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Beam–Plasma Instabilities in Electromagnetic Cascade from Laboratory to Astrophysical Contexts

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Suman Dey

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Gutachter/innen der Dissertation:

Prof. Dr. Günter Sigl

Prof. Dr. Marcus Brüggén

Zusammensetzung der Prüfungskommission:

Prof. Dr. Robi Banerjee

Prof. Dr. Dieter Horns

Prof. Dr. Johannes Haller

Prof. Dr. Marcus Brüggén

Prof. Dr. Günter Sigl

Vorsitzende/r der Prüfungskommission:

Prof. Dr. Johannes Haller

Datum der Disputation:

18.06.2026

Vorsitzender des Fach-Promotionsausschusses PHYSIK:

Prof. Dr. Johannes Haller

Leiter des Fachbereichs PHYSIK:

Prof. Dr. Markus Drescher

Dekan der Fakultät MIN:

Prof. Dr.-Ing. Norbert Ritter



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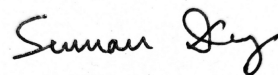
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Zusammenfassung

Die Ausbreitung von Gammastrahlung sehr hoher Energien aus fernen astrophysikalischen Quellen, wie etwa TeV-Blazaren, führt zur Entstehung elektromagnetischer Kaskaden im intergalaktischen Medium. Im Standardbild erzeugen diese Kaskaden sekundäre Gammastrahlung im GeV-Energiebereich durch inverse-Compton-Streuung von Elektron-Positron-Paaren. Beobachtungen haben jedoch eine Unterdrückung dieser erwarteten GeV-Emission gezeigt, die gemeinhin als “fehlende GeV-Kaskade” bezeichnet wird. Diese Diskrepanz wird häufig als Hinweis auf intergalaktische Magnetfelder (IGMF) interpretiert, wenngleich alternative Mechanismen, wie Plasma-Instabilitäten, eine ebenso bedeutende Rolle spielen könnten.

In dieser Arbeit untersuchen wir zunächst die Entwicklung von Strahl-Plasma-Instabilitäten als Funktion der Strahldichte in einem laborrelevanten Regime. Hierzu werden kinetische Particle-in-Cell-Simulationen mit **EPOCH** eingesetzt, um astrophysikalisch relevante Paarstrahlen unter kontrollierten Laborbedingungen zu modellieren. Diese Simulationen liefern eine vollständig kinetische Beschreibung des Systems und ermöglichen die Erfassung von Welle-Teilchen-Wechselwirkungen sowie Instabilitätsrückkopplungen. Die Ergebnisse zeigen, dass schräge Instabilitätsmoden effizient anwachsen und infolgedessen die elektrischen Felder verstärkt werden. Dieser Prozess führt zu signifikanten Energieverlusten des Strahls, einem Energietransfer in das Hintergrundplasma sowie zu einer Winkelaufweitung des Strahls. Wir identifizieren die physikalische Grenze im Dichtekontrast des Strahls, unterhalb derer ein warmer Strahl als verdünnt betrachtet werden kann, basierend auf Laborbedingungen, die astrophysikalische Paarstrahlen aus TeV-Blazaren nachbilden. Diese Ergebnisse werden anschließend auf astrophysikalische Szenarien, einschließlich Quellen wie 1ES 0229+200, extrapoliert. Die Instabilität führt zu einem Energieverlust von etwa 2 bis 4% eines astrophysikalisch relevanten Strahls, während nichtlineare Rückkopplungseffekte marginal bleiben und zu vernachlässigbarer transversaler Aufweitung führen.

Neben der numerischen Untersuchung analysieren wir die Realisierbarkeit solcher Strahl-Plasma-Wechselwirkungen in Laborumgebungen. Es werden experimentelle Aufbauten skizziert, die auf elektronenstrahlgetriebenen Konfigurationen in Laser-Wakefield-Beschleunigern sowie auf protonenstrahlgetriebenen Aufbauten basieren, welche relativistische Elektron-Positron-Strahlen erzeugen können. Zudem werden zentrale Herausforderungen adressiert, darunter die Strahlfilterung, die Ausgestaltung der Hintergrundplasmazelle sowie die Charakterisierung des Instabilitätswachstums. Diese vorgeschlagenen Experimente bieten eine vielversprechende Möglichkeit, die Dynamik von Instabilitäten unter kontrollierten Bedingungen direkt zu untersuchen und theoretische Vorhersagen zu überprüfen.

Anschließend untersuchen wir die Ausbreitung von Elektron-Positron-Paaren im extragalaktischen Raum sowie den Einfluss von Strahl-Plasma-Instabilitäten auf die Entwicklung elektromagnetischer Kaskaden mithilfe des Monte-Carlo-Simulationsframeworks **CRPropa 3.2**. Hierbei implementieren wir ein parametrisches Modell für energieverlustbedingte Effekte durch Plasma-Instabilitäten in **CRPropa 3.2** und simulieren die

Entwicklung elektromagnetischer Kaskaden in Anwesenheit intergalaktischer Magnetfelder (IGMF). Die Kaskadensimulationen unter Berücksichtigung von Instabilitäten und IGMF zeigen, dass die Einbeziehung von Instabilitätseffekten zu einer Unterdrückung des GeV-Flusses führt, wobei Plasma-Instabilitäten jedoch nur einen geringen Beitrag zum Gesamtenergieverlust des Strahls leisten, mit einem Anteil von $\lesssim 2\%$ für Elektronenenergien unterhalb von 4 TeV und $\lesssim 1\%$ im GeV-Bereich. Insbesondere ergibt sich unter Berücksichtigung der Kühlung durch Plasma-Instabilitäten, dass zur Konsistenz mit Beobachtungen des Fermi-LAT eine minimale Magnetfeldstärke von $B \gtrsim 2.7 \times 10^{-17}$ G für ein Sichtfeld von $\theta_{\text{FoV}} = 1.0^\circ$ erforderlich ist, welche sich für ein größeres Sichtfeld von $\theta_{\text{FoV}} = 4.5^\circ$ auf $B \gtrsim 5.4 \times 10^{-17}$ G erhöht. Zusammenfassend zeigen wir, dass Plasma-Instabilitäten die Kaskadenentwicklung nur geringfügig beeinflussen und dass die Kaskadenproduktion überwiegend durch magnetische Ablenkungen im intergalaktischen Magnetfeld bestimmt wird; die daraus abgeleiteten Einschränkungen an das IGMF bleiben daher robust.

Abstract

The propagation of very high-energy gamma rays from distant astrophysical sources, such as TeV blazars, gives rise to electromagnetic cascades in the intergalactic medium. In the standard picture, these cascades produce secondary gamma rays in the GeV energy range through inverse Compton scattering of electron–positron pairs. However, observations have revealed a suppression of this expected GeV emission, commonly referred to as the “missing GeV cascade”. This discrepancy is often interpreted as evidence for intergalactic magnetic fields (IGMF), although alternative mechanisms, such as plasma instabilities, may play an equally important role.

In this thesis, we first investigate the evolution of beam–plasma instabilities as a function of beam density in a laboratory-relevant regime, using kinetic Particle-in-Cell simulations in **EPOCH** to model astrophysically relevant pair beams under controlled laboratory conditions. These simulations provide a fully kinetic description of the system, allowing us to capture wave-particle interactions and instability feedback. The results show that oblique instability modes grow efficiently, and as a result, the electric fields grow. This process leads to significant beam energy loss, energy transfer into background plasma, and angular broadening of the beam. We identify the physical limit in beam density contrast below which a warm beam can be considered dilute, under laboratory conditions designed to replicate astrophysical pair beams produced by TeV blazars, and further extrapolate these findings to astrophysical scenarios, including sources such as 1ES 0229+200. The instability leads to an energy loss of approximately 2 to 4% of an astrophysically relevant beam, while nonlinear feedback remains marginal, yielding negligible transverse broadening.

In addition to the numerical study, we explore the feasibility of reproducing such beam–plasma interactions in laboratory environments. We outline experimental setups based on electron-beam-driven configurations in laser wakefield accelerators and proton-beam-driven setups that can generate relativistic electron–positron beams. We also address the main challenges related to beam filtering, the background plasma cell, and the characterization of instability growth. These proposed experiments offer a promising pathway to directly probe instability dynamics under controlled conditions and to validate theoretical predictions.

Next, we investigate the propagation of electron–positron pairs through the extragalactic space and the impact of beam–plasma instabilities on cascade development using a Monte Carlo **CRPropa 3.2** simulation framework. We incorporate a parametric model of plasma instability-induced energy losses into the **CRPropa 3.2** and simulate the development of electromagnetic cascades in the presence of IGMF. The cascade simulations with instability and IGMF show that including instability effects suppresses the GeV flux, but plasma instabilities contribute only marginally to the total energy loss of the beam, with a fractional loss of $\lesssim 2\%$ for electron energies below 4 TeV and $\lesssim 1\%$ in the GeV regime. In particular, accounting for plasma instability cooling, consistency with Fermi-LAT observations requires a minimum magnetic field strength of $B \gtrsim 2.7 \times 10^{-17}$ G for a field of view of $\theta_{\text{FoV}} = 1.0^\circ$, which becomes $B \gtrsim 5.4 \times 10^{-17}$ G

for $\theta_{\text{FoV}} = 4.5^\circ$. In summary, we find that plasma instabilities have only a minor impact on cascade evolution and that cascade production is predominantly governed by magnetic deflections in the IGMF; therefore, the IGMF constraints remain robust.

*“To succeed in life and achieve results, you must understand and master
three mighty forces — desire, belief, and expectation”.*
— A.P.J. Abdul Kalam, Wings of Fire

“আমাদের সমুদয় শিক্ষা-দীক্ষা কেবল মনুষ্যত্বলাভের উদ্দেশ্যে মাত্র।
(*All our education and training exist solely for the purpose
of attaining true humanity.*)”
— Jagadish Chandra Bose, অব্যক্ত (Abaktya)

List of Publications

Journal Papers

- **S. Dey**, S. Rossoni, and G. Sigl. (2026). *The impact of plasma instability cooling on intergalactic magnetic field constraints in GeV cascades for optimized instability cooling parameters*. **Astroparticle Physics**, 182, 103275. arXiv: 2604.08375 [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2026.103275>.
Author contributions: S. Dey performed the simulations, analyzed the simulation results, prepared the figures, and wrote the manuscript. S. Rossoni contributed to writing the manuscript, discussing the results, and reviewing the manuscript. G. Sigl supervised the research, provided numerical resources, contributed to the discussion of the results, and reviewed the manuscript.
- **S. Dey** and G. Sigl. (2025). *Simulations of astrophysically relevant pair beam instabilities in a laboratory context*. **Astroparticle Physics**, 173, 103153. arXiv:2501.14518 [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2025.103153>.
Author contributions: S. Dey performed the simulations, analyzed the simulation results, prepared the figures, and wrote the manuscript. G. Sigl supervised the research, contributed to the discussion of the results, and reviewed the manuscript.

Conference Proceedings

- **S. Dey** and G. Sigl. (2025). *Impact of plasma instabilities and of the intergalactic magnetic field on blazar-induced electromagnetic cascades*. **PoS (ICRC2025)** 627. <https://doi.org/10.22323/1.501.0627>.
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List of Abbreviations

AGN	Active Galactic Nucleus
CMB	Cosmic Microwave Background
COB	Cosmic Optical Background
CR	Cosmic Ray
CGB	Cosmic Gamma-ray Background
CIB	Cosmic Infrared Background
CXB	Cosmic X-ray Background
EBL	Extragalactic Background Light
GRB	Gamma-ray Burst
HEGR	High-energy gamma-ray
IC	Inverse Compton
IGM	Intergalactic Medium
IGMF	Intergalactic Magnetic Field
MHD	Magnetohydrodynamics
PIC	Particle-in-Cell
SFR	Star Formation Rate
UHEGR	Ultra-high-energy gamma-ray
VHEGR	Very-high-energy gamma-ray

Introduction

The large-scale structure of the Universe is dominated by regions of low matter density on cosmological scales known as cosmic voids. Although these regions may appear empty, they are filled with a tenuous plasma called the intergalactic medium, permeated by diffuse radiation fields, including the cosmic microwave background and the extragalactic background light. These environments are also considered to host extremely weak magnetic fields, commonly referred to as intergalactic magnetic fields, whose origin may be connected to processes in the early Universe [1–3]. Due to their extremely low field strengths, these magnetic fields cannot be measured directly, and their properties must be inferred from indirect astrophysical observations [4, 5].

One of the most effective ways to study the intergalactic medium and the intergalactic magnetic field is through high-energy gamma-ray emission from distant astrophysical sources, such as active galactic nuclei, particularly TeV blazars and gamma-ray bursts [6–8]. These events produce highly energetic photons that travel across cosmological distances before reaching Earth. During their propagation, very-high-energy gamma rays interact with the background photon field, producing electron-positron pairs and initiating electromagnetic cascades [9–11]. These secondary particles can further interact with ambient photons via inverse Compton scattering, generating a cascade of lower-energy gamma rays in the GeV range.

However, observations with instruments such as the Fermi-LAT have discovered a suppression of the GeV flux relative to predictions based on standard electromagnetic interactions in vacuum and in the absence of magnetic fields [7, 12]. This so-called *missing GeV cascade* has typically been interpreted as evidence for intergalactic magnetic fields, which can deflect charged particles and reduce the observed flux [13]. This interpretation has led to important constraints on the strength and coherence length of the intergalactic magnetic field, in which magnetic deflection is regarded as the dominant mechanism governing the spatial development of the cascade. Observational studies have placed constraints on the intergalactic magnetic field under different assumptions. For long coherence lengths, several works suggest lower bounds of $B \gtrsim 10^{-16}$ G, assuming that the observed TeV emission from blazars represents their typical activity over timescales of $\sim 10^6$ years [13, 14]. If the sources are instead considered active over time intervals typically ranging from years to decades, the constraint weakens to $B \gtrsim 10^{-17}$ G [15–17]. Analyses based on Fermi-LAT observations report limits at the level of $B \gtrsim 3 \times 10^{-16}$ G [12, 18], although these results still rely on assumptions regarding the time variability of TeV emis-

sion. Additional constraints arise from searches for extended emission around nearby blazars and from combined observations using instruments such as MAGIC¹ [19, 20], H.E.S.S.² [21], and LHAASO³ [6, 22], which explore different regions of energy range and angular scale. In particular, studies of sources such as 1ES 0229+200 and GRB 221009A have reported lower bounds extending down to $B \sim 10^{-19} - 10^{-16}$ G, depending on the coherence length and source properties [6, 7, 22, 23]. These results highlight that current constraints remain strongly dependent on underlying assumptions about source activity and cascade development.

An alternative explanation that has gained significant attention is the influence of collective plasma effects in cascade production [24–36]. The electron–positron pairs produced from gamma–gamma interactions form a highly relativistic, dilute beam propagating through the background plasma of the intergalactic medium. Such beam–plasma systems are known to be unstable and can give rise to various modes of instability [37–42]. These instabilities can transfer energy from the beam to the plasma via wave–particle interactions, potentially suppressing the cascade without requiring a background magnetic field. As a result, plasma instabilities may play a crucial role in interpreting gamma-ray observations and in determining reliable constraints on the intergalactic magnetic fields. Despite comprehensive analytical and numerical studies [24, 26, 43–45], the relative importance of plasma instabilities remains an open question. One of the main challenges is that realistic astrophysical pair beams are extremely dilute, making them difficult to simulate directly with current computational resources. Laboratory experiments provide a complementary approach to studying beam–plasma interactions under controlled conditions. In recent years, advances in high-intensity laser facilities, particularly laser wakefield accelerators (LWFA) [46], have made possible the generation of relativistic electron beams that can produce electron–positron pairs upon interaction with high-Z targets [47–50]. In addition to electron-driven setups, proton beam–driven experiments have been explored [51–54], in which high-energy protons interacting with a dense target initiate particle cascades that produce electrons and positrons. These approaches offer promising pathways to recreate key aspects of astrophysical pair beams on much smaller spatial and temporal scales. However, achieving conditions that accurately represent astrophysical scenarios remains challenging, primarily due to limitations in beam density, energy spread, and propagation length. Furthermore, the produced beams often contain secondary particles and exhibit significant divergence, necessitating careful beam handling and filtering.

In this thesis, we study the *missing GeV cascade* problem using a combination of analytical considerations and numerical simulations. We employ Particle-in-Cell simulations using EPOCH [55] to investigate beam–plasma instabilities under conditions that capture the essential physics of astrophysical pair beams while remaining computationally feasible. In addition, we use the CRPropa 3.2 Monte Carlo framework [56] to model the development of electromagnetic cascades in intergalactic space and study the impact of different physical processes on observable gamma-ray spectra. We have used SI units [57] in this thesis unless stated otherwise.

The structure of this thesis is as follows. In Chap. 2, we review the propagation of gamma

¹Major Atmospheric Gamma Imaging Cherenkov (MAGIC) observes gamma rays in the very-high-energy range 30 GeV–100 TeV (<https://magic.mpp.mpg.de/>).

²High Energy Stereoscopic System (H.E.S.S.) operates in the very-high-energy gamma-ray range from tens of GeV to several tens of TeV (<https://hess-experiment.eu/about-hess/>).

³Large High Altitude Air Shower Observatory (LHAASO) is a ground-based gamma-ray observatory designed to study very- and ultra-high-energy gamma rays (above 100 GeV) with high sensitivity and a wide field of view, enabling large-scale surveys of the sky (<https://english.ihep.cas.cn/lhaaso/>).

rays from extragalactic sources and the formation of electromagnetic cascades, including the role of background radiation fields and observational techniques. In Chap. 3, we discuss the theoretical framework of extragalactic magnetic fields, including their possible origins from primordial and astrophysical mechanisms, their subsequent evolution under magnetohydrodynamic processes, and the implications of current observational constraints. Chap. 4 introduces the fundamental physics of beam–plasma instabilities, including their kinetic description and dominant instability modes. In Chap. 5, we describe the numerical methods used in this work, focusing on the Particle-in-Cell approach and the simulation frameworks employed. Chap. 6 presents our main results on beam–plasma instabilities in laboratory-relevant regimes, including the evolution of fields, energy transfer, and implications for astrophysical scenarios. In Chap. 7, we discuss the generation of pair beams in laboratory experiments and the practical challenges associated with beam handling and diagnostics. In Chap. 8, we investigate the impact of plasma instabilities on the development of electromagnetic cascades and the resulting constraints on the intergalactic magnetic field, using the CRPropa 3.2 Monte Carlo framework. Finally, in Chap. 9, we summarize our findings and outline possible directions for future research.

Gamma-ray Propagation

This chapter outlines the physical concepts and observational framework relevant to TeV blazar-induced pair beams and the formation of electromagnetic cascades. In Sec. 2.1, we begin discussing the production of very-high-energy gamma-rays (VHEGRs) in extragalactic sources, with particular emphasis on blazars and gamma-ray bursts (GRBs). Sec. 2.2 addresses the interactions of these gamma rays with ambient radiation fields, such as the Extragalactic Background Light (EBL) and the cosmic microwave background (CMB), during their propagation through the intergalactic medium (IGM), which leads to the initiation of electromagnetic cascades. In Sec. 2.3, we examine proposed explanations for the non-detection of secondary cascade emission. Finally, in Sec. 2.4, we discuss the relevant observational techniques and current experimental results.

2.1 Extragalactic sources

Thermal radiation from various astrophysical components, such as stars, dust, and gas, is primarily observed at optical and infrared wavelengths. In addition to this thermal emission, we also detect non-thermal radiation across a range of wavelengths, including radio, X-ray, and gamma-ray bands. The acceleration of charged particles in extreme astrophysical environments gives rise to this non-thermal radiation. These high-energy particles produce radiation through mechanisms such as synchrotron emission, in which they gyrate around magnetic field lines, and IC scattering, in which they transfer energy to low-energy target photons [58]. Gamma rays in the energy range $100 \text{ MeV} \lesssim E \lesssim 100 \text{ GeV}$ are commonly referred to as high-energy gamma-ray (HEGR), while those with energies $E \gtrsim 100 \text{ GeV}$ are typically known as VHEGR [59].

Fig. 2.1 shows the number of observed extragalactic X-ray, HEGR, and VHEGR sources over time. The birth of high-energy astrophysics began with a breakthrough in X-ray detection that fundamentally altered our understanding of the ‘quiet’ universe. On June 18, 1962, a research team led by Giacconi et al. [60] detected the first evidence of X-ray emission from beyond the solar system, using an Aerobee sounding rocket launched from the White Sands Missile Range in New Mexico. This source, later identified as Scorpius X-1 (Sco X-1), was remarkably luminous, and the X-ray-dominated emission proved that high-energy processes were not limited to our

own Sun. This discovery paved the way for the first orbital surveys, such as the Uhuru satellite, which had cataloged 125 sources by 1972, and this number increased to 339 with its final release [61, 62]. Simultaneously, the GeV gamma-ray window opened in 1970 with the detection of the Crab Nebula by McBreen et al. [63], marking the start of systematic gamma-ray exploration.

A pivotal moment in the early 1970s was the launch of the second NASA Small Astronomy Satellite (SAS-2) on November 15, 1972 [64]. SAS-2 was dedicated to gamma-ray observations in the 35 MeV range and was equipped with a 32-level wire spark chamber. It underwent detailed calibration through studies conducted at the Deutsches Elektronen-Synchrotron (DESY) in Hamburg, which covers the energy range from 200 to 1000 MeV [64]. This work was continued by the Celestial Observation Satellite B (COS-B) mission (1975–1982) of the European Space Agency, which produced the first significant catalogs of gamma-ray sources: the CG catalog in 1977 with 13 sources and the 2CG catalog in 1981 with 25 sources [65–67]. These foundational missions set the stage for the 1989 ground-based detection of the Crab Nebula by the Whipple Observatory, which confirmed that the Imaging Atmospheric Cherenkov Technique (IACT) could open the Very High Energy (VHE) TeV frontier [68–70].

Today, high-energy astronomy is in a “golden age” of high-statistics catalogs and multi-wavelength discovery. Modern X-ray observatories such as Chandra [86] and XMM-Newton [87, 88] provide exceptional angular resolution and sensitivity, enabling the systematic identification and characterization of more than 1.3 million astrophysical sources. The Fermi Large Area Telescope (LAT) is the leading instrument in the GeV energy range. The updated source catalogs of Fermi-LAT, such as the recent 4FGL-DR4 (2024) release [78], list more than 7100 detected sources. The VHE frontier shows similar growth, as third-generation IACT arrays like High Energy Stereoscopic System (H.E.S.S.) [80], Major Atmospheric Gamma Imaging Cherenkov Telescope (MAGIC) [81], and Very Energetic Radiation Imaging Telescope Array System (VERITAS) [82], alongside wide-field detectors like High-Altitude Water Cherenkov Observatory (HAWC) [83] and Large High Altitude Air Shower Observatory (LHAASO) [84], have expanded the TeV catalog from a single source in 1989 to 274 sources as of early 2024. The Cherenkov Telescope Array (CTA) [85] is expected to revolutionize high-energy astrophysics by providing an order-of-magnitude improvement in sensitivity, enabling the expansion of the known TeV gamma-ray source catalog from a few hundred sources to several thousand.

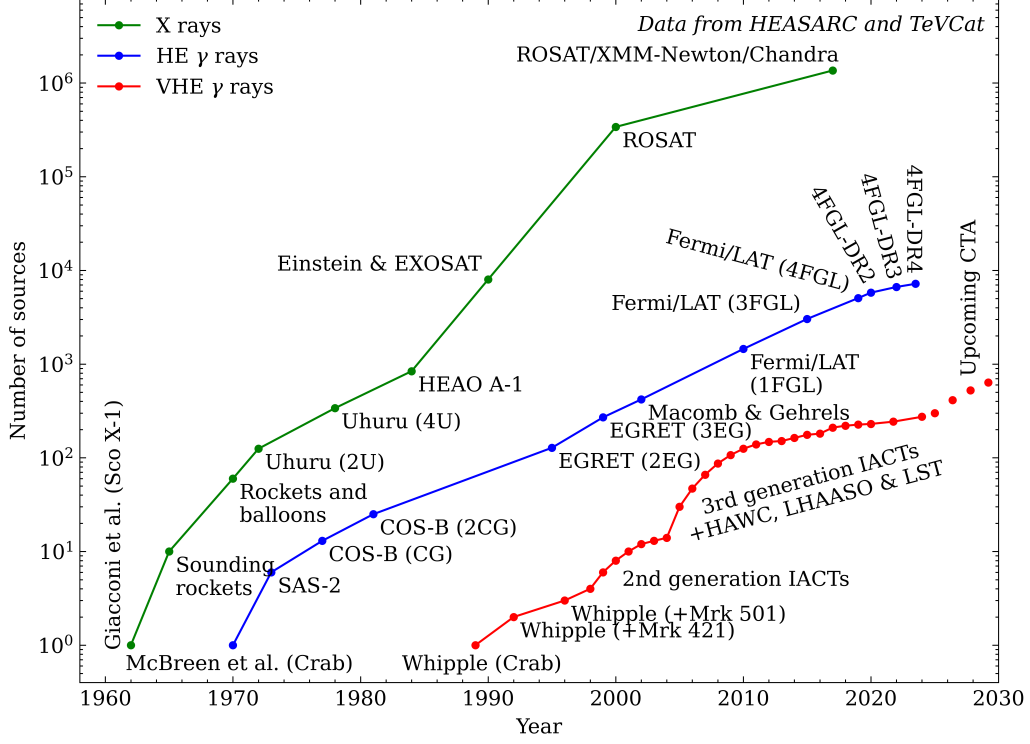


Figure 2.1: The number of extragalactic gamma-ray sources detected over time (Image credit: Stephen Fegan¹). The green line shows the historical growth in X-ray source detections, from early sounding rockets to major observatories such as ROSAT, XMM-Newton, and Chandra. The blue line represents detections by space-based **HEGR** telescopes, including the early SAS-2 and COS-B missions, followed by EGRET (2EG [71], 3EG [72]) and Fermi-LAT (1FGL [73], 3FGL [74], 4FGL [75], 4FGL-DR2 [76], 4FGL-DR3 [77], 4FGL-DR4 [78]). The red line represents detections from ground-based **VHEGR** detectors, starting with the detection of the Crab Nebula by Whipple Observatory [68, 79] in 1989. It shows the progression from second-generation IACTs to current third-generation arrays (H.E.S.S. [80], MAGIC [81], VERITAS [82]) and to HAWC [83], LHAASO [84], and the upcoming CTA [85].

2.1.1 AGN and TeV Blazars

An **AGN** is the compact, extremely luminous region at the center of a galaxy, powered by the accretion of matter onto a supermassive black hole (SMBH) with masses typically ranging from 10^5 to $10^9 M_\odot$ [89]. These systems launch collimated, relativistic jets of magnetized plasma along their axis of rotation. Fig. 2.2 shows the unified model of **AGN**, in which the different observed classes of **AGNs** are interpreted as the same physical system viewed at different orientations of the accretion disk and the jet with respect to the observer. The environment surrounding the SMBH is divided into distinct regions based on their distance from the center and their physical properties. The dense gas clouds located very close to the black hole (within light-days or light-weeks) are called the Broad-line Region (BLR). The BLR consists of gas orbiting the central SMBH at very high speeds, typically thousands of kilometers per second. This motion broadens the emitted spectral lines due to the Doppler effect [90]. Narrow-line Region (NLR)

¹Figure from <https://github.com/sfegan/kifune-plot>.

is located much further out (hundreds to thousands of light-years). The gas in this region is less dense and moves more slowly, and the spectral lines produced here are much thinner or ‘narrower’ than those from the BLR [91]. The dusty torus is the doughnut-shaped ring of gas and dust that surrounds the central engine and the BLR. This hot, dusty component emits radiation in the near- and mid-infrared wavelengths. The torus obstructs the line of sight to the BLR when an active galactic nucleus (AGN) is viewed edge-on, making only the clouds in the NLR visible [92]. The primary division in AGN is based on their radio emission, which depends on whether the central black hole has launched a powerful, large-scale jet. Radio-quiet AGNs, typically with luminosities $L < 5 \times 10^{41} \text{ erg} \cdot \text{s}^{-1}$, are a class of AGN that have accretion disks but lack strong relativistic jets [93]. Seyfert galaxies usually appear like normal spiral galaxies [94, 95]. AGN that exhibit both NLR and BLR emission lines in their optical spectra are classified as Seyfert 1 type sources. The NLR emission line spectrum includes both allowed transitions, such as the hydrogen Balmer lines ($\text{H}\alpha$, $\text{H}\beta$) and lines of helium, carbon, and magnesium (He I, He II, Mg II, and C IV), as well as forbidden transitions of doubly ionized oxygen (O III) [96]. However, the BLR emission is restricted to permitted transitions and exhibits high-velocity widths, typically exceeding 1500 km/s. Galaxies containing an AGN that exhibit only NLR allowed and forbidden emission lines are classified as Seyfert 2 galaxies. In these systems, the characteristic BLR emission lines are conspicuously absent from the optical spectrum. Radio-quiet quasars have spectral properties similar to Seyfert 1 galaxies but at much higher luminosities. They are often bright enough to outshine their entire host galaxy. Radio-loud AGNs, characterized by luminosities $L > 5 \times 10^{42} \text{ erg} \cdot \text{s}^{-1}$, launch powerful jets that emit strongly in radio wavelengths [97].

Radio galaxies are the radio-loud counterparts of Seyfert galaxies and are almost always of elliptical type. A Broad-Line Radio Galaxy is similar to a Seyfert 1 galaxy, in which both the NLR and BLR are visible. A Narrow-Line Radio Galaxy analogous to a Seyfert 2, with the BLR obscured by the torus. Radio-loud quasars are extremely luminous sources that emit a strong radio jet. If the jet is pointed directly at Earth, it is classified as a Blazar. In this thesis, we focus on TeV Blazars that exhibit significant emission at energies above 100 GeV. A significant benchmark in the study of extragalactic VHEGR sources is blazar 1ES 0229+200. This is regarded as one of the nearest known TeV blazars, located at a redshift of $z \approx 0.14$. The emitted jet radiation is confined to a narrow forward-directed cone with an opening angle of approximately $\theta \sim 1/\gamma$, as the jet plasma moves with a bulk Lorentz factor γ [29]. The observed photon energy is higher than its value in the jet rest frame due to relativistic boosting. The spectral energy distribution (SED), shown in Fig. 2.3 (bottom), extends over nearly 13 orders of magnitude and exhibits the characteristic “double-hump” structure that defines blazar emission spectra. The low-energy hump is constrained using data from SWIFT-X-ray (XRT) [99] and SWIFT-optical/UV (UVOT) [100, 101], together with archival BeppoSAX observations [102, 103]. The Swift-UVOT measurements include the Ultraviolet Wide band 1 (UVW1) band (effective wavelength $\sim 260 \text{ nm}$ and bandwidth $\sim 69 \text{ nm}$), which samples the near-ultraviolet regime. For comparison, the UVW2 band lies at shorter wavelengths within the ultraviolet, with an effective wavelength of $\sim 193 \text{ nm}$ and a bandwidth of $\sim 66 \text{ nm}$. The corresponding imaging counterparts are shown in Fig. 2.3 (top): panel (a) displays the Swift-UVOT UVW1 [104] image of 1ES 0229+200, while panel (b) presents the Sloan Digital Sky Survey Data Release 9 (SDSS-DR9) [105] color-composite image constructed from the green (g), red (r) and near-infrared (i) edge of optical bands (covering wavelengths of approximately 400–800 nm). The first peak in

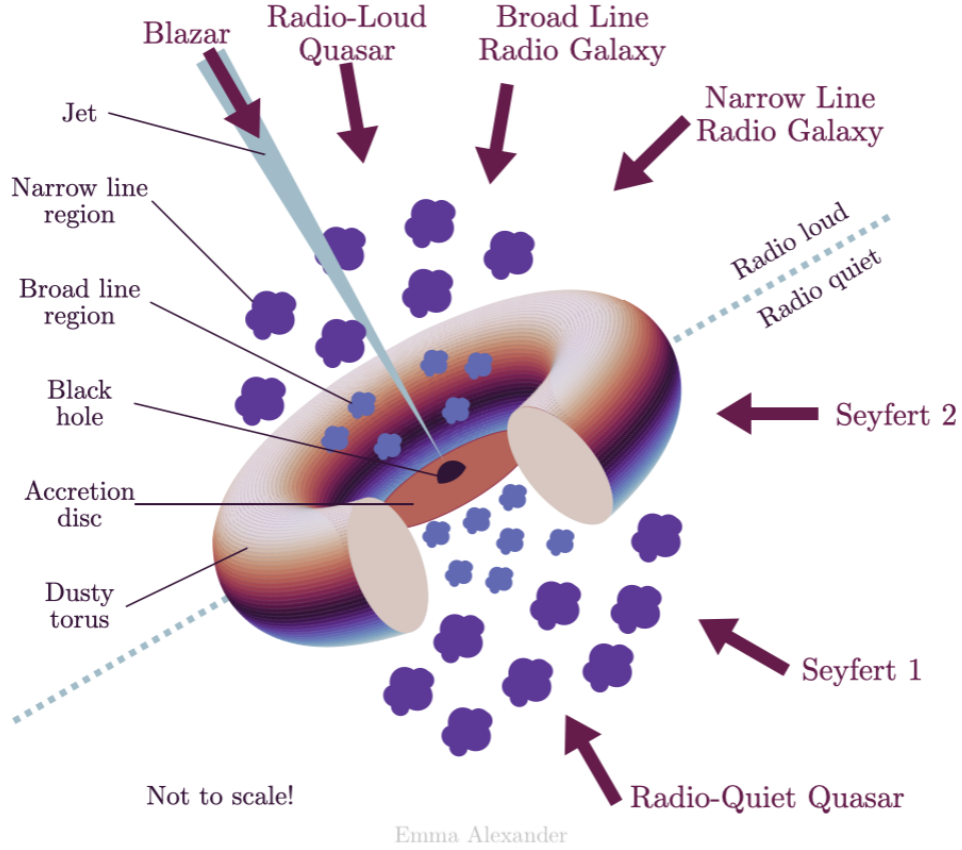


Figure 2.2: Schematic of the unified model of an AGN. The observational classification of an AGN is primarily determined by its orientation relative to the line of sight of the observer and the direction of relativistic jets. Blazars represent a subclass of radio-loud AGN in which the jet is oriented directly toward the Earth (Image courtesy: Emma Alexander²[98]). It is important to note that the asymmetry of the schematic is intended to illustrate different AGN classes (radio-loud/radio-quiet) and does not represent a single physical object.

the SED is typically located between the radio and optical/UV bands, arising from synchrotron radiation emitted by relativistic leptons gyrating in the magnetic fields of the jet. The second peak in the SED corresponds to the high-energy emission component, typically observed in the gamma-ray regime. This high-energy component is empirically well-defined by a synergy of space-based and ground-based observations. The Fermi LAT provides data across the MeV to GeV energies [106], while ground-based IACTs such as H.E.S.S. [107], MAGIC [20], and VERITAS [108], extend these observations into the VHEGR domain (> 100 GeV).

Leptonic and hadronic models typically provide physical interpretations of the high-energy peak in blazar spectra. In leptonic models, the high-energy peak is produced by inverse Compton (IC) scattering, in which the same population of relativistic leptons responsible for synchrotron emission at lower energies “up-scatters” those photons to the gamma-ray energy range. This process, commonly known as Synchrotron Self-Compton (SSC), provides a self-consistent mechanism

²<https://emmaalexander.github.io/resources.html>

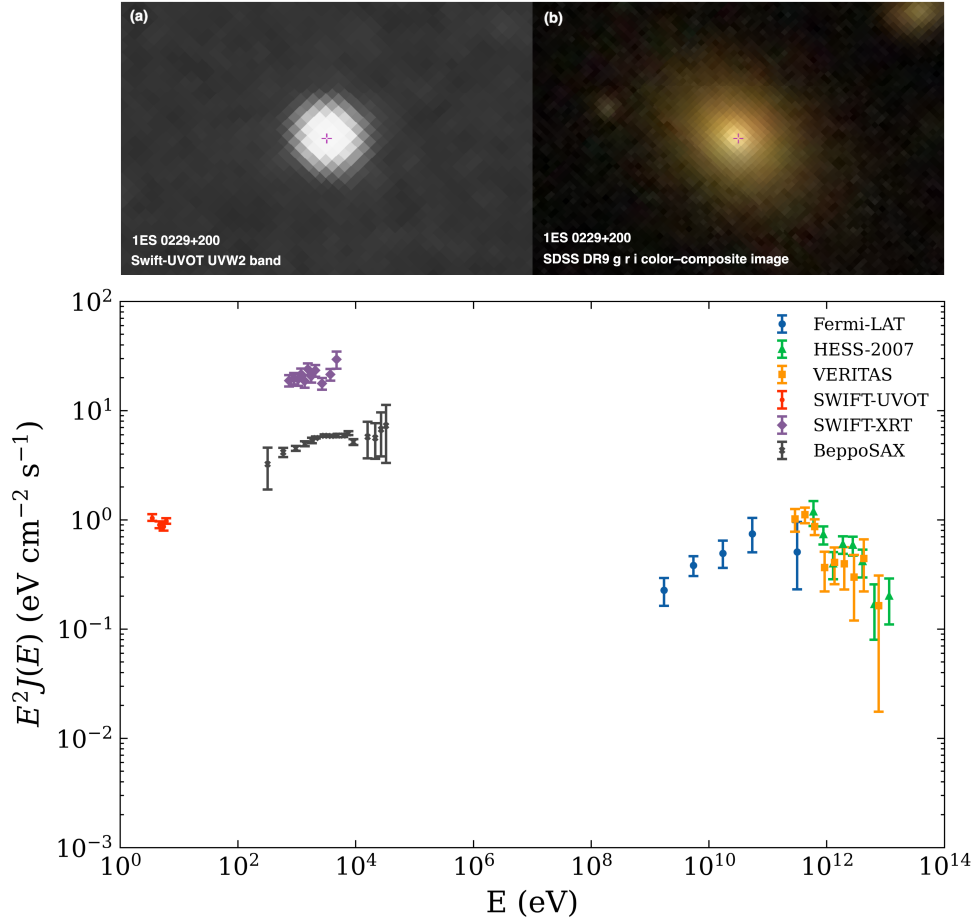


Figure 2.3: The bottom panel presents the multi-wavelength spectral energy distribution of 1ES 0229+200 measured between MJD 55000 and MJD 59000. Some data include corrections for propagation effects and background contributions. The right image in the top panel shows the Swift-UVOT UVW image of 1ES 0229+200. The left image presents the SDSS-DR9 color-composite image constructed from the green (g), red (r), and near-infrared (i) optical bands, covering wavelengths of approximately 400 – 800 nm.³

that links the two peaks of the spectral energy distribution. In contrast, hadronic models propose that the high-energy emission originates from ultra-relativistic protons and ions (hadrons) accelerated within the jet. These particles interact with the surrounding local ambient radiation fields, initiating photo-pion production processes that decay into gamma rays. While both models can explain observed gamma-ray emissions, they imply different physical conditions and particle populations within the jet.

³Data retrieved from the CDS Portal (<http://cdsportal.u-strasbg.fr>), which provides integrated access to the SIMBAD, VizieR, and Aladin databases maintained by the Centre de Données astronomiques de Strasbourg.

2.2 Radiation fields in intergalactic space

The **IGM** represents the primary baryonic reservoir of the universe and permeates the expansive voids between galactic structures. It is characterized by an exceptionally low particle density, with a typical electron number density of $n_e \approx 10^{-7} \text{ cm}^{-3}$ [34, 109]. This degree of rarefaction is extreme, surpassing even the vacuum conditions attainable in state-of-the-art laboratory environments. For context, ultra-high-vacuum systems at facilities such as CERN typically sustain particle densities of order $10^2 - 10^3 \text{ atoms/cm}^3$ [110], which are still many orders of magnitude higher than those of the **IGM**. The intergalactic medium not only contains diffuse baryonic matter but also follows the same large-scale gravitational scaffolding, shaped primarily by dark matter. The present-day fraction of baryons residing outside galaxies is believed to exceed the baryonic mass bound within galactic systems. The left panel of Fig. 2.4 represents this connec-

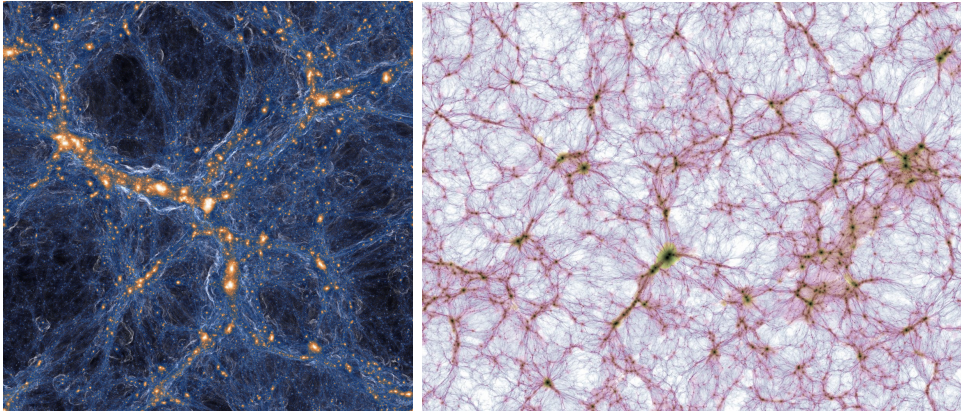


Figure 2.4: The left panel shows the connection between the intensity of shock waves in the cosmic gas (blue) and the surrounding collapsed dark matter structures (orange and light yellow). The right panel shows a thin slice through the cosmic large-scale structure from the largest IllustrisTNG simulation. The image brightness represents the baryonic mass density, while color represents the average baryonic gas temperature, across a region spanning approximately 1.2 billion light-years. Image credit: IllustrisTNG Collaboration (The Next Generation Illustris)⁴

tion by showing the intensity of shock waves in the cosmic gas (blue) surrounding collapsed dark matter structures (orange/light yellow) [111]. Analogous to a sonic boom, gas in these shock fronts is impulsively accelerated as it impacts cosmic filaments and galaxies. The right panel of Fig. 2.4 presents a thin cross-section through the cosmic large-scale structure from the largest simulation of the IllustrisTNG project [112]. In this image, brightness represents the mass density, while color indicates the average temperature of baryonic matter. The region shown spans approximately 1.2 billion light-years from left to right. This simulation is currently the most extensive magneto-hydrodynamic (**MHD**) simulation of galaxy formation, incorporating a total of 30 billion volume elements and particles [113].

Following the epoch of recombination at $z \approx 1100$, the **IGM** remained predominantly neutral until the first luminous structures began to form. The cosmic transition from darkness to the first light began at approximately $z \approx 70$, when the most extreme density peaks in the primordial universe collapsed to form the first stars within low-mass halos of 10^5 to $10^7 M_\odot$ [114, 115]. These

⁴IllustrisTNG Collaboration: <https://www.tng-project.org/media/>.

Population III stars served as the initial luminous sources of the universe, emerging during the “Dark Ages” while the IGM was still entirely neutral [116]. As star formation in these minihalos accelerated and reached saturation between $z \approx 20$ and $z \approx 30$, these massive, metal-free stars became the dominant source of radiative feedback [117]. This early luminous activity initiated the epoch of cosmological reionization, during which a uniform background of high-energy photons (> 13.6 eV) ionized the neutral hydrogen, converting the IGM into a warm-hot plasma with characteristic temperatures of order 10^4 K [34]. This evolutionary pathway is closely coupled to galaxy formation and growth, as both large-scale feedback from galaxies and the energetic influence of supermassive black holes progressively heat and chemically enrich the IGM.

Direct observation of the IGM is a significant challenge due to its extreme dilution. However, the properties of IGM can be inferred from the Lyman-alpha ‘forest’. When the light from distant, luminous quasars travels through the IGM, neutral hydrogen clouds along the line of sight absorb specific wavelengths, creating a dense ‘forest’ of absorption lines in the observed spectrum. At redshifts between $2 < z < 5$, these lines are shifted into the visible spectrum, allowing ground-based observatories like SDSS to map the thermal and chemical history of IGM [118–120]. Observing IGM at lower redshifts ($z < 2$) remains challenging because the continued expansion of the universe has further reduced the gas density, making it increasingly diffuse [121].

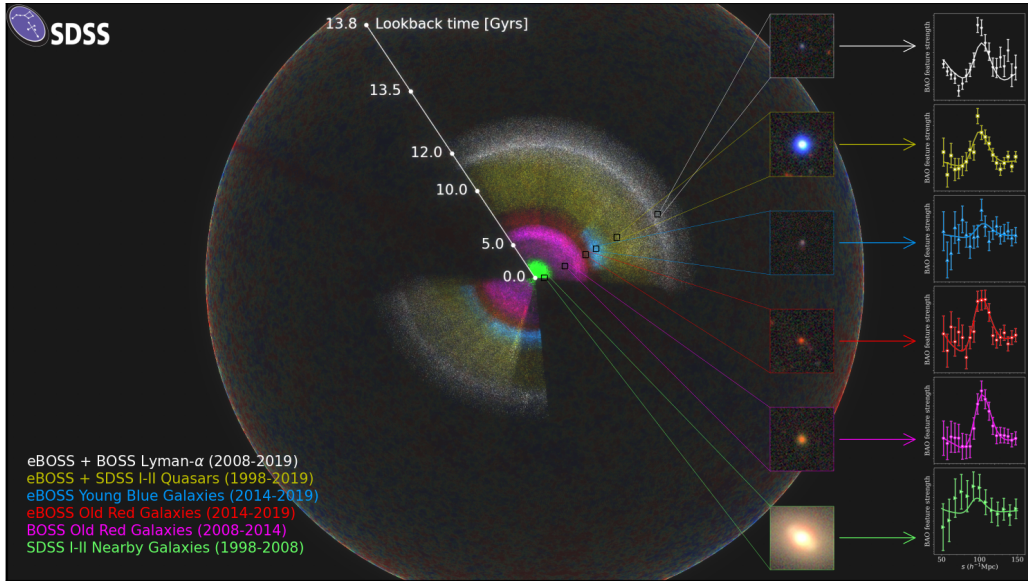


Figure 2.5: A map of the large-scale structure of the universe from the eBOSS (extended Baryon Oscillation Spectroscopic Survey), showing the distribution of galaxies, quasars, and the Lyman-alpha forest over ~ 11 billion years of cosmic history. The map is centered on Earth and uses lookback time as the radial scale, with different tracers highlighting the evolution of matter across cosmological epochs. The outer sphere, showing fluctuations in the CMB. Image credit: Anand Raichoor (EPFL), Ashley Ross (Ohio State University), and the SDSS Collaboration

The eBOSS (extended Baryon Oscillation Spectroscopic Survey)⁵ map, shown in Fig. 2.5 and produced by SDSS, provides a detailed 3D visualization of the large-scale structure of the

⁵eBOSS: <https://www.sdss.org/surveys/eboss/>.

observable universe, covering approximately 11 billion years of cosmic history [122]. Centered on Earth, the map uses a radial scale based on lookback time (in Gyrs) to show how the distribution of matter has evolved. The survey includes a range of astronomical tracers such as nearby galaxies shown in green, luminous red galaxies in pink and red, emission-line galaxies in blue, and distant quasars in yellow. The Lyman-alpha forest is shown in white, effectively linking the local universe to the primordial fluctuations of the CMB (discussed further in Sec. 2.6), which are represented on the outer sphere. The inset graphs show the Baryon Acoustic Oscillation (BAO) signals derived from each data set. These BAO features serve as ‘standard rulers’, allowing for precise measurements of the distance-redshift relationship and enabling researchers to trace the expansion history of the universe over various cosmological epochs.

In Sects. 2.2.1 and 2.2.2, we discuss the radiation components of the IGM, specifically the CMB and EBL. These radiation fields play a fundamental role in governing the propagation of TeV gamma rays through the IGM. In Sec 2.3, we discuss the missing GeV cascades from TeV blazar-induced beams. The detection techniques for gamma rays are addressed in the next Section. .

2.2.1 Cosmic microwave background radiation

In the early stages of the universe, the high temperatures sustained matter in the form of a fully ionized primordial plasma. In this regime, photons were not free to propagate independently but effectively ‘trapped’ or tightly coupled to free electrons. As the universe expanded and cooled significantly below the 13.6 eV ionization threshold of hydrogen (around 3000 K, or 0.26 eV), electrons and protons began to bind into neutral atoms ($e^- + p^+ \leftrightarrow H + \gamma$), a process known as recombination. This transition did not occur instantaneously; instead, it followed a freeze-out of the ionization fraction described by the Saha equation, at a redshift of $z \approx 1100$ [123]. This transition made the universe transparent, allowing photons to propagate freely for the first time. These primordial photons are observed today as the CMB, a nearly perfect black-body radiation field with a present-day temperature of $T = 2.725$ K [124]. The Fig. 2.6 shows the temperature fluctuations of the CMB (excluding the dipole component) as measured by the Planck satellite [125]. This map is created using the Spectral Matching Independent Component Analysis (SMICA) algorithm, which separates the primordial cosmic signal from astrophysical foreground [126, 127]. This map represents the oldest light in the universe, emitted approximately 400000 years after the Big Bang during the epoch of recombination. Although the map appears almost uniform at first glance, the subtle color variations correspond to extremely small temperature anisotropies, on the order of about $100 \mu\text{K}$. The power spectrum, which represents the two-point correlation function of these temperature fluctuations, provides a precise tool for studying the evolution of the early universe [128, 129].

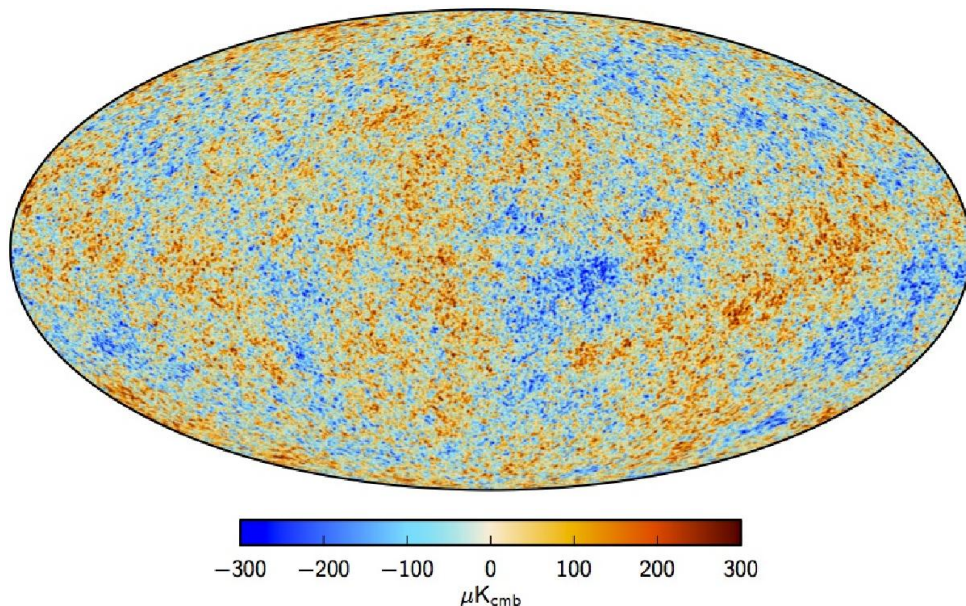


Figure 2.6: Full-sky map of the CMB temperature fluctuations, as measured by the Planck satellite. The monopole and dipole components have been subtracted. (Image credit: ESA⁶ and the Planck Collaboration [125])

2.2.2 Extragalactic background photons

The Extragalactic Background Light (EBL) represents the cumulative diffuse radiation produced by all cosmic sources beyond the Milky Way throughout the history of the universe. The total EBL is a composite of several wavelength-specific backgrounds, extending from the radio to the gamma-ray regime (the left panel of Fig. 2.7) [130]. Each component traces a different phase of matter and energy evolution in the cosmos. The Cosmic Gamma-ray Background (CGB) and Cosmic X-ray Background (CXB) are dominated by non-thermal emission processes and are produced primarily by astrophysical sources such as blazars and star-forming galaxies [131–135]. The Cosmic Optical backgrounds (COB) represent the direct, unabsorbed starlight emitted by all stars and galaxies. The Cosmic Infrared Background (CIB) contains the energy of starlight that was absorbed and re-radiated by cosmic dust [136, 137]. It peaks near 100 μm and is an essential tool for understanding the star-formation history of the universe, which is obscured at optical wavelengths. This particular portion of the EBL spectrum is characterized by a distinctive “double-hump” spectral distribution. The intensity of the COB and CIB remains orders of magnitude higher than the non-thermal CXB or CGB. This suggests that the post-recombination radiative history of the universe is fundamentally dominated by the birth, life, and thermal evolution of stars rather than the violent, intermittent activity of compact objects [138]. The right panel of Fig. 2.7 shows the magnified view of the COB and CIB intensity measurements over the wavelength range from 0.1 μm to 1000 μm . The EBL radiation field in this wavelength range is crucial because it facilitates photon-photon interactions that produce electron-positron pairs, thereby attenuating the observed flux from distant VHEGR sources. A detailed discussion

⁶European Space Agency (ESA): <https://www.cosmos.esa.int/>.

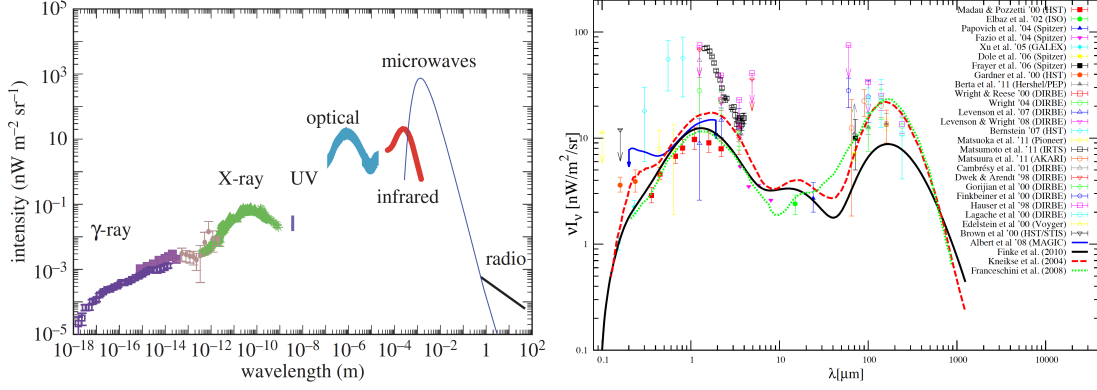


Figure 2.7: The left panel shows the intensity of the **EBL** across the electromagnetic radiation spectrum [130]. The **CGB** and **CXB** arise mainly from non-thermal emission in sources such as blazars and star-forming galaxies [131–135], while the **COB** traces direct starlight and the **CIB** corresponds to the radiation from dust [136, 137]. The right panel presents a zoomed-in view of the **COB-CIB** region (0.1 μm to 1000 μm), where **EBL** photons are responsible for pair-production interactions that contribute to the cascade emission. Different symbols denote measurements from HST [139–141], Infrared Space Observatory (ISO) [142], Galaxy Evolution Explorer (GALEX) [143], Spitzer [144–147], Pioneer [148], Herschel/PACS Evolutionary Probe (PEP) [149], InfraRed Telescope in Space (IRTS) [150], AKARI [151], DIRBE [152–160], Voyager [161], MAGIC [162], and the curves show representative **EBL** models [163–165]. Image source: [130] (left panel) and [166] (right panel).

of this process is provided in the next Sec. 2.3. In this figure, the different colored symbols (squares, circles, triangles) represent measurements from different satellite missions and surveys, such as the Hubble Space Telescope (HST) for optical wavelengths [139–141], Spitzer for infrared [144–147], and Diffuse Infrared Background Experiment (DIRBE) for far-infrared observations [152–160]. The colored solid and dashed lines represent theoretical models (see [163–165] for the models). The direct observation of the **EBL** remains a significant challenge due to strong foreground contamination from our own Galaxy, particularly the Zodiacal light. Therefore, theoretical models are essential for estimating the intrinsic **EBL** density, which is necessary to determine gamma-ray attenuation.

2.3 The missing GeV cascade

High-energy gamma-ray telescopes, such as Fermi-LAT, VERITAS, H.E.S.S., and MAGIC, have observed blazar emissions in the energy range from 1 GeV to 10 TeV, as discussed in Sec. 2.1. Some of these sources, such as the High-Frequency-Peaked BL Lacertae (HBL) object 1ES 0229+200 ($z \approx 0.14$), are located over a billion light-years from Earth [167]. Observing gamma rays from such extragalactic energetic sources provides an essential tool for studying the properties of the **IGM** and the density of **EBL** [168].

For a cosmic ray (**CR**) particle of energy E , mass m , and velocity β propagating through an isotropic radiation field composed of background photons of energy ϵ , characterized by the spectral density $n(\epsilon)$, the inverse of the interaction length is formally defined by the integral

$$\lambda^{-1}(E) = \frac{1}{8\beta E^2} \int_0^\infty \frac{n(\epsilon)}{\epsilon^2} \left[\int_{s_{th}}^{s_{max}} \sigma(s)(s - m^2 c^4) ds \right] d\epsilon, \quad (2.1)$$

where s is the center-of-mass energy squared, s_{th} is the threshold for the secondary particle production, and the maximum value $s_{max} = m^2 c^4 + 4E\epsilon$ denotes for the head-on collision between the **CR** particle and a background photon of energy ϵ , and $\sigma(s)$ is the cross section of the considered interaction [169].

