

**MUSICAL NOTATION BETWEEN
INDIVIDUALISATION AND STANDARDISATION
PROPORTION SIGNS IN THE MANUSCRIPTS
OF THE SO-CALLED *ARS SUBTILIOR***

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to Löhning (alias Löning, Loening, and Løning)

CONTENTS

Acknowledgements	xi
Abbreviations	xiii
List of Figures and Tables	xviii
Introduction	1
1 A New Approach to Proportion Signs	7
1.1 Proportions in Medieval Music and Music Treatises: an Introduction	9
1.1.1 Arithmetic, Geometric, and Harmonic Proportion	9
1.1.2 Proportions in Medieval and Early Renaissance Life and Culture	11
1.1.3 Pythagoras' <i>'Musica'</i> and Boethius' Proportions	13
1.1.4 Proportions in Rhythmic Contexts	22
1.2 <i>Ars subtilior</i>	30
1.3 Thinking about Proportion Signs	37
1.3.1 What Are Proportion Signs?	41
1.3.1.1 Definition	43
1.3.1.2 Extrinsic and Intrinsic Signs	43
1.3.1.3 The Minim as Central Level of Comparison	46
1.3.1.4 Visual Appearance	50
1.3.1.5 Unequivocal Proportional Meaning	51
1.4 A Reassessment of Baude Cordier's Picture Songs	53
1.4.1.1 A Frontispiece of the Chantilly Codex?	54
1.4.1.2 The Identity of the Composer Baude Cordier	56
1.4.2 The Rondeau <i>Belle, bonne, sage</i>	58
1.4.2.1 Proportion Signs in <i>Belle, bonne, sage</i>	63
1.4.2.2 Non-Cumulative, Cumulative, and Quasi-Cumulative Proportions	63
1.4.2.3 The Function of Signs in <i>Belle, bonne, sage</i>	71
1.4.3 The Rondeau <i>Tout par compas</i>	78
1.4.3.1 Proportion Signs Signalling Citation and Allusion	83
1.4.4 Recapitulation	84
1.4.5 Baude Cordier Reconsidered Once Again	89
2 Proportion Signs in the Manuscript <i>Mod A</i>	97
2.1 Overview of Proportion Signs in <i>Mod A</i>	102
2.2 Proportion Signs in the Compositions of Matteo da Perugia	108
2.3 Proportion Signs in the Compositions of Antonello da Caserta	125
2.4 Proportion Signs in the Compositions of Filippotto da Caserta and Johannes Ciconia's <i>Sus un' fontayne</i>	134

2.5	Other <i>Mod A</i> Compositions Containing Proportion Signs	143
2.6	Conclusion	153
3	Proportion Signs in the Manuscript <i>Turin</i>	157
3.1	Standardisation in <i>Turin</i>	162
3.2	Individualisation in <i>Turin</i>	169
3.2.1	Special Proportion Signs and Unusual Interpretations	172
3.2.2	Canons	177
3.2.3	A Musical Riddle?	183
3.3	Conclusion	188
4	Composer versus Theorist: A Comparison of Ugolino da Orvieto's Compositions and his Statements on Proportion Signs in the Manuscript <i>Rome, Biblioteca Casanatense, 2151</i>	191
4.1	The Manuscript <i>Cas</i> and its Contents	192
4.2	Proportion Signs in the <i>Cas</i> Compositions	197
4.3	Comparing Teachings and Songs	198
4.4	Conclusion	204
5	Proportion Signs in Music Treatises before c.1450	207
5.1	Searching Proportion Sign Treatises with the Help of the TML and Other Databases	215
5.2	Proportion Signs in Treatises up to c.1450: an Overview	221
5.2.1	Reviewing the Material	221
5.2.2	Proportion Signs in Treatises up to c.1450: Tables	226
5.3	Treatises and Manuscripts	233
5.3.1	Treatises in Favour of Standardisation	233
5.3.1.1	Prosdocimus de Beldemandis' Treatises <i>Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris</i> and <i>Tractatus practice cantus mensurabilis</i>	235
5.3.1.2	A 'New' Source Commenting on Diminution and Ø: the Treatise <i>Quoniam circa artem musice figurative seu mensuralis</i> of c.1450	240
5.3.1.3	Two Treatises from the <i>Tractatus de musica plana et mensurabili</i> (Coussemaker's Anonymous XI): <i>Sequitur hic aliqua declaratio atque denominatio</i> and <i>Proportio est duorum terminorum vel duorum numerorum</i>	243
5.3.1.3.1	The Treatise <i>Sequitur hic aliqua declaratio atque denominatio</i> (fols. 40r–40v) and the Treatise <i>Ars et practica cantus figurativi</i>	246
5.3.1.3.2	The Treatise <i>Proportio est duorum terminorum vel duorum numerorum</i> (fol. 41r)	253
5.3.1.4	A Mid-Fifteenth-Century Hebrew Treatise: <i>Student's notes of lectures on music</i>	257

5.3.2	Practice-oriented Treatises with Musical Examples	263
5.3.2.1	The Treatise <i>Proportio est duarum rerum</i>	265
5.3.2.2	Georgius Erber's Treatise <i>Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente</i>	274
5.3.2.3	The Treatise <i>Iste sunt proportiones</i>	278
5.3.2.4	The Treatise <i>Venerabiles domini mei</i>	281
5.3.3	The Treatise <i>Pars aliquota est illa quae aliquotiens sumpta</i>	285
5.4	Conclusion	292
	Conclusion	295
	Bibliography	305
	Sources	306
	Manuscripts (Music)	306
	Manuscripts (Treatises)	313
	Manuscripts (Others)	316
	Treatises (Including Respective Editions and Translations)	317
	Editions	330
	Editions (Music)	330
	Editions and Translations (Treatises)	332
	Secondary Literature	337
	Authors in Alphabetical Order	337
	No/Unknown Author	361
	Appendix	363
	TABLE A.1: Results of TML Search with Search Term ' <i>propor*</i> ' in Combination with the Selection 15 th Century	364
	Kurzfassungen der Ergebnisse der Dissertation in deutscher und englischer Sprache	377
	Liste der aus dieser Dissertation hervorgegangenen Vorveröffentlichungen	383
	Eidesstattliche Erklärung	385

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ABBREVIATIONS

Secondary Literature

- CMM* *Corpus Mensurabilis Musicae*.
For volumes and editors see <http://www.corpusmusicae.com/cmm.htm>
- DIAMM* *Digital Image Archive of Medieval Music*
Accessible online: <https://www.diamm.ac.uk/>
- LmL* *Lexicon musicum Latinum medii aevi*. Edited by Michael Bernhard. 2 vols. (19 fascicles). Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1995–2016.
Accessible online: <http://woerterbuchnetz.de/LmL/>
- MGG1* *Die Musik in Geschichte und Gegenwart: Allgemeine Enzyklopädie der Musik*. Edited by Friedrich Blume. 17 vols. Kassel and Stuttgart: Bärenreiter / J. B. Metzler, 1949–1986.
- MGG2* *Die Musik in Geschichte und Gegenwart: Allgemeine Enzyklopädie der Musik*. Edited by Ludwig Finscher. 2nd ed. Sachteil 9. vols. Personenteil 17 vols. Kassel and Stuttgart: Bärenreiter / J. B. Metzler, 1994–2007.
Accessible online: <https://www.mgg-online.com/>
- NG2* *The New Grove Dictionary of Music and Musicians*. Edited by Stanley Sadie. 2nd ed. 29 vols. London: MacMillan, 2001.
Accessible online: <https://www.oxfordmusiconline.com/>
- PMFC* *Polyphonic Music of the Fourteenth Century*. Leo Schrade, Frank Ll. Harrison, and Kurt von Fischer, general eds. 25 vols. (1–22, 23a, 23b, 24). Monaco: Éditions de l’Oiseau-Lyre, 1956–1991.
- RISM B III* *Répertoire International des Sources Musicales, The Theory of Music. Manuscripts from the Carolingian Era up to c. 1500*, 6 vols. Munich: G. Henle Verlag, 1961–2003.
Accessible online: <http://www.musmed.fr/RISM/rismindex01.htm>
- RISM B IV* *Répertoire International des Sources Musicales, Manuscripts of Polyphonic Music, 11th–16th centuries*, 6 vols. Munich: G. Henle Verlag, 1966–93.
- RISM B IX* *Répertoire International des Sources Musicales, Hebrew Sources*, 2 vols. Munich: G. Henle Verlag, 1975–89.

<i>SMI</i>	<i>Saggi musicali italiani</i> <i>Accessible online:</i> http://www.chmtl.indiana.edu/smi/
<i>TFM</i>	<i>Traité français sur la musique</i> <i>Accessible online:</i> http://www.chmtl.indiana.edu/tfm/
<i>TME</i>	<i>Texts on Music in English</i> <i>Accessible online:</i> http://www.chmtl.indiana.edu/tme/
<i>TML</i>	<i>Thesaurus Musicarum Latinarum</i> <i>Accessible online:</i> http://www.chmtl.indiana.edu/tml/

Manuscripts (Music)

<i>Autun 152</i>	Autun, Bibliothèque Municipale. MS 152.
<i>Bergamo 589</i>	Bergamo, Civica Biblioteca ‘Angelo Mai’. MA 589 (<i>olim</i> Σ VII 26).
<i>Bern 827</i>	Bern, Burgerbibliothek, Sammlung Bongarsiana. Fragment 827.
<i>Bodley 652</i>	Oxford, Bodleian Library. MS Bodl. 652.
<i>Bologna Q15</i>	Bologna, Civico Museo Bibliografico Musicale. MS Q15 (<i>olim</i> Liceo 37).
<i>Boorman</i>	Stanford, Stanford University Libraries, Special Collections. MLM 1346 (<i>olim</i> New York, private collection of Professor Stanley Boorman of New York University. parchment bifolium).
<i>Boverio</i>	Turin, Biblioteca Nazionale Universitaria. MS T.III.2.
<i>Budapest 298</i>	Budapest, Egetemi Könyvtár. U.Fr.l.m. 298.
<i>Cas</i>	Rome, Biblioteca Casanatense. MS 2151.
<i>Ch</i>	Chantilly, Musée Condé. MS 564 (<i>olim</i> 1047).
<i>Dartmouth</i>	Hanover, NH, Dartmouth College Library, Rauner Special Collections. MS 002387.
<i>Egidi</i>	Montefiore dell’Aso, Biblioteca-Archivio di Francesco Egidi. parchment bifolium.
<i>EscSL V.III.24</i>	San Lorenzo de El Escorial, Palacio Real, Monasterio de S Lorenzo. MS V.III.24.
<i>Fountains</i>	London, British Library. Add. Ms. 40011B.
<i>GR 197</i>	Grottaferrata, Biblioteca dell’Abbazia (Badia Greca). Kript. Lat. 224 (<i>olim</i> Collocazione provvisoria 197).
<i>Luc</i>	Lucca, Archivio di Stato Lucca. MS 184.

<i>Mod A</i>	Modena, Biblioteca Estense. MS α.M.5.24.
<i>Old Hall</i>	London, British Library. Add. MS 57950.
<i>Ox213</i>	Oxford, Bodleian Library. MS Canon. Misc. 213.
<i>Pad A</i>	Oxford, Bodleian Library. MS Canon. Pat. Lat. 229; Padua, Biblioteca Universitaria. MS 684; and Padua, Biblioteca Universitaria. MS 1475.
<i>Paris fr. 146</i>	Paris, Bibliothèque Nationale de France, fonds français 146 'Roman de Fauvel'.
<i>Paris NAF 22069</i>	Paris, Bibliothèque Nationale de France, nouvelles acquisitions françaises 22069.
<i>Parma</i>	Parma, Archivio di Stato, Raccolta Manoscritti, busta 75, n. 26 ex convento LXXXV (S. Servino di Piacenza) reg. n. 52.
<i>Pit</i>	Paris, Bibliothèque Nationale de France, fonds italien 568.
<i>Porto 714</i>	Porto, Biblioteca Pública Municipal. MS 714.
<i>PR</i>	Paris, Bibliothèque Nationale de France, nouvelles acquisitions françaises 6771 'Codex Reina'.
<i>Pz</i>	Paris, Bibliothèque Nationale de France, nouvelles acquisitions françaises 4917.
<i>SL</i>	Florence, Archivio del Capitolo di San Lorenzo. MS 2211.
<i>Sq</i>	Florence, Biblioteca Medicea-Laurenziana. Mediceo Palatino 87.
<i>Trem</i>	Paris, Bibliothèque nationale de France, Département des Manuscrits.
<i>Turin</i>	Turin, Biblioteca Nazionale Universitaria. MS J.II.9.
<i>Turin G.IV.31</i>	Turin, Biblioteca Nazionale Universitaria. MS G.IV.31.

Manuscripts (Treatises)

<i>A-Iu 962</i>	Innsbruck, Universitätsbibliothek. MS 962.
<i>A-KR 312</i>	Kremsmünster, Benediktinerstift Kremsmünster. MS 312.
<i>A-M 950</i>	Melk, Benediktinerstift Melk, Bibliothek. MS 950.
<i>D-Mbs Clm 18800</i>	Munich, Bayerische Staatsbibliothek. MS Clm 18800.
<i>D-Mbs Clm 19851</i>	Munich, Bayerische Staatsbibliothek. MS Clm 19851.
<i>D-Mbs Clm 26812</i>	Munich, Bayerische Staatsbibliothek. MS Clm 26812.
<i>D-Rp 98 th. 4°</i>	Regensburg, Bischöfliche Zentralbibliothek, Proschesche Musikabteilung. MS 98 th. 4°.

- F-Pn Lat. 7378A* Paris, Bibliothèque Nationale de France. MS Lat. 7378A.
- F-SDI 42* Saint-Dié, Bibliothèque municipale. MS 42.
- GB-Lbl Add. 33519* London, British Library. MS Add. 33519.
- GB-Lbl Add. 34200* London, British Library. MS Add. 34200.
- GB-Lbl Lansdowne 763* London, British Library. MS Lansdowne 763.
- GB-Ob Bodl. 842* Oxford, Bodleian Library. MS. Bodl. 842.
- GB-Ob Canon. Misc. 42* Oxford, Bodleian Library. MS Canon. Misc. 42.
- I-Bc A 29* Bologna, Civico Museo Bibliografico Musicale. MS A 29.
- I-Bc A 56* Bologna, Civico Museo Bibliografico Musicale. MS A 56.
- I-CATc D 39* Catania, Biblioteche Riunite Civica e Antonio Ursino Recupero. MS D 39.
- I-CR 238* Cremona, Biblioteca Governativa e Biblioteca Civica (Sala Cesari). MS 238.
- I-Fn Magliab. III 70* Florence, Biblioteca Nazionale Centrale. MS Magliab. III 70.
- I-Lg 359* Lucca, Biblioteca Statale. MS 359.
- I-Rc 2151* Rome, Biblioteca Casanatense. MS 2151.
- I-Sc L.V.30* Siena, Biblioteca Comunale degli Intronati. MS L.V.30.
- I-Tn G.IV.31* Turin, Biblioteca Nazionale Universitaria. MS G.IV.31
- I-Vnm Lat. VIII.85* Venice, Biblioteca Nazionale Marciana. MS Lat. VIII.85 (3579).
- PL-WRu IV.Q.16* Wrocław, Biblioteka Uniwersytecka. MS IV.Q.16.
- US-BEm 744* Berkeley, Jean Gray Hargrove Music Library. MS. 744
- V-CVbav Barb. lat. 307* Rome, Biblioteca Apostolica Vaticana. MS Barb. lat. 307.
= *I-Rvat Barb. lat. 307*
- V-CVbav lat. 5324* Rome, Biblioteca Apostolica Vaticana. MS lat. 5324.
= *I-Rvat lat. 5324*
- V-CVbav Reg. lat. 1146* Rome, Biblioteca Apostolica Vaticana. MS Reg. lat. 1146.
= *I-Rvat Reg. lat. 1146*
- V-CVbav Ross. 455* Rome, Biblioteca Apostolica Vaticana. MS Ross. 455.
= *I-Rvat Ross. 455*
- V-CVbav Urb. lat. 258* Rome, Biblioteca Apostolica Vaticana. MS Urb. lat. 258.
= *I-Rvat Urb. lat. 258*

Manuscripts (Others)

<i>D-Mbs Clm 2599</i>	Munich, Bayerische Staatsbibliothek. MS Clm 2599.
<i>D-W 334 Gud. Lat. 8°</i>	Wolfenbüttel, Herzog August Bibliothek. MS 334 Gud. Lat. 8°.
<i>GB-Cu Ii.3.12</i>	Cambridge, Cambridge University Library. MS Ii.3.12.
<i>I-Ta J.b.IX.10</i>	Turin, Archivio di Stato. MS J.b.IX.10.

Other Abbreviations

[2,2]	<i>tempus imperfectum</i> with <i>prolatio minor</i>
[2,3]	<i>tempus imperfectum</i> with <i>prolatio maior</i>
[3,2]	<i>tempus perfectum</i> with <i>prolatio minor</i>
[3,3]	<i>tempus perfectum</i> with <i>prolatio maior</i>
c.	<i>circa</i>
cf.	confer
chap./chaps.	chapter/chapters
col./cols.	column/columns
d.	died
ed.	edition or editor(s)
eds.	editors
e.g.	<i>exempli gratia</i>
etc.	<i>et cetera</i>
fol./fols.	folio/folios
i.a.	<i>inter alia</i>
i.e.	<i>id est</i>
m./mm.	measure/measures
n.	note
p./pp.	page/pages
<i>r</i>	<i>recto</i> (front)
s.v.	<i>sub voce</i>
<i>v</i>	<i>verso</i> (back)
vol./vols.	volume/volumes

LIST OF FIGURES AND TABLES¹

FIGURE 1.1:	Rithmomachy board with black and white pieces Source: https://en.wikipedia.org/wiki/Rithmomachia#/media/File:Ritho.jpg	12
FIGURE 1.2:	Depictions of Pythagorean proportions in sounding instruments in Gaffurius' <i>Theorica musicae</i> Source: Franchinus Gaffurius, <i>Theorica musicae</i> , Milan: Filippo Mantegazza, 1492, Book 1, fol. 18r. https://imslp.org/wiki/Theorica_musicae_(Gaffurius,_Franchinus)	14
FIGURE 1.3:	Pythagorean tetractys	15
TABLE 1.4:	Types of proportions of major inequality	19
FIGURE 1.5:	Basic note values in mensural notation	23
FIGURE 1.6:	<i>Dupla</i> (2:1) and <i>tripla</i> (3:1) proportion in <i>longa</i> division	24
FIGURE 1.7:	<i>Sesquialtera</i> (3:2) proportion in imperfection and alteration	25
FIGURE 1.8:	The four different mensurations of French notation	25
FIGURE 1.9:	Cantus of the virelai <i>Que pena maior agitanda menti (Mod A 73)</i> ; ex- tract (first two staves) Source: <i>Mod A</i> , fol. 37r ²	28
FIGURE 1.10:	Table of note shapes and their values in Johannes de Muris' <i>Notitia artis musicae</i> Source: Ulrich Michels, ed., <i>Johannis de Muris: Notitia artis musicae and Compen- dium musicae practicae; Petrus de Sancto Dionysio: Tractatus de musica</i> , Corpus scriptorum de musica 17 (Rome: American Institute of Musicology, 1972), 79.	37
FIGURE 1.11:	Rhythmic proportion caused by change in mensuration from [2,2] to [2,3]	48
FIGURE 1.12:	Baude Cordier's songs in <i>Ch</i> Source: <i>Ch</i> , fols. 11v–12r ²	54
FIGURE 1.13:	Cantus of <i>Belle, bonne, sage</i> : original notation, fictional version without proportion signs, and transcription	59
FIGURE 1.14:	Augmentation in <i>Belle, bonne, sage</i> : the change in mensuration does not influence the structure of the rhythmic whole due to the combination with <i>subdupla</i> (1:2) proportion at the minim level	60
FIGURE 1.15:	Impossibility of displaying <i>sesquialtera</i> (3:2) proportion at the semiminim level by using coloration	61
FIGURE 1.16:	Interpretation of void notation in <i>Belle, bonne, sage</i>	62
FIGURE 1.17:	Fictional voice with two proportion signs and void notation	64

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² See bibliography for online resource of this manuscript.

FIGURE 1.18: Non-cumulative proportions interpretation of fictional voice	64
FIGURE 1.19: Cumulative proportions interpretation of fictional voice	65
FIGURE 1.20: Quasi-cumulative proportions interpretation of fictional voice	65
TABLE 1.21: Ratios resulting from three different approaches to the interpretation of proportion signs in <i>Belle, bonne, sage</i>	66
FIGURE 1.22: Signs without stroke indicating <i>prolatio minor</i> in compositions by Cordier in <i>Ox213</i> Source: <i>Ox213</i> , fols. 108v, 110v, 123r, and 116r ³	76
FIGURE 1.23: Cut signs in <i>Amans amés secretement</i> by Cordier in <i>Ox213</i> Source: <i>Ox213</i> , fol. 123r ³	76
FIGURE 1.24: Original notation and transcription of <i>Tout par compas</i>	79
FIGURE 1.25: Proportional relationship between $\odot$ and $\ominus$ in <i>Tout par compas</i>	80
FIGURE 1.26: Proportional relationships in the cantus of <i>Tout par compas</i>	82
FIGURE 1.27: Beginning of cantus of <i>Tout par compas</i> (enhanced) Source: <i>Ch</i> , fol. 12r ³	83
FIGURE 1.28: Cantus of <i>Se fortune s'est tournee</i> (allusion to <i>Tout par compas</i> encircled) Source: <i>Ox213</i> , fol. 109r ³	84
TABLE 1.29: Interpretation of proportion signs in <i>Belle, bonne, sage</i> and <i>Tout par compas</i> (same voice quasi-cumulative proportions interpretation at the minim level)	84
FIGURE 1.30: Cantus of <i>De narcissus</i> in <i>Ch</i> , fol. 19v (mensuration signs encircled) Source: <i>Ch</i> , fol. 19v ³	92
FIGURE 1.31: Cantus of <i>De narcissus</i> in <i>PR</i> , fol. 81r Source: <i>PR</i> , fol. 81r ³	92
FIGURE 1.32: Cantus and tenor of <i>De narcissus</i> (third part only) in <i>Bergamo 589</i> , <i>recto</i> Source: <i>Bergamo 589</i> , <i>recto</i> ³	92
FIGURE 1.33: Cantus of <i>Era Venus</i> in <i>Pit</i> , fol. 54r (proportion sign encircled and enhanced) Source: <i>Pit</i> , fol. 54r ³	93
FIGURE 1.34: Interpretation of $\frac{\odot}{3}$ in <i>Era Venus</i> Source: Tiziana Sucato, "Alcune annotazioni sui madrigali di Paolo da Firenze e in particolare sull'uso dei segni C e $\odot$," in <i>Le notazioni della polifonia vocale dei secoli IX–XVII</i> , ed. Maria Caraci Vela, Daniele Sabaino, and Stefano Aresi (Pisa: Edizioni ETS, 2007), 259.	93
FIGURE 1.35: Cantus of <i>Era Venus</i> in <i>SL</i> , fol. 50v (position of missing proportion sign encircled and enhanced) Source: Andreas Janke, and John Nádas, eds., <i>The San Lorenzo Palimpsest Florence, Archivio del Capitolo di San Lorenzo Ms. 2211: Introductory Study and Multispectral Images</i> , Ars Nova: Nuova serie 4, 2 vols. (Lucca: LIM Editrice, 2016), 2: No. 142.	94

³ See bibliography for online resource of this manuscript.

FIGURE 2.1:	Fol. 2r of <i>Mod A_{I/IV}</i> (left) and fol. 18v of <i>Mod A_{II-IV}</i> (right) Source: <i>Mod A</i> , fols. 2r and 18v ⁴	97
TABLE 2.2:	Proportion signs in <i>Mod A</i>	102
TABLE 2.3:	Proportions achieved by simultaneous use of different mensurations in <i>Mod A</i>	107
TABLE 2.4:	Proportion signs in compositions by Matteo da Perugia	110
FIGURE 2.5:	Distribution of semiminims under $\bigcirc$	110
FIGURE 2.6:	Transcription of $\bigcirc$ passage in Gloria (No. 102)	111
FIGURE 2.7:	Transcription of $\bigcirc$ passage in Credo (No. 10) Source: Jos Haring and Kees Boeke, eds., <i>The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio</i> , Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019), 57.	112
FIGURE 2.8:	Transcription of $\bigcirc$ passage in <i>Pour Bel Acueil suy je, las, deceü</i>	112
FIGURE 2.9:	Transcription of $\bigcirc$ passage in <i>A qui Fortune ne se vult amer</i>	113
FIGURE 2.10:	Cantus of <i>A qui Fortune ne se vult amer</i> (fol. 43r) Source: <i>Mod A</i> , fol. 43r ⁴	114
FIGURE 2.11:	Transcription of $\bigcirc$ passage in <i>Helas, Avril, par ton doulz revenir</i>	114
FIGURE 2.12:	Transcription of $\bigcirc$ passage in Credo (No. 7)	115
FIGURE 2.13:	Transcription of $\bigcirc$ passage in <i>Se pour loyalement servir on puist merir</i>	116
FIGURE 2.14:	Transcription of $\bigcirc$ passage in <i>Dame souvrayne de beauté, d'onour</i> Source: Jos Haring and Kees Boeke, eds., <i>The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio</i> , Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019), 292.	116
FIGURE 2.15:	Transcription of $\bigcirc$ passage in Gloria (No. 2)	117
FIGURE 2.16:	Openings of Gloria (No. 2) (left) and <i>En attendant, souffrir m'estuet grief payne</i> (right)	117
FIGURE 2.17:	$\bigcirc$ passages of Gloria (No. 2) (left) and <i>En attendant, souffrir m'estuet grief payne</i> (right) Source: <i>Mod A</i> , fols. 2r and 20r ⁴	118
FIGURE 2.18:	Excerpt from cantus of <i>Le greygnour bien que nature</i> Source: <i>Mod A</i> , fol. 32r ⁴	119
FIGURE 2.19:	Examples of scribal changes in the contratenor of <i>Le greygnour bien que nature</i> Source: <i>Mod A</i> , fols. 31v and 32r ⁴	119
FIGURE 2.20:	Transcription of first $\bigcirc$ passage in <i>Già da rete</i>	121
FIGURE 2.21:	Transcription of second $\bigcirc$ passage in <i>Già da rete</i>	122

⁴ See bibliography for online resource of this manuscript.

FIGURE 2.22:	Transcription of $\bigcirc$ passage in <i>Serà quel zorno may, dolze madonna mia</i>	122
	Source: Jos Haring and Kees Boeke, eds., <i>The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio</i> , Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019), 361.	
FIGURE 2.23:	Variants of <i>sesquitertia</i> (4:3) proportion (at the minim level) and <i>dupla</i> (2:1) proportion (at the semibreve level) in Matteo da Perugia's compositions	124
TABLE 2.24:	Proportion signs in compositions by Antonello da Caserta	125
FIGURE 2.25:	$\bigcirc$ in <i>Tres nouble dame souverayne</i>	127
FIGURE 2.26:	Transcription of second $\bigcirc$ passage in <i>Tres nouble dame souverayne</i>	127
FIGURE 2.27:	Correction in the tenor of <i>Tres nouble dame</i> : A red B-B ligature and one or more black notes have been erased and replaced by a single red B-B-B ligature	129
	Source: <i>Mod A</i> , fol. 28v ⁵	
FIGURE 2.28:	Missing proportion sign in the contratenor of <i>Tres nouble dame</i>	129
	Source: <i>Mod A</i> , fol. 28v ⁵	
FIGURE 2.29:	Variants of <i>sesquitertia</i> (4:3) proportion in <i>Amour m'a le cuer mis en tel martire</i>	130
FIGURE 2.30:	Variants of <i>sesquialtera</i> (3:2) proportion in <i>Amour m'a le cuer mis en tel martire</i>	131
FIGURE 2.31:	Transcription of second $\frac{2}{2}$ passage in the cantus of <i>Dame d'onour, en qui tout mon cuer maynt</i>	133
TABLE 2.32:	Proportion signs in <i>Mod A</i> compositions by Filippotto da Caserta and Johannes Ciconia	134
FIGURE 2.33:	Transcription of $\bigcirc$ passage in <i>En attendant, souffrir m'estuet grief payne</i>	137
FIGURE 2.34:	Setting of the words "De la très noble puissance" in Machaut's <i>Ne say comment commencer</i>	137
FIGURE 2.35:	$\bigcirc$ in <i>En remirant vo douche portraiture</i>	138
FIGURE 2.36:	Transcription of longest $\bigcirc$ passage in <i>En remirant vo douche portraiture</i>	138
TABLE 2.37:	Mensuration and proportion signs in <i>Sus un' fontayne</i>	140
FIGURE 2.38:	Comparison of <i>En remirant</i> citation in <i>Sus un' fontayne</i> (cantus, top) and <i>En remirant</i> (cantus, bottom)	140
	Source: <i>Mod A</i> , fols. 27r and 34v ⁵	

⁵ See bibliography for online resource of this manuscript.

FIGURE 2.39: Comparison of <i>En remirant</i> citation in <i>Sus un' fontayne</i> (contratenor, top) and <i>En remirant</i> (contratenor, bottom) Source: <i>Mod A</i> , fols. 27r and 34v ⁶	141
FIGURE 2.40: Comparison of <i>De ma douleur</i> citation in <i>Sus un' fontayne</i> (contratenor, top) and <i>De ma douleur</i> (contratenor, bottom) Source: <i>Mod A</i> , fol. 26v ⁶	141
FIGURE 2.41: Cantus of <i>Sus un' fontayne</i> (proportion and mensuration signs highlighted) Source: <i>Mod A</i> , fol. 27r ⁶	142
TABLE 2.42: Proportion signs in compositions by Corrado da Pistoia, Bartolomeo da Bologna and two anonymous <i>Mod A</i> works	143
FIGURE 2.43: Cantus of <i>Se doulz espour ne me donne confort</i> Source: <i>Mod A</i> , fol. 31v ⁶	144
FIGURE 2.44: Transcription of proportion signs in all three voices in <i>Se doulz espour ne me donne confort</i> Source: Jos Haring and Kees Boeke, eds., <i>The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio</i> , Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019), 239.	145
FIGURE 2.45: Comparison of <i>Ch's Bien dire</i> (left) with <i>Mod A's Se doulz espour</i> (right) Source: <i>Ch</i> , fol. 51r and <i>Mod A</i> , fol. 31v ⁷	146
FIGURE 2.46: Excerpts from contratenor of <i>Que pena maior agitanda menti</i> : beginning (left) and end (right) Source: <i>Mod A</i> , fols. 36v and 37r ⁶	147
FIGURE 2.47: Beginnings of <i>En attendant souffrir</i> (left) and <i>Que pena maior</i> (right) Source: <i>Mod A</i> , fols. 20r and 37r ⁶	148
FIGURE 2.48: <i>Ma douce amour</i> in <i>Mod A</i> (left), <i>Ch</i> (middle), and <i>Ox213</i> (right) Source: <i>Ch</i> , fol. 34r and <i>Mod A</i> , fol. 28r, and <i>Ox213</i> , fol. 123r ⁷	149
FIGURE 2.49: Transcription of second part of <i>Ma douce amour</i> Source: Jos Haring and Kees Boeke, eds., <i>The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio</i> , Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019), 210.	150
FIGURE 2.50: First part of cantus of <i>En un vergier clos par mesure</i> Source: <i>Mod A</i> , fol. 18v ⁶	152
FIGURE 2.51: Final staff of tenor voice of <i>En un vergier clos par mesure</i> Source: <i>Mod A</i> , fol. 18v ⁶	152
FIGURE 2.52: Arabic numerals in the <i>Gloria</i> (No. 3): fol. 2v (top) and fol. 2r (bottom) Source: <i>Mod A</i> , fols. 2v and 2r ⁶	155

⁶ See bibliography for online resource of this manuscript.

⁷ See bibliography for online resources of these manuscripts.

FIGURE 2.53:	Arabic numerals in the <i>Gloria</i> (No. 4), fol. 4r	155
	Source: <i>Mod A</i> , fol. 4r ⁸	
FIGURE 3.1:	Fol. 152v of <i>Turin</i> (rondeaux <i>Qui n'a le cuer</i> and <i>Ie la remire, la belle</i>)	158
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 152v.	
FIGURE 3.2:	Coat of arms of the Avogadro family held by an angel displayed at the bottom of fol. 1r from <i>Turin</i>	160
	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 1r.	
TABLE 3.3:	Proportion signs and their interpretation in <i>Turin</i>	163
FIGURE 3.4:	Proportion sign $\frac{3}{1}$ in <i>Sous un bel abre</i>	164
	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 110r.	
TABLE 3.5:	Appearances of $\frac{3}{2}$ with either $\bigcirc$ or <i>dragmae</i> in <i>Turin</i>	165
FIGURE 3.6:	Rondeau <i>Il faut, pour trouver un bon port</i> (proportion sign and <i>dragmae</i> sections encircled)	166
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 153v.	
FIGURE 3.7:	Rondeau <i>Mener chiere lye</i> (proportion sign sections encircled)	166
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 155v.	
FIGURE 3.8:	Proportion sign section of <i>S'auncunne fois Fortune</i> (enhanced) with transcription	167
	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 138v; and Richard H. Hoppin, ed., <i>The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9</i> , CMM 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 3:176.	
FIGURE 3.9:	Cantus of the ballade <i>Sur toute fleur la rose est colourie</i>	169
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 137r.	
TABLE 3.10:	Copying order of all proportion sign pieces in <i>Turin</i> (special proportion sign pieces emphasised)	171
TABLE 3.11:	Interpretation of proportion signs in <i>Turin</i> compositions with special signs	172
TABLE 3.12:	Unusual proportion sign choices in <i>Turin</i>	173
TABLE 3.13:	Obvious proportion sign choices in the five <i>Turin</i> songs	175
FIGURE 3.14:	Tenor and contratenor of the ballade <i>Celle en qui j'ai mise m'amour</i>	179
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 131r.	
FIGURE 3.15:	Tenor and canon of the ballade <i>Se de mon mal delivré prestement</i>	180
REMOVED	Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 124v.	

⁸ See bibliography for online resource of this manuscript.

TABLE 3.16:	Terms for proportions used in the canons in <i>Turin</i>	180
FIGURE 3.17:	<i>Dupla</i> (2:1) proportion at the minim level in <i>Celle en qui j'ai mise m'amour</i> Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 131r; and Richard H. Hoppin, ed., <i>The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9</i> , CMM 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 3:144.	184
FIGURE 3.18:	<i>Quadrupla</i> (4:1) proportion at the minim level in <i>Se de mon mal delivré prestement</i> Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 124v; and Richard H. Hoppin, ed., <i>The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9</i> , CMM 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 3:119.	184
FIGURE 3.19:	Proportion sign melismata in <i>Je prens d'amour noriture</i> Source: Isabella Data and Karl Kügle, eds., <i>Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition</i> , Ars Nova 4 (Lucca: LIM Editrice, 1999), 154r; and Richard H. Hoppin, ed., <i>The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9</i> , CMM 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 4:53–54.	185
FIGURE 3.20:	Simultaneous ©-sections in cantus and contratenor of <i>Puis que amé</i> Source: Richard H. Hoppin, ed., <i>The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9</i> , CMM 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 3:44.	186
FIGURE 3.21:	Comparison of three <i>Turin</i> compositions by Daniel Leech-Wilkinson Source: Daniel Leech-Wilkinson, “The Cyprus Songs,” in <i>The Cypriot-French Repertory of the Manuscript Torino J.II.9</i> , Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 416.	190
TABLE 4.1:	Ugolino's compositions in <i>Cas</i>	194
FIGURE 4.2:	Cantus of <i>Chi solo a si</i> (<i>Cas</i> , fol. 345v)	195
REMOVED	Source: Elisabeth Hufnagel, “Adapting the Concept of <i>Proportio</i> to Rhythm in the <i>Ars subtilior</i> ” in <i>Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts</i> , ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 464.	
FIGURE 4.3:	Cantus and tenor of <i>Se videar</i> (<i>Cas</i> , fols. 343v (left) and 344r (right))	196
REMOVED	Source: Elisabeth Hufnagel, “Adapting the Concept of <i>Proportio</i> to Rhythm in the <i>Ars subtilior</i> ” in <i>Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts</i> , ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 460–61.	
FIGURE 4.4:	Cantus and tenor of <i>L'alta virtute</i> (<i>Cas</i> , fols. 344v (left) and 345r (right))	196
REMOVED	Source: Elisabeth Hufnagel, “Adapting the Concept of <i>Proportio</i> to Rhythm in the <i>Ars subtilior</i> ” in <i>Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts</i> , ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 462–63.	
TABLE 4.5:	Proportion signs and their interpretation in the <i>Cas</i> compositions	197

FIGURE 4.6: REMOVED	Fractions in Ugolino's <i>Declaratio</i> (left: <i>Cas</i> ; right: <i>V-CVbav Urb. lat.</i> 258; fractions encircled) Source: <i>Cas</i> , fol. 212r (416) ⁹ ; and <i>V-CVbav Urb. lat.</i> 258, fol. 226v ¹⁰	199
FIGURE 4.7: REMOVED	Diamond-shaped sign $\diamond$ in Ugolino's <i>Declaratio</i> (top: <i>Cas</i> ; bottom: <i>V-CVbav Urb. lat.</i> 258; sign encircled) Source: <i>Cas</i> , fol. 212r (416) ⁹ ; and <i>V-CVbav Urb. lat.</i> 258, fol. 227r ¹⁰	199
FIGURE 4.8:	Comparison of proportion signs in Ugolino's <i>Declaratio</i> and his compositions in <i>Cas</i>	200
FIGURE 4.9:	Proportion signs and their interpretation in <i>L'alta virtute</i> in the order of their appearance	203
FIGURE 5.1:	Screenshot of TML search Source: https://chmtl.indiana.edu/tml/quaero	217
FIGURE 5.2:	Illustration from Albert Seay's edition of Ugolino da Orvieto's <i>Tractatus monochordi</i> Source: Albert Seay, ed., <i>Ugolino of Orvieto: Declaratio Musicae Disciplinae</i> , Corpus scriptorum de musica 7, 3 vols. (Rome: American Institute of Musicology, 1959–62) 3:253.	219
FIGURE 5.3:	Reproduction of Seay's illustration in TML edition Source: http://www.chmtl.indiana.edu/tml/15th/UGOTRAM	219
FIGURE 5.4:	Annotated version of Table 6 of Busse Berger's <i>Mensuration and Proportion Signs</i>	222
FIGURE 5.5:	Illustrations of rhythmic proportion in the anonymous treatise <i>Aliquae demonstrationes in proportionibus</i> (excerpt) Source: <i>I-Sc L.V.30</i> , fol. 143v ¹¹	223
TABLE 5.6:	Proportion signs and their interpretation in treatises up to c.1450	227
TABLE 5.7:	Proportion sign treatises up to c.1450	228
FIGURE 5.8:	Fol. 36v of <i>London, British Library, MS Add. 34200</i> Source: Richard J. Wingell, "Anonymous XI (CS III): An Edition, Translation, and Commentary," PhD diss., University of Southern California, 1973, 154.	244
FIGURE 5.9:	Transcription of table on fol. 40v of <i>London, British Library, MS Add. 34200</i> Source: Richard J. Wingell, "Anonymous XI (CS III): An Edition, Translation, and Commentary," PhD diss., University of Southern California, 1973, 168.	246
FIGURE 5.10:	<i>I-Fn Magliab. III 70</i> , fol. 2b, excerpt (empty space (line 15) for presumed missing proportion sign encircled) Source: Israel Adler, ed., <i>Hebrew Writings Concerning Music in Manuscripts and Printed Books from Geonic Times up to 1800</i> , RISM B IX 2 (Munich: G. Henle Verlag, 1975), 70.	259

⁹ I thank Andreas Janke for providing me with a digitised version of the microform.

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¹¹ I thank Jan Herlinger for sharing his images of the manuscript with me.

FIGURE 5.11:	<i>I-Fn Magliab. III 70, fol. 3b, excerpt (proportion sign 4 (line 12) encircled)</i>	260
	Source: Israel Adler, ed., <i>Hebrew Writings Concerning Music in Manuscripts and Printed Books from Geonic Times up to 1800</i> , RISM B IX 2 (Munich: G. Henle Verlag, 1975), 72.	
FIGURE 5.12:	<i>Old Hall, fol. 17v, excerpt showing notes in blue ink</i>	261
	Source: <i>Old Hall</i> , fol. 17v ¹²	
FIGURE 5.13:	Incomplete musical example in <i>Incipiunt regule proporcionum</i>	263
	Source: <i>I-Sc L.V.30</i> , fol. 69r ¹³	
FIGURE 5.14:	Musical examples (fols. 123r–124v) in the treatise <i>Proportio est duarum rerum</i>	267
	Source: <i>GB-Lbl Lansdowne 763</i> , fols. 123r–124v ¹²	
FIGURE 5.15:	Musical example demonstrating <i>quadrupla</i> (4:1) proportion; minim units (according to tenor) marked and numbered; proportion sign encircled	268
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 123r ¹²	
FIGURE 5.16:	Musical example demonstrating <i>dupla sesquiquarta</i> (9:4) proportion; breve units marked and numbered; dot of division encircled	268
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 124v ¹²	
FIGURE 5.17:	Musical example demonstrating <i>trippla</i> (3:1) proportion; minim units (according to tenor) marked and numbered	269
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 123r ¹²	
FIGURE 5.18:	Musical example demonstrating <i>sesquitertia</i> (4:3) proportion; semi-breve units (according to tenor) marked and numbered; proportion sign encircled (blue); mensuration signs encircled (green)	270
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 124v ¹²	
FIGURE 5.19:	Musical example demonstrating <i>sesquialtera</i> (3:2) proportion; semi-breve units (according to tenor) marked and numbered; mensuration signs and dot of division encircled (green); dot of addition encircled (orange)	270
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 124v ¹²	
FIGURE 5.20:	Musical example demonstrating <i>dupla</i> (2:1) proportion; semibreve units (according to tenor) marked and numbered; proportion signs encircled (blue); dot of division in cantus encircled (green)	271
	Source: <i>GB-Lbl Lansdowne 763</i> , fol. 123r ¹²	
TABLE 5.21:	Proportion signs in Georgius Erber's treatise <i>Sequuntur proportiones</i>	275
FIGURE 5.22:	Musical example demonstrating <i>sesquialtera</i> (3:2) proportion in Georgius Erber's treatise <i>Sequuntur proportiones</i>	276
	Source: Renate Federhofer-Königs, "Ein Beitrag zur Proportionenlehre in der zweiten Hälfte des 15. Jahrhunderts," <i>Studia Musicologica Academiae Scientiarum Hungaricae</i> 11, no. 1 (1969): 154.	

¹² See bibliography for online resource of this manuscript.

¹³ I thank Jan Herlinger for sharing his images of the manuscript with me.

FIGURE 5.23:	<i>Siena, Biblioteca Comunale degli Intronati, MS L.V.30, fols. 142r–142v</i>	278
	Source: <i>I-Sc L.V.30, fols. 142r–142v</i> ¹⁴	
FIGURE 5.24:	<i>Iste sunt proportiones, fol. 142r (excerpt)</i>	279
	Source: <i>I-Sc L.V.30, fols. 142r–142v</i> ¹⁴	
FIGURE 5.25:	<i>Regensburg, Bischöfliche Zentralbibliothek, Proschesche Musikabteilung, MS 98 th. 4°, 372–73</i>	281
	Source: <i>D-Rp 98 th. 4°, 372–73</i> ¹⁵	
TABLE 5.26:	Proportion signs in <i>Venerabiles domini mei</i>	282
FIGURE 5.27:	Musical example demonstrating <i>sesquiquarta</i> (5:4) instead of <i>sesquitertia</i> (4:3) proportion	283
	Source: <i>D-Rp 98 th. 4°, 376</i> ¹⁵	
FIGURE 5.28:	Musical example intended to demonstrate <i>sesquioctava</i> (9:8) proportion	284
	Source: <i>D-Rp 98 th. 4°, 377</i> ¹⁵	
FIGURE 5.29:	<i>Rome, Biblioteca Apostolica Vaticana, MS Reg. lat. 1146, fols. 50v–51r</i>	285
REMOVED	Source: <i>V-CVbav Reg. lat. 1146, fols. 50v–51r</i> ¹⁶	
FIGURE 5.30:	Second musical example in <i>Pars aliquota est</i> (cantus upper staff, tenor lower staff)	286
REMOVED	Source: <i>V-CVbav Reg. lat. 1146, fol. 51r</i> ¹⁶	
FIGURE 5.31:	Transcription of second musical example	286
FIGURE 5.32:	Third musical example in <i>Pars aliquota est</i> (note that the tenor begins on the second staff right after the final <i>longa</i> of the cantus)	288
REMOVED	Source: <i>V-CVbav Reg. lat. 1146, fol. 51r</i> ¹⁶	
FIGURE 5.33:	Transcription of third musical example	288
FIGURE 5.34:	First musical example in <i>Pars aliquota est</i> (note that the tenor begins on the third staff right after the final <i>longa</i> of the cantus)	290
REMOVED	Source: <i>V-CVbav Reg. lat. 1146, fol. 50v</i> ¹⁶	
FIGURE 5.35:	Transcription of first musical example	290
FIGURE 6.1:	Final staff of cantus of <i>Bergerette</i> in <i>Pz</i> , fol. 12v	302
	Source: <i>Pz</i> , fol. 12v ¹⁶	
TABLE A.1:	Results of TML Search with Search Term ‘ <i>propor*</i> ’ in Combination with the Selection 15 th Century	364

¹⁴ I thank Jan Herlinger for sharing his images of the manuscript with me.

¹⁵ I thank Dr. Raymond Dittrich from the Proschesche Musikabteilung of the Bischöfliche Zentralbibliothek Regensburg for sending me a digital copy of these pages.

¹⁶ See bibliography for online resource of this manuscript.

INTRODUCTION

In the decades before and after the year 1400, so-called proportion signs were introduced in musical notation. By proportionally changing the duration of certain notes, these signs enabled the depiction of more complex rhythms and very small note values. Proportion signs appear in the form of geometric shapes, single Arabic numerals, and stacked Arabic numerals¹ and they can be found—not exclusively but for the greater part—in the secular repertoire transmitted in the French notational system.

The introduction of proportion signs into musical notation coincided with the emergence of a novel musical style: the so-called *Ars subtilior*.² Composers began to experiment with complex rhythms and used extensions of the French notational system—such as coloration (notes written in a different ink colour or hollowed notes) and special note shapes (notes with additional beams or flags), as well as proportion signs—in order to depict these rhythms. Given the importance of notational devices in the development of the *Ars subtilior*, it is surprising that most musicologists so far have treated proportion signs as a rather marginal aspect of their research on *Ars subtilior* manuscripts. Proportion signs have not yet been the sole focus of any broader study—a gap I am intending to fill, both empirically and theoretically. Therefore, Chapter 1 of my study systematically surveys the background as well as the ‘vocabulary’ and ‘language’ of proportion signs and gives the first ever definition of the concept. It should be noted that one scholar has dedicated an extensive chapter of more than 60 pages on the subject of proportion signs, namely Anna Maria Busse Berger in her distinguished monograph *Mensuration and Proportion Signs*.³ The present study also aims to re-evaluate the assertions and research results of this thirty-year-old publication.

The use of numbers is one of the most interesting techniques composers used to signify proportions. One of the significant advantages of proportion signs appearing as stacked Arabic numerals over coloration or special note shapes is that these signs can unequivocally convey information about the underlying proportion, which must be applied to the following notes. The

¹ Note that most scholars refer to stacked Arabic numerals as ‘fractions’. I have decided against the term ‘fractions’ since nearly all the proportion signs discussed in this study are missing the stroke between the two numbers. They appear as stacked numbers ($\frac{a}{b}$) and not as fractions ($\frac{a}{b}$). Moreover, the signs appearing as stacked Arabic numerals are not referred to as ‘fractiones’ in the treatises analysed in Chap. 5 of this study. The exception is Ugolino da Orvieto’s treatise *Declaratio musicae disciplinae*, which displays proportions as fractions ($\frac{a}{b}$) in at least two sources. See Chap. 4.3 for details.

² The term ‘*Ars subtilior*’ was coined by Ursula Günther (cf. “Das Ende der ars nova,” *Die Musikforschung* 16, no. 2 (1963): 105–20) and it has become a widely accepted designation of this particular musical style. However, its introduction has not gone without controversy. A detailed discussion of the term can be found in Chap. 1.2.

³ Cf. “Proportion Signs,” chap. 6 in *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 164–226.

sign $\frac{4}{3}$, for example, clearly shows in its two Arabic numerals that four notes of a particular value replace three of another section, i.e. that *sesquitertia* (4:3) proportion⁴ has to be applied. With special note shapes or coloration, one must establish this information through consistent use or explain their meaning in some other way, for example, in a verbal canon accompanying the composition. Through the consistent use of red ink coloration as a sign of *sesquialtera* (3:2) proportion, for instance, this means of rhythmic notation could be considered standardised for this particular proportion.⁵

The advantage that some proportion signs can convey information about the underlying proportion unequivocally seems to have been of no consequence, however. On the contrary, even a brief survey of the sources reveals that stacked Arabic numerals only rarely appear in music manuscripts of the *Ars subtilior*. Single Arabic numerals and geometric shapes are much more common, although their interpretation is less straightforward. The proportion sign 3, for example, can indicate *sesquialtera* (3:2) or *trippla* (3:1) proportion—it does not show which of the two by its visual appearance. Did a similar standardisation process occur here, comparable to that of red ink coloration? Or was it that the signs were used so consistently that an explanation was rendered unnecessary? With this question in mind, I set out to examine *Ars subtilior* music manuscripts as well as contemporary theoretical writings.

It seems that there was little standardisation concerning proportion signs. In her famous article on the early use of the proportion sign Ø, Margaret Bent has concluded:

Among a great variety of proportional signs and colorations used around and after 1400, very few notational usages could have been viewed as standard representations of particular temporal relationships; conversely, very few temporal relationships enjoyed monopoly of a single sign.⁶

Nevertheless, this study aims to investigate the exceptions from the otherwise highly individual approach to rhythmic notation, which the music manuscripts of the *Ars subtilior* reflect upon. Interestingly, even concordances sometimes show differences in the notation of the same rhythms, which raises the question of who was ultimately responsible for the notation in a composition: the composer, the scribe or some third party who added the signs to a now lost

⁴ I will refer to proportions in their Latin denominations, e.g. ‘*sesquitertia*’, throughout this study since these Latin—sometimes Greek—terms were used for proportions around 1400. The modern numeric relations, e.g. 4:3, will always be given in brackets following the Latin terms, but they are meant as an aid for the modern reader and are not historic. The visual appearance of proportions as a:b only came into use in the seventeenth century. Cf. Michael Baxandall, *Painting and Experience in Fifteenth Century Italy: A Primer in the Social History of Pictorial Style* (Oxford: Clarendon Press, 1972), 95.

⁵ A recent study of different colorations in the *Ars subtilior* (especially in the *Old Hall* manuscript) can be found in Margaret Bent, “Principles of Mensuration and Coloration: Virtuosity and Anomalies in the Old Hall Manuscript,” in *La notazione della polifonia vocale dei secoli IX-XVII. Antologia, Parte seconda: secoli XIV-XVII*, ed. Antonio Delfino and Francesco Saggio (Pisa: Edizioni ETS, 2022), 73–95.

⁶ Margaret Bent, “The Early Use of the Sign Ø,” *Early Music* 24, no. 2 (1996): 202.

exemplar?⁷ Or were there perhaps other reasons why one and the same person used different signs for the same proportion?

Furthermore, I intend to find out whether there is any significance to the position of a proportion sign within a certain piece. As extrinsic notational element, the proportion sign is clearly visible and stands out against the notes used in a composition. Does the sign mark something else apart from the beginning of a proportional rhythm, i.e. does it have more than one function?

Following the introductory chapter, the basic outline of this study follows the different materials and archival sources. The primary goal of this study is to examine the appearances, interpretations and functions of proportion signs in two main music sources of the *Ars subtilior*: *Modena, Biblioteca Estense, MS a.M.5.24* (henceforth *Mod A*) and *Turin, Biblioteca Nazionale Universitaria, MS J.II.9* (henceforth *Turin*). Chapters 2 and 3 respectively are devoted to detailed analyses of these two sources. While there are several other music manuscripts which could be considered ‘main sources of the *Ars subtilior*’, I chose these two manuscripts because they are as different as possible in certain aspects but also very similar in terms of their date and assumed place of origin.

Mod A is a collection of 104 mostly attributable compositions of sacred and secular polyphony. The manuscript consists of two distinct primary layers. The earlier one is assumed to have been compiled in Bologna around 1410, while the later one might not have been completed until the mid 1420s. *Mod A* shares concordances with other *Ars subtilior* sources, i.a. *Chantilly, Musée Condé, MS 564* (henceforth *Ch*), *Oxford, Bodleian Library, MS Canon. Misc. 213* (henceforth *Ox213*), and *Paris, Bibliothèque Nationale, nouvelles acquisitions françaises 6771* (henceforth *PR*). After a general evaluation of proportion signs within the manuscript, Chapter 2 is subdivided according to the *oeuvres* of different composers, most prominently Matteo da Perugia, Antonello da Caserta, and Filipotto da Caserta. The chapter focuses on additional functions of proportion signs as well as on the comparison of the rhythmic notation in concordant versions of the same piece. Therefore, other *Ars subtilior* manuscripts, especially *Ch*, also find their way into the discussion.

Turin, on the other hand, shares no concordance with any surviving *Ars subtilior* source. Moreover, this considerable music collection is transmitted completely anonymously. It has been suggested that the entire song repertoire of 166 songs is the work of just one composer. Due to this assertion, I have sought traces of standardisation and individualisation in those 31 songs containing proportion signs in Chapter 3. According to recent findings, the manuscript is

⁷ In this study, I will often refer to the composer as the author of the signs and will not always use phrases such as “the composer or the scribe or an unknown third party”. It would not make a very good read to always include all possibilities of authorship. This should be kept in mind.

assumed to have been compiled in Italy in the second or third decade of the fifteenth century. Hence, *Turin* is placed close to *Mod A* in terms of both their date and assumed place of origin. This circumstance makes a comparison of these two *Ars subtilior* manuscripts very promising and it was the reason for choosing these two sources for the analyses.

Following the examination of these two main sources, I will study statements on proportion signs in contemporary music theory. What did theorists say about proportion signs in treatises up to c.1450? A particularly interesting case is Ugolino da Orvieto, who was both a composer and a theorist. Chapter 4 compares his statements on proportion signs in his treatise *Declaratio musicae disciplinae* with the proportion signs found in his compositions. Do his descriptions of proportion signs in the *Declaratio* match the signs and their interpretation in his music?

In Chapter 5, I will discuss 13 other music treatises which roughly date to the first half of the fifteenth century. Scholars agree that the Italian theorist Prosdocimus de Beldemandis (c.1380–1428) was the first author who commented on proportion signs, although there are different understandings of when they were first mentioned.⁸ I argue that his 1404 treatise *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris* is the first datable source discussing proportion signs. An enlightening passage found in *Expositiones* hints towards a dispute concerning the notation of rhythmic proportion between theorists such as Prosdocimus on the one hand and composers on the other hand. After a lengthy discussion of coloration, Prosdocimus reports:

“However, a certain master of this art, presently considered master of all masters, who wants to maintain this figuration for good [because he thinks it is good], says that these figurations are sung in proportion, ignoring what is said about the proportion [in the treatises], because in reality anything which is actually sung is sung in any proportion. And further [this master] says that such figures should be sung in *sexquitercia* [4:3] proportion, and when I asked the reason for this, he could not give me any other reason than: I want it that way.”⁹

⁸ For example, F. Alberto Gallo and Laurie Koehler cite Prosdocimus’ 1404 treatise *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, while Anna Maria Busse Berger claims that his *Tractatus practice de cantus mesurabilis* of 1408 is the earliest surviving treatise, in which proportion signs are mentioned. Cf. F. Alberto Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” in *Die Mittelalterliche Lehre von der Mehrstimmigkeit*, ed. Hans Heinrich Eggebrecht et al. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1984), 340; Laurie Koehler, *Pythagoreisch-platonische Proportionen in Werken der ars nova und ars subtilior*, Göttinger musikwissenschaftliche Arbeiten 12, 2 vols. (Kassel: Bärenreiter, 1990), 1:57; and Anna Maria Busse Berger, *Mensuration and Proportion Signs*, 164. For a possible explanation for this inconsistency, see the discussion of Prosdocimus’ treatises in Chap. 5.3.1.1.

⁹ “Quidam tamen huius artis magister qui ad presens magister magistrorum reputatur, volens manutenere hanc figurationem pro bona, dicit has figurationes in proportione cantari, ignorans quid loquatur de proportionem, cum in rei veritate quicquid cantatur, in aliqua proportionem cantatur. Et dicit ulterius quod tales figure cantantur in proportionem sexquitercia, et dum huius causam quererem, nescivit mihi reddere aliam causam quam: sic volo.” Prosdocimus de Beldemandis, *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, chap. 61; edition in F. Alberto Gallo, ed., *Prosdocimi de Beldemandis opera 1: Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, *Antiquae musicae italicae scriptores* 3 (Bologna: Università degli Studi di Bologna, Istituto di Studi Musicali e Teatrali, 1966), 145; my translation.

The ‘master’ of this account might have been Johannes Ciconia, who had taken the post of *cantor et custos* at the cathedral in Padua in April 1403.¹⁰ This excerpt is exemplary of the conflict of standardisation versus individualisation, which I intend to study here. It seems that, while the theorist—in this case Prosdocius—advocates standardisation and even inquires why things are done differently from the theorist’s advice, the composer—presumably Ciconia—notates the music as he sees fit without caring about the theorist’s instructions. Is this conflict confirmed by the other treatises and the music repertoire studied here?

The analyses of proportion signs in the above-mentioned music manuscripts and treatises are preceded by a chapter that 1) introduces the cultural context of proportions in the late Middle Ages and the early Renaissance, 2) elucidates the emergence of complex rhythms in the French notational system, 3) explains the term ‘*Ars subtilior*’, 4) defines the term ‘proportion sign’, and 5) discusses Baude Cordier’s picture songs from *Ch* as an example to show which research questions and aspects are at the forefront of this study. With this extensive introductory chapter, I intend to give the reader a good grasp of the underlying concepts concerning the study of proportion signs. While my definition of the term ‘proportion sign’ and the re-evaluation of Cordier’s picture songs will provide a guideline for Chapters 2 and 3, which deal with proportion signs in music, the brief overview of the history of proportions in music will be a reference for the treatises discussed in Chapters 4 and 5.

¹⁰ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 333. It is noteworthy that Ciconia did not discuss proportion signs in his treatise *De proportionibus* of 1411, even though his compositions do contain proportion signs. See Chap. 5, 211 n. 13.

1

A NEW APPROACH TO PROPORTION SIGNS

This study will focus on rhythmical concepts and their graphic representation in music at the turn of the fifteenth century. Therefore, it seems in order to first provide the reader with a short introduction to medieval teachings of proportions and their connection to music and music teaching. In Chapter 1.1, I will briefly outline how Pythagorean and Neo-Pythagorean doctrine influenced medieval music theory. The classification and denomination of proportions according to Boethius will be discussed in more detail, because the author influenced the medieval conception of proportions, especially those applied to music. With the introduction of mensural notation in Europe, proportions increasingly appeared in contexts of rhythm and in discussions of rhythmic notation. Several notational devices were invented in order to depict the more and more complex rhythms found in fourteenth- and early fifteenth-century music, among them proportion signs.

After the introduction to proportions in medieval music and music treatises, I will elucidate my understanding of the term ‘*Ars subtilior*’ in Chapter 1.2, which includes a reading of *Ars subtilior* as movement and therefore exceeds the common interpretation of *Ars subtilior* as musical style of the late fourteenth and early fifteenth century. Understanding *Ars subtilior* as movement rather than only as musical style permits the inclusion of agents such as scribes, who had a direct influence on the notation of music, as well as theorists engaging with different aspects of *Ars subtilior* music in their treatises. For reasons specified below, I reject the understanding of the term as an epoch.

At the heart of this chapter stands what I will for want of a better term call the theoretical foundation for discussing proportion signs in the manuscripts of the *Ars subtilior*, as set forth in Chapter 1.3. What are proportion signs? They are often mentioned alongside mensuration signs but—as stated in the introduction—no scholar has ever attempted to clearly distinguish between these two terms in the context of the *Ars subtilior*. The consensus seems to be that *m e n s u r a t i o n* signs are geometric shapes, e.g. circles and semicircles, while *p r o p o r t i o n* signs are Arabic numerals. I will present a novel approach to proportion signs, including a definition of the term. This definition does not advance proportion signs from the perspective of their visual appearance, but instead takes into account their function, namely the proportional change of note values. I will explain why I think it important to regard mensuration signs and proportion signs to be indicators of two different rhythmical concepts.

Finally, I will demonstrate how to apply this new approach to proportion signs in music research by a detailed example in Chapter 1.4. Baude Cordier's picture songs *Belle, bonne, sage* and *Tout par compas* contain proportion signs in all three forms of visual appearance: single Arabic numerals, stacked Arabic numerals, and geometric shapes. I will discuss possible additional functions of proportion signs, including the function of signalling a certain position within a piece. Furthermore, I will address the question of the reference point for a proportion and argue for a linear reading direction (i.e. application within the same voice) against a vertical one (e.g. cantus against tenor). When the reference point lies within the same voice, there are—in some cases—still three different possible reference points, and I will explain the differences between these three interpretations (non-cumulative, cumulative, and quasi-cumulative). Finally, I will address different aspects of notational consistency and lack thereof within the *oeuvre* of one composer.

1.1 Proportions in Medieval Music and Music Treatises: an Introduction

1.1.1 Arithmetic, Geometric, and Harmonic Proportion

Before we approach the topic of proportions in medieval and early Renaissance culture and music, an essential remark concerning the word ‘proportion’ has to be made. The majority of texts—medieval and Renaissance music treatises as well as modern-day scholarly publications—exhibit the word ‘proportion’ or ‘*proportio*’ respectively when they refer to a relation of two numbers: $a:b$. However, the mathematical term for describing the relationship between two numbers ($a:b$) is ‘ratio’. The term ‘proportion’ in its correct mathematical meaning actually designates the similarity of two ratios: $a:b$ compared to $c:d$.

There are three different kinds of proportion: arithmetic, geometric, and harmonic proportion. In arithmetic proportion, the difference between denominator and numerator is the same: $b - a = d - c$. For example, the two ratios $2:3$ and $3:4$ can be described as arithmetic proportion, because the difference between denominator and numerator ($3 - 2$ and $4 - 3$) is 1 in both ratios. In geometric proportion, the quotient of numerator and denominator is the same: $a : b = c : d$. For example, $2:4$ and $4:8$ are called geometric proportion, because the quotient of both is 2. The third kind of proportion, namely harmonic proportion, is a little less straightforward. In harmonic proportion, the difference between denominator and numerator compared to the first numerator equals the difference between denominator and numerator compared to the second denominator: $(b - a) : a = (d - c) : d$ or, described another way, $a : d = (b - a) : (d - c)$. For example, $3:4$ and $4:6$ can be described as harmonic proportion. 4 is greater than 3 by one third of 3 and 6 is greater than 4 by one third of 6. When the denominator of the first ratio is the same as the numerator of the second ratio, one of the numbers can be omitted, which is why proportions are sometimes written as $a:b:d$, or as in the examples $2:3:4$ (arithmetic proportion), $2:4:8$ (geometric proportion), or $3:4:6$ (harmonic proportion) respectively.

The use of the two terms ‘ratio’ and ‘proportio’ as synonyms did not only commence in the Middle Ages but much earlier. The Ancient Greek word ‘ἀναλογία’ (analogia), meaning proportion, analogy, relation, correspondence, or resemblance,¹ for example, was used in the mathematical sense by Euclid, but can be found to have multiple meanings, including the de-

¹ Cf. *A Greek-English Lexicon*, comp. Henry George Liddell and Robert Scott, 9th ed. (Oxford: Clarendon Press, 1940), s.v. “ἀναλογία.”

notation ‘ratio’, in Nicomachus’ works.² The Roman philosopher Anicius Manlius Severinus Boethius, whose translations of and commentaries on Greek authorities greatly influenced medieval and Renaissance scholarship, used the term ‘proportion’ as synonym for ‘ratio’.³ As a consequence, this interpretation of ‘proportion’ can also be found in medieval and Renaissance treatises as well as modern publications on music and culture from that period.⁴ A more recent assessment of the subject can be found in Oscar João Abdounur’s study of the term ‘ratio’, in which the author has stated:

“Boethius adopted the Latin translation *proportio* and *proportionalitas* for *logos* [= ratio] and *analogia* [= proportion], respectively, which influenced tremendously either directly or indirectly the choice of terminology for such concepts throughout the Middle Ages, even among translators who worked with Arabic texts or directly consulted Greek works. For instance, through the version of Adalard, the Boethian terminology for ratio predominated during the High Middle Ages, as some later editions of the *Elements* illustrate. Contributing to the medieval misinterpretation of ratio and proportion, such terminology was superseded only with Zamberti’s translation of the *Elements* (Venice, 1505) directly from the Greek, which triggered the gradual re-establishment of the previous terminology for ratio and proportion.”⁵

In order to avoid misunderstandings, I will continue in the tradition of using the word ‘proportion’ for ‘ratio’ and speak of ‘arithmetic proportion’, ‘geometric proportion’, or ‘harmonic proportion’ when I wish to use the term in its mathematical meaning.

² Cf. Henry B. Fine, “Ratio, Proportion and Measurement in the Elements of Euclid,” *Annals of Mathematics*, Second Series, 19, no. 1 (1917): 70–76, esp. 73; and Peter H. Scholfield, *The Theory of Proportion in Architecture* (Cambridge: Cambridge University Press, 1958), 19–20. Euclid distinguished between ratio (*logos*) and proportion (*analogia*) and stated that “a proportion in three terms is the least possible.” *Elements*, Book 5, definition 8; translation taken from Thomas L. Heath, trans., *Euclid: The Thirteen Books of the Elements; Translated from the Text of Heiberg with Introduction and Commentary by Sir Thomas L. Heath*, 2nd ed., 3 vols. (New York: Dover Publications, 1956), 2:114.

³ Cf. Michael Walter, “Über den musikalischen Begriff *proportio*,” in *Musik – und die Geschichte der Philosophie und Naturwissenschaften im Mittelalter: Fragen zur Wechselwirkung von ‘Musica’ und ‘Philosophia’ im Mittelalter*, ed. Frank Hentschel (Leiden: Brill, 1998), 72; and Michael Masi, trans., *Boethian Number Theory: A translation of the De Institutione Arithmetica with Introduction and Notes*, Studies in Classical Antiquity 6 (Amsterdam: Rodopi, 1983), 26 n. 16.

⁴ For a detailed account of the development of the concept of proportionality in medieval Latin culture see Fabrizio della Seta, “Proportio: Vicende di un concetto tra scolastica e umanesimo,” in *In cantu et in sermone: For Nino Pirrotta on his 80th Birthday*, ed. Fabrizio della Seta and Franco Piperno (Florence: Olschki, 1989), 75–99.

⁵ Oscar João Abdounur, “Ratios and Music in the Late Middle Ages: a Preliminary Survey,” in *Music and Mathematics in Late Medieval and Early Modern Europe*, ed. Philippe Vendrix (Turnhout: Brepols, 2008), 40.

1.1.2 Proportions in Medieval and Early Renaissance Life and Culture⁶

Proportions were omnipresent in medieval and early Renaissance life and culture. They were not only fundamental to those four academic disciplines known as the *quadrivium*: arithmetic, geometry, astronomy, and music. Fascination with numbers and proportions is reflected in many aspects of medieval and Renaissance culture, from calculations in architecture to the introduction of linear perspective in art, as well as the engagement in magic squares or other mathematical puzzles as intellectual pastimes. Jan Herlinger has described the importance of proportions in medieval culture as follows:

“[T]he medievals came to see numeric relationships behind virtually every aspect of nature, art, and thought—in the motions of the heavenly spheres, in the harmony of soul and body, in the relationships of man to Christ; in the dimensions of cathedrals and wood carvings, in the octagonal shape of baptismal fonts; in poetic structures, metres, and rhyme schemes; and in musical tuning, melody, harmony, counterpoint, and rhythm.”⁷

Proportions also entered recreational activities. This is manifested in the game rithmomachy—also known as the ‘philosophers’ game’—, which is similar to chess and which is said to have been very popular in the Middle Ages. Once reportedly even competing with chess regarding its dissemination and popularity it fell into oblivion after the Renaissance and was only rediscovered when nineteenth- and twentieth-century historians found descriptions of its rules in various treatises.⁸

Unlike chess the pieces in rithmomachy do not have names but values (see FIGURE 1.1).⁹ Every piece in the game also moves in the same way and not in different ways according to its rank as in chess, where a pawn cannot move in as many different directions and as far as the

⁶ Some paragraphs in this and the following subchapter draw on a previous publication of mine, in which I have also outlined the importance of proportions in medieval culture and education. Cf. Elisabeth Hufnagel, “Adapting the Concept of *Proportio* to Rhythm in the *Ars subtilior*,” in *Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts*, ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 441–64.

⁷ Jan Herlinger, “Music Theory of the Fourteenth and Early Fifteenth Centuries,” in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie J. Blackburn (New York: Oxford University Press, 2001), 295.

⁸ For detailed accounts of the game’s history, its different versions and a list of treatises and other sources describing its rules see Arno Borst, *Das Mittelalterliche Zahlenkampfspiel* (Heidelberg: Carl Winter Universitätsverlag, 1986); and Ann E. Moyer, *The Philosophers’ Game: Rithmomachia in Medieval and Renaissance Europe* (Ann Arbor: University of Michigan Press, 2001).

⁹ The top row of the white pieces is made up of the first four even numbers 2, 4, 6, and 8. These pieces are faced by black ones bearing the numbers 3, 5, 7, and 9 on the opponent’s side. The values of the other pieces in the game are derived from these eight numbers in such a way that proportions between the numbers are created. The second row, for example, is comprised of the squares of the each number: 4, 16, 36, and 64 for the white pieces as well as 9, 25, 49, and 81 for the black pieces. The proportions between these pieces are of the multiplex type (for the different types of proportion see TABLE 1.4 below): $4:2 = 2:1$, $9:3 = 3:1$, $16:4 = 4:1$, and so on. The other numbers on the pieces in the game form superparticular and superpartient proportions with the numbers in the first row (for details see Moyer, *The Philosophers’ Game*, 10–12).

queen. However, rithmomachy and chess have in common that the first phase of the game entails capturing the opponent's pieces. In rithmomachy, this is done by matching the value of an opponent's piece by two or more of one's own pieces by addition or multiplication. For example, the white piece with the number eight can be captured by the black pieces 5 and 3, because the sum of 3 and 5 is 8. While in chess the object of the game is to checkmate the opponent's king, the ultimate goal of rithmomachy is assembling a line of arithmetic, geometric or harmonic proportion (in some versions one is allowed to use the captured pieces for this purpose).¹⁰ Rithmomachy can therefore be considered as game in which the understanding of basic mathematical concepts and proportions was trained in a playful manner.

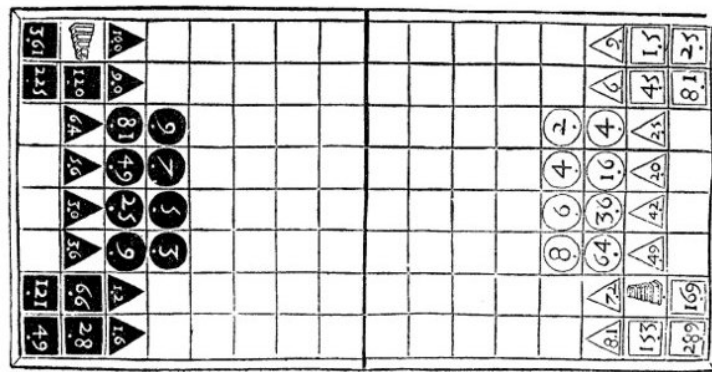


FIGURE 1.1: Rithmomachy board with black and white pieces

Proportions not only played a role in the life and works of intellectuals and artists, they were also vital to merchants who often travelled between different cities, each of which had its own currency as well as weights and measures.¹¹ Where proportions are concerned, the academic, cultural, and commercial worlds were closely connected and most probably exerted great influence on each other.¹² Mathematics—even commercial calculations—influenced the development of linear perspective in the fifteenth century, as did the study of geometry.¹³ The teachings of proportions, formed the basis of medieval and Renaissance architecture.¹⁴

¹⁰ For the difference between these three types of mathematical proportion see Chap. 1.1.1 above.

¹¹ An account of connections between late medieval and early Renaissance systems of measuring music and other measuring systems—especially commercial ones—is given by Anna Maria Busse Berger, “The Origins of the Mensural System and Mensuration Signs,” chap. 2 in *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 33–50.

¹² For a general appraisal of the role of proportions in various disciplines during the Middle Ages see Umberto Eco, “The Aesthetics of Proportion,” chap. 3 in *Art and Beauty in the Middle Ages*, trans. Hugh Bredin (New Haven: Yale University Press, 1986), 28–42.

¹³ See for example Michael Baxandall, *Painting and Experience in Fifteenth Century Italy: A Primer in the Social History of Pictorial Style* (Oxford: Clarendon Press, 1972), 86–108; and Kirsti Andersen, *The Geometry of an Art: The History of the Mathematical Theory of Perspective from Alberti to Monge* (New York: Springer, 2007), 17–80.

¹⁴ See for example Rudolf Wittkower, *Architectural Principles in the Age of Humanism*, Studies of the Warburg Institute 19, 2nd ed. (London: Alec Tirani Ltd., 1952), 89–135; and Scholfield, *Proportion in Architecture*, 33–

1.1.3 Pythagoras' *'Musica'* and Boethius' Proportions

According to legend, Pythagoras was the first to discover the close connection between music and mathematics.¹⁵ Due to the lack of authentic testimony we have to draw on Nicomachus of Gerasa's early second-century report of the incident.¹⁶ It tells the story of Pythagoras passing a blacksmith's shop one day. He is said to have been struck by the consonant harmony in which some of the different hammers thudded and hastened to find an explanation for the pleasing sound, which some pairs of hammers produced, and the dissonant sound created by other pairs. When he weighed the hammers he realised that the pitch levels were proportional to the weight of the hammers and that musical intervals could therefore be derived from numerical proportions: 2:1 will result in the octave (*diapason*), 3:2 in the perfect fifth (*diapente*), 4:3 in the perfect fourth (*diatessaron*), and 9:8 in the major second (*epogdoon*).¹⁷ While modern scholars have repeatedly remarked that pitch levels are not proportional to the weight of the hammers in the way described by Nicomachus,¹⁸ the tale of Pythagoras and his hammers was well received throughout the Middle Ages until the Renaissance. The tale is—among other treatises—recounted in Marchetto da Padova's *Lucidarium in arte musicae planae*, Johannes de Muris' *Musica speculativa secundum Boetium*, and Ugolino da Orvieto's *Declaratio musicae disciplinae*.¹⁹ The hammers are even depicted in Franchinus Gaffurius' influential

81. Wittkower has summarised this influence in Renaissance architecture as follows: "The conviction that architecture is a science, and that each part of a building, inside as well as outside, has to be integrated into one and the same system of mathematical ratios, may be called the basic axiom of Renaissance architects." *Architectural Principles*, 89.

¹⁵ It should be noted, however, that some medieval or Renaissance writers attributed the discovery of musical intervals to the biblical figure Jubal ("the ancestor of all those who play the lyre and pipe" Gen. 4:21 (New Revised Standard Version)). Cf. James W. McKinnon, "Jubal vel Pythagoras: Quis Sit Inventor Musicae?," *The Musical Quarterly* 64, no. 1 (1978): 1–28.

¹⁶ Nicomachus of Gerasa, *Manual of Harmonics*, chap. 6; translation in Andrew Barker, trans., *Greek Musical Writings II: Harmonic and Acoustic Theory* (Cambridge: Cambridge University Press, 1989), 256–58. No authentic writings of Pythagoras have survived, thus all modern knowledge about his life and teachings is retrieved from secondary sources.

¹⁷ The terms for the four musical intervals in brackets were used in treatises throughout the centuries. They originate from Ancient Greek: *διά* (*diá*), which means 'through' and *πᾶς* (*pas*) 'all', which might refer to all strings of an instrument with an octave range; *πέντε* (*pénte*) is the Ancient Greek word for 'five'; *τέσσαρες* (*téssares*) is 'four'; *ἐπὶ ὀγδοὺν* (*epógdoun*) is a combination of the terms *ἐπὶ* (*epi*, 'being or resting upon') and *ὀγδοὺς* (*ógdoos*, 'eighth'), i.e. being upon the eighth number. Cf. *A Greek-English Lexicon*, comp. Liddell and Scott, s.vv. "*διά*," "*πᾶς*," "*πέντε*," "*τέσσαρες*," "*ἐπὶ*," and "*ὀγδοὺς*."

¹⁸ Cf. for example Barbara Münxelhaus, *Pythagoras musicus: Zur Rezeption der pythagoreischen Musiktheorie als quadrivieraler Wissenschaft im lateinischen Mittelalter*, Orpheus-Schriftenreihe zu Grundfragen der Musik 19, ed. Martin Vogel (Bonn: Verlag für systematische Musikwissenschaft, 1976), 37 and 50–53; and Calvin M. Bower, "The Transmission of Ancient Music Theory into the Middle Ages," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 143. Amongst other scholars, Andrew Barker has raised doubts that the discovery of the fundamental intervallic ratios was made by Pythagoras or early Pythagoreans and has argued that instrument makers and practical musicians are much more likely candidates for making such a discovery. *Greek Musical Writings II*, 256 n. 43.

¹⁹ For specific details concerning chapters see Herlinger, "Music Theory," 296 n. 105. I have also found a retelling in the first chapter of Johannes de Grocheio's *De Musica*.

late-fifteenth-century treatise *Theorica musicae*, next to differently sized bells and pipes, glasses filled with different amounts of liquid, and strings with different weights attached to them (see FIGURE 1.2).²⁰



FIGURE 1.2: Depictions of Pythagorean proportions in sounding instruments in Gaffurius' *Theorica musicae*

Even though larger numbers are depicted in Gaffurius' treatise (4, 6, 8, 9, 12, and 16),²¹ what is said to have struck Pythagoras most was that consonant intervals (octave, perfect fifth, and perfect fourth) could all be determined by the ratios of small whole numbers, i.e. by the progression of the first four natural numbers 1, 2, 3, and 4, whose sum amounts to the 'per-

²⁰ Gaffurius, *Theorica musicae*, Book 1, fol. 18r. Remarkably, Gaffurius put Iubal (see n. 15 above) in the centre of the picture with the hammers and not Pythagoras. He explained (*Theorica musicae*, Book. 5, chap. 1) that Iubal discovered the intervals of the octave, fifth, fourth and major second but gave credit to Pythagoras for establishing music as science. Cf. Münxelhaus, *Pythagoras musicus*, 45. Depictions of Pythagoras with hammers can be found in several medieval manuscripts, for example, the early twelfth-century manuscript *Wolfenbüttel, Herzog August Bibliothek, Cod. Guelf 334 Gud. Lat.* (fols. 1v and 2r), the ca. 1160 English manuscript *Cambridge, Cambridge University Library, MS li.3.12* (fol. 61v), which reproduces Boethius' *De institutione musica*, and the ca. 1225–30 manuscript *Munich, Bayerische Staatsbibliothek, Clm 2599* (fol. 96v).

²¹ These are the numbers usually mentioned in connection with Pythagoras' hammers in medieval and Renaissance treatises. With this set all important intervals related to Pythagoras' discovery and his *tetractys* (see below) can be described by using natural numbers, including the dissonant major second (9:8), which cannot numerically be referred to by using the numbers 1 to 4. With the set of numbers found in Nicomachus' account of Pythagoras in the blacksmith's shop and depicted in Gaffurius' *Theorica musicae*, the octave (*dupla* (2:1) proportion) can be created by 8:4, 12:6, or 16:8, the fifth (*sesquialtera* (3:2) proportion) by 9:6 or 12:8, the fourth (*sesquitercia* (4:3) proportion) by 8:6 or 16:12, the twelfth (*trippla* (3:1) proportion) by 12:4, the double octave (*quadrupla* (4:1) proportion) by 16:4, and the major second (*sesquioctava* (9:8) proportion) by 9:8.

fect' number 10 and which together form the Pythagorean *tetractys* (see FIGURE 1.3).²² The first and second number gives the interval of the octave (2:1), the second and third number the fifth (3:2) and the third and fourth number the fourth (4:3). The series can produce two further consonances: the compound intervals of an octave and a fifth (3:1) and the double octave (4:1).

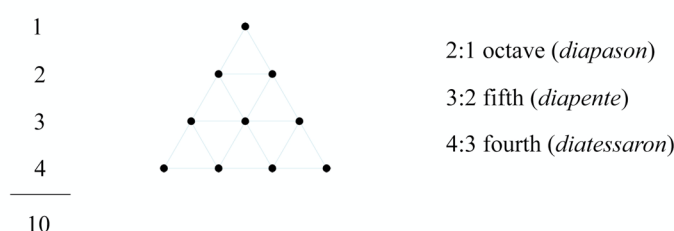


FIGURE 1.3: Pythagorean *tetractys*

Pythagoreans considered number and proportion to be the first principles of everything in existence and believed that there was mathematical and geometrical harmony in the universe. As Herlinger has put it: “Pythagoreanism may be defined as the doctrine that all being is governed by numbers.”²³ In the discovery that ‘simple’ proportions, i.e. ratios containing small natural numbers, produced consonant harmonies, Pythagoreans saw confirmation of their beliefs. In other words, Pythagorean doctrine proclaims that God has based the entire universe and everything within it on mathematic principles and that the proportional harmony is reflected in consonant intervals. Listening to or performing music as well as studying numbers were therefore regarded as acts of purification, in which the microcosm, i.e. the human soul, was set in harmony with the macrocosm, the universe, which were both determined by the same mathematical ratios.²⁴

²² Although ‘perfect’ numbers were later considered to be numbers, which equal the sum of their proper divisors (e.g. $6 = 1 + 2 + 3$), by Euclid and his successors, the number 10 was also called ‘perfect’ by Pythagoreans, because it forms the basis of the decimal system. Cf. Bartel L. van der Waerden, *Science Awakening*, trans. Arnold Dresden, 2 vols., 3rd ed. (Groningen: Wolters Noordhoff Publishing, 1969), 1:97–98 for an explanation of perfect numbers, which are sums of their proper divisors; and Thomas Heath, *Mathematics in Aristotle* (Oxford: Oxford University Press, 1949), 258–60 for the number 10 as fundamental to the decimal system and therefore perfect number in Pythagorean doctrine. Scholfield has outlined the differences in the conception of the ‘perfect number’ in *Proportion in Architecture*, 29. Later authorities—for example Remigius of Auxerre, who is quoted by Marchetto da Padova and Johannes Ciconia—considered 3, 6, and 9 to be perfect numbers. On disagreements on the matter between the contemporaries Johannes Ciconia and Prosdocius de Beldemandis see Jan Herlinger, “Prosdocius de Beldemandis contra Johannem Ciconiam?,” in *Johannes Ciconia: musicien de la transition*, ed. Philippe Vendrix (Turnhout: Brepols, 2003), 314–16.

²³ Herlinger, “Music Theory,” 294.

²⁴ For a more detailed account of Pythagorean doctrine see for example Calvin M. Bower, trans., introduction to *Fundamentals of Music: Anicius Manlius Severinus Boethius*, ed. Claude V. Palisca (New Haven: Yale University Press, 1989), xx–xxiv; David S. Chamberlain, “Philosophy of Music in the *Consolatio* of Boethius,” *Speculum* 45, no. 1 (1970), 80–97; and Edgar De Bruyne, *Etudes d’esthétique médiévale*, 3 vols. (Bruges: De Tempel, 1946), 1:3–26 and 306–38, 2:108–32, and 3:227–38.

Pythagorean doctrine influenced Plato, Aristotle, and Neo-Pythagoreans, such as Nicomachus of Gerasa, and led to the establishment of the *quadrivium*, comprising the four mathematical arts arithmetic,²⁵ geometry, astronomy, and music.²⁶ The combination of *trivium* (grammar, rhetoric, and logic) and *quadrivium* constituted the seven liberal arts, which formed the foundation of Western education until the Renaissance.²⁷ Plato had already described the four *quadrivium* subjects as fundamental disciplines for the education of statesmen in his *Republic*.²⁸ *Musica* was primarily considered to be a *scientia*, in which music was regarded from a mathematical and philosophical perspective. Number theory and the calculation of intervals were fundamental parts of the curriculum in this subject, while practical music making was hardly of interest.

Frieder Zaminer has described the view of music in antiquity as follows:

“Thinking about music and musical phenomena, namely the persistent examination of harmonic problems, was a matter of philosophy and science in antiquity. It was about recognition and interpretation of meaning of certain phenomena, which were ‘natural’ and independent from the arbitrariness of mankind and which were considered to be eternal, cosmic, and divine. Accordingly one was interested in the musical elements, the causes, the ‘substrate’, the ratios, the structures, i.e. the nature of the tone, the intervals, the consonants, the tetrachords, the modes, the construction of the tonal system, the keys etc., but also in the effect of music on people, the ethos of modes and rhythm. One was not interested in questions which lie closer to our modern understanding of music, for example ‘What do people do with these elements in practical music?’ and ‘How do people combine those elements in order to create melodies and compositions and what is the best way to do so?’ Understanding the act of composing as musicianship or art is beyond the scope of the ancient philosophers’ view of music. Following Greek conviction, Aristotle had once categorised every craft as philistine, including practical music making.”²⁹

²⁵ It should be noted that arithmetic—in the Pythagorean-Platonic sense as opposed to the Euclidean interpretation—had a meaning, which is different from today’s understanding of the term, in which arithmetic primarily comprises number theory and the traditional operations addition, subtraction, multiplication, division, raising to powers, and extraction of roots (cf. Thomas Heath, *A History of Greek Mathematics*, 2 vols. (Oxford: Clarendon Press, 1921), 1:13–16. Pythagoreans ascribed certain properties to particular numbers, which could be ‘perfect’ (see n. 22 above) or ‘amicable’—e.g. 220 and 284, each of which equals the sum of the proper divisors of the other (see Van der Waerden, *Science Awakening*, 1:98–99)—, thus number symbolism formed part of the curriculum in arithmetic as *quadrivial* discipline. For a (disputable) account of number symbolism in medieval music-related sources see Hermann Abert, *Die Musikanschauung des Mittelalters und ihre Grundlagen* (Halle: Max Niemeyer, 1905), 175–193.

²⁶ This obviously is a much-abbreviated account of the influence of Pythagorean doctrine on later philosophers as well as of ancient Greek music theory. For more on the transmission of Pythagorean philosophy and its influence on later authorities see Christiane L. Joost-Gaugier, *Measuring Heaven: Pythagoras and His Influence on Thought and Art in Antiquity and the Middle Ages* (Ithaca: Cornell University Press, 2006). Also see Bartel L. van der Waerden, “Die Harmonielehre der Pythagoreer,” *Hermes. Zeitschrift für klassische Philologie* 78, no. 2 (1943): 163–99. An overview of the traditions of ancient Greek music theory, including a table of primary Greek treatises, is given by Thomas J. Mathiesen, “Greek Music Theory,” in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 109–35. Pythagorean tradition is by no means the only ancient Greek music theory tradition, but it is the most influential were proportions are concerned, hence the focus on the Pythagorean tradition in this study.

²⁷ Some authorities included more than seven disciplines in the liberal arts. An overview can be found in Bower, “The Transmission of Ancient Music Theory,” 139.

²⁸ Plato, *Republic*, Book 7, 521c–541b.

²⁹ Frieder Zaminer, introduction to *Rezeption des antiken Fachs im Mittelalter*, ed. Michael Bernhard et al. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1990), 2; my translation. For a study of ‘*musica*’ as philosophical concept throughout the Middle Ages see Eva Hirtler, “Die *Musica* im Übergang von der *Scientia mathematica* zur *Scientia media*,” in *Musik – und die Geschichte der Philosophie und Naturwissenschaften im Mittelalter*:

Dorit Tanay has used even stronger words to describe Pythagorean influence on medieval music:

“The affiliation of music with the quadrivial-Pythagorean arithmetic took the form of the tyranny of an occult number theory over music. Musical correctness was determined by a set of simple harmonic proportions that allegedly tuned the sky and accounted for the world’s beauty. It was mathematics that dictated to music which harmonies ought to be reified in music and which not. To put it differently, the mathematization of music in antiquity and the Middle Ages turned music into a symbolic image of cosmic harmony. The evident physical presence of music became subordinated to the higher reality of transcendental perfection.”³⁰

The Ancient Greeks’ attitude towards music was often echoed in medieval music theory, where practical music (*musica practica*) regularly only came second.³¹ A distinction was made between a *musicus*, who had undergone a musical education in the quadrivial discipline, and a *cantor*, who was a mere practitioner.³² However, one can witness a departure from the Platonic-Pythagorean tradition from the fourteenth century onwards.³³ Late medieval and early Renaissance music treatises often contain both, speculative approaches and chapters on practical aspects, such as instructions on music notation, e.g. descriptions and illustrations of signs and note shapes, as well as counterpoint. Tanay has described in detail how fourteenth-century Nominalism³⁴ replaced Aristotelian views and made way for “new non-Pythagorean, demystified mathematics.”³⁵ At the turn of the fourteenth century Johannes de Grocheo described music thus:

“Certain people [however], considering its form and material, describe music by saying that it is a science of number related to sound. Others, looking at its performance, say that it is an art devoted to singing. We, however, intend to take it in both ways, considered as a tool and necessarily taken as one of the arts. Just as natural warmth (of the body) is a first tool through which the soul exercises its functions, so art is a principal tool [or a] rule through which the practical intellect explains and exposes its functions. We may say, therefore, that music is an art or science concerning numbered sound taken harmonically, designed for singing easily. I say both, a *science*, insofar as it treats the knowledge of principles, [but] an *art*, insofar as it rules the practical intellect in performing, concerning *harmonic sound*, since it is this basic material with which it is concerned. By *number* its form is

Fragen zur Wechselwirkung von ‘Musica’ und ‘Philosophia’ im Mittelalter, ed. Frank Hentschel (Leiden: Brill, 1998), 19–37.

³⁰ Dorit Tanay, “The Image of Music and the Bodies of Knowledge in the Late Middle Ages: Rhythmic Procedures as Cultural Representations,” *Science in Context* 9, no. 2 (1996): 124.

³¹ For an account of the differences between *musica practica* (or *musica activa*) and *musica speculativa* in music treatises see Herlinger, “Music Theory,” 246–300, esp. 297–300. An exhaustive bibliography of literature discussing the differentiation can also be found in Joseph Dyer, “The Place of *Musica* in Medieval Classifications of Knowledge,” *The Journal of Musicology* 24, no. 1 (2007): 3 n. 1.

³² On this distinction see for example Erich Reimer, “Musicus und Cantor: Zur Sozialgeschichte eines musikalischen Lehrstücks,” *Archiv für Musikwissenschaft* 35, no. 1 (1978): 18–28; and Oliver Strunk, ed., *Source Readings in Music History*, rev. ed. Leo Treitler (New York and London: W. W. Norton, 1998), 10 n. 5.

³³ For a summary of these developments see Dorit Tanay, introduction to *Noting Music, Marking Culture: The Intellectual Context of Rhythmic Notation, 1250–1400*, Musicological Studies & Documents 46 (Holzgerlingen: Hänssler-Verlag, 1999), 1–13 and pp. 35–36 below.

³⁴ For an explanation of the term see for example William J. Courtenay, “Nominalism and Late Medieval Religion,” in *The Pursuit of Holiness in Late Medieval and Renaissance Religion*, ed. Charles Trinkaus and Heiko A. Obermann (Leiden: Brill, 1974), 26–31.

³⁵ Tanay, *Noting Music, Marking Culture*, 10.

defined. But by *singing* performance is touched upon, to which it is properly *designated*. What music is has thus been discussed.”³⁶

In the widely disseminated fourteenth-century music treatise *Libellus cantus mensurabilis*, the alleged author Johannes de Muris stated in the incipit that he intended to steer a middle course between pure theory and pure practice.³⁷

Prior to these fourteenth-century developments, however, Pythagorean doctrine—as amongst others proclaimed by Plato in his *Timaeus*—had left its mark on medieval scholarship and culture.³⁸ It was transmitted to the Western medieval world by scholars who translated the Greek scripts into Latin. One of the most influential translators and writers was the Roman philosopher and statesman Anicius Manlius Severinus Boethius (ca. 480–525/26).³⁹ In the early sixth century Boethius had undertaken the mission of producing translations of and commentaries to the works of Aristotle, Euclid, Nicomachus, and Ptolemy and thus transmitted ancient Greek teachings of proportions to Western culture.⁴⁰ It is assumed that he wrote treatises on all four *quadrivium* subjects, although only his work on arithmetic and the greater

³⁶ “Describunt autem musicam quidam ad formam et materiam considerantes, dicentes eam esse scientiam de numero relato ad sonos. Alii autem ad eius operationem considerantes dicunt eam esse artem ad cantandum deputatam. Nos autem utroque modo notificare intendimus eandem, sicut notificatur instrumentum et quaelibet ars notificari debet. Sicut enim calidum naturale est primum instrumentum, mediante quo anima exercet suas operationes, sic ars est instrumentum principale sive regula, mediante qua intellectus practicus suas operationes explicat et exponit. Dicamus igitur, quod musica est ars vel scientia de sono numerato, harmonice sumpto, ad cantandum facilius deputata. Dico autem scientiam, in quantum principiorum tradit cognitionem, artem vero, in quantum intellectum practicum regulat operando. De sono vero harmonico, quia est materia propria, circa quam operatur. Per numerum etiam eius forma designatur. Sed per cantare tangitur operatio, ad quam est proprie deputata. Quid igitur sit musica, sic sit dictum.” Johannes de Grocheio, *De musica*, chap. 4; edition in Ernst Rohloff, ed., *Der Musiktraktat des Johannes de Grocheo nach den Quellen neu herausgegeben mit Übersetzung ins Deutsche und Revisionsbericht*, Media latinitas musica 2 (Leipzig: Gebrüder Reinecke, 1943), 46; translation taken from Albert Seay, trans., *Johannes de Grocheo: Concerning Music (De musica)*, Colorado College Music Press Translations 1, 2nd ed. (Colorado Springs: Colorado College Music Press, 1974), 9–10; emphasis in the original.

³⁷ Cf. F. Alberto Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” in *Die Mittelalterliche Lehre von der Mehrstimmigkeit*, ed. Hans Heinrich Eggebrecht et al. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1984), 299–300.

³⁸ However, it should be noted that some scholars, amongst them Christopher Page, have cautioned against exaggerated Pythagorean readings of medieval culture. Cf. *Discarding Images: Reflections on Music and Culture in Medieval France* (Oxford: Clarendon Press, 1993), 11–18. For comprehensive studies of Pythagorean and Platonic doctrine in medieval teachings on music see Abert, *Die Musikanschauung des Mittelalters*; Gerhard Pietzsch, *Die Klassifikation der Musik von Boetius bis Ugolino von Orvieto*, Studien zur Geschichte der Musiktheorie im Mittelalter 1 (Halle: Max Niemeyer, 1929); Münxelhaus, *Pythagoras musicus*; and Laurie Koehler, “Die Tradierung der pythagoreisch-platonischen Zahlenlehre und Musiktheorie,” chap. 1.3 in *Pythagoreisch-platonische Proportionen in Werken der ars nova und ars subtilior*, Göttinger musikwissenschaftliche Arbeiten 12, 2 vols. (Kassel: Bärenreiter, 1990), 1:26–59.

³⁹ For Boethius’ influence on medieval culture, especially the *quadrivium*, see Anja Heilmann, *Boethius’ Musiktheorie und das Quadrivium: Eine Einführung in den neuplatonischen Hintergrund von »De institutione musica«*, Hypomnemata: Untersuchungen zur Antike und zu ihrem Nachleben 171 (Göttingen: Vandenhoeck & Ruprecht, 2007).

⁴⁰ Again, this is an abbreviated account. For a detailed description of the transmission of several strands of ancient Greek music theory traditions see Bower, “The Transmission of Ancient Music Theory”. For other translators and commentators of ancient Greek music theory see Michael Bernhard, “Überlieferung und Fortleben der antiken lateinischen Musiktheorie im Mittelalter,” in *Rezeption des antiken Fachs im Mittelalter*, ed. Michael Bernhard et al. (Darmstadt: Wissenschaftliche Buchgesellschaft, 1990), 7–35.

part of his *De institutione musica* survived.⁴¹ Boethius' *De institutione musica* became an influential music treatise in the Middle Ages. His influence can most probably—at least partly—be explained by the fact that his *De institutione musica* became the primary textbook at late medieval universities.⁴²

In order to provide the reader with an overview of the medieval concept of proportions, I will briefly discuss the classification of proportions as it can be found in Boethius' *De institutione arithmetica*.⁴³ Drawing on Pythagorean teachings, especially on Nicomachus' work *Introduction to Arithmetic*, Boethius distinguished between three different kinds of proportions (a:b, where a and b are both natural numbers):⁴⁴ proportions of equality ($a = b$), proportions of major inequality ($a > b$), and proportions of minor inequality ($a < b$). Proportions of major inequality were further separated into five types (see TABLE 1.4).⁴⁵

Type	Description	Modern formula	Examples
1) <i>multiplex</i>	The larger number is a multiple (double, triple, etc.) of the smaller number.	$xn : n$ with $x > 1$	2:1, 4:2, 6:3, 8:4, [...] 3:1, 6:2, 9:3, 12:4, [...] etc.
2) <i>superparticularis</i>	The larger number is the sum of the smaller number and its smallest part or divisor, namely 1.	$(n+1) : n$ with $n > 1$	3:2, 4:3, 5:4, 6:5, [...]
3) <i>superpartiens</i>	The larger number is the sum of the smaller number and more than its smallest part.	$(n+m) : n$ with $n > m > 1$	5:3, 7:5, 9:7, 11:9, [...] 7:4, 8:5, 10:7, 11:8, [...] etc.
4) <i>multiplex superparticularis</i>	The larger number is the sum of a multiple of the smaller number and its smallest part.	$(xn+1) : n$ with $x > 1$ and $n > 1$	5:2, 7:2, 9:2, 11:2, [...] 7:3, 10:3, 13:3, 16:3, [...] etc.
5) <i>multiplex superpartiens</i>	The larger number is the sum of a multiple of the smaller number and more than its smallest part.	$(xn+m) : n$ with $x > 1$ and $n > m > 1$	8:3, 11:3, 14:3, 17:3, [...] 11:4, 15:4, 19:4, [...] etc.

TABLE 1.4: Types of proportions of major inequality

⁴¹ Cf. Michael Masi, "The Liberal Arts and Gerardus Ruffus' Commentary on the Boethian *De Arithmetica*," *The Sixteenth Century Journal* 10, no. 2 (1979): 24.

⁴² Cf. Bernhard, "Überlieferung der Musiktheorie im Mittelalter," 31; Max Haas, "Studien zur mittelalterlichen Musiklehre I: Eine Übersicht über die Musiklehre im Kontext der Philosophie des 13. und frühen 14. Jahrhunderts," in *Aktuelle Fragen der musikbezogenen Mittelalterforschung: Texte zu einem Basler Kolloquium des Jahres 1975*, ed. Wulf Arlt et al. (Winterthur: Amadeus, 1982), 343; and Matthias Hochadel, "Zur Rezeption der 'Institutio musica' von Boethius an der spätmittelalterlichen Universität," in *Musik – und die Geschichte der Philosophie und Naturwissenschaften im Mittelalter: Fragen zur Wechselwirkung von 'Musica' und 'Philosophia' im Mittelalter*, ed. Frank Hentschel (Leiden: Brill, 1998), 187–201.

⁴³ Cf. Boethius, *De institutione arithmetica*, chaps. 22–31; translation in Masi, *Boethian Number Theory*, 101–13.

⁴⁴ It is important to note that the visual appearance of proportions as a:b only came into use in the seventeenth century. Cf. Baxandall, *Painting and Experience*, 95. Before that time they were either referred to by their name (e.g. *proportio dupla* for 2:1) or displayed as stacked numbers ($\frac{a}{b}$) or fractions ($\frac{a}{b}$).

⁴⁵ Naturally, there were also five types of proportions of minor inequality (cf. Boethius, *De institutione arithmetica*, chap. 22), which only differed from the types named in the table by the prefix 'sub'. The proportion 1:2, for example, is of the type *submultiplex* and is called *proportio subdupla*.

The hierarchy among these five types of proportion reflects Pythagorean doctrine.⁴⁶ Proportions of the *multiplex* type are at the top because they are simple proportions, which create the most consonant sounds: octave (2:1), twelfth (3:1) and double octave (4:1). The Latin name of proportions of the *multiplex* type depends on the factor. 2:1, for example, is called *proportio dupla* ('double proportion'), 3:1 *proportio tripla* ('triple proportion'), 4:1 *proportio quadrupla* ('quadruple proportion'), and so forth. In Pythagorean theory, only intervals containing numbers up to 4 are considered to be consonant,⁴⁷ which is why the just major third (5:1) is not a consonant interval.⁴⁸

Then follow proportions of the *superparticularis* type, which create the consonant intervals of the fifth (3:2) and the fourth (4:3). The Latin names of these proportions are composites of the prefix *sesqui* (Lat.: 'one and a half') and a term derived from the smaller number in the proportion. 3:2, for example, is called *proportio sesquialtera*, 4:3 *proportio sesquitercia*, 5:4 *proportio sesquiquarta*, and so forth. Despite also belonging to this type of proportion, the major second (9:8, *proportio sesquioctava*) is not considered to be consonant because the proportion is made up of numbers larger than 4.

The third type of proportions of major inequality is the *superpartiens* type, which creates all other intervals within the octave, for example, the minor third (32:27), the major third (81:64), the minor sixth (128:81), and the major sixth (27:16).⁴⁹ The Latin names of *superpartiens* proportions consist of two words. The first is a composite of the term '*superpartiens*' and the difference between larger and smaller number. The second is a term derived from the

⁴⁶ "[A]ccording to the Pythagorean-Boethian tradition, proportions that depart from simplicity and singleness (all proportions other than multiple or superparticular ones) are excluded from consonance and harmony and have no place in a well-ordered musical system." Tanay, *Noting Music, Marking Culture*, 92.

⁴⁷ Cf. *Grove Music Online*, s.v. "Dissonance," accessed 6 December 2018, <https://doi.org/10.1093/gmo/9781561592630.article.07851>.

⁴⁸ A list of intervals, their corresponding proportions and calculation thereof can be found in Herlinger, "Music Theory," 251.

⁴⁹ As I understand it, the proportion will only be superpartient when it does not equal a superparticular proportion when fully reduced, i.e. the proportion 6:4 is superparticular ((n+1):n) and not superpartient ((n+m):n), because it equals 3:2 when reduced. Boethius discusses the proportions 6:4 and 9:6 in his chapter on superparticular proportions (cf. *De institutione arithmetica*, chap. 24), so I strongly believe that I am right in assuming that the type of inequality will be determined by the fully reduced proportion. Further evidence for this hypothesis can be found in chapter 28 of *De institutione arithmetica*, where Boethius calls the proportion 10:6 *superbipartiens*, which—according to the rules for proportions' denomination—refers to a proportion in which the difference between the larger and the smaller number is 2 and not 4. Therefore Boethius is most probably referring to the reduced proportion 5:3. Unfortunately, I was unable to find a reference dealing with the subject of reduction in Boethius' *De institutione arithmetica* or music-related treatises from the beginning of the fifteenth century. The same is true for literature of the late nineteenth, the twentieth or the twenty-first centuries. It was either clear to the authors I consulted that full reduction was obvious or they did not anticipate ambivalence when they notated modern formulas. Some fourteenth- and fifteenth-century treatises simply mention '*minimis numeris*' when discussing fully reduced proportions. However, Friedrich Wilhelm Marpurge's mid-eighteenth-century treatise gives information on the subject (*Anfangsgründe der theoretischen Musik* (Leipzig: Breitkopf, 1757), 17–24): The author explains the operation of reduction and states that only fully reduced ratios ("Radicalrationen"; *idem*, 22) are categorised. The above-mentioned problem of reduction also occurs in proportions of the *multiplex superpartiens* type. In this study, I will hence always categorise the fully reduced proportion.

smaller number in the proportion. 5:3, for example, is called *proportio superbipartiens tertias*, in which the syllable ‘bi’ in *superbipartiens* is derived from the difference 5–3 and *tertiens* describes the second term of the proportion. Thus, 8:5 is called *proportio supertripartiens quintas*, 9:5 is called *proportio superquadrupartiens quintas*, and so forth.

The last two types of proportions of major inequality are combinations of the first three groups. Since they contain proportions larger than 2:1, possible intervals deriving from the last two types will be greater than the octave. Proportions of the *multiplex superparticularis* type do, to my knowledge, not result in an interval in the Pythagorean tuning system. Since they occur in the rhythmic structures under examination in this study, however, they shall nonetheless be discussed here.⁵⁰ The Latin names of proportions of the *multiplex superparticularis* type can be derived from the rules of labelling proportions of the first two groups. They consist of two words. The first designates the factor, which describes how often the first number contains the second. The second then follows the rules for describing proportions of the *superparticularis* type. 5:2, for example, is called *proportio dupla sesquialtera*, 7:2 *proportio tripla sesquialtera*, 10:3 *proportio tripla sesquitercia*, and 9:4 *proportio dupla sesquiquarta*.

Lastly, proportions of the *multiplex superpartiens* type will create intervals larger than the octave, for example the augmented octave, whose ratio is 2187:1024, thereby containing 1024 twice and a part of it—namely 139—, which is larger than 1. Again, the Latin names of proportions of the *multiplex superpartiens* type can be derived from the rules of labelling proportions of the types it combines. As in the fourth group of proportions of major inequality, the first term designates the factor, which describes how often the larger number contains the smaller number. The second term then follows the rules for describing proportions of the *superpartiens* type. 8:3, for example, is called *proportio dupla superbipartiens tertias*, because 8 contains 3 twice (‘*dupla*’) and the difference between 8 and 6 is 2 (‘*bi*’). Accordingly, the proportion 18:5 is called *proportio tripla supertripartiens quintas*.

The classification and denotation of proportions constitutes a major part of Boethius’ *De institutione arithmetica*. Consequently, medieval treatises transmitting his teachings also often contain several chapters, which are concerned with this categorisation,⁵¹ which is why I intended to give the reader a brief overview of the subject. However, it can be assumed that—by way of the thorough treatment of this subject in writings—educated people at least were

⁵⁰ The rhythmic proportion 9:4 can be found in several manuscripts; 5:2, 7:2, and 10:3 occur in the manuscript *Turin, Biblioteca Nazionale Universitaria, MS J.II.9* (henceforth *Turin*). For more see Chaps. 2 and 3.

⁵¹ “Most medieval texts on ratio and proportion devoted a major section to this classification scheme and throughout the literature of the late Middle Ages right into the late seventeenth century, ratios were expressed by their denomination rather than by pairs of numbers.” Michael S. Mahoney, “Mathematics,” in *Science in the Middle Ages*, ed. David C. Lindberg (Chicago: University of Chicago Press, 1978), 164.

used to the representation of proportions in written-out Latin terms instead of pairs of Arabic numbers.⁵² This aspect should be kept in mind when pondering the question of how easily people in the late Middle Ages could interpret canons which accompany compositions.

As already stated (see above, p. 9), Boethius used the term ‘proportion’ for what would correctly be designated as ‘ratio’ of two numbers and not for the comparison of two ratios.⁵³ Moreover, Boethius conceived the combination of these two numbers in a proportion as one indivisible unit and thus not as ratio, in which two numbers are compared to each other: “*Proportio* does not refer to a ratio of two numbers but to an indivisible unit, which is constituted by two numbers. [...] It is neither a fraction nor a comparison.”⁵⁴ This is particularly relevant in view of this study because it might explain why proportions are rarely discussed in the context of rhythm. In his treatise *De institutione musica*, Boethius extensively discussed proportions and the derivation of musical intervals. It does not seem altogether unreasonable to assume that in this tradition a particular interval was regarded as sound and therefore as indivisible unit. In other words, the lengths of the strings were not compared to each other, they simply formed a particular sound that could be perceived as octave or any other interval respectively. In rhythmic contexts, however, the comparative aspect of different quantities is inevitable, since a certain quantity of rhythmic units is juxtaposed with a different quantity of rhythmic units. Consequently, proportions, which are interpreted as indivisible units, are not applicable in rhythmic contexts.

1.1.4 Proportions in Rhythmic Contexts

The first discussion of proportion in a rhythmic context coincided with the evolution of mensural notation in Europe in the last decades of the thirteenth century.⁵⁵ A treatise, generally attributed to Magister Lambertus and dated to the years 1265–75, contains a statement about two different types of proportion: “one [is measured] by location [placement] according to the proportion of sounds and voices, the other by time according to the proportion of longs and

⁵² In Chap. 5, I will discuss two treatises in which the authors give the impression of not being familiar with the rules of proportions’ denominations, however. Cf. Chaps. 5.3.1.3.2 and 5.3.1.4.

⁵³ Cf. Masi, *Boethian Number Theory*, 26 n. 16 and entries under ‘proportio’ and ‘ratio’ in Michael Bernhard, *Wortkonkordanz zu Anicius Manlius Severinus Boethius De institutione musica*, Bayerische Akademie der Wissenschaften: Veröffentlichungen der musikhistorischen Kommission 4 (Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1979).

⁵⁴ Walter, “Über den musikalischen Begriff *proportio*,” 72; my translation.

⁵⁵ For a detailed account of late thirteenth- and fourteenth-century treatises mentioning proportion in rhythmic contexts see Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 334–56; Walter, “Über den musikalischen Begriff *proportio*,” 84–86; and Koehler, *Pythagoreisch-platonische Proportionen*, 1:26–59.

breves.”⁵⁶ Franco of Cologne—in his influential treatise *Ars cantus mensurabilis* written in Paris c.1280—stated that different durations of sounds can be interpreted as being proportional: “Discant is a consonance of some different voices, in which these different voices are proportionally adjusted by *longae*, breves, and semibreves, and in writing must be proportioned against each other by corresponding notes.”⁵⁷

Proportional relationships are unquestionably the core element of the notational system called mensural notation, which evolved in the decades before 1300. The innovative feature of this notation was the measurability of note values, i.e. each note had a defined duration in terms of numerical proportions between the different note values. Mensural notation is based on the idea of a hierarchy of five different levels of basic note values—*maxima*, *longa*, *brevis*, *semibrevis*, and *minima* (see FIGURE 1.5)—in which the four latter note values are each worth a fraction of a note of a higher level.

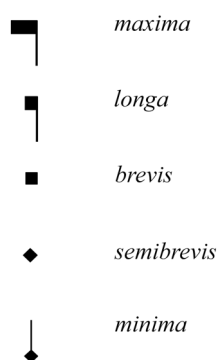


FIGURE 1.5: Basic note values in mensural notation⁵⁸

In fourteenth-century French notation⁵⁹ each note value could either be worth two or three of the next smaller note value. When the proportional relationship was binary, it was

⁵⁶ “una localis secundum proportionem sonorum vocumque, alia temporalis secundum proportionem longarum breviumque figurarum.” [Magister Lambertus?], *Tractatus de musica*; edition in Edmond de Coussemaker, ed., *Scriptorum de musica medii aevi nova series a Gerbertina altera*, 4 vols. (Paris: Durand, 1864–76; reprint ed., Hildesheim: Olms, 1963), 1:252; translation taken from Susan Fast, “Bakhtin and the Discourse of Late Medieval Music Theory,” *Plain-song and Medieval Music* 5, no. 2 (1996): 184.

⁵⁷ “Discantus est aliquorum diversorum cantuum consonantia, in qua illi diversi cantus per voces longas, breves vel semibreves proportionaliter adaequantur, et inscripto per debitas figuras proportionari adinvicem designantur.” Franco of Cologne, *Ars cantus mensurabilis*, chap. 2; edition in Gilbert Reaney and André Gilles, eds., *Franconis de Colonia: Ars cantus mensurabilis (ca. 1280)*, *Corpus scriptorum de musica* 18 (Rome: American Institute of Musicology, 1974), 26; my translation.

⁵⁸ These are the note values displayed at the very beginning of the influential fourteenth-century treatise *Libellus cantus mensurabilis* attributed to Johannes de Muris, on which numerous fourteenth- and fifteenth-century authors based their treatises.

⁵⁹ The established distinction between French notation and Italian notation is slightly misleading, since the terms imply that the sources written in the respective notation originated in either France or Italy. The attribute ‘French’, however, has been coined due to the fact that the notational system originated in France, primarily in Paris, following the treatises of Johannes de Muris and Philippe de Vitry (for doubts on Philippe de Vitry’s au-

termed imperfect, when it was ternary, it was called perfect. Thus *dupla* (2:1) and *tripla* (3:1) proportions were naturally present in this notation. For example, a *longa* could have the duration of two or three breves, depending on the mensuration, i.e. the rhythmic structure prevalent in the composition (see FIGURE 1.6).



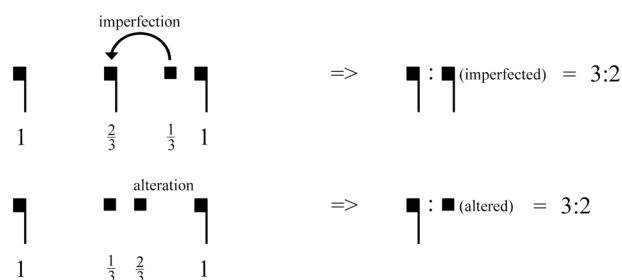
FIGURE 1.6: *Dupla* (2:1) and *tripla* (3:1) proportion in *longa* division

Johannes de Muris mentioned proportions between different note values as early as 1321. He described ternary relationships between note values in perfect time as “*proportio tripla*” (3:1) and the relationship between *longa* and semibreve as well as breve and minim in ternary structures as “*proportio nontupla*” (9:1).⁶⁰

Moreover, the concepts of alteration (i.e. prolongation of a note) and imperfection (i.e. reduction of a note) applied in triple division also created proportional relationships between the note values. A note value could be imperfected by a single note of the next smaller level, in which case it was reduced by a third of its value. The relationship between a perfect note and an imperfected note of the same level could thus be described by *sesquialtera* (3:2) proportion (see FIGURE 1.7). Similarly, when only two notes of the same level were placed in a perfection, the second one was prolonged to double its designated value—thus completing the perfection—, hence the relationship between a perfect note and an altered note of the next smaller level was also described by *sesquialtera* (3:2) proportion (see FIGURE 1.7).

thorship of the treatises see n. 82 below). The attribute is used in order to distinguish French notation from Italian notation, a contemporaneous system advocated by Marchetto da Padova, which used similar note shapes, but which differed from French notation in several important aspects. The French system eventually superseded the Italian system at the beginning of the fifteenth century (for details see for example Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 327). Many of the sources discussed in this study use French notation but originated in Italy. Karen Cook has recently suggested the terms ‘Murisian notation’ and ‘Marchettan notation’ as alternatives for ‘French notation’ and ‘Italian notation’ respectively. Cf. “Theoretical Treatments of the Semiminim in a Changing Notational World c. 1315–c. 1440,” PhD diss., Duke University, 2012, 4–5. However, I anticipate the problem that later developments of the respective notational systems (e.g. semiminims) could eventually be attributed to either Muris or Marchetto if the terms ‘Murisian notation’ and ‘Marchettan notation’ will gain acceptance. I have therefore decided to stick to the established terms ‘French notation’ and ‘Italian notation’ in this study.

⁶⁰ Johannes de Muris, *Notitia artis musicae*, Book 2, chap. 9 and chap. 11 respectively; edition in Ulrich Michels, ed. *Johannis de Muris: Notitia artis musicae and Compendium musicae practicae; Petrus de Sancto Dionysio: Tractatus de musica*, Corpus scriptorum de musica 17 (Rome: American Institute of Musicology, 1972), 89 and 102 respectively.

FIGURE 1.7: *Sesquialtera* (3:2) proportion in imperfection and alteration

Fourteenth-century French notation was based on four different mensurations, which are comparable to different metres in modern notation. The four mensurations (see FIGURE 1.8) resulted from different combinations of binary or ternary breve division (called *tempus imperfectum* or *tempus perfectum* respectively) and binary or ternary semibreve division (called *prolatio minor* or *prolatio maior* respectively).

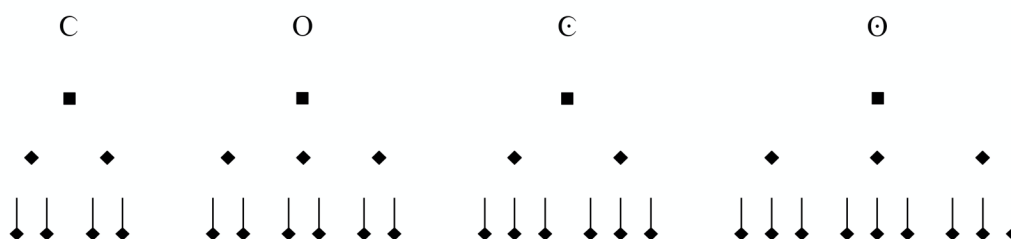


FIGURE 1.8: The four different mensurations of French notation

1) The combination of a binary breve with two binary semibreves was called *tempus imperfectum* with *prolatio minor* (in secondary literature sometimes abbreviated [2,2]) and most commonly indicated by the mensuration sign C. The semicircle signalled that the *tempus* was imperfect—hence the semicircle and not a full circle—and a missing dot within the circle indicated *prolatio minor*. 2) A ternary breve combined with binary semibreves resulted in a mensuration termed *tempus perfectum* with *prolatio minor* [3,2], often indicated by an empty circle O. 3) Accordingly, the opposing combination of a binary breve with ternary semibreves was called *tempus imperfectum* with *prolatio maior* [2,3]. This mensuration was most commonly indicated by C. 4) And finally, when breves and semibreves were both ternary one spoke of *tempus perfectum* with *prolatio maior* [3,3], which was indicated by a full circle with a dot: O.

