

Spectral and Vertical Decomposition of Oceanic Eddy Energy Pathways

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Abstract

The oceanic meso-scale eddy field, characterized by spatial scales of 10 – 200 km, is critical to the transport of heat, carbon, and momentum. It is well established that meso-scale energy is primarily generated via baroclinic instability and subsequently undergoes an inverse cascade toward larger horizontal scales, moving energy away from the small scale where dissipation takes place at $O(10^{-2} \text{ m})$. Consequently, a significant scale-gap exists between the energy-containing and dissipation scales, and how this scale is bridged remains uncertain. Energy-equation-based frameworks, like the Lorenz energy cycle, have proven valuable for deepening the understanding of energy reservoirs, conversions and transfers across scales. This study utilizes such frameworks in a hierarchy of ocean models to address the scale-gap conundrum and the uncertainties regarding meso-scale energy reservoirs and pathways in the ocean.

The first part of this thesis examines the spectral characteristics of the oceanic Lorenz energy cycle, focusing on the baroclinic conversion term. The analysis reveals a dipolar structure in spectral space, where baroclinic conversion acts as a source of eddy kinetic energy (EKE) at small horizontal scales and a sink at larger scales, partly balancing the inverse cascade of EKE. In realistic global simulations, this large-scale EKE sink is found predominantly poleward of 30° latitude in regions of eastward mean flow. In idealized models, the EKE sink appears in eastward mean flow and is absent for westward mean flow, confirming the link between the EKE sink and mean flow direction. Furthermore, the EKE sink disappears when β is set to zero, indicating that planetary Rossby waves are essential for its existence. These findings are explained by linear stability analysis, which confirms that the EKE sink only appears at large-scales for $\beta \neq 0$ and further reveals how the vertical structure of this sink is tied to the mean flow direction: In eastward mean flow, the EKE sink appears in the upper half of the water column, while it is found near the bottom for westward mean flow. Since EKE and baroclinic conversion are surface-intensified, the EKE sink near the bottom in westward flows is damped, whereas in eastward flows it is intensified, explaining the results found in the numerical models.

The second part of the thesis shifts the focus to analyzing the vertical structure of EKE using vertical Sturm-Liouville eigenmodes. Across all model configurations, and given sufficient resolution, most EKE is contained by the barotropic and first baroclinic mode, and the fundamental energy pathway remains consistent: EKE is produced by baroclinic instability in the baroclinic modes, while barotropic production remains weak. Subsequently, EKE is transferred to the barotropic mode by non-linear transfers, which are non-local in modal space with EKE transferred directly from higher modes to the barotropic mode. The presence of a pycnocline shifts baroclinic generation and non-linear transfer origins to higher modes and increases the complexity of

these transfers, by introducing an energy loop between the first and higher baroclinic modes that operates alongside the direct transfer to the barotropic mode. Within model configurations incorporating a rough bottom, topographic scattering of EKE can be locally significant, but remains small in the global integral and is of secondary importance relative to baroclinic generation and non-linear transfers. Applying a spectral decomposition of the modal EKE conversions and transfers reveals that while the baroclinic modes facilitate the large-scale EKE sink via negative baroclinic conversion, such a sink is absent in the barotropic mode. Simultaneously, non-linear transfers in modal and spectral space accumulate barotropic large-scale EKE. Consequently, the removal of large-scale barotropic EKE and its route to dissipation must rely on alternative processes. Finally, horizontal wavenumber spectra of modal eddy energy exhibit slopes that vary significantly with vertical mode and latitude, typically ranging between -2 and -5 . The analysis reveals that a clear scale separation between energy sources and sinks is absent, leading to a violation of classical inertial range assumptions and explaining the non-universality of the observed spectral slopes. By integrating modal and spectral perspectives, this work provides an unprecedentedly detailed analysis of meso-scale eddy energy pathways, representing a significant advancement in our fundamental understanding of the global ocean circulation.

Zusammenfassung

Die ozeanischen Wirbel der Mesoskala weisen räumliche Skalen zwischen 10 – 200 km auf und sind von zentraler Bedeutung für den Transport von Wärme, Kohlenstoff und Impuls. Es gilt als gesichert, dass mesoskalige Wirbelenergie primär durch barokline Instabilität generiert und durch eine inverse Kaskade zu großen horizontalen Skalen transferiert wird. Da molekulare Reibung auf kleinen Skalen ($O(10^{-2} \text{ m})$) wirkt, sind die Skala, auf der die inverse Kaskade Energie akkumuliert, und die Dissipationsskala durch mehrere Größenordnungen voneinander getrennt. Bisher ist ungewiss, wie die Wirbelenergie letztlich zur Dissipationsskala gelangt. Diagnostiken, die auf Energiegleichungen basieren, wie der Lorenz-Energiekreislauf, haben sich bereits als wertvoll erwiesen, um das Verständnis der Reservoirs, Umwandlungen und Transporte von Wirbelenergie zu vertiefen. Diese Arbeit verwendet solche energiebasierten Ansätze, um den Energietransport zwischen verschiedenen Skalen zu untersuchen und offene Fragen hinsichtlich der Pfade mesoskaliger Energie im Ozean zu klären.

Der erste Teil der Dissertation analysiert die spektralen Eigenschaften des ozeanischen Lorenz-Energiekreislaufs mit Fokus auf den baroklinen Produktionsterm. Die Analyse offenbart eine dipolare Struktur im spektralen Raum, bei der die barokline Umwandlung auf kleinen Skalen als Quelle und auf großen Skalen als Senke kinetischer Wirbelenergie fungiert. Diese Senke balanciert zum Teil die inverse Kaskade kinetischer Wirbelenergie zu großen Skalen. In realistischen globalen Modellen existiert diese Senke auf großen Skalen vorwiegend in Regionen polwärts von 30° geographischer Breite, in denen die Hintergrundströmung ostwärts fließt. In idealisierten Modellen zeigt sich die Senke kinetischer Wirbelenergie bei ostwärtiger Hintergrundströmung, fehlt jedoch bei westwärtiger Strömung, was den Zusammenhang zwischen der Energiesenke und der Strömungsrichtung bestätigt. Darüber hinaus existiert die Energiesenke nur für $\beta \neq 0$ und verschwindet, wenn $\beta = 0$ gesetzt wird. Diese Ergebnisse werden durch eine lineare Stabilitätsanalyse erklärt, welche bestätigt, dass die Existenz der Energiesenke auf großen Skalen an $\beta \neq 0$ gebunden ist. Zudem zeigt die lineare Stabilitätsanalyse den Zusammenhang zwischen der vertikalen Struktur der Energiesenke und der Richtung der Hintergrundströmung: Bei ostwärtiger Strömung befindet sich die Senke nahe der Oberfläche, während sie bei westwärtiger Strömung in der unteren Hälfte der Wassersäule liegt. Da Wirbelenergie und barokline Umwandlung oberflächenintensiviert sind, wird die Energiesenke in westwärtiger Strömung nahe des Bodens gedämpft, während sie in ostwärtiger Strömung verstärkt wird. Dies erklärt die Beobachtungen in den numerischen Modellen.

Der zweite Teil der Dissertation fokussiert sich mittels Sturm-Liouville-Eigenmoden

auf die vertikale Struktur der kinetischen Wirbelenergie und deren Transfers. Unabhängig von der Modellkonfiguration — eine ausreichende horizontale und vertikale Auflösung vorausgesetzt — konzentriert sich die meiste kinetische Wirbelenergie im barotropen und im ersten baroklinen Eigenmode. Der fundamentale Energiepfad bleibt dabei konsistent: Kinetische Wirbelenergie wird durch barokline Instabilität in den baroklinen Eigenmoden generiert und von dort durch nichtlinearen Transfer in den barotropen Eigenmode transferiert. Dieser Transfer ist im Raum der vertikalen Eigenmoden nicht lokal, da Energie direkt von den höheren baroklinen Eigenmoden in den barotropen Eigenmode transferiert wird. Die Präsenz einer Pyknokline verschiebt die Generation von kinetischer Wirbelenergie sowie den Transfer zum barotropen Eigenmode hin zu höheren baroklinen Eigenmoden und erhöht die Komplexität der Transfers, da parallel zum direkten Energietransfer in den barotropen Eigenmode ein Energiekreislauf zwischen dem ersten und den höheren baroklinen Eigenmoden entsteht. Der durch Topografie induzierte intermodale Energietransfer kann in Modellen mit rauem Boden lokal signifikant sein, ist global integriert jedoch klein im Vergleich zur baroklinen Produktion und zu nichtlinearen Transfers. Die modal-spektrale Analyse zeigt, dass die baroklinen Eigenmoden auf großen Skalen eine Senke für kinetische Wirbelenergie durch negative barokline Umwandlung aufweisen, der barotrope Eigenmode hingegen nicht. Gleichzeitig akkumuliert der nichtlineare Energietransfer im modalen und spektralen Raum barotrope kinetische Wirbelenergie auf großen Skalen. Daher müssen Senken barotroper kinetischer Wirbelenergie und ihr Pfad zur Dissipation auf anderen Prozessen beruhen. Schließlich zeigen horizontale Wellenzahlspektren der modalen Wirbelenergie Steigungen, die signifikant mit dem vertikalen Mode und der geographischen Breite variieren und typischerweise zwischen -2 und -5 liegen. Die Analyse verdeutlicht, dass eine klare Skalentrennung zwischen Energiequellen und -senken fehlt, was die Annahmen quasigeostrophischer Turbulenztheorie verletzt. Dies erklärt das Fehlen universeller Wellenzahlspektren in diesen Simulationen. Durch die Integration modaler und spektraler Perspektiven liefert diese Arbeit eine beispiellos detaillierte Analyse der Pfade mesoskaliger Wirbelenergie und stellt einen bedeutenden Fortschritt in unserem grundlegenden Verständnis der globalen Ozeanzirkulation dar.

