

**Development of an optical test facility with a
toolset for adjustable picometer-stable
interferometers for space-based
gravitational-wave detectors**

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“Curiosity is the root of all knowledge.”

Abstract

Space-based gravitational-wave detectors, such as the Laser Interferometer Space Antenna (LISA), employ picometer-precision laser interferometry to detect gravitational waves from 1 Hz down to below 0.1 mHz. These high-precision observatories enable diverse science cases, including detailed studies of massive black hole growth mechanisms and their pivotal role in galaxy formation and cosmic evolution. Ground-based prototypes of such interferometers are typically constructed by permanently bonding optics to an ultra-stable bench made of low-expansion glass ceramic, minimizing dominant temperature coupling at low frequencies.

This thesis presents the study of an alternative optomechanical concept in which optical components are mounted on an ultra-stable bench using adjustable and freely positionable mounts, while maintaining picometer-level length stability. The concept, referred to as *toolset for adjustable picometer-stable interferometers* (TAPSI), enables the realization of diverse interferometer configurations and facilitates a simplified and faster assembly process. In addition, an optical test facility was constructed to operate the interferometer setup within a seismically isolated and thermally insulated environment. Furthermore, a temperature sensor was developed that enables the characterization of the μK -stable thermal conditions and investigations of temperature coupling to the interferometer.

This thesis characterizes the length stability of the toolset by setting up a prototype cavity and measuring its displacement noise using two laser-cavity locking techniques, Heterodyne laser frequency Stabilization (HS) and Pound-Drever-Hall (PDH) locking. While the HS scheme was limited by demodulation phase noise, PDH demonstrated picometer-level stability, meeting LISA requirements. The interferometer toolset was subsequently employed to realize and investigate a low-frequency homodyne quadrature interferometer displacement sensing concept, a strong candidate for precursor missions of future gravitational-wave detection concepts like the Lunar Gravitational-Wave Antenna (LGWA), designed to measure gravitational waves via lunar seismic motion. The sensor achieved picometer-level performance that meets the precursor mission (LGWA *Soundcheck*) requirements and was limited only by electronic readout noise. Another interferometer, referred to as the *Concealed Noise* experiment, was set up to characterize the noise floor of sub-picometer-precision heterodyne interferometers operating at MHz frequencies. Combined theoretical modeling and experimental validation of this setup will contribute to a comprehensive noise budget, advancing understanding of individual noise sources essential for risk mitigation in space-based missions like LISA and for the design of future missions with higher sensitivity.

In conclusion, the picometer-level stability and rapid experimental commissioning demonstrated in this thesis confirm the suitability of the toolset for realizing flexible optical ground support equipment setups for space-based gravitational-wave detectors and other applications requiring ultra-stable, low-frequency laser interferometry.

Kurzfassung

Weltraumbasierte Gravitationswellendetektoren wie LISA nutzen pikometergenaue Laserinterferometrie, um Gravitationswellen von 1 Hz bis unter 0.1 mHz zu detektieren. Diese hochpräzisen Observatorien ermöglichen vielfältige wissenschaftliche Untersuchungen, einschließlich detaillierter Studien zu Wachstumsmechanismen massereicher Schwarzer Löcher und ihrer zentralen Rolle bei der Galaxienbildung und kosmischen Evolution. Bodenbasierte Prototypen solcher Interferometer werden üblicherweise durch dauerhaftes Verkleben der Optiken auf einer ultrastabilen Bank aus Glaskeramik mit geringer Ausdehnung gebaut, um dominante Temperaturkopplungen bei niedrigen Frequenzen zu minimieren.

Diese Arbeit untersucht ein alternatives optomechanisches Konzept, bei dem optische Komponenten mittels verstellbarer und frei positionierbarer Halterungen auf einer ultrastabilen Bank montiert werden, wobei Pikometer-Längenstabilität gewährleistet bleibt. Das Konzept, bezeichnet als *Toolset für verstellbare pikometer-stabile Interferometer* (TAPSI), ermöglicht die Realisierung vielfältiger Interferometerkonfigurationen und bietet einen vereinfachten und beschleunigten Montageprozess. Zusätzlich wurde eine optische Testeinrichtung aufgebaut, um das Interferometer in einem seismisch isolierten und thermisch gedämmten Umfeld zu betreiben. Zudem wurde ein Temperatursensor entwickelt, der die Charakterisierung der μ K-stabilen thermischen Bedingungen ermöglicht sowie die Untersuchung der Temperaturkopplung an das Interferometer.

Diese Arbeit charakterisiert die Längenstabilität des Toolsets durch Aufbau eines Prototyp-Resonators und Messung des Rauschens seiner Resonatorlänge mittels zweier Laser-Resonator-Stabilisierungstechniken, Heterodyne-Laserfrequenzstabilisierung (HS) und Pound-Drever-Hall (PDH)-Stabilisierung. Während das HS-Schema durch Demodulationsphasenrauschen limitiert war, konnte PDH eine Pikometerstabilität erreichen, die LISA-Anforderungen erfüllen. Das Interferometer-Toolset wurde anschließend eingesetzt, um ein Konzept eines homodyn Quadratur-Interferometer-Wegsensors für niedrige Frequenzen zu realisieren und zu untersuchen, welches ein starker Kandidat für Vorgängermissionen zukünftiger Gravitationswellenkonzepte wie die Lunare Gravitationswellen-Antenne (LGWA) ist, die Gravitationswellen über lunare seismische Bewegungen misst. Der Sensor erreichte Pikometerstabilität, die die Anforderungen der Vorgängermission (LGWA *Soundcheck*) erfüllt, und war ausschließlich durch das Rauschen der Ausleseelektronik limitiert. Ein weiteres Interferometer, das *Concealed Noise*-Experiment, wurde aufgebaut, um den Rauschuntergrund sub-pikometerpräziser Heterodyn-Interferometer, die im MHz-Frequenzbereich arbeiten, zu charakterisieren. Theoretisches Modellieren in Kombination mit experimenteller Validierung dieses Aufbaus wird zu einem umfassenden Rauschbudget beitragen, um das Verständnis einzelner Rauschquellen zu vertiefen. Dies ist für die Risikominimierung von weltraumbasierten Missionen wie LISA und für die Konzeption zukünftiger Missionen mit höherer Empfindlichkeit von entscheidender Bedeutung.

Zusammenfassend zeigen die in dieser Arbeit nachgewiesenen Stabilitäten im Pikometerbereich sowie die schnelle experimentelle Inbetriebnahme die Eignung des Toolsets für die Realisierung flexibler interferometerprototypen für weltraumbasierte Gravitationswellendetektoren und andere Anwendungen, die ultra-stabile Laserinterferometrie bei niedrigen Frequenzen erfordern.

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Introduction to gravitational-wave detection

The pursuit of knowledge, driven by an innate curiosity to explore and describe the world around us, is a fundamental aspect of the human nature. This quest extends beyond our immediate surroundings to the vastness of space where early telescopes, limited to visible light, provided humanity's first window into the universe. Over time, foundational principles such as Newtonian mechanics and electromagnetic theory expanded our understanding of nature and enabled new observational tools. These advances enabled the development of telescopes capable of probing the electromagnetic spectrum and cosmic particles, which was only the beginning ...

In 1916, Albert Einstein predicted the existence of gravitational waves (GWs), ripples in the fabric of spacetime generated by accelerating massive objects like merging black holes [1]. Nearly a century passed before their first direct detection. Detecting gravitational waves requires sensing incredibly tiny distortions in spacetime, characterized by strains on the order of 10^{-21} . Early measurement approaches included resonant bar detectors [2] which, however, showed insufficient sensitivity. The challenge was overcome by the Laser Interferometer Gravitational-Wave Observatory (LIGO), a sophisticated experimental approach based on laser interferometry, in particular on the principle of a long-baseline Michelson interferometer [3] [4]. These stabilized laser interferometers enable ultra-precise measurements of length variations induced by passing gravitational waves.

Ground-based gravitational wave detectors (LIGO, Virgo [5], and KAGRA [6]) face sensitivity limits below $f = 10$ Hz due to environmental noise, motivating the development of gravitational-wave observatories in space, such as the European Space Agency (ESA)'s LISA mission. The Laser Interferometer Space Antenna (LISA) features three identical spacecraft arranged in a near-equilateral triangle, exchanging laser beams over 2.5×10^6 km, where precise heterodyne interferometry measures distances between free-floating test masses serving as inertial reference points. These space missions demand rigorous testing of all components, including optical benches that perform the critical distance measurements. This ground-based prototyping is accomplished by optical ground support equipment (OGSE), which comprises the optical bench and also the test facility, providing essential environmental isolation from terrestrial noise sources. The construction of such optical test benches is a resource-intensive and time-consuming process. This motivated the development of a flexible, ultra-stable laser interferometer test-bench to serve as OGSE for LISA, while also allowing implementation for sub-hertz interferometer concepts for other future missions. We developed a 'toolset for adjustable picometer-stable interferometers' (TAPSI) to accelerate the interferometer construction process, while also allowing flexible positioning and reuse of optical components [7]. Our OGSE, comprising the

optical test facility and the toolset, accelerates experimental research and thus makes a substantial contribution to gravitational-wave detection.

Advancing GW detection is essential for exploring the Universe in greater detail, since GWs provide a unique window into celestial phenomena involving extremely dense objects, such as black holes. These objects emit little or no electromagnetic radiation and thus remain "invisible" to conventional telescopes, yet become clearly detectable through gravitational-wave observatories [8]. Moreover, electromagnetic waves cannot probe the very early Universe before it became transparent to light. In contrast, gravitational waves can carry information directly from this primordial, opaque era, including the moments immediately after the Big Bang [8]. Unlike electromagnetic radiation, which can be absorbed, scattered, refracted, or gravitationally lensed, gravitational waves interact extremely weakly with matter. This allows them to propagate across the cosmos almost unimpeded, preserving pristine information about their sources. Consequently, gravitational-wave astronomy provides a fundamentally new window into the Universe, complementing electromagnetic observations by revealing the dynamics, structure, and origins of astrophysical objects that would otherwise remain hidden.

1.1 Gravitational waves

The field of gravitational wave research has its origin with the publication of Albert Einstein's General Theory of Relativity in 1916 [1]. Einstein predicted that accelerated masses generate ripples in spacetime, so-called *gravitational waves*, that propagate almost undamped through the Universe at the speed of light. These waves cause periodic stretching and compression of spacetime perpendicular to their direction of propagation. The detection of gravitational waves opens a fundamentally new observational window into the universe, allowing direct investigation of astrophysical and cosmological phenomena via measurable fluctuations of spacetime itself. To fully exploit this new observational domain, a comprehensive understanding of gravitational waves, their origins and fundamental properties is essential. As this thesis focuses on the experimental detection of GWs, this chapter only provides a concise summary of their generation mechanisms and potential sources. The structure of this section follows the treatment in [9].

1.1.1 GWs in general relativity

In the theory of special relativity [10], the interval ds between two events in spacetime is defined within a four-dimensional space consisting of three spatial and one temporal dimension (x , y , z , and t). This formulation replaces the classical Newtonian concepts of absolute space and time with a unified four-dimensional spacetime geometry, expressed as

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \quad (1.1)$$

where c denotes the speed of light, which remains invariant under coordinate transformation. The equation can equivalently be expressed in tensor notation as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \quad (1.2)$$

where $\eta_{\mu\nu}$ denotes the Minkowski metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (1.3)$$

The differentials dx^μ and dx^ν represent infinitesimal changes in the spacetime coordinates, where the indices μ and ν range from 0 to 3, and $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$. The generalization of special relativity, known as general relativity (GR), describes gravity not as a force but as a manifestation of the curvature of spacetime induced by mass and energy. In this framework, spacetime is no longer flat, and Equation 1.2 can therefore be expressed in a more general form as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (1.4)$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (1.5)$$

represents a decomposition into the flat Minkowski metric $\eta_{\mu\nu}$ and a small perturbation $h_{\mu\nu}$. General relativity applies to all reference frames, including accelerating and non-inertial ones, allowing the use of coordinates that follow the trajectory of a free-falling mass (a locally inertial or free-falling frame). In such frames, Einstein's field equations¹ becomes a wave equation for the metric perturbation, expressed as

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0. \quad (1.6)$$

From this relation, it follows that the spacetime perturbation $h_{\mu\nu}$ can be expressed as a plane wave² of the form $h_{\mu\nu} \propto \exp[i(\omega t - kz)]$, propagating at the speed of light. This perturbation is known as the *gravitational wave*. For a gravitational wave traveling in the z -direction, the corresponding metric perturbation exhibits two independent polarization states, denoted by h_+ and $h_\times$, and can be written as

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1.7)$$

As the gravitational wave passes, it induces alternating stretching and compression of spacetime, causing relative displacements between the test masses (see Fig. 1.1). This effect is quantified by the dimensionless strain $h(t) = \Delta L/L$, representing the fractional change in their initial separation L . For example, consider a GW with purely h_+ polarization³, for which the spacetime interval becomes

$$ds^2 = -c^2 dt^2 + (1 + h_+) dx^2 + (1 - h_+) dy^2 + dz^2. \quad (1.8)$$

¹ Weak field slow motion approximation of Einstein's field equations in vacuum (see [11] p.81 for a detailed derivation or [12]).

² With $f = c/\lambda = c/(2\pi/|k|) = |k|/2\pi c$ and $\vec{k} = k/|k|$ denoting the direction of propagation.

³ The relative orientation of the detector and the gravitational wave is chosen such that the detector responds exclusively to the h_+ polarization mode.

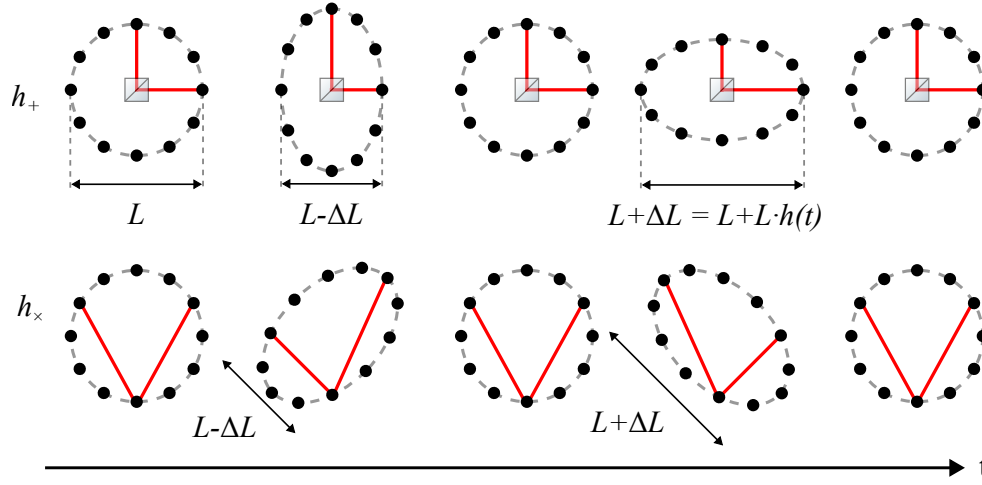


Figure 1.1: Illustration of a gravitational wave with two polarization states, h_+ and $h_\times$, passing through a ring of free-floating test masses. The unperturbed separation L between test masses undergoes periodic stretching and compression, such that the perturbed length becomes $L(1 + h(t))$, corresponding to a strain $h(t) = \Delta L/L$.

The corresponding strain results from the compression along the y -direction and simultaneous stretching along the orthogonal x -direction, yielding

$$h(t) = 2 \frac{\Delta L(t)}{L}. \quad (1.9)$$

1.1.2 Coalescence of binaries and other sources of gravitational waves

Coalescence of binary systems

The first confirmed detection of a gravitational wave was the merger of two black holes with masses $36 M_\odot$ and $29 M_\odot$ by LIGO and is shown in Fig. 1.2. The detected signal shows a typical binary coalescence that proceeds through three distinct phases: inspiral, merger, and ringdown. During the inspiral phase, the two massive objects orbit each other at gradually decreasing separation. According to GR (see Section 1.1.1), they lose orbital energy and angular momentum through the emission of gravitational waves, causing their orbit to decay until merger. The corresponding GW signal exhibits a characteristic *chirp*, a monotonic increase in both frequency and amplitude⁴. Following the merger, the remnant (typically a single black hole) undergoes the ringdown phase, during which it settles into a quasi-stable object.

Generalized equations of binary systems

The orbital frequency f_{orb} of two masses orbiting each other depends on their total mass $M = m_1 + m_2$ and orbital separation r , and follows Kepler's third law for circular orbits [14]. The GW frequency

⁴ The term *chirp* describes the distinctive waveform produced as two massive objects spiral inward and merge, with signal frequency and amplitude both increasing over time, somewhat like the sound of a bird chirping [13].

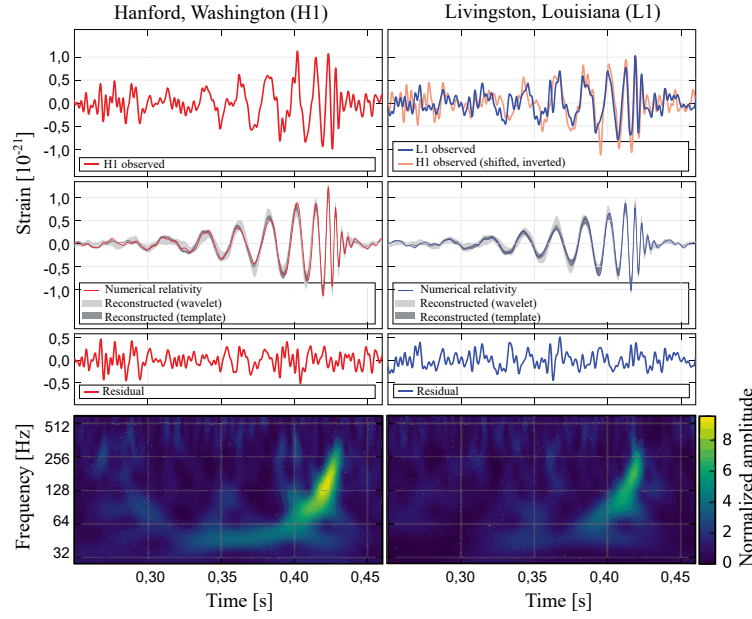


Figure 1.2: LIGO's first confirmed gravitational wave detection (14.09.2015) from a binary black hole merger (GW150914), shown alongside the corresponding theoretical prediction. The characteristic 'chirp' signal, exhibiting increasing frequency and amplitude, is clearly visible in both detectors during the binary coalescence. The Livingston (L1) detector signal appears in the right-hand panel, while the Hanford (H1) detector signal is shown on the left-hand panel. (Image credit: LIGO collaboration [3]).

f_{GW} arises from time variations in the mass quadrupole moment of the source⁵. For a circular binary, the quadrupole repeats every half-orbit period, yielding a dominant GW frequency that is twice the orbital frequency:

$$f_{\text{GW}} = 2f_{\text{orb}} = \frac{1}{\pi} \sqrt{\frac{GM}{r^3}}, \quad (1.10)$$

where $G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ denotes the gravitational constant. As GWs carry away energy, linear and angular momentum, the orbital separation decreases and the frequency $f_{\text{GW}}(t)$ increase over time, producing the characteristic chirp signal [15]:

$$f_{\text{GW}}(t) = \frac{1.8}{8\pi} \left(\frac{c^3}{G\mathcal{M}_c} \right)^{-5/8} (t_c - t)^{3/8}, \quad (1.11)$$

where t denotes the time before the coalescence t_c , and $\mathcal{M}_c$ is the chirp mass⁶, defined as

$$\mathcal{M}_c = \frac{(m_1 + m_2)^{3/5}}{(m_1 + m_2)^{1/5}}. \quad (1.12)$$

⁵ According to GR, isolated systems conserve mass-energy and momentum, eliminating monopole and dipole gravitational radiation. The quadrupole moment thus dominates.

⁶ The chirp mass $\mathcal{M}_c$ governs the inspiral chirp rate of the binary system and emerges naturally from the leading-order gravitational wave phase.

The time derivative $\dot{f}_{\text{GW}}$ ⁷ depends both on the chirp mass $\mathcal{M}_c$ and the instantaneous frequency $f_{\text{GW}}(t)$. This makes it possible to measure $\mathcal{M}_c$ directly from observational data. Furthermore, the luminosity distance r to a GW source can be determined directly from the observed signal evolution, including its time-dependent strain amplitude, frequency and phase. This becomes more complex for cosmologically redshifted signals⁸ as described in [16, 17]. The emitted GW strain for a compact binary inspiral is given by the leading-order quadrupole approximation [18]:

$$h_+(t) = \frac{4}{r} \left(\frac{G\mathcal{M}_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{\text{GW}}}{c} \right)^{2/3} \frac{1 + \cos^2 \theta}{2} \cos(2\pi f_{\text{GW}} t) \quad (1.13)$$

$$h_\times(t) = \frac{4}{r} \left(\frac{G\mathcal{M}_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{\text{GW}}}{c} \right)^{2/3} \cos \theta \sin(2\pi f_{\text{GW}} t) \quad (1.14)$$

where θ is the inclination angle between the binary's orbital plane and the observer's line of sight.

Sources of gravitational waves

Compact binary coalescences, such as binary black holes (BBH), binary neutron stars (BNS), and black hole-neutron star (BHNS) binaries, constitute the only confirmed sources detected by ground-based gravitational-wave observatories [19]. The detected component masses range from $\sim 1.2 M_\odot$ (BNS) [20] to total masses of $\sim 190\text{--}265 M_\odot$ (BBH) [21]. Their dominance stems from high mass density and tight orbits that generate intense GW emission during inspiral and merger within the sensitive frequency band ($\sim 10\text{ Hz--}1\text{ kHz}$) of current detectors [5, 6, 22]. Additionally, well-developed theoretical modeling and extensive matched-filter template banks efficiently identify their characteristic chirp signals.

However, other astrophysical sources such as (core-collapse) supernovae⁹, pulsars¹⁰ and the stochastic gravitational-wave background¹¹, remain promising targets that could probe diverse astrophysical phenomena. As gravitational waves from different sources span many orders of magnitude in frequency and amplitude, specialized detection methods (detectors) are required, each optimized for particular frequency bands. Figure 1.3 illustrates the landscape of detected and expected sources and their corresponding detectors. The focus of this thesis is on space-based observatories such as LISA and LGWA, designed for low-frequency gravitational-wave detection (0.1 mHz–1 Hz). These missions will observe high-mass, high-redshift sources, including supermassive black hole binary mergers.

⁷ $\dot{f}_{\text{GW}} = \frac{96}{5} \pi^{8/3} \left(\frac{G\mathcal{M}_c}{c^3} \right)^{5/3} f_{\text{GW}}^{11/3}$ derived from the leading-order post-Newtonian approximation [15].

⁸ Multiple $(\mathcal{M}_{c,\text{source}}, z)$ pairs can produce the same $\mathcal{M}_{c,\text{obs}}$. EM counterparts can help break the mass-redshift degeneracy.

⁹ A core-collapse supernova occurs when a massive star ($\gtrsim 8 M_\odot$) exhausts its nuclear fuel, leading to implosion and subsequent luminous explosion [23].

¹⁰ Pulsars are rapidly rotating, highly magnetized neutron stars that emit beams of electromagnetic radiation from their magnetic poles, which are observed as regular pulses when the beams sweep across the Earth [24].

¹¹ The stochastic background arises from the superposition of many weak, unresolved GW sources, including cosmological relics from the Big Bang [25].

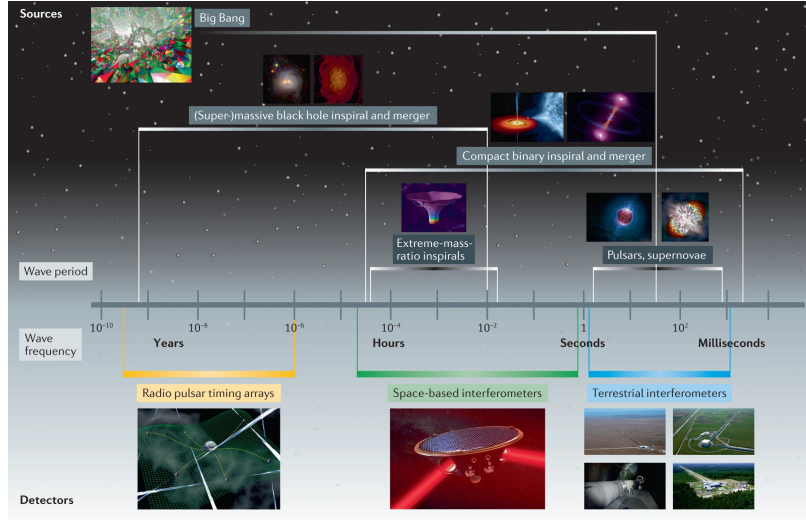


Figure 1.3: The gravitational wave spectrum, illustrating astrophysical sources alongside their targeted detectors and corresponding sensitivity bands. (Image credit: [26]).

1.2 Laser interferometric gravitational-wave detection

Gravitational waves induce spacetime ripples that alter the distance between free-falling (geodesic) test masses. Interferometric gravitational-wave detection (GWD) exploits this effect by measuring the relative displacements between free-falling test masses (reference objects) using laser interferometry. The first direct GW detection was made in 2015 by LIGO¹². Current GW observations rely exclusively on ground-based detectors, which employ pendulum-suspended mirrors as quasi-inertial test masses. These operate on the principle of a long-baseline Michelson interferometer (MI)¹³ with perpendicular arms typically several kilometers in length. When a GW passes the detector, it induces a differential arm-length change ΔL . In the case of a specific polarization and orientation relative to the detector arms, the GW stretches one arm while compressing the other. This induces a phase shift between the recombined laser beams:

$$\Delta\phi = \frac{2\pi}{\lambda}c \int_{t-T}^t h(t')dt' \approx \frac{2\pi}{\lambda}L \cdot h(t), \quad (1.15)$$

where λ is the laser wavelength, L is the arm length, $T = L/c$ is the light travel time, and the factor of 2 accounts for the round-trip light path. Upon recombination at the beamsplitter, the differential phase shift $\Delta\phi$ is converted into an amplitude modulation at the output port, where the measured power is directly proportional to the GW strain $h(t)$.

As of today, three ground-based GW detectors have successfully detected GWs: Virgo in Italy [27] and the two LIGO observatories in the United States, one in Livingston (Louisiana) and the other in Hanford (Washington) [22]. All three feature perpendicular arms (Virgo:3 km, LIGO:4 km), advanced

¹² The Laser Interferometer Gravitational-Wave Observatory (LIGO) detected the first GW signal (GW150914) on 14.09.2015. The signal is shown in Fig. 1.2 [3].

¹³ See Section 2.2.1 Homodyne interferometry.

sensitivity-enhancement techniques, including arm-cavities, power- and signal-recycling mirrors and seismic isolation systems [28]. Two additional terrestrial detectors are in operation: GEO600 in Germany [29] and Kamioka Gravitational-Wave Detector (KAGRA) in Japan [30]. GEO600's 600 m arm length limits its sensitivity, positioning it primarily as a technology testbed¹⁴. KAGRA, the world's first underground GW detector employing cryogenic mirrors, continues sensitivity improvements but has not yet contributed significantly to detection catalogs [6]. Having a network of detectors spread around the globe improves both the localization of sources and confirmation of genuine GW signals. Future planned ground-based detectors include LIGO-India, a near-identical copy of the existing LIGO observatories, and the Einstein Telescope (ET)¹⁵ from the European community, and the U.S. Cosmic Explorer (CE)¹⁶.

Ground-based detectors, such as those mentioned above, are limited by fundamental environment noise sources below a few Hz. These unshieldable¹⁷ terrestrial noise sources are seismic vibrations, gravity gradient noise (Newtonian noise), and thermal noise. At mid to high frequencies ($\sim 1 \times 10^2$ Hz up to a few kHz), photon shot noise dominates sensitivity. Frequency-dependent squeezed light [32] and increased laser power enhance performance but introduce challenges such as thermal lensing and quantum radiation pressure noise¹⁸. In contrast, space-based detectors avoid terrestrial noise sources, enabling gravitational-wave observations in the millihertz regime. Additionally, they allow significantly longer arm lengths, whereas realizing comparable baselines on the ground would incur prohibitive costs and would ultimately be constrained by the curvature of the Earth. For instance, the planned LISA mission (see Section 1.2.1.1) will feature 2.5×10^6 km arm lengths. Beyond LISA, the development of several space-based detectors is ongoing, such as Taiji [33], TianQin [34] [35], DECIGO [36], and the Beyond-LISA mission [37].

1.2.1 Laser Interferometer Space Antenna

The Laser Interferometer Space Antenna (LISA) mission, proposed by the international LISA consortium in 2017 [40], is a space-based gravitational-wave observatory led by ESA in collaboration with its member states and NASA. LISA will probe the millihertz band (~ 0.1 mHz to 1 Hz), bridging the nanohertz regime of pulsar timing arrays [41] and the decahertz band of ground-based detectors (see Fig. 1.4). This complementary network will enable multi-messenger tests of GW propagation and GR [42]. LISA will offer unique sensitivity to massive black hole binaries in the mass range $\sim 10^4 - 10^7 M_\odot$, with some sources expected to yield signal-to-noise ratios approaching $\mathcal{O}(10^3)$. These high-precision observations will enable diverse science cases, including detailed studies of massive black hole growth mechanisms and their pivotal role in galaxy formation and cosmic evolution [43].

¹⁴ GEO600 is currently being upgraded to observe gravitational waves at very high frequencies [31].

¹⁵ The ET will be located underground for superior seismic isolation and is planned with 10 km arm length. The detector configuration remains under discussion but will feature either an L-shaped or a triangle geometry.

¹⁶ The Cosmic Explorer employs a Michelson interferometer (MI) design similar to LIGO but with significantly longer arms: 40 km and 20 km for its two planned detectors.

¹⁷ The term 'unshieldable' signifies that, while noise sources can be mitigated, they cannot be entirely prevented.

¹⁸ Radiation-pressure noise dominates at low frequencies when using phase-squeezed light, as anti-squeezing appears in the amplitude quadrature. Frequency-dependent squeezing, implemented via filter cavities, rotates the squeezing ellipse to optimize noise reduction across the detection band.

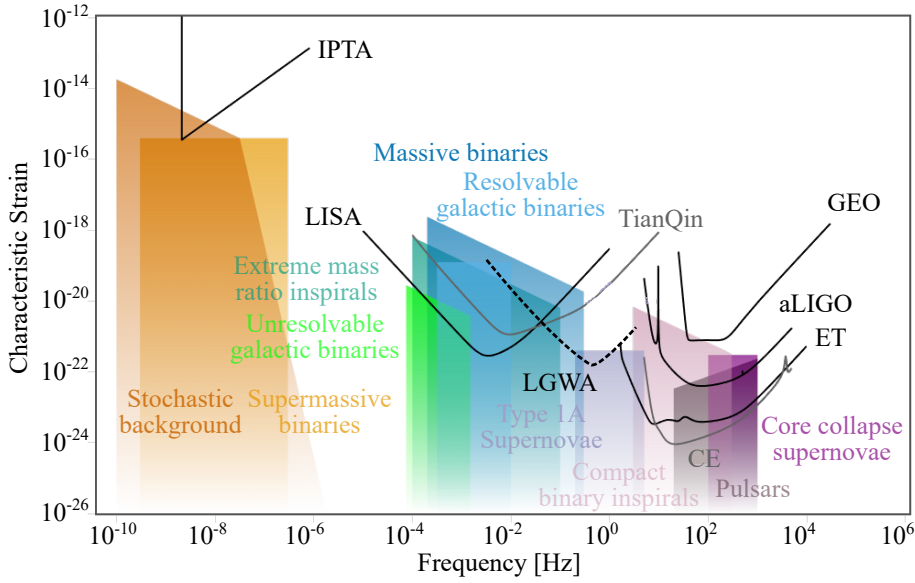


Figure 1.4: Design sensitivity curves for current (LIGO) and future (LISA, ET) gravitational-wave detectors alongside predicted astrophysical sources. Strain sensitivity (y-axis) is shown versus frequency (x-axis), spanning high-frequency ground-based detectors, millihertz space-based detectors like LISA, and nanohertz pulsar timing arrays (IPTA). (Plot created with: <https://gwplotter.com/>, credit: [38], LGWA added based on [39]).

1.2.1.1 LISA mission and instrument concept

The Laser Interferometer Space Antenna will consist of three identical spacecraft arranged in a near-equilateral triangle, exchanging laser beams over 2.5×10^6 km-long arms. Precise heterodyne laser interferometry will measure distances between free-floating test masses protected by the spacecraft from non-gravitational disturbances. LISA will continuously measure the strain induced by passing gravitational waves, allowing precise tracking of their amplitude, frequency, phase, and polarization as functions of time.

In each of the three spacecraft, two free-falling test masses¹⁹ serve as geodesic reference objects, shielded from external forces, such as solar radiation pressure and micrometeoroids. To achieve the required free-fall fidelity, the test masses are housed in vacuum within a cubic conductive gravitational reference system (GRS), which functions both as a capacitive position sensor and electrostatic force actuator. The employed technique, known as drag-free attitude control system (DFACS), measures the position and orientation of the test masses via a combination of capacitive and optical sensors and corrects them using a capacitive actuation system or spacecraft orientation²⁰ [44]. The primary performance constraints arise from internal acceleration noise on the test masses (force disturbances) such as residual gas pressure, thermal radiation pressure, electrostatic forces and magnetic forces, limiting low-frequency sensitivity. The test-mass acceleration noise requirements are at the femto-g level ($1 \times 10^{-14} \text{ m}/(\text{s}^2 \sqrt{\text{Hz}})$) at 0.1 Hz [45]. LISA Pathfinder (2017) has already successfully

¹⁹ The test masses are 46 mm cubes of a gold-platinum alloy.

²⁰ The DFACS controls the spacecraft via micropropulsion to follow the free-falling test mass along the sensitive axis in each LISA spacecraft, thereby achieving drag-free motion for the two test masses.

demonstrated this level of residual acceleration between two free-falling test masses in a single spacecraft [46].

LISA employs a transponder scheme, in which each spacecraft consists of two moving optical sub-assemblies (MOSAs). The MOSA comprises the telescope, optical bench, and the GRS with the free-falling test masses. Each MOSA transmits a high-power laser beam (2 W , $\lambda_{\text{Laser}} = 1\,064\text{ nm}$) to the distant spacecraft's MOSA, while simultaneously receiving a small fraction of the returning beam from that spacecraft. The relative phase between the transmitted and received beam is interferometrically measured on the optical bench of each MOSA. In total three interferometers are required to support the overall mission concept. First, the inter-satellite interferometer (ISI) measures the relative motion of the optical bench along the arm axis with respect to the incoming wavefront from the distant spacecraft. Second, the test mass interferometer (TMI) measures the relative longitudinal motion of the optical bench with respect to the free-falling test mass. The third interferometer is the reference interferometer (RFI), which measures the relative phase fluctuations between the local laser of one MOSA and that of the adjacent MOSA on the same spacecraft. Laser light exchange between the two benches is accomplished via an optical fiber, denoted as the *backlink fiber* [47]. The schematic of the LISA constellation and the three interferometers in a single arm are shown in Fig. 1.5 and 1.6.

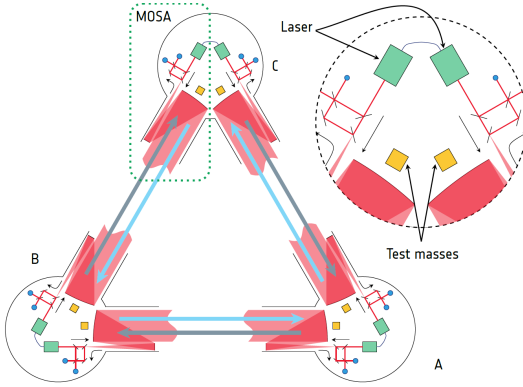


Figure 1.5: Schematic of the LISA constellation and the spacecraft to spacecraft measurement. Each spacecraft consists of two moving optical sub-assemblies (MOSAs), including the telescope, the optical bench, and the GRS with free-falling test masses. (Image credit: [45]).

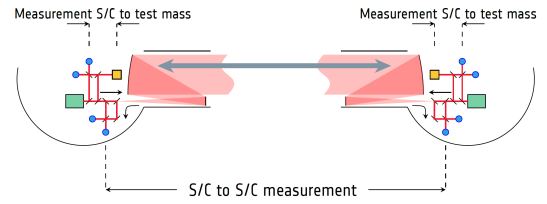


Figure 1.6: Simplified scheme of the full measurement of a single arm, divided into three segments: test mass to spacecraft (S/C), spacecraft to spacecraft, and spacecraft to test mass. On-ground data processing yields the corresponding test mass to test mass distance measurement. (Image credit: [45]).

The interferometer beat notes, in the range of $6\text{--}25\text{ MHz}$ ²¹, including the GW signals²², are measured by quadrant photodetectors. These signals are then forwarded to the phase measurement subsystem, which performs both the phase readout and auxiliary control functions²³. The phasemeter must not only track the heterodyne beat note to extract the GW signal with high fidelity, but also process additional auxiliary beat notes used for clock-transfer corrections²⁴ as well as ranging and data transfer [48]. The

²¹ The beat note frequency depends on the Doppler shift and the relative intensity noise spectrum.

²² The GW signal appears as a phase fluctuation in the weak beam arriving from the distant spacecraft.

²³ Auxiliary control uses the quadrant photodiodes to sense the beam pointing (tilt) of the incoming local oscillator beam, providing attitude control signals for the DFACS for maintaining telescope alignment.

²⁴ The pilot-tone correction method compensates for inherent sampling jitter in each Analog to digital converter (ADC).

individual phase measurements are continuously transmitted to Earth for on-ground data processing. These measurements contain the desired GW signal but also dominating instrumentation noise and large, slowly varying phase drifts caused by the long-term orbital motion and the associated Doppler shifts²⁵. Because the phase measurements are taken between separate spacecraft (MOSAs) there is no common-mode rejections as in balanced Michelson interferometers with equal arm length. Consequently, noise sources such as laser frequency noise (LFN) and clock phase noise would obscure any GW signal. However, since each laser pair interferes at both ends of an arm, every signal contains a time-delayed copy of the laser noise from the distant spacecraft. By combining these signals with properly chosen time delays, the LFN can be effectively cancelled while preserving the GW signal. This technique, known as time-delay interferometry (TDI), enhances sensitivity by several orders of magnitude [49]²⁶.

The noise budget of LISA's strain sensitivity, expressed in terms of amplitude spectral density (ASD), is shown in Fig. 1.7. At low frequencies ($f < 3$ mHz) the sensitivity is limited by acceleration noise, i.e., by the purity of the test masses' free fall. LISA Pathfinder has already demonstrated free-falling test masses in space, even surpassing the acceleration noise requirement for LISA [46]. In the mid-frequency range ($3 \text{ mHz} < f < 30 \text{ mHz}$), the sensitivity is constrained by the interferometric measurement of the relative motion between test masses. The dominant noise sources here are tilt-to-length coupling (TTLC) (see Section 3.5.2) and photon shot noise. The latter arises from the low power of the receiving beam from the distant spacecraft²⁷. This weak beam, containing the GW signal, is read out through interference with the local oscillator beam. At high frequencies, the sensitivity is limited by the 2.5×10^6 km arm length, which both suppresses the GW response through averaging along the long arms²⁸ and renders the shot noise non-white due to the frequency-dependent interferometric transfer function.

While LISA Pathfinder has already characterized test-mass acceleration noise to the required level, LISA's total interferometer noise performance still requires further investigation through dedicated testing with optical ground support equipment (OGSE).

1.2.1.2 Optical ground support equipment - Construction of the optical benches

At the core of each spacecraft (MOSA), is the optical bench comprising the individual interferometers. These measure changes in the beat note frequency and ultimately convert them into relative displacement. The required displacement sensitivity of the LISA optical system, denoted $u(f)$, is specified for the sub-Hz regime as [50]

$$u(f) = 1 \frac{\text{pm}}{\sqrt{\text{Hz}}} \times \sqrt{1 + \left(\frac{2 \text{ mHz}}{f}\right)^4} \quad (10^{-4} \text{ Hz} < f < 1 \text{ Hz}). \quad (1.16)$$

²⁵ The Doppler-shift arises by the relative motion of the spacecraft, which have line-of-sight velocities of approximately ± 10 m/s with respect to one other.

²⁶ TDI suppresses LFN by approximately 10^9 and clock phase noise by 10^6 .

²⁷ The received beam is expected to have a power of only a few hundred picowatts. Due to beam divergence, the wavefront arriving from the distant spacecraft spans several kilometers, and only a small fraction is captured by the 30 cm telescope aperture.

²⁸ The corresponding round-trip light-travel time of about 16.66 s defines a characteristic frequency near 0.06 Hz, where the first response minimum occurs as the GW period matches the round-trip light time. This feature appears in Fig. 1.7 as the first *wiggle*.

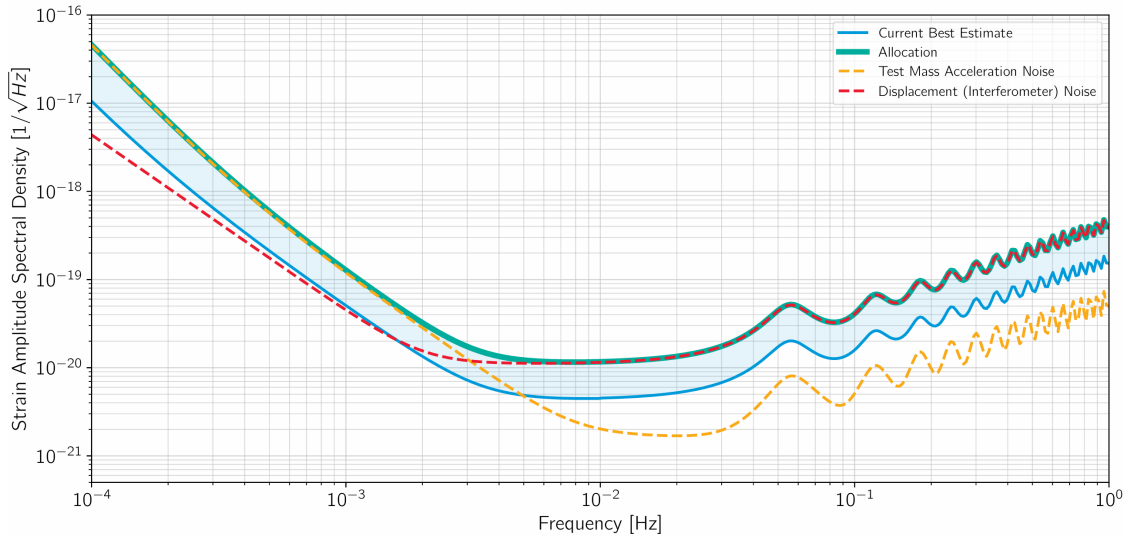


Figure 1.7: Noise budget for LISA’s strain sensitivity. The design allocation is shown alongside the current best estimate (noise prediction), derived from the latest subsystem designs, models, and test results. (Image credit: [45])

To achieve the required low-frequency displacement sensing noise, the optical benches for LISA will be constructed by hydroxide-catalysis bonding monolithic optics onto a low-expansion glass ceramic (e.g., Zerodur²⁹). This design minimizes the dominant low-frequency temperature coupling and is also used in optical flight systems, for example to withstand strong forces during rocket launch. The design and stability of these interferometers are tested beforehand on the ground using optical ground support equipment (OGSE), which generally comprises both the optical bench and the host optical test facility. These facilities are seismically isolated and thermally insulated vacuum chambers that provide a temperature-stable environment ($\sim \mu\text{K}/\sqrt{\text{Hz}}$).

The optical bench bonding process changed substantially over the past years since LISA Pathfinder [51]. Early alignment relied on manual methods or adjustment screws (see Fig. 1.8), whereas modern benches employ automated, robot-assisted techniques [52, 53] (see Fig. 1.9). In this process, components are held and precisely tilted by a hexapod with six degrees of freedom. A vertically passing beam monitors vertical angle and separation, while a horizontally reflected beam ensures pointing and positioning accuracy (see Fig. 1.10). This process achieves the required optical bench tolerances of $\pm 30 \mu\text{m}$ in position and $\pm 35 \mu\text{rad}$ in angle [54].

Precision bonding of optical components has become significantly faster over the years. State-of-the-art methods now complete the precision bonding of a single component in under 30 minutes [52].

OGSEs, particularly the optical bench test beds, investigate interferometer configurations and critical effects such as LISA imaging systems, tilt-to-length coupling, polarization, and spurious light beam mitigation [47, 55–58]. Furthermore, performance verification requires picometer-stable optical benches. Key tests include, e.g., *Three-Backlink Experiments* [59, 60] and characterization of phasemeter phase extraction linearity and noise contributions [61–63]. This shows that OGSEs play a pivotal role within the LISA mission. They enable a comprehensive investigation that

²⁹ The coefficient of thermal expansion (CTE) of Zerodur is on the order of $1 \times 10^{-9} \text{ 1/K}$.

ensures a thorough understanding of noise sources and their contribution to the overall interferometer performance, which is essential for further risk mitigation of the space mission. However, the construction of such optical test beds is an expensive and time-consuming process, necessitating the use of alignment machines and optical readouts during the bonding stage. Moreover, the stringent constraints imposed by the LISA bench prohibit any margin for errors.

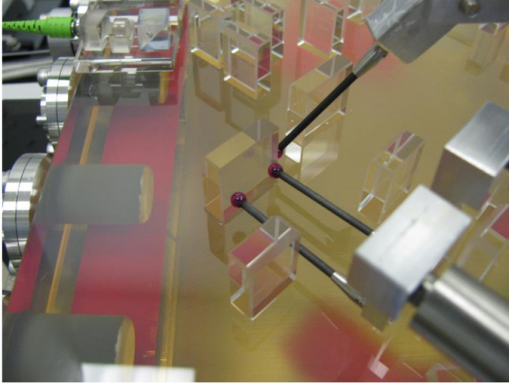


Figure 1.8: Photograph of the alignment for the LISA Pathfinder optical bench. 'Pointy-fingers' constrain the position of optical components during alignment. (Image credit: [64]).

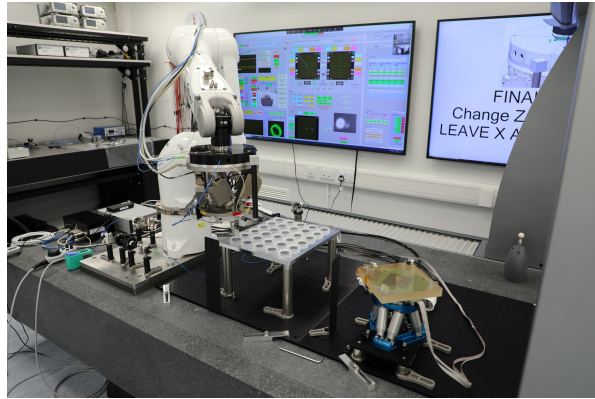


Figure 1.9: Robotically assisted bonding at the UK ATC in Edinburgh. (Image Credit: Science and Technology Facility Council: [65]).

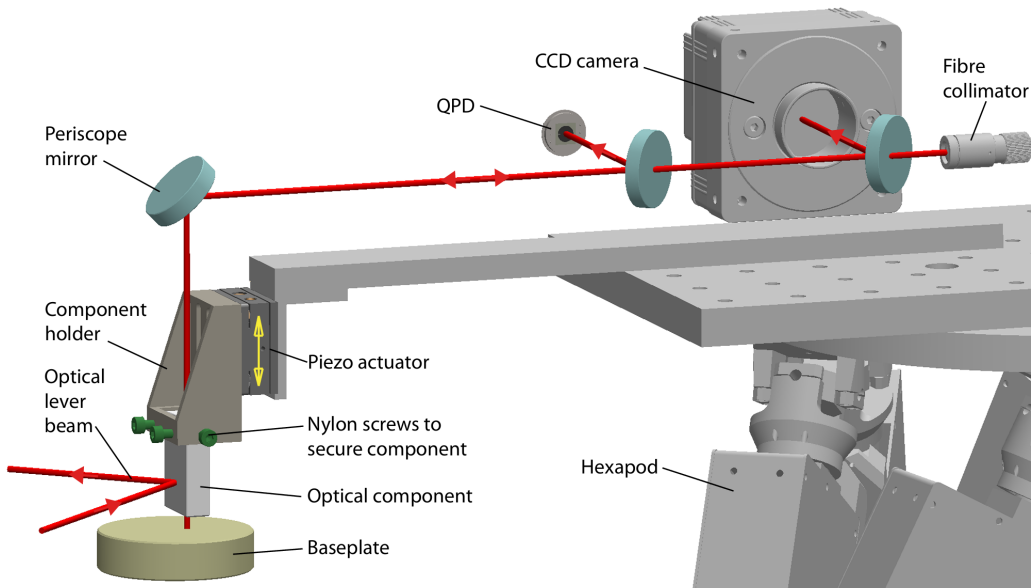


Figure 1.10: Schematic of an optical component being precision-bonded to a baseplate using a hexapod and alignment beams. The vertical and angular alignment of the optic is measured by a laser beam passing vertically through the component, while pointing and positioning is measured by a second beam in horizontal reflection (optical lever beam). The figure demonstrates the effort required to construct the optical benches of LISA's OGSE. (Image credit: [52])

1.2.2 Lunar Gravitational-wave Antenna

The Lunar Gravitational-Wave Antenna (LGWA) is a proposed mission concept that uses the Moon as a large-scale gravitational wave detector [39]. It aims to measure lunar vibrations caused by gravitational waves across the 1 mHz to 1 Hz frequency band, with peak sensitivity in the decihertz band at 0.1 Hz [66]. This frequency range bridges space-based detectors such as LISA (below decihertz frequencies) and terrestrial observatories (above a few Hz), including future detectors such as the Einstein Telescope and Cosmic Explorer. The LGWA builds on the work of Joseph Weber in the 1960s [2]³⁰, who attempted to measure the excitation of eigenmodes of an elastic body by GWs. Unlike his laboratory-scale experiment, which aimed to measure GWs in the kHz range, the LGWA will measure the Moon's eigenmodes below 1 Hz (1 mHz-1 Hz).

The LGWA will measure gravitational-wave signals as differential motion between a suspended inertial test mass and the rigid frame attached to the lunar surface, which responds elastically to passing GWs. It will deploy a kilometer-scale, star-shaped array of at least four³¹ cryogenic inertial sensors in a permanently shadowed lunar South Pole crater to minimize thermal and seismic noise (see Fig. 1.11). These sensors must detect test-mass accelerations or displacements with sub-femtometer precision. The lower frequency bound is set by the Moon's lowest-order quadrupole normal mode, while the upper bound ($f \gtrsim 1$ Hz) is limited by the weak lunar response to gravitational waves. At these high frequencies, instrumental and seismic background noise are expected to dominate the LGWA signal. To demonstrate and *check* these next-generation inertial sensors, the *Soundcheck* pathfinder will test them in the same frequency band as LGWA³². The conceptual overview of the Soundcheck seismic station is shown in Fig. 1.12. Requiring sub-picometer precision, the Soundcheck pathfinder mission motivates the development and testing of high-precision optical displacement sensors, such as low-frequency homodyne quadrature interferometers (see Section 8). The pathfinder and the subsequent LGWA mission demand picometer-stable optical benches and interferometry comparable to LISA's, underscoring the need for versatile OGSEs.



Figure 1.11: Star-like deployment configuration of four LGWA seismic stations in a kilometer-scale array. A laser-power beaming system is shown as a possible power system. (Image credit: [68]).

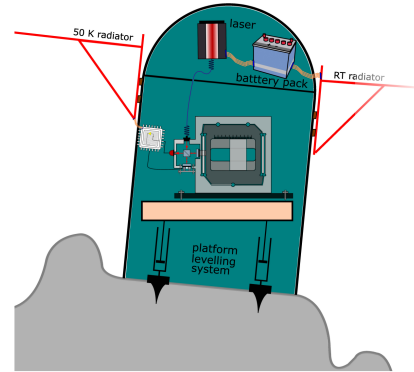


Figure 1.12: Conceptual overview of a Soundcheck seismic station on a tilted surface on the lunar regolith. (Image credit: [39]).

³⁰ Weber's bar detectors reached strain sensitivities of $\sim 10^{-16}$ at ~ 1 kHz, far above typical GW strain of $h \lesssim 10^{-21}$.

³¹ Although a single sensor suffices in theory, an array enhances SNR and helps to distinguish between local events like meteor impacts.

³² The mission was selected by the ESA in 2023 for the Reserve Pool of Science Activities for the Moon [67].

1.3 Thesis structure

This thesis presents the development of an optical test facility with a toolset for adjustable picometer-stable interferometers for space-based gravitational wave detectors. The toolset is also applicable to other domains requiring ultra-precise sub-Hz interferometry, such as satellite geodesy missions [69] and dark matter research [70]. This thesis reports on the development and length-stability characterization of the toolset, as well as its utilization for setting up further experiments. The developed toolset enables the implementation of diverse interferometer configurations; therefore, the first part of this thesis is presented in greater detail to establish the knowledge foundation for a wide range of concepts, with particular focus on heterodyne interferometers operating in the MHz range.

Part I provides the conceptual framework and terminology for discussing interferometer design, readout and noise mitigation in subsequent sections. It begins in Chapter 1.2 with a brief introduction to gravitational-wave detection. Chapter 2 provides an overview of homodyne and heterodyne interferometers, optical cavities, and laser frequency stabilization, and is complemented by an introduction to light field modulation. Chapter 3 introduces the fundamental noise sources relevant to this work. This includes intrinsic interferometer noises such as laser frequency and shot noise, spurious light signals, thermal and temperature noise, as well as fundamental electronic and readout noises. This detailed treatment is essential to enable diverse interferometer configurations, as demonstrated in Part III (optical cavities, homodyne (LF-HoQI), and heterodyne interferometer (Concealed Noise)).

Part II details the development of the optical ground support equipment. Chapter 4 presents the core of this thesis, the toolset for adjustable picometer-stable interferometers (TAPSI), covering its conceptual design, structural layout and materials, dominant noise sources (thermal expansion and tilt-to-length coupling), and the theoretical noise budget. Chapter 5 describes the construction of the optical test facility with emphasis on thermal insulation. It further details the development of a vacuum-compatible, microkelvin-precise temperature sensor and its associated readout electronics.

Part III presents the experimental validation and displacement sensing experiments. Chapter 6 characterizes the length stability of the toolset using optical cavities and two distinct laser-cavity locking schemes, the Heterodyne laser frequency Stabilization (HS) and the Pound-Drever-Hall (PDH) locking. The chapter discusses the obtained results and analyzes associated noise limitations. In the subsequent chapters, the interferometer toolset is employed to set up and investigate two distinct low-frequency interferometers. Chapter 7 introduces the *Concealed Noise project*, which aims to characterize the noise floor of heterodyne interferometers with sub-picometer precision operating at MHz frequencies through combined theoretical modeling and experimental validation. Here, only the initial setup and characterization of the experiment are described. The full exploitation of the experiment will be part of future research. Chapter 8 details the construction and testing of a displacement sensor concept, the low-frequency homodyne quadrature interferometer (LF-HoQI), which represents a strong candidate for precursor missions of future gravitational-wave detection concepts like the LGWA Soundcheck mission.

The final Chapter 9 summarizes the results of this thesis and outlines future work.

Part I

Laser Interferometry and Noise Sources

Basics of laser interferometry

Interferometric gravitational-wave detection employs laser interferometers to perform distance measurements. The laser wavelength, in the micrometer regime, serves as a fundamental unit of length, effectively transforming a laser beam into an optical ruler. The exceptional frequency stability and spatial coherence of lasers make them an ideal tool for measuring distances with high precision. This chapter introduces the essential concepts of laser interferometry that form the foundation for the subsequent chapters. It begins with a mathematical description of optical signals, followed by a brief introduction of the laser interferometers employed in this work. Subsequently, the chapter discusses various light-field modulation techniques as well as the nonplanar ring oscillator (NPRO) laser source used in the experiments (Part III).

2.1 Optical signals

A monochromatic laser beam propagating in free space along the z -direction can be represented by a complex electric field in the form of a plane wave, expressed as

$$\vec{E}(x, y, z, t) = \vec{E}_0 e^{i(\omega t - \vec{k}z + \phi)}, \quad (2.1)$$

where $\omega = 2\pi f$ denotes the angular frequency, t the time, $k = 2\pi/\lambda$ the wave vector, and ϕ the phase. The electric field amplitude vector, denoted by $\vec{E}_0$, also encodes the polarization¹. It should be noted that the physically measurable field corresponds to the real part of this complex representation. For convenience, the complex form is used in analytical calculations throughout this thesis.

Optical power measured by the photodiode

For an electromagnetic wave propagating in vacuum, the Poynting vector represents the directional energy flux density and is defined as

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}), \quad (2.2)$$

¹ $\vec{E}_0 = E_0 \hat{\epsilon}$, where E_0 is the complex scalar amplitude and $\hat{\epsilon}$ is the complex polarization vector with $|\hat{\epsilon}| = 1$, and $\hat{\epsilon} \cdot \vec{k} = 0$.

where $\vec{E}$ is the electric field, $\vec{B}$ the magnetic flux density, and $\vec{H}$ the magnetic field strength. The intensity of a light field impinging on a photodetector is proportional to the time-averaged magnitude of the Poynting vector. For a harmonic electric field $E = E_0 \cos(\omega t)$, the instantaneous and time-averaged intensities are

$$I(t) = c n \epsilon_0 E_0^2 \cos^2(\omega t), \quad \langle |\vec{S}| \rangle = \langle I \rangle = \frac{1}{2} c n \epsilon_0 E_0^2, \quad (2.3)$$

where c denotes the speed of light in vacuum, ϵ_0 the vacuum permittivity, and n the refractive index of the propagation medium. In complex notation, we can write

$$I = \frac{c \epsilon_0 n}{2} E E^*, \quad (2.4)$$

where E^* is the complex conjugate of the electric field. The response of a photodetector is determined by the total flux of incident radiation integrated over the detector's response time. When a photodiode detects photons, charged carriers are generated that drift through the diode under the influence of the applied bias voltage. The transit time of these charges, typically on the order of 10 ns for standard AC photodiodes, limits the detector's bandwidth to approximately 100 MHz. Consequently, such detectors cannot resolve the optical frequencies of laser light in the THz regime where conventional lasers operate (e.g., NPRO: $\lambda = 1\,064\text{ nm}$, $f = c/\lambda \approx 282\text{ THz}$). Thus, the photodetector effectively measures only the time-averaged optical power. The total power incident on the photodiode, P_{PD} , is obtained by integrating the optical intensity over the active area A of the photodiode:

$$P_{PD} = \int_A I \cdot dA = \frac{c \epsilon_0 n}{2} E_0^2 \cdot A. \quad (2.5)$$

The generated photocurrent i_{PD} is proportional to the incident optical power and to the photodiode's responsivity $R_{PD}(\lambda)$ at the respective wavelength:

$$i_{PD} = P_{PD} \cdot R_{PD}(\lambda). \quad (2.6)$$

This photocurrent is subsequently converted into a voltage signal V_{signal} using a transimpedance amplifier (TIA) with feedback resistor R_f :

$$V_{\text{signal}} = R_f \cdot i_{PD}. \quad (2.7)$$

Since typical interferometers measure laser power deviations, it is useful to recall the relationship between measured power and optical field amplitude. Assuming the detector captures the entire electric field, the power is then given by

$$P = \frac{c \epsilon_0 n}{2} |E E^*| \propto |E|^2. \quad (2.8)$$

This convention is used throughout this thesis for clarity and intuitive interpretation.

2.2 Laser interferometers

Laser interferometry is a precise measurement technique that exploits the interference of laser beams to resolve spatial patterns and the infinitesimal displacements essential to gravitational-wave detection (GWD). The performance relies on the coherent and monochromatic nature of the laser light, ensuring stable and well-defined interference. The phase difference between recombined beams after distinct optical paths encodes the displacement information. Owing to the short wavelength of laser light, interferometers can resolve path-length variations on the order of a fraction of a wavelength, enabling nanometer and even sub-nanometer precision in compact metrology systems. Dedicated large-scale instruments, such as ground-based gravitational-wave detectors, extend these techniques to achieve effective displacement sensitivities down to $10^{-19} \text{ m}/\sqrt{\text{Hz}}$ [22].

Interferometers can be realized in various optical configurations, using one or more laser frequencies, and are commonly categorized as *homodyne* or *heterodyne interferometers*. Another important configuration is the resonant (or multi-bounce) interferometer, typically referred to as an *optical cavity* or *resonator*. The following section provides a brief overview of the three interferometer configurations, all employed in this work. For a comprehensive review of compact interferometer designs, refer to [71].

2.2.1 Homodyne interferometry

In a homodyne interferometer, a single-frequency laser beam is divided into a reference and measurement path. Each beam is reflected from a mirror (the test object), and then recombined at a beamsplitter. The resulting interference pattern is detected by a photodetector (PD). Since both beams originate from the same unaltered laser source, they share an identical frequency. Any path length change, for instance due to mirror displacement, introduces a corresponding phase shift between the beams, leading to constructive or destructive interference. The most common homodyne interferometer configurations for displacement measurement are the Michelson, the Mach-Zehnder, and the Sagnac interferometer². The operating principles of these interferometers is similar. They rely on differential path delays for phase-sensitive readout.

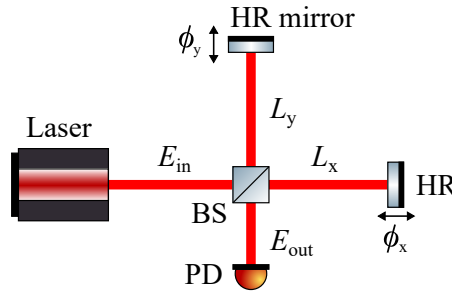


Figure 2.1: Sketch of a simple Michelson interferometer with input and output fields and E_{out} . The laser beam (E_{in}) is split at the beam splitter (BS) (50:50) into the x- and y-arms of lengths L_x and L_y . The returning fields interfere at the BS and are detected at the south port by a PD. A relative arm length change $\Delta L = L_x - L_y$ imprints a phase shift $\Delta\phi \propto \Delta L$ that modulates E_{out} .

² While the former two interferometers measure displacement, the Sagnac employs a common-path design with counter-propagating beams in a closed loop to detect rotation via the Sagnac effect (phase difference from rotational path asymmetry).

For a conventional Michelson interferometer (MI), as shown in Fig. 2.1, the laser beam is split by a (50:50) BS into two orthogonal arms (x, y), each terminated by high-reflection (HR) mirrors. The reflected fields are recombined (interfered) at the BS, and the resulting output field at the south port can be written as:

$$E_{\text{out}} = irtE_{\text{in}}(e^{i\phi_x} + e^{-i\phi_y}), \quad (2.9)$$

where r and t denote the reflection and transmission coefficients³, E_{in} the input field, and $\phi_{x,y}$ the round-trip phases in each arm. The arm phase is $\phi_i = 2kL_i$ ($i = x, y$), with arm lengths L_i and wave number $k = 2\pi/\lambda$, where λ is the laser wavelength. The differential phase follows as

$$\Delta\phi = 2k\Delta L, \quad \Delta L = L_x - L_y. \quad (2.10)$$

Defining a mean length⁴ $\bar{L} = \frac{L_x + L_y}{2}$ and substituting the expression in Equation 2.9, we obtain

$$E_{\text{out}} = \frac{i}{2}E_{\text{in}}e^{i2k\bar{L}}(e^{ik\Delta L} + e^{-ik\Delta L}) = iE_{\text{in}}e^{i2k\bar{L}}\cos(k\Delta L). \quad (2.11)$$

The detected optical power (P) on the PD, with P_{in} denoting the total input power, is then given by

$$P \propto E_{\text{out}}E_{\text{out}}^* = P_{\text{in}}\cos^2(k\Delta L) = \frac{P_{\text{in}}}{2}(1 - \cos(2k\Delta L)), \quad (2.12)$$

where we can express the differential phase $\Delta\phi$ as

$$\Delta\phi = \frac{4\pi\Delta L}{\lambda}. \quad (2.13)$$

The sinusoidal output power yields zero sensitivity at its extrema, and makes the motion direction ambiguous. Consequently, conventional homodyne interferometers are limited to an unambiguous range, typically less than half a wavelength. However, the optical phase can also be directly extracted using additional optical components and signal processing to realize a *homodyne phasemeter*, such as the homodyne quadrature interferometer, enabling unambiguous displacement measurements over multiple fringes.

Homodyne quadrature interferometer

The homodyne quadrature interferometer uses polarization optics, such as λ -waveplates, to generate at least two signals in quadrature, extending the operating range from half a fringe to multi-fringe readout by measuring phase rather than optical power. For instance, we assume that we measure the following three quadrature signals:

$$P_1 = P_0(1 + \sin(\Delta\phi)) \quad (2.14)$$

$$P_2 = P_0(1 + \cos(\Delta\phi)) \quad (2.15)$$

$$P_3 = P_0(1 - \cos(\Delta\phi)) \quad (2.16)$$

³ For a 50:50 BS: $r = t = 1/\sqrt{2}$

⁴ $2L_x = 2\bar{L} + \Delta L$ and $2L_y = 2\bar{L} - \Delta L$

where P_0 is the optical power on each PD and $\Delta\phi$ the differential optical phase encoding the displacement information. The phase is extracted via the $\arctan2$ function⁵ after subtracting the DC component as follows:

$$\Delta\phi = \arctan2\left(\frac{P_1 - P_2}{P_1 - P_3}\right) = \arctan2\left(\frac{\sqrt{2}P_0 \sin(\Delta\phi)}{\sqrt{2}P_0 \cos(\Delta\phi)}\right). \quad (2.17)$$

The phase extraction not only extends and linearizes the displacement sensing range but also achieves high common-mode rejection through ratiometric measurement.

