Supplementary Products in the Health Insurance Market and its Implications:
4 Essays in Health Economics

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To Malte and Lasse
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Chapter 1

Introduction

The best six doctors anywhere,
And no one can deny it,
Are sunshine, water, rest, and air,
Exercise and diet.
These six will gladly you attend,
If only you are willing.
Your mind they'll ease,
Your will they'll mend,
And charge you not a shilling.

Wayne Fields

Fields (1990) used a demonstrative metaphor in his book *What the River Knows*. This metaphor describes the significance of different factors that are closely related to the health care market. Sunshine, water, rest, air, exercise, and diet are called the *best doctors*. They certainly do not replace general practitioners or medical specialists, but there is no doubt that they play a very important role for our well-being and are therefore indeed closely related to the *the best doctors*. However, there still is an important difference: While doctors keep records of their treatments, individuals do not have such verifiable protocol about their exercise or diet. The utilization of these factors is a personal choice, i.e. individuals can choose how much they want to eat and how much they want to exercise. The utilization highly determines their health status. As an example, the body mass index affects the probability of falling ill. It is shown that obesity increases the risk for hypertension, diabetes, reflux, heart attacks, strokes and so on and so forth (Elias et al., 2005; Lenz et al., 2009). The health care expenditures for obesity are enormous.\footnote{Finkelstein (2003), for example, shows that the U.S. annual health care expenditures for obesity are about 147 billion dollars.} As a consequence, there are many campaigns that aim at increasing the
individuals' prevention effort. As an example, Michelle Obama established the initiative "Let's Move" (LetsMove, 2010).\footnote{http://www.letsmove.gov/} She tries to motivate children to eat healthy and get active. Putting things very simply, we can say that there are two kinds of producers in the health care markets: the individual who produces his own health and persons such as general practitioners or medical specialists who provide health care services to the individual. We therefore have uncertainty about the chosen effort level of at least two involved producers, i.e. the prevention effort of an individual and the treatment effort of the involved physicians. Hence, incentive compatible contracts are very important. It is well-known that the optimal contract design depends on the degree of uncertainty as well as on the number of involved parties. With uncertainty, we need to consider the physicians' incentive constraint. The optimal contract design then highly depends on the degree of information asymmetry. There might be hidden information or hidden action. The physician might have multiple tasks (e.g. educating the patient versus treating the patient) and therefore might substitute his effort. We might have multi-agent moral hazard problems (e.g. in Health Maintenance Organizations) Hence, we see that there are different contract designs for different relationships. For a thorough overview of optimal contract designs, see Bolton and Dewatripont (2005).

However, these simple optimizations are not necessarily applicable for the health care market due to the fact that we not only have individuals and physicians in the health care market who can write specific contracts, but also health insurance companies as intermediates and governments that regulate the market heavily. In the hospital sector, for instance, we often observe a prospective payment scheme which is called \textit{Diagnosis Related Groups} (DRG). The payment scheme is independent of the number of involved parties and the degree of uncertainty. DRG is a system to classify hospital cases into certain (homogeneous\footnote{DRGs have been used in the USA since 1982 in order to determine how much Medicare has to pay to a hospital for each service since patients within each category are clinically similar and are expected to use the same level of hospital resources. It is widely argued that the implementation of DRG was a very powerful innovation. As an example, Sheila Burke, Chief of staff of the former Senator Bob Dole legislation, claimed that "Medicare's traditional model of cost reimbursement was insanity. On the face of it, it encouraged people to do more: it paid them to to more and not in any particular rational way." (Mayes, 2007).}) groups. DRGs were first used in the US Medicare System. Mayes (2007) puts the impact of the implementation of DRGs in a nutshell:

"Inexorably rising medical inflation and deep economic deterioration forced policymakers in the late 1970s to pursue radical reform of Medicare to keep the program from insolvency. Congress and the Reagan administration eventually turned to the one alternative reimbursement system that analysts and academics had studied more than any other and had even tested with apparent success in New Jersey: prospective payment with diagnosis-related groups."
Rather than simply reimbursing hospitals whatever costs they charged to treat Medicare patients, the new model paid hospitals a predetermined, set rate based on the patient's diagnosis. The most significant change in health policy since Medicare and Medicaid's passage in 1965 went virtually unnoticed by the general public. Nevertheless, the change was nothing short of revolutionary."

Hence, it is obvious that the intention of the implementation of DRG was to increase efficiency. However, there is one main drawback as well. DRGs induce hospitals to manipulate the DRG due to the fact that DRG is a system that classifies diseases into different groups. The higher the group, the more money the hospital receives. Let us take a look at an example to emphasize this problem. In Germany, the payment amount is determined by the G-DRG. If a patient has an acute apoplexy with a neurological complex treatment, the hospital needs to decide whether there is a complicating development (G-DRG: B70A) or not (G-DRG: B70B). The monetary difference between these two DRGs is about 1,300 €. Therefore, the hospital has a high incentive for upcoding. Empirical research shows that upcoding is indeed a big problem. One example is that there is an accumulation of a certain birth weight that cannot be explained statistically, ranging from 740 to 749 grams, of newborn children (GKV-Spitzenverband, 2011). For premature infants weighting more than 749 grams, the hospital receives another DRG. The monetary difference between these groups is 23,000 €. Similar results can be found when people receive artificial respiration (GKV-Spitzenverband, 2011). Carter et al. (1990) finds that about one-third of the total expenditures in the hospital sector are due to fraud. For the US Medicare sector, this means about one billion dollars. The average health care expenditures for industrialized countries are about 9% of GDP (OECD, 2009). Clearly, already a small degree of inefficiency (like the possibility of committing fraud) has high monetary consequences. Therefore, adjusting the payment scheme should not be the sole instrument to increase efficiency. We also observe health care reforms that try to make the market more efficient. There are many possibilities to decrease inefficiency. These are, for instance, restructuring the contract design, lowering the switching costs, increasing the consumers' information, or enhancing the firms' competition. From past health care reforms, we can see that many governances pay attention to increase efficiency by increasing competition. In Germany, as an example, there was a big health care reform in 2007 in which the Social Health Insurance Competition Strengthening Act was adopted. One main part of this act was that supplementary health care can be provided by private health insurance companies as well as by non-profit sickness funds. The market for supplementary health care is a market where firms can provide differentiated products and compete for customers with a high preference for quality. We can deduct two things from this change in regulation: First, supplementary health care plays an important role. Second, we have a market in which non-profit firms play an important role. The
competition of non-profit firms might work very differently compared to the competition of for-profit firms, e.g. the firms might not fear price competition.

So far, we have seen that the health care market is characterized by a kind of competition that might work differently compared to other markets and a high degree of uncertainty. Therefore, this dissertation is divided into the parts "Competition" and "Uncertainty". In order to analyze the health care market, we need to understand the interaction of the involved players. It might sound like an oversimplification but roughly speaking, we can say that we have four major players in the market for health care. This is the physician, the patient, the insurance company, and the government. Analyzing the interaction of these players and its implications for the health care market is the core of this dissertation. A demonstration of these interactions and the topics of my papers can be seen in Figure 1.1. Let us take a look at this figure in order to emphasize the interaction: The insurance company has the possibility to control the physician (e.g. in Germany via Medizinischer Dienst der Krankenkassen (MDK)). The physician has to treat the patient and the government has the possibility to influence the physicians' behavior by regulating the market. This dissertation deals with all these interactions. In the following, I present an overview of each chapter in which I state the main results and explain the connection between the papers.

As mentioned at the beginning of this introduction, we observe health care reforms that try to increase efficiency by increasing competition. Due to the fact that non-profit firms play an important role in the health care market, the competition might be different. This is especially true for the supplementary health care market. Since this market is gaining more and more importance due to the demographic change and the epidemiological transition, Chapter 2 and 3 deal with the competition in the supplementary health care market and its implications. First we show the theoretical fundamentals of homogeneous non-profit firms and analyze the welfare implications of different organization types (private, public, or mixed competition) by considering an exemplary cost function. Then we analyze the firms' strategies when the firms are heterogeneous. In those chapters we assume that non-profit health insurance companies aim for output due to the high cross-selling potential in this market. Assuming a high cross-selling potential in the market for supplementary health care is reasonable for several reasons. One reason for the cross-selling potential is that the possibility of purchasing the supplementary health insurance can be conditional on being primarily insured by the same health insurance companies as well. Another reason is that the insured prefers to deal with only one firm instead of two.

The next Chapter, Chapter 2, examines the competition of two homogeneous non-profit health insurance companies. We investigate the product quality strategies when quality is costly and the sickness funds are competing for customers. As long as the sickness funds choose the qualities simultaneously, any equilibrium will be non-differentiated.
Only if total demand is increasing in quality, both sickness funds provide the maximum quality. For decreasing total demand the existence of an equilibrium depends on the consumers’ sensitivity. If there is no equilibrium in the simultaneous competition, sequential quality competition leads to a differentiated equilibrium with a first mover advantage. At the end of chapter 2, we also compare the resulting welfare when supplementary health insurance is provided either within a public, a private, or a mixed health care system. The target of this application is to answer the question of how the market for supplementary health insurance should be organized. We do not want to give any policy implications due to the fact that we concentrate on an exemplary cost function. However, answering this question of how the market should be organized is crucial for countries all over the world. And our application gives a very good intuition about resulting welfare effects. We show that a mixed competition, as it is present in Germany, is inefficient and that the market should be organized via competing private health insurance companies.

Chapter 3 examines the competition of two heterogeneous non-profit health insurance companies. If the firms are heterogeneous, the inefficient firm has to differentiate itself in order to obtain a positive demand. We show that entry deterrence is possible even without any fixed costs which is in contrast to profit maximizing firms. In a simultaneous
competition the efficient firm always responds with a lower quality and might deter entry. A Nash equilibrium only exists if consumers react sufficiently sensitive to quality changes and the cost inefficiency is sufficiently high. If the consumers' sensitivity is decreasing and the inefficient firm can act as a first mover, the inefficient firm always gains a positive demand.

Chapter 4 and 5 deal with uncertainty. In Chapter 4, we analyze the optimal audit strategy in a costly state verification model. The hospital has the possibility of committing fraud by upcoding. The hospital gets remunerated within a DRG system and eventually a fee for service system. This depends on whether a patient has supplementary health insurance. If a person has bought supplementary health insurance, the hospital gets remunerated by two different reimbursement schemes. This has an impact on the hospitals' fraud probability. The results are the following: If the health insurance company knows which patients have supplementary health insurance, asymmetric information only exists with respect to the diagnosis. We have a mixed equilibrium where cheating and auditing both occur with a positive probability. The health insurance company audits the hospital with a higher probability when the hospital treats a patient with supplementary health insurance. The hospital, on the other hand, has a higher probability of cheating if treating a patient with supplementary health insurance. If the health insurance company does not know which patients have supplementary health insurance, we have two kinds of asymmetric information. First, the health insurance company cannot observe the patients' state of health. Second, it cannot observe the hospitals' manipulation incentive. The chosen audit and cheating probabilities highly depend on the fraction of patients that have supplementary health insurance. For a high fraction of people with supplementary health insurance, the hospital never cheats if it treats a patient without supplementary health insurance and cheats with a positive probability that is below one if it treats a patient with supplementary health insurance. For a low fraction of people with supplementary health insurance, the hospital always cheats if it treats a patient with supplementary health insurance and cheats with a positive probability that is below one if it treats a patient without supplementary health insurance. The total cheating probability increases in the fraction of patients with supplementary health insurance if the fraction is sufficiently low and is independent of the fraction of patients with supplementary health insurance if the fraction is sufficiently high. Depending on the fraction of people who actually have supplementary health insurance, implementing a disclosure requirement for all people with supplementary health insurance might increase welfare due to the fact that audits are a waste of money.

The last chapter, Chapter 5, deals with ex ante Moral Hazard. Due to the fact that individuals have no verifiable protocol about their exercise or diet, it is broadly
assumed that prevention effort is unobservable for the other parties.\(^4\) It is well-known that prevention effort depends on insurance as well. Due to the fact that prevention effort is not observable in most of the cases, a person alters his prevention effort after having bought insurance. The seminal paper of Ehrlich and Becker (1972) analyzes the change in effort when effort influences either the probability of falling ill or the amount of the potential loss. It is shown that as long as effort only affects the amount of the potential loss, the existence of insurance decreases effort. If effort affects the probability of falling ill, the results are more ambiguous. It highly depends on the degree of risk aversion. However, assuming that prevention only affects the morbidity risk omits a very important fact: Prevention also affects the mortality risk. Hence, in Chapter 5, we assume that prevention influences the probability of falling ill as well as the survival probability. We are considering the whole life-time cycle. In the short-run, medical prevention decreases health care costs. In the long-run, prevention increases the likelihood of living up to a very high age and causing excessive end-of-life treatment costs. We derive conditions under which prevention either increases or decreases annualized health care expenditures. When considering the long-run effects, we show that moral hazard can increase preventive care compared to a situation with perfect information. The intuition of this result is relatively straightforward. If higher prevention increases the annual insurance premium (i.e. observable prevention effort), individuals take this into account when choosing their prevention effort. Accordingly, they prevent less than in a situation where they would only consider the impact of effort on the sickness probability and their longevity. Accordingly, unobservability of effort leads to more prevention. This is particularly the case with substantially higher treatment costs towards the end of life than during the second point in time, which does not sound too unlikely when considering current health care spendings over a life-time cycle.

Now let me give some brief comments on how to read this dissertation: The dissertation is divided into two parts. The first part is about competition and contains Chapter 2 and 3. The second part is about uncertainty and contains Chapter 4 and 5. Each chapter is relatively self-contained. Therefore, the reader is free to choose the order of reading the chapters according to his interests. However, I recommend to read Chapter 3 after having read Chapter 2. Chapter 2 explains the fundamentals of the competition of competing non-profit sickness funds in detail.

\(^4\) Of course, there are exceptions as well. These are, for instance, tooth precaution or back training in a medical gym.
Part I

Competition
Chapter 2

Competing non-profit Sickness Funds and the provision of supplementary health care

Joint work with Oliver Urmann.

Abstract

This paper examines the competition of homogeneous non-profit sickness funds in the market for supplementary health insurance. We investigate product quality strategies when quality is costly and the sickness funds are competing for customers. As long as the sickness funds choose the qualities simultaneously, any equilibrium will be non-differentiated. Only if total demand is increasing in quality, both sickness funds provide the maximum quality. For a decreasing total demand, the existence of an equilibrium depends on the consumers’ sensitivity. If there is no equilibrium in the simultaneous competition, sequential quality competition leads to a differentiated equilibrium with a first mover advantage. With an exemplary cost function we compare the welfare of a public, private, and mixed system in order to answer the question of how the market for supplementary health care should be organized. We show that a mixed system, as it is present in Germany, is inefficient.

Keywords: supplementary health insurance, vertical differentiation, output maximization

JEL: I11, L22, L30
2.1 Introduction

This study targets the research question of how competition in the market for supplementary health insurance works when the products are provided by competing homogeneous non-profit sickness funds. Our intention is to show the theoretical fundamentals for a fast-growing market. Furthermore, we compare the welfare of a public, private, and mixed system in order to answer the question of how the market for supplementary health care should be organized.

It is very surprising that this research question has not been answered so far due to the fact that the health care market makes up a substantial part of GDP.\(^1\) Furthermore, the market for supplementary health insurance in which firms can provide differentiated products is a very fast-growing market with a high strategic potential. Additionally, its relevance will increase even more due to demographic change and epidemiological transition. One reason why there is a lack of literature might be because each country’s precise organization of the health insurance market varies widely. However, there are three major organization types: The Beveridge model (e.g. UK), the Bismarck model (e.g. Germany) or a privately organized model (e.g. USA). Our model focuses on the Bismarck model in which we often observe competing (non-profit) health insurance companies.\(^2\) There are many countries that use the Bismarck model, such as Belgium, Germany, the Netherlands, Switzerland, Austria, France, Japan, Luxembourg, Romania, and, to some degree, Latin America.\(^3\)

The core business of non-profit health insurance companies in a Bismarck model can be divided into two parts. The first one is the market for primary health insurance in which the firms provide a homogeneous product, which is basic health care coverage. The second one is the market for supplementary health insurance in which the firms have the possibility of differentiating by providing different qualities (i.e. the benefit package covered by the supplementary health insurance product). Special kinds of products in the

\(^{1}\)The average health care expenditures for industrialized countries are about 9% of GDP (OECD, 2009).

\(^{2}\)Even though the focus of our model is the analysis of competing non-profit sickness funds we want to raise the following question: From a welfare perspective, what is the best way to organize the market for supplementary health care. This question is highly relevant since in some countries (e.g. Germany) we observe non-profit as well as for-profit health insurance companies in the market for supplementary health care.

\(^{3}\)In Germany, for instance, there are 144 sickness funds (date: October 2012) and people are allowed to switch between those sickness funds independent of their health care status, their income or their profession. Therefore, the market is highly competitive. In France, there are only four major sickness funds and a few minor sickness funds and the switching possibility depends on the citizens’ profession. In the Netherlands, as another example, there are privately operating health insurance companies. Even though this is uncommon for the Bismarck model, the health care system of the Netherlands still belongs to the Bismarck system. Due to a one-sided cross-selling potential in the market for supplementary health insurance (which will be explained later in detail), our model is applicable the for health care system of the Netherlands as well, but a few modifications are necessary.
market for supplementary health insurance might be the access to the best physicians’ network or to high cost technologies.\textsuperscript{4} The broader coverage can also include the level of care, the number of accessible doctors, the waiting time, and other amenities.

While the market for primary health insurance has a high volume, the market for supplementary health insurance has a low volume but a very high strategic potential. Hence, the goals of those business segments might be different.\textsuperscript{5} It is very likely that the goal in the market for supplementary health insurance is output maximization, which can be explained as follows.

If people are allowed to switch between health insurance companies, a company only gets new customers if it provides products with a high quality-cost ratio which can be achieved by quality differentiation in the market for supplementary health insurance. There is a one-sided complementarity in the market for supplementary health insurance which results in a (one-sided) high cross-selling potential.\textsuperscript{6} One reason for having a cross-selling potential is that the possibility of purchasing the supplementary health insurance can be conditional on being primarily insured by the same health insurance company. Another reason for having a high cross-selling potential is that the insured may prefer to deal with only one firm instead of two. Due to the fact that buyers of these high quality services might switch to the same firm for their primary health insurance, we assume that

\textsuperscript{4}Already today many medical treatments are not covered by primary health insurance and the legal foundations of many countries states that primary health insurance has to provide a basic coverage only. In Germany, for instance, legislation directs that primary health care coverage must not exceed the necessary health care (§12 German Social Security Code). Either a medical area is completely excluded from basic coverage, such as alternative medicine or some dental health services, or the method of treatment covered by the primary health insurance is not the best possible. As another example, in Germany magnetic resonance imaging for diagnosing breast cancer is only covered by primary health insurance if a lump was discovered via mammography or breast ultrasound beforehand. Medical research shows that MRI can discover lumps at an earlier stage and is therefore the better medical treatment (Kuhl et al., 2005). Another example is the dual energy X-ray absorptiometry. It is not covered by primary health insurance if it is used in a preventive medical examination.

\textsuperscript{5}The goal in the market for primary health insurance might be some kind of profit maximization (even though they are officially not allowed to make any profits), budget maximization, sales maximization, market share maximization, or maybe the managers’ income. For literature that focuses on these goals, see Lackman and Craycraft (1974); Niskanen (1968); Fershtman (1985); Sklivas (1987); Gannon (1973); Denzau et al. (1985); Hansmann (1987); Xu and Birch (1999); Newhouse (1970); Merrill and Schneider (1966).

\textsuperscript{6}It is worth mentioning that a health insurance company is a priori indifferent between high and low risk people due to the implemented risk adjustment schemes. In countries that organize the basic health care coverage via competing non-profit health insurance companies we often have community rating insurers. Since these community rating insurers must charge a uniform premium from all individuals, one could argue that there is a high incentive to get the low risk people only. But this argument is only valid as long as there is no risk adjustment scheme that is sufficient to remove the cause of risk selection by closing the gap between expected cost and premium income. Since this problem is well known, governments have developed very comprehensive risk adjustment schemes. In Germany, for instance, the risk adjustment scheme relies on age, gender, and 80 costly diseases. It is therefore very difficult for a health insurance company to distinguish between good and bad risks. Hence, if the risk adjustment scheme is sufficient to close the gap a health insurance company is a priori indifferent between high and low risk people. For papers that deal with cream skimming, see Kifmann (2002); Kifmann (2006); Hansen and Keiding (2002), or Danzon (2002).
the firms are trying to sell as many supplementary health insurance policies as possible, which means they are output maximizers.\footnote{Assuming output maximization as the goal of non-profit organizations is not uncommon. Xu and Birch (1999), for instance, show that almost two out of three non-profit firms aim for output maximization facing a maximum loss constraint.}

As a result, output maximization in the market for supplementary health insurance can be used strategically\footnote{Using the goal of output maximization strategically to supplement the main goal in the market for primary health insurance is closely related to the strategic delegation literature. As an example, Fershtman and Judd (1987) consider a mixture of profits and sales, while Jansen et al. (2007) and Ritz (2008) focus on profits and market share in the context of strategic incentivisation.} to supplement the main goal in the market for primary health insurance (with its high monetary volume) which might be, for instance, budget maximization.\footnote{Steinberg (1986) shows that budget maximization is the main goal of health care companies. However, the major goal in the market for primary health insurance does not affect the strategy in the market for supplementary health insurance as long as more customers are helpful for achieving the goal in the market for primary health insurance. More customers are helpful if the firm maximizes its budget, its sales or its profits (as long as a costumer does not have a negative contribution margin which is reasonable to assume due to the fact that there are sophisticated risk adjustment schemes in the health care market.)}

In contrast to standard economic theory of complementary goods (Telser, 1979), supplementary health insurance is not sold below marginal costs for two reasons. First, this is prohibited by regulation in many countries.\footnote{In Germany, for example, see §53(9) German Social Security Code.} Second, due to the fact that there is only a one-sided cross-selling potential,\footnote{If people buy supplementary health insurance from a firm, it is very likely that they have primary insurance at the same company as well. However, just because a person is primarily insured by a company does not have to mean that this person buys supplementary health insurance from that company as well.} a cross-subsidization from primary health insurance to supplementary health insurance does not make sense. Cross-subsidization makes all people who do not have supplementary health insurance switch to a company that calculates without cross-subsidization. Hence, the supplementary health care business has to be self-financing. Therefore, the firms are facing a no loss constraint.\footnote{We could assume a maximum loss constraint as well. This might be appropriate if a consumer has a positive contribution margin in the basic health insurance. Since this assumption does not change the results in a qualitative way, we stick to the assumption that there is a no-loss constraint.}

The competition of output maximizing firms works very differently compared to the competition of profit maximizing firms. In the market for supplementary health insurance, the firms can provide products for the different needs of the consumers.\footnote{As an example, the access to a physicians' network specialized on diabetes is very valuable to people suffering from diabetes while it is of no use to others.} It is very well known that profit maximizing firms use product differentiation in order to relax price competition.\footnote{Differentiation by quality was first analyzed by Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Tirole (1988) for-profit maximizing firms. They show that differentiation takes place in order to relax price competition even if quality improvement is costless. If quality improvement turns out to be costly, differentiation is still a valuable tool for-profit maximizing firms (Ronnen, 1991; Motta, 1993; Boom, 1995; Aoki and Prusa, 1997; Lehmann-Grube, 1997, among others).} However, output maximizing firms do not fear price competition. In our study, we therefore analyze whether product differentiation is a useful tool for output
maximizing health insurance companies as well (e.g. to deter entry).

To keep our model as simple as possible, we assume that there are only two health insurance companies in the market. Of course, this is a simplification, but it still captures a very important fact: We can model competition. These two competing health insurance companies need to position themselves in a market segment for supplementary health insurance. This means that if a health insurance company wants to be a high quality provider, it cannot provide a product that is below the quality of its competitor. To capture that point, we assume that each firm provides only one quality. We further assume that the provision of high quality supplementary health insurance is costly. This assumption is very intuitive. Otherwise, there would be no trade-off between price and quality and the product could belong to the basic health care coverage as well. Since the provision of high quality supplementary health care is costly, there is a trade-off between price and quality.\footnote{As an example, in Germany, legislation directs that primary health care coverage must not exceed the necessary health care (§12 German Social Security Code). This means that every treatment that exceed the necessary health care must belong to supplementary health insurance. Furthermore, the German Social Security Code states that expenditures have to be compensated by earnings. Otherwise, premiums have to be adjusted. Therefore, cost-subsidization is not allowed.} Furthermore, we focus on variable costs of quality improvement since the main part of the product costs in the market for supplementary health insurance accrues at the moment of purchase by consumers.\footnote{In the health market, there are obviously high fixed costs due to R & D, but the health insurance company only has to pay for each application. As an example, if supplementary health insurance includes the access to high cost technologies, the health insurance company pays a given price for each high quality treatment. A higher quality therefore leads to a higher price. The unit costs for supplementary health insurance are therefore independent of output.} In our model, we solely focus on vertical differentiation without considering horizontal differentiation.\footnote{For papers that focus on horizontal differentiation see, for instance, Che and Gale (1997), Gannon (1973), and Devletoglou and Demetriou (1967).} This is reasonable since the relative transportation costs can be seen as sufficiently low.\footnote{E.g. for a high quality screening with a shortened waiting time (e.g. a few weeks) a longer travel time (e.g. one hour) will be most likely not be preponderant.} Furthermore, we assume that there is no significant adverse selection problem, nor is there a moral hazard problem on the side of the consumers. While these two phenomena are important in the health care market, they are beyond the scope of the current paper. This assumption is in line with Che and Gale (1997). Absent adverse selection and moral hazard, we can, without loss of generality, focus on health insurance companies that offer insurance without any coinsurance. Of course, this is a simplification. However, these assumptions are consistent with many supplementary health insurance policies, since they often do not impose deductibles.\footnote{Furthermore, people with a high preference for costly supplementary health insurance are most likely those people who might need treatment. Self-selection leads to a homogeneous group. Then, a difference in preferences can be interpreted as a difference in income (Tirole, 1988).}

We further simplify our model by omitting risk aversion. At a first glance, this might
seem unusual for a paper that deals with health insurance companies, but it is justifiable for supplementary health care. Supplementary health care has to be seen rather as a product or service than a financial contract in which there is a simple money transfer in the case of a loss event. Those high quality products (e.g. the level of care, the number of accessible doctors, the waiting time, and other amenities) are bought because they generate a positive utility to the consumer and not because the consumer wants to minimize risk. Despite the fact that there might be risk neutrality in the market for supplementary health care, there are some arguments as to why we observe a high demand for supplementary health care instead of an out of pocket market. The most important one is the transaction cost argument. First of all, a health insurance company has an information advantage concerning the optimal treatment possibilities and therefore has lower search costs. Second, and even more important, having bought supplementary health insurance (instead of paying for the high quality treatment out of pocket) is beneficial. This is due to the fact that, in the case of illness, the customer's bargaining position is much worse (this especially holds for all acute diseases) and the acquisition costs are higher (especially the non-monetary costs). It even may be impossible for the consumer to buy the product when he needs it (e.g. in case of unconsciousness). Another argument is that supplementary health care might be sold exclusively by a health insurance company. Hence, a consumer buys the product if the individual quality-cost ratio is sufficiently high. This rather depends on his preference parameter (e.g. his income) than on his risk attitude. By assuming risk neutral consumers, we can omit uncertainty about the health status as well.\footnote{Accounting for uncertainty changes the results neither qualitatively nor quantitatively, except that firms calculate with expected costs.}

Our results are the following. As long as the homogeneous sickness funds choose the qualities for their supplementary health insurance policies simultaneously, any equilibrium will be non-differentiated. Only if total demand is increasing in quality, both sickness funds provide the maximum possible quality. For decreasing total demand, the existence of an equilibrium depends on the consumers' sensitivity. If there is no equilibrium in the case of simultaneous quality choice, sequential quality competition leads to a differentiated equilibrium with a first mover advantage. For the welfare comparison of a public, a private, and a mixed competition we show that the market for supplementary health care should be organized via for-profit health insurance companies. We want to be careful with any policy implications, since we use a specific cost function and there are many factors that influence the welfare.\footnote{This is, for instance, the total cost of quality, the marginal cost of quality improvement or the peoples' preference parameter.}

Still, our paper points out a very important fact: Allowing a mixed competition might be Pareto-inferior.