Different combinations of these mensurations could also cause rhythmic proportions. If one voice part, for example, was notated in *tempus imperfectum* with *prolatio maior* ([2,3] indicated by C) and the other voice was notated in *tempus imperfectum* with *prolatio minor*

([2,2], C) and we assume that the *mensura* had the same duration, i.e. the breve was equal,⁶¹ then six minims of one voice would have sounded in the same time as four minims of the other voice. The rhythmic proportion between the two voices would have been *sesquialtera* (3:2) proportion ($6:4 = 3:2$), or *subsesquialtera* (2:3) proportion respectively, depending on the perspective. It is perhaps not surprising that *sesquialtera* (3:2) proportion was the most common rhythmic proportion, since the combination of *tempus perfectum* with *prolatio maior* ([3,3], O) and *tempus imperfectum* with *prolatio maior* ([2,3], C) or *tempus perfectum* with *prolatio minor* ([3,2], O) respectively also resulted in *sesquialtera* (3:2) proportion ($9:6 = 3:2$). Indeed, the only other proportion that could be created by the combination of different mensurations under the assumption of breve equality was *dupla sesquiquarta* (9:4) proportion (or its inverse *subdupla sesquiquarta* (4:9) proportion) when *tempus perfectum* with *prolatio maior* ([3,3], O) was combined with *tempus imperfectum* with *prolatio minor* ([2,2], C).

By contrast, the contemporaneous Italian notational system knew more divisions of the breve—called *divisiones*—and thus could create more rhythmic proportions. While in the French system, the breve could at the maximum be divided into nine minims, Italian notation could divide the breve into twelve smaller units. This division, called *duodenaria*, was the point of departure for all other divisions of the breve into nine, eight, six, or four smaller units.⁶² More possibilities are the result: *dupla* (2:1) proportion (8:4 or 12:6), *trippla* (3:1) proportion (12:4), *sesquialtera* (3:2) proportion (6:4 or 12:8), *sesquitertia* (4:3) proportion (8:6 and 12:9), *sesquioctava* (9:8) proportion, and *dupla sesquiquarta* (9:4) proportion.

At the turn of the fourteenth century, Johannes de Grocheio had generally described mensural notation as veritable system of signs, which was used to intellectually work with music and transmit it.⁶³ Grocheio compared it to letters in language and numerals in arithmetic:

“Just as the grammarian can write any word from a few letters by their joining together and placement and can indicate any number into infinity, artificially numbering from a few figures by putting them in front or in back, so the musician can write out any measured song from these three figures.”⁶⁴

⁶¹ For more on the concept of breve equality see Chap. 1.3.1.3 below.

⁶² Cf. Oliver Huck, introduction to *Die mehrfach überlieferten Kompositionen des frühen Trecento: Anonyme Madrigale und Cacce sowie Kompositionen von Piero, Giovanni da Firenze und Jacopo da Bologna*, Musica Mensurabilis 2, 2 vols. (Hildesheim: Georg Olms Verlag, 2007), 1:XXXIX. The principles of breve division in the Italian system are described there (XXXIX–XLVI) as well as in Marco Gozzi, “New Light on Italian Trecento Notation,” *Recercare* 13 (2001): 5–78. For an overview of the different *divisiones* see the latter, ‘Table 8’ on p. 43.

⁶³ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 261. Gallo was one of the first to discuss mensural notation in connection with semiotics. Cf. “Figura and Regula: Notation and Theory in the Tradition of Musica mensurabilis,” in *Studien zur Tradition in der Musik. Kurt von Fischer zum 60. Geburtstag*, ed. Hans Heinrich Eggebrecht und Max Lütolf (Munich: Katzschichler, 1973), 43–48. For a more detailed study of this aspect see Tanay, “The Natural Foundation of Rhythmic Notation,” chap. 2 in *Noting Music, Marking Culture*, 48–63.

⁶⁴ “Et quemadmodum 26situation26s ex paucis litteris earum coniunctione et 26situation potest dictionem quamlibet designare et artificialiter numerans ex paucis figuris earum praepositione et postpositione numerum quemlibet infinitum designare, ita musicus ex tribus figuris cantum quemlibet mensuratum.” Johannes de

By “three figures” Grocheio meant *longa*, *breve* and *semibreve*. Only a few decades later, however, these three signs were no longer sufficient to depict the rhythms composers intended to notate.

First of all, the system had shifted from *longa*, *breve*, and *semibreve* being the central note values to a notation in smaller units. The *breve* now defined the duration of the *mensura* and it was divided into *semibreves*, which in turn were split into *minims*. Then again, as described above, the French notational system was theoretically restricted to two proportions in simultaneous use of different mensurations only.⁶⁵ In order to notate complex rhythms—as it was possible in Italian notation—new notational devices were invented and introduced into the French notational system.

The most common notational device used in order to indicate rhythmic proportion was coloration: red ink typically signalled *sesquialtera* (3:2) proportion. Rhythmically speaking, three red notes replaced two black ones, thus creating a three-against-two rhythm comparable to a triplet in modern notation. The earliest use of red ink for indication of rhythmic proportion can be found in the motet *Garrit gallus / In nova fert* attributed to Philippe de Vitry from the early fourteenth-century *Roman de Fauvel*.⁶⁶ Later compositions also contain hollowed red or hollowed black notes.⁶⁷ In manuscripts dating from the decades around 1400 one can also observe a large variety of novel note shapes, which exhibit additional stems or flags. These new note shapes usually indicated *sesquitercia* (4:3) or *dupla* (2:1) proportion.

The third innovative notational device encompassed proportion signs, which are the primary subject of this study. In the cantus of the virelai *Que pena maior agitanda menti* (see FIGURE 1.9) from the Modena Codex (*Mod A*), for example, they appear as two single Arabic numerals: 2 and 3. Proportion signs had the advantage that note shapes did not have to be modified or coloured. The proportion sign simply indicated that all notes following the sign should be proportionally altered until the sign was revoked. Proportion signs were used in order to indicate a large variety of proportions, including ratios such as 5:2, 5:3, 7:2, 7:3, or

Grocheio, *De musica*, chap. 14; edition in Rohloff, *Der Musiktraktat des Johannes de Grocheo*, 55; translation taken from Seay, *Johannes de Grocheo: Concerning Music*, 24.

⁶⁵ *Sesquialtera* (3:2) proportion and *dupla sesquiquarta* (9:4) proportion. See p. 26 above.

⁶⁶ Paris, *Bibliothèque Nationale de France*, MS fonds français 146, fol. 44v. Margaret Bent has identified two main types of coloration in practical use by c.1400, namely “‘imperfection’ coloration” for mensural change and “coloration for proportional change”. “Principles of Mensuration and Coloration: Virtuosity and Anomalies in the Old Hall Manuscript,” in *La notazione della polifonia vocale dei secoli IX-XVII. Antologia, Parte seconda: secoli XIV-XVII*, ed. Antonio Delfino and Francesco Saggio (Pisa: Edizioni ETS, 2022), 76.

⁶⁷ When I speak of ‘coloration’ in this study I am including hollowed notes in the term. Thus, a coloured note is a note that is not full black but, for example, red, hollowed, or red and hollowed. Although it may seem counter-intuitive to call a hollowed note in black ink ‘coloured’, the attribute—according to my understanding of the term—does not primarily refer to a change in note colour but to the process of proportionally altering the duration of a note by changing its visual appearance without changing its form, e.g. by the addition of stems or flags.

10:3, which could not be derived from the simultaneous use of different mensurations and which also went beyond the notational possibilities of Italian notation.



FIGURE 1.9: Cantus of the virelai *Que pena maior agitanda menti* (Mod A 73); extract (first two staves)

Although coloration was undoubtedly the earliest of these three notational innovations, it should be mentioned that there was no linear notational development that went from coloration to novel note shapes to proportion signs, each innovation replacing the former device along the way. On the contrary, all three notational devices coexisted during the period of the so-called *Ars subtilior*, as demonstrated by the extract from the cantus of *Que pena maior agitanda menti* in FIGURE 1.9.

Before continuing with a brief description of how *Ars subtilior* is defined in this study in the next subchapter, however, I would like to clarify what I mean by rhythmic proportion. When I speak of rhythmic proportion in the following chapters, I use the term in a broad sense to include all of the above-mentioned interpretations. Thus, the term rhythmic proportion can refer to the relationship between note values of the same hierarchic level, e.g. the duration of one minim compared to the duration of another minim, but it may also refer to proportional relationships between two notes of different hierarchic levels. This is in accordance with the comprehension of rhythmic proportions which can be found in music treatises from the fourteenth century.⁶⁸ Proportions can occur naturally due to the structure of the system of mensural notation and, for example, under the assumption of breve equality, as shown above. They can also be evoked by the application of proportion signs or other notational devices.

However, I will not consider proportions in the overall structure of compositions, so-called architectural proportions. Several scholars have put forward the question whether, for example, the durations of certain sections of a composition reflect on certain proportions.

⁶⁸ Cf. Koehler, *Pythagoreisch-platonische Proportionen*, 1:50.

From 1950 onwards, various musicologists have sought traces of the golden section⁶⁹ and/or Pythagorean proportions in the durational layout of compositions from the late Middle Ages and the Renaissance.⁷⁰ The expediency of this endeavour put aside, it is beyond the scope of this study to investigate these kinds of proportions.

⁶⁹ Anna Maria Busse Berger has searched for mentions of the golden section or the Fibonacci series—from which the golden section can be derived—in medieval and Renaissance music theory treatises and has concluded that “there is absolutely no evidence for the claim that fifteenth-century musicians knew about the Fibonacci series.” “Musical Proportions and Arithmetic in the Late Middle Ages and Renaissance,” *Musica Disciplina* 44 (1990): 101. She has therefore cast doubt on its application in music from the period. I understand that Busse Berger has also not found mentions of architectural proportions in medieval and Renaissance music theory treatises during her investigation.

⁷⁰ See Busse Berger, “Musical Proportions,” 90–92 for an overview of authors and studies.

1.2 *Ars subtilior*

The term *Ars subtilior* is commonly applied to describe a musical style which flourished in the decades surrounding the year 1400—roughly congruent with the time of the Western Schism (1378–1417)—mainly in the regions of today’s Southern France and Northern and Central Italy.⁷¹ *Ars subtilior* has also been used as a term to describe the epoch during which this style was practised.⁷² The corpus of *Ars subtilior* pieces that have survived is predominantly comprised of three-part secular French songs notated in black mensural notation and transmitted in decorated manuscripts. Although a large number of *Ars subtilior* compositions are transmitted anonymously or with attributions leaving doubts about the originators’ identities, those cases in which attributions can be considered verified show that composers of *Ars subtilior* music were often employed—primarily as singers—at Papal chapels or courts of the European aristocracy.⁷³ The level of education seems to be significantly high among *Ars subtilior* composers. Some belonged to a holy order, others seem to have undergone university training at the very least.⁷⁴

Emerging from the antecedent *Ars nova* style—a style of which Guillaume de Machaut can probably be called most famous representative—the compositions of the *Ars subtilior* are primarily still set in the so-called *formes fixes*: ballades, rondeaux, and virelais. However, *Ars subtilior* songs can most notably be distinguished from the *Ars nova* repertoire by the compositions’ rhythmic complexity, namely proportional rhythms and elaborate syncopations. This rhythmic complexity is reflected in a great variety of notational devices, including coloration,

⁷¹ Evidence gathered from music manuscripts or fragments, music treatises, or other historical documents points towards an even wider dissemination of the style, including Aragon, Catalonia, Flanders, and Paris. Cf. Maricarmen Gómez Muntané, *La música en la casa real catalano-aragonesa durante los años 1336–1437* (Barcelona: A. Bosch, 1979); David Catalunya, “¿*Ars subtilior* en Toledo? Un vestigio en el código M1361 de la Biblioteca Nacional de España,” *Anuario musical* 66 (2011): 3–46; Reinhard Strohm, “The Ars Nova Fragments of Ghent,” in *Tijdschrift van de Vereniging voor Nederlandse Muziekgeschiedenis* 34, no. 2 (1984): 109–31; and Anne Stone, “The Ars subtilior in Paris,” *Musica e storia* 10, no. 2 (2002): 373–404. Since many dedicatory *Ars subtilior* ballades can be connected to the courts of Foix and Aragon, as well as the Avignon popes, scholars have sometimes put forward that the *Ars subtilior* style was cultivated at these centres. However, Anne Stone has recently called this assessment a “widely disseminated truism.” “Ars Subtilior,” in *The Cambridge History of Medieval Music*, ed. Mark Everist and Thomas Forrest Kelly, 2 vols. (Cambridge: Cambridge University Press, 2018), 2:1129. Also see Yolanda Plumley, “An ‘Episode in the South’? Ars subtilior and the Patronage of the French Princes,” *Early Music History* 22 (2003): 103–68.

⁷² Cf. Maricarmen Gómez and Ursula Günther, “Ars nova – Ars subtilior: B. Ars subtilior,” in *MGG2*, Sachteil vol. 2 (Kassel: Bärenreiter, 1994), col. 892. Some scholars reject the term as definition for a period. See for example Jason Stoessel, “The Captive Scribe: The Context and Culture of Scribal and Notational Process in the Music of the Ars Subtilior,” 2 vols., PhD diss., University of New England, 2002, 1:4. I also reject the term as definition for a period on similar grounds. For more, see p. 31–32 below.

⁷³ Cf. Giuliano Di Bacco and John Nádas, “The Papal Chapels and Italian Sources of Polyphony during the Great Schism,” in *Papal Music and Musicians in Late Medieval and Renaissance Rome*, ed. Richard Sheer (Oxford: Clarendon Press, 1998), 44–92.

⁷⁴ Cf. Stoessel, “The Captive Scribe,” 1:290.

novel note shapes, and proportion signs. Additional distinguishing features include comparatively fast runs, a peculiar chromaticism, and a prioritisation of musical expression over the intelligibility of the text.⁷⁵

The term *Ars subtilior* was first introduced by Ursula Günther to describe these rhythmically complex compositions of the post-Machaut generation, which preceded the early-fifteenth-century composers of the so-called ‘simple style’.⁷⁶ Günther proposed the term as alternative to the *contradictio in adjecto* ‘late *Ars nova*’ and the negatively connoted term ‘mannerism’⁷⁷, coined by Willi Apel, who had previously described the stylistic developments thus:

“Toward the end of the fourteenth century the evolution of notation led to a phase of unparalleled complication and intricacy. Musicians, no longer satisfied with the rhythmic subtleties of the *Ars Nova*, began to indulge in complicated rhythmic tricks and in the invention of highly involved methods of notating them. It is in this period that musical notation far exceeds its natural limitations as a servant to music, but rather becomes its master, a goal in itself and an arena for intellectual sophistries.”⁷⁸

Günther derived the term ‘*subtilior*’ from several late-fourteenth-century treatises and complemented it with the Latin word ‘*Ars*’ to *Ars subtilior*, thereby following the expression for the antecedent *Ars nova* style.⁷⁹ This, incidentally, has also provoked criticism of the term: Max Haas has stated that the sequence of the terms *Ars antiqua*, *Ars nova*, and *Ars subtilior* formed a “terminologically construed linear course of events”⁸⁰ that suppressed different interpretations of historical events, as in the case of the different conceptual pairs “*Ars antiqua*”

⁷⁵ Cf. David Fallows, “The End of the Ars Subtilior,” in *Basler Jahrbuch für historische Musikpraxis* 20, ed. Peter Reidemeister (Winterthur: Amadeus, 1996), 21–23; and Gómez and Günther, “Ars subtilior,” cols. 901–11.

⁷⁶ Cf. Ursula Günther, “Das Ende der ars nova,” *Die Musikforschung* 16, no. 2 (1963): 105–20.

⁷⁷ Cf. Apel, *The Notation of Polyphonic Music*, 403–4. Apel had also proposed a time span of only 20 years between 1370–90 for the ‘manneristic style’. Cf. introduction to *French Secular Music of the Late Fourteenth Century* (Cambridge, MA: Mediaeval Academy of America, 1950), 10. This assessment can now be considered disproved. Apel was first criticised by Heinrich Bessler (cf. “Hat Matheus de Perusio Epoche gemacht?,” *Die Musikforschung* 8, no. 1 (1955): 21) and Ursula Günther (cf. “Der musikalische Stilwandel der französischen Liedkunst in der zweiten Hälfte des 14. Jahrhunderts, dargestellt an der Virelais, Balladen und Rondeaux von Machaut sowie datierbaren Kantilenensätzen seiner Zeitgenossen und direkten Nachfolger,” PhD diss., University of Hamburg, 1957, 269). Multiple studies published since confirm that many *Ars subtilior* sources originated well after 1390. See for example Fallows, “The End of the Ars Subtilior,” 21–40.

⁷⁸ Apel, *The Notation of Polyphonic Music*, 403.

⁷⁹ For an overview of treatises using the terms ‘subtilisque’, ‘subtilius’, and ‘subtilirer’ see Gómez and Günther, “Ars subtilior,” cols. 892–93. It has been pointed out, however, that “as admitted by Günther in her article [Das Ende der ars nova], the term *subtilis* was used in relation to music throughout the whole of the fourteenth century and not just towards the end of that century.” Stoessel, “The Captive Scribe,” 1:11. Furthermore, Stoessel documented that the term was also used in treatises in connection to a differentiation between the *Ars antiqua* and the *Ars nova*. Cf. *idem*, 11–19. Criticism of the term ‘*subtilis*’ has, less explicitly, previously been voiced by Max Haas. Cf. “Studien zur mittelalterlichen Musiklehre I,” 386 n. 294. For a recent assessment of the concept of *subtilitas* in the fourteenth century see Karen Desmond, “*Subtilitas* and the *ars nova*,” chap. 2 in *Music and the modern, 1300–1350: The ars nova in Theory and Practice* (Cambridge: Cambridge University Press, 2018), 35–69.

⁸⁰ Max Haas, “Studien zur mittelalterlichen Musiklehre I,” 385–86 n. 293; my translation.

and “*Ars nova*” (Jacobus of Liège⁸¹) and “*Ars vetus*” and “*Ars nova*” (Philippe de Vitry⁸²). Indeed, the sequence suggests that one style superseded the other. Günther’s famous article even ends on the note: “Its [= the *Ars subtilior*’s] emergence ended the *Ars nova*.”⁸³ Contradictory to this statement, Günther also repeatedly clarified that the two styles—*Ars nova* and *Ars subtilior*—always coexisted.⁸⁴ Therefore, one needs to differentiate between the epochal term and the designation of the style. In light of this, however, criticism of the term *Ars subtilior* as designation for an epoch seems to me well founded, since confusion between style and epoch seems to be inevitable. Should songs in the *Ars nova* style written between 1378 and 1417 be called *Ars subtilior* compositions due to their time of origin?

Nevertheless, the term *Ars subtilior*—at least as designation for a particular musical style—can now be regarded as generally accepted and has been adopted by the two large music encyclopaedias *The New Grove*⁸⁵ and *Die Musik in Geschichte und Gegenwart*⁸⁶ as well as compendia such as *The Cambridge History of Medieval Music*.⁸⁷ Amongst others, Anne Stone and Jason Stoessel have provided valuable reassessments of the term as originally proposed by Günther.⁸⁸ Both have argued that the term ‘*subtilior*’ should be interpreted as ‘more precise’ instead of ‘more subtle’, thereby referring to the refinements of musical notation at the turn of the fifteenth century.⁸⁹

On the other hand, Anne Stone has also put forward an argument that captures some aspects of Apel’s term ‘mannerism’:

“Günther proposed that the *Ars subtilior* began in the 1370s and she presented a litmus test for a song to be classified as ‘*Ars subtilior*’: the use of note shapes, not codified in the *Ars Nova* system, that caused a proportional change in the minim.

While there is a degree of technical accuracy about this definition, there is much more to the idea of the *Ars subtilior* that Günther does not address, something that is better captured by Apel’s ‘mannerism.’ Above all, the songs classified as ‘*Ars subtilior*’ bear witness to an extraordinary and self-conscious attention to music writing, and to the attendant creation of a new audience for song: the audience that receives the song in its written state. It is impossible to fully understand *Ars subtilior* songs without engaging with their written iterations, a fact that adds to their recondite reputation. This is ev-

⁸¹ For Margaret Bent’s recent research on Jacobus’ identity see n. 116 below

⁸² For doubts on Philippe de Vitry’s authorship of the treatises, which previous scholarship had deemed one unified treatise called ‘*Ars nova*’, see Sarah Fuller, “A Phantom Treatise of the Fourteenth Century? The *Ars Nova*,” *The Journal of Musicology* 4, no. 1 (1985): 23–50. Incidentally, Fuller agrees with Haas in that the *Ars antiqua* and the *Ars nova* should not be regarded as opposites. Cf. *idem*, 47.

⁸³ Günther, “Das Ende der *ars nova*,” 117; my translation.

⁸⁴ Cf. Gómez and Günther, “*Ars subtilior*,” col. 894.

⁸⁵ Cf. Nors S. Josephson, “*Ars subtilior*,” in *NG2* (London: MacMillan, 2001), 2:81–82.

⁸⁶ Cf. Gómez and Günther, “*Ars subtilior*.”

⁸⁷ Cf. Stone, “*Ars Subtilior*.”

⁸⁸ Cf. Anne Stone, “Che cosa c’è di più sottile riguardo l’*ars subtilior*?,” *Rivista Italiana di Musicologia* 31, no. 1 (1996): 3–31; and Stoessel, “What is the *ars subtilior*,” chap. 1 in “The Captive Scribe,” 1:10–23.

⁸⁹ Cf. Stone, “Che cosa c’è di più sottile riguardo l’*ars subtilior*?,” 9; and Stoessel, “The Captive Scribe,” 1:22–23.

ident not only in the often extravagant ways the songs are notated, but in song texts that refer self-reflexively to the music writing process.”⁹⁰

Indeed, as will be shown in this study, the notation of some compositions can be considered overcomplicated, i.e. there would have been simpler methods of notating the rhythms present in the respective songs. That said, although the term ‘*Ars subtilior*’ can be regarded as generally accepted, aspects of Apel’s ‘mannerism’ occasionally still reverberate in scholars’ descriptions of the style.⁹¹ Even Günther, first having criticised Apel’s term ‘mannerism’ for its negative connotations, has stated in the final paragraph of her article ‘Das Ende der ars nova’ that the *Ars subtilior* style is “often bizarre”.⁹²

Despite some reservations regarding Günther’s term I have decided to use it in this study. Needless to say, it should always be kept in mind that ‘*Ars subtilior*’ is not a historically documented term but a construct conceived by a twentieth-century musicologist in order to describe stylistic developments in music and its notation at the turn of the fifteenth century. I have decided to indicate this differentiation by referring to the ‘so-called *Ars subtilior*’ in the title of this study but refrained from using the expression throughout the text for improved readability.

My reading of the term ‘*Ars subtilior*’ follows Stoessel’s definition of it, in which Günther’s epochal designation is rejected, but which includes the designation of a particular cultural “movement”.⁹³ The definition therefore includes not only the composer of a piece but also other agents of said movement, such as the manuscript scribe(s) as well as authors of treatises who engage with the notational aspects of the style in their writings. When I speak of the manuscripts of the *Ars subtilior*, I therefore include not only music manuscripts or fragments thereof transmitting compositions in the *Ars subtilior* style, but also treatises in which

⁹⁰ Stone, “Ars Subtilior,” 1127.

⁹¹ Daniel Leech-Wilkinson for example has stated: “The notation, after all, was so complex that only a small circle of the most highly trained musicians could have performed from it; the style was so abstruse that only a handful of the most enlightened connoisseurs could have appreciated it or could have wished to support it.” “*Ars Antiqua – Ars Nova – Ars Subtilior*,” in *Antiquity and the Middle Ages: From Ancient Greece to the 15th Century*, Man and Music 1, ed. James McKinnon (London: MacMillan, 1990), 239. Dorit Tanay has spoken of “the chaotic notation of the late fourteenth-century *ars subtilior*.” “The Visible and the Invisible: Rhythmic Notation in the Late Middle Ages,” in *Die Schrift des Ephemerer: Konzepte musikalischer Notationen*, ed. Matteo Nanni (Basel: Schwabe, 2015), 60.

⁹² Günther, “Das Ende der ars nova,” 117; my translation. Criticism of Apel’s term ‘mannerism’ in *idem*, 106.

⁹³ Stoessel, “The Captive Scribe,” 1:4. The “central focus [of this movement] resided in overcoming the invariability of the French *minima* through various devices such as special notes shapes, Indo-Arabic numerals, coloration and canons. [...] [T]his occurred in response to the conceptual role that the organising principle of proportionality had in informing the new style. The very degree of separation in terms of the complexity of temporal subdivision which was finally achieved by the most advanced compositions in the *ars subtilior* style [...] from that occurring in Italian compositions, surely suggests proportionality and its clear representation was foremost in the minds of notators.” *Idem*, 1:22–23.

particular aspects of the style—mainly notational aspects, such as the description of signs—are discussed.

It is not the issue of this study to ponder the question why composers suddenly decided to write more complex music—or, in some cases, music that was made to look complex by the use of elaborate notation. Instead I will focus on different aspects of the representation of complex rhythms. Nevertheless, I wish to present some of the hypotheses scholars have put forward in the past for contextualisation.

Willi Apel has drawn parallels between the ‘international style’ in art, also known as ‘international Gothic’, and the *Ars subtilior*, which he has termed ‘manneristic style’. According to Apel, both developments can be attributed to social conditions in the late fourteenth century:

“Needless to say, the resemblance of general traits results to a large extent from the common soil of social conditions. Toward the end of the fourteenth century an aging feudal aristocracy, tenaciously clinging to outdated conventions, and desperately trying to infuse them with a semblance of new life, created a highly artificial style of living which is unique in its ‘amazing sophistication and extravagance in manners, dress, and appurtenances.’”⁹⁴

Similarly, Ursula Günther has suggested that the period of turmoil—marked by the Western Schism and the decay of the ecclesiastical order, the Hundred Years’ War, as well as the fear of the plague—encouraged the intellectual *l’art pour l’art*, which led to the *Ars subtilior* instead of compositions in simpler styles.⁹⁵ Richard Hoppin has associated the new style with a tendency towards secularisation in the papal court in Avignon and has even suggested an Avignones origin for the Modena Codex (*Mod A*).⁹⁶ Since Avignon can no longer be deemed the cradle of the *Ars subtilior* style,⁹⁷ however, this causal attribution needs adjustment at the very least. Such sociohistorical accounts have also been declared problematic for different reasons. As Dorit Tanay has remarked: “No one has ever demonstrated that there is an inherent, necessary relationship between a complex and decayed society and intricate rhythms of the music, or why there should be.”⁹⁸

⁹⁴ Apel, introduction to *French Secular Music*, 19. The quotation within the quotation stems from Erwin Panofsky’s lectures on the origin and character of early Flemish painting, given at Harvard University in 1947/48, later published as Erwin Panofsky, *Early Netherlandish Painting: Its Origins and Character*, 2 vols. (Cambridge, MA: Harvard University Press, 1953).

⁹⁵ Cf. Günther, “Das Ende der ars nova,” 117.

⁹⁶ Cf. Richard H. Hoppin, *Medieval Music* (New York: W. W. Norton, 1978), 472–76. Current research explicitly points towards an Italian origin of the Modena Codex, however (see introduction to Chap. 2).

⁹⁷ See n. 71 above.

⁹⁸ Tanay, *Noting Music, Marking Culture*, 211. Tanay has further objected: “Interestingly enough, as far as late fourteenth-century chansons style is concerned, such an association can even be countered, considering the coincidental cultivation of the new simple style in the complex and decadent Burgundian courts of the waning Middle Ages.” “‘Nos faysoms contre Nature...’: Fourteenth-Century Sophismata and the Musical Avant Garde,” *Journal of the History of Ideas* 59, No. 1 (1998): 31.

A different line of argument has been put forward by David Fallows who has suggested that professional musicians felt the need “to assert their superiority over a growing body of skilled musical amateurs” and that “part of the reason for the more complex notational style known as the *ars subtilior* was that this was music that could be read and performed by [only] those with a thoroughly professional grounding.”⁹⁹ It might be conceivable that composers—wishing to be considered as distinguished in elite circles—unwittingly set in motion a process that lead to some sort of competition of who would be able to present the most complex piece of music. Or at least pieces which were complex by all appearances. Indeed, the visual appeal of novel note shapes and proportion signs should not be disregarded when speaking about *Ars subtilior* compositions, as Anne Stone has pointed out.¹⁰⁰ Some pieces—as for example Baudé Cordier’s songs *Belle, bonne, sage* (Ch 1) and *Tout par compas* (Ch 2) (see FIGURE 1.12 on p. 54 below)—are presented in such a carefully devised layout that they have occasionally been called ‘Augenmusik’ (German: ‘music for the eyes’).

Dorit Tanay has presented a different narrative in *Noting Music, Marking Culture: The Intellectual Context of Rhythmic Notation, 1250–1400*. In offering a new reading of Johannes de Muris’ 1321 treatise *Notitia artis musicae*, she has hypothesised that it was the departure from the Pythagorean value system as well as from speculative thinking that allowed new experimentations with measurements—especially mathematics of the infinite—, which, applied to music, ultimately led to note values smaller than the minim.¹⁰¹ These new mathematics, originally conceived at Oxford University, found their way to Paris in the fourteenth century and were adapted and expanded by French scholars.¹⁰² The ensuing engagement with measurements, which John E. Murdoch has termed “measure mania”¹⁰³, might also have influenced music and its notation. Tanay has hypothesised: “By the end of the fourteenth century the mathematical techniques of dealing with simultaneous variable rates would account for

⁹⁹ Fallows, “The End of the *Ars Subtilior*,” 21.

¹⁰⁰ See n. 90 above.

¹⁰¹ Cf. Tanay, “Rhythm Mathematized and Demystified,” chap. 2 in *Noting Music, Marking Culture*, 67–101. For more on the division of the minim see p. 37–40 below. A recent assessment of Johannes de Muris’ scientific achievements can be found in Desmond, “Jean des Murs, Quadrivial Scientist,” chap. 3 in *Music and the modern*, 70–114. For a discussion of infinity and continuity in late medieval philosophy see John E. Murdoch, “Infinity and Continuity,” in *The Cambridge History of Later Medieval Philosophy: From the Rediscovery of Aristotle to the Disintegration of Scholasticism, 1100–1600*, ed. Norman Kretzmann, Anthony Kenny, Jan Pinborg, and Eleonore Stump (Cambridge: Cambridge University Press, 1982), 564–92.

¹⁰² Cf. Tanay, “Nos faysoms contre Nature,” 32; and John E. Murdoch, “*Subtilitates anglicanae* in Fourteenth-Century Paris: John of Mirecourt and Peter Ceignons,” in *Machaut’s World: Science and Art in the Fourteenth Century*, ed. Madeline P. Cosman and Bruce Chandler (New York: The New York Academy of Sciences, 1978), 51–56.

¹⁰³ John E. Murdoch, “From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning,” in *The Cultural Context of Medieval Learning: Proceedings of the First International Colloquium on Philosophy, Science, and Theology in the Middle Ages – September 1973*, ed. John E. Murdoch and Edith D. Sylla (Dordrecht and Boston: D. Reidel Publishing, 1975), 288.

the extended use of rhythmic proportions in the *Ars subtilior*.”¹⁰⁴ Furthermore, Tanay has suggested that the emergence of the *Ars subtilior* was linked to logical-mathematical sophisms (especially limit-decision sophisms), which she defines as “exercises in intricate reasoning”.¹⁰⁵ I wish to return to one such limit-decision sophism, namely the division of the formerly indivisible *minima* (the smallest unit), in the next subchapter.¹⁰⁶

¹⁰⁴ Tanay, *Noting Music, Marking Culture*, 101.

¹⁰⁵ Tanay, *Noting Music, Marking Culture*, 211. For *Ars subtilior* and sophisms see *idem*, 207–45; and *idem*, “Nos faysoms contre Nature,” 34–51. For a general discussion of sophisms see Edith D. Sylla, “The Oxford Calculators,” in *The Cambridge History of Later Medieval Philosophy: From the Rediscovery of Aristotle to the Disintegration of Scholasticism, 1100–1600*, ed. Norman Kretzmann, Anthony Kenny, Jan Pinborg, and Eleonore Stump (Cambridge: Cambridge University Press, 1982), 545–53; and Norman Kretzmann, “Syncategoremata, Exponibilia, Sophismata,” in *idem*, 218–40.

¹⁰⁶ However, I would like to already state at this point that I find Tanay’s argument of elaborate syncopations interrupting the so-called “rhythmic whole”, the *mensura*, being one such manifestation of these new philosophical approaches to music and rhythm convincing. Cf. Tanay, *Noting Music, Marking Culture*, 227. For a description of the term “rhythmic whole” see n. 124 below. That the notation of syncopations which exceeded one *mensura* was only possible in French notation can be regarded as primary reason for this notational system superseding Italian notation at the beginning of the fifteenth century. Cf. Apel, introduction to *French Secular Music*, 19.

1.3 Thinking about Proportion Signs

In the preceding subchapter, I have outlined my understanding of the term *Ars subtilior*. There is general consensus that one of the central aspects of the *Ars subtilior* was the overcoming of the minim as the smallest unit of the notational system. Dorit Tanay in *Noting Music, Marking Culture: The Intellectual Context of Rhythmic Notation, 1250–1400* and Karen Cook in *Theoretical Treatments of the Semiminim in a Changing Notational World c. 1315–c. 1440* have both demonstrated how the introduction of note values smaller than the minim was not merely a question of notational innovation but an intellectual achievement, which was rendered possible by a change in philosophical and mathematical doctrine.

The concept of a minimal duration in musical time goes back to Aristotle:

“The extreme low end of the musical continuum was [...] given the name of *minima* (sc. *nota*). The term itself came from Aristotle, who explained in his *De cealo* that not only is there a minimum time and a maximum speed for the motions of the heavens, but also for every action, for walking or for playing the lyre, there is a minimum time and a maximum speed for all such actions.”¹⁰⁷

In fourteenth-century treatises on mensural notation, the minim was given the value 1 from which the values of all other notes could be derived, e.g. 3 for the semibreve (containing three minims), 9 for the breve, and 27 for the *longa* (see FIGURE 1.10).¹⁰⁸

⚑	3	81	longissima		
⚑	2	54	longior		
⚑	1	27	longa	>	idem
⚑	3	27	perfecta		
⚑	2	18	imperfecta		
■	1	9	brevis	>	idem
■	3	9	brevis		
■	2	6	brevior		
◆	1	3	brevissima	>	idem
◆	3	3	parva		
◆	2	2	minor		
◆	1	1	minima		

primus gradus
 secundus gradus
 tertius gradus
 quartus gradus

FIGURE 1.10: Table of note shapes and their values in Johannes de Muris' *Notitia artis musicae*¹⁰⁹

¹⁰⁷ Tanay, *Noting Music, Marking Culture*, 90.

¹⁰⁸ See for example Muris, *Notitia*, Book 2, chap. 5.

¹⁰⁹ Image taken from Ulrich Michels, ed., *Johannis de Muris: Notitia artis musicae and Compendium musicae practicae; Petrus de Sancto Dionysio: Tractatus de musica*, Corpus scriptorum de musica 17 (Rome: American Institute of Musicology, 1972), 79.

The treatise *Quatuor principalia musicae* by the English theorist John of Tewkesbury shows that the minim was thought of as ‘unity’, analogous to the number 1, which was also considered to be indivisible: “For just as unity is not a number but the origin of number, so also the minim is not a time but the origin of measured time, for in this art unity and the minim are interchangeable.”¹¹⁰

The conceptual problem of the division of the so-called *minima* is obvious: How to divide something that is—by its name—defined as the smallest unit? Should a *semiminima* exist? Late medieval scholars were ambivalent about this *contradictio in adjecto*.

The Italian theorist Marchetto da Padova was the first to remark upon the possibility of a value smaller than the minim, although he did not mention the term ‘*semiminima*’ in his *Pomerium*.¹¹¹ However, there is no treatise of Italian provenance, which discusses possible designations for such a value before 1370. The first treatise of Italian provenance mentioning the semiminim is the *Tractatus figurarum*, which is commonly attributed to Philippus de Caserta.¹¹²

The fourteenth-century Nominalist debate about the term was primarily held in France. Two out of three versions of the *Ars nova* treatises¹¹³ containing the word *semiminima* comment on the problematic nature of the term.¹¹⁴ The manuscript *Paris, Bibliothèque Nationale de France, Lat. 7378A* contains the sentence: “It should be known that as a perfect long is worth three tempora, as the perfect breve [is worth] three semibreves and the perfect semibreve three minims, and a minim two semiminims, *if one can speak of semiminims*, as is shown here”¹¹⁵ as well as the suggestion to call the minim ‘*semiminor*’ and the semiminim ‘*minima*’ instead. A similar proposal can be found in the manuscript *Rome, Biblioteca Apostolica Vaticana, Barb. lat. 307*.

¹¹⁰ “Nam sicut unitas non est numerus, sed principum numeri; sic minima non est tempus, sed principum temporis mensurati, quia in ista arte unitas et minima convertuntur.” John of Tewkesbury, *Quatuor principalia musicae*, Book 4, chap. 1; edition in Coussemaker, *Scriptorum de musica*, 4:254; translation taken from Tanay, *Noting Music, Marking Culture*, 126.

¹¹¹ Cf. Cook, “Theoretical Treatments of the Semiminim,” 52–57.

¹¹² Cf. Cook, “Theoretical Treatments of the Semiminim,” 80–82. For more on the discussion surrounding Philippus’ authorship concerning the *Tractatus figurarum* see Chap. 2.4.

¹¹³ For an explanation of the term see n. 82 above.

¹¹⁴ Cf. Cook, “Theoretical Treatments of the Semiminim,” 58–59.

¹¹⁵ “Sciendum quod sicut longa perfecta tria valet tempora, sic brevis perfecta tres semibreves et semibrevis perfecta tres minimas et minima duas semiminimas, si dici possent semiminime, ut hic.” Edition in Gilbert Reaney, André Gilles, and Jean Maillard, eds., *Philippi de Vitriaco: Ars nova*, Corpus scriptorum de musica 8 (Rome: American Institute of Musicology, 1964), 63; translation taken from Cook, “Theoretical Treatments of the Semiminim,” 58 n. 14.

The perhaps most well-known criticism of the semiminim can be found in Jacobus of Liège's mid-1320s treatise *Speculum Musicae*.¹¹⁶ In it, Jacobus expressed his irritation over the *moderni*, who—according to his descriptions—modified note shapes and named them improperly.¹¹⁷ Amongst other disapprovals one can find this statement: “And [according to what was said] this name of minimity does not seem to be rational insofar since it is possible to place two semiminims for a minim. But there ought not be less than the least. Thus, the ancient names of notes given by the Ancients seem more rational than the Moderns.”¹¹⁸ Because of this and other remarks found in the seventh book of *Speculum Musicae* many scholars have deemed Jacobus a defender of the *Ars antiqua* style. However, Dorit Tanay and Karen Desmond have shown that Jacobus' judgement was based on philosophical grounds and was therefore quite unrelated to practical music.¹¹⁹

Johannes de Muris, on the other hand, is commonly portrayed as representative of the *moderni* and a visionary who “looked beyond the musical practice of this time, anticipating rhythmical progressions that would appear in actual musical compositions several decades later.”¹²⁰ His c.1340 treatise *Libellus cantus mensurabilis* already contains depictions of a semiminim and a semiminim rest without mentioning the philosophical dispute.¹²¹ That this debate might at some point have been dissociated from notational practice is suggested by a passage in the late-fourteenth-century treatise *Ars cantus mensurabilis mensurata per modos iuris* by Coussemaker's Anonymous V. Although semiminims as well as coloured minims, i.e. imperfect minims, are displayed in musical examples and discussed throughout the trea-

¹¹⁶ In a recent publication, Margaret Bent has suggested that the author—whom scholars have named Jacobus of Liège—might actually be James of Spain, a nephew of Eleanor of Castile, wife of King Edward I. Cf. *Magister Jacobus de Ispania, Author of the Speculum musicae*, Royal Musical Association Monographs 28 (Farnham: Ashgate Publishing, 2015).

¹¹⁷ *Moderni* in Jacobus' context are teachers and practitioners of the ‘new art’, the *Ars nova*, as opposed to the *antiqui*. For the perception of the term ‘*moderni*’ after Jacobus, see William J. Courtenay, “*Antiqui* and *Moderni* in Late Medieval Thought,” *Journal of the History of Ideas* 48, no. 1 (1987): 4–6.

¹¹⁸ “Adhuc secundum dicta nomen minimitalis non videtur usquequaque rationabile cum pro minima duae ponantur semiminimae. Minimo autem non est dare minus. Ideo notarum antiqua nomina saltem aliqua videntur rationabiliora quam moderna.” Jacobus of Liège, *Speculum Musicae*, Book 7, chap. 34; edition in Roger Bragard, ed., *Jacobi Leodiensis: Speculum musicae*, Corpus scriptorum de musica 3, 7 vols. (Rome: American Institute of Musicology, 1955–73), 7:69; translation taken from Karen Desmond, “Behind the Mirror: Revealing the Contexts of Jacobus's *Speculum musicae*,” PhD diss., New York University, 2009, 416.

¹¹⁹ Cf. Tanay, “Jacobus of Liège and William of Ockham,” chap. 5 in *Noting Music, Marking Culture*, 146–81; and Desmond, “Behind the Mirror,” *passim*. Tanay has summarised Jacobus' attitude thus: “Jacobus did not attack new developments in practical composition, as has sometimes been assumed. Indeed, Jacobus tacitly accepted the new reality of practical music, and on occasion he explicitly praised new compositions. It was at the level of theoretical discourse, and at this level only, that Jacobus attacked the *moderni*. [...] Jacobus labored energetically to identify weaknesses and deficiencies in the argumentation and terminology of the *moderni*.” *Noting Music, Marking Culture*, 147. I should remark, however, that Desmond has also criticised Tanay for oversimplifying and for relating individuals or schools of thought that might not have had much in common upon closer inspection. She refers especially to connections drawn by Tanay between Johannes de Muris and the Oxford calculators as well as Jacobus of Liège and William of Ockham. Cf. “Behind the Mirror,” 13–14.

¹²⁰ Tanay, *Noting Music, Marking Culture*, 194.

¹²¹ Cf. Cook, “Theoretical Treatments of the Semiminim,” 75–76.

tise, thereby implying that they were being used at the time, it also contains the statement that these note values “must not be given [...] because nothing is given beyond the least.”¹²²

The reason for laying out this Nominalist debate in such detail is that it is of course possible to visually avoid the problem of minim division by the application of coloration or proportion signs. By proportionally altering the minim by these two notational devices, it is possible to maintain the note shape of the minim as smallest note value and simultaneously assigning different durations to it. Thus, it can certainly be argued that the minim is not divided but only proportionally altered. There is a difference between a proportionally altered minim, which nevertheless maintains its general shape and thus remains a minim in the semantic sense, and an altered note shape, such as the semiminim, which needs a new semantic designation.

It stands to reason to suggest that proportion signs were introduced in order to expand the number of proportions used in one composition. As stated above, the avoidance of novel note shapes could be viewed as conform to Nominalist ideas. Since coloration in its three common forms—namely full red, hollowed red, and hollowed black—can only indicate three different proportions at a time, proportion signs could be regarded as accepted extension of notational devices. Alas, the corpus of *Ars subtilior* works analysed in this study paints a different picture. There are too many pieces which contain proportion signs as well as novel note shapes in order for this hypothesis to hold up for the whole *Ars subtilior* movement. Nevertheless, there are some compositions which only deploy proportion signs as device to alter the minim. I will return to this aspect in the respective discussions of the compositions. For now I wish to return to preliminary thoughts and observations, however, in order to provide the reader with an overview of the subjects discussed in this study.

¹²² The full passage reads: “But in truth, according to art, not only is the semiminima not given, neither is the imperfect minima. [Let’s see first: i]f the imperfect minima must not be given, as we saw earlier, the consequent also holds for the semiminima, because when arguing distributively from the greater to the lesser, there is good consequent. The assumption is proven because nothing is given beyond the least, as is shown by anything with the name ‘least’.” (“Sed vere, secundum artem, non solum semiminima non est danda, verum et minima imperfecta. Videamus primitus, si minima imperfecta non est danda, ergo nec semiminima tenet consequentia, quia de maiori ad minus distributive arguendo est bona consequentia. Assumptum probatur quia ultra minimum non datur, quod patet per quid nominis ipsius minimi.”) Anonymous V, *Ars cantus mensurabilis mensurata per modos iuris*, chap. 17; edition in C. Matthew Balensuela, ed. and trans., *Ars cantus mensurabilis mensurata per modos iuris*, Greek and Latin Music Theory 10 (Lincoln, NE: University of Nebraska Press, 1994), 250; translation taken from *idem*, 251. Cf. Cook, “Theoretical Treatments of the Semiminim,” 68–69. On this inconsistency Balensuela remarks: “Having previously posited imperfect minimae, the treatise appears self-contradictory in this statement. The author, however, has previously considered imperfect minimae and semiminimae only in groups proportional to black minimae. Although not explicitly clear, this paragraph implies that these note shapes should only be used as groups to replace groups of black minimae; they should not be used as individual note shapes.” *Ars cantus mensurabilis*, 251 n. 164.

1.3.1 What Are Proportion Signs?

This certainly is the primary question concerning the topic of my thesis. It is surprising that no scholar seems to have attempted to provide a definition of the term ‘proportion sign’ for the *Ars subtilior* repertoire yet. Furthermore, scholars usually do not differentiate between mensuration and proportion signs and use the term mensuration sign for what I would consider to be two different categories of signs.¹²³ Mensuration signs primarily define the basic structure of the “rhythmic whole”¹²⁴, i.e. the *mensura*. In French notation, there are four basic mensurations (see FIGURE 1.8 on p. 25 above). Proportion signs, on the other hand, proportionally alter the duration of note values within a composition and sometimes even disrupt the rhythmic whole. In the majority of cases, proportion signs indicate a deviation from the basic rhythmic structure of a composition. In that function, they are comparable to the combination of bracket with Arabic numeral indicating triplets, quadruplets, etc. in modern notation.

If—as stated repeatedly—the overcoming of the minim as the smallest unit of the notational system is the central aspect of the *Ars subtilior*, one ought to distinguish between these two functions of signs. A sign, which keeps the minim as stable unit within a composition and just changes the structure of the rhythmic whole, i.e. a mensuration sign, ought to be distinguished from a proportion sign, which represents variability of the minim and therefore one of the central achievements of the *Ars subtilior* movement. As will be demonstrated by various examples in this study, it is often the case that a proportion sign, which indicates a proportional alteration of the minim, also indicates a change in mensuration. Nevertheless, I am proposing here to then regard this sign as to having two separate functions.

Other scholars distinguish between signs indicating mensuration and signs indicating proportion but do not offer concrete definitions of the one or the other. Even Anna Maria Busse Berger avoids defining the two terms in her monograph *Mensuration and Proportion Signs*. From reading her chapter on proportion signs I got the impression that she only considers Arabic numerals as falling into the latter category.¹²⁵ Anne Stone seems to follow in this line of thinking and distinguishes between mensuration signs and proportional numbers.¹²⁶ Jason Stoessel appears to have a similar conception of the two terms. The title of his subchap-

¹²³ For example, Gordon K. Greene uses the term ‘mensural signature’ in his edition of *Ch*. “Critical Commentary,” in *PMFC* 18 & 19. Ursula Günther uses the term ‘Taktzeichen’ (‘time signatures’) in her analyses of the repertoire of the post-Machaut generation. “Der musikalische Stilwandel,” 268.

¹²⁴ Dorit Tanay has described the term ‘rhythmic whole’ as follows: “Since the thirteenth century, musical time was commonly divided into uniform cycles or periods. Each such cycle was conceived as a rhythmic whole (*totus perfectus*). In practice, this whole, the regulative principle of measuring time in music, was articulated and variegated through substituting the whole with its parts.” “Rhythmic Notation in the Late Middle Ages,” 60.

¹²⁵ Cf. Busse Berger, “Proportion Signs,” chap. 6 in *Mensuration and Proportion Signs*, 164–226.

¹²⁶ Cf. Stone, “Ars Subtilior,” 1131.

ter “Proportional uses of mensuration signs in the *ars subtilior*”¹²⁷ suggests that a mensuration sign can have an additional function as proportion sign in some *Ars subtilior* works. This approach is in accordance with Roger Bowers’ statement that the “proportional usage [in proportion signs is] generated by graphic modification of the standard mensuration signs.”¹²⁸ From the following assertion, I gather that Busse Berger also considers the indication of a mensuration to being the primary function of a sign: “In addition to fractions, composers and theorists of the period used three devices to indicate proportions: mensuration signs, coloration and Italian note-shapes.”¹²⁹ I will return to the question of functions of signs later in this chapter. However, I would like to point out that—from a semantic point of view—it is rather incongruous to state that the change of the duration of the minim is indicated by a mensuration sign, when one speaks about a system, in which the minim was the stable unit even if the mensuration changed. Instead, I would use the term ‘sign’ in general: Signs such as circles or semicircles, which are also used as mensuration signs in the *Ars nova* and *Ars subtilior* repertoires, can indicate rhythmic proportion. I would also like to note that single Arabic numerals, which appear far more frequently in the *Ars subtilior* repertoire than stacked Arabic numerals (called “fractions” in the quotation above), are not mentioned at all in Busse Berger’s above-cited description.

In summary, the consensus seems to be that mensuration signs are geometric shapes, e.g. circles and semicircles, while proportion signs are Arabic numerals (single or stacked). This categorisation is based on the visual appearance of signs. For example, the reversed semicircle $\bigcirc$ is considered to be a mensuration sign, while the Arabic numeral 4 is categorised as proportion sign—even if both signs indicate *sesquitercia* (4:3) proportion at the minim level. I disagree with this conventional reading. As stated above, my approach to proportion signs takes into account their function, namely the indication of the temporal change of the minim, and therefore acknowledges the intellectual achievement of the *Ars subtilior* movement.

I believe that one of the difficulties in defining the term ‘proportion sign’ lies in the continuously changing perception of what rhythmic proportion is and where it is reflected in music throughout the centuries. As stated earlier in this chapter, interpretations of the term proportion in rhythmic contexts can differ immensely depending on time, area, or authority in question. Since proportional relationships are the core element of mensural notation, many components of this notation could theoretically be called proportion sign. Mensuration signs, which indicate whether the relationships between different note values are binary or ternary,

¹²⁷ Chap. 5.5 in “The Captive Scribe,” 1:273–81.

¹²⁸ Roger Bowers, “Proportional Notation,” in *NG2* (London: MacMillan Publishers, 2001), 20:427.

¹²⁹ Busse Berger, *Mensuration and Proportion Signs*, 168.

could be labelled proportion sign and one could go so far as to term certain note shapes, e.g. coloured or void notes, proportion signs. In order to be able to circumscribe the signs connected to certain phenomena and concepts to be discussed in this study, I offer the following definition:

1.3.1.1 Definition

In musical notation, a proportion sign is an extrinsic notational element which indicates the change of the temporal duration of the minim—or under special circumstances, e.g. in case minim level is not relevant, a higher note value—in a certain ratio, which is applied at the minim or the respective level to all notes following the proportion sign until its revocation. This revocation is usually brought about by the re-installment of the initial mensuration or by the end of the composition. Proportion signs appear as single Arabic numerals, stacked Arabic numerals, or geometric shapes which may contain additional elements, such as strokes or dots. At each notational level, a proportion sign represents a definite and unchanging ratio within a composition, that designates which quantity of notes will replace another quantity of notes in the proportion sign section. Furthermore, the ratio represented by the proportion sign must be determinable directly from the composition or the composition's canon instructions.

This definition primarily serves the purpose to provide the reader with a clear idea which signs I have included in this discussion. It is tailored to reflect my findings in *Ars subtilior* repertoire and the treatises discussed in Chapter 5 and may be unsuitable for application in different contexts. The following subchapters are intended to address the central aspects of my definition.

1.3.1.2 Extrinsic and Intrinsic Signs

Following the Aristotelian tradition as proclaimed in the *Metaphysics*, some medieval theorists distinguished between intrinsic and extrinsic modes of signification in music notation.¹³⁰

¹³⁰ For a detailed account of the reception of the terminology of the *Metaphysics* in medieval music theory see Stoessel, “The Captive Scribe,” 1:186–91.

In his 1404 *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, Prosdocimus stated:

“Note that some might say: you said above that those rectangles, circles and semicircles were extrinsic signs and here you say that the variation of colour among notes shapes, their hollowing out and filling up are intrinsic signs; why therefore do you call these signs intrinsic and those others extrinsic? To this I respond: I called the first signs extrinsic since [they are] totally extraneous and extrinsic to the song and not the essence of the song. I called the second signs intrinsic since they are correctly the essence of the song. This is obvious because these second signs are the very figures thus varied in colour, hollowing out and fullness. But since the figures or notes themselves are rightly of the essence of their song in which they occur, it follows that these second signs are also of the essence of their song and as a consequence are intrinsic.”¹³¹

The difference between *essentia* and *accidentia* is thus explained. Prosdocimus calls note shapes and all their variations (e.g. through coloration or additional stems or flags) intrinsic elements of a song, i.e. the notes are the essence of a song. Mensuration and proportion signs on the other hand—described by Prosdocimus as “those rectangles, circles and semicircles”—are extrinsic elements and therefore not the essence of a song but nevertheless indispensable according to Prosdocimus because only they give meaning to the note shapes. As Stoessel puts it “knowledge of the essence of a thing can only be gained through *accidentie*”,¹³² i.e. Prosdocimus claims that a song can only be understood through additional signs. In his dissertation as well as his paper on the interpretation of unusual mensuration signs, Stoessel has already pointed out that many *Ars subtilior* compositions do not contain signs and can nevertheless be interpreted.¹³³

As stated above, I consider it necessary to distinguish between mensuration and proportion signs. This distinction is particularly important when pondering the question whether extrinsic signs are crucial for understanding the music. The general mensuration of a piece can and commonly has to be derived from intrinsic elements, such as groupings or combina-

¹³¹ “Item notandum, quod aliquis posset dicere: tu dixisti superius, quod illi quadranguli et illi circuli cum illis semicirculis erant signa extrinseca et hic dicis, quod variatio colorum inter figuras et ipsarum evacuatio et plenitudo sunt signa intrinseca; quare est ergo, quod ista signa nominas intrinseca et illa alia extrinseca? Ad hoc respondeo, quod ideo prima signa extrinseca nominavi, quoniam totaliter cantui extranea et extrinseca et non de essentia cantus; secunda vero signa intrinseca nominavi, quoniam bene sunt de essentia cantus. Quod patet, quia ista signa secunda sunt ipsemet figure sic variate in colore vel evacuatione et plenitudine. Sed cum ipse figure sive note sint bene de essentia ipsius cantus in quo sunt, sequitur quod ista signa secunda sunt etiam de essentia ipsius cantus et per consequens intrinseca.” Prosdocimus de Beldemandis, *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, chap. 57; edition in F. Alberto Gallo, ed., *Prosdocimi de Beldemandis opera 1: Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, *Antiquae musicae italicae scriptores 3* (Bologna: Università degli Studi de Bologna, Istituto di Studi Musicali e Teatrali, 1966), 131–32; translation taken from Jason Stoessel, “The Interpretation of Unusual Mensuration Signs in the Notation of the *Ars subtilior*,” in *A Late Medieval Songbook and Its Context: New Perspectives on the Chantilly Codex (Bibliothèque du Château de Chantilly, Ms. 564)*, ed. Yolanda Plumley and Anne Stone (Turnhout: Brepols, 2009), 182–83.

¹³² Stoessel, “The Captive Scribe,” 1:188.

¹³³ Cf. Stoessel, “The Interpretation of Unusual Mensuration Signs,” 183–84; and *idem*, “The Captive Scribe,” 1:190–91.

tion of certain notes. The combination of one semibreve and one minim, for example, points towards perfect prolation. Thus, the mensuration sign does not have to be present in order for the mensuration to be determinable. Proportion signs, on the other hand, indicate a certain rhythmic proportion, which is only applied to a small section of a composition. If not indicated intrinsically, for example by altered note shapes or coloration, it would be difficult for the reader to apprehend that certain notes should be rhythmically altered. I will discuss an example of an absent extrinsic sign, namely a ‘missing’ proportion sign in the madrigal *Era Venus al termin del suo giorno* by Paolo da Firenze, later in this chapter.¹³⁴ For the majority of the *Ars subtilior* repertoire in which proportion signs are present, however, I would follow Prosdocimus in arguing that these signs are indispensable elements for the comprehension of the musical notation.

As extrinsic notational elements, proportion signs provide information on the interpretation of intrinsic elements, namely the notes themselves. Proportion signs do not have their own rhythmic value, they influence the rhythmic value of the notes to which the proportion is applied. One could therefore ponder the question as to what extent proportion signs are still part of the musical text.

Elements, which appear together with the text but are not actually part of the text, are called ‘paratexts’ in literary interpretation. The term was coined by the French literary theorist Gérard Genette.¹³⁵ In the literary context, Genette considers textual elements, such as titles, subtitles, prefaces, introductions, epilogues and footnotes as paratexts, but also non-textual elements such as illustrations and book covers.¹³⁶ These elements, which are located close to the text, e.g. appearing in the same volume, are called “peritexts”.¹³⁷ Interviews, letters, diaries, and other publications referring to the text can also function as paratexts and are called “epitexts”.¹³⁸ The sum of peritexts and epitexts constitute the total of paratexts. Analogously, one could regard elements such as composers’ names and illuminations in *Ars subtilior* music manuscripts as peritexts and passages in music treatises, which refer to certain compositions, as epitexts.

Despite the distinction between intrinsic and extrinsic notational elements, I would not consider proportion and mensuration signs to be paratexts, however. Rather, I would compare

¹³⁴ See pp. 93–94 below.

¹³⁵ Cf. Gérard Genette, *Palimpsestes: La littérature au second degré* (Paris: Seuil, 1982); translated as: *Palimpsests: Literature in the Second Degree*, trans. Channa Newman and Claude Doubinsky (Lincoln, NE: University of Nebraska Press, 1997).

¹³⁶ Cf. Gérard Genette, introduction to *Paratexts: Thresholds of Interpretation*, trans. Jane E. Lewin (Cambridge: Cambridge University Press, 1997), 1–15.

¹³⁷ Genette, *Paratexts*, 5.

¹³⁸ Genette, *Paratexts*, 5.

them to punctuation in written language. Punctuation marks structure the text as represented by words displayed as groups of letters. A full stop marks the end of a sentence, a question mark indicates a question, quotation marks signal that the inserted words are taken from another source, and so on. Still, punctuation marks can be considered extrinsic elements in texts, since they are not words. It is often possible to derive the meaning of a text without punctuation marks. The beginning of a new phrase, for example, is marked by the intrinsic element of capitalisation of the first letter. A question can also be detected by the order of the words in a phrase. In this analogy, full stops and question marks can be compared to mensuration signs in *Ars subtilior* music. As Stoessel has shown, it is often possible to determine the mensuration of a composition from intrinsic signs. Proportion signs, on the other hand, may be compared to quotation marks, without which it is not possible to ascertain that the words in question stem from another source.

While proportion signs are not paratexts, canons—often containing instructions on the interpretation of proportion signs in a piece—could, in my opinion, well be regarded as peritexts. In this study, I will discuss accompanying canons when they are given. Music theory treatises can also be regarded as paratexts, namely epitexts.

To conclude: I have called proportion signs extrinsic notational elements in order to make a distinction between two different modes of signification. The change of the temporal duration of a note can be signalled intrinsically by alteration of the note shape, e.g. through coloration or additional stems or flags. If it is indicated extrinsically, however, the sign indicating this temporal change is called proportion sign.

1.3.1.3 The Minim as Central Level of Comparison

The question which note value remains the stable one in terms of duration when different mensurations are compared to each other, e.g. through a change of mensuration within a piece, has been posed repeatedly, not only by modern scholars and editors but also by medieval and Renaissance theorists.¹³⁹ Compositions as well as theoretical treatises suggest that composers and theorists can be divided into two camps: those who favour equality of the breve and others who advocate minim equality.

Breve equality seems to be an adoption from the Italian notational system in which the breve is the central unit, which is divisible into four, six, eight, nine, or twelve smallest units.

¹³⁹ For a summary of medieval and Renaissance theorists' positions see Busse Berger, *Mensuration and Proportion Signs*, 51–86.

In this top-down approach, the duration of the breve is stable throughout all different *divisiones*. Semibreves and minims have the value of fractions of the breve. In *octonaria*, for example, the minim has the duration of $\frac{1}{8}$ of a breve.

Breve equality means that the different number of minims in the six *divisiones* should sound simultaneously, i.e. *octonaria* and *duodenaria* are not just doublings or triplings of *quaternaria*. Instead, the *divisiones octonaria* and *duodenaria* should “be sung a bit more densely.”¹⁴⁰ Thus, *octonaria* is reduced to the duration of *senaria perfecta* or *imperfecta* and *duodenaria* to the duration of *novenaria*, thereby creating *sesquitertia* (4:3) proportion at the minim level.¹⁴¹ As stated above, possible rhythmic proportions created by two different *divisiones* sounding simultaneously were *dupla* (2:1) proportion (8:4 or 12:6), *tripla* (3:1) proportion (12:4), *sesquialtera* (3:2) proportion (6:4 or 12:8), *sesquitertia* (4:3) proportion (8:6 and 12:9), *sesquioctava* (9:8) proportion, and *dupla sesquiquarta* (9:4) proportion.

While the minim has a variable duration in the Italian notational system, it can be regarded as the central—and therefore stable—unit in French notation. In this bottom-up approach, the minim was given the value 1 and all other values derived from it (see FIGURE 1.10 on p. 37 above).¹⁴² This means that when the mensuration changes in French notation, either semibreve or breve or both change their value, but not the minim. Following this proposition, however, it is obvious that there is a need for determination of a certain level in the definition of proportion signs.

I have chosen the change of the duration of the minim as decisive criterion in order to make a distinction between mensuration and proportion signs. Rhythmic proportions are naturally caused by changes in mensuration with equal minims, as demonstrated in FIGURE 1.11. In this example, the mensuration changes from [2,2] to [2,3]. If the minim is equal in both mensurations, it follows that the duration of semibreves and breves will change in [2,3]. The semibreve in [2,3] lasts for three minims instead of two, thus two semibreves in [2,3] have the same duration as three in [2,2]. Hence the proportional relationship between the two semibreves is *subsesquialtera* (2:3) proportion. The same is true for the breve which lasts for six minims in [2,3] compared to four in [2,2]. Without the restriction to the change of the temporal duration of the minim, every sign indicating a change in mensuration with minim equality should be called proportion sign, since the values of semibreves and breves will change.

¹⁴⁰ “aliquantulum stricte cantabimus” Prosdocimus de Beldemandis, *Tractatus practice de musica mensurabili ad modum italicorum*; edition in Edmond de Coussemaker, ed., *Scriptorum de musica medii aevi nova series a Gerbertina altera*, 4 vols. (Paris: Durand, 1864–76; reprint ed., Hildesheim: Olms, 1963), 3:234; my translation.

¹⁴¹ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 330–31.

¹⁴² Cf. Koehler, *Pythagoreisch-platonische Proportionen*, 1:45.

Conversely, this means that I consider mensuration signs, which operate under breve equality and thereby cause a change in the duration of the minim, to also be proportion signs.¹⁴³

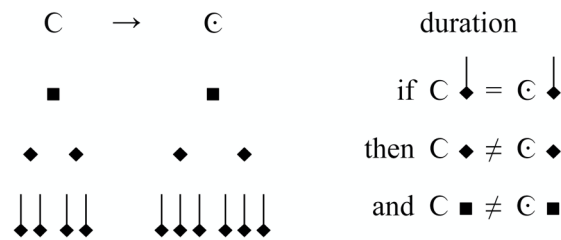


FIGURE 1.11: Rhythmic proportion caused by change in mensuration from [2,2] to [2,3]

For a handful of pieces I have decided to extend my definition to include proportions on higher note levels, for example Baude Cordier's *Pour le desfault du noble dieu bachus* (Ox213, fol. 108v). In this case, the piece is notated in larger note values, thus minim level becomes irrelevant. Therefore, the proportion has to be applied at a different level. In all exceptions, the smallest relevant level is used as reference for the comparison.

My approach of choosing minim level as decisive criterion in my definition is substantiated by a statement found in Prosdocimus de Beldemandis' 1404 treatise *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*: "Therefore you ought to know that whensoever notes are proportioned by such [proportion] signs, they are only proportioned with respect to their minims: with or without minims among them they are proportioned with respect to nothing else."¹⁴⁴ Furthermore, the author of the anonymous treatise *Pars aliquota est illa quae aliquotiens sumpta*, which might even predate Prosdocimus' *Expositiones*, has stated: "And bear in mind that proportion must always be computed only in minims and in no other way."¹⁴⁵

However, even more confirmation can be drawn from the musical sources themselves. Although proportion signs are often ambiguous, i.e. not representing the ratio as stacked Arabic numerals, accompanying canons in *Ars subtilior* music almost always refer to proportion at the minim level.

¹⁴³ For multiple functions of signs see Chap. 1.4 below.

¹⁴⁴ "Propter quod scire debes, quod quotienscumque proportionantur figure per talia signa, solum proportionantur in respectu ad minimas earum si minime non sunt et si minime sunt inter se proportionantur sine aliquo respectu." Prosdocimus, *Expositiones*, chap. 61; edition in Gallo, *Expositiones*, 142; translation taken from Stoessel, "The Captive Scribe," 1:297 n. 41.

¹⁴⁵ "Et nota quod proporcio semper debet computari in minimis tantum. et in nullo alio aliquo modo." Anonymous, *Pars aliquota est illa quae aliquotiens sumpta*, fol. 50v. Transcription taken from Peter M. Lefferts' TML edition at http://www.chmtl.indiana.edu/tml/14th/BERMAN4_MBAVR114. My translation. For the dating of this treatise see Chap. 5.3.3.

Canons in *Ars subtilior* music manuscripts also reveal that breve equality seems to have been an adopted concept in French notation. In *Mensuration and Proportion Signs*, Busse Berger has expounded on the Italian background of the central breve:

“In short, if my hypothesis is correct, the equal breve, amply attested in fifteenth-century theory, originated from the central breve of the Italian trecento [...]. The central breve developed into an equal breve in the late fourteenth century, in particular in the music of the *Ars subtilior*.”¹⁴⁶

Still, it appears that the idea of the equal minim was still in composers’ and performers’ minds. In some pieces analysed for this study, a change in mensuration involving an equal breve—and therefore not an equal minim—is accompanied by a canon explaining the proportional change.¹⁴⁷ This suggests that equal breve was not the norm but still an exception to be specially indicated.

Even if one establishes minim level as central point of comparison, however, the question remains which minims should be compared to each other. A comparison between minims of the same voice seems to be the most intuitive reading. However, a comparison between minims of different voices (e.g. minims of the cantus and minims of the tenor) is also conceivable. Since most *Ars subtilior* compositions proceed from the same mensuration in all voices, a comparison between minims within the same voice or in different voices will amount to the same thing.

However, there are even several possible points of reference within the same voice. I will address this aspect at great length in Chapter 1.4.2.2. In this study, I distinguish between non-cumulative, cumulative, and quasi-cumulative proportions within the same voice.

¹⁴⁶ Busse Berger, *Mensuration and Proportion Signs*, 83.

¹⁴⁷ See for example, the ballata *L’alta virtute* by Ugolino da Orvieto from the manuscript *Rome, Biblioteca Casanatense, MS 2151* (henceforth *Cas*), fols. 344v–345r. I discuss this composition in Chap. 4.3.

1.3.1.4 Visual Appearance

Proportion signs appear in three different types of shape: single Arabic numerals, stacked Arabic numerals, or geometric shapes. These are the signs found in the music manuscripts examined for this study: 2, 3, 4, 5, 6, 7, 8, 9, $\frac{2}{1}$, $\frac{2}{2}$, $\frac{3}{1}$, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{4}{2}$, $\frac{4}{3}$, $\frac{8}{6}$, $\frac{8}{9}$, $\frac{9}{6}$, C, $\mathcal{C}$, $\mathbb{C}$, O, $\odot$, $\oplus$, $\otimes$, and $\odot$. The fourteen treatises up to c.1450 studied in Chapter 5 describe or display the following signs: 2, 3, 4, 6, 9, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{3}{2}$, $\frac{4}{1}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{8}{3}$, $\frac{9}{4}$, $\frac{9}{6}$, $\frac{9}{8}$, $\mathcal{C}$, $\mathcal{C}$, $\cup$, $\cap$, Φ , $\ominus$, $\ominus$, Φ , $\emptyset$, $\emptyset$, $\ominus$, Φ , Φ , $\odot$, Φ , $\odot$, and $\diamond$.

The symbols used as proportion signs are mostly circles or semicircles, some of which contain additional visual elements, such as strokes or dots. I am not aware of proportion signs in triangular, quadrangular, or any other geometric shape appearing in music manuscripts, although Ugolino da Orvieto mentions and depicts a quadrangular proportion sign in his treatise *Declaratio musicae disciplinae*.¹⁴⁸

By far the most commonly used symbol is the reversed semicircle ($\mathcal{C}$), which indicates *sesquitertia* (4:3) proportion at the minim level in the majority of cases. Other examples for symbols are cut signs, with either horizontal, vertical, or angled stroke, or circles containing two or three dots ($\odot$ or $\oplus$). The four most common symbols to indicate mensuration (C, $\mathcal{C}$, O, and $\odot$) are also used as proportion signs. In the majority of cases, they simultaneously function as mensuration and proportion sign.¹⁴⁹

Single Arabic numerals appear much more frequently in music manuscripts than stacked Arabic numerals although the latter are more often described in treatises as a way to indicate proportion. Single Arabic numerals are naturally more ambiguous than stacked Arabic numerals. The Arabic numeral 3, for example, can indicate *sesquialtera* (3:2) or *tripla* (3:1) proportion, whereas the stacked numerals $\frac{3}{2}$ and $\frac{3}{1}$ have a distinct meaning. I am not aware of any multi-digit numbers (e.g. 10 or higher) being used as proportion signs in manuscripts around 1400, which is probably owing to the fact that the proportions based on larger numbers are too complex for application in rhythmic contexts.¹⁵⁰

¹⁴⁸ Cf. Ugolino da Orvieto, *Declaratio musicae disciplinae*, Book 3, chap. 6; edition in Albert Seay, ed., *Ugolino of Orvieto: Declaratio musicae disciplinae*, Corpus scriptorum de musica 7, 3 vols. (Rome: American Institute of Musicology, 1959–62), 2:210–11. As I will show in Chap. 4, the proportion signs discussed in this part of Ugolino's treatise do for the greater part not match the signs that Ugolino used in his own compositions. Furthermore, I doubt that the quadrangular sign ($\diamond$) described by Ugolino was used in music notation since it has the shape of a semibreve and therefore seems to be unsuitable to use for reasons of ambiguity (for more on this see Chap. 4.3).

¹⁴⁹ For more on multiple functions of signs see Chap. 1.4.2.3 below.

¹⁵⁰ The exception is *tripla sesquitertia* (10:3) proportion, which occurs in the virelai *Je prens d'amour noriture* in *Turin*. In this composition, the proportion is indicated by the double circle $\odot$, however.

It is noteworthy that no letters from the Latin alphabet appear in French notation in order to indicate proportion of the minim. This is all the more surprising since letters appear frequently in Italian notation, where they are used to indicate the different *divisiones*. Since the two notational systems co-existed, it would have stood to reason to adopt letters from the Latin alphabet into the French notational system, especially when, as explained above, the concept of the variable minim—indicated by proportion signs—was an adoption from the Italian system.

I am also not aware of any Roman numerals appearing as proportion signs in music manuscripts or treatises, although they remained a common form of representing numbers throughout the fourteenth century and were still used in treatises written in the fifteenth century.¹⁵¹

Finally, I would like to elaborate on the position of proportion signs within the musical text. Proportion signs are placed on the staff immediately before the first proportioned note, thereby distinctly indicating where the passage with proportioned notes begins. In some rare cases they are placed slightly above the staff, but never below. The end of the scope of a proportion sign section is usually indicated by a mensuration sign, which reinstalls either the initial mensuration or the mensuration which was in effect prior to the proportion sign section.

1.3.1.5 Unequivocal Proportional Meaning

The last two sentences of my definition address two different important aspects of the following discussions:

“At each notational level, a proportion sign represents a definite and unchanging ratio within a composition, that designates which quantity of notes will replace another quantity of notes in the proportion sign section. Furthermore, the ratio represented by the proportion sign must be determinable directly from the composition or the composition’s canon instructions.”¹⁵²

In the first phrase, I have formulated the basic premise that a sign has a definite meaning within a certain context. This premise is based on Nelson Goodman’s theory of symbols, in which he also considers musical notation: “A notational system is a system of signs, which is

¹⁵¹ Cf. Busse Berger, “Musical Proportions and Arithmetic,” 97. Note that Georgius Erber used Roman numerals to replace some Arabic numerals or Latin number terms in the text of his treatise *Sequuntur proportiones declarate et manifeste demonstrare ostense melodia vocum cantorum invente*. Still, they are not used as proportion signs in this work.

¹⁵² See p. 43 above.

free of ambiguities and which exhibits both syntactic as well as semantic disjunction and differentiation.”¹⁵³

In musical notation, a proportion sign always represents the same ratio within a composition when it appears more than once, e.g. the Arabic numeral 3 cannot represent *sesquialtera* (3:2) proportion in one section and *tripla* (3:1) proportion in another section of a piece. This premise is substantiated by the large corpus of *Ars subtilior* compositions in which this coherence can be observed. The premise also applies to intrinsic signs of proportion, i.e. coloration or altered note shapes. Without the premise there would hypothetically be several ‘solutions’ for pieces in which proportion signs appear more than once. This has a direct influence on the assertions made about proportion signs and the ratios they represent. The importance of this aspect will be demonstrated in the following discussion of Baude Cordier’s rondeau *Belle, bonne, sage* (Ch 1).

Finally, I have excluded signs from the discussion for which a proportional meaning can only be assumed but not directly determined. This is the case when all voices of a composition contain the same sign at the same position within the piece without a canon explaining the sign. Although this predominantly occurs in the repertoire of styles succeeding the *Ars subtilior*, there are also examples from the music manuscripts examined for this study.¹⁵⁴ The ballata *Serà quel zorno may, dolze madonna mia* (Mod A 98) by Matteo da Perugia, for example, contains a passage in which all three voices simultaneously change to $\bigcirc$.¹⁵⁵ In this composition, it is not determinable whether the change in mensuration also brings about a change of the duration of the minim. If a canon was present, the interpretation of $\bigcirc$ would of course be clear, which is why I have included a composition’s canon instructions in my definition.

I thank the reader for bearing with me through this very general and also technical part. I will now turn to the music itself and the proportion signs found therein.

¹⁵³ My translation of Oliver R. Scholz’s summary of Nelson Goodman’s theory (*Bild, Darstellung, Zeichen: Philosophische Theorien bildlicher Darstellung*, 2nd ed. (Frankfurt am Main: Klostermann, 2004), 124) as cited in Max Haas, “Mensuralnotation als Bild: Mathematik und Physik als Grundlagewissenschaften für das Visualisieren von Musik in mittelalterlicher Sicht,” in *Die Schrift des Ephemeren: Konzepte musikalischer Notationen*, ed. Matteo Nanni (Basel: Schwabe, 2015), 57.

¹⁵⁴ The interpretation of the sign $\bigcirc$ has been the point of departure for a longstanding debate among musicologists in the past decades. For details see n. 195 below.

¹⁵⁵ For more see discussion in Chap. 2.2.

1.4 A Reassessment of Baude Cordier's Picture Songs

I have chosen to discuss the two rondeaux *Belle, bonne, sage* and *Tout par compas* from the Chantilly codex at this point, because they allow me to address two important aspects, which will be central to this study of proportion signs, in an exemplary way. Much has been said and written about these two famous rondeaux in the past, yet a comprehensive analysis of the notation has brought forward details hitherto neglected or overlooked.

The first aspect concerns the question whether proportions are cumulative in two consecutive proportion sign sections. In other words: What is the reference point for the proportion, which the proportion sign represents? Is it the section directly preceding the proportion sign section in the same voice, which would be a cumulative interpretation, or is it, for example, the beginning of the same voice part or even a section in a different voice part? I will argue that it is often impossible to answer this question definitively. I also do not support the hypothesis of a linear development from non-cumulative proportions to cumulative proportions suggested by other scholars.¹⁵⁶ Therefore, I advise against using cumulative proportions as argument for stylistic assessments (e.g. for dating a composition), which has been done in the past in the case of these two rondeaux and other compositions.

The second aspect concerns functions of signs. Both rondeaux contain signs which have more than one function. The most common form of signs with multiple functions are signs, which simultaneously indicate a mensuration and a proportion. Thus, they indicate that the structure of the rhythmic whole, i.e. the *mensura*, changes (mensuration sign function) and that the duration of the minim in the subsequent section is different from that in the previous section (proportion sign function). As I will demonstrate, however, even more functions could be ascribed to the signs present in the two rondeaux. The cut circle sign $\ominus$, for example, can be said to function as *p o s i t i o n s i g n* in addition to its being a mensuration sign in *Belle, bonne, sage* and *Tout par compas*. Most notably, I wish to point out that I have not followed previous scholars in their interpretation of $\ominus$ as proportion sign representing *dupla* (2:1) proportion in both compositions.¹⁵⁷ On the contrary, in my reading of *Belle, bonne, sage* and *Tout par compass*, $\ominus$ does not function as proportion sign at all. An explanation for this interpretation will be given in the subsequent discussion of the compositions.

¹⁵⁶ I discuss this aspect below (see pp. 68–71). Meanwhile, cf. Busse Berger, *Mensuration and Proportion Signs*, 204.

¹⁵⁷ A selection of publications proposing the *dupla* (2:1) proportion interpretation is given in n. 196 below.

1.4.1.1 A Frontispiece of the Chantilly Codex?

Belle, bonne, sage (Ch 1) and *Tout par compas* (Ch 2) constitute the first two songs of the codex and they are particularly famous for their pictorial layout: *Belle, bonne, sage* is written in the shape of a heart and in *Tout par compas*, the music is arranged in a set of two circular staves (see FIGURE 1.12). There is general consensus that both songs are not part of the original collection and most probably stem from a later period than the rest of the manuscript's repertoire, hence they form a different codicological unit.¹⁵⁸ The somewhat paradoxical situation that the two Cordier songs are considered to be the most famous *Ch* songs has most trenchantly been described by Elizabeth R. Upton: "It is ironic that the two Baude Cordier works [...] are the most frequently reproduced images from this manuscript, as they are so clearly not part of the original corpus."¹⁵⁹



FIGURE 1.12: Baude Cordier's songs in *Ch*

¹⁵⁸ On the term 'codicological unit' see Chap. 2 n. 6 on p. 98.

¹⁵⁹ Elizabeth R. Upton, "The Chantilly Codex (F-Ch 564): The Manuscript, Its Music, Its Scholarly Reception," PhD diss., University of North Carolina at Chapel Hill, 2001, 100.

Several strong arguments have been put forward in support of this hypothesis: (1) The parchment, on which the Cordier songs are written, is different from that used for the other five gatherings of the manuscript; (2) the scribe of the Cordier songs is not the same as the Italian scribe of the main corpus; (3) the two folios of the Cordier songs are unfoliated and inserted between the main repertoire and the index, in which they are not recorded.¹⁶⁰ However, new evidence has emerged in the form of prick marks on the folios of the Cordier songs (fols. 11 and 12), which most probably stem from preparing the second page of the index on folio 10r.¹⁶¹ Thus, by all appearances, the folios of the Cordier songs were already part of the manuscript when the index was composed. As Plumley and Stone conclude, this new-found evidence suggests that the Cordier song folios “shared a much closer history with the main part of the manuscript than previously suspected.”¹⁶² Earlier, it had generally been assumed that the two Cordier pieces only became part of the manuscript after the index was written.¹⁶³ To solve this puzzle, Plumley and Stone suggested that the photogenic Cordier songs might have been inserted “to function as a kind of frontispiece, an iconic symbol of the music contained therein”¹⁶⁴, which could be the reason for them being part of the collection at the time the index was written without them being listed in it.

Indeed, the music of both compositions is said to be stylistically closer related to the later Burgundian style of composers such as Guillaume Du Fay and Gilles Binchois and their generation than the *Ars subtilior* repertoire. Nevertheless, they serve as good examples for the discussion of the above-named two aspects. In fact, it has been suggested that notational practices found in other *Ch* compositions were intentionally imitated in the two Cordier rondeaux, thereby strengthening the frontispiece-hypothesis.¹⁶⁵

¹⁶⁰ Cf. Yolanda Plumley and Anne Stone, “Cordier’s Picture-Songs and the Relationship between the Song Repertoires of the Chantilly Codex and Oxford 213,” in *A Late Medieval Songbook and its Context: New Perspectives on the Chantilly Codex (Bibliothèque du Château de Chantilly, Ms. 564)*, ed. Yolanda Plumley and Anne Stone (Turnhout: Brepols, 2008), 304.

¹⁶¹ Cf. Plumley and Stone, “Cordier’s Picture Songs,” 304–5. For a recent reassessment of Plumley’s and Stone’s findings see Margaret Bent, “The Absent First Gathering of the Chantilly Manuscript,” *Plainsong & Medieval Music* 26, no. 1 (2017): 28–34.

¹⁶² Plumley and Stone, “Cordier’s Picture Songs,” 303–4.

¹⁶³ See for example Upton, “The Chantilly Codex,” 101–2.

¹⁶⁴ Plumley and Stone, “Cordier’s Picture Songs,” 306.

¹⁶⁵ Cf. Ursula Günther, “Der Gebrauch des tempus perfectum diminutum in der Handschrift Chantilly 1047,” *Archiv für Musikwissenschaft* 17, no. 4 (1960): 282.

1.4.1.2 The Identity of the Composer Baude Cordier

The identity of the composer Baude Cordier remains uncertain. The only ascertained biographical information stems from the rondeau *Tout par compas*, whose text states that his music was known from his birthplace Reims to Rome: “De Reims dont est et jusqu’a Romme / Sa musique appert et a rode.”¹⁶⁶ The fame and popularity suggested by these verses certainly poses the question why Cordier remains an unidentified personality to this day.

In 1973, Craig Wright suggested that Baude Cordier could be identical with Baude Fresnel, who came from Reims and who was harper (hence the sobriquet ‘Cordier’) at the court of Philip the Bold (1342–1404), Duke of Burgundy.¹⁶⁷ However, Wright’s hypothesis has been questioned, amongst others, by Richard Hoppin as well as Ursula Günther. Hoppin has argued that one would expect to find the name ‘Cordier’ in archival sources if it was a sobriquet of Baude Fresnel’s, as it is the case with Fresnel’s colleague and friend Jean Tapissier, whose real name was Jean de Noyers.¹⁶⁸ For example, Tapissier is referred to as “Jehan de Noyers, called Tapissier” in a 1391 order of payment.¹⁶⁹ No document, in which the names Fresnel and Cordier appear side by side, has yet surfaced, however. Ursula Günther shares Hoppin’s scepticism and also doubts Gordon Greene’s hypothesis that Baude Cordier was involved in the process of compiling *Ch*.¹⁷⁰ A number of scholars, on the other hand, have accepted Wright’s hypothesis. Reinhard Strohm, for example, has stated: “[W]e can observe the breaking of new ground in the compositions of Magister Baude Cordier, since he must indeed be identifiable as Philip the Bold’s harper Baude Fresnel, who died in 1397 or 1398.”¹⁷¹

Cordier’s alleged ties to Avignon have also been questioned, namely by Gilbert Reaney, who has remarked that Cordier’s *Et in terra* in the manuscript *Apt*, one of the principle sources for sacred music from Avignon from the late fourteenth and early fifteenth century, is not attributed to him in that source. Thus, Reaney has concluded that Cordier “may have been a stranger to the district.”¹⁷² Yet, codicological evidence from the *Ch* manuscript suggests otherwise: The scribe of the Cordier songs in *Ch* also added three attributions to compositions by Matheus de Sancto Johanne, Johannes Hasprois, and Jacob de Senleches, and these three

¹⁶⁶ Transcription taken from Parrish, *The Notation of Medieval Music*, 193.

¹⁶⁷ Cf. Craig Wright, “Tapissier and Cordier: New Documents and Conjectures,” *The Musical Quarterly* 59, no. 2 (1973): 186–89.

¹⁶⁸ Cf. Hoppin, *Medieval Music*, 486 n. 21.

¹⁶⁹ Cf. Wright, “Tapissier and Cordier,” 179.

¹⁷⁰ Cf. Ursula Günther, “Unusual Phenomena in the Transmission of Late 14th Century Polyphonic Music,” *Musica Disciplina* 38 (1984): 89–92.

¹⁷¹ Reinhard Strohm, *The Rise of European Music, 1380–1500* (Cambridge: Cambridge University Press, 1993), 141. Also see Leech-Wilkinson, “Ars Antiqua – Ars Nova – Ars Subtilior,” 236.

¹⁷² Cf. Gilbert Reaney, *CMM* 11, *Early Fifteenth-Century Music* I, I.

composers can be linked to the papal court at Avignon.¹⁷³ Hence, it seems reasonable to assume that Cordier, too, belonged to an Avignon circle of composers known to the scribe.

Moreover, it has also been suggested that the two *Ch* picture songs are autographs because the text of *Tout par compas* refers to the composer in the first person (“J’ay fait ce rondelet”).¹⁷⁴ The hypothesis has been reiterated by Gilbert Reaney, who has stated: “It is very likely that Cordier himself wrote these two works in the manuscript, for there is a personal touch about their insertion in the already complete collection.”¹⁷⁵ In absence of proof of Baude Cordier’s identity and/or any other documents unquestionably written by him, however, I consider the autograph debate to be futile. Whether or not the two compositions are autographs is also irrelevant for the following discussion.

One of the major problems of Wright’s theory is Baude Fresnel’s date of death. According to Wright, Fresnel died in 1397 or 1398.¹⁷⁶ However, a *terminus ante quem* of 1398 has been considered to be irreconcilable with some stylistic aspects of Baude Cordier’s compositions, which apart from *Ch* can be found in *Ox213*, *Bologna Q15* and *Apt*. Instead, scholars have repeatedly suggested that it is much more plausible to date Cordier’s compositions to the first two decades of the fifteenth century. The reasons for this proposition are manifold. Imitation between voices, the use of cut signatures, composite signs, the use of what scholars have termed *tempus perfectum diminutum*, and cumulative proportions have all been adduced in support of the hypothesis that Cordier was active until c.1420.¹⁷⁷

As I will show in the following discussion, however, some of these arguments are considerably weakened by my analysis of the two rondeaux, thus I am inclined to suggest that a *terminus ante quem* of 1398 might be plausible for the composition of *Belle, bonne, sage* and *Tout par compas*. First, I do not agree with the frequently expressed assumption that the cut circle indicates *dupla* (2:1) proportion or *tempus perfectum diminutum* in the two rondeaux in *Ch*. Moreover, I propose that a proportion sign often read as composite sign in *Tout par compas* should actually be read as two separate signs, namely one mensuration sign and one pro-

¹⁷³ Cf. Plumley and Stone, “Cordier’s Picture-Songs,” 306–7.

¹⁷⁴ According to Plumley and Stone, this was first proposed by Léopold Delisle, one of three specialists to examine the manuscript on behalf of its nineteenth-century owner Henri d’Orléans (1822–1897), Duke of Aumale. Cf. “Cordier’s Picture-Songs,” 306 n. 7; also see Upton, “The Chantilly Codex,” 1–39 for a detailed account of nineteenth-century research on *Ch*.

¹⁷⁵ Cf. Reaney, *CMM* 11, *Early Fifteenth-Century Music I*, II. Ursula Günther first supported this proposition (cf. “Der Gebrauch des tempus perfectum diminutum,” 279) and it was later reiterated by Craig Wright (cf. “Tapissier and Cordier,” 187) and Gordon K. Greene (cf. *PMFC* 18, XI). Most recently, Margaret Bent has argued the possibility that the two pieces are autographs. Cf. “The Absent First Gathering,” 32 and 34.

¹⁷⁶ Cf. Wright, “Tapissier and Cordier,” 189.

¹⁷⁷ Cf. Heinrich Bessler, *MGGI*, vol. 2, cols. 1666–68; Reaney, *CMM* 11, *Early Fifteenth-Century Music I*, II; Günther, “Der Gebrauch des tempus perfectum diminutum,” 278–79; Margaret Bent, “The Early Use of the Sign Ø,” *Early Music* 24, no. 2 (1996): 223 n. 2; David Fallows, *A Catalogue of Polyphonic Songs, 1415–1480* (Oxford: Oxford University Press, 1999), 687; and Plumley and Stone, “Cordier’s Picture-Songs,” 308–10.

portion sign. And finally, I advise against using cumulative proportions as argument for stylistic assessments. It is true that some notational aspects of Cordier's compositions in *Ox213* still contradict Craig Wright's hypothesis concerning Baude Cordier's identification, still I wonder whether scribal influence could possibly explain the inconsistencies in this much later (c.1430) compiled manuscript.¹⁷⁸

1.4.2 The Rondeau *Belle, bonne, sage*

Returning to the two rondeaux it can be said that the music—aside from its notation—is not truly complex. Plumley and Stone have described the elaborate notation as “surface complexity” and concluded that there is “a fair amount of redundancy of result between the various signs.”¹⁷⁹ This has also been pointedly expressed by Carl Parrish who has stated: “[*Tout par compas*] is a sort of whimsical reversal of the trend toward rhythmical complexity which is to be seen in certain late fourteenth-century compositions: it is essentially simple in musical style, but is deliberately written in a highly complicated manner.”¹⁸⁰ The same is true for the facing rondeau *Belle, bonne, sage*. In fact, this composition could have been notated without proportion signs entirely, because the proportional rhythms (*sesquialtera* (3:2) and *sesquitertia* (4:3) proportion) could have been notated by the use of the two forms of coloration (red and void notation) also present in the piece, as demonstrated in FIGURE 1.13. In this illustration, I have juxtaposed the original notation (first staff) with a non-existent fictional version without proportion signs (second staff), which nevertheless depicts the same rhythms in *sesquialtera* (3:2) and *sesquitertia* (4:3) proportion. Since the two Cordier rondeaux from *Ch* are *unica*, we do not know whether such a version with simpler notation ever existed.

¹⁷⁸ For an assessment of notational and stylistic aspects of Cordier's songs in *Ox213* see Plumley and Stone, “Cordier's Picture-Songs,” 312–14.

¹⁷⁹ Plumley and Stone, “Cordier's Picture-Songs,” 311. It is certainly true that the two rondeaux contain an above-average number of signs. However, Plumley's and Stone's article—despite the focus on notational aspects in the pieces—regrettably contains a number of errors where the signs are concerned. One can find the introductory statement “Both songs [*Belle, bonne, sage* and *Tout par compas*] contain an arsenal of symbols for proportional notation ($\odot$, $\odot_1^3$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{3}{4}$)” (p. 310), which fails to inform the reader about the presence of the signs 3 and $\frac{8}{9}$ in *Belle, bonne, sage* as well as $\frac{2}{1}$ in *Tout par compas*. Surprisingly, the missing signs 3, $\frac{8}{9}$ (erroneously quoted as $\frac{9}{8}$ in the article (p. 311)), and $\ominus$ (interpreted as proportion sign by the authors) are depicted in the EXAMPLES 4 and 5 (pp. 310 and 311). Moreover, the sentence “[...] $\frac{3}{2}$ following $\ominus$ merely reinstates $\mathbb{C}_1^3$, and $\frac{3}{4}$ following $\frac{4}{3}$ merely reinstates $\mathbb{C}$.” (p. 311) should read $\odot_1^3$ for $\mathbb{C}_1^3$. Finally, EXAMPLE 5a (p. 312) displays an incorrect distribution of semibreves as well as a sign not present in the rondeaux: there should be four instead of three semibreves in *sesquitertia* (4:3) proportion, and $\odot$ does not appear in either *Belle, bonne, sage* or *Tout par compas*.

¹⁸⁰ Carl Parrish, *The Notation of Medieval Music*, 187.

Original notation
in *Ch*

Fictional version
(simpler notation)

Transcription

FIGURE 1.13: Cantus of *Belle, bonne, sage*: original notation, fictional version without proportion signs, and transcription¹⁸¹

¹⁸¹ Transcription in FIGURE 1.13 taken from Greene, *PMFC* 18, 1–2 (No. 1). Where note values and pitches are concerned it is the same as Reaney's edition, except *ficta* in m. 37 (see Reaney, *CMM* 11, *Early Fifteenth-Century Music I*, 9–10 (No. 8)).

First of all, the shift from *tempus perfectum* with *prolatio minor* [3,2] to *tempus imperfectum* with *prolatio maior* [2,3] happening twice in the cantus (mm. 10 and 42 in the transcription, see FIGURE 1.13) and once in the contratenor is at first glance completely unnecessary because it is combined with *subdupla* (1:2) proportion at the minim level, hence the structure of the rhythmic whole does not change, augmentation only calls for smaller note values to be used (see FIGURE 1.14). The significance of this shift will be addressed below. For now, it is sufficient to note that the shift causes notational difficulties, which are solved by the application of proportion signs. Without the shift from [3,2] to [2,3], however, no such difficulties would have occurred, as I will now demonstrate.

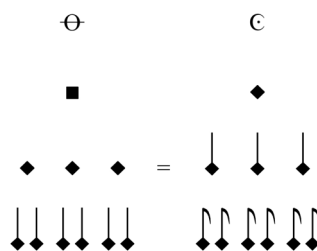


FIGURE 1.14: Augmentation in *Belle, bonne, sage*: the change in mensuration does not influence the structure of the rhythmic whole due to the combination with *subdupla* (1:2) proportion at the minim level

The red notation in the cantus indicates *sesquialtera* (3:2) proportion at several levels. The first group of red notes (m. 9, see FIGURE 1.13) indicates *sesquialtera* (3:2) proportion at the minim level while the group of three red breves occupies the space of two former black ones, thus indicating *sesquialtera* (3:2) proportion at the breve level (mm. 25–26). The concept of *sesquialtera* (3:2) proportion is used in two other places in the rondeau's cantus. However, coloration is not applied in these two cases. What is the explanation for this inconsistency?

Due to the prior shift to ⊙ in combination with a *subdupla* (1:2) proportion at the minim level (operating in mm. 10–11, 14–18, and 42), which—as described above and illustrated in FIGURE 1.14—equalises a former semibreve with the minim of the following part, coloration is not an option due to the conjuncture that minims cannot be perfect, i.e. divided into three semiminims (see FIGURE 1.15). The application of *sesquialtera* (3:2) proportion on semiminim level is therefore not possible, since minims should always be imperfect. Instead, *tripla* (3:1) proportion at the minim level is applied, which has the same effect as *sesquialtera* (3:2) proportion at the semiminim level. Three minims occupy the space of one minim and thus resolve the notational problem. *Tripla* (3:1) proportion is indicated by the Arabic numeral 3 in

Belle, bonne, sage (mm. 19 and 43). To sum up, the shift from [3,2] to [2,3] in combination with *subdupla* (1:2) proportion causes a notational problem, namely that red notation in the augmented section cannot be applied. Without the prior shift to a smaller notational level, the rhythm of mm. 19 and 43 could have been notated by using red notation (see mm. 19 and 43 of fictional version in FIGURE 1.13). Instead, the proportion sign 3 is applied in the two sections in the actual composition.

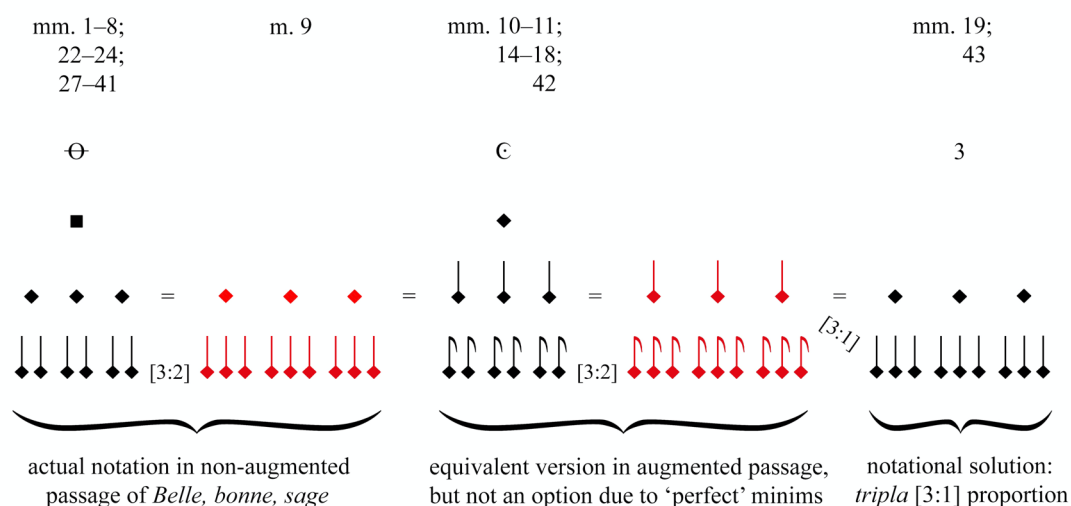


FIGURE 1.15: Impossibility of displaying *sesquialtera* (3:2) proportion at the semiminim level by using coloration

Void notation in *Belle, bonne, sage* indicates *sesquitertia* (4:3) proportion at the minim level and *dupla* (2:1) proportion at the semibreve level, most often indicated by the sign $\odot$ in other *Ars subtilior* works. In mm. 12–13, three semibreves and two minims in void notation occupy the space of two semibreves or six minims under $\odot$. Void notation could also have been applied without the prior shift from [3,2] to [2,3] in combination with *subdupla* (1:2) proportion. In this hypothetical case, void notation would indicate *sesquitertia* (4:3) proportion at the semibreve level (see mm. 12–13 in the fictional version in FIGURE 1.13). Void notation is also applied in mm. 20–21, however, since no minims are present—hypothetically four minims would replace three of the previous measure, thus indicating *sesquitertia* (4:3) proportion at the minim level—one has to assume *dupla* (2:1) proportion at the semibreve level, which yields the same result. As demonstrated in FIGURE 1.16, I have interpreted the groups of void notation in *Belle, bonne, sage* as cumulative proportion. This approach means that the number of minims in a section with proportional rhythms (in this case indicated by void notation) is compared to the number of minims in the section directly preceding this section. Thus, four void minims in mm. 12–13 replace three of the directly preceding section under $\odot$, but four void minims in mm. 20–21 replace

three of the directly preceding section under 3. In this cumulative approach, the number of void minims in one *mensura* changes (four compared to twelve), although the applied proportion at the minim level (*sesquitertia* (4:3) proportion) does not change. An explanation for the cumulative interpretation will follow below.

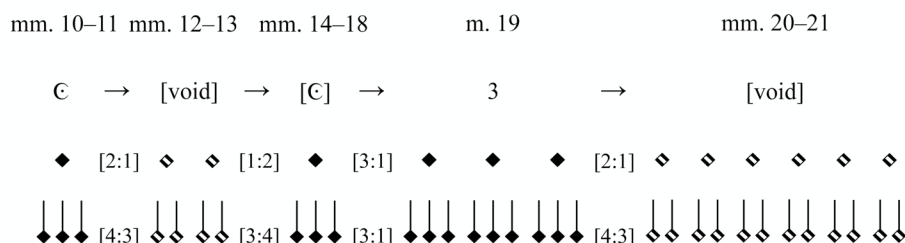


FIGURE 1.16: Interpretation of void notation in *Belle, bonne, sage*

In the fictional version with simpler notation and without proportion signs shown above, no void notation is necessary in mm. 20–21 (see FIGURE 1.13). After the red notation in m. 19 the scribe could have continued with five minims, three semibreves, and another minim in *tempus perfectum* with *prolatio minor* [3,2]. The penultimate measure, in which *subsesquioctava* (8:9) proportion is operating, could also have been notated differently in a simpler version. In this fictional version, proportions are not cumulative, therefore void notation could have been used in order to create the same rhythm (see m. 44 in FIGURE 1.13).

As just demonstrated and illustrated in FIGURE 1.13, the cantus of the rondeau *Belle, bonne, sage* could have been notated without proportion signs and the proportional rhythms present in the piece could have been displayed by coloration—red and void notation—which are present in the piece anyway. The same is true for the other two voices. Hence, Plumley’s and Stone’s conclusion that there is “a fair amount of redundancy of result between the various signs”¹⁸² seems to be an adequate assessment, at least where rhythmic possibilities are concerned. As I will argue below, however, the proportion signs in *Belle, bonne, sage* might have other functions in addition to indicating rhythmic proportion, which offer an explanation for their presence in the piece.

¹⁸² Plumley and Stone, “Cordier’s Picture-Songs,” 311.

1.4.2.1 Proportion Signs in *Belle, bonne, sage*

First of all, I would like to assess the situation. *Belle, bonne, sage* contains three different proportion signs, which appear as single Arabic numeral, stacked Arabic numerals, and geometric shape: the single Arabic numeral 3 is used twice in the cantus, the stacked Arabic numerals $\frac{8}{9}$ appear only once in the penultimate measure of the cantus, and the dotted semicircle C is used twice in the cantus and once in the contratenor. Additionally, coloration is applied in order to indicate rhythmic proportion: one can find red and void notation in the cantus as well as red notation in contratenor and tenor. As the cantus is the most interesting part from a notational point of view, the ensuing discussion will focus on this voice.

Since two proportion signs appear twice in the cantus, I would like to commence by restating an extract from the definition given earlier in this chapter concerning the unequivocal proportional meaning of proportion signs: “At each notational level, a proportion sign represents a definite and unchanging ratio within a composition.”¹⁸³ In other words, a proportion sign cannot represent a certain ratio in one section of a piece and another ratio in another section of the same piece. Furthermore, in order for different people to agree on the ratio, which a proportion sign represents at a certain notational level (e.g. minim level), these people also have to agree on the reference point for the application of the proportion. In other words, they also have to agree which note values—i.e. notes from which section of the piece and which voice—are compared to each other in the proportion. In the following discussion, I will mostly assume minim level as notational level for the application of the respective proportion and a reference point within the same voice (cantus). However, even if these two assumptions are agreed upon, there are still several different possible reference points within the same voice, which will result in different ratios. Particularly in pieces, in which two or more sections with proportional rhythms follow one another, one has to decide whether or not the proportions should be interpreted as cumulative or non-cumulative proportions.

1.4.2.2 Non-Cumulative, Cumulative, and Quasi-Cumulative Proportions

Let us, for a moment, abandon the cantus of *Belle, bonne, sage* and regard the fictional voice in FIGURE 1.17. There are three different approaches to the interpretation of the proportion signs and void notation respectively in this fictional example:

¹⁸³ See p. 43 above.

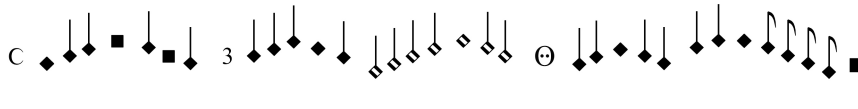


FIGURE 1.17: Fictional voice with two proportion signs and void notation

(1) **Non-cumulative proportions interpretation:** The reference point for the application of the proportion is the minim (or higher note value where necessary) in the initial mensuration.¹⁸⁴ The non-cumulative proportions interpretation would result in the following values in the fictional example (see FIGURE 1.18). The reference point will be the minim in *tempus imperfectum* with *prolatio minor* [2,2] (initial mensuration) for each section with proportional rhythms. In the section following the proportion sign 3, three minims replace two in the initial mensuration, hence 3 indicates *sesquialtera* (3:2) proportion. Furthermore, two void minims replace one in the initial mensuration, so void notation indicates *dupla* (2:1) proportion. Finally, in the last section, six minims replace two in the initial mensuration, hence Θ indicates *tripla* (3:1) proportion at the minim level.

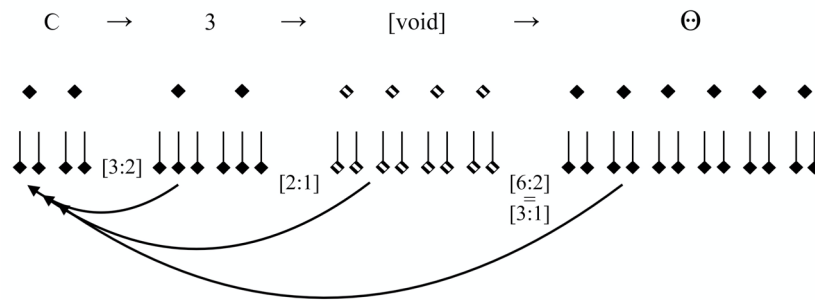


FIGURE 1.18: Non-cumulative proportions interpretation of fictional voice

(2) **Cumulative proportions interpretation:** The reference point for the application of the proportion is the minim (or higher note value where necessary) in the section directly preceding the proportion sign. The cumulative proportions interpretation would result in different values in the fictional example (see FIGURE 1.19). The reference point will be the minim of the preceding section, hence 3 indicates *sesquialtera* (3:2) proportion, void notation indicates *sesquitertia* (4:3) proportion, and Θ indicates *sesquialtera* (3:2) proportion at the minim level.

¹⁸⁴ In case the mensuration changes within a piece without influencing the duration of the minim (i.e. minim equivalence)—although this is not the case in *Belle, bonne, sage*—the reference point will still be the minim in the initial mensuration.

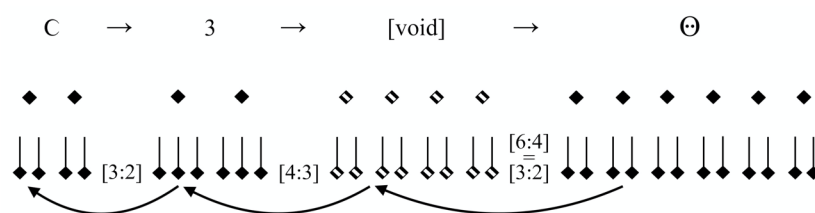


FIGURE 1.19: Cumulative proportions interpretation of fictional voice

(3) **Quasi-cumulative proportions interpretation:** The reference point for the application of the proportion is the last black minim in the section preceding the proportion sign. The quasi-cumulative proportions interpretation assumes a hierarchy within the notation of proportional rhythms, in which coloration is subordinate to mensuration and proportion signs: If coloration follows a proportion sign, the proportion indicated by said coloration is applied cumulatively. If a proportion sign follows a section notated in coloration, however, the reference point is the last black minim preceding the coloration section. While with this approach 3 would still indicate *sesquialtera* (3:2) proportion and void notation *sesquitertia* (4:3) proportion—as with the cumulative proportions interpretation—the minims of the last section (following the proportion sign $\ominus$) would be compared to the black minims of the section headed by the proportion sign 3 (see FIGURE 1.20).

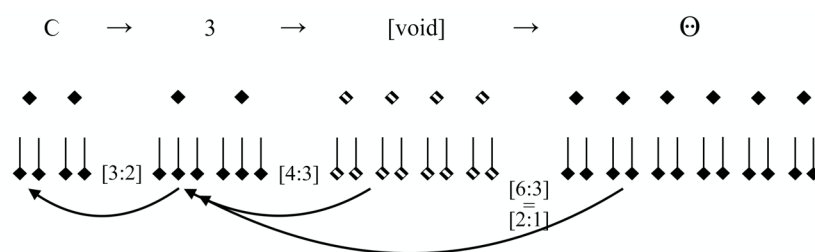


FIGURE 1.20: Quasi-cumulative proportions interpretation of fictional voice

All three different interpretation approaches ascribe different ratios to the proportion sign $\ominus$ and void notation. Furthermore, all three approaches result in perfectly plausible interpretations, namely simple ratios—2:1, 3:1, 3:2, and 4:3—which are the most common rhythmic proportions to be found in *Ars subtilior* music and are associated with consonant intervals in the Pythagorean tuning system. This fictional example thus demonstrates that one has to agree on a reference point (e.g. minim within the same voice) as well as on one of the three above-named approaches to the interpretation in order to agree on the ratio, which the proportion sign represents. TABLE 1.21 illustrates that the three different approaches also lead to different interpretations of the proportion signs present in *Belle, bonne, sage*.

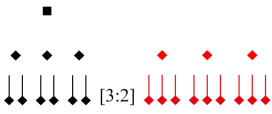
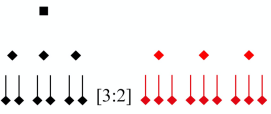
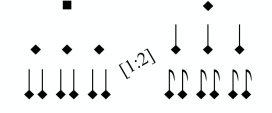
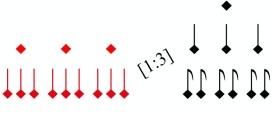
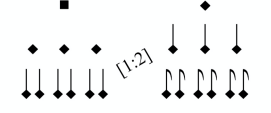
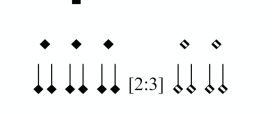
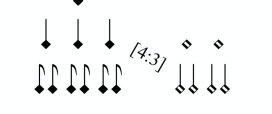
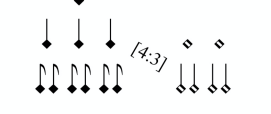
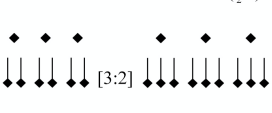
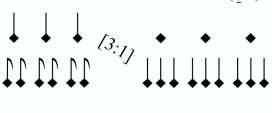
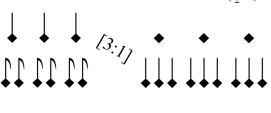
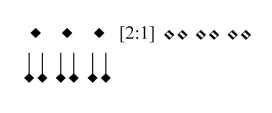
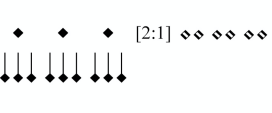
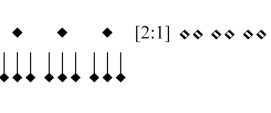
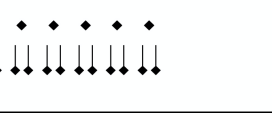
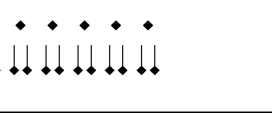
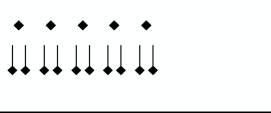
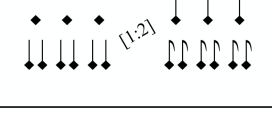
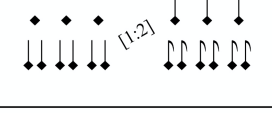
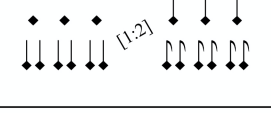
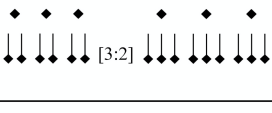
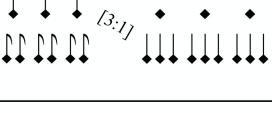
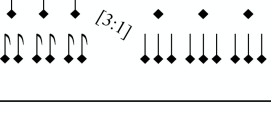
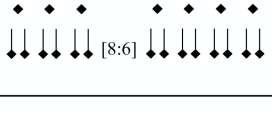
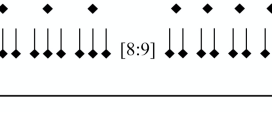
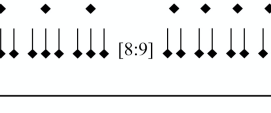
	non-cumulative proportions interpretation	cumulative proportions interpretation	quasi-cumulative proportions interpretation
red notation m. 9			
C mm. 10–11			
void notation mm. 12–13			
3 m. 19			
void notation mm. 20–21			
red notation mm. 25–26			
C m. 42			
3 m. 43			
8 9 m. 44			

TABLE 1.21: Ratios resulting from three different approaches to the interpretation of proportion signs in *Belle, bonne, sage*

The non-cumulative proportions interpretation (first column in TABLE 1.21), in which the reference point would in this case be the minim (or higher note value where necessary) in the

initial mensuration (*tempus perfectum* with *prolatio minor* [3,2]) of the cantus, does not seem to be the most suitable approach in *Belle, bonne, sage* for two reasons. First, void notation would indicate two different proportions in its two appearances: *subsesquialtera* (2:3) proportion at the minim and/or semibreve level on the one hand (mm. 12–13) and *dupla* (2:1) proportion on the other hand (mm. 20–21). Furthermore, the stacked Arabic numerals $\frac{8}{9}$ seem to be an odd choice for indicating the proportion 8:6 (i.e. *sesquitertia* (4:3) proportion).¹⁸⁵

The cumulative proportions interpretation (second column in TABLE 1.21), in which the reference point for the application of the proportion is the minim (or higher note value where necessary) in the section directly preceding the proportion sign, is also unsuitable in the case of *Belle, bonne, sage*, because it does not accord with the assumption that signs have an unequivocal proportional meaning within one composition.¹⁸⁶ In measure 10, C follows a group of red-coloured notes. With the cumulative proportions interpretation, three minims of C replace nine red minims, thus C would indicate *subtripla* (1:3) proportion at the minim level. In measure 42, on the other hand, C follows black notes in the initial mensuration [3,2]. In this case, three minims of C replace six minims of the preceding section, therefore C would indicate *subdupla* (1:2) proportion in the second part. The cumulative proportions interpretation thus ascribes two different ratios to the proportion sign C and is therefore inapt for *Belle, bonne, sage*.

It seems to be apparent that the quasi-cumulative proportions interpretation (third column in TABLE 1.21), in which the reference point for the application of the proportion is the last black minim in the section preceding the proportion sign, is best suited for *Belle, bonne, sage*. As stated above, this interpretation assumes a hierarchy within the notation of proportional rhythms, in which coloration is subordinate to mensuration and proportion signs. Only under this assumption can one explain that *dupla* (2:1) proportion—indicated by void notation in measures 25–26—is applied cumulatively to the preceding measure, in which *tripla* (3:1) proportion indicated by the proportion sign 3 is operating, while the proportion indicated by C (*subdupla* (1:2) proportion) is not applied cumulatively to the red-coloured notes in the first section (see mm. 9–10). Thus, the proportional values are consistent throughout the composition: C indicates *subdupla* (1:2) proportion at the minim level in both sections and 3 indicates *tripla* (3:1) proportion. I will therefore employ the quasi-cumulative proportions interpretation

¹⁸⁵ However, it should be noted that there are quite a few cases of single Arabic numerals indicating proportional rhythms that have nothing to do with the ratio they represent in the *Ars subtilior* repertoire, especially in *Turin*. See Chap. 3.2.1 for details.

¹⁸⁶ My definition of proportion signs states that “at each notational level, a proportion sign represents a definite and unchanging ratio within a composition.” See p. 43 above.

to describe the ratios represented by coloration or proportion signs in the following paragraphs.

It has to be noted that *Belle, bonne, sage* is a composition with particularly favourable characteristics, which is one of the reasons why I have chosen it as example piece to be discussed in this first chapter. One can make definite statements about whether proportions are cumulative or not, and if so, in which way, because the rondeau offers so many different clues to these particular questions. First, the proportion signs C and 3 appear twice in the cantus and they alternate with the two forms of coloration in such a way that it is possible to ascribe one particular interpretation to the signs. Second, the stacked numbers $\frac{8}{9}$ are following the proportion sign 3, thereby virtually hinting the fact that eight minims should replace nine of the preceding measure and that proportion signs in this composition should thus be interpreted cumulatively.¹⁸⁷

However, in most *Ars subtilior* compositions things are not equally obvious. Usually proportion signs only appear once in a piece or always in the same context. Furthermore, proportion signs are typically single Arabic numerals or geometric shapes, which opens the possibility of different interpretations. In the fictional example illustrated in FIGURE 1.17 above, all three interpretations of the proportion sign $\odot$ and void notation are plausible. It is important to stress that the question whether proportions are cumulative or not cannot be answered conclusively in many *Ars subtilior* pieces, in which two or more sections with proportional rhythms follow one another. Definite conclusions can only be drawn in three cases: (1) Either the proportion sign appears in the form of stacked Arabic numerals, in which case the ratio represented by the sign is explicitly indicated; (2) or the composition contains a canon, which names the proportion(s) indicated by the proportion sign(s), (3) or—as is the case in *Belle, bonne, sage*—the piece itself offers enough clues in order to make a definite statement.

The above-described characteristics cannot be found in the majority of *Ars subtilior* compositions, however. Therefore, the answer to the question whether proportion signs should be interpreted non-cumulatively, cumulatively, or quasi-cumulatively lies in the eye of the beholder. It is therefore surprising to find statements such as the following in the literature. Busse Berger writes: “A study of all numerical proportions encountered in ModA and Chan-

¹⁸⁷ It is all the more interesting that Anna Maria Busse Berger has stated that “Baude Cordier’s *Belle bonne, sage*, m. 44, uses an 8:9 fraction after the figure 3 that is not cumulative,” since it seems to be perfectly obvious that *subsesquioctava* (8:9) proportion has to be applied cumulatively here. *Mensuration and Proportion Signs*, 182 n. 76. The same remark can be found in *idem*, “The Origin and Early History of Proportion Signs,” 421 n. 49. TABLE 1.21 shows that a non-cumulative proportions interpretation of the proportion sign $\frac{8}{9}$ would result in an 8:6 ratio. As stated above, indicating *sesquitertia* (8:6 = 4:3) proportion by $\frac{8}{9}$ seems to be an odd choice.

tilly supports this view: none of the proportions appears to be cumulative.”¹⁸⁸ This assessment is certainly wrong for the two Cordier songs in *Ch.* And in *Mod A*, the proportion sign 2 has to be applied cumulatively in two pieces, namely Matteo da Perugia’s *A qui Fortune ne se vult amer* (*Mod A* 88) and Corrado da Pistoia’s *Se Doulz Espour ne me donne confort* (*Mod A* 61). However, as already stated, there are pieces for which a definite statement cannot be made for the reasons given above.