2.2.2 Heterodyne interferometry

Heterodyne interferometers employ two distinct frequencies to measure displacements. These frequency components can be generated either by using multiple lasers or by introducing a frequency shift to one of the beams via an acousto-optic modulator (AOM). In a simple heterodyne Mach-Zehnder interferometer, as shown in Fig. 2.2, the laser beam is split at BS_1 into a reference and measurement arm, the latter of which includes an AOM that shifts the optical frequency by Ω . The two beams are then recombined at BS_2 , producing a *beat note*⁶ at the frequency $f_{beat} = \Omega$, whose phase encodes the differential optical path length between the interferometer arms.

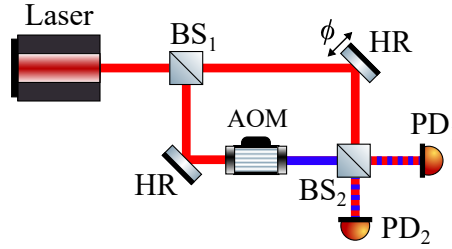


Figure 2.2: Schematic of a simple Mach-Zehnder heterodyne interferometer. The laser beam is split at the BS_1 into two arms with one arm including an AOM to shift the laser frequency by Ω . The two beams are then recombined at BS_2 , producing the beat note, measured by the two PDs.

The input beams before interference are expressed as

$$\mathcal{E}_i(\vec{r}, t) = \vec{E}_i e^{i(\omega_i t - \vec{k}\vec{r}_i + \phi_i)}, \quad i = 1, 2, \quad (2.18)$$

with frequencies $\omega_{1,2} = \omega \pm \Omega/2$ and phases ϕ_i . At BS_2 , the two beams interfere, i.e., their electric fields superpose:

$$\vec{E}(\vec{r}, t) = i\sqrt{t}\vec{E}_1 e^{i(\omega_1 t - \vec{k}\vec{r} + \phi_1)} + \sqrt{r}\vec{E}_2 e^{i(\omega_2 t - \vec{k}\vec{r} + \phi_2)}, \quad (2.19)$$

where r and t are the reflection and transmission coefficients. For equal amplitudes ($|\vec{E}_1| = |\vec{E}_2| = E_0$), identical phases ($\phi_1 = \phi_2 = 0$), matching profiles, and co-propagating beams, the electric field can be

⁵ The $\arctan2(y, x)$ function calculates the correct quadrant based on the signs of both arguments and returns the proper two-argument arctangent angle in the full $(-\pi, \pi]$ range, enabling continuous phase unwrapping without discontinuities.

⁶ The terms *beat note*, *beat signal*, and *beat* are used synonymously throughout this thesis.

expressed in the simple form:

$$E(t) = E_0 [\cos(\omega_1 t) + \cos(\omega_2 t)] . \quad (2.20)$$

The optical power measured at the photodetector is then given by:

$$\begin{aligned} P(t) \propto E^2(t) &= E_0^2 [\cos^2(\omega_1 t) + \cos^2(\omega_2 t) + 2\cos(\omega_1 t)\cos(\omega_2 t)] \\ &= E_0^2 [\cos^2(\omega_1 t) + \cos^2(\omega_2 t) + \cos(\omega_+ t) + \cos(\Delta\omega t)] , \end{aligned} \quad (2.21)$$

where $\omega_+ = \omega_1 + \omega_2$ and $\Delta\omega = \omega_1 - \omega_2$. Among these terms, $\cos(\Delta\omega t)$ represents the beat note, i.e., the signal of interest, and the only component within the detection bandwidth of the photodiode. The remaining terms oscillate at optical frequencies in the terahertz range and are therefore filtered out (i.e., averaged) by the finite bandwidth of typical AC-coupled photodiodes ($f_{BW} \lesssim$ several GHz). Thus, the detected power at the photodiode is given by

$$P_{PD}(t) = E_0^2 [1 + \cos(\Delta\omega t)] . \quad (2.22)$$

The expression can be extended to include unequal field amplitudes and phase offsets between the two interfering electric fields:

$$P_{PD}(t) = \frac{c\epsilon_0 n}{2} \left(\frac{E_1^2}{2} + \frac{E_2^2}{2} + 2|\langle E_1, E_2 \rangle| \cos(\Delta\omega t + \Delta\phi) \right) , \quad (2.23)$$

where $\Delta\phi = \phi_1 - \phi_2$. The photodiode then converts the incident optical power into a photocurrent according to its responsivity $R_{PD}(\lambda)$, yielding

$$i_{PD}(t) = R_{PD} \left(\frac{P_1 + P_2}{2} + \sqrt{\eta P_1 P_2} \cos(\Delta\omega t + \Delta\phi) \right) , \quad (2.24)$$

where η denotes the heterodyne efficiency that characterizes the interference contrast of the two beams (see Subsection 2.2.2). A concise and helpful representation reads [72]

$$i_{PD}(t) = i_{DC} (1 + \kappa \cos(\Delta\omega t + \Delta\phi)) = i_{DC} + i_{AC} , \quad (2.25)$$

where κ is the contrast, determining the fraction of the signal that is sensitive to fluctuations in the signal phase. The interference signal thus consists of a DC component and an AC component, the latter forming the beat note that carries the phase information and is read out by a phasemeter (see Section 2.2.3).

Contrast and heterodyne efficiency

The contrast κ of the heterodyne signal can be expressed in terms of photocurrent as

$$\kappa = \frac{i_{PD,max} - i_{PD,min}}{i_{PD,max} + i_{PD,min}} = \frac{P_{max} - P_{min}}{P_{max} + P_{min}} , \quad (2.26)$$

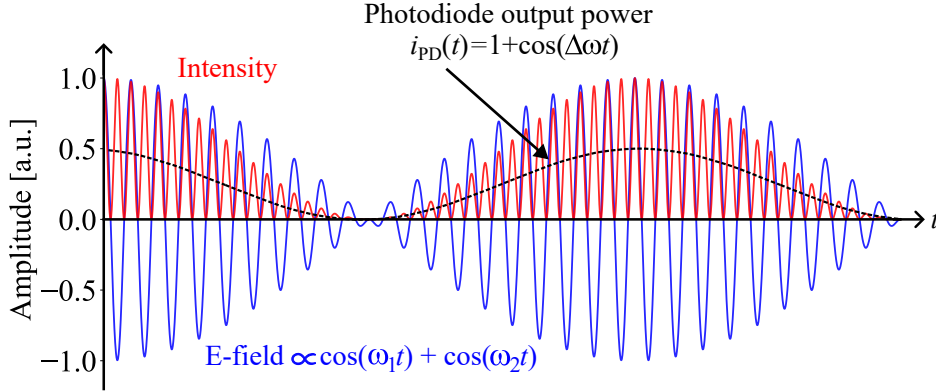


Figure 2.3: Time series plot of a heterodyne interferometer with two distinct frequency components, showing the resulting electric field, the corresponding intensity, and the measured interference signal (photodiode output power). The slowly oscillating term, referred to as the *beat note* ($\cos(\Delta\omega t)$), lies within the photodiode (PD) detection bandwidth, while the remaining terms oscillate at terahertz frequencies and are therefore filtered out (i.e., averaged).

where $i_{PD,max}$ (P_{max}) and $i_{PD,min}$ (P_{min}) denote the maximum and minimum photocurrent (optical power), respectively. Alternatively, the contrast can be interpreted as the modulation depth of the optical signal when normalized to the mean optical power $\bar{P} = \frac{1}{2}(P_1 + P_2)$. Rewriting Equation. 2.23 yields [57]

$$P_{PD} = \frac{c\epsilon_0 n}{2}(P_1 + P_2) \left(1 + \frac{2|\langle E_1, E_2 \rangle|}{P_1 + P_2} \cos(\Delta\omega t + \Delta\phi) \right), \quad (2.27)$$

and thus

$$P_{PD} \propto \bar{P} [1 + \kappa \cdot \cos(\Delta\omega t + \Delta\phi)], \quad (2.28)$$

with

$$\kappa = \frac{2|\langle E_1, E_2 \rangle|}{P_1 + P_2}. \quad (2.29)$$

The inner product $\langle E_1, E_2 \rangle$ quantifies the overlap of the two electric fields and thus how well their spatial, temporal, and polarization modes match. This overlap term likewise defines the heterodyne efficiency, given by

$$\sqrt{\eta} = \frac{|\langle E_1, E_2 \rangle|}{\sqrt{P_1 P_2}}. \quad (2.30)$$

The relationship between contrast and heterodyne efficiency follows directly from Equations. 2.29 and 2.30, differing only by a power-dependent scaling factor of the two interfering beams:

$$\kappa = \frac{2\sqrt{P_1 P_2}}{P_1 + P_2} \sqrt{\eta}. \quad (2.31)$$

When the optical powers in both beams are equal ($P_1 = P_2$), the expression simplifies to $\kappa = \sqrt{\eta}$. In summary, the heterodyne efficiency depends solely on the quality of the mode overlap (spatial, temporal, and polarization) between the two beams. Achieving high contrast, on the other hand,

additionally requires equal optical powers in both beams. In practical heterodyne detection, the local oscillator beam is typically made much stronger than the signal beam to ensure operation in the shot-noise-limited regime and thereby achieve optimal sensitivity⁷, rather than maximal contrast [73].

The case for heterodyne interferometers

Homodyne interferometers, as employed in earth-based GW detectors, achieve extremely low noise floors through strong circulating light fields and ultra-quiet test masses. However, their linear operating range is fundamentally limited to phase excursions $\Delta\phi \ll 2\pi$ around a chosen dark fringe working point. Heterodyne interferometers, by contrast, enable unambiguous measurement of large displacements ($\Delta\Phi \gg 2\pi$) by tracking the phase of the beat note. They provide inherently continuous readout and intrinsic direction sensing, as the sign of the beat note frequency shift encodes motion direction. Additionally, shifting the signal of interest from DC to the heterodyne beat frequency ($\sim$ MHz) provides strong suppression of low-frequency technical noise sources. For the LISA mission, a heterodyne interferometer design is adopted with beat frequencies in the low MHz range. This choice accommodates both arm-length fluctuations in the constellation, which induce Doppler shifts on the laser frequency, and the intrinsically weak far-field carrier received from distant spacecraft, whose phase is extracted by interfering it with a much stronger local oscillator beam [74]. Further details on LISA's heterodyne interferometer implementation is provided in Chapter 1, Section 1.2.1.1.

2.2.3 Phase readout – phasemeter

Heterodyne interferometers generate beat notes, AC signals typically in the MHz range, that require dedicated phase readout electronics. The optical beat note signal is detected by a photodiode, which generates a photocurrent that is converted into a voltage by a transimpedance amplifier and subsequently digitized by an analog to digital converter. Each of these components must provide sufficient bandwidth and resolution to capture the beat signal without distortion. The digitized signal is then processed by the phasemeter, which extracts the instantaneous phase as a function of time. From this phase information, the beat frequency is obtained as the temporal derivative of the phase. The phasemeter employs an in-phase and quadrature (IQ) demodulation technique, which forms the basis of its signal processing.

Consider a signal $v(t)$ with an angular beat frequency ω_{beat} , expressed in the form:

$$v(t) = v_0 \cos(\omega_{\text{beat}}t + \phi(t)), \quad (2.32)$$

where v_0 denotes the amplitude and $\phi(t)$ the phase to be extracted. The first demodulation step is to multiply the signal by a sine or cosine reference signal at the same frequency as the input signal ($\omega = \omega_{\text{beat}}$). A subsequent low-pass filter (LPF) removes the high-frequency component at 2ω . For

⁷ Most technical noise sources exhibit fixed noise levels independent of P_{LO} , such that their relative signal-to-noise ratio (SNR) degrades as $1/\sqrt{P_{\text{LO}}}$, while the shot-noise-limited SNR remains constant.

the cosine reference, this process yields

$$\begin{aligned}
 I_\omega &= v(t) \cdot \cos(\omega t) \\
 &= \frac{1}{2} v_0 [\cos(\phi(t)) + \cos(2\omega t + \phi(t))] \\
 &\stackrel{LPF}{=} \frac{1}{2} v_0 \cos(\phi(t)).
 \end{aligned} \tag{2.33}$$

The in-phase component I_ω and the quadrature component Q_ω are obtained as

$$I_\omega = \frac{1}{2} v_0 \cos(\phi(t)), \tag{2.34}$$

$$Q_\omega = \frac{1}{2} v_0 \sin(\phi(t)). \tag{2.35}$$

The instantaneous amplitude and phase of the signal can then be recovered from these components as

$$v_0(t) = \sqrt{I_\omega^2 + Q_\omega^2}, \tag{2.36}$$

$$\phi(t) = \arctan\left(\frac{Q_\omega}{I_\omega}\right). \tag{2.37}$$

This technique, commonly referred to as *IQ demodulation*, can be illustrated using a phasor diagram for visual clarity, as shown in Fig. 2.4.

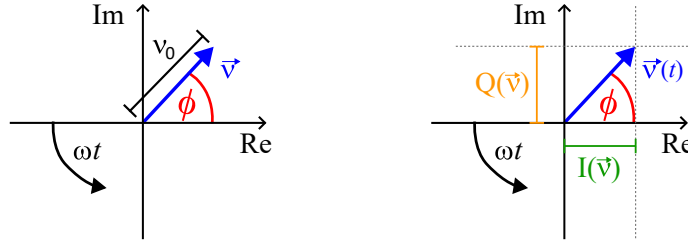


Figure 2.4: Phasor diagram of the IQ demodulation in a rotating reference frame at beat note frequency ω_{beat} . The in-phase component I_ω and the quadrature component Q_ω (Equation 2.34 and 2.35) are represented as projections onto the real and imaginary axes, forming the vector $\vec{v}$ with amplitude v_0 and phase ϕ . For clarity, the figure is displayed twice.

The IQ demodulation is implemented by an electronic readout system, the phasemeter. It is based on a field-programmable gate array (FPGA)⁸ and integrates ADCs [72]. In earlier phasemeter implementations, demodulation employed a fixed local oscillator (LO) frequency [76], which was sufficient to meet the performance requirements of LISA Pathfinder [77]. However, the LISA mission involves moving spacecraft and therefore demands a substantially larger dynamic phase range [50]. Consequently, the IQ-demodulation scheme was extended by incorporating a phase-locked loop (PLL) that dynamically controls the demodulation frequency and phase to track the received signal [78]. The LO frequency is governed by a feedback control loop, designed to minimize the quadrature

⁸ FPGAs enable real-time signal processing with minimal latency, which is essential for accurately measuring phase differences between signals [75].

component Q_ω . The instantaneous LO frequency is stored in the phase increment register (PIR), while the corresponding LO phase is stored in the phase accumulator (PA). Consequently, the phase information is derived from the actuator signal rather than directly from the quadrature component Q_ω . When the PLL is locked, the in-phase component I_ω provides a value directly proportional to the signal's amplitude (Equation 2.34). A detailed discussion of the PLL operation in conjunction with the phasemeter architecture can be found in [72, 79].

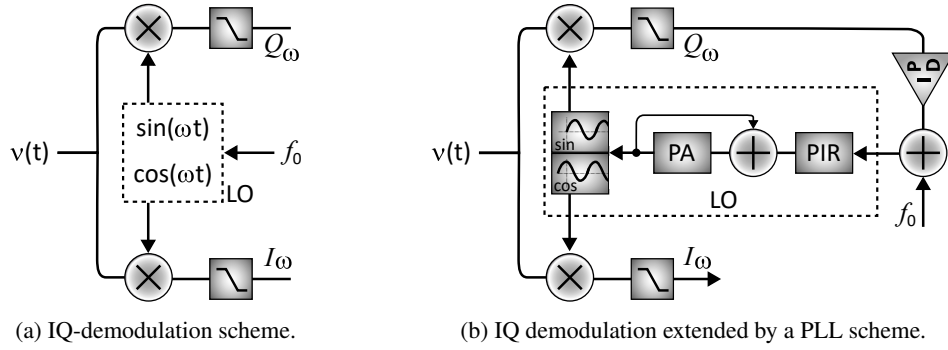


Figure 2.5: Phasemeter block diagrams illustrating IQ demodulation in (a) basic form with fixed LO and (b) extended implementation with a PLL. The input signal $v(t)$ is mixed with sine and cosine reference signal generated by a local oscillator (LO), then low-pass filtered to suppress the second-harmonic term. This yields the in-phase and quadrature components I_ω and Q_ω , from which the input signal phase and amplitude are recovered. In the PLL implementation (b), Q_ω drives a PID controller that corrects the LO phase via feedback within a closed-loop control system to correct the LO phase. The instantaneous LO frequency and phase are stored in the phase increment register (PIR) and phase accumulator (PA), respectively. (Image created based on [72]).

2.2.4 Optical cavities

An optical cavity is an arrangement of optical components that confines light to circulate along a closed path⁹. The simplest and most common configuration is the Fabry-Pérot interferometer (FPI) (or etalon¹⁰), a linear standing-wave resonator. It consists of two opposing mirrors between which the light undergoes repeated reflections. The mirrors are typically aligned such that the beam is incident normal to their surfaces. This geometry produces two counter-propagating waves that interfere to form a standing-wave pattern. Another common configuration is the ring resonator, in which the light circulates continuously along a closed loop. This configuration is realized using multiple mirrors arranged such that there are no end mirrors, allowing the optical field to propagate independently around the loop in either direction.

Static response of an optical linear cavity

The cavities employed in this work are optical linear cavities, specifically FPIs. A schematic representation of such a cavity, including the relevant electric fields, is shown in Fig. 2.6. The cavity consists of two partially reflecting mirrors (power reflectivity: $R = r^2$ and transmittance $T = t^2$),

⁹ The terms *cavity* and *resonator* are used synonymously throughout this thesis.

¹⁰ The term *etalon* refers to an optical cavity encountered typically in the context of unwanted/parasitic cavity effects.

separated by a distance L . For the following derivations, the cavity is assumed to be lossless, and a phase factor $i = \sqrt{-1}$ is conventionally assigned to a field upon transmission through an optical element.

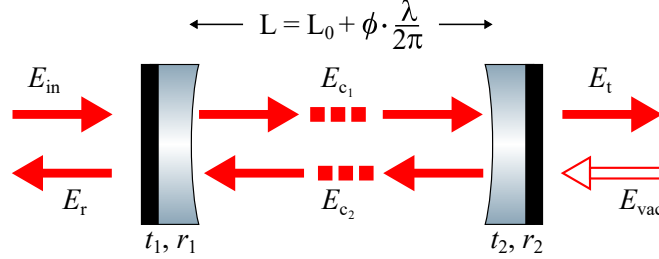


Figure 2.6: Optical linear cavity showing the incident, reflected, transmitted, circulating, and vacuum fields.

The following relationships¹¹ are derived from the schematic shown in Fig. 2.6:

$$E_r = E_{in}r_1 + i t_1 E_{c_2}, \quad (2.38)$$

$$E_t = E_{c_1} i t_2 e^{-2ikL} + r_2 E_{vac}, \quad (2.39)$$

$$E_{c_1} = E_{in} i t_1 + r_1 E_{c_2}, \quad (2.40)$$

$$E_{c_2} = E_{c_1} r_2 e^{-2ikL} + i t_2 E_{vac}. \quad (2.41)$$

Rearranging the expressions yields the reflected, transmitted, and circulating electric fields:

$$E_r = E_{in} \frac{r_1 - r_2 e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}}, \quad (2.42)$$

$$E_t = E_{in} \frac{t_1 t_2 e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}}, \quad (2.43)$$

$$E_{c_1} = E_{in} \frac{t_1}{1 - r_1 r_2 e^{-2ikL}}. \quad (2.44)$$

When a light field with frequency ω (wavelength λ) enters a cavity of given length L , it reproduces self-consistently after each round trip if the accumulated phase shift is an integer multiple of 2π . These resonance frequencies are termed resonator modes. Substituting the resonance condition into Equations 2.42 and 2.43 yields $E_r(kL = m\pi) = 0$ and $E_t(kL = m\pi) = E_{in}$ for integer m . The circulating field inside the cavity represents the stored optical power and is given by

$$P_{cav} \propto |E_{c_1}|^2 \propto P_{in} \frac{t_1^2}{(1 - r_1 r_2)^2} = P_{in} G_{cav}, \quad (2.45)$$

where G_{cav} denotes the cavity gain (or power building factor). The frequency spacing between adjacent

¹¹ For notational simplicity: $E(L) = E$.

resonator modes is termed the free spectral range (FSR), defined as

$$\text{FSR} = \frac{c}{2 L n_g}, \quad (2.46)$$

where c denotes the speed of light in vacuum, n_g the group refractive index¹² and L the physical length of the cavity. The ratio of the FSR to the full width at half maximum (FWHM) of the resonances defines the finesse $\mathcal{F}$, which quantifies the narrowness of the resonances relative to their frequency spacing (see Fig. 2.7). A high finesse results from highly reflective mirrors, yielding sharp resonances, and is given by¹³

$$\mathcal{F} = \frac{\text{FSR}}{\text{FWHM}} \approx \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}. \quad (2.47)$$

A cavity is impedance matched when both mirrors have the same reflectivity ($|r_1| = |r_2|$). When the mirror reflectivities do not match, the cavity operates in either the overcoupled ($|r_1| > |r_2|$) or undercoupled ($|r_1| < |r_2|$) regime. The amplitude and phase response for these cases are shown in Fig. 2.8. The figure also shows the coupling efficiency a , also known as fringe visibility, defined as

$$a = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}}, \quad (2.48)$$

where $P_{\max}$ is the off-resonance reflected power and $P_{\min}$ is the reflected power on resonance. For an impedance- and mode-matched cavity ($a = 1$) on resonance, all incident light is transmitted with zero reflection¹⁴.

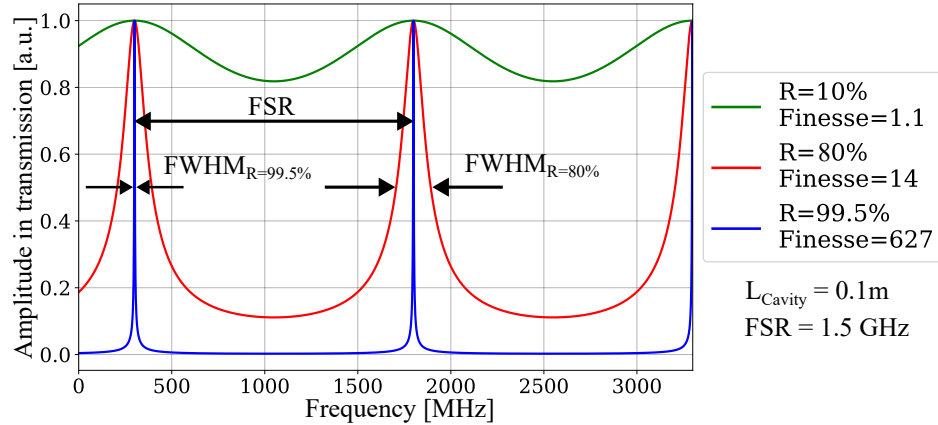


Figure 2.7: Transmission mode spectra of high- and low-finesse FPI cavities. The simulations assume impedance-matched cavities with a wavelength of $\lambda = 1\,064\text{ nm}$, cavity length $L = 0.1\text{ m}$ and a range of mirror reflectivities. The simulation was performed using Finesse3 software [80].

When designing an optical cavity, the operational regime must be tailored to the specific application.

¹² The group refractive index n_g accounts for chromatic dispersion in the cavity medium.

¹³ Equation 2.47 follows from a high-finesse approximation of the Airy distribution function. It assumes round-trip losses dominated by mirror transmission, neglecting absorption, scattering, clipping, and diffraction losses.

¹⁴ Strictly speaking, this condition holds after the circulating field inside the cavity reached a steady state.

For example, the GW detector LIGO requires high circulating power in the interferometer arms and therefore employs an overcoupled cavity design. Similarly, laser frequency stabilization (see Section 2.3) exploits the cavity's optical phase response, which also benefits from an overcoupled design due to its steep phase slope near resonance. An undercoupled design would be unsuitable, as it exhibits negligible phase response. For high finesse impedance-matched cavities, minor differences in mirror reflectivities or losses can easily cause undercoupling. Therefore, high-finesse cavities are typically designed overcoupled to avoid undercoupling. The optical phase shift accumulated from multiple reflections within the cavity greatly enhances sensitivity, making optical resonators ideal displacement sensors. The reflected or transmitted field encodes the cavity's optical phase information, which is converted into power variations and ultimately into displacement. However, since laser frequency (rather than phase directly) is typically measured, the following relationship between the frequency fluctuation δf and length fluctuation δL allows the conversion:

$$\frac{\delta f}{f} = -\frac{\delta L}{L}, \quad (2.49)$$

where f denotes the laser frequency and L the optical path length of the cavity.

The case for high-finesse optical cavities and their cavity response

High-finesse optical cavities are essential for precision interferometry in GW detectors. They enable laser frequency stabilization, power enhancement through multiple reflections, and filtering of unwanted frequency components (mode cleaner). While laser frequency stabilization is discussed in Section 2.3, here we derive the high-finesse cavity response, providing the theoretical foundation for the subsequent analysis in Section 6.2.1 and 6.3.1. The cavity (transfer function, or response) is characterized by the reflection coefficient:

$$R(f) = \frac{E_r(f)}{E_{in}(f)} = \frac{r_1 - r_2 e^{-i2\pi \frac{f}{\text{FSR}}}}{1 - r_1 r_2 e^{-i2\pi \frac{f}{\text{FSR}}}}, \quad (2.50)$$

derived from Equation 2.42 using the high finesse approximation from Equation 2.47. Far off resonance, the reflection coefficient is $|R(f)| \approx 1$ with constant phase. Near resonance (within the linewidth, $\text{FWHM} \gg |\delta f|$ where $\delta f = f - f_{\text{res}}$), the reflection coefficient is expressed as

$$R(\delta f) \approx \frac{r_1 - r_2}{1 - r_1 r_2} - i2\pi \frac{r_2(1 - r_1^2)}{(1 - r_1 r_2)^2 \text{FSR}} \frac{\delta f}{f}. \quad (2.51)$$

The imaginary part captures the dispersive phase response ($\propto \delta f$). For high-finesse (Equation 2.47) with $r_1 \approx r_2 \approx r$, it can be written as

$$\Im \{R(\delta f)\} \approx -\frac{2}{\pi} \left(\frac{1}{r} - r \right) \mathcal{F} \frac{\delta f}{\text{FWHM}}. \quad (2.52)$$

The expression simplifies with the introduction of an optical gain $\mathcal{G}$ to

$$\Im \{R(\delta f)\} \approx -\mathcal{G} \frac{\delta f}{\text{FWHM}}. \quad (2.53)$$

Thus, laser-cavity frequency detuning produces a linear dispersive response scaling with the optical gain $\mathcal{G}$, which serves as the error signal for laser frequency stabilization (see Section 2.3).

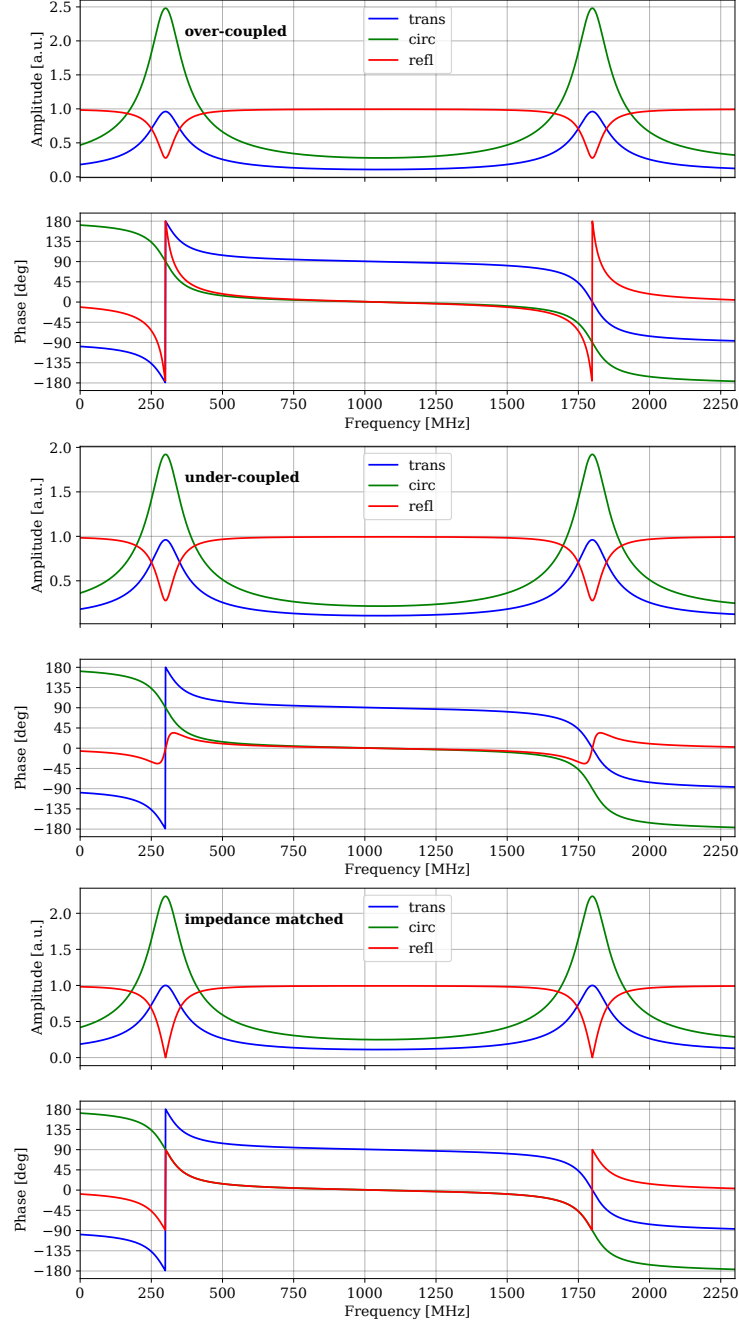


Figure 2.8: Simulation of the electrical fields in a Fabry-Pérot cavity, showing amplitude and phase responses of the transmitted, circulating and reflected fields. Results are presented for overcoupled ($R_1 = 0.75$, $R_2 = 0.85$), under-coupled ($R_1 = 0.85$, $R_2 = 0.75$), and impedance-matched ($R_1 = 0.8$, $R_2 = 0.8$) configurations. The simulation assumes a cavity length of $L = 0.1$ m, $\lambda = 1\,064$ nm, and considers only round-trip losses due to mirror transmission. The simulation was performed using *Finesse3* software.

2.3 Laser frequency stabilization

Laser interferometers measure differential phase between superimposed electric fields (laser beams). The monochromatic nature of laser light provides an ideal optical ruler for detecting displacements. However, practical lasers exhibit finite linewidth due to frequency noise from both fundamental limits and technical noise sources. The latter affect the laser cavity length or gain medium through temperature fluctuations, mechanical vibrations, electrical noise, acoustic disturbances, and variations in air pressure and humidity. While a well-designed laboratory mitigates these effects, they cannot be fully eliminated. Thus, laser frequency stabilization is essential for achieving the precision required in GW detection interferometry. Several established techniques achieve laser frequency stabilization by locking the laser's frequency to a stable reference, such as atomic transitions [81], unequal-arm interferometers [82], or stable reference cavities [83]. In this work, laser stabilization is achieved by locking the laser's frequency to the length of an ultra-stable optical cavity. Optical reference cavities constructed from Ultra-Low Expansion (ULE) materials exhibit minimal thermal drift and low mechanical noise [84]. Their well-defined resonance frequencies and narrow linewidths (for the case of high finesse cavities) produce steep slopes in the transmission and reflection spectra. The cavity serves as a frequency-selective filter that transmits only resonant frequencies, providing a stable reference for laser frequency stabilization.

Generally, the stabilization technique operates as a feedback loop: laser light interacts with the cavity, and the reflected (or transmitted) light is analyzed to generate an error signal quantifying frequency detuning from resonance. This error signal drives a proportional-integral-derivative (PID) controller that adjusts the laser frequency to maintain lock. Even with perfect stabilization, two fundamental quantum limits remain: the Schawlow-Townes linewidth from spontaneous emission phase noise [85], and shot noise in the photodetector measuring light from the cavity reflection (or transmission). This detection noise fundamentally limits feedback loop performance, thereby establishing the stabilization noise floor.

Several techniques exist for locking a laser to a cavity resonance. This work employs Pound-Drever-Hall (PDH) locking and Heterodyne laser frequency Stabilization (HS), described in detail in Chapter 6.

Control theory - feedback control loops

Laser frequency and amplitude stabilization, as well as active seismic isolation, rely on feedback control loops. These continuously monitor system output, compare it to a setpoint (reference), and apply corrections based on the error signal to maintain the desired operating condition. A typical feedback loop comprises a sensor to measure output, a controller to process the error, and an actuator to apply corrections, thereby stabilizing the output. For laser frequency stabilization using an optical reference cavity, the loop continuously measures laser frequency relative to the stable cavity resonance and adjusts it to maintain lock. The primary components are the photodetector to measure frequency deviations, a PID controller to compute corrections, and an actuator to adjust the laser frequency. A detailed introduction to control loops is given in [86].

Feedback control loop for a laser frequency stabilization

This subsection introduces a representative frequency-stabilization feedback control loop that forms the foundation for laser-cavity locking in Chapter 6¹⁵. Figure 2.9 shows a typical block diagram of such a control loop, including signals, noise sources, and transfer functions in the Laplace domain.

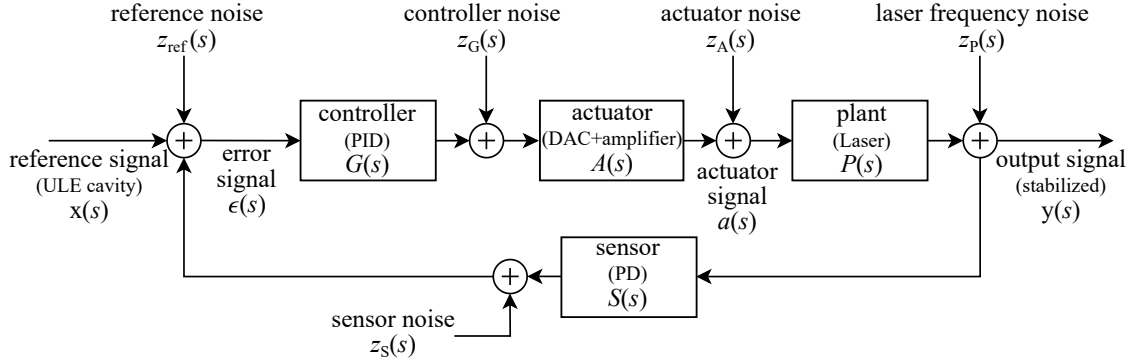


Figure 2.9: Block diagram of a feedback control loop for laser frequency stabilization using an optical reference cavity. Signals, noise sources (lowercase) and transfer functions (uppercase) are shown in the Laplace (s -domain) frequency domain.

The output signal $y(s)$ (laser frequency) is disturbed by noise $z_P(s)$. To stabilize the laser frequency, an error signal $\epsilon(s)$ is generated, proportional to the difference between the measured output $y(s)$ and the reference signal $x(s)$ from the ULE cavity. This error signal drives a PID controller, which computes the actuator signal $a(s)$ to adjust the laser frequency. From Fig. 2.9, the output signal for laser frequency stabilization is derived as follows ($y = y(s)$):

$$\begin{aligned}
 y = & \underbrace{\frac{GAP}{1 + GAPS}}_{\text{Reference signal}} x + \underbrace{\frac{GAP}{1 + GAPS}}_{\text{Reference noise}} z_{\text{ref}} + \underbrace{\frac{GAP}{1 + GAPS}}_{\text{Sensor noise}} z_S \\
 & + \underbrace{\frac{AP}{1 + GAPS}}_{\text{Controller noise}} z_G + \underbrace{\frac{P}{1 + GAPS}}_{\text{Actuator noise}} z_A + \underbrace{\frac{1}{1 + GAPS}}_{\text{Plant noise}} z_P
 \end{aligned} \tag{2.54}$$

For large loop gains $|G(s)| \gg 1$, all terms in the second row vanish, and the output $y(s)$ follows the reference signal $x(s)$ plus the reference $z_{\text{ref}}(s)$, and sensor noise $z_S(s)$ ¹⁶. This shows that noise from the reference/sensor cannot be suppressed by the loop and must instead be minimized through careful cavity and sensor design. Noise injected after the PID controller is less critical, as it gets suppressed by the loop gain $G(s)$.

¹⁵ This work employs laser frequency stabilization using both PDH locking and Heterodyne laser frequency Stabilization (HS). Although error-signal generation differs, both techniques are represented equivalently in the same feedback control loop block diagram.

¹⁶ Reference noise: reflected light from the optical reference cavity (e.g., shot noise). Sensor noise: electric noise from the PD and ADC.

2.4 Modulation of optical light fields

The key aspects of laser interferometers are the modulation and measurement of the optical light fields. In the context of GWDs, interferometric light fields can be distinguished into three categories. First, the carrier field, which originates from the laser source and operates in the terahertz regime (e.g., $\lambda = 1\,064\text{ nm}$ corresponds to $f = 282\text{ THz}$). Second, radio frequency (RF) sidebands are introduced for controlling the interferometer. These optical fields, offset from the carrier by $\sim 1\text{ MHz}–150\text{ MHz}$, are typically generated by electro-optic phase modulation. The RF sidebands serve as phase references for example, derive error signals for interferometer length and cavity control loops, such as laser frequency stabilization (see Section 2.3), or to enable signal post-processing, for instance, pilot-tone correction in LISA. Third, signal sidebands are produced by phase modulations resulting from variations in interferometer arm lengths. These modulations arise from mirror motion due to noise or, at sufficient sensitivity, from gravitational-wave strain signals.

This section provides a brief overview of the derivation of sideband components for amplitude, frequency, and phase modulation, serving as a foundation for the experimental chapters. The derivations follow [87] and assume the simple input electric field:

$$E_{\text{in}} = E_0 e^{i\omega_0 t}. \quad (2.55)$$

Before deriving the mathematical expressions, it is instructive to examine the time-domain representation of amplitude and phase modulation shown in Fig. 2.10. While amplitude modulation produces readily apparent intensity variations, phase modulation appears subtle in the time domain. In the frequency domain, however, the interpretation becomes intuitive: sinusoidal amplitude or phase modulation generates discrete sidebands symmetrically offset from the carrier frequency. This process redistributes the optical power from the carrier to these sidebands.

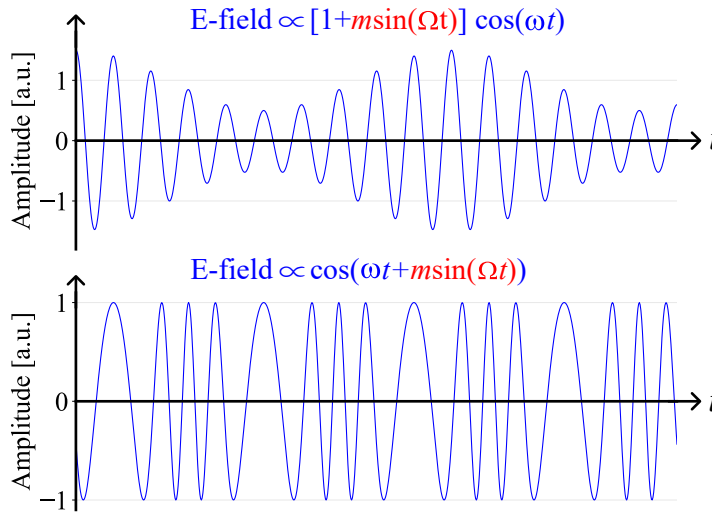


Figure 2.10: Time series of optical fields with carrier frequency ω_0 under amplitude modulation (AM) and phase modulation (PM) at modulation frequency Ω .

2.4.1 Amplitude modulation

Amplitude modulation (AM) of laser light varies the intensity of the optical carrier, generating sidebands symmetrically around the carrier frequency. Amplitude modulation of the field, $E_{\text{AM}}(t)$, can be implemented either by an active element, such as direct modulation of the laser diode current, or by a passive element, such as an AOM, which can only reduce the amplitude of the incident light. A sinusoidal amplitude modulation of the electric field $E_{\text{in}}(t)$ (Equation 2.55) by an *active* amplitude modulator is expressed as

$$\begin{aligned} E_{\text{AM}}(t) &= E_0 e^{i\omega_0 t} (1 + m \cos(\Omega t)) \\ &= E_0 e^{i\omega_0 t} \left(1 + \frac{m}{2} e^{i\Omega t} + \frac{m}{2} e^{-i\Omega t} \right), \end{aligned} \quad (2.56)$$

where E_0 is the carrier amplitude, ω_0 the carrier angular frequency, Ω the modulation frequency, and m the modulation index. For a *passive* modulator that attenuates the optical power, the field is expressed as

$$\begin{aligned} E_{\text{AM}}(t) &= E_0 e^{i\omega_0 t} \left(1 - \frac{m}{2} [1 - \cos(\Omega t)] \right) \\ &= E_0 e^{i\omega_0 t} \left(1 - \frac{m}{2} + \frac{m}{4} e^{i\Omega t} + \frac{m}{4} e^{-i\Omega t} \right). \end{aligned} \quad (2.57)$$

The modulation index m , also referred to as the modulation depth, quantifies how strongly the field amplitude (and thus the optical intensity) is varied. In pure amplitude modulation, the maximum value ($m = 1$) corresponds to a signal that varies from zero to twice its mean amplitude. For phase and frequency modulation, the modulation index instead quantifies the extent of phase or frequency variation, respectively.

2.4.2 Phase modulation

Phase modulation (PM) of laser light varies the phase of the electric field while keeping its amplitude constant¹⁷. This process generates sidebands symmetrically around the carrier frequency, whose power depends on the modulation index m . A sinusoidal phase modulation of the electric field $E_{\text{in}}(t)$ from Equation 2.55 is expressed as

$$\begin{aligned} E_{\text{PM}}(t) &= E_0 e^{i(\omega_0 t + m \cos(\Omega t))} \\ &= E_0 e^{i\omega_0 t} e^{i m \cos(\Omega t)}. \end{aligned} \quad (2.58)$$

This expression can be expanded using the Jacobi-Anger expansion [88]:

$$e^{iz \cos \theta} = \sum_{k=-\infty}^{\infty} i^k J_k(z) e^{ik\theta}, \quad (2.59)$$

¹⁷ The real amplitude $|E(t)| = E_0$ remains constant, whereas the complex amplitude is modulated.

where $J_k(z)$ denotes the Bessel functions of the first kind of order k . Substituting this expression into Equation 2.58 yields

$$E_{\text{PM}}(t) = E_0 e^{(i\omega_0 t)} \cdot \sum_{k=-\infty}^{\infty} i^k J_k(m) e^{ik\Omega t}. \quad (2.60)$$

The sidebands¹⁸ are indexed by k and occur symmetrically around the carrier frequency. In the small-modulation limit ($m \ll 1$), the Bessel functions ($J_k(m)$) drop off rapidly, and only the first-order terms contribute significantly:

$$J_0(m) \approx 1 - \frac{m^2}{4}, \quad J_1(m) \approx \frac{m}{2}. \quad (2.61)$$

Equation 2.60 then reduces to

$$\begin{aligned} E_{\text{PM}}(t) &= E_0 e^{(i\omega_0 t)} \cdot \left(J_0(m) + iJ_1(m)e^{(i\Omega t)} - iJ_{-1}(m)e^{(-i\Omega t)} \right) \\ &\approx E_0 e^{(i\omega_0 t)} + iE_0 J_1(m) e^{(i(\omega_0 + \Omega)t)} - iE_0 J_{-1}(m) e^{(i(\omega_0 - \Omega)t)}. \end{aligned} \quad (2.62)$$

Thus, a small phase modulation ($m \ll 1$) produces two additional electric field components next to the carrier: an upper sideband and a lower sideband.

Electro-optic phase modulator

Phase modulation of laser light can be achieved in several ways, the most common is the use of an electro-optic modulator (EOM). The EOM is based on the Pockels effect [89], in which an applied electric field alters the refractive index of a nonlinear material (e.g., a crystal)¹⁹. When a voltage is applied across the crystal, it generates an electric field that changes the refractive index. As the light passes through this material, it experiences a phase shift proportional to the applied voltage, thereby producing the desired phase modulation. In addition to phase modulation, an EOM can also be configured to operate as an AM (intensity) or polarization modulator.

Phase modulation through moving mirror

In general, we are using a laser interferometer to measure the displacement of our test-mass mirrors. The core principle underlying this measurement is the phase modulation of the laser beam induced by the motion of the mirrors. In a Michelson interferometer, for instance, displacement of the arm mirror introduces a phase modulation of the reflected light. Upon recombination at the beam splitter, this phase modulation is converted into an amplitude modulation, the interference signal detected by a photodetector. A detailed derivation of the phase modulation through a moving mirror and the resulting amplitude modulation at the beam splitter is provided in the Appendix B.

2.4.3 Frequency modulation

For small modulation indices, phase modulation and frequency modulation can be considered different descriptions of the same effect [90] (G. Heinzel, 1999).

¹⁸ Lower sidebands for $k < 0$ and upper sidebands for $k > 0$ appear with a frequency spacing of $k\Omega$ relative to the carrier.

¹⁹ Typical EOM implementations use nonlinear crystals, integrated waveguides, or waveplates.

A sinusoidal frequency modulation with modulation frequency Ω and amplitude m' is expressed as

$$\omega(t) = \omega_0 + m' \cos(\Omega t). \quad (2.63)$$

One might therefore be tempted to express the electric field as

$$E(t) = E_0 e^{i(\omega_0 + m' \cos(\Omega t))t} \quad \textbf{incorrect!}, \quad (2.64)$$

but this is incorrect because the instantaneous frequency must be integrated to obtain the phase. The electric field is defined as

$$E(t) = E_0 e^{i\phi(t)}, \quad \phi(t) = \int \omega dt. \quad (2.65)$$

Thus, for our expression in Equation 2.63 we obtain

$$\begin{aligned} \phi &= \int (\omega_0 + m' \cos(\Omega t)) dt \\ &= \omega_0 t + \frac{m'}{\Omega} \sin(\Omega t). \end{aligned} \quad (2.66)$$

The correct expression for the frequency-modulated electric field is

$$E = E_0 e^{i(\omega_0 t + \frac{m'}{\Omega} \sin(\Omega t))}. \quad (2.67)$$

The amplitude m' characterizes the magnitude of the frequency deviation, i.e., the instantaneous frequency shift from its mean value. The modulation frequency Ω determines how rapidly the frequency varies. Defining the modulation index $m = m'/\Omega$ shows that Equation 2.67 is formally identical to the phase-modulation expression in Equation 2.58. However, the two regimes differ in practice. For phase modulation, the index typically exceeds $m < 10$, whereas for frequency modulation it typically reaches values of $m > 10^4$. In this large-modulation regime, the small- m approximation used in the phase-modulation analysis is no longer valid, and many Bessel-function sidebands must be considered. A notable example is the deep-frequency modulation interferometry (DFMI) technique [91], where the applied modulation produces a broad frequency spectrum comprising numerous sidebands [92].

2.4.4 Phasor diagram

The previous subsections presented the mathematical description of light field modulation. These formulations can become excessively complex and difficult to interpret, particularly when multiple modulations are involved simultaneously. Phasor diagrams offer a concise and intuitive representation of sinusoidal optical fields, allowing clear visualization of their relative amplitudes and phases as illustrated in Fig 2.11. This approach greatly simplifies the analysis of interference and modulation effects, especially for heterodyne interferometers. In a phasor diagram, a complex scalar field is represented in the complex plane as a vector of length E_0 and phase $\omega_0 t$. For example, the electric field of a monochromatic light wave can be written as

$$E(t) = E_0 e^{i\omega_0 t}, \quad (2.68)$$

which can be interpreted as a vector of magnitude E_0 rotating around the origin with angular frequency ω_0 . When multiple light fields are considered (for example, the carrier and its sidebands), it is advantageous to adopt a *rotating reference frame* that rotates at the same frequency as the carrier ω_0 . In this frame, the carrier appears stationary, while sidebands appear as rotating vectors, clockwise for frequencies below the carrier and counterclockwise for those above it. Phasor diagrams are particularly useful for visualizing the coupling between optical sidebands and for understanding modulation type conversions, such as the transformation of phase modulation into amplitude modulation. A comprehensive treatment of complex optical-system simulations using phasor analysis is provided in [93].

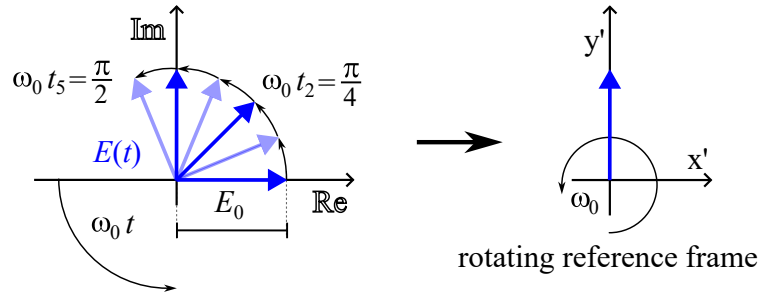


Figure 2.11: Left: Phasor diagram of a complex scalar field represented by a vector of magnitude E_0 rotating about the origin with an angular frequency ω_0 . Right: Corresponding static representation in the *rotating reference frame* at ω_0 .

2.5 Nonplanar ring oscillator laser

Nonplanar ring oscillator (NPRO) lasers are widely used in gravitational-wave detection due to their monolithic design, which provides exceptional frequency and amplitude stability, narrow linewidth, low noise, compactness, and ruggedness. A single crystal²⁰ serves as both the gain medium and the laser cavity. The laser light circulates along a nonplanar ring path within the crystal by total internal reflection at the facets and partial reflection from the output coupler on the front face. This face features a dielectric multilayer (V-coating) with partial reflection at the laser wavelength and high transmission for pump light. A critical feature of the ring resonator design is unidirectional travelling-wave operation, which suppresses standing-wave patterns and spatial hole burning to enable stable single-frequency emission even at high power. Unlike linear resonators with standing waves, travelling waves uniformly deplete all available gain, preventing multimode oscillations. Unidirectional operation inside the crystal is achieved via the Faraday effect, where an applied magnetic field rotates the polarization such that one propagation direction experiences much higher transmission through the partially polarizing output coupler coating.

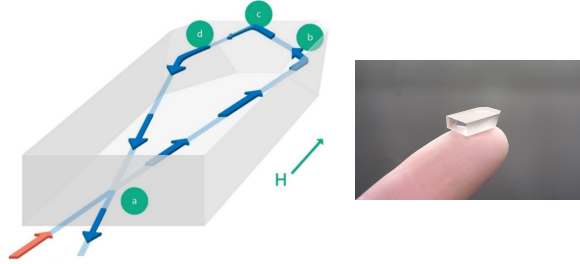


Figure 2.12: Schematic (left) and photograph (right) of the NPRO gain crystal. The crystal forms a monolithic laser cavity, where facet angles ensure high reflectivity through total internal reflection. The circulating optical mode (blue arrows) and pump light (orange arrow) are shown. Facet coatings, combined with polarization rotation from an external magnetic field (H), enable unidirectional traveling-wave operation. The photograph highlights the NPRO crystal size in Coherent's Mephisto lasers. (Image credit: Coherent [94])

Laser frequency tuning

Frequency tuning of the NPRO laser system²¹ is accomplished through two complementary mechanisms: piezoelectric transducer (PZT) and crystal temperature control. The PZT element, bonded directly to a large non-optical crystal face, enables fast, fine frequency adjustments. For instance, applying a voltage compresses the crystal, altering both its physical length and refractive index. These effects decrease the resonator's optical path length, which reduces the free spectral range (FSR) and shifts longitudinal modes (including the active lasing mode) to lower frequencies. The second frequency tuning mechanism is temperature tuning, which enables larger frequency excursions through combined refractive index and thermal expansion effects, but operates at a lower response time ($\sim$ Hz regime).

²⁰ Typically, Nd:YAG ($\lambda = 1\,064\text{ nm}$) or Yb:YAG ($\lambda = 1\,030\text{ nm}$).

²¹ All experiments in this thesis, utilized the *Mephisto 500 FAM* laser from Coherent.

Noise Sources in Laser Interferometers

This chapter introduces the fundamental and technical noise sources in laser interferometers, with a particular focus on the low-frequency interferometers used in this thesis. The optical noise sources include laser frequency noise, shot noise, and spurious light signals, while thermal and temperature noise, as well as electronic and readout noise, represent additional sources that limit interferometer performance. The presentation of the individual noise sources in this chapter establishes the foundation for the subsequent chapters and facilitates a concise experimental discussion in Section 5.3 and in Part III.

While the noise sources (shot noise, LFN, etc.) defined in this chapter are characterized in absolute terms, their noise contribution to the interferometer measurements (phase, displacement, temperature, voltage, etc.) determines their impact. To calculate the noise contribution, we determine the noise-induced, e.g., displacement noise $\tilde{S}_{\text{noise}}$ as the inverse of the configuration-specific SNR as follows:

$$\tilde{S}_{\text{noise}} = \frac{\text{noise}}{\text{signal}}, \quad (3.1)$$

where ‘noise’ denotes the noise sources defined in this chapter and “signal” the experiment-specific (linearized) signal.

3.1 Laser frequency noise

While lasers are often idealized as perfectly monochromatic light sources, in practice they exhibit inherent frequency fluctuations that manifest as phase and frequency noise. These fluctuations contribute substantially in precision interferometric measurements. Laser frequency noise (LFN) refers to random fluctuations in the instantaneous frequency of a laser’s output over time. The instantaneous frequency, defined as the temporal derivative of the optical phase ϕ , is given by

$$f(t) = \frac{1}{2\pi} \frac{d\phi}{dt} \quad [\text{Hz}]. \quad (3.2)$$

For a commercial NPRO laser (see Section 2.5), the frequency noise is typically $10 \text{ kHz}/\sqrt{\text{Hz}}$ at 1 Hz, following a $1/f$ slope. To quantify how laser frequency noise couples into the interferometer readout,

the optical phase difference of a Michelson interferometer can be expressed as:

$$\Delta\Phi = k\Delta L = \frac{2\pi f}{c}\Delta L \quad [\text{rad}], \quad (3.3)$$

where ΔL denotes the static arm-length difference, c the speed of light in vacuum, and f the laser frequency. Expressing the laser frequency as the sum of its mean value and fluctuation, $f = f_0 + \delta f$, and likewise the arm-length difference as $\Delta L = L_0 + \delta L$, where L_0 is the static arm imbalance and δL is the differential motion to be measured. Thus, the phase difference can be expressed as

$$\Delta\Phi = \left(\frac{2\pi}{c}\right)(f_0 + \delta f)(L_0 + \delta L) \quad [\text{rad}]. \quad (3.4)$$

Assuming $L_0 \gg \delta L$ and $f_0 \gg \delta f$, higher-order terms can be neglected and we obtain

$$\Delta\Phi \approx \left(\frac{2\pi}{c}\right)(f_0 L_0 + f_0 \delta L + \delta f L_0) \quad [\text{rad}]. \quad (3.5)$$

The first term, $f_0 L_0$, is static and determines the interferometer's operating point. The second term, $f_0 \delta L$, represents the desired signal arising from differential arm-length variations, while the third term, $\delta f L_0$, arises from laser frequency noise. Consequently, the contribution of laser frequency noise ($\delta f L_0$) is indistinguishable from the genuine differential path-length signal ($f_0 \delta L$). To suppress laser frequency noise coupling we can precisely match the interferometer arm lengths ($L_0 \approx 0$). Alternatively, the laser frequency can be stabilized, for instance, by laser-cavity locking using a stable reference ($\delta f_0 \approx 0$), as discussed in Section 2.3. The equivalent displacement noise induced by laser frequency noise follows from Equation 3.3 using the relation $\Delta\Phi/2\pi = \delta\lambda/\lambda$, yielding

$$\delta\lambda = \frac{\lambda}{c} \cdot \Delta L \cdot \delta f \quad [\text{m}]. \quad (3.6)$$

Substituting the spectral frequency noise $\tilde{S}_f$, we obtain the spectral displacement noise:

$$\tilde{S}_{\text{LFN}} = \frac{\lambda}{c} \cdot \Delta L \cdot \tilde{S}_f \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (3.7)$$

3.2 Photon shot noise

Shot noise (SN) represents the fundamental statistical fluctuation in a photocurrent (or charge current) arising from the discrete, Poisson-distributed arrival times of individual photons or electrons. It is white noise, with a power spectral density proportional to the average current. [95, 96]. The initial interpretation of photon shot noise attributes it to the random timing of photon absorption events in a photodetector. In other words, it was considered a property of the detection process rather than noise in the light field itself. However, the demonstration of amplitude-squeezed light, which exhibits intensity noise below the shot-noise limit, proved that shot noise is an intrinsic quantum property of the light field [32]. An ideal photodetector with unity quantum efficiency measures only photon shot noise. In practice, detection systems also exhibit electronic shot noise through amplifiers and readout electronics (see Section 3.6). The Schottky formula defines shot noise in terms of the power spectral

density (PSD) as

$$\tilde{S}_{\text{SN}_{xx}} = 2e\bar{i} = 2eR_{\text{PD}}\bar{P} \quad [\text{A}^2/\text{Hz}], \quad (3.8)$$

where e denotes the elementary charge, $\bar{i}$ the average photocurrent, and $\bar{P}$ the average optical power. The responsivity of the photodiode is given by

$$R_{\text{PD}} = \eta \frac{e}{h\nu} = \eta \frac{e\lambda}{hc} \quad [\text{A/W}], \quad (3.9)$$

where η denotes the quantum efficiency, $h\nu$ the photon energy, and λ the laser wavelength. By substituting Equation 3.9 into Equation 3.8, we obtain the *photon* shot noise in terms of the power-equivalent ASD as

$$\tilde{S}_{\text{SN}} = \sqrt{2 \frac{hc}{\lambda} \bar{P}} \quad [\text{W}/\sqrt{\text{Hz}}]. \quad (3.10)$$

3.3 Intensity noise

Intensity noise is typically quantified by the relative intensity noise (RIN), defined as the power noise $\delta P(t)$ normalized to the average power $\bar{P}$ and expressed as:

$$P_{\text{RIN}}(t) = \frac{\delta P(t)}{\bar{P}}. \quad (3.11)$$

In laser interferometers, RIN arises from various sources including the laser itself, optical components, photodetectors, and coupling through optical fibers or polarization optics. Polarization-maintaining (pm) fibers combined with a subsequent polarizing beamsplitter (PBS) are a significant source of amplitude noise at low frequencies. RIN, especially from technical noise sources affecting the laser system, decreases significantly at higher frequencies, demonstrating the advantage of heterodyne interferometers operating at MHz frequencies. The fundamental limit of RIN is set by shot noise, derived from Equation 3.10, and is given by [97]¹:

$$\tilde{S}_{\text{RIN,SN}}(f) = \frac{2h\nu}{\bar{P}} \quad [1/\text{Hz}]. \quad (3.12)$$

Above the shot noise limit, RIN can be canceled by balanced detection, a technique where both output ports of the recombination beam splitter are measured and subtracted from each other. This strongly rejects common-mode noise sources such as RIN. When shot noise limited, this technique can be used to measure the shot noise itself while rejecting other common-mode noise sources.

In heterodyne interferometers such as LISA, RIN couples into the phase readout through two distinct first-order mechanisms [98]: additive $1f$ -RIN near the heterodyne frequency and multiplicative $2f$ -RIN near twice the heterodyne frequency. $1f$ -RIN-induced phase noise scales inversely with contrast, depends on beam powers and splitter coefficients, and rejects via ideal balanced detection, while power mismatches amplify residuals. In contrast, $2f$ -RIN cannot be suppressed by balanced detection alone. It appears with identical phasing in both detection ports and is independent of beam powers and contrast. From the Taylor expansion one can see that half the $2f$ -RIN amplitude

¹ PSD of RIN: $\tilde{S}_{\text{RIN}}(f) = \frac{\tilde{P}_{\delta P}(f)}{\bar{P}^2}$, where $\tilde{P}_{\delta P}(f) = 2h\nu\bar{P}$.

down-converts to the heterodyne frequency (appearing as additive phase noise), while the other half up-converts beyond the phasemeter bandwidth. Thus, for heterodyne interferometers, balanced detection and precise power balancing of the interfering beams are essential to mitigate $1/f$ -RIN, while $2f$ -RIN requires laser power stabilization around $2f_{\text{het}}$.

3.4 Spurious light

In the previous sections, we assumed that the light beams propagate through the interferometer along a single, well-defined path. In practice, however, additional light fields, referred to as spurious light beam (SLB), are also present. These fields primarily originate from parasitic reflections and scattering at imperfect optical surfaces. When such beams re-enter the interferometer, they can interfere with the nominal beams², thereby producing spurious light signal (SLS). These unwanted signals not only reduce the interferometer contrast but may also introduce excess phase noise.

This section categorizes the different types of spurious light beam, outlines their physical origins, describes their impact on phase measurements and discusses possible mitigation strategies.

3.4.1 Categories of spurious light beams

Spurious light beams can be categorized according to their physical origin and properties into three main categories: ghost beams from secondary reflections, scattered light generated by surface imperfections or material contamination, and stray light originating from external optical sources. Figure 3.1 provides a schematic overview of the primary origin of these spurious light beams. In much of the literature, the terms *scattered light* and *stray light* are often used interchangeably to describe light that is diffusely reflected or scattered from optical surfaces. However, in this thesis, a distinction is maintained, with stray light specifically denoting contributions from external sources.

Ghost beams originate from unintended secondary reflections at optical components such as mirrors, beam splitters, and polarization optics.

Collinear ghost beams propagate the same, or nearly the same, optical path as the nominal beam. They are particularly problematic because they are difficult to spatially separate and can interfere coherently. Suppression of such ghost beams can be achieved by applying anti-reflection (AR) coatings, using wedged optics, or incorporating beam stops and spatial filters to block or separate unwanted reflections before they reach the detector (see Section 3.4.4).

Diffraction ghost beams are produced when light undergoes diffraction at optical elements such as gratings, pinholes, or surface defects. These beams typically propagate at discrete angles relative to the main beam, which reduces direct interference but can still introduce unwanted signals if they re-enter the detection path. Similar effects may also arise from scattering at sharp edges or slight misalignments of the nominal beam. Diffracted ghost beams can be mitigated by installing beam stops or spatial filters to prevent their re-coupling into the interferometer.

Polarization ghost beams occur when reflections or transmissions within the interferometer alter the light's polarization state. For instance, in a nonideal polarizing beamsplitter (PBS), a fraction of the orthogonal polarization may be inadvertently transmitted or reflected, leading to leakage of the undesired polarization component. If the detection system is not perfectly polarization-selective, these beams can interfere coherently with the main signal and degrade interferometer contrast.

² Nominal beams refer to the reference and measurement beams.

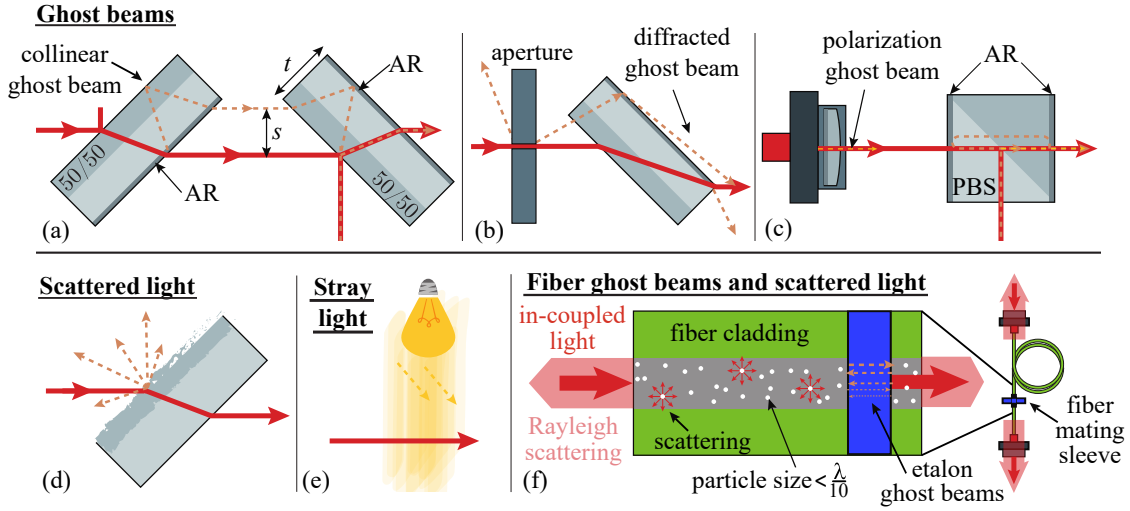


Figure 3.1: Schematic illustration of spurious light beams (SLBs). They can be categorized according to their physical origin and properties into three main categories: (a-c) ghosts beams from secondary reflections, (d) scattered light from surface imperfections or material contamination and (e) stray light originating from external optical sources. The final panel (f) shows multiple SLBs present in an optical fiber. (Illustration modified from: [57]).

Scattered light arises from imperfections in optical coatings, surface roughness, or contamination on optical components, leading to an unwanted redistribution of the optical field. In interferometric systems, scattered light can couple back into the main beam path, producing parasitic interference signals and thereby degrading phase stability and contrast. At the microscopic level, Rayleigh scattering occurs when light interacts with particles or inhomogeneities much smaller than the optical wavelength [99]. This process is relevant in optical fibers, where it contributes to signal attenuation and backscattering. Although Rayleigh scattering is typically weak in free-space optics, its cumulative effect across multiple surfaces can still become significant in precision interferometry.

Stray light refers to light from external sources, such as ambient illumination, or other laser systems, that scatters or reflects into the detection path. In most interferometric measurements, the influence of such light on the measured phase is negligible compared to other spurious contributions, provided that both the spatial mode overlap with the nominal beam and the optical power of the stray light remain sufficiently low. In addition, performing interferometric experiments within an enclosed environment, such as a vacuum chamber, inherently mitigates the influence of ambient light.

