Essays on Time-based Decision Making and the Performance of Lot-sizing Heuristics

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Chapter 1

Synopsis

1. Introduction

The productive use of limited managerial time presents a success factor for companies, especially those in highly competitive sectors with low margins, such as the logistics industry. In order to make the best possible use of the time, the following key questions arise for many business decision makers: Should existing opportunities be harvested and money earned now or should time be invested now to cultivate future opportunities? Is it useful to invest time now to save time later as the popular time management literature recommends? How does the time invested in the past influences the decision to continue or cancel a project? Similar questions arise for the investment of money. Economically, time and money are interchangeable via the wage rate, e.g. 1 hour = 12 $. It could therefore be assumed that the research results on the investment behavior of money could be transferred to the investment behavior of time. However, time and money evoke different behaviors. This dissertation consists of three papers, of which the first two papers address the questions above. Paper 1 introduces two stylistic models and analyzes with computerized laboratory experiments *when* and *how much* time or money decision makers invest to earn time or money.

While paper 1 considers the investment of time in the present, paper 2 looks at the influence of time invested in the past on current decisions. Time invested in the past is irretrievable and should therefore be irrelevant to deciding whether a project should be continued or not. However, practice shows that managers tend to delay the termination of projects (Long et al. 2020), continue investing in companies even when prospects of success are diminishing (Guler 2007) or continue new products in the market longer than optimal (Simester and Zhang 2010); all leading to considerable costs (Long et al. 2020). The sunk cost project is continued because decision makers have “too much invested to quit” or “throw good money after bad” to pull the project into the profit zone.

In the classical sunk cost situation, a choice can be made between the sunk cost project, in which investments have already been made, and a superior alternative. In addition, paper 2 also considers the case where the sunk cost project is the superior project. The laboratory experiments are based on the model presented in paper 1, extended by exogenous past investments.

In contrast to the first two papers, the third paper does not examine individual behavior, but rather the planning of lot sizes and thus represents an important area for minimizing production costs. Myopic lot-sizing heuristics are implemented in many modern Enterprise Resource Planning (ERP) systems and provide fairly good results if period demands are well above zero. However, modern ERP systems, represent demand daily rather than weekly, which leads to demand types where periods of no demand (sporadic demand) or small (close-to-zero) demands occur. For these demand types, we test common and specialized lot-sizing heuristics.

The dissertation contributes to the literature by (1) modeling dynamic decision situations and showing different investment behaviors of time and money, (2) presenting situations in which the participants leave the sunk cost projects, and (3) showing heuristics that perform well for different types of demand.

The papers are presented synoptically in the following Section. Following this, the research papers are presented in Chapters 2 to 4. Chapter 5 concludes the dissertation with a summary.

2. Research Papers

The three research papers in this dissertation are briefly presented and classified below, and an overview of the papers is provided in Table 1. Papers I and II belong to the experimental analysis of an individual's time and money investment decisions, while paper III evaluates the performance of different lot-sizing algorithms.

Table 1: Research Papers

I	Paper: Status: Authors:	When Subjects Fail to Invest First and Harvest Later: An Experimental Study Working paper available on request Johanna Dujesiefken, Guido Voigt, Charles Corbett
II	Paper: Status: Authors:	Should We Change the Decision Maker after Sunk Time Investments? Results from a Laboratory Experiment Working paper available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4537809 Johanna Dujesiefken
III	Paper: Status: Authors:	Performance of Myopic Lot-sizing Heuristics and an Improvement Heuristic in the Case of Regular, Sporadic, and Close-To-Zero Demand Working paper available on request Johanna Dujesiefken, Hartmut Stadtler, Guido Voigt

In Paper I "When Subjects Fail to Invest First and Harvest Later: An Experimental Study?" we present laboratory experiments to examine how time is invested compared to money. In the resource allocation problems considered, investing time or money first and harvesting the returns later is optimal. In the experimental evaluation, surprisingly, the timing of both time and money investments consistently deviates from the "invest first - harvest later" strategy. The key finding is that the timing of investments improves when time investments meet monetary rewards. In these cases, simple myopic rules do not seem to prevail, and cognitive reasoning kicks in. Our work is the first step in building a model-based theory of how individuals invest either time or money in dynamic contexts.

In Paper II " Should We Change the Decision Maker after Sunk Time Investments? Results from a Laboratory Experiment", we consider the behavioral sunk cost effect for time investments and conduct an incentivized dynamic laboratory experiment. In contrast to previous research, we examine the classical sunk cost situation, where a choice can be made between the sunk cost project and a superior alternative, and the situation where the sunk cost project is the superior project. The experiments are based on a model that clearly states the relations between the available time budget and the objective. The central finding is that decision makers leave the project with sunk time investments without responsibility for past unsuccessful investments – even if the project is superior.

The focus of paper III "Performance of Myopic Lot-sizing Heuristics and an Improvement Heuristic in the Case of Regular, Sporadic, and Close-To-Zero Demand", is not the study of individual behavior (like papers I and II), but the search for the heuristic that performs best across different demand patterns and types. For regular demand, if period demands are well above zero, myopic lot-sizing heuristics like those of Silver & Meal (1973) and Groff (1979) provide fairly good results. Many practitioners still prefer these heuristics which are implemented in many modern Enterprise Resource Planning (ERP) systems. However, modern ERP systems display demand daily rather than weekly leading to a precise demand representation where periods of no demand (sporadic demand) or small (close-to-zero) demands occur. In this paper, we show that many common myopic lot-sizing heuristics perform poorly in the cases of sporadic and close-to-zero demand. An extensive test compares ten lot-sizing heuristics applied in rolling schedules. Among them are two new and easy-to-implement heuristics that base on the heuristics by Silver & Meal (1973) and Groff (1979). This study shows the importance of considering realistic demand patterns when comparing lot-sizing heuristics. While all lot-sizing heuristics except IOQ perform equally well in the case of regular demand, we observe large differences between the heuristics in the case of sporadic and close-to-zero demand. Over all demand patterns and types, the Wagner-Whitin-Look-Beyond algorithm (Stadtler 2000) performs best. Further, we propose and test an easy-to-implement improvement heuristic that can be applied in conjunction with any solution method.

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Silver, E. A. & Meal, H.C. (1973). A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. *Production and Inventory Management*, 2, 64-74.

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Chapter 2

When Subjects Fail to Invest First and Harvest Later: An Experimental Study

When Subjects Fail to Invest First and Harvest Later: An Experimental Study

Johanna Dujesiefken, Guido Voigt, Charles Corbett

Abstract

We present laboratory experiments to compare how time is invested compared to money. Early investments are favorable because returns on investments accumulate over time. In the considered resource allocation problems, it is optimal to invest (time/money) and harvest rewards later. Surprisingly, the timing of both time and monetary investments consistently depart from the "invest first – harvest later" policy. The central finding is that the timing of investments improves when time investments meet monetary rewards. In these cases, it appears that simple myopic rules do not impose, and cognitive reflection sets in. Our work is the first step in building a model-based theory on how individuals invest either time or money in dynamic contexts.

1. Introduction

The productive use of managerial time presents a success factor for companies. The popular time management literature recommends investing time now to save time later (e.g., the 5S concept). In the same vein, traditional financial wisdom suggests that you need to invest before collecting the returns. Since there is often managerial discretion as to whether to invest time into a specific task or outsource it while paying for it, we aim to analyze and compare time investments and monetary investments.

There needs to be more scientific research that analytically models and empirically tests how decision makers behave, mainly when investing time. Yoo et al. (2016) are a notable exception. They normatively analyze the time allocation problem of an entrepreneur who may spend time improving processes to be more effective in crisis management versus harvesting revenue or ensuring future growth. In the same spirit, we present and test the following fundamental managerial resource allocation problem in laboratory experiments.

In our resource allocation problem, the decision maker has to decide whether to sort out "faulty" items or to return them. When picked, faulty items generate no revenues; sorting them out is considered an investment. It requires a time investment (while sorting out, one cannot select "faultless" items that generate revenues) or a monetary investment (e.g., one pays someone to perform this task). In turn, returning the item is considered as harvesting, i.e., the long-term benefit of not picking this item again in future periods is traded against the short-term gain of making immediate revenues. Intuitively, first sorting out items and then switching to returning faulty items towards the end appears normatively to be a good strategy, leading to our first research question.

Research Question 1: Do subjects invest first and collect the returns later if this is normatively the optimal policy?

A typical approach of economists is to convert time investments into monetary investments by the concept of opportunity cost (see Thaler & Johnson 1990, Thaler 1999, Friedman & Neumann 1980, Ebert & Prelec 2007). Intuitively, one may think of a time investment in making the investment oneself or hiring a person to do the job. By this means, one can translate the time investment to a monetary dimension (e.g., wage rate of the employed person), and the fundamental research of monetary investment decisions applies to time investments. Yet, considerable research shows that time and monetary investments trigger different behavioral phenomena (see literature review), which leads to our second research question.

Research Question 2: Does the investment behavior differ between the investment dimension (time vs. money)?

There is ample evidence that human decision makers apply simple heuristics for cognitively challenging tasks. Intuitively, the use of (too) simple heuristics is facilitated if simple comparisons or analogies suggest themselves (see, e.g., the discussion surrounding the cognitive reflection test, Toplak et al. 2011). In our context, this may apply if investment cost and consecutive rewards can be easily accounted for; however, more substantial cognitive reflection may set in if investments and rewards do not have identical unit measurements (i.e., time units or monetary units), leading to our third research question.

Research Question 3: Is there a difference between different unit measurements of investment cost and rewards?

We tackle our research questions with controlled, incentive-compatible laboratory experiments with a student subject pool. Laboratory experiments allow for determining the root-cause effects of problem settings and resource-specific attributes while ensuring internal validity. Although research on decision making concerning time and money can benefit from other empirical approaches (e.g., surveys and field studies), one key advantage of using experiments is the critical aspect of underlying economic incentives. Economic experiments are designed to be fully incentive-compatible, i.e., participants' decisions affect their payout.

In our first set of laboratory experiments, subjects have a fixed budget (measured in time or money) that subjects may invest in returning or sorting out items. Rewards are measured in monetary units. Our key findings are that (a) subjects fail to identify the optimal "first invest – then harvest"-policy under both budget dimensions (time/money), and (b) that this result is even more pronounced when the budget is presented in monetary units instead of time units.

In the second set of experiments, we test if this bias is still present if the investment dimension (i.e., the unit cost measurement of an investment is either a monetary unit or a time unit) is congruent with the reward dimension (time/money). Our key finding is that the timing of the investment decision does not significantly differ between the monetary and time unit conditions. A plausible explanation is that congruent dimensions (invest time to earn time) facilitate simple, myopic cost-benefit comparisons that do not factor in the timing aspects of investments.

In a third experiment, we show that the investment behavior aligns between monetary and time investments if the opportunity cost for time investments is displayed, providing evidence that our treatment manipulations are effective.

In sum, we find that subjects fail to identify the optimal "first invest, then harvest" strategy. This unfavorable behavior is slightly but significantly less pronounced when the investment and the time dimension are incongruent because myopic cost-benefit comparisons do not suggest themselves, leading to a more substantial consideration of the timing of investments.

Our findings have two implications for time and project management: First, "invest first – harvest later" is not a universally apparent policy to human subjects, even though such statements appear to be common wisdom. However, the cognitive reflection seems to set in if simple cost-benefit comparisons are not feasible. Management might use this fact when presenting/framing the cost and rewards of investments or other means to train or nudge decision makers towards more investments at the beginning of a task.

The remainder of the manuscript is organized as follows: We first review relevant literature on time vs. money decision making in §2. In §3, we introduce our investment models. In §4, we present and discuss the experiments, and §5 contains concluding comments.

2. Literature Review

Before we review existing literature that deals with differences in decision making between time and money, we explain two attributes in which time and money differ. First, the value of time is more ambiguous than the value of money. In the present, individuals have planned their concrete time use, and hence, the value of time is less ambiguous than in the future when time is unplanned.

Second, time is less fungible than money because the latter can easily be stored in an account, borrowed, lent, and saved for the future without knowing its concrete use ("a dollar is a dollar"). In the following, we review existing literature that finds differences between time-based and money-based decision making. In doing so, the observed differences are attributed to the different attributes of time and money, particularly ambiguity and fungibility.

Zauberman and Lynch (2005) provide evidence that differences in time and monetary investments are a result of the perceived abundance ("slack") of time and money in the future ("Resource Slack Theory"). They show that the perceived time slack grows over time: Free time is scarce in the near future and is abundant in the distant future. In contrast, individuals perceive monetary slack to be relatively constant over time. Similarly, Spiller and Lynch (2010) show that individuals plan their time much more in the short than in the long term. This is because time is perceived as scarce in the short term and abundant in the long term.

Leclerc et al. (1995) find that decisions involving time losses lead to risk-averse choices, while decisions involving money losses lead to risk-seeking choices. They analyze whether individuals choose the express option, which is faster but more expensive, versus the standard option, which is slower but cheaper. The authors note that the non-fungibility of time may explain this discrepancy. In contrast, Okada and Hoch (2004) observe that individuals are willing to spend more time on riskier higher return lotteries. When spending money, the pattern is reversed, and a more standard pattern of increasing risk aversion is observed. Further, as the variance of the outcomes increases, participants became relatively more risk averse when paying with money, whereas they became more risk-seeking when investing time. Further, individuals choose the ambiguous currency time over the explicit currency money to pay for uncertain outcomes. Okada and Hoch (2004) provide evidence that ambiguity leads to risk-taking by varying the purchasing power of money, thus making money's value more ambiguous. When the value of money is ambiguous, time-based decisions resemble money-based decisions. Besides lotteries with quantitative outcomes, Okada and Hoch (2004) also examine lotteries with qualitative outcomes (e.g., joy). They show that the ambiguity of time leads to a flexible evaluation of past time investments. The evaluation flexibility leads to higher satisfaction after a time than a monetary investment. In contrast, the unambiguity of money leaves no room for the subsequent valuation of money investments. (Okada and Hoch 2004)

Abdellaoui and Kemel (2013) use lotteries to elicit prospect theory components when consequences are the time dedicated to a specific task or activity. In line with Okada and Hoch (2004), they find that individuals tend to take more risks when they face time risk than when they face monetary risk. Conversely, Festjens et al. (2015) do not find different risk preferences for time and money when analyzing small and large stakes of time and money. They find that small time losses are less painful than losing the corresponding wage rate. However, this pattern reverses when stakes are high: Large

time losses are more painful than losing the corresponding wage rate. They explain the results with small time losses conflicting less with planned schedules than large time losses, while money supply and expenses are perceived to be stable over time. In contrast, the evaluation of rewards does not differ between time and money because rewards are additional and do not cause a conflict with existing schedules. (Festjens et al. 2015)
Soman (2001) conducts a series of experiments to analyze whether individuals choose the option in which they have invested before or the option that provides a better outcome (i.e., the so-called "sunk cost fallacy"). Soman (2001) frames the experiments for time and money and finds that individuals tend to choose the preferred or promising option in the time frame but the option with the sunk costs in the money frame. Different mental accounting processes explain the behavior. In particular, time investments are not perceived as investments ("time passes anyway"). Hence, time investments are not booked on a mental account and are therefore not considered when making decisions on the future. Similarly, Saini and Monga (2008) find that heuristics are used more for time because, compared to monetary expenditures, temporal expenditures are harder to account for (Saini & Monga 2008).

Key differences between former studies on time vs. money and ours: Previous research tackled time versus money decision making with hypothetical, static experiments such as surveys and lotteries. We, instead, focus on the dynamic nature of investments, i.e., investment costs occur in the short term, while benefits of investments prevail in the long term. We have an incentivized task in which the decisions have real monetary consequences. Additionally, we control for the alternative use of time/money since investments reduce the available resource for generating revenues (=drawing balls). By this means, we have tight control over the opportunity cost of time investments, allowing us to show differences in investment behavior based on the framing (time vs. money).

3. Theoretical Framework

3.1 Experimental Urn Scheme

We employ an urn scheme to model and experimentally analyze time investments compared to monetary investments. The urn contains black and white balls. Rewards result from a white ball colored black but not from a black ball drawn. After a possible coloring, the ball is black in any case. Individuals can sort out a black ball or return it to the urn. Sorting out takes either time or costs money. We denote this as an investment because the black ball cannot be drawn again in later periods. The black ball can be returned to the urn without additional costs or time. However, the black back ball can be drawn again in later periods.

As an intuitive example, consider an urn full of wrenches of different sizes. After using one (=drawing a white ball), one can invest (=sort out) by putting it back into an easy-to-find position or returning it unsorted. The next time the wrench is needed, one forgoes time to search for it or spends money for someone else to search for it. Another example is the preventive maintenance of machinery. “Sorting out” means investing time or money to reduce the likelihood of a machine failure in later periods (=drawing a black ball). In both examples, the key aspect is that the probability of being productive in upcoming periods is higher if you invest in sorting out black balls.

The individual’s productivity state is characterized by a triplet $[b, w, d]_k$, in which $b_k \in N$ $(w_k \in N)$ denotes the number of black (white) balls in the urn after draw $k = 1, \dots, K$ and $d_k \in \{0,1\}$ denotes, whether the k-th ball drawn is black $(d_k = 0)$ or white $(d_k = 1)$. Between k and $k+1$, individuals draw a ball (knowing the urn composition), color it, and decide on sorting the ball out or returning it. The latter choice determines the urn composition in $k+1$ (see Figure 1).

Figure 1: Sequence of Events

state $[b, w, d]_k$				Update state $[b, w, d]_{k+1}$
k	Draw ball	Coloring if $\mathrm{d} = 1$	Invest or harvest?	$k+1$

We denote the cost of drawing, coloring, and returning a ball by c_h. The cost of drawing, coloring, and sorting out a ball is denoted by c_i. Sorting out is more costly than returning it, i.e., we assume $c_i > c_h$. Both, c_i and c_h can be either measured in periods (e.g. seconds, hours, etc.) or monetary units.

We next formalize the urn scheme for arbitrary units and then formulate it for time and money, respectively.

3.2 The Current-Budget Model (CBM)

The current-budget model considers the allocation of a limited resource within one project. Consider that you rented a chainsaw for eight hours. In each hour, you can either chop a tree (=draw a white ball), maintain the chain saw (check for oil, sharpen the chain = sort out a black ball), or fail to chop the tree because of chainsaw failure (=draw a black ball). If you maintain the equipment, it is less likely to fail in one of the upcoming hours. Your goal is to chop as many trees as possible within eight hours. In monetary terms, consider that you have a budget of 80 € that you may spend on trying to chop a tree (hourly rate: 10 €) or hiring someone to do maintenance at a rate of 10 €.

The limited time or money budget is denoted by $B = \overline{K} \cdot c_h$ and offers $\overline{K}$ opportunities to draw and return a ball. Decision makers spend the budget (e.g. $B = 80$ sec. or $B = 80$ €) on returning (c_h) or sorting out (c_i) balls. The binary variable a_k denotes whether the k-th ball drawn is sorted out ($a_k = 1$) or returned ($a_k = 0$). Sorting out (returning) reduces the budget B by c_i (c_h).

The budget usage BU_k after the draw of ball $k-1$ is

$$BU_k = c_i \cdot \sum_{j=1}^{k-1} a_j + c_h \cdot \sum_{j=1}^{k-1} (1 - a_j) \leq B \qquad \forall\, k = 2, \dots, K, \tag{1}$$

and $BU_1 = 0$. The last draw $K \leq \bar{K}$ takes place when the last white ball has been drawn, i.e., $w_K = 1$, or the budget is used up, i.e., $BU_{K+1} = B$. When no investments take place, i.e., when all balls are returned to the urn, $K = \bar{K}$ balls are drawn. Sorting out a ball consumes more budget than returning it ($c_i > c_h$). As such, sorting out balls reduces the total number of balls drawn, K, and $K < \bar{K}$ follows.

While the budget B, the budget usage BU_k and the c_i, c_h are expressed in the investment unit (time / money), the reward r and the total rewards R_k are expressed monetarily in dollars ($).

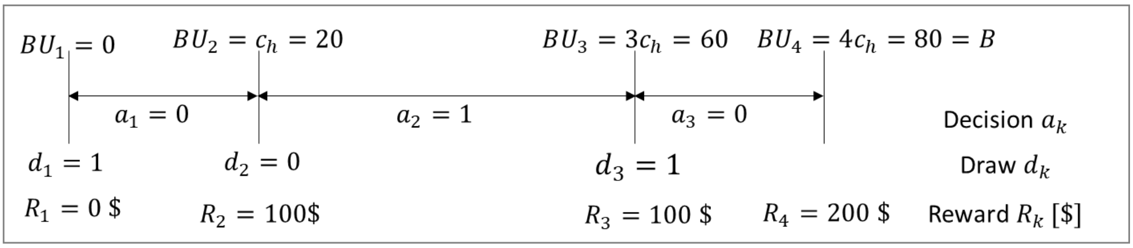
In the time treatment, the budget is expressed in a time unit and the reward in a monetary unit. Similarly, in the money treatment, we use different monetary units for the budget and the reward to illustrate that the budget cannot be offset against the reward. In other words, the unused budget will not increase rewards/compensation, as often observed in practice. To stay in the picture: you will only be rewarded for chopping trees, but not for savings in your budget. Individuals receive a reward r for each white ball drawn ($d_k = 1$). The total reward R_k after returning or sorting out the k-th ball drawn is

$$R_k = r \cdot \sum_{j=1}^{k-1} d_j \qquad \forall\, k = 2, \dots, K\,, \tag{2}$$

with R_1 being the initial reward / show-up fee.

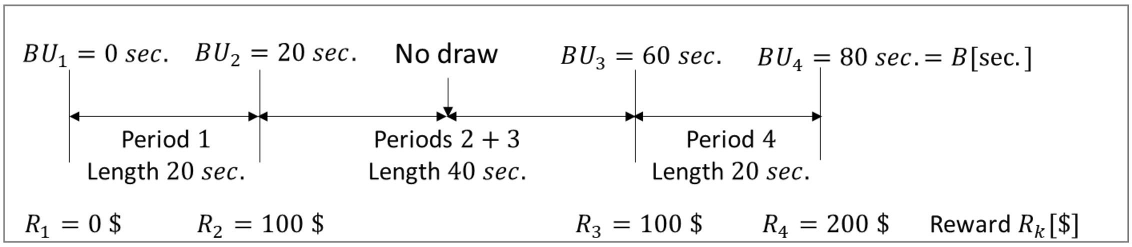
Examples: We formulate the mathematical model for time (example 1) and money investments (example 2), i.e., the budget is either in the dimension of time [seconds] or monetary units [€]. In both cases, we consider monetary rewards [$]. Suppose $R_0 = 0$ $. A reward $r = 100$ $ results from a white ball drawn colored black. We assume $\bar{K} = 4, c_h = 20, c_i = 40$. We consider the return of the first ball ($a_1 = 0$), sorting out the second ball ($a_2 = 1$) and returning the third ball ($a_3 = 0$). By sorting out a ball, one opportunity to draw remains unused, i.e. $K = 3 < 4 = \bar{K}$, see Figure 2.

Figure 2: Example of a Resource Allocation ($B = 4 \cdot 20; c_i = 40$)



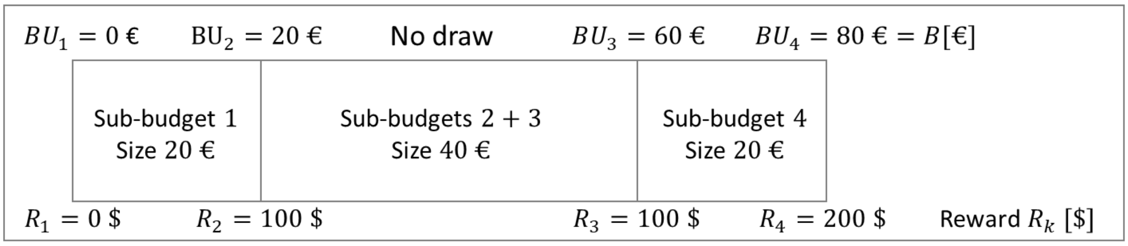
Example 1: Assume, a time budget of $B = 80$ seconds consisting of four periods of $c_h = 20$ seconds (sec.) each. The individual draws a white ball ($d_1 = 1$) and decides to harvest ($a_1 = 0$). After returning the ball, which takes 20 seconds, the next ball is drawn at the beginning of period 2 and sorted out ($a_2 = 1$). Since sorting out the ball consumes 20 seconds more than returning it, the investment takes $c_i = 40$ seconds, and no ball can be drawn in period 3. At the beginning of period 4, i.e. after $BU_3 = 60$ seconds, the last ball $K = 3$ is drawn and returned, and the budget $BU_4 = B = 80$ seconds is used up, see Figure 3.

Figure 3: Example of a Resource Allocation ($B = 4 \cdot 20\ sec.; c_i = 40\ sec.$)



Example 2: The monetary budget $B = 80$ € consists of four sub-budgets of $c_h = 20$ € each. One sub-budget is used to pay for the return of a ball, while two sub-budgets ($c_i = 40$ €) are used to sort out a ball (see Figure 4). Comparing Figures 3 and 4 shows that the money and the time frames are identical when accounting for an opportunity cost of 20 € for 20 $sec.$ time budget use.

Figure 4 Example of a Resource Allocation ($B = 4 \cdot 20$ €; $c_i = 40$ €)



3.3 The Future-Budget Model (FBM)

While the current-budget model considers the allocation of a limited resource within one project, the future-budget model considers the impact of the recent decision on the availability of future resources/budgets. To stay in the picture, assume that there are four trees. Only trees with a clean cut can be sold (=draw white ball), and the likelihood of frazzled cuts (=draw of a black ball) is higher if your equipment is not maintained. In any case, you will work on the four trees. However, the longer it takes because of maintenance (= sorting out black balls), the less time you have for future activities (e.g., selling the wood to the highest bidders). In monetary terms, the money you spend for maintenance cannot be used for upcoming activities.

In the future-budget model, we assume that the number of recent decisions (return/sort-out) is fixed to $K < w_0$, while we assume for the sake of exposition that there are always white balls in the urn. In contrast to the current-budget model, thus, sorting out balls has no consequences on the number of draws in the current task. In turn, sorting out balls results in more time/money spent on the task, limiting the available budget for future, unspecified tasks. As such, we capture the budget consequences of decisions in the future-budget model as budget-availability (BA), while the budget consequences of decisions in the current-budget model are captured by budget-usage (BU).

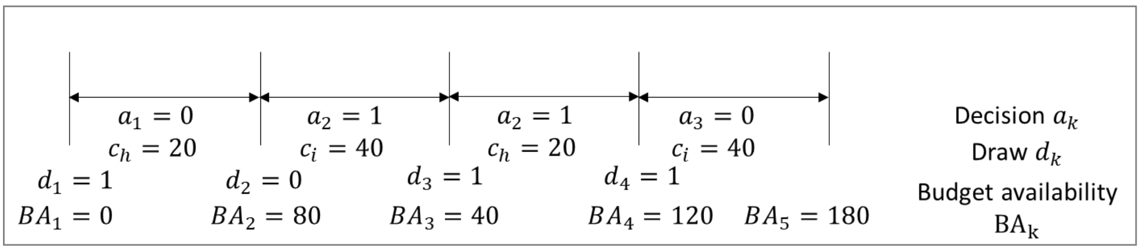
Formally, decision makers draw $k = 1, \dots, K$ balls from the urn and decide whether to sort out ($a_k = 1$) the ball or return it ($a_k = 0$). This decision influences the next urn composition $k + 1$ and the available budget BA in later periods. We assume that the available budget for future tasks is the difference of the reward (r) for coloring a white ball ($d_k = 1$) and the cost of returning/sorting out ($c_i \,/ c_h$) balls in the current task. Then, the available budget for future tasks after returning or sorting out the $k - 1$-th ball drawn is

$$BA_k = BA_1 + \sum_{j=1}^{k-1} r \cdot d_j - c_i \cdot a_j - c_h \cdot \left(1 - a_j\right) \qquad \forall\, k = 2, \dots, K, \tag{3}$$

with BA_1 being the initial amount of a resource / show-up fee. When the current project ends after K draws, the available resource BA_K is the available resource for future, unspecified tasks.

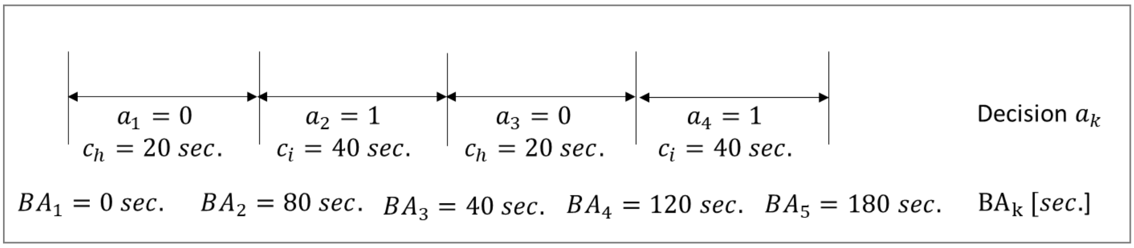
Examples: We formulate the mathematical model for time (example 1) and money investments (example 2). Suppose $BA_1 = 0$ \$. The available time or money budget BA increases by $r = 100$ time or monetary units if a white ball is drawn colored black. We assume $K = \overline{K} = 4, c_h = 20,\ c_i = 40$ and consider the return of the first ball ($a_1 = 0$), sorting out the second ball ($a_2 = 1$), returning the third ball ($a_3 = 0$) and sorting out the fourth ball ($a_4 = 1$), see Figure 5.