The rest of this article proceeds as follows. Section 2 gives a literature review. Sec-
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Section 3 introduces our model framework. Section 4 examines the reactions of the market participants. In section 5, we focus on two different market settings. We analyze the simultaneous as well as the sequential competition. Section 6 is an application. In this section we compare the resulting welfare of a public, a private, and a mixed system. The concluding section, Section 7, summarizes our main results and briefly discusses future research.

2.2 Literature review

This section gives a literature review and states the main distinctions to our article. Related literature can be found in different directions. As a starting point, we take a look at standard vertical differentiation models and models that focus on non-profit firms. After that, we review models that focus on supplementary health insurance. Differentiation by quality was first analyzed by Gabszewicz and Thissen (1979), Shaked and Sutton (1982), and Tirole (1988) for-profit maximizing firms. They show that differentiation takes place in order to relax price competition even if quality improvement is costless. If quality improvement turns out to be costly, differentiation is still a valuable tool for-profit maximizing firms (Ronen, 1991; Motta, 1993; Boom, 1995; Aoki and Prusa, 1997; Lehmann-Grube, 1997, among others). But if profit maximization is not the goal of a company as is the case in our analysis, there is no reason to fear price competition. Therefore, the results of our analysis are different.\footnote{While it is intuitive that output maximizing sickness funds will not differentiate in quality if a quality improvement is costless, it is not obvious if sickness funds can use quality differentiation as a strategic tool in order to gain customers if quality improvement is costly.}

Related literature concerning the competition of non-profit firms can be found in the hospital market, since we can observe heterogeneous products and non-profit firms in this market as well. Research was done on horizontal product differentiation (Cremer et al., 1991; Matsushima and Matsumura, 2003; Matsumura and Matsushima, 2004; Sanjo, 2009) as well as on vertical product differentiation (Grilo, 1994; Herr, 2011; Beitia, 2003; Brekke et al., 2010). But those papers are only helpful as a guidance, since hospitals are not solely competing for costumers.\footnote{Studies dealing with hospital competition often assume a mixed duopoly competition where one hospital maximizes its profits while the other hospital maximizes either social surplus (Matsushima and Matsumura, 2003; Cremer et al., 1991; De Fraja and Delbono, 1989; Grilo, 1994) or its output facing a budget constraint (Newhouse, 1970; Merrill and Schneider, 1966, among others).} There are also interesting papers dealing with supplementary health insurance; Kifmann (2002), for instance, presents a model of a competitive health insurance market with two risk types and two exogenously given health benefits where individuals have to buy a basic benefit package from a community rating insurer. The aim of his paper is to show the incentive of cream skinning.\footnote{Kifmann (2006) compares the integration approach to the separation approach in the market for}
community rating insurers must charge a uniform premium for all individuals, there is a high incentive to get the low risk people only.\footnote{Kifmann (2006) assumes that there is no sufficient risk adjustment scheme.} One way to avoid cream skimming is to regulate the benefit package so that community rating insurers are not allowed to provide any additional benefits. Therefore, in a benchmark situation, Kifmann assumes that community rating insurers offer the basic benefit only and risk rating insurers provide supplementary health insurance. It is shown that low risk types can only be better off at the expense of high risk types if community rating insurers are allowed to offer the additional benefit and no additional regulations are taken. Both risk types can only be made better off at the same time if community rating health insurers offering the additional benefit are subsidized while those selling only the basic benefit are taxed. A closely related paper that is concerned with asymmetric information is one by Hansen and Keiding (2002). Even though the question is similar to that of Kifmann (2002), the conclusion of this paper is very different. The authors conclude that the compulsory scheme with voluntary supplementation is likely to be welfare superior to the pure compulsory scheme. These contradictory findings are possible because the two papers differ in their basic assumptions. For a thorough comparison, see Danzon (2002).

Kifmann (2002) and Hansen and Keiding (2002) concentrate on cream skimming due to asymmetric information. Focussing on cream skimming is reasonable if the health insurance companies are obliged to charge a uniform premium for all individuals and if risk adjustment schemes are not sufficient to remove the cause of risk-selection by closing the gap between expected costs and premium income. Our focus is different. We concentrate on a homogeneous group with a high preference for quality. In our special case, this is plausible for two reasons. First, risk adjustment schemes are getting more and more sophisticated, making it very difficult for the firms to discriminate between good and bad risks.\footnote{Self-selection leads to a homogeneous group. If this is the case, a difference in preferences can be interpreted as a difference in income (Tirole, 1988).} Second, people with a high preference for costly supplementary health insurance are most likely those who might need treatment.\footnote{Pauly (2004) gives the example that the best hospital in town does not have to be the cheapest or supplementary health insurance in order to show the incentives to cream skimming. It is shown that under the integration approach, insurers cream skim by selling supplementary health insurance to low risks at a discount. The integration approach can still be Pareto-superior if the cost savings due to the integration of basic and supplementary health insurance are sufficiently large.} Pauly (2004) reviews the concept of optimal quality in medical care from an economic viewpoint. His paper coincides with our assumption that there might be a trade-off between price and quality and that people have different needs.\footnote{Pauly (2004) gives the example that the best hospital in town does not have to be the cheapest or} In our study, we continue to analyze this trade-off.
Since this trade-off is solely between price and quality, we will not allow for the possibility of horizontal differentiation. Gannon (1973) presents a model that is concerned with horizontal differentiation of market share maximizing non-profit firms. He shows that in a duopolistic market the non-profit firms always choose the geographical center regardless of the consumers' individual demand. Therefore, market share maximizing firms do not differentiate in taste.

In Section 2.6 we compare the welfare of a public, private, and mixed system. Hence, we also should take a look at literature that is dealing with mixed competition and state the main distinction to our approach. Related literature can be found in different research areas. Studies dealing with hospital competition often assume a mixed duopoly where one hospital maximizes its profits while the other hospital maximizes either social surplus (Matsushima and Matsumura, 2003; Cremer et al., 1991; De Fraja and Delbono, 1989; Grilo, 1994) or its output facing a budget constraint (Newhouse, 1970; Merrill and Schneider, 1966, among others). The first contributions in this domain have focused on a homogeneous good market (Newhouse, 1970; De Fraja and Delbono, 1989). Since most markets are in fact differentiated, further research was done on horizontal product differentiation (Cremer et al., 1991; Matsushima and Matsumura, 2003; Matsumura and Matsushima, 2004) as well as on vertical product differentiation (Grilo, 1994; Herr, 2011). Many studies show that welfare can be improved by allowing a public firm to enter the market (Cremer et al., 1991; Nishimori and Ogawa, 2002; Grilo, 1994).

There also has been a lot of research on the efficiency of health care systems where studies focus on the interaction between public and private health care provision in general. For instance, Brekke and Sørgard (2007) and Rickman and McGuire (1999) analyze the organization of the National Health Service. They consider the physicians' incentives if they are allowed to work in the public sector as well as in the private sector. Other studies analyze the effects of physician dual practice applying a principal-agent framework (Gonzalez, 2004; Barros and Olivella, 2005; Biglaiser and Ma, 2007; Barros and Martinez-Giralt, 2002). Those papers focus on potential moral hazard problems in public provision like an increase in waiting time, cream skimming or variations of quality that might arise due to the physicians' activities in the private sector. The interaction between public and private providers when consumers differ in income has been analyzed by Jofre-Bonet vice versa and he claims that it is certain that the optimal level of quality, given quantity, will be different for different people, depending on the value they attach to quality.

First research in this field stems from Devletoglou and Demetriou (1967). Following Devletoglou (1965), they assumed that a threshold for the consumers reaction exists. For profit maximizing firms, this only holds in a very special case (Hotelling, 1929).

There also has been research on the desirability of mixed health care systems when distributional aspects matter (Besley and Coate, 1991; Marchand and Schroen, 2005). They assess the equity grounds for a mixed health care system. Public provision can work as such a sorting device if low income citizens choose the publicly provided good while high income citizens go private.
(2000).\textsuperscript{32} She considers a consumer who allocates his income between a single composite good and health services.

In contrast to our application in Section 2.6, studies dealing with the efficiency of the health care market often assume free public care and costly private care and focus on health care in general instead of supplementary health insurance only. In studies of hospital competition it is often assumed that prices are regulated and firms therefore rather compete in quality or location than in price. Furthermore, those studies often assume a covered market. In our application we neither assume a covered market, since not every person wants to buy supplementary health insurance, nor do we assume price regulation except for a no loss condition. Last but not least and as already mentioned earlier, we alter the public firm’s objective function, since budget maximization or social surplus maximization is not reasonable for sickness funds in the market for supplementary health care (due to the high cross-selling potential).

2.3 Model

Our model framework builds on the following basic assumptions. Two output maximizing non-profit sickness funds compete in a duopolistic market for supplementary health insurance. In the first stage of the game, the sickness funds choose their respective quality \( S_1 \) and \( S_2 \) either simultaneously or in sequential order. With common knowledge about the chosen qualities, the sickness funds choose their prices \( P_1 \) and \( P_2 \) simultaneously in the second stage of the game under the constraint of non-negative profits. This constraint means that the firms run a self-financing business in this market.\textsuperscript{33} The interval \([S, \bar{S}]\), with \( S = 0 \), gives the possible qualities the sickness funds can choose for their products.\textsuperscript{34} If the two sickness funds provide the same quality at the same price, the total demand is split between the two firms in equal parts. Since the main part of the product costs in the market for supplementary health insurance accrues at the moment of purchase by consumers, we focus on variable costs of quality improvement.\textsuperscript{35} The unit costs for supplementary health insurance with quality \( S \) are therefore not dependent of output and are described by the twice continuously differentiable function \( C \) with \( C'(S) > 0 \) for all

\textsuperscript{32}The differentiation of consumers' income is equivalent to our assumption of different taste parameters. Both assumptions result in a vertical differentiation framework, since the preference parameter can be seen as the inverse of the rate of marginal substitution between income and quality (Tirole, 1988, p. 96).

\textsuperscript{33}As an example, in Germany the Social Security Code prohibits cross-subsidization.

\textsuperscript{34}The term product is to be seen in a broad sense. It includes all kinds of services, e.g. one firm provides access to a small physicians' network while the competitor provides access to a large physicians network with lots of specialist doctors.

\textsuperscript{35}In the health market, there are obviously high fixed costs due to R & D, but the sickness fund only has to pay for each application; for instance, if supplementary health insurance includes the access to high cost technologies, the sickness fund pays a given price for each high quality treatment. A higher quality therefore leads to a higher price.
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The cost function is exogenous and identical for both sickness funds.

The consumers are described via their valuation of quality $\theta \in [\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} = 0$. The net utility of a consumer with preference $\theta$ from buying a supplementary health insurance with quality $S$ at price $P \geq C(S)$ is given by the Mussa-Rosen utility function $u_\theta(S, P) = \theta \cdot S - P$ (Mussa and Rosen, 1978). Consumers maximize their individual utility and buy one supplementary health insurance at most.\(^{36}\) Only if the utility is non-negative, the consumer buys the product, meaning we might be facing an uncovered market. If the consumer is indifferent between two products, he buys the one with the higher quality. The marginal consumer who has utility zero from buying supplementary health insurance with quality $S$ at price $P$ is given by

$$\theta_0(S, P) = \frac{P}{S}. \hspace{1cm} (2.1)$$

The consumer with preference $\theta_{\text{ind}}$ who is indifferent between the two products, is determined by solving $u_{\theta_{\text{ind}}}(S_1, P_1) = u_{\theta_{\text{ind}}}(S_2, P_2)$. This leads to

$$\theta_{\text{ind}}(S_1, S_2, P_1, P_2) = \frac{P_2 - P_1}{S_2 - S_1}. \hspace{1cm} (2.2)$$

Let $D_1$ denote the demand for the supplementary health insurance provided by Sickness Fund 1 and $D_2$ the demand of Sickness Fund 2. Then the maximization problem is given by

$$\begin{align*}
\frac{D_1}{S_1, P_1} & \rightarrow \max & s.t. & & P_1 \geq C(S_1), \\
\frac{D_2}{S_2, P_2} & \rightarrow \max & s.t. & & P_2 \geq C(S_2).
\end{align*} \hspace{1cm} (2.3)$$

Total demand is $TD = D_1 + D_2$.

2.4 The Sickness Funds’ Reactions

We solve the game via backward induction. In the second stage, the sickness funds simultaneously choose their prices for given and known qualities of their supplementary health insurance products in order to maximize their respective output. If the sickness funds choose the same quality $S = S_1 = S_2$, the only stable price equilibrium will be at $P = C(S)$; otherwise, the sickness funds have the incentive to underbid each other. This way, we only have to focus on the situation $S_1 \neq S_2$ and without loss of generality, we

\(^{36}\) Of course, consumers can buy more than one supplementary health insurance for different segments. Buying more than one supplementary health insurance for the same segment obviously does not make any sense and the competition has to be analyzed for each segment individually.
assume $S_1 > S_2$ from which $P_1 > P_2$ follows. We then have

$$D_1 = \overline{\theta} - \theta_{\text{ind}}(S_1, S_2, P_1, P_2)$$
$$D_2 = \theta_{\text{ind}}(S_1, S_2, P_1, P_2) - \theta_0(S_2, P_2),$$

(2.4)

(2.5)

as long as $0 \leq \theta_0 \leq \theta_{\text{ind}} \leq \overline{\theta}$ holds. In this case, the total demand is $TD(S_2, P_2) = \overline{\theta} - \theta_0(S_2, P_2)$. As one can easily see, the demand decreases if the firm increases its price.

In this case, we also have $P_i = C(S_i)$ for $i = 1, 2$. Hence, as the solution of the second stage game, we always have a price equal to the unit costs. This result is very intuitive since the sickness funds try to sell as many supplementary health insurance policies as possible and a higher price c.p. decreases the consumer’s utility. A decrease in the consumer’s utility leads to a decrease in sales since then there will be consumers who are not willing to buy the product anymore. In order to simplify notation, we suppress prices as arguments from now on.

In the first stage, the sickness funds choose their qualities for their supplementary health insurance products. To choose their qualities optimally, the sickness funds need to know how the consumers react to changes in quality. Note that total demand is now $TD(S_2) = TD(S_2, C(S_2)) = \overline{\theta} - \frac{C(S_2)}{S_2}$. Depending on the slope of the cost function, total demand either is increasing or decreasing in quality.\(^{37}\)

**Proposition 1.** If total demand is increasing in quality, there is a unique sub game perfect Nash equilibrium in pure strategies with no quality differentiation. Both sickness funds provide supplementary health insurance with the highest quality.

**Proof.** Since the total demand $TD = \overline{\theta} - \theta_0$ is increasing in quality, we have $\frac{d\theta_0(S)}{dS} \leq 0$. Thus, an increase in quality leads to more consumers buying the product as long as $\theta_0(S) \leq \overline{\theta}$. This means that no consumers buy the low quality product, which is why both firms provide a product with the maximum possible quality $\overline{S}$. \(\square\)

As we can see in Proposition 1, both firms have an incentive to provide the maximum quality if total demand is increasing in quality. Let us now consider a strictly decreasing total demand. We assume $TD(S) = \overline{\theta}$ and $TD(\overline{S}) = 0$.\(^{38}\) Analogously to the proof of Proposition 1, we now have $\frac{d\theta_0(S)}{dS} > 0$ so that an improvement in quality leads to less consumers buying the product due to the disproportionately high increase of the price.

\(^{37}\)If the cost function $C$ is strictly convex, the price for supplementary health insurance increases disproportionately high when quality is increased. Thus, less people are willing to buy supplementary health insurance and total demand is strictly decreasing.

\(^{38}\)The latter equality is intuitive since even if a higher quality was possible, there would be no consumers willing to buy the product. The former equality is for ease of calculation. Although we have $\overline{S} = 0$, according to l'Hospital’s rule the equality $TD(\overline{S}) = \overline{\theta}$ holds as long as we also have $C(\overline{S}) = 0$ and $C'(S) \to 0$ for $S \to \overline{S}$. 

To derive the optimal strategies, the sickness funds need further information about the consumers’ reaction on variations of the quality. Not only is the direction of the consumers’ reaction important, i.e., decreasing total demand, but also consumers’ sensitivity measured by $\theta_0$. The relationship between total demand and consumers’ sensitivity is shown in Figure 2.1. The figure shows the slope of two different cost functions. From the linear price-demand function and the slope of the cost function, the demand-quality function can be calculated directly due to the results from the second stage price competition. From this we further see that consumers’ sensitivity can be either increasing or decreasing in quality.

In order to analyze the strategies of the firms, we now show how Sickness Fund 2 could react to the quality $S_1$ chosen by Sickness Fund 1. Basically, Sickness Fund 2 has three options how to react. It can either choose to provide supplementary health insurance with a higher quality ($S_2 > S_1$) which we will call “overbidding”, choose the same quality ($S_2 = S_1$), which we will call “equalizing”, or choose a lower quality ($S_2 < S_1$), which we will call “underbidding”. The resulting demand of Sickness Fund 2 is given by

\[
D_2(S_1, S_2) = \begin{cases} 
\bar{\theta} - \min \left( \bar{\theta}, \theta_{md}(S_1, S_2) \right), & S_1 < S_2 \\
\frac{\bar{\theta} - \theta_0(S_2)}{2}, & S_1 = S_2 \\
\min \left( \bar{\theta}, \theta_{md}(S_1, S_2) \right) - \theta_0(S_2), & S_1 > S_2.
\end{cases}
\]

(2.6)

Obviously, if Sickness Fund 2 equalizes, the two firms share the market equally according to the assumption on the consumers’ behavior. Now, we need to take a closer look at the strategies of “overbidding” and “underbidding”.

**Overbidding**

If Sickness Fund 1 chooses the quality $S_1$ for its supplementary health insurance, Sickness Fund 2 can overbid with every quality $S_2 \in (S_1, \bar{S})$. In this case, it is not possible to derive an optimal overbidding strategy. For every $S_2 > S_1$, there exists $\tilde{S}_2 \in (S_1, S_2)$
with $D_2(S_1, \tilde{S}_2) > D_2(S_1, S_2)$.\textsuperscript{39} Thus, the closer the overbidding quality is to $S_1$, the higher is the output of Sickness Fund 2. The limiting overbidding strategy leads to $\lim_{S_2 \searrow S_1} \theta_{ind}(S_1, S_2) = C'(S_1)$. We will denote this limiting strategy by $S_1^+$ and call it “marginal overbidding”.\textsuperscript{40} This strategy is only reasonable, as long as $C'(S_1) < \tilde{\theta}$, otherwise there will be no demand for the supplementary health insurance of the overbidding sickness fund.

**Underbidding**

If Sickness Fund 1 chooses to provide supplementary health insurance with quality $S_1$, Sickness Fund 2 can underbid with every quality $S_2$ from the set $[S, S_1)$. The first order condition $\frac{\partial D_2(S_1, S_2)}{\partial S_2} = 0$ again does not need to have an interior solution on $(S, S_1)$. If that is the case, either underbidding with $S_2 = S$ is optimal, which we call minimal underbidding, or underbidding with a slightly lower quality than $S_1$ is the best underbidding strategy. Analogously to the case of overbidding, this will be called “marginal underbidding”, denoted by $S_1^-$.\textsuperscript{41} In general, we define the optimal underbidding quality by

$$r_u(S_1) := \arg \max_{S_2 < S_1} D_2(S_1, S_2).$$

We always have $\theta_{ind}(S_1, r_u(S_1)) \leq \tilde{\theta}$ because for $S_2$ with $\theta_{ind}(S_1, S_2) > \tilde{\theta}$, it is

$$D_2(S_1, S_2) = \tilde{\theta} - \theta_0(S_2),$$

which is decreasing in $S_2$. It is also intuitively clear that once $\theta_{ind}(S_1, S_2) = \tilde{\theta}$, a further increase in the underbidding quality $S_2$ will lead to a smaller output of Sickness Fund 2 since the demand for the supplementary health insurance of Sickness Fund 1 is already zero. As long as the inequality $\frac{\partial D_2(S_1, S_2)}{\partial S_2} < 0$ holds for $S_2 < S_1$, which is equivalent to

$$\frac{\theta_0(S_1) - \theta_0(S_2)}{S_1 - S_2} < \theta_0'(S_2),$$

(2.7)

Sickness Fund 2 has the incentive to decrease its quality. Due to $TD = \tilde{\theta} - \theta_0$, this especially is the case for all combinations of $S_2 < S_1$ if consumers’ sensitivity is decreasing (see Figure 2.1). Then, for all $S_1$, the optimal underbidding strategy is $S_2 = S$. If

\textsuperscript{39}In the case of decreasing total demand for $\tilde{S}_2 \in (S_1, S_2)$, we have $\theta_{ind}(S_1, \tilde{S}_2) < \theta_{ind}(S_1, S_2)$.

\textsuperscript{40}Technically, no $S_2 \in (S_1, S)$ satisfies the first order condition $\frac{\partial D_2(S_1, S_2)}{\partial S_2} = 0$. If the overbidding quality had to be chosen from $[S_1 + \delta, S]$ for $\delta > 0$, $S_2 = S_1 + \delta$ would be the optimal overbidding strategy. $\delta$ can be interpreted as a threshold required for quality differentiation being recognized by the consumers. For sufficiently small $\delta$, the results remain valid while the formulas would become more complicated and less intuitive. In the further analysis, we therefore assume that the overbidding firm chooses marginal overbidding.

\textsuperscript{41}Again, an optimal underbidding strategy technically does not exist in this case, but we adopt our concept of the limiting strategy to keep the calculations simple.
consumers’ sensitivity is increasing, choosing a higher underbidding quality \( S_2 \) always leads to an increase in demand for Sickness Fund 2. Thus, marginal underbidding is the optimal underbidding strategy. If consumers’ sensitivity is constant, the resulting demand for Sickness Fund 2 is independent from the chosen underbidding quality. We then assume that Sickness Fund 2 chooses the marginal underbidding quality for its supplementary health insurance.

**Optimal Reaction**

To decide which reaction is optimal we have to compare the resulting outputs of the two sickness funds. Special attention has to be paid to those qualities which leave the competitor indifferent between two or more strategies. First, we will compare overbidding and equalizing. Let the quality at which the competitor is indifferent between those two strategies be called \( S_{OE} \).\(^{42}\) Comparing the respective outputs of Sickness Fund 2 and solving the equation \( D_2(S_{OE}, S_{OE}+) = D_2(S_{OE}, S_{OE}) \) yields

\[
\frac{\theta + \theta_0(S_{OE})}{2} = C'(S_{OE}). \tag{2.8}
\]

If the left hand side of (2.8) is greater, overbidding dominates equalizing and vice versa. Now we examine at which quality Sickness Fund 2 is indifferent between underbidding and equalizing. This quality is called \( S_{UE} \). Comparing the respective outputs leads to

\[
\theta_{ind}(S_{UE}, r_u(S_{UE})) - \theta_0(r_u(S_{UE})) = \frac{\theta - \theta_0(S_{UE})}{2}. \tag{2.9}
\]

Underbidding dominates equalizing, if in (2.9) the left hand side is greater. Finally, we derive the quality that leaves Sickness Fund 2 indifferent between overbidding and underbidding. This quality is called \( S_{OU} \). The comparison of the outputs leads to

\[
\bar{\theta} - C'(S_{OU}) = \theta_{ind}(S_{OU}, r_u(S_{OU})) - \theta_0(r_u(S_{OU})). \tag{2.10}
\]

Here, overbidding dominates underbidding if the left hand side of (2.10) is greater.

Having analyzed the possible reactions and identified the qualities that leave the competitor indifferent, we are now able to derive the reaction functions of the sickness funds. Based on these reaction functions we can examine the interaction between the quality choices. Here we have to distinguish between simultaneous and sequential competition in the first stage.

\(^{42}\) \( S_{OE} \) is, without loss of generality, the chosen quality of Sickness Fund 1, leaving Sickness Fund 2 indifferent between overbidding and equalizing. Furthermore, we assume that in the case of indifference, the sickness funds choose the same quality for their supplementary health insurance.
2.5 First Stage Quality Competition

2.5.1 Simultaneous Competition

In this section, we consider a simultaneous first stage quality competition, which means the sickness funds are able to adjust the quality of their supplementary health insurance. While marginal overbidding is the only relevant overbidding strategy, the optimal underbidding strategy \( r_u \) depends on the consumers’ sensitivity. From (2.7) we know that in the case of increasing consumers’ sensitivity it is \( r_u(S) = S^- \) for all \( S \) with \( \theta_{ind}(S, S^-) \leq \bar{\theta} \) and \( r_u(S) \) according to \( \theta_{ind}(S, r_u(S)) = \bar{\theta} \) otherwise, while in the case of strictly decreasing consumers’ sensitivity we have \( r_u(S) = S^- \) for all \( S \).