Fiber ghost beams and scattered light occur in optical fibers. Rayleigh scattering within the fiber core and ghost-beam reflections at fiber connectors, forming parasitic etalons. The resulting scattered or reflected light is indistinguishable from the nominal beam in terms of spatial mode, making it particularly difficult to isolate. Thermal or density fluctuations within the fiber can modify its refractive index, thereby inducing temporal variations in the amplitude and phase of the ghost beam. Moreover, many optical fibers used in precision interferometry are polarization-maintaining (pm)³, a property

³ All the fibers used in the experiments in this work are polarization-maintaining (pm)-fibers.

achieved through built-in stress rods, commonly referred to as *panda eyes*. Variation in mechanical tension, for example due to temperature fluctuations, can rotate the initial polarization axis. In the presence of subsequent polarization optics, such changes can enhance the formation of polarization ghost beams.

3.4.2 Small vector noise

When spurious light beams interfere with the nominal interferometer beams, they produce SLSs and introduce phase errors or phase noise in the case of non-stationary SLSs. The following derivation quantifies the SLB-induced phase error (based on [57]).

When the reference and measurement beams from the interferometer arms interfere at the recombination beam splitter, they generate a nominal beat note signal

$$S_{\text{nom}} \propto \hat{E}_1 \hat{E}_2 e^{i(\Delta\omega t + \Delta\phi)}, \quad (3.13)$$

where $\Delta\omega = \omega_1 - \omega_2$ is the beat-note frequency, and $\Delta\phi = \phi_1 - \phi_2$ the static differential arm-length phase. When a SLB interferes with one of the nominal beams *before* the recombination beam splitter, the measured signal becomes $S_{\text{meas}} = S_{\text{nom}} + S_{\text{SLS}}$. Such an example is shown in Fig. 3.2, in which the spurious light signal can be expressed as

$$S_{\text{SLS}} \propto \hat{E}_1 \hat{E}_{\text{SLB}} e^{i(\Delta\omega t + \phi_{\text{SLS}})}, \quad (3.14)$$

where $\phi_{\text{SLS}} = \phi_1 - \phi_{\text{SLS}}(t)$ is the SLS phase difference (assuming $\omega_{\text{SLB}} \approx \omega_2$ such that $\Delta\omega$ is preserved). The total measured signal becomes $S_{\text{meas}} = S_{\text{nom}} + S_{\text{SLS}}$. Since S_{nom} and S_{SLS} are in the same beat frequency regime ($\Delta\omega$), their vector addition can be visualized in a phasor diagram (Section 2.4.4) as shown in Fig. 3.3.

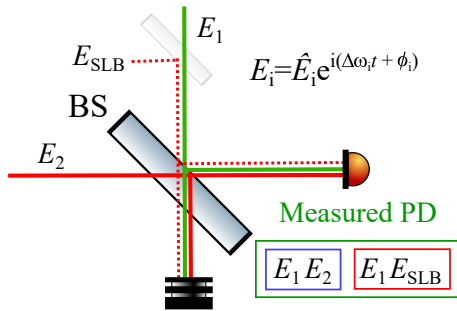


Figure 3.2: Heterodyne interferometer scheme of a spurious light beam (SLB) interfering with the measurement beam (E_1) *before* the recombination beam splitter (BS).

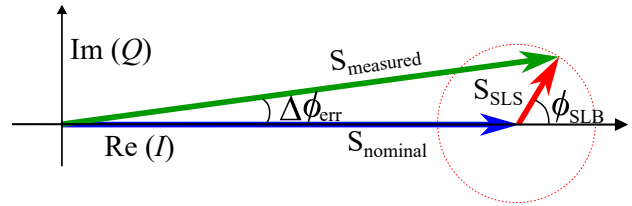


Figure 3.3: Phasor diagram of a SLS contaminating the nominal beat note, producing a phase error ϕ_{SLB} . The SLS phase ϕ_{SLS} and amplitude ($S_{\text{SLS}}/S_{\text{nom}}$) determine the error magnitude. The circle around the SLS indicates its non-stationarity, i.e., the deviation from the beat note frequency $\Delta\omega$.

The spurious light signal contaminates the nominal signal, inducing a phase error $\Delta\phi_{\text{err}}$ ⁴. Using

⁴ Assuming $\phi_{\text{nom}} = 0^\circ$ for simplicity.

trigonometric relations, the phase error is given by

$$\tan(\Delta\phi_{\text{err}}) = \frac{S_{\text{SLS}} \sin(\phi_{\text{SLS}})}{S_{\text{nom}} + S_{\text{SLS}} \cos(\phi_{\text{SLS}})}, \quad (3.15)$$

where S_{nom} denotes the nominal signal, S_{SLS} the spurious light signal, and S_{meas} the measured signal. For $S_{\text{SLS}} \ll S_{\text{nom}}$, Equation 3.15 simplifies⁵ to

$$\Delta\phi_{\text{err}} \approx \frac{S_{\text{SLS}} \sin(\phi_{\text{SLS}})}{S_{\text{nom}} + S_{\text{SLS}}} \approx \frac{S_{\text{SLS}}}{S_{\text{nom}}} \sin(\phi_{\text{SLS}}). \quad (3.16)$$

For n spurious signals, the phase error generalizes and can be expressed as

$$\Delta\phi_{\text{err}} = \frac{1}{S_{\text{nom}}} \cdot \sum_{i=1}^n S_{\text{SLS},i} \cdot \sin(\phi_{\text{SLS},i} - \phi_{\text{nom}}). \quad (3.17)$$

The phase error depends on the ratios of SLS to nominal signal amplitudes, as $2|\langle E_{\text{meas}} | E_{\text{ref}} \rangle| = \sqrt{\eta P_{\text{meas}} P_{\text{ref}}}$ and $2|\langle E_x | E_{\text{SLS},i} \rangle| = \sqrt{\eta_i P_{\text{SLS},i} P_x}$ with $P_x \in \{P_{\text{meas}}, P_{\text{ref}}, P_{\text{SLS},k}\}$ ($i \neq k$), where P_{meas} , P_{ref} are nominal signal powers, $P_{\text{SLS},i}$ are spurious signal powers, and η denotes interferometer heterodyne efficiency. Equation 3.16 can be rewritten in terms of power as

$$\Delta\phi_{\text{err}} = \frac{\sqrt{\eta_{\text{SLS}} P_{\text{meas}} P_{\text{SLS}}}}{\sqrt{\eta P_{\text{meas}} P_{\text{ref}}}} \sin(\phi_{\text{SLS}}) = \frac{\sqrt{\eta_{\text{SLS}} P_{\text{SLS}}}}{\sqrt{\eta P_{\text{ref}}}} \sin(\phi_{\text{SLS}}). \quad (3.18)$$

Assuming $\eta_{\text{SLS}} = \eta$ yields:

$$\Delta\phi_{\text{err}} = \sqrt{\frac{P_{\text{SLS}}}{P_{\text{ref}}}} \sin(\phi_{\text{SLS}}). \quad (3.19)$$

Spurious light signals are typically non-stationary in phase (ϕ_{SLS}) (or amplitude), thereby inducing noise. We can rewrite Equation 3.19 as

$$\tilde{S}_{\Delta\phi}(f) = \sqrt{\frac{P_{\text{SLS}}}{P_{\text{ref}}}} \cdot \tilde{S}_{\phi_{\text{SLS}}}(f) \quad [\text{rad}/\sqrt{\text{Hz}}]. \quad (3.20)$$

where $\tilde{S}_{\phi_{\text{SLS}}}(f)$ is (linearized) the spectral density of the SLS phase noise⁶. For worst-case analysis, we assume $\sin(\phi_{\text{SLS}}) = 1$ with white noise $\tilde{S}_{\phi_{\text{SLS}}}(f) \approx S_0$ [rad/ $\sqrt{\text{Hz}}$]. The SLS phase error contribution can critically limit high-precision interferometers like the optical benches of LISA including the OGSE, requiring displacement sensitivity below 1 pm/ $\sqrt{\text{Hz}}$. For a beam power of $P_{\text{ref}} = P_{\text{meas}} = 1$ mW, $\lambda = 1064$ nm, and $\tilde{S}_{\phi_{\text{SLS}}}(f) \approx S_0$, the 1 pm/ $\sqrt{\text{Hz}}$ (1.18×10^{-5} rad/ $\sqrt{\text{Hz}}$) LISA requirement limits SLS power to $P_{\text{SLS}} \approx 1.4 \times 10^{-10}$ mW.

Moreover, SLSs at frequencies outside the LISA measurement band cannot be neglected either.

⁵ $\tan(x \ll 1) \approx x$

⁶ The linearized formula applies to small fluctuations around the instantaneous SLS phase. Note that $\phi_{\text{SLS}}(t)$ effectively rotates over $0 \dots 2\pi$ (depending on the scatter source), so $\sin(\phi_{\text{SLS}}(t))$ spans $[-1, 1]$ and defies small-signal $\sin x \approx x$ approximations, potentially underestimating peak excursions.

Nonlinear effects, especially phase modulation from scattering off *moving* components (Sec. 2.4.2, Appendix B), up-convert low-frequency noise sources into the measurement band. A precise estimation of the SLS contribution to the overall interferometer performance is not only complex but also require computationally expensive time-domain simulations, as standard frequency-domain noise models fail [47].

3.4.3 Spurious light coupling in laser-cavity stabilization

In an optical cavity, the phase coupling error from SLSs is distinct from that of a simple interferometer due to multiple round-trips, resonant enhancement, and mode selectivity. While an incoherent⁷ SLS is heavily suppressed by the cavity (no buildup), a coherent SLS can interfere with the main signal and generate an error signal ϵ_{SLS} which is independent of the cavity phase ($\epsilon_{\text{SLS}} \not\propto \delta f$). Here, we assume the SLS interferes with the nominal signal after the reflection from the cavity. The summation of these components yields the total error signal:

$$\epsilon_{\text{total}}(\delta f) = \epsilon_{\text{nom}}(\delta f) + \epsilon_{\text{SLS}}, \quad (3.21)$$

where ϵ_{SLS} denotes the error signal SLS approximation as⁸

$$\epsilon_{\text{SLS}} \approx P_{\text{sig}} \cdot \sqrt{\frac{P_{\text{SLS}}}{P_{\text{sig}}}} \phi_{\text{SLS}}. \quad (3.22)$$

The coupling of this error depends on the SLS phase ϕ_{SLS} . When the phase error is non-stationary ($\phi_{\text{SLS}}(t)$), phase noise is directly induced into the feedback loop. For worst-case analysis, we assume S_0 ⁹. We can include the general expression of $\epsilon_{\text{nom}} = D \cdot \delta f$ and rewrite Equation 3.21 as

$$\epsilon_{\text{total}}(\delta f) = D \cdot \delta f + P_{\text{sig}} \sqrt{\frac{P_{\text{SLS}}}{P_{\text{sig}}}} S_0, \quad (3.23)$$

with the discriminator slope D as

$$D \propto P_{\text{sig}} \frac{\mathcal{G}}{\text{FWHM}} \propto P_{\text{sig}} \cdot \mathcal{F} \quad (3.24)$$

where $\mathcal{G}$ denotes the optical gain, FWHM the full width at half maximum (FWHM), and $\mathcal{F}$ the finesse of the cavity. When we lock the frequency of the laser to the length of the cavity, the feedback control loop drives the error signal ϵ_{total} to zero by adjusting the laser frequency. When locked, the loop forces:

$$\epsilon_{\text{total}} = 0. \quad (3.25)$$

⁷ An incoherent SLS has a different frequency and is off-resonant.

⁸ The error signal ϵ_{SLS} is an approximation using Equation 3.20 and assuming a linear relation between the phase error noise from the SLS and the nominal signal P_{signal} . In practice, the relation depends on the specific locking scheme. The approximation should solely be treated as a generalized description.

⁹ $\sin(\phi_{\text{SLS}}) = 1$ with white noise $\tilde{S}_{\phi_{\text{SLS}}}(f) \approx S_0 [\text{rad}/\sqrt{\text{Hz}}]$.

From this follows:

$$D \cdot \delta f = -P_{\text{sig}} \sqrt{\frac{P_{\text{SLS}}}{P_{\text{sig}}}} S_0, \quad (3.26)$$

and the resulting frequency error ($\delta f \rightarrow \delta f_{\text{err}}$) is obtained as

$$\delta f_{\text{err}}(t) = -\frac{P_{\text{sig}}}{D} \sqrt{\frac{P_{\text{SLS}}}{P_{\text{sig}}}} S_0 \quad [\text{Hz}] \quad (3.27)$$

Equation 3.27 reveals the critical role of the discriminator slope D : high-finesse cavities with steep error signal slopes (large D) provide intrinsic suppression of spurious beam phase errors. Furthermore, the phase error is proportional to the amplitude ratio of the SLS and the signal beam, emphasizing the importance of a high input power to the cavity.

In summary, we can conclude that the presence of SLS in a cavity locking scheme causes shifts of the lock point which, in the case of a time varying SLSs, ultimately translate into frequency noise. Mitigation of this noise can be achieved by increasing the power of the signal beam, suppressing the SLSs power, or by increasing the cavity finesse.

3.4.4 Mitigation of spurious light signals

As discussed in the previous sections, spurious light beams in an interferometer can induce excess phase noise that reduces measurement sensitivity and thus requires suppression. It should be noted that SLBs and the corresponding SLS are always present in interferometers. Their performance limitation depends solely on the coupling strength. Several mitigation strategies exist that aim to prevent the generation of SLSs, block their re-entry into the interferometer, or suppress their influence through post-processing. The most common approaches are summarized below, with emphasis on those relevant to this work.

- **High quality optical components**

High-quality optics feature extremely smooth and precisely polished surfaces, minimizing the number and amplitude of surface imperfections and thereby reducing light scattering. They also reduce the generation of higher order modes in e.g. optical cavities. Ghost beams arising from secondary reflections can be mitigated by applying a HR coating to the front surface and an AR coating to the rear surface. In addition to mirrors, care should be taken with lenses used for mode matching or for focusing light onto photodiodes. These elements are often positioned without angular tilt, which increases the likelihood of generating ghost beams [57].

- **Beam dumps / apertures**

Laterally displaced ghost beams can be effectively suppressed using beam dumps or apertures, as illustrated in Fig. 3.1(a), which depicts the following example: A light beam incident on a mirror at an angle of incidence (AOI) = 45° , whose surface coatings are neither perfectly reflective (HR) nor anti-reflective (AR), generates an additional spurious reflection that propagates parallel to the nominal beam. This ghost beam can re-enter the interferometer after reflection from another optical surface. To prevent such undesired coupling, an aperture is positioned along the beam path so that only the nominal beam can pass. The lateral displacement s of the ghost

beam relative to the nominal beam is given by [99]¹⁰

$$s = t \cdot \sin(\text{AOI}) \left[1 - \frac{\cos(\text{AOI})}{\sqrt{n^2 - \sin^2(\text{AOI})}} \right] \quad (3.28)$$

where n denotes the refractive index of the mirror substrate and t its thickness. For a typical fused-silica mirror ($n = 1.4496$ at 1 064 nm [100]), $t = 6.35$ mm, and $\text{AOI} = 45^\circ$, the resulting lateral shift is approximately 2.5 mm. Therefore, any aperture or beam dump placed in the path should have an opening of $d < 2.5$ mm to block the ghost beam while ensuring it does not clip the main Gaussian mode. A practical guideline is to use a pinhole diameter of at least three times the beam radius ($d > 3 \cdot 2w_0$), which allows most of the optical power to pass without introducing diffracted ghost beams.

- **Wedged components**

Parallel optical surfaces can form parasitic etalons due to multiple internal reflections. Introducing a wedge on the secondary surface breaks the parallelism and causes ghost beams to exit at an angle, preventing them from re-entering the interferometer. A common example is an optical fiber terminated with an angled physical connector (APC) connector. A disadvantage of wedged components is that they introduce beam angles, causing interferometer experiments aligned in air to experience a beam tilt when operated in vacuum.

- **Polarization optics**

Polarization optics can be employed to separate the signal beams from SLBs. Many ghost beams undergo additional reflections or transmissions, thereby acquiring a polarization state different from that of the nominal signal. This difference allows their separation/suppression using polarization-selective components such as polarizing beam splitters, wave plates, or polarizers.

- **Vibration and temperature fluctuation isolation**

Mechanical vibrations and temperature fluctuations cause small displacements of optical components such as mirrors, mounts, photodiodes, and optical benches. Even at low frequencies, spurious light signals reflected from these elements experience corresponding phase modulation, which can be up-converted into the measurement band and appear as excess phase noise. A well-isolated environment mitigates these effects, e.g. seismically isolated and thermally insulated vacuum chambers.

- **Balanced detection**

Balanced detection in heterodyne interferometry employs two matched photodiodes at the output ports of the recombination beam splitter (see Fig. 3.4). Subtracting their photocurrents suppresses spurious light signals while preserving the differential heterodyne phase, as visualized in the phasor diagram in Fig. 3.5. When a SLB interferes with one of the nominal beams, generating a beat note frequency near the nominal beat note, *before* the recombination beam splitter¹¹, both beams (E_1 , E_{SLB}) acquire a relative phase shift of π in reflection and none

¹⁰ Assuming $n_0 = 1$ in vacuum.

¹¹ SLBs interfering at the recombination beam splitter cannot be suppressed by balanced detection as the phase appears with opposite signs in both ports. Moreover, only SLS near the heterodyne beat note fall within the phasemeter's mixer+LPF

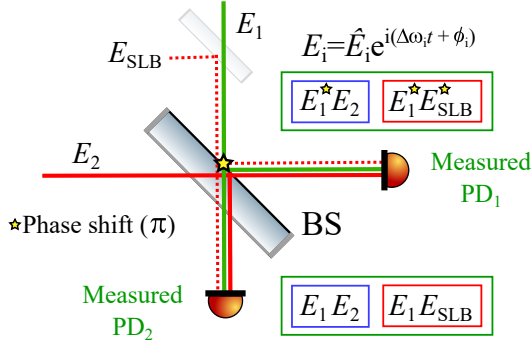


Figure note:

When the reference and measurement beams (E_2, E_1) from the interferometer arms interfere at the recombination beam splitter, they generate a beat-note signal with opposite phase (π shift) across PDs (opposite signs):

$$\phi_{PD_1} = [(\phi_1 + \pi) - \phi_2] + [(\phi_1 + \pi) - \phi_{SLB} + \pi]$$

$$\phi_{PD_2} = [\phi_1 - \phi_2] + [\phi_1 - \phi_{SLB}]$$

Figure 3.4: Balanced detection interferometer scheme, employing two matched photodiodes at the output ports of the recombination beam splitter. SLB interfering with a nominal beam (E_1) produce a SLS which appears with equal signs on both detectors.

in transmission¹². For nominal beams E_1 (reflected) and E_2 (transmitted) at PD_1 the signal acquires π phase; PD_2 sees E_1 transmitted and E_2 reflected (dense-to-air, no π phase shift). Thus, nominal signals appear with opposite phase (π shift) across PDs (opposite signs), while the SLS appear with equal signs on both detectors. Consequently, subtraction of the two signals recovers the pure differential signal, fully canceling SLS contributions. Previous experiments demonstrate that balanced detection suppresses SLS (collinear ghost beams), improving readout performance by up to two orders of magnitude [47]. Balanced detection (i.e., π -measurement) can also expose the SLS by adding the signals instead of subtracting them. This cancels the nominal beat note signals while the spurious light signals remain.

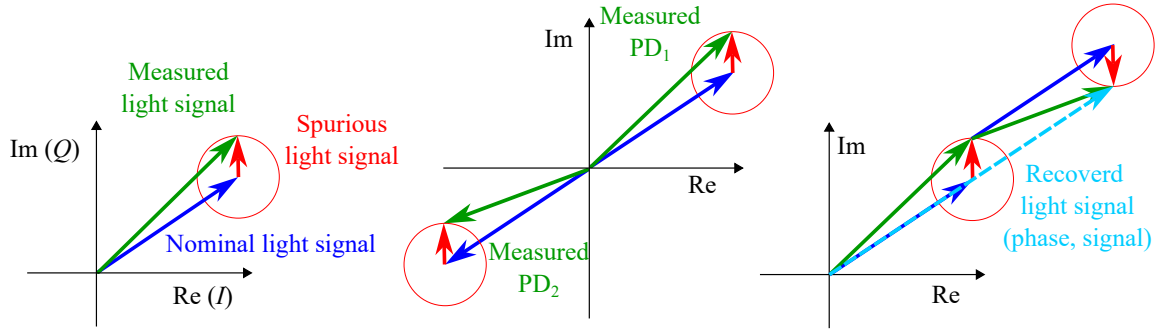


Figure 3.5: Phasor representation of the balanced detection technique in heterodyne interferometry suppressing SLSs. Left: Nominal beam corrupted by a SLS, resulting in measured phase error. Center: Detection at both output ports using matched photodiodes. Right: Signal subtraction cancels the SLS and recovers the nominal signal.

Balanced detection to suppress SLSs is also feasible in homodyne interferometers, although through distinct mechanisms. Deploying balanced homodyne detection at both the symmetric and antisymmetric ports of a Michelson interferometer enables subtraction of arm-specific

bandwidth and contribute noise.

¹² Convention-dependent; may be reversed.

scattered-light noise [101]. By linearly combining these outputs, phase fluctuations between arms can be isolated while SLS contributions are effectively canceled. However, complete elimination of SLS in both interferometer arms remains challenging.

- **Tunable coherence technique**

Tunable coherence, a recently developed technique, controls the coherence by modulating the laser light using pseudo-random-noise (PRN) phase modulation. Intentional interference of the arm beams (equal arm length) is preserved, while parasitic light traveling a different optical path becomes incoherent and thus suppressed [102]. Previous studies demonstrated suppression levels of up to 40 dB for scattered light in a Michelson interferometer [103]. Due to its considerable potential, it merits mention here despite not being implemented in this work. The short path lengths in experiments like LF-HoQI (Section 8) would require high PRN frequencies to efficiently suppress scattered light.

3.5 Thermal and temperature noise

Although related, thermal and temperature noise represent distinct mechanisms through which heat and temperature fluctuations degrade interferometer performance. Thermal noise, present at any non-zero temperature, refers to random fluctuations arising from microscopic thermal motion within materials at thermal equilibrium, setting the fundamental quantum limit via the fluctuation-dissipation theorem. Temperature noise, in contrast, stems from macroscopic temperature variations of components due to external environmental fluctuations. It typically dominates at lower frequencies ($f \lesssim 1$ Hz) and can be mitigated through active environmental stabilization and passive insulation.

3.5.1 Thermal noise

Thermal noise stems from the random thermal motion of atoms and molecules within interferometer components, particularly the test mass mirrors, their coatings, and suspensions. It is fundamentally linked to intrinsic mechanical dissipation via the fluctuation-dissipation theorem [104]. This theorem relates a system's Brownian motion directly to its dissipative properties, such as friction or internal loss. The power spectral density $\tilde{S}_{F,\text{thermal}}(\omega)$ of the fluctuating thermoelastic force acting on a system is given by

$$\tilde{S}_{F,\text{thermal}}(\omega) = 4k_B T \Re \{Z(\omega)\} \quad [\text{N}^2/\text{Hz}], \quad (3.29)$$

where k_B denotes the Boltzmann constant ($\approx 1.3801 \times 10^{-23}$ J/K), T the absolute temperature (in kelvin), and $\Re \{Z(\omega)\}$ represents the real (i.e. dissipative) part of the mechanical impedance¹³. For a simple mass-spring system characterized by mass m and resonant frequency f_0 , the real part of the mechanical impedance at resonance is approximated by

$$\Re \{Z(\omega_0)\} = \frac{m \omega_0}{Q} \quad [\text{N s/m}] \quad (3.30)$$

where Q denotes the quality factor, quantifying how underdamped the oscillator is. A high Q -factor corresponds to lower energy dissipation per oscillation cycle (slower amplitude decay), whereas a low Q -factor indicates stronger damping and faster energy loss. The displacement response (noise)

¹³ For a linear oscillator, $Z(\omega) = R + j(\omega m - k/\omega)$.

to a fluctuating force is obtained by multiplying the driving-force spectrum from the thermal noise $\tilde{S}_{F,\text{thermal}}$ with the mechanical susceptibility $\chi(\omega)$ ¹⁴. Thus, the PSD displacement noise is given by

$$\tilde{S}_{xx,\text{thermal}}(\omega) = |\chi(\omega)|^2 \cdot \tilde{S}_{F,\text{thermal}}(\omega) \quad [\text{m}^2/\text{Hz}]. \quad (3.31)$$

The mechanical susceptibility $\chi(\omega)$ depends on the specific component/configuration, e.g., suspension, substrate¹⁵, or coating thermal noise. While the latter requires detailed calculation (Section 6.2.3), we use here, as an example, the mechanical susceptibility of a simple suspension defined by

$$\chi(\omega) = \frac{1}{m(\omega_0^2 - \omega^2 + i\omega\omega_0/Q)} \quad [\text{m/N}]. \quad (3.32)$$

Near resonance $\omega \approx \omega_0$, the magnitude-squared susceptibility is given by the approximation

$$|\chi(\omega_0)|^2 \approx \frac{1}{m^2\omega_0^4}. \quad (3.33)$$

Substituting Equation 3.29 and 3.33 into Equation 3.31, we obtain the thermally induced displacement noise ASD as

$$\tilde{S}_{\text{thermal}} \propto \sqrt{\frac{k_B T}{m\omega_0^2 Q}} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (3.34)$$

This expression shows that, to minimize the displacement noise arising from thermal noise of a suspension, both the (test) mass m and the mechanical quality factor Q should be increased. In contrast, mirror coatings represent a design trade-off between mechanical loss (i.e., Q -factor), optical performance, and fabrication constraints¹⁶, making coating thermal noise one of the dominant noise sources in optical cavities [105, 106].

In LISA, residual gas (Brownian-force collisions on test masses) in the GRS is expected to dominate thermal noise [45, 107, 108]. Non-thermal coupling noise from temperature gradient fluctuations, manifesting as radiometer and the outgassing effect, is also expected to contribute significantly [45, 109].

3.5.2 Temperature coupling noise

Temperature coupling noise arises from environmental temperature fluctuations and dominates at lower frequencies ($f \lesssim 1$ Hz), typically following a $1/f$ slope¹⁷. It can be mitigated through appropriate environmental control such as dedicated thermally insulated vacuum chamber (see Section 5). These temperature fluctuations couple to interferometer performance through the primary coupling effects: thermal expansion, thermorefractive, and tilt-to-length coupling.

¹⁴ The magnitude-squared mechanical susceptibility $|\chi(\omega)|^2$ defines the transfer function from force to displacement.

¹⁵ This includes thermal noise from mirror substrates but also the cavity spacer.

¹⁶ Key constraints include layer deposition precision, material compatibility, and mechanical stress from multi-layer stacks.

¹⁷ This $1/f$ slope arises from thermally activated fluctuations with a broad distribution of relaxation timescales [110].

Thermal expansion coupling

Thermal expansion causes materials to expand and contract in response to temperature fluctuations, producing length changes in interferometer components such as mirror mounts, optics, and optical benches. These length variations modify the optical path length, expressed in ASD as

$$\tilde{S}_{TE} = \alpha \cdot L \cdot \tilde{T}_{\text{ambient}} \quad [\text{m}/\sqrt{\text{Hz}}], \quad (3.35)$$

where α denotes the coefficient of thermal expansion (CTE) of the material $[1/\text{K}]$ ¹⁸, L the nominal length, and $\tilde{T}_{\text{ambient}}$ the ambient temperature fluctuations $[\text{K}/\sqrt{\text{Hz}}]$.

Thermorefractive coupling

Temperature fluctuations alter the refractive index of optical materials, characterized by the thermo-optic coefficient dn/dT , thereby inducing path-length and displacement noise described by

$$\tilde{S}_{TR} = \frac{dn}{dT} \cdot L_{\text{opt}} \cdot \tilde{T}_{\text{ambient}} \quad [\text{m}/\sqrt{\text{Hz}}], \quad (3.36)$$

where L_{opt} denotes the nominal length of the optic.

Tilt-to-length coupling

Tilt-to-length coupling (TTLC) induces angular (tilt) and lateral beam jitter that cross-couples into the interferometric phase measurement, manifesting as apparent longitudinal displacement. This effect is particularly critical for high-precision interferometers such as LISA where it arises primarily from two sources. The first contribution stems from the free-floating test masses inside the spacecraft, making angular misalignments between the spacecraft unavoidable. The second contribution arises from the science interferometer receiving the laser beam from the distant spacecraft, where spacecraft tilt induces a wavefront tilt at the receiving telescope. Both sources make TTLC a key focus of optical bench design [55, 111]. Additional contributions, primarily relevant for OGSE, stem from seismic noise and temperature coupling, which are expected to dominate low-frequency measurements. We can categorize TTLC into two coupling mechanisms:

Geometric TTLC: Arises purely from ray geometry, where angular or lateral misalignments alter the optical path length. This includes the *lever-arm effect* and *piston effect*. The first effect describes a mirror rotation about an axis displaced from the beam spot, producing effective longitudinal motion, while the second effect describes the longitudinal displacement of reflecting surfaces. This geometric TTLC is further discussed in Section 4.4.2, with detailed derivations provided in the Appendix C.

Non-geometric TTLC: Arises from changes in the interference pattern across a finite-sized detector, primarily due to the interaction of interfering wavefronts and detector/aperture geometry [112]. Typical contributions include mismatched wavefront curvatures, aberrations, and clipping in the imaging system. This coupling mechanism is more complex to model analytically and is typically evaluated using simulation tools such as IfoCAD [113].

¹⁸ The CTE (1/K) of low thermal expansion materials is often expressed in units of $\mu\text{m}/\text{m}/\text{K}$ or ppm/K.

3.6 Electronic and readout noise

Electronic noise enters the interferometer readout chain at multiple stages, beginning at the laser source and control electronics and extending through the photodetector and to the digitization stage of the analog to digital converter (ADC). For picometer-stable measurements, these noise contributions must be thoroughly characterized and mitigated, as they can easily obscure optical signals and limit performance. Furthermore, auxiliary measurements used for monitoring and active control must employ low-noise electronics to avoid coupling via e.g., seismic and temperature noise, into the actual optical measurement. Consequently, this chapter discusses fundamental electronic noise sources such as shot noise and thermal noise but also readout noise, such as quantization noise. Amplifier noise is also addressed in the context of its contribution to the overall ADC noise floor.

This chapter establishes the theoretical foundation for the electronic noise coupling in Part III (Interferometer experiments) and for Section 5.3, where a temperature sensor and the associated readout electronics are presented.

3.6.1 Electronic shot noise

Shot noise (SN) arises from the discrete nature of electric charges, or from photons as previously discussed in Section 3.2. In electronic circuits, shot noise manifests as random fluctuations in the direct current that are independent of temperature and frequency (white noise). The Schottky formula defines shot noise in terms of the ASD as [95]

$$\tilde{U}_{\text{SN}} = \sqrt{2e\bar{i}} \quad [\text{A}/\sqrt{\text{Hz}}], \quad (3.37)$$

where e denotes the elementary charge and $\bar{i}$ the mean current. Electronic shot noise increases with the mean current. However, since the signal amplitude typically scales linearly with current, while the shot noise grows only with its square root, the resulting signal-to-noise ratio improves at higher operating currents. In this thesis, the photodetectors, temperature sensor, and corresponding electronic board operate at sufficiently high current that *electronic* shot noise remains well below the dominant thermal and quantization noise terms.

3.6.2 Electronic thermal noise

Thermal noise¹⁹ arises from the random fluctuations of charge carriers and is independent of any applied voltage. It is commonly referred to as Johnson-Nyquist (JN) noise [114, 115].

The JN noise root mean square (RMS) voltage across a resistor R at temperature T (in Kelvin) within a bandwidth Δf is given by

$$U_{\text{rms,JN}} = \sqrt{4k_{\text{B}}TR\Delta f} \quad [\text{V}], \quad (3.38)$$

where k_{B} denotes the Boltzmann constant ($\approx 1.3801 \times 10^{-23}$ J/K). The amplitude spectral density of the thermal noise is obtained by normalizing the RMS voltage by the measurement bandwidth $\sqrt{\Delta f}$,

¹⁹ Thermal noise of electronics and test mass mirrors both arise from the fluctuation-dissipation theorem, but through fundamentally different dissipation mechanisms. Johnson-Nyquist (JN) noise originates from microscopic charge carrier fluctuations (electron scattering); coating thermal noise arises from macroscopic lattice vibrations (mechanical damping) [104].

where Δf is half the ADC's sampling rate. Thus, we obtain

$$\tilde{U}_{\text{JN}}(f) = \sqrt{4k_{\text{B}}TR} \quad [\text{V}/\sqrt{\text{Hz}}]. \quad (3.39)$$

For purely passive, linear, and frequency-independent resistors where $R \neq R(f)$, JN noise is considered white noise. However, in circuits containing reactive components such as capacitors and inductors, the frequency-dependent real part of the complex impedance $\Re[Z(f)]$ must be considered, resulting in frequency-dependent thermal noise behavior.

Thermal noise in an ADC originates from resistive elements and the sampling network, manifesting as JN noise on the track-and-hold capacitor C . The total integrated RMS noise on the track-and-hold capacitor C after sampling is given by [116]

$$U_{\text{ADC,thermal}} \approx \sqrt{\frac{4k_{\text{B}}T}{C}}. \quad (3.40)$$

However, precise calculations of the ADC's thermal noise are complex and architecture-dependent. It is therefore typically measured rather than computed theoretically.

3.6.3 Quantization noise

Quantization noise in an ADC arises from mapping a continuous range of analog input voltages to a finite set of discrete digital codes. Each digital output code corresponds to an analog input range spanning one least significant bit (LSB), defined as

$$\text{LSB} = \frac{U_{\text{fsr}}}{2^N}, \quad (3.41)$$

where U_{fsr} denotes the full-scale range (fsr) reference voltage and N the ADC's bit resolution. Figure 3.6 shows an ideal ADC transfer function, unaffected by offset or gain errors. For an ideal ADC, the quantization error e is modeled as a uniform random variable between $-\text{LSB}/2$ and $+\text{LSB}/2$, with constant probability density $p(e) = 1/\text{LSB}$. The total quantization noise power is given by

$$U_{\text{quant}} = \int_{-\frac{\text{LSB}}{2}}^{\frac{\text{LSB}}{2}} e^2 p(e) \, de = \frac{\text{LSB}^2}{12} \quad [\text{V}^2]. \quad (3.42)$$

The corresponding ASD, assuming white noise distribution over the Nyquist bandwidth $f_s/2$, is obtained by normalizing the noise power:

$$\tilde{U}_{\text{quant}} = \frac{\text{LSB}}{\sqrt{6}f_s} \quad [\text{V}/\sqrt{\text{Hz}}]. \quad (3.43)$$

Substituting Equation 3.41 into the expression yields the generic spectral quantization noise, expressed as

$$\tilde{U}_{\text{quant}} = \frac{U_{\text{fsr}}}{2^N \sqrt{6}f_s} \quad [\text{V}/\sqrt{\text{Hz}}]. \quad (3.44)$$

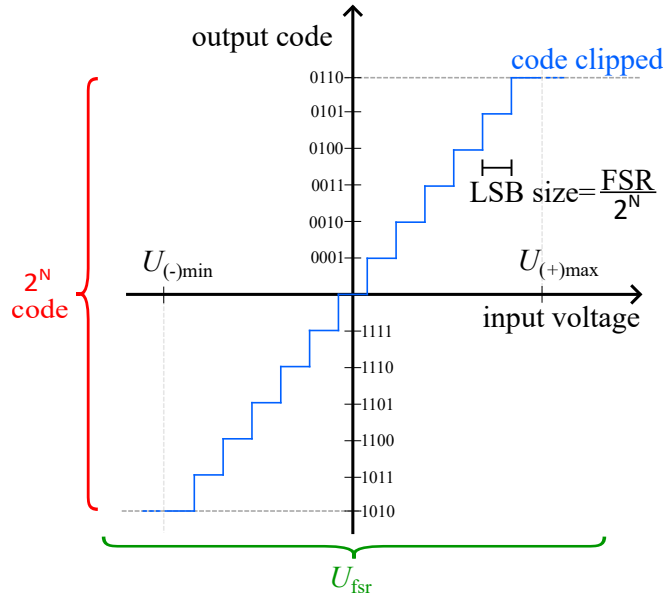


Figure 3.6: Ideal transfer function of a 4-bit ADC, showing quantization levels determined by the bit resolution (2^N codes) and full-scale reference voltage U_{fsr} . Offset and gain errors are neglected. (Image modified from: [117])

The case for analog, digital filters and oversampling

Analog antialiasing filters before the ADC are essential to prevent high-frequency noise above the Nyquist frequency $f_s/2$ from aliasing into the signal band, a corruption that no downstream digital filter can recover. After digitization, digital decimation filters reduce the data rate, while providing superior frequency selectivity, linear-phase response (minimal waveform distortion), and software-configurable cutoff frequencies that are impossible with analog circuits. Modern ADCs typically employ high sampling rates $f_s \gg \text{kHz}$ to mitigate aliasing effects and enable oversampling of the measured signal. The oversampling ratio (OSR) is defined as²⁰

$$\text{OSR} = \frac{f_s}{2 \times \text{data rate}}. \quad (3.45)$$

For example, a signal with $f = 1 \text{ Hz}$ is measured with $f_s = 1 \text{ kSa/s}$. Thus, it is oversampled by a factor of 500. Setting a digital filter at e.g., $f_{3dB} = 50 \text{ Hz}$ preserves the signal while reducing total effective noise power (RMS, [V]). However, the relevant quantity, the spectral density (noise floor) $\tilde{U}_{\text{quant}}$, remains constant and can only be reduced by increasing f_s ($\tilde{U}_{\text{quant}} \propto 1/\sqrt{f_s}$, Equation 3.44).

Sampling jitter

Sampling jitter, the timing uncertainty of ADC sampling instants, introduces phase noise. For an input signal at frequency f_A , the induced phase noise is

$$\tilde{S}_{\varphi, \text{jitter}} = 2\pi f_A \cdot \tilde{\tau}_A \quad [\text{rad}/\sqrt{\text{Hz}}] \quad (3.46)$$

²⁰ Note that the output data rate, set by the digital filter, differs from the ADC's sampling rate.

where $\tilde{\tau}_A$ is the timing jitter spectral density [$\text{s}/\sqrt{\text{Hz}}$]. The expression shows that phase jitter noise increases linearly with input signal frequency, significantly degrading the readout of heterodyne interferometers operating in the MHz regime. It contributes significant noise in the phasemeter readout of LISA, especially at low measurement frequencies [72].

3.6.4 ADC and amplifier noise

The primary noise sources of an ADC are thermal and quantization noise, as well as the reference voltage noise, which originates at the ADC's internal modulator. Since these contributions arise from independent physical mechanisms, they are uncorrelated, and the total ADC noise is expressed as

$$\tilde{U}_{\text{ADC}} = \sqrt{(\tilde{U}_{\text{ADC,thermal}})^2 + (\tilde{U}_{\text{ADC,quant}})^2 + (\tilde{U}_{\text{ADC,ref}})^2} \quad [\text{V}/\sqrt{\text{Hz}}]. \quad (3.47)$$

While the ADC's inherent thermal noise is determined by the device architecture, quantization noise depends on LSB size and can be influenced by the choice of the full-scale range reference voltage U_{fsr} . This dependency varies between low- and high-resolution ADCs, as illustrated in Fig. 3.7. For low-resolution ADCs, quantization noise dominates. Reducing the reference voltage²¹ decreases the LSB size, thereby lowering quantization noise and the total ADC noise level. For high-resolution ADCs, thermal noise typically dominates. Using a larger reference voltage extends the ADC input range, improves SNR, and keeps quantization noise below the thermal noise level.

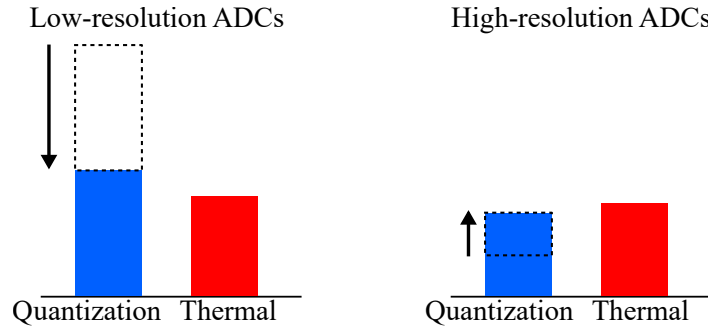


Figure 3.7: Quantization and thermal noise levels of a low- and high-resolution ADC. Appropriate adjustment of the quantization noise improves performance (signal-to-noise ratio). (Image modified from: [117])

The ADC is typically preceded by an amplifier that contributes thermal and flicker noise²², with the latter exhibiting $1/f$ slope. To understand the amplifier's noise contribution to the signal chain, the ADC and the amplifier must be considered together. A scheme of the signal chain is shown in Fig. 3.8, including an ideal (noiseless) amplifier and ADC with separate input-referred noise sources. Assuming uncorrelated noise sources, the output-referred noise is given by

$$\tilde{U}_{\text{out.r.}}(f) = \sqrt{(\tilde{U}_{\text{amp}}(f) \cdot G_{\text{amp}})^2 + (\tilde{U}_{\text{ADC}}(f))^2} \quad [\text{V}/\sqrt{\text{Hz}}], \quad (3.48)$$

where $\tilde{U}_{\text{amp}}(f)$ is the input-referred noise of the amplifier, G_{amp} is the amplification, and $\tilde{U}_{\text{ADC}}(f)$ is

²¹ Assuming the signal amplitude remains unchanged.

²² Flicker noise originates from charge carrier trapping and release in semiconductor materials [118].

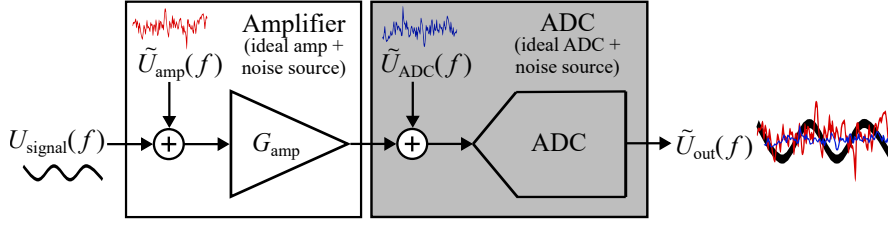


Figure 3.8: Schematic of a signal chain with ideal amplifier and ideal ADC, showing separate input-referred noise sources $\tilde{U}_{\text{amp}}$ and $\tilde{U}_{\text{ADC}}$. High amplifier gain G_{amp} makes amplifier noise dominant and suppresses the ADC noise contribution. (Image modified from: [117])

the input-referred noise of the ADC. The output-referred noise expression describes the total noise at the final measurement node ($\tilde{U}_{\text{out}}$) and depends explicitly on gain settings, increasing with G_{amp} . In contrast, input-referred noise $\tilde{U}_{\text{inp.r.}}(f)$ represents the equivalent input voltage that would produce the measured output noise after amplification and digitization. It is a fundamental metric for sensor performance, independent of gain distribution and directly specifying minimum resolvable input signals. It is obtained by scaling the ADC noise back to the amplifier input and combining both contributions:

$$\tilde{U}_{\text{inp.r.}}(f) = \sqrt{\left(\tilde{U}_{\text{amp}}(f)\right)^2 + \left(\frac{\tilde{U}_{\text{ADC}}(f)}{G_{\text{amp}}}\right)^2} \quad [\text{V}/\sqrt{\text{Hz}}]. \quad (3.49)$$

For a sufficiently large gain G_{amp} , the first term dominates and the ADC noise contribution becomes negligible. Overall performance is then effectively limited by amplifier noise. For a cascade of n amplifiers followed by an ADC, Equation 3.49 becomes

$$\tilde{U}_{\text{inp.r.}} = \sqrt{\left(\tilde{U}_{\text{amp},1}\right)^2 + \left(\frac{\tilde{U}_{\text{amp},2}}{G_1}\right)^2 + \cdots + \left(\frac{\tilde{U}_{\text{amp},n}}{G_1 \cdot G_2 \cdots G_{n-1}}\right)^2 + \left(\frac{\tilde{U}_{\text{ADC}}}{G_1 \cdot G_2 \cdots G_n}\right)^2} \quad [\text{V}/\sqrt{\text{Hz}}], \quad (3.50)$$

where each subsequent stage's noise is divided by the cumulative gain of all preceding stages. In a well-designed signal chain, the first amplifier thus dominates total input-referred noise, making it critical to select a low-noise first-stage amplifier with sufficient gain.

Part II

Development of Optical Ground Support Equipment

Toolset for adjustable picometer-stable interferometers

The toolset for adjustable picometer-stable interferometers (TAPSI) forms the core optical bench assembly of the optical ground support equipment developed and tested in this thesis. It facilitates the flexible and rapid construction of ultra-stable interferometers in the low-frequency regime. This chapter introduces the conceptual design of the toolset, describes its structure and materials, and discusses challenges and expected noise sources, including static friction, thermal expansion, and geometric TTLC. The chapter concludes with a generic noise budget of the toolset, evaluating its performance at the sub-picometer level.

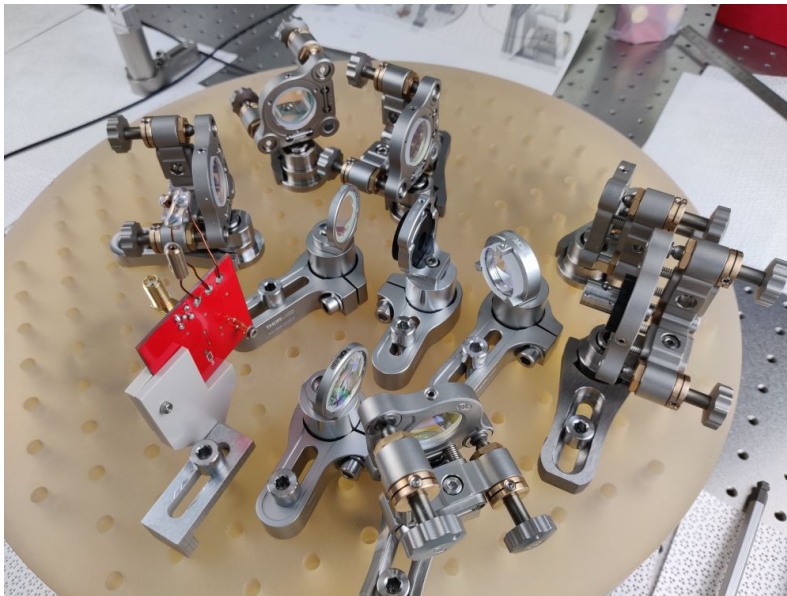


Figure 4.1: Photograph of the toolset for adjustable picometer-stable interferometers employed to set up a PDH locking scheme, featuring the optical cavity (visible in the two rear mirror mounts), a custom-designed photodetector and mount (red PCB), and commercial mounts for lenses and wave plates.

4.1 Conceptual design

The optical benches of optical ground support equipment for space missions such as LISA demand picometer-level displacement sensitivity in the sub-Hz regime [45]. Attaining such low-frequency displacement noise requires a dedicated optical bench design, including direct bonding of optics to the bench, as detailed in Section 1.2.1.2. It necessitates precision alignment equipment and optical readout during the bonding process, while allowing no margin for error and making construction both costly and time-intensive. However, Kulkarni *et al.* [119] demonstrated that alternative construction methods, beyond conventional bonding, can also achieve sub-picometer stability using commercial mirror mounts anchored to an ULE baseplate.

The aforementioned challenges, together with the prior work by Kulkarni *et al.* [119] and by [120], motivated the development of a fully adjustable interferometer toolset, denoted as toolset for adjustable picometer-stable interferometers (TAPSI). This toolset enables rapid reconfiguration of diverse interferometer topologies through free positioning of optical components, including mirror mounts, fiber couplers, lenses, wave plates, and PDs. Key features include adjustable mirror mounts that allow laser beam alignment at any construction stage. The positional and alignment freedom eliminates the need for robot-assisted construction or tedious manual assembly using coordinate measurement machines. Moreover, alignment errors can be corrected post-assembly, thereby substantially reducing planning and commissioning time required for experiments. Consequently, the overall setup period is reduced from the typical duration of weeks or months [52] to just a few hours. Another key advantage is the reusability of optical components including the baseplate, which significantly lowers construction costs. The toolset is designed to enable rapid prototyping of interferometers for testing individual components or entirely novel configurations.

To serve as OGSE for LISA, the toolset must provide picometer-level displacement stability (Equation 1.16) across the relevant frequency range. The primary low-frequency noise sources, thermal expansion and TTLC, guided our design choices. Accordingly, we selected thermally compensated mirror mounts with enhanced pointing stability and designed an optomechanics mounting concept that minimizes thermal expansion coupling. The TAPSI design incorporates mirror mount assemblies

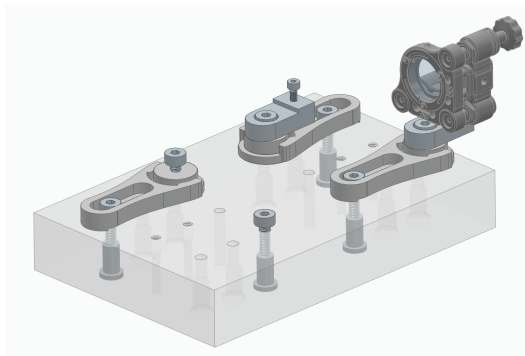


Figure 4.2: CAD model of the toolset for adjustable picometer-stable interferometers (TAPSI) mounting concept. The model illustrates the step-by-step assembly of the mirror mount clamped on the optical bench. Model created using Solid Edge.

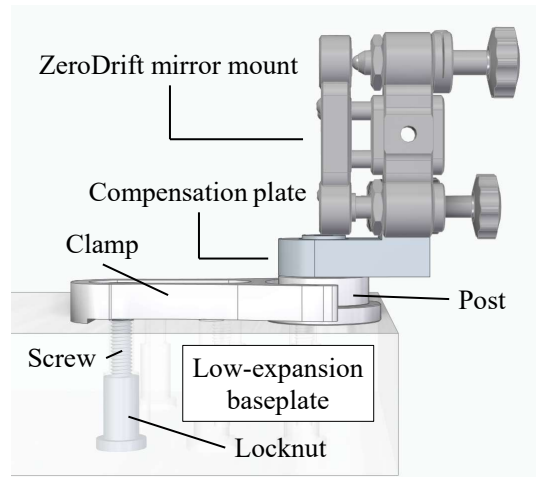


Figure 4.3: TAPSI components.

secured to a low-expansion baseplate using clamping forks. These clamps are secured by screws passing through baseplate holes and tightened with nuts on the opposite side. Figure 4.2 and Fig. 4.3 show the toolset's mounting design, with detailed descriptions of structure and materials given in the following Section 4.2.

4.2 Structure and materials

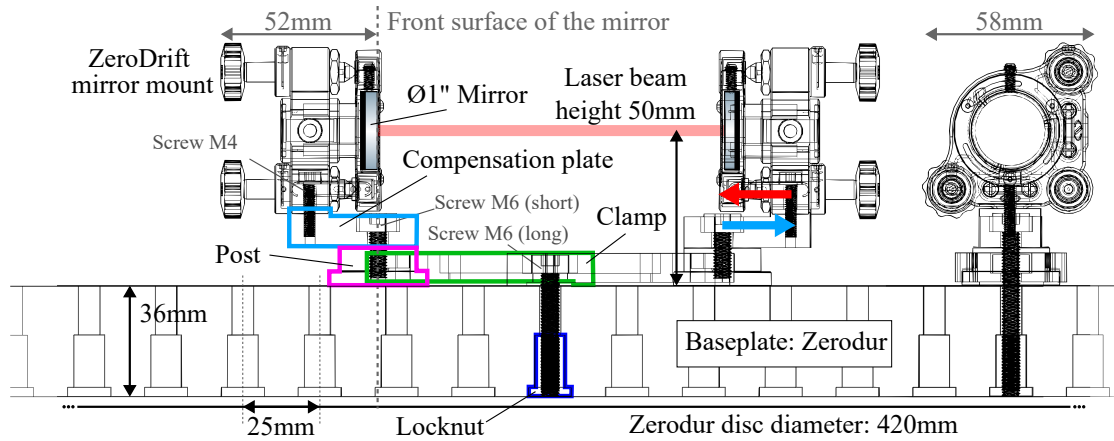


Figure 4.4: Schematic of the TAPSI mounting concept. The optomechanics (locknut, clamp, screws, post, compensation plate) are constructed from Invar 36 while the baseplate is made from Zerodur. The thermally compensated ZeroDrift™ mirror mount (beam height: 5 cm) is mounted such that its reflecting mirror surface is positioned precisely above the post fixture (short M6 screw). This design compensates for differential thermal expansion between the compensation plate and mirror mount in opposing horizontal directions (indicated by the red and blue arrow).

The toolset for adjustable picometer-stable interferometers design comprises the baseplate, optomechanics (locknut, clamp, screws, post, compensation plate) and thermally compensated ZeroDrift™ mirror mounts¹. Figure 4.4 shows the cross-sectional view of the toolset's mounting concept. The optomechanics are fabricated from Invar 36, which was selected for its ultra-low coefficient of thermal expansion ($CTE = 2 \text{ ppm/K}$) to minimize thermal expansion and the corresponding induced mirror displacement. The optical bench is manufactured from Zerodur², a glass-ceramic composite exhibiting an even lower $CTE = 0 \pm 7 \text{ ppb/K}$. The mounting architecture is designed around the principle of linear thermal expansion compensation. Horizontal expansions of the mirror mount and the compensation plate cancel at their opposing fixation points, thereby preserving the mirror position. However, the compensation plate is fabricated from Invar 36, whereas the ZeroDrift mirror mounts consist of various stainless steel alloys³ with typical values of $CTE = 17\text{-}10 \text{ ppm/K}$ [121]. This CTE mismatch degrades the thermal expansion compensation. The ZeroDrift mirror mounts were designed to minimize thermal

¹ Newport, *SU100TW-F2K*, vacuum compatible model.

² Schott, *ZERODUR Expansion Class 0 EXTREME* ($CTE = 0 \pm 7 \text{ nm/m/K}$)

³ Newport did not disclose any further information due to commercial confidentiality. Although the mirror mounts were primarily optimized for pointing stability, we also expect a smaller CTE compared to typical values of stainless alloys.

deflection through thermal expansion compensation. This innovation is termed ZeroDrift™ which, according to the manufacturer, improves optical pointing stability by 85 % compared to standard stainless steel designs (see Fig. 4.8). Within interferometric displacement sensing, this substantially reduces TTLC. The mount assembly⁴, in particular the post, is secured to the optical bench using a clamping fork with screw-and-locknut from below. This two-sided fastening strategy, combined with the flat locknut contact surface in the mounting hole, distributes bearing stress throughout the stiff bench material. A longitudinal slit machined into the locknut enables secure screw tightening. The chessboard-like hole pattern (2.5 cm spacing) of the baseplate allows flexible positioning of the mirror mounts to realize diverse interferometer configurations.

In addition to the aforementioned components such as the mount assembly and the baseplate, the toolset also includes fixed mirror mounts also fabricated from Invar 36, serving, for example, as beam splitters. Beam dumps (Aluminium) and photodetector (PCB) mounts (polyether ether ketone (PEEK) material) complete the toolset (see Appendix E.1). In future work, several additional components are planned, including pinhole beam dumps designed to block unwanted ghost beams, as well as dedicated fiber collimators, which represent a major source of TTLC noise in high-precision interferometers [122–124].

The TAPSI optomechanics, excluding the commercial ZeroDrift mirror mounts, were custom-designed using Solid Edge [125]. The designed models are shown in Fig. 4.2 and Fig. 4.3.

4.3 Static friction, thermal expansion and limited clamping force

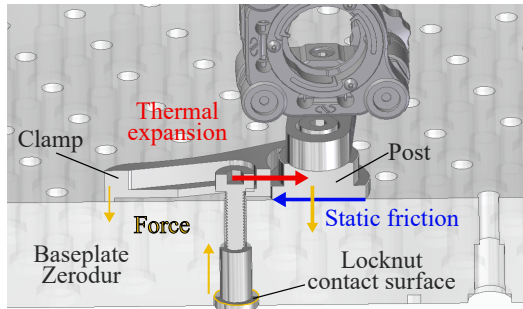


Figure 4.5: Cross-sectional view of the TAPSI CAD model. Thermal expansion (red arrow) is counteracted by static friction (blue arrow). Additional contact forces (orange arrows) indicate stress concentrations.

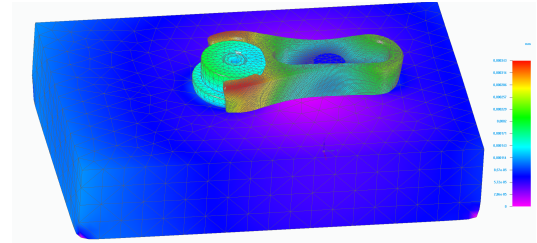


Figure 4.6: Thermal expansion simulation of the post (Invar 36) secured on the baseplate (ZeroDur) by a fork clamp (Invar 36) for $\Delta T = 10$ K. The simulation was performed using Solid Edge.

The mirror mount assembly, particularly the post, is clamped to the Zerodur baseplate using locknuts from the opposite side, as shown in Fig. 4.5. The materials, Zerodur (CTE = 7 ppb/K) and Invar 36 (CTE = 2 ppm/K) exhibit distinct CTEs, resulting in differential thermal expansion. When the temperature changes, it induces thermal expansion of the clamp, causing a displacement of the post relative to the baseplate. This temperature driven motion is opposed by static friction $F_{\text{friction}} = \mu_s \cdot F_{\eta}$, where F_{η} denotes the normal (orthogonal) clamping force and μ_s the static friction coefficient. The

⁴ The mount assembly is also referred to as the *mirror mount* throughout this thesis unless stated otherwise.

post slips when the horizontal thermal expansion force exceeds F_{friction} . Below this threshold, the mirror mount elastically deforms to follow clamp motion, as the simulation in Fig. 4.6 using Solid Edge⁵ shows. The position stability of the mirror mount, particularly that of the post, can be enhanced by increasing the clamping force F_{η} . This can be achieved either by advancing the screw further towards the post or by applying a higher torque to the screw. While the former approach may be impractical since flexible placement of the mirror mounts is desired, the latter poses a significant risk, as the high stiffness of the Zerodur material makes it susceptible to fracture under excessive mechanical stress. High stress concentrations in contact regions, particularly at the rear clamping area and the locknut/baseplate interface are critical points of concern. The locknut is therefore designed with a flat contact surface to distribute the force evenly. Static friction can alternatively be enhanced by increasing the coefficient μ_s , for example through controlled roughening of the baseplate or the post's contact surface. Notably, even below the macroscopic slip threshold, microscopic pre-sliding slip can occur at the contact interface [126, 127]. A further approach to reduce the effect of thermal expansion could involve an anisotropic interlayer at the clamp/post interface to decouple horizontal thermal expansion from vertical clamping force. Such a layer would exhibit greater stiffness perpendicular to the interface than parallel to it, ensuring high normal preload while accommodating differential expansions.

Although the prediction of macroscopic slippage remains constrained by current characterization, this discussion is nonetheless essential to support ongoing toolset development. Detailed theoretical analyses and simulations are left for future work, whereas this thesis primarily focuses on the development, testing, and implementation of the interferometer toolset.

4.4 Theoretical displacement noise of the toolset

This section addresses the primary low-frequency noise sources inherent to the toolset, namely thermal expansion and tilt-to-length coupling, and concludes with the discussion of the generic noise budget. Thermal expansion and thermorefractive coupling of the mirrors are excluded, as these effects are not inherent to the toolset.

4.4.1 Thermal expansion coupling

Displacement noise arising from thermal expansion coupling originates in both the mirror assembly and the baseplate and is defined as

$$\tilde{S}_{\text{TE}}(f) = \alpha \cdot L \cdot \tilde{T}_{\text{amb}} \quad [\text{m}/\sqrt{\text{Hz}}], \quad (4.1)$$

where α denotes the CTE, L the effective coupling length, and $\tilde{T}_{\text{amb}}$ the ambient temperature noise. The interferometer toolset is located within the optical test facility (described in the following Chapter 5), where the measured temperature noise is approximately $10 \mu\text{K}/\sqrt{\text{Hz}}$ at 10 mHz and follows a $1/f$ slope. The baseplate is manufactured from Zerodur⁶ featuring an ultra-low CTE (0 ± 7 ppb/K). As a worst-case scenario, we assume a maximum CTE of 7 ppb/K, resulting in the thermal expansion-induced

⁵ Visual representation only; simulated for $\Delta T = 10$ K with fixtures at baseplate edges, screw, and clamp/post interface.

⁶ Schott, ZERODUR Expansion Class 0 EXTREME (CTE = 0 ± 7 nm/m/K)

displacement noise of:

$$\tilde{S}_{\text{baseplate}}(f) = 7 \times 10^{-18} \cdot L \cdot \frac{1}{f} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (4.2)$$

The thermal expansion-induced displacement noise of the mirror mount assembly is determined by the difference in CTE between the ZeroDrift mirror mount and the Invar optomechanics, in particular the compensation plate, as described in the previous Section 4.2 and shown in Fig. 4.4⁷. For illustration purposes, the displacement noise from the mirror mount assembly (Equation 4.1) can be expressed as follows:

$$\begin{aligned} \tilde{S}_{\text{mount}}(f) &= \Delta\alpha \cdot 2.5 \times 10^{-17} \cdot \frac{1}{f} \quad [\text{m}/\sqrt{\text{Hz}}], \\ \tilde{S}_{\text{mount}}(10 \text{ mHz}) &= \Delta\alpha \cdot 2.5 \times 10^{-13} \quad [\text{m}/\sqrt{\text{Hz}}], \end{aligned} \quad (4.3)$$

where $\Delta\alpha$ denotes the differential CTE between the ZeroDrift mirror mount and compensation plate in units of $\mu\text{m}/\text{m}/\text{K}$ (ppm/K). The expression, particularly at lower frequencies (10 mHz), emphasizes the stringent CTE matching required between these two components to meet LISA's $\text{pm}/\sqrt{\text{Hz}}$ optical bench requirements. This is also evident from the noise budget in Section 4.4.3, Fig. 4.9, where $\Delta\alpha = 2 \mu\text{m}/\text{m}/\text{K}$ was assumed.

4.4.2 Tilt-to-length coupling

The second noise source inherent to the toolset arises from tilt-to-length coupling (TTLC), which is expected to be primarily temperature-driven through thermal deformations, including differential thermal expansion and thermoelastic distortion within the mirror mount assembly, in particular the ZeroDrift mounts. These thermally induced distortions produce angular (tilt) and lateral beam jitter, which cross-couple into the interferometric phase and manifest as apparent longitudinal displacement. To estimate the TTLC noise contribution, this analysis concentrates on the geometric TTLC (Appendix C), which originates from tilt-induced variations in the optical path length (OPL) of the beams. Non-geometric TTLC is neglected in this analysis, although it is known to have considerable influence [112, 128]. It arises from wavefront characteristics of the interfering beams and detector geometries, making it substantially more complex to model and describe.

For example, we consider a simple interferometer configuration as shown in Fig. 4.7, featuring a reference beam interfering with a measurement beam. The fiber collimator and the mirror mounts, set up using the toolset (TAPSI), are assumed to exhibit identical angular and lateral jitter noise levels. The BS, where both beams interfere, is treated as static, such that the resulting TTLC originates solely from the measurement beam. The corresponding unfolded beam path, shown on the right side of Fig. 4.7, visually clarifies the individual TTLC contributions.

Figure 4.7 (a) illustrates the case of an angular jitter in the fiber collimator, with all other optical components being static⁸. Figure (b) includes angular jitter of the mirror (M) at an incident angle $\beta = 45^\circ$. Here, the jittering beam from the collimator is treated as the nominal path, with the mirror

⁷ Fig. 4.4: Red and blue arrows indicate thermal expansion with an effective coupling length $L = 25 \text{ mm}$

⁸ The angular change is halved due to the absence of reflection. The collimator's TTLC is $\Delta\text{OPL}_{\text{lever,C}}^{2\text{D}} \approx \frac{d_{\text{lever}}\varphi^2}{2}$ and follows from the Taylor expansion of $d_{\text{lever}}[1/\cos(\varphi) - 1]$ where only half of the reflection angle is assumed, which differs to the reflection of a mirror.