Publications related to this dissertation

Jan Dettmer and Carsten Eden, (2025): “Mechanisms of Baroclinic Eddy Damping on Large Scales”, published in *Journal of Physical Oceanography*

Jan Dettmer and Carsten Eden, (2026): “Vertical Structure of Eddy Kinetic Energy Transfers”, submitted to *Journal of Physical Oceanography*

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Introduction

The ocean circulation is driven by surface fluxes of momentum, heat and freshwater predominantly at large horizontal scales on $O(1000 \text{ km})$. The horizontal scales of the circulation itself range from these largest scales over intermediate scales associated with equatorial, zonal and lateral boundary currents and various instabilities down to the smallest scale, given by the Kolmogorov scale, where molecular friction takes place and dissipates energy at $O(10^{-2} \text{ m})$. The ocean and its general circulation are a central component of the Earth system, as it stores and transports vast amounts of heat, carbon, freshwater, and momentum, thereby strongly influencing climate variability. Consequently, the fidelity of climate projections depends on our ability to understand and represent the transfer of tracers and energy across the vast range of dynamically active scales in the ocean - yet, the mechanisms governing these transfers are not yet fully elucidated. It is the goal of this thesis to quantify these multiscale interactions and transfers by developing a framework that decomposes the oceanic energy cycle into its horizontal scales and vertical structure, thereby bridging the gap between large-scale forcing and small-scale dissipation and providing a more complete picture of the pathways through which meso-scale energy is redistributed.

The different dynamically active scales of the ocean circulation are not separated. Rather, they interact and exchange energy, momentum, and tracers across a broad range of spatial and temporal scales. This scale coupling arises because oceanic flow is neither steady nor laminar, but predominantly turbulent in nature. Such turbulence results from the dominance of non-linear advection over weak molecular dissipation, leading to a strongly non-linear, multiscale flow. Turbulent flows contain eddies spanning a wide range of sizes, where the upper bound is often determined by the forcing scale or the domain scale, and the lower bound by viscosity. In the ocean, this turbulence occurs within a forced-dissipative system and is constrained by several properties such as planetary rotation and stratification, with the largest scales being eight orders of magnitude larger than the molecular dissipation scale. In steady state, energy at the large-scales must be dissipated at the small Kolmogorov scale and we presently do not know the processes by which energy cascades across this large-scale gap.

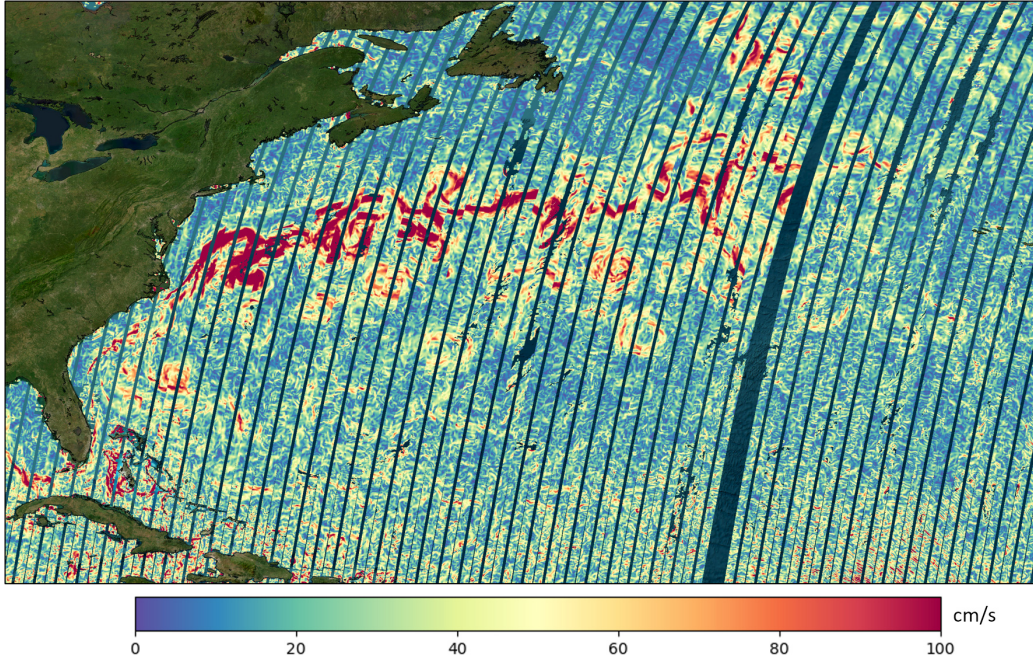


Figure 1.1: Geostrophic surface velocities in the North Atlantic region derived from KaRIn sea surface height anomaly measurements from the Surface Water and Ocean Topography (SWOT) mission. The Ka-band Radar Interferometer (KaRIn) instrument provides two-dimensional swath altimetry and is gridded in a multi-mission product. The lines arise from the 21-day sampling pattern of the orbit, where swaths from multiple passes are interwoven to construct a single composite image. From (Dibarboure et al., 2025).

This study focuses on the generation mechanisms, pathways and cascades of energy in the oceanic meso-scale ($O(10^2 \text{ km})$), which is characterized by eddies with horizontal scales of $O(10\text{--}100 \text{ km})$ and timescales ranging from days to months. Fig. 1.1 shows geostrophic surface velocities derived from sea surface height measurements and illustrates the spatial structure and variability of meso-scale eddies in the North Atlantic ocean. The behavior of these individual eddies is inherently unpredictable and the flow is therefore typically described in terms of statistical properties, such as energy spectra and scale-dependent energy transfers. To characterize these transfers, this work formulates spectrally and vertically decomposed energy equations and applies them within a hierarchy of numerical ocean models, thereby improving the understanding of how meso-scale energy is generated and redistributed across horizontal scales and vertical structure. Since this redistribution and conversion of energy across scales and between reservoirs is inherently linked to the turbulent nature of meso-scale dynamics, the following section reviews the fundamental concepts of oceanic turbulence relevant to this work.

1.1. Quasigeostrophic turbulence

Classical three-dimensional turbulence is characterized by a forward cascade of energy from large to small scales, where it is dissipated. However, the assumptions underlying 3D turbulence theory are often inadequate for oceanic flows on the meso-scale and larger, where planetary rotation, stable stratification and a small aspect ratio constrain the dynamics and suppress vertical velocity and render the flow approximately two-dimensional or quasigeostrophic. This motivates the need for different theories, which take these constraints into account. Two dimensional turbulence theory is based on the simultaneous conservation of kinetic energy and enstrophy, which is the square of vorticity. The conservation of both invariants constrain the non-linear interactions and lead to a forward cascade of enstrophy toward small scales and an inverse cascade of kinetic energy toward large-scales. The forward cascade of enstrophy can be understood as the increase of vorticity gradients as a fluid parcel is being stretched by the turbulent motion, until enstrophy is dissipated at small scales.

Similarly, quasigeostrophic (QG) turbulence theory, which applies for small Rossby numbers and horizontal scales close to the Rossby radius, can be considered quasi two dimensional, since vertical motion is very small compared to horizontal motion, but compared to two dimensional turbulence theory it allows for vertical structure, making it more realistic. It is based on the conservation of potential enstrophy and total energy, which is the sum of kinetic and available potential energy. Like two dimensional turbulence theory it predicts a forward cascade of now potential enstrophy and an inverse cascade of total energy, which reduces to kinetic energy in the barotropic limit.

Several key aspects of the quasigeostrophic (QG) turbulence regime are well understood. In particular, the primary generation mechanism of the coherent meso-scale eddies that populate this turbulent field, as well as their characteristic propagation properties, are well established within the theoretical and observational framework. While the meso-scale eddy field is characterized by non-linear turbulence, its primary generation mechanism is understood through linear stability theory, which describes how energy is first injected into the system baroclinic, or vertical shear instability. It is described by Eady (1949) for uniform stratification on a f -plane and by Charney (1947) on a β -plane and by Phillips (1954) for two discrete layers. The Eady problem admits a fully analytical solution, including a growth rate and a fastest-growing wavelength. Later, Smith (2007) demonstrated that the ocean is baroclinically unstable over most regions and that the Eady growth rate closely matches observed meso-scale variability timescales, supporting the view that baroclinic instability is the principal source of eddy kinetic energy.

Mesoscale eddies are ubiquitous in satellite observations of sea surface height (SSH). Their predominantly westward propagation and phase speeds are broadly consistent with the dispersion relation of linear Rossby waves (Chelton and Schlax, 1996). While this similarity motivates the idea to interpret the meso-scale eddies

as linear Rossby waves and to apply arguments of Rossby wave theory, they were later found to be inherently non-linear (Chelton et al., 2011), leading to systematic discrepancies between linear theory and observed propagation speeds, particularly poleward of 20° latitude. Nonetheless, the eddies retain important Rossby-wave-like characteristics, and coexist and are influenced by Rossby waves. This inherent non-linearity allows for the exchange of energy between different scales. Furthermore, the inverse energy cascade can be understood as a consequence of the non-linear (triad) interaction of Rossby waves and it is also found in satellite observations of sea surface height (SSH), as well as in numerical meso-scale eddy-resolving ocean models for scales between about 150 and 1000 km (Scott and Wang, 2005; Wang et al., 2015; Eden, 2007) suggesting that this behavior is a robust feature of QG turbulence.

The meridional variation of the Coriolis parameter introduces additional constraints on quasigeostrophic turbulence. In the presence of a planetary vorticity gradient β , Rossby waves can coexist and interact with the turbulent eddy field, as first described by Rhines (1975). Wave–turbulence interaction occurs when turbulent eddy and Rossby wave temporal and spatial scales overlap, allowing for a transfer of kinetic energy from eddies to waves. Equating the Rossby wave frequency with the inverse of a characteristic eddy timescale yields the scale of the wave–turbulence boundary, known as the Rhines scale. Rossby waves propagate predominantly in the zonal direction, making them inherently anisotropic (directionally dependent). When this characteristic is taken into account, a tendency toward the formation of zonal jets emerges. In two-dimensional wavenumber space, the wave–turbulence boundary forms a dumbbell-shaped region (see Fig. 3 in Vallis and Maltrud, 1993). Inside the dumbbell (i.e. at wavenumbers smaller than the wave–turbulence boundary), the Rossby wave frequency exceeds the characteristic turbulent frequency, so Rossby wave turbulence interactions are strongly inhibited. As a result, when turbulent eddy energy cascades inversely toward smaller wavenumbers, it cannot efficiently enter the interior of the dumbbell but instead continues to cascade toward a small zonal wavenumber k_x . In physical space, this spectral anisotropy manifests as the formation of zonal jets.