Figure 5: Example of a Resource Allocation ($K = \overline{K} = 4; c_h = 20, c_i = 40$)



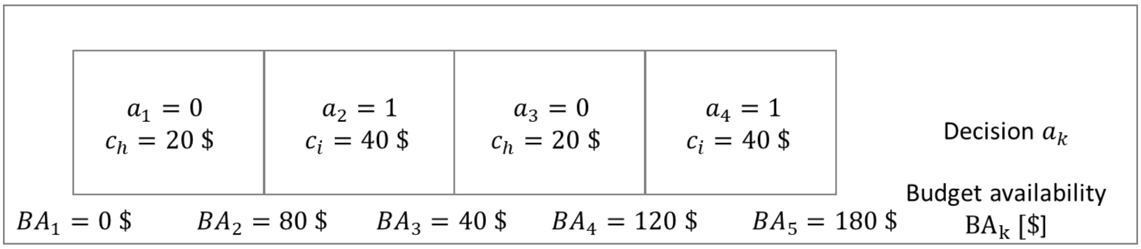
Example 1: Assume the decision maker successively draws $k = 1, \dots, 4$ balls from the urn. The draw of a white ball ($d_1 = 1$) leads to a time gain of $r = 100\ sec$. Since returning ($a_1 = 0$) takes 20 seconds, a time budget of $BA_2 = BA_1 + 100 - 20 = 80\ sec$. is available. The second ball drawn is black ($d_2 = 0$) which does not lead to a time gain. Sorting out the black ball ($a_2 = 1$) reduces the time budget by $c_h = 40\ sec.$, such that there are $BA_3 = BA_2 - 40 = 40\ sec$. available. The 3rd ball drawn is white ($d_3 = 1$) and leads to a time gain of $r = 100\ sec$. As the ball is returned ($a_3 = 0, c_h = 20\ sec.$), there are $BA_4 = BA_3 + 100 - 20 = 120\ sec$. available. The 4th ball drawn is white ($d_4 = 1, r = 100$ sec.) and sorted out ($a_4 = 1, c_i = 40$ sec). For future unspecified tasks, there are $BA_5 = BA_4 + 100 - 40 = 180\ sec$. available, see Figure 6.

Figure 6: Example of a Resource Allocation ($K = 4; c_h = 20\ sec., c_i = 40\ sec.$)



Example 2: The decision maker successively draws $k = 1, \dots, 4$ balls from the urn. The draw of a white ball ($d_1 = 1$) results in a win of $r = 100$ \$. Since returning ($a_1 = 0$) costs 20 \$, a monetary future budget of $BA_2 = 100 - 20 = 80$ \$ is available before the draw of the 2nd ball. As the second ball drawn is black ($d_2 = 0$), which does not result in a win, and sorted out ($a_2 = 1$), a budget of $BA_2 = BA_1 - 40 = 40$ \$ is available before the 3rd draw. The 3rd ball drawn is white ($d_3 = 1$) and leads to a win of $r = 100$ \$. As returning the ball ($a_3 = 0$) costs 20 \$, a budget of $BA_4 = BA_3 + 100 - 20 = 120$ \$ is available before the 4th draw. In the fourth draw a white ball is drawn ($d_4 = 1, r = 100$ \$) and sorted out ($a_4 = 1, c_i = 40$ \$). For future unspecified tasks, a budget of $BA_5 = BA + 100 - 40 = 180$ \$ is available, see Figure 7. Comparing Figures 6 and 7 shows that the money frame and the time frame are identical when accounting for an opportunity cost of 20 \$ for 20 sec. time budget availability.

Figure 7: Example of a Resource Allocation ($K = 4; c_h = 20\ \$, c_i = 40\ \$$)



The optimal decisions are state-dependent and trade-off the cost of sorting out a ball and the benefit of not drawing this ball in later periods. The Markov Decision Process is formalized in Appendix A1. We solved all instances numerically with MatLab R2017a.

4. Experiments

We conducted a computer-based laboratory experiment in which subjects had to decide whether to invest or harvest. We use a 2 (models) x 2 (time/money)-design that leads to four treatments. A total of 113 students were recruited via the Hamburg registration and organization online tool hroot (Bock et al. 2014). The experiment was conducted in four sessions. In total, 30 subjects were assigned to the CBM-T treatment, 30 to the CBM-M treatment, 28 to the FBM-T, and 25 to the FBM-M treatment. The sessions took place at WISO Experimental Lab at the University of Hamburg. Experiments were conducted using o-Tree (Chen et al. 2016) and z-Tree (Fischbacher 2007) and lasted no longer than 1 hour, with average subject earnings of 15.50 €.

Subjects were paid individually and discretely in cash after each session. Exchange rates are chosen such that payouts are similar across both experiments. In the CBM, the exchange rate is 145 (seconds/points) = 1 Euro. In the FBM, the budget for future tasks is exchanged by 90 thaler = 1 Euro. All sessions follow the same experimental protocol. The instructions were handed out to the subjects and were read aloud. (Instructions for the experiment are available in Appendix A3.) Then, after a short re-reading time, the individuals were allowed to ask questions that were answered privately. Communication between subjects and the pursuit of other activities during possible waiting times were prohibited. Subjects are required to pass comprehension questions. The programming allowed several trials, and subjects could ask questions, which were answered in private. The experiment began with a trial run. Then, the run relevant to the payout started. After subjects had made their investment decisions, they completed a post-experiential questionnaire, in which we asked questions regarding participants' attitudes and preferences and general questions about the experiment. We also collected demographic data.

We first present the experimental design, theoretical predictions, hypotheses, and results for the treatments based on the CBM, then for the treatments based on the FBM, and finally for the treatment FBM where the opportunity costs are given.

4.1 Experimental Design for the CBM

Table 2 summarizes the parameters and treatment variables. A current budget of $\overline{K} \cdot c_h = 400$ units is available. Harvesting consumes 10 units, while an investment consumes 20 units. In both frames, the draw of a white ball results in a monetary reward of $r = 100$ [thaler/ points], both being experimental monetary currencies. In the time treatment, the budget is expressed in a time unit. In the money treatment, different monetary units are used for the monetary budget (thaler) and the monetary reward (points). Final payouts only result from converting the experimental currency "points", i.e., the budget in thaler does not enter the final compensation.

Table 2: Parameter for the CBM

	Parameter	CBM-T	CBM-M
		Treatment Variable	
Initial white balls	$w_0 = 30$		
Initial black balls	$b_0 = 0$		
Current budget	$\overline{K} \cdot c_h = 400$		
Invest (sort out)	$c_i = 20$	Time (seconds)	Money (thaler)
Harvest (return)	$c_h = 10$		
Reward	$r = 100$	Money (thaler)	Money (points)

4.1.1 Time frame (CBM-T)

Individuals make decisions until the current budget of $\overline{K} \cdot c_h = 400\ sec$. is used up. An example decision making screen can be seen in Figure 8. Individuals are instructed that harvesting takes 10 seconds, while a time investment takes 20 seconds. At this time, the individual is busy sorting out the ball. By investing, she forgoes the draw of another ball and the possibility of generating a reward. For the duration that investing takes longer than harvesting, i.e., $c_i - c_h = 10$ seconds, the participant sees an hourglass on the screen (see Figure 8). Alternatively, participants could have been asked to sort out the ball physically. However, physically sorting out a ball may vary in duration and convenience between participants. With this design choice, we exclude the influence of these factors and synchronize the duration with the induced costs of sorting out (10 sec.).

Figure 8: CBM Sample Decision-Making Screen

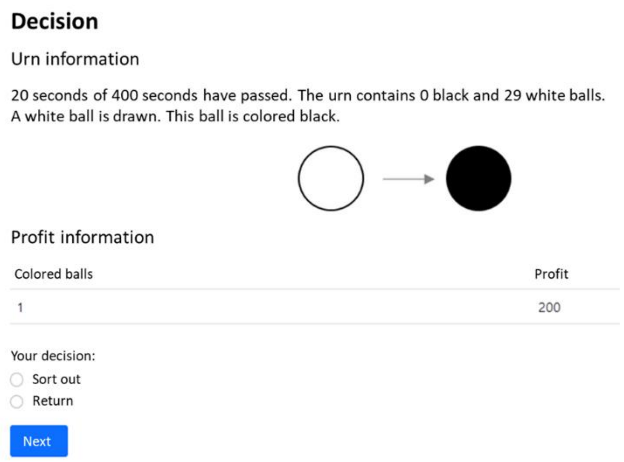
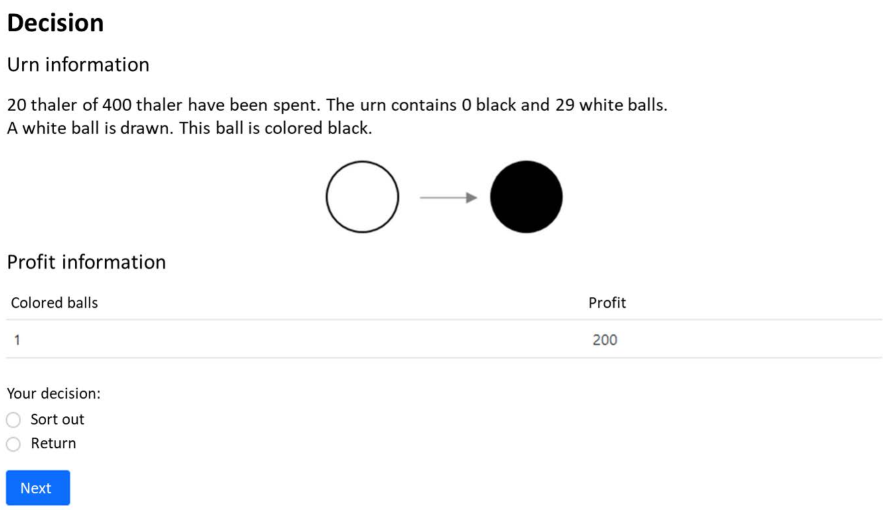


Figure 9: Screen After an Investment in CBM-T

4.1.2 Money frame (CBM-M)

Individuals make decisions until the current budget of $\overline{K} \cdot c_h = 400$ thaler is used up. An example decision-making screen can be seen in Figure 10. Individuals are instructed that harvesting costs 10 thaler, while a monetary investment costs 20 thaler. When investing, the individual is not occupied (i.e., an hourglass is not shown as in the time frame).

Figure 10: CBM Sample Decision-Making Screen



4.1.3 Theoretical Predictions and Hypotheses for the CBM

The policy $\pi^* = [\pi_1^* = 1, \dots, \pi_{10}^* = 1, \pi_{11}^* = 0 \ \dots, \pi_{30}^* = 0]$ maximizes the expected total reward R_k. Sorting out the balls drawn in the first 10 draws and then returning the balls drawn in the next 20 draws maximizes the expected total reward R_k.

The theoretical predictions for a rational, risk-neutral decision maker (the optimal decision in each state) do not differ between the frames. If participants make the same decisions regarding time and money, we expect the same decision behavior for time investments and monetary investments. Hence, we formulate the following hypothesis.

Hypothesis CBM1: The investment behavior does not differ between the investment dimensions.

Although the optimal solution does not differ between the frames, previous experiments show that changing the units from time to money can evoke different investment behavior (see Section 2). Hence, we formulate the competing hypothesis.

Hypothesis CBM2: The investment behavior differs between the investment dimensions.

To evaluate the hypotheses and characterize the decision behavior, we analyze (a) the overall investment quantity (i.e., number of sorted-out balls), (b) the timing of investments, and (c) the proportion of optimal decisions.

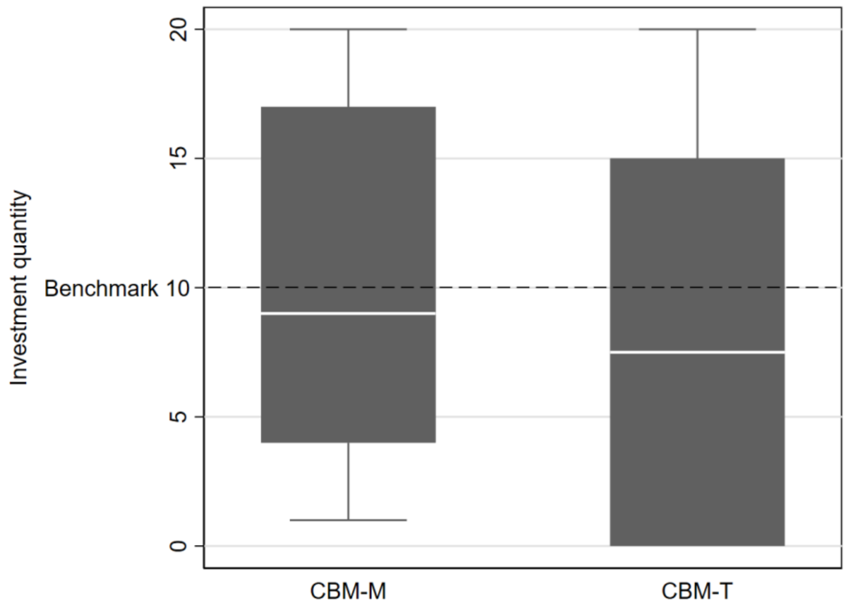
4.2 Results for the CBM

We test with two-sided Mann-Whitney U (MWU) whether the a) total investment quantities and c) the proportion of optimal decisions differ between the time and money frames. For each test, we report the p-values (p).

4.2.1. Overall Investment Quantity

We begin by examining the overall investment quantity defined as $q(i, K) = \sum_{j=1}^{K} a_{i,j}$ for each participant i. The median of overall time investments is 7.5, while it is 9 for monetary investments (see Figure 11). However, the difference is not statistically significant ($p = 0.12$, MWU).

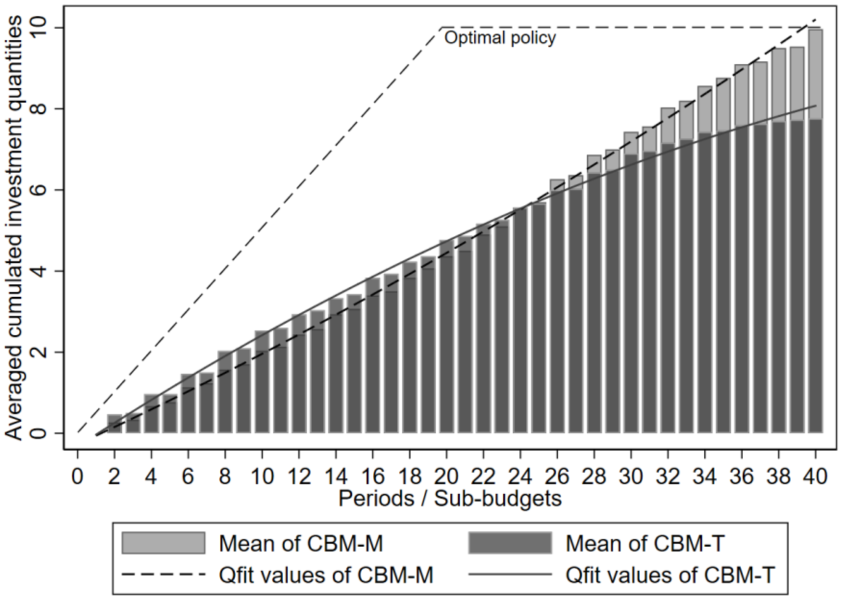
Figure 11: CBM Investment Quantities



4.2.2 Timing of Investments

We now focus on the order of investment decisions across the frames. We define the cumulated investment quantity up to period *t* for each participant i as $q(i,t) = \sum_{j=1}^{t} a_{ij}$, for $t = 1, \dots, 40 = \overline{K}$. To get an aggregate impression of the investment behavior over time, we consider the cumulated investment quantities averaged over all participants of a frame, as shown in Figure 12. Overall, we observe that subjects largely fail to identify the "invest first-harvest later"-policy, while this result is more pronounced in the money frame. On average, we observe more time than money investments until period / sub-budget $t = 24$. The reverse is true for period / sub-budget $t = 25, \dots, 40 = \overline{K}$. Here, we observe, on average, more monetary investments than time investments.

Figure 12: CBM Timing of Investments



A visual inspection of Figure 12 suggests a quadratic influence of periods on time investments but not on money investments. Therefore, we use a statistical model to examine the influence of t and t^2 on the cumulative investment quantities. For each frame, we run a random effects regression that accounts for individual heterogeneity:

$$q(\mathrm{i},\mathrm{t}) = \beta_0 + \beta_t \cdot t + \beta_{t^2} \cdot t^2 + u_i + \epsilon_{it}$$

The subscript i indicates the participant, and the t is the index for the decision periods. The dependent variables $q(\mathrm{i},\mathrm{t})$ are continuous. There are two error terms: u_i is pair-specific controlling for heterogeneity, and ϵ_{it} is independent across all observations. Table 3 shows the regression results.

Table 3: CBM Regression Results

	Time (CBM-T)	Money (CBM-M)
β_0	$+0.05\ (p = 0.91)$	$+0.02\ (p = 0.96)$
β_t	$+0.26\ (p = 0.00)$	$+0.18\ (p = 0.00)$
β_{t^2}	$-0.001\ (p = 0.01)$	$+0.002\ (p = 0.00)$
R^2(overall)	0.24	0.35
Wald χ^2	1364.43	2490.86
Prob $> \chi^2$	0.00	0.00
σ_u	2.12	2.30
σ_e	2.09	2.02
ρ	0.51	0.56

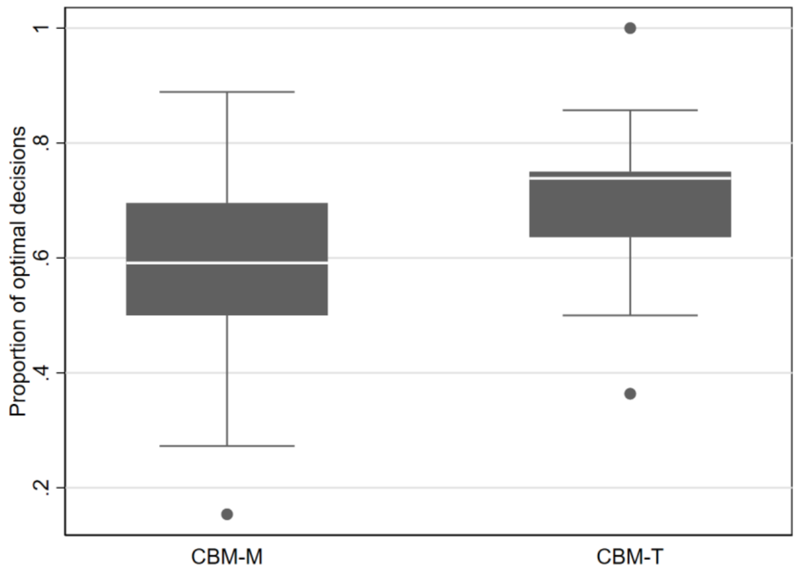
The overall regressions are significant (time: $\mathrm{R}^2(\text{overall}) = 0.24$, Wald $\chi^2 = 1364.43$, $\mathrm{p} = 0.00$ and money: $\mathrm{R}^2(\text{overall}) = 0.35$, Wald $\chi^2 = 2490.86$, $\mathrm{p} = 0.00$). The coefficients of t and t^2 are significant in both frames. However, the coefficient of t^2 is negative in the time frame and positive in the money frame, meaning that individuals invest slightly less time in later periods t, but more money. The overall investment quantities do not differ significantly (see 4.2.1), so we conclude that time investments are placed earlier than monetary investments.

4.2.3 Optimal Decision Making

We now focus on the state-based optimality of decisions, i.e., we compare each decision that participants make in a given state to the decision that is optimal in that state. We define the proportion of optimal decisions for each participant as $\sum_{j=1}^{K} 1_{\{a_{ij}=\pi_j^*\}} /K$. On average, 59% of decisions in the monetary frame are optimal, while $74\ \%$ of decisions in a time frame are optimal (see Figure 13). The difference is significant ($p = 0.00,\ MWU$).

As shown in *4.3.1,* the overall investment quantity does not differ between the frames. However, in the time frame, individuals place investments more favorably, namely more in the beginning and less towards the end. The better timing of time investments results in more optimal decisions in the time than in the money frame. However, better timing in the time frame does not result in a significantly higher payout ($p = 0.16$, MWU). On average, subjects earn 15.40 € in the money frame and 15.77 € in the time frame.

Figure 13:CBM Proportion of Optimal Decisions



4.3.4 Interpretation of Results

The investment behavior differs between the time and the money frame, which contradicts hypothesis CBM1 and supports hypothesis CBM2. The treatment differences show that the manipulation worked and time was perceived differently from money. We manipulated two parameters between the frames. First, in the time treatment, we showed an hourglass for the duration that investing takes longer than returning the ball. Second, we changed the investment dimension from time to money. The following explains how these treatment manipulations may explain the observed decision behavior.

First, it appears possible that individuals invested less toward the end because they were bored with the sight of the hourglass and did not want to wait again. Following this explanatory approach, we would expect less or as much time investment as money investment at the beginning. However, we observe more time than money investments initially, which is unlikely to be explained by the sight of the hourglass. Therefore, we do not consider it plausible that showing the hourglass drives decision behavior.

Second, in the money treatment, the budget and the reward are expressed monetarily, but in different monetary currencies (units), i.e., individuals invest money (unit: thaler) to earn money (unit: points). We conjecture that the congruence of the investment and the reward dimension facilitates too simple, myopic cost-benefit comparisons that neglect the temporal aspect to "first invest and then harvest". Given the complex dynamic decision problem, participants likely follow a myopic heuristic and decide after each ball is drawn rather than looking ahead, leading to constant investment. This conclusion is also reached by inspection of the handwritten notes. In contrast, individuals invest time to earn a monetary reward in the time frame. The incongruence of the investment and the reward dimension makes it difficult to perform (myopic) cost-benefit comparisons. We conjecture that this channels the attention towards the temporal dimension, such that individuals invest time earlier than money.

The follow-up experiment is based on a decision situation in which the investment dimension is congruent with the reward dimension. We consider two frames. In the time frame, time (in sec.) is invested to earn time (in sec.), while in the money frame, money (in thaler) is invested in making money (in thaler). The congruence of the investment dimension with the reward dimension enables the offsetting of investments and rewards.

4.3 Experimental Design for the FBM

The decision maker successively draws $k = 1, \dots, 18$ balls from the urn. The draw of a ball takes $10\ sec$. In total, $180\ sec$. are available. An investment reduces the available budget for later unspecified tasks by 30 units. The draw of a white ball results in a win of $r = 100$ units for later unknown jobs.

Table 4 shows the parameters and the unit change:

Table 4: Parameter for the FBM

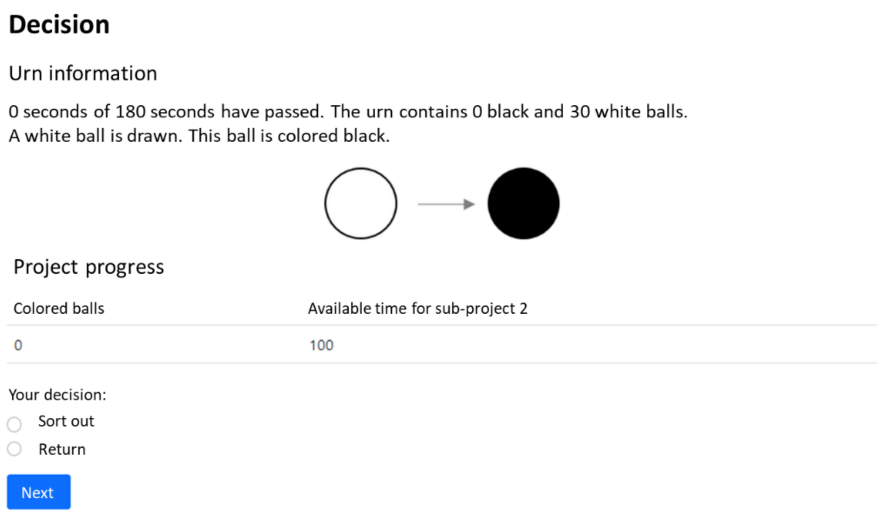
	Parameter	FBM-T	FBM-M
		Treatment Variable	
Initial white balls	$w_0 = 30$		
Initial black balls	$b_0 = 0$		
Draws	$K = 18$		
Available budget	$180\ sec.$		
Invest (sort out)	$c_i = 30$		
Harvest (return)	$c_h = 0$	seconds	thaler
Future budget	$r = 100$		

This experiment's treatment variable is the unit change from time to money. Changing the units from time to money leads to the following frames.

4.3.1 Time frame (FBM-T)

The decision maker successively draws $k = 1, \dots, 18$ balls from the urn. Figure 14 shows an exemplary decision-making screen. Participants are instructed that investments reduce the available budget for later unspecified tasks by 30 seconds. The draw of a white ball provides $r = 100$ seconds for the later unspecified tasks. In the experiment, the unknown tasks are referred to as sub-project 2. In contrast to CBM-T, investments affect the available time for the unspecified sub-project 2. Hence, the individual can draw the next ball right away and does not see an hourglass on the screen.

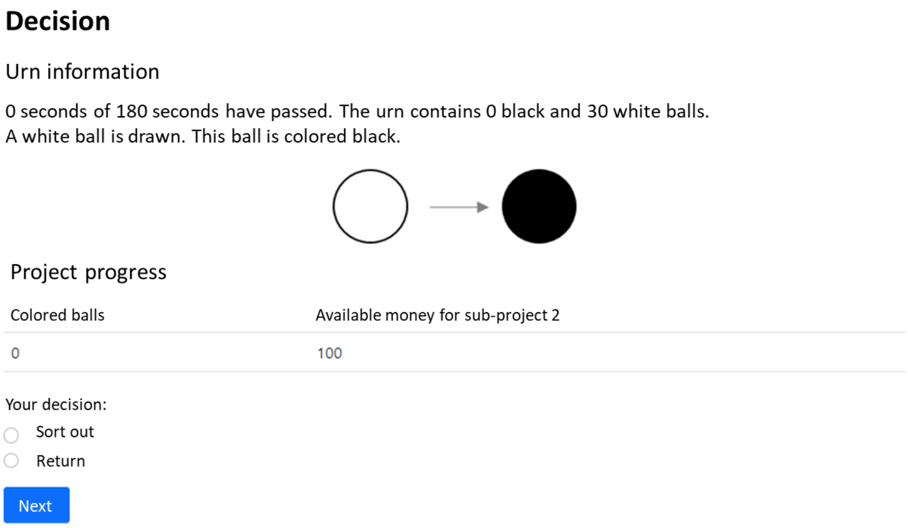
Figure 14: FBM-T Sample Decision-Making Screen



4.3.2 Money frame (FBM-M)

The decision maker successively draws $k = 1, \dots, 18$ balls from the urn. Figure 15 shows an exemplary decision-making screen. Participants are instructed that investments reduce the available budget for later unspecified tasks by 30 thaler. The draw of a white ball results in a win of $r = 100$ thaler for the later unspecified sub-project 2.

Figure 15: FBM-M Sample Decision-Making Screen



4.3.3 Theoretical predictions and hypotheses for the FBM

The policy $\pi^* = [\pi_1^* = 1, \dots, \pi_6^* = 1, \pi_7^* = 0 \dots, \pi_{18}^* = 0]$ maximizes the expected available budget BA_{18} for later unspecified tasks. Sorting out the balls drawn in the first 6 draws and then returning the balls drawn in the next 12 draws maximizes the expected total reward R_k. The mathematical model and the theoretical predictions for a rational, risk-neutral decision maker (the optimal decision in each state) do not differ between the frames. Normatively, the decision behavior should not significantly differ between the investment dimensions. We omit to formulate the hypothesis.

The CBM differs from the FBM in one central aspect: Investments do not affect the current budget. Instead, investments affect the future budget for unspecified tasks. In the time frame FBM-T, the reward dimension is congruent to the investment dimension, i.e., individuals invest time to earn time for unknown later tasks. *We hypothesize that the congruence of the investment and the reward dimension m*ay lead to a less favorable timing structure in FBM-T than in CBM-T. We formulate the corresponding hypotheses:

Hypothesis FBM1: The timing of investments is less favorable in FBM-T than in CBM-T.

Earning time for unspecified later tasks is more ambiguous than making money for unknown later tasks. This is because money is concrete without the concrete use being known (see Okada & Hoch 2004). We hypothesize that an ambiguous time reward attracts fewer investments than a concrete money reward and expect to observe fewer investments in the time than in the money frame.

Hypothesis FBM2: Fewer investments are made in the time frame than in the money frame.

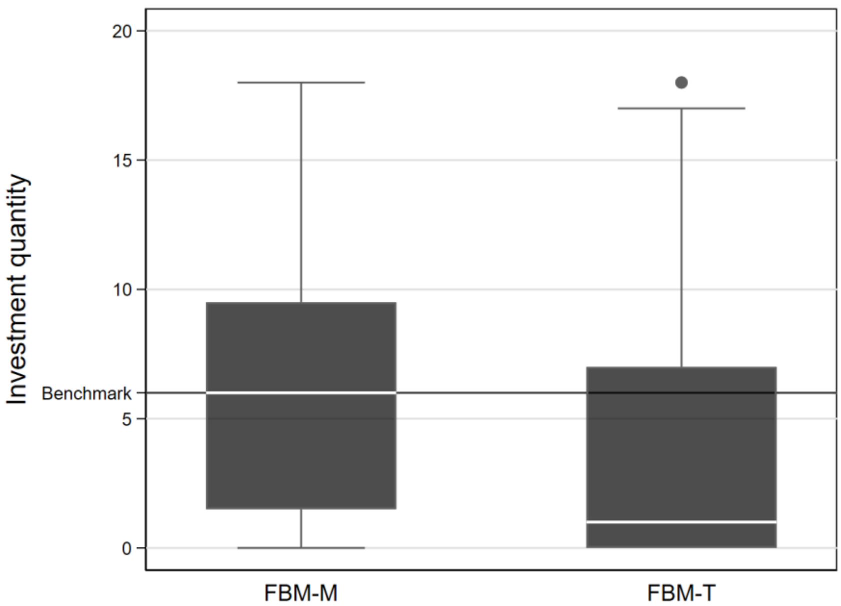
To evaluate the hypotheses and characterize the decision behavior, we analyze (a) the overall investment quantity (i.e., number of sorted-out balls), (b) the timing of investments, and (c) the proportion of optimal decisions.