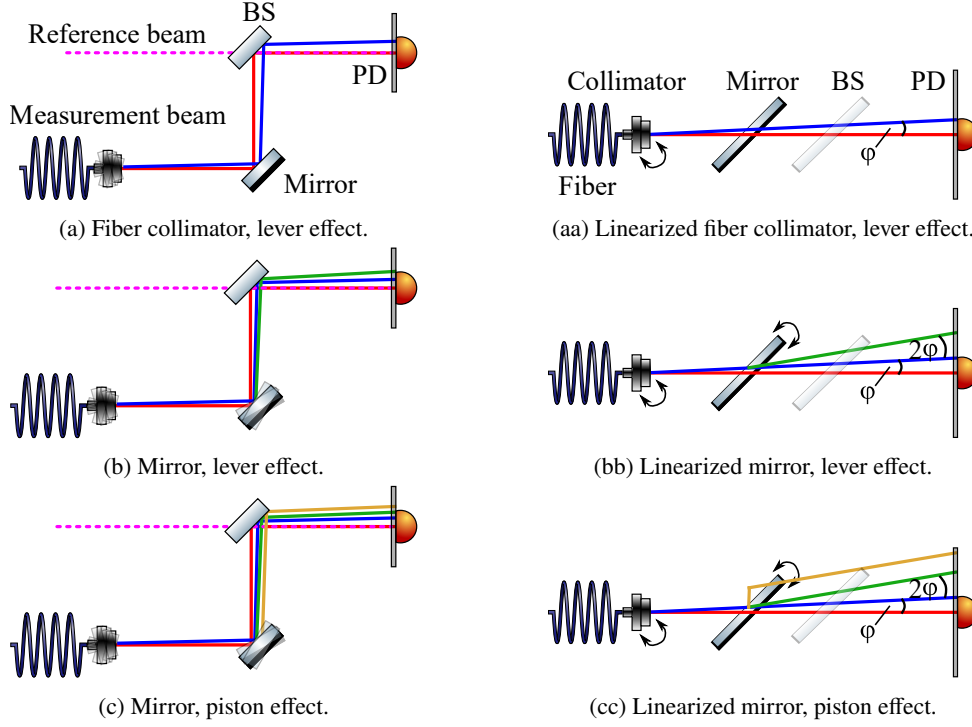


Figure 4.7: Schematic of the tilt-to-length coupling (TTLC) illustrating the lever and piston effect in a simple interferometer configuration, arising from angular jitter in the fiber collimator and mirror mount. Subfigures (a)-(c) show the interference between the measurement and reference beam at the beam splitter (BS). The reference beam is assumed stationary, so the TTLC arises solely from the motion of the measurement beam. This coupling can be represented linearly, as illustrated (aa)-(cc). The cumulative effect of all contributions results in an apparent optical path length deviation observed at the photodetector (PD).

motion adding both lever and piston effect contributions. For this estimation, we assume the *worst case scenario* where jitters from individual components add constructively. The total TTLC is then given by the sum of the individual contributions from the jittering components:

$$\Delta \text{OPL}_{\text{total}}^{2D} = \underbrace{\frac{d_{C,\text{lever}}}{2} \varphi^2}_{\text{Fiber coupler}} + \underbrace{2d_{M,\text{lever}} \varphi^2}_{\text{Mirror, lever}} + \underbrace{2d_{\text{lat}} \cos(\beta) \varphi + [2d_{\text{lat}} \sin(\beta) + d_{\text{long}} \cos(\beta)] \varphi^2}_{\text{Mirror, piston}}, \quad (4.4)$$

where $d_{C,\text{lever}}$ denotes the distance from the collimator to the PD, and $d_{M,\text{lever}}$ the distance from the mirror to the PD. The lateral d_{lat} and longitudinal d_{long} offsets arise from misalignment of the mirror's rotation point, assumed at the mirror center. Moreover, subsequent optical components experience shifted incident points, resulting in additional lateral offsets. Consequently, the lever effect of the fiber collimator amplifies the mirror's lateral offset and expands d_{lat} as follows:

$$d_{\text{lat}} = d_{\text{lat},M} + d_{\text{preceding}} \cdot \tan(\varphi) \approx d_{\text{lat},M} + d_{\text{LBM}} \varphi, \quad (4.5)$$

where $d_{\text{lat},M}$ is the initial offset at the mirror, and $d_{\text{preceding}}$ the additional offset induced by the preceding lever effect of the collimator. These expressions demonstrate that the strength of the TTLC depends

strongly on the beam alignment within the interferometer. Near-ideal alignment (contrast $\approx 100\%$) suppresses both primary and, more effectively, second-order coupling terms, as deviations around $\varphi \approx 0$ remain minor. Consequently, TTLC proves highly sensitive to interferometer misalignment. TTLC can also be mitigated using imaging systems, as demonstrated by [55, 111].

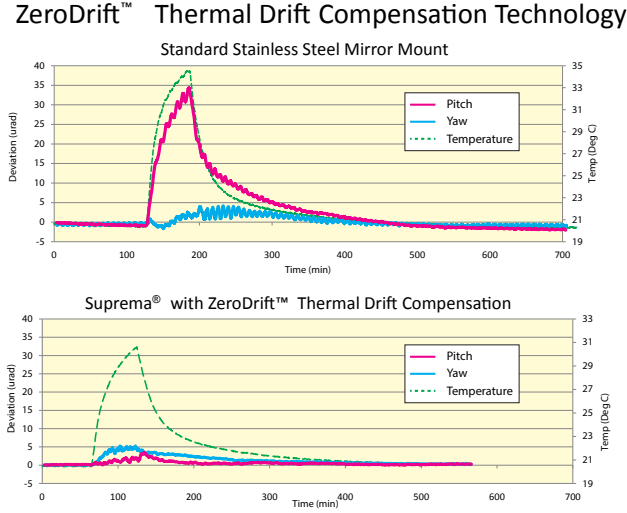


Figure 4.8: Newport measurement data demonstrating that ZeroDrift™ technology reduces thermal drift in mirror mounts by 85 % compared to standard stainless steel mirror mounts. (Image credit: [129]).

Deriving a complete analytical approximation for this setup is challenging, as it requires multiple assumptions regarding parameters such as the interferometer misalignment (contrast) and shifts in d_{lat} and d_{long} at the mirror mount. Moreover, the present analysis considers only the geometric TTLC. Depending on the beam parameters and photodetector geometry, non-geometric TTLC can contribute substantially to the overall noise budget.

Nevertheless, to assess the impact of the mirror mounts, we compare standard stainless steel mirror mounts with the toolset’s ZeroDrift mounts. This is done by linearizing the analytical expression from Equation 4.4 and assuming the standard mounts perform just above the LISA displacement noise requirement, which are then improved by substitution ZeroDrift mounts. For this estimation, we employ the temperature-to-tilt coupling coefficient S_{mount} [rad/K] provided by the manufacturer (Fig. 4.8) and multiply it by the ambient temperature noise spectrum $\tilde{T}_{\text{amb}}$ [K/ $\sqrt{\text{Hz}}$] inside the vacuum chamber (see Chapter 5, Fig. 5.15)⁹. The results, shown in Fig. 4.9, thus represent illustrative rather than absolute predicted noise levels. Nevertheless, it demonstrates the enhanced performance of the ZeroDrift mirror mounts, particularly for pitch motion.

Precise predictions would require simulation tools such as IfoCAD [113]. This software tool computes realistic heterodyne readout signals, emulating laboratory phasemeter responses, including path length, differential wavefront sensing (DWS), differential power sensing (DPS), contrast, and related quantities. From such simulations, an interferometer misalignment (e.g., 90 % contrast) can be identified, enabling linearization of second-order tilt coupling terms. This would enable direct incorporation of the temperature-dependent jitter noise of the mirror mounts (rad/ $\sqrt{\text{Hz}}$) into

⁹ $\tilde{S}_{\varphi} = \tilde{T}_{\text{amb}} \cdot S_{\text{mount}}$ [rad/ $\sqrt{\text{Hz}}$]

the linearized TTLC equation, yielding a more precise estimate of the TTLC contribution. These simulations are planned for future work and lie beyond the scope of this thesis.

4.4.3 Generic noise budget

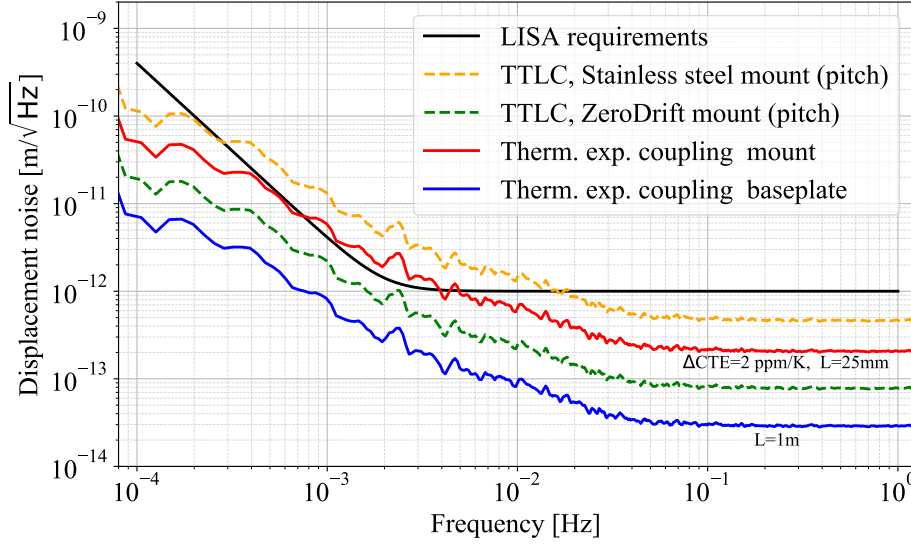


Figure 4.9: Displacement noise budget estimation of the toolset for adjustable picometer-stable interferometers, showing the contributions from tilt-to-length coupling (TTLC) and thermal expansion of the mirror mount assembly and baseplate alongside LISA requirements. TTLC noise evaluates standard stainless steel mirror mounts with ZeroDrift mirror mounts of the toolset. The TTLC estimations are illustrative rather than absolute.

At low frequencies ($f < 1$ Hz), temperature-driven noise sources, namely thermal expansion and tilt-to-length coupling, dominate the toolset's performance. Figure 4.9 presents the individual noise contributions, derived from the preceding subsections, alongside the LISA optical bench requirement. These contributions depend on the ambient temperature noise spectrum measured at the interferometer location (Measurements from Section 6.3.4.2 and shown in Fig. 5.15).

The Zerodur baseplate's thermal expansion-induced displacement noise remains far below requirement, even for $L = 1$ m. In contrast, the mirror mount assembly contribution depends critically on the CTE mismatch ($\Delta\alpha$) between the ZeroDrift mount and the Invar compensation plate (see Section 4.2). The presented displacement noise assumes $\Delta\alpha = 2$ ppm/K, which would exceed the LISA requirement at the corner frequency in the mHz range. The TTLC noise illustrates the performance improvement of ZeroDrift mounts over standard stainless steel mounts. However, these displacement noise estimates are illustrative rather than absolute and should be interpreted accordingly. Precise predictions require dedicated simulations (e.g., IfoCAD), planned for future work.

This thesis focuses on the design, testing and experimental implementation of the toolset (TAPSI), with comprehensive noise analysis reserved for future toolset iterations. The presented total noise budget was therefore experimentally validated in Chapter 6.

Optical test facility

The optical test facility provides a dedicated and well-isolated environment for the toolset for adjustable picometer-stable interferometers. It consists of a vacuum chamber with external and multiple internal thermal shields that ensure high passive thermal insulation. To monitor temperature stability with microkelvin-level precision, a dedicated temperature sensor and associated readout electronics were developed. The vacuum chamber is mounted on an optical table featuring passive seismic isolation, which mitigates vibrations and minimizes seismic coupling to the interferometer.

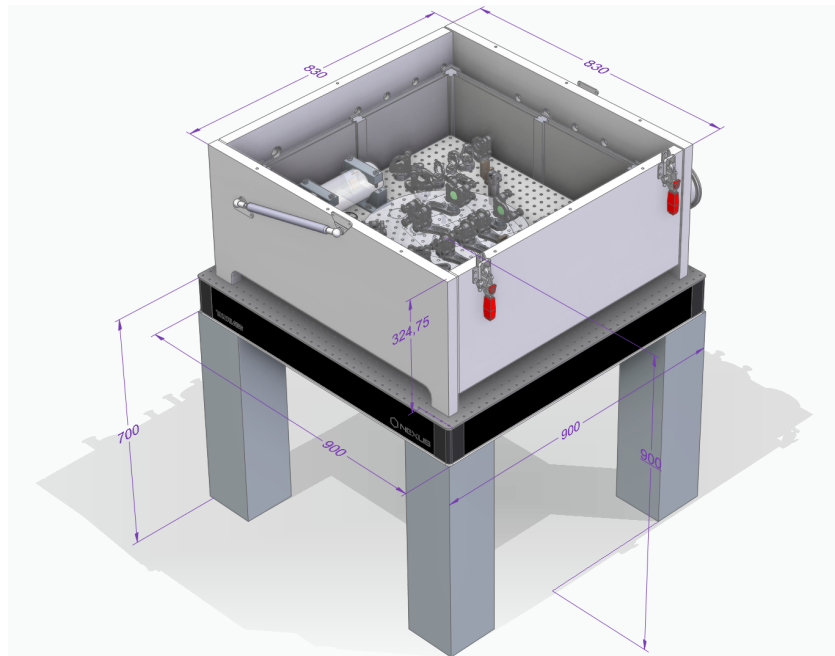


Figure 5.1: CAD model of the optical test facility (lids hidden for visibility). The facility, installed beneath a laminar flow box in a temperature- and humidity-controlled laboratory, provides a well-isolated environment for interferometric experiments. All dimensions are in millimeters.

5.1 Vacuum chamber and thermal insulation

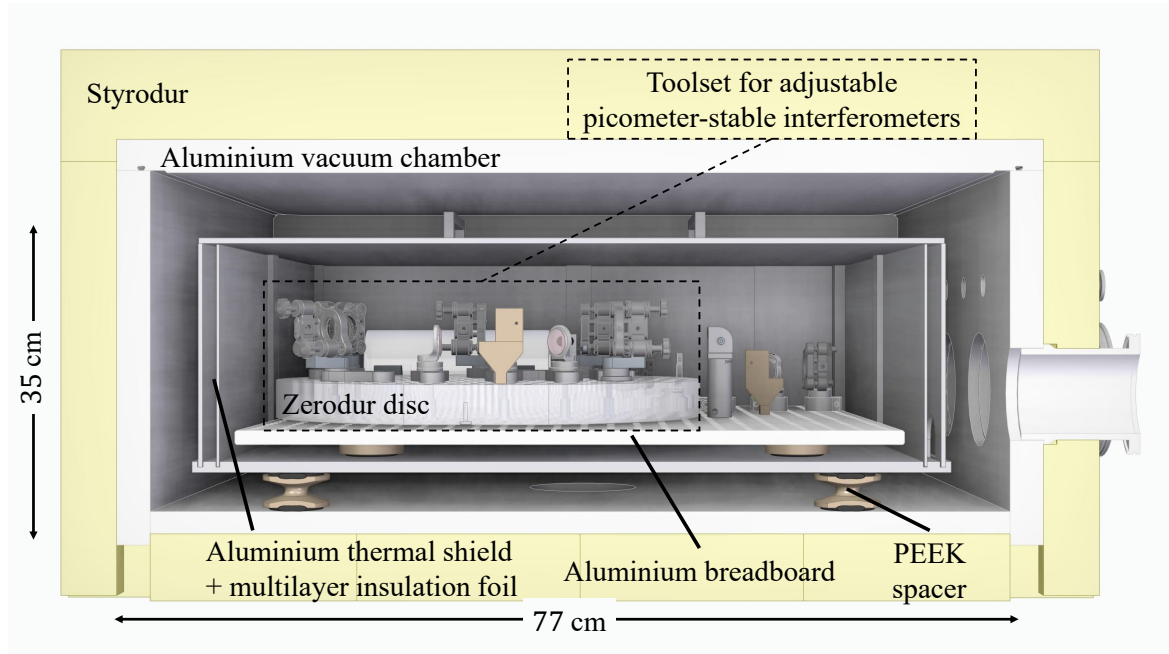


Figure 5.2: Cross-sectional view of the optical test facility comprising multiple layers of thermal isolation to achieve microkelvin-level temperature stability at the location of the toolset for adjustable picometer-stable interferometers (TAPSI). Model created using Solid Edge.

The optical test facility comprises multiple layers of thermal insulation to provide microkelvin-level temperature stability. A cross-sectional view of the design is shown in Fig. 5.2. The outermost layer enclosing the chamber consists of a 5 cm-thick Styrodur shell, which attenuates ambient thermal radiation and reduces temperature gradients across the chamber walls, thereby reducing conductive heat transfer.

The vacuum chamber¹, shown in Fig. 5.3, is fabricated from aluminium and has inner dimensions of $77 \times 77 \times 30$ cm. It features flanges for optical fiber and electrical feedthroughs (SMA, Sub-D) and connects to a scroll-pump for vacuum generation. An additional flange at the bottom of the chamber allows connection of a turbo pump, which was not used in any of the experiments presented in this work. To minimize seismic and acoustic coupling, the scroll pump is located in a separate room. A complete schematic of the vacuum pumping system is provided in Appendix E.2. The vacuum inside the chamber reaches an equilibrium pressure of approximately 5×10^{-3} mbar after two days of pumping, which is sufficient to suppress convective heat transfer [130].

Inside the chamber, an aluminium thermal shield is installed (see Fig. 5.4), consisting of double-layer walls inserted into the frame and a lid that rests in place under its own weight. Feedthroughs leading into the shield and toward the interferometer are implemented as cutouts at the bottom of the walls. This design allows all cabling (e.g., optical fibers and electrical cables) to be completed prior to inserting the walls into the frame. Furthermore, it enables rapid reconfiguration by simply

¹ Hositrad, *custom-design*.

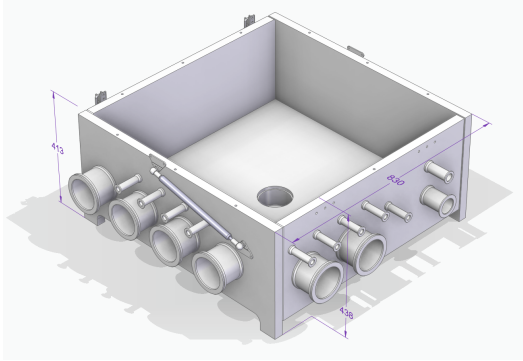


Figure 5.3: CAD model of the custom-made aluminium vacuum chamber with flanges for optical fibers and electrical feedthroughs (lid hidden for visibility). A scroll pump is connected to the rear side of the chamber (model right front), while the bottom flange allows connection of a turbo pump. The front of the chamber, which serves as the working area, is located at the model's back left.

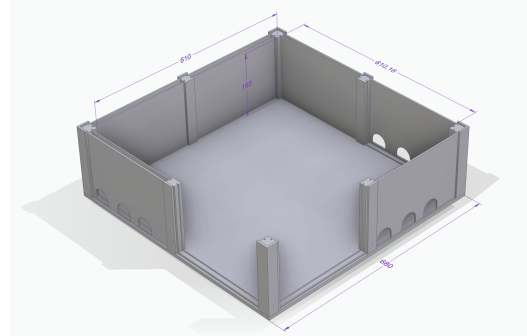


Figure 5.4: CAD model of the aluminium thermal shields inside the vacuum chamber (lid and several walls hidden for visibility). It consists of double-layer walls inserted into the frame, enabling flexible wall configurations for optical fibers and electrical cables to accommodate different experimental setups.

replacing the walls and their cutouts when transitioning to a new experimental setup. Additional thermal insulation is provided by a multilayer foil² applied to the outer surfaces of the lid, bottom plate, and walls of the thermal shield. The foil consists of twenty alternating layers of aluminium and woven fabric. The effectiveness of the implemented thermal insulation measures is shown in Fig. 5.5. The time series clearly highlights the progressive suppression of temperature fluctuations, from the laboratory environment, across the test facility, to the interior of the thermal shield, where the interferometer experiments are located. Inside the thermal shield, an aluminium breadboard mounted at the bottom serves as the primary structural platform for the interferometer toolset and provides additional space for auxiliary components or interferometers that do not require the length stability of TAPSI. Between the vacuum chamber, thermal shield, and breadboard, PEEK spacers provide thermal insulation while supporting the structural loads of the corresponding components. Rubber O-rings are mounted on both sides of each PEEK spacer to damp vibrations. To suppress spurious light beams, a beam dump tape³ was applied to the inner surface of the thermal shield at beam height.

5.2 Seismic isolation

The vacuum chamber is placed on a passively seismically isolated optical table⁴. The main sources of seismic and acoustic noise are expected to arise from the vacuum pump and the laboratory ventilation system (laminar flow box above the chamber). The seismic isolation performance was measured using

² Beyond Gravity, *Coolcat 2 NW sheet*.

³ Acktar, *LambertianBlack™*, adhesive, low-outgassing.

⁴ OPTA GmbH, custom design. Optical tabletop plate *HDT 100/25* with dimensions of $900 \times 900 \times 100$ mm and a weight of 101 kg. Optical table base *DMT 600* with a self-resonance frequency of 0.8 Hz and a maximum operational load of 600 kg. The aluminium vacuum chamber weighs 170 kg, and the total additional load from the experiments is expected to be 50 to 100 kg.

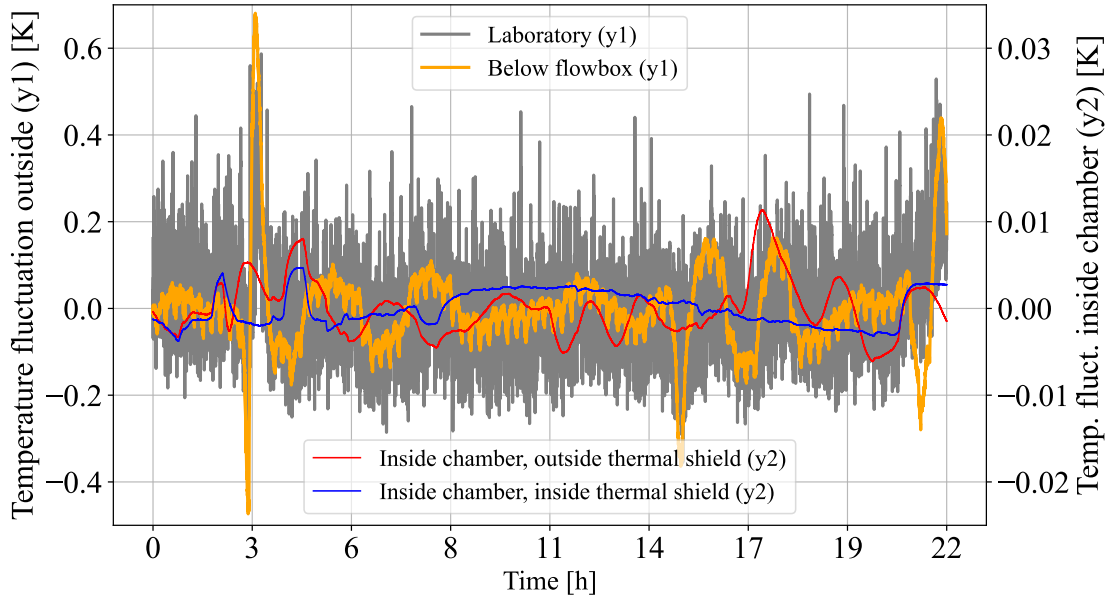


Figure 5.5: Temperature time series showing progressive temperature stabilization from the laboratory environment through the chamber to the thermal shield interior (interferometer location). Temperature fluctuations are mitigated from approximately 1 K down to the microkelvin level. The corresponding temperature spectral noise density is shown in Fig. 5.15.

two seismometers: one positioned on the laboratory floor and the other on top of the vacuum chamber (see Fig. 5.6). The measured seismic noise spectra in the vertical (Z), longitudinal (X), and transverse (Y) directions are shown in Fig. 5.7, Fig. 5.8, and Fig. 5.9, respectively. Further measurements, including those of the larger optical table used for the laser preparation setup, were conducted and are presented in Appendix E.2.3.

The seismic isolation measurements show seismic suppression in the range from approximately 15 to 100 Hz for all three measurement directions, with the (Z) direction exhibiting the most pronounced improvement. An ideal passively seismically isolated optical table should display a well-defined resonance peak followed by the characteristic $1/f^2$ attenuation. However, the measurements reveal multiple resonance peaks, with two distinct modes at approximately 8 Hz and 15 Hz. The distinct peak at 1.5 Hz originates from the running vacuum pump, as confirmed by its disappearance when the pump was turned off⁵. The remaining peaks likely arise from a combination of the vacuum pump and the laboratory ventilation system. Since ‘Chamber (pump, ventilation on/off)’ measurements were not conducted simultaneously, temporary excess seismic noise cannot be excluded.

Overall, the results indicate that the seismic isolation performance of the optical test facility can be further improved. In particular, shifting the resonance frequency of the optical table isolators to lower values would enhance isolation efficiency at higher frequencies. Although this frequency range lies outside the LISA measurement band, such an improvement would suppress resonance peaks of mirror mounts (see Chapter 6, Fig. 6.9) and mitigate other coupling effects, such as TTLC and spurious light signals, which are not fully mitigated by the rigid optical bench design.

⁵ Additional measurements were conducted to separate the contributions of the vacuum pump and the ventilation system.

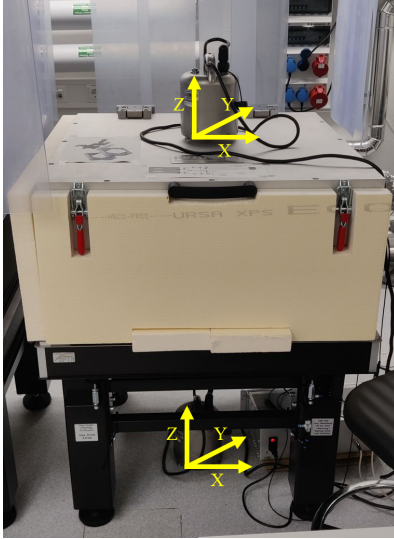


Figure 5.6: Seismometer measurement setup with one sensor placed on the laboratory floor and the second on top of the vacuum chamber.

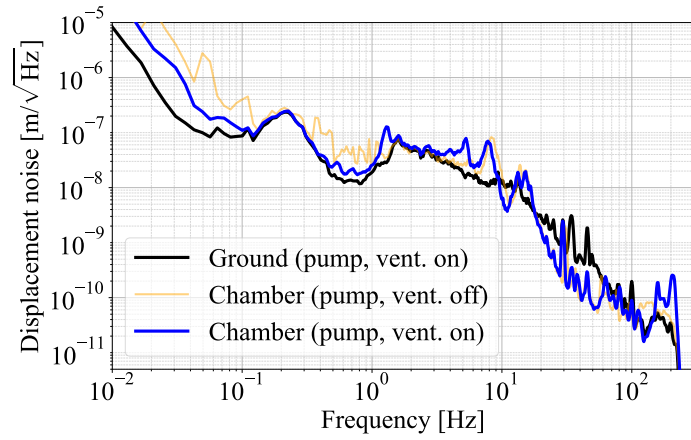


Figure 5.7: Seismic measurements of the optical test facility in the (Z) direction, measured with and without the ventilation system operating. (Measurement credit: Dr. Artem Basalaev.)

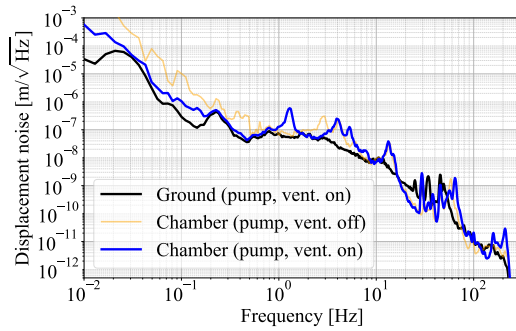


Figure 5.8: Seismic performance measurements of the optical test facility in the (X) direction.

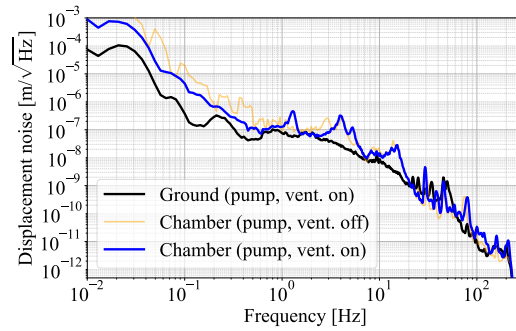


Figure 5.9: Seismic performance measurements of the optical test facility in the (Y) direction.

5.3 Temperature sensor and readout electronics

At sub-Hertz frequencies ($f < 1$ Hz) displacement noise shows significant contributions from thermally induced length changes. High-precision temperature monitoring is essential to distinguish these thermal effects from other interferometer noise sources. Such monitoring enables identification and quantification of temperature-driven displacement noise, and permits their removal during post-processing [131]. For this purpose, a dedicated vacuum-compatible temperature sensor and associated readout electronics were developed to measure microkelvin-level temperature fluctuations within the optical test facility [132]. The initial design of the temperature sensor was developed by G. Heinzl at the AEI Hannover [133].

5.3.1 Temperature sensor

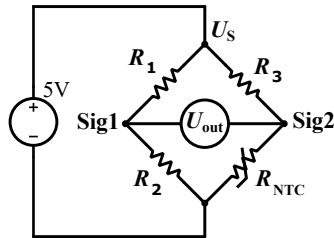


Figure 5.10: Schematic of the Wheatstone bridge (WB)-temperature sensor, supplied with a constant DC voltage (5 V). The bridge resistors R_1 , R_2 and R_3 have fixed resistance values, while $R_{NTC}(T)$ is the negative temperature coefficient (NTC) sensing resistor.

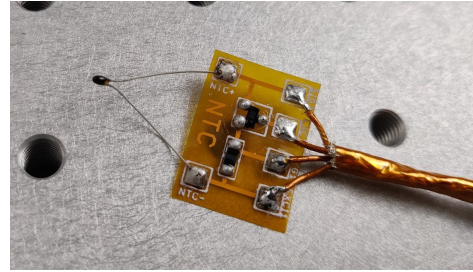


Figure 5.11: Photograph of the WB-temperature sensor, comprising the Kapton printed circuit board (PCB), low temperature coefficient (TC) resistors, and sensing NTC resistor.

The temperature sensor uses a Wheatstone bridge (WB) configuration, where a $50\text{ k}\Omega$ matched resistor pair⁶ ($R_1 = R_2$) forms the reference arm, while the measurement arm consists of a $83\text{ k}\Omega$ low-TC resistor (LTCR)⁷ (R_3) in series with the NTC sensing resistor⁸. The selection of an NTC thermistor over a platinum resistance temperature detector (PRTD) was based on its lower noise-equivalent temperature level, as demonstrated in the *Thermal diagnostics front-end electronics for LISA Pathfinder* [134]. Although PRTDs offer a wider operational temperature range, NTCs provide a much larger resistance change per degree, superior response time, adequate accuracy, and higher resistance values that eliminate lead-wire resistance effects [135]. Figure 5.11 shows the sensor schematic, while Fig. 5.10 presents a photograph of the assembled version with the corresponding resistors and NTC thermistor. The sensor's PCB is fabricated from Kapton-based material, which ensures vacuum compatibility and mechanical flexibility, allowing straightforward surface attachment.

⁶ Vishay, *Thin Film MPM Surface-Mount Resistor Networks*, temperature coefficient ratio (TCR) ± 2 ppm/K.

⁷ Vishay, *PLT1206Z8562LB, Ultra Precision Low TCR Thin Film Resistor, Surface Mount Chip*, temperature coefficient (TC) ± 5 ppm/K.

⁸ Semitec, *SEMI833ET*, $83\text{ k}\Omega$ at $T = 298.15\text{ K}$.

The WB-temperature sensor is powered by a 5 V supply from the custom-developed electronics, referred to as *temperature board*. The output voltage U_{out} of the WB is given by

$$U_{\text{out}}(T) = U_S \cdot \left(\frac{R_2}{R_1 + R_2} - \frac{R_{\text{NTC}}(T)}{R_3 + R_{\text{NTC}}(T)} \right) \quad [\text{V}], \quad (5.1)$$

where U_S denotes the supply voltage, $R_1 = R_2$ are the matched resistor pair, and R_3 is the low-TC resistor, and $R_{\text{NTC}}(T)$ the temperature-dependent NTC sensing resistor defined by

$$R_{\text{NTC}} = R_0 \cdot e^{\beta \left(\frac{1}{T} - \frac{1}{T_0} \right)} \quad [\Omega], \quad (5.2)$$

where $R_0 = 83 \text{ k}\Omega$ is the nominal resistance at temperature $T_0 = 298.15 \text{ K}$ (25°C), and $\beta = 4013 \text{ K} \pm 1$ the material constant of the selected NTC thermistor. The sensitivity of the Wheatstone bridge is given by

$$s_{\text{WB,NTC}}(T) = \frac{dU_{\text{out}}}{dT} = \frac{U_S \beta}{T^2} \frac{R_3 R_{\text{NTC}}(T)}{(R_3 + R_{\text{NTC}}(T))^2} \quad [\text{V/K}]. \quad (5.3)$$

The sensitivity reaches its maximum when $R_{\text{NTC}}(T = 25^\circ\text{C}) = R_3 = 83 \text{ k}\Omega$, yielding $s_{\text{WB,NTC}}(T) = 56.4 \text{ mV/K}$. The fundamental noise limit of the bridge is set by Johnson-Nyquist (JN) noise, with spectral voltage noise given by

$$\tilde{U}_{\text{WB,JN}}(T, f) = \sqrt{4k_B T \left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_{\text{NTC}}(T)}{R_3 + R_{\text{NTC}}(T)} \right)} \quad [\text{V}/\sqrt{\text{Hz}}], \quad (5.4)$$

where $k_B = 1.38 \times 10^{-23} \text{ J/K}$ is the Boltzmann constant. The corresponding temperature noise is obtained from

$$\tilde{s}_{\text{WB,JN}}(T, f) = \frac{\tilde{U}_{\text{WB,JN}}(T, f)}{s_{\text{WB,NTC}}(T)} \quad [\text{K}/\sqrt{\text{Hz}}]. \quad (5.5)$$

Increasing the supply voltage enhances the sensor sensitivity $s_{\text{WB,NTC}}(T)$ and reduces the relative contribution of JN noise. However, higher excitation voltages also increase power dissipation of the bridge, which may lead to self-heating of the sensing element and induce thermal effects in nearby optical components. The total power dissipated in the bridge can be estimated as the sum of the power dissipated in the individual resistors:

$$P_b = \sum_{n=1}^4 \frac{U_S^2}{R_{1,2,3,\text{NTC}}} \quad [\text{W}]. \quad (5.6)$$

Temperature sensors intended for use in the LISA mission are limited by strict power-dissipation constraints, restricting the maximum supply voltage to 0.632 V [134] due to thermal effects on nearby subsystems such as the GRS and the optical metrology system. However, these stringent limitations do not apply to the present OGSE, enabling operation at 5 V for higher sensor sensitivity.

Temperature coefficient of the WB sensor

In the previous section, an ideal Wheatstone bridge was assumed, where the fixed resistors are temperature-independent and only the sensing element R_{NTC} varies with temperature. In practice, however, all resistors exhibit temperature dependence, characterized by their temperature coefficient (TC), which quantifies the fractional resistance change per kelvin (typically in ppm/K). A total temperature coefficient can likewise be defined for the entire WB, denoted by $\alpha_{\text{WB}}(T)$. This coefficient quantifies undesired output-voltage variations per kelvin due to the intrinsic temperature drifts of all resistors, independent of the temperature sensitivity $s_{\text{WB,NTC}}(T)$ of the NTC thermistor. Such effects manifest as measurement errors by introducing apparent, non-existent temperature changes. The effective TC of the bridge is given by

$$\alpha_{\text{WB}}(T) = \frac{\partial U_{\text{out}}}{\partial T} = U_S \left(\frac{R_2 R_1}{(R_1 + R_2)^2} + \frac{R_3 R_{\text{NTC}}(T)}{(R_{\text{NTC}}(T) + R_3)^2} \right) \cdot \alpha_R \quad [\text{V/K}], \quad (5.7)$$

where α_R denotes the TC of the bridge resistors. For the case where $R_1 = R_2 = R_3 \approx R_{\text{NTC}}$, the expression simplifies to

$$\alpha_{\text{WB}}(T) = \frac{U_S}{2} \cdot \alpha_R \quad [\text{V/K}]. \quad (5.8)$$

The temperature-error noise contribution arising from resistor TC drift is expressed as

$$\Delta \tilde{T}_{\text{TC}} = \frac{\alpha_{\text{WB}}}{s_{\text{WB,NTC}}(T)} \cdot \tilde{T}_{\text{amb}} \quad [\text{K}/\sqrt{\text{Hz}}], \quad (5.9)$$

where $\tilde{T}_{\text{amb}}$ denotes the ambient temperature fluctuation (i.e., inside the vacuum chamber). This expression highlights a fundamental trade-off in analog bridge design: increasing the bridge voltage enhances the sensitivity $s_{\text{WB,NTC}}$ but proportionally amplifies the TC-related error α_{WB} .

For resistors with a temperature coefficient of $\alpha_R = 5$ ppm/K, the corresponding effective TC of the bridge is $\alpha_{\text{WB}} \approx 12.5 \mu\text{V/K}$. Inside the vacuum chamber, within the thermal shield, the ambient temperature noise is approximately $\tilde{T}_{\text{amb}} \approx 1 \text{ mK}/\sqrt{\text{Hz}}$ at 0.1 mHz, resulting in a temperature-error contribution at that frequency of about $\Delta \tilde{T}_{\text{TC}} = 0.2 \mu\text{K}/\sqrt{\text{Hz}}$, which is negligible. In contrast, for environments subject to larger temperature fluctuations ($\sim 0.1 \text{ K}/\sqrt{\text{Hz}}$) where μK -precision is required, such as the front-end electronics of the LISA mission, resistors with exceptionally low TC values become essential [134].

The TC-induced error in a signal can be measured in isolation using a *nil sensor*, where all resistors have fixed values, i.e., absence of the thermistor. Any voltage deviation thus arises solely from the individual TC values of the resistors. In other words, the nil sensor directly exposes the intrinsic TC-error signal in isolation. Accordingly, three nil sensors were constructed using fixed resistors with differing TC values (5 and 275 ppm/K). The sensors were operated in a laboratory environment with an ambient temperature noise of $\tilde{T}_{\text{amb}} \approx 0.5 \text{ K}/\sqrt{\text{Hz}}$ at 1 mHz. The measurement results in Fig. 5.12 show significant lower noise levels for resistors with lower TC values, as expected. Although the MELF and the LTCR resistors share the same TC, the measurements show a substantially lower noise for the LTCR resistor. Thus, the nil sensor measurement enables the selection of the low-TC resistors. However, batch-specific variations in TC values cannot be excluded, nor can temporary excess temperature noise during the measurements.

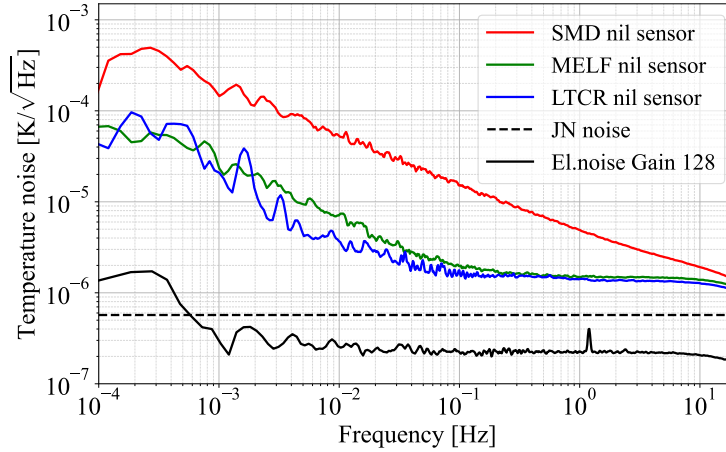


Figure 5.12: Nil sensor measurements using SMD (275 ppm/K), MELF (5 ppm/K), and LTCR resistors (5 ppm/K), alongside JN noise and electronic noise (ADS1261), at a gain of 128. The nil sensor consists solely of the respective fixed resistors.

5.3.2 Readout electronics

The WB sensor is read out by a custom-developed temperature-electronic board (hereafter referred to as the *temperature board*). The temperature board performs multiple functions: it provides the sensor with either DC or AC voltage excitation ($U_S = 5$ V), acquires the signals using a low-noise ADC⁹, and also interfaces the ADC with a microcontroller (Raspberry Pi) to set configuration parameters such as sampling rate, gain, and filter settings. The PCB schematic of the temperature board is shown in Fig. 5.13 where the main functional sections are highlighted. The temperature board uses a low-noise supply¹⁰ to power its electrical components and to provide the 5 V supply for the temperature sensor. Transmission from the board to the sensor, which is located inside the chamber, is achieved via Sub-D feedthrough flanges and a vacuum-compatible, Kapton-insulated cable¹¹. The four-wire cable supplies power to the sensor while also transmitting its measurement signals back to the temperature board, where they are acquired by the ADS1261. The ADS1261 is a low-drift, 24-bit, 10-channel $\Delta\Sigma$ ADC featuring an internal programmable gain amplifier (PGA) and an AC excitation capability. The 5 V reference voltage for the ADC is identical to the supply voltage for the temperature sensor; therefore, common-mode fluctuations are intrinsically rejected in the ratiometric measurement. The ADS1261 includes an internal analog anti-aliasing filter with a cutoff frequency at $f_c = 60$ kHz and an analog modulator (sampling) frequency $f_{\text{mod}} = f_{\text{clk}}/8 = 7.3728 \text{ MHz}/8 = 921.6 \text{ kHz}$. Communication with the ADS1261 is handled by the microcontroller¹², which configures parameters such as sampling rate, amplifier gain (PGA), and digital filter settings. The block diagram of the ADS1261 is shown in Fig. F.2 in the Appendix F. The corresponding Python library is available and described in the associated Git repository [136].

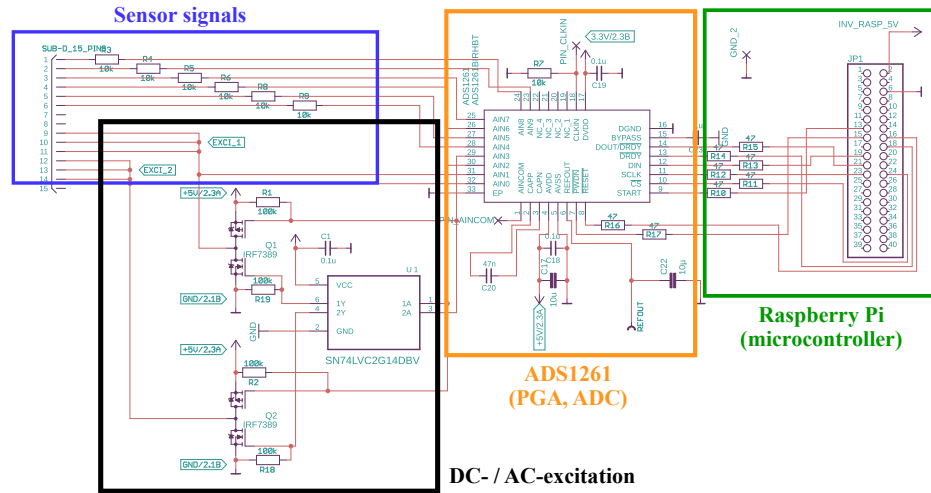
To minimize offset errors, such as thermoelectrical voltages and $1/f$ flicker noise from the

⁹ ADC: Texas Instrument, *ADS1261*.

¹⁰ Custom-made *GEO power supply*.

¹¹ Allectra, *4-pol, KAPTON wires for UHV*.

¹² *Raspberry Pi Zero 2 WH*.



$$U_{\text{AC-exc}} = \frac{U_{\text{P1}} - U_{\text{P2}}}{2}, \quad (5.10)$$

5.3.3 Noise analyses of temperature sensing

This subsection summarizes the noise-limiting values of the developed temperature sensor and its electronics board, in particular the noise from the ADC and the PGA. A detailed discussion of the underlying electronic noise mechanisms is provided in Section 3.6.

The noise of the WB temperature sensor is fundamentally limited by Johnson-Nyquist (JN) noise ($\tilde{S}_{\text{WB,JN}}$) in the bridge. The calculation¹³ follows directly from Equations 5.4 and 5.5, yielding $\tilde{U}_{\text{JN}} = 32 \text{ nV}/\sqrt{\text{Hz}}$, which corresponds to an equivalent temperature noise of $\tilde{S}_{\text{WB,JN}} = 0.57 \text{ } \mu\text{K}/\sqrt{\text{Hz}}$. ADC noise ($\tilde{S}_{\text{ADC}}$) arises from quantization, intrinsic thermal and reference-voltage noise. The latter is neglected in this analysis as it is expected to be canceled in the ratiometric measurement. Quantization noise¹⁴ follows directly from Equation 3.44¹⁵ and yields $\tilde{U}_{\text{quant}} = 127 \text{ pV}/\sqrt{\text{Hz}}$, which corresponds to an equivalent temperature noise of $\tilde{S}_{\text{ADC,quant}} = 2.25 \text{ pK}/\sqrt{\text{Hz}}$.

Each noise source can be characterized by its noise equivalent temperature (NET), defined as the input-referred noise voltage $\tilde{U}_{\text{inp.r.}}$ divided by the sensor sensitivity $s_{\text{WB,NTC}}(T)$:

$$\text{NET} = \frac{\tilde{U}_{\text{inp.r.}}}{s_{\text{WB,NTC}}(T)} \quad [\text{K}/\sqrt{\text{Hz}}]. \quad (5.11)$$

The total noise floor is obtained by combining all individual uncorrelated contributions in quadrature:

$$\tilde{S}_{\text{NET,total}} = \sqrt{(\tilde{S}_{\text{WB,JN}})^2 + (\tilde{S}_{\text{amp}})^2 + \left(\frac{\tilde{S}_{\text{ADC}}}{G_{\text{amp}}}\right)^2} \quad [\text{K}/\sqrt{\text{Hz}}], \quad (5.12)$$

where G_{amp} is the amplifier gain of the PGA. The amplifier noise $\tilde{S}_{\text{amp}}$ arises from the intrinsic thermal noise of the PGA circuitry, which makes analytical calculation impractical. However, the amplifier-noise contribution can be experimentally determined from direct measurements of the ADS1261 dark noise level¹⁶ at different amplifier-gain settings, as shown in Fig. 5.14. Bypassing the PGA amplification corresponds to a gain of one and measures directly the ADC noise ($\tilde{S}_{\text{ADC}}$). Increasing the gain reduces the relative noise contribution from the ADC and ultimately makes the total noise floor limited by the PGA noise itself.

5.3.4 Temperature noise of the optical test facility

Figure 5.15 presents the temperature noise spectra across the thermal layers of the optical test facility. The thermal layers are shown in Fig. 5.2. In addition, the theoretical Johnson noise limit and the measured electronic noise floor (PGA = 16) are shown, which were calculated in Section 5.3.3. The measurements labeled (a) were recorded simultaneously (or on the same day) and demonstrate

¹³ Values used for the calculation: $R_1 = R_2 = 50 \text{ k}\Omega$; $R_3 = R_{\text{NTC}} = 83 \text{ k}\Omega$; $T = 298.15 \text{ K}$.

¹⁴ For simplicity, the analysis assumes white quantization noise, which is typical for Nyquist-rate ADCs. For delta-sigma converters, such as the ADS1261 used in this work, quantization noise undergoes high-pass filtering, known as noise shaping, which reduces in-band noise, while increasing out-of-band noise. Further details are provided in the manufacturer's datasheet [137].

¹⁵ Values used for the calculation: $U_{\text{fsr}} = 5 \text{ V}$; $N = 24 \text{ bits}$; $f_s = f_{\text{mod}} = 921.6 \text{ kHz}$.

¹⁶ The dark noise measurement was performed by shorting the measurement input channels.

temperature noise suppression at each layer¹⁷. The temperature sensor performance¹⁸ achieved $10 \mu\text{K}/\sqrt{\text{Hz}}$ down to 5 Hz. The measured white noise floor exceeds the theoretical limitation given by JN noise and the measured electronic noise of the *ADS1261* (ADC and the preceding amplifier with a gain of 16). This excess noise likely originates from board-specific noise contributions, as nominally identical temperature boards exhibit different white noise levels. Furthermore, nil sensor measurements with the sensor directly connected to the board (Fig. 5.12) show a slightly lower noise floor. This indicates that sensor-to-board connections (e.g., cables, vacuum Sub-D feedthroughs) contribute additional noise. Another possible source of noise is the reference voltage, supplying the sensor and the ADC, which is only rejected in an ideal ratiometric measurement.

Laboratory temperature noise (gray traces) was measured at the NPRO/laser preparation area (see Fig E.10). Light thermal shielding (plastic enclosure and foam insulation, gray solid line,) suppressed presumed exhaust-induced fluctuations, revealing a dominant 25 mHz peak¹⁹. Measurements below the flow box and beneath the subsequent thermal insulation layer (Styrodur housing) reveal several dominant spectral peaks from laboratory temperature control oscillations, including humidity regulation. These strong oscillations propagate through the vacuum chamber walls, remaining detectable outside the thermal shield (solid orange line) but strongly suppressed inside (solid blue lines). Inside the thermal shield at the interferometer location, temperature noise reaches approximately $10 \mu\text{K}/\sqrt{\text{Hz}}$ down to 10 mHz. At lower frequencies, measurement during interferometer length stability measurements (laser-cavity PDH locking, Section 6.3.4.2) (b), where a constant local heat load (e.g., from AC-biased photodetectors) was present, exhibits a $1/f$ behavior (dark blue line). In contrast, measurements without active laser or PD operation (a) show a $1/f^2$ slope (light blue line). These different behaviors

¹⁷ The elevated white noise floor of the measurements outside the chamber results from a *multi-sensor readout*, which requires further optimization.

¹⁸ Determined by subtracting two simultaneous measurements of identical sensors at the same location (*Inside thermal shield (a)*).

¹⁹ Although originating outside the vacuum chamber, this 25 mHz oscillation couples into interferometer experiments via NPRO laser frequency noise and power fluctuations, as confirmed by the experiments in Part III.

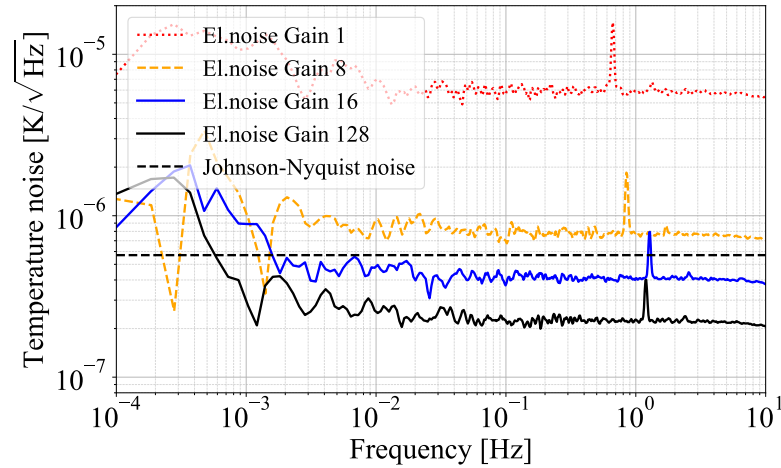


Figure 5.14: Noise budget of the temperature sensor, including the electronics board. Sensor noise is fundamentally limited by JN noise, while the temperature board's electronic dark noise floor decreases for higher amplifier gains.

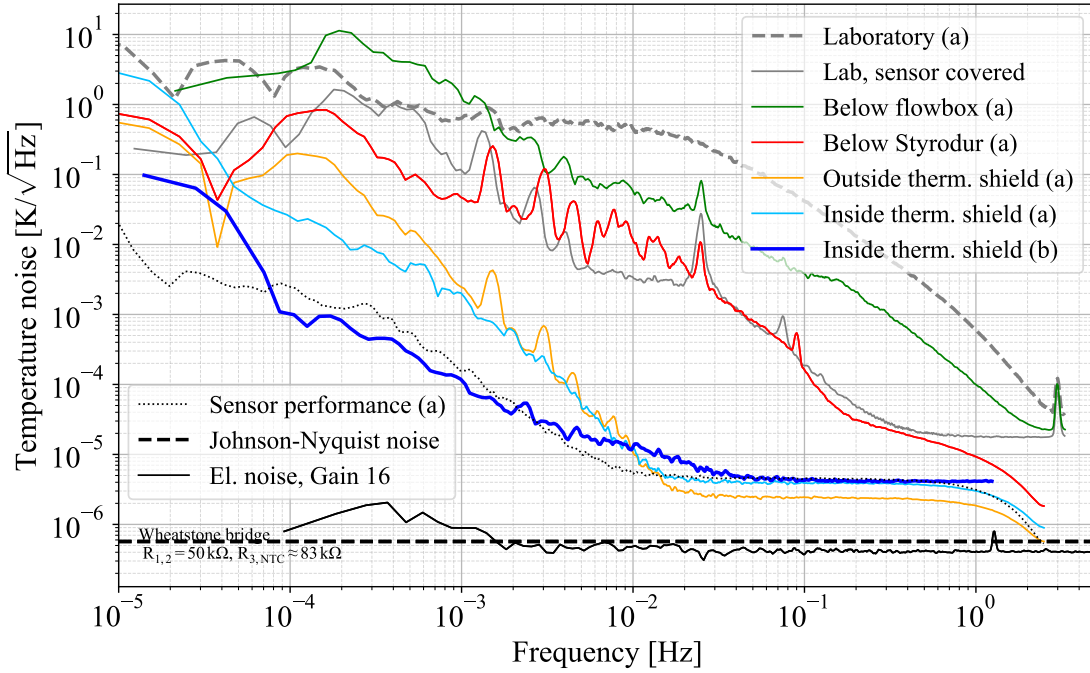


Figure 5.15: Temperature noise spectra across the thermal shield layers of the optical test facility, shown together with the theoretical Johnson noise limit and the measured electronic noise floor ($\text{PGA} = 16$). The laboratory environment (gray trace) exhibits significantly higher temperature noise, which is suppressed by several orders of magnitude within the facility at the interferometer location (blue lines). Measurements labeled (a) were acquired simultaneously or on the same day, whereas measurement (b) was recorded during the interferometer length stability measurements (Section 6.3.4.2).

require further investigation, as the laboratory ventilation has varied over the years. Moreover, dedicated sensor self-noise measurements (*sensor performance*, black trace) are needed, as the (dark blue) measurement falls below the sensor performance²⁰.

While the sensor performance is sufficient for temperature monitoring, noise levels outside the chamber, particularly below the flow box, require substantial improvement. Lowering these noise levels reduces thermal gradients across thermal shielding layers and ultimately decreases temperature fluctuations inside the thermal shield at the interferometer location.

²⁰ During the dark blue measurement (b), only a single sensor was available. The sensor performance measurement (a) was not optimized, with differing sensor head contact points that likely explains the elevated low-frequencies noise. In addition, the ventilation system of the flow chamber caused significantly more temperature noise during these measurements (a).

Part III

Experimental Validation and Displacement Sensing

Toolset length stability characterization

The primary objective of this work was the development of a toolset for adjustable picometer-stable interferometers (TAPSI) along with the associated optical test facility to provide an optical ground support equipment for space missions such as LISA [7]. The required level of length stability (displacement noise) within the LISA optical system, denoted as $u(f)$, in the sub-Hz range is specified in [40] as

$$u(f) = 1 \frac{\text{pm}}{\sqrt{\text{Hz}}} \times \sqrt{1 + \left(\frac{2 \text{ mHz}}{f}\right)^4} \quad (10^{-4} \text{ Hz} < f < 1 \text{ Hz}). \quad (6.1)$$

In order to validate the length stability of the toolset mounting concept, laser frequency stabilization using two optical cavities and two NPRO lasers was performed. The frequency of the first laser was locked to the length of a prototype cavity assembled using the toolset (TAPSI), while the second laser was locked to a reference cavity¹. Previous measurements of the reference cavity demonstrated a length stability of $7.5 \text{ fm}/\sqrt{\text{Hz}}$ ($20 \text{ Hz}/\sqrt{\text{Hz}}$) down to 3 mHz [138]. Optical and geometric specifications of the reference and prototype cavity are listed in Table 6.1. The relative length fluctuation between the two cavities was determined by measuring the change in the beat note frequency of the two locked lasers. For an ideal readout, the relative length fluctuation ∂L that exceeds the reference cavity's stability and is solely attributable to the prototype cavity, obtained from the relation:

$$\partial L = \frac{\partial f}{f} \cdot L, \quad (6.2)$$

where $L = 10 \text{ cm}$ is the length of the prototype cavity, $f \approx 282 \text{ THz}$ the frequency of the NPRO laser, and ∂f the frequency fluctuation of the beat note².

Length stability can be measured using various interferometer types, such as a Michelson or Mach-Zehnder interferometers [139]. However, an optical cavity such as the FPI achieves substantially higher sensitivity through multiple roundtrips, as determined by its finesse. The resulting steep error signal at cavity resonance provides strong discrimination against amplitude and phase perturbations, effectively rejecting noise sources such as spurious light beams (Section 3.4.3). Several techniques

¹ The reference cavity was manufactured from a ULE-glass spacer by Corning and built at the AEI Hannover.

² An alternative approach to calculate the displacement noise $\tilde{S}_L(f)$ from the frequency noise $\tilde{S}_f(f)$ is by the relation:
 $\tilde{S}_L(f) = \tilde{S}_f(f) \cdot (\lambda_{\text{laser}}/2 \text{FSR})$.

exist for locking the laser frequency to an optical cavity length, typically following the basic steps: feedback from the laser-cavity interaction generates an error signal, which drives PID control and actuator correction (see Section 2.3).

The length stability of the toolset mounting concept was experimentally characterized using two different locking schemes, the Heterodyne laser frequency Stabilization (HS) and the well-known Pound-Drever-Hall (PDH) locking technique. Both methods lock the laser frequency to the cavity resonance by exploiting the phase relationship between the intra-cavity field and the reflected input field. This chapter provides a comprehensive description of both locking techniques, including analytical signal derivations, noise analysis and the experimental setup, results and limitations. The final section compares both locking techniques and summarizes the measurement results.

Table 6.1: Optical and geometric specifications of the reference and prototype cavity.

Property	Reference cavity	Prototype cavity
Material	ULE glass	TAPSI mounting concept
Finesse $\mathcal{F}$	$\sim 9\,400$	~ 625
Cavity length L	21 cm	10 cm
Cavity Mirror RoC	flat, RoC=1 m	flat, RoC=0.5 m
Laser beam power P_1, P_2 onto the cavity	~ 10 mW	~ 10 mW

6.1 Laser preparation

Laser preparation denotes the optical preprocessing stage through which each NPRO laser beam passes before entering the vacuum chamber. This stage prepares the laser light through power regulation, polarization control, and mode matching to efficiently couple the light through the fiber collimator and feedthrough fiber into the vacuum chamber. A schematic of the laser preparation is presented in Fig. 6.1, with the corresponding laboratory photograph in the Appendix E.10. Although the ‘experimental specific setup’ part varied, such as beat note and dual fiber coupling, the initial laser preparation remained identical across all experiments in this thesis.

The initial beam from the NPRO laser was collimated, and its height adjusted from 73 mm to 50 mm using a periscope composed of two angled mirror mounts. Polarization was then cleaned using a quarter-wave plate ($\lambda/4$), converting the initial elliptical to linear polarization. The beam subsequently passed through a Faraday isolator to suppress back-reflections that could destabilize single-mode operation or damage the laser. Downstream, a beam splitter attenuated the optical power from 450 mW to 45 mW. The lasers were operated at maximum output power to ensure stable single-mode operation.

To transmit the laser beams into the vacuum chamber, fiber collimators³ and polarization-maintaining (pm) single-mode fibers⁴ were employed. Prior to fiber coupling, the beam polarization

³ Schäfter+Kirchhoff, 60FC-4-A4.5S-03-Ti.

⁴ Schäfter+Kirchhoff, V-KF16-PMC-980-6.6-NA012-APC/APC-150/300-P.

was aligned using a half-wave plate ($\lambda/2$)⁵ to match the slow axis of the pm-fiber⁶. Further polarization extinction ratio (PER) improvements⁷, were achieved by placing a PBS before the fiber collimator, ensuring clean linear polarization input to the pm-fiber. These single-mode fibers not only transmit the laser light into the vacuum chamber but also serve as spatial mode cleaners, delivering a clean Gaussian intensity profile to the interferometer experiments. Frequency tuning of the NPRO lasers was accomplished using PZT actuation and temperature control. The PZT enabled rapid frequency tuning, while the temperature control ensured that the PZT operated within its dynamic range and extended the overall frequency-tuning range.

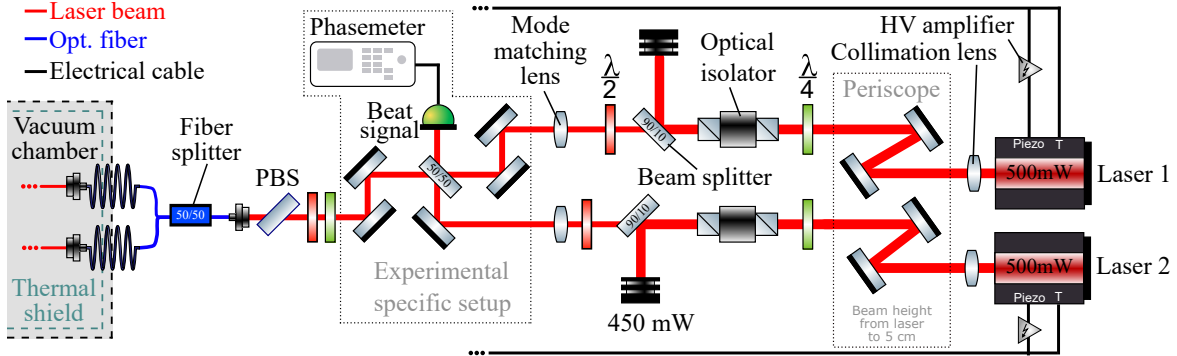


Figure 6.1: Schematic of the laser preparation stage, which conditions the NPRO laser light through power regulation, polarization control, and mode matching. The ‘prepared’ light passes through fiber collimators and pm feedthrough fibers into the vacuum chamber for the interferometer experiments. This initial preparation remained identical across all experiments, while ‘experimental specific setups’ varied.

⁵ Note: a slight rotation of the $\lambda/2$ waveplate around the vertical axis effectively converts it also to a quarter-wave plate ($\lambda/4$), changing circular to linear polarization.

⁶ The slow axis, aligned with the so-called *panda eyes*, maintains polarization orientation against external influences such as temperature-induced stress.

⁷ Implemented after the length stability measurements.

6.2 Heterodyne laser frequency stabilization

Heterodyne laser frequency Stabilization (HS) was selected as the initial locking technique due to its simple interferometer layout, which aligns well with the heterodyne architecture of LISA. Additionally, HS avoids EOMs, a significant noise source in PDH locking schemes [140–144].

6.2.1 Optical layout and signals

This section provides an analytical description of the Heterodyne laser frequency Stabilization scheme and the generation of its error signal. A comprehensive study of HS is available in [145]. A simple HS scheme is shown in Fig. 6.2, comprising two lasers, two PDs, an optical cavity, associated optics, and demodulation electronics.

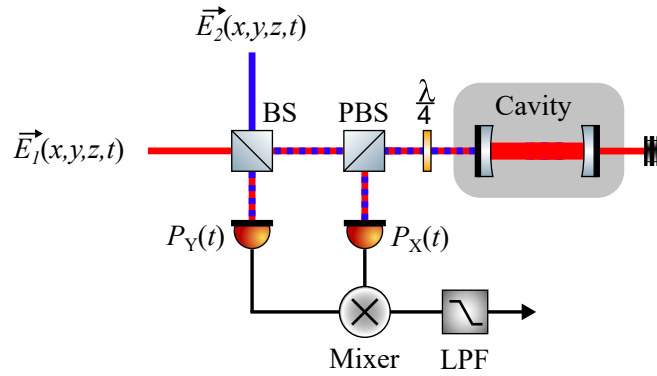


Figure 6.2: Simplified Heterodyne laser frequency Stabilization scheme. Two lasers interfere at the BS, producing a beat note measured by photodetector $P_Y(t)$ (*reference beat*). Part of the beat note enters the optical cavity where only one laser frequency (red) resonates with the cavity length. The reflected beat note is measured by $P_X(t)$ (*cavity beat*). Demodulation of the cavity beat against the reference beat isolates the cavity phase shift, encoding the length information.

