Imaging of seismic events.
The role of imaging conditions, acquisition geometry and source mechanisms

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Abstract

Localization of seismic events can provide us valuable information about structures activated by tectonic stresses, geothermal or volcanic activity, reservoir stimulation, and other subsurface activities. In the last few years automatic stacking-based localization methods, which do not require any picking, have proved to be reliable localization tools. Localization results obtained by such techniques are influenced by various circumstances. Influence of three key factors is studied in this work such as imaging conditions, acquisition geometry and source mechanisms. First, a commonly used imaging condition is discussed and alternatives are introduced. Then, I illustrate their advantages, limitations and sensitivity to velocity uncertainties.

Secondly, influence of acquisition geometry on localization results is examined. I illustrate impact of regularly and irregularly distributed receivers. Ways of acquisition footprint reduction are discussed.

Furthermore, localization of events with different source mechanisms is illustrated. Events with a double-couple source mechanism represent a challenge for stacking-based localization techniques due to waveform differences among receivers. An alternative stacking-based approach, especially suitable for the localization of double-couple dominant events, is introduced. As the majority of seismic events can be best characterized by a combination of explosive, double-couple (DC) and compensated linear vector dipole (CLVD) components, localization of such sources is also illustrated.

Finally, an application to field data from Southern California is presented. Despite the sparse and irregular receiver distribution, localization result obtained by a stacking-based localization technique deviates less than 1% of the maximum receiver offset to the location yielded by California Earthquake Data Center using a method requiring picking of phases.
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Chapter 1

Introduction

United States Geological Service (USGS (2013)) reported an average of 46 earthquakes, which could be felt on the surface, per day in the world in 2012. This seismic activity is a natural result of tectonic stresses that are present everywhere. Not all of these stress releases have to end in a devastating catastrophe. The most seismic events stay unnoticed as they cannot be felt on the Earth’s surface. Such events are called microseismic events.

There are two main scales in which microseismicity is defined: global and exploration scale. On global scale, microseismic events are generated by, for instance, tectonic stress releases, volcanic and geothermal processes. Whereas on the exploration scale, these events are triggered by hydraulic fracturing/stimulation, reservoir production, etc. For instance, during hydraulic fracturing treatment in oil and gas reservoirs a lot of microseismic events are induced along fractures. Localization of these events helps us to assess the effectiveness of the treatment.

In general, all seismic events as well as strong earthquakes as very weak microseismic events contain the valuable information not only about the Earth’s interior structure but also about processes going on inside. Thus, localization of these events can give us an important insight into properties and physical processes in the Earth.

Currently, there are different approaches to localize seismic events. Conventional localization methods are usually based on arrival times, thus requiring identification of different phases at each receiver position (see e.g. Thurber and Rabinowitz (2000)). Phase picking can be done manually or automatically. In general, automatic phase picking can be highly inaccurate and manual picking of phases is highly time and labor consuming. The accuracy and reliability of the localization by such approaches depends on the operator and different results may be obtained by different
operators (Bardainne et al. (2009)). Furthermore, data is often strongly affected by noise, which makes the phase identification at a single trace very difficult and thus, challenges such techniques extremely.

As an alternative to traveltime-based approaches requiring picking of different phases, various other methods were proposed. Gajewski and Tessmer (2005) introduced an automatic localization method based on reverse modeling, which does not require any picking of events. The advantage of this method is that the focusing of energy in the back projection process allows to image very weak events, which could not be identified in individual seismograms. Gajewski et al. (2007) proposed a diffraction stacking approach for localization of seismic events in 2D. The subsurface is discretized and each grid point is considered a possible event location, for which the traveltime trajectories are computed. Then, the amplitudes of the data are stacked along the calculated traveltime trajectories. The stack result for each time sample is squared and then all the stacks corresponding to one grid point are summed. The resulting value is assigned to the value of the so-called image function for one point. This procedure is repeated for each possible event location. The source location corresponds then to the maximum of the obtained image function.

Kao and Shan (2004) presented the Source-Scanning Algorithm where the so-called brightness function is computed by summing the absolute values of the amplitudes at predicted arrival times for a certain phase having the largest amplitude, usually the S phase, through the whole aperture. The brightest point in the image function represents the source location. Baker et al. (2005) suggested a similar method, especially designed for real time application. Unlike the Source-Scanning Algorithm, they use envelopes of seismograms, and the arrival times are computed using a P-wave velocity model. An image function at a certain time is constructed by stacking the amplitudes along the P-wave traveltime curves for each possible source position. The maximum value of the obtained function represents the summation of the signals and hence, the source location. Grigoli et al. (2013) proposed stacking of Short Time Average to Long Time Average ratio (STA/LTA) traces instead of stacking the amplitudes of the data. Grandi and Oates (2009) suggested applying diffraction stacking to the cross-correlated data to determine an event location. The drawback of this approach is that due to the cancellation of the origin times, an excitation time of an event cannot be obtained. Godwin et al. (2011) presented automatic location detection by cross-correlating reconstructed wavefields for small groups of geophones with different angular aperture for each time step. In other words, diffraction stacking is applied to the cross-correlated data for certain receivers. Reconstructed wavefields for different groups coincide in space and
time at the hypocenter of the microseismic event. Haldorsen et al. (2013) developed a totally
different approach by using projected traces in the frequency domain for stacking. Rentsch et al.
(2007) proposed to apply the stacking procedure only in a chosen time interval that includes a
few cycles of the P-wave arrival. Moreover, the energy of three-component data weighted with a
Gaussian-beam-type factor is stacked.

Hanafy et al. (2008) suggested a so-called Time Reversal Mirror (TRM) approach for location
of trapped miners inside a collapsed mine, which includes recording a natural Green’s function
prior to the collapse; after the collapse the vibrations caused by hammering of the trapped miners
recorded at the surface are correlated with the previously recorded natural Green’s function. The
maximum of the resulting correlation function corresponds thus to the location of the trapped
miners.

Murihead (1968) suggested to stack the n-th roots of the absolute signal value with the pre-
served sign. As a result, noise rejection in the image function is improved. This type of stacking
procedure is referred to as non-linear, whereas all the above described stacking-based localization
techniques may be referred to as linear as for the stacking process the pure amplitudes (except for
envelopes) are used.

Stacking-based localization techniques have become indispensable localization tools in the last
few years. However, the influence of the acquisition geometry, imaging conditions, source mecha-
nisms, etc. on the localization results remain to be studied. This thesis studies the role of three
key factors and illustrates their influence on localization results obtained by stacking techniques in
3D. First, I discuss advantages/disadvantages and limitations of existing and introduced imaging
conditions. Then, I study the effects of different acquisition geometries and show ways of reducing
the acquisition related artefacts. Furthermore, I discuss localization of double couple mechanism
dominated sources, which is a challenge for stacking-based localization techniques due to waveform
differences among receivers, and present a new approach for localizing such sources. Finally, an
application to a seismological field data from Southern California is presented.

Structure of the thesis

**Chapter 2** comprises an explanation of the diffraction stacking method for localization of
seismic events.

In **Chapter 3** theoretical fundamentals of different source mechanisms are described.

**Chapter 4** examines commonly used time collapsed imaging condition and introduces two
alternative imaging conditions. Their advantages and disadvantages are illustrated. Furthermore,
sensitivity of the imaging conditions to velocity uncertainties is discussed.

**Chapter 5** illustrates the impact of different surface acquisition geometries: as well regular as irregular on the localization results obtained by diffraction stacking techniques. Synthetic examples for both noise-free and noisy data are presented. Moreover, I suggest and discuss ways of reduction of the artefacts related to acquisition geometries.

**Chapter 6** shows how the radiation pattern of a double couple source influences the diffraction stacking localization results. The worst case scenario for the stacking-based techniques is presented. Different approaches to remove the radiation patter effects are discussed. Furthermore, compensated linear vector dipole source type, typical for geothermal and volcanic areas, is also considered. As the most seismic events can be best characterized by a combination of explosive, double-couple (DC) and compensated linear vector dipole (CLVD) components, localization of such sources is also examined. The synthetic tests illustrate that if the double-couple component is prevailing in the source mechanism, diffraction stacking fails to localize the event. I also introduce an alternative stacking-based approach, especially suitable for the localization of double-couple dominant events.

In **Chapter 7** a field data application is presented.

Finally, in **Chapter 8** I draw conclusions and discuss possible future research topics.
Chapter 2

Basic localization principles

In this chapter I will introduce basic concepts of seismic events localization by diffraction stacking.

2.1 Diffraction stacking

In passive seismic, different events are recorded during a certain period of time. The acquisition geometry (number of receivers and their positions) of the experiment is known. We assume that seismic events are caused by point sources, implying that the spatial dimensions of a source are significantly smaller than the prevailing wavelength, which is analogous to the case of diffractions. Diffraction stacking techniques have become well established for imaging of (micro)seismic event positions in time and space using the recordings from surface or borehole measurements. I start by explaining the concept of localization of seismic events by diffraction stacking.

First, the subsurface of the area of interest is discretized, whereas each grid point or so-called image point represents a possible event location (Figure 2.1 illustrates subsurface discretization in 2D).

![Figure 2.1: Discretization of the subsurface in 2D](image-url)
Then, I compute traveltimes from each image point to each receiver. In this work only P-waves are considered. However, the method can also be applied to S-waves. For homogeneous media, P-wave traveltimes can be calculated by:

\[ t_P = \frac{1}{V_P} \sqrt{(x_R - x_0)^2 + (y_R - y_0)^2 + (z_R - z_0)^2}, \]  

(2.1)

where \( V_P \) is the constant P-wave velocity in the given medium, \((x_0, y_0, z_0)\) represent the coordinates of an image point and \((x_R, y_R, z_R)\) are receiver coordinates. For heterogeneous media, traveltimes can be computed via ray tracing programs, e.g. Norsar-3D ray tracer (Norsar-3D (2013)).

The velocity model required for traveltime computations is assumed to be known. The next step is to stack the amplitudes along the precalculated traveltime curves for a given image point. Note that we use the real amplitude (with the preserved sign) of the data (this issue is discussed in Section 6.4.2). As the excitation time is not known, this procedure should be repeated for each time step. In other words, the traveltime curve, along which we stack the amplitudes, should be shifted through the whole time buffer of the given data. Stacking result along the traveltime curve is then squared and assigned to the time step and image point under consideration. The stacks has to be squared due to the following reason: the maximum amplitude values of the signal might be negative and thus, the stacking result for the right traveltime curve will be negative as well, whereas stacking of noise may lead to a positive value. To be able to compare the values, stacking results along the traveltime curves have to be squared. As an interim result, we get a 1D function of time for each image point. This procedure is then repeated for each image point. Finally, a 4D stacking function is obtained, which can be expressed as follows:

\[ S(x_0, y_0, z_0, t_i) = \left( \sum_{R=1}^{N} A(t_i + t^R_P) \right)^2, \]  

(2.2)

where \( S(x_0, y_0, z_0, t_i) \) is a 4D diffraction stacking function of space and time, \( t_i \) is the excitation time of the source, \( t^R_P \) is the computed P-wave traveltime from a chosen image point at \((x_0, y_0, z_0, t_i)\) to a receiver \( R \). \( N \) represents the total number of receivers and \( A \) stands for the amplitude for the given receiver \( R \) at the time \( t_i + t^R_P \).

Diffraction stacking procedure can be summarized into the following steps:

1. Discretize the subsurface and consider every grid point as a possible source position, so-called image point.
2. Compute traveltimes from each image point to each receiver.

3. Stack amplitudes of the data along computed traveltime curves for each time $t_i$.

4. Square the stacking result for each time sample.

5. Repeat the procedure for each image point.

6. Assign the values to the 4D image function $S(x_0, y_0, z_0, t_i)$ for each image point and time $t_i$.

7. Search the maximum of the image function. It corresponds to the source location in space and time.

The main idea of the diffraction stacking localization method may be formulated as follows: if the stacking of seismogram amplitudes is performed along the correct traveltime curve, the signal sums up constructively, whereas the uncorrelated noise is suppressed. Figure 2.2 illustrates the diffraction stacking algorithm. It represents the case where the image point coincides with the true source position. In Figure 2.2b, there are no values to stack along the traveltime curve, so the stacking result for this time step equals zero. Then the traveltime curve is shifted downward along the time axis and at some time step the best fit is achieved (see Figure 2.2c) and amplitudes are stacked constructively, which leads to the greatest contribution to the image function. Later the traveltime curve is shifted further and again the contribution for this time step and image point equals zero. Finally, we obtain a 1D stack function of time for each image point, which is shown in Figure 2.2d. Note that all values are positive as the stack results are squared.

The disadvantage of the 4D stacking function defined in Equation 2.2 is its size and thus, related high internal memory requirements and computational effort. For each dataset the number of elements of the image function can be computed by multiplying the number of image points by the number of time samples. To reduce this number and thus, speed up the diffraction stacking procedure, imaging conditions have to be introduced. An imaging condition describes a way of collapsing the time axis and turning the 4D image function into a 3D function.