Increasing Consumers’ Sensitivity

For the marginal underbidding strategy, (2.10) yields

\[
C'(S_{OU}) = \frac{\bar{\theta} + \theta_0(S_{OU})}{2}.
\]

According to (2.8), we then have \( S_{OU} = S_{OE} \). Obviously, this leads to \( S_{OU} = S_{OE} = S_{UE} \).

In order to analyze the sickness funds’ behavior, we need to derive the reaction functions.

**Lemma 2.** If consumers’ sensitivity is increasing, the reaction functions of the sickness funds are identical and given by

\[
\begin{align*}
\text{r}(S) = \begin{cases} 
S^+, & S < S_{OE} \\
S, & S = S_{OE} \\
r_u(S), & S > S_{OE}.
\end{cases}
\end{align*}
\]

(2.11)

**Proof.** According to (2.7), the only relevant underbidding strategy is given by the marginal underbidding \( r_u(S_1) = S_1^- \) on \( \{S_1 \mid \theta_{ind}(S_1, S_1^-) \leq \bar{\theta} \} \) and choosing the quality \( r_u(S_1) = \inf\{S_2 \mid S_2 < S_1, \theta_{ind}(S_1, S_2) \geq \bar{\theta}\} \) on the set \( \{S_1 \mid \theta_{ind}(S_1, S_1^-) > \bar{\theta}\} \). We further have \( S_{OE} \in \{S_1 \mid \theta_{ind}(S_1, S_1^-) \leq \bar{\theta}\} \) since otherwise there would be no demand in case of overbidding. Therefore, on \( \{S_1 \mid \theta_{ind}(S_1, S_1^-) > \bar{\theta}\} \), Sickness Fund 2 will never be indifferent between overbidding and equalizing. So on \( \{S_2, S_{OE}\} \), overbidding dominates underbidding and equalizing while on \( (S_{OE}, \bar{S}) \), underbidding dominates overbidding and equalizing. In \( S_{OE} \), all three strategies yield the same output and the sickness funds equalize. Thus, we yield the reaction function (2.11). \( \square \)

To improve readability, we denote the reaction function of Sickness Fund 1 and Sickness Fund 2 by \( r_1 \) and \( r_2 \), respectively, with \( r_1 = r_2 = r \). Now that we have derived the reaction function, we are able to examine whether equilibrium strategies exist.

---

43 Note that \( \lim_{S \to S_{OU}} \theta_0(S) = \theta_0(S_{OU}) \).
Proposition 3. If consumers’ sensitivity is increasing, there is a unique subgame perfect Nash equilibrium in pure strategies with no quality differentiation.

Proof. The two sickness funds have the same reaction function given by (2.11). Therefore, \( r_1(r_2(S)) = r_2(r_1(S)) = S \) holds if and only if \( S = S_{OE} \). Thus, \((S_{OE}, S_{OE})\) is the unique Nash equilibrium in pure strategies.

As an example, let the unit cost function be given by \( C(S) = S^\alpha \) with \( \alpha > 1 \), so the total demand is decreasing. Let further \( \bar{\theta} = 1 \) and \( \bar{S} = 1 \) so that we have \( \theta_0(\bar{S}) = \bar{\theta} \). Consumers’ sensitivity then is \( \theta_0'(S) = (\alpha - 1)S^{\alpha - 2} \) and \( S_{OE} = (1/(2\alpha - 1))^{1/(\alpha - 1)} \) is the equilibrium quality for \( \alpha \geq 2 \).

Let us take a look at the reaction function of Sickness Fund 2 in Figure 2.2: As we have seen before, if Sickness Fund 1 chooses a quality \( S_1 \in [\underline{S}, S_{OE}) = [0, \frac{1}{\sqrt{3}}) \) for its supplementary health insurance, marginal overbidding is the optimal reaction. If \( S_1 = S_{OE} = \frac{1}{\sqrt{5}} \), Sickness Fund 2 is indifferent between overbidding, underbidding, and equalizing and, according to (2.11), reacts with equalizing. In the case of \( S_1 \in (S_{OE}, \bar{S}] = \left(\frac{1}{\sqrt{3}}, 1\right] \), Sickness Fund 2 reacts with underbidding. On \( \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{3}}\right] \), marginal underbidding is the optimal strategy. If we have \( S_1 \in \left(\frac{1}{\sqrt{3}}, 1\right] \), marginal underbidding is not optimal anymore, since it is \( C'(S_1) > C'\left(\frac{1}{\sqrt{3}}\right) = 1 = \bar{\theta} \). Here we have \( r_2(S_1) = \frac{1}{2} \left(\sqrt{4 - 3S_1^2} - S_1\right) \), which leads to \( \theta_{ind}(S_1, r_2(S_1)) = \bar{\theta} \) for all \( S_1 \in \left(\frac{1}{\sqrt{3}}, 1\right] \). Since Sickness Fund 1 has the same reaction function as Sickness Fund 2, the two reaction functions intersect only in \((S_{OE}, S_{OE}) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\), which is the unique Nash equilibrium in pure strategies for the special cost function.

### Figure 2.2: Reaction functions of Sickness Fund 1 (solid) and Sickness Fund 2 (dashed) with \( \alpha = 3 \).
Strictly Decreasing Consumers’ Sensitivity

According to (2.7), in the case of strictly decreasing consumers’ sensitivity we have \( r_u(S) = \underline{S} \) for all \( S \in (\underline{S}, \overline{S}] \). From (2.10) we get

\[
\overline{\theta} - \theta_0(S_{OU}) = C'(S_{OU}).
\] (2.12)

**Lemma 4.** If consumers’ sensitivity is strictly decreasing, we have \( S_{UE} < S_{OU} < S_{OE} \) and the sickness funds’ reaction function is given by

\[
r(S) : [\underline{S}, \overline{S}] \rightarrow [\underline{S}, \overline{S}],
S \mapsto r(S) := \begin{cases} S+, & S < S_{OU}, \\ \underline{S}, & S \geq S_{OU}. \end{cases}
\] (2.13)

**Proof.** Since consumers’ sensitivity is strictly decreasing

\[
\theta'_0(S_1) < \frac{\theta_0(S_1) - \theta_0(S_2)}{S_1 - S_2}
\]
holds for all \( S_2 < S_1 \); especially for \( S_1 = S_{OU} \) and \( S_2 = \underline{S} \), we have

\[
\theta'_0(S_{OU}) < \frac{\theta_0(S_{OU})}{S_{OU}}
\]

\[
\iff C'(S_{OU}) - \theta_0(S_{OU}) < \theta_0(S_{OU})
\]

\[
\iff C'(S_{OU}) < 2\theta_0(S_{OU}).
\]

If we now had \( S_{OU} < S_{OE} \), we would get

\[
S_{OU} < S_{UE} \overset{(2.9)}{\iff} \theta(S_{OU}) < \frac{\overline{\theta}}{3}
\]

\[
\overset{(2.12)}{\iff} C'(S_{OU}) > \frac{2}{3}\overline{\theta} > 2\theta_0(S_{OU}),
\]

which is contradictory to a decreasing consumers’ sensitivity. Therefore, we have

\[
S_{UE} < S_{OU} \quad \iff \quad \theta_0(S_{UE}) < \theta_0(S_{OU})
\]

\[
\overset{(2.9),(2.12)}{\iff} \quad \frac{\overline{\theta} - \theta_0(S_{UE})}{2} < \overline{\theta} - C'(S_{OU})
\]

\[
\iff \quad \frac{\overline{\theta} - \theta_0(S_{OU})}{2} < \overline{\theta} - C'(S_{OU})
\]

\[
\iff \quad C'(S_{OU}) < \frac{\overline{\theta} + \theta_0(S_{OU})}{2}
\]

\[
\overset{(2.8)}{\iff} \quad S_{OU} < S_{OE}.
\]
Thus, for the optimal reactions of the sickness funds we now have the following rules for behavior: On $[S, S_{UE}]$, overbidding dominates equalizing and equalizing dominates underbidding. On $(S_{UE}, S_{OU})$, overbidding dominates underbidding and underbidding dominates equalizing. On $(S_{OU}, S_{OE})$, underbidding dominates overbidding and overbidding dominates equalizing. On $(S_{OE}, \overline{S})$, underbidding dominates equalizing and equalizing dominates overbidding. Thus, we yield the reaction function (2.13).

For the special cost function $C(S) = S^{\alpha}$ with $\alpha < 2$, we have $S_{UE} = (1/3)^{1/(\alpha-1)} < S_{OU} = (1/(\alpha + 1))^{1/(\alpha-1)} < S_{OE} = (1/(2\alpha - 1))^{1/(\alpha-1)}$. The shape of the reaction functions is shown in Figure 2.3. One can see that in this special case of $C(S) = S^{3/2}$ no equilibrium exists since the reaction functions do not intersect. In general, the following result holds.

**Proposition 5.** If consumers' sensitivity is strictly decreasing, there is no Nash equilibrium in pure strategies.

**Proof.** If Sickness Fund 1 chooses $S_1 \in [S, S_{OU}]$, overbidding is the dominant strategy. Thus, if $(S_1^*, S_2^*)$ was an equilibrium, it would have to be $(S_1^*, S_2^*) \in (S_{OU}, \overline{S})^2$. For $S_1^* \in (S_{OU}, \overline{S})$, according to (2.13), we have $r_2(S_1^*) = \overline{S} \notin (S_{OU}, \overline{S})$. Then again we have $r_1(r_2(S_1^*)) = r_1(\overline{S}) = S + \notin (S_{OU}, \overline{S})$. Hence, $S_1^*$ is no equilibrium strategy for Sickness Fund 1. Since this also holds for Sickness Fund 2, a Nash equilibrium in pure strategies does not exist. 

In this section we derived sufficient conditions for the existence of a unique Nash equilibrium in pure strategies in the case of simultaneous first stage quality competition. The sickness funds were able to adjust the quality of their supplementary health insurance...
policies whereas in the next section, the sickness funds have to commit themselves to a certain quality for their supplementary health insurance policies.

2.5.2 Sequential Competition

In this section, we consider a sequential first stage quality competition while we still assume a simultaneous second stage price competition, meaning the sickness funds enter the market simultaneously. We further assume that the sickness funds commit themselves to the chosen quality. This means the quality leader cannot adjust its quality after observing the quality chosen by the follower. We will again solve the problem via backward induction. The price competition in the second stage remains the same, while in the first stage, we now have a subgame of sequential quality choices. The leader will anticipate the follower’s reaction and therefore choose the quality that maximizes his output given the optimal reaction by the follower. Therefore, the leader’s output might differ from the follower’s. Without loss of generality, let Sickness Fund 1 be the leader and Sickness Fund 2 be the follower. Let the reaction function of Sickness Fund 2 be denoted by $r_2$ as before, then for any given quality choice $S_1$ of Sickness Fund 1, the optimal answer of Sickness Fund 2 is choosing the quality $S_2 = r_2(S_1)$ for its supplementary health insurance. Knowing this, Sickness Fund 1 faces the maximization problem

$$D_1(S_1, r_2(S_1)) \xrightarrow{S_1} \max.$$  \hfill (2.14)

In this section we will focus on a decreasing total demand.\textsuperscript{45}

Increasing Consumers’ Sensitivity

The reaction function of the follower corresponds to the reaction function derived in Lemma 2 in the preceding section.

**Proposition 6.** If consumers’ sensitivity is increasing, there is a unique subgame perfect Nash equilibrium in pure strategies with no quality differentiation. There is neither a first nor a second mover advantage.

*Proof.* If Sickness Fund 1 decides to be the high quality provider, it has to choose a quality $S_1 > S_{OE}$ for its supplementary health insurance. Of course the range of possible qualities in this case is limited by the condition $\theta_{\text{ind}}(S_1, r_2(S_1)) < \bar{\theta}$. For those qualities, we have $r_2(S_1) = S_1 - \sigma$ so that the resulting output is given by

$$D_1(S_1, r_2(S_1)) = \bar{\theta} - \theta_{\text{ind}}(S_1, S_1 - \sigma) = \bar{\theta} - C'(S_1),$$

\textsuperscript{45}If the total demand is increasing, the result from Proposition 1 remains valid.
which is obviously decreasing in $S_1$. If, on the other hand, Sickness Fund 1 decides to provide supplementary health insurance with a low quality, it has to choose $S_1 < S_{OE}$. Then, of course, it is $r_2(S_1) = S_1+$ and we have

$$D_1(S_1, r_2(S_1)) = \theta_{ind}(S_1, S_1+) - \theta_0(S_1) = C'(S_1) - \frac{C(S_1)}{S_1} = S_1 \theta_0'(S_1).$$

Since consumers’ sensitivity is increasing, derivation of this term shows that the output is increasing in $S_1$. Hence, Sickness Fund 1 will provide supplementary health insurance with the quality $S_1 = S_{OE}$. According to (2.11), Sickness Fund 2 also chooses the quality $S_2 = S_{OE}$. Thus, there is no quality differentiation and both sickness funds gain the same demand since they share the market equally.

![Figure 2.4: Output of Sickness Fund 1 (solid) and Sickness Fund 2 (dashed) with optimal reaction of Sickness Fund 2 with $\alpha = 3$.](image)

In Figure 2.4, we see the resulting outputs of the two sickness funds plotted against the quality choice of Sickness Fund 1. As we can see, Sickness Fund 1 maximizes its output by choosing the quality $S_1 = S_{OE} = \frac{1}{\sqrt{3}}$ for its supplementary health insurance, resulting in an output of $\frac{2}{5}$. If Sickness Fund 1 chooses a different quality, the resulting output would be less than $\frac{2}{5}$. A quality of $S_1 > \frac{1}{\sqrt{3}}$ would leave Sickness Fund 1 with no output because Sickness Fund 2 would provide the quality $S_2$ so that $\theta_{ind}(S_1, S_2) = \overline{\theta}$. Since Sickness Fund 2 provides the same quality as Sickness Fund 1, the resulting output is also $\frac{2}{5}$, which leaves the market uncovered.

**Strictly Decreasing Consumers’ Sensitivity**

Proposition 5 states that there is no equilibrium in pure strategies if the sickness funds choose their qualities simultaneously. In the sequential quality competition, the leader chooses his quality and commits himself. The follower reacts with his best response, so there will be an equilibrium. Now the reaction function of the follower corresponds to the reaction function given in Lemma 4.
Proposition 7. If consumers’ sensitivity is strictly decreasing, there is a unique subgame perfect Nash equilibrium in pure strategies with quality differentiation and a first mover advantage.

Proof. First we will show that Sickness Fund 1 chooses $S_1 = S_{OU}$. If Sickness Fund 1 decides to provide the high quality supplementary health insurance, the output is obviously decreasing in $S_1$, since we have

$$D_1(S_1, r_2(S_1)) = D_1(S_1, S) = \bar{\theta} - \theta_0(S_1)$$

for $S_1 \geq S_{OU}$; so Sickness Fund 1 will provide supplementary health insurance with a quality not higher than $S_{OU}$. It will also at least provide $S_{OU}$, since for $S_1 < S_{OU}$ and strictly convex $C$ we have

$$D_1(S_1, r_2(S_1)) = C'(S_1) - \theta_0(S_1) < C'(S_1)$$

$$< C'(S_{OU}) \overset{[2,12]}{=} \bar{\theta} - \theta_0(S_{OU})$$

$$= D_1(S_{OU}, r_2(S_{OU})).$$

Thus, Sickness Fund 1 chooses $S_1 = S_{OU}$ and, according to Lemma 4, Sickness Fund 2 responds with $S_2 = \underline{S}$. The resulting outputs in the equilibrium $(S_{OU}, S)$ are $D_2(S_{OU}, S) = \theta_0(S_{OU})$ and $D_1(S_{OU}, S) = \bar{\theta} - \theta_0(S_{OU})$. We have

$$D_1(S_{OU}, S) > D_2(S_{OU}, S)$$

$$\iff \bar{\theta} - \theta_0(S_{OU}) > \theta_0(S_{OU})$$

$$\overset{[2,12]}{\iff} \bar{\theta} - \theta_0(S_{OU}) > \bar{\theta} - C'(S_{OU})$$

$$\iff C'(S_{OU}) - \theta_0(S_{OU}) > 0.$$

Since $C$ is strictly convex, this shows the first mover advantage. 

Figure 2.5 shows the resulting outputs of the two sickness funds against the quality choice of Sickness Fund 1. As we can see, Sickness Fund 1 maximizes its output by choosing the quality $S_1 = S_{OU} = \frac{1}{20}$ for its supplementary health insurance, resulting in an output of $D_1(S_{OU}, S) = \frac{3}{5}$. At $S_1 = S_{OU}$, the output of Sickness Fund 1 is noncontinuous because at this quality, the optimal reaction of Sickness Fund 2 changes from marginal overbidding to underbidding with $S_2 = \underline{S}$. In equilibrium, we can clearly see the first mover advantage of Sickness Fund 1. Furthermore, the market is fully covered.
2.6 Application: Comparison of a Public, Private, and Mixed System

2.6.1 Motivation and specification

We now want to answer the question of how the market for supplementary health care should be organized. In order to do so, we compare the welfare of a public, a private and a mixed system. The competition of non-profit sickness funds is very different compared to the competition of for-profit health insurance companies. One major difference is that sickness funds do not fear price competition. This is in contrast to the competition of for-profit organizations. They use differentiation in order to relax price competition.\(^{46}\)

The provision of homogeneous products would lead to a zero profit equilibrium. As long as the people have different preferences for quality the answer to the question of how the market should be organized is not that simple, since there is a trade-off between product differentiation and taking a mark-up. People like differentiation, which they do not get if supplementary health insurance is provided by sickness funds. But people like cheap prices as well, and as long as the health insurance companies can differentiate they are able to take a mark-up. So at a first glance it is not clear at all how the market for supplementary health insurance should be organized. Another possible way to organize this market might be allowing a mixed competition.\(^{47}\)

Footnotes:

\(^{46}\) Quality differentiation was first analyzed by Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Tirole (1988) for-profit maximizing firms. They show that differentiation takes place in order to relax price competition even if a quality improvement is costless. If quality improvement turns out to be costly, differentiation is still a valuable tool for-profit maximizing firms (Ronen, 1991; Motta, 1993; Boom, 1995; Aoki and Prusa, 1997; Lehmann-Grube, 1997, among others).

\(^{47}\) A mixed competition is defined as competition in a market in which two or more firms with different objectives co-exist. For surveys of the literature on mixed oligopolies compare De Fraja and Delbono (1990) or Nett (1993). We can observe a mixed competition for instance in the German health care market. In 2007 the Social Health Insurance Competition Strengthening Act was adopted. One main part of this act was that supplementary health insurance can be provided by private health insurance...
for-profit organizations. In such a situation we obviously need to have differentiation, since a non-differentiated situation will never be optimal for the for-profit organization and therefore can not be a Nash equilibrium. Furthermore, not all the firms take a mark-up.

In order to answer the question of how the competition should be organized we use the same basic model as before. At the first stage of the game the firms simultaneously choose their respective qualities \( S_1 \) and \( S_2 \) for their supplementary health insurance products. At the second stage of the game the firms simultaneously\(^{46}\) choose their prices \( P_1 \) and \( P_2 \) under the constraint of nonnegative profits and under full information about the chosen qualities. The interval \( [\underline{S}, \overline{S}] = [0, 1] \) gives the possible qualities and the consumers are described via their valuation of quality \( \theta \in [\underline{\theta}, \overline{\theta}] = [0, 1] \). The unit costs \( C \) of supplementary health insurance are independent of the number of insured and is now given by \( C(S) = S^3 \). The cost function is exogenous and identical for both firms.\(^{49}\) Knowing that we have \( u_\theta(S, P) = \theta \cdot S - P, \ \theta_0(S, P) = \frac{P}{\overline{S}} \), and \( \theta_{\text{ind}}(S_1, S_2, P_1, P_2) = \frac{P_1 - P_2}{S_1 - S_2} \) the demand for the product of firm \( i \), with \( i \in \{1, 2\} \) and \( j \in \{1, 2\} \setminus \{i\} \), is described by\(^{50}\)

\[
D_i(S_1, S_2, P_1, P_2) := \begin{cases} 
\overline{\theta} - \max(\theta_{\text{ind}}(S_1, S_2, P_1, P_2), \theta_0(S_i, P_i)), & S_i > S_j \\
\frac{\theta_0(S_i, P_i)}{2}, & S_i = S_j \\
\theta_{\text{ind}}(S_1, S_2, P_1, P_2) - \theta_0(S_i, P_i), & S_i < S_j.
\end{cases} \tag{2.15}
\]

The profit of firm \( i, i \in \{1, 2\} \), is then given by

\[
\pi_i(S_1, S_2, P_1, P_2) := D_i(S_1, S_2, P_1, P_2) \cdot (P_i - C(S_i)). \tag{2.16}
\]

The resulting maximization problem highly depends on the firms’ objective functions and is therefore described in the respective sections.

\(^{46}\) In this section we focus on simultaneous competition without considering a sequential competition.\(^{46}\) We assume this special cost function in order to keep the analysis simple. Our results remain valid also for cost functions \( C(S) = S^\alpha \) with \( \alpha \) between 2 and 3.2. We choose \( \alpha = 3 \) since it is the only integer such that the solutions are robust against variations of \( \alpha \) in both directions.\(^{46}\) For \( S_i = S_j \) the mathematical correct notation is \( \frac{\theta_0(S_i, P_i)}{2} \left( 1 + 1_{(P_i < P_j)} - 1_{(P_i > P_j)} \right) \). The formula in (2.15) assumes \( P_1 = P_2 \). For technical reasons let \( \theta_0(S, P) = \min(\frac{P}{S}, 1) \) and \( \theta_{\text{ind}}(S_1, S_2, P_1, P_2) = \min \left( \max \left( \frac{P_1 - P_2}{S_1 - S_2}, 0 \right), 1 \right) \) in (2.15). If the two firms provide the same quality at the same price the total demand is split between the firms in equal parts.
2.6.2 Two competing sickness funds

In this section let firm 1 and firm 2 be output maximizing non-profit sickness funds. The optimization problem is then given by

\[ D_1(S_1, S_2, P_1, P_2) \xrightarrow{S_1, P_1} \max \quad P_1 \geq C(S_1), \]
\[ D_2(S_1, S_2, P_1, P_2) \xrightarrow{S_2, P_2} \max \quad P_2 \geq C(S_2). \]

Solving this maximization problem we yield

\[
\begin{align*}
S_1^* &= \frac{1}{\sqrt{5}} \\
S_2^* &= \frac{1}{\sqrt{5}} \\
P_1^* &= \frac{1}{\sqrt{125}} \\
P_2^* &= \frac{1}{\sqrt{125}} \\
D_1^* &= 0.4 \\
D_2^* &= 0.4 \\
\pi_1^* &= 0 \\
\pi_2^* &= 0.
\end{align*}
\]

While we have a non-differentiated equilibrium in the competition of two output maximizing sickness funds it is well known that profit maximizing firms differentiate to relax price competition (Gabszewicz and Thisse, 1979).

2.6.3 Two competing health insurance companies

In this section let firm 1 and firm 2 be profit maximizing private health insurance companies. The optimization problem is then given by

\[ \pi_1(S_1, S_2, P_1, P_2) \xrightarrow{S_1, P_1} \max \quad P_1 \geq C(S_1), \]
\[ \pi_2(S_1, S_2, P_1, P_2) \xrightarrow{S_2, P_2} \max \quad P_2 \geq C(S_2). \]

If without loss of generality firm 1 is the high quality provider. Solving this maximization problem we yield

\[
\begin{align*}
S_1^* &\approx 0.515784 \\
S_2^* &\approx 0.291495 \\
P_1^* &\approx 0.217703 \\
P_2^* &\approx 0.073901 \\
D_1^* &\approx 0.358856 \\
D_2^* &\approx 0.387619 \\
\pi_1^* &\approx 0.028883 \\
\pi_2^* &\approx 0.019045.
\end{align*}
\]

We now have analyzed the competition for the cases where the two firms aim for the same goal. While two output maximizing sickness funds do not differentiate in equilibrium, profit maximizing health insurance companies do differentiate. In the following section we analyze the competition, if the firms have different objectives.
2.6.4 Competition in a mixed duopoly

In this section let firm 1 be a profit maximizing health insurance company and firm 2 an output maximizing sickness fund. The optimization problem is given by

\[
\begin{align*}
\pi_1(S_1, S_2, P_1, P_2) & \frac{S_1, P_1}{\max} \quad \text{s.t.} \quad P_1 \geq C(S_1), \\
D_2(S_1, S_2, P_1, P_2) & \frac{S_2, P_2}{\max} \quad \text{s.t.} \quad P_2 \geq C(S_2).
\end{align*}
\]

In the second stage price competition the sickness fund chooses \( P_2^*(S_2) = C(S_2) \). The health insurance company chooses its price according to

\[
P_1^*(S_1, S_2) = \begin{cases} (S_1 - S_2)^2 [\bar{\theta} + C(S_2) + C(S_1)] \frac{2}{S_1C(S_1) + S_2C(S_1)} & , \quad S_1 \geq S_2 \\
\frac{S_2 - S_1}{4} \prod_{i=1}^{2} (\theta_{ind}(S_1, S_2, C(S_1), C(S_2)) - \theta_0(S_1, C(S_i))) & , \quad S_1 < S_2.
\end{cases}
\]

In the first stage quality competition we can now take a look at the reduced form objective functions. The reduced form profit is

\[
\pi_1^*(S_1, S_2) = \begin{cases} \frac{(S_1 - S_2)}{4} (\bar{\theta} - \theta_{ind}(S_1, S_2, C(S_1), C(S_2)))^2 & , \quad S_1 \geq S_2 \\
\frac{(S_2 - S_1)}{4} \prod_{i=1}^{2} (\theta_{ind}(S_1, S_2, C(S_1), C(S_2)) - \theta_0(S_1, C(S_i))) & , \quad S_1 < S_2.
\end{cases}
\]

and the reduced form demand is

\[
D_2^*(S_1, S_2) = \begin{cases} \frac{\bar{\theta} + \theta_{ind}(S_1, S_2, C(S_1), C(S_2)) - \theta_0(S_2, C(S_2))}{2} & , \quad S_2 \leq S_1 \\
\frac{-\bar{\theta} + \theta_{ind}(S_1, S_2, C(S_1), C(S_2)) + \theta_0(S_2, C(S_2))}{2} & , \quad S_2 > S_1.
\end{cases}
\]

For \( C(S) = S^3 \) this yields

\[
\pi_1^*(S_1, S_2) = \begin{cases} \frac{1}{4} (1 - S_1^2 - S_1S_2 - S_2^2)^2 & , \quad S_1 \geq S_2 \\
\frac{S_1 - S_2}{S_1S_2S_1 - S_1(S_2 + S_1)^2} & , \quad S_1 < S_2
\end{cases}
\]

\[
D_2^*(S_1, S_2) = \begin{cases} \frac{1 - S_2^2 + S_1S_2 + S_1^2}{2} & , \quad S_2 \leq S_1 \\
\frac{-2S_2^2 - S_1S_2 - S_1^2}{2} & , \quad S_2 > S_1
\end{cases}
\]

Figure 2.6 shows the reaction functions of the two firms derived from the reduced form objective functions.

