Essays on the Interdependencies and Linkages between the Real Economy and Financial Markets

Interactions of Monetary and Fiscal Policy and Asset Prices in General Equilibrium Models

Universität Hamburg

Fakultät für Wirtschafts- und Sozialwissenschaften

Dissertation

Zur Erlangung der Würde der Doktorin/des Doktors der Wirtschafts- und Sozialwissenschaften

(gemäß der PromO vom 18. Januar 2017)

vorgelegt von

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aus Düsseldorf

Hamburg 2023

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Datum der Disputation: 24.05.2023

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List of Abbreviations

BoJ Bank of Japan

CARES Act Coronavirus Aid, Relief and Economic Security Act

DGP Data Generating Process

DSGE Model Dynamic Stochastic General Equilibrium Model

FRED Federal Reserve Economic Data

FTPL Fiscal Theory of the Price Level

FOC First-Order Condition

GMM Generalized Method of Moments

HJB Hamilton-Jacobi-Bellman Equation

IRF Impulse Response Function

IQR Inter-Quartile Range

MEF Martingale Estimation Function

NK Model New Keynesian Model

NK-AC Model New Keynesian Model with Capital Adjustmen Costs

NK-AC-FTPL Model New Keynesian Model with Capital Adjustmen Costs and FTPL

NK-FTPL Model New Keynesian Model with FTPL

PDE Partial Differential Equation

SDF Stochastic Discount Factor

TFP Total Factor Productivity

ZLB Zero Lower Bound of the Nominal Interest Rate

bp Basis Points

Synopsis

S.1 Motivation

We live in a time of pandemics, unprecedented large-scale fiscal stimulus packages, central banks that where for years constrained by the Zero Lower Bound (ZLB) of the nominal interest rate, sky-rocketing debt levels and a recent surge in inflation, with rates not seen since the 1980s. These examples highlight the importance of developing and evaluating New Keynesian (NK) models not from a predominantly monetary driven point of view, but to reconcile the close interplay and linkages of monetary policy with fiscal policy and asset prices. A prominent example is the inflation rate, which has been largely centered around monetary policy in macroeconomic models in the last decades. Our task as macroeconomists is the aggregation of economic and financial variables in a way that enables us to reconcile basic patterns and linkages in the data. Against this background, the main motivation for this doctoral thesis is the conceptualization of linkages between monetary policy, asset prices and fiscal policy.

A guiding theme throughout this dissertation is the link between the price of government bonds and macroeconomic aggregates. The asset pricing of government bonds plays a crucial role in the estimation of structural parameters of macroeconomic models with financial data in Chapter 1. The interplay of bond prices and monetary and fiscal policy has important implications for the dynamic of macroeconomic models in Chapters 2 and 3. Interdependencies of macroeconomic and financial variables are especially highlighted when deriving the asset pricing kernel in Chapter 1 and when computing the term structure of the interest rate in Chapter 2. While the first essay considers a general macro-finance framework, the other two essays focus on monetary and fiscal linkages and interdependencies in New Keynesian models.

The first part of this doctoral thesis is devoted to exploiting financial data in the estimation of structural parameters of macroeconomic models. The motivation for doing so are limited availability, publication lags and revisions of macro data. Financial data, backed by the rapid emergence of availability and data science, offer an additional source of information. As a consequence, it is difficult to believe that macroeconomists can lightly dismiss financial data, which potentially contains information on the state of the economy.

One of the most obvious linkages of macro and finance in dynamic stochastic general equilibrium (DSGE) models is the stochastic discount factor (SDF), which allows to price any asset consistently with macro dynamics. Thus, in a joint macro-finance framework, the dynamics of observed macroeconomic aggregates and financial data should be replicated by the economic model. Consequently, estimating the model with macro and/or finance data should always yield the same structural parameter values. This motivates the comparison and evaluation of structural parameter estimates in Chapter 1.

The second part of this doctoral thesis addresses the relationship between monetary and fiscal policy in the New Keynesian framework. The Fiscal Theory of the Price Level (FTPL) is a relatively simple but powerful macroeconomic modeling framework that establishes a close link of macro variables to the price of government bonds with arbitrary maturities. In particular, the determination of the price level follows from tight and explicit interactions of bond prices as well as monetary and fiscal policy. The central equation in the FTPL framework is a government debt valuation equation, which for a given price level asserts a value to the supply of government bonds. The real value of debt, in turn, is anchored by the expected real present value of future primary surpluses. Consequently, every asset pricing re-evaluation of existing government bonds instantaneously affects the real value of debt. A no-arbitrage condition relates bond prices along the whole maturity spectrum and at any point in time to the nominal interest rate. Thus, inflation dynamics are driven by a mix of fiscal and monetary policy as well as the maturity structure of debt. Especially the last feature motivates the analysis of the term structure of the interest rates in the second chapter of this doctoral thesis.

While policy analysis in Chapter 2 mostly occurs within the boundaries of existing continuous-time NK models with FTPL, Chapter 3 offers an extension to the NK-FTPL framework in the literature by introducing capital. An explicit evaluation of FTPL in continuous-time NK-models with capital is basically a blank spot in the existing literature. Thus, the third part of this doctoral thesis is devoted to fill this gap and offers an extensive description of model features and applications. The focus of this section is more on the macro- rather than the financial dimension of the model.

The following sections offer a short description of the design, the results and the main contributions of the three essays of this doctoral thesis.

S.2 Macro-Finance Linkages in Structural Estimation

The first chapter of this doctoral thesis is based on the paper "Structural Estimation of Dynamic Macroeconomic Models using Financial Data", which is joint work with Olaf Posch and Michel van der Wel. This essay suggest a general approach to estimate the structural parameters of macroeconomic models using financial data. Starting from a class of dynamic equilibrium models, the essay shows how to derive the stochastic discount factor, which is used to price financial assets consistently with macroeconomic dynamics. Subsequently, the essay processes finance and macro variables into estimation equations that are used in different combinations in the estimation of the structural parameters. After evaluating various systems of macro and finance estimation equation in a simulation study, the essay builds on these insights in the empirical estimation. The latter exploits treasury bonds, macro variables, the S&P500 stock index as well corresponding future data.

A central contribution of this essay is the formulation of a general approach to estimate structural parameters for a class of dynamic equilibrium models using macro and finance data. Even though, analytical solutions are not required for this approach, a simple macroeconomic model with available close-form solution is utilized in order to illustrate each step. The corresponding simulation study suggest that one can in principal estimate all structural parameters from finance data alone. Thus, the paper contributes new insights on the feasibility and the benefits and drawbacks of using finance data in the structural estimation of macroeconomic models. As it turns out, using finance data either as a substitute or a complement improves parameter identification and increases the accuracy of the estimates. This holds especially true when using only first moments in the estimation. It is well established in the literature that the estimation of the considered Vasicek interest rate specification yields one biased parameter value. Against this background, another notably contribution of this essay is a nearly complete bias correction (in absolute terms) by simply using stock or future data in the estimation. In summary, the essay highlights that structural parameter estimation of macro models can benefit from exploiting different financial asset classes. This applies even in case of small and simple models without explicitly incorporated financial sector.

S.3 FTPL and the Maturity Structure of Government Debt

The underlying research paper to the second essay of this doctoral thesis is called "FTPL and the maturity structure of government debt in the New-Keynesian Model", which is a joint work with Olaf Posch. This essay revisits FTPL within the NK framework (see e.g. Sims (2011), Leeper and Leith (2016), Cochrane (2018) or Cochrane (2022b)). The focus is on the importance of the maturity of government bonds for macro dynamics. Considering fiscal and monetary policy shocks, this chapter emphasizes the analysis of model-implied expectations, the term structure of interest rate and transmission channels. After the theoretical evaluation, the essay takes the model to the data and addresses the US Coronavirus Aid, Relief and Economic Security (CARES) Act.

A minor contribution of this essay is a translation of the discrete-time FTPL and debt maturity analysis of Leeper and Leith (2016) into a continuous-time framework. This formulation allows for a more clear-cut analysis of maturity effects because inflation dynamics are partly driven by a pure asset pricing channel. While the existing FTPL literature mostly focuses on transitory shocks, this chapter contributes an elaborate evaluation of fully or partly permanent shocks. Furthermore, in contrast to existing simple NK-FTPL models in the literature, surplus rules are implemented as "Fiscal Taylor Rules" in the spirit of Kliem and Kriwoluzky (2014) and Kliem et al. (2016). By splitting primary surpluses into taxes and government expenditures, the essay contributes a novel evaluation of the CARES Act through the lens of FTPL. In particular, it quantifies the effects of the large-scale stimulus package and discusses conditions for a surge in inflation. Further examples are offered in terms of combinations of transitory and permanent shock components. Finally, this chapter contributes theoretical evaluations of the term-structure of interest rates and model-implied inflation expectation in the NK-FTPL framework with different maturities of government bonds.

S.4 Capital in the NK-FTPL Framework

The final essay of this doctoral thesis is based on my paper "The Fiscal Theory of the Price Level in New Keynesian Models with Capital". I start from the simple NK-FTPL framework of chapter 2 and introduce capital and capital adjustment costs. I show that in the absence of capital adjustment costs, the dynamics of the capital rental rate and the real interest rate coincide (cf. Dupor (2001) or Posch and Wang (2020)). This is a continuous-time specific feature. As a consequence, contractionary monetary policy shocks are expansionary and increase output and inflation. After introducing capital adjustment costs as a remedy, I evaluate model dynamics, determinacy conditions and tackle two puzzles in the literature.

Since continuous-time NK-FTPL models with capital are little covered in the existing literature, filling this gap is a central contribution of this essay. Thus, by developing and analyzing this novel framework, the essay contributes to the literature in various dimensions. The proposed model offers an important benchmark framework, as it allows analyzing interactions of fiscal and monetary policy, government debt, investments as well as capital, in a joint, simple and consistent framework. The essay presents an elaborate discussion on determinacy, transmission channels as well as the conceptual and theoretical underpinning of NK models with capital and FTPL. One of the main contributions of this essay is a novel approach to explicitly utilize FTPL in order to solve two economic puzzles. The first puzzle is the Crowding-In Consumption Puzzle, which refers to a mismatch of theoretically predicted and actual observed responses of consumption to changes in government expenditures. The essay shows that FTPL allows for either a crowding-in or a crowding-out effect of consumption. At the same time, the essay further contributes a consistent evaluation of the responses of investments. The model predicts a (at least temporary) crowding-in of investments. The second puzzle is the prediction of expansionary effects of capital destruction at the ZLB in the standard NK framework. Again, the NK model with FTPL and capital adjustment costs is able to explain either an expansionary or a contractionary output response. Thus, the essay contributes an explicit modeling framework to evaluate the puzzling behavior of NK models at the ZLB and offers a novel evaluation of the Great East Japan Earthquake of 2011 (*Tōhoku Earthquake*). Finally, FTPL models in the literature usually rely on long-term bonds in order to obtain a negative relationship between the nominal interest and the inflation rate. This essay contributes a new way to obtain the desired relationship with short-term bonds.

Chapter 1

Structural Estimation of Dynamic Macroeconomic Models using Financial Data

with Olaf Posch and Michel van der Wel

Abstract

In this paper we show how financial data can be used in a combined macro-finance framework to estimate the underlying structural parameters. For this purpose, we introduce a general estimation approach that is applicable to a whole class of macroeconomic models and translates them into systems of macro and finance estimation equations. Our formulation allows for consistently substituting macro variables by asset prices in a way that enables casting the relevant estimation equations partly (or completely) in terms of financial data. We illustrate the approach with a model specification with analytical solutions. We show that all structural parameters can basically be estimated from finance data alone, and discuss benefits and drawbacks of substituting and adding financial variables. In our simulation study, we find that financial data can improve the identification and accuracy of the parameter estimates. In the empirical application we use treasury bonds, macro variables, the S&P500 stock index as well corresponding future data. Our findings highlight that substituting and complementing macroeconomic variables by asset prices is not only feasible but in some cases also preferable. We achieve the best performance from a combination of bond, output and S&P500 data.

1.1 Introduction

The most vital macroeconomic data, such as output, consumption or inflation data, only appear with time lags and are subject to substantial revisions. This raises the question, whether readily available financial data is able to offer insights into the state of macroeconomic aggregates, and if so, how to exploit these information. Closely related to this question, is an important lesson learned from the financial crisis 2007/2008: The need for a joint framework, which overcomes the traditional separation of macroeconomics and finance. A large literature developing models at the intersection of macro and finance has emerged in which the asset pricing kernel is consistent with the macroeconomic dynamics (e.g. Rudebusch and Wu (2008), Rudebusch and Swanson (2016), Gürkaynak and Wright (2012), Joslin et al. (2014), Bauer and Rudebusch (2020)).[1] Although most of these papers do focus on the interaction of macro variables, fiscal and monetary policy, and their implications for the term structure of interest rates, an open question remains to what extent financial data eventually can be useful to complement or replace macroeconomic variables in structural estimation.

In this paper we shed light on how macroeconomic variables and financial data can be linked and what do we learn by connecting asset returns to macroeconomics. In a joint macro-finance framework both dynamics of observed aggregate and financial data should be replicated by the economic model. An important step in model comparison and evaluation is the estimation of its structural parameters. Against this background, we exploit the asset pricing implications of a macro-finance model in order to cast the relevant estimation equations partly or completely in terms of financial data. This allows us to estimate the structural parameters using financial data, macroeconomic data, and/or a combination of these. Our motivation for doing so is that macro data, in contrast to financial data, are usually available at lower frequencies, appear with significant publications lag and often with large measurement errors, which can result into poor data quality. These difficulties become further intensified by macro data being subject to substantial revisions. Finally, important variables, such as the aggregate capital stock or say the output gap are highly controversial objects, heavily debated and are not easily comparable across countries. Given the rapid emergence of data availability and data science, it is hard to imagine that macroeconomists can safely ignore financial data as an additional source of information. At the same time, for many applications it is important to keep the analysis linked to economic theory. In fact, macro-finance is typically understood as being the link between economic fluctuations and asset prices. Thus, given the relatively high volatility of financial data, we investigate the informational content of empirical financial

[1] There is a tradition of using the stochastic discount factor, implied by macro models, to derive asset prices that are consistent with macro dynamics (e.g. Cochrane (2005), Hansen and Scheinkman (2009), Christensen et al. (2016), Christensen (2017).

data. For this purpose, we utilize interest rates data as well as the prices of bonds, stocks and futures for the identification of parameters of the underlying macroeconomic model. Our aim is to provide new insights into the use of financial data in simple macro-finance models, and to derive implications for the estimation of more elaborated models.

Our approach to answer whether financial data is useful to complement (or replace) macroeconomic data is as follows. We introduce a generalized formulation that closely relates to Parra-Alvarez et al. (2021) and is applicable to a wide class of dynamic stochastic equilibrium models. This formulation allows representing macroeconomic variables in the form of a system of second-order partial differential equations (PDEs). Having found the solution of the model, the stochastic discount factor, i.e., the unique asset pricing kernel, can be used to consistently price any financial asset. The resulting system of macro and finance variables can then be utilized to estimate the structural parameters of the original DSGE model. We illustrate this approach by turning to a model specification with analytical solutions, so that each step remains fully traceable and comprehensible. It is important to stress, however, that analytical solutions are not required and the approach can readily be applied to elaborate models. After defining various financial claims, we compute their price dynamics and cast the model's equilibrium dynamics in terms of structural parameters as well as observed financial data alone, or combined with macro data. We then estimate the structural parameters of the model with different specifications and different types of financial data. We study the effects on parameter estimates both in a simulation study and empirically, using interest rate and macro as well as S&P500 stock index and future data. Our results, obtained from simulations and empirical estimations, indicate that using a combined macro-finance framework not only improves the identification of structural parameters, but also increases the accuracy of the estimates. Thus, our results indicate that financial data can be a useful substitute or complement. This holds especially true when macro variables remain unobserved, are only available at low frequencies or have a poor quality (e.g., due to revisions and publication lags). In some macro-finance formulations with stock or future data, the accuracy of estimates increases in a way that a widely known upward bias in one of the parameter of the Vasicek interest rate specification basically vanishes in absolute terms. Thus, there is no need for bias corrections as proposed by Tang and Chen (2009). All in all, we achieve the best performance from a macro-finance combination of bond, output and S&P500 data.

Structural parameter estimation is an ubiquitous topic in the empirical DSGE literature. While macroeconomic models become more complex, more elaborate methods are necessary to retrieve their structural parameters. What ones started by simple calibrations, along the lines of Kydland and Prescott (1982), now takes the form of highly sophisticated econometric methods. Our research questions focus less on the methodological but more on the data side. Although likelihood-based methods are common to estimate, we suggest to use Generalized Method of Moments (GMM) and Martingale Estimation Function (MEF).[2]

We borrow the methodological framework of Christensen et al. (2016), who show how to estimate the structural parameters of a stochastic AK model from a system of stochastic differential equations using bond, output and consumption data. Starting from our general framework, we use their model and show that alternative combinations of macro and financial estimation equations can improve identification and accuracy of the parameter estimates. We further deviate from Christensen et al. (2016) by comparing bootstrapped measures of dispersion (confidence intervals, medians and IQRs), which allows a in-depth comparison of empirical estimates. Our paper also relates to Tang and Chen (2009), who evaluate biased parameter estimates in diffusion processes. In particular, they analyze the Vasicek interest rate specification and show why estimates of the drift parameter κ, which measures the speed of mean reversion, can be upward biased by more than 200%. They highlight that a lack of interest rate dynamics (small values of κ) induce higher upward biases. As a remedy, they develop a parametric bootstrap procedure that corrects the bias. In our framework, we obtain comparable estimation results by utilizing combinations of interest rate, consumption and stock or future data. Even though, we still estimate a biased κ, there is a strong accuracy gain, which basically removes the bias in absolute terms.

Our strategy is conceptually similar to the one pursued by Guerrón-Quintana (2010), who analyses the effects of estimating DSGE models with different combinations of macroeconomic observables. In a first step Guerrón-Quintana (2010) uses a macroeconomic data set in order to estimate the structural parameters of a medium-scale DSGE model. In a second step, he uses the same data set to re-estimate the parameters, but always drops one of the variables at a time. The results indicate that the exclusion of certain variables can cause highly biased parameter estimates and even reverse model implications. Boivin and Giannoni (2006) also emphasize the role of data and highlight that the choice and especially the number of different macroeconomic time series can improve both accuracy and inference. Their study highlights that in some instances the use of comparably smaller data sets precludes the extraction of all relevant information. As suggested by our paper, smaller models, where only a limited number of macroeconomic time series are empirically observable, may profit from exploiting additional financial time series. Liew and Vassalou (2000) analyze the informational content of financial data on macroeconomic aggregates. Their study indicates that financial data can offer significant information on the growth rate of future GDP. Finally, Fernández-Villaverde and Rubio-Ramírez (2007) analyze whether structural parameters are indeed invariant to interventions, or if there is a drift in parameter values. For this purpose, they re-estimate a medium-scale DSGE

[2]See Fernández-Villaverde and Guerrón-Quintana (2020) for a discussion of recent advantages in the estimation of DSGE models, including fields like machine learning and tempered particle filters.

model several times, each time allowing one parameter at a time to drift. They conclude that there are substantial drifts in certain parameters throughout their sample period from 1955 to 2000, which casts doubt on their structural nature in the spirit of Hurwicz (1966). The sample size for macroeconomic variables is limited by the start and the end of the coverage as well as the frequency (e.g. annual or quarterly). When turning instead to financial data with a higher frequency (e.g. monthly or daily), one can increase the sample size without the need to increase the considered period length. This may provide sufficient observations to estimate the structural parameters for a comparably shorter period of time and for smaller sub-samples. This in turn may circumvent or at least unveil the problem of parameter drifts.

Our paper contributes to different fields in the literature. We propose a readily applicable approach to estimate structural parameters of macroeconomic models with financial data, and contribute new insights on benefits, drawbacks as well as the feasibility of substituting and complementing macro with finance data. We find that finance data can improve accuracy and identification, which contributes to the discussion on the informational content of financial data on macroeconomic aggregates. We also contribute to the literature on bias corrections of the mean-reverting Vasicek interest rate specification. Finally, we highlight that structural estimation can benefit from exploiting financial data even in case of small-scale macroeconomic models without explicitly modeled financial sector.

The rest of the paper is structured as follows. Section 1.2 describes the framework, derives the general equilibrium prices for different financial claims and outlines the systems of equilibrium equations that we use in the estimation. In section 1.3 we describe the estimation method and before turning to the empirical estimation, we conduct a simulation study to analyze the effects of using financial data in addition or as substitute in the parameter estimation. Section 1.4 concludes.

1.2 The Model

In what follows we present a class of dynamic equilibrium models. We show how to derive the stochastic discount factors and how to price financial assets consistently with macroeconomic dynamics. For reasons of clarity our approach will be illustrated by using analytical solutions. We do so to highlight the underlying mechanisms and to keep track of each step in processing the model for estimation. Hence, close-form analytical solutions are not required for the suggested approach and can readily be extended to more elaborate models.

1.2.1 Solution

Consider the class of dynamic equilibrium economies that can be represented in the form of a system of second-order partial differential equations (PDEs), which covers a wide range of dynamic equilibrium models and is studied (and solved) in Parra-Alvarez et al. (2021),

$$\mathcal{H}\left(\mathbf{x},\mathbf{y},\mathbf{y_x},\mathbf{y_{xx}};\phi\right)=\mathbf{0}, \tag{1.1}$$

where $\mathbf{x}$ is an $n_x \times 1$ vector of state variables, $\mathbf{y}$ is an $n_y \times 1$ unknown vector of endogenous variables, $\mathbf{y_x}$ is the $n_y \times n_x$ Jacobian matrix and $\mathbf{y_{xx}}$ is an $n_y \times n_x^2$ array of second-order derivatives, and ϕ is the $n_\phi \times 1$ vector of structural parameters of the model.[3]

The functional operator $\mathcal{H} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_y \times n_x} \times \mathbb{R}^{n_y \times n_x^2} \mapsto \mathbb{R}^{n_y}$ stacks the second-order PDEs associated with the endogenous variables. The dynamics of the state vector $\mathbf{x}$ can be compactly represented by the following system of controlled SDEs

$$d\mathbf{x}=\mathbf{b}\left(\mathbf{x},\mathbf{u}\right)dt+\boldsymbol{\sigma}\left(\mathbf{x}\right)d\mathbf{w}, \quad \mathbf{x}(0)=\mathbf{x}_0 \text{ given}, \tag{1.2}$$

where $\mathbf{b}$ is the $n_x \times 1$ drift vector, $\mathbf{u}$ the vector-valued function of controls $\mathbf{u}=\mathcal{U}(\mathbf{x},\mathbf{y},\mathbf{y_x})$ with $\mathcal{U} : \mathbb{R}^{n_x \times n_y \times n_y \times n_x} \mapsto \mathbb{R}^{n_u}$, and $\boldsymbol{\sigma}$ is the $n_x \times n_x$ diffusion matrix coupled with an $n_x \times 1$ vector of standard (uncorrelated) Brownian motions $\mathbf{w}$ such that $\boldsymbol{\Sigma}\left(\mathbf{x}\right)=\boldsymbol{\sigma}\left(\mathbf{x}\right)\boldsymbol{\sigma}\left(\mathbf{x}\right)^\top$ is the state vector's $n_x \times n_x$ variance-covariance matrix.

For notational convenience, we may focus on the family of rational expectations models where (1.1) takes the form of a system of second-order *quasilinear* PDEs

$$\mathcal{H}\left(\mathbf{x},\mathbf{y},\mathbf{y_x},\mathbf{y_{xx}};\phi\right):=\mathbf{a}\left(\mathbf{x},\mathbf{y},\mathbf{y_x}\right)+\mathbf{y_x}\mathbf{b}\left(\mathbf{x},\mathbf{u}\right)+\mathbf{y_{xx}}\mathbf{c}\left(\mathbf{x}\right), \tag{1.3}$$

in which $\mathbf{y}$ is an $n_y \times 1$ unknown vector of *only* costate variables, $\mathbf{a}$ is the $n_y \times 1$ vector, $\mathbf{b}$ is the $n_x \times 1$ drift vector of the state variables in (1.2), and $\mathbf{c}$ is an $n_x^2 \times 1$ vector associated with the variance-covariance matrix $\boldsymbol{\Sigma}$. Both systems (1.1) and (1.3) summarize the necessary conditions for optimality from the Hamilton-Jacobi-Bellman (HJB) equation.

The solution of the model in (1.1) is given by a set of policy functions

$$\mathbf{y}=\mathbf{g}\left(\mathbf{x};\phi\right), \tag{1.4}$$

where the vector-valued function $\mathbf{g} : \mathbb{R}^{n_x} \mapsto \mathbb{R}^{n_y}$ reduces (1.1) to an identity. Note that this function also determines $\mathbf{y_x}=\mathbf{g_x}\left(\mathbf{x};\phi\right)$ and $\mathbf{y_{xx}}=\mathbf{g_{xx}}\left(\mathbf{x};\phi\right)$.

[3] This simplified approach makes use of the optimality conditions and substitutes control variables from the problem formulation (cf. Parra-Alvarez et al., 2021).

1.2.2 Asset Pricing

Given the existence of an asset pricing kernel or stochastic discount factor consistent with the macro dynamics, we either use the general equilibrium shadow price $\mathbf{y}_i = \mathbf{g}_i(\mathbf{x};\phi)$ with $i \in \{1,...,n_y\}$ or $i \in \{\mathbf{x}, 1, ..., n_y, 1, ..., n_x\}$, or use $\mathbf{u}_j = \mathcal{U}_j(\mathbf{x}, \mathbf{y}, \mathbf{y_x})$ with $j \in \{1, ..., n_u\}$ and the first-order condition to compute the costate variable. Suppose that $\Lambda_t \equiv e^{-\rho t}\mathbf{y}_i$ such that the SDF follows

$$d\Lambda_t = -\rho e^{-\rho t}\mathbf{y}_i dt + e^{-\rho t} d\mathbf{y}_i, \tag{1.5}$$

where

$$\begin{aligned} d\mathbf{y}_i &= \mathbf{g}_{i\mathbf{x}}(\mathbf{x};\phi)\, d\mathbf{x} + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top \mathbf{g}_{i\mathbf{xx}}(\mathbf{x},\phi)^\top \\ &= \mathbf{g}_{i\mathbf{x}}(\mathbf{x};\phi)\left(\mathbf{b}(\mathbf{x},\mathbf{u})\, dt + \boldsymbol{\sigma}(\mathbf{x})\, d\mathbf{w}\right) + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top \mathbf{g}_{i\mathbf{xx}}(\mathbf{x},\phi)^\top dt \\ &= \left(\mathbf{g}_{i\mathbf{x}}(\mathbf{x};\phi)\,\mathbf{b}(\mathbf{x},\mathbf{u}) + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top \mathbf{g}_{i\mathbf{xx}}(\mathbf{x},\phi)^\top\right) dt + \mathbf{g}_{i\mathbf{x}}(\mathbf{x};\phi)\,\boldsymbol{\sigma}(\mathbf{x})\, d\mathbf{w} \end{aligned}$$

in which $\mathbf{g}_{i\mathbf{x}}$ denotes the $1 \times n_x$ Jacobian matrix belonging to $\mathbf{y}_i$, and $\mathbf{g}_{i\mathbf{xx}}$ the ith component in the $n_y \times n_x^2$ dimensional array of second-order derivatives.[4]

Applying conditional expectation we obtain

$$-\frac{1}{dt}E_t\left[\frac{d\Lambda_t}{\Lambda_t}\right] \equiv r_t^f, \tag{1.6}$$

together with the fundamental pricing equation for assets

$$E_t[d\mathbf{p}] - r_t^f \mathbf{p} dt = -E_t\left[d\mathbf{p}\frac{d\Lambda_t}{\Lambda_t}\right] \tag{1.7}$$

or more generally

$$E_t[d(\Lambda_t \mathbf{p})] + \Lambda_t \mathcal{D}(\mathbf{x},\mathbf{y})dt = 0 \tag{1.8}$$

when including dividend payments $\mathcal{D}$. To find the general equilibrium asset price we solve the resulting PDE, similar to finding the option price based on the celebrated Black-Scholes formula (henceforth *PDE approach*). Alternatively, we compute (e.g., Cochrane, 2005)

$$\mathbf{p} = E_t\left[\frac{\Lambda_s}{\Lambda_t}\mathcal{P}(\mathbf{x},\mathbf{y};s)\right] + E_t\left[\int_t^s \frac{\Lambda_u}{\Lambda_t}\mathcal{D}(\mathbf{x},\mathbf{y};u)\right] du, \quad s > t \tag{1.9}$$

that is for $s > t$ the pricing equation states that the equilibrium prices $\mathbf{p}$ of assets at time t is given by the conditional expectation of the product of the SDF and the future payoff $\mathcal{P}(\mathbf{x},\mathbf{y};s)$ and/or dividends $\mathcal{D}(\mathbf{x},\mathbf{y};s)$ (henceforth *expectations approach*). In this paper

[4] The vec operator transforms a matrix into a vector by stacking the columns of the matrix one underneath the other (cf. Magnus and Neudecker, 2019, p.34).

we focus on specific assets, where (1.9) is available analytically. Our results shed light on the potential efficiency gains for more elaborated models, where the numerical solution to the PDE implies the equilibrium prices $\mathbf{p} = \mathbf{h}(\mathbf{x}; \phi)$ as a function of the state variables.

1.2.3 Macro-Finance Framework

Steps in estimating given macro-finance models and initial ϕ_0, and possibly loop over i

1. Compute the solution $\mathbf{y} = \mathbf{g}(\mathbf{x}; \phi_i)$ in (1.4) to satisfy $\mathcal{H}(\mathbf{x}, \mathbf{y}, \mathbf{y}_{\mathbf{x}}, \mathbf{y}_{\mathbf{xx}}; \phi_i) = 0$
2. Compute the asset prices $\mathbf{p} = \mathbf{h}(\mathbf{x}; \phi_i)$ in (1.9)
3. Estimate parameters $\hat{\phi}_{i+1}$ from macro and financial data

In our applications below, where the model specifications allow for analytical solutions, it turns out that the most promising asset class are stock indices. While derivatives appear somewhat preferable over stock indices in the simulation study, there are in most cases no real-world analogues, or the data availability is limited, especially when considering periods prior to the year 2000. Hence, stocks are our preferred asset class in the structural estimation below. Because this conclusion might change with better availability of (high-frequency) financial data or in more elaborated models, below we will present a step-by-step derivations for different asset classes.

1.2.4 An Example: AK-Vasicek Model

To illustrate our estimation approach, we choose the AK-Vasicek model. In a nutshell, the model is as follows. At each instance in time, output Y_t is generated by a simple AK technology, which combines capital with the level of productivity:

$$Y_t = A_t K_t, \tag{1.10}$$

where K_t denotes the aggregate capital stock and A_t total factor productivity (TFP). We abstract from labor market dynamics for analytical tractability. Including hours worked does not pose a conceptional problem, but prevents us from obtaining analytical insights. In this economy TFP is driven by a standard Brownian motion, B_t, with $\mu(A_t)$ representing the generic drift- and $\eta(A_t)$ the generic volatility function:

$$dA_t = \mu(A_t)dt + \eta(A_t)dB_t. \tag{1.11}$$

Our specification of the macro model (1.10), allows us to link the macro model to the seminal finance models of the interest rate process. In general equilibrium, capital is rewarded by its marginal product $r_t \equiv Y_K = A_t$, so we may write $\eta(A_t) = \eta(r_t)$, and

$\mu(A_t) = \mu(r_t)$ interchangeably. In the Vasicek model, we specify $\mu(A_t) = \kappa(\gamma - A_t)$ and $\eta(A_t) = \eta$,

$$dr_t = \kappa(\gamma - r_t)dt + \eta dB_t. \tag{1.12}$$

Despite its simplicity, the one-factor model (1.12) is commonly used in finance, because if offers analytical calculations of the asset prices (Tang and Chen, 2009; Posch, 2009), hence, making it a natural starting point for our analysis.

If gross investments, I_t, are higher than capital depreciation, K_t increases according to

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t,$$

with σ measuring the standard deviation the stochastic depreciation shocks, δ representing the mean depreciation rate, and Z_t being another standard Brownian motion. The goods market clearing condition determines investment in general equilibrium, $I_t = Y_t - C_t$.

In this economy, households are represented by a representative consumer who exhibits additive separable utility and maximizes expected life time utility

$$U_0 \equiv E_0 \int_0^\infty e^{-\rho t} u(C_t) dt, \quad \text{where} \quad u' > 0,\ u'' < 0, \tag{1.13}$$

subject to

$$dK_t = ((r_t - \delta)K_t - C_t)dt + \sigma K_t dZ_t. \tag{1.14}$$

We restrict our focus to cases, where the aggregate capital stock cannot be observed by the econometrician or simply is not used in the estimation. Instead, in the AK model the capital stock can be recovered by observing macroeconomic and financial data, $K_t = Y_t/r_t$.

As shown in Christensen et al. (2016), the Euler equation is a necessary condition for optimality, and together with the output and interest rate dynamics recovers the capital stock. Thus our point of departure for structural estimation reads

$$\begin{aligned} d\ln C_t &= \left(\frac{u'(C_t)(\rho - r_t + \delta)}{u''(C_t)C_t} - \frac{C_K K_t}{C_t}\sigma^2 - \tfrac{1}{2}\frac{C_A^2 \eta(r_t)^2 + C_K^2 K_t^2 \sigma^2}{C_t^2}\frac{u'''(C_t)C_t + u''(C_t)}{u''(C_t)}\right)dt \\ &\quad + \frac{C_r \eta(r_t)}{C_t}dB_t + \frac{C_K \sigma K_t}{C_t}dZ_t \qquad \text{(1.15a)} \\ d\ln Y_t &= \left(\frac{\mu(r_t)}{r_t} + r_t - \delta - \frac{C_t}{K_t}\right)dt - \tfrac{1}{2}\frac{\eta(r_t)^2}{r_t^2}dt - \tfrac{1}{2}\sigma^2 dt + \frac{\eta(r_t)}{r_t}dB_t + \sigma dZ_t \qquad \text{(1.15b)} \\ dr_t &= \mu(r_t)dt + \eta(r_t)dB_t. \qquad \text{(1.15c)} \end{aligned}$$

For admissible instantaneous utility functions in (1.13) and interest rate dynamics, we obtain consumption $C_t = C(K_t, A_t)$, with derivatives $C_K = C_K(K_t, A_t)$ and $C_A = C_A(K_t, A_t)$. The simplest case is a linear approximation to the policy function, but also higher-order local approximations or global solutions can be used. Even for a linear approximation of the policy function, our approach is to preserve the nonlinear equilibrium dynamics for identification and casts the system in terms of macroeconomic variables and the interest rate. In what follows, we use (observable) macro and potentially higher-frequency financial data for the estimation of structural parameters, summarized by the $n_\phi \times 1$-vector ϕ.

To start with, the system in (1.15) is a subset of variables that can or need to be linked to the relevant state or to other observed variables. In what follows we show how the different asset classes can be used to complement or even to replace observed macroeconomic variables. Furthermore, we use the model specification to derive an alternative expression for the rental rate r_t in terms of observable variables r_t^f and model parameters. In our applications, we are using different combinations of financial and macro data to cast the relevant estimation equations partly and completely in terms of financial data.

Proposition 1 *For the case of logarithmic utility $u(C_t) = \ln C_t$, the AK-Vasicek model with $\mu(r_t) = \kappa(\gamma - r_t)$ and $\eta(r_t) = \eta$ implies $C_t = C(K_t, A_t) = \rho K_t$.*

Proof. See Appendix A.2 in Posch (2009) and Christensen et al. (2016). ■

Corollary 2 *For log-utility, the equilibrium dynamics in (1.15) simplify to*

$$
\begin{aligned}
d\ln C_t &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \sigma dZ_t && (1.16a)\\
d\ln Y_t &= \left(\kappa\gamma/r_t - \tfrac{1}{2}\eta^2/r_t^2 + r_t - \kappa - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \eta/r_t dB_t + \sigma dZ_t && (1.16b)\\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, && (1.16c)
\end{aligned}
$$

and the general equilibrium shadow price reads $\Lambda_t = e^{-\rho t}u'(C_t) = e^{-\rho t}/C_t$.

The general equilibrium SDF is easily obtained from the Euler equation (1.16a) and reads

$$
\Lambda_s/\Lambda_t = e^{-\int_t^s (r_v - \delta - \frac{1}{2}\sigma^2)dv - \sigma \int_t^s dZ_v}, \quad s > t \tag{1.17}
$$

in which (1.6) implies

$$
r_t^f \equiv r_t - \delta - \sigma^2. \tag{1.18}
$$

Even if the rental rate of capital r_t was not directly observable, it can be linked to an observed proxy for the risk-free rate (typically the 3-month yield of zero-coupon bonds). For simplicity, in this paper we neglect inflation dynamics, which should be included when considering the yield curve (cf. Posch and Van der Wel 2022).

Hence, the $n_\phi = 6$ structural parameters in the AK-Vasicek model (with log-utility) are

$$
\phi = (\kappa, \gamma, \eta, \rho, \delta, \sigma)^\top. \tag{1.19}
$$

Our macro-finance framework introduced in Section 1.2.3 is illustrated by using an AK model (macro) combined with the one-factor Vasicek specification (finance) for the interest rate. We estimate the structural parameters by following the suggested three-step procedure.

Step 1: Compute the Solution $\mathbf{y} = \mathbf{g}(\mathbf{x}; \phi_i)$ in (1.4)

It is known that the AK model with the Vasicek specification features a closed-form solution and we obtain an analytical expression for $\mathbf{y} = \mathbf{g} : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ in (1.4), in the simplified version with $n_y = 1$ relevant costate and its derivatives $\mathbf{y_x} = \mathbf{g_x}(\mathbf{x}; \phi)$ and $\mathbf{y_{xx}} = \mathbf{g_{xx}}(\mathbf{x}; \phi)$.

More compactly, the AK-Vasicek model has the state vector $\mathbf{x} = [A_t, K_t]^\top$ with $n_x = 2$, which from (1.11) and (1.14) follows

$$d\mathbf{x} = \begin{pmatrix} \kappa(\gamma - A_t) \\ A_t K_t - C_t - \delta K_t \end{pmatrix} dt + \begin{pmatrix} \eta & 0 \\ 0 & \sigma K_t \end{pmatrix} d\mathbf{w}, \quad \mathbf{x}(0) = \mathbf{x}_0 \text{ given,}$$

such that

$$\mathbf{b}(\mathbf{x}, \mathbf{u}) \equiv \begin{bmatrix} \kappa(\gamma - A_t) \\ A_t K_t - C_t - \delta K_t \end{bmatrix}, \quad \boldsymbol{\sigma}(\mathbf{x}) \equiv \begin{bmatrix} \eta & 0 \\ 0 & \sigma K_t \end{bmatrix},$$

and

$$\boldsymbol{\Sigma} = \boldsymbol{\sigma}(\mathbf{x}) \boldsymbol{\sigma}(\mathbf{x})^\top = \begin{bmatrix} \eta^2 & 0 \\ 0 & \sigma^2 K_t^2 \end{bmatrix},$$

in which the control $\mathbf{u}(\mathbf{x}, \mathbf{y}, \mathbf{y_x}) = C_t$ follows from the first-order condition.

In an expected utility framework, it is often sufficient to solve for $\mathbf{y} = V_K$ as the relevant costate variable, without explicitly computing the value function, because $C_t = \mathbf{y}^{-1}$, such that the model can be formalized as in (1.3) with $n_x \neq n_y \equiv 1$, and

$$\begin{aligned} \mathbf{y_x} &= \begin{bmatrix} \partial_A \mathbf{y} & \partial_K \mathbf{y} \end{bmatrix} = \begin{bmatrix} V_{KA} & V_{KK} \end{bmatrix} \\ \mathbf{y_{xx}} &= \begin{bmatrix} \partial_A \mathbf{y_x} & \partial_K \mathbf{y_x} \end{bmatrix} = \begin{bmatrix} V_{KAA} & V_{KKA} & V_{KAK} & V_{KKK} \end{bmatrix}. \end{aligned}$$

From the maximized HJB, the costate V_K must satisfy

$$\begin{aligned} \rho V_K &= ((A_t - \delta)K_t - C_t)V_{KK} + (A_t - \delta)V_K + \tfrac{1}{2}(\eta^2 V_{KAA} + \sigma^2 K_t^2 V_{KKK}) \\ &\quad + \sigma^2 K_t V_{KK} + \kappa(\gamma - A_t)V_{KA} \end{aligned}$$

or compactly,

$$\mathcal{H}(\mathbf{x}, \mathbf{y}, \mathbf{y_x}, \mathbf{y_{xx}}) = -\rho \mathbf{y} + \mathbf{a}(\mathbf{x}, \mathbf{y}, \mathbf{y_x}) + \mathbf{y_x} \mathbf{b}(\mathbf{x}, \mathbf{u}) + \mathbf{y_{xx}} \mathbf{c}(\mathbf{x}, \mathbf{y})$$

hence,

$$\mathcal{H}\left(\mathbf{x},\mathbf{y},\mathbf{y}_{\mathbf{x}},\mathbf{y}_{\mathbf{xx}}\right) := \mathbf{a}\left(\mathbf{x},\mathbf{y},\mathbf{y}_{\mathbf{x}}\right) + \mathbf{y}_{\mathbf{x}}\mathbf{b}\left(\mathbf{x},\mathbf{u}\right) + \mathbf{y}_{\mathbf{xx}}\mathbf{c}\left(\mathbf{x}\right),$$

where

$$\mathbf{a}\left(\mathbf{x},\mathbf{y},\mathbf{y}_{\mathbf{x}}\right) \equiv \left[\begin{array}{c} (\mathbf{x}_{11}-\delta)\mathbf{y}_{21} + \mathbf{y}_{\mathbf{x}}\sigma^2\mathbf{x}_{21} \end{array}\right], \quad \mathbf{b}\left(\mathbf{x},\mathbf{u}\right) \equiv \left[\begin{array}{c} \kappa(\gamma-\mathbf{x}_{11}) \\ (\mathbf{x}_{11}-\delta)\mathbf{x}_{21}-\mathbf{u} \end{array}\right],$$

and

$$\mathbf{c}\left(\mathbf{x}\right) \equiv \left[\begin{array}{cccc} \frac{1}{2}\eta^2 & 0 & 0 & \frac{1}{2}\sigma^2\mathbf{x}_{21}^2 \end{array}\right]^\top,$$

which is a (simplified) second-order quasilinear PDE.

Summarizing, from Proposition 1, the AK-Vasicek model (log-utility) implies $C_t = \rho K_t$ and the costate variable satisfies $V_K(A_t, K_t) = K_t^{-1}/\rho$ (cf. Posch, 2009). Hence,

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) = \left[\begin{array}{c} (\mathbf{x}_{21})^{-1}/\rho \end{array}\right]$$

together with

$$\begin{aligned} \mathbf{y}_{\mathbf{x}} &= \left[\begin{array}{cc} 0 & -(\mathbf{x}_{21})^{-2}/\rho \end{array}\right] \\ \mathbf{y}_{\mathbf{xx}} &= \left[\begin{array}{cccc} 0 & 0 & 0 & 2(\mathbf{x}_{21})^{-3}/\rho \end{array}\right] \end{aligned}$$

reduces $\mathcal{H}(\mathbf{x},\mathbf{y},\mathbf{y}_{\mathbf{x}},\mathbf{y}_{\mathbf{xx}}) = 0$ to an identity.

Step 2: Compute the Asset Prices $\mathbf{p} = \mathbf{h}\left(\mathbf{x};\phi_i\right)$ in (1.9)

As argued above, $\mathbf{y} \equiv \mathbf{y}_i = (\mathbf{x}_{21})^{-1}/\rho$ denotes the general equilibrium shadow price, such that $\Lambda_t = e^{-\rho t}\mathbf{y} = e^{-\rho t}(\mathbf{x}_{21})^{-1}/\rho$ and the SDF dynamics (1.5) are

$$d\Lambda_t = -\rho e^{-\rho t}\mathbf{y}dt + e^{-\rho t}d\mathbf{y},$$

where

$$d\mathbf{y} \;\; = \;\; \left(\mathbf{g}_{\mathbf{x}}\left(\mathbf{x};\phi\right)\mathbf{b}\left(\mathbf{x},\mathbf{u}\right) + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top\mathbf{g}_{\mathbf{xx}}(\mathbf{x},\phi)^\top\right)dt + \mathbf{g}_{\mathbf{x}}\left(\mathbf{x};\phi\right)\boldsymbol{\sigma}\left(\mathbf{x}\right)d\mathbf{w}$$

in which $\mathbf{g}_{\mathbf{x}}$ with $n_y \equiv 1$ denotes the $1\times n_x$ Jacobian matrix, and $\mathbf{g}_{\mathbf{xx}}$ the $1\times n_x^2$ dimensional array of second-order derivatives. In particular, the AK-Vasicek model implies

$$\mathbf{g}_{\mathbf{x}} = \left[\begin{array}{cc} 0 & -(\mathbf{x}_{21})^{-2}/\rho \end{array}\right],$$

which is the Jacobian matrix of $\mathbf{y}$, and

$$\mathbf{g}_{\mathbf{xx}} = \left[\begin{array}{cccc} 0 & 0 & 0 & 2(\mathbf{x}_{21})^{-3}/\rho \end{array}\right]$$

the $1 \times n_x^2$ dimensional array of second-order derivatives. Hence,

$$
\begin{aligned}
d\Lambda_t &= -\rho e^{-\rho t}\mathbf{y}dt + e^{-\rho t}\left(\mathbf{g}_\mathbf{x}(\mathbf{x};\phi)\,\mathbf{b}(\mathbf{x},\mathbf{u}) + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top \mathbf{g}_{\mathbf{xx}}(\mathbf{x},\phi)^\top\right)dt \\
&\quad + e^{-\rho t}\mathbf{g}_\mathbf{x}(\mathbf{x};\phi)\,\boldsymbol{\sigma}(\mathbf{x})\,d\mathbf{w} \\
&= -\rho e^{-\rho t}\mathbf{y}dt + e^{-\rho t}\begin{bmatrix} 0 & -(\mathbf{x}_{21})^{-2}/\rho \end{bmatrix}\begin{bmatrix} \kappa(\gamma - \mathbf{x}_{11}) \\ (\mathbf{x}_{11} - \delta)\mathbf{x}_{21} - \mathbf{u} \end{bmatrix} dt \\
&\quad + e^{-\rho t}\tfrac{1}{2}\begin{bmatrix} \eta^2 & 0 & 0 & \sigma^2\mathbf{x}_{21}^2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & 2(\mathbf{x}_{21})^{-3}/\rho \end{bmatrix}^\top dt \\
&\quad + e^{-\rho t}\begin{bmatrix} 0 & -(\mathbf{x}_{21})^{-2}/\rho \end{bmatrix}\begin{bmatrix} \eta & 0 \\ 0 & \sigma\mathbf{x}_{21} \end{bmatrix} d\mathbf{w} \\
&= -\rho\Lambda_t dt - \Lambda_t(A_t - \rho - \delta)dt + \sigma^2\Lambda_t dt - \Lambda_t\sigma dZ_t \\
&= -(A_t - \delta - \sigma^2)\Lambda_t dt - \Lambda_t\sigma dZ_t. \qquad (1.20)
\end{aligned}
$$

Below, we consider different classes of assets (bonds, stocks, futures) for which analytical solutions are available in order to illustrate our approach. It is shown that the AK-Vasicek model implies analytical solutions for a rich class of assets.

Proposition 3 (Bonds) *If an asset j pays continuously at r_t^f (floating rate note),*

$$\mathcal{P}_f(\mathbf{x},\mathbf{y};s) = 0, \quad \mathcal{D}_f(\mathbf{x},\mathbf{y};u) = e^{\int_t^s r_v^f dv},$$

then $\mathbf{p}_f = 1$. In contrast, a bond at $t = 0$ with unity payoff at maturity s (zero-coupon bond),

$$\mathcal{P}_b(\mathbf{x},\mathbf{y};s) = 1, \quad \mathcal{D}_b(\mathbf{x},\mathbf{y};u) = 0,$$

implies for given s

$$\mathbf{p}_b^{(s)} \equiv P_{b,t}^{(s)} = \mathbf{h}_b(\mathbf{x};\phi) = \exp\left(\mathcal{A}(s) - \mathcal{B}(s)\mathbf{x}_{11}\right)$$

where

$$\mathcal{A}(s) = -\left(\gamma - \delta - \sigma^2 - \frac{\eta^2}{2\kappa^2}\right)s + \left(\gamma - \frac{\eta^2}{2\kappa^2}\right)\mathcal{B}(s) - \frac{\eta^2}{4\kappa}\mathcal{B}^2(s), \quad \mathcal{B}(s) = \frac{1 - e^{-\kappa s}}{\kappa}.$$

such that the limiting case of maturity $s \to 0$ approaching zero yields $\mathbf{p}_b^{(0)} = 1$. See section 1.A.1 for a derivation of $\mathcal{A}$ and $\mathcal{B}$.

Proof. From (1.9) together with the SDF in (1.20), we get

$$
\begin{aligned}
\mathbf{p}_f &= E_t\left[\frac{\Lambda_s}{\Lambda_t}\mathcal{D}_f(\mathbf{x},\mathbf{y};u)\right] \\
&= E_t\left[e^{-\int_t^s (r_v - \delta - \frac{1}{2}\sigma^2)dv - \sigma\int_t^s dZ_v}\,{}^{\int_t^s (r_v - \delta - \sigma^2)ds}\right] = 1
\end{aligned}
$$

For the price of the zero-coupon bond $\mathbf{p}_b$ see Posch and Van der Wel (2022). ■

Proposition 4 (Stocks) *Consider a claim on the future dividends (stock market), with $s \to \infty$ (this is equivalent to a claim on the tree, not only the fruits), where*

$$\mathcal{P}_d(\mathbf{x}, \mathbf{y}; s) = 0, \quad \mathcal{D}_d(\mathbf{x}, \mathbf{y}; u) = Y_u$$

has an analytical price

$$\mathbf{p}_d \equiv P_{d,t} = \mathbf{h}_d\left(\mathbf{x}; \phi\right) = \mathbf{x}_{21}\left[\frac{\mathbf{x}_{11} - \gamma}{\rho + \kappa} + \frac{\gamma}{\rho}\right]$$

Proof. Appendix 1.A.2 ■

Proposition 5 (Futures) *Consider future contracts on the underlying state variables (future market), such as the claim on the capital stock or a future on output,*

$$\mathcal{P}_K(\mathbf{x}, \mathbf{y}; s) = K_s \quad \textit{or} \quad \mathcal{P}_Y(\mathbf{x}, \mathbf{y}; s) = Y_s,$$

or derivatives (derivatives market), such as the future contract on the stock price, where

$$\mathcal{P}_F(\mathbf{x}, \mathbf{y}; s) = P_{d,s}, \quad \mathcal{D}_F(\mathbf{x}, \mathbf{y}; u) = 0.$$

These assets have analytical prices $\mathbf{p}_j = \mathbf{h}_j\left(\mathbf{x}; \phi\right)$ for $j = \{K, Y, F\}$, i.e.,

$$\begin{aligned}
\mathbf{p}_K &= \mathbf{x}_{21} e^{-\rho(s-t)}, \\
\mathbf{p}_Y &= \mathbf{x}_{21}\left[(\mathbf{x}_{11} - \gamma)e^{-(\rho+\kappa)(s-t)} + \gamma e^{-\rho(s-t)}\right], \\
\mathbf{p}_F &= \mathbf{x}_{21}\left[\frac{(\mathbf{x}_{11} - \gamma)e^{-(\rho+\kappa)(s-t)}}{\rho + \kappa} + \frac{\gamma}{\rho} e^{-\rho(s-t)}\right].
\end{aligned}$$

Proof. Appendix 1.A.3 ■

Moreover, we are not restricted how many financial asset are being used in the structural estimation because $\mathbf{p}$ may contain alternative assets at the same time such that the dimension of $\mathbf{h}\left(\mathbf{x}; \phi_i\right)$ is $n_p \equiv \dim(\mathbf{p})$. Here, the set of assets primarily is restricted by the availability of data, and by the requirement of a clear mapping to the underlying model.

In general, we obtain for the prices of financial assets

$$d\mathbf{p} \;=\; \left(\mathbf{h}_{\mathbf{x}}\left(\mathbf{x}; \phi\right) \mathbf{b}\left(\mathbf{x}, \mathbf{u}\right) + \tfrac{1}{2}\mathrm{vec}(\mathbf{\Sigma}(\mathbf{x}))^\top \mathbf{h}_{\mathbf{xx}}(\mathbf{x}, \phi)^\top\right) dt + \mathbf{h}_{\mathbf{x}}\left(\mathbf{x}; \phi\right) \boldsymbol{\sigma}\left(\mathbf{x}\right) d\mathbf{w}$$

in which $\mathbf{h}_{\mathbf{x}}$ denotes the $n_p \times n_x$ Jacobian matrix, and $\mathbf{h}_{\mathbf{xx}}$ the $n_p \times n_x^2$ dimensional array of second-order derivatives.

Step 3: Estimate Parameters $\hat{\phi}_{i+1}$ from Macro and Financial Data

Below we construct martingale increments $\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i)$ using the solution $\mathbf{y} = \mathbf{g}(\mathbf{x}; \phi_i)$ and asset prices $\mathbf{p} = \mathbf{h}(\mathbf{x}; \phi_i)$ in terms of parameters and data (cf. Christensen et al., 2016). In most cases, a log-transformation, $\ln(\mathbf{y})$ and $\ln(\mathbf{p})$, forms the basis for constructing $\mathbf{m}$.

In general, we need Itô's formula to obtain n_m increments $d\mathbf{f}(\mathbf{x}, \mathbf{g}(\mathbf{x}; \phi), \mathbf{h}(\mathbf{x}; \phi))$, where $n_m \equiv \dim(\mathbf{m})$, from the transformation $\mathbf{f} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_p} \mapsto \mathbb{R}^{n_m}$ and integrate both sides of the equation from $t - \Delta$ through t, possibly solve components of the system analytically. We then define the $n_m \equiv \dim(\mathbf{m})$ martingale increments

$$\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i) \equiv \boldsymbol{\varepsilon}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i) \tag{1.21}$$

in which

$$\boldsymbol{\varepsilon}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i) = \left[\mathbf{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{y}, \mathbf{p}) + \mathbf{f}_{\mathbf{y}}(\mathbf{x}, \mathbf{y}, \mathbf{p})\, \mathbf{g}_{\mathbf{x}}(\mathbf{x}; \phi_i) + \mathbf{f}_{\mathbf{p}}(\mathbf{x}, \mathbf{y}, \mathbf{p})\, \mathbf{h}_{\mathbf{x}}(\mathbf{x}; \phi_i)\right] \boldsymbol{\sigma}(\mathbf{x})\, d\mathbf{w} \tag{1.22}$$

and where $\mathbf{f}_{\mathbf{x}}$ is the $n_m \times n_x$, $\mathbf{f}_{\mathbf{y}}$ the $n_m \times n_y$, and $\mathbf{f}_{\mathbf{p}}$ the $n_m \times n_p$ Jacobian matrix, respectively. The martingale increments can be written in terms of data and parameters as

$$\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i) = E_{t-\Delta}\left[d\mathbf{f}(\mathbf{x}, \mathbf{g}(\mathbf{x}; \phi_i), \mathbf{h}(\mathbf{x}; \phi_i))\right] \tag{1.23}$$

and we estimate the unknown parameters $\hat{\phi}_{i+1}$ based on macro and financial data.

For example, treating the triple (C_t, Y_t, r_t^f) in the AK-Vasicek model as being observable (see Christensen et al. (2016)), the transformation readily reads

$$\mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p}, \phi_i) = \begin{bmatrix} \ln \mathbf{y}^{-1} & \ln(\mathbf{x}_{11}\mathbf{x}_{21}) & \mathbf{x}_{11} - \delta - \sigma^2 \end{bmatrix}^\top .$$

such that

$$\boldsymbol{\varepsilon}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i) = \begin{pmatrix} \varepsilon_{C,t} \\ \varepsilon_{Y,t} \\ \varepsilon_{r,t} \end{pmatrix} = \begin{pmatrix} \sigma(Z_t - Z_{t-\Delta}) \\ \int_{t-\Delta}^{t} \frac{\eta}{(r_v^f + \delta + \sigma^2)} dB_v + \sigma(Z_t - Z_{t-\Delta}) \\ \eta e^{-\kappa\Delta} \int_{t-\Delta}^{t} e^{\kappa(v-(t-\Delta))} dB_v \end{pmatrix} \tag{1.24}$$

or written in terms of data and parameters

$$\mathbf{m}(\mathbf{x},\mathbf{y},\mathbf{p};\phi_i) \equiv m_t = \begin{pmatrix} \ln(C_t/C_{t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta \\ \\ \ln(Y_t/Y_{t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\kappa + \rho - \frac{1}{2}\sigma^2)\Delta \\ -\gamma\kappa \int_{t-\Delta}^{t} \left(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma\right)^{-1} dv \\ +\frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^{t} (\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^{-2} dv \\ \\ r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \end{pmatrix}.$$

In this paper, we suggest to use financial data to replace macro data, e.g., to replace output by stock market data, treating the triple $(C_t, P_{d,t}, r_t^f)$ as being observable,

$$\mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{p},\phi_i) = \begin{bmatrix} \ln \mathbf{y}^{-1} & \ln \mathbf{p}_d & \mathbf{x}_{11} - \delta - \sigma^2 \end{bmatrix}^\top.$$

such that

$$\boldsymbol{\varepsilon}(\mathbf{x},\mathbf{y},\mathbf{p};\phi_i) = \begin{pmatrix} \varepsilon_{C,t} \\ \varepsilon_{P_{d,t}} \\ \varepsilon_{r,t} \end{pmatrix} = \begin{pmatrix} \sigma(Z_t - Z_{t-\Delta}) \\ \int_{t-\Delta}^{t} \frac{\rho\eta}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} dB_v + \sigma(Z_t - Z_{t-\Delta}) \\ \eta e^{-\kappa\Delta} \int_{t-\Delta}^{t} e^{\kappa(v-(t-\Delta))} dB_v \end{pmatrix} \tag{1.25}$$

or written in terms of data and parameters

$$\mathbf{m}(\mathbf{x},\mathbf{y},\mathbf{p};\phi_i) \equiv m_t = \begin{pmatrix} \ln(C_t/C_{t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta \\ \\ \ln(P_{d,t}/P_{d,t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta \\ -\rho\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma}\right) dv \\ +\frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^{t} \frac{1}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} dv \\ \\ r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \end{pmatrix}.$$

Alternatively, we replace consumption by stock market data, treating the triple $(P_{d,t}, Y_t, r_t^f)$ in the AK-Vasicek model as being observable (see Christensen et al. (2016)), the transformation readily reads

$$\mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{p},\phi_i) = \begin{bmatrix} \ln \mathbf{p}_d & \ln(\mathbf{x}_{11}\mathbf{x}_{21}) & \mathbf{x}_{11} - \delta - \sigma^2 \end{bmatrix}^\top.$$

such that

$$\boldsymbol{\varepsilon}(\mathbf{x},\mathbf{y},\mathbf{p};\phi_i) = \begin{pmatrix} \varepsilon_{P_{d,t}} \\ \varepsilon_{Y,t} \\ \varepsilon_{r,t} \end{pmatrix} = \begin{pmatrix} \int_{t-\Delta}^{t} \frac{\rho\eta}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} dB_v + \sigma(Z_t - Z_{t-\Delta}) \\ \int_{t-\Delta}^{t} \frac{\eta}{(r_v^f + \delta + \sigma^2)} dZ_v + \sigma(Z_t - Z_{t-\Delta}) \\ \eta e^{-\kappa\Delta} \int_{t-\Delta}^{t} e^{\kappa(v-(t-\Delta))} dB_v \end{pmatrix} \tag{1.26}$$

or written in terms of data and parameters

$$\mathbf{m}(\mathbf{x},\mathbf{y},\mathbf{p};\phi_i) \equiv m_t = \begin{pmatrix} \ln(P_{d,t}/P_{d,t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta \\ -\rho\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f+\delta+\sigma^2)+\kappa\gamma}\right) dv \\ +\frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^{t} \frac{1}{(\rho(r_v^f+\delta+\sigma^2)+\kappa\gamma)^2} dv \\ \\ \ln(Y_t/Y_{t-\Delta}) - \int_{t-\Delta}^{t} r_v^f dv + (\kappa + \rho - \frac{1}{2}\sigma^2)\Delta \\ -\gamma\kappa \int_{t-\Delta}^{t} \left(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma\right)^{-1} dv \\ +\frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^{t} (\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^{-2} dv \\ \\ r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \end{pmatrix}.$$

In the above examples up to five parameters are identified from the martingale property of the estimation equations. To identify all structural parameters one can exploit the variances of the residuals of the martingale increments in equation (1.21) as separate moments. Alternatively, one can use MEF estimation techniques in order to identify the sixth structural parameter without turning to higher moment conditions. In this case, identification follows from the conditional variances as well as the conditional mean parameter derivatives of the martingale increments, which are used as optimally chosen weight matrices (instruments) in estimation. However, it is important to keep in mind that error terms are not necessarily traceable in more elaborate models. As a consequence, one has to rely on first moments only. In such cases the inclusion of additional estimation equations (additional data) may offers a possibility to identify more parameters. Regarding MEF estimation, untraceable error terms are especially troublesome because one does not know the parameter derivatives. Thus, while MEF turns out to be the preferable estimation technique in our framework, one has to keep in mind that its implementation in more general frameworks may be limited.