4.4 Results for the FBM

4.4.1 Investment Quantity

We begin by examining the overall investment quantity. The median investment quantity is 1 for the time frame and 6 for the money frame (see Figure 16). The median in the money frame coincides with the benchmark, the investment quantity of the optimal policy. In FBM-T, fewer investments than in the time frame and fewer investments than the benchmark were made. The difference is statistically significant ($p = 0.1$, MWU). Less investment is made in the time frame than in the money frame, supporting hypothesis FBM2.

Figure 16: FBM Investment Quantities

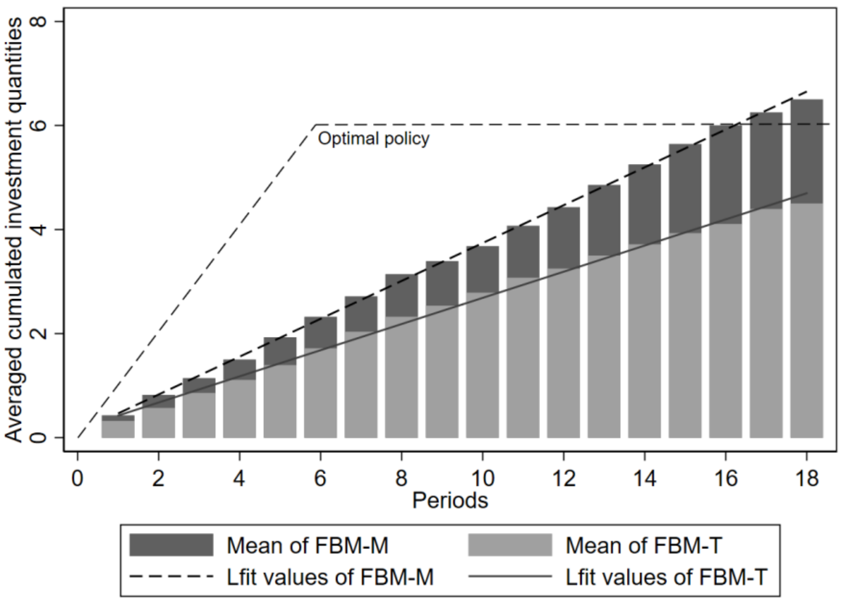


4.4.2 Timing of Investments

We now focus on the order of investment decisions across frames. We do not observe individuals investing earlier or less towards the end in both frames.

Figure 17 displays the averaged cumulated investment quantities after each draw k across frames. In contrast to the CBM, there are consistently fewer time investments than monetary investments, and time investments are not reduced towards the end.

Figure 17: FBM Timing of Investments



We estimate the cumulated investment quantities to evaluate the timing of investments as in Section 4.3. Table 5 shows the regression results.

Table 5: FBM Regression Results

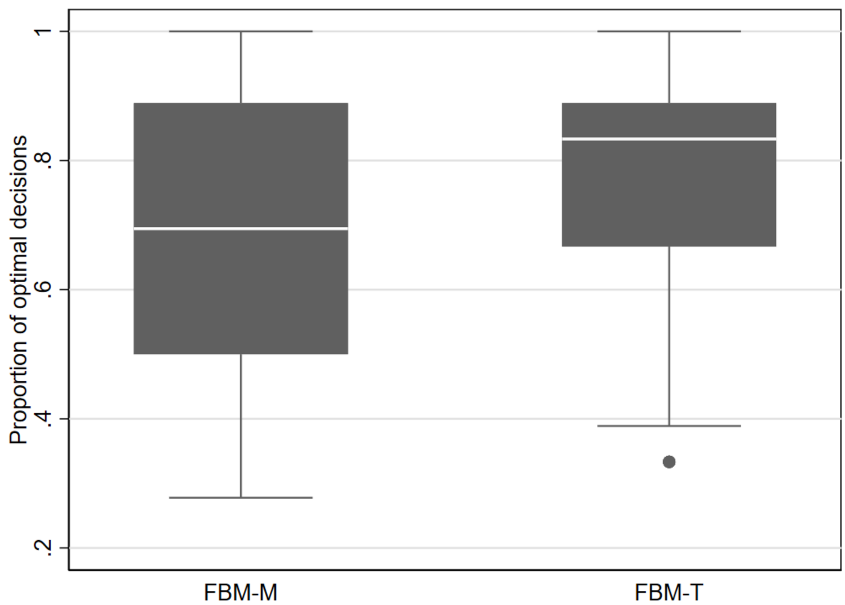
	Time (FBM-T)	Money (FBM-M)
Constant β_0	$+0.26$ ($p = 0.71$)	$+0.41$ ($p = 0.54$)
Coefficient β_t	$+0.31$ ($p = 0.00$)	$+0.38$ ($p = 0.00$)
Coefficient β_{t^2}	-0.004 ($p = 0.29$)	-0.001 ($p = 0.72$)
R^2(overall)	0.10	0.20
Wald χ^2	251.46	545.43
Prob $> \chi^2$	$p = 0.00$	$p = 0.00$
σ_u	3.43	3.39
σ_e	1.85	1.82
ρ	0.77	0.78

The overall regressions are significant (time: $\mathrm{R}^2(\text{overall}) = 0.10$, Wald $\chi^2 = 251.46$, $\mathrm{p} = 0.00$ and money: $\mathrm{R}^2(\text{overall}) = 0.20$, Wald $\chi^2 = 545.43$, $\mathrm{p} = 0.00$). In both frames, the coefficients of t are significant and positive, while the coefficients of t^2 are not significant. The slope does not change for large t. Individuals distribute the money investments evenly among all periods / sub-budgets t. If the investment and the reward dimensions are congruent, the temporal dimension is neglected, supporting hypothesis FBM1.

4.4.3 Optimal Decision Making

We now focus on the state-based optimality of decisions by comparing the participants' decisions with the optimal decisions in the corresponding states. As in the CBM, we observe a higher proportion of optimal decisions in the time than in the money frame. On average, $69\ \%$ of decisions in the monetary frame are optimal, while $83\ \%$ of decisions in the time frame are optimal (see Figure 18). The proportions of optimal decisions do not differ significantly ($p = 0.16$, MWU). Similarly, the average payouts do not vary significantly ($p = 0.96$, MWU). On average, subjects earn 16.18 € in the money frame and 16.13 € in the time frame.

Figure 18: FBM Proportion of Optimal Decisions



4.4.4 Interpretation of Results

We observe a constant investment rate in both frames. This supports our explanatory approach. If the investment dimension is congruent with the reward dimension, then the temporal dimension is neglected since the congruence allows the application of simple, myopic cost-benefit comparisons. In both money treatments (CBM-M and FBM-M), the investment dimension coincides with the reward dimension (congruence). As we observe a constant investment rate in both money treatments, we conclude that offsetting, which is only possible in FBM-M, does not cause the neglect of the temporal dimension. Instead, the congruence of the investment and reward dimensions drives the neglect of the temporal dimension. Individuals can perform simple, myopic cost-benefit comparisons when investments and rewards are of the same dimension (=congruent). Different units (e.g., EUR and USD) within a dimension (e.g., money), as in CBM-M, appear not to interfere with these cost-benefit-comparisons.

In contrast to the CBM treatment, we observe less time than monetary investments in the FBM treatments. We conjecture that the reward, expressed in terms of a future time budget for an unspecified later task, is more ambiguous without its concrete use stated than a future monetary budget. In other words, when the future budget is expressed in time rather than money, individuals take more risks, i.e., drawing a black ball while not generating revenue. Like Okada and Hoch (2004), we find that ambiguity triggers risk-taking.

The following experiment is designed to validate that an ambiguous future budget attracts fewer investments.

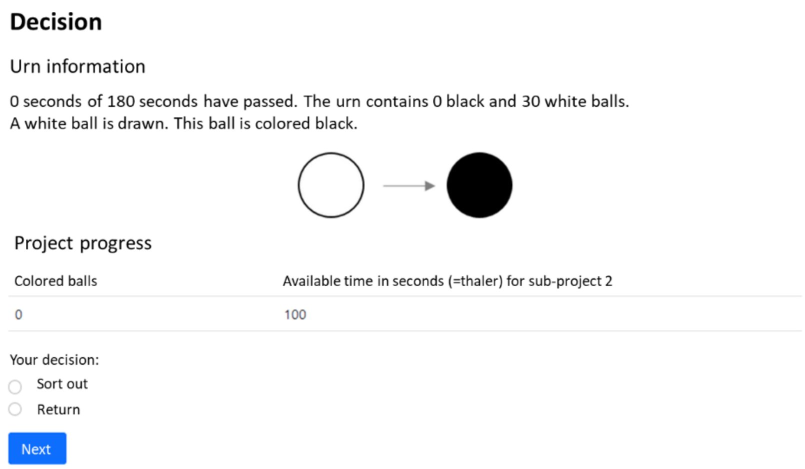
4.5 Experimental Design for the FBM-TM Opportunity Costs Saliency

To check if the ambiguity of future time rewards attracts fewer investments than future monetary rewards, we conduct a treatment that differs from the FBM-T treatment only in that the opportunity costs (in monetary units) are displayed after the time unit, see Figure 19 and Table 6. All other parameters and the normative benchmark remain identical. Twenty-five subjects were assigned to this treatment.

Table 6: Parameter for the FBM-TM

	Parameter	FBM-T	FBM-M	FBM-TM
		Treatment Variable		
Initial white balls	$w_0 = 30$			
Initial black balls	$b_0 = 0$			
Draws	$K = 18$			
Available budget	$180\ sec.$			
Invest (sort out)	$c_i = 30$			
Harvest (return)	$c_h = 0$	seconds	thaler	seconds (= thaler)
Future budget	$r = 100$			

Figure 19: FBM-TM Sample Decision-Making Screen



4.5.1 Theoretical Predictions and Hypotheses for the FBM-TM

The opportunity cost of time is shown for both investments and rewards. Because of the investment-reward congruence, we assume that simple, myopic cost-benefit comparisons are performed, and investments are not shifted towards the beginning compared to time FBM-T treatment without opportunity cost. We expect that the “first invest and then harvest”-strategy is not followed in FBM-TM and formulate the following hypothesis.

Hypothesis FBM-TM1: In FBM-TM, the investment quantity is not more pronounced in earlier periods than in later periods.

Earning time for unspecified later tasks is ambiguous unless concrete use is stated. In contrast, a money budget for unspecified later tasks is concrete without a known use. We hypothesize that specifying the opportunity cost of time makes time for unknown later tasks less ambiguous and increases the investment volume.

Hypothesis FBM-TM2: The investment quantity is higher in FBM-TM than in FBM-T.

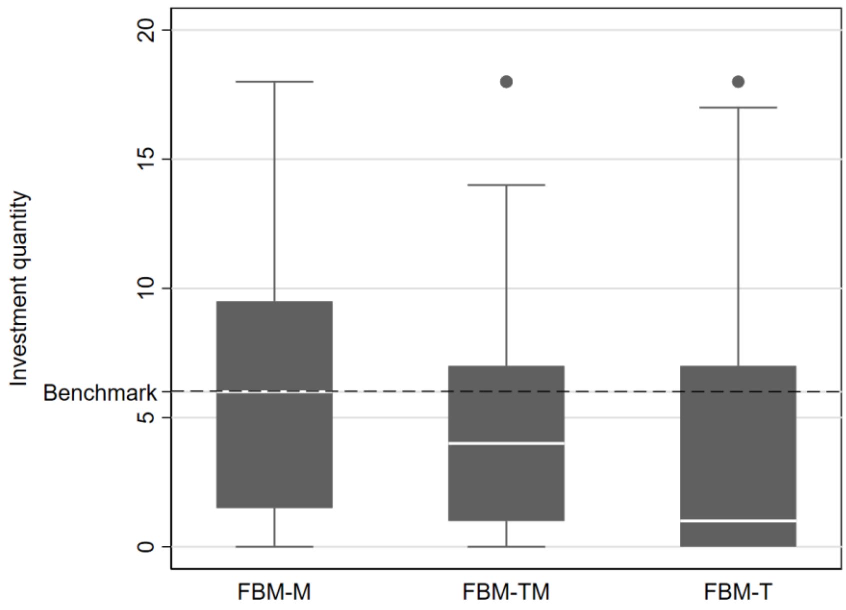
To evaluate the hypotheses and characterize the decision behavior, we analyze (a) the overall investment quantity (i.e., number of sorted-out balls), (b) the timing of investments, and (c) the proportion of optimal decisions.

4.6 Results for the FBM-TM

4.6.1 Investment Quantity

We now focus on the overall investment quantity displayed in Figure 20. The median of FBM-TM is 4 and lies between the median of FBM-T, which is 1, and the median of FBM-M, which is 6. While the difference between FBM-M and FBM-T is significant ($p = 0.1$, MWU), the differences of FBM-TM to FBM-M ($p = 0.37$, MWU) as well as to FBM-T ($p = 0.35$, MWU) are not significant, contradicting hypothesis FBM-TM2. We note that slightly more investments are made if the opportunity cost of time is given (as in FBM-TM) than if it is not (as in FBM-T).

Figure 20: FBM-TM Investment Quantity



4.6.2 Timing of Investments

We now focus on the order of investment decisions across the frames FBM-M, FBM-TM, and FBM-T. Figure 21 displays the cumulated investment quantities averaged over all participants. In all periods, $t = 1, .., 18$, more money is invested than time, regardless of whether the opportunity cost is specified or not. Comparing the time investments, we observe on average more investments until period $t = 13$. The reverse is true for period $t = 14, \dots, 18 = \bar{K}$. Here, we observe, on average, more time investments when the opportunity costs are stated. Compared to FBM-T, we observe fewer investments at the beginning and more investments toward the end.

Figure 21:FBM-TM Timing of Investments

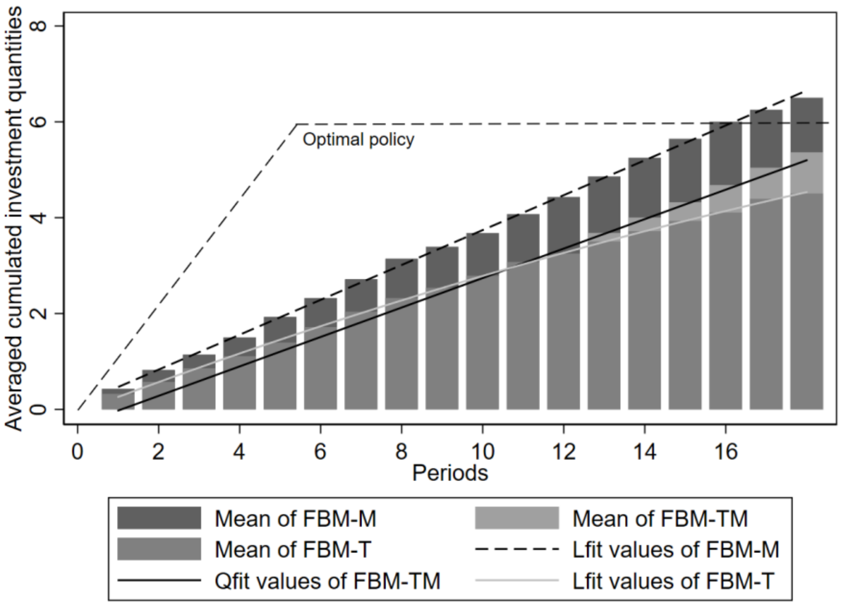


Table 7 shows the regression results in line with Section 4.3.

Table 7: FBM-TM Regression Results

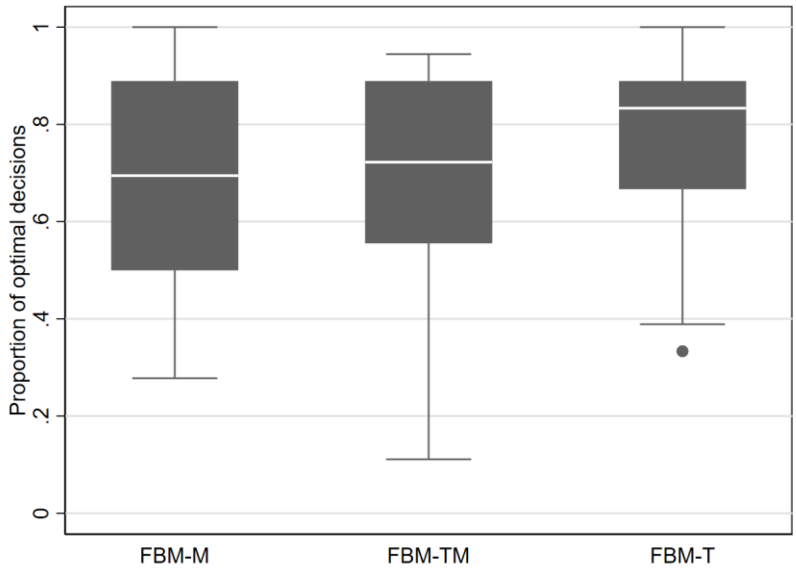
	Opportunity costs saliency: (FBM-TM)
Constant β_0	$+0.20$ ($p = 0.77$)
Coefficient β_t	$+0.22$ ($p = 0.00$)
Coefficient β_{t^2}	$+0.00$ ($p = 0.17$)
R^2(overall)	0.17
Wald χ^2	337.25
Prob $> \chi^2$	$p = 0.00$
σ_u	3.13
σ_e	1.85
ρ	0.74

The overall regression is significant ($R^2 = 0.17$, Wald $\chi^2 = 337.25$, $p = 0.00$). The coefficient of t is significant and positive ($p = 0.00$), while the coefficient of t^2 is not significant. Even if specifying the opportunity costs leads to slightly more investment overall, the additional investments should be made earlier. The investment quantity is not more pronounced in earlier than later periods, supporting hypothesis FBM-TM1. Stating opportunity costs do not nudge the favorable "first invest and then harvest"-strategy.

4.6.3 Optimal Decision Making

On average, 67.33 % of decisions are optimal in FBM-TM, see Figure 22. The proportion of optimal decisions in FBM-TM does not differ significantly from FBM-M ($p = 0.93$, MWU) and FBM-T ($p = 0.16$, MWU). Similarly, the payouts do not differ significantly ($p \geq 0.71$, MWU). On average, subjects earn 16.04 € when opportunity costs are shown compared to 16.18 € in FBM-M vs. 16.13 € in FBM-T.

Figure 22: FBM-TM Proportion of Optimal Decisions



4.6.4 Interpretation of Results

Adding a monetary reference (opportunity costs) increases the investment quantity slightly. This finding further supports that a future money budget attracts more investments than a future time budget. However, these investments should be placed earlier. Since the opportunity costs were shown for both the investment and the reward dimension, the congruence of the dimensions enables simple cost-benefit comparisons leading to the neglect of the temporal dimension.

4.7 Robustness Checks

We performed additional treatments with varying K to exclude the possibility that the favorable timing in CBM-T depends on the selected parameter, see Appendix A2. We conducted CBM-T treatments for $\mathrm{K} = 29, 40, 48$, i.e., we considered current budgets of 290, 400, and 480 time units. Similarly, we conducted FBM-M treatments for $\mathrm{K} = 14, 18, 22$ draws. The new data show that individuals invest rather constantly and do not follow the favorable "first invest and then harvest"-strategy (see changes and detailed results in Appendix A2). Further research is needed to examine whether individuals consider the time horizon differently in time and money investments.

5. Discussion and Conclusion

This Section summarizes our results and contributions and discusses our work's limitations and future research directions.

5.1 Key Results

Concerning our first research question, we find that subjects hardly follow the “invest first – harvest later” - policy, even if this is - by design - the optimal policy. While overall investment quantities appear reasonable, subjects fail to see that earlier investments trump later ones. Compared to the normative benchmark, poor performance is observed regardless of the model frame, i.e., if decisions impact current or future budgets and irrespective of the unit measurements (time/money). This finding calls for more research on how (management) interventions can improve decision behavior.
Concerning our second research question, we observe subtle differences between time and monetary investments, although the normative benchmarks are identical. Depending on the decision context (current budget vs. future budget), we observe differences in the investment quantity, the timing of investments, and the overall number of state-dependent, optimal decisions. One major root cause appears to be the congruence of the investment and reward dimensions, see research question 3. In particular, we observe that time investments tend towards the “first invest and then harvest”-strategy when time is invested to earn money. The reward dimension determines whether the temporal dimension is taken into account. If the reward is measured in time, i.e., congruent to the investment dimension, the temporal dimension is not considered. We argue that congruence facilitates the performance of simple, myopic cost-benefit heuristics. These heuristics neglect the temporal dimension and do not lead to the beneficial bundling of investments at the beginning. In contrast, when time is invested to earn money, participants avoid simple cost-benefit comparisons due to the different unit measurements.
We also provide an alternative explanation. The perception of a current budget differs between time and money. A current time budget is perceived as a temporal sequence of periods. As such, the time budget is ordered (e.g., budget[time] = {1st period, 2nd period, 3rd period}). In contrast, a current financial budget is not perceived as a sequence of sub-budgets but as an unordered set (e.g., budget[money] = {sub-budget 1, sub-budget 2, sub-budget 3} = {sub-budget 2, sub-budget 3, sub-budget 1}. When a current monetary budget is invested, participants focus on something other than the order of decisions because it is irrelevant whether actions are paid for with the first or third sub-budget.
On the contrary, when investing a current time budget, participants pay attention to the order of decisions because it is relevant in which period actions are performed.

While the current time budget is perceived as ordered, a future time budget is perceived as unordered. The future budget considers the number of white balls drawn but not when they were drawn.

Finally, in our experiments, time investments exhibit low ambiguity because they are concretely specified, e.g., sorting out, and occur in temporal proximity. In contrast, decision makers respond strongly to ambiguous reward dimensions. In line with Spiller & Lynch (2010), an unspecified time reward, i.e., 5 hours, triggers fewer investments than an unknown money reward, e.g., 5 $, because an unspecified time reward is more ambiguous than an unspecified time reward.

Besides ambiguity, literature discusses (1) mental accounting and (2) fungibility as drivers of time versus money decision-making differences.

(1) Mental accounting considers, e.g., *whether* investments and rewards are booked on separate mental accounts. According to the explanation approach by Soman (2001), time expenditures are not considered as such and, hence, not booked on a mental account. However, mental accounting does not explain our results. When time or money is invested to earn money, we observe similar overall investment quantities, suggesting that time and money investments are accounted for in the same manner. Further, the theory of mental accounting is not dynamic; i.e., it may not explain the differences in *the timing of* investments we observe. In sum, we do not find evidence that time and money, when quantified, are mentally accounted for differently.

(2) Our models capture decision-making situations where investments and rewards can be offset. Hence, they are fungible (exchangeable). However, offsetting requires congruence of the investment and reward dimension (e.g., time investment lead to time rewards). Our results show that not offsetting (fungibility) but the ambiguity of the reward dimension causes decision-making differences. Therefore, we conclude that fungibility can be neglected to organize our results in our specific context.

5.2 Future Research

There are several directions for future research. First, we consider a stylized investment model with an explicit statement of investment durations (costs) and rewards (future budgets). However, decision-making parameters are often not stated explicitly and must be estimated by decision makers. We know from behavioral science that individuals underestimate processing times (Kahneman & Tversky 1979). Hence, analyzing investment decisions when durations/costs are stochastic would be interesting. Second, in our experiment, individuals invest time or money. In reality, we often face make-or-buy decisions. Therefore, it would be interesting to let individuals (endogenously) choose to invest time or money. Third, an interesting area of further research is identifying factors that mitigate the ambiguity of future time, e.g., how much planning is necessary to increase the value of future time or how detailed future time use must be described to be concrete and unambiguous. Fourth, the investigation of how the influence of limited decision time affects performing cost-benefit comparisons in congruent decision-making situations would be interesting. Fifth, analyzing whether the favorable timing structure can be found in other models for time but not for money presents an interesting area for further research, as the payout differences are minor in the urn model.

Finally, the analysis of whether our key insights generalize to a) other time units than seconds, b) situations without a fixed ending, c) other subject pools, and d) framing contexts (e.g., preventive maintenance), and e) to experiments based on a real effort task present areas for further research.

5.3 Conclusion

The temporal dimension of investments is a crucial component of managing projects. Early investments are favorable because returns on investments accumulate over time. Our work is the first step in building a model-based theory on how individuals invest either time or money in dynamic contexts.

Surprisingly, the timing of both time and monetary investments consistently depart from the "invest first – harvest later" policy. The central finding in our series of experiments is that the timing of investments improves when time investments meet monetary rewards. In these cases, it appears that simple myopic rules do not impose, and cognitive reflection sets in. More research is needed to generalize the results further and identify appropriate behavioral interventions.

The research data is available at 10.25592/uhhfdm.12247 .

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Appendix

A1 Markov Decision Process

Assuming full rationality and without behavioral biases, optimal decisions are computed for each state with a Markov Decision Process (MDP).

A1.1 Markov Decision Process for the Current-budget model (CBM)

The state transitions are summarized in Table 8.

Table 8: Transitions from State $[\boldsymbol{b}, \boldsymbol{w}, \boldsymbol{d}, \boldsymbol{BU}]_k$

State $[b, w, d, BU]_k$	Action a_k	Draw d_{k+1}	Transition to state $[b, w, d, BU]_{k+1}$	Transition probability
$[b, w, 0, BU]_k$	Harvest $a_k = 0$	Black White	$[b, w, 0, BU + c_h]_{k+1}$ $[b, w, 1, BU + c_h]_{k+1}$	$\alpha = b/(b+w)$ $1 - \alpha$
$[b, w, 1, BU]_k$	Harvest $a_k = 0$	Black White	$[b+1, w-1, 0, BU + c_h]_{k+1}$ $[b+1, w-1, 1, BU + c_h]_{k+1}$	$\gamma = \frac{b+1}{b+w}$ $1 - \gamma$
$[b, w, 0, BU]_k$	Invest $a_k = 1$	Black White	$[b-1, w, 0, BU + c_i]_{k+1}$ $[b-1, w, 1, BU + c_i]_{k+1}$	$\beta = \frac{b-1}{b+w-1}$ $1 - \beta$
$[b, w, 1, BU]_k$	Invest $a_k = 1$	Black White	$[b, w-1, 0, BU + c_i]_{k+1}$ $[b, w-1, 1, BU + c_i]_{k+1}$	$\delta = \frac{b}{b+w-1}$ $1 - \delta$

For each state $(b, w, d, BU)_k$, we compute the optimal decision $a^*(b, w, d, BU)_k$. In the CBM, the optimal decision maximizes the expected total reward $E[R_K | (b, w, d, BU)_k, (a_k^*, a_{k+1}^*, \dots, a_K^*)]$ conditioned on further optimal decision making.

The optimal decisions are found by solving a dynamic program with the following terminal value functions. States are considered as terminal when there are no white balls left in the urn or the budget is exhausted. Let $\Omega = \{s = [b, w, d, BU]_k : w_k = 0 \lor BU = B\}$ be the set of all terminal states. For terminal states, we have

$$V([b, w, d, BU]_k) = r \cdot (w_0 - w_k) \ \forall s \in \Omega$$

In every non-terminal state, we look for the action that maximizes the state's value:

$$V([b, w, d, BU]_k) = max(Vi([b, w, d, BU]_k), Vh([b, w, d, BU]_k))$$

In this expression, $Vh([b, w, d, BU]_k)$ ($Vi([b, w, d, BU]_k$) denotes the expected future value when the ball drawn is returned to the urn (sorted out) under the condition of further optimal decision making.

In the CBM, $Vh([b,w,d,BU]_k)$ ($Vi([b,w,d,BU]_k)$) denotes the expected total reward that is achieved when the ball drawn in state $[b,w,d,BU]_k$ is returned (sorted out), i.e.

$$Vh([b,w,d,BU]_k) = E[R_K |\, a_k = 0, (b,w,d,BU)_k]$$
$$Vi([b,w,d,BU]_k)) = E[R_K |\, a_k = 1, (b,w,d,BU)_k]$$

If a black ball is drawn, i.e., $d_k = 0$, we have,

$$Vh([b,w,0,BU]_k) = \alpha \cdot V([b,w,0,BU + c_h]_{k+1}) + (1-\alpha) \cdot\ V([b,w,1,BU + c_h]_{k+1})$$
$$Vi([b,w,0,BU]_k) = \ \beta \cdot V([b-1,w,0,BU + c_i]_{k+1}) + (1-\beta) \cdot\ V([b-1,w,1,BU + c_i]_{k+1})$$

If a white ball is drawn, i.e., $d_{z_k} = 1$, we have,

$$Vh([b,w,1,BU]_k)$$
$$= \gamma \cdot V([b+1,w-1,0,BU + c_h]_{k+1}) + (1-\gamma) \cdot\ V([b+1,w-1,1,BU + c_h]_{k+1})$$
$$Vi([b,w,1]_k) = \delta \cdot V([b,w-1,0,BU + c_i]_{k+1}) + (1-\delta) \cdot\ V([b,w-1,1,BU + c_i]_{k+1})$$

A1.2 Markov Decision Process for the Future-budget model (FBM)

The state transitions are summarized in Table 9.