In the context of high-energy gamma-ray propagation, the interaction length λ_{pp} represents the mean free path of a photon before it undergoes pair production ($\gamma + \gamma_{\text{ebl}} \rightarrow e^+ + e^-$) against the **EBL**. The pair production cross-section can be expressed as a function of the velocity of the produced $e^\pm$ in the center-of-mass frame, $\beta^* = \sqrt{1 - (4m_e^2 c^4/s)}$, where m_e is the electron (or positron) rest mass. The pair production cross-section can be calculated analytically, and is given by [170]

$$\sigma_{pp}(\beta^*) = \frac{3}{16} \sigma_T (1 - \beta^{*2}) \left[(3 - \beta^{*4}) \ln \left(\frac{1 + \beta^*}{1 - \beta^*} \right) - 2\beta^*(2 - \beta^{*2}) \right], \quad (2.2)$$

where $\sigma_T = (8/3)\pi r_e^2$ is the Thomson cross-section and r_e is the classical electron radius. By differentiating the full expression for $\sigma_{pp}(\beta^*)$ with respect to velocity, we find a maximum at $\beta^* \approx 0.701$, which corresponds to $s \approx 2s_{th}$. Therefore, the maximum interaction probability of $\sigma_{pp} \approx 0.25\sigma_T$ occurs when the interacting photons satisfy the condition [171]

$$E_0 \epsilon_{\text{ebl}} (1 - \cos \theta) \approx 4m_e^2 c^4, \quad (2.3)$$

where E_0 is the energy of the primary high-energy photon, ϵ_{ebl} the energy of the background photon from **EBL**, and θ is the angle between the momentum vectors of the two interacting photons in the laboratory (or comoving) frame. In an isotropic radiation field (like the **EBL**), photons arrive from all directions, so the average value of $(1 - \cos \theta)$ is 1. Therefore, the pair production cross-section attains its maximum value when the background photon energy is $\epsilon_{\text{ebl}} \approx (E_0/\text{TeV})^{-1} \text{ eV}$. The resulting electron-positron pairs each carry approximately half the energy of the primary gamma-ray, moving in the original direction of propagation with a characteristic Lorentz factor $\gamma = E_0/2m_e c^2$ ranging from 10^6 to 10^8 [45, 172]. Although the exact pair-production mean free path for a gamma ray of energy E_0 depends on the description of the **EBL** density [173] and associated model uncertainties, it can be approximated as follows [174]

$$\lambda_{pp} \approx 80(1+z)^{-\xi} \left(\frac{E_0}{10 \text{ TeV}} \right)^{-1} \text{ Mpc}. \quad (2.4)$$

The parameter ξ is derived from the analysis of the **SFR** density, and for redshifts $z \leq 1$, it takes the value $\xi = 4.5$.

Once electron-positron pairs are produced, they undergo rapid energy loss through interactions with ambient photon fields. The primary mechanism for this interaction is called **IC** scattering, a process where a relativistic lepton transfers a significant fraction of its kinetic energy to a low-energy background photon ($e^\pm + \gamma_{\text{bg}} \rightarrow e^\pm + \gamma$). The cross-section can be calculated at leading order using the Klein-Nishina formula as [170]

$$\sigma_{\text{ic}}(\beta_{\text{ic}}^*) = \frac{3}{8} \sigma_T \frac{m_e^2 c^4}{s \beta_{\text{ic}}^*} \left[\frac{2(2 + 2\beta_{\text{ic}}^* - (\beta_{\text{ic}}^*)^2 - 2(\beta_{\text{ic}}^*)^3)}{\beta_{\text{ic}}^*(1 + \beta_{\text{ic}}^*)} - \frac{2 - 3(\beta_{\text{ic}}^*)^2 - (\beta_{\text{ic}}^*)^3}{(\beta_{\text{ic}}^*)^2} \ln \left(\frac{1 + \beta_{\text{ic}}^*}{1 - \beta_{\text{ic}}^*} \right) \right]. \quad (2.5)$$

In this expression, $\beta_{\text{ic}}^* = (s - m_e^2 c^4)/(s + m_e^2 c^4)$ represents the velocity of the outgoing lepton in the center-of-mass frame. While the **EBL** provides the primary target for the absorption of

TeV gamma rays, the CMB acts as the dominant field for IC scattering. The CMB dominates because of its extremely high photon density ($\sim 411 \text{ cm}^{-3}$), forming a pervasive target field that greatly surpasses the infrared and optical photon densities of the EBL [130].

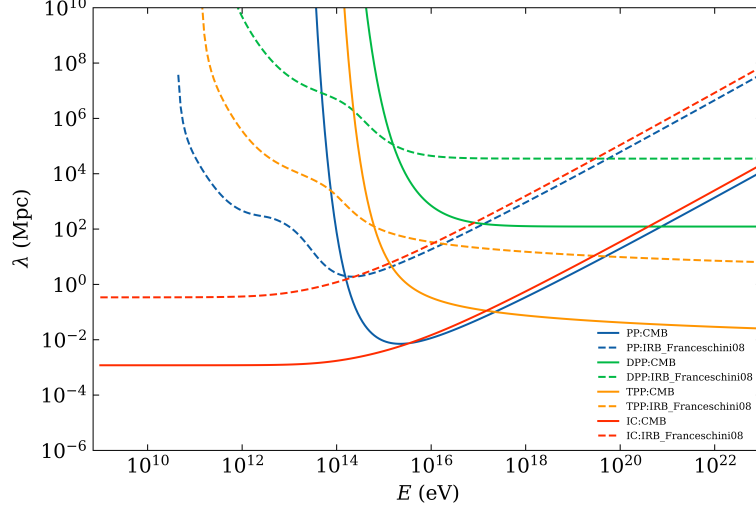


Figure 2.8: Interaction lengths of electromagnetic cascade processes as functions of the incident particle energy. Interactions with the CMB and CIB (based on the model described in [165]) are taken into account.

For primary photons, double pair production ($\gamma + \gamma_{\text{ebl}} \rightarrow e^+ + e^- + e^+ + e^-$) can be an additional interaction channel in which a single photon-photon collision leads to the simultaneous creation of two electron-positron pairs. For the double pair production interaction, the kinematic threshold is significantly higher than that of the standard pair production process, requiring $s \geq 16m_e^2 c^4$ to produce four leptons. The cross-section for double pair production can be approximated at high energies as [175]

$$\sigma_{\text{dpp}}(\beta_{\text{dpp}}^*) \approx 6.45 \mu\text{b} \cdot (\beta_{\text{dpp}}^*)^{12}, \quad (2.6)$$

where the center-of-mass velocity is defined as $\beta_{\text{dpp}}^* = \sqrt{1 - (16m_e^2 c^4/s)}$.

Similarly, triplet pair production can be another interaction channel where an incoming high-energy electron or positron interacts with a background photon, producing an additional electron-positron pair ($e^\pm + \gamma_{\text{ebl}} \rightarrow e^\pm + e^+ + e^-$). The threshold for triplet pair production is reached when the center-of-mass energy is sufficient to produce three leptons, corresponding to a threshold condition of $s \geq 9m_e^2 c^4$. The exact cross-section for triplet pair production is difficult to compute analytically because of the three-body phase space, but at high energies it can be approximated as [170]

$$\sigma_{\text{tpp}}(\beta_{\text{tpp}}^*) \approx \frac{3\alpha}{8\pi} \sigma_{\text{T}} \left[\frac{28}{9} \{ \ln(9) - \ln(1 - (\beta_{\text{tpp}}^*)^2) \} - \frac{218}{27} \right], \quad (2.7)$$

where we define the center-of-mass velocity for the triplet pair production process as $\beta_{\text{tpp}}^* = \sqrt{1 - (9m_e^2 c^4/s)}$ and $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ is the fine structure constant.

The interaction lengths (λ) as a function of energy, shown in Fig. 2.8, are computed using Eq. 2.1. These results demonstrate that the primary pair production is the dominant mechanism for photon attenuation in the GeV–TeV energy range. For energies between 10 GeV and 100 TeV, the interaction lengths for double pair production and triplet pair production remain several orders of magnitude higher than those for pair production. Since a longer mean free path corresponds to a lower probability of interaction, these higher-order interactions are effectively suppressed and negligible in this energy range. Once these high-energy photons have undergone pair production, the resulting electrons and positrons must dissipate their energy through IC scattering. In this figure, the interaction length for IC scattering on the infrared background is consistently much larger than that for the CMB. This clearly indicates that the CMB is the dominant target photon field responsible for producing the secondary gamma-ray emission. The secondary GeV emission from IC scattering is expected to reproduce the total observed photon spectrum in the GeV–TeV energy range [10, 174, 176, 177]. However, Fermi-LAT observations in the GeV energy range for several blazars show that this secondary GeV cascade is suppressed [13, 178], with measured GeV fluxes significantly lower than those predicted by standard electromagnetic cascades produced only via IC scattering, as shown by the red dashed line in Fig. 2.9. This indicates that the energy of electrons (and positrons) is either spread out in space or lost through non-radiative processes before undergoing IC scattering. The ‘missing’ GeV flux is typically explained by two alternative physical mechanisms: deflection of electrons and positrons by the IGMF, and energy loss of the pair beam due to beam–plasma instabilities before IC cooling occurs.

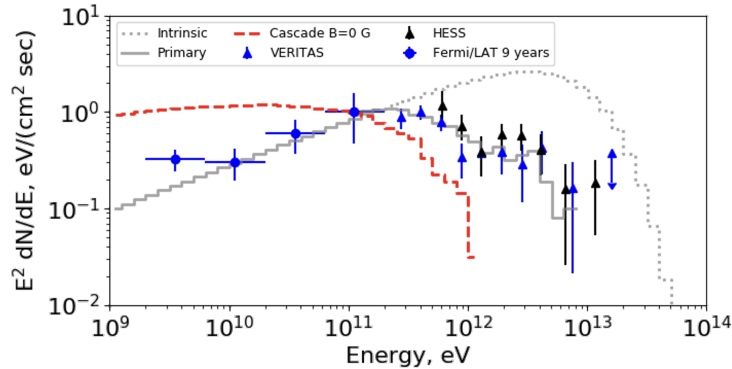


Figure 2.9: The spectral energy distribution of 1ES 0229+200 blazar. The grey curve presents the intrinsic spectrum, and the red dashed curve defines the total spectrum without considering the IGMF contribution in electromagnetic cascade development. The data points indicate the Fermi-LAT, H.E.S.S., and VERITAS measurements. Image source: Ievgen Vovk, TeVPA22 Kingston, on 10th August 2022⁷.

The most widely discussed explanation is the influence of the IGMF, where even a very weak magnetic field in cosmic voids (about $B \sim 10^{-18} - 10^{-16}$ G) can deflect the electrons and positrons produced by the primary gamma rays and deflect the trajectory of these charged particles [6, 16, 20, 106]. The secondary cascade can appear as an extended emission around the point-source spectrum if the deflection is large enough. In this case, the charged particles do not lose their energy; instead, the propagation is delayed and spread over a wider angular region [13, 174, 176, 177].

⁷https://indico.global/event/521/contributions/14878/attachments/4295/6679/Robust_IGMF_limit_MAGIC.pdf

An alternative explanation is that the suppression arises from collective plasma effects [24, 26–28, 30, 31, 33, 35, 36, 44, 45, 172, 179–185]. The relativistic pair beam traveling through the IGM can be viewed as a stream of relativistic charged particles that perturb the background plasma, leading to the growth of instabilities. Plasma waves can grow exponentially by extracting kinetic energy from the beam, eventually transferring it as heat into the IGM plasma, and the pair beam loses energy. If the beam energy loss due to instability growth occurs on a timescale much shorter than the IC cooling time, the secondary GeV cascade would be suppressed, which in turn affects constraints on the IGMF lower bound [26].

Nevertheless, the energy loss efficiency due to instability can be affected by its nonlinear evolution, which is not well understood in the literature. We provide a detailed discussion of the physics of beam–plasma instabilities in Chap. 4. In Chap. 5, we present the numerical methods used to study instability and evaluate its impact on the IGMF constraints. Finally, in Chap. 6, we examine the physical conditions required to study beam–plasma instability in laboratory settings that replicate relevant astrophysical environments.

2.4 Observational techniques

The Earth’s atmosphere acts as a natural barrier to high-energy radiation, necessitating space-borne platforms for the direct observation of X-rays and gamma rays. However, observations at the highest energies are intrinsically limited by the steeply declining photon flux. The relatively small effective areas of satellite detectors, typically a few square meters, are insufficient to obtain statistically significant measurements in the very-high-energy regime. To overcome this “photon-starvation” limitation at the very-high energy regime, the atmosphere is instead exploited as a giant calorimeter. Ground-based observatories detect the secondary particles produced in extensive air showers, the cascades that develop when primary gamma rays interact with atmospheric nuclei. This approach allows for effective collection areas several orders of magnitude larger than those of space-based telescopes, thereby providing the sensitivity necessary to explore the TeV universe.

In the HEGR regime, covering approximately 20 MeV to 300 GeV, observations are primarily performed by space-based instruments such as the Fermi-LAT [186, 187]. Since gamma rays in this range cannot be focused using conventional reflective optics, they are detected through their particle-like interactions within the instrument. These telescopes typically incorporate a tracker to reconstruct the trajectories of electron–positron pairs produced via pair production, a calorimeter to measure the deposited energy, and an anti-coincidence detector to suppress the background of charged CRs [59, 190]. Although space-based gamma-ray telescopes offer a wide field of view and an almost continuous all-sky monitoring capability, their size is fundamentally constrained by launch-vehicle payload limits, restricting their operation to energies where the photon flux remains sufficiently high [191].

Ground-based gamma-ray astronomy employs two complementary strategies to analyze the extensive air showers produced by VHEGRs: Surface Arrays (SAs) and IACTs. Surface arrays, such as the High-Altitude Water Cherenkov (HAWC) Observatory [192], use high-altitude tanks to directly measure particles from a cascade as they reach the ground. These facilities feature a wide field of view and operate with an almost continuous duty cycle, making them ideal for large-scale sky surveys and monitoring transient events [191]. In contrast, IACTs are represented by current-generation arrays such as H.E.S.S. [193–195], MAGIC [81, 196], and VERITAS [197,

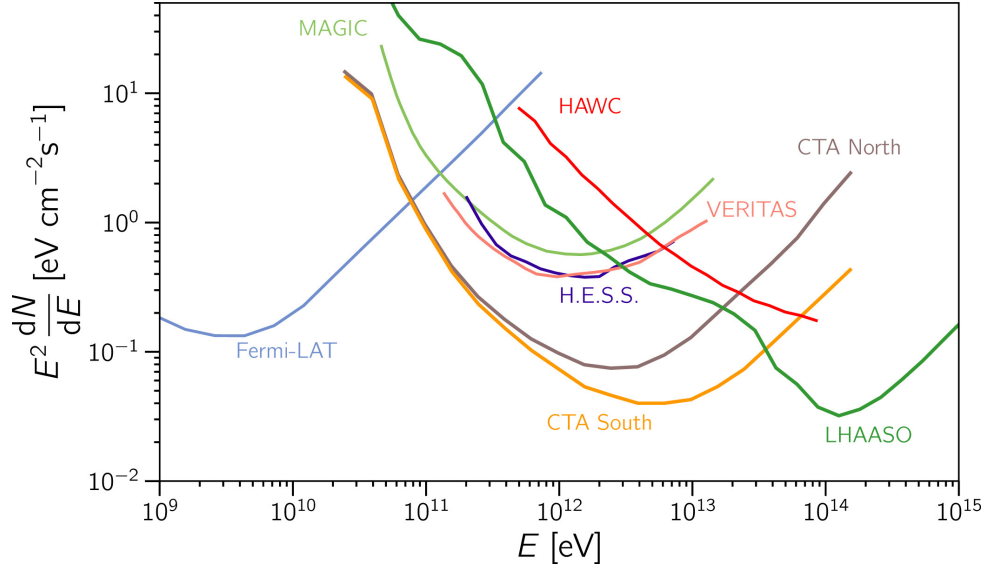


Figure 2.10: The differential sensitivities for various gamma-ray observatories across the GeV–PeV energy range (Image source: [188]). The performance of IACTs (CTA [85], H.E.S.S. [80], MAGIC [81], and VERITAS [82]) is shown for a 50-hour observation period. The sensitivities for HAWC [83] and LHAASO [84] correspond to approximately 3000 hours and 1 year of operation, respectively. The Fermi-LAT accounts for varying sky positions using the P8R3_SOURCE_V2 instrument response function (IRF) [189].

[198], as well as the forthcoming CTA [199], which capture the faint, nanosecond-scale Cherenkov radiation emitted by relativistic particles within atmospheric showers. Although IACTs are restricted to pointed observations under dark, cloudless conditions, they provide higher sensitivity and angular resolution. By analyzing the morphology and timing of the shower image, these telescopes can effectively discriminate between gamma-ray-induced electromagnetic cascades and the hadronic background from CRs, providing a high-precision window into the most energetic accelerators in the universe.

Fig. 2.10 shows the differential sensitivity of current and future gamma-ray observatories. The curves highlight the transition from space-based detection, conducted by Fermi-LAT in the HEGR regime, to ground-based observations by IACTs and SAs in the VHEGR and ultra-high-energy gamma-ray (UHEGR) regimes. While Fermi-LAT provides broad-sky coverage at lower energies, instruments like CTA and LHAASO [84] are designed to provide the high-sensitivity measurements necessary to detect the photon flux at the highest energy scales.

Theory of Extragalactic Magnetic Fields

In this chapter, we discuss the theoretical framework underlying the origin and evolution of extragalactic magnetic fields. Magnetic fields are observed across a wide range of astrophysical environments, from galaxies to galaxy clusters, yet their origin, particularly in the most underdense regions of the Universe, remains an open theoretical problem. The existence of magnetic fields in cosmic voids, potentially at the level of $\gtrsim 10^{-19}$ G, inferred from gamma-ray observations, has motivated a detailed investigation of their origin. Broadly, proposed scenarios fall into two categories: primordial (early-Universe) magnetogenesis and astrophysical (late-time) magnetization. Each scenario predicts distinct field strengths, coherence scales, and spectral properties, which are crucial when interpreting observational constraints.

3.1 Primordial magnetogenesis

Primordial magnetic fields are proposed to originate in the early Universe and subsequently evolve through cosmic expansion and magnetohydrodynamic (MHD) processes. Such fields are particularly plausible in explaining magnetic fields in cosmic voids, where astrophysical processes are expected to be inefficient.

Inflation provides a natural mechanism to generate magnetic fields with correlations on very large (super-horizon) scales. However, in standard electromagnetism, conformal invariance prevents the amplification of electromagnetic fields in an expanding Universe, causing any primordial fields to remain extremely weak. To generate magnetic fields of observational relevance, this symmetry must be broken, for instance, by coupling the electromagnetic field to a scalar field [200, 201]. These scenarios can produce nearly scale-invariant magnetic spectra [200, 202], but typically are constrained by strong-coupling or backreaction [1, 2, 4, 203–213]. From a phenomenological perspective, inflation-generated fields are expected to have very large coherence lengths but extremely small amplitudes, often far below observationally relevant levels unless model parameters are tuned. Consequently, current lower limits on intergalactic magnetic fields (IGMFs) place nontrivial constraints on viable inflationary magnetogenesis models [214, 215].

Magnetic fields may also be generated during cosmological phase transitions, such as the electroweak or quantum chromodynamics (QCD) transitions [216–220]. In these scenarios, causal

processes (e.g., bubble collisions, turbulence, or vorticity generation) produce magnetic fields with coherence scales limited by the horizon at the time of generation [221–228]. Such fields typically exhibit ‘blue spectra’ (i.e., most of the magnetic power is concentrated at small spatial scales or high wave number) and require subsequent evolution, particularly inverse-cascade processes, to transfer magnetic energy to larger scales [229–231]. In helical configurations, magnetic helicity conservation drives this inverse cascade, enabling the correlation length to grow over cosmic time [232–234]. The evolution of such fields is inherently nonlinear. As discussed in [235], decaying MHD turbulence leads to a redistribution of magnetic energy, with helicity playing a central role in transferring power to larger scales. This process is essential if phase-transition-generated fields are to remain relevant on cosmological scales today.

More recently, particle-physics-driven mechanisms have gained attention, particularly those involving chiral asymmetries in the early Universe [236–238]. In a relativistic plasma, a difference between right- and left-handed fermions induces an electric current via the chiral magnetic effect, leading to the amplification of magnetic fields. As reported in [235], this mechanism can generate helical magnetic fields through the chiral plasma instability. Importantly, these fields are not only amplified but also inherit helicity, which facilitates their long-term evolution to large scales via inverse-cascade processes. The efficiency of this mechanism depends on the persistence of chiral asymmetry, which may be sustained by out-of-equilibrium processes such as baryogenesis. These models are well-motivated because they establish connections among the origin of magnetic fields, fundamental particle physics, and the generation of baryon asymmetry.

3.2 Astrophysical sources of magnetic fields

In contrast to primordial scenarios, astrophysical models attribute intergalactic magnetic fields (IGMFs) to processes occurring after structure formation. Galaxies and active galactic nuclei (AGN) can expel magnetized plasma into the intergalactic medium via winds, jets, or feedback processes [14, 205, 207]. Over cosmic time, these outflows may seed magnetic fields in surrounding regions. However, their effectiveness depends on the volume-filling factor of such outflows [239]. As demonstrated in [240], current studies suggest that astrophysical outflows are insufficient to magnetize cosmic voids uniformly. The resulting fields are likely inhomogeneous and confined near structures, rather than pervasive across large volumes.

An alternative astrophysical proposal considers the cumulative effect of magnetic dipoles from individual galaxies. In principle, the superposition of many such dipoles could generate a large-scale field in voids [241]. However, this scenario faces significant theoretical challenges. Plasma effects in the intergalactic medium can lead to strong screening of magnetic fields on large scales. The characteristic screening length is many orders of magnitude smaller than void scales, resulting in an exponential suppression of dipole fields over megaparsec distances [242]. Consequently, purely dipolar contributions are unlikely to explain observed lower bounds on void magnetic fields.

3.3 Evolution of magnetic fields and observable signatures

Regardless of their origin, magnetic fields undergo significant evolution during cosmic history. Before recombination, their dynamics are governed by MHD (or chiral MHD at high temperatures), while after recombination, they are largely frozen into the expanding plasma. During

structure formation, several processes affect magnetic fields. Gravitational compression amplifies the field strength as the gas collapses, while turbulent motions redistribute magnetic energy across spatial scales via cascade processes. In addition, small-scale dynamo action can exponentially amplify initially weak seed fields, provided that the conditions for turbulent amplification are satisfied. Numerical studies (e.g., [240, 243–246]) show that turbulence generated during gravitational collapse can trigger a small-scale dynamo, significantly modifying the magnetic energy spectrum. Whether primordial signatures preserved depends on the competition between dynamo amplification and turbulence. In particular, small-scale processes tend to wash out the imprint of the initial conditions, while large-scale correlations may persist.

The difference between primordial and astrophysical scenarios is closely tied to observational constraints, especially those derived from gamma-ray observations of blazars. These constraints suggest lower limits on IGMFs in the range of $\sim 10^{-19} - 10^{-16}$ G on megaparsec scales [6, 7, 17, 23, 247]. These observational limits place stringent constraints on viable magnetogenesis scenarios. In particular, they require that any successful model produce magnetic fields with both sufficient strength and large coherence scales. While primordial mechanisms can naturally satisfy these requirements, especially in the presence of helicity and inverse-cascade processes, astrophysical scenarios are generally unable to account for the observed properties of void magnetic fields, primarily due to their limited volume filling and the suppression of large-scale fields by plasma effects. Consequently, current observational constraints tend to favor a primordial origin of IGMFs, although subdominant astrophysical contributions may still be present.

Physics of Beam–Plasma Instabilities

In this chapter, we present an overview of the theoretical aspects of the beam–plasma instabilities. This chapter begins with a discussion of the main properties of intergalactic medium (IGM) plasma, which is presented in Sec. 4.1. Beam–plasma instabilities are collective phenomena that, in their most general form, are described within the framework of plasma kinetic theory. This process is introduced in Sec. 4.2, and is based on the Vlasov equation for the particle distribution function coupled with Maxwell’s equations. By considering small perturbations around an equilibrium distribution, dispersion relations can be derived to determine whether these perturbations grow or decay. In particular, when the imaginary part of the wave frequency is positive, $\text{Im}(\omega(\mathbf{k})) > 0$, the perturbations grow by extracting energy from the particle beam, leading to the onset of instability. This process, known as the beam–plasma instability, is particularly relevant to blazar-induced pair beams, where the electrostatic instability in the kinetic regime plays a dominant role, as discussed in Sec. 4.3. The equations governing plasma dynamics are relatively simple, but analytical solutions can be obtained only for simplified cases. Consequently, a variety of numerical approaches have been developed to study more complex systems, ranging from fluid-based descriptions, such as magnetohydrodynamics (MHD), to fully kinetic methods, such as Particle-in-Cell (PIC) simulations. In addition, techniques for performing parametric studies using the CRPropa 3.2 framework are introduced in Sec. 4.4.

4.1 Properties of plasmas

A plasma is an ionized gas characterized by a significant population of free electrons and ions, which makes it electrically conductive. A key feature of plasmas is the presence of long-range collective effects governed by electromagnetic fields, rather than direct particle–particle interactions. In low-density environments, such as the IGM, collisions between particles are negligible, and the plasma can be considered collisionless. These collective effects determine the response of the plasma to small perturbations, leading to oscillations in the charge density and electromagnetic fields at a characteristic frequency that depends only on the particle density n_{IGM} . This

frequency, known as the plasma frequency ω_p , is defined as

$$\omega_p = \left(\frac{q_e^2 n_{\text{IGM}}}{m_e \epsilon_0} \right)^{\frac{1}{2}}. \quad (4.1)$$

Here, q_e is the electron charge, m_e is the electron rest mass, and ϵ_0 is the vacuum permittivity. A plasma does not permit the propagation of electromagnetic waves with frequencies below the plasma frequency; instead, such waves can only penetrate a finite distance characterized by the plasma skin depth c/ω_p . Because the charged particles in a plasma are free to move in response to electric fields, they rearrange themselves to reduce electric fields over sufficiently large length scales. As a result, electric fields are effectively screened beyond the Debye length λ_D , which is given by [248]

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_{\text{IGM}}}{n_{\text{IGM}} q_e^2}}, \quad (4.2)$$

where T_{IGM} is the *IGM* temperature.

In contrast, magnetic fields are not screened in a plasma. Instead, charged particles respond to magnetic fields by undergoing circular motion around the field lines at the gyro-frequency ω_L , given by [249]

$$\omega_L = \frac{|q_e| B}{m_e}. \quad (4.3)$$

The corresponding spatial scale associated with this motion is the Larmor (or gyro-) radius. It characterizes the radius of the circular trajectory of a charged particle in a magnetic field, and is given by

$$r_L = \frac{v_{\perp}}{\omega_L} = \frac{m_e v_{\perp}}{|q_e| B}, \quad (4.4)$$

where $v_{\perp}$ denotes the component of the particle velocity perpendicular to the magnetic field. The Larmor radius determines how strongly particles are tied to magnetic field lines: small values of r_L correspond to tightly bound motion, while large values indicate weak magnetic confinement.

In the *IGM*, where magnetic fields are typically very weak, the Larmor radius of relativistic particles can become much larger than the characteristic length scales of the system. As a result, particle trajectories may remain only weakly influenced by magnetic fields over large distances. The relative importance of collective plasma effects, electric field screening, and magnetic deflection is therefore governed by the interplay between characteristic scales such as the Debye length, the plasma skin depth, and the Larmor radius.

The intergalactic medium is characterized by extremely low particle densities, typically in the range $n_{\text{IGM}} \sim 10^{-8} - 10^{-6} \text{ cm}^{-3}$, and relatively high temperatures, which can vary from $T_{\text{IGM}} \sim 10^4 \text{ K}$ in photoionized regions to $T_{\text{IGM}} \sim 10^6 \text{ K}$ in shock-heated environments. The low density also implies a small plasma frequency and a correspondingly large plasma skin depth, which can reach macroscopic scales. The Debye length can be calculated as $\lambda_D \approx 6.9 - 690 \text{ km}$, which is many orders of magnitude smaller than the characteristic *IGM* system size, $L \sim 10 - 100 \text{ Mpc}$. Combined with the large number of particles per Debye sphere, $N_D = n_{\text{IGM}} \lambda_D^3 \sim 10^{18} - 10^{21}$, this implies a Coulomb mean-free path $l_{\text{mfp}} \propto \lambda_D N_D > L$. So, under these conditions, the plasma is highly ionized and remains effectively collisionless, as the Coulomb mean free path of particles greatly exceeds the characteristic system size.

Magnetic fields in the IGM are expected to be very weak, with typical strengths ranging from 10^{-18} to 10^{-9} G, depending on the environment and observational constraints. In particular, the weakest fields are expected in cosmic voids, where the magnetic field strength may be as weak as $10^{-18} - 10^{-15}$ G, while larger values are found in filaments and galaxy clusters. As a consequence, charged particles, especially relativistic ones, can have very large Larmor radii ($\sim 10^3 - 10^6$ Mpc), allowing them to propagate over cosmological distances with only weak deflection. This has important implications for the development of electromagnetic cascades and the transport of high-energy particles, particularly in void regions where magnetic deflections are minimal.

In contrast to the IGM, laboratory plasmas are characterized by much higher particle densities and stronger electromagnetic fields. Typical laboratory plasma densities range from $n \sim 10^{12} - 10^{16}$ cm $^{-3}$, with temperatures ranging from $10^4 - 10^6$ K depending on the experimental setup, similar to some astrophysical conditions. Under these conditions, the plasma frequency is significantly higher, often in the gigahertz to terahertz range. The corresponding Debye length is typically $\lambda_D \sim 10^1 - 10^4$ μ m, while the plasma skin depth lies in the range $\delta \sim 10^1 - 10^3$ μ m. As a result, collective effects occur on much shorter spatial and temporal scales compared to astrophysical plasmas.

Magnetic fields in laboratory experiments are also considerably stronger, typically in the millitesla to several tesla range, leading to small Larmor radii and strong particle confinement along magnetic field lines. In some regimes, collisional effects become relevant at higher densities, although many experiments are designed to operate in collisionless or weakly collisional conditions to mimic astrophysical environments. Recent experimental and theoretical studies have further demonstrated the capability of laboratory platforms to probe plasma processes under controlled magnetic field configurations, including regimes relevant to weakly magnetized astrophysical plasmas [250–252]. Controlled laboratory plasmas provide a useful platform for studying fundamental processes such as beam–plasma instabilities.

4.2 Kinetic description of plasma

In kinetic theory, plasma dynamics are described by the Vlasov equation together with Maxwell’s equations. Linearizing the governing equations about an equilibrium distribution leads to dispersion relations that describe the plasma wave spectrum and the growth or damping of modes. The distribution function plays a fundamental role in the kinetic description of plasma dynamics. The Vlasov equation describes the evolution of the distribution function $f_\alpha(\mathbf{p}, \mathbf{r}, t)$ of species α in phase space, assuming local conservation of the particle distribution,

$$\frac{\partial f_\alpha}{\partial t} + \frac{\mathbf{p}}{m_\alpha} \nabla f_\alpha + q_\alpha \left[\mathbf{E}(\mathbf{r}, t) + \frac{\mathbf{p}}{m_\alpha} \times \mathbf{B}(\mathbf{r}, t) \right] \frac{\partial f_\alpha}{\partial \mathbf{p}} = 0, \quad (4.5)$$

where m_α is the mass and q_α is the charge of the particle species α .

The evolution of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, in space and time is described by

Maxwell's equations,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad (4.6)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (4.7)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (4.8)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \quad (4.9)$$

The total charge and current densities for all species α can be expressed as,

$$\rho(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha} \int d^3p f_{\alpha}(\mathbf{p}, \mathbf{r}, t), \quad (4.10)$$

$$\mathbf{j}(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha} \int d^3p \frac{\mathbf{p}}{m_{\alpha}} f_{\alpha}(\mathbf{p}, \mathbf{r}, t), \quad (4.11)$$

To study plasma instabilities, one considers small perturbations in an equilibrium state. Let $f_{\alpha 0}$, $\mathbf{E}_0$, and $\mathbf{B}_0$ denote the equilibrium distribution function and electromagnetic fields, respectively, while δf_{α} , $\delta \mathbf{E}$, and $\delta \mathbf{B}$ represent their small perturbations. Assuming plane-wave perturbations, the distribution function and fields can be written as

$$f_{\alpha}(\mathbf{p}, \mathbf{r}, t) = f_{\alpha 0}(\mathbf{p}, \mathbf{r}) + \delta f_{\alpha}, \quad (4.12)$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}) + \delta \mathbf{E}, \quad (4.13)$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0(\mathbf{r}) + \delta \mathbf{B}, \quad (4.14)$$

where, for a homogeneous system, the perturbation terms can be expressed as plane waves,

$$\delta f_{\alpha} = f_{\alpha 1} \exp [i(\mathbf{k} \cdot \mathbf{r} - \omega t)], \quad (4.15)$$

$$\delta \mathbf{E} = \mathbf{E}_1 \exp [i(\mathbf{k} \cdot \mathbf{r} - \omega t)], \quad (4.16)$$

$$\delta \mathbf{B} = \mathbf{B}_1 \exp [i(\mathbf{k} \cdot \mathbf{r} - \omega t)]. \quad (4.17)$$

Substituting the first-order expansions from Eqs. (4.12)-(4.14) into the Vlasov-Maxwell equations, and keeping only terms linear in the perturbations gives

$$\frac{\partial(\delta f_{\alpha})}{\partial t} + \frac{\mathbf{p}}{m_{\alpha}} \cdot \nabla(\delta f_{\alpha}) + q_{\alpha} \left(\mathbf{E}_0 + \frac{\mathbf{p}}{m_{\alpha}} \times \mathbf{B}_0 \right) \cdot \frac{\partial(\delta f_{\alpha})}{\partial \mathbf{p}} + q_{\alpha} \left(\delta \mathbf{E} + \frac{\mathbf{p}}{m_{\alpha}} \times \delta \mathbf{B} \right) \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}} = 0. \quad (4.18)$$

The perturbed Maxwell equations are

$$\nabla \cdot \delta \mathbf{E} = \frac{\delta \rho}{\epsilon_0}, \quad (4.19)$$

$$\nabla \cdot \delta \mathbf{B} = 0, \quad (4.20)$$

$$\nabla \times \delta \mathbf{E} = -\frac{\partial(\delta \mathbf{B})}{\partial t}, \quad (4.21)$$

$$\nabla \times \delta \mathbf{B} = \mu_0(\delta \mathbf{j}) + \epsilon_0 \frac{\partial(\delta \mathbf{E})}{\partial t}. \quad (4.22)$$

The linearized Vlasov equation can be solved using the method of characteristics [253, 254], in which the evolution of the perturbation is followed along unperturbed particle trajectories in phase space. These trajectories are defined by the equations of motion,

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{m_\alpha}, \quad \frac{d\mathbf{p}}{dt} = q_\alpha \left(\mathbf{E}_0 + \frac{\mathbf{p}}{m_\alpha} \times \mathbf{B}_0 \right). \quad (4.23)$$

Along these trajectories, the linearized Vlasov equation can be written as a total time derivative

$$\frac{d}{dt}(\delta f_\alpha) = -q_\alpha \left(\delta \mathbf{E} + \frac{\mathbf{p}}{m_\alpha} \times \delta \mathbf{B} \right) \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}}. \quad (4.24)$$

This equation can then be integrated in time to obtain the perturbation of the distribution function,

$$\delta f_\alpha(\mathbf{p}, \mathbf{r}, t) = -q_\alpha \int_{-\infty}^t \left(\delta \mathbf{E} + \frac{\mathbf{p}}{m_\alpha} \times \delta \mathbf{B} \right) \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}} dt'. \quad (4.25)$$

The perturbation of the distribution function leads to a corresponding perturbation in the current density. It is obtained by integrating the contribution of all particles over momentum space and summing over all species,

$$\delta \mathbf{j} = \sum_{\alpha} q_{\alpha} \int \frac{\mathbf{p}}{m_{\alpha}} \delta f_{\alpha} d^3 p, \quad (4.26)$$

which further defines,

$$\delta \mathbf{j} = \boldsymbol{\sigma} \cdot \delta \mathbf{E}. \quad (4.27)$$

From Eq. (4.25), using the expression for δf_α allows the calculation of the perturbed charge density $\delta \rho$, and subsequently the electric field perturbation $\delta \mathbf{E}$ via Eq. (4.19). Substituting this into Eq. (4.26) and comparing with Eq. (4.27) then yields the conductivity tensor $\boldsymbol{\sigma}$. Since the perturbations can be expressed as plane waves using a Fourier representation (Eqs. (4.15)-(4.17)), the spatial and temporal derivatives are replaced by algebraic factors, $\nabla \rightarrow i\mathbf{k}$ and $\partial/\partial t \rightarrow -i\omega$, which significantly simplifies the governing equations. This transformation converts the differential Vlasov–Maxwell system into an algebraic system in $(\mathbf{k}, \omega)$ -space. This leads to the wave equation

$$(-\omega^2 + c^2 k^2) \mathbf{E} + c^2 (\mathbf{k} \cdot \mathbf{E}) \mathbf{k} + i\mu_0 \omega \boldsymbol{\sigma} \cdot \mathbf{E} = 0. \quad (4.28)$$

Writing this equation in index notation gives

$$[(-\omega^2 + c^2 k^2) \delta_{nj} + c^2 k_n k_j + i\mu_0 \omega \sigma_{nj}] E_j = 0, \quad (4.29)$$

where δ_{nj} represents the Kronecker delta. For notational convenience, we introduce the dielectric tensor $\phi_{nj}(\omega)$ and the components of vector $\mathbf{F}(\mathbf{k}, \omega)$ as

$$\phi_{nj}(\omega) = \delta_{nj} + \frac{i\sigma_{nj}}{\epsilon_0 \omega}, \quad (4.30)$$

$$F_n(\mathbf{k}, \omega) = \frac{i}{\omega} \left(E_n - \frac{c}{\omega} \epsilon_{npq} k_p B_q \right), \quad (4.31)$$

where ϵ_{npq} denotes the Levi-Civita symbol. We define a Maxwell operator $\mathbf{\Lambda}(\mathbf{k}, \omega)$, whose components are given by

$$\Lambda_{nj}(\mathbf{k}, \omega) = \frac{c^2 k^2}{\omega^2} \left(\frac{k_n k_j}{k^2} - \delta_{nj} \right) + \phi_{nj}. \quad (4.32)$$

Eq. (4.29) can be written as

$$\Lambda_{nj} E_j = F_n, \quad (4.33)$$

which constitutes a set of linear equations for the components of the electric field. Non-trivial solutions exist only when the determinant of the Maxwell operator vanishes

$$\det \mathbf{\Lambda}(\mathbf{k}, \omega) = 0, \quad (4.34)$$

which leads to the dispersion relation that characterizes the wave spectrum of the plasma. For longitudinal (electrostatic) modes, the electric field perturbation is parallel to the wave vector, i.e., $\delta \mathbf{E} \parallel \mathbf{k}$, while the magnetic field perturbation vanishes, $\delta \mathbf{B} = \mathbf{0}$. Under this assumption, the dynamics are governed purely by electrostatic interactions, and the Vlasov–Maxwell system simplifies considerably. In this limit, the linearized Vlasov equation takes the form

$$\begin{aligned} (-i\omega + i\mathbf{k} \cdot \mathbf{v}) \delta f_\alpha q_\alpha \mathbf{E} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}} &= 0, \\ \Rightarrow \delta f_\alpha &= \frac{-q_\alpha \mathbf{E} \cdot \nabla_{\mathbf{p}} f_{\alpha 0}}{i(\mathbf{k} \cdot \mathbf{v} - \omega)}. \end{aligned} \quad (4.35)$$

The perturbation in the charge density is obtained by integrating over momentum space and summing over all particle species,

$$\delta \rho(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha} \int \delta f_{\alpha}(\mathbf{p}, \mathbf{r}, t) d^3 p. \quad (4.36)$$

Substituting the expression for δf_{α} from Eq. (4.25) into the above equation of the charge density yields $\delta \rho$ in terms of the electric field perturbation. Using Poisson’s equation, Eq. (4.19), this leads to a self-consistent relation between the electric field and the equilibrium distribution function. The general dispersion relation for electrostatic perturbations can be expressed as

$$1 + \sum_{\alpha} \frac{q_{\alpha}^2}{\epsilon_0 k^2} \int d^3 p \frac{\mathbf{k} \cdot \frac{\partial f_{\alpha 0}}{\partial \mathbf{p}}}{\omega - \mathbf{k} \cdot \mathbf{v}} = 0. \quad (4.37)$$

For the background plasma, if we consider the cold limit, in which the thermal spread of particle velocities is neglected, the second term in the above expression reduces to the familiar expression,

$$\chi_p = -\frac{\omega_p^2}{\omega^2}, \quad (4.38)$$

where ω_p is the plasma frequency defined in Eq. (4.1). This term describes the restoring force associated with collective electron oscillations in the plasma.

For the beam component, substituting the spatially homogeneous normalized beam distribution $f_b(\mathbf{p}) \rightarrow f_b(\mathbf{p})/n_b$ into the general expression of the distribution function, the beam contribution to the dielectric response is given by

$$\chi_b = \frac{n_b q_e^2}{\epsilon_0 k^2} \int d^3 p \frac{\mathbf{k} \cdot \frac{\partial f_{b0}(\mathbf{p})}{\partial \mathbf{p}}}{\mathbf{k} \cdot \mathbf{v} - \omega}, \quad (4.39)$$

where n_b is the number density of the beam. The following expression represents the resonant interaction between beam particles and plasma waves, which governs the growth or damping of perturbations. A detailed discussion of beam–plasma instabilities is presented in Sec. 4.3. Combining the contributions from the background plasma and the beam, the dispersion relation can be written as,

$$1 - \frac{\omega_p^2}{\omega^2} - \frac{n_b e^2}{\epsilon_0 k^2} \int d^3 p \frac{\mathbf{k} \cdot \frac{\partial f_{b0}(\mathbf{p})}{\partial \mathbf{p}}}{\mathbf{k} \cdot \mathbf{v} - \omega} = 0. \quad (4.40)$$

4.2.1 Landau damping

A fundamental property of collisionless plasmas is the phenomenon of Landau damping, first identified by Landau in 1946 [255]. It arises naturally from the Vlasov equation and describes the exchange of energy between plasma waves and particles without requiring collisions. In particular, it explains how electrostatic waves, such as Langmuir waves, can be damped or amplified through resonant interactions with particles in the plasma. As discussed earlier in the section, we can express the dispersion relation in terms of integrals of the form

$$\int dv \frac{g(v)}{\omega - \mathbf{k} \cdot \mathbf{v}}, \quad (4.41)$$

which exhibit the resonance condition $\omega = \mathbf{k} \cdot \mathbf{v}$. In general, the wave frequency is complex and can be written as $\omega = \omega_r + i\omega_{im}$, where ω_r represents the real oscillation frequency and ω_{im} denotes the growth ($\omega_{im} > 0$) or damping ($\omega_{im} < 0$) rate. To evaluate the integral, Landau formulated the problem as an initial value problem and introduced a contour integration in the complex velocity plane. The integration contour is chosen such that it passes below the pole at $v = \omega/k$, ensuring causality. This allows the integral to be written as a principal value term plus a pole contribution,

$$\int dv \frac{g(v)}{\omega - \mathbf{k} \cdot \mathbf{v}} = \mathcal{P} \int dv \frac{g(v)}{\omega - \mathbf{k} \cdot \mathbf{v}} + i\pi \frac{g(v)}{k} \Big|_{v=\omega_r/k}. \quad (4.42)$$

For weak damping or growth, i.e. $|\omega_{im}| \ll |\omega_r|$, the imaginary part of the dispersion relation can be evaluated perturbatively. This yields an approximate expression for the growth (or damping) rate,

$$\omega_{im} \propto \frac{\partial f_{\alpha 0}}{\partial v} \Big|_{v=\omega_r/k}. \quad (4.43)$$

The sign of ω_{im} is determined by the slope of the distribution function at the resonant velocity.

$$\partial f_{\alpha 0} / \partial v < 0 \Rightarrow \omega_{im} < 0 \text{ (damping)}, \quad (4.44)$$

$$\partial f_{\alpha 0} / \partial v > 0 \Rightarrow \omega_{im} > 0 \text{ (growth)}. \quad (4.45)$$

The condition $\omega = \mathbf{k} \cdot \mathbf{v}$ identifies particles whose velocities match the phase velocity of the wave. These resonant particles interact most efficiently with the wave and exchange energy with it. If the distribution function decreases with velocity, more particles are accelerated by the wave than decelerated. As a result, energy is transferred from the wave to the particles, leading to damping. Conversely, if there is a positive slope in the distribution function, the wave gains

energy from the particles, resulting in instability (see Sec. 4.3 for a detailed discussion). At late times ($t \rightarrow \infty$), the electric field can be expressed as a superposition of wave modes,

$$\mathbf{E}(\mathbf{k}, t) = \sum_l \mathbf{R}_l(\mathbf{k}) \exp[-i\omega_{r,l}t + \omega_{im,l}t], \quad (4.46)$$

where $\mathbf{R}_l$ denotes the residue associated with each mode. The electric field consists of oscillatory components whose amplitudes either decay or grow exponentially in time, depending on the sign of $\omega_{im,l}$.

4.3 Beam–plasma instabilities

We consider a relativistic electron–positron pair beam propagating through a significantly denser, quasi-neutral background plasma composed of electrons and ions. The beam has zero net charge. The background plasma is assumed to be initially unmagnetized, which implies that at equilibrium, the electromagnetic fields vanish

$$\mathbf{E}_0 = 0, \text{ and } \mathbf{B}_0 = 0. \quad (4.47)$$

Under these conditions, the system is characterized by two key parameters: the Lorentz factor of the beam, $\gamma \gg 1$, and the density ratio, $\alpha = n_b/n_{bg} \ll 1$, where n_b and n_{bg} denote the beam and background plasma densities, respectively.

4.3.1 Beam distribution function

In the cold (monokinetic) limit, the momentum spread of both the beam and the background plasma is negligible. The corresponding equilibrium distribution functions are described by delta functions in momentum space. The beam distribution is written as [39]

$$f_{b0}(\mathbf{p}) = \alpha \delta(p_x - p_b) \delta(p_y) \delta(p_z), \quad (4.48)$$

where $p_b = \gamma m_e v_b$ with $v_b \simeq c$, while the background plasma is assumed to be at rest and described by

$$f_{bg0}(\mathbf{p}) = \delta(p_x) \delta(p_y) \delta(p_z). \quad (4.49)$$

While this approximation provides a useful starting point, it neglects the intrinsic momentum spread of realistic beams. A natural extension is to consider a relativistic Maxwellian (Maxwell–Jüttner) distribution, which describes a thermalized plasma. In the normalized form, it is expressed as [38]

$$f_{b0}(\mathbf{p}) = \frac{1}{4\pi\sigma_{\parallel,0}K_2(1/\sigma_{\parallel,0})} \exp\left(-\frac{\mathbf{p}}{\sigma_{\parallel,0}}\right), \quad (4.50)$$

where $\sigma_{\parallel,0}$ is the initial momentum spread along beam propagation direction, K_2 is the modified Bessel function of the second kind.

However, such a Maxwellian distribution is not representative of particle populations in astrophysical jets. In these environments, acceleration mechanisms such as shocks and magnetic reconnection produce non-Maxwellian beams with enhanced high-energy tails. To account for

this, one may consider a simplified suprathermal distribution which exhibits a power-law tail at high momenta of the form

$$f_{b0}(\mathbf{p}) = \frac{1}{\pi^2} \left(1 + \frac{\mathbf{p}^2}{\sigma_{\parallel,0}^2} \right)^{-2}. \quad (4.51)$$

Despite this difference in the high-energy behavior, the characteristic plasma screening length remains comparable for both distributions. For a Maxwellian beam defined in Eq. (4.50), the screening length can be computed as [250]

$$\lambda_{\text{scr}} \simeq \sigma_{\parallel,0}^{1/2} \frac{c}{\omega_p}. \quad (4.52)$$

For the Cauchy-type distribution function defined in Eq. (4.51), the corresponding expression becomes

$$\lambda_{\text{scr}} \simeq 0.88 \sigma_{\parallel,0}^{1/2} \frac{c}{\omega_p}. \quad (4.53)$$

The plasma screening-length expressions are approximately the same for both cases, indicating that the macroscopic screening properties are largely insensitive to the exact form of the distribution and instead depend primarily on the characteristic momentum scale. The key difference between the distributions, therefore, lies in their high-energy tails, which play a crucial role in wave-particle interactions and instability growth.

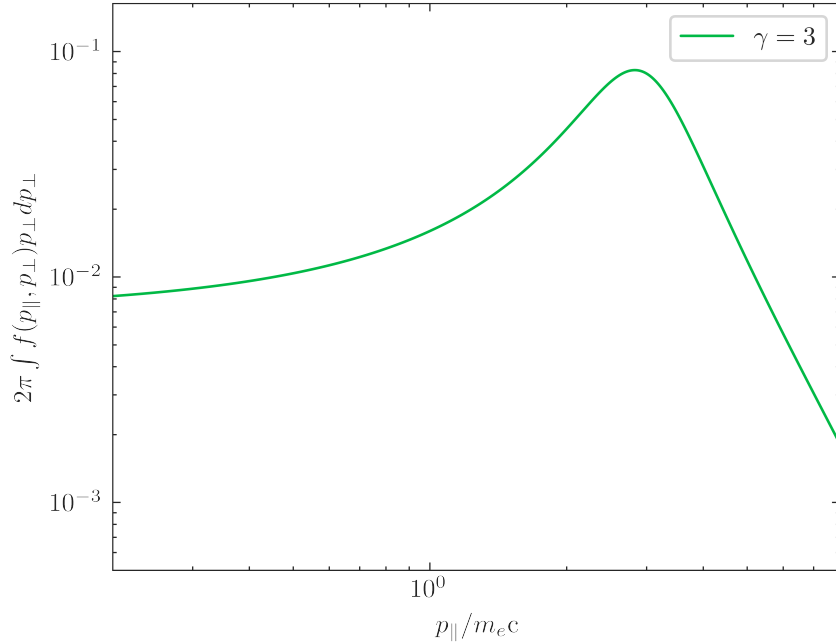


Figure 4.1: The distribution function given by Eq. (4.55) is employed in PIC simulations for a broad beam with $\gamma = 3$ and a range of α values. The beam is initialized with a longitudinal momentum spread, $\sigma_{\parallel,0} = 1.0$ MeV, a transverse momentum spread, $\sigma_{\perp,0} = 0.5$ MeV, and a mean longitudinal momentum, $\mu = 0.511(\gamma^2 - 1)^{1/2}$ MeV.

A more general framework to describe such non-thermal beams is provided by the relativistic

κ -distribution, which can be written as [256]

$$f_{b0}(\mathbf{p}) \propto \frac{\Gamma(\kappa + 1)}{2\pi\kappa^{3/2}\Gamma(3/2)\Gamma(\kappa - 1/2)} \left[1 + \gamma^2 \frac{(p_x - \mu)^2}{2\kappa\sigma_{\parallel}^2} + \gamma^2 \frac{(p_y^2 + p_z^2)}{2\kappa\sigma_{\perp}^2} \right]^{-(\kappa+1)}. \quad (4.54)$$

Here, κ determines the extent of the high-energy tail. In the limit $\kappa \rightarrow \infty$, the distribution converges to a Gaussian form, whereas for small values of κ it exhibits a power-law tail. Since the present work focuses on non-Maxwellian, astrophysically relevant beams, we adopt the limiting case $\kappa = 1$, which results in a distribution closely related to the Cauchy profile. We therefore employ an anisotropic Cauchy-type distribution (Breit-Wigner) for the beam, as shown in Fig. 4.1,

$$f_{b0}(\mathbf{p}) \propto \left[1 + \gamma^2 \frac{(p_x - \mu)^2}{2\sigma_{\parallel}^2} + \gamma^2 \frac{(p_y^2 + p_z^2)}{2\sigma_{\perp}^2} \right]^{-2}. \quad (4.55)$$

A transversely kinematically cold and a longitudinally warm, or broad beam, refers to a configuration in which the momentum spread in the perpendicular directions is small. In the case of astrophysical pair beams, this condition is typically expressed as $\sigma_{\perp} \ll \sigma_{\parallel}$, reflecting the strong collimation of particles along the beam propagation direction [33]. As a result, the particle momenta perpendicular to the beam are negligible compared to the parallel motion, and the beam remains highly collimated. Previous studies [31, 33, 44, 45, 54, 257, 258] investigating astrophysical plasma environments, particularly collisionless space plasmas, have mostly considered Maxwellian, non-monoenergetic beams with relatively small energy spreads. However, such distributions do not adequately capture the inherently broad and non-thermal nature of astrophysical pair beams. While Maxwellian beams are computationally convenient to implement in simulations, a more realistic representation can be achieved using broad (or warm) non-Maxwellian beams with a Cauchy distribution [250].

4.3.2 Beam-driven instabilities

The interaction between the relativistic pair beam and the background plasma gives rise to unstable collective modes, which can be broadly classified as electrostatic or electromagnetic. Electrostatic instabilities are characterized by fluctuations dominated by the electric field. The two-stream instability (TS) is a simplistic example of an electrostatic instability, characterized by the absence of magnetic fluctuations ($\delta\mathbf{B} = 0, \mathbf{k} \times \delta\mathbf{E} = 0$), and develops for wavevectors aligned with the beam propagation direction ($\mathbf{k} \parallel \mathbf{v}_b$). It is driven by the resonant interaction between beam particles and longitudinal plasma oscillations. In addition, oblique instability (Obl), for which the wavevector is oriented at an angle to the beam direction, is also predominantly electrostatic. Electromagnetic instabilities involve both electric and magnetic field perturbations. The filamentation instability (Cfi) is the primary example and develops when the wavevector is perpendicular to the beam velocity. This instability leads to the development of transverse current perturbations and the associated generation of magnetic fields.

In the cold limit, the theoretical growth rates of these instabilities can be expressed in terms

of the plasma frequency ω_p as [40]

$$\delta_{\text{TS}} \sim \left(\frac{\alpha}{\gamma}\right)^{1/3} \omega_p, \quad (4.56)$$

$$\delta_{\text{Cfl}} \sim \frac{\sqrt{3}}{2^{4/3}} \left(\frac{\alpha}{\gamma}\right)^{1/2} \omega_p, \quad (4.57)$$

$$\delta_{\text{Obl}} \sim \left(\frac{\alpha}{\gamma}\right)^{1/3} \omega_p, \quad (4.58)$$

where γ denotes the average Lorentz factor of the beam particles, and $\alpha = n_b/n_{\text{IGM}}$ is the ratio of the beam density n_b to the background plasma density n_{IGM} .

For astrophysically relevant scenarios, such as TeV pair beams produced by blazars in the IGM, the beam densities are typically of the order $n_b \sim [0.5-1] \times 10^{-23} \text{ cm}^{-3}$, while the background IGM density is $n_{\text{IGM}} \sim 10^{-7} \text{ cm}^{-3}$. This corresponds to a density ratio of $\alpha \sim [0.5-1] \times 10^{-16}$. The pair beams are produced with Lorentz factors $\gamma = 10^5 - 10^7$.

4.4 Numerical approaches

The fundamental equations governing plasma dynamics are, in principle, well established. However, analytical treatments are analytically solvable only for a limited range of idealized problems. In order to address more general and complex scenarios, a variety of numerical methods have been developed based on either kinetic or fluid descriptions of plasma behavior.

In MHD models, the plasma is treated as a conducting fluid by coupling Maxwell's equations with the equations of fluid dynamics. Such approaches are typically employed to investigate large-scale phenomena, where microscopic kinetic effects can be neglected. In contrast, kinetic models resolve the evolution of the particle distribution function in phase space. This can be achieved either by directly discretizing and solving the Vlasov equation or by representing the distribution function through a finite number of numerical particles that evolve under the self-consistent electromagnetic fields governed by Maxwell's equations.

The PIC method is a kinetic simulation framework that discretizes electromagnetic fields on a grid while evolving particles in continuous phase space. This method has become widely adopted due to its computational efficiency in handling high-dimensional kinetic problems. However, because the phase-space distribution is sampled by a finite number of macroparticles, PIC simulations inherently exhibit statistical noise. In contrast, Vlasov-based approaches are generally less noisy but computationally more demanding, particularly in multiple dimensions.

In certain applications, hybrid models are employed, in which some plasma species are treated using a fluid description while others are modeled kinetically, often via particle-based methods. Although kinetic approaches were originally developed for collisionless plasmas, extensions have been developed that incorporate collisional effects and other particle interactions within both the Vlasov and PIC frameworks.

The beam-plasma instabilities investigated in this work arise from perturbations in the momentum distribution of the beam particles. As a result, a kinetic description is essential to accurately capture the underlying physics. In this study, we adopt the PIC method, which will be described in detail in the next chapter.

Numerical Methods

This chapter presents the numerical methods used to study the propagation of blazar-induced particles through extragalactic space. We have to consider multiple physical processes in order to reconstruct the propagation of blazar-induced particles from their sources to Earth. These include stochastic energy losses, the generation of secondary particles, and magnetic deflections by intergalactic magnetic field (IGMF), each of which significantly influences the development and observable characteristics of the resulting cascade emission. Furthermore, given the uncertainties in modeling extragalactic propagation, numerical simulations are essential for testing predictions under different physical assumptions. In this thesis, two complementary simulation frameworks are employed. First, the plasma instability-driven energy losses are investigated using the kinetic Particle-in-Cell (PIC) code EPOCH-2D at laboratory scales. Afterward, a parametric energy-loss term is constructed to represent plasma instability cooling of electron-positron pairs, and its impact on the observed photon spectrum is studied within the Monte-Carlo framework CRPropa 3.2. This chapter provides an overview of these simulation tools and their implementation.

5.1 Overview of PIC simulations

The PIC simulation is one of the most widely used numerical frameworks for investigating the kinetic behavior of collisionless plasmas. The early implementations of PIC methods were limited to one-dimensional and purely electrostatic configurations [253, 254]. Contemporary PIC codes have extended their applicability to multidimensional domains in one-, two-, and three-dimensions [55, 259, 260]. In addition to increased dimensionality, modern implementations integrate advanced numerical techniques, such as higher-order particle shape functions and current-smoothing methods, to reduce numerical noise. Simulations are possible in three-dimensional momentum space and all three Cartesian components of the electric and magnetic fields, even when the spatial grid is restricted to one or two dimensions. Nevertheless, a fully three-dimensional approach is much more computationally expensive than this. In the context of beam-plasma instabilities, resolving two Cartesian spatial coordinates, with one in the direction parallel and the other in the perpendicular direction to the beam propagation, and three Cartesian dimensions of momentum is sufficient to capture the physical processes.