Once formed, zonal jets have a tendency to be self-sharpening. Eddies act to homogenize the meridional gradient of potential vorticity, which consists of contributions from planetary vorticity (β) and relative vorticity, but are not able to do so completely. Instead, bands of nearly constant potential vorticity form, creating a partially homogenized flow separated by sharp vorticity gradients. Within homogenized regions the potential vorticity gradient is weakened, facilitating Rossby wave breaking and enhanced mixing, whereas the sharp gradients act as barriers to further mixing. This feedback constitutes a self-reinforcing mechanism that maintains and sharpens zonal jets. In summary: as turbulent eddies cascade inversely toward larger spatial scales, they encounter the Rhines scale, where eddies and Rossby waves overlap in time and space, allowing energy to be transferred, channeling the inverse cascade into Rossby waves and self-sharpening zonal jets.

The Rossby wave frequency and phase propagation depend on the meridional

gradient of the Coriolis parameter β and therefore on latitude. As latitude increases, Rossby wave frequency decreases, and there exists a critical latitude poleward of which first-baroclinic Rossby waves are too slow to overlap in time with the turbulent eddy field, inhibiting wave–turbulence interaction. Theiss (2004) show that wave–turbulence interaction weakens as latitude increases and as the ratio of the Rossby radius to the Rhines scale decreases. At the critical latitude, the Rossby radius and Rhines scale are close. This theory is consistent with analyses of eddy length scales in the North Atlantic Ocean, where eddies are found to be elongated in the zonal direction and close to the Rhines scale equatorward of the critical latitude, but remain more isotropic and close to the Rossby radius poleward of it (Eden, 2007). A comparable transition emerges when the wavelength of linear first baroclinic Rossby waves is adjusted to match the observed phase speed of SSH signals. At low latitudes, such a match is possible, and the resulting adjusted length scale is close to the Rhines scale. At higher latitudes, however, the required length scale approaches the first baroclinic Rossby radius, or the observed speeds exceed the theoretical limit, making a match impossible (Tulloch et al., 2009). Further support for such a regime transition is provided by observations of Jupiter and Saturn, where alternating zonal jets are observed near the equator, while more isotropic turbulence dominates toward the poles (Theiss, 2006). On Earth, zonal jets are observed in the tropical and subtropical Atlantic, Indian, and Pacific Oceans (Schott et al., 2003; Maximenko et al., 2005), and although they weaken with increasing latitude, they can still be detected at higher latitudes (van Seville et al., 2011).

To summarize: the well known features of quasigeostrophic turbulence and meso-scale eddies include their generation by baroclinic instability, their Rossby wave like propagation characteristics, the importance of non-linear interactions leading to an inverse cascade of kinetic energy, and the ability of Rossby waves, given their time and space scales overlap with those of the turbulence, to redirect the inverse cascade toward zonal anisotropy and the formation of self-sharpening zonal jets.

1.2. Mesoscale eddy parameterizations

In today’s standard climate models with a horizontal resolution of about 50 – 100 km meso-scale eddies are not resolved, so their effect on the large-scale ocean circulation has to be parametrized. Their generation mechanism by baroclinic instability can be understood as the conversion of available potential energy stored in sloped large-scale isopycnals to kinetic energy by flattening of the isopycnal slope. This process can be parameterized as isopycnal thickness diffusion (Gent et al., 1995). At the same time baroclinic instability can be understood as a vertical downward flux of horizontal momentum, which can be parameterized by interior eddy form stress (Greatbatch and Lamb, 1990). Additionally climate models often implement backscatter parameterization (Bertoglio, 2005; Leith, 1990; Mason and Thomson, 1992), that re-inject energy removed by overly dissipative viscous schemes at the grid

scale back at the large-scale. Despite originally being motivated by over dissipation at the grid scale, backscatter parameterizations also represent in some sense the physical process of the inverse cascade in QG turbulence (Bachman, 2019).

1.3. Spectral scalings of quasigeostrophic turbulence

As discussed previously an important feature of quasigeostrophic turbulence are the spectral fluxes of energy and a central objective of quasigeostrophic turbulence theory is to predict spectral slopes, flux directions, and turbulent eddy mixing. The first deformation (Rossby) radius plays a fundamental role as the scale at which available potential energy is converted into kinetic energy by baroclinic instability. If sufficient scale separation exists between forcing, dissipation, and domain scales, inertial ranges may emerge in which energy is neither generated nor removed and non-linear interactions dominate. Consequently, universal power-law spectra can be predicted (Kraichnan, 1967; Leith, 1968; Batchelor, 1969). In statistically steady state, energy must be removed at large-scales (e.g. bottom drag) and enstrophy at small scales (e.g. viscous or biharmonic dissipation).

One of the earliest theoretical treatments of these ideas is due to Charney (1971), who predicts a k^{-3} kinetic energy spectrum for quasigeostrophic turbulence at scales smaller than the deformation radius, with approximate equipartition between kinetic and available potential energy. Subsequent work (Salmon, 1980; Larichev and Held, 1995; Held and Larichev, 1996) placed this result in a forced–dissipative framework, where baroclinic instability injects energy near the deformation radius. This triggers a dual cascade: a downscale enstrophy transfer with a k^{-3} spectrum, and an upscale total energy cascade yielding a $k^{-5/3}$ spectrum. The inverse cascade continues until it is arrested by large-scale friction or modulated by the Rhines scale on a β -plane. The former sets the characteristic energy-containing eddy scale, while the latter can lead to the emergence of zonostrophic turbulence with $E(k) \propto \beta^2 k^{-5}$ (Galperin et al., 2001; Danilov and Gurarie, 2002; Vallis, 2017).

These theoretical predictions raise the question of how the power-law spectral scalings and spectral fluxes are found to be in ocean observations. Analyses of satellite measurements of sea surface height (SSH) spectra confirm the existence of an inverse kinetic energy cascade and a forward enstrophy cascade (Scott and Wang, 2005). However the reported spectral slopes vary substantially. Steep slopes near -5 have been reported (Stammer, 1997), while other studies find slopes between -3 and $-5/3$ in limited ranges (Khatri et al., 2018; Storer et al., 2022, 2023). Wavenumber spectra of velocity and temperature from acoustic Doppler current profiler measurements in the Gulf Stream indicate a k^{-3} slope in the meso-scales (Wang et al., 2010). Recent results from SWOT indicate even steeper balanced spectra, between approximately -4 and -6 (Zhang and Callies, 2025). Thus, although the observed slopes are often in the predicted range, a robust, universal spectrum with two well separated inertial ranges ($-5/3$ and -3) is rarely observed. This variability is commonly attributed to

insufficient scale separation between forcing, arrest, and dissipation scales, which are not sharply localized in wavenumber space, and therefore prevents the formation of classical inertial ranges.

The spectral inertial-range framework therefore rests on assumptions that are often only marginally satisfied in the ocean. A key assumption is spectral locality of the non-linear interactions. This can be assessed by considering the contribution of motions at all wavenumbers smaller than a given k to the strain rate. However, for a $-5/3$ spectrum the contribution from each octave increases with wavenumber and peaks near k , consistent with locality. For a -3 spectrum, however, each octave contributes equally, indicating marginal or non-local behavior Vallis (2017).

However, the physical presence of coherent structures often violates this theoretical prediction: Danilov and Gurarie (2001) demonstrate that the inverse energy cascade is intrinsically non-universal and highly sensitive to the form of friction. While local interactions drive the classical forward cascade, the inverse transfer is often associated with spectrally non-local interactions. These interactions directly couple the forcing scale to the scale where energy accumulates, primarily through the emergence of coherent vortices. Such vortices cause deviations from the $-5/3$ slope and only when they are suppressed by stronger friction does the spectral slope approach the classical prediction. On a β -plane, jets form only when the Rhines scale is smaller than the frictional arrest scale (Danilov and Gurarie, 2002). If β is weak or the friction is strong the turbulence remains isotropic. Thus, β redistributes energy anisotropically but cannot halt the inverse cascade without friction.

These limitations motivate alternative theoretical descriptions that do not rely on inertial-range assumptions. Vortex-gas theory (Onsager, 1949; Thompson and Young, 2006; Gallet and Ferrari, 2021) represents the flow as an ensemble of interacting coherent vortices, whose non-linear interactions drive an inverse transfer of energy. In this framework, characteristic length scales and eddy mixing rates are not determined by universal inertial-range arguments, but emerge from vortex statistics and are strongly controlled by bottom friction and the planetary vorticity gradient β .

In summary, classical quasigeostrophic turbulence theory predicts dual energy–enstrophy cascades with characteristic slopes (-3 , $-5/3$, or zonostrophic -5 regimes), organized around the deformation and Rhines scales. Observations confirm the direction of cascades but do not always reveal universal inertial ranges. Spectral slopes and energy containing scales are highly sensitive to the type and strength of friction and β .

1.4. Mesoscale eddy energy pathways

The inverse cascade of kinetic energy in quasigeostrophic turbulence leads to accumulation of kinetic energy at the large-scale. Consequently a large-scale gap emerges between the large energy containing scale and the small scale, where viscous dissipation becomes effective. This separation raises a fundamental question: by which

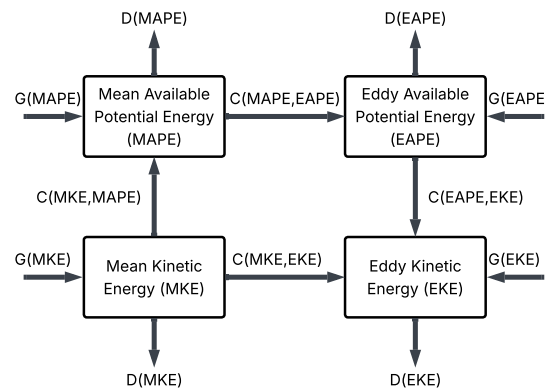


Figure 1.2: Schematic of the oceanic Lorenz energy cycle showing its four energy reservoirs: mean available potential energy (MAPE), mean kinetic energy (MKE), eddy available potential energy (EAPE), and eddy kinetic energy (EKE). The arrows indicate the pathways for energy generation (G), conversion between reservoirs (C), and dissipation (D).

mechanisms is large-scale kinetic energy ultimately dissipated? Several pathways have been proposed for the dissipation of meso-scale eddy energy. These include spontaneous emission of internal gravity waves by balanced meso-scale eddies, loss of balance via symmetric or convective instabilities, lee-wave generation over rough topography, frictional losses in surface and bottom boundary layers, and the interaction of eddies with lateral boundaries (Müller et al., 2005). Numerical studies suggest that spontaneous gravity wave emission is likely of minor importance (Eden et al., 2019). Other interactions between internal waves and the balanced flow may contribute to dissipation (Barkan et al., 2021), but their dynamical mechanisms and global relevance remain poorly constrained. At the same time, commonly employed dissipation schemes in ocean models are only loosely connected to the underlying physics. While these models must dissipate energy to maintain numerical stability and achieve a steady state, schemes such as (bi-)harmonic viscosity are frequently implemented for these practical reasons rather than as a representation of specific physical processes. Consequently, the fact that models effectively remove energy numerically does not resolve the physical scale-gap problem. Instead, this discrepancy highlights the need for a detailed investigation of scale-dependent pathways of oceanic meso-scale energy.