Table 9: Transitions from state $[\boldsymbol{b},\boldsymbol{w},\boldsymbol{d}]_{\boldsymbol{k}}$

State $[b,w,d]_k$	Action a_k	Draw d_{k+1}	Transition to state $[b,w,d]_{k+1}$	Transition probability
$[b,w,0]_k$	Harvest $a_k = 0$	Black White	$[b,w,0]_{k+1}$ $[b,w,1]_{k+1}$	$\alpha = \ b/(b+w)$ $1-\alpha$
$[b,w,1]_k$	Harvest $a_k = 0$	Black White	$[b+1,w-1,0]_{k+1}$ $[b+1,w-1,1]_{k+1}$	$\gamma = \frac{b+1}{b+w}$ $1-\gamma$
$[b,w,0]_k$	Invest $a_k = 1$	Black White	$[b-1,w,0]_{k+1}$ $[b-1,w,1]_{k+1}$	$\beta = \frac{b-1}{b+w-1}$ $1-\beta$
$[b,w,1]_k$	Invest $a_k = 1$	Black White	$[b,w-1,0]_{k+1}$ $[b,w-1,1]_{k+1}$	$\delta = \frac{b}{b+w-1}$ $1-\delta$

For each state $(b,w,d)_k$, we compute the optimal decision $a^*(b,w,d)_k$. In the FBM, the optimal decision maximizes the expected available budget $E[BA_K |\, (b,w,d)_k, (a_k^*, a_{k+1}^*, \dots, a_K^*)]$ conditioned on further optimal decision making.

The optimal decisions are found by solving a dynamic program with the following terminal value functions. States are considered as terminal when there are no white balls left in the urn or all possible draws have taken place. Let $\Omega = \{s = [b,w,d]_k : w_k = 0 \lor k = K\}$ be the set of all terminal states. For terminal states, we have

$$V([b,w,d]_k) = r \cdot (w_0 - w_k) \ \forall s \in \Omega$$

In every non-terminal state, we look for the action that maximizes the state's value:

$$V([b,w,d]_k) = max(Vi([b,w,d]_k), Vh([b,w,d]_k))$$

In this expression, $Vh\big([b,w,d]_{z_k}\big)$ $(Vi\big([b,w,d]_{z_k}\big)$ denotes the expected future value when the ball drawn is returned to the urn (sorted out) under the condition of further optimal decision making.

In the FBM, $Vh([b,w,d]_k)$ $(Vi([b,w,d]_k))$ denotes the expected available budget that is achieved when the ball drawn in state $[b,w,d]_k$ is returned (sorted out), i.e.

$$Vh([b,w,d]_k) = E[BA_K|\ a_k = 0, (b,w,d)_k]$$
$$Vi([b,w,d]_k)) = E[BA_K|\ a_k = 1, (b,w,d)_k]$$

If a black ball is drawn, i.e., $d_k = 0$, we have,

$$Vh([b,w,0]_k) = \alpha \cdot V([b,w,0]_{k+1}) + (1-\alpha) \cdot V([b,w,1]_{k+1}) - c_h$$
$$Vi([b,w,0]_k) = \beta \cdot V([b-1,w,0]_{k+1}) + (1-\beta) \cdot V([b-1,w,1]_{k+1}) - c_i$$

If a white ball is drawn, i.e., $d_k = 1$, we have,

$$Vh([b,w,1]_k) = \gamma \cdot V([b+1,w-1,0]_{k+1}) + (1-\gamma) \cdot V([b+1,w-1,1]_{k+1}) - c_h$$
$$Vi([b,w,1]_k) = \delta \cdot V([b,w-1,0]_{k+1}) + (1-\delta) \cdot V([b,w-1,1]_{k+1}) - c_i$$

The MDPs were implemented in MATLAB R2017a.

A2 Additional Treatments: Varying time horizon

We conducted two additional CBM treatments for time and two additional FBM treatments for money. The CBM treatments vary in the currently available budget, while the FBM treatments vary in the number of draws K. Table 10 gives an overview:

Table 10: Parameter for the Additional Treatments

Treatment	K	Participants	Treatment	K	Participants
CBM-T1	29	29	FBM-M1	14	30
CBM-T	40	30	FBM-M	18	25
CBM-T3	49	30	FBM-M3	22	30
Invest (sort out)	$c_{invest} = 20$		Invest (sort out)	$c_{invest} = 30$	
Harvest (return)	$c_{harvest} = 10$		Harvest (return)	$c_{harvest} = 0$	

The new data show that our key insights from the original experiments continue to hold. We use a statistical model to examine the influence of t and t^2 on the cumulative investment quantities. For each treatment, we run a random effects regression that accounts for individual heterogeneity:

$$q(\mathrm{i,t}) = \beta_0 + \beta_t \cdot t + \beta_{t^2} \cdot t^2 + u_i + \epsilon_{it}$$

The subscript i indicates the participant, and the t is the index for the time periods. The dependent variables $q(\mathrm{i,t})$ are continuous. There are two error terms: u_i is pair-specific controlling for heterogeneity, and ϵ_{it} is independent across all observations. Table 11 shows the regression results.

Table 11: Additional Treatments Regression Results

	Time		Money	
	CBM-T1	CBM-T3	FBM-M1	FBM-M3
Constant β_0	$+0.01$	-0.37	-0.18	-0.19
	$(p = 0.97)$	$(p = 0.43)$	$(p = 0.80)$	$(p = 0.85)$
Coefficient β_t	$+0.10$	$+0.34$	$+0.53$	$+0.44$
	$(p = 0.00)$	$(p = 0.00)$	$(p = 0.00)$	$(p = 0.00)$
Coefficient β_{t^2}	$+0.002$	-0.0002	-0.006	$+0.0009$
	$(p = 0.01)$	$(p = 0.61)$	$(p = 0.36)$	$(p = 0.76)$
R^2(overall)	0.24	0.50	0.19	0.22
Wald χ^2	982.76	5824.89	404.33	815.11
Prob $> \chi^2$	$(p = 0.00)$	$(p = 0.00)$	$(p = 0.00)$	$(p = 0.00)$
σ_u	1.29	2.29	3.29	4.89
σ_e	1.14	2.21	1.82	2.62
ρ	0.56	0.52	0.76	0.78

The overall regressions are significant. In all treatments, the constants are not significant ($p > 0.1$), and the coefficients of t are positive and significant ($p = 0.00$). The coefficient of t^2 is only significant in the short time treatment and positive, indicating that slightly more investments are made towards the end. We do not find additional evidence for time investments made earlier than money investments. Figure 23 and Figure 24 illustrate the averaged cumulated investments for the additional time and money treatments. Further research is needed to examine differences in timing between time and money investments.

Figure 23: Timing in Additional Time Treatments

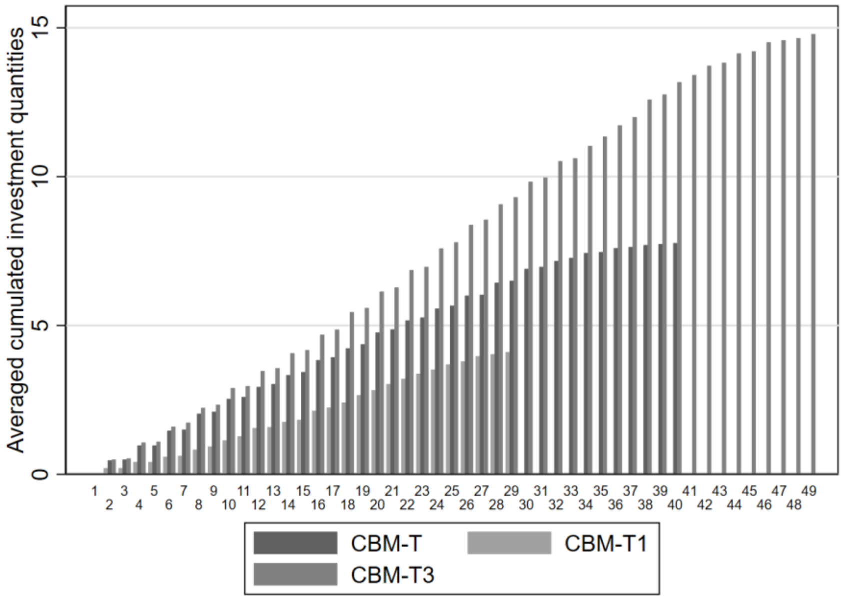
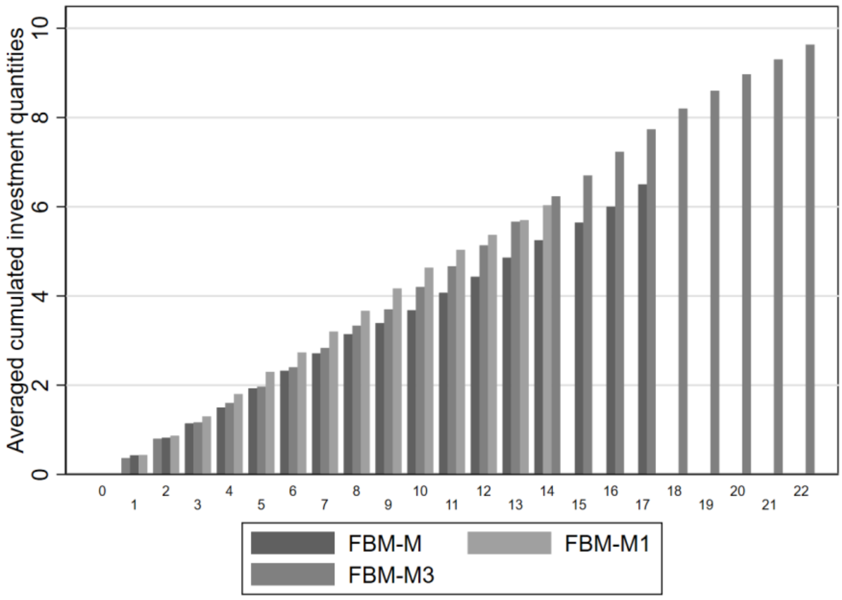


Figure 24: Timing in Additional Money Treatments



A3 Instructions for the Experiments

CBM

Welcome to the experimental laboratory. You are now taking part in an economic experiment.

The duration of this experiment is about 60 minutes.

- For the duration of the experiment, we ask you to observe a few rules: From now on, communication is strictly forbidden. If you have any questions, please address them to us. Raise your hand, and we will come to your cabin.
- The use of mobile phones and other technical devices is not permitted during the entire duration of the experiment. Switch them off or to silent mode and place them in the pockets on the curtain rod.
- It is forbidden to pursue other occupations during any waiting times that may occur.

Please comply with these rules to avoid immediate exclusion from the experiment and all payments.

In total, the experiment consists of two runs and one survey. The entire experiment is anonymous.

The second run is relevant for the payout. Your decisions in the second run determine the amount of your payout.

The profit is expressed in the experimental currency thaler and converted to 145 thaler = 1 Euro. The 50-Cent coin is the smallest that is issued. Payouts are rounded so that no one is worse off.

The payment takes place individually and anonymously after the experiment.

Please read the instructions carefully.

We thank you in advance for your participation and wish you good luck!

CBM-T

You start with a reward of **100 thaler**.

Initially, a hidden urn contains **30 white** balls **and 0 black** balls.

The procedure is as follows: You draw a ball from the urn.

If you draw a white ball, this ball will be dyed black, and you will receive a one-time **reward** of **100 thaler**.

You will not receive any reward if you draw a ball that is already black.

In total, the drawing of a ball as well as the eventual coloring of the ball takes **10 seconds**.

After drawing and possibly coloring the ball, you have **two choices**: Either return the ball to the urn or sort out the ball.

- If you return the ball to the urn, you may draw the already black-colored ball again and thus do not generate any additional reward. Instead, you can draw the next ball right away.
- If you sort out the ball, you will not draw that ball again. It takes **another 10** seconds to sort out a ball. During this time, you cannot draw a ball or make a decision.

You have a budget of **400** seconds.

The time it takes you to decide is independent of the time available.

CBM-M

You start with a reward of **100 points**.

Initially, an urn contains **30 white** balls **and 0 black** balls.

The procedure is as follows: You draw a ball from the urn.

If you draw a white ball, this ball will be dyed black, and you will receive a one-time **reward** of **100 points**.

You will not receive any reward if you draw a ball that is already black.

After drawing and possibly coloring the ball, you have **two choices**: Either return the ball to the urn or sort out the ball.

- If you return the ball to the urn, you may draw the already black-colored ball again and thus do not generate any additional reward. Returning a ball costs **10 thaler**. This cost reduces your budget.
- If you sort out the ball, you will not draw that ball again. Sorting out a ball costs **20 thaler**. This cost reduces your budget.

You have a budget of **400** thaler.

FBM

Welcome to the experimental laboratory. You are now taking part in an economic experiment.

The duration of this experiment is about 60 minutes.

For the duration of the experiment, we ask you to observe a few rules:

- From now on, communication is strictly forbidden. If you have any questions, please address them to us. Raise your hand, and we will come to your cabin.
- The use of mobile phones and other technical devices is not permitted during the entire duration of the experiment. Switch them off or to silent mode and place them in the pockets on the curtain rod.
- It is forbidden to pursue other occupations during any waiting times that may occur.

Please comply with these rules to avoid immediate exclusion from the experiment and all payments.

In total, the experiment consists of two runs and one survey. The entire experiment is anonymous.

The second run is relevant for the payout. Your decisions in the second run determine the amount of your payout.

The profit is expressed in the experimental currency thaler and converted to 90 thaler = 1 Euro. The 50-Cent coin is the smallest that is issued. Payouts are rounded so that no one is worse off.

The payment takes place individually and anonymously after the experiment.

Please read the instructions carefully.

We thank you in advance for your participation and wish you good luck!

FBM-T

You are working on subproject 1.

The project progress of subproject 1 influences the available time for subproject 2.

The procedure for subproject 1 is as follows:

You start with an available budget for subproject 2 of **100 seconds**.

Initially, a hidden urn contains **30 white** balls **and 0 black** balls.

The procedure is as follows: You draw a ball from the urn.

If you draw a white ball, this ball will be colored black. This project progress achieves **100 additional seconds** for subproject 2.

If you draw a ball that is already black, **no additional time** is obtained for subproject 2.

After drawing and possibly coloring the ball, you have **two choices**: Either return the ball to the urn or sort out the ball.

- If you return the ball to the urn, you may draw the black ball again and thus do not generate additional time for subproject 2.
- If you sort out the ball, you will not draw that ball again. Sorting out a ball reduces the available time for subproject 2 by **30 seconds**.

In total, drawing a ball, possibly coloring it, and sorting it out or returning it takes **10 seconds.** This time reduces the time for subproject 1.

You have **180 seconds** for subproject 1.

The time it takes you to decide is independent of the time available.

FBM-M

You are working on subproject 1.

The project progress of subproject 1 influences the money available for subproject 2.

The procedure for subproject 1 is as follows:

You start with an available budget for subproject 2 of **100 thaler**.

Initially, an urn contains **30 white** balls **and 0 black** balls.

The procedure is as follows: You draw a ball from the urn.

If you draw a white ball, this ball will be colored black. This project progress achieves **100 additional thaler** for subproject 2.

If you draw a ball that is already black, **no additional money** is obtained for subproject 2.

After drawing and possibly coloring the ball, you have **two choices**: Either return the ball to the urn or sort out the ball.

- If you return the ball to the urn, you may draw the black ball again and thus do not generate additional money for subproject 2.
- If you sort out the ball, you will not draw that ball again. Sorting out a ball reduces the money available for subproject 2 by **30 thaler**.

In total, drawing a ball, possibly coloring it, and sorting it out or returning it takes **10 seconds.** This time reduces the time for subproject 1.

You have **180 seconds** for subproject 1.

The time it takes you to decide does not affect the time available.

FBM-TM: Opportunity Costs Saliency

You are working on subproject 1.

The project progress of subproject 1 influences the available time for subproject 2.

100 seconds of your available time for subproject 2 equals 100 thaler.

The procedure for subproject 1 is as follows:

You start with an available time budget for subproject 2 of **100 seconds (=100 thaler).**

Initially, a hidden urn contains **30 white** balls **and 0 black** balls.

The procedure is as follows: You draw a ball from the urn.

- If you draw a white ball, this ball will be colored black. This project progress achieves **100 additional seconds (=100 thaler)** for subproject 2.
- If you draw a ball already black, no additional time is obtained for subproject 2.

After drawing and possibly coloring the ball, you have two choices: Either return the ball to the urn or sort out the ball.

- If you return the ball to the urn, you may draw the black ball again and thus do not generate additional time (=money) for subproject 2.
- If you sort out the ball, you will not draw that ball again. Sorting out a ball reduces the time available for subproject 2 by **30 seconds (=30 thaler).**

In total, drawing a ball, possibly coloring it, and sorting it out or returning it takes **10 seconds**. This time reduces the time for subproject 1.

You have **180 seconds** for subproject 1.

The time it takes you to decide is independent of the time available.

The time available for subproject 2 is converted to 90 seconds = 90 thaler = 1 Euro. The 50-Cent coin is the smallest that is issued. Payouts are rounded so that no one is worse off.

Chapter 3

Should We Change the Decision Maker after Sunk Time Investments? Results from a Laboratory Experiment

Should We Change the Decision Maker after Sunk Time Investments? Results from a Laboratory Experiment

Johanna Dujesiefken

Abstract

We consider the behavioral sunk cost effect for time investments and conduct an incentivized dynamic laboratory experiment. In contrast to previous research, we examine the classical sunk cost situation, where a choice can be made between the sunk cost project and a superior alternative, and the situation where the sunk cost project is the superior project. Without responsibility for past unsuccessful investments, decision makers leave the project with sunk time investments – even if the project is superior. The experiments are based on a model that makes the relationships between the available time budget and the objective clear and suggests itself to further research in this area.

1. Introduction

The sunk cost effect is the “greater tendency to continue an endeavor once an investment in money, effort or time has been made” (Arkes & Blumer 1985) and one of many potential sources for escalating commitment (Staw & Ross 1989, Staw 1997). In the following, persisting to a failing course of action is termed as escalation. The study of sunk time investments is important because sunk investments often occur in practice, and early termination of unsuccessful projects saves companies resources and reduces disruption (Meredith 1988). Previous research has focused on classic sunk cost situations where the decision maker can choose between the project with sunk investments and a superior one. If the decision maker persists in a failing course of action, a sunk cost effect is documented, and a reverse effect otherwise. However, the advice to limit escalation is only appropriate when escalation should be limited (Heath 1995). Therefore, we compare two decision-making situations: In one, investments are made in the project with a lower expected profit, while in the other, investments are made in the project with a higher expected profit. The comparison reveals whether decision makers persist in a failing course of action or another behavior dominates. In particular, we analyze 1) whether people stay with the previous project or leave it in favor of the alternative and 2) how much is invested in the chosen project further down the line. Additionally, we investigate how an emphasis on the opportunity cost of time affects 1) project choice and 2) subsequent investment behavior.

The decision-making setting is as follows: We consider an environment with complete information and an absence of responsibility for past unsuccessfulness. These assumptions provide a baseline that can be relaxed in further experiments. The time investment is mapped in the experiment so that it neither gives particular pleasure nor is perceived as tedious, but is experienced as approximately neutral. Based on these assumptions, our results lay a foundation for deciding whether project managers should stay on the project or be replaced after past unsuccessful time investments.

The remainder of this paper is organized as follows: Section 2 summarizes the literature on the sunk cost effect of past time investments. In Section 3, a description of the hypotheses, the theoretical model, and the experiment is given. Section 4 outlines experimental results. Section 5 presents limitations of our work. Finally, Section 6 presents managerial insights, future research and summarizes our findings. The Appendix contains the instructions and sample decision screens from the experiment.

2. Literature Review

Escalation behavior after past time investments has been studied in various contexts: research & development (Schmidt & Calantone 2002, Manez et al. 2009), consumption (Ülkü et al. 2020), project management (Long et al. 2020), and auctions (Herrmann et al. 2015). In escalation situations, after irretrievable investments, either a dichotomous decision is made between "project A" and "project B", "terminate" and "continue", or a quantitative decision is made, such as how much to continue to invest in a project. In hypothetical surveys, participants imagine themselves in the situation description, while in non-hypothetical experiments, participants experience real decisions with real consequences. Hypothetical and non-hypothetical studies that have analyzed escalation following past time investments are described subsequently.

Soman (2001) analyzes choices between two projects and observes escalation for money but not for time. Escalation for time reappears only if a wage rate as a monetary reference for time is given or when subjects have received instruction about economic approaches to time (e.g., classroom discussion about consumers' scarcity and cost of time). The results are interpreted as follows: For time investments, individuals behave rationally until they are aware about the economic value of time.

Navarro & Fantino (2009) analyze decisions between "dig the ten more days to collect the 10 pounds of copper" or "abandon this 'Shady Creek' mine and go home" (Navarro & Fantino 2009). This is a dichotomous "continue vs. terminate" decision, where the "continue" option is quantitatively described. How the decision maker earns money when choosing "terminate" remains ambiguous. In their questionnaire studies, no tangible objective is stated. Escalation, i.e., project continuation, is observed in the presence of personal responsibility. However, they note that personal responsibility plays only a minor role. Further, they varied the qualitative description of sunk time and found no impact on the occurrence of a sunk cost effect.

In Coleman's experiment (2009), participants invested time to arrange a date online. They were then given the choice of either attending the date set online or going on a (better) blind date. A sunk cost effect for time is observed. (Coleman 2009) In another experiment by Coleman (2010), participants invested time or money to schedule chiropractic sessions. Then they decided how much time they wanted to spend on those chiropractic sessions or an alternative treatment with a better chance of success. Here, only a sunk cost effect for money, but not for time, is found. (Coleman 2010)

In Braverman & Blumenthal-Barby's (2012) experiment, participants are asked to recommend treatment based on one of four hypothetical clinical scenarios differing in source and type of prior investment. The ineffective treatment is recommended to be terminated, consistent with the normative rational response. (Braverman & Blumenthal-Barby 2012)

Herrmann et al. (2015) analyze market transaction data from approximately 7,000 pay-per-bid auctions concerning normatively irrational decisions. They show that participants with a higher behavioral investment are more likely to escalate. Delegating bidding to an automated bidding agent reduces escalation in subsequent decisions by the same participant. (Herrmann et al. 2015)

Nash et al. (2019) find that short-term physical investments (real or imagined) in tasks (e.g., card sorting task) either reveal a reverse sunk cost effect or fail to find an effect. However, the replication of the hypothetical experiments by Arkes & Blumer (1985) reveals a sunk cost effect (Nash et al. 2019).

Ülkü et al. (2020) find in a series of laboratory and field experiments and analysis of transaction data that people tend to purchase more when people spend a longer time waiting in line. Individuals try to amortize waiting times by larger or more expensive purchases. (Ülkü et al. 2020)

In the domain of time, four experimental studies of sunk time involve non-hypothetical choices. First, in Soman's 6th experiment, participants complete a long and a short questionnaire. For each completed questionnaire, they receive a gift as compensation. Escalation was observed only when the gift's monetary value was reported. However, the value of the gift was not linked to the time invested. (Soman 2001) Second, in Navarro & Fantino's (2009) experiment, two puzzles were played. The 2x2 design varies whether the start of the main puzzle is voluntary or compulsory and whether a long or short time has already been spent on the main puzzle. The voluntary group had the choice of working on the main puzzle after the preliminary puzzle, while the compulsory group was obliged to do so. The main puzzle was continued more often in the voluntary than in the compulsory group. It is shown that personal responsibility (voluntary decision) increases the probability of escalation, whereas no escalation is observed in the absence of personal responsibility (compulsory group). This is consistent with previous studies on escalation after monetary investments. From these, it is known that the willingness to invest is significantly greater if the decision maker is personally responsible for negative consequences. (Staw 1976). Third, in Cunha & Caldieraro's experiments (2009), participants were asked to rate products to decide which one to buy (effort manipulation). After the evaluation, a superior alternative was offered. The authors observe a greater tendency to switch to the new product with a better overall rating in the less effort than in the high effort condition when opportunity costs are low. For high opportunity costs, there was no effect. However, the results could not be replicated (Otto 2010).

Different results on whether a sunk cost effect occurs after past time investments may be attributed to the response options offered. Escalation is likely to be observed when a quantitative decision is made, e.g., investments in a mine (Navarro & Fantino 2009), the number of times an automated bidding agent is used (Herrmann et al. 2015), and the purchase amount (Ülkü et al. 2020). If time has been invested in the past, a higher investment quantity leads to a higher return per time invested and the individual perceives it as a better deal according to transaction utility theory (Thaler 1980). Dichotomous questions are asked to choose either the benefit-maximizing answer or the answer that includes sunk costs. However, neglecting the counterpart of escalation - examples of de-escalation - may lead us to misunderstand the way people invest and can result in giving the wrong advice to decision makers because advice to limit escalation is only appropriate when escalation should be limited. (Heath 1995)

The inconsistent results of the hypothesized studies could be due to participants using their own preferences instead of the preferences stated in the questionnaire. In Soman's first experiment (2001), participants are told that attending the rock concert is preferred. However, if the subject wants to attend both events, it is better to attend the theater performance for which he worked more and work again for the less labor-intensive rock concert ticket. The decision for the theater performance is evaluated as a sunk-cost response. (Friedman et al. 2007) It is assumed that in non-hypothetical studies, the stated objective is not replaced by an own objective.

Escalation can be explained by self-justification theory, prospect theory, and mental accounting. Internal justification is the psychological need to justify past decisions to oneself. In contrast, external justification is the desire to justify past decisions to others, either to save face or to fulfill the social norm that consistent decision makers are better executives (Staw 1976, Staw & Ross 1980, Berg 2009). Responsibility and the aversion to writing off past investments as a waste (Arkes & Blumer 1985) are motives for self-justification. Responsibility is assumed to increase escalation after sunk monetary cost but does not seem to be necessary for escalation to occur (Schoorman & Holahan 1996).

According to prospect theory, sunk investments induce a loss frame, causing risk-seeking behavior and escalation (Kahneman & Tversky 1979, Thaler 1980, Arkes & Blumer 1985, Whyte 1986). As sunk costs rise, a subject becomes less wealthy, increasing her risk aversion until she reaches the point of injection at the reference point. Then as the subject becomes less wealthy, she will be progressively more risk-seeking. Prospect theory predicts that the deeper subjects move into loss territory, the stronger the escalation.

Zeelenberg & van Dijk (1997) analyze risk attitudes in the gain and the loss domain after the investments of time and effort. As payment, participants choose either the sure win or a gamble. More risk-averse choices are made in the presence versus absence of prior work. Hence, in line with prospect theory, participants were more risk averse in gain than loss situations. Moreover, sunk investments of time and effort appear to produce more risk-averse decisions, i.e., a reverse sunk cost effect. (Zeelenberg & van Dijk 1997)

According to the theory of mental accounting (Thaler 1985), past investments are recorded in a mental account. If the mental account is not balanced by income, i.e., it is in the negative, efforts are made to close the account positively (in the black). As a result, past costs influence future decisions. Mental accounting would allow small changes to be of meaningful magnitude within a particular account (Weigel 2018).

3. Experiment

3.1 Hypotheses Development

Our experiment consists of a control group with no sunk time investments and three treatment groups with sunk time investments. The first treatment depicts a classical sunk cost situation in which a choice between the sunk-cost option and a superior option can be made. In the second treatment, a choice can be made between a sunk-cost option and an *inferior* option. The third treatment is similar to the first treatment, with the only change being that opportunity costs are emphasized. The experiments are described in detail in Section 3.3.

The first three competing hypotheses relate to project choice. According to the first hypothesis, we expect a sunk cost effect after past time investments.

Hypothesis 1A: Participants choose the option with sunk costs in all three treatments.

Decision situations in real life differ in the way the objective is formulated. From research on the consideration of past monetary investments, it is known that the following factors limit escalation. First, stating an objective, an explicit estimate of future returns, or pointing out the limited remaining time shifts attention from sunk costs to the future. Second, the presence of information that allows the decision maker to set an investment limit reduces escalation. Third, the ability to offset past investments and future profits reduces escalation. (Heath 1995, Tan & Yates 1995, Strough et al. 2014)

Our experiment contains factors that mitigate escalation: The objective function is explicitly stated, and a time budget, the use of which participants decide, is given. Hence, we formulate the following competing hypothesis:
Hypothesis 1B: Participants choose the option without sunk costs in all three treatments.

Another factor that reduces escalation is the absence of responsibility. Without responsibility, participants do not need to justify the unsuccessfulness internally or externally. According to self-justification theory, we would expect most participants to choose the profit-maximizing project. We state hypothesis 1C. Hypotheses 1A, 1B and 1C are competing hypotheses.
Hypothesis 1C: The occurrence of sunk investments drives participants to choose the profit-maximizing project. This means that participants in the classic sunk cost situation choose the option without sunk costs and choose the option with sunk costs in the other situation.

Previous literature states that decision makers fail to book past time investments in a mental account (Soman 2001). Emphasizing the alternative use of time is assumed to facilitate the mental accounting for time investments. Hence, we formulate the following hypothesis.
Hypothesis 2: Emphasis on alternative time use leads to an increased choice of the sunk cost option.

The following hypothesis relates to investment behavior after project choice. According to prospect theory, sunk investments induce a loss frame, which leads to risk-seeking behavior. In our experiment, less investment increases the profit variance and is therefore considered risk-taking behavior. Consequently, we expect fewer investments after past investments and formulate the following hypothesis.
Hypothesis 3: Participants who have experienced unsuccessful investments invest less than participants who have not experienced unsuccessful investments.

Our design allows linking the dichotomous decision of the ballot-box choice and the amount invested in the chosen project. According to mental accounting, participants who choose the sunk cost option escalate their commitment to close the account in the black. Hence, we formulate the following hypotheses:
Hypothesis 4: Participants who have experienced unsuccessful investments (sunk cost) and stay with the sunk cost option invest more than participants who have not experienced unsuccessful investments.

Analogously, participants leaving the sunk cost option do not need additional investment to bring the mental account into the black.