The initial laser beams, with distinct angular frequencies ω_1, ω_2 , phases ϕ_1, ϕ_2 , and electric field amplitudes $\vec{E}_1, \vec{E}_2$, are interfered at a beamsplitter, expressed as:

$$\vec{E}(x, y, z, t) = \vec{E}_1 e^{i(\omega_1 t - \vec{k}z + \phi_1)} + \vec{E}_2 e^{i(\omega_2 t - \vec{k}z + \phi_2)}. \quad (6.3)$$

Assuming matched spatial profiles, collinear propagation, and no initial differential phase, the expression simplifies to

$$E(t) = E_1 e^{i(\omega_1 t)} + E_2 e^{i(\omega_2 t)}. \quad (6.4)$$

The power, measured with an ideal PD, is proportional to the intensity of the electric fields, given as

$$P_Y(t) \propto |E(t)|^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos(\Delta\omega), \quad (6.5)$$

where $\Delta\omega = \omega_1 - \omega_2$ denotes the heterodyne beat note frequency. Upon reflection from the optical cavity, the two electric fields experience frequency-dependent filtering through the cavity transfer function (Equation 2.50). When operating near resonance, one laser acquires an amplitude and phase shift $R(\delta f)$ that encodes its frequency detuning from resonance ($\delta f = f - f_{\text{res}}$). The other laser is

detuned far off-resonance where $R(\delta f) \approx 1$ and experiences nearly complete reflection independent of cavity dynamics. This asymmetry forms the core of the HS scheme. The reflected fields from the cavity are then expressed as

$$E_r(t) = R(\delta f) \cdot E_1 e^{-i(\omega_1 t)} + E_2 e^{-i(\omega_2 t)}. \quad (6.6)$$

A quarter-wave plate ($\lambda/4$) and PBS direct the reflected light to the photodetector P_X , which detects the power given by

$$P_X(t) \propto |E_r(t)|^2 = |R(\delta f)|^2 E_1^2 + E_2^2 + 2E_1 E_2 [\Re \{R(\delta f)\} \cos(\Delta\omega t) + \Im \{R(\delta f)\} \sin(\Delta\omega t)]. \quad (6.7)$$

The phase information in $R(\delta f)$ is encoded in both real and imaginary parts of the equation. However, in the case of a high-finesse cavity near resonance (Section 2.2.4), the real part remains approximately constant, while the imaginary part exhibits a linear dependence on the frequency detuning δf . Demodulation of $P_X(t)$ against $P_Y(t)$, followed by low-pass filtering, isolates this dispersive signal. A $\pi/2$ phase shift in one path generates a symmetric zero-crossing error signal. This process is described in analytical terms as follows:

We express the ($\pi/2$ phase-shifted) AC part of P_Y as

$$P_{Y,AC}(t) = 2E_1 E_2 \sin(\Delta\omega t). \quad (6.8)$$

The mixing of the two PD signals $P_X(t)$ and $P_Y(t)$ yields

$$P_{X,AC}(t) \times P_{Y,AC}(t) = 2E_1 E_2 [\Re \{R(\delta f)\} \cos(\Delta\omega t) + \Im \{R(\delta f)\} \sin(\Delta\omega t)] \times 2E_1 E_2 \sin(\Delta\omega t). \quad (6.9)$$

After some algebra we obtain

$$\begin{aligned} P_{X,AC}(t) \times P_{Y,AC}(t) = & 2E_1^2 E_2^2 [\Re \{R(\delta f)\} (\sin(2\Delta\omega t) + \sin(0))] \cdot l \\ & + 2E_1^2 E_2^2 [\Im \{R(\delta f)\} (\cos(0) - \cos(2\Delta\omega t))] \cdot l, \end{aligned} \quad (6.10)$$

where $l \leq 0.5 \text{ V}^{-1}$ denotes the mixing conversion factor⁸. Subsequent low-pass filtering (LPF) removes the high-frequency terms ($2\Delta\omega$), yielding

$$P_{X,AC}(t) \times P_{Y,AC}(t) \stackrel{\text{LPF}}{=} l 2E_1^2 E_2^2 \Im \{R(\delta f)\}. \quad (6.11)$$

The imaginary part of the cavity's transfer function for high finesse cavities (Section 2.2.4, Equation 2.53) is defined as

$$\Im \{R(\delta f)\} \approx -\frac{2}{\pi} \left(\frac{1}{r_1} - r_1 \right) \mathcal{F} \frac{\delta f}{\text{FWHM}} = -\mathcal{G} \frac{\delta f}{\text{FWHM}}, \quad (2.53)$$

where $\mathcal{G}$ is the optical gain of the cavity⁹. Substituting Equation 2.53 into Equation 6.11 yields the

⁸ Mixing multiplies two sinusoidal voltages. Using $\cos(A) \cdot \cos(B) = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$, this generates sum and difference frequency terms. The mixed product amplitude scales as $E_1 E_2 \propto \sqrt{P_1 P_2}$, yielding units of [W] rather than [W²]. It follows that the mixer applies a conversion factor $l \approx 0.5 \text{ V}^{-1}$.

⁹ In overcoupled cavities ($T_2 \ll T_1$) $G \lesssim 4$.

HS error signal

$$\epsilon(\delta f) = \frac{-2\mathcal{G}lE_1^2E_2^2}{\text{FWHM}}\delta f = \frac{-2\mathcal{G}lP_1P_2}{\text{FWHM}}\delta f, \quad (6.12)$$

where $P \propto |E|^2$. The error signal can also be written in the compact form

$$\epsilon(\delta f) = D_{\text{HS}} \cdot \delta f, \quad (6.13)$$

where D_{HS} is the discriminator slope¹⁰ defined as

$$D_{\text{HS}} = \frac{-2\mathcal{G}lP_1P_2}{\text{FWHM}} \quad [\text{W/Hz}]. \quad (6.14)$$

In a well-designed HS setup, the error signal depends solely on the frequency detuning δf between laser and cavity resonance. Laser power fluctuation $\tilde{P}_0$ does not corrupt frequency stabilization to first order, since it has no effect on the zero-crossing lock point of the demodulated error signal.

6.2.2 Simulating HS signals

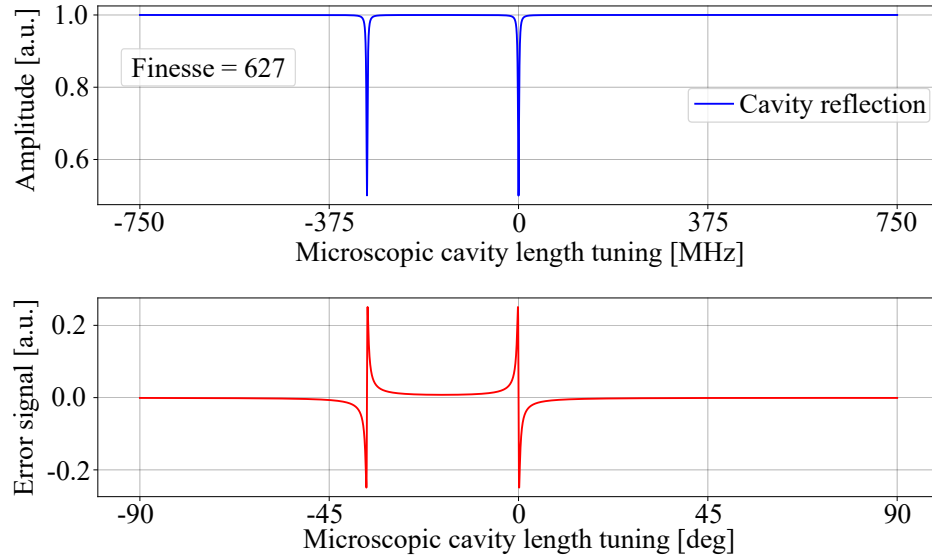


Figure 6.3: Finesse3 simulation of the Heterodyne laser frequency Stabilization (HS). The simulation comprised two laser sources, with equal power and a frequency offset $f_{\text{beat}} = 300$ MHz between them, and a 10 cm cavity with $\mathcal{F} = 625$.

To validate the theoretical derivations of the HS signals in Section 6.2.1, a simulation was conducted using the Python-based optical simulation software Finesse3 [80]. The simulation employed two laser sources with equal power and a frequency offset $f_{\text{beat}} = 300$ MHz between them. The laser beams were interfered and directed onto a 0.1 m impedance-matched cavity with a finesse of 627. The cavity length was tuned, while the reflected light was detected using DC and AC photodetectors. It

¹⁰ The introduction of the discriminator slope D_{HS} allows a straightforward comparison with the D_{PDH} of the PDH lock, addressed in the following section in Equation 6.51.

should be noted that the cavity response remains identical whether the length of the cavity is tuned or the frequency of the laser is scanned.

The signal detected by the DC photodetector is displayed in the upper panel of Fig. 6.3. The microscopic cavity length tuning is expressed in both MHz and degrees, using the conversion $\text{FSR}_{\text{cavity}} = 1.5 \text{ GHz} = 180^\circ$ ¹¹. When one laser is on resonance, the other laser is off-resonance and the power drops by half of its maximum value. After an additional tuning of 300 MHz the second laser is on resonance.

The signal detected by the AC photodetector is demodulated at the offset frequency $f_{\text{beat}} = 300 \text{ MHz}$ and displayed in the lower panel of Fig. 6.3. This demodulated signal constitutes the error signal, which exhibits a zero-crossings at each resonance. In our laser-cavity locking setup, where each laser locks to a single optical cavity, the two zero-crossings enable independent locking. However, this requires sufficiently separated resonances for effective demodulation.

6.2.3 Noise analysis

This section investigates the dominant noise sources in the Heterodyne laser frequency Stabilization scheme to evaluate the maximum achievable displacement sensitivity. Noise couples into the measurement via the laser-cavity locking loop and the beat-note frequency detection. The secondary laser, locked to the reference cavity and serving as the frequency reference for the heterodyne beat, is treated as ideal and contributes no noise. Consequently, the analysis in this section is restricted to noise arising purely from the prototype cavity. The HS noise budget is shown in Fig. 6.4, while individual noise sources are analyzed subsequently.

In this analysis, we neglect noise sources that do not couple into the error signal to first order. These include fluctuations in laser power, photodiode response, and the relative phase of the two mixer inputs. The HS is (theoretically) insensitive to all these factors, as the locking occurs on resonance where the reflected beat note vanishes.

6.2.3.1 Noise sources influencing the laser-cavity locking loop

Shot noise-induced displacement noise

Error signal noise is indistinguishable from laser frequency noise and has a fundamental quantum limit set by the shot noise of the laser (Section 3.2). When locking the laser to the cavity length using the HS technique, one laser (P_1) is on resonance, while the off-resonant second laser (P_2) is fully reflected and detected by the photodetector. The fundamental error signal shot noise consequently arises from the reflected power P_2 and is given by

$$\tilde{S}_\epsilon = \sqrt{2 \frac{hc}{\lambda}} (P_2) \quad [\text{W}/\sqrt{\text{Hz}}]. \quad (6.15)$$

¹¹ Since one round-trip within the cavity is twice the physical length, the total optical path length change is twice the tuning length.

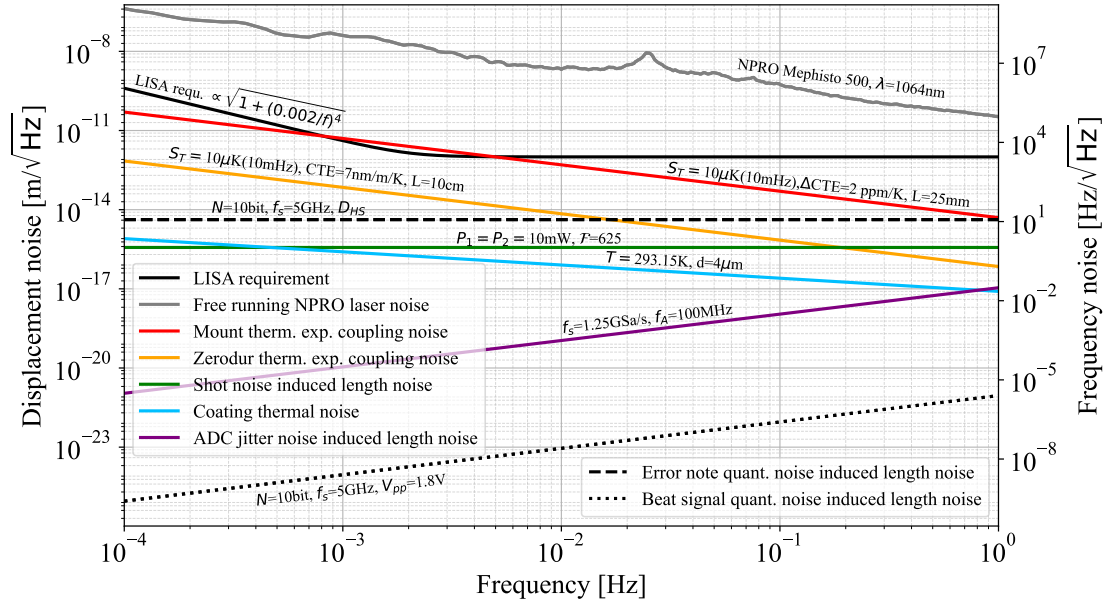


Figure 6.4: Noise budget estimation of the Heterodyne laser frequency Stabilization (HS), comprising the noise sources discussed in Section 6.2.3. The HS locks the laser frequency (free running NPRO laser noise) to the length of the prototype cavity. The expected dominant noise contribution arises from thermal expansion coupling of the mirror mount assembly, in particular the CTE mismatch between the ZeroDrift mirror mount and the Invar optomechanics.

Dividing the expression by the HS discriminator slope D_{HS} (Equation 6.14) yields the corresponding frequency noise spectrum:

$$\tilde{S}_f = \frac{\sqrt{hc^3}}{-2\mathcal{G}l\mathcal{F}L\sqrt{\lambda}\sqrt{P_2P_1}} \quad [\text{Hz}/\sqrt{\text{Hz}}], \quad (6.16)$$

which depends on the power of both lasers (P_1 and $\sqrt{P_2}$, respectively). The equivalent displacement noise is obtained by Equation 6.2 as

$$\tilde{S}_L = \frac{L}{f}\tilde{S}_f = \frac{\sqrt{hc}}{-2\mathcal{G}l\mathcal{F}\sqrt{P_2P_1}} \frac{\sqrt{\lambda}}{\sqrt{P_2P_1}} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.17)$$

Substituting the cavity parameters¹² listed in Table 6.1¹³ and laser powers $P_1 = P_2 = 10$ mW into Equation 6.17, we obtain the shot noise-induced displacement noise:

$$\tilde{S}_L \approx 3.7 \times 10^{-16} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.18)$$

This white noise floor lies well below LISA's $1 \text{ pm}/\sqrt{\text{Hz}}$ requirement.

¹² $\mathcal{G} \approx 2$, $l \approx 0.5 \text{ V}^{-1}$.

¹³ $\mathcal{F} = 625$, $L = 0.1 \text{ m}$, $\lambda = 1064 \text{ nm}$

Coating thermal noise

Although the coating accounts for only a negligible proportion of the mirror's total mass, it dominates the mirror's total thermal noise. Coating thermal noise is defined as the random, thermally driven fluctuations in the position of a mirror's surface that are caused by mechanical losses in the thin dielectric coatings and predicted by the Fluctuation-Dissipation Theorem [104] (Section 3.5.1). It states that the thermal noise power spectrum is proportional to the real part of the mechanical admittance $\mathbb{Y}(f)$ ¹⁴ of the test mass (mirror) and can be written as

$$\tilde{S}_{xx}(f) = \frac{k_B T}{\pi^2 f^2} \Re\{\mathbb{Y}(f)\} \quad [1/\text{Hz}]. \quad (6.19)$$

For an interferometer with fused silica coated mirrors that have alternating layers of tantala (Ta_2O_5) and fused silica (SiO_2), the amplitude spectral density of the coating thermal noise is given by [146, 147]

$$\tilde{S}_x(f) = \sqrt{\frac{2k_B T}{\pi^2 f} \frac{1 - \sigma_{\text{sub}} - 2\sigma_{\text{sub}}^2}{Y_{\text{sub}} w^2} \sum_j b_j d_j \phi_j} \quad [1/\sqrt{\text{Hz}}], \quad (6.20)$$

where w denotes the laser beam waist radius, d the coating thickness and ϕ_j the coating mechanical loss angle. Y_{sub} and σ_{sub} are the Young's modulus and the Poisson ratio of the substrate. The index j refers to the material parameters of the j th layer in the coating. b_j is a weighting factor described by

$$b_j = \frac{[(1 - 2\sigma_j)(1 + \sigma_j)]}{[(1 - 2\sigma_s)(1 + \sigma_s)]} \frac{1}{1 - \sigma_j} \times \left[\left(1 - n_j \frac{\partial \theta_c}{\partial \theta_j} \right)^2 \frac{Y_s}{Y_j} + \frac{(1 - \sigma_s - 2\sigma_s^2)^2}{(1 + \sigma_j)^2 (1 - 2\sigma_j)} \frac{Y_j}{Y_s} \right] \frac{Y_j}{Y_s} \quad (6.21)$$

where σ_j and σ_s are the Poisson's ratio of each coating layer and substrate. n_j is the refractive index of the j th layer and $\partial \theta_{\text{coat}} / \partial \theta_j$ describes the sensitivity of the total coating phase θ_{coat} to fluctuations in the round-trip phase θ_j in each layer.

The cavity mirrors used for both HS and PDH-locking were fused silica concave mirrors¹⁵ with a high-reflective front and anti-reflective back surface. In order to calculate the coating thermal with equation 6.20 the following assumptions were made:

- The standard laboratory temperature was 293 K. However, previous sections have shown that the temperature inside the vacuum chamber increases by some degrees due to the heat dissipation of the PDs.
- The cavity under investigation was built with a concave and flat mirror so that the beam waist was slightly different at each mirror ($w_{\text{flat}} = 260 \mu\text{m}$, $w_{\text{concave}} = 290 \mu\text{m}$). We can assume an average beam waist of $w = 275 \mu\text{m}$.
- The mirror substrate is made of fused silica with a Young's modulus of $Y_{\text{sub}} = 72 \text{ GPa}$, a Poisson ratio $\sigma_{\text{sub}} = 0.17$ and a refractive index of $n_{\text{SiO}_2} = 1.44$ [147].
- The coating deposited on the substrate surface is typically made of a multilayer of alternating

¹⁴ For comparison with Section 3.5.1, $\mathbb{Y}(\omega) = 1/Z(\omega)$.

¹⁵ Transmission = 0.5; $\lambda = 1064 \text{ nm}$; Substrate: Fused Silica-LO-V from Laseroptik.

low refraction index fused silica (SiO_2) and high refraction index tantala (Ta_2O_5). Usually the optical thickness of each layer is $\lambda/4$ where $\lambda = 1\,064\text{ nm}$ is the laser wavelength. For our cavity mirrors with a reflectivity of about $R = 99.5\%$ we can assume a number of 15 layers¹⁶ which results in a total coating thickness of $\sum d_j \approx 4\text{ }\mu\text{m}$.

- The Young's modulus of the substrate and coatings is $Y_s = 72\text{ GPa}$ for fused silica and $Y_j = 140\text{ GPa}$ for tantala [147] with a refractive index of tantala of $n_{\text{Ta}_2\text{O}_5} \approx 2$.
- For the effective substrate and coating mechanical losses, we assume $\sigma_{s,\text{sub}} = 0.17$ and $\sigma_j = 0.23$ [146].
- The experimental setup under investigation is an optical cavity, which requires us to multiply $S_{xx}(f)$ by a factor of 2 to account for the total contribution of the coating thermal noise from both mirrors.

Assuming the values previously enumerated, and a weighting factor of $b_j \approx 1.5$, the total coating thermal noise-induced displacement noise of our 0.1 m prototype cavity is

$$\tilde{S}_x(f) \approx 8 \times 10^{-18} \cdot \frac{1}{\sqrt{f}} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.22)$$

Despite the necessity for assumptions, the results remain within an order of magnitude and well below the pm-requirement, even when slight variations are made to the experimental setup.

Temperature coupling noise

The prototype cavity ($L = 10\text{ cm}$), assembled using the interferometer toolset (TAPSI), combines the Zerodur¹⁷ optical baseplate, thermally compensated ZeroDrift mirror mounts, and Invar¹⁸ optomechanics (Section 4.2). Temperature coupling noise, especially at low frequencies, is expected to be one of the dominant noise sources inherent to the interferometer toolset. The thermal expansion coupling-induced displacement noise of the optical cavity is given by

$$\tilde{S}_L(f) = \alpha \cdot L \cdot \tilde{S}_T(f) \quad [\text{m}/\sqrt{\text{Hz}}], \quad (6.23)$$

where α denotes the CTE of the material and L the cavity length. To estimate the displacement noise we approximate the temperature noise inside the optical test facility (location of the prototype cavity) with $\tilde{T}_{\text{ambient}}(f) \approx 10\text{ }\mu\text{K}/\sqrt{\text{Hz}}$ at 10 mHz, following a $1/f$ dependency (see measurements in Section 5.3). For the displacement noise of the baseplate follows $\tilde{S}_L(10\text{ mHz}) \approx 7\text{ fm}/\sqrt{\text{Hz}}$. The displacement noise arising from the mirror assembly depends on $\Delta\alpha$, the CTE mismatch between the Invar optomechanics, particularly the compensation plate and the ZeroDrift mirror mount, as discussed in Section 4.4.1. As an approximation, we assume a thermal expansion of one mirror mount and no compensation from the opposing mount. The results are plotted in Fig. 6.4.

¹⁶ Typical amount of layers for this reflectivity [28].

¹⁷ Zerodur by Schott (CTE = 7 nm/m/K)

¹⁸ Invar36 (CTE = 2 $\mu\text{m}/\text{m/K}$)

Tilt-to-length coupling (TTLC) of the optical cavity, particularly the mirror mounts, also contribute noise. However, the noise contribution is expected to be minor due to the short cavity length and the usage of thermally compensated ZeroDrift mirror mounts (pointing stability), and was therefore neglected. Nevertheless, a fully comprehensive noise budget requires accounting for TTLC. As mentioned in Section 4.4.2, analytical calculations are complex and require simulation software such as IfoCAD.

Common-mode noise rejection

Both cavities share the same vacuum chamber and thus experience identical environmental noise, including temperature, pressure and seismic noise. These common-mode noise sources would be rejected by the differential cavity length measurement, preventing their contribution to the overall displacement noise. For example, temperature noise expands both cavities equally, leaving the beat note unchanged and thus masking the individual displacement of each cavity. However, the reference cavity uses a ULE spacer while the prototype uses the TAPSI mounting concept. These distinct materials and mechanical designs yield uncorrelated length responses to common environmental noise, rendering residual common-mode coupling negligible compared to cavity-specific noise sources.

Pressure-refractive-index coupling is the sole exception, affecting both cavities equally. As this noise is not intrinsic to the prototype cavity itself, we exclude it from the analysis. Note, however, that this noise source becomes dominant when cavities reside in separate vacuum chambers. An example of this is given in the Appendix E.3.2.

Quantization noise-induced displacement noise

Quantization noise from signal digitization affects the laser-cavity lock through two primary paths: error signal generation and heterodyne beat note measurement¹⁹. The analog signals (beat notes) are detected by a photodetector and subsequently digitized by a 10-bit ADC ($f_s = 5$ GHz) from the Moku:Pro (Liquid Instruments [148])²⁰. The quantization noise is given by

$$\tilde{U}_{\text{quant}} = \frac{U_{\text{fsr}}}{2^N \sqrt{6} f_s} = 7 \times 10^{-7} \quad [\text{V}/\sqrt{\text{Hz}}], \quad (6.24)$$

where $U_{\text{fsr}} = 4$ V denotes the full-scale range voltage of the ADC²¹. The quantization noise $\tilde{U}_{\text{quant}}$ remains identical in both the *error signal* and the *heterodyne beat note quantization noise-induced displacement noise*, differing only in the respective signal characteristics.

Error signal quantization noise-induced displacement noise

The error signal arises from demodulating the reference beat against the cavity beat (Section 6.2.1). Dividing the quantization noise $\tilde{U}_{\text{quant}}$ by the discriminator slope D_{HS} (Equation 6.14) yields the quantization noise-induced frequency noise. Since the discriminator slope D_{HS} was derived in units

¹⁹ For clarity: the *beat note measurement* refers to the measurement of the beat note between the lasers outside the vacuum chamber.

²⁰ Initial measurements used a Red Pitaya, which we neglect in this analysis for simplicity.

²¹ The 2 Vpp input range was typically used for all HS and PDH-locking experiments, including the error signal detection and the heterodyne beat note measurement

of W/Hz, we multiply it by the photodiode responsivity $R_{PD} = 0.7 \text{ A/W}$ and the feedback resistor $R_f = 1 \text{ k}\Omega$ to obtain D_{HS} in units of V/Hz. Thus, it follows:

$$\tilde{S}_{f,\text{quant}} = \frac{\tilde{U}_{\text{quant}}}{D_{HS} \cdot R_{PD} \cdot R_f} \quad [\text{Hz/Hz}], \quad (6.25)$$

with

$$D_{HS} = \frac{-2\mathcal{G} l P_1 P_2}{\text{FWHM}} \approx 8.3 \times 10^{-11} \quad [\text{W/Hz}], \quad (6.26)$$

where $\text{FWHM} = 2.4 \text{ MHz}$, $P_1 = P_2 = 10 \text{ mW}$ and $\mathcal{G} \approx 2$. Substituting the values into Equation 6.25, we obtain the frequency noise

$$\tilde{S}_{f,\text{quant}} \approx 12 \text{ Hz}/\sqrt{\text{Hz}}, \quad (6.27)$$

and the corresponding displacement noise as

$$\tilde{S}_{L,\text{quant}} = \frac{L}{f_0} \cdot \tilde{S}_{f,\text{quant}} \approx 4.2 \times 10^{-15} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.28)$$

These quantization noise estimates are conservative. The effective SNR can be further improved by amplifying the error signal to completely utilize the available ADC dynamic range. In the first iteration of the HS experiment, demodulation was analog (Section 6.2.4.2) and the analog generated error signal was digitized. Later iterations (Section 6.2.4.3) digitized raw signals post measurement and demodulated digitally, increasing quantization noise by a factor of $\sqrt{2}$.

6.2.3.2 Noise sources influencing the beat note detection

The second coupling path of noise into the displacement sensing occurs through the heterodyne beat-note frequency measurement (P_Y) between the two lasers, each locked to its respective cavity.

Heterodyne beat note quantization noise-induced displacement noise

The beat note is detected by a photodetector and subsequently amplified to a peak-to-peak voltage of $V_{pp,\text{signal}} = 1.8 \text{ V}$. Substituting the values into Equation 6.24 and dividing by the signal gives us the heterodyne beat note quantization induced noise:

$$\tilde{S}_{\phi,\text{quant}} = \frac{\tilde{U}_{\text{quant}}}{V_{pp,\text{signal}}} \approx 4 \times 10^{-7} \quad [\text{rad}/\sqrt{\text{Hz}}]. \quad (6.29)$$

The equivalent frequency noise is obtained by multiplication with $2\pi f^{22}$. For the corresponding noise-induced displacement noise follows:

$$\tilde{S}_{L,\text{quant}} = \frac{L}{f_0} \cdot \tilde{S}_{\phi,\text{quant}} \cdot 2\pi f \approx 9 \times 10^{-22} \cdot f \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.30)$$

The contribution of the quantization noise-induced displacement noise is negligible small.

²² Differentiating in time ($\delta\varphi/\delta t$) corresponds to multiplication by $2\pi f$ in the frequency domain. f represents the Fourier frequency.

ADC sampling jitter

Sampling jitter, the timing uncertainty of ADC sampling instants, introduces phase noise that degrades displacement sensitivity, particularly at MHz frequencies [72]. For input frequency f_A , the induced phase noise is

$$\tilde{S}_{\varphi, \text{jitter}} = 2\pi f_A \cdot \tilde{\tau}_A \quad [\text{rad}/\sqrt{\text{Hz}}], \quad (6.31)$$

where $\tilde{\tau}_A$ denotes the (ASD) timing sampling jitter. We estimate the sampling jitter $\tilde{\tau}_A = 8 \text{ ps}/\sqrt{\text{Hz}}$ from the Moku:Pro specifications (1.25 GSa/s), where linear interpolation reduces jitter by 100 [149]. At a heterodyne beat-note frequency of $f_A = 100 \text{ MHz}$, the phase noise becomes $\tilde{S}_{\varphi, \text{jitter}} = 5 \times 10^{-3} \text{ rad}/\sqrt{\text{Hz}}$, corresponding to a displacement noise of

$$\tilde{S}_{L, \text{jitter}} \approx 1.1 \times 10^{-17} \cdot f \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.32)$$

The contribution of the ADC sampling jitter-induced displacement noise is negligible small.

Theoretical noise budget of the HS

Free-running laser noise from the NPRO laser²³ has not yet been addressed in this section. Its spectral density of $20 \text{ kHz}/\sqrt{\text{Hz}}$ at 1 Hz , following a $1/f$ slope, exceeds picometer displacement requirements. However, the HS feedback loop actively stabilizes the laser frequency to the cavity length, suppressing LFN within the servo bandwidth. The total noise budget $S_{f, \text{NoiseBudget}}$ is calculated as the quadrature sum of the uncorrelated individual noise sources as²⁴

$$S_{f, \text{NoiseBudget}} = \sqrt{\left(\frac{S_{f, \text{free}}}{1 + G(s)}\right)^2 + \sum_i \left(S_{(i, \text{NoiseSource})} \cdot \frac{G(s)}{1 + G(s)}\right)^2} \quad [\text{Hz}/\sqrt{\text{Hz}}], \quad (6.33)$$

where the index i denotes the uncorrelated noise contributions $S_{(i, \text{NoiseSource})}$. For high loop gain ($G(s) \gg 1$) and sufficient loop bandwidth ($\gtrsim 100 \text{ kHz}$), free-running laser noise is effectively suppressed. Downstream noise (post PID) is also suppressed by $\frac{1}{1+G(s)}$, while upstream noise couples directly into the error signal and the heterodyne beat note measurement. The total noise budget of the HS scheme is shown in Fig. 6.4, comprising all previously analyzed noise sources affecting the laser-cavity lock and the readout.

As demonstrated in the noise budget in Section 4.4.3, thermal expansion of the mirror mount assembly, particularly the CTE mismatch between ZeroDrift mount and Invar optomechanics, dominates. Mitigation requires either improved CTE matching or reduced temperature noise within the optical test facility. Additional noise sources, including non-ideal electronics, spurious light signals (SLSs) and cavity contrast degradation can also limit performance. These are omitted from the theoretical budget due to complex nonlinear modeling, especially SLS coupling (Section 3.4). Thus, to validate the picometer-level length stability of the TAPSI mounting concept, the laser-cavity locking scheme was set up and measured.

²³ Coherent, NPRO *Mephisto 500*.

²⁴ The expression is a simplification of Equation 2.54 from the feedback control loop in Section 2.3.

6.2.4 Experimental setup

The length stability of the mounting concept (see Chapter 4) was investigated using laser-cavity stabilization via the Heterodyne laser frequency Stabilization technique. A prototype cavity was constructed using the toolset, in which two mirror-mount assemblies formed a linear cavity with a length of 10 cm (see Fig. 4.2²⁵). The initial version of the baseplate consisted of a rectangular Zerodur plate, whereas the final design employed the large Zerodur disc.

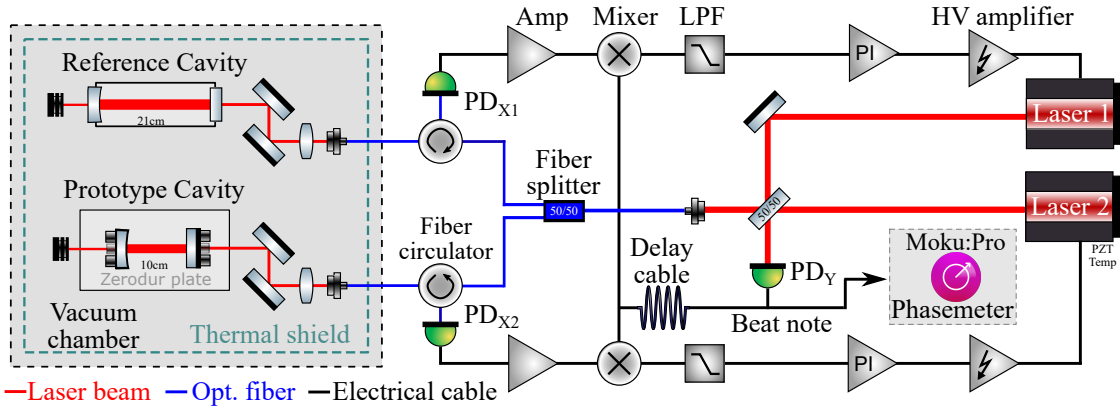


Figure 6.5: Initial Heterodyne laser frequency Stabilization scheme. Two lasers are interfered at the BS to generate a beat note which is measured by PD_Y (reference beat). The beat note from the second output port of the BS is split, and directed onto the optical cavities where only one of the lasers is resonant in each respective cavity. The reflections from each cavity (cavity beat) are measured by PD_{X1} and PD_{X2} , respectively. Subsequent demodulation (mixing +LPF) of the cavity beat notes against the reference beat note generates an error signal, which drives an feedback control loop to stabilize the laser frequency to the length of the respective cavity. The relative length fluctuation of the cavities is derived from the measured beat note signal PD_Y .

The HS experimental setup is shown in Fig. 6.5. Two NPRO-lasers were interfered at a BS to generate a beat note which was detected by the photodetector PD_Y ²⁶ and subsequently measured by the phasemeter instrument of a Moku:Pro²⁷. The beat note from the second output port of the BS was coupled into a pm-fiber and split into two equal parts, each propagating through a fiber circulator²⁸ before entering the vacuum chamber via pm-fiber feedthroughs. Inside the chamber, the beams were coupled from fiber to free space and directed onto the respective cavities. When the laser-cavity lock is active, only one laser resonates while the second laser is reflected from the cavity. The reflected beams traverse the same path before being directed by the circulator towards photodetectors PD_{X1} and PD_{X2} , respectively. The detected cavity beat notes, were demodulated by analog mixing against the reference beat note and subsequent low pass filtering. Additional phase-shifters in front of each mixer allowed fine adjustment of the demodulation phase. The resulting error signals were digitized by an ADC and processed by a PID-controller²⁹ to generate actuation signals that stabilize each laser frequency to

²⁵ Fig. 4.2 shows only one side of the linear cavity structure.

²⁶ Coherent, ET-3000A, wavelength: 800-1 700 nm, bandwidth: 30 kHz-1.5 GHz.

²⁷ Liquid Instruments, Moku:Pro, phasemeter-instrument with 300 MHz bandwidth.

²⁸ Fibermart, P-Grade, 3 Ports, minimum isolation: 40 dB.

²⁹ Early iterations employed a Red Pitaya running the PyRPL-software package to perform digitization and realize the PID controller, while later iterations employed the corresponding instruments of the Moku:Pro.

the length of its respective cavity. Fast frequency tuning was implemented via laser-integrated PZT actuators, while slow laser-temperature tuning ensured that the PZT operated within its dynamic range.

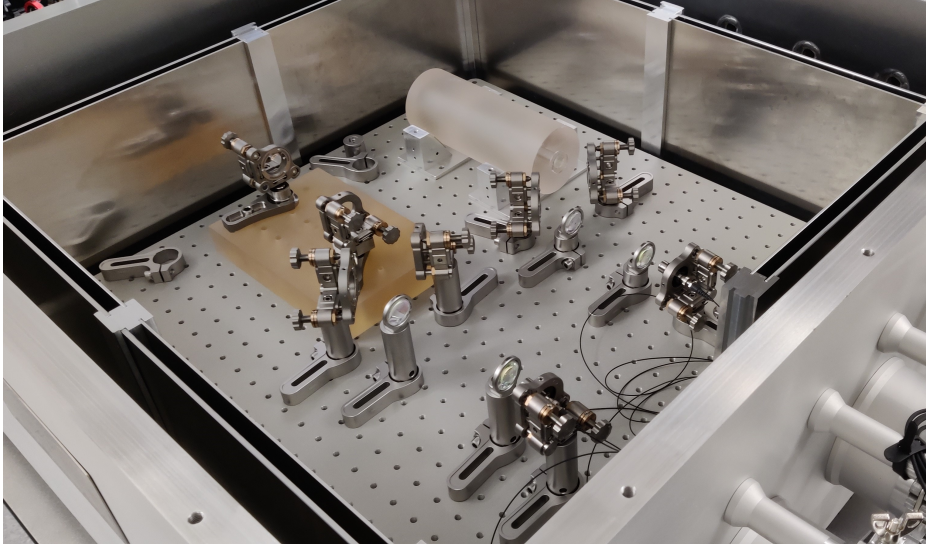


Figure 6.6: Photograph of the initial HS locking setup inside the vacuum chamber. The ULE reference cavity is located at the rear of the chamber, while the prototype cavity, implemented using the toolset for adjustable picometer-stable interferometers, is at the front.

6.2.4.1 Demodulation phase and path length mismatch

The detuning frequency of the laser frequency from the resonance of the cavity is encoded as a phase shift onto the *cavity beat note* (PD_X). Extracting this phase information was done by demodulation against the *reference beat note* (PD_Y). As discussed in Section 6.2.1, the signals require a relative phase shift of $\pi/2$ to generate a symmetric zero-crossing error signal. Any deviation from this ideal demodulation phase distorts the symmetric error signal and exhibits a reduced linear (locking) range, thereby decreasing the tolerance for frequency deviations around the locking point (typically zero) and increasing the likelihood of the system to lose lock. A simulation was conducted for different demodulation phases and is shown in Fig. 6.7.

HS path length mismatch

In theory, HS generates an error signal with a zero-crossing at cavity resonance and a lineshape that depends on the demodulation phase (see simulated in Fig. 6.7). However, scanning the laser frequency revealed an oscillating error signal that masked the true resonance response (see Figure 6.8). This distortion occurred due to different path lengths of the cavity and reference beat signals (PD_X , PD_Y). The theoretical derivation assumed the demodulation of identical beat note frequencies. However, in the presence of a path length mismatch and during laser frequency scan, the beat notes differ at the mixer, producing the observed oscillations. The reference beat was detected outside the vacuum chamber and has only traveled a short distance from the detection to the mixer. In contrast, the cavity

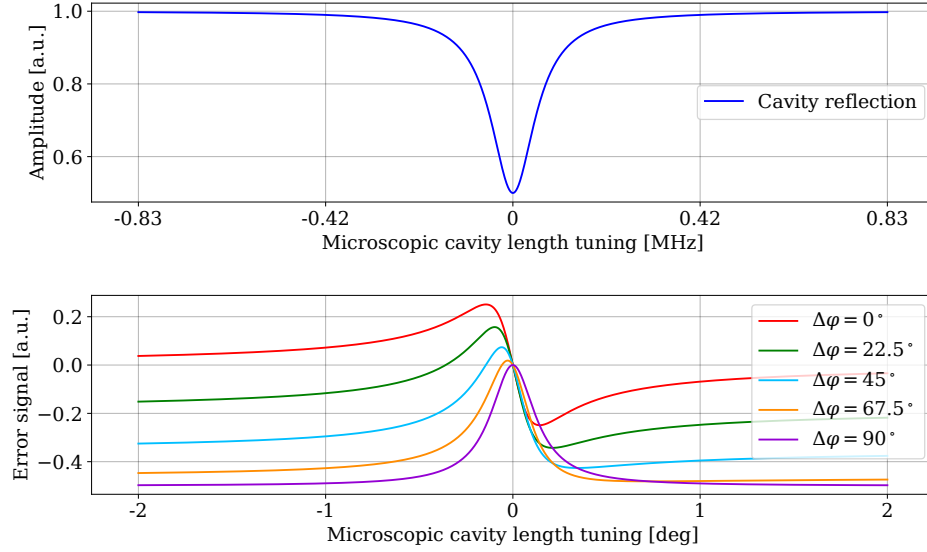


Figure 6.7: Simulation of the cavity response and corresponding error signal at resonance using the Heterodyne laser frequency Stabilization scheme. The upper panel shows the amplitude of the reflected cavity field, while the lower panel displays the error signal for deviating demodulation phases $\Delta\varphi$. Any deviation from the ideal demodulation phase of $\pi/2$ ($\Delta\varphi = 0^\circ$) distorts the symmetric error signal shape. The simulation was performed using the Finesse3 software [80].

beat entered the vacuum chamber via optical fibers, was coupled to free space, reflected from the cavity, recoupled into the fiber, then detected, and forwarded to the mixer, resulting in a substantially longer total path length. To compensate for this difference, we added electrical cables (*delay cables*) to the reference path, which reduced but did not fully suppress the oscillations³⁰.

Furthermore, the demodulation phase change is also frequency-dependent. Even with constant path length mismatch Δl , the demodulation phase $\Delta\varphi$ varies with beat note frequency f_{beat} , as given by

$$\Delta\varphi_{\text{static}}(f_{\text{beat}}) = \Delta l \cdot \frac{2\pi f_{\text{beat}}}{c_n}, \quad (6.34)$$

where c_n is the signal speed in the medium³¹. To address this problem, a tunable demodulation phase that changes in accordance with the beat note frequency is required. The analog demodulation scheme lacked this capability. The challenge was later addressed (Section 6.2.4.3) with the implementation of a digital phasemeter [79, 151].

6.2.4.2 Preliminary results of the HS scheme based on analog demodulation

Despite the limited options to optimize the demodulation phase across the full frequency spectrum using analog electronics, a proof-of-principle demonstration of the HS was performed. For a fixed

³⁰ Precise path length matching required both correct cable length and phase shifter fine-tuning, which proved challenging to implement practically.

³¹ The signal speed (light) in an optical fiber is c/n where n denotes the refractive index of the medium and c the speed of light in vacuum; the signal speed in an electrical (coaxial) cable is $\approx 0.67c$ [150].

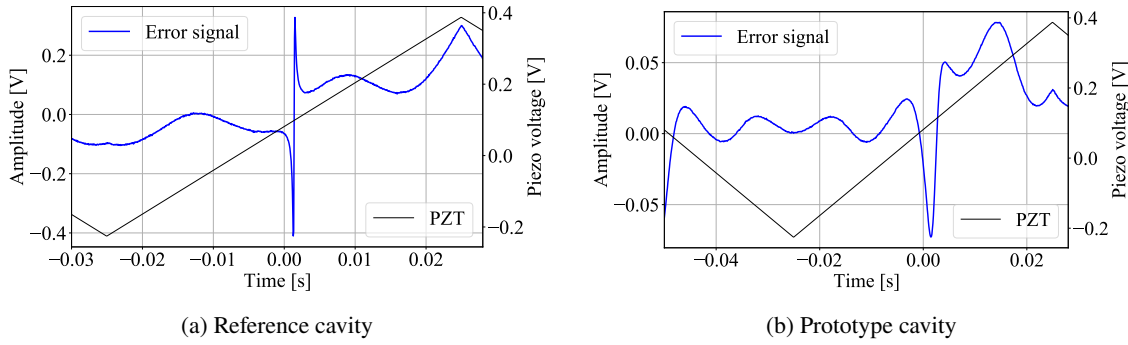


Figure 6.8: Laser frequency scan across reference and prototype cavity resonances in the presence of a path length mismatch between the cavity and reference beat note signal. The scan shows an oscillating error signal that masks the true resonance response and shifts the zero-crossing. Error signals for various path length mismatches are shown in the Appendix E.3.3.

beat note³², the optimal electrical cable length was determined by scanning the laser frequency and minimizing off-resonance error-signal oscillations.

The displacement noise sensitivity is shown in Fig. 6.9, alongside the LISA requirement. Additionally, the free-running NPRO laser noise (gray line) is included as a reference for unlocked operation. This ‘virtual’ displacement noise is calculated from the laser’s frequency noise spectrum, assuming a nominal laser-cavity stabilization, via Equation 6.2. The actual laser-cavity stabilization (colored lines) suppresses frequency noise by several orders of magnitude. Peaks outside the band of interest ($f > 1$ Hz) arise from resonances in the prototype cavity mirror mounts, likely due to seismic coupling. Substituting the *ZeroDrift* with *Polaris* mirror mounts³³ slightly shifted the resonance peaks but did not result in differing displacement heights. These peaks lie beyond the LISA bandwidth and thus do not impact the toolset’s performance. However, they necessitated a phasemeter tracking bandwidth of 1 MHz, as 100 kHz proved insufficient.

The presented measurements (Fig. 6.9) also document the progress achieved through several iterations, including thermal insulation³⁴ of fiber optics and electronic components located outside the vacuum chamber (see Fig. E.11 in the appendix.). Nevertheless, below 40 mHz, displacement noise remained well above the LISA requirement. This limitation is attributed to the phase coupling from the path length mismatch in HS, which, in practice, cannot be fully suppressed using analog demodulation.

6.2.4.3 Digital demodulation and free-space optics

The initial HS configuration based on analog demodulation and fiber-optic components was limited by path length mismatches (see Section 6.2.4.1) and therefore substantially redesigned. One of the two primary modification involved replacing fiber-optic components with free-space optics wherever feasible, effectively suppressing spurious light beams typically generated by optical fibers and their interconnections (formation of etalons). The optical layout of the HS scheme based on digital

³² The beat note arises from the detuning between cavity resonances to which the lasers are locked, and its frequency is adjustable only in steps matching the FSR of each cavity.

³³ Thorlabs, Polaris, *POLARIS-K1S4*.

³⁴ Optical fiber and electronic components (cables, PID controller, amplifier, etc.) were placed inside thermal boxes.

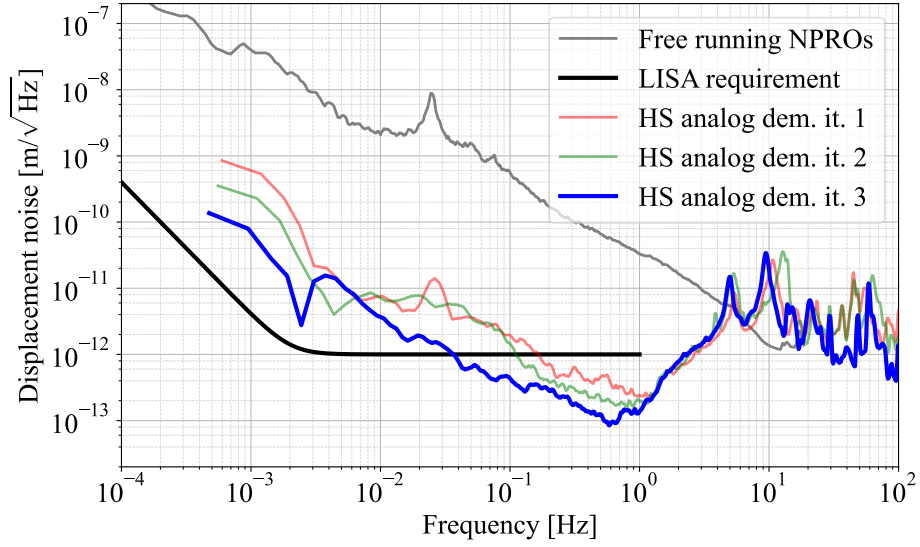


Figure 6.9: Initial displacement noise measurements of the prototype cavity using the HS scheme based on analog demodulation and fiber-optic components along with the $\text{pm}/\sqrt{\text{Hz}}$ requirements of LISA and the free running NPRO laser noise. Several iterations (it.) improved the sensitivity, including thermal isolation of fiber optics and electronic components located outside the vacuum chamber. Nevertheless, the sensitivity remained limited by path length mismatches in HS.

demodulation and free-space optics is shown in Fig. 6.10. Unlike the previous fiber-optic setup, the free-space implementation utilized the full toolset for adjustable picometer-stable interferometers to set up both the prototype cavity and the associated HS scheme optics. Furthermore, the photodetectors³⁵ to detect the cavity beat notes were shifted inside the vacuum chamber.

The second modification implemented a digital phasemeter³⁶. The phasemeter acquired the beat note signals (reference and the two cavity beats notes) and provided the digital heterodyne controller with frequency-dependent demodulation phase adjustment. The phase adjustment allowed the correction (stabilization) of the demodulation phase error arising from the path length mismatch in the HS scheme, as discussed in Section 6.2.4.1. The resulting error signal was forwarded to the Moku:Pro lockbox instrument, comprising the PID controller and the generation of the laser frequency actuation signal.

Despite utilization of the digital heterodyne controller, the demodulation phase error could not be fully suppressed. The best error signals for the reference and prototype cavities are shown in Fig. 6.8. The controller's discrete step size to adjust frequency and phase limited the error-signal optimization. The achieved displacement noise sensitivity is shown in Fig. 6.12, alongside the LISA requirement and the previous sensitivity of HS scheme based on analog demodulation and fiber-optic components (Figure 6.9). The results of the HS based on digital demodulation and free-space optics did not improve performance. The limitations are discussed in the following Section 6.2.4.4.

³⁵ Photodetector: Thorlabs, *DET08CFC/M*.

³⁶ Digital phasemeter by S. C. Subrahmanya [79, 151]

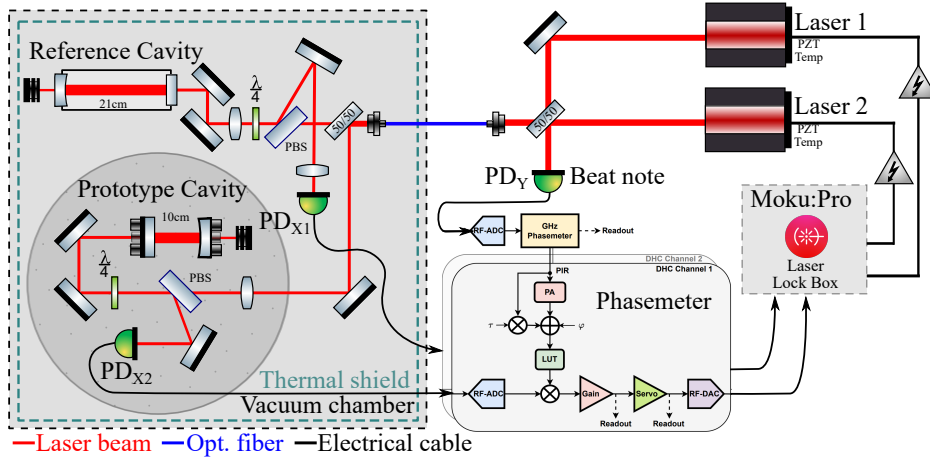


Figure 6.10: Heterodyne laser frequency Stabilization scheme using a digital phasemeter [151] for demodulation and free-space optics. Two lasers are interfered at a BS and generate a beat note which is measured by $PD_Y(t)$ (reference beat). The beat note from the second BS output is forwarded into the chamber, coupled to free space and directed onto the respective optical cavities. The reflection off the cavity (cavity beat) is measured by $PD_X(t)$ and forwarded to the digital phasemeter for digitization and demodulation. The resulting error signal is used by the Laser Box instrument (PID controller) of the Moku:Pro to generate an actuation signal which adjust and thereby stabilizes the laser frequency to length stability of the respective cavity.

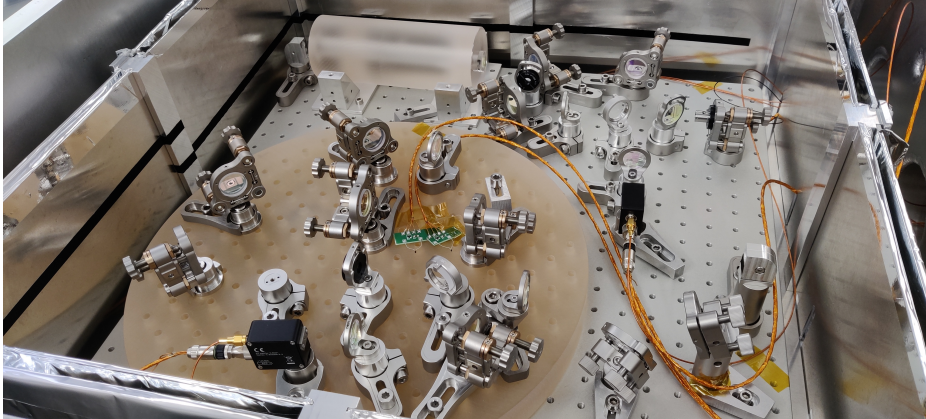


Figure 6.11: Photograph of the HS based on free-space optics inside the vacuum chamber. The lasers were locked to their respective cavities: reference cavity (rear) and prototype cavity (front). The locking scheme was set up using the toolset (TAPSI). Temperature sensors (green PCBs) were placed in close proximity to the prototype cavity.

6.2.4.4 Experimental limitations

Despite the apparent simplicity of the HS locking scheme, which involves only the generation of a beat note and the acquisition of three signals, certain challenges remain. The main limitation arises from the demodulation phase error due to optical and electrical path length mismatches between the reference and cavity beat notes. Each path length spans from the beam splitter, generating the beat note, to the point of demodulation (signal mixer). In the presence of a path length mismatch, two

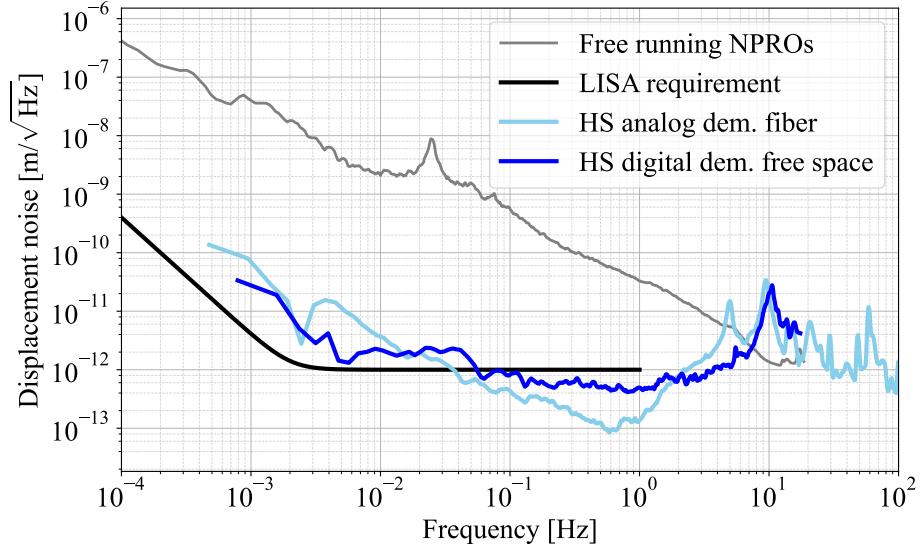


Figure 6.12: Displacement noise measurements of the prototype cavity using the HS scheme based on digital demodulation and free-space optics together with the measurement results from the previous scheme based on analog demodulation. Both HS schemes are well above the LISA requirement for frequencies above 40 mHz.

primary effects contribute to a change of the demodulation phase. The first effect arises from the different propagation time of each beat note. When the beat note frequency changes (interference at the BS), the reference path's frequency at the mixer shifts immediately (short path), while the cavity path's beat note remains unchanged (longer path). This results in two different instantaneous beat note frequencies at the mixer producing a dynamic demodulation phase error, expressed as

$$\Delta\phi_{\text{dynamic}} = 2\pi \Delta f_{\text{beat}} \Delta t = 2\pi \Delta f_{\text{beat}} \cdot \frac{\Delta l}{c_n}, \quad (6.35)$$

where Δf_{beat} denotes the frequency difference between the reference and cavity beat note, and Δt the propagation time difference. The expression is also given in terms of path length mismatch Δl , and signal propagation velocity c_n . The second effect is a static phase offset. Even with constant beat note frequency, unequal optical/electrical paths cause the longer path's signal to accumulate extra phase. This produces a fixed delay between reference and cavity signals, given by

$$\Delta\phi_{\text{static}} = 2\pi f_{\text{beat}} \Delta t = 2\pi f_{\text{beat}} \cdot \frac{\Delta l}{c_n}, \quad (6.36)$$

where f_{beat} is the constant beat note frequency. A third effect that should be noted arises from the cross-coupling between the two locked cavities. When one laser adjust its frequency to follow the cavity length, the beat note frequency changes accordingly, triggering the first effect in *both* feedback loops. However, this cross-coupling mechanism requires further investigation to be fully understood and characterized. Additionally, non-flat transfer functions in signal chain components introduce phase shifts, e.g., analog mixers [72]. We can generally refer to all these contributions as *demodulation phase error* or *demodulation phase noise*.

Estimation of the demodulation phase noise coupling in HS

The previous sections discussed the sources of the demodulation phase errors. Here, we examine how demodulation phase noise couples into the error signal. The reference and cavity demodulation signal was derived in Section 6.2.1, Equation 6.10. We can now expand the equation by including the demodulation phase error $\Delta\varphi(\omega)$, and write the expression as

$$P_{X,AC}(t) \times P_{Y,AC}(t) = 2E_1^2 E_2^2 \{ [\Re(R(\delta f)) \sin(2\Delta\omega t) + \sin(0 + \Delta\varphi(\omega))] [\Im(R(\delta f)) (\cos(0 + \Delta\varphi(\omega)) - \cos(2\Delta\omega t))] \}. \quad (6.37)$$

After low-pass filtering we obtain

$$P_{X,AC}(t) \times P_{Y,AC}(t) = 2E_1^2 E_2^2 \{ [\Re(R(\delta f)) \cdot \sin(\Delta\varphi(\omega))] + [\Im(R(\delta f)) \cdot \cos(\Delta\varphi(\omega))] \}. \quad (6.38)$$

At cavity resonance, the real and imaginary part vanish, as $R(\delta f) = 0$. This condition holds only true for an ideal cavity with perfect impedance matching and 100 % contrast. Any deviation from this ideal case yields $\Re\{R(\delta f)\} \neq 0$, leaving a residual offset in Equation 6.38 at cavity resonance. This residual offset can be seen visually in the simulation in Fig. 6.13, where a cavity contrast of 90 % was assumed. While this behavior is present in all locking schemes such as PDH locking (Section 6.3), it is particularly critical in HS, since the phase error $\Delta\varphi(\omega)$ depends on both the beat note frequency $f_{\text{beat}}(t)$ and the path length mismatch $\Delta l(t)$.

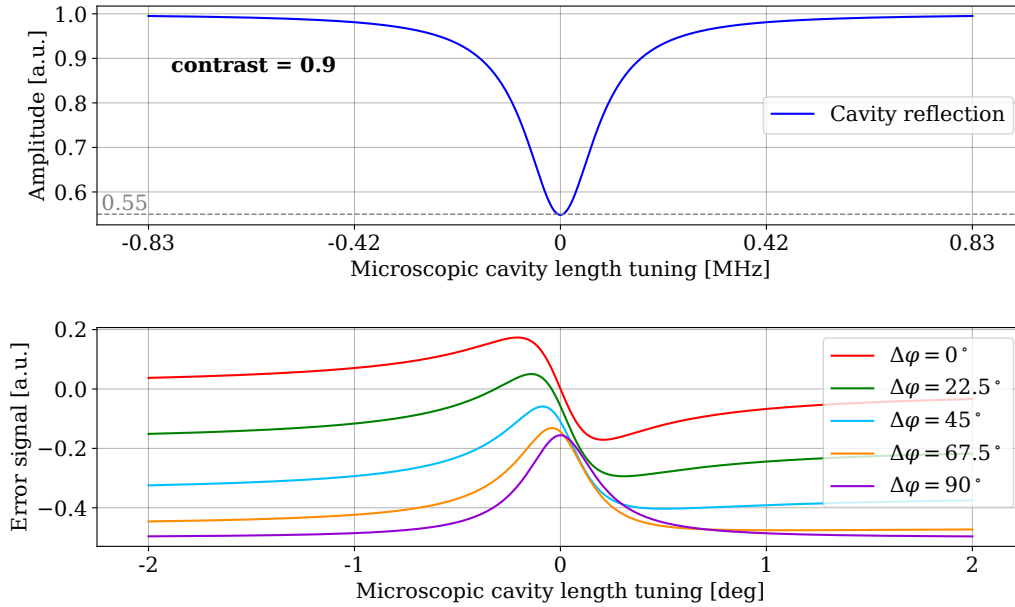


Figure 6.13: Simulation of the cavity response and the corresponding error signals in the HS scheme at resonance with a cavity contrast of 90 %. The upper panel shows the amplitude of the reflected cavity field, while the lower panel displays the error signals for different demodulation phases $\Delta\varphi$. Any deviation from the ideal demodulation phase of $\pi/2$ ($\Delta\varphi = 0^\circ$) alters both the slope and the zero-crossing of the error signal. The simulation was performed using the `Finesse3` software [80].

Numerical estimation

The following numerical estimation is intended to be illustrative rather than to provide exact noise levels for the conducted HS experiment, since several parameters could not be verified with sufficient precision.

We assume an electrical cable³⁷ path length mismatch of 1 m and a 220 MHz beat note frequency drifting by 20 MHz over a time span of about 11 h (40 ks)³⁸. Using Equation 6.35, this corresponds to a demodulation phase shift of 36° . The resulting offset in the error signal can be approximated from the simulation in Fig. 6.14³⁹ where $\kappa = 90\%$. For simplicity, a linear error signal is assumed, even though its slope depends on the demodulation phase. When the feedback loop is closed (locked), the PID controller drives the error signal to zero. This produces a laser frequency offset from the true resonance of the cavity by

$$\Delta f = \text{FSR} \cdot (\Delta\phi/\pi) \approx 0.3 \text{ MHz}, \quad (6.39)$$

where $\text{FSR} = 1.5 \text{ GHz}$ is the free spectral range of the prototype cavity and $\Delta\phi \approx 0.035^\circ$ the microscopic cavity length tuning. Using Equation 6.2, we can calculate the corresponding displacement as $\Delta L \approx 100 \text{ pm}$.

The estimation above was performed for a fixed path length mismatch and a varying beat note. However, temporal variations in the path length must also be considered. Environmental factors, such as temperature fluctuations, induce thermal refractive index changes in the optical fibers and thereby alter the optical path length. A precise prediction of all the aforementioned demodulation phase error effects and the resulting displacement noise is complex and beyond the scope of this work.

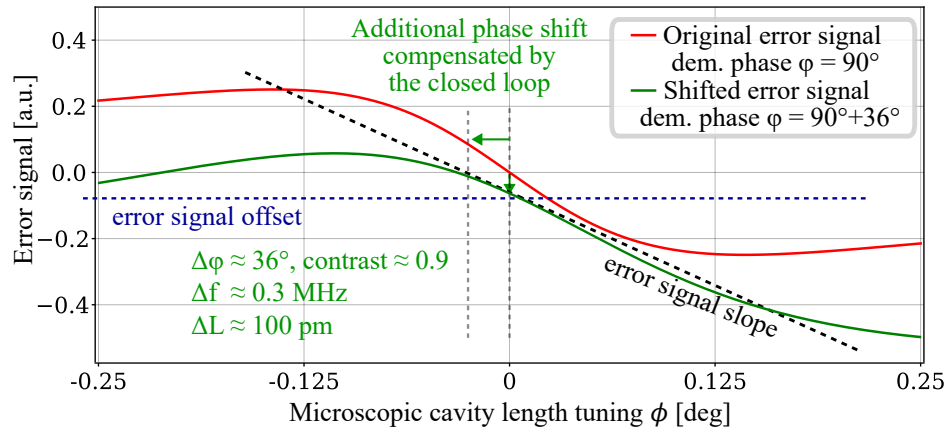


Figure 6.14: Simulation of the cavity response and corresponding error signal obtained from the HS at resonance with a cavity contrast of 90 %. The error signal is shown for the ideal demodulation phase $\Delta\phi = 0^\circ$ ($\pi/2$) (red) and the demodulation phase error $\Delta\phi = 36^\circ$, which alters both the slope and the zero-crossing of the error signal (green). The closed control loop compensates for this phase shift $\Delta\phi$, resulting in laser frequency deviation Δf from the true resonance. The corresponding displacement error is denoted as ΔL . The simulation was performed using the Finesse3 software [80].

³⁷ The signal propagation speed in an electrical (coaxial) cable is approximately $0.67c$ [150].

³⁸ The assumption is based on the measurement shown in Figure 6.24.

³⁹ See also Fig. 6.7 for various demodulation phase errors $\Delta\phi$.

Conclusion

In summary, the experimental implementation of the Heterodyne laser frequency Stabilization scheme has shown that the current state of the analog demodulation with fiber-optic components cannot achieve the required picometer-level displacement sensitivity across the LISA frequency band. The HS scheme based on digital demodulation and free-space optics, also requires further improvement, particularly the resolution of the heterodyne controller to enable further optimization of the error signal⁴⁰. The primary limitation was the demodulation phase error arising from path length mismatches between the reference and cavity beat note signals in the presence of nonideal cavity contrast. A contrast of 90 % for the prototype cavity and 60 % for the reference cavity⁴¹ was the maximum achieved in any of the measurements, most likely due to insufficient mode matching. Thus, measurement limitations stemmed from the HS readout rather than from the prototype cavity's length stability, i.e., the intrinsic toolset (TAPSI) length stability. Consequently, an alternative locking scheme was required to validate the picometer-level length stability of the toolset across the LISA band.

⁴⁰ It should be noted that the HS scheme, when implemented in an optimized setup with two ULE cavities, has demonstrated sub-picometer displacement noise [152].

⁴¹ The borrowed reference cavity (ULE spacer) also exhibited a maximum contrast of 60 % in previous experiments [138].

6.3 Pound-Drever-Hall locking

Given the persisting limitations of the Heterodyne laser frequency Stabilization (HS) scheme (Section 6.2.4.4), the length stability of the toolset (TAPSI) was further characterized using the Pound-Drever-Hall (PDH) locking scheme. In contrast to HS, PDH locking is limited by the demodulation-phase noise, as its demodulation frequency is given by a fixed, frequency-stable local oscillator (LO). The PDH technique represents the most widely used laser frequency stabilization technique and achieves state-of-the-art performance, with a noise of 4×10^{-17} m/ $\sqrt{\text{Hz}}$ at 0.8 Hz extending up to a few hertz [153]⁴².

6.3.1 Optical layout and signals

This section provides an analytical understanding of the Pound-Drever-Hall locking scheme and the generation of its error signal. The derivations presented here focus on the specific configurations implemented in this work, whereas a comprehensive treatment of the PDH technique can be found in [154]. A simple Pound-Drever-Hall (PDH) scheme is shown in Fig. 6.15, consisting of a laser, EOM, PD, an optical cavity, LO, associated optics and demodulation electronics.

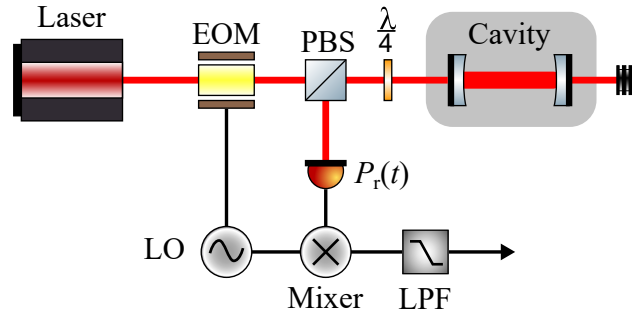


Figure 6.15: Simplified Pound-Drever-Hall scheme. The laser beam is phase-modulated by an EOM at modulation frequency Ω set by the local oscillator (LO), producing sidebands around the carrier ($\pm\Omega$). Only the carrier resonates in the cavity, while off-resonant sidebands reflect. Demodulation of the reflected light with the LO isolates the cavity phase shift, encoding length information.

The laser beam is phase-modulated by an EOM at modulation frequency Ω set by a signal generator (LO), which drives both the EOM and serves as the LO reference for demodulation. The phase-modulated electric field is expressed as⁴³

$$E(t) = E_0 e^{i\omega_0 t} \cdot \left(J_0(m) + iJ_1(m)e^{i\Omega t} - iJ_{-1}(m)e^{-i\Omega t} \right) \quad (6.40)$$

$$\approx E_0 J_0(m) e^{i\omega_0 t} + iE_0 J_1(m) e^{i(\omega_0 + \Omega)t} - iE_0 J_{-1}(m) e^{i(\omega_0 - \Omega)t},$$

where E_0 denotes the electric field amplitude, ω_0 the angular frequency of the laser, Ω the modulation frequency and m the phase modulation index, also known as modulation depth or strength. It should be noted that for small modulation indices ($m < 1$), nearly all power P_0 resides in the carrier P_C and

⁴² Note: This state-of-the-art performance was achieved under highly specialized conditions, including cryogenic operation, ultra-high-finesse cavities, and comprehensive noise mitigation techniques.