The source imaging process can be generalized as a process consisting of 2 steps: traveltime computation for the defined grid of image points and the imaging condition. The grid of image points can be defined in different ways, which is discussed in the following section.
Figure 2.2: The concept of the stacking procedure for the case that the image point corresponds to the true source position. (a) shows the input signal form. In Figure (b) we can see that there are no amplitudes to stack along the red traveltime curve, whereas in (c) there is a perfect match of the traveltime curve and the event, which leads to the greatest contribution for the image function. (d) represents the final stacking result for all time steps for one image point.
2.2 Grid search

The most intuitive way of the search grid definition is to use an equidistant grid in all directions consisting of all points in the area of interest and apply the line grid search. The advantage of such an approach is that it reveals properties of the image function and it always finds the global maximum (see e.g., Zimmer and Jin (2011)). The disadvantage of such a grid choice is that the computational effort is directly proportional to the number of image points. To speed up the search, one could use at first a coarser grid to detect a potential area of the event location and then, to refine the grid in this particular area. The spatial sampling or grid step choice of the final localization grid should resemble the prevailing wavelength of the seismic events under consideration.

Another possibility of an accelerated grid search is simulated annealing (SA). In this approach a random distribution of image points in a certain area of the region of interest is applied, at first. Then the image function is evaluated at these points and the image point with the maximum value is then chosen as the center of a new bunch of grid points. This process is repeated iteratively until the set maximum number of iterations is reached. (see also Zimmer and Jin (2011)).

Gharti et al. (2010) used differential evolution (DE) method to define a grid of image points for localization purposes. At first, one chooses an initial population of grid points. Then a reference point is set and three randomly distributed image points from its neighborhood are taken to compute a new grid point \( \vec{X}_{\text{new}} \) by using the Equation 2.3.

\[
\vec{X}_{\text{new}} = \vec{X}_3 + F \cdot (\vec{X}_2 - \vec{X}_1),
\]

where \( \vec{X}_1, \vec{X}_2, \vec{X}_3 \) represent the coordinates of the three randomly chosen image points. \( F \) stands for a so-called mutation factor and it controls the rate of the population evolvement. If the value of the image function at the new point \( \vec{X}_{\text{new}} \) is higher than at the reference point, it replaces the reference point and the procedure is repeated until a stop criterion like, for example, a predefined number of iterations is reached (for more details see e.g., Gharti et al. (2010), Zimmer and Jin (2011) or Feoktisov (2006)).

In this work I exploit the ability of the line grid search to always find the global maximum of the image function. Thus, at first a coarse grid is chosen to detect the possible area of the source location, which is then refined.

At the end of the diffraction stacking procedure we not only know the location of the seismic...
event in space in time, but also the source function. This information gives us a clue about source characteristics, which are discussed in the following section.
Chapter 3

Seismic sources

Seismic events are the result of a sudden release of energy within the Earth or on the surface. During explosions either chemical or nuclear energy is released, this leads to the formation of a cavity around the source and compaction of the zone around the original charge. In case of earthquakes it is usually associated with tectonic stress release. Gas and oil production may also induce or trigger seismic events due to the pore pressure increase caused by fluid injection during hydraulic fracturing or due to poroelastic stress transfer caused by reservoir depletion (see e.g. Grasso (1992)). Only a fraction of the total radiated energy reaches the surface and thus, can be recorded. Despite this fact, it contains valuable information about the source position, excitation time, source function and mechanism, and additional information about properties of the traversed environment (different layers of rocks or fluids).

In the immediate area around the source, a shock wave is spread out into the medium and the released stress is not linearly related to the considerable displacements. The size of the region for which non-linear effects are of importance, scales with the size of the source. At some distance from the source, the displacements become small enough to be described linearly. The models of seismic sources can be simplified and characterize the radiation in a far field, at distances of several wavelengths away from the source, without attempting to describe the source process.

3.1 Moment tensor

Seismic events can be approximated as point sources, as their spatial dimensions are small enough in comparison to the wavelength of the radiated energy. The most simple model of a seismic source in terms of moment tensor, which will be introduced in the next paragraph, is a single point force.
The next step in complexity is to represent a source by a single couple (dipole) of forces. In the early 1950s it was discovered that P-waves radiated by earthquakes have a spatial distribution similar to one produced by single couples of forces. Unfortunately such a source could not explain S-wave radiation. So a more complex source model was introduced - a double couple source, a source without resultant force or moment. In the 1960s, the physical origin of the double couple model was established, thanks to the work of numerous seismologists and the crucial theoretical breakthrough of Maruyama (1963) and Burridge et al. (1964), who managed to prove that a fault in an elastic model was equivalent to a double couple source, (see historical overview in Madariaga (2007)).

The equilibrium conditions for a finite volume $V$ in a deformable body require that the resulting force $\mathbf{f}$ and resulting angular moment disappear:

\[ \sum \mathbf{f} = 0, \]
\[ \sum \mathbf{r} \times \mathbf{f} = 0. \] (3.1)

The simplest model of a source that fulfills both conditions (3.1) and (3.2) is a dipole (a single couple of forces). For instance, the so-called linear dipole is constrained of two single point sources, that act in opposite directions at two points. It is possible to combine three orthogonal linear dipoles in order to form a general seismic source. Any bipolar seismic source can be simulated by adjusting the strength of these three dipoles. These three dipoles represent the principal directions of a tensor of rank 2 that is called a seismic moment tensor:

\[
\begin{pmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{pmatrix}
\]

The seismic moment tensor has to be symmetric ($M_{xy} = M_{yx}, M_{xz} = M_{zx}, M_{yz} = M_{zy}$), so that the conservation of angular momentum is fulfilled. Graphic representation of the nine seismic moment tensor components is illustrated in Figure 3.1

The diagonal elements of the moment tensor $M_{xx}, M_{yy}, M_{zz}$ correspond to dipoles (see Figure 3.1) and have a bi-lobed radiation pattern for both P- and S-waves. That means that the radiation pattern is compound of 2 rounded structures looking similar to leaves. The radiation pattern describes the directional dependency of the P- or S-wave amplitude variation. The other components
of the moment tensor describe couples with a four-lobed radiation pattern, which consists of 2 rounded structures (lobes), for P- and a bilobate pattern for S-wave (Kennett (2001)). The trace of the moment tensor (sum of the diagonal elements) describes volume changes that accompany the event (Shearer (1999)).

3.2 Explosion

An explosive source with constant energy radiation in all directions is the easiest source model to imagine as its P-wave radiation pattern is spherical (see Figure 3.2). In exploration seismic, perforation and calibration shots can be characterized by an explosive source type. Theoretically, for the explosion case, there are no S-waves, but as a pure explosion is not possible, real explosions generate some S-waves.

Such source can be described by an isotropic moment tensor

\[
\begin{pmatrix}
M_0 & 0 & 0 \\
0 & M_0 & 0 \\
0 & 0 & M_0
\end{pmatrix}
\]

where \(M_0\) is the product of the volume times the pressure change in the source. The trace of the moment tensor for an explosive source is nonzero, which implies a volume change (Stein and
The force system of such a source consists of three perpendicular dipoles of equal strength (see Figure 3.3).

3.3 Double couple

It is obvious that the complex processes causing earthquakes cannot be described by an explosive source as that is only a crude approximation. We can model earthquakes resulting from a shear or slip on a fault by a double-couple (DC) seismic source. This is still a simple model but more realistic for natural seismic events than the explosive one. In general, a double couple source represents a shear dislocation source without any volume change.

We simplify earthquakes as movements across a planar fault of arbitrary orientation. The equivalent force system for a fault slip can be illustrated by two opposing force couples whose torques compensate each other, which is illustrated in Figure 3.4).

Let us consider the dextral movement on a vertical fault oriented in the x direction, the so-called
strike-slip (see Figure 3.5). The corresponding moment tensor reads

\[
\begin{pmatrix}
0 & M_0 & 0 \\
M_0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

where \( M_0 \) is defined as the scalar seismic moment and is given by

\[
M_0 = \mu D A, \quad (3.3)
\]

where \( \mu \) is the shear modulus, \( D \) is the average slip (or displacement) on the fault and \( A \) represents the area of the fault (Stein and Wysession (2007)). The relation between the seismic moment \( M_0 \) and the amount of released energy, the so-called magnitude \( M_w \) of an event derived by Hanks and Kanamori (1979), is described by

\[
M_w = 2/3 \log_{10} M_0 - 10.7. \quad (3.4)
\]

Wells and Coppersmith (1994) derived empirical scaling relations between the magnitude of an earthquake and the geometrical extension of the rupture by studying 421 historical earthquakes. The resulting relations for the strike-slip source mechanism are given by:

\[
\begin{align*}
\log_{10} RL &= -2.57 + 0.62 \ast M_w, \\
\log_{10} RW &= -0.76 + 0.27 \ast M_w,
\end{align*}
\]

where RL stands for the rupture length, RW - rupture width. Whereas the relation for all
rupture types can be expressed as:

\[
\log_{10} RL = -2.44 + 0.59 * M_w, \quad (3.7)
\]

\[
\log_{10} RW = -1.01 + 0.32 * M_w, \quad (3.8)
\]

The trace of the moment tensor is equal to zero, which means that it does not imply any volume changes.

![Dextral fault](image)

Figure 3.5: Dextral (right-lateral) strike-slip fault

Radiation pattern for P-waves in the far field is given by

\[
A^P = \sin 2\theta \cos \varphi, \quad (3.9)
\]

where \(\theta\) is the angle measured from the z axis and \(\varphi\) is measured in the xy - plane (Aki and Richards (2002)) as shown in Figure 3.6. The P-wave radiation pattern is illustrated in Figure 3.7.

The P-wave radiation of such a double-couple source results in a pattern of alternating quadrants around the focus, which reveal different directions of motion, as can be seen in Figure 3.8. The quadrants are separated through two planes: the real fault plane (solid line); and an indistinguishable auxiliary (dashed line) plane. Describing the orientations of these planes is essential for determination of the earthquake’s focal mechanism.

P-wave amplitudes are equal to zero along the nodal planes, whereas they are positive along the so called Tension-axis (T-axis); in this area the motion is compressional. Along the Pressure-axis (P-axis) the amplitudes are negative, the motion is then dilatational. It is clearly visible that the P-wave radiation pattern shows the strongest compressions and dilatation at 45° angle measured from the planes.

Not all seismic events can be explained by a double couple source, especially in volcanic and geothermal areas (Julian (1998)). Moreover, assuming the purely double-couple nature of the microseismic events induced during the hydrofrac treatment are claimed to be too confining (Baig
Figure 3.6: Explanation of the angles $\theta$ and $\phi$ (Stein and Wyssession (2007))

Figure 3.7: Radiation pattern for P-waves for the case of double couple. Red colored lobes represent positive amplitudes and blue colored - negative.

and Urbancic (2010)). The third type of a source type is introduced in the next section.
3.4 Compensated linear vector dipole

Another non-double-couple source beside the explosive one is a so-called compensated linear vector dipole (CLVD) source type. It represents a situation where strain along one axis is compensated by contraction or expansion along the other axis (see Figure 3.9).

The corresponding moment tensor looks as follows:

\[
\begin{pmatrix}
2M_0 & 0 & 0 \\
0 & -M_0 & 0 \\
0 & 0 & -M_0
\end{pmatrix}
\]

The trace of the moment tensor \( tr(M) = M_0 - 2M_0 + M_0 \) is zero, which means that there is no
isotropic component or volume change. There are two general explanations for CLVD mechanisms. Firstly, especially in volcanic areas, where an inflating magma dike can be thought of as a crack opening under tension. Secondly, CLVDs may occur due to almost simultaneous earthquakes or close-by faults with different geometry. In other words, as a superposition of two double couple sources. (Stein and Wysession (2007)):

\[
\begin{pmatrix}
2M_0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -2M_0
\end{pmatrix}
+ 
\begin{pmatrix}
0 & 0 & 0 \\
0 & -M_0 & 0 \\
0 & 0 & M_0
\end{pmatrix} 
\]

resulting in a seismic moment tensor corresponding to a compensated linear vector dipole:

\[
\begin{pmatrix}
2M_0 & 0 & 0 \\
0 & -M_0 & 0 \\
0 & 0 & -M_0
\end{pmatrix}
\]
In reality, most seismic sources can be best expressed as a combination of an explosive, double-couple and compensated linear vector dipole components (see e.g. Baig and Urbancic (2010)).

In the following chapters different source mechanisms will be considered. Next chapter is dealing with imaging conditions and for simplicity, an explosive source mechanism is assumed.
Chapter 4

Diffraction stacking: imaging conditions

In this chapter I will at first describe a synthetic dataset, which was generated to test different imaging conditions. An imaging condition describes a way of collapsing the time axis and turning the 4D image function into a 3D function. I will examine the most commonly used imaging condition. Furthermore, I propose two alternative imaging conditions for diffraction stacking localization methods and evaluate their advantages and limitations.

4.1 Synthetic dataset

To illustrate and evaluate different imaging conditions of the diffraction stacking localization technique, a synthetic seismogram was generated. The acquisition geometry consisted of 144 regularly distributed receivers at the surface (see Figure 4.1). The distance between the receivers is 100 m in both x- and y-directions. An explosive seismic source is located at (860/1120/2500) m. The aperture of the acquisition geometry is 1100 m, which is a relatively small aperture compared to those used for microseismic monitoring in the field (3000 - 5000 m and more). The larger the aperture the more focused image function can be obtained (see e.g. Gajewski et al. (2005) or Zou et al. (2009)). It is known that stacking enhances the signal-to-noise ratio of the data with the square root of the number of receivers (\(\sqrt{\text{number of receivers}}\)). Therefore, theoretically for the white noise we can expect an improvement of the signal-to-noise ratio of a factor 12 after applying the diffraction stacking to this dataset. This is an optimal value, which is unlikely to be achieved
for field data. If we consider surface microseismic monitoring experiments for evaluation of the hydrofrac effectiveness, there are often more than 500 receivers involved (see e.g. Chambers et al. (2010)). Thus, an optimal signal-to-noise ratio improvement of a factor of 22 and more is possible. So the chosen acquisition geometry is rather disadvantageous for diffraction stacking as it performs at its best with a large aperture and high number of receivers, which are both not given in this synthetic example. However, this disadvantageous acquisition geometry was chosen on purpose, to test the limits of various imaging conditions of diffraction stacking. In addition, I will show that even under such conditions we can obtain reliable localization results using diffraction stacking.