Describing the reaction functions we begin with the sickness fund. On 1 the provision of a supplementary health insurance with a marginally higher quality is dominant. On 2 and 3 underbidding is the dominant strategy. On 2 the optimal underbidding strategy is determined by solving \( \frac{\partial D_2^*}{\partial S_2} = 0 \) and on 3 the underbidding quality \( S_2 \) is chosen such that \( P_1^*(S_1, S_2) = C(S_1) \) which leaves the health insurance company with no profit. 4, 5 and
Figure 2.6: Reaction functions of the health insurance company (dashed) and the sickness fund (solid).

6 show the reaction of the health insurance company on a given quality of the sickness fund. On 4 overbidding is dominant. Starting with the monopoly quality the optimal overbidding quality is increasing in $S_2$. On 5 underbidding is dominant. The higher $S_2$ is, the higher is $S_1$ until $S_1$ reaches the monopoly quality.\textsuperscript{51} On 6 underbidding with the monopoly quality is the optimal reaction. Since the sickness fund still gains a positive demand, the optimal price is lower than the monopoly price but increasing in $S_2$. For $S_2 = \bar{S} = 1$ the monopoly price is reached again. As the solution of \eqref{2.21} we yield a differentiated subgame perfect Nash equilibrium in pure strategies with

$$
\begin{align*}
S_1^* &\approx 0.485071 & P_1^* &\approx 0.185468 \\
S_2^* &\approx 0.242536 & P_2^* &\approx 0.014267 \\
D_1^* &\approx 0.294118 & \pi_1^* &\approx 0.020981 \\
D_2^* &\approx 0.647059 & \pi_2^* & = 0.
\end{align*}
$$

\textsuperscript{(2.22)}

So in equilibrium the health insurance company provides a higher quality than the sickness fund.

\subsection*{2.6.5 Welfare Implications}

From a policy perspective it is important to understand the welfare implications of different kinds of competition. For our special cost function we want to show which competition leads to the highest welfare. As a benchmark case we first have a look at the welfare when

\textsuperscript{51}Note that due to $P_1 > C(S_1)$ the sickness fund gains a positive demand in this situation. Only for $P_1 = C(S_1)$ we would have $\theta_{ind}(S_1, S_2, P_1, C(S_2)) = \bar{\theta}$. 

supplementary health care is provided by a social planner. We assume a social planner whose objective is the maximization of the gross benefit of the consumers reduced by the costs of the supplementary health care.\footnote{Ghosh and Morita (2007) also used this definition of welfare in their work.} For any given combination of $S_1, S_2, P_1$ and $P_2$ with $S_2 < S_1$ and $P_2 < P_1$ the total surplus is described by\footnote{In the following we denote $\theta_0 = \theta_0(S_2, P_2)$ and $\theta_{ind} = \theta_{ind}(S_1, S_2, P_1, P_2)$.} \begin{equation}
 W := \int_{\theta_0}^{\theta_{ind}} u_\theta(S_2, P_2) \, d\theta + \int_{\theta_{ind}}^{\bar{\theta}} u_\theta(S_1, P_1) \, d\theta + \pi_1 + \pi_2. \tag{2.23}
 \end{equation}

The social optimum

As a benchmark we derive the first best solution which is given by maximizing\footnote{As usual, we assume that the social planner does not allow a mark-up. We therefore have $C(S_i) = P_i$.}

\begin{equation}
 W_{soc}(S_1, S_2, C(S_1), C(S_2)) = \int_{\theta_0}^{\theta_{ind}} u_\theta(S_2, C(S_2)) \, d\theta + \int_{\theta_{ind}}^{\bar{\theta}} u_\theta(S_1, C(S_1)) \, d\theta. \tag{2.24}
 \end{equation}

Maximizing (2.24) with respect to $S_1$ and $S_2$ yields

\begin{align*}
 S_1^* &\approx 0.503186 & P_1^* &\approx 0.127404 \\
 S_2^* &\approx 0.322234 & P_2^* &\approx 0.033459 \\
 D_1^* &\approx 0.480826 & \pi_1^* & = 0 \\
 D_2^* &\approx 0.415339 & \pi_2^* & = 0.
\end{align*}

The social optimum then is

\begin{equation}
 W_{soc}(S_1^*, S_2^*, P_1^*, P_2^*) \approx 0.150312.
 \end{equation}

In the competition of two sickness funds there is no differentiation in equilibrium. $S_1^*$, $S_2^*$, $P_1^*$ and $P_2^*$ are given in (2.18) and welfare is

\begin{equation}
 W_\circ(S_1^*, S_2^*, P_1^*, P_2^*) = \int_{\theta_0}^{\bar{\theta}} u_\theta(S_2^*, C(S_2^*)) \, d\theta \approx 0.143108.
 \end{equation}

In the competition of two health insurance companies the firms differentiate too much in equilibrium. $S_1^*$, $S_2^*$, $P_1^*$ and $P_2^*$ are given in (2.20) and the welfare is

\begin{equation}
 W_\circ(S_1^*, S_2^*, P_1^*, P_2^*) = \int_{\theta_0}^{\theta_{ind}} u_\theta(S_2^*, C(S_2^*)) \, d\theta + \int_{\theta_{ind}}^{\bar{\theta}} u_\theta(S_1^*, C(S_1^*)) \, d\theta \approx 0.143584.
 \end{equation}

In the mixed duopoly $S_1^*$, $S_2^*$, $P_1^*$ and $P_2^*$ are given in (2.22) and the welfare is

\begin{equation}
 W_{mix}(S_1^*, S_2^*, P_1^*, P_2^*) = \int_{\theta_0}^{\theta_{ind}} u_\theta(S_2^*, C(S_2^*)) \, d\theta + \int_{\theta_{ind}}^{\bar{\theta}} u_\theta(S_1^*, C(S_1^*)) \, d\theta \approx 0.138892.
 \end{equation}
So in our non-cooperative framework with the special cost function the provision of supplementary health insurance by two competing profit maximizing health insurance companies is second best. The mixed competition, which is observed e.g. in Germany in the market for supplementary health insurance, yields the lowest welfare.

2.7 Conclusion

In this paper we have analyzed a duopolistic competition of output maximizing non-profit sickness funds in the market for supplementary health insurance. The solution of the second stage price competition has shown, that the sickness funds choose their prices according to their unit costs. Therefore, in the case of output maximization, quality differentiation is not used to relax price competition as it is in the case of profit maximization. We have shown that the equilibrium in the first stage quality competition highly depends on the slope of the cost function and therefore on the consumers' sensitivity. The equilibrium quality for supplementary health insurance is the maximum quality if and only if the sickness funds face concave unit costs, which leads to a total demand increasing in quality. This holds for the simultaneous as well as for the sequential first stage quality competition.

If the unit cost function is convex, the sickness funds will never choose the highest quality for their supplementary health insurance. This is because an increase in quality leads to a disproportionally higher price and therefore to a decrease in market coverage. We have taken a look at the sickness funds' reactions of overbidding, underbidding, and equalizing. Since the output maximizing sickness funds do not fear price competition, equalizing might be optimal. The analysis has shown that there is a unique subgame perfect Nash equilibrium in pure strategies when the sickness funds face an increasing consumers' sensitivity. This is independent of whether the quality competition is either simultaneous or sequential. There is no quality differentiation and no sickness fund has the opportunity to achieve quality leadership. Hence, there is neither a first nor a second mover advantage and both sickness funds gain the same demand regardless of the game's structure. If the sickness funds face a strictly decreasing consumers' sensitivity, there is no Nash equilibrium in pure strategies in the simultaneous first stage quality competition. A possible way to cope with this fact is to act first and commit oneself to a certain quality, since a first mover advantage exists if the quality competition is sequential. The quicker moving sickness fund then receives a higher demand than the competitor. Therefore, the quality competition might tend to be sequential in the case of strictly decreasing consumers' sensitivity.

The underbidding behavior is important for the existence of an equilibrium. For increasing total demand, there is no incentive to underbid at all. Therefore, the existence
of an equilibrium with both sickness funds providing supplementary health insurance with the highest quality is intuitive. For decreasing total demand, the existence depends on the consumers’ sensitivity. If the consumers’ sensitivity is increasing, enforcing quality competition leads to a higher demand since the consumers with a low preference for quality react insensitively. Thus, the sickness fund has no incentive to deviate substantially from the competitor’s quality choice, which results in a stable market outcome. If the consumers’ sensitivity is decreasing, the consumers with a weak preference for quality react highly sensitively. Thus, there is an incentive to deviate a lot from the competitor’s quality choice, since the gain in demand of consumers with a weak preference for quality outweighs the loss of demand due to the relaxed competition. This substantial deviation leads to an adjustment of the competitor’s quality choice and therefore no stable market outcome is achieved.

This paper shows the theoretical fundamentals of the competition of non-profit sickness funds in the market for supplementary health insurance. Those fundamentals are necessary in order to do applied research. One very important applied research question is the question of how the market for supplementary health care should be organized. For an exemplary cost function we answered that question. Based on our analysis supplementary health insurance should be provided by health insurance companies only. A mixed competition, as it is present e.g. in Germany, might be inefficient. As a consequence the parts of the Social Health Insurance Competition Strengthening Act which allow sickness funds to provide high quality supplementary health insurance should be reviewed, since this instrument of competition might make the consumers worse off. Of course further research has to be done to verify this policy implication. While our results are robust against changes in the cost function to some extent, it needs to be analyzed whether they still hold if market entrance is allowed and numerous firms compete. Also in this paper the firms were assumed to be homogeneous. Results might change if firms have different cost functions or can differentiate horizontally as well. Another extension can be the introduction of quality uncertainty. All mentioned aspects can also be analyzed for fixed costs of quality improvement, different consumer utility or changes in the distribution of preferences.
Chapter 3

Entry deterrence when aiming for customers: Strategies of heterogeneous health insurance companies

*Joint work with Oliver Urmann.*

**Abstract**

We analyze the strategies of output maximizing firms when one firm is inefficient and firms can differentiate in quality. We show that entry deterrence is possible even without any fixed costs, which is in contrast to the competition of profit maximizing firms. In a simultaneous quality competition, the efficient firm provides a lower quality and might deter entry. An equilibrium only exists if consumers react sufficiently sensitive on quality changes and cost inefficiency is sufficiently high. If consumers’ sensitivity is decreasing and the inefficient firm can act as a first mover, the inefficient firm always gains a positive demand.

*Keywords:* strategy of the firm, output maximization, heterogeneous firms, health care market, vertical differentiation

*JEL:* D21, I11, L11, L21
CHAPTER 3. ENTRY DETERRENCE

3.1 Introduction

This study targets the research question of how competition in the market for health insurance works when vertically differentiated products\(^1\) are provided by competing health insurance companies that aim for costumers and might differ in cost structure. There are many possible explanations why the cost structure of different health insurance companies might differ. This is, for instance, a different bargaining power or different management skills. Furthermore, there might be differences in administration costs.\(^2\)

The intention of our paper is to determine the strategies of the firms. In particular, we analyze whether an inefficient health insurance company can gain positive demand in a market where firms can provide differentiated products.

As an example, an efficient firm might try to deter entry while an inefficient firm might try to position itself in a niche. In an output maximizing framework in which the firms try to attract as many costumers as possible, we show that the firms' strategies and the market outcome highly depend on the consumers' sensitivity and on the degree of inefficiency.

This Chapter builds on the same assumptions as Chapter 2, except that we now assume that one firm might be inefficient. Our results are the following: We show that the strategies of the firms and the market outcome highly depend on the consumers' sensitivity and the degree of inefficiency. When the firms face the same cost function an increasing consumers' sensitivity makes the health insurance companies fight for the same customers while for a decreasing consumers' sensitivity differentiation becomes more attractive. The former results in a stable equilibrium with both firms provide the same quality. The latter leads to a situation where there is no equilibrium in a simultaneous competition and a first mover advantage in a sequential competition. If inefficiency occurs, the inefficient firm has to differentiate in order to gain positive demand. If consumers' sensitivity is increasing and the firms enter the market simultaneous, the efficient firm always chooses a quality that results in zero demand for the inefficient firm. In the sequential competition each firm can try to be either the first or the second mover. We find that the inefficient firm never acts as a first mover if consumers' sensitivity is increasing in quality. However, if the consumers' sensitivity is decreasing and the inefficient firm is the first mover, it always gains a positive demand. If the inefficient firm acts as a second mover and the consumers' sensitivity is decreasing, the efficient firm might deter entry. If the inefficiency

---

1. The health care market is divided into two sub-markets. These are the market for primary health insurance and the market for supplementary health insurance. In the latter the firms have the possibility of providing differentiated products. We therefore focus on the market for supplementary health insurance. A more detailed explanation will follow.

2. One further possible explanation for differences in the cost structure might be a difference in the customer's risk structure (athlete versus diabetic). Since in many countries we have a thoroughly risk classification system we will refrain from this possibility. For a detailed explanation, see Chapter 2.
is sufficiently high, the efficient firm deters entry and therefore is the sole provider leaving the market partially uncovered.

The rest of this article proceeds as follows. The next section, Section 3.2, gives a literature review. Section 3.3 introduces our model framework. In Section 3.4 we analyze the firms' strategies when the firms face the same cost structure. In Section 3.5 we analyze the firms' strategies when one firm is inefficient. In Section 3.6 the sequential competition in order to determine entry deterrence strategies of the efficient firm and strategies of the inefficient firm to enter the market. The concluding section, Section 3.7, summarizes our results and states the main implications.

3.2 Literature Review

This section gives a brief literature review about entry deterrence of profit maximizing firms and states the main distinctions to our article. Schmalensee (2011) assumes that an incumbent firm can provide more than one quality so that an entrant cannot gain any profits. Hung and Schmitt (1988) and Hung and Schmitt (1992) derive conditions under which it is feasible and profitable for a single incumbent to deter entry. They also show that if entry deterrence is not profitable the incumbent will always be the high quality provider. In their model the marginal cost of quality improvement is L-shaped, i.e. the marginal cost is zero up to a certain quality level and infinite above that level. The firms only differ in their fixed costs that can be seen as entry costs that are sunk afterwards. Ronnen (1991) analyzes a vertical quality competition in which there are no production costs but quality-development costs that are increasing in quality. Lutz (1997) has similar assumptions. Two firms have a cost function that is composed of a fixed setup cost and a quality-development cost that is increasing in quality. There are no unit costs of production. He shows that the incumbent will always deter entry if possible as long as the firms face the same marginal quality-development cost. Entry deterrence is possible if the fixed setup cost of the potential entrant is sufficiently high. If the firms face different quality-development cost the profitability of entry deterrence depends on both fixed setup cost and quality-development cost. The entrant can either have lower or higher quality-development cost. It is shown that a difference in the cost structure might make it more profitable for the incumbent to accommodate entry, even if entry deterrence is possible.

Due to the fact that variable costs might increase as well if a firm increases its quality, recent models deal with quality-dependent variable costs. Higher quality goods may be more expensive to manufacture because of, for instance, requirements of more skilled labor or more expensive raw materials and inputs. Lambertini (1996) and Wang (2003) note that the high-quality advantage may fail to hold if there are variable costs that depend
on quality. Noh and Moschini (2006) analyze entry deterrence strategies when there are quality-dependent marginal production costs and market coverage is endogenous. They assume that variable costs are strictly convex in quality, but for a given quality, the unit production costs are constant. In the case of deterred entry, the incumbent modifies its behavior by either increasing or decreasing its quality in order to deter entry. Compared to costless quality improvements, a costly quality improvement leads to less differentiation, which enforces price competition and therefore reduces profits.

However, if profit maximization is not the goal of a company, as in our analysis, there is no reason to fear price competition. Therefore, the results of our analysis are different. Furthermore, we assume that marginal cost of quality improvement are the only relevant costs. Assuming that there are no fixed costs of quality improvement in the market for supplementary health insurance is reasonable for two reasons. First, due to the high treatment expenses in the health care market that are variable for a health insurance company, fixed costs are relatively unimportant. Second, the health insurance companies are already established in the primary health care market so that they are already well-known.\footnote{Assuming no fixed costs for an established firm is in line, for instance, with Noh and Moschini (2006). They assume that an incumbent firm can change its product quality without incurring any fixed costs.}

### 3.3 Model

Our model builds on the following basic assumptions. There are two output maximizing health insurance companies, Firm 1 and Firm 2, which compete in a duopolistic market. We assume that Firm 2 might be inefficient. Therefore, let $C_i$ denote the two times continuously differentiable and strictly convex unit cost function of Firm $i$ with $C_2 \geq C_1$. Let further $\Delta C := C_2 - C_1$ denote the additional costs, where $\Delta C$ might be quality-dependent. At the first stage of the game, the firms choose whether to enter the market or not. At the second stage of the game, the firms choose their respective qualities either simultaneously or in sequential order.\footnote{To be specific, quality is the vertical element of a service. As mentioned in the introduction, the vertical element includes the benefit package covered by the health insurance company (e.g. the access to the best physicians’ network or to high cost technologies) as well as the level of care, the number of accessible doctors, the waiting time, or other amenities. For reasons of simplicity, we represent quality as a one-dimensional variable, $S \geq 0$. This assumption is in line with Che and Gale (1997).} The firms choose the quality $S_i$ of their respective products from the interval $[\underline{S}, \bar{S}]$ with $\underline{S} = 0$ being a mass market product.\footnote{The term product is to be seen in a broad sense. It especially includes all kinds of services.}

With common knowledge of the chosen qualities, the firms choose their respective prices $P_1$ and $P_2$ simultaneously at the third stage of the game under the constraint of nonnegative profits. This constraint means that the firms run a self-financing business in this market. The solution of the third stage is straightforward. The output maximizing
firms choose their prices equal to their unit costs, i.e. $P_1 = C_1(S_1)$ and $P_2 = C_2(S_2)$, since an increase in price c.p. leads to a decrease in output.

The consumers are described via their valuation of quality $\theta \in [\underline{\theta}, \bar{\theta}]$, with $\theta$ normalized to zero. The net utility of a consumer with preference parameter $\theta$ from buying a product of quality $S$ provided by Firm $i$ is given by the Mussa-Rosen utility function (Mussa and Rosen, 1978)

$$u_{\theta,i}(S) := \theta S - C_i(S).$$ \hspace{1cm} (3.1)

Consumers maximize their individual utility and buy one unit at most.\(^6\) Only if the utility is nonnegative does the consumer buy the product, meaning that we might face an uncovered market. If he is indifferent between two products he buys the one with the higher quality. The marginal consumer who has utility zero from buying a product of quality $S$ from Firm $i$ is given by\(^7\)

$$\theta_i(S) = \frac{C_i(S)}{S}. \hspace{1cm} (3.2)$$

The preference parameter indicating indifference between the products of the two firms can be derived by solving $u_{\theta_{\text{ind}},1}(S_1) = u_{\theta_{\text{ind}},2}(S_2)$ and is given by

$$\theta_{\text{ind}}(S_1, S_2) = \frac{C_1(S_1) - C_2(S_2)}{S_1 - S_2}. \hspace{1cm} (3.3)$$

For $\theta_{\text{ind}} \notin [\underline{\theta}, \bar{\theta}]$, no consumer is indifferent between the two products. The resulting demand of Firm $i$ is denoted by $D_i$.

Before starting to analyze the firms’ strategies, we go back to our research question. In the introduction, we claimed that the intention of our paper is to determine the strategies of the firms. In particular, we analyze whether an inefficient health insurance company can gain positive demand in a market in which the firms can provide differentiated products. This leads us to the following definition.

**Definition 8.** A Two-Firm Solution is a subgame-perfect Nash equilibrium in pure strategies $(S_1^*, S_2^*)$ in which both firms gain a positive demand, i.e. $D_1(S_1^*, S_2^*) > 0$ and $D_2(S_1^*, S_2^*) > 0$.

As a benchmark case, we first analyze the strategies of homogeneous firms.

\(^6\)Of course, consumers can buy more than one supplementary health care product for different segments. Buying more than one supplementary health care product for the same segment obviously does not make any sense and the competition has to be analyzed for each segment individually.

\(^7\)If we had accounted for uncertainty with probability of health loss $\pi$, firms would have chosen $P_i = \pi C_i(S_i)$ and consumers’ expected utility would have been $E[u_{\theta,i}(S)] = \pi \theta S - \pi C_i(S)$. Hence, the marginal consumer is still $\theta_i(S) = C_i(S)/S$. 
3.4 Homogeneous Firms: The Benchmark

As mentioned in Section 3.3, the firms choose their prices equal to their unit costs since their sole goal is output maximization. In the following analysis we therefore focus on the second stage of the game. This section examines the benchmark case in which the firms have identical cost structures, i.e. $\Delta C \equiv 0$. The benchmark allows us to analyze how the firms adjust their strategies when inefficiency occurs. The demand for Firm $i$'s product is

$$D_i(S_1, S_2) = \begin{cases} 
\bar{\theta} - \min (\bar{\theta}, \theta_{\text{ind}}(S_1, S_2)), & S_i > S_j \\
\frac{\bar{\theta} - \theta_1(S_i)}{2}, & S_i = S_j \\
\min (\bar{\theta}, \theta_{\text{ind}}(S_1, S_2)) - \theta_1(S_i), & S_i < S_j.
\end{cases}$$ (3.4)

We define the sensitivity of consumers to quality variation as $\left| \frac{d(\bar{\theta} - \theta_1(S))}{dS} \right| = \theta'_1(S)$. An analogous definition was used by Dorfman and Steiner (1954). Hence, an increasing sensitivity means that the reaction of consumers is inelastic for small qualities while it is elastic for higher qualities, while analogously, a decreasing sensitivity leads to an elastic demand for small qualities and an inelastic demand at the higher qualities as it can be seen in Figure 3.1.

![Figure 3.1: Cost function $C_1$ of Firm 1 (left), demand-quality function $TD(S) = \bar{\theta} - C(S)/S$ (center), and consumers' sensitivity (right). $C(S) = S^\alpha$ with $\alpha = 1.5$ (solid) and $\alpha = 2.5$ (dashed).](image)

**Lemma 9.** If consumers’ sensitivity is increasing and firms are homogeneous, no differentiated Two-Firm Solution exists.

**Proof.** See Appendix.

According to Lemma 9, an increasing consumers’ sensitivity makes the firms fight for the same customers. In the low quality area, for instance, the low quality provider has the

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8 Since now $C_1 \equiv C_2$ we also have $\theta_1 \equiv \theta_2$. We therefore formulate all equations in this section in terms of $\theta_1$.

9 In our case $\frac{d(\bar{\theta} - \theta_1(S))}{dS}$ is negative, due to the anticipation of the change in price. We therefore use the absolute value to avoid misunderstandings with the terms "increasing" or "decreasing" sensitivity.
incentive to increase his own quality and therefore reduce differentiation. For him, it is worth giving up some customers with a weak preference for quality since this is overcompensated by the gain of customers with a stronger preference for quality. Therefore, any stable market outcome has to be non-differentiated.

**Proposition 10.** If consumers' sensitivity is increasing and firms are homogeneous, the quality combination \((S^*, S^*)\) with \(C_1'(S^*) = \frac{\theta + \theta_1(S^*)}{2}\) is the unique Two-Firm Solution.

*Proof.* See Appendix.

If consumers' sensitivity is decreasing, the firms' strategies change. In the low quality area, for instance, the low quality provider has the incentive to decrease his own quality and therefore increase differentiation. With a lower quality he gains more customers with a weak preference for quality than he loses customers with strong preference for quality to his competitor. But still, there is no differentiated equilibrium.

**Lemma 11.** If consumers' sensitivity is decreasing and firms are homogeneous, there is no differentiated Two-Firm Solution.

*Proof.* See Appendix.

However, the change in strategy of the low quality provider leads to a situation in which there is no non-differentiated equilibrium, either.

**Lemma 12.** If consumers' sensitivity is decreasing and firms are homogeneous, there is no non-differentiated Two-Firm Solution.

*Proof.* See Appendix.

When simultaneous quality choice does not lead to a stable market outcome, the firms might enter the market sequentially\(^{10}\). This especially is the case when there is a first mover advantage, since any firm has the incentive to act first and commit itself to a certain quality.

**Proposition 13.** When qualities are chosen sequentially and consumers' sensitivity is decreasing, a differentiated Two-Firm Solution with a first mover advantage exists.

*Proof.* See Appendix.

In this section, we analyzed the benchmark case without inefficiency. Entering the market first is a weakly dominant strategy for both firms. This does not have to be the case when firms are heterogeneous.

\(^{10}\)When there is a Two-Firm Solution in the case of simultaneous quality choice, sequential quality competition leads to the same results.
3.5 Heterogeneous Firms

Now let \( \Delta C > 0 \). Therefore, Firm 2 now is inefficient and has to differentiate in order to gain positive demand since otherwise the provision of a homogeneous product at a higher price than the competitor leads to zero demand for the inefficient firm. The demand for Firm 1 is now given by

\[
D_1(S_1, S_2) = \begin{cases} 
\theta - \max \left( \frac{C_1(S_1)}{S_1} , \frac{C_1(S_1) - C_2(S_2)}{S_1 - S_2} \right) , & S_1 > S_2 \\
\theta - \frac{C_1(S_1)}{S_1} , & S_1 = S_2 \\
\min \left( \theta , \frac{C_2(S_2) - C_1(S_1)}{S_2 - S_1} \right) - \frac{C_1(S_1)}{S_1} , & S_1 < S_2 
\end{cases}
\]  

(3.5)

and the demand for Firm 2 by

\[
D_2(S_1, S_2) = \begin{cases} 
\max \left( \frac{C_1(S_1) - C_2(S_2)}{S_1 - S_2} , \frac{C_2(S_2)}{S_2} \right) , & S_1 > S_2 \\
0 , & S_1 = S_2 \\
\theta - \min \left( \theta , \frac{C_2(S_2) - C_1(S_1)}{S_2 - S_1} \right) , & S_1 < S_2 
\end{cases}
\]  

(3.6)

In the following, we derive the reaction functions in order to analyze the competition.