⁴³ The expression only considers the first-order sidebands.

the first-order sidebands P_S , yielding:

$$P_0 \approx P_C + 2P_S, \quad (6.41)$$

where each term has the corresponding power

$$P_C = J_0^2(m)P_0, \quad (6.42)$$

$$P_S = J_1^2(m)P_0. \quad (6.43)$$

Upon reflection from the optical cavity, the phase-modulated optical field experiences frequency-dependent filtering via the cavity's transfer function (Equation 2.42). When operating near resonance, the carrier acquires an amplitude and phase shift $R(\delta f)$, that encodes its frequency detuning from resonance ($\delta f = f - f_{\text{res}}$) onto the reflected light. In the fast modulation regime ($\Omega \gg \text{FWHM}$), the sidebands are far off-resonance, experience nearly complete reflection $R(f \pm \Omega) \approx 1$, and are thus independent of cavity dynamics. The near-resonant carrier, however, acquires both amplitude and phase shifts from $R(\delta f)$, encoding its detuning. The reflected electric field thus comprises three distinct terms:

$$E_r(t) = R(\delta f) \cdot E_0 J_0(m) e^{i\omega_0 t} + iE_0 J_1(m) e^{i(\omega_0 + \Omega)t} - iE_0 J_{-1}(m) e^{i(\omega_0 - \Omega)t}. \quad (6.44)$$

The photodetector (PD) measures the reflected power $P \propto |E|^2$, yielding

$$\begin{aligned} P_r(t) = & R(\delta f)^2 P_C + P_S \\ & + 2\sqrt{P_C P_S} \cdot \Re \{R(\delta f) - R(\delta f)^*\} \cos(\Omega t) \\ & + 2\sqrt{P_C P_S} \cdot \Im \{R(\delta f) - R(\delta f)^*\} \sin(\Omega t) \\ & + (2\Omega\text{-terms}). \end{aligned} \quad (6.45)$$

The first term represents the DC component of the power. The second and third terms produce Ω -frequency beat notes from carrier-sideband interference, encoding $R(\delta f)$ in both their real and imaginary components. For high-finesse cavities near resonance, the real part $\Re \{R(\delta f)\}$ remains constant while the imaginary part $\Im \{R(\delta f)\}$ varies linearly with detuning δf . Demodulation of $P_r(t)$ against the local oscillator ($\Omega_{\text{LO}} = \Omega$), followed by low-pass filtering, isolates the dispersive signal. A $\pi/2$ phase shift in one path generates a symmetric zero-crossing error signal. The LO signal from the signal generator is expressed as

$$P_{\text{SG}} = \hat{P}_{\text{SG}} \sin(\Omega t), \quad (6.46)$$

where we define the amplitude $\hat{P}_{\text{SG}} = 1$ for simplicity.

The first step of the demodulation is the multiplication of the two signals $P_r(t)$ and P_{SG} . After some

algebra we obtain

$$\begin{aligned}
 P_r(t) \times P_{SG} = & R(\delta f)^2 P_C \sin(\Omega t) + P_S \sin(\Omega t) \\
 & + \sqrt{P_C P_S} \cdot \Re \{ R(\delta f) - R(\delta f)^* \} (\sin(0) + \sin(2\Omega t)) \\
 & + \sqrt{P_C P_S} \cdot \Im \{ R(\delta f) - R(\delta f)^* \} (\cos(0) - \cos(2\Omega t)) \\
 & + (2\Omega\text{-terms}) \cdot \sin(\Omega t).
 \end{aligned} \tag{6.47}$$

Subsequent low-pass filtering (LPF) removes the high-frequency 2Ω terms, yielding

$$\begin{aligned}
 P_r(t) \times P_{SG} \stackrel{\text{LPF}}{=} & \sqrt{P_C P_S} \cdot \Im [R(\delta f) - R(\delta f)^*] \\
 \approx & -2\sqrt{P_C P_S} \Im [R(\delta f)].
 \end{aligned} \tag{6.48}$$

The imaginary part of the cavity transfer function for high-finesse cavities (Section 2.2.4, Equation 2.53) is given by

$$\Im \{ R(\delta f) \} \approx -\frac{2}{\pi} \left(\frac{1}{r_1} - r_1 \right) \mathcal{F} \frac{\delta f}{\text{FWHM}} = -\mathcal{G} \frac{\delta f}{\text{FWHM}}, \tag{2.53}$$

where $\mathcal{G}$ is the optical gain of the cavity⁴⁴. Substituting Equation 2.53 into Equation 6.11 yields the error signal $\epsilon(\delta f)$:

$$\epsilon(\delta f) = \frac{-2\mathcal{G}\sqrt{P_C P_S}}{\text{FWHM}} \delta f. \tag{6.49}$$

The error signal can also be written in the compact form:

$$\epsilon_{\text{PDH}}(\delta f) = D_{\text{PDH}} \cdot \delta f, \tag{6.50}$$

where D_{PDH} is the discriminator slope defined as

$$D_{\text{PDH}} = \frac{-2\mathcal{G}\sqrt{P_C P_S}}{\text{FWHM}} \quad [\text{W/Hz}]. \tag{6.51}$$

In a well-designed PDH setup, the error signal depends solely on the laser-cavity detuning δf . Laser power fluctuation $\tilde{P}_0$ does not corrupt frequency stabilization to first order, since it has no effect on the zero-crossing lock point of the demodulated error signal. However, unequal sideband power introduces DC offsets that shift the lock point from true resonance of the cavity. This effect is analyzed further in Section 6.3.3.

The preceding derivation of the PDH error signal ϵ_{PDH} assumed a high-finesse cavity, fast modulation ($\Omega \gg \text{FWHM}$), and low modulation depth ($m \ll 1$) which is the standard configuration for PDH locking. However, slow modulation ($\Omega < \text{FWHM}$), where the sidebands remain within the cavity's FWHM, also generates an error signal but with reduced slope. This demands higher feedback loop gain and bandwidth to achieve comparable lock stability. Higher modulation frequencies are typically preferred in PDH locking, as they position the error signal at frequencies where $1/f$ noise from thermal, mechanical, and electronic sources is suppressed [154].

⁴⁴ In overcoupled cavities ($T_2 \ll T_1$) $G \lesssim 4$.

6.3.2 Simulating PDH signals

To validate the theoretical derivations of the PDH locking signals in Section 6.3.1, a simulation was conducted using the Python-based optical simulation software *Finesse3* [80]. The simulation employed a laser source which was modulated by an EOM at modulation frequency $\Omega = 10$ MHz. Subsequently, the laser beam was directed onto a 0.1 m impedance-matched cavity with a finesse of 627 and 6282 in a second iteration. The cavity length was tuned while the reflected light was detected using DC and AC photodetectors. It should be noted that the cavity response remains identical whether the length of the cavity is tuned or the frequency of the laser is scanned.

The signal detected by the DC photodetector is displayed in the upper panel of Fig. 6.16. The microscopic cavity length tuning is expressed in both MHz and degrees, using the conversion from $\text{FSR}_{\text{cavity}} = 1.5 \text{ GHz} = 180^\circ$ ⁴⁵. When the carrier is on-resonance, the sidebands are off-resonance and the reflected power drops according to the modulation index $m = 0.7$. The power in the carrier and the sidebands is determined by the Bessel functions $J_i(m)$ which yields for $P_{\text{carrier}} \propto |J_0(0.7)|^2 \approx 0.88$ ² and $P_{\text{sideband}} \propto |J_1(0.7)|^2 \approx 0.33$ ² [155]. The cavity length tuning by the modulation frequency $\Omega = 10$ MHz, and the corresponding decrease in power reflection at resonance, matches the analytical calculations.

The signal detected by the AC photodetector is demodulated at the modulation frequency $\Omega = 10$ MHz and is displayed in the lower panel of Fig. 6.16. This demodulated signal constitutes the error signal, which exhibits zero-crossings at each resonance. Although any zero-crossing can theoretically be used to lock the laser frequency to the cavity length, the zero-crossing at the carrier resonance possesses the largest linear slope and is therefore the optimal locking point.

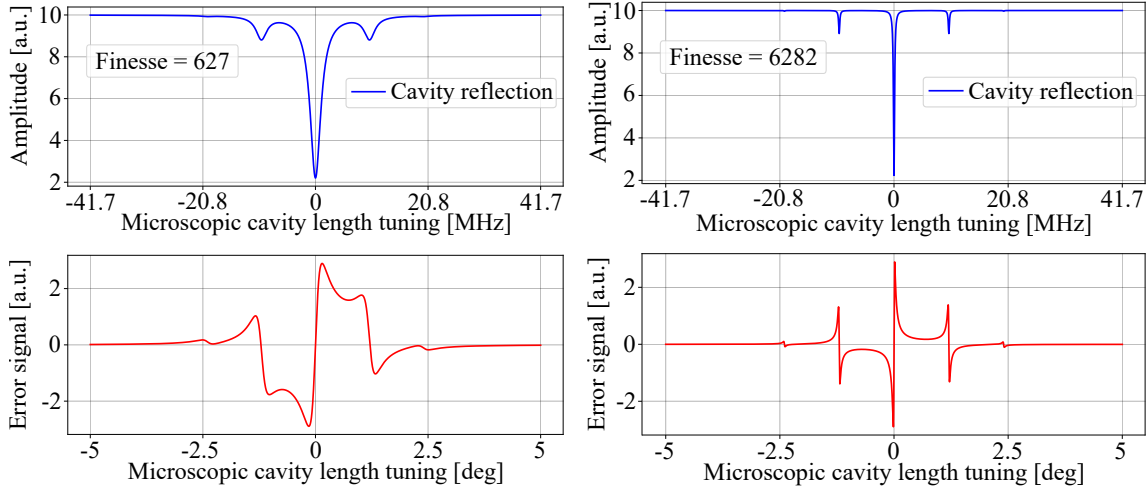


Figure 6.16: *Finesse3* simulation of the Pound-Drever-Hall (PDH) lock. The simulation comprises a laser source, a 10 cm cavity, and an EOM with a modulation frequency $\Omega = 10$ MHz. Results are shown for a low-finesse and high-finesse cavities (panels on the left and right, respectively).

⁴⁵ Since one round-trip within the cavity is twice the length, the total optical path length change is twice the tuning length.

6.3.3 Noise analysis

Theoretical noise budget of the PDH lock

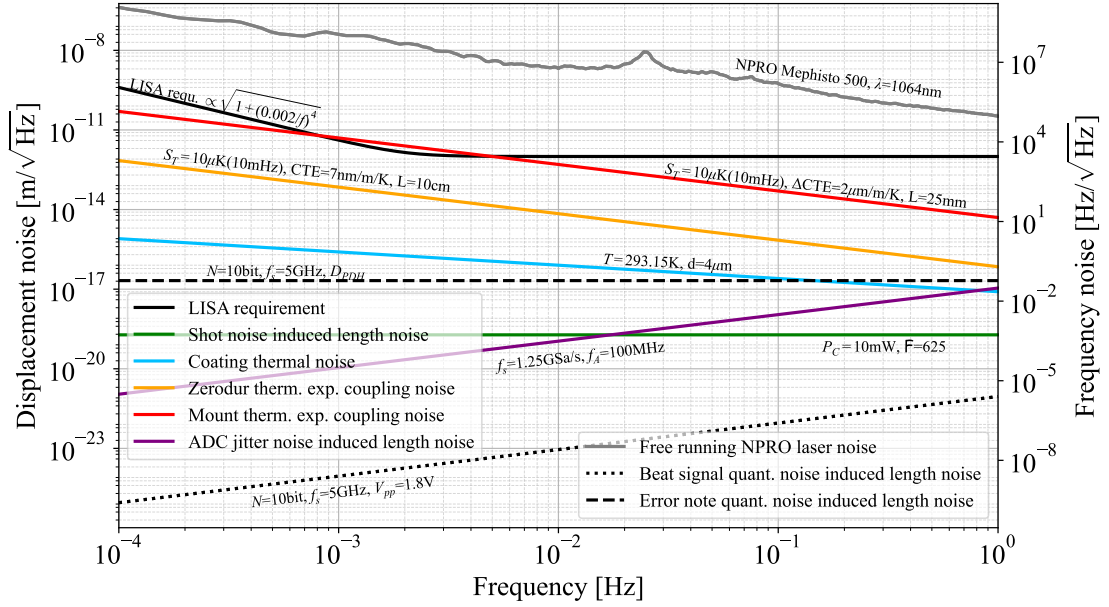


Figure 6.17: Theoretical noise budget of the Pound-Drever-Hall (PDH) lock, comprising the noise sources discussed in Section 6.3.3. The PDH locks the laser frequency (free running NPRO laser noise) to the length of the prototype cavity. The dominant noise source arises from thermal expansion of the mirror mount assembly, in particular the CTE mismatch between the ZeroDrift mirror mount and the Invar optomechanics.

This section investigates the dominant noise sources in the PDH locking scheme to evaluate the maximum achievable displacement sensitivity. Both the HS and PDH locking techniques were employed using the same prototype and reference cavity. Common noise sources addressed in the HS Section 6.2.3 are therefore not reiterated in this section. The theoretical noise budget of the PDH locking scheme is shown in Fig. 6.17, while individual noise sources are analyzed subsequently. The noise contributions closely match those of the HS scheme. While negligible contributions differ (e.g., shot noise), thermal expansion coupling noise of the TAPSI mirror mount assembly remains the dominant limitation.

In this analysis, we neglect noise sources that do not couple into the error signal to first order. These include fluctuations in laser power, photodiode response, and the relative phase of the two mixer inputs. The PDH lock is insensitive to all these factors, as the locking occurs on resonance where the reflected beat note vanishes. However, the PDH lock is first-order sensitive to sideband power imbalance at the modulation frequency Ω , generally referred to as residual amplitude modulation (RAM) noise.

Residual amplitude modulation noise

Although RAM noise is a well-documented source of noise in PDH locking [140–144], its theoretical prediction remains challenging. RAM, generated by imperfect sideband symmetry during EOM phase

modulation, shifts the PDH zero-crossing (error signal locking point) from true cavity resonance. An ideal cavity with perfect mode matching and impedance matching (contrast = 100 %) reflects no carrier light on resonance, yielding a RAM independent zero-crossing. In reality, non-ideal cavities couple RAM noise via residual carrier reflection. Since RAM levels fluctuate temporally, they directly degrade long-term laser frequency stability and thereby induce displacement noise.

Residual amplitude modulation arises from multiple sources. For example, input polarization misalignment with the EOM extraordinary axis causes output polarization rotation, which a subsequent PBS converts to amplitude noise [140]. An additional major noise source is the temperature-induced birefringence, where polarization axes shift at different rates, producing a phase retardation independent of the applied modulation voltage. Temperature stabilization via thermal shielding and active Peltier feedback mitigates this contribution. An additional strategy is RAM monitoring, enabling subtraction during post-processing.

The EOMs used in this work employ an annealed proton-exchanged (APE) waveguide technology that guides the transverse electrical (TE) mode [156]. Input polarization misalignment thus increases insertion loss, which inherently suppresses RAM via polarization filtering, a standard practice in RAM suppression [140].

Shot noise limitation

The fundamental limit of the error signal is set by photon shot noise. On resonance, the carrier P_C vanishes and power onto the PD comes exclusively from the two sidebands P_S . The shot noise amplitude spectral density of the error signal is given by

$$\tilde{S}_\epsilon = \sqrt{2 \frac{hc}{\lambda} (2P_S)} \quad [\text{W}/\sqrt{\text{Hz}}]. \quad (6.52)$$

Dividing the expression by the discriminator slope D_{PDH} (Equation 6.51) yields the corresponding frequency noise spectrum:

$$\tilde{S}_f = \frac{\sqrt{hc^3}}{-2\mathcal{G}\mathcal{F}L\sqrt{\lambda P_C}} \quad [\text{Hz}/\sqrt{\text{Hz}}], \quad (6.53)$$

which depends solely on the power in the carrier and not on the power in the sidebands. The equivalent displacement noise is obtained by Equation 6.2 as

$$\tilde{S}_L = \frac{L}{f} \tilde{S}_f = \frac{\sqrt{hc}}{-2\mathcal{G}\mathcal{F}} \frac{\sqrt{\lambda}}{\sqrt{P_C}} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.54)$$

Substituting the cavity parameters⁴⁶ listed in Table 6.1⁴⁷ and carrier power $P_C = 10 \text{ mW}$ into Equation 6.54, we obtain the shot noise-induced displacement noise:

$$\tilde{S}_L \approx 1.9 \times 10^{-18} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.55)$$

⁴⁶ $\mathcal{F} = 625$, $L = 0.1$, $\lambda = 1064 \text{ nm}$, $\mathcal{G} \approx 2$

⁴⁷

This white noise floor lies well below LISA's $1 \text{ pm}/\sqrt{\text{Hz}}$ requirement.

Quantization noise-induced displacement noise

Quantization noise arising from heterodyne beat note digitization is identical for HS and PDH. The noise coupling through the error signal is similar, but it differs in the discriminator slope D_{PDH} . Thus, for PDH locking, the error signal quantization noise-induced displacement noise is expressed as

$$\tilde{S}_{f,\text{quant}} = \frac{\tilde{U}_{\text{quant}}}{D_{\text{PDH}} \cdot R_{\text{PD}} \cdot R_f}, \quad (6.56)$$

with

$$D_{\text{PDH}} = \frac{-2 \mathcal{G} \sqrt{P_C P_S}}{\text{FWHM}} \approx 1.7 \times 10^{-8} \text{ W/Hz}, \quad (6.57)$$

where $P_C = 10 \text{ mW}$ and $P_S = 1.5 \text{ mW}$, $R_{\text{PD}} = 0.7 \text{ A/W}$, $R_f = 1 \text{ k}\Omega$ and $\mathcal{G} = 2$. Substituting these values into Equation 6.57, we obtain the frequency noise:

$$\tilde{S}_{f,\text{quant}} \approx 0.06 \text{ Hz}/\sqrt{\text{Hz}}, \quad (6.58)$$

and the corresponding displacement noise as

$$\tilde{S}_{L,\text{quant}} = \frac{L}{f_0} \cdot \tilde{S}_{f,\text{quant}} \approx 2.1 \times 10^{-17} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.59)$$

As with the HS analysis, these quantization noise estimates are conservative. The effective SNR can be further improved by amplifying the error signal to fully utilize the ADC dynamic range.

6.3.4 Experimental setup

The initial PDH locking scheme, based on fiber-optic components (Section 6.3.4.1), followed the initial HS scheme (Section 6.2.4). Performance limitations in the fiber-based setup then motivated a redesign toward a free-space implementation (Section 6.3.4.2).

6.3.4.1 Scheme based on fiber-optic components

The schematic illustration of the PDH locking scheme is shown in Fig. 6.18. The NPRO lasers were each coupled into pm-fiber splitters, where half of the power was used to produce the beat note, detected by photodetector PD_1 ⁴⁸ and subsequently measured by a phasemeter⁴⁹. The other half of each beam was phase-modulated by an EOM⁵⁰, producing two sidebands around the carrier. The modulated beams were then directed through fiber circulators into the vacuum chamber via pm-fiber feedthroughs. Inside the chamber, the beams were coupled from fiber to free-space and directed onto the cavities. When the laser-cavity lock was active only the carrier resonated while the off-resonant

⁴⁸ Coherent, *ET-3000A*, wavelength range: 800-1 700 nm, bandwidth: 30 kHz-1.5 GHz.

⁴⁹ Liquid Instruments, *Moku:Pro*, phasemeter instrument with 300 MHz bandwidth.

⁵⁰ Thorlabs, *LYN1010A*, fiber-coupled EOM with an operating frequency range from DC to 10 GHz.

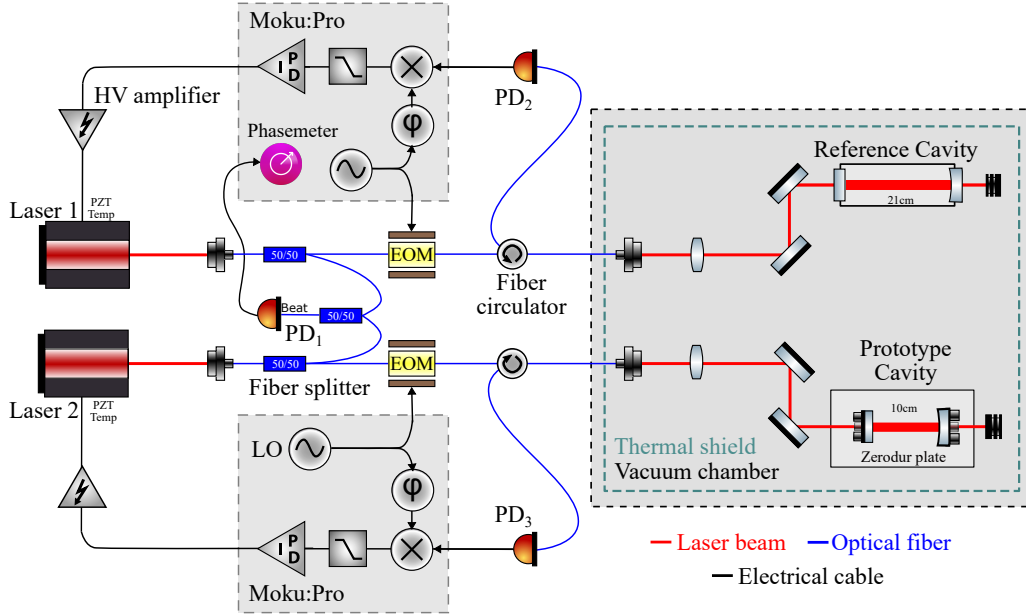


Figure 6.18: PDH locking scheme based on fiber-optic components. The lasers, each coupled into pm -fibers, are interfered via a fiber splitter, producing a beat note, detected by PD_1 and measured by the phasemeter instrument of a Moku:Pro. The second part of each laser beam is phase modulated by an EOM, producing a carrier and two sidebands, and directed onto the respective optical cavities, where only the carrier resonates while the off-resonant sidebands are reflected. The reflections from each cavity are detected by PD_2 and PD_3 , respectively. Subsequent demodulation (mixing + LPF) generates the error signal, which drives an feedback control loop to stabilize the laser frequency to the length of the respective cavity. The relative length fluctuation of the cavities is derived from the measured beat note signal PD_1 .

sidebands were reflected off the cavity. The reflected beams propagated along the same optical path before being routed by the circulators toward the photodetectors $PD_2(t)$ and $PD_3(t)$, respectively. The Moku:Pro lockbox-instrument demodulated the signals at the EOM's modulation frequency⁵¹ set by the LO⁵². The instrument also included the PID controller and digital to analog converter (DAC) to generate the actuation signals that stabilize each laser frequency to the length of its respective cavity. Fast frequency scanning was implemented via laser-integrated PZT actuators, while slow laser-temperature tuning ensured that the PZT operated within its dynamic range.

The displacement noise measurements are shown in Fig. 6.19 and also document the progress achieved through several iterations, including polarization alignment and improved thermal isolation of the fiber-optic and electronic components located outside the vacuum chamber. Nevertheless, for frequencies below 0.1 Hz, the displacement noise remained several orders of magnitude above the LISA requirement. This excess noise manifested as a characteristic *scatter shoulder*, attributed to spurious light beams inherent to the fiber-based configuration [157] (see Section 3.4 Spurious light). The setup comprised multiple fiber-optic components such as fiber splitters, fiber coupled EOMs

⁵¹ Reference cavity: $\Omega \approx 1$ MHz

Prototype cavity: $\Omega \approx 10$ MHz

⁵² The local oscillator (LO) signal originated from a signal generator (Rohde & Schwarz SMC100A) and was synchronized to the 10 MHz clock of the Moku:Pro.

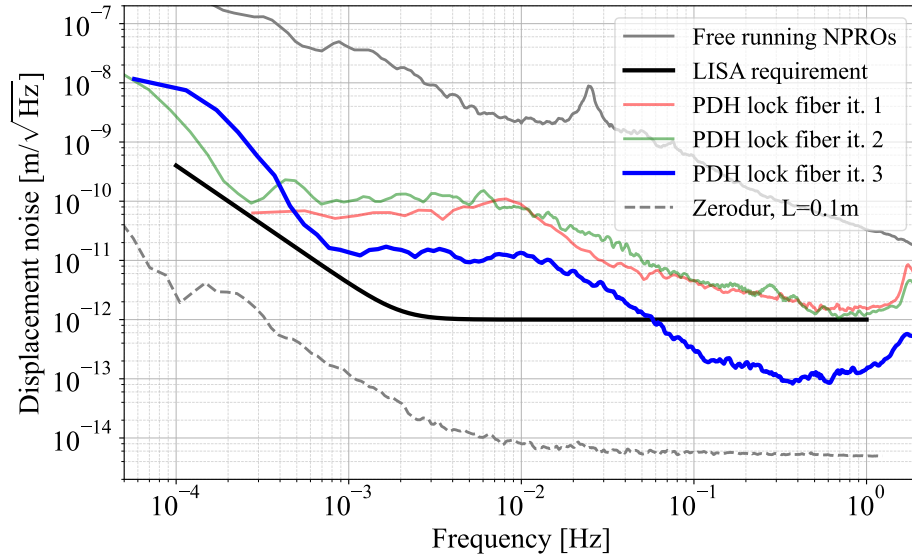


Figure 6.19: Displacement noise of the prototype cavity using PDH scheme based on fiber components along with the $\text{pm}/\sqrt{\text{Hz}}$ LISA requirements, free-running NPRO laser noise and displacement noise induced by the thermal expansion of the 0.1 m Zerodur baseplate. Several iterations (it.) improved the sensitivity, including better polarization alignment and enhanced thermal isolation of the fiber and electronic components located outside the vacuum chamber. However, the characteristic scatter shoulder, attributed to spurious light signal inherent to the fiber-based configuration, persisted.

and fiber circulators, all interconnected via mating sleeves. The employed fibers featured angled physical connectors (APCs), which minimize reflections along the nominal path and consequently mitigate ghost beams [158]. Nevertheless, each fiber-to-fiber connection constitutes a potential etalon, capable of generating spurious light signal (SLS). Another potential source of noise stems from RAM (see Section 6.3.3), primarily arising from the EOMs. These were located outside the vacuum chamber in a simple plastic enclosure providing only modest thermal insulation. Furthermore, parasitic etalon interference between the leakage field of the fiber circulator and the cavity-reflected field can induce additional RAM [144]. The excess measured noise likely originated from one or more of the aforementioned noise sources. Due to time constraints, detailed investigations and targeted noise mitigation strategies were not pursued. Instead, the experimental configuration was substantially redesigned.

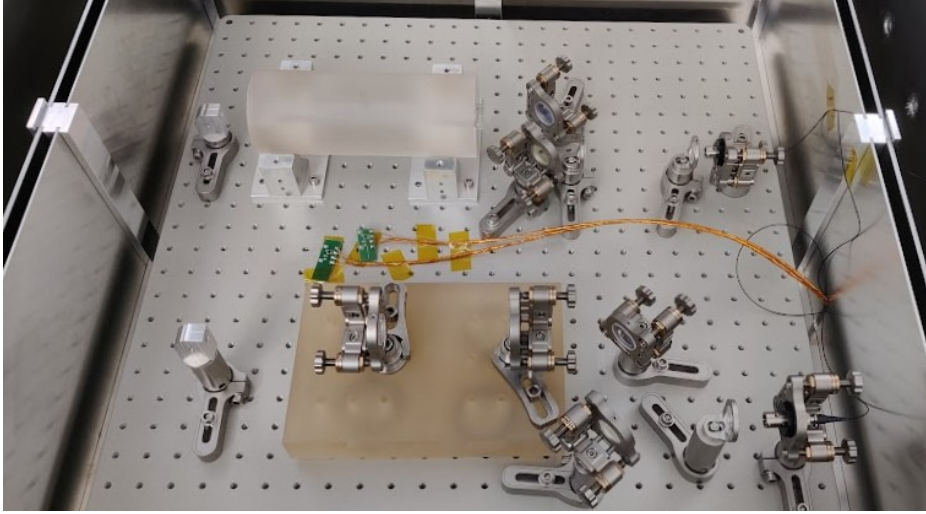


Figure 6.20: Photograph of the PDH locking setup inside the vacuum chamber, with fiber-optic components located outside the chamber. The ULE reference cavity is positioned at the rear of the chamber, while the prototype cavity, implemented using the toolset for adjustable picometer-stable interferometers, is located at the front. Temperature sensors (green PCBs) were placed between the cavities.

6.3.4.2 Scheme based on free-space optics

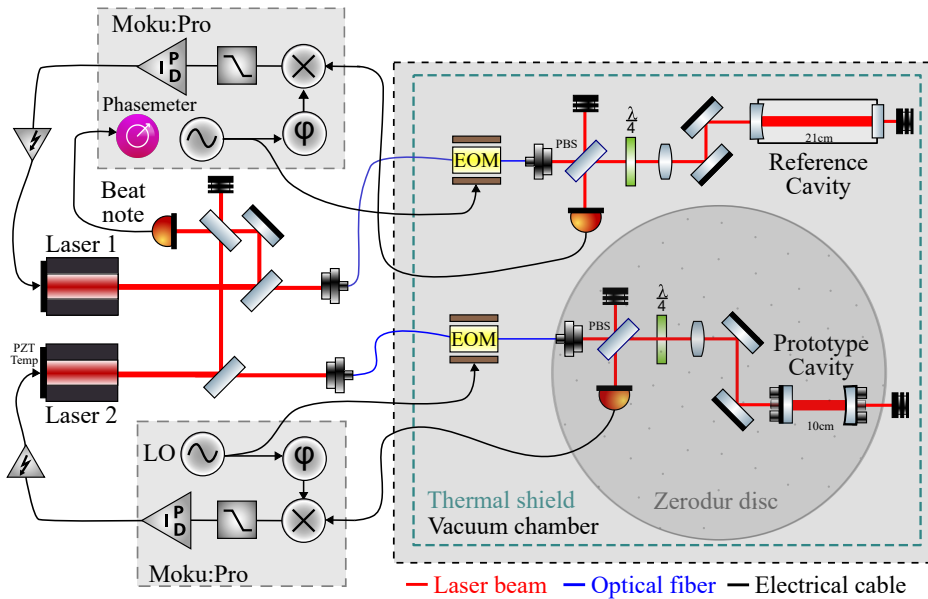


Figure 6.21: PDH locking scheme based on free-space optics. The laser beam is phase modulated by an EOM with a modulation frequency Ω from a signal generator. The laser beam now consists of three frequencies: the carrier frequency and two sidebands, each frequency-shifted by $\pm\Omega$. The beam is then sent to an optical cavity, where only the carrier frequency is resonant, while the sidebands are off resonance and therefore reflected. The reflection from the cavity is measured using a PD and contains the cavity interaction phase. Subsequent demodulation against the modulation frequency (LPF) isolates this phase.

The noise limitations of the initial PDH scheme based on fiber-optic components (Section 6.3.4.1) motivated a substantial redesign of the experimental setup. The primary modification involved replacing fiber-optic components with free-space optics wherever feasible. This transition effectively suppresses spurious light beams and RAM noise typically generated by optical fibers and their interconnections. The complete PDH locking scheme, comprising a PBS, $\lambda/4$ -waveplate and photodetector (PD), was set up inside the vacuum chamber for both cavities. In contrast to the fiber-optic setup, the free-space setup utilized the full toolset for adjustable picometer-stable interferometers to construct both the prototype cavity and the associated optics for PDH locking. Photodetectors⁵³ and the EOMs were placed inside the vacuum chamber, where the latter aimed to suppress temperature-induced RAM noise.

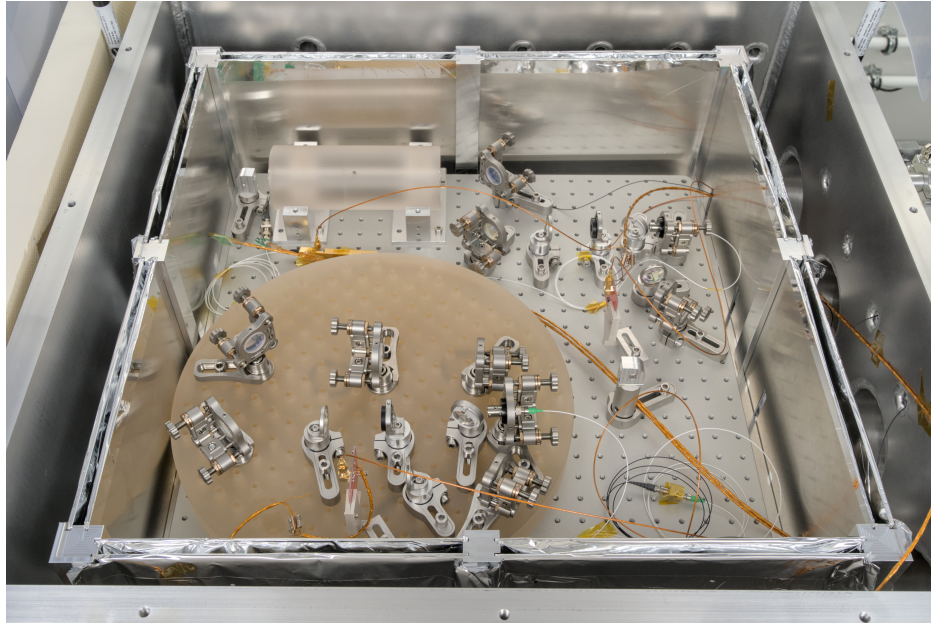


Figure 6.22: Photograph of the PDH locking setup inside the vacuum chamber after replacing the fiber-optic components with free-space optics wherever feasible. The ULE reference cavity is positioned at the rear of the chamber, while the prototype cavity is located at the front. The prototype cavity and the associated optics for PDH locking are set up using the toolset for adjustable picometer-stable interferometers. Temperature sensors (green PCBs) were placed in close proximity to the prototype cavity (position not shown here).

Measurement results

The displacement noise of the prototype cavity using the free-space PDH locking scheme is shown in Fig. 6.23, alongside the previous fiber-based measurement from Fig. 6.19. The transition from fiber-optic components to free-space optics substantially suppresses spurious light and eliminates the characteristic scatter shoulder. The free-space configuration meets the LISA displacement noise requirements across most frequencies, exceeding them only slightly at the linear-to-slope transition at 0.8-3 mHz. This demonstrates the picometer-level length stability of the TAPSI mounting concept.

⁵³ PCB schematics and layouts are shown in the Appendix F in Fig. F.4.

Additional measurements confirmed the repeatability of these results, all conducted during periods of low laboratory activity (i.e., outside working hours).

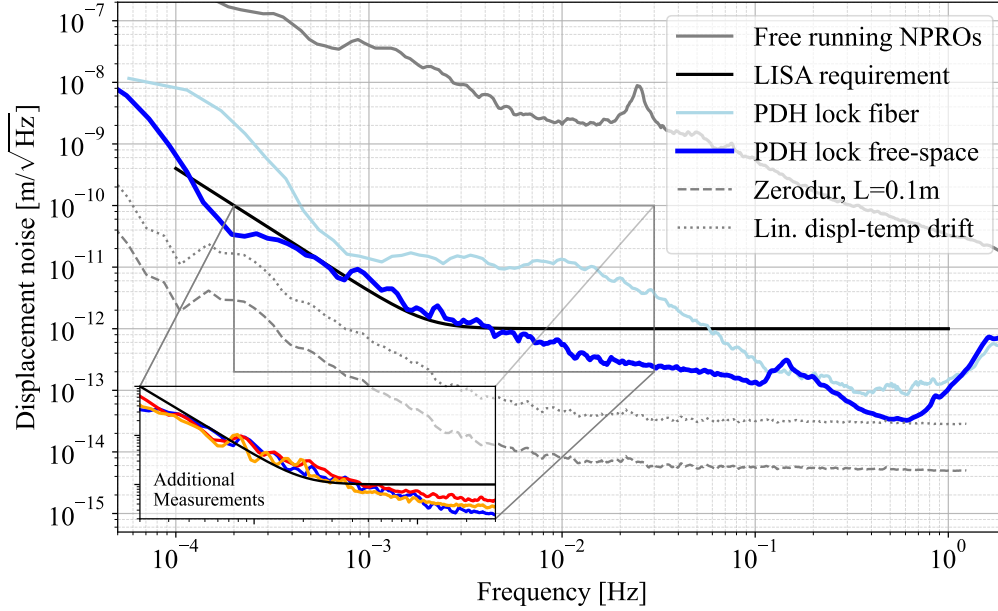


Figure 6.23: Displacement noise of the prototype cavity using the free-space PDH locking scheme together with the previous configuration based on fiber-optic components. The displacement noise meets the LISA requirements, confirmed by additional measurements. The linearized displacement-temperature drift, calculated from the linear rise in frequency caused by the constant temperature rise, demonstrates that temperature noise is not the limitation but rather the readout, particularly scattered light.

6.3.4.3 Experimental limitations

Although the PDH locking achieved the target noise performance, a linear frequency drift was observed and attributed to the gradual temperature increase caused by the photodetectors located inside the vacuum chamber. The photodiode converts laser power to a photocurrent, which is subsequently converted to voltage by a transimpedance amplifier (TIA). For AC signals ($f \gg \text{kHz}$), the TIA is integrated in close proximity to the photodiode, i.e., on the same PCB. During operation, the TIA (particularly its operational amplifier (OpAmp)) generates substantial heat [159], resulting in a continuous temperature rise within the vacuum chamber. This steady thermal gradient induces a corresponding thermal expansion of the prototype cavity, manifesting as a linear drift in the beat note frequency as it can be observed in Fig. 6.24. This linear displacement-temperature relationship is given by

$$\frac{dL}{dT} = \frac{df}{dT} \cdot \frac{L}{f_0} \quad [\text{m/K}], \quad (6.60)$$

where df is the linear rise in beat note frequency, dT the linear rise in the temperature, f_0 the laser frequency, and L the length of the prototype cavity. Determining the values from Fig. 6.24 yields

$$\frac{dL}{dT} \approx 4 \times 10^{-8} \quad [\text{m/K}]. \quad (6.61)$$

Multiplying this linear displacement-temperature coupling by the measured temperature noise within the chamber quantifies its contribution to the total displacement noise. The projected noise from this coupling ('Lin. displ-temp drift') is included in Fig. 6.23 and shows that thermal drift does not limit measurement performance. Furthermore, no correlation appears between temperature fluctuations

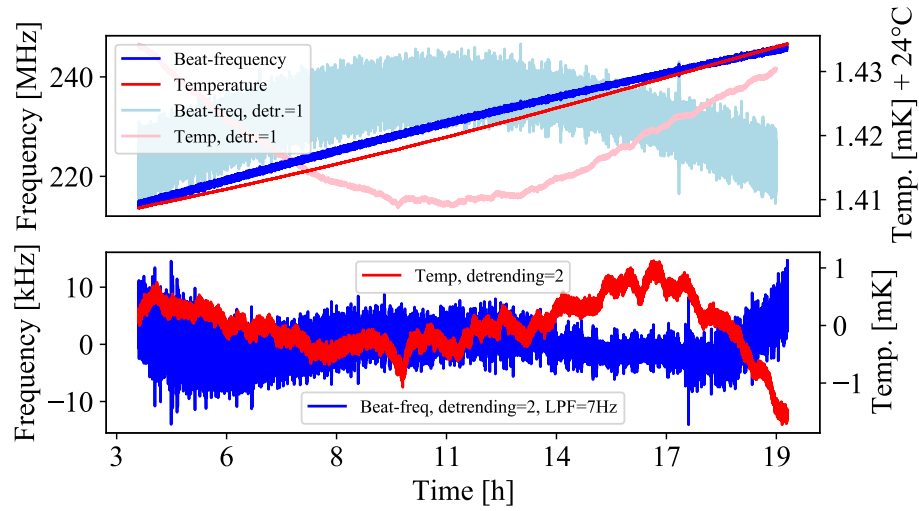


Figure 6.24: Time series of the beat note frequency alongside the temperature inside the vacuum chamber at the prototype cavity position. The upper panel shows the beat note and temperature time series of parallel measurements, together with their respective time series after the removal of the linear drift. In the lower panel the quadratic drift was removed, while an additional low-pass filter with a cut-off frequency of 7 Hz was explicitly applied to highlight the presence of non-stationary noise such as parasitic beams.

and cavity length after detrending the linear rise (Fig. 6.24, lower panel). The coherence analysis in Fig. 6.25 confirms no significant displacement-temperature coupling across the LISA frequency band.

The lower panel of Fig. 6.24 shows the beat note frequency time series, after the removal of the quadratic drift and the application of a low-pass filter with a cut-off frequency of $f_{3\text{dB}} = 7 \text{ Hz}$ ⁵⁴. The signal filtering reveals the level of non-stationarity in the measurements, a phenomenon which may be attributed to factors such as spurious light signals and could explain the observed limitation in the measurement. Spurious light couples in a non-linear fashion, but generally the maximum phase error can be estimated by the ratio of the square root of the parasitic beam powers (P_{SLS}) and the signal beam power (P_{sig}) as discussed in Section 3.4. The SLS-induced frequency error is given by (Section 3.4.3):

$$\delta f_{\text{err}}(t) = -\frac{P_{\text{sig}}}{D_{\text{PDH}}} \sqrt{\frac{P_{\text{SLS}}}{P_{\text{sig}}}} \sin(\phi_{\text{SLS}}(t)) \quad [\text{Hz}], \quad (6.62)$$

⁵⁴ The cut-off frequency was chosen at 7 Hz to filter out the resonance peaks from the mirror mounts which would mask the lower-frequency noise

where D_{PDH} denotes the discriminator slope of the PDH lock, and $\sin(\phi_{\text{SLS}}(t))$ is the phase-coupling factor of the spurious light signal. Based on this expression we can estimate the noise contribution. The prototype cavity has a finesse of $\mathcal{F} = 625$ and a cavity length $L = 0.1$ m. $P_{\text{sig}}/D_{\text{PDH}} \approx 0.6 \times 10^{-6}$ Hz. We assume a signal power of $P_{\text{sig}} = 10$ mW, a SLS power of $P_{\text{SLS}} = 1$ μ W and the phase-coupling factor as a constant maximum $\sin(\phi_{\text{SLS}}(t)) = 1$. This results in a SLS-induced frequency offset in the error signal is $\delta f_{\text{err}} \approx 6$ kHz. Assuming $\sin(\phi_{\text{SLS}}(t)) = S_0$, the corresponding displacement noise is

$$\tilde{S}_L \approx 2.1 \times 10^{-12} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (6.63)$$

The expression and the estimated displacement noise level depend on simplified assumptions and should be treated as approximate rather than absolute. Nevertheless, the expression demonstrates the impact of SLSs and also highlights how their contribution can be mitigated by an increase in the discriminator slope D_{PDH} , which can be achieved by increasing the finesse of the cavity.

Subsequent experiments revealed that the local oscillator (signal generator) driving the EOMs at modulation frequency exhibited a phase drift relative to the Moku:Pro's internal demodulation LO ($\approx \pi$ per day). This demodulation phase drift distorted the PDH error signal over time (see Fig. 6.7), eventually causing signal asymmetry that led to a loss of lock and prevented measurements over long time periods. For a perfect impedance and mode matched cavity (contrast = 100 %), demodulation phase shifts only distort the error signal shape while preserving the zero-crossing at true cavity resonance. However, in practice, the contrast was approximately 90 % and allowed error signal offset coupling. Feedback loop compensation of these offsets produced apparent displacement noise. The LO phase drift's contribution to excess noise remains unquantified, as this coupling was unknown at the time of the measurements in this chapter.

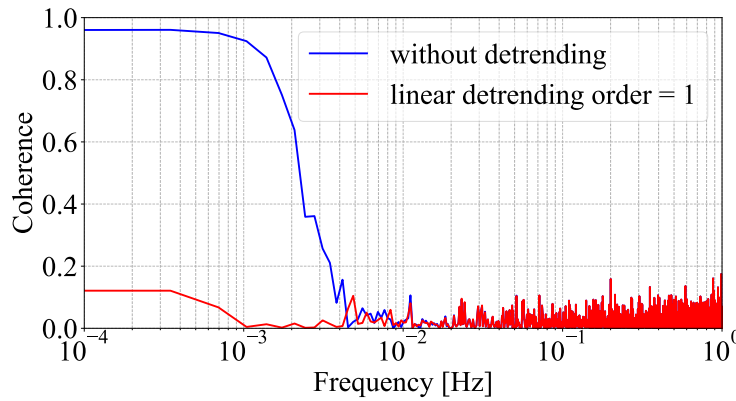


Figure 6.25: Coherence between beat note frequency and temperature. After detrending (removal of the linear frequency-temperature drift), values below 0.2 confirm no significant displacement-temperature coupling across the LISA frequency band.

6.4 Comparison of the HS and the PDH locking schemes

Although the Heterodyne laser frequency Stabilization (HS) and the Pound-Drever-Hall (PDH) locking techniques employ distinct optical configurations, both encode the cavity response as a phase shift on the reflected light from the cavity, which is measured by detecting the *beat note*. In HS, the beat note arises from interference between two independent lasers, while in PDH locking, phase modulation of a single laser generates the carrier-sideband beat notes after interaction with the cavity. Demodulation recovers the cavity phase information (detuning from resonance) to produce an error signal that drives a feedback loop to stabilize the laser frequency at the cavity resonance. In theory, both techniques lock the laser frequency to the cavity length, enabling length-stability measurements of the prototype cavity and thus of the TAPSI mounting concept. However, HS proves less straightforward than it may initially appear, due to several technical challenges.

In HS, the beat note itself serves as the demodulation reference signal. Asynchronous frequency changes between the reference beat note and the cavity beat note, due to path-length mismatches, lead to a demodulation-phase error. For non-ideal cavity contrast, this demodulation-phase error couples into the error signal as an offset shift, detuning the laser from the true cavity resonance. A time-varying beat-note frequency or path-length mismatch produces nonstationary apparent displacement noise. To minimize path-length changes, for example arising from temperature fluctuations, requires thermal insulation of the entire signal chain, adding experimental complexity. Beat-note frequency changes, on the other hand, require corresponding demodulation phase tuning. This can be accomplished by a heterodyne controller in a digital demodulation scheme, but also adds experimental complexity.

In contrast, PDH locking uses a constant demodulation reference, the LO, which also drives the EOM modulation frequency. Another advantage is the lower shot noise of PDH. On resonance, where the laser is locked, the reflected light in the PDH locking scheme comes from the sidebands, while in HS the off-resonant laser light is reflected, resulting in substantially larger power on the photodiode and thus higher shot noise.

The primary advantage of HS is its simple interferometer layout, which avoids EOMs and their RAM noise [160]. However, the inherent demodulation-phase error in the presented experimental schemes makes PDH locking the superior technique for laser-cavity stabilization and thus for characterizing the length stability of the toolset.

6.5 Toolset length stability summary

The length stability of the toolset's mounting concept was characterized by locking the laser frequency to the length of a prototype cavity, which was set up using the toolset for adjustable picometer-stable interferometers (TAPSI). A second laser was locked to an ultra-stable reference cavity. By interfering the two lasers and measuring their beat-note frequency, the information about the relative displacement noise was obtained. Both cavities were located within the multilayer, thermally insulated optical test facility. Inside the thermal shield at the interferometer location, a temperature noise of $\tilde{T} = 10 \mu\text{K}/\sqrt{\text{Hz}}$ at 10 mHz was measured, following a $1/f$ slope (see Section 5).

The laser-cavity lock was implemented first using the Heterodyne laser frequency Stabilization and subsequently the Pound-Drever-Hall locking scheme. The displacement noises from each locking scheme and their corresponding experiments are presented in Fig. 6.26. The transition from fiber-optic components to free-space optics improved the performance of both locking schemes. However, HS

remained limited by the demodulation-phase error arising from a path-length mismatch between the reference and cavity beat notes. The implementation of a digital demodulation scheme enabled frequency-dependent demodulation phase adjustment but could not fully suppress the phase error due to the limited resolution of the heterodyne controller and did not improve the performance. In contrast, free-space PDH locking achieved LISA-compatible performance, demonstrating picometer-level stability of the toolset. Temperature coupling was not identified as the dominant limitation. Instead, other noise sources, such as spurious light signals, likely represent the primary performance limit. Employing a higher-finesse prototype cavity is expected to mitigate these noise contributions.

Nevertheless, the results demonstrate that the toolset, when operated in a thermally well-isolated vacuum environment, meets LISA's $1 \text{ pm}/\sqrt{\text{Hz}}$ -level stability requirements. Current performance limitations arise from readout noise rather than the toolset's intrinsic length stability. These findings establish the toolset as a viable approach for realizing adjustable, picometer-stable interferometers with flexible optical-component placement at sub-hertz frequencies.

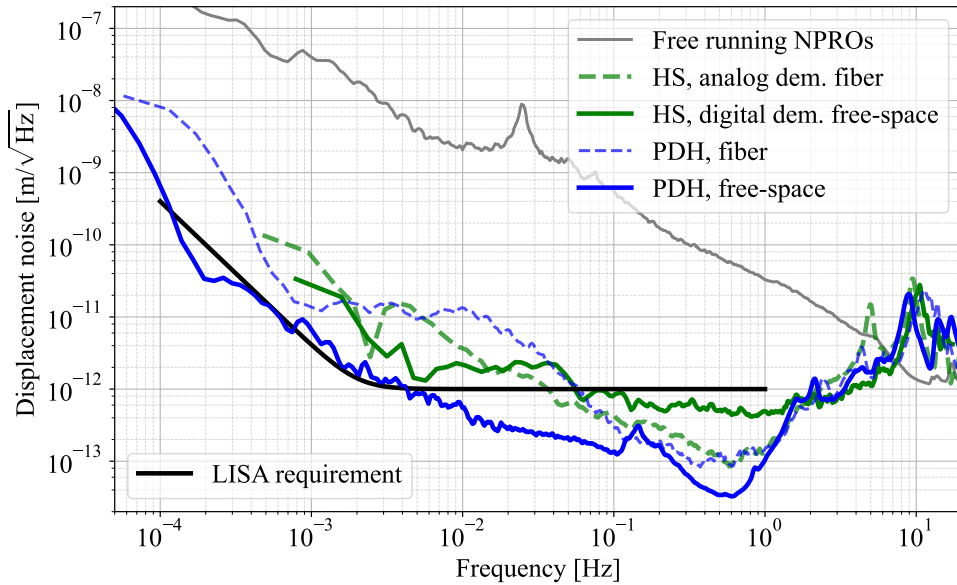


Figure 6.26: Displacement noise measurements of the prototype cavity using HS and PDH locking schemes alongside LISA requirement and free-running laser noise of the NPRO lasers as a reference. Both locking techniques were set up with two distinct configurations, including the transition from fiber-optic components to free-space optics. PDH locking based on free-space optics achieved performance compatible with the LISA requirement and successfully demonstrated picometer-level stability of the toolset.

Concealed Noise project

High-precision heterodyne interferometers are limited by noise contributions arising from both the optical setup and the associated readout electronics. While breadboard experiments for the LISA mission have already demonstrated sufficient performance [58, 61, 63, 120, 139, 161], unexplained noise contributions from the phasemeter and interferometer below the picometer level remain [162]. Understanding these noise contributions is essential for further risk mitigation in LISA and for guiding the design of future missions beyond LISA that demand even higher sensitivity [37, 163, 164]. The Concealed Noise project aims to characterize the noise floor of picometer-precision heterodyne interferometers operating at MHz frequencies through combined theoretical modeling and experimental validation. The Concealed Noise experiment employs an interferometer with a simple optical layout that produces four optical signals. Subtracting the measured phase signals isolates the underlying interferometer and readout noise floor. By using high optical power and multiple phasemeter channels per photodiode segment, we can probe this noise floor at sub-picometer precision. We then assess whether established noise models, including shot noise, quantization noise, relative intensity noise, and photodetector noise, match the measured values, or whether additional modeling or mitigation techniques are required. The developed toolset for adjustable picometer-stable interferometers allows us to quickly set up the experiment and provides the required low-frequency length stability. A multichannel electrical ground support equipment (EGSE) phasemeter [165], under development in our research group, provides phase readout for this experiment. It features up to 64 readout channels within a single device, allowing multiple readouts per photodetector signal to suppress the impact of phasemeter noise. The initial step measures the experiment's noise floor to identify dominant contributions and establish system performance limits. Individual noise sources are then isolated and investigated to validate theoretical predictions against measurements. Here, only the initial setup and characterization of the experiment are described. The full exploitation of the experiment will be pursued in future research.

A key implementation will be the multichannel phasemeter readout alongside the use of high optical power ($P > 10$ mW). In addition, dedicated photodetectors are developed, with their noise performance investigated alongside other electronic contributions such as RF reflections from impedance mismatches in electrical cables. Beyond electronics, optical noise sources will be studied, including relative intensity noise coupling through intentional amplitude modulations near the beat frequency, building on [98]. Complementing these investigations, a simulation of the interferometer readout noise floor will be performed using simulation tools such as IfoCAD [113]. Together, the

theoretical modeling and experimental validation will establish a comprehensive noise budget for MHz heterodyne interferometers to consistently realize sub-picometer sensitivity at low frequencies ($f < 1$ Hz).

7.1 Optical layout and signals

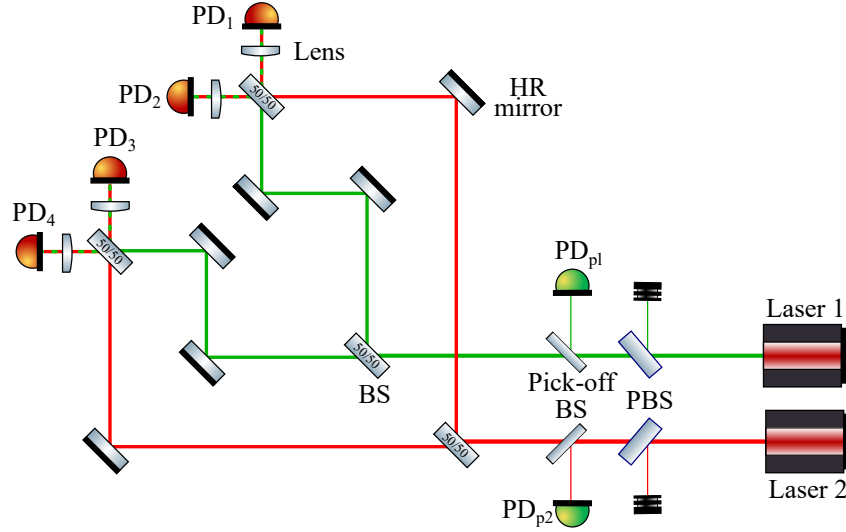


Figure 7.1: Schematic of the Concealed Noise experiment. The simple interferometer optical layout employs two distinct laser frequencies to produce four beat note signals. Subtraction of these signals cancels common-mode noise and reveals the underlying interferometer and readout noise floor. Preceding PBSs provide polarization filtering, while pick-off beam splitters (BS) enable laser power monitoring (PD_{p1} , PD_{p2}).

The optical layout of the Concealed Noise experiment is shown in Fig. 7.1. Laser beams with distinct frequencies pass through polarization filtering via a PBS and a pick-off BS for power monitoring ($PD_{p1/2}$). The beams are then split at non-polarizing BSs and recombined at a second pair of BSs, producing four beat note signals, expressed as

$$P_{PD_x} = \frac{c\epsilon_0 n}{2} \left(\frac{E_1^2}{2} + \frac{E_2^2}{2} \pm 2|\langle E_1, E_2 \rangle| \cos(\Delta\omega t + \Delta\phi) \right), \quad (7.1)$$

where $E_{1,2}$ denotes the electric field amplitude of the respective beams, $\Delta\omega = |\omega_1 - \omega_2|$ the beat note frequency, $\Delta\phi = |\phi_1 - \phi_2|$ the static differential phase, and $\pm$ corresponds to a π measurement. The generated photocurrent takes the conventional form:

$$i_{PD} = R_{PD} \left(\frac{P_1 + P_2}{2} \pm \sqrt{\eta P_1 P_2} \cos(\Delta\omega t + \Delta\phi) \right), \quad (7.2)$$

where R_{PD} denotes the responsivity of the photodiode, and $\eta \leq 1$ the heterodyne efficiency accounting for imperfect polarization and mode overlap. The photocurrent is then converted into a voltage by the

TIA, given in the abbreviated form as

$$u_{PD} = u_{DC} + u_{AC}, \quad (7.3)$$

where the beat note signal u_{AC} is expressed as

$$u_{AC} = \underbrace{A_{\text{gain}} \cdot R_f \cdot R_{PD}}_{\text{photodetector}} \cdot \underbrace{\sqrt{\eta P_1 P_2} \cos(\Delta\omega t + \Delta\phi)}_{\text{interferometer}}. \quad (7.4)$$

Here, A_{gain} denotes the amplifier gain and R_f the feedback resistor of the TIA. The signal is subsequently digitized and measured by the phasemeter.

7.2 Experimental setup

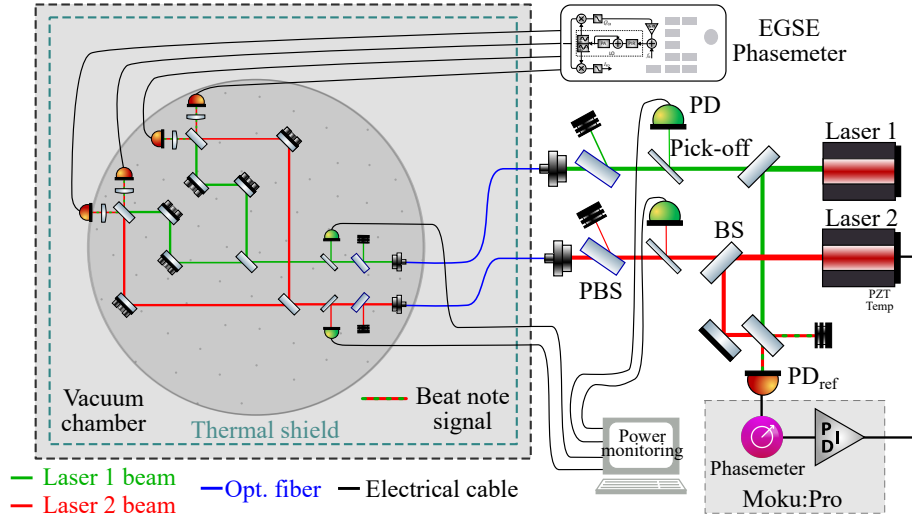


Figure 7.2: Schematic of the Concealed Noise experiment configuration. The laser beams enter the vacuum chamber, where they are collimated and polarization-filtered via a PBS before being split and subsequently interfered to generate four theoretically identical beat note signals. The beat note signals are detected by PDs (AC) and subsequently measured by the electrical ground support equipment (EGSE) Phasemeter. The reference beat note (PD_{ref}), generated outside the vacuum chamber, is used to lock the laser frequency to a fixed beat note. Subtraction of the beat note signals cancels common-mode noise, enabling characterization of the heterodyne interferometer noise floor. Additional DC signals are measured for laser power monitoring and data post-processing.

The Concealed Noise experiment was set up using the toolset for adjustable picometer-stable interferometers and placed inside the vacuum chamber (optical test facility) as shown in Fig. 7.3, while the full experimental schematic is displayed in Fig. 7.2. The laser beams were attenuated to 50 mW and polarization-filtered before entering the vacuum chamber through pm-fiber feedthroughs. Inside, the beams pass through polarization filtering via a PBS and power monitoring via a pick-off BS. Power measurements outside and inside the chamber enable precise monitoring of fiber noise contributions such as RIN. The two laser beams are then split and recombined to produce the four beat

note signals. The interferometer arm path lengths¹ were matched to $L \approx 33$ cm. The initial setup and rough alignment of the interferometer were completed in under two hours using the toolset (TAPSI). The adjustable mirror mounts enabled beam walking to maximize contrast, while fixed mounts (Invar²) were employed for the beam splitters. The beat notes were detected by photodetectors that were preceded by focusing lenses ($f = 25$ mm). PCB schematics of the photodetectors are provided in Appendix F.6. The beat note signals were measured using the EGSE phasemeter [165]. The reference beat note (PD_{ref}), generated outside the vacuum chamber, locks the frequency of one laser to the second, producing a stable beat note frequency. This was accomplished using the phasemeter³ and the PID controller instrument of a Moku:Pro. In future iterations, the EGSE phasemeter will perform this frequency stabilization task. Additional DC signals were acquired for laser power monitoring and data post-processing.

Subtracting pairs of extracted phase signals cancels common-mode interferometer noise and, depending on the combination, isolates the readout noise floor. For instance, subtracting PD_1 and PD_2 (π measurement) cancels interferometer noise, including differential arm length signals, and $1f$ RIN, leaving primarily shot noise and residual photodetector/phasemeter contributions. In contrast, $PD_1 - PD_3$ (opposite arms) primarily cancels common-mode laser frequency and amplitude noise for matched path lengths, remaining sensitive to differential arm-lengths noise alongside readout noise. A detailed overview of the present noise sources in the heterodyne interferometer is given in the following section.

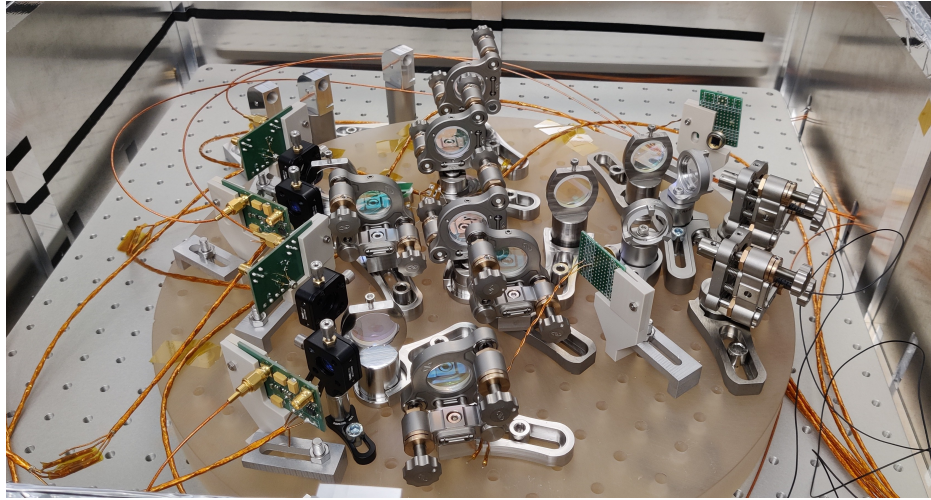


Figure 7.3: Photograph of the Concealed Noise experiment inside the vacuum chamber, set up using the toolset for adjustable picometer-stable interferometers. The interferometer produces four beat note signals, detected by photodetectors (green PCBs) and measured by the EGSE phasemeter.

¹ The term *interferometer arm* refers to the optical path from the first to the second BS.

² The Invar material used for the fixed BS mirror mounts is identical to that employed in other optomechanical components of TAPSI (see Appendix E.1).

³ The phasemeter's frequency offset output signal was used as an error signal for the feedback loop.

7.3 Theoretical expected noise sources

The Concealed Noise project comprises theoretical modeling and experimental validation to characterize the noise floor of MHz heterodyne interferometers achieving sub-picometer precision. This subsection provides an overview of the primary noise sources expected in the Concealed Noise experiment, following the structure presented in [161]. Given the rigorous construction and isolation of the interferometer, seismic and acoustic noise contributions are neglected in this analysis. Detailed discussions of individual sources are given in Section 3, while a comprehensive analysis and noise budget is part of future research.