5.1.1 Simulation box and boundary conditions

The simulation box defines the spatial domain in which the particles and electromagnetic fields evolve. We use a Cartesian coordinate system, with both the simulation domain and the computational grid square or rectangular. In general, the simulation domain and the corresponding grid are characterized in each spatial direction i by the box length L_i and the number of grid cells N_i , assuming that the grid is equally spaced in every direction. The grid spacing $\Delta x_i = L_i/N_i$ determines the smallest spatial scale that can be resolved. Since the plasma skin depth c/ω_p represents the fundamental length scale of the system, the resolution must satisfy the condition $\Delta x_i \ll c/\omega_p$ in order to capture the relevant physical processes. These two quantities determine the range of wavevectors that can be resolved. The minimum resolvable wavenumber is $\Delta k_i = \pi/L_i$, while the maximum resolvable wavenumber (Nyquist limit) is $k_{i,\text{max}} = \pi/\Delta x_i$, where $\mathbf{k} = (k_x, k_y, k_z)$ denotes the wavevector of a plasma mode. In our simulation, we define $k_{\parallel} = k_x$ and $k_{\perp} = (k_y^2 + k_z^2)^{1/2}$, considering the beam is propagating along x-axis. For electrostatic oblique instability, the fastest growing modes are localized around $k_{\parallel} \approx c \cdot \omega_p^{-1}$ and a wide range in $k_{\perp}$. Whereas, the filamentation instability is a purely transverse electromagnetic instability characterized by $k_{\parallel} \approx 0$ and finite $k_{\perp}$. The simulation domain must be sufficiently large to resolve the instability.

We need to specify boundary conditions in each spatial direction of the simulation box to determine how particles and electromagnetic fields behave at these boundaries. Boundary conditions must be chosen according to the physical setup being modeled and can be assigned independently of the following options:

- **Periodic:** Particles and fields leaving one side of the simulation box re-enter from the opposite side. This effectively models a spatially repeating system.
- **Reflecting:** Particles are reflected at the boundary, while the electromagnetic fields are set to zero there (Dirichlet boundary conditions).
- **Conducting:** Particles are reflected at the boundary, and the electromagnetic fields satisfy perfectly conducting boundary conditions, meaning the tangential component of the electric field vanishes at the boundary.
- **Open (or absorbing):** Particles and fields are propagated through the boundary and removed from the computational domain.
- **Thermal:** These boundary conditions model a thermal particle reservoir at the boundary. Particles leaving the simulation are replaced by incoming particles whose velocities are obtained from a Maxwellian distribution corresponding to the initial plasma temperature.

When studying large-scale astrophysical phenomena in the laboratory, periodic boundary conditions along the beam propagation direction are often used because they effectively mimic an infinitely extended system. However, to realistically model an astrophysical beam in a laboratory setup, one must consider a beam of finite width with open boundary conditions in the transverse direction.

5.1.2 Particle shape functions

The phase-space distribution function $f_\alpha(\mathbf{p}, \mathbf{r}, t)$ of a particle species α is represented by a finite ensemble of macro-particles. Each macro-particle evolves according to the characteristics of the Vlasov equation, as described in the previous section (see Eq. (4.5)). If each macro-particle evolves along a characteristic of the Eq. 4.5, the total distribution function for a particle species $f_\alpha(\mathbf{p}, \mathbf{r}, t)$ can be expressed as a sum over the contributions of the macro-particles representing that species. Substituting this representation into Eq. 4.5 and taking the first moments with respect to position $\mathbf{r}$ and momentum $\mathbf{p}$ yields the equations of motion for the macro-particles for particle species α [253, 254],

$$\dot{\mathbf{r}} = \frac{\mathbf{p}}{m_\alpha}, \quad (5.1)$$

$$\dot{\mathbf{p}} = q_\alpha \left[\mathbf{E}(\mathbf{r}, t) + \frac{\mathbf{p}}{m_\alpha} \times \mathbf{B}(\mathbf{r}, t) \right]. \quad (5.2)$$

Together, the Vlasov equation and Maxwell's equations form the Maxwell-Vlasov system, which provides the fundamental kinetic framework for collisionless plasma dynamics. In the PIC framework, the continuous distribution function f_α appearing in the kinetic description (Eqs. (4.10) and (4.11)) is represented by a finite number of macro-particles. Therefore, an interpolation scheme is required to consistently deposit particle charge and current densities onto the simulation grid. The charge and current densities are obtained by summing over these macro-particles, where the particle shape function S replaces the spatial dependence of the distribution function. Each macroparticle is characterized by a weight factor w_i and a shape function $S(\mathbf{r} - \mathbf{r}_i(t))$, where w_i represents the number of physical particles contained within a single macroparticle, and $\mathbf{r}$ corresponds to the grid-point position and $\mathbf{r}_i$ denotes the position of the i -th particle. The charge density and current density can be expressed using Eqs. (4.10) and (4.11) as

$$\rho(\mathbf{r}, t) = \sum_{i=1}^{N_p} q_i \int w_i S(\mathbf{r} - \mathbf{r}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t)) d^3p, \quad (5.3)$$

$$\mathbf{j}(\mathbf{r}, t) = \sum_{i=1}^{N_p} q_i \int w_i \frac{\mathbf{p}_i}{m_i} S(\mathbf{r} - \mathbf{r}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t)) d^3p. \quad (5.4)$$

The trajectories of the macro-particles in the simulations are treated continuously, whereas $\rho(\mathbf{r}, t)$ and $\mathbf{j}(\mathbf{r}, t)$ are defined on a discrete grid [261, 262]. The shape function S is generally defined as a B-spline function. The simplest choice is the top-hat shape function, corresponding to a zeroth-order B-spline, in which each particle is treated as a delta function, and its entire charge is assigned to the nearest grid point. This approach is computationally inexpensive, but it introduces significant numerical noise due to the discrete nature of the particle representation. More advanced schemes distribute particle quantities over multiple grid points, resulting in smoother charge and current densities. For instance, the cloud-in-cell method corresponds to a first-order shape function, while higher-order schemes such as the triangular-shaped cloud and higher-order B-splines further improve smoothness and accuracy. The B-spline is a piecewise polynomial function defined by the convolution of lower-order splines, commonly used as a smooth, compact shape function [263],

$$b_l(\mathbf{r}) = \int_{-\infty}^{\infty} d\mathbf{r}' b_{l-1}(\mathbf{r}') b_0(\mathbf{r} - \mathbf{r}'), \quad (5.5)$$

$$b_0(\mathbf{r}) = \begin{cases} 1, & \text{for } |\mathbf{r}| < \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases} \quad (5.6)$$

Since the electromagnetic fields are defined on a discrete grid, they must be interpolated to the particle positions in order to evaluate the Lorentz force. For consistency and energy conservation, the same shape function used in charge and current deposition is employed as [264]

$$\mathbf{E}(\mathbf{r}_i) = \sum_j \mathbf{E}_j S(\mathbf{r}_i - \mathbf{r}_j), \quad (5.7)$$

$$\mathbf{B}(\mathbf{r}_i) = \sum_j \mathbf{B}_j S(\mathbf{r}_i - \mathbf{r}_j), \quad (5.8)$$

where $\mathbf{r}_j$ denotes the position of the grid point indexed by j .

5.1.3 Field solver

The electromagnetic fields are updated according to Maxwell's equations. In kinetic simulations, Eqs. (4.8) and (4.9) are represented on a staggered Yee lattice configuration using the finite-difference time-domain (FDTD) scheme [265, 266]. For the scenario of a neutral beam propagating through a background plasma, these constraints are automatically satisfied when both the electric and magnetic fields are initially set to zero. The Yee lattice [267] (shown in Fig. 5.1) places the electric and magnetic field components on staggered spatial grids, which allows the spatial derivatives required for the curl operators to be evaluated using centered finite differences with second-order accuracy. To update the magnetic field component at the grid location $(i + 1/2, j + 1/2, k)$, the curl of the electric field, $\nabla \times \mathbf{E}$, must be evaluated at that position. This requires computing the spatial derivatives of the electric field components at the same location. As an example, for B_z the required terms to compute are $\partial E_y / \partial x$ and $\partial E_x / \partial y$. The second-order accurate derivatives can be updated by

$$\left(\frac{\partial E_y}{\partial x} \right)_{i+1/2, j+1/2, k} = \frac{(E_y)_{i+1, j+1/2, k} - (E_y)_{i, j+1/2, k}}{\Delta x}, \quad (5.9)$$

$$\left(\frac{\partial E_x}{\partial y} \right)_{i+1/2, j+1/2, k} = \frac{(E_x)_{i+1, j+1/2, k} - (E_x)_{i, j+1/2, k}}{\Delta y}. \quad (5.10)$$

The sixth-order accurate derivatives are computed as follows [25]

$$\begin{aligned} \left(\frac{\partial E_y}{\partial x} \right)_{i+1/2, j+1/2, k} = & \frac{1}{64\Delta x} \left[\frac{3}{10} (E_y)_{i+3, j+1/2, k} - \frac{25}{6} (E_y)_{i+2, j+1/2, k} + 75 (E_y)_{i+1, j+1/2, k} \right. \\ & \left. - 75 (E_y)_{i, j+1/2, k} + \frac{25}{6} (E_y)_{i-1, j+1/2, k} - \frac{3}{10} (E_y)_{i-2, j+1/2, k} \right], \end{aligned} \quad (5.11)$$

$$\begin{aligned} \left(\frac{\partial E_x}{\partial y} \right)_{i+1/2, j+1/2, k} = & \frac{1}{64\Delta y} \left[\frac{3}{10} (E_x)_{i+3, j+1/2, k} - \frac{25}{6} (E_x)_{i+2, j+1/2, k} + 75 (E_x)_{i+1, j+1/2, k} \right. \\ & \left. - 75 (E_x)_{i, j+1/2, k} + \frac{25}{6} (E_x)_{i-1, j+1/2, k} - \frac{3}{10} (E_x)_{i-2, j+1/2, k} \right]. \end{aligned} \quad (5.12)$$

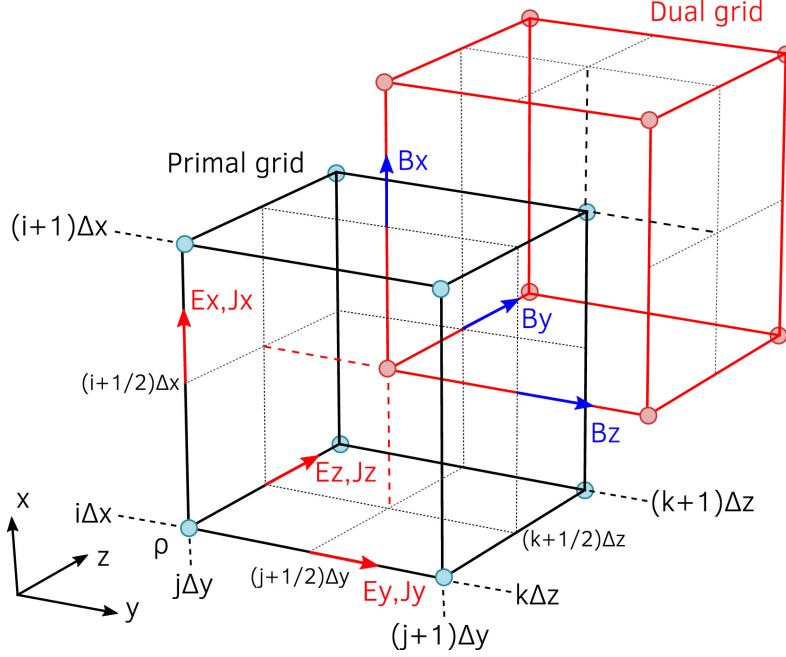


Figure 5.1: Geometric representation of the three-dimensional Staggered Yee lattice used in the FDTD method. In each simulation cell, the charge density ρ is defined at the cell corners, while the current density $\mathbf{j}$ and the electric field $\mathbf{E}$ are defined at the centers of the cell edges. The magnetic field $\mathbf{B}$ is defined at the centers of the cell faces [268]. The figure is taken from [268].

The electric and magnetic fields are updated using a temporal leapfrog scheme (as shown in Fig. 5.2) that computes the electric field component $\mathbf{E}^{n+1/2}$ and magnetic field component $\mathbf{B}^{n+1/2}$ at time $t_{n+1/2} = (n + 1/2)\Delta t$,

$$\mathbf{E}^{n+1/2} = \mathbf{E}^n + \frac{c^2 \Delta t}{2} \left(\nabla \times \mathbf{B}^n - \frac{\mathbf{j}^n}{c^2 \epsilon_0} \right), \quad (5.13)$$

$$\mathbf{B}^{n+1/2} = \mathbf{B}^n - \frac{\Delta t}{2} \left(\nabla \times \mathbf{E}^{n+1/2} \right). \quad (5.14)$$

Then the particle pusher (see Sec. 5.1.4 for detail description) updates the particle position $\mathbf{r}^n$ and momentum $\mathbf{p}^n$ over a full time step using the half-step field values. From the updated particle quantities, the current density $\mathbf{j}^{n+1}$ is computed and used to advance the electromagnetic fields, $\mathbf{E}^{n+1}$ and $\mathbf{B}^{n+1}$, at time $t_{n+1} = (n + 1)\Delta t$,

$$\mathbf{B}^{n+1} = \mathbf{B}^{n+1/2} - \frac{\Delta t}{2} \left(\nabla \times \mathbf{E}^{n+1/2} \right), \quad (5.15)$$

$$\mathbf{E}^{n+1} = \mathbf{E}^{n+1/2} + \frac{c^2 \Delta t}{2} \left(\nabla \times \mathbf{B}^{n+1} - \frac{\mathbf{j}^{n+1}}{c^2 \epsilon_0} \right). \quad (5.16)$$

The time step Δt defines the temporal grid spacing. Since the computational effort increases as the time step decreases for a given physical time, it is desirable to choose Δt as large as possible. In beam-plasma instability simulations, the time step must be smaller than the plasma

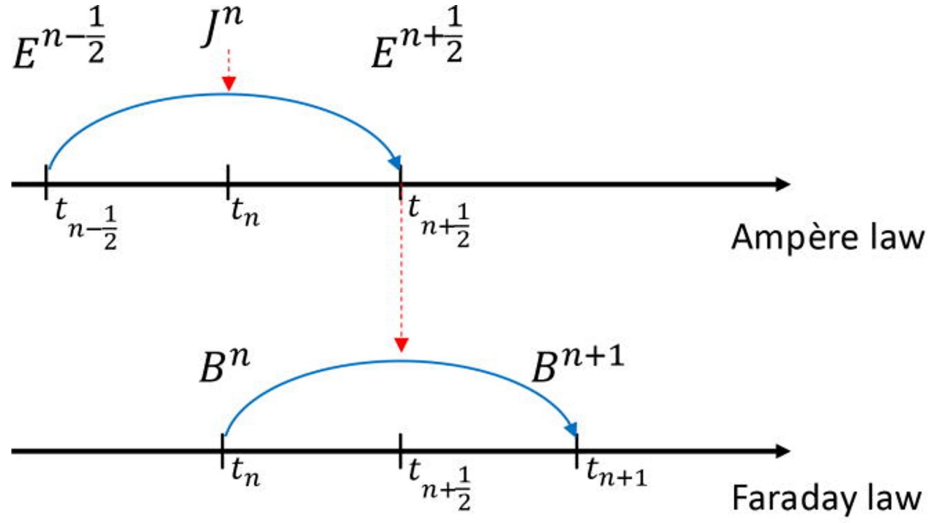


Figure 5.2: Leapfrog update scheme for electromagnetic fields in the kinetic simulations, where the electric field is advanced via Ampère’s law and the magnetic field via Faraday’s law, with $\mathbf{E}$ and $\mathbf{B}$ staggered by half a timestep to achieve time-centered, second-order accurate evolution. The figure is adapted from [269].

oscillation time scale, although the grid spacing already imposes an upper bound through the Courant–Friedrichs–Lewy (CFL) condition. Substituting Eq. (5.14) into Eq. (5.15) gives the magnetic field update, whereas adding Eq. (5.16) at n with Eq. (5.13) provides the electric field on the staggered Yee grid,

$$\mathbf{B}^{n+\frac{1}{2}} = \mathbf{B}^n - \Delta t \left(\nabla \times \mathbf{E}^{n+\frac{1}{2}} \right), \quad (5.17)$$

$$\mathbf{E}^{n+\frac{1}{2}} = \mathbf{E}^{n-\frac{1}{2}} + c^2 \Delta t \left(\nabla \times \mathbf{B}^n - \frac{\mathbf{j}^n}{c^2 \epsilon_0} \right). \quad (5.18)$$

The y -component of $(\nabla \times \mathbf{E})_y = -\partial E_z / \partial x$ for a one-dimensional system $\mathbf{E} = (0, 0, E_z)$ and $\mathbf{B} = (0, B_y, 0)$. In one dimension, and in the absence of particles ($\mathbf{j} = \mathbf{0}$), the second-order derivative can be expressed as [25],

$$(B_y)_j^{n+1} = (B_y)_j^n + \frac{\Delta t}{\Delta x} \left((E_z)_{j+1}^{n+\frac{1}{2}} - (E_z)_j^{n+\frac{1}{2}} \right), \quad (5.19)$$

$$(E_z)_j^{n+\frac{1}{2}} = (E_z)_j^{n-\frac{1}{2}} + \frac{c^2 \Delta t}{\Delta x} \left((B_y)_j^n - (B_y)_{j-1}^n \right). \quad (5.20)$$

Using a plane-wave ansatz $(A_y)_j^n = A_0 \exp[i(kj\Delta x - \omega n\Delta t)]$, where n and j denote the time and grid indices, respectively, and substituting into Eq. (5.19) and Eq. (5.20), the dispersion relation for the one-dimensional second-order scheme is obtained

$$\sin^2 \left(\frac{\omega \Delta t}{2} \right) = \left(\frac{c \Delta t}{\Delta x} \right)^2 \sin^2 \left(\frac{k \Delta x}{2} \right). \quad (5.21)$$

For numerical stability, the left-hand side of the dispersion relation must satisfy $\sin^2(\omega \Delta t / 2) \leq 1$, while the maximum value of the right-hand side occurs when $\sin^2(k \Delta x / 2) = 1$. This leads to

the CFL stability condition or the one-dimensional second-order scheme,

$$\Delta t \leq \frac{\Delta x}{c}. \quad (5.22)$$

Similarly, the CFL criterion [270] in three dimensions for second-, fourth-, and sixth-order schemes can be expressed as,

$$\Delta t \leq \frac{1}{c} \frac{1}{\sqrt{(\Delta x)^{-2} + (\Delta y)^{-2} + (\Delta z)^{-2}}}, \quad (5.23)$$

$$\Delta t \leq \frac{6}{7c} \frac{1}{\sqrt{(\Delta x)^{-2} + (\Delta y)^{-2} + (\Delta z)^{-2}}}, \quad (5.24)$$

$$\Delta t \leq \frac{120}{149c} \frac{1}{\sqrt{(\Delta x)^{-2} + (\Delta y)^{-2} + (\Delta z)^{-2}}}, \quad (5.25)$$

where $\Delta x, \Delta y, \Delta z$ are the grid spacings in the three spatial directions.

5.1.4 Particle pusher scheme

The particle pusher solves the equation of motion governed by the Lorentz force to update the position and momentum using a leapfrog scheme shown in Fig. 5.3. The equation of motion Eq. (5.1) can be written as,

$$\frac{\mathbf{r}^{n+1} - \mathbf{r}^n}{\Delta t} = \frac{\mathbf{p}^{n+\frac{1}{2}}}{m_\alpha}, \quad (5.26)$$

$$\frac{\mathbf{p}^{n+\frac{1}{2}} - \mathbf{p}^{n-\frac{1}{2}}}{\Delta t} = q_\alpha \left[\mathbf{E}^n(\mathbf{r}^n, t) + \frac{\mathbf{p}^n}{m_\alpha} \times \mathbf{B}^n(\mathbf{r}^n, t) \right], \quad (5.27)$$

where Δt represents the time-step size. The electric field is responsible for particle acceleration, while the magnetic field only modifies the direction of the particle momentum. In the standard Boris scheme [271], the electric and magnetic effects are handled separately: half of the electric field acceleration is applied first, followed by the magnetic rotation, and then the remaining half of the electric field acceleration. A more advanced scheme by Higuera and Cary [272] introduces an alternative velocity-averaging method that improves accuracy in the highly relativistic regime.

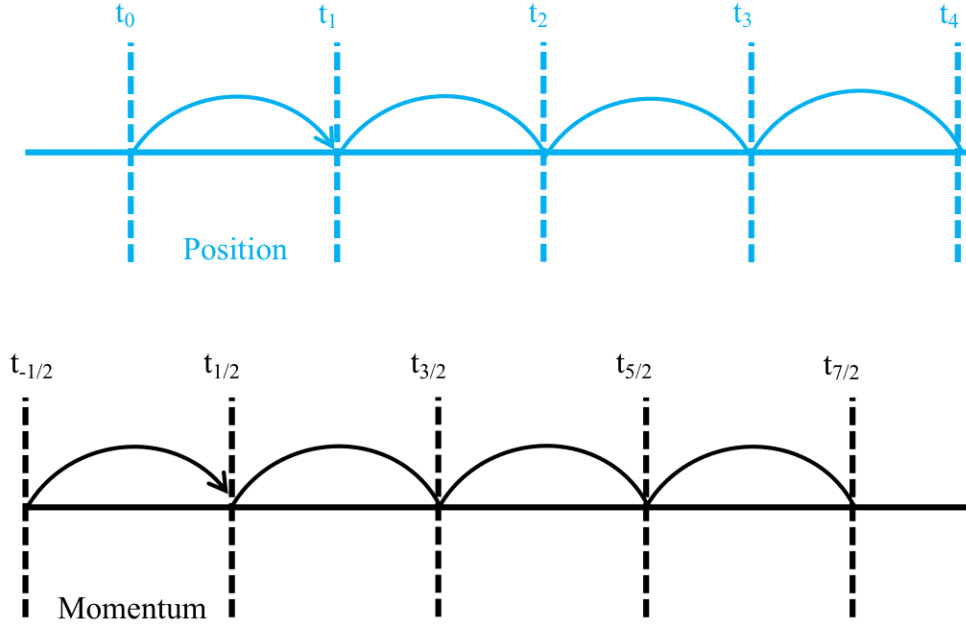


Figure 5.3: Schematic of the leapfrog method for advancing the particles.

5.1.5 Current smoothing method

To reduce numerical noise, particularly at high wavenumbers in the current density $\mathbf{j}$, smoothing or filtering methods are applied. This convolution-based smoothing acts as a low-pass filter, suppressing high-frequency components while preserving large-scale structures. In practice, the current at a given grid point is averaged with the values at neighboring points separated by a distance h , resulting in a filtered current $\mathbf{j}^f$ [273]. The parameter ξ controls the strength of the damping by determining the relative contribution of the local and neighboring values. Repeated application of the filter, possibly with different stride lengths, reduces noise across multiple spatial scales. The filtered current for d -dimensional can be expressed as,

$$\mathbf{j}_{\mathbf{i}}^f = \xi \mathbf{j}_{\mathbf{i}} + \frac{1 - \xi}{2d} \sum_{m=1}^d (\mathbf{j}_{\mathbf{i} + h\mathbf{e}_m} + \mathbf{j}_{\mathbf{i} - h\mathbf{e}_m}). \quad (5.28)$$

In the general formulation of the filtered current in Eq. (5.28), $\mathbf{e}_m$ denotes the unit vector along the m -th coordinate direction, where $m = 1, \dots, d$ and d is the number of spatial dimensions. The grid index vector is defined as $\mathbf{i} = (i_1, \dots, i_d)$, where each component corresponds to an integer index along a coordinate direction. The term $\mathbf{i} \pm h\mathbf{e}_m$ therefore represents a shift of the grid index by a distance h in the positive or negative direction along the m -th axis. The summation over m accounts for all coordinate directions, with each direction contributing two neighboring points. Eq. (5.28) can be written explicitly for one-, two-, and three-dimensional

cases as

$$\mathbf{j}_i^f = \xi \mathbf{j}_i + \frac{1-\xi}{2} (\mathbf{j}_{i+h} + \mathbf{j}_{i-h}), \quad (5.29)$$

$$\mathbf{j}_{i,j}^f = \xi \mathbf{j}_{i,j} + \frac{1-\xi}{4} (\mathbf{j}_{i+h,j} + \mathbf{j}_{i,j+h} + \mathbf{j}_{i-h,j} + \mathbf{j}_{i,j-h}), \quad (5.30)$$

$$\mathbf{j}_{i,j,k}^f = \xi \mathbf{j}_{i,j,k} + \frac{1-\xi}{6} (\mathbf{j}_{i+h,j,k} + \mathbf{j}_{i,j+h,k} + \mathbf{j}_{i,j,k+h} + \mathbf{j}_{i-h,j,k} + \mathbf{j}_{i,j-h,k} + \mathbf{j}_{i,j,k-h}), \quad (5.31)$$

In one dimension, this reduces to the two nearest neighbors j_{i+h} and j_{i-h} . In two dimensions, it yields four neighbors, $j_{i\pm h,j}$ and $j_{i,j\pm h}$, corresponding to shifts along the x - and y -directions. In three dimensions, the expression extends to six neighbors, $j_{i\pm h,j,k}$, $j_{i,j\pm h,k}$, and $j_{i,j,k\pm h}$, accounting for all directions in the three-dimensional grid. The parameter ξ controls the relative weight between the central grid point and its neighboring values in the filtering operation. The explicit forms of the filter in one, two, and three dimensions are given in Eqs. (5.29), (5.30), and (5.31). The choice of stride and the number of applications must be made carefully, as excessive filtering can lead to physically unrealistic results, while insufficient filtering fails to adequately suppress noise.

5.1.6 Core algorithm of PIC

The core components of the PIC algorithm comprise the particle pusher and the field solver, which are discussed in the subsequent sections. As shown in Fig. 5.4, the computational procedure is initialized with macroparticles distributed across a discretized spatial domain and evolved over discrete time steps. At each time step, the electromagnetic fields are interpolated from the grid nodes to the particle positions, following which the Lorentz force is evaluated. The particle velocities and positions are then updated accordingly. Subsequently, the resulting particle currents are deposited onto the grid nodes, thereby ensuring a self-consistent coupling between the particles and the electromagnetic fields.

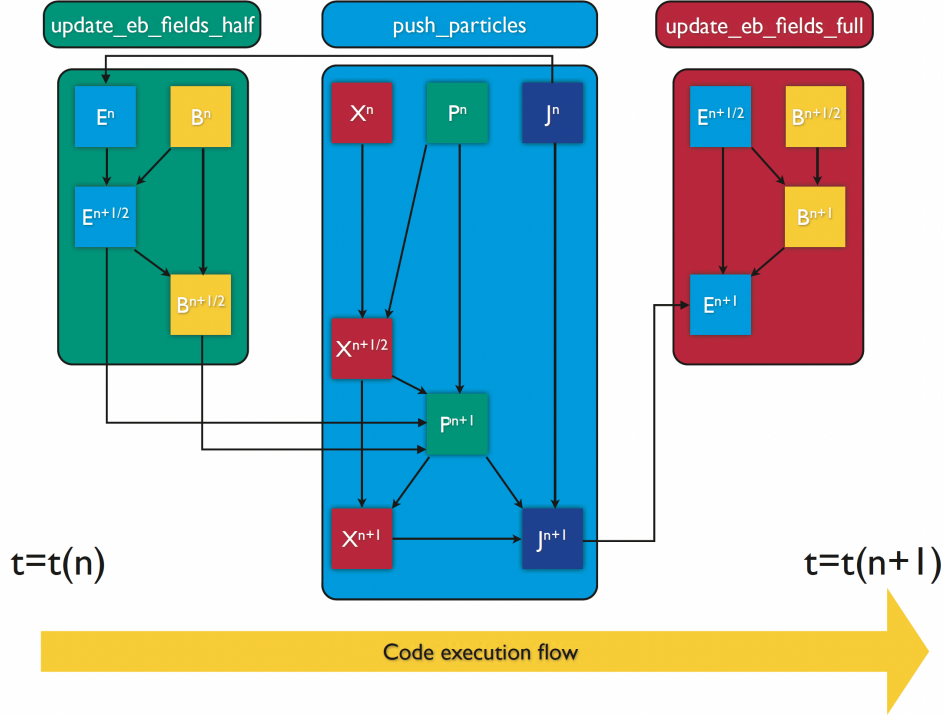


Figure 5.4: Schematic of the PIC algorithm showing field interpolation, particle push, and current deposition on the computational grid, adapted from [274].

5.1.7 Simulation output

PIC simulations generate a large amount of data, as both particle properties (positions and momenta) and electromagnetic fields must be stored across the computational domain. Even storing these quantities for a single time step can be memory-intensive, making it impractical to record their full evolution at every time step. To address this limitation, more efficient data reduction strategies are employed. In this work, we refer to any quantity saved at a given time step, or averaged over a time interval, as an observable. For instance, one may consider a field component at a given time or the positions of particles of a specific species.

A simple approach to reduce storage requirements is to record output quantities only at selected time steps, either at regular intervals $\Delta t_{\text{out}} \gg \Delta t$ or at predefined times. Here, Δt_{out} denotes the time interval between successive data outputs, and the simulation time step is defined by Δt . In addition, data output can be restricted to times beyond a specified starting point t_{start} . Further efficiency can be achieved by constructing observables that require less memory. For instance, scalar quantities such as the total field energy density or the particle kinetic energy density can be stored with minimal storage requirements. Particle properties can also be averaged over grid cells, yielding observables comparable in magnitude to field data, or represented as histograms to capture their statistical distribution.

Another strategy is to consider only a subset of particles, selected either randomly or based on specific criteria. Random sampling reduces storage requirements at the expense of increased statistical noise, while conditional selection enables focused analysis of particular regions. When

the same subset of particles is followed over multiple time steps, it is called a persistent subset.

5.2 Configuration of simulations in EPOCH

PIC method has been implemented in a wide range of numerical simulation codes. In this work, we employ the EPOCH code [55], in particular, version 4.18.0. EPOCH is a fully electromagnetic, relativistic PIC code that supports simulations in multiple spatial dimensions. It is written in Fortran and employs OpenMPI (Message Passing Interface)¹ for parallelization, allowing efficient execution on distributed computing systems. The simulation setup is controlled by a configuration file, referred to as the input deck, together with compiler options. A representative example of the input deck we use in our simulation is provided in Appendix B.

EPOCH is available in three variants, EPOCH-1D, EPOCH-2D, and EPOCH-3D, corresponding to simulations in one, two, and three spatial dimensions, respectively. In all cases, particle momentum is resolved in three dimensions. Apart from the dimensionality of the spatial grid, the different versions share the same underlying implementation and functionality. In this work, simulations are performed using the two-dimensional version of the code EPOCH-2D. Although EPOCH includes a variety of advanced physics modules, such as laser-plasma interactions, particle collisions, quantum electrodynamics (QED) effects, radiation reaction, and ionization processes, these features are not relevant for our study.

5.2.1 Compiler configuration

At compile time, a range of options can be specified to tailor the code to the problem at hand. In particular, unnecessary features can be disabled to reduce computational overhead. Two key choices that must be made during compilation are the particle pusher and the particle shape function. EPOCH provides implementations of both the Boris pusher [271] and the Higuera and Cary pusher [272]. In addition, several particle shape functions are available, including top-hat, triangular, and higher-order B-spline shapes, corresponding to different orders of particle weighting.

5.2.2 Input Deck parameters

The input deck in EPOCH is a text-based configuration file that defines the simulation setup in a flexible and user-friendly manner. It allows defining constants, enabling straightforward parameterization and facilitating the execution of multiple simulations with varying parameters. The parser also supports a limited set of mathematical expressions, which can simplify the specification of input parameters.

The input deck provides control over several aspects of the simulation, including the size of the computational domain and the number of grid cells in each direction. The total runtime can be specified either in terms of the number of time steps or the physical simulation time. It is also possible to define a moving simulation window with a prescribed velocity, optionally activating it after a specified start time.

The choice of field solver can be specified, including Yee's scheme [267] with second-, fourth-, or sixth-order accurate derivatives, as well as alternative second-order schemes designed to

¹For details: <https://www.open-mpi.org/>.

optimize dispersion properties under specific constraints. The corresponding CFL condition [270] is automatically evaluated based on the selected solver and grid configuration, and a scaling factor between 0 and 1 can be applied to further reduce the time step if required.

Additional options include applying stride current filtering, in which both the stride length and the number of filter passes per time step can be specified. For each particle species, the spatial density profile and momentum distribution can be specified explicitly. For thermal populations, it is also possible to specify the temperature rather than provide a full distribution function. Output quantities can be written at regular intervals, with independent control over the output frequency for each quantity. All data for a given time step are stored in a single file in the Self-Describing File (SDF) format.

5.2.3 Unit system

The code internally operates using SI units [57]. However, the characteristic time and length scales relevant to plasma phenomena are typically determined by the electron plasma frequency. For example, in laboratory plasmas with densities in the range $10^{12} - 10^{16} \text{ cm}^{-3}$, the plasma frequency lies in the terahertz (THz) range, whereas in astrophysical environments with densities of $10^{-8} - 10^{-6} \text{ cm}^{-3}$, it is of the order of hertz (Hz). To ensure a consistent description across these widely different regimes, all quantities in this work are normalized to plasma parameters.

Specifically, time is scaled by the inverse plasma frequency, $t \rightarrow t\omega_p$, and spatial coordinates are normalized by the corresponding plasma length scale, $x \rightarrow x\omega_p/c$. In this normalization, only the background plasma density is taken into account, while the contribution of the dilute pair beam is not considered. Additionally, energies, particle momenta, and temperatures are expressed in energy units, which simplifies the interpretation of relativistic quantities. This leads to a normalized unit system, often referred to as 'plasma units', in which $\omega_p = c = k_B = 1$. While this normalization facilitates comparison across different density regimes, it is necessary to convert back to physical units when making predictions for specific physical environments.

5.3 Structure of CRPropa 3.2 framework

CRPropa [56, 275] is a Monte-Carlo simulation framework designed to model the propagation of cosmic rays, electromagnetic particles, and neutrinos in a variety of astrophysical environments. In this work, version CRPropa 3.2² is used. CRPropa is implemented in C++ and provides a Python interface through SWIG³, enabling flexible user interaction and scripting. From version CRPropa3 onward, it adopts a fully modular architecture (as seen from Fig. 5.5), in which different aspects of a simulation, such as particle interactions, source definitions, and observer configurations, are handled by independent modules. This design not only improves flexibility but also allows users to extend the code by implementing custom modules tailored to specific applications. A simulation consists of propagating particles through the specified environment. As each particle is treated independently, the computation can be efficiently parallelized. In CRPropa 3.2, this is achieved using OpenMP⁴, which enables shared-memory parallelization and significantly reduces the overall computational time.

²CRPropa latest release: <https://crpropa.desy.de/> and <https://crpropa.github.io/CRPropa3/>.

³For more information see <https://www.swig.org/> and <https://pypi.org/project/swig/>.

⁴See <https://www.openmp.org/specifications/> for specifications.

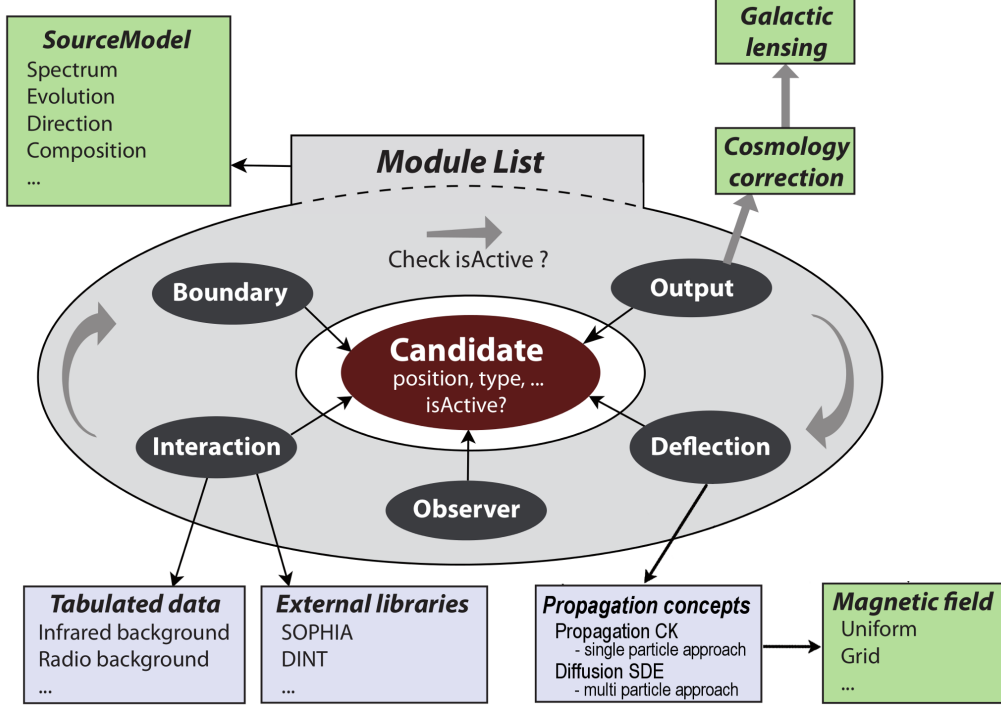


Figure 5.5: The modular structure of CRPropa. At each propagation step, the `isActive` flag determines whether the candidate continues to propagate. The figure is taken from [275].

5.3.1 Source modeling and observer configuration

The collimated jets of TeV blazars can be approximated in simulations as conical emission regions to simplify their propagation and detection. For a realistic representation of the gamma-ray emission from a blazar, the jet can be modeled as an emission cone following a von Mises–Fisher distribution (vMF) [276], which corresponds to the generalization of a normal distribution on a sphere. The implementation of the von Mises-Fisher distribution in CRPropa 3.2 can be found in [56, 277]. Given the unit vector $\hat{n}$, the von Mises-Fisher distribution of $\hat{n}$ on the unit sphere is given by

$$\Pi_{\text{vMF}}(\hat{n}; \hat{\mu}, \kappa) = \frac{\kappa}{2\pi(1 - e^{-2\kappa})} \exp[\kappa(\hat{n} \cdot \hat{\mu} - 1)], \quad (5.32)$$

where $\hat{\mu}$ is a unit vector representing the mean injection direction and $\kappa > 0$ is the concentration parameter, which parameterizes the width of the distribution. We can see that for $\kappa \ll 1$, the vMF distribution converges to the isotropic distribution $1/4\pi$. The concentration parameter κ can be used to parameterize the opening angle δ_{jet} of the jet. Assuming that a certain fraction P of emitted photons is contained within the jet axis and the opening angle δ_{jet} , from Eq. (5.32) we obtain that

$$P = \frac{1 - e^{\kappa(\cos \delta_{\text{jet}} - 1)}}{1 - e^{-2\kappa}}. \quad (5.33)$$

For $\kappa \gg 1$, the concentration parameter is given by

$$\kappa \simeq \frac{\ln(1 - P)}{\cos \delta_{\text{jet}} - 1}. \quad (5.34)$$

For instance, a value of $\kappa = 196$ yields an angular distribution in which approximately 95% of

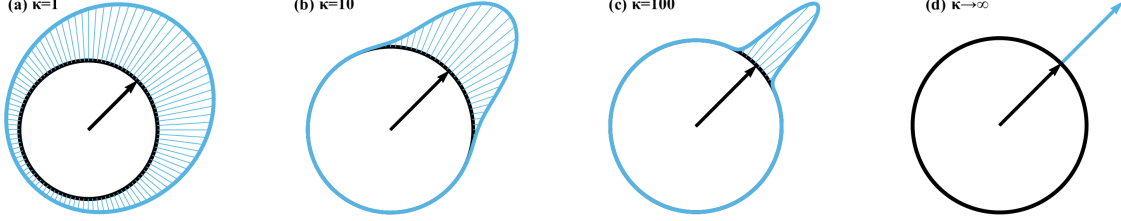


Figure 5.6: Two-dimensional representation of the angular distribution. The blue contour shows the vMF probability density in two dimensions as a function of the targeting parameter κ . The mean emission direction is indicated by an arrow. The parameter values from (a) to (d) are $\kappa = 1, 10, 100, \infty$. The figure is adapted from Ref. [278].

the injected photons are contained within the jet opening angle $\delta_{\text{jet}} = 10^\circ$. Photons are injected as primary particles, and their energies are sampled from a power-law spectrum characterized by a minimum energy, a maximum energy, and a spectral index. In **CRPropa**, observers are implemented as surfaces, such as spheres, that register particles upon crossing and record both their propagated-state properties (energy, position, momentum, particle type) and their initial source parameters in the output file.

5.3.2 Modeling of a turbulent magnetic field

In **CRPropa**, the turbulent magnetic field is modeled as a homogeneous field following a Kolmogorov power spectrum with spectral index $\mathcal{K} = 5/3$, concentrated around a characteristic wavenumber λ_c^{-1} . The magnetic field coherence length, λ_c , is determined by the maximum and minimum turbulence scales, λ_{max} and λ_{min} , respectively, and is expressed as [279]

$$\lambda_c = \frac{1}{2\lambda_{\text{max}}} \cdot \frac{\mathcal{K} - 1}{\mathcal{K}} \cdot \frac{1 - (\lambda_{\text{min}}/\lambda_{\text{max}})^{\mathcal{K}}}{1 - (\lambda_{\text{min}}/\lambda_{\text{max}})^{\mathcal{K}-1}}. \quad (5.35)$$

Since λ_{max} and λ_{min} serve as direct inputs to the simulation, this relation must be solved numerically to obtain a target coherence length λ_c . The discretized nature of the magnetic field grid in **CRPropa**, however, imposes physical limits on the turbulence scales: for a grid with N number of cells of size Δx , the turbulent field is fully resolved only if $\lambda_{\text{max}} < N\Delta x$ and $\lambda_{\text{min}} > 2\Delta x$. These constraints limit the achievable range of λ_c . This field strength can be approximated by $B \simeq E_e \delta / (|q_e| c \sqrt{L \lambda_c})$, where L is the distance travelled by the electron, δ is its deflection angle with respect to the direction of the primary photon, and E_e is the electron energy with charge q_e [280]. As an example, for a grid with $N = 256$, the values of λ_{min} and λ_{max} are chosen in accordance with these constraints to ensure a physically consistent realization of the turbulent magnetic field.

5.3.3 Particle propagation and interactions

The evolution of the electromagnetic cascade in **CRPropa** is modeled using a Monte Carlo approach that simulates the production of secondary particles via interactions. At each step of the simulation, interaction probabilities are evaluated based on the particle energy and the selected

background photon fields. These probabilities determine the generation of secondary particles, which are then incorporated into the simulation through the corresponding interaction modules.

There are integrated modules for interaction processes, including pair production, double-pair production, triplet-pair production, and inverse Compton scattering in **CRPropa**. These modules require, as input, the choice of photon background and options for tracking secondary particles, such as electron–positron pairs or photons. For each interaction, the code first computes the interaction rates, defined as the inverse of the interaction lengths (as shown in Fig. 2.8), based on numerical cross-section data. The energies of the secondary particles are then determined using different approaches depending on the interaction type. For pair production and inverse Compton scattering, the kinematics are obtained by sampling the center-of-mass energy and energy fractions of the secondary particles according to the corresponding differential cross-sections, while ensuring energy conservation. For other processes, approximate relations from the literature [281] and [282] are used to assign the energies of the secondary particles. After each interaction, the energy of the primary particle is updated accordingly, provided it remains part of the cascade. Both interaction lengths and particle energies are scaled by redshift to account for the expansion of the Universe, which effectively modifies the energy of background photons and, consequently, the interaction rates.

Particle propagation can be modeled under different regimes, including rectilinear propagation [283], diffusive transport [284], and propagation in the presence of magnetic fields [285, 286]. These regimes are implemented through dedicated modules, for which appropriate step sizes must be specified. In this work, rectilinear propagation is used in the absence of magnetic fields, while simulations including turbulent magnetic fields employ a pre-existing module based on the Cash-Karp algorithm [283], a Boris push method [271] for solving ordinary differential equations. To terminate particle propagation, stopping conditions must be defined. These typically include a minimum particle energy or a maximum propagation distance, and multiple criteria can be combined to improve computational efficiency without affecting the relevant physical observables.

5.3.4 Output format

CRPropa provides several output modules that allow flexible storage and analysis of simulation results. The simplest option is the **ShellOutput** module, which prints basic information directly to the console during runtime. For more structured data storage, the **TextOutput** module enables writing simulation results to plain text (`.txt`) files, with configurable presets that define the information stored. These presets include **Event1D**, **Event3D**, **Trajectory1D**, **Trajectory3D**, and **Everything**. In addition, the output can be further customized by selectively enabling or disabling individual data columns, allowing fine control over the stored quantities. If the output filename ends with `.gz`, the data are automatically compressed to reduce storage requirements. In addition, **CRPropa** supports the **HDF5Output** module, which stores data in the hierarchical HDF5 format, allowing efficient handling of large datasets. For temporary in-memory storage, the **ParticleCollector** module can be used to store particle candidates during runtime; however, this approach must be used cautiously due to its high memory demand.

Beam–Plasma Instabilities in Laboratory-Relevant Regimes

This chapter is based on the publication:

- **S. Dey** and G. Sigl. (2025). *Simulations of astrophysically relevant pair beam instabilities in a laboratory context*. **Astroparticle Physics**, 173, 103153. arXiv:2501.14518 [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2025.103153>.

This chapter investigates the beam–plasma instabilities of relativistic electron–positron pair beams produced by TeV blazars, in a laboratory context. We use kinetic simulations to study the beam–plasma instability. The simulation parameters are chosen to mimic the astrophysical systems within a laboratory-accessible regime. In the astrophysical limit of dilute and anisotropic beams, electromagnetic instabilities are suppressed due to the anisotropy in longitudinal and transverse momentum. As a result, electrostatic modes are expected to dominate. In Sec. 6.1, we discuss the criteria for setting up a physically relevant simulation in laboratory scale. In Sec. 6.2 and A.1, we present the configuration of the simulation and the modifications in the initialization of EPOCH code. In Sec. 6.3, we study the growth rates of plasma instabilities and the range of density contrasts for which electrostatic instabilities dominate over electromagnetic modes in the presence of anisotropic beams under laboratory-relevant conditions, consistent with expectations for blazar-induced pair beams. In Sec. 6.4, we estimate the amount of total energy of the system allocated to beam kinetic energy, total field energy, and the energy of background particles. In Sec. 6.5, we investigate the angular broadening in the nonlinear regime, interpreting it as a result of instability-driven feedback. Finally, in Sec. 6.6, we estimate the fractional energy loss of the beam and its transverse broadening induced by this feedback, with particular emphasis on sources similar to the 1ES 0229+200 blazar source. The discussion and conclusions on the growth of the beam–plasma instability in astrophysically relevant pair beams in the laboratory regime are presented in Sec. 6.7.

6.1 Conditions for beam–plasma instability in laboratory

We consider a laboratory-relevant parameter space to investigate the evolution of instability as a function of density contrast in both the linear and nonlinear phases using numerical simulations. For an astrophysical pair beam, [24] established the criteria for setting up a physically relevant simulation are:

1. The kinetic energy density ratio of the beam to the background, $\epsilon = \alpha(\gamma - 1)m_e c^2 / k_B T_{bg}$ should be less than unity, where γ is the Lorentz factor and T_{bg} defines the temperature of the background plasma;
2. The electrostatic instability growth should dominate over electromagnetic instabilities, as the beam has a momentum anisotropy and a large longitudinal momentum spread.

In our study, we fix the kinetic energy density ratio to a value less than 1 that satisfies the first condition and conduct an in-depth investigation of the density-contrast range in which electrostatic instability dominates over electromagnetic instability, thereby satisfying the second criterion, for a broad (or warm) non-Maxwellian beam under laboratory conditions. We then extrapolate the nonlinear regime from laboratory to astrophysical scales to investigate beam energy loss due to instabilities and to examine the effect of nonlinear feedback on angular broadening for parameters representative of realistic blazar sources. We assume that instability is the dominant mechanism in comparison to the IC cooling for astrophysical pair beams in order to capture the feedback of beam–plasma instability on the pair beam. We investigate the evolution of a relativistic beam with a realistic Cauchy distribution (as described in Eq. (4.55)) in our simulation. In this study, the distribution function of the produced electron-positron (e^+e^-) pairs is non-thermal. To replicate an astrophysical pair beam in the laboratory, we must account for it producing electron-positron pairs with an opening angle of approximately $\theta_0 \sim \gamma^{-1}$. Considering a longitudinally warm or broad and transversely kinematically cold beam, the initial transverse momentum spread can be scaled as,

$$\sigma_{\perp,0} = p \sin(\theta_0) = \gamma m_e \sin(\gamma^{-1}) \sim m_e. \quad (6.1)$$

This suggests that the highly relativistic particle will exhibit a wide range of longitudinal momenta, and a transverse momentum spread approximately equal to the electron mass. This configuration closely resembles the characteristics of an astrophysical pair beam.

6.2 Simulation setup

In order to model the propagation of a warm beam through a background plasma in a two-dimensional Cartesian configuration, we employ EPOCH-2D Particle-in-Cell (PIC) simulation code [55]. In the initial condition setup, the length of simulation box of the longitudinal direction is defined by $L_{\parallel,box} = 120c \cdot \omega_p^{-1}$ and the transversal direction is represented by $L_{\perp,box} = 120c \cdot \omega_p^{-1}$. The number of cells along x - and y -direction is $N_x = N_y = 880$ and each cell contains $N_p = 200$ particles per species. The total number of particles per species is $N_{tot} = 1.5488 \times 10^8$. We use four types of particles: beam electrons and beam positrons exhibiting relatively broad Cauchy momentum distribution, background plasma consisting of electrons with no bulk velocity, and background protons, which are immobile due to their larger mass. To sample particles from

the specified distribution, an acceptance–rejection method is employed, and the implementation details are provided in Appendix A. The initial beam density profile is given by

$$n_b = n_{b0} \left[1 + \left(\frac{y}{R_{y,0}} \right)^2 \right]^{-2}, \quad (6.2)$$

where $R_{y,0} = 110c \cdot \omega_p^{-1}$ denotes the initial root mean square (rms) beam transverse spatial width, the initial peak beam density $n_{b0} = \alpha n_{bg}$, and the background particles have a density of $n_{bg} = 10^{16} \text{ cm}^{-3}$ for different values of α in the simulations. In laboratory experiments, the transverse spatial width of relativistic pair beams is typically on the order of mm to cm [250–252, 287]. In our study, where the simulations are performed on laboratory scales, the skin depth $c/\omega_p \sim 0.53 \text{ mm}$, and the transverse beam width we have used is greater than the skin depth, which is consistent in the laboratory regime. In our series of simulations, we explore α values of 0.0005, 0.005, and 0.05 while maintaining a fixed Lorentz factor, $\gamma = 3$ for all cases. Although these α values are significantly higher than those relevant to real TeV Blazars, the chosen range within the laboratory regime provides ample scope for reliable extrapolation to lower values. We employ periodic boundary conditions in the longitudinal direction and open boundary conditions in the transverse direction to simulate a finite-width beam. Periodic boundary conditions ensure that fields and particles arriving at one side of the simulation box reappear at the opposite side, but for the open boundary system, particles simply transmit through the boundary and leave the system. The beam width is smaller than the transverse size of the simulation box, $L_{y,box}$, and does not reach the boundaries. Therefore, the use of open (or absorbing) boundary conditions does not significantly influence beam-edge effects, as it provides the physical consistency of the system and closely resembles a beam with a finite width. In contrast, a periodic boundary represents an infinitely extended system, and a reflecting boundary reflects particles at the boundary, potentially causing artificial field enhancements within the simulation box. Neither is appropriate for a finite-width beam configuration. Since we do not want to lose the background plasma particles, we apply thermal boundary conditions for the background in both x and y directions. In the case of a thermal boundary condition, whenever a particle exits the simulation domain, it is replaced by an incoming particle with the same temperature. Simulations are conducted up to the total time $4500\omega_p^{-1}$ to capture the linear, the nonlinear growth phase, and the saturation region. The criteria for a physically relevant configuration of the beam–plasma system are that the kinetic energy density ratio must satisfy the condition $\epsilon < 1$. In accordance with the previous section, the warm neutral pair beam is characterized by maintaining an initial longitudinal momentum spread, $\sigma_{\parallel,0} = 1.0 \text{ MeV}$ with a mean longitudinal momentum, $\mu = 0.511(\gamma^2 - 1)^{1/2} \text{ MeV}$ and an initial transverse momentum spread, $\sigma_{\perp,0} = 0.5 \text{ MeV}$ for each sub-beams. This indicates that in the background plasma rest frame, the beam is initially transversely cold and longitudinally warm as the initial angular spread, $\theta_0 \equiv \sigma_{\perp,0}/\sigma_{\parallel,0} = 0.5$, which is less than 1. A comprehensive overview of the simulation parameters is presented in Table 6.1, and Table 6.2 outlines the sub-beam parameters.

Parameters	Value
Number of Dimensions	2 (x beam direction, y transverse)
Simulation box Length along x -direction $L_{x,box}(L_{\parallel,box})$	$120 c \cdot \omega_p^{-1}$
Simulation box Length along y -direction $L_{y,box}(L_{\perp,box})$	$120 c \cdot \omega_p^{-1}$
No. of cells along x -direction (N_x)	880
No. of cells along y -direction (N_y)	880
No. of particles per cell N_p (per species)	200
Total no. of particles (N_{tot})	1.5488×10^8
Timestep Δt	0.95 CFL-Criterion
Maxwell Solver	Yee (2nd order)
Finite difference scheme	6th order
Particle Pusher	Higuera & Cray
Particle Shape Function	Third Order B-Spline
Current Filtering	5-fold (1-2-3-4 steps)
n_{bg}	10^{16} cm^{-3}
T_{bg}	$\alpha(\langle\gamma\rangle - 1)600 \text{ keV}$
Background particles	Immobile Protons & e^-
Beam particles	e^- & e^+
Distribution Function	Eq. (4.55)
ϵ	0.85
Total time T	$4500 \omega_p^{-1}$
Boundary conditions	
For the beam and fields (along x)	periodic
For the beam and fields (along y)	open
For the background (along x & y)	thermal

Table 6.1: The summary of the PIC Simulation configuration for the simulation run.

Parameters	Beam
γ	3
μ (MeV)	$m_e(\gamma^2 - 1)^{1/2}$
$\sigma_{\parallel,0}$ (MeV) (alias $\Delta p_{x,0}$)	1.0
$\sigma_{\perp,0}$ (MeV) (alias $\Delta p_{y,0}$)	0.5

Table 6.2: The overview of the beam parameters for the Cauchy beam distribution function.

6.3 Growth of plasma instabilities and evolution of fields

The primary numerical measurable quantity that we can access in a simulation run is the growth of the fields. The initial noise due to the thermal fluctuation of the background plasma can influence the evolution of the fields. The initial noise of the system is inversely proportional to ϵ , i.e., Initial noise $\propto 1/(N_{tot}\epsilon)$, where N_{tot} is the total number of simulation particles [53]. Increasing the background plasma temperature results in a decrease of the parameter ϵ . Therefore, we increase N_{tot} to offset the significant noise caused by high background plasma temperatures. Throughout these simulations, we maintain a very low initial noise level. We employ a Yee

Maxwell field solver with 6th-order field interpolation and a 3rd-order B-spline shape function (yielding a 5th-order weighting) for placing particles on the grid. The default multiplying factor for this field solver is set to 0.95 of the Courant–Friedrichs–Lewy (CFL) criterion¹ on time steps. A particle pusher is implemented following the method described by [272]. To reduce numerical noise, we apply a 5-fold current smoothing, following the approach outlined by [273]. The wave number is expressed as having a component parallel to the direction of propagation of the beam, denoted as $k_{\parallel} = k_x$ and the component perpendicular to the beam propagation is given by $k_{\perp} = (k_y^2 + k_z^2)^{1/2}$. In Figs. 6.1a, 6.1c, and 6.1e, we present the growth rates of electrostatic instability, which is plotted against wavevectors for different α values with beam parameters specified in Table 6.2. The Figs. 6.1b, 6.1d, and 6.1f show the growth rates of electromagnetic instability, which is plotted against wavevectors for the same set of α values. The growth rate is defined as a function of $\delta(k_{\parallel}, k_{\perp})$, extracted by performing a Fourier transform of the E_x and E_y fields, followed by computing the Fourier amplitude for each mode. The resonant electric field mode is the fastest growing mode for $\alpha = 0.0005$ and 0.005 . The maximum oblique growth is observed around $k_{\parallel} \sim c \cdot \omega_p^{-1}$. In contrast, the magnetic field modes appear comparatively weak for $\alpha = 0.0005$ and 0.005 . The electric and magnetic field modes become comparable for $\alpha = 0.05$. The maximum filamentation growth is observed around $k_{\parallel} \sim 0$. In Fig. 6.1e, resonant modes around $k_{\parallel} \sim 0$ are also observed, as the E_y component is also associated with the electromagnetic filamentation mode. The transition between the dominant current filamentation modes to oblique modes is immediately observed while decreasing α from 0.05 to 0.005. Fig. 6.2a illustrates the time evolution of the longitudinal electric fields for different values of α . As α increases, a significantly smaller fraction of the beam kinetic energy is converted into longitudinal electric fields, indicating that the oblique mode faints as the beam becomes relatively denser. Fig. 6.2b shows that the beam loses less than 1% of its kinetic energy during the linear growth phase, thereby contributing to the growth of electromagnetic fields. We find that the beam energy loss rate reduces after the linear growth phase ends, and the system continues to evolve nonlinearly. By the end of saturation, the diluted beam lost approximately 7% of its kinetic energy, contributing to the growth of electromagnetic fields. Fig. 6.3 shows that as α decreases, the magnetic field strength increases compared to the electric field. The magnetic field amplification is triggered between $\alpha = 0.005$ and 0.05 , as electromagnetic instability begins to dominate. Consequently, for ultra-relativistic dilute beams (i.e., $\alpha \ll 1$), the condition of growth rate, $\delta \ll 1$ is satisfied for the electrostatic modes. For instance, when $\alpha = 0.005$ and Lorentz factor $\gamma = 3$, the theoretical electrostatic growth rate is calculated as $\delta_{\text{Obl,theo}} \sim 0.071\omega_p$. The maximum electrostatic growth rate measured in the simulation is approximately $\delta_{\text{Obl,sim}} \sim 0.064\omega_p$. This fulfills the consistency of the simulation with the analytical results.