3.5.1 The Firms’ Reactions

For any given \( S_1 \) chosen by the efficient firm, the inefficient firm can either respond by choosing a higher or a lower quality. Qualities in the neighborhood of \( S_1 \) result in an output equal to zero since the inefficient firm’s quality-cost-ratio is unattractive compared to its competitor’s. Therefore, the inefficient firm has to differentiate itself substantially either by underbidding or overbidding.\(^{11}\) The first order condition for the optimal overbidding reaction \( r_2^o \)

\[
\frac{\partial D_2(S_1, S_2)}{\partial S_2} \bigg|_{S_2 > S_1} = \frac{1}{S_2 - S_1} \left[ \frac{C_2(S_2) - C_1(S_1)}{S_2 - S_1} - C_2'(S_2) \right] \equiv 0
\]

(3.7)

can be rearranged to

\[
C_2(S_1) = C_2(S_2) + C_2'(S_2)(S_1 - S_2) + \Delta C(S_1).
\]

(3.8)

The optimal underbidding reaction of the inefficient firm \( r_2^u \) is determined by solving the first order condition

\(^{11}\)Firm 2 chooses a quality \( S_2 \) out of the union of two disjoint compact subsets of \([S, \overline{S}]\) and since \( D_2(S_1, \cdot) \) is continuous on those subsets, an optimal reaction exists.
\[
\frac{\partial D_2(S_1, S_2)}{\partial S_2} \bigg|_{S_2 < S_1} = \frac{C_2(S_1) - C_2(S_2) - (S_1 - S_2)C'_2(S_2) - (\Delta C(S_1) + (S_1 - S_2)^2\theta'_2(S_2))}{(S_1 - S_2)^2} = 0.
\]

Utilizing the Taylor formula yields a closed form for the optimal overbidding reaction \(r_2^o\) of Firm 2.

**Lemma 14.** For a given \(S_1\) the optimal overbidding reaction \(r_2^o\) of the inefficient Firm 2 is given by

\[
r_2^o(S_1) = F_{S_1}^{-1}(\Delta C(S_1)) + S_1,
\]

with \(F_{S_1}(x) := \int_0^x tC''_2(t + S_1) \, dt\). The optimal underbidding reaction \(r_2^u\) is the solution of

\[
(S_1 - S_2)^2\theta'_2(S_2) + \Delta C(S_1) = F_{S_1}(S_2 - S_1),
\]

The overall reaction function \(r_2\) of the inefficient firm is then given by

\[
r_2(S_1) := \arg\max_{S_2 \in \{r_2^o(S_1), r_2^u(S_1)\}} D_2(S_1, S_2).
\]

**Proof.** See Appendix. \(\square\)

Note that for increasing inefficiency, represented by \(\Delta C\), differentiation increases, since \(F_{S_1}^{-1}\) is strictly increasing.\(^{12}\)

Now we analyze the reaction of the efficient Firm 1. For Firm 1, overbidding is always dominated by equalizing, since for \(S_1 > S_2\) and strictly convex \(C_1\), we have

\[
D_1(S_1, S_2) \leq \bar{\theta} - \frac{C_1(S_1)}{S_1} < \bar{\theta} - \frac{C_1(S_2)}{S_2} = D_1(S_2, S_2).
\]

Then, no consumers will buy the product of the inefficient Firm 2. Furthermore, underbidding with \(\tilde{S}\), where \(\tilde{S}\) satisfies \(\theta_{ind}(\tilde{S}, S_2) = \bar{\theta}\), dominates equalizing. Although in this situation Firm 2 has a quality advantage, the cost disadvantage is too high and no consumer would prefer the quality cost ratio of Firm 2. Therefore, Firm 1 will choose some underbidding quality \(S_1 \in [\underline{S}, \tilde{S}]\). \(\tilde{S}\) obviously depends on \(S_2\) and therefore has to be understood as a function of \(S_2\).\(^{13}\) Thus, Firm 1 reacts with an underbidding quality \(S_1 \in [\underline{S}, \tilde{S}(S_2)]\). Since the function \(S_1 \mapsto D_1(S_1, S_2)\) is continuous on the compact interval \([\underline{S}, \tilde{S}(S_2)]\) an optimal reaction \(r_1(S_2) := \arg\max_{S_1 \in [\underline{S}, \tilde{S}(S_2)]} D_1(S_1, S_2)\) exists for every \(S_2\).

\(^{12}\)Also note that \(F_{S_1}(S_2 - S_1)\) is negative for \(S_2 < S_1\) and \(\theta'_2(S_2)\) is also negative for small \(S_2\).

\(^{13}\)For the closed form solution of \(\tilde{S}\), see Appendix.
Partial derivation of $D_1$ with respect to $S_1$ yields

$$\frac{\partial D_1(S_1, S_2)}{\partial S_1} \bigg|_{S_1 < S_2} = \frac{\Delta C(S_2)}{(S_2 - S_1)^2} + \frac{S_2}{S_2 - S_1} \left( \frac{\theta_1(S_2) - \theta_1(S_1)}{S_2 - S_1} - \theta_1'(S_1) \right). \quad (3.11)$$

The first order condition $\frac{\partial D_1(S_1, S_2)}{\partial S_1} = 0$ can be rearranged to

$$C_1(S_2) = C_1(S_1) + (S_2 - S_1)C_1'(S_1) - (\Delta C(S_2) - (S_2 - S_1)^2\theta_1'(S_1)). \quad (3.12)$$

From these equations, we can derive the following result for the reaction of the efficient Firm 1.

**Lemma 15.** The efficient firm will always respond with a lower quality and for its reaction function, the following holds: $r_1(S_2) \in \{S, \bar{S}(S_2)\}$ for all $S_2 \in [\underline{S}, \bar{S}]$.

**Proof.** See Appendix. \[ \square \]

The intuition behind this reaction is the following. Given the quality $S_2$ chosen by the inefficient Firm 2, the efficient Firm 1 has to decide whether to fight or not to fight. The former results in no output for Firm 2, but Firm 1 leaves a part of the market unserved. Consumers with preference $\theta < \theta_1(\bar{S}(S_2))$ would suffer a negative net utility and therefore would not buy the product. If Firm 1 does not fight and provides $\underline{S}$, the whole market is covered, but Firm 2 has a positive output. So Firm 1 has to trade-off the potentially unserved consumers against the ones left to Firm 2. Based on the reaction functions, we analyze the competition of the two firms and search for pure Nash equilibria in the next section.

### 3.5.2 Quality Competition

In the previous section, we derived the reaction functions $r_1$ of the efficient Firm 1 and $r_2$ of the inefficient Firm 2. Now we analyze whether and under which conditions a Two-Firm Solution exists. An equilibrium quality is a fixed point of the composition of the two reaction functions, i.e. $r_1(r_2(S_1)) = S_1$ or, equivalently, $r_2(r_1(S_2)) = S_2$. The simple structure of $r_1$ as derived in section 3.5.1 puts the focus on the former formulation $r_1(r_2(S_1)) = S_1$. According to Lemma 15, the efficient firm reacts either with the minimum quality $\underline{S}$ or with the quality $\bar{S}(S_2)$, which leaves no output for the inefficient Firm 2. It is clear that Firm 2 has no incentive to enter the market and provide a quality to which Firm 1 reacts with $\bar{S}$, since the resulting demand $D_2$ is zero. Thus, Firm 1 necessarily provides the minimum quality $\underline{S}$ in any equilibrium.

Two factors determine whether an equilibrium exists. First, the extent of the cost difference between the two firms and, second, the sensitivity of consumers to variations
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of quality. Figure 3.2 shows the reaction functions of the two firms. If the sensitivity of consumers to a quality variation is increasing in quality, Firm 1 reacts with \( \tilde{S}(S_2) \) for any \( S_2 \). This is due to the fact that Firm 1 only has to give up a few consumers with a low preference for quality in order to gain the demand that would have been left to Firm 2 as can be seen in (3.11). An increasing consumers’ sensitivity is represented by \( \theta_1 \) being convex and according to the proof of Lemma 15, the reaction function of Firm 1 then is \( r_1(S_2) = \tilde{S}(S_2) \) for all \( S_2 \). As already stated earlier, in this case a Two-Firm Solution does not exist since \( r_2(\tilde{S}(S_2)) \neq S_2 \) for all \( S_2 \). So Firm 2 would not enter the market in the first stage. Therefore, a necessary condition for the existence of a Two-Firm Solution is a decreasing consumers’ sensitivity. Furthermore, equation \( r_1(r_2(S)) = S \) has to hold.

From Lemma 14 with \( S_1 = \tilde{S} \) we yield that Firm 2 chooses its quality \( r_2(S) \) according to \( C'_2(S_2) = \theta_2(S_2) \).

The inequality \( D_1(S, r_2(S)) \geq D_1(\tilde{S}(r_2(S)), r_2(S)) \), so that \( r_1(r_2(S)) = S \) holds, is a sufficient condition for the existence of a pure Nash equilibrium.\(^{14}\) Figure 3.2 shows \( \frac{D_1(\tilde{S}(r_2(S)), r_2(S))}{D_1(S, r_2(S))} \) plotted against an increasing inefficiency and for varying consumers’ sensitivity. An equilibrium exists if this quotient is smaller than one. For a given sensitivity, the inefficiency needs to be sufficiently high.\(^{15}\) A higher sensitivity, represented by a smaller \( \alpha \), allows a lower inefficiency.

**Proposition 16.** If consumers react sufficiently sensitive on quality changes (in the low quality area) and the cost inefficiency is sufficiently high, a unique Two-Firm Solution exists.

**Proof.** See Appendix.

So far, we have seen that the existence of an equilibrium highly depends on consumers’ sensitivity and the degree of inefficiency. If the firms choose their qualities simultaneously, there are many situations without a stable market solution in which the inefficient firm would gain any demand. In the next section, we therefore analyze whether a sequential quality competition enables the inefficient firm to obtain a positive demand.

\(^{14}\)The inequality is equivalent to \( \tilde{S} \cdot \frac{S_2}{S(S_2) + S_2} \leq \theta_2(S_2) \). The greater \( \theta_2(S_2) \), the more likely it is that Firm 1 will react with \( S \) and that we will have an equilibrium. Keep in mind that the consumers’ sensitivity influences \( S_2 = r_2(S) \), which was determined by solving \( \theta_2(S_2) = C'_2(S_2) \). Further, the consumers’ sensitivity influences \( \tilde{S} \). Both these qualities, \( S_2 \) and \( \tilde{S} \), are also heavily dependent on \( \Delta C \). Note that ceteris paribus a higher \( \Delta C \) leads to the existence of an equilibrium since \( r_2(S) \) is increasing in \( \Delta C \) and Firm 1 has less incentive to fight.

\(^{15}\)Of course, the inefficiency must not be too high. For our example, \( \beta \) must be lower than \( \alpha^{-\frac{1}{1-\alpha}}(1 - \alpha) \).
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Figure 3.2: In this figure, the cost function is given by $C_1(S) = S^\alpha$ and $\Delta C \equiv \beta$. The first figure shows the reaction functions of the firms for $\alpha = 2.5$ and $\beta = 0.01$. Here $r_1(S_2) = \hat{S}(S_2)$ for all $S_2$ and thus the reaction functions do not intersect. The second figure shows the reaction functions of the firms for $\alpha = 1.5$ and $\beta = 0.01$. Here the inefficiency is not sufficiently high so that the reaction functions do not intersect. The third figure shows the reaction functions of the firms for $\alpha = 1.5$ and $\beta = 0.01$. Here the inefficiency is sufficiently high and therefore the reaction functions do intersect. The last figure shows $D_1(\hat{S}(r_2(S)), r_2(S))/D_1(S, r_2(S))$ plotted against $\beta$ with $\alpha = 1.3$ (solid), $\alpha = 1.5$ (dashed), and $\alpha = 1.7$ (dotted).

3.6 Entry Deterrence and Strategies of the Inefficient Firm

In the sequential quality competition, the inefficient firm can try to act either as the first or the second mover. If it is the first mover, it needs to choose a quality so that the efficient firm has no incentive to put the inefficient firm out of the market. If it is the second mover, the inefficient firm needs to find a niche.

3.6.1 The Inefficient Firm as the First Mover

We know that once Firm 2 has decided to provide a certain quality $S_2$, Firm 1 will choose either $S$ or $\hat{S}(S_2)$. If now the consumers’ sensitivity to quality variations is increasing, Firm 1 will still react with $\hat{S}(S_2)$ as it was the case with the simultaneous competition. Obviously, Firm 2 then obtains zero demand, as can be seen in the left part of figure 3.3. Hence, the inefficient firm will not be the first mover if consumers’ sensitivity is increasing in quality. If consumers’ sensitivity is decreasing it might be optimal for Firm 1 to provide the minimum quality $S$, as we have already seen in section 3.5.2. This is a necessary condition in order to make sure that Firm 2 can gain a positive demand. For a given consumers’ sensitivity, the existence of an equilibrium depends on the degree of inefficiency in the simultaneous competition. If the inefficiency is sufficiently high and an equilibrium exists in the simultaneous competition, the same qualities are provided when the inefficient firm enters the market first.

Let us now discuss the case with no equilibrium in the simultaneous competition.
Figure 3.3: For given $S_2$, this figure shows the demand for Firm 1 when choosing $S$ (dotted), $\tilde{S}(S_2)$ (solid), and the optimal reaction $r_1(S_2)$ (solid, thick), and it shows the resulting demand for Firm 2 (dashed, thick) given the optimal reaction of Firm 1. On the right hand side we also see $D_2(S_2, \tilde{S}_2)$ (dashed). The cost function is given by $C_1(S) = S^\alpha$ and $\Delta C \equiv \beta = 0.01$, with $\alpha = 2.5$ (left) and $\alpha = 1.5$ (right).

Firm 2 has to make sure that the efficient Firm 1 provides $S$ and therefore provides a quality $S_2$, maximizing $D_2(r_1(S_2), S_2)$ under the constraint $D_1(S, S_2) \geq D_1(\tilde{S}(S_2), S_2)$. This constraint is binding and Firm 2 will choose its quality $S_2$ according to $D_1(S, S_2) = D_1(\tilde{S}(S_2), S_2)$, which is indicated by the right dashed vertical line in the right part of figure 3.3. One can see that this quality is higher than $r_2(S)$, which is indicated by the left dashed vertical line. So Firm 2 gives up some market share in order to avoid competition and to ensure that Firm 1 provides $S$.

Proposition 17. If the consumers’ sensitivity is decreasing and the inefficient firm is the first mover, the inefficient firm always gains a positive demand.

Proof. Clear from the above. \hfill \Box

We now take a look at the case in which the inefficient firm is the second mover.

3.6.2 The Inefficient Firm as the Second Mover

The efficient firm, as the first mover, anticipates the optimal reaction of the inefficient Firm 2. Hence, Firm 1 will choose the quality $S_1$ that maximizes $D_1(S_1, r_2(S_1))$. If the inefficiency $\Delta C$ is sufficiently high, it can be possible for Firm 1 to deter entry. Yet, this does not have to be the optimal choice for the efficient firm. If entry is not deterred, the inefficient firm can choose either a higher or a lower quality according to its reaction function $r_2$. If Firm 2 responds with a higher quality, the resulting demand for the firms can be written as $D_2(S_1, r_2^o(S_1)) = \overline{\theta} - C_2'(r_2^o(S_1))$ and $D_1(S_1, r_2^o(S_1)) = C_2'(r_2^o(S_1)) - \theta_1(S_1)$, respectively. When $\Delta C$ is sufficiently small and $S_1$ sufficiently high, Firm 2 could also respond with a lower quality according to $r_2^u(S_1)$, which results in
\[ D_2(S_1, r_2^a(S_1)) = S_1\theta_2'(r_2^a(S_1)) \text{ and } D_1(S_1, r_2^a(S_1)) = \theta_2(r_2^a(S_1)) - S_1\theta_2'(r_2^a(S_1)). \]

Which of these two reactions is optimal and, subsequently, which quality will be chosen by Firm 1, depends on the consumers’ sensitivity and the degree of inefficiency, as can be seen in figure 3.4. If consumers’ sensitivity is decreasing, represented by \( \alpha = 3/2 \) in figure 3.4,

![Figure 3.4](image)

Figure 3.4: In this figure, the cost function is given by \( C_1(S) = S^\alpha \) and \( \Delta C \equiv \beta \). The figures show the output of Firm 1 (solid) and Firm 2 (dashed) plotted against the quality chosen by Firm 1 with optimal reaction of Firm 2. In the first figure, we have \( \alpha = 3/2 \) and \( \beta = 0.01 \). In the second figure, we have \( \alpha = 3/2 \) and \( \beta = 0.04 \). In the third figure, we have \( \alpha = 5/2 \) and \( \beta = 0.01 \). In the last figure, we have \( \alpha = 5/2 \) and \( \beta = 0.04 \).

and the inefficiency is low, entry cannot be deterred. Firm 1 as the first mover will then be the high quality provider. Firm 1 chooses \( S_1 \) so that Firm 2 is indifferent between underbidding with \( r_2^a(S_1) \) and overbidding with \( r_2(S_1) \). We assume that Firm 2 then underbids since otherwise Firm 1 would choose a only marginally higher quality \( S_1 + \epsilon \) with \( r_2(S_1 + \epsilon) = r_2^a(S_1 + \epsilon) \). While Firm 2 is indifferent between \( r_2^a(S_1) \) and \( r_2(S_1) \), the demand of Firm 1 is higher if Firm 2 chooses \( r_2^a(S_1) \) since then market coverage is higher. If consumers’ sensitivity is still decreasing but the inefficiency is sufficiently high, represented by \( \Delta C \equiv \beta = 0.04 \) in figure 3.4, entry deterrence is possible. However, it does not have to be optimal for Firm 1 to deter entry, as can be seen in figure 3.4. Instead, Firm 1 provides the lowest quality and the inefficient firm is the high quality provider. The market then is fully covered and Firm 1 gains a higher demand than Firm 2. If consumers’ sensitivity is increasing, represented by \( \alpha = 5/2 \) in figure 3.4, and the inefficiency is low, entry deterrence is not possible and the situation is essentially the same as in the first case with \( \alpha = 3/2 \) and \( \beta = 0.01 \). If the inefficiency is sufficiently high, Firm 1 deters entry and therefore is the sole provider, leaving the market partially uncovered.

### 3.6.3 Discussion

As we have seen in Section 3.5.2, there are many situations in which no equilibrium exists if competition is simultaneous. However, a stable market outcome could often be obtained if one of the firms commits itself to a certain quality, leading to a sequential competition. If consumers’ sensitivity is increasing, waiting is a credible commitment made by the inefficient firm.\(^{16}\) If the inefficiency is high, entry is deterred. If the inefficiency is sufficiently

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\(^{16}\)If Firm 2 as the first mover chooses \( S_2 \), Firm 1 always reacts with \( \hat{S}(S_2) \) leaving Firm 2 with no demand.
low, Firm 2 can find a niche in the low quality segment. If consumers’ sensitivity is decreasing, Firm 2 can always obtain a positive demand by entering the market as the first mover, as seen in figure 3.3.

In the case of low inefficiency, the inefficient firm has a first mover advantage and the efficient firm has a second mover advantage.\(^{17}\) Hence, the incentives regarding the entry order are congruent. However, there are situations in which the incentives regarding the entry order might be contrary. For \(\alpha = 3/2\) and \(\beta = 0.03\), we have \(D_1 = 0.605405\) and \(D_2 = 0.394595\) if the inefficient firm acts as the first mover and \(D_1 = 0.58723\) and \(D_2 = 0.41277\) if the efficient firm acts as the first mover, as illustrated in figure 3.5. Therefore, both firms have a second mover advantage. Such a situation is known as the chicken game.

Figure 3.5: In this figure we have \(C_1(S) = S^{3/2}\) and \(\Delta C \equiv \beta = 0.03\). The left figure shows the resulting demands when the efficient firm acts as the first mover and Firm 2 reacts according to \(r_2\). The right figure shows the resulting demands when the inefficient firm acts as the first mover and Firm 1 reacts according to \(r_1\).

\[ \text{Figure 3.5: In this figure we have } C_1(S) = S^{3/2} \text{ and } \Delta C \equiv \beta = 0.03. \text{ The left figure shows the resulting demands when the efficient firm acts as the first mover and Firm 2 reacts according to } r_2. \text{ The right figure shows the resulting demands when the inefficient firm acts as the first mover and Firm 1 reacts according to } r_1. \]

3.7 Conclusion

In this paper, we analyze a duopolistic competition with quality differentiation in the market for supplementary health care in order to determine the strategies of the firms. As an example, an efficient firm might try to deter entry or an inefficient firm might try to position itself in a niche. In an output maximizing framework in which the firms try to attract as many customers as possible, we show that the strategies of the firms and the market outcome highly depend on the consumers’ sensitivity and the degree of inefficiency.

In a benchmark case, the firms face the same cost function. An increasing consumers’ sensitivity makes the firms fight for the same customers. In the low quality area, for instance, the low quality provider has the incentive to increase its own quality and therefore

\(^{17}\) Compare the right-hand side of figure 3.3 and the left part of figure 3.4, where \(\alpha = 3/2\) and \(\beta = 0.01\).
reduce differentiation. For him, it is worth giving up some customers with a weak preference for quality since this is overcompensated by the gain of customers with a stronger preference for quality. We show that there is a unique Two-Firm Solution in which both firms provide the same quality. If consumers' sensitivity is decreasing, the firms' strategies change. In the low quality area, for instance, the low quality provider has the incentive to decrease its own quality and therefore increase differentiation. With a lower quality, he gains more customers with a weak preference for quality than he loses customers with strong preference for quality to his competitor. For a decreasing consumers' sensitivity, there is neither a differentiated nor a non-differentiated equilibrium in the case of simultaneous competition. However, when qualities are chosen sequentially, a differentiated Two-Firm Solution with a first mover advantage exists.

If inefficiency occurs, the inefficient firm has to differentiate in order to gain positive demand since otherwise, the provision of a homogeneous product at a higher price than the competitor leads to zero demand for the inefficient firm. Two factors determine whether an equilibrium exists or not. First, the extent of cost difference between the two firms and, second, the sensitivity of consumers to variations of quality. If consumers' sensitivity is increasing and the firms enter the market simultaneously, the efficient firm always chooses a quality that results in zero demand for the inefficient firm. This is due to the fact that the efficient firm has to give up only a few consumers with a low preference for quality in order to gain the demand that would have been left to the inefficient firm. Hence, a necessary condition for the existence of a Two-Firm Solution in a simultaneous competition is a decreasing consumers' sensitivity.

In the sequential competition, we analyze under which conditions a stable Two-Firm Solution exists. Each firm can try to be either the first or the second mover. If the inefficient firm is the first mover, it needs to choose a quality so that the efficient firm has no incentive to put the inefficient firm out of the market. If the inefficient firm is the second mover, it needs to find a niche. We find that the inefficient firm never acts as a first mover if consumers' sensitivity is increasing in quality. However, if the consumers' sensitivity is decreasing and the inefficient firm is the first mover, it always gains a positive demand.

The inefficient firm might also act as a second mover. If consumers' sensitivity is decreasing and the inefficiency is low, entry cannot be deterred. If consumers' sensitivity is decreasing but the inefficiency is high, entry can be deterred. However, it does not have to be optimal for the efficient firm to deter entry. If consumers' sensitivity is increasing and the inefficiency is low, entry deterrence is not possible. If the inefficiency is sufficiently high, the efficient firm deters entry and therefore is the sole provider, leaving the market partially uncovered.

The results of our analysis are in contrast to the results of entry deterrence of profit
maximizing firms. In the case of profit maximization, entry deterrence is possible if and only if fixed costs are sufficiently high. For output maximizing firms, entry deterrence even is possible if there are no fixed costs at all. In some situations, even a small difference in variable costs leads to entry deterrence. Furthermore, the analysis has shown that there are many situations in which no equilibrium exists if the competition is simultaneous. However, a stable market outcome could often be obtained if one of the firms committed itself to a certain quality, leading to a sequential competition. In the case of sequential competition, the incentives regarding the entry order might be congruent or contrary. There are scenarios in which both firms have an incentive to wait. In such a situation, a welfare loss might occur due to the fact that none of the firms might enter the market.

There are two main conclusions a health insurance company can draw from our study. First, understanding consumer behavior is crucial for assessing the right strategy. Therefore, the firms need to identify how sensitive consumers react in certain market segments. Clearly, demand characteristics might be different when analyzing patients with diabetes or patients in need of an artificial hip joint. Second, firms need to analyze their cost structure for each segment individually since a certain degree of inefficiency has different consequences, depending on the consumers’ sensitivity. Even a slight difference in efficiency might result in entry deterrence; especially in important segments with a high strategic value, firms need to know how to position themselves.

Appendix

Proof of Lemma 9. Assume \((S_1^*, S_2^*)\) equilibrium strategy combination with \(S_1^* \neq S_2^*\) and w.l.o.g. \(S_2^* < S_1^*\). Then \(\frac{\partial D_2(S_1^*, S_2)}{\partial S_2} \bigg|_{S_2 = S_2^*} = 0\) has to hold. If now \(\theta_1\) is strictly convex, the equation

\[
\theta'_1(S_2) < \frac{\theta_1(S_1) - \theta_1(S_2)}{S_1 - S_2}
\]

holds for all \(S_2 < S_1\), which is equivalent to \(\frac{\partial D_2(S_1, S_2)}{\partial S_2} > 0\). This is a contradiction. Thus, any equilibrium will be non-differentiated. \(\square\)

Proof of Proposition 10. (I) In the first step it is shown that

\[
S^* = \arg \max_{S_2 \in [S, S]} D_2(S^*, S_2)
\]

with \(S^*\) satisfying \(C'_1(S^*) = \frac{\bar{\theta} + \theta_1(S^*)}{2}\).

i) Let \(S_2 > S^*\):

\[
D_2(S^*, S_2) = \bar{\theta} - \frac{C_1(S_2) - C_1(S^*)}{S_2 - S^*} < \bar{\theta} - C'_1(S^*) = \frac{\bar{\theta} - \theta_1(S^*)}{2} = D_2(S^*, S^*)
\]
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(II) Now the uniqueness of $S^*$ is shown.

i) $\forall S_1 > S^* \exists S_2 < S_1 : D_2(S_1, S_2) > D_2(S_1, S_1)$.

Let $\epsilon := C'_1(S_1) - \frac{\bar{\theta} + \theta_1(S_1)}{2} > 0$ and $S_2 := \theta_1^{-1}(\theta'_1(S_1) - \frac{\epsilon}{S_1}) < S_1$, then

$$D_2(S_1, S_2) = S_1 \frac{\theta_1(S_1) - \theta_1(S_2)}{S_1 - S_2} > S_1 \theta'_1(S_2) = S_1 \theta'_1(S_1) - \epsilon$$

$$= C'_1(S_1) - \theta_1(S_1) - \epsilon = \frac{\bar{\theta} - \theta_1(S_1)}{2} = D_2(S_1, S_1).$$

ii) $\forall S_1 < S^* \exists S_2 > S_1 : D_2(S_1, S_2) > D_2(S_1, S_1)$.