We choose the above examples to illustrate the steps that are needed to derive the different model setups. In section 1.A.4 in the appendix, we provide the corresponding equilibrium equations for all considered variables. In the subsequent section 1.A.5, we show how to construct the corresponding martingale increments. Building on this, we prepare the different model setups for estimation.

1.3 Estimation

In this section, we show how the different model setups of the previous section can be used to estimate the six structural parameters ϕ in (1.19) of the AK-Vasicek model (log-utility). Posch and Van der Wel (2022) show how to estimate the AK-Vasicek model using the term structure of interest rate. In this paper, we follow the short-cut and proxy the short rate by the 3-month rate in order to study a broader class of financial assets including stock market and derivatives market data (cf. Christensen et al., 2016).

After introducing the estimation methods, we proceed by conducting a simulation study for all considered setups and offer some rationale for bringing the model to the data. Building on these insights, we then turn to the empirical estimation.

1.3.1 Estimation Method

We focus on two alternative estimation methods. As already pointed out in the introduction, these are GMM and MEF estimation techniques. We adopt both methods from Christensen et al. (2016) so that we only sketch the basic ideas of the estimation approaches and refer the reader to Christensen et al. (2016) for a detailed description.

For the GMM estimation approach we start from the vector of martingale increments, $\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i)$, that is given by the model equations and stated in terms of parameters and data. As common in practice, we consider instruments that belong to the information set and, consequently, are known at time $t - \Delta$. In particular, we choose lagged variables from the vector of martingale increments in order to obtain instruments that contain both data and parameters. We are considering a two-stage GMM approach where we select the identity matrix as positive definite weighting matrix in the first stage. Using the error terms and the estimator from the first stage, we derive a new weighting matrix and use it in the second stage estimation.

In contrast to the GMM approach, the instruments enter in form of a matrix when using (optimal) MEF estimation techniques. One obtains the optimal weighting matrix from the conditional variances and the conditional mean parameter derivatives of the martingale increments. Using the weighting matrix and the vector of martingale increments, $\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i)$, we compute the martingale estimation function. The parameter vector is then estimated by finding the combination where the martingale estimation equation is equal to zero.

Utilizing financial claims turns out to be more demanding than using the risk-free rate or macro variables. This is particularly true for the computation of the conditional covariance matrix in the MEF estimation. Because iterating over the covariance matrix seems computationally cumbersome, we turn in some instances to a simplified two-stage MEF approach, as described in Christensen et al. (2016). The computational difficulties of using financial claims in the optimal MEF approach are less severe at a monthly frequency. However, they become more evident for daily frequencies, because the increased sample size drastically increases the dimensions of the conditional covariance matrix.

In our empirical estimation, for some specifications, asymptotic t-statistics tend to diverge and in most cases produce high values. Hence, we apply a bootstrapping method, which yields more stable and more plausible values that are inline with the insights from our simulation study. Since the true data generating process in the empirical application is unknown, we exploit the sieve bootstrap method (see e.g. Bühlmann (1997), Kreiss and Lahiri (2012), Meyer and Kreiss (2015)). The central idea of this approach is to fit a high-order VAR(p) (with p large) to the data. This higher-order model then allows us to generate the bootstrapped data. Thus, the high-order VAR is treated as an approximation to the unknown true model. In settings where we utilize daily financial data together with monthly macro data, the estimation is based on the lower frequency of macro data and we approximate integrals over daily variables by Riemann sums. Thus, we do not bootstrapped mixed-frequency data, but the lower-frequency variables including those constructed from higher-frequency data. The central drawback of using monthly financial data in the bootstrapped estimation is a comparably smaller sample size and consequently fewer structural parameters appearing in the estimation functions. However, the simulation study suggest that there is essentially no difference when directly using monthly data. Further note that our bootstrap approach preserves the linear correlations of macroeconomic and financial aggregates.[5] In our empirical application we consider VAR(30) models (hence, 30 lags) to approximate the true data generating process.

After estimating the VAR model parameters on the data, we construct bootstrapped pseudo-data sets using residuals from the VAR model. We start by initializing the pseudo-data set with a random draw from the set of observations. Then we draw (with replacement) a vector of residuals from the high-order VAR process and use it together with the fitted parameters of the VAR model to construct the successive values of the pseudo-depended variables. We repeat this procedure until we obtain a whole time series for these variables. Using this approach we compute 550 bootstrapped pseudo-data sets and each time estimate the structural parameters of the model with as input the bootstrapped time series. We store the corresponding estimates and obtain bootstrapped probability density functions for all structural parameters. We then use these bootstrapped densities to compute medians, interquartile ranges and 95% confidence intervals. We chose this simple measures of dispersion because the bootstrapped probability density functions are in most cases non-normal and heavily skewed. An important reason for this behavior are non-negativity restrictions that we use in the empirical estimation. Consequently, bootstrapped density functions become truncated in some cases, which results in a disproportionate accumulation of mass on the left-hand side. In order to compute bootstrapped 95% confidence intervals, we apply the percentile method (Efron (1982)). For this purpose, we use the bootstrapped distribution of parameter estimates and define the confidence interval as the lower and upper limit corresponding to the 2.5th and 97.5th

[5] An alternative approach would be fitting independent high-order AR(ρ) processes to the actual data. While this preserves the frequencies of the data, one would lose the macro-finance correlations. When one intends to directly work with mixed frequencies, a promising approach is the use of mixed-frequency VARs (e.g. Ghysels (2016) or Götz et al. (2016)) or a block-bootstrap approach (e.g. Lahiri (1999) or Andrews (2004)). We leave mixed-frequency bootstrapping as an interesting separate direction for further research.

percentiles. While some bootstrapped distributions are heavily skewed and have relatively large outliers, we use the interquartile range as additional measure of dispersion.

Note that starting with the financial crisis, the nominal interest rate hits the zero lower bound. This evolution is also reflected in our risk-free rate proxy but not in all bootstrapped samples. In some samples, however, the ZLB poses difficulties for our bootstrap method, since the counterfactual risk-free rate can in principle become negative. To avoid working with a negative interest rate, we set it close to zero whenever drawing errors in the construction of the bootstrap sample would imply a negative level. In Table 1.A.9 we provide evidence from simulation that this simple reflective approach suffice for our model framework and does not essentially interfere with the accuracy of the parameter estimates.[6]

1.3.2 Simulation Study

In order to examine the small sample properties of our estimation procedures, we conduct simulation studies for the different model setups. In a first step, we apply simple Euler approximations to the differential equations. In a second step, we use these approximations as data generating process (DGP) with known parameter values and simulate 25 years of data for the short rate, consumption, output and the financial claims. In case of the output derivative, $P_{Y,t}$, and the stock future, $P_{F,t}$, we chose a time to maturity of 1 month ($T = 1/12$) and 6 months ($T = 0.5$), respectively. Starting from the martingale increments, $\mathbf{m}(\mathbf{x}, \mathbf{y}, \mathbf{p}; \phi_i)$, we use the estimation methods of the previous section and retrieve the structural parameters from the simulated data sets. Whenever a parameter is unidentified or the estimation methods fail to consistently estimate the corresponding parameter, we fix it at its true level and estimate the remaining parameters. As in our bootstrap implementation, we repeat this procedure 550 times in order to obtain a simulated distribution for each parameter.

In the literature (cf. Christensen et al. (2016)) the model parameters are estimated by using bond, consumption and output data (C_t, Y_t, r_t^f). Thus, we denote the macro-finance combination

$$\mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p}, \phi_i) = \begin{bmatrix} \ln \mathbf{y}^{-1} & \ln(\mathbf{x}_{11}\mathbf{x}_{21}) & \mathbf{x}_{11} - \delta - \sigma^2 \end{bmatrix}^\top$$

as our benchmark model.

A standard finding in the literature on structural parameter estimation of the Vasicek interest rate specification (see e.g. Tang and Chen (2009)) is an upward bias in the parameter of the speed of mean reversion, κ. Since we do not apply bias correction

[6]We apply this reflective approach in the simulation study and re-calibrate the parameter values in the data generating process in a way that the interest rate would turn negative in many instances with high probability. The accuracy of median estimates as well as the interquartile ranges are similar to our baseline parametrization. However, there tend to be a relatively small upward bias in the median estimates for γ and an increase in the bias of κ.

methods, this feature is also prominent in all setups that we consider. Tables 1.A.1 to 1.A.8 show the estimation results from the simulation study. Each table contains the median estimates and the interquartile range (IQR) when using only finance or a combination of macro and finance variables. We denote the derivative on the stock index as future, and label the futures on capital and output more generally as *derivatives*. Recall that the evolution of consumption is identical to the one of the capital derivative in the AK-Vasicek model. Thus,

$$d\ln(C_t/C_{t-\Delta}) = d\ln(P_{K,t}/P_{K,t-\Delta}), \tag{1.27}$$

so that the corresponding estimation results can be interpreted interchangeably.

For the remainder of this section, we summarize the central insights from the simulation study for the GMM and MEF estimations and discuss benefits and drawbacks of substituting macro by finance variables. For this purpose, we start from the benchmark model and replace or add finance variables. We provide additional insights by comparing the corresponding histograms. In order to make the histograms comparable, we normalize their mass to 1 and plot them jointly, using the same bin width. Dark gray histograms are always obtained from the benchmark model, while light blue histograms correspond to parameter estimates that we obtain from using either purely finance data or a combination of macro and finance data. As encountered in Christensen et al. (2016), there are many cases where the estimation methods turn out to be unable to retrieve all theoretically identified parameters. As a brief overview, Table 1.1 summarizes the robustly retrievable parameter constellations for various systems of macro and finance estimation equations.

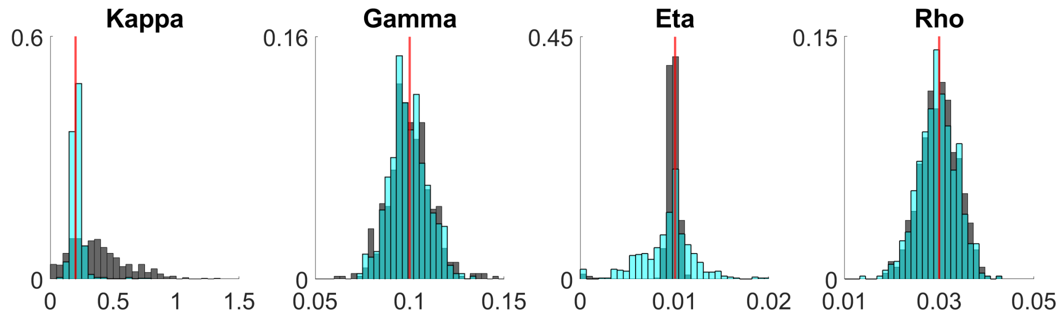
Using First Moments

Figure 1.1a plots the histograms of parameter estimates from the benchmark model, (C_t, Y_t, r_t^f), against the corresponding estimates obtained from the triple $(C_t, P_{d,t}, r_t^f)$. In theory, the benchmark model is able to identify 5 parameters through the martingale property of the estimation equations without exploiting second moments. However, this setting experiences numerical problems. In particular, the estimate of σ frequently diverges towards zero. Thus, we decide to apply two parameter restrictions (on δ and σ) in this setting. When we replace output, Y_t, in the benchmark model by stock data, $P_{d,t}$, or future data, $P_{F,t}$, we can in theory identify 4 parameters using only first moments. In this case we also rely on restricting δ and σ. Utilizing the systems $(C_t, P_{d,t}, r_t^f)$ and $(C_t, P_{F,t}, r_t^f)$ results in a strong accuracy gain in the κ estimates. In fact, the bias nearly vanishes in absolute terms. However, this advantage comes at the cost of less accurate estimates of η. Nevertheless, Tables 1.A.1 and 1.A.2shows that all medians of the parameter estimates are closely centered around their true values used in the data generating process. Because the dynamics of consumption and the derivative on capital coincide (see equation (1.27)), Figure 1.1a can also be interpreted as comparing the benchmark model to a purely fi-

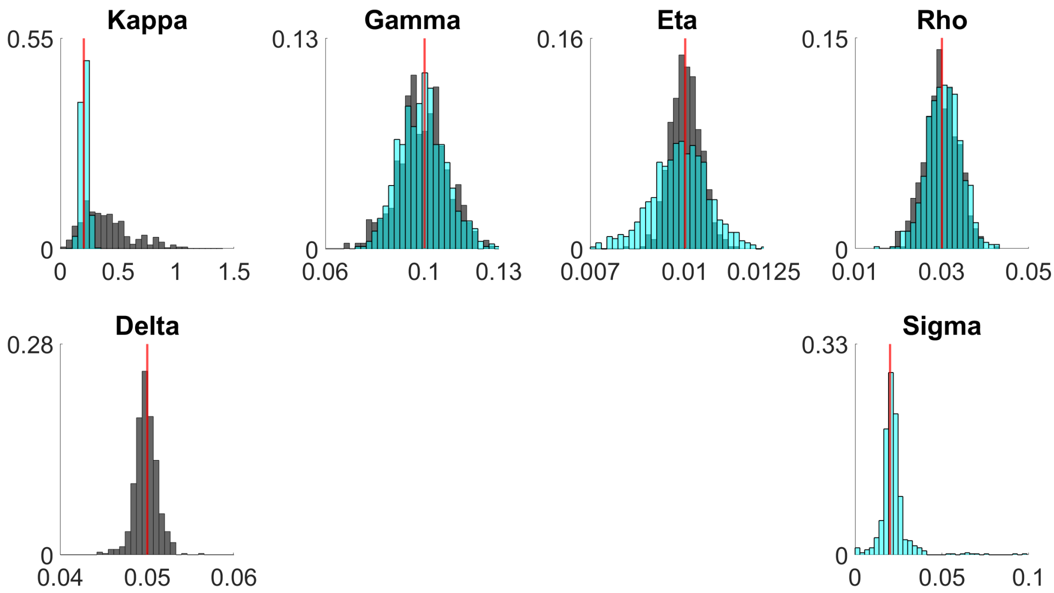
Table 1.1: Robust parameter estimates using different combinations of macro and finance data. Green crossed circles: Parameters obtained when using only first moments. Red circles: Additional parameters obtained when exploiting second moments (otherwise fixed at true values). Empty entries: Parameters fixed at true values. "Deriv." denotes the output derivative. Cons* denotes either consumption or the capital derivative (cf. equation 1.27).

GMM	**Finance Model**						**Macro-Finance Model**						
	Bond	**Bond Stock**	**Bond Future**	**Bond Deriv.**	**Bond Stock Future**	**Bond Stock Deriv.**	**Bond Cons***	**Bond Cons* Stock**	**Bond Cons* Future**	**Bond Cons* Deriv.**	**Bond Cons* Output**	**Bond Stock Output**	**Bond Future Output**
κ	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
γ	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
η	○	○	○	○	⊗	⊗	○	⊗	⊗	⊗	⊗	⊗	⊗
ρ		⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
δ				○		○				○	○	○	○
σ		○	○		○	⊗	○	○	○	○	○	⊗	⊗
MEF	**Finance Model**						**Macro-Finance Model**						
κ	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
γ	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
η	○	○	○	○	⊗	⊗	○	⊗	⊗	⊗	⊗	⊗	⊗
ρ		⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗	⊗
δ				⊗		⊗				⊗	⊗	⊗	⊗
σ		⊗	⊗		⊗	○	○	⊗	⊗	○	○	⊗	⊗

nance formulation. Thus, finance and macro data are perfect substitutes in the structural estimation. When replacing consumption in the benchmark model (C_t, Y_t, r_t^f) with stock or future data, the resulting macro-finance combinations $(P_{d,t}, Y_t, r_t^f)$ and $(P_{F,t}, Y_t, r_t^f)$ are able to robustly estimate σ as additional parameter. Thus, we are able to reliably estimate 5 structural parameters (see Table 1.1). The same holds true for the complete finance combinations $(P_{d,t}, P_{Y,t}, r_t^f)$ and or $(P_{F,t}, P_{Y,t}, r_t^f)$ where we further substitute output by the derivative of output. The reason for the accuracy gains in the estimation of σ is the identification of additional parameter combinations that enter the system through the financial claims. This in turn improves the identification of σ and allows more robust estimates. In all considered models, we observe a skewed distribution of σ estimates, with a median slightly above the true parameter value used in the data generating process.



a) GMM estimation using only first moments.



b) MEF estimation using only first moments.

Figure 1.1: Histograms: Simulation study results using GMM and MEF and first moments only. Benchmark model (gray histograms) vs. Benchmark model when replacing output by stock data (blue histograms).

Regarding MEF, one can, in theory, utilize the benchmark model (C_t, Y_t, r_t^f) and identify all 6 structural parameters without relying on second moments. As in the corresponding GMM estimation, 5 parameters are identified through the martingale property of the estimation equations. Identification of the sixth parameter follows from the conditional variance and mean parameter derivatives. However, we again experience numerical problems in practice. Similar to GMM estimation, retrieving σ is troublesome and results in estimates frequently converging towards zero. Consequently, we only estimate 5 parameters in this setting (see Table 1.1). In order to improve the accuracy of σ estimates, we replace consumption with stock or future data. By utilizing the triples $(P_{d,t}, Y_t, r_t^f)$ or $(P_{F,t}, Y_t, r_t^f)$ additional parameter combinations are identified, which, in turn, improves the identification of σ. As a consequence, we obtain reliable estimates of all 6 parameters without turning to higher moments. In case of the baseline model, we are free to either restrict δ or σ. Our simulation study suggests that fixing σ results in more reliable estimates. Figure 1.1b highlights that there is relatively strong upward bias in κ, whereas the median estimates of γ, η, ρ and δ are close to their true values and have low IQRs. When we replace output by stock data, $P_{d,t}$, or by future data, $P_{d,t}$, we are no longer able to accurately estimate δ but instead obtain stable estimates for σ. As in the GMM case, the accuracy of the κ estimates drastically improves. At the same time, the distribution of the η estimates becomes wider bell-shaped but still with unbiased median estimate and with low IQR (see Tables 1.A.3 and 1.A.4). Similar to GMM, we obtain a skewed distribution of σ in all considered systems of estimation equation. Thus, the median estimate turns out to be slightly above the true parameter value used in the data generating process.

Using Second Moments

In our exemplary model, error terms are traceable and analytical expressions for conditional covariance and parameter derivative matrices are readily available. Consequently, extending our analysis by turning to second moments is straightforward.

Using either GMM or MEF, the volatility parameters σ and η are readily identified through the second moments of consumption (or the capital derivative) and the risk-free rate, respectively (see equation (A.18)). Thus, whenever we utilize one or both of the corresponding estimation equations, there is little variation in these parameters among the considered models. Most notably, in all analyzed combinations of macro and finance data, the distribution of σ estimates is no longer skewed when exploiting second moments. A crucial advantage of second moments is the possibility to already identify 5 structural parameters by only using two different time series. In fact, as suggested by Tables 1.A.7 and 1.A.8, the combination of bond and consumption data is already able to accurately (except for the upward bias in κ) estimate 5 parameters. In some of the larger systems of estimation equations, GMM is now able to retrieve all structural parameters. In case of MEF, second moments also improve identification, and consequently yield more robust parameter estimates with little dispersion. In particular, we are able to estimate all structural parameters in most of the larger models. When exploiting second moments in GMM estimation, the benchmark model is able to additionally retrieve δ and σ. Thus, we are able to accurately estimate all structural parameters. The same holds true when replacing consumption by stock or future data. When further replacing output by the derivative on output, we obtain a complete finance formulation, $(P_{d,t}, P_{Y,t}, r_t^f)$, that retrieves all six parameters. In contrast, despite exploiting second moments, the triples $(C_t, P_{d,t}, r_t^f)$ and $(C_t, P_{F,t}, r_t^f)$ can only retrieve 5 parameters because the additionally identified parameter combinations are unable to disentangle γ and δ. In comparison to the benchmark model, this inability is a critical drawback. We decide to restrict δ and estimate the remaining 5 parameters. As in the corresponding estimations with first moments, there is again

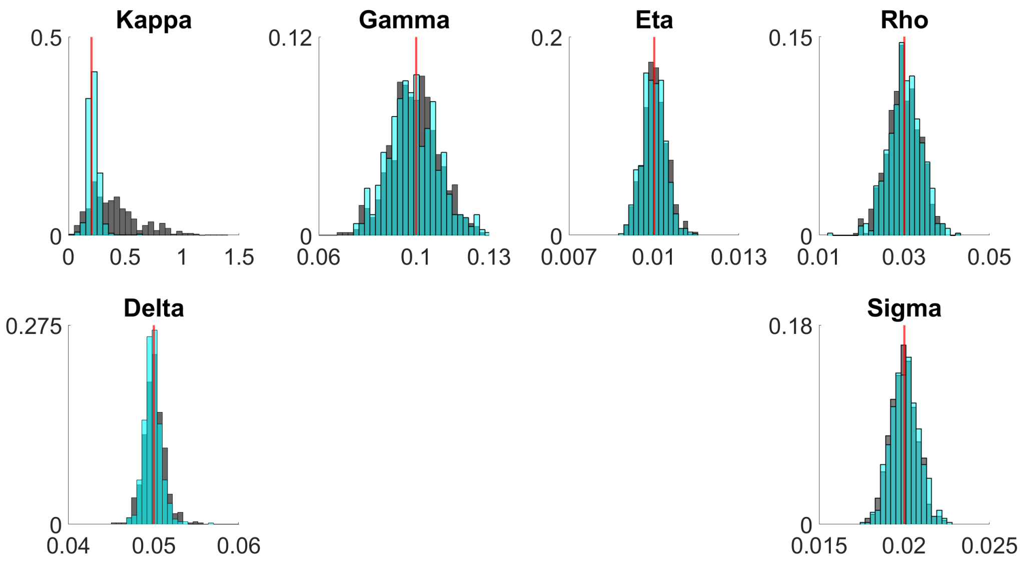


Figure 1.2: Histograms: Simulation study results using MEF and second moments. Benchmark model (grey histograms) vs. Benchmark model when replacing output by derivative data and additionally introducing stock data (blue histograms).

the strong increase in the accuracy of the κ estimate. As before, we can interpret these settings as complete finance formulations.

In most cases, there are only marginal differences between parameter estimates that are obtained at a monthly or at a daily frequency in the simulation study. However, there are some instances where a higher frequency improves the accuracy of volatility parameter estimates. This appears especially important for the empirical estimation. One possible explanation is the greater sample size and the fact that more parameters show up during estimation. In contrast to MEF, a daily frequency does not pose a problem for GMM estimation. In particular, MEF experiences numerical problems because the relatively higher sample size blows up the dimension of the conditional covariance matrix. The joint stock, bond and future setting, $(P_{d,t}, P_{F,t}, r_t^f)$, can identify 5 parameters with GMM. When turning to a daily frequency, there is not only an increase in the accuracy of the κ, γ and σ estimates but also a large accuracy gain in the estimate of η (see Figure 1.A.1).

Finally, consider the macro-finance data set $(C_t, P_{d,t}, P_{Y,t}, r_t^f)$. As highlighted by Figure 1.2, this setting is able to identify all parameters. At the same time it preserves the accuracy gain in the estimation of κ, which we encounter when utilizing the triple $(C_t, P_{d,t}, r_t^f)$. Thus, again the bias nearly vanishes in absolute terms. Again, we can substitute consumption by the price of the capital derivative and cast the estimation equation completely in terms of financial data.

Simulation Study: Summary and Concluding Remarks

In summary, the simulation study suggest that replacing macro by finance data as well as adding additional financial variables can improve the identification and accuracy of parameter estimates. Thus, finance data can be a substitute and/or a complement to macro data. The results offer important implications for more elaborate models. First of all, the simulations suggest that we are not comparing apples with oranges. The closed-form solutions of the financial claims highlight that the SDF incorporates additional parameters when deriving asset prices from macro economic variables. This can improve both identification and accuracy. Finance data turns out to be especially useful when relying solely on first moment conditions. In fact, replacing macro data in the benchmark model with financial data (stocks, futures, derivatives) offers a remedy to encountered numerical problems and allows us to retrieve more parameters. In particular, finance data increases the accuracy of estimates and results in more reliable estimates.

Before turning to the empirical estimation, we briefly discuss some additional insights from simulation. Note that the estimation equations for the prices of the stock future and of the derivatives on capital and output explicitly account for a time to maturity effect. As a consequence, one can not only utilize one specific asset but also exploit the whole maturity spectrum of available data. Even though, the time to maturity effect appears to be relatively small in our framework, it probably becomes more relevant for larger models where risk plays a greater role (e.g. term premia or default risk). While macroeconomic data is usually observed (if at all) at an annual or quarterly basis, we can use these (lower frequency) aggregates to readily derive financial claims and evaluate them at any desired (available) frequency. While the effects from going from a monthly to a daily frequency appears less relevant in our simulation study, it might play a bigger role in more elaborate models. In this respect, a crucial benefit is the increase in the sample size, which is especially useful when considering relatively shorter periods of time. In our empirical estimation, a higher frequency improves the accuracy (as measured by bootstrapped confidence intervals) when estimating the volatility and the time preference parameters. Furthermore, utilizing the stock price estimation equation with monthly stock price data appears to be more intuitive than utilizing monthly proxy variables for unobservable macroeconomic aggregates like the capital stock or the output gap.

1.3.3 Taking the Model to the Data

In order to estimate the different systems of equilibrium equations of our model, we need data on consumption, the short-term interest rate and on the financial claims. We consider the period from January 1982 to December 2019.[7] Data on consumption, output

[7]In case of MEF, we stop the sample period in all systems of estimation equations containing stock data at December 2016. There are convergence problems in this cases. Since GMM has no difficulties

and the short rate are obtained from the Federal Reserve Economic Data (FRED). We use the monthly level of real personal consumption expenditures (PCE) as a proxy for consumption, and monthly industrial production (IP) as a measure of output. Following Chapman et al. (1999), we use the 3-month interest rate, derived from US treasury bonds, as proxy for the risk-free rate. For stock data, we use the daily and monthly value-weighted returns (including dividends) of the S&P500 obtained from the Center for Research in Security Prices (CRSP). Finally, for data on future contracts, we consistently utilize data on the S&P 500. For this purpose, we use Bloomberg data on the *generic S&P500 mini future contract*, which is available for a time to maturity of 6 months. Thus, this time series allows us to account for maturity, without explicitly dealing with maturity dates and roll-overs of future contracts. Unfortunately, the data is only available since 1997. Thus, we have to consider a shorter sub-sample when using future data in the estimation. In section 1.3.4, we evaluate related drawbacks of the shorter sample period.

1.3.4 Estimation Design

In the empirical estimation we use the GMM and the MEF approach of the previous sections. For the evaluation of the estimation results, we turn to the sieve bootstrap method of section 1.3.1. For this purpose, we generate 550 pseudo data sets and each time estimate the structural parameters. We then use the resulting bootstrapped probability density functions in order to compute 95% confidence intervals, medians as well as the first and the third quartile in order to obtain the inter-quartile range. Regarding the implementation of the bootstrap, fitting the high-order VAR to the data turns out to be troublesome when jointly considering stock and future data. The reason for this is the high correlation of the generic 6-month S&P500 mini future contract with its underlying (the S&P500 stock index). Thus, despite the promising implications from the simulation exercise, we do not estimate systems that contain combinations of both contracts. Furthermore, we abstract from capital and output derivatives because finding convincing real world analogs remains, at least to some extent, ambiguous and challenging.

To avoid identification issues encountered in Christensen et al. (2016), we directly exploit second moments in our empirical estimation, consider more sophisticated minimization routines and impose more parameter restrictions. As it turns out, these adjustments make estimation results less prone to chosen starting values. In appendix 1.A.7, we relax some of these restrictions and in case of MEF consider its simplified 2-step approach as mentioned in section 1.3.1. By doing so, we are able to estimate all but one parameter from finance data alone. However, the estimation of some parameters strongly depends on the starting values and the chosen minimizing routine in this case. Thus, for the sake of comparability, we impose a relatively stricter set of restrictions in the main text. When

with the full sample, it appears to be a MEF-specific numerical problem.

Table 1.2: Empirical GMM estimation: Model overview and comparison. Bold parameter estimates lie within the bootstrapped 95% confidence interval. First and third quartiles, obtained from the bootstrapped parameter distributions are given below the estimates.

Empirical parameter estimates - GMM							
	Pure finance			**Macro-finance**			
Setup	Bond	Bond Stock	Bond Future	Bond Cons	Bond Cons Out	Bond Cons Stock	Bond Cons Future
	(Daily)	(Daily)	(Daily)	(Monthly)	(Monthly)	(Monthly)	(Monthly)
$\boldsymbol{\kappa}$	**0.268**	**0.186**	**0.173**	**0.154**	**0.033**	**0.189**	**0.006**
	[0.371, 0.638]	[0.279, 0.550]	[0.185, 0.595]	[0.131, 0.252]	[0.020, 0.069]	[0.161, 0.326]	[0.103, 0.389]
$\boldsymbol{\gamma}$	**0.074**	**0.075**	**0.060**	**0.069**	**0.045**	**0.072**	**0.030**
	[0.073, 0.081]	[0.070, 0.085]	[0.063, 0.078]	[0.071, 0.081]	[0.051, 0.074]	[0.068, 0.079]	[0.060, 0.068]
$\boldsymbol{\eta}$	*0.0090*	**0.012**	**0.009**	**0.008**	*0.003*	**0.008**	*0.005*
	[0.011, 0.011]	[0.011, 0.012]	[0.008, 0.009]	[0.008, 0.008]	[0.007, 0.008]	[0.008, 0.008]	[0.006, 0.007]
$\boldsymbol{\rho}$		**0.013**	**0.007**	**0.000**	**0.000**	**0.000**	**0.000**
		[0.001, 0.022]	[0.005, 0.027]	[0.000, 0.005]	[0.000, 0.004]	[0.001, 0.004]	[0.000, 0.000]
$\boldsymbol{\delta}$	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$\boldsymbol{\sigma}$	0.02	0.02	0.02	**0.013**	**0.015**	*0.012*	**0.012**
				[0.014, 0.015]	[0.014, 0.016]	[0.014, 0.152]	[0.011, 0.012]

applying GMM, we exploit daily data in our pure finance formulations. While the role of the frequency appears to be small in the simulation study, it turns out to be more relevant in the empirical estimation and results in more robust estimates (see Table 1.A.10 for the corresponding results at a monthly frequency). In setups with only 2 variables, we consider 4 conditional moment restrictions. In larger system we only exploit the second moments of the bond and consumption. We do this for reasons of comparability and to address apparently numerical difficulties when dealing with the second moments of output, stocks and futures in some macro-finance settings. Finally, in Tables 1.A.10 and 1.A.12, we estimate the structural parameters of different macro-finance models for the shorter sub-period starting 1997. As suggested by the simulation study, the smaller number of observations in the sub-sample is sufficiently large for applying GMM and MEF techniques. The aim of this exercise is a better understanding of how the shorter time series affects the quality of parameter estimates when turning to the S&P500 future data (unavailable prior to 1997). In most cases, the estimates only slightly differ from the full sample. Differences are primarily related to κ, γ, and η, which probably results from the relatively higher proportion of observations at the ZLB.

1.3.5 Estimation Results

Table 1.2 shows the empirical GMM estimates obtained from different combinations of finance and macroeconomic estimation equations. Even though, the simulation study suggests that we can estimate up to 5 parameters from the complete finance models, we decide to restrict δ and σ. Otherwise, there tend to be convergence problems that result in most parameters simultaneously approaching zero or extremely large values in both the empirical and the bootstrapped estimation. Table 1.A.10 states the corresponding results when estimating 5 instead of 4 parameters from the bond/stock and the bond/future models. The estimates are similar to the corresponding 4 parameter case. However, the results are less reliable, as they critically depend on the starting values and the tolerance level of the minimizing routine. Nevertheless, confidence intervals and medians of the bootstrapped distributions are in most cases similar to the ones obtained in other macro-finance settings in Tables 1.2.

A striking feature of all larger-scale as well as the bond/consumption model is the GMM estimate of the time preference parameter. In all of these models ρ turns out to be approximately equal to zero in the empirical and most of the bootstrapped estimates. Thus, even though we obtain 5 GMM parameter estimates in the macro-finance models in Table 1.2, the identification of ρ appears troublesome. In contrast, the pure finance settings may only yield 4 parameter estimates, but non of these parameters converges towards zero and all lie within their corresponding bootstrapped 95% confidence intervals. Thus, a complete finance setting allows for an identification of ρ, with values comparable to the ones obtained from MEF estimation. As it turns out, the convergence of ρ towards zero remains an exclusive feature of GMM estimation and in most considered settings does not occur when using the MEF approach (see Table 1.3).

In line with the insights from the simulation study, the second moments of the bond and consumption estimation equations appear to be the central drivers for the estimates of the two variance parameters η and σ. Furthermore, the bootstrap analysis also suggest that the speed of mean reversion parameter κ has a relatively large upward bias. In contrast to the simulation study, however, there appears to no large bias reduction in macro-finance models with stock or future data. In fact, bootstrapped confidence intervals, interquartile ranges and medians are similar to the ones obtained from the other model formulations.

In all considered models, the GMM estimate of γ lies withing the bootstrapped 95% confidence interval and in most cases even between the third and first quartiles of the bootstrapped distribution. When using S&P500 future data, however, the empirical GMM estimate of γ in the larger-scale macro-finance model is relatively small. This is also the case when estimating the model with MEF (see table 1.3). The relatively low estimate of γ probably results from using the smaller sub-sample, which is characterized by overall lower interest rates with little variation. However, when using the shorter sub-sample in the bond/consumption/stock model, the empirical estimate of γ turns out to be only marginally smaller than the one obtained from the full sample (see Table 1.A.12). Nevertheless, in both settings, the shorter sub-sample is accompanied by significantly lower estimates of η, which suggests that the prolonged ZLB period affects parameter estimates. Note that the empirical estimate of γ in the bond/consumption/future model lies significantly below its bootstrapped median. The ZLB is the likely reason for differences between the empirical estimate and the bootstrapped median, because we do not model a persistently binding lower bound in the bootstrapped samples. Thus, even though there is a high likelihood of drawing a large proportion of ZLB observations in the construction of the pseudo-data sets, the pseudo-times series are, on average, more volatile (higher η) and overall imply higher levels of the risk-free rate (higher γ).

The empirical parameter estimates from the bond/consumption/output model are basically the same as in Christensen et al. (2016)(Table E5). Our bootstrap analysis unveils that the estimate for $\eta = 0.003$ is significantly smaller than the bootstrapped median estimates and lies relatively far outside the bootstrapped 95% confidence interval. The corresponding confidence interval for σ is relatively wide, which shows that we frequently observe large outliers. Comparing the law of motions of consumption (A.17a) and of output (A.17g), there are many combinations of parameter values that suggest a relatively high difference in the volatility of consumption and output. However, the volatility is of similar magnitude in the empirical data so that drawing residuals for the bootstrap can result in significant differences in the volatility of the two pseudo-time series. Consequently, in some instances, either consumption or output turn out to be more volatile, which explains the relatively large proportion of outliers.

The empirical parameter estimates from the bond/consumption/stock setting are similar to the ones obtained from the bond/consumption setting. In case of the former, the estimate of γ nearly coincides with the bootstrapped median, whereas it lies in the first quartile in the bond/consumption case. However, the estimate of σ is relatively small when using bond, consumption and stock data. In fact, the estimate does not even fall within the bootstrapped 95% confidence interval, whereas σ lies more closely to the bootstrapped median when using only bond and consumption data.

Table 1.3 shows the estimation results when using the MEF approach. As suggested by the simulation study, MEF allows us to estimate all parameters when utilizing macro-finance estimation equations. In the complete finance settings, in contrast to GMM, we do not consider a daily frequency, as the increase in the sample size is computationally burdensome and appears to exhibit numerical difficulties. Thus, we consider a monthly frequency, restrict δ and σ and estimate the remaining 4 parameters. Nevertheless, the MEF method turns out to have problems to retrieve ρ, which converge towards zero in both the empirical and most of the bootstrapped estimation. As highlighted by Table

Table 1.3: Empirical MEF estimation: Model overview and comparison. Bold parameter estimates lie within the bootstrapped 95% confidence interval. First and third quartiles, obtained from the bootstrapped parameter distributions are given below the estimates.

Empirical parameter estimates - MEF							
	Pure finance			**Macro-finance**			
Setup	Bond	Bond Stock	Bond Future	Bond Cons	Bond Cons Out	Bond Cons Stock	Bond Cons Future
	(Daily)	(Daily)	(Daily)	(Monthly)	(Monthly)	(Monthly)	(Monthly)
κ	**0.165**	**0.135**	**0.115**	**0.170**	**0.022**	**0.007**	**0.068**
	[0.299, 0.514]	[0.059, 0.129]	[0.076, 0.139]	[0.088, 0.215]	[0.015, 0.056]	[0.007, 0.014]	[0.016, 0.037]
γ	**0.071**	**0.062**	**0.057**	**0.071**	**0.128**	*0.110*	**0.035**
	[0.072, 0.081]	[0.064, 0.081]	[0.061, 0.074]	[0.069, 0.084]	[0.052, 0.176]	[0.079, 0.093]	[0.057, 0.073]
η	*0.012*	*0.011*	**0.007**	**0.011**	**0.007**	**0.003**	**0.006**
	[0.011, 0.012]	[0.008, 0.009]	[0.006, 0.007]	[0.008, 0.008]	[0.008, 0.009]	[0.002, 0.003]	[0.004, 0.007]
ρ		**0.000**	**0.000**	**0.009**	**0.009**	**0.012**	**0.005**
		[0.000, 0.000]	[0.000, 0.000]	[0.000, 0.006]	[0.000, 0.005]	[0.004, 0.010]	[0.002, 0.006]
δ	0.05	0.05	0.05	0.05	**0.027**	0.05	0.05
					[0.031, 0.171]		
σ	0.02	0.02	0.02	**0.017**	**0.017**	**0.018**	**0.013**
				[0.015, 0.016]	[0.015, 0.016]	[0.016, 0.017]	[0.012, 0.013]

1.A.10, GMM estimation experiences the same behavior when using a monthly frequency. In case of GMM, the remedy to this problem is switching to a daily frequency. Due to the numerical and computational problems, increasing the data frequency tuns out to be no reliable option in MEF estimation. In Table 1.A.11 we apply the simplified 2-step MEF approach (see Christensen et al. (2016)) and estimate 5 parameters in the bond/stock and bond/future settings at a daily frequency. The results are in line with the larger macro-finance models and all empirical parameter estimates lie withing their 95% confidence intervals and in most cases close to their corresponding bootstrapped medians. However, similar to estimating 5 parameters from the pure finance models with GMM (see Table 1.A.10), the results are less reliable because estimates strongly depend on the starting values and the tolerance levels used in the minimization routines. These findings suggest that the differences in the complete finance estimations in Tables 1.2 and 1.3 are not necessarily a matter of the applied estimation routines but of the different sample frequencies used in the MEF (monthly) and the GMM (daily) estimations. This again highlights one of the strengths of financial data; switching to a higher frequency can improve identification and yield more significant estimates as measured in term of lower statistical dispersion. It is important to stress, however, that the convergence problems of ρ are not a specific feature of using finance data. In fact, the small-scale macro finance setting with bond and output data exhibits even more severe convergence problems.

In the small-scale bond/consumption setting, the obtained values for κ, γ and η are nearly identical to the ones when using only bond data, suggesting that identification of these parameters primarily occurs via the bond price. In contrast to the corresponding GMM estimation, ρ does not converges towards zero. Even though the parameter estimate of $\rho = 0.009$ lies within its bootstrapped 95% confidence interval, its size is three times bigger than the median (0.003) of its bootstrapped distribution. When additionally exploiting stock data, the empirical estimate for γ increases from 0.071 to 0.110 so that the parameter value slightly exceeds its bootstrapped 95% confidence interval. The parameter estimate of γ appears to be quite high (around 11%). However, one has to keep in mind that this simple model is very likely miss-specified and that our sample period already starts in the early eighties, which were characterized by relatively high interest rates. Nevertheless, this does not explain the significantly lower estimates of γ when applying GMM (see Table 1.3), and thus points in the direction of an MEF-related property. In comparison to other MEF estimates of κ, the bond/consumption/stock model has the smallest empirical estimate with a narrow bootstrapped confidence interval and the smallest bootstrapped interquartile range. Recall that this accuracy gain is also suggested by the simulation study. Furthermore, the estimate of the volatility parameter η turns out to be only half as large as the ones obtained in the other MEF settings and nearly coincides with the median of the bootstrapped distribution. In comparison to the corresponding GMM estimation result, the parameter ρ no longer converges towards zero. In fact, the parameter has the highest value among all MEF estimates in Table 1.3 and is nearly identical to the one obtained with GMM in case of the pure finance formulation with daily data.

In case of the macro-finance system with bond, consumption and future data, most parameter estimates are similar to the ones obtained with GMM. However, as in most other large-scale models in Table (1.3), ρ does not converge towards zero. The ZLB again appears to have a relatively large impact on the estimation results. In particular, compared to the other settings, we obtain the lowest estimates for γ and η when utilizing S&P500 future data. Furthermore, as suggested by the simulation study and in line with the bond/consumption/stock model, the empirical estimate of κ is relatively small, has a narrow 95% confidence interval and lies close to its bootstrapped mean.

Using the benchmark bond/consumption/output setting, we are able to estimate all 6 parameters. The obtained parameters are within their corresponding bootstrapped 95% confidence intervals and, except for ρ and δ, lie relatively close to the medians of their bootstrapped distributions. The empirical estimates of ρ and σ and their bootstrapped statistics are nearly identical to the ones obtained from the smaller-scale bond/consumption model, which suggest that identification primarily results from bond and consumption data. The absolute value of the parameter estimate of γ nearly doubles, when adding output data to the bond/consumption model. Similar to the bond/consumption/stock model, a value of nearly 13% appears to be relatively large. The depreciation rate parameter δ has a relatively large bootstrapped 95% confidence interval and the bootstrapped distribution suggest a relatively high degree of dispersion as measured in terms of a large interquartile range. The empirical parameter estimate of δ (0.026) is more than 3 times smaller than the median (0.087) of the bootstrapped distribution. Taking everything together, we conclude that the benchmark model outperforms the other systems in case of MEF estimation. However, as argued above, this system performs relatively poorly in case of GMM estimation.

Our results indicate that the AK-Vasicek specification with logarithmic preferences does not match the data very well and is very likely miss-specified. However, despite the simplicity of the model, our estimation results highlight benefits and drawbacks of utilizing financial data in the empirical estimation of structural estimation. Furthermore, macro-finance and complete finance settings yield similar parameter estimates, which suggest that we are not comparing apples and oranges. Hence, our findings are encouraging to apply this approach to the estimation of more elaborate macroeconomic models. Recall that we choose the above modeling framework as an illustrative example of our general estimation approach. The practical evaluation of larger models is beyond the scope of this paper but remains part of our future research agenda.

1.4 Conclusion

We discuss a general framework to estimate the structural parameters of macroeconomic models, by utilizing a combination of macro variables and consistently priced financial assets. To highlight this approach, we turn to a simple model with closed-form solution that allows us to keep track of each step and to closely monitor macroeconomic and financial dynamics. By doing so we can attribute identification of parameters to specific estimation equations and moment conditions. We show how to estimate the structural parameters with various different combination of macro as well as finance data and point out drawbacks and benefits of the considered settings. Our simulation study results justify a critical assessment of the informational content of financial data on the state of the economy. By utilizing the stochastic discount factor, any model implied asset price can be derived consistently with macroeconomic dynamics. We show how the stochastic discount factor allows for the computation of various financial asset prices even in very simplistic macroeconomic frameworks. This enables us to use rich financial data sets in our estimation, including S&P500 index and S&P500 future data with any available frequency. We highlight that this allows substituting non-observable variables like the risk-free rate or the capital stock with observable financial data. Furthermore, we can use higher-frequency financial data to replace lower-frequency macro data (e.g. GDP). Our results suggest that a higher frequency can increase the accuracy of structural parameter estimates. Additionally, due to the relative increase in the sample size one does not have to go back decades in time in order to obtain a sufficiently large data set needed for estimation. Furthermore, we show that certain combinations of macro and finance data offer a nearly complete correction of the well established upward bias in the drift parameter κ in the Vasicek interest rate specification. Strictly speaking, κ remains biased in these macro-finance models but due to a strong accuracy gain, the bias basically vanishes in absolute terms.

Despite the promising implications from simulation, actual empirical estimation exhibits various problems and limitations. Similar to Christensen et al. (2016), these are in some instances a relatively strong dependence on initial guesses and the used minimization routines. The prolonged ZLB period also appears to be problematic. Nevertheless, empirical estimation results appear to be relatively sound, given the highly stylized nature of our exemplary model.

We believe that this paper is a promising starting point on how to utilize financial variables in the estimation of more elaborate macroeconomic models. First, our considered generalized approach is readily applicable to a whole class of macroeconomic models and does not stop at the exemplary AK-model. Second, while small-scale macroeconomic models usually rely on unrealistic simplifications, they offer important benchmark cases and insights. As a consequence, our analytical solutions can turn out useful for understanding transmission channels and relationships of macro and financial data, when introducing more realistic features. Third, even though, this paper does not advocate casting the estimation equations completely in terms of financial variables, it shows that this is possible and, at least in theory, is equivalent to considering combinations of macro and financial variables. A complete finance representation of macroeconomic models might turn out to be an interesting alternative for policymakers. Especially to circumvent publication lags and revisions of macro data, which can turn out to be useful if there is a quick need for information on the state of the economy (e.g. policy responses to crises).

Chapter 2

FTPL and the Maturity Structure of Government Debt in the New-Keynesian Model

with Olaf Posch

Abstract

In this paper, we revisit the fiscal theory of the price level (Fiscal Theory of the Price Level (FTPL)) within the New Keynesian model. We show in which cases the average maturity of government debt matters for the transmission of policy shocks. The central task of this paper is to shed light on the theoretical predictions of the maturity structure on macro dynamics with an emphasis on model-implied expectations. In particular, we address the transmission channels of monetary and fiscal policy shocks on the interest rate and inflation dynamics. Our results illustrate the role of the maturity of existing debt in the wake of skyrocketing debt-to-GDP ratios and increasing government expenditures. We highlight our results by quantifying the effects of the large-scale US fiscal packages (CARES) and predict a surge in inflation if the deficits are not sufficiently backed by future surpluses.

2.1 Introduction

In response to the global coronavirus pandemic, governments around the world tried to cushion the economic downturn by financing large-scale fiscal support and relief packages such as the US Coronavirus Aid, Relief, and Economic Security (CARES) Act, with unprecedented volumes. For example, when including loan guarantees, the CARES Act amounts to about $2 trillion (or 10% of US GDP) with substantial budgetary effects. The Congressional Budget Office (CBO) projects CARES to add $1.7 trillion to deficits over the next decade.[1] In order to alleviate a deep recession, policy makers have implemented further stimulus packages (e.g., the American Rescue Plan, the Next Generation EU fund, NGEU). The funding of these unprecedentedly large fiscal programs drastically increased debt levels with yet unknown consequences (e.g., accounting for distributional effects, CARES increases the debt-to-GDP ratio by 12% in Kaplan et al., 2020).

In the macroeconomic literature, there are, however, open questions and ongoing debates about the effects of sovereign debt on macro aggregates, inflation, the term structure, and inflation expectations where no consensus has been reached. One central question here is how the structure of outstanding government debt affects the transmission channels of fiscal and monetary policy. Clearly, governments face a challenging task to maintain a sustainable level and maturity structure of outstanding sovereign debt. On the one hand, fiscal policy faces a financing decision on whether to either increase the level of public debt or to raise taxes today. On the other hand, fiscal policy needs to decide on whether to issue bonds with longer maturities, or to simply roll-over maturing debt with short-term bonds. What will be the effect of those large-scale fiscal programs, in particular, how does the maturity structure of outstanding debt affect those outcomes? This paper fills this gap in the macroeconomic analysis of fiscal and monetary policy.

In this paper we address the transmission of fiscal and monetary policy shocks on interest rates and inflation dynamics in a framework which combines the fiscal theory of the price level (FTPL) with the traditional New Keynesian model of inflation. Our central aims are the theoretical predictions of transitory and permanent policy shocks, which offer empirical testable implications for the role of the maturity structure of debt on the transmission of fiscal and monetary policy. Our application studies the effects of the recent CARES Act trough the lens of fiscal theory. We depart from the existing literature on the effects of the maturity structure of government debt in three dimensions. First, our formulation allows us to link the macro model easily to term-structure models in finance (Vasicek, 1977; Cox et al., 1985) and model-implied inflation expectations. Our approach allows us to compute the term structure of interest rates and inflation expectations by solving a partial differential equation, which is easily extended to nonlinear

[1] Congressional Budget Office, CARES Act, https://www.cbo.gov/publication/56334

solutions, default risk, and term premia. Second, in contrast to existing approaches[2], we directly compute zero-coupon bond prices for arbitrary maturities and states and then show bounds for the effects of the maturity structure of government debt on macro dynamics and inflation decomposition. Finally, we show that the fiscal theory in the continuous-time version works through two distinct channels: (i) a direct FTPL effect through a discrete jump in the price of existing bonds and (ii) an indirect effect through changing the path of future real interest rates. While the first channel is a pure asset pricing channel, the second channel is the traditional effect present in forward-looking rational expectations models. Hence, even in the model with short-term debt, the fiscal theory has implications on the future path of the real interest rate, in particular, the term structure of interest rate, inflation expectations, and the real economy.

We calibrate a simple NK-FTPL model to match the average maturity of outstanding US government debt and study aggregate dynamics. We find that the average maturity of debt has important implications for the transmission channels of both monetary and fiscal policy. Our results show how the maturity of existing sovereign debt significantly shapes the inflation response to fiscal and monetary policy shocks. First, following a transitory monetary policy shock, a longer maturity structure translates to a larger response in the real interest rate. In cases where outstanding government debt consists solely of short-term debt, the traditional negative correlation of the nominal interest rate and current inflation is reversed and term structure and inflation expectations are more sensitive to shocks. Similarly, based on the underlying maturity structure of government debt, expansionary fiscal policy leads to higher inflation and more accumulation of debt with short-term debt. Our inflation decomposition shows that with perpetuities, the inflation response to transitory shocks is dictated solely by future fiscal policy with changes in future monetary policy being soaked up by an immediate asset pricing effect. Second, we illustrate how inflation expectations and the term structure helps in identifying permanent policy shocks. Here, the maturity structure often produces some unpleasant short-term side effects. For example, a permanently lower inflation target increases current inflation and interest rates, but reduces long-term bond yields due to the re-evaluation of existing bonds.

Our findings confirm the hypothesis that the CARES Act with its unprecedented large-scale fiscal stimulus programs, i.e., the large cuts in primary surplus and hikes in government debt, has generated a market response with strong inflationary effects but effectively helped stimulating the real economy. However, the recent surge in inflation and medium-term inflation expectations indicate that markets do *not* expect that the newly issued debt is backed by subsequent higher future surpluses. This seems in contrast to the aftermath of the global financial crisis and raises cautionary flags as hyperinflations are widely believed to have fiscal origins (cf. Leeper and Leith, 2016).

[2]Among others see Leeper et al. (2019), Lustig et al. (2008), Faraglia et al. (2013) or Faraglia et al. (2019).

In line with the existing literature on the fiscal theory, we confirm a prominent role of those ideas in the NK-FTPL model with a plausible maturity structure of sovereign debt (cf. Cochrane, 2001; Leeper and Leith, 2016).[3] Most theoretical studies, such as Sims (2011, 2013), Leeper and Leith (2016), and Cochrane (2018), highlight important insights, e.g., the role of long-term bonds in the simple NK model causing a 'boomerang inflation' response to monetary policy shocks. In these models, long-term bonds are used to offset an otherwise initial positive co-movement of the inflation and the interest rates.[4] Other studies focus on the low-frequency relationship between the fiscal stance and inflation in a model with long-term debt (see Kliem et al., 2016) or the government spending multiplier (see Leeper et al., 2017a). We are not aware of a comprehensive study on the effects of fiscal and monetary policy shocks on inflation and inflation expectations, or generally about the role of fiscal theory in the NK model with an empirically calibrated average maturity of existing sovereign debt. Unfortunately, an inflation decomposition into a direct FTPL effect and an indirect effect is tricky and less clear-cut in the discrete-time model because the price level can jump (which in the continuous-time version is determined by past inflation). Hence, a continuous-time version of the NK-FTPL model (see also Sims, 2011; Cochrane, 2018) helps identifying the effects of the maturity structure because in the model with short-term debt, as in traditional NK models with fiscal policy and sovereign debt, the direct bond pricing effect is zero and the fiscal theory would work solely through the indirect effect.

Many theoretical and empirical studies recognize an important effect of the maturity structure of government in a broader context of optimal monetary and fiscal policies.[5] Leeper et al. (2019) show how high sovereign debt levels and the debt maturity structure can increase the 'inflationary bias'. In this setup, higher debt levels and shorter maturities increase the temptation of the policy maker to use surprise inflation and to decrease the real value of government debt. Similarly, Lustig et al. (2008) study the optimal policy if the fiscal authority is constrained by its ability to lend and only issues non-contingent nominal debt. In this case, optimal policy is achieved by almost the exclusive use of long-term debt. Even though the holding return on long-term debt is more volatile in contrast to short-term debt, it offers a hedge against fiscal shocks. Faraglia et al. (2013) analyze how inflation is affected by the maturity of sovereign debt and debt levels when fiscal and monetary policy are coordinated. They conclude that higher debt levels cause higher inflation, while a longer maturity structure increases its persistence.

[3] In this paper we focus on the fiscal regime and neglect potential fiscal-monetary coordination problems which may arise in a regime-switching model as in Bianchi (2012) or Bianchi and Melosi (2019).

[4] Cochrane (2022b) and Liemen (2022) discuss alternative ideas and show that long-term debt is not necessary to address this counterfactual response for short-term debt in the NK-FTPL model.

[5] Other papers study the optimal debt-maturity management (cf. Buera and Nicolini, 2004; Shin, 2007; Faraglia et al., 2010; Debortoli et al., 2017; Bigio et al., 2019). For example, Bigio et al. (2019) show how liquidity costs can prevent an instantaneous re-balancing across maturities and identify different forces that ultimately shape the optimal debt-maturity distribution.

More recently, Kaplan et al. (2020) and Bayer et al. (2021) also evaluate the role of skyrocketing debt levels, following the large-scale fiscal stimulus programs within the NK models with heterogeneous agents (HANK). Focusing on the role of public debt as private liquidity, Bayer et al. (2021) find that the expansionary stimulus programs decreased the liquidity premium of government bonds over less liquid assets.

The rest of the paper is organized as follows. In Section 2 we formalize the simple perfect-foresight NK-FTPL model and study dynamics of transitory and permanent structural zero-probability shocks. In Section 3 we provide a thorough analysis and simulation of the CARES Act of 2020 and discuss the recent surge in inflation and differences to the aftermath of the global financial crisis in 2008. Section 4 concludes.

2.2 The Model

In this section, we show how the FTPL mechanism outlined in Sims (2011) and Cochrane (2018) is embedded in the continuous-time NK model (cf. Posch, 2020). For reasons of clarity, we shortly discuss the main channels of FTPL in the linear NK framework and abstract from the effects of uncertainty and nonlinearities.

2.2.1 Monetary Policy or Fiscal Theory of Monetary Policy

As shown in Cochrane (2018), the presence of longer-term debt has effects on both the real economy and on how monetary policy is conducted, and more generally how government policies affect inflation. Consider the three-equation perfect-foresight NK model

$$\begin{aligned} \mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && (2.1)\\ \mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t && (2.2)\\ \mathrm{d}i_t &= \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, && (2.3) \end{aligned}$$

in which x_t is the output gap, y_t is output, i_t is the nominal interest rate, ρ the rate of time preference, π_t is inflation, where κ controls the degree of price stickiness with $\kappa \to \infty$ as the frictionless (flexible price) and $\kappa \to 0$ perfectly inelastic (fixed price) limits, θ controls interest rate smoothing with $\theta \to \infty$ implying the traditional feedback rule, $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1)$, and with π_t^* and i_t^* being parametric values.