1.4.1. Lorenz energy cycle

A common approach for diagnosing oceanic energy pathways is to derive energy equations from the momentum and buoyancy equations. A prominent example is the Lorenz energy cycle (LEC) (Lorenz, 1955), which comprises four energy reservoirs: mean and eddy available potential energy, and mean and eddy kinetic energy. Within each reservoir, energy may be generated, dissipated, or exchanged with the other reservoirs. Originally developed for the atmosphere, the LEC has subsequently

proven to be a useful diagnostic framework for the ocean as well. Using a global eddy-resolving ocean model, von Storch et al. (2012) diagnose the LEC in the ocean as shown in Fig. 1.2. The dominant energy pathway is found to be associated with baroclinic instability. In this pathway, eddy energy is generated from available potential energy stored in sloping isopycnals, which are maintained by large-scale, wind-driven Ekman pumping. The conversion associated with baroclinic instability is from eddy available potential energy to eddy kinetic energy and occurs simultaneously with the conversion from mean available potential energy to eddy available potential energy. In contrast, the conversion from mean kinetic energy to eddy kinetic energy, which is linked to barotropic instability, is found to be much smaller than the eddy kinetic energy production due to baroclinic instability.

A key conceptual difficulty in applying the LEC to the ocean concerns the choice of averaging framework. Averaging at fixed depth, as done by von Storch et al. (2012), can lead to spurious mixing between different water masses (McDougall and McIntosh, 2001; Eden, 2012), whereas isopycnal-thickness-weighted averaging avoids unphysical mixing and provides a physically more consistent description of tracer and mass budgets. This approach introduces a mean advection velocity that differs from the Eulerian mean velocity used in the z-level based formulation, with consequences for the definition of mean kinetic energy in the LEC. In practice, applying such a formulation to z-level ocean models requires interpolation onto density surfaces, which introduces additional errors and ambiguities and complicates the diagnosis of energy conversions (Aiki and Richards, 2008; Aiki et al., 2016). While analysis in density space reveals pathways hidden in the z-level formulation, such as the role of layer-thickness form drag in removing the energy of vertically sheared geostrophic currents that balances the work of the eddy induced overturning circulation in the Southern Ocean (Aiki and Richards, 2008), the fundamental energetic description remains consistent across frameworks: baroclinic instability, here manifested through the layer-thickness form drag, remains the primary energy pathway that generates the meso-scale eddy field. Consequently, while recognizing the conceptual advantages of thickness-weighted averaging, this work adopts the z-level formulation of the LEC to ensure methodological simplicity and comparability with previous studies, and to maintain consistency with the vertical coordinate system of the underlying ocean model.

1.4.2. Spectrally decomposed energy pathways

Wang et al. (2019) extend the analysis of the LEC to spectral space and diagnose it in an eddy-resolving ocean model. They find that baroclinic conversion (Fig. 1.2 C(EAPE,EKE)) exhibits a dipolar structure in wavenumber space, with energy being generated at scales close to the first baroclinic Rossby radius and removed at larger scales. This large-scale sink of eddy kinetic energy is not accounted for by classical theory and has two important implications. Firstly, it partially counteracts the inverse cascade of kinetic energy and thereby, secondly, provides a potential pathway to

dissipation, since potential energy is expected to cascade forward to smaller scales where dissipative processes may take place. A similar dipolar structure in the exchange between potential and kinetic energy is found by Roullet et al. (2012) in an idealized channel model of a baroclinically unstable flow. They put forward the idea of a closed energy loop between kinetic energy, cascading upscale and potential energy cascading downscale, connected by the baroclinic instability term. Lobo (2025) finds a similar structure of the conversion term between eddy available potential energy and eddy kinetic energy in a two-layer QG model and similarly Tro et al. (2025) in a continuously stratified QG model of decaying turbulence for the conversion term between available potential energy and kinetic energy. While these studies demonstrate that scale-dependent diagnostics of the LEC reveal processes hidden in physical-space averages, several open questions regarding the large-scale eddy kinetic energy sink via negative baroclinic conversion remain. A comprehensive understanding of the geographical distribution and the underlying physical mechanisms of negative baroclinic conversion in a global, realistic ocean model is still lacking. This study addresses this gap by investigating the spectral LEC in a hierarchy of ocean models ranging from global, realistic to highly idealized configurations.

1.4.3. Vertically decomposed energy pathways

These studies resolve the structure of the eddy energy conversions and transfers in horizontal spectral space and naturally raise the questions of how eddy energy is distributed and converted across the vertical structure of geostrophically turbulent flows, motivating an analysis in terms of vertical modes. The vertical modes often used for such an analysis are the Sturm-Liouville eigenmodes derived from the linearized Boussinesq equations under the assumptions of a rigid lid and a flat bottom Olbers and Eden (2013). It has been shown that oceanic horizontal kinetic energy is mostly contained by the barotropic and first baroclinic vertical mode (Wunsch, 1997).

Vallis (2017) argue that in stratified quasigeostrophic turbulence the three-dimensional wavenumber is relevant. As a consequence, the energy cascade to larger horizontal scales is generally accompanied by a cascade to larger vertical scales, corresponding to a barotropization of the flow Vallis (2017). This inverse cascade in the vertical and its dependence on stratification is studied by Smith and Vallis (2001), who analyze numerical simulations of freely decaying and later forced quasigeostrophic turbulence (Smith and Vallis, 2002). They show that kinetic energy is transferred efficiently toward lower vertical modes in the case of uniform stratification, whereas this transfer is significantly reduced when a strong pycnocline is present. For uniform stratification, energy within each vertical mode cascades to its corresponding deformation radius, where it is transferred directly to the barotropic mode. In contrast, for nonuniform, surface-intensified stratification, energy contained in higher modes is first transferred to the first baroclinic mode near its deformation radius, before subsequently being transferred to the barotropic mode at the first

baroclinic deformation radius. Shevchenko and Berloff (2017) partially confirm this modal pathway in multi-layer quasi-geostrophic simulations, where energy in high baroclinic modes can also be transferred directly to the barotropic mode without transfer through the first baroclinic mode. Furthermore, while an inverse energy transfer dominates, in configurations with high vertical resolution a simultaneous forward energy transfer from lower to higher modes is identified.

Mesoscale eddy energetics and energy transfers between vertical modes have also been examined by Tedesco et al. (2022) in a regional model of the Agulhas system using a vertical mode eddy kinetic energy budget. By separating barotropic and baroclinic contributions, they show that meso-scale variability along the path of the strong mean currents is dominated by the barotropic and first baroclinic modes, whereas higher baroclinic modes become increasingly important in regions of strong eddy-topography interaction. Their modal EKE budget reveals several distinct energy pathways, including transfers toward bottom-intensified mean flows, dissipation through bottom friction, and non-linear transfers to higher baroclinic modes. Away from the continental shelf, they identify an inverse cascade in modal space, consistent with Smith and Vallis (2001). Owing to strong local eddy generation, the Agulhas Current region remains a net source of meso-scale eddy energy, in contrast to the prevailing view that meso-scale eddies predominantly decay near western boundaries.

The vertical structure of meso-scale eddy energetics has also been analyzed in a resolution based model hierarchy by Yankovsky et al. (2022). It is shown that as resolution increases from $1/4^\circ$ resolution to $1/32^\circ$ resolution and eddies become increasingly resolved, kinetic energy shifts toward the barotropic mode reflecting more efficient barotropization of the flow. It is proposed that at coarse resolution, unresolved eddy dynamics lead to suppressed vertical energy fluxes and trapping of energy in higher baroclinic modes. The sensitivity of this partitioning is especially pronounced for the eddy part of kinetic energy and depends strongly on the underlying dynamical regime: regions characterized by weak, baroclinic mean flows exhibit pronounced resolution dependence, whereas strongly baroclinic regimes, such as the Antarctic Circumpolar Current, are comparatively insensitive. Despite these insights into modal eddy kinetic energy distribution, conversion and transfers, the specific nature of these transfers, their global relevance, in particular of topographic scattering and their sensitivity to stratification remain insufficiently explored.

Kinetic energy spectra are traditionally diagnosed using Fourier transforms applied to horizontal subdomains. This approach is adopted here, as the division into subdomains allows for the analysis of horizontal variability across a large domain, while maintaining direct comparability with the vast majority of previous studies. While the coarse graining framework (Aluie et al., 2018), which diagnoses scale-dependent energy fluxes by spatial filtering, offers an alternative that avoids the assumptions of homogeneity and periodicity required by Fourier-based methods, both approaches have been shown to yield consistent results for oceanic meso-scale

flow (Storer et al., 2022). Since geographically resolved coarse-graining diagnostics remain technically challenging and are rarely implemented in oceanic applications, the Fourier approach is the most suitable framework for the large-scale analysis conducted in this study.

Similarly, multiple approaches exist for the vertical decomposition of oceanic flow. Empirical orthogonal functions (EOFs) provide a statistically driven basis that optimally represents variance and can efficiently capture dominant vertical structures. However, EOF modes depend on the specific dataset and domain, and lack a direct physical motivation. An alternative is the use of Sturm–Liouville eigenmodes Olbers et al. (2012), obtained from the vertical structure eigenvalue problem that arises from separation of variables in the linearized, hydrostatic, rotating Boussinesq equations, typically under the assumptions of a rigid lid and a flat bottom. These modes form a consistent basis and are directly linked to the deformation radii. Here, the Sturm–Liouville framework is adopted, as it provides a physically interpretable modal decomposition that allows for comparison with previous studies (Smith and Vallis, 2001, 2002; Tedesco et al., 2022; Yankovsky et al., 2022).