Let $\epsilon := \frac{\bar{\theta} + \theta_1(S_1)}{2} - C'_1(S_1) > 0$ and $S_2 := C'_1(S_1) + \epsilon > S_1$, then

$$D_2(S_1, S_2) = \frac{C_1(S_2) - C_1(S_1)}{S_2 - S_1} > \bar{\theta} - C'_1(S_2) = \frac{\bar{\theta} - \theta_1(S_1)}{2} = D_2(S_1, S_1).$$

Proof of Lemma 11. Assume $(S^*_1, S^*_2)$ equilibrium strategy combination with $S^*_1 \neq S^*_2$ and w.l.o.g. $S^*_2 < S^*_1$. Let $\mathcal{S} := \{S \mid \bar{\theta} - C'_1(S) > \theta_1(S)\}$ and $\overline{\mathcal{S}} := \{S \mid \bar{\theta} - C'_1(S) \leq \theta_1(S)\}$.

We split the proof into three parts:

(i) Assume $S^*_1 \in \mathcal{S}$ and $0 < S^*_2 < S^*_1$: $\exists S_2 > S^*_1 : D_2(S^*_1, S_2) > D_2(S^*_1, S^*_2)$

Let $S_2 > S^*_2$ so that $\theta_1(S_2) = \theta_1(S^*_1) + (1 - S^*_1/S_2) \theta_1(S^*_2)$, then

$$D_2(S^*_1, S^*_2) = \frac{C_1(S_2) - C_1(S^*_1)}{S_2 - S^*_1} - \theta_1(S^*_2) < C'_1(S^*_1) - \theta_1(S^*_2)$$

$$= \frac{\bar{\theta} - \theta_1(S^*_1) - \frac{S_2}{S_2 - S^*_1}(\theta_1(S_2) - \theta_1(S^*_1))}{S_2 - S^*_1} = D_2(S^*_1, S_2).$$

(ii) Assume $S^*_1 \in \overline{\mathcal{S}}$ and $0 < S^*_2 < S^*_1$: $D_2(S^*_1, 0) > D_2(S^*_1, S^*_2)$
We have for all $S_2 < S_1$
\[
\frac{\partial D_2(S_1, S_2)}{\partial S_2} = -\frac{C'_1(S_2)(S_1 - S_2) + C_1(S_1) - C_1(S_2)}{(S_1 - S_2)^2} - \theta'_1(S_2)
\]
\[
= \frac{S_1\theta_1(S_1) - S_2\theta_1(S_2) - C'_1(S_2)}{(S_1 - S_2)^2} - \frac{C'_1(S_2)}{S_1 - S_2} - \theta'_1(S_2)
\]
\[
= \frac{S_1}{S_1 - S_2} \left( \frac{\theta_1(S_1) - \theta_1(S_2)}{S_1 - S_2} - \theta'_1(S_2) \right) < 0
\]

since $\theta_1$ is (strictly) concave. So especially $D_2(S_1^*, S_2)$ is decreasing in $S_2$ and due to continuity, $D_2(S_1^*, 0) > D_2(S_1^*, S_2)$ for all $0 < S_2 < S_1^*$.

(iii) Assume $S_2^* = 0$: $D_1(S_1, 0)$ is decreasing in $S_1$ and therefore, \( \frac{\partial D_1(S_1, 0)}{\partial S_1} \bigg|_{S_1 = S_1^*} = 0 \) does not hold for any $S_1^*$. Thus, $(S_1^*, 0)$ cannot be a Nash equilibrium.

The parts (i)-(iii) show the proposition.

**Proof of Lemma 12.**  
(i) Let $S_1 \in \mathcal{S}$ (i.e. $\bar{\theta} - \theta_1(S_1) < C'_1(S_1)$). Since $\theta_1$ is concave, we have $\theta'_1(S) \leq \theta_1(S)/S$ for all $S$. This is equivalent to $C'_1(S)/2 \leq \theta_1(S)$ since

\[ S \theta'_1(S) \leq \theta_1(S) \iff C'_1(S) - \theta_1(S) \leq \theta_1(S) \iff C'_1(S) \leq 2 \theta_1(S) \iff \frac{C'_1(S)}{2} \leq \theta_1(S). \]

Thus,

\[ D_2(S_1, S_1) = \frac{\bar{\theta} - \theta_1(S_1)}{2} < \frac{C'_1(S_1)}{2} \leq \theta_1(S_1) = D_2(S_1, 0). \]

(ii) Let $S_1 \in \mathcal{S}$ (i.e. $\bar{\theta} - \theta_1(S_1) > C'_1(S_1)$) and $S_2 := C'^{-1}_1((\bar{\theta} + \theta_1(S_1))/2)$. Since $\theta_1$ is concave and therefore $C'_1(S_1)/2 + \theta_1(S_1) \geq C'_1(S_1)$, we have

\[ C'_1(S_2) = \frac{\bar{\theta} + \theta_1(S_1)}{2} = \frac{\bar{\theta} - \theta_1(S_1)}{2} + \theta_1(S_1) > \frac{C'_1(S_1)}{2} + \theta_1(S_1) \geq C'_1(S_1). \]

So $S_2 > S_1$, from which follows

\[ \frac{\bar{\theta} + \theta_1(S_1)}{2} = C'_1(S_2) > \frac{C_1(S_2) - C_1(S_1)}{S_2 - S_1}. \]
This yields
\[ D_2(S_1, S_2) = \bar{\theta} - \frac{C_1(S_2) - C_1(S_1)}{S_2 - S_1} > \bar{\theta} - \frac{\theta_1(S_1)}{2} = D_2(S_1, S_1). \]

Proof of Proposition 13. Let \( S_1^* = \inf \mathcal{S} \). For all \( S_1 \in \mathcal{S} \), Firm 2 responds with \( S_2 = 0 \) and \( D_1(S_1^*, 0) > D_1(S_1, 0) \) for all \( S_1 \in \mathcal{S} \) \( \setminus \{ S_1^* \} \). For every \( S_1 \in \mathcal{S} \), Firm 2 can choose \( S_2 > S_1 \) with \( D_2(S_1, S_2) = \theta_1(S_1^*) \) due to continuity. Then, \( D_1(S_1, S_2) = \bar{\theta} - \theta_1(S_1) - \theta_1(S_1^*) < D_1(S_1^*, 0) \). Therefore, Firm 1, as the first mover, chooses \( S_1^* = \inf \mathcal{S} \) and Firm 2 chooses \( S_2^* = 0 \). This yields
\[ D_1(S_1^*, S_2^*) = \bar{\theta} - \theta_1(S_1^*) > \bar{\theta} - C_1'(S_1^*) = \theta_1(S_1^*) = D_2(S_1^*, S_2^*), \]
which shows the first mover advantage.

Proof of Lemma 14. Rearranging the first order condition \( \frac{\partial D_2(S_1, S_2)}{\partial S_2} \bigg|_{S_2 > S_1} = 0 \) yields
\[ C_2(S_1) = C_2(S_2) + C_2'(S_2)(S_1 - S_2) + \Delta C(S_1). \]

Therefore, \( \Delta C(S_1) \) has to be equal to the remainder of the first order Taylor approximation of \( C_2(S_1) \) in \( S_2 \), which is
\[ \int_0^{S_2 - S_1} tC_2''(t + S_1) \, dt. \]

So with \( F_{S_1}(x) := \int_0^x tC_2''(t + S_1) \, dt \) from \( \Delta C(S_1) = F_{S_1}(S_2 - S_1) \) it follows that
\[ S_2 = F_{S_1}^{-1}(\Delta C(S_1)) + S_1. \]

Since \( C_2 \) is strictly convex, \( F_{S_1} \) is strictly increasing with \( F_{S_1}^{-1}(0) = 0 \) and also \( F_{S_1}^{-1} \) strictly increasing.\(^{18}\) Therefore, the solution of the first order condition is unique and gives the global maximum since
\[ \frac{\partial^2 D_2(S_1, S_2)}{\partial S_2^2} \bigg|_{S_2 = \bar{r}_2(S_1)} = -\frac{C_2''(S_2)}{S_2 - S_1} + \frac{C_2'(S_2) + \frac{\partial \theta_{ind}(S_1, S_2)}{\partial S_2} \cdot (S_2 - S_1) - \theta_{ind}(S_1, S_2)}{(S_2 - S_1)^2} \]
\[ = -\frac{C_2''(S_2)}{S_2 - S_1} + 2 \cdot \frac{\frac{\partial \theta_{ind}(S_1, S_2)}{\partial S_2} \cdot S_2 - S_1}{S_2 - S_1} = -\frac{C_2''(S_2)}{S_2 - S_1} < 0. \]

So \( \bar{r}_2 \), according to (3.9), is the well-defined overbidding reaction function of Firm 2.

Applying the Taylor formula on the first order condition for the optimal underbidding

\(^{18}\)Note that \( F_{S_1}(S_2 - S_1) \) is strictly decreasing in \( S_1 \).
reaction yields $(S_1 - S_2)^2\theta'(S_2) + \Delta C(S_1) = F_{S_1}(S_2 - S_1)$. Unfortunately, there is no closed form solution for $r_u^2$, but given the optimal underbidding reaction, the resulting demand for Firm 2 is $D_2(S_1, r_u^2(S_1)) = S_1\theta'(r_u^2(S_1))$.}

For the proof of Lemma 15, we need the following result:

**Lemma 18.** For strictly concave $\theta_1$ and given $S_2$ the unique solution of (3.12) is the global minimum.

**Proof.** Using the first order Taylor formula for given $S_2$, the interior solution $S_1$ needs to satisfy

$$\Delta C(S_2) - (S_2 - S_1)^2\theta'(S_1) = -\int_{S_1}^{S_2} (S_2 - t)C''_1(t) dt$$

and substituting with $S_2 - t$ and switching the integration limits yields

$$-\int_{S_1}^{S_2} (S_2 - t)C''_1(t) dt = -\int_0^{S_2 - S_1} tC''_1(S_2 - t) dt.$$

With $G_{S_2}$ denoting the anti-derivative of $tC''_1(S_2 - t)$ this yields

$$\Delta C(S_2)^\triangledown = -G_{S_2}(S_2 - S_1) + (S_2 - S_1)^2\theta'(S_1) =: H_{S_2}(S_1).$$

$H_{S_2}$ is strictly decreasing in $S_1$ if $\theta_1$ is concave since

$$\frac{\partial H_{S_2}(S_1)}{\partial S_1} < 0 \Leftrightarrow C''_1(S_1) - 2\theta'(S_1) + (S_2 - S_1)\theta''(S_1) < 0 \tag{\ast}$$

and $(\ast) < 0$ directly follows from $\theta''(S_1) < 0$. Thus $H_{S_2}$ is invertible and it follows with $G_{S_2}(0) = 0$

$$S_1 = H_{S_2}^{-1}(\Delta C(S_2)).$$

The second partial derivative of $D_1$ with respect to $S_1$ yields

$$\frac{\partial^2 D_1(S_1, S_2)}{\partial S_1^2} \bigg|_{S_1 = H_{S_2}^{-1}(\Delta C(S_2))} = -\frac{S_2\theta''(S_1) - S_2\theta''(S_1) + S_2\theta'(S_1) + S_2S_1\theta''(S_1)}{(S_2 - S_1)^2}$$

$$+ \frac{\partial D_1(S_1, S_2)}{\partial S_1} \bigg|_{S_1 = H_{S_2}^{-1}(\Delta C(S_2))} \frac{2}{S_2 - S_1}$$

$$= -\frac{S_2}{S_2 - S_1}\theta''(S_1) > 0.$$

The uniqueness of the solution of the first order condition ensures that this is a local and global minimum. 

\[\blacksquare\]
Proof of Lemma 15. For convex $\theta_1$, the term in brackets in (3.11) is always positive and therefore $D_1(S_1, S_2)$ is strictly increasing in $S_1$ on $[\underline{S}, \hat{S}(S_2)]$. In this case, the reaction function $r_1$ of Firm 1 is $r_1(S_2) = \hat{S}(S_2)$ for all $S_2$. For strictly concave $\theta_1$, the term in brackets in (3.11) is always negative and therefore a unique interior solution satisfying the first order condition might exist. According to Lemma 18, this interior solution gives a local and global minimum. Therefore, the optimal reaction will again be a corner solution, i.e. $r_1(S_2) \in \{\underline{S}, \hat{S}(S_2)\}$ for all $S_2 \in [\underline{S}, \overline{S}]$.

Proof of Proposition 16. Let $S_2$ be given by $\theta_2(S_2) = C'_2(S_2)$ with $\theta_2(S_2) < \overline{\theta}$ and $\hat{S}$ by $\frac{C_2(S_2) - C_1(\hat{S})}{S_2 - \hat{S}} = \overline{\theta}$. A necessary condition for the existence of a Two-Firm Solution is a decreasing consumers’ sensitivity, which yields a high sensitivity for small qualities. For a given sensitivity, the existence depends only on $\Delta C$. The higher inefficiency $\Delta C$ is, the less incentive Firm 1 has to enforce quality competition, and thus the more likely $r_1(r_2(\underline{S})) = \underline{S}$ holds. Of course, $\Delta C$ must not be too high, i.e. $\theta_2(S_2) < \overline{\theta}$ has to hold, since otherwise Firm 2 cannot gain any demand.

Closed form of $\hat{S}$

In order to obtain a closed form for $\hat{S}$, let $G(x) := \overline{\theta}x - C_1(x)$ and $F(x) := \overline{\theta}x - C_2(x)$. Then from $\frac{C_2(S_2) - C_1(\hat{S})}{S_2 - \hat{S}} = \overline{\theta}$ we get $\hat{S}(S_2) = G^{-1}(F(S_2)) = G^{-1}(\overline{\theta}S_2 - C_2(S_2)) = G^{-1}(S_2(\overline{\theta} - \theta_2(S_2)))$. $G$ is strictly increasing on $\{x \mid C'_1(x) \leq \overline{\theta}\}$ and strictly decreasing on $\{x \mid C'_1(x) > \overline{\theta}\}$. We only need to focus on the area in which $G$ is strictly increasing, since Firm 1 will always choose the smaller solution of the equation $G(\hat{S}) = F(S_2)$.
Part II

Uncertainty
Chapter 4

The Impact of Supplementary Health Insurance on Upcoding in Hospitals

Abstract

This paper studies the impact of supplementary health insurance on the amount of fraudulent behavior in hospitals. In this model, prices are regulated and hospitals act as experts. In a non-commitment costly state verification setting, health insurance companies can only detect fraudulent claims by performing costly audits. We show that the cheating probability as well as the audit probability are higher when a patient has supplementary health insurance as long as the health insurance company knows which patients do have supplementary health insurance. If this information is private to the hospital, the health insurance company chooses either a low or a high audit probability, depending on the fraction of people with supplementary health insurance. Implementing a disclosure requirement for all people with supplementary health insurance might increase welfare. This depends on the fraction of people who actually have supplementary health insurance.

Keywords: Fraud, Costly State Verification, Costly State Falsification, Upcoding, Health Care Market, Supplementary Health Insurance

JEL: I11, I13, D82, G22
4.1 Introduction

This article targets the research question whether supplementary health insurance has an effect on the fraudulent behavior of hospitals. Understanding the influence of supplementary health insurance on upcoding in hospitals is very important. In many countries hospitals receive activity-based payments (Diagnosed Related Groups - DRG) and people are permitted to buy supplementary health insurance.\textsuperscript{1} In Germany, for instance almost 6 million people bought supplementary health insurance for the hospital sector. The expenditures for such supplementary treatments were about 6 billion EUR in 2009 (PKV-Verband, 2010).

Due to the fact that the institutional background of the health care systems varies from country to country, the model setup focus on a kind of supplementary health insurance that is bought in order to receive a better treatment (e.g., treatment by the chief physician as is the case in Germany\textsuperscript{2}). With a few modifications, the model is applicable for other forms of supplementary health insurance (e.g., a decrease in the coinsurance rate as is the case in the United States for all people who are in a Medicare program) as well.\textsuperscript{3}

It is often argued that activity-based payments such as DRG are beneficial because they encourage hospitals to increase efficiency (Kuhn and Siciliani, 2008). However, one disadvantage is that they also induce hospitals to manipulate the DRG due to the fact that DRG is a system that classifies diseases into different groups. The higher the group, the more money the hospital receives. Let us take a look at an example to emphasize

\textsuperscript{1}Diagnosed Related Groups or better known as DRG is a system to classify hospital cases into one group. For instance, it is used in the United States in Medicare (Medicare Severity-DRG). Similar DRG systems are used in France (Groupes homogènes de malades), Australia (Australian National-DRG), Germany (G-DRG), Switzerland (SwissDRG), Scandinavia (NorDG), and Denmark (DkDRG). Furthermore, some countries have developed a system that is comparable with the DRG-System such as UK (Health Care Resource Groups), Canada (Case Mix Groups), Austria (Leistungsbezogene Diagnose-Fallgruppen), Hungary (Homogén Beteégés-Csoporth), or Japan (Diagnosis Procedure Combinations). In many countries, people are allowed to buy supplementary health insurance in order to either receive better treatment or to lower their coinsurance rate. An example can be found in the United States: those people who are in a Medicare program are permitted to buy Medicare supplement policies (Medigap) in order to decrease their deductible or coinsurance rates. In Germany, people are allowed to buy supplementary health insurance in order to receive certain extra treatments such as treatments by a chief physician and in Denmark, people can buy supplementary health insurance in order to skip waiting lists.

\textsuperscript{2}In Germany, most of the people are insured via Statutory Health Insurance. When a patient receives treatment in a hospital, the sickness fund has to pay the fee for the DRG. If the patient wants to be treated by the chief physician, he can buy supplementary health insurance from one of over 40 private health insurance companies. The private health insurance company then has to pay the hospital an extra fee for service. The exact amount for each service is determined by the medical fee schedule (Gebührenordnung für Ärzte).

\textsuperscript{3}In order to make this model applicable for other forms of supplementary health insurance, we could assume that patients have the opportunity to search for a second opinion. As an example, the lower the coinsurance rate, the higher the relative transportation costs. This affects the likelihood to ask for a second opinion and therefore also affects the fraudulent behavior of the hospital. For a paper that deals with second opinions, see Sülzle and Wambach (2005). We could also assume that the patient might complain about the fee and therefore act as a kind of fraud detection system. For a paper that deals with fraud detection, see Schiller (2006).
this problem. In Germany, the amount of payment is determined by the G-DRG. If a patient has an acute apoplexia with a neurological complex treatment, the hospital needs to decide whether there is a complicating development (G-DRG: B70A) or not (G-DRG: B70B). The monetary difference between these two DRGs is about 1300 €. Therefore, the hospital has a high incentive for upcoding, especially if one considers the fact that the patient might not complain about the fee charged for several reasons. First, it is very difficult for the patient to assess the accuracy of the diagnosis. Second, in countries such as Germany, the patient does not care about the fee because he is fully insured. A third reason that holds for almost all acute diseases is the fact that the patient is not able to receive a second opinion. Due to these facts one could argue that upcoding is a dominant strategy for hospitals. Empirical research shows that upcoding is indeed a big problem. One example is that there is an accumulation that cannot be explained statistically of a certain birth weight, ranging from 740 to 749 grams, of newborn children (GKV-Spitzenverband, 2011). For premature infants weighting more than 749 grams, the hospital receives another DRG. The monetary difference between these groups is 23,000 €. Similar results can be found where people get artificial respiration (GKV-Spitzenverband, 2011).

In order to reduce upcoding, the health insurance company (e.g., the sickness funds in Germany) that has to pay the fee for the DRG has the possibility of auditing the hospital.\footnote{In Germany, a sickness fund can authorize an external company to provide auditing services (Medizinischer Dienst der Krankenkassen (MDK)). In 2010, the MDK had an audit probability of around 11\% and almost every second audit detected discrepancies in the claimed DRG (GKV-Spitzenverband, 2011). Since not every discrepancy can be seen as upcoding, a hospital only is punished in some cases. The hospitals only are punished if they act wantonly negligent. Otherwise they just have to pay back the monetary difference between the claimed DRG and the correct DRG.}

For an optimal audit policy, the health insurance company has to evaluate the validity of the reported DRG. The upcoding and audit strategies depend on a few well known factors. The fine for detected upcoding, the cost of an audit and the potential upcoding gains are all good examples for factors that influence the upcoding and audit strategies. But fraud in the hospital sector depends on one more determinant that has not been analyzed in the literature so far: the patient may have supplementary health insurance. Analyzing the influence of this factor is the core of this paper. The information concerning the existence of supplementary health insurance might be common knowledge or private information to the hospitals.\footnote{To be more precise, if we assume that the information is private, we assume that the information is private to the hospital and the patient. Due to our assumption that the patient does not complain about the price, the only relevant counterpart to the hospital is the sickness fund. Hence, if we assume that the information is private, we assume that the health insurance company does not know which patient has supplementary health insurance. As already mentioned above, the model is applicable for other forms of supplementary health insurance as well. We then require the assumption that the patient might complain about the price or at least ask for a second opinion.} To analyze the effect, let us go back to our first example.
We assume that there is no complicating development so that the correct code is G-DRG: B70B. The reporting of G-DRG: B70A is risky but profitable, if cheating is not detected.

The hospital can influence the detection probability by manipulating in order to make it more difficult for the health insurance company to detect cheating. Of course, manipulation comes at a cost, since the hospital needs to engage in extra treatments that are not necessary to treat the patient (e.g., expend the clinical pathway, higher usage of drugs, increased residence time).

As is typical for a prospective payment system, the reimbursement is a fixed amount. Hence, for a given DRG, any additional treatment lowers the profit. If a patient has supplementary health insurance, the hospital is authorized to charge extra money for every service it provides as long as it is remunerated by a fee for service system, e.g., like in Germany. Therefore, additional treatments might even increase overall profits or at least lead to a smaller decrease in profits. As a consequence, the existence of supplementary health insurance makes manipulation more attractive.\(^6\)

In order to react to the fact that manipulation becomes more attractive, the health insurance company needs to know whether a patient has supplementary health insurance or not. However, this information is private in most cases.\(^7\) I therefore introduce one more kind of asymmetric information, since I assume that the information about the existence of supplementary health insurance might be private to the hospital.

To the best of my knowledge, there have not been any studies about the optimal audit policy with patients having supplementary health insurance. The results are the following: if the health insurance company knows which patients have supplementary health insurance, asymmetric information only exists with respect to the diagnosis. We have a mixed equilibrium where cheating and auditing both occur with a positive probability. The health insurance company audits the hospital with a higher probability when the hospital treats a patient with supplementary health insurance.

If the health insurance company does not know which patients have supplementary health insurance, the results change dramatically. We then have two kinds of asymmetric information. First, the health insurance company cannot observe the patients state of health. Second, it cannot observe the hospitals manipulation incentive. This is due to the fact that the manipulation incentive depends on the existence of supplementary health insurance. In this game nature decides in the first stage whether or not a patient has supplementary health insurance. The hospital then chooses its overall cheating probability. The hospital either has a dominant strategy when it treats a patient with supplementary

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\(^6\) Another example that shows that the existence of supplementary health insurance makes manipulation more attractive is the hospitalization insurance for single rooms. The hospital then receives an extra fee for every day the patient stays in the hospital.

\(^7\) In Germany, for instance people can buy supplementary health insurance from over 40 companies. Hence, the existence of supplementary health insurance is only common knowledge if it is bought from a cooperation partner.
health insurance and a mixed strategy when it treats a patient without supplementary health insurance or vice versa. The insurers’ chosen audit probability depends on the fraction of patients that have supplementary health insurance. The higher that fraction, the more likely it is that the health insurance company audits with the same probability as if there were no people without supplementary health insurance. This is due to the fact that the insurance company can only choose one overall audit probability. If the fraction of patients that have supplementary health insurance is higher than a critical level, welfare could be increased if the government implemented a disclosure requirement for supplementary health insurance due to the fact that audits are a waste of money. In any case, the health insurance company is better off with a disclosure requirement.

The rest of this article is organized as follows: the next section gives a literature review and states the main distinctions in comparison with my article. Section 3 explains the model. In Section 4, the analysis is made under the assumption that the existence of supplementary health insurance is common knowledge. In Section 5, the analysis is made under the assumption that the existence of supplementary health insurance is private information. This means that only the hospital knows which patients do have supplementary health insurance while the health insurance company only knows the overall fraction of people with supplementary health insurance. Section 6 discusses the results.

4.2 Related Literature

Literature related to this topic can be found in three directions: the first is the market for credence goods, the second is literature about general fraud models, and the third direction is literature on upcoding. The latter is closely related to literature on fraud, but the studies focus explicitly on the health care market.

**Credence Goods:** Several articles deal with credence goods (De Jaegher and Jegers, 2001; Emons, 1997, 2001; Wolinski, 1993, 1995; Sülzle and Wambach, 2005; Dionne, 1984, among others). Since the study of Sülzle and Wambach (2005) is closely related to my study, I briefly explain their results. The study examines the impact of variations in the degree of coinsurance on the amount of fraud in a hospital-patient relationship where prices are regulated and patients can search for second opinions. As patients do not have any information about the degree of their illness, hospitals can claim that the illness is very severe even if only a small treatment is necessary. The only choice variable of the patients is to accept or reject the treatment. In the case of rejection, the patients go to another hospital and trust the second diagnosis. In equilibrium, the extent of fraud

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8The intuition behind that is very simple: If the insurance company chooses an audit strategy that is lower than the audit probability when everyone has supplementary health insurance but higher than the audit probability when everyone is without supplementary health insurance, the hospital would always have a dominant strategy. In such a situation the health insurance company would always be worse off.
has to be just sufficient to keep patients indifferent. Sülzle and Wambach (2005) show that a higher coinsurance rate may lead to either less fraud and a lower probability of patients searching for second opinions or more fraud and more searches. This is due to the fact that patients are c.p. less willing to accept a high diagnosis and search for a second opinion if the coinsurance rate increases. This makes fraudulent behavior less attractive. On the other hand, if patients are more willing to reject the first diagnosis, chances that a patient coming to a hospital on a second visit are higher. In this case, the patient would accept a high diagnosis as a confirmation of the first diagnosis. This makes fraudulent behavior more attractive.

The assumptions that Sülzle and Wambach (2005) make are reasonable for health care services where people have coinsurance and where the diagnosis is not acute. However, there are many situations, especially in the hospital market, where at least one of the assumptions does not apply (e.g. heart attack, aches and pains, unconsciousness). I therefore assume that patients do not search for a second opinion.