The path length sensitivity of the interferometer, expressed as an amplitude spectral density, is given by

$$\tilde{S}_x = \frac{\lambda}{2\pi} \frac{\text{noise}}{\text{signal}} \quad [\text{m}/\sqrt{\text{Hz}}], \quad (7.5)$$

where $\lambda = 1\,064\text{ nm}$ denotes the laser wavelength, and both the noise and signal are expressed in amplitude spectral density. For our heterodyne interferometer, we consider the combined RMS photodiode signals from each output port, expressed as

$$\text{signal} = R_{\text{PD}} \cdot \sqrt{\eta_h P_{\text{sig}} P_{\text{LO}}} \sqrt{2} \quad [\text{A}/\sqrt{\text{Hz}}], \quad (7.6)$$

where R_{PD} denotes the photodiode responsivity, and η_h the heterodyne efficiency. The dominant noise sources, referred to the photodiode input, comprise shot noise (SN), relative intensity noise (RIN), and electrical noise (EL). The electrical contribution includes readout noise, including quantization noise, jitter noise, and ADC thermal/flicker noise. These noise terms (in units $\text{A}/\sqrt{\text{Hz}}$) are expressed as

$$\text{noise}_{\text{SN}} = \sqrt{2e R_{\text{PD}} (P_{\text{sig}} + P_{\text{LO}})}, \quad (7.7)$$

$$\text{noise}_{\text{RIN}} = \gamma_{\text{RIN}} R_{\text{PD}} (P_{\text{sig}} + P_{\text{LO}}), \quad (7.8)$$

$$\text{noise}_{\text{EL}} = \sqrt{2} n_{\text{el}}, \quad (7.9)$$

where e denotes the elementary charge, γ_{RIN} the relative intensity noise coupling of signals and LO beams, and n_{el} the electrical noise per photodetector, including the phasemeter contributions. Substituting these noise expressions and the signal from Equation 7.6 into Equation 7.5 yields the corresponding path length noise (in units $\text{m}/\sqrt{\text{Hz}}$), expressed as

$$\tilde{S}_{\text{SN}} = \frac{\lambda}{2\pi} \frac{\sqrt{2e R_{\text{PD}} (P_{\text{sig}} + P_{\text{LO}})}}{R_{\text{PD}} \cdot \sqrt{\eta_h P_{\text{sig}} P_{\text{LO}}} \sqrt{2}}, \quad (7.10)$$

$$\tilde{S}_{\text{RIN}} = \frac{\lambda}{2\pi} \frac{\gamma_{\text{RIN}} R_{\text{PD}} (P_{\text{sig}} + P_{\text{LO}})}{R_{\text{PD}} \cdot \sqrt{\eta_h P_{\text{sig}} P_{\text{LO}}} \sqrt{2}}, \quad (7.11)$$

$$\tilde{S}_{\text{EL}} = \frac{\lambda}{2\pi} \frac{\sqrt{2} n_{\text{el}}}{R_{\text{PD}} \cdot \sqrt{\eta_h P_{\text{sig}} P_{\text{LO}}} \sqrt{2}}. \quad (7.12)$$

The total interferometric readout path length noise, $\tilde{S}_{\text{readout}}$, is derived from the quadrature sum of the

individual uncorrelated noise sources and is expressed as

$$\tilde{S}_{\text{readout}} = \sqrt{\tilde{S}_{\text{SN}}^2 + \tilde{S}_{\text{RIN}}^2 + \tilde{S}_{\text{EL}}^2} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (7.13)$$

Laser frequency noise (LFN) couples into the differential measurement (PD₁,PD₂ vs. PD₃,PD₄) as a consequence of the arm length mismatch ΔL , and is given by

$$\tilde{S}_{\text{LFN}} = \left| \frac{\lambda}{c} \cdot \Delta L \cdot \tilde{S}_f \right| \quad [\text{m}/\sqrt{\text{Hz}}], \quad (7.14)$$

where $\tilde{S}_f$ denotes the frequency noise from the laser source (e.g., NPRO laser). Temperature fluctuations couple into the optical path length through several mechanisms, including thermal expansion, thermorefractive noise, and tilt-to-length coupling (TTLC). The optics, made of fused silica, are influenced by thermal expansion, thermorefractive noise and coating thermal noise (CTN). The total temperature-induced path length noise for optics is therefore given by

$$\tilde{S}_{\text{temp, optic}} = \left| \alpha \cdot L_{\text{optic}} \cdot \tilde{T}_{\text{ambient}} \right| + \left| \frac{dn}{dT} \cdot L_{\text{optic}} \cdot \tilde{T}_{\text{ambient}} \right| \quad [\text{m}/\sqrt{\text{Hz}}], \quad (7.15)$$

where $\tilde{T}_{\text{ambient}}$ is the ambient spectral temperature noise, $L_{\text{optic}} = L_{\text{geom}} \cdot n$ denotes the optical path length of the component with n as the refractive index of fused silica, dn/dT is the thermorefractive coefficient, and α represents the material-specific CTE. Coating thermal noise is treated separately (see Section 6.2.3) and summarized by the expression:

$$\tilde{S}_{\text{CTN}} = \sqrt{\frac{2k_{\text{B}}T}{\pi^2 f} \frac{1 - \sigma_{\text{sub}} - 2\sigma_{\text{sub}}^2}{Y_{\text{sub}} w^2} \sum_j b_j d_j \phi_j} \quad [1/\sqrt{\text{Hz}}]. \quad (7.16)$$

Temperature fluctuations affect the toolset (TAPSI) through thermal expansion of the baseplate (Zerodur) and optomechanics, with the latter also experiencing TTLC. The path length noise of the Zerodur baseplate is given by

$$\tilde{S}_{\text{temp, bench}} = \left| \alpha \cdot L_{\text{bench}} \cdot \tilde{T}_{\text{ambient}} \right| \quad [\text{m}/\sqrt{\text{Hz}}] \quad (7.17)$$

where L_{bench} denotes the optical path length along the Zerodur baseplate between the mounted optics. The path length noise of the optomechanics is expressed as

$$\tilde{S}_{\text{optomech}} = \tilde{S}_{\text{therm. expan.}} + \tilde{S}_{\text{TTLC}} \quad [\text{m}/\sqrt{\text{Hz}}], \quad (7.18)$$

where the thermal expansion coupling noise $\tilde{S}_{\text{therm. expan.}}$ and the TTLC noise $\tilde{S}_{\text{TTLC}}$ of the toolset are described in detail in Chapter 4. The total temperature-related path length noise is given by⁴

$$\tilde{S}_{\text{temp}} = \sqrt{[\tilde{S}_{\text{temp, optic}} + \tilde{S}_{\text{temp, bench}} + \tilde{S}_{\text{optomech}}]^2 + \tilde{S}_{\text{CTN}}^2} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (7.19)$$

The expression needs be treated with caution. Thermal noise (CTN) is uncorrelated with temperature

⁴ For simplicity, CTN is included in $\tilde{S}_{\text{temp, optic}}$, even though it originates from thermal rather than temperature fluctuations.

fluctuations and correctly added linearly. However, temperature noise sources sharing the same $\tilde{T}_{\text{ambient}}$ must be combined coherently rather than linearly, as linear addition assumes perfect correlation and overestimates the total noise. TTLC contributions further complicate the coupling. An additional interferometric noise contribution arises from spurious light signals (SLSs) (e.g., scattered light, ghost beams). Their total induced path length noise is given by

$$\tilde{S}_{\text{SLS}} = \frac{\lambda}{2\pi} \sqrt{\frac{P_{\text{SLS}}}{P_{\text{ref}}}} \cdot S_0 \quad [\text{m}/\sqrt{\text{Hz}}], \quad (7.20)$$

where P_{SLS} denotes the spurious light beam power, P_{ref} the reference beam power, and S_0 denotes the white noise phase coupling factor. The total optical path length noise of the heterodyne interferometer is summarized in the following expression⁵:

$$\tilde{S}_{\text{total}} = \sqrt{\underbrace{\tilde{S}_{\text{SN}}^2 + \tilde{S}_{\text{RIN}}^2 + \tilde{S}_{\text{EN}}^2}_{\text{readout noise}} + \underbrace{\tilde{S}_{\text{temp}}^2 + \tilde{S}_{\text{LFN}}^2 + \tilde{S}_{\text{SLS}}^2}_{\text{interferometer noise}}} \quad (7.21)$$

Subtracting pairs of beat note signals cancels common-mode noise and, depending on the PD combination, either isolates the pure readout noise floor (π -measurement) or includes interferometer noise contributions as described in Equation 7.21. These signal subtractions reveal the *Concealed Noise* floor.

7.4 Preliminary results

The Concealed Noise experiment was set up using the toolset for adjustable picometer-stable interferometers and located inside the vacuum chamber (optical test facility). The beat note frequency was locked to 8 MHz using a Moku:Pro's phasemeter and lock box instrument. The four interferometer signals were measured by the EGSE phasemeter, including pilot tone correction. The first measurement results are shown in Fig. 7.4⁶ displaying both signal subtraction combinations and phasemeter zero measurements⁷ at 8 MHz. It should be noted that the phasemeter noise scales with beat note frequency effects such as sampling jitter [162] (see Section 3.6 sampling jitter).

Our measurements exceed LISA requirements and exhibit a distinct 25 mHz peak matching the laboratory ventilation oscillation (see Section 5.3.4). While definitive noise identification awaits further optimization, the measurements already demonstrate interferometer noise dominance over readout noise, as evidenced by the lower noise floor of the π -measurement. The measurements should be treated as preliminary results, with further research in progress. Previous and ongoing LISA interferometry breadboard experiments operating at MHz frequencies have already demonstrated LISA-level performance, although they typically require extensive commissioning efforts [58, 61, 139, 161]. The Concealed Noise experiment is expected to achieve comparable interferometric noise levels after further commissioning. Combined with the multichannel phasemeter readout, this will allow operation in a regime where the underlying noise sources can be revealed and studied in greater detail.

⁵ Caveat: SLS and shot noise are correlated since both scale with laser power, requiring coherent combination.

⁶ Coherent subtraction of all four signals, is shown in Appendix E.4, Fig. E.15.

⁷ The phasemeter zero measurements were taken during non-ideal conditions, including intermitting ventilation, missing temperature sensors, and open ADC housing.

Low-frequency homodyne quadrature interferometer

Precision displacement sensing is a fundamental requirement for next-generation gravitational wave detectors such as the Lunar Gravitational-Wave Antenna (LGWA), targeting signals in the sub-hertz regime. Achieving sub-picometer-level stability¹ over large displacements at these low frequencies ($f < 1$ Hz) presents substantial technical challenges due to environmental disturbances, optical noise, and electronic readout noise. To address these challenges, we developed a homodyne quadrature interferometer (HoQI)-based displacement sensor set up using the toolset for adjustable picometer-stable interferometers (TAPSI). This sensor represents a promising candidate for the LGWA precursor mission, referred to as *Soundcheck*, whose peak sensitivity is in the decihertz band (approximately 0.1-1 Hz) [66]. Furthermore, this configuration offers another opportunity to assess the performance of the toolset.

State-of-the-art homodyne quadrature interferometer (HoQI)s developed at the University of Birmingham achieve sub-picometer sensitivity down to 0.1 Hz [166]. This represents an approximately three orders of magnitude improvement in displacement sensitivity compared to seismic displacement sensors currently deployed in ground-based gravitational-wave detectors such as LIGO, including the capacitive position sensor (CPS) [167] and the Birmingham optical sensor and electromagnetic actuator (BOSEM) [168]. The current HoQI design from the University of Birmingham employs three photodiodes to measure the optical displacement signal via its quadrature phase components, with each photodiode receiving one-eighth of the incident optical power.

In this work, we extend the design by adding a fourth photodiode and demonstrate measurements at sub-hertz frequencies. In contrast to the three-photodiode configuration, our approach enables simultaneous measurement of all optical quadratures while monitoring total incident power distributed uniformly across the photodetectors. Our displacement sensor, denoted as the low-frequency homodyne quadrature interferometer (LF-HoQI), permits accurate characterization of photodetector gain imbalances and relative phase misalignment through simultaneous quadrature measurements. Mitigating these nonideal parameters has the potential to reduce nonlinearities that currently limit HoQI performance [166, 169, 170].

This chapter presents the LF-HoQI optical layout, derives the four quadrature signals and identifies

¹ LGWA requires femtometer-level stability, while the precursor mission, referred to as *Soundcheck*, requires picometer-level stability.

their principal noise sources. The displacement sensor was then set up using the toolset (TAPSI) and located in the optical test facility to characterize its low-frequency length stability. Experimental results and limitations are subsequently discussed.

Further investigation of displacement noise mitigation through post-processing is deferred to future work. This thesis primarily focuses on measuring the performance of the LF-HoQI configuration implemented with the toolset for adjustable picometer-stable interferometers and identifying current noise limitations at sub-hertz frequencies.

8.1 Optical layout and signals

The LF-HoQI employs an Michelson interferometer (MI) principle with a polarization-based scheme to generate four orthogonal quadratures encoding the differential phase shifts. Figure 8.1 shows the optical layout including the polarization states at each position throughout the interferometer.

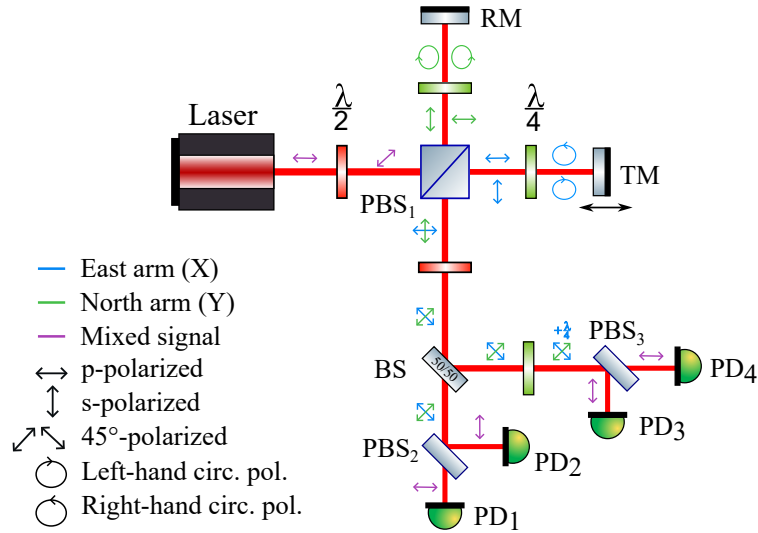


Figure 8.1: Optical layout of the LF-HoQI. The input beam is split at **PBS₁** and interferometrically recombined at **PBS₂** and **PBS₃**. A quarter-wave plate ($\lambda/4$) in the east readout arm introduces a relative phase shift to generate the sine or cosine quadratures for arctangent demodulation. This quadrature signal readout (homodyne phasemeter) enables multi-fringe measurement of the differential arm length $\Delta L = L_X - L_Y$. Double-headed arrows denote polarization states, with colors corresponding to the polarization from individual interferometer arms. Reference Mass/Mirror (RM), Test Mass/Mirror (TM).

Linearly polarized light from the laser source is rotated by a 45° by a half-wave plate ($\lambda/2$), resulting in equal power splitting at **PBS₁** into the interferometer arms. In each arm, a quarter-wave plate ($\lambda/4$) rotates the polarization by 90° , directing the full laser power toward the south port and thereby preventing back-reflections towards the laser. The beams returning from the two arms possess orthogonal polarizations and thus co-propagate without interference. A subsequent half-wave plate ($\lambda/2$) rotates both beam polarizations by 45° enabling equal power splitting at the non-polarizing beamsplitter (NPBS) into the two readout arms. In one readout arm, a quarter-wave plate ($\lambda/4$) introduces an additional 90° phase shift, required for generating the relative sine and cosine signals. The

orthogonal polarization components then interfere at PBS₂ and PBS₃, producing the four quadrature outputs.

Signal derivation of the four photodiodes

The output signals of the LF-HoQI are derived by calculating the polarization states using Jones matrix calculus [171]. For brevity, only key steps of the derivation are presented here, while the complete mathematical treatment is provided in the Appendix E.5. The input electric field from the laser source, normalized to the input amplitude E_0 , after the first half-wave plate ($\lambda/2$) is defined as²

$$E_1 = \frac{1}{E_0} \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix} = \begin{pmatrix} e^{i\phi_x} \\ e^{i\phi_y} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (8.1)$$

The beam is split into two equal parts at PBS₁. Each part undergoes a polarization rotation due to the double pass through the quarter-wave plate ($\lambda/4$) and subsequently exits the interferometer through the south port. Assuming a differential arm-length change ΔL in one arm (TM, p-polarized), which corresponds to a differential phase shift $\Delta\phi$, the light field exiting the interferometer is given by

$$E_{10,X,Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\Delta\phi} \end{pmatrix}. \quad (8.2)$$

The beam is subsequently rotated by the second half-wave plate ($\lambda/2$) and equally split by the non-polarizing beamsplitter (NPBS). The east readout arm contains a quarter-wave plate ($\lambda/4$) oriented along one of the polarization axes, introducing a phase shift of $\pi/2$ to the signal. For the two beams in their respective readout arms, just before the PBS₂ and PBS₃, we obtain

$$E_{12} = M_{\text{NPBS}} \cdot E_{10} = i \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 1 - e^{i\Delta\phi} \end{pmatrix}, \quad (8.3)$$

$$E_{13} = M_{\text{NPBS}} \cdot E_{10} = \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + i e^{i\Delta\phi} \\ 1 - i e^{i\Delta\phi} \end{pmatrix}. \quad (8.4)$$

The PBS_{2,3} separates the s- and p-polarization components and interferes the orthogonal polarized beams from the two interferometer arm. Thus, the field incident on PD₁ is

$$E_{\text{PD}_1} = \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 0 \end{pmatrix}. \quad (8.5)$$

The photodiode signal PD_x is proportional to the squared magnitude of the electric field, $\text{PD} \propto |E|^2 = EE^*$. Incorporating the relation between the laser field amplitude and the source output power, $E_{\text{in}} \propto \sqrt{P_{\text{in}}}$, the photodiode signals are expressed as

$$\text{PD}_1 = \frac{P_{\text{in}}}{4} (1 - \sin(\Delta\phi)), \quad (8.6)$$

² The index numbers indicate the laser light at each interferometer position with a detailed description in the Appendix E.5, Fig. E.16.

$$PD_2 = \frac{P_{in}}{4}(1 + \sin(\Delta\phi)), \quad (8.7)$$

$$PD_3 = \frac{P_{in}}{4}(1 + \cos(\Delta\phi)), \quad (8.8)$$

$$PD_4 = \frac{P_{in}}{4}(1 - \cos(\Delta\phi)). \quad (8.9)$$

The differential optical phase $\Delta\phi$ is defined in terms of interferometer arm lengths³ L_X , L_Y , and laser wavelength λ as

$$\Delta\phi = \frac{4\pi(L_X - L_Y)}{\lambda}. \quad (8.10)$$

Displacement signal derivation

Each of the four measured signals corresponds to one of the quadrature components. As a first step, these signals are combined to remove the DC offset, yielding purely AC signals. For example, we can do the following subtraction/combination:

$$S_{1,2,\sin} = PD_1 - PD_2 = \frac{\sqrt{2}P_{in}}{4}(\cos(\Delta\phi - \frac{\pi}{4})), \quad (8.11)$$

$$S_{3,4,\cos} = PD_3 - PD_4 = \frac{\sqrt{2}P_{in}}{4}(\cos(\Delta\phi + \frac{\pi}{4})). \quad (8.12)$$

The optical phase is subsequently extracted by arctangent demodulation, using the signal combinations from Equations 8.11 and 8.12:

$$\Delta\phi = \arctan\left(\frac{S_{1,2,\sin}}{S_{3,4,\cos}}\right). \quad (8.13)$$

The resulting differential arm length, $\Delta L = L_X - L_Y$, is then obtained by the conversion

$$\Delta L = \frac{\lambda}{4\pi}\Delta\phi. \quad (8.14)$$

This ratiometric calculation effectively suppresses common-mode noise for *ideal* signals (i.e., in the absence of nonlinearities).

Introduction to nonlinearities

An ideal homodyne quadrature interferometer produces four quadrature signals with equal amplitudes, 90° phase shifts, and zero offsets. However, practical experimental implementations introduce significant deviations from this ideal behavior. These deviations arise from sources such as unequal laser power levels $\tilde{P}_x$, incident on each PD; dark noise offsets, $\tilde{P}_{DN}$, from the photodetector electronics; and variations in TIA gain, $\tilde{g}_j$, across readout channels. Furthermore, these noise sources often exhibit non-stationary and frequency-dependent characteristics, rendering low-frequency measurements ($f < 1$ Hz), such as those required for LGWA, particularly susceptible to performance degradation. Consequently, the measured signal deviate from the ideal case described by Equation 8.6.

³ The north arm is denoted L_Y , and the east arm L_X

The actual measured $\text{PD}x_{\text{meas}}$ signal, including noise sources, is therefore expressed as

$$\text{PD}x_{\text{meas}} = \tilde{\text{PD}}_{\text{DN}} + \frac{\tilde{P}_x}{4} \tilde{g}_j (1 + a \cdot \cos(\Delta\phi + \tilde{\delta})), \quad (8.15)$$

where a denotes the interferometer contrast and $\tilde{\delta}$ represents an additional phase shift that deviates from the orthogonal quadrature relation. As a result, the quadrature signals are no longer equal in power and orthogonal, rendering the phase-extraction method (Equation 8.13) invalid. These distortions introduce nonlinearities in the measurement, arising from the discrepancy between the extracted optical phase and the true phase of the interferometer. Furthermore, the resulting amplitude and phase imbalance reduces the achievable level of common-mode noise rejection [172, 173].

When plotting all four quadrature signals on separate axes, a Lissajous figure emerges. In the ideal case of perfectly orthogonal signals with equal amplitudes, this figure appears as a circle. Deviations such as phase offsets or amplitude imbalances distort it into an ellipse. By fitting an ellipse to the measured data, the ellipse parameters can be extracted and corrections applied to compensate for these imperfections⁴. This technique, first demonstrated by Heydemann [174], substantially suppresses nonlinearities and is widely employed in homodyne quadrature interferometry [170].

Nonideal alignment of quarter- and half-wave plates contributes to nonlinearities, alongside other sources. This represents a potential drawback of the proposed interferometer configuration (see Section 8.1), which, unlike common HoQI configurations (e.g., Birmingham), incorporates an additional pair of quarter-wave plates. These can amplify sensitivity to birefringent fluctuations, as documented in previous studies [169, 170].

8.2 Noise analysis

This section presents the noise analysis of the primary noise sources expected to limit LF-HoQI sensitivity. Noise contributions inherent to the toolset (TAPSI) are excluded here, as they have been addressed in previous chapters (Section 4.4.3 and 6.2.3) and demonstrated performance at picometer-level LISA requirement (Section 6.5).

Note: For simplicity, the LF-HoQI was depicted in the previous sections as an equal-arm-length interferometer. The experimental setup, however, employed unequal arm lengths, as described in the following Section 8.3.

Optical noise:

The dominant optical noise source in the LF-HoQI is laser frequency noise (LFN), which couples into the phase measurement through the unequal-arm lengths difference, given by

$$\tilde{S}_{\text{LFN},\phi} = \frac{\lambda}{c} \cdot \Delta L \cdot \tilde{f}_{\text{Laser}} \quad [\text{rad}/\sqrt{\text{Hz}}], \quad (8.16)$$

where $\tilde{f}_{\text{Laser}}$ denotes the frequency noise of the NPRO laser (in units $\text{Hz}/\sqrt{\text{Hz}}$), and $\Delta L = 13 \text{ cm}$ the static differential arm length. The corresponding displacement noise is obtained from the relation:

$$\tilde{S}_{\text{LFN}} = \frac{\lambda}{4\pi} \tilde{S}_{\text{LFN},\phi} \quad [\text{m}/\sqrt{\text{Hz}}]. \quad (8.17)$$

⁴ Comprehensive treatments of this topic are provided by [170] (S. Cooper, PhD-thesis, Chapter 3.2).

The LFN-induced displacement noise exceeds the targeted picometer-level by several orders of magnitude. However, the experimental setup also employed laser-cavity locking via PDH locking to stabilize the laser frequency before coupling into the interferometer (see Section 8.3). Consequently, LFN is suppressed and does not limit the sensitivity. Instead, the fundamental shot noise (SN) sets the optical noise floor and is given by

$$\tilde{S}_{\text{SN}} = \sqrt{2eR_{\text{PD}}(P_{\text{PD}})} \quad [\text{A}/\sqrt{\text{Hz}}], \quad (8.18)$$

where e denotes the elementary charge and $R_{\text{PD}} = 0.3 \text{ A/W}$ the photodetector responsivity. Assuming a power of $P_{\text{PD}} = 10 \text{ mW}$, this results in the corresponding phase-induced shot noise level of

$$\tilde{S}_{\text{SN},\phi} = \frac{\tilde{S}_{\text{SN}} \cdot R_f}{\text{signal}} \approx 5.4 \times 10^{-8} \text{ rad}/\sqrt{\text{Hz}}, \quad (8.19)$$

where $R_f = 1 \text{ k}\Omega$ is the feedback resistor of the TIA and the signal is approximated⁵ by

$$\text{signal} = \left| \frac{dV}{d\phi} \right| \approx \frac{1.8V}{\pi} \quad [\text{V}/\text{rad}]. \quad (8.20)$$

The corresponding shot noise-induced displacement noise is

$$\tilde{S}_{\text{SN}} \approx 4.6 \times 10^{-15} \text{ m}/\sqrt{\text{Hz}}, \quad (8.21)$$

and is well below our targeted picometer-level sensitivity of the LGWA Soundcheck mission requirement.

Electronic noise:

The dominant electronic noise sources in the LF-HoQI experimental setup arise from the photodetectors, specifically the TIA⁶ at low frequencies ($f \leq 1 \text{ Hz}$) and the ADC of the Moku:Pro for data acquisition. To isolate purely electronic contributions, the photodiodes were replaced with an ideal photocurrent source. This was realized using a low-noise constant voltage source⁷ and appropriate resistors. We found that the electronic noise floor is not limited by the photodetector circuitry but primarily by the ADC noise of the Moku:Pro. The observed noise increases with higher input voltage levels, despite constant signal amplitude, indicating scaling of the ADC reference voltage noise, as shown in Fig. 8.2. The input voltage levels were chosen based on interferometer measurements (Fig. 8.5), where amplitudes of 1.5 V and 0.3 V correspond to a specific optical phase.

The phase noise of the individual PDs (dashed lines) was then obtained from the measured voltage noise using the linearized interferometer response

$$\tilde{S}_{\phi} = \tilde{S}_V \cdot \left| \frac{d\phi}{dV} \right| \quad (8.22)$$

where $\tilde{S}_V$ denotes the voltage noise spectral density from the individual PDs, $dV = -1.8 \text{ V}$ as the

⁵ Linearization of the voltage amplitude (see Section 8.3) across the phase excursion of π .

⁶ The TIA employed an *OPA197* operational amplifier.

⁷ Analog Devices / Maxim Integrated, *MAX6350*, 5 V voltage reference.

signal amplitude, and $d\phi = \pi$ the corresponding phase excursion. The extracted phase noise (solid lines) was calculated using Equation 8.13.

The measurements demonstrate that ratiometric phase extraction (solid lines in Fig. 8.2) effectively suppresses common-mode noise components, including the ADC (reference voltage) $1/f$ flicker noise. Selecting the lower-amplitude 0.3 V signals from both PDs for the phase extraction via the $\arctan2(\sin/\cos)$ demodulation, significantly reduces the noise floor. Thus, electronic noise-limited measurements, particularly those dominated by input voltage scaling, benefit from the four-PD configuration, which enables selection of the *least noisy* quadrature signals.

However, lacking detailed specifications of the Moku:Pro's internal electronic architecture and its common-mode rejection during ratiometric phase extraction, further analysis of this noise source was not possible. Additionally, these measurements assume perfectly matched and constant signal amplitude⁸ across all photodiodes. Amplitude mismatches degrade the demonstrated common-mode suppression. The conservative worst-case phase noise floor is given by the root-mean-square sum of the individual phase noise contributions from all four photodiodes, neglecting any residual common-mode rejection.

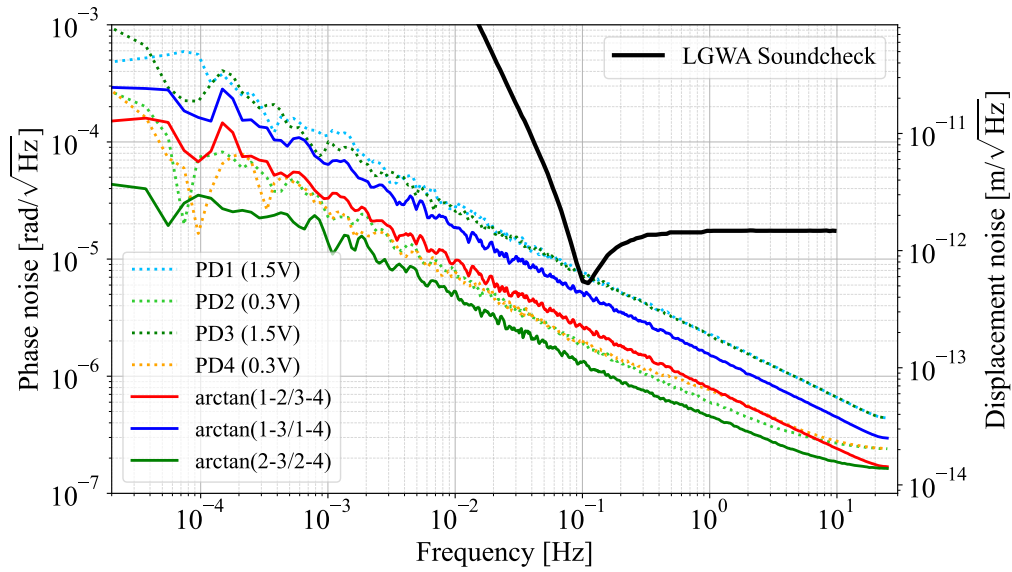


Figure 8.2: Electronic noise of the LF-HoQI alongside the LGWA Soundcheck requirement [66], plotted in phase noise and displacement noise. The ADC noise of the Moku:Pro instrument exhibits flicker ($1/f$) noise and input voltage scaling. The phase noise (solid lines) is obtained from the ratiometric $\arctan2(\sin/\cos)$ combination of the individual photodetector signals (dashed lines), which suppresses common-mode noise components. Choosing photodetector combinations with lower output voltages, and thus lower individual noise levels, reduces the overall noise floor.

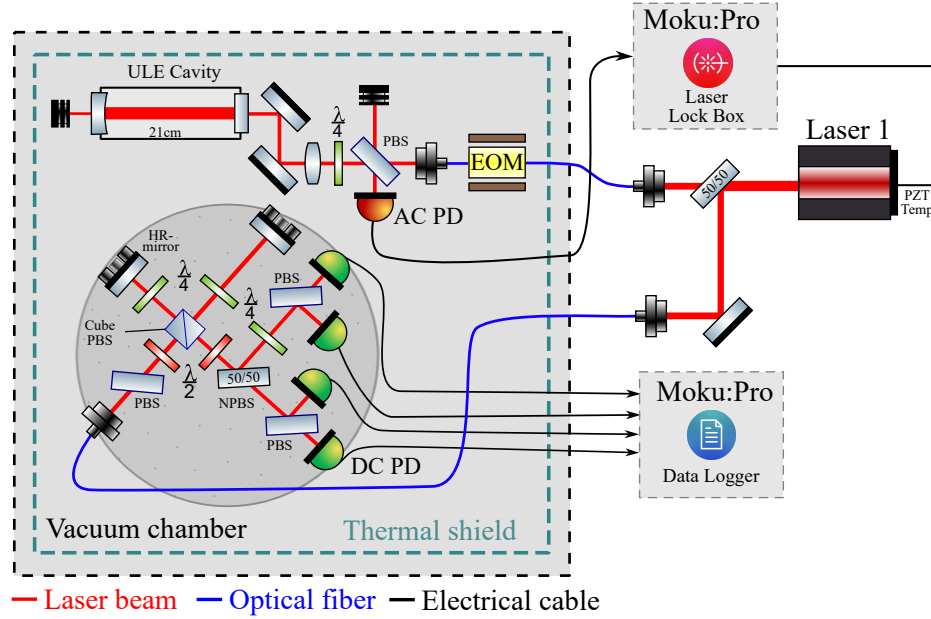


Figure 8.3: Optical layout of the LF-HoQI experimental setup. The laser beam splits into two paths: one to the LF-HoQI interferometer and the other to an ULE cavity for laser frequency stabilization via PDH locking (Section 6). Four (DC) photodetectors detect the quadrature signals from the LF-HoQI, which are recorded using the Moku:Pro Data Logger instrument.

8.3 Experimental setup

The LF-HoQI experimental setup included the laser preparation⁹, the interferometer configuration, and the DC readout electronics. The interferometer was set up using the toolset for adjustable picometer-stable interferometers and located inside the vacuum chamber (optical test facility). The laser beam was split, attenuated to approximately 50 mW, and polarization-filtered¹⁰ before entering the vacuum chamber via pm-fiber feedthroughs. Inside the chamber, the beam was collimated and further polarization-filtered using a PBS before reaching the LF-HoQI setup. The LF-HoQI followed the optical layout, described in Section 8.1, with the key modification of unequal arm lengths. This design choice enables fringe scanning calibration via frequency tuning but also introduces laser frequency noise (LFN) coupling. The coupling was suppressed by employing an PDH locking scheme, where the laser frequency was stabilized to the length of an ULE cavity (Section 6). This unequal-arm frequency tuning design replaced the prior heating-element method¹¹, which used a heating element on one arm-mirror mount for optical path length modulation. In the ‘Michelson interferometer’ setup part, a cube polarization beamsplitter¹² was utilized, while the readout arms

⁸ *Matched amplitude* refers to signal amplitudes, not instantaneous power.

⁹ Laser source: NPRO laser, Coherent, *Mephisto 500*.

¹⁰ Commonly known as *polarization cleaning*.

¹¹ The *unequal-arm frequency tuning* design enables faster, uniform fringe scanning, avoiding the hysteresis and long-term heating effects associated with the *heating-element method*.

¹² The ‘MI’ beamsplitter requires single-surface (polarization) reflection to avoid collinear (polarization) ghost beams. The plate-PBS (Brewster-angle: 55.4°) featured back-surface AR coating solely optimized for p-polarization. Consequently,

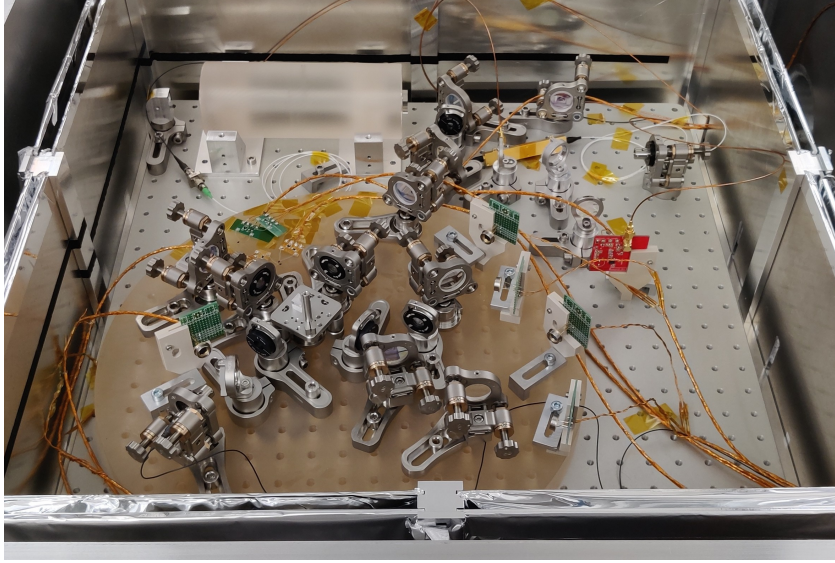


Figure 8.4: Photograph of the low-frequency homodyne quadrature interferometer (LF-HoQI) inside the vacuum chamber. The interferometer was implemented using the toolset for adjustable picometer-stable interferometers. The unequal-arm-lengths design choice enables fringe-scanning calibration via frequency tuning. Laser frequency noise coupling from this design is suppressed by PDH. The laser frequency is locked to the resonance of the ULE cavity, located at the rear of the chamber.

featured plate-polarizing beamsplitters. The four quadrature signals from the readout arms were detected by DC photodetectors (PD_x) and recorded using the Moku:Pro Data Logger. The schematics and PCB layouts of the employed photodetectors are shown in Fig. F.5 in Appendix F. Initially, the full photodetector assembly, including the TIA was located inside the vacuum chamber. However, TIA heating prompted relocation of the TIA board outside, retaining only the photodiodes inside¹³. An additional improvement was achieved by replacing the InGaAs photodiodes¹⁴ with Si photodiodes¹⁵, which offer a significantly larger active area. This prevented clipping effects and mitigated beam-jitter noise by spatial averaging¹⁶.

Before each measurement, the LF-HoQI was calibrated via fringe scanning, accomplished by laser-frequency tuning, to determine each PD's amplitude. The resulting amplitude values enabled power normalization for equal voltage-to-phase scaling across photodetectors. Small offsets from non-ideal fringe contrast were neglected. The laser was then PDH-locked to the ULE cavity prior to starting the measurement (time series: Fig. 8.5). The phase was extracted via Equation 8.13 using Python's $\arctan2$ function¹⁷, and displacement noise conversion from Equation 8.14.

we used a cube beamsplitter, accepting etalon effects from its parallel surfaces.

¹³ Unlike high-frequency AC-coupled readouts, requiring TIA-photodiode proximity to preserve bandwidth and minimize noise pickup, DC signals tolerate long cables without significant roll-off.

¹⁴ Roithner LaserTechnik, *LAPD-1-09-17-TO46*, active area: $\varnothing = 1$ mm

¹⁵ Hamamatsu, *S2386-8K* [175], active area: 5.8 x 5.8 mm

¹⁶ Beam jitter moves the focused spot across the detector. Responsivity and dark current inhomogeneities cause intensity fluctuations, introducing noise [176].

¹⁷ $\arctan2(y, x)$ computes $\phi \in (-\pi, \pi)$ correctly handling all quadrants.

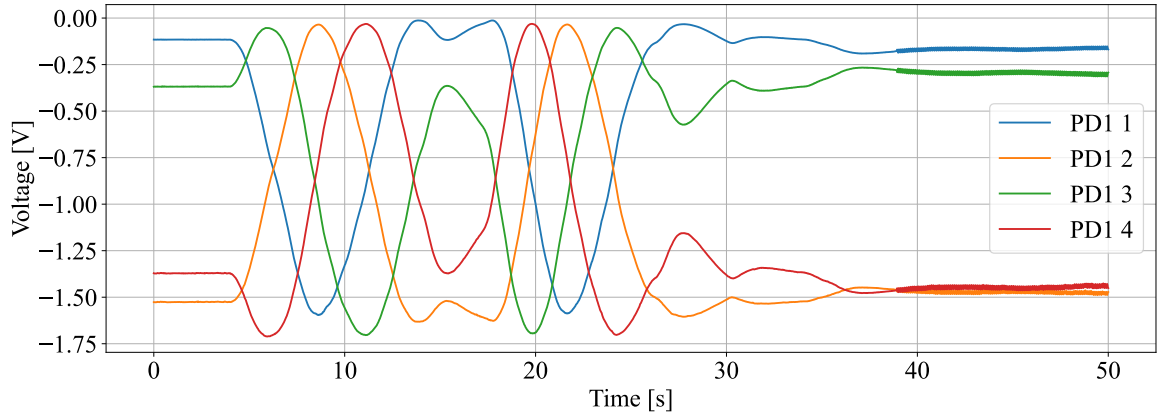


Figure 8.5: Pre-measurement calibration process: First, the fringes were scanned via laser-frequency tuning to determine PD amplitudes for normalization. Subsequently, the laser was locked to a ULE cavity to suppress LFN coupling from the unequal arm interferometer.

8.4 Results and limitations

The sensitivity of the low-frequency homodyne quadrature interferometer (LF-HoQI), expressed in phase noise and equivalent displacement noise, is shown in Fig. 8.6 alongside modeled and measured noise sources from Section 8.2 and the LGWA Soundcheck sensitivity requirement [66]. The primary optical noise sources are shot noise¹⁸ and LFN coupling due to unequal interferometer arm lengths. Phase-induced LFN, the primary limitation without mitigation, was effectively suppressed by locking the laser to an ULE cavity via PDH locking.

The LF-HoQI sensitivity measurements (solid lines) were obtained via arctangent demodulation of the quadrature PD signals (dashed lines). These individual PD signals¹⁹ exhibit elevated noise by several orders of magnitude, including a 25 mHz peak. This excess noise stemmed from the power fluctuations arising at the PBS preceding the interferometer, which converted polarization rotation into power fluctuation, ideally affecting all PDs equally. The polarization rotation was attributed to the pm feedthrough fibers. Despite >25 dB polarization extinction ratio (PER)²⁰, residual polarization rotation persisted in the pm-fiber due to laboratory temperature variations (outside the chamber). This is evidenced by the prominent peak at 25 mHz matching laboratory temperature oscillations (see Fig. 5.15). Temperature coupling inside the vacuum chamber was excluded, as phase-temperature coherence is negligible. However, this common-mode noise was largely suppressed by the ratiometric arctangent demodulation, reducing the individual PD signals by several orders of magnitude.

The performance of LF-HoQI is below the LGWA Soundcheck requirement for a large part of the relevant frequencies. The excess noise above 1 Hz stems from mirror mount resonances and is less critical as it can be mitigated by mirror mount design or improved seismic isolation. The measurement towards lower frequencies exhibits a linear scaling closely following the electrical noise

¹⁸ Calculated for 10 mW per PD.

¹⁹ The individual quadrature PD noise signals are derived by the linear approximation of the signal amplitude across the phase excursion (Equation 8.22).

²⁰ The PER of the pm-fiber was maximized by using a PBS and a quarter-wave plate in front of the fiber collimator to align the beam's polarization along the fiber's slow-axis.

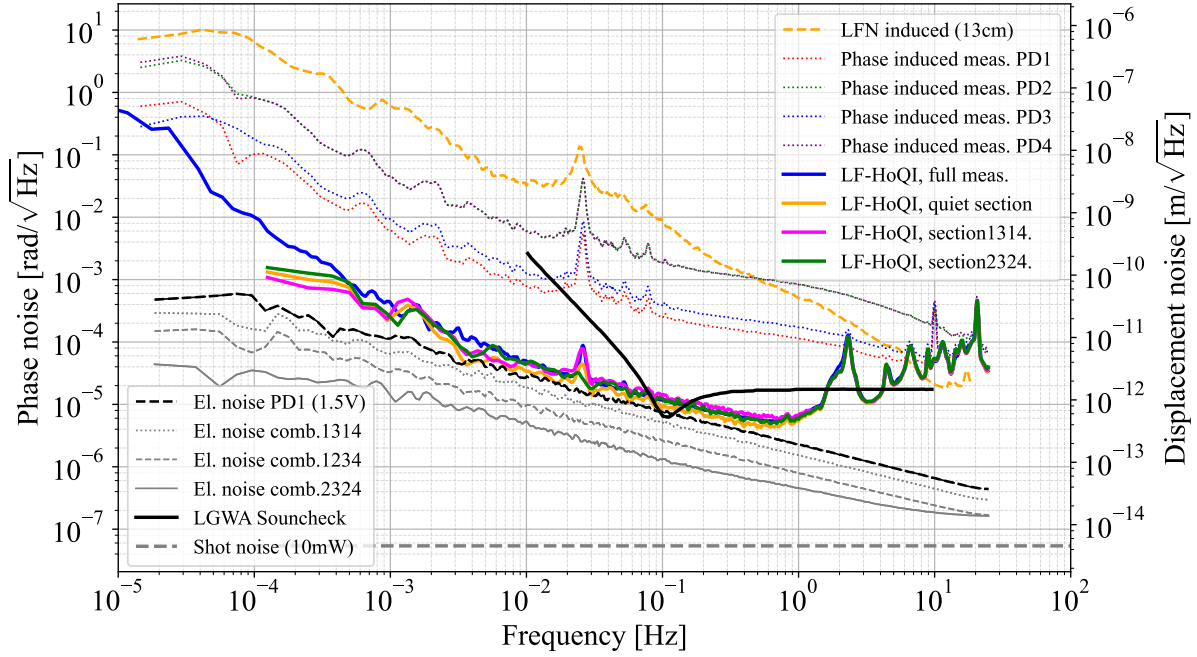


Figure 8.6: Performance measurement results of LF-HoQI (solid colored lines), along with the associated noise sources, the LGWA Soundcheck requirements and the individual PD signals (dashed lines). The ratiometric arctangent phase reconstruction provides strong common-mode rejection, reducing the noise of the individual PDs by orders of magnitude. However, amplitude (and more general ellipse) mismatch allows the leakage of common-mode noise into the reconstructed phase. The common-mode rejection is reduced, and the performance limited by the electronic noise floor of the PDs.

floor of an individual PD (black solid line). In an ideal ratiometric measurement, the electrical noise is reduced by the common-mode suppression as described in Section 8.2 and shown in Fig. 8.2. However, mismatched quadrature amplitudes and phase offsets degrade rejection, leaking noise into the reconstructed phase. This may explain the measurement limitation being determined by the individual noise contribution of one detector. Beyond initial amplitude calibration of the individual PDs, no further optimization, such as gain matching or contrast offset removal, was performed. Heydemann ellipse correction was applied but yielded no measurable improvement. For visual clarity, we can decompose the measured phase noise into the following noise contributions:

$$\tilde{S}_{\phi, \text{meas}}^2 \approx \underbrace{\tilde{S}_{\phi, \text{signal}}^2}_{\text{arm length fluctuation}} + \underbrace{\tilde{S}_{\phi, \text{el}}^2 + \tilde{S}_{\phi, \text{opt}}^2}_{\text{uncorrelated noise}} + \underbrace{\left(\epsilon \cdot \sigma_{\phi, \text{opt}, \text{el}} \right)^2}_{\text{common-mode noise leaked by mismatch}}, \quad (8.23)$$

where $\tilde{S}_{\phi, \text{signal}}$ represents the displacement signal, while $\tilde{S}_{\phi, \text{el}}^2$ and $\tilde{S}_{\phi, \text{opt}}^2$ are uncorrelated noise contributions persisting through ratiometric demodulation. The ellipse mismatch factor ϵ , from quadrature amplitude imbalance or phase offset, determines common-mode leakage into the reconstructed phase.

A strong indicator of common-mode leakage is the 25 mHz peak matching laboratory temperature oscillations. This laser power fluctuation (common-mode noise) affects all photodetectors equally but fails to be fully rejected by the ratiometric demodulation. Furthermore, the apparent limiting electrical

noise should, for ideal signals (matching amplitudes and phase relations), reject common-mode noise as shown beforehand (Fig. 8.2). Different combinations only yielded marginal performance improvements²¹. Even a quieter measurement segment from the full measurement showed modest noise reduction, insufficient for definitive conclusions. LF-HoQI performance appears primarily limited by electronic noise, with quadrature mismatches in amplitude and phase preventing further ratiometric rejection.

Immediate improvements can be achieved by deploying a low-noise ADC alongside laser power stabilization to reduce reliance on ratiometric rejection. Further mitigation of common-mode noise leakage requires precise amplitude and gain matching of individual PDs and orthogonal quadrature alignment. Maintaining these parameters over extended durations, critical for low-frequency performance, poses significant experimental challenges beyond this thesis scope, and will be addressed in future work. The fourth PD in our LF-HoQI configuration also enables comprehensive quadrature measurement and total-power monitoring, facilitating advanced nonlinearity correction, which will also be addressed in future work.

Nevertheless, the LF-HoQI already demonstrates performance compatible with the LGWA Soundcheck mission [67]. Additionally, the experiment further tested the developed toolset (TAPSI), which not only demonstrated picometer-level length stability but also provided a rapid commissioning of the experiment (initial interferometer setup in under two hours). This confirmed the practical implementation of the developed OGSE, including the toolset and optical test facility.

²¹ Fig. 8.6, e.g., ‘section1234’ = $\arctan\left(\frac{PD1-PD2}{PD3-PD4}\right)$; ‘section2324’ = $\arctan\left(\frac{PD2-PD3}{PD2-PD4}\right)$ demodulation. Evaluation algorithms that treat all four results equally were not examined, but are the subject of future work.

Summary and Outlook

Space-based gravitational-wave detectors, such as the LISA mission and the LGWA, extend the accessible frequency band of gravitational-wave observations toward the sub-hertz regime, enabling the study of sources from earlier epochs of the universe. Achieving sensitivity at these low frequencies requires dedicated interferometers with exceptional long-term stability and presents significant technological challenges. Moreover, these space missions require extensive on-ground testing and qualification of key subsystems such as the optical bench, whose development and assembly consume substantial effort and time to meet the sub-hertz, picometer-level stability requirements.

Keeping these requirements in mind, we have designed and built an OGSE featuring the optical test facility and a toolset for adjustable picometer-stable interferometers (TAPSI) for space-based gravitational-wave detectors. Our OGSE provides a testbed and toolset for realizing diverse interferometer configurations while significantly simplifying and accelerating the optical bench construction. Part I gives a brief introduction to gravitational-wave detection and provides the foundation knowledge on laser interferometry, including intrinsic interferometer noise, as well as electronic and readout noise sources. This detailed treatment is essential for the diverse configurations set up in this thesis using the toolset (homodyne LF-HoQI, heterodyne Concealed Noise, optical cavities).

Part II describes the development of the optical ground support equipment, comprising the toolset (TAPSI) and the optical test facility. The toolset was designed to enable flexible interferometer design with picometer-level-sensitivity at sub-hertz frequencies. It comprises low-CTE optomechanics and thermally compensated mirror mounts (ZeroDrift) to mitigate the main temperature-related noise sources: thermal expansion and tilt-to-length coupling (TTLC) noise. These noise sources were analyzed, and a general noise budget was established. The primary noise source arises from the CTE mismatch between the mirror mount and the optomechanics, particularly the compensation plate, which connects the mirror mount and the post. The optical test facility provides a seismically isolated and thermally insulated environment for the interferometers by featuring multiple thermal isolation layers. This design mitigates temperature fluctuations and achieved temperature stability within the chamber and thermal shield of $10 \mu\text{K}/\sqrt{\text{Hz}}$ at 10 mHz, following a $1/f$ slope. The temperature noise was measured using a custom-built vacuum-compatible temperature sensor with dedicated readout electronics featuring a low-frequency noise ADC (ADS1261) and AC excitation/readout.

Part III presented the experimental length stability validation and the construction of two distinct interferometer experiments. The length stability of the toolset's mounting concept was characterized by setting up a prototype cavity and measuring its displacement noise using laser-cavity locking. One

laser's frequency was locked to the length of the prototype cavity, while a second laser was locked to a reference ULE cavity. By interfering the lasers and measuring their beat note frequency, the relative displacement noise was obtained. Laser-cavity locking was performed using two techniques, the Heterodyne laser frequency Stabilization (HS) and Pound-Drever-Hall (PDH) locking. Both techniques operate on a similar principle, deriving an error signal from the detuning between the cavity resonance and the laser frequency, which is then used in a feedback loop to stabilize the laser frequency. The initial HS scheme employed analog demodulation, which was ultimately limited by path-length mismatches between the reference beat note and the cavity reflection beat note. This mismatch introduced demodulation-phase noise into the error signal and degraded picometer-level performance at frequencies below 40 mHz. Switching to digital demodulation reduced error signal distortion but could not fully suppress the issue due to insufficient resolution in the heterodyne controller's frequency-dependent demodulation-phase adjustment.

Therefore, the PDH locking technique was employed to measure the toolset's length stability. The initial locking scheme was limited by spurious light signals and featured a characteristic *scatter shoulder*, which exceeded the picometer-level displacement noise below 50 mHz. This was likely due to fiber-optic components used in this scheme. After a substantial redesign, including replacing fiber components with free-space optics wherever feasible, picometer-level length stability meeting LISA requirements was successfully demonstrated. The PDH laser-cavity lock was ultimately limited by readout noise rather than the toolset's intrinsic length stability. The readout noise likely originated from spurious light beams, which can be mitigated in future iterations by using a higher-finesse prototype cavity. After characterizing the toolset's length stability, two distinct interferometers were constructed using the toolset.

The *Concealed Noise project* aims to characterize the noise floor of picometer-precision heterodyne interferometers operating at MHz frequencies through combined theoretical modeling and experimental validation. The project will establish a comprehensive noise budget, quantifying the relative contributions of individual noise sources to overall system performance. Here, only the expected noise sources are described, along with the initial setup of the experiment. The full exploitation of the experiment is part of future research. Initial measurements are well above the LISA requirements but are expected to reach noise levels comparable to previous and ongoing LISA interferometry breadboard experiments [58, 61, 139, 161].

The second interferometer constructed was the low-frequency homodyne quadrature interferometer (LF-HoQI). Compared to conventional HoQIs [170], our approach incorporated a fourth photodiode and an unequal-arm-length interferometer, where laser frequency noise coupling was suppressed via PDH locking using an ULE cavity. Our displacement sensor extends the HoQI concept to the low-frequency regime and thereby making it a strong candidate for precursor missions of future gravitational-wave detection concepts such as the LGWA. Our displacement sensor achieved picometer-level performance that meets the precursor mission (LGWA *Soundcheck*) requirements. Future work will improve the readout electronics and investigate nonlinearity suppression in post-processing using the fourth PD for comprehensive quadrature signal and total-power monitoring.

In conclusion, this work has designed and constructed an OGSE featuring the optical test facility and the toolset for adjustable picometer-stable interferometers, demonstrating picometer-level displacement noise that meets LISA requirements. The rapid experimental commissioning demonstrated in this thesis confirms the toolset's suitability for realizing flexible optical ground support equipment setups for space-based gravitational-wave detectors such as LISA and LGWA, as well as missions beyond LISA [37, 163, 164] and other applications requiring ultra-stable, low-frequency laser interferometry.

Appendix

Noise analysis methodology

To characterize experimental noise performance, we analyze displacement time series in the frequency domain by calculating the power spectral density (PSD) according to [9]. For a recorded signal $x(t)$ with units of [m], the autocorrelation function is defined as the correlation of $x(t)$ with its time-delayed version $x(t + \tau)$:

$$R_{xx}(\tau) = x \star x = \int_{-\infty}^{+\infty} x(t) x(t + \tau) dt \quad \left[\text{m}^2 \right]. \quad (\text{A.1})$$

The PSD is then obtained via the Fourier transform of $R_{xx}(\tau)$:

$$\tilde{S}_{xx}(f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-i2\pi f\tau} d\tau \quad \left[\text{m}^2/\text{Hz} \right]. \quad (\text{A.2})$$

The PSD $\tilde{S}_{xx}(f)$ quantifies the power distribution of the time series as a function of frequency. Equivalently, it measures the contribution to total signal power from sinusoidal components at frequency f . Conventionally, the *single-sided PSD* is used, which accounts only for positive frequencies by doubling the spectral content at those frequencies:

$$\tilde{S}_{xx}(f) = \begin{cases} 2 \tilde{S}_{xx}(f) & f \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Interferometer experiments typically record *phase* measurements [rad], which are subsequently converted to *displacement* [m]. The amplitude spectral density (ASD) with units of [m/ $\sqrt{\text{Hz}}$] is therefore more intuitive than the PSD. It is defined as

$$\tilde{S}_x(f) = \sqrt{\tilde{S}_{xx}(f)} \quad \left[\text{m}/\sqrt{\text{Hz}} \right]. \quad (\text{A.3})$$

Although the units [m/ $\sqrt{\text{Hz}}$] appear unconventional, they arise naturally because independent (uncorrelated) noise sources combine in quadrature (not linearly) in the ASD domain:

$$\tilde{S}_{\text{total}}(f) = \sqrt{\tilde{S}_1^2(f) + \tilde{S}_2^2(f)} \quad \left[\text{m}/\sqrt{\text{Hz}} \right]. \quad (\text{A.4})$$

Logarithmic amplitude spectral density

This thesis employs the Logarithmic frequency axis Power Spectral Density (LPSD) method [177] to compute the ASDs. The logarithmic frequency axis provides more informative results than conventional spectra, particularly for broadband noise analysis. The LPSD was implemented using the `lpsd` Python package to directly estimate spectra from recorded time series.

Interferometer signal derivation from signal sidebands

Laser interferometer gravitational wave detectors measure the displacement of test-mass mirrors. In simple terms: a passing gravitational wave causes these test masses to move relative to each other, and this relative motion constitutes the measurable signal. It is therefore advisable to understand mathematically how the GW signal is encoded in the light reflected from a moving mirror. This appendix derives the interferometer signal for a Michelson interferometer (MI) in detail, starting from a phase modulation of a moving mirror in one of the interferometer arms and continuing with the subsequent amplitude modulation at the recombination beamsplitter. The following derivations are based on [87].

Phase modulation through a moving mirror

Although a variety of optical components can induce phase modulation, the length change that is typically measured is that of the end mirror of the interferometer. In a Michelson interferometer, for instance, a moving (end-)mirror in one of the interferometer arms causes a phase modulation of the light. This modulation is subsequently converted into an amplitude modulation at the BS, which constitutes the detected signal and is measured by a photodetector. It is therefore worthwhile to take a closer look at the phase modulation through a moving mirror. A sketch of such an example is shown in Figure B.1.

When the laser light beam strikes a mirror (hard reflection), the reflected light experiences a phase shift of π while the transmitted light is unchanged. We assume a sinusoidal modulation of the mirror position, expressed as

$$x_m = a_m \cos(\omega_m t + \phi_m), \quad (\text{B.1})$$

where a_m denotes the amplitude, ω_m the modulation frequency and ϕ_m the phase of the mirror's position. This results in the reflected field from the mirror as follows:

$$E_r = r E_0 e^{-i2(\phi_0 + \phi_m)}, \quad (\text{B.2})$$

where $E_0 = \hat{E}_0 \cos(\omega_0 t) = \hat{E}_0 e^{i\omega_0 t}$ is the electric field and ϕ_0 the initial phase of the incident laser light. The accumulated phase from the moving mirror is expressed by ϕ_m .

Subsequently, the formula for the reflected field can be rewritten, including the mirror modulation, and we obtain:

$$\begin{aligned}
 E_r &= rE_0 e^{-i2(\phi_0 + \phi_m)} \\
 &= rE_0 e^{-i2(\phi_0 + k_0 x_m)} \\
 &= rE_0 e^{-i2\phi_0} e^{i2k_0 x_m} \\
 &\approx rE_0 e^{-i2\phi_0} e^{i2k_0 a_m \cos(\omega_m t + \phi_m)}.
 \end{aligned} \tag{B.3}$$

Rewriting the equation with the modulation index, defined as $m = 2k_0 a_m$, ultimately yields the following result:

$$E_r = rE_0 e^{-i2\phi_0} e^{im \cos(\omega_m t + \phi_m)}. \tag{B.4}$$

The equation can be expressed using the Bessel functions¹ (see Section 2.4.2, Equation 2.58) as

$$E_r = rE_0 e^{-i2\phi_0} \left(1 + i\frac{m}{2} e^{-i(\omega_m t + \phi_m)} + i\frac{m}{2} e^{i(\omega_m t + \phi_m)} \right). \tag{B.5}$$

For the initial electric field, we assume a phase of $\phi_0 = 0$, and $\phi_m = 0$ for the initial mirror position. For a high-reflectivity mirror, $r \approx 1$, we can write:

$$\begin{aligned}
 E_r &= E_0 \left(1 + i\frac{m}{2} e^{-i(\omega_m t)} + i\frac{m}{2} e^{i(\omega_m t)} \right) \\
 &= \hat{E}_0 e^{i\omega_0 t} \left(1 + i\frac{m}{2} e^{-i(\omega_m t)} + i\frac{m}{2} e^{i(\omega_m t)} \right) \\
 &= \hat{E}_0 e^{i\omega_0 t} + i\frac{m}{2} \hat{E}_0 e^{i((\omega_0 - \omega_m)t)} + i\frac{m}{2} \hat{E}_0 e^{i((\omega_0 + \omega_m)t)}.
 \end{aligned} \tag{B.6}$$

The expression shows that the reflected light E_r from a moving mirror can be represented by the carrier and a pair of phase modulation sidebands. In the case of an interferometer, the beam that experienced the phase modulation through a moving mirror is interfered with a reference beam. For a homodyne interferometer (same frequency ω_0), the interference at the beam splitter converts the phase modulation into an amplitude modulation which is subsequently measured by a DC photodetector. For a heterodyne interferometer ($\omega_1 \neq \omega_2$), the interference of the two beams at the beam splitter produces a beat note $\Delta\omega = |\omega_1 - \omega_2|$, detected by an AC photodetector and measured by a phasemeter. The phase information from the moving mirror is encoded as a phase shift in this beat signal.

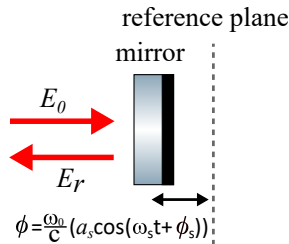


Figure B.1: Phase modulation of a laser beam through a moving mirror.

¹ We assume $m \ll 1$ and only keep the carrier and first-order sidebands of the Bessel functions.

Arm fields of a Michelson interferometer with Bessel expansion

For a standard Michelson convention, the (nominally dark) output port field is proportional to the difference of the arm fields $E_x(t)$ and $E_y(t)$:

$$E_{\text{out}}(t) \propto E_x(t) - E_y(t). \quad (\text{B.7})$$

The electric field for the unmodulated arm x is given by

$$E_x(t) = \hat{E}_0 e^{i(\omega_0 t + \phi_x)}, \quad (\text{B.8})$$

while the modulated arm y , expanded to first order with the modulation index m is given by

$$E_y(t) \approx \hat{E}_0 e^{i(\omega_0 t + \phi_y)} \left[1 + i \frac{m}{2} e^{-i(\omega_m t)} + i \frac{m}{2} e^{i(\omega_m t)} \right]. \quad (\text{B.9})$$

Substituting the equations from above in Equation B.7 yields

$$E_{\text{out}}(t) \propto \hat{E}_0 e^{i\omega_0 t} [e^{i\phi_x} - e^{i\phi_y}] \left(1 + i \frac{m}{2} e^{-i(\omega_m t)} + i \frac{m}{2} e^{i(\omega_m t)} \right). \quad (\text{B.10})$$

The carrier term is given by

$$E_c \propto \hat{E}_0 e^{i\omega_0 t} (e^{i\phi_x} - e^{i\phi_y}). \quad (\text{B.11})$$

By choosing the static differential phase $\phi_x \approx \phi_y$, so that the carrier is almost at the dark point, we obtain:

$$E_c \propto \hat{E}_0 e^{i\omega_0 t} (1 - 1) \approx 0. \quad (\text{B.12})$$

The sidebands do not cancel because only one arm is modulated. The total field at the output port results in:

$$E_{\text{out}}(t) \propto -i \frac{m}{2} \hat{E}_0 e^{i(\omega_0 t + \phi_y)} [e^{-i(\omega_m t)} + e^{i(\omega_m t)}]. \quad (\text{B.13})$$

With $e^{i\alpha} + e^{-i\alpha} = 2 \cos \alpha$ we obtain:

$$E_{\text{out}}(t) \propto -i m \hat{E}_0 e^{i(\omega_0 t + \phi_y)} \cos(\omega_m t + \phi_m). \quad (\text{B.14})$$

Beat signal from the leakage carrier field and modulation sidebands

Now, we consider the case in which the dark port is not perfectly dark but has a small carrier amplitude from the unmodulated arm, or in other words: it is leaking. We can write:

$$E_{\text{leak}}(t) = \epsilon \hat{E}_0 e^{i(\omega_0 t + \phi_x)}, \quad (\text{B.15})$$

with $\epsilon \ll 1$, which describes the offset from the perfect destructive interference. The total output field can be expressed as

$$E_{\text{out}}(t) = E_{\text{leak}}(t) + E_{\text{mod}}(t). \quad (\text{B.16})$$

The intensity of the output port measured by a photodetector is expressed as

$$I(t) \propto |E_{\text{out}}(t)|^2 = |E_{\text{leak}}|^2 + |E_{\text{mod}}|^2 + 2 \Re\{E_{\text{leak}} E_{\text{mod}}^*\}. \quad (\text{B.17})$$

For $\epsilon \ll 1$ and $m \ll 1$, the first two terms can be neglected, and only the cross term is dominant.

$$\begin{aligned} E_{\text{leak}} E_{\text{mod}}^* &= \epsilon \hat{E}_0^2 e^{i(\omega_0 t + \phi_x)} \left[+i \frac{m}{2} \hat{E}_0 e^{-i(\omega_0 t + \phi_y)} \left(e^{-i(\omega_m t + \phi_m)} + e^{i(\omega_m t + \phi_m)} \right) \right] \\ &= i \epsilon \frac{m}{2} \hat{E}_0^2 e^{i(\phi_x - \phi_y)} \left(e^{-i(\omega_m t + \phi_m)} + e^{i(\omega_m t + \phi_m)} \right). \end{aligned} \quad (\text{B.18})$$

The real part of this expression is:

$$2 \Re\{E_{\text{leak}} E_{\text{mod}}^*\} \propto \epsilon m \hat{E}_0^2 \cos(\omega_m t + \phi_m + \delta), \quad (\text{B.19})$$

where $\delta = \phi_x - \phi_y$ is the constant phase between the two arms. The intensity at the detector can be approximated by a constant offset I_0 plus a sinusoidal amplitude modulation at the modulation frequency.

$$I(t) \approx I_0 + I_1 \cos(\omega_m t + \phi_m + \delta), \quad (\text{B.20})$$

where I_1 denotes the amplitude of the beating term, i.e., the relative strength of the modulation. In summary, the beating of the leakage carrier field and the modulation field converts the initial single-arm phase modulation into a linear amplitude modulation at the frequency ω_m , from which we read out the differential displacement signal. When the sensitivity of our detector is sufficiently high, this modulation originates from the gravitational wave to be measured.

Geometrical tilt-to-length coupling

This appendix briefly describes the geometric tilt-to-length coupling (TTLC), particularly the *lever arm* and *piston* effect. The following derivations are based on [128]. A detailed description of the non-geometric TTLC can be found in [112].

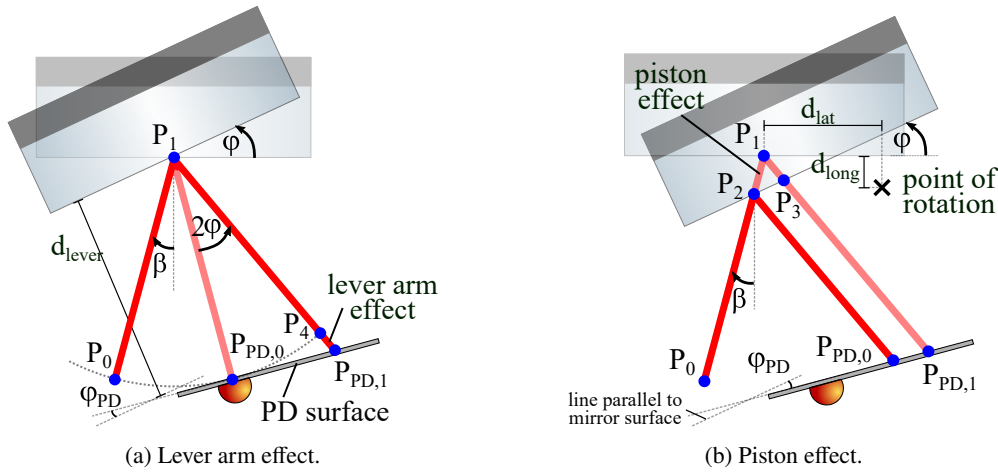


Figure C.1: Sketch of the geometrical tilt-to-length coupling (TTLC) showing the nominal and tilted beams which are reflected by a mirror and subsequently measured using a PD. The points P_1 and $P_{PD,0}$ mark the reflection on the mirror and the point of incidence on the PD for the nominal beam. The points P_2 and $P_{PD,1}$ mark the respective points when the mirror is tilted. Figure (a) shows the *lever arm effect*, where a change in path length is due to the rotation of the reflected beam axis. While P_1 is the same for the nominal and tilted case, $P_{PD,1}$ is shifted on the PD due to the tilt angle φ . The *piston effect*, shown in figure (b), accounts for additional changes resulting from the rotation of the reflective surface into or out of the beam path. The mirror rotates around a center that is shifted laterally by d_{lat} and longitudinally by d_{long} relative to P_1 . The diverted beam is sketched with a dashed line. In the sketched example, the optical beam path is thereby shortened by P_1 to P_2 to P_3 .