An exception is the ballade *Amour m’a le cuer mis en tel martire* (*Mod A* 63) by Antonello da Caserta, in which stacked numbers ($\frac{4}{2}$, $\frac{8}{6}$, and $\frac{9}{6}$) are used in order to indicate proportions. These unambiguous values make it clear that the proportions in this piece are not to be applied cumulatively.¹⁸⁹ However, the song is a special case, for which one can make a definite statement concerning the question whether the proportion signs are cumulative or not, since it contains stacked Arabic numerals. As mentioned above, stacked Arabic numerals are rare in *Ars subtilior* compositions. Furthermore, the numbers in stacked Arabic numerals do not always indicate the proportion that has to be applied to the music. For example, the ballade *Sous un bel abre* (*Turin* 196) exhibits the proportion sign $\frac{3}{1}$ in the cantus, which does not indicate *tripla* (3:1) proportion, however, but *dupla sesquiquarta* (9:4) proportion.¹⁹⁰

Despite the fact that songs for which definite statements can be made are scarce, a prevailing opinion is that proportion signs in early *Ars subtilior* pieces were applied non-cumulatively while later compositions contain cumulative proportions. The above-mentioned ballade *Amour m’a le cuer mis en tel martire* has, for example, been adduced in order to support this view. Jason Stoessel states: “This work demonstrates an early phase in the use of proportions in which they are non-cumulative.”¹⁹¹ In *Mensuration and Proportion Signs*, Busse Berger has summarised the situation as follows:

“What impact did these [fifteenth-century] treatises have on music? The first area that they illuminate is the transition from non-cumulative to cumulative proportions. I suggested earlier that originally proportions were not cumulative because they were used interchangeably with mensuration signs, which by their very nature could not be cumulative. If one compares Boethian number ratios, which were called proportions, with the arithmetic described in the algorism treatises, another reason becomes apparent: in Boethius the proportion is not considered a fraction, but a ratio. Ratios refer to the comparative relation of numbers, while a fraction is an indicated quotient of whole numbers. Multiplication of fractions is not an issue in Boethian number theory. The new mathematical treatises, on the other hand, include detailed instructions on how to multiply and divide fractions. A good example is Johannes de Lineriis’ treatise *Algorismus de minutiis* (published together with Prosdocimus’ treatise *Algorismus*), which is entirely devoted to mathematical operations with fractions. Tinctoris, who introduced the idea that proportions should be cumulative, must have been exposed to the new mathematics either in Italy or possibly already during his studies at the University of Orleans. On the basis

¹⁸⁸ Busse Berger, *Mensuration and Proportion Signs*, 182.

¹⁸⁹ See discussion in Chap. 2.3, as well as Busse Berger, *Mensuration and Proportion Signs*, 183–84, and Stoessel, “The Captive Scribe,” 1:297–99.

¹⁹⁰ See Chap. 3.1 for details.

¹⁹¹ Stoessel, “The Captive Scribe,” 1:298.

of this acquaintance [sic], he must have concluded that a figure written over another one looked like a fraction and should be treated as such that is, that they should be multiplied when occurring successively. On the other hand, Anthonello de Caserta probably still thought in terms of Boethian number ratios, possibly was not familiar with the new mathematics, and might not have known how to multiply fractions.”¹⁹²

As enticing as the suggested connection to the “new mathematics” sounds, Busse Berger has based her entire argument on the assumption that one needs to be familiar with the multiplication of fractions in order to interpret cumulative proportions. However, that is not the case. Cumulative proportions in a composition simply call for a *d i f f e r e n t r e f e r e n c e p o i n t* for the application of the proportion. As demonstrated in FIGURE 1.19 above, the reference point for cumulative proportions is the minim in the directly preceding section instead of the minim in the initial mensuration. I would thus argue that the composer, scribe, or reader of a composition with cumulative proportions did not and does not need to know how fractions are multiplied. He simply needs to relate the new minim to the minim in the previous section. Thus no knowledge of any kind of advanced mathematics is required.

To sum up, I have explained in detail in which cases definite statements concerning the question of cumulative proportions can be made (stacked Arabic numerals, canon instructions explaining the proportion signs, and/or sufficient clues in the composition). The majority of *Ars subtilior* pieces containing proportion signs do not match these criteria, however. Since the question is only relevant for compositions, in which a minimum of two sections with proportional rhythms follow one another, it is not surprising that the topic is raised in discussions of *Ars subtilior* music, in which new rhythmic possibilities were explored. However, with so few compositions, for which definite statements can be made, I find it difficult to see sufficient proof for a linear development from non-cumulative to cumulative proportions, as suggested by Stoessel.¹⁹³ Moreover, I would decisively question the direct influence of the “new mathematics”, which Busse Berger has proposed, since, as explained above, no knowledge of multiplication of fractions is required for the interpretation of cumulative proportions. I would therefore argue that cumulative proportions should not be put forward as conclusive argument for the indication of a composition’s most likely time of origin.

This, however, has been done in case of Cordier’s *Belle, bonne, sage* and *Tout par compas*: “Their notation contains devices found in later sources but not in the main corpus of *Ch*, in particular cut signs to indicate a transfer of the *integer valor* from the semibreve to the breve (‘diminution’), and the use of cumulative proportions.”¹⁹⁴ As already stated in the in-

¹⁹² Busse Berger, *Mensuration and Proportion Signs*, 204.

¹⁹³ Cf. Stoessel, “The Captive Scribe,” 1:298.

¹⁹⁴ Plumley and Stone, “Cordier’s Picture-Songs,” 309.

introduction to this subchapter, these stylistic aspects have been considered to be irreconcilable with Baude Fresnel's death in 1397 or 1398. I hope that I have been able to demonstrate why I would caution against cumulative proportions being used as indicator towards a composition's time of origin. But what about the cut signs mentioned in the above citation?

1.4.2.3 The Function of Signs in *Belle, bonne, sage*

Returning to *Belle, bonne, sage*, the reader might already have wondered why I have not mentioned the cut circle sign $\ominus$ yet. Cut signs and their interpretation are at the core of a longstanding debate amongst musicological scholars, and the two rondeaux by Cordier are presumed to be the earliest surviving pieces with a cut circle.¹⁹⁵ Hence, these two pieces have received much scholarly attention in the past. There is general consensus that $\ominus$ indicates *dupla* (2:1) proportion at the minim level in these two rondeaux.¹⁹⁶ However, I would like to offer a different approach to the interpretation of $\ominus$ in the following paragraphs.

Most notably, scholars, when discussing *Belle, bonne, sage*, often describe $\ominus$ as proportion sign indicating *dupla* (2:1) proportion, in this context often called *tempus perfectum diminutum*.¹⁹⁷ In their opinion, $\textcircled{C}$ is the mensuration sign which sets the *integer valor* at the

¹⁹⁵ The debate has primarily evolved around the question whether cut signs in fifteenth-century music indicate a faster tempo compared to signs without strokes. See for example Eunice Schroeder, "The Stroke Comes Full Circle: $\emptyset$ and $\textcircled{C}$ in Writings on Music, ca. 1450–1540," *Musica Disciplina* 36 (1982): 133–137; Rob Wegman, "What Is 'Acceleratio Mensurae'?", *Music & Letters* 73, no. 4 (1992): 522–23; Margaret Bent, "The Early Use of the Sign $\emptyset$," *Early Music* 24, no. 2 (1996): 199–225; Anna Maria Busse Berger, "Cut Signs in Fifteenth-Century Musical Practice," in *Music in Renaissance Cities and Courts: Studies in Honor of Lewis Lockwood*, ed. Jessie A. Owens and Anthony M. Cummings (Michigan: Harmonie Park Press, 1997), 101–12; Margaret Bent, "The Use of Cut Signatures in Sacred Music by Ockeghem and His Contemporaries," in *Johannes Ockeghem: Actes du XI^e Colloque internationale d'études humansites*, ed. Philippe Vendrix (Paris: Klincksieck, 1998), 641–80; Alexander Blanchy, "Reading Tinctoris for Guidance on Tempo," in *Antoine Busnoys: Method, Meaning, and Context in Late Medieval Music*, ed. Paula Higgins (Oxford: Clarendon Press, 1999), 399–427; Margaret Bent, "The Use of Cut Signatures in Sacred Music by Binchois," in *Binchois Studies*, ed. Andrew Kirkman and Dennis Slavin (Oxford: Oxford University Press, 2000), 277–312; Rob C. Wegman, "Different Strokes for Different Folks? On Tempo and Diminution in Fifteenth-Century Music," *Journal of the American Musicological Society* 53, no. 3 (2000): 461–505; and Margaret Bent, "On the Interpretation of $\emptyset$ in the Fifteenth Century: A Response to Rob Wegman," *Journal of the American Musicological Society* 53, no. 3 (2000), 598–612.

¹⁹⁶ Cf. Heinrich Bessler, *Bourdon und Fauxbourdon: Studien zum Ursprung der niederländischen Musik* (Leipzig: Breitkopf und Härtel, 1950), 132; Gilbert Reaney, introduction to *Early Fifteenth-Century Music I* (Rome: American Institute of Musicology, 1955), II; Ursula Günther, "Der Gebrauch des tempus perfectum diminutum," 279; Busse Berger, *Mensuration and Proportion Signs*, 155; Bent, "The Early Use of the Sign $\emptyset$," 199; Wegman, "Different Strokes for Different Folks?" 462; Stoessel, "The Captive Scribe," 1:307; and Plumley and Stone, "Cordier's Picture Songs," 311.

¹⁹⁷ The term is not contemporary. Cf. Michael Collins' remark as quoted in Bobby Wayne Cox, "'Pseudo-Augmentation' in the Manuscript Bologna, Civico Museo Bibliografico Musicale, Q 15 (BL)," *The Journal of Musicology* 1, no. 4 (1982): 421 n. 14a. According to Wegman, the term *tempus perfectum diminutum* was not used before 1490: "Adam of Fulda, writing in 1490, appears to be the first theorist to describe $\ominus$ as the sign of 'tempus perfectum per diminutionem' (rather than 'per semi' or 'per medium')." "Different Strokes for Different Folks?" 494 n. 64. Charles E. Hamm has instead coined the term "pseudo-augmentation" (*A Chronology of the*

semibreve.¹⁹⁸ However, the piece commences in a different mensuration, and $\textcircled{C}$ first appears in m. 10 of the cantus. Indeed, all three voices begin with *tempus perfectum* with *prolatio minor* [3,2] (as reinstated by $\ominus$ in m. 22 in the cantus), setting the perfect breve as the central unit of the *mensura*. Moreover, the tenor stays in that mensuration throughout the entire composition. I would therefore argue that the initial mensuration *tempus perfectum* with *prolatio minor* [3,2] is the *integer valor* and that all other signs ($\textcircled{C}$, 3, and $\frac{8}{9}$) and coloration change this initial mensuration.¹⁹⁹ As already discussed above, with this approach, $\textcircled{C}$ is interpreted as proportion sign indicating *subdupla* (1:2) proportion at the minim level (i.e. augmentation) as well as mensuration sign indicating *tempus imperfectum* with *prolatio maior* [2,3].²⁰⁰ The sign itself therefore has more than one function: it is proportion sign and mensuration sign combined in one sign.

For the remainder of the discussion of *Belle, bonne, sage*, I would like to explore possible functions of the signs found therein. I will include the mensuration sign $\ominus$ in this discussion since I would like to suggest that it also has more than one function. Signs with several functions will be called ‘multi-function signs’ throughout this study. Some might say that the term is similar to Margaret Bent’s proposal of calling $\ominus$ a “general-purpose sign.”²⁰¹ In her article, Bent hypothesised that “at first [2:1 proportion] was only one meaning of a more general-purpose sign, and a meaning that was only weakly established before 1430. [...] Where present, the strokes are signposts, repeat marks and co-ordination signs.”²⁰² Bent has been criticised for the term by Rob C. Wegman, who—drawing on Karl Popper’s criteria of non-falsifiability—argued that it would be next to impossible to disproof this hypothesis:

“No matter how many pieces one might choose to examine, there is in principle no limit to the number of purposes that a ‘general-purpose sign’ may be found to have served. [...] So long as the sign $\ominus$ can

Works of Guillaume Dufay Based on a Study of Mensural Practice (Princeton: Princeton University Press, 1964), 43), which does not seem to have become widely accepted.

¹⁹⁸ See for example Plumley and Stone, “Cordier’s Picture Songs,” 311.

¹⁹⁹ Among all scholars studied for this dissertation, only Willi Apel and Carl Parrish seem to have taken this view. Both have spoken of augmentation by $\textcircled{C}$ instead of diminution by $\ominus$. Furthermore, Apel also does not interpret $\ominus$ as proportion sign: “The sign $\ominus$ at the end of this staff indicates (or confirms) the return to normal *tempus perfectum*. In spite of the dash the sign has no proportional meaning. The explanation of this uncommon usage probably lies in the fact that this sign serves merely as a time signature which is understood at the beginning of all the parts.” Apel, *The Notation of Polyphonic Music*, 425–26. Parrish has summarised: “[T]he semicircle-and-dot [...] indicates that the S[emibreve] of the previous section, which had been the *integer valor*, is now replaced by the M[inim], which becomes the tactus through augmentation.” *The Notation of Medieval Music*, 189.

²⁰⁰ The *subdupla* (1:2) proportion interpretation of $\textcircled{C}$ was already suggested by Hugo Riemann in 1920. Cf. *Handbuch der Musikgeschichte I*, 2 vols. (Leipzig: Breitkopf & Härtel, 1919–20), 2:353–54. However, Riemann also interpreted $\ominus$ as indicating *dupla* (2:1) proportion.

²⁰¹ Bent, “The Early Use of the Sign $\textcircled{O}$,” 219.

²⁰² Bent, “The Early Use of the Sign $\textcircled{O}$,” 219.

be construed to have *some* meaning, therefore, whatever that meaning may happen to be, the hypothesis need not be invalidated at all. [...] Either Bent's hypothesis is valid or Θ is meaningless."²⁰³

I have chosen the term multi-function sign instead of adopting Bent's suggestion of general-purpose sign. Moreover, I understand multi-function sign in the sense that a certain sign has more than one function within the same piece. This shall not be confused with the same sign having several different meanings in different pieces. That the latter assertion is true for almost all the proportion signs discussed in this study will be shown throughout the following chapters.

Having said this, let us quickly recapitulate what happens in *Belle, bonne, sage* from a notational point of view.²⁰⁴ The rondeau has a canonic opening, which starts with the contratenor. The first five notes are repeated by the tenor after two measures and, a fifth above, by the cantus after four measures. All three voices start in *tempus perfectum* with *prolatio minor* [3,2]. The tenor stays in that mensuration throughout the piece, while the contratenor has $\textcircled{C}$ in the third measure indicating a change to *tempus imperfectum* with *prolatio maior* [2,3] and *subdupla* (1:2) proportion at the minim level. As already described above, the rhythmic structure of the piece does not change; augmentation only calls for smaller note values to be used (see FIGURE 1.14 on p. 60 above). Again, $\textcircled{C}$ has a double function: it changes the mensuration (mensuration sign) as well as the duration of the minim (proportion sign).

In the A section of the cantus (mm. 1–22) the melismata seem to fall together with the sections with proportional rhythms. Red notation—indicating *sesquialtera* (3:2) proportion at the minim level—coincides with the first melisma on the syllable “sant” of the word “plaisant” (see m. 9). The long melisma on the penultimate syllable of the first verse (“gen” of “gente”) is notated as two measures of $\textcircled{C}$ and two measures of white notation (mm. 10–13). One can observe a similar arrangement between music and text in the second verse. The melisma on the syllable “nou” of the word “renouvelle” (see m. 19) coincides with the triple rhythm indicated by the proportion sign 3.²⁰⁵ And the penultimate syllable of the second verse (“vel” of “renouvelle”) is notated as two measures of white notation (mm. 20–21). Thus, melismata are always underlined by proportional rhythms in the A section of the rondeau.

²⁰³ Wegman, “Different Strokes for Different Folks?” 466.

²⁰⁴ I have based the following discussion on Greene's edition (*PMFC* 18, 1–2 (No. 1)). If not at hand, I advise the reader to consult FIGURE 1.12 on p. 54, which shows the manuscript, and FIGURE 1.13 on p. 59, which shows the original notation with accompanying transcription.

²⁰⁵ As shown in the distribution of breves and semibreves respectively in TABLE 1.21 I have interpreted 3 as double-function sign indicating *tripla* (3:1) proportion at the minim level as well as a change to *tempus imperfectum* (only two semibreves to a breve). This can be derived from the breve present in m. 19, which is imperfect, but it has no further consequences for the music.

Curiously, melismata are not treated as consequently in the B section (mm. 22–45). The melisma on the syllable “vel” of the word “renouvelle” in the third verse (see mm. 28–30) is not highlighted by proportional rhythms. Neither is the beginning of the final melisma of the fourth verse on “sen” (m. 37) of “presente” indicated by a proportion sign. Proportional rhythms first reappear in m. 42, introduced by C and then followed by 3 (m. 43) and $\frac{8}{9}$ (m. 44).²⁰⁶ It is curious that the last syllable “te” of the text-underlay of the fourth verse in the manuscript is not placed on the final *longa*, which is why one could generally question the correctness of the underlay by the scribe in the fourth verse. However, I see no point in arguing that the melisma should only begin in m. 42, since too many syllables would be stretched over several notes in the otherwise syllabic piece in this scenario.²⁰⁷ Nevertheless, even if the B section does not support the hypothesis that proportional rhythms always indicate the beginnings of all melismata in *Belle, bonne, sage*, it cannot be denied that proportion signs only appear in melismatic passages.

It is not unusual that melismata are accompanied by proportional rhythms in *Ars subtilior* music. As stated above, however, the proportional rhythms present in *Belle, bonne, sage* could have been notated by using coloration only. Considering the carefully devised layout of the song, it seems reasonable to assume that the originator used the proportion signs on purpose. So why are they present? I would like to suggest that the proportion signs have an additional function, namely signalling to the singer that he should not lose tempo in the melisma by the use of smaller note values. Thus, proportion signs could combine the want for rhythmic variety during the melismata with this additional hint for the singer.

To sum up, three functions could be ascribed to the signs C , 3, and $\frac{8}{9}$ in *Belle, bonne, sage*: 1) change of the duration of the minim (proportion sign); 2) change of the mensuration (mensuration sign); 3) instruction for the singer to sing fast enough in the melismata. As already stated, C even indicates the beginning of the final melisma of the first verse in the A section of the rondeau.²⁰⁸

Finally, I would like to return to the interpretation of the sign $\ominus$ in *Belle, bonne, sage*. Its position within the piece is noteworthy since it marks the measure in which the B section be-

²⁰⁶ The proportion sign $\frac{8}{9}$ also changes the mensuration from *prolatio maior* to *prolatio minor*, i.e. the semibreve is now imperfect and only contains two minims. This is inevitable since 8 cannot be divided evenly by 3. Thus, $\frac{8}{9}$ can also be said to have at least two functions: mensuration sign and proportion sign.

²⁰⁷ A comprehensive study concerning text underlay in early fifteenth-century music can be found in Graeme M. Boone, *Patterns in Play: A Model for Text Setting in the Early French Songs of Guillaume Dufay*, American Musicological Society Monographs 1, Lincoln and London: University of Nebraska Press, 1999.

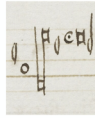
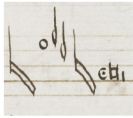
²⁰⁸ Incidentally, the contratenor only has the words “Belle bonne” written as text underlay. If one assumes that the contratenor is a voice part and that it is sung mostly on one syllable, C would also fall together with the melisma on “ne” of the only word “bonne”. Analogously, the tenor has ligatures for almost the entire voice part starting on “ne” of the word “bonne”, thereby also possibly indicating a melisma.

gins. Thus, $\ominus$ not only functions as mensuration sign but also as sign to indicate the position of the beginning of the new section (position sign). Therefore, it can also be called multi-function sign. In its capacity as position sign, $\ominus$ ends the passage with four different proportional rhythms (indicated by red notation, C , 3, and void notation). It is somewhat remarkable that the sign so closely resembles the ‘end of all restrictions’ road sign used in many European countries, although I am of course not suggesting a connection here.

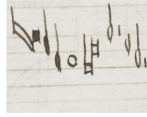
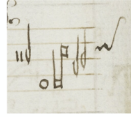
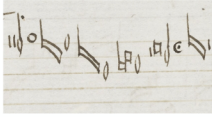
As stated above, scholars have commonly interpreted $\ominus$ as proportion sign indicating *dupla* (2:1) proportion in *Belle, bonne, sage* and *Tout par compas*. I have already explained that I reject this reading and that I have instead followed Apel’s contrary interpretation of the sign, according to which it is a mensuration sign reinstalling the initial mensuration after a section with proportional rhythms (m. 22).²⁰⁹ Since the stroke is the only way in which *prolatio minor* is indicated in both Cordier rondeaux in *Ch* one might even suggest that the cut circle was just an alternative mensuration sign to the circle without a dot. In other words, a circle or semicircle with a dot ($\odot$ or $\odot$) indicates *prolatio maior*, a circle or semicircle with a stroke (C or $\ominus$ instead of C or O) indicates *prolatio minor*. Maybe Cordier himself or the scribes responsible for copying his works only used these alternative signs to indicate *prolatio minor*.

Alas, this hypothesis is not supported by the compositions by Cordier found in *Ox213*. This c.1430 Italian manuscript contains the largest collection of chansons by Cordier, altogether seven pieces, four of which contain circles or semicircles without strokes, i.e. empty (semi)circles, which indicate *prolatio minor* (see FIGURE 1.22): the contratenors of the rondeaux *Pour le desfault du noble dieu bachus* (fol. 108v) and *Que vaut avoir qui ne vit liement* (fol. 110v), all three voices of the rondeau *Amans amés secretement* (fol. 123r), and all four voices of the ballade *Dame excellent ou sont bonté scavoir* (fol. 116r).

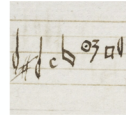
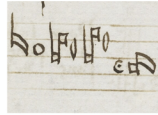
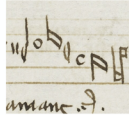
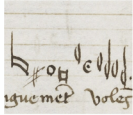
²⁰⁹ See pp. 71–72 above.



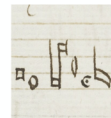
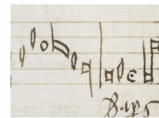
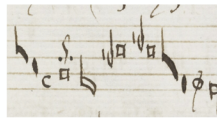
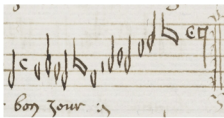
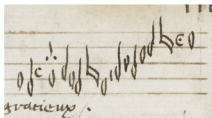
Contratenor of *Pour le desfault du noble dieu bachus* (fol. 108v), extracts



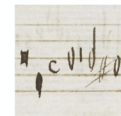
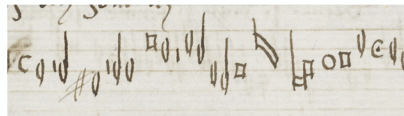
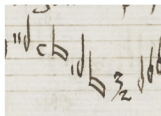
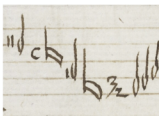
Contratenor of *Que vaut avoir qui ne vit liement* (fol. 110v), extracts



Cantus, tenor, and contratenor of *Amans amés secretement* (fol. 123r), extracts



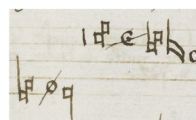
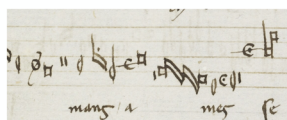
Cantus 1, tenor,



cantus 2, and contratenor of *Dame excellent ou sont bonté scavoir* (fol. 116r), extracts

FIGURE 1.22: Signs without stroke indicating *prolatio minor* in compositions by Cordier in *Ox213*

Then again, one can also find cut signs in two of Cordier's *Ox213* compositions and here they are indeed signs of diminution. In the ballade *Dame excellent ou sont bonté scavoir*, the minim of $\textcircled{C}$ (initial mensuration) is equated with the semibreve of $\textcircled{\Theta}$ in the second cantus and the tenor. Finally, in the rondeau *Amans amés secretement*, one can observe an even larger variety of cut signs (see FIGURE 1.23), namely not only the full circle $\textcircled{\Theta}$, but also a cut semi-circle $\textcircled{C}$ and a cut semicircle with a dot $\textcircled{\cdot}$, the latter of which contradicts the idea of the stroke being a sign of *prolatio minor*.



Cantus and tenor of *Amans amés secretement* (fol. 123r), extracts

FIGURE 1.23: Cut signs in *Amans amés secretement* by Cordier in *Ox213*

However, I would like to point out that the interpretation of $\ominus$ as a sign of diminution—i.e. *dupla* (2:1) proportion—in *Ch* by modern scholars is probably influenced by the meaning of the sign in later manuscripts, among them *Ox213*. This interpretation has been uttered repeatedly:

“In *Belle, bonne, sage* [...] t[empus] p[erfectum] dim[inutum] is clearly demanded by the sign $\ominus$.”²¹⁰

“ $\emptyset$ appears in the top part to signal perfect diminished time [...] in *Belle, bonne sage* [sic] and *Tout par compas* from Chantilly.”²¹¹

“Only one actual use of $\emptyset$ may date from earlier than 1420. It occurs in the unique pair of pieces [...] attributed to Baude Cordier and copied at an uncertain date in the first quarter of the 15th century as an addendum to the Chantilly manuscript [...]. Both pieces equate the semibreve of $\emptyset$ with the minim of a signature without stroke in another simultaneously sounding part. [...] $\emptyset$ means 2:1 [...] in simultaneous use.”²¹²

“When $\emptyset$ is combined with a different mensuration in another voice part (as is the case in the earliest known pieces using the sign [*Belle, bonne, sage* and *Tout par compas*]), the ‘vertical’ relationship between the voice parts confirms that the stroke calls for diminution by half.”²¹³

“In Cordier’s works found in CH 564 and Ob 213, [$\ominus$] always signifies that the duration of written note values must be diminished by one half.”²¹⁴

“*Belle, bonne, sage* has a mensural structure and notational setup very similar to that of *Tout par compas* [...]. It shares the comparison between $\textcircled{C}$ and $\emptyset$ in which a minim of the former is made equal to a semibreve of the latter.”²¹⁵

What seems to have been overlooked is that this retrospective interpretation is an anachronistic approach that does not quite fit the situation in *Ch*. As I have demonstrated, the order of appearance of the signs in *Belle, bonne, sage* (see FIGURE 1.13 on p. 59) calls for $\textcircled{C}$ (and not $\ominus$) to be interpreted as proportion sign. Again, $\ominus$ simply reinstalls the initial mensuration, which incidentally is maintained by the tenor throughout the composition. The medieval reader who neither knew the pieces in *Ox213* nor much later uses of cut circles most probably assumed a *sequential* direction of reading and therefore presumably arrived at the conclusion that $\textcircled{C}$ was a sign of *subdupla* (1:2) proportion in *Belle, bonne, sage*. Just because $\ominus$ indicates diminution in later sources one should not assume that it always bore that meaning.

Earlier in this chapter, I have explained that, in order for two people to agree on the ratio that a certain proportion sign represents, these two people also have to agree on the point of reference of the proportion. I have established that—since the proportions in *Belle, bonne, sage* clearly have to be applied quasi-cumulatively—this reference point must

²¹⁰ Günther, “Der Gebrauch des tempus perfectum diminutum,” 279; my translation.

²¹¹ Busse Berger, *Mensuration and Proportion Signs*, 155.

²¹² Bent, “The Early Use of the Sign $\emptyset$,” 199.

²¹³ Wegman, “Different Strokes for Different Folks?” 462.

²¹⁴ Stoessel, “The Captive Scribe,” 1:307.

²¹⁵ Plumley and Stone, “Cordier’s Picture Songs,” 311.

lie within the same voice. It is therefore all the more surprising that both Margaret Bent and Rob Wegman have adduced the vertical relationship between voices in their interpretation of Θ .²¹⁶ If the points of reference for all the signs in *Belle, bonne, sage* lie within the same voice, why should one make an exception for Θ ? Even more unlikely, the vertical relationship between Θ and $\mathbb{C}$ in mm. 22–41 would have had to be established between cantus and contratenor, and not even the tenor—which could be considered the rhythmically stable voice part—since the tenor stays in *tempus perfectum* with *prolatio minor* for the entire composition. Moreover, no 2:1 vertical relationship can be established for Θ in *Tout par compas*. For this canonic rondeau, which I would like to discuss next, Bent’s and Wegman’s assertion is even incorrect.

1.4.3 The Rondeau *Tout par compas*

Tout par compas is notated on two circular sets of staves, visually capturing the idea of the rondeau. The text underlay emphasises: “Tout par compas suy composee en ceste rode prement pour moy chanter plus seurement.” (“I am properly written with a compass in this round so that one may sing me more accurately.”²¹⁷) Only cantus and tenor are given. The third voice, a canonic voice, is deduced from the cantus and starts three measures after the cantus begins (see FIGURE 1.24). The entrance of the canonic voice is hinted at by the text of the rondeau, which states: “Trois temps entiers par toy posés chacer me pues joyeusement” (“After three perfect beats you can chase me joyously”²¹⁸).

But the entry of the canonic voice is also marked by Θ (m. 4 in FIGURE 1.24). This visual arrangement is worth taking a closer look at. While the tenor commences with Θ indicating *tempus perfectum* with *prolatio minor*, the cantus has three measures of Θ (*tempus perfectum* with *prolatio maior*) before also switching to minor prolation. Remarkably, the relationship between semibreve and minim (i.e. the prolation) is completely irrelevant in the first three measures since minims first appear in m. 8 of the cantus (see FIGURE 1.24). Thus, a change of prolation would have no consequence in m. 4.

²¹⁶ See n. 212 and 213 above.

²¹⁷ Translation taken from Parrish, *The Notation of Medieval Music*, 187.

²¹⁸ Translation taken from Gordon K. Greene, “From Mistress to Master: The Origins of Polyphonic Music as a Visible Language,” *Visible Language* 6, no. 3 (1972): 250. It must be noted that scholars disagree on the reading of this verse. Parrish, amongst others, translated it as “Three times around, put by you, you can chase me merri-ly”, referring to the number of times the rondeau should be sung. Cf. *The Notation of Medieval Music*, 219. This interpretation has expressly been criticised by Ralph Leavis: “Bergsagel has followed Parrish in a strange mis-translation. [...] Troys temps [...] means: When you have waited three *tempora*, you may pursue me joyfully.” “Review of ‘Cordier’s circular canon’ by John Bergsagel,” *Musical Times* 114, no. 2 (1973): 144.

The image displays a musical score for the piece "Tout par compas". It is organized into two main systems of staves. The first system includes staves for Cantus (original notation), Cantus 1 (transcription), Cantus 2 (transcription), Tenor (original notation), and Tenor (transcription). The second system continues the transcription for the same parts. The original notation is written on a single staff with a large circle containing a dot at the beginning. The transcription is written on multiple staves, with measure numbers 5, 10, 15, 20, 25, and 30 indicated. The score is in 3/4 time and features various musical notations including notes, rests, and accidentals. The transcription includes a circular notation at the beginning, similar to the original notation.

FIGURE 1.24: Original notation and transcription of *Tout par compas*²¹⁹

So why do we find this strange arrangement of signs at the beginning of the piece? It bears mentioning that a dot within a larger circle somewhat mirrors the visual arrangement of the music on the page, namely two sets of staves written as concentric circles, one within the other. Thus, $\odot$ could be interpreted as yet another allusion to the form of the rondeau, just as

²¹⁹ The transcription follows John Bergsagel's edition of the piece (except tenor in m. 10 and *ficta* in m. 27). Cf. "Cordier's circular canon," *Musical Times* 113, no. 12 (1972): 1177. Gordon Greene's 1980 edition (*PMFC* 18, 3–6) has been criticised in the past. Cf. Ursula Günther, "Fourteenth-Century Music with Texts Revealing Performance Practice," in *Studies in the Performance of Late Medieval Music*, ed. Stanley Boorman (Cambridge: Cambridge University Press, 1982), 262–63.

the text (“I am properly written with a compass in this round so that one may sing me more accurately.”).²²⁰

My hypothesis concerning the opening of the rondeau is, however, that Θ has been put at the beginning of the cantus instead of $\ominus$, so that $\ominus$ can indicate the entry of the canonic voice, as a *signum congruentiae* would. While the tenor has $\ominus$ at the b e g i n n i n g, the cantus has Θ three measures later, indicating by the same sign that something else b e g i n s in this position. The primary function of $\ominus$ is to mark this entry, thus it is principally a position sign, just as in *Belle, bonne, sage*.²²¹ Only theoretically does it change the prolation (mensuration sign) under an equal breve and therefore also the duration of the minim (proportion sign), as demonstrated in FIGURE 1.25. Since minim level is completely irrelevant in the first three measures and since the duration of the semibreve and breve does not change, however, it seems debatable whether $\ominus$ should be interpreted as proportion sign here. If it were, the proportion would be *subsesquialtera* (2:3) proportion at the minim level, but definitely not *dupla* (2:1) proportion, as suggested by other scholars. The tenor voice supports this view: *dupla* (2:1) proportion is indicated by $\frac{2}{1}$ in measure 25 and $\ominus$ indicates the initial mensuration. Why should $\ominus$ be interpreted differently in the cantus?

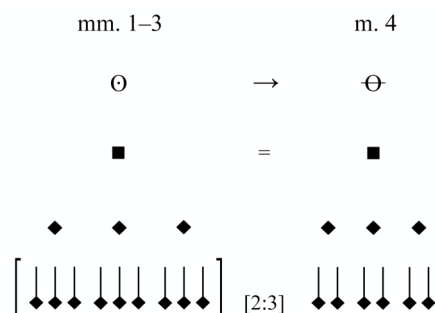


FIGURE 1.25: Proportional relationship between Θ and $\ominus$ in *Tout par compas*

As stated above, the beginning of the rondeau also reveals that even the comparison with another simultaneously sounding voice part does not result in a 2:1 interpretation of $\ominus$, as claimed by Bent and Wegman.²²² The tenor commences in $\ominus$ while the cantus has Θ . FIGURE 1.25 shows that even the vertical relationship is *subsesquialtera* (2:3) proportion at the minim level. When $\ominus$ appears in the cantus three measures later, the mensuration in the tenor

²²⁰ Self-reflexive references between notated music and its text are a common feature of *Ars subtilior* compositions. On this aspect see Anne Stone, “Self-Reflexive Songs and Their Readers in the Late 14th Century,” *Early Music* 31, no. 2 (2003), 180–94, esp. 183.

²²¹ I have thereby followed Parrish’s interpretation of $\ominus$: “The entrance of the imitating voice is marked by the circle with a horizontal line through it [...]. This symbol also marks the point at which all the voices come to rest when the canon is brought to an end.” *The Notation of Medieval Music*, 191.

²²² See n. 212 and 213 above.

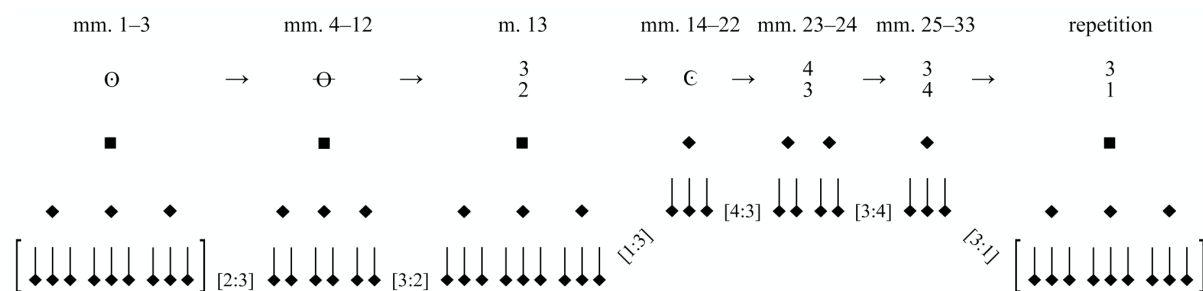
has not changed (see m. 4 in FIGURE 1.24), so no vertical relationship can be ascertained (except 1:1). It is true that a *subdupla* (1:2) relationship could be established for $\mathbb{C}$ in the cantus in mm. 14–22 when compared to Θ in the tenor, analogous to the situation in *Belle, bonne, sage* (see FIGURE 1.14 on p. 60 above). This long-winded approach would finally result in a 2:1 relationship between Θ and $\mathbb{C}$, if the *subdupla* (1:2) proportion of $\mathbb{C}$ is conceived reversed. However, to me at least, this interpretation seems to be far-fetched. Picking a reference point for the interpretation of Θ that lies halfway within the piece and in another voice part is a bit arbitrary.²²³

The *dupla* (2:1) proportion interpretation of Θ can probably be ascribed to findings in later sources, such as *Ox213* and *Bologna Q15*. However, it does not suit the situation in the two Codier rondeaux in *Ch*. I consider the interpretation according to this anachronistic approach false, since, just as in *Belle, bonne, sage*, the proportion signs present in *Tout par compas* clearly indicate a cumulative interpretation within the same voice part, as I will now demonstrate.

Tout par compas contains six different proportion signs, which appear as geometric shape ($\mathbb{C}$) and stacked Arabic numerals: $\frac{3}{2}$, $\frac{4}{3}$, $\frac{3}{4}$, $\frac{3}{1}$, and $\frac{2}{1}$. It is rather unusual that the tenor contains proportion signs ($\frac{2}{1}$ and $\mathbb{C}$). Since their interpretation is relatively straightforward, the following discussion will focus on the cantus. I have decided against interpreting Θ as proportion sign, since, as mentioned above, in the cantus this sign only involves a change in the duration of the minim in a section where minim level is irrelevant. It is primarily a position sign indicating the entry of the canonic voice and a mensuration sign indicating [3,2] (*tempus perfectum* with *prolatio minor*). In the tenor, Θ indicates the initial mensuration [3,2].

While *Belle, bonne, sage* contains two kinds of coloration (red and void notation), the rondeau *Tout par compas* only uses red notation as an indicator of *sesquialtera* (3:2) proportion at two different levels: breve level (mm. 6–7 of the cantus and mm. 16–17 of the tenor) and semibreve level (mm. 29–30 of the cantus). Since no proportion sign immediately follows sections with coloration in *Tout par compas*, the cumulative and the quasi-cumulative interpretation are congruent.

²²³ As I have demonstrated in FIGURES 1.18, 1.19, and 1.20 on pp. 64–65 above, almost any simple proportion can be ascribed to a sign if the reference point is changed. Thus, if one wants Θ to mean 2:1, there will be a reference point which supports this hypothesis.

FIGURE 1.26: Proportional relationships in the cantus of *Tout par compas*

As stated above, the stacked Arabic numerals $\frac{3}{4}$, which follow the short section of *sesquitertia* (4:3) proportion (mm. 23–24) signalled by $\frac{4}{3}$, are a clear indicator that the proportion signs in *Tout par compas* should be interpreted cumulatively.²²⁴ This sequence clarifies that *sesquitertia* (4:3) proportion is revoked by *subsesquitertia* (3:4) proportion. Thus, $\frac{3}{4}$ only reinstates the rhythmic structure of C.²²⁵ $\frac{3}{4}$ following $\frac{4}{3}$ is a rather remarkable feature of the piece, since proportions in *Ars subtilior* compositions are more commonly revoked by a re-statement of the sign that indicates the rhythmic structure prior to the proportion sign section in question, which would be C in this case.²²⁶ An additional function could thereby be ascribed to the proportion sign $\frac{3}{4}$: just as the stacked numbers $\frac{8}{9}$ in *Belle, bonne, sage* were hinting the fact that proportions had to be interpreted cumulatively in the rondeau,²²⁷ so the stacked numbers $\frac{3}{4}$ not only signal rhythmic proportion in *Tout par compas* but also provide instructions on the correct interpretation of proportion signs in the piece.

A cumulative interpretation of proportion signs in *Tout par compas* also results in a *subtripla* (1:3) proportion being ascribed to C, since one minim replaces three minims of the previous section (see FIGURE 1.26). This *subtripla* (1:3) proportion is finally revoked by *tripla* (3:1) proportion for the repetition of the rondeau, which is indicated by $\frac{3}{1}$.

It is remarkable that the two signs at the beginning of the cantus, Θ and $\frac{3}{1}$ (see FIGURE 1.27), have so often been interpreted as *c o m p o s i t e s i g n*.²²⁸ Bergsagel, for example, states: “The first [proportional] signature in the cantus, [Θ] above 3-over-1, is a sign of triple diminution.”²²⁹ However, I would argue that these two signs must be interpreted

²²⁴ Cf. Stoessel, “The Captive Scribe,” 1:300.

²²⁵ Cf. Plumley and Stone, “Cordier’s Picture Songs,” 311.

²²⁶ Note that the situation in Corrado da Pistoia’s *Se doulz espour ne me donne confort* (Mod A 61) is similar. 2 following 3 simply reinstalls the initial mensuration. See Chap. 2.5 for details.

²²⁷ See p. 68 above.

²²⁸ Cf. Reaney, *CMM* 11, *Early Fifteenth-Century Music I*, XI; Greene, *PMFC* 18, 146; Bergsagel, “Cordier’s circular canon,” 1175; Günther, “Der Gebrauch des tempus perfectum diminutum,” 281; Stoessel, “The Captive Scribe,” 1:313; and Plumley and Stone, “Cordier’s Picture Songs,” 310.

²²⁹ Bergsagel, “Cordier’s circular canon,” 1175.

as two separate signs, which—due to the arrangement of the rondeau on the circular stave—happen to be in the same place. In the beginning, Θ indicates the initial mensuration [3,3] and the proportion sign $\frac{3}{1}$ should be ignored. As demonstrated in FIGURE 1.26, the cantus then has several different proportional changes. Since $\frac{3}{4}$ revokes $\frac{4}{3}$ in m. 25, *subtrippla* (1:3) proportion (indicated by $\ominus$ in m. 14) is still operating when the cantus arrives at the place where the music is about to be repeated. For the repetition, i.e. reading the circle for the second time, this *subtrippla* (1:3) proportion is revoked by $\frac{3}{1}$, which indicates *trippla* (3:1) proportion. The initial mensuration (first indicated by Θ) is thus re-established.²³⁰



FIGURE 1.27: Beginning of cantus of *Tout par compas* (enhanced)

1.4.3.1 Proportion Signs Signalling Citation and Allusion

Before closing the discussion of the two rondeaux by Cordier, I would like to draw the reader's attention towards a musical allusion to the beginning of *Tout par compas* in the manuscript *Ox213*. The anonymous composition *Se fortune s'est tournée* (fol. 109r) exhibits a melodic line similar to the first notes of Cordier's rondeau in *Ch* in its final measures (see FIGURE 1.28).²³¹ There are notable differences: The melodic line of *Se fortune s'est tournée* starts on a' and not c''. Furthermore, it contains an extra note in the penultimate *mensura* (three semibreves: d' – c' – b; instead of breve and semibreve: e' – d'). Still, the resemblance is there. The most noteworthy part is that the beginning of the allusion is marked by Θ . The cut circle is thus again used as position sign, this time highlighting the beginning of the allusion to *Tout par compas*.

²³⁰ Of all publications I consulted for this study only Parrish offers the interpretation of Θ and $\frac{3}{1}$ being two separate signs. Cf. *The Notation of Medieval Music*, 190.

²³¹ Cf. Plumley and Stone, "Cordier's Picture Songs," 316.

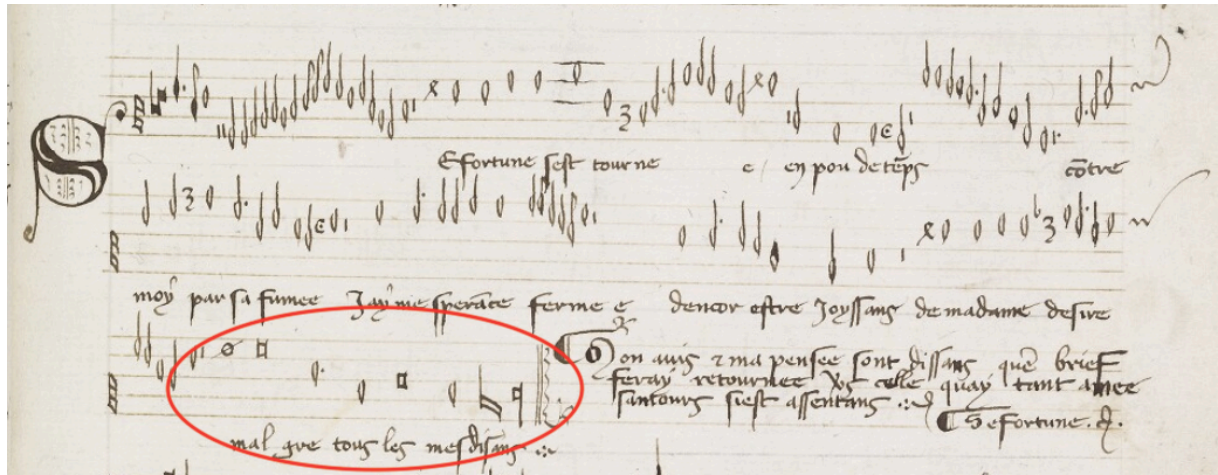


FIGURE 1.28: Cantus of *Se fortune s'est tournee* (allusion to *Tout par compas* encircled)

In the following chapters, I will present several other examples in which signs simultaneously function as proportion signs as well as as position signs marking the beginning of a citation or allusion. These can be musical as well as textual citations.²³² With many musical sources from the beginning of the fifteenth century assumed to be lost, it might be possible that signs in *Ars subtilior* music often signalled quotations and allusions, only we do not know the original piece in question.

1.4.4 Recapitulation

The two rondeaux *Belle, bonne, sage* and *Tout par compas* by Baude Cordier, which constitute the first two songs of *Ch*, contain eight different proportion signs. In a same voice quasi-cumulative proportions interpretation at the minim level, they can be said to represent eight different proportions, as demonstrated in TABLE 1.29.

	1:2	2:1	1:3	3:1	3:2	3:4	4:3	8:9
<i>Belle, bonne, sage</i> (Ch 1)	C			3				8 9
<i>Tout par compas</i> (Ch 2)		2 1	C	3 1	3 2	3 4	4 3	

TABLE 1.29: Interpretation of proportion signs in *Belle, bonne, sage* and *Tout par compas* (same voice quasi-cumulative proportions interpretation at the minim level)

²³² For more on the subject of citation and allusion in *Ars subtilior* music see Yolanda Plumley, "Citation and Allusion in the Late *Ars Nova*: The Case of *Esperance* and the *En Attendant* Songs," *Early Music History* 18 (1999): 287–363. Plumley has already remarked that citations are often visually highlighted: "Where the citation is pre-echoed most clearly in *En attendant*, *Esperance* [...] void red notation is used to eye-catching effect on the page. The use of coloration for visual effect can also be found in two other works from *Ch*." *Ibid*, 317.

The comparison shows that the originator—either Cordier himself or the scribe—used two different signs for *tripla* (3:1) proportion, namely the single Arabic numeral 3 in *Belle, bonne, sage* and the stacked Arabic numerals $\frac{3}{1}$ in *Tout par compas*. Furthermore, the originator used © in two different capacities: In *Belle, bonne, sage*, © indicates *subdupla* (1:2) proportion at the minim level, while the same sign indicates *subtripla* (1:3) proportion at the minim level in *Tout par compas*. Moreover, *Belle, bonne, sage* contains all three forms of visual appearance of proportion signs (geometric shape, Arabic numeral and stacked Arabic numerals), while *Tout par compas* only exhibits two (geometric shape and stacked Arabic numerals). It is noteworthy that *Tout par compas* contains twice as many proportion signs as *Belle, bonne, sage*, namely six compared to three, however, the latter composition makes more use of coloration (red and void notation).

In this comparison of the two rondeaux, something else becomes apparent: We do not find consistency in the meaning of proportion signs or in the visual representation of the same proportion. Even in these two closely related rondeaux—same composer, same scribe, same codicological unit—there are two different ways in which *tripla* (3:1) proportion is indicated. Furthermore, © has two different meanings as a proportion sign. *Belle, bonne, sage* and *Tout par compas* thus already demonstrate the highly individual approach to the representation of rhythmic proportion in *Ars subtilior* music, which can also be observed in the other manuscripts analysed for this study. It is something more, which—as antonym to standardisation—goes beyond the individual: every composition seems to require its own specific interpretation of the signs it contains. Once having abandoned the idea that signs have to bear the same meaning throughout a specific manuscript or a specific period of time, in the works of a particular composer or in all sources copied by the same hand, a novel perspective on pieces even as famous as the two Cordier rondeaux in *Ch* can lead to new conclusions.

These new conclusions in the two Cordier rondeaux especially concern the interpretation of ©. As I have argued, the prevailing opinion that © indicates *dupla* (2:1) proportion (or *tempus perfectum diminutum*²³³) in *Belle, bonne, sage* and *Tout par compas* should be revised. This reading could be ascribed to scholars' operating under the assumption that signs were used in a standardised manner. Interpreting © as sign of *dupla* (2:1) proportion because it bears that meaning in some works in *Ox213*, *Bologna Q15* and other later sources, or because

²³³ For more information on the term see n. 197 above. I have not only refrained from using the term *tempus perfectum diminutum* in this study, because it is not contemporary, but also because I want to distinguish between the two functions of signs. The '*tempus perfectum*' part of the term refers to the indicated mensuration, the '*diminutum*' to the proportion indicated. Instead of *tempus perfectum diminutum*, I would say that a multi-function sign indicates a particular mensuration and a particular proportion. To © this does not apply, because it does not have a proportional meaning in *Belle, bonne, sage* and *Tout par compas*.

theorists from the late fifteenth century ascribe this meaning to the sign is a prejudice of hindsight. However, the *dupla* (2:1) proportion interpretation of Θ can hardly be established by a comparison between simultaneously sounding voice parts in the two rondeaux, contrary to other scholars' statements.²³⁴ Furthermore, the proportion signs found in both pieces clearly indicate a same voice quasi-cumulative proportions interpretation at the minim level. If one insisted on the interpretation of Θ as proportion sign—which I do not—the ratio could not be *dupla* (2:1) proportion in either composition, as I have shown above.

Instead, I have offered the following interpretation: Θ is used as a mensuration sign for *tempus perfectum* with *prolatio minor* [3,2] in both Cordier compositions in *Ch. In Belle, bonne, sage*, Θ indicates the return to the initial mensuration. The same mensuration is also indicated by Θ in *Tout par compas*. However, Θ seems to have been used primarily as a position sign in these two rondeaux, signalling the beginning of the B section in *Belle, bonne, sage* and the start of the canonic voice in *Tout par compas*.

By rejecting Margaret Bent's term "general-purpose sign"²³⁵, Rob C. Wegman may have been too quick in discarding the idea of multiple functions of Θ . His standpoint becomes even more evident in another passage of his reply to Bent:

[T]he claim that the 2:1 relationship between $\emptyset$ and Θ occurs 'in no early manuscript except the two Cordier songs,' and 'was only weakly established before 1430' ([Bent, "The Early Use of the Sign $\emptyset$,"] p. 219), appears odd to say the least. One cannot help but ask what other use of Θ could have been 'strongly' established by comparison."

Here, I would argue that Θ as sign of position could be considered established. This function cannot only be attributed to Θ in the two rondeaux by Cordier in *Ch*, but also to Θ in the anonymous composition *Se fortune s'est tournée* (fol. 109r) in *Ox213*, where the sign indicates the beginning of a musical allusion to Cordier's *Tout par compas*. In her article, Bent has listed even more examples of different meanings of the cut circle before 1430, which could all be summarised under the general category of position sign: In the *Fountains* fragments, Θ functions as repetition sign (*signe de renvoi*);²³⁶ the same function can be attributed to Θ in a Cambrai Antiphoner from the thirteenth century;²³⁷ in *Bodley 652*, Θ signals that

²³⁴ Cf. Bent, "The Early Use of the Sign $\emptyset$," 199; and Wegman, "Different Strokes for Different Folks?" 462. Bent has even stated: "For the Cordier songs, and for all cases of simultaneous use of $\emptyset$ and Θ , a 2:1 relationship is inescapable" ("The Early Use of the Sign $\emptyset$," 199), which is odd, since Θ does not appear in either *Belle, bonne, sage* or *Tout par compas*.

²³⁵ Bent, "The Early Use of the Sign $\emptyset$," 219. For Wegman's criticism see pp. 72–73 above.

²³⁶ Θ can be found on a strip (used as binding material for another manuscript) now catalogued as *London, British Library, Add. Ms. 40011B*, 4v. Cf. Bent, "The Early Use of the Sign $\emptyset$," 204–5. Admittedly, the high-resolution images on DIAMM leave doubts on the presence of the stroke, or at least on the assumption that the stroke was intended by the scribe. I think it might be possible that the ink has run in that spot, since half of the circle is blurred.

²³⁷ Cf. Barbara Haggh's finding as quoted in Bent, "The Early Use of the Sign $\emptyset$," 224 n. 12.

musical material copied on the right-hand margin of the page should be inserted where Θ is positioned;²³⁸ in a Gloria by Guillaume Legrant from *Bologna Q15*, Θ functions as “scoring co-ordinator”²³⁹ signalling where the first cantus is joined by contratenor and tenor.

To sum up, prior to 1430, Θ seems to have been used as a sign to mark where a) music is repeated (*Fountains* fragments and Cambrai Antiphoner), or b) a different section begins (*Belle, bonne, sage*), or c) one or more voice parts join the cantus (Legrant’s Gloria in *Bologna Q15* and *Tout par compas*) or d) music, that was left out during the copying process, has to be inserted (*Bodley 652*). All these different functions can be summarised under the general category of position sign, but it also becomes apparent that no universal interpretation can be established for Θ . Furthermore, Θ also functions as a mensuration sign indicating *tempus perfectum* with *prolatio minor* [3,2] in *Belle, bonne, sage*, *Tout par compas*, and some of the other examples. In those cases, Θ is used as a multi-function sign.

Bent’s insistence on interpreting Θ as a sign of *dupla* (2:1) proportion in the two Cordier rondeaux is remarkable considering that her entire article focuses on the argument that Θ seldomly bears that meaning prior to 1430. Nevertheless, in light of my interpretation of Θ , Bent’s suggestion of revisiting the practice of reducing note values in editions of pieces, in which all voices simultaneously change to Θ , seems to me well founded.²⁴⁰ The interpretation of Θ as a position sign in *Belle, bonne, sage*, where the sign marks the beginning of the B section, and *Tout par compas*, where the sign marks the beginning of the canonic voice entry, strengthens Bent’s hypothesis that the cut circle was sometimes used as scoring-co-ordinator before 1430 and had no proportional meaning.

Another remark on editions of *Ars subtilior* music seems to be in order at this point. In the majority of editions from the series *Polyphonic Music of the Fourteenth Century* (PMFC) and *Corpus mensurabilis musicae* (CMM), it is standard practice to omit the proportion signs in the edition.²⁴¹ I have often wondered about this inconsistent approach: While coloration is

²³⁸ Cf. Bent, “The Early Use of the Sign Θ ,” 205. Since the folio in question was used as binding material, the two cut circles are now positioned on two different strips. There are discrepancies between the folio numbers given in Bent’s article, where two black and white facsimiles are shown (“ff. i-ii”) and DIAMM’s foliation of these two folios. The point of insertion in the music (Bent’s fol. i) can be found on DIAMM’s fol. 69r, where Θ is faintly visible in the middle of the second staff. The material that should be inserted (Bent’s fol. ii) can be found on the picture DIAMM has named “Folio back board offset”. Even more cut circles appear in the fragments, namely on fols. i and ii according to DIAMM’s foliation, but these are not the folios displayed in Bent’s article. Due to the fragmented state of the manuscript it is probably impossible to tell what the cut circles mean in those instances.

²³⁹ Bent, “The Early Use of the Sign Θ ,” 205.

²⁴⁰ Cf. Bent, “The Early Use of the Sign Θ ,” 223.

²⁴¹ A change can be observed in more recent publications of editions of *Ars subtilior* music, e.g. Jason Stoessel’s collection of editions published as “Appendix A” to his PhD dissertation “The Captive Scribe” and Jos Haring and Kees Boeke’s complete edition of the Modena codex (*The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio*, Olive

indicated by some form of brackets in most editions, the position of proportion and mensuration signs is not marked and their presence has to be deduced from the critical commentary. Of course, in most cases it is possible to discern that something is going on from the unexpected rhythms appearing as duplets, triplets or quadruplets in modern editions, but considering other functions, which can be attributed to the signs in the music, I would argue that important information is lost in the edition when the position of proportion signs is not indicated.

In *Belle, bonne, sage*, these additional functions include the indication of the beginning of a melisma paired with a reminder for the singer to sing fast enough by the use of small note values in the melisma. The visual appearance of some proportion signs as stacked Arabic numerals in *Belle, bonne, sage* and *Tout par compas* also hint to the interpreter that the proportion signs have to be interpreted cumulatively. One could argue that melismata are indicated by other means in modern editions and that the interpreter does not need to interpret proportion signs non-cumulatively, cumulatively, or quasi-cumulatively, because the editor has already made the decision. However, at least in the case of the anonymous composition *Se fortune s'est tournée* in *Ox213*, where Θ indicates the beginning of the allusion to *Tout par compas* (see FIGURE 1.28 on p. 84 above), the musical allusion is further obscured when Θ is omitted from the edition.

The additional functions of the signs (including Θ) in *Belle, bonne, sage* and *Tout par compas* discussed above also call for a re-evaluation of Plumley and Stone's assessment, in which they have described the notation as "surface complexity" and concluded that there is "a fair amount of redundancy of result between the various signs."²⁴² As I have shown in FIGURE 1.13 (p. 59 above), *Belle, bonne, sage* could have been notated without any signs and only with the aid of coloration. The same is true for the rondeau *Tout par compas*. However, this simpler version would not supply the information on structure and interpretation given by the signs, which is why I would argue that they are not redundant. In a reading, in which these signs only function as mensuration and/or proportion signs, they might rightly be called thus, but the multi-function-sign perspective can—at least partially—explain their presence.

Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019)). Both publications display proportion and mensuration signs in the modern editions of *Ars subtilior* compositions.

²⁴² Plumley and Stone, "Cordier's Picture-Songs," 311.

1.4.5 Baude Cordier Reconsidered Once Again²⁴³

I would like to close the discussion of the two famous rondeaux by returning to the debate concerning Baude Cordier's identity. As described above, some stylistic aspects of the compositions have been called irreconcilable with a date of death in 1397 or 1398, reported as the year Baude Fresnel died.²⁴⁴ As stylistic aspects such as counterpoint and imitation are not the focus of this study, I will not comment on these aspects here.²⁴⁵ However, many arguments adduced against Craig Wright's hypothesis that Baude Cordier is Baude Fresnel, harper of Philipp the Bold, focussed on the signs and issues related to rhythmic proportion: 1) the use of cut signatures; 2) composite signs; 3) cumulative proportions.²⁴⁶

As reported by Margaret Bent, the cut circle sign $\ominus$ is already used in a Cambrai Antiphoner from the thirteenth century.²⁴⁷ Thus cut signatures should not be adduced as strong argument for dating a composition to the fifteenth century or a later period, especially when—as in the *Ch* rondeaux by Cordier—they are not used as proportion but as position signs.

Furthermore, I have explained why I do not think that the $\ominus$ over $\frac{3}{1}$ (see FIGURE 1.27 on p. 83 above) at the beginning of *Tout par compas* should be interpreted as composite sign. I have argued that these two signs can be interpreted as two separate signs, which—due to the arrangement of the rondeau on the circular stave—happen to be in the same place.

The two compositions by Cordier also served as good examples for demonstrating that different approaches to the interpretation of proportion signs result in different proportions being ascribed to the signs, i.e. non-cumulative, quasi-cumulative or cumulative interpretation yields different results. As I have demonstrated in FIGURES 1.18–1.20 and TABLE 1.21 on pp. 64–66 above, the reference point for the application of the proportion is crucial and should therefore be stated when proportion signs are interpreted. Definite statements concerning the question of cumulative proportions in a composition can only be made in some cases, e.g. if a piece contains stacked Arabic numerals, or canon instructions explaining the proportion signs, or sufficient clues in order to rule out other interpretations. I have explained that I do not see sufficient proof for a linear development from non-cumulative to cumulative proportions, as

²⁴³ In his subchapter “Baude Cordier reconsidered” (chap. 6.3 in “The Captive Scribe,” 1:306–15), Jason Stoessel has already re-evaluated the arguments adduced in favour of and against Craig Wright's hypothesis concerning Baude Cordier's identity in some detail. However, I hope to be able to contribute some new arguments, which offer a new perspective on some of the aspects discussed by Stoessel.

²⁴⁴ Cf. Wright, “Tapissier and Cordier,” 189.

²⁴⁵ However, as Plumley and Stone have put it: “[W]e have very little data between those *termini* [1398 and 1420] with which to make informed style-based assessments; we find ourselves here at the juncture between what we might call ‘late medieval’ and ‘early Renaissance’ song.” “Cordier's Picture-Songs,” 309.

²⁴⁶ See n. 177 above.

²⁴⁷ Cf. Barbara Haggh as quoted in Bent, “The Early Use of the Sign $\emptyset$,” 224 n. 12.

suggested by Stoessel.²⁴⁸ This is why I advise against using cumulative proportions as argument for stylistic assessments.

Does that mean that there is now good reason to believe that Baude Cordier is Baude Fresnel who died in 1397 or 1398? While the analyses of *Belle, bonne, sage* and *Tout par compas* presented here weaken the arguments of the critics who have adduced signs in *Ch*, it is still true that some notational aspects of Cordier's compositions in *Ox213* seem to contradict Craig Wright's hypothesis. For example, $\ominus$ does indicate *dupla* (2:1) proportion in the *Ox213* compositions *Dame excellent ou sont bonté scavoir* and *Amans amés secretement*. Furthermore, Baude Cordier's compositions in *Ox213* also contain composite signs. In the rondeaux *Pour le desfault du noble dieu bachus*, O2 indicates *tempus perfectum* with *prolatio minor* [3,2] combined with *dupla* (2:1) proportion at the minim level and in *Amans amés secretement*, O3 indicates *tempus perfectum* with *prolatio minor* [3,2] combined with *tripla* (3:1) proportion at the minim level.²⁴⁹

Here we touch upon an important issue concerning signs in compositions, which will reverberate throughout this study: Who is the originator of the notation we find in *Ars subtilior* compositions? In scholarly publications, the notation of the composition is often attributed to the composer. In Baude Cordier's case, this opinion is echoed in many of the publications cited in this discussion. For example, Anna Maria Busse Berger has wondered about notational inconsistencies in Cordier's works and has stated: "What is striking is that Cordier's interpretation of O2 and Ø are [sic] identical [...]. One wonders why he needed both. But the important fact remains that he invented signs to indicate diminution, which represented an important step in the development of notation."²⁵⁰ Leaving aside that it seems a little audacious to claim as "fact" that Cordier himself "invented [these] signs", I am surprised that Busse Berger did not consider scribal or other influence as explanation for this inconsistency. Even in "The Captive Scribe", in which Jason Stoessel has sought traces of scribal influence in *Ars subtilior* manuscripts, he has declared: "[I]t can be stated that the composer's [= Cordier's] mind was firmly entrenched in the arithmetic mentality and that *he used* [my italicisation] Indo-Arabic numerals as an unambiguous expressions [sic] of his intent."²⁵¹

It seems to be a natural conclusion that the composer is the originator of the notation we find in *Ars subtilior* compositions. This conjecture often goes unchallenged where composi-

²⁴⁸ Cf. Stoessel, "The Captive Scribe," 1:298.

²⁴⁹ Cf. Busse Berger, *Mensuration and Proportion Signs*, 154. Busse Berger calls these composite signs "*modus cum tempore* signs" in her monograph (see esp. chap. 5 "Diminution by Stroke and by Mode Signs", section V "*Modus cum tempore* Signs", 148–59).

²⁵⁰ Busse Berger, *Mensuration and Proportion Signs*, 155.

²⁵¹ Stoessel, "The Captive Scribe," 1:312.

tions only survived in a single copy, because there are no concordant sources to which the notation can be compared. The majority of Cordier's compositions are *unica*. However, there is precedent for challenging this view in the form of compositions that appear in several sources, in which the notation is considerably different in each version. A famous example, which will be discussed in Chapter 2.4 below, is Johannes Ciconia's *Sus un fontayne*, which is transmitted in two sources with differing notation.

An example from the Chantilly codex—the cantus of the ballade *De narcissus, home tres ourgilleus* by Magister Franciscus, which can be found in no less than eight sources²⁵², shall illustrate these possible contrasting notations. A change from *tempus perfectum* to *tempus imperfectum* is indicated in three different ways: standard mensuration signs in *Pit*, *Autun 152*, *Trem*, and *Ch* (see FIGURE 1.30); red coloration in *PR* (see FIGURE 1.31); and void notation in the Bergamo fragment (see FIGURE 1.32).

On an interesting side note: the mensuration signs are absent from the version in the San Lorenzo codex. However, one can also rule out the possibility that the change in *tempus* is indicated by coloration, since red ink would reflect differently on the multispectral images of the palimpsest.²⁵³ There is also no indication of void notation. Thus, one has to assume that the performers were able to gain knowledge of the change in *tempus* from intrinsic signs, e.g. groupings of notes.

Admittedly, the notation of the ballade *De narcissus, home tres ourgilleus* is simpler than the notation in the two Cordier rondeaux in *Ch*. The complexity of the notation is also one of the arguments adduced by Jason Stoessel, which leads him to the conclusion that re-notation is unlikely in the case of Cordier's compositions:

“While re-notation at a minor level is a feature of some works in the *ars subtilior* repertoire, there is no surviving evidence to suggest that complex notation, as in the works of composers such as Senleches and Suzoy, was rewritten using Indo-Arabic proportions and cut sign notation. [...] Furthermore, the similarity of notational devices in his [= Cordier's] Chantilly inserts and works in Ob 213 [= *Ox213*] tend to indicate that the notation found in these sources reflects Cordier's idiom.”²⁵⁴

²⁵² The Chantilly codex (fol. 19v), the manuscript *Pit* (fols. 33v–34r), the codex Reina (fol. 81r), the manuscript *Autun 152* (fol. 160v, cantus only), the manuscript *Trem* (fols. 21v–22r), the fragment *Budapest 298* (*verso*, beginning of cantus only), the San Lorenzo palimpsest (fol. 90v), and the recently discovered fragment *Bergamo 589* (*recto*, third part of cantus and tenor only). I thank Andreas Janke for bringing this last new source to my attention.

²⁵³ Compare, for example, red notes showing up blue in the multispectral image of the unique ballade *Adieu plaisir* (fol. 66v) and the notation of the ballade *De narcissus, home tres ourgilleus* (fol. 90v), which is full black throughout all voices, in Andreas Janke and John Nádas, eds., *The San Lorenzo Palimpsest Florence, Archivio del Capitolo di San Lorenzo Ms. 2211: Introductory Study and Multispectral Images*, Ars Nova: Nuova serie 4 (Lucca: LIM, 2016), 2: fols. 66v and 90v.

²⁵⁴ Stoessel, “The Captive Scribe,” 1:314.

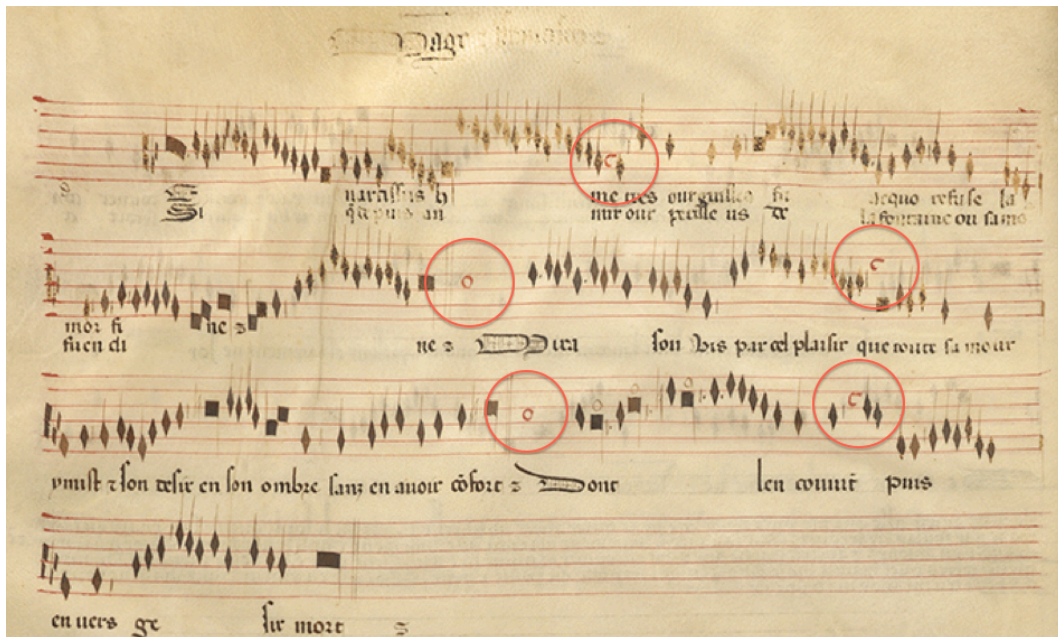


FIGURE 1.30: Cantus of *De narcissus* in Ch, fol. 19v (mensuration signs encircled)

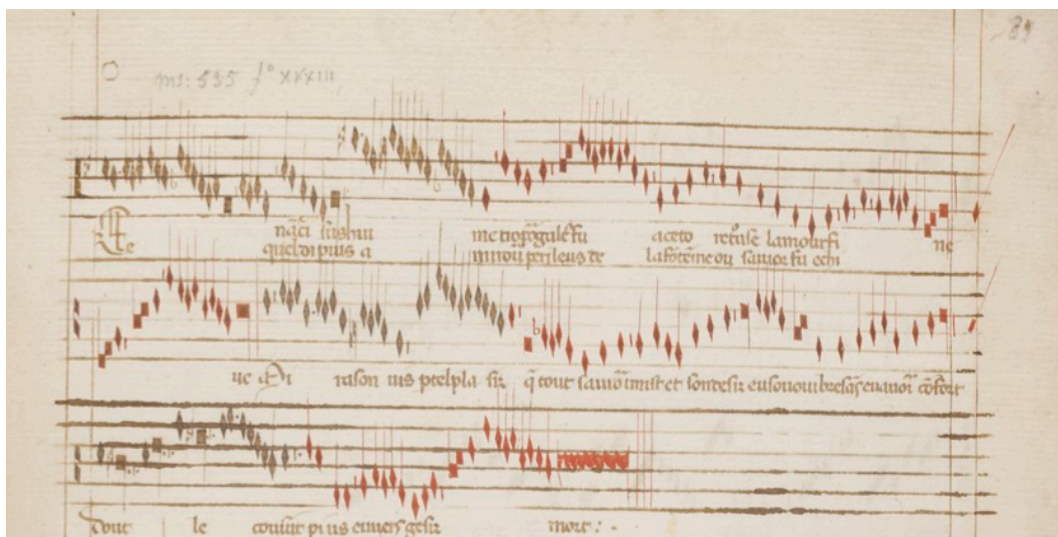


FIGURE 1.31: Cantus of *De narcissus* in PR, fol. 81r

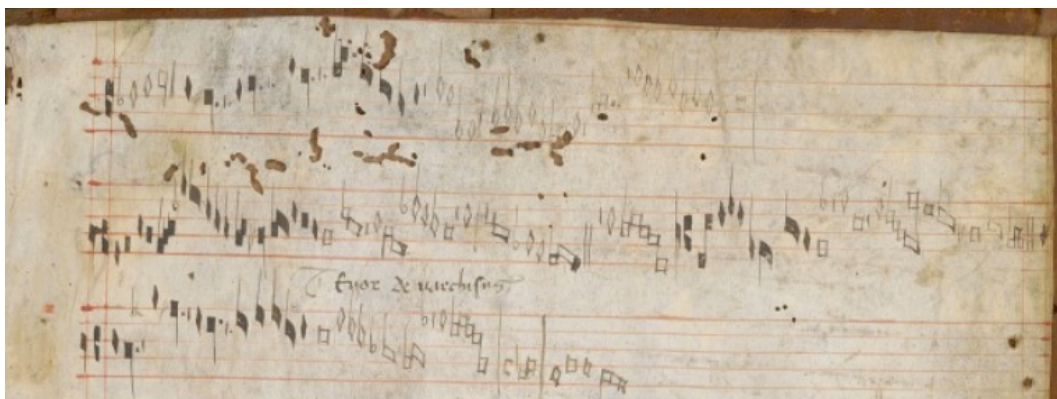


FIGURE 1.32: Cantus and tenor of *De narcissus* (third part only) in Bergamo 589, recto

However, there is another example of re-notation from the San Lorenzo codex, which shows that there are different versions of compositions with complex notation. Paolo da Firenze's madrigal *Era Venus al termin del suo giorno* contains a multi-function sign in *Pit* (see FIGURE 1.33).



FIGURE 1.33: Cantus of *Era Venus* in *Pit*, fol. 54r (proportion sign encircled and enhanced)

The combination of a reversed semicircle (C) and the Arabic numeral 3 indicates a change from *tempus perfectum* to *tempus imperfectum*, but also a perfect *modus* (hence the Arabic numeral 3) in diminution, i.e. *dupla* (2:1) proportion at the minim level (see FIGURE 1.34). In Italian notation, the voices would change from *senaria perfecta* (.p.) to *duodenaria* (.d.). The combination of symbol and Arabic numeral seems to be an attempt to express Italian Trecento notation in the French style.

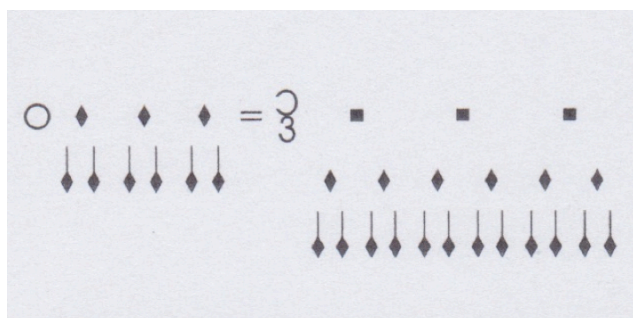


FIGURE 1.34: Interpretation of $\frac{C}{3}$ in *Era Venus*²⁵⁵

²⁵⁵ Illustration taken from Tiziana Sucato, "Alcune annotazioni sui madrigali di Paolo da Firenze e in particolare sull'uso dei segni C e C," in *Le notazioni della polifonia vocale dei secoli IX–XVII*, ed. Maria Caraci Vela, Daniele Sabaino, and Stefano Aresi (Pisa: Edizioni ETS, 2007), 259.

Again, this mensural change is not indicated by a sign or by coloration in the San Lorenzo codex (see FIGURE 1.35). This example shows that there are compositions with complex rhythmic notation—such as Paolo’s madrigal *Era Venus* containing the composite sign $\frac{3}{3}$ in *Pit*—of which differently notated versions seem to have circulated.

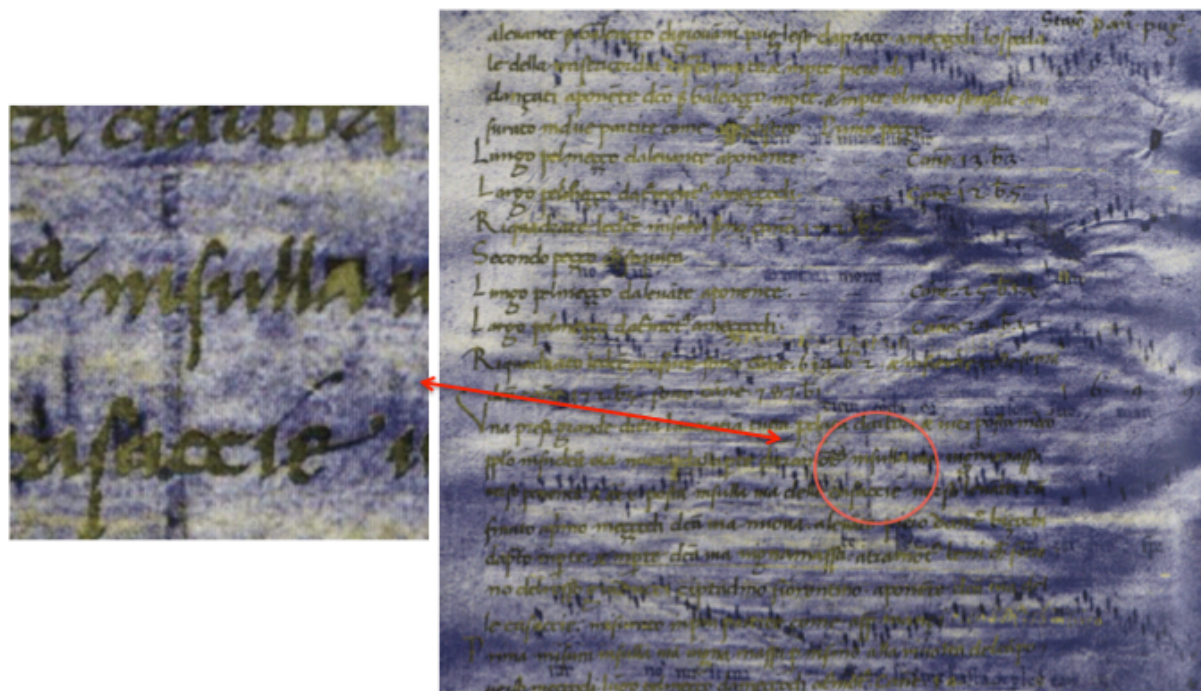


FIGURE 1.35: Cantus of *Era Venus* in *SL*, fol. 50v (position of missing proportion sign encircled and enhanced)²⁵⁶

At this point I would like to raise the question in how far it is advisable at all to adduce the notation of rhythmic proportion (proportion signs and coloration) as argument for dating compositions. I will explore this aspect in the next chapter by analysing the proportion signs used in the oeuvre of a particular composer within a particular manuscript. Are they used in a consistent manner? Do we find differently notated versions in concordant sources? Can we find traces of scribes’ attempts to standardise the notation within a particular manuscript?

In his re-evaluation of Craig Wright’s hypothesis concerning Baude Cordier’s identity, Jason Stoessel has followed other scholars in primarily adducing notational aspects in support of counterarguments.²⁵⁷ He has come to the conclusion that Cordier must still have been active after the end of the fourteenth century, since his works contain cut signs and stacked Arabic numerals, which have to be interpreted cumulatively.²⁵⁸ My concerns about the argument

²⁵⁶ Multispectral images taken from Andreas Janke, and John Nádas, eds., *The San Lorenzo Palimpsest Florence, Archivio del Capitolo di San Lorenzo Ms. 2211: Introductory Study and Multispectral Images*, Ars Nova: Nuova serie 4, 2 vols. (Lucca: LIM Editrice, 2016), 2: No. 142.

²⁵⁷ Cf. Stoessel, “Baude Cordier reconsidered,” chap. 6.3 in “The Captive Scribe,” 1:306–15. “[T]he most telling indications of Cordier’s chronology lies in his notation.” *Idem*, 313.

²⁵⁸ Cf. Stoessel, “The Captive Scribe,” 1:313.

of cumulative proportions being a late development left aside, I have shown in the previous paragraphs that differently notated versions of *Ars subtilior* compositions—even pieces containing complex notation—have survived and leave room for doubt concerning the assumption that the composer is also the originator of the notation.

Jason Stoessel has even remarked upon the similarity of the signs found in the gatherings of *Ox213*, which contain works by Cordier: “In general, many unusual mensuration signs found in especially the older repertoire of the seventh and eighth gatherings of *Ob 213* rely on the same processes [my italicisation] used to alter mensural signs in [Cordier’s] *Amans, ames secretement*.”²⁵⁹ In other words, the notation found in those gatherings is unusual and strikingly similar. A closer analysis of Cordier’s *Ox213* works in comparison with other repertoire from these gatherings might shed light on the issue, whether we can suspect some scribal influence in the notation of the compositions. Plumley and Stone’s 2009 article “Cordier’s Picture-Songs” certainly provides a starting point for such an endeavour.

I agree with Stoessel in that “the evidence is at best circumstantial for Wright’s duly cautious conjecture.”²⁶⁰ However, for reasons given throughout this subchapter, I would also caution against basing all counterarguments on notational aspects. Certainly, I see no ground for Karl Kügle’s statement concerning the two Chantilly rondeaux: “[T]here is now solid evidence that two of the most iconic “ars subtilior” songs, Cordier’s *Belle bonne sage* and *Tout par compas*, were composed much later than previously assumed, namely during the 1420s.”²⁶¹ Unfortunately, the note accompanying that assertion only reads: “This suspicion was first voiced by Bent, ‘Early Use,’ 223n2.” So regrettably, the reader does not learn about the “solid evidence” for that claim. If Kügle relied on Bent’s statements from “The Early Use of the Sign Ø”, it should be noted that Bent has recanted her previous claim in a recent publication:

“I would now row back from the claim [Plumley and Stone] cite on p. 309 where I proposed a date in the late 1410s or early 20s for the Cordier songs, and would now settle with them for a dating of Cordier’s songs and others in *Ox* with cut signs for diminution in the second decade of the fifteenth century, that is, nearly coeval with the main compilation of *Ch*.”²⁶²

Finally, I would like to comment on another argument that Baude Cordier is probably not identical with Baude Fresnel given by Jason Stoessel:

²⁵⁹ Stoessel, “The Captive Scribe,” 1:309.

²⁶⁰ Stoessel, “The Captive Scribe,” 1:306. Previous scholars’ criticism of Wright is summarised above (see pp. 56–57).

²⁶¹ Karl Kügle, “Glorious Sounds for a Holy Warrior: New Light on Codex Turin J.II.9,” *Journal of the American Musicological Society* 65, no. 3 (2012): 672.

²⁶² Bent, “The Absent First Gathering,” 31 n.32.

“[T]here is the absence of Cordier’s secular works in the older layer (Layer I) of Codex Chantilly and any other compilations of French music before the third decade of the fifteenth century. While chance or scribal taste obviously mitigate this observation to a lower status, it is strange that a composer who is accorded so much space in Ob 213 [= *Ox213*] should not have at least one work transmitted in the sources of the *ars subtilior* repertoire proper if he was active before their compilation.”²⁶³

As I will show in the next chapter, however, a similar situation applies to Matteo da Perugia, who is represented by 37 compositions in *Mod A* but at the same time is not represented in any other *Ars subtilior* collection even though he clearly composed songs before their compilation. Hence Stoessel’s argument is considerably weakened by this and other cases of composers who are only represented in one manuscript.

In conclusion to this extensive study of the two Cordier pieces, I would hence like to state that—although I go against the prevailing scholarly opinion—I see no compelling reason why Baude Cordier could not be Baude Fresnel.

²⁶³ Stoessel, “The Captive Scribe,” 1:315.

2

PROPORTION SIGNS IN THE MANUSCRIPT *MOD A*

The manuscript *Modena, Biblioteca Estense, a.M.5.24* (hereafter *Mod A*) is one of the principle sources of *Ars subtilior* music and contains 104 compositions of sacred and secular polyphony, 66 of which are unique to the manuscript.¹ Amongst these unique compositions are numerous works by Matteo da Perugia and the majority of surviving French settings of Antonella da Caserta. 21 of the compositions of *Mod A* contain proportion signs—16 of the *unica*² and five songs with concordances³ in other *Ars subtilior* manuscripts or fragments.

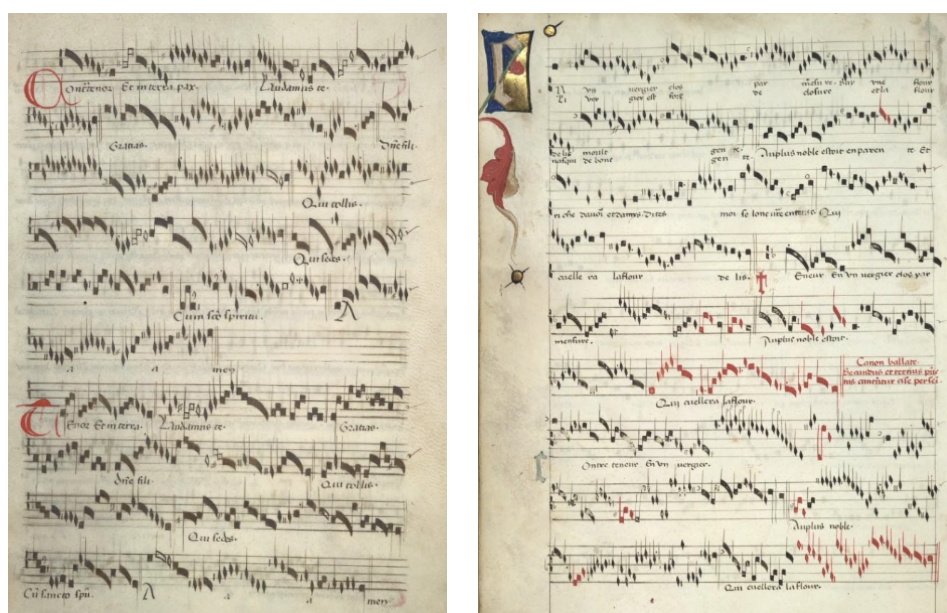


FIGURE 2.1: Fol. 2r of *Mod A_{I/IV}* (left) and fol. 18v of *Mod A_{II-IV}* (right)

It is common understanding that the five gatherings of *Mod A* consist of two distinct primary layers (see FIGURE 2.1 for two example folios from these different layers), an earlier one comprising gatherings 2 to 4 (fols. 11r–40v, hereafter *Mod A_{II-IV}*), and a later one (hereafter *Mod A_{I/IV}*) consisting of gatherings 1 (fols. 1r–10v) and 5 (fols. 41r–50v) as well as two front

¹ A complete inventory of all compositions as well as a comprehensive study of the manuscript's contents and history can be found in Anne Stone, *The Manuscript Modena, Biblioteca Estense, a.M.5.24: Commentary*, Ars Nova: Nuova serie 1 (Lucca: LIM Editrice, 2005).

² Eight compositions (sacred and secular) by Matteo da Perugia, two anonymous Credos likely by him, three songs by Antonella da Caserta, a ballade by Corrado da Pistoia, a virelai by Bartolomeo da Bologna, and an anonymous ballade (see respective chapters below for details).

³ A rondeau by Matteo da Perugia, two ballades by Filippotto da Caserta, a virelai by Johannes Ciconia, and an anonymously transmitted ballade, which is attributed to Hasprois in *Ch* (see respective chapters below for details).

and back flyleaves (fols. av and zr).⁴ Both layers seem to represent unrelated individual projects that were brought together at a certain point.⁵ They are therefore two different codicological units.⁶ I will nevertheless regard *Mod A* as one manuscript in this chapter, because my focus lies on the notation of proportion signs within the *oeuvre* of one respective composer. Nevertheless, I have indicated the gathering of each proportion sign song in the tables in this chapter so that it is discernible which proportion signs are used in what layer.

In her 2005 study supplementing the colour facsimile of *Mod A*, Anne Stone has already recapitulated that

“the manuscript lacks any obvious signs of ownership or patronage [...] and no archival evidence documenting its commission has yet surfaced” hence “any conclusion about its precise origins must remain speculative, based largely upon inference from what we know of composers’ biographies, clues in song texts, and circulation of the repertory.”⁷

Evaluating scholars’ findings and hypotheses throughout the twentieth century, many have come to the conclusion that *Mod A* seems to have originated in circles surrounding Pietro Filargo da Candia—who in 1409 was elected pope Alexander V of Pisa—, his successor John XXIII (antipope from 1410 to 1415), and in the vicinity of the Visconti court in Pavia.⁸ Pointing towards the “problematic nature of music manuscript studies that rely on repertory for determining a source’s origin”⁹, Jason Stoessel has instead tried to identify the illuminator of the gatherings *Mod AII* and *Mod AIII*. There is strong evidence that the illuminations were crafted by the ‘Master of 1411’, an anonymous illuminator who was active in Bologna from 1404 to 1411.¹⁰ Stoessel has further suggested that the ‘Master of 1411’ could be Giacomo da Padova, who is documented at the Olivetan abbey of San Michele in Bosco near Bologna

⁴ Cf. Stone, *The Manuscript Modena*, 23.

⁵ Cf. Jason Stoessel, “The Angevin Struggle for the Kingdom of Naples (c. 1378–1411) and The Politics of Repertoire in *Mod A*: New Hypotheses,” *Journal of Music Research Online* 5 (2014), 2.

⁶ A codicological unit is defined as “a volume, part of a volume or group of volumes, whose production can be considered as a unitary process, realized in the same conditions of place, time and technique” (“Volume, partie de volume ou ensemble de volumes dont l’exécution peut être considérée comme une opération unique, réalisée dans les mêmes conditions de lieu, de temps et de technique.” Denis Muzerelle, *Vocabulaire codicologique: répertoire méthodique des termes français relatifs aux manuscrits* (Paris: Editions CEMI, 1985); accessible online: <http://codicologia.irht.cnrs.fr/accueil/vocabulaire> [last accessed 20 January 2023]; translation taken from the *Glossary of Manuscript Cultures* of the Centre for the Study of Manuscript Cultures at the University of Hamburg. Also see Peter Gumbert, “Codicological Units: Towards a Terminology for the Stratigraphy of the Non-Homogeneous Codex”, in: *Il codice miscellaneo. Tipologie e funzioni. Atti del convegno internazionale (Cassino, 14–17 maggio 2003)*, Segno e testo 2, ed. Edoardo Crisci and Oronzo Pecere (Cassino: Università degli Studi di Cassino, 2004), 33.

⁷ Stone, *The Manuscript Modena*, 62.

⁸ A summary of these hypotheses can be found in Stone, *The Manuscript Modena*, 62–66.

⁹ Jason Stoessel, “Arms, A Saint and *Inperial sedendo fra più stelle*: The Illuminator of *Mod A*,” *The Journal of Musicology* 31, no. 1 (2014), 3.

¹⁰ Cf. Stoessel, “The Illuminator of *Mod A*,” 4–15.

from 1407 to 1409 and still active in Bologna in 1413.¹¹ He therefore concludes that “Mod A may have been illuminated when the papal entourage of John XXIII visited San Michele in Bosco in the fall of 1410, although further compositions were added after the illuminator had finished his work.”¹² In a further article, Stoessel has also hinted at possible connections between the repertoire of *Mod A_{II-IV}* and the house of Valois-Anjou, especially surrounding political events concerning Louis II of Anjou and his presence in Pisa and Bologna from 1409 to 1411.¹³

While an increasing number of links thus place the earlier layer *Mod A_{II-IV}* in Bologna around 1410, the dating of *Mod A_{I/V}* has proven to be more difficult. Little is known about the movements of Matteo da Perugia—the primary composer of these two gatherings—between 1407 and 1414 as well as after 1416 (for more see Chapter 2.2 below). A scribal connection between *Mod A_{I/V}* and the Parma fragments¹⁴ in combination with the absence of two late works by Johannes Ciconia in the Lucca codex, one of which is contained in *Parma*, might suggest a *terminus post quem* for *Mod A_{I/V}* of between 1409 and 1412.¹⁵ By connecting Matteo da Perugia’s ballade *Pres du soloil* with Filippo Maria Visconti and an emblem, which the latter adopted at a certain point in the 1420s, especially, Anne Stone has recently proposed a date of compilation as late the mid 1420s.¹⁶ When and why the later layer, which almost exclusively transmits works by Matteo da Perugia, was joined with the earlier one remains speculative.

Our lack of knowledge concerning dates of compilation and places of origin of both *Mod A* layers notwithstanding, it can be concluded from *Mod A_{II-IV}*’s repertoire that the compiler had “Francophile interests”¹⁷ as well as a preference for *Ars subtilior* music. Many of the 69 compositions of these three gatherings exhibit extraordinarily complex notation, not necessarily only those containing proportion signs. Furthermore, Italian repertoire is clearly underrepresented. Only one ballata by Francesco Landini was copied into the anthology and none of Johannes Ciconia’s or Antonello da Caserta’s Italian songs were included.

Stone has identified one scribe for the music and the text of *Mod A_{II-IV}*, who also seems to have been musically literate.¹⁸ He edited the notation in several compositions by erasing notes he had already copied and changing them in their visual appearance—probably with the aim

¹¹ Cf. Stoessel, “The Illuminator of Mod A,” 15–23.

¹² Stoessel, “The Illuminator of Mod A,” 42.

¹³ Cf. Stoessel, “The Angevin Struggle,” 5–20.

¹⁴ *Parma, Archivio di Stato, Raccolta Manoscritti, busta 75, n. 26 ex convento LXXXV (S. Servino di Piacenza) reg. n. 52* (henceforth *Parma*).

¹⁵ Cf. Stone, *The Manuscript Modena*, 108.

¹⁶ Cf. Stone, *The Manuscript Modena*, 102–109; for more see Chap. 2.2 below.

¹⁷ Stone, *The Manuscript Modena*, 61.

¹⁸ Cf. Stone, *The Manuscript Modena*, 42–43.

to reduce ambiguities.¹⁹ It might therefore not be a coincidence that the proportion signs in *Mod A_{II-IV}*, as will be shown, seem to be rather standardised, comprising—apart from two compositions by Antonello da Caserta, which feature stacked Arabic numerals—only $\bigcirc$ and the single Arabic numerals 2, 3, and 4.

Although the scribe of *Mod A_{II-IV}* seems to have had great influence on the notation in this part of the manuscript, it is noteworthy that there are still ambiguities. For example, the proportion sign 2 indicates *dupla* (2:1) proportion in three compositions but *subsesquialtera* (2:3) proportion in *Se doulz espour ne me donne confort*. I have therefore decided to regard the songs of different composers separately in this chapter in order to see whether signs are used consistently in the *Mod A* compositions by one composer but not in those by another.

There is, however, another important aspect to the songs of *Mod A*, namely citation and allusion. Much attention has been drawn to the fact that the three ‘*En attendant* songs’ of *Mod A*²⁰ seem to be part of some kind of citation game.²¹ Each song contains musical as well as textual references to other songs. The most obvious musical and textual citations are to be found in Johannes Ciconia’s virelai *Sus un’ fontayne*, which cites three songs by Filippotto da Caserta. This made me wonder whether there are other examples of musical citation and allusion in the *Mod A* repertoire and I particularly kept an eye out for these in the proportion sign passages, as these are sections of the song, which are visually highlighted by the sign. I have yielded some promising results in that search and can only encourage other scholars to look for similar examples in other manuscripts.

After a general overview of proportion signs and their visual appearance and interpretation in Chapter 2.1, I will discuss each of the 21 proportion sign pieces in subchapters, which are divided according to composer. Matteo da Perugia’s compositions naturally deserve special attention since all proportion signs in his *oeuvre* are uniquely transmitted in *Mod A* works. Following this, I will discuss Antonello da Caserta’s virelai *Tres nouble dame souverayne* as well as his ballades *Amour m’a le cuer mis en tel martire* and *Dame d’onour, en qui tout mon cuer maynt*, all unique to *Mod A* and also the only three pieces within his *oeuvre* which contain proportion signs. Filippotto da Caserta’s ballades *En attendant, souffrir*

¹⁹ For more on this see “Chapter Three: The Scribe as Editor” in Anne Stone, “Writing Rhythm in Late Medieval Italy: Notation and Musical Style in the Manuscript Modena, Biblioteca Estense, Alpha.M.5.24,” PhD diss., Harvard University, 1994, 107–165. I discuss the changes in *Le greygnour bien* in Chap. 2.2 below, but do not think that these changes were necessarily made by the main scribe.

²⁰ *En attendant, Esperance conforte* by Jacob Senleches, *En attendant d’avoir la douce vie* by Galiot and *En attendant, souffrir m’estuet grief payne* by Filippotto da Caserta.

²¹ For more on this see Reinhard Strohm, “Filippotto da Caserta, ovvero i francesi in Lombardia,” in *In cantu et in sermone: For Nino Pirrotta on his 80th Birthday*, ed. Fabrizio Della Seta and Franco Piperno (Florence: Olschki, 1989), 69–70; *id.*, *The Rise of European Music 1380–1500* (Cambridge: Cambridge University Press, 1993), 59–60; and Yolanda Plumley, “Citation and Allusion in the Late *Ars nova*: The Case of *Esperance* and the *En attendant* songs,” *Early Music History* 18 (1999), 287–363.

m'estuet grief payne and *En remirant vo douche portraiture* will be reviewed alongside Johannes Ciconia's virelai *Sus un' fontayne*. Before coming to a conclusion, I will briefly look at Corrado da Pistoia's *Se Doulz Espour ne me donne confort*, Bartolomeo da Bologna's *Que pena maior agitanda menti* and the two anonymously transmitted ballades *Ma douce amour, je me doi bien complayndre* and *En un vergier clos par mesure*.

2.1 Overview of Proportion Signs in *Mod A*

The manuscript exhibits a great variety of complex notational devices. Rhythmical proportions are created by special note shapes (e.g. semiminims and *dragmae*), coloration (including black and red void notation), simultaneous use of different mensurations, diminution by canonic instruction, and proportion signs.²² The latter appear in all three forms of visual appearance: geometric shapes, single Arabic numerals and stacked Arabic numerals. *Mod A* features eight different proportion signs which result in five different rhythmic proportions: *sesquitertia* (18 compositions; represented by the three different signs $\complement$, $\frac{2}{2}$, and $\frac{8}{6}$), *sesquialtera* (four compositions; represented by the two different signs 3 and $\frac{9}{6}$), *dupla* (four compositions; represented by the two different signs 2 and $\frac{4}{2}$), *tripla* (one composition; represented by 4), and *subsesquialtera* (one composition, represented by 2).

TABLE 2.2²³ gives an overview of the proportion signs in *Mod A*. It shows the different interpretations of each proportion sign grouped by their visual appearance (first geometric shapes, then single Arabic numerals, followed by stacked Arabic numerals).

Sign	Related to ²⁴	Interpretation ²⁵	Incipit	Composer	No.	fol(s). gathering	Comment(s)
$\complement$	$\complement$	4:3	<i>Et in terra</i> ST	Matteo da Perugia	2	1 ^v –2 ^r <i>Mod A</i> ₁	Proportion sign only in contratenor.
$\complement$	$\complement$	4:3	<i>Patrem omnipotentem</i>	Matteo da Perugia?	7	5 ^v –6 ^v <i>Mod A</i> ₁	
$\complement$	$\complement$	4:3	<i>Patrem omnipotentem</i>	Matteo da Perugia?	10	7 ^v –9 ^r <i>Mod A</i> ₁	

TABLE 2.2: Proportion signs in *Mod A*²⁶ (note that the table continues on the following pages)

* = Piece has concordance(s) in other source(s).

ST = Proportion sign mentioned by Stone, “Writing Rhythm” (see n. 23).

BB = Proportion sign mentioned by Busse Berger, *Mensuration and Proportion Signs* (see n. 23).

²² A comprehensive study of the notational devices used in *Mod A* can be found in Anne Stone, “Writing Rhythm,” 72–106.

²³ This table aims to complement the tables that can be found in Stone, “Writing Rhythm,” 75–76 (“Table I-a: Pieces using $\complement$ in *Mod A* and *Ch*”); 85–86 (“Table II: Notational devices in pieces by Anthonello da Caserta”); 88–89 (“Table III-a: Numbers without canon”); and 90–91 (“Table III-b: Numbers with canon”); as well as Anna Maria Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 171 (“Table 7: Sesquialtera on the semibreve level”); 172 (“Table 8: Sesquialtera on the minim level”); 174 (“Table 9: Sesquitertia on the semibreve and minim level”), which—concerning *Mod A*—are incomplete. Pieces that are mentioned by the authors are marked with a small ST (for Stone, “Writing Rhythm”) and BB (for Busse Berger, *Mensuration and Proportion Signs*) respectively in the incipit column.

²⁴ “Related to” generally meaning mensuration preceding the proportion sign.

²⁵ Proportions at the minim level, unless otherwise stated.

²⁶ Incipits, composers, and numberings are taken from Stone, *The Manuscript Modena*. “Incipits”, 151–54; “Index of Composers”, 133–6; “Inventory of *Mod A*”, 112–31.

TABLE 2.2 continued

Sign	Related to	Interpretation	Incipit	Composer	No.	fol(s). gathering	Comment(s)
○	☉	4:3	<i>En un vergier clos par mesure</i> ST, BB	Anonymous	31	18 ^v <i>Mod A_{II}</i>	Piece comes with a canon, which does, however, not give advice on the interpretation of ○.
○	☉	4:3	<i>*En attendant, souffrir m'estuet grief payne</i> ST, BB	Filippotto da Caserta (attributed to Galiot in <i>Ch</i>)	35	20 ^r <i>Mod A_{II}</i>	<i>Mod A</i> version is the only one with proportion sign. <i>Dragmae</i> indicate <i>sesquiertia</i> (4:3) proportion in <i>Ch</i> , <i>PR</i> and <i>GR197/Dartmouth</i> . Possible allusion to Machaut in proportion sign passage.
○	☉ in cantus and tenor; ☉ and ○ in contratenor	4:3	<i>*Sus un' fontayne</i> ST, BB	Johannes Ciconia	49	27 ^r (26 ^v) <i>Mod A_{III}</i>	Virelai cites three compositions by Filippotto da Caserta.
○	○	4:3	<i>Tres nouble dame souverayne</i> ST, BB	Antonello da Caserta	53	28 ^v <i>Mod A_{III}</i>	Piece comes with a canon. Proportion sign is missing in the contratenor.
○	☉	4:3	<i>Le greygnour bien que nature</i>	Matteo da Perugia	62	32 ^r (31 ^v) <i>Mod A_{IV}</i>	Proportion signs only in contratenor. The proportion sign sections exhibited special note shapes in the original version, which was then edited by a scribe.
○	○ in cantus and tenor; ○ and ☉ in contratenor	4:3	<i>*En remirant vo douche portraiture</i> ST, BB	Filippotto da Caserta	68	34 ^v – 35 ^r <i>Mod A_{IV}</i>	Only <i>Mod A</i> composition in which ○ is combined with ○. Red void notation indicates <i>sesquiertia</i> (4:3) proportion in <i>Ch</i> and <i>PR</i> .
○	☉	4:3	<i>Dame souvrayne de beauté, d'onour</i> ST, BB	Matteo da Perugia	75	38 ^r <i>Mod A_{IV}</i>	Proportion sign only in contratenor.

TABLE 2.2 continued

Sign	Related to	Interpretation	Incipit	Composer	No.	fol(s). gathering	Comment(s)
○	O	4:3	<i>A qui Fortune ne se vult amer</i> BB	Matteo da Perugia	88	43 ^r <i>Mod Av</i>	Proportion is applied cumulatively in addition to <i>dupla</i> (2:1) proportion, which is applied in the cantus from the beginning. This results in a 4:3 (<i>sesquitertia</i>) relation within the cantus and a 8:3 (<i>dupla superbipartiens tertias</i>) relation between the cantus and the other two voices.
○	Ⓒ	4:3	<i>Se pour loyalement servir on puist merir</i> ST, BB	Matteo da Perugia	89	43 ^v <i>Mod Av</i>	
○	Ⓒ	4:3	<i>*Pour Bel Acueil suy je, las, deceü</i>	Matteo da Perugia	91	44 ^v <i>Mod Av</i>	○ only in <i>Mod A</i> , because concordance (<i>Bern</i>) only transmits tenor and contratenor.
○	Ⓒ	4:3	<i>Helas, Avril, par ton doulz revenir</i>	Matteo da Perugia	93	45 ^r <i>Mod Av</i>	
○	O	4:3	<i>Già da rete d'amor libera et sciolta</i> ST	Matteo da Perugia	96	46 ^v <i>Mod Av</i>	
○	Ⓒ	4:3	<i>Et in terra</i>	Matteo da Perugia	102	49 ^v – 50 ^r <i>Mod Av</i>	
2	O	2:1	<i>*Ma douce amour, je me doi bien complayndre</i> ST	Anonymous (attributed to Hasprois in <i>Ch</i> and <i>Turin J.b.IX.10</i>)	52	28 ^r <i>Mod Am</i>	Piece comes with a canon. Proportion is not applied cumulatively but refers to the initial mensuration, although 2 (<i>dupla</i> (2:1) proportion) passage succeeds 3 (<i>sesquialtera</i> (3:2) proportion) passage.
2	C with tripla (3:1) proportion	2:3	<i>Se doulz espour ne me donne confort</i> ST	Corrado da Pistoia	61	31 ^v <i>Mod Am</i>	Proportion is applied cumulatively, restoring the initial mensuration.
2	O	2:1	<i>Que pena maior agitanda menti</i> ST	Bartolomeo da Bologna	73	37 ^r (36 ^v) <i>Mod Am</i>	Piece comes with a canon.
2	tenor and contratenor in Ⓒ	2:1	<i>A qui Fortune ne se vult amer</i>	Matteo da Perugia	88	43 ^r <i>Mod Av</i>	The whole cantus uses <i>dupla</i> (2:1) proportion compared to tenor and contratenor.

TABLE 2.2 continued

Sign	Related to	Interpretation	Incipit	Composer	No.	fol(s). gathering	Comment(s)
3	O	3:2	<i>*Ma douce amour, je me doi bien com-playndre</i> ST	Anonymous (attributed to Hasprois in <i>Ch</i> and <i>Turin J.b.IX.10</i>)	52	28 ^r <i>Mod A_{III}</i>	Piece comes with a canon.
3	C	3:2	<i>Se Doulz Espour ne me donne confort</i> ST	Corrado da Pistoia	61	31 ^v <i>Mod A_{IV}</i>	
3	O	3:2	<i>Que pena maior agitanda menti</i> ST	Bartolomeo da Bologna	73	37 ^r (36 ^v) <i>Mod A_{IV}</i>	Piece comes with a canon. Proportion is not applied cumulatively but refers to the initial mensuration, although 3 (<i>sesquialtera</i> (3:2) proportion) passages sometimes succeed 2 (<i>dupla</i> (2:1) proportion) passages.
4	O	3:1	<i>*Ma douce amour, je me doi bien com-playndre</i> ST	Anonymous (attributed to Hasprois in <i>Ch</i> and <i>Turin J.b.IX.10</i>)	52	28 ^r <i>Mod A_{III}</i>	Piece comes with a canon.
$\frac{2}{2}$	$\frac{3}{2}$ (= C) in cantus; $\frac{3}{2}$ (= C) and $\frac{2}{3}$ (= O) in contratenor	4:3	<i>Dame d'onour, en qui tout mon cuer maynt</i> ST	Antonello da Caserta	82	40 ^v <i>Mod A_{IV}</i>	Piece comes with a canon, which does, however, not give advice on the interpretation of the alternative mensuration and proportion signs.
$\frac{4}{2}$	O	4:2 (= 2:1)	<i>Amour m'a le cuer mis en tel martire</i> ST	Antonello da Caserta	63	32 ^v – 33 ^r <i>Mod A_{IV}</i>	Proportion is not applied cumulatively but refers to the initial mensuration, although $\frac{4}{2}$ (<i>dupla</i> (2:1) proportion) passage succeeds $\frac{9}{6}$ (<i>sesquialtera</i> (3:2) proportion) passage.
$\frac{8}{6}$	O	8:6 (= 4:3)	<i>Amour m'a le cuer mis en tel martire</i> ST	Antonello da Caserta	63	32 ^v – 33 ^r <i>Mod A_{IV}</i>	
$\frac{9}{6}$	O	9:6 (= 3:2)	<i>Amour m'a le cuer mis en tel martire</i> ST	Antonello da Caserta	63	32 ^v – 33 ^r <i>Mod A_{IV}</i>	

The table reveals that the interpretation of the proportion signs contained in *Mod A* is pretty straightforward in the majority of cases. As proportion sign, $\circ$ always indicates *sesquitertia* (4:3) proportion at the minim level. $\circ$ is also used in Matteo da Perugia's ballata *Serà quel zorno may, dolze madonna mia* where it functions as a mensuration sign.²⁷ In Antonello da Caserta's virelai *Tres nouble dame souverayne* and Corrado da Pistoia's ballade *Se Doulz Espour ne me donne confort* it could be argued that the semibreve might be the point of reference for the application of the proportion. In only two pieces—both by Antonello da Caserta—is *sesquitertia* (4:3) proportion indicated in an alternative way. In the ballade *Dame d'onour, en qui tout mon cuer maynt*, which also uses alternative mensuration signs ($\frac{3}{2} = \text{C}$ and $\frac{2}{3} = \text{O}$), *sesquitertia* (4:3) proportion is indicated by the stacked Arabic numerals $\frac{2}{2}$ and in his ballade *Amour m'a le cuer mis en tel martire* the proportion is expressed by the stacked Arabic numerals $\frac{8}{6}$.²⁸

Proportion signs in the form of single Arabic numerals appear in four pieces in *Mod A*. The already mentioned ballade *Se Doulz Espour* by Corrado da Pistoia uses 3 to indicate *sesquialtera* (3:2) proportion, which is followed by 2 indicating *subsesquialtera* (2:3) proportion, thereby restoring the initial mensuration. The cantus of Matteo da Perugia's *A qui Fortune ne se vult amer* has 2 in the beginning, indicating that *dupla* (2:1) proportion is applied in the cantus in relation to tenor and contratenor, i.e. the cantus moves twice as fast as the other two voices.²⁹ Bartolomeo da Bologna's virelai *Que pena maior agitanda menti* and the anonymous ballade *Ma douce amour, je me doi bien complayndre* provide a canon to explain the Arabic numerals contained in the piece. Although the interpretation of 3 as *sesquialtera* (3:2) proportion and 2 as *dupla* (2:1) proportion might be considered straightforward, the use of 4 to indicate *tripla* (3:1) proportion calls for an explanation.

It should be mentioned that *Mod A* also contains four compositions in which rhythmic proportions are achieved by the simultaneous use of different mensurations without minim equivalence. TABLE 2.3 provides an overview. The following chapters are intended to discuss the compositions and some *Mod A* composers in more detail.

²⁷ For more see discussion in Chap. 2.2 below.

²⁸ Antonello da Caserta is the only composer in *Mod A* who uses stacked Arabic numerals to indicate proportions. Busse Berger names *Amour m'a le cuer* as earliest piece in which fractions are used in this context. Cf. Busse Berger, *Mensuration and Proportion Signs*, 164. For more see Chap. 2.3 below.

²⁹ This is the only piece in which Matteo da Perugia uses a proportion sign other than $\circ$. For more see Chap. 2.2 below.

Sign	Related to	Interpretation	Incipit	Composer	No.	fol(s). gathering	Comment(s)
Θ	C	9:4	<i>Une dame requis l'autrier d'amer</i>	Johannes de Janua	17	12 ^r <i>Mod A_{II}</i>	Θ in cantus against C in tenor and contratenor with breve equivalence resulting in <i>dupla sesquiquarta</i> (9:4) proportion.
O	C	3:2	<i>Plus onques dame n'ameray</i>	Matteo da Perugia?	11	9 ^r (8 ^v) <i>Mod A_I</i>	O in cantus against C in tenor and contratenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion.
O	C	3:2	<i>Dame que j'aym sour toutes de ma enfance</i>	Matteo da Perugia	14	10 ^v – 11 ^r <i>Mod A_I</i> (<i>Mod A_{II}</i>)	O in cantus against C in tenor and contratenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion.
O	C	3:2	<i>Une dame requis l'autrier d'amer</i>	Johannes de Janua	17	12 ^r <i>Mod A_{II}</i>	O in cantus against C in tenor and contratenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion.
O	C	3:2	<i>Le greygnour bien que nature</i>	Matteo da Perugia	62	32 ^r (31 ^v) <i>Mod A_{III}</i>	O in contratenor against C in cantus and tenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion
Є	C	3:2	<i>Plus onques dame n'ameray</i>	Matteo da Perugia?	11	9 ^r (8 ^v) <i>Mod A_I</i>	Є in cantus against C in tenor and contratenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion
Є	C	3:2	<i>Une dame requis l'autrier d'amer</i>	Johannes de Janua	17	12 ^r <i>Mod A_{II}</i>	Є in cantus against C in tenor and contratenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion
Є	C	3:2	<i>Le greygnour bien que nature</i>	Matteo da Perugia	62	32 ^r (31 ^v) <i>Mod A_{III}</i>	Є in contratenor against C in cantus and tenor with breve equivalence resulting in <i>sesquialtera</i> (3:2) proportion

TABLE 2.3: Proportions achieved by simultaneous use of different mensurations in *Mod A*

2.2 Proportion Signs in the Compositions of Matteo da Perugia

With 30 attributed compositions and seven works likely by him, Matteo da Perugia could constitute as much as one third of the whole *Mod A* collection and is thus the best represented composer of the entire manuscript.³⁰ His compositions are found in all five gatherings, furthermore, the later layer *Mod A_{IV}* is almost exclusively devoted to works by or assumed to be by him. In her study on *Mod A*, Anne Stone has called the “unique survival of a single composer’s musical corpus of this dimension [...] one of the most intriguing aspects of the manuscript”.³¹ Apart from *Mod A*, Matteo’s works only survive in three fragments which for the most part transmit additional contratenors for the songs of other composers.³²

From 1402 to 1407 Matteo da Perugia was the first *magister capellae* at the cathedral of Milan which by then was still under construction. He was associated with Pietro Filargo da Candia who in 1409 was elected Pope Alexander V of Pisa. However, it is uncertain whether Matteo followed him to Pisa. Matteo’s name reappears in the salary records of the Milanese cathedral in 1414 with continuing payments until 1416.³³ It has previously been assumed that Matteo died in 1418.³⁴ However, it has more recently been suggested by Stone that the text of his ballade *Pres du soloil* might refer to the emblem of Filippo Maria Visconti, Duke of Milan from 1412 to 1447, which Filippo Maria adopted in the mid 1420s—a fact that would provide a considerably later *terminus post quem* for Matteo’s death.³⁵ It seems at least plausible that Matteo resigned his post at the cathedral of Milan and passed into Visconti patronage.³⁶

³⁰ Cf. Ursula Günther and Anne Stone, “Matteo da Perugia [Matheus de Perusio, de Perusiis, Perusinis],” in *NG2* (London: MacMillan, 2001), 16:136. Anne Stone’s ‘Index of Composers’ (cf. Stone, *The Manuscript Modena*, 134–35) names only 41 works instead of 42, because the additional contratenor for Grenon’s *Je ne requier de ma dame et m’amie* is missing from that list.

³¹ Stone, *The Manuscript Modena*, 26.

³² Bern, *Burgerbibliothek, Sammlung Bongarsina, Fragment 827* (Bern 827), New York, private collection of Professor Stanley Boorman of New York University, parchment bifolium (Boorman), and Parma. Reinhard Strohm has called these “complicated contratenors” Matteo’s “speciality”. Strohm, *The Rise of European Music*, 61.

³³ Cf. Günther and Stone, “Matteo da Perugia,” 136.

³⁴ Some scholars have relied on Ambrogio Nava’s statement in his history of the Milanese cathedral (*Memorie e documenti storici intorno all’origine, alle vicende, ed ai riti del Duomo di Milano* (Milan: Borroni e Scotti, 1854), 190). However, the mentioning of Matteo in the cathedral’s documents in January 1418 is only connected with a complaint by his successor Ambrogio da Pessano who intended to raise his salary, as has been stated repeatedly in recent studies.

³⁵ Cf. Stone, *The Manuscript Modena*, 102–8. Stone further concludes that this would even leave open the possibility of a personal encounter between Matteo and Pierre Fontaine as well as Beltram Ferragut, which could have resulted in Matteo’s composing contratenors for works by these two composers. Cf. Stone, *The Manuscript Modena*, 107. Jason Stoessel has shown some reserve concerning Stone’s hypothesis: “Falcons in Trecento poetry are not uncommon, especially in erotic poetry in which the falcon’s snatching away or piercing prey with its talons formed part of the discourse belonging to the hyper-masculinised allegory of the ‘love hunt’ that features in the courtly lyric tradition.” Stoessel, “The Angevin Struggle,” 3.

³⁶ Cf. Stone, *The Manuscript Modena*, 106.

In his 1950 edition of many *Mod A* works, Willi Apel has suggested that Matteo's French secular songs were composed before 1402 because their composition would not seem likely during his employment at the Milanese cathedral.³⁷ This hypothesis has been challenged by Heinrich Besseler as early as 1955 and more recently by Anne Stone who, in her 1994 dissertation, proposed that Matteo might even have been the scribe of *Mod A_{IV}*.³⁸ Although Stone has not repeated this view in her 2005 study on *Mod A*, she argues convincingly that the scribe of *Mod A_{IV}* must have had "good access to the works of Matteo da Perugia and exhibited a marked concern over their correct written representation."³⁹ It seems possible that Matteo composed (and perhaps edited) his works after 1402—and in case of the above-mentioned ballade *Pres du soloil* perhaps as late as the mid 1420s. Nevertheless, Apel's observation that the majority of Matteo's works exhibit a more traditional style compared to the more complex *Ars subtilior* works found in *Mod A_{II-IV}* is partly supported by Matteo's use of proportion signs as will be shown below.⁴⁰

As can be seen in TABLE 2.4, eleven of Matteo da Perugia's 37 compositions in *Mod A* contain proportion signs.⁴¹ He is also the only composer of *Mod A* who uses proportion signs in mass sections: one can find $\bigcirc$ in two of his Glorias and two Credos from the later layer *Mod A_{IV}*.⁴² With the exception of the rondeau *A qui Fortune ne se vult amer*, in which *dupla* (2:1) proportion is applied to the whole cantus, Matteo's works only contain $\bigcirc$ as proportion sign, always indicating *sesquitertia* (4:3) at the minim level. In the majority of cases, $\bigcirc$ is only applied once in the piece and only to a short section of the part, usually of a duration between one and three breves in the initial mensuration. Thus, the proportional section can be grasped easily and does not call for an explanatory canon.