I assume a homogeneous velocity model with the P-wave velocity of 2500 m/s. Events observed in surface acquisitions usually have peak frequencies below 50 Hz (see e.g. Duncan and Eisner (2010)). The higher the peak frequency of the event, the more focused image function can be obtained (Zhebel (2010)). To challenge the diffraction stacking technique I took a minimum-phase Ricker wavelet with a peak frequency of 10 Hz to generate a synthetic seismogram. Thus, the prevailing wavelength is 250 m. The resulting normalized synthetic seismogram is shown in Figure 4.2a.

In general, microseismic events with the signal-to-noise ratio greater than 2 are processed using conventional localization techniques (Bardainne et al. (2009)). As conventional localization methods, which require phase picking, fail in the presence of a high noise level, we need to test different imaging conditions for diffraction stacking also for noisy data. In our context, noise is

![Figure 4.1: Surface acquisition geometry consisting of 144 regularly distributed receivers (black triangles) and the hypocenter of a seismic event (red star). Note that the event hypocenter with the coordinates (860/1120) m is placed asymmetrically in-between the receivers. The aperture is 1100 m. Blue arrows indicate receiver lines, along which seismogram excerpts are displayed in Figure 4.2.](image-url)
Figure 4.2: Excerpts of normalized synthetic seismograms along four receiver lines (indicated by blue arrows in Figure 4.1) for a seismic event with the coordinates (860/1120/2500) m (a) without any noise, (b) with the signal-to-noise ratio of 2 and (c) with the signal-to-noise ratio of 0.5. Note that in the seismogram with the signal-to-noise ratio of 2 the seismic event is still recognizable, whereas for the ratio of 0.5 it becomes impossible to detect an event.
regarded to be a part of the wavefield which is not relevant for the seismic event we are localizing. White Gaussian noise was added to the numerical seismograms described above using the Seismic Unix function suaddnoise. The noisy output data were constrained as follows:

\[ \text{Output} = \text{Signal} + \text{scale} \cdot \text{Noise}, \quad (4.1) \]

where the scaling factor is defined as:

\[ \text{scale} = \frac{1}{\text{SNR}} \cdot \frac{A_{\text{max}}}{\sqrt{2}} \cdot E, \quad (4.2) \]

where SNR stands for the desired signal-to-noise ratio, \( A_{\text{max}} \) represents the maximum amplitude of the input signal and \( E \) is the energy per sample (SeismicUnix (2013)).

The resulting synthetic seismograms with the signal-to-noise ratio of 2 and 0.5 are shown in Figures 4.2b and 4.2c, respectively. Note that in the seismogram with the signal-to-noise ratio of 2 the seismic event is still recognizable, whereas for the ratio of 0.5 it becomes impossible to visually detect the event.

### 4.2 Time collapsed imaging condition

In Section 2.1 the diffraction stacking approach was described. The disadvantage of the 4D stacking function defined in Equation 2.2 is its size and thus, related high internal memory requirements and computational effort. Let us assume a subsurface volume of 5000 m x 5000 m x 5000 m to investigate and a dataset with a single seismogram of 1000 time samples. If we choose a discretization step of 10 m in all directions, we would get \( 501^3 \) image points and multiplied by the time samples we would get a 4D matrix of 501 x 501 x 501 x 1000 elements, the total of 125,751,501,000 matrix elements. It demands high memory requirements (in the given case it requires 125,751,501,000 x 4 bytes \( \approx 468 \) Gb).

We can reduce the high internal memory and computational effort demanded for the 4D diffraction stacking function by applying alternative imaging conditions of diffraction stacking. The first imaging condition lets us reduce the number of the matrix elements from the example above by a factor of 1000. In this case the values stacked along the traveltime curves for one image point assigned to different time steps are summed and form the value of the 3D image function. I will call this procedure collapsing the time axis of the 4D image function. This imaging condition was suggested by Gajewski et al. (2007).
As a result, I obtain a 3D diffraction stacking function of space (see also Zhebel et al. (2011)):

$$S_1(x_0, y_0, z_0) = \sum_{t_{0,n} = 0}^{t_{END}} \left( \sum_{R=1}^{N} A(t_0 + t_{R,n}^P) \right)^2,$$  \hspace{1cm} (4.3)

I denote this kind of diffraction stacking the "time collapsed diffraction stacking".

Figure 4.3 shows horizontal and vertical slices through the maximum of the collapsed diffraction stacking image function obtained using the noise-free synthetic data described above. The discretization step was set to 10 m in both horizontal and vertical directions. The values of the image function are normalized by dividing each value by the maximum of the image function to make the image functions comparable. The resolution of the image function is better in the horizontal plane rather than in the vertical (see also Vermeer (1999)). This effect can be explained by the surface acquisition: as all the receivers are placed in the horizontal plane. Furthermore, focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

Corresponding horizontal and vertical slices through the maximum of the time collapsed diffraction stacking image function are presented in Figure 4.4. We still get a distinct maximum at the true source position, but the image function is contaminated with the stacking noise. However, the amplitude of noise is less than 50% of the maximum. If we have a look at the resulting image function for the seismogram with the signal-to-noise ratio of 0.5 shown in Figure 4.5 (see also Gajewski et al. (2007) and Zhebel (2010), mind the scales), we cannot distinguish between the noise and the maximum of the image function. Even though there is an absolute maximum corresponding to the true source position, the level of the stacking noise around the focal area has increased dramatically compared to the results for the data with the better signal-to-noise ratio of 2. This effect is the result of collapsing the time axis: we not only stack the signal but also a lot of noise. Moreover, there are also local maxima visible in the horizontal slice, the values of which are less than 20% different from the global maximum at the true source position.

Time collapsed diffraction stacking was previously tested on data contaminated with white Gaussian noise with the signal-to-noise ratio of 0.5 in 2D with 198 receivers by Anikiev et al. (2007) and further, by Zhebel (2010) in 3D with 10201 receivers. Both authors report distinct maxima in resulting image functions with the same accuracy as for the noise-free data. These conclusions do not contradict results presented in this work. I normalize image functions and use the same color scale (from 0 to 1) for each image function to make them easy to be compared. As a result, images in this work seem less focused than obtained by Anikiev et al. (2007) or Zhebel
(2010), as the first author did not normalize image functions and the second did not use the same color scales starting from 0. I also observed maximum of the image function at the true source position even for the data with the signal-to-noise ratio of 0.5, but the values of the stacking noise were less than 20% different from the global maximum. The higher level of stacking noise present in the image function compared to the one presented by Zhebel (2010) can be also explained by a much lower number of receivers used in this work.

Synthetic tests using the explosive source type showed that time collapsed diffraction stacking works reliably for signal-to-noise ratios down to 2, in case of a lower signal-to-noise ratio stacking noise leads to ambiguous results. The presence of noise makes this imaging condition unstable.
4.3 Maximum imaging condition

Instead of collapsing the time axis (see Section 4.2) we can search for the maximum stack along the time axis for each image point and use only this value for our image function. In other words, the maximum value of all the stacks corresponding to one moveout/image point is assigned to the image function for this very image point. This imaging condition should also resolve the problem with the stacking noise occurring for the time collapsed diffraction stacking, as we do not sum all the stacked values for each image point. The maximum imaging condition for the diffraction stacking can be formulated as follows:
Figure 4.5: (a) Horizontal slice and (b) vertical slice through the image function obtained by time collapsed diffraction stacking applied to the dataset with the SNR=0.5. Note that level of the stacking noise around the focal area has increased dramatically (compare with Figure 4.4).

\[ S_{\text{max}}(x_0, y_0, z_0) = \max \left( \sum_{R=1}^{N} A(t_0 + t_R^P) \right)^2, \]  

(4.4)

Vertical and horizontal slices of the resulting image function with the maximum imaging condition is shown in Figure 4.6 for the noise-free data. We find a distinct maximum at the true source position. Compared to the image function obtained using time collapsed diffraction stacking (compare Figure 4.3), the image function is more focused both in vertical and horizontal directions. This is also true for both datasets with signal-to-noise ratios of 2 and 0.5 (see Figures 4.7 and 4.8).

As we do not collapse the time axis we avoid stacking noise and thus, get a better resolved image function. In the case of a very low signal-to-noise ratio (less than 1) noise effects become visible in the image function, but the noise amplitude values are lower than 50% of the maximum, which still allows us to unambiguously distinguish between the noise and signal in the image function.
Figure 4.6: (a) Horizontal and (b) vertical slice through the image function obtained by maximum diffraction stacking. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

Figure 4.7: (a) Horizontal and (b) vertical slice through the normalized image function obtained by maximum diffraction stacking for the data with SNR=2. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

4.4 Sliding time window imaging condition

In the presence of a high level of coherent noise in the seismogram (with the signal-to-noise ratio less than 1), maximum diffraction stacking might fail and lead to a wrong event location by focusing the coherent noise and discarding the desired seismic event. For instance, if only P-waves are taken into account during localization procedure for the data where S-wave arrivals are present, S-wave arrivals represent coherent noise. This limitation can be overcome if we stack the data amplitudes along the traveltime curve within a sliding time window:
Figure 4.8: (a) Horizontal and (b) vertical slice through the normalized image function obtained by maximum diffraction stacking for the data with SNR=0.5. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

\[
S_{\text{slide}}(x_0, y_0, z_0) = \max \left( \sum_{t_1}^{t_2} \left( \sum_{R=1}^{N} A(t_i + t_R^P) \right)^2 \right),
\]

(4.5)

where \( t_2 - t_1 \) is the length of the applied sliding window. In other words, we choose a time window within which the amplitudes are stacked along the traveltime curves for each image point and slide this window through the whole dataset. Then, the maximum value of the stacks within the sliding time window forms the value of an image function. The general concept of this imaging condition is illustrated in Figure 4.9. Blue lines represent the limits of the time window, their moveout corresponds to the true source location in this case. The amplitudes are stacked along the traveltime curves for each time step within the chosen time window, the stack results are squared and summed. In other words, time collapsed diffraction stacking is performed within the chosen time window. Figure 4.9a demonstrates the case where the chosen time window contains the whole signal and thus, the resulting value for this time step is maximal for the given image point. Then, the window is shifted downward, whereby only a part of the signal is captured. Figure 4.9c illustrates the case, where the window contains no signal information and thus, the contributed value equals zero. After the time window is shifted through the whole length of the seismogram, we search for the maximum value. This value is assigned to the image function for this very image point.

By choosing the length of the sliding time window equal to the duration of the signal \( T \) we stack the whole signal and thus, use more information compared to the maximum imaging condition. The duration of the signal \( T \) in seconds) can be defined as \( T = 1/f \), where \( T \) represents.
Figure 4.9: The concept of sliding time window stacking procedure for the case that the image point corresponds to the true source position. In Figure (a) we obtain a perfect match of the time window and the event, which leads to the greatest contribution for the image function. Whereas in (b) only a part of the signal is inside the time window, as a result, in this step we get a lower contribution compared to the previous one. In (c) the time window contains no signal and its contribution equals zero.
the duration of the signal in seconds and $f$ stands for the peak frequency of the signal (in Hertz). Peak frequency can be determined from the frequency spectrum of the input data. One shifts the chosen time window with the step of one sample along the seismogram. As a result, we get one value for each time shift.

In our case for the data with the peak frequency of 10 Hz the signal duration accounts for 0.1 seconds. The corresponding result for the noise-free data are presented in Figure 4.10.

If we shift the sliding time window not every time sample but every n-th time samples, whereby $n = T/2$, we would reduce computational effort by approximately 15%. But the disadvantage of such an approach is that the sliding window within we stack the amplitudes does not include the whole signal but just a part of it. Let us assume we have a signal with the duration of 0.1 seconds with the $t_0 = 1.23$ seconds and we choose the time window of the same length and shift it every 0.05 seconds. In this case we would stack only the part of the signal at first, between 1.2 and 1.3 seconds and then between 1.25 and 1.30 seconds. In both cases, only a part of the signal is present in the sliding time window. In both cases we would stack only 80% of the desired signal. In case of the low signal-to-noise ratio (less than 1) we would lose extremely valuable information and it may lead to a wrong localization result. To overcome this limitation, we can either choose a larger sliding time window, for example, the window with the size of one and a half times of the signal duration ($3T/2$) or reduce the shifting step to one fourth of the signal duration ($T/4$). The disadvantage of the first possibility is that we would not only stack the wanted signal but also noise, which would lead to a less focused image function. The second variant with the smaller shifting step ($T/4$) would increase the computational effort compared to the $T/2$ shift by approximately 13%.

Let us compare these variants for the dataset with the signal-to-noise ratio of 0.5. If we choose to shift the sliding time window by one fourth of the signal duration, the image function would look as shown in Figure 4.13a. There are no visible changes of the image function compared to the image function obtained using one sample shift of the sliding time window in Figure 4.12. Figure 4.13b demonstrates the difference between the both image functions, its highest value is less than 4%.

Applying $T/2$ shift of the sliding time window leads to the image function shown in Figure 4.14a. The difference to the image function obtained by applying one sample shift is shown in Figure 4.14b. The maximal difference accounts for 14%.