Following Cochrane (2018) we implement the fiscal theory of the price level (FTPL) by closing the system with a fiscal block

$$\begin{aligned} \mathrm{d}a_t &= ((i_t - \pi_t)a_t - s_t)\mathrm{d}t && (2.4)\\ \mathrm{d}s_t &= f(s_t, y_t, a_t)\mathrm{d}t, && (2.5) \end{aligned}$$

in which a_t is the real value of sovereign debt (held by households), and $s_t \equiv T_t - g_t$ is the primary surplus, where T_t denotes lump-sum tax revenues and g_t government spending other than interest payments. It represents the net payments to holders of bonds, both through interest and retirement of outstanding debt (cf. Sims, 2011). In what follows, we use the notion of 'sovereign debt' and 'government bonds' interchangeably, which after all can be considered as a medium of exchange (paper money).

The central equation in the NK-FTPL model links primary surpluses to the real value of sovereign debt. In fact, solving forward (2.4), the future path of primary surpluses imposes a 'constraint' for fiscal policy (government budget constraint), because

$$a_t \equiv \frac{n_t p_t^b}{p_t} = \mathbb{E}_t \int_t^\infty e^{-\int_t^u (i_v - \pi_v)\mathrm{d}v} s_u \mathrm{d}u, \tag{2.6}$$

where n_t denotes the number of outstanding bonds, p_t^b the bond price, and p_t the price level, which must equal its (expected) real present value.[6] In this paper, we focus on bounded solutions and $\lim_{T\to\infty} e^{-\int_t^T (i_v - \pi_v)\,\mathrm{d}v} a_T = 0$.[7] Rather than being a budget constraint or limiting fiscal capacity, equation (2.6) should be thought of as being a valuation formula as it asserts a value p_t^b to the supply of government bonds n_t and a given price level p_t.

Similar to assuming perfectly flexible prices, it is unrealistic assuming that government debt is either floating debt or perpetual debt (cf. Sims, 2011). In what follows, we refer to floating debt as short-term and to long-term debt as perpetuities. We introduce bonds with decaying coupon payments (similar to Woodford, 2001), and assume that longer-term bonds (average duration) are amortized at rate δ and pay a nominal coupon $\chi+\delta$ such that at steady state the bonds sell at par and results compare to Sims (2011). No-arbitrage requires (see PDE approach Cochrane, 2005, chap. 19.4),

$$\mathrm{d}p_t^b = (i_t - ((\chi+\delta)/p_t^b - \delta))p_t^b \mathrm{d}t + \mathrm{d}\delta_{p_t^b}, \quad \mathbb{E}_t(\mathrm{d}\delta_{p_t^b}) = 0 \tag{2.7}$$

in which $\mathrm{d}\delta_{p_t^b}$ captures discrete changes in the bond price due to zero-probability structural shocks, with $\chi = i_{ss}$ such that $p_{ss}^b = 1$ is identical to floating debt. Note that (2.7) is *not* a stochastic differential equation (SDE) because the 'shocks' have zero probability. Following the literature, $\mathrm{d}\delta_{p_t^b}$ reminds us that the variable p_t^b can jump (forward-looking). In theory, we can issue floating debt which pays at $\chi = i_t$ and with $\delta \to \infty$ average duration approaches zero such that $p_t^b \equiv 1$. In contrast, for long-term bond we set $\delta = 0$ (cf. Sims, 2011). By integrating the linear approximation of equation (2.7), we obtain

$$p_t^b = 1 - \mathbb{E}_t \int_t^\infty e^{-(\chi+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u, \tag{2.8}$$

[6] Cochrane (2018) as well as Sims (2011) abstract from government consumption, g_t, in their framework, such that primary surpluses correspond to taxes, $s_t = T_t$.

[7] Hence, we focus on the standard no-bubble solution (e.g., Sims, 2011; Cochrane, 2018). There is a literature showing that a 'bubble term' can be important for the budget constraint (cf. Reis 2021).

which shows that the initial response of the bond price is determined entirely by the discounted and maturity-adjusted path of the nominal interest rate. If we use the average duration of 6.8 years from the central bank's Security Open Market Account (SOMA), we calibrate $\delta = 1/6.8$ and $\chi = 0.03$ (see Del Negro and Sims, 2015).[8]

In contrast to the discrete-time model, the price level p_t cannot jump and is given by past price quotations (Calvo's insight).[9] Because the number of outstanding bonds in (2.6) is fixed and cannot jump either, only the bond price p_t^b, which is determined in general equilibrium, can jump due to changes in either future surplus s_u or the future discount rate $i_u - \pi_u$ for $u \geq t$ (direct FTPL effect). Because with short-term debt $p_t^b \equiv 1$, the direct FTPL requires the presence of longer-term debt. The bond price effect then passes on to the value of debt, inducing a jump in a_t (market value), i.e., a forward-looking variable. Hence, the average duration δ of the maturity structure of government debt determines the strength of the direct FTPL effect, such that $\delta \to \infty$ eliminates jumps in p_t^b.

The path of the primary surplus on the right-hand side of equation (2.6) is determined by fiscal policy, so by assumption, surpluses typically do not jump if the value of sovereign debt changes (we discuss different scenarios below). Hence, changes in fiscal policy are accommodated by the real interest rate (indirect FTPL effect) such that (2.6) is not violated. So even without the presence of long-term debt, monetary policy must accommodate future changes in fiscal policy. Although households are indifferent with respect to the maturity of government debt because of arbitrage, the bottom line of this paper is to show that it has important implications for inflation dynamics, the term structure, inflation expectations, and the real economy. Thus, for ease of illustration, we focus on a fiscal regime (or fiscal dominance) throughout the paper, while the insights are useful for a more realistic regime-switching approach, as in Bianchi and Melosi (2019).

2.2.2 Simple Fiscal Policy Rules Versus Policy Inertia

There seems to be a consensus among economists that there is a systematic response of fiscal policy to the state of the economy. While theoretical papers often assume contemporaneous responses using simple fiscal policy rules (Sims, 2011; Cochrane, 2018), most empirical studies suggest that there is a time lag (inertia) between the relevant variables and the policy response, such as changes in the tax code or a revised public expenditure budget plan (cf. Kliem et al., 2016; Bianchi and Melosi, 2019). In this paper, we provide a general framework, where the specifications can be coherently studied and which allows us to investigate the effects of temporary and permanent shocks. Starting with the central

[8]Below we use a zero-coupon bond with time-to-maturity of $1/\delta$ years interchangeably.

[9]Because no mass of firms can change prices instantaneously, the NK Phillips curve allows a jump in the inflation rate but not in the price level (cf. Cochrane, 2018, Online Appendix). Here, the price-level jump of the discrete-time model rather translates into a smooth change by affecting inflation.

NK-FTPL equation in (2.5), $s_t \equiv T_t - g_t$, and specifying a tax rule as

$$\mathrm{d}T_t = \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t, \tag{2.9}$$

where ρ_τ controls the degree of inertia with $\rho_\tau \to \infty$ as the flexible limit (feedback rule), in which $T_t = T_t^* + \tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss})$. For $\rho_\tau \to 0$ we obtain the inelastic limit where $T_t \equiv T_t^*$. This fiscal policy is accompanied by a rule for government spending

$$\mathrm{d}g_t = \rho_g \left(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_t^*)\right) \mathrm{d}t, \tag{2.10}$$

where ρ_g controls the degree of inertia with $\rho_g \to \infty$ as the flexible limit (feedback rule), in which $g_t = g_t^* + \varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss})$. For $\rho_g \to 0$ we obtain the inelastic limit where $g_t \equiv g_t^*$. In what follows, we refer to the model parameters, or more generally, to the levels of government expenditures, taxes, and debt as 'fiscal policy', such that

$$\begin{aligned} \mathrm{d}s_t &= \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t \\ &\quad - \rho_g \left(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_t^*)\right) \mathrm{d}t. \end{aligned}$$

Note that we could add others variables such as the inflation rate, π_t, which will be a function of the relevant state variables.[10] In a linearized version, such addition of variables gives rise to different parametrization of the responses to the state variables. Our results thus shed light on reasonable fiscal policy rules, which ultimately is an empirical question and beyond the scope of our analysis (e.g., Kliem and Kriwoluzky, 2014).

Kliem and Kriwoluzky (2014) show that the standard fiscal policy rules, in which tax rates respond to the level of output, are not supported by the data. Most contributions in the FTPL literature, such as Sims (2011) and Cochrane (2018), study models with an output response only.[11] Kliem et al. (2016) find that there is only weak empirical evidence in favor of output in fiscal policy rules, but rather evidence in favor of responses with respect to the fiscal stance (such as the level of debt or debt-to-GDP ratios). We follow the conventional approach and focus on (locally) determinate solutions only. As shown in Leith and von Thadden (2008), this has important implications for the admissible parameter set for a particular regime, in particular the size of parameters τ_a and φ_a.[12]

Our benchmark parametrization closely follows Kliem and Kriwoluzky (2014), which allows for inertia in the fiscal policy rule for tax revenues. Since our focus is on the effects of maturity on the transmission of shocks, we abstract from introducing distortionary taxes. We focus on a tax rule (2.9) with an output response $\tau_y > 0$ and an inelastic fiscal

[10] With a fiscal policy rule responding to inflation, a higher interest rate may produce lower inflation even with short-term debt (cf. Cochrane, 2022b, Chap. 5.7).

[11] Note that Sims (2011) and Cochrane (2018) impose $\rho_\tau \to \infty$ (feedback rule), and the fiscal policy rule $g = s_g(y/y_{ss} - 1)$ can be replicated for $\rho_g \to \infty$ (feedback rule) and by setting $\varphi_y = s_g$.

[12] See Section 3.3.2 for an elaborate discussion on determinacy regions.

Table 2.1: Parametrization 1 (benchmark, similar to Kliem and Kriwoluzky 2014).

ρ	0.03	subjective rate of time preference
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady state output
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	1	output response fiscal tax rule (Sims, 2011; Cochrane, 2018)
τ_a	0	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	0	output response fiscal expenditure rule
φ_a	0	debt response fiscal expenditure rule
ρ_g	0	inertia of fiscal expenditure rule
s_g	0.1534	government consumption to output ratio (Bilbiie et al., 2019)
s_{ss}	0.0324	steady-state surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

expenditure target such that $g_t \equiv g_t^*$ with $\rho_g \to 0$, and a corresponding T_t^* to match the US debt-to-GDP ratio of about 108% right before the pandemic (2020Q1).[13] We follow Bilbiie et al. (2019) and set the steady-state government consumption-to-output ratio equal to 15.34%. A higher share of government consumption-to-output of about 20%, similar to Justiniano et al. (2013) and Eichenbaum et al. (2020), only slightly affects the model dynamics.

Our benchmark parametrization is summarized in Table 2.1 such that the implied fiscal rule $f(s_t, y_t, a_t)$, in the law of motion for primary surplus (2.5), takes the form

$$f(s_t, y_t, a_t) \equiv y_t/y_{ss} - 1 - (s_t - s_t^*). \tag{2.11}$$

Market clearing and the fiscal policy rule then imply (cf. Appendix 2.A.1):

$$y_t/y_{ss} - 1 = (1 - s_g)x_t. \tag{2.12}$$

[13]U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, Federal Debt: Total Public Debt as Percent of Gross Domestic Product [GFDEGDQ188S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GFDEGDQ188S, January 13, 2022.

Hence, the equilibrium dynamics can be summarized as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(2.13a)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t && \text{(2.13b)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\mathrm{d}t && \text{(2.13c)}\\
\mathrm{d}a_t &= ((i_t - \pi_t)a_t - s_t)\mathrm{d}t && \text{(2.13d)}\\
\mathrm{d}s_t &= ((1 - s_g)x_t - (s_t - s_t^*))\mathrm{d}t && \text{(2.13e)}
\end{aligned}$$

in which x_t, π_t are forward-looking (jump) variables, and a_t satisfies (2.6).[14]

2.2.3 Solution to the Linearized Equilibrium Dynamics

Following the FTPL literature, we solve a linearized system around the steady state for the initial values π_0 and x_0 given the state variables i_0, a_0, and s_0.[15] To this end, we use an eigenvalue-decomposition on the Jacobian matrix of the set of differential equations and study the local dynamics induced by an unexpected (zero-probability) shock on the stable manifold back to a steady state. Technically, we solve the system using the stable eigenvalues in order to find the unique (backward) solution. The jumps in forward-looking variables π_t and x_t, together with zero-probability shocks to the state variables i_t, a_t, and s_t, determine the initial values of the endogenous model variables.

In case of long-term debt, we use the bond price equation (2.7) and the dependence of a_t on the price in p_t^b from the valuation equation (2.6). Note that we need the bond price equation (2.7) only to pin down the initial price jump (direct FTPL effect), which translates to a shock to a_t. For example, consider a monetary policy shock $\mathrm{d}\varepsilon_i \equiv i_t - i_{t-}$ in the model with longer-term debt and store the implied initial price jump $\mathrm{d}\delta_{p_t^b} \equiv p_t^b - p_{t-}^b$. Consider then the same monetary policy shock $\mathrm{d}\varepsilon_i$ in the model with short-term debt, without bond price effects (no direct FTPL effect), and a contemporaneous shock $\mathrm{d}\varepsilon_a \equiv a_t - a_{t-} = \mathrm{d}\delta_{p_t^b}$, i.e., use the stored price jump as an additional structural shock to a_t, the short-term debt model has exactly the same solution as the model with long-term debt.

Proposition 6 (Linear solution) *The linear approximation to the system of the model's equilibrium dynamics (2.13) implies a set of functions for given states (i_t, a_t, s_t)*

$$\begin{aligned}
x_t &= \bar{x}_i(i_t - i_{ss}) + \bar{x}_a(a_t - a_{ss}) + \bar{x}_s(s_t - s_{ss}), && \text{(2.14a)}\\
\pi_t &= \pi_{ss} + \bar{\pi}_i(i_t - i_{ss}) + \bar{\pi}_a(a_t - a_{ss}) + \bar{\pi}_s(s_t - s_{ss}), && \text{(2.14b)}\\
p_t^b &= p_{ss}^b + \bar{p}_i^b(i_t - i_{ss}) + \bar{p}_a^b(a_t - a_{ss}) + \bar{p}_s^b(s_t - s_{ss}), && \text{(2.14c)}
\end{aligned}$$

[14] For an alternative parametrization, $f(s_t, y_t, a_t) \equiv (\tau_a - \varphi_a)(a_t - a_{ss}) - (s_t - s_t^*)$ together with a slightly changed Phillips curve (2.13b).

[15] Alternative approaches, which can account for non-linearities and risk, either solve the boundary value problem for a grid of state variables to approximate the policy function (cf. Posch, 2020), or use perturbation (cf. Parra-Alvarez et al., 2021) to obtain the policy functions.

where bars denote the partial derivatives (slopes), evaluated at (i_{ss}, a_{ss}, s_{ss}):

$$\begin{aligned}
\bar{x}_i &= x_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}^b_i v_{ss}\bar{x}_a/(1 - v_{ss}\bar{p}^b_a), \\
\bar{x}_a &= x_v(i_{ss}, v_{ss}, s_{ss})p^b_{ss}(1 - v_{ss}\bar{p}^b_a)/(1 - v_{ss}\bar{p}^b_a + p^b_{ss}v_{ss}\bar{p}^b_a), \\
\bar{x}_s &= x_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}^b_s v_{ss}\bar{x}_a/(1 - v_{ss}\bar{p}^b_a), \\
\bar{\pi}_i &= \pi_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}^b_i v_{ss}\bar{\pi}_a/(1 - v_{ss}\bar{p}^b_a), \\
\bar{\pi}_a &= \pi_v(i_{ss}, v_{ss}, s_{ss})p^b_{ss}(1 - v_{ss}\bar{p}^b_a)/(1 - v_{ss}\bar{p}^b_a + p^b_{ss}v_{ss}\bar{p}^b_a), \\
\bar{\pi}_s &= \pi_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}^b_s v_{ss}\bar{\pi}_a/(1 - v_{ss}\bar{p}^b_a), \\
\bar{p}^b_i &= p^b_i(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}^b_a), \\
\bar{p}^b_a &= p^b_v(i_{ss}, v_{ss}, s_{ss})/(1 + v_{ss}p^b_n(i_{ss}, v_{ss}, s_{ss})/p^b_{ss}), \\
\bar{p}^b_s &= p^b_s(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}^b_a).
\end{aligned}$$

Here, $v_t \equiv n_t/p_t$ defines the real number of bonds because the partial derivatives in terms of a_t (market value) reflect the indirect effects only, keeping fixed the price of government debt, p^b_t, while the total effects are visible only in terms of v_t (face value).

Proof. Appendix 2.A.3 ■

Our linearized solution (2.14) thus gives the policy functions in terms of v_t in Figure 2.1. For illustration, we also show the policy functions in terms of a_t (cf. Figure 2.2). Except for the bond price p^b_t, the policy functions coincide for different maturity structures and correspond in terms of a_t to the short-term debt case in terms of v_t. Figure 2.1 sheds light on how the maturity structure of government debt matters for the responses of macro aggregates with changes in the state variables. Probably the most striking result is the link between inflation and interest rates: For the average duration of government bonds in the data (blue solid), we obtain the traditional negative link between interest rates and current inflation rates. This shows that the fiscal regime is crucial to the traditional effect of monetary policy. A knife-edge case exists in which the direct FTPL effect offsets the indirect effect and interest rates would have no contemporaneous effect on inflation.

2.2.4 Term Structure of Interest Rates

The term structure of interest rate, defined as the yield of zero-coupon bonds as a function of their maturity, reveals important insights on expectations about the future path of macro aggregates and inflation. Given the equilibrium prices, we can price any asset. The no-arbitrage condition implies that the asset prices adjust such that the households will be indifferent in their portfolio decision. Let us consider a nominal (zero-coupon) bond with unity payoff at maturity N:

$$p^{(N)}_t = \mathbb{E}_t\left(e^{-\rho N}\lambda_{t+N}/\lambda_t e^{-\int_t^{t+N}\pi_u du}\right), \qquad (2.15)$$

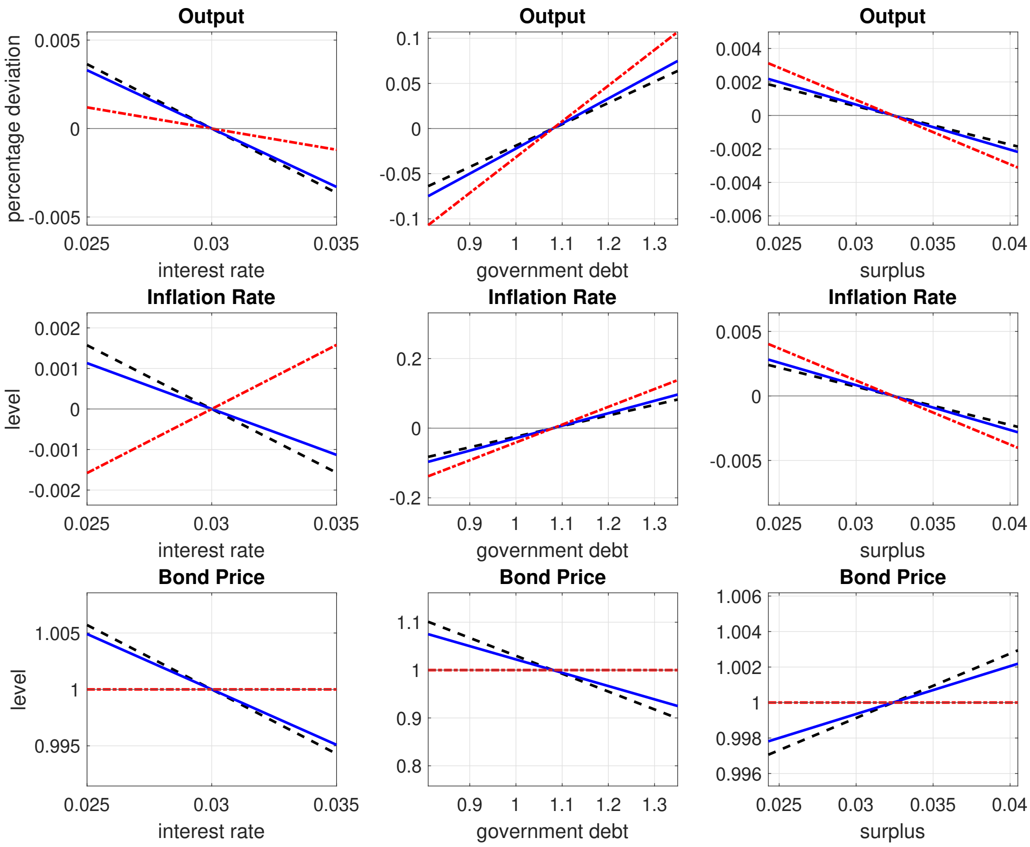


Figure 2.1: Policy functions for the parametrization in Table 1, showing the total response in terms of v_t (indirect and direct effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

where λ_t is the marginal value of wealth, or the current value shadow price, consistent with equilibrium dynamics of macro aggregates. Note that the equilibrium price p_t^b can be computed along the same lines (because the maturity distribution is approximately exponential with a duration of $1/\delta$, the average-maturity bonds will share the same properties as zero-coupon bonds at maturity $1/\delta$). The equilibrium bond price can be obtained from the fundamental pricing equation for the price $p_t^{(N)}$ (Cochrane, 2005, chap. 19.4):

$$\mathbb{E}_t\left(\left(\mathrm{d}p_t^{(N)}\right)/p_t^{(N)}\right) - \left(1/p_t^{(N)}(\partial p_t^{(N)}/\partial N) + i_t\right)\,\mathrm{d}t = 0. \tag{2.16}$$

Observe that in equilibrium, the bond price $p_t^{(N)}$ is a function of the state variables, so $p_t^{(N)} = p_t^{(N)}(i_t, a_t, s_t)$, where from (2.13c), (2.13d), and (2.13e) we get

$$\begin{aligned}\mathrm{d}p_t^{(N)} &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t)\,\mathrm{d}t\\ &\quad+(\partial p_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t\\ &\quad+(\partial p_t^{(N)}/\partial s_t)((1-s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t\end{aligned}$$

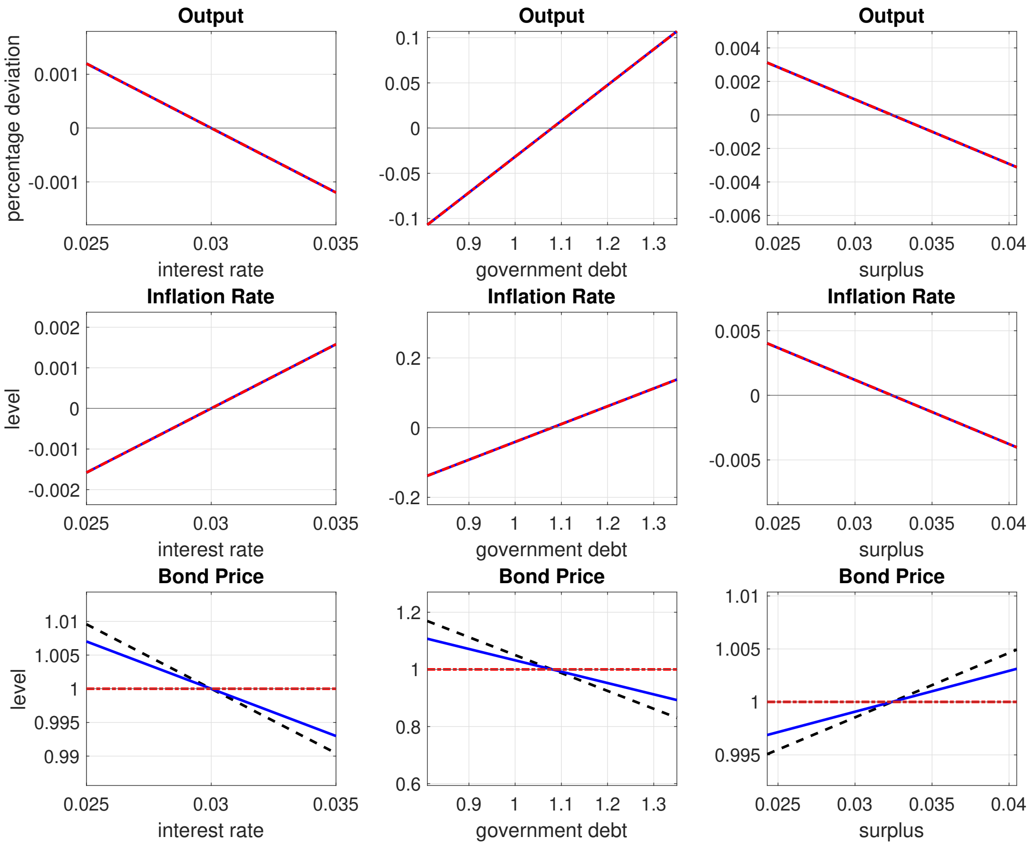


Figure 2.2: Policy functions for the parametrization in Table 1, showing the partial response in terms of a_t (indirect effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

together with the solution (2.14) and thus the PDE (henceforth *PDE approach*) reads:

$$(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t) + ((1 - s_g)x_t - (s_t - s_t^*))(\partial p_t^{(N)}/\partial s_t)$$
$$+((i_t - \pi_t)a_t - s_t)(\partial p_t^{(N)}/\partial a_t) = (\partial p_t^{(N)}/\partial N) + i_t p_t^{(N)}. \tag{2.17}$$

The solution to the pricing equation implies the complete term structure of interest rate for any given interest rate and maturity:

$$y_t^{(N)} \equiv y^{(N)}(i_t, a_t, s_t) = -\log p_t^{(N)}(i_t, a_t, s_t)/N. \tag{2.18}$$

Our strategy is to use collocation to approximate the function $p_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$, in which v is an n-vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a *linear* interpolation equation:

$$(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2'(N, i_t, a_t, s_t)v + ((i_t - \pi_t)a_t - s_t)\Phi_3'(N, i_t, a_t, s_t)v$$
$$+((1 - s_g)x_t - (s_t - s_t^*))\Phi_4'(N, i_t, a_t, s_t)v = \Phi_1'(N, i_t, a_t, s_t)v + i_t\Phi(N, i_t, a_t, s_t)v,$$

or

$$\big((1-s_g)x_t-(s_t-s_t^*)\big)\Phi_4'+((i_t-\pi_t)a_t-s_t)\Phi_3' \\ +(\phi_\pi(\pi_t-\pi_t^*)-(i_t-i_t^*))\Phi_2'-\Phi_1'-i_t\Phi\big)v=0_n, \qquad (2.19)$$

where $n=n_1\cdot n_2\cdot n_3\cdot n_4$ with boundary condition $\Phi(0,i_t,a_t,s_t)v=1_n$. So we concatenate the two matrices and solve the linear system for the unknown coefficients. While in this paper, we focus on the expectation channel and abstract from other determinants such as risk premia and liquidity, an extension to include risk and term premia in the analysis is straightforward (cf. Posch, 2020). In particular we want to study the effects of temporary and permanent shocks on the term structure of interest rates.

2.2.5 Inflation Decomposition and Expected Inflation

Inflation and expected inflation are key determinants of monetary policy. In what follows we decompose the total effects of structural shocks on those key variables from their theoretical impulse response functions (IRFs). By the decompositions we answer the question how much such shocks contribute to the observed response.

For our decomposition based on the IRFs, we start with the linearized debt evolution using $r\equiv i_{ss}-\pi_{ss}=\rho$ and $s_{ss}=\rho a_{ss}$ (our decomposition follows Cochrane, 2022a,b)

$$\mathrm{d}(a_t/a_{ss}-1)=(i_t-\pi_t+r(a_t/a_{ss}-1)-s_t/a_{ss})\mathrm{d}t$$

and

$$a_t/a_{ss}-1=\mathbb{E}_t\int_t^\infty e^{-r(u-t)}s_u/a_{ss}\mathrm{d}u-\mathbb{E}_t\int_t^\infty e^{-r(u-t)}(i_u-\pi_u)\mathrm{d}u,$$

which is the linearized present value formula corresponding to (2.6). The real value of debt is the present value of surpluses, discounted at the real interest rate.

From the linearized definition (2.6), the real value of sovereign debt (market value) can be decomposed into

$$a_t/a_{ss}-1=v_t/v_{ss}-1+p_t^b/p_{ss}^b-1, \qquad (2.20)$$

either by changes in debt issued or valuation (direct effects). Hence, we get the identity

$$\int_t^\infty e^{-r(u-t)}\pi_u\mathrm{d}u = \int_t^\infty e^{-r(u-t)}i_u\mathrm{d}u-\int_t^\infty e^{-r(u-t)}s_u/a_{ss}\mathrm{d}u \\ +p_t^b/p_{ss}^b-1+v_t/v_{ss}-1 \qquad (2.21)$$

in the perfect-foresight model, which allows us, for example, to decompose the effects of zero-probability shocks on present values of future inflation into changes in the present value of future interest rates (monetary policy), the present value of changes in future surpluses (fiscal policy), and the direct effects (real debt decomposition).

Moreover, from (2.8) and with $\chi \equiv r$ and $v_t \equiv v_{ss}$ in the perfect-foresight model

$$p_t^b = 1 - \int_t^\infty e^{-(r+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u,$$

we conclude that the strength of the direct FTPL bond price effect depends on both the average maturity $1/\delta$ and the expected future path of monetary policy, at $t = 0$,

$$\int_0^\infty e^{-ru}(\pi_u - \pi_{ss})\mathrm{d}u = \int_0^\infty e^{-ru}\left(1 - e^{-\delta u}\right)(i_u - i_{ss})\mathrm{d}u - \int_0^\infty e^{-ru}(s_u - s_{ss})/a_{ss}\mathrm{d}u.$$

The effect is strongest for perpetuities with $\delta \to 0$, where all changes in future interest rates (monetary policy) will be soaked up in an initial re-evaluation of sovereign debt, and fiscal policy fully determines inflation. In contrast, in the short-term model with $\delta \to \infty$, changes in future monetary policy affect future expected inflation most.

Similarly, inflation expectations are at the core of monetary policy, often considered even as a separate variable. Hence, we can study the effects of monetary and fiscal policy shocks on the model-implied expected inflation, e.g., to confront the rational expectation forecast results with survey data. From the Phillips curve in (2.13b) it follows

$$\pi_t - \pi_t^* = \kappa \int_t^\infty e^{-\rho(v-t)} x_u \mathrm{d}u.$$

The inflation rate, π_t, denotes *current* expected inflation measured as deviation from its policy target rate π_t^*. Multiplying the differential equation for the inflation rate by the integrating factor and evaluating from t to $t+N$, we obtain

$$\pi_t^{(N)} \equiv \mathbb{E}_t(\pi_{t+N}) = \pi_t^* + e^{\rho N}(\pi_t - \pi_t^*) - \kappa e^{\rho N} \int_t^{t+N} e^{-\rho(u-t)} x_u \,\mathrm{d}u. \tag{2.22}$$

Intuitively, the model-implied inflation forecast is a forward contract to inflation, which can be more informative than using forward rates (Gürkaynak et al., 2007). We compute the rational expectation forecast π_{t+N} as a function of the current state variables (i_t, a_t, and s_t) and the fixed forecasting horizon N. Hence, for the N-year ahead future expected inflation rate, we compute $\pi_t^{(N)}$ from (using Feynman-Kac)

$$\begin{aligned} \partial\pi_t^{(N)}/\partial N &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial\pi_t^{(N)}/\partial i_t)\,\mathrm{d}t \\ &\quad + (\partial\pi_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + (\partial\pi_t^{(N)}/\partial s_t)((1 - s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t \end{aligned}$$

together with the known solution (2.14) and by imposing the boundary condition $\pi_t^{(0)} = \pi_t$. Similar to the term structure of interest rates, the solution to the PDE then implies the

Table 2.2: Inflation decomposition (2.21) for the monetary policy shock in Figure 2.3.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_v \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-rv} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	-0.29	-1.14	0.29	1.14
Average	-0.48	-1.25	0.21	0.98
Short-Term	-1.62	-1.91	-0.29	0

N-years ahead inflation expectations for a given state variable as

$$\pi_t^{(N)} = \pi^{(N)}(i_t, a_t, s_t). \tag{2.23}$$

Our strategy is to use collocation to approximate the function $\pi_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$. The n-vector v is a vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from the *linear* interpolation equation

$$\begin{aligned}\big(((1-s_g)x_t - (s_t - s_t^*))\Phi_4' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\ +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1'\big)v = 0_n,\end{aligned}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$ with the boundary condition $\Phi(0, i_t, a_t, s_t)v = 1_n \cdot \pi_t$. So we concatenate the two matrices and solve the linear system for the unknown coefficients.

Because the model time unit is years, the N-year ahead inflation forecast $\pi_t^{(N)}$ refers to the empirical NY1Y measure. As a simple approximation, we may define the weighted sum of N-year ahead inflation forecast for the successive k years $\pi_t^{(N,k)}$ as

$$\pi_t^{(N,k)} \approx (1/k)\ln\Big(\sum_{i=N}^{k}\big(1+\pi_t^{(i)}\big)\Big). \tag{2.24}$$

2.2.6 Monetary and/or Fiscal Policy and Transitional Dynamics

Defining monetary policy shocks as changes in monetary policy with no exogenous changes in surplus (cf. Cochrane, 2018), we can answer the question of how maturity matters in the model for the transition of unexpected (zero-probability) shocks. Similarly, we consider unexpected changes in fiscal policy without changing the nominal interest rate.

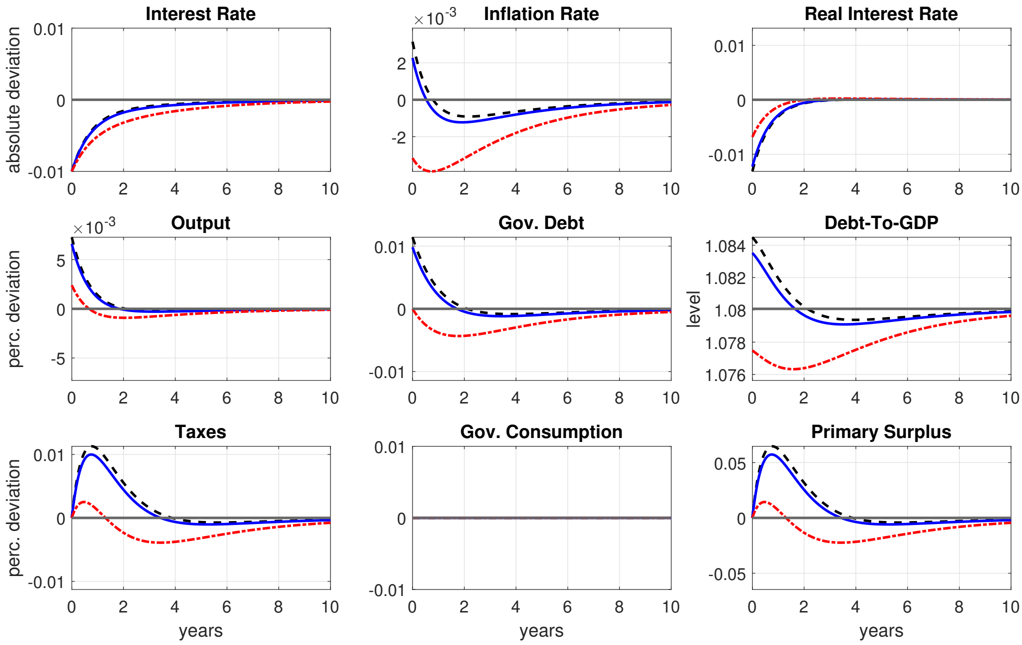


Figure 2.3: Transitory monetary policy shock for the parametrization in Table 2.1. Decrease in nominal interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Transitory Shocks

Consider an expansionary transitory monetary policy shock of 100 basis points (bp), i.e., the policy rate i_t decreases unexpectedly by 1 percentage point. That unexpected decrease in nominal interest rates i_t initially has expansionary effects on output because the real interest rate decreases (cf. Figure 2.3). This effect is larger the longer the average maturity of government debt (i.e., 'stepping on a rake effect of inflation' for perpetuities). Here, the maturity structure matters because the monetary policy shock decreases the real interest rate even more for long-term bonds (black dashed) than with only short-term debt (red dotted). Because with short-term debt the direct FTPL effect is missing, the real debt does not respond immediately and we are left with the indirect FTPL effect, which unambiguously lowers inflation on impact (cf. Cochrane, 2018).

Fiscal authorities now habitually react following the specified fiscal rule and respond to the increased output by higher surpluses from increased tax receipts. A higher surplus then lowers inflation (cf. Figure 2.1), which again slowly increases the real interest rate. While the sign of the initial response of inflation depends on the maturity structure, which is basically dictated by the policy functions, future expected inflation turns negative for all maturities (as shown in Figure 2.4). In fact, the net present value of future expected inflation is negative, ranging from -0.29 to -1.62 percentage points depending on the maturity of government debt (cf. Table 2.2). Here, the negative effect on inflation can be

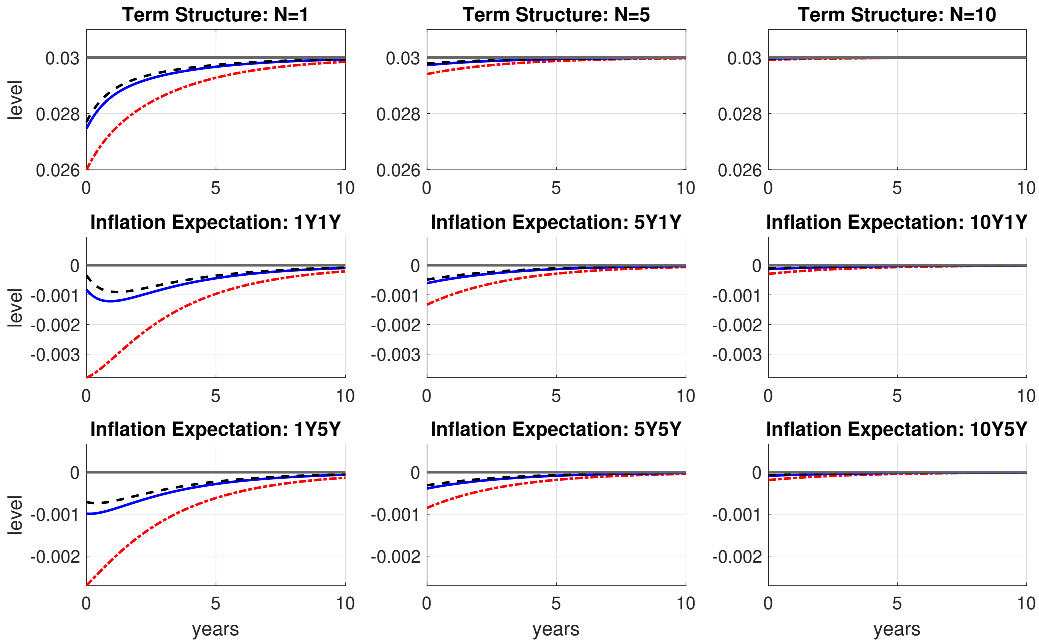


Figure 2.4: Transitory monetary policy shock for the parametrization in Table 2.1. Decrease in nominal interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.3: Inflation decomposition (2.21) for the fiscal policy shock in Figure 2.5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	0.29	0.17	−0.29	−0.17
Average	0.34	0.20	−0.27	−0.12
Short-Term	0.48	0.28	−0.20	0

attributed to either fiscal policy (black dashed), where future monetary policy is soaked up by higher bond prices, or a mix of monetary and fiscal policy, which is buffered by *lower* net present value of future tax receipts (solid blue and red dotted).

The direct FTPL effect increases the value of government debt as bonds appreciate, even more than output in the case of perpetuities such that lower interest rates initially lead to a higher debt-to-GDP ratio. With short-term debt only, essentially the picture is reversed: government debt initially is reduced because of higher output, which in turn leads to a substantially lower debt-to-GDP ratio.

Along the same line, defining fiscal policy as a change in the surplus (or its compo-

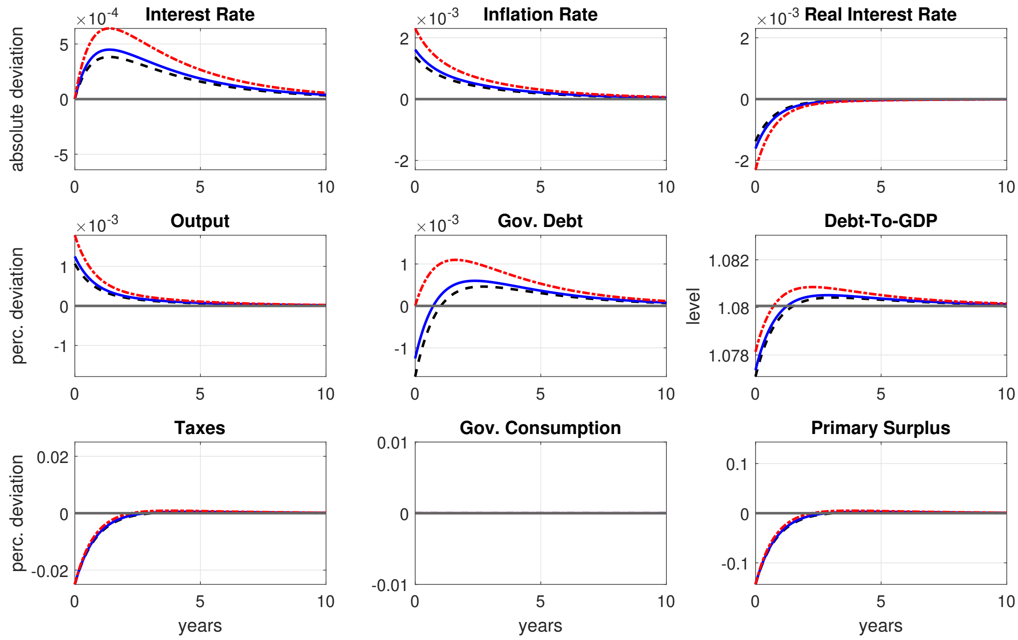


Figure 2.5: Transitory fiscal policy shock for the parametrization in Table 2.1. Decrease in taxes (surplus) by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

nents), with no change in monetary policy, we can answer the question of how maturity matters in the model for the transition of zero-probability fiscal policy shocks. Consider an expansive fiscal policy shock (cut T_t by 2.5 percent). That unexpected cut in taxes (decreases surplus s_t) has expansionary effects on output and thus unambiguously increases inflation and leads to higher inflation expectations, such that for a given short-term rate, the real interest rate is lower (cf. Figures 2.5 and 2.6).

Hence, expansive fiscal policy (decreased surplus) leads to more inflation and lowers the real interest rate (cf. Figure 2.1). This in turn causes the monetary authority, following a Taylor rule, to slightly increase nominal rates, whereas the effects on 5-year bond yields are being driven mainly by higher inflation expectations. Lower primary surpluses, after an initial devaluation of real government debt, lead to further accumulation of debt and are accompanied by higher future inflation. In fact, the net present value of future inflation is positive, ranging from 0.29 to 0.48 percentage points depending on the maturity structure of government debt (cf. Table 2.3). Again, the total effect on inflation can be attributed to either fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or a mix of monetary and fiscal policy (blue solid and red dotted).

After all, the maturity structure of government debt matters most for the direct FTPL effect, which dampens the effects on interest rates, inflation, and output dynamics. The direct FTPL effect decreases the real value of government debt as bonds depreciate and output increases, which initially leads even to a lower debt-to-GDP ratio. Here, the initial

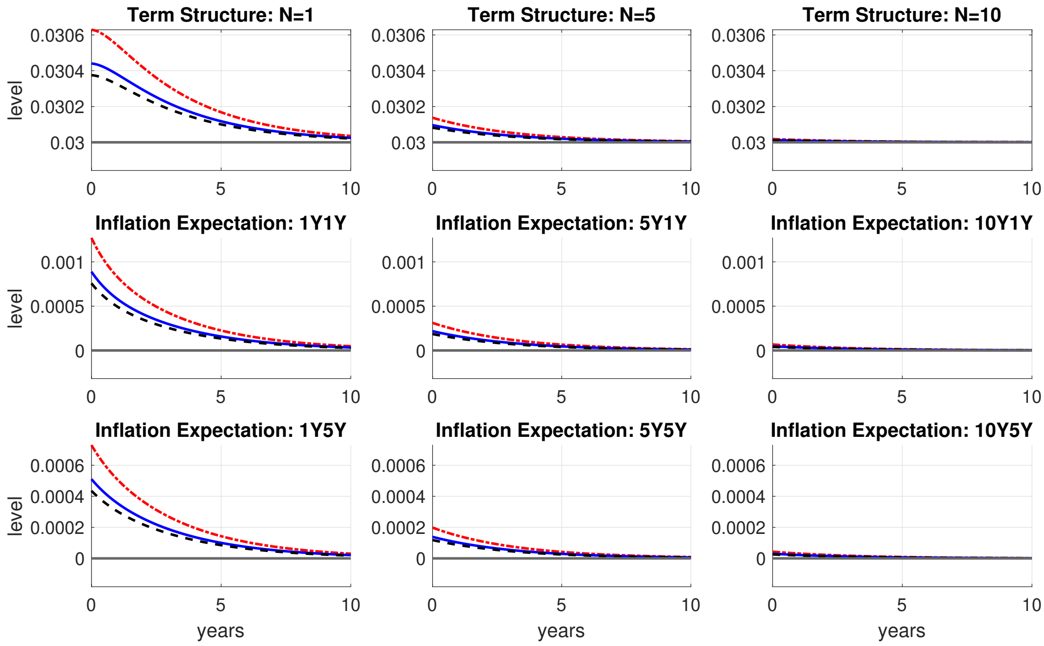


Figure 2.6: Transitory fiscal policy shock for the parametrization in Table 2.1. Decrease in taxes (surplus) by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.4: Inflation decomposition (2.21) for the fiscal policy shock in Figure 2.7.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	2.08	1.21	0.92	-1.21	3.00
Average	2.44	1.42	1.08	-0.90	3.00
Short-Term	3.49	2.03	1.54	0	3.00

deficits are not repaid by subsequent surpluses or output growth but at the cost of higher inflation and more nominal debt, which is inflated away by subsequent *unexpected* inflation with no permanent changes in the real value of debt. This in fact is like a 'partial default' on nominal debt. For the case of short-term debt, higher output leads after a decrease in the debt-to-GDP ratio to more debt accumulation because the direct effect is missing, all deficits are being inflated away. What may seem like a deal, "the trick is to convince people that sinning once does not portend a dissolute life; that this is a once-and-never-again devaluation or at best a rare state-contingent default, not the beginning of a bad habit." (p.245 Cochrane, 2022b).

Finally, consider a fiscal policy shock of issuing new debt (increase n_t by 3 percent).

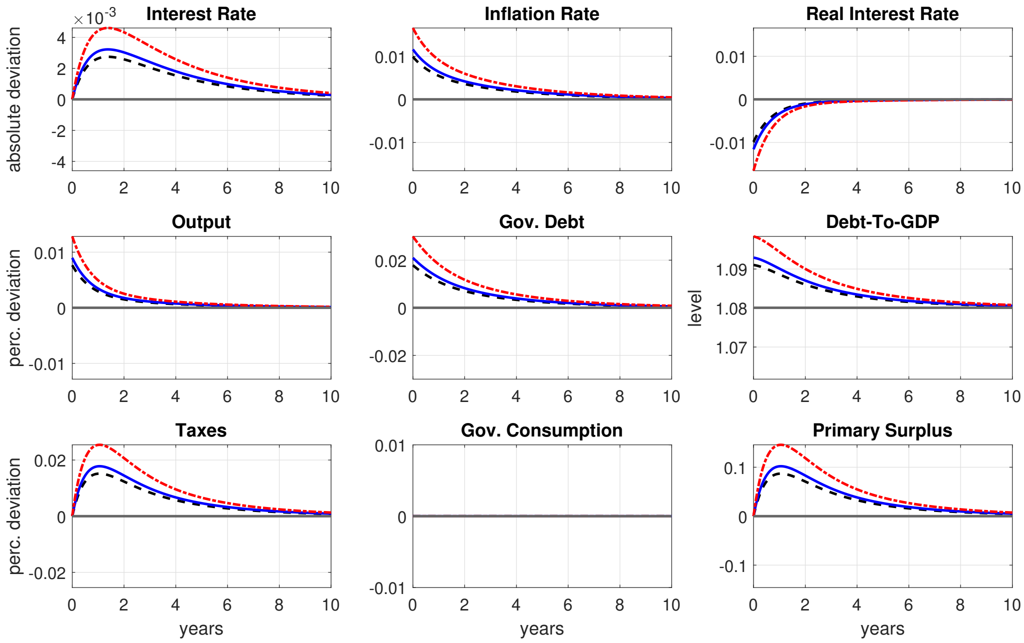


Figure 2.7: Transitory fiscal policy shock for the parametrization in Table 2.1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

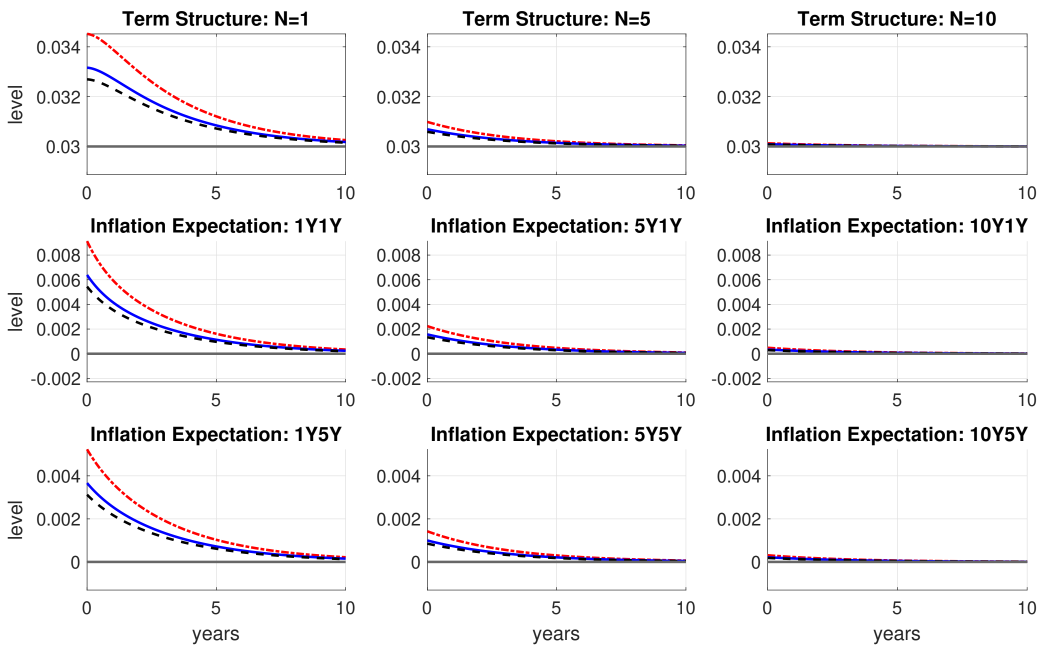


Figure 2.8: Transitory fiscal policy shock for the parametrization in Table 2.1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Suppose that this increase in government debt leaves the average maturity unchanged, and that this unexpected change is without changes in long-run surpluses. Then, the

Table 2.5: Inflation decomposition (2.21) for the monetary policy shock in Figure 2.9.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^{b,new} - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.02	5.16	3.34	−4.91	11.11
Average	2.39	1.88	0.85	−1.24	2.60
Short-Term	0.81	0.96	0.15	0	0

newly issued debt creates unexpected inflation and higher inflation expectations because the debt is not fully paid back by subsequent surpluses (inflate away the debt) and has expansionary effects through a lower real interest rate (cf. Figures 2.7 and 2.8). In fact, the net present value of future expected inflation ranges from 2.08 to 3.49 percentage points depending on the maturity structure of government debt (cf. Table 2.4). It is most striking for long-term debt, where the total effect on inflation and on inflation expectations is smallest as one third of the initial debt shock is repaid by higher surpluses. Only the remainder creates unexpected future inflation, and future monetary policy is soaked up by lower bond prices (black dashed). For the case of short-term debt, the direct effect does not offset monetary policy, which results in the highest net present value of future inflation, even higher than the initial debt shock (red dotted).

Again, the maturity structure of government debt matters because the direct FTPL effect devaluates long-term debt such that the initial increase in real debt (market value) is lower and the effect on inflation is largest for short-term debt. The indirect effect rises inflation and inflation expectations, which forces the monetary authority to increase nominal interest rates. Though the higher output also leads to higher tax receipts and implies a larger future primary surplus, the stimulus only partially accounts for the increased liabilities. Eventually, the unexpected increase in real debt (face value) is inflated away by unexpected future inflation and is only partially repaid by higher surpluses. However, the number of outstanding bonds increases permanently to $n_{ss} = v_{ss} e^{\int_t^\infty \pi_u \mathrm{d}u}$.

Permanent Shocks

Consider a monetary policy shock decreasing the inflation target by 50 bp, or equivalently, the policy interest rate target (which is isomorphic to the inflation target), $i_{ss}^{new} = \rho + \pi_{ss}^{new}$, decreases by 0.5 percentage points. Suppose for the moment that the policy change is fully credible and fully observed, i.e., does not require learning and filtering. An unexpected lower long-term interest rate or inflation target then has an expansionary effect on output because it creates inflation and the real interest rate decreases (cf. Figure 2.9, solid blue).

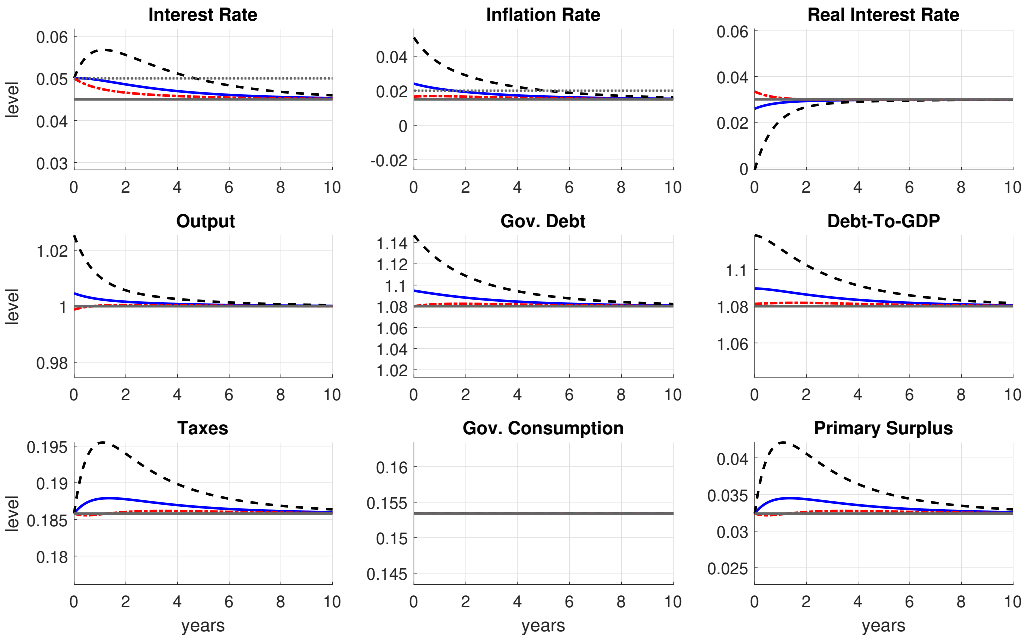


Figure 2.9: Permanent monetary policy shock for the parametrization in Table 2.1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

In all models, independent of the maturity structure, the permanent shock clearly shows up in the 10-year ahead inflation expectations and bond yields (cf. Figure 2.10). While the permanent shock increases the 1-year bond yields up to 50 bp, it decreases 10-year bond yields by 50 bp (cf. Figure 2.10, dashed black). However, in the model with short-term debt only, the permanent lower inflation target would be even contractionary because lower current inflation increases the real interest rate. Most importantly, the maturity structure matters because the permanent shock even *increases* current expected inflation and decreases the real interest rate (solid blue and black dashed). Because the direct FTPL effect is missing in the model with short-term debt, real debt does not respond immediately and we are left with the indirect effect. However, the direct FTPL effect substantially increases the real value of existing long-term government debt such that the lower inflation target leads to a higher debt-to-GDP ratio, higher tax receipts and thus higher primary surpluses. With short-term debt, the picture is different: initially lower tax revenues (primary surpluses) and lower output with only small changes in real debt lead to negligible effects on the debt-to-GDP ratio. Hence, the maturity effect is more pronounced the longer the average maturity of government debt (cf. Table 2.5). In fact, current inflation increases by more than 300 bp in the model with perpetuities with net present value of future inflation of about 8 percent. How can we understand this dramatic response for inflation dynamics in the model with long-term debt?

The simple answer is that the response of inflation is due to a price or valuation effect on existing longer-term bonds, which (still) pay a nominal coupon $\chi + \delta$. Hence, a monetary policy shock in form of a lower inflation target $\pi_t^* \equiv \pi_{ss}^{new} = \pi_{ss} - 0.005$ translates into a higher price $p_{ss}^{b,new}$, and with no change in fiscal surplus results into a lower steady-state value of sovereign debt v_{ss}^{new}. From the decomposition (2.21), we get

$$\begin{aligned}\int_t^\infty e^{-r(u-t)}(\pi_u - \pi_{ss}^{new})\mathrm{d}u \;&=\; \int_t^\infty e^{-r(u-t)}(i_u - i_{ss}^{new})\mathrm{d}u - \int_t^\infty e^{-r(u-t)}(s_u - s_{ss})/a_{ss}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,\end{aligned}$$

or

$$\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u = \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,$$

with a new

$$p_{ss}^{b,new} = \frac{\chi + \delta}{i_{ss}^{new} + \delta}, \quad \text{and} \quad v_{ss}^{new} = a_{ss}/p_{ss}^{b,new}. \tag{2.25}$$

Hence, a permanent monetary policy shock leads to a debt shock $v_t/v_{ss}^{new} - 1$ because of existing longer-term bonds do no longer sell at par in steady state. Relative to the lower new steady state level of government debt v_{ss}^{new} (face value), the current debt level v_t now is above its steady-state level – because debt v_t does not jump, which thus can be interpreted as an 'implicit' expansionary fiscal policy shock (compare to Figure 2.7). This shock is inflationary and its size depends on the maturity structure (cf. Table 2.5). The effect is already sizable with average maturity (by 2.60 percent), and is substantial with longer maturities (up to more than 11 percent for perpetuities). Both direct effects give the change in the market value of government debt. Even the price effect is negative of about -1.24 percent (p_0^b increases, but p_{ss}^b increases even more), the implied debt shock by 2.60 percent leads to an increase of the market value by 1.36 percent.