In summary, the studies discussed above demonstrate that an energy-equation-based framework provides valuable insight into meso-scale energy reservoirs and pathways in the ocean. They consistently identify baroclinic instability as the primary source of eddy kinetic energy, with energy transferred from mean available potential energy to eddy available potential energy and subsequently to eddy kinetic energy. This eddy kinetic energy undergoes an inverse cascade toward larger horizontal scales, where it is arrested. Beyond horizontal transfers, an inverse cascade is also found in vertical mode space, corresponding to a barotropization of the flow whose efficiency depends sensitively on the stratification profile. Additionally at sufficiently high resolution, interactions with topography redistribute energy toward higher baroclinic modes and enhance modal exchanges. Spectral analyses of the baroclinic conversion term show a dipolar structure in wavenumber space, with kinetic energy generated near the deformation radius and removed at larger scales. This large-scale sink could potentially relieve the problem of energy accumulation at the large-scales by the inverse cascade and the unclear route of eddy energy to dissipation. However, as identified above, the global relevance, geographical variability, dynamical regime dependence, and underlying mechanism of this large-scale eddy kinetic energy sink remain insufficiently understood and are the primary focus of this work.

Similarly, significant gaps remain in the understanding of vertical mode energy pathways. While kinetic energy production by baroclinic instability and inverse transfers in modal space with the tendency toward barotropization have been documented, the understanding of the underlying interactions of these transfers, topographic scattering and sensitivity to stratification, as well as global relevance remains insufficient. It is the focus of this study to address these questions and uncertainties in the current understanding of meso-scale energy pathways.

1.5. Thesis overview

This thesis comprises three parts: Two publications, each focusing on a different approach to study meso-scale energy pathways, one chapter that presents and discusses additional results and a concluding chapter.

Mechanisms of baroclinic eddy damping on large-scales

In this chapter the oceanic LEC and in particular the baroclinic conversion term are resolved spectrally in a hierarchy of ocean models ranging from a realistic global model to idealized channel models. Thereby addressing the current uncertainties regarding the physical mechanisms and dynamical regime dependence of the dipolar spectral structure in baroclinic conversion term that acts to remove kinetic energy at the large-scales found by Wang et al. (2019) and Roullet et al. (2012). In a global model the spatial variability of this large-scale eddy kinetic energy sink is described and linked to dynamical regime parameters. The findings are explained by the use of idealized simulations and linear stability analysis, thus providing relevant new insights into this large-scale eddy kinetic energy sink.

Vertical Structure of Eddy Kinetic Energy Transfers

In this chapter, the oceanic LEC is projected onto vertical Sturm-Liouville Eigenmodes and diagnosed across a hierarchy of ocean models. By resolving the vertical structure of meso-scale energy reservoirs and transfers, this work uncovers a robust modal energy pathway that emerges consistently across all considered model configurations. Furthermore, the non-linear inter-modal energy transfers are detailed, with a specific focus on how their structure is modulated by the stratification profile.

Modal-Spectral eddy kinetic energy properties and transfers

In this chapter the spectral slopes of eddy energy are diagnosed and discussed and additional details of the modal-spectral eddy energy pathway are presented.

Conclusions

Finally the thesis is concluded by discussing the insights presented in the previous chapters in an integrated way. The findings are put into the broad perspective and ideas for future research are presented.

2

Mechanisms of baroclinic eddy damping on large scales

This chapter is a reprint of the publication:

Jan Dettmer and Carsten Eden (2025): “Mechanisms of baroclinic eddy damping on large scales” in Journal of Physical Oceanography

Abstract

The oceanic Lorenz energy cycle and its baroclinic eddy conversion B are analysed in realistic global and idealized models. We find a dipolar structure of B in spectral space, acting as source of eddy kinetic energy at smaller, and as sink ($B < 0$) at larger scales, partly balancing the inverse kinetic energy cascade. Together with a forward potential energy cascade, this opens a potential pathway for small scale eddy dissipation.

In the realistic model, $B < 0$ is found predominantly poleward of 30° latitude, where the zonal mean current $\bar{u}$ tends to be eastward. Simulations using idealized models also show a connection between $B < 0$ and the sign of $\bar{u}$: $B < 0$ consistently appears for $\bar{u} > 0$, but diminishes for $\bar{u} < 0$. When the meridional gradient of planetary vorticity β is set to zero, $B < 0$ disappears, suggesting that planetary Rossby waves are also essential for its existence.

Linear stability analysis can explain the findings: it shows no $B < 0$ for $\beta = 0$, but $B < 0$ at large-scales when $\beta \neq 0$. It also shows that the vertical structure of $B < 0$ changes with the sign of $\bar{u}$: for eastward currents, $B < 0$ is located in the upper half of the water column, whereas for westward currents, it shifts to the bottom. Since eddy energy and B are surface intensified, the near-bottom energy sink at large-scales by B for $\bar{u} < 0$ is damped and becomes unimportant, while the near-surface $B < 0$ for $\bar{u} > 0$ is amplified, which we see in turn in the models.

2.1. Introduction

Satellite observations of the ocean surface are dominated by eddies with a diameter of about 5 – 150 km and time scales of days to months. They are also seen in numerical models with a sufficient spatial resolution. These so called meso-scale eddies are relevant for transporting energy, temperature, salinity and other tracers in the ocean. Many question regarding their generation and propagation characteristics are well understood. It however remains mostly unclear how their energy is dissipated. The goal of this study is to improve the understanding of eddy energy pathways in the ocean, by a spectral analysis of energy reservoirs, transports, and exchanges.

Comparing the spatial scale of meso-scale eddies to predictions from analytical models (Eady, 1949; Phillips, 1954; Charney, 1947) suggests that they are mostly generated by baroclinic instability (Smith, 2007; von Storch et al., 2012; Böning and Budich, 1992). This instability can be understood as the release of available potential energy through the flattening of sloping isopycnals, often parameterized as isopycnal thickness diffusion (Gent et al., 1995), which converts potential energy into eddy kinetic energy. In parallel, baroclinic instability leads to the vertical downward redistribution of horizontal momentum, parameterized by interior eddy form stress (Greatbatch and Lamb, 1990).

The meso-scale eddies are geostrophically balanced and can to first order be described and understood by the quasigeostrophic (QG) approximation. QG theory predicts an forward cascade of (potential) enstrophy and an inverse cascade for kinetic energy. Conversely to the forward cascade of kinetic energy in the three-dimensional turbulence regime (Kolmogorov, 1941), here kinetic energy cascades away from the small scales, where dissipation takes place. The inverse cascade of eddy kinetic energy is found in observations of sea surface height and meso-scale eddy resolving ocean models for scales between about 150 – 1000 km (Scott and Wang, 2005; Wang et al., 2015; Schlösser and Eden, 2007). As a consequence of the inverse cascade, kinetic energy accumulates at the large scales, which raises two questions: What process arrests the cascade and sets the scale of the energy maximum, and how is the kinetic energy at the large scale dissipated?

There are several possible mechanisms to dissipate the eddy energy: Spontaneous emission of gravity waves by the geostrophically balanced meso-scale eddies, loss of balance by convective or symmetric instabilities, lee wave generation at rough topography, boundary layer friction at the surface and bottom or breakdown of eddies at lateral boundaries are among them. More recent studies of spontaneous gravity emission using numerical models suggest that this process is of little importance (Eden et al., 2019). Other interactions between gravity waves and balanced flow might be important for eddy dissipation (Barkan et al., 2021), but the mechanism and the global relevance remains unclear.

The dissipation parameterisations implemented in state of the art ocean models, however, are quite likely wrong. For example the widely used (bi-)harmonic friction is not related to any of the processes mentioned above and often only used for numerical

reasons. To conclude, the unknown route of eddy energy to dissipation and the partly unphysical dissipation parameterisations used in ocean models motivates the need for a better understanding of energy pathways in the ocean.

One way to analyse the energy pathways in ocean and atmosphere is the Lorenz energy cycle (LEC) (Lorenz, 1955). The LEC consists of four energy reservoirs: Mean and eddy available potential energy, and mean and eddy kinetic energy. For each reservoir, energy can be generated, dissipated, or exchanged with other reservoirs. The concept was developed for the atmosphere and later proved useful for the ocean as well. von Storch et al. (2012) diagnose the LEC from a global eddy resolving ocean model and find the dominant energy pathway to be related to baroclinic instability, which produces eddy energy from available potential energy stored in sloping isopycnals, that are maintained by large scale wind driven Ekman pumping. The conversion related to baroclinic instability is from eddy available potential energy to eddy kinetic energy and occurs hand in hand with the conversion from mean available potential energy to eddy available potential energy. The conversion from mean kinetic energy to eddy kinetic energy is related to barotropic instability, which is found to be much smaller than eddy kinetic energy production by baroclinic instability by von Storch et al. (2012).

Once generated, the turbulent meso-scale eddy field is influenced by Rossby waves (Rhines, 1975) and also the inverse energy cascade can be understood as a consequence of the non-linear (triad) interaction of Rossby waves. Rhines (1975) shows that for stratified two-dimensional isotropic turbulence on a β -plane, Rossby waves create self-sharpening zonal flows of alternating sign and an associated background potential vorticity (PV) gradient (McIntyre, 2015). The Rossby waves, however, can only interact with the turbulent eddies if their temporal and spatial scale overlaps (Tulloch et al., 2009). The spatial scale at which this interaction takes place is estimated by the Rhines scale. Theiss (2004) shows that, since the Rossby wave frequency depends on latitude, the ability of Rossby waves to interact with the turbulent eddies ceases as latitude and the ratio of the Rhines scale to the Rossby radius increases. At the critical latitude, polewards of which this interaction does not take place anymore, the Rhines scale matches the Rossby radius. On celestial bodies like Jupiter and Saturn, alternating zonal winds are indeed observed around the equator which decrease with latitude, and isotropic meso-scale turbulence is found in the polar regions (Theiss, 2006).