¹In this work, we use the CFL condition corresponding to sixth-order field interpolation, given by $\Delta t \leq (120/149) \times (\Delta x \Delta y / \sqrt{\Delta x^2 + \Delta y^2}) c^{-1}$.

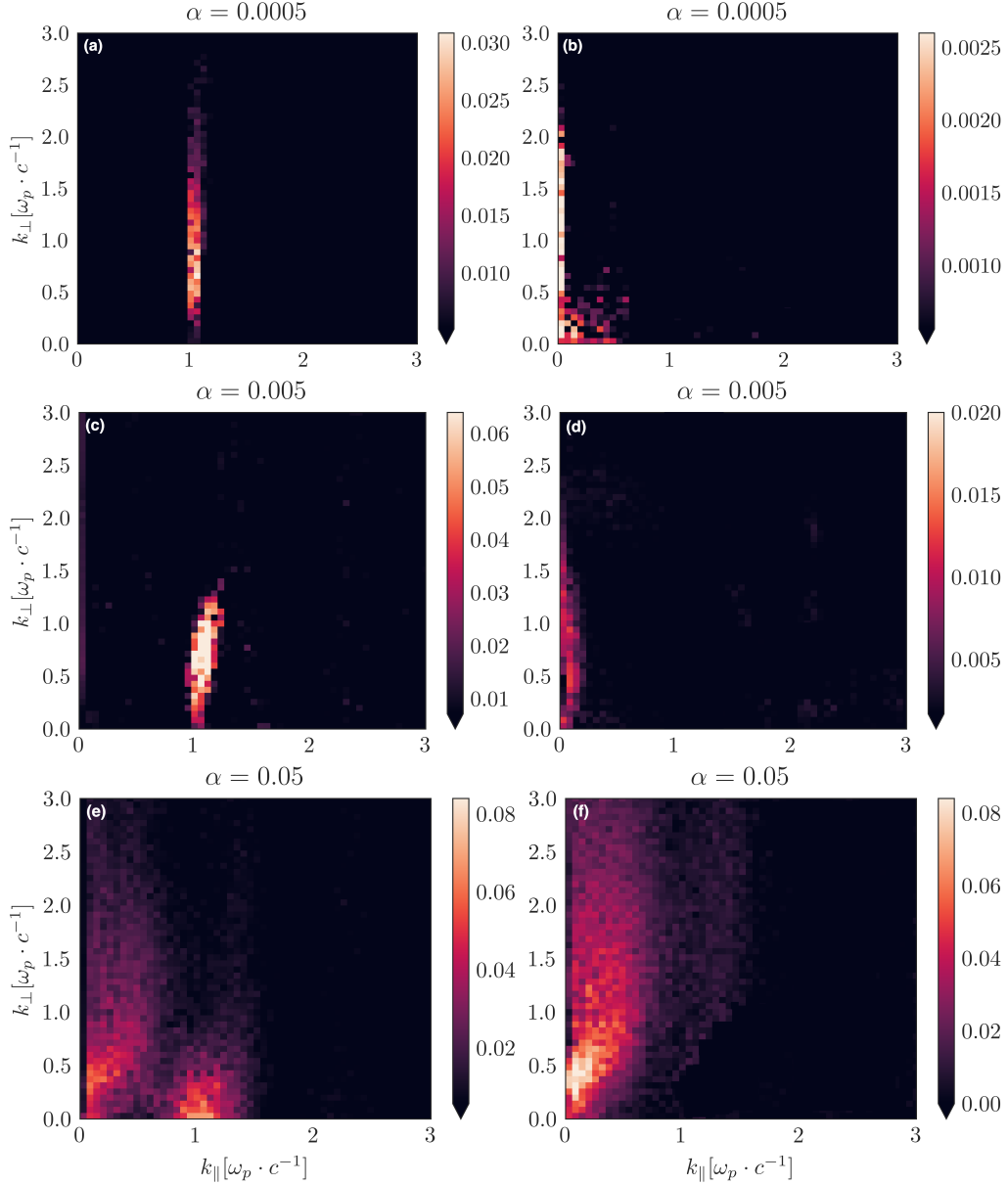


Figure 6.1: Two-dimensional snapshots of the growth rate of instability in $\mathbf{k}$ space. Figs. (a), (c), and (e) along the **left** column show the growth rates of electrostatic instability, which are obtained from the Fourier transformation of E_x and E_y fields. Figs. (b), (d), and (f) along the **right** column show the growth of B-modes, which are obtained from the Fourier transformation of the B_x and B_z fields. The color scale defines the growth rate which is plotted as a function of $\delta(k_{\parallel}, k_{\perp})$ [in units of ω_p] for $\alpha = 0.0005$ at $t\omega_p \sim 670$, $\alpha = 0.005$ at $t\omega_p \sim 310$, and $\alpha = 0.05$ at $t\omega_p \sim 170$. With increasing α , the electrostatic and magnetic modes become comparable, and the system gradually transitions to a regime where electromagnetic modes dominate.

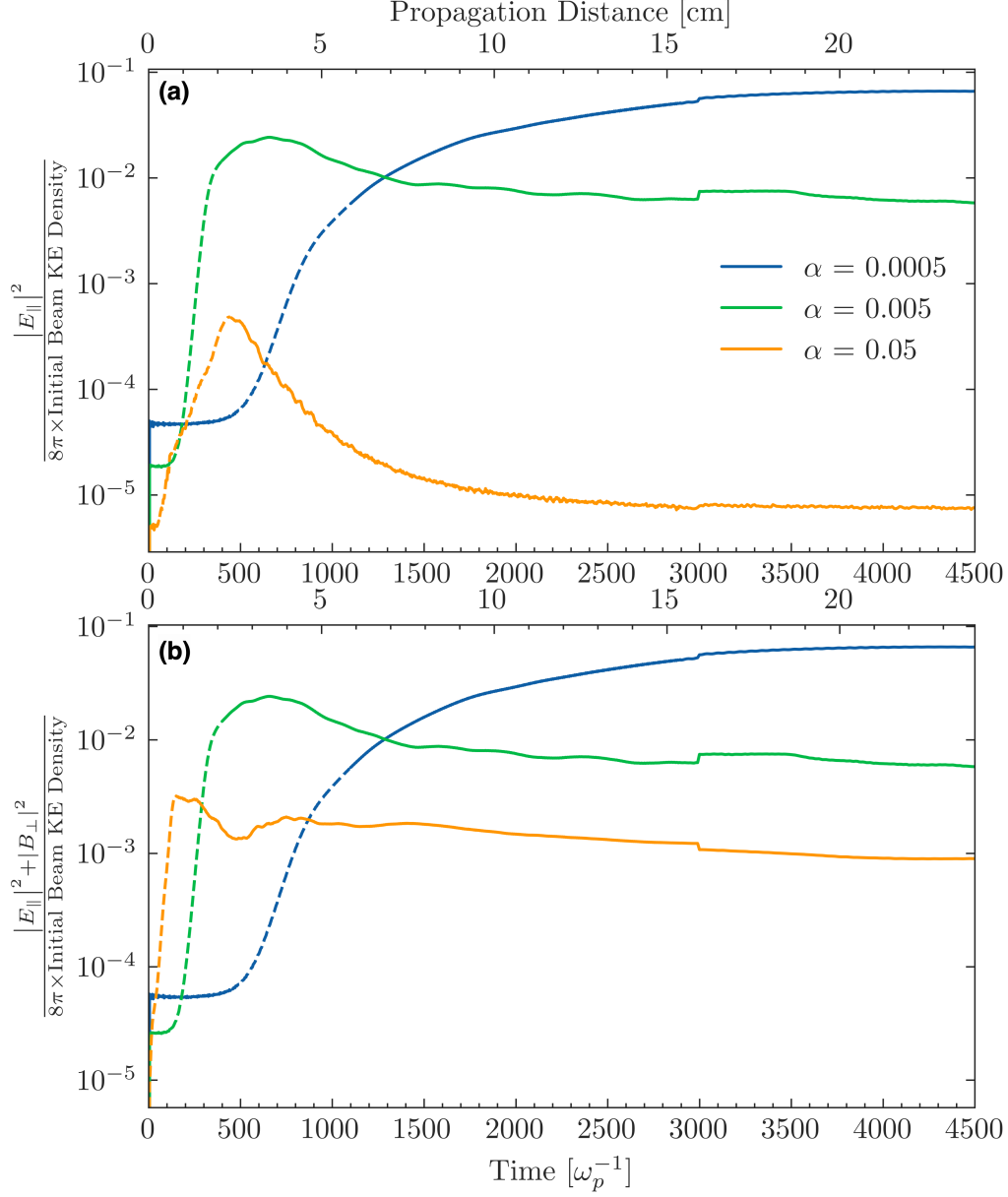


Figure 6.2: The fraction of beam kinetic energy converted into the (a) longitudinal electric field, (b) electromagnetic field across all modes is shown with respect to time on the bottom x -axis and beam propagation distance on the top x -axis. The beam parameters outlined in Table 6.2 are applied in every simulation for different density contrast α . The dashed lines represent the linear growth phase for each α , with lower values of α leading to a greater dominance of the electric field modes. Whereas, for $\alpha = 0.05$, electromagnetic modes begin to dominate.

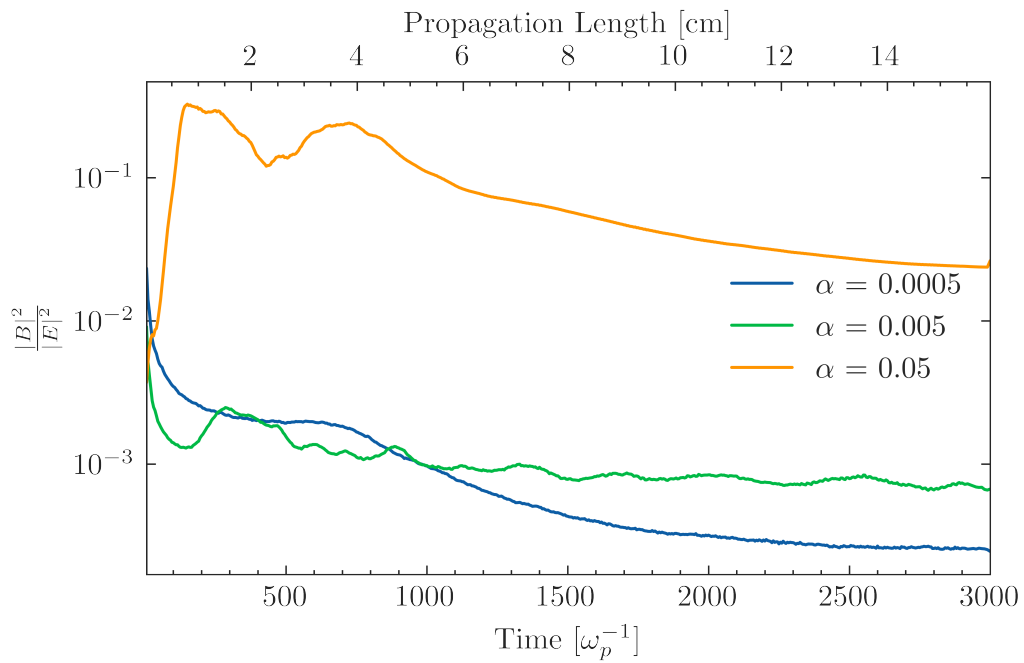


Figure 6.3: The comparison of magnetic field to electric field strength across all modes is shown as a function of time (bottom x -axis) and beam propagation distance (top x -axis). The beam parameters specified in Table 6.2 are used for all simulations with varying density contrast α , similar to Fig. 6.2. As α decreases, the magnetic field strength becomes weaker than the electric field strength.

6.3.1 Interplay between magnetic field and beam momentum

When a neutral-pair beam propagates through a plasma, small perturbations can arise, leading to spatial separation of electrons and positrons within the beam and the formation of localized currents. The separation can be on the order of the beam skin depth. The localized currents can create filaments, leading to the generation of magnetic fields. Fig. 6.4 shows the fraction of beam kinetic energy converted into the magnetic field for different α values. For increasing values of α , the magnetic field strength increases because the electron and positron filaments start to separate spatially, producing self-generated localized currents [287]. The electromagnetic instability grows until the wavelength of unstable modes is comparable to the Larmor radius of the particles in the self-generated magnetic field, causing them to become trapped in the magnetic fields [288, 289]. Consequently, an electromagnetic counterpart emerges even though the overall beam remains neutral. Accordingly, at higher density contrast, magnetic fields are generated, and the filamentation instability develops because of the transverse magnetic pressure linked to the filaments, causing the total magnetic field to predominantly align in the transverse direction ($|B| \sim |B_\perp|$). However, after the linear growth phase, there is a secular growth of filamentation

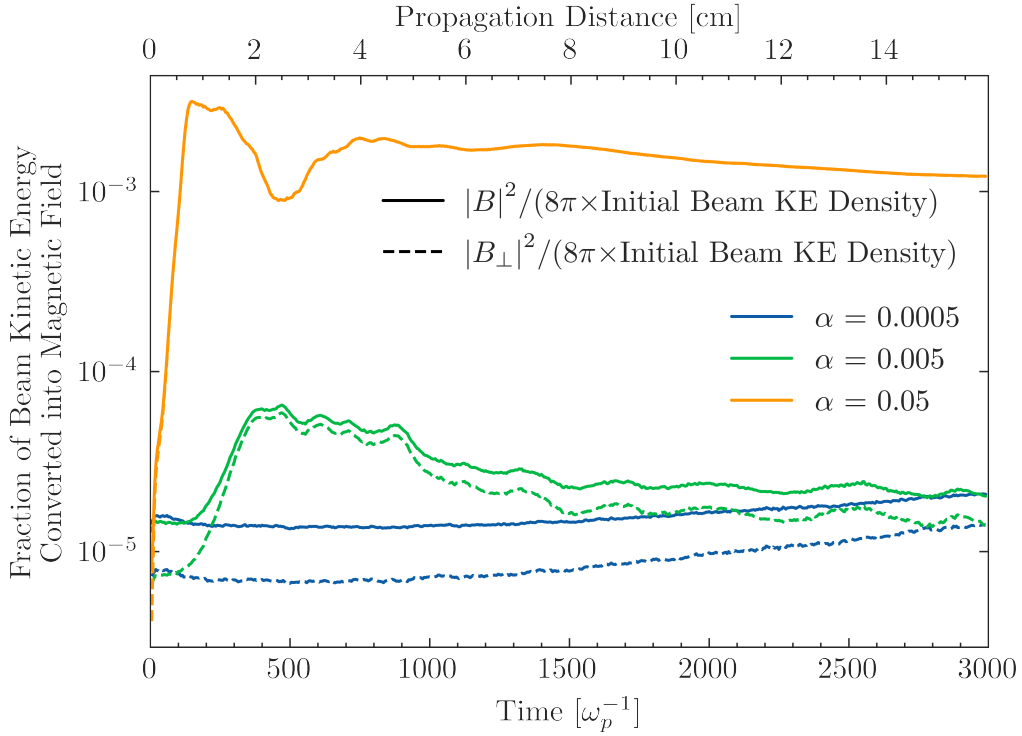


Figure 6.4: The fraction of beam kinetic energy transferred into the magnetic field across all modes is shown with respect to time (bottom x -axis) and the distance the beam travels during this time (top x -axis). For every simulation with varying density contrast α , the beam parameters provided in Table 6.2 are applied similarly as Fig. 6.2 and 6.3. The solid lines represent the total magnetic field, and the dashed lines depict only the transverse component of the magnetic field.

instability before saturation (which starts at $t \sim 510\omega_p^{-1}$, as shown in Fig. 6.4), particularly for $\alpha = 0.05$. This is due to the formation of a small cavity, which is more clearly observed in the

spatial structure of the magnetic field in a 3D simulation [290, 291]. As the magnetic pressure within these unscreened cavities causes them to expand, more current is exposed, leading to secular growth. The secondary growth will eventually saturate as the net current in the cavity decreases. Fig. 6.6 depicts the evolution of the spatial structure of the transverse magnetic field at the initial state and during the primary linear growth phase. As α increases, the filamentation instability gives rise to the development of distinct transverse magnetic filament structures. The phenomenon of generation of current filamentation has been studied explicitly using both analytical and semi-analytical methods by [44, 292–297].

Fig. 6.5 illustrates the fraction of beam kinetic energy transferred to the transverse magnetic field for different initial transverse beam momentum spreads. As the initial transverse beam momentum increases, with $\sigma_{\parallel,0} = 1.0$ MeV fixed, the induced transverse magnetic field decreases, suggesting a decrease in transverse current filamentation instability in the linear growth regime. However, a warm beam can drive current filamentation instability during the linear growth phase and also enhance conditions for secondary filamentation growth in the nonlinear regime.

Fig. 6.7 shows the two-dimensional momentum distribution at different simulation timestamps

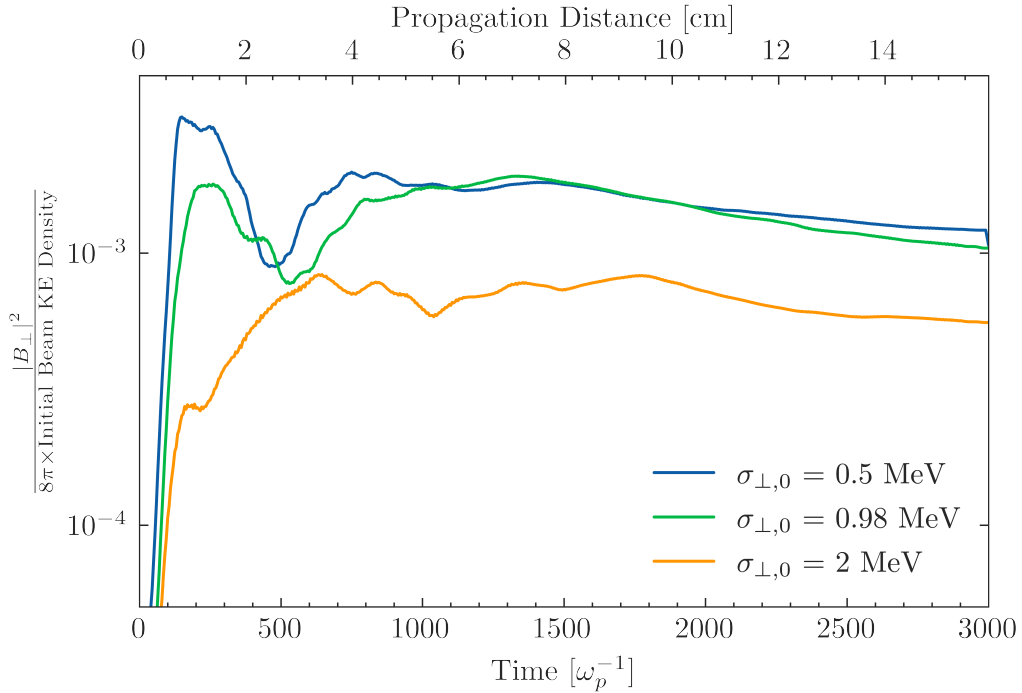


Figure 6.5: The fraction of beam kinetic energy transferred into the transverse magnetic field across all modes with respect to time is represented on the bottom x -axis, and the propagation distance is shown on the top x -axis. In every simulation, the density contrast is fixed at $\alpha = 0.05$ and $\sigma_{\parallel,0} = 1.0$ MeV, while the initial transverse momentum of the beam is varied. The beam parameters listed in Table 6.2 are used. As the initial transverse momentum increases or the beam becomes colder, both the current filamentation instability and the secondary growth are diminished.

for various values of α . Due to the different growth rates associated with varying α values, the linear growth phase begins and ends at different times. At the start of the linear growth phase, the beam is focused and remains stable, with no significant perturbations affecting the

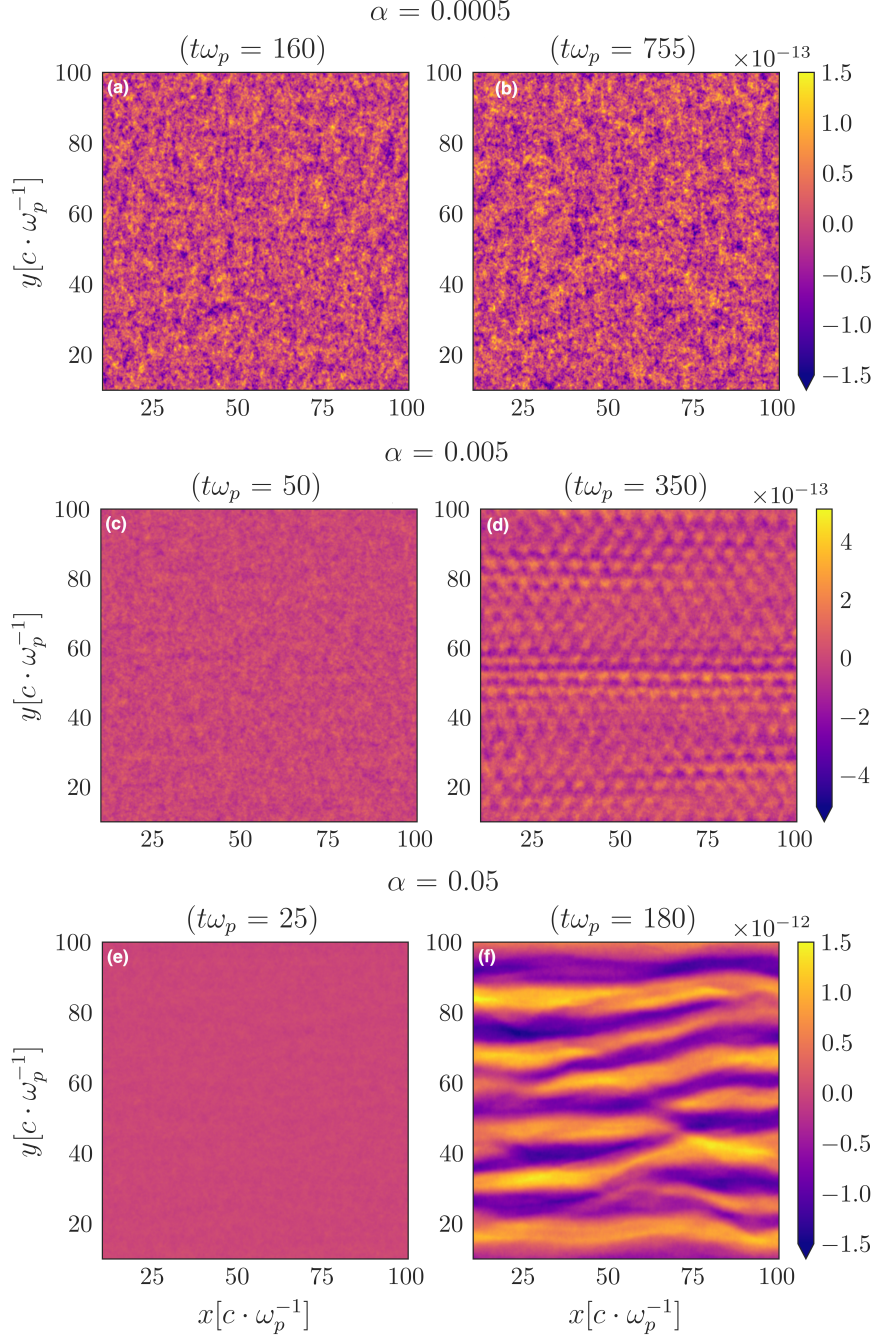


Figure 6.6: The evolution of the transverse magnetic field with varying density contrast α . The color scale represents the fraction of beam kinetic energy converted into each mode of the associated transverse magnetic field, i.e., $(B_{\perp})_k / \sqrt{\text{Beam KE Density}}$. The Figs. (a), (c), and (e) along the **left** column illustrate the state before the linear growth phase of the instability sets in, whereas Figs. (b), (d), and (f) along the **right** column show the evolution of the transverse magnetic field at the instability growth phase.

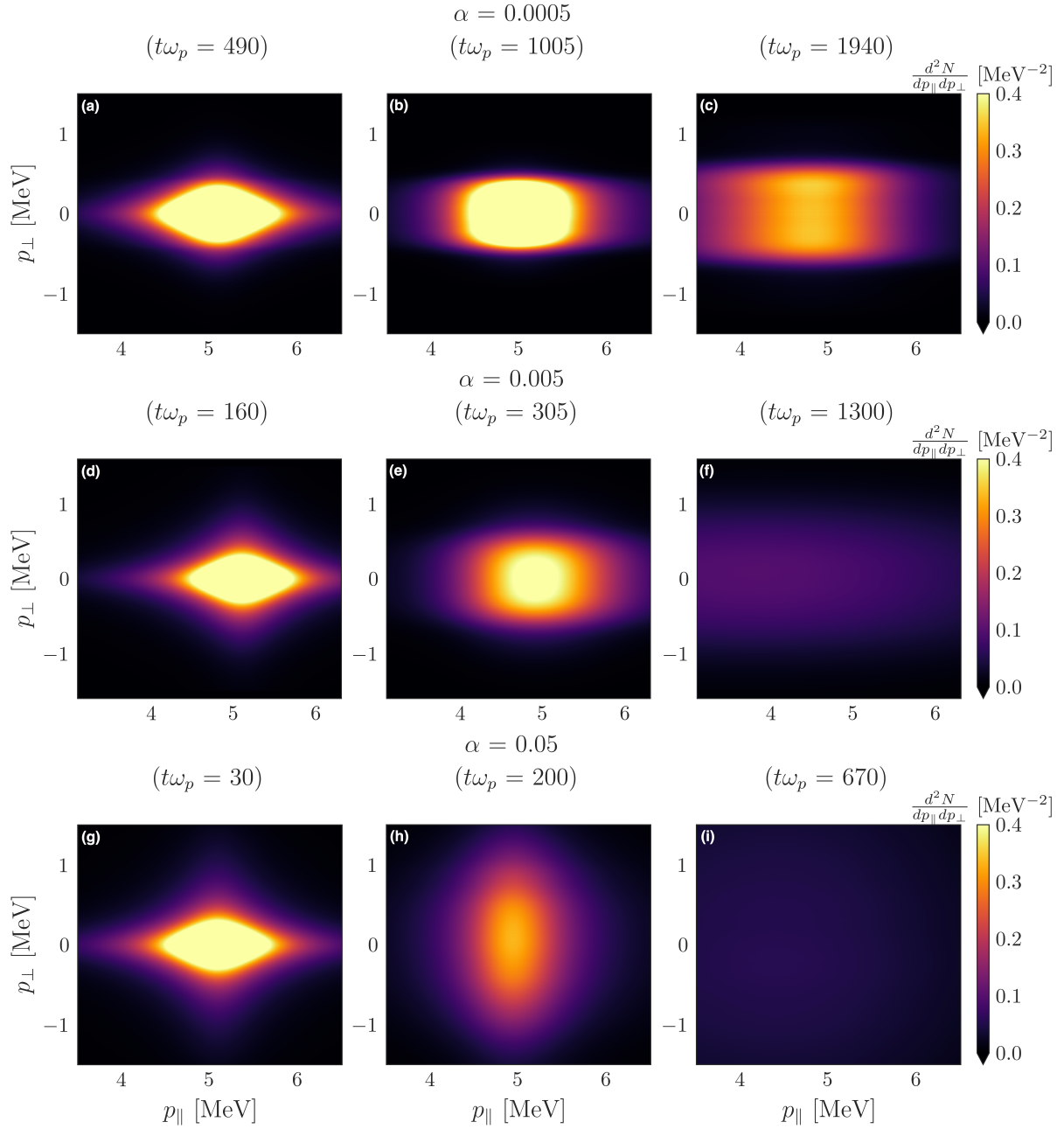


Figure 6.7: Momentum distribution in two dimensions for different stages of instability growth for different values of α . The Figs. (a), (d), and (g) on the **left row**: before starting the linear growth, Figs. (b), (e), and (h) on the **middle row**: during the linear growth phase, and Figs. (c), (f), and (i) on the **right row**: at the end of the nonlinear regime (starts to saturate). The color scale represents the beam distribution function in momentum space and is in units of MeV^{-2} . The different timestamps for instability growths depend on the dominant instability modes, as shown in Table 6.3.

longitudinal ($p_{\parallel}$) or transverse ($p_{\perp}$) beam momentum. As instability growth progresses, the beam spreads energetically in both the transverse and longitudinal directions, resulting in an overall

broadening of its distribution. The transverse broadening in the nonlinear phase is reduced as the beam is diluted because the electromagnetic modes are suppressed. As previously explained, in the filamentation-instability-driven scenario, a secondary filamentation instability develops during the nonlinear phase. This leads to nonlinear feedback, causing significant transverse-momentum broadening. Table 6.3 summarizes the dominant instabilities that are responsible

θ_0 [rad]	γ	α	Dominant instability
0.5	3	0.0005	Oblique
		0.005	Oblique
		0.05	Transverse current filamentation

Table 6.3: Summary of the dominant instabilities in the linear growth phase that are responsible for momentum broadening in nonlinear regions for different α runs.

for momentum broadening in the nonlinear phase across the range of α values considered in our simulation. The beam with $\alpha = 0.005$ is at the threshold on a laboratory scale at which the nonlinear feedback of electrostatic instability is observed, leading to the energetic broadening of the beam over time.

6.4 Energy budget and beam energy-loss

We have studied the quantitative estimation of the amount of total energy of the system allocated to beam kinetic energy, total field energy, and the energy of background particles. For instance, based on the parameters listed in Table 6.2, Fig. 6.8 indicates that initially, around 45.8% of the total energy is allocated to the kinetic energy of the beam, while approximately 54.2% is allocated to the energy of the background particles. This reflects the initial condition of our simulation setup, where the ratio of the beam to background kinetic energy density is $\epsilon \sim 0.85$. The growth of plasma instabilities can facilitate energy transfer from the beam to the background. The energy transfer continues even after the instability growth phase ends. For beams with $\alpha = 0.0005$, 0.005, and 0.05, the total beam energy loss relative to the initial beam energy is approximately 29%, 36%, and 38%, respectively. Of the initial beam energy, the contribution to field energy growth is about 7% for $\alpha = 0.0005$, 1% for $\alpha = 0.005$, and less than 1% for $\alpha = 0.05$. We define the fractional beam energy loss as $\Delta U/U_{\text{beam},0}$, where ΔU represents the total energy lost by the beam and $U_{\text{beam},0}$ is the initial kinetic energy of the beam. This quantity measures how much of the initial beam energy is dissipated into the fields or the background plasma. We obtain the power-law scaling of the fractional beam energy loss with α

$$\left(\frac{\Delta U}{U_{\text{beam},0}} \right) \sim 0.48 \cdot \alpha^{0.07}. \quad (6.3)$$

Fig. 6.9 shows the power-law scaling of the fractional beam energy loss with α . This suggests that even at very small density contrasts, the energy loss remains non-negligible.

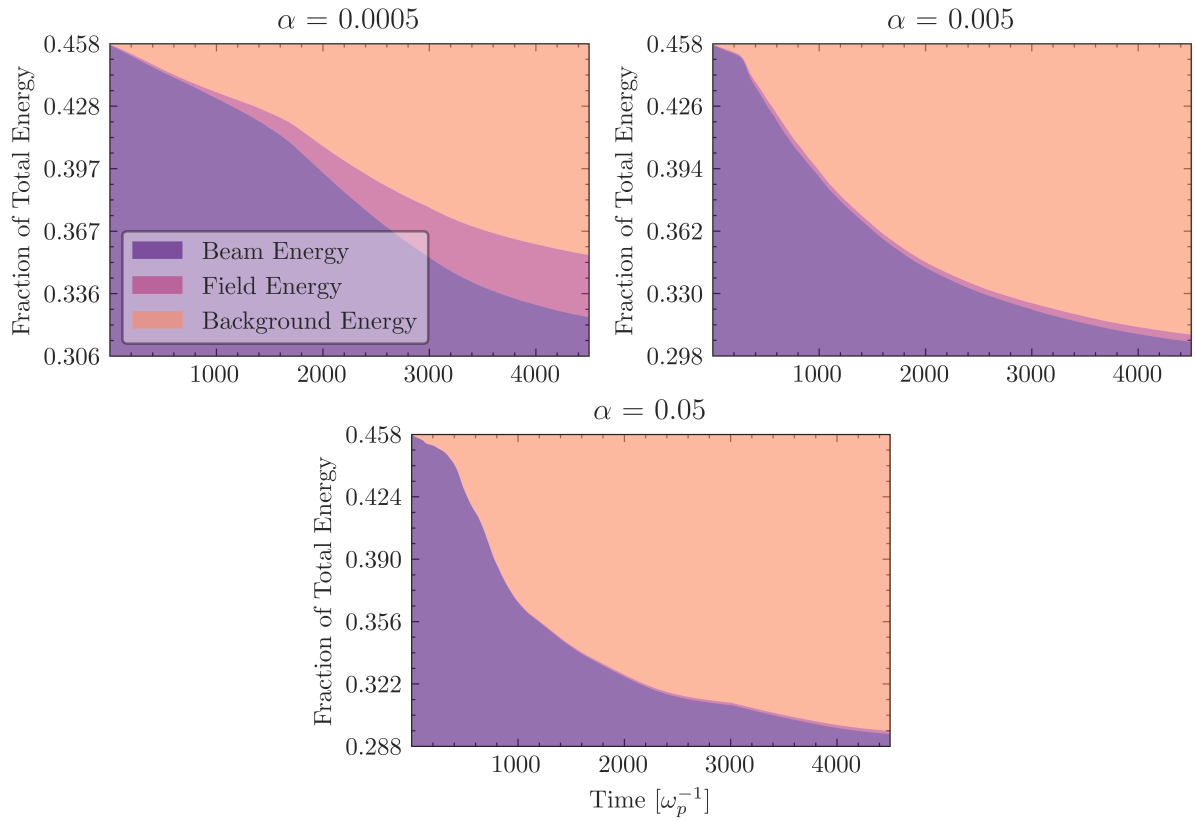


Figure 6.8: The allocation of the total energy of the system between the beam kinetic energy, the total electromagnetic field energy, and the energy of the background medium is presented over time with different density contrast α . Each simulation run uses beam parameters as detailed in Table 6.2.

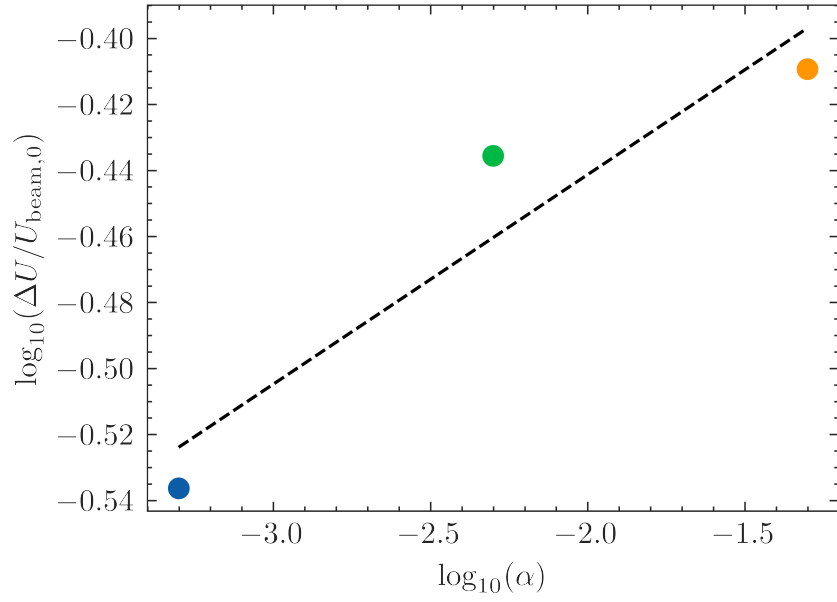


Figure 6.9: The power-law scaling of the fractional beam energy loss, $\Delta U/U_{\text{beam},0}$ with α at the saturation. The black dashed line represents the fitted line. The fitted power-law index for the α scaling is 0.07.

6.5 Angular broadening of the beam

Here we establish a power-law scaling relationship between the angular broadening ($\Delta\theta_{\text{non-lin}}$) due to nonlinear instability feedback and α , with the other parameters fixed as specified in Table 6.2. The power-law scaling is estimated at times when the beam has already formed its shape and is entering the saturation region. Fig. 6.10 illustrates the scaling of the $\Delta\theta_{\text{non-lin}}$ ($\equiv \Delta p_{\perp}/p_{\parallel}$) with α and the errorbars have been estimated with a deviation of $\Delta(t\omega_p) = \pm 10$, is shown in the inset plot. Table 6.4 represents the data used to obtain the linear plot. The power-law scaling can be expressed as

$$\left(\frac{\Delta\theta_{\text{non-lin}}}{1 \text{ rad}} \right) \sim 0.75 \cdot \alpha^{0.19}. \quad (6.4)$$

The dependence of $\Delta\theta_{\text{non-lin}}$ on α is relatively weak.

γ	$\log_{10}(\theta_0)$ [rad]	$t\omega_p$	α	$\log_{10}(\Delta\theta_{\text{non-lin}})$ [rad]
3	-0.3	1940	0.0005	$-0.708^{+0.03}_{-1.37}$
		1300	0.005	$-0.625^{+0.16}_{-1.11}$
		670	0.05	$-0.334^{+0.11}_{-0.86}$

Table 6.4: The overview of the $\Delta p_{\perp}$ data for different α values and the fitting parameters are obtained from the simulation. The errors are estimated for $\Delta(t\omega_p) = \pm 10$.

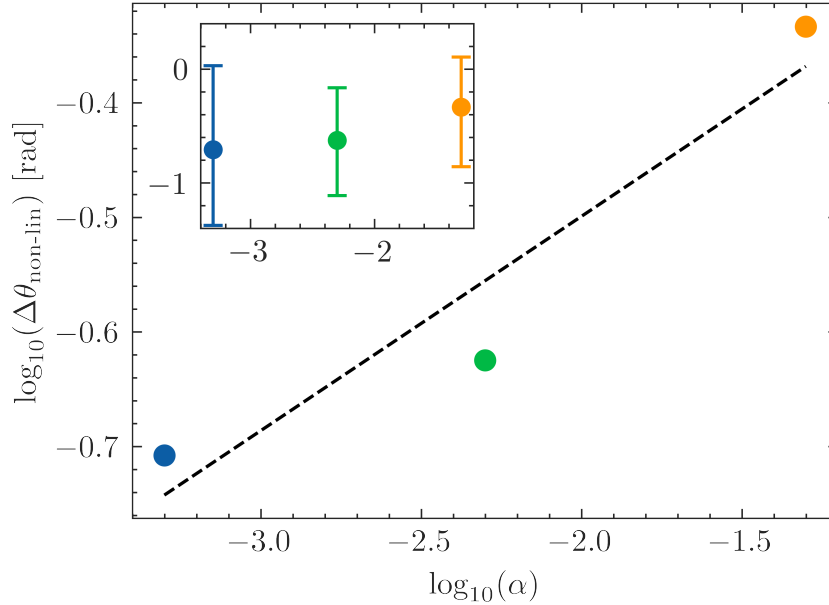


Figure 6.10: The power-law scaling of $\Delta\theta_{\text{non-lin}}$ approximately at the end of nonlinear phase with α is observed at $t\omega_p \simeq \{1940, 1300, 670\}$ respectively for $\alpha = \{0.0005, 0.005, 0.05\}$. This indicates the moment when the beam starts to enter the saturation phase. The black dashed line represents the fitted line. The embedded plot displays the error bars associated with the main plot (also shown in Table 6.4), with the errors calculated for $\Delta(t\omega_p) = \pm 10$.

6.6 Beam-plasma instabilities for 1ES 0229+200-like sources

The total isotropic-equivalent luminosity ($\mathcal{L}$) of the blazar and photon energies influence the density of electron-positron pairs. The upper limit of the pair density can be derived assuming a balance between pair production and the IC cooling mechanism [33] as

$$n_b \simeq 1.9 \cdot 10^{-21} \text{cm}^{-3} \left(\frac{1+z}{2} \right)^{3\zeta-4} \left(\frac{\mathcal{L}}{10^{45} \text{erg/s}} \right) \left(\frac{\gamma}{10^7} \right). \quad (6.5)$$

The parameter $\zeta = 4.5$ for $z < 1$ can be derived from the local star formation rate analysis [173]. As a benchmark scenario, we consider the blazar source 1ES 0229+200 located approximately at $z \approx 0.14$, fixing other parameters $n_{\text{IGM}} = 10^{-7} \text{cm}^{-3}$ and the fiducial value of $\mathcal{L} = 10^{45} \text{erg/s}$, consistent with [35]. The density ratio can be expressed as

$$\alpha \simeq 9.1 \cdot 10^{-17} \left(\frac{\gamma}{10^7} \right). \quad (6.6)$$

For a pair beam with a Lorentz factor of 10^7 , the corresponding density contrast is $\simeq 10^{-17}$. Using Eq. (6.3), the beam energy loss due to plasma instabilities can be estimated as

$$\left(\frac{\Delta U}{U_{\text{beam},0}} \right) \times 100\% \simeq 4.0 \times \left(\frac{\gamma}{10^7} \right)^{0.07} \%. \quad (6.7)$$

This implies that for a beam with $\gamma = 10^7$, the energy loss of the beam due to instabilities is approximately 4%.

The intrinsic opening angle of the pair beam with a Lorentz boost 10^7 is determined by $\theta_0 \sim 10^{-7}$ rad. The angular spread at the nonlinear stage can be expressed from Eq. (6.4) as

$$\Delta\theta_{\text{non-lin}} = \left. \frac{\Delta p_{\perp}}{p_{\parallel}} \right|_{\text{non-lin}} \sim 6.7 \cdot 10^{-4} \left(\frac{\gamma}{10^7} \right)^{0.19} \text{rad}. \quad (6.8)$$

The resulting angular spread due to the instability feedback for a beam with a Lorentz boost 10^7 is nearly $6.7 \cdot 10^{-4}$ rad, which leads to a negligible angular broadening of the pair beam, considering the fact that the IC cooling rate is slower than the instability growth.

6.7 Discussion and conclusions

In this chapter, we investigate the evolution of blazar-induced neutral pair beams under laboratory conditions using PIC simulations. We aim to determine the density-contrast range over which the beam can be considered dilute, in the sense that electromagnetic instabilities are subdominant. In this parameter regime, the laboratory experiments can closely mimic the astrophysical case. The conclusions of this study can be summarized as follows:

- In the absence of an external magnetic field, the dominant instability depends on the peak beam density. For a warm beam, at significantly higher beam density contrast (~ 0.05), the beam is more prone to transverse current filamentation growth during the linear growth phase. The magnetic field amplification results from the combined effects of momentum anisotropy and beam-plasma instabilities.

- Our results suggest that in laboratory experiments, a longitudinally broad (or warm) beam with an initial angular spread, $\theta_0 = 0.5$ and $\alpha \leq 0.005$ can be classified as a ‘dilute beam’, achieving a regime where oblique instability dominates over the electromagnetic instabilities (as specified in Table 6.3). In the nonlinear regime, the instability feedback leads to transverse broadening of the beam. Thus, $\alpha \leq 0.005$ represents the physical limit for observing these effects in laboratory conditions.
- In the laboratory-scale simulations, the diluted beam is losing around 29% of its kinetic energy due to instability. Of this, approximately 7% of the beam kinetic energy contributes to the electromagnetic field growth. We find that the beam energy loss rate slows down after the linear growth phase ends, and the system continues to evolve nonlinearly. Despite that, the nonlinear feedback of the instability has only a marginal effect on the angular broadening of the beam.
- In a very dilute beam, electromagnetic instabilities are subdominant, allowing electrostatic oblique instabilities to take precedence. This leads to a decrease in the emerging magnetic field strength, suggesting that the rates of transverse beam broadening are also reduced. In astrophysical scenarios, the background IGM has a density of approximately $n_{\text{IGM}} \sim 10^{-7} \text{cm}^{-3}$, which translates to the density contrast of $\alpha = 10^{-21} - 10^{-15}$ for lower redshift ($z < 1$) TeV sources. This indicates that the beam is very dilute. The beam lost approximately 2 – 4% of its energy due to instability. Moreover, the nonlinear feedback from instability effects is almost marginal, resulting in negligible transverse broadening.
- A related study by [27] presents a quantitative assessment of the blazar 1ES 0229+200 using an alternative numerical approach to investigate the angular spread of the beam induced by the instability. The study concluded that the angular broadening is very minimal. In a steady-state scenario, the angular spread of the beam is approximately $5 \cdot 10^{-6}$ rad for a Lorentz factor of 10^7 , with a gamma-ray photon mean-free path of about 13 kpc. However, for the same source, we note that our extrapolated estimation of the angular broadening of a beam with bulk Lorentz factor of 10^7 in the nonlinear phase (approaching saturation regime) is $\sim 6.7 \cdot 10^{-4}$ rad, which is about two orders of magnitude larger than what was estimated in [27].

In conclusion, we have identified the physical limit of the beam density contrast below which a warm beam can be considered ‘dilute’, using laboratory experiments designed to replicate astrophysical pair beams produced by TeV blazars. Although we have studied the evolution of pair beams in an unmagnetized background plasma, it is also worthwhile to explore their behavior in a magnetized plasma, as this can suppress instability growth and modify the condition for dilute beams. Another important consideration is collisional effects, particularly if the collisional frequency becomes comparable to the growth rate of the filamentation instability, which can further suppress it.

Pair Beam in Laboratory Experiments

Several approaches have been proposed to reproduce this phenomenon under laboratory conditions; however, a key challenge remains in generating a charge-neutral pair beam. In particular, most conventional beam optics are not designed to handle beams that simultaneously contain electrons and positrons, making beam control and transport more difficult. Previous experimental studies at high-intensity laser facilities have made significant progress in this direction. Measurements of the positron-to-electron ratio (N_{e^+}/N_{e^-}) have shown a steady improvement across different platforms, ranging from about 10% in early experiments at OMEGA-EP and Orion/OMEGA-EP [298, 299], to approximately 50% at the Texas-Petawatt laser [300], and up to nearly unity in more recent experiments at ASTRA-GEMINI, OMEGA-EP, and (High-Radiation to Materials) HiRadMat [250, 252, 287, 301, 302]. These results demonstrate that producing near-charge-neutral pair beams is becoming increasingly feasible. Laboratory experiments, therefore, provide a valuable platform to directly compare experimental observations with numerical simulations. In this context, our work contributes to identifying physically realistic parameter ranges that allow astrophysical pair-beam scenarios, such as those found in jets, to be reproduced under controlled laboratory conditions.

In this chapter, we describe laboratory experiments and discuss their applicability for testing the beam–plasma instability, based on [53, 250–252, 303], which have already demonstrated successful neutral beam generation or conceptual experimental setups. Such an experiment would allow direct, in situ measurements of electromagnetic fields and particle-beam properties. In particular, it would enable the study of pair-beam behavior in the nonlinear post-saturation regime and in the transition region of a mildly relativistic beam. These regimes are difficult to access through simulations alone and would benefit from experimental verification. Two possible setups for such an experiment are shown in Figs. 7.1 and 7.2. In these configurations, a high-energy particle beam can be produced either by extracting a proton beam from an accelerator facility or by using a purely electron beam. When a high-energy primary beam interacts with a dense target, it initiates a cascade of secondary particles. In the case of an electron beam, the cascade is dominated by bremsstrahlung and subsequent pair production, leading to the generation of electrons and positrons [48–50, 287]. For a proton beam, the interaction proceeds via hadronic processes, producing neutral pions that decay into high-energy photons, which in turn generate electron–positron pairs through electromagnetic cascades. In both cases, the result

is a beam containing electrons and positrons that can be used for further study. The produced pair beam, which typically has a broad energy distribution and finite angular spread, can then be injected into a plasma cell. Inside the plasma, beam-plasma instabilities develop, leading to the growth of electromagnetic fields and heating of both the background plasma and the beam.

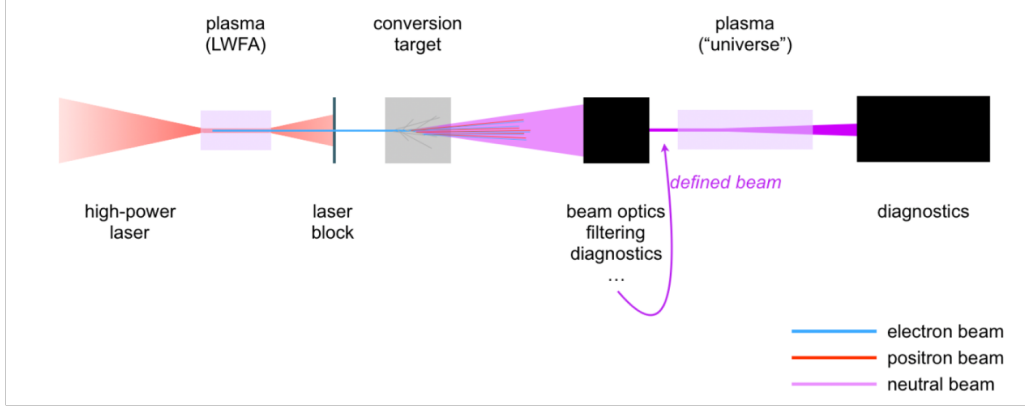


Figure 7.1: Schematic of a conceptual laser-based laboratory setup where an initial electron beam is converted into a neutral electron-positron pair beam using a high-density target in laser wakefield acceleration (LWFA). Image courtesy: Beck *et al.* [53] and Benno Zeitler.

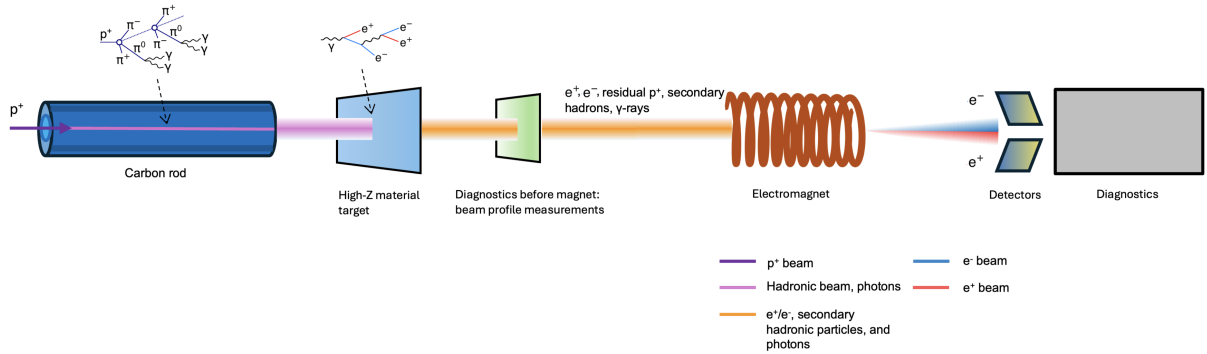


Figure 7.2: An accelerator-based experimental setup for the creation of electron-positron pair beams from a high-energy proton beam, using a low-Z material (e.g., Carbon) and a high-Z converter target (e.g., Tantalum). This figure is redrawn and adapted from [250, 252].

7.1 Primary beam to pair-beam generation

Modern accelerator beams are typically characterized by parameters such as average energy, momentum spread, charge, emittance [304], and bunch length. However, in experiments involving high-density targets to produce the beam and to develop the cascade, most of these qualities are significantly degraded by radiative losses and multiple scattering. As a result, only a few input parameters remain physically relevant for the downstream pair beam: the initial beam energy, which sets the upper energy scale of secondaries; the beam charge, which determines the

total particle number and hence the cascade density; and the bunch length, which controls the temporal extent of the beam and its ability to drive collective effects.

A promising approach to generating such beams is laser wakefield acceleration (LWFA), as shown in Fig. 7.1, which is based on [303, 305, 306]. LWFA systems can produce ultra-short electron bunches (tens of femtoseconds) with high peak currents, nanocoulomb-level charge, and energies up to hundreds of MeV [307–310]. When these relativistic electrons interact with a high-Z target, they undergo bremsstrahlung, emitting high-energy photons as they are decelerated in the electric field of atomic nuclei,

$$e^- + Z \rightarrow e^- + \gamma + Z.$$

These emitted gamma rays can then convert into electron–positron pairs via Bethe–Heitler pair production in the same material

$$\gamma + Z \rightarrow e^+ + e^- + Z.$$

Through repeated bremsstrahlung and pair-production processes, an electromagnetic cascade develops, leading to a high-density population of electrons and positrons. These beams are suitable because they are compact and inherently synchronized. However, their extremely short bunch length is a fundamental limitation for plasma instabilities such as the oblique mode. The instability growth time typically exceeds the LWFA bunch duration by several orders of magnitude, meaning that, even if sufficient energy and density are achieved, the beam cannot interact with the background plasma long enough to efficiently drive the instability. While techniques such as bunch compressors using dipole magnets can stretch the bunch, this comes at the cost of reducing beam density, and the scaling of the growth rate with density ($\propto n_b^{1/3}$) makes this trade-off inefficient. Consequently, LWFA-generated beams are generally too short-lived to sustain the desired collective processes.

An alternative and more suitable approach (shown in Fig. 7.2) is to use high-energy proton beams (~ 440 GeV/c) as the primary driver, as proposed in [250–252, 311]. Due to their much larger mass, protons can be accelerated to hundreds of GeV energies with comparatively less radiative loss and can be delivered in long bunches (hundreds of picoseconds) with very high particle numbers. When such a proton beam interacts with a high-density target, it produces a hadronic cascade that generates large numbers of neutral pions (π^0). These pions decay into high-energy gamma rays, which subsequently initiate electromagnetic cascades, producing dense populations of electrons and positrons

$$p + N \rightarrow \pi^0 \rightarrow \gamma + \gamma,$$

where N refers to an atomic nucleus in the target material. The charged pions ($\pi^\pm$) continue interacting or decay into muons, but they are less relevant for the electromagnetic signal being measured. A high-Z target (usually tantalum or lead) is placed downstream to facilitate the conversion of gamma rays into charged particles via Bethe–Heitler pair production,

$$\gamma + Z \rightarrow e^+ + e^- + Z.$$

A high-Z target maximizes the conversion probability within a short distance (~ 2.5 radiation lengths). This creates a compact and intense source of e^+/e^- pairs. The resulting pair beams

can reach densities exceeding 10^{13} cm^{-3} [251] and, crucially, inherit the long temporal structure of the proton bunch, making them far more suitable for studying plasma instabilities. Facilities such as AWAKE ($\sim 400 \text{ GeV/c}$ with up to 3×10^{11} protons per bunch and a bunch duration of $\sim 400 \text{ ps}$) [312], and HiRadMat ($\sim 440 \text{ GeV/c}$ with up to 3×10^{11} protons per bunch and a bunch duration of $\sim 250 \text{ ps}$) [313] exemplify this regime, providing beams with both the energy and duration required to generate and sustain relevant pair cascades experimentally.

7.2 Beam filtering

In the previous section, we described the conversion of a primary beam into an electron-positron pair beam using a converter target. Moreover, since the resulting beam is not a purely neutral electron-positron pair beam and contains secondary particles, beam filtering is required to produce a purely electron-positron pair beam.

In primary electron-beam-driven experiments (e.g., LWFA-based setups), the pair beam emerging from the converter is not a pure electron-positron plasma, but rather a mixture of positrons, electrons, and secondary particles (electrons and photons) generated during the cascade. As a result, the beam's density is rapidly decreasing due to cascade development and multiple scattering. In addition, the beam is typically transversely hot, and desirable features such as approximate charge neutrality can only be achieved through beam focusing and filtering. Magnetic elements such as solenoid magnets [314], quadrupole doublets [314, 315], or active plasma lenses [316–318] can be used for this purpose. Among these, solenoids are particularly advantageous for pair beams, as they provide charge-independent focusing through the azimuthal motion induced by the Lorentz force, resulting in a net transverse confinement for both electrons and positrons. In the thin-lens approximation, the focusing strength of a solenoid of length L and magnetic field strength B is given by [314],

$$\frac{1}{f_{\text{sol}}} \approx \left(\frac{q_e}{2m_e c \beta \gamma} \right)^2 B^2 L. \quad (7.1)$$

For a beam with an energy of 100 MeV, the focal length can be estimated as

$$f_{\text{sol}} \approx 0.45 \times \left(\frac{B}{1 \text{ T}} \right)^{-2} \left(\frac{L}{1 \text{ m}} \right)^{-1} \text{ m}. \quad (7.2)$$

That means focusing a 100 MeV electron beam to a focal length of $f_{\text{sol}} \sim 1 \text{ m}$ with a solenoid of length $L = 10 \text{ cm}$ requires a magnetic field of $B \sim 2.1 \text{ T}$. In the thin-lens approximation, the focal strength of a quadrupole doublet is given by [314]

$$\frac{1}{f_d} \approx \left(\frac{q_e}{m_e c \beta \gamma} \right)^2 \frac{B^2 l^2 s}{a^2}, \quad (7.3)$$

where l is the quadrupole length, s the separation between their centers, and a the pole radius. Under the assumption that the inter-magnet gap equals the quadrupole length, $s = 2l$, the beam with an energy of 100 MeV, the focal length can be estimated as

$$f_d \approx 5.6 \times 10^{-2} \times \left(\frac{B}{1 \text{ T}} \right)^{-2} \left(\frac{l}{1 \text{ m}} \right)^{-3} \left(\frac{a}{1 \text{ m}} \right)^2 \text{ m}. \quad (7.4)$$

This implies that focusing a 100 MeV electron beam to a focal length of $f_d \sim 1$ m, using quadrupoles of length $l = 10$ cm and a pole radius $a = 50$ mm, requires a magnetic field of approximately $B \sim 0.4$ T. Unlike solenoids, quadrupoles focus in one plane and defocus in the other, requiring complex multi-element configurations to achieve net focusing.