Enlarging the sliding time window to the one and a half of signal duration and sliding with the
shift equals to half of the signal duration leads to the image function presented in Figure 4.15a. The comparison to the image function shown in Figure 4.12 is illustrated in Figure 4.15b. The maximum difference accounts for 21%. Note that we get higher level of stacking noise, but not in the focal area of the function, but outside of it.

These results bring us to the conclusion that by choosing the sliding time window length of one fourth of the signal duration ($T/4$) represents the best compromise between minimizing the computational effort and obtaining reliable localization results with a well focused image function even in case of a very low signal-to-noise ratio (less than 1).

All in all, synthetic tests have shown that time collapsed imaging condition works reliably for signal-to-noise ratios down to 2. Unfortunately, presence of high level of noise (signal-to-noise ratios less than 2) makes it unstable, leading to ambiguous results. The introduced maximum and sliding time window condition proved to provide reliable localization results even for signal-to-noise ratios under 1. The latter imaging condition leads to less focused image compared to the maximum imaging condition. I suggest using both imaging conditions at the same time. This does not require significantly higher computational effort compared to sliding time window imaging condition but increases the probability of the reliable localization.

![Figure 4.10: (a) Horizontal and (b) vertical slice through the normalized image function obtained by applying sliding time window imaging condition with one sample shift for noise-free data. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).](image)

In the next section I will test robustness of imaging conditions to velocity uncertainties.
Figure 4.11: (a) Horizontal and (b) vertical slice through the normalized image function obtained by applying sliding time window imaging condition with one sample shift for the data with SNR=2. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

Figure 4.12: (a) Horizontal and (b) vertical slice through the normalized image function obtained by applying sliding time window imaging condition with one sample shift for the data with SNR=0.5. Note that focal area of the image function is inclined in the vertical direction due to asymmetrical location relatively to receivers distribution (see Figure 4.1).

4.5 Velocity uncertainties

As a diffraction stacking approach requires computation of traveltimes, the knowledge of the velocity model is inevitable. In exploration seismic velocity model can be constrained from the previously acquired active source data. But it is less common in seismology. Nevertheless, velocity models are characterized by numerous uncertainties. The accuracy of the localization results strongly depends on the accuracy of the velocity model. A poor velocity model can lead to a dislocation of an event (see e.g. Warpinski (2009)). In this section I illustrate the effects of incorrect velocity model on the localization results. For synthetic tests, the noise-free data was used described in Section 4.1.
Figure 4.13: (a) Horizontal slice through the image function obtained by sliding window diffraction stacking with $T/2$ shift and (b) shows the absolute difference between the image function obtained by sliding time window diffraction stacking with one sample shift and the $T/2$ shift for the data with SNR=0.5.

Figure 4.14: (a) Horizontal slice through the image function obtained by sliding window with $T/2$ shift diffraction stacking and (b) shows the absolute difference between the image function obtained by sliding time window diffraction stacking with one sample shift and the one with $T/2$ shift for the data with SNR=0.5.
Figure 4.15: (a) Horizontal slice through the image function obtained by sliding window diffraction stacking with $T/2$ shift with the time window of $3T/2$ and (b) is the absolute difference between the image function obtained by sliding time window diffraction stacking with one sample shift and the one with $T/2$ shift with the sliding time window equals to $3T/2$ for the data with SNR=0.5.

The true homogeneous P-wave velocity accounts for 2500 m/s. To compute traveltime curves for the diffraction stacking procedure velocity model was changed by ±5% and ±10%. In microseismic reservoir characterization standard uncertainty in the velocity model is usually less than 5% (Maxwell (2009)), but it is often not the case in seismology. On one hand, these variations were applied to exaggerate the impact of the velocity model accuracy on localization results and, on the other hand, to test robustness of the diffraction stacking method to velocity uncertainties.

4.5.1 Time collapsed imaging condition

At first, time collapsed diffraction stacking was applied to the data. Figure 4.16 shows resulting horizontal slices through the maximum of the normalized image functions for different P-wave velocities. The lateral position of the source was correctly obtained for velocity variations by ±5%. For velocity models with ±10% variations the resulting x-position is 20 m shifted relatively to the true location, whereas the y-position is correctly computed. This effect can be explained be the relative position of the the source to acquisition geometry (see Figure 4.1). The source is located more centered along the y-direction and thus, is better illuminated in this direction.

Corresponding vertical slices are presented in Figure 4.17. The higher the variation of the velocity, the higher the location uncertainty in the vertical direction. For velocities varying by ±5% to the true velocity, vertical dislocation accounts for 150 m, whereas for ±10% velocity variations dislocation reaches 200 m. Higher sensitivity of the vertical coordinate to velocity changes can be explained by the surface acquisition geometry. For wrong velocity models visible artefacts are present, where the maximum areas are discontinuous. There is an area of a global
maximum and areas of local maxima stretched in the vertical directions. This might be the effect of a limited aperture and high source depth to aperture ratio of approximately 2.3 : 1. If I increase the aperture to 2500 m, so that the source depth to aperture ratio equals to 1 : 1, these discontinuity artefacts disappear. Distance between receivers in both x- and y-directions was set to 209 m to keep the number of receivers equal to 144. Figure 4.18 shows vertical slice through maximum of the image function obtained by time collapsed diffraction stacking for the wrong P-wave velocity with −10% variation to the correct velocity. Note also that the focal area has become wider due to sparser receiver distribution.

Anikiev et al. (2006) suggested to stack image functions for different velocity models. The idea of such a concept is that the Figure 4.19 demonstrates the result of summing the image functions for different velocities models. The maximum of the obtained function corresponds to the true source location. Artefacts are not visible any more. Nevertheless, the obtained image function is less focused and the maximum area is more extended in the vertical direction compared to the one obtained by using the correct velocity model (compare with Figures 4.16c and 4.16c). To obtain a more focused image I suggest the following procedure: at first, each two image functions obtained by using batched velocity models are zero-lag cross-correlated. It means that for instance, the image functions corresponding to the velocity model with 5% and 4% variance are cross-correlated, then the ones for the 4% and 3% variations and so on. Next, all out-coming cross-correlation functions are summed and the result represents the final image function. The idea of this approach is based on the definition of cross-correlation. Cross-correlation function represents a measure of similarity of two functions. As a result, by zero-lag cross-correlating two batched image functions, the overlaps of maximum areas are amplified, whereas the amplitudes in the areas where one function has higher values and the other low values are suppressed.

The resulting image function obtained by pairwise zero-lag cross-correlated image functions for different P-wave velocities is shown in Figure 4.20.

As the velocity uncertainty mainly influences the accuracy of depth location as a consequence of a surface acquisition, only vertical slices are demonstrated for the following tests.

4.5.2 Maximum imaging condition

Next, maximum imaging condition was applied to the data. Corresponding vertical slices for different velocity models are shown in Figure 4.21. The image functions are more focused compared to the time collapsed imaging, which is consistent with the previous synthetic tests (see Section
Figure 4.16: Horizontal slices through maximum of the normalized image functions obtained by time collapsed diffraction stacking for the P-wave velocities: (a) $V_{P1} = 2250 \text{ m/s} = 0.9V_{true}$, b) $V_{P2} = 2375 \text{ m/s} = 0.95V_{true}$, c) $V_{P3} = 2500 \text{ m/s} = V_{true}$, d) $V_{P4} = 2625 \text{ m/s} = 1.05V_{true}$, c) $V_{P5} = 2750 \text{ m/s} = 1.1V_{true}$. The lateral position of the location was correctly obtained for velocity variations by $\pm 5\%$. For $V_{P1}$ and $V_{P5}$ with 10% variations the resulting x-position is 20 m shifted relatively to the true location, whereas the y-position is correctly determined.
Figure 4.17: Vertical slices through maximum of the normalized image functions obtained by time collapsed diffraction stacking for the P-wave velocities: (a) \( V_{P1} = 2250 \text{ m/s} = 0.9V_{true} \), b) \( V_{P2} = 2375 \text{ m/s} = 0.95V_{true} \), c) \( V_{P3} = 2500 \text{ m/s} = V_{true} \), d) \( V_{P4} = 2625 \text{ m/s} = 1.05V_{true} \), c) \( V_{P5} = 2750 \text{ m/s} = 1.1V_{true} \). For velocity variations by ±5% vertical position of the source was shifted 150 m up- and downwards, respectively, whereas for \( V_{P1} \) and \( V_{P5} \) with 10% variations the shift along the z-axis accounts for 200 m up- and downwards.
Figure 4.18: Vertical slice through maximum of the normalized image function obtained by time collapsed diffraction stacking for the wrong P-wave velocity with $-10\%$ variation to the correct velocity for the dataset, where acquisition geometry has an aperture of 2500 m, so that the ratio of source depth to aperture equals $1 : 1$. Note that discontinuity artefacts disappear.

4.3). The location uncertainty is comparable to the one for time collapsed diffraction stacking. Further, all the image functions were stacked and the resulting vertical slice through the maximum of the normalized image function is presented in Figure 4.22. After summing the images over the range of velocities, the image has become less focused. However, the maximum area is better defined compared to the one in Figure 4.19b. Unfortunately, discontinuous artefacts are still present in the image. Applying the pairwise zero-lag cross-correlation of the batched images for different velocity models leads to a more focused image function and removes the discontinuous artefacts, which is demonstrated in Figure 4.23.

### 4.5.3 Sliding time window imaging condition

Finally, the robustness of the sliding time window condition to velocity uncertainties was tested. Vertical slices for different velocity model variations are shown in Figure 4.24. As the data is noise-free, the results are comparable to those for the time collapsed imaging condition (see also Section 4.4). This also applies to the image obtained by stacking the image functions over the range of velocities (see Figure 4.25). Stacking pairwise zero-lag cross-correlated batched images for different velocity models results in a more focused image function (see Figure 4.26).

### 4.5.4 Conclusions

Synthetic tests have shown that the depth location is more sensitive to velocity uncertainties due to the surface acquisition geometry. The higher the variation of the velocity model, the higher location uncertainty is obtained. An approach of stacking the images over the range of velocities
Figure 4.19: a) Horizontal and b) vertical slices through maximum of the normalized image function obtained by stacking image function for different P-wave velocities, time collapsed imaging condition was applied. Note that the image function is less focused compared to the one for the correct velocity model.
Figure 4.20: a) Horizontal and b) vertical slices through maximum of the normalized image function obtained by stacking the pairwise zero-lag cross-correlated image functions for different P-wave velocities, time collapsed imaging condition was applied. Note that the resulting image function is better focused in both horizontal and vertical directions than in Figure 4.19.
Figure 4.21: Vertical slices through maximum of the normalized image functions obtained applying maximum imaging condition for the P-wave velocities: (a) $V_{P1} = 2250\, \text{m/s} = 0.9V_{true}$, (b) $V_{P2} = 2375\, \text{m/s} = 0.95V_{true}$, (c) $V_{P3} = 2500\, \text{m/s} = V_{true}$, (d) $V_{P4} = 2625\, \text{m/s} = 1.05V_{true}$, (e) $V_{P5} = 2750\, \text{m/s} = 1.1V_{true}$. For velocity variations by $\pm 5\%$ vertical position of the source was shifted 150 m up- and downwards, respectively, whereas for $V_{P1}$ and $V_{P5}$ with $10\%$ variations the shift along the $z$-axis accounts for 200 m up- and downwards.
Figure 4.22: Vertical slice through maximum of the normalized image function obtained by stacking image functions for different P-wave velocities, maximum imaging condition was applied. Note that the image function is less focused compared to the one for the correct velocity model.

Figure 4.23: Vertical slice through maximum of the normalized image function obtained by pairwise zero-lag cross-correlation of the batched images for different P-wave velocities, maximum imaging condition was applied. Note that the image function is more focused compared to the one obtained by stacking image functions for different P-wave velocities (compare with Figure 4.22).

was illustrated. It led to less focused image function, but the maximum area corresponds to the true source location. On the contrary, the suggested procedure of stacking zero-lag cross-correlated batched images for different velocity variations results in a well focused image function. It means that even if the velocity model is not precisely known, scanning over different velocities in a given range may contribute to a problem solution.
Figure 4.24: Vertical slices through maximum of the normalized image functions obtained by sliding time window diffraction stacking for the P-wave velocities: (a) $V_{P1} = 2250 \text{ m/s} = 0.9V_{true}$, b) $V_{P2} = 2375 \text{ m/s} = 0.95V_{true}$, c) $V_{P3} = 2500 \text{ m/s} = V_{true}$, d) $V_{P4} = 2625 \text{ m/s} = 1.05V_{true}$, c) $V_{P5} = 2750 \text{ m/s} = 1.1V_{true}$. For velocity variations by $\pm 5\%$ vertical position of the source was shifted $150 \text{ m}$ down- and upwards, respectively, whereas for $V_{P1}$ and $V_{P5}$ with $10\%$ variations the shift along the z-axis accounts for $200 \text{ m}$ up- and downwards.
Figure 4.25: Vertical slice through maximum of the normalized image functions obtained by stacking image function for different P-wave velocities, sliding time window imaging condition was applied. Note that the image function is less focused compared to the one for the correct velocity model.