³⁷ Cf. Willi Apel, ed., *French Secular Music of the Late Fourteenth Century* (Cambridge, MA: Medieval Academy of America, 1950), 24.

³⁸ Cf. Heinrich Besseler, "Hat Matheus de Perusio Epoche gemacht?," *Die Musikforschung* 8, no. 1 (1955), 19–23; and Stone, "Writing Rhythm," 26–35.

³⁹ Stone, *The Manuscript Modena*, 55.

⁴⁰ Cf. Apel, *French Secular Music*, 13. Apel introduces the term 'modern style' which is "characterized by an abandoning of the intricacies and complexities of the Manneristic [= *Ars subtilior*] period, and by the discovery of new musical values, such as simplicity of design and naturalness of expression. [...] The tendency of the Modern Style toward greater simplicity results in a certain similarity to the Machaut style." *French Secular Music*, 13.

⁴¹ The number 37 excludes the five single parts and is only valid if the unattributed works assumed to be by Matteo da Perugia are included. Accordingly, the Credos No. 7 and 10 will be treated as his compositions in the following discussion.

⁴² It has already been remarked upon that the canti of some of Matteo's mass sections bear stylistic resemblance to his secular songs. Cf. Uri Smilansky, "Rethinking *Ars Subtilior*: Context, Language, Study and Performance," PhD diss., University of Exeter, 2010, 129.

Incipit	Genre	No.	fol(s). gather- ing	2:1	4:3	Comment(s)
<i>Et in terra</i>	Gloria	2	1v–2r <i>Mod A_I</i>		○	Proportion sign only in contratenor.
<i>Et in terra</i>	Gloria	102	49v– 50r <i>Mod A_V</i>		○	
<i>Patrem omnipotentem</i>	Credo	7	5v–6v <i>Mod A_I</i>		○	uncertain attribution
<i>Patrem omnipotentem</i>	Credo	10	7v–9r <i>Mod A_I</i>		○	uncertain attribution
<i>A qui Fortune ne se vult amer</i>	Rondeau	88	43r <i>Mod A_V</i>	2	○	Proportion is applied cumulatively in addition to <i>dupla</i> (2:1) proportion, which is applied in the cantus from the beginning. This results in a 4:3 (<i>sesquitertia</i>) relation within the cantus and a 8:3 (<i>dupla superbipartiens tertias</i>) relation between the cantus and the other two voices.
<i>Se pour loyalement servir on puist merir</i>	Rondeau	89	43v <i>Mod A_V</i>		○	
<i>Pour Bel Accueil suy je, las, deceü</i>	Rondeau	91	44v <i>Mod A_V</i>		○	○ only in <i>Mod A</i> , because concordance (<i>Bern</i>) only transmits tenor and contratenor.
<i>Dame souvrayne de beauté, d'onour</i>	Virelai	75	38r <i>Mod A_{IV}</i>		○	Proportion sign only in contratenor.
<i>Helas, Avril, par ton doulz revenir</i>	Virelai	93	45r <i>Mod A_V</i>		○	
<i>Le greygnour bien que nature</i>	Ballade	62	32r (31v) <i>Mod A_{IV}</i>		○	Proportion signs only in contratenor. The proportion sign sections exhibited special note shapes in the original version and were edited by a scribe.
<i>Già da rete d'amor libera et sciolta</i>	Ballata	96	46v <i>Mod A_V</i>		○	

TABLE 2.4: Proportion signs in compositions by Matteo da Perugia

In three works, ○ is used in combination with semiminims in the cantus in order to create very small note values. Compared with 12 semiminims in C, 16 semiminims in ○ fit into the duration of one breve in C or O—as demonstrated in FIGURE 2.5—and the combination causes even faster ornamental melismata.

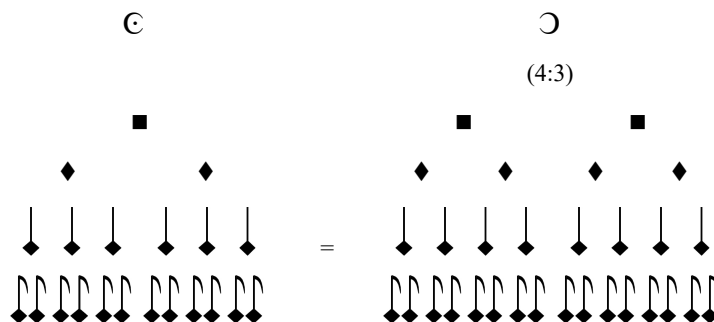


FIGURE 2.5: Distribution of semiminims under ○

This happens in the cantus in one of Matteo's seven Glorias (No. 102, fols. 49v–50r) over the word “Dei” of the phrase “Domine Deus, Agnus Dei, Filius Patris” (see FIGURE 2.6).⁴³ The only succession (more than two notes) of even smaller note values present in the Gloria are red void semiminims which are found in the final melisma over the word “Amen”.⁴⁴ Nevertheless, the “Dei”-setting is certainly the most striking passage in the Gloria from a rhythmical point of view, not only due to the fast melisma but also because of the complex notation following it, including a half black / half red *dragma* and two red void semiminims which are open on the right side. The word is thus clearly highlighted.



FIGURE 2.6: Transcription of $\circ$ passage in Gloria (No. 102)

The cantus of the second Credo of *Mod A* (No. 10, fols. 7v–9r), too, combines $\circ$ with semiminims (altogether eight), here in the final “Amen” melisma (see FIGURE 2.7).⁴⁵ Von Fischer's and Gallo's edition of the passage in $\circ$ is debatable, because they transcribe the semiminims as minims, thereby prolonging the passage to last a breve in C instead of a semibreve. It is therefore half as fast (see upper staff in FIGURE 2.7).⁴⁶ A scribal error has to be assumed because otherwise the cantus is a semibreve too long. I find Jos Haring's and Kees Boeke's edition (FIGURE 2.7) more convincing, however, because it avoids the rather unpleasant dissonance $c\sharp-d$ between cantus and tenor.⁴⁷

⁴³ Measure 50 in the modern edition of the Gloria. Cf. Kurt von Fischer and F. Alberto Gallo, eds., *Italian Sacred and Ceremonial Music*, Polyphonic Music of the Fourteenth Century 13 (Monaco: L'Oiseau-Lyre, 1987), 75). Transcription of underlying text taken from the edition.

⁴⁴ Measure 116 (von Fischer and Gallo, *Italian Sacred and Ceremonial Music*, 77). The red void semiminim divide the minim into three instead of two, therefore 18 red void semiminims fit into the duration of a breve in C or O .

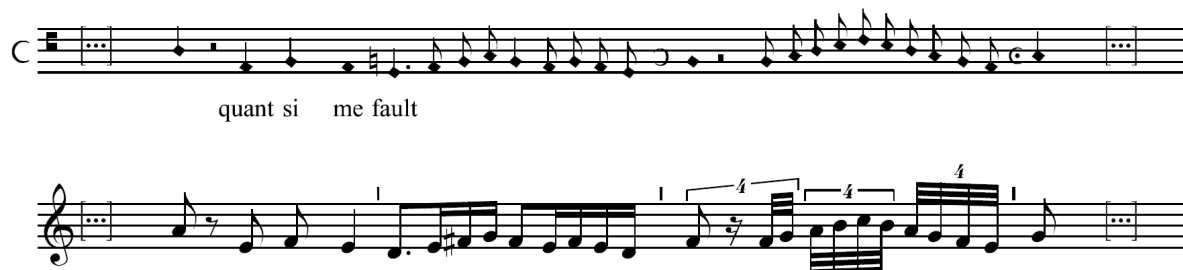
⁴⁵ Smilansky has stated that *sesquitertia* (4:3) proportion is achieved by special note shapes in this composition. Cf. “Rethinking *Ars Subtilior*,” 129. However, only $\circ$ results in *sesquitertia* (4:3) proportion in this Credo. Moreover, compared to other *Mod A* works, the notation of this Credo is rather simple—the only ‘specialities’ being void notation (resulting in *sesquialtera* (3:2) proportion at the semibreve level) as well as the $\circ$ passage that shall be discussed here.

⁴⁶ Von Fischer and Gallo, *Italian Sacred and Ceremonial Music*, 162. This is not remarked upon in the critical apparatus (see *id.*, 278).

⁴⁷ Jos Haring and Kees Boeke, eds., *The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio*, Olive Music Editions 1

FIGURE 2.7: Transcription of $\circ$ passage in Credo (No. 10)⁴⁸

The cantus of the rondeau *Pour Bel Accueil suy je, las, deceü* (No. 91, fol. 44v) also contains a passage in $\circ$ of the duration of one breve in $\mathbb{C}$, containing a semibreve, a minim rest and ten semiminims resulting in a fast melisma shortly before the end of the second part (see FIGURE 2.8).⁴⁹ The proportion sign passage as well as the preceding *mensura*—together constituting the most interesting rhythmical part of the composition—highlight the word “fault” from the phrase “Quant si me fault ce que plus ay creü”.⁵⁰

FIGURE 2.8: Transcription of $\circ$ passage in *Pour Bel Accueil suy je, las, deceü*

(Amsterdam and Arezzo: Olive Music, 2019), 57. Haring and Boeke assume the scribal error to lie in the rests following shortly after the $\circ$ passage and they transmit the semiminims with their correct value rather than prolonging them.

⁴⁸ Transcription taken from Haring and Boeke, *The Manuscript a.M.5.24*, 57.

⁴⁹ Measure 27 in the modern editions (Apel, *French Secular Music*, 19*; or *id.*, ed., *French Secular Compositions of the Fourteenth Century*, Corpus Mensurabilis Musicae 53, 3 vols. (Rome: American Institute of Musicology, 1970), 1:127; or Gordon K. Greene, ed., *French Secular Music: Rondeaux and Miscellaneous Pieces*, Polyphonic Music of the Fourteenth Century 22 (Monaco: L'Oiseau-Lyre, 1989), 25).

⁵⁰ The sixth line of the rondeau has the word “sont” from the phrase “Mais a moy sont tout ses bien retenu” in that position. Transcriptions of underlying text taken from Apel’s editions.

The rondeau *A qui Fortune ne se vult amer* (No. 88, fol. 43r) combines C with minims (see FIGURE 2.9).⁵¹ Since *dupla* (2:1) proportion is applied to the whole cantus, however, all 16 minims are equal to the duration of one breve in C in the tenor or contratenor, i.e. the proportion in the C passage compared to the other two voices is *dupla superbipartiens tertias* ($8:3 = 2:1$ combined with $4:3$).⁵² Although the resulting note value equals the semiminims in the three above-mentioned works, it should not automatically be assumed that the cantus moves equally fast. Therefore it might be inappropriate to speak of a fast melisma. More likely is the intention to highlight the beginning of the second verse of the refrain by a rhythmically striking passage.

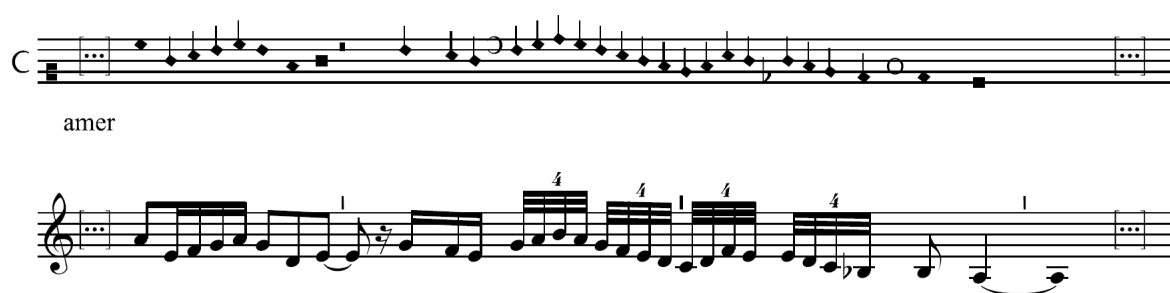


FIGURE 2.9: Transcription of C passage in *A qui Fortune ne se vult amer*

As already mentioned, *A qui Fortune* is also the only piece by Matteo in which an Arabic numeral (2) is used to indicate a proportion. It is placed next to the ledger line at the beginning of the piece (see FIGURE 2.10). As proportion sign it is rather inconspicuous because of its small size—it bears more resemblance to the *custodes* found in the piece—and the fact that no canon is provided to explain its meaning. Its presence is even more puzzling due to the circumstance that the rondeau could have been notated with longer note values in the tenor and contratenor to avoid confusion. It is possible that a virtuosic cantus was intended, hence the diminution. Future research on performance practice can perhaps shed more light on this work, which with its proportional application to an entire voice part remains a *unicum* in Matteo's surviving *oeuvre*.

⁵¹ Measures 16–17 in Apel's editions (Apel, *French Secular Music*, 25*; or *id.*, *French Secular Compositions*, 1:119); measure 8 in Greene's edition (Greene, *French Secular Music*, 15).

⁵² Interestingly, the cantus of this work is in O rather than in C as the other two voices. This, however, has no impact on the interpretation of C .

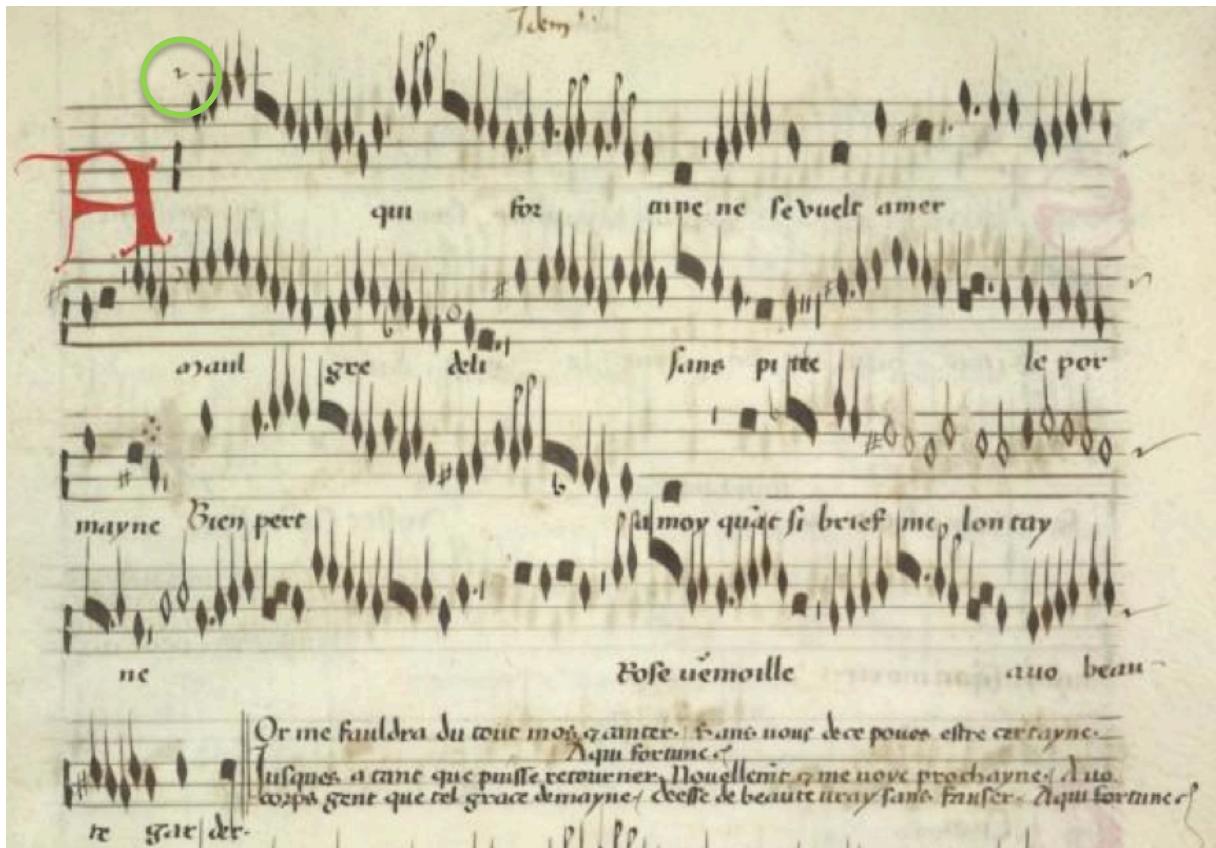


FIGURE 2.10: Cantus of *A qui Fortune ne se vult amer* (fol. 43r)

Another piece in which the $\bigcirc$ passage highlights a section by its rhythmic uniqueness is the virelai *Helas, Avril, par ton doulz revenir* (No. 93, fol. 45r), where eight minims mark the penultimate *mensura* of the second part (see FIGURE 2.11).⁵³

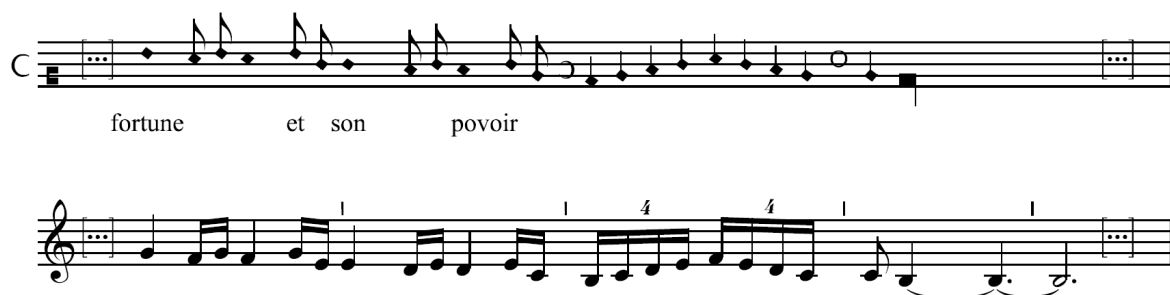


FIGURE 2.11: Transcription of $\bigcirc$ passage in *Helas, Avril, par ton doulz revenir*

⁵³ Measure 66 in the modern editions (Apel, *French Secular Music*, 14*; or *id.*, *French Secular Compositions*, 1:113; or Gordon K. Greene, ed., *French Secular Music: Virelais*, Polyphonic Music of the Fourteenth Century 21 (Monaco: L'Oiseau-Lyre, 1987), 31). The problematic text underlay of this work has been discussed elsewhere. Cf. Apel, *French Secular Music*, 29.

The use of proportion signs at the end of sections is continued in another Credo (No. 7, fols. 5v–6v) where the cantus features *sesquitertia* (4:3) proportion (indicated by $\oslash$) for the duration of seven breves in C , thereby highlighting the phrase “Qui cum Patre et Filio simul adoratur, et conglorificatur: qui locutus est per Prophetas.” (see FIGURE 2.12).⁵⁴ The $\oslash$ passage features several different note values (not only semibreves and minims but also two semiminims and a *longa*) and is outstanding in its length among Matteo’s works.⁵⁵

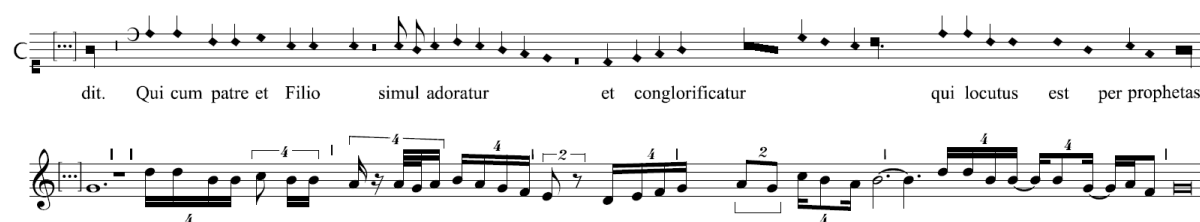


FIGURE 2.12: Transcription of $\oslash$ passage in Credo (No. 7)

A shorter $\oslash$ passage, which similar to the Credo (No. 7) also features different note values (semibreves, minims and a single breve), is found in the cantus of the rondeau *Se pour loyalement servir on puist merir* (No. 89, fol. 43v) and lasts for three breves in C (see FIGURE 2.13).⁵⁶ However, it is positioned in the middle of the second section and also in the middle of the line. Furthermore, the semiminims found in other places in the rondeau constitute much faster note values than those found in the $\oslash$ passage. The motive for introducing *sesquitertia* (4:3) proportion in this position hence leaves room for speculation. It stands to reason that the words “Vostre suy et seray jusqu’au morir” might be a citation. Unfortunately, however, I have been unable to find similar wordings in contemporary repertoire.⁵⁷ One could argue, of course, that the central theme of loyalty transmitted by the line (“I am yours and will

⁵⁴ Measures 172–78 in the modern edition (von Fischer and Gallo, *Italian Sacred and Ceremonial Music*, 151). Transcription of underlying text taken from the edition.

⁵⁵ The $\oslash$ passage in the *Le greygnour bien que nature* lasts for five measures in C but was originally notated differently and has possibly been reworked by a scribe (see pp. 119–20 below). The ballata *Serà quel zorno may, dolze madonna mia* also features a long passage in $\oslash$ in which all three voices, however, move into that mensuration (see p. 122 below).

⁵⁶ Measures 26–28 in the modern editions (Apel, *French Secular Music*, 23*; or *id.*, *French Secular Compositions*, 1:130; or Greene, *French Secular Music*, 26).

⁵⁷ The line “Domna, vostre sui e serai” appears in the cançon *Pel doutz chan que-l rossinhols fai* (No. 33) by the twelfth-century troubadour Bernart de Ventadorn (for the complete text and overview of sources see Carl Appel, *Bernart von Ventadorn: Seine Lieder mit Einleitung und Glossar* (Halle: Max Niemeyer Verlag, 1915), 194–98). Despite the unlikelihood of an intended citation from this early cançon, it is noteworthy that Ventadorn’s works were apparently well known in Northern Italy. In 1210, the scholar Boncompagno da Signa—former professor at the University of Bologna—praised Bernart de Ventadorn in one of his writings: “Quanti nominis quanteve fame sit Bernardus e Ventator, et quam gloriosas fecerit cançiones et dulcisonas invenerit melodias, multe orbis provincie recognosunt.” (see ‘Boncompagnus, chapter 6.8.1’ in the online edition by Steven M. Wight (<http://scrineum.unipv.it/wight/index.htm>) and Roger Boase, *The Origin and Meaning of Courtly Love: A Critical Study of European Scholarship* (Manchester: Manchester University Press, 1977), 5).

be until I die”) is especially highlighted by the rhythmical change, even if no fast melisma occurs in that place.



FIGURE 2.13: Transcription of $\circ$ passage in *Se pour loyalement servir on puist merir*

Thus far the discussed works contained a proportion passage in the cantus. Several of Matteo’s compositions contain proportion signs in the contratenor, however. Here especially, $\circ$ is not used to create fast melismata but rhythmically striking passages highlighting the dual-binary nature of the proportion section. The contratenor of the virelai *Dame souvrayne de beauté, d’onour* (No. 75, fol. 38r) features a syncopated semibreve-minim-semibreve rhythm in the $\circ$ passage which is repeated twice (see FIGURE 2.14).⁵⁸ While the cantus has a breve on the word “dieu” the contratenor highlights the section with this syncopated figure.

FIGURE 2.14: Transcription of $\circ$ passage in *Dame souvrayne de beauté, d’onour*⁵⁹

Matteo’s Gloria (No. 2, fols. 1v–2r) also uses $\circ$ in the contratenor for the same duration (one and a half breves in C) as in *Dame souvrayne* (see FIGURE 2.15).⁶⁰ Since the passage does not feature a syncopation (it simply consists of six semibreves) and the cantus is also moving (on the word “unigenite”) and does not have a long note as in *Dame souvrayne* it

⁵⁸ Measures 8–9 in the modern editions (Apel, *French Secular Music*, 15*; or *id.*, *French Secular Compositions*, 1:110; or Greene, *French Secular Music*, 26).

⁵⁹ Transcription taken from Haring and Boeke, *The Manuscript a.M.5.24*, 292.

⁶⁰ Measures 56–57 in the modern edition (von Fischer and Gallo, *Italian Sacred and Ceremonial Music*, 57).

would be rather farfetched to speak of highlighting in this case. Here is another work in which the intentions for the introduction of $\textcircled{\circ}$ are not obvious.



FIGURE 2.15: Transcription of $\textcircled{\circ}$ passage in Gloria (No. 2)

A discovery made in recent years might add a new perspective on the matter. It has been suggested that this Gloria is connected with the ballade *En attendant, souffrir m'estuet grief payne* (No. 35, fol. 20r) attributed to Filippotto da Caserta in *Mod A*, but to *Galiot* in *Ch*.⁶¹ Pedro Memelsdorff has discovered that the Gloria might cite the two opening measures of the ballade (rhythmically slightly altered and transposed by a fifth) at the beginning (see FIGURE 2.16).⁶²

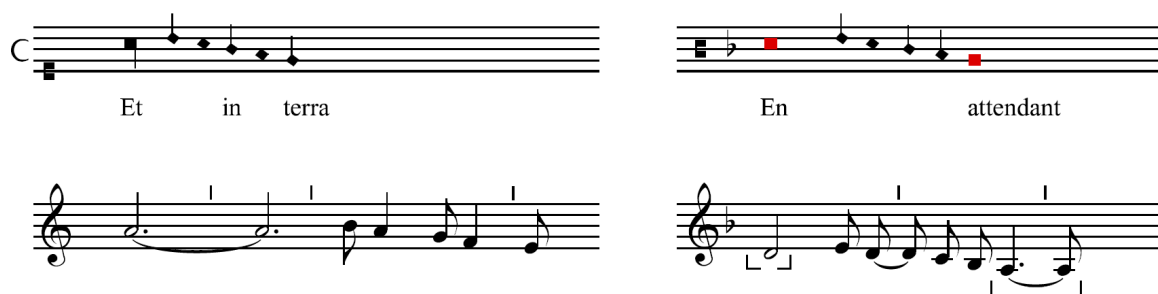


FIGURE 2.16: Openings of Gloria (No. 2) (left) and *En attendant, souffrir m'estuet grief payne* (right)

Although the intention of citation seems to be debatable at least due to the rhythmic alteration and commonness of a stepwise descending melodic line, it is noteworthy that *En attendant* also contains a $\textcircled{\circ}$ passage with the exact same length and featuring the same note values (six

⁶¹ Despite the attribution to Galiot in *Ch*, Filippotto da Casterta is assumed to be the composer (for more see Chap. 2.4 below).

⁶² The work is even titled “Gloria [En attendant]” in the booklet of Mala Punica’s record *Missa Cantilena* (Erato, 1997).

semibreves) as the Gloria (see FIGURE 2.17).⁶³ Considering the citation game⁶⁴ surrounding the three *En attendant* songs in *Mod A* it almost seems to be too much of a coincidence that the opening line and a distinct rhythmic feature (if not audibly then certainly visibly in the case of the $\circ$ passage of Filippotto's ballade reappear in Matteo's Gloria. Did Matteo join the citation game by citing Filippotto's *En attendant*, just as Ciconia did in his *Sus un' fontayne*? Assuming that the citation was intended this would strengthen the hypothesis that Matteo was physically close to *Mod A*, since the concordant versions of Filippotto's ballade all transmit *dragmae* instead of $\circ$.



FIGURE 2.17: $\circ$ passages of Gloria (No. 2) (left) and *En attendant, souffrir m'estuet grief payne* (right)

$\circ$ is also used twice in the ballade *Le greygnour bien que nature* (No. 62, fol. 32r (31v)), commonly referred to as the most complex work in Matteo's surviving *oeuvre*.⁶⁵ It not only features several methods of coloration (void notation, red notation and red void notation) to create proportional change (see FIGURE 2.18) but also uses different mensuration signs simultaneously without minim equivalence for that effect (see TABLE 2.3 above). The cantus contains more rhythmically altered passages than sections in the initial mensuration and even the tenor contains coloration (red ink and void notation). The interpretation of the different colorations and note shapes in cantus and tenor have been explained in detail elsewhere, which is why I will limit the discussion to the notation of the contratenor, which is the only voice part that contains the proportion sign $\circ$.⁶⁶ $\circ$ indicates *sesquitertia* (4:3) proportion at the minim

⁶³ It is noteworthy that the opening line of *En attendant* also reappears rhythmically altered in several places in Filippotto's ballade. (cf. Plumley, "Citation and Allusion," 315), though none of these alterations exactly match the opening of Matteo's Gloria. The opening from Filippotto's *En attendant* can also be found in two ballades by Jacob Senleches: The second section of his ballade *En attendant, Esperance conforte* (*Mod A*, fols. 39v–40r, No. 79) and in his double-texted ballade *Je me merveil/J'ay plusieurs fois* (*Ch*, fol. 44v, No. 69), as reported by Plumley (cf. *id.*, 320–25). It is, of course, possible that Filippotto's and Senleches' songs as well as Matteo's Gloria cite from yet another piece of music, which is either lost or not yet discovered: "Whether Philippus was the original composer of the theme or whether he himself borrowed it from another source is difficult to say." (*id.*, 325). The proportion sign passage in Filippotto's ballade is an allusion to Machaut's lai *Ne say comment commencer* (for more see Chap. 2.4 below).

⁶⁴ For more on this see Chap. 2.4 below.

⁶⁵ For modern editions of this work see Apel, *French Secular Music*, 1*–3*; or *id.*, *French Secular Compositions*, 1:98–101; or Gordon K. Greene, ed., *French Secular Music: Ballades and Canons*, Polyphonic Music of the Fourteenth Century 20 (Monaco: L'Oiseau-Lyre, 1987), 60–64.

⁶⁶ See, for example, Maria Tereza Rosa Barezani, "Una rilettura di *Le Greygnour Bien* di Matteo da Perugia," *Philomusica* on-line 1, no. 1 (2001). <http://dx.doi.org/10.6092/1826-9001.1.106>.

level.⁶⁷ One section appears in the middle of the first part and the other towards the end of the second part of the ballade. As the contratenor is untexted and since the piece contains so many rhythmically altered passages altogether, it seems not very promising to search for citations in the two proportion sign sections, especially since the notation appears to be altered by scribal agency.



FIGURE 2.18: Excerpt from cantus of *Le greygnour bien que nature*

Anne Stone summarises these notational changes in the contratenor as follows:

“First, a passage in sesquitertia proportion originally notated with Italian note shapes^[68] was erased and replaced with standard note shapes under O [see FIGURE 2.19 left]. Two other emendations involve black void notation. The song was originally copied using black void notation to serve in two different capacities: hemiola proportion and sesquitertia proportion. In the first case, the scribe erased the black void notation and replaced it with the more common solid coloration [see FIGURE 2.19 middle (two excerpts)], while for sesquitertia he replaced black void with the more common red void notes [see FIGURE 2.19 right].”⁶⁹



FIGURE 2.19: Examples of scribal changes in the contratenor of *Le greygnour bien que nature*

Stone has assumed that the scribe of *Mod AII-IV* changed the notation but I have some problems with that theory.⁷⁰ The scribe of *Mod AII-IV* has worked very neatly throughout the three gatherings but the person who changed the notation in *Le greygnour bien que nature* has done

⁶⁷ Anna Maria Busse Berger has claimed that O indicates *dupla* (2:1) proportion in this composition. Cf. “The Origin and Early History of Proportion Signs,” *Journal of the American Musicological Society* 41, no. 3 (1988): 410 n. 21. Stoessel has already pointed out that this is wrong. Cf. Stoessel, “The Captive Scribe,” 1:279.

⁶⁸ To me it seems as if at least some of these notes were void or partly void, in addition to their Italian elements such as stems or flags.

⁶⁹ Stone, *The Manuscript Modena*, 44.

⁷⁰ I do not reject Stone’s hypothesis that the scribe of *Mod AII-IV* made changes in the notation altogether and I think that she has made a good argument for some of the other notational changes in *Mod AII-IV* being the principal scribe’s work. Cf. Stone, *The Manuscript Modena*, 41–48.

a rather crude job. The second section in $\bigcirc$ (see FIGURE 2.19 left) is probably the best example. To me it seems as if some of these notes were black void notes before someone filled the void spaces with black ink. This person does not seem to have had the tools or the expertise of a trained music scribe, however, because a trained scribe would probably have drawn new black notes over the void ones. The person changing notation in *Le greygnour bien que nature* seems to have used a writing tool of medium thickness to fill in black ink or red ink respectively and to frame the black void notes with red ink.⁷¹ As stated above, this has not been done neatly and in addition to the inadequate tool with which these changes seem to have been made I do not think that it is likely that the scribe of *Mod A_{II-IV}* was responsible for them.

Nevertheless, in light of these obvious changes to the notation it is necessary to ask who the author of the proportion sign really was. Was it the composer Matteo da Perugia or was it a scribe's work? And if it was the latter, as seems to be the case in *Le greygnour bien que nature*, is it possible to speak of the composer's intention at all? There are quite a few other examples of *Ars subtilior* compositions in which the notation was either changed or where the notation is dissimilar in different sources of the pieces.⁷² Emily Zazulia, who has discussed some of these examples in her dissertation on verbal canons, concludes that:

"It is highly unlikely that the composer [Matheus de Sancto Johannes] conceived of two different ways of notating the same piece [*Inclide flos*]; one—if not both—of these copies has been altered by scribal preference. [...] [T]he only surviving copy of Hasprois's *Puis que je suis fumeux* (in Ch) includes the canon: "...notule vacue balade in proporcione dupla cantetur" ("the void notes of the balade are sung in duple proportion"). In the accompanying music, however, there are no void notes; the notes in question are instead written in red ink. There are several ways this may have come about, and all involve some scribal agency, whether willful or due to an error or misunderstanding."⁷³

As I will show throughout this chapter, the proportion signs in *Mod A* are rather standardised compared to other *Ars subtilior* sources. It is not only possible but highly likely that scribal agency attributed to that state, whether with the intent of reducing ambiguities or whether to standardise the notation is anyone's guess. Even if some of the proportion signs in *Mod A* are the work of a scribe, however, it does not mean that my search for citations in passages with proportions signs is redundant, because though the notation might have been different in the autograph, it would still be visually different from the notes in the initial mensuration, either by coloration or by special note shapes. Therefore, I will treat the proportion

⁷¹ Compare in particular the thickness of the stems of the minim and the semibreve ligature preceding the proportion sign $\bigcirc$ with the thickness of the stems of the $\bigcirc$ section in FIGURE 2.19 left.

⁷² See Chap. 2.4 for examples in *Mod A*.

⁷³ Emily Carolyn Zazulia, "Verbal Canons and Notational Complexity in Fifteenth-Century Music," PhD diss., University of Pennsylvania, 2012, 52–53.

signs in the *unica* without visible notational alterations in this chapter as if they were the composer's idea.

The manuscript *Mod A* also contains two Italian ballatas by Matteo, both of which contain passages in $\textcircled{\circ}$. The song *Già da rete d'amor libera et sciolta* (No. 96, fol. 46v) features a passage in which both cantus and contratenor change from $\textcircled{\circ}$ to $\textcircled{\circ}$ (see FIGURE 2.20).⁷⁴ While the cantus could not have been notated in another mensuration the contratenor could have been notated with semibreves in $\textcircled{\circ}$, a change, that is also found in the cantus in other places of the piece. Why use the proportion sign in the contratenor in that position instead of changing from $\textcircled{\circ}$ to $\textcircled{\circ}$? Since both voices change to $\textcircled{\circ}$ at the same time for almost the same duration, I would argue that the proportion sign $\textcircled{\circ}$ functions as coordination sign as well as as proportion sign in this section of the composition. This would at least explain why $\textcircled{\circ}$ was chosen over $\textcircled{\circ}$ in the contratenor.



FIGURE 2.20: Transcription of first $\textcircled{\circ}$ passage in *Già da rete*

This hypothesis, however, is weakened by the second $\textcircled{\circ}$ passage of *Già da rete* (contratenor only, see FIGURE 2.21), which only occurs in the contratenor. Nothing particularly special happens in the cantus at the same time, so the argument that $\textcircled{\circ}$ was used for coordination purposes does not hold true for at least this passage. Moreover, the combination of $\textcircled{\circ}$ with breves to this extreme extend is unique among the works found in *Mod A*.⁷⁵ Due to the long note values it is questionable whether the listener would perceive the passage as rhythmically striking. The proportion sign in combination with the long ligature is visually striking though. It is

⁷⁴ Measures 54–56 in the modern edition (W. Thomas Marrocco, ed., *Italian Secular Music*, Polyphonic Music of the Fourteenth Century 10 (Monaco: L'Oiseau-Lyre, 1977), 96–97). The transcriptions of the $\textcircled{\circ}$ passages in the contratenor in Marrocco's edition are debatable.

⁷⁵ Measures 62–65 (Marrocco, *Italian Secular Music*, 97).

possible that the melodic line is cited from somewhere else and intentionally highlighted by $\circ$.

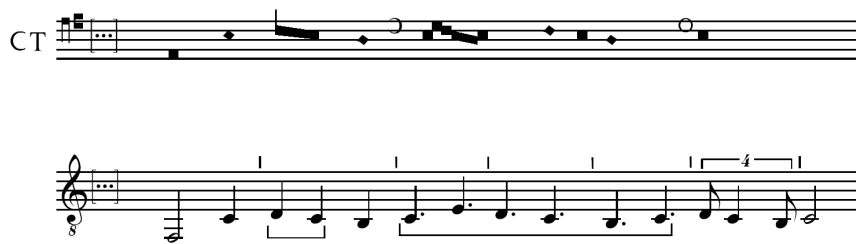


FIGURE 2.21: Transcription of second $\circ$ passage in *Già da rete*

Finally, I would like to discuss Matteo's ballata *Serà quel zorno may, dolze madonna mia* (No. 98, fols. 47v–48r) in which all three voices move to $\circ$ for an entire passage before returning to C (see FIGURE 2.22).⁷⁶ Since no part stays in C (or O), I would argue that $\circ$ is used as mensuration sign here and I have therefore not included it in the tables above. Nevertheless, the longer note values used in the $\circ$ section (a long in the cantus) calls for a proportional relationship of 4:3 compared to the previous section in all voices. It is unclear why $\circ$ was used instead of C , with which the same dual-binary division could have been achieved. In her 1986 dissertation on proportions in the *Ars nova* and the *Ars subtilior*, Laurie Koehler has suggested that augmented notations (which is the case here, because a breve in $\circ$ has half the length of a breve in C) were often favoured over non-augmented notations in order to avoid the semiminim.⁷⁷ Since the semiminim appears frequently in Matteo's compositions, however—and especially in this ballata—, this explanation does not seem to be the reason in this case.

FIGURE 2.22: Transcription of $\circ$ passage in *Serà quel zorno may, dolze madonna mia*⁷⁸

⁷⁶ Measures 50–58 in the modern edition (Marrocco, *Italian Secular Music*, 100).

⁷⁷ Cf. Laurie Koehler, *Pythagoreisch-platonische Proportionen in Werken der ars nova und ars subtilior*, Göttinger musikwissenschaftliche Arbeiten 12 (Kassel: Bärenreiter, 1990), 151.

⁷⁸ Transcription taken from Haring and Boeke, *The Manuscript a.M.5.24*, 361.

As initially stated, Apel's observation that the majority of Matteo's works exhibit a more traditional style compared to the more complex *Ars subtilior* works found in *Mod A* (with the exception of his ballade *Le greygnour bien que nature*) is confirmed by the use of proportion signs. They are not used in order to 'complicate' the music, i.e. using these notational devices to express something that could have been notated in a simpler manner, but—in almost all cases—to achieve rhythms that could not have been achieved by conventional notation. It has been shown that—in the majority of works— $\bigcirc$ is used in connection with minims or semi-minims in order to create fast melismata at the end of parts or rhythmically striking passages in structurally important positions.⁷⁹ In these cases, the duration of the $\bigcirc$ passage has a length of between one and two breves in the initial mensuration (C or O) and can easily be grasped by the reader. Furthermore, $\bigcirc$ is often combined with only one certain note value (semiminims in two mass sections (Gloria No. 102 and Credo No. 10), minims in the rondeau *A qui Fortune ne se vult amer* and the virelai *Helas, Avril, par ton doulz revenir*, semibreves in the Gloria No. 2 and the cantus of the ballata *Già da rete d'amor libera et sciolta*, as well as breves in the contratenor of that ballata). Since $\bigcirc$ is only used once in the pieces (except in the ballata *Già da rete d'amor libera et sciolta*) it can be said that these passages are highlighted by the proportion sign, if not audibly (as doubted in the case of the above-mentioned ballata and the Gloria No. 2) than certainly visually. In the case of the last two mentioned works as well as in the rondeau *Se pour loyalement servir on puist merir* the reason for the positioning of the $\bigcirc$ passages leaves room for speculation. This is also true for the introduction of $\bigcirc$ instead of C in the ballata *Serà quel zorno may, dolze madonna mia*.

Before turning to other composers of *Mod A* and their works, I would like to briefly draw attention to the fact that Matteo da Perugia's works exhibit several alternative ways to express rhythms otherwise achieved by $\bigcirc$ despite the scribe's or scribes' apparent efforts to standardise the notation (most clearly observable in the ballade *Le greygnour bien que nature*). The virelai *Dame souvrayne de beauté, d'onour*, for example, applies $\bigcirc$ in order to create *sesquitertia* (4:3) proportion for a short syncopated sequence but also features four *dragmae*—which are equal to the duration of one breve in C —in the same voice (contratenor) closer to the beginning.⁸⁰ These *dragmae* could have been replaced by four semibreves in $\bigcirc$ (the same rhythm thus expressed can be found in the contratenor of the Gloria No. 2 and the cantus of the ballata *Già da rete d'amor libera et sciolta*) if it had indeed been the scribe's wish to replace special noteshapes. Furthermore, the cantus of *Dame souvrayne* even features a down-

⁷⁹ Strohm has termed these melismata "more modern mannerisms". Strohm, *The Rise of European Music*, 61.

⁸⁰ This has already been observed by Laurie Koehler. Cf. *Pythagoreisch-platonische Proportionen*, 152–53.

wardly flagged note () which has the same value as the normal *dragma* in the contratenor, namely one fourth of a breve in $\mathbb{C}$. This flagged note appears in several of Matteo's works⁸¹ and is almost always combined with a semiminim in order to form the value of two minims in $\mathbb{C}$. It seems that the combination of the two notes was a standard way to express the punctuated rhythm. However—and most inconsequently—, two of the flagged notes appear consecutively at the very end of the tenor in the rondeau *Helas! merci, merci, pour Dieu merci* (No. 77, fols. 38v–39r), thus the apparent 'rule' that they are only to be used in combination with semiminims is broken. We end up with three different ways to express the value of one fourth of a breve in $\mathbb{C}$: The *dragma* and the flagged note () in $\mathbb{C}$ and the semibreve in $\mathbb{O}$ (see FIGURE 2.23).

Even more remarkable is another note shape also found in the rondeau *Helas! merci*, which is unique to the manuscript: . This minim, which is open on the lower right side, has the value of three quarters of a minim in $\mathbb{C}$ —and therefore the exact same value as a minim in $\mathbb{O}$ (see FIGURE 2.23). It is curious that $\mathbb{O}$ was not applied in this piece, as it would have been easy for the scribe to colour the edges of the notes and introduce $\mathbb{O}$ for the section.

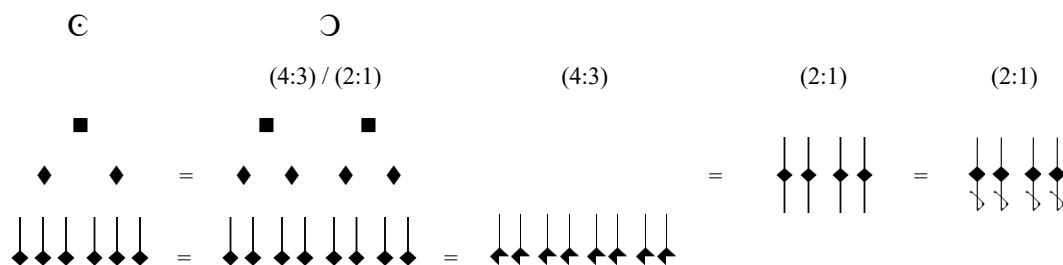


FIGURE 2.23: Variants of *sesquitertia* (4:3) proportion (at the minim level) and *dupla* (2:1) proportion (at the semibreve level) in Matteo da Perugia's compositions

Notational inconsistencies are common in *Mod A* compositions and perhaps most distinct in the works of Antonello da Caserta, which will be discussed in the next chapter.

⁸¹ *Dame que j'aym sour toutes de ma enface* (No. 14, fols. 10v–11r), *Pres du soloil deduisant s'esbanoye* (No. 28, fol. 16r), *Plus liés des liés, plus joieux et plus gay* (No. 42, fol. 23r), *Le grant desir que j'ay du retourner* (No. 65, fol. 33v), *Helas! merci, merci pour Dieu merci* (No. 77, fols. 38v–39r), *Pour Dieu vous pri, haulte dame d'honneur* (No. 84, fol. 41r), *Se je me plaing de furtune, j'ay droit* (No. 87, fols. 42v–43r), *Puisque je sui pour loyauté tenir* (No. 90, fol. 44r (43v)), *Trover ne puis aucunement confort* (No. 95, fol. 46r), and the additional contratenor to *El non me zova né val, donna, fuzire* (No. 5, fols. 3v–4r).

2.3 Proportion Signs in the Compositions of Antonello da Caserta

For the current study, the *Mod A* compositions of Antonello da Caserta⁸² constitute the most interesting collection of works, since they represent many different methods to create rhythmic proportions.⁸³ His compositions contain a variety of different proportion signs, sometimes even indicating the same proportion: *sesquitertia* (4:3) proportion, for example, is indicated by $\complement$, $\frac{2}{2}$, and $\frac{8}{6}$. However, one can never observe different proportion signs for the same proportion in the same piece. With eight compositions, Antonello is the best represented composer in *Mod A_{II-IV}* and the manuscript is the principle source for his French songs.⁸⁴

Incipit	Genre	No.	fol(s). gathering	2:1	3:2	4:3	Comment(s)
<i>Amour m'a le cuer mis en tel martire</i>	Ballade	63	32v–33r <i>Mod A_{IV}</i>	4 2	9 6	8 6	Proportions are not applied cumulatively.
<i>Dame d'onour, en qui tout mon cuer maynt</i>	Ballade	82	40v <i>Mod A_{IV}</i>			$\frac{2}{2}$	Piece comes with a canon, which does, however, not give advice on the interpretation of the alternative mensuration and proportion signs.
<i>Tres nouble dame souverayne</i>	Virelai	53	28v <i>Mod A_{III}</i>			$\complement$	Canon: “Canon virelarie: ubicunque i(n) venieris signum i(m)perfecti minoris cantetur de modo epitrito.”

TABLE 2.24: Proportion signs in compositions by Antonello da Caserta

By stating that “Anthonellus, despite the fact that he might be regarded as *the* Italian master of the *ars subtilior*, remains largely an enigmatic figure in relation to his biography”,⁸⁵ Jason Stoessel has aptly summarised what little we know about the man Antonello da Caserta. Judging by his name, he came from the town Caserta north of Naples. John Nádas and Agostino Ziino have argued that he was probably associated with the Visconti court in Pavia in the late 1390s.⁸⁶ Stone reports the discovery of a 1402 document which refers to a “frater An-

⁸² *Du val prilleus ou pourpris de jeunesse* (No. 18, fol. 12v), *Beauté parfait, bonté sovrayne* (No. 19, fol. 13r), *Notés pour moi ceste ballade* (No. 21, fol. 13v), *Dame d'onour, c'on ne puet esprixier* (No. 33, fol. 19v), *Tres nouble dame souverayne* (No. 53, fol. 28v), *Amour m'a le cuer mis en tel martire* (No. 63, fols. 32v–33r), *Dame zentil, en qui est ma sperance* (No. 76, fol. 38v), and *Dame d'onour, en qui tout mon cuer maynt* (No. 82, fol. 40v).

⁸³ An excellent and exhaustive study of notational devices found in the works of Antonello and Filippotto da Caserta is given by Carla Vivarelli, ed., *Le composizioni francesi di Filippotto e Antonello da Caserta trãdite nel Codice Estense a.M.5.24*, «Diverse voci...» 6 (Pisa: Edizioni ETS, 2005), which is why I will strictly limit the discussion to proportion signs used in both composers' *Mod A* works here.

⁸⁴ Cf. Ursula Günther and Anne Stone, “Antonello [Anthonello, Anthonellus, (An)tonelus] da Caserta,” in *NG2* (London: MacMillan, 2001), 1:761.

⁸⁵ Jason Stoessel, “The Captive Scribe,” 1:130.

⁸⁶ His madrigal *Del glorioso titolo* (transmitted in *Luc*) might honour the coronation of Giangaleazzo Visconti as Duke of Milan (1395) and his ballata *Più chiar che 'l sol* (*Luc* and *Parma*) could stand in connection with the 1399 marriage of Giangaleazzo's niece Lucia with Frederick of Thuringia. Cf. John Nádas and Agostino Ziino, eds., *The Lucca Codex (Codice Mancini): Introductory Study and Facsimile Edition*, *Ars Nova* 1 (Lucca: LIM Editrice, 1990), 38–39.

toniello de Caserta” but casts doubt on whether this person is identical with the *Mod A* composer, because the *Mod A_{II-IV}* scribe added the title “frater” in many other ascriptions but not in Antonello’s case.⁸⁷ Nigel Wilkens has gone so far as to suggest that the composer Antonello da Caserta, who composed the French songs in *Mod A*, is not identical with the composer of the Italian songs from the Lucca codex: “Any such connection should surely be discounted, for there seems to be no point of contact at all between these two musicians apart from a coincidence of names.”⁸⁸ While it is true that the French and Italian songs by Antonello exhibit different styles (not least in their rhythmic complexity), these traditional differences are not uncommon, e.g. in the repertoire of the contemporary Johannes Ciconia. It will be difficult to settle the dispute before further biographical evidence surfaces. Nevertheless, judging from the number of compositions by Antonello contained in *Mod A_{II-IV}*, he seems to have been a composer of some importance to the compiler. Stone has even put forward the hypothesis that the second gathering of *Mod A* “might originally have been planned to collect the songs of Antonello”.⁸⁹

The most straightforward piece that uses a proportion sign is Antonello’s virelai *Tres nouble dame souverayne* (No. 53, fol. 28v), in which $\bigcirc$ indicates *sesquitertia* (4:3) proportion. The proportion can be applied at both the minim and the semibreve level, as demonstrated in FIGURE 2.25.⁹⁰ The interpretation at the minim level in *tempus perfectum* with *prolatio minor* does of course assume a division of the middle group of minims (dividing the *mensura* into 2+1 and 1+2 minims), therefore the interpretation of 4:3 at the semibreve level might be preferable, especially since no minims are present in the proportion sign sections of *Tres nouble dame*. However, *Mod A* contains several pieces, in which $\bigcirc$ is applied in minor prolation, and two of these exhibit minims in the proportional sections.⁹¹

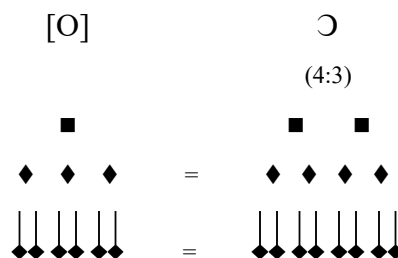
⁸⁷ Cf. Stone, “Writing Rhythm,” 63; and *id.*, *The Manuscript Modena*, 79.

⁸⁸ Nigel Wilkens, “Some Notes on Philipoctus de Caserta (c.1360?–c.1435),” *Nottingham Medieval Studies* 8 (1964): 84. These doubts have more tentatively been recast by Jason Stoessel. Cf. “The Angevin Struggle,” 4.

⁸⁹ Stone, *The Manuscript Modena*, 38.

⁹⁰ A recent edition can be found in Vivarelli, *Le composizioni francesi*, 140–42 (cantus measures 20–24 and contratenor measures 19–26; as well as cantus measures 53–54). Also see the diplomatic transcription of the proportion sign passages (Example 16 a–c) in *id.*, 69–70. Older editions include Apel, *French Secular Music*, 40*–41*; or *id.*, *French Secular Compositions*, 1:12–13; and Greene, *French Secular Music*, 1–3.

⁹¹ $\bigcirc$ following O in *Mod A* also occurs in Ciconia’s *Sus un’ fontayne* and Perugia’s *A qui fortune* and *Già da rete*. Jason Stoessel has stated that “[In cases of $\bigcirc$ following O] only semibreves or breves are found in passages affected by this sign.” Stoessel, “The Captive Scribe,” 1:275. However, *Sus un’ fontayne* and *A qui fortune* do contain minims in $\bigcirc$ sections. Nevertheless, a tendency towards longer note values can be observed when $\bigcirc$ is combined with O, the exception being Perugia’s *A qui fortune* in which only minims occur in the $\bigcirc$ passage (for a discussion of the composition, see Chap. 2.2 above).

FIGURE 2.25: $\circ$ in *Tres nouble dame souverayne*

In the refrain of Antonello's virelai, the proportional sections in both cantus and contratenor highlight the phrase "je vous supli très unblement"⁹² (see FIGURE 2.26). Furthermore, the tenor also exhibits its only mensural change in this part of the song. Could this be a citation, audibly and visually highlighted by the proportion sign section?⁹³

FIGURE 2.26: Transcription of second $\circ$ passage in *Tres nouble dame souverayne*

⁹² Transcription taken from Vivarelli, *Le composizioni francesi*, 95.

⁹³ Unfortunately, I have been unable to find a musical setting of these words, which could have preceded Antonello's composition and thus function as a model for a possible citation. It is noteworthy, however, that the text line appears in the envoy of the ballade *Au court jeu de tables jouer* by Charles d'Orléans, the son of Louis I, Duke of Orléans, and Valentina Visconti, daughter of Giangaleazzo. Although Charles d'Orléans' ballade was certainly written after Antonello had composed his virelai, it does not seem altogether implausible that poetry or music containing the phrase circulated at the Visconti court around 1400.

It is curious and unique to *Mod A* in the case of $\circ$ that a canon was added. While the meaning of Arabic numerals and special geometric shapes are sometimes described in a canon, although not in the other works by Antonello, the explanation of $\circ$ in *Tres nouble dame* is indeed striking.⁹⁴ The canon states: “ubicunque invenieris signum imperfecti minoris cantetur de modo epitrito”.⁹⁵ $\circ$ is not described by its visual shape as reversed semicircle but referred to as “signum imperfecti minoris”, i.e. a sign indicating *tempus imperfectum* with *prolatio minor*, thereby presuming some advanced knowledge on the reader’s side. Despite its more common interpretation as proportion sign, $\circ$ is also used as alternative mensuration sign for C in some works from Italian music manuscripts—most likely of Florentine origin—as well as in at least two treatises.⁹⁶ Could Antonello’s *Tres nouble dame* possibly be an early work, written in a period when the proportional use of $\circ$ was not yet well established? On first glance, the explanatory canon points towards an early date of composition and the description of $\circ$ as “signum imperfecti minoris” hints towards an Italian origin of the virelai. However, it is just as conceivable that the canon is not the work of Antonello—a likely scenario in light of the absence of canons in his other *Mod A* compositions—and that the *Mod A_{II-IV}* scribe was copying the virelai from an exemplar—perhaps even of Florentine origin—in which a third party had added the canon in order to avoid confusion with the mensuration sign $\circ$. This does, of course, not establish any grounds for conclusions about Antonello’s whereabouts during his lifetime. Moreover, Antonello uses C and not $\circ$ to indicate *tempus imperfectum* with *prolatio minor* in his ballade *Beauté parfait, bonté sovrayne* (No. 19, fol. 13r).⁹⁷

It is indeed and a bit puzzling that the *Mod A_{II-IV}* scribe copied the canon at all. If we are correct in assuming that he was musically literate, why did he not add canons to other works

⁹⁴ This has already been remarked upon by Anne Maria Busse Berger who states: “Because [$\circ$] was almost exclusively associated with the *sesquitercia* proportion, few composers found it necessary to clarify its meaning by additional signs: Anthonello de Caserta (*Tres nouble dame*) adds a canon.” Busse Berger, *Mensuration and Proportion Signs*, 172–73. Stoessel, too, supports this view: “It is noteworthy that in Anthonellus de Caserta’s *Tres nouble dame* [...] the meaning of the sign in the context of [3,2] *dim* is explained by the canon.” Stoessel, “The Captive Scribe,” 1:275.

⁹⁵ *Mod A*, fol. 28v. “Wheresoever you find the imperfect minor sign, it must be sung in the proportion of 4:3.” Translation taken from Stoessel, “The Captive Scribe,” 1:275.

⁹⁶ Cf. Jason Stoessel, “The Interpretation of Unusual Mensuration Signs in the Notation of the *Ars subtilior*,” in *A Late Medieval Songbook and Its Context: New Perspectives on the Chantilly Codex (Bibliothèque du Château de Chantilly, Ms. 564)*, ed. Yolanda Plumley and Anne Stone (Turnhout: Brepols, 2009), 185–89. As Stoessel states, $\circ$ indicates *tempus imperfectum* with *prolatio minor* in two *Pit* compositions and in Andrea da Firenze’s *Donna se per te moro* in *Sq*. It also appears as a mensuration sign in the *Pad A* version of Ciconia’s virelai *Sus un’ fontayne* (for more see Chap. 2.4 below). Furthermore, it is listed as a sign for *tempus imperfectum* in two treatises: Johannes Ciconia’s *De proportionibus* (1411) and Johannes Hanboys’ *Summa* (c.1375). After studying the San Lorenzo Codex (*SL*), I can now add two more compositions to Stoessel’s list (‘Table 4: Non-sesquitercial uses of [$\circ$] in early 15th-century manuscripts without a qualifying canon’, 189): Paolo da Firenze’s madrigal *Ventila con tumulto la gran fama* (*Pit*, fols. 57v–58r, concordance in *SL*, fol. 51r (No. 144, tenor only) lacking all mensuration signs) and Piero Mazzuoli’s three part ballata *A Febo Damn’e a Marte Venere mai* (*SL*, fols. 91v–92r (No. 185)).

⁹⁷ The concordant version in *PR* (fol. 46v) also features C.

in which $\circ$ is used as proportion sign? On the other hand, if he copied the manuscript for a learned audience, which would know how to interpret $\circ$, why copy the canon in Antonello's virelai at all?

It is also curious that the proportion sign $\circ$ in the contratenor is missing in *Tres nouble dame*, although the piece has been impeccably copied into the manuscript otherwise—and even corrected in the tenor (see FIGURE 2.27).⁹⁸ FIGURE 2.28 shows the missing position. Apart from the fact that the note values would not match the other voices in any other constellation, the initial mensuration is reinstalled after eight *mensurae* by O. If no proportion were applied to the preceding part, the sign would be utterly superfluous. There is a possibility that the proportion sign was left out intentionally in order to create some sort of puzzle. I do not find this course of action likely, however, and assume that the missing sign is a scribal error.



FIGURE 2.27: Correction in the tenor of *Tres nouble dame*: A red B-B ligature and one or more black notes have been erased and replaced by a single red B-B-B ligature



FIGURE 2.28: Missing proportion sign in the contratenor of *Tres nouble dame*⁹⁹

⁹⁸ *Mod A*, fol. 28v, staff 4.

⁹⁹ *Mod A*, fol. 28v, staff 6.

In the cantus of Antonello's ballade *Amour m'a le cuer mis en tel martire* (No. 63, fols. 32v–33r) one can observe three stacked Arabic numerals, which indicate proportions: $\frac{9}{6}$ (=3:2) for *sesquialtera* proportion, $\frac{8}{6}$ (=4:3) for *sesquitertia* proportion, and $\frac{4}{2}$ (=2:1) for *dupla* proportion.¹⁰⁰ According to Anna Maria Busse Berger, this composition is probably the earliest piece in which proportion signs are applied.¹⁰¹ In light of the lack of biographical evidence of Antonello as well as the problems concerning the dating of manuscripts however, such assertions should be approached with caution.

The composition has certainly received much scholarly attention in the past and I refrain from repeating most but the essential of it here.¹⁰² Busse Berger as well as Jason Stoessel have shown that the proportions are not applied cumulatively but always refer to the initial mensuration (*tempus perfectum* with *prolatio minor*).¹⁰³ Stone has shown that the scribe of *Mod AII–IV* edited the work in order to avoid the ambiguity of red coloration in the original reading.¹⁰⁴ The full red semibreves in the tenor indicate *subsesquialtera* (2:3) proportion at the semibreve level. In the contratenor, the full red notes were erased and replaced by red void notation which indicates *sesquitertia* (4:3) proportion. This proportion is also applied in the cantus by use of the proportion sign $\frac{8}{6}$. Finally, the tenor has two groups of four *dragmae* which have the same value as three semibreves or six minims respectively, effectively creating the same rhythm. Theoretically, these three representations of *sesquitertia* (4:3) would be interchangeable (see FIGURE 2.29).¹⁰⁵

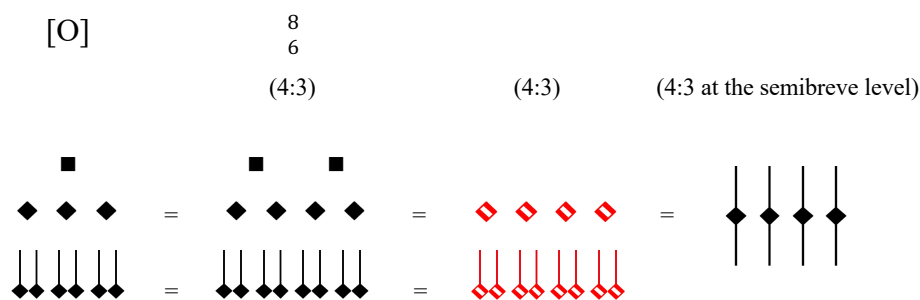


FIGURE 2.29: Variants of *sesquitertia* (4:3) proportion in *Amour m'a le cuer mis en tel martire*

¹⁰⁰ A recent edition can be found in Vivarelli, *Le composizioni francesi*, 143–46. Also see the diplomatic transcriptions of the proportion sign passages (Examples 20 and 21) in *id.*, 73–74. Older editions include Apel, *French Secular Music*, 37*–38*; or *id.*, *French Secular Compositions*, 1:3–5; and Greene, *French Secular Music*, 3–6.

¹⁰¹ Cf. Busse Berger, *Mensuration and Proportion Signs*, 164. In her assertion, Busse Berger probably referred to stacked Arabic numerals in particular, possibly building on Willi Apel's statement: "A notational detail of some interest are the signs 9/6, 8/6, 4/2, found in the discant of Anthonello's *Amour m'a le cuer mis* [...]. They indicate proportions and may well be the earliest known instance of this device [...]." *French Secular Music*, 9. Apel is not talking about proportion signs in general here but refers to stacked Arabic numerals in this context.

¹⁰² See, for example, Smilansky, "Rethinking *Ars Subtilior*," 227–38; and Vivarelli, *Le composizioni*, 73–74.

¹⁰³ Cf. Busse Berger, *Mensuration and Proportion Signs*, 183; and Stoessel, "The Captive Scribe," 1:298–99.

¹⁰⁴ Cf. Stone, "Writing Rhythm," 111–12; and *id.*, *The Manuscript Modena*, 44.

¹⁰⁵ This has already been remarked upon by Apel. Cf. *French Secular Music*, 30.

The full red notes in the cantus have been erased by the scribe and replaced by black void semibreves which indicate *sesquialtera* (3:2) proportion at the semibreve level.¹⁰⁶ Here one has to differentiate between the application at the minim level and at the semibreve level respectively (see FIGURE 2.30). In the $\frac{9}{6}$ sections, the semibreves are perfect, which becomes apparent by the arrangement of minim rests, semibreve-minim pairs, and several dots of division. The black void semibreves, on the other hand, are imperfect.

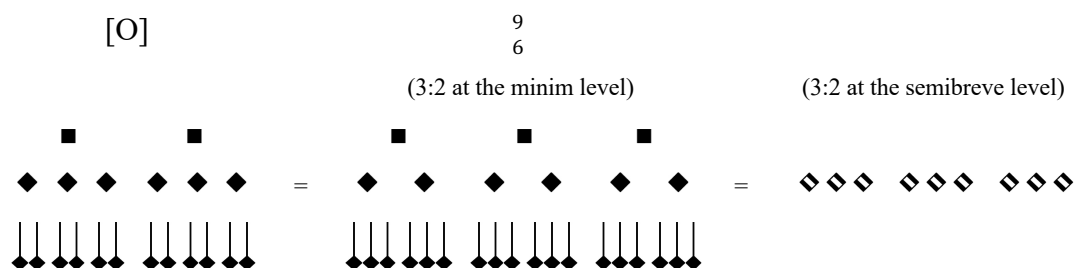


FIGURE 2.30: Variants of *sesquialtera* (3:2) proportion in *Amour m'a le cuer mis en tel martire*

A yet disregarded aspect of the stacked Arabic numerals in *Amour m'a le cuer* are the numbers used to display the proportion. Instead of the stacked Arabic numerals $\frac{3}{2}$ and $\frac{4}{3}$, $\frac{9}{6}$ and $\frac{8}{6}$ are used in order to display *sesquialtera* (3:2) and *sesquitercia* (4:3) proportion respectively, revealing a tendency to regard the whole *mensura* (i.e. six minims) as unit instead of cancelling numerator and denominator down to their lowest possible values. Admittedly, the stacked Arabic numerals of $\frac{4}{2}$ do not really fit into this picture, but this sign also describes a unit, namely that of an imperfect semibreve rather than a single minim. There remains a possibility that the numbers were picked randomly or that they were chosen in order to complicate matters, just as the reader has to figure out the three different interpretations of the original red coloration. The prospect that the figures in the denominators were chosen for their representation of rhythmical units (i.e. 6 for a breve and 2 for a semibreve in O) might, however, shed light on the perception of rhythmic proportions in Antonello's circle. It seems that proportions were not necessarily derived from the teachings of harmonic intervals (in which case $\frac{3}{2}$, $\frac{4}{3}$ and $\frac{2}{1}$ would have been the likeliest candidates for the stacked Arabic numerals in *Amour m'a le cuer*) but from the simultaneous use of different mensurations without minim equivalence. While $\frac{9}{6}$ and $\frac{8}{6}$ create special rhythms, however, it must be said the introduction of $\frac{4}{2}$ seems a bit superfluous in light of semiminims also being present in the piece.

¹⁰⁶ I agree with Smilansky who remarks: "Without the erasure, red would have to signify shortening in the cantus and lengthening in the tenor. The scribe clearly decided that an unusual non-proportional interpretation of coloration is more understandable than a proportional one, and changed the reading of the cantus to accommodate the use in the tenor." Smilansky, "Rethinking *Ars Subtilior*," 229.

Thus, the whole $\frac{4}{2}$ section could have been notated with minims and semiminims instead of semibreves and minims in *dupla* (2:1) proportion. *Amour m'a le cuer* can therefore be regarded as composition in which proportions are used in an experimental fashion, with several notational devices representing the same proportion and a seemingly redundant introduction of *dupla* (2:1) proportion.

Stacked Arabic numerals are also used in Antonello's ballade *Dame d'onour, en qui tout mon cuer maynt* (No. 82, fol. 40v).¹⁰⁷ Here they are used as alternative mensuration signs, which can easily be determined because of their position at the very beginning of cantus and contratenor, where mensuration signs would be expected to stand. The composition exhibits a canon, which does, however, not give instructions on the interpretation of the alternative mensuration signs but concerns certain applications of diminution or augmentation in the contratenor. In this ballade, $\frac{3}{2}$ indicates *tempus imperfectum* with *prolatio maior* (usually indicated by C) and $\frac{2}{3}$ indicates *tempus perfectum* with *prolatio minore* (usually indicated by O). Although stacked Arabic numerals are only used in this function in one other work that has survived, namely Goscalch's ballade *En nul estat*,¹⁰⁸ these alternative mensuration signs are described by at least two treatises: The Berkley manuscript¹⁰⁹ of 1375 and Johannes Ciconia's *De proportionibus*¹¹⁰ of 1411.¹¹¹ It is interesting that the number in the numerator indicates the *prolatio* and the number in the denominator indicates the *tempus*. The top-down reading of the stacked Arabic numerals coupled with the common practice of naming the *tempus* first and the *prolatio* second would suggest the opposite interpretation.

The change of *prolatio* and *tempus* indicated by the stacked Arabic numerals $\frac{2}{2}$ in Antonello's ballade is applied proportionally. Thus, $\frac{2}{2}$ is an alternative for C indicating *sesquitertia* (4:3) proportion.¹¹² Most curious is the notation in the cantus in the second part of the ballade. The sections starts in *sesquitertia* (4:3) proportion as indicated by $\frac{2}{2}$. After four *mensurae* in the relative mensuration ($\frac{3}{2}$ = C), the cantus features six red *dragmae*, which divide the rela-

¹⁰⁷ A recent edition can be found in Vivarelli, *Le composizioni francesi*, 150–52. Older editions include Apel, *French Secular Music*, 33*–4*; or *id.*, *French Secular Compositions*, 1:7–8; and Greene, *French Secular Music*, 11–13.

¹⁰⁸ *Ch*, fol. 39v and *PR*, fol. 79v.

¹⁰⁹ See Oliver B. Ellsworth, ed., *The Berkeley Manuscript University of California Music Library, MS. 744 (olim Phillipps 4450): A New Critical Text and Translation on Facing Pages*, Greek and Latin Music Theory 2 (Lincoln: University of Nebraska Press, 1984), 170.

¹¹⁰ See Oliver B. Ellsworth, ed., *Johannes Ciconia: Nova musica and De proportionibus: A New Critical Text and Translation on Facing Pages*, Greek and Latin Music Theory 9 (Lincoln: University of Nebraska Press, 1993), 443.

¹¹¹ Cf. Anne Stone, "The *Ars subtilior* in Paris," *Musica e storia* 10, no. 2 (2002), 383–85.

¹¹² Cf. Stoessel, "The Captive Scribe," 1:274.

tive *mensura* into six equal parts (see FIGURE 2.31).¹¹³ For the next *mensura*, the cantus switches back to $\frac{3}{2}$ (=C), thus the red *dragmae* could have been substituted by six black minims and a shift of the mensuration sign to the previous *mensura*. It seems to me that Antonello was intent on highlighting this particular *mensura*, since the tenor also features *dragmae*—an absolute exception in the otherwise steady voice part. The underlying words “tout dis” from the phrase “Et tout dis plus me double l’esperance / qu’en aucun temps dire pourai: «Amie»”¹¹⁴ do, however, not seem to carry an important enough meaning to explain the visual (cantus) and audible (tenor) accentuation. The phrase could, of course, be yet another citation which has not yet been discovered.¹¹⁵ It is long since known that Antonello cites Vailant’s virelai *Par maintes fois*¹¹⁶ in the refrain of his ballade.¹¹⁷ It may well be that he wanted to draw the attention to another citation by the prominent notation. Citation also plays an important role in the *Mod A* works of Philippotto da Caserta and in Johannes Ciconia’s *Sus un’ fontayne*, which will be discussed in the next chapter.

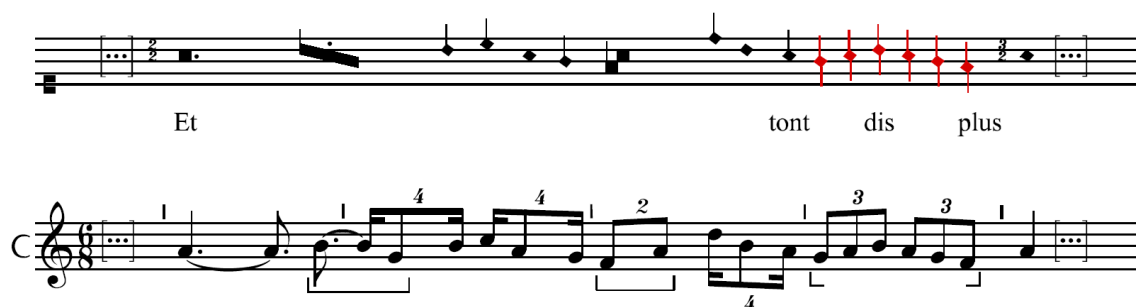


FIGURE 2.31: Transcription of second $\frac{2}{2}$ passage in the cantus of *Dame d'onour, en qui tout mon cuer maynt*

¹¹³ Measure 34 in Vivarelli’s edition (*Le composizioni francesi*, 151). According to Vivarelli, these are the only red *dragmae* to be found in Antonello’s *oeuvre*. Cf. *id.*, 27.

¹¹⁴ Transcription taken from Vivarelli, *Le composizioni francesi*, 98.

¹¹⁵ I thank Michael Scott Cuthbert for his search in his database (private communication 13 August 2017). Neither music nor text yielded promising search results: The words “tout dis” also appear in Bartholomeus Brollo’s *Nulx ne pouroit ymaginer* (*Ox 213*, fols. 37v–38r) and another anonymous *Mod A* work: the rondeau *Hors sui je bien de trestoute ma joye*—which is intriguingly situated on the bottom of two facing folios (12v–13r), which otherwise transmit works by Antonello.

¹¹⁶ *Ch*, fol. 60r. For other concordances see Yolanda Plumley and Anne Stone, eds., *Codex Chantilly, Bibliothèque du Château de Chantilly, Ms. 564: Introduction* (Turnhout: Brepols, 2008), 197.

¹¹⁷ Cf. Apel, *French Secular Compositions*, 1:XXXIII.

2.4 Proportion Signs in the Compositions of Filippotto da Caserta and Johannes Ciconia's *Sus un' fontayne*

Composer	Incipit	Genre	No.	fol(s). gather- ing	4:3	Comment(s)
Filippotto da Caserta	<i>En remirant vo douche portraiture</i>	Ballade	68	34v– 35r <i>Mod A_{IV}</i>	○	Only <i>Mod A</i> composition in which ○ is combined with ○. Red void notation indicates <i>sesquitertia</i> (4:3) proportion in <i>Ch</i> and <i>PR</i> .
Filippotto da Caserta (attributed to Galiot in <i>Ch</i>)	<i>En attendant, souffrir m'estuet grief payne</i>	Ballade	35	20r <i>Mod A_{II}</i>	○	<i>Mod A</i> version is the only one with proportion sign. <i>Dragmae</i> indicate <i>sesquitertia</i> (4:3) proportion in <i>Ch</i> , <i>PR</i> and <i>GR197/Dartmouth</i> . Possible allusion to Machaut in proportion sign passage.
Johannes Ciconia	<i>Sus un' fontayne</i>	Virelai	49	27r (26v) <i>Mod A_{III}</i>	○	Ballade cites three compositions by Filippotto da Caserta.

TABLE 2.32: Proportion signs in *Mod A* compositions by Filippotto da Caserta and Johannes Ciconia

The biographical evidence concerning Filippotto da Caserta is almost as non-existent as that of his namesake Antonello da Caserta. As in the case of Antonello, he probably came from the town Caserta near Naples. Further biographical evidence is scarce. Only seven French chansons and one Credo by Filippotto survive. However, he is generally believed to be the author of two theoretical treatises.¹¹⁸ Although Gilbert Reaney as well as Reinhard Strohm have stated that Filippotto's authorship of the *Tractatus figurarum* is partly confirmed by the fact that his surviving music exhibits the note shapes in a manner described by the treatise, his authorship has also been questioned on the same grounds by Philip Schreur.¹¹⁹ In support of Schreur's view, proportion signs are not described in *Tractatus figurarum* but do appear in the *Mod A* versions of *En attendant, souffrir m'estuet grief payne* and *En remirant vo douche portraiture* as well as in the ballade *Par le grant senz d'Adriane* transmitted in *Ch* and *Pit*. It speaks against Schreur, however, that all three pieces are transmitted using different means of rhythmic notation (see comments in TABLE 2.32), therefore it is possible that the proportion signs might not stem from Filippotto's hand. Furthermore, as I will show in Chapter 4 of this

¹¹⁸ *Tractatus figurarum* (see Philip E. Schreur, ed., *Tractus Figurarum – Treatise on Noteshapes: A New Critical Text and Translation on Facing Pages*, Greek and Latin Music Theory 6 (Lincoln: University of Nebraska Press, 1989)) and *Regule contrapuncti* (see Pier Paolo Scattolin, "Le *Regule contrapuncti* di Filippotto da Caserta," in *L'Ars nova italiana del Trecento V*, ed. Agostino Ziino (Certaldo: Edizioni centro di studi sull'Ars Nova italiana del Trecento, 1985), 231–44). For a recent discussion on authorship and circulation of these treatises see Giuliano Di Bacco, "Original and Borrowed, Authorship and Authority. Remarks on the Circulation of Philipoctus de Caserta's Theoretical Legacy," in *A Late Medieval Songbook and its Context: New Perspectives on the Chantilly Codex (Bibliothèque du Château de Chantilly, Ms. 564)*, eds. Yolanda Plumley and Anne Stone (Turnhout: Brepols, 2009), 329–64.

¹¹⁹ Cf. Strohm, *The Rise of European Music*, 59; and Gilbert Reaney, "Caserta, Philippus de [Philipoctus, Filipoctus]," *NG2* (London: MacMillan, 2001), 5:237; as well as Schreur, *Tractus Figurarum*, 7–8.

study, there is not necessarily any congruence between what a person says as theorist and what this person does as composer, as can be seen in the case of Ugolino da Orvieto.¹²⁰

Filippotto's ballade *Par les bons Gedeons et Sanson delivré*, which is a tribute to the Antipope Clement VII, has led some scholars to believe that Filippotto resided in Avignon at some point during Clement VII's reign from 1378 to 1394.¹²¹ However, Clement VII was elected pope at Fondi on 20 September 1378 and resided in and around Naples for the following eight months. And it seems just as plausible that Filippotto composed his ballade in Italy around that time, as has been suggested by Reinhard Strohm as well as by Giuliano Di Bacco and John Nádas.¹²² Another composition by Filippotto, his ballade *Par le grant senz d'Adriane*, is connected with the imprisonment of Queen Johanna of Naples and defends her adopted son Louis of Anjou's campaign to reclaim the Kingdom of Naples as well as Rome for Clement VII (1382–84).¹²³ Hence, it seems reasonable to doubt that Filippotto resided in France for a longer period or, as has been suggested by Wilkins, spend the majority of his life there.¹²⁴

Despite being attributed to Galiot in *Ch, En attendant, souffrir m'estuet grief payne* (No. 35, fol. 20r)¹²⁵ is generally believed to be by Filippotto da Caserta.¹²⁶ Gagnéviève Thibault's claim that Filippotto's ballade *En attendant, souffrir m'estuet grief payne* cites the motto 'Souffrir m'estuet' of Bernabò Visconti, ruler of Milan from 1354 to 1385, is generally ac-

¹²⁰ See Chap. 4.3.

¹²¹ Cf. Nino Pirrotta, "Scuole polifoniche italiane durante il sec. XIV: di una pretesa scuola napoletana," *Collettanea Historiae Musicae* 1 (1953): 14–16; Wilkins, "Some Notes on Philipoctus de Caserta," 86 and 88; and Reaney, "Caserta, Philippus de," 237.

¹²² Cf. Strohm, "Filippotto da Caserta," 69; and Giuliano Di Bacco and John Nádas, "The Papal Chapels and Italian Sources of Polyphony during the Great Schism," in *Papal Music and Musicians in Late Medieval and Renaissance Rome*, ed. Richard Sherr (Oxford: Clarendon Press, 1998), 46.

¹²³ Cf. Wilkins, "Some Notes on Philipoctus de Caserta," 84–86; and Stoessel, "The Interpretation of Unusual Mensuration Signs," 192–94.

¹²⁴ Wilkins, "Some Notes on Philipoctus de Caserta," 84–89.

¹²⁵ A recent edition can be found in Vivarelli, *Le composizioni francesi*, 116–17. Older editions include Apel, *French Secular Music*, 92*–93*; or *id.*, *French Secular Compositions*, 1:56–57; or Greene, *French Secular Music*, 125–27. Apel has remarked that the sign $\odot$ visible at the beginning of the cantus in *Ch* cannot be applied to the entire ballade, not realising that it is shine through from the *recto* of fol. 33. Cf. Apel, *French Secular Music*, 31. Greene has followed Apel's erroneous reading in his edition by beginning in a 9/8 metre.

¹²⁶ In his 1950 edition of French chansons, Willi Apel had argued for the Galiot attribution on stylistic grounds. Cf. Apel, *French Secular Music*, 31. After Suzanne Clercx had found that Ciconia cited the incipits of this composition and those of two other ballades by Filippotto in his virelai *Sus un' fontayne* (cf. Suzanne Clercx, *Johannes Ciconia: Un musicien liégeois et son temps (Vers 1335–1411)*, 2 vols. (Brussels: Palais des Académies, 1960), 1:57), Richard Hoppin was the first to announce that the music is also cited (for more see below) and therefore argued that Filippotto's authorship is more likely than Galiot's. Cf. Richard Hoppin, "Review of *Johannes Ciconia: Un musicien*," *The Musical Quarterly* 47, no. 3 (1961): 417. Ursula Günther suggested that the *Ch* scribe might have erroneously added the Galiot attribution to this particular ballade due to the fact that there are two other compositions beginning with the words "En attendant" which are both attributed to him. Cf. Ursula Günther, "Zitate in französischen Liedsätzen der Ars nova und Ars subtilior," *Musica Disciplina* 26 (1972): 63. Reinhard Strohm has offered the possible explanation that "'Jean Galiot' represents a mishearing of 'Jean-Galéas' Visconti", the French spelling of Bernabò Visconti's nephew and co-ruler Giangaleazzo, therefore the attribution could refer to the patron rather than to the composer. Strohm, *The Rise of European Music*, 60.

cepted.¹²⁷ Reinhard Strohm has suggested that the ballade is a tribute to the Milanese ruler, which Filippotto composed in the year of Bernabò's death (1385).¹²⁸ However, it is not confirmed that Filippotto was actually employed at the Visconti court.¹²⁹ The ballade seems to be part of some kind of citation game complex¹³⁰ involving the songs *En attendant, Esperance conforte* by Jacob Senleches¹³¹ and *En attendant d'avoir la douce vie* by Galiot.¹³² While Senleches' and Galiot's compositions exhibit complex *Ars subtilior* notation, Filippotto da Caserta's ballade is notated in a simpler manner. However, *En attendant, souffrir m'estuet grief payne* is the only piece of the citation game complex which contains proportion signs. Furthermore, only the *Mod A* version of the ballade features $\bigcirc$ in order to create *sesquitertia* (4:3) proportion at the minim level—the concordant versions in *Ch*, *PR* and the Grottaferrata/Dartmouth fragments¹³³ exhibit *dragmae*. The proportion sign passage lasts for only one and a half breves in the initial mensuration and accompanies the words “et très” from the phrase “Celle vertu li a Dieüs donée / qu'el puet souplir cascuns a soufissance / per dignité **et très** nouble pousance” (see FIGURE 2.33).¹³⁴ While these two words do not seem particularly special, a glance into the concordant sources reveals that a longer text part (“et très noble puissance”) accompanies the *dragmae* passage in *Ch*.¹³⁵ Taking all four words and the different spelling in *Ch* into account reveals a hitherto unrecognised Machaut citation: the phrase “De la très noble puissance” appears in Machaut's lai *Ne say comment commencer*.¹³⁶ Furthermore, the music of the proportion sign passages bears at least some resemblance to Machaut's musical setting (see FIGURE 2.34). Admittedly, pitch and rhythm deviate from the original, but the combination of text and music seem to be too much of a coincidence not to assume an

¹²⁷ G  n  v  e Thibault, “Embl  mes et devises des Visconti dans les oeuvres musicales du Trecento,” in *Atti del secondo convegno internazionale 17-22 luglio 1969, L'Ars nova italiana del Trecento* 3, ed. F. Alberto Gallo (Certaldo: Edizioni centro di studi sull'Ars Nova italiana del Trecento, 1970), 152.

¹²⁸ Cf. Strohm, “Filippotto da Caserta,” 71.

¹²⁹ Contrary to Strohm's suggestion, Anne Stone has put forward the hypothesis that Filippotto might have been seeking employment at Bernab  's court and interprets the ballade *En attendant, souffrir m'estuet grief payne* as application for a position, thus suggesting an earlier date of composition. Cf. Anne Stone, “A Singer at the Fountain: Homage and Irony in Ciconia's ‘Sus une fontayne’,” *Music & Letters* 82, no. 3 (2001): 374–75.

¹³⁰ For more on this see Strohm, “Filippotto da Caserta,” 69–70; *id.*, *The Rise of European Music*, 59–60; and Plumley, “Citation and Allusion,” 287–363.

¹³¹ *Mod A*, fols. 39v–40r (No. 79) and *Ch*, fol. 40r (attribution reading “Galiot” at the top but “Jacob de Senleches” at the end of the text residuum).

¹³² *Mod A*, fol. 40r (No. 81) and *Ch*, fol. 44r (attribution only in *Ch*).

¹³³ *Grottaferrata, Biblioteca dell'Abbazia (Badia Greca)*. Kript. Lat. 224 olim *Collocazione provvisoria* 197 (GR 197) and *Hanover, NH, Dartmouth College Library, Rauner Special Collections MS 002387* (Dartmouth). The latter fragments once belonged to the same manuscript and fit between fols. 3v and 4r of the Grottaferrata fragments. Filippotto's *En attendant* is thus situated on GR 197, fol. 3v and Dartmouth, fol. 1r.

¹³⁴ Measures 49–50 in Vivarelli's edition (Vivarelli, *Le composizioni francesi*, 117). Also see the diplomatic transcription of the proportion sign passage (Example 17) in *id.*, 70. Transcription of underlying text also taken from Vivarelli (*id.*, 86).

¹³⁵ “[digni]t   et tr  s noble” in *PR*, fol. 84v; and “et tr  s no[ble]” in *Dartmouth*, fol. 1r.

¹³⁶ Alternative title: *Le lay de l'ymage*. For sources and further information see Lawrence Earp, *Guillaume de Machaut: A Guide to Research* (New York and London: Garland Publishing, 1995), 348–49.

intended reference.¹³⁷ It appears that Filippotto made use of the special notational devices (proportion sign or *dragmae* respectively) to highlight the citation.

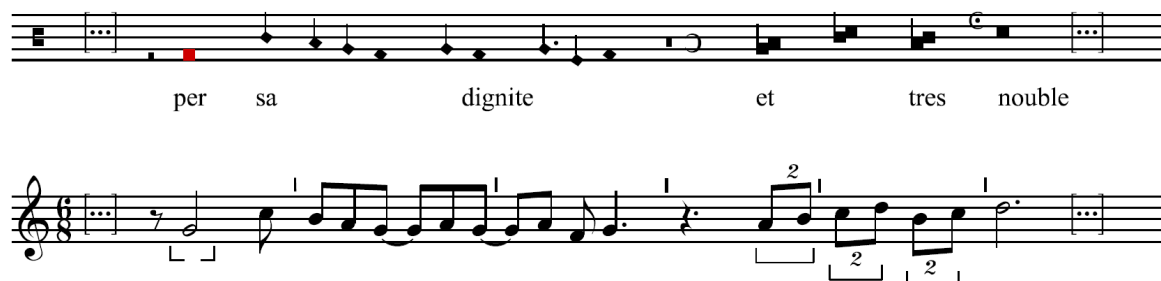


FIGURE 2.33: Transcription of $\circ$ passage in *En attendant, souffrir m'estuet grief payne*

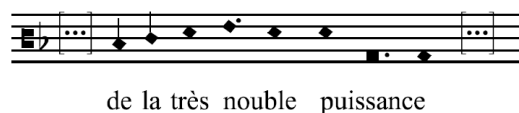


FIGURE 2.34: Setting of the words “De la très noble puissance” in Machaut’s *Ne say comment commencer*

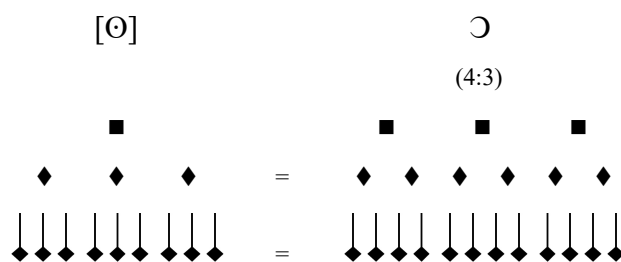
Machaut citations appear even more frequently in Filippotto’s other *Mod A* proportion sign ballade *En remirant vo douche portraiture* (No. 68, fols. 34v–35r).¹³⁸ The ballade also applies $\circ$, though in no less than twelve cases in cantus (once), tenor (once), and contratenor (ten times).¹³⁹ In addition to this extraordinarily frequent use, the ballade is also the only composition in *Mod A* in which $\circ$ is applied in combination with *tempus perfectum* with *prolatio maior* (see FIGURE 2.35).¹⁴⁰ Furthermore, only the *Mod A* version of the ballade features $\circ$ in order to create *sesquitertia* (4:3) proportion at the minim level—the concordant versions in *Ch* and *PR* have red void notes.

¹³⁷ That citations in *Ars subtilior* music may not always be precise has already been stated by Plumley: “The musical citations may be tucked away in the body of the new work (rather than in the traditional positions at the opening or close of the musical form) and they may be separated from, or lack entirely, their original text. Moreover, the reference may be less than precise, more in the nature of an allusion than a clear citation.” Yolanda Plumley, “Ciconia’s *Sus un’ fontayne* and the Legacy of Philipoctus de Caserta,” in *Johannes Ciconia: musicien de la transition*, ed. Philippe Vendrix (Turnhout: Brepols, 2003), 142.

¹³⁸ For the connections between Filippotto’s *En remirant* and Machaut see Plumley, “Ciconia’s *Sus un’ fontayne* and the Legacy,” 146–56.

¹³⁹ A recent edition can be found in Vivarelli, *Le composizioni francesi*, 123–25. Older editions include Apel, *French Secular Music*, 98*–99*; or *id.*, *French Secular Compositions*, 1:148–50; or Greene, ed., *French Secular Music*, 21–24.

¹⁴⁰ Although not being the case here, $\circ$ in $\circ$ can also be interpreted as *subsesquioctava* (8:9) proportion at the minim level. Cf. Busse Berger, *Mensuration and Proportion Signs*, 55; and Vivarelli, *Le composizioni francesi*, 68.

FIGURE 2.35: O in *En remirant vo douche portraiture*

While the O passages in the contratenor of *En remirant* only last for short durations of up to six minims in O in the majority of cases, there is one longer passage during which all three voices change to O at a certain point, thus highlighting the phrase “en la quel est tous doulz ymaginer” (see FIGURE 2.36).¹⁴¹ It has already been observed that the same text line appears in Matheus de Sancto Johanne’s ballade *Sans vous ne puis, tres douche creature*.¹⁴² However, Plumley has argued that Matheus de Sancto Johanne is more likely to have cited from Filippotto rather than the other way around.¹⁴³ In addition to Plumley’s line of argument, one could reason that the coloration in Matheus de Sancto Johanne’s ballade might just as well be signalling a citation. In that case, of course, it remains unclear if and from which source Filippotto is citing here or if perhaps both composers cite from an unknown source.

FIGURE 2.36: Transcription of longest O passage in *En remirant vo douche portraiture*

¹⁴¹ Measures 12–15 in the editions.

¹⁴² *Mod A*, fol. 15v (No. 27) and *Ch*, fol. 35r. For textual references between the ballades as well as connections with Machaut’s *En remirant vo gracieus viaire* see Plumley, “Ciconia’s *Sus un’ fontayne*,” 150–56.

¹⁴³ Cf. *id.*, 155.

While it remains speculative that the citations and allusions discussed so far were indeed intended, it is indisputable that Johannes Ciconia has cited three ballades by Filippotto da Caserta in his virelai *Sus un' fontayne*, namely *En remirant vo douche portraiture*, *En attendant, souffrir m'estuet grief payne*, and *De ma dolour ne puis trouver confort*.¹⁴⁴ Richard Hoppin was the first to suggest a student-teacher relationship between Ciconia and Filippotto based on this evidence.¹⁴⁵ This hypothesis has been challenged by Yolanda Plumley who suggests that it is “more plausible that Ciconia’s song was prompted by a desire to participate in [the above mentioned] musical [citation] game, rather than by some personal encounter with Philipoctus at the Visconti court.”¹⁴⁶

Ciconia’s virelai *Sus un' fontayne* (No. 49, fol. 27r(26v)) is certainly remarkable in several aspects.¹⁴⁷ First, the citations—musical as well as textual—are extensive and make up 20 per cent of the entire composition.¹⁴⁸ Second, *Sus un' fontayne* is the only work by Ciconia in the *Ars subtilior* style and one of only three French-texted compositions. Third, the virelai survives in two sources which feature different proportion and mensuration signs.¹⁴⁹ Since the notational features of the composition have been discussed extensively in the past, I will limit the discussion to those aspects relevant to the argument.¹⁵⁰

At first glance, Ciconia’s composition looks extremely complex in both its versions due to the assortment of signs present in all voices. Because of its unusual notation it has been

¹⁴⁴ See n. 126 above.

¹⁴⁵ Cf. Hoppin, “Review of *Johannes Ciconia*,” 417. Reinhard Strohm has gone so far as to consider this conclusion “inescapable” and has suggested that “the two men met under the patronage of the Visconti ‘fountain’.” Strohm, *The Rise of European Music*, 60. A comprehensive review of what is known about Johannes Ciconia’s biography can be found in Philippe Vendrix, “Johannes Ciconia, cantus et musicus,” in *Johannes Ciconia: musicien de la transition*, ed. Philippe Vendrix (Turnhout: Brepols, 2003), 7–37.

¹⁴⁶ Plumley, “Ciconia’s *Sus un' fontayne*,” 132. Giuliano Di Bacco and John Nádas have discovered that Ciconia was employed by Cardinal Philippe d’Alençon in Rome until 1397 (cf. Giuliano Di Bacco and John Nádas, “Verso uno ‘stile internazionale’ della musica nelle cappelle papali e cardinalizie durante il Grande Scisma (1378–1417): il caso di Johannes Ciconia da Liège,” in *Collectanea I, Capellae Apostolicae Sixtinaeque Collectanea Acta Monumenta* 3, ed. Adalbert Roth (Vatican City: Biblioteca Apostolica Vaticana, 1994), 32–33) thus considerably limiting the timespan for a possible employment at Giangaleazzo Visconti’s court in Pavia to 1397–1401 instead of 1390–1401, as has previously been suggested (cf. Nádas and Ziino, *The Lucca Codex*, 42–44). Plumley’s doubts concerning Filippotto’s association with the court during these years seem reasonable in light of the prominent position of Bernabò’s motto in *En attendant*. Cf. Plumley, “Ciconia’s *Sus un' fontayne*,” 159.

¹⁴⁷ For modern editions of this work see Apel, *French Secular Music*, 108*–9*; or *id.*, *French Secular Compositions*, 1:25–27; or Margaret Bent and Anne Hallmark, eds., *The Works of Johannes Ciconia*, Polyphonic Music of the Fourteenth Century 24 (Monaco: L’Oiseau-Lyre, 1985), 170–74.

¹⁴⁸ Cf. Günther, “Zitate in französischen Liedsätzen,” 62. Note that not only the cantus but all three voices from the original are cited (except for the *En attendant* citation, where only the cantus is borrowed from the original).

¹⁴⁹ The concordant version can be found on fol. 38v of the manuscript *Oxford, Bodleian Library, MS Canon. Pat. Lat. 229* (henceforth *Pad A*). It is unique in the case of *Mod A* that a concordant version transmits alternative proportion signs and not special note shapes. For more on this see below.

¹⁵⁰ Cf. Stone, “Writing Rhythm,” 115–30; *id.*, “A Singer at the Fountain,” 379–90; Stoessel, *The Captive Scribe*, 1:156–59 and 264–66; Bent and Hallmark, *The Works of Johannes Ciconia*, 216; Margaret Bent, “The Early Use of the Sign Ø,” *Early Music* 24, no. 2 (1996): 202–3; Anne Hallmark, “Some Evidence for French Influence in Northern Italy, c. 1400,” in *Studies in the Performance of Late Mediaeval Music*, ed. Stanley Boorman (Cambridge: Cambridge University Press, 1983), 207–9.

suggested that the virelai is the work of a student who is struggling with *Ars subtilior* notation.¹⁵¹ Ursula Günther, on the other hand, has argued that the composition is a real masterpiece, which demonstrates that even the most complex syncopations and rhythms can be displayed by using three standard mensurations and *sesquitertia* (4:3) proportion, thus abandoning coloration and special note shapes (see TABLE 2.37).¹⁵² The new mensuration is sometimes applied for the stretch of a single semibreve only, therefore mensural changes appear most frequently (see FIGURE 2.41 below).

Sign in <i>Mod A</i>	Sign in <i>Pad A</i>	Interpretation
O	O	<i>tempus perfectum</i> with <i>prolatio minor</i> [3,2]
C	⊖	<i>tempus imperfectum</i> with <i>prolatio minor</i> [2,2]
⊕	C	<i>tempus imperfectum</i> with <i>prolatio maior</i> [2,3]
⊖	3	<i>sesquitertia</i> (4:3) proportion at the minim level

TABLE 2.37: Mensuration and proportion signs in *Sus un' fontayne*¹⁵³

I agree with Günther in that the frequent mensural changes replace other means of notating rhythmic proportion, such as coloration or special note shapes. Ciconia demonstrates that all special rhythms in the virelai can be displayed by using three mensuration and one proportion sign. This can be observed most clearly in the citation sections of the virelai, where the coloration from the original piece is replaced by changes from ⊕ to C or O (see FIGURES 2.38–2.40).

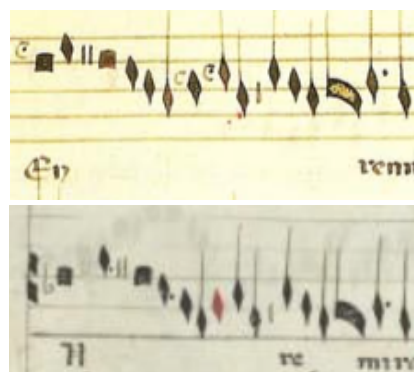


FIGURE 2.38: Comparison of *En remirant* citation in *Sus un' fontayne* (cantus, top) and *En remirant* (cantus, bottom)

¹⁵¹ Cf. Bent and Hallmark, *The Works of Johannes Ciconia*, 216.

¹⁵² Cf. Günther, “Zitate in französischen Liedsätzen,” 67.

¹⁵³ A similar table can be found in Stone, “Writing Rhythm,” 117; or *id.*, “A Singer at the Fountain,” 368. Note that C and ⊖ are mixed up in the *Pad A* column of Bent’s table (see Bent, “The Early Use of the Sign Ø,” 203).

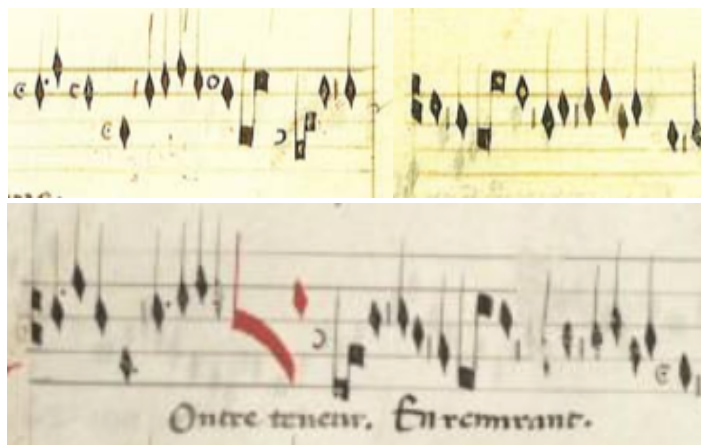


FIGURE 2.39: Comparison of *En remirant* citation in *Sus un' fontayne* (contratenor, top) and *En remirant* (contratenor, bottom)



FIGURE 2.40: Comparison of *De ma douleur* citation in *Sus un' fontayne* (contratenor, top) and *De ma douleur* (contratenor, bottom)

The compositions by Filippotto da Caserta under discussion in this chapter so far have suggested that—in the *Mod A* versions of pieces at least—proportion signs were used to signal citations or musical allusions. At first glance, this hypothesis is weakened by the case of *Sus un' fontayne*, in which the obviously intended citations are not signalled by the proportion sign $\circ$. However, I would argue that it is impossible to unite both ambitions. One cannot use $\circ$ in order to signal citations, if, on the other hand, the goal is to avoid coloration or special note shapes, as these would have to be used combined with $\circ$ to create the rhythms from the original. If one wants to cite as directly as Ciconia does in *Sus un' fontayne*, it is impossible to use $\circ$ without the aid of additional coloration, as the original music is in C or C .

All of the three citations are, however, signalled by mensuration signs (and a concomitant change in mensuration) in at least one of the voices. For the *En remirant* citation, cantus and contratenor change from O to C , for the *En attendant* citation, the cantus moves from C to C , and the tenor moves from O to C for the *De ma douleur* citation. It could be argued that Cico-

nia deliberately used a different mensuration prior to the citation so that its beginning could be marked by the sign.

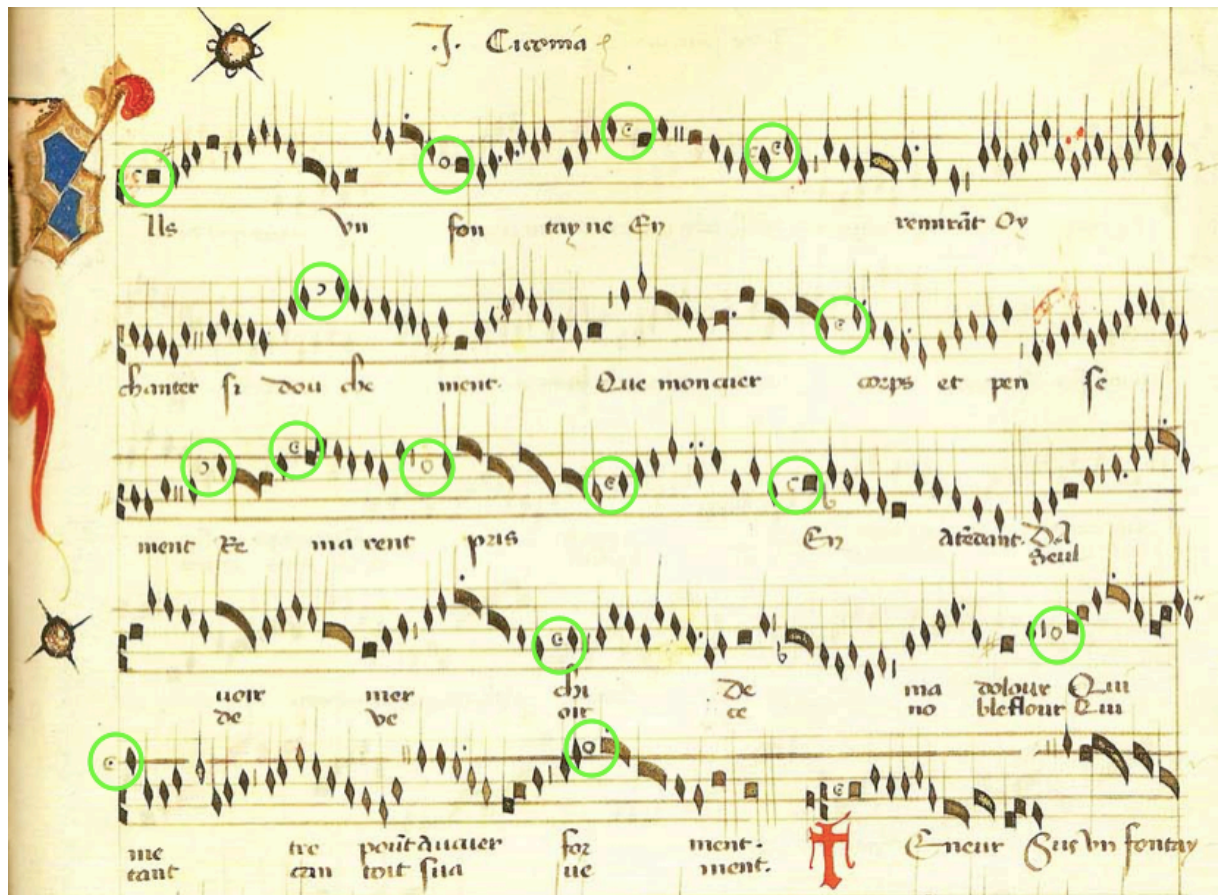


FIGURE 2.41: Cantus of *Sus un' fontayne* (proportion and mensuration signs highlighted)

There are, of course, more mensural changes in *Sus un' fontayne* than hitherto discovered citations (see FIGURE 2.41). This made me wonder whether there are perhaps even more citations to be found in the piece. Plumley reports a “possible reference to the ‘Esperance’ theme in *Sus une fontayne* [which] occurs immediately after the quotation of the opening phrases of *En attendant souffrir*.”¹⁵⁴ Changes from *prolatio maior* to *prolatio minor* for the duration of only one or a few notes have been a good indicator to detect citations in *Sus un' fontayne*, as these replace the coloration in the original. Unfortunately, they occur only in the already discussed citations. Although I have been unable to find other citations in *Sus un' fontayne*, I do not want to rule out the possibility that there are more citations and allusions in this piece. Given the probable loss of much of the original *Ars subtilior* repertoire, it is conceivable that the original music will never resurface. However, the evidence so far suggests that it is likely that these citations are announced by the mensural changes in the composition.

¹⁵⁴ Plumley, “Citation and Allusion,” 363 n. 130.

2.5 Other *Mod A* Compositions Containing Proportion Signs

In this last subchapter, I will discuss the four remaining *Mod A* proportion sign compositions: Corrado da Pistoia's *Se Doulz Espour ne me donne confort*, Bartolomeo da Bologna's *Que pena maior agitanda menti*, and the two anonymously transmitted ballades *Ma douce amour, je me doi bien complayndre* and *En un vergier clos par mesure*. TABLE 2.42 gives an overview over the signs found in these four pieces.

Composer	Incipit	Genre	No.	fol(s). gather- ing	2:1	2:3	3:1	3:2	4:3	Comment(s)
Corrado da Pistoia	<i>Se Doulz Espour ne me donne confort</i>	Ballade	61	31v <i>Mod An</i>		2		3		2 is applied cumulatively, restoring the initial mensuration.
Bartolomeo da Bologna	<i>Que pena maior agitanda menti</i>	Virelai	73	37r (36v) <i>Mod An</i>	2			3		Canon: "canon virelarie: ad figuram 2 i[n] dupla p[ro]portione cantetur. Ad 3 vero i[n] p[ro]p[or]tione emiola."
Anonymous (attributed to Hasprois in <i>Ch</i> and <i>Turin J.b.IX.10</i>)	<i>Ma douce amour, je me doi bien complayndre</i>	Ballade	52	28r <i>Mod Am</i>	2		4	3		Canon: "Canon ballate: ad figuram ternariam in proportionem sexquialtera cantetur. Ad binariam dupla. Ad quaternariam vero tripla cantetur."
Anonymous	<i>En un vergier clos par mesure</i>	Ballade	31	18v <i>Mod Am</i>					○	Piece comes with a canon, which does, however, not give advice on the interpretation of ○.

TABLE 2.42: Proportion signs in compositions by Corrado da Pistoia, Bartolomeo da Bologna and two anonymous *Mod A* works

Little is known about Corrado da Pistoia, the composer of altogether two songs in *Mod A*, one of which contains proportion signs. The ascriptions identify him as "frater" and he has therefore been identified with an Augustinian monk who is named in two documents in Flor-

ence from 1385 and 1410 respectively.¹⁵⁵ Due to striking similarities between Corrado da Pistoia's *Veri almi pastoris* (No. 72, fol. 36v) and Bartolomeo da Bologna's *Arte psallentes* (No. 74, fol. 37v–38r), it has been suggested that the two composers met at some point, the Council of Pisa being the likeliest occasion for such an encounter.¹⁵⁶ They might also have belonged to the papal chapel.¹⁵⁷

Proportion signs are not found in the above mentioned Latin-texted ballade *Veri almi pastoris*, a praise of the pope, but in Corrado da Pistoia's other *Mod A* work: *Se doulz espour ne me donne confort* (No. 61, fol. 31v). The piece contains two Arabic numerals as proportion signs: 3 indicating *sesquialtera* (3:2) proportion and 2 indicating *subsesquialtera* (2:3) proportion at the minim level. The ballade is remarkable as well as unique in *Mod A* due to the only cumulative proportion: the proportion sign 2 has to be applied cumulatively to the preceding proportion sign sections, which operate under 3.

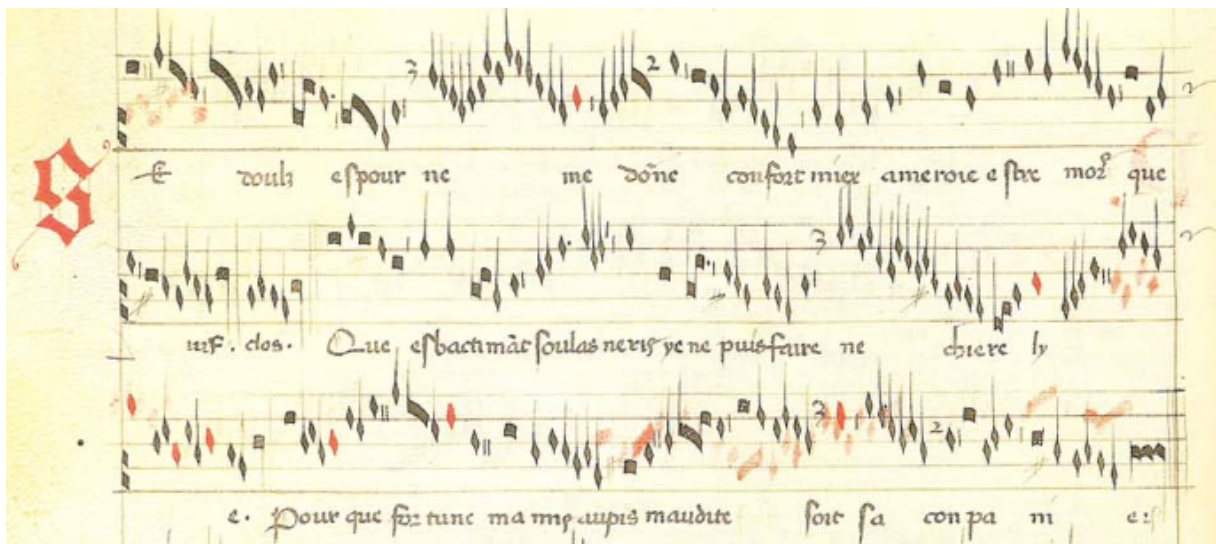


FIGURE 2.43: Cantus of *Se doulz espour ne me donne confort*

The proportion sign 2 follows 3 in the first and the third part of the ballade (see FIGURE 2.43). By indicating the reverse *subsesquialtera* (2:3) proportion, it cancels out *sesquialtera* (3:2) proportion and restores the initial mensuration *tempus imperfectum* with *prolatio minor* [2,2]. Usually in *Ars subtilior* repertoire, a proportion sign section is revoked by a mensuration sign, which would have to be C in this case, assuming that Corrado da Pistoia would have

¹⁵⁵ Cf. Michael P. Long, "Francesco Landini and Florentine Cultural Élite," in *Early Music History* 3 (1983): 98; and Stone, *The Manuscript Modena*, 71.

¹⁵⁶ Cf. Nino Pirrotta, "Il codice Estense Lat. 568 e la musica francese in Italia al principio del '400," *Atti della Reale Accademia di Scienze: Lettere e Arti di Palermo* 4, no. 5 (1946): 138–39; Günther, "Das Manuskript Modena, Biblioteca Estense, a.m.5,24 (olim Lat. 568 = Mod)," *Musica Disciplina* 24 (1970): 25–29; and Stone, *The Manuscript Modena*, 69–71.

¹⁵⁷ Pirrotta, "Il codice estense lat 568," 152–53.

used the standardised signs. Since he uses 2 instead of C, one could argue that 3 and 2 are simply alternative mensuration signs to C and C. Unfortunately, the only other piece by Corrado da Pistoia (*Veri almi pastoris*) does not use mensuration signs, so we do not know if 3 and 2 were indeed the mensuration signs he would have used instead of the standardised ones C and C. This alternative mensuration sign hypothesis, however, would call for minim equivalence to be lifted between cantus and the other two voices when the cantus has 3 alone. Parallel mensurations without minim equivalence do occur in other pieces in *Mod A* (see TABLE 2.3 on p. 107 above for details). Corrado da Pistoia's other piece *Veri almi pastoris*—even if it does not exhibit mensuration signs—gives a clue on the composer's habits in the use of simultaneous mensurations, however. The Latin-texted ballade has *tempus imperfectum* with *prolatio maior* [2,3] in tenor and contratenor but *tempus imperfectum* with *prolatio minor* [2,2] in the cantus. Here, minim equivalence is given so that the *mensurae* of the cantus are shorter than those of the other two voices. As *Veri almi pastoris* does hint that minims are generally equivalent in Corrado da Pistoia's compositions, I would argue that 3 should be interpreted as proportion sign in this ballade. And as 2 has the same visual appearance as 3, it too should be interpreted as proportion sign in this context. The proportion (2:3) has to be applied cumulatively then, which—as stated above—is unique in *Mod A*, assuming the interpretation within the same voice.

Another interesting feature is the proportion sign 3 in the tenor.¹⁵⁸ It occurs simultaneously with 3 in the contratenor, though not with 3 in the cantus (see FIGURE 2.44). Due to this coinciding of the only proportion signs in tenor and contratenor, I would suggest that the proportion sign 3 in these two voices has a double function, indicating proportion on the one hand, but the beginning of the rhythmical change in both voices on the other hand. The two singers can use the sign as point of orientation within the second part of the ballade.



FIGURE 2.44: Transcription of proportion signs in all three voices in *Se doulz espour ne me donne confort*¹⁵⁹

¹⁵⁸ Note that the tenor is unique in that it is not named “teneur” by the scribe, as he has done throughout *Mod AII–IV* for all French songs, but simply “tenor”. Cf. Stone, *The Manuscript Modena*, 53.

¹⁵⁹ Transcription taken from Haring and Boeke, *Modena Codex*, 239.

Interestingly, *Se doulz espour* shares parts of its refrain with another piece, namely the ballade *Bien dire et sagement parler* from *Ch* (fol. 51r), which is attributed to Solage.¹⁶⁰ The line “maudite soit sa companie” from *Se doulz espour* is very closely related to the entire text of the refrain of *Bien dire* which has “Maudite soit dont leur conpaignie.” Given the connection, I hoped to find a direct citation from the music of *Bien dire* in *Se doulz espour* as well but was unlucky. However, one could argue that there is at least an allusion to the beginning of the refrain of *Bien dire* in the proportion sign section of *Se doulz espour* (see FIGURE 2.45). Both excerpts start on *g'*.

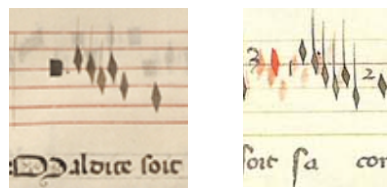


FIGURE 2.45: Comparison of *Ch*'s *Bien dire* (left) with *Mod A*'s *Se doulz espour* (right)

If there is musical allusion to another song in this section, it is of course reasonable to suspect that the other proportion sign sections in the cantus also cite or allude to other material. I discovered that the line “faire ne chiere lye” of the second proportion sign section appears in a very similar version (“ne feray chiere lye”) in the residuum of Pierre Fontaine's *J'ayme bien celui*, which is transmitted in altogether three sources.¹⁶¹ I could not establish any musical connection between the two songs, however. And I have also been unable to find the line “ne me donne confort”¹⁶² from the first proportion sign section anywhere but in Corrado da Pistoia's song. Still, *Se doulz espour* might yet be another piece to show that rhythmically emphasised song sections sometimes cite or allude to other song material.

Bartolomeo da Bologna has been identified as a Benedictine monk who was prior of the convent of San Niccolò of Ferrara.¹⁶³ He is documented as the cathedral's organist there from 1405 to 1427. As stated above, he might have been a singer in the papal chapel and it is likely that an encounter took place between Corrado da Pistoia and him during the Council of Pisa. His altogether seven works are transmitted in *Mod A*, *Ox 213*, and *PR*.

¹⁶⁰ Cf. Stone, *The Manuscript Modena*, 125.

¹⁶¹ *Bologna Q15*, fol. 281r; *Ox213*, fols. 17v–18r; and *EscSL V.III.24*, fols. 49v–50r. For different readings of the text in the three sources see David Fallows, “French as a Courtly Language in Fifteenth-Century Italy: The Musical Evidence,” *Renaissance Studies* 3, no. 4 (1989): 436.

¹⁶² I also searched for the version with “comfort” instead of “confort”.

¹⁶³ Cf. Hans Schoop, “Bartolomeo da Bologna [Bartholomeus de Bononia],” in *NG2* (London: MacMillan, 2001), 2:822.

The notation of his Latin-texted virelai *Que pena maior agitanda menti* (No. 73, fol. 37r (36v)) is very complex, not necessarily due to the two proportion signs 2 and 3, but due to special note shapes being used in the song, especially in the contratenor (see FIGURE 2.46). Two note shapes are even unique within the *Mod A* repertoire, a half black/half hollowed *sem-idragma* (see first note in FIGURE 2.46) and a breve with a downward tail (see fourteenth note in the right part of FIGURE 2.46).¹⁶⁴

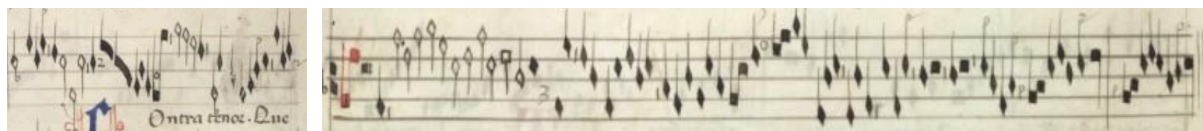


FIGURE 2.46: Excerpts from contratenor of *Que pena maior agitanda menti*: beginning (left) and end (right)

The meaning of the proportion signs is explained in a canon accompanying the composition: “canon vielarie: ad figuram 2 i[n] dupla p[ro]portione cantetur. Ad 3 vero i[n] p[ro]po[r]tione emiola.”¹⁶⁵ That 2 indicates *dupla* (2:1) and 3 indicates *sesquialtera* (3:2) proportion respectively could probably have been figured out without the canon’s explanation. The canon instructions are perhaps even more surprising in light of the partly unique special note shapes, which are the true difficulty regarding the interpretation of the song. It has been suggested that the absence of instructions on the interpretation of the special note shapes further underlies the message of the text, in which a musician complains about an ignorant audience.¹⁶⁶ Smilansky has summarised that

“Th[e] ability to ‘figure out’ the notation seems to have been a part of the musical currency of the time, and was perhaps used to show the authoritative ability of the composer in creating personal notational devices, and to differentiate between readers of different abilities. This fits well with the examples in *Que pena maior*, as the skill of the misunderstood musician of the text is enhanced by the special notational use, and readers are accepted into his circle only after demonstrating that they can perform the work and figure out the notation. The song further highlights such attitudes towards special note-shapes, as its canon instruction explains only the two proportion-indicating numerals, leaving the interpretation of the myriad special note-shapes to the reader.”¹⁶⁷

¹⁶⁴ Cf. Haring and Boeke, *Modena Codex*, 287.