Zonal jets can also be identified in the tropical and subtropical Atlantic, Indian, and Pacific Oceans (e.g. Schott et al., 2003; Maximenko et al., 2005). In the zonal jet regime of the North Atlantic Ocean, the Rossby waves deform the eddy length scales and elongate them in the zonal direction (Eden, 2007). There, the eddy length scale is close to Rhines scale, while polewards of the critical latitude, the eddy length scales are close to the Rossby radius. A similar regime change is found when assuming linear QG dynamics and adjusting the horizontal length scale of first baroclinic Rossby waves to match the phase speed of observed SSH signals (Tulloch et al., 2009). At lower latitudes, the phase speed can be matched and the adjusted spatial scale is close to the Rhines scale, while at higher latitudes, the adjusted spatial scale is close to the

first baroclinic Rossby radius or the adjustment becomes impossible. In agreement to this regime change, zonal jets are less prominent as latitude increases, but they can be still present (e.g. van Seville et al., 2011).

Wang et al. (2019) extend the analysis of the LEC of an eddy-resolving ocean model to spectral space. They find that eddy kinetic energy production by baroclinic instability shows a dipolar structure in wavenumber space: It produces eddy kinetic energy at scales close to the first baroclinic Rossby Radius, but removes eddy kinetic energy at larger scales. The eddy kinetic energy sink at larger scales is not explained by the classical theory and has two consequences: First, it partly balances the inverse cascade of K , and second, it opens a possible route to dissipation, since potential energy is expected to have a forward cascade to smaller scales where dissipative processes are more likely. Rouillet et al. (2012) find a similar dipolar structure of the exchange term between potential and kinetic energy in an idealised channel model of baroclinic turbulence. They propose the idea of a closed energy loop between kinetic energy and its inverse cascade and potential energy and its forward cascade, connected by the baroclinic instability term.

To summarise: The ocean surface is dominated by meso-scale eddies whose dynamics can be described as the interplay of turbulence and Rossby waves. Two important features are first the latitude dependent interaction between the turbulent eddies and Rossby waves, and second the inverse eddy kinetic energy cascade from the scale where eddy kinetic energy is generated by baroclinic instability (close to the Rossby radius) to the larger scales where the energy has to be dissipated or be converted to other forms of energy. There are, however, not many possible mechanism that can produce dissipation at large scales.

The aim of this study is to spectrally resolve the LEC and in particular the baroclinic instability conversion as in Wang et al. (2019) and Rouillet et al. (2012), but in a hierarchy of ocean models ranging from a realistic global model to idealised channel models, including linear stability analysis. Section 2 gives the definition of the spectrally resolved LEC and a description of the ocean models. The results derived from these models and the linear stability analysis are presented in Section 3. We first confirm the dipolar structure of the baroclinic instability conversion in a realistic global ocean model in Section 3 a). Then, we use less complex but still similar aquaplanet simulations in Section 3 b) to develop hypotheses for understanding the mechanism behind the eddy energy sink at larger scales. We test these hypotheses using idealized channel model simulations representative of particular latitudes of the aquaplanets in Section 3 c), and finally perform a linear stability analysis of the baroclinic conversion term in Section 3 d). A summary and discussion is given in Section 4.

REAL	eddy permitting global ocean model with horizontal resolution of $1/10^\circ$, 42 vertical levels, realistic topography.
AQUA_FLAT	global aqua planet model, same resolution as REAL, flat bottom, single meridional barrier, open Southern Ocean.
AQUA_ROUGH	same as AQUA_FLAT but rough bottom.
CHANNEL	set of idealised forced baroclinically unstable periodic zonal channel models.

Table 2.1: Name and short description of the different ocean model experiments discussed below.

2.2. Methods

2.2.1. Numerical model experiments

The LEC is diagnosed from a hierarchy of different ocean models described in this section. The models consist of an eddy resolving global ocean model (REAL), two aqua-planet models (AQUA_FLAT and AQUA_ROUGH), and a set of idealised channel models (CHANNEL), all based on the code found at <https://github.com/ceden/pyOM2>. Tab. 2.1 gives a short description of each model experiment and more details are given below.

Experiment REAL

Experiment REAL has a realistic topography resembling the world’s ocean. The same model is used by Eden (2016) and Eden et al. (2021). The model has 42 vertical levels and a horizontal resolution of $1/10^\circ \times 1/10^\circ \cos(\phi)$ with latitude ϕ . The horizontal resolution varies between ≈ 11 km at the equator and ≈ 3 km at the poles, the grid consists of 3600×2163 grid points. Equatorwards of about 20° latitude, the first baroclinic Rossby radius L_r is resolved by eight grid points, decreasing to three to seven grid points in the mid latitudes, and polewards of about 40° , L_r is resolved only by three or less grid points. The vertical resolution decreases from about 10 m in the upper 100 m to 250 m in the deep ocean below 1250 m. The vertical grid and topographic mask in the southern hemisphere are identical to the one used by Griesel et al. (2015). To avoid the singularity at the north pole, Griesel et al. (2015) use a tripolar grid in the northern hemisphere attached to a regular latitude-longitude grid in the southern hemisphere. Here, we use a regular latitude-longitude grid everywhere, but cutting out the Arctic Ocean north of 70°N . The northern hemisphere topography, forcing, etc is interpolated from the tripolar grid on our grid.

The heat and momentum forcing is taken from a monthly climatology from Barnier et al. (1995). A restoring boundary condition with a timescale of 90 days is used for surface salinity. The forcing for the boundary condition is provided by the World Ocean Atlas by Zweng et al. (2019). Instead of using an explicit sea ice

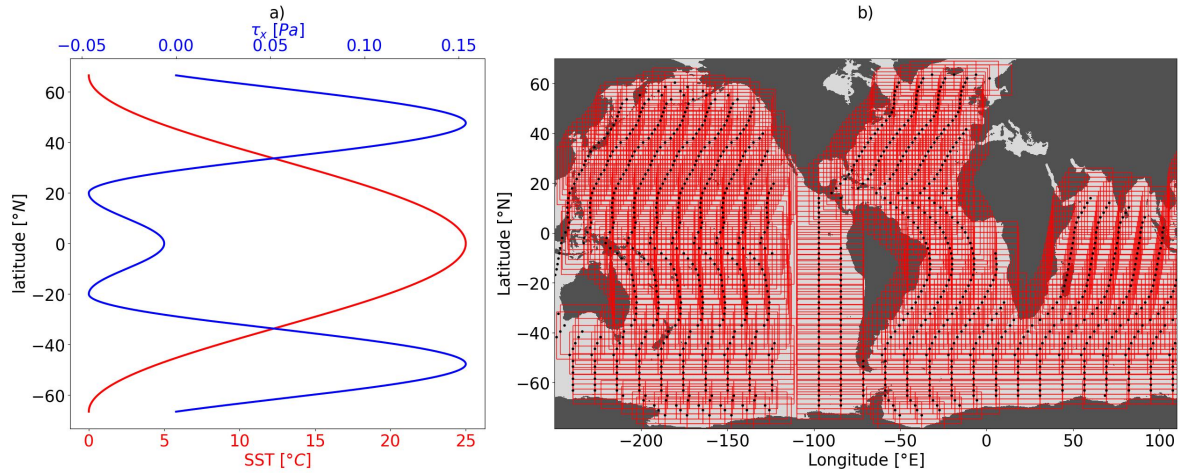


Figure 2.1: a) Idealised surface forcing functions for AQUA_FLAT and AQUA_ROUGH. Zonal wind stress $\tau_x(y)$ in Pa (blue line) and target temperature in $^{\circ}C$ for SST (red line). b) Windows used for Fourier transform in REAL. The windows are half overlapping in the zonal direction. In the meridional direction the overlap is larger since a window is placed every two degree latitude. The windows are 256 grid points large and contain up to 25% land in order to resolve boundary current dynamics.

model the surface heat flux from ocean to atmosphere is set to zero when the surface temperature falls below the freezing point. The model is initialised with a state of rest, where the temperature and salinity are taken from the combined WOCE/ARGO dataset by Gouretski (2018). The model uses biharmonic lateral friction and a second order central difference advection scheme with superbee flux limiter for temperature and salinity and the TEOS-10 equation of state. There is no additional lateral diffusion. Vertical mixing and friction induced by small scale turbulence in the surface layer and the ocean interior are parameterised using the scheme by Gaspar et al. (1990). The effects of breaking internal gravity waves are parameterised with the IDEMIX closure Olbers and Eden (2013). No explicit bottom friction is included in the model. The spinup period is 15 years long, after which the model is run for 5 more years to carry out the diagnosis, using 3-daily snapshots. This spinup period is too short to expect the large scale circulation to be in steady state, but the meso-scale eddy field can be expected to be in quasi-stationary state.

Experiments AQUA_FLAT and AQUA_ROUGH

AQUA_FLAT has constant depth of 4750 m, a single narrow land part which extends from the northern boundary at $66.5^{\circ}N$ to $35^{\circ}S$, zonally periodic boundary conditions, and a meridionally closed domain. Most model details are the same as in REAL if not otherwise stated. The horizontal and vertical resolution are the same as in REAL, but the number of gridpoints is reduced to 1800×1800 . Different to REAL, a linear equation of state is used. AQUA_FLAT is forced by an idealised wind stress vector $(\tau_x(y), 0)$ shown in Fig. 2.1. Sea surface temperature (SST) is restored in the uppermost model level with 10 m thickness with a time scale of 15 days towards the

target temperature also shown in Fig. 2.1.

AQUA_ROUGH is identical to AQUA_FLAT, but has a rough bottom topography, which is randomized and aims to match the topography spectrum of REAL ocean. The topography spectrum is given by

$$E \sim \left[1 + \left(L_0 \sqrt{k_x^2 + k_y^2} \right)^3 \right]^{-1}$$

with $L_0 = 50$ km and horizontal wavenumber vector components k_x and k_y . E is normalized such that the root mean square topographic height equals 500 m. The bottom topography is calculated from E by applying an inverse Fourier transform with a random phase. Linear bottom friction is used in AQUA_FLAT, which is not used in AQUA_ROUGH or REAL. Both models are integrated for 15 year, after which averages are calculated based on an integration until year 20 using 3-daily snapshots.