In proton-driven experiments [250, 252, 311], beam filtering is implemented through a combination of passive shielding, spatial collimation, and magnetic spectrometry, due to the mixture of hadrons, muons, photons, and electron-positron pairs produced in the cascade. After the pair-beam is produced, a Chromox scintillation screen is used to measure the transverse beam profile, while a thin aluminum blocker foil (~ 50 μm) suppresses stray optical light and background. The screen is imaged from a distance of several meters, providing a spatial resolution on the order of 100 μm [250]. To further suppress unwanted background from hadronic secondaries, concrete shielding blocks are placed along the beamline, leaving a narrow aperture that effectively collimates the beam and reduces contamination from large-angle particles. At a distance of approximately 2 m downstream, the filtered beam enters a magnetic spectrometer, where electrons and positrons are spatially separated, and their energy distribution is measured. The spectrometer typically consists of an electromagnet and downstream scintillation screens positioned off-axis (e.g., ~ 240 mm). This setup enables simultaneous charge separation and spectral resolution of the pair beam. The high-density pair beam preserves the long temporal structure of the proton bunch and is suitable for detailed studies of plasma interactions.

7.3 Beam–plasma interaction cell

The pair-beam diagnostics are a key component of our experiment, where the properties of the electron-positron beam are measured and characterized. The most important quantities to determine are the spatial profile, energy distribution, and charge composition of the beam, which are essential for understanding its suitability for instability growth. Following the filtering stages, the initially mixed particle beam is transformed into a beam effectively dominated by electrons and positrons, with the secondary hadrons and photon components largely removed. The growth of the instability is governed by the properties of the background plasma, which must be properly defined. The background plasma can be produced by ionizing a neutral gas, such as hydrogen, using either a current discharge or a laser, resulting in an electron–proton plasma whose density is determined by the initial gas conditions [319–321]. For hydrogen gas, the background plasma density is determined from the ideal gas law,

$$n_{\text{bg}} = x \frac{p}{k_B T_{\text{gas}}} \approx 2.4 \times 10^{16} x \left(\frac{p}{\text{mbar}} \right) \left(\frac{T_{\text{gas}}}{300 \text{ K}} \right)^{-1} \text{ cm}^{-3}, \quad (7.5)$$

where x is the ionization fraction. In practical laboratory conditions, for a fully ionizing Hydrogen gas, background plasma densities in the range $10^{13} - 10^{15} \text{ cm}^{-3}$, corresponding to gas pressures of $10^{-4} - 10^{-2}$ mbar at room temperature, are typically achievable and suitable for instability studies. In order to observe the full development of the instability, the plasma must extend over a length sufficient to contain the entire growth process, i.e.,

$$L_{\parallel} \gtrsim c \cdot \delta_{\text{growth}}^{-1}, \quad (7.6)$$

and must remain stable and homogeneous over this distance. In addition, the transverse size of the beam must exceed the characteristic plasma scale to ensure collective plasma behavior,

requiring

$$L_{\perp} \gg 2\pi c \cdot \omega_p^{-1}. \quad (7.7)$$

For a characteristic instability growth time of order 1 ps, using Eq. (7.6), the plasma must extend over a length of approximately 300 μm to allow the instability to develop.

Impact of Instability on IGMF Constraints

This chapter is based on the publication:

- **S. Dey**, S. Rossoni, and G. Sigl. (2026). *The impact of plasma instability cooling on intergalactic magnetic field constraints in GeV cascades for optimized instability cooling parameters*. **Astroparticle Physics**, 182, 103275. arXiv: [2604.08375](https://arxiv.org/abs/2604.08375) [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2026.103275>.

The chapter is organized as follows: Sec. 8.3 introduces the parametric model of plasma instability. Sec. 8.4 describes the cascade simulation configuration, including the jet emission scheme, observer setup, intergalactic magnetic field (IGMF) configuration, and other relevant interactions for cascade production. Sec. 8.5 presents the output analysis methods, including cascade spectrum construction, field-of-view analysis, and test statistics. Sec. 8.6 reports the simulation results for electromagnetic cascades with IGMF only and with both IGMF and plasma instability. Sec. 8.7 discusses the impact of plasma instabilities on IGMF constraints. Finally, Sec. 8.9 offers a discussion and conclusion.

8.1 Electromagnetic cascade

The propagation of TeV gamma rays through the intergalactic medium (IGM) is strongly affected by interactions with the extragalactic background light (EBL) [322]. This interaction produces electron–positron pairs via the process $\gamma + \gamma_{\text{EBL}} \rightarrow e^+ + e^-$, commonly referred to as electromagnetic pair production. The resulting pairs are typically highly relativistic, with Lorentz factors Γ in the range 10^5 to 10^8 [281].

The produced relativistic electrons and positrons then interact with cosmic microwave background (CMB) photons [323, 324] via inverse Compton (IC) scattering, $e^\pm + \gamma_{\text{CMB}} \rightarrow e^\pm + \gamma$. This process generates secondary photons at lower energies, which may undergo further interactions of the same type, thereby giving rise to an electromagnetic cascade [9, 176]. Besides these dominant mechanisms, higher-order processes such as double pair production (DPP), $\gamma + \gamma \rightarrow e^+ + e^- + e^+ + e^-$, and triple pair production (TPP), $e^\pm + \gamma \rightarrow e^\pm + e^+ + e^-$, are also possible. However, their contribution remains negligible at energies below approximately 100, TeV [169].

Measurements of photon fluxes from electromagnetic cascades are carried out both directly with space-based instruments, such as the Fermi-LAT [325], and indirectly using ground-based Cherenkov telescopes, including H.E.S.S. [326], MAGIC [327], and VERITAS [198]. Observations indicate a suppression of the photon flux at GeV energies compared to the expectations from standard cascade models. This effect is generally explained by the deflection of electron–positron pairs in the presence of the IGMF [5, 13, 16, 17, 174, 328]. As a result, the secondary emission forms an extended halo around the source, thereby reducing the observed gamma-ray flux along the line of sight. The non-detection of extended or delayed GeV emission from several extragalactic TeV sources has been used to place lower bounds on the strength of the IGMF in cosmic voids [6, 8, 18, 20, 106, 247].

An alternative explanation involves energy losses of the e^+e^- pairs due to beam–plasma instabilities, which can reduce the efficiency of IC cooling. In this scenario, the interaction between the relativistic pair beam produced by blazars and the ambient intergalactic plasma excites electrostatic waves, providing an additional channel for energy dissipation that may compete with IC losses. The role and efficiency of these instabilities remain subjects of ongoing theoretical and numerical studies [28, 31, 33, 35, 44, 45, 257, 258]. Previous works [35, 329] have investigated the growth of plasma instabilities in the IGM under different beam and environmental conditions, such as the temperature and density of the medium. These studies show that energy losses induced by plasma instabilities can account for the observed suppression of GeV emission in blazar gamma-ray spectra. In Chap. 6, we have studied the fractional energy loss of the beam due to beam–plasma instability feedback alone, using a kinetic simulation approach, with particular emphasis on sources similar to blazar-like sources. We find that the resulting fractional energy loss is approximately $\lesssim 4\%$. In this chapter, we use a Monte Carlo approach using CRPropa 3.2 to investigate the combined effects of plasma instabilities and the IGMF on the development of the electromagnetic cascade. The effect of the instability is included via a parametric energy-loss rate function. The propagation of injected and secondary electrons and positrons is treated in three dimensions to account for their deflection by the IGMF.

8.2 Existing constraints on the IGMF

A number of studies have placed constraints on long-coherence-length IGMFs. In particular, [13, 14] inferred a lower bound of $B \gtrsim 10^{-16}$ G under the assumption that the observed TeV emission from selected blazars represents their typical activity over timescales of $\sim 10^6$ yr. If this assumption is relaxed, and the sources are instead considered active only during the observational period, the limit weakens to $B \gtrsim 10^{-17}$ G [15–17]. Using 7.5 years of Fermi-LAT data [12], the analysis presented in [18] derived a constraint of $B \gtrsim 3 \times 10^{-16}$ G, although it still depends on assumptions regarding the stability of TeV emission over decade-long timescales. Further constraints arise from searches for extended emission around Mrk 421 and Mrk 501 with MAGIC, which exclude field strengths in the range $[0.4\text{--}1] \times 10^{-14}$ G, assuming persistent emission above 20 TeV [19]. Similarly, H.E.S.S. observations exclude IGMF strengths below 3×10^{-16} G and up to 3×10^{-15} G under comparable assumptions [21]. A combined analysis of MAGIC and Fermi-LAT observations of the blazar 1ES 0229+200 yielded a lower bound of $B \sim 1.8 \times 10^{-17}$ G [20]. In addition, observations of the exceptionally bright “BOAT” event, GRB 221009A, with LHAASO and Fermi-LAT suggest $B \gtrsim 10^{-19}$ G for coherence lengths $\lambda_c > 1$ Mpc [6]. More recently, [22] reported an improved constraint based on the same Gamma-ray Burst (GRB),

excluding $B < 2.5 \times 10^{-17}$ G at the 95% confidence level for $\lambda_c \gtrsim 1$ Mpc. This corresponds to one of the weakest limits currently reported. Another study by Webar *et al.* [330] reports the detection of extended GeV emission around Mkn 501, interpreting it as evidence for an IGMF strength $\sim 1.5 \times 10^{-15}$ G with a coherence length of ~ 10 kpc. Their analysis is based on searches for extended emission using the point-spread function (PSF) of the Fermi-LAT telescope.

If a TeV blazar emits gamma rays continuously over timescales much longer than the typical delay time of cascade photons, these delays become effectively ‘washed out’, and the observer detects the total emission spectrum, including all cascade components. Since 1ES 0229+200 does not exhibit significant short-term variability (see [20]), we focus our analysis on the angular extension of the emission rather than on time-delay techniques, as employed in [6, 20].

In this work, we introduce a two-parameter energy-loss term (described in Sec. 8.3), following a similar approach to [35], to model the cooling of electron–positron pairs due to plasma instabilities. The propagation of very-high-energy gamma rays is simulated using the Monte Carlo framework CRPropa 3.2 [56], incorporating all relevant electromagnetic interactions, the additional energy-loss contribution from plasma instabilities, and the effects of the IGMF. We then examine how plasma instabilities affect measurements of the IGMF from Fermi-LAT observations of the blazar 1ES 0229+200.

8.3 Parametric modeling of plasma instability

High-energy gamma rays interacting with low-energy background photon fields, such as the EBL, produce relativistic and low-density electron–positron pair beams. As these beams propagate through the IGM, perturbations in their momentum distribution can disturb the ambient plasma and excite various plasma instability modes. In this regime, Langmuir waves generated in the background plasma can resonate with fluctuations in the beam, leading to the exponential growth of electrostatic perturbations [39]. The characteristic length scale associated with the growth of beam–plasma instabilities is set by the plasma skin depth, defined as $\lambda_s = 2\pi c/\omega_p$, where the plasma frequency is $\omega_p = (4\pi e^2 n_{\text{IGM}}/m_e)^{1/2}$ and n_{IGM} denotes the number density of the IGM plasma. Physically, collective plasma effects become important when the number of beam particles within a volume of radius λ_s is much larger than unity [33], i.e.,

$$\lambda_s^3 n_b \gg 1, \quad (8.1)$$

where n_b is the pair-beam number density. For typical astrophysical conditions, with $n_b \sim 10^{-22} \text{ cm}^{-3}$ and $n_{\text{IGM}} \sim 10^{-7} \text{ cm}^{-3}$ [33], this condition is satisfied, indicating that plasma instabilities can be relevant in such environments. Astrophysical pair beams produced through interactions of TeV photons with background radiation fields exhibit a strongly anisotropic momentum distribution [33]. This anisotropy arises because the longitudinal momentum, aligned with the beam direction, is much larger, reflecting the high energy of the parent photon, while the transverse momentum remains comparatively small. As a result, the angular spread of the beam is narrow, since pair production largely preserves the direction of the incident photon. This configuration suppresses electromagnetic instabilities, and the growth of electrostatic modes becomes dominant [31, 331]. Of the various electrostatic modes, the oblique and modulation instabilities are the most relevant in this context, as they exhibit the fastest growth rates under these conditions [31, 40, 331]. The oblique instability arises from the coupling between beam-driven electrostatic waves and density perturbations in the background plasma, with wave

vectors oriented at an angle $0 < \theta < \pi/2$ relative to the beam direction. It can be viewed as a variant of the two-stream instability in which the wave vector is not parallel to the beam velocity. The modulation instability, in contrast, corresponds to an oscillatory form of the two-stream-like instability, where the interaction occurs between forward and backward Langmuir sidebands rather than between two particle streams.

The propagation of electron–positron pairs is influenced by both IC scattering with CMB photons [332] and beam–plasma instabilities, as described above. Two characteristic length scales are therefore important: the instability cooling length, λ_0 , defined as the typical distance over which an electron or positron loses a significant fraction of its energy due to plasma processes, and the IC interaction length, λ_{IC} , which represents the mean free path for IC scattering with CMB photons. When λ_0 becomes comparable or smaller than λ_{IC} , energy losses due to plasma instabilities can dominate, leading to a suppression of secondary GeV photon production.

We define a two-parameter energy-loss term (following the approach described in [333]) to parameterize the plasma instability cooling of electron–positron pairs. Therefore, the energy-loss rate for electrons with energy E_e due to plasma instabilities is given by

$$E_e^{-1} \left(\frac{dE_e}{dt} \right) = c\lambda_0^{-1} \left(\frac{E_e}{\tilde{E}_e} \right)^{-s} \text{ sec}^{-1}. \quad (8.2)$$

In this work, we perform a parametric study in terms of instability cooling length, λ_0 , and instability power-law index, s , without explicitly resolving the microscopic coupling between individual particles and plasma waves. Instead, the instability is treated at the single-particle level through the effective energy-loss rate given in Eq. (8.2). The propagation of electromagnetic particles and the development of cascades are simulated using the Monte Carlo framework CRPropa 3.2 [56]¹. A similar strategy was adopted in [35], where instability-induced energy losses were derived from both analytical considerations [33, 45, 257, 258] and kinetic simulations [31, 44]. However, the assumption in [35] that the electron energy-loss timescale, τ_{inst} , is equal to the instability growth timescale, τ_{gr} , differs from the treatment adopted in [31, 44]. In particular, equating these two timescales corresponds to assuming that the beam energy loss proceeds on the same timescale as the instability growth. In this work, we generalize the parametrization of the energy-loss term according to [333] and estimate the fractional energy loss associated with the instability, and therefore do not rely on the previous assumption. According to previous studies [31, 33, 44, 45, 257, 258], the energy dependence for both cold- and warm-plasma beams ranges from a power-law index range of $-2.0 \leq s \leq 1.6$. For warm plasma beams where the oblique instability dominates, typical values of s lie in the negative range $-1 \leq s \leq 0$. In this work, we adopt a weak energy dependence with $s = -0.5$ for the energy-loss term in our simulations, consistent with the warm-plasma beam regime where oblique instability is the dominant mode.

Under realistic astrophysical conditions, the energy-loss length scale is typically larger than the instability growth length scale. For typical astrophysical parameters, the minimum growth length is estimated to be $\lambda_{\text{gr}} \approx 1.43\text{--}2.85$ pc, assuming beam densities $n_b \sim [0.5\text{--}1] \times 10^{-23} \text{ cm}^{-3}$ and $n_{\text{IGM}} \sim 10^{-7} \text{ cm}^{-3}$ [26]. Taking into account the nonlinear evolution of electrostatic waves [26, 31], the corresponding energy-loss length scale is $\lambda_0 \simeq 5 \times 10^4 \lambda_{\text{gr}} \simeq [0.7\text{--}1.4] \times 10^2$ kpc. Motivated by this, we choose $\lambda_0 = 120$ kpc to evaluate the fractional energy loss due to instability over one IC scattering length, λ_{IC} . This λ_0 value is reported by [333] as the best-fit value for cascades produced purely by plasma instabilities.

¹<https://crpropa.desy.de/>

Our treatment of beam energy losses due to plasma instabilities is based on the following assumptions:

1. The energy losses are assumed to be spatially uniform, such that all electrons lose energy independently of their distance from the source.
2. Although plasma instability eventually saturates after the linear growth due to nonlinear effects caused by the instability feedback, we assume that our implementation remains valid within the regime where the instability undergoes growth between two different IC scatterings. That means, the instability saturation time, τ_{sat} , is longer than the IC interaction time, τ_{IC} , i.e., $\tau_{\text{IC}} < \tau_{\text{sat}}$, ensuring that instability remains in its growth phase between two different IC scatterings. For electrons with energies in the range $10^{-3} \leq E_e/\text{TeV} \leq 10^2$, IC interaction length is nearly independent of energy, with $\lambda_{\text{IC}} \sim 1.2$ kpc, corresponding to an IC interaction time of $\tau_{\text{IC}} \sim 1.23 \times 10^{11}$ sec [13]. According to [28], the instability saturation time is approximately $\tau_{\text{sat}} \sim 5 \times 10^{13}$ sec, which justifies our assumption.

8.4 Cascade simulation setup

This section outlines the numerical framework used in this work to model the injection of blazar emission and the propagation of secondary particles generated in electromagnetic cascades. In this work, the emission from the blazar is modeled as a collimated jet with a finite opening angle, as discussed in Sec. 5.3.1 of Chap. 5. The angular distribution of the injected photons follows a von Mises–Fisher profile (as given in Eq. (5.32)), which provides a physically motivated description of anisotropic emission around a preferred direction. The concentration parameter κ determines the degree of collimation and can be directly related to the jet opening angle, ensuring that a given fraction of the emitted photons is confined within the jet cone.

The IGMF is implemented as a turbulent field with a Kolmogorov-type power spectrum, characterized by a spectral index, $\mathcal{K} = 5/3$, as described in Sec. 5.3.2 of Chap. 5. The magnetic field is generated as a homogeneous random field with fluctuations across a range of spatial scales, limited by minimum and maximum turbulence lengths, according to Eq. (5.35). The resulting coherence length sets the scale over which the field remains correlated and controls how charged particles are deflected during their propagation.

We define an observer as a spherical surface of radius $d = 585$ Mpc centered on the source, corresponding to the comoving distance to 1ES 0229+200 at redshift $z = 0.14$. To obtain both the injected and propagated photon spectra for subsequent analysis, we additionally place a second detection surface around the source with a radius of 1 pc, which is smaller than the pair-production interaction length for high-energy injected photons and therefore sufficient to capture only the source spectrum.

8.4.1 Interactions for cascade development

TeV photons emitted by extragalactic sources propagate through ambient radiation fields, such as the CMB and the EBL, thereby initiating electromagnetic cascades. The cascade process begins when high-energy photons interact with background photons, producing electron–positron pairs. These secondary particles carry a large fraction of the initial photon energy and continue to move primarily along the original direction of propagation. During their transport, they

may experience energy losses due to plasma instabilities, be deflected by the weak IGMF, and subsequently scatter background photons via IC interactions.

The resulting secondary photons can again produce pairs, leading to a repeated sequence of interactions that continues until particle energies fall below the relevant interaction thresholds. In our simulations, we include the interaction rates for pair production and IC scattering on both the CMB and Cosmic Infrared Background (CIB) photons, using the Franceschini08 model [165] as implemented in CRPropa 3.2. We also account for higher-order processes, such as double- and triple-pair production. However, these contributions are negligible at photon energies of ~ 100 TeV, since their interaction lengths are considerably larger, leading to a strong suppression of these processes (see [169] for further details).

8.5 Spectrum predictions

We employ a reweighting approach to reproduce the observed emission spectra of blazars. In this framework, we define a cascade signal function, $G(E_0, E; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c)$, which connects the energy of the primary photon, E_0 , to the observed photon energy, E . This function depends on the IGMF coherence length, the observational field of view, and the parameters that describe the plasma instability. For each combination of IGMF strength and coherence length, a distinct cascade signal function is obtained for a given instability scenario.

During the cascade development, a single injected photon can produce multiple secondary photons in the GeV range. As a result, an observed photon with energy E may originate from any of several secondary photons generated by a primary photon of energy E_0 . Using this relation, we construct the photon spectrum for fixed plasma instability parameters and identify the values of IGMF strength and coherence length that best reproduce the observational data for a given field-of-view. In our simulations, the propagation of injected and secondary electrons and positrons is treated differently depending on the physical process. A one-dimensional treatment is used to model energy losses due to plasma instability cooling, while a full three-dimensional propagation is employed to account for their deflection in the presence of the IGMF.

8.5.1 Cascade spectrum construction

Observations from H.E.S.S. and VERITAS do not show significant spectral variability of 1ES 0229+200 above 100 GeV [20]. Motivated by this, we model the intrinsic source emission using a power-law spectrum with an exponential cutoff,

$$J_0(E_0) = A \left(\frac{E_0}{\text{TeV}} \right)^{-\beta} \exp \left(-\frac{E_0}{E_{\text{cut}}} \right), \quad (8.3)$$

where E_0 denotes the energy of the primary photon, β is the spectral index, E_{cut} represents the cutoff energy, and A is a normalization factor. The photon spectrum observed at Earth, $J(E; \lambda_0, s; \theta_{\text{FoV}}, B)$, is obtained by convolving the cascade signal function $G(E_0, E; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c)$ with the initial injection spectrum $J_0(E_0)$ [20],

$$J(E; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c) = \int_{E_0 \geq E} G(E_0, E; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c) \cdot J_0(E_0) dE_0. \quad (8.4)$$

In our simulations, we propagate 10^4 primary photons with energies in the range $10^{-3} \leq E_0/\text{TeV} \leq 10^2$. The propagation is performed with a step size of $\Delta x \simeq 10^{14}$ cm, up to a comoving distance of $d = 585$ Mpc.

8.5.2 Cascade geometry and observables

The emission timescale of a TeV blazar is much longer than the typical delay time of cascade photons, i.e., $\tau_{\text{emission}} > \tau_{\text{delay}}$, so the time-delay effects become effectively washed out, and the observed signal is no longer influenced by propagation delays. Since the blazar 1ES 0229+200 does not exhibit rapid variability, we base our analysis on the angular extension of the observed emission rather than on time-delay methods as used in [6, 20].

The production of electron-positron pairs through the interaction of a primary TeV photon with an [EBL](#) photon is governed by relativistic kinematics. In the rest frame of the [IGM](#), a high-energy photon with four-momentum $P_0 = E_0(1, \hat{\mathbf{p}}_0)$ interacts with a low-energy [EBL](#) photon of four-momentum $P_1 = E_{\text{EBL}}(1, \hat{\mathbf{p}}_1)$, where $\hat{\mathbf{p}}_0$ and $\hat{\mathbf{p}}_1$ denote the corresponding unit momentum vectors. This interaction produces an electron-positron pair with four-momenta $P_{\pm} = (E_{e\pm}, \mathbf{q}_{\pm})$ [43]. The angle between the direction of the incoming photon and that of the produced electron or positron (often referred to as the initial beam opening angle) is approximately $\sim \Gamma^{-1}$, where $\Gamma \sim 10^5 - 10^7$ is the Lorentz factor of the pairs. This angle is therefore extremely small and can be considered negligible [334]. After their production, the electrons and positrons propagate through the [IGM](#), where they may be deflected by the [IGMF](#) and simultaneously lose energy due to plasma instabilities, before undergoing inverse Compton scattering with [CMB](#) photons. The deflection angle with respect to the original photon direction is given by $\delta = \cos^{-1}(\hat{\mathbf{p}}_0 \cdot \hat{\mathbf{q}}'_{\pm})$, where $\hat{\mathbf{q}}'_{\pm}$ is the unit vector of the particle momentum after deflection, obtained directly from the simulation output. The secondary photons produced through [IC](#) scattering, with energy E_{γ} , are observed within an angle θ_{obs} . This angle depends on the gamma-ray mean free path, λ_{γ} , the comoving distance between the source and the observer, d , and the deflection angle δ of the electrons [174],

$$\sin(\theta_{\text{obs}}) = \frac{\lambda_{\gamma}(E_0)}{d} \sin \delta. \quad (8.5)$$

The mean free path of a primary photon with energy E_0 for pair production on [EBL](#) photons can be approximated as $\lambda_{\gamma} \approx 44.36 \times (E_0/10 \text{ TeV})^{-1}$ Mpc for redshifts $z < 1$ [28, 173, 174]. Using Eq. (8.5), we estimate the corresponding observation angle θ_{obs} . Based on this, we define the observer's field-of-view angle, θ_{FoV} , as a free parameter to quantify the cascade emission within a given angular region. A schematic illustration of the electromagnetic cascade, including the deflection of charged particles by the [IGMF](#), is shown in Fig. 8.1. The angular resolution of the Fermi-LAT, characterized by its point-spread function (PSF), depends on energy, with a 68% containment angle of about 0.7° at 1 GeV and 0.08° at 1 TeV [12, 335]. In our analysis, we use representative values of $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° , consistent with the condition $\theta_{\text{psf}} < \theta_{\text{obs}} < \theta_{\text{FoV}}$ [3]. Within these field-of-view angles, we simulate the photon spectra for different magnetic field strengths and coherence lengths, while also accounting for energy losses due to plasma instabilities.

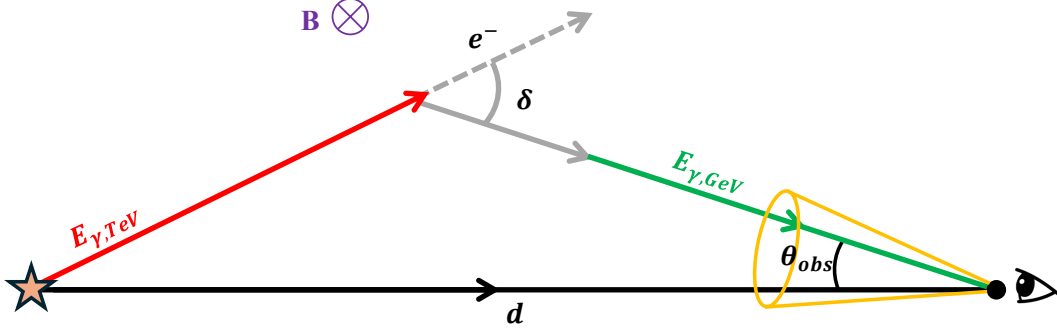


Figure 8.1: Schematic representation of the production of secondary GeV photons from the deflection of charged electrons (or positrons) by IGMF. The solid gray line denotes the deflected electron due to the IGMF, and the dashed gray line represents the original direction of the electron without any deflection. The yellow region shows the field of view of the observer, defined by the angle θ_{FoV} .

8.5.3 Statistical method for spectrum fitting

We use a χ^2 test to compare the simulated photon spectrum with the observed data. The test statistic is defined as

$$\chi^2(A, \beta, E_{\text{cut}}; \lambda_c) = \sum_i \left(\frac{J_{\text{data}}(E_i) - J_{\text{sim}}(E_i; A, \beta, E_{\text{cut}}; \lambda_c)}{\sigma_{\text{data},i}} \right)^2, \quad (8.6)$$

where $J_{\text{data}}(E_i)$ and $J_{\text{sim}}(A, E_i; \beta, E_{\text{cut}}; \lambda_c)$ are the observed and simulated spectra at energy E_i , and $\sigma_{\text{data},i}$ is the uncertainty in the data. The sum is taken over all energy bins. The analysis is carried out for field-of-view angles $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . We first consider the case without plasma instabilities and then include their effect using the best-fit parameters from [333]. For each combination of magnetic field strength B and coherence length λ_c , we determine the best-fit intrinsic spectral parameters A , β , and E_{cut} at the 90% confidence level. The χ^2 value is normalized by the number of degrees of freedom, n_{dof} , defined as the number of data points minus the number of fitted parameters. The parameter space is scanned over $\beta = 1.0 - 2.0$ with step size $\Delta\beta = 0.01$, $E_{\text{cut}} = 1.0 - 15.0$ TeV with $\Delta E_{\text{cut}} = 0.1$ TeV, and $\log_{10} A = -24.000$ to -23.200 $\text{eV}^{-1}\text{cm}^{-2}\text{s}^{-1}$ with $\Delta \log_{10} A = 0.001$ $\text{eV}^{-1}\text{cm}^{-2}\text{s}^{-1}$. Using the corresponding best-fit values of A, β, E_{cut} , we then calculate $\chi^2_{\text{min}} = \min_{\lambda_c} [\chi^2_{\text{min}}(\lambda_c)] = \min_{\lambda_c} (\min_{A, \beta, E_{\text{cut}}} \chi^2(A, \beta, E_{\text{cut}}; \lambda_c))$. The fit is performed to the observed GeV data, as cascade emission is predominant in this energy range, while the TeV component of the spectrum remains unaffected. The total $n_{\text{dof}} = 2$ when the GeV-band data from Fermi-LAT are considered. We require that the minimum χ^2 satisfies the condition for a 2σ confidence level. This corresponds to $\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}} \leq 8$. In normalized form, this becomes $\Delta\chi^2/n_{\text{dof}} \leq 4$, which is the standard normalized threshold used in this work.

8.6 Simulation results

In this section, we discuss the propagated photon spectrum of the 1ES 0229+200 blazar under different propagation scenarios. We first consider the case without plasma instabilities, in which the pairs experience only magnetic deflection by the IGMF. We then present the propagated

photon spectrum, accounting for both the effects of plasma instabilities and **IGMF**. We determine the combination of **IGMF** and plasma instability parameters that result in the minimum test statistic $\chi^2(A_i, \beta, E_{\text{cut}}; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c)$, according to Eq. (8.6).

8.6.1 Electromagnetic cascade with IGMF only

We begin by simulating the propagated photon spectrum without including the effects of plasma instabilities and magnetic deflections. In this case, all secondary GeV photons are produced solely through **IC** scattering of electrons and positrons. We then extend the calculation by introducing the **IGMF**. In the presence of the **IGMF**, electrons and positrons experience magnetic deflections before generating GeV photons via **IC** interactions. For each configuration of magnetic field strength B and coherence length λ_c , we compute the cascade signal $G(E_0, E; \theta_{\text{FoV}}, \lambda_c)$, which is independent of plasma instability parameters.

In the first panel of Fig. 8.2, the gray dashed-dotted curve represents the propagated spectrum without both plasma instabilities and magnetic fields. We then introduce a magnetic field of strength 10^{-15} G and vary the coherence length to identify the spectrum that best matches the observed data from Fermi-LAT, H.E.S.S., and VERITAS (shown as blue, orange, and green data points, respectively). The corresponding spectra for different values of λ_c are shown as colored curves. In the second and third panels of Fig. 8.2, we consider weaker magnetic field strengths and adjust the coherence lengths accordingly to reproduce spectra consistent with observations. The parameter space is explored systematically. For the first panel, we vary $B \in [0.5, 15.5] \times 10^{-15}$ G with a step size $\Delta B = 0.1 \times 10^{-15}$ G, and $\lambda_c \in [10^{-4}, 10^{-2}]$ Mpc with a step size $\Delta \log_{10} \lambda_c = 2 \times 10^{-3}$ Mpc. For the second panel, we use $B \in [0.5, 15.5] \times 10^{-16}$ G with $\Delta B = 0.1 \times 10^{-16}$ G, and $\lambda_c \in [10^{-4}, 10^{-1}]$ Mpc with $\Delta \log_{10} \lambda_c = 3 \times 10^{-3}$ Mpc. For the third panel, the ranges are $B \in [0.5, 15.5] \times 10^{-17}$ G with $\Delta B = 0.1 \times 10^{-17}$ G, and $\lambda_c \in [10^{-2}, 10^1]$ Mpc with $\Delta \log_{10} \lambda_c = 3 \times 10^{-3}$ Mpc. These analyses are performed for two field-of-view angles, $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . For each combination of B and λ_c , the intrinsic source parameters A , β , and E_{cut} are determined using a χ^2 minimization procedure (Eq. (8.6)) at a 90% confidence level, as described in Sec. 8.5.3. The lower panels in Fig. 8.2 show the cumulative $\chi^2(\lambda_c)/n_{\text{dof}}$ as a function of λ_c , assuming one degree of freedom. The results presented in this figure correspond to $\theta_{\text{FoV}} = 1.0^\circ$. The same analysis is repeated for a larger field of view, $\theta_{\text{FoV}} = 4.5^\circ$, as shown in Fig. 8.3. Increasing the field of view allows the observer to include photons from particles with larger deflection angles. As a result, a slightly stronger magnetic field is required to reproduce the observed spectrum.

From the first panels of Figs. 8.2 and 8.3, we find that the best-fit spectra correspond to larger coherence lengths for a fixed magnetic field strength of 10^{-15} G. This behavior arises because the deflection angle scales with $B\sqrt{\lambda_c}$. A larger spatial extent of the emission implies larger deflections, which in turn requires a larger coherence length. In the second and third panels of Figs. 8.2 and 8.3, we explore different combinations of B and λ_c to identify configurations that reproduce the observed data. In particular, the third panels show that for $\lambda_c \gtrsim 0.2$ Mpc, the cascade spectra become independent of the coherence length at a fixed magnetic field strength. In this regime, particle propagation is quasi-rectilinear (nearly ballistic), and the deflection angle is proportional to B but independent of λ_c . Therefore, these parameter combinations provide a lower limit on the **IGMF**. A more detailed discussion of this limit is given in Sec. 8.7. The minimum values of $\chi^2_{\text{min}}/n_{\text{dof}}$ for each case are summarized in Table 8.1.

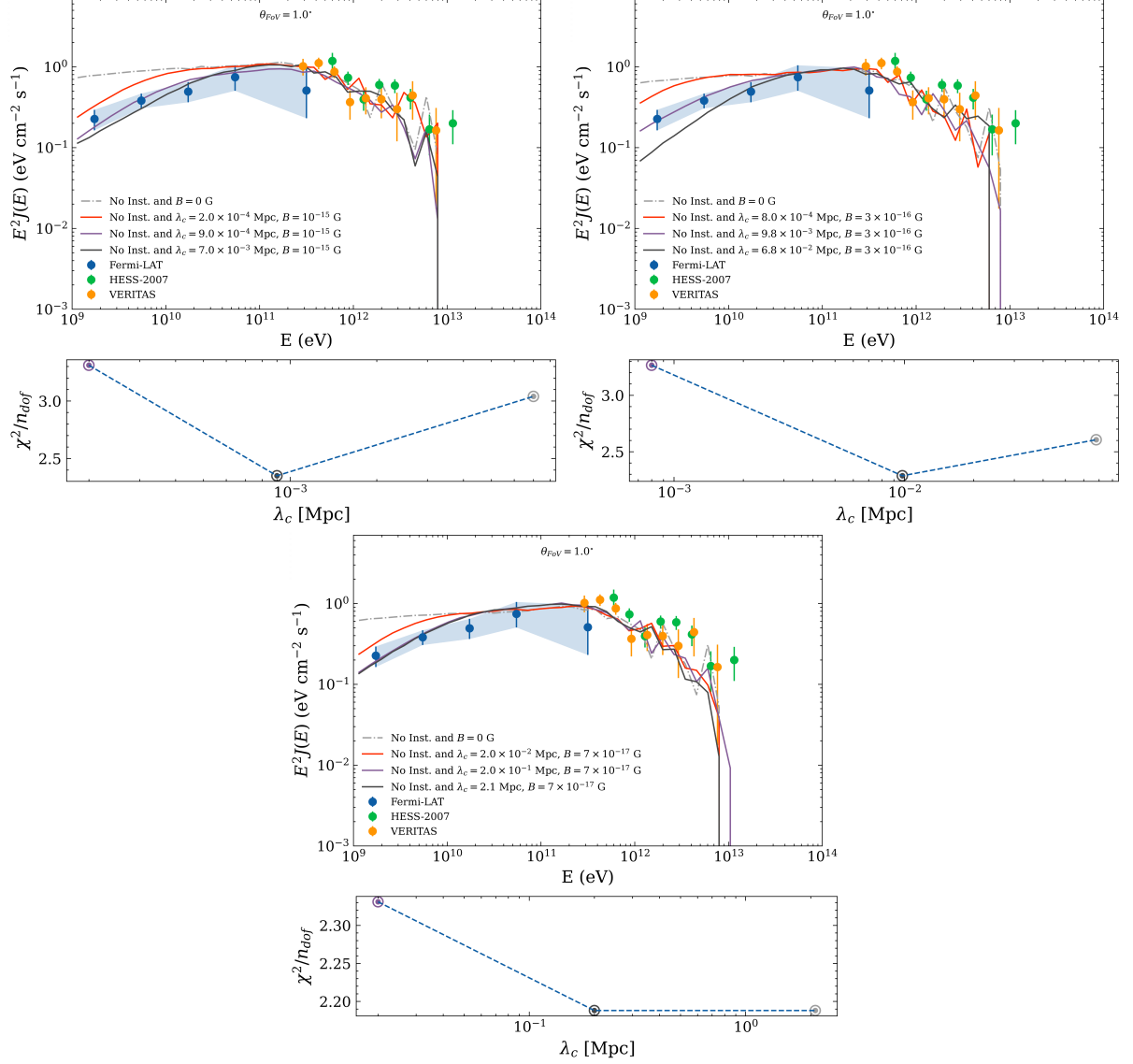


Figure 8.2: The energy spectrum of 1ES 0229+200 in $10^{-3} \leq E/\text{TeV} \leq 10^2$. The colored solid curves represent the propagated photon spectrum for different IGMF strengths and coherence lengths, without plasma instability cooling. The blue data points represent the Fermi-LAT [20], the green data points show the H.E.S.S. [167], and the orange data points show the VERITAS [336] spectrum. We investigate the energy spectrum, which is consistent with the Fermi-LAT, H.E.S.S., and VERITAS observational data in each plot. The energy spectrum is obtained for a fixed $\theta_{\text{FoV}} = 1.0^\circ$. The χ^2/n_{dof} test statistic is obtained for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$). Different panels correspond to different field strengths, as indicated in the legends.

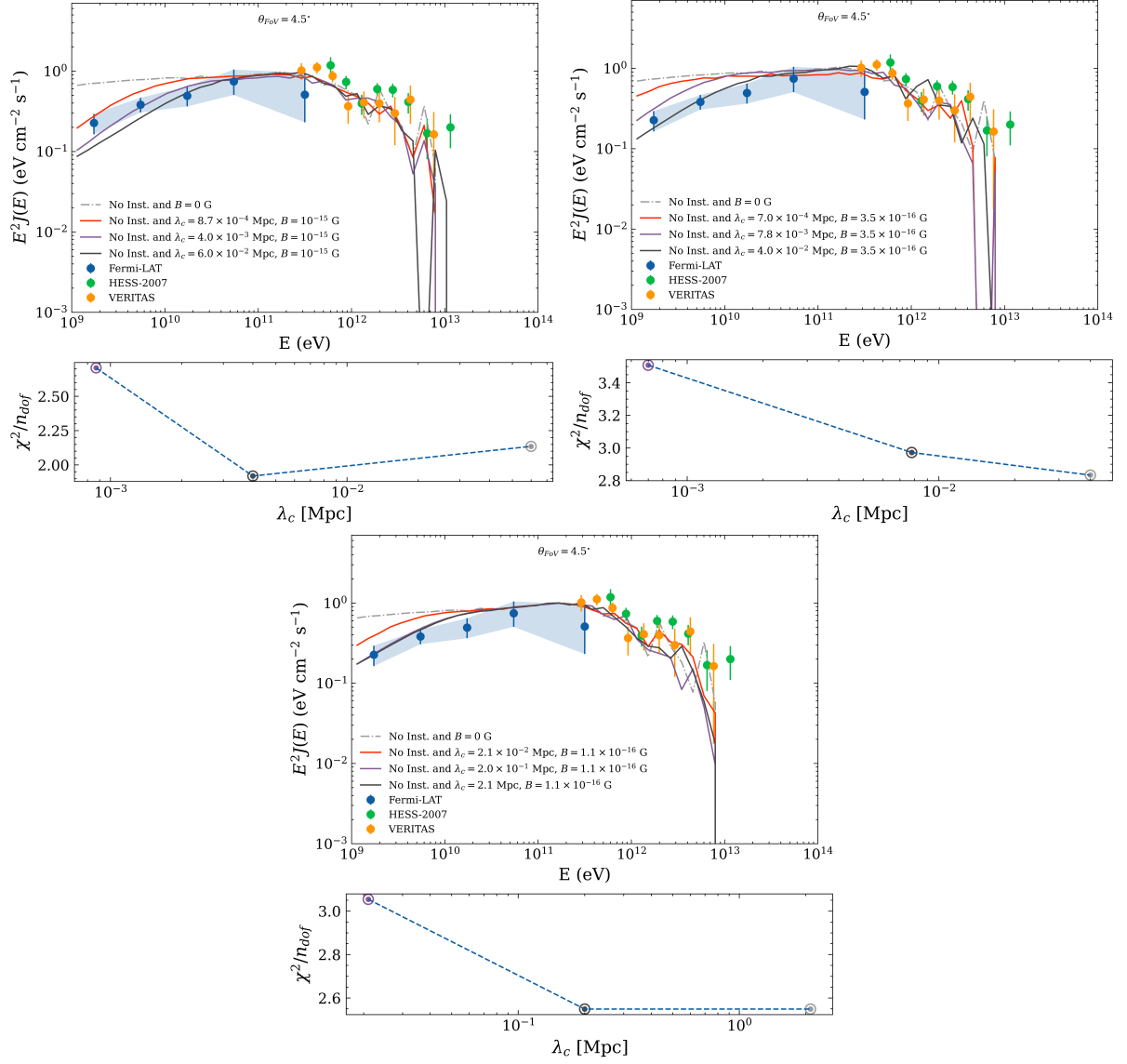


Figure 8.3: Same as Fig. 8.2, in this case, the spectral energy distribution is obtained for $\theta_{\text{FoV}} = 4.5^\circ$. The χ^2/n_{dof} test statistic is obtained for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$).

λ_0 [kpc]	θ_{FoV} [deg]	B [G]	λ_c [Mpc]	β	E_{cut}	$\frac{\log_{10} A}{[\text{eV}^{-1} \text{cm}^{-2} \text{sec}^{-1}]}$	$\chi^2_{\text{min}}/n_{\text{dof}}$
No Inst.	1.0	10^{-15}	2.0×10^{-4}	1.39	6.3	-23.717	3.311
			9.0×10^{-4}	1.48	5.3	-23.708	2.350
			7.0×10^{-3}	1.54	6.4	-23.711	3.040
		3.0×10^{-16}	8.0×10^{-4}	1.46	5.3	-23.708	3.266
			9.8×10^{-3}	1.57	4.6	-23.718	2.287
			6.8×10^{-2}	1.41	5.8	-23.712	2.606
		7.0×10^{-17}	2.0×10^{-2}	1.43	5.0	-23.722	2.331
			2.0×10^{-1}	1.51	5.5	-23.719	2.188
			2.1	1.53	5.2	-23.711	2.188
	4.5	10^{-15}	8.7×10^{-4}	1.40	5.5	-23.718	2.707
			4.0×10^{-3}	1.45	5.0	-23.726	1.919
			6.0×10^{-2}	1.50	5.2	-23.714	2.135
		3.5×10^{-16}	7.0×10^{-4}	1.36	5.1	-23.717	3.508
			7.8×10^{-3}	1.42	5.9	-23.711	2.972
			4.0×10^{-2}	1.51	5.7	-23.723	2.833
		1.1×10^{-16}	2.1×10^{-2}	1.47	5.6	-23.714	3.054
			2.0×10^{-1}	1.56	4.9	-23.720	2.549
			2.1	1.58	4.8	-23.718	2.549

Table 8.1: $\chi^2_{\text{min}}/n_{\text{dof}}$ values for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$), for $\theta_{\text{FoV}} = 1.0^\circ, 4.5^\circ$. The $\chi^2_{\text{min}}/n_{\text{dof}}$ value is highlighted in bold.

8.6.2 Modified electromagnetic cascade with IGMF and plasma instability

In this section, we study the combined effects of plasma instability-induced energy losses and the propagation of electrons and positrons in the presence of the IGMF. Our goal is to reproduce the observed photon spectrum of the blazar 1ES 0229+200. The propagated photon spectrum is obtained from the cascade signal $G(E_0, E; \lambda_0, s; \theta_{\text{FoV}}, \lambda_c)$, as defined in Eq. (8.4), for each configuration of magnetic field strength B and coherence length λ_c , while also including the plasma instability parameters λ_0 and s . We choose the instability length scale, $\lambda_0 = 120$ kpc, as the key parameter determining whether the cascade is dominated by plasma instabilities or by IC scattering, based on the fractional energy loss due to instabilities relative to a single IC interaction length. Following the procedure described in the previous Sec. 8.6.1, we first consider the propagated spectrum without including either magnetic effects or instability losses, shown as the gray dashed curve in Fig. 8.4. We then include only plasma-instability cooling, without any contribution from the IGMF, as represented by the red solid curve. This shows that plasma instabilities alone already suppress the photon spectrum in the GeV energy range.

In the first panel of Fig. 8.4, we include both magnetic deflections and instability cooling. We compute the propagated spectra for different values of λ_c at a fixed magnetic field strength of $B = 10^{-15}$ G, following the same method as in the previous Sec. 8.6.1. This allows us to examine how the combined effects modify the spectrum and to identify the parameter combinations that best match the observational data. We explore the parameter space systematically. For the first panel, we use $B \in [0.5, 15.5] \times 10^{-15}$ G with a step size $\Delta B = 0.1 \times 10^{-15}$ G, and $\lambda_c \in [10^{-5}, 10^{-3}]$ Mpc with $\Delta \log_{10} \lambda_c = 2 \times 10^{-3}$ Mpc. For the second panel, we consider $B \in [0.5, 15.5] \times 10^{-16}$ G with $\Delta B = 0.1 \times 10^{-16}$ G, and $\lambda_c \in [10^{-3}, 10^{-2}]$ Mpc with $\Delta \log_{10} \lambda_c = 10^{-3}$ Mpc. For the third panel, we take $B \in [0.5, 15.5] \times 10^{-17}$ G with $\Delta B = 0.1 \times 10^{-17}$ G, and $\lambda_c \in [10^{-2}, 10^1]$ Mpc with $\Delta \log_{10} \lambda_c = 3 \times 10^{-3}$ Mpc. These calculations are performed for two field-of-view angles, $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . In the same manner as in the previous section, the second and third panels of Fig. 8.4 correspond to smaller magnetic field strengths, with coherence lengths chosen such that the resulting spectra remain consistent with observations. For each pair (B, λ_c) , the intrinsic source parameters A , β , and E_{cut} are obtained through a χ^2 minimization (Eq. (8.6)) at the 90% confidence level (see Sec. 8.5.3). The lower panels show the cumulative $\chi^2(\lambda_c)/n_{\text{dof}}$ as a function of λ_c , assuming one degree of freedom. The spectra shown in Fig. 8.4 correspond to $\theta_{\text{FoV}} = 1.0^\circ$. We repeat the same analysis for a larger field of view, $\theta_{\text{FoV}} = 4.5^\circ$, as presented in Fig. 8.5. The minimum values of $\chi^2_{\text{min}}/n_{\text{dof}}$ for each configuration are listed in Table 8.2.

We find that the plasma instability scenario with $\lambda_0 \geq 120$ kpc (for $s = -0.5$) represents the minimum instability length scale required to reproduce the observed spectrum. For smaller values of λ_0 , the predicted spectra fall below the observational data in the GeV energy range, even when magnetic effects are not included. We evaluate the fractional energy loss due to instability over a distance of a single IC interaction length in the presence of the IGMF. If $E_e(x)$ is the energy of an electron at a distance x from the point of injection, then the energy loss of the electron within an IC interaction length, λ_{IC} , is given by $\Delta E_e(\lambda_{\text{IC}}) = E_e(x_0) - E_e(\lambda_{\text{IC}})$, where x_0 is the injection point. Then, the fractional energy loss due to instability over a distance of a single IC interaction length is given by [333]

$$\left(\frac{\Delta E_e(\lambda_{\text{IC}})}{E_e(x_0)} \right)_{\text{Inst.}} = \frac{\lambda_{\text{IC}}}{\lambda_0} \left(\frac{E_e(x_0)}{1 \text{ TeV}} \right)^{1/2}. \quad (8.7)$$

For $\lambda_0 = 120$ kpc (with $s = -0.5$), the fractional energy loss within one IC interaction length

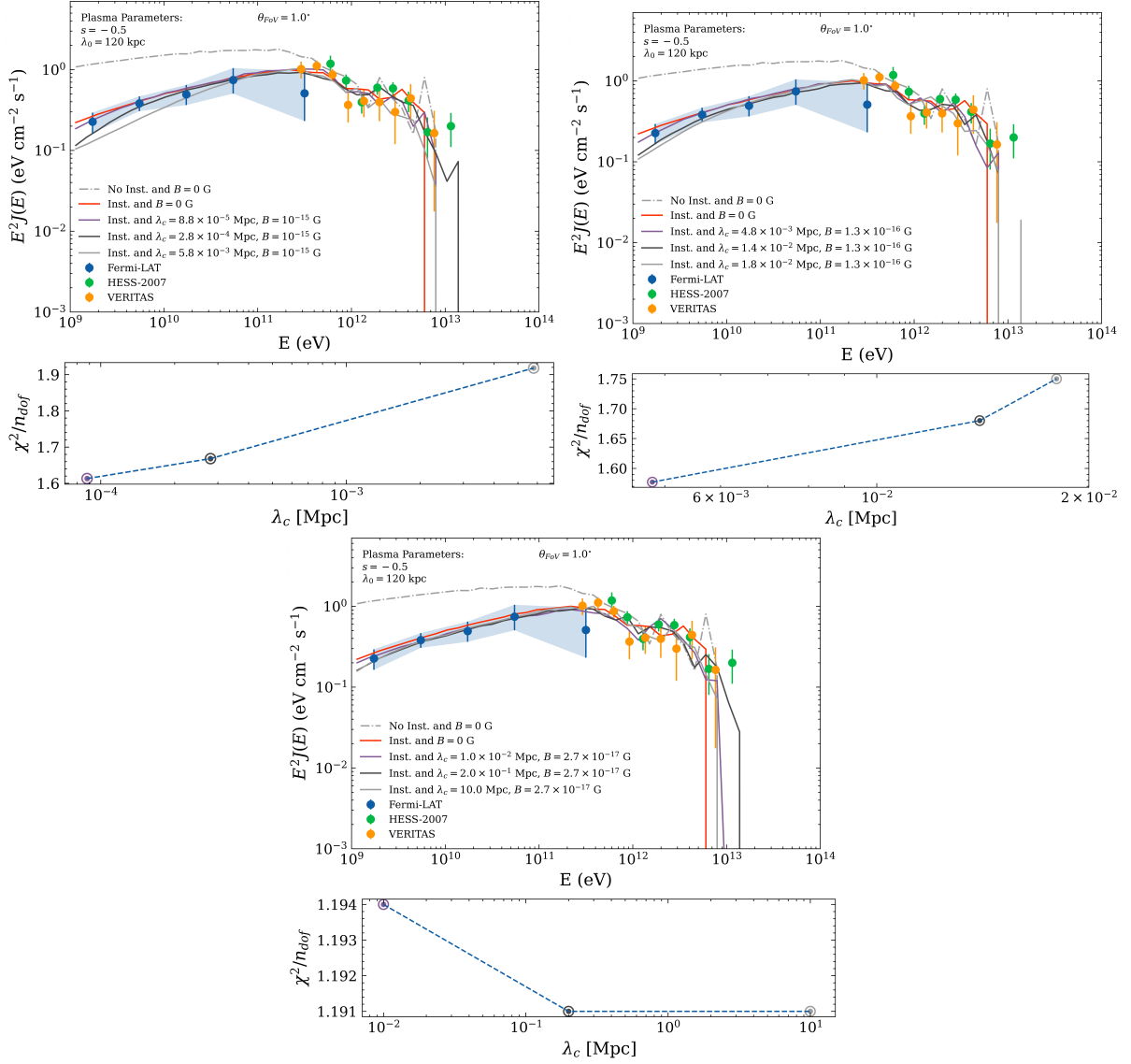


Figure 8.4: The energy spectrum of 1ES 0229+200 in $10^{-3} \leq E/\text{TeV} \leq 10^2$. The grey dash-dotted curve shows the propagated photon spectrum without any contribution of the plasma instability cooling (Inst. stands for Instability in the plot legends) and IGMF. The colored solid curves represent the propagated photon spectrum for different IGMF strengths and coherence length combinations, as well as plasma instability cooling for $\lambda_0 = 120$ kpc and $s = -0.5$. The plasma instability parameters considered in this study are taken from the best-fit scenario in [333]. The blue data points indicate the Fermi-LAT [20], the green data points show the H.E.S.S. [167], and the orange data points show the VERITAS [336] spectrum. We investigate the energy spectrum, which is consistent with the Fermi-LAT, H.E.S.S., and VERITAS observational data in each plot. The energy spectrum is obtained for a fixed $\theta_{\text{FoV}} = 1.0^\circ$. The χ^2/n_{dof} test statistic is obtained for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$).

remains below approximately 2% for electron energies below 4 TeV, and below about 1% in the GeV range. This indicates that, even in the best-fit case, the effect of plasma instability is relatively small.

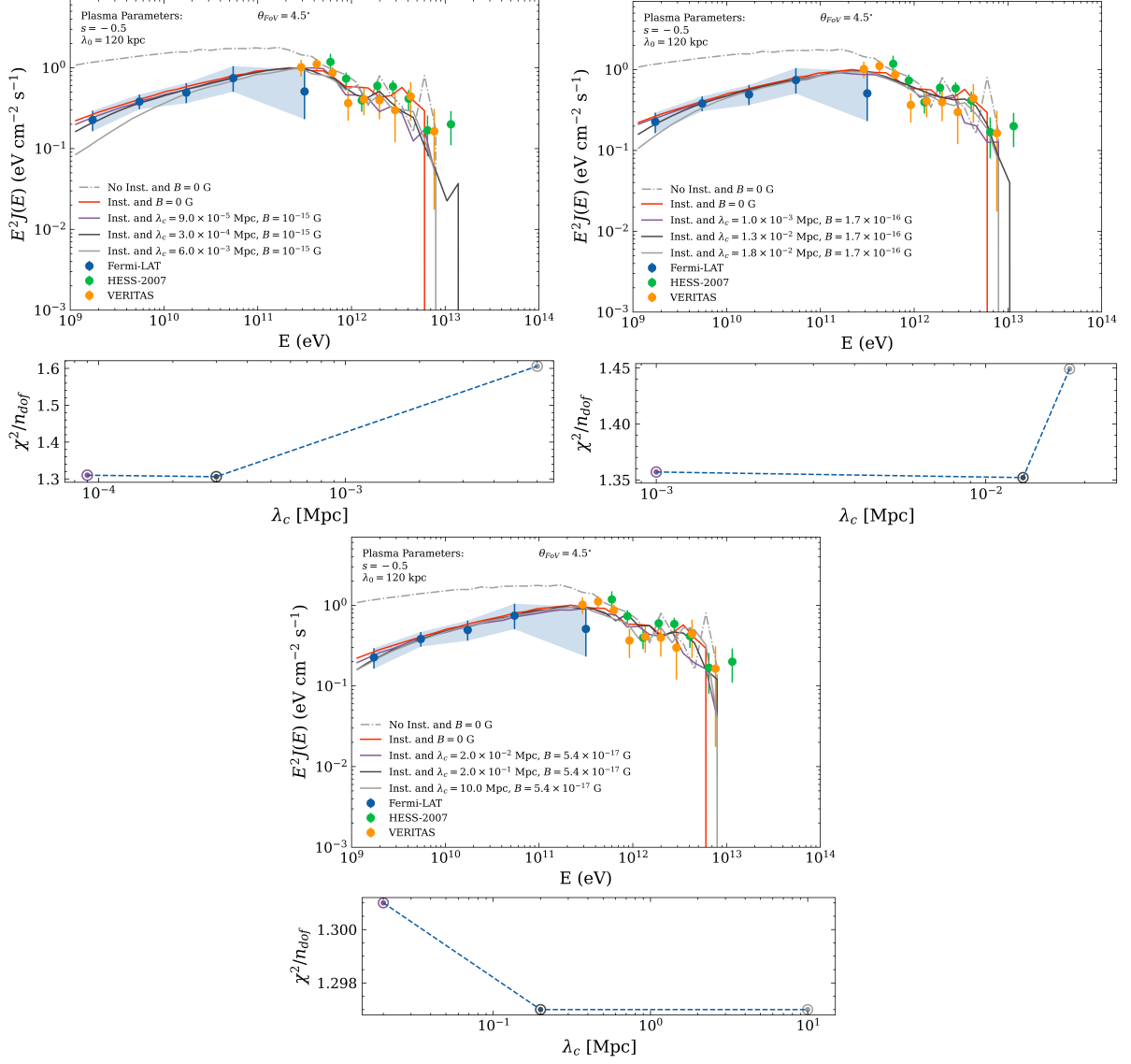


Figure 8.5: Same as Fig. 8.4, in this case, the spectral energy distribution is obtained for $\theta_{\text{FoV}} = 4.5^\circ$. The χ^2/n_{dof} test statistic is obtained for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$).

λ_0 [kpc]	θ_{FoV} [deg]	B [G]	λ_c [Mpc]	β	E_{cut}	$\log_{10} A$ [eV $^{-1}$ cm $^{-2}$ sec $^{-1}$]	$\chi^2_{\text{min}}/n_{\text{dof}}$
120	1.0	10^{-15}	9.0×10^{-5}	1.27	10.5	-23.705	1.613
			3.0×10^{-4}	1.20	9.5	-23.720	1.688
			6.0×10^{-3}	1.55	7.7	-23.710	1.918
		1.3×10^{-16}	4.8×10^{-3}	1.28	10.7	-23.711	1.577
			1.4×10^{-2}	1.31	9.9	-23.719	1.680
			1.8×10^{-2}	1.49	10.1	-23.712	1.750
		2.7×10^{-17}	1.0×10^{-2}	1.22	8.9	-23.713	1.194
			2.0×10^{-1}	1.31	9.5	-23.711	1.191
			10.0	1.33	10.1	-23.710	1.191
	4.5	10^{-15}	9.0×10^{-5}	1.51	9.2	-23.713	1.310
			3.0×10^{-4}	1.48	9.7	-23.710	1.306
			6.0×10^{-3}	1.46	8.7	-23.717	1.606
		1.7×10^{-16}	1.0×10^{-3}	1.20	9.0	-23.711	1.357
			1.3×10^{-2}	1.24	9.6	-23.702	1.352
			1.8×10^{-2}	1.31	10.3	-23.719	1.449
		5.4×10^{-17}	2.0×10^{-2}	1.21	8.9	-23.715	1.301
			2.0×10^{-1}	1.27	10.1	-23.713	1.297
			10.0	1.25	9.7	-23.711	1.297

Table 8.2: $\chi^2_{\text{min}}/n_{\text{dof}}$ values for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 2$), for $\theta_{\text{FoV}} = 1.0^\circ, 4.5^\circ$. The $\chi^2_{\text{min}}/n_{\text{dof}}$ value is highlighted in bold.