**Insurance Fraud:** Since the health insurance company has the opportunity of auditing the hospital, models that focus on insurance fraud are closely related to my study as well. Insurance fraud is a well-studied research topic. Most dominant in this line of research are costly state verification models (Townsend, 1979; Mookherjee and Png, 1989; Khalil, 1997; Fagart and Picard, 1999, among others). First studies analyzed principal-agent relationships where the principal can commit to an audit policy ex ante.\(^9\) Under commitment, the revelation principle\(^10\) applies, and therefore, the agent truthfully reports his private information. Hence, insurance fraud can be prevented entirely. However, in many situations the principal is not able to credibly commit himself to an audit policy ex ante. In such a situation, the revelation principle is not applicable anymore (Bester and Strausz, 2001). Khalil (1997) was one of the first who used a costly state verification model in which the principal cannot commit to an audit policy. The model is solved by first characterizing the principal’s second-stage equilibrium auditing strategy and the agent’s second-stage equilibrium compliance strategy given the terms of the incentive contract chosen in the first-stage of the game. There is a mixed strategy equilibrium in the second-stage where noncompliance and auditing both occur with a positive probability. As is typical for a mixed strategy equilibrium, the audit probability makes the agent indifferent between complying and not complying. Analogously, the probability of compliance makes the expected penalty collected by the principal equal to the audit costs. In the first-stage, the optimal contract is designed, given the mixed equilibrium in the second stage. In my

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\(^9\)For articles that assume commitment, see Baron and Besanko (1984), Dunne and Loevenstein (1995), Graetz et al. (1986) or Mookherjee and Png (1989).

\(^10\)The revelation principle reduces the complexity of contracting problems under hidden information. It states that an uninformed principal can use a direct revelation mechanism where each informed agent is asked to reveal his private information and in which truth telling is an optimal strategy for each informed agent (Gibbard, 1973; Green and Laffont, 1977; Dasgupta et al., 1979; Myerson, 1979, among others).
study, the equilibrium strategies are analyzed as well but considering the optimal contract design is not necessary since prices are exogenously determined by DRG.

There also have been many modifications of the costly state verification model. Schiller (2006), for instance has analyzed the impact of fraud detection systems. The study shows that auditing becomes more effective and that an overcompensation of agents can be reduced as long as the insurance company is able to condition its audits on the information provided by the detection system. And Boyer (2000) has constructed a model with two types of agents, one completely honest and one opportunistic. The types therefore differ in their behavior in so far as that the former never exaggerates his claim, while the latter plays a non-cooperative game with the insurer.\textsuperscript{11} His results have strong implications. If the insurer can not distinguish between honest and opportunistic types, he has to provide a pooling contract. This pooling contract is set up so that the opportunistic type ends up with the same contract as in an economy in which there are no honest people, provided that there are not too many honest people in the economy. If this is the case, then the probability that a fraudulent claim is filed is independent of the exact population share of each type. The opportunistic people just alter their cheating probability in order to keep the insurance company indifferent between auditing and not auditing.

Another interesting line of research in the field of insurance fraud are costly state falsification models (Lacker and Weinberg, 1989; Crocker and Morgan, 1998; Maggi and Rodriguez-Clare, 1995; Crocker and Slemrod, 2011; Kuhn and Siciliani, 2009, among others). Lacker and Weinberg (1989) analyzed an exchange economy in which the realization of certain endowments can be falsified by the agent at a certain cost. Kuhn and Siciliani (2009) used the costly falsification approach in order to explain how the usage of performance indicators influences the quality of health care. The hospitals have private information about their own ability and can engage in costly manipulation of quality measures. The reported quality is the sum of quality effort and manipulative effort.

In this study, the hospital can engage in manipulation as well. As is typical for costly state falsification models, manipulation effort is costly. In this particular setting, manipulation effort is less costly if the patient has supplementary health insurance due to the fact that the hospital gets remunerated by a supplementary fee for service from the private health insurance company. Since this study targets the question whether supplementary health insurance has an effect on fraudulent behavior of hospitals, the engagement in manipulation can be kept very simple. For any given diagnosis, the optimal manipulation effort is chosen such that marginal costs are equal to marginal monetary benefit.

\textbf{Upcoding:} The third direction of related literature is literature on upcoding. Essentially, it is about fraud in the health insurance market. Since activity-based payments are

\textsuperscript{11}A similar setup can be found in the study by Picard (1996).
common in many OECD countries (Mossialos and Le Grand, 2000, among others), the relevance of upcoding is significant due to the fact that one main disadvantage of activity-based payments is that they induce hospitals to manipulate the payment to their advantage. The explanation of upcoding is closely related to the explanation of fraud in the market for credence goods. A number of empirical studies point out that upcoding is a big problem in the health care system (Carter et al., 1990; Silverman and Skinner, 2004; Dafny, 2011, among others). Carter et al. (1990), for instance find that one third of the change in the Medicare’s case-mix index between 1986 and 1987 was due to upcoding.

A theoretical paper about upcoding and optimal auditing has been written by Kuhn and Siciliani (2008). The authors investigate the optimal contractual arrangements between a purchaser and a provider of health services where the reported output can either be additive or multiplicative in quality and manipulation. The paper combines the costly state verification approach with the costly state falsification approach. Although the basic idea is similar, my approach is very different. First of all, they assume that a hospital can increase the demand by increasing quality but might also increase demand through a manipulation effort. Increasing demand through manipulation is reasonable if patients are encouraged to be readmitted to the hospital several times. Increasing demand through a higher quality is reasonable if the treatment is not a credence good. In this paper I assume that a patient needs to be treated and goes to a certain hospital. It is irrelevant whether he goes there because of its good reputation or due to a readmission which means that I do not account for any demand side effects. Hence, the hospital has to decide for each patient whether it wants to code up or not. Furthermore, I focus on the impact of supplementary health insurance on upcoding. Kuhn and Siciliani (2008) solely focus on basic health insurance.

4.3 Model

The following framework is assumed. There is one risk neutral hospital that maximizes its surplus $S$ and one risk neutral health insurance company that acts on behalf of the patients and therefore minimizes its costs $C$ for a given quality. There are only two possible states of health in this world. The patient comes to the hospital either with a moderate disease ($L$) or with a severe disease ($H$). This information is private to the hospital. The health insurance company only knows that indication $H$ occurs with

Assuming that the hospital maximizes its surplus is a simplification in order to keep the calculation as simple as possible. Of course, there are various layers of incentives within the hospital (e.g. the involved physician, the department or the hospital as a unit). As an example, we could assume that the physician might have some disutility from fraudulent behavior. However, this simplification does not change the results in a qualitative way (e.g. the physicians’ disutility from fraudulent behavior could be subsumed into the penalty that the hospital has to pay if fraudulent behavior is detected. It also could be subsumed into the netprice $P_H$ in the case of fraud.).
probability $\pi$. The price $P$ that a hospital gets for treating $L$ or $H$ is exogenously given by the DRG with $P_L < P_H$.\footnote{To keep the model simple, $P_L$ and $P_H$ can be seen as the net prices, this being the DRG minus the treatment costs.} Some patients $\xi \in [0, 1]$ have supplementary health insurance ($\Xi = 1$), some $(1 - \xi)$ do not ($\Xi = 0$). This information is either common knowledge or private to the hospital.

Due to the asymmetric information about the disease, the hospital has the possibility of upcoding. While for a patient with a severe diagnosis cheating\footnote{For a patient with a severe diagnosis cheating means that the hospital claims $P_L$. This could only be a reasonable strategy if there is a significant capacity constraint which we do not consider here.} of course, is a dominated strategy, the hospital can upcode a patient with a moderate disease.\footnote{If the hospital claims $P_H$, we assume that the hospital makes a surplus that is independent of the real indication $\{L, H\}$. This is equivalent to the assumption that a patient with indication $L$ then gets treatment $H$ instead of treatment $L$. The patient does not receive any additional utility or disutility from the higher treatment. Of course one could argue that the hospitals’ surplus is higher if a $L$-type is treated, but that would not change the result. Hence, this assumption just keeps the model simple.} Being caught defrauding is costly for the hospital. Let $k$ be a fixed penalty that is sunk and not collected by the health insurance company. The health insurance company audits with probability $\nu \in [0; 1]$ at cost $c$ and the hospital cheats with probability $\eta \in [0; 1]$. The hospital can influence the conditional probability of detection $d(s)$ by manipulation $s$.\footnote{$d(s) = d(s|\nu = 0)$ is the detection probability under the condition that the hospital cheats and the health insurance company audits. Without loss of generality we assume $d(s) = d(s|\nu = 0) = 0$.} The cost of manipulation is given by $m(s)$ with $m'(s) > 0$ and $m''(s) > 0$. We further have $d''(s) < 0$ and $d''(s) > 0$.

The sequence of play is the following: In the first stage, the contract is signed. If the contract is not signed, the game ends. Otherwise it continues with given prices. In the next stage, nature chooses an indication $\{L, H\}$ and decides whether there is a patient with or without supplementary health insurance. After the hospital has observed the state of health, it decides whether to claim $P_L$ or $P_H$. Then the health insurance company decides whether to audit or not. After an audit, the payoffs are distributed. To analyze the game it is sufficient to analyze the fraud and audit strategies.

It is a game of incomplete information. A game of this sort is known as a Bayesian game.

### 4.4 Common Knowledge of Type

If the health insurance company knows which patients have supplementary health insurance, the only information that is asymmetric is about the health state of the patient. As mentioned in the introduction, the hospital can influence the detection probability by manipulation\footnote{E.g. expend the clinical pathway, usage of drugs, increased residence time. Since the hospital needs to engage in extra treatments that are not necessary to treat the patient, we assume that manipulation} in order to make it harder for the health insurance company to detect...
cheating. As long as the hospital is not cheating, we assume that there is no incentive to manipulate. In the case of upcoding, the optimal amount of manipulation $s^*$ is chosen so that the marginal costs of manipulation are equal to the marginal monetary benefit because of the lower detection probability. Obviously, any additional treatment, i.e. manipulation, lowers the detection probability. On the other hand, if a patient does not have supplementary health insurance, any additional treatment lowers the profit for a given DRG. We need to compare the chosen $s^*_{\Xi=0}$ with the optimal amount of manipulation $s^*_{\Xi=1}$ when a patient has supplementary health insurance. If a patient has supplementary health insurance, the hospital is authorized to charge extra money for every service it provides as long as it gets a fee for service (as in Germany).

**Proposition 19.** C.p. there is a higher incentive to manipulate if the patient has supplementary health insurance. Therefore, for any positive audit probability the detection probability in the case of cheating is lower compared to the situation in which the patient does not have supplementary health insurance.

**Proof.** While the marginal monetary benefit is independent of $\Xi$, the marginal cost of manipulation $m(s)$ depends on $\Xi$. For $\Xi = 0$, the hospital only gets remunerated by DRG but for $\Xi = 1$ the hospital gets the DRG plus a fee for service. This fee for service makes manipulation more attractive, since the hospital receives extra payment for all supplementary treatments. We therefore have $m'(s)_{\Xi=1}(s) < m'(s)_{\Xi=0}(s)$. Resulting from this inequality, we further have $s^*_{\Xi=1} > s^*_{\Xi=0}$ and $d(s^*_{\Xi=1}) < d(s^*_{\Xi=0})$.

Without loss of generality we assume $0 < d = d(s^*_{\Xi=1}) < d(s^*_{\Xi=0}) = 1$. Knowing about this optimal manipulation strategy, we can now focus on the commonly used costly state verification approach.

We first analyze the optimal audit and cheating strategy for $\Xi = 0$. The aim of the hospital is to maximize its surplus $S$. For a severe disease (diagnose $H$) the hospital always claims $P_H$. We therefore only need to concentrate on the situation in which the patient has a moderate disease (diagnose $L$). The maximization problem of the hospital is given by

\[
\text{is costly to the hospital.}
\]

To emphasize the manipulation effort we go back to our example where the person has an acute apoplexia with a neurological complex treatment. We now assume that the person has supplementary health insurance and that the development is no complicating development (G-DRG: B70B). Due to the fee for service system in the supplementary health insurance the total amount the hospital gets can be increased if the patient gets more treatment. If the hospital cheats by claiming G-DRG: B70A more additional treatments can be done (due to the complicating development.) If those additional treatments are remunerated manipulation is more attractive.

Of course, the detection probability is likely to be smaller than one and differs from disease to disease. Therefore, one could assume $d_i(s^*_{\Xi=0} \in (0,1)) < 1$ for all $i$, where $i$ is a certain disease. Due to the fact that a normalization, i.e. $0 < d(s^*_{\Xi=1}) < d(s^*_{\Xi=0}) = 1$, does not change results in a qualitative way, we keep the model as simple as possible. Furthermore, we can omit the index $i$ since the hospital as well as the health insurance company have to derive the optimal audit and fraud strategies for every disease $i$. 


\[ \max_{\eta} \quad (1 - \eta) \cdot P_L + \eta \cdot [P_H \cdot (1 - \nu) + (P_L - k) \cdot \nu]. \]  

(4.1)

Optimization yields

\[ \frac{\partial S}{\partial \eta} = -P_L + P_H \cdot (1 - \nu) + (P_L - k) \cdot \nu \overset{!}{=} 0 \]

\[ \Leftrightarrow \nu_{\Xi=0} = \frac{P_H - P_L}{P_H - (P_L - k)}. \]

The aim of the health insurance company is to minimize its costs \( C \) for a given treatment. As long as the hospitals claims \( P_L \), the health insurance company does not audit. We therefore only need to concentrate on the situation in which the hospital claims \( P_H \). In \( \frac{(1 - \pi)\eta}{\pi + (1 - \pi)\eta} \) situations, the health insurance company detects a false claim. The maximization problem is given by

\[ \min_{\nu} \quad (1 - \nu) \cdot P_H + \nu \cdot \left[ P_H + c - \frac{(1 - \pi)\eta}{\pi + (1 - \pi)\eta} \cdot (P_H - P_L + k) \right]. \]

(4.2)

Optimization yields

\[ \frac{\partial C}{\partial \nu} = -P_H + P_H + c - \frac{(1 - \pi)\eta}{\pi + (1 - \pi)\eta} \cdot (P_H - P_L + k) \overset{!}{=} 0 \]

\[ \Leftrightarrow \eta_{\Xi=0} = \left( \frac{\pi}{1 - \pi} \right) \left( \frac{c}{(P_H - P_L + k) - c} \right). \]

We now need to do the same for \( \Xi = 1 \). In this case we have \( 0 < d < 1 \). We therefore need to alter the objective functions. The hospitals’ maximization problem is now given by

\[ \max_{\eta} \quad (1 - \eta) \cdot P_L + \eta \cdot [P_H \cdot (1 - \nu) + P_H \cdot \nu \cdot (1 - d) + (P_L - k) \cdot \nu \cdot d] \]

(4.3)

Optimization yields

\[ \frac{\partial S}{\partial \eta} = -P_L + P_H \cdot (1 - \nu) + P_H \cdot \nu \cdot (1 - d) + (P_L - k) \cdot \nu \cdot d \overset{!}{=} 0 \]

\[ \Leftrightarrow \nu_{\Xi=1} = \frac{P_H - P_L}{P_H - (P_L - k)} \cdot \frac{1}{d}. \]

The new maximization problem of the hospital is now given by

\[ \min_{\nu} \quad (1 - \nu) \cdot P_H + \nu \cdot \left[ P_H + c - \frac{(1 - \pi)\eta d}{\pi + (1 - \pi)\eta} \cdot (P_H - P_L + k) \right] \]

(4.4)
Optimization yields
\[
\frac{\partial C}{\partial \nu} = -P_H + P_H + c - \frac{(1 - \pi) \eta d}{\pi + (1 - \pi) \eta} \cdot (P_H - P_L + k) = 0
\]
\[
\iff \eta_{\Xi=1} = \left( \frac{\pi}{1 - \pi} \right) \left( \frac{c}{d(P_H - P_L + k) - c} \right).
\]

In order to analyze the effect of supplementary health insurance we need to compare \(\nu_{\Xi=1}\) with \(\nu_{\Xi=0}\) and \(\eta_{\Xi=1}\) with \(\eta_{\Xi=0}\).

**Proposition 20.** Supplementary health insurance is costly for the health insurance company even if the patient has bought the supplementary health insurance from a different health insurance company. This is due to the fact that there is more manipulation as well as more audits.

**Proof.** Due to \(0 < d < 1\), we have \(\nu_{\Xi=1} > \nu_{\Xi=0}\) and \(\eta_{\Xi=1} > \eta_{\Xi=0}\).

\(\nu_{\Xi=1} > \nu_{\Xi=0}\) and \(\eta_{\Xi=1} > \eta_{\Xi=0}\) are due to the fact that an audit is not that effective anymore. The health insurance company needs to audit with a higher frequency in order to keep the hospital indifferent, while the hospital can cheat with a higher frequency in order to keep the health insurance company indifferent. The existence of supplementary health insurance therefore increases the cost of the health insurance company due to more fraud and more audits. In this section, we assume that the health insurance company knows which persons have supplementary health insurance. But this assumption does not have to be fulfilled. In Germany, for instance the market for supplementary health insurance is highly competitive so that sickness funds (that has to pay the fee DRG) do not know whether their policy holders (i.e. patients to the hospital) have supplementary health insurance or not. We now analyze the game in which the information about the existence of supplementary health insurance \(\Xi \in \{0, 1\}\) is only available to the hospital.

### 4.5 Private Knowledge of Type

We now have asymmetric information about \(\Xi\). The extensive form game is given in Figure 4.1. While the hospital of course knows which patients have supplementary health insurance \(\Xi \in \{0, 1\}\), the health insurance company only knows that the share \(\xi \in [0, 1]\) of the population has supplementary health insurance. If a high claim is made, the health insurance company has to form its beliefs. If the patient has a severe disease (diagnose \(H\)), the hospital always claims \(P_H\). Since diagnose \(H\) occurs with probability \(\pi\), the health insurance company believes that with probability

\[
\frac{\pi}{\pi + (1 - \pi)[\xi \eta_{\Xi=1} + (1 - \xi) \eta_{\Xi=0}]}.
\]
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Figure 4.1: Extensive form game where $\Xi$ is private information

the claim is justified. Further, the health insurance company detects a fraudulent claim if the hospital treats a patient without supplementary health insurance with probability

$$\frac{(1 - \pi)(1 - \xi)\eta_{\Xi=0}}{\pi + (1 - \pi)[\xi\eta_{\Xi=1} + (1 - \xi)\eta_{\Xi=0}]}.$$ 

It detects a fraudulent claim if the hospital treats a patient with supplementary health insurance with probability\(^{20}\)

$$\frac{(1 - \pi)\xi\eta_{\Xi=1}d}{\pi + (1 - \pi)[\xi\eta_{\Xi=1} + (1 - \xi)\eta_{\Xi=0}]}.$$

As before, the aim of the health insurance company is to minimize its costs $C$

$$\min \nu \quad (1 - \nu) \cdot P_H + \nu \cdot \left[ P_H + c - \frac{(1 - \pi)[\xi\eta_{\Xi=1}d + (1 - \xi)\eta_{\Xi=0}]}{\pi + (1 - \pi)[\xi\eta_{\Xi=1} + (1 - \xi)\eta_{\Xi=0}]} \cdot (P_H - P_L + k) \right]. \quad (4.5)$$

From (4.5) we get

$$\frac{\partial C}{\partial \nu} = -P_H + P_H + c - \frac{(1 - \pi)[\xi\eta_{\Xi=1}d + (1 - \xi)\eta_{\Xi=0}]}{\pi + (1 - \pi)[\xi\eta_{\Xi=1} + (1 - \xi)\eta_{\Xi=0}]} \cdot (P_H - P_L + k) \overset{!}{=} 0$$

\(^{20}\)Remember that the health insurance company believes that with a probability of

\(\frac{(1 - \pi)\xi\eta_{\Xi=1}}{\pi + (1 - \pi)[\xi\eta_{\Xi=1} + (1 - \xi)\eta_{\Xi=0}]}\) the claim is fraudulent; but due to $d < 1$, the detection probability is lower.
\[ \Leftrightarrow \eta_{\Xi=0} = \eta_{\text{CK}} \cdot \frac{1}{1 - \xi} - \eta_{\Xi=1} \frac{\xi}{1 - \xi} D, \] (4.6)

where \( \eta_{\text{CK}} := \left(\frac{\pi}{1 - \pi}\right) \left(\frac{c}{(P_H - P_L + k) - c}\right) \) is the probability of fraud from a hospital that treats a patient without supplementary health insurance in the situation where the type is common knowledge and \( D = \frac{d(P_H - P_L + k) - c}{(P_H - P_L + k) - c} \) is a positive parameter with \( D \in (0, 1) \) that considers the lower detection probability. Equation (4.6) is a necessary condition in order to keep the health insurance company indifferent between auditing and not auditing.

**Proposition 21.** An increase in fraud for a patient with supplementary health insurance leads to a direct decrease in fraud for a patient without supplementary health insurance. Hence, the hospitals’ fraud strategies are strategic substitutes.

**Proof.** For a given \( \xi \), we clearly see that \( \frac{\partial \eta_{\Xi=0}}{\partial \eta_{\Xi=1}} < 0. \)

We can now derive the new \( \nu \). In order to do so, we need to maximize the hospital’s surplus \( S \) again. In \( (1 - \xi) \) situations the hospital maximizes

\[
\max_{\eta_{\Xi=0}} (1 - \eta_{\Xi=1}) \cdot P_L + \eta_{\Xi=0} \cdot [P_H \cdot (1 - \nu) + (P_L - k) \cdot \nu]
\]

\[
\frac{\partial S}{\partial \eta_{\Xi=0}} \bigg|_{\nu_{\Xi=0}} = 0 \Leftrightarrow \nu_{\Xi=0} = \frac{P_H - P_L}{P_H - (P_L - k)}.
\]

In \( \xi \) situations the hospital maximizes

\[
\max_{\eta} (1 - \eta) \cdot P_L + \eta \cdot [P_H \ast (1 - \nu) + P_H \cdot \nu \cdot (1 - d) + (P_L - k) \cdot \nu \cdot d]
\]

\[
\frac{\partial S}{\partial \eta} \bigg|_{\nu_{\Xi=1}} = 0 \Leftrightarrow \nu_{\Xi=1} = \frac{P_H - P_L}{P_H - (P_L - k)} \cdot \frac{1}{d}.
\]

Since the health insurance company does not know whether the hospital treats a patient with or without health insurance, it can only choose one audit strategy \( \nu_{\text{PT}} \) instead of two.

**Proposition 22.** The chosen audit probability is in the interval of the chosen audit probability when no patient has supplementary health insurance and the chosen audit probability when all patients have supplementary health insurance.

**Proof.** \( \nu_{\text{PT}} \in [\nu_{\Xi=0}, \nu_{\Xi=1}] \) needs to hold, since for \( \nu_{\text{PT}} < \nu_{\Xi=0} \) the hospital would always cheat and for \( \nu_{\text{PT}} > \nu_{\Xi=1} \) the hospital would never cheat. If the hospital always (never) cheats the optimal reaction of the health insurance company is (not) to audit. Therefore, \( \nu_{\text{PT}} \notin [\nu_{\Xi=0}, \nu_{\Xi=1}] \) cannot be an equilibrium. \( \square \)
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For \( \nu_{PT} = \nu_{\Xi=0} (\nu_{PT} = \nu_{\Xi=1}) \), the hospital will play a mixed strategy (will never cheat) if the patient has no supplementary health insurance and will always cheat (will play a mixed strategy) if the patient has supplementary health insurance. For \( \nu_{\Xi=1} > \nu_{PT} > \nu_{\Xi=0} \), the hospital always has a dominant strategy with \( \eta_{\Xi=0} = 0 \) and \( \eta_{\Xi=1} = 1 \). The chosen audit probability highly depends on the fraction of people that have supplementary health insurance.

As a starting point, we assume that \( \xi = 1 \). Obviously, we then have the same optimization problem as in Section 4.4. In such a situation, we have \( \nu_{PT} = \nu_{\Xi=1} \) and

\[
\eta_{\Xi=1} = (\frac{\pi}{1-\pi}) \left( \frac{c}{d(PH - P_L + k) - c} \right).
\]

If some people then terminate their supplementary health insurance the hospital has to treat patients without supplementary health insurance. It therefore might be necessary to adjust the cheating probability as well as the audit probability in order to keep the other indifferent. Let us start analyzing the audit strategy of the hospital. If it lowers its audit probability, the hospital chooses \( \eta_{\Xi=1} = 1 \) and \( \eta_{\Xi=0} = 0 \) (for \( \nu_{\Xi=1} > \nu_{PT} > \nu_{\Xi=0} \)) or \( 0 < \eta_{\Xi=0} < 1 \) (for \( \nu_{PT} = \nu_{\Xi=0} \)). As long as \( \xi \) is sufficiently large (\( \xi > \frac{\eta_{CK}}{D} \)), the health insurance company will never be indifferent between auditing and not auditing for any \( \nu_{PT} \in [\nu_{\Xi=0}, \nu_{\Xi=1}] \), which we can see from (4.6). It will therefore stick to its initial audit probability \( \nu_{PT} = \nu_{\Xi=1} \).

**Proposition 23.** For sufficiently large \( \xi \), the hospital never cheats if it treats a patient without supplementary health insurance and cheats with a positive probability that is below one if it treats a patient with supplementary health insurance.

**Proof.** From the maximization problem (4.1) we get \( \eta_{\Xi=0} = 0 \) for \( \nu_{PT} = \nu_{\Xi=1} \). The expected loss is higher than the expected gain. Therefore not cheating is a dominant strategy. If the patient has supplementary health insurance, the hospital has a positive cheating probability. From (4.6) we get

\[
\eta_{\Xi=1} = \frac{\eta_{CK}}{\xi D}.
\]

According to \( \eta_{\Xi=1} = \frac{\eta_{CK}}{\xi D} \), the hospital chooses \( \eta_{\Xi=1} = 1 \) as soon as \( \xi = \frac{\eta_{CK}}{D} \). Knowing that the hospital now has a dominant strategy, the health insurance company adjusts its audit probability as well. Obviously, \( \nu_{PT} = \nu_{\Xi=1} \) is not optimal anymore, since \( \eta_{\Xi=1} = 1 \) and \( \eta_{\Xi=0} = 0 \) can be obtained with any \( \nu_{PT} \in (\nu_{\Xi=0}, \nu_{\Xi=1}) \). Hence, the health insurance

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21 The optimization problem of the hospital is given by equation (4.3) and the optimization problem of the health insurance company is given by (4.4).