¹⁶⁵ Transcription taken from Stone, *The Manuscript Modena*, 129.

¹⁶⁶ “The subject of this virelai distances itself from the former Latin-texted ballades, taking up the familiar vein of the ‘musician’s complaint’ with heavy emphasis on the poetic and musical ‘I’. [...] In *Que pena maior*, the poet-musician, in concert with the Muses, sings and plays to an indifferent audience: only those that understand that music is the way to virtue shall see the glorious wisdom of Apollo.” Stoessel, *The Captive Scribe* 1:126. A translation of the song can be found in *id.*, 125–26.

¹⁶⁷ Uri Smilansky, “Rethinking *Ars Subtilior*,” 163.

The presence of special note shapes together with proportion signs within the same piece leads to different displays of the same rhythm, which have been discussed elsewhere.¹⁶⁸ It is noteworthy, however, that special note shapes and proportion signs never coincide in this composition. Even though some proportion sign sections last for several *mensurae*, special note shapes are not to be found in these parts. It is therefore not surprising that the contratenor uses the proportion signs 2 and 3 only once, as this voice contains many special note shapes. In the cantus, on the other hand, 2 and 3 appear frequently, altogether 13 times. Interestingly, the appearances of 2 and 3 in the contratenor always coincide with the same proportion in the cantus, hence the signs could also function as coordination signs, at least in the contratenor.¹⁶⁹ It should be mentioned that the proportions are not applied cumulatively in this virelai, although one sign follows the other in many places in the cantus. In that, the virelai differs from Corrado da Pistoia's *Se doulz espour*, which uses the same signs but in which 2 is applied cumulatively to 3.¹⁷⁰

I have not found citations or allusions in the proportion sign sections of the song. I also do not see a pattern for the use of proportion signs regarding melismata in this composition. Some proportion sign sections coincide with melismata, other sections are fully texted. The use of *dupla* (2:1) proportion seems a bit pointless in light of semiminims being present in the piece anyway. Why not use these semiminims instead of taking the circuit with the proportion? All the rhythms in the *dupla* (2:1) proportion sections could have been displayed in the initial mensuration (*tempus perfectum* with *prolatio minor* [3,2]). This makes it seem even more likely that the passages of *dupla* (2:1) proportion are meant to highlight the music or the text. Alas, I was unable to find a satisfying solution to this puzzle. I should mention that Haring and Boeke report a “free citation of the famous opening of *En attendant souffrir*”¹⁷¹ in the beginning of *Que pena maior* (see FIGURE 2.47). I have to admit, however, that I do not see much of a resemblance apart from the red-colored note in the beginning.



FIGURE 2.47: Beginnings of *En attendant souffrir* (left) and *Que pena maior* (right)

¹⁶⁸ Cf. Stoessel, *The Captive Scribe*, 1:191.

¹⁶⁹ Cf. measures 3 and 54 in modern editions, for example, Haring and Boeke, *Modena Codex*, 284 and 286.

¹⁷⁰ See discussion above for details.

¹⁷¹ Haring and Boeke, *Modena Codex*, 287.

The ballade *Ma douce amour, je me doi bien complayndre* (No. 52, fol. 28r) is transmitted anonymously in *Mod A*, but attributed to Hasprois in *Ch* and *Turin J.b.IX.10*, the latter transmitting only the text of the ballade. With altogether three concordant sources *Ma douce amour* is one of the most widely transmitted songs among the proportion sign pieces in *Mod A*.¹⁷² While *Mod A* and *Ch* have full black notes, the *Ox213* version has hollowed notes (see FIGURE 2.48). It is noteworthy that all three music sources exhibit the same proportion signs: 2 for *dupla* (2:1), 3 for *sesquialtera* (3:2), and 4 for *tripla* (3:1) proportion and none of the versions uses coloration.



FIGURE 2.48: *Ma douce amour* in *Mod A* (left), *Ch* (middle), and *Ox213* (right)

The piece comes with a canon which is transmitted in all three music sources: “Canon ballate: ad figuram ternariam in proportione sexquialtera cantetur. Ad binariam dupla. Ad quaternariam vero tripla cantetur.”¹⁷³ These verbal instructions were probably necessary because using 4 in order to indicate *tripla* (3:1) proportion is counterintuitive and therefore confusing. The Arabic numeral 3 was no option, as it was already used to indicate *sesquialtera* (3:2) proportion. The composer is very consistent in using Arabic numerals for proportions in *Ma douce amour*. It would have been possible to use red ink in order to indicate *sesquialtera* (3:2) proportion and use 3 for *tripla* (3:1) proportion but apparently the aim was to strictly use numbers to indicate rhythmic proportion in the ballade.

The proportion signs only appear in the cantus. Tenor and contratenor remain in *tempus perfectum* with *prolatio minor* [3,2] throughout the piece. In the cantus, the signs always appear in succession towards the end of the ballades’ three different parts, thereby creating

¹⁷² *Ch*, fol. 34r; *Ox213*, fol. 123r; and *Turin J.b.IX.10*, fol. 5v.

¹⁷³ Transcription taken from Stone, *The Manuscript Modena*, 121.

stepwise faster notes in the melismata, moving from *sesquialtera* (3:2) to *dupla* (2:1) proportion—and to *tripla* (3:1) proportion in the third part (see transcription of FIGURE 2.49 for an example).¹⁷⁴

The image shows a musical score transcription of the second part of 'Ma douce amour'. It consists of two systems of music, each with three staves (treble, alto, and bass clefs). The first system starts at measure 23. The lyrics are: '3. Tant cum vi - vray Las! n'a-ray bien né sou - las Quant ye ne' and '7. Si ne puis mais s'ain - si sou-vent m'a - noi - e Quant si long [lon-gue]'. Above the first staff, there are tempo markings: 'Ch, Ox:' and 'F'. Above the second staff, there is a marking 'Ms: F'. The second system starts at measure 30. The lyrics are: 'voy vo gen - te por - tray - re.' and 'sui de vo bel - le fi - gu - re.'. Above the first staff, there are tempo markings: 'Ch, Ox:' and '2/4'. Above the second staff, there is a marking '4/4'. The score includes various musical notations such as notes, rests, and bar lines.

FIGURE 2.49: Transcription of second part of *Ma douce amour*¹⁷⁵

Although the proportion sign sections follow one after the other, the proportions are not applied cumulatively in this ballade. The canon instructions leave no doubt about this. Otherwise, there would, perhaps, have been the possibility of a different interpretation of 4 other than *tripla* (3:1) proportion.

Due to the successive use of proportion sign sections to create increasingly faster notes in the melismata of *Ma douce amour*, I have refrained from searching for possible citations in the proportion sign passages. It seems clear to me that the composed accelerando in the cantus is the function of the proportion signs in the ballade.

¹⁷⁴ Note that the *Ch* and *Ox* 213 versions move to *tripla* (3:1) proportion in the second part already, while *Mod A* remains in *dupla* (2:1) proportion (see FIGURE 2.49). Cf. Haring and Boeke, *Modena Codex*, 211.

¹⁷⁵ Transcription taken from Haring and Boeke, *Modena Codex*, 210.

The final piece to be discussed in this chapter is the anonymous ballade *En un vergier clos par mesure* (No. 31, fol. 18v), which is unique to *Mod A*. Its poetic topic is a lily, the ‘*fleur de lis*’, which most likely alludes to the emblem of the French royal family. A connection in form of the possibility of a dedicatory ballade has been suggested before by Stone.¹⁷⁶ Stoessel has presented an even more specific hypothesis by identifying several textual relationships between *En un vergier* and Philipotto da Caserta’s *Par le grant senz d’Adriane*, thereby connecting the song to the Angevin struggle for Naples.¹⁷⁷ He claims that

“*En un vergier* is a song in praise of a socially desirable figure, including a veiled allusion to a possible alliance with its dedicatee. It is plausible to imagine Alexander V, John XXIII, or someone in their circle commissioning a composer to write this song for the newly arrived Louis II as an invitation to reaffirm his alliance with the conciliar popes.”¹⁷⁸

Stoessel also suggests, that the “[compositional] techniques used in *En un vergier* [...] are typical of the Ars subtilior style that Louis II, as a child of the late fourteenth century, might have appreciated.”¹⁷⁹

As throughout *Mod A*, $\circ$ indicates *sesquitertia* (4:3) proportion at the minim level in *En un vergier*. While the proportion sign only appears once in the cantus (see FIGURE 2.50, second staff), the contratenor has four prolonged passages in $\circ$ and even starts with this proportioned rhythm (compared to cantus and tenor). The rhythmic complexity in this ballade is not caused by the proportion sign and the mensural changes in the cantus in the first part, however, but by diminution in the cantus in the second and third part.¹⁸⁰ Verbal canon instructions explain how this diminution should be executed by the singer: “Secundus et tertius pu[n]ctus canta[n]tur cise per semi.”¹⁸¹ That no sign is present to indicate this diminution, while all the other mensural changes are indicated by an array of mensuration signs, is remarkable.

It is noteworthy that $\circ$ never occurs in the same place in cantus and contratenor. On the contrary, the two voices seem to alternate rhythmical changes. As can be seen in FIGURE 2.50, the cantus has frequent changes of mensuration, which do not cause proportional rhythms (except for $\circ$), but which nevertheless may be perceived as audibly different from the tenor, which is in *tempus imperfectum* with *prolatio maior* [2,3]. These changes almost always coincide with the contratenor’s changes from $\circ$ to $\textcircled{C}$. It seems that one voice part at a time takes

¹⁷⁶ Cf. Stone, *The Manuscript Modena*, 100. For other compositions containing allusions to the *fleur de lis* and their connection to the French royal family see Stone, *The Manuscript Modena*, 100 n. 191.

¹⁷⁷ Cf. Stoessel, “The Angevin Struggle for the Kingdom of Naples,” 8–9.

¹⁷⁸ Stoessel, “The Angevin Struggle for the Kingdom of Naples,” 9.

¹⁷⁹ Stoessel, “The Angevin Struggle for the Kingdom of Naples,” 9.

¹⁸⁰ For details on this mensural change in the cantus see discussion of *En un vergier clos* in Nors S. Josephson, “Vier Beispiele der Ars Subtilior,” *Archiv für Musikwissenschaft* 27, no. 1 (1970): 47–48.

¹⁸¹ Transcription taken from Haring and Boeke, *Modena Codex*, 133.

over the rhythmically interesting part while the other joins the tenor's pace. Hence, the words “clos par mesure” might be taken literally, meaning that the musical flourishes of the piece (the sections in $\circ$) are enclosed by ordinary mensurations.¹⁸²



FIGURE 2.50: First part of cantus of *En un vergier clos par mesure*

It can also be observed that the changes of mensuration in the cantus always coincide with melismata in the first part. Since these are only single words and as the contratenor is untexted, I will not begin to search for possible citations indicated by the signs here. However, even if it is not likely that the $\circ$ -passages contain citations on a textual level, it has been suggested that the tenor might cite the music of a popular song, because its refrain is prominently written in red ink (see FIGURE 2.51).¹⁸³ Still, a possible reference has not yet been identified.

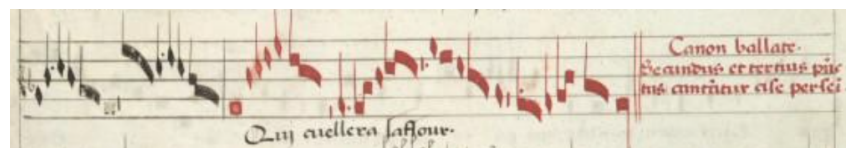


FIGURE 2.51: Final staff of tenor voice of *En un vergier clos par mesure*

¹⁸² I thank Oliver Huck for pointing out the possibility of a textual reference to the proportion sign sections.

¹⁸³ Cf. Stone, *The Manuscript Modena*, 101 n. 192.

2.6 Conclusion

The discussion has shown that proportion signs in *Mod A* are used in various different ways and capacities. They most often appear in melismata where they are used to create small note values. We can frequently find these melismata in the works of Matteo da Perugia, especially his sacred works, where $\bigcirc$ is often combined with minims or semiminims to create fast notes. The best example outside Matteo's *oeuvre* is certainly the anonymous ballade *Ma douce amour, je me doi bien complayndre* (No. 52), in which the proportion signs are used to create increasingly faster notes for the melisma, even using *tripla* (3:1) proportion, which is unique in *Mod A* and also the proportion with the greatest capacity to create small note values among all the five rhythmic proportions being used in the manuscript. We will encounter a similar composed accelerando as in *Ma douce amour* in the virelai *Je prens d'amour noriture* from *Turin* in the next chapter.

There are several possible functions which the proportion signs can have in addition to the indication of rhythmic proportion. One such function is the indication of a citation by the visual (proportion sign) and audible (change of rhythm) highlighting of certain words and/or notes. As I have shown throughout this chapter, there are several incidents in which an intended citation is likely or at least possible: (1) the text citation of and musical allusion to Machaut in the proportion sign section of Filippotto da Caserta's *En attendant souffrir m'estuet grief payne* (No. 35), (2) the highlighting of the phrase "en la quel est tous doulz ymaginer" in *En remirant vo douche portraiture* (No. 68), which also appears in the probably later *Sans vous ne puis, tres douche creature* by Matheus de Sancto Johanne, (3) and finally the two sections in Corrado da Pistoia's *Se doulz espour ne me donne confort* (No. 61), in which text and/or music of other works are used.

The coordination between voices due to the simultaneous beginning of proportion sign sections might also have been a reason to introduce these proportion signs to the songs, especially when they appear in the contratenor or even the tenor. Examples for these simultaneous proportion sign sections are Matteo da Perugia's *Già da rete d'amor libera et sciolta* (No. 96), Corrado da Pistoia's *Se doulz espour ne me donne confort* (No. 61), and Bartolomeo da Bologna's *Que pena maior agitanda menti* (No. 73).

In some pieces, the proportion signs seem to complicate matters unnecessarily. Proportion signs can appear in contexts, in which the same rhythm could have been displayed by conventional notation or where other means to display the same rhythms are also present in the composition. The inconspicuous number 2 at the beginning of Matteo da Perugia's *A qui*

Fortune ne se vult amer (No. 88) is an example, as well as Antonello da Caserta's *Amour m'a le cuer mis en tel martire* (No. 63), in which proportion signs are strictly speaking redundant because the coloration in the piece achieves the same rhythms.¹⁸⁴ This playing around with musical notation—which often makes things more complicated—has already been observed in Baude Cordier's *Belle, bonne, sage*.¹⁸⁵ I will address these riddle-like tendencies in the musical notation of the *Ars subtilior* in the next chapter, because there are some pieces in *Turin*, which carry these notational games to extremes.

One can observe that the majority of proportion signs in *Mod A* are used in a rather standardised manner. $\circ$ always indicates *sesquitertia* (4:3) proportion and there is only slight variation in the interpretation of the single Arabic numerals 2 and 3. The consistency is especially given in the later layer of the manuscript (*Mod A_{I-V}*) but this is not surprising as it almost exclusively transmits compositions by Matteo da Perugia, who only uses $\circ$ in his compositions (with the exception of the above-mentioned inconspicuous number 2 at the beginning of *A qui Fortune ne se vult amer*).

My impression that $\circ$ was already well established as proportion sign which indicates *sesquitertia* (4:3) proportion when *Mod A* was compiled is reinforced by the fact that the sign is seldomly explained in a canon. The only exception in *Mod A* is Antonello da Caserta's *virelai Tres nouble dame souverayne* (No. 53), which does give instructions on the interpretation of $\circ$. Arabic numerals, on the other hand, are more often explained in canons. Perhaps this is due to these single Arabic numerals being less standardised than $\circ$. Another reason could be that Arabic numerals also appear elsewhere in the manuscript, but not as proportion signs:¹⁸⁶

- (1) The tenor of the *Gloria* (No. 3)—probably by Matteo da Perugia—contains the numbers 8 and 3 (see FIGURE 2.52), which indicate the number of breves that the contratenor has to wait before its canonic pursuit, which is implied by the instruction “Tenor faciens contratenorem”.
- (2) The Arabic numerals 2, 3, 4, and 5 are found in the *Gloria* (No. 4) where they number the *talee* of the isorhythmic composition in the two upper voices (see FIGURE 2.53).¹⁸⁷ As these Arabic numerals appear in the first gathering of the later layer (*Mod A_{I-V}*) and the canons can be found in the earlier layer (*Mod A_{II-IV}*), however, a connection is not particularly likely.

¹⁸⁴ The same is partly true for Bartolomeo da Bologna's *Que pena maior agitando menti*. This is discussed in Stoessel, *The Captive Scribe*, 1:191.

¹⁸⁵ See Chap. 1.4.2.

¹⁸⁶ Cf. Fischer and Gallo, *Italian Sacred and Ceremonial Music*, 271; and Stoessel, *The Captive Scribe*, 1:291 n. 28.

¹⁸⁷ Stoessel reports that the Arabic numerals 1–5 are used in this piece, but I have so far failed to locate the number 1 anywhere in the different voice parts. Cf. Stoessel, *The Captive Scribe*, 1:291 n. 28.

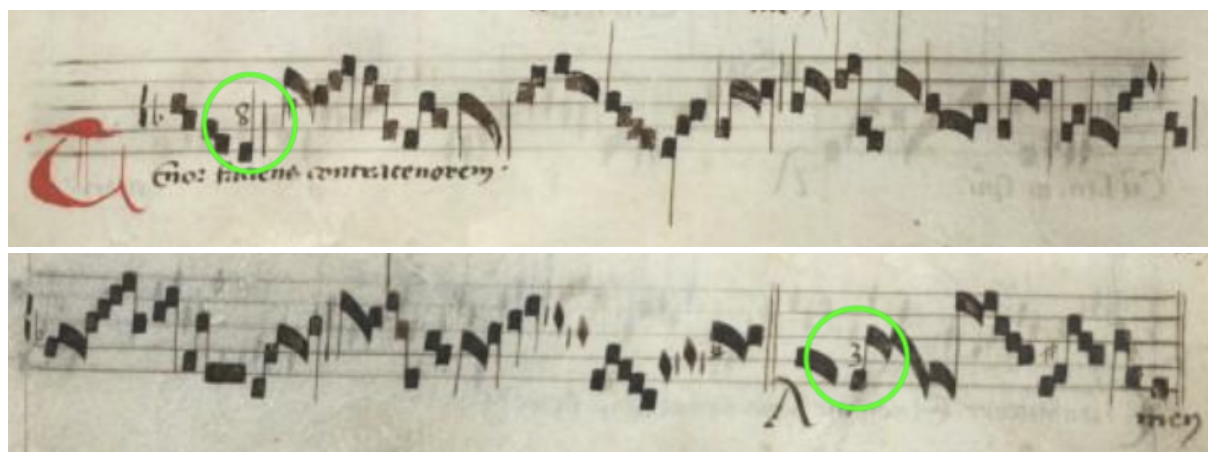


FIGURE 2.52: Arabic numerals in the *Gloria* (No. 3): fol. 2v (top) and fol. 2r (bottom)

The image shows a full page of musical notation from fol. 4r of the Gloria (No. 4). The page contains seven staves of square neumes on four-line red staves, with Latin text written below each staff. Several Arabic numerals are circled in green: '2' on the second staff, '7' on the fourth staff, '4' on the fifth staff, and '9' on the seventh staff. The text includes: "Et in terra pax hominibus bone uoluntatis Laudamus te Benedicimus te Adoramus te Glorificamus te Gratias agimus tibi propter magnam gloriam tuam Domine deus rex celestis deus pater omnipotens Domine fili unigenite ihu xpe domine deus agnus dei filius patris Qui tollis peccata mundi miserere nobis Qui tollis peccata mundi suscipe deprecationes nostras Qui sedes ad dexteram patris miserere nobis Quomodo tu solus sanctus Tu solus dominus Tu solus altissimus ihu xpe Cu sancto spiritu in gloria dei patris Amen".

FIGURE 2.53: Arabic numerals in the *Gloria* (No. 4), fol. 4r

While there is standardisation concerning the three proportion signs $\circ$, 2, and 3 on the one hand, *Mod A* also bears signs of individualisation, especially in the two ballades *Amour m'a le cuer mis en tel martire* and *Dame d'onour, en qui tout mon cuer maynt* by Antonello da Caserta, which contain different stacked Arabic numerals. The three discussed *Mod A* compositions by Antonello give the impression that the composer experimented with different visual appearances of proportional rhythms. The anonymous ballade *Ma douce amour, je me doi bien complayndre* also contains a proportion sign which only appears once in the manuscript, namely the Arabic numeral 4. As stated above, the indication of *tripla* (3:1) by that particular number is not the most obvious choice.

But even if the signs themselves or their interpretation are not particularly special, some proportion signs point towards individualisation due to the context in which they are used. (1) The application of *dupla* (2:1) proportion to an entire voice part in Matteo da Perugia's *A qui Fortune ne se vult amer* is unique in *Mod A*. (2) Johannes Ciconia's effort to display all rhythmic proportion in *Sus un' fontayne* by mensuration and proportion signs is also rather special. (3) And finally, the use of a cumulative proportion in Corrado da Pistoia's *Se doulz espour ne me donne confort* is unique to the manuscript.

All things considered, *Mod A* with its eight different proportion signs and five different rhythmic proportions is not the manuscript with the most elaborate rhythmic notation where proportion signs are concerned. The lack of geometric shapes other than $\circ$ is particularly noteworthy when *Mod A* is compared to other *Ars subtilior* manuscripts. As will be shown in the next chapter, there is a large variety of circles and semicircles to indicate rhythmic proportion in *Turin* and variations of geometric shapes are also rather common in other *Ars subtilior* sources. It will most probably remain speculation whether the scribe of *Mod A_{II-IV}* had such a great influence on the notation that he changed all signs appearing as geometric shapes in his exemplars to more standardised signs or whether the manuscript simply collects works of composers who favoured $\circ$ and single Arabic numerals in their works. Given the assumed close proximity of some of *Mod A*'s composers to one another—for example as members of the papal chapel—they might also have influenced each other.

3

PROPORTION SIGNS IN THE MANUSCRIPT *TURIN*¹

The codex *Turin*, *Biblioteca Nazionale Universitaria*, *J.II.9*—henceforth called *Turin*—is a music manuscript from the early fifteenth century. It consists of 159 parchment folios and contains musical repertoire which can stylistically be divided into four groups: 1. plainchant (fascicle I); 2. polyphonic mass movements (fascicle II); 3. Latin and French motets (fascicle III); and 4. a collection of French ballades, virelais, and rondeaux (fascicles IV and V).²

Many pieces from the fourth group can be said to exhibit characteristics of the *Ars subtilior*, containing, for instance, semiminims or other special note shapes (*dragmae*). Proportion signs can be found in 31 of the 166 songs³ of this group (see TABLE 3.3 on p. 163 below for details). The reversed semicircle $\bigcirc$ indicating the *sesquitertia* (4:3) proportion can furthermore be found in two polyphonic mass movements from the second group.⁴ Five pieces from the collection of French songs stand out among the 31 containing proportion signs because they use special proportion signs and rare proportions, such as *dupla sesquialtera* (5:2) and *dupla sesquitertia* (7:3) proportion. I will discuss these five songs in a separate subchapter (see Chap. 3.2 below).

Turin is a luxury manuscript with beautiful illuminations and gold ornaments. As an example, FIGURE 3.1 depicts folio 152v from the fourth group containing the two rondeaux *Qui n'a le cuer* and *Je la remire, la belle*. As one can see, many staves are left empty on the parchment which appears to be of the best quality—there are no wholes on any of the folios—and must have been costly. The manuscript survived a ravaging fire in the Biblioteca Nazionale in Turin in 1904, but the edges of the parchment folios are scorched and there is water damage on many folios as result of the firefighting. This affected the red ink in particular and there are stains of said ink—as those on the two lower staves in FIGURE 3.1—to be found on facing folios in many places.

¹ Parts of this chapter draw on my unpublished master's thesis "Proportion Signs in the Manuscript *Turin*, *Biblioteca Nazionale Universitaria*, *J.II.9*" submitted at the Humboldt-Universität zu Berlin in December 2015.

² Cf. Karl Kügle, "Some Notes on the Structure of the Manuscript," in *Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition*, Ars Nova 4, ed. Isabella Data and Karl Kügle (Lucca: LIM Editrice, 1999), 26.

³ 102 ballades, 43 rondeaux, and 21 virelais.

⁴ The mass cycle, which is inserted on fols. 139r–141v between the ballade section and the rondeaux and virelais section, contains the Arabic numerals 2 and 3, which are used as proportion signs. I have, however, chosen to ignore the mass cycle in this study, since it seems to be a later addition to *Turin*: "The Mass cycle, No. 11, was obviously added after the entire manuscript had been completed and assembled. [...] In my opinion, the scribe of the text had nothing to do with any other part of Tu B, and certainly the music was copied by a new and different scribe." Richard H. Hoppin, ed., *The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9, Corpus Mensurabilis Musicae* 21, 4 vols. (Rome: American Institute of Musicology, 1960–63), 1:VI. This view has been repeated by Jean Widaman, Andrew Wathey, and Daniel Leech-Wilkinson in *id.*, "The Structure and Copying of Torino," in *The Cypriot-French Repertory of the Manuscript Torino J.II.9*, Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 99.

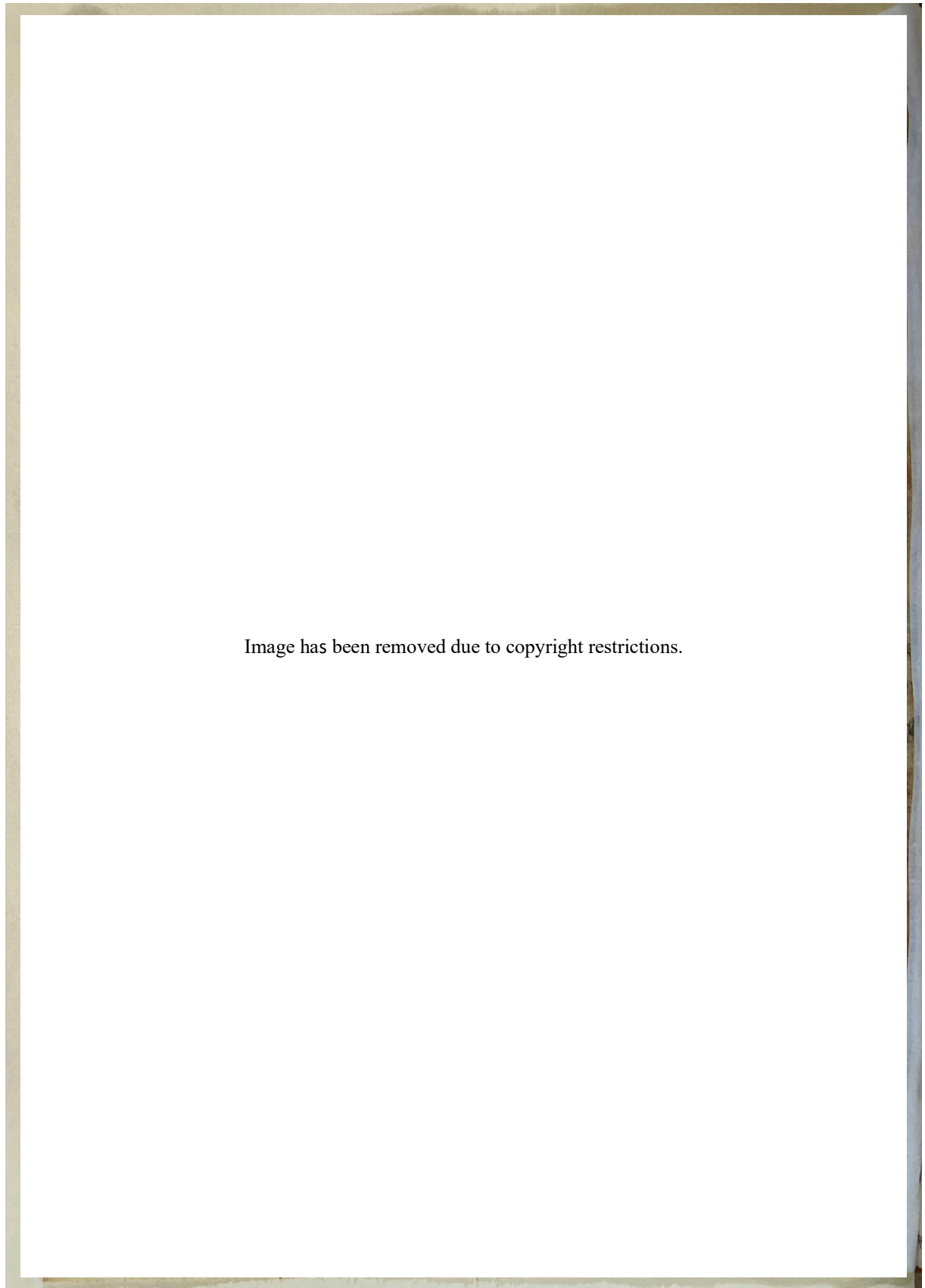


FIGURE 3.1: Fol. 152v of *Turin* (rondeaux *Qui n'a le cuer* and *Je la remire, la belle*)

While many pieces in comparable *Ars subtilior* manuscripts such as *Ch* or *ModA* can be attributed to certain composers, the repertoire from *Turin* is completely anonymous and—perhaps even more surprising—not a single concordance between the music of *Turin* and the contemporary repertoire of western Europe could yet be established. This circumstance makes it very difficult to narrow down possible times and places of the compilation of the manuscript and has led scholars to believe that it was compiled in an isolated place.⁵

In 1925, Heinrich Besseler suggested that *Turin* was produced at the Cyprian royal court of Lusignan, because two offices in the first fascicle of the manuscript can be linked to Cyprus: *In festo Sancti Ylarionis Abbatis* (Office and Mass for Saint Hylarion) and *In festo beate Anne matris Virginis Marie* (Office for Saint Anne).⁶ A bull from Antipope John XXIII dated 23 November 1413 was reproduced and attached to *Turin* on a flyleaf. In this bull, the pope granted permission to King Janus of Cyprus to have an office composed in honour of St. Hylarion, which is believed to be the office on the first folios of *Turin*.⁷ Moreover, Anne was the name of King Janus of Cyprus's only daughter, most probably born in 1419 and betrothed to Prince Amédée of Piedmont from the house of Savoy in 1431 and—in 1432, after Amédée's premature death—to his younger brother Louis, whom she married in 1433.⁸ Due to the lack of concordances between *Turin* and the contemporary repertoire of western Europe, Besseler concluded: "It can be assumed with certainty that not only the codex but also its content originated in Cyprus, namely from a circle of French-taught musicians."⁹ However, Besseler's hypothesis has been challenged by Karl Kügle, because only two other possible connections between *Turin* and Cyprus—in addition to the above-named offices—could be established: The refrain "Pour leaulté maintenir" from the ballade *Par doulceur refréner m'ire* is the motto of the Cyprian Order of the Sword.¹⁰ And in the rondeau *Puis que sans vous querons nostre plaisir*, an absent person is begged to return home from Nicosia.¹¹ Karl Kügle has argued that "one would [...]"

⁵ However, as we have seen in the last chapter, Matteo da Perugia's *oeuvre* is almost exclusively transmitted in *Mod A* (one of the small number of concordances only being a serendipitous discovery on a flea market in the 1980s (cf. Christian Berger, "'Pour Doulez Regard ...': Ein neu entdecktes Handschriftenblatt mit französischen Chansons aus dem Anfang des 15. Jahrhunderts," *Archiv für Musikwissenschaft* 51, no. 1 (1994): 51)) and *Mod A* was probably compiled in Bologna, or at least Italy. I see no reason why *Turin* should not have originated in Italy either, as suggest by Karl Kügle (see below), despite the lack of concordances.

⁶ Cf. Heinrich Besseler, "Studien zur Musik des Mittelalters I: Neue Quellen des 14. und beginnenden 15. Jahrhunderts," *Archiv für Musikwissenschaft* 7, no. 2 (1925): 209–10.

⁷ Cf. Hoppin, *The Cypriot-French Repertory*, 1:I.

⁸ Cf. Isabella Data, "Anne of Cyprus and Louis of Savoy: the marriage, the Ducal Library and the Franco-Cypriot codex," in *Il Codice J.II.9 Torino, Biblioteca Nazionale Universitaria: Facsimile Edition*, Ars Nova 4, ed. Isabella Data and Karl Kügle (Lucca: LIM Editrice, 1999), 67–68.

⁹ "Es kann somit als sicher angenommen werden, daß nicht nur der Kodex, sondern auch sein Inhalt aus Zypern, und zwar aus einem französisch geschulten Musikerkreis stammt." Besseler, "Studien zur Musik des Mittelalters," 212. My translation.

¹⁰ Cf. Hoppin, *The Cypriot-French Repertory*, 3:X.

¹¹ Cf. Hoppin, *The Cypriot-French Repertory*, 4:VIII.

expect significant traces of Greek influence in the hands and in particular the illuminations of codex J.II.9, if indeed it were copied in Cyprus. These, however, are nowhere to be noted.”¹²

Studies of *Turin* and its repertoire, which are more recent than Besseler’s early study and Hoppin’s edition from the early 1960s, have revealed a stylistic resemblance to music found in sources from northern France and the Low Countries in the early fifteenth century, particularly Cambrai.¹³ Margaret Bent, Giulio Cattin, and Karl Kügle have also argued the possibility of a northern Italian influence and even provenance.¹⁴ As Kügle has stated in an even more recently published article, “the primary corpus was copied by a team of Italian, or at least Italian-trained scribes.”¹⁵ In said article, Kügle established a connection between a coat of arms—displayed on the opening folio of the manuscript (see FIGURE 3.2)—and the Avogadro family of Brescia, which moved to Venice in 1438.¹⁶



FIGURE 3.2: Coat of arms of the Avogadro family held by an angel displayed at the bottom of fol. 1r from *Turin*

The question of a probable date of compilation of *Turin* seems to be as difficult as that of origin. Besseler has suggested a rather large timespan: “The codex Turin [...] can only have

¹² Karl Kügle, “Glorious Sounds for a Holy Warrior: New Light on Codex Turin J.II.9,” *Journal of the American Musicological Society* 65, no. 3 (2012): 645.

¹³ Cf. Daniel Leech-Wilkinson, “The Cyprus Songs,” in *The Cypriot-French Repertory of the Manuscript Torino J.II.9*, Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 398; and Karl Kügle, “The Repertory of Manuscript Torino, Biblioteca Nazionale J.II.9, and the French Tradition of the 14th and Early 15th Centuries,” in *ibid.*, 156.

¹⁴ Margaret Bent lists features characteristic of the Italian motet which she has detected in the *Turin* motet collection. Cf. “Some Aspects of the Motets in the Cyprus Manuscript,” in *The Cypriot-French Repertory of the Manuscript Torino J.II.9*, Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 359–61. Also cf. Giulio Cattin, “The Texts of the Offices of Sts. Hylarion and Anne in the Cypriot Manuscript Torino J.II.9,” in *ibid.*, 268–69; and Kügle, “The Repertory of Torino,” 174–77.

¹⁵ Kügle, “New Light on Codex Turin J.II.9,” 643.

¹⁶ Cf. Kügle, “New Light on Codex Turin J.II.9,” 649.

been compiled in Cyprus between 1413 and 1434.”¹⁷ Besseler based his theory on the date of 1413 provided by the papal bull reproduced on the flyleaf of the codex, which he took as a *terminus post quem*, and Anne de Lusignan’s departure to Chambéry in Savoy in 1434 as *terminus ante quem*. In an article leading up to his edition of *Turin*, Hoppin significantly narrowed the timespan for a probable date of compilation to the years between 1413 and 1420 due to stylistic characteristics of the manuscript’s repertoire.¹⁸ However, Kügle has recently argued a possible date of compilation as late as the early 1430s.¹⁹ In addition to identifying the coat of arms displayed on the opening folio of the manuscript as that of the Avogadro family of Brescia, Kügle found convincing proof for a connection between the Avogadro family and Jean Hanelle, the chapel master to King Janus of Cyprus, who travelled to northern Italy on many occasions in the 1430s and might even be the composer or one of the composers of the *Turin* repertoire.²⁰ The involvement of Jean Hanelle, who links Cyprus to the musical circles of northern Italy, which would explain why some parts of the repertoire have a strong connection to the Cyprian court although the manuscript was probably not compiled there, is a ground-breaking new finding. The scenario of an Italian origin of *Turin* now seems very likely.

This chapter is divided into three parts. In the first subchapter, I will give an overview of all the proportion signs found in *Turin*, but the focus will be on standardisation, namely the interpretation of the reversed semicircle $\bigcirc$ and the stacked Arabic numerals $\frac{3}{2}$ indicating *sesquitertia* (4:3) and *sesquialtera* (3:2) proportion respectively. Furthermore, as mentioned above, five songs from the collection of French songs stand out because of their use of unusual proportion signs and rare proportions. These will be discussed in the second subchapter. Finally, I would like to present my conclusions based on my analysis of the proportion signs used with the codex.

¹⁷ “[D]er Turiner Kodex [...] kann [...] nur in Zypern zwischen 1413 und 1434 entstanden [...] sein.” Besseler, “Studien zur Musik des Mittelalters,” 210. My translation.

¹⁸ Cf. Richard H. Hoppin, “The Cypriot-French Repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9,” *Musica Disciplina* 11 (1957): 92–93.

¹⁹ Cf. Kügle, “New Light on Codex Turin J.II.9,” 672.

²⁰ Cf. Kügle, “New Light on Codex Turin J.II.9,” 669–80. The involvement of other composers has also been suggested in the past, most prominently Gilet Velut whom Daniel Leech-Wilkinson considers to be the composer of *Turin*. Cf. Leech-Wilkinson, “The Cyprus Songs,” 408.

3.1 Standardisation in *Turin*

Turin contains 16 different proportion signs, namely six geometric shapes ($\bigcirc$, C, $\ominus$, $\oplus$, $\odot$, $\odot$), eight single Arabic numerals (2, 3, 4, 5, 6, 7, 8, 9), and two stacked Arabic numerals ($\frac{3}{1}$, $\frac{3}{2}$). These 16 signs indicate altogether 15 different proportions, which already shows by numbers alone that at least one proportion must be indicated by two different signs. As will be shown below, almost all 15 proportions are indicated by more than one sign, except those, of course, which only appear once in the manuscript: *tripla* (3:1), *dupla superbipartiens tertias* (8:3), and *dupla sesquiquarta* (9:4) proportion. Since that is the case, it follows that one and the same sign has two or more different meanings, which—as discussed below—is also true for *Turin*. Again, of course, except those signs, which only appear once (as proportion signs) in the manuscript: C, $\ominus$, 5, 6, 7, and $\frac{3}{1}$. I have summarised my findings in a comprehensive table (TABLE 3.3²¹) that intends to show, which proportion signs can be found in which pieces and how they are to be interpreted there.

TABLE 3.3 casts a different light on what might a moment ago have sounded like there is great variety of signs and interpretations throughout all the pieces. On the contrary, one can see that two signs appear frequently and that they almost always have the same meaning: $\frac{3}{2}$ indicates *sesquialtera* (3:2) proportion in 17 songs and $\bigcirc$ indicates *sesquitertia* (4:3) proportion in 18 compositions. The interpretation of $\bigcirc$ only deviates in the two pieces *Puis que amé sui doucement* and *Je prens d'amour noriture*, which are among the five special cases discussed in the next subchapter.²² With this degree of unambiguousness it seems, in my eyes, fair to speak of established, standardised signs for *sesquialtera* (3:2) and *sesquitertia* (4:3) proportion respectively.

²¹ The numbers of the compositions in TABLE 3.3 (brackets behind each title) follow the inventory in Data and Kügle, *Il Codice J.II.9 Torino*, 50–63. Note that Hoppin uses different numbers in his editions due to his numbering each piece according to genre.

²² As I argue there, the use of $\bigcirc$ for a proportion other than *sesquitertia* (4:3) proportion is in all probability intentional because it appears alongside other obscure choices of signs (e.g. mensuration signs C and $\ominus$ used as proportion signs) and unusual proportions, so that one might conclude that it was the composer's goal to make the interpretation as hard as possible. A canon helps the singer with the interpretation in these cases.

Composition	fol(s).	○	C	⊙	☺	☹	⊗	2	3	4	5	6	7	8	9	$\frac{3}{1}$	$\frac{3}{2}$
<i>Et in terra</i> (No. 108)	29v–30r	4:3															
<i>Patrem</i> (No. 109)	30v–32r	4:3															
<i>Pour haut et liement chanter</i> (No. 184)	104v	4:3															
<i>Puis que amé sui doucement</i> (No. 190)	107r	3:2	7:3				4:3 2:1			5:2				8:3			
<i>Quiconques veult user</i> (No. 191)	107v–108r																3:2
<i>Sous un bel arbre</i> (No. 196)	110r															9:4	
<i>Je ne quid pas</i> (No. 201)	112r	4:3															
<i>Se vrai secours</i> (No. 210)	114v–115r	4:3															3:2
<i>Moult longtemps a qu'amer</i> (No. 214)	117r																3:2
<i>Puis que m'amour</i> (No. 222)	121r																3:2
<i>Se de mon mal</i> (No. 231)	124v–125r						4:1		3:2	4:3							
<i>Celle en qui j'ai mise</i> (No. 245)	131r							2:1	3:1	4:3				3:2			
<i>L'esperer sans aucun</i> (No. 250)	132v	4:3															
<i>La belle et la gente rose</i> (No. 252)	133v																3:2
<i>Je la veul toustans servir</i> (No. 255)	135r																3:2
<i>Tout homme veult aus biens</i> (No. 256)	134v–135r	4:3															3:2
<i>Je ne desir fors</i> (No. 257)	135v	4:3															
<i>La belle qu'ai chierre lie</i> (No. 259)	135v–136r	4:3															3:2
<i>Esvellon nous</i> (No. 260)	136v	4:3															
<i>Sur toute fleur la rose</i> (No. 261)	137r			2:3	3:4	10:3	7:3	5:3	3:2	4:3	5:2		7:2		9:8		
<i>S'aucunne fois</i> (No. 264)	138v	4:3															3:2
<i>Viaire gent, veuillés moi</i> (No. 275)	144r																3:2
<i>Mon cuer m'en rit</i> (No. 278)	145r																3:2
<i>Mon cuer s'enfuit</i> (No. 286)	147r																3:2
<i>Il n'est amant qui n'a</i> (No. 307)	152r	4:3															
<i>La dame ou mon cuer</i> (No. 308)	152r	4:3															3:2
<i>Il faut, pour trouver</i> (No. 313)	153v	4:3															3:2
<i>Je prens d'amour noriture</i> (No. 315)	154r	5:2		3:4			10:3	2:3	5:3	4:1		7:2		7:3	9:8		
<i>Il n'aime pas, celui qui</i> (No. 316)	154r	4:3															3:2
<i>Souventes fois asprement</i> (No. 318)	154v																3:2
<i>Mener chiere lye</i> (No. 322)	155v	4:3															
<i>Combien que tout homme</i> (No. 324)	156r	4:3															3:2
<i>Parle qui veul, je veul</i> (No. 331)	158r	4:3															

TABLE 3.3: Proportion signs and their interpretation in *Turin*

The proportion sign $\frac{3}{2}$ always indicates *sesquialtera* (3:2) proportion in *Turin*. It is therefore the only proportion sign in the manuscript that appears more than once but always has the same interpretation. In its form of appearance as stacked Arabic numerals one would expect the sign to have this meaning: Three minims replace two in the previous mensuration as indicated by the two Arabic numerals 3 and 2 used in the sign.

So far, so obvious, were it not for the other proportion sign that appears as stacked Arabic numerals: $\frac{3}{1}$. This sign appears in the cantus of the ballade *Sous un bel abre* (see FIGURE 3.4), but does not indicate *tripla* (3:1) proportion as one might think, but *dupla sesquiquarta* (9:4) proportion at the semibreve level. The 18 semibreves in the extract in FIGURE 3.4 replace eight semibreves of the previous mensuration (*tempus imperfectum* with *prolatio maior* [2,3]). I was so very surprised by the fact that $\frac{3}{1}$ does not indicate *tripla* (3:1) proportion that I tried to establish a level (breve or minim) at which the proportion would be *tripla*, even if it was *dupla sesquiquarta* (9:4) at the semibreve level (all notes following the proportion sign are semibreves), but had to find that it was mathematically impossible. Since stacked Arabic numerals were even so often praised by contemporary theorists for their unambiguousness, it seems all the more peculiar that $\frac{3}{1}$ was used to indicate *dupla sesquiquarta* (9:4) proportion. One hypothesis could be that the scribe simply forgot to copy some of the semibreves. There is this rather suspiciously large gap between the penultimate and the last semibreve ligature of the proportion sign section but I also think that it would be very difficult to cram six semibreves—the number of semibreves needed to make the proportion *tripla* (3:1) and not *dupla sesquiquarta* (9:4) work—into that tiny space. The ballade *Sous un bel abre* with its proportion sign, which does not indicate by numbers which proportion should be applied, is a special case. As mentioned above there are other cases of peculiar choices for signs in the manuscript (see Chap. 3.2) but in all five pieces there is a canon explaining the signs. *Sous un bel abre* is exceptional in that aspect. The proportion would have had to be ascertained by trial and error.



FIGURE 3.4: Proportion sign $\frac{3}{1}$ in *Sous un bel abre*

Returning to the proportion sign $\frac{3}{2}$, it is noteworthy that the sign always appears in the context of *tempus imperfectum* with *prolatio maior* [2,3], i.e. in a context in which the

semibreve is divided into three and not two minims. Hence the three minims that replace two minims are only a part (two thirds to be exact) of that perfect semibreve. That means that in the proportion sign section there are nine minims to a breve instead of the former six. While the calculation is not difficult, I personally find it harder to grasp that the numbers in the proportion sign do not refer to a whole rhythmic unit (semibreve or breve) but to parts of that unit, namely two minims out of three minims to a perfect semibreve. I would prefer proportion signs that refer to a whole rhythmic unit, as, for example, the proportion sign $\frac{9}{6}$ in Antonello da Caserta's *Amour m'a le cuer*.²³ Perhaps the fifteenth-century singers were simply more skilled.

The proportion sign $\frac{3}{2}$ appears in 17 pieces in *Turin*. It is used only in the cantus in 16 of these, only in the ballade *Moult longtemps a qu'amer* does $\frac{3}{2}$ also appear once in the contratenor. No definite pattern can be observed in the use of that proportion. Sometimes it is used in melismata but in other cases there is almost complete text underlay. Other times it appears together with $\textcircled{\circ}$, but in other pieces it is the only proportion sign used in the composition. However, $\frac{3}{2}$ always appears in the same context of initial mensuration (*tempus imperfectum* with *prolatio maior* [2,3]) and always indicates *sesquialtera* (3:2) proportion.

One inconsistency can be observed in the use of $\frac{3}{2}$ and $\textcircled{\circ}$. While these two signs appear together in eight compositions, there are other pieces in which *dragmae* are used in order to indicate *sesquitertia* (4:3) proportion and not $\textcircled{\circ}$ (see TABLE 3.5). That the two ways of indicating *sesquitertia* (4:3) proportion—proportion sign and special note shape—are more or less interchangeable is further suggested by the rondeau *Il faut, pour trouver un bon port*, which appears in both columns because the cantus uses $\frac{3}{2}$ and $\textcircled{\circ}$, while *dragmae* are used in tenor and contratenor (see FIGURE 3.6).²⁴

$\frac{3}{2}$ appearing with $\textcircled{\circ}$	$\frac{3}{2}$ appearing with <i>dragmae</i>
<i>Se vrai secours</i> (No. 210)	<i>Moult longtemps a qu'amer</i> (No. 214)
<i>Tout houe veut aus biens</i> (No. 256)	<i>Puis que m'amour</i> (No. 222)
<i>La belle qu'ai chierre lie</i> (No. 259)	<i>La belle et la gente rose</i> (No. 252)
<i>S'auncunne fois</i> (No. 264)	<i>Viaire gent, veulliés moi</i> (No. 275)
<i>La dame ou mon cuer</i> (No. 308)	<i>Mon cuer m'en rit</i> (No. 278)
<i>Il faut, pour trouver</i> (No. 313)	<i>Mon cuer s'enfuit</i> (No. 286)
<i>Il n'aime pas, celui qui</i> (No. 316)	<i>Il faut, pour trouver</i> (No. 313)
<i>Combien que tout houe</i> (No. 324)	

TABLE 3.5: Appearances of $\frac{3}{2}$ with either $\textcircled{\circ}$ or *dragmae* in *Turin*

²³ See Chap. 2.3 for details on that composition.

²⁴ The only other example of $\textcircled{\circ}$ and *dragmae* appearing simultaneously in one piece is the rondeau *Il n'aime pas, celui qui* (No. 316). Cf. Laurie Koehler, *Pythagoreisch-platonische Proportionen in Werken der ars nova und ars subtilior*, Göttinger musikwissenschaftliche Arbeiten 12, 2 vols. (Kassel: Bärenreiter, 1990), 1:219.

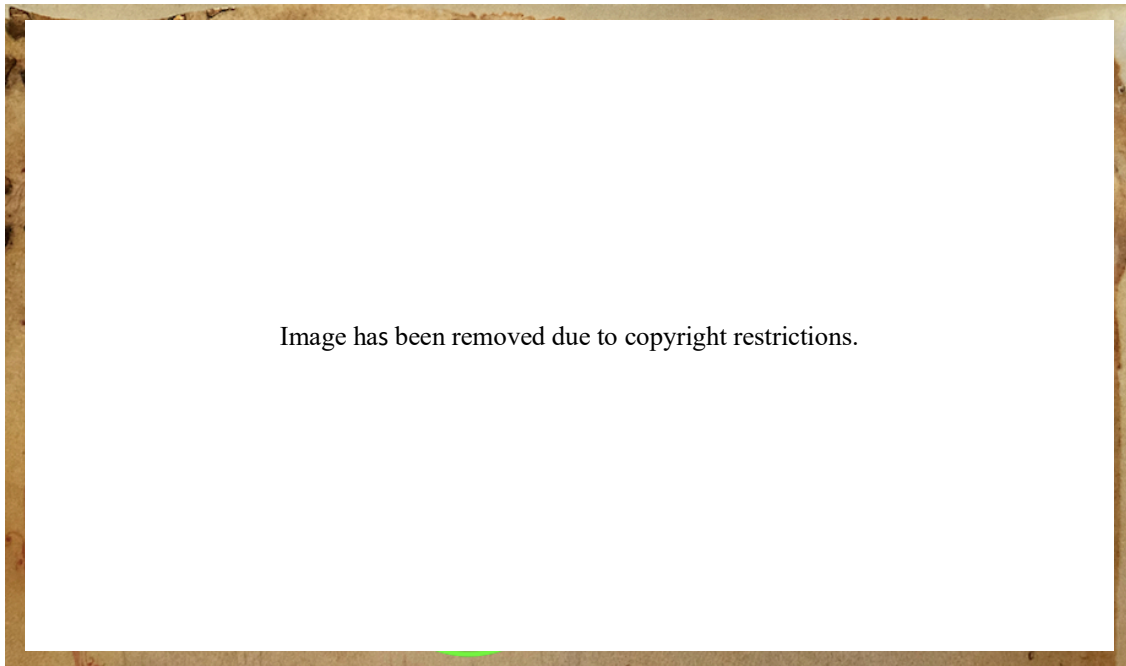


FIGURE 3.6: Rondeau *Il faut, pour trouver un bon port* (proportion sign and *dragmae* sections encircled)

Although $\circ$ and *dragmae* coexist in *Turin* it is in my opinion still adequate to speak of standardisation where $\circ$ is concerned because the sign itself almost always has the same meaning. $\circ$ appears in altogether 20 compositions of the manuscript and indicates *sesquitertia* (4:3) proportion in 18 of them. It is noteworthy that contrary to the sign $\frac{3}{2}$, which only appears in the secular repertoire, $\circ$ also appears in two polyphonic mass movements from the second fascicle (see TABLE 3.3 above). While $\circ$ usually appears in the cantus in all 20 compositions, there is a significant exception: The rondeau *Mener chiere lye* has $\circ$ in both tenor and contratenor but not in the cantus (see FIGURE 3.7).

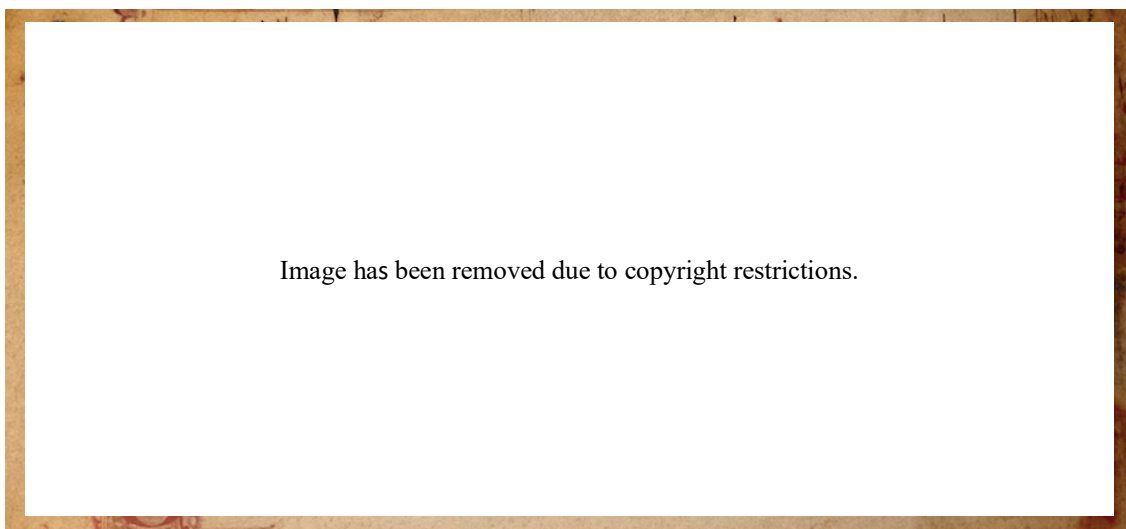


FIGURE 3.7: Rondeau *Mener chiere lye* (proportion sign sections encircled)

from non-cumulative to cumulative proportions took place, since *S'auncunne fois Fortune* as well as *Se de mon mal* would then be very late examples for non-cumulative proportions.²⁸ Admittedly, my argument is partly based on the hypothesis that $\bigcirc$ was used in a standardised manner in *S'auncunne fois Fortune*, but the case of *Se de mon mal* with its canon is unmistakable. However, there is precedent in *Turin* that $\bigcirc$ does not always indicate *sesquitertia* (4:3) proportion. These exceptions will be discussed in the next subchapter, which focuses on individualisation.

²⁸ Cf. Kügle, “New Light on Codex Turin J.II.9,” 672.

3.2 Individualisation in *Turin*

In this subchapter, I will take a closer look at four ballades²⁹ and a virelai³⁰ from *Turin*, because these five songs deviate strongly from the repertoire of the manuscript in their use of proportion signs and also proportions used in the music. A brief glance at TABLE 3.3 (see p. 163 above) reveals that these are the only pieces containing single Arabic numerals as well as special geometric shapes for proportion signs (other than $\bigcirc$).³¹ FIGURE 3.9 depicts the cantus of the ballade *Sur toute fleur la rose est colourie*, in which four different geometric shapes ($\ominus$, $\ominus$, $\oplus$, $\odot$) and six different Arabic numerals (2, 3, 4, 5, 7, 9) indicate altogether ten different rhythmic proportions.

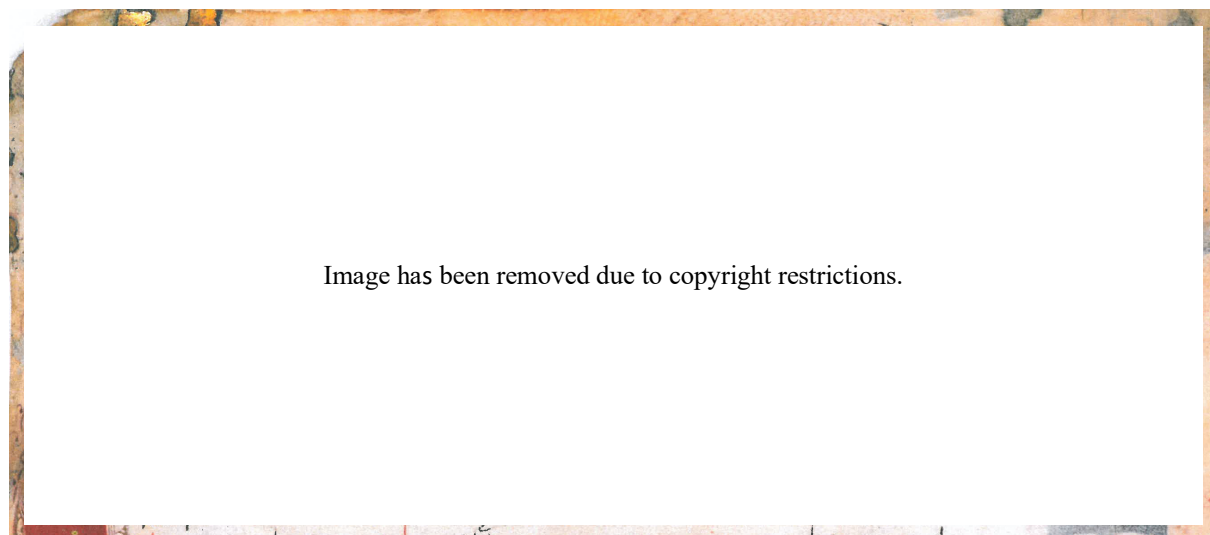


FIGURE 3.9: Cantus of the ballade *Sur toute fleur la rose est colourie*

In these five pieces, one can detect a strong tendency towards individualisation concerning the use of signs. The standardised sign for *sesquialtera* (3:2) proportion $\frac{3}{2}$, for instance, which can be found in altogether 17 pieces of the manuscript, is not present in any of the compositions. Naturally, the proportion itself is used in four of the five. However, it is indicated by the Arabic numerals 3 and 8 and, in one instance, by $\bigcirc$, which seems to be an odd choice considering that the sign indicates *sesquitertia* (4:3) proportion in almost all the other compositions in *Turin* and in other manuscripts.³² The reversed semicircle $\bigcirc$ also appears in one other of the five pieces,

²⁹ *Puis que amé sui doucement* (No. 190), *Se de mon mal delivré prestement* (No. 231), *Celle en qui j'ai mise m'amour* (No. 245), and *Sur toute fleur la rose est colourie* (No. 261).

³⁰ *Je prens d'amour noriture* (No. 315).

³¹ The Arabic numerals 2 and 3 also appear in the mass cycle on fols. 139r–141v, which, as stated earlier, I chose to ignore due to its being a later addition to the manuscript in all probability (see n. 4 above).

³² For meanings of $\bigcirc$ other than *sesquitertia* (4:3) proportion see Jason Stoessel, "The Interpretation of Unusual Mensuration Signs in the Notation of the *Ars subtilior*," in *A Late Medieval Songbook and Its Context: New*

but does not indicate *sesquitertia* (4:3) proportion there either, but *dupla sesquialtera* (5:2) proportion.

Moreover, other signs are also not used consistently in these five pieces but vary in each piece. The double circle ©, for instance, has four different meanings in four different pieces, indicating *dupla* (2:1), *quadrupla* (4:1), *dupla sesquitertia* (7:3), and *tripla sesquitertia* (10:3) proportion respectively. In order to help with the interpretation, each piece is accompanied by a canon explaining the meaning of the proportion signs. But even in the canons, the terminology is not used in a consistent manner. In the ballade *Sur toute fleur*, 10:3 proportion is called “*tripla sexquitercia*”, whereas it is called “*tripla epitrita*” in the virelai *Je prens d’amour*.³³

Nevertheless, the similarities between the songs, namely the special signs, the sometimes strange choices of signs for particular proportions, and the canons are striking. In all these characteristics, the five pieces seem to form a group within the manuscript. However, they are not grouped closely together on consecutive folios but are spread through fascicles four and five.³⁴ TABLE 3.10 gives the order of copying for all the pieces containing proportion signs in *Turin*.³⁵ While the music scribe (M2) is the same for all five compositions, two different text scribes were involved in copying the text underlay and the canons: The texts of the two ballades *Puis que amé* and *Celle en qui* are in a different script (T8) than those of the songs *Sur toute fleur*, *Je prens d’amour*, and *Se de mon mal* (T9).³⁶ Since the pieces were apparently not entered into the manuscript as a group they were probably not composed at the same time, even if—as I will suggest below—they might all stem from the same composer. These intervals would explain why there are small differences in the use of proportion signs and terms of the canon, even if the composer was one person. In this subchapter, I will explore similarities and differences between all five compositions where proportion signs and their interpretation as well as use and function within the compositions are concerned.

Perspectives on the Chantilly Codex (Bibliothèque du Château de Chantilly, Ms. 564), ed. Yolanda Plumley and Anne Stone (Turnhout: Brepols, 2009), 185–92.

³³ It is remarkable that so many Greek terms are used for rhythmic proportions in the five songs. I will discuss this in Chap. 3.2.2 below.

³⁴ Fascicle four: *Puis que amé sui doucement* (fol. 107r), *Se de mon mal delivré prestement* (fols. 124v–125r), *Celle en qui j’ai mise m’amour* (fol. 131r), and *Sur toute fleur la rose est colourie* (fol. 137r). Fascicle five: *Je prens d’amour noriture* (fol. 154r).

³⁵ According to the suggested timeline of the copying process provided by Jean Widaman, Andrew Wathey, and Daniel Leech-Wilkinson. Cf. “The Structure and Copying of Torino,” 114–15.

³⁶ For an assessment of scripts and scribes in *Turin* see Kügle, “Some Notes,” 29–31. According to Widaman, Wathey, and Leech-Wilkinson, the text of *Puis que amé* and *Celle en qui* was copied into the manuscript at an early stage while the music was copied later. Cf. “The Structure and Copying of Torino,” 115–16. The order of music copying is assumed to be the following for the five special pieces: *Je prens d’amour* (fol. 154r), *Puis que amé* (fol. 107r), *Celle en qui* (fol. 131r), *Se de mon mal* (fols. 124v–125r), and *Sur toute fleur* (fol. 137r).

Composition	fol(s).	Proportion signs
text of <i>Et in terra</i> (No. 108)	29v–30r	(proportion signs below, since only text was prepared here)
text of <i>Patrem</i> (No. 109)	30v–32r	
text of <i>Puis que amé sui doucement</i> (No. 190)	107r	
text of <i>Quiconques veult user</i> (No. 191)	107v–108r	
text of <i>Sous un bel arbre</i> (No. 196)	110r	
text of <i>Je ne quid pas</i> (No. 201)	112r	
text of <i>Moult longtemps a qu'amer</i> (No. 214)	117r	
text of <i>Puis que m'amour</i> (No. 222)	121r	
text of <i>Celle en qui j'ai mise</i> (No. 245)	131r	
music of <i>Et in terra</i> (No. 108)	29v–30r	○
music of <i>Patrem</i> (No. 109)	30v–32r	○
text and music of <i>Mener chiere lye</i> (No. 322)	155v	○
text and music of <i>Conbien que tout houme</i> (No. 324)	156r	○ and $\frac{3}{2}$
text and music of <i>Viaire gent, veulliés moi</i> (No. 275)	144r	$\frac{3}{2}$
text and music of <i>Mon cuer m'en rit</i> (No. 278)	145r	$\frac{3}{2}$
text and music of <i>Mon cuer s'enfuit</i> (No. 286)	147r	$\frac{3}{2}$
text and music of <i>Il n'est amant qui n'a</i> (No. 307)	152r	○
text and music of <i>La dame ou mon cuer</i> (No. 308)	152r	○ and $\frac{3}{2}$
text and music of <i>Il faut, pour trouver</i> (No. 313)	153v	○ and $\frac{3}{2}$
text and music of <i>Je prens d'amour noriture</i> (No. 315)	154r	○, ○, ⊙, 2, 3, 4, 6, 8, and 9
text and music of <i>Il n'aime pas, celui qui</i> (No. 316)	154r	○ and $\frac{3}{2}$
text and music of <i>Souventes fois asprement</i> (No. 318)	154v	$\frac{3}{2}$
text and music of <i>Parle qui veut, je veuil ...</i> (No. 331)	158r	○
music of <i>Puis que amé sui doucement</i> (No. 190)	107r	○, C, ⊙, ⊙, 4, and 8
music of <i>Quiconques veult user</i> (No. 191)	107v–108r	$\frac{3}{2}$
music of <i>Sous un bel arbre</i> (No. 196)	110r	$\frac{3}{1}$
music of <i>Je ne quid pas</i> (No. 201)	112r	○
music of <i>Moult longtemps a qu'amer</i> (No. 214)	117r	$\frac{3}{2}$
music of <i>Puis que m'amour</i> (No. 222)	121r	$\frac{3}{2}$
music of <i>Celle en qui j'ai mise</i> (No. 245)	131r	2, 3, 4, and 8
music and text ³⁷ of <i>Pour haut et liement chanter</i> (No. 184)	104v	○
music and text of <i>Se vrai secours</i> (No. 210)	114v–115r	○ and $\frac{3}{2}$
music and text of <i>Se de mon mal</i> (No. 231)	124v–125r	⊙, 3, and 4
music and text of <i>L'esperer sans aucun</i> (No. 250)	132v	○
text and music of <i>La belle et la gente rose</i> (No. 252)	133v	$\frac{3}{2}$
text and music of <i>Je la veul toustans servir</i> (No. 255)	135r	$\frac{3}{2}$
text and music of <i>Tout houme veut aus biens</i> (No. 256)	134v–135r	○ and $\frac{3}{2}$
text and music of <i>Je ne desir fors</i> (No. 257)	135v	○
text and music of <i>La belle qu'ai chierre lie</i> (No. 259)	135v–136r	○ and $\frac{3}{2}$
text and music of <i>Esvellon nous</i> (No. 260)	136v	○
text and music of <i>Sur toute fleur la rose</i> (No. 261)	137r	⊙, ⊙, ⊙, ⊙, 2, 3, 4, 5, 7, 9
text and music of <i>S'auncunne fois</i> (No. 264)	138v	○ and $\frac{3}{2}$

TABLE 3.10: Copying order of all proportion sign pieces in *Turin* (special proportion sign pieces emphasised)

³⁷ According to Widaman, Wathey, and Leech-Wilkinson, the music was copied first and the text was added second in this and the following three compositions. Cf. “The Structure and Copying of Torino,” 115 n. 27.

3.2.1 Special Proportion Signs and Unusual Interpretations

As stated earlier, the five songs are the only *Turin* compositions containing single Arabic numerals as well as special geometric shapes for proportion signs (other than $\odot$). It is important to make this distinction because many more *Turin* songs do contain geometric shapes such as $\mathbb{C}$, $\mathbb{C}$, $\mathbb{O}$, and $\mathbb{O}$, but they are used as *m e n s u r a t i o n* signs. TABLE 3.11 aims to provide an overview over the signs found in these special five pieces. It is noteworthy that stacked Arabic numerals do not appear although they are used in many other compositions in the manuscript.

Composition	$\mathbb{C}$	$\odot$	$\oplus$	$\ominus$	$\odot$	$\odot$	2	3	4	5	6	7	8	9
<i>Puis que amé sui doucement</i>	7:3	3:2	4:3			2:1			5:2				8:3	
<i>Se de mon mal delivré prestement</i>						4:1		3:2	4:3					
<i>Celle en qui j'ai mise m'amour</i>							2:1	3:1	4:3				3:2	
<i>Sur toute fleur la rose est colorie</i>			10:3	3:4	2:3	7:3	5:3	3:2	4:3	5:2		7:2		9:8
<i>Je prens d'amour noriture</i>		5:2			3:4 ³⁸	10:3	2:3 ³⁸	5:3	4:1		7:2		7:3	9:8

TABLE 3.11: Interpretation of proportion signs in *Turin* compositions with special signs

Two of the pieces contain the largest variety of proportion signs among all the songs analysed for this study, with ten different signs in the ballade *Sur toute fleur la rose est colourie* and nine signs in the virelai *Je prens d'amour noriture*. The ballade *Se de mon mal delivré prestement*, on the other hand, only contains three signs.

Although single Arabic numerals are not particularly special as proportion signs in *Ars subtilior* manuscripts in general, the numerals 5, 6, and 7 are very rare and in *Turin* they also indicate proportions which are seldomly used, namely *dupla sesquialtera* (5:2) and *tripla sesquialtera* (7:2) proportion. In using 5, 6, and 7 as proportion signs, *Turin* is unique among all the music manuscripts analysed for this study.³⁹ As TABLE 3.11 shows, all single Arabic numerals except the numeral 1 appear in the five songs. It almost seems as if the composer(s) or scribe wanted to make sure to have the complete set.

³⁸ The proportions given for the proportion signs $\odot$ and 2 in *Je prens d'amour* are mixed up in the canon, according to which $\odot$ indicates *subsesquialtera* (2:3) and 2 indicates *subsesquitercia* (3:4) proportion. This has been previously remarked upon by Hoppin, *The Cypriot-French Repertory*, 4:XXI.

³⁹ The Arabic numeral 6 appears as proportion sign indicating *subsesquitercia* (3:4) proportion in the composition *Se videar* by Ugolino da Orvieto, which is attached to his music theory treatise *Declaratio musicae disciplinae* in the manuscript *Rome, Biblioteca Casanatense*, 2151. For a discussion of proportion signs in this manuscript, see Elisabeth Hufnagel, "Adapting the Concept of *Proportio* to Rhythm in the *Ars subtilior*," in *Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts*, ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 441–64 or Chap. 4.2 below.

Nevertheless, there is no song containing all these single Arabic numerals. Four of the five pieces contain a mixture of single Arabic numerals and geometric shapes. Only the ballade *Celle en qui j'ai mise m'amour* displays merely numerals, namely 2, 3, 4, and 8. While the interpretation of the first three proportion signs—2 for *dupla* (2:1) proportion, 3 for *tripla* (3:1) proportion, and 4 for *sesquitertia* (4:3) proportion—is rather straightforward, choosing the Arabic numeral 8 to indicate *sesquialtera* (3:2) proportion is most unusual.⁴⁰ Then again, the more obvious choice, namely 3, was already used for *tripla* (3:1) proportion. Other compositions, however, do use signs which are not straightforward options, although the more obvious choice would have been available. The virelai *Je prens d'amour* is an example for that phenomenon: 6 is used to indicate *tripla sesquialtera* (7:2) proportion although the Arabic numeral 7 would have been an option.

Incipit	C	⊙	⊖	2	3	4	6	8
<i>Puis que amé sui doulcement</i>	7:3	3:2				5:2		
<i>Celle en qui j'ai mise m'amour</i>								3:2
<i>Sur toute fleur la rose est colorie</i>			2:3	5:3				
<i>Je prens d'amour noriture</i>		5:2	3:4		5:3		7:2	7:3

TABLE 3.12: Unusual proportion sign choices in *Turin*

TABLE 3.12 aims to show all the unusual proportion sign choices in *Turin*. The table includes more examples for peculiar choices of Arabic numerals for particular proportions as in the examples just given. Moreover, it is noteworthy that the standardised mensuration signs C and ⊖, which appear in their capacity as mensuration signs in many *Turin* compositions—and, of course, other *Ars subtilior* manuscripts—are used as proportion signs here, indicating *dupla sesquitertia* (7:3), *subsesquialtera* (2:3), and *subsesquitertia* (3:4) proportion respectively. Why was the Arabic numeral 7 not used for *dupla sesquitertia* (7:3) in *Puis que amé* instead of C? Or the numeral 2 instead of ⊖ for *subsesquialtera* (2:3) in *Sur toute fleur*? And 3 instead of ⊖ in *Je prens d'amour*? It should be noted that ⊖ as well as O and C are used as m e n s u r a t i o n signs in one of the five pieces, namely the ballade *Celle en qui j'ai mise*

⁴⁰ I am speaking of unusual choices or options here based on my analysis of *Ars subtilior* repertoire and of the contents of music theory treatises from the first half of the fifteenth century. Choosing single Arabic numerals as proportion signs that have nothing to do with the numbers in the proportion they represent, for example 4 for *dupla sesquialtera* (5:2) proportion, is not logic and many music manuscripts as well as treatises confirm that 4 is mostly used to represent *sesquitertia* (4:3) or *quadrupla* (4:1) proportion, i.e. proportions, in which the number four plays a role. I do acknowledge that certainty about what was usual or unusual in the *Ars subtilior* movement will never be reached for obvious reasons, not least because of the small body of source material.

m'amour, which makes it seem even more striking that they are used as proportion signs in the other two pieces. Using standard mensuration signs as proportion signs would probably have confused the singers, despite the explanatory canon. Assuming that these five pieces all stem from one composer or a close group of composers, it is remarkable that the same geometric shape is used as mensuration sign in one piece and as proportion sign in another.

Moreover, the reversed semicircle $\oslash$, which almost always indicates *sesquitertia* (4:3) proportion and, in fact, does so in 18 other *Turin* compositions, indicates *sesquialtera* (3:2) proportion in *Puis que amé* and *dupla sesquialtera* (5:2) proportion in *Je prens d'amour*. Given these unusual proportion sign choices, it is quite evident that a canon is essential for interpretation of the four pieces given in TABLE 3.12 above. Since some aspects concerning the canons are quite noteworthy, I would like to discuss them in more detail in the next subchapter.

Finally, I would like to draw attention to three rare geometric shapes used in the five pieces, namely the circle with two dots $\odot$, the circle with three dots $\odot$, and the double circle $\odot$.

The first two signs ($\odot$ and $\odot$) also appear in J. Galiot's ballade *Le sault perilleux* from *Ch.* However, their meaning there is incongruent with the interpretations found in *Turin*. In J. Galiot's ballade, $\odot$ indicates *sesquialtera* (3:2) proportion at the breve level and $\odot$ indicates *sesquioctava* (9:8) proportion at the semibreve level. Also, the interpretation of the proportion signs in *Le sault perilleux* is not as straight forward as those of *Turin*, because, for example, the equation of $\odot$ (nine semibreves equal eight semibreves) takes place between the cantus or contratenor and the tenor. This is unusual, because conventionally the proportion relates to the preceding passage in the same voice. That, however, is not possible in *Le sault perilleux*, if the proportion signs are interpreted according to the canon's instructions. Furthermore, $\odot$ is also used as proportion sign in the ballata *L'alta virtute* by Ugolino da Orvieto in the *Cas* manuscript, where it indicates *quadrupla sesquialtera* (9:2) proportion.⁴¹ It should also be mentioned that the two signs $\odot$ and $\odot$ appear in many music theory treatises as alternative signs to the mensuration signs $\circ$ and $\oslash$, although they are only rarely found in their capacity as mensuration signs in actual music.⁴²

⁴¹ Cf. Hufnagel, "Adapting the Concept of *Proportio* to Rhythm," 454.

⁴² A list of treatises describing circles with three or two dots as signs for *tempus perfectum* with *prolatio maior* or *minor* respectively can be found in Busse Berger, *Mensuration and Proportion Signs*, 236–37. Busse Berger also names a number of treatises in which the signs $\odot$ and $\odot$ are listed as signs indicating *modus* and *tempus*. Cf. *id.*, 13. As I have explained in the above-named article, it makes perfect sense to use the mensuration sign $\odot$ in the context of Ugolino's song due to the numerical relationship, although it is used in a proportional capacity. Therefore one could argue, that $\odot$ is used as mensuration sign in at least one piece, namely Ugolino's *L'alte virtute*. Cf. "Adapting the Concept of *Proportio* to Rhythm," 454–56.

The double circle $\odot$ is also described as mensuration sign in treatises, where it is mostly given as sign for *modus*, the level indicating how many breves fit into a *longa*.⁴³ Given that the sign, if used as sign for *modus*, indicates relationships one or two rhythmic levels above the level used in the five *Turin* songs, the choice of $\odot$ indicating *dupla* (2:1) proportion in *Puis que amé* and *quadrupla* (4:1) proportion in *Se de mon mal* may not be as unusual as one might think at first.⁴⁴ One could argue that the sign suggests to the singer that the standard rhythmic unit is shifted from breve to *longa* or *duplex longa* respectively, i.e. that diminution is applied. Therefore, all small note values following the sign are proportioned according to the diminution. I would even be tempted to argue along this train of thought—as Busse Berger has done—were it not for the other meanings of $\odot$ in *Sur toute fleur* and *Je prens d’amour*, where the sign indicates *dupla sesquitertia* (7:3) and *tripla sesquitertia* (10:3) proportion respectively. No diminution between rhythmic levels can be suggested by these odd numeric relationships. Furthermore, the other choices of signs in *Puis que amé* make it seem unlikely that the composer aimed for clarity in using certain signs and not others. As written above, *Puis que amé* also contains the reversed semicircle $\oslash$, which indicates *sesquialtera* (3:2) and not *sesquitertia* (4:3) proportion, so I doubt that he deliberately used $\odot$ in order to hint that some sort of diminution is applied (see TABLE 3.12 above for other unusual proportion sign choices in this ballade).

I have listed the more obvious proportion sign choices in the five pieces in TABLE 3.13 below in order to illustrate that *Puis que amé* with its altogether six proportion signs contains only one more or less obvious choice of sign for a particular proportion, namely 8 for *dupla superbipartiens tertias* (8:3) proportion. All five songs contain a certain number of proportion signs, which could probably have been interpreted without the aid of an explanatory canon:

Incipit	2	3	4	5	7	8	9
<i>Puis que amé sui doucement</i>						8:3	
<i>Se de mon mal delivré prestement</i>		3:2	4:3				
<i>Celle en qui j’ai mise m’amour</i>	2:1	3:1	4:3				
<i>Sur toute fleur la rose est colorie</i>		3:2	4:3	5:2	7:2		9:8
<i>Je prens d’amour noriture</i>			4:1				9:8

TABLE 3.13: Obvious proportion sign choices in the five *Turin* songs

⁴³ For an overview of treatises containing $\odot$, see Busse Berger, *Mensuration and Proportion Signs*, 236.

⁴⁴ Busse Berger probably had the same thought when she wrote: “The sign $\odot$ occurs in *Puis que amé sui doucement* in Turin J.II.9. It seems logical that the mode sign indicates here a 2:1 diminution.” *Mensuration and Proportion Signs*, 25.

The mixture of these more obvious as well as less obvious choices of signs made me wonder whether the more unusual proportion sign choices were a later addition to an existing song already containing some proportion signs, with the goal of deliberately making the song more complicated. As I will show in the next subchapter, however, the canons seem to suggest otherwise: The proportion signs in the canons are listed in their order of appearance in the *cantus* in every song. If signs were added at a later point, it would be likely that this order would be reflected in the canon, i.e. the canon listing obvious choices first and less obvious choices second. That is assuming that a canon existed in the first place, of course.

Putting the canons aside for a moment, I do not think that the idea of proportion signs being added at a later stage is a likely scenario. In case of the ballade *Puis que amé*, it would mean that there was a version only containing 8 as proportion sign. Assuming that the less obvious choices followed the more obvious ones, that is. And since 8 indicates *dupla superbipartiens tertias* (8:3) proportion it would mean that this would have been the only rhythmic proportion in the piece (disregarding here the common *sesquialtera* (3:2) proportion indicated by red ink). No special note shapes representing other rhythmic proportions are present in the ballade. Given that *sesquitertia* (4:3) proportion is so commonly used in the *Ars subtilior* repertoire, I find it unlikely that a composition would contain *dupla superbipartiens tertias* (8:3) proportion but not *sesquitertia* (4:3) proportion. It seems that the mixture of more obvious as well as less obvious choices of signs was intentional and that the scenario of different versions I suggested above did not exist.

The signs and proportions used in the five *Turin* songs create the impression that one deals with some sort of musical riddle here. Only if the proportion signs are interpreted correctly, the voices come together at the right moment. The large variety of signs (ten different signs in *Sur toute fleur* and nine signs in *Je prens d'amour*) reinforce this impression and lend a playful character to the pieces. The optics should also not be neglected here. This many signs certainly look impressive and in this way, the five pieces stand out when leafing through the manuscript. Using standard mensuration signs as proportion signs (meaning something completely different) in three of the five pieces as well as the unusual choices of Arabic numerals for certain proportions also add a touch of irony to the songs. Almost as if the composer is winking at us and saying: "Careful here. Do read the canon."

3.2.2 Canons

All five songs with special proportion signs in *Turin* contain a canon explaining the meaning of the signs used in the respective piece. I am giving the canon of *Sur toute fleur* here as an example for these instructions:

Canon balade talis est.

Ad figuram 9^{am} in proporcione epogdoa

ad 3^{am} in emiolia

ad 4^{am} in epitrita

ad circulum cum puncto in supsexquialtera

ad circulum cum duobus punctis in supsexquitercia

ad figuram 5^{am} in dupla emiolia

ad figuram 7^{am} in tripla emiolia

ad circulum duplicem in dupla sexquitercia

ad circulum cum tribus punctis in tripla sexquitercia et

ad figuram 2^{am} in superbiparciens tercias

residuum sicut iacet.⁴⁵

The rule of this ballade is such:

in 9:8 proportion at 9,

in 3:2 at 3,

in 4:3 at 4,

in 2:3 at the dotted circle,

in 3:4 at the twice-dotted circle,

in 5:2 at 5,

in 7:2 at 7,

in 7:3 at the double circle,

in 10:3 at the thrice-dotted circle and

in 5:3 at 2,

the rest as is.⁴⁶

The following observations refer to the canon of *Sur toute fleur* but they are also true for all five canons. The first line specifies the text as canon. Then follow explanations of each sign. The word “*proporcione*”⁴⁷ is only used once (or not at all in case of *Celle en qui*⁴⁸). It is fairly obvious that all the terms refer to proportions anyway. With the medieval mindset of using as little space as possible, omitting the word proportion is the logical thing to do. It is therefore even more surprising to me that the geometric shapes are described in words rather than displayed: Instead of “ad $\odot$ in supsexquitercia” the respective line reads “ad circulum cum duobus punctis in supsexquitercia”. The same is true for all geometric shapes used in the five pieces and respective canons.⁴⁹ Is describing the signs rather than displaying them part of the riddle I

⁴⁵ My transcription. Note that the number 7 is written as Λ in the canon as well as in the music. Transcriptions of all five canons can be found in Hoppin, *The Cypriot-French Repertory*, 3:XXII, XXIX, XXXII, XXXV, and 4:XXI. I have used my own transcription here since neither Hoppin’s nor Stoessel’s (see note below) transcriptions are completely accurate.

⁴⁶ Translation taken from Jason Stoessel, “Looking Back Over the ‘Missa L’Ardant desir’: Double Signatures and Unusual Signs in Sources of Fifteenth-Century Music,” *Music & Letters* 91, no. 3 (2010): 342.

⁴⁷ Spelled as “*proporcione*” with the letter ‘c’ in *Se de mon mal*, *Sur toute fleur*, and *Je prens d’amour*, and as “*proportione*” with the letter ‘t’ in *Puis que amé*. The different spellings go together with the different text scribes (see p. 170 above or TABLE 3.16 below.).

⁴⁸ The canon of *Celle en qui* uses an alternative phrasing to the other canons: The second line reads: “Nam ad figuram 2^{am} ca[n]tatur in dyapason.” Afterwards it is “Ad [description of proportion sign] in [respective proportion].” as in the other four canons.

⁴⁹ The way of describing proportion signs instead of displaying them is more susceptible to error. There is indeed an error in the canon of *Je prens d’amour*, in which the scribe has forgotten to add the word ‘*retrogradum*’: The line “Ad cemiculum in dupla emiola” should read “Ad cemiculum retrogradum in dupla emiola.” While the Arabic numerals are displayed, they are also often accompanied by the word “*figuram*”. Note that the word “*figuram*” is omitted in line 3 and 4 of *Sur toute fleur*’s canon but used again in lines 7, 8, and 11.

suggested above? An attempt to make the deciphering of the music more complicated than necessary?

In Chapter 1, I have already explained that canons, which provide information on the interpretation of signs, are paratexts.⁵⁰ The singer would be able to figure out the meaning of the signs without the canon, for example by calculating, by intrinsic visual elements such as groupings of notes, or by comparing the intervals which the voice in question forms with the tenor. The musical text can be interpreted without the canon, but reading the canon is probably quicker than the approaches suggested above, especially with the unusual proportion sign choices (see TABLE 3.12 on p. 173 above) found in four of the five pieces, which can mislead the singer.

For scholars studying the pieces round about 600 years after their composition—as I am now—the canons also provide helpful additional information on the pieces. It turns out that the five songs are not only similar in their use of proportion signs which are not used anywhere else in the manuscript, the canons are also connecting these five compositions because there are striking similarities between them.

For example, the last line of each canon is almost the same in every piece: “Residuum sicut iacet.” in *Celle en qui* and *Sur toute fleur* and “Residuum vero sicut iacet.” in *Puis que amé*, *Se de mon mal*, and *Je prens d’amour*. A similar phrasing occurs only in three other *Ars subtilior* pieces, but it is remarkable that the phrasing is almost identical in all five pieces in *Turin*, linking them together further.⁵¹ When translated as “the rest however as is” or “the rest as is” respectively—as Jason Stoessel does⁵²—the hint seems to be a bit superfluous, for is it not obvious that all non-proportion sign sections are sung “as is”? However, having this somewhat redundant remark in common is by far not the only similarity the canons share.

The first lines of the ballade canons, for instance, all begin with a version of the same phrase: “Canon bal(l)ade talis est.” in *Se de mon mal* and *Sur toute fleur* as well as “Canon huius balade talis est.” in *Celle en qui* and “Canon balade cantus et contratenor talis est.” in *Puis que amé*. In describing which voices contain proportion signs, *Puis que amé* is unique among the five pieces, although proportion signs are also found in cantus and contratenor of all three other ballades. Only *Je prens d’amour* has proportion signs in the cantus alone.

⁵⁰ See Chap. 1, p. 46.

⁵¹ Philipus de Caserta’s *Par les bons Gedeon* has the phrase “alie sicut iace(n)t” in the canon in two of its three sources, the anonymous *Medee fu* has the phrase “cantetur relique pro ut iacent” also in two of its three sources and another anonymous composition contains the phrase “figuras nigras ut iacent” in its canon. Cf. Emily Carolyn Zazulia, “Verbal Canons and Notational Complexity in Fifteenth-Century Music,” PhD diss., University of Pennsylvania, 2012, 71, 78, and 80.

⁵² Stoessel, “Looking Back Over the ‘Missa L’Ardant desir’,” 341–42.

The tenor never contains proportion signs in any of the pieces. It is therefore noteworthy that the canon of *Celle en qui* is written directly after the word tenor and also beneath the tenor voice (see FIGURE 3.14), when—as Hoppin has already noted—the tenor “is the only voice that makes no use of proportions.”⁵³ Therefore the choice of position for the canon is a bit peculiar, especially since there is enough space left on the empty staves beneath the contratenor (see FIGURE 3.14). It cannot be argued that someone intended to make the canon harder to find, since text underlay in the tenor voice is most probably an eye catcher for any trained singer. The canon’s position beneath the tenor as well as the alternative phrasing of the second line of the canon (“Nam ad figuram 2^{am} ca[n]tatur in dyapason” instead of “Ad figuram 2^{am} in dyapason.”) sets *Celle en qui* slightly apart from the other four pieces. In *Se de mon mal*, the canon is also written in the space after the tenor (see FIGURE 3.15), but in this case it is most probably due to it being the only space left on this and the facing page.

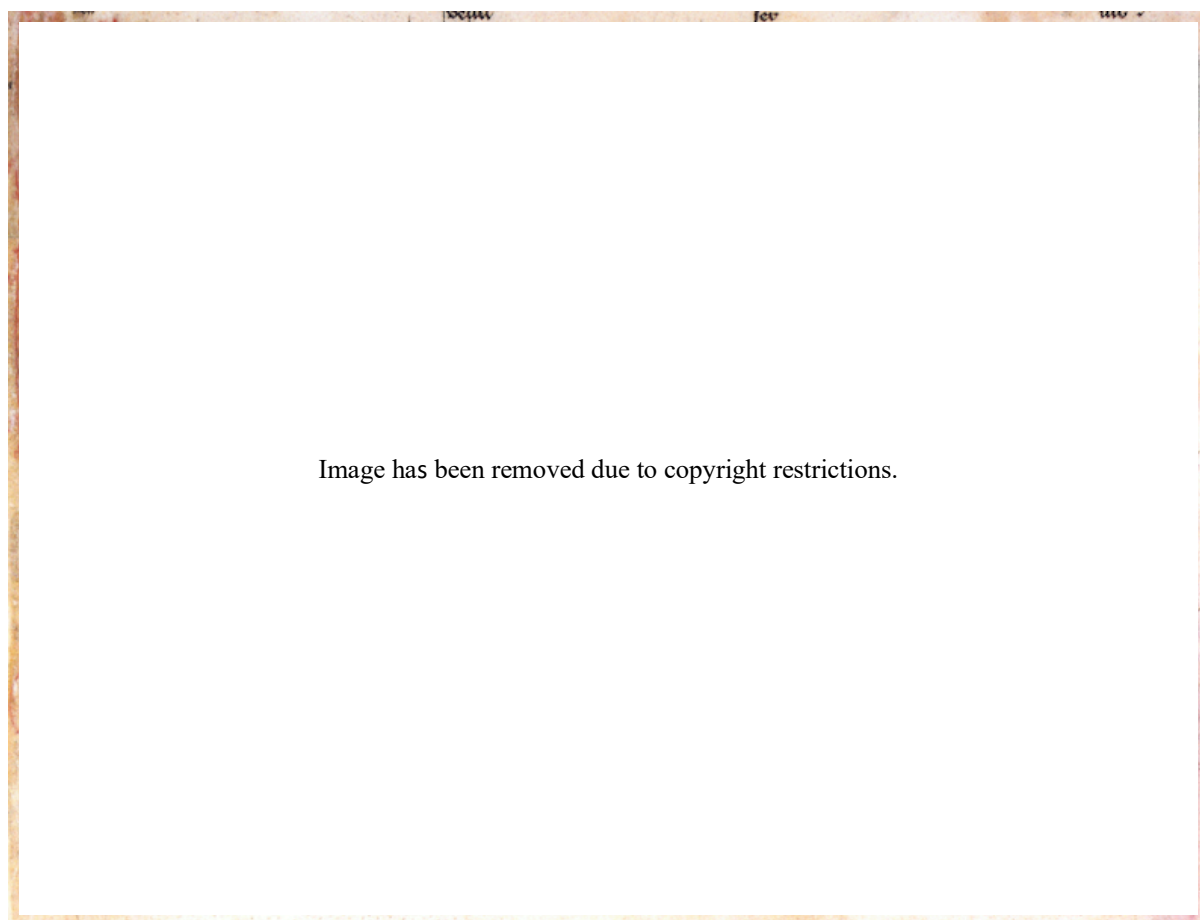


FIGURE 3.14: Tenor and contratenor of the ballade *Celle en qui j'ai mise m'amour*

⁵³ Hoppin, *The Cypriot-French Repertory*, 3:XXXII. Hoppin should probably have used the term ‘proportion signs’ instead of ‘proportions’ here, since the tenor does use coloration in order to indicate rhythmic proportion.

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FIGURE 3.15: Tenor and canon of the ballade *Se de mon mal delivré prestement*

Returning to the similarities in the five canons, it should be noted that all five list the proportion signs in the order in which they appear in the cantus. Of course, it is the logical approach for the person who composes the canon writing the meanings of the signs down in this order, as it is the most helpful for the singer of the cantus. Hence, this similarity between the canons could just be pure coincidence. On top of the other above-named parallels however, I do not think that it is.

Finally, the canons almost always use the same mixture of Greek and Latin terms for the proportions, as demonstrated in TABLE 3.16. In this table, Greek terms are set in black while Latin terms are in grey.

	<i>Puis que amé</i> (text script 8)	<i>Se de mon mal</i> (text script 9)	<i>Celle en qui</i> (text script 8)	<i>Sur toute fleur</i> (text script 9)	<i>Je prens d'amour</i> (text script 9)
2:1	<i>dyapason</i>		<i>dyapason</i>		
2:3				<i>supsexquialtera</i>	<i>supsexquialtera</i>
3:1			<i>dyapason</i> <i>dyapente</i>		
3:2	<i>emyolia(m)</i>	<i>emiolia</i>	<i>hemiolia</i>	<i>emiolia</i>	
3:4				<i>supsexquitercia</i>	<i>supsexquitercia</i>
4:1		<i>quadrupla</i>			<i>quadrupla</i>
4:3	<i>epitrita(m)</i>	<i>epitrita</i>	<i>epitrita</i>	<i>epitrita</i>	
5:2	<i>dupla emyolia</i>			<i>dupla emiolia</i>	<i>dupla emiola</i>
5:3				<i>superbiparciens</i> <i>tercias</i>	<i>superbipartiens</i> <i>tercias</i>
7:2				<i>tripla emiolia</i>	<i>tripla emiolia</i>
7:3	<i>dupla sexquitercia</i>			<i>dupla sexquitercia</i>	<i>dupla epitrita</i>
8:3	<i>dupla superbipartiens</i> <i>tercias</i>				
9:8				<i>epogdoa</i>	<i>epogdoa</i>
10:3				<i>tripla sexquitercia</i>	<i>tripla epitrita</i>

TABLE 3.16: Terms for proportions used in the canons in *Turin*

First of all, it is noteworthy that so many Greek terms are used in the canons. The *Turin* canons are not unique in this, but canons with all Latin terms are definitely in the majority in the *Ars*

subtilior repertoire studied here.⁵⁴ Greek terms are, of course, closely related to the teachings of intervals.⁵⁵ When the concept of ‘proportio’ was adapted to rhythm, the Greek terms sometimes lingered for those proportions forming the basic intervals while Latin terms were used for all other proportions. Something similar goes on in the *Turin* canons. Greek terms are used for 2:1 proportion (octave), 3:2 proportion (perfect fifth), 4:3 proportion (perfect fourth), 3:1 proportion (perfect twelfth), and 9:8 proportion (major second).

Interestingly, the double octave (4:1 proportion) is described in Latin (“*quadrupla*”) in both, *Se de mon mal* and *Je prens d’amour* and not as *dyapason dyapason*. Moreover, inverted proportions, in which the numerator is smaller than the denominator (here 2:3 and 3:4 proportion) are in Latin (“*supsesquialtera*” and “*supsesquitercia*” respectively).⁵⁶ Also, more complex proportions of the *superpartiens*, *multiplex superparticularis*, and *multiplex superpartiens* type⁵⁷ (e.g. 5:3, 7:2 or 8:3) are in Latin, with a few notable exceptions: Some proportions are described by a mixture of Greek and Latin terms, namely 5:2 proportion, which is not called *dupla sesquialtera* but “*dupla emiolia*”⁵⁸ in all three pieces in which it appears. The same is true for 7:2 proportion, which is called “*tripla emiolia*” and not *tripla sesquialtera*. As TABLE 3.16 shows, the mixture of Greek and Latin terms is not used consistently, however. For example, the canon of the ballade *Puis que amé* describes 5:2 proportion as “*dupla emyolia*”, but then uses Latin terms only for 7:3 proportion (“*dupla sexquitercia*”), which—if consequently described by the Latin and Greek mixture—should be called *dupla epitrita*. The canon of the ballade *Sur toute fleur* exhibits the same inconsistencies: “*dupla emiolia*” and “*tripla emiolia*” for 5:2 and 7:2 proportion respectively, but “*dupla sexquitercia*” and “*tripla sexquitercia*” for 7:3 and 10:3 proportion respectively. I find it all the more surprising that the canon of the virelai *Je prens d’amour* then uses a mixture of Latin and Greek for these two last proportions, calling them “*dupla epitrita*” and “*tripla epitrita*”.

With all the consistencies between the canons, I would have suggested that all five were created by the same person. But here we stumble across an inconsistency that seems to be more

⁵⁴ For example, the canon of J. Galiot’s ballade *Le sault perilleux* also uses Greek terms: “In proportione epitri ad cemi circulum cantetur. Ad circulum cum duobus punctis in proportione emiolus. Et ad circulum cum tribus [punctis] in proportione epo[g]doy.” Transcription taken from Gordon K. Greene, ed., *French Secular Music. Manuscript Chantilly, Musée Condé 564*, PMFC XIX (Monaco: Éditions de l’Oiseau-Lyre, 1982), 181. Furthermore, note that the proportion signs are also described (“circulum cum duobus punctis”) rather than displayed (⊕) in this canon. Further examples of canons using Greek terms can be found in Ursula Günther, “Some Polymetric Songs in the Manuscript Torino J.II.9,” in *The Cypriot-French Repertory of the Manuscript Torino J.II.9*, Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 477.

⁵⁵ See Chap. 1, p. 13 for more details.

⁵⁶ The spelling with ‘p’ in the prefix ‘sup’ (instead of sub) is consistent in both *Sour toute fleur* and *Je prens d’amour*.

⁵⁷ For the different types of proportions, see Chap. 1, p. 19.

⁵⁸ Note that *emiolia* is spelled with a ‘y’ in *Puis que amé*.

of an obstacle in view of that hypothesis. This and the fact that there are three different spellings of the term *emyolia* (see TABLE 3.16). The spellings might easily be attributed to an inattentive scribe. But using different terms for the same proportion is a different case. On the other hand, the two ballades *Puis que amé* and *Sur toute fleur* use both versions for the more complex proportions, a mixture of Latin and Greek terms for *dupla* and *tripla emiolia* and only Latin terms for *dupla* and *tripla sesquitercia*. So perhaps the author of the canons—again assuming one person was responsible for all five—had not yet made up his mind about using Greek or Latin terms for the more complex proportions when he created the canons of *Puis que amé* and *Sur toute fleur*, but had decided to use Greek terms by the time he wrote the canon of *Je prens d’amour*. This hypothesis would suggest that some time passed between the composition of the ballades on the one hand and the virelai on the other hand.

3.2.3 A Musical Riddle?

In this last subchapter on *Turin*'s five special pieces, I would like to return to the idea of musical riddles. At the end of the first subchapter, I have already suggested that the interpretation of the proportion signs is deliberately made more difficult than necessary. Using standardised mensuration signs such as C and O for proportion signs, or the reversed semicircle $\cap$ for proportions other than *sesquitercia* (4:3) proportion, or single Arabic numerals that have nothing to do with the proportion they represent (see TABLE 3.12 on p. 173 above for details) are examples for this scheme. The canons as paratexts then give the solutions to this riddle. The situation is similar to Baude Cordier's *Belle, bonne, sage*, which I discussed in Chapter 1.4.2. There, the piece is made more complicated than necessary, because all the proportions could have been displayed by coloration (red ink and hollowing) that are already present in the piece. I will not argue that all proportions in the five special pieces could have been displayed by using special note shapes or coloration, as I have done in the case of *Belle, bonne, sage*, but it seems to me that things could have been notated in a simpler manner.⁵⁹

As the five special pieces are the last group of musical pieces out of music manuscripts to be discussed in this study, I would like to put some aspects of that thought into a broader perspective so that the conclusion I draw here might also be transferred to other pieces I have discussed in previous chapters. I would like to explore different hypotheses to the question whether there is a special purpose to the proportion signs in the five special pieces: Or simply put: Why proportion signs? Why use proportion signs if there are other means of notating complex rhythms (special note shapes, coloration or hollowing)?

The first hypothesis is that proportion signs are used to avoid the division of the minim. In Chapter 1.3, I have described in detail why the division of the minim was undesirable. It was thought of as the smallest musical unit, hence the name *minima*, and therefore should not be further divided. Special note shapes indicating a smaller note value than the minim were rejected on this ground. A way around that problem for composers who still intended to use smaller units was the application of proportion signs. By applying a proportion to a certain number of minims, smaller note values could be created without violating the rule that the minim should not be divided. The ballade *Celle en qui* is an example for such an approach. It uses neither *dragmae* nor semiminims, but creates all rhythmic proportion through proportion

⁵⁹ This has occasionally been remarked upon in passing by other scholars. Günther, for example, writes in her discussion of *Celle en qui*: "[The indication of *sesquialtera* (3:2) proportion] would have been possible with the usual sign 3/2", which is, however, not used in the ballade. Günther, "Some Polymetric Songs in the Manuscript Torino J.II.9," 478.

signs (with the exception of coloration for *sesquialtera* (3:2) proportion that is). FIGURE 3.17 shows how smaller note values than the minim are created by the application of *dupla* (2:1) proportion. However, I have already argued that it seems likely that all special proportion sign pieces in *Turin* stem from the same composer and I will also elaborate on that point in the conclusion. All other four special proportion sign pieces do contain special note shapes, among others semiminims, therefore it is does not seem likely that avoidance of *minima* division was the primary goal when using proportion signs in *Turin*.

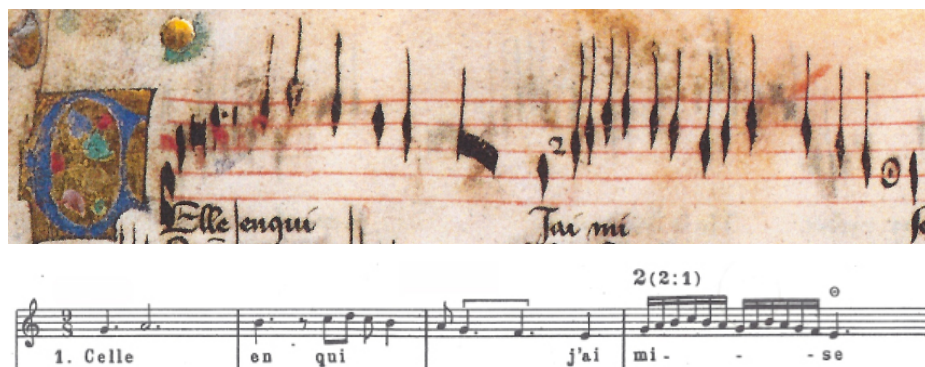


FIGURE 3.17: *Dupla* (2:1) proportion at the minim level in *Celle en qui j'ai mise m'amour*⁶⁰

Another hypothesis could be that proportion signs are used to signal melismata so that the singer knows to speed up when seeing a proportion sign. I have made a similar argument in the discussion of *Belle, bonne, sage*.⁶¹ In *Se de mon mal*, the proportion sign © indicates *quadrupla* (4:1) proportion at the minim level, which results in very small note values (see FIGURE 3.18). The minims are grouped together very closely, but it could be argued that the proportion sign additionally signals: “You have to sing these notes fast.”

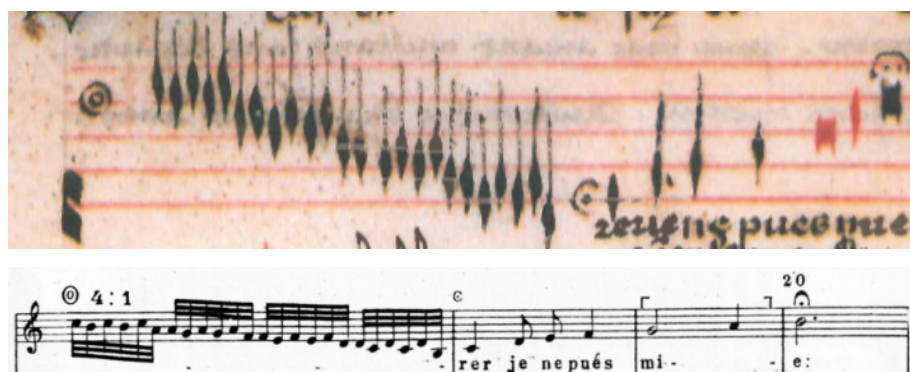


FIGURE 3.18: *Quadrupla* (4:1) proportion at the minim level in *Se de mon mal delivré prestement*⁶²

⁶⁰ Transcription taken from Hoppin, *The Cypriot-French Repertory*, 3:144.

⁶¹ See Chap. 1.4.2.3, p. 73–74.

⁶² Transcription taken from Hoppin, *The Cypriot-French Repertory*, 3:119.

A similar situation can be observed in the virelai *Je prens d'amour*, in which proportion signs are used to create successively smaller note values, which culminates in a *trippla sesquiter-tia* (10:3) proportion also indicated by the proportion sign © (see FIGURE 3.19). Ursula Günther has stated that *Je prens d'amour* exhibits “the first precisely calculated and notated acceleration in the history of music.”⁶³ And Leeman Perkins has called this piece “a dizzying display of proportional pyrotechnics” and has concluded that it “must have been particularly difficult in performance [...] and was intended [...] as an impressive demonstration of compositional – and of course performance – skills.”⁶⁴

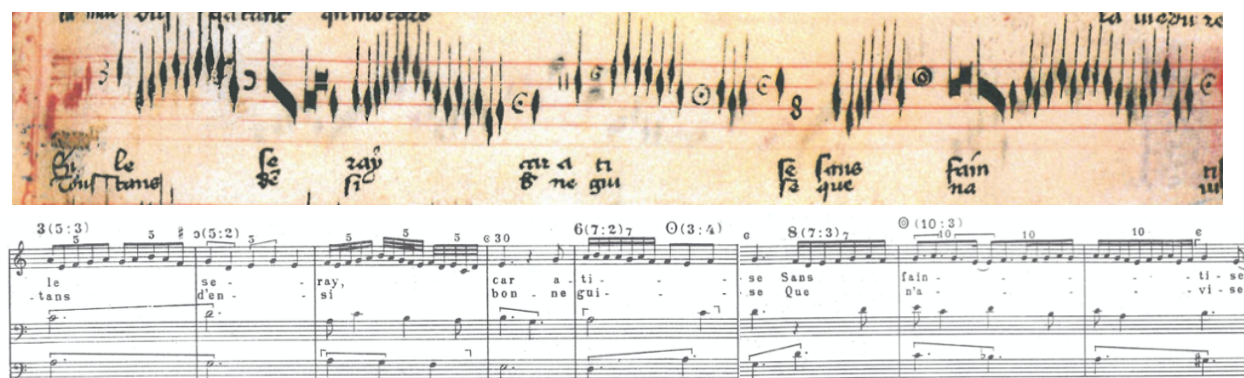


FIGURE 3.19: Proportion sign melismata in *Je prens d'amour noriture*⁶⁵

On the other hand, there are many examples within the five special pieces in which the proportion signs either do not coincide with melismata or where long and fast melismata occur without the use of proportion signs. Hence, even if there is a great congruence of melismata and proportion signs within the pieces, it is difficult to argue that proportion signs were used only in order to signal these melismata.

A third hypothesis could be that some of the proportion signs help the singers with the coordination in the piece. In *Puis que amé*, one can observe that the double circle © is used simultaneously in cantus and contratenor in two sections of the ballade (see FIGURE 3.20). Hence, the sign © might have been used intentionally in these places to coordinate the two voices. In my eyes, it is noteworthy that a special geometric shape is applied in these places of simultaneous proportional rhythms and not a sign using Arabic numerals, because it does remind me of the situation in *Belle, bonne, sage* and *Tout par compas*, where the cut circle ⊖ serves as multi-function sign, signalling proportion on the one hand and points of structural importance to the piece on the other hand.⁶⁶

⁶³ Günther, “Some Polymetric Songs in the Manuscript Torino J.II.9,” 481.

⁶⁴ Perkins, “At the Intersection of Social History and Musical Style,” 449–50.

⁶⁵ Transcription taken from Hoppin, *The Cypriot-French Repertory*, 4:53–54.

⁶⁶ Cf. Chap. 1.4.2.3.



FIGURE 3.20: Simultaneous ©-sections in cantus and contratenor of *Puis que amé*⁶⁷

Unfortunately, however, this argument is weakened by the fact that © also appears in other places in *Puis que amé*, where it does not seem to have a function as coordination sign. And it does not seem to have this function at all in the other three pieces in *Turin* in which it occurs. Using a sign as position or coordination sign is, of course, only useful if it serves this function with some regularity. And this does not seem to be the case in *Turin*.

What does it mean if no general pattern can be determined in the use of the proportion signs in the five special pieces in *Turin*? Perhaps no general pattern can be determined because the composer deliberately tried to avoid a general pattern. The interpretation is made difficult for the singer, not only by the use of rare signs or proportions but also because every sign seems to be its own special case in every piece. For example, consistent interpretation of the same sign does not work in the five special pieces. © has four different meanings in four songs, the same is true for other signs. Standardised signs are suddenly used outside the realms of their conventional interpretation, for example the mensuration signs C and O, or the proportion sign O for *sesquialtera* (3:2) and *dupla sesquialtera* (5:2) proportion.

I can only deduce that there is a purpose behind the deception and that is the deception itself. These five songs are meant to be musical riddles. I would argue that in the *Ars subtilior*, we can see the beginnings of what will a few decades later become a whole riddle culture, as described by Katelijne Schiltz in her book *Music and Riddle Culture in the Renaissance*. Schiltz, however, regards the more complex *Ars subtilior* pieces as “rather isolated examples that do not represent the musical state of affairs at their time.”⁶⁸ In that, I would disagree with Schiltz, because I think that there are too many of these pieces to speak of “isolated examples”. Many *Ars subtilior* compositions contain notation that is far more complex than necessary. Among the compositions discussed in this study, there are the five special pieces in *Turin*, pieces such as Johannes Ciconia’s *Sus un fontayne* and Matteo da Perugia’s *Le greygnour bien* in *Mod A*, and Baude Cordier’s two pieces from *Ch*. And there are several more pieces which

⁶⁷ Transcription taken from Hoppin, *The Cypriot-French Repertory*, 3:44.

⁶⁸ *Music and Riddle Culture in the Renaissance* (Cambridge: Cambridge University Press, 2015), 3.

have not been discussed in this study in *Ch*, *Ox213*, *Paris 22069*, the *Panciatichi Codex*, and the *Utrecht fragments*.

It is true that the canon instructions are very clear and explicit in these *Ars subtilior* riddle pieces. They are not yet the riddle canons they will become from around 1500 onwards, in which the instructions themselves are riddles. However, I would argue that the notation of some *Ars subtilior* compositions—especially the five pieces in *Turin*—is complex enough to serve as sophisticated musical riddle. There are choices of signs and proportions that do not seem to be logic in any way, and, as I hope to have been able to show in this chapter, obvious choices seem to have been intentionally avoided by the composer of the pieces.

3.3 Conclusion

As I have shown throughout this chapter on *Turin*, there is strong consistency in the notation of rhythmic proportion between the pieces. The proportion signs $\oslash$ and $\frac{3}{2}$ are with only two exceptions used in a standardised manner, always indicating *sesquitertia* (4:3) or *sesquialtera* (3:2) proportion respectively.⁶⁹ On the other hand, there are the five special pieces, which have a strong tendency towards individualisation, by special signs such as $\oplus$, $\ominus$, and $\odot$ being used for proportions or by unusual proportions. What these five pieces have in common, however, are exactly these special signs as well as the phrasing of the canons. So is it fair to speak of individualisation at all in this context? Are there different individuals working on these five pieces or is it one composer? And if all five special pieces stem from one composer, does it necessarily have to be a different composer from that of some or all of the other *Turin* pieces?

Daniel Leech-Wilkinson has indeed suggested that the entire repertoire might be the work of just one composer or at least a small group of composers composing in a very similar style.⁷⁰ Leech Wilkinson has based his hypothesis on striking similarities between many *Turin* compositions, especially the songs. His examples mainly compare melodic shapes. What is most interesting, however, is that his analyses link the five special pieces to the rest of the repertoire.⁷¹

For example, he has found that many cantus openings in *d*-based compositions are based on the melodic outline *a' b' a' g' f' e'*: Among these altogether 20 songs are the proportion sign pieces *Quiconques veult user* and *Tout houme veut aus biens*, as well as the special proportion sign *virelai Je prens d'amour*.⁷² Other shared characteristics, which link proportion sign pieces to the rest of the repertoire, are 1) cantus openings based on the melodic outline *c'' d'' c'' b' a' g'* in *f*-based compositions (four proportion sign pieces (*Je ne quid pas*, *Esvellon nous*, *Il n'aime pas celui qui triche*, and *Souventes fois asprement*) and six non-proportion sign pieces); 2) decorated descents from *a'* to *d* (two proportion sign pieces (*Quiconques veult user* and *Tout houme veut aus biens*) and three non-proportion sign pieces); 3) decorated descents from *c''* to *f'* (*Je ne quid pas* and three non-proportion sign pieces); and 4) characteristic descending chains of thirds

⁶⁹ The two exceptions are the compositions *Puis que amé* and *Je prens d'amour*, in which $\oslash$ does not indicate *sesquitertia* (4:3) proportion.

⁷⁰ Cf. Leech-Wilkinson, "The Cyprus Songs," 395.

⁷¹ Note that Günther has also established some stylistic connections between the special pieces and the general song repertoire of *Turin*, though her article does not focus on these resemblances in the way Leech-Wilkinson's article does. Cf. Günther, "Some Polymetric Songs in the Manuscript Torino J.II.9," 478.

⁷² Cf. Leech-Wilkinson, "The Cyprus Songs," 398. Leech-Wilkinson mentions that this is also "a common feature in Baude Cordier songs" and other works of northern composers, leading up to his hypothesis that the *Turin* composer might come from that circle of composers. His Example 5 (pp. 414–15) shows cantus openings based on *a' b' a' g' f' e'* in compositions by Baude Cordier, Lebertoul, Malbecque, Loqueville, and Grossin.

(two proportions sign pieces (*Moult longtemps a qu'amer* and *Mon cuer s'enfuit*) and 13 non-proportion sign pieces).⁷³

All things considered, Leech-Wilkinson has found compelling examples to support his hypothesis that the entire *Turin* repertoire might be the work of as little as one composer. Leeman Perkins has drawn the same conclusions on notational grounds, but his argument only extends to the extremely consistent use of the proportion signs $\bigcirc$ and $\frac{3}{2}$ and not to the special proportion signs, which I would very much like to include in that argument, since there are remarkable similarities between the five special pieces as well.⁷⁴

But why are there proportion signs in some pieces, especially as elaborate as those in the five special pieces, but not in others? Leech-Wilkinson works around that problem as follows: In his article, he has compared the A-sections of the two virelais *Je prens d'amour* and *Tres purement je amerai* as well as the rondeau *Puisque je voi que pour son vrai servant* with one another (see FIGURE 3.21 below). The similarity in the melodic gestures is admittedly rather remarkable. He then writes:

“[T]hese three pieces are more likely – though by no means certain – to be the work of one composer elaborating one mental model. And this in no way contradicted by the complex proportions found in Virelai 17 [*Je prens d'amour*] but not in the other two pieces, for these proportions produce nothing but superficial melodic decoration: they are clearly imposed on a simple structure for effect and cannot therefore be taken as reliable indicators of a different composer let alone of a fundamentally different style. (I suspect that this may be true of all Cyprus proportions, at least in the songs: they are there to impress somebody, not because they are an essential ingredient in style).”⁷⁵

What Leech-Wilkinson is suggesting is in my opinion an extremely likely scenario for *Je prens d'amour* and, in fact, many other proportion sign compositions discussed in this study. Namely, that proportion sign sections are ornamental elements, which are imposed upon a pre-existing melodic structure. I also agree with Leech-Wilkinson's hypothesis that the proportion signs “are there to impress somebody”. They make the composition look interesting when it is perhaps not as interesting as the optics imply. The *Turin* song repertoire is based on melodic models and ideas so similar that Leech-Wilkinson has even called the music “uninspired”.⁷⁶ He has, indeed, proposed the following scenario of one composer working very hard in a relatively short period of time:

⁷³ Cf. Leech-Wilkinson, “The Cyprus Songs,” 400, 402, and 426–29.

⁷⁴ Cf. Leeman L. Perkins, “At the Intersection of Social History and Musical Style: The Rondeaux and Virelais of the Manuscript Torino J.II.9,” in *The Cypriot-French Repertory of the Manuscript Torino J.II.9*, Musicological Studies & Documents 45, ed. Ursula Günther and Ludwig Finscher (Neuhausen-Stuttgart: Hänssler-Verlag, 1995), 438 and 450–51.

⁷⁵ Leech-Wilkinson, “The Cyprus Songs,” 399.

⁷⁶ Leech-Wilkinson, “The Cyprus Songs,” 407.

The requirement to produce a lot of music fast was met by the formulaic model-based approach to composition [...], a method which would make a lot of sense under these circumstances. Masterpieces are unlikely, of course; he [the composer] would be more likely to produce competent but uninspired music notable for consistency of style rather than for startling or exceptional ideas. Which is exactly what we get.”⁷⁷

And here I would argue that the proportion sign pieces stand out. Because even if they are not “masterpieces”, they certainly appear to be with their special signs and the canons. They suggest that it needs an extremely skilled singer to perform these pieces and that in turn purports that an equally skilled composer was at work here, with a commissioner who had the means and resources to employ such a person.



FIGURE 3.21: Comparison of three *Turin* compositions by Daniel Leech-Wilkinson⁷⁸

In addition to their playful riddle manner or perhaps their didactic purpose these pieces give the impression that all the music contained within the manuscript is interesting and individual. I would even go so far in arguing that the many ways in which the application of proportion signs is varied in the codex is intentional. Here, a single composer might make an effort to feign individualisation in order to gloss over the fact that only a single composer was at work. But the similar if unconventional signs and the phrasings of the canons suggest otherwise. Future scholarship can hopefully shed light on who this individual was and whether he worked in Cyprus, France, or Italy when composing the *Turin* repertoire.