Hypothesis 5: Participants who have experienced unsuccessful investments (sunk cost) and choose the option without sunk cost do not invest more than participants who have not experienced unsuccessful investments.

The emphasis on the alternative use of time may lead to less investment. Hence, we formulate the following hypothesis.

Hypothesis 6: Emphasis on the alternative use of time results in less investment.

3.2 Theoretical Model

The theoretical model consists of a framework provided by the current-budget model (CBM), into which an experimental ballot-box model is inserted. The CBM was first introduced by Dujesiefken et al. (2022).

In the CBM, a project manager distinguishes between two types of time expenditures. Both may lead to expected profits. The shorter type leads to an unchanged or worse overall state of the project. It is referred to as a *harvest profit*. The longer type improves the project state overall, and is, hence, called an *investment*. For a project, a limited time budget is available. The project manager decides at what times he harvests profit or invests time.

An experimental ballot-box model is being inserted into the current-budget model as a current project. Let a ballot box contain black and white balls. Decision makers draw a ball, whereupon it is automatically colored black. Then decision makers decide whether this ball is sorted out or returned. Returning the ball reduces the budget by c_h time units, sorting out by $c_i > c_h$ time units. A ball returned, which is now black in any case, can be drawn again in the further course, while this is impossible with a ball sorted out.

In the CBM formulation, sorting out a black ball is equivalent to making an investment. The budget B is constructed so that there are $\overline{K}$ opportunities to draw and return a ball, i.e.,

$$B[\text{time}] = \overline{K} \cdot c_h.$$

Let the binary variable a_k denote whether the k-th ball drawn is sorted out ($a_k = 1$) or returned ($a_k = 0$). The budget usage BU_k after the draw of ball $k-1$ is

$$BU_k = c_i \cdot \sum_{j=1}^{k-1} a_j + c_h \cdot \sum_{j=1}^{k-1} (1 - a_j) \leq B \qquad \forall\, k = 2, \dots, K, \tag{1}$$

with $BU_1 = 0$.

The maximum number of balls, $K = \overline{K}$, will be drawn, when all balls are returned to the ballot box. For sorting out a ball the decision maker misses an opportunity to draw a ball, and $K < \overline{K}$ follows.

The last draw $K \leq \overline{K}$ takes place when the last white ball has been drawn, i.e., $w_K = 1$, or the budget is used up, i.e., $BU_{K+1} = B$.

The budget is expressed in time units to answer the research question of how past time investments are considered. The CBM can also be formulated for monetary investments and thus allows model-based comparisons between time investments and monetary investments (see Dujesiefken et al. 2022).

While the budget is expressed in a time unit, the profit is expressed in a monetary unit. The advantage of a monetary profit over a time-based profit is that it is concrete without specifying its concrete use.

Profits r result from drawing and blacking a white ball. Total profits $R_k[\text{money}]$ after returning or sorting out the k-th ball drawn are

$$R_k = r \cdot \sum_{j=1}^{k-1} d_j \qquad \forall\, k = 2, \ldots, K\,, \tag{2}$$

with R_1 being the initial profit / show-up fee.

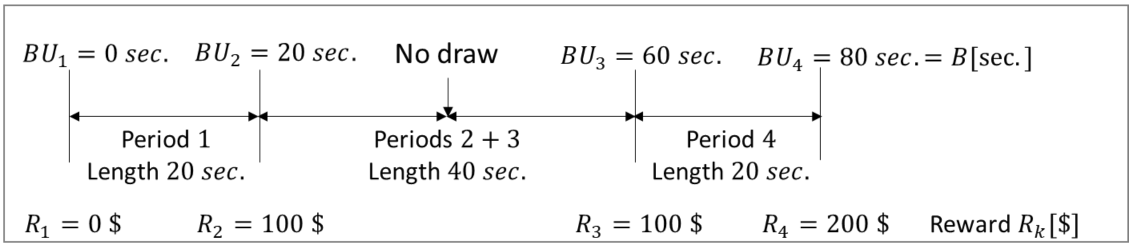
The decision maker's productivity state is characterized by a triplet $[b, w, d]_k$, in which $b_k \in N$ $(w_k \in N)$ denotes the number of black (white) balls in the ballot box after draw $k = 1, \ldots, K$ and $d_k \in \{0,1\}$ specifies, whether the k-th ball drawn is black $(d_k = 0)$ or white $(d_k = 1)$. Between k and $k+1$, decision makers draw a ball (knowing the ballot box composition), color it, and decide on sorting the ball out or returning it. The latter choice determines the ballot box composition in $k + 1$ (see figure 1).

Figure 1: Sequence of events

state $[b, w, d]_k$				Update state $[b, w, d]_{k+1}$
k	Draw ball	Coloring if $\text{d} = 1$	Invest or harvest?	$k + 1$

Figure 2 illustrates the following example**:** The decision maker has a time budget of 4 periods of 20 seconds each, i.e., $B = 80$ seconds. At the beginning, the decision maker draws a white ball ($d_1 = 1$) and decides to harvest it ($a_1 = 0$). After returning the ball, which takes 20 seconds, the next ball is drawn and sorted out at the beginning of period 2 ($a_2 = 1$). Since sorting out the ball takes 20 seconds longer than returning it ($c_i = 40$ seconds), no ball can be drawn in period 3. At the beginning of period 4, i.e. after $BU_3 = 60$ seconds, the last ball $K = 3$ is drawn and returned. Thus, the budget is used up, i.e., $BU_4 = B = 80$ seconds.

Figure 2: Example for a resource allocation ($B = 4 \cdot 20\ sec.; c_i = 40\ sec.$)



3.3 Experimental Protocol

The experiment was programmed and conducted in oTree (Chen et al. 2016). A total of 133 students were recruited via the Hamburg registration and organization online tool hroot (Bock et al. 2014), of which 33 participants were assigned to the control group, 32 participants each to T1 and T2, and 36 to T1-OC. The experiments were digitally run by the Experimental Laboratory of the University of Hamburg in four sessions, May 6-8, 2020, and May 20, 2022.

After participants read the instructions, which are available in Appendix A1, they were asked to answer comprehension questions. These comprehension questions are programmed such that several attempts are possible. A trial run was then held to ensure understanding of the ballot-box model.

After subjects selected a ballot box and made their investment decisions, they completed a post-experiential questionnaire, in which we asked questions regarding participants' attitudes and preferences and general questions about the experiment. We also collected demographic data. Each treatment lasted no longer than 30 minutes, with average subject earnings of 7.11 €. (9.44 € in the control group, 6.36 € in T1, 5.81 € in T2 and 6.81 € in T1-OC.) The exchange rate is 200 thaler = 1 Euro. Earnings were transferred directly after the experiment.

3.4 Treatment Overview

In the experiment, the current-budget model is used with a ballot-box model and the following parameters: One round takes 10 seconds. Drawing, possible coloring, and sorting out takes $c_i = 20$ seconds, while drawing, possible coloring, and returning takes $c_h = 10$ seconds. A budget of $B = 290$ seconds is available in the treatments. Sunk time investments are exogenously predetermined and consume 120 seconds of the budget, leaving 170 seconds for participants to decide how to spend. In the control group, participants could decide on a time budget of $B = 290$ seconds. The draw of a white ball results in a win of $r = 100$ thaler.

Participants choose either the ballot box with 8 black and 40 white balls or the one with 16 black and 80 white balls. In the treatments, before choosing the ballot box, participants invested time in either the ballot box with 8 black and 46 white balls or the one with 16 black and 86 white balls (see Figure

4). For all four ballot boxes, it is optimal to return all balls. The profit-maximizing ballot box is the one with 16 black and 80 white balls. Based on an available time budget of $B = 170$ seconds, the ballot box with 16 black and 80 white balls results in an expected profit of 1390 thaler (=9.09 €), whereas the ballot box with 8 black and 40 white balls yields an expected profit of 1215 thaler (=8.38 €). After sorting out, participants in all groups observe the hourglass for 10 seconds, see Figure 3. For sorting out, the participant forgoes the opportunity to draw a white ball and generate profit, representing the opportunity cost for a time investment.

Figure 3: Participants see the hourglass for 10 seconds after sorting out a ball

3.4.1 Control Group

In the trial run, 8 black and 46 white balls are in the initial ballot box (see Figure 5 in Appendix A2). After the trial run, participants are asked: *"Imagine the same situation as before. Which ballot box would you choose?"* Decision makers choose between a ballot box with 16 black and 80 white balls and one with 8 black and 40 white balls (see Figure 6 in Appendix A2). Then, decision makers continue the experiment with the chosen ballot box (see Figure 7 in Appendix A2). The control treatment is designed to detect how often the profit-maximizing ballot box with 16 black and 80 white balls is chosen and how much is invested in it further on. Example decision screens can be found in Appendix A2.

3.4.2 Treatment Group 1

After the trial run in which the initial ballot box contains 8 black and 46 white balls (see Figure 8 in Appendix A3), decision makers in treatment group 1 (T1) start with a ballot box containing 8 black and 46 white balls (see Figure 9 in Appendix A3). Then, decision makers are asked to use 120 seconds of their budget to sort out the first 6 balls. Due to a failure of the coloring machine, individuals do not receive the profit. Participants are told:

"Unfortunately, you receive a notification that the quality control has declared the previous 6 colored balls as rejects due to quality defects. You will not make any profit with these 6 balls. The ballot box in your project contains 8 black and 40 white balls.

So far, the project has not been successful because the investments are not matched by any profits and you have already invested 120 seconds in coloring and sorting out 6 balls.

Therefore, you consider not to continue the previous project and start a new one. At your disposal is a project where the ballot box contains 16 black and 80 white balls. You have not yet invested any time in this new project.

Quality control has changed the settings of the machine for coloring the balls. Now the colored balls meet the requirements of quality control. Since quality control has already been convinced of the quality of the colored balls, in the future you will no longer have to bring balls to quality control - regardless of your project choice. This means that from now on you will make profits as soon as you draw a white ball, as it will then be colored automatically."

After the sunk time experience, the same ballot boxes as in the control group are offered: A ballot box with 16 black and 80 white balls or one with 8 black and 40 white balls (see Figure 10 in Appendix A3). Then, decision makers continue the experiment with the chosen ballot box (see Figure 11 in Appendix A3). When decision makers stay with the former ballot box, the decision screen differs from that of the control group only by the sunk time (120 seconds instead of 0 seconds).

The order of the ballot boxes was not randomized. The ballot box of the previous project was stated first followed by that of the alternative project (see Figure 10 in Appendix A3).

This treatment depicts a classic escalation/sunk cost situation. The profit-maximizing decision is to *leave* the ballot box in which investment has already been made *and move* to the ballot box with 16 black and 80 white balls. Participants succumb to the sunk cost effect when they *stay* with the ballot box with 8 black balls and 40 white balls.

3.4.3 Treatment Group 2

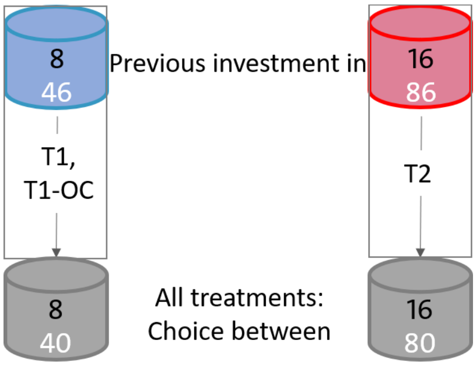
In contrast to T1, investments are made in the profit-maximizing ballot box in treatment group 2 (T2) (see Figure 13 in Appendix A4). In detail, the trial run starts with a ballot box containing 16 black and 86 white balls (see Figure 12 in Appendix A4). After that, decision makers start with a ballot box containing 16 black and 86 white balls. After being asked to sort out the first 6 balls, deciders choose between a ballot box with 16 black and 80 white balls and one with 8 black and 40 white balls (see Figure 14 in Appendix A4). Then, decision makers continue the experiment with the chosen ballot box (see Figure 15 in Appendix A4). Note that the decision screen is the same as for treatment group 1 (T1) if the same ballot box is selected.

As a counterpart to treatment group T1, the profit-maximizing response option simultaneously contains sunk costs in treatment T2. The profit-maximizing decision is to *stay* with the ballot box with 16 black and 80 white balls in which investment has already been made.

3.4.4 Treatment Group T1-OC

This treatment corresponds to treatment T1 with an additional emphasis on the alternative use of time. Whenever participants sort out a ball, the hourglass screen says " During this time you could have drawn a white ball and generated a profit of 100 thalers."

Figure 4: Treatment Overview



4. Results

We first present the results of the ballot-box choice for each group, followed by an analysis of investment behavior. Then, we discuss the implications of emphasizing the alternative use of time. Finally, we provide explanation approaches.

4.1 Ballot-Box Choice

The proportion of participants who chose the ballot box in which no investment had been made is presented in Table 1 for each treatment.

We start with the ballot-box choice if neither ballot box has already been invested in (C). In the absence of past investments, 63.63 % of participants chose the profit-maximizing ballot box with 16 black and 80 white balls, while only 36.36 % chose the ballot box with 8 black balls and 40 white balls. Whether the choice of a ballot box differs systematically from a random choice ($p_{\text{assumed}} = 0.5$) is determined using a binomial test at a significance level of 0.1. The difference is not statistically significant ($p = 0.16$, two-sided binomial test).

Table 1: Proportion of participants who chose a ballot box in which no investment had yet been made.

Group	Ballot box without sunk costs	Choice of the ballot box without sunk costs
Control	(16,80)	21/33 (63.63 %)
T1	(16,80)	30/32 (93.75 %)
T1-OC	(16,80)	31/36 (87.11 %)
Control	(8,40)	12/33 (36.36 %)
T2	(8,40)	25/32 (78.13 %)

Remark: The profit-maximizing ballot box contains 16 black and 80 white balls.

We proceed with the ballot-box choice if one of the two ballot boxes has been invested in. Chi-squared tests are performed at a significance level of 0.05 to determine the significance of the influence of past investments on ballot-box choice.

In T1, where investment was made in the ballot box with 8 black balls and 40 white balls, 93.75 % of participants selected the ballot box that had not been invested in, i.e., the ballot box with 16 black and 80 white balls. The difference to the control group is significant ($\chi^2 = 8.72, p < 0.01$). In this treatment, the majority of participants *left* the ballot box in which they had already invested and *moved* to the profit-maximizing ballot box. We observe a reverse sunk cost effect. The following treatment provides information on whether decision makers decide *against* the ballot box in which they have already invested or *in favor of* the profit-maximizing ballot box.

In T2, where investment had already been made in the ballot box with 16 black and 80 white balls, 78.13 % of participants selected the ballot box that had not yet been invested in, i.e., the ballot box with 8 black and 40 white balls. The difference to the control group is significant ($\chi^2 = 11.55, p < 0.01$). As before, the majority of participants *left* the ballot box in which they had already invested. However, they moved to the ballot box, which leads to a lower expected profit.

In T1-OC, 87.11 % of participants *left* the ballot box with sunk costs. The difference to T1 is not statistically significant ($\chi^2 = 1.07, p > 0.30$). Hence, emphasizing the alternative use of time does not impact the ballot-box choice, which contradicts hypothesis 2.

In all treatments, the majority of decision makers *left* the ballot box in which they have invested. Hence, we find support for hypothesis 1B and not for the competing hypotheses 1A and 1C. Taken together, we observe neither a sunk cost effect nor a reverse sunk cost effect but an escape from the ballot box in which past investments did not lead to success.

4.2 Investment Quantity after Ballot-Box Choice

After selecting a ballot box, participants drew balls and decided whether each would be sorted out or returned. The following analysis shows how ballot-box choice and past investments affect the number of balls sorted out. These influences are tested for significance at a significance level of 0.1 using the Mann-Whitney test (MWU) and the Kruskal-Wallis test (KW). The MWU is used to test whether two groups each differ significantly in investment behavior, while the KW is used to test whether there are significant differences in investment behavior among all four groups. Table 2 shows the mean number of balls sorted out for each group and ballot box. Due to an error in the experimental procedure, the time budget B of participants in the control group was too high, namely 290 seconds instead of 170 seconds. We approximate the participants' investments in the control group using two methods. 1) Only investments up to period 170 are considered, as most participants show a relatively constant investment rate. 2) The investment quantity after using up the budget of 290 seconds is multiplied by $(170/290) = 0.59$. The following analysis is based on the application of approximation method 1. Any deviations when using the second approximation method are indicated.

Table 2: Mean number of balls sorted out for each group and ballot box

Group	Ballot box (16,80)	Ballot box (8,40)
Control	1.90 (3.05)	3.83 (6.33)
T1	1.27 (*left*)	6.00 (*stayed*)
T1-OC	0.32 (*left*)	0.80 (*stayed*)
T2	2.57 (*stayed*)	1.64 (*left*)

Remark: In all groups, it is optimal not to invest. In the control group, the results using approximation method 2 are given in parentheses. In the treatment groups, whether participants *stayed* with the sunk investment ballot box or *left* it is indicated in parentheses.

First, we consider the number of balls sorted out after selecting the ballot box with 16 black and 80 white balls. Participants in the control group sort out on average 1.90 balls. In T1, where there was no investment in this ballot box, 1.27 balls were sorted out on average. When opportunity costs are emphasized, on average 0.32 investments are made. In T2, where this ballot box has already been invested, 2.57 balls were sorted out. Most investments were made when the sunk cost urn was chosen. Overall differences are statistically significant ($KW, \chi^2 = 7.57, p = 0.06$), but pairwise differences are not ($MWU, p \geq 0.26$). Hence, we observe a treatment effect. However, this effect is weak, so the pairwise differences are not significant. Using approximation method 2 also yields overall significant differences ($KW, \chi^2 = 9.71, p = 0.02$) and insignificant pairwise differences ($MWU, p \geq 0.18$). The individual p-values are listed in Appendix A5.

Next, we consider the number of balls sorted out after selecting the ballot box with 8 black balls and 40 white balls. Participants in the control group sort out on average 3.83 balls. In T1, where this ballot box has already been invested, 6 balls were sorted out on average. When opportunity costs are emphasized, on average 0.8 investments are made. In T2, where this ballot box has not been invested, an average of 1.64 balls were sorted out. Overall and pairwise differences are statistically significant ($KW, \chi^2 = 8.80, p = 0.03$; $MWU, p \leq 0.05$). Significant overall and pairwise differences also result from using approximation method 2 ($KW, \chi^2 = 12.08, p = 0.01$; $MWU, p \leq 0.01$). The individual p-values are listed in Appendix A5.

Taken together, participants who chose the sunk cost ballot box made the most overinvestment, i.e., they escalated their commitment, which contradicts hypothesis 3 and supports hypothesis 4. Participants who left the ballot box with sunk costs made the least overinvestment, i.e., they kept their commitment constant or de-escalated, supporting hypothesis 5. Emphasizing the alternative use of time results in fewer investments, which supports hypothesis 6.

In this experiment, a tendency toward early investment is not observed (see Appendix A6). Another experiment found that time investments are made slightly earlier than money investments (Dujesiefken et al. 2022). The experiments differ mainly in the size of the time budget and the initial ballot box composition.

4.3 Behavioral interpretation

Mental accounting best explains our results. Since most participants left the ballot box with past time investments, we conclude that past investments are booked and tracked. The explicit statement of expenses and income supports the mental accounting of time investments. Participants who chose the sunk cost ballot box escalated their commitment to balance the mental account. In contrast, de-escalating behavior is observed for participants who left the sunk cost ballot box.

Without responsibility, participants do not need to justify the unsuccessfulness internally or externally. According to self-justification theory, we would expect most participants to choose the profit-maximizing ballot box. However, in our experiment, most participants *left* the ballot box with sunk costs, even if this is the profit-maximizing one. According to self-justification theory, participants who *stayed* with the sunk cost ballot box justify their choice with escalating commitment, while participants who *left* the sunk cost ballot box do not need to escalate their commitment.

Prospect theory does not lend itself to explaining our results. According to prospect theory, sunk investments induce a loss frame, resulting in risk-seeking behavior. However, the majority of participants *left* the sunk cost ballot box. Those who *stayed* with the sunk cost ballot box overinvested more, avoiding the risk of drawing a black ball for which there is no profit.

5. Limitations

The following four aspects of the experimental design might have contributed to the result that participants in T1 and T2 leave the ballot box with sunk investments. Changing these aspects may lead to past investments influencing the decision. 1) Participants may be concerned about the coloring machine failing again, although they were told it had been repaired and would not fail again. Redacting this concern may lead more participants in T1 and T2 to stay with the ballot box with past investments, and past investments would influence decision making.
2) After sunk investments participants do not receive profits. In the current-budget model, time investments and profits cannot be offset (such as deposits and withdrawals). Probably, individuals do not perceive the mental account to be in the red and, hence, do not perceive the lack of profits as a loss. The offsetting of investments and profits is possible with the future-budget model (see Dujesiefken et al. 2022).
3) Past investments were not based on voluntary decisions by participants (in T1 and T2), but were exogenously assigned. Exogenous assignment may cause participants to feel irresponsible (see Staw 1976, Weigel 2018). Therefore, individuals tend to leave the ballot box with previous investments.
4) The reason for unsuccessful past investments was explained in detail to the participants. Possibly, a causal link between the broken machine and the ballot box with past investments was established and therefore the ballot box with past investments was abandoned.
Finally, the ballot boxes available for selection may differ too slightly in their expected profits. If the ballot boxes differed more in expected profits, more participants in T2 would likely stay with the profit-maximizing ballot box despite sunk costs, while this change would likely not affect participants' decisions in T1. This would lead to the overall result that participants would not leave the ballot box with past investments, but would also not result in a sunk cost effect.

6. Discussion

6.1 Managerial insights

We consider the behavioral sunk-cost effect for time investments and conduct an incentivized dynamic laboratory experiment where decision makers are not responsible for past unsuccessful investments. In contrast to previous studies, we consider not only the classical sunk-cost situation in which a choice can be made between the sunk-cost project and a superior alternative but also the situation in which the sunk-cost project is the superior project. In both situations, decision makers leave the project with past unsuccessful investments - even if the previous project is superior. Knowledge of the projects' economics and the tendency to leave the project with past unsuccessful investments is necessary to make good decisions. If the project with no previous investments is seen as more promising, the project manager will likely behave optimally and change the project after unsuccessful investments. No escalation is expected. If the previous project is seen as more promising despite unsuccessful investments and the project manager stays on the project, escalation of commitment is likely. This escalation can potentially be prevented by bringing in a new project manager. As other research has shown, appointing a new project manager to prevent escalation has proven effective (Boulding et al. 2017).

6.2 Future research

In this paper, the CBM from Dujesiefken (2022) was taken as the baseline model. The model-based investigation of the impact of past investments has the advantage that essential relations between available time and the objective are given. This leads to the following directions for future research. First, we propose to insert experimental models that are more practical than the ballot-box model (e.g., project management) into the CBM to verify and generalize our results. In particular, investments should be assigned endogenously. This way, individuals invest voluntarily and are responsible for the investments. Further, the reason for unsuccessfulness should be given in less detail. Second, the CBM can be formulated for other dimensions, such as money and effort, for which the sunk cost effect is documented and thus enables comparative model-based research. Third, our experiment did not account for possible errors that arise in time estimates because the durations of the actions were explicitly stated (see Kahneman & Tversky 1979). Fourth, research could analyze whether the presence versus absence of an explicit objective impacts the consideration of past time investments. Fifth, analyzing the generalizability of our key findings to a) units of time other than seconds, b) situations in which a goal is to be attained, c) other subject pools, and d) experiments based on a real effort task, present opportunities for further research.

6.3 Summary

We consider the behavioral sunk cost effect for time investments and conduct an incentivized dynamic laboratory experiment. In contrast to previous research, we consider the classical sunk cost situation, where a choice can be made between the sunk cost project and a superior alternative, and the situation where the sunk cost project is the superior project. Considering both situations leads to the conclusion that most participants leave the sunk cost project - even if this is the superior project. After choosing the project with sunk time, participants escalate their commitment, while after choosing the project without sunk time, participants keep their commitment constant or de-escalate. Additionally, emphasizing the opportunity costs of time does not impact project choice but leads to less commitment in the further course. Our results are valid in an environment where the decision maker is not responsible for past unsuccessful investments and where the decision maker uses a time budget to generate a monetary profit. In such an environment, our results suggest the following managerial implications: First, it is not necessary to force the appointment of a new project manager. After all, if the project manager can decide by himself, he will switch the project after unsuccessful investment. In the further course, he does not tend to escalate the commitment. Second, if the project management is not changed regarding personnel, the previous project manager must be convinced to continue managing the project. However, he will escalate his commitment. Emphasizing alternative uses of time mitigates the escalation of engagement. We discuss different explanation approaches and find that mental accounting best explains our results. Directions for future model-based research to further explore the consideration of past time investments are given.

All research data in this paper can be accessed at 10.25592/uhhfdm.12248.

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Appendix

A1 Instructions

A warm welcome to you! Welcome to the online experimental laboratory! You are now taking part in an economic experiment. The experiment takes about 30 minutes. Please observe the following rules for the duration of the experiment:

- Sit down with your notebook in an undisturbed place.
- Let other people know that you do not want to be disturbed for the next 30 minutes. If necessary, hang a sign on the door.
- Communication is prohibited in experiments. Therefore, do not communicate with other persons during the experiment.
- Close all applications on your notebook except for this experiment. In particular, close email and chat notifications.
- The use of telephones and other technical devices is not permitted in experiments. Switch your telephone(s) and other technical devices off or silent.
- Do not engage in other activities during any waiting periods that may occur.
- Please use a browser other than the Internet Explorer.

Failure to comply with these rules will result in immediate exclusion from the experiment and all payments. You may take notes. Have a sheet of paper and a pen ready. You may use a non-programmable calculator.

In total, the experiment consists of four comprehension questions, two runs, and one survey. The entire experiment is anonymous.

The second run is relevant for the payout. Your decisions in the second run determine the amount of your payout. The profit is expressed in the experimental unit thaler and converted in the ratio 200 thaler = 1 Euro. The 50-Cent coin is the smallest that is issued. Payouts are rounded so that no one is worse off. The payout will be transferred to you after the experiment.

We wish you good luck!

Instructions

Please read the instructions carefully.

Your starting budget is 100 thalers. In the beginning, there are a certain number of white and black balls in a ballot box that cannot be seen.

The procedure is as follows: You draw a ball from the ballot box.

- If you draw a white ball, this ball will be colored black, and you will receive a one-time profit of 100 thalers.
- If you draw a ball that is already black, you will not get any profit.

In total, the drawing of a ball and the eventual coloring of the ball takes 10 seconds. After drawing and possibly coloring the ball, you have two options: You return the ball to the ballot box, or you sort out the ball.

- If you return the ball to the ballot box, it may happen in the further course that you draw the already black-colored ball again and thus generate no additional profit. Instead, you can draw the next ball directly.
- When you sort out the ball, you will not draw that ball again. Sorting out a ball takes another 10 seconds. During this time, you may not draw a ball or make a decision.

You have 290 seconds. The time it takes you to decide is independent of the time available. You will find these instructions at the bottom of each page for reference.