22 With an audit probability \( \nu_{PT} \in [\nu_{\Xi=0}, \nu_{\Xi=1}] \), the hospital always chooses \( \eta_{\Xi=1} = 1 \). In order to satisfy equation (4.6), the hospital needs to choose \( \eta_{\Xi=0} = \frac{\eta_{CK}}{1-\xi} \). Since \( 0 \leq \eta_{\Xi=0} \leq 1 \) needs to hold, (4.6) can never be satisfied for \( \xi > \frac{\eta_{CK}}{D} \). Due to the high cheating probability, the best response of the health insurance company would always be an increase to the initial audit probability \( \nu_{PT} = \nu_{\Xi=1} \) again.

23 To be precise, the equation is given by \( \eta_{\Xi=1} = Min \left( \frac{\eta_{CK}}{\xi D}, 1 \right) \).
company will at most audit with probability $\nu_{PT} = \nu_{\Xi=0} + \epsilon$ with $\epsilon > 0$ being as small as possible.

Additionally, it is easy to show that $\nu_{PT} = \nu_{\Xi=0} + \epsilon$ cannot be an equilibrium. This can be seen from (4.6). For $\nu_{PT} = \nu_{\Xi=0} + \epsilon$, we have $\eta_{\Xi=0} = 0$, which cannot satisfy the equation. Given $\xi < \frac{\eta_{CK}}{D}$ and therefore $\eta_{\Xi=1} = 1$, $\eta_{\Xi=0}$ needs to be equal to

$$\eta_{\Xi=0} = \frac{\eta_{CK} - \xi D}{1 - \xi},$$

(4.7)

which is strictly positive. Hence, for $\nu_{PT} > \nu_{\Xi=0}$ equation (4.6) does not hold and the overall cheating probability is too low such that the health insurance company has an incentive to decrease its audit probability to $\nu_{PT} = \nu_{\Xi=0}$.

**Proposition 24.** For sufficiently low $\xi$, the hospital always cheats if it treats a patient with supplementary health insurance and cheats with a positive probability that is below one if it treats a patient without supplementary health insurance.

**Proof.** In equilibrium, the health insurance company will directly decrease its audit probability from $\nu_{PT} = \nu_{\Xi=1}$ to $\nu_{PT} = \nu_{\Xi=0}$ and the hospital cheats with $\eta_{\Xi=1} = 1$ and

$$\eta_{\Xi=0} = \frac{\eta_{CK} - \xi D}{1 - \xi}.$$ 

So far, we have seen that the fraud strategies highly depend on the fraction of people that have supplementary health insurance. The fraction of people that have supplementary health insurance also determine the total cheating probability.

**Proposition 25.** The total cheating probability increases in the fraction of patients with supplementary health insurance for low $\xi$ and is independent of the fraction of patients with supplementary health insurance for high $\xi$.

**Proof.** For sufficiently large $\xi$, we have $\eta_{\Xi=0} = 0$ and $\eta_{\Xi=1} = \frac{\eta_{CK}}{\xi D}$. Due to $\frac{\partial \eta_{\Xi=1}}{\partial \xi} < 0$, the hospital increases its cheating probability if $\xi$ decreases. Since the proportion of patients with supplementary health insurance is $\xi$, the total cheating probability remains constant at $0 \cdot (1 - \xi) + \frac{\eta_{CK}}{\xi D} \cdot \xi = \left(\frac{\pi}{1 - \pi}\right) \left(\frac{\eta_{CK}}{\xi D} \cdot \frac{\xi}{1 - \xi}\right)$. For sufficiently low $\xi$, we have $\eta_{\Xi=0} = \frac{\eta_{CK} - \xi D}{1 - \xi}$ and $\eta_{\Xi=1} = 1$. We can see that $\frac{\partial \eta_{\Xi=0}}{\partial \xi} < 0$. The hospital therefore lowers its cheating probability for the patients without supplementary health insurance if $\xi$ increases. Since the proportion of patients without supplementary health insurance is $(1 - \xi)$, the total cheating probability is $1 \cdot \xi + \frac{\eta_{CK} - \xi D}{1 - \xi} \cdot (1 - \xi) = \eta_{CK} + \xi(1 - D)$ which is strictly increasing in $\xi$. 

Figure 4.2 shows the cheating probability. For large $\xi$, the total cheating probability is independent of $\xi$. This result is consequential since otherwise, the health insurance company would not be indifferent any longer. The interpretation is the following. For
patients without supplementary health insurance there is no cheating. For patients with supplementary health insurance the hospital chooses its cheating probability such that the health insurance company is indifferent between auditing and not auditing. If the fraction of people with supplementary health insurance decreases there is ceteris paribus less incentive to audit. This is anticipated by the hospital. It therefore increases its cheating probability for the treatment of people with supplementary health insurance in order to keep the health insurance company indifferent. For small $\xi$, the total cheating probability depends on $\xi$. This result is consequential as well since the higher the fraction of patients with supplementary health insurance, the more attractive manipulation is (due to lower manipulation costs). This makes an audit less efficient and therefore allows a higher manipulation probability.

4.6 Conclusion

Brief Summary - Main Results

This study analyzes the implications of supplementary health insurance on fraudulent behavior. It shows that fraudulent behavior depends on the existence of supplementary health insurance. If the patient has supplementary health insurance and this information is common knowledge, the hospital has a higher cheating probability and the health insurance company has a higher audit probability compared to the situation in which the hospital does not adjust its cheating probability for the treatment of people with supplementary health insurance the health insurance company would not audit. Of course, this cannot be an equilibrium since the optimal reaction is cheating.
patient has no supplementary health insurance. When the existence of supplementary health insurance is not common knowledge the results change dramatically. The health insurance company either audits with a low probability or with a high probability. This depends on the fraction of people that have supplementary health insurance. Depending on the audit strategy the hospital either has a dominant strategy for the treatment of a patient without supplementary health insurance and a mixed strategy for the treatment of a patient with supplementary health insurance or vice versa. The total cheating probability increases in the fraction of people with supplementary health insurance as long as the fraction is sufficiently low. If the fraction reaches a critical level, it remains constant. At this critical level the health insurance company as well as the hospital change their strategy. The health insurance company increases its audit probability from low to high. The hospital changes from a mixed (dominant) to a dominant (mixed) strategy for patients without (with) supplementary health insurance. This transition is smooth which is in contrast to the strategy change of the health insurance company.

Implications

There are two main factors that influence the results. The first factor is the type of information about the existence of supplementary health insurance. The second factor is the fraction of people that actually have supplementary health insurance. If the information concerning the existence of supplementary health insurance is common knowledge, the health insurance company audits more often when the hospital treats a patient with supplementary health insurance. This wastes resources. The higher the fraction of people with supplementary health insurance, the more expensive it gets for the health insurance company. If the information concerning the existence of supplementary health insurance is private, the health insurance company is worse off, independent of the fraction of people with supplementary health insurance. This is due to the fact that the health insurance company cannot differentiate anymore and audits only with one intensity instead of two. Hence, the health insurance company should demand a disclosure requirement for all people with supplementary health insurance. The implementation costs will most likely be sufficiently small (compared to the potential gains for the health insurance company). A database management system in which everyone with supplementary health insurance is automatically registered can easily be set up.

From a welfare perspective a disclosure requirement might make sense as well. Due to the demographic change and the rapid technological transition, the fraction of people with supplementary health insurance is increasing. Hence, sooner or later the health insurance company will increase its audit probability. This wastes resources. In a situation like this, less resources will be wasted if a disclosure requirement exists.
Chapter 5

Consumption, Savings and Medical Prevention in the Long Run

Joint work with Martin Nell and Petra Steinorth

Abstract

This paper investigates the impact of savings and prevention effort accounting for the long term effects of medical prevention. In the short run, medical prevention decreases health care costs. In the long run, prevention increases the likelihood of living up to a very high age and causing excessive end-of-life treatment costs. We derive conditions under which prevention either increase or decrease annualized health care expenditures. When considering the long-run effects, we show that moral hazard may actually increase preventive care compared to a situation with perfect information which is in stark contrast to previous findings.

Keywords: Long- vs. short-term effects of prevention, Consumption, Life cycle

JEL: D14, H24, H31, I11
5.1 Introduction

In the short run, medical prevention decreases health care costs. In the long run, prevention increases the likelihood of living up to a very high age and causing excessive end-of-life treatment costs. Taking the long term effects into consideration may lead to a situation where prevention does not have to decrease the overall health care costs over the lifetime as often assumed. Hence, we are considering the whole lifetime cycle in our analysis which is crucial in order to have a holistic approach of the prevention effect. Empirical evidence suggests that this is necessary since it is shown that there is a significant long run impact of prevention. As an example, it is shown that smoking increases short term health care expenditures (Oster, 1984; Izumi et al., 2001; Halpern et al., 2001; Max et al., 2004; Welte et al., 2000, among others). Considering the long term effects (which is the impact on mortality) changes the results substantially. It is shown that smoking does not increase the overall health care expenditures (Hayashida et al., 2012; Bearman, 2012, among others) which indicates that the effect of prevention (i.e. quit smoking) on overall health care expenditures is ambiguous.

Another example that emphasizes the trade-off between short term and long term effects is obesity which increases short term medical costs. On the other hand, Allison et al. (1999) show that when accounting for higher mortality the health care expenditures of overweight individuals are even lower than the average health care expenditures. Accordingly, it seems natural to include the long term perspective to the analysis since it influences the optimal contract design.

Our results are the following: Adding the long term perspective, we derive conditions under which prevention increases annualized health care costs. This is especially the case if prevention has a substantial impact on life expectancy. In the second step, we introduce a moral hazard problem. We show that moral hazard can actually increase the individual effort level. This is the case when higher prevention increases the annualized health care premium. Furthermore, we show that prevention and savings can either be substitutes or complements.

The remainder of this article is organized as follows: The second section illustrates the impact of medical prevention under observability of the chosen effort level. Section three investigates the impact of prevention effort under asymmetric information. The paper ends with a brief summary of results.

\footnote{See Burton et al. (1998), Sander and Bergemann (2003), Finkelstein et al. (2005), Durden et al. (2008), and Finkelstein et al. (2010)}
5.2 Observable prevention effort

5.2.1 Model set-up

We assume that individuals pass through three stages in their lifetime. At the first stage which is denoted by $t_1$ individuals are assumed to be young and healthy.\(^2\) Health risk becomes apparent in the second period ($t_2$) when individuals fall ill with probability $\pi_2$ and then have to cover their medical expenses. Only a fraction $\pi_3$ of the population actually reaches the point in time $t_3$ which we assume to be a very old age. At this time, individuals have a very high frailty rate meaning that they will have significant health care and long term care costs.

Individuals have to make two decisions: First, they have to determine their level of prevention $e$ (e.g. stopping to smoke, eating healthily, or exercising). We assume that $e$ comes at a disutility in $t_1$, reduces the probability of falling ill in $t_2$ ($\pi_2'(e) < 0, \pi_2''(e) > 0$) and increases life expectancy ($\pi_3'(e) > 0, \pi_3''(e) < 0$).\(^3\) Second they had to decide how to spend their lifetime earnings $W$. This decision is influenced by two factors: First, future consumption is financed out of the portion of their earnings which is not spent when young. Second, there is, depending on the health insurance contract, a threat of medical expenses in the future which they can finance with their savings.

We assume in this section that $e$ is observable for health insurers and medical cost insurance is available at a fair premium $P(e)$ and a deductible $D$.$^4$ In addition to the risk of medical costs, individuals also suffer from uncertainty about their life expectancy as they do not live up to the last period for sure. As the focus of our model is on prevention and savings for medical costs, we assume that there is an efficient market for eliminating longevity risk. After the health state in $t_2$ is revealed, individuals make a decision about how much of their remaining assets they want to spend in $t_2$ and how much they want to invest into a fairly priced life annuity, i.e. a life annuity where the expected return equals the premium paid and pays back the invested amount divided by the survival probability in $t_3$ in case of survival. We assume that individuals are rational expected utility maximizers without bequest motives.

Due to consumption smoothing, individuals choose the same deductible at both points in time. The utility in $t_2$ (in case of being healthy) and $t_3$ (in case of survival) is given by $u\left(\frac{W - c_1 - P(e) - D\pi_3(e)}{1 + \pi_3(e)}\right)$. In the situation where the person is healthy in $t_2$ his wealth in $t_2$ is $W - c_1 - P(e)$. If a person spends the amount $Y$ on a life annuity, he receives

\[^2\]We, of course, acknowledge the fact that there can be significant health expenditures when young. However, we assume increasing medical costs over the life-cycle and therefore normalize the costs when young to zero.

\[^3\]Without loss of generality $e$ assume that prevention takes place in $t_1$.

\[^4\]If the insurance premium is fair the optimal deductible is of course zero. However, since many health insurance programs in different countries entail positive minimum deductibles we allow for $D \geq 0$. 
\( Y \pi_3(e) \) in case of survival. Due to a high frailty at older ages, we assume that high medical costs are inevitable and the individual has to pay the deductible at \( t_3 \). We then have 
\[
u(W - c_1 - P(e) - Y) = u(\frac{Y}{\pi_3(e)} - D).
\]
Hence, 
\[
Y = (W - c_1 - P(e) + D) \pi_3(e) \quad \text{and therefore}
\]
\[
u(W - c_1 - P(e) - Y) = u(\frac{(W - c_1 - P(e)) - D \pi_3(e)}{1 + \pi_3(e)}).
\]
The same argument holds for the situation where the person is ill in \( t_2 \). The only difference is that the person has to pay the deductible in \( t_2 \) and \( t_3 \). We therefore can write the utility as 
\[
u \left( \frac{W - c_1 - P(e)}{1 + \pi_3(e)} - D \right).
\]

The total premium for life time insurance is given by 
\[
P(e) = \pi_2(e)(L_2 - D) + \pi_3(e)(L_3 - D).
\]
The annualized premium therefore is given by 
\[
P(e) = \frac{\pi_2(e)(L_2 - D) + \pi_3(e)(L_3 - D)}{2^{1+\pi_3(e)}}.
\]
There are two possible ways to implement the insurance premium in the maximization problem. Either we subtract the annualized premium in each period or we subtract the total premium in the first period. As we do not assume differing time preferences and interest rates, it basically does not matter when the premium is formally deducted. We choose the more common latter approach for convenience. This is in line with Leland (1968) and Kimball (1990). If the person pays the insurance premium \( P(e) \) and consumes \( c_1 \) in \( t_1 \) his net wealth in \( t_2 \) is given by \( W - c_1 - P(e) \). We can summarize the maximization problem in the following way: 
\[
E(U)
\]
\[
= u(c_1 - e) + (1 - \pi_2(e)) \left[ u \left( \frac{W - c_1 - P(e) - D \pi_3(e)}{1 + \pi_3(e)} \right) + \pi_3(e) u \left( \frac{W - c_1 - P(e) - D \pi_3(e)}{1 + \pi_3(e)} \right) \right] + \pi_2(e) \cdot u \left( \frac{W - c_1 - P(e)}{1 + \pi_3(e)} - D \right) + \pi_3(e) \cdot u \left( \frac{W - c_1 - P(e)}{1 + \pi_3(e)} - D \right).
\]

We can simplify this expression to\(^5\)
\[
E(U) = u \left( c_1 - e \right) + (1 - \pi_2(e)) (1 + \pi_3(e)) \left[ u \left( \frac{W - c_1 - P(e) - D \pi_3(e)}{1 + \pi_3(e)} \right) \right] + \pi_2(e) (1 + \pi_3(e)) \left[ u \left( \frac{W - c_1 - P(e)}{1 + \pi_3(e)} - D \right) \right].
\]

We define an increase of savings to be equivalent to a decrease in consumption in \( t_1 \).

\(^5\)The assumption that \( \pi_3 \) is independent of the realization of the health state in \( t_2 \) may be critical at first sight. An individual that e.g. has cancer in early years is probably less likely to arrive at a very old age. Accounting for this dependency would not change our results qualitatively, as we would simply use different longevity probabilities depending on the health state in \( t_2 \). However, the analyses would become less clear. Therefore, we refrain from adding this perspective.
5.2.2 The impact of prevention on the insurance premium

It is often argued that prevention decreases health care expenditures. But those studies solely focus on short term effects and do not take into account that prevention increases life expectancy. When considering long term effects of prevention on the insurance premium, we need to have a closer look at the annualized insurance premium.\footnote{Analyzing the effect prevention has on the total health insurance premium is straightforward. Derivation of total premium yields $\frac{\partial P}{\partial e} = (\pi'_{2}(e)(L_{2} - D) + \pi'_{3}(e)(L_{3} - D))$ where the first term is always negative (impact of the probability of falling ill in the second period) while the second term is positive. However, analyzing the annualized premium is much more appropriate. Since we consider the long term effect as well, we might observe a decreasing annualized premium while the total premium is already increasing.}

For the annualized health insurance premium, we have

$$P = \frac{\pi_{2}(e) \cdot (L_{2} - D) + \pi_{3}(e) \cdot (L_{3} - D)}{2 + \pi_{3}(e)}$$

and therefore $\frac{\partial P}{\partial e}$

$$= \frac{(\pi'_{2}(e)(L_{2} - D) + \pi'_{3}(e)(L_{3} - D))(2 + \pi_{3}(e)) - (\pi_{2}(e)(L_{2} - D) + \pi_{3}(e)(L_{3} - D))\pi'_{3}(e)}{(2 + \pi_{3}(e))^{2}}.$$

From this derivative we yield the following proposition.

**Proposition 26.** If $L_{3} < L_{2}$, prevention decreases the annual insurance premium as long as the prevention effect on the survival probability is not greater than the prevention effect on the probability of falling ill in the second period. However, if $L_{3}$ is (much) greater than $L_{2}$, even a relatively small effect on the survival probability increases the annual insurance premium. Furthermore, for $L_{3} > L_{2}$ an increase in the deductible makes it more likely that prevention increases the insurance premium.

**Proof.** The denominator $(2 + \pi_{3}(e))^{2}$ is always positive. Hence, we focus on the nominator which can be rearranged to

$$\pi'_{2}(e)(L_{2} - D)(2 + \pi_{3}(e)) + \pi'_{3}(e)(2(L_{3} - D) - \pi_{2}(e)(L_{2} - D)).$$

The first term $\pi'_{2}(e)(L_{2} - D)(2 + \pi_{3}(e))$ is always negative (impact of the probability of falling in in $t_{2}$). The nominator is always negative (sufficient condition), when the second term is not positive. That is

$$L_{3} \leq D + \frac{\pi_{2}(L_{2} - D)}{2}.$$

In such a situation the insurance payments in $t_{3}$ are so low, that an increase in $\pi_{3}$ due to an increase in prevention does not increase the annually health care expenditures.
Let us now assume that \( L_3 > D + \frac{\pi_2(L_2 - D)}{2} \). We still might have a decrease in the annually health care expenditures.

\[
\frac{\partial P}{\partial e} < (>)0 \iff -\frac{\pi_2'(e)}{\pi_3'(e)} > (>)\frac{2(L_3 - D) - \pi_2(e)(L_2 - D)}{(L_2 - D)(2 + \pi_3(e))}.
\]

The right side is \( > (>)1 \) for

\[
\frac{L_3 - D}{L_2 - D} > (>)\frac{2}{2 + \pi_2 + \pi_3}.
\]

**5.3 Unobservable prevention effort**

In the following we assume that the chosen level of prevention is not observable in \( t_1 \) for insurance companies. Accordingly, the (individual) price for insurance does not change when a person changes his prevention effort. Hence, the insurance premium depends only on the chosen deductible and therefore is not calculated on the individual level, but rather depends on the loss experience in the group of insured.

As before individuals make the decision about how much to invest in the life annuity after the health status in \( t_2 \) is revealed. Thus we assume that prevention is observable ex post and fairly priced annuities are still available.\(^7\)

Knowing this, the maximization problem changes to

\[
E(U) = u(c_1 - e) + (1 - \pi_2(e))(1 + \pi_3(e)) \left[ u \left( \frac{W - c_1 - P}{1 + \pi_3(e)} - D \right) \right] + \pi_2(e)(1 + \pi_3(e)) \left[ u \left( \frac{W - c_1 - P}{1 + \pi_3(e)} - D \right) \right].
\]

From analyzing the maximization problem (5.1) we yield

**Proposition 27.** Savings and prevention can either be substitutes or complements when prevention is priced in a life annuity. This highly depends on the effect prevention has on the survival probability, i.e. the long term effect, as well as on the effect prevention has on the probability of falling ill, i.e. the short term effect. The higher the long term effect it is more likely (c.p.) that savings and prevention are complements. On the other hand, the higher the short term effect it is the more likely (c.p.) that savings and prevention are substitutes.

\(^7\)This assumption is in line with the literature on enhanced annuities (see Steinorth (2012)).
Proof. We have that $\frac{\partial EU}{\partial c}$

$$\begin{align*}
&= -u'(c_1 - e) \\
&+ \left[-\pi_2'(e)(1 + \pi_3(e)) + (1 - \pi_2(e))\pi_3'(e)\right] \cdot \left[u\left(\frac{(W - c_1 - P) - D\pi_3(e)}{1 + \pi_3(e)}\right)\right] \\
&- (1 - \pi_2(e))(1 + \pi_3(e)) \cdot \left[u'\left(\frac{(W - c_1 - P) - D\pi_3(e)}{1 + \pi_3(e)}\right)\right] \\
&\cdot \frac{\pi_3'(e) \cdot (W - c_1 - P + D)}{(1 + \pi_3)^2} \\
&+ \left[\pi_2'(e)(1 + \pi_3(e)) + \pi_2(e)\pi_3'(e)\right] \cdot \left[u\left(\frac{(W - c_1 - P - D) - D\pi_3(e)}{1 + \pi_3(e)}\right)\right] \\
&- \pi_2(1 + \pi_3(e)) \left[u'\left(\frac{(W - c_1 - P - D) - D\pi_3(e)}{1 + \pi_3(e)}\right)\right] \cdot \frac{\pi_3'(e) \cdot (W - c_1 - P)}{(1 + \pi_3)^2} \equiv 0
\end{align*}$$

which leads to $\frac{\partial^2 EU}{\partial c_1 \partial c}$

$$\begin{align*}
&= -u''(c_1 - e) \\
&+ \pi_2'(e) \cdot \left[u'\left(\frac{(W - c_1 - P) - D\pi_3}{1 + \pi_3}\right) - u'\left(\frac{(W - c_1 - P - D) - D\pi_3}{1 + \pi_3}\right)\right] \\
&+ \pi_3'(e) \cdot (1 - \pi_2) \cdot u''\left(\frac{(W - c_1 - P) - D\pi_3}{1 + \pi_3}\right) \cdot \frac{W - c_1 - P + D}{(1 + \pi_3)^2} \\
&+ \pi_2 \cdot u''\left(\frac{(W - c_1 - P - D) - D\pi_3}{1 + \pi_3}\right) \cdot \frac{W - c_1 - P}{(1 + \pi_3)^2} \geq 0.
\end{align*}$$

The sign of $\frac{\partial^2 EU}{\partial c_1 \partial c}$ is ambiguous. Savings and prevention can either be substitutes or complements. The first term is always positive and shows the direct utility effect of prevention. Since prevention occurs in $t_1$ the individual increases his consumption in order to compensate the negative prevention effect. The second term is always positive as well and shows the short term effect of prevention. An increase in prevention decreases the likelihood of falling ill in $t_2$. Hence, the individual can increase his consumption and therefore decrease his savings. The third term is always negative and shows the long term effect of prevention. An increase in prevention increases the likelihood of reaching the third period. Furthermore, an increase in prevention increases the premium for the annuity.

Comparing the optimal effort level under moral hazard to the situation with perfect information, we have that

**Proposition 28.** Moral Hazard increases prevention activities if $\frac{\partial P}{\partial c} > 0$ under an observable effort level. If $\frac{\partial P}{\partial c} < 0$ under observable effort, moral hazard decreases prevention activities.
Proof. If prevention is observable, we have that

\[
\frac{\partial EU}{\partial e} = -u'(c_1 - e) \\
+ \left[ -\pi'_2(e)(1 + \pi_3(e)) + (1 - \pi_2(e))\pi'_3(e) \right] \cdot u \left( \frac{(W - c_1 - P(e)) - D\pi_3(e)}{1 + \pi_3(e)} \right) \\
- (1 - \pi_2(e))(1 + \pi_3(e)) \cdot u' \left( \frac{(W - c_1 - P(e)) - D\pi_3(e)}{1 + \pi_3(e)} \right) \\
\cdot \frac{(\frac{\partial P}{\partial e} + D) \cdot (1 + \pi_3(e)) + \pi'_3(e) \cdot (W - c_1 - P(e) - D\pi_3(e))}{(1 + \pi_3)^2} \\
+ \left[ \pi'_2(e)(1 + \pi_3(e)) + \pi_2(e)\pi'_3(e) \right] \cdot u \left( \frac{(W - c_1 - P(e) - D - D\pi_3(e))}{1 + \pi_3(e)} \right) \\
- \pi_2(1 + \pi_3(e)) \left[ u' \left( \frac{(W - c_1 - P(e) - D - D\pi_3(e))}{1 + \pi_3(e)} \right) \right] \\
\cdot \frac{(\frac{\partial P}{\partial e} + D) \cdot (1 + \pi_3(e)) + \pi'_3(e) \cdot (W - c_1 - P(e) - D - D\pi_3(e))}{(1 + \pi_3)^2} \right] = 0.
\]

The only difference to the foc under moral hazard (2) is that there is a term for how effort impacts the premium. If \( \frac{\partial P}{\partial e} > 0 \), the optimal prevention level is smaller under observability. This implies that moral hazard leads to an increase of prevention activities. If \( \frac{\partial P}{\partial e} < 0 \), the standard result holds, that moral hazard reduces prevention.

The intuition of this result is straightforward. If higher prevention increases the annual insurance premium, individuals take this into account when choosing their prevention effort. Accordingly, they prevent less than in a situation where they would only consider the impact of effort on the sickness probability and their longevity. Consequently, moral hazard leads to more prevention in this situation. This is particularly the case if prevention has a high impact on life expectancy and if treatment costs at a very old age are considerable high. This does not seem unlikely when considering current health care spendings over a life cycle.

5.4 Conclusion

This paper investigates the often overlooked fact that prevention decreases medical cost in the short run, but it also increases the life expectancy which may make excessive end-of-life treatment costs more likely. We show that in this situation, it may actually be that a higher prevention level does not have to lower the overall or annualized health care expenditures. They increase it if the long term effect is sufficiently strong. The second part of the paper shows that this phenomenon also implies that moral hazard may increase prevention activities. This is stark contrast to previous literature which usually assumes that moral hazard always decreases preventive effort.
Bibliography


Eidesstattliche Versicherung


Hamburg, February 3, 2013