Figure 4.26: Vertical slice through maximum of the normalized image function obtained by stacking pairwise zero-lag cross-correlated batched images for different P-wave velocities, sliding time window imaging condition was applied. Note that the image function has become better focused (compare with Figure 4.25).
Chapter 5

Acquisition footprint

In passive seismic we usually have control over the number and position of the receivers. Thus, we can design the acquisition geometry so that we get a large aperture and proper illumination for the area of interest (hundreds of geophones, see, e.g. Chambers et al. (2010) or Eisner et al. (2010)). But if our target area is located in limited accessible environments, e.g. mountainous environment, or close to communities, we often do not get a chance to place our receivers in a regular net. In global seismology we generally have sparse irregular distribution of geophones (often less than hundred receivers).

So far the impact of the acquisition geometry on the stacking-based localization results was studied in terms of the aperture (see e.g. Gajewski et al. (2005) or Zou et al. (2009)). The larger the aperture the better focused image function and thus, more reliable source location can be obtained. Behura et al. (2013) developed an optimized acquisition geometry for microseismic monitoring to yield better event imaging. They stated that the hypocenter has to be illuminated evenly from all directions to obtain the best possible imaging result and also to decipher the source radiation pattern correctly. To design such an acquisition one needs the information about an approximate center of the microseismicity zone, maximum possible aperture and number of receivers, and an approximate smooth velocity model of the subsurface. By taking this information into account, rays from the center of the microseismic zone are shot at equally spaced take-off angles. The optimal location of the surface receivers is estimated by the intersection of these rays with the subsurface. Furthermore, Eisner et al. (2010) compared event locations from surface and borehole acquisition geometries. Their study revealed that focal areas of an image function obtained using surface receivers are less scattered in both vertical and horizontal directions.
However, the effect of the irregular receiver distribution on localization results has not been addressed yet. In seismic imaging a phenomenon of so-called acquisition footprint, whereby amplitudes variations are related to the surface geometry and not only to the physical properties of the subsurface, is well known (see e.g., Gardner and Canning (1994)). Marfurt et al. (1998) defined acquisition footprint as 'any pattern of noise that is highly correlated to the geometric distribution of sources and receivers on the earth’s surface'. As a result, acquisition footprint leads to artificial focusing of the function and thus, makes it difficult to distinguish between the artefacts related to the experiment geometry and the subsurface response.

The impact of the acquisition footprint on localization results obtained by diffraction stacking is studied in this chapter. Here I will present the comparison between the effects of the regular and irregular acquisition geometries on source locations techniques for different imaging conditions.

5.1 Regular acquisition geometry

To illustrate the effect of the regular acquisition geometry, let us consider an example. At first, a regular acquisition geometry consisting of 529 receivers equidistantly distributed on the surface with 200 m spacing in x- and y-directions is assumed. The maximum receiver offset is about 6500 m. The explosive seismic event occurred at (1210/1620/2500) m. The area of interest accounts for 25 km$^2$. Top view of the acquisition geometry and event hypocenter are shown in Figure 5.1a. A homogeneous velocity model with the P-wave velocity of 2500 m/s is assumed. For the synthetic seismogram we use a Ricker wavelet with a peak frequency of 10 Hz. Thus, the prevailing wavelength is 250 m. Note that the source is placed asymmetrically in-between receivers, but nevertheless is well illuminated from various directions.

5.1.1 Time collapsed imaging condition

First, time collapsed diffraction stacking (see Section 4.2) was applied to the synthetic seismogram. Horizontal slice through the maximum of the normalized resulting image function is presented in Figure 5.1b. We get a distinct maximum at the true position of the event. Moreover, the focal area (area around the maximum of the image function) is symmetrical relative to the source position because of the proper illumination from different takeoff angles/directions. The diameter of the focal area of the normalized image function accounts for 120 m in both x- and y-directions, which corresponds approximately to the half of the prevailing wavelength. Note that the focal area has
Figure 5.1: (a) Acquisition geometry with 529 regularly distributed receivers and (b) represents the corresponding horizontal slice through the normalized resulting image function through its maximum obtained applying time collapsed diffraction stacking. Note the symmetric form of the image function focal area a form of a circle.

Furthermore, the number of receivers was reduced to 81. The resulting regular acquisition geometry is shown in Figure 5.2a. The receivers spacing is 500 m in both x- and y-direction. The maximum receiver offset is approximately 5800 m. The resulting image function obtained by applying time collapsed diffraction stacking is presented in Figure 5.2b. The focal area of the function is well defined and the distinct maximum is visible. Note that the focal area has the same circle form like in Figure 5.1b. Furthermore, the image shows an artificial pattern outside the focal area, which can be assigned to a sparser receiver distribution. We can recognize 2 dominating directions (arms going from the focal area) of the higher values radiation. One of the arms is allocated along the x- and the other - along the y-axis. There are 4 other radiation arms, 2 of them are stretched along the angle bisectors of the angle between the main radiation directions.

To better understand the nature of these artefacts, I chose an acquisition geometry of 52 regularly distributed receivers aligned along angle bisectors, which is shown in Figure 5.3a. The source location stayed the same with the coordinates (1210/1620/2500) m. The corresponding time collapsed diffraction stacking image function is shown in Figure 5.3b. Note the angular shape of the focal area (compare with Figure 5.2b). This is the result of an uneven illumination of the source. We get 2 main directions of the smearing of the image function and these directions are parallel to the receivers lines.

Let us further reduce the number of receivers to 26 and choose one main direction along
Figure 5.2: (a) Regular acquisition geometry consisting of 81 receivers (black triangles), the distance between the receivers is 500 m in both x- and y-direction, the coordinates of the source (red star) are \((1210/1620/2500)\) m. Note that the source is located in between the receivers and (b) represents the corresponding image function obtained applying time collapsed diffraction stacking with \(r\).

Figure 5.3: (a) Regular acquisition geometry consisting of 52 receivers (black triangles), oriented along the angle bisectors, the coordinates of the source (red star) are \((1210/1620/2500)\) m and (b) shows the image function obtained by time collapsed diffraction stacking.
Figure 5.4: (a) Acquisition geometry consisting of 26 regularly distributed receivers (black triangles) the coordinates of the source (red star) are (1210/1620/2500)m and (b) shows the image function obtained by time collapsed diffraction stacking which the receivers are aligned to see if the image function is smeared along this direction. The new acquisition geometry is shown in Figure 5.4a. The corresponding time collapsed diffraction stacking image function can be seen in Figure 5.3b. The maximum is stretched perpendicular to the direction of the acquisition geometry. Note also that the focal area is more smeared above the maximum area, as there are less receivers and thus, weaker illumination from this side. The smearing of the focal area is the result of an uneven illumination of the source.

The synthetic tests with different regular acquisition geometries have shown that if the receivers are distributed sparsely or if the source is unevenly illuminated, acquisition footprint artefacts occur in the image function obtained by time collapsed diffraction stacking. Moreover, the smearing directions are oriented perpendicular to the main allocation directions of the receivers due to the lacking illumination of the event.

5.1.2 Acquisition footprint reduction: sliding time window imaging condition

For the sliding time window imaging condition not all the stacks corresponding to one image point for each time sample contribute to the value of the image function, but only the stack for the time window with the maximum value for each image point (see Section 4.4). As a result, a more focused image function is obtained. Thus, applying the sliding time window imaging condition to the data obtained with sparsely distributed receivers leads to reduction of the acquisition footprint.

The image function for the regular acquisition geometry of 81 sparsely distributed receivers
obtained by applying the sliding time window diffraction stacking is presented in Figure 5.5. Note that the acquisition footprint present in Figure 5.2b vanished. Moreover, the image function looks the same like for the acquisition geometry consisting of 529 receivers with a two an a half (2.5) times smaller receiver distance. Unfortunately, the application of the sliding time window imaging condition does not change the form of the focal area and thus, does not reduce the illumination artefacts of the acquisition geometry.

So we can conclude that in case of sparsely but regularly distributed receivers we can reduce the related acquisition footprint by using the sliding window imaging condition, but cannot influence the illumination artefacts.

Figure 5.5: Image function obtained applying sliding window diffraction stacking with $T/4$ shift with regular acquisition geometry consisting of 81 receivers. Note that the acquisition effects present in Figure 5.2b disappeared. This image function looks similar to the one obtained by using the acquisition geometry of 529 regularly distributed receivers.

5.2 Irregular acquisition geometry

This section should help us to understand what effects occur if we have very few (less than 100) receivers which are not only irregular but also sparsely distributed over the area of interest. An acquisition geometry of 81 irregularly and sparsely distributed receivers was assumed (see Figure 5.6). Note that the area of interest can be divided into 3 zones due to the receiver concentration. In the first zone (on the left side) we observe a cluster with high concentration of receivers, whereas in the middle there are a few receivers which are sparsely distributed. On the right side of the area of interest there are almost no receivers. The design of the acquisition geometry was chosen...
by having in mind the common distribution of the receivers in seismology (see e.g. Baker et al. (2005)). But on the other hand, we have a large aperture (the maximum receiver offset is 5777 m), which is a key factor for successful localization by diffraction stacking techniques. As diffraction stacking technique is based on stacking of amplitudes along traveltime curves, i.e., their move-outs, it benefits from large apertures a lot. The larger the aperture, the larger event moveout can be observed.

For synthetic tests 3 explosive source positions were chosen (see Figure 5.6). The first source, located at (1210/1620/2500) m, is situated in the zone with the highest receiver concentration, its average distance to the geophones is approximately 3050 m. The second one is placed at (2000/2500/2500) m in the area, where we hardly have a receiver, but nevertheless it is well illuminated from various directions and the average distance to the receivers accounts for 2915 m. The third event occurred at the edge of the acquisition geometry at (4000/4000/2500) m. It is illuminated only from one side, the average distance to the geophones is about 4085 m. For the synthetic seismogram I used a minimum-phase Ricker wavelet with a peak frequency of 10 Hz. We chose a homogenous velocity model with the P-wave velocity of 2500 m/s. Thus, the prevailing wavelength is 250 m. A normalized noise-free seismogram corresponding to the source position of the first source with the coordinates (1210/1620/2500) m is presented in Figure 5.7.

For synthetic tests 3 explosive source positions were chosen (see Figure 5.6). The first source, located at (1210/1620/2500) m, is situated in the zone with the highest receiver concentration, its average distance to the geophones is approximately 3050 m. The second one is placed at (2000/2500/2500) m in the area, where we hardly have a receiver, but nevertheless it is well illuminated from various directions and the average distance to the receivers accounts for 2915 m. The third event occurred at the edge of the acquisition geometry at (4000/4000/2500) m. It is illuminated only from one side, the average distance to the geophones is about 4085 m. For the synthetic seismogram I used a minimum-phase Ricker wavelet with a peak frequency of 10 Hz. We chose a homogenous velocity model with the P-wave velocity of 2500 m/s. Thus, the prevailing wavelength is 250 m. A normalized noise-free seismogram corresponding to the source position of the first source with the coordinates (1210/1620/2500) m is presented in Figure 5.7.

![Figure 5.6: Acquisition geometry of 81 irregularly distributed receivers (black triangles). Notice that on the left side we have a cluster of receivers, whereas the right side of the area is hardly covered. Source position relative to the acquisition geometry, the coordinates of the source position are (1210/1620/2500) m (red star), (2000/2500/2500) m (green star) and (4000/4000/2500) m (magenta star). Note that the sources are located asymmetrically relative to the acquisition geometry. Resulting image functions obtained by time collapsed diffraction stacking for the three different sources positions](image)

Resulting image functions obtained by time collapsed diffraction stacking for the three different sources positions.
source locations are presented in Figures 5.8 and 5.10. All image function have a distinct maximum, the sources were localized at the true positions. Focal areas of the functions are inclined because of asymmetrical source locations relative to the acquisition geometry. The dimensions of the focal area for the first and second sources are comparable, whereas for the third one the focal area is much more smeared and stretched in both horizontal and vertical directions. This effect can be explained by the poor illumination of the source at the edge of the acquisition geometry. The focal area of the image function for the irregular acquisition geometry displays a more complex shape in comparison to the one for a regular acquisition geometry (compare with 5.2b). Furthermore, all three images feature artefacts around the focal area resulting from the sparse receiver distribution, we get higher values of the image function compared to the background values radiating in different directions. These artefacts have values varying between 1% and 15% of the maximum value. To clearly observe these effects, the artefacts were amplified and the focal area was set to zero (see Figure 5.9).

If we plot the acquisition geometry over the the image function (see Figure 5.11), at first sight it seems that we get higher image function values in directions where we have very few or even no receivers, whereas in the area with a high concentration of receivers we do not observe these effects. Should this assumption be valid, in the image function for the source located at (4000/4000/2500) we should get these artefacts more or less evenly in all directions, but it does not happen. A more thorough look at the image functions and the the acquisition geometry reveals that the occurring artefacts are mostly aligned perpendicularly to the main direction of the receivers (see also Figure 5.9).
Figure 5.8: Horizontal slices through the maximum of normalized image functions obtained applying time collapsed diffraction stacking for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m. Note the acquisition related artefacts.
Figure 5.9: Horizontal slice through the maximum of the normalized image function obtained applying time collapsed diffraction stacking for the source with the coordinates (1210/1620/2500) m, where the artefacts are amplified and the focal area values are set to zero. Note the different scale limits from 0 to 14%.

Application of the sliding time window diffraction stacking with the $T/4$ shift leads to the results presented in Figures 5.12 and 5.13. Note that the acquisition footprint related to the sparse receiver distribution was removed. Focal area has become significantly better focused in vertical direction (compare with Figure 5.10). But the orientation and form of the focal area in horizontal plane, which is affected by the illumination of the event, has not changed. The results are consistent with those for the regular acquisition geometry with the same number of receivers (compare with Figures 5.2b and 5.5). The image functions for the data with added Gaussian noise with the resulting signal-to-noise ratio of 0.5 are shown in Figure 5.14. The form of the focal area has not changed, it even falsely seems to be more focused because of the noise effects present in the image function. The images for the the sources located at (1210/1620/2500) m and at (2000/2500/2500) m are comparable, the difference between the maximum area and the background noise is approximately 80%, whereas for the event at the edge of the acquisition geometry the difference accounts just 55%.

