that, even though \(\theta_1^+\) is monotone increasing and \(\theta_2^+\) is monotone decreasing, but still
we can not prove that they will have a cross point. we also need to prove that \( F_{1,\beta\to\infty} > F_{2,\beta\to\infty} \) and \( F_{1,\beta_{\min}} < F_{2,\beta_{1,\min}} \), only in these cases, they will have, and only have one cross point. From Equation A.22 and Equation A.23, we know that

\[
\begin{align*}
F_{1,\beta\to\infty} &= +\infty \\
F_{2,\beta\to\infty} &= (R_2/R_1)^{1/2} < 1
\end{align*}
\]  
(A.30)

\[
\begin{align*}
F_{1,\beta_{1,\min}} &= \exp\left\{ \frac{2(\phi_2 - \phi_1)}{\pi} \ln\left(1 + \frac{1}{R_1^2} - 1\right) \right\} = \frac{1}{2} \left[ \frac{1}{R_1^2} - 1\right]^{1/2} + 1 \\
F_{2,\beta_{1,\min}} &= \frac{1}{1 - \exp(\beta \pi)} \frac{R_2^2}{R_1^2} \left(1 - \exp(-\beta \pi)\right) = \frac{1}{R_2 - R_1} \frac{R_2^2}{R_1^2} \beta \pi
\end{align*}
\]  
(A.31)

if \( F_{1,\beta_{1,\min}} = F_{2,\beta_{1,\min}} \), we have

\[
R_2^2 = \frac{1}{2} \left[ \frac{1}{R_1^2} - 1\right]^{1/2} + 1
\]  
(A.32)

For example, \( \phi_2 = 250.5^\circ, \phi_1 = 205.5^\circ, R_1 = 0.3 \), from Equation A.32, we calculate the threshold of \( R_{2,\min} = 0.5272 \). So we discuss the value of \( R_2 \) and \( R_{2,\min} \) in three conditions:

If \( R_2 = R_{2,\min} \), the cross point will be at \( (\beta_{1,\min}, \phi_1) \), in Figure A.7a, the \( F_1 \) and \( F_2 \) are given to illustrate the monotone increasing of \( F_1(\beta) = \exp[2\beta(\phi_2 - \phi_1)] \) and monotone decreasing of \( F_2(\beta) \) as given in Equation A.23, as discussed before, the \( \beta \geq \max(\beta_{1,\min}, \beta_{2,\min}) \), here \( \beta_{\min} = \beta_{1,\min} \), Figure A.7b illustrates the cross point of curves \( \theta_1^* \) and \( \theta_2^* \), the cross point locates at the start point of curve \( \theta_1^* \).

If \( R_2 > R_{2,\min} \), we define \( R_2 = 0.7272 \), as shown in Figure A.8, the curves \( \theta_1^* \) and \( \theta_2^* \) will have, but only have one cross point, Figure A.8a gives the curve of \( F_1(\beta) \) and \( F_2(\beta) \), because of the characteristic of monotone-varying of \( F_1(\beta) \) and \( F_2(\beta) \), and we have proved at \( \beta_{\min} \), \( F_1(\beta_{\min}) < F_2(\beta_{\min}) \), and when \( \beta \to +\infty \), \( F_1(\beta \to \infty) > F_2(\beta \to \infty) \). So these two curves just have one cross point. Figure A.8b shows the curves \( \theta_1^* \) and \( \theta_2^* \) and the cross point, we have \( \beta = 0.478 \) and the wind wave direction is \( \theta = 175^\circ \).

If \( R_2 < R_{2,\min} \), we define \( R_2 = 0.3272 \), as we seen in Figure A.9, the curves \( \theta_1^* \) and \( \theta_2^* \) will not have a cross point, Figure A.9a gives the curve of \( F_1(\beta) \) and \( F_2(\beta) \), because of the characteristic of monotone-varying of \( F_1(\beta) \) and \( F_2(\beta) \), and we have proved at \( \beta_{\min} \), \( F_1(\beta_{\min}) > F_2(\beta_{\min}) \), and when \( \beta \to +\infty \), \( F_1(\beta \to \infty) > F_2(\beta \to \infty) \). So these two curves just have no cross point. Figure A.9b shows the curves \( \theta_1^* \) and \( \theta_2^* \) and the cross point is located at the curve \( \theta_1^* \) and \( \theta_2^* \), but not \( \theta_1^* \) and \( \theta_2^* \), the cross point is also presented, we have \( \beta = 0.44 \) and the wind wave direction \( \theta = 226^\circ \).

Considering all conditions, the cross point for the two curves are detailed in Table A.4. From which, we know that the pattern fitting method using the hyperbolic secant squared function gives only one cross point for the wind direction and the spreading parameter of Bragg resonant waves.
Figure A.7: Threshold for $R_2$ having a cross point of $\theta_1^\pm$ and $\theta_2^\pm$ ($R_2 = R_{2,\text{min}}$), the cross point is $(\beta_{1,\text{min}}, \phi_1)$.

Figure A.8: Threshold for $R_2$ having a cross point of $\theta_1^\pm$ and $\theta_2^\pm$ ($R_2 > R_{2,\text{min}}$).

Figure A.9: Threshold for $R_2$ having a cross point of $\theta_1^\pm$ and $\theta_2^\pm$ ($R_2 < R_{2,\text{min}}$).
### Table A.4: The possible cross points analysis for hyperbolic secant squared function

|  |  
|---|---|
| $R_1 < R_2$ | $R_1 > R_2$ |
| $R_1 > R_2$ | $R_1 < R_2$ |
| $R_1 = R_2$ | $R_1 = R_2$ |

The analysis considers the following conditions:

- $0 < R_1 < R_2 < 1$
- $1 < R_1 < R_2$
- $R_1 = R_2$
- $R_1 > R_2$
- $R_1 < R_2$

Note: The table entries are placeholders for the actual mathematical expressions and conditions.

**Appendix A: Wind Direction and Power Ratio of Radar First-order Peaks**
Bibliography