8.7 Constraints on the IGMF

Interactions of TeV photons with CMB photons produce electron–positron pairs with energies in the range $0.6 \lesssim E_e/\text{TeV} \lesssim 5.6$ [174]. These electrons lose energy through IC scattering over a characteristic cooling length scale of $0.1 \lesssim l_{\text{IC}}/\text{Mpc} \lesssim 0.6$ [174]. If the magnetic coherence length is longer than the length scale of electron cooling via IC ($l_{\text{IC}} < \lambda_c$), then the deflection angle of the e^+e^- pairs is given by,

$$\delta = \frac{l_{\text{IC}}}{r_L} = \frac{eBl_{\text{IC}}}{E_e}. \quad (8.8)$$

The deflection is therefore independent of the coherence length. Here, $r_L = E_e/eB$ is the Larmor radius in a magnetic field B . In contrast, in the regime $l_{\text{IC}} > \lambda_c$, the deflection follows a random walk, and particles experience random magnetic field orientations, causing a diffusive angular spread. In this case, the field geometry becomes turbulent (many small cells), causing random-walk deflection with a deflection angle, $\delta_{\text{cell}} \simeq \lambda_c/r_L$. After $n = l_{\text{IC}}/\lambda_c$ cells, the cumulative effect of these small deflections over l_{IC} distance leads to an overall deflection angle [6, 337],

$$\delta \simeq \delta_{\text{cell}} \sqrt{\frac{l_{\text{IC}}}{\lambda_c}} = eB \sqrt{\frac{l_{\text{IC}} \lambda_c}{E_e}} \propto B \sqrt{\lambda_c}. \quad (8.9)$$

Thus, the lower bound can be interpreted when the field is independent of λ_c when $\lambda_c > l_{\text{IC}}$, while for shorter coherence lengths ($\lambda_c < l_{\text{IC}}$) the field strength scales as $\lambda_c^{-1/2}$.

		$\theta_{\text{FoV}} = 1.0^\circ$	$\theta_{\text{FoV}} = 4.5^\circ$
No Inst.	$B \gtrsim$	$7.0 \times 10^{-17} \text{ G},$ $7.0 \times 10^{-17} (\lambda_{\text{c}}/0.20 \text{ Mpc})^{-1/2} \text{ G},$	$1.1 \times 10^{-16} \text{ G},$ $1.1 \times 10^{-16} (\lambda_{\text{c}}/0.20 \text{ Mpc})^{-1/2} \text{ G},$
$\lambda_0 = 120 \text{ kpc}$	$B \gtrsim$	$2.7 \times 10^{-17} \text{ G},$ $2.7 \times 10^{-17} (\lambda_{\text{c}}/0.20 \text{ Mpc})^{-1/2} \text{ G},$	$5.4 \times 10^{-17} \text{ G},$ $5.4 \times 10^{-17} (\lambda_{\text{c}}/0.20 \text{ Mpc})^{-1/2} \text{ G},$

Table 8.3: The summary of IGMF limits depending on plasma instability parameters.

Figure 8.6 shows that, in the absence of plasma instabilities, the lower limit on the IGMF is $B \simeq 7 \times 10^{-17} \text{ G}$ for an observational field of view of $\theta_{\text{FoV}} = 1.0^\circ$, and increases to $B \simeq 1.1 \times 10^{-16} \text{ G}$ for $\theta_{\text{FoV}} = 4.5^\circ$. When plasma instabilities are included, the lower bound becomes weaker. This is because beam–plasma instabilities transfer the energy of the pair beam into IGM as heat before the electrons can up-scatter CMB photons to GeV energies. As a result, the observable cascade flux is suppressed, leading to a weaker lower limit on the IGMF. The corresponding limits for different instability scenarios are summarized in Table 8.3. The red lines in Fig. 8.6 represent the lower limits on the IGMF derived from observations of the blazar 1ES 0229+200 by the MAGIC+Fermi-LAT collaboration [20], using a time-delay method with a maximum geometric delay of $\tau_{\text{delay}} = 10$ years and assuming no instability-induced energy losses. The black lines show the corresponding limits for GRB 221009A, obtained using LHAASO+Fermi-LAT data [6] and based on the same method and assumptions.

As shown in the first panel of Figure 8.6, the angular broadening analysis provides slightly stronger constraints than the time-delay analysis. This can be understood using the analytical expressions for the time delay and the angles relative to the line of sight. Analytically, for $\lambda_c < l_{\text{IC}}$ and at redshifts $z \ll 1$, the delayed cascade emission exhibits a characteristic energy

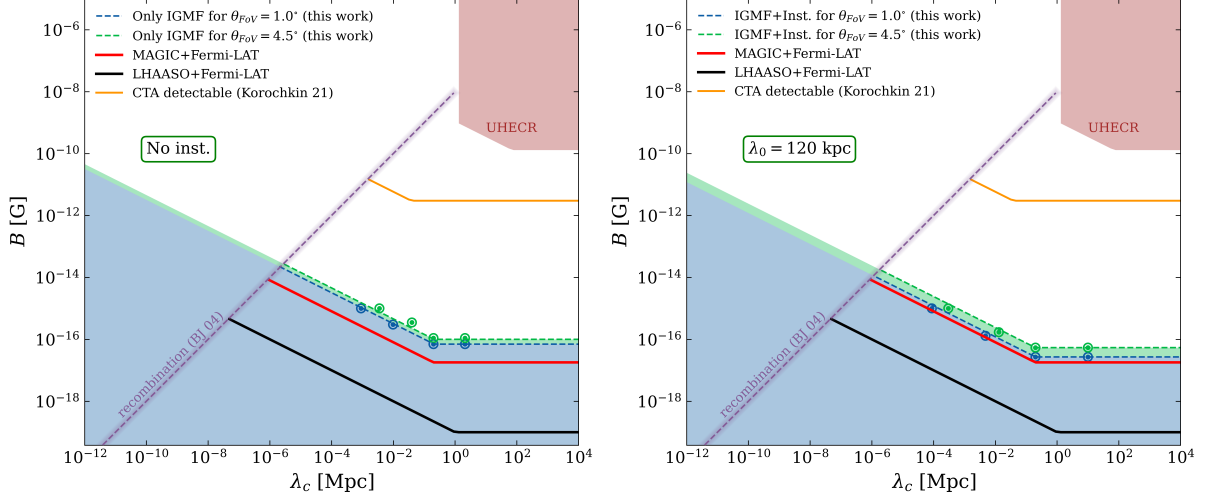


Figure 8.6: Lower limits on the IGMF derived from the blazar 1ES 0229+200, shown without (left panel) and with (right panel) plasma instabilities. The results are compared with existing constraints from MAGIC+Fermi-LAT [20] (red line), LHAASO+Fermi-LAT observations of GRB 221009A [6] (black line), Ultra-high-energy cosmic ray (UHECR) measurements [338] (light brown shaded region), predictions from primordial magnetic-field evolution models [339] (purple dashed line), and projected sensitivities of the Cherenkov Telescope Array (CTA) [337] (orange line). The green and blue dashed curves correspond to observational field-of-view angles $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° , respectively. The right panel includes plasma instability effects with an instability length scale $\lambda_0 = 120$ kpc (approximately two orders of magnitude larger than the IC scattering length) and spectral index $s = -0.5$.

dependence of $\tau_{\text{delay}} \propto E^{-5/2}$, while the angular extent of the emission scales as $\theta_{\text{obs}} \propto E^{-1}$ [3, 16, 174]. As an example, for a source at $z \sim 0.14$, a cascade photon with energy $E \sim 10$ GeV and an observed angular extent $\theta_{\text{obs}} \sim 0.3^\circ$ implies a lower limit of $B \sim 10^{-17}$ G when using the angular extension method. In contrast, using a typical time delay of $\tau_{\text{delay}} \sim 10$ years for the same photon energy leads to a weaker constraint of $B \sim 10^{-18}$ G. This difference arises from the different energy scalings of the two methods. The blue and green curves in Fig. 8.6 show the lower limits on the IGMF with and without plasma instabilities, respectively, for observational field-of-view angles $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . For the plasma instability parameters $\lambda_0 = 120$ kpc, $s = -0.5$, and $\tilde{E} = 1.0$ TeV, we find that the minimum magnetic field strength required to remain consistent with the Fermi-LAT data is $B \gtrsim 2.7 \times 10^{-17}$ G for $\theta_{\text{FoV}} = 1.0^\circ$.

8.8 IGMF constraints from other blazar sources

We further extend our analysis to three additional blazar sources for which the available data are of sufficiently high quality to detect secondary emission and derive constraints on the IGMF. The blazar 1ES 1101-232 ($z \sim 0.186$) was used in an earlier study by [13] to constrain the IGMF in cosmic voids. Similarly, H 1426+428 ($z \sim 0.129$) has been analyzed by [8] to obtain a lower bound on the magnetic field strength, while H 2356-309 ($z \sim 0.165$) has been studied in the context of IGMF constraints by [106]. We explore the parameter space over $B \in [1.0, 10.0] \times 10^{-17}$ G with a step size $\Delta B = 0.01 \times 10^{-17}$ G, and $\lambda_c \in [10^{-2}, 10^1]$ Mpc with a step size $\Delta \log_{10} \lambda_c = 3 \times 10^{-3}$. The analysis is performed for two observational field-of-view angles, $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . Fig. 8.7

presents the resulting energy spectra for these sources. The top, middle, and bottom rows correspond to 1ES 1101–232, H 1426+428, and H 2356–309, respectively. The left and right columns show the results for $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° .

λ_0 [kpc]	Source	θ_{FoV} [deg]	$B/10^{-17}$ [G]	λ_c [Mpc]	β	E_{cut}	$\log_{10} A$ [$\text{eV}^{-1} \text{cm}^{-2} \text{sec}^{-1}$]	$\chi^2_{\text{min}}/n_{\text{dof}}$
120	1ES 1101–232	1.0	5.32	2.3×10^{-2}	1.51	2.2	–23.390	1.175
				2.0×10^{-1}	1.45	2.8	–23.371	1.129
				2.0	1.40	3.2	–23.340	1.129
		4.5	8.05	2.1×10^{-2}	1.51	3.6	–23.453	0.967
				2.0×10^{-1}	1.41	3.5	–23.391	0.844
				2.0	1.43	4.1	–23.434	0.844
	H 1426+428	1.0	3.92	2.0×10^{-2}	1.76	4.5	–23.585	2.349
				2.0×10^{-1}	1.74	3.9	–23.580	1.881
				2.0	1.73	3.2	–23.553	1.881
		4.5	5.60	2.0×10^{-2}	1.76	3.8	–23.586	2.333
				2.0×10^{-1}	1.73	4.6	–23.583	1.840
				2.0	1.71	4.3	–23.541	1.840
	H 2356–309	1.0	4.63	1.9×10^{-2}	1.64	1.6	–23.483	2.058
				2.0×10^{-1}	1.60	1.4	–23.442	1.551
				2.0	1.58	1.8	–23.438	1.551
		4.5	5.80	2.0×10^{-2}	1.64	1.9	–23.459	2.677
				2.0×10^{-1}	1.60	1.3	–23.432	1.615
				2.0	1.61	1.2	–23.435	1.615

Table 8.4: $\chi^2_{\text{min}}/n_{\text{dof}}$ values for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 7$), for $\theta_{\text{FoV}} = 1.0^\circ, 4.5^\circ$. The $\chi^2_{\text{min}}/n_{\text{dof}}$ value is highlighted in bold.

Sources	z	λ_c [Mpc]	$B/10^{-17}$ [G]	
			$\theta_{\text{FoV}} = 1.0^\circ$	$\theta_{\text{FoV}} = 4.5^\circ$
1ES 1101–232	0.186	0.2	5.32	8.05
H 1426+428	0.129		3.92	5.60
H 2356–309	0.165		4.63	5.80

Table 8.5: Summary of constraints on the IGMF for other blazar sources.

We model the total spectra of 1ES 1101–232 and H 2356–309 using the Fermi-LAT and H.E.S.S. data sets from [106], while for H 1426+428 we use the Fermi-LAT and VERITAS data reported in [340, 341]. The same χ^2 analysis as employed for 1ES 0229+200 (as described in Sec. 8.5.3 and Sec. 8.6) is applied here to determine the best-fit parameters, which are listed in Table 8.4. Fig. 8.8 presents the χ^2/n_{dof} profiles derived from the Fermi-LAT data for all three sources, for different observational fields of view. The corresponding constraints on the IGMF are summarized in Table 8.5. We find that the constraints obtained from these additional sources are consistent with those derived from the primary analysis of 1ES 0229+200. In particular, the lower limits on the IGMF are found within the range $[3.9\text{--}8.1] \times 10^{-17}$ G, showing a similar dependence on the field of view and plasma instability cooling. This consistency suggests that the inferred IGMF constraints remain robust even when plasma instability effects are taken into account.

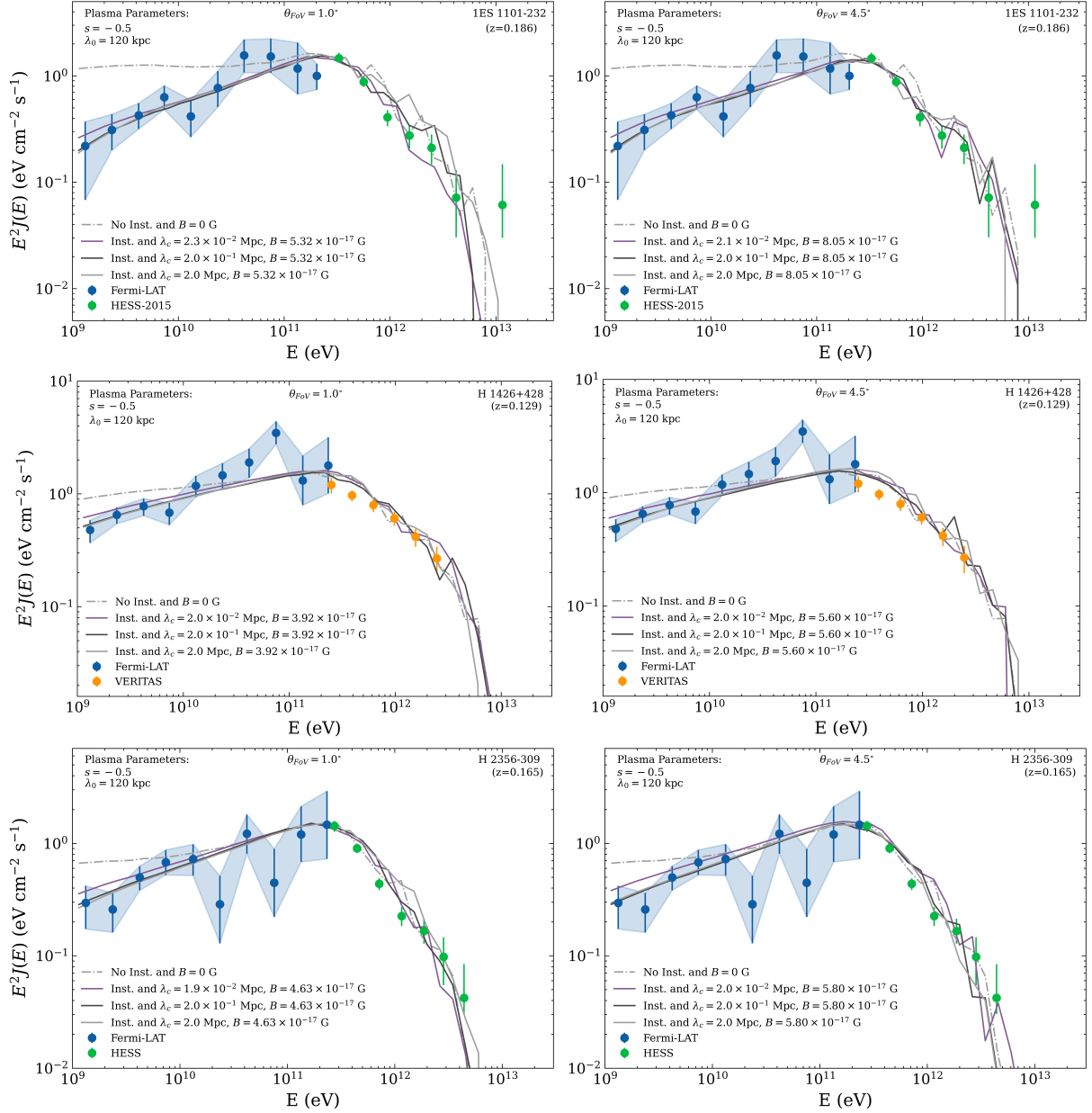


Figure 8.7: Simulated gamma-ray spectra of 1ES 1101-232 ($z \sim 0.186$, top row), H 1426+428 ($z \sim 0.129$, middle row), and H 2356-309 ($z \sim 0.165$, bottom row) in the energy range $10^{-3} \leq E/\text{TeV} \leq 10^2$. The grey dash-dotted curve shows the propagated photon spectrum without including plasma-instability cooling or the IGMF. The colored solid curves represent propagated spectra that include plasma instability cooling with $\lambda_0 = 120$ kpc and $s = -0.5$, for different combinations of magnetic field strength and coherence length. Blue data points correspond to Fermi-LAT observations in all panels [106, 340]. Green data points show H.E.S.S. observations for 1ES 1101-232 and H 2356-309 [106], while orange data points correspond to VERITAS observations for H 1426+428 [341]. The left and right columns correspond to observational fields of view $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° , respectively.

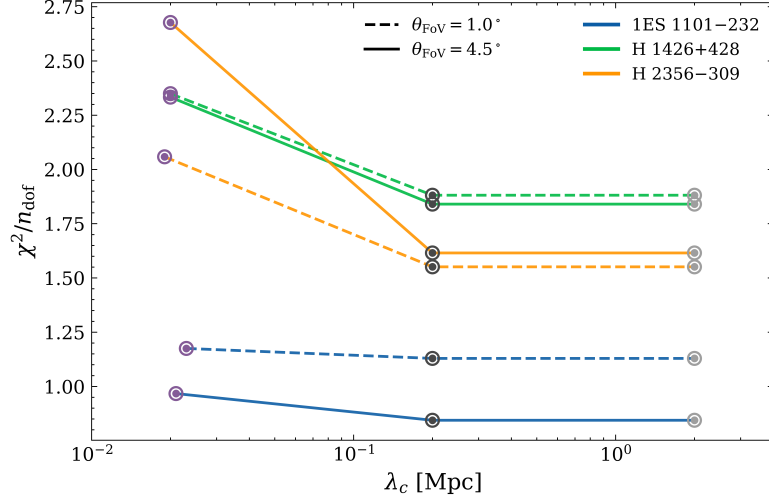


Figure 8.8: The χ^2/n_{dof} test statistic is shown for the GeV-band Fermi-LAT data ($n_{\text{dof}} = 7$). The dashed and solid curves correspond to fields of view $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° , respectively. Different colors indicate the three blazar sources.

8.9 Discussion and conclusions

In this work, we perform a parametric study of energy losses in pair beams driven by plasma instabilities in the presence of the **IGMF**. We simulate the extragalactic propagation of high-energy photons from the blazar 1ES 0229+200 using the Monte Carlo framework **CRPropa 3.2**. The propagated photon spectra are obtained for two cases: (i) cascades produced only by the **IGMF**, and (ii) cascades produced by both plasma instability cooling and the **IGMF**. We use the angular extension of the emission as the main observable. The cascade signal is constructed as a function of plasma parameters, magnetic field strength, coherence length, and the observer's field-of-view angle. For each configuration, we minimize χ^2/n_{dof} to determine the magnetic field that best reproduces the observed spectrum at a 2σ confidence level. The test statistic is evaluated using only the GeV-band Fermi-LAT data, where the cascade contribution is dominant. The main findings of this work can be summarized as follows:

1. For the instability parameters $\lambda_0 = 120$ kpc, $s = -0.5$, and $\tilde{E} = 1.0$ TeV, consistency with the Fermi-LAT observations requires a minimum magnetic field strength of $B \gtrsim 2.7 \times 10^{-17}$ G for an observational field of view $\theta_{\text{FoV}} = 1.0^\circ$. Increasing the instability length scale to $\lambda_0 = 360$ kpc leads to stronger lower limits, with $B \gtrsim 6.6 \times 10^{-17}$ G for $\theta_{\text{FoV}} = 1.0^\circ$ and $B \gtrsim 3.8 \times 10^{-17}$ G for $\theta_{\text{FoV}} = 4.5^\circ$, as presented in Appendix F. We find that $\lambda_0 \geq 120$ kpc (for $s = -0.5$) represents the minimum instability-loss scale required to reproduce the observed photon spectrum. For smaller values of λ_0 , the cascade is strongly suppressed, and the predicted spectra fall below the Fermi-LAT data in the GeV range, even in the absence of magnetic deflections. This indicates that very short instability cooling lengths efficiently quench the cascade and prevent the production of GeV secondary photons.
2. In the best-fit scenario, the contribution of plasma instability to the total energy loss

remains small. The fractional energy loss within one IC interaction length is $\lesssim 2\%$ for electrons with energies below 4 TeV and $\lesssim 1\%$ in the GeV range. Therefore, while instability effects modify the cascade, they do not dominate the energy loss in the preferred parameter regime.

3. We also compare our results with constraints derived in previous studies. The lower limit derived by [20], based on time-delay analysis of MAGIC and Fermi-LAT data, is weaker than the limit obtained here without plasma instabilities. This difference arises from the distinct energy dependences of the two observables. For $\lambda_c < l_{IC}$, the time delay scales as $\tau_{\text{delay}} \propto E^{-5/2}$, whereas the angular extent scales as $\theta_{\text{obs}} \propto E^{-1}$. As a result, lower-energy photons are more difficult to detect in time-delay studies due to large delays, but are more accessible in angular-broadening analyses because of their larger spatial extent. The constraints obtained from additional blazar sources (1ES 1101–232, H 1426+428, and H 2356–309) are consistent with those derived from 1ES 0229+200. In all cases, the lower limits on the IGMF lie at the level of a few $\times 10^{-17}$ G and show a similar dependence on the field of view and plasma instability parameters. This agreement across multiple sources supports the robustness of the derived IGMF constraints.

Fig. 8.6 compares our results with existing bounds on the IGMF. Cosmological magnetic fields may have originated during inflation or during the electroweak and QCD phase transitions [228, 342]. Their initial coherence lengths are limited by the horizon size at the time of generation, and can grow over time due to turbulent processes [339]. Observations of GRBs provide lower bounds on the IGMF, while UHECR measurements set upper limits. For example, UHECR observations constrain the field strength to $B \sim 10^{-10}$ G for coherence lengths of order 70 Mpc [338]. More recent analyses of GRB 221009A using LHAASO and Fermi-LAT data suggest lower limits of $B \sim 10^{-19}$ G [6], with improved constraints excluding $B < 2.5 \times 10^{-17}$ G at 95% confidence level for $\lambda_c \gtrsim 1$ Mpc [22]. These limits are also based on time-delay methods and differ due to the energy range of the observed cascade emission.

Finally, we note some limitations of the current approach. In this work, we assume spatially uniform energy losses, meaning that all electrons lose energy in the same way regardless of their distance from the source. A more realistic treatment would include distance-dependent instability cooling, which could provide a better description of the cascade evolution. In addition, recent studies [30] suggest that MeV cosmic-ray electrons may suppress the TeV pair-beam instability through linear Landau damping. Incorporating these effects will be an important step toward building a more complete and predictive framework for cascade evolution.

Conclusions and Outlook

In the final chapter, we present the conclusive remarks of this thesis and discuss potential directions for future investigations.

9.1 Conclusions

We began this thesis by investigating the role of beam–plasma instabilities in the evolution of relativistic electron-positron pair beams, with the aim of attempting to replicate the astrophysical phenomena associated with TeV blazars in laboratory-scale experiments. Using kinetic simulations with the **EPOCH** Particle-in-Cell code, we have investigated the unstable behavior of a broad, neutral, finite-width pair beam propagating through a homogeneous, unmagnetized background plasma. In this context, we studied the growth of beam–plasma instabilities under laboratory-relevant conditions, including their implications for the energy-loss fraction at astrophysical scales for 1ES 0229+200-like blazar sources. Furthermore, employing the Monte Carlo framework **CRPropa** 3.2, we analyzed the influence of plasma instabilities in cascade production using a parametric energy-loss model in the presence of the intergalactic magnetic field. This allowed us to evaluate the impact of instability-induced cooling on the development of electromagnetic cascades and to derive corresponding constraints on the intergalactic magnetic field from observations of 1ES 0229+200 and other TeV blazar sources.

A key outcome of this work is the determination of a parameter regime in which laboratory-generated pair beams can consistently reproduce astrophysical conditions. In particular, the concept of a dilute, broad beam has been quantified in terms of density contrast and beam anisotropy. Within this regime, electrostatic modes, especially oblique instabilities, dominate the beam evolution, while electromagnetic instabilities remain subdominant. This result lays the groundwork for identifying optimal parameters for laboratory experiments on beam density contrast and for designing experiments aimed at reproducing beam–plasma interactions relevant to the astrophysical regime. The kinetic simulations further show that, even in the presence of instabilities, the overall impact of beam–plasma instabilities on the beam energy loss and angular broadening remains moderate. While a fraction of the beam energy is transferred to plasma waves and electromagnetic fields, the resulting energy loss and angular broadening are

limited. When extrapolated to realistic astrophysical beams, these effects become even weaker, indicating that plasma instabilities alone are unlikely to significantly affect the propagation of pair beams over cosmological distances, with the beam losing only $\lesssim 4\%$ of its energy due to the instability.

The second part of this thesis focuses on the impact of instability on the development of electromagnetic cascades in the presence of the intergalactic magnetic field. For the best-fit instability case, defined by instability cooling length, $\lambda_0 = 120$ kpc, and instability power-law index, $s = -0.5$, consistency with the Fermi-LAT observations requires a minimum magnetic field strength of $B \gtrsim 2.7 \times 10^{-17}$ G for a field of view of $\theta_{\text{FoV}} = 1.0^\circ$. A larger field of view leads to slightly stronger constraints, with $B \gtrsim 5.4 \times 10^{-17}$ G for $\theta_{\text{FoV}} = 4.5^\circ$. In the preferred parameter regime, the contribution of plasma instabilities to the total energy loss remains small, with a fractional energy loss of $\lesssim 2\%$ for electron energies below 4 TeV and $\lesssim 1\%$ in the GeV range. This indicates that, while instability effects modify the cascade development, they do not dominate the energy-loss process. Importantly, the inclusion of plasma instability effects does not substantially modify these constraints on the magnetic field within the preferred parameter regime.

Overall, the results of this thesis suggest that plasma instabilities have a marginal impact on the production of the electromagnetic cascade. The suppression of the secondary gamma-ray flux is therefore better explained by magnetic deflections in the intergalactic magnetic field rather than by instability-induced cooling.

9.2 Outlook

Although this work provides a coherent picture of beam-plasma interactions across laboratory and astrophysical scales, several open questions remain, motivating further investigation.

A natural extension of the present study is to include background magnetic fields in kinetic simulations. Weak magnetization in the background plasma might modify the growth of instabilities, potentially influencing the transition between dilute and non-dilute regimes. Exploring this transition would improve the connection between controlled laboratory experiments and realistic astrophysical environments.

Another important direction is the development of a more realistic energy-loss parameterization model. In this thesis, plasma instability cooling is treated as spatially uniform, whereas in reality, the efficiency of energy loss may depend on distance from the source and on local plasma conditions. Incorporating spatially and temporally varying cooling rates could provide a more accurate description of cascade evolution and its observational signatures.

From an experimental perspective, advances in high-intensity laser facilities and beam generation techniques offer promising opportunities to test these ideas in the laboratory. The production of relativistic pair beams, together with improved beam control and diagnostics, could enable direct measurements of instability growth and energy transfer under controlled conditions. Such experiments would provide valuable benchmarks for simulation results and help validate theoretical predictions.

Finally, future gamma-ray observatories (e.g., Cherenkov Telescope Array Observatory) with improved sensitivity and angular resolution will play a crucial role in constraining the properties of electromagnetic cascades and the intergalactic magnetic field. A combined effort integrating observations, simulations, and laboratory experiments will be essential to reduce current uncer-

tainties and achieve a more comprehensive understanding of plasma processes in extragalactic propagation.

References

- [1] Kandaswamy Subramanian. “The origin, evolution and signatures of primordial magnetic fields”. In: *Rept. Prog. Phys.* 79.7 (2016), p. 076901. DOI: [10.1088/0034-4885/79/7/076901](#). arXiv: [1504.02311 \[astro-ph.CO\]](#) (cit. on pp. [1](#), [21](#)).
- [2] Ruth Durrer and Andrii Neronov. “Cosmological magnetic fields: their generation, evolution and observation”. In: *Astron. Astrophys. Rev.* 21 (2013), pp. 1–109. DOI: [10.1007/s00159-013-0062-7](#) (cit. on pp. [1](#), [21](#)).
- [3] Rafael Alves Batista and Andrey Saveliev. “The Gamma-ray Window to Intergalactic Magnetism”. In: *Universe* 7.7 (2021), p. 223. DOI: [10.3390/universe7070223](#) (cit. on pp. [1](#), [83](#), [94](#)).
- [4] Andrii Neronov and Ievgen Vovk. “Evidence for strong extragalactic magnetic fields from Fermi observations of TeV blazars”. In: *Science* 328.5974 (2010), pp. 73–75. DOI: [10.1126/science.1184192](#) (cit. on pp. [1](#), [21](#)).
- [5] A. Neronov and Dmitry V. Semikoz. “A method of measurement of extragalactic magnetic fields by TeV gamma ray telescopes”. In: *JETP Lett.* 85 (2007), pp. 473–477. DOI: [10.1134/S0021364007100013](#). arXiv: [astro-ph/0604607](#) (cit. on pp. [1](#), [78](#)).
- [6] Ie. Vovk et al. “Constraints on the intergalactic magnetic field from Fermi/LAT observations of the ‘pair echo’ of GRB 221009A”. In: *Astron. Astrophys.* 683 (2024), A25. DOI: [10.1051/0004-6361/202347310](#). arXiv: [2306.07672 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [2](#), [18](#), [23](#), [78](#), [79](#), [83](#), [93](#), [94](#), [98](#)).
- [7] Victor A Acciari et al. “A lower bound on intergalactic magnetic fields from time variability of 1ES 0229+ 200 from MAGIC and Fermi/LAT observations”. In: *Astron. Astrophys.* 670 (2023), A145. DOI: [10.1051/0004-6361/202244126](#) (cit. on pp. [1](#), [2](#), [23](#)).
- [8] J. Blunier, A. Neronov, and D. Semikoz. *Revision of conservative lower bound on intergalactic magnetic field from Fermi and Cherenkov telescope observations of extreme blazars*. June 2025. arXiv: [2506.22285 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [78](#), [94](#)).
- [9] V. Berezhinsky and O. Kalashev. “High energy electromagnetic cascades in extragalactic space: physics and features”. In: *Phys. Rev. D* 94.2 (2016), p. 023007. DOI: [10.1103/PhysRevD.94.023007](#). arXiv: [1603.03989 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [77](#)).

- [10] A. Elyiv, A. Neronov, and D. V. Semikoz. “Gamma-ray induced cascades and magnetic fields in intergalactic medium”. In: *Phys. Rev. D* 80 (2009), p. 023010. DOI: [10.1103/PhysRevD.80.023010](#). arXiv: [0903.3649 \[astro-ph.CO\]](#) (cit. on pp. [1](#), [18](#)).
- [11] E. Aliu et al. “Very-High-Energy Gamma Rays from a Distant Quasar: How Transparent Is the Universe?” In: *Science* 320.5884 (2008), p. 1752. DOI: [10.1126/science.1157087](#). arXiv: [0807.2822 \[astro-ph\]](#) (cit. on p. [1](#)).
- [12] William Atwood, Aous Abdo, Markus Ackermann, et al. “The large area telescope on the Fermi gamma-ray space telescope mission”. In: *The Astrophysical Journal* 697.2 (2009), p. 1071. DOI: [10.1088/0004-637X/697/2/1071](#) (cit. on pp. [1](#), [78](#), [83](#)).
- [13] A. Neronov and I. Vovk. “Evidence for strong extragalactic magnetic fields from Fermi observations of TeV blazars”. In: *Science* 328 (2010), pp. 73–75. DOI: [10.1126/science.1184192](#). arXiv: [1006.3504 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [18](#), [78](#), [81](#), [94](#)).
- [14] F. Tavecchio et al. “Extreme TeV blazars and the intergalactic magnetic field”. In: *Mon. Not. Roy. Astron. Soc.* 414 (2011), p. 3566. DOI: [10.1111/j.1365-2966.2011.18657.x](#). arXiv: [1009.1048 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [22](#), [78](#)).
- [15] Charles D. Dermer et al. “Time Delay of Cascade Radiation for TeV Blazars and the Measurement of the Intergalactic Magnetic Field”. In: *Astrophys. J. Lett.* 733 (2011), p. L21. DOI: [10.1088/2041-8205/733/2/L21](#). arXiv: [1011.6660 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [78](#)).
- [16] A. M. Taylor, I. Vovk, and A. Neronov. “Extragalactic magnetic fields constraints from simultaneous GeV-TeV observations of blazars”. In: *Astron. Astrophys.* 529 (2011), A144. DOI: [10.1051/0004-6361/201116441](#). arXiv: [1101.0932 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [18](#), [78](#), [94](#)).
- [17] Ievgen Vovk et al. “Fermi/LAT observations of 1ES 0229+200: implications for extragalactic magnetic fields and background light”. In: *Astrophys. J. Lett.* 747 (2012), p. L14. DOI: [10.1088/2041-8205/747/1/L14](#). arXiv: [1112.2534 \[astro-ph.CO\]](#) (cit. on pp. [1](#), [23](#), [78](#)).
- [18] M. Ackermann et al. “The Search for Spatial Extension in High-latitude Sources Detected by the *Fermi* Large Area Telescope”. In: *Astrophys. J. Suppl.* 237.2 (2018), p. 32. DOI: [10.3847/1538-4365/aacdf7](#). arXiv: [1804.08035 \[astro-ph.HE\]](#) (cit. on pp. [1](#), [78](#)).
- [19] J. Aleksic et al. “Search for an extended VHE gamma-ray emission from Mrk 421 and Mrk 501 with the MAGIC Telescope”. In: *Astron. Astrophys.* 524 (2010), A77. DOI: [10.1051/0004-6361/201014747](#). arXiv: [1004.1093 \[astro-ph.HE\]](#) (cit. on pp. [2](#), [78](#)).
- [20] V. A. Acciari et al. “A lower bound on intergalactic magnetic fields from time variability of 1ES 0229+200 from MAGIC and Fermi/LAT observations”. In: *Astron. Astrophys.* 670 (2023), A145. DOI: [10.1051/0004-6361/202244126](#). arXiv: [2210.03321 \[astro-ph.HE\]](#) (cit. on pp. [2](#), [9](#), [18](#), [78](#), [79](#), [82](#), [83](#), [86](#), [90](#), [93](#), [94](#), [98](#)).
- [21] A. Abramowski et al. “Search for Extended \gamma-ray Emission around AGN with H.E.S.S. and Fermi-LAT”. In: *Astron. Astrophys.* 562 (2014), A145. DOI: [10.1051/0004-6361/201322510](#). arXiv: [1401.2915 \[astro-ph.HE\]](#) (cit. on pp. [2](#), [78](#)).

-
- [22] Lea Burmeister et al. “Constraints on the intergalactic magnetic field from Fermi-LAT observations of GRB 221009A”. In: *Phys. Rev. D* 113.4 (2026), p. 043041. DOI: [10.1103/g8rr-g7p4](https://doi.org/10.1103/g8rr-g7p4). arXiv: [2512.11128](https://arxiv.org/abs/2512.11128) [[astro-ph.HE](#)] (cit. on pp. [2](#), [78](#), [98](#)).
 - [23] Ténémán Keita, Renaud Belmont, and Thierry Stolarczyk. “Impact of Observational and Modelling Assumptions on Intergalactic Magnetic Field Constraints from TeV Gamma-Ray Bursts with the Cherenkov Telescope Array Observatory”. In: *60th Rencontres de Moriond on Very High Energy Phenomena in the Universe: Moriond VHEPU 2026*. Apr. 2026. arXiv: [2604.10097](https://arxiv.org/abs/2604.10097) [[astro-ph.HE](#)] (cit. on pp. [2](#), [23](#)).
 - [24] Iman Rafighi et al. “Plasma effects on relativistic pair beams from TeV blazars: PIC simulations and analytical predictions”. In: *Astron. Astrophys.* 607 (2017), A112. DOI: [10.1051/0004-6361/201731127](https://doi.org/10.1051/0004-6361/201731127) (cit. on pp. [2](#), [19](#), [54](#)).
 - [25] Iman Rafighi. “Plasma Instabilities from blazar-induced pair beams propagating through IGM”. doctoralthesis. Universität Potsdam, 2018, pp. ix, 114 (cit. on pp. [2](#), [40](#), [42](#)).
 - [26] Mahmoud Alawashra and Martin Pohl. “Suppression of the TeV Pair-beam–Plasma Instability by a Tangled Weak Intergalactic Magnetic Field”. In: *Astrophys. J.* 929.1 (2022), p. 67. DOI: [10.3847/1538-4357/ac5a4b](https://doi.org/10.3847/1538-4357/ac5a4b). arXiv: [2203.01022](https://arxiv.org/abs/2203.01022) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [80](#)).
 - [27] Mahmoud Alawashra, Ievgen Vovk, and Martin Pohl. “Marginal Role of the Electrostatic Instability in the GeV-scale Cascade Flux from 1ES 0229+200”. In: *Astrophys. J.* 978.1 (2025), p. 95. DOI: [10.3847/1538-4357/ad98f9](https://doi.org/10.3847/1538-4357/ad98f9). arXiv: [2412.01406](https://arxiv.org/abs/2412.01406) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [70](#)).
 - [28] Mahmoud Alawashra and Martin Pohl. “Nonlinear Feedback of the Electrostatic Instability on the Blazar-induced Pair Beam and GeV Cascade”. In: *Astrophys. J.* 964.1 (2024), p. 82. DOI: [10.3847/1538-4357/ad24ea](https://doi.org/10.3847/1538-4357/ad24ea). arXiv: [2402.03127](https://arxiv.org/abs/2402.03127) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [78](#), [81](#), [83](#)).
 - [29] Mahmoud S. A. Alawashra. “Plasma Instabilities of TeV Pair Beams Induced by Blazars”. PhD thesis. Astroparticle Physics Theory Group, Deutsches Elektronen-Synchrotron DESY, Zeuthen, Germany, GRID:grid.7683.a, 2025. DOI: [10.1007/978-3-031-98568-3](https://doi.org/10.1007/978-3-031-98568-3) (cit. on pp. [2](#), [8](#)).
 - [30] Mahmoud Alawashra et al. “MeV Cosmic-Ray Electrons Modify the TeV Pair-beam Plasma Instability”. In: *Astrophys. J.* 989.1 (2025), p. 37. DOI: [10.3847/1538-4357/ade9e](https://doi.org/10.3847/1538-4357/ade9e). arXiv: [2507.02423](https://arxiv.org/abs/2507.02423) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [98](#)).
 - [31] S. Vafin et al. “The electrostatic instability for realistic pair distributions in blazar/EBL cascades”. In: *Astrophys. J.* 857.1 (2018), p. 43. DOI: [10.3847/1538-4357/aab552](https://doi.org/10.3847/1538-4357/aab552). arXiv: [1803.02990](https://arxiv.org/abs/1803.02990) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [34](#), [78–80](#), [147](#)).
 - [32] Dahai Yan et al. “Impact of Plasma Instability on Constraint of the Intergalactic Magnetic Field”. In: *Astrophys. J.* 870.1 (2019), p. 17. DOI: [10.3847/1538-4357/aaef7d](https://doi.org/10.3847/1538-4357/aaef7d). arXiv: [1810.07013](https://arxiv.org/abs/1810.07013) [[astro-ph.HE](#)] (cit. on p. [2](#)).

- [33] Avery E. Broderick, Philip Chang, and Christoph Pfrommer. “The Cosmological Impact of Luminous TeV Blazars I: Implications of Plasma Instabilities for the Intergalactic Magnetic Field and Extragalactic Gamma-Ray Background”. In: *Astrophys. J.* 752 (2012), p. 22. DOI: [10.1088/0004-637X/752/1/22](https://doi.org/10.1088/0004-637X/752/1/22). arXiv: [1106.5494](https://arxiv.org/abs/1106.5494) [[astro-ph.CO](#)] (cit. on pp. [2](#), [19](#), [34](#), [69](#), [78–80](#), [147](#)).
- [34] Philip Chang, Avery E. Broderick, and Christoph Pfrommer. “The Cosmological Impact of Luminous TeV Blazars II: Rewriting the Thermal History of the Intergalactic Medium”. In: *Astrophys. J.* 752 (2012), p. 23. DOI: [10.1088/0004-637X/752/1/23](https://doi.org/10.1088/0004-637X/752/1/23). arXiv: [1106.5504](https://arxiv.org/abs/1106.5504) [[astro-ph.CO](#)] (cit. on pp. [2](#), [11](#), [12](#)).
- [35] Rafael Alves Batista, Andrey Saveliev, and Elisabete M. de Gouveia Dal Pino. “The Impact of Plasma Instabilities on the Spectra of TeV Blazars”. In: *Mon. Not. Roy. Astron. Soc.* 489.3 (2019), pp. 3836–3849. DOI: [10.1093/mnras/stz2389](https://doi.org/10.1093/mnras/stz2389). arXiv: [1904.13345](https://arxiv.org/abs/1904.13345) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [69](#), [78–80](#)).
- [36] Avery E. Broderick et al. “Implications of Plasma Beam Instabilities for the Statistics of the Fermi Hard Gamma-ray Blazars and the Origin of the Extragalactic Gamma-Ray Background”. In: *Astrophys. J.* 790 (2014), p. 137. DOI: [10.1088/0004-637X/790/2/137](https://doi.org/10.1088/0004-637X/790/2/137). arXiv: [1308.0340](https://arxiv.org/abs/1308.0340) [[astro-ph.CO](#)] (cit. on pp. [2](#), [19](#)).
- [37] A Bret, M-C Firpo, and C Deutsch. “Electromagnetic instabilities for relativistic beam-plasma interaction in whole k space: Nonrelativistic beam and plasma temperature effects”. In: *Phys. Rev. E* 72.1 (2005), p. 016403. DOI: [10.1103/PhysRevE.72.016403](https://doi.org/10.1103/PhysRevE.72.016403) (cit. on p. [2](#)).
- [38] Antoine Bret, Laurent Gremillet, and Didier Benisti. “Exact relativistic kinetic theory of the full unstable spectrum of an electron-beam–plasma system with Maxwell-Jüttner distribution functions”. In: *Phys. Rev. E* 81.3 (2010), p. 036402. DOI: [10.1103/PhysRevE.81.036402](https://doi.org/10.1103/PhysRevE.81.036402) (cit. on pp. [2](#), [32](#)).
- [39] Antoine Bret, Laurent Gremillet, and Mark Eric Dieckmann. “Multidimensional electron beam-plasma instabilities in the relativistic regime”. In: *Phys. Plasmas* 17.12 (2010). DOI: [10.1063/1.3514586](https://doi.org/10.1063/1.3514586) (cit. on pp. [2](#), [32](#), [79](#)).
- [40] A. Bret. “Weibel, Two-Stream, Filamentation, Oblique, Bell, Buneman... which one grows faster?” In: *Astrophys. J.* 699 (2009), pp. 990–1003. DOI: [10.1088/0004-637X/699/2/990](https://doi.org/10.1088/0004-637X/699/2/990). arXiv: [0903.2658](https://arxiv.org/abs/0903.2658) [[astro-ph.HE](#)] (cit. on pp. [2](#), [35](#), [79](#)).
- [41] Antoine Bret and Mark Eric Dieckmann. “Hierarchy of instabilities for two counter-streaming magnetized pair beams: influence of field obliquity”. In: *Phys. Plasmas* 24.6 (2017), p. 062105. DOI: [10.1063/1.4985321](https://doi.org/10.1063/1.4985321). arXiv: [1705.07974](https://arxiv.org/abs/1705.07974) [[physics.plasm-ph](#)] (cit. on p. [2](#)).
- [42] Antoine Bret. “Hierarchy of instabilities for two counter-streaming magnetized pair beams”. In: *Phys. Plasmas* 23 (2016), p. 062122. DOI: [10.1063/1.4954307](https://doi.org/10.1063/1.4954307). arXiv: [1605.09372](https://arxiv.org/abs/1605.09372) [[physics.plasm-ph](#)] (cit. on p. [2](#)).
- [43] R. Schlickeiser et al. “The pair beam production spectrum from photon-photon annihilation in cosmic voids”. In: *Astrophys. J.* 758 (2012), p. 101. DOI: [10.1088/0004-637X/758/2/101](https://doi.org/10.1088/0004-637X/758/2/101) (cit. on pp. [2](#), [83](#)).

-
- [44] Lorenzo Sironi and Dimitrios Giannios. “Relativistic Pair Beams from TeV Blazars: A Source of Reprocessed GeV Emission rather than Intergalactic Heating”. In: *Astrophys. J.* 787 (2014), p. 49. DOI: [10.1088/0004-637X/787/1/49](https://doi.org/10.1088/0004-637X/787/1/49). arXiv: [1312.4538](https://arxiv.org/abs/1312.4538) [[astro-ph.HE](#)] (cit. on pp. [2](#), [19](#), [34](#), [62](#), [78](#), [80](#), [147](#)).
 - [45] Francesco Miniati and Andrii Elyiv. “Relaxation of Blazar Induced Pair Beams in Cosmic Voids: Measurement of Magnetic Field in Voids and Thermal History of the IGM”. In: *Astrophys. J.* 770 (2013), p. 54. DOI: [10.1088/0004-637X/770/1/54](https://doi.org/10.1088/0004-637X/770/1/54). arXiv: [1208.1761](https://arxiv.org/abs/1208.1761) [[astro-ph.CO](#)] (cit. on pp. [2](#), [16](#), [19](#), [34](#), [78](#), [80](#), [147](#)).
 - [46] Eric Esarey, Carl B Schroeder, and Wim P Leemans. “Physics of laser-driven plasma-based electron accelerators”. In: *Reviews of modern physics* 81.3 (2009), pp. 1229–1285. DOI: [10.1103/RevModPhys.81.1229](https://doi.org/10.1103/RevModPhys.81.1229) (cit. on p. [2](#)).
 - [47] Gianluca Sarri et al. “Overview of laser-driven generation of electron–positron beams”. In: *Journal of Plasma Physics* 81.4 (2015), p. 455810401. DOI: [10.1017/S002237781500046X](https://doi.org/10.1017/S002237781500046X) (cit. on p. [2](#)).
 - [48] JR Warwick et al. “General features of experiments on the dynamics of laser-driven electron–positron beams”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 909 (2018), pp. 95–101 (cit. on pp. [2](#), [71](#)).
 - [49] Gianluca Sarri et al. “Table-top laser-based source of femtosecond, collimated, ultrarelativistic positron beams”. In: *Physical review letters* 110.25 (2013), p. 255002 (cit. on pp. [2](#), [71](#)).
 - [50] Gianluca Sarri et al. “Spectral and spatial characterisation of laser-driven positron beams”. In: *Plasma Physics and Controlled Fusion* 59.1 (2017), p. 014015 (cit. on pp. [2](#), [71](#)).
 - [51] Yan-Jun Gu et al. “Generation of highly stable electron beam via the control of hydrodynamic instability”. In: *Scientific Reports* 14.1 (2024), p. 31162. DOI: [10.1038/s41598-024-82304-y](https://doi.org/10.1038/s41598-024-82304-y) (cit. on p. [2](#)).
 - [52] FM Foerster et al. “Stable and high-quality electron beams from staged laser and plasma wakefield accelerators”. In: *Physical Review X* 12.4 (2022), p. 041016. DOI: [10.1103/PhysRevX.12.041016](https://doi.org/10.1103/PhysRevX.12.041016) (cit. on p. [2](#)).
 - [53] Marvin Beck. “Numerical Studies for a Laboratory Astrophysics Experiment of Unstable Electron-Positron Beams”. PhD thesis. Staats-und Universitätsbibliothek Hamburg Carl von Ossietzky, 2023 (cit. on pp. [2](#), [56](#), [71](#), [72](#)).
 - [54] Marvin Beck et al. 2023. arXiv: [2306.16839](https://arxiv.org/abs/2306.16839) [[astro-ph.HE](#)] (cit. on pp. [2](#), [34](#)).
 - [55] C. P. Ridgers et al. “Contemporary particle-in-cell approach to laser-plasma modelling”. In: *Plasma Phys. Control. Fusion* 57.11 (2015), p. 113001. DOI: [10.1088/0741-3335/57/11/113001](https://doi.org/10.1088/0741-3335/57/11/113001) (cit. on pp. [2](#), [37](#), [47](#), [54](#)).
 - [56] Rafael Alves Batista et al. “CRPropa 3.2 — an advanced framework for high-energy particle propagation in extragalactic and galactic spaces”. In: *JCAP* 2022.09 (2022), p. 035. DOI: [10.1088/1475-7516/2022/09/035](https://doi.org/10.1088/1475-7516/2022/09/035) (cit. on pp. [2](#), [48](#), [49](#), [79](#), [80](#)).
 - [57] International Bureau of Weights et al. *The international system of units (SI)*. Physical Measurement Laboratory National Institute of Standards and Technology, 2019 (cit. on pp. [2](#), [48](#)).

- [58] Gabriele Ghisellini. *Radiative Processes in High Energy Astrophysics*. Vol. 873. Springer Cham, 2013. DOI: [10.1007/978-3-319-00612-3](#). arXiv: [1202.5949 \[astro-ph.HE\]](#) (cit. on p. 5).
- [59] Bernard Degrange and Gérard Fontaine. “Introduction to high-energy gamma-ray astronomy”. In: *Comptes Rendus Physique* 16 (2015), pp. 587–599. DOI: [10.1016/j.crhy.2015.07.003](#). arXiv: [1604.05488 \[astro-ph.HE\]](#) (cit. on pp. 5, 19).
- [60] Riccardo Giacconi et al. “Evidence for x Rays From Sources Outside the Solar System”. In: *Phys. Rev. Lett.* 9 (1962), pp. 439–443. DOI: [10.1103/PhysRevLett.9.439](#) (cit. on p. 5).
- [61] R. Giacconi et al. “The Uhuru catalog of X-ray sources.” In: *Astrophys. J.* 178 (1972), pp. 281–308. DOI: [10.1086/151790](#) (cit. on p. 6).
- [62] W. Forman et al. “The fourth Uhuru catalog of X-ray sources.” In: *Astrophys. J. Suppl. Ser.* 38 (1978), pp. 357–412. DOI: [10.1086/190561](#) (cit. on p. 6).
- [63] B McBreen et al. “Pulsed high-energy gamma rays from the Crab Nebula”. In: *Astrophys. J., v. 184, no. 2, pp. 571-580* (Sept. 1973). ISSN: ISSN ASJOA. DOI: [10.1086/152350](#). URL: <https://www.osti.gov/biblio/4348073> (cit. on p. 6).
- [64] C. E. Fichtel et al. “High-energy gamma-ray results from the second Small Astronomy Satellite.” In: *Astrophys. J.* 198 (1975), pp. 163–182. DOI: [10.1086/153590](#) (cit. on p. 6).
- [65] K. Bennett et al. “Preliminary results from the European Space Agency’s COS-B satellite for gamma-ray astronomy.” In: *NASA Conference Publication*. Ed. by C. E. Fichtel and F. W. Stecker. Vol. 2. NASA Conference Publication. 1977, pp. 27–64 (cit. on p. 6).
- [66] W. Hermsen et al. “New High-Energy Gamma-Ray Sources Observed by COS-B”. In: *Nature* 269 (1977), pp. 494–495. DOI: [10.1038/269494a0](#) (cit. on p. 6).
- [67] B. N. Swanenburg et al. “Second Cos B Catalog of High-energy gamma-ray Sources”. In: *Astrophys. J. Lett.* 243 (1981), pp. L69–L73. DOI: [10.1086/183445](#) (cit. on p. 6).
- [68] T. C. Weekes et al. “Observation of TeV gamma rays from the Crab nebula using the atmospheric Cerenkov imaging technique”. In: *Astrophys. J.* 342 (1989), pp. 379–395. DOI: [10.1086/167599](#) (cit. on pp. 6, 7).
- [69] M. Punch et al. “Detection of TeV photons from the active galaxy Markarian 421”. In: *Nature* 358 (1992), pp. 477–478. DOI: [10.1038/358477a0](#) (cit. on p. 6).
- [70] J. Quinn et al. “Detection of gamma-rays with $E > 300$ -GeV from Markarian 501”. In: *Astrophys. J. Lett.* 456 (1996), pp. L83–L86. DOI: [10.1086/309878](#) (cit. on p. 6).
- [71] C. E. Fichtel et al. “The first energetic gamma-ray experiment telescope (EGRET) source catalog”. In: *Astrophys. J. Suppl.* 94 (1994), pp. 551–581. DOI: [10.1086/192082](#) (cit. on p. 7).
- [72] R. C. Hartman et al. “The Third EGRET catalog of high-energy gamma-ray sources”. In: *Astrophys. J. Suppl.* 123 (1999), p. 79. DOI: [10.1086/313231](#) (cit. on p. 7).
- [73] A. A. Abdo et al. “Fermi Large Area Telescope First Source Catalog”. In: *Astrophys. J. Suppl.* 188 (2010), pp. 405–436. DOI: [10.1088/0067-0049/188/2/405](#). arXiv: [1002.2280 \[astro-ph.HE\]](#) (cit. on p. 7).

-
- [74] F. Acero et al. “Fermi Large Area Telescope Third Source Catalog”. In: *Astrophys. J. Suppl.* 218.2 (2015), p. 23. DOI: [10.1088/0067-0049/218/2/23](https://doi.org/10.1088/0067-0049/218/2/23). arXiv: [1501.02003](https://arxiv.org/abs/1501.02003) [[astro-ph.HE](#)] (cit. on p. 7).
 - [75] S. Abdollahi et al. “*Fermi* Large Area Telescope Fourth Source Catalog”. In: *Astrophys. J. Suppl.* 247.1 (2020), p. 33. DOI: [10.3847/1538-4365/ab6bcb](https://doi.org/10.3847/1538-4365/ab6bcb). arXiv: [1902.10045](https://arxiv.org/abs/1902.10045) [[astro-ph.HE](#)] (cit. on p. 7).
 - [76] J. Ballet et al. *Fermi Large Area Telescope Fourth Source Catalog Data Release 2*. May 2020. arXiv: [2005.11208](https://arxiv.org/abs/2005.11208) [[astro-ph.HE](#)] (cit. on p. 7).
 - [77] Soheila Abdollahi et al. “Incremental Fermi Large Area Telescope Fourth Source Catalog”. In: *Astrophys. J. Suppl.* 260.2 (2022), p. 53. DOI: [10.3847/1538-4365/ac6751](https://doi.org/10.3847/1538-4365/ac6751). arXiv: [2201.11184](https://arxiv.org/abs/2201.11184) [[astro-ph.HE](#)] (cit. on p. 7).
 - [78] Jean Ballet et al. “Fermi Large Area Telescope fourth source catalog data release 4 (4FGL-DR4)”. In: *arXiv preprint arXiv:2307.12546* (2023) (cit. on pp. 6, 7).
 - [79] F. Krennrich et al. “Stereoscopic observations of gamma-rays at the Whipple observatory”. In: *Astropart. Phys.* 8 (1998), pp. 213–221. DOI: [10.1016/S0927-6505\(97\)00058-3](https://doi.org/10.1016/S0927-6505(97)00058-3) (cit. on p. 7).
 - [80] Markus Holler et al. “Photon Reconstruction for H.E.S.S. Using a Semi-Analytical Model”. In: *PoS ICRC2015* (2016), p. 980. DOI: [10.22323/1.236.0980](https://doi.org/10.22323/1.236.0980). arXiv: [1509.02896](https://arxiv.org/abs/1509.02896) [[astro-ph.IM](#)] (cit. on pp. 6, 7, 20).
 - [81] J. Aleksić et al. “The major upgrade of the MAGIC telescopes, Part II: A performance study using observations of the Crab Nebula”. In: *Astropart. Phys.* 72 (2016), pp. 76–94. DOI: [10.1016/j.astropartphys.2015.02.005](https://doi.org/10.1016/j.astropartphys.2015.02.005). arXiv: [1409.5594](https://arxiv.org/abs/1409.5594) [[astro-ph.IM](#)] (cit. on pp. 6, 7, 19, 20).
 - [82] VERITAS Collaboration. *VERITAS Specifications*. <https://veritas.sao.arizona.edu/about-veritas/veritas-specifications>. Accessed: 2026-01-04 (cit. on pp. 6, 7, 20).
 - [83] A. U. Abeysekara et al. “Observation of the Crab Nebula with the HAWC Gamma-Ray Observatory”. In: *Astrophys. J.* 843.1 (2017), p. 39. DOI: [10.3847/1538-4357/aa7555](https://doi.org/10.3847/1538-4357/aa7555). arXiv: [1701.01778](https://arxiv.org/abs/1701.01778) [[astro-ph.HE](#)] (cit. on pp. 6, 7, 20).
 - [84] Andrea Addazi et al. “The Large High Altitude Air Shower Observatory (LHAASO) Science Book (2021 Edition)”. In: *Chin. Phys. C* 46. arXiv: [1905.02773](https://arxiv.org/abs/1905.02773) (2022), pp. 035001–035007 (cit. on pp. 6, 7, 20).
 - [85] B. S. Acharya et al. *Science with the Cherenkov Telescope Array*. WSP, Nov. 2018. ISBN: 978-981-327-008-4. DOI: [10.1142/10986](https://doi.org/10.1142/10986). arXiv: [1709.07997](https://arxiv.org/abs/1709.07997) [[astro-ph.IM](#)] (cit. on pp. 6, 7, 20).
 - [86] Martin C Weisskopf et al. “Chandra X-ray Observatory (CXO): overview”. In: *X-ray optics, instruments, and missions iii* 4012 (2000), pp. 2–16 (cit. on p. 6).
 - [87] Frederik BS Paerels and Steven M Kahn. “High-resolution x-ray spectroscopy with Chandra and XMM-Newton”. In: *Annual Review of Astronomy and Astrophysics* 41.1 (2003), pp. 291–342 (cit. on p. 6).