Synthetic tests have shown that as well irregular as sparse receiver distributions lead to the occurrence of an acquisition footprint in the image function obtained by diffraction stacking techniques. The effects related to the sparse distribution can be removed by applying the sliding time window imaging condition. But the illumination artefacts could not be reduced by this approach.
Figure 5.10: Vertical slices through the maximum of the normalized image function obtained applying time collapsed diffraction stacking for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m.
Figure 5.11: Horizontal slices through the maximum of image functions obtained applying time collapsed diffraction stacking for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m with the acquisition geometry overlying. Note the orientation of acquisition related artefacts relative to receivers distribution.
Figure 5.12: Horizontal slices though maxima of image functions obtained applying sliding time window diffraction stacking with $T/4$ shift for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m.
Figure 5.13: Vertical slices through the normalized image functions obtained applying sliding time window diffraction stacking with $T/4$ shift for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m.
Figure 5.14: Image function obtained applying sliding time window diffraction stacking with $T/4$ shift, SNR=0.5 for different source positions: a) $(1210/1620/2500)$ m, b) $(2000/2500/2500)$ m and c) $(4000/4000/2500)$ m. Note the acquisition related artefacts.
Following is an attempt to remove these effects by weighting each trace.

5.2.1 Acquisition footprint reduction: Voronoi cells

Canning and Gardner (1998) suggested to reduce an acquisition footprint by applying trace weighting according to the relative portion of the space that it represents. The weighting factors are estimated by constructing so-called Voronoi cells (Voronoi (1908)) for each trace.

Voronoi cells represent a way of dividing space into a number of areas bounded by polygons. The computation of such areas is illustrated in Figure 5.15 and can be summarized into following steps:

1. Choose one point (receiver position)
2. Find closest neighbors (receivers) of this point
3. Draw lines connecting the chosen point with all its neighboring points (dashed lines)
4. Construct perpendicular bisectors to all dashed lines drawn in the previous step (solid lines)
5. The intersections of the perpendiculars define the vertices of the polygon
6. Compute the area embedded between the vertices (see Figure 5.16)
7. Repeat for each point

![Figure 5.15: Geometry of polygon construction, (Canning and Gardner (1998))](image)

For this work Voronoi cells were constructed with the *Matlab function voronoiDiagram*. Figure 5.17 illustrates Voronoi cells for the irregular acquisition geometry consisting of 81 sparsely distributed receivers shown in Figure 5.6. One can easily notice that in the areas of high receiver concentration the areas are very small compared to the areas with very low receiver coverage. The computed areas are used as weights for each trace. The marginal receivers obtain infinite weights.
as they do not have any neighboring receivers on one or several sides. These weights are excluded and the corresponding traces are not weighted.

Figure 5.17: Voronoi cells for the acquisition geometry consisting of 81 sparsely irregularly distributed receivers

Sliding time window diffraction stacking with the $T/4$ shift was applied to the synthetic data described above and weighted by Voronoi cells. Resulting image functions for each three source positions are shown in Figures 5.18 and 5.19. Note that the image function is much more focused compared to those without Voronoi cells weighting (see Figure 5.12). The focal area shrunk significantly in both horizontal and vertical directions. But on the other hand, new effects occurred in the images. The resulting functions for the first two source positions with the coordinates $(1210/1620/2500)$ m and $(2000/2500/2500)$ m resemble the main direction of the receivers alignment. Note also that the best focusing is achieved for the second source because of the best illumination. On one side, weighting the traces according to the portions of the space they represent leads to a better focused image function. On the other side, the direction aligned with the
main receiver distribution direction is over-weighted and thus, evokes other artefacts with higher energy break-outs around the focal area. If there is a coherent noise present in this data, this effect may amplify the noise.

Furthermore, the weighting was also applied to the data with the signal-to-noise of 2 (white Gaussian noise). The resulting image functions obtained by the sliding time window diffraction stacking for each three source locations are presented in Figure 5.20. The first two images (corresponding to sources with the coordinates (1210/1620/2500) m and (2000/2500/2500) m, respectively) retained the resolution of the focal area, whereas for the third source the focal area became more smeared and stretched. Note also that the artefacts caused by weighting procedure become stronger.

The results obtained for the data with the signal-to-noise ratio of 0.5 after the weighting procedure are shown in Figure 5.21. Note that only the first event located in zone with the highest receiver concentration could be properly localized. For the other two events the noise level increased significantly and the source location became not distinguishable from the noise. Moreover, a strange pattern consisting of lines with higher amplitudes arises. To understand the nature of the this pattern, I conducted tests with a seismogram which contained no signal, but just white Gaussian noise extracted out of the weighted by Voronoi cells seismogram with the signal-to-noise ratio of 0.5. The corresponding horizontal slice through the normalized image function is shown in Figure 5.22. The image function reveals that the artefact consisting of lines with higher amplitudes arose due to the weighting of noise.

If we compare the results with those without any weighting (see Figure 5.14), it becomes obvious that the artefacts caused by the weighting disturb the image and thus, localization. This happens because of the presence of large areas without any receivers, which results in large "holes". This is a problem of spatial under-sampling and resulting spatial aliasing (Canning and Gardner (1998)), which cannot be completely solved by the weighting procedure and thus, leads to the failure of the stacking-based localization for very low signal-to-noise ratios.

Synthetic tests have revealed that the sliding time window imaging condition reduces the acquisition footprint caused by the sparse receiver distribution and leads to reliable localization results even if the signal-to-noise ratio is less than 1. Furthermore, weighting the data according to the areas they represent results in a more focused image function if the signal-to-noise ratio is over 1. However, it also adds artefacts which may lead to an artificial focusing. It compounds the problem of distinguishing between the artificial noise and the desired signal. Unfortunately,
Figure 5.18: Horizontal slices through maximum of normalized image functions obtained applying sliding time window diffraction stacking with $T/4$ shift step after weighting the seismograms with the Voronoi cells for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m
Figure 5.19: Vertical slices through normalized image functions obtained applying sliding time window diffraction stacking with $T/4$ shift step after weighting the seismograms with the Voronoi cells for different source positions: a) $(1210/1620/2500)$ m, b) $(2000/2500/2500)$ m and c) $(4000/4000/2500)$ m.
Figure 5.20: Horizontal slices through maximum of image functions obtained applying sliding time window diffraction stacking with $T/4$ shift step after weighting the seismograms with the Voronoi cells SNR=2 for different source positions: a) (1210/1620/2500) m, b) (2000/2500/2500) m and c) (4000/4000/2500) m.
Figure 5.21: Horizontal slices through maximum of image functions obtained applying sliding time window diffraction stacking with $T/4$ shift step after weighting the seismograms with the Voronoi cells SNR=0.5 for different source positions: a) $(1210/1620/2500)$ m, b) $(2000/2500/2500)$ m and c) $(4000/4000/2500)$ m.
for the data with the signal-to-noise ratio lower than $1$ weighting disturbs the image function by amplifying various directions and thus, localization for the events occurred in areas with a low receiver concentration fails.
Chapter 6

Influence of different source mechanisms on localization

The majority of seismic events are non-explosive (see e.g. Rutledge and Phillips (2003) or Baig and Urbancic (2010)). The polarity of the direct P-wave depends on the take-off angle. As a result, the polarities of the data amplitudes can be both positive and negative. To illustrate the impact of different source radiation patterns on localization results, four different source types are presented. At first, localization of a pure double-couple (DC) source, which represents the worst case scenario for diffraction stacking localization techniques as none of the polarities are balanced, is discussed. Further, pure compensated linear vector dipole (CLVD) sources and two sources constrained of explosive, DC and CLVD components, which were described in Chapter 3.4, are considered.

6.1 Double couple source: strike-slip

As mentioned in Section 3.3 double couple source represents a good model for earthquakes caused by a shear or slip on a fault. A strike-slip source type was chosen. This source type was also observed for microseismic events (see e.g. Rutledge and Phillips (2003)). A surface acquisition geometry of 441 equidistantly placed receivers with an aperture of 5000 m was considered. Receiver distance is 250 m in both x- and y-directions. The source was placed in the center of the acquisition geometry with the coordinates of (2500/2500/2500) m. A minimum-phase Ricker wavelet with the peak frequency of 10 Hz was assumed for the source. A homogeneous velocity model with the
P-wave velocity of 2500 m was considered. The corresponding seismogram excerpt is shown in Figure 6.1. One can see that on the left side of the seismogram the polarity of the first arrival is positive whereas for the right side of the seismogram the polarity flips to the negative. If we stack the amplitudes along the traveltime curve corresponding to the true source position (see Figure 6.2), the amplitudes cancel each other. Actually, a strike-slip source type represents the worst case scenario for diffraction stacking localization techniques as none of the polarities are balanced. Any non purely double-couple source type is easier to stack, then in such case, one of the two polarities dominates.

Diffraction stacking procedure with the time collapsed imaging condition was applied to the synthetic seismogram for the strike-slip source type. The resulting image function is shown in Figure 6.4. The vertical slice is oriented along the angle bisector between the x- and y-axes, crossing the two of four maxima. Note that we not only get a zero value at the true source position but also 4 maxima concentrated around it. The maxima coordinates are: (2400/2400/2500) m, (2600/2400/2500) m, (2400/2600/2500) m and (2600/2600/2500) m, each maximum is about 100 m distant from the true source location in x- and y-directions. If we overlay the traveltime curve corresponding to one of the maxima shown in Figure 6.3, it becomes obvious that along this curve we stack the amplitudes with the same polarity and they thus, do not cancel each other.

Figure 6.1: Seismogram for a strike-slip source type

Unfortunately, the diffraction stacking localization technique does not provide the right localization for a double-couple source. But on the other side, the image function resembles the radiation pattern of the source (compare with Figure 3.7). This image function pattern may be used as an indicator for a double-couple source. Let us have a look at the image functions for noisy data. At first, white Gaussian noise was added to the data with a resulting signal-to-noise ratio.
Figure 6.2: Seismogram of a strike-slip source type and the blue traveltime curve corresponding to the true source position at (2500/2500/2500) m.

Figure 6.3: Seismogram of a strike-slip source type with the traveltime curve corresponding to one of the maxima of the image function with the coordinates (2600/2600/2500) m. The true source location is at (2500/2500/2500) m.
Figure 6.4: (a) Horizontal and (b) vertical slices through the image function obtained by time collapsed diffraction stacking for noise-free data.
of 2. The corresponding horizontal slice of the normalized image function is shown in Figure 6.5a. Despite the presence of noise image function still resembles the radiation pattern of the source. But if the signal-to-noise ratio decreases to 0.5, localization fails (see Figure 6.5b).

Applying maximum imaging condition to the noise-free data leads to results comparable to the ones obtained by time collapsed diffraction stacking (see Figure 6.6a). But the deviation of the true source position to the four maxima reduces to about 50 m in both x- and y-directions. The imaging function is also more focused. However, it fails for the data with the signal-to-noise ratio of 0.5, the corresponding image function is shown in Figure 6.6b. The source was localized at (2700/2350/2170)m and not at the true position with the coordinates (2500/2500/2500) m. The noise content increases drastically, which makes it impossible to recognize the source position unambiguously.

Kao and Shan (2004) proposed to solve the problem with the changing polarities by using the absolute values of the amplitudes. Baker et al. (2005) suggested using envelopes of the data to avoid the destructive summation of the amplitudes at the true source position. Envelope or analytical signal can be described by

\[ E(t) = \sqrt{g^2(t) + g_2(t)}, \quad (6.1) \]

where \( g(t) \) is a seismic trace and \( g_2(t) \) is a so-called quadrature trace of \( g(t) \), which is the imaginary part of the complex seismic trace and can be computed by Hilbert transformation to the complex seismic trace (Sheriff and Geldart (1995)). The envelope of the signal has only positive values, but it has a lower frequency content compared to the input signal.

The resulting image functions obtained with time collapsed imaging condition applied to the data with the signal-to-noise ratio of 2 using absolute values and envelopes are shown in Figure 6.7. Both results look alike. Despite the fact that the focal areas of the functions correspond to the true source location area, the values of the image function outside the focal area lay within 20% of the maximum value, which makes the localization not unique. This effect might be explained by the fact that we turn not only the signal amplitudes to positives but also the present noise. As a result, the noise does not stack destructively but constructively and thus, contaminates the image function. The synthetic example shows that even for the signal-to-noise ratio of 2 time collapsed diffraction stacking fails while using absolute values or envelopes of the signal.

Applying the maximum imaging condition to the absolute values and envelopes of the data to the seismogram with the signal-to-noise ratio of 2 leads to the more focused image functions with
Figure 6.5: (a) Horizontal slice for the data with SNR= 2 and (b) horizontal slice for the data with SNR= 0.5 through the image function obtained by time collapsed diffraction stacking.
Figure 6.6: (a) Horizontal slice for noise-free data and (b) represents a horizontal slice for the data with SNR= 0.5 through the image function obtained by maximum diffraction stacking.
Figure 6.7: Horizontal slices for (a) absolute values and (b) envelopes of the data with SNR= 2 through the image function obtained by time collapsed diffraction stacking.
an unambiguous maximum shown in Figure 6.8. Note that the presence of noise is visible in the image functions. The result for the envelopes of the data contains higher level of noise compared to the one for the absolute values. Unfortunately, this imaging condition does not provide reliable localization results for lower signal-to-noise ratios.