Experiment CHANNEL

CHANNEL is a zonally open and meridionally closed channel with zonally periodic boundary conditions. The domain covers $1000 \text{ km} \times 1000 \text{ km}$ with a flat bottom at a depth of 2000 m, with 512×512 horizontal and 80 vertical gridpoints and horizontal and vertical resolution of 1.95 km and 25 m, respectively. Dissipation is given by biharmonic mixing and friction, Rayleigh damping and linear bottom friction. Lateral biharmonic viscosity and diffusivity are set to $3.91 \times 10^7 \text{ m}^4 \text{ s}^{-1}$, vertical viscosity and diffusivity are set to $6.25 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and the bottom friction coefficient is set to $3.0 \times 10^{-5} \text{ s}^{-1}$. The β -plane approximation is used with varying f and β (see Tab. 2.2) and a linear equation of state. There is no forcing except for restoring of the zonal mean temperature gradient towards the initial conditions with a restoring timescale of $t_{res} = 50$ days, resembling a large scale forcing. Note that perturbations from the zonal mean like meso-scale eddies are not damped by this restoring, only the zonal mean flow. The initial conditions feature a meridional buoyancy gradient M^2 and a vertical buoyancy gradient N_0^2 , which vary between the models (see Tab. 2.2). The models are integrated for 20 years, after which 10-year long averages are taken.

2.2.2. Diagnostics

Spectrally resolved Lorenz Energy cycle

The equations for eddy kinetic energy and eddy available potential energy are derived from the primitive equations. The derivation of the spectrally resolved LEC is described in the Appendix and is standard (e.g. Olbers et al., 2012), except that we allow for large-scale variations of the background state related to the available potential energy and for the spectral resolution. Assumptions are also standard, frictional and diabatic processes are neglected. Eddy kinetic energy and eddy available potential energy as a function of the horizontal wavenumber vector $\mathbf{k}$ are

given by

$$K(\mathbf{k}) = \overline{\hat{\mathbf{u}}'^* \cdot \hat{\mathbf{u}}'} / 2, \quad P(\mathbf{k}) = \overline{\hat{b}'^* \hat{b}'} / (2N^2) \quad (2.1)$$

where the hat ($\hat{\cdot}$) indicates the two-dimensional Fourier transform, the star ($\cdot^*$) indicates the complex conjugate and the overbar denotes a time average over the diagnosis period. All vectors are two-dimensional here, $\mathbf{u}$ denotes horizontal velocity, b buoyancy perturbation relative to a reference state, and N the stability frequency as defined in the Appendix. The tendency equations for K and P are given by

$$\partial_t K = B - \underbrace{\text{Re} \left[\hat{\mathbf{u}}'^* \cdot \left(\overline{\mathbf{u}' \cdot \nabla \mathbf{u}} \right) \right]}_{\text{I}} - P_K + F_K, \quad \partial_t P = -B - \underbrace{N^{-2} \text{Re} \left[\hat{b}'^* (\overline{\mathbf{u}' \cdot \nabla b}) \right]}_{\text{II}} + F_P \quad (2.2)$$

where Re indicates the real part. Eq. 2 contains the energy exchange terms that are the key of the LEC. Most prominent is the exchange of P and K related to baroclinic instability or vertical shear instability given by

$$B(\mathbf{k}) = \text{Re} \left[\hat{w}'^* \hat{b}' \right] \quad (2.3)$$

The term I in Eq. 2 denotes exchange of mean kinetic energy and K , and is related to barotropic instability or lateral shear instability. The term II in Eq. 2 denotes the exchange of mean available potential energy and P , and is together with B part of the baroclinic energy pathway, which is thought to be the main source of K in the ocean (von Storch et al., 2012). In addition to these energy exchange terms, there are flux divergence terms in physical and spectral space: $P_K(\mathbf{k}) = \text{Re} \left[\rho_0^{-1} \partial_z \hat{p}' \hat{w}'^* \right]$ denotes the divergence of the vertical pressure induced energy flux, F_K the divergence of the spectral flux of K , and F_P the divergence of the spectral flux of P . The latter two are given by

$$\begin{aligned} F_K(\mathbf{k}) &= -\text{Re} \left[\hat{\mathbf{u}}'^* \cdot \left(\overline{\mathbf{u} \cdot \nabla \mathbf{u}'} \right) + \hat{\mathbf{u}}'^* \cdot \left(\overline{\mathbf{u}' \cdot \nabla \mathbf{u}} \right) \right] \\ F_P(\mathbf{k}) &= -N^{-2} \text{Re} \left[\hat{b}'^* \left(\overline{\mathbf{u} \cdot \nabla b'} \right) + \hat{b}'^* \left(\overline{\mathbf{u}' \cdot \nabla b} \right) \right] \end{aligned} \quad (2.4)$$

and will be discussed together with B in the following.

F_P depends on N and therefore on the definition of the reference density ρ_{ref} (see Appendix), which is ambiguous. It is possible to argue that P is only a leading order approximation of (eddy) available potential energy, and that ρ_{ref} should really be the state of minimal potential energy found after adiabatically resorting all water parcels of the world's ocean. However, this precise definition and practical calculation under the influence of a non-linear equation of state is difficult (e.g. Hieronymus and Nycander, 2015), such that we use here the simple form of P . Note that B , which will be the main focus of this study, and also F_K , are independent of N .

Windowing for Fourier transform

To determine the spatial variation of the spectrally resolved quantities, the horizontal domain of REAL and AQUA_FLAT/ROUGH is covered with windows in which the Fourier transform is performed, as illustrated in Fig. 2.1. All model levels are covered with windows as in Fig. 2.1, but windows with more than 25% land grid points are neglected. For AQUA_FLAT/ROUGH (not shown) such a criterion is not needed since the boundaries are much simpler. The windows are 256×256 grid points large. In CHANNEL, no windowing is used and the analysis is carried out for the whole model domain. Before a two-dimensional Fourier transform is applied to each window, the land grid points are set to zero and a Hamming window is applied to the data, similar to what is done in Scott and Wang (2005).

Vertical mode projection

To diagnose the model experiments we also use vertical mode projection. The vertical eigenmodes (Olbers et al., 2012; Robey and Grooms, 2024) are calculated from

$$\phi_n + c_n^2 \frac{d}{dz} \left(\frac{1}{N^2} \frac{d\phi_n}{dz} \right) = 0 \quad \text{with} \quad \frac{d\phi}{dz} = 0 \quad \text{at} \quad z = 0, -h \quad (2.5)$$

ϕ_n are the eigenfunctions with their eigenvalues c_n^2 . We use the approximate solutions for the vertical modes given by

$$\phi_n = \sqrt{\frac{2\tilde{N}}{h\tilde{N}}} \cos \left(\int_{-h}^0 N/c_n dz \right) \quad (2.6)$$

with $\tilde{N} = 1/h \int_{-h}^0 N dz$ and $c_n = h\tilde{N}/(n\pi)$ and Rossby Radius $L_n = c_n/f$. Using the orthogonality of ϕ_n , the horizontal velocity perturbation $\mathbf{u}'$ is projected on the vertical modes as $\mathbf{u}'_n = \int_{-h}^0 \mathbf{u}' \phi_n dz$. From the modal eddy kinetic energy $E_n = \overline{\mathbf{u}'_n^2}/2$, a modal eddy speed $U_n = \sqrt{E_n}/H$ with water depth H can be defined, and from that a modal Rhines scale $\lambda_n = \sqrt{U_n/\beta}$.

2.3. Results

2.3.1. Experiment REAL

In this section we search in REAL for the same dipole structure of the baroclinic eddy conversion term B in the spectrally resolved LEC as in Wang et al. (2019) and Roulet et al. (2012). We will then proceed in this section to relate the geographical distribution of $B < 0$ at large scales to properties of the mean flow. We examine these relationships in more detail in the less complex but still similar aquaplanet simulations in Section 3 b), and the idealized channel model experiments in Section 3 c), before we simplify the most using linear stability analysis in Section 3 d).

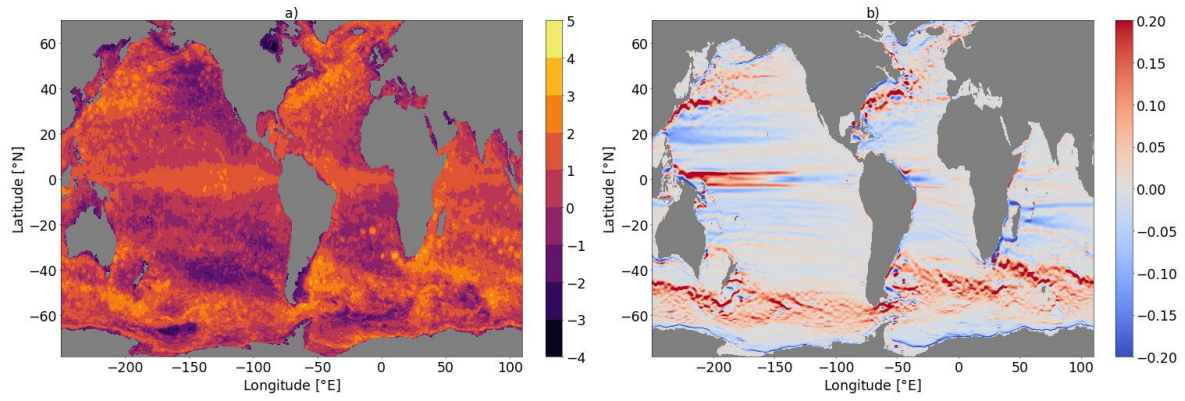


Figure 2.2: a) $\log_{10}\left(\int_{-h}^0 \overline{|\mathbf{u}'|^2} / 2 \, dz / [\text{m}^3 \text{s}^{-1}]\right)$ in REAL. b) $\bar{u}$ at 268 m depth $[\text{m s}^{-1}]$ in REAL.

To display the eddy activity in REAL and the large-scale circulation which will later become important, Fig. 2.2 (a) shows the vertically integrated eddy kinetic energy, and the near surface $\bar{u}$ (b). Large eddy kinetic energy is found in the western boundary currents, the equatorial current system, the Alghulas and Brazil currents, and in the Antarctic Circumpolar Current (ACC). Smaller eddy kinetic energy is found in the eastern parts of the basins, especially in the Pacific Ocean. In particular, it is lower in the westward return flows of the subtropical gyres between 10° and 30° – 35° , compared to the poleward part with eastward currents between 30° and 50° latitude.

In Fig. 2.2 (b), several zonal jets can be seen superimposed on the zonal background flow. As for eddy kinetic energy, the highest values of $\bar{u}$ are associated to the western boundary currents, their extensions, the equatorial current system, and the ACC. The westward return flow of the subtropical gyres is in general weaker than the eastwards flowing currents poleward of it, except for the North Pacific, where the return flow has a similar strength as the eastward currents. In the southern hemisphere, the return flow is less pronounced as in the northern hemisphere.