- [88] KO Mason et al. “The XMM-Newton optical/UV monitor telescope”. In: *Astronomy & Astrophysics* 365.1 (2001), pp. L36–L44 (cit. on p. 6).
- [89] Stefano Bianchi, Vincenzo Mainieri, and Paolo Padovani. “Active galactic nuclei and their demography through cosmic time”. In: *Handbook of X-ray and Gamma-ray Astrophysics*. Springer, 2023, pp. 1–32 (cit. on p. 7).
- [90] Bradley M Peterson. “The broad-line region in active galactic nuclei”. In: *Physics of Active Galactic Nuclei at all Scales*. Springer, 2006, pp. 77–100 (cit. on p. 7).
- [91] Thomas Wilbur Hartquist, John Edward Dyson, and Deborah P Ruffle. *Blowing bubbles in the cosmos: astronomical winds, jets, and explosions*. Oxford University Press, 2004, pp. 124–141 (cit. on p. 8).
- [92] Silvia Carolina Rueda-Vargas. “Characterization of the structure of the AGN torus through time-resolved X-ray spectroscopy”. MA thesis. U. Potsdam, 2024 (cit. on p. 8).
- [93] Ari Laor and Ehud Behar. “On the origin of radio emission in radio-quiet quasars”. In: *Monthly Notices of the Royal Astronomical Society* 390.2 (2008), pp. 847–862 (cit. on p. 8).
- [94] P Véron. “The luminosity function of Seyfert 1 galaxy nuclei and BL Lac objects, and the X-ray background”. In: *Astronomy and Astrophysics, vol. 78, no. 1, Sept. 1979, p. 46-52*. 78 (1979), pp. 46–52 (cit. on p. 8).
- [95] Maša Lakićević, Luka Č Popović, and Jelena Kovačević-Dojčinović. “Narrow vs. broad-line Seyfert 1 galaxies: X-ray, optical, and mid-infrared AGN characteristics”. In: *Monthly Notices of the Royal Astronomical Society* 478.3 (2018), pp. 4068–4083 (cit. on p. 8).
- [96] Randi Renae Ludwig. “Emission-line properties of active galactic nuclei and an experiment in integrated, guided-inquiry science classes and implications for teaching astronomy”. PhD thesis. Austin, Texas, United States: The University of Texas at Austin, 2012 (cit. on p. 8).
- [97] Jack Frederick Radcliffe. *Nowhere to hide: identifying AGN in the faint radio sky*. The University of Manchester (United Kingdom), 2019 (cit. on p. 8).
- [98] C. Megan Urry and Paolo Padovani. “Unified schemes for radio-loud active galactic nuclei”. In: *Publ. Astron. Soc. Pac.* 107 (1995), p. 803. DOI: [10.1086/133630](https://doi.org/10.1086/133630). arXiv: [astro-ph/9506063](https://arxiv.org/abs/astro-ph/9506063) (cit. on p. 9).
- [99] F. Tavecchio et al. “The hard TeV spectrum of 1ES 0229+200: new clues from Swift”. In: *Mon. Not. Roy. Astron. Soc.* 399 (2009), p. 59. DOI: [10.1111/j.1745-3933.2009.00724.x](https://doi.org/10.1111/j.1745-3933.2009.00724.x). arXiv: [0905.0899](https://arxiv.org/abs/0905.0899) [[astro-ph.CO](https://arxiv.org/archive/astro)] (cit. on p. 8).
- [100] T. S. Poole et al. “Photometric Calibration of the Swift Ultraviolet/Optical Telescope”. In: *Mon. Not. Roy. Astron. Soc.* 383 (2008), pp. 627–645. DOI: [10.1111/j.1365-2966.2007.12563.x](https://doi.org/10.1111/j.1365-2966.2007.12563.x). arXiv: [0708.2259](https://arxiv.org/abs/0708.2259) [[astro-ph](https://arxiv.org/archive/astro)] (cit. on p. 8).
- [101] S. Kaufmann et al. “1ES 0229+200: An extreme blazar with a very high minimum Lorentz factor”. In: *Astron. Astrophys.* 534 (2011), A130. DOI: [10.1051/0004-6361/201117215](https://doi.org/10.1051/0004-6361/201117215). arXiv: [1109.3628](https://arxiv.org/abs/1109.3628) [[astro-ph.HE](https://arxiv.org/archive/astro)] (cit. on p. 8).

-
- [102] D. Donato, R. M. Sambruna, and M. Gliozzi. “Six years of BeppoSAX observations of blazars: A Spectral catalog”. In: *Astron. Astrophys.* 433 (2005), p. 1163. DOI: [10.1051/0004-6361:20034555](#). arXiv: [physics/0412114](#) (cit. on p. 8).
 - [103] L. Costamante et al. *The bepposax view of extreme bl lacs*. June 2002. arXiv: [astro-ph/0206482](#) (cit. on p. 8).
 - [104] VN Yershov. “Serendipitous UV source catalogues for 10 years of XMM and 5 years of Swift”. In: *Astrophysics and Space Science* 354.1 (2014), pp. 97–101 (cit. on p. 8).
 - [105] Christopher P. Ahn et al. “The Ninth Data Release of the Sloan Digital Sky Survey: First Spectroscopic Data from the SDSS-III Baryon Oscillation Spectroscopic Survey”. In: *Astrophys. J. Suppl.* 203 (2012), p. 21. DOI: [10.1088/0067-0049/203/2/21](#). arXiv: [1207.7137 \[astro-ph.IM\]](#) (cit. on p. 8).
 - [106] F. Aharonian et al. “Constraints on the Intergalactic Magnetic Field Using Fermi-LAT and H.E.S.S. Blazar Observations”. In: *Astrophys. J. Lett.* 950.2 (2023), p. L16. DOI: [10.3847/2041-8213/acd777](#). arXiv: [2306.05132 \[astro-ph.HE\]](#) (cit. on pp. 9, 18, 78, 94–96).
 - [107] Carlo Romoli et al. “Intrinsic spectra of H.E.S.S. blazars : what would we see without EBL absorption”. In: *PoS ICRC2017* (2018), p. 605. DOI: [10.22323/1.301.0605](#) (cit. on p. 9).
 - [108] Pedro Ivo Silva Batista. “Characterizing the VHE emission of the extreme HBLs 1ES 1218+304 and 1ES 0229+200 with VERITAS”. In: *PoS Gamma2022* (2023), p. 083. DOI: [10.22323/1.417.0083](#) (cit. on p. 9).
 - [109] Piero Madau. *The intergalactic medium*. May 2000. arXiv: [astro-ph/0005106](#) (cit. on p. 11).
 - [110] Vincent Baglin and Roberto Kersevan. “II.8 - Vacuum systems”. In: *CERN Yellow Rep. School Proc.* 3 (2024), pp. 1259–1338. DOI: [10.23730/CYRSP-2024-003.1259](#) (cit. on p. 11).
 - [111] Annalisa Pillepich et al. “First results from the IllustrisTNG simulations: the stellar mass content of groups and clusters of galaxies”. In: *Mon. Not. Roy. Astron. Soc.* 475.1 (2018), pp. 648–675. DOI: [10.1093/mnras/stx3112](#). arXiv: [1707.03406 \[astro-ph.GA\]](#) (cit. on p. 11).
 - [112] Dylan Nelson et al. “First results from the IllustrisTNG simulations: the galaxy colour bimodality”. In: *Mon. Not. Roy. Astron. Soc.* 475.1 (2018), pp. 624–647. DOI: [10.1093/mnras/stx3040](#). arXiv: [1707.03395 \[astro-ph.GA\]](#) (cit. on p. 11).
 - [113] Dylan Nelson et al. “The IllustrisTNG simulations: public data release”. In: *Comput. Astrophys. Cosmol.* 6.1 (2019), p. 2. DOI: [10.1186/s40668-019-0028-x](#). arXiv: [1812.05609 \[astro-ph.GA\]](#) (cit. on p. 11).
 - [114] Smadar Naoz, Shay Noter, and Rennan Barkana. “The first stars in the universe”. In: *Mon. Not. Roy. Astron. Soc.* 373 (2006), p. L98. DOI: [10.1111/j.1745-3933.2006.00251.x](#). arXiv: [astro-ph/0604050](#) (cit. on p. 11).
 - [115] Ralf S Klessen and Simon CO Glover. “The first stars: formation, properties, and impact”. In: *Annual Review of Astronomy and Astrophysics* 61.1 (2023), pp. 65–130 (cit. on p. 11).

- [116] Volker Bromm and Richard B. Larson. “The First stars”. In: *Ann. Rev. Astron. Astrophys.* 42 (2004), pp. 79–118. DOI: [10.1146/annurev.astro.42.053102.134034](#). arXiv: [astro-ph/0311019](#) (cit. on p. 12).
- [117] M. Trenti and M. Stiavelli. “The Formation Rates of Population III Stars and Chemical Enrichment of Halos during the Reionization Era”. In: *Astrophys. J.* 694 (2009), pp. 879–892. DOI: [10.1088/0004-637X/694/2/879](#). arXiv: [0901.0711 \[astro-ph.CO\]](#) (cit. on p. 12).
- [118] Joop Schaye et al. “The Thermal history of the intergalactic medium”. In: *Mon. Not. Roy. Astron. Soc.* 318 (2000), p. 817. DOI: [10.1046/j.1365-8711.2000.03815.x](#). arXiv: [astro-ph/9912432](#) (cit. on p. 12).
- [119] Massimo Ricotti, Nickolay Y. Gnedin, and J. Michael Shull. “The evolution of the effective equation of state of the igm”. In: *Astrophys. J.* 534 (2000), pp. 41–56. DOI: [10.1086/308733](#). arXiv: [astro-ph/9906413](#) (cit. on p. 12).
- [120] Khee-Gan Lee et al. “The BOSS Lyman-alpha Forest Sample from SDSS Data Release 9”. In: *Astron. J.* 145 (2013), p. 69. DOI: [10.1088/0004-6256/145/3/69](#). arXiv: [1211.5146 \[astro-ph.CO\]](#) (cit. on p. 12).
- [121] Romeel Dave et al. “The intergalactic medium over the last 10 billion years I: Lyman alpha absorption and physical conditions”. In: *Mon. Not. Roy. Astron. Soc.* 408 (2010), p. 2051. DOI: [10.1111/j.1365-2966.2010.17279.x](#). arXiv: [1005.2421 \[astro-ph.CO\]](#) (cit. on p. 12).
- [122] Shadab Alam et al. “Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory”. In: *Phys. Rev. D* 103.8 (2021), p. 083533. DOI: [10.1103/PhysRevD.103.083533](#). arXiv: [2007.08991 \[astro-ph.CO\]](#) (cit. on p. 13).
- [123] Barbara Ryden. *Introduction to cosmology*. Cambridge University Press, 2016 (cit. on p. 13).
- [124] D. J. Fixsen. “The Temperature of the Cosmic Microwave Background”. In: *Astrophys. J.* 707 (2009), pp. 916–920. DOI: [10.1088/0004-637X/707/2/916](#). arXiv: [0911.1955 \[astro-ph.CO\]](#) (cit. on p. 13).
- [125] R. Adam et al. “Planck 2015 results. I. Overview of products and scientific results”. In: *Astron. Astrophys.* 594 (2016), A1. DOI: [10.1051/0004-6361/201527101](#). arXiv: [1502.01582 \[astro-ph.CO\]](#) (cit. on pp. 13, 14).
- [126] Jacques Delabrouille, J. -F. Cardoso, and G. Patanchon. “Multi-detector multi-component spectral matching and applications for CMB data analysis”. In: *Mon. Not. Roy. Astron. Soc.* 346 (2003), p. 1089. DOI: [10.1111/j.1365-2966.2003.07069.x](#). arXiv: [astro-ph/0211504](#) (cit. on p. 13).
- [127] Jean-François Cardoso et al. “Component separation with flexible models—application to multichannel astrophysical observations”. In: *IEEE Journal of Selected Topics in Signal Processing* 2.5 (2008), pp. 735–746 (cit. on p. 13).
- [128] Martin Bucher. “Physics of the cosmic microwave background anisotropy”. In: *Int. J. Mod. Phys. D* 24.02 (2015), p. 1530004. DOI: [10.1142/S0218271815300049](#). arXiv: [1501.04288 \[astro-ph.CO\]](#) (cit. on p. 13).

- [129] Ruth Durrer. “The cosmic microwave background: the history of its experimental investigation and its significance for cosmology”. In: *Class. Quant. Grav.* 32.12 (2015), p. 124007. DOI: [10.1088/0264-9381/32/12/124007](https://doi.org/10.1088/0264-9381/32/12/124007). arXiv: [1506.01907](https://arxiv.org/abs/1506.01907) [[astro-ph.CO](#)] (cit. on p. [13](#)).
- [130] Asantha Cooray. “Extragalactic background light measurements and applications”. In: *Royal Society Open Science* 3.3 (2016), p. 150555 (cit. on pp. [14](#), [15](#), [17](#)).
- [131] H. Abdalla et al. “Measurement of the EBL spectral energy distribution using the VHE γ -ray spectra of H.E.S.S. blazars”. In: *Astron. Astrophys.* 606 (2017), A59. DOI: [10.1051/0004-6361/201731200](https://doi.org/10.1051/0004-6361/201731200). arXiv: [1707.06090](https://arxiv.org/abs/1707.06090) [[astro-ph.HE](#)] (cit. on pp. [14](#), [15](#)).
- [132] A. U. Abeysekara et al. *Measurement of the extragalactic background light spectral energy distribution with VERITAS*. Oct. 2019. DOI: [10.3847/1538-4357/ab4817](https://doi.org/10.3847/1538-4357/ab4817). arXiv: [1910.00451](https://arxiv.org/abs/1910.00451) [[astro-ph.HE](#)] (cit. on pp. [14](#), [15](#)).
- [133] Abraham Loeb and Eli Waxman. “Cosmic γ -ray background from structure formation in the intergalactic medium”. In: *Nature* 405.6783 (2000), pp. 156–158 (cit. on pp. [14](#), [15](#)).
- [134] A Moretti. “A New Measurement of the Cosmic X-ray Background”. In: *AIP Conference Proceedings*. Vol. 1126. 1. American Institute of Physics. 2009, pp. 223–226 (cit. on pp. [14](#), [15](#)).
- [135] Pilar Ruiz-Lapuente et al. “The origin of the cosmic gamma-ray background in the MeV range”. In: *The Astrophysical Journal* 820.2 (2016), p. 142 (cit. on pp. [14](#), [15](#)).
- [136] Michael G Hauser and Eli Dwek. “The cosmic infrared background: measurements and implications”. In: *Annual Review of Astronomy and Astrophysics* 39.1 (2001), pp. 249–307 (cit. on pp. [14](#), [15](#)).
- [137] Edward L Wright. “COBE observations of the cosmic infrared background”. In: *New Astronomy Reviews* 48.5-6 (2004), pp. 465–468 (cit. on pp. [14](#), [15](#)).
- [138] Nick Kaiser and Anthony N Lasenby. *The Post-Recombination Universe*. Vol. 240. Springer Science & Business Media, 2012 (cit. on p. [14](#)).
- [139] Lucia Pozzetti and Piero Madau. “The optical extragalactic background light from resolved galaxies”. In: *Symposium-International Astronomical Union*. Vol. 204. Cambridge University Press. 2001, pp. 71–85 (cit. on p. [15](#)).
- [140] Jonathan P Gardner et al. “The Hubble Deep Field South: STIS Imaging”. In: *The Astrophysical Journal* 119.2 (2000), p. 486 (cit. on p. [15](#)).
- [141] Rebecca A Bernstein, Wendy L Freedman, and Barry F Madore. “The first detections of the extragalactic background light at 3000, 5500, and 8000 ÅI Results”. In: *The Astrophysical Journal* 571.1 (2002), p. 56 (cit. on p. [15](#)).
- [142] D Elbaz et al. “The bulk of the cosmic infrared background resolved by ISOCAM”. In: *Astronomy & Astrophysics* 384.3 (2002), pp. 848–865 (cit. on p. [15](#)).
- [143] C Kevin Xu et al. “Number counts of GALEX sources in far-ultraviolet (1530 Å) and near-ultraviolet (2310 Å) bands”. In: *The Astrophysical Journal* 619.1 (2005), p. L11 (cit. on p. [15](#)).

- [144] Casey Papovich et al. “The 24 micron source counts in deep Spitzer Space Telescope surveys”. In: *The Astrophysical Journal Supplement Series* 154.1 (2004), p. 70 (cit. on p. 15).
- [145] GG Fazio et al. “Number counts at $3\text{ }\mu\text{m} < \lambda < 10\text{ }\mu\text{m}$ from the Spitzer space telescope”. In: *The Astrophysical Journal Supplement Series* 154.1 (2004), p. 39 (cit. on p. 15).
- [146] Herve Dole et al. “Far-infrared source counts at 70 and 160 microns in Spitzer deep surveys”. In: *The Astrophysical Journal Supplement Series* 154.1 (2004), p. 87 (cit. on p. 15).
- [147] David T Frayer et al. “Spitzer 70 micron source counts in GOODS-North”. In: *The Astrophysical Journal* 647.1 (2006), p. L9 (cit. on p. 15).
- [148] Yoshiaki Matsuoka et al. “Cosmic optical background: The view from Pioneer 10/11”. In: *The Astrophysical Journal* 736.2 (2011), p. 119 (cit. on p. 15).
- [149] S Berta et al. “Building the cosmic infrared background brick by brick with Herschel/PEP”. In: *Astronomy & Astrophysics* 532 (2011), A49 (cit. on p. 15).
- [150] T Matsumoto et al. “Reanalysis of the near-infrared extragalactic background light based on the IRTS observations”. In: *The Astrophysical Journal* 807.1 (2015), p. 57 (cit. on p. 15).
- [151] S Matsuura et al. “Detection of the cosmic far-infrared background in AKARI deep field south”. In: *The Astrophysical Journal* 737.1 (2011), p. 2 (cit. on p. 15).
- [152] EL Wright and ED Reese. “Detection of the Cosmic Infrared Background at 2.2 and 3.5 Microns Using DIRBE Observations”. In: *The Astrophysical Journal* 545.1 (2000), p. 43 (cit. on p. 15).
- [153] Edward L Wright and Benjamin D Johnson. “Dirbe minus 2mass: the cosmic infrared background at 3.5 microns”. In: *arXiv preprint astro-ph/0107205* (2001) (cit. on p. 15).
- [154] LR Levenson, EL Wright, and BD Johnson. “DIRBE minus 2MASS: confirming the CIRB in 40 new regions at 2.2 and $3.5\text{ }\mu\text{m}$ ”. In: *The Astrophysical Journal* 666.1 (2007), p. 34 (cit. on p. 15).
- [155] LR Levenson and EL Wright. “Probing the $3.6\text{ }\mu\text{m}$ CIRB with Spitzer in three DIRBE dark spots”. In: *The Astrophysical Journal* 683.2 (2008), p. 585 (cit. on p. 15).
- [156] L Cambr sy et al. “The cosmic infrared background at 1.25 and 2.2 microns using DIRBE and 2MASS: A contribution not due to galaxies?” In: *The Astrophysical Journal* 555.2 (2001), p. 563 (cit. on p. 15).
- [157] E Dwek and RG Arendt. “A tentative detection of the cosmic infrared background at $3.5\text{ }\mu\text{m}$ from COBE/DIRBE observations”. In: *The Astrophysical Journal* 508.1 (1998), p. L9 (cit. on p. 15).
- [158] V Gorjian, EL Wright, and RR Chary. “Tentative detection of the cosmic infrared background at 2.2 and 3.5 microns using ground-based and space-based observations”. In: *The Astrophysical Journal* 536.2 (2000), p. 550 (cit. on p. 15).
- [159] Douglas P Finkbeiner, Marc Davis, and David J Schlegel. “Detection of a Far-Infrared Excess with DIRBE at 60 and 100Microns”. In: *The Astrophysical Journal* 544.1 (2000), p. 81 (cit. on p. 15).

-
- [160] Michael G Hauser et al. “The COBE diffuse infrared background experiment search for the cosmic infrared background. I. Limits and detections”. In: *The Astrophysical Journal* 508.1 (1998), p. 25 (cit. on p. 15).
 - [161] Jerry Edelstein, Stuart Bowyer, and Michael Lampton. “Reanalysis of Voyager Ultraviolet Spectrometer Limits to the Extreme-Ultraviolet and Far-Ultraviolet Diffuse Astronomical Flux”. In: *The Astrophysical Journal* 539.1 (2000), p. 187 (cit. on p. 15).
 - [162] J Albert et al. “MAGIC observations of PG 1553+ 113 during a multiwavelength campaign in July 2006”. In: *Astronomy & Astrophysics* 493.2 (2009), pp. 467–469 (cit. on p. 15).
 - [163] Justin D Finke, Soebur Razzaque, and Charles D Dermer. “Modeling the extragalactic background light from stars and dust”. In: *The Astrophysical Journal* 712.1 (2010), p. 238 (cit. on p. 15).
 - [164] Tanja M Kneiske et al. “Implications of cosmological gamma-ray absorption-II. Modification of gamma-ray spectra”. In: *Astronomy & Astrophysics* 413.3 (2004), pp. 807–815 (cit. on p. 15).
 - [165] Alberto Franceschini, Giulia Rodighiero, and Mattia Vaccari. “Extragalactic optical-infrared background radiation, its time evolution and the cosmic photon-photon opacity”. In: *Astronomy & Astrophysics* 487.3 (2008), pp. 837–852 (cit. on pp. 15, 17, 82).
 - [166] Yoshiyuki Inoue and Kunihito Ioka. “Upper Limit on the Cosmological Gamma-ray Background”. In: *Phys. Rev. D* 86 (2012), p. 023003. DOI: [10.1103/PhysRevD.86.023003](https://doi.org/10.1103/PhysRevD.86.023003). arXiv: [1206.2923](https://arxiv.org/abs/1206.2923) [astro-ph.HE] (cit. on p. 15).
 - [167] F. Aharonian et al. “New constraints on the Mid-IR EBL from the HESS discovery of VHE gamma rays from 1ES 0229+200”. In: *Astron. Astrophys.* 475 (2007), pp. L9–L13. DOI: [10.1051/0004-6361:20078462](https://doi.org/10.1051/0004-6361:20078462). arXiv: [0709.4584](https://arxiv.org/abs/0709.4584) [astro-ph] (cit. on pp. 15, 86, 90).
 - [168] Alberto Franceschini. “Photon–Photon Interactions and the Opacity of the Universe in Gamma Rays”. In: *Universe* 7.5 (2021), p. 146. DOI: [10.3390/universe7050146](https://doi.org/10.3390/universe7050146) (cit. on p. 15).
 - [169] Christopher Heiter et al. “Production and propagation of ultra-high energy photons using CRPropa 3”. In: *Astropart. Phys.* 102 (2018), pp. 39–50. DOI: [10.1016/j.astropartphys.2018.05.003](https://doi.org/10.1016/j.astropartphys.2018.05.003). arXiv: [1710.11406](https://arxiv.org/abs/1710.11406) [astro-ph.IM] (cit. on pp. 16, 77, 82).
 - [170] Sangjin Lee. “Propagation of extragalactic high energy cosmic and γ rays”. In: *Physical Review D* 58.4 (1998), p. 043004 (cit. on pp. 16, 17).
 - [171] Robert J. Gould and Gerard P. Schreder. “Pair Production in Photon-Photon Collisions”. In: *Phys. Rev.* 155 (1967), pp. 1404–1407. DOI: [10.1103/PhysRev.155.1404](https://doi.org/10.1103/PhysRev.155.1404) (cit. on p. 16).
 - [172] R. Schlickeiser, D. Ibscher, and M. Supsar. “Plasma Effects on Fast Pair Beams in Cosmic Voids”. In: *Astrophys. J.* 758.2 (2012), p. 102. DOI: [10.1088/0004-637X/758/2/102](https://doi.org/10.1088/0004-637X/758/2/102) (cit. on pp. 16, 19).
 - [173] Tanja M. Kneiske et al. “Implications of cosmological gamma-ray absorption. 2. Modification of gamma-ray spectra”. In: *Astron. Astrophys.* 413 (2004), pp. 807–815. DOI: [10.1051/0004-6361:20031542](https://doi.org/10.1051/0004-6361:20031542). arXiv: [astro-ph/0309141](https://arxiv.org/abs/astro-ph/0309141) (cit. on pp. 16, 69, 83).

- [174] A. Neronov and D. V. Semikoz. “Sensitivity of gamma-ray telescopes for detection of magnetic fields in intergalactic medium”. In: *Phys. Rev. D* 80 (2009), p. 123012. DOI: [10.1103/PhysRevD.80.123012](https://doi.org/10.1103/PhysRevD.80.123012). arXiv: [0910.1920](https://arxiv.org/abs/0910.1920) [[astro-ph.CO](#)] (cit. on pp. [16](#), [18](#), [78](#), [83](#), [93](#), [94](#)).
- [175] RWea Brown et al. “Role of $\gamma + \gamma \rightarrow e^+ + e^- + e^+ + e^-$ in photoproduction, colliding beams, and cosmic photon absorption”. In: *Physical Review D* 8.9 (1973), p. 3083 (cit. on p. [17](#)).
- [176] F. A. Aharonian, P. S. Coppi, and H. J. Voelk. “Very high-energy gamma-rays from AGN: Cascading on the cosmic background radiation fields and the formation of pair halos”. In: *Astrophys. J. Lett.* 423 (1994), pp. L5–L8. DOI: [10.1086/187222](https://doi.org/10.1086/187222). arXiv: [astro-ph/9312045](https://arxiv.org/abs/astro-ph/9312045) (cit. on pp. [18](#), [77](#)).
- [177] R. Plaga. “Detecting intergalactic magnetic fields using time delays in pulses of γ -rays”. In: *Nature* 374.6521 (1995), pp. 430–432. DOI: [10.1038/374430a0](https://doi.org/10.1038/374430a0) (cit. on p. [18](#)).
- [178] Fermi-LAT. “Spectral Properties of Bright Fermi-detected Blazars in the Gamma-ray Band”. In: *Astrophys. J.* 710 (2010), pp. 1271–1285. DOI: [10.1088/0004-637X/710/2/1271](https://doi.org/10.1088/0004-637X/710/2/1271). arXiv: [1001.4097](https://arxiv.org/abs/1001.4097) [[astro-ph.HE](#)] (cit. on p. [18](#)).
- [179] Reinhard Schlickeiser, Steffen Krakau, and Markus Supsar. “Plasma Effects on Fast Pair Beams. II. Reactive versus Kinetic Instability of Parallel Electrostatic Waves”. In: *Astrophys. J.* 777 (2013), p. 49. DOI: [10.1088/0004-637X/777/1/49](https://doi.org/10.1088/0004-637X/777/1/49). arXiv: [1308.4594](https://arxiv.org/abs/1308.4594) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [180] Philip Chang et al. “The Effect of Nonlinear Landau Damping on Ultrarelativistic Beam Plasma Instabilities”. In: *Astrophys. J.* 797.2 (2014), p. 110. DOI: [10.1088/0004-637X/797/2/110](https://doi.org/10.1088/0004-637X/797/2/110). arXiv: [1410.3797](https://arxiv.org/abs/1410.3797) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [181] Markus Supsar and Reinhard Schlickeiser. “Plasma effects on fast pair beams. III. Oblique electrostatic growth rates for perpendicular Maxwellian pair beams”. In: *The Astrophysical Journal* 783.2 (2014), p. 96 (cit. on p. [19](#)).
- [182] Philip Chang et al. “The Linear Instability of Dilute Ultrarelativistic $e^\pm$ Pair Beams”. In: *Astrophys. J.* 833.1 (2016), p. 118. DOI: [10.3847/1538-4357/833/1/118](https://doi.org/10.3847/1538-4357/833/1/118). arXiv: [1610.02040](https://arxiv.org/abs/1610.02040) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [183] A. Kempf, P. Kilian, and F. Spanier. “Energy loss in intergalactic pair beams: Particle-in-cell simulation”. In: *Astron. Astrophys.* 585 (2016), A132. DOI: [10.1051/0004-6361/201527521](https://doi.org/10.1051/0004-6361/201527521). arXiv: [1512.00662](https://arxiv.org/abs/1512.00662) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [184] S. Vafin et al. “Revisit of Nonlinear Landau Damping for Electrostatic Instability Driven by Blazar-induced Pair Beams”. In: *Astrophys. J.* 873.1 (2019), p. 10. DOI: [10.3847/1538-4357/ab017b](https://doi.org/10.3847/1538-4357/ab017b). arXiv: [1901.09640](https://arxiv.org/abs/1901.09640) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [185] Mohamad Shalaby et al. “The growth of the longitudinal beam–plasma instability in the presence of an inhomogeneous background”. In: *J. Plasma Phys.* 86.2 (2020), p. 535860201. DOI: [10.1017/S0022377820000215](https://doi.org/10.1017/S0022377820000215). arXiv: [2003.02849](https://arxiv.org/abs/2003.02849) [[astro-ph.HE](#)] (cit. on p. [19](#)).
- [186] A. A. Abdo et al. “Fermi Large Area Telescope Observations of the Crab Pulsar and Nebula”. In: *Astrophys. J.* 708 (2010), pp. 1254–1267. DOI: [10.1088/0004-637X/708/2/1254](https://doi.org/10.1088/0004-637X/708/2/1254). arXiv: [0911.2412](https://arxiv.org/abs/0911.2412) [[astro-ph.HE](#)] (cit. on p. [19](#)).

-
- [187] Markus Ackermann et al. “The Fermi large area telescope on orbit: event classification, instrument response functions, and calibration”. In: *The Astrophysical Journal Supplement Series* 203.1 (2012), p. 4 (cit. on p. 19).
 - [188] A. Addazi et al. “Quantum gravity phenomenology at the dawn of the multi-messenger era—A review”. In: *Prog. Part. Nucl. Phys.* 125 (2022), p. 103948. DOI: [10.1016/j.pnpnp.2022.103948](https://doi.org/10.1016/j.pnpnp.2022.103948). arXiv: [2111.05659](https://arxiv.org/abs/2111.05659) [hep-ph] (cit. on p. 20).
 - [189] Fermi-LAT Collaboration. *Fermi LAT Performance*. http://www.slac.stanford.edu/exp/glast/groups/canda/lat_Performance.htm. Accessed: 2026-01-04 (cit. on p. 20).
 - [190] Alessandro De Angelis and Manuela Mallamaci. “Gamma-Ray Astrophysics”. In: *Eur. Phys. J. Plus* 133 (2018), p. 324. DOI: [10.1140/epjp/i2018-12181-0](https://doi.org/10.1140/epjp/i2018-12181-0). arXiv: [1805.05642](https://arxiv.org/abs/1805.05642) [astro-ph.HE] (cit. on p. 19).
 - [191] Stefan Funk. “Ground-and space-based gamma-ray astronomy”. In: *Annual Review of Nuclear and Particle Science* 65.1 (2015), pp. 245–277 (cit. on p. 19).
 - [192] Andrew J Smith. “HAWC: Design, operation, reconstruction and analysis”. In: *arXiv preprint arXiv:1508.05826* (2015) (cit. on p. 19).
 - [193] Felix Aharonian et al. “Observations of the Crab nebula with HESS”. In: *Astronomy & Astrophysics* 457.3 (2006), pp. 899–915. DOI: [10.1051/0004-6361:20065351](https://doi.org/10.1051/0004-6361:20065351) (cit. on p. 19).
 - [194] K Bernlöhner et al. “The optical system of the HESS imaging atmospheric Cherenkov telescopes, Part 1: Layout and components of the system”. In: *Astropart. Phys.* 20 (2003), pp. 111–128. DOI: [10.1016/S0927-6505\(03\)00171-3](https://doi.org/10.1016/S0927-6505(03)00171-3). arXiv: [astro-ph/0308246](https://arxiv.org/abs/astro-ph/0308246) (cit. on p. 19).
 - [195] R. Cornils et al. “The optical system of the HESS imaging atmospheric Cherenkov telescopes, Part 2: Mirror alignment and point spread function”. In: *Astropart. Phys.* 20 (2003), pp. 129–143. DOI: [10.1016/S0927-6505\(03\)00172-5](https://doi.org/10.1016/S0927-6505(03)00172-5). arXiv: [astro-ph/0308247](https://arxiv.org/abs/astro-ph/0308247) (cit. on p. 19).
 - [196] Juan Cortina. “Status and first results of the MAGIC telescope”. In: *Astrophysics and Space Science* 297.1 (2005), pp. 245–255 (cit. on p. 19).
 - [197] Jamie Holder et al. “The first VERITAS telescope”. In: *Astropart. Phys.* 25 (2006), pp. 391–401. DOI: [10.1016/j.astropartphys.2006.04.002](https://doi.org/10.1016/j.astropartphys.2006.04.002). arXiv: [astro-ph/0604119](https://arxiv.org/abs/astro-ph/0604119) (cit. on p. 19).
 - [198] T. C. Weekes et al. “VERITAS: The Very energetic radiation imaging telescope array system”. In: *Astropart. Phys.* 17 (2002), pp. 221–243. DOI: [10.1016/S0927-6505\(01\)00152-9](https://doi.org/10.1016/S0927-6505(01)00152-9). arXiv: [astro-ph/0108478](https://arxiv.org/abs/astro-ph/0108478) (cit. on pp. 19, 78).
 - [199] Orel Gueta. “The Cherenkov Telescope Array: layout, design and performance”. In: *arXiv preprint arXiv:2108.04512* (2021) (cit. on p. 20).
 - [200] Sagarika Tripathy et al. “Challenges in the choice of the nonconformal coupling function in inflationary magnetogenesis”. In: *Phys. Rev. D* 105.6 (2022), p. 063519. DOI: [10.1103/PhysRevD.105.063519](https://doi.org/10.1103/PhysRevD.105.063519). arXiv: [2111.01478](https://arxiv.org/abs/2111.01478) [astro-ph.CO] (cit. on p. 21).

- [201] Orlando Luongo, Antonino Giacomo Marino, and Tommaso Mengoni. *Inflationary magnetogenesis from non-minimal coupling in large- and small-field potentials*. Apr. 2026. arXiv: [2604.10143 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [202] Tomohiro Fujita and Ruth Durrer. “Scale-invariant Helical Magnetic Fields from Inflation”. In: *JCAP* 09 (2019), p. 008. DOI: [10.1088/1475-7516/2019/09/008](#). arXiv: [1904.11428 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [203] Rainer Beck. “Magnetic fields in spiral galaxies”. In: *Astron. Astrophys. Rev.* 24.1 (2015). [Erratum: *Astron. Astrophys. Rev.* 24, 7 (2016)], p. 4. DOI: [10.1007/s00159-015-0084-4](#). arXiv: [1509.04522 \[astro-ph.GA\]](#) (cit. on p. [21](#)).
- [204] C. L. Carilli and G. B. Taylor. “Cluster magnetic fields”. In: *Ann. Rev. Astron. Astrophys.* 40 (2002), pp. 319–348. DOI: [10.1146/annurev.astro.40.060401.093852](#). arXiv: [astro-ph/0110655](#) (cit. on p. [21](#)).
- [205] Lawrence M. Widrow. “Origin of galactic and extragalactic magnetic fields”. In: *Rev. Mod. Phys.* 74 (2002), pp. 775–823. DOI: [10.1103/RevModPhys.74.775](#). arXiv: [astro-ph/0207240](#) (cit. on pp. [21](#), [22](#)).
- [206] Russell M. Kulsrud et al. “The Protogalactic origin for cosmic magnetic fields”. In: *Astrophys. J.* 480 (1997), p. 481. DOI: [10.1086/303987](#). arXiv: [astro-ph/9607141](#) (cit. on p. [21](#)).
- [207] Steven Furlanetto and Abraham Loeb. “Intergalactic magnetic fields from quasar outflows”. In: *Astrophys. J.* 556 (2001), p. 619. DOI: [10.1086/321630](#). arXiv: [astro-ph/0102076](#) (cit. on pp. [21](#), [22](#)).
- [208] Alan H. Guth. “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems”. In: *Phys. Rev. D* 23 (1981). Ed. by Li-Zhi Fang and R. Ruffini, pp. 347–356. DOI: [10.1103/PhysRevD.23.347](#) (cit. on p. [21](#)).
- [209] Michael S. Turner and Lawrence M. Widrow. “Inflation Produced, Large Scale Magnetic Fields”. In: *Phys. Rev. D* 37 (1988), p. 2743. DOI: [10.1103/PhysRevD.37.2743](#) (cit. on p. [21](#)).
- [210] Lawrence M. Widrow et al. “The First Magnetic Fields”. In: *Space Sci. Rev.* 166 (2012), pp. 37–70. DOI: [10.1007/s11214-011-9833-5](#). arXiv: [1109.4052 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [211] John D. Barrow and Christos G. Tsagas. “Cosmological magnetic field survival”. In: *Mon. Not. Roy. Astron. Soc.* 414 (2011), p. 512. DOI: [10.1111/j.1365-2966.2011.18414.x](#). arXiv: [1101.2390 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [212] Martin Teuscher et al. “Gravitational waves sourced by gauge fields during inflation”. In: *JCAP* 03 (2026), p. 043. DOI: [10.1088/1475-7516/2026/03/043](#). arXiv: [2510.00869 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [213] Daniel Green and Takeshi Kobayashi. “Constraints on Primordial Magnetic Fields from Inflation”. In: *JCAP* 03 (2016), p. 010. DOI: [10.1088/1475-7516/2016/03/010](#). arXiv: [1511.08793 \[astro-ph.CO\]](#) (cit. on p. [21](#)).
- [214] Yuri Shtanov and Mykhailo Pavliuk. “Model-independent constraints in inflationary magnetogenesis”. In: *JCAP* 08 (2020), p. 042. DOI: [10.1088/1475-7516/2020/08/042](#). arXiv: [2004.00947 \[astro-ph.CO\]](#) (cit. on p. [21](#)).

-
- [215] Tomohiro Fujita and Shuichiro Yokoyama. “Critical constraint on inflationary magnetogenesis”. In: *JCAP* 03 (2014). [Erratum: *JCAP* 05, E02 (2014)], p. 013. DOI: [10.1088/1475-7516/2014/03/013](https://doi.org/10.1088/1475-7516/2014/03/013). arXiv: [1402.0596](https://arxiv.org/abs/1402.0596) [[astro-ph.CO](#)] (cit. on p. 21).
 - [216] Craig J. Hogan. “Magnetohydrodynamic Effects of a First-Order Cosmological Phase Transition”. In: *Phys. Rev. Lett.* 51 (1983), pp. 1488–1491. DOI: [10.1103/PhysRevLett.51.1488](https://doi.org/10.1103/PhysRevLett.51.1488) (cit. on p. 21).
 - [217] Chiara Caprini, Ruth Durrer, and Geraldine Servant. “The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition”. In: *JCAP* 12 (2009), p. 024. DOI: [10.1088/1475-7516/2009/12/024](https://doi.org/10.1088/1475-7516/2009/12/024). arXiv: [0909.0622](https://arxiv.org/abs/0909.0622) [[astro-ph.CO](#)] (cit. on p. 21).
 - [218] Alexander G. Tevzadze et al. “Magnetic Fields from QCD Phase Transitions”. In: *Astrophys. J.* 759 (2012), p. 54. DOI: [10.1088/0004-637X/759/1/54](https://doi.org/10.1088/0004-637X/759/1/54). arXiv: [1207.0751](https://arxiv.org/abs/1207.0751) [[astro-ph.CO](#)] (cit. on p. 21).
 - [219] Sampurn Anand, Ujjal Kumar Dey, and Subhendra Mohanty. “Effects of QCD Equation of State on the Stochastic Gravitational Wave Background”. In: *JCAP* 03 (2017), p. 018. DOI: [10.1088/1475-7516/2017/03/018](https://doi.org/10.1088/1475-7516/2017/03/018). arXiv: [1701.02300](https://arxiv.org/abs/1701.02300) [[hep-ph](#)] (cit. on p. 21).
 - [220] Tigran Kalaydzhyan and Edward Shuryak. “Gravity waves generated by sounds from big bang phase transitions”. In: *Phys. Rev. D* 91.8 (2015), p. 083502. DOI: [10.1103/PhysRevD.91.083502](https://doi.org/10.1103/PhysRevD.91.083502). arXiv: [1412.5147](https://arxiv.org/abs/1412.5147) [[hep-ph](#)] (cit. on p. 21).
 - [221] Jarkko Ahonen and Kari Enqvist. “Magnetic field generation in first order phase transition bubble collisions”. In: *Phys. Rev. D* 57 (1998), pp. 664–673. DOI: [10.1103/PhysRevD.57.664](https://doi.org/10.1103/PhysRevD.57.664). arXiv: [hep-ph/9704334](https://arxiv.org/abs/hep-ph/9704334) (cit. on p. 22).
 - [222] Jing Yang and Ligong Bian. “Magnetic field generation from bubble collisions during first-order phase transition”. In: *Phys. Rev. D* 106.2 (2022), p. 023510. DOI: [10.1103/PhysRevD.106.023510](https://doi.org/10.1103/PhysRevD.106.023510). arXiv: [2102.01398](https://arxiv.org/abs/2102.01398) [[astro-ph.CO](#)] (cit. on p. 22).
 - [223] John Ellis et al. “Intergalactic Magnetic Fields from First-Order Phase Transitions”. In: *JCAP* 09 (2019), p. 019. DOI: [10.1088/1475-7516/2019/09/019](https://doi.org/10.1088/1475-7516/2019/09/019). arXiv: [1907.04315](https://arxiv.org/abs/1907.04315) [[astro-ph.CO](#)] (cit. on p. 22).
 - [224] Chiara Caprini and Ruth Durrer. “Gravitational waves from stochastic relativistic sources: Primordial turbulence and magnetic fields”. In: *Phys. Rev. D* 74 (2006), p. 063521. DOI: [10.1103/PhysRevD.74.063521](https://doi.org/10.1103/PhysRevD.74.063521). arXiv: [astro-ph/0603476](https://arxiv.org/abs/astro-ph/0603476) (cit. on p. 22).
 - [225] Alberto Roper Pol et al. “Numerical simulations of gravitational waves from early-universe turbulence”. In: *Phys. Rev. D* 102.8 (2020), p. 083512. DOI: [10.1103/PhysRevD.102.083512](https://doi.org/10.1103/PhysRevD.102.083512). arXiv: [1903.08585](https://arxiv.org/abs/1903.08585) [[astro-ph.CO](#)] (cit. on p. 22).
 - [226] Tina Kahniashvili et al. “Numerical simulations of the decay of primordial magnetic turbulence”. In: *Phys. Rev. D* 81 (2010), p. 123002. DOI: [10.1103/PhysRevD.81.123002](https://doi.org/10.1103/PhysRevD.81.123002). arXiv: [1004.3084](https://arxiv.org/abs/1004.3084) [[astro-ph.CO](#)] (cit. on p. 22).
 - [227] Nickolay Y. Gnedin, Andrea Ferrara, and Ellen G. Zweibel. “Generation of the primordial magnetic fields during cosmological reionization”. In: *Astrophys. J.* 539 (2000), pp. 505–516. DOI: [10.1086/309272](https://doi.org/10.1086/309272). arXiv: [astro-ph/0001066](https://arxiv.org/abs/astro-ph/0001066) (cit. on p. 22).

- [228] Dario Grasso and Hector R. Rubinstein. “Magnetic fields in the early universe”. In: *Phys. Rept.* 348 (2001), pp. 163–266. DOI: [10.1016/S0370-1573\(00\)00110-1](#). arXiv: [astro-ph/0009061](#) (cit. on pp. 22, 98).
- [229] P. Olesen. “On inverse cascades in astrophysics”. In: *Phys. Lett. B* 398 (1997), pp. 321–325. DOI: [10.1016/S0370-2693\(97\)00235-9](#). arXiv: [astro-ph/9610154](#) (cit. on p. 22).
- [230] Martin Arteaga Tupia, Anish Ghoshal, and Alessandro Strumia. “Primordial magnetogenesis from a supercooled dynamical electroweak phase transition”. In: *JHEP* 10 (2025), p. 135. DOI: [10.1007/JHEP10\(2025\)135](#). arXiv: [2506.16387 \[hep-ph\]](#) (cit. on p. 22).
- [231] Tina Kahniashvili et al. “Evolution of Primordial Magnetic Fields from Phase Transitions”. In: *Phys. Rev. D* 87.8 (2013), p. 083007. DOI: [10.1103/PhysRevD.87.083007](#). arXiv: [1212.0596 \[astro-ph.CO\]](#) (cit. on p. 22).
- [232] George B. Field and Sean M. Carroll. “Cosmological magnetic fields from primordial helicity”. In: *Phys. Rev. D* 62 (2000), p. 103008. DOI: [10.1103/PhysRevD.62.103008](#). arXiv: [astro-ph/9811206](#) (cit. on p. 22).
- [233] Axel Brandenburg et al. “Primordial magnetic helicity evolution with a homogeneous magnetic field from inflation”. In: *Phys. Rev. D* 102.2 (2020), p. 023536. DOI: [10.1103/PhysRevD.102.023536](#). arXiv: [2005.06449 \[astro-ph.CO\]](#) (cit. on p. 22).
- [234] Leonardo Campanelli. “Evolution of Magnetic Fields in Freely Decaying Magnetohydrodynamic Turbulence”. In: *Phys. Rev. Lett.* 98 (2007), p. 251302. DOI: [10.1103/PhysRevLett.98.251302](#). arXiv: [0705.2308 \[astro-ph\]](#) (cit. on p. 22).
- [235] Murman Gugenidze et al. *Primordial magnetic field from chiral plasma instability with sourcing*. Dec. 2025. arXiv: [2512.09177 \[hep-ph\]](#) (cit. on p. 22).
- [236] Alexey Boyarsky, Jurg Frohlich, and Oleg Ruchayskiy. “Self-consistent evolution of magnetic fields and chiral asymmetry in the early Universe”. In: *Phys. Rev. Lett.* 108 (2012), p. 031301. DOI: [10.1103/PhysRevLett.108.031301](#). arXiv: [1109.3350 \[astro-ph.CO\]](#) (cit. on p. 22).
- [237] Axel Brandenburg et al. “The turbulent chiral-magnetic cascade in the early universe”. In: *Astrophys. J. Lett.* 845.2 (2017), p. L21. DOI: [10.3847/2041-8213/aa855d](#). arXiv: [1707.03385 \[astro-ph.CO\]](#) (cit. on p. 22).
- [238] Alexey Boyarsky, Jurg Frohlich, and Oleg Ruchayskiy. “Magnetohydrodynamics of Chiral Relativistic Fluids”. In: *Phys. Rev. D* 92 (2015), p. 043004. DOI: [10.1103/PhysRevD.92.043004](#). arXiv: [1504.04854 \[hep-ph\]](#) (cit. on p. 22).
- [239] Oindrila Ghosh et al. “Can galactic magnetic fields diffuse into the voids?” In: *Phys. Rev. D* 113.2 (2026), p. 023523. DOI: [10.1103/4114-mgsp](#). arXiv: [2510.26918 \[astro-ph.CO\]](#) (cit. on p. 22).
- [240] Jennifer Schober et al. *Imprints of primordial magnetic fields on the late-time Universe*. Feb. 2026. arXiv: [2602.23263 \[astro-ph.CO\]](#) (cit. on pp. 22, 23).
- [241] Deepen Garg, Ruth Durrer, and Jennifer Schober. *Are magnetic fields in cosmic voids primordial?* May 2025. arXiv: [2505.14774 \[astro-ph.CO\]](#) (cit. on p. 22).
- [242] Károly Sella and Günter Sigl. *On the contribution of galaxies to the magnetic field in cosmic voids*. Oct. 2025. arXiv: [2510.08025 \[astro-ph.CO\]](#) (cit. on p. 22).

-
- [243] Jennifer Schober et al. “Small-Scale Dynamo Action in Primordial Halos”. In: *IAU Symp.* 294 (2013). Ed. by Alexander G. Kosovichev, Elisabete de Gouveia Dal Pino, and Yihua Yan, pp. 237–248. DOI: [10.1017/S1743921313002585](https://doi.org/10.1017/S1743921313002585). arXiv: [1210.7751](https://arxiv.org/abs/1210.7751) [[astro-ph.CO](#)] (cit. on p. [23](#)).
 - [244] Dominik R. G. Schleicher et al. “Small-scale dynamo action during the formation of the first stars and galaxies. I. The ideal MHD limit”. In: *Astron. Astrophys.* 522 (2010), A115. DOI: [10.1051/0004-6361/201015184](https://doi.org/10.1051/0004-6361/201015184). arXiv: [1003.1135](https://arxiv.org/abs/1003.1135) [[astro-ph.CO](#)] (cit. on p. [23](#)).
 - [245] Sharanya Sur et al. “Magnetic field amplification during gravitational collapse - Influence of initial conditions on dynamo evolution and saturation”. In: *Mon. Not. Roy. Astron. Soc.* 423 (2012), p. 3148. DOI: [10.1111/j.1365-2966.2012.21100.x](https://doi.org/10.1111/j.1365-2966.2012.21100.x). arXiv: [1202.3206](https://arxiv.org/abs/1202.3206) [[astro-ph.SR](#)] (cit. on p. [23](#)).
 - [246] M. A. Latif et al. “The small scale dynamo and the amplification of magnetic fields in massive primordial haloes”. In: *Mon. Not. Roy. Astron. Soc.* 432 (2013), p. 668. DOI: [10.1093/mnras/stt503](https://doi.org/10.1093/mnras/stt503). arXiv: [1212.1619](https://arxiv.org/abs/1212.1619) [[astro-ph.CO](#)] (cit. on p. [23](#)).
 - [247] Ievgen Vovk. “Intergalactic magnetic field lower limits up to the redshift $z \approx 3$ ”. In: (Dec. 2025). arXiv: [2512.11397](https://arxiv.org/abs/2512.11397) [[astro-ph.HE](#)] (cit. on pp. [23](#), [78](#)).
 - [248] EV Stenson et al. “Debye length and plasma skin depth: two length scales of interest in the creation and diagnosis of laboratory pair plasmas”. In: *Journal of Plasma Physics* 83.1 (2017), p. 595830106. DOI: [10.1017/S0022377817000022](https://doi.org/10.1017/S0022377817000022) (cit. on p. [26](#)).
 - [249] RJ Hastie and R Dendy. “Plasma particle dynamics”. In: *Plasma Physics: an Introductory Course* 1 (1993), p. 5 (cit. on p. [26](#)).
 - [250] C. D. Arrowsmith et al. “Laboratory realization of relativistic pair-plasma beams”. In: *Nature Commun.* 15.1 (2024), p. 5029. DOI: [10.1038/s41467-024-49346-2](https://doi.org/10.1038/s41467-024-49346-2). arXiv: [2312.05244](https://arxiv.org/abs/2312.05244) [[physics.plasm-ph](#)] (cit. on pp. [27](#), [33](#), [34](#), [55](#), [71–73](#), [75](#)).
 - [251] Charles D Arrowsmith et al. “Generating ultradense pair beams using 400 GeV/c protons”. In: *Phys. Rev. Res.* 3.2 (2021), p. 023103. DOI: [10.1103/PhysRevResearch.3.023103](https://doi.org/10.1103/PhysRevResearch.3.023103) (cit. on pp. [27](#), [55](#), [71](#), [73](#), [74](#)).
 - [252] Charles D. Arrowsmith et al. “Suppression of pair beam instabilities in a laboratory analogue of blazar pair cascades”. In: *Proc. Nat. Acad. Sci.* 122.45 (2025), e2513365122. DOI: [10.1073/pnas.2513365122](https://doi.org/10.1073/pnas.2513365122). arXiv: [2509.09040](https://arxiv.org/abs/2509.09040) [[astro-ph.HE](#)] (cit. on pp. [27](#), [55](#), [71–73](#), [75](#)).
 - [253] Charles K Birdsall, A Bruce Langdon, and AB Langdon. *Plasma physics via computer simulation*. CRC press, 2018 (cit. on pp. [29](#), [37](#), [39](#)).
 - [254] Roger W Hockney and James W Eastwood. *Computer simulation using particles*. crc Press, 2021 (cit. on pp. [29](#), [37](#), [39](#)).
 - [255] Lev Davidovich Landau. “On the Vibrations of the Electronic Plasma”. In: *J. Phys. (USSR)* 10 (1946). Ed. by D. ter Haar. DOI: [10.1016/b978-0-08-010586-4.50066-3](https://doi.org/10.1016/b978-0-08-010586-4.50066-3) (cit. on p. [31](#)).
 - [256] George Livadiotis. “Statistical origin and properties of kappa distributions”. In: *Journal of Physics: Conference Series*. Vol. 900. 1. IOP Publishing. 2017, p. 012014. DOI: [10.1088/1742-6596/900/1/012014](https://doi.org/10.1088/1742-6596/900/1/012014) (cit. on p. [34](#)).

- [257] R Schlickeiser, D Ibscher, and M Supsar. “Plasma effects on fast pair beams in cosmic voids”. In: *Astrophys. J.* 758.2 (2012), p. 102. DOI: [10.1088/0004-637X/758/2/102](https://doi.org/10.1088/0004-637X/758/2/102) (cit. on pp. [34](#), [78](#), [80](#), [147](#)).
- [258] Reinhard Schlickeiser, Steffen Krakau, and Markus Supsar. “Plasma effects on fast pair beams. II. Reactive versus kinetic instability of parallel electrostatic waves”. In: *Astrophys. J.* 777.1 (2013), p. 49. DOI: [10.1088/0004-637X/777/1/49](https://doi.org/10.1088/0004-637X/777/1/49) (cit. on pp. [34](#), [78](#), [80](#), [147](#)).
- [259] Julien Derouillat et al. “Smilei: A collaborative, open-source, multi-purpose particle-in-cell code for plasma simulation”. In: *Computer Physics Communications* 222 (2018), pp. 351–373. DOI: [10.1016/j.cpc.2017.09.024](https://doi.org/10.1016/j.cpc.2017.09.024) (cit. on p. [37](#)).
- [260] Ricardo A Fonseca et al. “OSIRIS: A three-dimensional, fully relativistic particle in cell code for modeling plasma based accelerators”. In: *International conference on computational science*. Springer. 2002, pp. 342–351. DOI: [10.1007/3-540-47789-6_36](https://doi.org/10.1007/3-540-47789-6_36) (cit. on p. [37](#)).
- [261] John Villasenor and Oscar Buneman. “Rigorous charge conservation for local electromagnetic field solvers”. In: *Computer Physics Communications* 69.2-3 (1992), pp. 306–316. DOI: [10.1016/0010-4655\(92\)90169-Y](https://doi.org/10.1016/0010-4655(92)90169-Y) (cit. on p. [39](#)).
- [262] T Zh Esirkepov. “Exact charge conservation scheme for particle-in-cell simulation with an arbitrary form-factor”. In: *Computer Physics Communications* 135.2 (2001), pp. 144–153. DOI: [10.1016/S0010-4655\(00\)00228-9](https://doi.org/10.1016/S0010-4655(00)00228-9) (cit. on p. [39](#)).
- [263] Roch Smets et al. “A new method to dispatch split particles in Particle-In-Cell codes”. In: *Computer Physics Communications* 261 (2021), p. 107666. DOI: [10.1016/j.cpc.2020.107666](https://doi.org/10.1016/j.cpc.2020.107666) (cit. on p. [39](#)).
- [264] A Pukhov. “Particle-In-Cell Codes for Plasma-based Particle Acceleration”. In: *CERN Yellow Reports* 1 (2016), pp. 181–181. DOI: [10.5170/CERN-2016-001.181](https://doi.org/10.5170/CERN-2016-001.181) (cit. on p. [40](#)).
- [265] Stephen D Gedney. *Introduction to the finite-difference time-domain (FDTD) method for electromagnetics*. Vol. 27. Morgan & Claypool Publishers, 2011 (cit. on p. [40](#)).
- [266] Tony D Arber et al. “Contemporary particle-in-cell approach to laser-plasma modelling”. In: *Plasma Physics and Controlled Fusion* 57.11 (2015), p. 113001 (cit. on p. [40](#)).
- [267] Kane Yee. “Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media”. In: *IEEE Transactions on antennas and propagation* 14.3 (1966), pp. 302–307. DOI: [10.1109/TAP.1966.1138693](https://doi.org/10.1109/TAP.1966.1138693) (cit. on pp. [40](#), [47](#)).
- [268] Arnaud Beck et al. “Adaptive SIMD optimizations in particle-in-cell codes with fine-grain particle sorting”. In: *Computer Physics Communications* 244 (2019), pp. 246–263 (cit. on p. [41](#)).
- [269] Maxence Antoine et al. “Embedded grid refinement for semi-Lagrangian parallelized relativistic Vlasov–Maxwell solver”. In: *Physics of Plasmas* 32.4 (2025) (cit. on p. [42](#)).
- [270] Carlos A De Moura and Carlos S Kubrusly. “The courant–friedrichs–lewy (cfl) condition”. In: *AMC* 10.12 (2013), pp. 45–90 (cit. on pp. [43](#), [48](#)).
- [271] Jay P Boris et al. “Relativistic plasma simulation-optimization of a hybrid code”. In: *Proc. Fourth Conf. Num. Sim. Plasmas*. 1970, pp. 3–67 (cit. on pp. [43](#), [47](#), [51](#)).