A2 Decision screens for the Control group

Figure 5: Sample decision screen for the trial run

Decision

Ballot box information

0 seconds of 290 seconds have passed. There are 8 black and 46 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Profit
0	100

Your decision

- ○ Sort out
- ○ Return

Next

Figure 6: Sample decision screen for choosing a ballot box in the control group

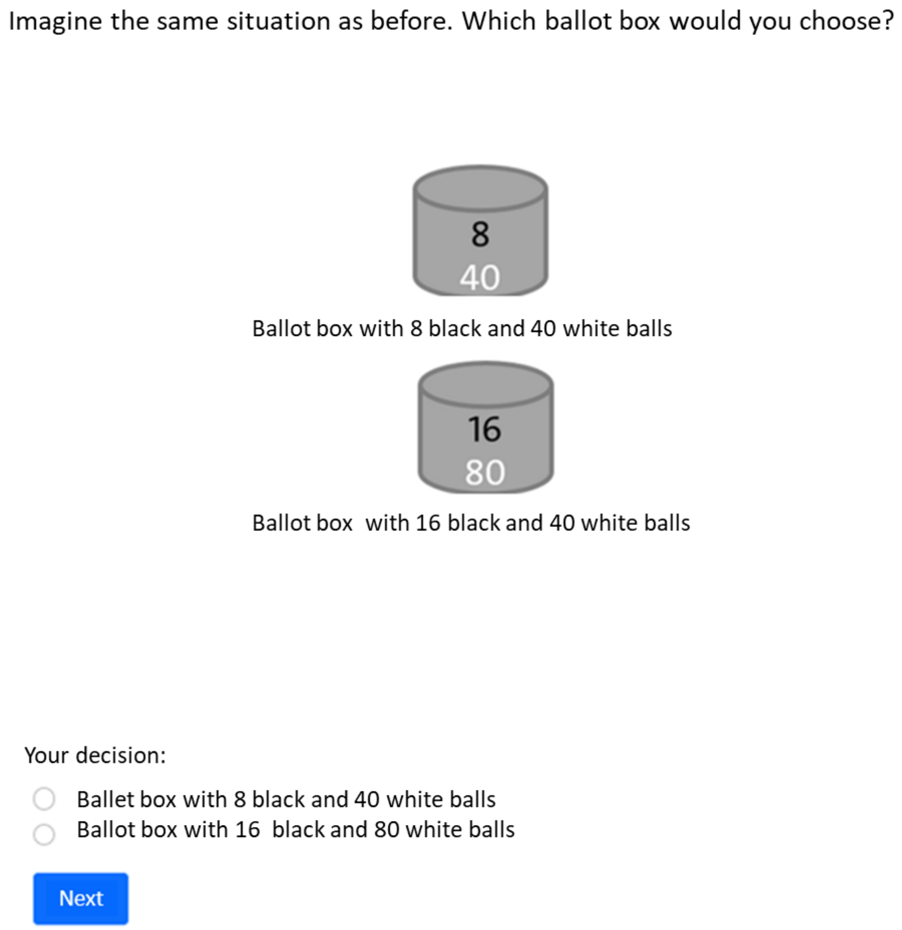
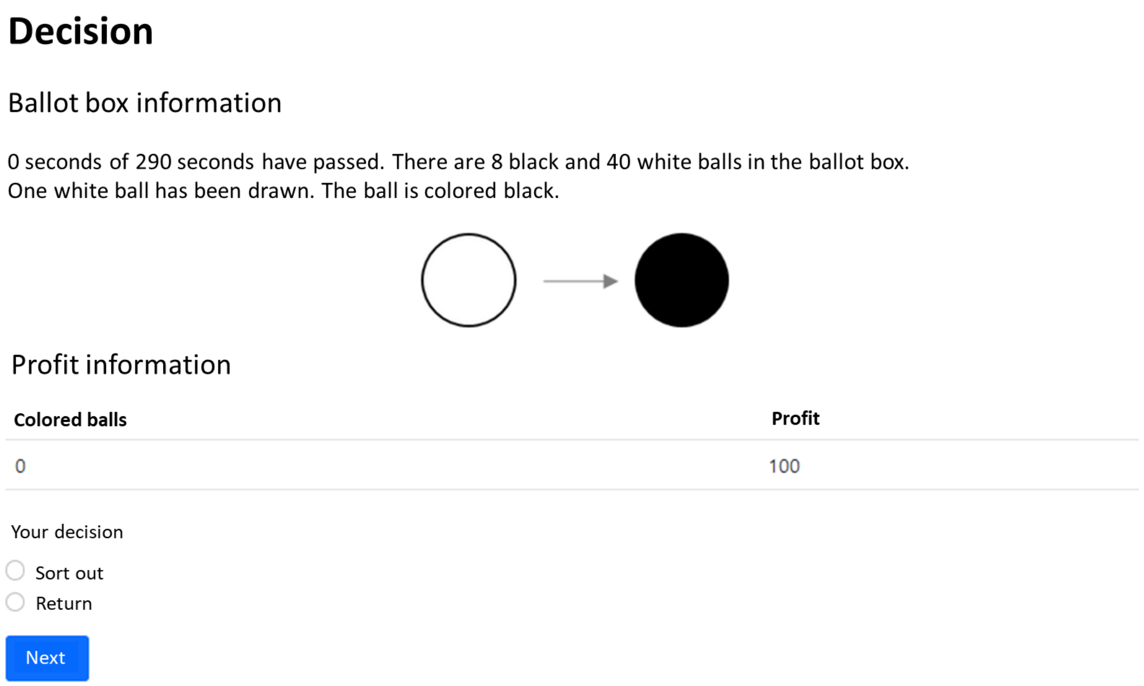


Figure 7: Sample decision screen after choosing the ballet box with 8 black and 40 white balls



A3 Decision screens for T1

Figure 8: Sample decision screen for the trial run

Decision

Ballot box information

0 seconds of 290 seconds have passed. There are 8 black and 46 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Profit
0	100

Your decision

- ○ Sort out
- ○ Return

Next

Figure 9: Sample decision screen for the phase of past time investments

Decision

Ballot box information

0 seconds of 290 seconds have passed. There are 8 black and 46 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Balls in quality control	Profit
0	0	100 thaler

The ball is requested by quality control. Please bring the ball to quality control:

- ○ Sort out into quality control
- ○ Return

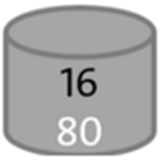
Next

Figure 10: Sample decision screen for selecting a ballot box

Quality control message

Unfortunately, you receive a notification that the quality control has declared the previous 6 colored balls as rejects due to quality defects. You will not make any profit with these 6 balls. The ballot box in your project contains 8 black and 40 white balls.
So far, the project has not been successful because the investments are not matched by any profits and you have already invested 120 seconds in coloring and sorting out 6 balls. Therefore, you consider not to continue the previous project and start a new one. At your disposal is a project where the ballot box contains 16 black and 80 white balls. You have not yet invested any time in this new project.

Ballot box of the previous project



Ballot box of the new project

Quality control has changed the settings of the machine for coloring the balls. Now the colored balls meet the requirements of quality control. Since quality control has already been convinced of the quality of the colored balls, in the future you will no longer have to bring balls to quality control - regardless of your project choice. This means that from now on you will make profits as soon as you draw a white ball, as it will then be colored automatically.

Your decision:

- ○ Your previous project
- ○ New project

Next

Figure 11: Sample decision screen after choosing the ballot box with 8 black and 40 white balls

Decision

Ballot box information

120 seconds of 290 seconds have passed. There are 8 black and 40 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Balls in quality control	Profit
6	6	100 thaler

Your decision:

- Sort out
- Return

Next

A4 Decision screens for T2

Figure 12: Sample decision screen for the trial run

Decision

Ballot box information

0 seconds of 290 seconds have passed. There are 16 black and 86 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Profit
0	100

Your decision

- ○ Sort out
- ○ Return

Next

Figure 13: Sample decision screen for the phase of past time investments

Decision

Ballot box information

0 seconds of 290 seconds have passed. There are 16 black and 86 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Balls in quality control	Profit
0	0	100 thaler

The ball is requested by quality control. Please bring the ball to quality control:

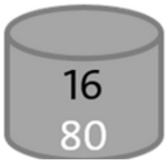
- ○ Sort out into quality control
- ○ Return

Next

Figure 14: Sample decision screen for selecting a ballot box

Quality control message

Unfortunately, you receive a notification that the quality control has declared the previous 6 colored balls as rejects due to quality defects. You will not make any profit with these 6 balls. The ballot box in your project contains 8 black and 40 white balls.
So far, the project has not been successful because the investments are not matched by any profits and you have already invested 120 seconds in coloring and sorting out 6 balls. Therefore, you consider not to continue the previous project and start a new one. At your disposal is a project where the ballot box contains 16 black and 80 white balls. You have not yet invested any time in this new project.



Ballot box of the previous project

Ballot box of the new project

Quality control has changed the settings of the machine for coloring the balls. Now the colored balls meet the requirements of quality control. Since quality control has already been convinced of the quality of the colored balls, in the future you will no longer have to bring balls to quality control - regardless of your project choice. This means that from now on you will make profits as soon as you draw a white ball, as it will then be colored automatically.

Your decision:

- ◯ Your previous project
- ◯ New project

Next

Figure 15: Sample decision screen after choosing the ballot box with 8 black and 40 white balls

Decision

Ballot box information

120 seconds of 290 seconds have passed. There are 8 black and 40 white balls in the ballot box.
One white ball has been drawn. The ball is colored black.

Profit information

Colored balls	Balls in quality control	Profit
6	6	100 thaler

Your decision:

- Sort out
- Return

Next

A5 Reported p-values

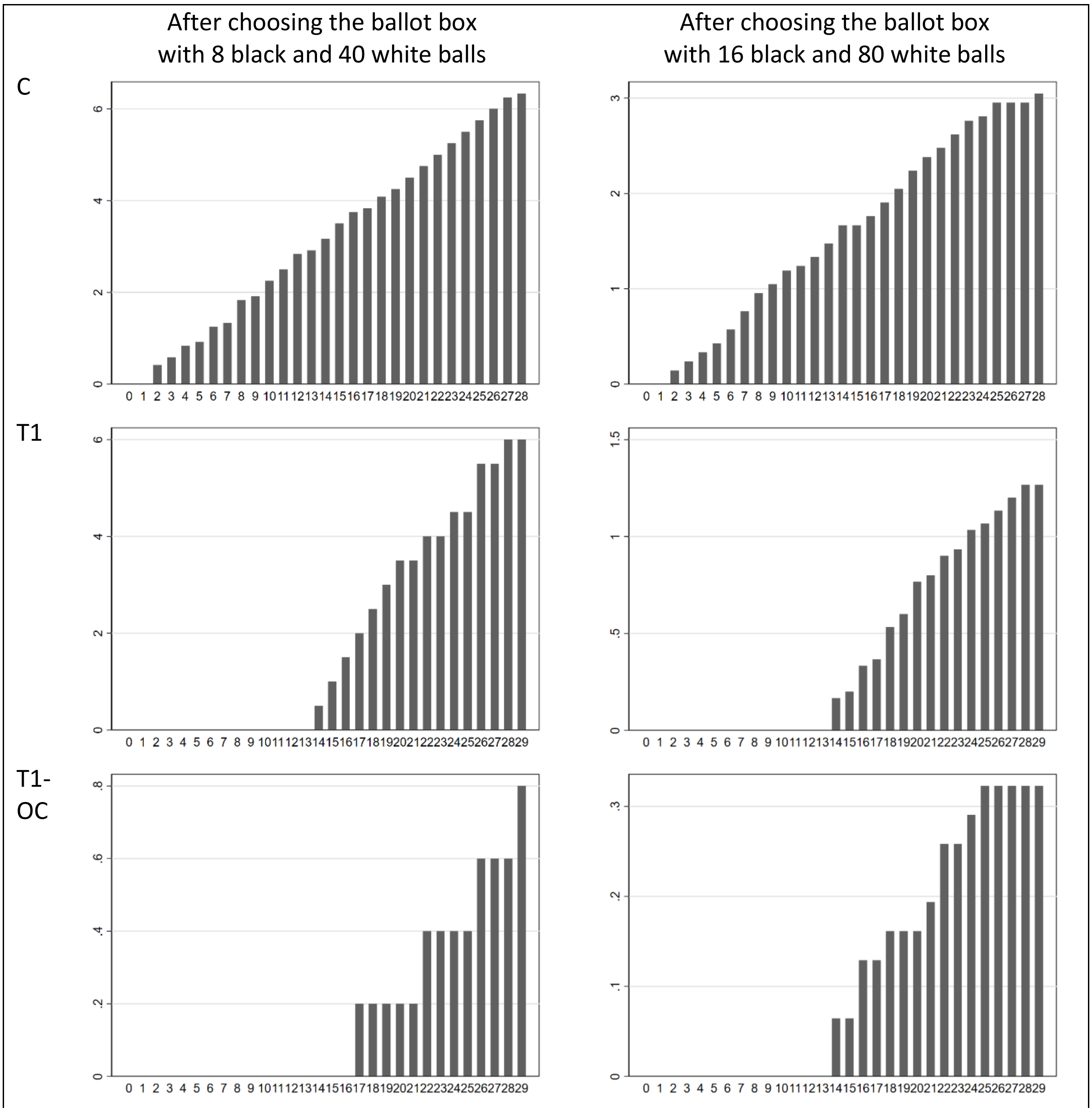
Table 3. Reported p-values

Comparison	Test	p-value
Project choice within the control group	Binomial test	p=0.163
Choice of ballot box (16,80) between T1 and Control	Chi-squared test	p=0.003
Choice of ballot box (8,40) between T2 and Control	Chi-squared test	p=0.001
Choice of ballot box (16,80) between T1 and T1-OC	Chi-squared test	p=0.301
Escalation between T1 and T2: Ballot box (16,80)	Mann-Whitney test	p=0.388
Escalation between T1 and control: Ballot box (16,80)	Mann-Whitney test	p=0.434
Escalation between T2 and control: Ballot box (16,80)	Mann-Whitney test	p=0.755
Escalation between T1 and T1-OC: Ballot box (16,80)	Mann-Whitney test	p=0.265
Escalation between all treatments: Ballot box (16,80)	Kruskal-Wallis test (with ties)	p=0.056
Escalation between T1 and T2: Ballot box (8,40)	Mann-Whitney test	p=0.042
Escalation between T1 and control: Ballot box (8,40)	Mann-Whitney test	p=0.015
Escalation between T2 and control: Ballot box (8,40)	Mann-Whitney test	p=0.030
Escalation between T1 and T1-OC: Ballot box (8,40)	Mann-Whitney test	p=0.003
Escalation between all treatments: Ballot box (8,40)	Kruskal-Wallis test (with ties)	p=0.032
Approximation method 2:		
Escalation between T1 and control: Ballot box (16,80)	Mann-Whitney test	p=0.183
Escalation between T2 and control: Ballot box (16,80)	Mann-Whitney test	p=0.334
Escalation between all treatments: Ballot box (16,80)	Kruskal-Wallis test (with ties)	p=0.021
Escalation between T1 and control: Ballot box (8,40)	Mann-Whitney test	p=0.001
Escalation between T2 and control: Ballot box (8,40)	Mann-Whitney test	p=0.003
Escalation between all treatments: Ballot box (8,40)	Kruskal-Wallis test (with ties)	p=0.007

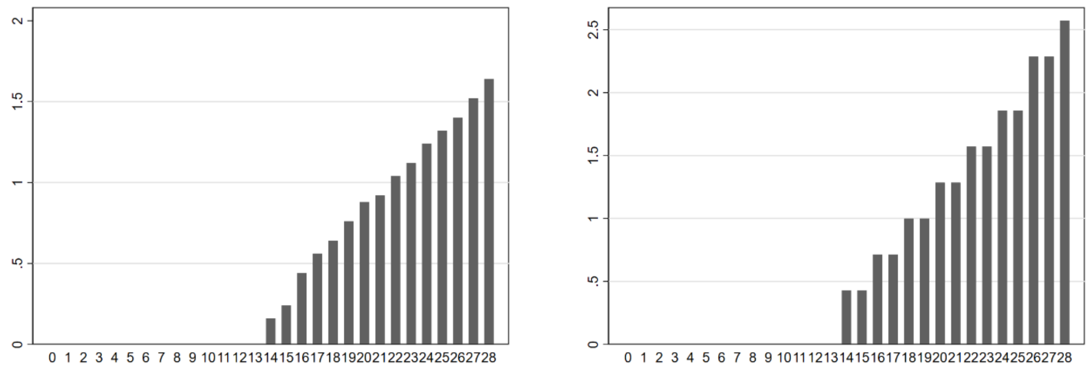
A6 Timing of investments

Figure 16 shows each group's mean cumulative investment quantities by ballot box choice. The x-axis shows the rounds (=past time/10), and the y-axis shows the mean cumulative investment quantities. The investment rate looks more constant (even slightly increasing) than decreasing in all charts.

Figure 16: Investment quantities after choosing a ballot box



T2



To support the observation of a constant investment rate, we use a statistical model. For each group and ballot-box choice, we run a random effects regression that accounts for individual heterogeneity. The subscript i indicates the participant and the t is the index for the decision period. The dependent variables $q(\mathrm{i,t})$ are continuous. The influence of t and t^2 on the cumulative investment quantities $q(\mathrm{i,t})$ is estimated by:

$$q(\mathrm{i,t}) = \beta_0 + \beta_t \cdot t + \beta_{t^2} \cdot t^2 + u_i + \epsilon_{it}.$$

There are two error terms: u_i is pair-specific controlling for heterogeneity and ϵ_{it} is independent across all observations. Table 4 and Table 5 show the regression results.

Table 4: Regression results after choosing the ballot box with 8 black and 40 white balls

	Control	T1	T1-OC	T2
β_0	-0.13 ($p = 0.82$)	-0.05 (p=0.93)	+0.05 (p=0.76)	0.01 (p=0.93)
β_t	$+0.24$ ($p = 0.00$)	-0.07 (p=0.12)	-0.02 (p=0.09)	-0.03 (p=0.08)
β_{t^2}	-0.0001 ($p = 0.87$)	+0.11 (p=0.00)	+0.001 (p=0.00)	+0.003 (p=0.00)
R^2(overall)	0.29	0.85	0.21	0.18
Wald χ^2	471.03	442.26	60.74	260.26
Prob $> \chi^2$	0.00	0.00	0.00	0.00
σ_u	2.72	0.67	0.29	0.74
σ_e	1.68	0.79	0.36	0.93
ρ	0.72	0.42	0.39	0.38

Table 5: Regression results after choosing the ballot box with 16 black and 80 white balls

	Control	T1	T1-OC	T2
β_0	-0.13 ($p = 0.80$)	-0.02 (p=0.91)	-0.01 (p=0.87)	+0.04 (p=0.93)
β_t	$+0.13$ ($p = 0.00$)	-0.01 (p=0.28)	-0.002 (p=0.70)	-0.04 (p=0.25)
β_{t^2}	-0.0005 ($p = 0.60$)	+0.002 (p=0.00)	+0.0005 (p=0.00)	+0.005 (p=0.00
R^2(overall)	0.13	0.15	0.07	0.23
Wald χ^2	291.32	256.88	118.01	99.52
Prob $> \chi^2$	0.00	0.00	0.00	0.00
σ_u	2.24	0.72	0.31	1.02
σ_e	1.43	0.87	0.35	1.19
ρ	0.71	0.41	0.44	0.42

The regressions are significant (Wald χ^2 test, $\mathrm{p} = 0.00$). Except for T1 after choosing the ballot box with 8 black and 40 white balls (2 participants, $\mathrm{R}^2(\mathrm{overall}) = 0.85$), the coefficients of determination are low ($\mathrm{R}^2(\mathrm{overall}) \leq 0.28$). Here, at most $28\ \%$ of the variance of the cumulated investment quantities is explained by the explanatory variables t and t^2.
In the control group, regardless of the ballot box choice, the coefficients of t, but not of t^2, are significant, which indicates a constant investment rate. In the treatment groups, the coefficients of t^2 are significant (p=0.00) and positive ($0 < \beta_{t^2} \leq 0.11$) indicating a slightly increasing investment rate. Only in the treatments T1-OC and T2 after choosing the ballot box with 8 black and 40 white balls (5 participants), the slight increase in the investment rate is mitigated by the significant negative coefficient of t at a significance level of 0.1. In sum, the statistical regression favors a constant or slightly increasing investment rate and not a decreasing investment rate.

Chapter 4

Performance of Myopic Lot-sizing Heuristics and an Improvement Heuristic in Case of Regular, Sporadic, and Close-To-Zero Demand

Performance of Myopic Lot-sizing Heuristics and an Improvement Heuristic in the Case of Regular, Sporadic, and Close-To-Zero Demand

Johanna Dujesiefken, Hartmut Stadtler, Guido Voigt

Abstract

Myopic lot-sizing heuristics like that of Groff and Silver/Meal are often used in ERP systems due to their simplicity. These lead to fairly good solutions when period demands are well above zero. ERP systems now often show demand on a daily basis rather than on a weekly basis. This daily demand representation leads to periods in which there is little or no demand. This paper presents and tests lot-sizing heuristics that perform well for regular (weekly) and sporadic and close-to-zero (daily) demand. An extensive test compares ten heuristics for regular and irregular demand in a rolling planning environment and demonstrates that the originally known myopic heuristics, with the exception of Groff-zero, perform poorly and the Wagner-Whitin-Look-Beyond algorithm performs best for daily demand. Furthermore, we present an improvement heuristic consisting of lot splitting, lot combination, and lot shifting.

1. Introduction

Myopic lot-sizing heuristics are implemented in many modern Enterprise Resource Planning (ERP) systems and are still preferred by many practitioners. Indeed, myopic lot-sizing heuristics provide fairly good results if period demands are well above zero.

In modern ERP systems, demand is displayed daily rather than weekly. This precise demand representation leads to demand types where periods of no demand (sporadic demand) or small (close-to-zero) demands occur. Also, if lot sizing is done for final products, it may follow that there is no secondary demand for intermediate products in many periods. Close-to-zero demand occurs when an Original Equipment Manufacturer (OEM) places large orders at longer intervals and small (daily) orders for spare parts. Actually, we will show that myopic lot-sizing heuristics, except for Groff-zero, perform poorly for sporadic and close-to-zero demand.

Although some heuristics adapted to sporadic demand are known in the literature, these have not been tested in an extensive study with sporadic and regular demands. Hence, it is unknown which heuristic leads to good solutions for regular and sporadic, as well as close-to-zero demand.

This paper will show two new and easy-to-implement heuristics, namely Groff-zero (Gr-z) and Silver/Meal-close-to-zero (SM-ctz), intended to yield solutions at least as good as well-known heuristics for regular demand while also performing well for sporadic and close-to-zero demands.

These lot-sizing heuristics are compared with eight well-known lot-sizing heuristics, such as those of Silver & Meal (SM, 1973) and Groff (Gr, 1979). In addition, we propose and test an easy-to-implement improvement heuristic that can be applied in conjunction with any solution method.
The lot-sizing heuristics are applied in rolling schedules. In this process, only the decision of the first period is implemented, after which the model is rerun with an updated data set. Using rolling schedules allows future developments to be included in the decision(s) of the current period, while future decisions are postponed as long as possible. This common planning practice accounts for uncertain data about the future (not considered in this paper). For example, the efficiency of rolling schedules is analyzed by Baker (1977).
The remainder of this paper is organized as follows: Section 2 summarizes the literature. The newly proposed heuristics, Gr-z and SM-ctz, and the improvement heuristic are presented in the third Section. Section 4 describes the testbed used to compare the performance of the lot-sizing heuristics tested and shows our test results. Finally, Section 5 summarizes our findings and outlines future research directions. The pseudocodes for the proposed heuristics, the termination criteria of the other heuristics tested, and the reported p-values are listed in Appendices A2-A4. Appendix A5 lists the detailed relative additional costs for close-to-zero demand, and Appendix A6 lists the portion of modified first lots using the Wagner-Whitin-Look-Beyond algorithm.

2. Literature Review

The model of the Single-Level-Lotsizing-Problem (SLLSP) considers lot-sizing decisions of a single item over a multi-period horizon ($t = 1,...,T$) with given net demands d_t. A fixed and period-independent setup cost sc is incurred in each period of production, and a linear holding cost hc is attributed to each unit of the end of the period inventory. Although the cost parameters of the SLLSP may be time-dependent, we restrict ourselves to time-independent cost parameters since known heuristics are designed for this case. The SLLP is given in the shortest path representation following Eppen and Martin (1987):

$$\text{Min } Z = \sum_{t=1}^{T}\sum_{t\prime=t}^{T} K_{tt\prime} \cdot W_{tt\prime} \quad (1)$$

subject to

$$\sum_{t\prime=1}^{T} W_{1t\prime} = 1 \quad (2)$$

$$\sum_{p=1}^{t-1} W_{pt-1} = \sum_{t\prime=t}^{T} W_{tt\prime} \ \forall\, t = 2, \dots, T \quad (3)$$

$$W_{tt\prime} \in \{0,1\} \ \forall\, t, t' \text{ for } t' \geq t. \quad (4)$$

The objective function (1) seeks to minimize total cost. Decision variables $W_{tt'}$ take the value of 1 if there is a production in period t covering demand from period t to period t' and $K_{tt'}$ depicts the cost incurred with this lot-sizing decision consisting of the setup cost in period t and the cost of holding inventory up to period t'. Constraints (2) and (3) ensure that the periods' demands will be satisfied. Constraints (4) restrict variables $W_{tt'}$ to values of either 0 or 1. The costs $K_{tt'}$ are defined as

$$K_{tt'} := \left(sc + \sum_{p=t}^{t'} hc \cdot (p - t) \cdot d_p \right). \tag{5}$$

The dynamic single-level lot-sizing problem (SLLSP) has been studied extensively in the literature concerning the performance of different solution heuristics applied in rolling schedules for regular demand. For example, myopic lot-sizing heuristics have been subject to extensive numerical tests by Zoller & Robrade (1987), using test data with different regular demand patterns. A regular demand exists if (almost) all period demands are positive, while sporadic demand is characterized by a high portion of zero demand periods. The heuristics SM and Gr as well as the combination algorithm K-Gr (Zoller & Robrade 1987) stood out from the study due to their very good solution quality.

In order to reduce the end-of-horizon effect occurring in the exact solution of the model presented above, Stadtler (2000) has proposed a Wagner-Whitin algorithm that is able to look beyond the planning horizon (Wagner-Whitin-Look-Beyond, WW-lb). Note that the exact model for a finite horizon applied in rolling schedules, like WW-lb, is also a heuristic. Tests have shown that WW-lb performs better than well-known heuristics like that of Silver & Meal and Groff for regular demand (Stadtler 2000).

If demand is regular and normally distributed with varying standard deviations, Groff's heuristic outperforms that of Silver & Meal (Baciarello et al. 2013).

A lot-sizing heuristic is called myopic if it is based on a forward heuristic where the time between orders (tbo) is increased period by period until a given stopping criterion is fulfilled for the first time. Myopic heuristics like that of Silver & Meal and Groff stop in the first local minimum. If demand is regular, the cost differences between the lot sizes determined by the first local minimum and the global minimum are negligible. In case demand is sporadic, terminating at the first local minimum results in a smaller lot size than that determined by the global minimum. (Silver & Meal 1973, Knolmayer 1987).

Silver & Peterson (1979), Blackburn & Millen (1980), Silver & Miltenburg (1984), as well as Knolmayer (1987) adapt the standard SM heuristic to sporadic demand, while Yilmaz (1992) proposes the use of the Incremental Order Quantity (IOQ).

Silver & Peterson (1979) suggest overcoming the first local minimum by comparing the average costs per period only for *periods with positive demand* (Silver/Meal-zero (SM-z), Silver & Peterson 1979). The same modification is suggested by Blackburn & Millen (1980), noting that comparing the average cost per period only for periods with positive demand may result in merging too many demands into one lot (Blackburn & Millen 1980).

Two-phase SM heuristics are suggested by Silver & Miltenburg (1984) and Knolmayer (1987). Silver & Miltenburg (1984) adapted the standard SM heuristic to sharply decreasing and sporadic demand. In phase 1, the SM is used to find a local minimum. If a local minimum is found or other criteria are met, the modified heuristic checks whether separating the current lot into two lots reduces cost. Phase 1 is iterated until the planning horizon is reached. The solution from phase 1 is improved by phase 2, where two adjacent lots are combined into one common lot if this reduces cost.

Phase 1 of the two-phase SM heuristic suggested by Knolmayer (1987) continues SM *period-by-period* beyond the first local minimum. The number of periods considered after the first local minimum is determined empirically and depends on the mean demand per period. Looking 2 to 8 periods beyond the first local minimum has shown the best results for sporadic demand. In the improvement phase, the solution from phase 1 is modified if a combination of lots (analogous to phase 2 by Silver & Miltenburg 1984) or a shift of the replenishment point a few periods forward or backward reduces cost (see Aucamp & Fogarty 1982).

An obvious property of reasonable solutions for the sporadic and close-to-zero demand types is that each lot arrives in a period with positive demand. Ensuring this property already reduces cost (Knolmayer 1987).

Yilmaz (1992) shows with a few examples that IOQ may lead to good solutions in the case of sporadic demand. However, tests by Zoller & Robrade (1987) have demonstrated that IOQ is already inferior to other heuristics such as SM and Gr in the case of regular demand (see Zoller & Robrade 1987).

Tempelmeier (2003) suggests calculating the average cost per period for any period until the end of the planning horizon and then selecting the global minimum in the case of strongly fluctuating demand. In this way, the termination at the first local minimum can be avoided (Tempelmeier 2003).

In sum, there are approaches in the literature to apply the SM heuristic beyond the first local minimum. Still, a meaningful termination criterion is missing without prior empirical analysis of the demand structure. The following Section proposes two lot-sizing heuristics for the regular, sporadic, as well as close-to-zero demand.

Improvement steps to refine solutions from any construction heuristic are suggested by Aucamp & Fogarty (1982), Silver & Miltenburg (1984), and Zoller & Robrade (1987). Aucamp & Fogarty (1982) propose shifting the replenishment point step by step, one period at a time, forward or backward, as long as savings are achieved. Silver & Miltenburg (1984) calculate savings that can be achieved by lot splitting and lot combination. Zoller & Robrade (1987) compute the costs per period when shifting the replenishment point a few periods forward or backward. The costs for different replenishment points are not calculated step by step as in Aucamp & Fogarty (1982) but are calculated first, and then the minimum is selected. Except for Zoller & Robrade (1987), a comprehensive computational study is missing to evaluate the improvement steps. An improvement heuristic based on these approaches is presented in the next chapter.

3. Heuristics for Regular, Sporadic, and Close-to-zero Demand

3.1 Groff-zero

The standard heuristic by Groff (1979) extends a lot until the period in which the increase in holding costs per period is greater than the savings in fixed setup costs per period which leads to the following termination criterion:

$$\frac{hc \cdot d_{t+1}}{2} > \frac{sc}{\bar{\tau} \cdot (\bar{\tau} + 1)} \, , \tag{6}$$

where $\bar{\tau}$ denotes the number of periods covered by the current lot size. If a period of positive demand occurs after one or several zero demand periods, holding costs will likely increase more than fixed setup costs will decrease. Gr-z compares the marginal costs only for periods with positive demand to overcome the first local minimum. Each period of positive demand generates a demand cycle. A demand cycle starts with positive demand followed by zero demand period(s). The length of the demand cycle τ is the time between two positive demands tbd_τ. The heuristic Gr-z by Stadtler (2022) is described below. Gr-z (Stadtler 2022) extends a lot until the demand cycle in which the increase in holding costs exceeds the savings in fixed setup costs per demand cycle. The termination criterion of Gr-z is similar to that of Gr (cf. (6) and (16)). A pseudocode of Gr-z is illustrated in Figure 1 in Appendix A1 and is described in the following. The range of a lot that covers the demand cycles $1, \dots, \tau$ is denoted by

$$tbd_\tau^{cum} = \sum_{i=1}^{\tau} tbd_i \tag{7}$$

with $tbd_0^{cum} = 0$.