Along the same line, consider an expansive fiscal policy shock (cut T_t^* by 1 percent).[16] An unexpected change in future tax revenues (decreases surplus s_t^*) has expansionary effects on output today and thus increases current inflation and inflation expectations, which lowers real interest rates (cf. Figures 2.11 and 2.12). The stimulus to output quickly leads to higher tax revenues in the short run at the cost of higher inflation. In this case, the net present value of future inflation is positive, ranging from 4.02 to 6.66 percentage points depending on the maturity structure of government debt (cf. Table 2.6). Our fiscal policy shock leads to an instantaneous devaluation of long-term debt and dampens the effects on interest rate and inflation dynamics. Again, the total effect on inflation can be attributed either to fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or to a mix of monetary and fiscal policy (solid blue and red dotted).

[16] A contemporaneous fiscal policy shock $T_t = 0.99T_{t-}$ with permanent effects, $T_{ss}^{new} = 0.99T_{ss}$ has a similar decomposition and would create more unexpected inflation (cf. Figure 2.A.2 and Table 2.A.2).

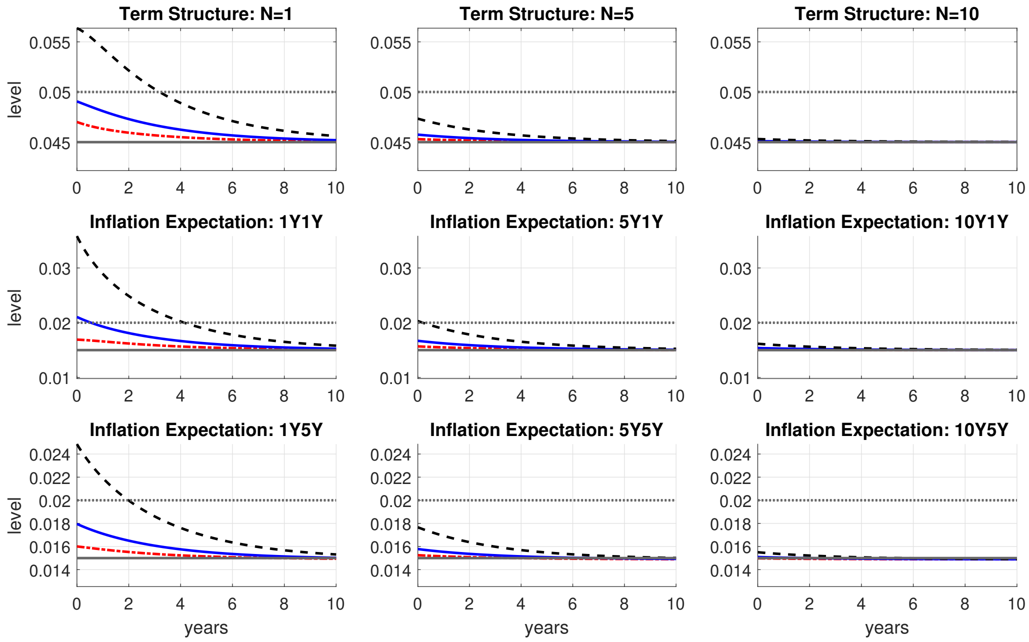


Figure 2.10: Permanent monetary policy shock for the parametrization in Table 2.1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.6: Inflation decomposition (2.21) for the fiscal policy shock in Figure 2.11.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	4.02	2.34	2.07	−2.34	6.08
Average	4.70	2.74	2.38	−1.74	6.08
Short-Term	6.66	3.88	3.31	0	6.08

The indirect effect unambiguously rises inflation (decreases the real interest rate), which causes the monetary authority to adjust the nominal interest rates. Temporarily higher tax revenues (higher surplus) then lead to a further decline of government debt, and the debt-to-GDP ratio converges to its lower steady-state level.

In particular, the change in the target tax receipts, $T_t^* \equiv T_{ss}^{new} = 0.99T_{ss}$ translates into changes in the steady-state values of primary surplus, $s_{ss}^{new} = T_{ss}^{new} - g_{ss}$, and sovereign

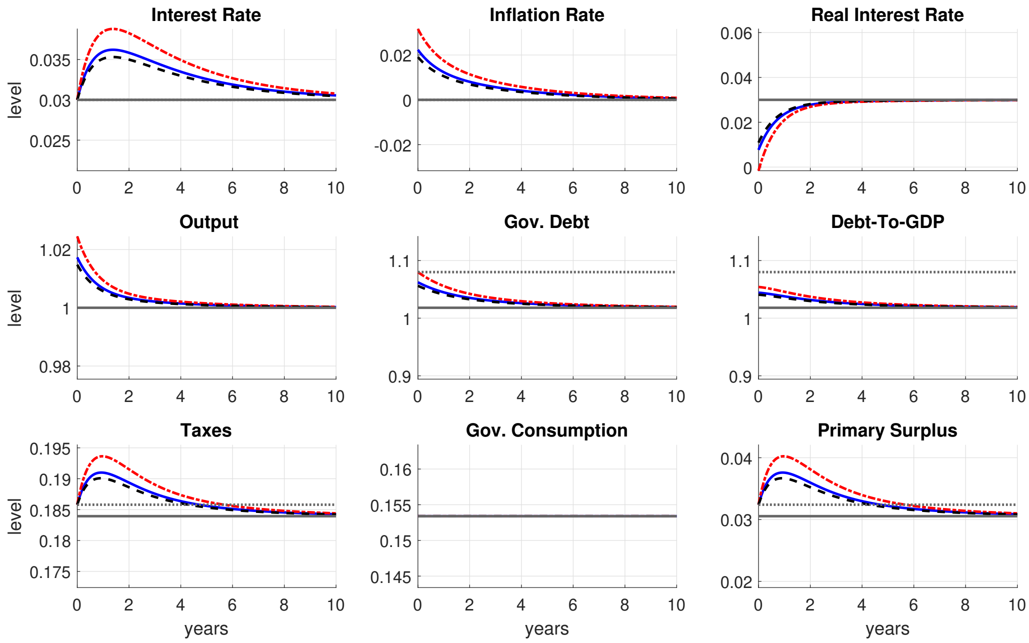


Figure 2.11: Permanent fiscal policy shock for the parametrization in Table 2.1. Decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

debt, $a_{ss}^{new} = s_{ss}^{new}/\rho$ or $v_{ss}^{new} = a_{ss}^{new}/p_{ss}^b$, and from the identity (2.21),

$$\begin{aligned}\int_t^\infty e^{-r(u-t)}(\pi_u - \pi_{ss})\mathrm{d}u &= \int_t^\infty e^{-r(u-t)}(i_u - i_{ss})\mathrm{d}u - \int_t^\infty e^{-r(u-t)}(s_u - s_{ss}^{new})/a_{ss}^{new}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^b - 1 + v_t/v_{ss}^{new} - 1\end{aligned}$$

or

$$\begin{aligned}\int_t^\infty e^{-r(u-t)}\pi_u\mathrm{d}u &= \int_t^\infty e^{-r(u-t)}i_u\mathrm{d}u - \int_t^\infty e^{-r(u-t)}s_u/a_{ss}^{new}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^b - 1 + v_t/v_{ss}^{new} - 1\end{aligned}$$

such that our permanent fiscal policy shock leads to an ‘implicit’ debt shock $v_t/v_{ss}^{new} - 1$, because debt v_t does not jump and is ‘too high’ relative to the new and lower v_{ss}^{new}. More generally, with similar arguments – because of government debt being backed by taxes – any (austerity) measure leading to *higher* tax receipts, T^*, and/or *lower* government consumption, g_t^*, such that the steady-state primary surplus, $s_t^* = T_t^* - g_t^*$, increases, eventually need to *increase* the long-run real bond supply and the real value of government debt (increase the market and face value debt-to-GDP ratio).

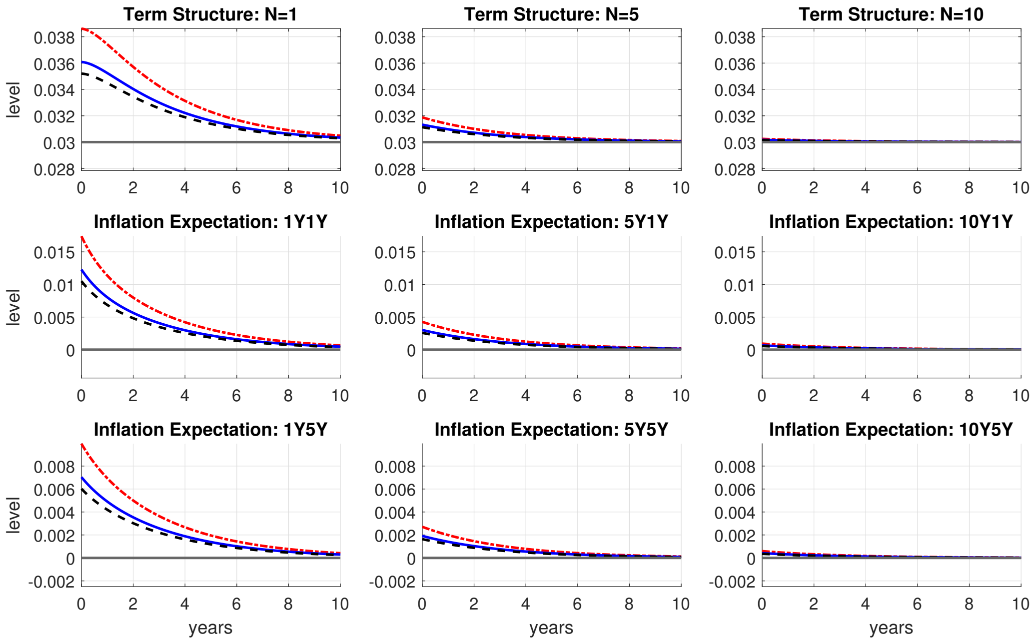


Figure 2.12: Permanent fiscal policy shock for the parametrization in Table 2.1. Decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

2.3 The CARES Act

The Coronavirus Aid, Relief, and Economic Security (CARES) Act is an extensive US economic stimulus package that was signed into law on March 27, 2020, in response to the COVID-19 pandemic. Its central objective was a direct and fast assistance for the real economy in order to keep it afloat and as functioning as possible. The unprecedented volume of the act is estimated to be more than \$2 trillion (10% of US GDP). However, since CARES includes loan guarantees, the Congressional Budget Office (CBO) projects smaller budgetary effects. Still, the CBO estimates that CARES will add \$1.7 trillion to deficits between 2020 and 2030, but most effects take place until 2022.

2.3.1 Taking the Model to the Data

In this section, we translate the empirical data to model variables and assume them to arrive as (structural) zero-probability shocks. Table 2.7 shows the CBO's breakdown of the \$1.7 trillion into outlays and receipts. The size of the budgetary relevant part of the CARES Act exceeds more than 8% of US GDP. Following Kaplan et al. (2020), we presume that the increased outlays (6.1% of GDP) together with decreased revenues (1.9% of GDP) are going to increase the US debt-to-GDP ratio by 12% in the first eighteen months. The lower part of Table 2.7 shows how we transfer the CARES Act into zero-probability shocks in the NK-FTPL model. We attribute the increase in outlays to an unexpected rise in

Table 2.7: Upper Part: Predictions of the CARES Act by the Congressional Budget Office (CBO), the Joint Committee on Taxation (JCT), and estimated effect on debt-to-GDP ratio from Kaplan et al. (2020). Lower part: Translation to NK-FTPL model.

CARES Act: Empirical Figures

		Billions of Dollars	as % of GDP	as % of Outlays (receipts) 2019
A	Increased Mandatory Outlays	988	4.6%	22.2%
B	Increased Discretionary Outlays	326	1.5%	7.3%
C	Decreased Revenues	408	1.9%	11.8%
D	Estimated Increase of debt-to-GDP Ratio: 12% (cf. Kaplan et al., 2020)			

CARES Act: NK-FTPL Model

			abs. Change	as % of GDP	as % of Steady State Value
A + **B**	$\equiv$	Shock g_t	0.061	6.1%	39.8%
C	$\equiv$	Shock T_t	-0.019	-1.9%	-10.2%
D	$\equiv$	Shock v_t by 12% (either temporary and/or permanent)			

Sources: Congressional Budget Office (2020).

g_t by 6.1% of GDP (cf. Table 2.7). Here, the shock in g_t corresponds to an increase in government consumption by about 39.8%. In the empirical data, the rise in mandatory and discretionary outlays amounts to 29.5% of total expenditures in 2019. Analogously we attribute the decrease in revenues as a revenue shock by 1.9% of GDP, which translates to a decrease in tax receipts by 10.2%. Empirically, the decrease in revenues was about 11.8% of total receipts in 2019. It shows that the order of magnitude of shocks in our stylized model is roughly in line with the empirical figures.

For the simulation (see Section 2.3.2 below), we employ our benchmark parametrization in Table 2.1, except for the government expenditures (and thus surplus dynamics). Because we want to model a persistent shock to government consumption with own dynamics, we set $\rho_g \equiv 1$ and assume a counter-cyclical output response of $\varphi_y = -s_g$,

$$\mathrm{d}g_t = (\varphi_y(y_t/y_{ss} - 1) - (g_t - g_t^*))\,\mathrm{d}t, \tag{2.26}$$

e.g., example policies like food stamps, unemployment insurance, or predictable stimulus programs, such that surplus reacts pro-cyclically (cf. Sims, 2011; Cochrane, 2022b).

Moreover, keep in mind that monetary policy was not silent in response to the global coronavirus pandemic, but responded to the large drop in output growth and fears of deflationary pressures. In March 2020, the Federal Reserve decreased the federal funds rate in two steps from 1.58% to 0.05%. Since the timing of the rate cuts and the introduction of the CARES Act was about the same time, we study the additional effects of a temporary expansionary monetary policy shock by 150 bp (see Section 2.3.3). Finally, we consider the case where the unprecedented value of newly issued debt – at least to some degree – permanently increases the debt-to-GDP ratio in both face value v_{ss}^{new}/y_{ss} and market value a_{ss}^{new}/y_{ss} because $p_{ss}^{b} = 1$ (see Section 2.3.4). Our experiment sheds light on the debate of permanent vs. temporary changes in the debt-to-GDP ratio and gives important insights into the predictions of the NK-FTPL model. Recall that debt is backed by taxes such that a higher level of real debt requires a higher future surplus. Hence, we assume that tax receipts ultimately have to rise in the future, while future government consumption remains unchanged (higher value of surplus s_{ss}^{new}). We set T_{ss}^{new} to match a fraction α of the 12% projected increase (face value) in the current *and* the permanent debt-to-GDP ratio. Subsequently, we compute the predicted responses and also analyze a combination of the fiscal shocks together with the contemporaneous monetary policy shock.

2.3.2 The CARES Act Shock

We are mainly interested in quantifying the effects of the large scale fiscal policy operation to which we refer as the CARES Act shock (cf. Table 2.7). Suppose that the economy is at steady state. Without a contemporaneous response of the monetary authority we now study the effects of the shocks to government consumption ($\mathbf{A} + \mathbf{B} = 6.1\%$ of GDP), and to tax receipts ($\mathbf{C} = -1.9\%$ of GDP), such that the steady state primary surplus turns into a large deficit of roughly $s_t = -8.0\%$ of GDP and amounts to nearly -250%. Finally, the CARES Act is projected to increase the debt-to-GDP ratio ($\mathbf{D} = 12\%$ of GDP). In our model, the initial increase in debt also increases output on impact. We define $\mathbf{D}$ as a shock to debt (or equivalently v_t/y_{ss}) rather than a shock to the debt-to-GDP ratio. Both shocks to the primary surplus and to the debt-to-GDP ratio are expansionary and create unexpected current inflation between 6 and 8 percent, and increase, e.g., the 5-year ahead inflation expectations about 1 percent, such that for a given short-term rate, the real interest rate drops substantially (cf. Figures 2.13 and 2.14).

Hence, the CARES Act shock (decreased surplus and increased debt) unambiguously leads to higher inflation, inflation expectations, short-term bond yields, and lowers the real interest rate, which forces the monetary authority to increase nominal rates. Through the lens of fiscal theory, this unprecedented large-scale fiscal program, which is not followed by sufficiently higher subsequent surpluses, is expected to spur inflation and inflation

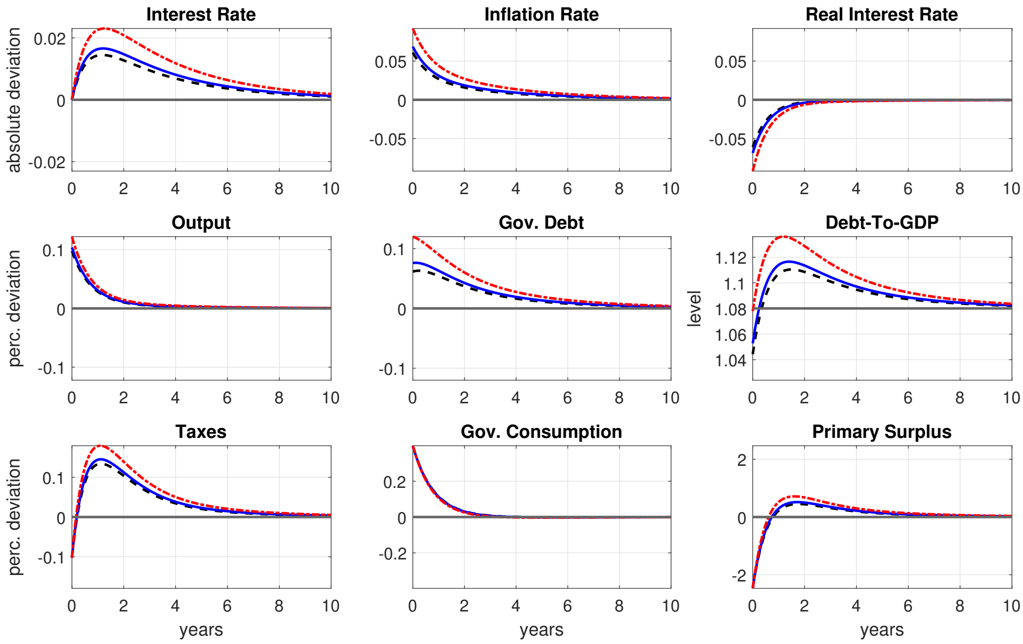


Figure 2.13: Transitory CARES Act shock for the parametrization in Table 2.1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.8: Inflation decomposition (2.21) for the CARES Act shock in Figure 2.13.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	10.05	5.85	1.95	-5.85	12.00
Average	11.68	6.81	2.71	-4.41	12.00
Short-Term	16.68	9.71	5.03	0	12.00

expectations. In particular, the net present value of future inflation is even about the same size of the increase in the debt-to-GDP ratio (11.68 percentage points), and depending on the maturity structure of government debt ranges from 10 to more than 16 percentage points (cf. Table 2.8). Some of the newly issued debt v_t thus can be repaid by a higher net present value of future surpluses between 1.95 and 5.03 percentage points, but most of it will be deflated away by future inflation. Hence, the total effect on inflation can be fully attributed to fiscal policy and the large build-up of government debt (black dashed), where future monetary policy is soaked up by lower bond prices of -5.85 percentage points, or a mix of monetary and fiscal policy with either a slightly smaller response of bond prices

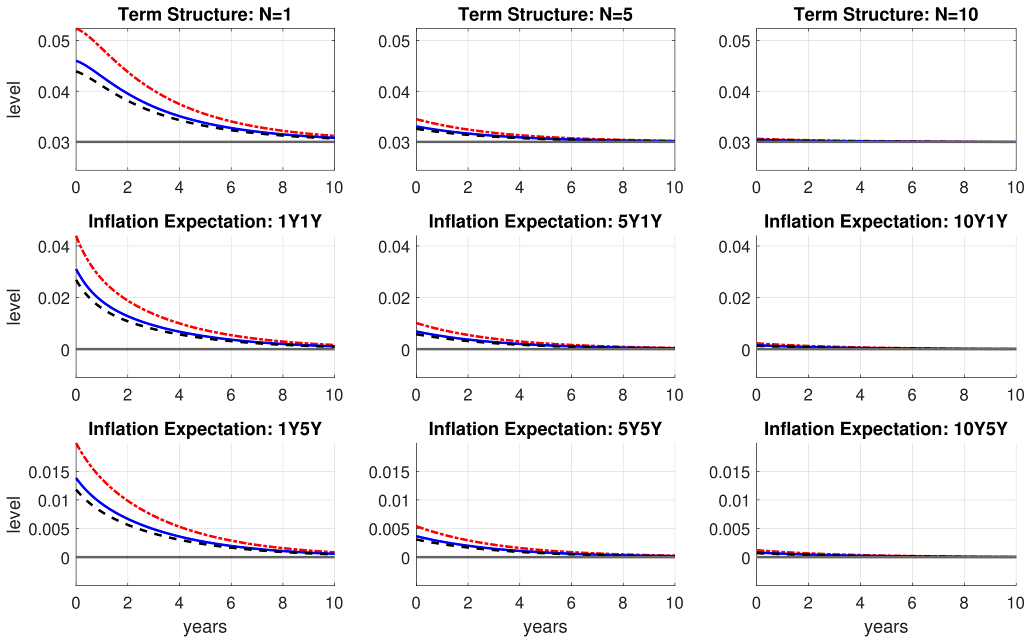


Figure 2.14: Transitory CARES Act shock for the parametrization in Table 2.1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

(blue solid), or no response of bond prices (red dotted). Here, the shorter the maturity of government debt, the larger the effect on future inflation.

The main take-away from this experiment is a predicted surge in inflation because of the large unexpected build-up of government debt and the expansionary increase of outlays (and decrease of taxes). In the next section, we contrast these results to a situation in which the fiscal policy shock is accompanied by a monetary policy shock.

2.3.3 The CARES Act and Monetary Policy Shock

In this section we quantify the effects of the CARES Act shock (cf. Table 2.7) together with an expansionary monetary policy shock decreasing nominal rates by 150 bp. While this shock typically increases current inflation, the net present value of future inflation is negative (cf. Section 2.2.6). Hence, the contemporaneous monetary policy shock accompanying the CARES Act might help reducing the large inflationary effects.

As a result, an accompanying monetary policy shock of 150 bp creates slightly less inflation for all maturities with similar dynamics (cf. Figure 2.A.3 and Table 2.A.3). Now the net present value of future interest rates is smaller because of initially lower interest rates, which translate to even more negative real interest rates and even more expansionary effects. Moreover, the net present value of future surpluses (fiscal policy) is higher for average maturity and perpetuities but slightly decreases for short-term bonds. In contrast, the direct FTPL effect is smaller because it offsets a smaller net present value of future interest rates (monetary policy). Overall, the picture does not change dramatically when assuming that the CARES Act shock was accompanied by an expansionary monetary policy shock. Though a profound analysis, which requires estimating the structural parameters and potentially latent state variables, is beyond the scope of the paper, the experiment mimics a low interest rates environment, a situation which seems more plausible for the US at the outset of the great pandemic. It shows that fiscal theory identifies the large-scale fiscal packages as the source of the recent surge in inflation.

2.3.4 A Permanent Shock Scenario?

A key question is whether agents 'believe' that the observed large-scale fiscal operations will be backed by subsequent higher future surpluses. What do responses to inflation and inflation expectations tell us about such beliefs at the core of the fiscal theory? From the fiscal theory point of view, this question translates to whether the increase in debt is followed by a subsequent higher future surplus. While the higher future surplus does not necessarily have to be permanent, possibly the cleanest analysis is to ask whether the CARES Act shock is considered permanent or transitory. In what follows, we consider a scenario in which the CARES Act shock does have a permanent component causing a permanently higher debt-to-GDP ratio. Because the debt level is ultimately determined by future surpluses, a permanently higher debt level $a_{ss}^{new} \equiv s_{ss}^{new}/\rho$ requires higher surpluses s_{ss}^{new}. Put differently, the *real* debt level or debt-to-GDP ratio can increase permanently only if economic agents presume that additional debt is financed by either higher revenues and/or lower government consumption (i.e., backed by higher future surpluses).

Suppose that a fraction α of the newly issued debt is followed by permanently higher tax revenues, so that $v_{ss}^{new} = v_{ss} + \alpha(v_0 - v_{ss})$. Hence, we may interpret α as the fraction of the newly issued debt $v_0 - v_{ss} = \mathbf{D}v_{ss}$ that is backed by higher future surpluses. If the observed shock to debt v_t (face value) was permanent, i.e., the fiscal expansion was backed by higher future surpluses, we set $\alpha = 1$. If only a fraction of the newly issued debt $\alpha\mathbf{D}$ is backed by higher future surpluses, we may set $0 \leq \alpha < 1$. Here, the case of $\alpha = 1$ shows that from the fiscal theory point of view, an initial shock to v_t which is fully backed by higher future surpluses does not lead to an unexpected 'debt shock'. In fact, the effective 'debt shock' size in our inflation decomposition (2.21) is $(1 + \mathbf{D})/(1 + \alpha\mathbf{D}) - 1 \geq 0$.

For illustration, suppose for the moment that half of the newly issued debt are backed by subsequent higher future surpluses, $\alpha = 0.5$, which for $\mathbf{D} = 0.12$ implies a debt shock of $((1+0.12)/(1+0.5 \cdot 0.12) - 1) \cdot 100 = 5.66$ percent (cf. Figure 2.A.4 and Table 2.A.4). We then contrast our results to both a permanent CARES Act shock scenario with $\alpha = 1$ (cf. Figure 2.A.5 and Table 2.A.5) and the transitory scenario with $\alpha = 0$ (see Section 2.3.2). Comparing Table 2.8 to the permanent scenarios highlights that only the CARES Act shock in which the newly issued debt is *not* sufficiently backed by higher future surpluses leads to a surge in future expected inflation similar to the observed response.

In particular, the effects of the CARES Act on future discounted inflation with $\alpha = 1$ would be moderate between 1.72 and 2.92 percentage points. Here, the debt component of the CARES Act shock (an increase in v_t by 12 percentage points) is soaked up by higher future tax revenues such that $v_{ss}^{new} = v_0$ and $v_0/v_{ss}^{new} - 1 = 0$. Without this permanent shock, the debt shock directly would add up to 12 percentage points for $\alpha = 0$ to the net present value of future inflation, as shown in Table 2.8. The maturity structure matters because longer maturities dampen the response of the real value of debt through the direct effect (changes in bond prices). Similar to the temporary case with $\alpha = 0$, even in the case of $\alpha = 1$ the permanent CARES Act scenario would be expansionary and thus temporarily increases output. Consequently, the debt-to-GDP ratios (market value) for all maturities initially only increase by roughly 3.5 percentage points before gradually approaching the higher steady state value of about 120 percent.

2.4 Conclusion

We revisit the fiscal theory and extend the simple NK model with a fiscal block in order to analyze the role of the maturity structure of sovereign debt on interest rates and inflation dynamics. Our results suggest that the average maturity of existing debt has a prominent role for the propagation of transitory and permanent policy shocks in the NK-FTPL model. We show how the effects translate to the term structure of interest rate and to model-implied inflation expectations. Our finding justifies a critical assessment of neglecting the direct FTPL effect in the traditional NK framework. Through the lens of the fiscal theory, we decompose the present value of future inflation into indirect effects (changes in future monetary policy and fiscal policy) and a direct FTPL effect, which basically is an asset pricing re-evaluation of existing bonds. In particular, we highlight that sovereign debt, with an empirically plausible average maturity for the US, largely offsets the impact of monetary policy on the present value of future inflation.

Our application simulates the CARES Act of 2020, which we translate to shocks to the primary surplus of about 8 percent of GDP and to the debt (face value) by 12 percent. Without a credible future (s-shaped) policy change, the NK-FTPL model predicts a surge in inflation, which amounts to an increase of the net present value of future inflation about the same size as the increase of newly issued debt. We show how this dramatic inflation response not only depends on the average maturity of existing bonds, but also primarily on the perception of agents whether the large-scale fiscal operations are ultimately backed by a higher future surplus or not. In contrast to the aftermath of the global financial crisis of 2008, where the inflation response was not as strong or inflation even declined, the recent surge in inflation and medium-term inflation expectations indicates that the newly issued debt is not considered as being backed by subsequent higher surpluses.

We believe that this paper is a promising starting point for the fiscal theory in more elaborate models, including regime-switching, nonlinearities, and stochastic shocks. First, our results for the term structure of interest rates and inflation expectations would be much more informative. Our setup is a natural starting point and benchmark for models with term premia (cf. Posch, 2020), convenience yield, or default risk. Second, more research is needed for the surplus dynamics, e.g., estimating parameters of the fiscal policy rule (cf. Kliem et al., 2016). Third, we need to study the effects of maturity in medium-size NK models including regime switches (see Bianchi and Melosi, 2019), financial frictions (cf. Brunnermeier and Sannikov, 2014), and productive capital (cf. Brunnermeier et al., 2021; Liemen, 2022), and to study the effects and transmission in models with heterogeneous agents (cf. Kaplan et al., 2018; Bayer et al., 2021). This opens the path toward a more profound fiscal policy evaluation and to address questions of fiscal limits and sovereign defaults (fiscal sustainability).

Chapter 3

The Fiscal Theory of the Price Level in New Keynesian Models with Capital

Abstract

In this paper, I embed the fiscal theory of the price level (FTPL) in a simple continuous-time New Keynesian model with capital and capital adjustment cost. I offer an elaborate analysis of determinacy, model dynamics, transmission channels and the importance of capital adjustment costs in the continuous-time NK-FTPL framework. My results indicate that FTPL lives up to its name, as the exact specification of fiscal policy is crucial for model implications and predictions. Equipped with the fiscal theory, I evaluate the Great East Japan Earthquake of 2011 (*Tōhoku Earthquake*) and show how to explain and solve the puzzling behavior of expansionary effects of capital destruction at the Zero Lower Bound of the nominal interest rate. I then address and solve the Crowding-In Consumption Puzzle that refers to a discrepancy between the empirically observed and theoretically predicted responses of consumption to government consumption shocks. My model supports consumption dynamics in either direction and at the same time suggest a crowding-in of investment. Finally, FTPL models in the literature usually introduce long-term debt in order to obtain a negative correlation between inflation and the nominal interest rate. I show that the inclusion of capital and its effect on fiscal policy rules is able to induce the negative correlation even under short-term debt.

3.1 Introduction

Pandemics, unprecedented large-scale fiscal stimulus packages, central banks that where for years constrained by the Zero Lower Bound of the nominal interest rate, sky-rocketing debt levels and a recent surge in inflation, with rates not seen since the 1980s, are just some of the omnipresent challenges of our time. These developments highlight the need for a joint framework that offers a clear and consistent understanding of monetary and fiscal policies as well as their interactions with government debt and inflation.

One promising framework to address these developments is the fiscal theory of the price level (FTPL), which attracted a growing interest in the academic literature in recent years (see e.g. Sims (2011), Cochrane (2018), Brunnermeier et al. (2021), Cochrane (2022b) or Liemen and Posch (2022)). At the core of FTPL is the government debt valuation equation, which links primary surpluses to the real value of government debt. Satisfying this relationship pins down the path of future inflation rates. This in turn, makes fiscal policy one of the central driving forces for inflation dynamics.

Smaller New Keynesian models as in Sims (2011), Werning (2012), Cochrane (2017), Cochrane (2018), Posch (2020), Cochrane (2022b) or Liemen and Posch (2022) usually abstract from capital dynamics. However, capital and labor are commonly considered the central input factors needed to produce output. Consequently, abstracting from capital is not necessarily an innocuous simplification both from an empirical and from a theoretical point of view. Thus, medium- and larger-scale macroeconomic models frequently introduce capital stock dynamics (see e.g. Smets and Wouters (2003), Christiano et al. (2005) or Kaplan et al. (2018)). Since the capital stock can be considered a physical form of wealth, the synthesis of capital and FTPL is an appealing framework to simultaneously analyze the interactions between capital, government debt as well as monetary and fiscal policy. Note that I prefer the evaluation of FTPL in a continuous-time framework because, in contrast to discrete-time NK-FTPL models, the price level cannot jump. As shown by Liemen and Posch (2022) this feature allows a more clear-cut analysis of FTPL effects and transmission channels.

With the above considerations in mind, I analyze the fiscal theory of the price level in the continuous-time NK framework with capital. To that end, I start from the model of Dupor (2001), which is a continuous-time formulation of the three equation NK model, augmented with a simple rule for capital dynamics. As highlighted by Dupor (2001) this framework implies that contractionary monetary policy shocks are expansionary and increase inflation. Despite this counterintuitive result, I introduce FTPL to the model, discuss determinacy issues and evaluate model implications as well as transmission channels of monetary policy shocks. My analysis highlights that this framework not only inherits the shortfalls of the model of Dupor (2001) but also produces counterintuitive results and predictions on its own. Hence, one can think of this framework more like a theoretical construct, which for obvious reasons, so far, attained little attention in the literature. However, I evaluate this model for reasons of comparison and in order to justify the introduction of capital adjustment costs. Because the model is add odds with conventional economic thinking, I do not use it for actual policy evaluation. What I am are after, is a simple NK model with capital and FTPL that maintains the central features of the simple NK-FTPL framework of Sims (2011) and Cochrane (2018). I show that the introduction of capital adjustment costs (cf. Dupor (2002) or Posch and Wang (2020)) offers a simple and effective remedy to the counterintuitive implications of the continuous-time NK model with capital and FTPL. Thus, in order to obtain my baseline model, I introduce capital and capital adjustment costs to the NK-FTPL framework of Sims (2011), Cochrane (2018) and Cochrane (2022b). My results suggest that model predictions are similar to the ones of the simple NK-FTPL model without capital. At the same time, regarding capital specific variables, such as the capital rental rate or investment, the model dynamics are, at least initially, closely related to the ones of the corresponding NK model without FTPL. My findings indicate that the fiscal theory of the price level lives up to its name, as the exact specifications of the processes for taxes and government consumption are crucial for the implications and predictions of the model. After extensively discussing determinacy, model dynamics, transmission channels and the importance of capital adjustment costs in the continuous-time NK framework with capital, I use my baseline model to understand and solve two puzzles in the literature.

The first puzzle is the expansionary effect of capital destruction at the ZLB, which is a frequently encountered feature within the traditional NK framework[1]. The fiscal theory is a promising starting point because an interest rate peg is already nested as limiting case in the passive monetary policy specification. To address this puzzle, my application simulates the Great East Japan Earthquake of 2011 (*Tōhoku Earthquake*), which I translate to an exogenous shock that destroys 1 percent of the capital stock (cf. Wieland (2019)). My results suggest that solving the puzzle depends less on the distinction between a pegged and a variable interest rates but more on the specification of fiscal policy rules. I then solve the *Crowding-In Consumption Puzzle*[2], which refers to a discrepancy between theoretical and empirically observed responses of consumption to increases in government consumption. While empirical studies frequently point to a crowding-in effect, the traditional NK framework implies a crowding-out effect. I show that the NK-FTPL framework with capital adjustment costs delivers the theoretical underpinning for either a crowding-in or a crowding-out effect. In both cases, the model implies, at least initially, a crowding-in of investments. Finally, I also contribute to the literature on the implementation of the fiscal theory in NK models. The most commonly applied approach to obtain a (tempo-

[1]See e.g. Eggertsson (2011), Eggertsson et al. (2014), Kiley (2016), Cochrane (2017) or Wieland (2019).

[2]See e.g. Linnemann (2006), Galí et al. (2007), Bilbiie (2011), Iwata (2013), Ambler et al. (2017), Lewis and Winkler (2017) or Rüth and Simon (2022) (forthcoming).

rary) negative inflation response to contractionary monetary policy shocks in the FTPL literature, is the introduction of long-term debt. Using the NK-FTPL model with capital adjustment cost, I show how to obtain the desired response without introducing long-term bonds.

I am are not aware of a comprehensible evaluation of FTPL in the simple NK framework with capital. Thus, this paper is novel in various dimensions and justifies an elaborate discussion of basic transmission channels and dynamics before turning to actual policy analysis that I consider to be the main contribution of this paper. There are at least three crucial contributions. First, my framework allows to analyze the interactions between debt and capital, through the lens of FTPL, in a simple and traceable continuous-time framework. Second, to the best of my knowledge I am the first to solve the above puzzles by utilizing FTPL in the continuous-time NK framework. Third, I present a novel approach of calibrating the fiscal policy rules in order to obtain the negative relationship between the inflation and the nominal interest rate in the FTPL framework, even in the presence of short-term debt.

Regarding related literature, the probably most extensive single-authored treatment of the fiscal theory of the price level is the eponymous book of Cochrane (2022b) (forthcoming). Closely related to my model is Sims (2011) who introduces FTPL in a simple continuous-time model and highlights how the government debt valuation equation becomes the central mechanism for equilibrium selection. Building on this paper, Cochrane (2018) further evaluates this modeling framework and shows how to simplify the model without losing its main insights. Brunnermeier et al. (2021) analyze how the introduction of a bubble term in the FTPL framework can explain why countries that persistently run negative primary surpluses, are able to achieve low inflation rates. The bubble occurs via uninsurable idiosyncratic risk on capital returns and government bonds taking the function of a safe asset. Permanently negative primary surpluses are possible as long as they are accompanied by a positive bubble term. More closely related to my paper is Dupor (2001), who highlights that the continuous-time formulation of the model is a central problem for introducing capital to the NK framework. In the basic NK model with capital, there is a direct relation between the rental rate on physical capital and the real interest rate. This dependence produces the counterintuitive result of contractionary monetary policy shocks being expansionary. Dupor (2001), Carlstrom and Fuerst (2005) and Leith and von Thadden (2008) discuss this observation extensively. While not explicitly referring to FTPL, Leith and von Thadden (2008), analyze determinacy regions when considering government debt and capital in a joint framework. Carlstrom and Fuerst (2005) solve a model similar to Dupor (2001) and highlight that both the passive monetary policy requirement as well as the counterintuitive policy implications are indeed a continuous-time specific phenomenon. As already pointed out above, the standard NK framework is known to generate counterintuitive predictions under a nominal interest rate peg. For instance, wasteful government spending, capital and output destruction as well as technical regress, are usually not only highly expansionary but also produce large multipliers at the ZLB (see for instance Eggertsson (2011), Eggertsson et al. (2014), Kiley (2016), Cochrane (2017) or Wieland (2019)). There are various ways to address these issues. For example, Kiley (2016) shows that using sticky information instead of sticky prices is able to solve the puzzle, whereas Boneva et al. (2016) solve the puzzle by allowing for additional non-linearities in the solution method of the model considering alternative parametrizations. Regarding the crowding-in consumption puzzle, Galí et al. (2007) show how the introduction of rule-of-thumb investors produces model dynamics that are consistent with the considered data. Bilbiie (2011) highlights that the puzzle can be solved by a combination of Edgeworth substitutability in the utility function along with shifts in labor demand. Turning to the negative relationship between the inflation and the nominal interest rate in the FTPL framework with short-term debt, Cochrane (2022b) shows that allowing the fiscal authority to directly respond to the inflation rate is able to generate the desired response.

The rest of the paper is organized as follows. I derive the perfect-foresight NK-FTPL model with capital and capital adjustment costs in Section 2. Subsequently, I evaluate the baseline model and its limiting special cases and study transitory zero-probability shocks in Section 3. Section 4 confronts the baseline model with the above puzzles. Finally, Section 5 concludes.

3.2 The Framework

In this section, I describe the continuous-time NK model with capital and capital adjustment costs (abbreviated *NK-AC model*). The model is basically borrowed from Posch and Wang (2020). I then outline the Fiscal Theory of the Price Level and merge it with the NK-AC model in order to obtain my baseline model (abbreviated *NK-AC-FTPL model*) for the subsequent analysis.

3.2.1 The NK Model With Capital Adjustment Costs

The core of my framework is the simple perfect-foresight continuous-time NK model[3]. I extend the model by introducing capital, capital adjustment costs and the Fiscal Theory of the Price Level. This model nests the simple NK model and the NK model with capital, both either with or without FTPL as special (limiting) cases. I only focus on the most central equations and offer a complete derivation of the model and its limiting cases in appendix 3.A.4. For the sake of clarity, throughout this paper I limit my analysis to

[3]See e.g. Dupor (2001), Sims (2011), Werning (2012), Cochrane (2017), Cochrane (2018), Posch (2020), Wieland (2019) or Cochrane (2022b).

perfect-foresight solutions, where only initial conditions have to be determined. Hence, I analyze "MIT shocks", which are one-time unexpected shocks at time 0 that are responded to under perfect foresight. I denote predetermined variables (the variables that cannot jump endogenously) as state variables and forward-looking variables as jump variables. Despite considering only linearized models (simple first-order linearizations around steady states), I state non-linearized equations throughout this paper for better traceability and clarity. Furthermore, I denote steady state values either with a *star* (for exogenously determined steady states) or the subscript *ss*.

I proceed by shortly wrapping up the central features of the model. I consider a representative household, which saves, supplies labor, and consumes. There is one competitive representative final-good producer, who uses intermediate goods to create the final output in the economy. The required inputs (a continuum of intermediate goods) are obtained from intermediate good producers that engage in monopolistic competition. Each of these producers manufactures its good by renting capital and labor, given price adjustment costs à la Rotemberg (1982). I consider a fiscal authority that levies taxes and consumes. By allowing for long-run primary surpluses, government debt and its maturity structure enter the framework. Finally, there is a central bank that engages in open market operations to steer the nominal interest rate.

Households

Let ρ, ξ and ϑ denote the subjective rate of time preference, the inverse of the Frisch labor supply elasticity and preference for leisure, respectively. Households optimize their reward function

$$\int_0^\infty e^{-\rho t}\left[\log(c_t) - \vartheta \frac{l_t^{1+\xi}}{1+\xi}\right] \mathrm{d}t, \tag{3.1}$$

and can invest in physical capital, k_t (illiquid asset), and government bonds (liquid asset). Let n_t denote the number of bonds with price p_t^b and a geometric maturity structure. Thus, *nominal financial wealth* of the household, b_t, amounts to

$$b_t = n_t p_t^b. \tag{3.2}$$

Together with the price level, p_t (equal to the price of the final good), one obtains the *real financial wealth* of the household as

$$a_t = b_t/p_t = n_t p_t^b/p_t. \tag{3.3}$$

No-arbitrage between the nominal interest rate and the bond return implies (cf. Liemen and Posch (2022))

$$i_t \mathrm{d}t = ((\chi + \delta^b)/p_t^b - \delta^b)\mathrm{d}t + \mathrm{d}p_t^b/p_t^b. \tag{3.4}$$

Similar to Woodford (2001), the bond pays a nominal coupon of $\chi + \delta^b$ that declines geometrically at rate δ^b. One can interpret δ^b as 1 over the maturity in years. Consequently, one obtains perpetuities for $\delta^b \to 0$ and short-term debt for $\delta_b \to \infty$. When setting χ equal to the steady state level of the nominal interest rate, i_{ss}, the bonds sell at par at the steady state, regardless of the maturity structure of debt. By rearranging equation (3.4) one obtains the evolution of the bond price (a forward-looking variable) as

$$\mathrm{d}p_t^b = (i_t p_t^b - \chi - \delta^b(1 - p_t^b))\mathrm{d}t. \tag{3.5}$$

Households earn labor income $w_t l_t$, where w_t denotes the real wage and l_t is labor in terms of hours worked. Furthermore, households obtain profits from firm ownership, $\digamma_t$, and coupon payments from government bonds, $n_t(\chi + \delta^b)$. The income of the households has to finance consumption, c_t, lump-sum taxes, T_t, the purchase of new government bonds, $\mathrm{d}n_t$, at price p_t^b, as well as price adjustment costs, $\Theta(\pi_t)$. The savings of the households are used to accumulate financial wealth (the liquid asset) and physical capital (the illiquid asset). Consequently, households can withdrawal from or deposit into the illiquid account, where $d_t = x_t - r_t^k k_t$ denotes the net value. One can interpret r_t^k as the percentage gross dividend payments on the illiquid asset and x_t as investments into the illiquid asset.

From the government's perspective, issued debt has to cover coupon payments and the amortization of outstanding debt. At the same time, the government's revenue are primary surpluses, s_t, which are defined (see equation (3.45)) as taxes, T_t, less government consumption, g_t. Hence, the nominal value of outstanding debt follows

$$\mathrm{d}n_t = \left(\left((\delta^b + \chi) n_t - p_t s_t \right) / p_t^b - \delta^b n_t \right) \mathrm{d}t. \tag{3.6}$$

In continuous-time, the price level, p_t, is a predetermined variable that evolves according to[4]

$$\mathrm{d}p_t = \pi_t p_t \mathrm{d}t. \tag{3.7}$$

Equation (3.7) can be understood as realized inflation over the interval $[t, t + \mathrm{d}t]$ at rate π_t.

Differentiate equation (3.3) and substitute equations (3.5)-(3.7) from above in order to obtain the evolution of government debt in real terms (market value) as

$$\begin{aligned} \mathrm{d}a_t &= (p_t^b \, \mathrm{d}n_t + n_t \, \mathrm{d}p_t^b - n_t p_t^b / p_t \, \mathrm{d}p_t)/p_t \\ &= ((i_t - \pi_t)a_t - T_t + g_t) \, \mathrm{d}t. \end{aligned} \tag{3.8}$$

[4]See e.g. Sims (2011), Posch (2020) or Cochrane (2018).

Similarly, the household's budget constraint in real terms can be written as

$$\mathrm{d}a_t \quad = \quad ((i_t - \pi_t)a_t + w_t l_t - d_t - c_t - \Theta(\pi_t)/p_t - T_t + \digamma_t)\,\mathrm{d}t.$$

The market clearing condition for output[5], y_t, implies

$$y_t = c_t + g_t + x_t + \Theta(\pi_t)/p_t. \tag{3.9}$$

As a consequence, the evolution of real government debt and the real financial wealth of the households coincide. In an economy without capital, d_t is equal to zero and the evolution of real debt becomes (cf. Sims (2011))

$$\mathrm{d}a_t = ((i_t - \pi_t)a_t + w_t l_t - c_t - \Theta(\pi_t)/p_t - T_t + \digamma_t)\,\mathrm{d}t. \tag{3.10}$$

Turning back to the NK model with capital adjustment costs, the savings of the households are not only used to accumulate financial wealth but also physical capital. The latter increases if the investment in the capital stock exceeds capital depreciation, which occurs at the constant rate δ. Let capital be subject to adjustment costs so that one obtains the law of motion of k_t as

$$\mathrm{d}k_t = \left(\Phi\left(\frac{x_t}{k_t}\right) - \delta\right) k_t\ \mathrm{d}t. \tag{3.11}$$

I follow Parra-Alvarez et al. (2021) and define the capital adjustment costs function, Φ, as

$$\Phi\left(\frac{x_t}{k_t}\right) = \frac{\delta^{1/\kappa}}{1 - 1/\kappa}\left(\frac{x_t}{k_t}\right)^{1-1/\kappa} + \frac{\delta}{1-\kappa}. \tag{3.12}$$

The functional form is based on Jermann (1998) and Boldrin et al. (2001). Equation (3.12) implies that the steady state level of capital, k_{ss}, equals steady state investment, x_{ss}, over the capital depreciation rate. Consequently, the steady states of the model are invariant to the choice of κ (see derivations in Section 3.A.4). The parameter κ, which has to be positive and not equal to 1, denotes the elasticity of the ratio of investments over capital w.r.t. Tobin's q. For $\kappa \to \infty$ one obtains the perfectly elastic limit, which corresponds to the case without capital adjustment costs (cf. Dupor (2001)), as

$$\mathrm{d}k_t = \left(\frac{x_t}{k_t} - \delta\right) k_t\ \mathrm{d}t. \tag{3.13}$$

[5] Note, when linearizing the model around a zero-inflation target, the price adjustment costs term, $\Theta(\pi_t)/p_t$, drops out.

The Government Problem

Through open market operations, the monetary authority sets the nominal interest rate according to a partial adjustment interest rate rule as in Posch (2020). Hence,

$$\mathrm{d}i_t = \theta\left(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_{ss})\right)\mathrm{d}t, \tag{3.14}$$

where ϕ_π and ϕ_y denote inflation and output response parameters, where θ controls interest rate smoothing, and where π_t^* denotes the inflation target. For $\theta \to \infty$ the partial adjustment rule converges towards a feedback interest rate rule of the form $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1)$. While specification (3.14) comes of the cost of an additional state variable, it facilitates the analysis of monetary policy shocks.

Following Liemen and Posch (2022), let government consumption evolve according to

$$\mathrm{d}g_t = \rho_g(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_{ss}))\mathrm{d}t. \tag{3.15}$$

The parameters φ_y and φ_a govern the responsiveness of government consumption towards changes in output and debt, while ρ_g determines the degree of inertia. One obtains the inelastic limit $g_t = g_{ss}$ by letting $\rho_g \to 0$. Analogously to the monetary policy rule, by letting $\rho_g \to \infty$ one obtains the flexible limit (feedback rule) as $g_t = g_{ss} + \varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss})$. Similar to Kliem et al. (2016) and Leith and von Thadden (2008), I allow for a response of government consumption to changes in the real value of debt. Finally, I assume that the government consumes a constant share, s_g, of output in equilibrium.

Final Good Producers

There is one final good with price p_t that is produced by a competitive representative firm using intermediate goods. The production function reads

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.16}$$

where ε denotes the elasticity of substitution across intermediate goods. Subject to the production function (3.16) the final good producer maximizes its profit, taking as given the price of the final good as well as the prices p_{it} of all intermediate goods. The corresponding input demand functions are

$$y_{i\tau} = \left(\frac{p_{i\tau}}{p_\tau}\right)^{-\varepsilon} y_t \qquad \forall i, \tag{3.17}$$

and

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon}\mathrm{d}i\right)^{\frac{1}{1-\varepsilon}}, \tag{3.18}$$

where p_t is readily interpreted as the aggregate price level.

Intermediate Good Producers

There is a continuum of intermediate good producers that engage in monopolistic competition. Producer i maximizes its profit by choosing its price p_{it} given price adjustment costs à la Rotemberg (1982). Recall that I only consider first-order linear approximations around a zero-inflation target so that Calvo (1983) pricing would yield identical inflation dynamics. I consider Rotemberg pricing along the lines of Kaplan et al. (2018). Being quadratic in the rate of price changes, $\dot{p}_{it}/p_{it}$, price adjustment costs are expressed in terms of a fraction of nominal output, $p_t y_t$, and read

$$\Theta_t \left(\frac{\dot{p}_{it}}{p_{it}} \right) = \frac{\gamma}{2} \left(\frac{\dot{p}_{it}}{p_{it}} \right)^2 p_t y_t, \tag{3.19}$$

where γ denotes the degree of price stickiness. The intermediate good producers use the same technology to produce differentiated goods:

$$y_{it} = k_{it}^{\alpha} l_{it}^{1-\alpha}. \tag{3.20}$$

This is a Cobb-Douglas-type production function, where I impose $0 < \alpha < 1$. As a consequence, α and $1 - \alpha$ are readily interpreted as the capital and the labor share, respectively. Thus, in the absence of capital, $\alpha = 0$, and one obtains $y_{it} = l_{it}$. Perfectly competitive factor markets imply $r_t^k = \partial y_{it}/\partial k_{it}\, mc_t$ and $w_t = \partial y_{it}/\partial l_{it}\, mc_t$, where mc_t are marginal costs that are the same across firms. Thus, one obtains

$$r_t^k = \alpha mc_t y_{it}/k_{it} \qquad \text{and} \qquad w_t = (1-\alpha) mc_t y_{it}/l_{it}. \tag{3.21}$$

Combining terms,

$$k_{it}/l_{it} = \alpha/(1-\alpha) w_t/r_t^k. \tag{3.22}$$

Substitute equation (3.22) back into the rental rate representation in equation (3.21) and rearrange terms to obtain marginal costs as

$$mc_t = (1/(1-\alpha))^{1-\alpha} (1/\alpha)^{\alpha} w_t^{1-\alpha} (r_t^k)^{\alpha}, \tag{3.23}$$

where in the absence of capital, marginal costs are equal to wages. With the above considerations in mind, the real profit maximization problem of intermediate good producers reads

$$\max \mathbb{E}_t \int_t^{\infty} \frac{\lambda_\tau}{\lambda_t} e^{-\rho(\tau - t)} \left(\frac{p_{i\tau}}{p_\tau} y_{i\tau} - mc_\tau y_{i\tau} - \Theta_t \left(\frac{\dot{p}_{i\tau}}{p_{i\tau}} \right) / p_\tau \right) \mathrm{d}\tau, \tag{3.24}$$

subject to the input demand function (3.17). Since I consider a symmetric equilibrium, it follows that $p_{it} = p_t$ and the forward-looking NK Philips-Curve (cf. Kaplan et al. (2018))

reads

$$\mathrm{d}\pi_t = \left(\frac{\mathrm{d}c_t}{c_t} - \frac{\mathrm{d}y_t}{y_t}\right)\pi_t + \left(\rho\pi_t - \frac{\varepsilon-1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}mc_t - 1\right)\right)\mathrm{d}t \tag{3.25}$$

Note that in a continuous-time framework the expected rate of inflation coincides with the current rate of inflation (cf. Posch (2020)).

Aggregation

I consider a symmetric equilibrium. As a consequence, one can express output in terms of the production function of the intermediate good producers (3.20) as

$$y_t = k_t^{\alpha} l_t^{1-\alpha}, \tag{3.26}$$

or alternatively in terms of aggregate demand

$$y_t = c_t + g_t + x_t + \Theta(\pi_t)/p_t = \frac{c_t + g_t + x_t}{1 - \gamma\pi_t^2/2}, \tag{3.27}$$

where I substitute the price adjustment costs function (3.19). Note that one can also express output as the sum of labor- and capital income as well as real profits of the firms, giving rise to

$$y_t = r_t^k k_t + w_t l_t + \digamma_t. \tag{3.28}$$

Due to the symmetric equilibrium assumption, equation (3.22) implies to

$$r_t^k = \frac{\alpha\vartheta c_t}{(1-\alpha)k_t} l_t^{1+\xi}. \tag{3.29}$$

Furthermore, one obtains the real interest rate, r_t, as

$$r_t = i_t - \pi_t. \tag{3.30}$$

The Hamilton-Jacobi-Bellman Equation

The *Hamilton-Jacobi-Bellman* equation (HJB) to the above problems reads

$$\begin{aligned} \rho V(a_t, i_t, g_t, k_t) \quad = \quad & \max_{(c_t, l_t, d_t)} \left(\log(c_t) - \vartheta l_t^{1+\xi}/(1+\xi)\right) \\ & + V_a\left(a_t(i_t - \pi_t) + w_t l_t - d_t + \digamma_t - c_t - \Theta_t(\pi_t)/p_t - T_t\right) \\ & + V_i\left(\theta\left(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_{ss})\right)\right) \\ & + V_g\left(\rho_g(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_{ss}))\right) \\ & + V_k\left(\left(\left(\delta^{1/\kappa}(r_t^k + d_t/k_t)^{1-1/\kappa}/(1 - 1/\kappa) + \delta/(1-1/\kappa)\right) - \delta\right) k_t\right), \end{aligned}$$

where the value function, V, is a function of the four state variables a_t, i_t, g_t and k_t, with corresponding partial derivatives V_a, V_i, V_g and V_k. The control variables are consumption, c_t, labor, l_t, and net changes in the illiquid account[6], d_t.

The first-order condition (FOC) of the HJB w.r.t. consumption yields

$$V_a = 1/c_t \equiv \lambda_t, \tag{3.31}$$

where λ_t denotes the marginal value of wealth. One obtains from the FOC w.r.t. labor that

$$\lambda_t w_t = \vartheta l_t^{\xi}. \tag{3.32}$$

When combining equations (3.31) and (3.32), one can eliminate the co-state λ_t and arrives at

$$w_t = \vartheta l_t^{\xi} c_t. \tag{3.33}$$

Taking the derivative of the HJB w.r.t. financial wealth, a_t, one obtains the law of motion of the marginal value of wealth as

$$\mathrm{d}\lambda_t = \lambda_t(\pi_t + \rho - i_t)\mathrm{d}t, \tag{3.34}$$

which together with (3.31) yields the consumption Euler equation

$$\mathrm{d}c_t = c_t(i_t - \rho - \pi_t)\mathrm{d}t. \tag{3.35}$$

The FOC of the HJB w.r.t. d_t yields

$$\mu_t = \lambda_t \delta^{-1/\kappa} \left(\frac{k_t^{\alpha} l_t^{1-\alpha} - c_t - g_t - \Theta(\pi_t)/p_t}{k_t} \right)^{1/\kappa}, \tag{3.36}$$

where I denote the marginal value of capital as $\mu_t \equiv V_k$ and use $d_t = x_t - r_t^k$. Without capital adjustment cost, $\kappa \to \infty$, equation (3.36) reduces to

$$\mu_t = \lambda_t. \tag{3.37}$$

In this case, the marginal values of wealth and capital coincide (cf. Dupor (2001)). Starting from equation (3.36), I show in appendix 3.A.4 that the co-state μ_t evolves according to

$$\mathrm{d}\mu_t = \left((\rho + \delta)\mu_t - \left(\frac{\delta(\mu_t/\lambda_t)^{\kappa-1} - \delta}{\kappa - 1} \right) \mu_t - r_t^k \lambda_t \right) \mathrm{d}t. \tag{3.38}$$

[6]Note that one can equivalently substitute $d_t = x_t - r_t^k k_t$ so that investment, x_t, instead of d_t becomes the relevant control variable.