![Image](image.png)

Figure 6.8: Horizontal slices for (a) absolute values and (b) envelopes of the data with SNR= 2 through the normalized image functions obtained by applying maximum imaging condition

Afterward, the sliding time window imaging condition with $T/4$ shift was applied to both absolute values and envelopes of the signal ($T$ was computed for absolute values and envelopes under consideration of their frequency content), corresponding results are presented in Figure 6.9. The image functions are more focused than for the time collapse imaging condition and are less focused compared to the maximum one. The values around the focal area of the function

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differ from the maximum only within 30 – 35%, which might lead to ambiguous interpretations. Unfortunately, both applying absolute values or the envelopes of the data for the stack do not solve the problem of the localization of double-couple sources.

Figure 6.9: (a) Horizontal slice for the absolute values of the amplitudes and (b) presents the horizontal slice for the envelopes of the data with SNR= 2 obtained by applying moving window imaging condition with $T/4$ shift

6.2 Pairwise cross-correlation diffraction stacking

An alternative stacking-based procedure for localization of seismic sources which reduces the destructive summation effect of the radiation pattern in the image function (see Figure 6.4) is presented. This approach includes pairwise zero-lag cross-correlation of the signal within the
sliding time window. In other words, the signal of each two neighboring traces are zero-lag cross-correlated, which is a simple multiplication, within the sliding time window. For the right moveout of the computed travelt ime curve and the right excitation time we get the maximum values for the cross-correlation function. In the next step we stack the values of the cross-correlation function within the chosen time window, then the window is shifted through the whole seismogram. This is repeated for each image point. This approach exploits the fact that the form of the event arriving at different stations is similar and thus, correlating the desired signal within the appropriately chosen time window we amplify the desired signal and suppress the uncorrelated noise. Moreover, the cross-correlation solves the problem with the different polarities, as by cross-correlating the negative polarized amplitudes, positive values are obtained. As a result, the amplitudes do not cancel each other, but are amplified while stacking. The advantage of this method is that zero-lag cross-correlation is a simple multiplication and in case of a source type with different polarities, we would not get a low or even zero value at the source position like the above described approaches (see Figure 6.4).

Figure 6.10 illustrates the procedure. Figure 6.10a shows the seismogram for the strike-slip source, the blue curves represent the time window contours within which the stacking is applied. This time window has the moveout corresponding to the true source position at \((2500/2500/2500)\) m and the length of the time window is chosen according to the length of the signal so that the whole signal fits in within the time window. To visualize the process, the moveout correction according to the moveout of the blue traveltime curve was applied to the data (see Figure 6.10b. Then, each of 2 traces are correlated (multiplied with each other), the result can be seen in Figure 6.10c. As the signal form at each trace is similar, the multiplication along the correct travelt ime curve within the time window, leads to the strengthening of the signal and also it flips the negative polarities to the positive. The next step includes stacking the correlated amplitudes along the traveltime curve, the result is presented in Figure 6.10d. Finally, we collapse the time axis within the chosen time window and the out-coming value forms the value of the image function corresponding to the image point. This procedure is repeated for each image point. I denote this technique "pairwise cross-correlation diffraction stacking".

The suggested approach was applied to the strike-slip source data with the signal-to-noise ratio of 2 described above. The resulting image function can be seen in Figure 6.11. The maximum of the image function corresponds to the true source location and we do not have four maxima at wrong positions any more (compare with Figure 6.4). The image function is well focused. The
Figure 6.10: (a) shows the seismogram for the strike-slip source, the blue curves represent the time window contours within which the stacking is applied. This time window has the moveout corresponding to the true source position at (2500/2500/2500) m and the length of the time window is chosen according to the length of the signal so that the whole signal fits within the time window. (b) shows the result after the moveout correction according to the moveout of the blue traveltime curve. (c) represents the outcome of the pairwise zero-lag cross-correlation (multiplication) of each 2 neighboring traces within the chosen time window marked in blue. (d) shows the stacking result of the correlated amplitudes along the blue traveltime curve.
Figure 6.11: Sliding time window imaging condition with pairwise zero-lag cross-correlation diffraction stacking applied to the data with the SNR = 2 and peak frequency of 10 Hz. White arrows indicate shadows of a double-couple radiation pattern.

The focal area is smeared in the surroundings within the radius corresponding to half of the prevailing wavelength (\(\lambda/2\)). The vertical slice resembles the shadows of the originally double couple radiation pattern, indicated by white arrows in Figure 6.4.

The pairwise cross-correlation diffraction stacking was also applied to the data with the signal-to-noise ratio of 0.5 and the peak frequency of 10 Hz. The resulting horizontal slice though the maximum of the image function is shown in Figure 6.12. The maximum of the function is located one grid step away from the true source position. The noise effects become very clear and the focal area of the image function is asymmetrically smeared, but nevertheless the difference between the values of the maximum and the background noise outside the focal area is over 50%.
It was demonstrated that the stacking of data amplitudes fails to localize the source position. The suggestions by Kao and Shan (2004) to use the absolute values and by Baker et al. (2005) to use the envelopes of the signal do not lead to the reliable localization results in the presence of noise. Furthermore, the synthetic examples showed that the proposed pairwise cross-correlation diffraction stack with the sliding time window imaging condition solves the problem of the changing polarities for the double-couple source.

6.3 Compensated linear vector dipole source

Compensated linear vector dipole (CLVD) source types are typical for geothermal and volcanic areas (Stein and Wyssession (2007)). A CLVD source described by the following moment tensor was tested:

\[
\begin{pmatrix}
2 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{pmatrix}
\]

Synthetic dataset was created using the same acquisition geometry, velocity model and signal frequency like in Section 6.1. The source has also the coordinates of (2500/2500/2500) m. Resulting image function obtained by time collapsed diffraction stacking is shown in Figure 6.13. The image function partly resembles the radiation pattern of a CLVD source type (see Figure 3.10). The negative polarized part of the radiation pattern has more influence on the image as it is radiating directly upward. That is why the focal area of the image function resembles mainly
the form of the blue marked part in Figure 3.10. The maximum area of the image function is
stretched along the y-axis. Moreover, in the xz-plane it is narrower than in the yz, analogous to
the radiation of the negative polarized P-waves.

However, both negative and positive polarities are present, one of them is dominant. As a result,
time collapsed diffraction stacking leads to well focused image function at the true source position
and thus, reliable localization results. Maximum and sliding time window imaging condition
influence the focusing of the image function analogously to the explosive source shown in Section
4.5.4.

6.4 Mixed source type

6.4.1 Combination of an explosive (35%), CLVD (35%) and DC (30%)
components

As in the reality most seismic sources can be best explained by a combination of explosive, double-
couple and compensated linear vector dipole components (see e.g. Baig and Urbancic (2010)), a
source with 35% explosive, 35% CLVD and 30% double-couple was chosen. All three modes are
present in approximately equal portions. The corresponding moment tensor looks as follows:

\[
\begin{pmatrix}
2 & 1 & 0 \\
1 & 0.5 & 0 \\
0 & 0 & 0.5
\end{pmatrix}
= \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
+ \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
+ \begin{pmatrix}
1 & 0 & 0 \\
0 & -0.5 & 0 \\
0 & 0 & -0.5
\end{pmatrix}
\]

(6.2)

The first moment tensor after the equals sign represents the DC, the middle one - explosive
and the last one - CLVD component. Synthetic dataset was created using the same acquisition ge-
ometry, velocity model and signal frequency as in Section 6.1. The source has also the coordinates
of (2500/2500/2500) m. Slices through the resulting image function obtained by time collapsed
diffraction stacking in shown in Figure 6.14. As all source components (explosive, DC and CLVD)
are more or less balanced, the image function has its maximum at the true source position despite
the changing polarities. Note that the focal area is inclined even though the source is placed
symmetrically relative to the acquisition geometry. It means that the inclination of the focal area
is not only evoked by an asymmetrical source position relative to the acquisition, but also by a
source type as in our case a combination of three different radiation pattern components is given.
Figure 6.13: (a) Horizontal, (b) vertical slice along the x-axis and (c) represents a vertical slice along the y-axis through maximum of the normalized image function obtained applying time collapsed diffraction stacking for the source position (2500/2500/2500) m and CLVD source mechanism.
Figure 6.14: (a) Horizontal, (b) vertical slice along the x-axis and (c) vertical slice along the y-axis through maximum of the normalized image function obtained applying time collapsed diffraction stacking for the source position (2500/2500/2500) m. Source mechanism is constrained by 35% explosive, 35% CLVD and 30% double-couple components.
6.4.2 Combination of CLVD (30%) and DC (70%) components

An interesting issue represents a source type, where the double-couple component is dominating. That is why a source with 30% CLVD and 70% double-couple was chosen. The corresponding moment tensor looks as follows:

\[
\begin{pmatrix}
1 & 3 & 0 \\
3 & -0.5 & 0 \\
0 & 0 & -0.5 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 3 & 0 \\
3 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}
+ \begin{pmatrix}
1 & 0 & 0 \\
0 & -0.5 & 0 \\
0 & 0 & -0.5 \\
\end{pmatrix}
\]

The first moment tensor after the equals sign represents the DC and the second one - CLVD component. Assumed synthetic test conditions are described above, in the Section 6.1. At first, time collapsed diffraction stacking was applied to the data. The resulting horizontal slice through the maximum of the image function is shown in Figure 6.15a. As the DC-component is dominant, the image function resembles its radiation pattern (see Figure 3.8). Note that there are only two maxima present (compare with Figure 6.4). Furthermore, the focal area shrunk compared to the one for a pure double-couple source. This effect can be explained by the presence of a CLVD component. I chose to display two vertical slices along the angle bisectors between the x- and y-axes, one of them crosses both maxima and the other one is oriented perpendicularly to the first one. The bisector lines are shown in Figure 6.15b. The corresponding vertical slices are presented in Figure 6.16. In the vertical plane, the image function is as well more focused compared to the pure DC. This brings me to a conclusion, that a source mechanism constrained by the combination of a CLVD and DC component leads to a more focused image function. Unfortunately, time collapsed diffraction stacking does not lead to the correct location. Application of the maximum and sliding time window imaging conditions results in analogous images as in the pure DC case (see Section 6.1).

Next, pairwise cross-correlation diffraction stacking was applied to the data. Resulting image function is shown in Figure 6.17. The maximum of the image function corresponds to the true source position. The vertical slices resemble the shadows of the DC component, indicated by white arrows in Figure 6.17.

If a pure double-couple source or a mechanism with the dominant double-couple component is given, synthetic test have shown that conventional diffraction stacking fails to localize the event. Image function has low or zero values at the true source position surrounded by several maxima. On the contrary, pairwise cross-correlation diffraction stacking relaxes the problem and
Figure 6.15: a) Horizontal slice through the maximum of the normalized image function obtained applying time collapsed diffraction stacking. The source position is (2500/2500/2500) m. Two maxima are visible around the true source position.
Figure 6.16: Vertical slices through the image function along the a) magenta and b) white lines marked in Figure 6.15b. The images were obtained by applying time collapsed diffraction stacking. The source position is (2500/2500/2500) m, source mechanism is constrained by 30% CLVD and 70% DC mechanisms.
Figure 6.17: a) Horizontal slice, b) vertical slice along the magenta line and c) vertical slice along the white line marked in Figure 6.15b of the normalized image function obtained applying pairwise cross-correlation diffraction stacking. The source position is (2500/2500/2500) m, source mechanism is constrained by 30% CLVD and 70% DC mechanisms. White arrows indicate shadows of the double-couple component.
the maximum of the image function corresponds to the true source location. Moreover, it was illustrated that the inclination of the focal area is not only evoked by an asymmetrical source position relative to the acquisition, but also by a source type represented by a combination of explosive, DC and CLVD components. I have also shown that if we have a CLVD source type or a more or less balanced combination of the three modes, conventional diffraction stacking localization can locate these events.
Chapter 7

Field data application

This chapter illustrates the application of diffraction stacking localization method to seismological field data from Southern California.

7.1 Input data and acquisition geometry

Diffraction stacking-based localization was applied to seismological field data. For this purpose, vertical component seismograms were downloaded from the Southern California Earthquake Data Center database (SCEDC (2013)). The selected event occurred a few miles east-northeast of San Fernando, California with a magnitude $M_w$ of 4.05. According to the the scaling relation between the magnitude and the area of the rupture given in Equations 3.7 and 3.8 (Wells and Coppersmith (1994)), rupture width is 0.89 km and length is 1.93 km. Thus, the rupture area is $1.89 \times 10^3$ km$^2$.

The downloaded data includes 74 $z$-component waveforms in sac-format (see SAC (2009)) recorded by broadband stations. Each waveform had different time window limits. That is why, the waveforms had to be shifted according to the individual station delay. The resulting seismograms with normalized trace amplitudes by dividing each trace by its absolute maximum are shown in Figure 7.1. The sampling increment of the data is $\Delta t = 10$ milliseconds. The frequency spectrum of the data is displayed in Figure 7.2. The peak frequency of the data is 4 Hz. Thus, the signal duration $T$ equals to 0.25 seconds. The total duration of the recorded data accounts approximately 100 seconds. To remove the disturbing very low and high frequency components, the data were filtered by a bandpass filter with the limits of 1 and 15 Hz. The filtered seismograms are shown in Figure 7.3.