Adding demand cycle $\tau + 1$ to a lot covering the demand cycles $1, \dots, \tau$ results in fixed setup costs savings $\Delta\, sc_{\tau,\tau+1}$ per demand cycle:

$$\Delta\, sc_{\tau,\tau+1} = \frac{sc}{tbd_\tau^{cum}} - \frac{sc}{tbd_{\tau+1}^{cum}} = \frac{tbd_{\tau+1} \cdot sc}{tbd_{\tau+1}^{cum} \cdot tbd_\tau^{cum}} \, . \tag{8}$$

Analogous to the assumption of constant demand in the derivation of the standard Groff termination criterion (see Baciarello et al. 2013), we assume constant demand and equally long time between demands, i.e., $tbd_\tau = tbd \; \forall \; \tau$, and receive:

$$\Delta\, sc_{\tau,\tau+1} \;=\; \frac{tbd \cdot\; sc}{tbd_{\tau+1}^{cum} \cdot\; tbd_{\tau}^{cum}} = \frac{tbd \cdot\; sc}{tbd \cdot (\tau+1) \cdot tbd \cdot \tau} = \frac{sc}{tbd \cdot (\tau+1) \cdot\; \tau}. \tag{9}$$

Holding costs for a lot issued in period t covering τ demand cycles are

$$H_{t+\tau} = \sum_{i=1}^{\tau} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}}\,. \tag{10}$$

Adding demand cycle $\tau + 1$ increases holding costs to

$$H_{t+\tau+1} = \sum_{i=1}^{\tau+1} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}} = H_{t+\tau} +\; tbd_{\tau}^{cum} \cdot hc \cdot d_{t+tbd_{\tau}^{cum}}\,. \tag{11}$$

In case of equally long tbd and constant demand, holding costs for a lot issued in period t covering τ demand cycles are

$$\begin{aligned} H_{t+\tau} &= \sum_{i=1}^{\tau} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}} = \sum_{i=1}^{\tau}\sum_{k=1}^{i-1} tbd_k \cdot hc \cdot d_{t+tbd_{i-1}^{cum}} \\ &= \sum_{i=1}^{\tau} (i-1) \cdot tbd \cdot hc \cdot d_{t+tbd_{i-1}^{cum}} = \frac{1}{2} \cdot \tau \cdot (\tau-1) \cdot tbd \cdot hc \cdot d\,. \end{aligned} \tag{12}$$

In case of equally long tbd and constant demand, holding costs for a lot issued in period t covering $\tau + 1$ demand cycles are

$$H_{t+\tau+1} = \frac{1}{2} \cdot \tau \cdot (\tau+1) \cdot tbd \cdot hc \cdot d\,. \tag{13}$$

Adding demand cycle $\tau + 1$ to a lot covering the demand cycles $1, \ldots, \tau$ results in an increase in holding cost $\Delta\, H_{\tau,\tau+1}$ per demand cycle:

$$\begin{aligned} \Delta\, H_{\tau,\tau+1} &= \frac{H_{t+\tau+1}}{tbd_{\tau+1}^{cum}} - \frac{H_{t+\tau}}{tbd_{\tau}^{cum}} \\ &= \frac{tbd_{\tau}^{cum} \cdot H_{t+\tau+1} - tbd_{\tau+1}^{cum} \cdot H_{t+\tau}}{tbd_{\tau}^{cum} \cdot tbd_{\tau+1}^{cum}} = \frac{\tau \cdot tbd \cdot H_{t+\tau+1} - (\tau+1) \cdot tbd \cdot H_{t+\tau}}{\tau \cdot tbd \cdot (\tau+1) \cdot tbd} \\ &= \frac{\tau \cdot tbd \cdot \frac{1}{2} \cdot \tau \cdot (\tau+1) \cdot tbd \cdot hc \cdot d - (\tau+1) \cdot tbd \cdot \frac{1}{2} \cdot \tau \cdot (\tau-1) \cdot tbd \cdot hc \cdot d}{\tau \cdot (\tau+1) \cdot tbd^2} \\ &= \frac{d \cdot hc \cdot \tau \cdot (\tau+1) \cdot tbd^2}{2 \cdot (\tau \cdot (\tau+1) \cdot tbd^2)} = \frac{d \cdot hc \cdot tbd^2}{2 \cdot tbd^2} = \frac{d \cdot hc}{2}\,. \end{aligned} \tag{14}$$

The termination criterion is reached when adding the demand of demand cycle $\tau + 1$ results in an increase in holding cost per demand cycle that exceeds the decrease in setup cost per demand cycle:

$$\Delta\, H_{\tau,\tau+1} > \Delta\; sc_{\tau,\tau+1}$$
$$\Leftrightarrow \frac{hc \cdot d}{2} > \frac{sc}{tbd \cdot \tau \cdot (\tau+1)}$$
$$\Leftrightarrow \frac{tbd \cdot hc \cdot d}{2} > \frac{sc}{\tau \cdot (\tau+1)}. \tag{15}$$

Let τ^* denote the number of demand cycles covered by the current lot and tbd_{τ^*} the duration of the last demand cycle covered by the current lot. Furthermore, let $d_{t(\tau^*+1)}$ denote the positive demand of demand cycle $\tau^* + 1$. Adjusting for dynamic demand leads to the termination criterion of the Gr-z heuristic:

$$\frac{tbd_{\tau^*} \cdot\; hc \cdot\; d_{t(\tau^*+1)}}{2} > \frac{sc}{\tau^* \cdot\; (\tau^*+1)}. \tag{16}$$

In period t, a lot with lot size x_t is issued, which covers the demand of τ^* demand cycles (positive demands):

$$Q_t = \sum_{i=1}^{\tau^*} d_{t+tbd_{i-1}^{cum}}\,. \tag{17}$$

(Stadtler 2022)

3.2 Silver/Meal-close-to-zero

While SM-z and Gr-z check the termination criterion only for each demand cycle, SM-ctz proceeds *period-by-period*. The pseudocode is illustrated in Figure 1 in Appendix A2 and is described in the following.

Periods with zero demand are included in the current lot, while for periods with positive demand the total cost per period of periods t and $t + 1$ are compared. Demand d_{t+1} of period $t + 1$ is included in the current lot, if

$$TCP_{t+1} \leq TCP_t \tag{18}$$

$$\Rightarrow \frac{sc + hc \cdot Icum_{t+1}}{\bar{\tau}+1} \leq \frac{sc + hc \cdot Icum_t}{\bar{\tau}}, \tag{19}$$

where $\bar{\tau}$ denotes the number of periods covered by the current lot size and $Icum_t = Icum_{t-1} + (\bar{\tau} - 1) \cdot d_t$ measures the cumulative demand covered by a lot weighted by holding periods.

If the total cost per period of period $t + 1$ have increased compared to period t, i.e., $TCP_{t+1} > TCP_t$, inequality (19) is not fulfilled. In this case, the heuristic does not stop in the first local minimum, but evaluates several local minima. For this, the total cost per period until period t, TCP_t , are compared with the local minimum TCP^*.

- If the total cost per period until period t are lower or equal than the previous local minimum, i.e., $TCP_t \leq\ TCP^*$, a new local minimum is set, i.e., $t^* = t, TCP^* = TCP_t$.
- If the additional holding costs for demand d_{t+1} do not exceed the setup costs, i.e., $d_{t+1} \cdot hc \cdot \bar{\tau} \leq\ sc$, the current lot is extended by period $t+1$. Otherwise, the lot cycle ends in the current local minimum $t = t^*$.
- The lot cycle ends in the local minimum t^* if the total cost per period TCP_t until period $t > t^*$ exceeds the previous local minimum $TCP_t >\ TCP^*$.

The lot created in period t_{lot} has the size

$$Q_{t_{lot}} = \sum_{i=t_{lot}}^{t^*} d_i \ . \tag{20}$$

3.3 Improvement Heuristic

The improvement heuristic is designed to improve an initial lot-sizing solution obtained from any construction heuristic. It is based on the two steps of lot splitting and lot combination along the lines of Silver & Miltenburg (1984) and a third step named lot shifting following Knolmayer (1987).

First, each lot with $\bar{\tau} > 2$ and total holding costs that exceed setup cost is examined whether splitting the lot will result in cost savings. Suppose the lot is created in period t_{lot} and covers τ periods. Starting with $J^+ = \{\}$, we test for all periods $j = t_{lot} + 1, \ldots., t_{lot} + \bar{\tau} - 1$ whether the following inequality is fulfilled:

$$S_{t_{lot}+j} = (t_{lot} + j - 1) \cdot \sum_{i=t_{lot}+j}^{t_{lot}+j+\bar{\tau}-1} d_i > \frac{sc}{hc} . \tag{21}$$

If the inequality above is fulfilled, then $\{j\} \cup J^+$. A replenishment is added at the start of period

$$t_{lot} + j^* = \arg\max_{j \in J^+} S_{t_{lot}+j} \ . \tag{22}$$

Then, the preceding lot covers demand from periods $t_{lot}, \ldots, t_{lot} + j^* - 1$, while the added lot satisfies the demand from periods $t_{lot} + j^*, \ldots, t_{lot} + \bar{\tau} - 1$ (see Silver & Miltenburg 1984).

Second, two consecutive lots will be reviewed to determine if combining two lots will result in cost savings. For two consecutive lots, created in periods t_{lot_1} and t_{lot_2} , the saving is computed with

$$\text{Saving} = \ sc \ - hc \cdot Q_{t_{lot_2}} \cdot \left(t_{lot_2} - t_{lot_1}\right) . \tag{23}$$

If the “Saving” is positive, combine the replenishments, $Q_{t_{lot_1}} = Q_{t_{lot_1}} + Q_{t_{lot_2}}$ (see Silver & Miltenburg 1984). While Silver & Miltenburg propose to conduct lot combination rolling backward (starting in the last period of the planning horizon PH), we start the second improvement step in the penultimate lot of the planning horizon in order to prevent the planning horizon effect from being included in the rolling schedules.

Third, two consecutive lots are examined to determine if moving a lot a few periods forward or backward will result in cost savings. For two consecutive lots, the total cost of the two lots to expand or reduce the coverage of the first lot is compared. The first lot is extended or shortened by $r^w \in \{-2, -1, 1, 2\}$ periods in the case of regular demand (see Zoller & Robrade 1988) and by $r^d \in \{-14, -13, \dots, -1, 1, 2, \dots, 13, 14\}$ in the case of close-to-zero and sporadic demand.

4. Numerical Tests

4.1 Testbed

Many testbeds have been developed and utilized for testing solution heuristics for the SLLP (e.g., Berry 1972, Blackburn & Millen 1980, Carlson et al. 1982, Zoller & Robrade 1988, Russell & Urban 1993, Federgruen & Tzur 1994). To verify our results with previous ones, we do not propose a totally new testbed but refer to that of Zoller & Robrade (1987), which is outlined below, together with our modifications and extensions.

A test instance is defined by a given planning horizon, a part-period ratio, a deterministic demand pattern, and in the case of daily demand, the portion of low to zero demands (POLD). These parameters of a test instance are explained subsequently.

Assuming that relatively reliable demand data are available for a quarter, a planning horizon of $PH^w = 13$ periods (weeks) for regular (weekly) demand and a planning horizon of $PH^d = 91$ periods (days) for sporadic and close-to-zero (daily) demand is chosen. Zoller & Robrade (1987) tested planning horizons of 7, 10, 13 and 16 periods and considered a planning horizon of one quarter as reasonable.

Six different setup and holding costs ratios - known as part periods - have been tested. For regular (weekly) demand, the part periods are $\frac{sc}{hc} \in \{1{,}000;\ 2{,}500;\ 5{,}000;\ 7{,}500;\ 10{,}000; 15{,}000\}$. Based on an average demand of 1,000 units per week, these part periods result in a mean time between orders tbo in the range of one to six periods. (see Zoller & Robrade (1987)). For sporadic and close-to-zero (daily) demand, the part periods are $\frac{sc}{hc} \in$ $\{7{,}000;\ 17{,}500;\ 35{,}000;\ 52{,}500;\ 70{,}000;\ 105{,}000\}$ and result in a mean time between orders tbo in the range of 7 to 42 periods based on an average period demand of $\frac{1{,}000}{7} = 142.86$ units.

The following regular demand pattern have been generated as in Zoller & Robrade (1987):

I. constant demand of 1,000 units per period;
II. systematic demand patterns, with (1) a positive and (2) a negative linear trend, (3) a progressive and (4) a degressive trend, (5) stepwise decreasing demands, (6) constant

demand plus an additive seasonal component, (7) constant demand plus a multiplicative seasonal component, (8) a positive, and (9) a negative linear trend plus an additive seasonal component, (10) a progressive, and (11) a degressive linear trend plus an additional multiplicative seasonal component;

III. erratic demands with an average of 1,000 units per period superimposed by five levels of uniformly distributed demand fluctuations within an interval of ±20%, ±40%, ±60%, ±80%, ±100%.

For regular demand, just one test interval of constant demand covering 50 periods has been generated, whereas 10 test intervals cover 500 periods for each pattern of (II), and 80 test intervals each with 50 periods cover 4000 periods for each pattern of (III). For more details, see Robrade (1990).

To obtain the close-to-zero and sporadic demand, the weekly demand in the testbed of Robrade & Zoller (1987) is spread over 7 days.

In the case of close-to-zero demand, there is a high demand on one day of each week, covering 90%, 80%, or 60% of the weekly demand. The day of the week the high demand occurs is determined randomly on the basis of a uniform distribution. The remainder of the demand in that week is evenly distributed among the 4 working days of the week and is equal to 10 %, 20 %, or 40 %, i.e., $POLD_{ctz} = \{10\ \%, 20\ \%, 40\ \%\}$. Demand on weekends (multiples of periods 6 and 7) is zero.

In the case of sporadic demand, there are 1, 2 or 3 days with the same level of positive demand every week, while demand on the remaining days and weekends (multiples of periods 6 and 7) is zero. This results in a portion of zero demand periods equal to 86 %, 71 %, or 57 %, i.e., $POLD_{sporadic} = \{86\ \%, 71\ \%, 57\ \%\}$. The day of the week on which the high demand occurs follows a uniform distribution.

As three test instances for daily demand are generated from one test instance for weekly demand, both sporadic and close-to-zero demand, each contains three test instances of constant demand (I), and three test instances for each pattern of (II) and (III). Each test interval of constant demand covers 350 periods, whereas 10 test intervals cover 3,500 periods for each pattern of (II), and 80 test intervals cover 28,000 periods for each pattern of (III).

Table 1 summarizes the number of parameters, test instances, and test intervals.

Table 1. Overview of the Number of Parameters, Test Instances, and Test Intervals

	Regular (weeks)			Close-to-zero & sporadic (days)		
	Constant	Systematic	Erratic	Constant	Systematic	Erratic
Planning horizon	1	1	1	1	1	1
Part periods	6	6	6	6	6	6
Demand pattern	1	11	5	1	11	5
POLD	-	-	-	3	3	3
Test instances	6	66	30	18	198	90
Test intervals per test instance	1	10	80	1	10	80
Test intervals	6	660	2400	18	1980	7200

With the above parameters and test instances for each lot-sizing heuristic tested, there are 3,066 test intervals with regular demand and 9,198 test intervals, each with sporadic and close-to-zero demand. The test instances are also used to test the performance of the improvement heuristic. The elimination of the end-of-horizon effect resulting from the finite time interval to which a rolling schedule is applied is described below. The proposal of Zoller & Robrade (1987) eliminates the end-of-horizon effect for much smaller intervals of time than that of other researchers who have chosen a rather long time interval (e.g., Blackburn & Millen (1980) consider 300 periods). This allows more test intervals to be generated with the same computational effort. Let the application of a specific lot-sizing heuristic to a test instance be defined as an experiment e. For regular (weekly) demand, the stopping rule of Zoller & Robrade (1987) for finding the interval of time I^e to be evaluated for an experiment e is applied:

$$\sum_{k=1}^{m-1} \tau_k^* < 40 \leq \sum_{k=1}^{m} \tau_k^* < 50 \text{ and } I^e \coloneqq \sum_{k=1}^{m} \tau_k^* \, . \tag{24}$$

On sections of 50 periods each, each heuristic generates range-sequences $\tau_1^*, \tau_2^*, \ldots, \tau_m^*$, where τ_k^* depicts the tbo resulting from the first lot-sizing decision of the kth plan ($k = 1, \ldots, m$) with m being the number of rolling schedules generated. The interval of time I^e to be evaluated for an experiment e covers a minimum of 40 and a maximum of 49 periods.

For daily demand, the stopping rule is modified to:

$$\sum_{k=1}^{m-1} \tau_k^* < 280 \leq \sum_{k=1}^{m} \tau_k^* < 350 \text{ and } I^e \coloneqq \sum_{k=1}^{m} \tau_k^* \, . \tag{25}$$

On sections of 350 periods each, each heuristic generates range-sequences $\tau_1^*, \tau_2^*, \ldots, \tau_m^*$, where τ_k^* depicts the tbo resulting from the first lot-sizing decision of the kth plan ($k = 1, \ldots, m$) with m being the number of rolling schedules generated. The interval of time I^e to be evaluated for an experiment e covers a minimum of 280 and a maximum of 349 periods.

The experiments are evaluated as follows: Each experiment yields an objective function value $C^e(\tau_1, \tau_2, \dots, \tau_m)$. For comparison, the optimal objective function value C^{e^0} for I^e periods is calculated using the Wagner/Whitin algorithm (Wagner & Whitin 1958).

$$RC(\theta, D, P, V) := \frac{C^e(\tau_1, \tau_2, \dots, \tau_m) - C^{e^0}}{C^{e^0}}. \tag{26}$$

$RC(\theta, D, P, V)$ depicts the relative additional cost of an experiment with a given combination of parameters θ, a set of demands D, a lot-sizing heuristic P, and V applications of the improvement heuristic.

4.2 Lot-Sizing Heuristics Tested

Our test compares the performance of 10 lot-sizing heuristics. These lot-sizing heuristics are distinguished into 1) simple myopic, 2) advanced myopic, and 3) non-myopic heuristics. While simple myopic heuristics are based on elementary termination rules, advanced myopic heuristics involve more sophisticated termination structures. The simple myopic heuristic Gr-z and the advanced myopic heuristic SM-ctz have already been described in Section 3. Furthermore, the following heuristics are included in the test:

1) The simple myopic heuristics in the test are Gr (Groff 1979), SM (Silver & Meal 1973), the IOQ (Yilmaz 1992), as well as SM-z (Silver & Peterson 1979), which is SM adapted to sporadic demand.

2) Besides SM-ctz, the advanced myopic heuristics K-Gr (Zoller & Robrade 1987) and K-Gr-z are included in the test.

3) The non-myopic Silver-Miltenburg heuristic (SiMi, Silver & Miltenburg 1984) as well as the non-myopic WW-lb heuristic (Stadtler 2000) are included in the test.

The termination criteria of the heuristics, which are not described in the previous chapter, are presented in Appendix A3.

The heuristic K-Gr (see Zoller & Robrade 1987) is included since this heuristic has turned out to show the best performance in a rolling horizon environment with erratic demands (even dominating heuristics that have been mainly designed for rolling horizon environments like those of Chand (1982) and Blackburn & Millen (1980)). While the K-Gr heuristic is based on Groff's heuristic, the variant K-Gr-z considered in this paper for the first time uses the Gr-z heuristic. Both combination heuristics require the input parameter (r, s) meaning that two consecutive lots are considered, $s = 2$. Furthermore, for regular demand, the number of periods a replenishment point is at most moved forward or backward is $r^w = 2$, while for close-to-zero and sporadic demand, the replenishment point can be shifted at most $r^d = 14$ periods forward or backward if demand in that period is positive.

SiMi (1984) is included in the test as it was specially designed for sharply decreasing and sporadic demand. WW-lb (Stadtler 2000) showed the least minimal relative additional cost in a rolling horizon environment for regular demand and hence is included in our test.

4.3 Test Results

The following criteria will evaluate the performance of the above lot-sizing heuristics:

- the relative additional costs $RC(\theta, D, P, V)$ (see (26));
- a rank $RK(\theta, D, P, V)$ with respect to the relative additional costs $RK(\theta, D, P, V)$ compared with those values $RC(\theta, D, P, V)$ achieved by the other lot-sizing heuristics P tested in the same test interval averaged over all test intervals. Ties are solved such that the rank attributed to a value $RC(\theta, D, \cdot, V)$ is granted to all lot-sizing heuristics yielding the same relative additional cost. For example, when comparing the quality of solutions of 10 lot-sizing heuristics for a particular test instance, rank 1 will be granted when nine lot-sizing heuristics yield the same best value $RC(\theta, D, \cdot, V)$ in the test interval considered while the tenth lot-sizing heuristic with a higher relative additional cost obtains rank 10;

Differences in relative additional costs between 10 different lot-sizing heuristics P lead to 45 pairwise comparisons that have been analyzed by the Wilcoxon matched-pairs signed rank test for significance. For this, we use the function *signrank* available in Matlab and correct for multiple comparisons by implementing the Benjamini-Hochberg Procedure (Benjamini & Hochberg 1995) at an overall significance level of 0.05. The p-values for each comparison and demand type can be viewed in Appendix A4.

To validate our test implementation, we first compare our test results with those of Robrade (1990) and Stadtler (2000) for a few selected lot-sizing heuristics—namely, SM, Gr, K-Gr-2,2, and WW-lb with a planning horizon of $PH = 10$. (Table 2).

Table 2: Mean Relative Additional Costs for Selected Lot-Sizing Heuristics in the Case of Erratic Demands and a Planning Horizon of $PH = 10$

	SM	Gr	K-Gr-2,2	WW-lb
Robrade (1990)	1.126	1.076	0.360	-
Stadtler (2000)	1.184	1.117	0.530	0.158
Dujesiefken, Stadtler, Voigt	1.144	1.118	0.637	0.163

Except for K-Gr-2,2, mean relative additional costs are relatively close for both test implementations. Zoller & Robrade (1988) achieved lower mean relative additional costs for K-Gr-2,2 because they did not impose a finite planning horizon on the heuristics. When considering two consecutive lots, K-Gr-2,2 frequently exceeds the planning horizon of $PH = 10$ (see Stadtler 2000).

In the following, we present our test results for a fixed planning horizon of $PH^w = 13$ periods (weeks) for regular demand and $PH^d = 91$ periods (days) for sporadic and close-to-zero demand. The performance of the different lot-sizing heuristics will be discussed for the regular, close-to-zero, and sporadic demands separately.

Table 3: Mean Relative Additional Costs for Different Demand Patterns of Regular Demand in [%]

Demand	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
Constant	19.072	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Systematic	11.894	0.016	0.016	0.021	0.004	0.004	0.029	0.041	0.017	0.042
Erratic	18.479	1.144	1.144	1.226	1.086	1.086	0.386	0.408	0.883	0.077

Table 4: Mean Ranks in the Case of Regular Demands for Different Demand Patterns

Demand	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
Constant	8.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Systematic	7.5	1.2	1.2	1.4	1.0	1.0	1.7	1.7	1.2	2.2
Erratic	9.0	4.4	4.4	5.0	4.5	4.5	2.6	2.7	3.8	1.5

Starting with regular demand, Table 3 and Table 4 show that all heuristics except IOQ provide the optimal solutions for constant demand. In the case of systematic demand, all heuristics except for IOQ are equally good since the differences are smaller than 0.01 percentage points. For erratic demand, WW-lb performs best. The next best lot-sizing heuristics are K-GR and K-Gr-zero, which are statistically significantly different from WW-lb (see Appendix A4). The heuristics SM, Gr-z, SM-z, SM-ctz, and SiMi follow. The heuristic IOQ performs significantly worse than all other heuristics (see Appendix A4).

Table 5: Mean Relative Additional Costs in the Case of Close-to-Zero Demands for Different Portions of Low Demands in [%]

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
$10\ \%$	23.914	33.697	29.582	1.372	20.354	11.675	2.837	2.235	7.093	0.142
$20\ \%$	28.801	29.734	25.644	1.329	16.950	9.912	2.262	1.810	6.279	0.151
$40\ \%$	41.395	20.668	16.888	6.266	10.349	5.552	1.584	1.229	5.155	0.147
Overall	31.370	28.033	24.038	2.989	15.884	9.046	2.228	1.758	6.176	0.147

For close-to-zero demand, we again have evaluated the lot-sizing heuristics using its mean relative additional costs (Table 5) and mean rank (Table 6) for different portions of low demand. The last row of Table 5 shows the performance of the heuristics over all portions of low demand. Over all portions of low demand, WW-lb performs best with respect to relative additional cost ($\leq 1.51\ \%$) and mean rank, followed by the advanced myopic heuristics, namely SM-ctz and the combination heuristics K-Gr-z and K-Gr. The best simple myopic heuristic is Gr-z, which outperforms the other heuristics, namely the standard heuristics by Silver & Meal and Groff, as well as SM-z and IOQ. All differences except that between IOQ and SM are statistically significant (see Appendix A4). Actually, the extent of additional costs is substantial for all simple myopic heuristics, e.g., more than 15.88 % for SM-z and Gr, thus motivating the use of more sophisticated lot-sizing heuristics.

Further, we have observed that the relative additional cost decreases when the portion of small individual demands increases (when demand is more uniform) except for IOQ and SM-ctz, and WW-lb (see Table 5). These findings also hold when considering constant, systematic, and erratic demand patterns separately (see Appendix A5).

Table 6: Mean Ranks in the Case of Close-to-Zero Demands for Different Portions of Low Demands

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
10 %	7.4	9.3	8.7	2.8	7.3	6.0	3.5	3.1	4.6	1.1
20 %	8.5	9.2	8.5	2.7	7.1	5.9	3.6	3.1	4.7	1.1
40 %	9.7	8.7	8.0	4.2	6.5	5.3	3.3	2.8	4.9	1.2

As before, we first provide an aggregate view of the different lot-sizing heuristics for the case of sporadic demands concerning mean relative additional costs (Table 7) and mean rank (Table 8). The last row of Table 7 shows the performance of the heuristics over all portions of zero demand. WW-lb performs best with respect to mean relative additional costs and mean rank. The next best lot-sizing heuristic is K-Gr-z, the best advanced myopic lot-sizing heuristic, followed by K-Gr. Again, the best simple myopic heuristic is Gr-z, which outperforms the other heuristics (SM-ctz, SiMi, Gr, SM, SM-z). As before, IOQ performs worst. All differences except SM-ctz and SiMi, and SM-z and SiMi, are statistically significant (see Appendix A4).

With a higher portion of zero demand periods, the relative additional costs of the heuristics SM, Gr, K-Gr, SiMi, and WW-lb increase. In contrast, the relative additional costs of IOQ, SM-z, SM-ctz, and K-Gr-z decrease. Only Gr-z and K-Gr-z show the lowest relative additional costs with a portion of zero demand periods of 71 %.

Table 7: Mean Relative Additional Costs in the Case of Sporadic Demands for Different Portions of Zero Demands in [%]

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
57 %	71.565	15.525	9.314	9.321	8.998	4.843	1.406	0.817	6.336	0.092
71 %	53.204	25.923	8.329	8.463	14.108	4.440	1.670	0.642	6.787	0.133
86 %	20.087	38.518	1.562	1.616	23.845	5.611	2.030	0.644	8.029	0.146
Overall	48.285	26.656	6.402	6.467	15.650	4.964	1.702	0.701	7.051	0.124

Table 8: Mean Ranks in the Case of Sporadic Demands for Different Portions of Zero Demands

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
57 %	9.7	8.8	6.1	6.1	6.8	4.6	2.9	2.3	5.3	1.1
71 %	9.8	9.0	5.6	5.7	7.7	4.4	3.1	1.9	5.2	1.2
86 %	7.4	9.6	3.6	3.4	8.4	5.7	4.3	2.2	6.0	1.3

Finally, we consider the performance of lot-sizing heuristics aggregated over all demand patterns and demand types. $\underline{RC}$, and $\underline{RK}$ indicate mean relative additional costs and mean rank of a specific lot-sizing heuristic P over all parameter combinations and demand types and demand patterns (Table 9). Over all demand patterns and demand types, the originally known myopic heuristics except for Gr-z have high relative additional costs, which motivates using better, more sophisticated heuristics. WW-lb performs best. The next best heuristic with respect to mean relative additional cost and mean rank is K-Gr-z, followed by K-Gr and SM-ctz. Again, the best simple myopic heuristic is Gr-z, and the worst is IOQ. All differences except for Gr and SM-ctz are statistically significant (see Appendix A4).

For regular demand, we observe higher relative additional costs for erratic demand than for systematic demand; this difference is much smaller when demand is sporadic or close-to-zero. Although the heuristics have not been implemented with the aim of minimizing computation time, we checked the computation time and observed that they are low for all heuristics except for K-Gr, K-Gr-z, and SiMi.

Table 9: Mean Relative Additional Costs in [%] and Mean Rank over all Demands Patterns and Types

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
$\underline{RC}$	32.046	18.358	10.275	3.290	10.633	4.791	1.356	0.870	4.509	0.103
$\underline{RK}$	8.6	6.8	5.2	3.6	5.6	4.3	2.9	2.3	4.1	1.3

The WW-lb heuristic performs best across all demand types and patterns. However, this heuristic is not myopic, which can lead to nervousness. In the context of rolling planning, nervousness means that a lot size changes from one planning run to the next. We investigate whether the second lot of a planning run alters its quantity in the next planning run, becoming the first lot. On average, $8.84\ \%$ of (first) lots change their quantity with respect to the previous planning run (where they were the second lot) across all demand types and patterns. The lot size varies on average by $25.08\ \%$. For regular demand, an average of $5.71\ \%$ of (first) lots change their quantity with respect to the previous planning run; for close-to-zero demand $10.02\ \%$, and sporadic demand $8.63\ \%$. Table 10 shows the average portion of lots whose quantity has changed compared to the previous planning run for each demand type and setup cost rate. The tendency is for nervousness to increase with higher setup costs.