Without capital adjustment cost, equation (3.38) reduces to

$$\mathrm{d}\mu_t = (\rho + \delta - r_t^k)\mu_t \mathrm{d}t, \tag{3.39}$$

which together with equations (3.34) and (3.37) implies (cf. Dupor (2001))

$$r_t^k = r_t + \delta. \tag{3.40}$$

Thus, in the absence of capital adjustment costs, the dynamics of the real interest rate and the capital rental rate concur. In order to obtain an expression for the rental rate in the model with capital adjustment costs, rearrange equation (3.36) to obtain an alternative representation for labor,

$$l_t = \left(k_t^{1-\alpha} (\mu_t/\lambda_t)^\kappa \delta + k_t^{-\alpha} (c_t + g_t + \Theta(\pi_t)/p_t)\right)^{\frac{1}{1-\alpha}}, \tag{3.41}$$

which after substituting in equation (3.29) yields

$$r_t^k = \frac{\alpha\vartheta c_t}{(1-\alpha)k_t} \left(k_t^{1-\alpha} \left(\frac{V_k}{V_a}\right)^\kappa \delta + k_t^{-\alpha} (c_t + g_t + \Theta_t(\pi_t)/p_t)\right)^{\frac{1+\xi}{1-\alpha}}. \tag{3.42}$$

3.2.2 The Fiscal Theory of the Price Level

Following Sims (2011), I introduce FTPL by augmenting the NK-AC model with a fiscal policy block, which consists of government debt, primary surpluses and the bond price. I implement government debt in terms of its face value[7]

$$v_t = n_t/p_t. \tag{3.43}$$

Differentiate equation (3.43) and use equations (3.6) and (3.7) to obtain the evolution of government debt as

$$\mathrm{d}v_t = (((\delta^b + \chi)/p_t^b - \delta^b - \pi_t)v_t - s_t/p_t^b)\mathrm{d}t, \tag{3.44}$$

where primary surpluses are defined as the difference of taxes and government consumption[8]

$$s_t = T_t - g_t. \tag{3.45}$$

Note that T_t and g_t can enter the system either as (predetermined) state or as (forward-looking) jump variables. Since I want to study exogenous shocks to taxes and government

[7]One can equivalently let government debt enter the model in terms of the market value, $a_t = n_t p_t^b/p_t$ (see for instance Sims (2011), Cochrane (2018) or Liemen and Posch (2022)). This formulation implies the same dynamics and refers to the same model.

[8]Without government consumption, as in Sims (2011), primary surpluses correspond to taxes.

consumption, I implement both of them as additional state variables.

I follow Liemen and Posch (2022) and let taxes evolve according to

$$\mathrm{d}T_t = \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t. \tag{3.46}$$

The parameter τ_y governs the responsiveness of taxes towards changes in the output gap, while, similar to Kliem et al. (2016), τ_a introduces a dependence of taxes on the real value of debt. The parameter ρ_τ determines the degree of inertia. One obtains the inelastic limit $T_t = T_t^*$ by letting $\rho_\tau \to 0$. By letting $\rho_\tau \to \infty$, one obtains the flexible limit (feedback rule) with $T_t = T_t^* + \tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss})$.

The values of the debt response parameters τ_a and φ_a in the government consumption rule (3.15) and the tax rule (3.46) play a crucial role for model determinacy and are vital for the transmission of fiscal policy. See Section 3.3.2 for a discussion on determinacy as well as the role and the interdependence of debt responses in the fiscal policy rules.

By integrating[9] the law of motion of real debt (3.8), one obtains the government debt valuation equation as

$$a_t \equiv \frac{n_t p_t^b}{p_t} = v_t p_t^b = \mathbb{E} \int_t^\infty e^{-\int_t^u (i_v - \pi_v)\mathrm{d}v} s_u \mathrm{d}u. \tag{3.47}$$

This valuation equation is the core of FTPL. The price level, p_t, as well as the number of outstanding government bonds, n_t, are predetermined variables and, consequently, do not jump in a continuous-time framework with price stickiness (see e.g. Posch (2020), Cochrane (2018), Cochrane (2022b)). For given paths of the state variables i_t and s_t, the right-hand side of equation (3.47) pins down the path for the inflation rate. Due to equation (3.5) any discrete change in p_t^b directly translates to a jump in a_t on the left-hand side of equation (3.47). Inherited jumps in a_t are higher the longer the maturity of government bonds and vanish in case of instantaneous debt. To highlight this maturity channel, linearize and integrate the bond price equation (3.4) to arrive at

$$p_t^b = 1 - \int_t^\infty e^{-(\chi+\delta^b)(v-t)}(i_v - i_{ss})\mathrm{d}v. \tag{3.48}$$

In case of short-term bonds, $\delta^b \to \infty$, the integral term in equation (3.48) drops out and the bond price is always equal to one. For longer maturity bonds, $\delta^b \to 0$, the exponent in equation (3.48) becomes bigger and approaches $-\chi$. Thus, $\delta^b \to \infty$ and $\delta^b = 0$ are the lower and upper bounds for the size of the maturity effect.

To highlight how FTPL relates to the inflation rate, I use the inflation decomposition

[9] I restrict my focus on bounded solutions, where $\lim_{T\to\infty} e^{-\int_t^T r_s \mathrm{d}s} a_T = 0$ so that I rule out *bubble* solutions. See e.g. Brunnermeier et al. (2021) for an analysis of FTPL, where the transversality condition does not necessarily rules out a *bubble* component.

from Section 2.2.5, which gives rise to

$$\int_t^\infty e^{-\rho(v-t)}(\pi_v - \pi_t^*)\mathrm{d}v = \int_t^\infty e^{-\rho(v-t)}(i_v - i_{ss})\mathrm{d}v - \int_t^\infty e^{-\rho(v-t)}(s_v - s_t^*)/a_{ss}\mathrm{d}v + (p_t^b/p_{ss}^b - 1) + (v_t/v_{ss} - 1). \quad (3.49)$$

According to equation (3.49), changes in the present value of future inflation can be attributed to the weighted present value of changes in future surpluses (fiscal policy), to changes in the present value of future interest rates (monetary policy) and two direct effects (cf. Liemen and Posch (2022)). In particular, the penultimate term of equation (3.49) accounts for a maturity effect (as described by equation (3.48)) and the last term captures the effect of a potential exogenous shock to the face value of debt (which is a predetermined variable). Further note that the size of the weighting factor $1/a_{ss} = \rho/s_t^*$ for the present value of changes in future surpluses depends on the (exogenous) choice of the equilibrium value of surpluses (see Section 3.A.4).

As in Sims (2011), I introduce the fiscal policy block by augmenting the model of the previous section with the three additional differential equations for the bond price (3.5), taxes (3.46) and the face value of government debt (3.44).

3.2.3 Equilibrium Dynamics

In summary, I obtain a system with 5 differential equations for the state variables (the face value of government debt v_t, the nominal interest rate i_t, taxes T_t, government consumption g_t and the capital stock k_t) and 4 differential equations for the forward-looking jump variables (the inflation rate π_t, consumption c_t, the marginal value of wealth μ_t and the bond price p_t^b). Thus, the model reads

$$\begin{aligned}
\mathrm{d}\pi_t &= (\mathrm{d}c_t/c_t - \mathrm{d}y_t/y_t)\,\pi_t + (\rho\pi_t - (\varepsilon-1)/\gamma\,(\varepsilon/(\varepsilon-1)mc_t - 1))\,\mathrm{d}t \\
\mathrm{d}c_t &= c_t\,(i_t - \pi_t - \rho)\,\mathrm{d}t \\
\mathrm{d}i_t &= \theta\,(\phi_\pi\,(\pi_t - \pi_t^*) + \phi_y\,(y_t/y_{ss} - 1) - (i_t - i_{ss}))\,\mathrm{d}t \\
\mathrm{d}g_t &= \rho_g\,(\varphi_y\,(y_t/y_{ss} - 1) - (g_t - g_t^*))\,\mathrm{d}t \\
\mathrm{d}k_t &= k_t(\Phi(x_t/k_t) - \delta)\mathrm{d}t \\
\mathrm{d}\mu_t &= \left(\mu_t(\rho+\delta) - \mu_t\left(\left(\delta\,(\mu_t c_t)^{\kappa-1} - \delta\right)/(\kappa-1)\right) - r_t^k/c_t\right)\mathrm{d}t,
\end{aligned} \quad (3.50)$$

together with the fiscal block

$$\begin{aligned}
\mathrm{d}T_t &= \rho_\tau\,(\tau_y\,(y_t/y_{ss} - 1) - (T_t - T_t^*))\,\mathrm{d}t \\
\mathrm{d}v_t &= (((\delta^b + \chi)/p_t^b - \delta^b - \pi_t)v_t - s_t/p_t^b)\mathrm{d}t \\
\mathrm{d}p_t^b &= \left(i_t p_t^b - \chi - \delta^b\left(1 - p_t^b\right)\right)\mathrm{d}t,
\end{aligned} \quad (3.51)$$

where

$$y_t = k_t^{\alpha} l_t^{1-\alpha},$$

$$l_t = ((k_t^{1-\alpha}\delta(V_k c_t)^{\kappa} + k_t^{-\alpha}c_t + k_t^{-\alpha}g_t)/(1-\gamma/2\pi_t^2))^{1/(1-\alpha)},$$

$$x_t = y_t - c_t - g_t - \frac{\gamma}{2}\pi_t^2 y_t,$$

and

$$\Phi(x_t/k_t) = \delta^{1/\kappa}/(1-1/\kappa)(x_t/k_t)^{1-1/\kappa} + \delta/(1-\kappa).$$

Thus, one can use the 9 variables in the above differential equations to determine aggregate output (3.9), the capital adjustment costs function (3.12), marginal costs (3.23), the production function (3.26), wages (3.33), the capital rental rate (3.42), labor (3.41) and primary surpluses (3.45). See appendix 3.A.4 for steady-state values and a complete derivation of the model.

In the following sections I use the simple NK- and the NK model with capital (both with and without FTPL) as reference models. I derive both models in the appendix in Section 3.A.4.[10] Both models are limiting cases of the NK-AC-FTPL framework. To obtain the NK model with capital, start from the NK-AC-FTPL model, remove the fiscal policy block, and let $\kappa \to \infty$ so that capital adjustment costs approach zero. In this case, one arrives at the framework of Dupor (2001). When additionally setting $\alpha = 0$, one obtains the simple NK model. By augmenting these models with the fiscal policy block of the previous section, one obtains the *simple NK-FTPL model* and the *NK-FTPL model with capital*.

3.3 Fiscal Theory and Capital: Model Evaluations and Comparisons

In what follows, I linearize all models around their steady states by applying simple first-order linear approximations to all variables and differential equations. Regarding the inflation rate, I assume a zero-inflation target, $\pi_t^* = 0$. For reasons of clarity, I derive and present all steady states in Section 3.A.4 in the appendix. After the linearization, I solve each model along the lines of Cochrane (2018). To this end, I apply an eigenvalue-decomposition to the Jacobian matrix of the linearized system of differential equations. I then use this decomposition together with the (exogenous) initial values of the state variables (i.e. the shocks) to determine the initial values of all remaining variables. Equipped with the initial values, I use the stable eigenvalues of the system to find the unique backward solution of the model. See e.g. Cochrane (2018), Cochrane (2022b) or Liemen and

[10]See e.g. Dupor (2001), Cochrane (2018), Posch (2020) or Liemen and Posch (2022) for an elaborate description of these frameworks.

Posch (2022) for a detailed description of this method.

I restrict my analysis to perfect foresight solutions. Consequently, in order to obtain impulse-response functions I rely on analyzing non-recurring unexpected shocks (MIT shocks). Economic agents address these shocks with perfect foresight responses. I follow Sims (2011) and define monetary policy as changes in the short-term interest rate. Hence, I can compare how different model specifications and parametrizations affect the transmission of exogenous unexpected (zero-probability) monetary policy shocks. In the same way, I define fiscal policy as changes in surpluses, via taxes and/or government consumption (cf. Sims (2011)). Finally, I introduce an unexpected negative shock to the capital stock, which I define along the lines of Wieland (2019) as an unexpected event that destroys parts of the capital stock (e.g. natural disaster).

Similar to Chapter 2 of this doctoral thesis, some arguments in this section are based on policy functions. Recall that these functions follow directly from the model's solution and express any forward-looking variable as function of the state variables (see e.g. Posch (2020)). Consequently, in a linearized model environment, policy functions have to be linear functions of the states. Thus, for an arbitrary forward-looking variable z_t one obtains

$$\begin{aligned} z_t - z_{ss} = z(i_t, g_t, k_t, v_t, T_t) \quad &= \quad z_i(i_t - i_{ss}) + z_g(g_t - g_{ss}) + z_k(k_t - k_{ss}) \\ &\quad + z_v(v_t - v_{ss}) + z_T(T_t - T_{ss}) \end{aligned} \tag{3.52}$$

where z_i, z_g, z_k, z_v and z_T are constant coefficients that correspond to the partial derivatives evaluated at the steady state. Thus, these coefficients are functions of the states (e.g. $z_i = z_i(i_{ss}, g_{ss}, k_{ss}, v_{ss}, T_{ss})$). Since I consider a linearized model, the partial derivatives in equation (3.52) are constant coefficients. Hence, similar to Leeper et al. (2017b), one can interpret the policy function coefficients as *impact multipliers*, as they directly show the proportional initial responses of forward-looking variables to changes in the state variables of the model.

3.3.1 Model Parametrization

Despite its simplicity, the FTPL framework allows one to depict a wide variety of economic dynamics. In this context, the parametrization plays a crucial role since many mechanisms and features are captured by single parameter values. For example, δ^b captures the underlying debt maturity structure, s_t^* defines the equilibrium debt-to-GDP ratio and τ_y, τ_a, φ_y and φ_a not only determine fiscal dynamics but also determinacy regions of the model. There are in total 22 structural parameters. If not stated otherwise, I always use the *baseline parametrization*, which I describe below and summarize in Section 3.A.1. Most of the fundamental parameters are borrowed from Kaplan et al. (2018). For the debt level I follow Liemen and Posch (2022) and choose the equilibrium value of primary surpluses in a way to obtain the US Q1 2020 debt-to-GDP ratio[11] of 108%. I calibrate ϑ in a way to normalize equilibrium output ($y_{ss} = 1$). As a consequence, one obtains different values for ϑ in settings with and without capital. When normalizing output this way, one can use the same value of s_t^* in models with and without capital and always obtains identical debt-to-GDP ratios. By doing so, interest rate shocks produce the same initial response of the real value of debt (measured in terms of deviation from the steady state). This facilitates the comparison of settings with and without capital. I follow Parra-Alvarez et al. (2021) and set the degree of capital adjustment costs equal to 0.326 (Jermann (1998)). I consider a partial adjustment tax rule with parameters similar to the ones in Sims (2011), Davig and Leeper (2011) and Liemen and Posch (2022). Furthermore, I implement a partial adjustment rule for government consumption without direct debt response, $\varphi_a = 0$. Regarding the output response parameter, I calibrate φ_y in a way that in the flexible limit, $g_t = g_t^* + \varphi_y(y_t/y_{ss} - 1)$, government consumption responds one-to-one to changes in the output gap with the negative of its constant share on output, $\varphi_y = -s_g$. Overall, as emphasized by Cochrane (2022b) and Sims (2011), my parametrization lets primary surpluses and taxes respond pro- and government consumption counter-cyclically to changes in output. Regarding monetary policy, I use standard values from the literature and implement a zero-inflation target. Finally, the decay parameter δ^b in the bond price equation is defined as 1 over the average debt maturity in years. I follow Del Negro and Sims (2015), who assume an average debt maturity of 6.8 years for the US. Finally, for the inflation response parameter, ϕ_π, I parameterize active and passive monetary policy as in Bianchi and Melosi (2017), whose parameter estimates are in line with Leeper et al. (2017b).

I analyze the Great East Japan Earthquake of 2011 in Section 3.4.1. For this purpose, I consider an alternative parametrization for the Japanese economy, as described below and summarized in Section 3.A.1. In particular, I adjust my baseline parametrization and closely follow Braun and Körber (2011), who specifically calibrate a NK model for Japan. In general, most parameters are similar to Kaplan et al. (2018). In terms of empirical data, I use the 2011 average debt maturity of 7 years[12] and the 2011 debt-to-GDP ratio[13] of 175.9%. Finally, I use an alternative surplus rule specification, which is based on Kliem et al. (2016).

More generally, finding appropriate fiscal policy rules remains a challenging task both from a empirical and a theoretical point of view because the mandate of fiscal authorities tends to be less clear than the mandate of most central banks. Possible reasons are

[11]Source: FRED Data (GFDEGDQ188S), "*Federal Debt: Total Public Debt as Percent of Gross Domestic Product*".

[12]Source: *Ministry of Finance, Japan - Debt Management Report 2020.*

[13]Source: *World Bank, Central government debt, total (% of GDP) for Japan [DEBTTLJPA188A], retrieved from FRED, Federal Reserve Bank of St. Louis.*

institutional details, political orientation or (temporary) voter preferences. My take on the current discussion on fiscal policy rules is a missing consensus on their appropriate specification, which points toward the need to intensify research in this area. Thus, one has to keep in mind that even though I orientate on empirical parameter estimates, I primarily chose surplus rules for illustrative reasons. Nevertheless, I only consider parametrizations that are based on estimates in the literature and establish and match observable empirical patterns, such as higher deficits in recessions.

3.3.2 Model Determinacy

The intention of my paper is not an elaborate discussion of determinacy regions. However, since I analyze various models where active/passive monetary and fiscal policy specifications do affect transmission channels and implications, determinacy needs to be defined and addressed properly. This holds especially true when working around the model of Dupor (2001) because this framework is known to imply determinacy regions that, at first glance, appear to contradict the commonly presumed understanding of active/passive policy regimes along the lines of Leeper (1991). The results in this section follow from a determinacy analysis similar to the ones in Dupor (2001) and Leith and von Thadden (2008). To that end, I only consider bounded solutions and examine determinacy in terms of stable and unstable Eigenvalues of the Jacobian matrix of the model. I present the determinacy analysis in detail in appendix 3.A.2.

Table 3.1 summarizes the insights from Section 3.A.2 and shows necessary monetary and fiscal policy stances needed for determinacy in the simple NK model, the NK model with capital and the NK model with capital adjustment costs. I consider these models with and without the fiscal block of Section 3.2.2. Note that the mere existence of the fiscal policy block does not determine whether fiscal policy is active or passive. By considering Ricardian households (see e.g. Leith and von Thadden (2008) or Bayer et al. (2021)), one can always calibrate the fiscal policy block in a way that its implementation leaves determinacy conditions of the underlying model unchanged. In this case, however, the fiscal policy block just becomes dragged along and remains irrelevant for the dynamic of other model variables (see e.g. Liemen and Posch (2022)). Since I am after the role of FTPL in models with capital, I focus on situations where the introduction of the fiscal policy block does matter for the dynamics of the underlying NK model. Thus, I always analyze the implementation of an active fiscal policy block throughout this paper (*fiscal regime*).

As a standard finding in the literature, monetary policy in the NK framework is either considered to be active ($\phi_\pi > 1$) or passive ($\phi_\pi < 1$). The central mechanism to render fiscal policy active or passive is its role in stabilizing debt (see e.g. Leeper (1991), Leith and von Thadden (2008) or Bai and Leeper (2017)). A fiscal policy rule that stabilizes

Table 3.1: Determinacy requirements for bounded solutions in considered NK models. The table summarizes the results of the determinacy analysis in appendix 3.A.2. Fiscal policy blocks are always chosen so that they imply active fiscal policy.

Determinacy Requirements

Model	**Monetary Policy**	**Fiscal Policy**
NK-Simple	active	passive
NK-Simple + Fiscal Block	passive	active
NK-Capital	passive	passive
NK-Capital + Fiscal Block	active	active
NK-Capital + Adj. Cost	active	passive
NK-Capital + Adj. Cost + Fiscal Block	passive	active

debt is considered to be passive while a destabilizing one is considered to be active. Similar to the value of the inflation response parameter in the Taylor rule, there is a comparable condition for the debt response parameters τ_a and φ_a in the fiscal policy rules (3.15) and (3.46). When setting τ_y and φ_y equal to zero, fiscal policy is active if $\tau_a - \varphi_a < \rho$. This demarcation line, however, can become blurred[14] under certain parametrizations that allow for a direct output response in the fiscal policy rules. Regarding this property, note that government consumption is a direct component of output so that taxes and government consumption are both explicitly and implicitly linked via their responses to output. An elaborate analysis of all special cases that result from the introduction of government consumption and all possible combinations of τ_a, τ_y, φ_a and φ_y is beyond the scope of my paper. Thus, I generally circumvent these special cases[15] by imposing parametrizations where either $\tau_a = \varphi_a \equiv 0$ or $\tau_y = \varphi_y \equiv 0$.

As a standard finding in the literature, determinacy in the simple NK model requires active monetary policy. Sims (2011) shows how the introduction of the fiscal policy block to the simple NK model changes the determinacy requirement from active- to passive monetary policy. By doing so, one obtains the NK-FTPL model along the lines of Sims (2011) and Cochrane (2018) (in both papers s_g, ρ_g and τ_a are equal to zero). When introducing capital to the simple NK model, one has to consider (similar to the NK-FTPL model) one additional state variable (the capital stock) as well as an additional no-arbitrage condition that links the marginal values of wealth and capital (equation (3.37)).

[14] Also see the discussion in Leith and von Thadden (2008).

[15] I consider simultaneous responses to y_t and a_t in Section 3.4.3. However, I directly choose unambiguous combinations of parameter values that are distinctly within the active fiscal/passive monetary policy region.

Dupor (2001) elaborates that this model specification requires passive monetary policy for determinacy. At the same time, fiscal policy also has to be passive as highlighted by Leith and von Thadden (2008) (Proposition 2.1, p. 293). Now, introduce the fiscal policy block. In line with the findings of Leith and von Thadden (2008) (Proposition 2.2, p. 293), a determinate solution requires monetary and fiscal policies to be simultaneously *active*. In fact, a determinate bounded solution always requires ϕ_π to be slightly larger than one (see Leith and von Thadden (2008)). Not least because of the famous determinacy analysis in Leeper (1991), it is widely perceived that situations, where monetary and fiscal policy are simultaneously active or passive, induce instability and deem the model indeterminate. However, the above results do not contradict Leeper (1991), as he neither considers capital accumulation[16] nor the continuous-time specific determinacy conditions. Further note that environments with non-Ricardian consumers can also blur the demarcation lines of active and fiscal policies (see Leith and von Thadden (2008)). Strictly speaking, due to the simultaneously active monetary and fiscal policy requirement, the system is *not* a FTPL model in the sense of Sims (2011) or Cochrane (2018), as fiscal policy is no longer the single main driving force in the determination of the price level. Nevertheless, for the sake of clarity and because fiscal policy is active, I denote this model as the NK model with capital and FTPL.

In line with Dupor (2002) and Posch and Wang (2020), the introduction of capital adjustment costs to the continuous-time NK-capital model with capital changes the determinacy requirements from passive to active monetary policy. This time, however, fiscal policy remains passive. This results follows from breaking the strict one-to-one relationship between the marginal values of wealth and capital in equation (3.37) when making adjustments of capital costly. As a consequence, the law of motion of the marginal value of capital (3.38) no longer coincides with equation (3.34) so that it has to explicitly enter the model as additional equilibrium variable. Finally, the determinacy region becomes reversed once more when introducing the fiscal policy block to the NK model with capital adjustment costs. Thus, one obtains a New Keynesian FTPL framework along the lines of Sims (2011) and Cochrane (2018) with active fiscal- and passive monetary policy.

The results of my determinacy analysis are in line with standard findings in the literature. What is new, however, is the explicit analysis of determinacy when adding capital adjustment costs to the simple continuous-time NK model with capital and FTPL.

3.3.3 Model Comparisons

This section compares and analyzes model dynamics and predictions of the NK-AC-FTPL model and its limiting cases discussed in Section 3.2.3.

[16]See discussion on determinacy and capital accumulation in Schmitt-Grohe and Uribe (2007).

NK Model with Capital and FTPL

Consider a contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Figure 3.1 shows the corresponding IRFs for the NK model with capital and/or FTPL. Regarding the dynamics of the simple NK-FTPL model, I refer to Chapter 2 of this doctoral thesis. For later reference, Table 3.2 shows the inflation decomposition (3.49) for the monetary policy shock. I proceed with a short recap on the dynamics of the continuous-time NK framework with capital (cf. Dupor (2001)). Turning to Figure 3.1, the unexpected increase in the nominal interest rate induces an instantaneous upward jump in the real interest rate. Due to overall higher future real interest rates, household immediately decrease consumption through inter-temporal substitution. According to equation (3.40), the dynamics of the capital rental rate and the real interest rate coincide. Due to optimizing firms, the capital rental rate is equal to the marginal product of capital times marginal costs. Consequently, in order to match the increase in the rental rate, there has to be a rise in the marginal product of capital and/or a rise in marginal costs. In particular, the optimization problem of the firms gives rise to equation (3.29), which shows that an instantaneous increase in the rental rate has to be matched by an immediate upward jump in consumption and/or labor. Because consumption initially decreases, labor has to increase instantaneously. This effect not only raises total output but also unambiguously the inflation rate (cf. Dupor (2001)).

Having laid out the central dynamics of the frameworks of Dupor (2001) and Sims (2011), I now turn to the NK-FTPL framework with capital. Obtaining a determinate solution requires that fiscal and monetary policy are simultaneously active (see Section 3.3.2). As in Dupor (2001), equation (3.40) establishes the one-to-one relationship between the dynamics of the real interest and the capital rental rate. At the same time, despite monetary policy being active, the government debt valuation equation (3.47) has to be satisfied as well. Combining equations (3.40) and (3.49) underscores the explicit relation between capital and the fiscal policy block in this framework

$$\begin{aligned} \int_t^\infty e^{-\rho(v-t)}(r_v^k - r_{ss}^k)\mathrm{d}v \quad &= \quad \int_t^\infty e^{-\rho(v-t)}(s_v - s_t^*)/a_{ss}\mathrm{d}v \\ &\quad -(p_t^b/p_{ss}^b - 1) - (v_t/v_{ss} - 1). \end{aligned} \tag{3.53}$$

Thus, changes in the present value of future capital rental rates have to be equal to the changes in the weighted present value of future surpluses less direct maturity and debt effects. Figure 3.1 illustrates that the inclusion of the fiscal policy block in the NK model with capital causes essentially all variables to respond in wavelike motions to the monetary policy shocks. As in Leith and von Thadden (2008), the model is indeterminate for parametrizations with ϕ_π only slightly larger than 1. As it turns out, if θ and ϕ_π are close to the lower bound of determinacy, shocks induce extremely slow decaying dynamics

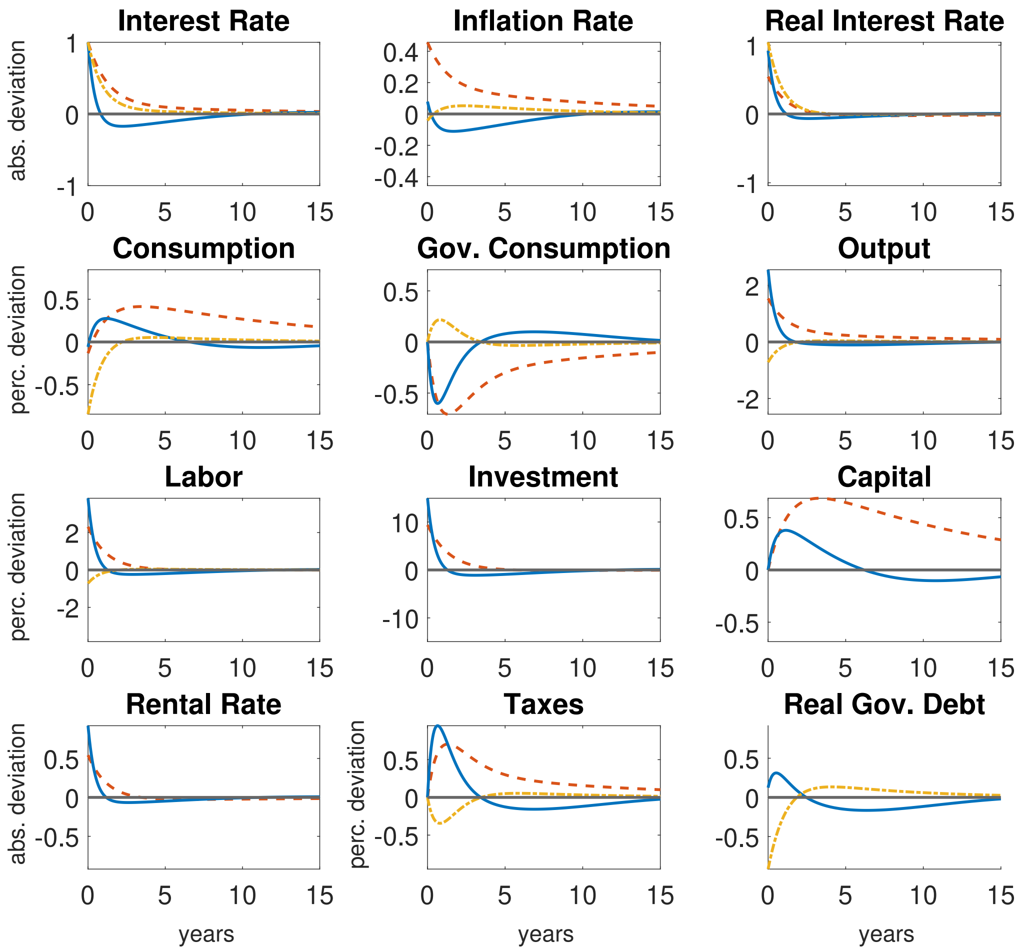


Figure 3.1: IRFs: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization. NK-FTPL model (dotted yellow lines), NK model with capital (dashed red lines) and NK model with capital and fiscal policy block and with $\theta = 3$ (solid blue lines).

Table 3.2: Inflation decomposition: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization. Simple NK-FTPL model. Entries given by equation (3.49).

$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
0.30	1.14	-0.08	-0.92

with high frequencies and amplitudes. But even for higher values of ϕ_π, the wavelike responses of variables persist. My findings suggest that the inertia of the Taylor rule with respect to the nominal interest rate is the critical factor to smooth out the wavelike IRFs. Put differently, the central bank has to commit to a high degree of interest rate smoothing as captured by high values of θ in the Taylor rule[17]. Therefore, I deviate from the baseline parametrization in this framework and set $\theta = 3$, as this value already smooths out most of the undulating dynamics.

As in the frameworks of Dupor (2001) and Sims (2011), the unexpected increase in the nominal interest rate induces an immediate upward jump in the real interest rate. The marginal values of wealth and capital coincide so that the dynamics of the real interest rate are equal to the ones of the capital rental rate (cf. Dupor (2001)). The existence of optimizing firms pins down the capital rental rate as marginal costs times the marginal product of capital. With an unambiguous initial rise in the rental rate, equation (3.29) dictates that the upward jump in the rental rate has to be matched by an immediate upward jump in consumption and/or labor. Due to overall higher future real interest rates, households substitute inter-temporally and thus decrease consumption. Consequently, firms have to increase labor demand to make up for the drop in consumption and the rise in the rental rate. Higher labor demand, in turn, causes an upward jump in wages, which together with the rise in the rental rate induces an immediate upward jump in marginal costs. The change in marginal costs is smaller than the change in the rental rate so that the upward jump in labor causes an upward jump in the marginal product of capital (cf. Dupor (2001)). Thus, investment increase immediately and induce an upward jump in output, which reflects the initial rise in labor (recall that capital cannot jump endogenously). Due to the initial rise in wages and the rental rate, the present value of future marginal costs increases, reflecting an upward in the inflation rate.

Model responses in the framework of Dupor (2001) and in the NK-FTPL model with capital initially only differ in terms of magnitudes (see Figure 3.1). Contrary to the frameworks of Dupor (2001) and Sims (2011), the interest rate hike in the NK-FTPL model with capital is short-lived and followed by a prolonged period of lower nominal interest rates. In fact, the change in the maturity weighted present value of future nominal interest rates is negative so that the no-arbitrage condition (3.48) requires an upward jump in the bond price, which translates to an immediate increase in real debt. In contrast to the simple NK-FTPL framework, the contractionary monetary policy shock is highly expansionary. Following a pro-cyclical surplus rule, the fiscal authority addresses higher output by gradually raising taxes and by lowering government consumption. Consequently, as highlighted by the inflation decomposition equation (3.49), the weighted present value of

[17]It is important to keep in mind that the model remains determinate even for lower values of θ. Also see Schmitt-Grohe and Uribe (2007) for a discussion on determinacy and interest rate smoothing in discrete-time models with capital accumulation.

Table 3.3: Inflation decomposition: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization. NK-FTPL model with capital and $\theta = 3$. Entries given by equation (3.49).

$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
-0.43	-0.34	0.21	0.12

future surpluses increases by 0.21 percentage points. Hence, fiscal policy has a negative impact on the change in the present value of future inflation rates. Similar to the model of Dupor (2001), the monetary policy shock is not only expansionary but initially raises inflation. The central bank starts to decrease the nominal interest rate in order to bring down the increased real interest rate (equivalently the capital rental rate). In contrast to Dupor (2001), however, the path of future inflation follows from a combination of (active) monetary and (active) fiscal policy as well the debt decomposition. As it turns out, satisfying the government debt valuation equation (3.47) requires that the inflation rate and the nominal interest rate persistently drop below their corresponding steady states during transition. In particular, the inflation decomposition shows that the present value of future nominal interest rates decreases by 0.34 percentage point so that monetary policy also contributes to overall lower inflation rates. This effect, however, is partly soaked up by higher bond prices as reflected in the direct maturity effect of 0.12 percentage points. Taking everything together, the present value of future inflation declines by 0.43 percentage points. This in turn, implies an increase in the present value of future real interest rates by 0.09 percentage points, which is equal to the change in the present value of future capital rental rates. Consequently, as captured by equation (3.54), changes in the weighted present value of future surpluses less the initial maturity effect fully account for the change in the present value of future capital rental rates.

The above evaluation of the NK model with capital and FTPL highlights the various and partly competing interactions of fiscal and monetary policy. As a consequence, the dynamics of all model variables experience wavelike dynamics. I conclude that the NK-FTPL model with capital is unsuitable for actual policy analysis, as it inherits undesired properties from the model in Dupor (2001), and consequently is ad odds with conventional economic thinking in many different dimensions. In particular, the model's predictions turn out to be even less plausible than the ones of the simple NK capital model without FTPL (e.g. the overall decline in nominal interest rates or the wavelike dynamics). In summary, if one is interested in analyzing FTPL in a simple continuous-time framework with capital, one has to rely on additional ingredients to avoid ending up with the shortcomings of the NK-FTPL model with capital. Hence, I reject the model and do not analyze the propagation of other shocks.

NK Model with Capital Adjustment Costs and FTPL

I build on the insights of the previous section, and show how the introducing of capital adjustment costs resurrects the counterintuitive implications of the continuous-time NK-FTPL model with capital. Analogously to the previous section, I initially abstract from FTPL and start the analysis with a short recap on the effects of a contractionary monetary policy shock in the NK model with capital adjustment costs (cf. Posch and Wang (2020)). Figure 3.2 shows the IRFs for an interest rate hike by 1 percentage point in the simple NK-FTPL, the NK-AC and the NK-AC-FTPL model. The dynamics of the simple NK-FTPL model are as described in Section 3.3.3.

In case of the NK model with capital adjustment costs, monetary policy is active and an interest rate hike is accompanied by an increase in the real interest rate. In line with the standard NK framework, the inflation rate moves inversely to the nominal interest rate. Thus, the inflation rate initially jumps downwards before steadily converging back towards its equilibrium level. Due to persistently higher real interest rates, households respond by initially decreasing their consumption expenditures through inter-temporal substitution. The first-order condition (3.31) of the HJB unveils that percentage changes in the marginal value of wealth are the mirror image of percentage changes in consumption. Thus, there is a steep upward jump in the marginal value of wealth. Firms address the decreased demand for goods by reducing investments and labor, which is reflected in a downward jump in the marginal value of capital. The rental rate and wages decrease, which in turn induces a downward jump in marginal costs. The capital stock decreases for as long as the marginal value of capital exceeds the marginal value of wealth. Since both marginal values initially jump in opposite directions, there is a gradual and prolonged decline in capital. The drawn out response of capital and its relative slow convergence back to steady state, are, above all, consequences of the relatively costly adjustment of capital. In summary, the introduction of capital adjustment costs resurrects the model implications of Dupor (2001). Hence, adjustment costs not only offer a possibility to change determinacy conditions form passive to active monetary policy but also a way to offset the counterintuitive implications of the expansionary nature of contractionary monetary policy shocks in the simple continuous-time NK model with capital (cf. Dupor (2002) and Posch and Wang (2020)).

In the NK-AC-FTPL model, the unexpected increase in the nominal interest rate translates to an immediate upward jump in the real interest rate. Responding to overall higher real interest rates, households initially decrease consumption through inter-

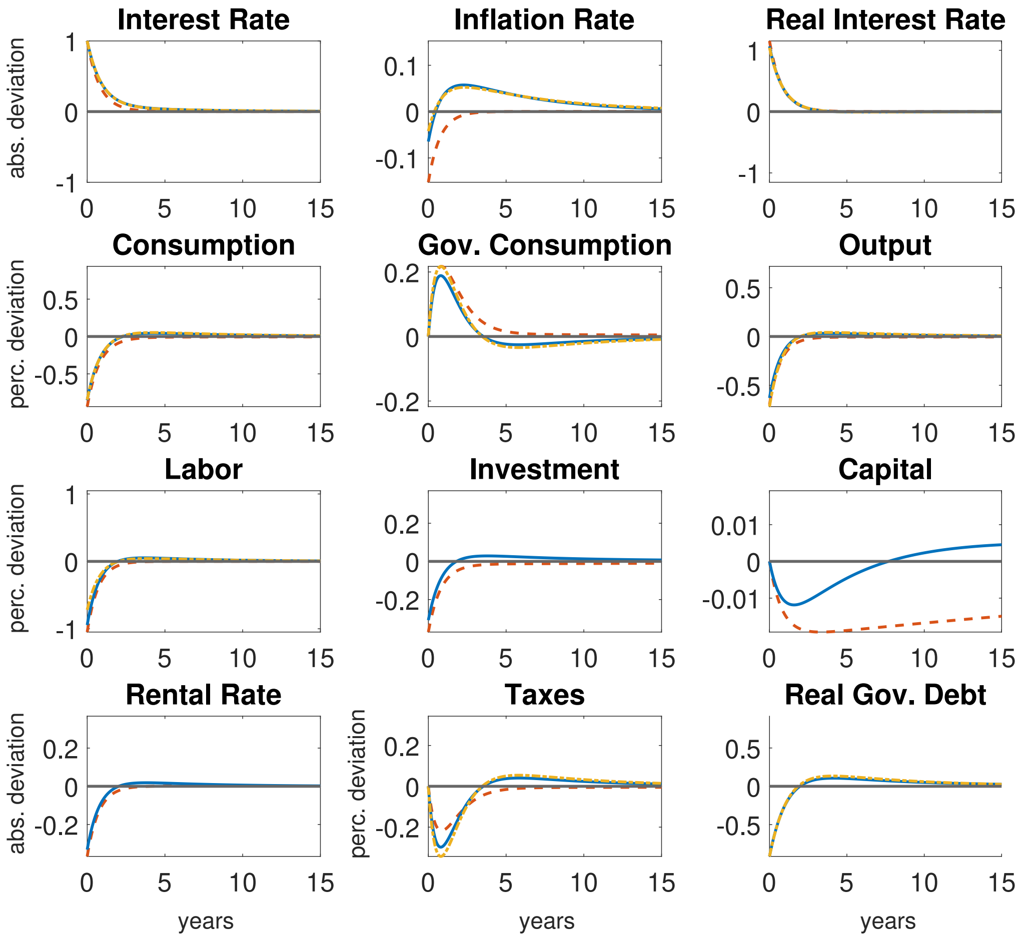


Figure 3.2: IRFs: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization. NK-FTPL model (dotted yellow lines), NK model with capital adjustment costs (dashed red lines) and NK model with capital adjustment costs and FTPL (solid blue lines).

temporal substitution. Firms address declining demand by decreasing labor inputs and investment, which corresponds to an immediate downward jump in the marginal value of capital. Hence, wages and the capital rental rate also decrease initially. As in the NK-AC model, percentage changes in the marginal value of wealth are the mirror image of percentage changes in consumption. Thus, the marginal values of capital and wealth initially jump in opposite directions so that the capital stock gradually decreases for as long as the marginal value of wealth exceeds the marginal value of capital. The fiscal authority cannot instantaneously adjust taxes and government consumption (both variables are predetermined). Consequently, the initial declines in investments and consumption induce an immediate contraction of output. As captured by the no-arbitrage condition for the bond price (3.48), overall higher nominal interest rates require an instantaneous decline in the bond price. This in turn, induces an immediate devaluation of government

Table 3.4: Inflation decomposition: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization. NK-AC-FTPL model. Entries given by equation (3.49).

$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
0.30	1.13	-0.08	-0.91

debt, which causes a downward jump in the real value of debt. The fiscal authority follows a pro-cyclically surplus rule. Thus, it lowers taxes and at the same time provides an additional fiscal stimulus by increasing government consumption. Surpluses have an s-shaped response so that preceding deficits are partly payed back by future surpluses. In fact, most variables in the NK-AC-FTPL have a slightly s-shaped response, which, as argued below, primarily results from the evolution of the real interest rate. According to the inflation decomposition equation (3.49), the increase in the present value of future inflation rates (0.3 percentage points) follows from an increase in the present value of future nominal interest rates (1.13 percentage points), which is partly soaked up by a revaluation of debt (-0.91 percentage points), and a decrease in the weighted present value of surpluses by 0.08 percentage points. In order to satisfy the government debt valuation equation (3.47), this requires a path of the inflation rate with an initial drop, followed by a relative larger and more drawn-out increase in future inflation rates (*stepping on a rake effect*). Monetary policy is passive and the central bank adjusts the nominal interest less than one-to-one to changes in the inflation rate. As a consequence, the change in the inflation rate exceeds the change in the nominal interest rate during transition. Thus, the real interest rate has a slightly s-shaped response. However, the present value of future real interest rates increases by 0.83 percentage points, which reflects that the temporary decline in the real interest rate is relatively small compared to the preceding period of higher real interest rates. Nevertheless, the subsequent decline in the real interest rate induces a small expansionary effect, which causes s-shaped responses of most variables in the model. Note that this feature is not specific to the NK-AC-FTPL model but a general property of the NK-FTPL framework.

The above analysis and the IRFs in Figure 3.2 highlight that the FTPL mechanisms directly translate to the NK model with capital adjustment costs. In case of my baseline parametrization, the dynamics of all variables in the NK-AC-FTPL model that are also present in the simple NK-FTPL model closely resemble the dynamics of their corresponding counterparts in the simple NK-FTPL model. The remaining variables, at least initially, respond to the shock as suggested by the NK-AC model without FTPL. In the longer-run, however, the fiscal theory kicks in and model predictions deviate from the NK-AC model without FTPL. A relatively large part of these differences can be attributed to the effects of FTPL on the evolution of the inflation rate as well as the related dynamics of the real interest rate and the marginal value of wealth. Due to the s-shaped responses of the marginal values of wealth and capital in the NK-AC-FTPL model, initial decreases in the capital stock are short-lived and capital becomes replaced relatively quick. In fact, the capital stock even overshoots its equilibrium, which reflects the above described temporary expansionary effect of the monetary policy shock during transition. Hence, in contrast to the NK-AC model, the initial declines in investment and the rental rate also have to be followed by s-shaped responses. Recall that the Philips curve relates changes in inflation to changes in the present value of marginal costs. Since the government debt valuation equation implies a boomeranging path of the inflation rate, the initial drop in marginal costs ultimately has to be followed by a period of higher marginal costs. Overall, even though transmission channels differ, the responses of the NK-AC-FTPL model to the monetary policy shock preserve the initial and medium-term dynamics of the underlying NK-AC model.

In case of the baseline parametrization, the adjustment of capital is relatively costly. As a consequence, there are in most cases only minor differences between common variables in the simple NK-FTPL and the NK-AC-FTPL model. In fact, the respective inflation decomposition in Tables 3.2 and 3.4 are basically identical. Keeping all else equal, a less costly adjustment of capital (higher value of κ) induces significant differences in the dynamics of the two models. Consequently, by lowering capital adjustment costs, most of the IRFs of the two FTPL models in Figure 3.2 are basically no longer on top of each other. However, it is important to keep in mind that if $\kappa \to \infty$, the NK-AC-FTPL model approaches the simple NK-FTPL with capital (see Section 3.3.3) and inherits its undesired features.

I conclude that the combination of capital adjustment costs and the fiscal theory offers a simple and intuitive framework to study the role of FTPL in NK models with capital. One appealing feature of this approach is the ability to (at least temporary) maintain model implications and predictions of the NK-AC and the NK-FTPL model. This property also holds when considering other policy shocks. For an elaborate discussion on shocks to capital and government consumption, I refer to Sections 3.4.1 and 3.4.2, respectively. Furthermore, as I argue in these sections, the combination of FTPL and capital adjustment costs is able to solve at least two puzzles in the literature. Thus, the NK-AC-FTPL model offers an important benchmark framework, as it allows analyzing interactions of fiscal and monetary policy, government debt, investments as well as capital, in a joint, simple and consistent framework.

3.4 NK-AC-FTPL Model: Policy Experiments, Puzzles and Properties

This section evaluates and solves two puzzles in the literature. Strictly speaking, I show that these are no puzzles at all in a FTPL framework. Furthermore, I revisit the short-term debt implications of the FTPL framework.

3.4.1 The Great East Japan Earthquake

It was on March 11 in 2011 that a magnitude 9.0 earthquake struck the northeast part of the Japan Trench. Within 30 minutes the resulting Tsunami hit the east cost of Japan, devastating critical infrastructure, the power grid as well as production facilities. The earthquake (known as the *Great East Japan Earthquake* or the *Tōhoku Earthquake*) turned out to be extremely damaging for Japan's economy. In particular, available evidence suggest that the earthquake was contractionary and accounted for a substantial drop in output (see e.g. Tokui et al. (2017), Wieland (2019) or Carvalho et al. (2021)). For instance, Tokui et al. (2017) estimate that resulting supply chain disruptions induced a production loss of at least 0.35 percent of GDP, whereas Carvalho et al. (2021) estimate that the earthquake resulted in a 0.47 percentage points drop in GDP in the subsequent year.

Following Wieland (2019), I define the Great East Japan Earthquake of 2011 as an unexpected shock to the capital stock. Equipped with the fiscal theory and capital, I confront the NK-AC-FTPL model with the natural disaster and compare predictions and implications with standard NK models in the literature.

Frequently encountered features of some models within the standard NK framework are counterintuitive predictions under a nominal interest rate peg. In particular, wasteful government spending, capital and output destruction as well as technical regress turn out to be highly expansionary and produce large multipliers at the ZLB (see for instance Eggertsson (2011), Eggertsson et al. (2014), Kiley (2016), Cochrane (2017) or Wieland (2019)). Since the standard NK model relies on active monetary policy, certain assumptions and modifications are necessary to obtain determinate solutions. One widely used approach in the literature[18] exploits that the standard NK model, despite being build around the assumption of active monetary policy, can under a zero-inflation target and a temporary binding interest rate peg still select a locally unique and forward-bounded equilibrium. Pinning down this equilibrium requires that among all forward-bounded solutions, the central bank is able to restore its zero inflation target after a shock occurred. Cochrane (2017) shows that the counterintuitive predictions of the standard NK models

[18] See for instance Eggertsson (2011), Christiano et al. (2011), Werning (2012) or Wieland (2019).

at the ZLB are a direct consequence of ultimately letting the monetary authority commit to its zero-inflation target. Due to its wide adoption in the literature, I take the uniquely defined equilibrium path and the resulting counterintuitive implications of the above models as given, and use them in direct comparison to my framework. For this purpose, I compare the NK-AC-FTPL model with the Smets and Wouters (2007) model as specified in Wieland (2019). In this framework Wieland (2019) directly assumes an interest rate peg (or tautologically the ZLB) that binds for 25 years and defines the Great East Japanese Earthquake of 2011 as an unexpected negative shock to the capital stock. The paper highlights the expansionary effect of capital destruction at the ZLB and concludes that the model implications fail to reproduce the empirically observed drop in output.

As highlighted in the previous sections, FTPL offers an alternative equilibrium selection mechanism. Since monetary policy is *passive* in the NK-AC-FTPL model, one can readily analyze an interest rate peg[19] (such as the ZLB) by setting θ in the Taylor rule (3.14) equal to zero. Thus, the NK-FTPL framework remains determinate even under the limiting case of a pegged interest rate. To motivate my approach, note that the basic loan rate of the Bank of Japan (BoJ) between 1960 and 2021 remains basically pegged at a lower-bound since 1995.[20] In particular, the official policy target rate even became negative during this time. Despite a recent rise in global interest rates, the BoJ (in contrast to the FED or the ECB) confirms its commitment to keep its policy rate unchanged[21]. This suggests that the Japanese economy is indeed in a passive monetary policy regime, which is characterized by a prolonged ZLB episode for nearly 30 years. Thus, directly imposing passive monetary policy (or even a peg) appears justified by the data.

Figure 3.3 shows the IRFs for output, the capital stock and the inflation rate following a temporary shock that destroys one percent of the capital stock. For reasons of comparability, the layout of the figure is borrowed from Wieland (2019). The left-hand side panels show the IRFs of the Smets and Wouters (2007) model as replicated by Wieland (2019). The term "*normal times*", which I also borrow from Wieland (2019), denotes the situation without interest rate peg. The right-hand side panels show the corresponding IRFs of the NK-AC-FTPL model. In contrast to the previous sections, I closely follow the parametrization of Braun and Körber (2011), who explicitly parameterize a NK model for Japan for the period prior to the earthquake. I further deviate from my baseline parametrization and consider fiscal response parameters similar to the ones in Kliem et al. (2016). Hence, government consumption and taxation are now explicitly driven by debt- instead of output dynamics. I change the law of motion of surpluses for illustrative reasons. As already pointed out throughout the previous sections, no clear consensus has yet emerged regarding the specification of surplus rules.

[19]Note that it is unimportant for simple NK models whether pegging the interest rate at zero or another non-negative constant (see Wieland (2019)).

[20]See e.g. Bank of Japan: Basic Loan Rate (*FRED time series: [IRSTCB01JPM156N].*)

[21]Source: *Bank of Japan, Statement on Monetary Policy, July 21, 2022.*

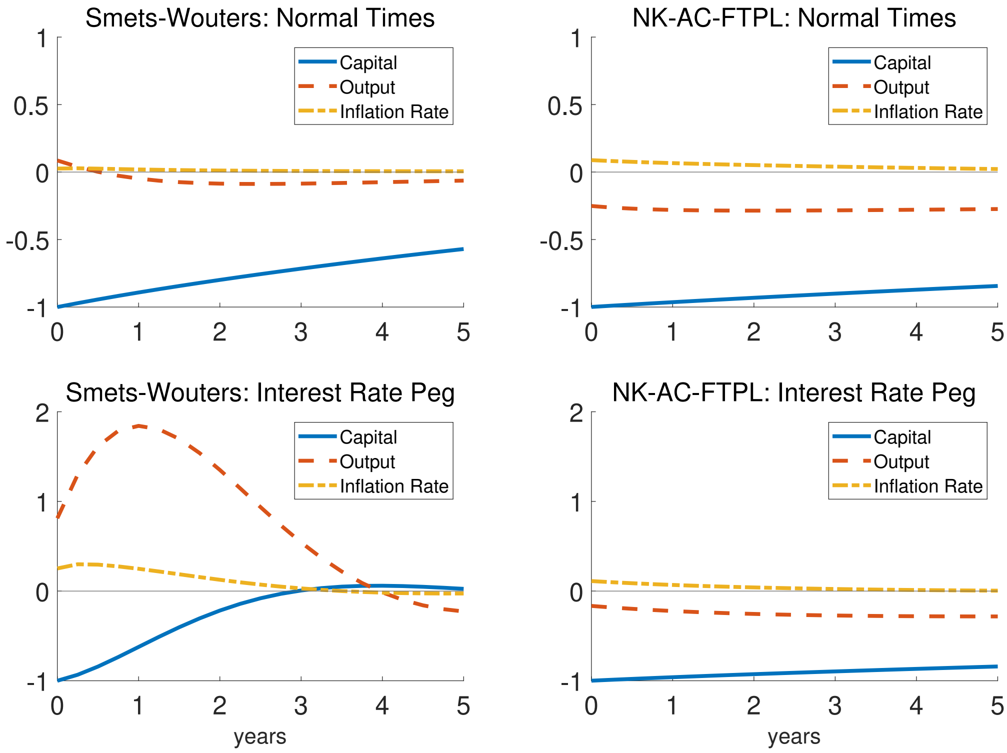


Figure 3.3: IRFs: Negative shock to capital by 1 percent (capital destruction). Left-hand side panels show IRFs of the Smets and Wouters (2007) model as replicated by Wieland (2019). Right-hand side panels show the corresponding IRFs of the NK-AC-FTPL model using parametrization 2.

In the standard NK framework in *normal times* (upper left-hand side panel of Figure 3.3), capital destruction raises marginal costs, which in turn increases inflation. Because monetary policy is active, the central bank reacts by increasing the nominal interest rate more than one-to-one with inflation. Thus, the real interest rate starts to rise so that a small initial increase in output is followed by a prolonged contraction. With an interest rate peg (lower left-hand side panel of Figure 3.3), however, the monetary authority cannot adjust the nominal interest rate. Hence, the rise in inflation consequently decreases the real interest rate substantially. Due to overall lower future real interest rates, households substitute inter-temporally and increase consumption. Firms build up investment, which in turn also fuels the expansionary effect. Consequently, the capital stock is replaced at a relatively high rate compared to the *normal times* scenario. Due to the overall expansionary nature of the shock, the initial upward jump in output is followed by a further gradual increase, which results in a rise in output by nearly 2 percent. Thus, in line with the theoretical literature discussed above, capital destruction turns out to be highly expansionary at the ZLB.

Turning to the NK-AC-FTPL framework (right-hand side panels of Figure 3.3), the inflation decomposition equation (3.49) implies that the path of future inflation rates is

Table 3.5: Inflation decomposition: Negative shock to capital by 1 percent (capital destruction). Parametrization 2. NK-AC-FTPL model in *normal times* and at the ZLB. Entries given by equation (3.49).

Setting	$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
Normal Times	-0.25	-0.21	-0.08	-0.12
ZLB	0.08	0	-0.08	0

again determined by an interplay of monetary and fiscal policy as well as the composition of government debt. Since the earthquake destroys a part of the capital stock, there is less capital available for production. The rental rate sharply jumps upwards and becomes the central driving force for overall higher future marginal costs. Following the rise in the capital rental rate, there is an instantaneous drop in investment. Due to overall higher marginal costs, the inflation rate immediately increases as highlighted by the integrated Philips curve. The central bank cannot instantaneously adjust the nominal interest rate so that the real interest rate drops on impact. Responding to the rise in the inflation rate, the central bank then starts to increase the nominal interest rate, which in turn gradually elevates the decreased real interest rate. However, due to the passive monetary policy regime, the monetary authority adjusts the nominal interest rate less than one-to-one with inflation. In order to satisfy the government debt valuation equation (3.47), the nominal interest rate as well the inflation rate have to drop below their corresponding steady states during transition. In particular, the decline in the inflation rate has to temporary exceed the drop in the nominal interest rate. Inflation dynamics again follow from the interplay of monetary and fiscal policy as well as the maturity structure of debt. As captured by the inflation decomposition equation, the path of the inflation rate has to imply an increase in the present value of future real interest rates by 0.04 percentage points (see Table 3.5). This increase can be attributed to changes in fiscal policy and to the direct maturity effect. Regarding the latter, despite the drop in the present value of future nominal interest rates by 0.21 percentage points, the maturity weighted present value of future nominal interest rates in the no-arbitrage condition (3.48) rises. Consequently, the bond price jumps downwards, which in turn induces an immediate devaluation of real debt. In contrast to my baseline parametrization, surpluses respond positively to changes in debt. Hence, the fiscal authority decreases taxes and increases government expenditures. The size of the corresponding debt response parameter, however, is relatively small compared to the output response parameters in the surplus rule

Table 3.6: Inflation decomposition: Negative shock to capital by 1 percent (capital destruction). Parametrization 2 with fiscal policy parameters from the baseline parametrization. NK-AC-FTPL model in *normal times* and at the ZLB. Entries given by equation (3.49).

Setting	$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
Normal Times	1.68	1.41	-1.08	-0.81
ZLB	0.96	0	-0.96	0

in the baseline parametrization. Therefore, changes in surpluses are relatively small in absolute terms. Thus, fiscal policy stabilizes the real interest rate because the weighted present value of futures surpluses only contributes a relative small decrease of 0.08 percentage points to the overall change in the present value of future real interest rates. The maturity structure of debt, therefore, accounts for the rise in the present value of future real interest rates. Due to overall higher future real interest rates, households initially decrease consumption through inter-temporal substitution. The fiscal authority cannot instantaneously adjust government consumption. Therefore, in contrast to the Smets and Wouters (2007) model[22], the initial declines in consumption and investments translate to a instantaneously downward jump in output. Note that labor initially increases, despite a drop in wages. This occurs because the decrease in real wages is smaller than the drop in consumption so that total labor income ($w_t l_t$) slightly increases. However, higher labor and lower wages neither offset the rise in marginal costs nor the drop in output. To obtain further insights into transmission channels and effects of the earthquake in the *normal times* setting, I additionally evaluate two alternative surplus rules. Tables 3.6 and 3.7 highlight that the surplus process is a crucial factor for obtaining an initial drop in consumption and output. To evaluate the underlying mechanisms, I again consider Parametrization 2 but replace the tax and government consumption response parameters with the ones from the baseline parametrization. Thus, taxes and government consumption now directly respond to changes in output instead of debt. Figure 3.4 shows the corresponding IRFs. Because surpluses react relatively strong to changes in output, there is a prolonged period of lower tax revenues and higher government consumption. As a consequence, the weighted present value of future surpluses now contributes a rise by 1.08 percentage points to the change in the present value of future inflation (versus a

[22]In general, the initial output response can also be negative in the traditional NK framework. See for instance Figure 3.A.1.

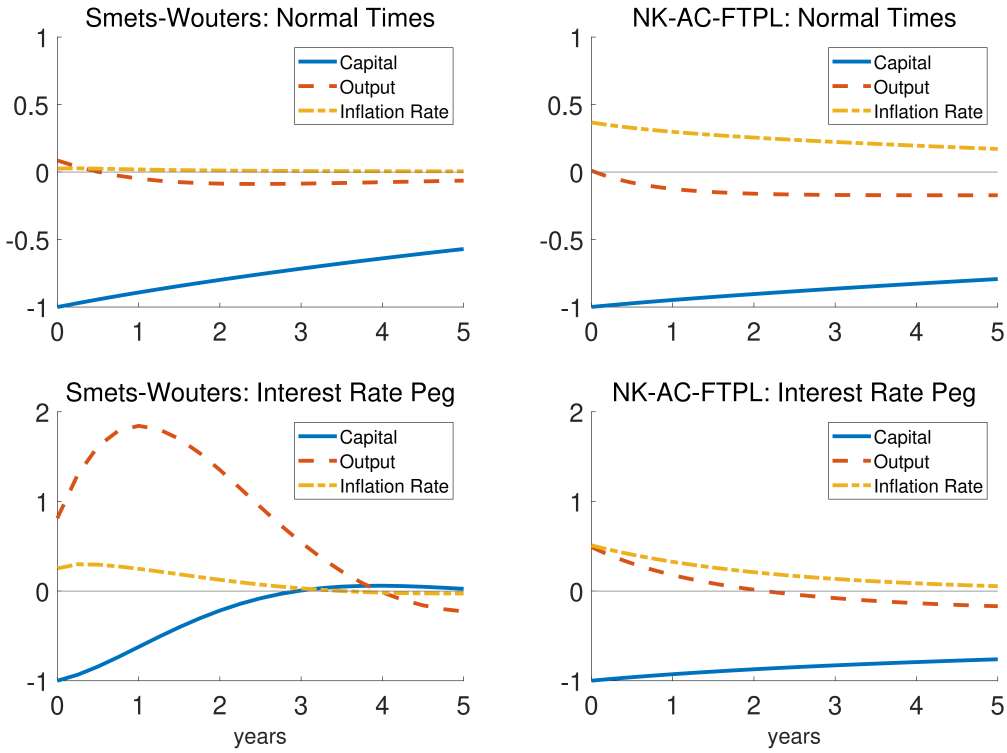


Figure 3.4: IRFs: Negative shock to capital by 1 percent (capital destruction). Left-hand side panels show IRFs of the Smets and Wouters (2007) model as replicated by Wieland (2019). Right-hand side panels show the corresponding IRFs of the NK-AC-FTPL model when using Parametrization 2 but the fiscal policy parameters from the baseline parametrization.