⁷⁷ Leech-Wilkinson, “The Cyprus Songs,” 407.

⁷⁸ Example taken from Leech-Wilkinson, “The Cyprus Songs,” 416.

4

COMPOSER VERSUS THEORIST

A COMPARISON OF UGOLINO DA ORVIETO'S COMPOSITIONS
AND HIS STATEMENTS ON PROPORTION SIGNS IN THE
MANUSCRIPT *ROME, BIBLIOTECA CASANATENSE, 2151*¹

Before turning to treatises and the statements on proportion signs found therein in Chapter 5, I would like to discuss Ugolino da Orvieto who is composer and theorist combined in one person. He has written a comprehensive treatise on music but also composed songs, some of which contain proportion signs. This is a singular case, as no other composer-theorist using proportion signs, for example Johannes Ciconia or Filippotto da Caserta, left instructions on the use of proportion signs in their theoretical works.² A comparison of proportion signs contained in Ugolino's songs and his statements on music notation will explore the question if teachings on music reflect on actual notational practice and vice versa. Research on this question in general has to be conducted with great caution because there are many uncertainties in the equation, as for example the origin of a manuscript, the influence of the copyist, and so forth. Since we have to assume regional and cultural differences in the notation of mensural music, different sources of (sometimes) unknown origin are not very well suited for the comparison of music treatises and music manuscripts.

The manuscript *Rome, Biblioteca Casanatense, 2151* (henceforth *Cas*), however, constitutes an exception. It contains the music treatise *Declaratio musicae disciplinae* (henceforth *Declaratio*),³ which includes a discussion of notational devices to indicate rhythmic proportion, as well as three *Ars subtilior* songs (see FIGURES 4.2–4.4 below) featuring such devices, namely

¹ This chapter has been published as chapter with the title “Adapting the Concept of *Proportio* to Rhythm in the *Ars subtilior*” in *Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts*, ed. Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden (Berlin: De Gruyter, 2021), 441–64. This version contains a couple of alterations to the text in order to fit into the present study. I would like to take the opportunity of thanking Andreas Janke again for bringing the manuscript *Cas* and its compositions to my attention and for providing me with the images and other useful material. Without his generous help neither version would exist.

² Johannes Ciconia wrote an extensive treatise on proportions in 1411 (*De proportionibus*) but did not include proportion signs in the discussion. And Filippotto da Caserta is believed to be the author of two theoretical treatises, one of which (*Tractatus figurarum*) discusses notational means of rhythmic proportion but not proportion signs. For more on the discussion surrounding Filippotto's authorship concerning the *Tractatus figurarum* see Chap. 2.4.

³ With 427 chapters the *Declaratio* is one of the most comprehensive music treatises of the fifteenth century. It comprises five books: 1: *musica plana* (fundamentals of music and the modes based on Boethius and Marchetto da Padova); 2: *melodiatae musicae seu contrapuncti ratio* (counterpoint based on Prosdocimus de Beldemandis); 3: *musica mensurata* (mensural notation based on Johannes de Muris); 4: *omnium generum proportiones* (teachings of proportions based on Boethius); 5: *musica speculativa*. Cf. Jan Herlinger, “Music Theory of the Fourteenth and Early Fifteenth Centuries,” in *Music as Concept and Practice in the Late Middle Ages*, ed. Reinhard Strohm and Bonnie J. Blackburn (New York: Oxford University Press, 2001), 255.

coloration and proportion signs. As stated above, both treatise and songs are attributed to the same person, namely Ugolino di Francesco da Orvieto, who was a cleric, musical theorist, composer and singer. Due to this congruent authorship of treatise and compositions, *Cas* is a particularly suitable candidate for a comparison of statements from the music treatise and actual music notation.

Ugolino lived and worked in the Italian cities of Forlì, Florence, and Ferrara between c.1380 and 1452 and it is assumed that he wrote his treatise around the year 1430, although the autograph is lost.⁴ Ugolino seems to have enjoyed a high reputation within the Italian musical world, as can be deduced from the praise of his contemporary Flavio Biondo: “And what shall I say of Ugolino Urbevetano? Born and raised in Forlì, by universal consent he surpasses all the musicians of our time, and the book he has published on music will eclipse the labors of all who have written before him.”⁵

4.1 The Manuscript *Cas* and its Contents

The manuscript *Cas* is one of only two complete copies of the *Declaratio* and it is the only copy transmitting Ugolino’s three compositions.⁶ Unfortunately, neither scribe nor origin of the *Cas* copy of the treatise have yet been identified but Albert Seay has stated that the scribe’s Italian humanistic book hand points towards a copying date around the middle of the fifteenth century.⁷ The three songs are part of a gathering of six folios (fols. 340r–345v), which also contains a part of the index to the treatise preceding the compositions.⁸ Therefore, the three songs were

⁴ Cf. Albert Seay, “Ugolino of Orvieto, Theorist and Composer,” *Musica Disciplina* 9 (1955): 118 and Evan A. MacCarthy, “The Sources and Early Readers of Ugolino of Orvieto’s *Declaratio Musice Discipline*,” in *Beyond 50 Years of Ars Nova Studies at Certaldo 1959–2009*, ed. Marco Gozzi, Agostino Ziino, and Francesco Zimei (Lucca: Libreria Musicale Italiana, 2014), 408. Amongst others, Lewis Lockwood also considers a later date of completion of the treatise possible. Cf. *Music in Renaissance Ferrara 1400–1505: The Creation of a Musical Center in the Fifteenth Century* (Oxford: Oxford University Press, 2009), 85. The latest biographical account of Ugolino da Orvieto can be found in Andreas Janke, *Die Kompositionen von Giovanni Mazzuoli, Piero Mazzuoli und Ugolino da Orvieto im San-Lorenzo-Palimpsest (ASL 2211)*, *Musica Mensurabilis* 7 (Hildesheim: Georg Olms Verlag, 2016), 127–134, esp. 127–128. MacCarthy gives c.1390 as the year of his birth.

⁵ “Quid quod Ugolinus cognomine Urbevetanus Forlivii genitus et nutritus ornnes aetatis nostrae musicos sine contradictione superat, editusque ab eo de musica liber haud secus omnium qui ante se scripserunt labores obscurabit [...]” This commendation was first reported by Franz X. Haberl, “Bio-bibliographische Notizen über Ugolino von Orvieto,” *Kirchenmusikalisches Jahrbuch* 10 (1895): 43. Translation taken from MacCarthy, “The Sources and Early Readers of Ugolino,” 402.

⁶ For a detailed description of the manuscript see Seay, “Ugolino of Orvieto, Theorist and Composer,” 128–133. *Cas* also contains Ugolino’s monochord treatise *Tractatus monochordi*, which is only transmitted in *Cas*, *London, British Library, Add. 33519*, and *Rome, Biblioteca Apostolica Vaticana, Urb. lat. 258*. The latter manuscript is the only other complete copy of the *Declaratio*.

⁷ Cf. Seay, “Ugolino of Orvieto, Theorist and Composer,” 129.

⁸ Cf. David Fallows, “A Catalogue of Polyphonic Songs, 1415–1480: Update,” published online (2010), 19. <https://personalpages.manchester.ac.uk/staff/david.fallows/appendix.pdf>.

presumably entered after the index was completed.⁹ It can be ascertained that the music scribe is the same for all three compositions and that he was in all likelihood also responsible for copying the musical examples within the treatise.¹⁰ This would imply that treatise and compositions form one codicological unit.¹¹ Hence, the songs are in all probability not as far removed from the copying of the treatise as their position after the index as appendix to the manuscript might suggest. Furthermore, it can be assumed with reasonable certainty that the author of the treatise and the composer are the same person, namely Ugolino, since each song carries the attribution ‘Idem Ugolinus’ and he is identified as author in *Cas* as well as in several other surviving copies of the treatise.

Cas contains the following three compositions: a Latin ballade and two Italian *ballate* (see TABLE 4.1). The two voice parts are juxtaposed on facing pages. The cantus is written on the *verso* side and the tenor is notated on the *recto* of the following folio. As already stated, all three works are unique to *Cas* and do not appear in other copies of the *Declaratio* nor in any other surviving music anthology.¹² Until the discovery of the San Lorenzo palimpsest, which contains five further pieces by Ugolino, the *Cas* compositions were believed to be the only surviving music by the composer.¹³

⁹ Cf. Janke, *Die Kompositionen*, 136.

¹⁰ A comparison of the clefs and custodes in the musical example on fol. 102v of the treatise with the compositions in *Cas* strongly suggest that the same scribe was at work in these different sections of the manuscript. See n. 14 below for a comment on the manuscript’s foliation.

¹¹ It should be noted, however, that the text scribe of the three songs is not the text scribe of the treatise.

¹² Three other copies of the *Declaratio* contain music, though none of the compositions are attributed to Ugolino as in *Cas*. The manuscript *Oxford, Bodleian Library, Canon. Misc. 42* includes three anonymous two-voice Italian songs (fols. 185v–188r), *Turin, Biblioteca Nazionale Universitaria, G.IV.31* contains a textless anonymous three-voice song (fol. 97v), and *Porto, Biblioteca Pubblica Municipale, 714* contains 19 polyphonic songs of various composers (fols. 51v–79r).

¹³ The discovery of musical compositions in the manuscript *Florence, Archivio del Capitolo di San Lorenzo, 2211* (henceforth *SL*) was first reported by Frank A. D’Accone, “Una nuova fonte dell’Ars nova italiana: il codice di San Lorenzo 2211,” *Studi Musicali* 13 (1984): 3–31. Due to overwriting on almost all folios the majority of music contained in the manuscript was undecipherable. Recently, Andreas Janke and John Nádas (cf. *The San Lorenzo Palimpsest Florence, Archivio del Capitolo di San Lorenzo Ms. 2211: Introductory Study and Multispectral Images* (Lucca: LIM, 2016)) published a volume of multispectral images of all folios from the codex, which provide (partly) legible reconstructions of the original layer, thereby enabling further research on the repertoire. The compositions by Ugolino contained in *SL* are edited and discussed in Janke, *Die Kompositionen*, 135–42.

Incipit	Genre	Folios (Pages) ¹⁴	Mise-en-page ¹⁵	
<i>Se videar</i>	ballade	343v–344r (679–680)	cantus 1–6 canon 6–7	tenor 1–5
<i>L’alta virtute</i>	ballata	344v–345r (681–682)	cantus 1–8 residuum 8	tenor 1–5 canon 5
<i>Chi solo a si</i>	ballata	345v–[346r] (683–[684])	cantus 1–8 residuum 8	[tenor]

TABLE 4.1: Ugolino’s compositions in *Cas*¹⁶

All three compositions are notated in black mensural notation. However, all three songs also contain additional visual elements, namely coloration (notes in red ink) and proportion signs, which in all probability indicate the complex rhythms typical for *Ars subtilior* repertoire. In the case of the ballade *Se videar* and the ballata *L’alta virtute* this can be stated with certainty because they exhibit canons giving instructions on the interpretation of the proportion signs contained in the compositions. In the case of the last composition, the ballata *Chi solo a si*, the interpretation of the cantus, which features red and void red notation, is not straightforward, because the composition is incomplete. Folio 346r, which presumably contained the tenor, is missing from the manuscript.¹⁷ Due to water damage faint imprints of the note shapes from the *recto* of the missing folio are visible on folio 345v (see FIGURE 4.2).¹⁸ It might even be possible to reconstruct some parts of the second voice with the aid of multispectral imaging. Until then the meaning of red and void red notation in *Chi solo a si* remains in the realm of speculation.¹⁹

¹⁴ The folios of the *Cas* manuscript contain three different sets of numbers. There is complete (most likely stamped) foliation in the lower right-hand margin of each recto, which is used by the Casanatense Library and which I therefore decided to refer to in this paper. Moreover, there is complete pagination in the upper right-hand corner of each recto, which Albert Seay used for his edition of the manuscript and which is given in brackets here. And finally, there is an early foliation, which has been cut away on many folios and therefore disregarded here.

¹⁵ The numbers in the two columns refer to the staves (five lines in red ink) on the folios.

¹⁶ A similar table can be found in Janke, *Die Kompositionen*, 135 (table IV.1). It contains an error concerning the *mise-en-page* of *Se videar*, however, which I have corrected here.

¹⁷ Usually the tenor determines the duration of a piece but in this case, we do not know into how many rhythmic units the upper voice should be fitted. Moreover, due to strict counterpoint rules we can usually reject alternative interpretations on grounds that the resulting intervals are not acceptable. In absence of a second voice, we are denied this approach.

¹⁸ Cf. Janke, *Die Kompositionen*, 135.

¹⁹ Seay, who first discussed this composition in detail (cf. “Ugolino of Orvieto, Theorist and Composer,” 152–162), turned towards Ugolino’s *Declaratio* for guidance. There, Ugolino states: “Item nota plena si evacuetur tertiam partem perdit per auctorem in textu, igitur novem rubeae vacuae sex rubeas plenas valent, se sex rubeae plenae quatuor nigras plenas valent. Igitur novem rubeae vacuae quatuor nigras planes valent.” *Declaratio*, Book 3, chap. 6; edition in Albert Seay, ed., *Ugolino of Orvieto: Declaratio Musicae Disciplinae*, Corpus scriptorum de musica 7, 3 vols. (Rome: American Institute of Musicology, 1959–62), 2:211–12. In other words, six full red notes replace four black ones, thus indicating *sesquialtera* (3:2) proportion, and nine void red notes replace four black ones, indicating *dupla sesquiquarta* (9:4) proportion. Seay assumed that the coloured notes in the ballata *Chi solo a si* indicated rhythmic proportion, namely those two rhythmic proportions described in the above given excerpt from Ugolino’s *Declaratio*. Cf. “Ugolino of Orvieto, Theorist and Composer,” 160–161. Seay’s evaluation that coloured notation in *Chi solo a si* indicates proportional rhythms has recently been repeated by Janke (cf. *Die Kompositionen*, 135). As stated above, however, in absence of the tenor we cannot make definite statements on the correct rhythmic interpretation of the cantus. There are other ways of approaching the question what the

The following discussion therefore focuses on the two complete compositions, *Se videar* (see FIGURE 4.3) and *L'alta virtute* (see FIGURE 4.4).



FIGURE 4.2: Cantus of *Chi solo a si* (Cas, fol. 345v)

coloured notes are indicating in *Chi solo a si*, however. One possibility, apparently one that has not been considered by Seay, is counting the number of coloured notes and assessing whether the resulting numbers can be linked to a certain proportion. In the case of full red notes this is rather straightforward. They always appear in groups which add up to three imperfect breves. Three combinations are found in the composition: (1) three breves in ligature, (2) two breves in ligature and two semibreves, and (3) a single long and two semibreves in ligature (see FIGURE 4.2). Due to this constant number of breves it stands to reason that full red notation indicates *sesquialtera* (3:2) proportion, in which three red breves replace two black ones. Since the indication of *sesquialtera* (3:2) proportion is also the most common interpretation of full red notation in the *Ars subtilior* repertoire, it is highly likely that the full red notes indicate *sesquialtera* (3:2) proportion, just as Ugolino says in the *Declaratio*. The void red notation paints a different picture, however. According to Ugolino's statement in the *Declaratio*, void red notes indicate *dupla sesquiquarta* (9:4) proportion in comparison with full black notes. However, counting the number of breves and semibreves in the two passages, in which void red notation is applied in *Chi solo a si*, shows that they do not relate to numbers divisible by nine. The first section commencing in the second staff adds up to 16 imperfect breves (32 semibreves) and the second section in the fifth staff adds up to 20 imperfect breves (40 semibreves). That means that if void red notation indicated a proportional rhythm, the numerator of the ratio would have to be divisible by two or four at both the breve and the semibreve level. The number nine is therefore not a candidate and it follows that *dupla sesquiquarta* (9:4) proportion cannot be the proportion indicated by void red notation in *Chi solo a si*. However, the second most common rhythmic proportion in the *Ars subtilior* repertoire, namely *sesquitertia* (4:3) proportion, would be applicable to 32 and 40 semibreves and it is, in my opinion, most likely that the void red notation indicates this proportion at the semibreve level in *Chi solo a si*. The indication of a proportional rhythm is also made probable by the position of the passages in void red notation within the song, namely in melismatic passages, i.e. passages in which one syllable is stretched over several notes.



FIGURE 4.3: Cantus and tenor of *Se videar* (Cas, fols. 343v (left) and 344r (right))

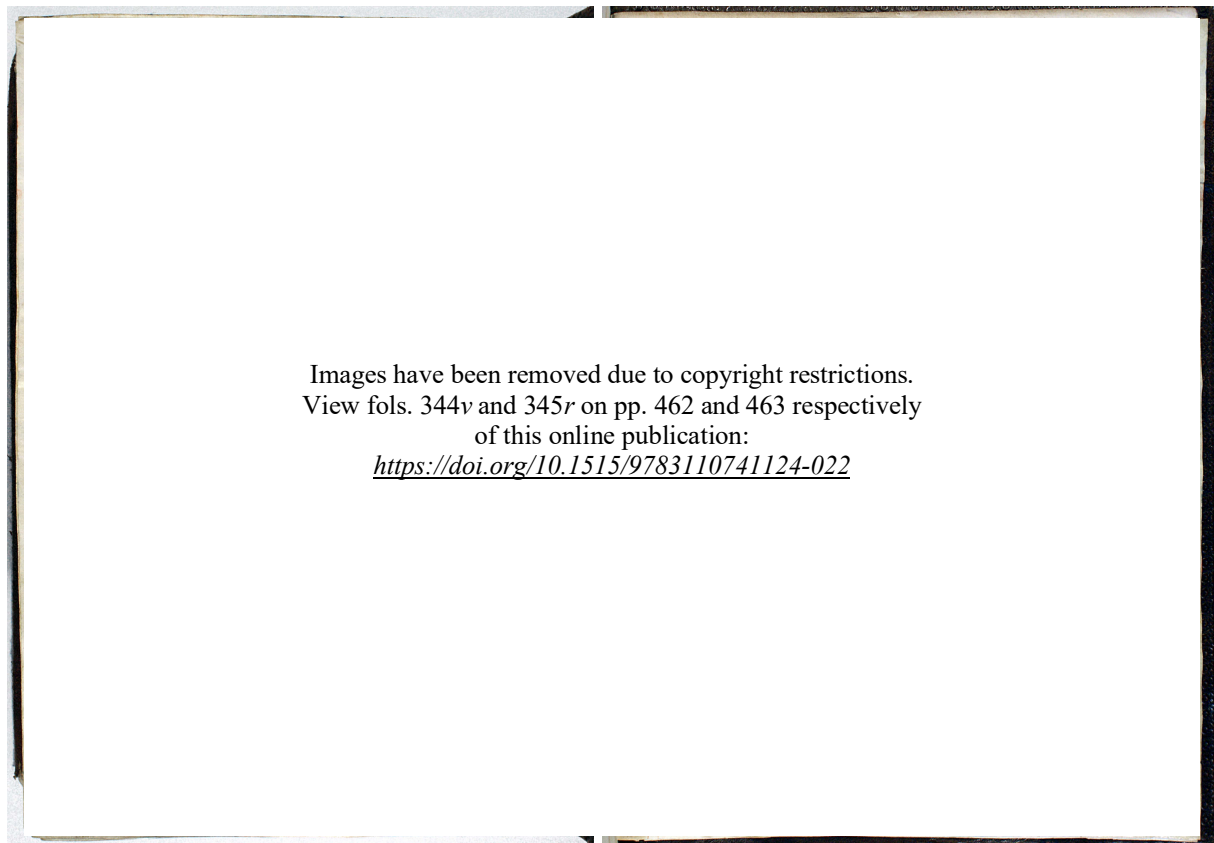


FIGURE 4.4: Cantus and tenor of *L'alta virtute* (Cas, fols. 344v (left) and 345r (right))

4.2 Proportion Signs in the *Cas* Compositions

The two songs contain ten different proportion signs. Several observations can be made from a comparison of the proportion signs and their interpretation²⁰ as illustrated in TABLE 4.5:

	2:1	1:2	3:1	3:2	2:3	4:3	3:4	9:2	9:4
<i>Se videar</i>	2	4	3	Ⓒ and ⊙	9	8	6		
<i>L'alta virtute</i>	2		O	Ⓒ		4		⊙	⊙

TABLE 4.5: Proportion signs and their interpretation in the *Cas* compositions

1) Appearance: The proportion signs in the *Cas* compositions appear as six different single Arabic numerals and four different geometric shapes, namely circles and semicircles.²¹ As can be seen from TABLE 4.5, both songs contain single Arabic numerals as well as geometric shapes, i.e. neither composition strictly uses either one form or the other. 2) Congruence: Only two proportion signs have a concordant interpretation in the two pieces. The Arabic numeral 2 indicates *dupla* (2:1) proportion in both cases and the semicircle Ⓒ indicates *sesquialtera* (3:2) proportion. 3) Divergence: Two proportions appearing in both songs are indicated by different proportion signs. *Tripla* (3:1) proportion is indicated by the Arabic numeral 3 in *Se videar* but by the circle O in *L'alta virtute*. *Sesquitertia* (4:3) proportion is indicated by 8 in *Se videar* but by 4 in *L'alta virtute*. Furthermore, 4 is used to signal another proportion in *Se videar*, namely *subdupla* (1:2) proportion. And finally, the circle with one dot indicates *sesquialtera* (3:2) proportion in *Se videar* but *dupla sesquiquarta* (9:4) proportion in *L'alta virtute*.

It can be noted that the use of signs in the two compositions is not consistent in the sense that one particular proportion sign always indicates one rhythmic proportion. This accords with findings in other *Ars subtilior* manuscripts. The pieces by Ugolino confirm that the use of proportion signs is not necessarily consistent even in music by one composer from the same manuscript. In that light, it seems even more plausible that the five special pieces in *Turin* were the output of just one individual. There are also similarities between *Turin's* special pieces and Ugolino's compositions, namely the unusual proportion sign choices in *Se videar* (9 for *subsesquialtera* (2:3) proportion, 8 for *sesquitertia* (4:3) proportion, and 4 for *subdupla* (1:2) proportion) and the use of the circle with three dots ⊙, as well as explanatory canons which contain detailed instructions on the interpretation of the signs.

²⁰ The ratios given as interpretation of the proportion signs in the two songs operate at the minim level. The two canons given in the two compositions also refer to minim level.

²¹ Anna Maria Busse Berger erroneously mentions stacked Arabic numerals ('fractions') in pieces by Ugolino. Cf. *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 183. However, stacked Arabic numerals neither appear in the *Cas* songs nor in Ugolino's *SL* compositions.

4.3 Comparing Teachings and Songs

What does the theorist Ugolino say about proportion signs? In his *Declaratio*, he writes:

“If modern cantors want to show the proportions of notes in their chants, they write some signs which are like the proportions, so if they want to make a *sesquitertia* proportion between notes they write this sign $\frac{4}{3}$ in the canon. [...] If they want to indicate *sesquialtera* proportion, then they write $\frac{3}{2}$. [...] Even such *multiplex* proportions can thus be signified by *multiplex* signs, so that if the proportion in the notes are *tripla*, *tripla* proportion shall be demonstrated in numbers in this way $\frac{3}{1}$, if *quadrupla*, in this way $\frac{4}{1}$, and so on. [...] The modern cantors also use other signs to show diminution. One of these is the semicircle, who with his semicircle looks back to the left, which is [written] like this $\ominus$, for this sign is placed by the *moderni* [...] and they attribute *sesquitertia* proportion to it. Certain people use other signs, namely semicircles, of whom one faces the upper part and the other one the lower part, and in them they distinguish different proportions, so that in this semicircle $\smile$ they make *dupla* proportion and in this $\frown$ *subsesquialtera* [proportion]. Again, they place another sign, namely this $\diamond$, of four sides, for which they use *sesquitertia* proportion. But we prefer the use of numbers, with which the proportions are shown more clearly. For in them there is no deception, but in these others there can happen ambiguity and error.”²²

Ugolino’s description of proportion signs begins with the two most common rhythmic proportions, namely *sesquitertia* (4:3) and *sesquialtera* (3:2) proportion, which are indicated by the fractions $\frac{4}{3}$ and $\frac{3}{2}$ respectively. It is remarkable that these two proportions are given as fractions with a stroke between the two numbers, since this visual appearance is unique among the sources discussed in this study. As stated before, proportions are more commonly displayed as stacked Arabic numerals ($\begin{smallmatrix} a \\ b \end{smallmatrix}$). The fractions can be found in at least two sources of the treatise (see FIGURE 4.6), however, hence the stroke does not seem to be attributable to one particular scribe. Ugolino resumes with a description of proportions of the *multiplex* type: *tripla* (3:1) proportion indicated by $\frac{3}{1}$ and *quadrupla* (4:1) proportion signalled by $\frac{4}{1}$. The supplement *et sic de ceteris* suggests that all rhythmic proportions may be indicated by fractions. Ugolino then continues by stating that the reversed semicircle $\ominus$ indicates diminution and that this diminution also results in *sesquitertia* (4:3) proportion. Other semicircles are also mentioned: $\smile$ indicates *dupla* (2:1) proportion and $\frown$ signals *subsesquialtera* (2:3) proportion. Finally, Ugolino

²² “Moderni enim cantores volentes in suis cantibus notarum proportionem ostendere, signa quaedam proportionibus conformia scribunt, nam si inter notas comparationem seu proportionem facere volunt sexquiterciam hoc in canone signum $\frac{4}{3}$ subscribunt. [...] Si sexquialteram volunt significare proportionem hoc signum describunt $\frac{3}{2}$. [...] Possunt etiam multiplicatae proportionem huiusmodi per signa multiplicata significari, ut si fiat proportio in notis tripla, tripla in cifris proportio demonstratur hoc modo $\frac{3}{1}$, si quadrupla hoc modo $\frac{4}{1}$, et sic de ceteris. [...] Aliis etiam signis moderni cantores utuntur ostendendae diminutionis causa, quorum unus est semicirculus sua semicirculatione partem sinistram respiciens qui talis est $\ominus$, hoc enim signum ponunt moderni [...] et eum sexquiterciae proportioni attribuunt. Quidam vero alia signa ponunt, scilicet, semicirculos, quorum unus partem superiorem, alter partem inferiorem respicit, sub quibus diversas intelligunt proportionem, ut sub hoc semicirculo $\smile$ duplam faciunt proportionem, sub hoc $\frown$ subsexquialteram. Aliud etiam signum apponunt hoc, scilicet, $\diamond$, quatuor laterum pro quo sexquitercia utuntur proportionem, sed nobis plus placet cifrarum positio qua proportionum clarior ostenditur demonstratio. In eis namque nulla deceptio, in his autem ambiguitas cadere potest et error.” *Declaratio*, Book 3, chap. 6; edition in Seay, *Ugolino of Orvieto: Declaratio Musicae Disciplinae*, 210–11; my translation.

declares that the diamond-shaped sign $\diamond$ (see FIGURE 4.7) may also indicate *sesquitertia* (4:3) proportion but that he would recommend the use of numerals, i.e. stacked Arabic numerals, because they are less ambiguous than the other signs.

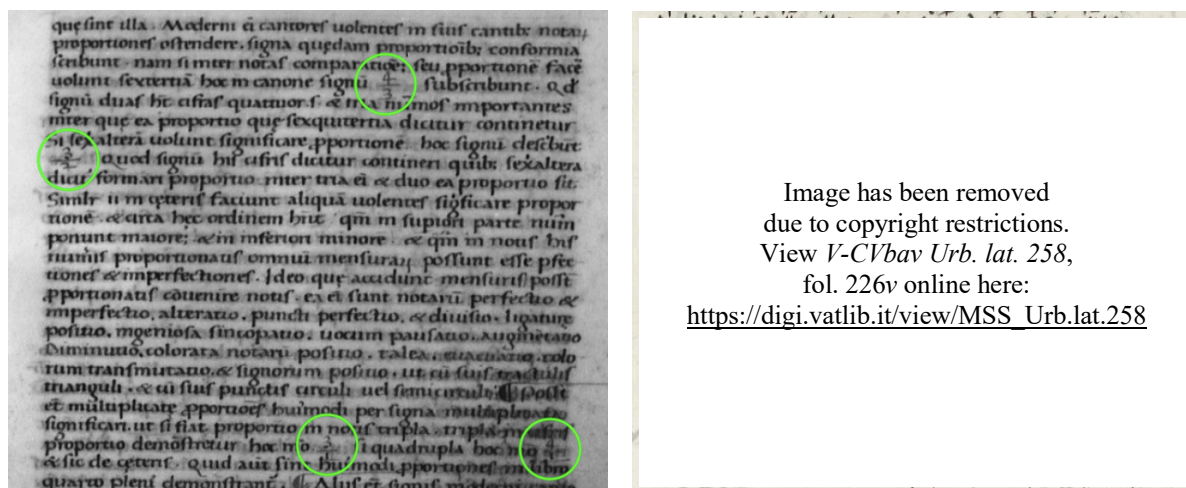


FIGURE 4.6: Fractions in Ugolino's *Declaratio* (left: *Cas*; right: *V-CVbav Urb. lat. 258*; fractions encircled)

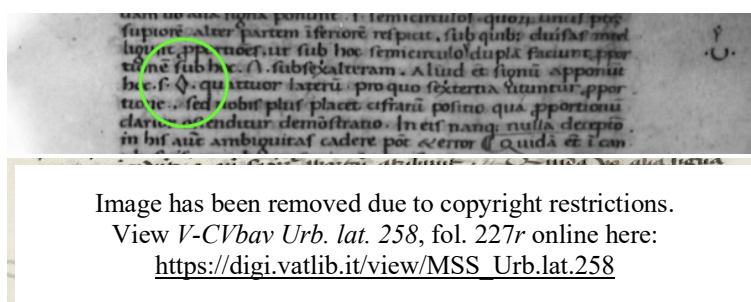


FIGURE 4.7: Diamond-shaped sign $\diamond$ in Ugolino's *Declaratio* (top: *Cas*; bottom: *V-CVbav Urb. lat. 258*; sign encircled)

A direct comparison with the statements on proportion signs from the *Declaratio* reveals that none of the signs described in the treatise matches those found in the two pieces (see TABLE 4.8):

	2:1	1:2	3:1	4:1	3:2	2:3	4:3	3:4	9:2	9:4
<i>Declaratio</i>	$\smile$		$\frac{3}{1}$	$\frac{4}{1}$	$\frac{3}{2}$	$\frown$	$\frac{4}{3}$ or $\bigcirc$ or $\diamond$			
<i>Se videar</i>	2	4	3		$\odot$ and $\ominus$	9	8	6		
<i>L'alta virtute</i>	2		O		$\odot$		4		$\oplus$	$\ominus$

TABLE 4.8: Comparison of proportion signs in Ugolino's *Declaratio* and his compositions in *Cas*

Out of the eight proportion signs depicted in the *Declaratio* only one sign, namely the reversed semicircle $\bigcirc$, frequently appears in *Ars subtilior* compositions, though not in the three songs in *Cas*. In the majority of cases, $\bigcirc$ indicates *sesquitertia* (4:3) proportion – this interpretation is

also given in the treatise – and it can even be found in Ugolino’s ballata *La vista di costei* from the San Lorenzo codex (*SL*), though its meaning there is unclear due to poor legibility.²³

Stacked Arabic numerals—given as fractions by Ugolino—only appear in a handful of *Ars subtilior* compositions and they are much less common than single Arabic numerals.²⁴ It is telling that no stacked Arabic numerals or fractions can be found in Ugolino’s surviving *oeuvre* and that the *Cas* compositions contain six different single Arabic numerals, although these are not mentioned in the treatise.

Most remarkably, we can find two semicircles ($\smile$ and $\frown$) in Ugolino’s description which seldom if ever appear in surviving *Ars subtilior* music manuscripts.²⁵ *Dupla* (2:1) proportion is usually expressed by the single Arabic numeral 2—as in the two *Cas* compositions—and *subsesquialtera* (2:3) proportion is most often applied in order to revoke *sesquialtera* (3:2) proportion, in which case the sign for the initial mensuration is used again. In *Se videar*, *subsesquialtera* (2:3) proportion is indicated by the Arabic numeral 9. And finally, the diamond-shaped sign ($\diamond$) would be most unsuitable for the indication of proportion in notated music because it has the exact same shape as the semibreve and would therefore not be distinguishable from the notes.²⁶ Ugolino even remarks upon this ambiguity: “In [numbers] there is no deception, in these others there can be ambiguity and error.”²⁷ But why does Ugolino describe seven proportion signs, which do not reflect on contemporary notational practice, and only one sign ($\circ$), which is actually being used in *Ars subtilior* music?

The third book of the *Declaratio*, in which we can find the above description on the different ways to notate proportional rhythms,²⁸ is in large parts a commentary on the famous music treatise *Libellus cantus mensurabilis* (henceforth *Libellus*) of c.1340, generally attributed to

²³ Cf. Janke, *Die Kompositionen*, 46–47.

²⁴ Among these are Petrus de Goscalch’s *En nul estat*, Anthonello de Caserta’s *Dame d’onour, en qui tout mon cuer maynt* and *Amour m’a le cuer mis en tel martire*, and Baude Cordier’s two rondeaux *Tout par compas* and *Belle, bonne, sage*. In Goscalch’s ballade and Caserta’s *Dame d’onour*, stacked Arabic numerals are (partly) used as alternative mensuration signs.

²⁵ I am only aware of one *Ars subtilior* music source using the semicircle opened at the top ($\smile$), namely the Boverio codex (*Turin, Biblioteca Nazionale Universitaria, T.III.2*), where $\smile$ appears in Johannes Suzoy’s ballade *Pythagoras, Jobal et Orpheus*—indeed indicating *dupla* (2:1) proportion as stated in Ugolino’s treatise. Cf. Jason Stoessel, “Looking Back Over the ‘Missa L’Ardant desir’: Double Signatures and Unusual Signs in Sources of Fifteenth-Century Music,” *Music & Letters* 91, no. 3 (2010): 325. Stoessel has already remarked upon the fact that the transmissions of Suzoy’s ballade in *Ch* and *Paris, Bibliothèque nationale de France, nouvelles acquisitions françaises 22069* contain written instructions underneath the music instead of $\smile$. Cf. *ibid.* 342. The semicircle $\frown$ does—to my knowledge—not appear in any surviving *Ars subtilior* music manuscript.

²⁶ It should be noted that the sign ($\diamond$) is void and not full black—at least in the *Cas* and *V-CVbav Urb. lat. 258* versions of the *Declaratio* (see FIGURE 4.7 above). One wonders whether Ugolino meant that void notation in general would indicate *sesquitercia* (4:3) proportion. But then he would more likely have given a minim instead of a semibreve. Moreover, coloration of notes and its interpretation is discussed elsewhere in the same chapter. See n. 19 above.

²⁷ Reference in n. 22 above.

²⁸ The full version of the third book of Ugolino’s *Declaratio*, which other than the abridged version contains this description, is transmitted in five different manuscripts (see TABLE 5.7 in Chap. 5.2.2 or MacCarthy, “The Sources and Early Readers of Ugolino,” 424–25 for details).

Johannes de Muris, which was written almost a century before the *Declaratio*. Proportion signs do not appear in the music of Johannes de Muris' time. Hence, Ugolino's description of proportion signs represents an adaptation to the *Libellus*, which takes into account recent developments in music notation, namely the use of proportion signs in *Ars subtilior* music.²⁹ However, in practice, these proportion signs were not used in a standardised manner, which is also reflected in the two *Cas* compositions. It is conceivable that descriptions of proportion signs in music treatises were a reaction to their inconsistent use in music notation. These discussions of notational devices to indicate rhythmic proportion might actually have been intended to standardise music notation.

It stands to reason that Ugolino's adaptation in the *Declaratio* should be regarded as a suggestion for a future notational practice rather than representation of a contemporary practice, which was not standardised. The fact that he recommends the use of numbers rather than other signs by arguing that numbers are less ambiguous points towards the intention to achieve more clarity and consistency in music notation.³⁰ After elaborating on these preferred numbers he moves on to describing the only proportion sign in *Ars subtilior* music, which has a rather consistent meaning, namely the reversed semicircle $\oslash$, which commonly indicates *sesquitertia* (4:3) proportion. The semicircle $\circ$ also exists in music notation, but it usually does not have a proportional meaning. It indicates the mensuration *tempus imperfectum* with *prolatio minor* [2,2]. With $\oslash$ and $\circ$ already established in music notation, is it not conceivable that Ugolino simply added the other two semicircles ($\smile$ and $\frown$) while acting under systemic coercion and attributed contrived interpretations to these signs? They might also have been derived from diagrams of intervals, in which proportions are often displayed as arches. In any case, the description of the two semicircles $\smile$ and $\frown$ may also be regarded as suggestion for future music notation. As to why he decided to include a sign ($\diamond$) in his description, which is—as already stated—completely unsuitable for the use in practical music remains a mystery to me. Unfortunately, I have been unable to find this sign in contexts other than music.

As established above, the proportion signs described in the *Declaratio* do not appear in the compositions in *Cas*, and—with the exception of $\oslash$ —very rarely or not at all in other *Ars subtilior* music manuscripts. However, there is an overlap between the *Declaratio* and the ballata

²⁹ Even before 1430, $\oslash$ is discussed in music treatises as a proportion sign indicating *sesquitertia* (4:3) proportion. Ugolino is therefore not the first person to mention $\oslash$ in writings on music even though the mention of the sign is an adaptation to the *Libellus*. The same is true for stacked Arabic numerals. Stacked Arabic numerals as proportion signs as well as the reversed semicircle $\oslash$ are already discussed in Prosdocimus de Beldemandis' *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris* of 1404.

³⁰ "But we prefer the use of numbers, with which the proportions are shown more clearly." Reference in n. 22 above.

L'alta virtute concerning the notation of rhythmic proportions which I would like to discuss in the remainder of this chapter.

L'alta virtute contains a proportion sign which is rarely used in *Ars subtilior* music, namely the circle with three dots $\odot$ indicating *quadrupla sesquialtera* (9:2) proportion. As shown in Chapter 3.2.1, the sign only appears in two anonymous ballades in *Turin* and the ballade *Le sault perilleux* by J. Galiot in *Ch*. However, $\odot$ is regularly featured in music treatises, where it is often discussed as mensuration sign indicating *tempus perfectum* with *prolatio maior* [3,3].³¹ In fact, it is described by Ugolino as preferable to the more common sign $\circ$ for this particular mensuration:

“Certain people, [however,] wanting in skill, in order to show perfection in minor modus, employ only one tail in a quadrangle instead of three, and, in order to show imperfection in minor modus, employ none. Similarly in signifying prolatio maior, instead of three points they use one and in showing prolatio minor [instead of two] use none. [For t]hese people [not taught in theory] do not know [what] they do [...], for a ternary number placed for the signifying of perfection is perfect, and a binary one for imperfection [is] imperfect. Therefore these numbers contain and impart perfection and imperfection, but unity [= 1], which is a part of a number and not a number, can in no way signify perfection or imperfection. Therefore one tail, which they use instead of three, or one point signify neither perfection of the modus nor [of the] prolatio. Similarly nothing is made from nothing, therefore from a sign showing nothing no perfection of mensuration can be signified, whose opposite these people put forward.”³²

According to Ugolino, the indication of major prolation by a single dot in a circle or semicircle is refutable because the value 1—thought of as ‘unity’—is not a number. Similarly, a void space in a circle or semicircle cannot represent minor prolation because a void space is ‘nothing’ and therefore cannot indicate anything. The customary signs $\odot$, $\circ$, $\odot$ and $\odot$, which are even found in Ugolino’s own compositions, are rejected. In this example, we can observe another discrepancy between music notation described in treatises and actual notational practice. The statements in the treatise are clearly influenced by philosophical doctrine, especially the statement according to which a void space in a circle cannot indicate anything. The existence of the more common form of mensuration signs with one or no dot is acknowledged, but their use is attributed to less talented composers.

³¹ A list of treatises describing $\odot$ as sign for *tempus perfectum* with *prolatio maior* can be found in Busse Berger *Mensuration and Proportion Signs*, 236–37. Among the authors are some of Ugolino’s contemporaries: Johannes Ciconia (d. 1412), Prosdocimus de Beldemandis (d. 1428), and Giorgio Anselmi (d. c.1440–1443).

³² “Quidam vero ignari peritiae pro tribus in quadrangulo tractulis ad modi minoris ostendendam perfectionem, uno duntaxat utuntur tractulo et ad minoris imperfectionem modi nullo penitus utuntur. Similiter in significanda prolatione maiori pro punctis tribus uno utuntur et in minori pro duobus nullo. Hi namque nulla ratione fundati quod agunt penitus ignorant, nam numerus ternarius pro significanda perfectione positus perfectus est, et binarius pro imperfectione imperfectus. Perfectionem igitur et imperfectionem ii numeri continent et important, sed unitas quae pars numeri est et non numerus perfectionem vel imperfectionem nullatenus significare potest, ergo unus tractulus quem isti ponunt pro tribus, vel unus punctus nec modi nec prolationis significant perfectionem. Similiter ex nihilo nihil fit, igitur ex nullo signo nulla mensurae imperfectio potest significari, cuius oppositum isti ponunt.” *Declaratio*, Book 3, chap. 6; edition in Seay, *Ugolino of Orvieto: Declaratio Musicae Disciplinae*, 2:200–1; translation taken from *idem*, “Ugolino of Orvieto, Theorist and Composer,” 154.

By using $\ominus$ in *L'altra virtute*, is Ugolino taking the above statements into consideration? The answer is: probably not. Regarding the proportion signs used in *L'altra virtute* in the order of their appearance, we can observe that the more common forms of circles and semicircles with one or no dot are used first (see FIGURE 4.9).

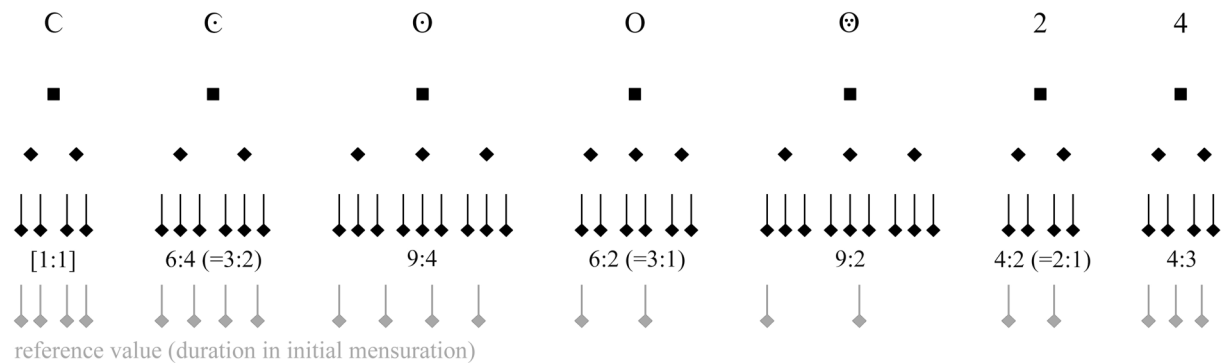


FIGURE 4.9: Proportion signs and their interpretation in *L'altra virtute* in the order of their appearance

All geometric shapes in the ballata function as proportion signs as well as mensuration signs, i.e. the general distribution of semibreves and minims is changed under each sign in addition to them having a proportional meaning. The mensuration first changes from the initial *tempus imperfectum* with *prolatio minor* (C) to *tempus imperfectum* with *prolatio maior* (Ċ). Breve equivalence leads to a proportional change at the minim level: Three minims under Ċ replace two in C. Subsequently, the mensuration changes to *tempus perfectum* with *prolatio maior*, which is indicated by the common sign Θ. With breve equivalence still operating, the proportional change at the minim level is 9:4. After a change to yet another mensuration (*tempus perfectum* with *prolatio minor* indicated by O) the ballata returns to *tempus perfectum* with *prolatio maior*. This time, however, diminution is also involved, i.e. the breve under ⊕ is equivalent to a semibreve in the initial mensuration in C. Ugolino therefore needed an alternative sign for Θ, because he already used the circle with the single dot for the indication of *dupla sesquiquarta* (9:4) proportion. He reverts to the sign ⊕, which the treatise praises as superior to the circle with the single dot, only in need of an alternative to Θ. This demonstrates that despite the statements in the treatise, the sign ⊕ is still only number-two choice in music notation.

Incidentally, although some proportion sign passages follow one after another, the canon makes it clear that the proportions have to be applied to the initial mensuration, i.e. in a non-cumulative manner. With Ugolino's date of birth given as 1380 or even 1390 by Evan MacCarthy, *L'altra virtute* is another argument against a development from non-cumulative to cumulative proportions.

4.4 Conclusion

The comparison of music treatise and songs in the *Cas* manuscript has shown that the introduction of proportion signs and the discussion thereof in music treatises are not as interrelated as might be expected. We can observe a high amount of divergence between Ugolino's statements on proportion and mensuration signs on the one hand and the signs used in the compositions transmitted in *Cas* on the other hand.

The proportion signs discussed in the sixth chapter of the third book of the *Declaratio* seem for the greater part to have been adopted from other contexts. They may have a speculative background, as stacked Arabic numerals or fractions respectively have their origin in arithmetic and the two unusual semicircles resemble arches used in the depiction of intervals. Out of the eight proportion signs, which Ugolino mentions, only the reversed semicircle $\oslash$ is commonly used in *Ars subtilior* compositions, though not in the *Cas* songs. The discussion of fractions could be interpreted as an attempt to achieve more clarity in the future, but we can observe that the use of single Arabic numerals is still the prevalent custom in music notation. Disregard of the instructions in the treatise can also be discerned in the use of mensuration signs with one dot or without a dot instead of signs with three or two dots. As has been shown, the sign with three dots is only reverted to on rare occasions. In the case of *Cas*, it was the need for an alternative sign for *tempus perfectum* with *prolatio maior* ($\odot$), because $\oslash$ was already used for another purpose. This example as well as the inconsistent use of signs in the two compositions demonstrate that proportion signs seem to have been chosen more or less arbitrarily according to their availability. This pragmatic approach contrasts with the theoretical approach in the *Declaratio*.

The discrepancies between statements in music treatises and notational practice discussed in this chapter should particularly be taken into account when treatises are consulted for the purpose of interpreting means of rhythmic notation in compositions. Several sources of *Ars subtilior* music are incomplete or partly illegible, for example because of water damage or scraping, in which case scholars often draw on contemporary treatises in order to provide partial transcriptions and editions. This is even true for *Cas*. As shown above, Albert Seay turned towards Ugolino's *Declaratio* for guidance on the interpretation of coloured notes in the incomplete ballata *Chi solo a si* transmitted on the last folio of the manuscript. I have explained why void red notes cannot indicate *dupla sesquiquarta* (9:4) proportion in this composition.³³ Not least to caution against such approaches, this chapter has aimed to establish that discussions

³³ See n. 19 above.

of music notation in treatises and actual notational practice should be regarded in their context and are not necessarily congruent.

While I have addressed the situation, in which pieces are transmitted incompletely, such as *Chi solo a si*, the approach of reverting to theory if in doubt about a sign's interpretation may likewise be inadvisable for compositions in which all voices simultaneously change to a proportion at a certain point in the composition and where the interpretation of a sign may therefore be unclear.³⁴ In his *NG* article on proportional notation, Roger Bowers has stated:

“[W]hen all the constituent voices of a polyphonic composition progress simultaneously from *integer valor* to proportion (or vice versa), the temporal relationship cannot be resolved by inspection and resort must be made to contemporary theory. At this stage, distinction must always be made between those writers who were faithfully and objectively describing current usages, and those who took up the pen to advocate novel systems and approaches consciously divergent from the contemporary practice, revised and reformed in ways commendable to the author but not necessarily to anyone else. Both are illuminating in academic terms, only the former in practical terms.”³⁵

Again, the case of Ugolino has shown that the theorist's instructions are not even applicable to his own compositions. Hence, I would advise against Bowers' approach of consulting contemporary theory in these instances. However, Bower seems to distinguish between two kinds of authors. Those “describing current usages” and those advocating “novel systems [...] divergent from contemporary practice.” But how can we tell the one from the other? And do the latter really describe novel systems or are these texts—as seems to be the case with Ugolino—strongly influenced by quadrivial scholarship and speculative paradigms and therefore not novel but simply not practical? Furthermore, there is the description of the reversed semicircle $\bigcirc$ in Ugolino's *Declaratio* and this sign is part of contemporary practice, while some of the other signs are not. Where do we draw the line between ‘helpful’ and ‘unhelpful’ theory? By analysing the statements on proportion signs in thirteen other treatises up to c.1450 in the next chapter, I wish to further explore this question.

³⁴ As discussed in Chap. 2.2, this is the case in the ballata *Serà quel zorno may, dolze madonna mia* (Mod A 98) by Matteo da Perugia, which contains a passage in which all three voices simultaneously change to $\bigcirc$.

³⁵ Roger Bowers, “Proportional Notation,” in *NG2* (London: MacMillan Publishers, 2001), 20:428; my emphasis.

5

PROPORTION SIGNS IN MUSIC TREATISES BEFORE C.1450

It is apparent that many music treatises originated from a context of teaching and learning. The treatise served as instruction, in which rules were laid out, sometimes accompanied by musical examples. In addition to verbal instructions offered in lessons and the act of learning by imitation—both from the category of oral transmission of knowledge—treatises can be regarded as central medium for teaching and learning about music and its notation throughout the Middle Ages.

As already stated in the first chapter of this study, Johannes de Grocheio, who lived at the turn of the fourteenth century, wrote that only three signs—namely *longa*, *brevis*, and *semi-brevis*—were needed to notate any melody:

“Just as the grammarian can write any word from a few letters by their joining together and placement and can indicate any number into infinity, artificially numbering from a few figures by putting them in front or in back, so the musician can write out any measured song from these three figures.”¹

However, just as the grammarian needed a grammar textbook and as the calculator needed tuition in arithmetic, the musician required instructions on how to interpret those three note signs in the process of reading music, and the composer needed a manual on how to arrange the signs when notating melodies.

Treatises played an important role in providing this knowledge. Even Franco of Cologne—in the prologue of his influential treatise *Ars cantus mensurabilis* written in Paris c.1280—stated that this was his primary aim:

“And let not somebody say that we have undertaken this work out of arrogance, or perhaps even for our own advantage, but truly for the sake of evident necessity, and for the sake of readiest understanding by students, and also, indeed, of the most perfect instruction of all notators of the same measurable music.”²

¹ “Et quemadmodum grammaticus ex paucis litteris earum coniunctione et situatione potest dictionem quamlibet designare et artificialiter numerans ex paucis figuris earum praepositione et postpositione numerum quemlibet infinitum designare, ita musicus ex tribus figuris cantum quemlibet mensuratum.” Johannes de Grocheio, *De musica*, chap. 14; edition in Ernst Rohloff, ed., *Der Musiktraktat des Johannes de Grocheo nach den Quellen neu herausgegeben mit Übersetzung ins Deutsche und Revisionsbericht*, Media latinitas musica 2 (Leipzig: Gebrüder Reinecke, 1943), 55; translation taken from Albert Seay, trans., *Johannes de Grocheo: Concerning Music (De musica)*, Colorado College Music Press Translations 1, 2nd ed. (Colorado Springs: Colorado College Music Press, 1974), 24.

² “Nec dicat aliquis nos hoc opus propter arrogantiam, vel forte propter propriam tantum commoditatem incepisse, sed vere propter evidentem necessitatem et auditorum facillimam apprehensionem necnon et omnium notatorum ipsius mensurabilis musicae perfectissimam instructionem.” Franco of Cologne, *Ars cantus mensurabilis*, prologue; edition in Gilbert Reaney and André Gilles, eds., *Franconis de Colonia: Ars cantus mensurabilis (ca. 1280)*, Corpus scriptorum de musica 18 (Rome: American Institute of Musicology, 1974), 24; translation taken from Rob. C. Wegman, trans., “Franco of Cologne: The Art of Measurable Song (c.1280),” 1.

However, what also becomes apparent in several treatises is that there was disagreement among theorists as well as practitioners and that different teachings circulated simultaneously:

“To these figures they attributed a significance in various ways. Hence, knowing how to sing and to read a song is done by one method according to some, according to another by others. The diversity of all these methods will appear to those looking into the various treatises of others.”³

“Since we touched lightly and briefly on the theory of music in the preceding discourse[s], it now remains to [also] inquire at greater length into its practice, that part, which is measurable, since different practitioners think differently about this.”⁴

“We have still to show by what figures, signs, or notes the things which we have said may be appropriately indicated or represented, and by what words or names they may be called, for at this very time our doctors of music dispute daily with one another about this. And although signs are arbitrary, yet, since all things should somehow be in mutual agreement, musicians ought to devise signs more appropriate to the sounds signified.”⁵

Thus, there seems to be dissent concerning musical notation between different agents. “[O]ur doctors of music” appear to dispute with one another, but different practitioners also seem to have different opinions on mensural notation. When we find contradictory statements concerning musical notation, which theorist has greater authority? And to what extent are statements that are found in treatises valid and applicable to music transmitted in music manuscripts of the same or an earlier period? As I have shown in Chapter 4, the proportion signs described by Ugolino da Orvieto in his treatise *Declaratio musicae disciplinae* are completely different from those that he used in his own compositions.

In the fifteenth century, we can—among various other matters—observe disagreement concerning the instructions on notating complex rhythms, either in the interpretation of various special note shapes or in the descriptions of proportion signs. Sometimes, one can find inconsistent or even contradictory statements concerning proportion signs within the same treatise.⁶ In addition to that, the proportion signs mentioned by the authors of treatises and their

³ “Istis autem figuris diversimode significationem tribuerunt. Unde sciens cantare et exprimere cantum secundum quosdam, secundum alios non est sciens. Omnium autem istorum diversitas apparebit diversos tractatus aliorum intuenti.” Johannes de Grocheio, *De musica*, chap. 14; edition in Rohloff, *Der Musiktraktat des Johannes de Grocheio*, 56; translation taken from Seay, *Johannes de Grocheio: Concerning Music*, 25.

⁴ “Quoniam in antepositis sermonibus theoricam musicae leniter tetigimus et in brevi, nunc quoque de eius practica, ea parte, qua mensurabilis est, restat diffusius inquirendum, cum de ipsa diversi diversimode sentiant practi-cantes.” Johannes de Muris, *Notitia artis musicae*, Book 2, chap. 1; edition in Ulrich Michels, ed. *Johannis de Muris: Notitia artis musicae and Compendium musicae practicae; Petrus de Sancto Dionysio: Tractatus de musica*, Corpus scriptorum de musica 17 (Rome: American Institute of Musicology, 1972), 65; translation taken from Oliver Strunk, ed., *Source Readings in Music History*, rev. ed. Leo Treitler (New York and London: W. W. Norton, 1998), 262.

⁵ “Restat quoque, quibus figuris, signis, notulis, quae dicta sunt, convenienter debeant designari quibusque sermonibus vel vocibus appellari, cum modo tempore nostro super hoc cotidie nostri doctores musicae ad invicem convixantur. Et licet signa sint ad placitum, tamen quoniam omnia sibi invicem consonant quodammodo signa convenientiora vocibus signandis debent a musicis inveniri.” Johannes de Muris, *Notitia artis musicae*, Book 2, chap. 4; edition in Michels, *Notitia artis musicae*, 74; translation taken from Strunk, *Source Readings*, 264.

⁶ For example, the anonymous treatise *Venerabiles domini mei* lists the same Arabic numeral as indicating three different rhythmic proportions. For more, see Chap. 5.3.2.4 below.

interpretation often do not match what is found in music of the same period. For example, while many authors advocate the use of stacked Arabic numerals for the sake of clarity, we only rarely find stacked Arabic numerals in *Ars subtilior* compositions.

In Chapter 1 of this study, I have already described how the concept of proportion and proportionality was gradually adapted to rhythmic contexts with the evolution of mensural notation in Europe in the last decades of the thirteenth century. The term ‘proportion’ was no longer only used within discussions of musical intervals and their relating numerical relationships (e.g. *dupla* (2:1) proportion for the octave etc.), but was also applied in the context of durations of note values. This adaptation can already be observed in the treatise *Tractatus de musica* (written between 1265 and 1275 and attributed to Magister Lambertus), in Franco of Cologne’s influential treatise *Ars cantus mensurabilis* of c.1280, and in Johannes de Muris’ *Notitia artis musicae* of 1321. The anonymous treatise *Les règles de la seconde rhétorique* (1411–32) claims that Philippe de Vitry is the discoverer of these new kinds of proportions: “And after that came Philippe de Vitry, who in music found the four prolations, the red notes, and the new proportions.”⁷

Eventually, discussions of notational devices to indicate rhythmic proportion also found their way into music treatises. By including descriptions of special note shapes, coloration, and proportion signs, authors composed new texts or sometimes updated texts by other authors—for example the *Libellus cantus mensurabilis*—to include some of the devices composers used for indicating complex rhythms.

However, where rhythmic proportion is concerned, there appear to be significant time gaps between musical practice, the invention of notational devices to depict these rhythms, and the adaptation thereof in music theory treatises. It seems that minims of different durations were already established in musical practice before notational devices to indicate such rhythms appeared in music or music theory manuscripts. The 1351 treatise *Quatuor principalia musicae* by the English theorist John of Tewkesbury, for example, contains a remark on *sesquitertia* (4:3) proportion. The author rejects the idea of a variable minim but reports of certain people believing that four minims can be sung against three:

“[Therefore it is to remark that] whenever four minims are pronounced separately [...] they are the equipollent of an imperfect brevis of minor prolation. [For i]f three minims are pronounced equally, they are the equipollent of a semibreve of major prolation. Many believe the measure

⁷ “Après vint Philippe de Vitry, qui [...] en la musique trouva les .iiij. prolacions, et les notes rouges, et la noveleté des proporcions.” Transcription taken from Ernest Langlois, ed., *Recueil d’arts de seconde rhétorique* (Paris: Imprimerie nationale, 1902), 12; my translation.

to be one when someone pronounces four distinct minims [while] another pronounces three. In this they are deceived, for reason contradicts them, as there is no equipollence between these minims.”⁸

In other words, four minims in *tempus imperfectum* with *prolatio minor* [2,2] were sometimes ‘pronounced’ or sung in the same time as three minims in *prolatio maior* (either [2,3] or [3,3]). As Anne Stone has concluded, John of Tewkesbury’s remark implies that:

“already by 1351, *sesquitertia* proportion, 4:3, was being performed, a practice that was outside the scope of *Ars nova* doctrine, and in fact one of the principal stylistic features of the *Ars subtilior*. No music from this period survives that shows this proportion notated, but it seems clearly to have been practiced.”⁹

While this may be true for *Ars nova* notation, the Italian system could display *sesquitertia* (4:3) proportion in the *divisio octonaria* which was already practiced and notated at that time. Since John of Tewkesbury uses French terms for the prolations, however, he seems to refer to the French notational system. Hence, the first time gap occurs between musical practice and the reflection of this practice in notated music. John of Tewkesbury’s treatise indicates that there was a musical practice of *sesquitertia* (4:3) proportion in French notation around the middle of the fourteenth century, which, nonetheless cannot be found in *Ars nova* music manuscripts of that period.¹⁰ Furthermore, the idea of a variable minim is clearly rejected by the theorist when he writes: “they are deceived, for reason contradicts them.” A denial of rhythmical phenomena on grounds of philosophical doctrine can also be observed in theoretical treatments of the semiminim, as outlined in Chapter 1.3.¹¹

A second time gap occurs between notational practices in notated music and the description of these practices in music theory. The rejection of certain notational signs due to philosophical doctrine, for example, still pertains to Ugolino da Orvieto’s c.1430 treatise *Declaratio musicae disciplinae*, in which the author explains why one should use of the mensuration signs ☉ and ☼ instead of Θ and O for *tempus perfectum* with *prolatio maior* or *minor* respectively, as shown in Chapter 4.3.

⁸ “Unde notandum quod quandocunque quatuor minimae separatim pronuntiantur [...] aequipollent brevi imperfectae de minori prolatione. Si enim tres aequaliter pronuntiantur, semibrevis de majori prolatione aequipollent. Tamen multi credunt unam esse mensuram, cum quis quatuor distinctas pronuntiat minimas, dummodo alius pronuntiat tres; in hoc enim decepti sunt, quia ratio eis contradicit, cum aequipollentia inter illas non est.” John of Tewkesbury, *Quatuor principalia musicae*, Book 4, chap. 9; edition in Coussemaeker, *Scriptorum de musica*, 4:277; translation taken from Luminita Florea Aluas, “The *Quatuor principalia musicae*: A Critical Edition and Translation with Introduction and Commentary,” 2 vols., PhD diss., Indiana University, 1996, 1:703.

⁹ Anne Stone, “The *Ars subtilior* in Paris,” *Musica e storia* 10, no. 2 (2002): 396.

¹⁰ For an explanation of the terms ‘French notation’ and ‘Italian notation’, see Chap. 1, n. 59 on p. 23.

¹¹ For a detailed study of this particular aspect see Karen Cook, “Theoretical Treatments of the Semiminim in a Changing Notational World c. 1315–c. 1440,” PhD diss., Duke University, 2012.

Ugolino was by no means the only theorist describing signs with three or two dots respectively as mensuration signs during a time when the signs Θ , O , $\mathbb{C}$ and $\mathbb{C}$ were customarily used. Among other treatises, these signs can be found in Prosdocimus de Beldemandis' two works *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris* (1404) and *Tractatus practice cantus mensurabilis* (1408) as well as Johannes Ciconia's early-fifteenth-century *Nova musica*.¹² Johannes Ciconia, incidentally, also seems to be following the canon rather than including novel notational devices to indicate rhythmic proportion. It is most noteworthy that the composer, whose compositions contain an array of different proportion signs, does not mention proportion signs in his treatise *Nova musica* at all, and not even in his treatise *De proportionibus* of 1411.¹³

When regarding individuals, which acted as theorist as well as as composer, we seem to have to distinguish between the theorist, who follows the canon of earlier authorities, and the composer, who applies note values smaller than the minim or notational devices disapproved of by music theorists. In the cases of Ciconia and Ugolino, the theorist seems to be a different agent than the composer.

On the other hand, there are treatises, which seem to have been intended as manuals for composers and notators, in which the authors have disregarded any reservations concerning minima division and signs with one or no dot. The wish of being able to notate what can be sung is, for example, stated clearly by the author—some believe him to be Philippus de Caserta—of the late fourteenth-century treatise *Tractatus figurarum*: “Since it would be very incongruous for that which is able to be performed not to be able to be written, I took care to arrange this little treatise to show this clearly.”¹⁴

¹² A list of treatises describing circles with three or two dots as signs for *tempus perfectum* with *prolatio maior* or *minor* respectively can be found in Anna Maria Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 236–37.

¹³ As Philippe Vendrix has described, Ciconia seems to have aimed at re-establishing music as university discipline rather than describing notational practices of the day in his two treatises. Cf. “Johannes Ciconia, *cantus et musicus*,” in *Johannes Ciconia: musicien de la transition*, ed. *idem* (Turnhout: Brepols, 2003), 34. In the preface of the edition and translation of the composer's two theoretical works, Oliver B. Ellsworth has noted: “The purpose of [*Nova musica*], as stated clearly in the prologue, is to return to the writings of earlier authors (through the eleventh century) and, using their material as a basis, to redefine the scope of the discipline of music so that it may be classified and function as one of the literary arts, as well as a mathematical one. This new view of music can be regarded, as a clear indication of a new humanistic approach to the arts.” *Johannes Ciconia: Nova Musica and De Proportionibus: New Critical Texts and Translations by Oliver B. Ellsworth*, *Greek and Latin Music Theory* 9 (Lincoln, NE and London: University of Nebraska Press, 1993), ix. For a contextualisation of Ciconia's theoretical output see Mark André, “Johannes Ciconia, théoricien,” / “L'Œuvre théorique de Johannes Ciconia,” *Revue de la Société liégeoise de Musicologie* 4 (1996), 23–40.

¹⁴ “Quia esset multum inconueniens quod illud quod potest pronuntiari non posset scribe et clare ostendere tractatum hunc parvulum ordinare curavi.” Anonymous, *Tractatus figurarum*, chap. 2; edition in Philip E. Schreier, ed. and trans., *Tractatus figurarum: Treatise on Noteshapes*, *Greek and Latin Music Theory* 6 (Lincoln, NE: University of Nebraska Press, 1989), 72; translation taken from Anne Stone, “Ars Subtilior,” in *The Cambridge History of Medieval Music*, ed. Mark Everist and Thomas Forrest Kelly, 2 vols. (Cambridge: Cambridge University Press,

Naturally, of interest to this study are all those treatises, which either display or describe proportion signs, which the *Tractatus figurarum*, for example, does not. I am making the distinction between display and description, because some treatises describe the visual appearance of signs, without displaying the actual signs. A treatise labelled *Student's notes of lectures on music*, which is written in Hebrew, for example, describes $\mathcal{C}$ and $\emptyset$ respectively as “a half circle cut in half or a full circle broken”¹⁵ without displaying the geometric shapes. I have included these signs in this study, even if they are only described. It is noteworthy that canons accompanying *Ars subtilior* compositions sometimes also describe signs rather than displaying them. For example, the canon of the ballade *Sur toute fleur la rose est colorie* from *Turin* describes the signs $\odot$, $\odot$, and $\odot$ respectively as “*circulum cum tribus punctis*”, “*circulum cum duobus punctis*”, and “*circulum duplicem*”.¹⁶

An overview of treatises mostly originating from the first half of the fifteenth century, which mention proportion signs, was given by F. Alberto Gallo as early as 1984.¹⁷ All those treatises can be found in Anna Maria Busse Berger's chapter on proportion signs, in which she has greatly drawn on Gallo's study.¹⁸ Both Gallo's study “Die Notationslehre im 14. und 15. Jahrhundert” and Busse Berger's chapter on proportion signs, however, should be treated as preliminary reports on proportion signs in theoretical treatises in the context of the *Ars subtilior* movement. For example, neither author has described the actual signs found in the respective treatises. No comprehensive overview of treatises from the first half of the fifteenth century containing descriptions of proportion signs (including a table of the signs described or displayed) has yet been published, which is why I intend to provide such a table (see TABLE 5.6 on p. 227 below) in this study.¹⁹ Beyond that, I will take a closer look at the teachings transmitted in the respective treatises and at the musical examples given in five of them.

Despite the common topic of the text passages under discussion here, namely proportion signs, one can observe strikingly different approaches to the subject. Some authors, for example Prosdocimus de Beldemandis and Ugolino of Orvieto, have treated proportion signs only very

2018), 2:1128. For more on the discussion surrounding Philippus' authorship concerning the *Tractatus figurarum* see Chap. 2.4.

¹⁵ Anonymous, [*Student's notes of lectures on music*], chap. 4; translation taken from Stone, “The *Ars subtilior* in Paris,” 398. I have refrained from citing the original phrase in Hebrew. An edition can be found in Israel Adler, ed., *Hebrew Writings Concerning Music in Manuscripts and Printed Books from Geonic Times up to 1800*, RISM B IX 2 (Munich: G. Henle Verlag, 1975), 58–66. The publication also contains a b&w facsimile of the treatise.

¹⁶ *Turin*, fol. 137r. I have discussed this in Chap. 3.2.2.

¹⁷ Cf. F. Alberto Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” in *Die Mittelalterliche Lehre von der Mehrstimmigkeit*, ed. Hans Heinrich Eggebrecht *et al.* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1984), 340–48.

¹⁸ Cf. Busse Berger, “Proportion Signs,” chap. 6 in *Mensuration and Proportion Signs*, 164–226.

¹⁹ I also wish to resolve some misunderstandings or errors concerning Busse Berger's “TABLE 6. Proportions in theoretical treatises, c.1400–1450” in *Mensuration and Proportion Signs*, 167.

briefly in their works, mentioning different signs in a few sentences, usually in the context of discussing mensuration signs. In that case, the discussion is part of a comprehensive treatise on music. In other cases, there are texts of various, often anonymous authors, which only discuss proportion signs in short treatises comprising a few folios. In yet other examples, proportion signs are discussed in the context of treatises on proportion in general, sometimes attached at the very end as if in an afterthought. It stands to reason that the subject of proportion signs was added to already existing texts on proportions in these cases.

One can definitely observe different degrees of standardisation of musical notation in the altogether 14 treatises under discussion here. There are treatises defining only one sign—in case of *dupla* (2:1) or *sesquiertia* (4:3) proportion sometimes two signs—for a particular proportion, as if to say: “It is done thus and cannot be done differently.” This tendency can especially be observed in the comprehensive treatises on music, e.g. the works by Prosdocimus de Beldemandis and Ugolino of Orvieto, and in those shorter treatises, which do not contain musical examples.

Other treatises, namely those seemingly more oriented towards notational practice, do contain musical examples, e.g. Georgius Erber’s *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente* or the anonymous treatise *Venerabiles domini mei*. These treatises frequently offer more than one sign for a particular proportion, thereby—intentionally or not—acknowledging their manifold appearances in music. Musical examples and/or text seem to state: “You can do it this or that or another way. These are different possibilities and the choice is yours.”

One treatise discussed in this chapter does not fit into either category, however, namely the anonymous *Pars aliquota est illa quae aliquotiens sumpta*. The musical examples in this work are so complex that they are in no way inferior to the more challenging compositions found in music manuscripts. As I will show below, I have been unable to find satisfactory transcription solutions to all three musical examples. Hence, one needs at least some prior knowledge to read and understand the section on proportion signs in this treatise. Furthermore, the section is more of a portrayal of current musical practice than any helpful instruction on how rhythmic proportion works and how it is indicated. More on this treatise can be found in Chapter 5.3.3 below.

To recapitulate: The aspect of notating proportional rhythms is approached in different ways throughout the treatises under discussion here. One can discern different degrees of standardisation of musical notation in the texts. This chapter is subdivided accordingly: I will discuss the treatises in favour of standardisation without musical examples first, followed by those which do contain musical examples and which allow some degree of individualisation. This

chapter will close with the above-described anonymous *Pars aliquota est illa quae aliquotiens sumpta*, which contains three examples in the *Ars subtilior* style within its text. Preceding the discussion of individual treatises are two subchapters. First, I will explain my approach to finding all edited fifteenth century treatises mentioning proportion signs with the help of the TML and other databases. Following this, I will provide a general overview of treatises, which most probably originate from the first half of the fifteenth century, describing or displaying proportion signs. The reader interested in a general overview of what is advised in which treatise is referred to Chapter 5.2, while the person looking for a more detailed discussion of the respective contents should regard Chapter 5.3.

5.1 Searching Proportion Sign Treatises with the Help of the TML and Other Databases

F. Alberto Gallo's study on the teachings of proportions in the fourteenth and fifteenth century has been available since 1984 and Anna Maria Busse Berger published her monograph on mensuration and proportion signs in 1993.²⁰ Both studies are good points of departure for anyone interested in proportion sign treatises from the fifteenth century. However, neither Gallo nor Busse Berger had access to the research tools available via online databases such as the *Lexicon musicum Latinum medii aevi* (LmL)²¹ or the *Thesaurus Musicarum Latinarum* (TML).²² With research tools such as full-text search available in many databases today, it is possible to begin from scratch, i.e. searching all edited treatises mentioning proportion signs, thereby confirming other scholars' findings concerning treatises and adding new treatises to the list. Moreover, the procedure is not only possible, but also necessary, since many editions of medieval music treatises have been published during the past three decades. My search for proportion sign treatises has mostly confirmed the findings in the above-named studies but also unearthed a treatise, which has not been considered by either Gallo or Busse Berger.

Unfortunately, the editors of the LmL have not included '*proportio*' as lemma in their publication: the term '*properchant*' is followed by '*proprietas*'.²³ The TML, on the other hand, offers full-text search of all editions of Latin music treatises from the third to the seventeenth century. Thus, searching for a particular term within all treatises originating from a certain century is possible today, for example, searching for mentions of the term '*prolatio*' in treatises of the fourteenth century. However, searching for particular signs, e.g. specific proportion signs, is—most unfortunately—not as straightforward.

Entering Arabic numerals, e.g. the Arabic numeral 2, into the search box yields many results not at all connected to proportion signs. For example, this search will include all numerals 2 in the results which are present in the text. This includes all numbers containing the numeral 2, e.g. 12 or 24 (which often appear in discussions of musical intervals), chapter numbers, and even editors' notes, such as "[C on staff 2]".

²⁰ Cf. Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 257–356; and Busse Berger, *Mensuration and Proportion Signs*.

²¹ Online at <http://www.lml.badw.de/lml-digital/woerterbuchnetz.html>.

²² Online at <http://www.chmtl.indiana.edu/tml/>.

²³ Cf. Michael Bernhard, ed., *Lexicon musicum Latinum medii aevi*, 2 vols. (Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1995–2016), 2: col. 928. Probably, proportion is too broad and too unspecific a term to assign explicit interpretations to it. Any terms related to the word '*signum*' also do not yield fruitful results where proportion signs are concerned, neither do the searches for the terms '*dupla*', '*tripla*', '*sesquialtera*', or '*sesquitercia*' respectively.

Searching for the code of a particular sign is possible, though. For example, the reversed semicircle $\oslash$ is encoded as [CL], a circle with three dots $\odot$ is encoded as [O3d], a circle with two dots $\odot$ as [O2d], and so on.²⁴ Entering '[CL]' in the search box will list only those treatises as search results, however, in which this particular code is used. The problem is that some signs are not given as codes in the edition, but rather included in the illustrations attached to the edition as .gif files. In that case, the search for '[CL]' would not include the treatise in the search results. Moreover, as mentioned above, some treatises only describe the signs but do not display them. Thus, the search for the code would also miss those treatises.

Eventually, only a search for the root '*propor*' would ensure that I would find all edited treatises mentioning proportion signs. I assumed that any description of proportion signs in a Latin treatise would contain a term with this root, since it would have to describe the proportion, which the sign indicates. The addition of an asterisk in the search term would include words that begin with '*propor*', hence variations such as *proportione*, *proportionem*, *proportionibus*, and even spellings with c instead of t (e.g. *proporcione*) would all be included in the search results. The search term '*propor**' in combination with the selection '15th century' (see FIGURE 5.1) yielded 158 results, i.e. 158 different TML documents containing variations of the word *proportio* or *proporcione*, sometimes as often as over 1000 times.²⁵ I set forth to analyse the individual hits in each treatise presumed to originate from the first half of the fifteenth century. I have enclosed the table with the results of my search in the Appendix (see p. 364).²⁶

As mentioned above, my search has unearthed a treatise neither considered by Gallo nor Busse Berger, namely the anonymous mid-fifteenth-century treatise *Quoniam circa artem musicæ figurative seu mensuralis* contained in the manuscript *Munich, Bayerische Staatsbibliothek, MS Clm 26812* (344r–355v). The treatise seems to have received little scholarly attention hitherto, but contains a description of proportion signs as well as several comments on diminution. As I will elucidate below, the treatise might be one of the earliest sources describing cut signs in the context of diminution.

²⁴ A list of codes for signs used in the TML can be found at <http://www.chmtl.indiana.edu/tml/documents/codes>.

²⁵ Original search conducted in September and October 2016. Number of 158 search results last confirmed on 27 July 2020. This number does not account for 158 individual treatises, since some treatises are catalogued several times, either due to different editions or sources respectively or because a single treatise has been split into multiple files (e.g. Ugolino da Orvieto's *Declaratio musicae disciplinae* is divided into its separate five books).

²⁶ This table maintains the order in which the results came up in my 2016 TML search. It remains unclear to me whether the algorithm follows any apparent logic, because the search results are neither ordered alphabetically, nor according to the total number of hits, nor do they appear to be ordered chronologically. My approach was as follows: If the treatise originated from the second half of the fifteenth century, I did not analyse the individual '*propor**' hits and instead entered 'irrelevant' into the fields of the last two columns. If the author was anonymous or if the treatise was difficult to date due to other reasons, I analysed each individual '*propor**' hit within the treatise to see whether proportion signs were mentioned. If this was not the case, I entered 'irrelevant' into the fields 'Date' and 'First Half of 15th Century?'. Treatises mentioning proportion signs are set in bold type.

Search the TML

The search is mainly used to look for a specific text or phrase or to discover all the instances of a particular term across the corpus. Note that the TML reproduces texts exactly as in the sources, so it is always advisable to try alternative spellings.

Through the "Search options" panel, you can select any number of centuries in which to search, or limit your search to digitized editions or manuscript transcriptions. You can choose to have matched lines of the text shown by default in the search results (otherwise they are hidden until you click "See matches" to show them).

Search options

CENTURIES

☐ All ☐ 13th

☐ 3rd–5th ☐ 14th

☐ 6th–8th ☒ 15th

☐ 9th–11th ☐ 16th

☐ 12th ☐ 17th

TEXTS

☒ All

☐ Digitized editions

☐ TML MS transcriptions

LINES WITH A MATCH

☐ show by default

1 line(s) of context

Rules for searching

- By default, this utility finds indexed words that match *or end with* the searched string.
- The Boolean operators AND/OR can be used, in all capital letters. A space between two terms is interpreted as the Boolean operator AND.
- The addition of the asterisk (*) at the end of a string has the effect of searching for words that *begin with* that string (not necessarily a full word) followed by zero or more alphanumerical characters. It is mostly used for searching inflected forms, or any word with a given root. Note that the asterisk tells the utility to add to the basic results all words with the same root of *the exact string*; in other words, the utility does not find words with extra characters on both sides of the searched string.

FIGURE 5.1: Screenshot of TML search

I also searched for proportion signs in non-Latin treatises from the first half of the fifteenth century. There are only four Italian treatises from the *quattrocento* listed in the database *Saggi musicali italiani* (SMI):²⁷ an anonymous treatise, which only discusses proportions in harmonic contexts; two treatises by Johannes Hothby, which in all likelihood originate from the second half of the fifteenth century; and the treatise *Regulae de contrapunto* by Antonius de Leno, which discusses rhythmic proportions but does not display or describe proportion signs.

The database *Traité français sur la musique* (TFM)²⁸ contains ten treatises in the category ‘*Sources avant 1600*’, most of which, namely seven, originate from the sixteenth century.²⁹ The other three are comprised of a late fourteenth century treatise and two versions of the anonymous treatise *Traité de deschant*, which do not mention rhythmic proportions. Unfortunately, the database does not list Georgius Erber’s French treatise *Sequuntur proportionones declarate et manifeste demonstrate ostense melodia vocum cantorum invente*³⁰ copied c.1460, which includes detailed descriptions of proportion signs. Since this treatise is known since at least 1969, when Renate Federhofer-Königs published her article³¹ with an edition and facsimile prints, the TFM database does not seem to be up to date.

The database *Texts on Music in English* (TME)³² lists 34 documents in the category ‘Fifteenth-Century Sources’, many of which are up to four different versions of the same treatise.

²⁷ Online at <http://www.chmtl.indiana.edu/smi/>. Original search conducted in October 2016. Number of four treatises last confirmed on 27 July 2020.

²⁸ Online at <http://www.chmtl.indiana.edu/tfm/>.

²⁹ Original search conducted in October 2016. Number of ten treatises last confirmed on 27 July 2020.

³⁰ The treatise is written in French but has a Latin title.

³¹ Cf. Renate Federhofer-Königs, “Ein Beitrag zur Proportionenlehre in der zweiten Hälfte des 15. Jahrhunderts,” *Studia Musicologica Academiae Scientiarum Hungaricae* 11, no. 1 (1969): 145–57.

³² Online at <http://www.chmtl.indiana.edu/tme/>.

Although there are several treatises dealing with proportions, no proportion signs are mentioned in any of the texts.

The collection of three treatises in English beginning with *Her beginneth tretises diverse of musical proporcions* transmitted in the manuscript *London, British Library, MS Lansdowne 763* (fols. 117r–122r) does not contain descriptions of rhythmic proportion or proportion signs, despite being included in Busse Berger’s table. However, a Latin treatise (*Proportio est rerum*, see Chap. 5.3.2.1 below) copied into the same manuscript does contain musical examples displaying proportion signs. It was slightly disheartening to find that my TML search had not unearthed this treatise, since it contains the term ‘*proporcio*’ in almost any other sentence. The TML link given in the *RISM B III* entry of the manuscript led to a TML document categorised as *f o u r t e e n t h* century, thus explaining why I did not find this treatise in my ‘*propor**’ search of manuscripts from the fifteenth century.³³ I see no strong indication that the treatise is a fourteenth century work, however.³⁴ The manuscript, in which it is contained, was in all likelihood copied in the fifteenth century, since another treatise from the same manuscript is attributed to the composer Leonel Power.³⁵ Unfortunately, another treatise contained from yet another English manuscript (*Pars aliquota est illa quae aliquotiens sumpta* from the manuscript *Rome, Biblioteca Apostolica Vaticana, MS Reg. lat. 1146*, see Chap. 5.3.3 below) is also categorised as fourteenth-century manuscript.³⁶ Although the dating is more difficult in this case, most scholars seem to agree that this treatise was written at the beginning of the fifteenth century.³⁷

The examples of the TFM not listing Georgius Erber’s treatise and the TML containing two documents probably falsely attributed to the fourteenth century is somewhat disappointing, because it raises the question whether the search of these databases is a reliable approach to find all editions of treatises describing or displaying proportion signs. Yet, consulting all manuscripts known to contain (edited or unedited) treatises discussing rhythmic proportion is beyond the scope of this study, since—unfortunately—many of these manuscripts are not available online or in any other easily accessible (digitised) format.

The possible incompleteness of search results put aside, I would like to use the opportunity to make a suggestion for databases such as the TML. In my opinion, it would be advantageous for future scholars to being able to search the entire TML for particular signs (note shapes or

³³ Cf. https://www.chmtl.indiana.edu/tml/14th/ANODUA_MLBLL763.

³⁴ For more see Chap. 5.3.2.1 below.

³⁵ Cf. Sanford B. Meech, “Three Musical Treatises in English from a Fifteenth-Century Manuscript,” *Speculum* 10, no. 3 (1935): 236.

³⁶ Cf. http://www.chmtl.indiana.edu/tml/14th/BERMAN4_MBAVR114.

³⁷ See Chap. 5.3.3 below.

the documents in the respective databases. For example, it would be beneficial to add information about mensuration or proportion signs in musical examples to the edition in form of comments, for example: “musical example inserted at this point contains [CL]” in addition to the .gif files. Thinking ahead, it would be helpful to display Arabic numerals mentioned in the text in square brackets, e.g. [2] for 2, analogous to the encoded representation of mensuration and proportion signs, so they can also be found.⁴⁰ These measures—although involving tedious effort—would greatly facilitate future scholarship on notational aspects of medieval and Renaissance music as described in music treatises of the time.

⁴⁰ The problems involved in searching the TML for Arabic numerals have been described on p. 215 above.

5.2 Proportion Signs in Treatises up to c.1450: an Overview

5.2.1 Reviewing the Material

As mentioned in the introduction to this chapter on treatises, there are two preliminary studies giving an overview of writings mostly originating from the first half of the fifteenth century, which mention proportion signs.⁴¹ Gallo's and Busse Berger's discussions of individual treatises, however, only briefly discuss these treatises' contents, which is why I aim to provide more detailed analyses in the subsequent chapter. Nevertheless, an overview summarising my findings will be helpful to those readers seeking information in a condensed format. Hence, I will provide a table, which displays the actual signs described or displayed in the respective treatises (see TABLE 5.6 on p. 227 below).

I can only suspect that Anna Maria Busse Berger had similar intentions for her table "Proportions in theoretical treatises, c.1400–1450" (see FIGURE 5.4 for an annotated version), which is given at the beginning of her chapter on proportion signs.⁴² However, it is not entirely clear by which criteria Busse Berger has included treatises in her table. The caption reads "Proportions in theoretical treatises, c.1400–1450", yet many more treatises than the eleven treatises listed in the table discuss *p r o p o r t i o n s*, not to be confused with *proportion signs* here. A famous example is Johannes Ciconia's 1411 treatise *De proportionibus*, which is not mentioned. But the table does not seem to only list treatises, which mention proportion signs, either, since the anonymous treatise *Aliquae demonstrationes in proportionibus*⁴³ is also included. This treatise of possibly French origin, however, only gives diagrams, in which a certain number of minims is juxtaposed a different number of minims (see FIGURE 5.5 below).⁴⁴ Proportion signs are neither mentioned nor displayed in this short work, hence it is unclear to me why Busse Berger has included the treatise in her table. Busse Berger also lists the collection *Her beginneth tretises diverse of musical proporcions*, which also does not mention or display proportion signs despite treating the subject of proportions. As I will explain below, I assume Busse Berger referred to the Latin treatise *Proportio est duarum rerum* found in the same manuscript as *Her beginneth tretises*.

⁴¹ Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 340–48 and Busse Berger, "Proportion Signs," chap. 6 in *Mensuration and Proportion Signs*, 164–226.

⁴² Cf. Busse Berger, "TABLE 6. Proportions in theoretical treatises, c.1400–1450" in *Mensuration and Proportion Signs*, 167.

⁴³ Note that this treatise is called *Minima in musica dicitur figura* (according to its incipit) in Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 344.

⁴⁴ Cf. Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 344–45.

Busse Berger's table does not show which signs are actually described or displayed in the treatises, but only states the proportions represented by the proportion signs, e.g. 4:3. There are also quite a few mistakes concerning the proportions discussed, which I have corrected in red colour, while annotations are written in different colours. Besides, only a few treatises are discussed in Busse Berger's monograph beyond being mentioned in the table. The following paragraphs will discuss the errors in the table and elaborate on the annotations.

<i>Proportion Signs</i>									
TABLE 6. <i>Proportions in theoretical treatises, c.1400–1450</i>									
Theorist	2:1	3:1	4:1	3:2	4:3	9:8	9:4	8:3	other proportions
Prosdocimus	x*	x*		x	x		x*		
Ugolino	x	x	x	x	x		x		2:3
<i>Iste sunt proportiones</i>	x	x		x	x	x	x	x*	
[<i>Alique demonstrationes</i>	x	x		x	x	x	x	x *	
<i>Pars aliquota est</i>	x	x		x	x	x	x		
<i>Venerabiles domini</i>	x	x		x	x	x			8:1, 9:1
Georg Erber	x	x		x	x	x	x	x	
<i>Regule proportionum</i>	[x]	x	x	x					
<i>Her beginneth tretises</i>	x	x	x	x	x	(x)*	(x)*		
<i>Exposition</i>	x	x	x	x	x	x			17:8?
Anonymus XI	x	x	x	x	x	x		x*	5:4, 5:3

* as ~~16:6~~ * 1408 only * does not discuss proportion signs
 * example shows 9:4 but text reads 9:8

FIGURE 5.4: Annotated version of TABLE 6 of Busse Berger's *Mensuration and Proportion Signs*

The first datable treatise mentioning and displaying proportion signs is Prosdocimus de Beldemandis 1404 work *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, in which the author discusses the proportion signs $\frac{3}{2}$, $\frac{4}{3}$, and $\odot$. As I will describe in Chap. 5.3.3 below, it is possible that the anonymous treatise *Pars aliquota est illa quae aliquotiens sumpta* was written even earlier. It should be noted that Busse Berger's often cited statement that Prosdocimus' 1408 treatise *Tractatus practice cantus mensurabilis* is the first treatise

mentioning proportion signs is not correct.⁴⁵ In this second work, Prosdocimus has only added three proportion signs to those mentioned above, namely $\frac{2}{1}$, $\frac{3}{1}$ and $\frac{9}{4}$.

It is also not true that Ugolino da Orvieto describes the same rhythmic proportions as Prosdocimus, as suggested by the table. As shown in Chapter 4, he discusses 4:1, but not 9:4, and also the rather unusual 2:3.

The anonymous treatise *Iste sunt proportiones* uniquely transmitted in the manuscript *Siena, Biblioteca Comunale degli Intronati, MS L.V.30* discusses the seven different proportions as shown in Busse Berger's table. It is not correct, however, that the sign for *dupla superbipartiens tertias* (8:3) proportion is given as $\frac{16}{6}$. It is given as $\frac{8}{3}$. The numerical relation 16:6 is only given in the accompanying text as alternative to 8:3, which is not unusual.⁴⁶ The same is true for the treatise *Proportio est duorum terminorum vel duorum numerorum*, which is part of the collection known as Coussemaker's Anonymous XI's *Tractatus de musica plana et mensurabili*.

The treatise *Aliquae demonstrationes in proportionibus*—also in *Siena, Biblioteca Comunale degli Intronati, MS L.V.30*—does not discuss proportion signs after all, but only gives diagrams with different amounts of minims to illustrate *dupla* (2:1), *sesquialtera* (3:2), *tripla* (3:1), *sesquitercia* (4:3), and *sesquioctava* (9:8) proportion (see FIGURE 5.5), while *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion, as listed in Busse Berger's table, are not discussed in this treatise.

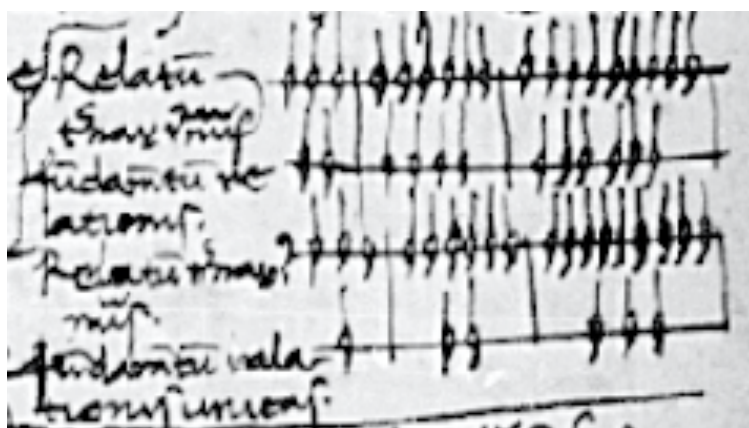


FIGURE 5.5: Illustrations of rhythmic proportion in the anonymous treatise *Aliquae demonstrationes in proportionibus* (excerpt)⁴⁷

⁴⁵ “[T]he earliest theoretical explanation of rhythmic proportions—or, to be more specific, fractions in which a certain number of notes in the numerator are made equal to a different number of notes of the same type in the denominator—was given by Prosdocimus de Beldemandis in his *Tractatus practice cantus mensurabilis* of 1408.” Busse Berger, *Mensuration and Proportion Signs*, 164. I discuss this in more detail in Chap. 5.3.1.1 below.

⁴⁶ “sedecim ad sex ut otto ad tres.” *Siena, Biblioteca Comunale degli Intronati, MS L.V.30*, fol. 142r.

⁴⁷ *Siena, Biblioteca Comunale degli Intronati, MS L.V.30*, fol. 143v. I am most grateful to Jan Herlinger who gave me access to his images of the manuscript during the corona pandemic lockdown.

The entries concerning the anonymous treatise *Venerabiles domini mei* and Georgius Erber's *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente* are mostly correct. *Venerabiles domini mei* also gives a sign for *trippla* (3:1) proportion and Erber's text also includes a discussion of *dupla superbipartiens tertias* (8:3) proportion.⁴⁸

The anonymous treatise *Incipiunt regule proporcionum in quantum pertinent ad musica*—called *Regule proportionum* by Busse Berger and Gallo—transmitted in the manuscript *Venice, Biblioteca Nazionale Marciana, Lat. VIII.85 (3579)* is presumed to have been copied near Mantova between 1463 and 1464.⁴⁹ The treatise mentions the stacked Arabic numerals $\frac{4}{1}$ as “most intelligible sign” (“signum magis intelligibile”⁵⁰) to indicate *quadrupla* (4:1) proportion.⁵¹ The treatise also contains an incomplete musical example showing Ø (see FIGURE 5.13 in Chap. 5.3.2 below), but it is not clear what this example was supposed to demonstrate. It reads “dyapason” and “proportio binaria” underneath the example, so I assume that Ø is used in the context of *dupla* (2:1) proportion, but this is by no means certain. Therefore, *dupla* (2:1) proportion is put in brackets in the annotated version of Busse Berger's table above (see FIGURE 5.4). While there are musical examples accompanying the descriptions of other proportions in *Incipiunt regule*, I was unable to discern any other proportion sign on the microfilm digitisations I received.⁵² Therefore, these proportions are crossed out in the annotated version of Busse Berger's table. It is at least questionable whether the treatise refers to *Ars subtilior* practices at all. Jan Herlinger has concluded that “the contents of [the manuscript] Marciana VIII.85—including, of course, the treatise on how to make counterpoint using the remote hexachords with all their accidentals—provide a tantalizing glimpse into the musical interests, tastes, and practices of the city during that decade”⁵³, thereby referring to the 1460s. Nevertheless, it is conceivable that the proportion treatise predates that period.

The collection of three treatises on proportions written in English⁵⁴ beginning with *Her beginneth tretises diverse of musical proporcions* transmitted in the manuscript *London, British*

⁴⁸ That this proportion (8:3) is missing from Busse Berger's table in case of Erber's treatise has been remarked upon previously. Cf. Rob C. Wegman, “Different Strokes for Different Folks? On Tempo and Diminution in Fifteenth-Century Music,” *Journal of the American Musicological Society* 53, no. 3 (2000): 486 n. 42.

⁴⁹ Cf. Jan Herlinger, “Biblioteca Nazionale Marciana, MS Latini, Cl. VIII.85: A Preliminary Report,” *Philomusica on-line* 4, no. 2 (2005).

⁵⁰ *Venice, Biblioteca Nazionale Marciana, Lat. VIII.85 (3579)*, margin of fol. 69r.

⁵¹ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 347.

⁵² I would like to thank Cathrin Dux and Lukas Nussbaumer from the microfilm archive in Basel for sending me the images.

⁵³ Herlinger, “Biblioteca Nazionale Marciana, MS Latini, Cl. VIII.85: A Preliminary Report.”

⁵⁴ *Her beginneth tretises diverse of musical proporcions* (fols. 117r–118v); *Here foluyth a breue tretise of proporcions and of their denominacions* (fols. 118v–120r); and *Thus ouerpassid the rwlis of Porporcions and of their*

Library, MS Lansdowne 763 (fols. 117r–122r) does not contain any descriptions of rhythmic proportion or proportion signs. The first treatise explains the five types of proportions of major inequality.⁵⁵ The second treatise deals with denominations of particular proportions. It includes a table (fols. 119v–120r), in which all kinds of different proportions and their designations are listed. Afterwards, the author goes on to explain the differences between geometric, arithmetic, and harmonic proportion.⁵⁶ However, the Latin treatise *Proportio est duarum rerum*, which follows the three English treatises does describe rhythmic proportions and contains proportion signs in the musical examples and hence will be discussed below (see Chap. 5.3.2.1). Therefore, Busse Berger was right to include the collection of treatises in the list but should perhaps have given the Latin title of the last treatise.⁵⁷ While several versions of the English texts exist in different manuscripts, which all seem to derive from a common source, the Latin treatise *Proportio est duarum rerum* is uniquely transmitted in the Lansdowne manuscript.⁵⁸

The Hebrew treatise labelled *Student's notes of lectures on music*—called *Exposition* by Busse Berger—is a very corrupt source. As I will describe below, some proportion signs, for example the sign for *tripla* (3:1) proportion, can be assumed to be missing from the text.⁵⁹ However, I see no evidence that this is the case for *sesquialtera* (3:2) and *sesquitercia* (4:3) proportion, which is why these proportions are crossed out in the annotated version of Busse Berger's table (see FIGURE 5.4). Furthermore, as I will describe in Chap. 5.3.1.4 below, I doubt that the author meant 17:8 proportion when he wrote '*dupla sesquioctava*', because there are other instances in which his denominations appears to be incorrect. I have therefore added a question mark to 17:8 in Busse Berger's table.

Finally, I treat Anonymous XI's *Tractatus de musica plana et mensurabili* as collection of individual texts, two of which, namely *Sequitur hic aliqua declaratio atque denominatio* and *Proportio est duorum terminorum vel duorum numerorum*, contain proportion signs. Therefore, the two treatises are listed in different rows in my table (see Table 5.6 on p. 227 below).

Denominacions (fols. 120v–122v). These three treatises are often perceived as one unit. The manuscript also contains two other treatises in English. For more see Meech, "Three Musical Treatises in English," 235–69.

⁵⁵ See TABLE 1.4 on p. 19 in Chap. 1 for an explanation.

⁵⁶ See Chap. 1, pp. 9–10 for an explanation.

⁵⁷ It should be noted, however, that Busse Berger has given fols. 117r–123r as source for the treatise. The description of rhythmic proportion including the musical examples displaying proportion signs does not start before fol. 123v, however.

⁵⁸ Cf. Theodor Dumitrescu, *The Early Tudor Court and International Musical Relations* (Aldershot: Ashgate, 2007), 184–86.

⁵⁹ See discussion in Chap. 5.3.1.4, especially FIGURE 5.10 on p. 259.

5.2.2 Proportion Signs in Treatises up to c.1450: Tables

Altogether 14 treatises (see TABLE 5.7 for details) are candidates for transmitting *Ars subtilior* teachings during the first half of the fifteenth century, including the mid-fifteenth-century treatise *Quoniam circa artem musice figurative seu mensuralis*, which is not mentioned in the preliminary studies conducted by Gallo and Busse Berger. A handful of these manuscripts can be dated to an exact year or a time span of a few years due to the identification of their authors. However, the majority of treatises analysed for this study have anonymous authors, hence even a rough estimate is difficult. Some manuscripts were even copied later than 1450 but it seems highly probable that the texts originate from the first half of the fifteenth century and were simply copied at a later date. Short treatises without musical examples, mostly found in collections of several texts, are especially difficult to date since there is often not enough context to evaluate the time or place of origin.

As can be seen in TABLE 5.6 there is a wide range of proportion signs reaching from single Arabic numerals to geometric shapes representing different rhythmic proportions in the 14 treatises. There are six different single Arabic numerals, most commonly 2, 3, and 4 representing *dupla* (2:1), *tripla* (3:1), and *quadrupla* (4:1) proportion respectively. The Arabic numerals 6 and 9 most often refer to the number of minims compared to four minims in one *mensura* in *sesquialtera* (3:2) or *dupla sesquiquarta* (9:4) proportion.

The stacked numerals given in nine of the fourteen treatises always indicate the proportion represented by the two numbers, i.e. $\frac{2}{1}$ indicates *dupla* (2:1) proportion, $\frac{3}{1}$ indicates *tripla* (3:1) proportion and so on. The most commonly mentioned sign is $\frac{3}{2}$ indicating *sesquialtera* (3:2) proportion, which is given in half of the treatises.

Geometric shapes can be found in twelve of the 14 treatises and they are always variants of a semicircle or a circle with the exception of $\diamond$ in Ugolino's *Declaratio musicae disciplinae*. As I have explained in Chapter 4.3, I doubt that the sign was ever used in actual music due to its similarity to a semibreve.

It should be noted that nearly all the authors have given at least two different of the above-named forms of visual appearance of proportion signs in their texts, thereby at least partially acknowledging their manyfold representation in music, with the exception of the anonymous treatise *Iste sunt proportiones* which gives only stacked Arabic numerals.

Treatise and author ⁶⁰	2:1	3:1	4:1	3:2	4:3	9:8	9:4	8:3	other	
Anonymous: <i>Ars et practica</i>	[2] ⁶¹	3	4	$\frac{3}{2}$	$\frac{4}{3}$ or $\circ$	$\frac{9}{8}$ or $\ominus$		$\frac{8}{3}$ or $\ominus$	5:4 $\circ$	
Ugolino: <i>Declaratio musicae disciplinae</i>	$\smile$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{3}{2}$	$\frac{4}{3}$ or $\circ$ or $\diamond$				2:3 $\smile$	
Prosdocimus: <i>Expositiones</i>				$\frac{3}{2}$	$\frac{4}{3}$ or $\circ$					
Anonymous: <i>Incipiunt regule</i>	[Ø] ⁶²		$\frac{4}{1}$							
Anonymous: <i>Iste sunt</i>	$\frac{2}{1}$	$\frac{3}{1}$		$\frac{3}{2}$	$\frac{4}{3}$	$\frac{9}{8}$	$\frac{9}{4}$	$\frac{8}{3}$		
Anonymous: <i>Pars aliquota est</i>	4	3		6	2 or $\circ$	$\ominus$	9			
Anonymous (Chilston?): <i>Proportio est</i>	2* ⁶³		4*		$\circ$ *					
Anonymous: <i>Proportio est</i> ⁶⁴				$\frac{9}{6}$				$\ominus$		
Anonymous: <i>Quoniam circa</i>	$\frac{2}{\emptyset}$ or $\mathcal{C}$	3	4							
Anonymous: <i>Sequitur hic aliqua</i> ⁶⁴	2	3	4	$\frac{3}{2}$	$\circ$	$\frac{9}{8}$ or $\ominus$			5:4 $\frac{5}{4}$ or $\circ$	5:3 $\mathcal{D}$
Georgius Erber: <i>Sequuntur</i>	$\emptyset$ or $\mathcal{C}$ or 2	$\ominus$ * or $\frac{3}{1}$ *		$\frac{\circ}{6}$ or $\frac{3}{4}$ or $\frac{3}{2}$	$\circ$	$\frac{9}{8}$	$\frac{9}{4}$ *	$\ominus$ or $\frac{8}{3}$ *		
Anonymous: [<i>Student's notes</i>]	[$\mathcal{C}$] or [Ø] ⁶⁵	[] ⁶⁶	4			$\ominus$			[17:8] ⁶⁷ $\ominus$	
Prosdocimus: <i>Tractatus practice</i>	$\frac{2}{1}$	$\frac{3}{1}$		$\frac{3}{2}$	$\frac{4}{3}$ or $\circ$		$\frac{9}{4}$			
Anonymous: <i>Venerabilis domini</i>	2 or $\mathcal{C}$ or Φ	3		$\frac{3}{2}$ or 9 or Φ or $\mathcal{C}$ *	$\frac{4}{3}$ or $\circ$ or $\mathcal{C}$ or 2* or Φ *	9 or 3			8:1 8* or $\mathcal{C}$ *	9:1 9*

TABLE 5.6: Proportion signs and their interpretation in treatises up to c.1450

⁶⁰ Short titles (in alphabetical order) and authors' names are given here. For full reference see bibliography.

⁶¹ Although the actual sign 2 is missing in the original text it can be assumed that this figure should have been written there due to the striking similarities of this treatise and the treatise *Sequitur hic aliqua declaratio atque denominati*. For more, see Chap. 5.3.1.3.1 below.

⁶² Sign only in incomplete musical example (see FIGURE 5.13 below), so perhaps this is not a proportion sign.

⁶³ An asterisk (*) means that the sign only appears in the accompanying musical example but not in the text.

⁶⁴ Part of Anonymous XI's *Tractatus de musica plana et mensurabili*. For more see Chap. 5.3.1.3 below.

⁶⁵ Signs are described but not displayed: "a half circle cut in half or a full circle broken." Anonymous, [*Student's notes of lectures on music*], chap. 4; translation taken from Stone, "The Ars subtilior in Paris," 398.

⁶⁶ Sign is missing. See Adler, *Hebrew Writings*, 70, facsimile of fol. 2b, line 15; also see FIGURE 5.10 below.

⁶⁷ The term that the anonymous author of this treatise uses ('*dupla sesquioctava*') refers to this proportion (17:8). However, as I explain below, this is in all probability not the proportion he meant. It is my conclusion that the author had a false understanding of the rules of proportions' denominations. See Chap. 5.3.1.4 below.

Treatise	Author	Date	TML / LML / other reference(s)	Source(s)	Language	Treatise on proportion (signs) or part of treatise on music?	Incipit / Explicit (of treatise or of chapter discussing proportion signs)	Musical examples?	Proportion signs only or other means of rhythmic notation discussed in (this part of the) treatise?
<i>Ars et practica cantus figurativi</i>	Anonymous	c.1450	TML: ANOAPC LML: ANON. Tegems. II	• D-Mbs CIm 19851	Latin	Music treatise; proportion signs are discussed in chap. “De proportionibus”.	“Octo sunt proportiones musicales in cantu mensurali [...]” / “[...] ut sic $\odot$ vel sic $\frac{9}{8}$.”	No	Proportion signs only.
<i>Declaratio musicae disciplinae</i>	Ugolino of Orvieto	c.1430	TML: UGODEC1A LML: UGOL. URB	• I-Bc A 29 • I-Rc 2151 • V-CVbav lat. 5324 • V-CVbav Ross. 455 • V-CVbav Urb. lat. 258 [9 others] ⁶⁸	Latin	Music treatise; proportion signs are discussed in Book 3, chap. 6 “De signis”.	“De signis modum, tempus et prolationem distinguuntibus.” / “[...] et sic est finis quinti capituli musicae mensuratae magistri Iohannis de Muris.”	No, but I-Rc 2151 contains two compositions with proportion signs at the end of the manuscript.	Mensuration signs, coloration, and proportion signs.
<i>Expositiones tractatus practice cantus mensurabilis magistri Iohannis de Muris</i>	Prosdocimus de Beldemandis	1404	TML: PROEXP LML: PROSD. exp.	• I-Bc A 56 • I-CATc D 39	Latin	Music treatise; proportion signs are discussed in chap. 4 “Quartum capitulum de signis mensurarum”.	“Cum duplex sit modus tempus et prolatio [...]” / “Et sic sit finis sententie huius quarti capituli.”	No	Mensuration signs, coloration, and proportion signs.

TABLE 5.7: Proportion sign treatises up to c.1450 (table continues on the following pages)

⁶⁸ I have only listed those sources which contain the proportion sign section of the *Declaratio*. For a complete list of sources see LML or Evan A. MacCarthy, “The Sources and Early Readers of Ugolino of Orvieto’s *Declaratio Musicae Discipline*,” in *Beyond 50 Years of Ars Nova Studies at Certaldo 1959–2009*, ed. Marco Gozzi, Agostino Zino, and Francesco Zimei (Lucca: Libreria Musicale Italiana, 2014), 424–25.

Treatise	Author	Date	TML / LML / other reference(s)	Source(s)	Language	Treatise on proportion (signs) or part of treatise on music?	Incipit / Explicit (of treatise or of chapter discussing proportion signs)	Musical examples?	Proportion sings only or other means of rhythmic notation discussed in (this part of the) treatise?
<i>Incipiunt regule proporcionu m in quantum pertinent ad musica</i>	Anonymous	c.1450		• I-Vnm Lat. VIII.85	Latin	Treatise on proportions	“Incipiunt regule proporcionum in quantum pertinent ad musica.” / “[...] ut sunt quindecim ad 4.”	Yes, but complete examples do not exhibit proportion signs. One incomplete example shows Ø.	Proportion signs only.
<i>Iste sunt proportiones</i>	Anonymous	c.1450		• I-Sc L.V.30	Latin	Treatise on proportion signs	“Iste sunt proportiones que utuntur communit[er] [...]” / “[...] tale signum $\frac{2}{1}$ ut hic.”	Yes, 13 musical examples demonstrating seven different proportions.	Proportion signs only.
<i>Pars aliquota est illa quae aliquotiens sumpta</i>	Anonymous	late 14 th or early 15 th century	TML: BERMAN4_MBAVR114	• V-CVbav Reg. lat. 1146	Latin	Treatise on proportions	“Pars aliquota est illa quae aliquotiens sumpta [...]” / “Sesque enim grece est idem quod totum latine.”	Yes, three musical examples demonstrating six different proportions.	Proportion signs only, but musical examples contain some red coloured notes.
<i>Proportio est duarum rerum</i>	Anonymous (Chilston?)	c.1450	TML: ANODUA_MLBL763 Busse Berger: “Her beginneth tretises diverse of musical proporcions”	• GB-Lbl Lansdowne 763	Latin	Treatise on proportions	“Proportio est duarum rerum equalium vel inequalium [...]” / “[...] vt nouem ad octo.”	Yes, six musical examples demonstrating six different proportions.	Musical examples exhibit proportion signs and coloration.

TABLE 5.7: Proportion sign treatises up to c.1450 (continued)

Treatise	Author	Date	TML / LML / other reference(s)	Source(s)	Language	Treatise on proportion (signs) or part of treatise on music?	Incipit / Explicit (of treatise or of chapter discussing proportion signs)	Musical examples?	Proportion signs only or other means of rhythmic notation discussed in (this part of the) treatise?
<i>Proportio est duorum terminorum vel duorum numerorum</i>	Anonymous (part of Anonymous XI's <i>Tractatus de musica plana et mensurabili</i>)	c.1450	TML: ANO11TDM LML: PROP. Prop. est duorum	• GB-Lbl Add. 34200	Latin	Treatise on proportion	“Proporcio est duorum terminorum vel duorum numerorum [...]” / “Epogdous dicitur sesquioctavus.”	No	Proportion signs only.
<i>Quoniam circa artem musice figurative seu mensuralis</i>	Anonymous	c.1450	TML: ANOTDMM LML: ANON. Monac.	• D-Mbs Clm 26812	Latin	Music treatise; proportion signs are discussed in chap. 4.	“Circa quartum capitulum videndum est de signis [...]” / “Sequitur ultimum capitulum.”	No	Mensuration signs and proportion signs.
<i>Sequitur hic aliqua declaratio atque denominatio</i>	Anonymous (part of Anonymous XI's <i>Tractatus de musica plana et mensurabili</i>)	c.1450	TML: ANO11TDM LML: PROP. MENS. Sequitur	• D-Rp 98 th. 4° • GB-Lbl Add. 34200	Latin	Treatise on proportion signs	“Sequitur hic aliqua declaratio atque denominatio [...]” / “Et sic est finis.”	No, only a certain amount of minims (without staves) set against a different amount of minims, which are integrated in the text in seven examples.	Proportion signs and coloration.

TABLE 5.7: Proportion sign treatises up to c.1450 (continued)

Treatise	Author	Date	TML / LML / other reference(s)	Source(s)	Language	Treatise on proportion (signs) or part of treatise on music?	Incipit / Explicit (of treatise or of chapter discussing proportion signs)	Musical examples?	Proportion signs only or other means of rhythmic notation discussed in (this part of the) treatise?
<i>Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente</i>	Georgius Erber	c.1460		• A-lu 962	French	Treatise on proportion signs	“Sequuntur proportiones declarate et manifeste demonstrate [...]” / “[...] des proportions, nouvellement trouvees pour l’habilité dex voix des puntament chantans.”	Yes, seven musical examples demonstrating seven different proportions.	Proportion signs only, but musical examples contain some coloured notes.
<i>[Student’s notes of lectures on music]</i>	Anonymous	c.1450	Busse Berger: “Exposition”	• I-Fn Magliab. III 70	Hebrew	Music treatise; proportion signs are discussed in chaps. 4, 5, and 6.	[Incipit and explicit in Hebrew, cf. edition]	No, but treatise refers to a composition from <i>Ch.</i>	Proportion signs and coloration.
<i>Tractatus practice cantus mensurabilis</i>	Prosdocimus de Beldemandis	1408	TML: PROTRAP I LML: PROSD. mens.	• I-Bc A 56 • I-CR 238 (fragm.) • I-Lg 359 [1 other] ⁶⁹	Latin	Music treatise; proportion signs are discussed in chap. 4 “De signis mensure”.	“Sequitur capitulum de signis per que mesure distinguntur [...]” / “Et hec de signis pro nunc sufficient.”	No	Mensuration signs, coloration, and proportion signs

TABLE 5.7: Proportion sign treatises up to c.1450 (continued)

⁶⁹ Prosdocimus de Beldemandis’ *Tractatus practice cantus mensurabilis* is transmitted in altogether four sources, but Prosdocimus’ autograph of the treatise—transmitted as fragment on only two folios of I-FI Ashburnham 206—does not contain the proportion sign section. See F. Alberto Gallo, *La tradizione dei trattati musicali di Prosdocimo de Beldemandis*, Biblioteca di Quadrivium: serie musicologica 5, Bologna: [Forni], 1964, 10b for a black and white facsimile of the two I-FI Ashburnham 206 folios.

Treatise	Author	Date	TML / LML / other reference(s)	Source(s)	Language	Treatise on proportion (signs) or part of treatise on music?	Incipit / Explicit (of treatise or of chapter discussing proportion signs)	Musical examples?	Proportion sings only or other means of rhythmic notation discussed in (this part of the) treatise?
<i>Venerabiles domini mei proportiones</i>	Anonymous	c.1450	Venerabilis dominici proportiones (incipit in F-SDI 42)	• D-Rp 98 th. 4^o • F-SDI 42	Latin	Treatise on proportion.	“Venerabiles domini mei proportiones per Dei gratiam intendo declarare [...]” / “[...] de cantu per medium facto.”	Yes, 30 musical examples demonstrating six different proportions.	Proportion signs only.

TABLE 5.7: Proportion sign treatises up to c.1450 (continued)

5.3 Treatises and Manuscripts

As described in the introduction to this final chapter, the authors of the altogether 14 different treatises have approached the subject of proportion signs quite differently in their respective texts. Some theorists, e.g. Prosdocimus de Beldemandis and Ugolino of Orvieto, seem to favour standardisation of musical notation by defining only one sign—in some cases two signs—for a particular proportion. Others offer the reader more options for displaying rhythmic proportions without necessarily favouring one sign over the other and thereby giving the notators of music a free choice which sign to use. These texts often include musical examples and seem to be more oriented towards common notational practice, in which proportion signs appear in many different forms.

I have refrained from trying to establish a timeline for the 14 treatises and discussing them in the assumed chronological order. As mentioned above, most of the texts can only roughly be dated to a certain decade due to anonymous authorship. Instead, I have grouped the treatises according to their author's approach to the subject. In Chapter 5.3.1, I will discuss all those texts in favour of standardisation, while the more practice-oriented treatises are considered in Chapter 5.3.2. The chapter will be concluded by a discussion of the extraordinary treatise *Pars aliquota est illa quae aliquotiens sumpta*, which does not fit into either category because of its challenging musical examples.

5.3.1 Treatises in Favour of Standardisation

It is my understanding that a treatise favours standardisation if the texts promote the message: “You write it this and not another way.” According to my analysis, eight of the altogether 14 proportion sign treatises of the present study belong to this first category (see enumeration in brackets following the titles in next paragraph). This tendency towards standardisation can especially be observed in the comprehensive treatises on music, e.g. the works by Prosdocimus de Beldemandis and Ugolino of Orvieto, and in shorter treatises without musical examples.

Ugolino's *Declaratio* (1) and Prosdocimus' two treatises *Expositiones* (2) and *Tractatus practice* (3) are Italian sources.⁷⁰ Two sources of German origin shed light on the transmission of proportion sign teachings in the German tradition. The first is the anonymous treatise

⁷⁰ Ugolino da Orvieto's *Declaratio Musicae Disciplinae* will not again be discussed here. See Chap. 4 for details on this treatise.

Quoniam circa artem musice figurative seu mensuralis (4) from the manuscript *Munich, Bayerische Staatsbibliothek, MS Clm 26812*. The second is the *Tractatus de musica plana et mensurabili* (Coussemaker's Anonymous XI) transmitted in *London, British Library, MS Add. 34200*. There are great differences in the amount of scholarly attention both sources have received: While the *Tractatus de musica plana et mensurabili* (Anonymous XI) is well known and has been available in translation since 1973,⁷¹ the anonymous treatise *Quoniam circa artem musice figurative seu mensuralis* has hardly been recognised yet.⁷² It is this latter treatise that has made my TML search worthwhile, because it is a most interesting source, which not only mentions proportion signs, but also elaborates on the subject of diminution and might be the earliest source discussing Ø and Ç in that context. Being dated to c.1450, both works can still be considered to stem from the first half of the fifteenth century, and are therefore also included in TABLE 5.6 above. Rob C. Wegman has suggested that the *Tractatus de musica plana et mensurabili* might have been written as early as 1420.⁷³ This assertion is based on the observation that a composite sign, namely O2, was allegedly only used in the capacity described in the *Tractatus de musica plana et mensurabili* in the 1420s and 1430s. I will explain below why I think that a date of origin closer to the middle of the fifteenth century might be plausible after all. Two treatises from the *Tractatus de musica plana et mensurabili* are relevant to this study, namely *Sequitur hic aliqua declaratio atque denominatio* (5) and *Proportio est duorum terminorum vel duorum numerorum* (6). Both will be discussed subsequent to the anonymous treatise *Quoniam circa artem musice figurative seu mensuralis*. The treatise *Ars et practica cantus figurativi* (7) is so closely related to *Sequitur hic aliqua* that I have decided to discuss it alongside this treatise. The subchapter is concluded by the discussion of a rather fascinating treatise of French origin written in Hebrew (8), which not only mentions proportion signs but also gives intriguing advice on coloration.

⁷¹ Cf. Richard J. Wingell, "Anonymous XI (CS III): An Edition, Translation, and Commentary," PhD diss., University of Southern California, 1973.

⁷² The edition of this treatise (Bernhold Schmid, "Der Musiktraktat aus Clm 26812," in *Quellen und Studien zur Musiktheorie des Mittelalters I*, ed. Michael Bernhard (Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1990), 82–98) has been available since 1990.

⁷³ Cf. Rob C. Wegman, "Different Strokes for Different Folks?" 485 n. 40.

5.3.1.1 Prodocimus de Beldemandis' Treatises *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris* and *Tractatus practice cantus mensurabilis*

Prodocimus de Beldemandis was an influential early fifteenth century music theorist, who is the author of eight different treatises on music written between 1404 and 1425. He was a student at the universities of Padua and Bologna and received a doctorate in arts at Padua in 1409 and in medicine in 1411. Prodocimus also wrote treatises on arithmetic, geometry, as well as astronomy, and eventually became a professor at Padua, teaching a variety of subjects from 1422 until his death in 1428.⁷⁴

Prodocimus devoted several treatises—including both treatises under discussion here—to the French notational system, especially to the teachings of Johannes de Muris as transmitted in the *Libellus cantus mensurabilis*. However, he eventually turned towards the Italian notational system, which was gradually superseded in Italy by French notation. In his 1412 treatise *Tractatus practice de musica mensurabili ad modum italicorum*, he repeatedly—sometimes polemically—pointed out the superiority of Italian notation over the French system. However, contemporary and succeeding music theorists did not join him in his attempts, leaving him to remain the last advocate for Italian notation.⁷⁵

It is unknown whether Prodocimus practiced music. He was a friend of Luca da Lendenara, successor of Johannes Ciconia as *cantor* at the cathedral of Padua after Ciconia's death in 1412.⁷⁶ Moreover, his treatises reveal thorough knowledge of contemporary notational practices and, as will be shown below, he even seems to have discussed musical notation with at least one composer. Thus, it is conceivable that he also was a practitioner. Certainly, no music written by Prodocimus—if it ever existed—seems to have survived.