As already mentioned above, the dataset originated from Southern California. Figure 7.4 dis-
Figure 7.1: Resulting seismograms of the field data.

plays an excerpt of a world map, where the red rectangular indicates the area of interest. Figure 7.5 shows a map with the surface network. The area of about 25 km$^2$ is covered by just 74 receivers. The maximum receiver offset accounts for approximately 441 km. Receiver distribution is not only irregular but also extremely sparse. As a result, we get a problem of spatial under-sampling and weighting by areas of Voronoi cells will not solve the problem and therefore, was not applied. The Southern California Earthquake Data Center (SCEDC (2013)) applied conventional localization techniques requiring picking the phases and localized the event at the following position: $(34.289^\circ/ -118.403^\circ/8.4km)$. Distances between some receivers and the source are so large that we get almost horizontal ray paths there.


**7.2 Localization results**

For this experiment, a constant P-wave velocity of $6 \text{ km/s}$ was chosen after Baker et al. (2005). The peak frequency of the data is $4 \text{ Hz}$ and thus, the prevailing wavelength is $\lambda = \frac{6 \text{ km/s}}{4 \text{ Hz}} = 1.5 \text{ km}$. At first, localization process for an epicenter was conducted, whereas all receiver and image point positions were projected onto the surface. This procedure should give us an idea of the approximate position of the earthquake epicenter. The spatial discretization step was set to $0.5 \text{ km}$ in x- and y-directions, which corresponds to one third of the prevailing wavelength $\lambda$. At first, maximum imaging condition was applied and the resulting normalized image function can be seen in Figure 7.6a. Image function obtained by sliding time window imaging condition with the shift step of $T/4$ and pairwise cross-correlation diffraction stacking are shown in Figures 7.6b and 7.6c, respectively. The images resemble the irregular and sparse acquisition geometry. The first two image functions look very similar, whereas the last one has only similar acquisition geometry related artefacts. Cross-correlating the images results in an image function with a well defined maximum area (see Figure 7.7).

Further, I applied 3D localization procedures. In the horizontal plane the area in the following latitude and longitude range was thoroughly searched through: $(34.1^\circ : 34.5^\circ / -118.6^\circ : -118.3^\circ)$ (indicated by a black rectangular in Figure 7.5) and in the depth between 5 and 12 km. The discretization step of $0.2 \text{ km}$ in all directions was chosen. Velocity scanning over the range of $\pm 7.5\%$ variation of the homogenous velocity model was conducted. Afterward, pairwise zero-lag
cross-correlated images were stacked and formed the resulting image function.

First, maximum imaging condition was applied to the filtered data and the outcome is presented in Figure 7.8. Note that a distinct maximum area is visible. The vertical dimension of the focal area accounts about 2 km, which corresponds to $1.33\lambda$. In the horizontal plane the focal area has the dimensions of $1 \times 1.6$ km, which is comparable with the computed rupture width and length (see Section 7.1). The absolute maximum has the following coordinates: $(34.33^\circ / -118.478^\circ / 8.1$ km).

Next, sliding time window diffraction stacking was applied. The shifting step of 0.0625 seconds was chosen, which corresponds to approximately one fourth of the signal length. Resulting image function is shown in Figure 7.9. A distinct maximum area is also visible. There are also local maximum areas, one on them correlates with the maximum of the image function obtained by applying the maximum imaging condition. These local maxima can be aligned with the S-waves.
Figure 7.4: World map. Red rectangular represents the area of interest.

Figure 7.5: Map of Southern California. Red triangles represent receiver positions. Black rectangular, pointed at with a black arrow, represents an area within which it was thoroughly searched through with a discretization step of 0.2 km in all directions.
Figure 7.6: Image function obtained applying a) maximum and b) sliding time window with $T/4$ shift step imaging conditions, whereas c) was obtained by applying pairwise cross-correlation diffraction stacking.
arrivals. In other words, some P-wave traveltime curves partly coincide with S-wave moveout, leading to false positives. As it becomes clear from the seismogram shown in Figure 7.3, the S-wave arrivals have a larger amplitude compared to the P-waves. S-wave traveltime curves have a different moveout and thus, fitting the traveltime curves corresponding to the P-waves we cannot achieve the best fit of the curve and maximal arrivals. As a result, stacking of values along P-wave traveltime curves which are partly aligned with the S-waves, leads to an artificial maximum at the wrong position.

Application of the pairwise cross-correlation diffraction stacking leads to unambiguous localization results (see Figure 7.10). Maxima of the image function can be aligned to P- and S-wave amplitudes. Unfortunately, pairwise cross-correlation diffraction stacking fails to localize an event due to low number and sparse distribution of receivers.

7.2.1 Comparison of the results

To be able to compare the results, I picked manually P-wave traveltimes. In the next step, P-wave traveltimes for the homogeneous velocity model with $V_P = 6\text{km/s}$ corresponding to the localization result given by SCEDC were computed. Figure 7.11 shows the difference between the picked traveltimes and the computed traveltimes. The bigger the circle, the bigger the deviation is. Black triangles represent receiver positions, where the deviation was less than a sampling increment of $\Delta t = 0.01\text{ seconds}$. There are two outlier in the left upper corner, these are the receiver positions, for which unambiguous phase picking was not possible. The maximum deviation
Figure 7.8: (a) Horizontal and (b) vertical slice through the normalized image function obtained applying maximum imaging condition. Extension of the horizontal axis along which latitude values are plotted corresponds to approximately 44 km, whereas the extension of the axis along which longitude values are plotted is equivalent to approximately 28 km.
Figure 7.9: a) Horizontal and b) vertical slice through the maximum of the normalized image function obtained applying sliding time window imaging condition (shift step is $T/4$). Extension of the horizontal axis along which latitude values are plotted corresponds to approximately 44 km, whereas the extension of the axis along which longitude values are plotted is equivalent to approximately 28 km.
value accounts about 3.5 seconds. These large deviations may have been caused by the following factors: first, velocity uncertainties. Unfortunately, we do not know which velocity model was used and presence of a homogenous velocity model over such a large region is very unlikely. Let us assume a wave has to travel 400 km. If we set a velocity of the medium to 6 km/s, the wave needs around 66.67 seconds to reach the destination. If the assumed velocity model features a 5% variation to the true one, then the difference between the computed traveltine and the true traveltine would account about 3.3 seconds. Secondly, we also do not know how many stations were used to obtain the location, as it can be seen that the stations around the obtained source position feature hardly any traveltine difference. Another uncertainty factor is introduced by my lack of experience in manual phase picking.

Figure 7.12 presents the difference plot between the manually picked traveltimes and the computed traveltimes for the location obtained by the sliding time window imaging condition. The maximum value corresponds to 1.8 seconds. The event location by the sliding time window diffraction stacking is given by the following coordinates: (34.292°/ − 118.4680°/8.74km). It is approximately 0.33 km in x-direction, 6.03 km in y-direction and 0.34 km in z-direction away from the location obtained by SCEDC. Lower deviation value in x-direction compared to y-direction can be explained by a better illumination along x-axis. The average deviation accounts for 2.23 km, which corresponds to 0.5% of the maximum receiver offset.

In conclusion, I would like to point out that application the sliding time window diffraction
Figure 7.11: Differences between picked traveltimes and the computed traveltimes corresponding to the localization result given by SCEDC, traveltimes were computed for the homogeneous velocity model with $V_P = 6\text{km/s}$. Green star indicates the located source position. Size of each circle is proportional to the deviation: the bigger the circle, the bigger the deviation is. Circle colors do not imply any function except for a better distinction between the circle sizes. Black triangles represent receiver positions, where the deviation was less than a sampling increment of $\Delta t = 0.01\text{ seconds}$. There are two outlier in the left upper corner, these are the receiver positions, for which unambiguous phase picking was not possible.

Figure 7.12: Differences between picked traveltimes and the computed traveltimes corresponding to the localization result obtained by sliding time window diffraction stacking for the homogeneous velocity model with $V_P = 6\text{km/s}$. Green star indicates the located source position. Size of each circle is proportional to the deviation: the bigger the circle, the bigger the deviation is. Circle colors do not imply any function except for a better distinction between the circle sizes. Black triangles represent receiver positions, where the deviation was less than a sampling increment of $\Delta t = 0.01\text{ seconds}$. There are two outlier in the left upper corner, these are the receiver positions, for which unambiguous phase picking was not possible.
stacking technique led to unambiguous localization result with the deviation of less than 1% of the maximum receiver offset compared to the localization results obtained by conventional techniques requiring picking of the arrivals. The focusing of the image function and location can be improved if we use a more precise velocity model. Furthermore, applying diffraction stacking techniques involving both P- and S-waves at the same time would improve the image and localization result.
Chapter 8

Conclusions and Discussions

Passive seismic monitoring has great potential to provide an insight into characteristics of the subsurface structures, activated by either tectonic stress, volcanic or geothermal processes or well exploration and production. Especially, in exploration seismic, knowledge of the spatial dimensions of the fractures is extremely valuable in terms of effectiveness of the hydraulic fracturing and thus, inevitable for optimization of the field development and production. Stacking based localization techniques have proved themselves in the last few years. Without proper imaging conditions, accounting for acquisition geometry related artefacts and different source radiation patterns, they may lead to misinterpretations and thus, provide misinformation about the area of interest. In this thesis, I studied three key factors, which influence stacking-based localization results: various imaging conditions, acquisition geometries and radiation patterns.

At first, three different imaging conditions were examined. Time collapsed imaging condition performed reliably for signal-to-noise ratios down to 2. Unfortunately, presence of high level of noise (signal-to-noise ratio less than 2) makes it unstable, as the presence of migration noise in the image function leads to ambiguous results. On the contrary, maximum and sliding time window conditions introduced in this thesis proved to provide reliable localization results even for signal-to-noise ratio under 1. Maximum imaging condition leads to a better focused image function compared to the sliding time imaging condition, as only the maximum values of the waveforms contribute to the image function. Furthermore, robustness of three considered imaging conditions to velocity perturbations was studied. Synthetic tests have shown that the vertical resolution is more sensitive to velocity uncertainties due to the surface acquisition. The higher the variation of the velocity model, the higher location uncertainty is obtained. An approach of stacking the
images over the range of velocities was presented. It leads to less focused image function, but the maximum area corresponds to the true source location. It means that even if the velocity model is not precisely known, scanning over different velocities in a given range may solve the problem.

Surface acquisition with sparsely and/or irregularly distributed receivers leave a footprint on the image functions. Weighting the data according to the areas they represent results in a more focused image function if the signal-to-noise ratio is over 1. However, it also adds artefacts which may lead to false positives, or so-called artificial focusing. It compounds the problem of distinguishing between the artificial noise and the desired signal. For case of a signal-to-noise ratio lower than 1, weighting disturbs the image function by amplifying various directions and thus, localization for the events occurred in areas with a low receiver concentration fails. Furthermore, synthetic tests have revealed that the sliding time window imaging condition reduces the acquisition footprint caused by the sparse receiver distribution and leads to reliable localization results even if the signal-to-noise ratio is less than 1.

Furthermore, the importance of taking the source radiation pattern into account is inevitable for reliable localization. It was demonstrated that the stacking of data amplitudes fails to localize the source position of a pure double-couple source. The balanced positive and negative amplitudes add up to a zero value at the true source position. Existing suggestions to eliminate the effect of the radiation pattern by using absolute values or envelopes of the signal do not lead to the reliable localization results in the presence of noise. The pairwise cross-correlation diffraction stack with the sliding time window imaging condition was introduced. It was shown that it solves the problem of the changing polarities for the double-couple source. A compensated linear vector dipole source type, typical for geothermal and volcanic areas, was also considered. Synthetic results showed that in this case, conventional diffraction stacking provides reliable location. As in the reality, seismic events can be best characterized by a mixture of explosive, double-couple (DC) and compensated linear vector dipole (CLVD) components, localization of such sources was also examined. At first, a mixture constrained by 35% explosive, 35% CLVD and 35% DC source was considered. Localization results revealed that conventional diffraction stacking leads to focusing at the true source position. Also this example has shown that the inclination of the focal area is not only evoked by an asymmetrical source position relative to the acquisition, but also by a source type as in our case a combination of three different radiation pattern components is given. Another seismic source contained 30% of the CLVD and 70% of the DC components. This composition, where DC part is dominant, also challenged diffraction stacking and two maxima
occurred around the true source position. The application of the pairwise correlation diffraction stack within the sliding time window imaging condition to the data led to a well focused function with the maximum corresponding to the true source position.

Field data application demonstrates the necessity of the well constrained velocity model. Sliding time window imaging condition led to a localization result which is less than 1% of the maximum receiver offset away from the location yielded by California Earthquake Data Center.

8.1 Outlook

In this thesis I studied three key factors which influence the localization results obtained by stacking-based techniques. However, it is necessary to investigate the influence of other factors on localization results in the future. It needs to be studied how precisely velocity model should be known to be able to localize seismic events reliably. In other words, how well can we resolve our event if we simplify a complex overburden to a homogenous velocity model. Furthermore, the influence of the velocity model in terms of anisotropy on localization results has to be considered.

Irregular acquisition geometry, high presence of noise and other factors may lead to false positives in the image function. Research should be conducted to determine thresholds to distinguish the maximum corresponding to the desired event from the false positives in the image function.

Another possible future research topic might be to find out to what extent taking as well P- as S-waves into account might improve focusing of an image function and localization results. It should also be investigated if one should proceed diffraction stacking for P- and for S-waves separately or simultaneously to obtain better localization results.


Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Hamburg, den