0.08 percentage points increase with parametrization 2). Higher inflation requires that the monetary authority increases the nominal interest rate more aggressively as reflected by a rise in the present value of future nominal interest rates by 1.41 percentage points (versus a drop by 0.21 percentage points increase with parametrization 2). Hence, the direct maturity effect is larger and decreases the present value of future inflation by 0.81 percentage points (versus a 0.12 percentage point decline with parametrization 2). In total, the present value of future inflation increases by 1.68 percentage points (versus a decrease of 0.25 percentage points with parametrization 2). Thus, satisfying the government debt valuation equation (3.47) requires a path with higher inflation rates, characterized by a sharp initial upward jump. Consequently, the real interest rate decreases substantially. Due to overall lower future real interest rates, households initially increase consumption. Thus, in contrast to parametrization 2, there is a rise in consumption, which exceeds the initial drop in investments so that there is an overall short lived expansionary effect on output as in the Smets and Wouters (2007) model.

To conclude the analysis of the *normal times* setting, I again consider Parametrization 2 but this time make surpluses constant ($T_t = T_{ss}$ and $g_t = g_{ss}$). In this case, the

Table 3.7: Inflation decomposition: Negative shock to capital by 1 percent (capital destruction). Parametrization 2 with constant surpluses ($g_t = g_{ss}$ and $T_t = T_{ss}$). NK-AC-FTPL model in *normal times* and at the ZLB. Entries given by equation (3.49).

Setting	$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
Normal Times	-0.42	-0.35	0	-0.07
ZLB	0	0	0	0

model dynamics are closely related to the ones of parametrization 2. This follows directly from the role of surpluses in the inflation decomposition equation (3.49). In the constant surplus case, there are no changes in taxes and government consumption. Consequently, the weighted present value of future surpluses is equal to zero and, therefore, in absolute terms, only slightly smaller than the one obtained with Parametrization 2 (0 versus -0.08 percentage points). With constant surpluses, changes in the present value of future real interest rates are fully accounted for by the direct maturity effect. In the previous two examples, there are pro-cyclical surplus rules, which induce a decrease to the weighted present value of future surpluses. Since this (expansionary) effect vanishes in the constant surplus setting, the constant surplus rule specification is associated with the biggest instantaneous drop in output among all considered parametrizations.

When turning to the NK-AC-FTPL model at the ZLB, basically the same mechanisms as in *normal times* apply. This result directly follows from the passive monetary policy regime, which already nests the limiting case of an immobile nominal interest rate. In order to satisfy the no-arbitrage relation between the nominal interest rate and the bond price (3.48), all debt has to be short-term at the ZLB. As a consequence, there is no immediate (endogenous) revaluation of government debt. Thus, at the ZLB, the change in the present value of future inflation is fully determined by the weighted present value of future surpluses as captured by the inflation decomposition equation (3.49).

I start my analysis by turning to Parametrization 2 and implement an interest peg by setting $\theta \equiv 0$. The model responses to the earthquake closely follow the ones in *normal times*, as the model mainly operates through the same channels. Thus, one again obtains the initial rise in marginal costs and inflation. As before, investments decrease, which reflects the rise in the capital rental rate. Surplus dynamics depend positively on the evolution of real debt, which (in the absence of long-term bonds) starts to decrease gradually. At the ZLB, the weighted present value of future surpluses falls by 0.08 percentage points (see Table 3.5). In order to satisfy the government debt valuation equation (3.47), this requires an equivalent increase in the present value of future inflation. Thus, the total effect on inflation can be attributed to fiscal policy. Since the nominal interest rate remains pegged, the dynamic of the real interest rate is equal to the negative of the dynamic of the inflation rate. The change in the present value of future real interest rates is negative (in contrast to *normal times*), which is fully accounted for by the decrease in the present value of future surpluses. The real interest rate has an s-shaped response with a relatively steep initial decline. Since the central bank cannot adjust the nominal interest rate, the subsequent rise in the real interest rate is relatively slow compared to *normal times*. In fact, the integral over (non-discounted) changes in the real interest rate is positive so that the drop in present value of future real interest rates results from discounting. Thus, overall higher real interest rates induce a drop in consumption through inter-temporal substitution. Together with the downward jump in investments, one obtains an initial drop in output as in *normal times*. However, keeping all else equal, the contractionary effect has to be smaller at the ZLB compared to *normal times* (see Figure 3.3). On the one hand, the maturity channel ceased to exist. On the other hand, the central bank cannot adjust the nominal interest rate. As a consequence, the present value of future inflation rate increases in *normal times* but decreases at the ZLB. Thus, future real interest rates are lower at the ZLB, which induces a smaller initial drop in consumption, and consequently output.

At the ZLB, higher changes in the weighted present value of future surpluses translate to higher changes in the present value of future inflation. To further evaluate this feature, I consider Parametrization 2 with an interest rate peg and the fiscal policy parameters of Parametrization 1. The NK-AC-FTPL model now implies a decrease in the present value of future real interest rates by 0.96 percentage points (see Table 3.6). Responding to overall lower real interest rates, households substitute inter-temporally and instantaneously increase consumption expenditures. The initial rise in consumption is more substantial that the initial drop in investment. Consequently, the immediate output response to the earthquake is highly expansionary as implied by the Smets and Wouters (2007) model. In contrast to this model, there is no further increase in output in the NK-AC-FTPL framework. Thus, output directly starts to decline and persistently falls below its equilibrium level during transition.

Finally, I consider Parametrization 2 with an interest rate peg and constant primary surpluses. As in *normal times*, this specification amplifies the contractionary effect of the shock through the same transmission channels. Table 3.7 highlights that all components of the inflation decomposition have to be equal to zero in this case. In line with the insights from parametrization 2, the absence of monetary policy and long-term debt induces a increase in the (non-discounted) integral over future real interest rates. Thus, through inter-temporal substitution, the constant surplus induces the biggest contractionary effect among considered parametrizations.

The above analysis suggest that the predictions of the NK-AC-FTPL model are closer to the actual data than those implied by the above formulations of the standard NK framework. Most notably, in line with the existing empirical evidence and economic intuition, the NK-AC-FTPL model hints that the great east Japan earthquake was indeed contractionary. In regards to policy analysis at the ZLB, one crucial benefit of the FTPL over the standard NK framework is the passive monetary policy specification, which nests the special case of an unresponsive nominal interest rates. Hence, one does not rely on additional assumptions or alternative equilibrium selection mechanisms. The fiscal theory, therefore, offers an elegant and simple way to overcome the counterintuitive predictions of the standard NK framework at the ZLB. However, model implications are prone to the parametrization and obtaining an initial contractionary output response depends less on the distinction between *peg* or *normal times* but more on the specification of fiscal policy. In *normal times*, even though transmission channels differ, the standard NK framework and the NK-AC-FTPL model imply similar dynamics.

3.4.2 The Crowding-In Consumption Puzzle

Understanding the effects of an increase in government spending on private consumption is a much debated topic in both empirical and theoretical macroeconomics. One particular aspect of the discussion is the *Crowding-in consumption puzzle*[23]. This puzzle refers to a frequently encountered discrepancy of a theoretically implied crowding-out but an empirically observed crowding-in of consumption in response to an increase in government spending. In this section, I provide a simple theoretical framework that allows consumption responses in either direction.

There exist different approaches in the literature to address the theoretically side of the Crowding-in Consumption Puzzle. For instance, Galí et al. (2007) show how the introduction of rule-of-thumb investors can produce a crowding-in effect that is consistent with their considered data. Bilbiie (2011) suggest that a combination of Edgeworth substitutability in the utility function and shifts in labor demand is able to solve the puzzle. For additional examples, I refer to Lewis and Winkler (2017), who offer an extensive literature review and discussion on the Crowding-in Consumption Puzzle, in both empirical and theoretical studies. While the existence of a crowding-in effect of consumption seems to be widely accepted, there appears to be no clear consensus, neither in the empirical nor in the theoretical literature, on how investments respond to increases in government consumption (see e.g. Burnside et al. (2004), Galí et al. (2007), Mountford and Uhlig (2009), Lewis and Winkler (2017) or Bayer et al. (2021)). I also contribute to this strand of literature because the NK-AC-FTPL framework allows me to simultaneously analyze

[23]See e.g. Linnemann (2006), Galí et al. (2007), Bilbiie (2011), Iwata (2013), Ambler et al. (2017), Lewis and Winkler (2017) or Rüth and Simon (2022) (forthcoming).

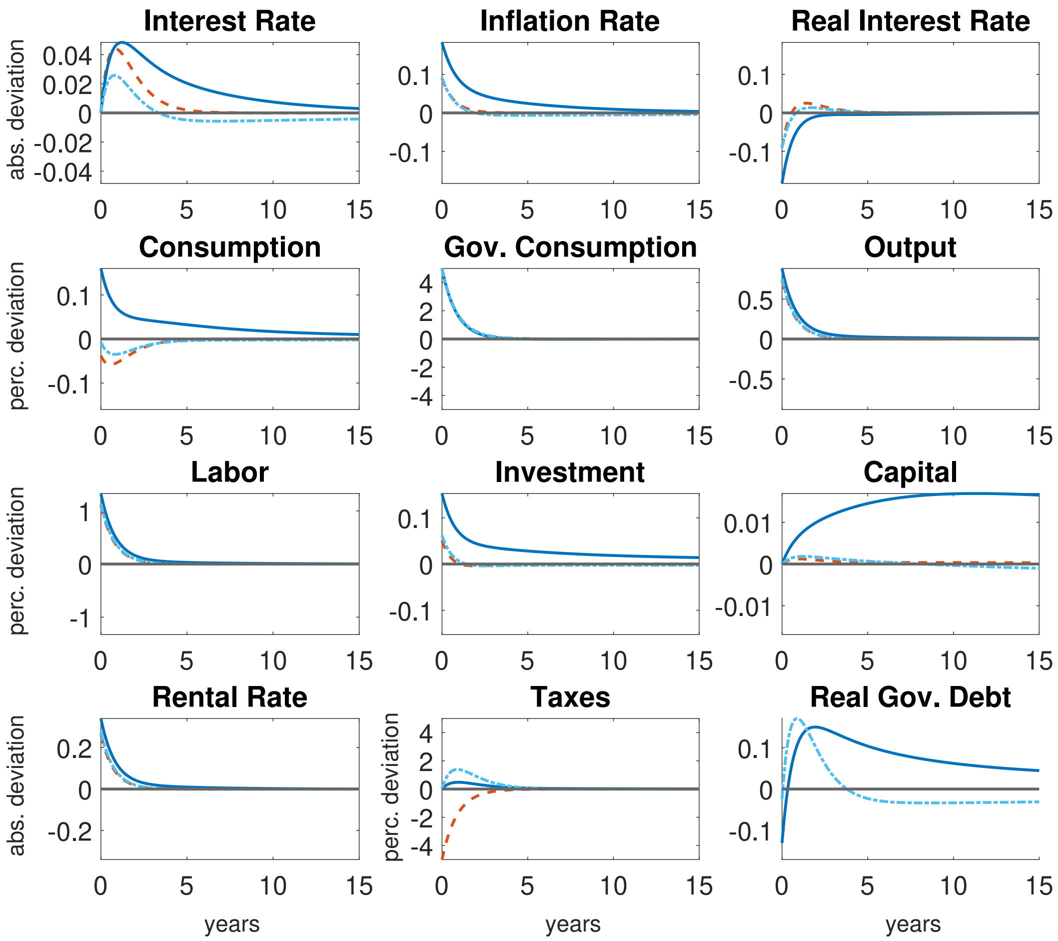


Figure 3.5: IRFs: Fiscal policy shock that increases government consumption by 5 percent. Baseline parametrization. NK-AC model (dashed red lines), NK-AC-FTPL model (solid dark blue lines) and recalibrated NK-AC-FTPL model with $\tau_y = 1.15$ and $\phi_\pi = 0.95$ (dotted light blue lines).

the responses of investments and the capital stock to changes in government consumption.

Consider a fiscal policy shock that increases government consumption by 5 percent. Figure 3.5 shows the corresponding IRFs for the NK-AC model as well as the NK-AC-FTPL model when using either the baseline or an alternative parametrization, which I choose for illustrative reasons and describe in detail below. Government consumption is a component of output so that the fiscal policy shock has an expansionary effect on aggregate output. As a consequence, output and the inflation rise immediately in the NK-AC model. Since the central bank cannot instantaneously adjust the nominal interest rate, the real interest rate jumps downwards. Monetary policy is active and the central bank responds more than one-to-one to changes in inflation. Thus, the nominal interest rate catches up with the higher inflation rate at a relatively fast pace, compared to the NK-AC-FTPL model, so that there is a s-shaped response of the real interest rate. Despite the initial drop, there is an overall increase in future real interest rates. Households respond to higher future real interest rates by initially decreasing consumption, due to inter-temporal substitution. In particular, households even further decrease consumption for as long as the real interest rate stays below its equilibrium level. Thus, in line with the theoretical literature above, the simple NK-AC model predicts a crowding-out of consumption. As stated by equation (3.31), percentage changes in the marginal value of wealth are the mirror image of percentage changes in consumption. Thus, the marginal value of wealth jumps upwards initially. Addressing the increased demand for output, firms increase investments and labor inputs, which reflects an upward jump in marginal value of capital. As a consequence, production costs (wages and the rental rate on capital) jump upwards as well. The capital stock then gradually increases for as long as investments exceed capital depreciation, or put differently, for as long as the marginal value of capital exceeds the marginal value of wealth. Both marginal values initially jump upwards but the increase in the marginal value of wealth is smaller so that the capital stock starts to increase gradually. In line with the s-shaped response of the real interest rate, the marginal value of wealth increases further and quickly exceeds the decreasing marginal value of capital. Consequently, this brings the rise in capital to an end and the initial increase in investment is followed by an s-shaped dynamic, where investment undershoot its equilibrium level for a prolonged period of time. Thus, the NK-AC model responds to a increase in government consumption with a temporary crowding-in of investment, which is followed by a period of lower investments. Nevertheless, the integral over changes in future investment remains positive.

Turning to the baseline NK-AC-FTPL model, government consumption is again a part of output so that the fiscal policy shock has expansionary effects on output today, and thus increases the inflation rate. The monetary authority cannot immediately adjust the nominal interest so that the increase in inflation induces an instantaneous decrease in the real interest rate. Following a Taylor rule, the monetary authority then addresses the upsurge in the inflation rate by gradually increasing the nominal interest rate. Due to overall higher future nominal interest rates, the bond price jumps downwards initially, as required by the no-arbitrage relation (3.48). The decline in the bond price immediately devalues the real value of debt, which consequently jumps downwards. Taxes are predetermined so that the unexpected rise in government consumption instantaneously decreases primary surpluses. Lower primary surpluses, in turn, induce an accumulation of government debt and are accompanied by higher future inflation rates. This feature again follows directly from the government debt valuation equation (3.47) and is reflected in the inflation decomposition equation (3.49). In absolute terms, the weighted present value of future primary surpluses has the highest impact on the change in the present value of future inflation rates, and thus the highest impact on the change in the present value of future real interest rates (see Table 3.8). In particular, most of the changes in future nominal interest are soaked up by the initial re-valuation of debt, effectively leaving an increase of 0.09 percentage points. Thus, the greatest part of the increase in the present value of future inflation (0.36 percentage points) can be attributed to fiscal policy as measured by the decline in the weighted present value of future surpluses (-0.27 percentage points). Put differently, changes in the weighted present value of future surpluses and the maturity effect fully account for the decrease in the present value of future real interest rates by -0.14 percentage points. In response to overall lower future real interest rates, households substitute inter-temporally and instantaneously increase consumption expenditures. Hence, in contrast to the NK-AC model, there is a crowding-in of consumption as suggested by the empirical literature above. Obtaining the crowding-in effect relies on the specification and interactions of monetary and fiscal policy as well as the maturity structure of government debt. I further elaborate this model feature below. The response of investment to the expansionary fiscal policy shock is similar to the one in the NK-AC model. In line with the upward jump in output, the marginal value of capital increases instantaneously. Hence, firms increase labor inputs and investments, which in turn raises wages and the rental rate of capital. Higher consumption corresponds to a lower marginal value of wealth. In contrast to the NK-AC model, this means that the marginal values of wealth and capital jump in opposite directions. Economic agents invest in the capital stock for as long as the marginal value of capital exceeds the marginal value of wealth. Thus, the increase in the capital stock is significantly higher and more drawn out in the baseline NK-AC-FTPL. In particular, the integral over future changes in the capital stock is nearly 50 times higher than the one in the underlying NK-AC model. In contrast to the NK-AC model, there is no s-shaped response of investment so that the initial upward jump of investment in the NK-AC-FTPL model is followed by a relatively slow and gradual convergence back to the steady state. Thus, the baseline NK-AC-FTPL suggest a relative strong crowding-in effect of investment, which consequently increases the capital stock and further contributes to the rise in output. In fact, as given by equation (3.52), the (impact) fiscal multiplier of output in the baseline NK-AC-FTPL model turns out to be greater than one (equal to 1.18), whereas the corresponding impact fiscal multiplier is smaller than one (equal to 0.98) in the NK-AC model without FTPL.

Figure 3.5 shows that the dynamics of consumption and investment in the reparametrized NK-AC-FTPL model closely resemble the ones in the NK-AC model. This highlights that (depending on its parametrization) the NK-AC-FTPL framework is able to predict either a crowding-in or a crowding-out of consumption (or no initial change in consumption at all). In fact, the inflation decomposition equations (3.49) suggest that one can easily switch off the crowding-in effect through various channels. Starting from the baseline parametrization, I increase the inflation response parameter in the Taylor rule ($\phi_\pi = 0.95$) and assume a stronger output response of taxes ($\tau_y = 1.15$). That is, I set ϕ_π close to the demarcation line between passive and active monetary policy regimes. With these

Table 3.8: Inflation decomposition: Expansionary fiscal policy shock that increases government consumption by 5 percent. Baseline parametrization. NK-AC-FTPL model and reparametrized NK-AC-FTPL model with $\phi_\pi = 0.95$ and $\tau_y = 1.15$. Entries given by equation (3.49).

Setting	$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus	$(p_0^b/p_{ss}^b - 1)$ direct effect
Baseline	0.36	0.22	-0.27	-0.13
Reparameterized	-0.014	-0.013	-0.024	-0.025

adjustments, Figure 3.5 illustrates that consumption and investment dynamics behave similar to their counterparts in the NK-AC model. Thus, there is no crowding-in of consumption in the reparametrized model. Therefore, keeping all else equal, one can obtain a crowding-out effect in the FTPL framework by adjusting the surplus process. In particular, the path of the inflation rate, which is determined by the government debt valuation equation (3.47), has to result in overall lower future inflation rates. This in turn, corresponds to overall higher future real interest rates so that households substitute inter-temporally consumption from today into the future. Thus, in order to obtain the desired crowding-out effect, the change in the weighted present value of future surpluses has to be sufficiently large. Put differently, the initial drop in surpluses, which results from the increase in government consumption, has to be offset by a quick and aggressive tax hike. This in turn, requires a surplus process with a substantial s-shaped response. In case of the reparameterized NK-AC-FTPL model, the response of taxes to changes in output is larger compared to the baseline parametrization. Consequently, the fiscal authority adjusts taxes more aggressively to changes in the business cycle. Thus, the overall increase in the weighted present value of future tax receipts is twice as big as the corresponding response under the baseline parametrization. In particular, the increase in taxes induces a much more substantial s-shaped surplus in the reparameterized model. Because the s-shape does not completely nets out the preceding decrease in surpluses, one is left with a drop in the weighted present value of future surpluses by 0.02 percentage points (compared to a 0.27 percentage points drop under the baseline parametrization). Thus, one obtains that a stronger pro-cyclical tax response is able to counteract the effect of increasing government consumption on primary surpluses in equation (3.49). The second change in the reparameterized model is a "*more active*" monetary policy rule, which consequently suggest an overall "*less passive*" fiscal policy regime. Thus, compared to the baseline NK-AC-FTPL model, the central bank reacts more aggressively to changes in the inflation rate so that the nominal interest rate drops below its equilibrium level for a prolonged period of time during transition. This in turn, dampens the effect of the preceding increases of the nominal interest rate and ultimately lowers its present value by 0.01 percentage points. Due to no-arbitrage between the nominal interest rate and bond returns, the initial drop in the bond price is smaller than the one obtained in the baseline model. Taking everything together, equation (3.49) implies a small increase in the present value of future real interest rates by 0.001 percentage points. Facing overall higher future real interest rates, households immediately decrease consumption due to inter-temporal substitution. Hence, as in the NK-AC model, there is a crowding-out of consumption in the reparametrized model. Firms initially behave similar to the ones in the baseline NK-AC-FTPL model and there is also an upward jump in the marginal value of capital. At the same time, the marginal value of wealth evolves inversely to consumption. Hence, in contrast to the baseline NK-AC-FTPL model, the marginal values of wealth and capital both jump upwards initially. As a consequence, the dynamics of investment and the capital stock are less drawn out and similar to the corresponding dynamics in the NK-AC model as illustrated in Figure 3.5.

I conclude that the fiscal theory is able to capture the positive co-movement of consumption and government consumption in the data. Therefore, FTPL offers a simple and intuitive way to solve the crowding in puzzle. The NK-AC-FTPL model delivers the theoretical underpinning for either a crowding-in or crowding-out effect of consumption. This flexibility makes it especially useful to evaluate identified crowding-in and crowding-out of consumption and/or investment in the empirical literature. It is important to stress that the FTPL framework does not require capital to generate a crowding-in of consumption. In fact, this effect can also occur in the simple NK-FTPL framework through the (non capital-specific) transmission channels discussed above. However, the benefit of introducing capital to the model is the possibility to simultaneously analyze the responses of investments and the capital stock under the fiscal theory.

3.4.3 Resurrecting FTPL Model Implications for Short-Term Debt

A frequently encountered feature of simple NK models with FTPL and short-term debt is the inability to generate a drop in the inflation rate in response to contractionary monetary policy shocks[24]. Sims (2011) highlights that the introduction of long-term bonds is (at least temporary) able to fix this counterintuitive result (*stepping on a rake effect*). Cochrane (2022b) stresses that the fiscal theory does not necessarily relies on longer-term debt in order to generate a temporary drop in the inflation rate. To that end, he proposes to implement a direct inflation response in the surplus rule. In my framework, this translates

[24]See e.g. Sims (2011), Leeper and Leith (2016), or Cochrane (2018).

to adding the term $\tau_\pi(\pi_t - \pi_t^*)$ to the surplus rule. Cochrane (2022b) is after highlighting the mechanisms to produce the negative inflation response. While this specification might be considered controversial in the empirical literature, its implications are intriguing and justify further research on fiscal policy rules.

I highlight that one can obtain the negative inflation response to contractionary monetary policy shocks by turning to a larger NK framework with FTPL and capital. To that end, I show how to get the desired response in the NK-AC-FTPL model with short-term debt by using only the fiscal policy rules for government consumption and taxes as specified by equations (3.15) and (3.46). Thus, my approach does not relies on the introduction of additional inflation response parameters or other adaptions. In fact, all rests on the parametrization of the policy rules. Consequently, I only need direct responses to output and debt, which are the typically encountered variables in the estimation of fiscal policy rules in the empirical literature[25]. I start my analysis from the NK-AC-FTPL model with short-term debt, apply the baseline parametrization but alter parameter values in the fiscal policy rules and impose a lower equilibrium Debt-to-GDP ratio. In particular, I set $\tau_a = 0.029$, $\varphi_a = -0.038$, $\tau_y = 0.69$ and $s_t^* = 0.02$. It is important to stress that this is just one exemplary parametrization, which I chose to highlight central mechanism for obtaining the desired response. Based on these channels, there is a wide range of suitable parameter constellations within the FTPL framework with capital adjustment costs.

Consider a contractionary monetary policy shock that increases the nominal interest rate by 1 percentage point. A comparison of Figures 3.2 and 3.6 unveils that replacing long- by short-term debt in the baseline NK-AC-FTPL model causes an upward jump in the inflation rate. In this case, the bond price is constant so that debt does not become immediately revalued and FTPL only operates through the real interest rate channel (cf. Liemen and Posch (2022)). In the reparameterized model, however, satisfying the inflation decomposition equation (3.49) requires an initial drop in the inflation rate, followed by a less drawn out increase. Consequently, the dynamics are closely related to the baseline NK-AC-FTPL model with long-term debt. The model implications essentially only differ in terms of magnitudes, where the most evident differences are observed for fiscal policy block variables and the inflation rate. These variations, in turn, especially result from the absence of the direct maturity effect in the reparameterized model.

It is important to stress that the maturity structure of debt still plays a crucial role in the reparameterized model. In particular, if one introduces long-term debt, FTPL operates through the same channels as in the baseline setting. To highlight this model property, Figure 3.7 shows the policy functions for the baseline and the reparameterized NK-AC-FTPL. In each case, I consider either perpetuities, the baseline maturity or short-term debt (cf. Liemen and Posch (2022)). The two left-hand side panels depict the inflation response

[25] See e.g. Chung and Leeper (2007), Traum and Yang (2011), Davig and Leeper (2011), Kliem and Kriwoluzky (2014), Kliem et al. (2016), Ricco et al. (2016).

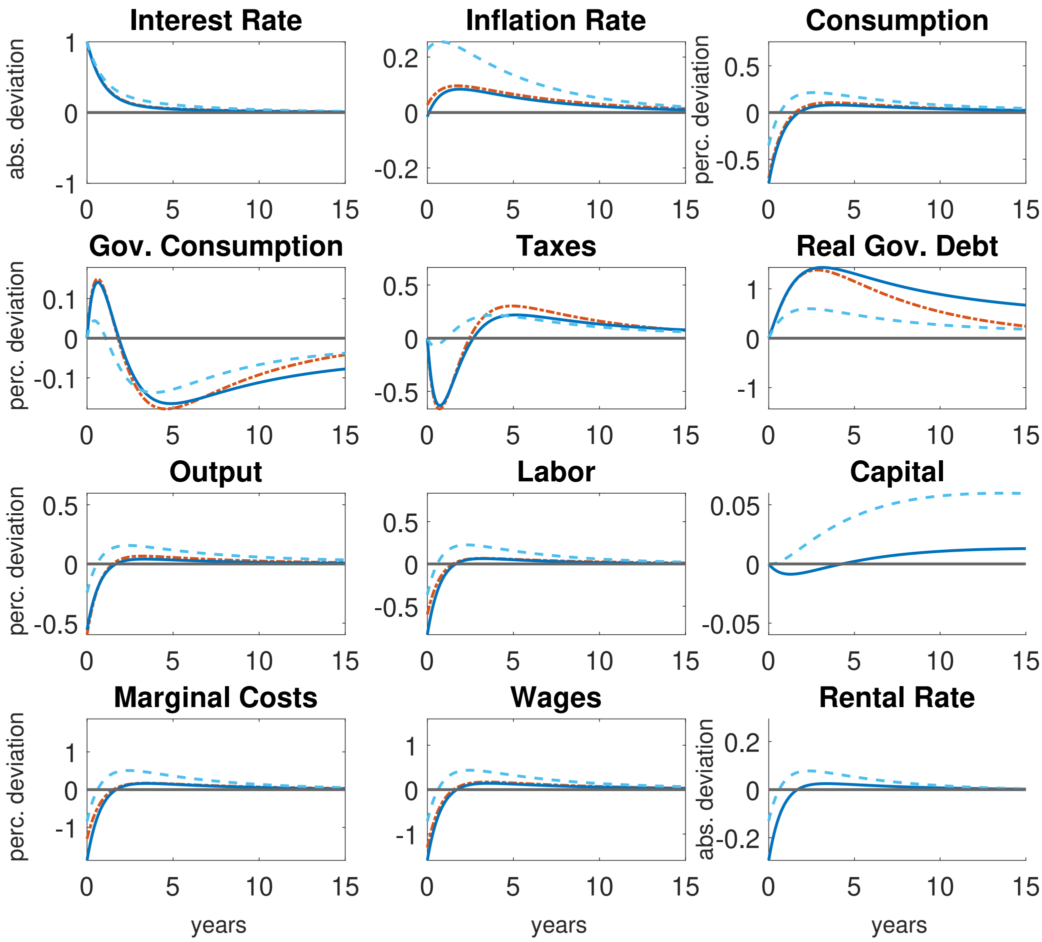


Figure 3.6: IRFs: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization with short-term debt. NK-AC-FTPL model (dashed light blue lines) and reparametrized NK-FTPL model (dotted red lines) and reparametrized NK-AC-FTPL model (solid dark blue lines). Reparametrization uses $\tau_a = 0.029$, $\varphi_a = -0.038$, $\tau_y = 0.69$ and $s_t^* = 0.02$.

to changes in the nominal interest rate. In case of the baseline parametrization, the NK-AC-FTPL model exhibits a positive correlation between the inflation- and the nominal interest rate for shorter maturities (upper panel). In contrast, in the reparameterized model (lower panel), the inflation rate coefficient is always negatively correlated to the nominal interest rate, independent of the underlying maturity structure. However, longer maturities are still associated with steeper downward jumps in the inflation rate.

To evaluate the reasons for the negative inflation response in the reparametrized NK-AC-FTPL model, Table 3.9 shows the inflation decomposition (3.49) for the baseline and the reparameterized NK-AC-FTPL model with short-term debt (see Figure 3.6). Since I consider short-term debt, there is no direct maturity effect. Consequently, changes in the present value of future real interest rates must corresponds to changes in the weighted

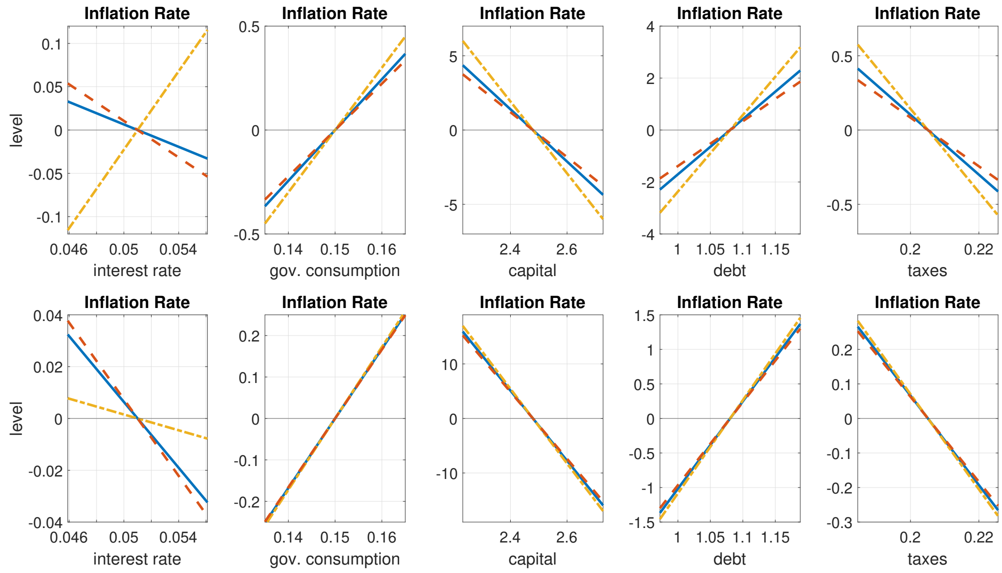


Figure 3.7: Policy functions for the inflation rate in the NK-AC-FTPL model for different maturity structures: Long-term (dashed red lines), longer-term (solid blue lines) and short-term (dotted yellow lines). Upper panels: Baseline parametrization. Lower panels: Baseline parametrization with $\tau_a = 0.029$, $\varphi_a = -0.038$, $\tau_y = 0.69$ and $s_t^* = 0.02$.

present value of future surpluses. Comparing both parametrizations, the increase in the weighted present value of future surpluses is larger in the reparameterized NK-AC-FTPL model. To see where this difference comes from, note that primary surpluses in the reparameterized framework react more strongly to the monetary policy shock and respond with higher up- and down swings (compare Figures 3.2 and 3.6). A crucial reason for this dynamic is the additional objective of the fiscal authority to directly address changes in the real value of debt. As highlighted by Figure 3.6, debt rises substantially during transition so that the increase in the weighted present value of future surpluses becomes larger in the reparameterized model and accumulates to 0.76 percentage points (versus a 0.42 percentage points increase in the baseline setting with short-term debt). Note that part of this difference also results form a higher weighting factor, $1/a_{ss} = \rho/s_t^*$, in the reparametrized model, which follows from the lower equilibrium Debt-to-GDP ratio. Compared to the baseline model with short-term debt, the increases in the present value of future nominal interest rates (1.25 versus 1.78 percentage points) and the present value of future inflation (0.48 versus 1.36 percentage points) are significantly smaller. Since there is no maturity effect, the present value of future real interest rates must also increase by 0.76 percentage points in the reparametrized model. In the baseline NK-AC-FTPL model with short-term debt, the relatively large increase in the present value of future inflation (1.36 percentage points) requires a path of the inflation rates with an initial upward jump. In contrast, the relatively small increase in the present value of future inflation in the reparametrized NK-AC-FTPL model (0.48 percentage points) requires overall lower future inflation with an initial downward jump. Thus, comparing the inflation decomposition for both parametrizations, the direct dependence on debt in the surplus rule as well as the higher weighting factor for the present value of surpluses turn out to be the relevant factors for the initial negative inflation response.

The further illustrate the above findings, I turn to a comparison between reparameterized versions of the simple NK-FTPL (without capital) and the NK-AC-FTPL model with short-term debt. Figure 3.6 shows the corresponding model responses to the contractionary monetary policy shock. The dynamics of the NK-AC-FTPL model again produce the desired inflation response. In the simple NK-FTPL model, however, the inflation rate jumps upwards, despite applying the alternative parametrization. This is notably because with the baseline parametrization, inflation dynamics in the NK-FTPL and the NK-AC-FTPL model are more closely related (see e.g. Figure 3.2). Nevertheless, the upward jump in the inflation rate in the reparametrized NK-FTPL model is smaller than the one implied by my baseline parametrization. In general, the simple NK-FTPL model with short-term debt turns out to be unable (for reasonable parameter values) to produce the desired inflation response (cf. Sims (2011) or Cochrane (2022b)). To understand the differences between the inflation responses in the reparametrized versions of the simple NK-FTPL and the NK-AC-FTPL model, I again apply the inflation decomposition equation (3.49) to the models (see Table 3.9). In comparison to the simple NK-FTPL framework, the change in the present value of future inflation is 0.13 percentage points lower in the NK-AC-FTPL setting. This corresponds to a higher increase in the present value of future real interest rates (and consequently in the weighted present value of future surpluses) by 0.05 percentage points. In both models, overall higher real interest rates initially cause households to decrease consumption. This behavior follows directly form inter-temporal substitution considerations and the increased marginal value of wealth. Facing a decreased demand for output, firms lower production. In case of the simple NK-FTPL model, firms decrease their labor demand, which in turn initially decreases the real wage. Thus, output adjusts immediately because it is equal to labor in the NK-FTPL model. In the NK-AC-FTPL model, however, output consists of both labor (jump variable) and capital (predetermined variable). Hence, firms address the decreased demand of the households by cutting investment and by reducing labor. As a consequence, wages and the capital rental rate instantaneous decline. The initial drop in output is smaller in the NK-AC-FTPL model because, unlike labor, capital cannot be adjusted immediately. In both frameworks (see Section 3.3.3), the initial decline in output is ultimately followed by a temporary increase during transition. The adjustment of capital is relatively costly. Hence, due to the dynamics of the capital stock, the subsequent increase in output is more dampened in the NK-AC-FTPL model. As in the previous sections, the output response parameters are the central driving factors for the dynamics of the fiscal policy rules. Thus,

Table 3.9: Inflation decomposition: Contractionary monetary policy shock that raises the nominal interest rate by 1 percentage point. Baseline parametrization with short-term debt. Simple NK-FTPL and NK-AC-FTPL model. Reparametrized versions of the models are marked with a star. In this case $\tau_a = 0.029$, $\varphi_a = -0.038$, $\tau_y = 0.69$ and $s_t^* = 0.02$. Simple NK-FTPL model and NK-AC-FTPL model. Entries given by equation (3.49).

Setting	$\int_0^\infty e^{-\rho u}(\pi_u - \pi_t^*)\mathrm{d}u$ inflation	$\int_0^\infty e^{-\rho u}(i_u - i_{ss})\mathrm{d}u$ interest rate	$\int_0^\infty e^{-\rho u}(s_u - s_t^*)/a_{ss}\mathrm{d}u$ surplus
NK-FTPL*	0.61	1.32	0.71
NK-AC-FTPL*	0.48	1.25	0.76
NK-AC-FTPL	1.36	1.78	0.42

the differences in the dynamics of output are also reflected in the weighted present values of future surpluses. In particular, this property results in a relatively lower increase in the present value of future inflation in the reparametrized NK-AC-FTPL model (0.48 versus 0.61 percentage points), which in turn requires a path of future inflation with an initially lower inflation rate. In contrast, in the reparametrized NK-FTPL model, the increase in the present value of future inflation still remains too high to imply a path of future inflation rates with the desired downward jump. To sum things up, in Sims (2011) long-term debt is able to induce the desired initial downward in the inflation rate, whereas in Cochrane (2022b) the additional dependence of surpluses on the inflation rate does the trick. In my framework, one has an additional capital channel, which circumvents the need for long-term debt or the introduction of additional responses in the surplus rule. Even though, my results are intriguing, actual government bonds are neither all short-term nor all perpetual. Thus, assuming some type of average maturity as in Liemen and Posch (2022) or in my baseline parametrization is probably an empirically more valid approach. However, the reparameterized model sheds light on the importance of surplus processes and its role in the propagation of shocks within joint frameworks with capital and FTPL.

3.5 Conclusion

In this paper I introduce capital and capital adjustment costs to the continuous-time NK-FTPL framework of Sims (2011) and Cochrane (2018). Due to limited or non-existing coverage in the related literature, I start with an elaborate description of determinacy conditions and model dynamics in the continuous-time NK framework with capital and

FTPL. I then show that the NK-FTPL model with capital and capital adjustment costs maintains the basic predictions of the simple NK-FTPL model, and thus consistently extends the FTPL framework of Sims (2011) by capital dynamics. My results show that the combination of FTPL and capital adjustment costs offers an intuitive and simple way to understand and address puzzling implications of the standard NK framework. Using the Great East Japan Earthquake of 2011 as illustrative example, I discuss conditions under which the NK-AC-FTPL model predicts a contractionary effect of capital destruction at the ZLB. I highlight that the effect critically depends on the specification of fiscal policy, and that it is also possible to obtain an expansionary effect, which is frequently encountered in the standard NK literature. I then address the Crowding-In Consumption Puzzle and present in detail how the FTPL framework allows for either a crowding-in or a crowding-out of consumption. By doing so, I also find a (at least temporary) crowding-in of investment. Again, the decisive factor is the design of fiscal policy. Finally, I show that contractionary monetary policy shocks in larger-scale FTPL models with capital adjustment costs can induce an initial drop in the inflation rate, even in the absence of long-term debt.

Complex macroeconomic problems highlight the need for more involved economic models. However, smaller macroeconomic models offer important benchmarks before turning to more elaborate models. While the simple NK-FTPL model is already widely covered in the literature, I believe that the inclusion of capital is an important step towards a medium-scale continuous-time NK-FTPL model. Hence, I consider my analysis to be helpful for obtaining intuition, for conceptualization and for understanding basic transmission channels in NK models with capital and FTPL, which in turn facilitates the translation to more evolved frameworks. In fact, the introduction of capital opens the door for a wide field of further economic applications. First, my setup is a promising benchmark and starting point for stochastic models featuring different types of risk. Promising approaches are term premia (cf. Posch (2020)) or the introduction of uninsurable idiosyncratic return risk for physical capital along the lines of Brunnermeier et al. (2021). Second, allowing for regime switches between active and passive monetary policy (see e.g. Bianchi and Melosi (2017)) is a promising extension, which allows for a more profound fiscal policy evaluation and is especially interesting from an empirical point of view. Regarding empirical evaluations, one ultimately has to estimate the parameters of the surplus rule because of its strong impact on the predictions of the model. Third, one needs to evaluate the effects of alternative surplus rules, which arise endogenously from the underlying NK model, such as the implementation of distortionary taxes. This facilitates the evaluation of fiscal limits and sovereign default (fiscal sustainability), which is a crucial endeavor of my research agenda. Finally, since my setting already contains many features of the continuous-time HANK model in Kaplan et al. (2018), it is a natural starting point to analyze the effects of FTPL in heterogeneous agent models.

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Appendices of Essays

1.A Appendix, Chapter 1

1.A.1 Properties and Derivations for the SDF

Starting from equation (1.5) we apply Itô's formula to obtain the evolution of $\ln(\Lambda_t)$:

$$\begin{aligned} d\ln(\Lambda_t) &= \frac{1}{\Lambda_t}(d\Lambda_t) - \frac{1}{2}\frac{1}{\Lambda_t^2}(d\Lambda_t)^2 \\ &= -(r_t - \delta - \tfrac{1}{2}\sigma^2)dt - \sigma dZ_t. \end{aligned}$$

Integration yields

$$\int_t^s d\ln(\Lambda_v)dv = -\int_t^s (r_v - \delta - \tfrac{1}{2}\sigma^2)dv - \sigma\int_t^s dZ_v,$$

from which we obtain the stochastic discount factor as the process given by equation (1.17).

Now to compute the expected value of the SDF we start from equation (1.12). Since this is an Ornstein-Uhlenbeck process we can find the solution by using a standard technique in differential equations as shown below.

$$\begin{aligned} e^{\kappa t}(dr_t + \kappa r_t)dt &= e^{\kappa t}\kappa\gamma dt + e^{\kappa t}\eta dB_t \\ \int_t^s (dr_t e^{\kappa u}) &= \int_t^s (d\gamma e^{\kappa u}) + \eta\int_t^s e^{\kappa u}dB_u \\ e^{\kappa s}r_s - e^{\kappa t}r_t &= e^{\kappa s}\gamma - e^{\kappa t}\gamma + \eta\int_t^s e^{\kappa u}dB_u \\ r_s &= e^{-\kappa(s-t)}r_t + (1 - e^{-\kappa(s-t)})\gamma + \eta e^{-\kappa(s-t)}\int_t^s e^{\kappa(u-t)}dB_u. \end{aligned}$$

Note that in order to obtain the expected value of the stochastic discount factor we employ log-normality and compute

$$\ln E_t\left[e^{\ln(\Lambda_s)-\ln(\Lambda_t)}\right] = E_t[\ln(\Lambda_s) - \ln(\Lambda_t)] + \tfrac{1}{2}Var_t[\ln(\Lambda_s) - \ln(\Lambda_t)]. \tag{A.1}$$

We can now plug our solution for r_s into our log expression for the stochastic discount factor and obtain

$$\begin{aligned}\ln(\Lambda_s) - \ln(\Lambda_t) &= -\int_t^s r_v dv + \int_t^s (\delta + \tfrac{1}{2}\sigma^2) dv - \sigma \int_t^s dZ_v \\ &= -\int_t^s (e^{-\kappa(v-t)} r_t + (1 - e^{-\kappa(v-t)})\gamma - \delta - \tfrac{1}{2}\sigma^2) dv \\ &\quad -\eta \int_t^s e^{-\kappa(v-t)} \int_t^v e^{\kappa(u-t)} dB_u dv - \sigma \int_t^s dZ_v.\end{aligned}$$

Reversing the order of integration and evaluating the ds integrals yield

$$\ln(\Lambda_s) - \ln(\Lambda_t) = -\frac{r_t - \gamma}{\kappa}(1 - e^{-\kappa(s-t)}) - (\gamma - \delta - \tfrac{1}{2}\sigma^2)(s-t) - \frac{\eta}{\kappa}\int_t^s (1 - e^{-\kappa(s-u)}) dB_u - \sigma \int_t^s dZ_v.$$

Inspection of the last two integrals give rise to a normally distributed random variable with mean zero and variance

$$\begin{aligned}Var_t[\ln(\Lambda_s) - \ln(\Lambda_t)] &= \int_t^s \left(\frac{\eta}{\kappa}(1 - e^{-\kappa(s-u)})\right)^2 du + \int_t^s \sigma^2 du \\ &= \left(\left(\frac{\eta}{\kappa}\right)^2 + \sigma^2\right)(s-t) - 2\frac{\eta^2}{\kappa^3}(1 - e^{-\kappa(s-t)}) + \frac{\eta^2}{2\kappa^3}(1 - e^{-2\kappa(s-t)}),\end{aligned}$$

and

$$E_t[\ln(\Lambda_s) - \ln(\Lambda_t)] = -\frac{r_t - \gamma}{\kappa}(1 - e^{-\kappa(s-t)}) - (\gamma - \delta - \frac{1}{2}\sigma^2)(s-t).$$

Thus by plugging in we conclude

$$\begin{aligned}\ln E_t\left[e^{\ln\Lambda_s - \ln\Lambda_t}\right] &= -\left(\frac{r_t - \gamma}{\kappa} + \frac{\eta^2}{\kappa^3}\right)(1 - e^{-\kappa(s-t)}) \\ &\quad -\left(\gamma - \delta - \sigma^2 - \tfrac{1}{2}\frac{\eta^2}{\kappa^2}\right)(s-t) + \frac{\eta^2}{4\kappa^3}(1 - e^{-2\kappa(s-t)}).\end{aligned}$$

1.A.2 Proof of Proposition 4

One challenge is the incorporation of the empirically observed data into the theoretical framework. For example, we have alternative approaches to asset pricing. Starting from (1.9), one way of interpreting the data is to consider a claim on future dividends (in an endowment economy this is equivalent to a claim on the tree, *not* only on the next period's fruit), that is

$$P_{d,t} = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} Y_s ds\right]. \tag{A.2}$$

To find the equilibrium price of this asset we have to compute an expression for Y_s, the rate of output at time s. First, we obtain an expression for the capital stock in period

$s > t$ as

$$K_s = K_t e^{\int_t^s (r_v - \rho - \delta - \frac{1}{2}\sigma^2)dv + \sigma \int_t^s dZ_v}. \tag{A.3}$$

In combination with consumption we obtain for output

$$Y_s = \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{\int_t^s (r_v - \delta - \rho - \kappa - \frac{1}{2}\sigma^2)dv + \sigma \int_t^s dZ_v}. \tag{A.4}$$

Now, we may find the price of the claim on future dividends. We insert into (A.2) and get

$$P_{d,t} = E_t \left[\int_t^\infty \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{-\int_t^s (\rho+\kappa)dv} ds \right].$$

Solving the integrals yields

$$P_{d,t} = K_t \left[\frac{r_t - \gamma}{\rho + \kappa} + \frac{\gamma}{\rho} \right]. \tag{A.5}$$

This is an important, and intuitive result. The price of the claim is based on the sum of (two) annuities multiplied with the capital stock. Recall that in the AK-Vasicek model the parameter γ can be interpreted as the mean of the interest rate, or, since $A_t = r_t$, the mean productivity level. Therefore, the price of the claim is the current capital stock multiplied by average productivity γ (annuity of mean output), plus the current capital stock times the current deviation of A_t from γ, properly accounted for the speed of mean reversion κ.

We may be interested in the holding return of this asset from t to s. The dynamics of the equilibrium price (A.5) are then given by

$$dP_{d,t} = P_{d,t}[r_t - \rho - \delta]dt + P_{d,t}\sigma dZ_t + P_{d,t}\frac{[\rho\kappa(\gamma - r_t)]}{\rho r_t + \kappa\gamma}dt + P_{d,t}\frac{\rho\eta}{\rho r_t + \kappa\gamma}dB_t. \tag{A.6}$$

Or applying Itô's formula to find an expression for the log price change of the claim

$$\begin{aligned} d\ln P_{d,t} &= \left[r_t - \delta - \rho - \tfrac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \tfrac{1}{2}\frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt \\ &\quad + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t + \sigma dZ_t, \end{aligned} \tag{A.7}$$

or

$$d\ln P_{d,t} = d\ln C_t + \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \tfrac{1}{2}\frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t, \tag{A.8}$$

which shows how stock market data can be used to recover the consumption dynamics. This allows us to estimate the structural parameters completely by financial market data.

1.A.3 Proof of Proposition 5

The market for derivatives offer additional information about (potentially) latent variables. For example, a claim on capital can be defined as an asset whose payoff is the future capital stock K_s (the replacement cost of the firm). Using the SDF we can find the price for a claim on the capital stock (a future contract) using the basic pricing equation:

$$P_{K,t} = E_t \left[\frac{\Lambda_s}{\Lambda_t} K_s \right], \tag{A.9}$$

in which K_s is the underlying asset and $P_{K,t}$ is the price of the future contract at time t. Now using (A.3) and the SDF given by (1.17) we get

$$P_{K,t} = K_t e^{-\rho(s-t)}. \tag{A.10}$$

If we are interested in the price movement of this asset, or to be more precisely in the price dynamics of the asset class, the prices follow

$$dP_{K,t} = \frac{dK_t}{K_t} P_{K,t} \tag{A.11}$$

or

$$\begin{aligned} d\ln P_{K,t} &= d\ln K_t = d\ln C_t \\ &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \sigma dZ_t, \end{aligned} \tag{A.12}$$

which shows that the log price of the claim on capital behaves like the log change of the capital stock or consumption. In other words, the instantaneous return on the claim can be interpreted as percentage changes in the capital stock or consumption. Another interesting derivative is a claim on output, i.e., an asset which pays Y_s at time s (in an endowment economy this is equivalent to the claim on the next period's fruit). We may obtain the price on the claim on output, $P_{Y,t}$, by using (1.9):

$$P_{Y,t} = E_t \left[\frac{\Lambda_s}{\Lambda_t} Y_s \right], \tag{A.13}$$

in which Y_s is the underlying asset and $P_{Y,t}$ is the price of the future contract at time t.

Plugging in the expressions for Y_s and the stochastic discount factor, we arrive at:

$$P_{Y,t} = K_t \left[(r_t - \gamma) e^{-(\rho+\kappa)(s-t)} + \gamma e^{-\rho(s-t)} \right],$$

where

$$
\begin{aligned}
d\ln P_{Y,t} &= \left[r_t + \frac{\kappa(\gamma - r_t)}{(r_t + \gamma e^{\kappa(s-t)} - \gamma)} - \rho - \delta - \tfrac{1}{2}\sigma^2 - \tfrac{1}{2}\frac{\eta^2}{[r_t + \gamma e^{\kappa(s-t)} - \gamma]^2}\right] dt \\
&\quad + \sigma dZ_t + \eta/(r_t + \gamma e^{\kappa(s-t)} - \gamma) dB_t.
\end{aligned}
$$

Obviously, when $s = t$, the price dynamic simplifies to the law of motion of log output, $d\ln Y_t$.

Consider a future contract on the stock price in (A.5), which must satisfy

$$
P_{F,t} = E_t\left[\frac{\Lambda_s}{\Lambda_t} P_{d,s}\right].
$$

Plugging in and rearranging terms

$$
P_{F,t} = K_t\left[\frac{(r_t - \gamma)e^{-(\rho+\kappa)T}}{\rho + \kappa} + \frac{\gamma}{\rho} e^{-\rho T}\right] \tag{A.14}
$$

where $s - t = T$ denotes the time to maturity of the stock future contract. Differentiating yields the price change as

$$
\begin{aligned}
dP_{F,t} &= P_{F,t}\left(r_t - \rho - \delta + \frac{\kappa(\gamma - r_t)}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}}\right) dt \\
&\quad + P_{F,t}\left(\sigma dZ_t + \frac{\eta}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} dB_t\right).
\end{aligned} \tag{A.15}
$$

Finally, using Itô's formula the log price change follows

$$
\begin{aligned}
d\ln P_{F,t} &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\kappa(\gamma - r_t)}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}}\right. \\
&\quad \left. - \frac{\tfrac{1}{2}\eta^2}{((r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T})^2}\right) dt + \sigma dZ_t + \frac{\eta}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} dB_t.
\end{aligned} \tag{A.16}
$$

Observe that when the time to maturity, T, approaches 0, the price of the stock future collapse to the price of the claim on future dividends.

1.A.4 Equilibrium Dynamics

Our aim is to investigate different possibilities of replacing macro and financial variables in this setup. In what follows we consider either a pure finance or a combined macro-finance framework. First, we study the benefits and drawbacks of these approaches in a simulation study. Second, we then use the insights to empirically estimate the structural model parameters. As in the main text $T = (s - t)$. In summary our relevant equilibrium equations are

$$
\begin{aligned}
d\ln C_t &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \sigma dZ_t && \text{(A.17a)}\\
dr_t^f &= \kappa(\gamma - r_t)dt + \eta dB_t && \text{(A.17b)}\\
d\ln P_{d,t} &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{(\rho r_t + \kappa\gamma)} - \tfrac{1}{2}\frac{(\rho\eta)^2}{(\rho r_t + \kappa\gamma)^2}\right) dt && \text{(A.17c)}\\
&\quad + \frac{\rho\eta}{(\rho r_t + \kappa\gamma)} dB_t + \sigma dZ_t\\
d\ln P_{K,t} &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \sigma dZ_t && \text{(A.17d)}\\
d\ln P_{Y,t} &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\kappa(\gamma - r_t)}{(r_t + \gamma e^{\kappa(s-t)} - \gamma)} - \tfrac{1}{2}\frac{\eta^2}{(r_t + \gamma e^{\kappa(s-t)} - \gamma)^2}\right) dt\\
&\quad + \frac{\eta}{(r_t + \gamma e^{\kappa(s-t)} - \gamma)} dB_t + \sigma dZ_t && \text{(A.17e)}\\
d\ln P_{F,t} &= \left(r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\kappa(\gamma - r_t)}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}}\right.\\
&\quad \left. - \tfrac{1}{2}\frac{\eta^2}{((r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T})^2}\right) dt\\
&\quad + \frac{\eta}{(r_t - \gamma) + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} dB_t + \sigma dZ_t && \text{(A.17f)}\\
d\ln Y_t &= \left(\frac{\kappa\gamma}{r_t} - \tfrac{1}{2}\frac{\eta^2}{r_t^2} + r_t - \kappa - \rho - \delta - \tfrac{1}{2}\sigma^2\right) dt + \frac{\eta}{r_t} dB_t + \sigma dZ_t && \text{(A.17g)}
\end{aligned}
$$

where $r_t = r_t^f + \delta + \sigma^2$. We use different combinations of these equations to estimate the structural parameters. In our empirical estimation, however, we only consider a smaller set of financial claims, with unequivocal real world analogs and readily available data.

1.A.5 Derivations of the Estimation Equations

In this section we will derive, in a first step, the discrete time formulations for the system of equilibrium equations given by (A.17). In a second step, we will then set up the equations necessary for the GMM and MEF estimations. For the sake of clarity, we keep the derivations as general as possible and show a system of 5 variables, with their corresponding 2nd moments. This leads to a system with 10 estimation equations. To obtain a specific set of estimation equations, one can simply drop the remaining equations and remove the corresponding rows and columns in the matrices provided in this section. As in the main text $T = (s - t)$. Integrating the differential equations in system (A.17) over $(t-\Delta)$ to t we obtain

$$
\begin{aligned}
\ln(C_t/C_{t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \varepsilon_{C,t} \\
\ln(P_{K,t}/P_{K,t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \varepsilon_{P_K,t} \\
\ln(P_{d,t}/P_{d,t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \rho\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&\quad -\tfrac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^{t} \frac{1}{\left[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma\right]^2} dv + \varepsilon_{P_d,t} \\
\ln(P_{Y,t}/P_{Y,t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{(r_v^f + \delta + \sigma^2) + \gamma e^{\kappa T} - \gamma} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \frac{1}{\left[(r_v^f + \delta + \sigma^2) + \gamma e^{\kappa T} - \gamma\right]^2} dv + \varepsilon_{P_Y,t} \\
\ln(P_{F,t}/P_{F,t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta \\
&\quad +\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{(r_v^f + \delta + \sigma^2) - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \frac{1}{\left[(r_v^f + \delta + \sigma^2) - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T}\right]^2} dv + \varepsilon_{P_F,t} \\
\ln(Y_t/Y_{t-\Delta}) &= \int_{t-\Delta}^{t} r_v^f dv - (\rho + \kappa - \tfrac{1}{2}\sigma^2)\Delta + \kappa\gamma \int_{t-\Delta}^{t} \left(\frac{1}{r_v^f + \delta + \sigma^2} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \frac{1}{\left[r_v^f + \delta + \sigma^2\right]^2} dv + \varepsilon_{Y,t} \\
r_t^f &= e^{-\kappa\Delta} r_{t-\Delta}^f + (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) + \varepsilon_{r,t}
\end{aligned}
$$

where the martingale increments are defined by

$$
\begin{aligned}
\varepsilon_{C,t} &\equiv \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{P_K,t} &\equiv \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{P_d,t} &\equiv \int_{t-\Delta}^{t} \frac{\rho\eta}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} dB_v + \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{P_Y,t} &\equiv \int_{t-\Delta}^{t} \frac{\eta}{(r_v^f + \delta + \sigma^2) + \gamma e^{\kappa T} - \gamma} dZ_v + \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{P_F,t} &\equiv \int_{t-\Delta}^{t} \frac{\eta}{(r_v^f + \delta + \sigma^2) - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} dZ_v + \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{Y,t} &\equiv \int_{t-\Delta}^{t} \frac{\eta}{(r_v^f + \delta + \sigma^2)} dZ_v + \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{r,t} &\equiv \eta e^{-\kappa\Delta} \int_{t-\Delta}^{t} e^{\kappa(v-(t-\Delta))} dB_v.
\end{aligned}
$$

Recall that the estimation equations for the claim on capital and for consumption coincide (see equation (1.27)). Thus, for the sake of clarity we do not explicitly consider $P_{K,t}$ in the subsequent computations and keep in mind that we can interpret consumption and the claim on capital interchangeably.