Two treatises by Prodocimus mention proportion signs, namely his 1404 treatise *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris* and his 1408 treatise *Tractatus practice cantus mensurabilis*. Both works are well structured and similar in their arrangement of subjects.⁷⁷

⁷⁴ Cf. Jan Herlinger, "Prodocimus de Beldemandis [Prodocimo de' Beldomandi]," in *NG2* (London: MacMillan Publishers, 2001), 20:431–32; and Michele Calella, "Prodocimus, de Beldemandis," in: *MGG2*, Personenteil vol. 13 (Kassel and Stuttgart: Bärenreiter / J. B. Metzler, 2005), cols. 990–92.

⁷⁵ Cf. Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 327–28, and 333.

⁷⁶ Cf. Gallo, "Die Notationslehre im 14. und 15. Jahrhundert," 327.

⁷⁷ Chapters in *Expositiones* (1404):

- 1) De perfectione et imperfectione [sic] notarum
- 2) De alteratione
- 3) De puncto
- 4) De signis mensurarum
- 5) De ligaturis
- 6) De sincopa

Chapters in *Tractatus practice* (1408):

- 1) De perfectione et imperfectione notarum
- 2) De alteratione
- 3) De puncto
- 4) De signis mesure
- 5) De ligaturis
- 6) De pausis

The discussion of proportion signs in *Expositiones* can be found in the epilogue of the section on signs in the fourth chapter, which also addresses coloration and special note shapes, i.e. intrinsic signs of proportional rhythms.⁷⁸ The preceding subsections of the fourth chapter contain explanations on how to recognise *modus*, *tempus*, and *prolatio* by either extrinsic or intrinsic signs.⁷⁹

Prosdocimus discussed three proportion signs in *Expositiones*, namely two fractions and the reversed semicircle, which indicate two different proportions:

“[T]he fraction behaves with respect to the minims in *sesquitertia* [4:3] proportion as in four to three and the fraction behaves with respect to the minims in *sesquialtera* [3:2] proportion as in three to two. One can recognise the first fraction by the sign $\frac{4}{3}$; the second by the sign $\frac{3}{2}$. [...] The first fraction can also be recognised according to some by another sign, namely by the reversed semicircle or right-facing with the opening to the left, like this: $\ominus$. And this sign is commonly placed by the *moderni*.⁸⁰

In short: *sesquitertia* (4:3) proportion can be indicated by either $\frac{4}{3}$ or $\ominus$, and *sesquialtera* (3:2) proportion can be indicated by $\frac{3}{2}$. $\ominus$ is the sign commonly used by the *moderni*.⁸¹

The reversed semicircle is also mentioned in Prosdocimus’ 1408 treatise *Tractatus practice*, accompanied by the remark that this sign is used by ‘others’:

“In the same vein others want that the right semicircle, like this $\ominus$, shall be the universal sign to show us that we must sing the notes following it in *sesquitertia* [4:3] proportion [compared] to notes in their own values, like four to three, or eight to six.”⁸²

-
- 7) De pausis
 - 8) De diminutione

- 9) De colore et talea

- 7) De sincopa
- 8) De diminutione
- 9) De augmentatione
- 10) De colore et talea
- 11) De modo cognoscendi mensuras cantuum

⁷⁸ The fourth chapter of the treatise is called “Quantum capitulum de signis mensurarum” and is subdivided into nine subsections (LV–LXIII) according to subject matter in Gallo’s edition. Cf. Gallo, *Prosdocimi de Beldemandis opera* 1, 11. The subsection discussing proportion signs (epilogus) is no. LXI. Cf. *ibid.*, 138–52.

⁷⁹ The position of the discussion of proportion signs in the 1408 *Tractatus practice* is very similar.

⁸⁰ “[S]icut fractio se habens ad minimas in proportione sexquitercia sicut quatuor pro tribus et fractio se habens ad ipsas minimas in proportione sexquialtera sicut tres pro duabus. Prima namque fractio cognosci potest per signum tale: $\frac{4}{3}$; secunda vero per tale: $\frac{3}{2}$. [...] Prima etiam fractio potest cognosci secundum aliquos per aliud signum, scilicet per semicirculum transversum sive dextrum respicientem partem sinistram, ut hic: $\ominus$. Et hoc signum ponitur comuniter a modernis” Prosdocimus de Beldemandis, *Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, chap. 61; edition in F. Alberto Gallo, ed., *Prosdocimi de Beldemandis opera 1: Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris*, *Antiquae musicae italicae scriptores* 3 (Bologna: Università degli Studi de Bologna, Istituto di Studi Musicali e Teatrali, 1966), 141–42; my translation.

⁸¹ For Jacobus of Liège, the *moderni* had been practioners of the *Ars nova* as opposed to the *antiqui*. Cf. Chap. 1.3. According to William J. Courtenay, even after Jacobus, the term was often used in a deprecating way: “inasmuch as a writer rarely mentioned contemporary opinion except to attack it, the term had a slightly negative connotation.” “*Antiqui and Moderni in Late Medieval Thought*,” *Journal of the History of Ideas* 48, no. 1 (1987): 4. Also see Karen Desmond, *Music and the moderni, 1300–1350: The ars nova in Theory and Practice* (Cambridge: Cambridge University Press, 2018), 7.

⁸² “Item vol[u]nt aliqui alii, quod semicirculus dexter ut iste $\ominus$ sit signum universale nobis demonstrans quod cantare debemus figuras ipsum sequentes in proportione sexquitercia ad figuras in suis propriis valoribus, sicut quatuor pro tribus, vel octo pro sex.” Prosdocimus de Beldemandis, *Tractatus practice de musica mensurabili*;

In his 1408 treatise, Prosdocimus also discussed the proportion signs $\frac{2}{1}$, $\frac{3}{1}$, and $\frac{9}{4}$:

“By this premise I say, that if we want to sing in *dupla* [2:1] proportion or two to one, we must place this sign $\frac{2}{1}$. If we want to sing in *tripla* [3:1] proportion or three to one, we must place this sign $\frac{3}{1}$. If we want to sing in *sesquialtera* [3:2] proportion or three to two, we must place this sign $\frac{3}{2}$. If we want to sing in *sesquitercia* [4:3] proportion or four to three, we must place this sign $\frac{4}{3}$. If we want to sing in *dupla sesquiquarta* [9:4] proportion or nine to four, we must place this sign $\frac{9}{4}$.”⁸³

Prosdocimus two treatises exhibit a strong aspiration for standardisation of musical notation where proportion signs are concerned. The two treatises do not contradict each other, the 1408 version simply contains three more proportion signs for the proportions *dupla* (2:1), *tripla* (3:1), and *dupla sesquiquarta* (9:4), which were not contained in the 1404 *Expositiones*. Prosdocimus reveals his preference for clarity in the visual representation of rhythmic proportions by advocating stacked Arabic numerals: “[T]hose signs are most accommodating since they suit all notes, namely *maxime*, *longe*, *breves*, *semibreves*, *minime* and *semiminime* as well as all rests of these (note values).”⁸⁴ It is not clear whether Prosdocimus favoured stacked Arabic numerals over geometric signs such as the reversed semicircle $\bigcirc$ or whether he preferred stacked Arabic numerals as opposed to coloration. Naturally, both forms of notating proportional rhythms can also be applied to all rhythmic levels. It is a question of how the respective notational device is defined, for example, by canon instructions. As I have shown, there are quite a few examples of proportions being applied on semibreve instead of minim level in *Mod A* and *Turin*. In the majority of those cases, the means of notating rhythmic proportion are explained by such a canon. In that, Prosdocimus has a point, however: While stacked Arabic numerals unequivocally specify the number of notes compared to each other in the proportion—at least in theory—, geometric signs and coloration might require additional explanation.⁸⁵ In *Expositiones* he criticised the “peculiar types of proportional figures used by

edition in Edmond de Coussemaker, ed., *Scriptorum de musica medii aevi nova series a Gerbertina altera*, 4 vols. (Paris: Durand, 1864–76; reprint ed., Hildesheim: Olms, 1963), 3:216; my translation.

⁸³ “Isto premissis dico, quod si cantare volumus in proportione dupla vel duas pro una, tale debemus ponere signum $\frac{2}{1}$. Si vero cantare volumus in proportione tripla vel tres pro una, tale debemus ponere signum $\frac{3}{1}$. Si vero discantare volumus in proportione sexquialtera vel tres pro duabus, tale debemus ponere signum $\frac{3}{2}$. Si vero discantare volumus in proportione sexquitercia vel quatuor pro tribus, tale debemus ponere signum $\frac{4}{3}$. Si vero discantare volumus in proportione dupla sexquiquarta vel novem pro quatuor, tale debemus ponere signum $\frac{9}{4}$.” Prosdocimus, *Tractatus practice*; edition in Coussemaker, ed., *Scriptorum de musica*, 3:218; my translation.

⁸⁴ “[I]sta sunt signa comunissima, quoniam conveniunt omnibus figuris, scilicet maximis, longis, brevibus, semibrevis, minimis et semiminimis atque omnibus earum pausis.” Prosdocimus, *Expositiones*, chap. 61; edition in Gallo, ed., *Prosdocimi de Beldemandis opera I*, 141; translation taken from Jason Stoessel, “The Captive Scribe: The Context and Culture of Scribal and Notational Process in the Music of the Ars Subtilior,” 2 vols., PhD diss., University of New England, 2002, 1:296. Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 341–42.

⁸⁵ It is not true that stacked Arabic numerals in music sources always indicate the proportion as represented by the numbers used in the sign, however. As discussed in Chap. 3.1, there is the peculiar case of *Sous un bel abre* in *Turin*, in which the stacked Arabic numerals $\frac{3}{1}$ indicate *dupla sesquiquarta* (9:4) proportion, without that

the *moderni*”⁸⁶ and stated that “something is done to no avail which could happen with less effort.”⁸⁷ It is remarkable that he still acknowledges $\circ$ as “universal sign” to indicate *sesquitertia* (4:3) proportion in the *Tractatus practice* and also calls it thus, as it seemingly contradicts his argument that stacked Arabic numerals are preferable.

Prosdocimus was not the only theorist advocating stacked Arabic numerals in his treatise. Ugolino da Orvieto revealed a similar preference in his *Declaratio* and there will be other examples in this chapter.⁸⁸ For example, the anonymous treatise *Iste sunt proportiones* only contains stacked Arabic numerals as proportion signs.

Before moving on to these other treatises, I would like to comment on Busse Berger’s claim that Prosdocimus’ 1408 *Tractatus practice cantus mensurabilis* is the first to mention proportion signs:

“[T]he earliest theoretical explanation of rhythmic proportions—or, to be more specific, fractions in which a certain number of notes in the numerator are made equal to a different number of notes of the same type in the denominator—was given by Prosdocimus de Beldemandis in his *Tractatus practice de cantus mensurabilis* of 1408.”⁸⁹

This is not correct, as proportion signs appearing as stacked Arabic numerals (called “fractions” by Busse Berger) are already mentioned in his earlier work *Expositiones*. It is possible that Busse Berger has based her assertion on the fact that the earliest transmitted copy of *Expositiones* is dated to the year 1412.⁹⁰ The year 1404 is given in the explicit of the 1437 copy of *Expositiones*.⁹¹ As the earliest completely transmitted copy of the later *Tractatus practice* originated in the year 1409, however, Busse Berger should have given that year if her argument why the *Tractatus practice* should be the first treatise containing proportion signs is based on the dating of transmitted copies.⁹² In my opinion, the fact that *Expositiones* only contains three proportion signs while the *Tractatus practice* contains six points towards the likely scenario that the original 1404 version already contained the proportion sign discussion and that this

circumstance being explained in a canon. And the stacked Arabic numerals $\frac{2}{2}$ in Antonello da Caserta’s *Dame d’onour, en qui tout mon cuer maynt in Mod A* indicate *sesquitertia* (4:3) proportion. See Chap. 2.3 for details.

⁸⁶ “extraneos modos proportionandi figuras a modernis positos” Prosdocimus, *Expositiones*, chap. 61; edition in Gallo, *Prosdocimi de Beldemandis opera 1*, 151; my translation.

⁸⁷ “[F]rustra fit per plura quod fieri potest per pauciora.” Prosdocimus, *Expositiones*, chap. 20; edition in Gallo, ed., *Prosdocimi de Beldemandis opera 1*, 59; my translation.

⁸⁸ For Ugolino’s statement on stacked Arabic numerals, see Chap. 4.2.

⁸⁹ Busse Berger, *Mensuration and Proportion Signs*, 164.

⁹⁰ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 302.

⁹¹ “Et sic sit finis totius huius operis per musicorum minimum Prosdocimum de Beldemandis patavum **anno domini 1404** Padue complitati.” *Bologna, Civico Museo Bibliografico Musicale. MS A 56*, fol. 36v (p. 72). Transcription taken from F. Alberto Gallo, *La tradizione dei trattati musicali di Prosdocimo de Beldemandis*, Biblioteca di Quadrvium: serie musicologica 5, Bologna: [Forni], 1964, 11. See pp. 14–15. for the dating of this manuscript.

⁹² For datings of the different sources of the *Tractatus practice* see Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 302.

discussion is not an addition in the 1412 version of the treatise. I think that it is probable that Prosdocimus first discussed three proportion signs in *Expositiones* in 1404 and extended that list to contain three more proportion signs four years later in the *Tractatus practice*.⁹³ If—as I can only assume Busse Berger thinks—the proportion sign section was added in the 1412 copy of *Expositiones* and was not contained in the original version, it is inscrutable why Prosdocimus would only have included three signs when he discussed altogether six signs in the 1408 *Tractatus practice*. Hence, I consider his 1404 *Expositiones* to be the first datable treatise containing proportion signs.⁹⁴

⁹³ Stoessel has made a similar argument in “The Captive Scribe,” 1:295–96 n. 38.

⁹⁴ This view is supported by Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 340; Koehler, *Pythagoreisch-platonische Proportionen*, 1:57; and Stoessel, “The Captive Scribe,” 1:295.

5.3.1.2 A ‘New’ Source Commenting on Diminution and Ø: the Treatise *Quoniam circa artem musice figurative seu mensuralis* of c.1450

The mid-fifteenth-century treatise *Quoniam circa artem musice figurative seu mensuralis* is the only music-related text contained in the manuscript *Munich, Bayerische Staatsbibliothek, MS Clm 26812* (344r–355v), which primarily contains texts on astronomy and theology.⁹⁵ In 1876, the codex was given to the Bavarian State Library by the Dominican Monastery Saint Blaise in Regensburg, who in turn had received it in 1498.⁹⁶ Dates added to other treatises in the collection point towards a copying date of *Quoniam circa artem musice figurative seu mensuralis* between 1443 and 1453.⁹⁷

In the introduction to his edition, Bernhold Schmid has concluded that the treatise in all likelihood originated from the region of today’s Southern or Eastern Germany, based on its terminology: the text exhibits parallels to the Melk Anonymous⁹⁸ (ANON. Mell.), the Wrocław Anonymous⁹⁹ (ANON. Vratisl.), and the Kremsmünster Anonymous¹⁰⁰ (ANON. Kellner).¹⁰¹ More recently, Christian Meyer has also found parallels to the anonymous treatise *Circa musicam est notandum* from another manuscript at the Bavarian State Library (*Clm 18800*).¹⁰² However, all these parallels do not extend to the treatise’s teachings on diminution and proportion signs.

Although Schmid’s edition has been available since the year 1990, the treatise does not seem to have received much scholarly attention yet. This is surprising, since the text deals with the subject of diminution and might be one of the earliest sources discussing the matter in connection to cut signs. Margaret Bent’s assertion that “[a]ll theoretical evidence of Ø dates from the 1470s or later”¹⁰³ should certainly be reviewed in more detail.¹⁰⁴ One can also find

⁹⁵ Cf. Bernhold Schmid, “Der Musiktraktat aus Clm 26812,” 77.

⁹⁶ Cf. David Juste, ed., *Catalogus codicum astrologorum latinorum I: Les manuscrits astrologiques latins conservés à la Bayerische Staatsbibliothek de Munich* (Paris: CNRS Éditions, 2011), 171.

⁹⁷ Cf. Karl Halm, Georg von Laubmann, and Wilhelm Meyer, eds., *Catalogus codicum latinorum Bibliothecae Regiae Monacensis II.4: Codices num. 21406–27268 complectens* (Munich: 1881; reprint ed., Wiesbaden: Otto Harrassowitz, 1969), 215–16. The copyist is identified as Conrad Hebenpecher, ‘cappellanus’ at the hospital of Landau (fol. 147v). Cf. Juste, *Catalogus codicum astrologorum latinorum I*, 171.

⁹⁸ Melk, *Benediktinerstift Melk, Bibliothek, MS 950*, fols. 188v–204v.

⁹⁹ Wrocław, *Biblioteka Uniwersytecka. MS IV.Q.16*, fols. 151v–160r.

¹⁰⁰ Kremsmünster, *Benediktinerstift Kremsmünster. MS 312*, fols. 210v–212v.

¹⁰¹ Cf. Schmid, “Der Musiktraktat aus Clm 26812,” 77–78.

¹⁰² Cf. Christian Meyer, “Une ‘dissertation’ sur la musique autour de 1400 «Circa musicam est notandum...» (München, BSB, Clm 18800, f. 134r–138r),” 11 and 43–45.

¹⁰³ Margaret Bent, “The Early Use of the Sign Ø,” *Early Music* 24, no. 2 (1996): 202.

¹⁰⁴ A good starting point for such an endeavour is Rob C. Wegman’s criticism (cf. “Different Strokes for Different Folks?” 484–89), in which he has already elaborated on the existence of three treatises mentioning Ø prior to 1470, namely Anonymous XII’s *Tractatus de musica*, Anonymous XI’s *Tractatus de musica plana et mensurabili* and Georgius Erber’s treatise *Sequuntur proportionones declarate et manifeste demonstrate ostense melodia vocum cantorum invente*. I do not agree with Wegman’s dating the latter two treatises to a date as early as the 1420s,

contradictory statements concerning the earliest appearances of Ø in Busse Berger's *Mensuration and Proportion Signs*. In her chapter on cut signs, Busse Berger has written: "The earliest German theorist to discuss Ø is Anonymus XII, whose treatise was written before 1471."¹⁰⁵ But only a few pages later, she has elaborated on the earlier work of Anonymous XI: "The last group of northern theorists unequivocally favoured the diminution of Ø by one-half. The earliest treatise is Anonymus XI's *Tractatus de musica plana et mensurabili*, from c.1450."¹⁰⁶

As shown throughout this chapter on treatises, there are not only the two c.1450 German sources of this subchapter discussing Ø before 1470. As previously stated by Rob C. Wegman, cut signs indicating diminution (i.e. *dupla* (2:1) proportion) can already be found in Georgius Erber's treatise of c.1460 (see Chap. 5.3.2.2).¹⁰⁷ Not mentioned by either Wegman nor Busse Berger, but certainly strengthening the hypothesis that Ø can be found in treatises predating 1470, is the fact that the anonymous treatise *Venerabiles domini mei* probably written before 1457 (see Chap. 5.3.2.4) also contains cut signs to indicate diminution. And finally, the Hebrew treatise labelled *Student's notes of lectures on music* also describes a cut semicircle and a cut circle as indicating *dupla* (2:1) proportion, even if the signs themselves are not displayed (see Chap. 5.3.1.4).

Returning to *Quoniam circa artem musice figurative seu mensuralis*, I would now like to discuss the author's remarks on diminution and proportion. In the third of altogether five chapters of the treatise, diminution is described as common feature of modern chants. Moreover, the author discusses both diminution by half and by one third.¹⁰⁸ The mention of diminution by one third in this treatise accords with Busse Berger's findings, namely that diminution by one third is a trait of German music theory:

"I have found that, contrary to common opinion, not a single Italian theorist advocated diminution by one-third, and conclude therefore that this must be a German tradition that probably resulted from a misreading of Johannes de Muris' rules of diminution. Moreover, a careful reading of German treatises

however. For more, see Chap. 5.3.2.2 for Georgius Erber's treatise and Chap. 5.3.1.3 for the writings of Anonymous XI.

¹⁰⁵ Busse Berger, *Mensuration and Proportion Signs*, 134. The year 1471 is the copying date of the manuscript, the treatise itself has been dated to c.1460. Cf. Alejandro E. Planchart, "The Relative Speed of *Tempora* in the Period of Dufay," *Royal Musical Association Research Chronicle* 17 (1981), 35. A timespan of these eleven years (1460–1471) as time of origin of Anonymous XII's *Tractatus de musica* is given in most recent publications. See for example Ruth I. DeFord, *Tactus, Mensuration and Rhythm in Renaissance Music* (Cambridge: Cambridge University Press, 2015), 17.

¹⁰⁶ Busse Berger, *Mensuration and Proportion Signs*, 137–38.

¹⁰⁷ Cf. Wegman, "Different Strokes for Different Folks?" 485–88.

¹⁰⁸ "Et ille modus diminuendi occurrit iam communiter in multis carminibus modernorum cantorum, sic, quod ipsi suas canciones componunt **per diminucionem**. Et ideo videatur solum ad quid diminuatur quelibet nota an scilicet **pro sua parte media aut tercia** et cetera et secundum hoc ipse cantentur, quia omnia illa solum stant in beneplacito componencium." Anonymous, *Quoniam circa artem musice figurative seu mensuralis*, chap. 3; edition taken from Schmid, "Der Musiktraktat aus Clm 26812," 96; my emphasis.

raises serious doubts whether the diminution of Ø by one-third was ever observed in practice. It seems more likely that German theorists felt compelled to discuss diminution by one-third because this is what was done in older treatises.”¹⁰⁹

Judging from Busse Berger’s table, which lists northern authors discussing diminution in connection with Ø,¹¹⁰ *Quoniam circa artem musice figurative seu mensuralis* might actually be the earliest treatise mentioning diminution by one third. However, the signs for diminution described and displayed in the fourth chapter (Ø and ℄) are then said to indicate diminution by half, therefore endorsing Busse Berger’s hypothesis that diminution by one third might be an idiosyncrasy of German music theory rather than an actual musical practice:

“Nota, quod signa iam dicta, scilicet circulus et semicirculus, quandoque ponuntur in aliquo cantu et habent tractum per eius medietatem et illud signum est, quod talis cantus, in quo ponitur **circulus vel semicirculus cum tractu, dicitur per diminutionem, quod alii dicunt per semi**, idest, quod note diminuuntur ultra suum proprium valorem, ut patuit in capitulo praecedenti.”¹¹¹

At the end of the fourth chapter, the author describes three proportion signs: 2 indicates *dupla* (2:1) proportion, 3 indicates *tripla* (3:1) proportion, and 4 indicates *quadrupla* (4:1) proportion:

“Sunt eciam alia signa, que non respiciunt tempus nec prolacionem ita simpliciter, sed ponuntur propter variaciones proporcionum, ut si in cantu ponitur **proporcio dupla**, tunc circa eandem ponitur huiusmodi **signum 2**. Si autem **proporcio tripla**, tunc huiusmodi **signum ponitur 3**. Si vero **quadrupla**, tunc huiusmodi **signum ponitur 4**, et sic de aliis signis.”¹¹²

While the use of single Arabic numerals accords with my findings in *Ars subtilior* music, it should be noted, that the author only mentions proportions of the *multiplex* type. Those proportions most common in *Ars subtilior* music, namely *sesquialtera* (3:2) and *sesquitercia* (4:3) proportion, are not mentioned. Since proportions of the *multiplex* type do not necessarily require the figure ‘1’ in the denominator—3 equals $\frac{3}{1}$ —, the exclusion of proportions of any other type might be the reason for the author’s use of single instead of stacked Arabic numerals.

I have not further analysed the non-proportion-sign-related contents of *Quoniam circa artem musice figurative seu mensuralis* for this study, but I think that the treatise should certainly be examined more closely for future research on early appearances of cut signs in music theory. As described above, I have only found it through my search of the TML and would otherwise not have been aware of it.

¹⁰⁹ Busse Berger, *Mensuration and Proportion Signs*, 10.

¹¹⁰ Cf. Busse Berger, “TABLE 5. Diminution of Ø: northern theorists” in *Mensuration and Proportion Signs*, 127.

¹¹¹ Anonymous, *Quoniam circa artem musice figurative seu mensuralis*, chap. 4; edition taken from Schmid, “Der Musiktraktat aus Clm 26812,” 97; bold type my emphasis.

¹¹² Anonymous, *Quoniam circa artem musice figurative seu mensuralis*, chap. 4; edition taken from Schmid, “Der Musiktraktat aus Clm 26812,” 98; bold type my emphasis.

5.3.1.3 Two Treatises from the *Tractatus de musica plana et mensurabili* (Coussemaker's Anonymous XI): *Sequitur hic aliqua declaratio atque denominatio* and *Proportio est duorum terminorum vel duorum numerorum*

Although the *Tractatus de musica plana et mensurabili*—attributed to Coussemaker's Anonymous XI and transmitted in the manuscript *London, British Library, MS Add. 34200* (fols. 1r–41r)—is often cited as one treatise, because Coussemaker considered it as such, the work should actually be viewed as collection of several different texts by different anonymous authors. In his detailed study of the collection's contents and scribes, Richard J. Wingell has divided the *Tractatus de musica plana et mensurabili* into seven different treatises with additional—sometimes subject-wise unrelated—material interspersed throughout the texts.¹¹³

The collection as a whole is generally dated to c.1450,¹¹⁴ although—as mentioned above—Rob C. Wegman has suggested a date of origin as early as 1420 for the *Tractatus de musica plana et mensurabili*.¹¹⁵ According to Wegman, the sign O2, which is described by Anonymous XI as a sign of diminished perfect tempus—equivalent to Ø—, was no longer used in this capacity by 1450.¹¹⁶ Notwithstanding Wegman's possibly contradictory analysis of when the interpretation of the sign O2 changed, I would counter that the process of adaptation of notational innovations into music theory treatises always took some time—even decades.¹¹⁷ In Chapter 1 of this study, I have summarised Karen Cook's findings concerning the semiminim, which appeared in music long before being accepted by theorists and included in music theory treatises.¹¹⁸ Hence, a ten-year gap between the alleged changing of O2's interpretation and the

¹¹³ Cf. Wingell, "Anonymous XI," 358–59. Note that the RISM editors have divided the work into nine different sections while the LML editors have separated the text into 14 different units. This is due to the fact that the RISM and LML editors have included most of the miscellaneous material in their count while Wingell only counted the larger text units.

¹¹⁴ Cf. C. Matthew Balensuela, "Anonymous theoretical writings," in *NG2*, 1:697–98 (no. 39); and Busse Berger, *Mensuration and Proportion Signs*, 137–38.

¹¹⁵ Cf. Wegman, "Different Strokes for Different Folks?" 485 n. 40.

¹¹⁶ Wegman refers to this passage of the work: "And the sign of diminution is equal to this: $\mathcal{O}$; Likewise this $\mathcal{O}$ is equal to this O2 according to most recent singers." ("Eciam illud signum diminucionis equivalet huic $\mathcal{O}$; Similiter illud $\mathcal{O}$ huic O2 secundum modernissimos cantores.") Edition and translation taken from Wingell, "Anonymous XI," 155 and 330.

¹¹⁷ Wegman has stated: "**By the 1440s, however, O2 was generally understood** [bold type my emphasis] to be a sign of perfect modus [...]." But in the next sentence he gives only two examples for this new interpretation: "**The earliest known use of O2** as a sign of perfect modus is in the Trent 88 Mass Propers, composed by Guillaume Dufay in the 1440s [...] and Petrus de Domarto's *Missa Spiritus almus* of ca. 1450." "Different Strokes for Different Folks?" 485 n. 40. That two examples of a different interpretation of a particular sign already account for a new and different *general* understanding of said sign seems at least questionable to me. And as Wegman himself has written, Dufay's composition of the 1440s appears to be the *e a r l i e s t* example of O2 indicating perfect *modus*.

¹¹⁸ Cf. Chap. 1.3, pp. 37–40, which summarise Karen Cook's findings, "Theoretical Treatments of the Semiminim in a Changing Notational World c. 1315–c. 1440," PhD diss., Duke University, 2012.

assumed compilation of the *Tractatus de musica plana et mensurabili* does not constitute an impossibility in my view.

Perhaps the debate surrounding this particular phrase is completely redundant in the context of this study, however, because it is written on a page, which does not seem to belong to the original collection. It appears well towards the end of the *Tractatus de musica plana et mensurabili*, namely on fol. 36v (see FIGURE 5.8), which contains miscellaneous material on mensuration and diminution signs.

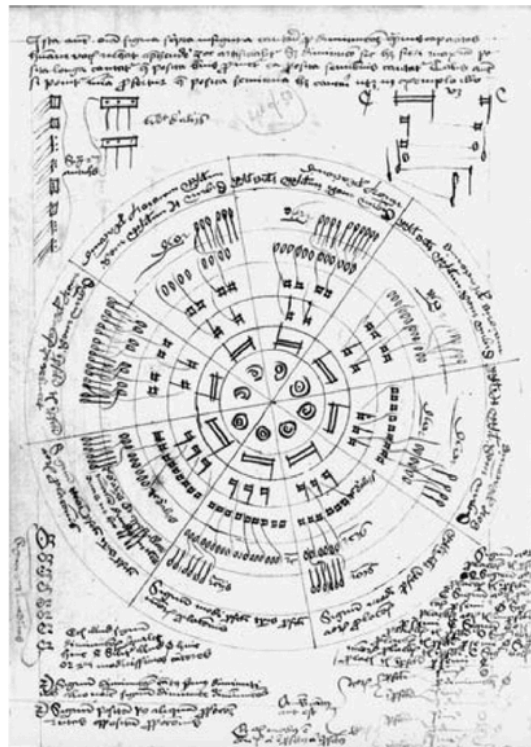


FIGURE 5.8: Fol. 36v of London, British Library, MS Add. 34200

The content of the page is concerned with different ways of signifying mensuration. Prominently featured is a round diagram, which also appears elsewhere in music theory, showing different combinations of perfect/imperfect *maximodus*, perfect/imperfect *modus*, perfect/imperfect *tempus*, and major/minor *prolatio*.¹¹⁹ One can also find an explanation of diminution and a comparison of signs according to the *antiqui* and the *moderni*. Several signs in square boxes with dots—also found in earlier music theory treatises—labelled “*signa secundum antiquos*” (left side above the circle) are juxtaposed various *modus cum tempore* signs labelled “*signa secundum modernos*” (left side under the circle).¹²⁰ And finally, there is

¹¹⁹ Cf. Wingell, “Anonymous XI,” 408.

¹²⁰ For an explanation of the term *modus cum tempore* signs, see Busse Berger, *Mensuration and Proportion Signs*, esp. chap. 5 “Diminution by Stroke and by Mode Signs”, section V “*Modus cum tempore* Signs”, 148–59.

an illustration of simple mensuration signs and variations thereof for displaying diminution (right side above and beneath the circle).¹²¹

The majority of the material found on that folio (36v) can also be found in Anonymous XII's *Tractatus de musica*¹²² of 1460–71 and Wingell has concluded that “the whole folio may be misplaced from that treatise.”¹²³ It may well be that the material was copied onto an empty page *after* the collection was compiled. I am suggesting here that space was left out between certain treatises and later filled with either the intended or other material. An argument in favour of that scenario can be that the contents of pages (fols. 38v–39r) following shortly after fol. 36v also seems to be gap-filling material, namely musical examples, and that the scribe of those pages (Hand C) was also responsible for adding other material on the bottom of folios in different sections of the *Tractatus de musica plana et mensurabili*.¹²⁴ For those reasons, I have decided to exclude the contents of fol. 36v from this study. In any case—apart from the signs of diminution—no proportion signs are mentioned.

Two treatises from the collection do mention proportion signs explicitly, however. They are found at the very end of the compilation and were also written by different scribes.¹²⁵ The first has the incipit *Sequitur hic aliqua declaratio atque denominatio*¹²⁶ (fols. 40r–40v) and is also transmitted in the manuscript *Regensburg, Bischöfliche Zentralbibliothek, Proskesche Musikabteilung, MS 98 th. 4°* (fols. 311r–314r); the second begins with *Proportio est duorum terminorum vel duorum numerorum*¹²⁷ (fol. 41r) and is uniquely transmitted in *London, British Library, MS Add. 34200*.

¹²¹ Ruth I. DeFord has pointed out some errors in Wingell's transcription of these illustrations, which, unfortunately, are continued shown in the TML version. Cf. “On Diminution and Proportion in Fifteenth-Century Music Theory,” *Journal of the American Musicological Society* 58, no. 1 (2005): 16 n. 39.

¹²² Cf. Edmond de Coussemaker, ed., *Scriptorium de musica medii aevi nova series a Gerbertina altera*, 4 vols. (Paris: Durand, 1864–76; reprint ed., Hildesheim: Olms, 1963), 3:489; and Jill M. Palmer, ed., *Tractatus et compendium cantus figurati (Mss. London, British Libr., Add. 34200; Regensburg, Proskesche Musikbibl., 98 th. 4°)*, *Corpus scriptorum de musica* 35 (Rome: American Institute of Musicology, 1990), 80.

¹²³ Wingell, “Anonymous XI,” 409. However, DeFord has pointed out: “[B]oth authors might have taken the diagram from another source. The appendix is not part of the original text of the treatise of Anonymous 12, and the explanation of diminution signs in the appendix is incompatible with the one in the main text.” “On Diminution and Proportion,” 16 n. 39.

¹²⁴ For a description of the scribe called “Hand C” see Wingell, “Anonymous XI,” 355–56. According to Wingell, “the polyphony seems to have no connection with any part of the treatise.” 356. He has therefore concluded that “the example[s] seem to be a later addition.” 409. I agree with Wingell in that most of the examples do not seem to make much musical sense: “All efforts by the present writer to work out a satisfactory transcription have failed. Some sections seem to work out well, but others are mathematically impossible, even leaving aside the normal rules of consonance and dissonance.” 409.

¹²⁵ Cf. Wingell, “Anonymous XI,” 413.

¹²⁶ “VI. De proporcionibus” according to Wingell (cf. “Anonymous XI,” 163–69); “*Tractatus de proportionibus musicae mensuralis* (PROP. MENS. Sequitur)” according to LML.

¹²⁷ “VII. De proporcionibus” according to Wingell (cf. “Anonymous XI,” 171–73); “*Tractatus de proportionibus* (PROP. Prop. est duorum)” according to LML.

5.3.1.3.1 The Treatise *Sequitur hic aliqua declaratio atque denominatio* (fols. 40r–40v) and the Treatise *Ars et practica cantus figurativi*

The first proportion treatise of the collection of treatises known as *Tractatus de musica plana et mensurabili* constitutes an intelligible and well organised text, which describes eight different rhythmic proportions and their respective signs. Towards the end of the treatise, the proportions and their respective signs are even given in a table (see FIGURE 5.9), thus summarising the detailed statements.

Signa proportionum	Duplae	2
	Triplae	3
	Quadruplae	4
	Sesquialterae	$\frac{3}{2}$
	Sesquiterciae	$\ominus$
	Sesquiquartae	$\odot$
	Superbiparciens	Φ
	Sesquioctava	$\frac{9}{8}$

FIGURE 5.9: Transcription of table on fol. 40v of London, British Library, MS Add. 34200¹²⁸

The treatise exhibits all three different forms of visual appearance of proportion signs: single Arabic numerals, stacked Arabic numerals, and geometric shapes. According to the text, *dupla* (2:1), *tripla* (3:1), and *quadrupla* (4:1) proportion are indicated by single Arabic numerals: 2, 3, and 4 respectively. *Sesquialtera* (3:2) proportion is indicated by the stacked Arabic numerals $\frac{3}{2}$, while *sesquitercia* (4:3) proportion is indicated by the reversed semicircle $\ominus$. *Sesquiquarta* (5:4) proportion is indicated by either $\frac{5}{4}$ or $\odot$,¹²⁹ *superbipartiens tertias*¹³⁰ (5:3) proportion is indicated by Φ , and *sesquioctava* (9:8) proportion is indicated by either $\frac{9}{8}$ or $\odot$.

¹²⁸ Illustration taken from Wingell, “Anonymous XI,” 168.

¹²⁹ Note that the table at the end of the treatise only displays the geometric shape, while the text mentions both $\frac{5}{4}$ and $\odot$.

¹³⁰ This proportion is called *superbipartiens* without the addition ‘*tertias*’ in the treatise. According to the rules of proportions’ denominations, however, the term *superbipartiens* refers to all proportions of the *superpartiens* type, in which the difference between first and second term in the proportion is 2, e.g. *superbipartiens quintas* (7:5) proportion (7–5 = 2) and so on (see Chap. 1, p. 20). The reason for leaving out the addition ‘*tertias*’ might be that *superbipartiens tertias* (5:3) proportion is the very first of this type of proportions, ‘first’ meaning proportion with the smallest number as second term. The only other proportions, in which the difference between first and second

Each proportion is more or less described in the same manner, although the detailedness decreases towards the end of the descriptions, i.e. the statements at the beginning of the treatise contain more numerical examples and variations of denominations than the descriptions towards the end. As an example, I am giving the description of *dupla* (2:1) proportion:

“[Therefore, o]ne should know that duple proportion [has to become] the first [in] order of the [things about which I wrote] above, when the larger number contains the smaller, or thus, when two are sung against the previous one; or thus, when the larger number doubles the smaller, for example, two against one, or four against two, and so forth. Applying this proportion in the musical sense, duple proportion is when, in mensural music, two similar notes are sung against a similar one, for example, two *minimae* against one *minima*, or four against two, etc. And this [has to] be applied to [the single] notes, that is, to *semibreves* in relation to the *semibrevis*, *breves* to *brevis*, *longae* to *longa*, *maximae* to *maxima*. Or it can be understood this way: duple proportion is when double the value of any note or notes is set or sung against that note or notes. And this proportion is called duple from *duplo*, *duplare*, that is, one number contains the other twice, since it occurs when the larger number of notes is double in relation to the smaller number of notes against which it is sung. The sign for this proportion is the following: the number two, set down as in this example: 2 ↓ ↓ .”¹³¹

Two observations are most noteworthy. First, we learn that proportions should be applied in a linear manner, because a number of notes “are sung against the *previous* [my emphasis] one”. This accords with the application of proportions in the *Ars subtilior* music discussed in this study. Where it is possible to draw such conclusions, proportions are most often applied to the previous section within the same voice. Second, the author states that notes of all values—minims, semibreves, breves, *longae*, and even *maximae*—can be compared to each other in a proportion. Although the notes given in the examples are always minims and the statement of note values also begins with minims, thereby implying that these are the standard note values which should be used for the comparison, the author is strangely insistent about the point that all note values can be compared to each other, repeating it in every description.¹³²

term in the proportion is 2, are *tripla* (3:1) proportion and *dupla* (4:2 = 2:1) proportion, which are of another, namely the *multiplex* type.

¹³¹ “Pro quo sciendum: proporcio dupla ex praescriptis in ordine prima habet fieri, quum maior numerus continet brevioris his, aut sic quum duae contra praevisam unam proferuntur; aut sic quum maior numerus duplicat brevioris, ut in exemplo duae ad unam aut quattuor ad duo, et caetera. Sic intelligendo hanc proporcionem ad sensum musicalem, proporcio dupla est quum in cantu mensurali duae similes notae contra similem proferuntur, ut duae minimae contra unam minimam, vel quattuor contra duas, et caetera. Et hoc de singulis notis intelligendum est, scilicet de semibrevis ad semibrevis, de brevis ad brevis, aut longis ad longam, aut maximis ad maximam. Aut sic intelligendo, proporcio dupla est quum bis valor alicuius notae vel notarum contra notam seu notas componitur sive profertur. Et dicitur haec proporcio dupla a duplo, -as, -are, id est numerus numerum bis continere, quia fit quum ut numerus maior brevioris duplicat, aut quum maior numerus notarum duplus est ad brevioris notarum contra quas profertur. Et huic proporcioni praeponitur tale signum, videlicet binarius numerus algoristice positus, ut hic in exemplo.” Anonymous, *Sequitur hic aliqua declaratio atque denominatio*; edition and translation taken from Wingell, “Anonymous XI,” 163–64 and 338–39.

¹³² In the description of *tripla* (3:1) proportion: “And this can apply to any value of note against the same value, namely *semibreves* against *semibrevis*, *breves* against *brevis*, etc.” *Quadrupla* (4:1) proportion: “This can apply to notes of any value—*semibreves* to *semibrevis*, etc.” *Sesquialtera* (3:2) proportion: “This applies to notes of any value, as is clear.” *Sesquitercia* (4:3) proportion: “[A]nd this can apply to notes of any value, as above.” *Sesquiquarta* (5:4) proportion: “[A]nd this applies to notes of any value.” *Superbipartiens tertias* (5:3) proportion: “[A]nd this applies to any notes of the same value, as above.” *Sesquioctava* (9:8) proportion: “[A]nd this applies

The treatise, in addition to describing and displaying proportion signs, also provides a few interesting glimpses into the transmission of teachings on rhythmic proportion. There are indications which suggest that the author of *Sequitur hic aliqua declaratio atque denominatio* has based his treatise on older material or alternatively updated such material. After the discussion of *dupla* (2:1), *tripla* (3:1), and *quadrupla* (4:1) *sesquialtera* (3:2), and *sesquitertia* (4:3) proportion as well as their respective signs, one can find the remark: “regula Muris. Likewise, according to the opinion of some, no more proportions should follow.”¹³³ In the subsequent paragraphs, the author has described further three proportions: *sesquiquarta* (5:4), *superbipartiens tertias* (5:3), and *sesquioctava* (9:8) proportion. The remark inserted between these five and three descriptions suggests that the latter three were at some previous stage not included in the list of rhythmic proportion.

Although attributed to him by the author of *Sequitur hic aliqua declaratio atque denominatio*, I am not aware of an existing treatise by Johannes de Muris, in which Muris has stated that the above-mentioned five are the only proportions in mensural music, especially not in the rhythmic context. Since a large number of treatises on mensural music from the fourteenth and fifteenth century were based on the *Libellus cantus mensurabilis*, however, it seems reasonable to assume that one of those treatises ‘*secundum* Muris’ was updated by the author of *Sequitur hic aliqua declaratio atque denominatio* and therefore contains the remark about Muris.

It is noticeable, that the proportions and respective signs labelled “regula Muris” accord with what we might call ‘standardised signs’ in *Ars subtilior* music. The reversed semicircle $\circ$ is the most common sign for *sesquitertia* (4:3) proportion, while *sesquialtera* (3:2) proportion is frequently indicated by the stacked Arabic numerals $\frac{3}{2}$, if not indicated by red ink, that is.¹³⁴ As I have shown in the previous chapters, single Arabic numerals for *dupla* (2:1), *tripla* (3:1), and *quadrupla* (4:1) proportion are used in *Ars subtilior* music, consistent with the statements of the author. Hence, I propose that this first part (“regula Muris”) harks back to a now lost original earlier source on *Ars subtilior* notation, which was written by an author describing the appearance of proportion signs in actual music sources.

Two of the additions by our present anonymous author, however, do not seem to reflect on conventional music notation. At least the signs for the two uncommon proportions,

to notes of any value, as above.” Anonymous, *Sequitur*; translations taken from Wingell, “Anonymous XI,” 339–42.

¹³³ “regula Muris. Item sub uno colore secundum opinionem aliquorum non debent sequi plures porporciones.” Anonymous, *Sequitur*; edition and translation taken from Wingell, “Anonymous XI,” 166 and 341.

¹³⁴ For $\frac{3}{2}$ indicating *sesquialtera* (3:2) proportion, see especially Chap. 3.1, describing the sign in *Turin*.

sesquiquarta (5:4) and *superbipartiens tertias* (5:3) proportion ($\ominus$ and $\mathfrak{D}$ respectively), were not used in the capacity described in *Sequitur hic aliqua declaratio atque denominatio*.¹³⁵ The reversed cut semicircle $\mathfrak{D}$, for example, would usually be interpreted as sign for *sesquitertia* (4:3) proportion in diminution, i.e. *dupla superbipartiens tertias* (8:3) proportion.¹³⁶ There are also appearances of $\ominus$ in music sources from the fifteenth century, but in none of these compositions does the sign indicate *sesquiquarta* (5:4) proportion.¹³⁷ In fact, I am not aware of a single *Ars subtilior* song applying this proportion.¹³⁸ However, there are at least two examples of *superbipartiens tertias* (5:3) proportion in *Ars subtilior* music: The ballade *Sur toute fleur la rose est colorie* and the virelai *Je prens d'amour noriture* from *Turin. Superbipartiens tertias* (5:3) proportion is not indicated by $\mathfrak{D}$ in those examples, however, as shown in Chapter 3 of this study.

It is rather remarkable that the signs for the two uncommon proportions in *Sequitur hic aliqua declaratio atque denominatio* are variants of the semicircle: $\ominus$ and $\mathfrak{D}$, because it looks like systemic coercion in that semicircles should be used for indicating rhythmic proportion. It seems that the author has then taken a common sign, namely $\ominus$, and altered it. One can observe a similar situation in Ugolino's *Declaratio musicae disciplinae*, in which the semicircles $\frown$ and $\smile$ described by him also have no or very little precedent in actual music notation, but might simply be new variants of a conventional sign.¹³⁹ I would also suggest that these additions are more likely the invention of an author keen on presenting novel proportions, which do not actually have much foundation in musical practice. On the other hand, *sesquioctava* (9:8) proportion is definitely used in *Ars subtilior* music, as shown in the previous chapters. Furthermore, the circle with two dots is also mentioned as sign for *sesquioctava* (9:8) proportion in the Hebrew treatise [*Student's notes of lectures on music*] and the anonymous treatise *Ars et practica cantus figurativi* from a manuscript which is dated to 1483.¹⁴⁰

¹³⁵ It should be noted, however, that Adam of Fulda gives the same signs for these two proportions in his 1490 treatise (cf. *Musica, pars quarta*, chap. 8). Cf. Busse Berger, *Mensuration and Proportion Signs*, 177 n. 50; and Jason Stoessel, "Looking Back Over the 'Missa L'Ardant desir': Double Signatures and Unusual Signs in Sources of Fifteenth-Century Music." *Music & Letters* 91, no. 3 (2010): 334 nn. 53–54.

¹³⁶ This is the interpretation given by Georgius Erber in his treatise *Sequuntur proportionones declare et manifeste demonstrate ostense melodia vocum cantorum invente* of c.1460 (see Chap. 5.3.2.2) and also in the second proportion treatise *Proportio est duorum terminorum vel duorum numerorum* of the *Tractatus de musica plana et mensurabili* (see Chap. 5.3.1.3.2).

¹³⁷ Cf. Stoessel, "Looking Back Over the 'Missa L'Ardant desir'," 334–38.

¹³⁸ According to Busse Berger, John Hothby has used *sesquiquarta* (5:4) proportion in his motet *Ora pro nobis*, however. Cf. *Mensuration and Proportion Signs*, 167–68.

¹³⁹ There is only one piece applying $\smile$. See Chap. 4, n. 25 on p. 200.

¹⁴⁰ For the sign $\ominus$ in the Hebrew treatise [*Student's notes of lectures on music*] see Chap. 5.3.1.4, for the treatise *Ars et practica cantus figurativi* see p. 251 below.

I cannot think of a simple explanation as to why the author has added the two uncommon proportions to the treatise. The fact that they were included, while proportions such as *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion were left out, is rather striking, because the author has made it clear at the very beginning that “there are eight proportions in mensural music” and that “no more are used.”¹⁴¹ However, there are many examples of *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion in mensural music. Thus, if the author intended to include proportions which were used in actual music, these two would have been the more obvious choice.

Is it the relative speed of those two proportions that has led the author to exclude them from his treatise? I am suggesting this, because an enlightening remark on the possibility to perform proportional rhythms is found in the introduction: “Still, there are more geometric¹⁴² proportions, three of which are not given here, and are not used in mensural music, and cannot be used, *because of the difficulty of singing them* [my emphasis], namely *quintupla* (5:1), *sextupla* (6:1), and *septupla* (7:1).”¹⁴³ Hence, it might be for that reason that the author has excluded the relatively common proportions *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion, while including the uncommon proportions *sesquiquarta* (5:4) and *superbipartiens tertias* (5:3) proportion. It has to be noted however, that while *dupla sesquiquarta* (9:4) proportion is naturally faster than *sesquiquarta* (5:4) proportion, and *dupla superbipartiens tertias* (8:3) proportion is faster than *superbipartiens tertias* (5:3), both unmentioned proportions do not produce shorter minims than *trippla* (3:1) as well as *quadrupla* (4:1) proportion, which the author *has* included. However, it is true, that *quintupla* (5:1), *sextupla* (6:1), and *septupla* (7:1) proportion are not to be found in the *Ars subtilior* repertoire discussed in this study. Where speed is concerned, *quadrupla* (4:1) proportion on minim level is definitely producing the smallest note values. Nonetheless, it should be noted that there is at least one treatise, which gives signs, namely the Arabic numerals 5 and 6, for *quintupla* (5:1) and *sextupla* (6:1) proportion respectively.¹⁴⁴

Before moving on to the treatise *Proportio est duorum terminorum vel duorum numerorum*, I would like to point out that another text from a manuscript dated to 1483, namely the

¹⁴¹ “[O]cto sunt proporcionones musicales cantus mensuralis [...] et plures in cantu mensurali non ponuntur.” Anonymous, *Sequitur*; edition and translation taken from Wingell, “Anonymous XI,” 163 and 338.

¹⁴² I suspect that the author meant to write proportions of the *multiplex* type, because geometric proportions are something else (see Chap. 1, p. 9).

¹⁴³ “Non obstante quando plures proporcionones geometricas sunt, quarum denominationes tres non scribuntur, nec in cantu mensurali ponuntur, nec poni licet, ratione difficultatis modulacionis, videlicet quintupla, sextupla, septupla.” Anonymous, *Sequitur*; edition and translation taken from Wingell, “Anonymous XI,” 163 and 338.

¹⁴⁴ Cf. Anonymous I, *Tractatus de musica figurata et de contrapuncto ab anonymo auctore*; edition in Edmond de Coussemaker, ed., *Scriptorum de musica medii aevi nova series a Gerbertina altera*, 4 vols. (Paris: Durand, 1864–76; reprint ed., Hildesheim: Olms, 1963), 4:434–69, proportion signs on p. 438.

anonymous treatise *Ars et practica cantus figurativi*, mentions almost the exact same signs as *Sequitur hic aliqua declaratio atque denominatio*, also in the same order.¹⁴⁵ Since the descriptions of proportions and their respective signs in *Ars et practica cantus figurativi* are much shorter, I am giving the full proportion sign section here. Similarities to *Sequitur hic aliqua declaratio atque denominatio* are set in bold type.

“De proportionibus

Octo sunt proportiones musicales in cantu mensurali, scilicet proportio **dupla, tripla, quadrupla, sesquialtera, sesquitercia, sesquiquarta**, dupla superbipartiens et **sesquioctava**.

Proportio dupla est quando maior numerus continet minorem bis, ut quatuor ad duo. Musicaliter quando in aliquo cantu duae miniae contra unam proferuntur aut quatuor contra duas. Cuius proportionis signum proprium est figura binaria algoristica, ut sic: [2]¹⁴⁶.

Proportio tripla est quando maior numerus continet minorem ter, ut tria ad unum. Musicaliter quando tres miniae contra unam proferuntur. Cuius proportionis signum: 3.

Quadrupla proportio est quando maior numerus continet minorem quatuor, ut 4 ad 1. Musicaliter quando quatuor miniae contra unam proferuntur. Signum: 4.

Proportio sesquialtera est quando maior numerus continet minorem semel et eius alteram partem seu mediam partem, ut tria ad duo. Musicaliter quando tres miniae contra duas proferuntur. Signum: $\frac{3}{2}$.

Proportio sesquitercia est quando maior numerus continet minorem semel et eius tertiam partem, ut 4 ad 3. Musicaliter quando quattuor miniae contra tres proferuntur. Signum sic $\frac{4}{3}$, vel sic: $\ominus$.

Proportio sesquiquarta est quando maior numerus continet semel minorem et eius quintam partem, ut 5 ad 4. Musicaliter quando quinque miniae [contra] quattuor proferuntur, quidem tamen raro contingit propter eius difficultatem. Cuius signum est $\frac{5}{4}$, vel sic: $\ominus$.

Proportio dupla superbipartiens est quando maior numerus bis continet minorem et insuper duas eius partes, ut decem ad quatuor. Musicaliter quando decem miniae contra quattuor proferuntur. Cuius signum $\frac{8}{3}$, vel tale $\ominus$ sed raro.

Ultima proportio sesquioctava est quando maior numerus continet minorem semel et eius octavam partem, ut novem ad octo. Musicaliter quando novem miniae contra octo proferuntur, quidem rarissime contingit propter eius nimiam difficultatem pronuntiandi. Cuius proportionis signum est circulus cum duobus punctis obliquo modo in eo positus, ut sic $\ominus$ vel sic: $\frac{9}{8}$.¹⁴⁷

It stands to reason that, due to the striking similarities, both *Sequitur hic aliqua declaratio atque denominatio* and *Ars et practica cantus figurativi* were ultimately based on the same source. The only obvious discrepancy is that the author of *Ars et practica cantus figurativi* mentions *dupla superbipartiens tertias* (8:3) proportion (instead of *superbipartiens tertias* (5:3) proportion) as interpretation of $\ominus$, which, as explained above, is the more obvious reading, namely *sesquitercia* (4:3) proportion in diminution. Perhaps the inclusion of *superbipartiens tertias* (5:3) proportion in *Sequitur hic aliqua declaratio atque denominatio* was based on a misreading of the original text.

I will now move on to the second proportion sign treatise of the *Tractatus de musica plana et mensurabili*. While the first treatise *Sequitur hic aliqua declaratio atque*

¹⁴⁵ Similarities between *Sequitur hic aliqua declaratio atque denominatio* and *Ars et practica cantus figurativi* regarding the sign $\ominus$ have previously been remarked upon by Stossel, “Looking Back Over the ‘Missa L’Ardant desir’,” 334 n. 54.

¹⁴⁶ It should be noted that the figure 2 is missing from the manuscript and has been inserted by the editor.

¹⁴⁷ Edition taken from Christian Meyer, ed., *Anonymi Tractatus de cantu figurativo et de contrapuncto* (c. 1430–1520), Corpus scriptorum de musica 41 (Rome: American Institute of Musicology, 1997), 40–41.

denominatio was only concerned with rhythmic proportion, i.e., the text did not mention the significance of proportions for the derivation of musical intervals, the second treatise *Proportio est duorum terminorum vel duorum numerorum* is a mixed text on intervals and proportion signs.

5.3.1.3.2 The Treatise *Proportio est duorum terminorum vel duorum numerorum* (fol. 41r)

The shorter second proportion sign treatise of the *Tractatus de musica plana et mensurabili* can be divided into three different sections written by three different scribes.¹⁴⁸ It can therefore be assumed that the treatise originally consisted only of the first part and that the other two parts constitute additions, which the respective scribes deemed thematically fitting. Since the third section written by the third scribe is not thematically relevant, I will not discuss it here, but only refer to the first two sections.¹⁴⁹ Only two proportion signs are mentioned in the treatise, namely in the section added by the second scribe: $\ominus$ and $\frac{9}{6}$.

The assumed original treatise, which is the largest part of the text and written by the first scribe, is concerned with musical intervals. It names six different proportions, their denominations in Greek and Latin, and the musical intervals, which are formed thereof, in the following order: 9:8; 4:3, 3:2, 2:1, 3:1, 4:1.¹⁵⁰ As an example, I am giving the description of *sesquialtera* (3:2) proportion:

“The third proportion is *emiolia*, in Latin *sesquialtera*, named from *emi*, half, and *olii*, the whole, since it contains the whole plus its half part. From this number comes the concord called *diapente*, from *dia*, [which is de,] and *penta*, five, since it consists of five notes, and contains in itself three tones and a small semitone.”¹⁵¹

This part has a clear explicit after the sixth proportion: “Et haec sufficient.”¹⁵²

Another scribe has then added two further sentences, which—where the structure is concerned—resemble the explanations of the preceding proportions. The author of these phrases has even continued the numeration, calling his additions the seventh and eighth proportion:

“The seventh is called *dupla superbipartiens*, or double the *epitrita* proportion, since it is formed of half the *epitrita* proportion; it contains sixteen to six, as the *epitrita* contains eight to six, and its sign is $\ominus$. The eighth and final proportion is called the *sesquitercia in gravi superparticulari*, as for example nine to six, and it is the subduple (half) of the triple proportion, since the triple has eighteen to six; the sign for this proportion is $\frac{9}{6}$.”¹⁵³

¹⁴⁸ On the different scribes on this folio see Wingell, “Anonymous XI,” 417.

¹⁴⁹ For a discussion of the third section, see Wingell, “Anonymous XI,” 417.

¹⁵⁰ According to Wingell, order and terminology of the treatise closely resembles the c. 1470 *Tractatus modi, temporis et prolationis* written by the Benedictine monk Christian Sage. Cf. “Anonymous XI,” 416.

¹⁵¹ “Tercia proporcio est emiolia, latine sesquialtera, et dicitur ab emi et olii, totum, quasi continens totum et eius mediam partem. Et ex hoc numero nascitur symphonia quae dicitur diapenthe, a dia, quod est de, et penta, quinque, quasi de quinque vocibus constituta; et continet in se tres tonos cum semitonio breviori.” Anonymous, *Proportio est duorum terminorum vel duorum numerorum*; edition and translation taken from Wingell, “Anonymous XI,” 171–72 and 347.

¹⁵² Anonymous, *Proportio est*; edition taken from Wingell, “Anonymous XI,” 172.

¹⁵³ “Septima vocatur dupla superbiparciens, seu dupla proporcionis epitritae, qui habet cantari per semi proporcionis epitritae; et continet 16 ad 6, sicut epitrite continet 8 ad 6; et eius signum est tale: $\ominus$. Octava et ultima sesquitercia in gravi superparticulari, ut 9 ad 6; et est subdupla proporcionis triplae, quia tripla habet 18 ad 6; et

However, the focus in these additional phrases has clearly shifted from intervals to rhythmic proportion. Still, it is noteworthy that the second scribe has just added two more proportions, adding up to eight proportions altogether—one of the proportions is not exactly new though, as shown below. The sum of eight proportions is reminiscent of the previous treatise, which clearly states that “there are eight proportions in mensural music” and that “no more are used.”¹⁵⁴

The two added phrases give the impression that their author had only mediocre knowledge of the teachings of proportions. For example, while *dupla superbipartiens tertias* is the correct denomination of 8:3 proportion, the term for the second proportion (9:6) is absolutely puzzling: *sesquitertia in gravi superparticulari*. As 9:6 proportion is an expansion of *sesquialtera* (3:2) proportion, the term ‘*sesquitertia*’ does not seem to make any sense in this context. Since *sesquialtera* (3:2) is also already mentioned as fourth proportion in the previous part of the treatise, its description as eighth proportion also seems to be incorrect.

Two comments should be made about the signs given by the treatise. First, the reversed cut semicircle $\ominus$ is the obvious choice for *dupla superbipartiens tertias* (8:3) proportion, especially when the proportion is thought of as *sesquitertia* (4:3) proportion in diminution—reflected in the text by the term *dupla porporcionis epitritae*.¹⁵⁵ As already mentioned, this interpretation deviates from the interpretation given in the previous treatise *Sequitur hic aliqua declaratio atque denominatio*, where $\ominus$ is said to indicate *superbipartiens tertias* (5:3) proportion. This discrepancy constitutes further indication that the *Tractatus de musica plana et mensurabili* is a collection of different treatises by different authors.¹⁵⁶ That the proportion is given as 16 to 6 and not 8:3 suggests that the assertions are based on findings in actual music notation: the 6 most probably refers to six minims in [2,3] (C) or [3,2] (O), hence the total number of minims in a *mensura*. These are replaced by 16 minims under $\ominus$.

Second, the stacked Arabic numerals $\frac{9}{6}$ for *sesquialtera* (3:2) proportion are uncommon, but not necessarily exceptional. Again, the 6 most probably refers to six minims in [2,3] or [3,2], which are replaced by nine minims to form *sesquialtera* (3:2) proportion. As shown in Chapter 2.3, the stacked Arabic numerals $\frac{9}{6}$ are actually used as proportion sign in Antonello da Caserta’s *Amour m’a le cuer mis en tel martire* to indicate *sesquialtera* (3:2) proportion.

huic porporcioni appropriatur tale signum: 9/6.” Anonymous, *Proportio est*; edition and translation taken from Wingell, “Anonymous XI,” 172–73 and 347–48.

¹⁵⁴ “[O]cto sunt porporciones musicales cantus mensuralis [...] et plures in cantu mensurali non ponuntur.” Anonymous, *Sequitur*; edition and translation taken from Wingell, “Anonymous XI,” 163 and 338.

¹⁵⁵ Not to be confused with *dupla epitrita* or *dupla sesquitertia* in Latin, which is 7:3 proportion. In this example one can observe that proportions are multiplied: *dupla porporcionis epitritae* would be equal to the mathematical operation $2:1 \times 4:3 = 8:3$.

¹⁵⁶ This has already been remarked upon by Stossel, “Looking Back Over the ‘Missa L’Ardant desir’,” 334 n. 53.

However, as shown in Chapter 3.1, the stacked Arabic numerals $\frac{3}{2}$ are the much more common sign to indicate this proportion when stacked Arabic numerals are used.

Most noteworthy of the treatise in my view, however, is the idea of *sesquialtera* (3:2) proportion as half *tripla* (3:1) proportion. The text clearly states *subdupla proporcionis triplae*, i.e. $1:2 \times 3:1 = 3:2$. This unconventional approach of deriving *sesquialtera* (3:2) proportion is unique of all the treatises discussed within this chapter. While the step from *sesquitercia* (4:3) proportion to *dupla superbipartiens tertias* (8:3) proportion by diminution of the first is easily comprehensible, the idea of slowing down *tripla* (3:1) proportion to arrive at *sesquialtera* (3:2) proportion seems outlandish, especially since *sesquialtera* (3:2) is much more common than *tripla* (3:1) proportion. Hence, the more obvious direction of thought would be to read *tripla* (3:1) proportion as *sesquialtera* (3:2) proportion in diminution, perhaps indicated by the stacked Arabic numerals $\frac{3}{2}$ with a stroke drawn through them.¹⁵⁷

In summary, the second proportion sign treatise *Proportio est duorum terminorum vel duorum numerorum* of the *Tractatus de musica plana et mensurabili* does not seem to be as suitable for teaching purposes as the first proportion sign treatise, where proportion signs are concerned. While the first treatise is clearly structured and contains detailed explanations of the eight different rhythmic proportions, we can observe a peculiar shift from proportions forming intervals to rhythmic proportion in the second treatise. This shift coincides with the change of hands in the text, which only explains two proportion signs.

While, as mentioned above, the total number of eight proportions in the second treatise is reminiscent of the previous treatise *Sequitur hic aliqua declaratio atque denominatio*, the author of the additions in second treatise does not seem to have taken the first as a model for his addenda, or otherwise he would have stated that the reversed cut semicircle $\ominus$ indicates *superbipartiens tertias* (5:3) proportion, as asserted by the author of *Sequitur hic aliqua declaratio atque denominatio*. Did the author of the additions in the second treatise intend to correct the teachings of the first treatise by giving an alternative interpretation of $\ominus$ and an alternative sign for *sesquialtera* (3:2) proportion? If so, why did he not comment on the previous folio?

The circumstances of at least four different scribes being involved in the copying of these two last treatises on fols. 40r–41r of the *Tractatus de musica plana et mensurabili* leave much room for speculation as to when and in which order these two treatises entered the manuscript. As described above, space seems to have been left out, which was later filled with other material

¹⁵⁷ Note that this is done by Georgius Erber who gives $\ominus$ as sign for *tripla* (3:1) proportion, which is his sign for *sesquialtera* (3:2) proportion ($\odot$) in diminution (indicated by the cut through the sign). See Chap. 5.3.2.2 below.

by Hand C.¹⁵⁸ Fol. 36v might be a misplaced folio from Anonymous XII's *Tractatus de musica* of 1460–71.¹⁵⁹ The fact that the cut semicircle $\ominus$ is not described as a sign of diminution in *Sequitur hic aliqua declaratio atque denominatio* seems to accord with early *Ars subtilior* conventions, the application of proportions to notes of all values and not only the minim, on the other hand, points towards a later date of origin of that treatise, not to mention the striking similarity to the 1483 anonymous treatise *Ars et practica cantus figurativi*. That $\ominus$ is discussed as diminution sign in the second treatise *Proportio est duorum terminorum vel duorum numerorum* seems to be an indicator for it being written after *Sequitur hic aliqua declaratio atque denominatio*. What seems to be an inevitable conclusion is that the *Tractatus de musica plana et mensurabili* is not the work of one anonymous author (Anonymous XI) but several different individuals. Certainly, it is advisable to not refer to it as one treatise, as has been done in the past.¹⁶⁰ Further research on the compilation promises to unearth further insights into the transmission of teachings found therein but is beyond the thematic scope of this study.

¹⁵⁸ See p. 245 above.

¹⁵⁹ Cf. Wingell, "Anonymous XI," 409.

¹⁶⁰ See, for example, Busse Berger, *Mensuration and Proportion Signs*, 167.

5.3.1.4 A Mid-Fifteenth-Century Hebrew Treatise: *Student's notes of lectures on music*

The treatise labelled *Student's notes of lectures on music*¹⁶¹ definitely constitutes the most unusual source of this chapter. It is written in Hebrew and seems to be comprised of a student's notes on a lecture held in Provençal in the south of France, possibly Avignon.¹⁶² The text is written on paper, whose watermark suggests that it was composed or copied c.1450.¹⁶³

There are two reasons for assuming an earlier date of origin: (1) The text mentions Jean Vaillant as one of several teachers in Paris ("great and wise teachers of the city of Paris [...] Maître Jean Vaillant"¹⁶⁴), who is assumed to have lived in the fourteenth century.¹⁶⁵ (2) The minim is the smallest note value which is mentioned.¹⁶⁶ On the other hand, the notes or short musical examples interspersed throughout the treatise are in white notation, suggesting a date more towards the middle of the fifteenth century.¹⁶⁷ Furthermore, since the treatise describes cut signs as indicating diminution, there is strong reason to believe that the lectures were held closer to the date of c.1450 and not in the late fourteenth century as suggested by Don Harrán.¹⁶⁸

The fact that Vaillant's name is mentioned five times throughout the treatise has led scholars to hypothesise that the author was his student. Adler has concluded: "The reverential form and context in which Vaillant's name is always quoted in our text seems to indicate that the anonymous author had been a disciple of Jean Vaillant in Paris."¹⁶⁹ Given the assumption

¹⁶¹ Adler, *Hebrew Writings*, 55. The treatise does not have a Hebrew title as it commences with a text fragment on fol. 1a, which Adler has described as "[f]ragment of the end (?) of a lecture which was probably devoted to the permissible (melodic) intervals." *Hebrew Writings*, 56.

¹⁶² Cf. Adler, *Hebrew Writings*, 55–57.

¹⁶³ Cf. Adler, *Hebrew Writings*, 57.

¹⁶⁴ Anonymous, [*Student's notes of lectures on music*], chap. 4; translation taken from Stone, "The Ars subtilior in Paris," 398.

¹⁶⁵ The identity of Jean Vaillant remains uncertain. An excellent summary of scholars' findings and positions can be found in Stone, "The Ars subtilior in Paris," 386 n. 28. Vaillant is also the composer of a handful of songs transmitted in *Ch*, but these do not contain proportion signs. His composition *Dame, doucement/Doulz amis* in *Ch* is dated "Paris 1369". An authority with the name Jean Vaillant is mentioned in the treatise *Les règles de la seconde rhétorique* (1411–32) as a "master who ran a school of music in Paris" ("maistre [...] le quel tenoit a Paris escolle de musique"; edition taken from Langlois, *Recueil d'arts de seconde rhétorique*, 13; my translation) and this is most probably the Jean Vaillant the Hebrew treatise is referring to. It is not true that Vaillant is "named immediately after Machaut" in *Les règles de la seconde rhétorique*, as reported by Ursula Günther (cf. "Vaillant [Vayllant], Jehan [Johannes]," in *NG2*, 26:199). There are at least three other persons named in between: Jehan Lissans Draps, Jaquemart Le Cuvelier, and Hanequin d'Odenarde. Cf. Langlois, *Recueil d'arts de seconde rhétorique*, 13. Since the treatise *Les règles de la seconde rhétorique* mentions Vaillant in the past tense, the year 1432 constitutes a *terminus ante quem* of 1432 for his death.

¹⁶⁶ Cf. Adler, *Hebrew Writings*, 58.

¹⁶⁷ Cf. fols. 2b, 3a, 4a, and 4b. It is true that the music transmitted on fols 14a, 14b, and 15a is in black square notation, but—as Adler has acknowledged—these "notational exercises do not seem to be in direct relationship with the text of f. 1a–4b [= the Hebrew treatise]." Adler, *Hebrew Writings*, 57.

¹⁶⁸ Cf. Don Harrán, *Three Early Modern Hebrew Scholars on the Mysteries of Song*, Studies in Jewish History and Culture 47 (Leiden: Brill, 2015), 15. I do not agree with this suggestion for the reasons given above.

¹⁶⁹ Adler, *Hebrew Writings*, 57. This opinion has been seized by Ursula Günther. Cf. "Vaillant [Vayllant], Jehan [Johannes]," 199.

that the student was listening to a lecture in the south of France, however, it seems more likely in my opinion that the person *giving* that lecture had been a student of Vaillant in Paris and moved south afterwards. That this anonymous teacher was one or even more steps removed from Vaillant is, of course, also conceivable given the assumed date of c.1450.¹⁷⁰ In any case, the “great and wise teachers of the city of Paris”—among them Jean Vaillant—seem to have been regarded as knowledgeable authorities by both teacher and student.

The quality of the text is rather poor. As Anne Stone has put it: “The text is somewhat corrupt, and if these are student notes, we might speculate that the student was not terribly attentive.”¹⁷¹ Judging by the layout on the page and the organisation of the text, the treatise might actually be comprised of the original notes and not copies thereof, which could probably be expected to be more structured. Although the text has been subdivided into seven chapters by Adler, there are sudden changes of subject and, as I will explain below, a missing proportion sign, which hint towards the author not keeping pace with the lecture he was attending.¹⁷² Another possible scenario is that the treatise reflects on a poorly structured lecture.

For example, consonances are first dealt with in the second chapter of the treatise. After an explanation of elementary rules of counterpoint in chapter 3 and proportions in chapter 4, the text returns to consonances—which had been the subject of chapter 2—in chapter 5. Another example indicating that these are notes from an actual lecture in which a teacher might have jumped back and forth or the student did not keep pace with the lecture is the description of the terms for specific proportions discussed (especially in the context of intervals):

“It is true that music has 8 ratios and song has 8 tones [...] That is: 3:2, 4:3, 5:4, 6:5, 7:6, 8:7, 9:8. The ratio *sesquiertia* for the first one, then *sesquiquarta* [...] or hemiola for 3:2 or *epogdoi* for the fourth one or *epitrita* for the 5th one, *se-epitrita* for the 6th one, toicut for the 7th one [ed: 3:2], then *sesquioctava* for the 8th one and *semiepitrita* for the 9th one when *sesquiertia* is not present.”¹⁷³

The Latin and Greek terms do not fit the order in which the proportions are written down as ratios. First, there are only seven proportions given as ratios, as *dupla* (2:1) for the octave is missing from the list, but nine terms according to the description of the terms. However, the order according to the terms listed is completely different and some proportions are left out while others are described twice: The order is the following: (1) 4:3 [“*sesquiertia* for the first one”], (2) 5:4 [“*sesquiquarta*”], (3) 3:2 [“*hemiola*”], (4) 9:8 [“*epogdoi* for the fourth one”], (5)

¹⁷⁰ However, Anne Stone has noted: “Throughout the text the writer speaks of Vaillant as though he is living, rather than as a distant, long-dead authority, suggesting a relatively straight line of pedagogical ancestry.” “The Ars subtilior in Paris,” 386.

¹⁷¹ Stone, “The Ars subtilior in Paris,” 386.

¹⁷² For an overview of contents of the seven different chapters see Adler, *Hebrew Writings*, 56–57.

¹⁷³ Anonymous, [*Student’s notes of lectures on music*], chap. 4; translation taken from Stone, “The Ars subtilior in Paris,” 398.

4:3 [again (cf. no. 1), now in Greek: “*epitrita* for the 5th one”], (6) ?? [“*se-epitrita* for the 6th one”], (7) 3:2? [again (cf. no. 3) “*toicut*¹⁷⁴ for the 7th one”], (8) 9:8 [again (cf. no. 4), now in Latin: “*sesquioctava* for the 8th one”], (9) ?? [“*semiepitra* for the 9th one when *sesquitertia* is not present”]. This passage is exemplary for many rather confusing sections of the treatise.

Proportions are not only discussed in the context of intervals, however, but also in the context of musical notation, including proportion signs. The descriptions of proportion signs are not really grouped together and are located in the chapters 4, 5, and 6 according to Adler’s subdivision. Discussions of intervals and the mensurations can be found interspersed throughout. The treatise contains three displayed proportion signs (4, $\odot$, and $\ominus$), two signs which are described in words but not displayed ($\mathcal{C}$ and $\emptyset$), and an empty space which presumably should have contained a proportion sign indicating *trippla* (3:1) proportion (see FIGURE 5.10).

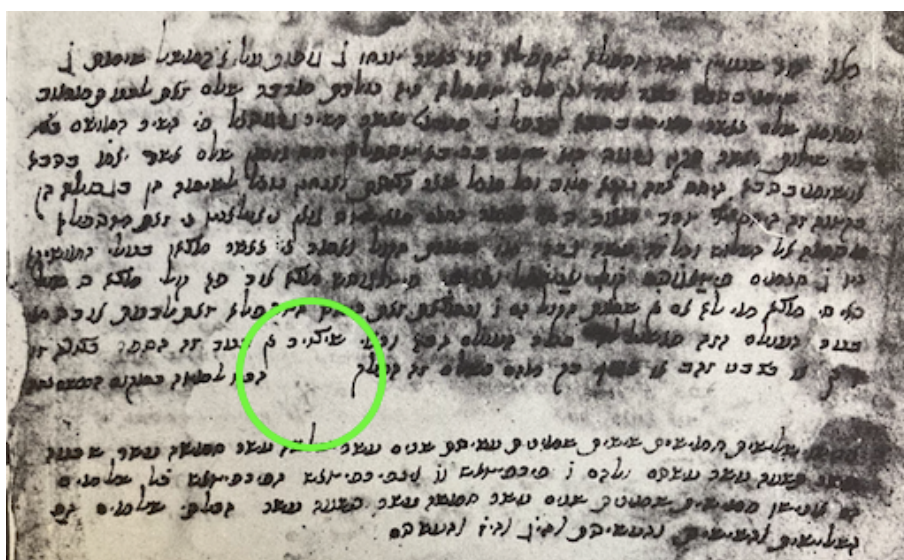


FIGURE 5.10: *I-Fn Magliab. III 70, fol. 2b, excerpt (empty space (line 15) for presumed missing proportion sign encircled)*

The author starts by discussing *dupla* (2:1) proportion in the fourth chapter. According to the text, the proportion can be indicated by “a half circle cut in half [$\mathcal{C}$] or a full circle broken [$\emptyset$].”¹⁷⁵ It is noteworthy that the signs are described but not displayed, as this is also the case in many verbal canons accompanying music containing proportion signs.¹⁷⁶ He continues with the discussion of *trippla* (3:1) proportion, which “should be delineated in the following way:

¹⁷⁴ I have to rely on Stone’s interpretation of the term as editor since I do not know which language it is.

¹⁷⁵ Anonymous, [*Student’s notes of lectures on music*], chap. 4; translation taken from Stone, “The Ars subtilior in Paris,” 398.

¹⁷⁶ See Chap. 3.2.2 for details.

[empty space (see FIGURE 5.10)].”¹⁷⁷ Since there is space left out in the manuscript in that exact position, I can only assume that the author intended to insert a sign there but maybe the lecturer had already moved on to a different subject and/or the author eventually forgot to write the sign down. The size of the space is comparable to the space for the proportion sign 4 for *quadrupla* (4:1) proportion in the sixth chapter two folios later (see FIGURE 5.11).

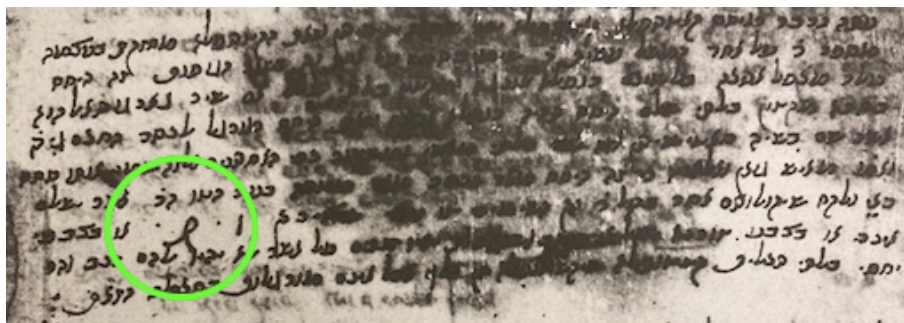


FIGURE 5.11: *I-Fn Magliab. III 70, fol. 3b, excerpt*
(proportion sign 4 (line 12) encircled)

It is most noteworthy that the treatise not only contains instructions on the notation of proportion signs but also on coloration. According to the author, *dupla* (2:1) proportion can be indicated “in azure color, or if all the notes are black for major prolation, yellow and green would be used for minor prolation”; *tripla* (3:1) proportion should be written “in a gold color”; and *quadrupla* (4:1) proportion may be displayed by using “indy color or tornasole or [also] gold”¹⁷⁸ Even if there is precedent of blue coloration—indeed indicating *dupla* (2:1) proportion—in the English *Old Hall* manuscript (see FIGURE 5.12), these instructions sound a bit too phantastic to have a basis in actual music notation, especially the costly gold colour.¹⁷⁹

The explanation of *tripla* (3:1) proportion also contains the remark that notes may be written “in a thinner note shape”¹⁸⁰ in order to indicate the proportion, but I have not encountered that practice in *Ars subtilior* music sources, at least if one interprets “thinner” to mean smaller. It is true that notes are sometimes written closer together if their values are small,

¹⁷⁷ Anonymous, [*Student's notes of lectures on music*], chap. 4; translation taken from Stone, “The *Ars subtilior* in Paris,” 399.

¹⁷⁸ Anonymous, [*Student's notes of lectures on music*], chap. 4 and 6; translation taken from Stone, “The *Ars subtilior* in Paris,” 398–99.

¹⁷⁹ In addition to the example of FIGURE 5.12 (*Gloria* by Leonel Power), blue ink colour can also be found in an anonymous *Credo* on fol. 62v of the *Old Hall* manuscript. Cf. Stone, “The *Ars subtilior* in Paris,” 387. According to Margaret Bent, these “are the only extant practical uses of colours other than black and red.” “Principles of Mensuration and Coloration: Virtuosity and Anomalies in the Old Hall Manuscript,” in *La notazione della polifonia vocale dei secoli IX-XVII. Antologia, Parte seconda: secoli XIV-XVII*, ed. Antonio Delfino and Francesco Saggio (Pisa: Edizioni ETS, 2022), 91.

¹⁸⁰ Anonymous, [*Student's notes of lectures on music*], chap. 4; translation taken from Stone, “The *Ars subtilior* in Paris,” 399.

but I gather that this is not what the author means by “thinner”. It is possible that the author means void notation, as these notes would have had to be written with a sharper writing tool and might therefore have been thinner compared to full ink notes.



FIGURE 5.12: *Old Hall*, fol. 17v, excerpt showing notes in blue ink

At least the suggestions of extravagant ink colours makes it seem as if this treatise does not truly reflect on musical practice of the time. But it does not seem to have a quadrivial background either because one can also observe a strange interpretation of some Latin terms, which would probably not have happened if the author or teacher had been trained at a university. For example, the term ‘*sesquiquarta*’ is explained in numbers by 6:4 (i.e. 3:2), but it naturally is 5:4.¹⁸¹ I can see how a French speaking person might derive 6:4 from ‘*sesquiquarta*’ as the French ‘*six*’ (6) and ‘*quatre*’ (4) are phonetically very close to the ‘*ses*’ and ‘*quarta*’ of the Latin term. Another example is the interpretation of ‘*dupla sesquiquarta*’ as 8:6 (i.e. 4:3), which is also not correct, as *dupla sesquiquarta* is 9:4.¹⁸² Again, I can only assume that the author did not know better than to double the number ‘*quatre*’ to make eight and also put ‘*six*’ in the mix. This confusion of Latin terms might explain why the sign Θ is said to indicate ‘*dupla sesquioctava*’, which is 17:8.¹⁸³ That this is an unusual proportion without precedent in musical notation has already been remarked upon by Busse Berger, but she has not drawn the conclusion that the author simply used a false term.¹⁸⁴ However,

¹⁸¹ Cf. Anonymous, [*Student’s notes of lectures on music*], chap. 6, sentence 17. I thank Raphael Isaac Landzbaum for translating this passage for me, since it is not translated in Stone, “The *Ars subtilior* in Paris.” This particular erroneous interpretation has already been reported by Busse Berger, *Mensuration and Proportion Signs*, 167 n. 12. See Chap. 1., pp. 20–21 for an explanation of proportions’ Latin terms.

¹⁸² Cf. Anonymous, [*Student’s notes of lectures on music*], chap. 4, sentence 12. Again, my gratitude to Raphael Isaac Landzbaum for the translation.

¹⁸³ Cf. Anonymous, [*Student’s notes of lectures on music*], chap. 4, sentence 28.

¹⁸⁴ “The author described 5:4 (but the subsequent explanations make clear that he meant 6:4) and the very unusual 17:8, *dupla sesquioctava*, indicated by Θ .” Busse Berger, *Mensuration and Proportion Signs*, 167 n. 12. I am surprised that Busse Berger has taken the author’s word for the proportion, as she had observed that his description of *sesquiquarta* is also wrong.

considering the other two misreadings of terms, it seems highly likely that the author meant a different proportion instead of 17:8. Analogous to the interpretation of *dupla sesquiquarta* as 8:6 as described above, he might actually have meant 16:6 (i.e. 8:3). Still, the description of a standardised mensuration sign as proportion sign also seems a bit peculiar, hence I am uncertain what to make of this passage of the treatise.

While the ink colours make it seem as if the treatise is not connected to notational practice where rhythmic proportion is concerned, the proportion signs 4, $\mathcal{C}$ and $\emptyset$ do appear in music manuscripts. Thus a connection to actual notational practice cannot be denied. Furthermore, the treatise does contain a reference to an actual composition, namely the ballade *Le sault perilleux* by Galiot from *Ch*. The description of this piece is located in the fourth chapter, succeeding a paragraph on the notation of prolations. According to the treatise, the sign $\ominus$ indicates *sesquioctava* (9:8) proportion in that composition, which is, however, not correct according to the only surviving source (*Ch*).¹⁸⁵ $\ominus$ does appear in this ballade, but it indicates *sesquialtera* (3:2) proportion at the breve level, while it is $\oplus$ that indicates *sesquioctava* (9:8) proportion at the semibreve level in this piece. This connection to real compositions of the time puts the treatise into a different light. It might not be as far removed from musical practice as the analysis has suggested so far.

While it is not unusual that theoretical treatises mention actual compositions, it is singular in the case of proportion signs among all the treatises examined for this study. The Hebrew treatise thus is a peculiar mixture of seemingly pure fiction (e.g. gold ink for proportional rhythms) and actual music. Treatises with more foundation in musical practice will be examined in the next subchapter.

¹⁸⁵ Cf. Stone, “The Ars subtilior in Paris,” 388.

5.3.2 Practice-oriented Treatises with Musical Examples

A handful of treatises analysed for this study tend to be more oriented towards notational practice by including musical examples demonstrating the use of proportion signs. Most of these treatises also offer more than one sign for a particular proportion, which matches notational practice where a large variety of signs are used for the same proportion(s), as for example demonstrated in Chapter 2 on *Mod A*. I consider five of the altogether fourteen different treatises as belonging to this more practice-oriented category: the anonymous treatises *Incipiunt regule proporcionum in quantum pertinent ad musica*¹⁸⁶, *Proportio est duarum rerum*, *Iste sunt proportiones*, and *Venerabiles domini mei*, as well as Georgius Erber's *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente*. This last treatise stands out because it was written in medieval French, with the exception of its Latin title. It is also a French source in a manuscript most likely compiled in Paris. *Proportio est duarum rerum* is contained in a mid-fifteenth-century English source, while the other three treatises most probably originated in Italy.

I will not discuss *Incipiunt regule proporcionum* in detail as this treatise only mentions the stacked Arabic numerals $\frac{4}{1}$ as “most intelligible sign” (“signum magis intelligibile”¹⁸⁷) to indicate *quadrupla* (4:1) proportion.¹⁸⁸ It also includes an incomplete musical example showing $\emptyset$ probably intended to demonstrate *dupla* (2:1) proportion, as it says “dyapason” and “proportio binaria” underneath the example (see FIGURE 5.13). This, however, is by no means certain.

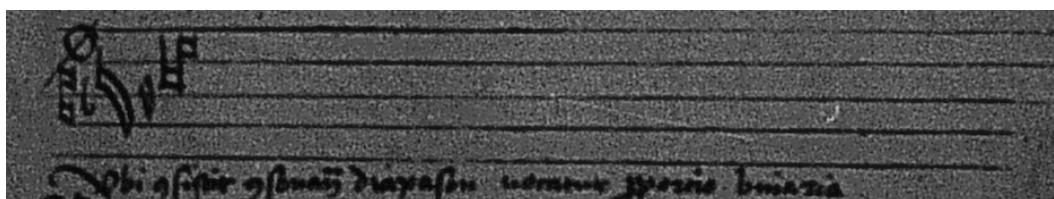


FIGURE 5.13: Incomplete musical example in *Incipiunt regule proporcionum*

While all four other treatises contain musical examples, I will discuss these at length only for the treatise *Proportio est duarum rerum*. This discussion shall be exemplary for all the musical examples in this subchapter. It is noteworthy that only *Iste sunt proportiones* contains examples in which the proportion is applied within one voice part, i.e. the part begins in a

¹⁸⁶ Note that this treatise is called *Regule proportionum* by Busse Berger and Gallo.

¹⁸⁷ Venice, Biblioteca Nazionale Marciana, Lat. VIII.85 (3579), margin of fol. 69r.

¹⁸⁸ Cf. Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 347.

certain mensuration to which a proportion is then applied in a linear manner. All three other treatises compare a steady tenor against a proportioned upper voice. In *Proportio est duarum rerum* and Georgius Erber's *Sequuntur proportiones* this is done in a very clear form of visual organisation using different staves for each voice (Erber's treatise even contains lines resembling modern bar lines to separate the rhythmic units). In *Venerabiles domini mei*, the very short musical examples are written one after another on the same staff. All three treatises have in common that certain proportion signs only appear in the musical examples but not in the text. Moreover, what *Proportio est duarum rerum* and *Venerabiles domini mei* have in common is that there are some major mistakes in the musical examples. The need for alteration is ignored in two incidents in *Proportio est duarum rerum*, and in *Venerabiles domini mei*, some examples simply do not add up. In contrast to that, the musical examples in Erber's *Sequuntur proportiones* and in *Iste sunt proportiones* work out well.

The four treatises also differ in the amount and variety of proportion signs given. While *Iste sunt proportiones* only gives stacked Arabic numerals, the other three contain different forms of visual appearance: geometric signs and (stacked) Arabic numerals. And while *Proportio est duarum rerum* only shows three proportion signs in its musical examples, Georgius Erber's *Sequuntur proportiones* gives altogether thirteen different signs. The other two average at seven proportion signs in *Iste sunt proportiones* and ten in *Venerabiles domini mei*. This last treatise will also be discussed last as it is the only one of all treatises discussed in this chapter which gives the same sign for different proportions. While I have shown that it is not unusual for several proportion signs being given for one particular proportion, it is an exception that the same sign is said to indicate different proportions. In this aspect, *Venerabiles domini mei* is perhaps as close to notational practice as any of the other treatises, because in *Ars subtilior* music this happens very frequently, especially with single Arabic numerals.

As laid out, there are many different aspects under which these four treatises can be examined. I attempt to do so in the following four subchapters examining the more practice-oriented treatises with musical examples.

5.3.2.1 The Treatise *Proportio est duarum rerum*

The treatise *Proportio est duarum rerum* is the last treatise (no. 20) of the manuscript *London, British Library, MS Lansdowne 763*, which altogether contains 20 treatises on music, including writings by Guido of Arezzo and Leonel Power. The manuscript was copied c.1450 by John Wyldem, a preceptor of the Waltham Abbey of the Holy Cross.¹⁸⁹

The treatise *Proportio est duarum rerum* bears a remarkable connection to the three previous treatises (nos. 17, 18, and 19), which are in English and which are usually perceived as one unit with the title *Her beginneth tretises diverse of musical proporcions*.¹⁹⁰ This connection manifests itself in an announcement of a Latin text following the English texts in the manuscript itself. The first treatise of the collection of three (no. 17) begins with “Here beginneth Tretises diuerse of musical proporcions. And of their naturis. and denominacions. Ferst in englissh. and than in latyne”¹⁹¹. This means that *Proportio est duarum rerum* (no. 20) is not just another treatise added at the end of a manuscript but that its copying was already planned when treatise no. 17 was entered. Hence, one could also argue that there is a collection of four proportion treatises (nos. 17–20), which should be considered as one unit despite their difference in language.

I am emphasising this connection, because there is an attribution in the closing of treatise no. 19, which reads: “But this sufficith for knowleche of Proporcions. Secundum. Chilston.”¹⁹² If these four last treatises are one unit, in how far does this attribution extend to the last Latin treatise? Naturally, the attribution could mean one of two things: (1) Chilston could either be the author of the English treatises or (2) the author of the treatises followed Chilston’s teachings. Chilston remains hitherto unidentified, not least because of the lack of his Christian name.¹⁹³ But if the last treatise is regarded as some sort of translation of the preceding teachings by Chilston (“Ferst in englissh. and than in latyne”), the person might also have been the author or originator of the teachings transmitted in this last treatise under discussion here.

¹⁸⁹ Cf. Dumitrescu, *The Early Tudor Court*, 178–79. Note that the London British Library Catalogue gives c.1460 as copying date.

¹⁹⁰ See n. 54 above. The titles of treatises are taken from *RISM B III*. Note that there are discrepancies between the incipits given as titles according to *RISM* and the incipits given in the editions on TME.

¹⁹¹ *London, British Library, MS Lansdowne 763*, fol. 117r. Transcription taken from Peter M. Lefferts’ edition on TME (DEPRPA1B_MLBLL763).

¹⁹² *London, British Library, MS Lansdowne 763*, fol. 122v. Transcription taken from Peter M. Lefferts’ edition on TME (DEPRPC1B_MLBLL763).

¹⁹³ In a letter to the editor published in the *Musical Times* in 1927, W. H. Grattan Flood suggested that Chilston might be John Chelston, author of a treatise on Psalms in a 1446 manuscript given as *British Museum, Royal MS 5 A vi*. Cf. “Who Was Chilston?,” *Musical Times* 68, no. 10 (1927): 933. Unfortunately, I have been unable to identify the manuscript in question and hence could not investigate this lead further.

On the other hand, the compiler of the collection of proportion treatises does not truly abide by his promise in relying the same teachings of proportions first in English then in Latin. While the English texts are transmitted on altogether twelve pages of the manuscript, the Latin treatise only comprises three pages (fols. 123r–124r). And while the English treatises deal with the five types of proportions of major inequality, the denominations of particular proportions, and the differences between geometric, arithmetic, and harmonic proportion in all detail, the Latin treatise does not contain more than a one-and-a-half-page summary of the different types of major inequality and denominations of very few particular proportions. The differences between geometric, arithmetic, and harmonic proportion are not explained.

Thus, the content of the Latin treatise *Proportio est duarum rerum* is remarkably different from those of the English treatises. The Latin treatise is certainly not the Latin equivalent of the English texts preceding it, as proclaimed at the beginning of the collection of the four proportion treatises. Yet, the differences also go the other way: the Latin text contains a description of rhythmic proportion—including six two-voice musical examples (see FIGURE 5.14)—, which the English texts do not contain.

These six musical examples are introduced by the phrase: “The proportions, through their pronounciation, pertain to *musica practica*, namely to singers. There are six known types thereof: *dupla*, *tripla*, *quadrupla*, *sesquialtera*, *sesquitercia*, *sesquioctava*. Following are examples of all of these.”¹⁹⁴ Although the late copying date of c.1450 would suggest that this treatise does not refer to the *Ars subtilior* movement, the fact that the examples are written in full black mensural notation (the meaning of red ink will be explained presently) might point towards a much earlier time of origin of the text.

The six musical examples demonstrate *dupla* (2:1), *tripla* (3:1), *quadrupla* (4:1), *sesquialtera* (3:2), *sesquitercia* (4:3), and *dupla sesquiquarta* (9:4) proportion, although—as will be discussed below—the text next to this last example reads “*proportio sesquioctava*”, referring to 9:8 proportion. Each musical example is comprised of two voices: a stable tenor and an upper voice with proportioned note values. The red ink of the upper voice does not have any meaning of its own (e.g. indicating *sesquialtera* (3:2) proportion), but only indicates the duration of the proportion. Note that the last *longa* in each example is written in black ink to match up with the tenor. At first glance, this seems to be a deviation from *Ars subtilior* practice, in which red notation almost always indicates *sesquialtera* (3:2) proportion. However—as

¹⁹⁴ “Proporciones pronounciatione pertinent ad Musicam practicam scilicet ad Cantores. quarum sex sunt species scilicet Dupla. Tripla. Quatrupla. Sesquealtera Sesquetercia Sesqueoctaua. Que omnes exemplarie sequuntur.” Anonymous (Chilston?), *Proportio est duarum rerum*, fol. 123v; transcription taken from Peter M. Lefferts’ edition on TML (ANODUA_MLBLL763); my translation.

shown in Chapter 2—there are examples in which red notation was used in other capacities, e.g. indicating the duration of a quotation.



FIGURE 5.14: Musical examples (fols. 123r–124v) in the treatise *Proportio est duarum rerum*

Only three of the musical examples actually contain proportion signs: the Arabic numeral 2 is displayed in the example demonstrating *dupla* (2:1) proportion, the Arabic numeral 4 is included in the *quadrupla* (4:1) proportion example, and the reversed semicircle $\circ$ can be found in the example demonstrating *sesquitercia* (4:3) proportion. Four mensuration signs (C, C, O, and O) are also featured in some of the examples, although their meaning is not entirely clear, as will be explained below. Since the musical examples exhibit varying degrees of complexity, I will not discuss them chronologically.

The simplest example is the one demonstrating *quadrupla* (4:1) proportion, which can be found on the bottom staff of fol. 123r (see FIGURE 5.15). It is the only example, which is comprised of only one staff. All other examples are written in quasi-score format, in which the tenor notes are (at least roughly) vertically aligned with the notes simultaneously sounding in the proportioned upper voice. In this example, three minims in [3,3] (indicated by Θ) in the tenor are set against twelve minims in the cantus. The cantus contains the Arabic numeral 4 at the beginning of the staff (encircled in blue in the illustration). The numerical examples given in the text accompanying the music are 4 to 1 and 16 to 4.

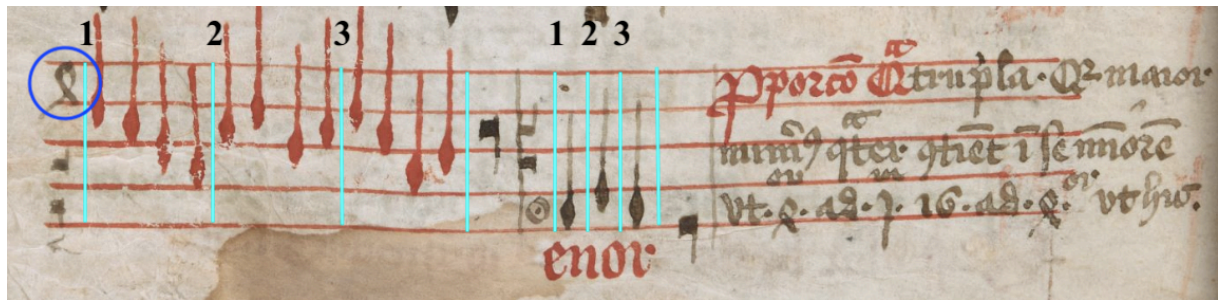


FIGURE 5.15: Musical example demonstrating *quadrupla* (4:1) proportion; minim units (according to tenor) marked and numbered; proportion sign encircled

The musical example demonstrating *dupla sesquiquarta* (9:4) proportion is also quite straightforward (see FIGURE 5.16). The tenor is in [2,2] indicated by C, thus a breve contains four minims. The four semibreves (= two breve units) in the tenor add up to eight minims in the tenor. The cantus has to be in a different mensuration since it is impossible to fit nine minims into a mensuration with minor prolation. Major prolation is also indicated intrinsically by a dot of division (encircled in green in the illustration). The mensuration of the cantus seems to be [3,3]. Each breve unit in the cantus thus contains nine minims, so the two breve units in the example add up to 18 minims in the cantus. 18:8 equals 9:4. As remarked above, the text to the right of the example states, that a different proportion is demonstrated in the music, namely *sesquioctava* (9:8) proportion, referred to as 9 to 8 in the numerical example. This proportion could have been made possible by eliminating every other semibreve in the tenor. As we shall see, this is only the first of several small errors and inconsistencies to be found in this treatise.

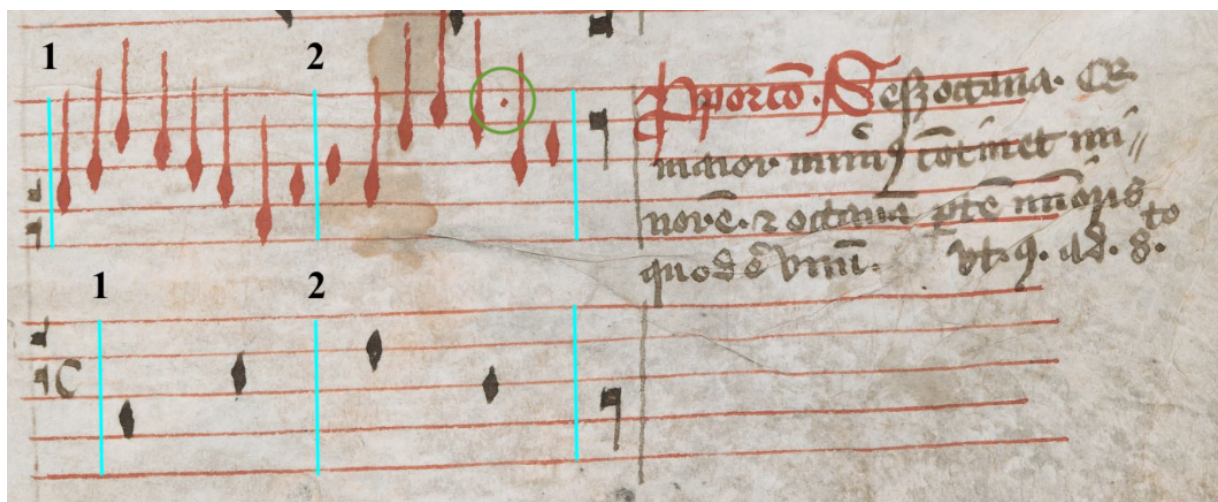


FIGURE 5.16: Musical example demonstrating *dupla sesquiquarta* (9:4) proportion; breve units marked and numbered; dot of division encircled

The musical example demonstrating *tripla* (3:1) proportion is also rather simple (see FIGURE 5.17). The only difficulty here is finding the correct value of the two void minims in the tenor. It is easily discernible that these have to be semiminims, since there are six units à three minims in the cantus, therefore, the tenor must contain altogether six minims to form *tripla* (3:1) proportion. Moreover, six minims make up a *mensura* of [2,3], which is the mensuration indicated by C in the tenor. No Arabic numeral, e.g. 3, or other proportion sign is given in this example. The sign at the beginning of the staff in the cantus is a flat sign. The numerical examples given in the text accompanying the music are 3 to 1 and 9 to 3.

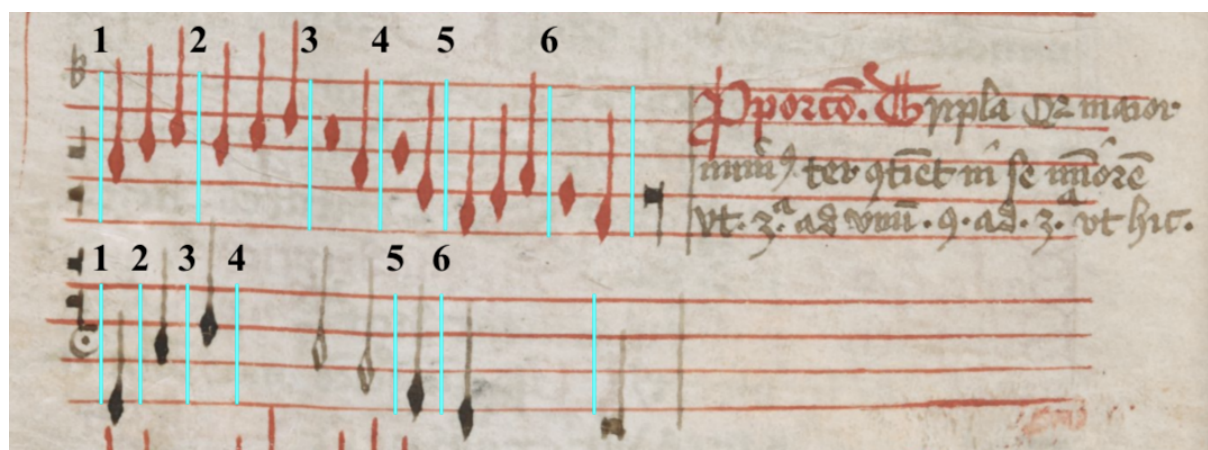


FIGURE 5.17: Musical example demonstrating *tripla* (3:1) proportion; minim units (according to tenor) marked and numbered

The tenor of the musical example demonstrating *sesquiertia* (4:3) proportion (see FIGURE 5.18) oddly has four different mensuration signs marked at the beginning of the staff (O, O, C, and C). What to make of these four different options? The cantus can easily be divided into five units à four minims, adding up to 20 minims altogether. Since the proportion has to be *sesquiertia* (4:3) proportion, 15 minims distributed onto the five semibreves are required in the tenor. That means that only major prolation is an option. Furthermore, the tempus cannot be perfect, since the second semibreve (in the first ligature) would have to be altered in this case, adding up to altogether six semibreves in the tenor. However, only five semibreves are needed, hence [2,3] indicated by C is the only possible solution here. I assume that the author intended the musical examples to include some aspects of a riddle, since the remaining examples to be discussed here also contain mensuration signs, which can be excluded on logical grounds. Still, it seems odd that no even number of semibreves, e.g. 4 or 6, was used in the tenor, thus leaving an incomplete *mensura*.

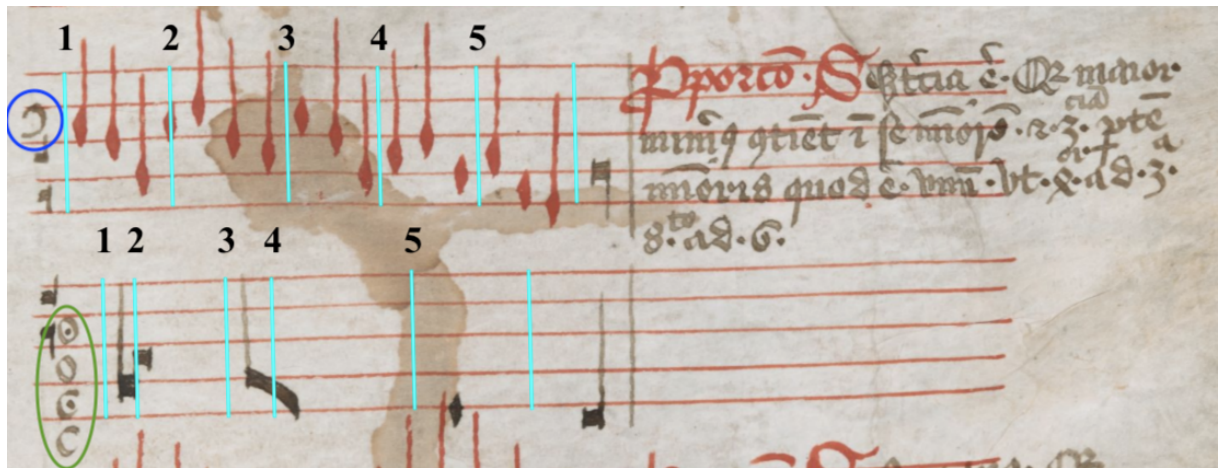


FIGURE 5.18: Musical example demonstrating *sesquitercia* (4:3) proportion; semibreve units (according to tenor) marked and numbered; proportion sign encircled (blue); mensuration signs encircled (green)

An incomplete *mensura* can also be found in the tenor of the musical example demonstrating *sesquialtera* (3:2) proportion (see FIGURE 5.19). This example is the most complex of the treatise, since it also contains void notation at the beginning of the cantus. It is easiest to work this example out by reading the upper voice backwards. Reading from the final *longa*, one can find four perfect semibreve units à three minims each (units 5–8). Major prolation is clearly indicated intrinsically by a dot of division (encircled in green in the illustration). The other dot in the cantus (encircled in orange) is a dot of addition, hence the dotted minim and the semiminim (again in void notation, as in the example demonstrating *tripla* (3:1) proportion) form a unit of two minims. Working backwards to the beginning of the cantus, one ends up with two semiminims and a void semibreve in the first unit. If void notation means diminution, the void semibreve (with the value of a minim in diminution) has to be altered to form a perfection with the two void minims.

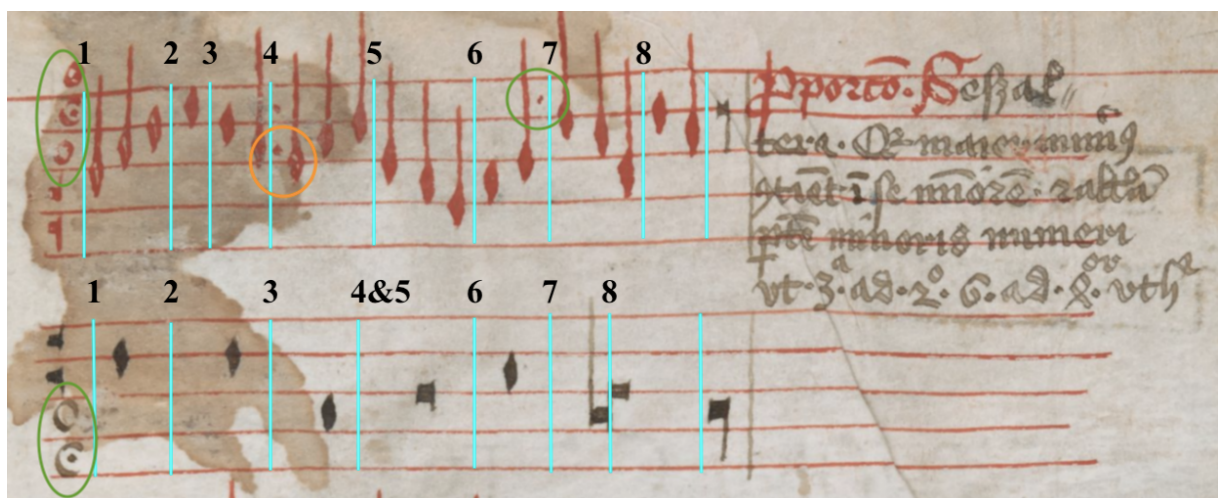


FIGURE 5.19: Musical example demonstrating *sesquialtera* (3:2) proportion; semibreve units (according to tenor) marked and numbered; mensuration signs and dot of division encircled (green); dot of addition encircled (orange)

Three mensuration signs (Θ, C, and O) are given at the beginning of the cantus, but only C works for the upper voice. This is not the case for the tenor, however, which also has two mensuration signs (O and C) given at the beginning of the staff. If the tenor would be in C, it would contain just as many minims as the cantus and hence would not demonstrate any rhythmic proportion. Minor prolation with two minims per semibreve seems to be the solution. [3,2] indicated by O is the only other option given by the mensuration signs. While this works for units 1–6, however, units 7 and 8 are a problem. In *tempus perfectum*, the second semibreve of the ligature would have to be altered. An incomplete *mensura* of only two semibreves is the only solution for the tenor. If the second semibreve was altered—following the standard rules of mensural notation—the tenor would contain altogether 18 minims (= 9 units à two minims) against 24 minims (= 8 units à three minims) in the cantus. Hence the example would demonstrate *sesquitercia* (4:3) proportion in a very unusual way. Actually, [2,2] indicated by C would have been the correct mensuration for this musical example's tenor. C is not given, however, which is another inconsistency of the treatise.

The last inconsistency appears in the musical example demonstrating *dupla* (2:1) proportion (see FIGURE 5.20). There, we find two proportion signs (encircled in blue in the illustration) in the form of the Arabic numeral 2, one in each voice. However, if rhythmic proportion is created between voices in all the examples, it makes no sense to have two proportioned voices in this example. I can only assume that the author or scribe intended to indicate by the number 2 in the tenor that these two staves belonged to the same example, namely the example indicating *dupla* (2:1) proportion. Note that 'tenor' is not written underneath this second staff and that the text does not refer to two voices. Since this is the first example, the Arabic numeral 2 in the tenor can be interpreted as hint that these examples are in quasi-score format.



FIGURE 5.20: Musical example demonstrating *dupla* (2:1) proportion; semibreve units (according to tenor) marked and numbered; proportion signs encircled (blue); dot of division in cantus encircled (green)

The tenor's mensuration is [2,3] as indicated by © and contains four perfect semibreve units. Accordingly, the cantus contains eight perfect semibreve units, equalling four imperfect breve units. The dot of division between unit 3 and 4 indicates major prolation, hence only [2,3] (©) is an option. Although [3,2] (O) would also produce six minims per unit, the dot between unit 3 and 4 could only be interpreted as dot of addition in minor prolation, hence adding the value of a semiminim, which cannot be fitted into the unit.

To sum up, the treatise contains only three examples with proportion signs: 2 indicates *dupla* (2:1) proportion, 4 indicates *quadrupla* (4:1) proportion, and the reversed semicircle ⊔ indicates *sesquiertia* (4:3) proportion. These signs are in accordance with proportion signs found in the *Ars subtilior* sources discussed in this study, namely single Arabic numerals and ⊔ as most common sign for *sesquiertia* (4:3) proportion. It is not necessarily surprising not to find proportion signs in two of the other examples: *sesquialtera* (3:2) proportion is commonly indicated by coloration and *dupla sesquiquarta* (9:4) proportion is seldomly used. Following in this line of thinking, however, it is not clear why the Arabic numeral 3 is not used in the example indicating *tripla* (3:1) proportion. It is also noteworthy, that the text written on the right side of the staves does not refer to any of the signs or musical examples.

As described above, the treatise contains one major error: While the text of the treatise refers to *sesquioctava* (9:8) proportion, the musical example unmistakably demonstrates *dupla sesquiquarta* (9:4) proportion. Then there are several inconsistencies, namely an incomplete *mensura* in the tenors of two musical examples (*sesquialtera* (3:2) and *sesquiertia* (4:3) proportion). In *tempus perfectum*, the second semibreve of the respective ligature would have to be altered according to the standard rules of mensural notation. However, the number of minims in the respective proportioned cantus dictates that these rules have to be suspended in these examples. Another inconsistency is comprised of the Arabic numeral 2 appearing not only in the proportioned voice (cantus) of the musical example demonstrating *dupla* (2:1) proportion, but also in the tenor, which is not proportioned. As suggested above, this could be interpreted as reference to the visual organisation of the musical examples, i.e. that two staves belong to the same example.

And finally, there is the question of the superfluous or even erroneous mensuration signs in the examples demonstrating *dupla* (2:1), *sesquialtera* (3:2), and *sesquiertia* (4:3) proportion. Were they intended as riddles or do they rather accord with the mediocre quality of the treatise? The first interpretation would naturally match the general propensity for musical riddles typical of the *Ars subtilior* movement. However, the above-mentioned errors and inconsistencies found

in the treatise more likely point towards an author who had not truly thought through the musical examples he had invented for the treatise.

Altogether, the treatise *Proportio est duarum rerum* does not seem to be particularly suited for teaching purposes, at least when regarded individually. First, it is not very detailed regarding the explanations of the five types of proportions of major inequality, the denominations of proportions, and the differences between geometric, arithmetic, and harmonic proportion. It has to be noted though, that—as explained above—the English treatises (nos. 17–19) preceding the Latin text (no. 20) contain detailed explanations of said matter. Hence, when regarded as collection of four treatises on proportions, the shortcomings of *Proportio est duarum rerum* regarding the mathematical teachings are not grave. However, the musical examples found in the Latin treatise are not actually connected to the text. The text passages neither refer to proportion signs nor to different numbers of minims being compared in rhythmic proportion. Therefore, the reader is required to have some prior knowledge of rhythmic proportions in mensural music in order to being able to comprehend the examples.

This last aspect could naturally point towards a rather late compilation date of the treatise, maybe even much closer to the copying date of c.1450 than the full black mensural notation exhibited in the musical examples would suggest. Perhaps rhythmic proportion was already common enough, hence the examples did not necessarily require an explanation. It should be observed that rhythmic proportion between voices, i.e. a certain number of minims in the cantus set against a different number of minims in the tenor, is not particularly typical of *Ars subtilior* music. The reference point of the application usually lies within the same voice.

In short, considering the absence of proportion signs from three of the six examples as well as the strange array of mensuration signs at the beginning of three examples, the focus of this treatise does not seem to be the transmission of teachings on musical notation. Nevertheless, the musical examples allow us to catch a glimpse of notational practices in England toward the middle of the fifteenth century.

5.3.2.2 Georgius Erber's Treatise *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente*

The manuscript *Innsbruck, Universitätsbibliothek, MS 962* is a collection of several treatises on philosophy, history, grammar, literature, and music.¹⁹⁵ The 18 music treatises of the manuscript, which are located on folios 128r–164r, concern three different subjects: plainchant, proportions, and mensural notation.¹⁹⁶ The manuscript is assumed to be of Parisian origin due to a number of references to the city on several folios.¹⁹⁷ The dates that can be found on seven folios of the manuscript hint towards a date of compilation around 1460.¹⁹⁸ Folios 128r–151r of the manuscript are comprised of a collection of treatises presumably compiled by a certain Georgius Erber, who is believed to be identical with one Georius [sic] Erber de Aybling, who was enrolled at the University of Vienna in 1455 and who got a title from the University of Paris in 1463.¹⁹⁹

Georgius Erber's French treatise with the Latin title *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente* is located on folios 142r–

¹⁹⁵ Cf. Federhofer-Königs, "Ein Beitrag zur Proportionenlehre," 145.

¹⁹⁶ For a study of all music treatises contained in this manuscript see Christian Meyer, "L'enseignement de la musique à Paris au XVe siècle. Un témoin inattendu: la compilation de Georgius Erber," in *Quellen und Studien zur Musiktheorie des Mittelalters III*, ed. Michael Bernhard (Munich: Verlag der Bayerischen Akademie der Wissenschaften, 2001), 305–28. A list of incipits and concordances is given in the appendix of Meyer's article (321–24). Editions of these treatises can be found in *idem*, "La compilation sur la musique de Georgius Erber (Paris, 1460–1462) Innsbruck, Universitätsbibliothek, Cod. 962, f. 128r–163r," 8–70.

¹⁹⁷ "Parisiensis" (fol. 95) and "Parisius" (fols. 149 and 172). Cf. Federhofer-Königs, "Ein Beitrag zur Proportionenlehre," 145.

¹⁹⁸ "1457" (fol. 66), "1460" (fols. 10, 95, 149, and 172), "1462" (fol. 151), and "1466" (fol. 7). Cf. Federhofer-Königs, "Ein Beitrag zur Proportionenlehre," 145. It should be noted that one scholar has proposed a significantly earlier date for Erber's treatise. Rob C. Wegman has remarked upon the fact that the proportions described in the text are called "lately invented" ("nouvellement trouvées"). Cf. Wegman, "Different Strokes for Different Folks?" 485–86 and 488. Wegman has argued: "This would have been a strange remark in 1460, when the earliest songs featuring these proportions had been out of the repertory for at least two generations, and their composers would have been regarded as 'ancients' in relation to such recent masters as Dufay, Binchois, and Ockeghem." *Ibid.*, 486. His hypothesis that the treatise might actually originate from the 1420s is based on the cut signs described by the text. I would counter, however, that rhythmic proportions were already used in compositions long before cut signs indicating diminution entered the picture, hence the remark about the novelty of proportions in general might already have been outdated in 1420, which is why I would not set too much store by that ending comment of the treatise. Instead, I have focussed on the notational evidence: (1) void notation is used throughout the examples; (2) the musical examples demonstrate a vertical relationship between two voices as proportion, not a linear proportional change within the same voice; (3) and perhaps most notably, the examples are written in score-format and contain bar lines to indicate the beginning and end of a *mensura* (see FIGURE 5.22 below). The sum of all these indicators do, in my view, point towards a date of origin towards the middle of the century, certainly not 1420. And if it is true that Georgius Erber was in Vienna until at least 1455, this year might actually be a *terminus post quem* for a text written in French from Paris, as Erber would have had to leave Vienna to get to Paris to write it. Nevertheless, I have included the treatise here, especially due to its similarities to the two treatises *Sequitur hic aliqua declaratio atque denominatio* and *Ars et practica cantus figurativi* (see below).