Table 10: Nervousness of the WW-lb Heuristic for each Demand Type and Setup Costs in [%]

Demand type / sc *	100/700	250/1750	500/3500	700/5250	750/7000	1000/10500
Regular	0.428	2.016	2.016	7.669	11.035	11.103
Close-to-zero	0.155	1.087	5.037	10.492	15.406	27.968
Sporadic	0.153	0.824	4.554	9.530	13.372	23.333

Remark: * sc in case of regular (weekly) demand / sc in case of close-to-zero and sporadic (daily) demand

With regular constant demand, there is no nervousness. If demand is regular systematic, 5.85 % of (first) lots are changed, while 7.35 % of (first) lots are changed when demand is regular erratic compared to the previous planning run. The higher the demand fluctuation, the fewer lots are changed. This is not particularly pronounced for close-to-zero and sporadic demand. Detailed results can be found in Appendix A6.

Next, we analyze the effect of applying the improvement heuristic once, i.e., $V = 1$, to each rolling schedule. Table 11 shows the mean relative additional costs after applying the improvement heuristic once for regular, close-to-zero, and sporadic demand. Furthermore, the performance of the improvement heuristic is evaluated using optimality gap closure, i.e.,

$$OGC(P) = \frac{RC(\theta,D,P,V)-RC(\theta,DS,P,V-1)}{RC(\theta,D,P,V)} \cdot 100 \;. \tag{27}$$

Table 12 shows the OGC after applying the improvement heuristic for each demand type. The improvement heuristic has not been applied to solutions of WW-lb because these are nearly optimal and thus make improvements almost impossible.

For regular demand, the improvement heuristic closes the optimality gap by 48.63 % for IOQ. Initial solutions obtained by the other heuristics, namely SM, SM-z, Gr, Gr-z, SM-ctz, and K-Gr, are improved by up to 19 %.

In the case of close-to-zero and sporadic demand, the improvement heuristic closes the optimality gap by at least 55.86 % for IOQ. Initial solutions obtained by the other heuristics, namely SM, SM-z, Gr, Gr-z, SM-ctz, and K-Gr, are improved by up to 89.74 %.

However, the closure of the optimality gap by the improvement heuristic is accompanied by high computation times which exceed the computational times of the WW-lb substantially. Therefore, the improvement heuristic cannot be recommended.

Table 11. Mean Relative Additional Costs after Applying the Improvement Heuristic in the Case of Regular, CTZ, and Sporadic Demand in [%]

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
Regular	6.563	0.185	0.185	0.170	0.200	0.200	0.134	0.145	0.178	-
CTZ	13.843	3.146	2.986	0.717	1.982	1.808	2.092	1.689	1.652	-
Sporadic	27.292	3.093	1.218	1.286	2.456	1.952	1.690	0.657	1.842	-

Table 12. Optimality Gap Closure after Applying the Improvement Heuristic in the Case of Regular, CTZ, and Sporadic Demand in [%]

	IOQ	SM	SM-z	SM-ctz	Gr	Gr-z	K-Gr	K-Gr-z	SiMi	WW-lb
Regular	48.63	17.56	17.56	18.29	14.72	14.72	0.40	0.46	15.40	-
CTZ	69.92	89.74	88.60	53.89	87.99	79.72	4.58	3.82	71.70	-
Sporadic	55.86	88.88	77.72	77.50	82.35	52.81	0.50	6.12	72.14	-

5. Summary

Modern ERP systems allow demand to be forecasted for short time periods (daily instead of weekly). This has an impact on demand patterns. Generally speaking, weekly demand may contain only a few periods with zero demand, while periods with zero demand occur more frequently when demand is daily. Close-to-zero demand means that large demands occur at longer intervals, while relatively small demands occur on all other days. For example, consider an OEM mainly facing demand from a large customer and a small portion of additional demands at a nearly constant rate to be used as spare parts.

Myopic lot-sizing heuristics like that of Silver & Meal (1973) and Groff (1979) are still preferred by many practitioners and implemented in ERP systems. However, these heuristics do not perform well for irregular (daily) demand. This paper aims to present and test lot-sizing heuristics that perform well for different types of demand, namely regular, close-to-zero, and sporadic. Therefore, we introduce a new advanced myopic lot-sizing heuristic, especially addressing close-to-zero demands, named Silver/Meal-close-to-zero.

Ten lot-sizing heuristics are compared in a rolling horizon environment, including well-known simple myopic heuristics like that of Silver & Meal (1973) and Groff (1979), as well as other simple myopic heuristics developed specifically for irregular demand, namely Groff-zero (Stadtler 2022), Incremental-Order-Quantity (Yilmaz 1992) and Silver-Meal-zero (Silver & Peterson 1979). In addition to the proposed Silver/Meal-close-to-zero heuristic, the combination heuristics K-Gr (Zoller & Robrade 1987) and its variant K-Gr-zero are also included in the test. Together with two non-myopic heuristics, namely the heuristic by Silver & Miltenburg (1984) and Wagner-Whitin-Look-Beyond (Stadtler 2000), these are applied to regular, close-to-zero, and sporadic demand.

This study shows the importance of considering realistic demand patterns when comparing lot-sizing heuristics. While all lot-sizing heuristics except IOQ perform equally well in the case of regular demand, we observe large differences between the heuristics in the case of irregular demand. The originally known myopic heuristics, except for Groff-zero, have high relative additional costs, which motivates using more sophisticated heuristics.

Over all demand patterns and types, the Wagner-Whitin-Look-Beyond algorithm has shown the least mean relative additional costs (0.103 %). The next best heuristic is the combination heuristic K-Gr-zero that consists of the Groff-zero heuristic and an iteratively applied improvement step, in which replenishment points are shifted forward or backward a few periods (mean relative additional costs of $0.87\ \%$). The best simple myopic heuristic is Groff-zero (mean relative additional costs of $4.791\ \%$), which outperforms Silver/Meal-zero in the case of close-to-zero and sporadic demand. In addition, we have presented an improvement heuristic consisting of the three building blocks, namely lot splitting, lot combination, and lot shifting. The improvement heuristic refines the initial solutions obtained by other heuristics. With the improvement heuristic, the optimality gap can be closed up to 90% when demand is close-to-zero and sporadic. Further research can address integrating the first two improvement steps, lot splitting and lot combination, into the K-Gr-z combination algorithm.

The research data is available at 10.25592/uhhfdm.13470.

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Appendix

A1 Pseudocode for Groff-zero

Figure 1: Pseudocode for the Gr-z heuristic

```
//Determine the first period with positive demand
SET t = first period of the planning horizon
FOR each period t of the planning horizon
|         IF d_t > 0
|         |         SET quantity_counter = t
|         |         SET tbegin = t
|         |         BREAK
|         END IF
END FOR

//Determine the time between demands per cycle
SET τ = 1
FOR each period t from tbegin to the last period of the planning horizon
|         IF d_t > 0
|         |         FOR each period i from t to the last period of the planning horizon
|         |         |         IF i is the last period of the planning horizon
|         |         |         |         IF d_i > 0
|         |         |         |         |         //Cycle of length 1
|         |         |         |         |         SET tbd(τ) = 1
|         |         |         |         ELSE
|         |         |         |         |         //Extend the length of the cycle by 1
|         |         |         |         |         SET tbd(τ) = PH - t + 1
|         |         |         |         END IF
|         |         |         ELSE
|         |         |         |         IF d_{i+1} > 0
|         |         |         |         |         //Compute time between demands for the current cycle
|         |         |         |         |         SET tbd(τ) = i + 1 - t
|         |         |         |         |         INCREMENT τ
|         |         |         |         |         BREAK
|         |         |         |         END IF
|         |         |         END IF
|         |         END FOR
|         END IF
END FOR

//Determine lot sizes
SET τ̄= 1 //range of the current lot
SET lot = 0
SET τ = 1
FOR each period t from tbegin to the last period of the planning horizon
|         IF d_t > 0
|         |         FOR each period i from t to the last period of the planning horizon
|         |         |         IF i is not the last period of the planning horizon
|         |         |         |         IF d_{i+1} > 0
|         |         |         |         |         IF 1/2 * tbd(τ) * hc · d_{i+1} <= sc / (τ̄ ·(τ̄ + 1))
|         |         |         |         |         |         //Lot continuation
|         |         |         |         |         |         INCREMENT τ̄
|         |         |         |         |         |         INCREMENT τ
|         |         |         |         |         |         SET lot = lot + d_i
```

```
|       |       |       |       |       ELSE
|       |       |       |       |       |       //Lot termination in i
|       |       |       |       |       |       SET lot = lot + d_i
|       |       |       |       |       |       SET Q(quantity_counter) = lot
|       |       |       |       |       |       SET quantity_counter = i + 1
|       |       |       |       |       |       SET lot = 0
|       |       |       |       |       |       SET τ̄ = 1
|       |       |       |       |       |       INCREMENT τ
|       |       |       |       |       END IF
|       |       |       |       BREAK
|       |       |       |       ELSE //d_{i+1} = 0
|       |       |       |       |       SET lot = lot + d_i
|       |       |       |       END IF
|       |       |       ELSE
|       |       |       |       IF i is the last period of the planning horizon
|       |       |       |       |       SET Q(quantity_counter) = lot + d_i
|       |       |       |       |       BREAK
|       |       |       |       END IF
|       |       |       END IF
|       |       END FOR
|       END IF
END FOR
```

Remark: The Gr-z heuristic can be implemented in linear time $O(\mathrm{t})$ (Stadtler 2023). The pseudocode exhibits quadratic computation time following the implementation used in the computational study.

A2 Pseudocode for Silver/Meal-close-to-zero

Figure 2: Pseudocode for the SM-ctz heuristic

```
//Initialize:
SET Icum(t) = 0; TCP(t) = sc; τ̄ = 1, t_lot = 1, TCP* = bigM, sh = sc/hc

WHILE t ≤last period of the planning horizon
        SET Icum(t + 1) = Icum(t) + τ̄ · d_{t+1}
        SET TCP(t + 1) = (sc+hc·Icum_{t+1}) / (τ̄+1)

        IF d(t+1)=0
                //Lot continuation
                INCREMENT τ̄
                INCREMENT t
        ELSE
                IF TCP(t+1) <= TCP(t)
                        //Lot continuation
                        INCREMENT τ̄
                        INCREMENT t
                ELSE
                        IF TCP(t) ≤ TCP*
                                SET TCP* = TCP(t), t* = t //Update local minimum
                                IF d_{t+1} · hc · τ̄ ≤ sc
                                        //Lot continuation
                                        INCREMENT τ̄
                                        INCREMENT t
                                ELSE
                                        SET Q_{t_lot} = Σ_{i=t_lot}^{t*} d_i //Calculate lot size
                                        SET TC_{tlot} = TCP* · (t* − t_lot + 1) //Calculate lot costs
                                        CALL Initialize_new_lot with t*, bigM
                                END
                        ELSE
                                SET Q_{t_lot} = Σ_{i=t_lot}^{t*} d_i //Calculate lot size
                                SET TC_{tlot} = TCP* · (t* − t_lot + 1) //Calculate lot costs
                                CALL Initialize_new_lot with t*, bigM
                        END IF
                END IF
        END IF
NEXT

SUBROUTINE Initialize_new_lot with t*, bigM
        SET
        t = t* + 1
        t_lot = t* + 1
        TCP* = bigM
        Icum = 0
        τ̄ = 1
```

A3 Termination Criteria of the Heuristics

IOQ:

$$sc \leq \; hc \cdot \bar{\tau} \cdot d_{t+1} \, , \tag{28}$$

where $\bar{\tau}$ denotes the number of periods covered by the current lot size.

SM:

$$\bar{\tau}^2 \cdot \; d_{t+1} > sh + Icum_t \tag{29}$$

with $sh = \frac{sc}{hc}$ and $Icum_t = Icum_{t-1} + (\bar{\tau} - 1) \cdot d_t$.

SM-z:

$$\frac{sc + H_{t+\tau}}{tbd_{\tau}^{cum}} > \frac{sc + H_{t+\tau-1}}{tbd_{\tau-1}^{cum}} \, , \tag{30}$$

where $H_{t+\tau}$ denotes the holding costs for a lot issued in period t covering τ demand cycles.

A4 Reported p-values

Pairwise comparison between		Adjusted p-values			
		Regular erratic	CTZ	Sporadic	Overall
IOQ	SM	0.000	0.810	0.000	0.000
IOQ	SM-z	0.000	0.001	0.000	0.000
IOQ	Gr	0.000	0.000	0.000	0.000
IOQ	Gr-z	0.000	0.000	0.000	0.000
IOQ	SM-ctz	0.000	0.000	0.000	0.000
IOQ	K-Gr	0.000	0.000	0.000	0.000
IOQ	K-Gr-z	0.000	0.000	0.000	0.000
IOQ	SiMi	0.000	0.000	0.000	0.000
IOQ	WW-lb	0.000	0.000	0.000	0.000
SM	SM-z	0.000	0.000	0.000	0.000
SM	Gr	0.022	0.000	0.000	0.000
SM	Gr-z	0.021	0.000	0.000	0.000
SM	SM-ctz	0.285	0.000	0.000	0.000
SM	K-Gr	0.000	0.000	0.000	0.000
SM	K-Gr-z	0.000	0.000	0.000	0.000
SM	SiMi	0.000	0.000	0.000	0.000
SM	WW-lb	0.000	0.000	0.000	0.000
SM-zero	Gr	0.021	0.000	0.000	0.010
SM-zero	Gr-z	0.020	0.000	0.000	0.000
SM-zero	SM-ctz	0.278	0.000	0.000	0.000
SM-zero	K-Gr	0.000	0.000	0.000	0.000
SM-zero	K-Gr-z	0.000	0.000	0.000	0.000
SM-zero	SiMi	0.000	0.000	0.389	0.000
SM-zero	WW-lb	0.000	0.000	0.000	0.000
Gr	Gr-z	0.000	0.000	0.000	0.000
Gr	SM-ctz	0.312	0.000	0.000	0.957
Gr	K-Gr	0.000	0.000	0.000	0.000
Gr	K-Gr-z	0.000	0.000	0.000	0.000
Gr	SiMi	0.001	0.000	0.000	0.000
Gr	WW-lb	0.000	0.000	0.000	0.000
Gr-zero	SM-ctz	0.305	0.000	0.000	0.000
Gr-zero	K-Gr	0.000	0.000	0.000	0.000
Gr-zero	K-Gr-z	0.000	0.000	0.000	0.000
Gr-zero	SiMi	0.001	0.000	0.000	0.000
Gr-zero	WW-lb	0.000	0.000	0.000	0.000
SM-ctz	K-Gr	0.000	0.000	0.000	0.000
SM-ctz	K-Gr-z	0.000	0.000	0.000	0.000
SM-ctz	SiMi	0.000	0.000	0.411	0.000
SM-ctz	WW-lb	0.000	0.000	0.000	0.000
K-Gr	K-Gr-z	0.137	0.000	0.000	0.000
K-Gr	SiMi	0.000	0.000	0.000	0.000
K-Gr	WW-lb	0.000	0.000	0.000	0.000
K-Gr-z	SiMi	0.000	0.000	0.000	0.000
K-Gr-z	WW-lb	0.000	0.000	0.000	0.000
SiMi	WW-lb	0.000	0.000	0.000	0.000

Remark: Shaded areas show significant results.

A5 Detailed Relative Additional Cost for Close-To-Zero Demand

Demand type	POLD	IOQ	SM	SM-z	Gr	Gr-z	SM-ctz	K-Gr	K-Gr-z	SiMi	WW-lb
Constant	10 %	28.75	35.45	30.75	21.85	12.36	1.12	2.57	2.06	7.61	0.16
Constant	20 %	33.95	32.16	27.44	18.26	10.86	0.90	1.84	1.51	6.55	0.19
Constant	40 %	44.96	22.00	17.42	10.50	4.50	0.56	1.14	0.95	5.28	0.19
Systematic	10 %	17.97	26.99	24.17	16.25	9.34	1.24	1.87	1.52	5.46	0.11
Systematic	20 %	22.23	23.59	20.74	13.57	8.08	1.36	1.62	1.37	5.05	0.09
Systematic	40 %	34.67	16.39	13.51	8.23	5.07	1.15	1.22	0.96	4.23	0.11
Erratic	10 %	25.02	38.64	33.83	22.97	13.32	1.75	4.08	3.12	8.20	0.16
Erratic	20 %	30.22	33.46	28.76	19.02	10.79	1.73	3.33	2.54	7.24	0.17
Erratic	40 %	44.56	23.62	19.74	12.32	7.09	17.08	2.39	1.78	5.96	0.14

A6 Nervousness of Wagner-Whitin-Look-Beyond

Demand type	Demand pattern	POLD	Demand fluctuation	Changed first lots
regular	constant	-	-	0,00%
regular	systematic	-	-	5,85%
regular	erratic	-	20	8,39%
regular	erratic	-	40	8,32%
regular	erratic	-	60	7,52%
regular	erratic	-	80	6,56%
regular	erratic	-	100	5,98%
close-to-zero	constant	10 %	-	11,86%
close-to-zero	constant	20 %	-	12,42%
close-to-zero	constant	40 %	-	14,93%
close-to-zero	systematic	10 %	-	6,92%
close-to-zero	systematic	20 %	-	6,92%
close-to-zero	systematic	40 %	-	7,91%
close-to-zero	erratic	10 %	20	14,61%
close-to-zero	erratic	10 %	40	13,46%
close-to-zero	erratic	10 %	60	14,26%
close-to-zero	erratic	10 %	80	12,69%
close-to-zero	erratic	10 %	100	12,56%
close-to-zero	erratic	20 %	20	14,19%
close-to-zero	erratic	20 %	40	14,60%
close-to-zero	erratic	20 %	60	14,40%
close-to-zero	erratic	20 %	80	14,77%
close-to-zero	erratic	20 %	100	13,98%
close-to-zero	erratic	40 %	20	15,44%
close-to-zero	erratic	40 %	40	19,77%
close-to-zero	erratic	40 %	60	20,32%
close-to-zero	erratic	40 %	80	19,79%
close-to-zero	erratic	40 %	100	17,91%
sporadic	constant	57 %	-	15,48%
sporadic	constant	71 %	-	11,58%
sporadic	constant	86 %	-	13,11%
sporadic	systematic	57 %	-	7,08%
sporadic	systematic	71 %	-	6,20%
sporadic	systematic	86 %	-	6,38%
sporadic	erratic	57 %	20	13,77%
sporadic	erratic	57 %	40	13,88%
sporadic	erratic	57 %	60	12,95%
sporadic	erratic	57 %	80	12,32%
sporadic	erratic	57 %	100	12,86%
sporadic	erratic	71 %	20	12,98%
sporadic	erratic	71 %	40	13,45%
sporadic	erratic	71 %	60	12,38%
sporadic	erratic	71 %	80	12,84%
sporadic	erratic	71 %	100	10,95%
sporadic	erratic	86 %	20	12,91%
sporadic	erratic	86 %	40	11,79%
sporadic	erratic	86 %	60	11,53%
sporadic	erratic	86 %	80	9,78%

sporadic	erratic	86 %	100	9,10%

Chapter 5

Appendix

1. Summary

The productive use of managerial time and the timing of investments present success factors for companies. The popular time management literature recommends investing time now to save time later (e.g., the 5S concept). In finance, early investing is recommended, too, as returns of investments accumulate over time. Managers are often faced with the decision of whether to invest time in a particular task or pay for its completion. In many economic approaches, time is set equal to money, since these are exchangeable via labor wages. However, time is perceived differently than money. In particular, time is perceived to be more ambiguous and less easily fungible than money. Although past investments in a project are irrelevant to deciding whether to invest in the project in the future or not, the occurrence of past investments often influences investment behavior. The tendency to continue with a project when money, effort, or time has already been invested is termed the sunk cost effect and can lead to escalating commitment. The sunk cost effect has been demonstrated in various contexts. However, the results of previous research on the consideration of past investments are inconsistent.

In addition to investing time and money, lot sizing represents another topic of operations management and this dissertation. Modern ERP systems allow demand to be forecasted for short time periods. The daily instead of weekly representation impacts demand patterns. Weekly demand may contain only a few periods with zero demand, while periods with zero demand occur more frequently when demand is daily. Close-to-zero demand means that large demands occur at longer intervals, while relatively small demands occur on all other days. In many ERP systems, myopic lot-sizing heuristics like that of Silver & Meal (1973) and Groff (1979) are implemented. However, these heuristics do not perform well for irregular (daily) demand.

The dissertation “Essays on Time-based Decision Making and the Performance of Lot-sizing Heuristics” includes 3 research papers. The first two papers analyze how time versus money is invested in a dynamic situation and how time is invested after past time investments occur. In the first paper, we theoretically model and empirically investigate how time versus money is invested in dynamic decision-making situations. In the considered resource allocation problems, early investments are favorable because returns on investments accumulate over time. It is optimal to invest (time/money) and harvest rewards later. However, individuals fail to invest first and harvest later. The central finding is that the timing of investments improves when time investments meet monetary rewards. In these cases, it appears that simple myopic rules do not impose, and cognitive reflection sets in.

Paper 2 investigates the effect of the occurrence of past unsuccessful investments. We examine the classical sunk cost situation, where a choice can be made between the sunk cost project and a superior alternative, and the situation where the sunk cost project is the superior project. We

analyze whether individuals abandon a project they have unsuccessfully invested time. In the setting considered, without responsibility for past unsuccessful investments, decision makers leave the project with sunk time investments – even if the project is superior.

In paper 3, we compare ten lot-sizing heuristics, including well-known simple myopic heuristics like that of Silver & Meal (1973) and Groff (1979), in a rolling horizon environment and find that over all demand patterns and types, the Wagner-Whitin-Look-Beyond algorithm has shown the least mean relative additional costs (0.103 %). The next best heuristic is the combination heuristic K-Gr-zero that consists of the Groff-zero heuristic and an iteratively applied improvement step, in which replenishment points are shifted forward or backward a few periods (mean relative additional costs of 0.87 %).

2. Zusammenfassung

Die produktive Nutzung der Zeit von Führungskräften und das Timing von Investitionen sind Erfolgsfaktoren für Unternehmen. In der gängigen Literatur zum Zeitmanagement wird empfohlen, jetzt Zeit zu investieren, um später Zeit zu sparen (z. B. das 5S-Konzept). Auch in der Finanzwelt wird empfohlen, frühzeitig zu investieren, da sich die Erträge von Investitionen im Laufe der Zeit ansammeln. Manager stehen oft vor der Entscheidung, ob sie Zeit in eine bestimmte Aufgabe investieren oder für deren Erledigung bezahlen sollen. In vielen wirtschaftlichen Ansätzen wird Zeit mit Geld gleichgesetzt, da diese über Arbeitslöhne austauschbar sind. Zeit wird jedoch anders wahrgenommen als Geld. Insbesondere wird Zeit als mehrdeutig und weniger leicht fungibel als Geld wahrgenommen.

Obwohl frühere Investitionen in ein Projekt für die Entscheidung, ob in das Projekt in der Zukunft investiert wird oder nicht, irrelevant sind, beeinflusst das Auftreten früherer Investitionen häufig das Investitionsverhalten. Die Tendenz, ein Projekt fortzusetzen, wenn bereits Geld, Mühe oder Zeit investiert wurde, wird als Sunk-Cost-Effekt bezeichnet und kann zu eskalierendem Engagement führen. Der Sunk-Cost-Effekt ist in verschiedenen Zusammenhängen nachgewiesen worden. Die Ergebnisse früherer Untersuchungen zur Berücksichtigung vergangener Investitionen sind jedoch inkonsistent.

Neben der Investition von Zeit und Geld stellt die Losgrößenbestimmung ein weiteres Thema der Betriebsführung und dieser Dissertation dar. Moderne ERP-Systeme erlauben es, den Bedarf für kurze Zeiträume zu prognostizieren. Die tägliche statt der wöchentlichen Darstellung wirkt sich auf die Nachfragemuster aus. Wöchentlicher Bedarf kann nur wenige Perioden mit Nullbedarf enthalten, während Perioden mit Nullbedarf häufiger vorkommen, wenn der Bedarf täglich dargestellt wird. Ein Nahe-Null-Bedarf bedeutet, dass große Bedarfe in längeren Abständen auftreten, während an allen anderen Tagen relativ kleine Bedarfe auftreten. In vielen ERP-Systemen werden myopische Losgrößenheuristiken wie die von Silver & Meal (1973) und Groff (1979) eingesetzt. Diese Heuristiken sind jedoch bei unregelmäßigem (täglichem) Bedarf nicht gut geeignet.

Die Dissertation "Essays on Time-based Decision Making and the Performance of Lot-sizing Heuristics" umfasst 3 Forschungsarbeiten. In den ersten beiden Artikeln wird analysiert, wie Zeit im Vergleich zu Geld in einer dynamischen Situation investiert wird und wie Zeit investiert wird, nachdem vergangene Zeitinvestitionen erfolgt sind. Im ersten Beitrag wird theoretisch modelliert und empirisch untersucht, wie Zeit im Vergleich zu Geld in dynamischen Entscheidungssituationen investiert wird. In den betrachteten Ressourcenallokationsproblemen sind frühe Investitionen vorteilhaft, da sich die Erträge aus Investitionen im Laufe der Zeit akkumulieren. Es ist optimal, (Zeit/Geld) zu investieren und die Erträge später zu ernten. Allerdings scheitern Individuen daran, zuerst zu investieren und später zu ernten. Das zentrale Ergebnis ist, dass sich der Zeitpunkt der

Investitionen verbessert, wenn Zeitinvestitionen auf monetäre Belohnungen treffen. In diesen Fällen scheinen sich einfache kurzsichtige Regeln nicht durchzusetzen, und kognitive Reflexion setzt ein.
In Beitrag 2 wird die Auswirkung des Auftretens früherer erfolgloser Investitionen untersucht. Wir untersuchen die klassische Sunk-Cost-Situation, in der eine Wahl zwischen dem Sunk-Cost-Projekt und einer überlegenen Alternative getroffen werden kann, sowie die Situation, in der das Sunk-Cost-Projekt das überlegene Projekt ist. Wir analysieren, ob Individuen ein Projekt aufgeben, in das sie erfolglos Zeit investiert haben. In der betrachteten Situation, in der die Entscheidungsträger keine Verantwortung für frühere erfolglose Investitionen tragen, verlassen sie das Projekt mit versunkenen Zeitinvestitionen - selbst wenn das Projekt überlegen ist.
In Beitrag 3 vergleichen wir zehn Losgrößenheuristiken, darunter bekannte einfache myopische Heuristiken wie die von Silver & Meal (1973) und Groff (1979), in einer Umgebung mit rollierendem Horizont und stellen fest, dass der Wagner-Whitin-Look-Beyond-Algorithmus über alle Nachfragemuster und -typen hinweg die geringsten durchschnittlichen relativen Zusatzkosten aufweist (0,103 %). Die nächstbeste Heuristik ist die Kombinationsheuristik K-Gr-Null, die aus der Groff-Null-Heuristik und einem iterativ angewandten Verbesserungsschritt besteht, bei dem die Auffüllpunkte um einige Perioden nach vorne oder hinten verschoben werden (mittlere relative Zusatzkosten von 0,87 %).

3. The applicant's contribution

In the following, I (the applicant) explain my contribution to the three research papers of this dissertation. In doing so, I address the papers' conception, realization, and documentation.

Johanna Dujesiefken, Guido Voigt, and Charles Corbett were involved in paper I. The basic concept of the research project resulted from a research discussion between Guido Voigt and Charles Corbett. Guido Voigt and Charles Corbett are the main idea givers. The applicant developed an experimental design and incorporated feedback provided by the co-authors. The applicant programmed the experiment in the experimental software z-Tree and the Python framework oTree, and conducted the experiment at the WiSo Research Laboratory of the University of Hamburg. Further, the applicant performed the statistical analysis, derived the results, and implemented feedback and input provided by the co-authors. In the documentation, Guido Voigt and the applicant shared the work in the form that Guido Voigt wrote down a proposal for the introduction and contributed extensive feedback and input on the other chapters. The applicant's task was to implement the feedback and suggestions into the manuscript.

Johanna Dujesiefken has single authorship for Paper II. The basic idea of the paper resulted from a conversation between Charles Corbett, Guido Voigt, and Johanna Dujesiefken. Guido Voigt and Johanna Dujesiefken further elaborated on the experimental concept. The applicant implemented the experiment in the Python framework oTree. Guido Voigt continued to provide input on the statistical analysis of the experiment. The applicant prepared a proposal for the manuscript and then incorporated feedback from Guido Voigt.

Johanna Dujesiefken, Hartmut Stadtler, and Guido Voigt were involved in Paper III. The basic idea of the paper goes back to a discussion between Hartmut Stadtler, Guido Voigt, and representatives of SAP. They identified the need for research in the area of lot sizing when demand is sporadic and close-to-zero because common lot-sizing algorithms lead to poor solutions for these demand types. Hartmut Stadtler developed two algorithms: Groff-zero and Silver/Meal-close-to-zero. The applicant's task was to implement the algorithms and extensively test them in comparison with eight other algorithms and evaluate the results with respect to regular, sporadic, and close-to-zero demand. The software Matlab was used. Hartmut Stadtler's valuable input was included in the testing and preparation of the results. The applicant prepared a proposal for the documentation and subsequently incorporated Hartmut Stadtler's input.

Eidesstattliche Erklärung

Hiermit erkläre ich, Johanna Dujesiefken, an Eides statt, dass ich die Dissertation mit dem Titel " Essays on Time-based Decision Making and the Performance of Lot-sizing Heuristics " selbstständig – und bei einer Zusammenarbeit mit anderen Wissenschaftlern gemäß der beigefügten Darlegung – nach § 6 Abs. 4 der Promotionsordnung der Fakultät für Betriebswirtschaft vom 9. Juli 2014 verfasst und keine anderen als die von mir angegebenen Hilfsmittel benutzt habe. Die den herangezogenen Werken wörtlich oder sinngemäß entnommenen Stellen sind als solche gekennzeichnet.

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____

Johanna Dujesiefken

Hamburg, 21. Februar 2024