Now, to derive the vector of martingale difference sequences, m_t, let

$$
\begin{aligned}
\mathbb{M}_C &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta \\
\mathbb{M}_{P_d} &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \rho\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \left[r_v^f + \delta + \sigma^2 + \kappa\gamma/\rho \right]^{-2} dv \\
\mathbb{M}_{P_F} &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta \\
&\quad +\kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{r_v^f + \delta + \sigma^2 - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T}} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \left[r_v^f + \delta + \sigma^2 - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T} \right]^{-2} dv \\
\mathbb{M}_Y &= \int_{t-\Delta}^{t} r_v^f dv - (\rho + \kappa - \tfrac{1}{2}\sigma^2)\Delta + \kappa\gamma \int_{t-\Delta}^{t} \left[r_v^f + \delta + \sigma^2 \right]^{-1} dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \left[r_v^f + \delta + \sigma^2 \right]^{-2} dv \\
\mathbb{M}_{P_Y} &= \int_{t-\Delta}^{t} r_v^f dv - (\rho - \tfrac{1}{2}\sigma^2)\Delta + \kappa \int_{t-\Delta}^{t} \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{(r_v^f + \delta + \sigma^2) + \gamma e^{\kappa T} - \gamma} \right) dv \\
&\quad -\tfrac{1}{2}\eta^2 \int_{t-\Delta}^{t} \left[r_v^f + \delta + \sigma^2 + \gamma e^{\kappa T} - \gamma \right]^{-2} dv + \varepsilon_{P_Y,t}
\end{aligned}
$$

such that m_t reads

$$
\left[
\begin{array}{c}
\ln(C_t/C_{t-\Delta}) - \mathbb{M}_C \\
\ln(P_{d,t}/P_{d,t-\Delta}) - \mathbb{M}_{P_d} \\
r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \\
\ln(Y_t/Y_{t-\Delta}) - \mathbb{M}_Y \\
\ln(P_{F,t}/P_{F,t-\Delta}) - \mathbb{M}_{P_F} \\
\ln(P_{Y,t}/P_{Y,t-\Delta}) - \mathbb{M}_{P_Y} \\
(\ln(C_t/C_{t-\Delta}) - \mathbb{M}_C)^2 - \sigma^2\Delta \\
(\ln(P_{d,t}/P_{d,t-\Delta}) - \mathbb{M}_{P_d})^2 - \sigma^2\Delta - \int_{t-\Delta}^{t} \eta^2 (r_v^f + \delta + \sigma^2 + \kappa\gamma/\rho)^{-2} dv \\
(r_t^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) - e^{-\kappa\Delta} r_{t-\Delta}^f)^2 - \eta^2(1 - e^{-2\kappa\Delta})/(2\kappa) \\
(\ln(Y_t/Y_{t-\Delta}) - \mathbb{M}_Y)^2 - \sigma^2\Delta - \int_{t-\Delta}^{t} \eta^2 (r_v^f + \delta + \sigma^2)^{-2} dv \\
\Big((\ln(P_{F,t}/P_{F,t-\Delta}) - \mathbb{M}_{P_F})^2 - \sigma^2\Delta \\
- \int_{t-\Delta}^{t} \eta^2 ((r_v^f + \delta + \sigma^2) - \gamma + (\gamma + \gamma\kappa/\rho)e^{\kappa T})^{-2} dv \Big) \\
\Big((\ln(P_{Y,t}/P_{Y,t-\Delta}) - \mathbb{M}_{P_Y})^2 - \sigma^2\Delta \\
- \int_{t-\Delta}^{t} \eta^2 ((r_v^f + \delta + \sigma^2) + \gamma e^{\kappa T} - \gamma)^{-2} dv \Big)
\end{array}
\right] . \qquad \text{(A.18)}
$$

1.A.6 Simulation Study Results

Table 1.A.1: Simulation Study: GMM estimation using only first moments. Median estimates of structural parameters for 550 runs of the simulation study. Corresponding interquartile ranges are given in brackets below the estimates. Blank spaces indicate parameters that are fixed at their value used in the DGP. All results involving consumption data can be interpreted as using the derivative on the capital stock instead (e.g. finance or macro-finance).

Simulation Study: GMM First Moments

	Finance or Macro-Finance					**Macro-Finance**			
Setup	Bond Cons*	Bond Cons* Stock	Bond Cons* Future	Bond Cons* Derivative	Bond Cons* Derivative Stock	Bond Out	Bond Cons* Out	Bond Stock Out	Bond Future Out
$\kappa = 0.2$	**0.356**	**0.202**	**0.201**	**0.388**	**0.202**	**0.358**	**0.341**	**0.300**	**0.432**
	(0.290)	(0.041)	(0.033)	(0.262)	(0.137)	(0.285)	(0.285)	(0.151)	(0.217)
$\gamma = 0.1$	**0.099**	**0.099**	**0.099**	**0.099**	**0.100**	**0.099**	**0.100**	**0.102**	**0.099**
	(0.012)	(0.013)	(0.013)	(0.013)	(0.013)	(0.012)	(0.014)	(0.015)	(0.013)
$\eta = 0.01$		**0.010**	**0.010**	**0.010**	**0.010**		**0.010**	**0.010**	**0.009**
		(0.003)	(0.003)	(0.001)	(0.002)		(0.001)	(0.002)	(0.003)
$\rho = 0.03$	**0.030**	**0.030**	**0.030**	**0.030**	**0.022**	**0.030**	**0.030**	**0.027**	**0.030**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.014)	(0.005)	(0.006)	(0.010)	(0.006)
$\delta = 0.05$									
$\sigma = 0.02$								**0.022**	**0.021**
								(0.005)	(0.002)

Table 1.A.2: Simulation Study: GMM estimation using only first moments. Purely financial models. Setting as described in Table 1.A.1.

Simulation Study: GMM First Moments

	Finance					
Setup	Bond	Bond Stock	Bond Future	Bond Derivative	Bond Stock Future	Bond Stock Derivative
$\kappa = 0.2$	**0.359**	**0.374**	**0.342**	**0.370**	**0.201**	**0.360**
	(0.277)	(0.260)	(0.244)	(0.269)	(0.075)	(0.242)
$\gamma = 0.1$	**0.099**	**0.099**	**0.099**	**0.099**	**0.098**	**0.100**
	(0.012)	(0.013)	(0.013)	(0.012)	(0.013)	(0.013)
$\eta = 0.01$					**0.010**	**0.010**
					(0.002)	(0.001)
$\rho = 0.03$		**0.030**	**0.030**	**0.029**	**0.030**	**0.030**
		(0.005)	(0.005)	(0.006)	(0.005)	(0.007)
$\delta = 0.05$						
$\sigma = 0.02$						**0.022**
						(0.003)

Table 1.A.3: Simulation Study: MEF estimation using only first moments. Median estimates of structural parameters for 550 runs of the simulation study. Corresponding interquartile ranges are given in brackets below the estimates. Blank spaces indicate parameters that are fixed at their value used in the DGP. All results involving consumption data can be interpreted as using the derivative on the capital stock instead (e.g. finance or macro-finance).

Simulation Study: MEF First Moments

	Finance or Macro-Finance					**Macro-Finance**			
Setup	Bond Cons*	Bond Cons* Stock	Bond Cons* Future	Bond Cons* Derivative	Bond Cons* Derivative Stock	Bond Out	Bond Cons* Out	Bond Stock Out	Bond Future Out
$\kappa = 0.2$	**0.353**	**0.207**	**0.204**	**0.311**	**0.348**	**0.357**	**0.358**	**0.280**	**0.218**
	(0.283)	(0.043)	(0.034)	(0.272)	(0.277)	(0.287)	(0.286)	(0.238)	(0.157)
$\gamma = 0.1$	**0.099**	**0.099**	**0.100**	**0.097**	**0.099**	**0.099**	**0.099**	**0.102**	**0.100**
	(0.012)	(0.013)	(0.014)	(0.013)	(0.013)	(0.018)	(0.013)	(0.017)	(0.015)
$\eta = 0.01$		**0.010**	**0.010**	**0.010**	**0.010**		**0.010**	**0.010**	**0.010**
		(0.001)	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
$\rho = 0.03$	**0.030**	**0.031**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)
$\delta = 0.05$				**0.049**	**0.050**	**0.050**	**0.050**	**0.053**	**0.050**
				(0.003)	(0.002)	(0.012)	(0.001)	(0.008)	(0.006)
$\sigma = 0.02$		**0.021**	**0.021**					**0.023**	**0.021**
		(0.005)	(0.005)					(0.013)	(0.003)

Table 1.A.4: Simulation Study: MEF estimation using only first moments. Purely financial models. Setting as described in Table 1.A.3.

Simulation Study: MEF First Moments

	Finance					
Setup	Bond	Bond Stock	Bond Future	Bond Derivative	Bond Stock Future	Bond Stock Derivative
$\kappa = 0.2$	**0.355**	**0.273**	**0.271**	**0.336**	**0.256**	**0.314**
	(0.310)	(0.175)	(0.176)	(0.272)	(0.160)	(0.264)
$\gamma = 0.1$	**0.099**	**0.100**	**0.100**	**0.097**	**0.099**	**0.101**
	(0.013)	(0.014)	(0.013)	(0.018)	(0.013)	(0.016)
$\eta = 0.01$					**0.010**	**0.010**
					(0.002)	(0.001)
$\rho = 0.03$		**0.030**	**0.030**	**0.030**	**0.031**	**0.030**
		(0.006)	(0.005)	(0.005)	(0.004)	(0.005)
$\delta = 0.05$				**0.048**		**0.053**
				(0.012)		(0.006)
$\sigma = 0.02$		**0.021**	**0.020**		**0.020**	
		(0.011)	(0.011)		(0.011)	

Table 1.A.5: Simulation Study: GMM estimation when exploiting second moments. Median estimates of structural parameters for 550 runs of the simulation study. Corresponding interquartile ranges are given in brackets below the estimates. Blank spaces indicate parameters that are fixed at their value used in the DGP. All results involving consumption data can be interpreted as using the derivative on the capital stock instead (e.g. finance or macro-finance).

Simulation Study: GMM First and Second Moments

	Finance or Macro-Finance					**Macro-Finance**			
Setup	Bond Cons*	Bond Cons* Stock	Bond Cons* Future	Bond Cons* Derivative	Bond Cons* Derivative Stock	Bond Out	Bond Cons* Out	Bond Stock Out	Bond Future Out
$\kappa = 0.2$	**0.357**	**0.202**	**0.201**	**0.311**	**0.211**	**0.331**	**0.264**	**0.314**	**0.321**
	(0.285)	(0.052)	(0.047)	(0.181)	(0.220)	(0.302)	(0.261)	(0.220)	(0.296)
$\gamma = 0.1$	**0.099**	**0.100**	**0.100**	**0.100**	**0.100**	**0.101**	**0.106**	**0.108**	**0.107**
	(0.013)	(0.013)	(0.013)	(0.012)	(0.013)	(0.015)	(0.023)	(0.025)	(0.023)
$\eta = 0.01$	**0.010**	**0.010**	**0.010**	**0.010**	**0.007**	**0.010**	**0.010**	**0.009**	**0.009**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.002)	(0.002)
$\rho = 0.03$	**0.030**	**0.030**	**0.030**	**0.030**	**0.022**	**0.030**	**0.030**	**0.028**	**0.028**
	(0.005)	(0.006)	(0.005)	(0.006)	(0.012)	(0.006)	(0.006)	(0.008)	(0.007)
$\delta = 0.05$						**0.051**	**0.051**	**0.055**	**0.054**
						(0.003)	(0.003)	(0.009)	(0.011)
$\sigma = 0.02$	**0.020**	**0.020**	**0.020**	**0.020**	**0.021**		**0.020**	**0.020**	**0.020**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)		(0.001)	(0.003)	(0.003)

Table 1.A.6: Simulation Study: GMM estimation when exploiting second moments. Purely financial models. Setting as described in Table 1.A.5.

Simulation Study: GMM First and Second Moments

	Finance					
Setup	Bond	Bond Stock	Bond Future	Bond Derivative	Bond Stock Future	Bond Stock Derivative
$\kappa = 0.2$	**0.364**	**0.374**	**0.360**	**0.287**	**0.252**	**0.339**
	(0.277)	(0.275)	(0.231)	(0.249)	(0.075)	(0.220)
$\gamma = 0.1$	**0.099**	**0.099**	**0.099**	**0.099**	**0.100**	**0.105**
	(0.012)	(0.013)	(0.013)	(0.016)	(0.013)	(0.019)
$\eta = 0.01$	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
$\rho = 0.03$		**0.030**	**0.030**	**0.029**	**0.030**	**0.028**
		(0.005)	(0.005)	(0.006)	(0.005)	(0.008)
$\delta = 0.05$				**0.050**		**0.052**
				(0.004)		(0.006)
$\sigma = 0.02$		**0.022**	**0.022**		**0.019**	**0.020**
		(0.002)	(0.002)		(0.002)	(0.003)

Table 1.A.7: Simulation Study: MEF estimation when exploiting second moments. Median estimates of structural parameters for 550 runs of the simulation study. Corresponding interquartile ranges are given in brackets below the estimates. Blank spaces indicate parameters that are fixed at their value used in the DGP. All results involving consumption data can be interpreted as using the derivative on the capital stock instead (e.g. finance or macro-finance).

Simulation Study: MEF First and Second Moments

	Finance or Macro-Finance					**Macro-Finance**			
Setup	Bond Cons*	Bond Cons* Stock	Bond Cons* Future	Bond Cons* Derivative	Bond Cons* Derivative Stock	Bond Out	Bond Cons* Out	Bond Stock Out	Bond Future Out
$\kappa = 0.2$	**0.358**	**0.204**	**0.225**	**0.349**	**0.209**	**0.359**	**0.358**	**0.281**	**0.279**
	(0.288)	(0.044)	(0.045)	(0.286)	(0.055)	(0.288)	(0.287)	(0.205)	(0.205)
$\gamma = 0.1$	**0.099**	**0.098**	**0.099**	**0.098**	**0.099**	**0.100**	**0.100**	**0.099**	**0.099**
	(0.013)	(0.013)	(0.014)	(0.013)	(0.014)	(0.013)	(0.013)	(0.013)	(0.013)
$\eta = 0.01$	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\rho = 0.03$	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**	**0.030**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
$\delta = 0.05$				**0.049**	**0.050**	**0.050**	**0.050**	**0.050**	**0.050**
				(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma = 0.02$	**0.020**	**0.020**	**0.020**	**0.020**	**0.020**		**0.020**	**0.021**	**0.021**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)		(0.001)	(0.003)	(0.003)

Table 1.A.8: Simulation Study: MEF estimation when exploiting second moments. Purely financial models. Setting as described in Table 1.A.7.

Simulation Study: MEF First and Second Moments

	Finance					
Setup	Bond	Bond Stock	Bond Future	Bond Derivative	Bond Stock Future	Bond Stock Derivative
$\kappa = 0.2$	**0.358**	**0.290**	**0.260**	**0.348**	**0.251**	**0.256**
	(0.286)	(0.251)	(0.220)	(0.287)	(0.137)	(0.166)
$\gamma = 0.1$	**0.099**	**0.099**	**0.100**	**0.098**	**0.098**	**0.100**
	(0.012)	(0.013)	(0.012)	(0.013)	(0.012)	(0.014)
$\eta = 0.01$	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\rho = 0.03$		**0.030**	**0.029**	**0.030**	**0.031**	**0.030**
		(0.005)	(0.005)	(0.005)	(0.003)	(0.005)
$\delta = 0.05$				**0.049**		**0.050**
				(0.004)		(0.001)
$\sigma = 0.02$		**0.021**	**0.021**		**0.021**	**0.021**
		(0.003)	(0.003)		(0.002)	(0.003)

Table 1.A.9: Simulation Study: Robustness Check for Bootstrap. We re-calibrated the DGP in a way that interest rate easily can become negative and apply the reflective approach used in the bootstrap method of section 3. That is, we artificially set the interest rate to a small positive value, whenever it would turn negative. Blank spaces indicate parameters that are fixed at their value used in the DGP.

Simulation Study: Robustness Check for Bootstrap

	GMM			**MEF**		
Setup	Bond	Bond Cons	Bond Cons Stock	Bond	Bond Cons	Bond Cons Stock
$\kappa = 0.20$	**0.470**	**0.460**	**0.229**	**0.463**	**0.463**	**0.222**
	(0.371)	(0.373)	(0.119)	(0.375)	(0.377)	(0.085)
$\gamma = 0.03$	**0.032**	**0.032**	**0.033**	**0.032**	**0.032**	**0.029**
	(0.009)	(0.010)	(0.010)	(0.009)	(0.009)	(0.011)
$\eta = 0.01$	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**	**0.010**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\rho = 0.03$		**0.030**	**0.029**		**0.030**	**0.030**
		(0.005)	(0.007)		(0.005)	(0.005)
$\delta = 0.01$						
$\sigma = 0.02$		**0.020**	**0.020**		**0.020**	**0.020**
		(0.001)	(0.001)		(0.001)	(0.001)

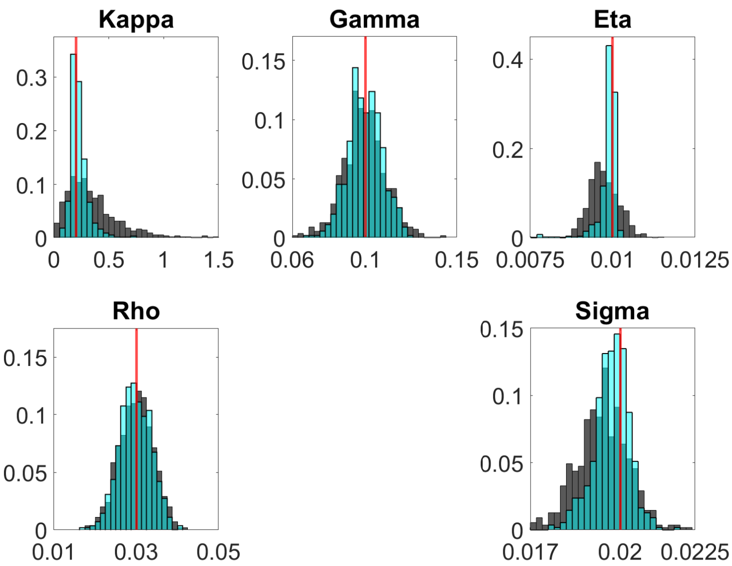

Figure 1.A.1: Histograms: Simulation study results using GMM and second moments (Daily). Benchmark model (gray histograms) at monthly frequency vs. Benchmark model when we replace output by stock and consumption by future data (blue histograms) at daily frequency.

1.A.7 Additional Empirical Estimation Results

Table 1.A.10: Empirical GMM estimation: Pure finance models. Bold parameter estimates lie within the bootstrapped 95% confidence interval. First and third quartiles, obtained from the bootstrapped parameter distributions are given below the estimates.

Empirical GMM: Additional Finance Models

	Sample: 1982-2017			**Sample: 1997-2017**			**Sample: 1997-2020**	
Setup	Bond	Bond Stock	Bond Stock	Bond	Bond Stock	Bond Stock	Bond Future	Bond Future
	(Monthly)	(Monthly)	(Daily)	(Daily)	(Monthly)	(Daily)	(Monthly)	(Monthly)
$\boldsymbol{\kappa}$	*0.029*	**0.041**	**0.118**	**0.169**	**0.075**	**0.119**	**0.069**	**0.118**
	[0.157, 0.286]	[0.125, 0.276]	[0.075, 0.129]	[0.340, 0.737]	[0.001, 0.725]	[0.082, 0.133]	[0.197, 0.424]	[0.072, 0.120]
$\boldsymbol{\gamma}$	*0.033*	*0.042*	**0.074**	**0.070**	**0.059**	**0.068**	**0.065**	**0.102**
	[0.071, 0.081]	[0.069, 0.083]	[0.075, 0.129]	[0.064, 0.074]	[0.061, 0.067]	[0.066, 0.102]	[0.062, 0.070]	[0.092, 0.148]
$\boldsymbol{\eta}$	**0.008**	**0.008**	**0.008**	**0.008**	**0.007**	**0.007**	**0.007**	**0.006**
	[0.008, 0.008]	[0.008, 0.008]	[0.009, 0.011]	[0.008, 0.009]	[0.006, 0.007]	[0.006, 0.008]	[0.006, 0.007]	[0.007, 0.008]
$\boldsymbol{\rho}$		**0.000**	**0.014**		**0.000**	**0.015**	**0.001**	**0.009**
		[0.000, 0.000]	[0.018, 0.026]		[0.000, 0.000]	[0.021, 0.029]	[0.000, 0.002]	[0.011, 0.020]
$\boldsymbol{\delta}$	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
$\boldsymbol{\sigma}$	0.02	0.02	**0.037**	0.02	0.02	**0.038**	0.02	**0.043**
			[0.028, 0.055]			[0.027, 0.049]		[0.028, 0.050]

Table 1.A.11: Empirical MEF estimations: Additional pure finance models. Bold parameter estimates lie within the bootstrapped 95% confidence interval. First and third quartiles, obtained from the bootstrapped parameter distributions are given below the estimates.

Empirical MEF: Additional Finance Models

Setup	Bond (Monthly)	Bond Stock (Daily)	Bond Future (Daily)	Bond Output Stock (Monthly)
κ	**0.170**	**0.009**	**0.010**	**0.025**
	[0.000, 0.360]	[0.005, 0.008]	[0.004, 0.009]	[0.017, 0.097]
γ	**0.071**	**0.063**	*0.096*	**0.136**
	[0.069, 0.081]	[0.048, 0.074]	[0.043, 0.070]	[0.096, 0.178]
η	*0.011*	**0.007**	**0.014**	**0.022**
	[0.008, 0.008]	[0.002, 0.006]	[0.005, 0.010]	[0.014, 0.026]
ρ		**0.017**	*0.013*	**0.000**
		[0.012, 0.026]	[0.018, 0.029]	[0.000, 0.003]
δ	**0.05**	**0.05**	**0.05**	**0.031**
				[0.012, 0.056]
σ	**0.0200**	**0.021**	**0.0447**	*0.024*
		[0.024, 0.041]	[0.0233, 0.0392]	[0.035, 0.090]

Table 1.A.12: Empirical GMM estimation: Shorter samples starting 1997. Bold parameter estimates lie within the bootstrapped 95% confidence interval. First and third quartiles, obtained from the bootstrapped parameter distributions are given below the estimates.

Empirical GMM: Shorter Sample

	(1997-2020)	**(1997-2017)**
Setup	Bond Cons	Bond Cons Stock
κ	*0.064*	**0.379**
	[0.299, 0.560]	[0.332, 0.613]
γ	**0.063**	**0.063**
	[0.062, 0.067]	[0.062, 0.067]
η	*0.005*	**0.006**
	[0.006, 0.007]	[0.006, 0.007]
ρ	**0.000**	**0.000**
	[0.000, 0.000]	[0.000, 0.000]
δ	**0.05**	**0.05**
σ	**0.012**	**0.012**
	[0.011, 0.012]	[0.012, 0.013]

2.A Appendix, Chapter 2

2.A.1 Technical Details FTPL Model

In this paper, we use a linear version of the micro-founded NK model (cf. Posch, 2020). The basic structure of the model is as follows. A representative household consumes, saves, and supplies labor. The final output is assembled by a final good producer, who uses as inputs a continuum of intermediate goods manufactured by monopolistic competitors. The intermediate good producers rent labor to manufacture their good and face the constraint that they can only adjust the price following Calvo's pricing rule (Calvo, 1983). Finally, there is a monetary authority that fixes the short-term nominal interest rate through open market operations following a Taylor rule and a detailed government sector with a fiscal authority that issues debt, taxes, and consumes following fiscal policy rules.

Households

Let the reward function of the households be given as

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left\{ \log c_t - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right\} dt, \qquad \psi > 0, \tag{A.1}$$

where ρ denotes the subjective rate of time preference, ϑ is the inverse of the Frisch labor supply elasticity, and ψ scales the disutility from working by supplying labor in terms of hours l_t (we use ψ to normalize $l_{ss} = 1$). Let n_t denote the number of shares of government bonds; assuming that each bond has a nominal value of one unit, whereas p_t^b is the equilibrium price of bonds. Suppose the household earns a disposable income of

$$\delta^c n_t + p_t w_t l_t - p_t T_t + p_t \digamma_t$$

where δ^c are coupon payments, p_t is the price level (or price of the consumption good), w_t is the real wage, T_t are lump-sum taxes, and $\digamma_t$ are the profits of the firms in the economy. Hence, the household's budget constraint reads

$$\mathrm{d}n_t = \left(\left(\delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t \right) / p_t^b - \delta n_t \right) \mathrm{d}t, \tag{A.2}$$

in which p_t^b denotes the bond price. Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ.

The first-order condition for households to maximize (A.1) subject to (A.2) is

$$\psi l_t^\vartheta c_t = mc_t, \tag{A.3}$$

which is the standard static optimality condition between labor and consumption. Hence, for the given preferences (A.1), the Euler equation for consumption reads (cf. Posch, 2020)

$$\mathrm{d}c_t = (i_t - \pi_t - \rho)c_t \, \mathrm{d}t, \tag{A.4}$$

or the linearized version

$$\mathrm{d}c_t \approx (i_t - \rho - \pi_t)c_{ss} \, \mathrm{d}t, \tag{A.5}$$

with π_t being determined in general equilibrium.

The Final Good Producer

There is one final good, produced using intermediate goods with

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.6}$$

where ε is the elasticity of substitution.

Final good producers are perfectly competitive and maximize profits subject to the production function (A.6), taking as given all intermediate goods prices p_{it} and the final good price p_t. Hence, the input demand functions associated with this problem are:

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t \qquad \forall i,$$

and

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

is the (aggregate) price level.

Intermediate Good Producers

Each intermediate firm produces differentiated goods out of labor using:

$$y_{it} = l_{it}, \tag{A.8}$$

where l_{it} is the amount of the labor input rented by the firm. Therefore, the marginal cost of the intermediate good producer is the same across firms:

$$mc_t = w_t. \tag{A.9}$$

The monopolistic firms engage in price setting à la Calvo, which then gives rise to the NK Phillips curve (see, e.g., Leith and von Thadden, 2008; Posch, 2020)

$$\mathrm{d}(\pi_t - \pi_{ss}) \approx (\rho(\pi_t - \pi_{ss}) - \kappa_0(mc_t/mc_{ss} - 1)) \, \mathrm{d}t. \tag{A.10}$$

Note that from (A.3) $\psi y_t^{\vartheta} c_t = mc_t$ such that a linearized version is

$$mc_t/mc_{ss} - 1 \approx (c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1).$$

Moreover, for the parametrization in Table 2.1, we have that $g_t \equiv g_{ss}$ and thus

$$\begin{aligned} \mathrm{d}(\pi_t - \pi_{ss}) &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((y_t/y_{ss} - 1)y_{ss}/c_{ss} + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &\equiv (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t \qquad \text{(A.11)} \end{aligned}$$

as in (2.2), where $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$ is the output gap and $\kappa \equiv \kappa_0(1 + \vartheta(1 - s_g))$ captures 'price stickiness'. Our definition of the output gap is to formulate the benchmark model as close as possible to the one used in the literature, where typically $s_g \equiv 0$.

Note that with this definition of the output gap, we obtain (2.1) from (A.5) as

$$\begin{aligned} \mathrm{d}(y_t - g_{ss}) &= (i_t - \rho - \pi_t)(y_{ss} - g_{ss})\,\mathrm{d}t \\ &= (i_t - \rho - \pi_t)(1 - s_g)y_{ss}\,\mathrm{d}t \end{aligned}$$

after inserting our definition $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$.

Or with variable government consumption,

$$\begin{aligned} mc_t/mc_{ss} - 1 &= (1 + \vartheta(1 - s_g))(y_t/y_{ss} - 1)/(1 - s_g) - (g_t/g_{ss} - 1)s_g/(1 - s_g) \\ &= (1 + \vartheta(1 - s_g))x_t - (g_t/g_{ss} - 1)s_g/(1 - s_g) \end{aligned}$$

and thus the Phillips curve in the generalized version obeys

$$\mathrm{d}(\pi_t - \pi_{ss}) = (\rho(\pi_t - \pi_{ss}) - \kappa x_t + \kappa_0 s_g/(1 - s_g)(g_t/g_{ss} - 1))\,\mathrm{d}t. \qquad \text{(A.12)}$$

Government

We assume that the monetary authority sets the nominal interest rate i_t of short-term bonds through open market operations according to either the feedback model,

$$i_t - i_t^* = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1), \quad \phi_\pi > 0,\ \phi_y \geq 0, \qquad \text{(A.13a)}$$

or the partial adjustment model (cf. Posch, 2020):

$$\mathrm{d}i_t = \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, \quad \theta > 0, \qquad \text{(A.13b)}$$

which includes a response to inflation and output, and a desire to smooth interest rates.

The fiscal authority trades a nominal non-contingent bond. Let n_t be the outstanding stock of nominal government bonds, i.e., the total nominal value of outstanding debt (alternative assets are priced using arbitrage arguments but are in net zero supply). The government incurs a real primary surplus $s_t \equiv T_t - g_t$ where revenues T_t and expenditure g_t rules are given in (2.9) and (2.10). Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ. Hence, the government faces the constraint that the newly issued debt must cover amortization plus coupon payments of outstanding debt, net of the primary surplus such that the nominal value of outstanding debt follows

$$\mathrm{d}n_t = \big(\left((\delta+\chi)n_t - p_t s_t\right)/p_t^b - \delta n_t\big)\ \mathrm{d}t, \tag{A.14}$$

where p_t^b is the bond price.

Aggregation

First, market clearing demands:

$$y_t = c_t + g_t = c_t + T_t - s_t, \tag{A.15}$$

and suppose aggregate output is produced according to (e.g., in the linearized model)

$$y_t = l_t$$

in which we normalized to $y_{ss} = l_{ss} \equiv 1$ in the benchmark parametrization, and the income is generated through

$$y_t = w_t l_t + \digamma_t.$$

All outstanding sovereign debt is owned by households, so (A.2) and (A.14) yield

$$(\delta+\chi)n_t - p_t s_t = \delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t.$$

Recall that the real value of sovereign debt is defined as in (2.6), $a_t = n_t p_t^b / p_t$. In equilibrium,

$$i_t \mathrm{d}t = ((\chi+\delta)/p_t^b - \delta)\mathrm{d}t + (1/p_t^b)\,\mathrm{d}p_t^b$$

such that the bond price follows (2.7). We define the inflation rate π_t such that

$$\mathrm{d}p_t = \pi_t p_t \mathrm{d}t \tag{A.16}$$

and the (realized) rate of inflation is locally non-stochastic.

Hence, the budget constraint of the fiscal authority (2.6) can be written as

$$\begin{aligned}
\mathrm{d}a_t &= (p_t^b\,\mathrm{d}n_t + n_t\,\mathrm{d}p_t^b - n_t p_t^b/p_t\,\mathrm{d}p_t)/p_t \\
&= ((\delta+\chi)n_t/p_t - s_t)\ \mathrm{d}t - \delta n_t p_t^b/p_t + n_t\,\mathrm{d}p_t^b/p_t - n_t p_t^b/p_t(1/p_t)\,\mathrm{d}p_t \\
&= ((\delta+\chi)n_t/p_t - s_t)\ \mathrm{d}t - \delta a_t\,\mathrm{d}t + a_t i_t\,\mathrm{d}t - ((\delta+\chi)n_t/p_t - \delta a_t)\,\mathrm{d}t - a_t\pi_t\,\mathrm{d}t,
\end{aligned}$$

which is equation (2.4) in the fiscal block.

Similarly, the household's budget constraint (A.2) can be written as

$$\begin{aligned}
\mathrm{d}a_t &= (p_t^b\,\mathrm{d}n_t + n_t\,\mathrm{d}p_t^b - n_t p_t^b/p_t\,\mathrm{d}p_t)/p_t \\
&= \big((\delta+\chi)a_t/p_t^b - s_t\big)\ \mathrm{d}t - \delta a_t + a_t(1/p_t^b)\,\mathrm{d}p_t^b - a_t\pi_t\,\mathrm{d}t \\
&= \big((\delta+\chi)a_t/p_t^b - s_t\big)\ \mathrm{d}t - \delta a_t + (-((\delta+\chi)/p_t^b - \delta) + i_t)a_t\mathrm{d}t - a_t\pi_t\,\mathrm{d}t \\
&= -s_t\,\mathrm{d}t + i_t a_t\mathrm{d}t - a_t\pi_t\,\mathrm{d}t \\
&= ((i_t - \pi_t)a_t + w_t l_t - c_t - T_t + \digamma_t)\,\mathrm{d}t,
\end{aligned}$$

which again shows that the household's budget constraint coincides with the government budget constraint. Using (A.2) and (A.14), together with market clearing (A.15), the coupon payments cover payouts and amortization such that $\delta^c \equiv \delta + \chi$.

Steady-State Values

From (2.1), (2.4), and (2.7), we obtain $i_{ss} = \rho + \pi_{ss}$, $a_{ss} = s_{ss}/\rho$, and $p_{ss}^b = 1$. In this model

$$mc_{ss} = w_{ss} = \frac{\varepsilon - 1}{\varepsilon},$$

where ε is the elasticity of substitution between intermediate goods. Moreover, condition (A.3) implies together with the market clearing condition (A.15) that

$$\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}.$$

Observe that $c_{ss} = y_{ss} - g_{ss} = l_{ss} - g_{ss}$, defining $s_g = g_{ss}/y_{ss}$ such that

$$\psi l_{ss}^{1+\vartheta}(1 - s_g) = w_{ss}.$$

Hence, we parameterize

$$\psi \equiv w_{ss} l_{ss}^{-(1+\vartheta)}/(1 - s_g)$$

to normalize the steady-state output $y_{ss} = l_{ss} = 1$, such that $\digamma_{ss} = 1/\varepsilon$, $c_{ss} = 1 - g_{ss}$, $T_{ss} = s_{ss} + g_{ss}$ (s_{ss} and s_g are calibrated using US targets).

2.A.2 Linearized Dynamics

In this paper use the linearized NK model, so we need to linearize the equations (A.4), (2.4), and (2.7). Let us summarize the equilibrium dynamics for our parametrization.

Benchmark Parametrization (Table 2.1)

Using $\pi_t^* = \pi_{ss}$, $i_t^* = i_{ss} = \rho + \pi_{ss}$, and $s_t^* = s_{ss}$, together with the parametrization of the benchmark model (cf. Table 2.1), the linearized equilibrium dynamics can be written as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(A.17)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t && \text{(A.18)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_{ss}) - (i_t - i_{ss}))\mathrm{d}t && \text{(A.19)}\\
\mathrm{d}a_t &= (a_{ss}(i_t - \pi_t - \rho) + \rho(a_t - a_{ss}) - (s_t - s_{ss}))\mathrm{d}t && \text{(A.20)}\\
\mathrm{d}s_t &= ((y_t/y_{ss} - 1) - (s_t - s_{ss}))\,\mathrm{d}t && \text{(A.21)}\\
\mathrm{d}p_t^b &= \big((i_t - i_{ss}) + (\chi + \delta)(p_t^b - 1)\big)\,\mathrm{d}t, && \text{(A.22)}
\end{aligned}$$

where

$$y_t/y_{ss} - 1 = (c_t - c_{ss} + g_t - g_{ss})/y_{ss}$$

such that with $g_t = g_{ss}$ we get $\kappa \equiv (1 + \vartheta(1 - s_g))\kappa_0$, and

$$x_t = (y_t/y_{ss} - 1)/(1 - s_g) = (c_t/c_{ss} - 1)(c_{ss}/y_{ss})/(1 - s_g) = (c_t/c_{ss} - 1),$$

i.e., the consumption Euler equation can be written in terms of the output gap.

2.A.3 Proof of Proposition 6

Recall that in the model with long-term debt, a proper predetermined state variable (which does not jump) is v_t rather than a_t, hence, we linearize

$$a_t - a_{ss} \quad = \quad p_{ss}^b(v_t - v_{ss}) + v_{ss}(p_t^b - p_{ss}^b)$$

such that the real value of government debt changes due to two channels

$$\mathrm{d}a_t = p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b. \tag{A.23}$$

The partial derivatives of the policy function $x(i_t, a_t, s_t)$ show the indirect FTPL effect for a given bond price, p_t^b, such that we need to isolate the direct FTPL effect due to the re-evaluation of sovereign debt. Now, evaluating the effect of a change to i_t at some reference point, say $\bar{x}_i = x_i(i_{ss}, a_{ss}, s_{ss})$, the slope of the policy function in terms of a_t

would only include the indirect effect, keeping fix the price of government debt. Note that our solution implies both $p_t^b = p^b(i_t, v_t, s_t)$ or $p_t^b = p^b(i_t, a_t, s_t)$ such that

$$\mathrm{d}p_t^b = p_i^b(i_t, v_t, s_t)\,\mathrm{d}i_t + p_v^b(i_t, v_t, s_t)\,\mathrm{d}v_t + p_s^b(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.24}$$

and $\mathrm{d}p_t^b = p_i^b(i_t, a_t, s_t)\,\mathrm{d}i_t + p_a^b(i_t, a_t, s_t)\,\mathrm{d}a_t + p_s^b(i_t, a_t, s_t)\,\mathrm{d}s_t$ and thus using (A.23)

$$\mathrm{d}p_t^b = p_i^b(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}i_t + p_a^b(i_{ss}, a_{ss}, s_{ss})(p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b) + p_s^b(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}s_t$$

or equivalently

$$\begin{aligned}\mathrm{d}p_t^b \;&=\; \frac{p_i^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}i_t + \frac{p_{ss}^b p_a^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}v_t \\ &\quad+\frac{p_s^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}s_t\end{aligned} \tag{A.25}$$

and by matching coefficients with (A.24)

$$\begin{aligned}p_i^b(i_t, v_t, s_t) &= \frac{p_i^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})} \\ p_v^b(i_t, v_t, s_t) &= \frac{p_{ss}^b p_a^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})} \\ p_s^b(i_t, v_t, s_t) &= \frac{p_s^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})},\end{aligned}$$

we can conclude that

$$\begin{aligned}\bar{p}_i^b &\equiv p_i^b(i_{ss}, a_{ss}, s_{ss}) = p_i^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b) \\ \bar{p}_a^b &\equiv p_a^b(i_{ss}, a_{ss}, s_{ss}) = \frac{p_v^b(i_{ss}, v_{ss}, s_{ss})}{1 + v_{ss}p_n^b(i_{ss}, v_{ss}, s_{ss})/p_{ss}^b} \\ \bar{p}_s^b &\equiv p_s^b(i_{ss}, a_{ss}, s_{ss}) = p_s^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b).\end{aligned}$$

Similarly, for the inflation rate we can utilize

$$\mathrm{d}\pi_t = \pi_i(i_t, v_t, s_t)\,\mathrm{d}i_t + \pi_n(i_t, v_t, s_t)\,\mathrm{d}n_t + \pi_s(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.26}$$

or, equivalently,

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)\,\mathrm{d}a_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t. \tag{A.27}$$

We substitute equation (A.23)

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)(p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b) + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t$$

or

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)p^b_{ss}\,\mathrm{d}v_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t + v_{ss}\pi_a(i_t, a_t, s_t)\,\mathrm{d}p^b_t.$$

Substitute by equation (A.25)

$$\begin{aligned}
\mathrm{d}\pi_t &= \left(\pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}i_t \\
&\quad + \left(\pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}v_t \\
&\quad + \left(\pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}s_t
\end{aligned}$$

and matching coefficients with equation (A.26)

$$\begin{aligned}
\pi_i(i_{ss}, v_{ss}, s_{ss}) &= \pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_v(i_{ss}, v_{ss}, s_{ss}) &= \pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_s(i_{ss}, v_{ss}, s_{ss}) &= \pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}.
\end{aligned}$$

Rearranging terms we arrive at

$$\begin{aligned}
\bar{\pi}_i &\equiv \pi_i(i_{ss}, a_{ss}, s_{ss}) = \pi_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{\pi}_a &\equiv \pi_a(i_{ss}, a_{ss}, s_{ss}) = \pi_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{\pi}_s &\equiv \pi_s(i_{ss}, a_{ss}, s_{ss}) = \pi_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a}.
\end{aligned}$$

We proceed analogously for the output gap, $x(i_t, v_t, s_t)$ and $x(i_t, v_t, s_t)$. Except for notation the derivations are identical to the inflation rate. Thus,

$$\begin{aligned}
\bar{x}_i &\equiv x_i(i_{ss}, a_{ss}, s_{ss}) = x_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{x}_a &\equiv x_v(i_{ss}, a_{ss}, s_{ss}) = x_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{x}_s &\equiv x_s(i_{ss}, a_{ss}, s_{ss}) = x_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a},
\end{aligned}$$

which closes the proof (inflation rates and output gap analogously).

2.A.4 Figures and Tables

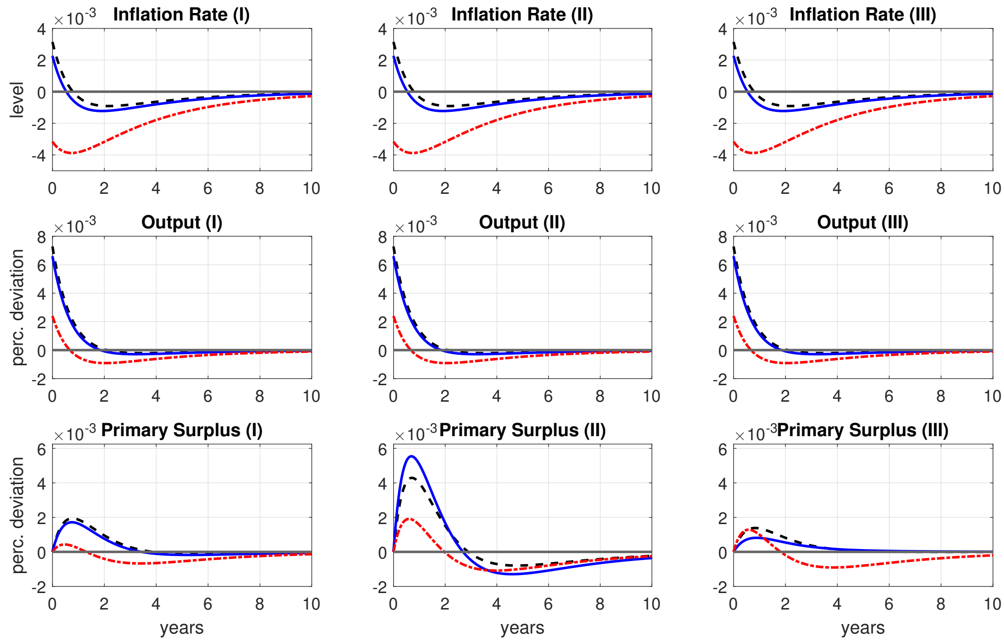

Figure 2.A.1: Transitory monetary policy shock for the parametrization in Table 2.1 and different surplus dynamics. Decrease nominal interest rate by 1 percentage point. Left-hand panel: Baseline scenario, $\tau_\pi = 0$ and $\tau_y = 1$. Middle panel: $\tau_\pi = 1.02$ and $\tau_y = 3.08$. Right-hand panel: $\tau_\pi = 0.5$ and $\tau_y = -0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.A.1: Inflation decomposition (2.21) for the monetary policy shock in Figure 2.A.1.

Surplus Rule	Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
I	Average	-0.48	-1.25	0.21	0.98
II	Average	-0.48	-1.25	0.21	0.98
III	Average	-0.48	-1.25	0.21	0.98

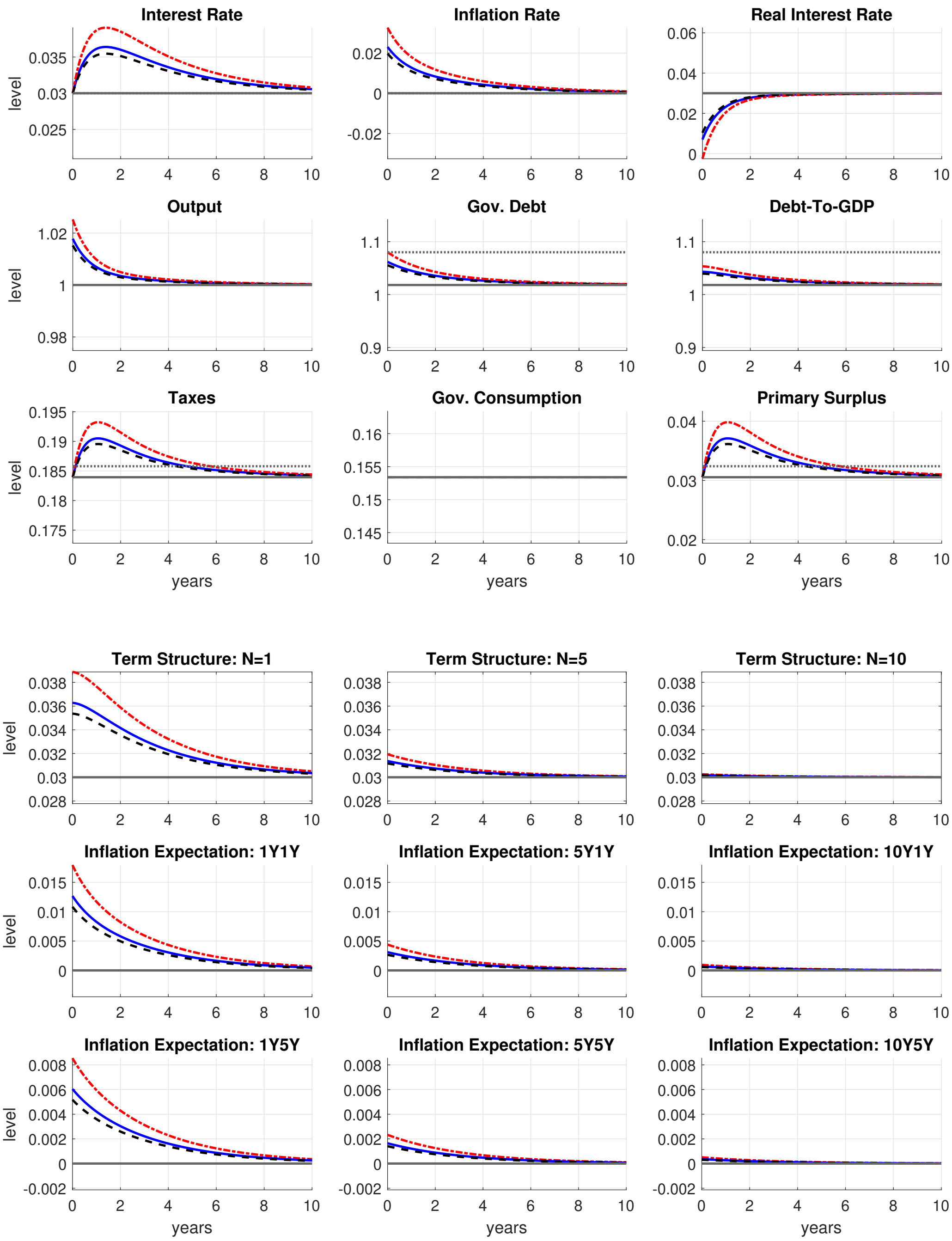

Figure 2.A.2: Permanent fiscal policy shock for parametrization in Table 2.1. Permanent decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$, together with a transitory shock that decreases taxes by 1 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

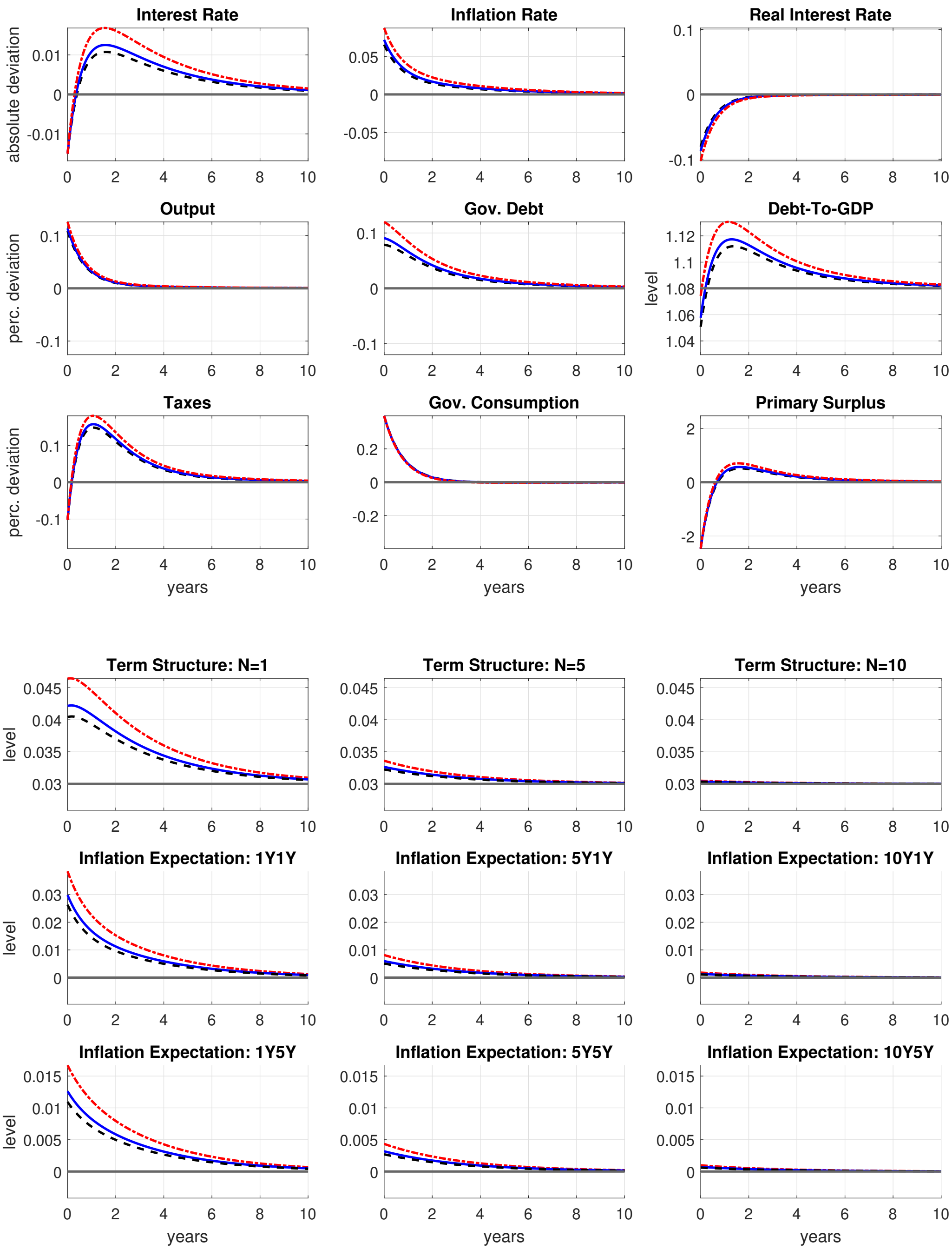

Figure 2.A.3: CARES Act and monetary policy shock using parametrization in Table 2.1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, and increase in debt (face value) by 12 percent, and decrease interest rates by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

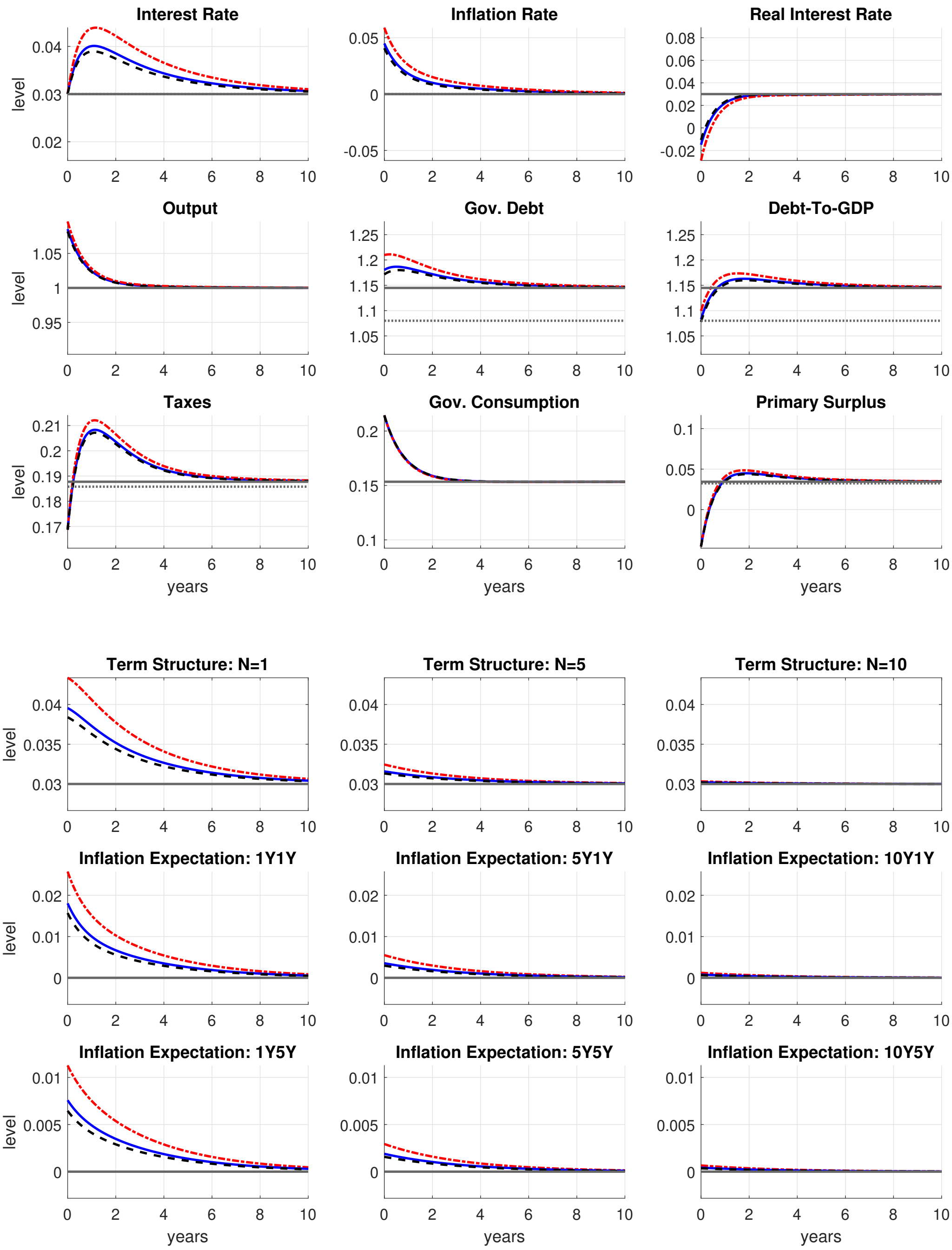

Figure 2.A.4: CARES Act shock with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 2.1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

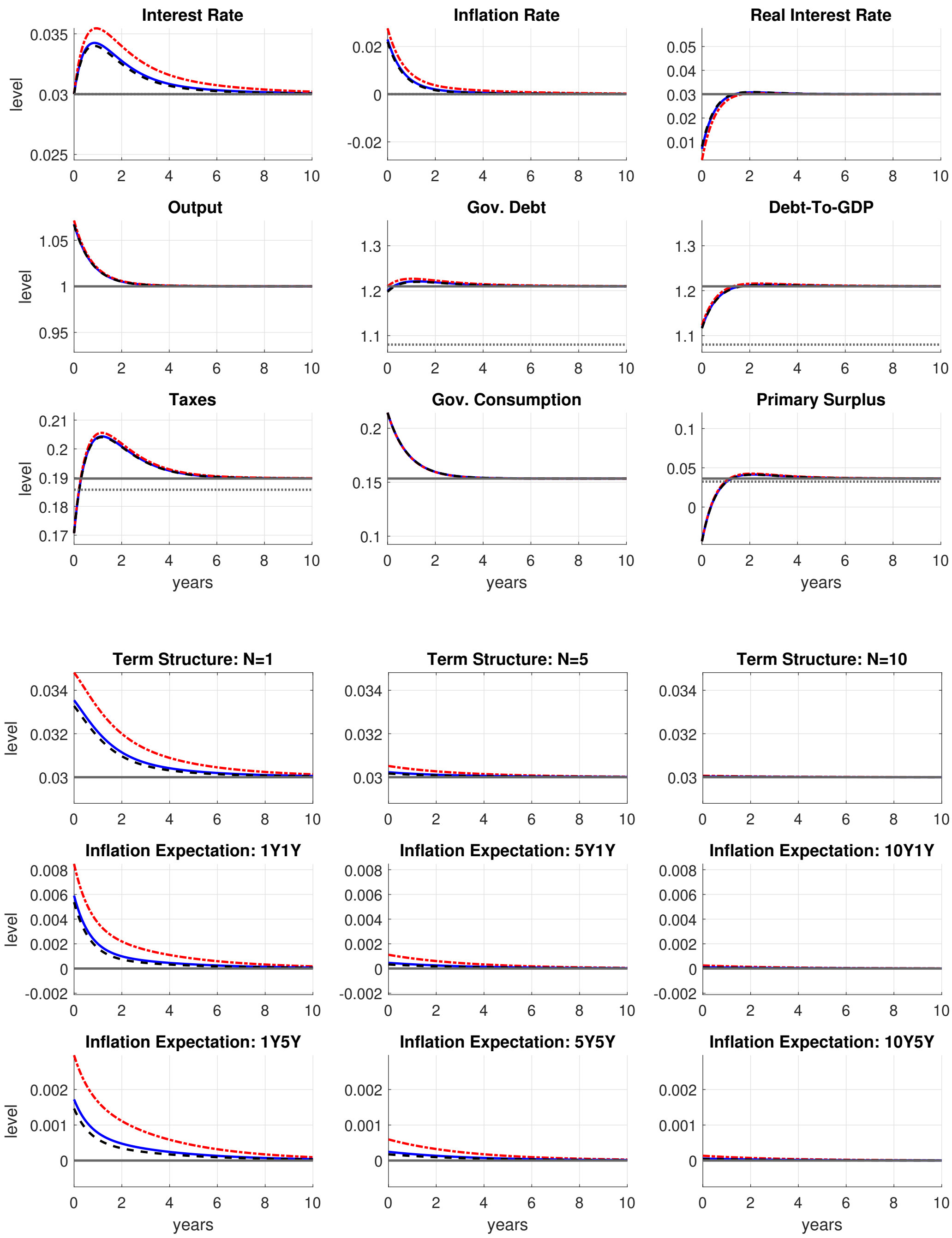

Figure 2.A.5: CARES Act shock with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 2.1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2.A.2: Inflation decomposition (2.21) for the fiscal policy shock in Figure 2.A.2.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	4.14	2.41	1.94	−2.41	6.08
Average	4.84	2.82	2.28	−1.79	6.08
Short-Term	6.86	3.99	3.22	0	6.08

Table 2.A.3: Inflation decomposition (2.21) for the CARES Act in Figure 2.A.3.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	9.60	4.14	2.40	−4.14	12.00
Average	10.96	4.93	3.04	−2.93	12.00
Short-Term	14.28	6.86	4.58	0	12.00

Table 2.A.4: Inflation decomposition (2.21) for the CARES Act shock in Figure 2.A.4.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	5.72	3.33	−0.06	−3.33	5.66
Average	6.63	3.86	0.34	−2.56	5.66
Short-Term	9.61	5.60	1.65	0	5.66

Table 2.A.5: Inflation decomposition (2.21) for the CARES Act shock in Figure 2.A.5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	1.72	1.00	−1.72	−1.00	0
Average	1.93	1.12	−1.63	−0.83	0
Short-Term	2.92	1.70	−1.22	0	0

3.A Appendix, Chapter 3

3.A.1 Parametrization

Parametrization 1: Baseline (United States)

Table 3.A.1: Baseline Parametrization, Chapter 3

		General Model Parameters	
γ	100	Price adjustment cost	(Kaplan et al. (2018))
ε	10	Demand elasticity	(Kaplan et al. (2018))
α	1/3	labor share of production function	(Kaplan et al. (2018))
δ	0.07	depreciation rate of physical capital	(Kaplan et al. (2018))
ρ	0.051	time preference	(Kaplan et al. (2018))
ξ	1	labor supply elasticity	(Kaplan et al. (2018))
ϑ	2.2	preference for leisure (capital models)	normalizes output $y_{ss} = 1$
ϑ	1.06	preference for leisure (non-capital models)	normalizes output $y_{ss} = 1$
s_g	0.15	gov. consumption output ratio	(cf. Bilbiie et al. (2019))
s_t^*	0.0551	long-run surplus[a]	FRED[b]
δ^b	1/6.8	avg. debt maturity	(Del Negro and Sims (2015))
κ	0.3261	degree of adjustment cost	(Jermann (1998))
		Monetary Policy Parameters	
ϕ_π	1.6	inflation response Taylor Rule (active)	(Bianchi and Melosi (2017))
ϕ_π	0.64	inflation response Taylor Rule (passive)	(Bianchi and Melosi (2017))
θ	1	interest rate smoothing Taylor Rule	(Kaplan et al. (2018))
ϕ_y	0	response to output gap in Taylor Rule	(Kaplan et al. (2018))
π_t^*	0	inflation target	(Kaplan et al. (2018))
		Fiscal Policy Parameters	
ρ_g	1	inertia of adjustment of gov. consumption	(Liemen and Posch (2022))
φ_y	-0.15	gov. cons. responsiveness to output gap	(Liemen and Posch (2022))
ρ_τ	1	inertia of adjustment of taxes	(Liemen and Posch (2022))
τ_y	0.324	tax responsiveness to output gap	(Davig and Leeper (2011))

[a] Matches the US debt-to-GDP ratio of about 108% in 2020Q1.