-
- [272] Adam V Higuera and John R Cary. “Structure-preserving second-order integration of relativistic charged particle trajectories in electromagnetic fields”. In: *Physics of Plasmas* 24.5 (2017). DOI: [10.1063/1.4979989](https://doi.org/10.1063/1.4979989) (cit. on pp. 43, 47, 57).
 - [273] Jean-Luc Vay and Brendan B Godfrey. “Modeling of relativistic plasmas with the Particle-In-Cell method”. In: *Comptes Rendus Mécanique* 342.10-11 (2014), pp. 610–618. DOI: [10.1016/j.crme.2014.07.006](https://doi.org/10.1016/j.crme.2014.07.006) (cit. on pp. 44, 57).
 - [274] Keith Bennett et al. *Users Manual for the EPOCH PIC codes*. 2017 (cit. on p. 46).
 - [275] Lukas Merten et al. “CRPropa 3.1—a low energy extension based on stochastic differential equations”. In: *JCAP* 06 (2017), p. 046. DOI: [10.1088/1475-7516/2017/06/046](https://doi.org/10.1088/1475-7516/2017/06/046). arXiv: [1704.07484](https://arxiv.org/abs/1704.07484) [[astro-ph.IM](#)] (cit. on pp. 48, 49).
 - [276] R. A. Fisher. “Dispersion on a sphere”. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 217.1130 (1953), pp. 295–305. DOI: [10.1098/rspa.1953.0064](https://doi.org/10.1098/rspa.1953.0064) (cit. on p. 49).
 - [277] Jens Jasche, Arjen van Vliet, and Jörg P. Rachen. “Targeting Earth: CRPropa learns to aim”. In: *PoS ICRC2019* (2020), p. 447. DOI: [10.22323/1.358.0447](https://doi.org/10.22323/1.358.0447). arXiv: [1911.05048](https://arxiv.org/abs/1911.05048) [[astro-ph.HE](#)] (cit. on p. 49).
 - [278] Julian Straub. *Bayesian inference with the von-Mises-Fisher distribution in 3d*. 2017 (cit. on p. 50).
 - [279] Andrej Dundović and Günter Sigl. “Anisotropies of Ultra-high Energy Cosmic Rays Dominated by a Single Source in the Presence of Deflections”. In: *JCAP* 2019.01 (2019), p. 018. DOI: [10.1088/1475-7516/2019/01/018](https://doi.org/10.1088/1475-7516/2019/01/018) (cit. on p. 50).
 - [280] Günter Sigl. *Astroparticle Physics: Theory and Phenomenology*. Vol. 1. Atlantis Studies in Astroparticle Physics and Cosmology. Springer, 2017. DOI: [10.2991/978-94-6239-243-4](https://doi.org/10.2991/978-94-6239-243-4) (cit. on p. 50).
 - [281] Sangjin Lee. “On the propagation of extragalactic high-energy cosmic and gamma-rays”. In: *Phys. Rev. D* 58 (1998), p. 043004. DOI: [10.1103/PhysRevD.58.043004](https://doi.org/10.1103/PhysRevD.58.043004). arXiv: [astro-ph/9604098](https://arxiv.org/abs/astro-ph/9604098) (cit. on pp. 51, 77).
 - [282] Apostolos Mastichiadis, Alan P Marscher, and Kenneth Brecher. “Electron-positron pair production by ultrarelativistic electrons in a soft photon field”. In: *Astrophysical Journal, Part 1 (ISSN 0004-637X)*, vol. 300, Jan. 1, 1986, p. 178-189. 300 (1986), pp. 178–189. DOI: [10.1086/163792](https://doi.org/10.1086/163792) (cit. on p. 51).
 - [283] Jeff R Cash and Alan H Karp. “A variable order Runge-Kutta method for initial value problems with rapidly varying right-hand sides”. In: *ACM Transactions on Mathematical Software (TOMS)* 16.3 (1990), pp. 201–222. DOI: [10.1145/79505.79507](https://doi.org/10.1145/79505.79507) (cit. on p. 51).
 - [284] Lukas Merten and Sophie Aerdker. “Modeling Cosmic-Ray Transport: A CRPropa based stochastic differential equation solver”. In: *Comput. Phys. Commun.* 311 (2025), p. 109542. DOI: [10.1016/j.cpc.2025.109542](https://doi.org/10.1016/j.cpc.2025.109542). arXiv: [2410.01472](https://arxiv.org/abs/2410.01472) [[astro-ph.HE](#)] (cit. on p. 51).
 - [285] Hong Qin et al. “Why is Boris algorithm so good?” In: *Physics of Plasmas* 20.8 (2013). DOI: [10.1063/1.4818428](https://doi.org/10.1063/1.4818428) (cit. on p. 51).

- [286] Mathias Winkel, Robert Speck, and Daniel Ruprecht. “Does Boris-SDC conserve phase space volume?” In: *PAMM* 15.1 (2015), pp. 687–688. DOI: [10.1002/pamm.201510333](https://doi.org/10.1002/pamm.201510333) (cit. on p. 51).
- [287] Gianluca Sarri et al. “Generation of neutral and high-density electron–positron pair plasmas in the laboratory”. In: *Nat. Commun.* 6.1 (2015), p. 6747. DOI: [10.1038/ncomms7747](https://doi.org/10.1038/ncomms7747) (cit. on pp. 55, 61, 71).
- [288] Ronald C Davidson et al. “Nonlinear development of electromagnetic instabilities in anisotropic plasmas”. In: *The Physics of Fluids* 15.2 (1972), pp. 317–333 (cit. on p. 61).
- [289] A Achterberg, J Wiersma, and CA Norman. “The Weibel instability in relativistic plasmas-II. Nonlinear theory and stabilization mechanism”. In: *Astronomy & Astrophysics* 475.1 (2007), pp. 19–36 (cit. on p. 61).
- [290] J. R. Peterson, S. Glenzer, and F. Fiuza. “Magnetic field amplification by a nonlinear electron streaming instability”. In: *Phys. Rev. Lett.* 126.21 (2021), p. 215101. DOI: [10.1103/PhysRevLett.126.215101](https://doi.org/10.1103/PhysRevLett.126.215101). arXiv: [2104.08246](https://arxiv.org/abs/2104.08246) [[physics.plasm-ph](#)] (cit. on p. 62).
- [291] J. Ryan Peterson, Siegfried Glenzer, and Frederico Fiuza. “Magnetic Field Amplification by a Plasma Cavitation Instability in Relativistic Shock Precursors”. In: *Astrophys. J. Lett.* 924.1 (2022), p. L12. DOI: [10.3847/2041-8213/ac44a2](https://doi.org/10.3847/2041-8213/ac44a2). arXiv: [2201.03547](https://arxiv.org/abs/2201.03547) [[astro-ph.HE](#)] (cit. on p. 62).
- [292] Erich S Weibel. “Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution”. In: *Phys. Rev. Lett.* 2.3 (1959), p. 83. DOI: [10.1103/PhysRevLett.2.83](https://doi.org/10.1103/PhysRevLett.2.83) (cit. on p. 62).
- [293] Peter H Yoon and Ronald C Davidson. “Exact analytical model of the classical Weibel instability in a relativistic anisotropic plasma”. In: *Phys. Rev. A* 35.6 (1987), p. 2718. DOI: [10.1103/PhysRevA.35.2718](https://doi.org/10.1103/PhysRevA.35.2718) (cit. on p. 62).
- [294] Jun-ichi Sakai et al. “Magnetic field generation and subsequent field dissipation with plasma heating in relativistic streaming pair plasmas”. In: *J. Phys. Soc. Jpn.* 69.8 (2000), pp. 2503–2513. DOI: [10.1143/jpsj.69.2503](https://doi.org/10.1143/jpsj.69.2503) (cit. on p. 62).
- [295] Luís Silva et al. “On the role of the purely transverse Weibel instability in fast ignitor scenarios”. In: *Phys. Plasmas* 9.6 (2002), pp. 2458–2461. DOI: [10.1063/1.1476004](https://doi.org/10.1063/1.1476004) (cit. on p. 62).
- [296] CH Jaroschek, H Lesch, and RA Treumann. “Ultrarelativistic plasma shell collisions in γ -ray burst sources: dimensional effects on the final steady state magnetic field”. In: *Astrophys. J.* 618.2 (2005), p. 822 (cit. on p. 62).
- [297] Daniel Groselj, Lorenzo Sironi, and Anatoly Spitkovsky. “Long-term Evolution of Relativistic Unmagnetized Collisionless Shocks”. In: *Astrophys. J. Lett.* 963.2 (2024), p. L44. DOI: [10.3847/2041-8213/ad2c8c](https://doi.org/10.3847/2041-8213/ad2c8c). arXiv: [2401.02392](https://arxiv.org/abs/2401.02392) [[astro-ph.HE](#)] (cit. on p. 62).
- [298] Hui Chen et al. “Magnetic collimation of relativistic positrons and electrons from high intensity laser–matter interactions”. In: *Phys. Plasmas* 21.4 (2014). DOI: [10.1063/1.4873711](https://doi.org/10.1063/1.4873711) (cit. on p. 71).
- [299] Hui Chen et al. “Scaling the yield of laser-driven electron-positron jets to laboratory astrophysical applications”. In: *Phys. Rev. Lett.* 114.21 (2015), p. 215001. DOI: [10.1103/PhysRevLett.114.215001](https://doi.org/10.1103/PhysRevLett.114.215001) (cit. on p. 71).

-
- [300] E Liang et al. “High e^+/e^- ratio dense pair creation with 1021W. cm- 2 laser irradiating solid targets”. In: *Sci. Rep.* 5.1 (2015), p. 13968. DOI: [10.1038/srep13968](https://doi.org/10.1038/srep13968) (cit. on p. 71).
 - [301] CJ Hooker et al. “The Astra Gemini project—A dual-beam petawatt Ti: Sapphire laser system”. In: *J. Phys. IV (Proceedings)*. Vol. 133. EDP sciences. 2006, pp. 673–677. DOI: [10.1051/jp4:2006133135](https://doi.org/10.1051/jp4:2006133135) (cit. on p. 71).
 - [302] JL Peebles et al. “Magnetically collimated relativistic charge-neutral electron–positron beams from high-power lasers”. In: *Phys. Plasmas* 28.7 (2021). DOI: [10.1063/5.0053557](https://doi.org/10.1063/5.0053557) (cit. on p. 71).
 - [303] Benno Zeitler. “Phase Space Linearization and External Injection of Electron Bunches into Laser-Driven Plasma Wakefields at REGAE”. PhD thesis. Hamburg: Hamburg U., 2016. DOI: [10.3204/PUBDB-2017-00801](https://doi.org/10.3204/PUBDB-2017-00801) (cit. on pp. 71, 73).
 - [304] Klaus Floettmann. “Some basic features of the beam emittance”. In: *Physical Review Special Topics—Accelerators and Beams* 6.3 (2003), p. 034202 (cit. on p. 72).
 - [305] R. Weingartner et al. “Ultralow emittance electron beams from a laser-wakefield accelerator”. In: *Phys. Rev. ST Accel. Beams* 15 (2012), p. 111302. DOI: [10.1103/PhysRevSTAB.15.111302](https://doi.org/10.1103/PhysRevSTAB.15.111302) (cit. on p. 73).
 - [306] R. Weingartner et al. “Imaging laser-wakefield-accelerated electrons using miniature magnetic quadrupole lenses”. In: *Phys. Rev. ST Accel. Beams* 14 (2011), p. 052801. DOI: [10.1103/PhysRevSTAB.14.052801](https://doi.org/10.1103/PhysRevSTAB.14.052801) (cit. on p. 73).
 - [307] JP Couperus et al. “Demonstration of a beam loaded nanocoulomb-class laser wakefield accelerator”. In: *Nature communications* 8.1 (2017), p. 487 (cit. on p. 73).
 - [308] Stuart PD Mangles et al. “Laser-wakefield acceleration of monoenergetic electron beams in the first plasma-wave period”. In: *Physical review letters* 96.21 (2006), p. 215001 (cit. on p. 73).
 - [309] Wim P. Leemans et al. “GeV electron beams from a cm-scale accelerator”. In: *Nature Phys.* 2 (2006), pp. 696–699. DOI: [10.1038/nphys418](https://doi.org/10.1038/nphys418) (cit. on p. 73).
 - [310] G. Sarri et al. “Overview of laser-driven generation of electron–positron beams”. In: *J. Plasma Phys.* 81.04 (2015), p. 455810401. DOI: [10.1017/S002237781500046X](https://doi.org/10.1017/S002237781500046X) (cit. on p. 73).
 - [311] JWD Halliday et al. “Laboratory observation of collective beam-plasma instabilities in a relativistic pair jet”. In: *arXiv preprint arXiv:2602.16542* (2026) (cit. on pp. 73, 75).
 - [312] Edda Gschwendtner et al. “AWAKE, the advanced proton driven plasma wakefield acceleration experiment at CERN”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 829 (2016), pp. 76–82 (cit. on p. 74).
 - [313] I Efthymiopoulos et al. *HiRadMat: A new irradiation facility for material testing at CERN*. 2011 (cit. on p. 74).
 - [314] Ingo Hofmann. “Performance of solenoids versus quadrupoles in focusing and energy selection of laser accelerated protons”. In: *Physical Review Special Topics—Accelerators and Beams* 16.4 (2013), p. 041302 (cit. on p. 74).

- [315] Riccardo de Maria. “General method for final focus system design for circular colliders”. In: *Physical Review Special Topics—Accelerators and Beams* 11.3 (2008), p. 031001 (cit. on p. 74).
- [316] KN Sjobak et al. “Strong focusing gradient in a linear active plasma lens”. In: *Physical Review Accelerators and Beams* 24.12 (2021), p. 121306 (cit. on p. 74).
- [317] Tong Yang et al. “Designing of active plasma lens for focusing laser-plasma-accelerated pulsed proton beams”. In: *Physical Review Accelerators and Beams* 24.3 (2021), p. 031301 (cit. on p. 74).
- [318] J Van Tilborg et al. “Active plasma lensing for relativistic laser-plasma-accelerated electron beams”. In: *Physical review letters* 115.18 (2015), p. 184802 (cit. on p. 74).
- [319] DJ Spence and Simon Mark Hooker. “Investigation of a hydrogen plasma waveguide”. In: *Physical Review E* 63.1 (2000), p. 015401 (cit. on p. 75).
- [320] Jonathan Warwick et al. “Experimental observation of a current-driven instability in a neutral electron-positron beam”. In: *Physical review letters* 119.18 (2017), p. 185002 (cit. on p. 75).
- [321] CV Pieronek et al. “Laser-heated capillary discharge waveguides as tunable structures for laser-plasma acceleration”. In: *Physics of plasmas* 27.9 (2020) (cit. on p. 75).
- [322] Michael G. Hauser and Eli Dwek. “The cosmic infrared background: measurements and implications”. In: *Ann. Rev. Astron. Astrophys.* 39 (2001), pp. 249–307. DOI: [10.1146/annurev.astro.39.1.249](https://doi.org/10.1146/annurev.astro.39.1.249) (cit. on p. 77).
- [323] Robert J Gould and Gérard P Schröder. “Opacity of the universe to high-energy photons”. In: *Phys. Rev.* 155.5 (1967), p. 1408. DOI: [10.1103/PhysRev.155.1408](https://doi.org/10.1103/PhysRev.155.1408) (cit. on p. 77).
- [324] George R Blumenthal and Robert J Gould. “Bremsstrahlung, synchrotron radiation, and compton scattering of high-energy electrons traversing dilute gases”. In: *Rev. Mod. Phys.* 42.2 (1970), p. 237. DOI: [10.1103/RevModPhys.42.237](https://doi.org/10.1103/RevModPhys.42.237) (cit. on p. 77).
- [325] W. B. Atwood et al. “The Large Area Telescope on the Fermi Gamma-ray Space Telescope Mission”. In: *Astrophys. J.* 697 (2009), pp. 1071–1102. DOI: [10.1088/0004-637X/697/2/1071](https://doi.org/10.1088/0004-637X/697/2/1071). arXiv: [0902.1089](https://arxiv.org/abs/0902.1089) [astro-ph.IM] (cit. on p. 78).
- [326] W. Hofmann. “The High Energy Stereoscopic System (HESS) project”. In: *AIP Conf. Proc.* 515.1 (2000). Ed. by B. L. Dingus, M. H. Salamon, and D. B. Kieda, p. 500. DOI: [10.1063/1.1291416](https://doi.org/10.1063/1.1291416) (cit. on p. 78).
- [327] Javier Rico. “Review of fundamental physics results with the MAGIC telescopes”. In: *AIP Conf. Proc.* 1792.1 (2017), p. 060001. DOI: [10.1063/1.4968984](https://doi.org/10.1063/1.4968984) (cit. on p. 78).
- [328] F. A. Aharonian. “TeV blazars and cosmic infrared background radiation”. In: *27th International Cosmic Ray Conference*. Dec. 2001, pp. 250–261. arXiv: [astro-ph/0112314](https://arxiv.org/abs/astro-ph/0112314) (cit. on p. 78).
- [329] Andrey Saveliev, Carmelo Evoli, and Guenter Sigl. *The Role of Plasma Instabilities in the Propagation of Gamma-Rays from Distant Blazars*. Nov. 2013. arXiv: [1311.6752](https://arxiv.org/abs/1311.6752) [astro-ph.HE] (cit. on p. 78).

-
- [330] Matías Sotomayor Webar, Yosef Abed, and Dieter Horns. *Discovery of femto Gauss intergalactic magnetic fields towards Mkn 501*. Sept. 2025. arXiv: [2509.11996 \[astro-ph.HE\]](#) (cit. on p. [79](#)).
 - [331] Suman Dey and Günter Sigl. “Simulations of astrophysically relevant pair beam instabilities in a laboratory context”. In: *Astropart. Phys.* 173 (2025), p. 103153. DOI: [10.1016/j.astropartphys.2025.103153](#). arXiv: [2501.14518 \[astro-ph.HE\]](#) (cit. on p. [79](#)).
 - [332] Arno A. Penzias and Robert Woodrow Wilson. “A Measurement of excess antenna temperature at 4080-Mc/s”. In: *Astrophys. J.* 142 (1965), pp. 419–421. DOI: [10.1086/148307](#) (cit. on p. [80](#)).
 - [333] Luis Enrique Espinosa Castro, Simone Rossoni, and Günter Sigl. 2024. arXiv: [2405.15390 \[astro-ph.HE\]](#) (cit. on pp. [80](#), [84](#), [89](#), [90](#), [139](#), [141](#)).
 - [334] Roy Perry and Yuri Lyubarsky. “The role of resonant plasma instabilities in the evolution of blazar induced pair beams”. In: *Mon. Not. Roy. Astron. Soc.* 503.2 (2021), pp. 2215–2228. DOI: [10.1093/mnras/stab324](#) (cit. on p. [83](#)).
 - [335] W. Atwood et al. Mar. 2013. arXiv: [1303.3514 \[astro-ph.IM\]](#) (cit. on p. [83](#)).
 - [336] E. Aliu et al. “A three-year multi-wavelength study of the very-high-energy γ -ray blazar 1ES 0229+200”. In: *Astrophys. J.* 782.1 (2014), p. 13. DOI: [10.1088/0004-637X/782/1/13](#). arXiv: [1312.6592 \[astro-ph.HE\]](#) (cit. on pp. [86](#), [90](#)).
 - [337] Alexander Korochkin et al. “Sensitivity reach of gamma-ray measurements for strong cosmological magnetic fields”. In: *Astrophys. J.* 906.2 (2021), p. 116. DOI: [10.3847/1538-4357/abc697](#). arXiv: [2007.14331 \[astro-ph.CO\]](#) (cit. on pp. [93](#), [94](#)).
 - [338] Andrii Neronov, Dmitri Semikoz, and Oleg Kalashev. “Limit on the intergalactic magnetic field from the ultrahigh-energy cosmic ray hotspot in the Perseus-Pisces region”. In: *Phys. Rev. D* 108.10 (2023), p. 103008. DOI: [10.1103/PhysRevD.108.103008](#) (cit. on pp. [94](#), [98](#)).
 - [339] Robi Banerjee and Karsten Jedamzik. “The Evolution of cosmic magnetic fields: From the very early universe, to recombination, to the present”. In: *Phys. Rev. D* 70 (2004), p. 123003. DOI: [10.1103/PhysRevD.70.123003](#). arXiv: [astro-ph/0410032](#) (cit. on pp. [94](#), [98](#)).
 - [340] A. A. Abdo et al. “Fermi observations of TeV-selected AGN”. In: *Astrophys. J.* 707 (2009), pp. 1310–1333. DOI: [10.1088/0004-637X/707/2/1310](#). arXiv: [0910.4881 \[astro-ph.HE\]](#) (cit. on pp. [95](#), [96](#)).
 - [341] Kayla Farrell. “VERITAS Observations of the Blazar H 1426+ 428”. PhD thesis. University College Dublin, 2022 (cit. on pp. [95](#), [96](#)).
 - [342] Ruth Durrer and Andrii Neronov. “Cosmological Magnetic Fields: Their Generation, Evolution and Observation”. In: *Astron. Astrophys. Rev.* 21 (2013), p. 62. DOI: [10.1007/s00159-013-0062-7](#). arXiv: [1303.7121 \[astro-ph.CO\]](#) (cit. on p. [98](#)).
 - [343] George EP Box and Mervin E Muller. “A note on the generation of random normal deviates”. In: *The annals of mathematical statistics* 29.2 (1958), pp. 610–611 (cit. on p. [129](#)).

A

Modifications in PIC code

This appendix is based on the publication:

- **S. Dey** and G. Sigl. (2025). *Simulations of astrophysically relevant pair beam instabilities in a laboratory context*. **Astroparticle Physics**, 173, 103153. arXiv:2501.14518 [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2025.103153>.

In this appendix, we present the acceptance–rejection sampling technique used to generate particles from the specified relativistic anisotropic Cauchy distribution. The algorithm used for particle initialization in the simulations is described in detail.

A.1 Particle initialization in EPOCH code

In the analytical framework discussed in Sec. 4.3.1 of Chap. 4, the beam is described using a prescribed distribution function in phase space. A consistent initialization of the particle distribution is therefore also required in kinetic simulations. In the standard implementation of the EPOCH-2D code, particle momenta are initialized using a non-relativistic Maxwellian distribution generated via the Box–Muller method [343]. This approach is sufficient for thermal background plasmas. However, in the present study, we consider a relativistic pair beam with non-thermal features, thereby making the default initialization scheme insufficient for this purpose.

To address this, we implement a custom particle loader in the input of EPOCH that samples from an anisotropic relativistic Cauchy distribution, consistent with the beam model introduced in the Chap. 4. This choice is motivated by the need to accurately represent the broad, non-Maxwellian momentum distribution expected in astrophysical pair beams. The implementation is based on an acceptance-rejection sampling technique, which allows efficient generation of particles from the desired distribution without requiring an explicit inversion of the cumulative distribution function.

The pseudo code shown in Table 1 proceeds by first constructing an auxiliary or envelope distribution that is straightforward to sample. Specifically, a radial variable is generated via a transformation that yields a power-law form, and the direction is assigned isotropically in spherical coordinates. The sampled momenta are then scaled anisotropically, with separate

longitudinal and transverse widths, and shifted by a drift μ to represent the beam propagation. Subsequently, the acceptance probability is evaluated by comparing the target distribution,

$$f_{\text{true}} \propto (1 + \gamma^2 A)^{-2}, \quad (\text{A.1.1})$$

with the chosen envelope distribution,

$$f_{\text{env}} \propto (1 + A)^{-2}, \quad (\text{A.1.2})$$

where, $A = (p_x - \mu)^2/2\sigma_{\parallel}^2 + (p_y^2 + p_z^2)/2\sigma_{\perp}^2$. A candidate particle is accepted if a uniformly distributed random number, X_4 satisfies the acceptance criterion $X_4 \leq f_{\text{true}}/f_{\text{env}}$. This procedure ensures that the resulting particle ensemble follows the desired relativistic Cauchy distribution. The acceptance-rejection method requires more computational effort than direct sampling techniques. However, since the loader is used only during the initialization stage of the simulation, this additional cost has a negligible impact on the overall runtime. In contrast, the improved representation of the beam distribution provides a more accurate description of the system, particularly in capturing the contribution of high-energy particles to the development of plasma instabilities.

Algorithm 1 Particle loader from a relativistic anisotropic Cauchy distribution using the Acceptance-rejection method

```

repeat
  Generate  $X_1, X_2, X_3, X_4 \sim \mathcal{U}(0, 1]$ 
  // Step 1: radius and direction
   $r \leftarrow \sqrt{\frac{X_1}{1 - X_1}}$ 
   $\cos \theta \leftarrow 2X_2 - 1$ 
   $\sin \theta \leftarrow \sqrt{1 - \cos^2 \theta}$ 
   $\phi \leftarrow 2\pi X_3$ 
  // Step 2: scaling with drift
   $p_x \leftarrow \mu + \sigma_{\parallel} r \cos \theta$ 
   $p_y \leftarrow \sigma_{\perp} r \sin \theta \cos \phi$ 
   $p_z \leftarrow \sigma_{\perp} r \sin \theta \sin \phi$ 
  // Step 3: envelope function
   $A \leftarrow \frac{(p_x - \mu)^2}{2\sigma_{\parallel}^2} + \frac{p_y^2 + p_z^2}{2\sigma_{\perp}^2}$ 
   $f_{\text{env}} \leftarrow (1 + A)^{-2}$ 
  // Step 4: true distribution
   $f_{\text{true}} \leftarrow (1 + \gamma^2 A)^{-2}$ 
  // Step 5: acceptance probability
   $P \leftarrow \frac{f_{\text{true}}}{f_{\text{env}}}$ 
until  $X_4 \leq P$ 
return  $p_x, p_y, p_z$ 

```

B

Example input file for PIC simulation

This appendix is based on the publication:

- **S. Dey** and G. Sigl. (2025). *Simulations of astrophysically relevant pair beam instabilities in a laboratory context*. **Astroparticle Physics**, 173, 103153. arXiv:2501.14518 [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2025.103153>.

B.1 Input file of EPOCH code

In this appendix, we present an example of the input deck used for a two-dimensional Particle-in-Cell (PIC) simulation of a beam-plasma instability, corresponding to the setup in Chap. 6 with parameters $\gamma = 3$ and $\alpha = 0.005$. The remaining results in that chapter can be reproduced by varying α in the constant block. The acceptance-rejection method, following the Appendix A, is used to sample particles from the relativistic Cauchy-type momentum distribution.

```
1 # -----
2 # Define constants for the simulation
3 # -----
4 begin:constant
5   # Grid resolution
6   ncell_x = 880
7   ncell_y = 880
8
9   # Background plasma density in SI units (m-3)
10  dens = 1e16*1e6
11
12  # Beam Lorentz factor
13  gamma_peak = 3
14
15  # Plasma frequency
16  omega_p = sqrt(dens*qe^2/(me*epsilon0))
17
18  # Domain size (scaled with plasma skin depth)
19  Length_x = 120*c/omega_p
20  Length_y = 120*c/omega_p
21
```

```

22 # Initial beam transverse spatial width
23 dlimlen = 110*c/omega_p
24
25 # Particles per cell
26 ppercell = 200
27
28 # Beam-to-background density ratio
29 alpha = 0.005
30
31 # Reference momentum of the beam
32 p_ref = sqrt(gamma_peak^2 - 1)*me*c
33
34 # Momentum spread (parallel and transverse)
35 sigma_par = 1.0/0.511
36 sigma_trans = 0.5/0.511
37
38 # Derived momentum widths
39 p0 = p_ref
40 p1 = sigma_par*me*c
41 p2 = sigma_trans*me*c
42 end:constant
43
44
45 # -----
46 # Simulation control parameters
47 # -----
48 begin:control
49   physics_table_location = /home/sumandey/epoch/epoch2d/src/physics_packages/
      TABLES
50
51 # Number of grid cells
52 nx = ncell_x
53 ny = ncell_y
54
55 # Final simulation time
56 t_end = 3500/omega_p
57
58 # Domain boundaries
59 x_min = 0
60 x_max = Length_x
61 y_min = 0
62 y_max = Length_y
63
64 # Numerical parameters
65 dt_multiplier = 0.95
66 dlb_threshold = 0.1
67 field_order = 6
68
69 # Output and runtime settings
70 stdout_frequency = 2000
71 use_random_seed = T
72 smooth_currents = T
73 smooth_iterations = 5
74 use_accurate_n_zeros = F
75 print_constants = T
76 end:control

```

```

77
78
79 # -----
80 # Boundary conditions
81 # -----
82 begin:boundaries
83     bc_x_min = periodic
84     bc_x_max = periodic
85     bc_y_min = open
86     bc_y_max = open
87 end:boundaries
88
89 # -----
90 # Electron beam definition
91 # -----
92 begin:species
93     name = ElectronBeam
94     charge = -1
95     mass = 1.0
96
97     # Beam density
98     number_density = if(abs(y) lt dlimlen,
99                        alpha*dens*(1 + (y^2 /dlimlen^2))^(-2), 0.0)
100
101     # Number of macro-particles
102     nparticles = ppercell * nx * ny
103
104     # Beam distribution (Non-Maxwellian (Cauchy-type))
105     dist_fn = (1/(1+gamma_peak*((px - p0)^2/(2*p1^2))
106                + gamma_peak*(py^2/(2*p2^2))
107                + gamma_peak*(pz^2/(2*p2^2)))^2)
108
109     # Sampling ranges for acceptance-rejection method
110     # Longitudinal range centered around peak momentum
111     dist_fn_px_range = (p_ref - 3*p1, p_ref + 3*p1)
112
113     # Transverse ranges
114     dist_fn_py_range = (-3*p2, 3*p2)
115     dist_fn_pz_range = (-3*p2, 3*p2)
116 end:species
117
118 # -----
119 # Positron beam definition
120 # -----
121 begin:species
122     name = PositronBeam
123     charge = 1
124     mass = 1.0
125
126     number_density = if(abs(y) lt dlimlen,
127                        alpha*dens*(1 + (y^2 /dlimlen^2))^(-2), 0.0)
128     nparticles = ppercell * nx * ny
129
130     dist_fn = (1/(1+gamma_peak*((px - p0)^2/(2*p1^2))
131                + gamma_peak*(py^2/(2*p2^2))
132                + gamma_peak*(pz^2/(2*p2^2)))^2)

```

```

133
134     dist_fn_px_range = (p_ref - 3*p1, p_ref + 3*p1)
135     dist_fn_py_range = (-3*p2, 3*p2)
136     dist_fn_pz_range = (-3*p2, 3*p2)
137 end:species
138
139 # -----
140 # Background electrons (thermal by default)
141 # -----
142 begin:species
143     name:ElectronBG
144     charge = -1
145     mass = 1.0
146
147     # Temperature scaled with beam energy
148     temperature_ev = 6e5*alpha*(gamma_peak - 1)
149
150     number_density = dens
151     nparticles = ppercell * nx * ny
152 end:species
153
154 # -----
155 # Background ions (immobile)
156 # -----
157 begin:species
158     name:IonBG
159     charge = 1
160     mass = 1836.0
161
162     temperature_ev = 6e5*alpha*(gamma_peak - 1)
163
164     number_density = dens
165     nparticles = ppercell * nx * ny
166
167     # Ions treated as stationary background
168     immobile = T
169 end:species
170
171 # -----
172 # Output: total energy diagnostics
173 # -----
174 begin:output
175     dt_snapshot = 1/omega_p
176     dump_last = T
177     name = o1
178
179     total_energy_sum = always + single + species
180 end:output
181
182 # -----
183 # Output: fields and distributions (early phase)
184 # -----
185 begin:output
186     dt_snapshot = 5/omega_p
187     dump_last = F
188     name = o2

```



```

189
190     ex = always + single
191     ey = always + single
192     ez = always + single
193     bx = always + single
194     by = always + single
195     bz = always + single
196
197     distribution_functions = always + single
198
199     time_stop = 1500/omega_p
200 end:output
201
202 # -----
203 # Output: fields and distributions (late phase)
204 # -----
205 begin:output
206     dt_snapshot = 10/omega_p
207     dump_last = T
208     name = o3
209
210     ex = always + single
211     ey = always + single
212     ez = always + single
213     bx = always + single
214     by = always + single
215     bz = always + single
216
217     distribution_functions = always + single
218 end:output

```

Listing B.1: EPOCH input deck for beam-plasma instability simulation

C

Numerical convergence test in kinetic simulations

In PIC simulations with finite grid resolution and a limited number of macro-particles, ensuring numerical convergence is essential. In our setup, the plasma skin depth (c/ω_p) is resolved using 8 grid cells in both the longitudinal and transverse directions. However, the number of macro-particles used to sample the distribution function can introduce artificial numerical heating. In particular, an insufficient number of macro-particles per cell leads to enhanced field fluctuations as particles traverse grid cells. It is therefore crucial to control numerical heating to maintain the physical reliability of the simulation. To this end, we examine the stability of the plasma

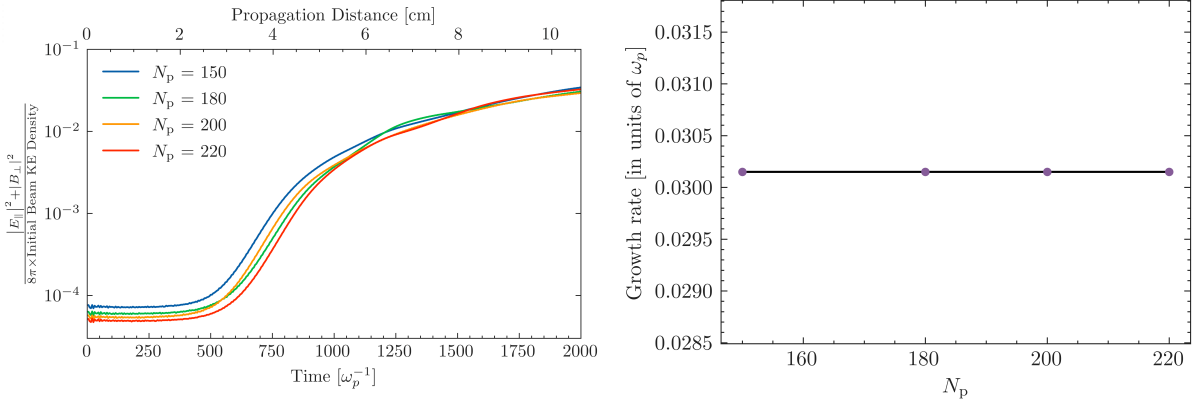


Figure C.0.1: Left panel: The fraction of beam kinetic energy converted into the total electromagnetic field across all modes is shown as a function of time (bottom axis) and propagation distance (top axis). Each simulation assumes a density contrast $\alpha = 0.0005$ and a Lorentz factor $\gamma = 3$, while varying the number of macroparticles per grid cell, N_p . The other beam parameters used in the simulation are listed in Tables 6.1 and 6.2. **Right panel:** The growth rate of instability, derived from the left panel, is shown as a function of N_p .

instability growth rate as a function of the number of macro-particles per cell, as shown in Fig. C.0.1. We find that the growth rate converges for values exceeding 150 macro-particles per

cell. Furthermore, the total energy of the system is conserved to better than 1% accuracy, with improved conservation at higher macro-particle numbers.

D

CRPropa module for plasma instabilities in electromagnetic cascades

This appendix describes the implementation of a plasma instability module within the CRPropa 3.2 framework, following the approach of [333], as introduced in Chap. 8. The module is designed to incorporate instability-induced energy losses of electrons and positrons during the development of electromagnetic cascades.

D.1 Instability energy-loss rate module

This section presents the main implementation of the plasma instability module, including both the source file and its corresponding header file, as shown in Listings D.1 and D.2.

```
1 #include "crpropa/module/PlasmaInstability.h"
2
3 PlasmaInstability::PlasmaInstability(double a, double Energyscale, double
   Lengthscale)
4     : crpropa::Module() {
5     setPlasmaIndex(a);
6     setPlasmaEnergyScale(Energyscale);
7     setPlasmaLengthScale(Lengthscale);
8     setDescription("PlasmaInstability");
9 }
10
11 void PlasmaInstability::setPlasmaIndex(double s) {
12     index = s;
13 }
14
15 void PlasmaInstability::setPlasmaEnergyScale(double Escale) {
16     tildeE = Escale;
17 }
18
19 void PlasmaInstability::setPlasmaLengthScale(double lambdascale) {
20     lambda = lambdascale;
21 }
```

```
22
23 double PlasmaInstability::getPlasmaIndex() const {
24     return index;
25 }
26
27 double PlasmaInstability::getPlasmaEnergyScale() const {
28     return tildeE;
29 }
30
31 double PlasmaInstability::getPlasmaLengthScale() const {
32     return lambda;
33 }
34
35 void PlasmaInstability::process(crpropa::Candidate *candidate) const {
36     int id = candidate->current.getId();
37
38     // only works for electrons and positrons
39     if (fabs(id) != 11)
40         return;
41
42     double dx = candidate->getCurrentStep();
43     double z = candidate->getRedshift();
44     double E = candidate->current.getEnergy();
45     double dEdx = coolingPower(E, z);
46
47     if (dEdx < 0)
48         dEdx = 0;
49
50     double Enew = E - dEdx * dx;
51     candidate->current.setEnergy(Enew);
52
53     candidate->limitNextStep(0.1 * E / dEdx);
54 }
55
56 double PlasmaInstability::coolingPower(double E, double z) const {
57     E /= (1 + z);
58     return (tildeE / lambda) * pow(E / tildeE, 1 - index);
59 }
60
61 std::string PlasmaInstability::getDescription() const {
62     std::stringstream ss;
63     ss << "PlasmaInstability";
64     return ss.str();
65 }
```

Listing D.1: Source file PlasmaInstability.cpp

```
1 #ifndef CRPROPA_PLASMAINSTABILITY_H
2 #define CRPROPA_PLASMAINSTABILITY_H
3
4 #include <crpropa/Cosmology.h>
5 #include <crpropa/Module.h>
6 #include <crpropa/ParticleID.h>
7 #include <crpropa/ParticleMass.h>
8 #include <crpropa/Random.h>
9 #include <crpropa/Units.h>
```

```

10
11 class PlasmaInstability : public crpropa::Module {
12 protected:
13     double index;
14     double tildeE;
15     double lambda;
16
17 public:
18     PlasmaInstability(double index, double tildeE, double lambda);
19
20     void setPlasmaIndex(double indexplasma);
21     void setPlasmaEnergyScale(double tildeEplasma);
22     void setPlasmaLengthScale(double lambdaplasm);
23
24     double getPlasmaIndex() const;
25     double getPlasmaEnergyScale() const;
26     double getPlasmaLengthScale() const;
27
28     double coolingPower(double energy, double redshift) const;
29     std::string getDescription() const;
30
31     void process(crpropa::Candidate *candidate) const;
32 };
33
34 #endif

```

Listing D.2: Header file PlasmaInstability.h

D.2 Installation procedure of instability energy-loss rate module

The modular structure of CRPropa 3.2 allows additional physical processes to be included through user-defined modules. In this work, we implement a plasma instability module, following the approach of [333] that modifies the energy evolution of electrons and positrons by introducing a parametric cooling term.

The implementation consists of two files: a source file (PlasmaInstability.cpp) and a corresponding header file (PlasmaInstability.h). Following the standard CRPropa 3.2 directory¹, the source file is placed in the `src/module` directory, while the header file is included in `include/crpropa/module`.

The integration of the PlasmaInstability module within the CRPropa3 framework is carried out through the following steps:

1. Create the module files, consisting of a source file (PlasmaInstability.cpp) and a corresponding header file (PlasmaInstability.h),
2. Place the source file in the directory CRPropa3/src/module,
3. Place the header file in the directory CRPropa3/include/crpropa/module,
4. Register the module in the Python interface by adding the line:
`%include "crpropa/module/PlasmaInstability.h"` to the file `python/2_headers.i`.

¹<https://github.com/CRPropa/CRPropa3>

5. Include the module in the main header by adding:
`#include "crpropa/module/PlasmaInstability.h"` to the file `include/CRPropa.h`.
6. Add the source file to the build configuration by inserting:
`src/module/PlasmaInstability.cpp` into the `CMakeLists.txt` file.
7. The code is then compiled within the `build` directory using the command `make`.
8. Install the updated build by running `make install`.

Once these steps are completed, the module can be compiled and accessed through the Python interface of `CRPropa`. The module introduces an energy-loss prescription characterized by three parameters: the plasma index (s), a normalization energy ($\tilde{E}$), and a characteristic instability length scale (λ_0). These parameters define the cooling rate applied to electrons and positrons during propagation. At each simulation step, the module evaluates the energy-loss rate and updates the particle energy accordingly. The implementation is restricted to electrons and positrons, ensuring that only relevant particles are affected. In practice, the module is included in a simulation by calling it within the `CRPropa` Python pipeline using:

```
add(PlasmaInstability(index, energy_scale, length_scale))
```

An example simulation script with plasma instability is shown in Appendix [E](#).

E

An example of cascade simulation code

This appendix is based on the publication:

- **S. Dey**, S. Rossoni, and G. Sigl. (2026). *The impact of plasma instability cooling on intergalactic magnetic field constraints in GeV cascades for optimized instability cooling parameters*. **Astroparticle Physics**, 182, 103275. arXiv: [2604.08375](https://arxiv.org/abs/2604.08375) [astro-ph.HE]. <https://doi.org/10.1016/j.astropartphys.2026.103275>.

In this appendix, we present an example CRPropa 3.2 code snippet for three-dimensional simulations of electromagnetic cascade signal production is presented in Listing E.1, for a plasma instability length scale of $\lambda_0 = 120$ kpc, a spectral index $s = -0.5$, a magnetic field strength of $B = 2.7 \times 10^{-17}$ G, and a coherence length of $\lambda_c = 0.2$ Mpc.

```
1 from crpropa import *
2 import numpy as np
3
4 #-----
5 # INITIAL VALUES
6 #-----
7 # Total number of injected photons
8 N = 1e5
9
10 #Injection spectrum parameters
11 minE = 1.0*GeV
12 maxE = 100.0*TeV
13 spectral_index = -1.2
14
15 #Distance to source and maximum propagation distance
16 distance = 580*Mpc
17 maxdist = 585*Mpc
18
19 #Magnetic field strength
20 Bamplitude = 2.7e-17*gauss
21
22 #Magnetic field grid parameters (these parameters define correlation length
    according to Eq. (4.35) in Sec. 4.3.2)
23 gridSize = 256
```

```

24 gridSpacing = 50.0*kpc
25 randomSeed = 42
26
27 #-----
28 # MAGNETIC FIELD
29 #-----
30 #Creating a turbulent field
31 turbSpectrum = SimpleTurbulenceSpectrum(Bamplitude, 400.0*kpc, 410.0*kpc, 5./3.)
32 gridprops = GridProperties(Vector3d(0.)*Mpc, gridSize, gridSpacing)
33 BField = SimpleGridTurbulence(turbSpectrum, gridprops, randomSeed)
34
35 #Printing the values
36 print('Lc = {:.4f} Mpc'.format(BField.getCorrelationLength() / Mpc))
37 print('sqrt(<B^2>) = {:.11f} nG'.format(BField.getBrms() / nG))
38 print('<|B|> = {:.11f} nG'.format(BField.getMeanFieldStrength() / nG))
39 print('Bfield amplitude: ')
40 print(Bamplitude)
41
42 #-----
43 # INTERACTIONS
44 #-----
45 sim = ModuleList()
46
47 #Propagation
48 #sim.add(SimplePropagation()) #For rectilinear propagation. This can be used
   #for the simulations without a magnetic field
49 sim.add(PropagationCK(BField, 1e-12, 1e10*cm, 1.0*Mpc)) #With B-Field
50
51 #Interactions
52 thinning = 0.0 #Minimum thinning 0, maximum thinning 1
53 sim.add(EMInverseComptonScattering(CMB(), True, thinning))
54 sim.add(EMInverseComptonScattering(IRB_Franceschini08(), True, thinning))
55 sim.add(EMPairProduction(CMB(), True, thinning))
56 sim.add(EMPairProduction(IRB_Franceschini08(), True, thinning))
57 sim.add(EMDoublePairProduction(CMB(), True, thinning))
58 sim.add(EMDoublePairProduction(IRB_Franceschini08(), True, thinning))
59 sim.add(EMTripletPairProduction(CMB(), True, thinning))
60 sim.add(EMTripletPairProduction(IRB_Franceschini08(), True, thinning))
61
62 #Limits
63 sim.add(MaximumTrajectoryLength(maxdist))
64 sim.add(MinimumEnergy(minE))
65
66 #-----
67 # PLASMA INSTABILITY
68 #-----
69 #Instability parameters as defined in the thesis
70 s = -0.5
71 tildeE = 1.0*TeV
72 lambda_0 = 120*kpc
73 sim.add(PlasmaInstability(s, tildeE, lambda_0))
74
75 #-----
76 # SOURCE
77 #-----
78 source = Source()

```

```

79 source.add(SourcePosition(0.0*Mpc))
80
81 #Target emission of photons using vMF distribution
82 angle = 0.17453 # in radian
83 P = 0.95
84 kappa = np.log(1-P)/(np.cos(angle)-1)
85 source.add(SourceDirectedEmission(Vector3d(1.0,0.0,0.0),kappa))
86 source.add(SourceParticleType(22))
87
88 #Injection spectrum
89 source.add(SourcePowerLawSpectrum(minE,maxE,spectral_index))
90 print(source)
91
92 # -----
93 # OBSERVERS
94 # -----
95 #First observer sphere around source (for initial spectrum)
96 obs1 = Observer()
97 obs1.add(ObserverSurface(Sphere(Vector3d(0.),1.0*pc)))
98 obs1.setDeactivateOnDetection(False)
99 output1 = TextOutput("initial-spectrum-B-2-7-17-lc-0-2-Mpc-lam0-120-kpc-10
    maxdist.h5", crpropa.Output.Everything)
100 obs1.onDetection(output1)
101 sim.add(obs1)
102
103 #Second observer sphere around source (for final spectrum)
104 obs2 = Observer()
105 obs2.add(ObserverSurface(Sphere(Vector3d(0.),distance)))
106 obs2.setDeactivateOnDetection(True)
107 output2 = TextOutput("output-B-2-7-17-lc-0-2-Mpc-lam0-120-kpc-10maxdist.h5",
    crpropa.Output.Everything)
108 obs2.onDetection(output2)
109 sim.add(obs2)
110 print(obs2)
111
112 # -----
113 # RUNNING
114 # -----
115 sim.setShowProgress(True)
116 sim.run(source,int(N))
117 output1.close()
118 output2.close()

```

Listing E.1: An example CRPropa 3.2 code snippet for three-dimensional simulations of electromagnetic cascade signal production is shown for a plasma instability length scale of $\lambda_0 = 120$ kpc, a spectral index $s = -0.5$, a magnetic field strength of $B = 2.7 \times 10^{-17}$ G, and a coherence length of $\lambda_c = 0.2$ Mpc.

F

Spectral predictions for an additional instability length scale

This appendix is based on the publication:

- **S. Dey**, S. Rossoni, and G. Sigl. (2026). *The impact of plasma instability cooling on intergalactic magnetic field constraints in GeV cascades for optimized instability cooling parameters*. **Astroparticle Physics**, 182, 103275. arXiv: [2604.08375](https://arxiv.org/abs/2604.08375) [[astro-ph.HE](https://arxiv.org/archive/hep)]. <https://doi.org/10.1016/j.astropartphys.2026.103275>.

This appendix presents spectral predictions for an additional plasma instability length scale discussed in Chap. 8.

F.1 Energy loss fraction for different instability parameters

If $E_e(x)$ represents the energy of an electron at a distance x from its injection point x_0 . The energy loss of the electron over one inverse Compton (IC) interaction length, λ_{IC} , is given by $\Delta E_e(\lambda_{\text{IC}}) = E_e(x_0) - E_e(\lambda_{\text{IC}})$. Then the fractional energy loss due to plasma instability within one IC interaction length is defined as $\Delta E_e(\lambda_{\text{IC}})/E_e$. Fig. F.1.1 shows the fractional energy loss due to instabilities over one IC interaction length for $\lambda_0 = 120$ kpc and 360 kpc for different instability spectral indices $s \leq 0$. We focus on the regime $s \leq 0$ because these spectral indices correspond to energy-loss rates due to electrostatic instability growth, as discussed in previous studies [31, 33, 44, 45, 257, 258].

For an energy-independent instability spectrum ($s = 0$), the instability-driven losses dominate over IC cooling with the CMB for electron energies $\lesssim 2$ TeV. As s decreases, the instability-induced fractional energy loss becomes comparable to the energy loss fraction by IC with CMB photons.

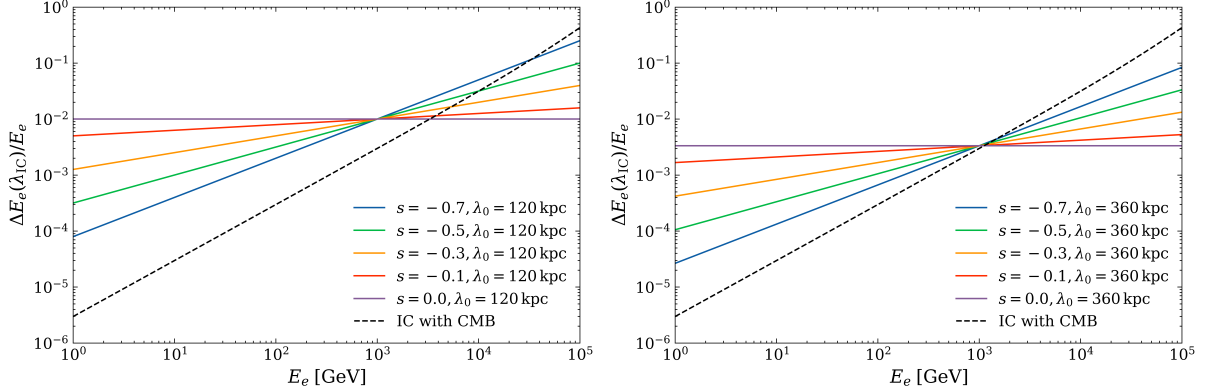


Figure F.1.1: Fractional energy loss due to plasma instability and IC scattering with cosmic microwave background (CMB) as a function of electron energy. Plasma instability energy loss fractions over a distance of a single IC interaction length for $\lambda_0 = 120$ kpc and 360 kpc for different instability spectral indices ($s \leq 0$). The normalization energy is $\tilde{E}_e = 1$ TeV. For comparison, the fractional energy loss due to IC scattering with CMB photons is shown as the black dashed line.

F.2 Propagated photon spectrum for instability length scale $\lambda_0 = 360$ kpc

The analysis for $\lambda_0 = 360$ kpc (with corresponding plasma index, $s = -0.5$) follows the same procedure as in Sec. 8.6. The first and second panels of Fig. F.2.1 include both magnetic deflections and plasma instability cooling, with the propagated spectra computed over the (B, λ_c) parameter space to determine the corresponding lower limits. For each (B, λ_c) , the intrinsic source parameters are obtained via χ^2 minimization at the 90% confidence level, as described in Sec. 8.5.3. Fig F.2.2 shows $\chi^2(\lambda_c)/n_{\text{dof}}$ as a function of λ_c for $\theta_{\text{FoV}} = 1.0, 4.5^\circ$, with the corresponding minimum $\chi^2_{\text{min}}/n_{\text{dof}}$ values listed in Table F.1.

λ_0 [kpc]	Source	θ_{FoV} [deg]	$B/10^{-17}$ [G]	λ_c [Mpc]	β	E_{cut}	$\log_{10} A$ [$\text{eV}^{-1} \text{cm}^{-2} \text{sec}^{-1}$]	$\chi^2_{\text{min}}/n_{\text{dof}}$
360	1ES 0229+200	1.0	3.8	2.0×10^{-2}	1.47	7.2	-23.680	2.920
				2.0×10^{-1}	1.38	6.4	-23.694	1.668
				2.0	1.36	6.7	-23.702	1.668
		4.5	6.6	2.0×10^{-2}	1.61	8.1	-23.793	1.662
				2.0×10^{-1}	1.59	7.3	-23.810	1.497
				6.0	1.58	7.9	-23.808	1.497

Table F.1: The $\chi^2_{\text{min}}/n_{\text{dof}}$ values for the GeV-band data from Fermi-LAT ($n_{\text{dof}} = 4$), for $\theta_{\text{FoV}} = 1.0^\circ, 4.5^\circ$, with plasma instability length scale $\lambda_0 = 360$ kpc and plasma index $s = -0.5$. The $\chi^2_{\text{min}}/n_{\text{dof}}$ value is highlighted in bold.

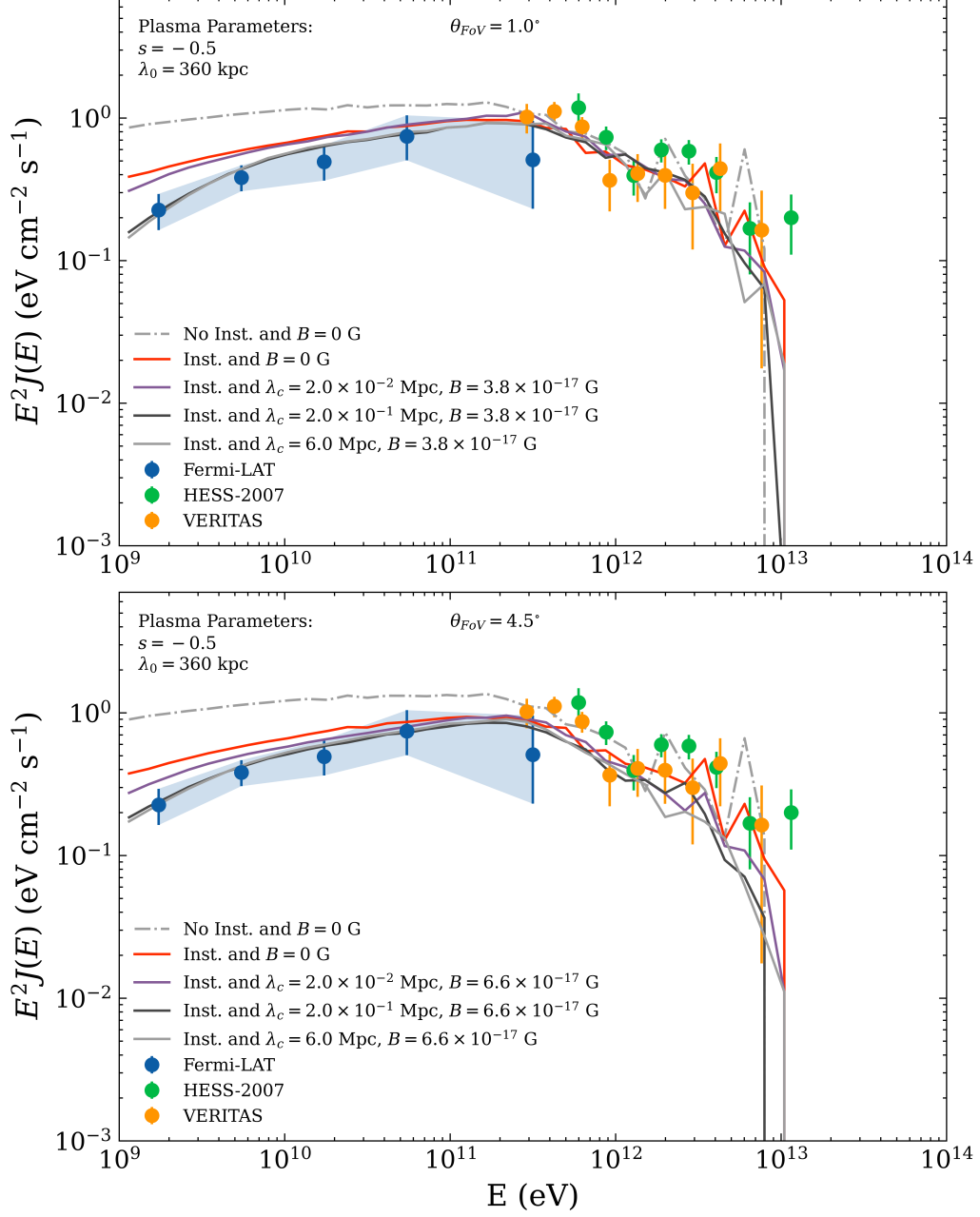


Figure F.2.1: Propagated photon spectrum of 1ES 0229+200 over $10^{-3} \leq E/\text{TeV} \leq 10^2$. The grey dash-dotted curve shows the propagated photon spectrum without plasma instability cooling or intergalactic magnetic field (IGMF). Solid curves include different IGMF strength-coherence length combinations with plasma instability cooling for $\lambda_0 = 360$ kpc and $s = -0.5$. Data points are from Fermi-LAT, H.E.S.S., and VERITAS. Spectra are computed for $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° . The test statistic χ^2/n_{dof} is evaluated using Fermi-LAT GeV data ($n_{\text{dof}} = 4$).

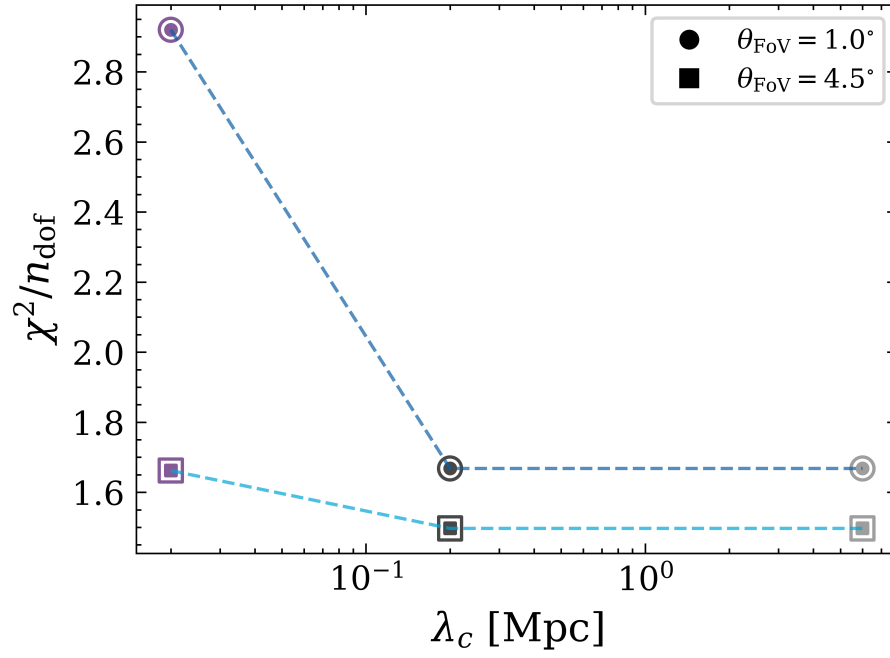


Figure F.2.2: The χ^2/n_{dof} test statistic is shown for the GeV-band Fermi-LAT data ($n_{\text{dof}} = 4$). The dashed lines correspond to fields of view $\theta_{\text{FoV}} = 1.0^\circ$ and 4.5° , respectively.

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