¹⁹⁹ Cf. Federhofer-Königs, "Ein Beitrag zur Proportionenlehre," 146; and Meyer, "La compilation sur la musique de Georgius Erber," 2.

144r.²⁰⁰ The treatise exhibits a variety of signs—geometric signs and (stacked) Arabic numerals—to indicate seven different proportions: *dupla* (2:1), *sesquiertia* (4:3), *sesquialtera* (3:2), *dupla superbipartiens tertias* (8:3), *dupla sesquiquarta* (9:4), *tripla* (3:1), and *sesquioctava* (9:8) proportion (see TABLE 5.21). Each explanation of a proportion is accompanied by a musical example in void notation (see FIGURE 5.22 below).

Proportion	Designation	Proportion sign(s)
2:1	<i>dupla</i>	∅ or ∅ or 2
4:3	<i>sesquiertia</i> or <i>epitrita</i>	⊖
3:2	<i>sesquialtera</i> or <i>sesquidupla</i> ²⁰¹ or <i>emiolia</i>	⊖ or $\frac{6}{4}$ or $\frac{3}{2}$
8:3	<i>dupla superbipartiens</i>	⊖ or $\frac{8}{3}$ ²⁰²
9:4	<i>dupla sesquiquarta</i>	$\frac{9}{4}$ *
3:1	<i>tripla</i>	⊖* or $\frac{3}{1}$
9:8	<i>epogdoy</i>	$\frac{9}{8}$

TABLE 5.21: Proportion signs in Georgius Erber’s treatise *Sequuntur proportiones*

The inclusion of *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion in Erber’s collection has been described as unusual by Christian Meyer.²⁰³ When derived by the number of minims in simultaneously sounding mensurations (in diminution), these proportions are not far to seek and are also featured in contemporary treatises, as is shown throughout this chapter (see TABLE 5.6 above for details).²⁰⁴ The text even specifies that proportions are derived from simultaneous mensurations, as for each proportion the text gives an explanation for the number of minims. For example, *sesquiertia* (4:3) proportion is derived from “a time [= breve] of minor prolation, which has the value of four minims, against a semibreve of major prolation, which has the value of three.”²⁰⁵ Hence, the presence of *dupla sesquiquarta* (9:4) and *dupla superbipartiens tertias* (8:3) proportion follows logically.

²⁰⁰ A facsimile of the treatise can be found in Federhofer-Königs, “Ein Beitrag zur Proportionenlehre,” 153–57. Note that the order of the folios in the facsimile is mixed up, as it is the following: 142r on p. 153, 143r on p. 154, 144r on p. 155, 143v on p. 156, and 142v on p. 157.

²⁰¹ The term ‘*sesquidupla*’ is very unusual. According to a TML search, it only appears in three other treatises from the fourteenth and fifteenth century [number of search results last confirmed 25 February 2023].

²⁰² An asterisk (*) means that the sign only appears in the accompanying musical example but not in the text.

²⁰³ Meyer, “L’enseignement de la musique à Paris,” 311.

²⁰⁴ *Dupla sesquiquarta* (9:4) proportion is even included in Prosdocimus’ *Tractatus practice*, hence its presence in *Sequuntur proportiones* is neither surprising nor an innovation.

²⁰⁵ “ung temps de minor prolation, qui vault quatree minimes, contre une semibreve de maior prolation, qui vault troys.” Georgius Erber, *Sequuntur proportiones declarate et manifeste demonstrate ostense melodia vocum cantorum invente*, fol. 142v; transcription taken from Federhofer-Königs, “Ein Beitrag zur Proportionenlehre,” 149; my translation.

The explanations of the seven different proportions always have the same structure: (1) an introductory phrase “Here follows another proportion”²⁰⁶; (2) the explanation of how the numbers in the proportion are derived from simultaneous mensurations; (3) the terms used for the proportion in Latin and/or in Greek; (4) the signs; and (5) a musical example. As a demonstration, I am giving the description of *sesquialtera* (3:2) proportion:

“Item s’ensuit une aultre proportion, en la quelle on prononce le tiers plus tost que la teneur, comme qui chanteroit ung temps [parfait de minor prolation]²⁰⁷, le quell vault 6 minimes, auxi toust comme on chanteroit ung temps imparfait de minour prolation. Et est apellee ceste proportion en latin proportio sesquialtera ou selon aucuns sesquidupla. Et en grec elle est apellee emiolia, et se cognoit par ceste figure $\ominus \mid \frac{6}{4} \mid \frac{3}{2}$; exemple s’ensuit la dicte proportion: [musical example, see FIGURE 5.22]”²⁰⁸



FIGURE 5.22: Musical example demonstrating *sesquialtera* (3:2) proportion in Georgius Erber’s treatise *Sequuntur proportiones*²⁰⁹

The sign $\ominus$ for *sesquialtera* (3:2) proportion is truly noteworthy, because it seems to be the equivalent of $\ominus$ as opposed to C. The reversed sign $\ominus$ shows that the minims in the mensuration (*tempus imperfectum* with *prolatio minor* [2,2]) are proportioned, while the minims under the conventional sign are not. Analogous, $\ominus$ shows that minims under the reversed sign for *tempus imperfectum* with *prolatio maior* [2,3] are proportioned, but they maintain their mensuration. It

²⁰⁶ “Item s’ensuit une aultre proportion.” Georgius Erber, *Sequuntur proportiones*; transcription taken from Federhofer-Königs, “Ein Beitrag zur Proportionenlehre,” 149; my translation. Naturally, this phrase is not used for the first proportion, but only for the following six ‘others’. Federhofer-Königs has suggested that repeating the same phrase helps memorising the content (cf. “Ein Beitrag zur Proportionenlehre,” 151) but I cannot relate to that argument. On the contrary, the identical beginning makes it—in my eyes—even harder to memorise six different continuations of the sentence. I think that the reoccurring phrase gives the text its structure without the need of paragraphs.

²⁰⁷ Federhofer-Königs’ transcription reads “[im]parfait de majour³³” with note 33 explaining that the original “minor” is wrong. What she failed to recognise is that the original is right, namely a breve in *tempus perfectum* with *prolatio minor* is worth six minims. It is Federhofer-Königs’ addition of an [im] to the word ‘parfait’ which makes the change from ‘minor’ the ‘majour’ necessary. Federhofer-Königs was probably led by the musical example which features $\ominus$, but the musical examples in *Sequuntur proportiones* do not necessarily always demonstrate what is said in the preceding explanations of the derivation from simultaneous mensurations.

²⁰⁸ Georgius Erber, *Sequuntur proportiones*, fol. 143r; transcription taken from Federhofer-Königs, “Ein Beitrag zur Proportionenlehre,” 149.

²⁰⁹ Innsbruck, Universitätsbibliothek, MS 962, fol. 143r (excerpt). Image taken from Federhofer-Königs, “Ein Beitrag zur Proportionenlehre,” 154.

is in my view a perfectly logical step to interpret $\oslash$ as indicating *sesquialtera* (3:2) proportion—as Georgius Erber does—, when $\oslash$ indicates *sesquitertia* (4:3) proportion, even if $\oslash$ is not frequently used in music manuscripts.²¹⁰ And it is the more obvious interpretation of the sign as opposed to *sesquiquarta* (5:4) proportion, which is given in the two proportion sign treatises *Sequitur hic aliqua declaratio atque denominatio* and *Ars et practica cantus figurativi* and the 1490 *De musica* by Adam of Fulda.²¹¹ Furthermore, if $\oslash$ is the sign for *sesquialtera* (3:2) proportion it also follows logically that $\ominus$ is the sign for *tripla* (3:1) proportion, as *tripla* is *sesquialtera* in diminution ($3/2 \times 2/1 = 6/2 = 3/1$). This is the interpretation given by Erber. In summary, Erber's treatise exhibits an expansion of the concept of reversing signs to indicate rhythmic proportion in a certain mensuration, and this expansion is consistent and reasonable.

Where the explanation of rhythmic proportion and the different signs are concerned, *Sequuntur proportiones* is probably the most helpful treatise for students among all the works discussed in the chapter.²¹² First, there are the comprehensible derivations of each proportion from the number of minims in simultaneously sounding mensurations. Second, the Latin and Greek terms are correct and there is no (scribal) error in the musical examples demonstrating the proportions. Third, by giving more than one sign for most of the proportions, the text prepares the reader for their manifold appearance in music manuscripts. Compared to most other treatises discussed in this chapter, especially those in favour of standardisation, *Sequuntur proportiones* is the most helpful manual for a student learning about proportions without much prior knowledge.

²¹⁰ For appearances of $\oslash$ in music manuscripts see Stoessel, "Looking Back Over the 'Missa L'Ardant desir'," 334–38. It is noteworthy that Gaffurius and Tinctoris stated that the interpretation of $\oslash$ as indicating *sesquialtera* (3:2) proportion is an error. Cf. *id.*, 334 n. 54.

²¹¹ See Chap. 5.3.1.3.1 for details, especially n. 135. The similarities between *Sequitur hic aliqua declaratio atque denominatio* and Erber's treatise have already been acknowledged by Federhofer-Königs. Cf. "Ein Beitrag zur Proportionenlehre," 151.

²¹² Renate Federhofer-Königs has put forward the hypothesis that the treatise served as manual for choral students at a Parisian Cathedral, especially due to its special feature of being written in French. Cf. "Ein Beitrag zur Proportionenlehre," 147.

5.3.2.3 The Treatise *Iste sunt proportiones*

The treatise *Iste sunt proportiones* is uniquely transmitted in the manuscript *Siena, Biblioteca Comunale degli Intronati, MS L.V.30* (see FIGURE 5.23).²¹³ Although the manuscript originates from the end of the fifteenth century according to *RISM*, I believe that it is very likely that the original text was written earlier.²¹⁴ This is due to the musical examples demonstrating the altogether seven different proportions in the treatise (*dupla* (2:1), *tripla* (3:1), *sesquialtera* (3:2), *sesquitercia* (4:3), *sesquioctava* (9:8), *dupla superbipartiens tertias* (8:3), and *dupla sesquiquarta* (9:4) proportion). They do not consist of two different parts (one steady and one proportioned), the proportion is applied within the same voice. This linear interpretation of rhythmic proportion is very close to that of the *Ars subtilior* compositions analysed for this study, hence I am inclined to suggest that this source could even stem from the first half of the fifteenth century.

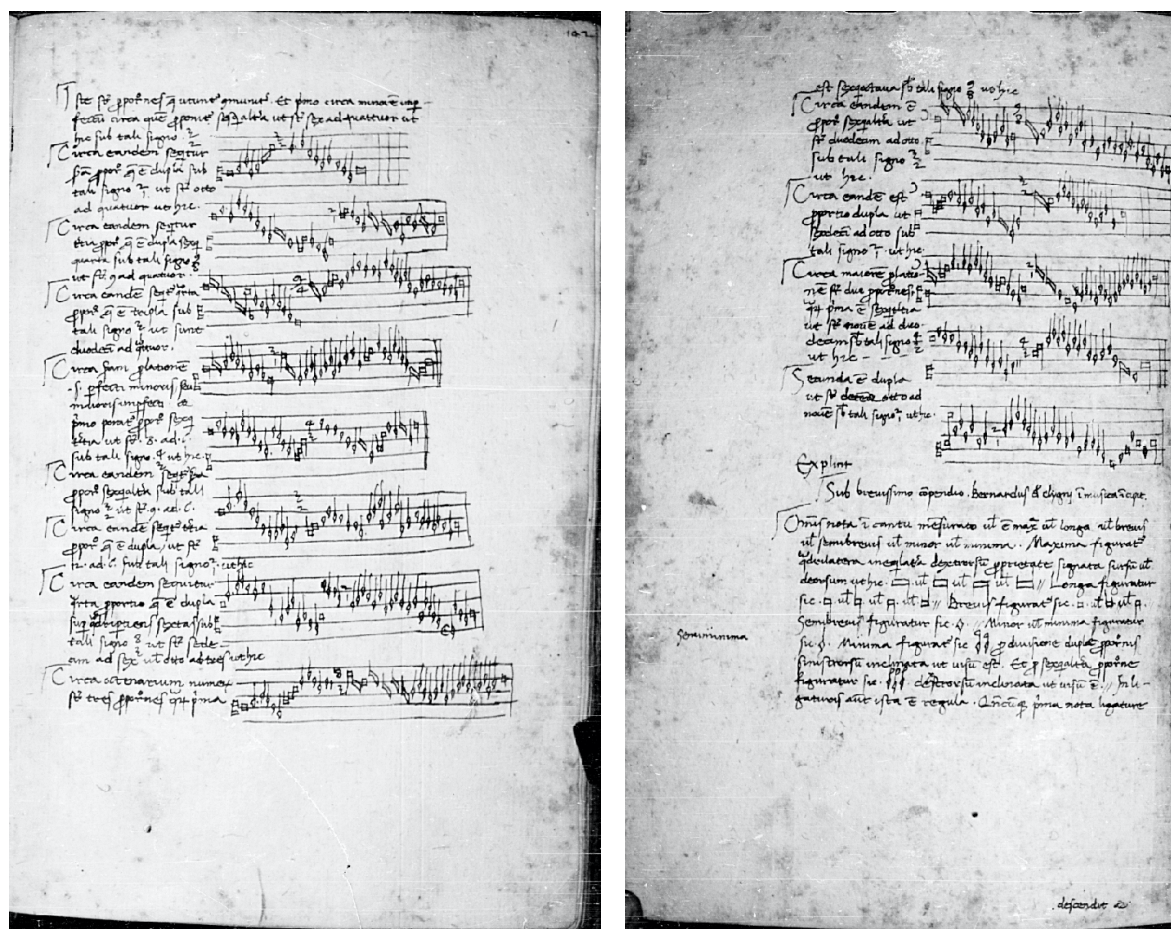


FIGURE 5.23: *Siena, Biblioteca Comunale degli Intronati, MS L.V.30, fols. 142r–142v*

²¹³ I am most grateful to Jan Herlinger who gave me access to his images of the manuscript during the corona pandemic lockdown.

²¹⁴ “Fin du XV^e s. Orig. Italie” *RISM B III*, <http://www.musmed.fr/RISM/rismindex01.htm>.

The treatise is very short and only comprises two folios (142r–142v) including the musical examples. It gives seven different proportion signs all appearing as stacked Arabic numerals. Although altogether thirteen different musical examples with accompanying text are included, the total of proportions discussed also adds up to only seven, because there are multiple examples for *dupla* (2:1), *sesquialtera* (3:2), and *sesquitertia* (4:3) proportion. The reason for some proportions being demonstrated in multiple examples is that the author systematically moves through the different mensurations and shows how the proportions are applied in each particular mensuration. For example, he begins by demonstrating how *sesquialtera* (3:2), *dupla* (2:1), *dupla sesquiquarta* (9:4) and *trippla* (3:1) proportion are applied in *tempus imperfectum* with *prolatio minor* [2,2] (see FIGURE 5.24).²¹⁵ Hence, a certain number of minims are compared to four minims of one breve in that mensuration. This interpretation is confirmed by the accompanying text, which first gives the Latin term of each proportion and then the numerical relation of the proportion.²¹⁶ In the first four examples, the number is always 4: “sex ad quatuor [sic]” (6:4 = 3:2), “otto ad quatuor” (8:4 = 2:1), “9 ad quatuor” (9:4), and “duodecim ad quatuor” (12:4 = 3:1).

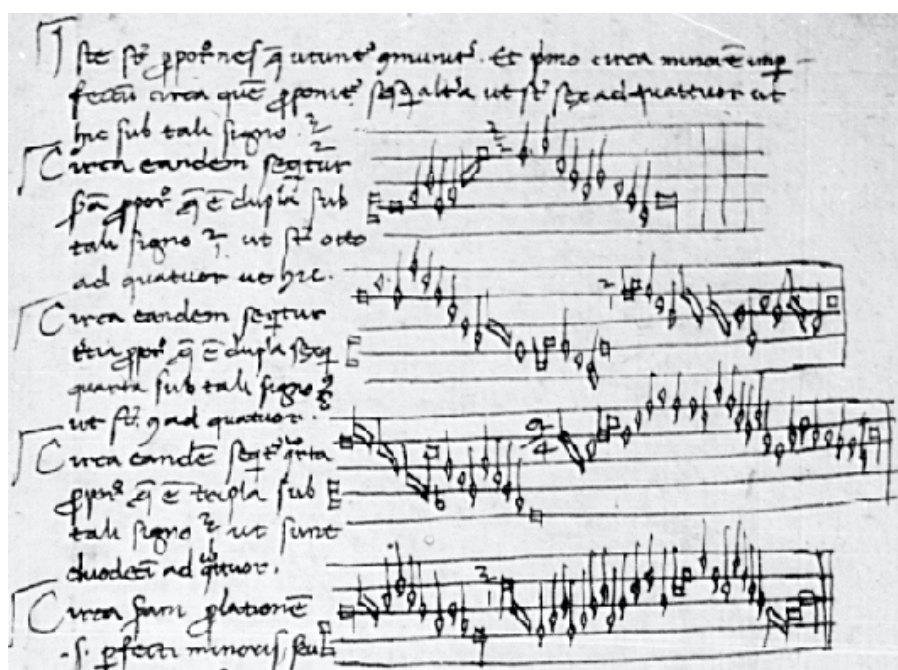


FIGURE 5.24: *Iste sunt proportiones*, fol. 142r (excerpt)

Analogously in examples five to eight, four different proportions are applied to *tempus imperfectum* with *prolatio maior* [2,3], which has six minims to the breve. According to the

²¹⁵ *Tempus imperfectum* with *prolatio minor* [2;2] is implied intrinsically in each example.

²¹⁶ Note that the musical examples are not always aligned with the text as they take up more space.

text accompanying these examples, the four proportions result from 8 to 6 minims for *sesquitertia* (4:3) proportion, 9 to 6 for *sesquialtera* (3:2), 12 to 6 for *dupla* (2:1) and 16 to 6 for *dupla superbipartiens tertias* (8:3) proportion.²¹⁷

Most remarkably, $\bigcirc$ is used in three musical examples (see first three examples on fol. 142v in FIGURE 5.23), but not as proportion sign but as *m e n s u r a t i o n* sign against which a proportion sign is set ($\frac{9}{8}$ in the first example, $\frac{3}{2}$ and $\frac{2}{1}$ respectively in the second and third). One could argue that $\bigcirc$ is simply used as alternative sign for C in this text, suggesting that the examples begin with *tempus imperfectum* with *prolatio minor* [2,2]. But this mensuration was already used in the first four examples of the treatise, so it is unlikely that it would reoccur in this otherwise systematically structured text. Moreover, the number of minims given in the texts accompanying the examples is eight, namely. “duodecim ad otto” ($12:8 = 3:2$) and “sedecim ad otto” ($16:8 = 2:1$).²¹⁸ Hence, it seems that for the author, $\bigcirc$ was an established sign for *tempus imperfectum* with *prolatio minor* [2,2] in diminution, which could be used as a mensuration as much as as a proportion. This is noteworthy because while other treatises still explain the meaning of $\bigcirc$, for this author the sign does not seem to need explaining. While still relating to *Ars subtilior* practices in applying the proportion in a linear manner then, it is likely that the text stems from the mid rather than the early fifteenth century.

The treatise is outstanding among all the sources discussed within this chapter as it advocates stacked Arabic numerals throughout and does not give any alternative signs. The author thus probably favoured standardisation, yet the musical examples show that he also had thorough knowledge of how proportions are applied in actual music, hence the text is much more practice-oriented than the treatises without musical examples discussed in Chapter 5.3.1. Moreover, demonstrating that the same proportion can be applied in different *m e n s u r a t i o n*s, the author moves closer to musical practice than the anonymous author of *Proportio est duarum rerum* and Georgius Erber in *Sequuntur proportiones*. That proportions can be applied in different mensurations is also demonstrated in the musical examples of the anonymous treatise *Venerabiles domini mei*, which will be discussed next.

²¹⁷ Busse Berger’s table (see FIGURE 5.4 above for an annotated version) suggests that *dupla superbipartiens tertias* (8:3) proportion is given as the s i g n $\frac{16}{6}$. However, only the text gives the relation “sedecim ad sex”, but not as sign but in words. As shown above, this is true for all the examples.

²¹⁸ No numbers are given in the text for *sesquioctava* (9:8) proportion.

5.3.2.4 The Treatise *Venerabiles domini mei*

The anonymous treatise *Venerabiles domini mei* is transmitted in two sources: a manuscript of assumed Franco-Italian origin, which is now kept in Saint-Dié, and a collection of treatises from South-Germany, now part of the *Proskesche Musikabteilung* in Regensburg (see FIGURE 5.25 for the opening pages of the latter source).²¹⁹ Copying dates found in the Regensburg source suggest that the manuscript was compiled between 1457 and 1476, but an earlier origin of the text is conceivable. Unfortunately, no edition nor a translation is available yet.



FIGURE 5.25: Regensburg, Bischöfliche Zentralbibliothek, Proskesche Musikabteilung, MS 98 th. 4°, 372–73

The treatise contains ten different proportion signs for seven different rhythmic proportions (see TABLE 5.26) and altogether 30 short musical examples in void notation demonstrating six

²¹⁹ I thank Dr. Raymond Dittrich from the Proskesche Musikabteilung of the Bischöfliche Zentralbibliothek Regensburg for sending me a digital copy of the relevant pages, as it was not possible to visit the library during the corona pandemic. The following discussion is based on the Regensburg version (*Bischöfliche Zentralbibliothek, Proskesche Musikabteilung, MS 98 th. 4°, 372–79*) as I was unable to acquire images of the Saint-Dié manuscript (*Bibliothèque municipale, MS 42, fols. 131r–131v*). I suspect that the Saint-Dié version is shorter as it only comprises two folios. Perhaps this source does not contain as many musical examples as the Regensburg version. Also see Gallo, “Die Notationslehre im 14. und 15. Jahrhundert,” 265–66 and 346.

different proportions. The musical examples consist of a tenor-triplum couple, in which the triplum is always proportioned.

Proportion	Designation	Proportion sign(s)
2:1	<i>dupla</i>	2 or $\mathbb{C}$ or Φ
3:1 ²²⁰	<i>tripla</i>	3
3:2	<i>sesquialtera</i>	3 or 9 or Φ or $\mathbb{C}^{*221}$
4:3	<i>sesquitertia</i>	4 or $\mathbb{O}$ or $\mathbb{C}$ or $\mathbb{C}$ or 2^{*} or Φ^{*}
9:8	<i>sesquioctava</i>	9 or 3
8:1	<i>octupla</i>	8 or $\mathbb{C}^{*}$
9:1	<i>nontupla</i>	9^{*}

TABLE 5.26: Proportion signs in *Venerabiles domini mei*

Venerabiles domini mei is the only one of all fourteen treatises discussed in this chapter, which gives the same sign for several proportions. While it is not unusual that several proportion signs are given as options for one proportion, it is singular that the same sign is listed for different proportions. For example, the Arabic numeral 3 is said to indicate *tripla* (3:1), *sesquialtera* (3:2), and *sesquioctava* (9:8) proportion. Similarly, the Arabic numeral 9 also indicates three different proportions, namely *sesquialtera* (3:2), *sesquioctava* (9:8), and *nontupla* (9:1) proportion. TABLE 5.26 shows that there are even more examples for the same sign indicating different proportions.

By stating that the same sign can represent different proportions, the author is very close to capturing the situation in actual musical notation, where signs are rarely unambiguous and are used for different proportions all the time. This aspect sets *Venerabiles domini mei* apart from the other treatises discussed in this chapter, as the advice given in the text (and musical examples) is as far removed from the possible goal of standardisation as that found in any of the others.

While *Venerabiles domini mei* may be close to actual music notation where the ambiguity of signs is concerned, *octupla* (8:1) and *nontupla* (9:1) proportion do not appear in music manuscripts. Notes in this proportion would be extraordinarily fast (if used for small note values) and I have not encountered these proportions in any composition or, for that matter,

²²⁰ This proportion is missing from Gallo's discussion of the treatise. Cf. "Die Notationslehre im 14. und 15. Jahrhundert," 346. It is true that this proportion is not discussed in the context of the musical examples, but it is nevertheless given in the text as a sign for *tripla* (3:1) proportion (see sixth and fifth to the last line on p. 372 in FIGURE 5.25 above).

²²¹ An asterisk (*) means that the sign only appears in the accompanying musical example but not in the text.

other proportion sign treatise. Still, even these two proportions are accompanied by musical examples, suggesting that the author does mean that they are actually being used.²²²

It should be noted, however, that the focus of the musical examples seems to lie on those three proportions, which are most often found in music, namely *dupla* (nine examples), *sesquialtera* (ten examples), and *sesquitertia* proportion (five examples). There are only two examples for each *sesquioctava* (9:8), *octupla* (8:1), and *nontupla* (9:1) proportion. As in the first treatise discussed in this subchapter (*Proportio est duarum rerum*), there are some problems concerning the musical examples. Some of them actually show a different proportion, for example *sesquiquarta* (5:4) instead of *sesquitertia* (4:3) proportion created by four semibreves in Θ (= 12 minims) in the tenor against 15 minims in the triplum (see FIGURE 5.27). Here, the author or the scribe seems to have forgotten to insert one minim to make it 16 minims against 12. Another instance is the first example demonstrating *nontupla* (9:1) proportion, in which 15 minims in the triplum are set against two in the tenor. Here, the author probably just forgot to add three more minims in the triplum, so that the proportion would be 18:2 (=9:1).



FIGURE 5.27: Musical example demonstrating *sesquiquarta* (5:4) instead of *sesquitertia* (4:3) proportion

Some other examples do not seem to work out at all, for instance the first example demonstrating *sesquioctava* (9:8) proportion (see FIGURE 5.28). Here, the triplum has two breves set against two breves in the tenor. It is possible to achieve nine semibreves in the triplum by assuming perfect modus, as the second breve would be altered in this case. These nine semibreves would then be set against eight minims in C in the tenor. Since all other musical examples in this treatise compare the same note values against one another (minims compared to minims, semibreves compared to semibreves and so on), however, I assume that there is a mistake in this example.²²³ Another possibility is that the author wanted to demonstrate that

²²² In that light, it is even more curious that there are no musical examples for *tripla* (3:1) proportion.

²²³ Two more examples do not add up in a similar way in the treatise: The last example demonstrating *sesquialtera* (3:2) proportion and the fourth example of *sesquitertia* (4:3) proportion.

two breves in C (8 minims) can be set against one breve in Θ (nine minims), but then he accidentally wrote down two breves instead of one in the triplum.



FIGURE 5.28: Musical example intended to demonstrate *sesquioctava* (9:8) proportion

The discussion of *Venerabiles domini mei* concludes the subchapter on the more practice-oriented treatises. While the musical examples of *Proportio est duarum rerum*, Georgius Erber's *Sequuntur proportiones* and *Venerabiles domini mei* are set in two voices, *Iste sunt proportiones* seems to be closer to musical convention in having the proportional change in the same voice part. Although this meets the situation as found in many music manuscripts, it is less helpful in a treatise teaching the use of proportion as there is no certain number of minims set against a different number of minims, as clearly demonstrated by the two-voice examples in the other three treatises (in Erber's *Sequuntur proportiones* even indicated by lines between the rhythmic units (see FIGURE 5.22 above for an example). Two voices are also found in *Pars aliquota est illa quae aliquotiens sumpta*, but, as we will see, this treatise is not at all helpful to a student intent on learning about proportions as the musical examples contained therein are as complex as the music of some *Ars subtilior* works.

5.3.3 The Treatise *Pars aliquota est illa quae aliquotiens sumpta*

The anonymous treatise *Pars aliquota est illa quae aliquotiens sumpta* is part of the collection titled [De discantu, proportionibus et tonis]²²⁴ on fols. 46r–55r from the manuscript *Rome, Biblioteca Apostolica Vaticana, MS Reg. lat. 1146*. *Pars aliquota est* is a rather comprehensive proportion treatise. It explains the five types of proportions of major inequality in great detail on altogether six folios (fols. 48r–50v), giving many numerical examples to the Latin terms for the proportions. After discussing proportions of the *multiplex superpartiens* type, the text abruptly leads over to rhythmic proportion by stating: “And bear in mind that proportion must always be computed only in minims and in no other way.”²²⁵ This remark is followed by three musical examples including proportion signs with three explanatory texts (see FIGURE 5.29).

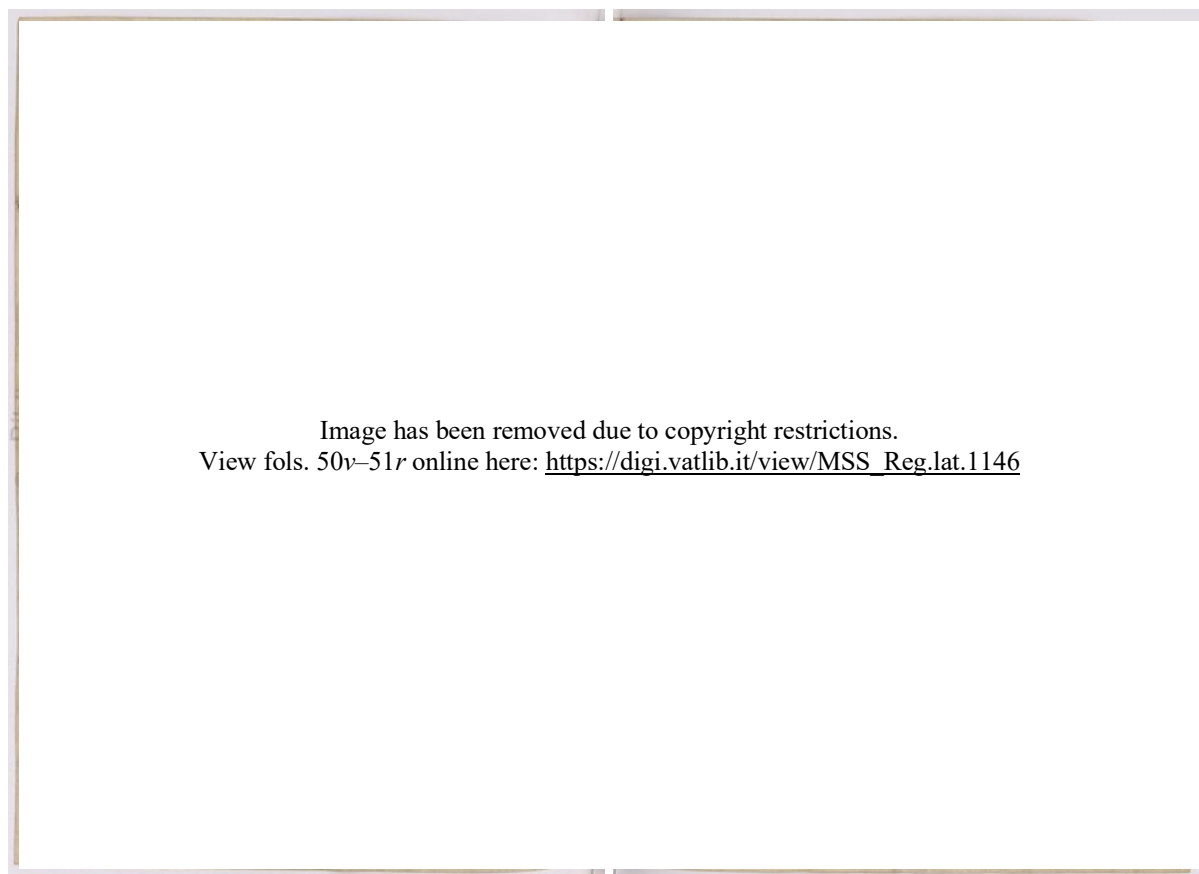


FIGURE 5.29: *Rome, Biblioteca Apostolica Vaticana, MS Reg. lat. 1146*, fols. 50v–51r

²²⁴ Cf. https://chmtl.indiana.edu/tml/14th/BERMAN4_MBAVR114. The proportion part of the collection commences with the words “[P]ars aliquota est illa” (this and all following transcriptions from the treatise taken from Peter M. Lefferts’ TML edition) on fol. 48r and ends with “Sesque enim grece est idem quod totum latine” on fol. 51r.

²²⁵ “Et nota quod proporcio semper debet computari in minimis tantum. et in nullo alio aliquo modo.” Anonymous, *Pars aliquota est illa quae aliquotiens sumpta*, fol. 50v. My translation.

The musical examples are in two voices but only the upper cantus contains proportion signs. While this matches the situation in the treatises discussed in the previous subchapter, the combination of music and text in *Pars aliquota est* is fundamentally different from any treatise analysed for this study. In *Pars aliquota est*, the musical example is not a demonstration of the proportion as given in the text, rather the text is a paratext to explain the musical examples, comparable to a canon accompanying some of the more complex *Ars subtilior* works. This is already signalled by the order in which both elements appear: the musical examples precede the text in each instance. Moreover, the first and the third musical example contain several proportion signs in one example, thereby not only demonstrating one particular proportion but several different proportions. In this, they are rather complex and resemble real *Ars subtilior* compositions compared to the short musical examples of other treatises.

Alas, the two voices do not work out very well in each of the three examples. I will start by discussing the second musical example at the top of fol. 51r (see FIGURE 5.30), as it is the shortest of all three and only contains one rhythmic proportion, namely *sesquitertia* (4:3), indicated by both 2 and O.

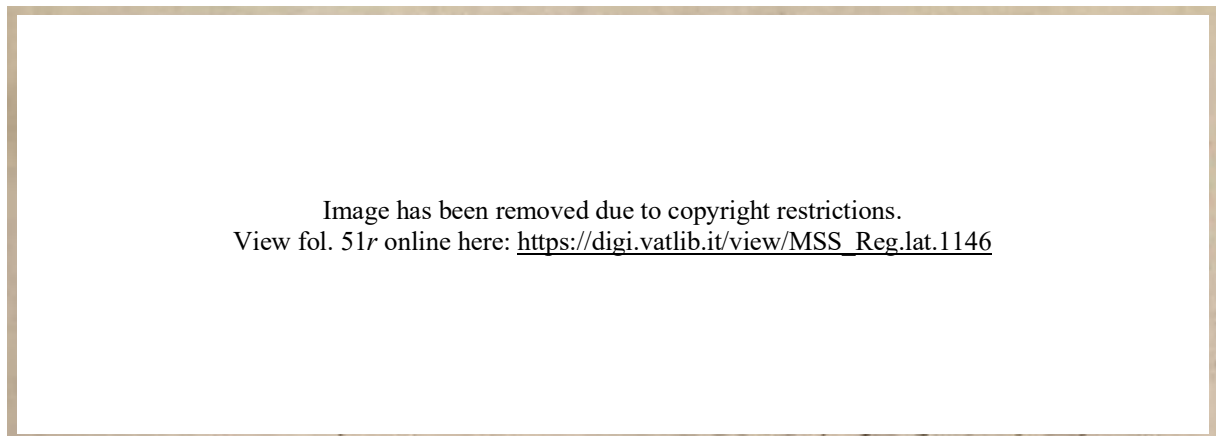


FIGURE 5.30: Second musical example in *Pars aliquota est* (cantus upper staff, tenor lower staff)

FIGURE 5.31: Transcription of second musical example

As indicated by the mensuration sign C in the cantus, the music is in *tempus imperfectum* with *prolatio maior* [2,3]. After three minims, the cantus part changes to *sesquitertia* (4:3) proportion until the final *longa* in the fourth measure of the modern transcription. The tenor, however, has more note values than the cantus, as it has altogether eight *longae*, which last for 16 measures instead of four. The only way to make the tenor suit the cantus is to shorten the *longae* radically to one fourth their normal length as in the part labelled ‘Alternative tenor’ in FIGURE 5.31. This way, the two voices would end simultaneously, but this does not seem to be a solution either due to the clash of *f*’ against *g* on the final note. Does the tenor have to be read backwards in addition to the suggested diminution?

The accompanying text does not help with this puzzle as it only states:

“This cantus planus is simple, that means of two and imperfect major prolation and likewise threefold up to the cipher. And this sign which is called .2. or this sign O which is called reversed C is a figure to use for this proportion that in Latin is called *sesquitertia*, which the Greeks called *epitrita*. That means 4 minims per three.”²²⁶

The only ambiguous part in this explanation is the very first phrase stating that the cantus is “simple, that means of two”. I assumed that the author was talking about the *modus* meaning imperfect *modus* in which the *longa* is worth two breves, as information on *tempus* and *prolatio* follows immediately. This explanation, however, does not really fit with the other two accompanying texts, which start similarly. The text from the first example reads: “This cantus planus is double, that means of two and imperfect major prolation.”²²⁷ What exactly is the difference between ‘simple’ (as in the second example) and ‘double’ (first example) if both terms apparently mean that something is ‘of two’? The third text then states: “This [cantus] planus is fourfold, that means of four and imperfect minor prolation.”²²⁸ While imperfect *modus* seemed to be a possible explanation for ‘of two’, this hypothesis does not work out if something can be ‘of four’ as in the third text.

Unfortunately, the tenor voice is too long in each example. In the third musical example, the tenor exceeds the cantus for only one *longa* (see FIGURE 5.32 and transcription in FIGURE 5.33), but in the first example (see FIGURE 5.34 and transcription in FIGURE 5.35), it extends for

²²⁶ “Iste planus cantus est simplex id est de duobus et imperfectus maioris prolacionis et triplex similiter usque ad ciphram Et hec figura que dicitur ciphra .2. vel ista figura O que dicitur C retorta est propria figura ad libitum ad istam proporcionem que dicitur Sesquetercia apud latinos Apud Grecos Epitrita hoc est dictu .4. minime pro tribus.” Anonymous, *Pars aliquota est*, fol. 51r. My translation.

²²⁷ “Iste planus cantus est duplex id est de duobus et imperfectus minoris prolacionis.” Anonymous, *Pars aliquota est*, fol. 50v. My translation.

²²⁸ “Iste planus est quadriplex id est de quatuor et imperfectus minoris prolacionis.” Anonymous, *Pars aliquota est*, fol. 51r. My translation.

seven more measures in the modern transcription. No diminution of the tenor (as suggested for the second example) can give a solution with this odd amount of measures.

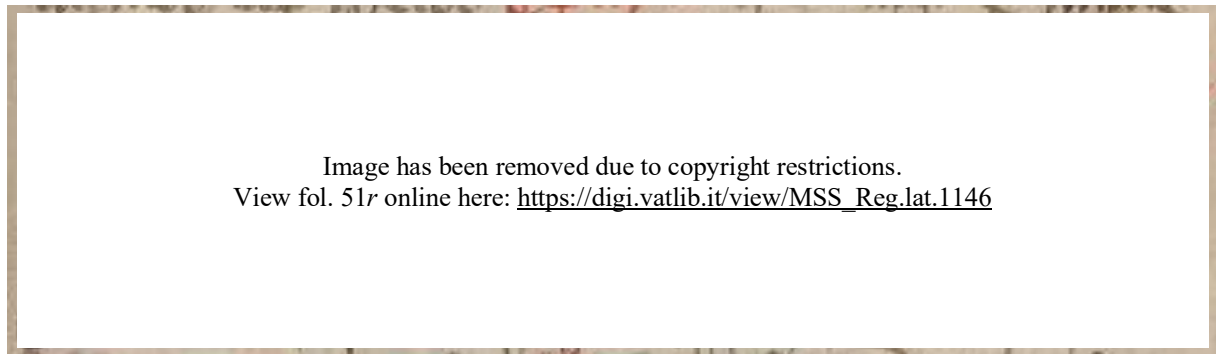


FIGURE 5.32: Third musical example in *Pars aliquota est* (note that the tenor begins on the second staff right after the final *longa* of the cantus)

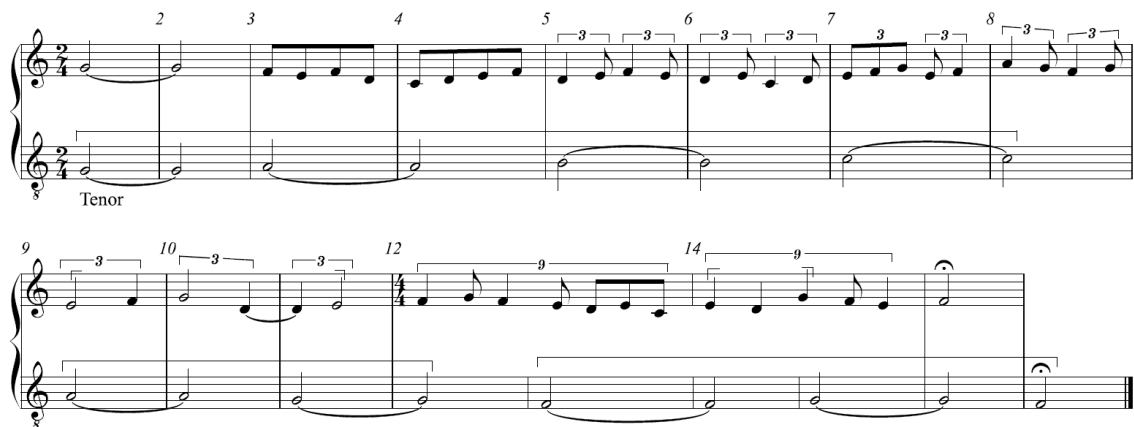


FIGURE 5.33: Transcription of third musical example

It would be a simple solution to suggest a scribal error but as the cantus works out very well despite the numerous proportions throughout the examples, I do not think that this explains the problem of the exceeding tenors.

Returning to the third example, this one contains two proportion signs, namely the Arabic numeral 6 indicating *sesquialtera* (3:2) proportion and the geometric sign Θ indicating *sesquioctava* (9:8) proportion. The accompanying text states:

“This [cantus] planus is fourfold, that means of four and imperfect minor prolation and likewise threefold up to this figure .6. which is called *sesquialtera*. And from *sesquialtera* up to the sign with a dot in the middle Θ the proportion is *sesquioctava* and in Greek is *epogdoy*. And from this figure .6. the proportion is *sesquialtera* as above. And from this sign with a dot in the middle it is *sesquioctava* proportion obviously nine minims per eight.”²²⁹

²²⁹ “Iste planus est quadriplex id est de quatuor et imperfectus minoris prolacionis et triplex similiter usque ad hanc figuram .6. que dicitur Sesquealtra Et a sesquealtra usque ad hanc figuram cum puncto in medio [Θ] est proporcio Sesquioctava et apud grecos Epogdoy et ab hac figura .6. est proporcio sesquealtra ut supra Et ab hac figura cum

For *sesquioctava* (9:8) proportion, I had to switch to a modern 4/4 (see measure 12 in FIGURE 5.33) in order to display nine quavers replacing the former eight. The change in metre got me wondering whether this was what the author meant by “fourfold, that means of four”, since *sesquioctava* (9:8) is the only proportion which requires the combination of two *mensurae* of *tempus imperfectum* with *prolatio minor* [2;2] for the calculation. But the addition “and likewise threefold up to this figure .6.” does not seem to make sense in this context. The example also shows that proportions are not applied cumulatively in the examples, as it would not be possible to apply *sesquioctava* (9:8) to the previous *sesquialtera* (3:2) proportion without arriving at very strange subdivisions of the *mensura*.

It is noteworthy that Θ is used as a proportion sign in this last example as the use of C and $\textcircled{C}$ as mensuration signs suggest that the author of the examples habitually used standard mensuration signs, which would include Θ . As the other proportion signs in this treatise are all single Arabic numerals (except $\textcircled{O}$ which is given as an alternative to 2 in the second example), Θ seems to be a bit out of the ordinary. However, the obvious choice 9 was already used in the first example for *dupla sesquiquarta* (9:4) proportion, hence it seems that the author reverted to mensuration signs for proportion signs in need of an alternative for 9. In this case, Θ for *sesquioctava* (9:8) proportion is clearly the best choice of all four standard mensuration signs, as Θ usually represents nine minims to the breve.

The first musical example (see FIGURE 5.34 and transcription in FIGURE 5.35) is the most complex as it is the longest and contains the largest number of proportion signs. 6 indicates *sesquialtera* (3:2) proportion, 4 indicates *dupla* (2:1) proportion, 9 indicates *dupla sesquiquarta* (9:4) proportion, and 3 indicates *tripla* (3:1) proportion, as explained by the accompanying text:

“This cantus planus is double, that means of two and imperfect minor prolation and likewise threefold from the semicircle up to this sign. And this figure .6. which is called sign, is used for the proportion *sesquialtera* in Latin and *emyolia* in Greek this is six minims per four. From the sign which is called *sesquialtera* up to the sign .4. that follows immediately, up to the proportion *dupla sesquiquarta* up to the sign 9, the proportion is *dupla*, obviously .4. minims per two. And from this figure .9. up to this sign .3. it is *dupla sesquiquarta* proportion, obviously nine minims per four. And from this sign .3. up to the sign .4. or the semicircle without dot of division [sic] .C. it is *tripla* proportion, obviously but three minims per one.”²³⁰

puncto in medio est proporcio Sesqueoctaua videlicet nouem minime pro octo.” Anonymous, *Pars aliquota est*, fol. 51r. My translation.

²³⁰ “Iste planus cantus est duplex id est de duobus et imperfectus minoris prolacionis. et triplex similiter a semicirculo usque ad hanc ciphram Et hec figura .6. qui dicitur ciphra ponitur pro proporcione apud latinos Sesquealtra et apud grecos Emyolia hoc est sex minime pro quatuor. a figura qui dicitur sesquealtra usque ad hanc figuram immediate sequente .4. usque ad proporcionum duplam sesquartam usque ad hanc figuram .9. est proporcio Dupla videlicet .4. minime pro duabus Et ab hac figura .9. usque ad hanc figuram .3. est proporcio Duplasesquequarta videlicet nouem minime pro quatuor Et ab hac figura .3. usque ad hanc figuram .4. vel semicirculum sine punctu diuisionis .[C]. est proporcio Tripla videlicet sed tres minime pro vna.” Anonymous, *Pars aliquota est*, fol. 50v. My translation.

Image has been removed due to copyright restrictions.
View fol. 50v online here: https://digi.vatlib.it/view/MSS_Reg.lat.1146

FIGURE 5.34: First musical example in *Pars aliquota est* (note that the tenor begins on the third staff right after the final *longa* of the cantus)

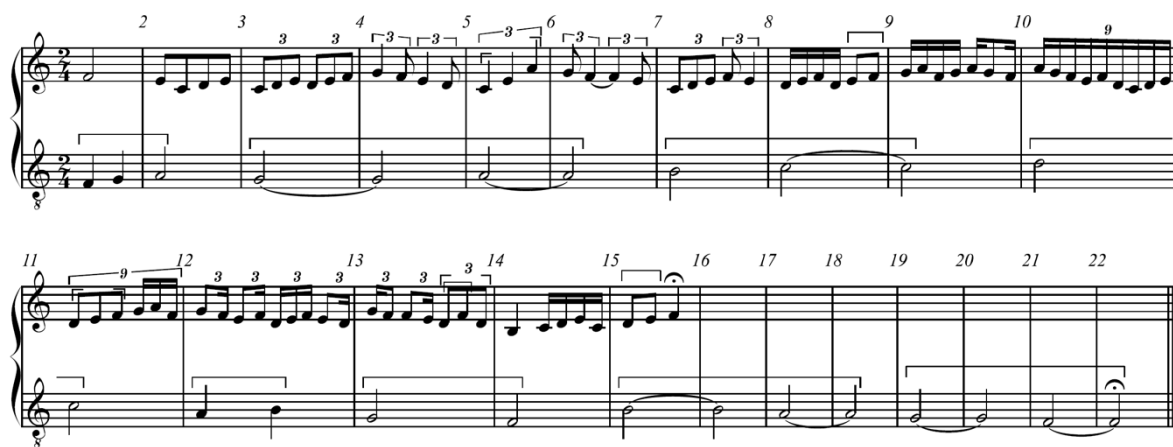


FIGURE 5.35: Transcription of first musical example

The composed accelerando in the cantus (first four minims to the breve, then six in *sesquialtera* (3:2) proportion, then eight in *dupla* (2:1), then nine in *dupla sesquiquarta* (9:4) and finally twelve in *tripla* (3:1) proportion) is reminiscent of the ballade *Ma douce amour, je me doi bien complayndre* in *Mod A* and the virelai *Je prends d'amour* from *Turin*. It is not entirely clear whether the 4 written over the mensuration sign C in the last staff means that the last section of the cantus should also be in *dupla* (2:1) proportion, as the explaining texts ends after the description of the preceding *tripla* (3:1) proportion. I have interpreted it in *dupla* (2:1) proportion in the transcription, but a return to the initial mensuration without proportion is also conceivable. It is curious that 4 indicates *dupla* (2:1) proportion while 2 is said to indicate *sesquitertia* (4:3) proportion in the second example, as it would be more logical the other way around. Still, the use of proportion signs is consistent throughout the examples, as 6 returns in the third example also indicating *sesquialtera* (3:2) proportion as in the first.

Taken as a whole, this last part of *Pars aliquota est* which concerns rhythmic proportion does not really seem to be suited for teaching purposes with its complex musical examples.

This is in contrast to the preceding part, which extensively explains the five types of proportions of major inequality. It seems as if the author intended to give the reader a glimpse of the riddle culture surrounding rhythmic proportion by including complex examples that needed some skill to resolve. As demonstrated above, I have been unable to solve these riddles but I hope that someone in the future will find a satisfying solution to the three examples.

What remains to be mentioned is the origin and date of the treatise. Jan Herlinger has stated that the manuscript is from England and roughly dates it to the fourteenth or fifteenth century.²³¹ The TML edition has labelled the treatise as 14th-century work. Therefore, *Pars aliquota est* might actually be the earliest proportion sign treatise, even predating Prosdocimus' *Expositiones*. In this light, more research on the manuscript and its treatise *Pars aliquota est* seems to be worthwhile. It is curious why this treatise has not received more attention in the past, especially due to its curious musical examples.

²³¹ Cf. Jan Herlinger, "A Fifteenth-Century Italian Compilation of Music Theory," *Acta Musicologica* 53, no. 1 (1981): 104.

5.4 Conclusion

The analysis of the discussed fourteen proportion sign treatises has shown that not all texts are suited for teaching students with little prior knowledge about rhythmic proportion. On a scale ranking the different treatises for suitability according to their comprehensibility to such students, I would assign Georgius Erber's treatise *Sequuntur proportiones* the top rank and *Pars aliquota est* with its *Ars subtilior* examples the lowest rank. The more practice-oriented treatises with musical examples discussed in Chapter 5.3.2 are closer to Erber's treatise on that scale while those treatises without musical examples discussed in Chapter 5.3.1 are closer to the less suitable *Pars aliquota est*. The Hebrew treatise does not seem to be suited for teaching purposes at all as the author gives the impression of not having grasped the subject himself, as shown above.

It is, of course, questionable whether teaching was the main purpose of (some of) the texts or whether they did not serve a completely different purpose. According to the observations in this chapter, it can at least be stated that the aspiration for standardisation does not increase the comprehensibility of the text for pupils, with the exception of *Iste sunt proportiones*, which shows a clear tendency towards standardisation, but at the same time contains helpful musical examples.

One can observe a clear tendency towards unambiguousness in the signs used to indicate proportion among all treatises, independent from some treatises' tendency towards standardisation. Unambiguous stacked Arabic numerals, which represent the respective proportion in their two numbers, appear most frequently. Some authors, for example Prosdocius, even commented on that very quality of stacked Arabic numerals, as shown above. The number of stacked Arabic numerals in treatises is remarkable compared to their rare use in music manuscripts. Single Arabic numerals are also popular among theorists and almost always are closely related to the proportion they represent, as for example 4 for *sesquiertia* (4:3) proportion or 2 for *dupla* (2:1) proportion. This is not always the case in *Ars subtilior* music, particularly not in the special five *Turin* pieces.²³² It should be noted, however, that the treatise *Pars aliquota est* also contains two less straightforward choices for proportion signs, as shown in Chap. 5.3.3. Geometric signs appear least frequently, with the exception of cut signs. It is apparent that—at least among theorists writing c.1450—the stroke had a standardised meaning of indicating diminution, i.e. *dupla* (2:1) proportion, as almost all the signs with strokes in treatises are connected to that proportion. This includes signs such as $\ominus$ for *dupla*

²³² See TABLE 3.12 on p. 173 in Chap. 3.2.1 for details.

superbipartiens tertias (8:3) proportion, as in Erber's treatise and the anonymous *Ars et practica*, since the interpretations of $\oslash$ for *sesquitertia* (4:3) and the stroke through the sign for *dupla* (2:1) proportion are combined to indicate *dupla superbipartiens tertias* (8:3) proportion in $\oslash$. The exception is $\oslash$ in the treatise *Sequitur hic aliqua*, since the given interpretation as *superbipartiens tertias* (5:3) proportion cannot be achieved by a combination of *dupla* (2:1) and another proportion. It should be noted that some of the interpretations of cut signs in the treatise *Venerabilis domini mei* also deviate from this approach, but this concerns those signs which are given for more than one proportion (see TABLE 5.26 on p. 282 above).

It is striking that the large majority of treatises only discusses proportion signs but not other means of notating rhythmic proportion, such as special note shapes or coloration. This is, in my view, surprising as these different means of displaying rhythmic proportion so often appear together in the same piece in *Ars subtilior* music. Theorists, on the other hand, seem to strictly separate these different methods. The exception is the Hebrew treatise labelled *Student's notes of lectures on music*, in which coloration is discussed alongside proportion signs. As explained above, however, the instructions on coloration in this treatise (including gold ink) do not seem to have much foundation in actual music notation. Furthermore, it seems as if the discussion of proportion signs in treatises is still linked to the discussion of musical intervals, even if none of the texts addresses this connection clearly. In most texts, the order, in which the proportions are being discussed—2:1, (3:1), (4:1), 3:2, 4:3, 9:8, 5:4, and 5:3—closely resembles the order of proportions in treatises on the intervals.²³³

Finally, I would like to point out that there are most probably more proportion sign treatises from the first half of the fifteenth century than the 14 treatises discussed in this chapter. With my TML search I could only find treatises which were already edited. However, I expect there to be many more texts on proportion signs, most probably just comprising a few folios and therefore not yet edited. Searching for these texts is beyond the scope of the present study, but I hope that future research will include new sources in the discussion.

²³³ Usually, proportions of the *multiplex* type are discussed first, followed by proportions of the *superparticularis* type, thus following the hierarchy among the five types of proportion according to Pythagorean doctrine. See Chap. 1, p. 20. The intervals deriving from the proportions discussed in the above named order would be octave (2:1), compound fifth (3:1), double octave (4:1), fifth (3:2), fourth (4:3), major second (9:8), major third (5:4) (note: 5:4 according to Ptolemaic tuning, in Pythagorean tuning the proportion is given as 81:64), and major sixth (5:3) (note: 5:3 in Ptolemaic tuning, Pythagorean tuning is 27:16).

CONCLUSION

Proportions played an essential role in medieval music and medieval music theory. While there was a long tradition of discussing proportions in the context of musical intervals, which harks back to the Ancient Greeks, the concept of *proportio* was only gradually adapted to rhythm in the late Middle Ages. The music of the so-called *Ars subtilior* exhibits a great variety of signs to indicate proportional rhythms. While these include coloration and special note shapes, proportion signs have been the focus of this study. By understanding the *Ars subtilior* as a movement instead of only as a musical style, I have included agents such as scribes and authors of treatises in the discussion.

Where proportion signs are concerned, the analysis of *Ars subtilior* manuscripts for this study speaks in favour of a certain amount of standardisation. The reversed semicircle $\circ$ is the most frequently appearing proportion sign and it is described or used as a sign for *sesquitercia* (4:3) proportion in both music manuscripts and treatises.¹ As proportion signs, the stacked Arabic numerals $\frac{3}{2}$ are also used exclusively for *sesquialtera* (3:2) proportion.² Other proportion signs, for example the single Arabic numerals 2 und 3, do have more than one possible interpretation, nevertheless, the meaning is always the one or the other, namely *dupla* (2:1) or *subsesquialtera* (2:3) respectively for 2 as well as *tripla* (3:1) or *sesquialtera* (3:2) proportion respectively for 3.³ Hence, it is reasonable to also speak of standardisation in these two cases. The proportion sign 4, however, is not only used for *sesquitercia* (4:3) and *quadrupla* (4:1) proportion, but also for *tripla* (3:1) proportion in *Ma douce amour, je me doi bien complayndre*, which is a less obvious choice.⁴ There is reason to believe that single Arabic numerals were used one after the other according to their availability for the proportions in a piece, even though they did not necessarily represent one of the numbers in the proportion. Although I assumed a different scenario for the five special pieces in *Turin*, which contain the single Arabic numerals

¹ The only exceptions are two of the five special pieces in *Turin*, in which $\circ$ indicates *sesquialtera* (3:2) and *dupla sesquialtera* (5:2) proportion respectively. In my eyes, these two exceptions do not speak against standardisation of $\circ$ in general. As I have argued in Chap. 3.2.3, I suspect that the use of $\circ$ for other proportions than *sesquitercia* (4:3) in these pieces is intentional and part of the musical riddle.

² It should be noted, however, that the stacked Arabic numerals $\frac{3}{2}$ are used as a mensuration sign without additional proportional meaning for *tempus imperfectum* with *prolatio maior* [2,2] in Antonello da Caserta's *Dame d'onour, en qui tout mon cuer maynt*.

³ Again, exceptions can be found in two of the five special pieces in *Turin*, in which 3 and 2 respectively indicate *superbipartiens tertias* (5:3) proportion. Moreover, the treatise *Pars aliquota est illa quae aliquotiens sumpta* gives 2 as a sign for *sesquitercia* (4:3) proportion, while 4 is used for *dupla* (2:1) proportion. Finally, the treatise *Venerabiles domini mei* offers several different interpretations for 2 and 3 respectively, among them *sesquitercia* (4:3) proportion for 2 and *sesquioctava* (9:8) proportion for 3. Also note that 3 is used to indicate *sesquitercia* (4:3) proportion in the *Pad A* version of Johannes Ciconia's *Sus un' fontayne*.

⁴ This is true for all three sources of the piece. See Chap. 2.5 for details. 4 also appears as a sign for *dupla sesquialtera* (5:2) proportion in one of the five special pieces in *Turin*.

2, 3, 4, 5, 6, 7, 8 and 9, this might be an alternative explanation for some of the unusual proportion sign choices in these compositions.

In this study, I have argued why I consider signs in all three forms of visual appearance (single Arabic numerals, stacked Arabic numerals and geometric shapes) to be proportion signs. In my definition of the term, I have made the change of the duration of the minim (or under special circumstances a higher note value) the decisive criterion in order to make a distinction between a mensuration and a proportion sign. This approach has worked out well for the music studied here. The prevailing scholarly opinion that mensuration signs are geometric shapes, while proportion signs are Arabic numerals (single or stacked) is indefensible, not only but especially in light of the signs given in treatises. Even though a preference for stacked Arabic numerals is evident and even expressed by Prosdocius de Beldemandis and Ugolino da Orvieto, the authors of proportion sign treatises up to c.1450 do not seem to differentiate between the three forms of visual appearance. Hence, modern scholars should also refrain from making a distinction on that ground. Moreover, as I have shown throughout Chapters 1 to 4, a sign can simultaneously function as a mensuration sign and as a proportion sign, i.e. changing the structure of the rhythmic whole (mensuration sign function) and the duration of the minim (proportion sign function). Therefore, it is only natural that proportion signs also appear as geometric shapes.

While I have demonstrated what proportion signs are in the five chapters of this study, I would also like to devote a paragraph to what they are not. When proportion signs appear as geometric shapes, they are always circles or semicircles, often with modifications in the form of dots or strokes. I have not encountered triangles or quadrangles or any other form of geometric shape in *Ars subtilior* manuscripts. The exception is the treatise *Declaratio musicae disciplinae* by Ugolino da Orvieto, in which a diamond-shaped sign ($\diamond$) is given for *sesquitertia* (4:3) proportion. As I have explained in Chapter 4, I doubt that this sign was ever used in actual notation of music due to its resemblance to the semibreve, which is an intrinsic sign. Nevertheless, I am still curious to know the origin of Ugolino's suggestion and look forward to further investigation into that question. I have also not encountered numbers larger than 9, i.e. multi-digit numbers in proportion signs appearing as either single or stacked Arabic numerals. This could be owing to the fact that proportions based on large numbers are rare in music notation. The only proportion including a multi-digit number in this study is *trippla sesquitertia* (10:3) proportion, which is used in two pieces in *Turin*, but indicated by different circles in these compositions. It is also remarkable that no letters seem to have been used for proportion signs,

although the concept of rhythmic proportion in *Ars subtilior* music is derived from the Italian notational system, in which the *divisiones* are indicated by letters.

What I also have not found in the music is a clear tendency of composers—or scribes—to favour one form of visual appearance over the other. This is especially true for a preference of Arabic numerals over geometric shapes. Only in music treatises do we find a clear preference for stacked Arabic numerals, as outlined in Chapter 5. In music, there are just a handful of pieces using only single or stacked Arabic numerals as proportion signs without geometric shapes also being present at the same time.⁵ This small number faces 14 compositions in *Mod A* and *Turin*, in which the two forms (Arabic numerals and geometric shapes) appear side by side, and 25 compositions, which only use $\circ$.

In Chapter 1.3, I have—among other hypotheses—suggested that proportion signs and coloration might have been a way for composers or scribes to avoid minim division in conformity with Nominalism. Although these two notational devices can create values smaller than the minim, the *minima*—the so-called ‘smallest unit’—can maintain its visual and semantic form and is not divided into semiminims. As already stated in that chapter, the *Ars subtilior* repertoire analysed for this study paints a different picture though. Semiminims are often used as a notational device in addition to coloration and proportion signs. However, a handful of proportion sign pieces do not use semiminims, among them *Puis que amé sui doucement* in *Turin*, and Baude Cordier’s *Tout par compas* in *Ch*. As these two pieces appear alongside compositions, which do contain semiminims, however, it is difficult to argue that the avoidance of minim division was the motivation behind the use of proportion signs by the composer or scribe in these two cases. Yet, this is no reason to reject the hypothesis that the general idea of using coloration (already in *Ars nova* notation) and proportion signs (in *Ars subtilior* repertoire) was not born out of the volition to avoid minim division. *Ars subtilior* composers might simply not have cared about or defied music theorists’ advice on purpose.

That composers often ignored the instructions on musical notation given in treatises has been demonstrated in Chapter 4, in which I have shown that Ugolino as a composer has not followed his very own instructions on the use of proportion signs as given in his *Declaratio*. Other discrepancies between notational practice and music treatises have been addressed in

⁵ The anonymous *Ma douce amour, je me doi bien complayndre*, Corrado da Pistoia’s *Se doulz espour ne me donne confort*, Bartolomeo da Bologna’s *Que pena maior agitanda menti*, and two compositions by Antonello da Caserta. See Chaps. 2.3 and 2.5 for details. Naturally, there are quite a few pieces only using the stacked Arabic numerals $\frac{3}{2}$ in *Turin*, but this sign also appears alongside $\circ$ in many compositions in this manuscript’s *Ars subtilior* repertoire, which—as argued in Chap. 3—might be the output of just one composer, and which certainly is the work of just one music scribe. I would therefore argue that it is coincidence that $\frac{3}{2}$ appears as only proportion sign in some pieces, and that it is not for reasons of a preference of Arabic numerals over symbols of the composer or the scribe.

Chapter 5, in which I have analysed thirteen additional proportion sign treatises up to c.1450. It is remarkable that the two above-mentioned concepts for avoidance of minim division—namely coloration and proportion signs—are rarely discussed within the same treatise, or in case of comprehensive music treatises in the same chapter.⁶ The evidence suggests that theorists perceived these two means of rhythmic notation as different concepts, even though they appear side by side in compositions. Concordances even show that these two means of rhythmic notation were interchangeable in notational practice.⁷

Further differences between music treatises and musical practice can be observed in the preferred form of visual appearance of signs. While it is true that the general forms of visual appearance of proportion signs in treatises are congruent with those found in music—that means treatises do not recommend forms of visual appearance which have never been seen in musical notation—there is a discrepancy concerning the use of stacked Arabic numerals. Although theorists clearly favour these, they are only rarely used in music. Antonello da Caserta is the only composer using different stacked Arabic numerals in *Mod A* while *Turin* only contains $\frac{3}{2}$ and $\frac{3}{1}$. Furthermore, many treatises contain proportion signs appearing as variations of the semicircle (such as $\cup$, $\cap$, $\ominus$, Φ , $\mathcal{C}$, $\odot$, $\ominus$, and $\mathbb{C}$), which are nevertheless seldomly found in music. As I have suggested in Chapters 4 and 5, authors might have acted under systemic coercion when giving these variations. However, the five special pieces in *Turin* show that the composer(s) or the scribe reverted to standard mensuration signs or single Arabic numerals for rare proportions such as *superbipartiens tertias* (5:3) proportion and not for special variations of the semicircle.

Most of the musical examples in the more practice-oriented treatises do show how proportion signs are applied in music, hence they have a connection to actual musical notation. Therefore these treatises were probably more suited for students intent on learning about proportion signs with the aim of using them in music. Notably, *Iste sunt proportiones* is the only treatise in which proportion signs are applied within the same voice, which corresponds with the linear approach that one can find in the more complex *Ars subtilior* compositions, especially those in which proportions are applied cumulatively or quasi-cumulatively. Those treatises showing how proportion signs are applied between different voice parts, however, might be more suitable for teaching purposes, as the reader can comprehend how the different amounts of minims sound simultaneously. Another remarkable feature of one of the more practice-oriented treatises

⁶ See last column in Table 5.7 on pp. 228–32 for details.

⁷ On this aspect see Chap. 2.4 on the compositions by Filippotto da Caserta, which exhibit proportion signs in *Mod A* but coloration in the concordances in *Ch*.

is that the author of *Venerabiles domini mei* offered the same sign for several different proportions. While this corresponds to the situation in music sources, giving several possible interpretations of the same sign is a unique feature among all the treatises analysed for this study. In all other thirteen treatises, each sign has an unequivocal meaning, even if more than one sign is given for a particular proportion. Their authors are more decisive concerning the use of one particular sign.

When comparing *Turin* to the instructions given in treatises, one might even say that the composer(s) or the scribe of *Turin* abide by the notational rules found therein. As shown in Chapter 3, *Turin* shows a significant tendency towards standardisation in large parts of the repertoire and $\bigcirc$ and $\frac{3}{2}$ are used in accordance with the signs given by most treatises. If ‘the one composer’ hypothesis holds true, however, then this individual also composed the five special pieces. And in these five compositions, we find that one and the same sign seldom has the same meaning, i.e. a particular sign has different interpretations within the *oeuvre* of one composer. These unusual proportion sign choices go beyond the tastes or preferences of a particular individual. Rather, each particular piece has to be looked at and interpreted in its own unique manner.

I have attributed these notational peculiarities to the emerging riddle culture that would become a movement in the Renaissance.⁸ *Ars subtilior* composers experimented with musical notation and turned its interpretation into a challenging game. This riddle culture is not only manifested in the unusual proportion sign choices in *Turin* but also in other *Ars subtilior* pieces, which were notated in a more complex manner than strictly necessary. I have demonstrated this by the fictional simple version of Baude Cordier’s *Belle, bonne, sage* in Chap. 1.4.2. These examples show that it is time to abandon the idea that signs have to bear the same meaning throughout a specific manuscript or a specific period of time, in the works of a particular composer or in all sources copied by the same hand.

Including the music scribes in the debate prompts the question of who was ultimately responsible for the rhythmic notation of a composition. It seems to be a foregone conclusion that the composer is the originator of the notation we find in *Ars subtilior* compositions. However, the analysis of the manuscript *Mod A* reveals that scribes may have had significant influence on the musical notation. This is not only manifested in erasures and corrections done by the scribe in *Mod A* but also by different proportion signs or even means of rhythmic proportion in concordances, as shown in Chapters 1.4.5 and 2.4.

⁸ Cf. Katelijne Schiltz, *Music and Riddle Culture in the Renaissance* (Cambridge: Cambridge University Press, 2015), *passim*.

Due to the influence of scribes, I advise against adducing the notation of rhythmic proportion in a particular piece as argument for dating this composition, as it might not have been the composer but the scribe introducing the sign to the music when copying the piece. I have shown throughout this study that signs are not used consistently in the *oeuvre* of one composer, even within in the same manuscript.

An exception is the ballade *Ma douce amour, je me doi bien complayndre*, in which all three proportion signs (2, 3, and 4) stay the same in the three concordant music sources. As there is an accompanying canon to this piece, the presence of a canon likely prevented the scribes from changing the notation. In that scenario, the music scribes would have been able to read and understand the canon. Moreover, it is also conceivable that certain scribes were the authors of accompanying canons. Although this would explain the close resemblance of the canons of the five special pieces in *Turin*, two different text scribes copied these canons, hence the theory does not hold up in this case. In Chapter 3.2.2, I have shown that an analysis of accompanying canons can shed new light on certain aspects of compositions. This analysis has led to the conclusion that one individual composed all five special pieces, despite differences in the notation of rhythmic proportion within these five compositions.

In complex compositions such as these five songs, canons can be regarded as peritexts, i.e. paratexts, which are located close to the musical text.⁹ They are not part of the musical text themselves but can play an important role in its interpretation. The possibility of such canons explaining the meaning of proportion signs was not considered by Anna Maria Busse Berger when she stated: “The invention of the fraction to show rhythmic proportions therefore presented a true innovation because it permitted the indication of proportions not naturally inherent in the mensural system.”¹⁰ After my analysis of *Ars subtilior* music in this study, I cannot agree with Busse Berger here. On the contrary, the most exceptional proportions (especially those in *Turin*) are not indicated by stacked Arabic numerals—‘fractions’ for Busse Berger—but by single Arabic numerals and geometric shapes. The solution for the composer or scribe was to add a canon explaining the meanings of the signs. Hence, canons appearing as peritext next to the composition are the true innovation here.

If canons can be paratexts, music treatises discussing proportion signs might also be regarded as paratexts, namely epitexts. As I have shown in Chapters 4 and 5, however, treatises

⁹ I have explained the terms ‘paratext’ and ‘peritext’, which were coined by the French literary theorist Gérard Genette, in Chapter 1.3.1.2. There, I have argued, that I do not consider proportion signs to be paratexts. They are extrinsic notational elements, but I have compared them to punctuation in written language, which is also part of the actual text.

¹⁰ Anna Maria Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 228.

are not very well suited to advise on the interpretation of proportion signs in actual compositions. They describe a desired state of musical notation, not necessarily the situation found in *Ars subtilior* music. This is why they should not be used as an argument to explain certain phenomena in musical notation. The comparison of Ugolino's compositions with his statements on proportion signs is probably the best example for the discrepancy.

In Chapter 1.4, I have explained that proportion signs sometimes do not only indicate a particular proportion but that they may have multiple functions. They can indicate musical or textual citations and allusions (for example Θ in *Se fortune s'est tournée* in *Ox213* and different signs in several pieces in *Mod A*), the beginning of a melisma, or positions of structural importance to a piece (for example the entry of a canonic voice as in *Tout par compas*).

As I have also shown in this chapter, the reference point for a proportion is crucial for the interpretation of its sign. Even when a proportion refers to a preceding minim within the same voice, the non-cumulative, the cumulative, and the quasi-cumulative proportions interpretations each yield different results. An important uncovering was that the quasi-cumulative proportions interpretation had to be applied in Baude Cordier's *Belle, bonne, sage*, because this finding confirmed that—at least in this case—there seems to be a hierarchy in musical notation, in which coloration is subordinate to proportion and mensuration signs. In the discussion of cumulative proportions, I have explained why I reject the hypothesis of a linear development from non-cumulative to cumulative proportions and I have substantiated my argument with further musical examples in other chapters.

Based on the quasi-cumulative proportions interpretation, I have not interpreted the cut circle sign Θ as a proportion sign in the two Cordier songs in *Ch*, but as a mensuration sign. By doing so, I have contradicted many scholars' previous assumptions that Θ indicates *tempus perfectum diminutum* in these pieces. While I reject this interpretation for *Ch*, there is no denying that the sign indicates diminution in later sources and even in compositions by Cordier in *Ox213*. The music treatises seem to confirm that there was a standardised interpretation of the stroke as a sign of diminution (i.e. *dupla* (2:1) proportion) from c.1450 onwards—at least among music theorists—since almost all the signs with strokes given by these treatises appear in the context of diminution.¹¹ Here, I agree with Margaret Bent who has put forward the hypothesis that at an early stage, Θ had different functions (e.g. indicating position), and that the interpretation of it as a sign of diminution only came later.¹² As suggested in Chapter 5.3.1.2,

¹¹ That includes the interpretation of signs such as $\curvearrowright$ for *dupla superbipartiens tertias* (8:3) proportion, as it would be *sesquitercia* (4:3) proportion (indicated by $\curvearrowright$) in diminution (indicated by the stroke). The only exceptions are the two treatises *Sequitur hic aliqua declaratio atque denominatio* and *Venerabilis domine mei*, which also give different interpretations to signs with strokes. See Chaps. 5.3.1.3.1 and 5.3.2.4 or TABLE 5.6. above for details.

¹² Cf. Margaret Bent, "The Early Use of the Sign Θ ," *Early Music* 24, no. 2 (1996): 199–225.

the treatise *Quoniam circa artem musice figurative seu mensuralis* might even be the earliest treatise discussing cut signs and this source certainly deserves more scholarly attention. It was fortunate that this treatise came up in my TML search because it does not seem to have been discussed as a source containing signs of diminution before this study.

It is precisely because proportion and mensuration signs can have multiple functions that I suggest that their positions and their visual appearances are included in future editions of *Ars subtilior* music. Without signalling their position, additional information (e.g. indication of a citation) from the original text gets lost in the modern transcription. Editions such as Haring's and Boeke's complete edition of *Mod A* do that already.¹³ And I hope that future revised editions of the comprehensive collections (*PMFC*, *CMM*, etc.) will also include proportion signs.

Finally, I would like to draw attention to the fact that proportion signs can also appear outside the repertoire that is usually considered to be *Ars subtilior*. For example, there is a proportion sign, namely the single Arabic numeral 2 indicating *dupla* (2:1) proportion, in the manuscript *Pz*, which has hitherto gone unnoticed.¹⁴ Proportion signs may not be limited to the *Ars subtilior* movement. Therefore, the study of proportion signs should be expanded to all music manuscripts from around 1400. That, as well as the study of further sources of *Ars subtilior* music (e.g. *Reina*, *Pit*, *Ox213*, etc.) is beyond the scope of the present study, as is the search of further hitherto unedited music treatises discussing proportion signs. I nevertheless hope that my contribution to the discussion of *Ars subtilior* manuscripts encourages further

¹³ Cf. Jos Haring and Kees Boeke, eds., *The Manuscript a.M.5.24 Modena Codex: New Complete Edition with Commentary Including All Known Works Written or Expanded by Matheus de Perusio*, Olive Music Editions 1 (Amsterdam and Arezzo: Olive Music, 2019).

¹⁴ *Pz* contains a collection of 32 French and Italian songs and is believed to be of Venetian provenience with a presumed compilation date between 1420 and 1430. Cf. Christian Berger, ed., introduction to *Ein venezianisches Liederbuch aus dem Anfang des 15. Jahrhunderts: Die Handschrift Paris, Bibliothèque nationale, nouv. acq. fr. 4917 [Pz]*, *Musikalische Denkmäler* 12 (Mainz: Schott, 2016), V. The anonymous rondeau *Bergerette* contains the proportion sign 2 in the final melisma:

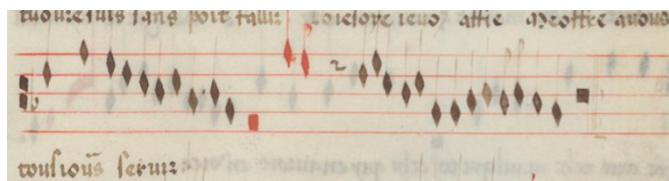


FIGURE 6.1: Final staff of cantus of *Bergerette* in *Pz*, fol. 12v

Instead of interpreting the sign as proportion sign, both previous editors have found elaborate explanations for the otherwise resulting additional measure in the cantus compared to the tenor. In 1944, Philipp Möllner suggested that the sign indicated that the three notes in red ink preceding the sign should be sung soloistically while the other two voices rested for the duration of a breve. Cf. Philipp Möllner, “Die französischen Lieder der Handschrift Paris, Bibl. nat. nouv. acq. fr. 4917,” PhD diss., Johann Wolfgang Goethe-Universität Frankfurt am Main, 1944, 125–26. And in his 2016 edition of *Bergerette*, Christian Berger interprets the sign as Tironian note for the Latin word ‘et’ and concludes that the three red notes and the succeeding six minims should be sung simultaneously. Cf. Berger, *Ein venezianisches Liederbuch*, 137. If the sign is interpreted as a proportion sign indicating *dupla* (2:1) proportion, however, the two voices add up perfectly and end at the same time.

research on the subject of proportion signs that might bring to light new aspects of late fourteenth- and early fifteenth-century music.

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This bibliography first gives sources (starting on p. 306), then editions and translations (starting on p. 330), and finally other secondary literature (starting on p. 337).

Where possible for respective music manuscripts, I have given information on facsimiles (online and print) as well as on editions. Furthermore, I have always included the respective *DIAMM* link. I have refrained from using *RISM* sigla for the music manuscripts, since abbreviations such as *Ch* for the Chantilly codex or *Mod A* for codex Modena are much more common in secondary literature. Entries are ordered alphabetically according to location. Alphabetical order according to sigla (for all manuscripts) can be found in the list of abbreviations (starting on p. xiii) at the beginning of this study. It should be noted that *RISM* sigla are used for all other manuscripts, however.

Moreover, I have included information on facsimiles (online and print) for the manuscripts containing treatises. The section on treatises (starting on p. 317) includes information on respective editions and translations for practical reasons. The editions and translations listed there are the ones I consulted for this publication.² Spellings as well as order of first and last names in this list are taken from the respective articles in *NG2*.

And finally, a general remark concerning links to online resources in this bibliography: I have decided against adding the date of the last access behind every link given below. I have checked every link before submitting this thesis, thus all websites in this bibliography were last accessed on 5 April 2023, unless otherwise stated.

¹ This bibliography contains more information than what might be called the ‘average bibliography’ because I have endeavoured to include additional information I considered to be useful. I have done so because during the years of my B.A. and M.A. studies, I sometimes had difficulties finding the material I needed. Even during my PhD years, I sometimes found that I had not been aware that a certain collection of transcriptions or editions existed. Occasionally, I serendipitously found print facsimiles of manuscript fragments I had been looking for months earlier. From time to time, I was looking for a translation of a treatise and did not realise it was part of a compilation—sometimes I even accidentally stumbled across translations only published online. In order to save fellow scholars the trouble, I have included information I myself would have found helpful at the time. For example, I have grouped references concerning the availability of printed or online facsimiles underneath each manuscript. I have followed a similar strategy in the list of treatises, for which I added information about editions and translations. However, please note that I have not been aiming to provide a comprehensive research guide, which is why I cannot guarantee that the information given is complete. In light of increasing digitisation, I also took the liberty of adding links to papers and articles available on digital platforms, e.g. jstor, oxford-musiconline, mgg-online, etc., which I only find appropriate in the year 2023, especially after the experience of closed libraries during the coronavirus pandemic. I invite everyone to share the information in this bibliography with others and do hope that the reader will find it helpful.

² For additional information on treatises, editions, and translations see C. Matthew Balensuela and David Russell, eds., *Music Theory from Boethius to Zarlino: A Bibliography and Guide*, Harmonologia 14, ed. Thomas Christensen (Hillsdale, NY: Pendragon Press, 2007); and Ullrich Scheideler and Felix Wörner, eds., *Musiktheorie von der Antike bis zur Gegenwart*, Lexikon Schriften über Musik 1, ed. Hartmut Grimm and Melanie Waldfuhrmann (Kassel and Stuttgart: Bärenreiter / J. B. Metzler, 2017).

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Selected images online:
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APPENDIX

TABLE A.1: Results of TML Search with Search Term ‘*proportio*’ in Combination with the Selection 15th Century

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of ‘ <i>proportio</i> ’ Hits	Mentions Proportion Signs?
1	Hothby, Johannes	De proportionibus	HOTDEP	De proportionibus	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
2	Hothby, Johannes	Regulae super proportionem	HOTREGP	Regulae super proportionem	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
3	Hothby, Johannes	Regulae super proportionem	HOTRSP	Regulae super proportionem	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
4	Anonymous	Compendium breve de proportionibus	ANOCBP	Compendium breve de proportionibus	irrelevant	irrelevant	5	no
5	Ciconia, Johannes	De proportionibus	CICPROP	De proportionibus	1411	yes	97	no
6	Tinctoris, Johannes	Proportionale musices	TINPRO	Proportionale musices	1472–73	no	irrelevant	irrelevant
7	Tinctoris, Johannes	Proportionale musices	TINPROM	Proportionale musices	1472–73	no	irrelevant	irrelevant
8	Prosdocimo de’ Beldomandi	Brevis summula proportionum quantum ad musicam pertinet	PROBRE1	Brevis summula proportionum quantum ad musicam pertinet	1409	yes	48	no
9	Prosdocimo de’ Beldomandi	Brevis summula proportionum quantum ad musicam pertinet	PROBRE2	Brevis summula proportionum quantum ad musicam pertinet	1409	yes	48	no
10	Ciconia, Johannes	Nova musica, liber tertius de proportionibus	CICNM3	Nova musica	1408	yes	61	no
11	Ramus de Pareia, Bartholomeus	Musica practica, tertia pars, tractatus secundus	RAMMP3T2	Musica practica	probably post 1450, because author was born c. 1435/40	no	irrelevant	irrelevant
12	Guillermus de Podio	Ars musicorum liber VIII	GUIARS8	Ars musicorum	1495	no	irrelevant	irrelevant

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
13	Anonymous	[Regulae de contrapuncto]	ANORDC	[Regulae de contrapuncto]	irrelevant	irrelevant	1	no
14	Anonymous	[Tractatus de musica cum glossis]	ANOHOL3_MVNB4774	[Tractatus de musica cum glossis]	irrelevant	irrelevant	18	no
15	Gaffurio, Franchino	Practica musice	GAFFPM4	Practica musice	1496	no	irrelevant	irrelevant
16	Ciconia, Johannes	Nova musica, liber primus de consonantiis	CICNM1	Nova musica	1408	yes	73	no
17	Ciconia, Johannes	Nova musica, liber secundus de speciebus	CICNM2	Nova musica	1408	yes	4	no
18	Ciconia, Johannes	Nova musica, liber quartus de accidentibus	CICNM4	Nova musica	1408	yes	37	no
19	Adamus de Fulda	Musica, pars quarta	FULMUS4	Musica	1490	no	irrelevant	irrelevant
20	Gaffurio, Franchino	Theorica musice, liber quartus	GAFTM4	Theorica musice	1492	no	irrelevant	irrelevant
21	Keck, Ioannes	Introductorium musicae	KECKINMU	Introductorium musicae	1442	yes	38	no
22	Anonymous	Tractatus de contrapuncto	ANOTDC	Tractatus de contrapuncto	irrelevant	irrelevant	*1	no
23	Antonius de Luca	Ars cantus figurati	LUCARSC	Ars cantus figurati	irrelevant	irrelevant	*1	no
24	Anonymous	Tractatulus de mensurationum	ANOTRAM	Tractatulus de mensurationum	irrelevant	irrelevant	*1	no
25	Anonymous	Capitulum de quattuor mensuris	ANOCQM	Capitulum de quattuor mensuris	irrelevant	irrelevant	*1	no
26	Anonymous	[Mensura monochordi]	ANOEIN	[Mensura monochordi]	irrelevant	irrelevant	*1	no
27	Anonymous	Tractatulus prolationum cum tabulis	ANOTPT	Tractatulus prolationum cum tabulis	irrelevant	irrelevant	*1	no
28	Prosdocimo de' Beldomandi	Parvus tractatulus de modo monacordum dividendi	PROPAR	Parvus tractatulus de modo monacordum dividendi	1413	yes	37	no

¹ 'fuzzy match'. This treatise does not contain 'propor*', but the (book) title of the edition does, which is why it is included in the search results.

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
29	Marchetus de Padua, Ciconia, and Anonymous	Lucidarium, Regulae de tonis, Tractatus figurarum, and Nova musica	LAFVAR	*2	*2	*2	*2	*2
30	Antonius de Luca	Ars cantus figurati	LUCARS	Ars cantus figurati	1475–1500	no	irrelevant	irrelevant
31	Anonymous	Tractatus de musica compendium cantus figurati	ANOCOMP	Tractatus de musica compendium cantus figurati	late 15 th century	no	irrelevant	irrelevant
32	Johannes de Olomons	Palma choralis	JOHPAL	Palma choralis	1425–43	yes	8	no
33	Johannes de Olomons	Palma choralis	JOHPALM_M MBAL20I	Palma choralis	1425–43	yes	8	no
34	Guillermus de Podio	Enchiridion de principiis musicae discipline	GUIENC	Enchiridion de principiis musicae discipline	1495–1500	no	irrelevant	irrelevant
35	Herbenus Traiectensis	De natura cantus ac miraculis vocis	HERDEN	De natura cantus ac miraculis vocis	author born 1451	no	irrelevant	irrelevant
36	Heritius, Erasmus	Musica speculativa	HERSPE	Musica speculativa	1498	no	irrelevant	irrelevant
37	Hothby, Johannes	Dialogus in arte musica	HOTDIA	Dialogus in arte musica	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
38	Hothby, Johannes	Excitatio quaedam musicae artis per refutationem	HOTEXC	Excitatio quaedam musicae artis per refutationem	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
39	Hothby, Johannes	Regulae cantus mensurati secundum Johannem Otteby	HOTRCM2	Regulae cantus mensurati secundum Johannem Otteby	probably post 1450, because author was born 1430	no	irrelevant	irrelevant
40	Anonymous	Opusculum de musica	ANOTRHO	Opusculum de musica	irrelevant	irrelevant	23	no
41	Anselmus, Georgius	De musica	ANSDEM	De musica	1434	yes	180	no

² This is a collection of editions of treatises. Ciconia's *Nova musica* also appears individually in this search, the other treatises originate from the fourteenth century.

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
42	Bonaventura da Brescia	Brevis collectio artis musicae	BONBRE	Brevis collectio artis musicae	1497	no	irrelevant	irrelevant
43	Burtius, Nicolaus	Musices opusculum, tractatus primus	BURMUS1	Musices opusculum	1487	no	irrelevant	irrelevant
44	Burtius, Nicolaus	Musices opusculum, tractatus tertius	BURMUS3	Musices opusculum	1487	no	irrelevant	irrelevant
45	Monachus Carthusiensis	Tractatus de musica plana	CARTRA	Tractatus de musica plana	irrelevant	irrelevant	20	no
46	Ciconia, Johannes	Nova musica	CICNM	Nova musica	1408	yes	9	no
47	Guilielmus monachus	De preceptis artis musicae et prattice compendiosus libellus	MONPRE	De preceptis artis musicae et prattice compendiosus libellus	probably post 1450	no	irrelevant	irrelevant
48	Guilielmus monachus	De preceptis artis musicae et prattice compendiosus libellus	MONPREC	De preceptis artis musicae et prattice compendiosus libellus	probably post 1450	no	irrelevant	irrelevant
49	Gaffurio, Franchino	Glossemata quaedam super nonnullas partes theoricæ præscriptæ Iohannis de Muris	MURMGAFG_MMBAH165	Glossemata quaedam super nonnullas partes theoricæ præscriptæ Iohannis de Muris	post 1450	no	irrelevant	irrelevant
50	Anonymous	Quæstiones et solutiones avidendum tam mensurabilis cantus quam immensurabilis musica	ANOQS	Quæstiones et solutiones avidendum tam mensurabilis cantus quam immensurabilis musica	irrelevant	irrelevant	1	no
51	Anonymous	[Tractatus de musica mensurabili]	ANOTDMM	Quoniam circa artem musicae figurative seu mensuralis	c.1450	yes	19	yes
52	Anonymous	Terminorium musicalium diffinitorium anonymi	ANOTER	Terminorium musicalium diffinitorium anonymi	irrelevant	irrelevant	11	no
53	Anonymous	Summula	ANOSUM	Summula	irrelevant	irrelevant	13	no
54	Anonymous	[Excerpta de consonantiis et de contrapuncto]	ANOEXC	[Excerpta de consonantiis et de contrapuncto]	irrelevant	irrelevant	1	no

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
55	Anonymous	[Compendium secundum famosiores musicos]	ANOFMUS_MSAVI44	[Compendium secundum famosiores musicos]	1490	no	irrelevant	irrelevant
56	Anonymous	De musica	ANOFRA4	De musica	irrelevant	irrelevant	13	/
57	Gaffurio, Franchino	Theorica musice, liber primus	GAFTM1	Theorica musice	1492	no	irrelevant	irrelevant
58	Gaffurio, Franchino	Theorica musice, liber secundus	GAFTM2	Theorica musice	1492	no	irrelevant	irrelevant
59	Gaffurio, Franchino	Theorica musice, liber tertius	GAFTM3	Theorica musice	1492	no	irrelevant	irrelevant
60	Gaffurio, Franchino	Theorica musice, liber quintus	GAFTM5	Theorica musice	1492	no	irrelevant	irrelevant
61	Johannes Gallicus dictus Carthusiensis seu de Mantua	Tacita nunc inchoatur stupendaque numerorum musica	GALLJB	Tacita nunc inchoatur stupendaque numerorum musica	1473	no	irrelevant	irrelevant
62	Johannes Gallicus	Ritus canendi [Pars prima]	GALRC1	Ritus canendi	1458–64	no	irrelevant	irrelevant
63	Johannes Gallicus dictus Carthusiensis seu de Mantua	Ritus canendi vetustissimus et novus, liber primus	GALRIT1	Ritus canendi	1458–64	no	irrelevant	irrelevant
64	Johannes Gallicus dictus Carthusiensis seu de Mantua	Ritus canendi vetustissimus et novus, liber secundus	GALRIT2	Ritus canendi	1458–64	no	irrelevant	irrelevant
65	Johannes Gallicus dictus Carthusiensis seu de Mantua	Ritus canendi vetustissimus et novus, liber tertius	GALRIT3	Ritus canendi	1458–64	no	irrelevant	irrelevant
66	Johannes Gallicus dictus Carthusiensis seu de Mantua	Incipit praeftationcula in tam admirabilem quam tacitam et quietissimam novorum concinentiam	GALTRA	Incipit praeftationcula in tam admirabilem quam tacitam et quietissimam novorum concinentiam	irrelevant	irrelevant	9	no

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
67	Dionysius Lewis de Ryckel	De arte musicali, secunda pars: Musica practica	GENTPRA_M GRU70	De arte musicali	irrelevant	irrelevant	4	no
68	Dionysius Lewis de Ryckel	De arte musicali, prima pars: Musica speculativa	GENTSPE_M GRU70	De arte musicali	irrelevant	irrelevant	374	no
69	Guerson, Guillaume	Utilissime musicales regule	GUEUT	Utilissime musicales regule	c.1495	no	irrelevant	irrelevant
70	Anonymous	[Opusculum de musica ex traditione Iohannis Hollandrini]	ANOHOL6A_MVNB4774	[Opusculum de musica ex traditione Iohannis Hollandrini]	irrelevant	irrelevant	23	no
71	Anonymous	Tractatus de musica	ANOLEU	Tractatus de musica	irrelevant	irrelevant	1	no
72	Anonymous	[De modis cantandi]	ANOMCANT MSAVI44	[De modis cantandi]	1490	no	irrelevant	irrelevant
73	Anonymous	Musica compendium	ANOMCOMP MSAVI44	Musica compendium	irrelevant	irrelevant	9	no
74	Anonymous	[Tractatus compilatus de musica plana et mensurabili e traditione franconiana]	ANOMPM_M BER267	[Tractatus compilatus de musica plana et mensurabili e traditione franconiana]	irrelevant	irrelevant	5	no
75	Anonymous	De origine et effectu musicae	ANOOREF	De origine et effectu musicae	early 15 th century	yes	15	no
76	Anonymous	De origine et effectu musicae	ANOORI_ML BLL763	De origine et effectu musicae	early 15 th century	yes	14	no
77	Anonymous	[Tractatus de musica]	ANOPHIL	[Tractatus de musica]	irrelevant	irrelevant	4	no
78	Anonymous	[Tractatus de musica]	ANOPRO	[Tractatus de musica]	irrelevant	irrelevant	5	no
79	Gaffurio, Franchino	Extractus parvus musicae	GAFEXT	Extractus parvus musicae	late 15 th century	no	irrelevant	irrelevant
80	Gaffurio, Franchino	Practica musicae	GAFPM1	Practica musicae	1496	no	irrelevant	irrelevant
81	Gaffurio, Franchino	Practica musicae	GAFPM2	Practica musicae	1496	no	irrelevant	irrelevant
82	Gaffurio, Franchino	Practica musicae	GAFPM3	Practica musicae	1496	no	irrelevant	irrelevant
83	Anonymous	Tractatus de musica	AGANONT	Tractatus de musica	irrelevant	irrelevant	1	no

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
84	Anonymous XI	[Tractatus de musica plana et mensurabili]	ANO11TDM	[Tractatus de musica plana et mensurabili]	c.1450	yes	67	yes
85	Anonymous XI	Tractatus de musica plana et mensurabili	ANO11TRA	Tractatus de musica plana et mensurabili	c.1450	yes	67	yes
86	Anonymous [XII]	Tractatus cantus figurati	ANO12TCF	[= Tractatus de musica]	1460–71	no	irrelevant	irrelevant
87	Anonymous XII	Tractatus de musica	ANO12TRA	Tractatus de musica	1460–71	no	irrelevant	irrelevant
88	Anonymous	Ars musice	ANOAM	Ars musice	irrelevant	irrelevant	1	no
89	Anonymous	Ars et practica cantus figurati	ANOAPC	Ars et practica cantus figurati	copied 1483	uncertain	15	yes
90	Anonymous	[De musica mensurata]	ANOBRI	[De musica mensurata]	c.1400	yes	3	no
91	Anonymous	[De musica mensurata]	ANOBRIg	[De musica mensurata]	c.1400	yes	3	no
92	Anonymous	[Compendium breve artis musicae]	ANOCBAM	[Compendium breve artis musicae]	irrelevant	irrelevant	1	no
93	Prosdocimo de' Beldomandi	Tractatus musice speculative	PROSPE	Tractatus musice speculative	1425	yes	365	no
94	Prosdocimo de' Beldomandi	Musica speculativa	PROSPEC	Musica speculativa	1425	yes	347	no
95	Prosdocimo de' Beldomandi	Tractatus de contrapuncto	PROTRAC	Tractatus de contrapuncto	1412	yes	4	no
96	Prosdocimo de' Beldomandi	Tractatus practice de musica mensurabili	PROTRAP1	Tractatus practice de musica mensurabili	1408	yes	16	yes
97	Prosdocimo de' Beldomandi	Tractatus practice de musica mensurabili ad modum italicorum	PROTRAP2	Tractatus practice de musica mensurabili ad modum italicorum	1412	yes	23	no
98	Prosdocimo de' Beldomandi	Tractatus practice cantus mensurabilis ad modum ytallicorum	PROTRAY	Tractatus practice cantus mensurabilis ad modum ytallicorum	1412	yes	22	no

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
99	Ramus de Pareia, Bartholomeus	Musica practica, prima pars, tractatus primus	RAMMP1T1	Musica practica	probably post 1450 because author was born c.1435/40	no	irrelevant	irrelevant
100	Ramus de Pareia, Bartholomeus	Musica practica, prima pars, tractatus secundus	RAMMP1T2	Musica practica	probably post 1450 because author was born c.1435/40	no	irrelevant	irrelevant
101	Ramus de Pareia, Bartholomeus	Musica practica, secunda pars, tractatus primus	RAMMP2	Musica practica	probably post 1450 because author was born c.1435/40	no	irrelevant	irrelevant
102	Ramus de Pareia, Bartholomeus	Musica practica, tertia pars, tractatus primus	RAMMP3T1	Musica practica	probably post 1450 because author was born c.1435/40	no	irrelevant	irrelevant
103	Prosdocimo de' Beldomandi	Contrapunctus	PROCON	Contrapunctus	1412	yes	2	no
104	Prosdocimo de' Beldomandi	Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris	PROEXP	Expositiones tractatus practice cantus mensurabilis magistri Johannis de Muris	1404	yes	14	yes
105	Prosdocimo de' Beldomandi	Libellus monocordi	PROLIB	Libellus monocordi	1413	yes	36	no
106	Tinctoris, Johannes	Tractatus de notis et pausis	TINTDN	Tractatus de notis et pausis	c.1475	no	irrelevant	irrelevant
107	Tinctoris, Johannes	Tractatus de punctis	TINTDP	Tractatus de punctis	c.1475	no	irrelevant	irrelevant
108	Tinctoris, Johannes	Tractatus de regulari valore notarum	TINTDR	Tractatus de regulari valore notarum	1474–75	no	irrelevant	irrelevant
109	Tinctoris, Johannes	Tractatus de notis et pausis	TINTRAN	Tractatus de notis et pausis	c.1475	no	irrelevant	irrelevant
110	Tinctoris, Johannes	Tractatus super punctis musicalibus	TINTRAP	Tractatus super punctis musicalibus	c.1475	no	irrelevant	irrelevant
111	Tinctoris, Johannes	Tractatus de regulari valore notarum	TINTRAR	Tractatus de regulari valore notarum	1474–75	no	irrelevant	irrelevant

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'proportio' Hits	Mentions Proportion Signs?
112	Anonymous	Tractatus de musica figurata et de contrapuncto ab anonymo auctore	TRADEM	Tractatus de musica figurata et de contrapuncto ab anonymo auctore	1474–84	no	irrelevant	irrelevant
113	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber primus	UGODEC1A	Declaratio musicae disciplinae	c.1430	yes	115	no
114	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber primus	UGODEC1B	Declaratio musicae disciplinae	c.1430	yes	2	no
115	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber secundus	UGODEC2	Declaratio musicae disciplinae	c.1430	yes	12	no
116	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber tertius	UGODEC3A	Declaratio musicae disciplinae	c.1430	yes	18	no
117	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber tertius	UGODEC3B	Declaratio musicae disciplinae	c.1430	yes	18	yes
118	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber quartus	UGODEC4	Declaratio musicae disciplinae	c.1430	yes	984	no
119	Ugolino Urbevitanis	Declaratio musicae disciplinae, liber quintus	UGODEC5	Declaratio musicae disciplinae	c.1430	yes	1101	no
120	Ugolino Urbevitanis	Musica disciplina	UGOMUS	Declaratio musicae disciplinae	c.1430	yes	64	no
121	Ugolino Urbevitanis	Tractatus monochordi	UGOTRAM	Tractatus monochordi	c.1430	yes	65	no
122	Wenceslaus de Prachatitz	[Commentum in musicam speculativam Iohannis de Muris]	WENCOM	[Commentum in musicam speculativam Iohannis de Muris]	first half of 15 th century	yes	23	no
123	Weyts, Carnelite, Nicastus	Regule	WEYREG	Regule	irrelevant	irrelevant	2	no

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
124	Anonymous	Tractatus de musica mensurabili	WFANON4	Tractatus de musica mensurabili	irrelevant	irrelevant	3	no
125	Wylde, Johannes	Musica manualis cum tonale	WYLMUS	Musica manualis cum tonale	irrelevant	irrelevant	26	no
126	Wylde, Johannes	Musica guidonis	WYLMUSI_ML763	Musica guidonis	irrelevant	irrelevant	26	no
127	Guido d'Arezzo [Ps.]	Tractatus Guidonis correctorius multorum errorum, qui fiunt in cantu gregoriano in multis locis	ANOTRA2	Tractatus Guidonis correctorius multorum errorum, qui fiunt in cantu gregoriano in multis locis	irrelevant	irrelevant	2	no
128	Guido d'Arezzo [Ps.]	Tractatus Guidonis correctorius multorum errorum, qui fiunt in cantu gregoriano in multis locis	ANOTRAC2	Tractatus Guidonis correctorius multorum errorum, qui fiunt in cantu gregoriano in multis locis	irrelevant	irrelevant	2	no
129	Capuanus, Nicolaus	Compendium musicale	NICCOM	Compendium musicale	1415	yes	1	no
130	Paulirinus, Paulus	Tractatus de musica	PAUTRA	Tractatus de musica	c.1460	no	irrelevant	irrelevant
131	Person, Gobelinus	Tractatus musicae scientiae	PERTRA	Tractatus musicae scientiae	irrelevant	irrelevant	7	no
132	Zabern, Conrad von	De modo bene cantandi	CONCAN	De modo bene cantandi	1474	no	irrelevant	irrelevant
133	Zabern, Conrad von	Opusculum de monochordo	COMMON	Opusculum de monochordo	1462–74	no	irrelevant	irrelevant
134	Zabern, Conrad von	Novellus musicae artis tractatus	CONNOV	Novellus musicae artis tractatus	1462–74	no	irrelevant	irrelevant
135	Anonymous	[Tractatus de musica plana]	EICANON_M EUB685	[Tractatus de musica plana]	irrelevant	irrelevant	3	no
136	Sadze de Flandria, Christianus	Tractatus modi, temporis et prolationis	FLATRA	Tractatus modi, temporis et prolationis	c.1470	no	irrelevant	irrelevant
137	Adamus de Fulda	Musica, pars prima	FULMUSI	Musica	1490	no	irrelevant	irrelevant

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
138	Adamus de Fulda	Musica, pars secunda	FULMUS2	Musica	1490	no	irrelevant	irrelevant
139	Adamus de Fulda	Musica, pars tertia	FULMUS3	Musica	1490	no	irrelevant	irrelevant
140	Faber Stapulensis, Jacobus	Elementa musicalia	STAPMUS	Elementa musicalia	1496	no	irrelevant	irrelevant
141	Ladislaus de Zalka	[Musica]	SZAMUS	[Musica]	1490	no	irrelevant	irrelevant
142	Szydlow, Johannes de	Musica	SZYDMUS	Musica	1475	no	irrelevant	irrelevant
143	Tallanderius, Petrus	Lectura	TALLEC	Lectura	irrelevant	irrelevant	3	no
144	Tinctoris, Johannes	Complexus effectuum musices	TINCOM	Complexus effectuum musices	1472-75	no	irrelevant	irrelevant
145	Tinctoris, Johannes	Complexus viginti effectuum nobilis artis musices	TINCOM2	Complexus viginti effectuum nobilis artis musices	1472-75	no	irrelevant	irrelevant
146	Tinctoris, Johannes	Liber de arte contrapuncti, Liber primus	TINCON1	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant
147	Tinctoris, Johannes	Liber de arte contrapuncti, Liber secundus	TINCON2	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant
148	Tinctoris, Johannes	Liber de arte contrapuncti, Liber tertius	TINCON3	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant
149	Tinctoris, Johannes	Liber de arte contrapuncti, Liber primus	TINCPT1	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant
150	Tinctoris, Johannes	Liber de arte contrapuncti, Liber secundus	TINCPT2	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant

#	Author	TML Treatise Name	TML Reference	Treatise	Date	First Half of 15 th Century?	Number of 'propor*' Hits	Mentions Proportion Signs?
151	Tinctoris, Johannes	Liber de arte contrapuncti, Liber tertius	TINCPT3	Liber de arte contrapuncti	1477	no	irrelevant	irrelevant
152	Tinctoris, Johannes	Diffinitorium musicae	TINDIF	Diffinitorium musicae	1495	no	irrelevant	irrelevant
153	Tinctoris, Johannes	De inventione et usu musicae	TININV	De inventione et usu musicae	c.1481	no	irrelevant	irrelevant
154	Tinctoris, Johannes	De inventione et usu musicae	TININV4	De inventione et usu musicae	c.1481	no	irrelevant	irrelevant
155	Tinctoris, Johannes	Liber de natura et proprietate tonorum	TINLDN	Liber de natura et proprietate tonorum	1476	no	irrelevant	irrelevant
156	Tinctoris, Johannes	Liber imperfectionum notarum musicalium	TINLIB	Liber imperfectionum notarum musicalium	1472-75	no	irrelevant	irrelevant
157	Tinctoris, Johannes	Liber imperfectionum notarum musicalium	TINLIMP	Liber imperfectionum notarum musicalium	1472-75	no	irrelevant	irrelevant
158	Tinctoris, Johannes	Liber de natura et proprietate tonorum	TINNAT	Liber de natura et proprietate tonorum	1476	no	irrelevant	irrelevant

KURZFASSUNG DER ERGEBNISSE

Proportionszeichen treten in den Handschriften der sogenannten *Ars subtilior* (einschließlich Musikhandschriften und musiktheoretischen Abhandlungen) in drei Erscheinungsformen auf: als einzelne arabische Ziffern, als übereinander geschriebene arabische Ziffern und als geometrische Formen. Da der Begriff ‘Proportionszeichen’ in bisherigen Abhandlungen nur unzureichend abgegrenzt wurde, habe ich in dieser Studie die erste Definition des Begriffs erarbeitet. Dieser Definition liegt die Änderung der Dauer der *Minima* (oder unter besonderen Umständen eines höheren Notenwerts) als entscheidendes Kriterium zugrunde, um zwischen einem Mensurzeichen und einem Proportionszeichen zu unterscheiden. Die vorherrschende Meinung, dass es sich bei Mensurzeichen um geometrische Formen und bei Proportionszeichen um (einzelne oder übereinander geschriebene) arabische Ziffern handelt, ist nicht nur, aber insbesondere vor dem Hintergrund musiktheoretischer Abhandlungen, in denen Proportionszeichen in allen drei Erscheinungsformen auftreten, nicht haltbar. Die Unterscheidung zwischen den beiden Begriffen Mensurzeichen und Proportionszeichen aufgrund der Erscheinungsform ist eine moderne Fehlvorstellung. Ein Zeichen kann jedoch Proportions- und Mensurzeichen kombiniert in einem Zeichen sein. Daher habe ich in meinen Analysen verschiedene Funktionen der Zeichen betrachtet. Meiner Beurteilung nach ist die graphische Darstellung eines bestimmten proportionalen Verhältnisses nicht die einzige Funktion, die einige der Proportionszeichen haben. Sie können auch musikalische oder textliche Zitate und Anspielungen markieren, den Beginn eines Melismas anzeigen oder Stellen von struktureller Bedeutung für ein Stück hervorheben. Ich habe bisher unentdeckte musikalische oder textliche Zitate und Anspielungen in einigen Kompositionen des Manuskripts *Mod A* gefunden.

Die Ergebnisse der Analyse der *Ars subtilior* Handschriften für diese Studie sprechen für ein gewisses Maß an Standardisierung bestimmter Proportionszeichen. Der umgekehrte Halbkreis $\bigcirc$ ist das am häufigsten vorkommende Proportionszeichen und er wird sowohl in Musikmanuskripten als auch in Traktaten als Zeichen für die Proportion *sesquitertia* (4:3) beschrieben oder verwendet. Auch die übereinander geschriebenen arabischen Ziffern $\frac{3}{2}$ werden als Proportionszeichen ausschließlich für die Proportion *sesquialtera* (3:2) verwendet. Andere Proportionszeichen, z. B. die arabischen Ziffern 2 und 3, haben zwar mehr als eine mögliche Interpretation, die Bedeutung ist jedoch fast immer die eine oder andere, nämlich Proportion *dupla* (2:1) bzw. *subsesquialtera* (2:3) für 2 sowie *tripla* (3:1) bzw. *sesquialtera* (3:2) für 3. Eine deutliche Tendenz zur Standardisierung ist vor allem in den Musiktraktaten zu beobachten, in denen eindeutige, übereinander geschriebene arabische Ziffern häufiger beschrieben werden als

mehrdeutige geometrische Formen. In den eher praxisorientierten Abhandlungen, die auch musikalische Beispiele enthalten, werden jedoch oft mehrere Möglichkeiten der Darstellung einer bestimmten Proportion angegeben, was die Vielfältigkeit der Zeichen in den Musikhandschriften widerspiegelt.

Diese Vielfältigkeit kann sicherlich auf Individualisierung zurückgeführt werden, d. h. auf Komponisten oder Schreiber, die mit der rhythmischen Notation nach eigenem Ermessen experimentierten. Einige Kompositionen enthalten seltene Proportionszeichen. In anderen Fällen wurden konventionelle Proportionszeichen für unkonventionelle Proportionen verwendet, die sich nicht aus musikalischen Intervallen ableiten lassen. Einige dieser ungewöhnlichen Proportionszeichen können jedoch nicht nur auf den Geschmack oder die Vorlieben eines bestimmten Individuums zurückgeführt werden, da sie selbst in den Werken desselben Komponisten unterschiedlich verwendet werden. Vielmehr muss jedes Stück einzeln und auf seine eigene Art und Weise betrachtet und interpretiert werden. Ich habe die Besonderheiten in der Notation mit der aufkommenden Rätselkultur begründet, die sich später in der Renaissance zu einer Bewegung entwickelte. Die Komponisten und/oder Schreiber der *Ars subtilior* verwandelten die Notation und ihre Interpretation manchmal in ein Rätselspiel und notierten die Stücke absichtlich komplizierter als nötig. Aufgrund dieser Erkenntnisse habe ich argumentiert, dass es an der Zeit ist, sich von der Vorstellung zu verabschieden, dass Zeichen in einem bestimmten Manuskript oder einem bestimmten Zeitraum, in den Werken eines bestimmten Komponisten oder in allen von derselben Hand kopierten Quellen stets dieselbe Bedeutung haben müssen.

Die Einbeziehung der Schreiber in die Debatte wirft die Frage auf, wer letztlich für die Notation in einer Komposition verantwortlich war. Es scheint oft wie selbstverständlich davon ausgegangen worden zu sein, dass der Komponist der Urheber der Notation ist, die wir in den Kompositionen der *Ars subtilior* finden. Die Analyse des Manuskripts *Mod A* hat jedoch ergeben, dass die Schreiber einen erheblichen Einfluss auf die Notation gehabt haben könnten. Dies zeigt sich nicht nur in den Korrekturen des Schreibers in *Mod A*, sondern auch in der unterschiedlichen Darstellung von proportionalen Rhythmen in Konkordanzen. Aufgrund des Einflusses der Schreiber rate ich davon ab, die Notation proportionaler Rhythmen in einem bestimmten Stück als Argument für die Datierung dieser Komposition anzuführen, da es möglicherweise nicht der Komponist, sondern der Schreiber war, der das Proportionszeichen bei der Abschrift des Stückes in die Musik einfügte. Ich habe in dieser Studie gezeigt, dass Proportionszeichen sogar im Werk desselben Komponisten nicht immer einheitlich verwendet werden, nicht einmal in denjenigen Werken, die in ein und demselben Manuskript zu finden sind.

Ich habe auch gezeigt, dass eine Analyse des begleitenden Kanons neue Perspektiven auf bestimmte Aspekte der Kompositionen eröffnen. Ein Kanon kann als Peritext, d. h. als Paratext, betrachtet werden, der sich in unmittelbarer Nähe zu dem Notentext befindet. Er ist nicht Teil des Notentextes selbst, kann aber eine wichtige Rolle bei dessen Interpretation spielen. Musiktraktate, die Proportionszeichen behandeln, können ebenfalls als Paratexte, nämlich als Epitexte, angesehen werden. Wie ich gezeigt habe, sind Traktate jedoch nur sehr eingeschränkt dafür geeignet, um eine Hilfestellung für die Interpretation von Proportionszeichen in konkreten Kompositionen zu geben. Die meisten Traktate beschreiben musikalische Notation in einem erwünschten Zustand, der nicht notwendigerweise der Realität in den Kompositionen der *Ars subtilior* entspricht. Deshalb sollten Traktate nicht als Argument zur Erklärung bestimmter Phänomene der Musiknotation herangezogen werden. Der Vergleich von Ugolinos Kompositionen mit seinen Aussagen über Proportionszeichen ist das beste Beispiel für diese Diskrepanz.

SUMMARY

In the manuscripts of the so-called *Ars subtilior* (including music manuscripts and music theory treatises), proportion signs appear in three forms of visual appearance: single Arabic numerals, stacked Arabic numerals and geometric shapes. In the first ever definition of the term ‘proportion sign’, which I have offered in this study, I have made the change of the duration of the minim (or under special circumstances a higher note value) the decisive criterion in order to make a distinction between a mensuration and a proportion sign. The prevailing scholarly opinion that mensuration signs are geometric shapes, while proportion signs are Arabic numerals (single or stacked) is indefensible, not only but especially in light of music theory treatises, in which proportion signs are given in all three forms of visual appearance. The distinction between the two terms ‘mensuration sign’ and ‘proportion sign’ on grounds of visual appearance is a modern misconception. However, a sign can be proportion sign and mensuration sign combined in one sign. Therefore, I have regarded different functions of one sign in my analyses. I have suggested that indicating a certain proportion is not the only function that some of the proportion signs have. They can also indicate musical or textual citations and allusions, the beginning of a melisma, or positions of structural importance to a piece. I have found hitherto undiscovered musical or textual citations and allusions in some compositions from the manuscript *Mod A*.

The analysis of *Ars subtilior* manuscripts for this study speaks in favour of a certain amount of standardisation concerning certain proportion signs. The reversed semicircle $\bigcirc$ is the most frequently appearing proportion sign and it is described or used as a sign for *sesquitertia* (4:3) proportion in both music manuscripts and treatises. As proportion signs, the stacked Arabic numerals $\frac{3}{2}$ are also used exclusively for *sesquialtera* (3:2) proportion. Other proportion signs, for example the single Arabic numerals 2 and 3, do have more than one possible interpretation, nevertheless, the meaning is almost always the one or the other, namely *dupla* (2:1) or *subsesquialtera* (2:3) respectively for 2 as well as *trippla* (3:1) or *sesquialtera* (3:2) proportion respectively for 3. A clear tendency towards standardisation can especially be observed in music treatises, in which unequivocal stacked Arabic numerals are described more often than ambiguous geometric shapes. However, the more practice-oriented treatises, which include musical examples, often give more options of signs for one particular proportion, thereby acknowledging their manifold appearance in music manuscripts.

This manifold appearance can with some certainty be attributed to individualisation, i.e. composers or scribes experimenting with rhythmic notation as they saw fit. Some compositions

contain rare proportion signs. In other cases, conventional proportion signs were used for unconventional proportions, which cannot be derived from musical intervals. Some of these unusual proportion sign choices go beyond the tastes or preferences of a particular individual. Rather, each particular piece has to be looked at and interpreted in its own unique manner. I have attributed these notational peculiarities to the emerging riddle culture that would become a movement in the Renaissance. *Ars subtilior* composers and/or scribes sometimes turned musical notation and its interpretation into a challenging game, intentionally notating pieces in a more complex manner than necessary. Due to these findings, I have argued that it is time to abandon the idea that signs have to bear the same meaning throughout a specific manuscript or a specific period of time, in the works of a particular composer or in all sources copied by the same hand.

Including the music scribes in the debate prompts the question of who was ultimately responsible for the rhythmic notation of a composition. It seems to be a foregone conclusion that the composer is the originator of the notation we find in *Ars subtilior* compositions. However, the analysis of the manuscript *Mod A* reveals that scribes may have had significant influence on the musical notation. This is not only manifested in erasures and corrections done by the scribe in *Mod A* but also by different proportion signs or even means of rhythmic proportion in concordances. Due to the influence of scribes, I advise against adducing the notation of rhythmic proportion in a particular piece as argument for dating this composition, as it might not have been the composer but the scribe introducing the sign to the music when copying the piece. I have shown throughout this study that signs are not used consistently in the *oeuvre* of one composer, even within in the same manuscript.

I have also shown that an analysis of accompanying canons can shed new light on certain aspects of compositions. Canons can be regarded as peritexts, i.e. paratexts, which are located close to the musical text. They are not part of the musical text themselves but can play an important role in its interpretation. Music treatises discussing proportion signs might also be regarded as paratexts, namely epitexts. As I have shown, however, treatises are not very well suited to advise on the interpretation of proportion signs in actual compositions. The majority of treatises describes a desired state of musical notation, not necessarily the situation found in *Ars subtilior* music. This is why they should not be used as an argument to explain certain phenomena in musical notation. The comparison of Ugolino's compositions with his statements on proportion signs is probably the best example for the discrepancy.

LISTE DER AUS DIESER DISSERTATION HERVORGEGANGENEN VORVERÖFFENTLICHUNGEN

Aus dieser Dissertation ist folgende Veröffentlichung hervorgegangen:

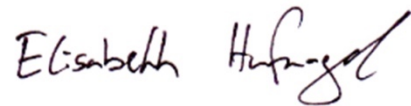
Hufnagel, Elisabeth. “Adapting the Concept of *Proportio* to Rhythm in the *Ars subtilior*.” In *Education Materialised: Reconstructing Teaching and Learning Contexts through Manuscripts*, edited by Stefanie Brinkmann, Giovanni Ciotti, Stefano Valente, and Eva Maria Wilden, 441–64. Berlin: De Gruyter, 2021.

Übereinstimmungen mit Kapitel 1.1 “Proportions in Medieval Music and Music Treatises: an Introduction” und Kapitel 4 “Composer versus Theorist: A Comparison of Ugolino da Orvieto’s Compositions and his Statements on Proportion Signs in the Manuscript *Rome, Biblioteca Casanatense, 2151*” wurden als solche kenntlich gemacht.

EIDESSTATTLICHE ERKLÄRUNG

Hiermit versichere ich an Eides statt, dass ich die Arbeit selbstständig angefertigt, andere als die von mir angegebenen Quellen und Hilfsmittel nicht benutzt und die den herangezogenen Werken wörtlich oder inhaltlich entnommenen Stellen als solche kenntlich gemacht habe.

Hamburg, den 11. April 2023

A handwritten signature in dark ink, reading 'Elisabeth Hufnagel'. The script is cursive and fluid, with the first name 'Elisabeth' written in a more legible hand than the last name 'Hufnagel', which is more stylized.

Elisabeth Hufnagel