[b] U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, Federal Debt: Total Public Debt as Percent of Gross Domestic Product [GFDEGDQ188S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GFDEGDQ188S, January 13, 2022.

Parametrization 2: Japan

Table 3.A.2: Alternative Parametrization for Japan, Chapter 3

		General Model Parameters	
γ	80	Price adjustment cost	(Braun and Körber (2011))
ε	7.5	Demand elasticity	(Braun and Körber (2011))
α	0.362	labor share of production function	(Braun and Körber (2011))
δ	0.085	depreciation rate of physical capital	(Braun and Körber (2011))
ρ	0.029	time preference	(Braun and Körber (2011))
ξ	1	labor supply elasticity	(Kaplan et al. (2018))
ϑ	3.028	preference for leisure	normalizes output $y_{ss} = 1$
s_g	0.19	government consumption output ratio	(Braun and Körber (2011))
s_t^*	0.051	yields Debt-2-GDP of 175.9% (in 2011)	FRED[a]
δ^b	1/7	avg. debt maturity of 7 years (in 2011)	Ministry of Finance Japan[b]
κ	0.326	degree of adjustment cost	(Jermann (1998))
		Monetary Policy Rule Parameters	
ϕ_π	1.6	inflation response Taylor Rule (active)	(Bianchi and Melosi (2017))
ϕ_π	0.86	inflation response Taylor Rule (passive)	(Davig and Doh (2008))
θ	1	interest rate smoothing (use $\theta = 0$ for peg)	(Kaplan et al. (2018))
ϕ_y	0	response to output gap	(Kaplan et al. (2018))
π_t^*	0	inflation target	(Kaplan et al. (2018))
		Fiscal Policy Parameters	
ρ_g	1	inertia of adjustment of gov. consumption	(Liemen and Posch (2022))
φ_y	0	gov. cons. responsiveness to output gap	(Kliem et al. (2016))
φ_a	-0.01	gov. cons. responsiveness to debt	(Kliem et al. (2016))
ρ_τ	1	inertia of adjustment of taxes	(Liemen and Posch (2022))
τ_y	0	tax responsiveness to output gap	(Kliem et al. (2016))
ρ_a	0.01	tax responsiveness to debt	(Kliem et al. (2016))

[a]Source: *World Bank, Central government debt, total (% of GDP) for Japan [DEBTTLJPA188A], retrieved from FRED, Federal Reserve Bank of St. Louis.*

[b]Source: *Ministry of Finance, Japan - Debt Management Report 2020.*

3.A.2 Determinacy Analysis

In this section, I evaluate determinacy of the models in this paper. It is important to stress that I only consider bounded solutions. To start my analysis, I break all considered models down to their most basic formulation. For this purpose, I implement a feedback interest rate rule of the form

$$i_t = i_{ss} + \phi_\pi(\pi_t - \pi_t^*),$$

where $\phi_\pi > 1$ corresponds to active monetary policy. Furthermore, I introduce a feedback rule for surpluses

$$s_t = s_t^* + \tau_a(v_t - v_{ss})$$

so that $\tau_a < \rho$ corresponds to active fiscal policy. As in the main text, I always consider the implementation of active fiscal policy rules. Thus, to keep things as simple as possible, I impose constant surpluses by setting $\tau_a \equiv 0$. Finally, I abstract from government consumption and assume debt to be short-term. As a consequence, the fiscal policy block reduces to

$$\mathrm{d}v_t = ((\rho - \tau_a)(v_t - v_{ss}) + v_{ss}(\phi_\pi - 1)(\pi_t - \pi_t^*))\mathrm{d}t. \tag{A.1}$$

Note that the determinacy conditions in this section can (except for government consumption) readily be translated to the more sophisticated model formulations in the main text. As discussed in Section 3.3.2, the introduction of government consumption is troublesome because it affects determinacy in various dimensions and results in less straightforward conditions.

To evaluate determinacy, I analyze the Jacobian matrices of the linearized models along the lines of Leeper (1991), Dupor (2001) and Leith and von Thadden (2008). To illustrate my findings, I additionally provide some numerical examples when using the baseline parametrization (Table 3.A.1). Note that obtaining a determinate and unique solution in a continuous-time framework requires that each predetermined (state) variable has to correspond to exactly one negative (stable) eigenvalue of the Jacobian matrix of the linearized model. At the same time, each forward-looking (jump) variable has to correspond to exactly one positive (explosive) eigenvalue of the Jacobian. If these conditions hold true, the model is considered determinate (see for instance Dupor (2001) or Leith and von Thadden (2008)).

Note that in case of the more sophisticated model specifications in the main text, the partial adjustment interest rate rule makes the nominal interest rate a state. In this case, the Jacobian needs one negative eigenvalue that corresponds to the nominal interest rate. While a determinate solution requires that each state variable correspond to exactly one negative eigenvalue, the active/passive interest rate specification, in general, determines whether the nominal interest rate contributes a negative or a positive eigenvalue to the Jacobian. Regarding taxes and government consumption, I only consider situation where both variables enter the model as state variables and where their inclusion is associated with corresponding negative eigenvalues of the Jacobian.

For a Jacobian-based determinacy analysis for the simple NK and the NK-FTPL model with or without capital see e.g. Leith and von Thadden (2008). The simplified version of the NK-AC model becomes

$$
\begin{aligned}
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \varepsilon/\gamma(mc_t - mc_{ss}))\,\mathrm{d}t \\
\mathrm{d}c_t &= c_{ss}(\phi_\pi - 1)(\pi_t - \pi_t^*)\mathrm{d}t \\
\mathrm{d}k_t &= ((x_t - x_{ss}) - \delta(k_t - k_{ss}))\mathrm{d}t \\
\mathrm{d}\mu_t &= (\rho(\mu_t - \mu_{ss}) + \rho\mu_{ss}(c_t/c_{ss} - 1) - \mu_{ss}(r_t^k - r_{ss}^k))\mathrm{d}t.
\end{aligned}
\tag{A.2}
$$

Hence, the Jacobian reads

$$
J_4 = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & 0 & 0 & 0 \\ 0 & f_{32} & 0 & f_{34} \\ 0 & f_{42} & f_{43} & f_{44} \end{bmatrix} = \begin{bmatrix} 0.051 & -0.145 & 0.014 & -0.002 \\ 0.535 & 0 & 0 & 0 \\ 0 & 0.068 & 0 & 0.054 \\ 0 & -0.289 & 0.088 & 0.041 \end{bmatrix}
$$

where

$$
\begin{aligned}
f_{11} &= \rho \\
f_{12} &= \frac{mc_{ss}\varepsilon(c_{ss}\xi + \kappa\xi x_{ss} + \alpha(c_{ss} + \kappa x_{ss} - y_{ss}) + y_{ss})}{(-1+\alpha)c_{ss}\gamma y_{ss}} \\
f_{13} &= \frac{mc_{ss}\varepsilon(\xi x_{ss} + \alpha(x_{ss} - (1+\xi)y_{ss}))}{(\alpha-1)\gamma k_{ss} y_{ss}} \\
f_{14} &= \frac{\kappa mc_{ss}\varepsilon(\alpha+\xi)x_{ss}}{(\alpha-1)\gamma\mu_{ss}y_{ss}} \\
f_{22} &= (\phi_\pi - 1)c_{ss} \\
f_{32} &= \delta\kappa k_{ss}\mu_{ss} \\
f_{34} &= c_{ss}\delta\kappa k_{ss} \\
f_{42} &= -\delta\mu_{ss}^2 - \frac{\alpha l_{ss}^{1+\xi}\vartheta(1+\kappa x_{ss}\mu_{ss})(1+\xi)}{(\alpha-1)^2 k_{ss} y_{ss}} \\
f_{43} &= \frac{\alpha l_{ss}^{1+\xi}\vartheta((1+\alpha)y_{ss} - (1+\xi)x_{ss})}{((-1+\alpha)^2 k_{ss}^2 y_{ss}} \\
f_{44} &= \rho - \frac{\alpha\delta\kappa\vartheta(1+\xi)l_{ss}^{1+\xi}}{(\alpha-1)^2\mu_{ss}y_{ss}}.
\end{aligned}
$$

For the sake of clarity, I abstain from a meticulous mathematical characterization and instead offer a more intuitive approach. The model with capital adjustment costs consists of one state and three jump variables. Thus, model determinacy requires one negative and three positive eigenvalues. To evaluate the effect of capital adjustment costs, consider the limiting case (no capital adjustment costs) by letting $\kappa \to \infty$. For this purpose, turn to the Jacobian matrix, J_4, and assume passive monetary policy, $\phi_\pi < 1$. Then, with κ approaching infinity, one eigenvalue, ν_4, converges towards negative infinity, while the remaining three eigenvalues ν_1, ν_2, ν_3, converge towards the ones of the (determinate) Jacobian of the simple NK model with capital. Thus, one obtains one negative and two positive eigenvalues.

$$\begin{bmatrix} \boldsymbol{\nu}_{1,\text{NK-AC}} \\ \boldsymbol{\nu}_{2,\text{NK-AC}} \\ \boldsymbol{\nu}_{3,\text{NK-AC}} \\ \boldsymbol{\nu}_{4,\text{NK-AC}} \end{bmatrix} \quad \text{for} \quad \kappa \to \infty \quad \begin{bmatrix} \boldsymbol{\nu}_{1,\text{NK-AC}} \to \boldsymbol{\nu}_{1,\text{NK-Capital}} \\ \boldsymbol{\nu}_{2,\text{NK-AC}} \to \boldsymbol{\nu}_{2,\text{NK-Capital}} \\ \boldsymbol{\nu}_{3,\text{NK-AC}} \to \boldsymbol{\nu}_{3,\text{NK-Capital}} \\ \boldsymbol{\nu}_{4,\text{NK-AC}} \to -\boldsymbol{\infty} \end{bmatrix} \quad \text{thus} \quad \begin{bmatrix} \boldsymbol{\nu}_1 < 0 \\ \boldsymbol{\nu}_2 > 0 \\ \boldsymbol{\nu}_3 > 0 \\ \boldsymbol{\nu}_4 < 0 \end{bmatrix}.$$

To render the model determinate, one negative eigenvalues has to flip its sign. For this to be case, the model requires active monetary and passive fiscal policy. In this case, there is exactly one negative eigenvalue corresponding to the single state variable, k_t.

I introduce the fiscal policy block in the simple NK model with capital adjustment costs (A.3) and obtain a system with five differential equations. The corresponding Jacobian matrix reads

$$J_4 \quad = \quad \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} & 0 \\ f_{21} & 0 & 0 & 0 & 0 \\ 0 & f_{32} & 0 & f_{34} & 0 \\ 0 & f_{42} & f_{43} & f_{44} & 0 \\ f_{51} & 0 & 0 & 0 & f_{55} \end{bmatrix} = \begin{bmatrix} 0.051 & -0.145 & 0.014 & -0.002 & 0 \\ -0.321 & 0 & 0 & 0 & 0 \\ 0 & 0.068 & 0 & 0.0542 & 0 \\ 0 & -0.289 & 0.088 & 0.041 & 0 \\ -0.389 & 0 & 0 & 0 & 0.051 \end{bmatrix}$$

where

$$\begin{aligned} f_{51} &= v_{ss}(\phi_\pi - 1) \\ f_{55} &= (\rho - \tau_a). \end{aligned}$$

Note that all but one entry in the last column of the Jacobian are equal to zero. Thus, it directly follows that there is one positive eigenvalue, which is equal to $f_{55} = \rho$ (recall I assume $\tau_a \equiv 0$). The remaining variables are identical to the simple 4 equation NK model with capital adjustment costs. As argued above, in case of passive monetary policy, $\phi_\pi < 1$, there are two negative and two positive eigenvalues, which one obtains from the 4×4 matrix of the non-fiscal part of the model. Hence, there are, in total, three positive and two negative eigenvalues, corresponding to two states and three jump variables. Thus, model determinacy requires passive monetary- and active fiscal policy.

3.A.3 Great East Japanese Earthquake, Additional Figure

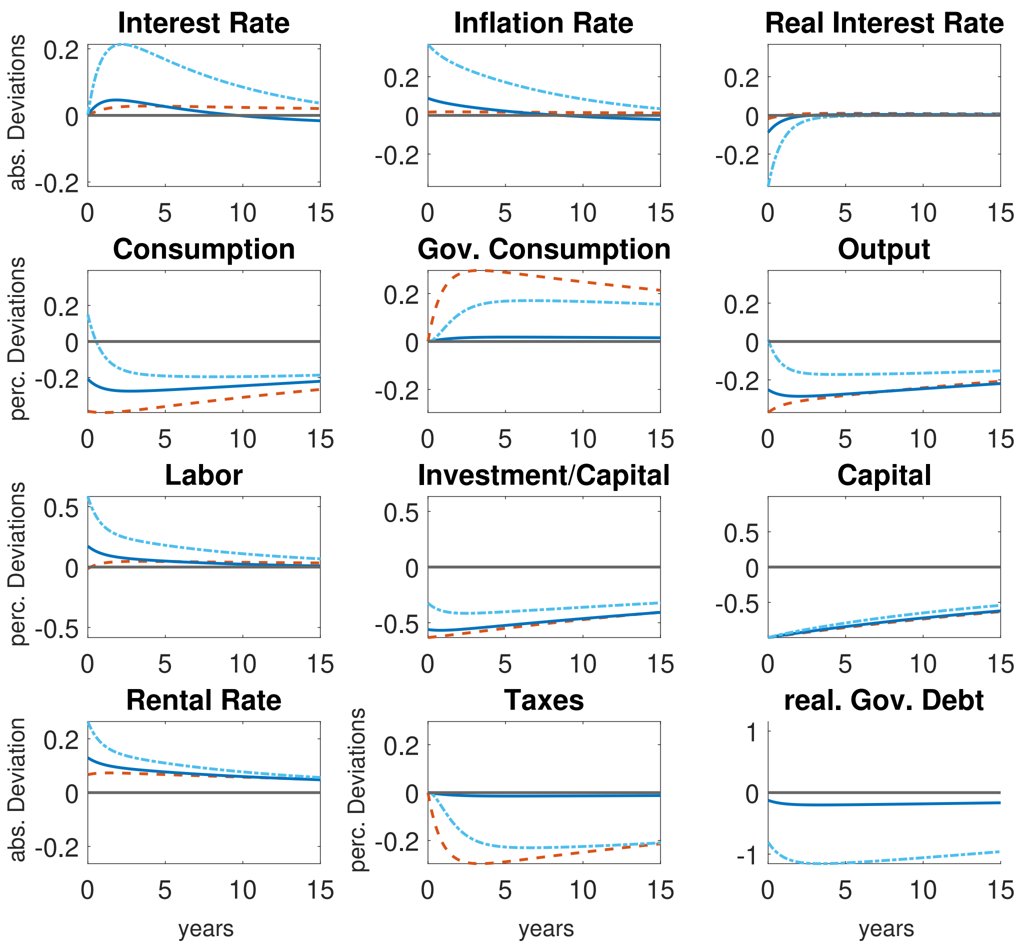

Figure 3.A.1: IRFs: Negative shock to capital by 1 percent (capital destruction). Parametrization 2. NK-AC model (dashed red lines). NK-AC-FTPL model (solid blue lines) and NK-AC-FTPL model when using Parametrization 2 but the fiscal policy parameters from the baseline parametrization.

3.A.4 Model Derivations

Derivation of the simple NK Model (with and without FTPL)

The starting point in the derivation of the simple NK model is the same one described in Section 3.2.1. As shown in Section 3.2.1, the simple NK framework is a limiting case of the NK-AC-FTPL model. Households maximize

$$\int_0^\infty e^{-\rho t}\left[\log(c_t) - \vartheta \frac{l_t^{1+\xi}}{1+\xi}\right] \mathrm{d}t.$$

The HJB of the simple NK model reads

$$
\begin{aligned}
\rho V(a_t, i_t, g_t) &= \max_{(c_t, l_t)} \left(\log(c_t) - \vartheta \frac{l_t^{1+\xi}}{1+\xi} \right) \\
&+ V_a \left(a_t(i_t - \pi_t) + w_t l_t + F_t - c_t - \Theta_t(\pi_t)/p_t - T_t \right) \\
&+ V_i \left(\theta \left(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_{ss}) \right) \right) \\
&+ V_g \left(\rho_g(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_{ss})) \right)
\end{aligned}
$$

where the value function, V, is a function of the three state variables a_t, i_t and g_t, with corresponding partial derivatives V_a, V_i and V_g. One obtains the first-order conditions as

$$V_a = 1/c_t \tag{A.3}$$

and

$$V_a w_t = \vartheta l_t^{\xi}. \tag{A.4}$$

Define

$$\lambda_t \equiv V_a = 1/c_t.$$

Combine equations (A.3) and (A.3), to obtain

$$w_t = \vartheta l_t^{\xi} c_t. \tag{A.5}$$

Since labor is the single input factor in the production of output, y_t, marginal costs equal wages

$$mc_t = w_t. \tag{A.6}$$

To obtain the consumption Euler equation, differentiate equation (A.3). Hence,

$$\mathrm{d}\lambda_t/\lambda_t = -\mathrm{d}c_t/c_t. \tag{A.7}$$

Taking the derivative of the HJB w.r.t. a_t yields

$$\rho V_a = V_a(i_t - \pi_t) + V_{aa}\mathrm{d}a_t + V_{ai}\mathrm{d}i_t + V_{ag}\mathrm{d}g_t.$$

Itô calculus implies

$$\mathrm{d}V_a = V_{aa}\mathrm{d}a_t + V_{ai}\mathrm{d}i_t + V_{ag}\mathrm{d}g_t.$$

Hence,

$$\mathrm{d}\lambda_t = (\rho - i_t - \pi_t)\lambda_t \mathrm{d}t.$$

Together with equation (A.7) one obtains the consumption Euler equation as

$$\mathrm{d}c_t = (i_t - \pi_t - \rho)c_t \mathrm{d}t.$$

The output identity reads

$$y_t = c_t + g_t + \gamma\pi_t^2/2 = l_t,$$

which one can combine with equations (A.5) and (A.6) to obtain

$$mc_t = \vartheta(c_t + g_t + \gamma\pi_t^2/2)^\xi c_t.$$

As in Kaplan et al. (2018) the Phillips curve reads

$$\mathrm{d}\pi_t = \left(\frac{\mathrm{d}c_t}{c_t} - \frac{\mathrm{d}y_t}{y_t}\right)\pi_t + \left(\rho\pi_t - \frac{\varepsilon-1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}mc_t - 1\right)\right)\mathrm{d}t$$

Finally, putting everything together, one obtains the simple NK model, augmented with the fiscal policy block of Section 3.2.2 as

$$\begin{aligned}
\mathrm{d}\pi_t &= \left(\frac{\mathrm{d}c_t}{c_t} - \frac{\mathrm{d}y_t}{y_t}\right)\pi_t + \left(\rho\pi_t - \frac{\varepsilon-1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}mc_t - 1\right)\right)\mathrm{d}t \\
\mathrm{d}c_t &= c_t\left(i_t - \pi_t - \rho\right)\mathrm{d}t \\
\mathrm{d}i_t &= \theta\left(\phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_y\left(y_t/y_{ss} - 1\right) - \left(i_t - i_{ss}\right)\right)\mathrm{d}t \\
\mathrm{d}g_t &= \rho_g\left(\varphi_y\left(y_t/y_{ss} - 1\right) - \left(g_t - g_t^*\right)\right)\mathrm{d}t \\
\mathrm{d}T_t &= \rho_\tau\left(\tau_y\left(y_t/y_{ss} - 1\right) - \left(T_t - T_t^*\right)\right)\mathrm{d}t \\
\mathrm{d}a_t &= \left(\left(i_t - \pi_t\right)a_t - s_t\right)\mathrm{d}t \\
\mathrm{d}p_t^b &= \left(i_t p_t^b - \chi - \delta_B\left(1 - p_t^b\right)\right)\mathrm{d}t
\end{aligned}$$

where

$$s_t = T_t - g_t$$

Starting from the Phillips curve one can successively derive the steady states of the model

$$mc_{ss} = \frac{\rho\pi_t^*\gamma}{\varepsilon-1} + \frac{\varepsilon-1}{\varepsilon},$$

$$w_{ss} = mc_{ss},$$

$$mc_{ss} = \vartheta l_{ss}^\xi c_{ss},$$

$$l_{ss} = y_{ss},$$

$$y_{ss} = c_{ss} + g_{ss} + \tfrac{\gamma}{2}(\pi_t^*)^2 y_{ss},$$

$$g_{ss} = s_g y_{ss},$$

$$c_{ss} = y_{ss}(1 - s_g - \tfrac{\gamma}{2}(\pi_t^*)^2),$$

$$c_{ss} = (mc_{ss}/\vartheta)^{1/(1+\xi)} \left(\frac{1}{1 - s_g - \frac{\gamma}{2}(\pi_t^*)^2}\right)^{-\xi/(1+\xi)}.$$

s_t^* is an exogenous steady state value for primary surpluses. Hence,

$$T_{ss} = s_t^* + g_{ss}$$

$$p_{ss}^b = (\chi + \delta^b)/(i_{ss} + \delta^b)$$

$$a_{ss} = s_t^*/\rho$$

Derivation of the NK Model with Capital (with and without FTPL)

The starting point in the derivation of the simple NK model with capital is the same one described in Section 3.2.1. As highlighted in Section 3.2.1, the simple NK framework with capital is a limiting case of the NK-AC-FTPL model. To avoid repetitions of common equations (e.g. consumption Euler equation), I refer the reader to Section 3.A.4.

Households maximize

$$\int_0^\infty e^{-\rho t}\left[\log(c_t) - \vartheta\frac{l_t^{1+\xi}}{1+\xi}\right] \mathrm{d}t.$$

The HJB of the simple NK model with capital reads

$$\begin{aligned}
\rho V(a_t, i_t, g_t, k_t) \quad = \quad & \max_{(c_t, l_t, x_t)} \left(\log(c_t) - \vartheta\frac{l_t^{1+\xi}}{1+\xi}\right) \\
& + V_a\left(a_t(i_t - \pi_t) + w_t l_t + r_t^k k_t + F_t - c_t - \Theta_t(\pi_t)/p_t - T_t\right) \\
& + V_i\left(\theta\left(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_{ss})\right)\right) \\
& + V_g\left(\rho_g(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_{ss}))\right) \\
& + V_k\left(x_t - \delta k_t\right)
\end{aligned}$$

where the value function, V, is a function of the four state variables a_t, i_t, g_t and k_t, with corresponding partial derivatives V_a, V_i, V_g and V_k. The FOCs in the NK model with capital are

$$\lambda_t = 1/c_t, \tag{A.8}$$

$$V_a w_t = \vartheta l_t^\xi, \tag{A.9}$$

and

$$V_a = V_k. \tag{A.10}$$

Combine equations (A.8) and (A.9) to obtain

$$w_t = \vartheta l_t^{\xi} c_t. \tag{A.11}$$

Define $\lambda_t \equiv V_a$ and $\mu_t \equiv V_k$. Analogously to Section 3.A.4 one obtains

$$\mathrm{d}\lambda_t = (\rho - i_t + \pi_t)\lambda_t \mathrm{d}t \tag{A.12}$$

and

$$\mathrm{d}c_t = (i_t - \pi_t - \rho)c_t \mathrm{d}t.$$

Taking the derivative of the HJB w.r.t. k_t yields

$$\rho V_k = r_t^k V_a + V_{ka}\mathrm{d}a_t - \delta V_k + V_{kk}\mathrm{d}k_t + V_{ki}\mathrm{d}i_t + V_{kg}\mathrm{d}g_t. \tag{A.13}$$

Itô calculus implies that V_k follows

$$\mathrm{d}V_k = V_{ka}\mathrm{d}a_t + V_{kk}\mathrm{d}k_t + V_{ki}\mathrm{d}i_t + V_{kg}\mathrm{d}g_t, \tag{A.14}$$

which after substituting equations (A.10) and (A.13) yields

$$\mathrm{d}\mu_t = (\rho + \delta - r_t^k)\mu_t \mathrm{d}t. \tag{A.15}$$

Combine this expression with equations (A.10) and (A.12) to obtain

$$r_t^k = i_t - \pi_t + \delta. \tag{A.16}$$

The output identity reads

$$y_t = k_t^{\alpha} l_t^{1-\alpha} = c_t + g_t + x_t + \Theta(\pi_t)/p_t = \frac{c_t + g_t + x_t}{1 - \gamma\pi_t^2/2}.$$

Let mpk_t and mpl_t denote the marginal product of capital and labor, respectively. Then the optimization problem of the firms implies

$$r_t^k = mpk_t\, mc_t = \alpha mc_t y_t / k_t \quad \text{and} \quad w_t = mpl_t\, mc_t = (1-\alpha) mc_t y_t / l_t,$$

Substituting and equating terms, one obtains

$$w_t mpk_t = r_t^k mpl_t,$$

and

$$k_t / l_t = \alpha/(1-\alpha) w_t / r_t^k.$$

Substituting the latter expression back into equation (3.21) and rearranging terms yields

$$mc_t = (1/(1-\alpha))^{1-\alpha}(1/\alpha)^{\alpha}w_t^{1-\alpha}(r_t^k)^{\alpha}.$$

Combine equations (A.11), (3.21) and (A.17) to arrive at

$$r_t^k = \frac{\alpha\vartheta c_t}{(1-\alpha)k_t}l_t^{1+\xi}. \tag{A.17}$$

Then, rearrange equation (A.17) to obtain

$$l_t = ((1-\alpha)/(\alpha\vartheta))^{(1/(1+\xi))}(k_t/c_t)^{1/(1+\xi)}(r_t^k)^{(1/(1+\xi))}.$$

One can substitute r_t^k from equation (A.16) to express labor as

$$l_t = ((1-\alpha)/(\alpha\vartheta))^{(1/(1+\xi))}(k_t/c_t)^{1/(1+\xi)}(i_t-\pi_t+\delta)^{(1/(1+\xi))}.$$

Rearrange the output identity to obtain investments, x_t as

$$x_t = y_t - c_t - g_t - \gamma\pi_t^2 y_t/2.$$

Finally, one obtains the NK model with capital along the lines of Dupor (2001) and fiscal policy block as

$$\begin{aligned}
\mathrm{d}\pi_t &= \left(\frac{\mathrm{d}c_t}{c_t} - \frac{\mathrm{d}y_t}{y_t}\right)\pi_t + \left(\rho\pi_t - \frac{\varepsilon-1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}mc_t - 1\right)\right)\mathrm{d}t \\
\mathrm{d}c_t &= c_t\left(i_t - \pi_t - \rho\right)\mathrm{d}t \\
\mathrm{d}i_t &= \theta\left(\phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_y\left(y_t/y_{ss} - 1\right) - \left(i_t - i_{ss}\right)\right)\mathrm{d}t \\
\mathrm{d}g_t &= \rho_g\left(\varphi_y\left(y_t/y_{ss} - 1\right) - \left(g_t - g_t^*\right)\right)\mathrm{d}t \\
\mathrm{d}k_t &= \left(x_t - \delta k_t\right)\mathrm{d}t \\
\mathrm{d}T_t &= \rho_\tau\left(\tau_y\left(y_t/y_{ss} - 1\right) - \left(T_t - T_t^*\right)\right)\mathrm{d}t \\
\mathrm{d}a_t &= \left(\left(i_t - \pi_t\right)a_t - s_t\right)\mathrm{d}t \\
\mathrm{d}p_t^b &= \left(i_t p_t^b - \chi - \delta_B\left(1 - p_t^b\right)\right)\mathrm{d}t.
\end{aligned}$$

To shut of FTPL, simply drop the fiscal policy block variables. To compute the steady states, start again from the Philips Curve and successively compute

$$\begin{aligned}
0 &= \rho\pi_t^* - \frac{\varepsilon-1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}mc_{ss} - 1\right) \\
mc_{ss} &= \frac{\rho\pi_t^*\gamma}{\varepsilon-1} + \frac{\varepsilon-1}{\varepsilon}
\end{aligned}$$

so that

$$mc_{ss} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right) w_{ss}^{1-\alpha}(r_{ss}^k)^{\alpha}.$$

Hence,

$$w_{ss} = \left(mc_{ss}\left(\frac{1}{1-\alpha}\right)^{-(1-\alpha)}\left(\frac{1}{\alpha}\right)^{-\alpha}(r_{ss}^k)^{-\alpha}\right)^{\frac{1}{1-\alpha}},$$

and

$$y_{ss} = c_{ss} + x_{ss} + s_g y_{ss} + \frac{\gamma}{2}(\pi_t^*)^2 y_{ss}$$

where $g_{ss} = s_g y_{ss}$. Rearranging terms yields

$$\begin{aligned} c_{ss} + x_{ss} &= y_{ss}(1 - s_g - \tfrac{\gamma}{2}(\pi_t^*)^2) \\ y_{ss} &= \frac{c_{ss} + x_{ss}}{(1 - s_g - \frac{\gamma}{2}(\pi_t^*)^2)}. \end{aligned}$$

Alternatively, one can use

$$y_{ss} = k_{ss}^{\alpha} l_{ss}^{1-\alpha},$$

together with

$$x_{ss} = \delta k_{ss},$$

in order to rewrite the steady state value of consumption as

$$c_{ss} = y_{ss}((1 - s_g) - \tfrac{\gamma}{2}(\pi_t^*)^2) - \delta k_{ss}.$$

Optimal factor rewards imply

$$\begin{aligned} r_{ss}^k &= \frac{\alpha y_{ss} mc_{ss}}{k_{ss}} \\ y_{ss} &= \frac{r_{ss}^k}{\alpha mc_{ss}} k_{ss}. \end{aligned}$$

Consequently,

$$c_{ss} = \left(\frac{r_{ss}^k}{\alpha mc_{ss}}((1 - s_g) - \tfrac{\gamma}{2}(\pi_t^*)^2) - \delta\right) k_{ss}.$$

The FOCs of the HJB imply

$$w_{ss} = \vartheta l_{ss}^{\xi}\left(\frac{r_{ss}^k}{\alpha mc_{ss}}(1 - s_g - \tfrac{\gamma}{2}(\pi_t^*)^2) - \delta\right) k_{ss}.$$

Note that $(y_{ss}k_{ss}^{-\alpha})^{\frac{1}{1-\alpha}}$. Thus,

$$l_{ss} = \left(\left(\frac{r_{ss}^k}{\alpha mc_{ss}}k_{ss}\right)k_{ss}^{-\alpha}\right)^{\frac{1}{1-\alpha}}$$

which after some algebra yields

$$l_{ss}^{\xi} = \left(\left(\frac{r_{ss}^k}{\alpha mc_{ss}}\right)\right)^{\frac{\xi}{1-\alpha}} k_{ss}^{\xi}.$$

A combination with the above steady state expressions yields for wages

$$w_{ss} = \vartheta\left(\left(\frac{r_{ss}^k}{\alpha mc_{ss}}\right)\right)^{\frac{\xi}{1-\alpha}}\left(\frac{r_{ss}^k}{\alpha mc_{ss}}((1-s_g)-\tfrac{\gamma}{2}(\pi_t^*)^2)-\delta\right)k_{ss}^{1+\xi}.$$

Thus, one obtains the equilibrium value of the capital stock as

$$k_{ss} = \left(\vartheta\left(\left(\frac{r_{ss}^k}{\alpha mc_{ss}}\right)\right)^{\frac{\xi}{1-\alpha}}\left(\frac{r_{ss}^k}{\alpha mc_{ss}}((1-s_g)-\tfrac{\gamma}{2}(\pi_t^*)^2)-\delta\right)/w_{ss}\right)^{-\frac{1}{1+\xi}}.$$

Finally,

$$l_{ss} = \left(\left(\frac{r_{ss}^k}{\alpha mc_{ss}}\right)\right)^{\frac{1}{1-\alpha}} k_{ss}.$$

Derivation of the NK Model with Capital Adjustment Costs (with and without FTPL)

In this section, I derive the NK-AC-FTPL model. The starting point is the one described in Section 3.2.1. To avoid repetitions of common equations (e.g. consumption Euler equation), I refer the reader to sections 3.A.4 and 3.A.4.

Households maximize

$$\int_0^{\infty} e^{-\rho t}\left[\log(c_t) - \vartheta\frac{l_t^{1+\xi}}{1+\xi}\right]\mathrm{d}t.$$

In contrast to the main text, I directly substitute $d_t = x_t - r_t^k k_t$. Then, the HJB reads

$$\begin{aligned}
\rho V(a_t, i_t, g_t, k_t) \quad = \quad & \max_{(c_t, l_t)} \left(\log(c_t) - \vartheta \frac{l_t^{1+\xi}}{1+\xi} \right) \\
& + V_a \left(a_t(i_t - \pi_t) + w_t l_t + r_t^k k_t + F_t - c_t - \Theta_t(\pi_t)/p_t - T_t \right) \\
& + V_i \left(\theta \left(\phi_\pi (\pi_t - \pi_t^*) + \phi_y (y_t/y_{ss} - 1) - (i_t - i_{ss}) \right) \right) \\
& + V_g \left(\rho_g (\varphi_y (y_t/y_{ss} - 1) + \varphi_a (a_t - a_{ss}) - (g_t - g_{ss})) \right) \\
& + V_k \left(\left(\left(\frac{\delta^{1/\kappa}}{1 - 1/\kappa} \left(\frac{x_t}{k_t} \right)^{1-1/\kappa} + \frac{\delta}{1-\kappa} \right) - \delta \right) k_t \mathrm{d}t \right)
\end{aligned}$$

where the value function, V, is a function of the four state variables a_t, i_t, g_t and k_t, with corresponding partial derivatives V_a, V_i, V_g and V_k. The FOCs of the HJB w.r.t. consumption and labor are

$$\lambda_t = 1/c_t, \tag{A.18}$$

$$V_a w_t = \vartheta l_t^\xi. \tag{A.19}$$

The derivative of the HJB w.r.t x_t reads

$$\begin{aligned}
V_k \quad & = \quad V_a \delta^{-1/\kappa} \left(\frac{x_t}{k_t} \right)^{1/\kappa} \\
& = \quad V_a \delta^{-1/\kappa} \left(\frac{k_t^\alpha l_t^{1-\alpha} (1 - \gamma \pi_t^2/2) - c_t - g_t}{k_t} \right)^{1/\kappa},
\end{aligned}$$

which after some algebra yields

$$l_t = \left(\frac{k_t^{1-\alpha} \left(V_k/V_a \right)^\kappa \delta + k_t^{-\alpha} (c_t + g_t)}{1 - \gamma \pi_t^2/2} \right)^{\frac{1}{1-\alpha}},$$

and

$$x_t = \left(\frac{V_k}{V_a} \right)^\kappa \delta k_t = k_t^\alpha l_t^{1-\alpha} (1 - \gamma \pi_t^2/2) - c_t - g_t.$$

When taking the derivative of the HJB with respect to k_t, one obtains

$$\rho V_k = \left(\frac{\delta^{1/\kappa} (x_t/k_t)^{(\kappa-1)/\kappa} - \kappa \delta}{\kappa - 1} \right) V_k + r_t^k V_a + \mathrm{d}k_t V_{kk} + \mathrm{d}a_t V_{ak} + V_{ik} \mathrm{d}i_t + V_{gk} \mathrm{d}g_t.$$

The co-state has to obey

$$\mathrm{d}V_k = \mathrm{d}k_t V_{kk} + \mathrm{d}a_t V_{ak} + V_{ik} \mathrm{d}i_t + V_{gk} \mathrm{d}g_t.$$

By combining the last two expressions and by rearranging terms, one obtains

$$\begin{aligned} \mathrm{d}V_k &= (\rho+\delta)V_k - \left(\frac{\delta^{1/\kappa}\left(\frac{x_t}{k_t}\right)^{(\kappa-1)/\kappa} - \delta}{\kappa - 1} \right) V_k - r_t^k V_a \\ &= (\rho+\delta)V_k - \left(\frac{\delta(V_k c_t)^{\kappa-1} - \delta}{\kappa-1} \right) V_k - r_t^k / c_t. \end{aligned}$$

Firm optimization pins down the rental rate of capital as

$$r_t^k = \frac{\alpha \vartheta c_t}{(1-\alpha)k_t} l_t^{1+\xi}$$

where one can substitute labor to obtain

$$r_t^k = \frac{\alpha\vartheta c_t}{(1-\alpha)k_t}\left(k_t^{1-\alpha}\left(\frac{V_k}{V_a}\right)^{\kappa}\delta + k_t^{-\alpha}\left(c_t + g_t + \Theta_t(\pi_t)/p_t\right)\right)^{\frac{1+\xi}{1-\alpha}}.$$

Finally,

$$\begin{aligned} \mathrm{d}V_k &= (\rho+\delta)V_k - \left(\frac{\delta(V_k c_t)^{\kappa-1} - \delta}{\kappa-1} \right) V_k \\ &\quad - \frac{\alpha\vartheta}{(1-\alpha)k_t}\left(k_t^{1-\alpha}\left(\frac{V_k}{V_a}\right)^{\kappa}\delta + k_t^{-\alpha}\left(c_t + g_t + \Theta(\pi_t)/p_t\right)\right)^{\frac{1+\xi}{1-\alpha}}. \end{aligned}$$

Wages can be expressed as

$$w_t = \vartheta l_t^{\xi} c_t.$$

As in the simple NK model with capital, marginal costs read

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w_t^{1-\alpha}(r_t^k)^{\alpha}.$$

Putting everything together, one obtains the NK-AC-FTPL model as stated in Section (3.2.3). Due to the specification of capital adjustment costs, the steady states of the NK models with adjustment costs, coincide with the ones of the corresponding NK models with capital and no adjustment costs (see Section 3.A.4).

General Appendix

A.1 Abstract

Chapter 1

The first essay of this doctoral thesis with the title *Structural Estimation of Dynamic Macroeconomic Models using Financial Data* evaluates the benefits, drawbacks and limitations of using financial data as substitute or complement for macro data in the estimation of macroeconomic models. To that end, the essay introduces a generalized estimation framework that is applicable to a whole class of macroeconomic models. There are 4 necessary steps that consists of (I) solving the model, (II) deriving the stochastic discount factor, (III) pricing financial assets and (IV) setting up the estimation equations. The required steps are illustrated by a small-scale macroeconomic model with analytical solutions. The essay proposes the use of GMM and MEF estimation techniques. Estimating the structural parameters with different combinations of macro and financial data, the essay evaluates and compares the different settings empirically and in a simulation study. The essay argues that there is a lack of suitable *real world analogues* for certain derived assets prices. Thus, corresponding systems of estimation equations are only analyzed in the simulation study for illustrative purpose. In particular, the essay utilizes data on treasury bonds, real personal consumption expenditures, industrial production and the S&P500 stock index in the empirical estimation.

This essay offers new insights into the use of financial data in the estimation of macroeconomic models. To that end, the essay contributes a generalized formulation to introduce financial estimation equations, which are consistent with the macroeconomic dynamics of the model. This approach is applicable to a whole class of DSGE models. By choosing a simple macroeconomic model with close-form solutions as an illustrative example, the essay discusses benefits, drawbacks and limitations of utilizing asset prices in the structural parameter estimation. By doing so, the essay contributes new insights into the informational content of financial data on macroeconomic aggregates, both empirically and theoretically. A crucial result is an increased accuracy and improved identification

when substituting or complementing macro with financial data. In the related literature, the estimate of the parameter of the speed of mean reversion of the Vasicek interest rate specification is known to be biased. The essay contributes to this literature by showing that certain combinations of macro and finance estimation equations basically remove the model inherent parameter bias. Thus, no further bias correction methods are required. Finally, the essay highlights that exploiting financial data in structural parameter estimation can be advantageous even in case of small-scale macroeconomic models without explicit financial sector.

Chapter 2

The second essay of this doctoral thesis with the title *FTPL and the Maturity Structure of Government Debt in the New-Keynesian Model* evaluates the impact of the maturity structure of government bonds on the inflation rate and other macroeconomic aggregates. For this purpose, the essay uses the continuous-time NK-FTPL framework (see e.g. Sims (2011), Cochrane (2018) or Cochrane (2022b). In particular, the price level is determined by explicit interactions of bond prices with monetary and fiscal policy. These interactions follow from the assumption that the real value of debt has to be equal to the present value of future surpluses discounted by the real interest rate. The bond maturity plays a vital role because it determines the magnitude of endogenous changes in bond prices in response to exogenous shocks. These price changes induce an immediate revaluation of existing debt, and consequently establishes a direct link to the path of future inflation rates. While considering different bond maturities, this essay emphasizes the effects of monetary- and fiscal policy shocks on the term structure of the nominal interest rate and model-implied inflation expectations. The theoretical evaluation of the model is followed by an analysis of the US Coronavirus Aid, Relief and Economic Security (CARES) Act, which is a unprecedentedly large fiscal stimulus from 2020.

The modeling framework in this essay is similar to the discrete-time NK-FTPL model in Leeper and Leith (2016). Thus, it directly replicates their maturity related results within the continuous-time NK-FTPL model. This chapter emphasizes that a continuous-time formulation has important benefits regarding the interpretation of FTPL effects. However, these are not the main contributions of this essay. The focus is on the implementation of fiscal Taylor rules along the lines of Kliem and Kriwoluzky (2014) and Kliem et al. (2016), which divides primary surpluses into taxes and government consumption and allows for a quantifiable evaluation of the CARES Act. Furthermore, these rules imply s-shaped surplus dynamics without the need for further assumptions or ingredients, such as latent state variables as in Cochrane (2022b). While the analysis of policy shocks in the FTPL literature mostly focuses on transitory dynamics, this essay explicitly evaluates permanent changes. Another crucial contribution of the essay is an analysis of the impact of the bond maturity on the terms structure of the nominal interest rate and model-implied inflation expectation, which fills a gap in the corresponding literature.

Chapter 3

The third essay of this doctoral thesis is named *The Fiscal Theory of the Price Level in New Keynesian Models with Capital.* It starts from the model of the preceding chapter and extends it by capital and capital adjustment costs. Simple continuous-time NK model with capital (see Dupor (2001)) predict that contractionary monetary policy shocks are expansionary and increase both output and inflation. This however, is at odds with conventional economic reasoning, which predicts a decrease in output and inflation. In order to obtain a FTPL framework along the lines of Sims (2011), this essay additionally introduces capital adjustment costs. An elaborate description of model dynamics and features is followed by an evaluation of two economic puzzles. Finally, the essay evaluates necessary ingredients to obtain a negative correlation between the nominal interest- and the inflation rate in the FTPL framework.

Continuous-Time NK models with capital and FTPL attain, at best, marginal coverage in the existing literature. This essay fills the gap and highlights the versatility of the NK-FTPL model with capital adjustment costs. By developing this novel framework, the essay contributes to the literature in various dimensions, such as new insights into determinacy issues, transmission channels or the role of parametrization. The derived model is then used to address two economic puzzles in the literature. The first one is the Crowding-In Consumption Puzzle, which refers to a mismatch of empirically observed and theoretically predicted responses of private consumption to changes in government expenditures in conventional NK models. The essay contributes a novel solution to this problem and highlights that the FTPL framework is able to predict either a crowding-in, a crowding-out or no initial change at all. At the same time, it predicts a consistent (and at least temporary) crowding-in of investment. The second puzzle are counterintuitive predictions of traditional NK models at the ZLB. In particular, the destruction of capital turns out to be highly expansionary and increases output in these models. The essay shows how FTPL solves this Puzzle and illustrate the result with an extensive and novel analysis of the Great East Japan Earthquake of 2011 (*Tōhoku Earthquake*). The analysis and the solution of the two puzzles with an explicit NK-FTPL model is one of the main contributions of this chapter. Finally, FTPL models in the literature usually rely on long-term bonds in order to establish a negative correlation between the inflation and the nominal interest rate. This chapter of the doctoral thesis suggest a novel approach to obtain the desired correlation in the presence of short-term debt.

A.2 Zusammenfassung

Die Zusammenfassung beschreibt die Inhalte sowie die zentralen Ergebnisse der vorliegenden Dissertation.

Kapitel 1

Der erste Essay dieser Dissertation mit dem Namen "Structural Estimation of Dynamic Macroeconomic Models Using Financial Data" (dt. "Strukturelle Schätzung von dynamischen makroökonomischen Modellen unter der Verwendung von Finanzdaten") untersucht die Vor- und Nachteile sowie die Einschränkungen der Finanzdatenverwendung als Komplemente oder Substitute für makroökonomische Variablen in der strukturellen Schätzung makroökonomischer Modelle. Hierzu wird in dem Essay ein allgemeiner Rahmen zur Schätzung vorgestellt, welcher sich auf eine ganze Klasse von makroökonomischen Modellen anwenden lässt.

Das allgemeine Vorgehen besteht aus 4 Schritten. Diese sind (I) die Lösung des Modells, (II) die Herleitung des stochastischen Diskontfaktors, (III) die Bepreisung von Vermögenswerten sowie (IV) die Herleitung der benötigten Schätzgleichungen. Zur Veranschaulichung wird hierzu ein einfaches makroökonomisches Modell mit analytischer Lösung herangezogen. Der Essay verwendet zur strukturellen Parameterschätzung sowohl die Generalisierte Momentenmethode (GMM) als auch Martingale Schätzfunktionen (MEF). Über die Schätzung der strukturellen Parameter mittels unterschiedlicher Kombinationen von makroökonomischen und finanziellen Variablen finden theoretische (mittels Simulationsstudie) sowie empirische Evaluationen und Vergleiche der unterschiedlichen Schätzsysteme statt. Der Essay legt dar, dass es für einige der hergeleiteten Finanzvariablen keine überzeugenden empirischen Gegenstücke gibt. Nichtsdestotrotz werden die dazugehörigen Schätzsysteme zur Veranschaulichung in der Simulationsstudie analysiert. In der empirischen Schätzung werden US Daten von Treasury Bonds, von realen persönlichen Konsumausgaben, von der industriellen Produktion sowie des S&P500 Aktienindex verwendet.

Dieser Essay bietet neue Einblicke in die Verwendung von Finanzdaten in der Schätzung makroökonomischer Modelle. Ein wichtiger Beitrag ist die allgemeine Formulierung der Vorgehensweise, welche die Aufnahme von Vermögenswerten ermöglicht. Letztere sind über den stochastischen Diskontfaktor konsistent mit der makroökonomischen Dynamik bepreist. Ein entscheidender Vorteil, des im Essay entwickelten allgemeinen Ansatzes, ist die Anwendbarkeit auf eine ganze Klasse von DSGE Modellen. Die beispielhafte Analyse eines analytisch lösbaren makroökonomischen Modells zeigt die Vor- und Nachteile sowie die Einschränkungen der Verwendung von Finanzdaten auf. Dadurch bietet der Essay neue Einblicke in den theoretischen und empirischen Informationsgehalt von Finanzdaten über den Zustand makroökonomischer Größen. Ein weiterer zentraler Beitrag des Essays ist das Resultat, dass Finanzdaten sowohl die Genauigkeit als auch die Identifikation von geschätzten Modellparametern erhöhen können. Ein in der zugehörigen Literatur etabliertes Resultat ist ein verzehrter Schätzer des Parameters, der die Geschwindigkeit der Mittelwertrückkehr in der Vasicek Zinssatzspezifizierung determiniert. Der Essay zeigt, dass diese Verzerrung in einigen Schätzsystemen mit Finanzdaten nahezu vollständig korrigiert wird. Somit sind keine zusätzlichen Korrekturverfahren nötig. Zudem zeigt der Essay, dass selbst in einfachen makroökonomischen Modellen, ohne explizit modelliertem Finanzsektor, die Aufnahme von Finanzdaten die Genauigkeit und Parameteridentifikation in strukturellen Schätzungen erhöhen kann.

Kapitel 2

Im zweiten Kapitel dieser Dissertation mit dem Titel "FTPL and the maturity structure of government debt in the New-Keynesian Model" (dt. "Die fiskalische Theorie des Preisniveaus (FTPL) und die Fälligkeitsstruktur von Staatsverschuldung im neukeynesianischen (NK) Modell") geht es um den Einfluss der Laufzeit von Staatsanleihen auf die Entwicklung der Inflationsrate und anderer makroökonomischer Größen. Zu diesem Zweck findet der zeitstetige NK-FTPL Modellrahmen (siehe z. B. Sims (2011), Cochrane (2018) oder Cochrane (2022b)) Verwendung. Das Preisniveau wird demnach durch ein enges und explizites Zusammenspiel von Anleihepreisen sowie geld- und fiskalpolitischen Entscheidungen determiniert. Diesem Zusammenspiel liegt die Annahme zugrunde, dass der reale Wert der Staatsverschuldung dem mit dem realen Zinssatz diskontierten Gegenwartswert aller zukünftigen Primärüberschüssen entsprechen muss. Hierbei spielt die Laufzeit von Anleihen eine entscheidende Rolle, da diese das Ausmaß der endogenen Preisanpassung der Anleihen in Folge exogener Shocks bestimmt. Da Preisanpassungen von Anleihen eine sofortige Neubewertung der realen Schulden zufolge haben, besteht ein direkter Einfluss auf die Entwicklung der zukünftigen Inflationsraten.

Ein besonderes Augenmerk liegt in diesem Essay auf den Effekten von geld- und fiskalpolitischen Schocks auf die Zinsstrukturkurve und den modellimplizierten Inflationserwartungen unter Berücksichtigung unterschiedlicher Laufzeiten von Anleihen. Im Anschluss an die theoretische Evaluation erfolgt eine Analyse des Coronavirus Aid, Relief and Economic Security (CARES) Act, bei dem es sich um ein von der Größe her präzedenzloses Konjunkturprogramm der USA im Jahr 2020 handelt.

Der Modellrahmen im zweiten Essay dieser Dissertation ist ähnlich zu dem zeitdiskreten NK-FTPL Modell in Leeper und Leith (2016). Somit werden deren Ergebnisse bezüglich der Effekte der Laufzeitstruktur von Staatsverschuldung direkt in dem zeitstetigen NK-FTPL Modell dieses Essays repliziert. Es wird gezeigt, dass die zeitstetige Formulierung entscheidende Vorteile bei der Interpretation von FTPL Effekten bietet. Dieses ist jedoch nicht der eigentliche Beitrag des zweiten Kapitels. Vielmehr wird der NK-FTPL Rahmen um fiskalische Taylor Regeln im Sinne von Kliem und Kriwoluzky (2014) sowie Kliem et al. (2016) erweitert, welche Primärüberschüsse in Steuern und Staatsausgaben aufschlüsseln und dadurch eine quantifizierbare Analyse des CARES Act ermöglichen. Zudem erlauben die fiskalischen Regeln eine S-förmige Dynamik von Primärüberschüssen, ohne dass weitere Anpassungen, wie beispielsweise die Einführung von zusätzlichen latenten Zustandsvariablen wie in Cochrane (2022b), vorgenommen werden müssen. Während sich die Untersuchung von Schocks in der FTPL Literatur weitestgehend auf temporärere Effekte konzentriert, werden in diesem Essay explizit permanente Änderungen analysiert. Ein weiterer wichtiger Beitrag dieses Kapitels ist die Untersuchung des Einflusses der Fälligkeit von Anleihen auf die Zinsstrukturkurve und den modellimpliziten Inflationserwartungen, welche ebenfalls eine existierende Lücke in der diesbezüglichen Literatur schließt.

Kapitel 3

Der dritte Essay dieser Dissertation mit dem Namen "The Fiscal Theory of the Price Level in New Keynesian Models with Capital" (dt. "Die Fiskalische Theorie des Preisniveaus in neukeynesianischen Modellen mit Kapital") greift das Modell des vorangegangenen Kapitels auf und erweitert dieses um Kapital und Kapitalanpassungskosten. Im einfachen zeitstetigen NK Modell mit Kapital (siehe Dupor (2001)) führen restriktive geldpolitische Schocks zu einem Anstieg von Inflation und Output. Dies widerspricht jedoch den konventionellen ökonomischen Grundannahmen, nach denen Output und Inflation sinken. Um ein typisches FTPL Modell im Sinne von Sims (2011) zu erhalten, werden in diesem Essay Kapitalanpassungskosten eingeführt. Es folgt eine ausführliche Beschreibung der Modelldynamiken und eine anschließende Analyse von zwei ökonomischen Puzzles. Abschließend wird der entwickelte Modellrahmen dazu verwendet, um die nötigen Bedingungen zu untersuchen, die einen negativen Zusammenhang zwischen Inflation und dem nominalen Zinssatz in FTPL Modellen herstellen.

In der bestehenden Literatur finden zeitstetige NK Modelle mit FTPL und Kapital bislang weitestgehend keine Beachtung. Dieser Essay füllt die entsprechende Lücke und unterstreicht die Vielseitigkeit des NK-FTPL Modells mit Kapitalanpassungskosten. Durch die Entwicklung dieses neuartigen Modellrahmens trägt der Essay in vielen unterschiedlichen Dimensionen zur Literatur bei. So werden beispielsweise neue Einblicke in die Bestimmbarkeit des Modells, die Transmissionskanäle und die Bedeutung der Parametrisierung gegeben. Das hergeleitete Modell wird anschließend dazu verwendet zwei ökonomische Puzzle in der Literatur zu lösen. Das erste Puzzle ist das "Crowding-In Consumption Puzzle", welches sich auf die Unstimmigkeiten in empirischen und theoretischen

Vorhersagen bezüglich des Effekts von Staatsausgabenerhöhungen auf privaten Konsum im einfachen neukeynesianischen Modellrahmen bezieht. Der Essay schlägt eine neuartige Lösung des Puzzles durch das hergeleitete Modell vor und unterstreicht dessen Fähigkeit, einen anfänglichen Crowding-In oder Crowding-Out Effekt vorherzusagen. Das Modell erlaubt zudem eine konsistente Vorhersage bezüglich der Dynamik von Investitionen, bei denen (wenigstens anfänglich) ein Cowding-In Effekt stattfindet. Das zweite Puzzle betrifft die kontraintuitive Vorhersage von vielen neukeynesianischen Modellen, nach denen die Zerstörung von Kapital am ZLB einen extrem starken Anstieg von Output zufolge hat. Der Essay zeigt, wie FTPL dieses Puzzle lösen kann und illustriert die Ergebnisse anhand einer extensiven und neuartigen Analyse der großen Erdbebenkatastrophe von 2011 in Japan (*Tōhoku-Erdbeben*). Einer der wichtigsten Beiträge dieses Essays zur relevanten Literatur ist daher die neuartige Analyse und Lösung dieser beiden Puzzles mittels eines expliziten NK-FTPL Modells. FTPL Modelle in der Literatur benötigen in der Regel langfristige Anleihen, um eine negative Korrelation von Inflation und des nominalen Zinssatzes zu erhalten. Dieser Essay der Dissertation stellt einen neuen Ansatz vor, mit dem die gewünschte Korrelation ebenfalls mit kurzfristigen Anleihen erzielt werden kann.

Eidesstattliche Versicherung

Ich, Max Ole Liemen, versichere an Eides statt, dass ich die Dissertation mit dem Titel:

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selbst und bei einer Zusammenarbeit mit anderen Wissenschaftlerinnen oder Wissenschaftlern gemäß den beigefügten Darlegungen nach § 6 Abs. 3 der Promotionsordnung der Fakultät Wirtschafts - und Sozialwissenschaften vom 18. Januar 2017 verfasst habe. Andere als die angegebenen Hilfsmittel habe ich nicht benutzt.

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Liste der aus der Dissertation hervorgegangenen Veröffentlichungen

Bislang wurde keiner der in dieser Dissertation enthaltenen Essays in einem Journal mit Peer-Review Verfahren veröffentlicht.