The TTLC due to mirror tilt is obtained by comparing the optical path length of the beam in the tilted configuration with that of the nominal beam,

$$\Delta OPL_{TTL} = OPL_{TTL} - OPL_{nominal}. \quad (C.1)$$

Fig. C.1 illustrates and describes the geometrical TTLC which is split into a *lever arm effect* and a *piston effect*. The lever arm effect is defined as the change in path length due to the rotation of the reflected beam axis. The piston effect, on the other hand, accounts for additional changes resulting from the rotation of the reflective surface into or out of the beam path.

A phase change measured on and by the PD¹ occurs because the beam coming from the tilted mirror has a different OPL compared to the nominal beam. Accordingly, any jitter caused by angular or lateral movement of the mirror leads to phase noise. However, it should be mentioned that lateral jitter only causes a path length change at an angle of incidence other than zero degrees.

The beams accumulate an optical path length change between their defined origin P_0 and the incidence point P_{PD} on the photodiode. This can be expressed by assigning a position vector $\vec{p}$ to each point P_i . We can express the nominal optical path length as

$$\text{OPL}(\varphi = 0) = |\vec{p}_1 - \vec{p}_0| + |\vec{p}_{PD,0} - \vec{p}_1|, \quad (\text{C.2})$$

and the tilted mirror as

$$\text{OPL}(\varphi) = |\vec{p}_2 - \vec{p}_0| + |\vec{p}_{PD} - \vec{p}_2|. \quad (\text{C.3})$$

The optical path length difference OPLD is thereby

$$\Delta\text{OPL}(\varphi) = \text{OPL}(\varphi) - \text{OPL}(\varphi = 0). \quad (\text{C.4})$$

As illustrated in Fig. C.1, the ΔOPL is the result of the lever arm and piston effect, and it is naturally independent of the choice of the starting point P_0 :

$$\begin{aligned} \Delta\text{OPL} &= \Delta\text{OPL}_{\text{lever}} + \Delta\text{OPL}_{\text{piston}} \\ &= (|\vec{p}_{PD,1} - \vec{p}_4|) + (|\vec{p}_2 - \vec{p}_1| + |\vec{p}_3 - \vec{p}_1|) \end{aligned} \quad (\text{C.5})$$

We can use the fact that the distance from $\vec{p}_1$ to $\vec{p}_{PD,0}$ is the same as $\vec{p}_1$ to $\vec{p}_4$ and write

$$\Delta\text{OPL}_{\text{lever}} = |\vec{p}_{PD,1} - \vec{p}_1| - |\vec{p}_{PD,0} - \vec{p}_1| \quad (\text{C.6})$$

The vector notation can also be expressed in two dimensions (2D) as²

$$\Delta\text{OPL}_{\text{lever}}^{2D} = d_{\text{lever}} [\sec(2\varphi - \varphi_{PD}) \cos(\varphi_{PD}) - 1]. \quad (\text{C.7})$$

In Equation C.5, the piston effect is described by a change in absolute length. However, the path length can increase or decrease in comparison to the non-tilted case ($\varphi = 0$). By accounting for this possibility, we can arrive at the following equation:

$$\begin{aligned} \Delta\text{OPL}_{\text{piston}}^{2D} &= 2\sec(2\varphi - \varphi_{PD}) \cos(\beta + \varphi - \varphi_{PD}) \\ &\quad \cdot \{-d_{\text{lat}} \sin(\varphi) + d_{\text{long}} [1 - \cos(\varphi)]\} \end{aligned} \quad (\text{C.8})$$

where the lateral and longitudinal displacements of the center of rotation from the nominal reflection

¹ The presence of the required reference beam for a phase measurement is not shown in order to keep the sketch simple.

² $L_{\text{tilt}} = L_{\text{nom}}/\cos(\varphi)$, $\Delta\text{OPL} = L_{\text{tilt}} - L_{\text{nom}} = L_{\text{nom}}/\cos(\varphi) - L_{\text{nom}} = a \left(\frac{1}{\cos(\varphi)} - 1 \right)$ without considering the tilt of the photodiode.

point P_1 are represented by d_{lat} and d_{long} , respectively.

In addition to the TTLC effects for reflections at a mirror, further effects occur for transmissive components (e.g. beam splitter) or rotating systems, where a freely propagating beam is perfectly stable and is incident on a jittering receiving system. An example for this would be a remote satellite in a space mission like LISA. The ΔOPL is expressed as

$$\begin{aligned} \Delta\text{OPL}_{\text{RS}}^{2D} = & -d_{\text{lat}} \sec(\varphi_{\text{PD}}) \sec(\varphi_{\text{RS}} + \varphi_{\text{PD}}) \sin(\varphi_{\text{RS}}) \\ & + d_{\text{long}} [\cos(\varphi_{\text{PD}}) \sec(\varphi_{\text{RS}} + \varphi_{\text{PD}}) - 1]. \end{aligned} \quad (\text{C.9})$$

In the case of high-precision interferometry we typically have a well aligned configuration and small angular jitter. Therefore, we can approximate the Equations C.7 and C.8 by a Taylor expansion³ to second-order around zero ($\varphi \rightarrow 0$). For simplicity, we also assume a zero photodiode tilt ($\varphi_{\text{PD}} = 0^\circ$) for the lever and piston effect. Thus, we obtain the approximated primary TTLC effects:

$$\Delta\text{OPL}_{\text{lever}}^{2D} \approx 2d_{\text{lever}}\varphi^2 \quad (\text{C.10})$$

$$\Delta\text{OPL}_{\text{piston}}^{2D} \approx -2d_{\text{lat}}\cos(\beta)\varphi + [2d_{\text{lat}}\sin(\beta) + d_{\text{long}}\cos(\beta)]\varphi^2 \quad (\text{C.11})$$

$$\Delta\text{OPL}_{\text{RS}}^{2D} \approx -2d_{\text{lat}}\varphi_{\text{RS}} + d_{\text{long}}(\varphi_{\text{RS}}^2/2 + \varphi_{\text{RS}}\varphi_{\text{PD}}). \quad (\text{C.12})$$

To describe these jitter effects, we distinguish between yaw (rotation around the vertical Z-axis, which determines horizontal facing direction) and pitch (rotation around the lateral Y-axis, which governs up/down tilt).

³ $\frac{1}{\cos(x)} = 1 + \frac{x^2}{2} + \dots$; $\frac{1}{\cos(2x)} = 1 + 2x^2 + \dots$

Mode matching

This appendix outlines the steps for mode matching of the NPRO laser, fiber collimator, and cavities. In Chapter 6, the length stability (displacement noise) of the TAPSI toolset was characterized. This involved coupling the free-space NPRO lasers into optical fibers via collimators, followed by recollimation inside the vacuum chamber and coupling into the optical cavities.

Precise spatial matching of a laser beam to another beam or optical mode is required for efficient coupling, and is referred to as *mode matching* [178]. Mode matching to an optical fiber maximizes the amount of optical power that can be coupled into the fiber. For cavities, this is particularly critical, as the buildup is directly proportional to the mode overlap and thus determines the achievable interferometric contrast. Mathematically, the quality of mode matching is quantified via the overlap integral between the input and target field modes. The coupling efficiency η is then given by the squared modulus of this integral [178]:

$$\eta = \frac{|\int E_1^* E_2 dA|^2}{\int |E_1|^2 dA \int |E_2|^2 dA}, \quad (\text{D.1})$$

where E_1 , E_2 are the complex electric fields, for example, of the input laser beam and the cavity mode¹. The integration area dA spans over the whole beam cross-section. A visualization of the expression is shown in Fig. D.1, where a lens of suitable focal length aligns the laser beam mode (red) with that of the cavity (blue).

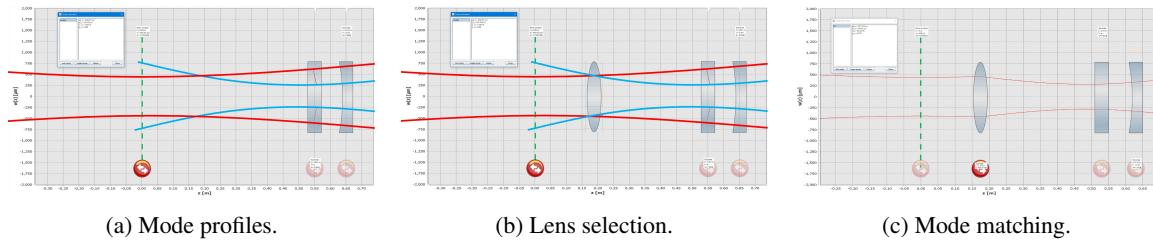


Figure D.1: Simplified illustration of mode matching. The laser source mode (red) is matched to the cavity mode (blue) via a lens of suitable focal lengths. The simulation was conducted using JamMT.

¹ The expression assumes matching temporal and polarization modes. See also Equation 2.30.

The first step toward mode matching was the measurement of the input beam profile from either the NPRO laser² or from the fiber collimator (Schäfer+Kirchhoff, *60FC-4-A4.5S-03-Ti*). This was performed using a scanning slit optical beam profiler (Thorlabs, *BP209-IR/M*). The beam radius ($1/e^2$) was measured in x and y direction at various positions in front of the laser/collimator. The data was then fitted to a Gaussian beam profile using the Python package *openqlab*, yielding the waist position and size (see Fig. D.2).

Cavity mode parameters (reference and prototype cavity, Table 6.1) were obtained using *jscav* (Linear Cavity Calculator) [179]. With both input and target modes determined, suitable lenses were identified using the simulation tool *JamMT* (Just another mode Matching Tool) [180] (see Fig. D.1). This simulation tool propagates the input beam through optics and provides a mode-matching assistant for lens selection (position and focal length). Two lenses theoretically enable 100 % mode matching in waist position *and* size. However, these dual-lens systems impose stringent positional requirements and degrade coupling efficiency significantly for even small deviations. Experiments showed a single lens is practically sufficient, maintaining >98 % efficiency over a misalignment tolerance of several centimeters.

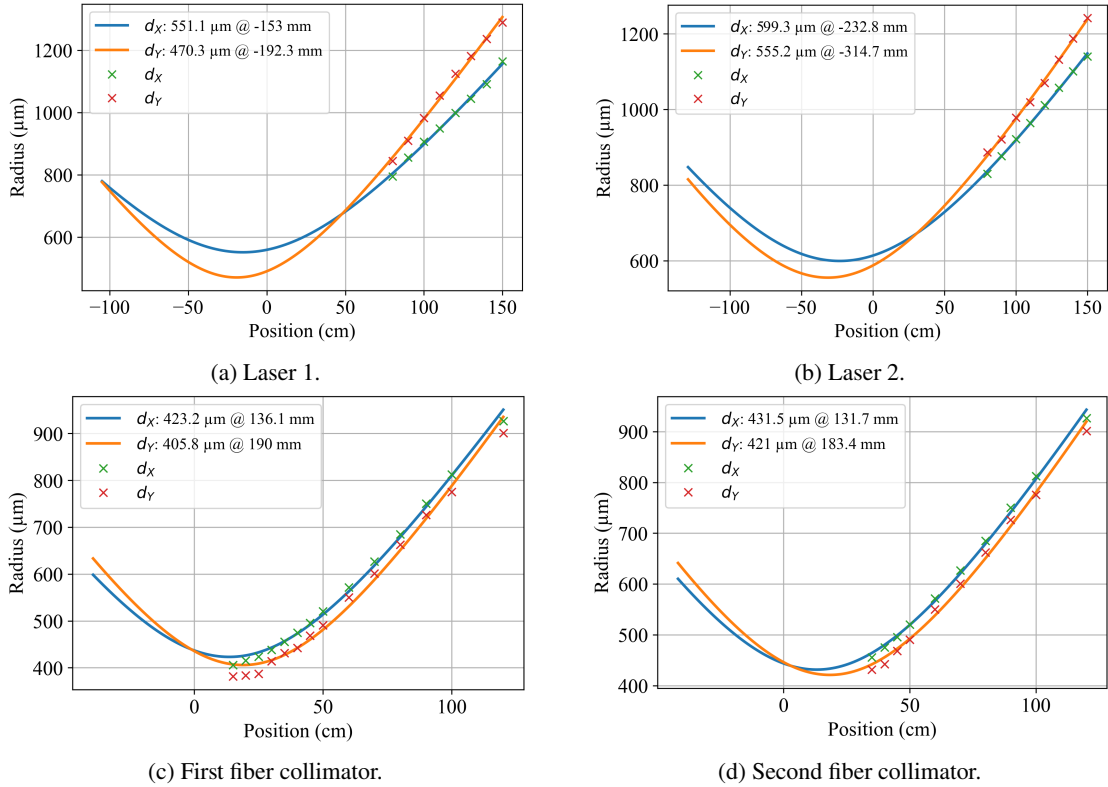


Figure D.2: Beam radius measurements (x) and corresponding Gaussian beam profile fits (solid lines). The output beam radius of both NPRO lasers and fiber collimators was measured in x and y directions at various z positions. Although identical models, the collimators showed distinct beam profiles, likely due to internal lens positioning. The Gaussian fit was computed using the Python package *openqlab* and the function `analysis.gaussian_beam.fit_beam_data(data, 1064e-9)`.

² The output beam from the laser also includes a collimation lens, as the initial NPRO divergence is excessive.

Additional Information by Chapter

E.1 Toolset for adjustable picometer-stable interferometers

This appendix provides additional information on the toolset for adjustable picometer-stable interferometers (TAPSI), particularly 3D CAD models of the constructed components, using Solid Edge. In addition to the components of TAPSI introduced in Chapter 4, such as the mount assembly and the baseplate, there are fixed mirror mounts (beam splitters), beam dumps, and photodetector mounts (PCB holders) that complete the toolset.

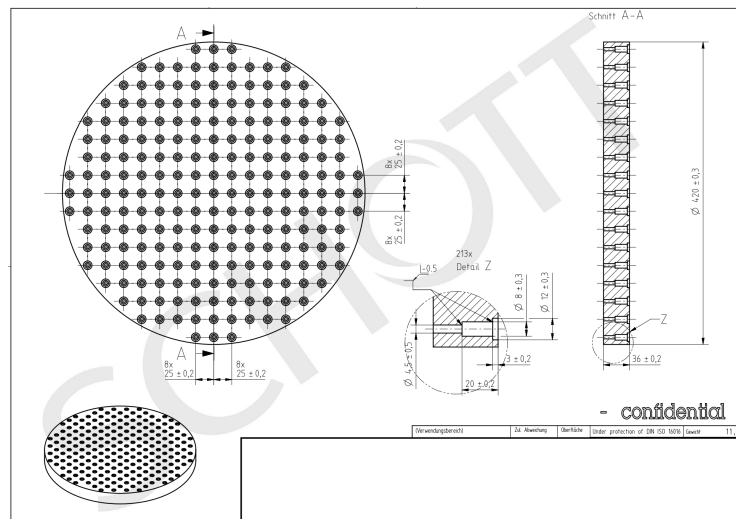


Figure E.1: CAD drawing of the Zerodur baseplate manufactured by Schott.

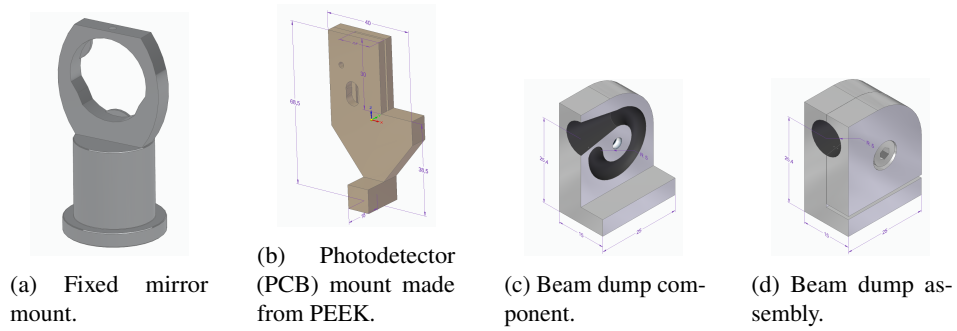


Figure E.2: CAD models of additional components of the TAPSI. The models were designed using Solid Edge. Credit for beam dump design: AEI Hannover. Dimensions in millimeters.

E.2 Optical test facility

E.2.1 Vacuum pump system

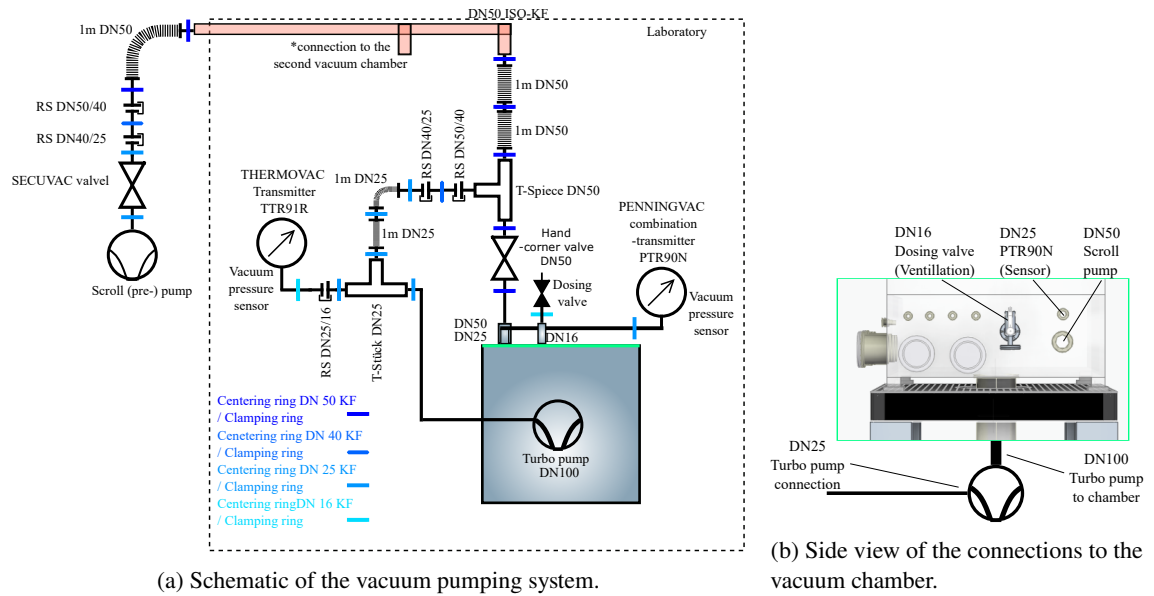


Figure E.3: The vacuum pumping system consists of a scroll pump that generates a preliminary vacuum of 1×10^{-3} mbar for the turbo pump, to achieve of high-vacuum conditions. The scroll pump is located in the room adjacent to the laboratory to minimize seismic and acoustic noise coupling to the experiment inside the vacuum chamber. Two sensors are used to monitor the vacuum pressure. The entire pumping setup is duplicated for a second vacuum chamber located in the laboratory.

E.2.2 Optical test facility 3D CAD models

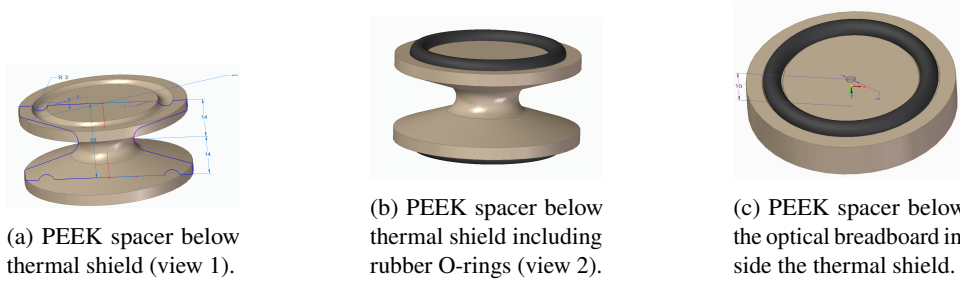


Figure E.4: CAD models of PEEK spacers, placed between thermal layers to minimize conductive heat transfer. The models were designed using Solid Edge.

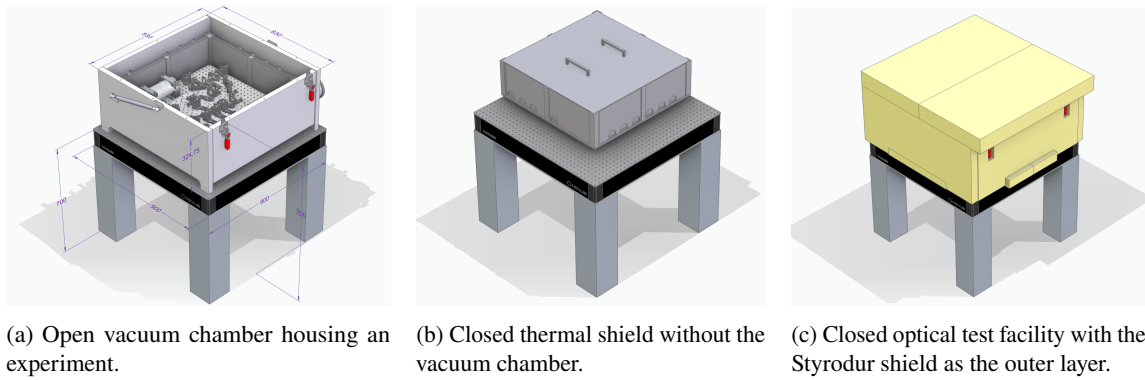


Figure E.5: CAD drawing (Solid Edge) of the optical test facility displaying the different layers.

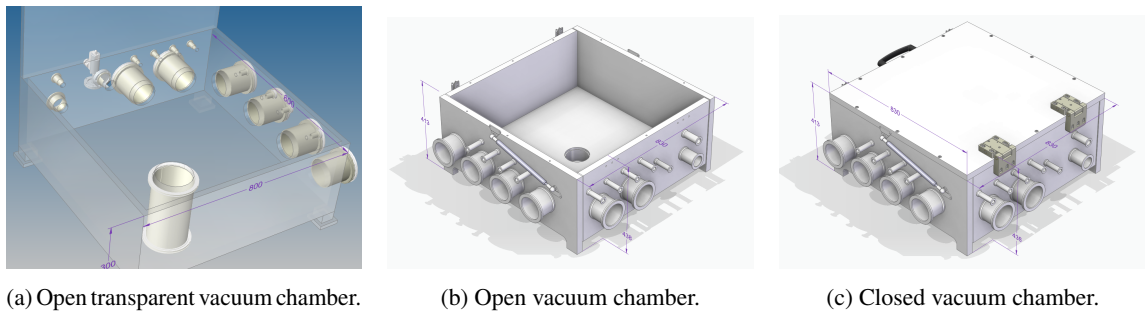
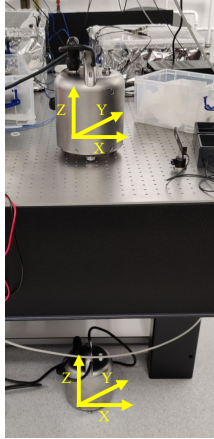


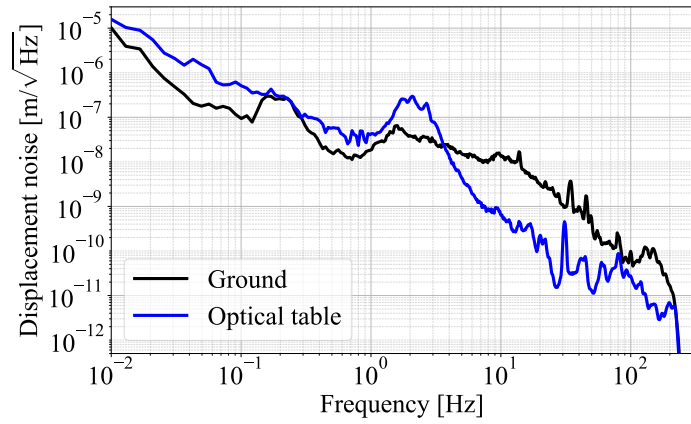
Figure E.6: CAD drawing (Solid Edge) of the vacuum chamber.

E.2.3 Seismic isolation of the laser preparation optical table

This subsection presents the seismic isolation measurements of the (large) optical table where the laser preparation setup was located next to the optical test facility. The seismic isolation was measured using two seismometers, one on the ground and the second on top of the optical table (see Fig. E.7(a)). The measurement shows a resonance peak at 1 Hz, followed by the seismic suppression at higher frequencies for all three axes.



(a) Seismometer measurement setup with one sensor placed on the laboratory floor and the other on top of the optical table.



(b) Seismic performance measurements of the (large) optical table in Z direction. (Measurement credit: Dr. Artem Basalaev.)

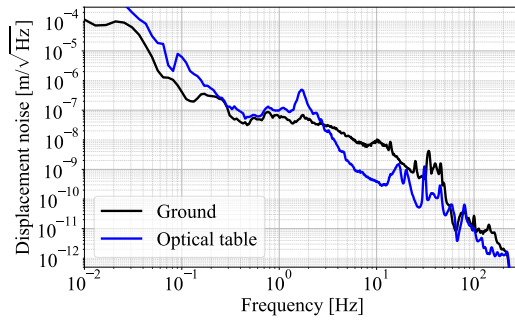


Figure E.8: Seismic performance measurements of the large optical table in X direction.

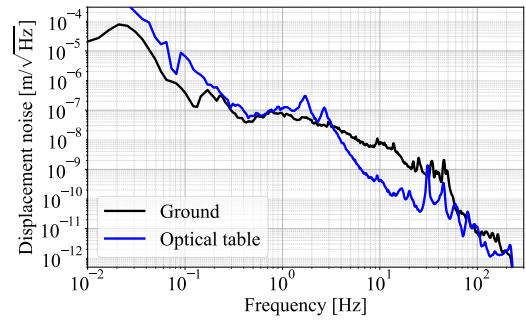


Figure E.9: Seismic performance measurements of the large optical table in Y direction.

E.3 Toolset length stability characterization

E.3.1 Laser preparation

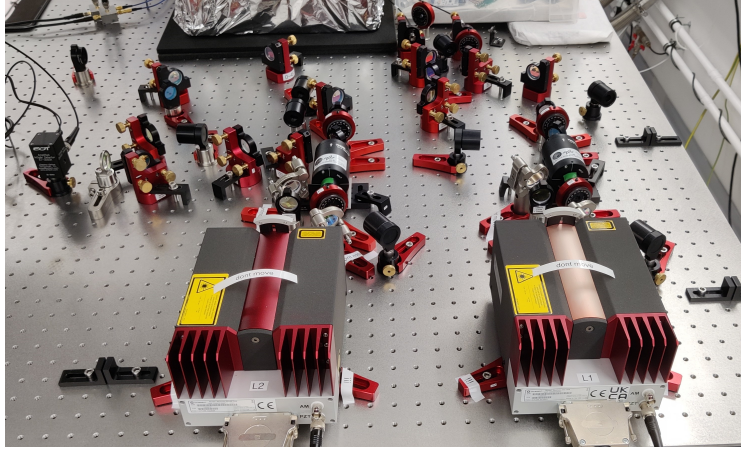
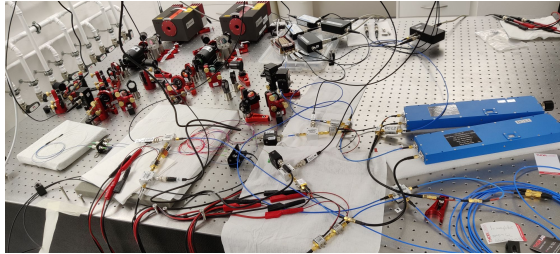
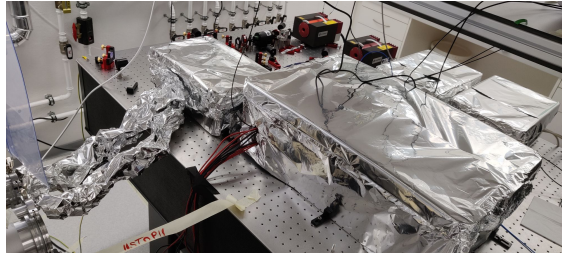


Figure E.10: Photograph of the laser preparation setup, which provides power regulation, polarization cleaning and mode matching of the NPRO lasers before coupling the laser light into the fiber collimator and feedthrough fiber into the chamber and onto the interferometer experiments. Interference between both lasers provided the beat note signal for the respective experiments (see Section 6).



(a) Initial configuration.



(b) Thermally isolated iteration.

Figure E.11: The initial configuration outside the vacuum chamber (a) was improved (b) by placing all optical fiber and electrical components (cables, PID controller, amplifier a.o.) inside thermal boxes. The thermal boxes consisted of a *useful plastic box* with foam covering on the inside and aluminium foil on the outside.

E.3.2 Pressure-refractive index coupling

The optical cavities are located in the same vacuum chamber and therefore experience the same common-mode pressure fluctuations. Nevertheless, it is important to note that pressure-refractive-index coupling is a significant limitation when the cavities are located in two different vacuum chambers. We can estimate this limitation using the following derivation:

The optical path length of the optical cavity depends on the geometrical distance and the medium between the mirrors which is air for all cavities, or more precisely vacuum. Fluctuations of such

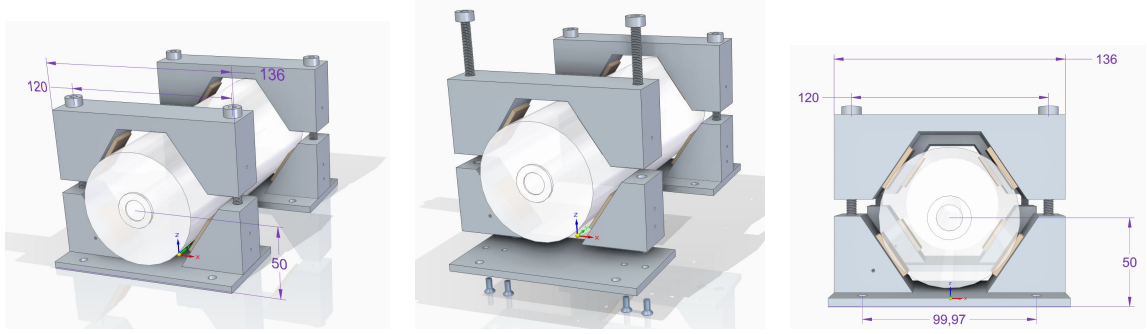


Figure E.12: CAD model (Solid Edge) of the ULE cavity holder. Dimensions given in millimeters.

vacuum causes pressure-to-refractive-index coupling. The optical path length change is given by

$$\Delta L_{\text{OPL}} = L \cdot \Delta n, \quad (\text{E.1})$$

where $L = 0.1$ m is the length of the cavity and δn the change in refractive index. The refractive index is calculated according to [181] as

$$n = 1 + 7.86 \cdot 10^{-4} \frac{p}{T} - 1.5 \cdot 10^{-11} \text{RH}(T^2 + 160), \quad (\text{E.2})$$

where T denotes the temperature in Kelvin, RH the relative humidity and p the pressure in Pascal. If we assume an ideal gas and that the change in refractive index is solely caused by a pressure change Δp we obtain:

$$\Delta n = \frac{\partial n}{\partial p} \Delta p = \frac{7.86 \cdot 10^{-4}}{298 \text{ K}} \Delta p. \quad (\text{E.3})$$

Over the course of a day the pressure deviation in the vacuum chamber is on the order of $\Delta p = 1 \times 10^{-4}$ mbar = 1 Pa. Substituting Δn into Equation E.1 and assuming a linear pressure drift over the day, lead to a pressure-refractive-index coupling of $\Delta L_{\text{OPL}} \approx 1.8 \text{ nm}/\sqrt{\text{Hz}}$ at 1 mHz following a $1/f$ slope. This length coupling would be well above the picometer-level requirement of LISA and emphasizes the importance of sharing the same vacuum chamber with the reference cavity.

E.3.3 Error signal distortion in HS

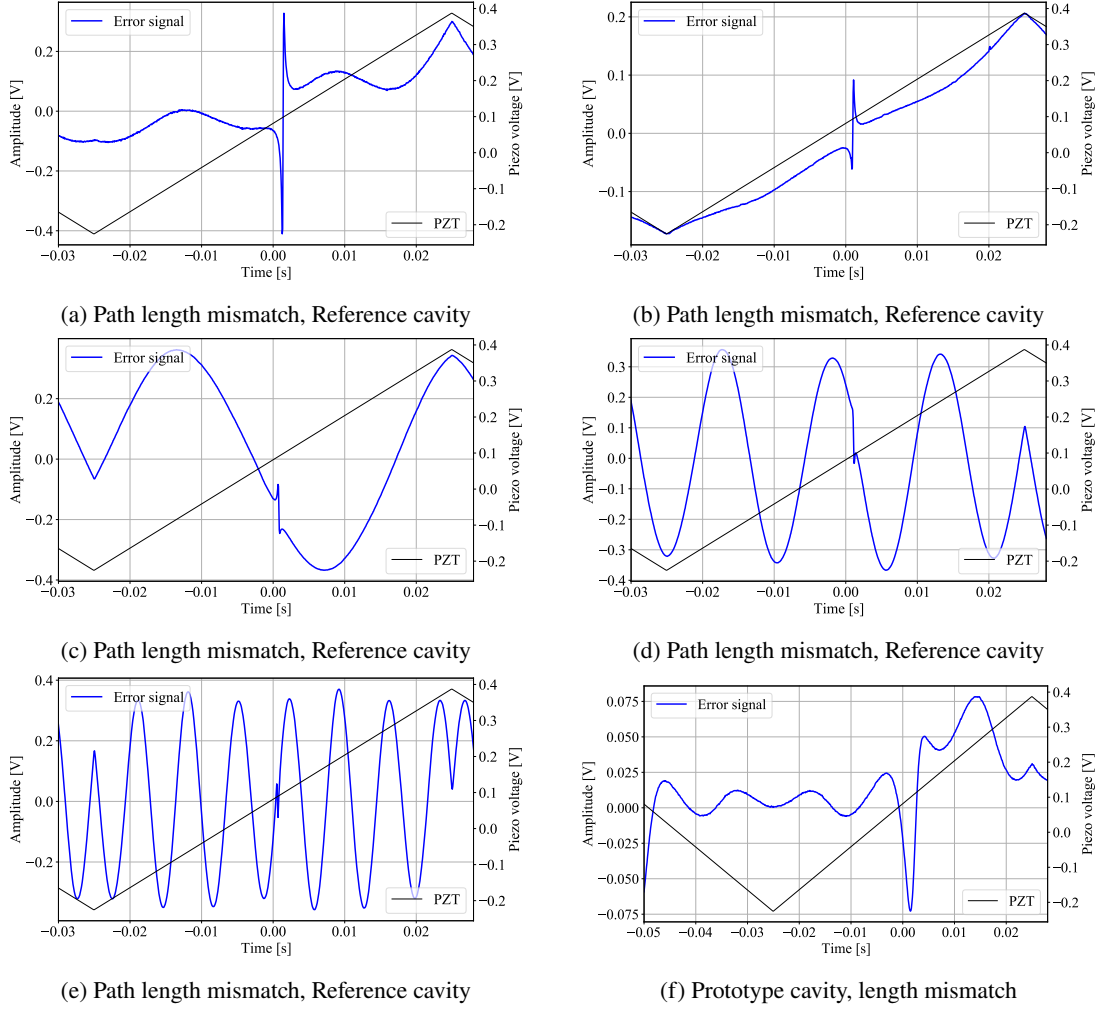


Figure E.13: Laser frequency scanning over the resonances of the cavity shows a distorted (oscillating) error signal. The path length mismatch increases from (a) to (e) in the laser-reference cavity lock. Panel (f) shows the best achievable error signal of the prototype cavity, which was set up using TAPSI.

E.4 Concealed Noise project

The measurement results from the first commissioning are shown once more in this appendix, together with the coherent subtraction results from all four signals, for comprehensive documentation. Coherent subtraction, based on [182], was performed using `spicypy` [183] and is shown in Fig. E.15. Both plots reveal excess noise contributions from both the readout and the interferometer. While further commissioning is necessary, the measurement results present the current state of the experiment as of that time.

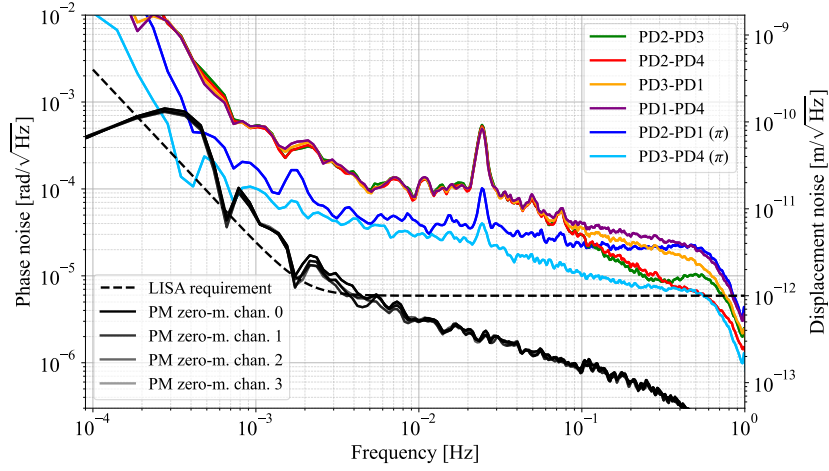


Figure E.14: Preliminary results of the Concealed Noise experiment. The subtraction of $PD_1 - PD_2$ is the π -measurement and solely contains the readout noise. The subtraction of, e.g. $PD_1 - PD_3$ is between two readout arms and includes additional interferometer noise.

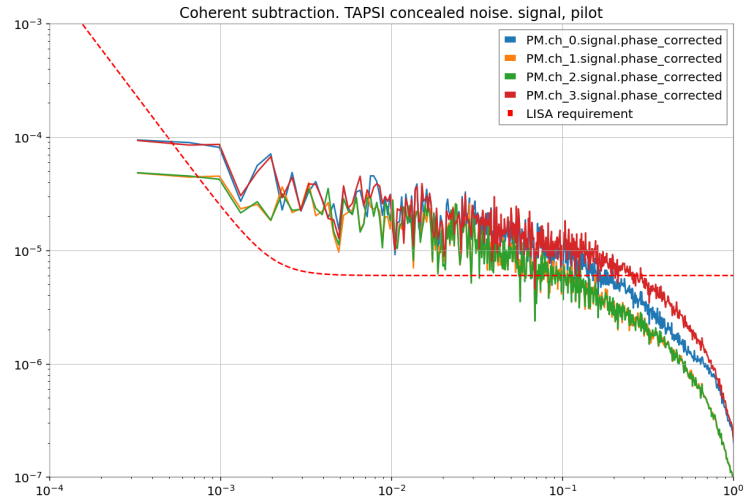


Figure E.15: Preliminary results from coherent subtraction of all four signals in the Concealed Noise experiment. The coherent subtraction cancels all common-mode noise, isolating the individual readout noise floor of the EGSE phasemeter channels. (Figure credit: [165]).

E.5 LF-HoQI signal derivation

To derive the PD signals of LF-HoQI, we can use the Jones matrix calculus which is a matrix formulation of polarized light. The formulation consists of 2x1 Jones vectors to describe the field components and 2x2 Jones matrix to describe the polarizing components. The following table (see Table E.1) lists the Jones matrices of the polarized components used.

Table E.1: Table of common Jones matrices.

Optic	General Jones matrix	Jones matrix for xx°
$\lambda/2$ -waveplate (45°)	$\begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$
$\lambda/4$ -waveplate (22.5°)	$\begin{pmatrix} \cos^2(\theta) + i \sin^2(\theta) & (1-i) \sin(\theta) \cos(\theta) \\ (1-i) \sin(\theta) \cos(\theta) & \sin^2(\theta) + i \cos^2(\theta) \end{pmatrix}$	$\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$
NPBS	$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \pm \frac{1}{\sqrt{2}} \end{pmatrix}$	
PBS_p	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	
PBS_s	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	

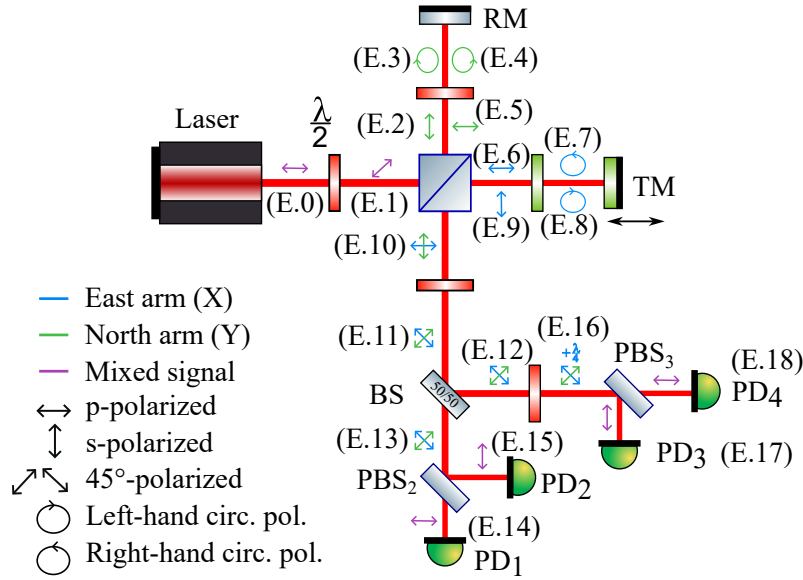


Figure E.16: Optical layout of LF-HoQI including a labeling of the electric fields at different points in the interferometer. Double-headed arrows denote polarization states, with colors corresponding to the polarization from individual interferometer arms. Reference Mass/Mirror (RM), Test Mass/Mirror (TM).

The initial electric field (E.0) from the laser is linearly polarized and rotated by the $\lambda/2$ -waveplate

by 45° . The resulting normalized electric field is defined as

$$E_1 = \frac{1}{E_0} \begin{pmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{pmatrix} = \begin{pmatrix} e^{i\phi_x} \\ e^{i\phi_y} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (\text{E.4})$$

where E_0 is the electric field amplitude and ϕ_x, ϕ_y are the respective phases from each arm. The beam polarized at (45°) is split at the cube PBS into the p- and s-polarization components. The electric field passes a $\lambda/4$ -waveplate (45°) which converts the linear polarization into a circular polarization. The reflection at the end mirror changes the sign and passing through the $\lambda/4$ -waveplate a second time converts it back into linear polarization but now with a 90° rotation. The light therefore leaves the interferometer arm in the opposite direction. The electric fields during this process are described by the following:

$$E_3 = M_{\lambda/4} \cdot E_2 = \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad (\text{E.5})$$

$$E_4 = M_{\text{mirror}} \cdot E_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad (\text{E.6})$$

$$E_5 = M_{\lambda/4} \cdot E_4 = \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{E.7})$$

$$E_{10,Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (\text{E.8})$$

The calculation for the east arm (X) is analogous.

$$E_{5,X} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{E.9})$$

At the PBS₁, the polarizations of the recombined beams from the interferometer-arms are orthogonal and therefore co-propagating without interfering. We can also introduce a phase shift $\Delta\phi$ in one of the arms (TM, p-pol.) or in other words, the arm length is changing. The relation between phase and length is the following

$$\Delta\phi = \frac{4\pi\Delta L}{\lambda} \quad (\text{E.10})$$

The electric field leaving the interferometer is therefore

$$E_{10,X,Y} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\Delta\phi} \end{pmatrix} \quad (\text{E.11})$$

where ΔL is the relative change in arm length and λ is the laser wavelength ($\lambda_{NPRO}=1\,064\text{ nm}$). The subsequent $\lambda/2$ -waveplate (22.5°) rotates the polarization by 45° .

$$E_{11} = M_{\lambda/2} \cdot E_{10,X,Y} = \frac{1}{2} \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} e^{i\Delta\phi} \\ -e^{i\Delta\phi} \end{pmatrix} \right) \quad (\text{E.12})$$

The beam is divided by the NPBS into the two readout arms.

$$E_{12} = M_{\text{NPBS}} \cdot E_{11} = i \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 1 - e^{i\Delta\phi} \end{pmatrix} \quad (\text{E.13})$$

$$E_{13} = M_{\text{NPBS}} \cdot E_{11} = \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 1 - e^{i\Delta\phi} \end{pmatrix} \quad (\text{E.14})$$

The south readout arm's electric field after the PBS₂ at the PDs is

$$E_{14} = \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 0 \end{pmatrix}, \quad E_{15} = i \frac{\sqrt{2}}{4} \begin{pmatrix} 0 \\ 1 - e^{i\Delta\phi} \end{pmatrix} \quad (\text{E.15})$$

The east readout arm is phase shifted by the $\lambda/4$ -waveplate (45°) by 90°.

$$E_{16} = M_{\lambda/4} \cdot E_{12} = \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + ie^{i\Delta\phi} \\ 1 - ie^{i\Delta\phi} \end{pmatrix} \quad (\text{E.16})$$

The y-readout arm's electric field after the PBS₂ at the PDs is

$$E_{17} = i \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + ie^{i\Delta\phi} \\ 0 \end{pmatrix}, \quad E_{18} = \frac{\sqrt{2}}{4} \begin{pmatrix} 0 \\ 1 - ie^{i\Delta\phi} \end{pmatrix} \quad (\text{E.17})$$

The measured power on the photodiode is proportional to the square of the electric field. This results in the signal of PD₁.

$$\text{PD}_3 \propto |E_{17}E_{17}^*| = |i^2 \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{i\Delta\phi} \\ 0 \end{pmatrix} \cdot \frac{\sqrt{2}}{4} \begin{pmatrix} 1 + e^{-i\Delta\phi} \\ 0 \end{pmatrix}| = \frac{1}{8}(1 + e^{-i\Delta\phi} + e^{i\Delta\phi} + 1) \quad (\text{E.18})$$

$$\text{PD}_3 \propto \frac{1}{8}(2 + e^{-i\Delta\phi} + e^{i\Delta\phi}) = \frac{1}{8}(2 + 2\cos(\Delta\phi)) = \frac{1}{4}(1 + \cos(\Delta\phi)) \quad (\text{E.19})$$

The calculation of the other three signals is done similarly, giving us all four PD signals.

$$\text{PD}_1 \propto \frac{1}{4}(1 - \sin(\Delta\phi)) \quad (\text{E.20})$$

$$\text{PD}_2 \propto \frac{1}{4}(1 + \sin(\Delta\phi)) \quad (\text{E.21})$$

$$\text{PD}_3 \propto \frac{1}{4}(1 + \cos(\Delta\phi)) \quad (\text{E.22})$$

$$\text{PD}_4 \propto \frac{1}{4}(1 - \cos(\Delta\phi)) \quad (\text{E.23})$$

Electronics – Schematics and PCBs

Table F.1: List of photodiodes and receivers used in the corresponding experiments.

Photodetector (photodiode)	Company	Experiment	Bandwidth
ET-3000A	Coherent	HS and PDH-locking	30 kHz–1.5 GHz
DET08CFC/M	Thorlabs	HS	DC–5 GHz
HomoV3-PCB (LAPD-1-09-17-TO46)	Roithner Lasertechnik	PDH locking	DC–40 MHz
TinyPD (LAPD-1-09-17-TO46)	Roithner Lasertechnik	LF-HoQI	DC–40 MHz
TinyPD (S2386-8K)	Hamamatsu	LF-HoQI	DC–35 kHz
LISA-PD (LAPD-1-09-17-TO46)	Roithner Lasertechnik	Concealed Noise	DC–40 MHz

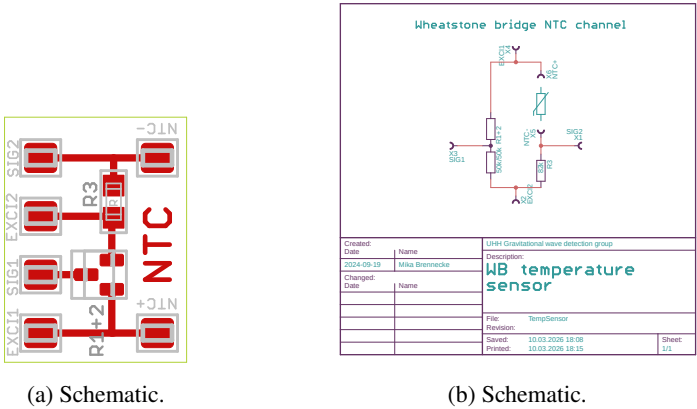


Figure F.1: PCB schematics and board (Eagle /Autodesk Fusion) of the temperature sensor. Modified from, credit: AEI Hannover.

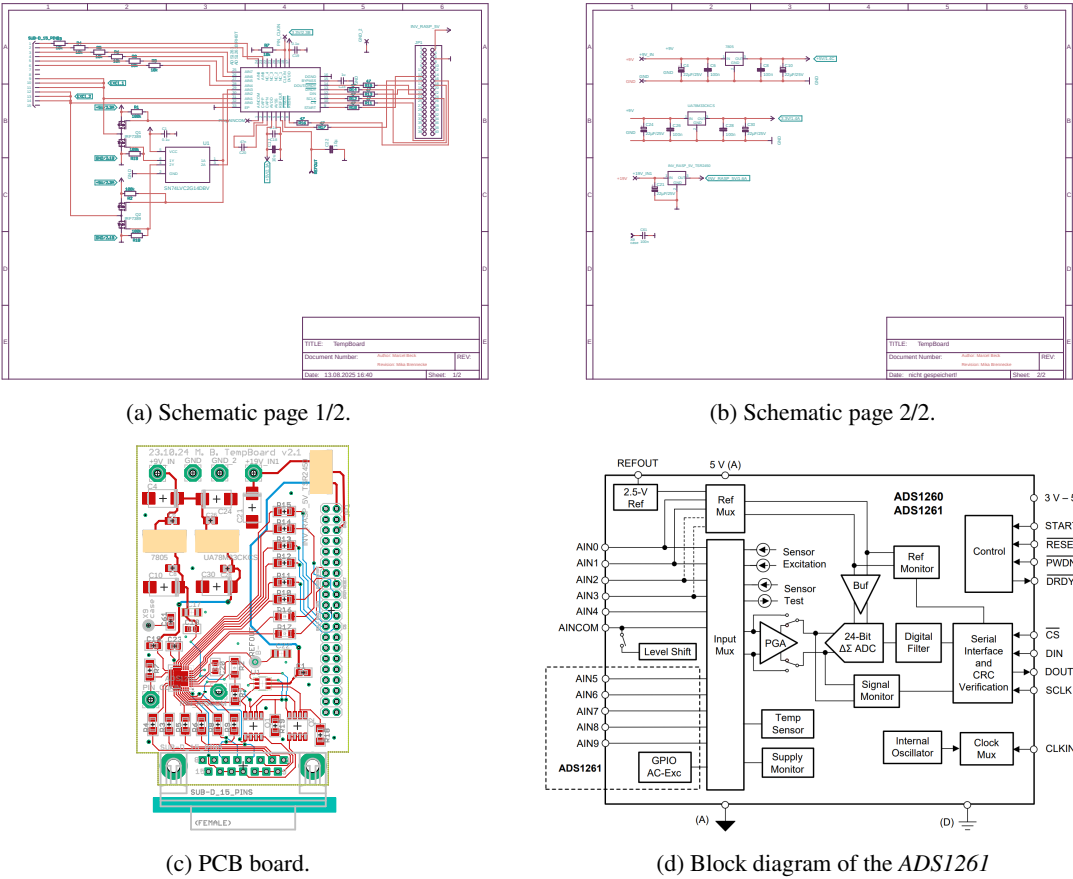
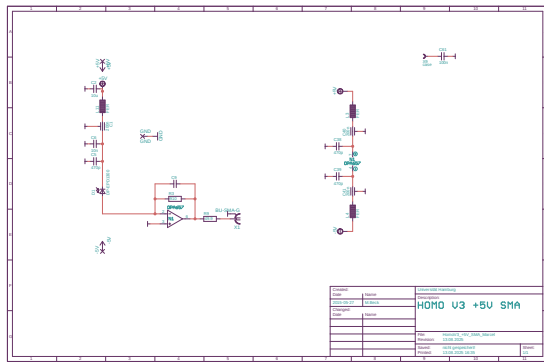
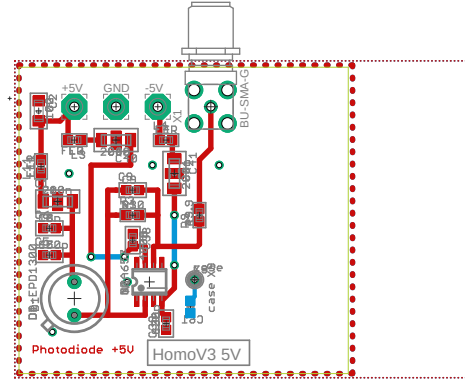


Figure F.2: PCB schematics and board (Eagle /Autodesk Fusion) of the temperature board alongside the block diagram of the *ADS1261* from Texas Instruments.

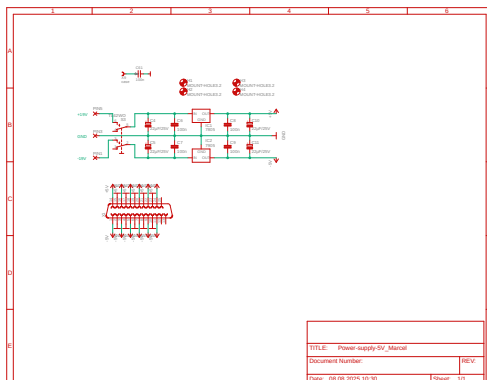


(a) Schematic.

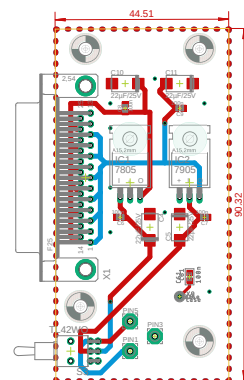


(b) PCB board.

Figure F.3: PCB schematic (Eagle /Autodesk Fusion) of the PDH signal photodiode.



(a) Schematic.



(b) PCB board.

Figure F.4: PCB schematic (Eagle /Autodesk Fusion) of power supply for the PDH signal photodetectors.

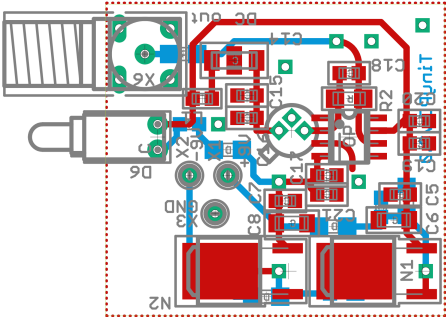
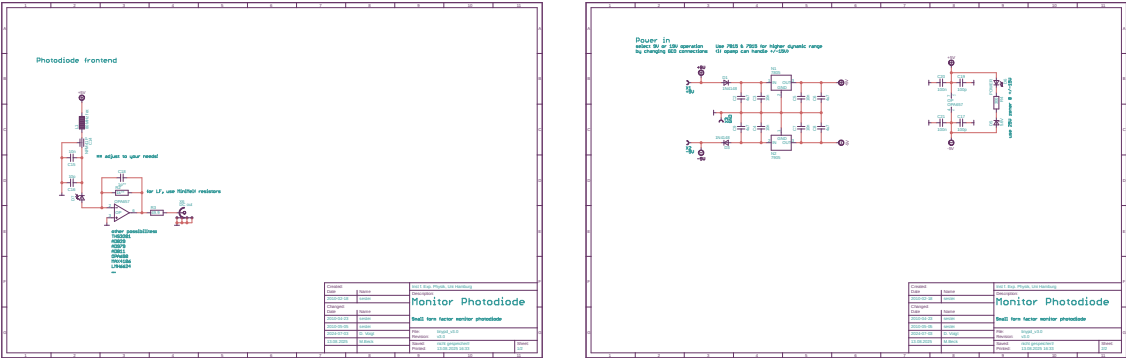


Figure F.5: PCB schematic (Eagle /Autodesk Fusion) of the LF-HoQI photodetectors.

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Acronyms

ADC analog to digital converter 10, 26, 27, 34, 55–59, 81–85, 99–102, 118, 133, 135, 142, 143, 148, 149, 204

AM amplitude modulation 35–37, 203

AOI angle of incidence 49, 50

AOM acousto-optic modulator 23, 36

APC angled physical connector 50, 119

APE annealed proton-exchanged 117

AR anti-reflection 44, 49, 144

ASD amplitude spectral density 11, 43, 53–56, 101, 133, 153, 154

BOSEM Birmingham optical sensor and electromagnetic actuator 137

BS beam splitter 21–23, 46, 68, 69, 92, 102, 107, 108, 130–132, 155, 165

CPS capacitive position sensor 137

CTE coefficient of thermal expansion 12, 54, 65–68, 71, 96, 98, 101, 116, 134, 149

DAC digital to analog converter 118

DFACS drag-free attitude control system 9, 10

DFMI deep-frequency modulation interferometry 38

EGSE electrical ground support equipment 129, 131, 132, 135, 172

EOM electro-optic modulator 37, 92, 112, 114–119, 121, 124–126

ESA European Space Agency 1, 8, 14

ET Einstein Telescope 8, 9

FPGA field-programmable gate array 27

FPI Fabry-Pérot interferometer 28, 30, 89, 203

FSR free spectral range 30, 40, 105, 110

fsr full-scale range 56, 58, 99

FWHM full width at half maximum 30, 48, 114

GR general relativity 3–5, 8

GRS gravitational reference system 9, 10, 53, 79

GW gravitational wave 1–8, 10, 11, 14, 26, 31, 33, 155, 158

GWD gravitational-wave detection 7, 21, 35, 155

Acronyms

- HoQI** homodyne quadrature interferometer 137, 140, 141, 150
- HR** high-reflection 22, 49
- HS** Heterodyne laser frequency Stabilization 15, 33, 34, 90, 92–97, 99–112, 115, 117, 118, 125, 126, 150, 177, 204, 205, I, III
- JN** Johnson-Nyquist 55, 56, 79, 81–83, 85
- KAGRA** Kamioka Gravitational-Wave Detector 8
- LF-HoQI** low-frequency homodyne quadrature interferometer 15, 137–139, 141–148, 150, 173, 205, 206
- LFN** laser frequency noise 11, 41, 42, 101, 134, 141, 142, 144–146, 150
- LGWA** Lunar Gravitational-Wave Antenna 6, 9, 14, 15, 137, 140, 142, 143, 146–150, 203, 205, I, III
- LIGO** Laser Interferometer Gravitational-Wave Observatory 1, 4, 5, 7–9, 31, 137, 203
- LISA** Laser Interferometer Space Antenna 1, 6, 8–14, 26, 27, 35, 43, 47, 53, 54, 58, 64, 68, 70, 71, 76, 79, 80, 89, 92, 96, 105, 106, 108, 111, 117, 119–121, 123, 125–127, 129, 135, 141, 149, 150, 161, 170, 203, I, III
- LO** local oscillator 27, 28, 112, 113, 118, 124, 125, 133
- LPF** low-pass filter 26, 27, 50, 82, 93, 102, 114, 119
- LPSD** Logarithmic frequency axis Power Spectral Density 154
- LSB** least significant bit 56, 58
- MI** Michelson interferometer 7, 8, 11, 21, 22, 138, 144, 155, 157, 203
- MOSA** moving optical sub-assembly 10, 11
- NASA** National Aeronautics and Space Administration 8
- NET** noise equivalent temperature 83
- NPBS** non-polarizing beamsplitter 138, 139, 175
- NPRO** nonplanar ring oscillator 19, 20, 40, 41, 85, 89–91, 96, 101, 102, 105, 106, 116, 118, 120, 126, 134, 141, 143, 163, 164, 169, 203
- NTC** negative temperature coefficient 78–80
- OGSE** optical ground support equipment 1, 11–15, 54, 63, 64, 79, 89, 148–150, I
- OpAmp** operational amplifier 122
- OPL** optical path length 68, 160
- PBS** polarizing beamsplitter 43, 44, 91, 93, 116, 121, 130, 131, 138, 139, 144, 146, 174, 175
- PCB** printed circuit board 63, 66, 78, 81, 82, 107, 120–122, 132, 144, 178, 179, 206
- PD** photodetector 21–23, 25, 34, 51, 64, 69, 92, 93, 97, 112, 113, 117, 121, 131, 135, 139, 140, 142, 143, 145–148, 150, 156, 159, 160, 173, 175
- PDH** Pound-Drever-Hall 15, 33, 34, 63, 85, 90, 92, 94, 97, 99, 109, 112, 114–127, 142, 144–146, 150, 177, 205, I, III
- PEEK** polyether ether ketone 66, 75
- PER** polarization extinction ratio 91, 146
- PGA** programmable gain amplifier 81–84
- PID** proportional-integral-derivative 33, 34, 90, 101, 102, 106, 107, 110, 118, 132
- PLL** phase-locked loop 27, 28
- PM** phase modulation 35, 36, 203
- pm** polarization-maintaining 45, 90, 91, 102, 118, 119, 131, 144, 146

- PRTD** platinum resistance temperature detector 78
- PSD** power spectral density 42, 43, 53, 153
- PZT** piezoelectric transducer 40, 91, 103, 118
- RAM** residual amplitude modulation 116, 117, 119, 121, 126
- RF** radio frequency 35, 129
- RIN** relative intensity noise 10, 43, 44, 131–133
- RMS** root mean square 55–57, 133
- SLB** spurious light beam 44–46, 49–51, 75, 89, 150, 203
- SLS** spurious light signal 41, 44, 46–52, 76, 101, 119, 120, 123, 124, 127, 135, 203
- SN** shot noise 42, 55, 133, 142
- SNR** signal-to-noise ratio 8, 14, 26, 41, 55, 58, 100, 118
- TAPSI** toolset for adjustable picometer-stable interferometers 1, 15, 63–66, 68, 71, 73–75, 89, 90, 98, 99, 101, 103, 106, 107, 111, 112, 116, 120–122, 125, 126, 129, 131, 132, 134, 135, 137, 138, 141, 143, 145, 148–150, 163, 165, 166, 171, 204, 205, I, III
- TC** temperature coefficient 78–80
- TCR** temperature coefficient ratio 78
- TDI** time-delay interferometry 11
- TE** transverse electrical 117
- TIA** transimpedance amplifier 20, 26, 122, 131, 140, 142, 144, 145
- TTLC** tilt-to-length coupling 11, 12, 15, 53, 54, 63, 64, 66–71, 76, 99, 134, 135, 149, 159–161, 204, 205
- ULE** Ultra-Low Expansion 33, 34, 64, 89, 99, 103, 111, 120, 122, 144–146, 150, 170, 206
- WB** Wheatstone bridge 78–81, 83, 204
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This thesis used the ComponentLibrary by A. Franzen:
(<https://www.gwoptics.org/ComponentLibrary/>)

Curriculum vitae

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Education

2001–2005	Grundschule an der Schäferwiese 81245 München
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2013–2017	Hochschule München 80335 Fakultät 06 Physikalische Technik – Bachelor (2,5)
2017–2020	Hochschule München 80335 Photonik – Master (1,5)

Professional experience

2016 (6 months)	Internship semester at TOPTICA Photonics
2017 (4 months)	Bachelor's thesis at Hochschule München 80335 <i>Fabrication and characterization of regenerated fiber Bragg gratings in 240 μm SM fibers</i>
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2020 (9 months)	Master's thesis at TOPTICA Photonics <i>Design and characterization of a diode-based ring resonator laser</i>
2020 (3 months)	Student Assistant at TOPTICA Photonics
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Languages

German	Mother tongue
English	Fluent
French	Basic

Publications

Peer-reviewed journal articles

M. Beck, S. Chalathadka Subrahmanya, and O. Gerberding

“Adjustable picometer-stable interferometers for testing space-based gravitational wave detectors”

Classical and Quantum Gravity, Volume 42, Number 13 (2025)

DOI 10.1088/1361-6382/ade1c7

Co-supervision

Projects co-supervised:

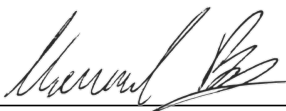
- Nurmi Sawlanski (2024)
Bachelor thesis
Unequal arm-length interferometer for laser frequency pre-stabilization
- Mika Tjoerben Brennecke (2024)
Bachelor thesis
Construction of ULE-cavities and high precision temperature sensors for laser frequency stabilisation
- Marie Christin Methling (2025)
Bachelor thesis
Investigating Homodyne Quadrature Interferometer for low frequencies
- Julian Priedemann (2025)
Bachelor thesis
Investigation of a homodyne Quadrature Interferometer for low frequency displacement sensing

Declaration of oath

I hereby declare and affirm that this doctoral dissertation is my own work and that I have not used any aids and sources other than those indicated.

If electronic resources based on generative artificial intelligence (gAI) were used in the course of writing this dissertation, I confirm that my own work was the main and value-adding contribution and that complete documentation of all resources used is available in accordance with good scientific practice. I am responsible for any erroneous or distorted content, incorrect references, violations of data protection and copyright law or plagiarism that may have been generated by the gAI.

Hamburg, 21.07.2026



Signature Marcel Beck