Energy reservoirs and transfers in spectral space

Fig. 2.3 shows $K(\mathbf{k})$ and the baroclinic conversion term $B(\mathbf{k})$ from Eq. 3 in REAL for three different windows in the North Atlantic at 293 m depth. K and B are also averaged over the wavenumber angle α , with $\mathbf{k} = k(\cos \alpha, \sin \alpha)$, and shown as function of the wavenumber magnitude k , i.e. $K(k) = (2\pi)^{-1} \int_0^{2\pi} K(k, \alpha) d\alpha$ with $K(k, \alpha) = kK(\mathbf{k})$, and similar for the other LEC terms. $K(\mathbf{k})$ in Fig. 2.3 (a) is positive definite without large dependencies on α , such that the averaging over α does not hide important details. For $B(\mathbf{k})$ on the other hand, the averaged view can be problematic. While $B(\mathbf{k})$ has no large α dependency in Fig. 2.3 (b), in (c) and (d) positive and negative values are added together in the integral over α . In Fig. 2.3 (c) and (g), some negative signal is still found after the integration, but in Fig. 2.3 (d) and (h), the negative signal is lost completely. This complication becomes even more problematic

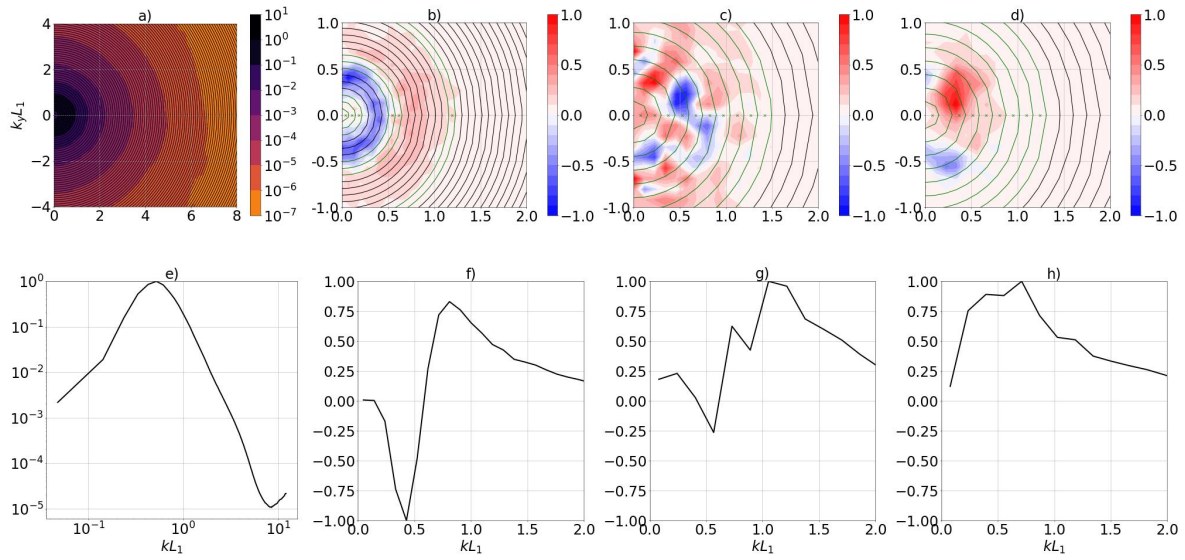


Figure 2.3: $K(\mathbf{k})$ (a) and the baroclinic conversion term $B(\mathbf{k})$ (b-d) for different windows at 293 m depth for REAL. e) to h) show $K(k)$ and $B(k)$ averaged over wavenumber angle α , respectively. K and B are normalised by $|K|_{max}$ and $|B|_{max}$, respectively, for each window. The rings indicate the averaging at $k = const$. Green rings indicate that positive and negative values are averaged together.

if a vertical integral is applied in addition. In the following, results are often shown as average over α since the reduction in dimension is useful and the averaging is justified for many windows and regions. These results should, however, be interpreted with care.

Fig. 2.4 shows $K(k)$ and $B(k)$ for REAL vertically and zonally integrated as a function of latitude and k , and zonally and meridionally integrated as a function of depth and k . $K(k)$ shows three distinct maxima. The first one at about 35°N is associated with the western boundary currents. The second one around the equator is associated with the equatorial current system. The third one between 20°S and 60°S is associated with the Agulhas current, the Brazil current and the ACC. The wavenumber k_{max} of the maximum in K increases with latitude, which indicates that the eddy scale decreases towards the poles. As k_{max} , the inverse Rossby radius $1/L_1$ increases with latitude, but more quickly. K is maximal close to the surface and decreases with depth. The maximum of $K(k)$ gets narrower with depth, but k_{max} does not depend on depth.

$B(k)$ shows a distinct dipolar structure in k between 30° and 50° latitude: B is negative for $k < k_{max}$ and positive for $k > k_{max}$. Equatorward of 30° and poleward of 50° , B is much weaker. Large magnitudes of B occur at the same latitudes as high values of K . The minimum of B in the southern hemisphere is slightly weaker than in the northern hemisphere. The dipolar structure of $B(k)$ shows up also in the (k, z) plane, where $B(k)$ shows two peaks at different depths. The first peak is close to the surface at about 30 m depth. The second peak has the maximum at about 300 m depth and extends from 60 m down to about 1500 m. Both peaks are similar in magnitude,

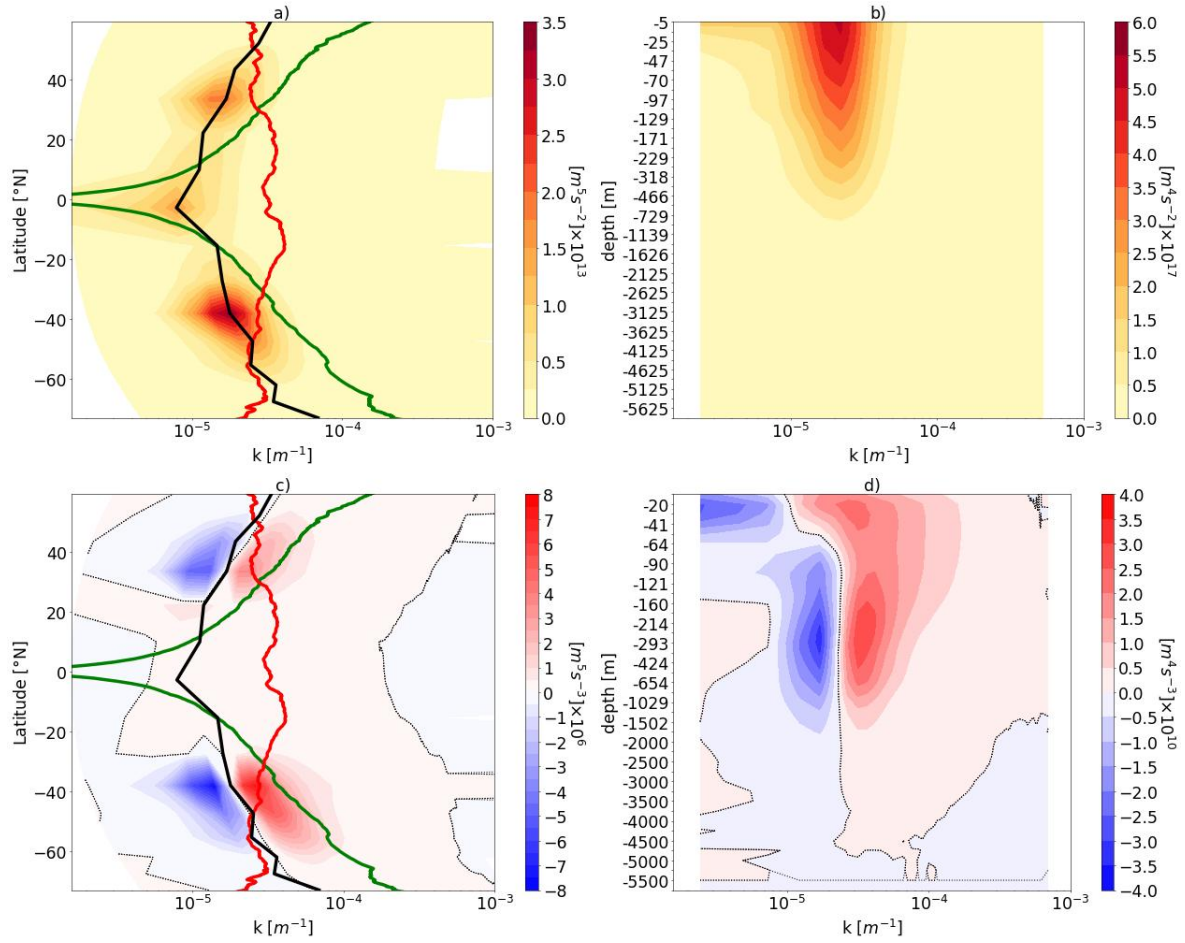


Figure 2.4: $K(k)$ and baroclinic conversion $B(k)$ for REAL. (a): Vertically and zonally integrated $K(k)$. (b): Zonally and meridionally integrated $K(k)$. (c): Vertically and zonally integrated $B(k)$. (d): Zonally and meridionally integrated $B(k)$. The solid lines indicates the wavenumber $k_{\max}$ of the maximum in $K(k)$ (black), the first Rossby radius $1/L_1$ (green) and the first mode Rhines scale $1/\lambda_1$ (red) as defined in Section 2.b.3.

but the surface peak is located at smaller wavenumbers than the deeper one. For the latter, $k_{\max}$ separates the zero crossing between maximum and the minimum well. The upper peak is associated with the equatorial region, while the lower peak is associated with the off-equatorial region (not shown). Note that because $\int B(k)dk$ is positive at all depths and latitudes, the net effect is eddy kinetic energy production. The barotropic production term (not shown) is much weaker than the baroclinic conversion term. It is predominantly positive, generating eddy kinetic energy at scales near $k_{\max}$. At smaller scales it becomes weakly negative polewards of about 30° latitude.

Fig. 2.5 shows the divergence of the spectral flux of K and P , $F_K(k)$ and $F_P(k)$, respectively, for REAL in the same way as Fig. 2.4. Positive $F_K(k)$ and $F_P(k)$ are a source of K and P . $F_K(k)$ shows a similar dipolar structure as B , but with opposite

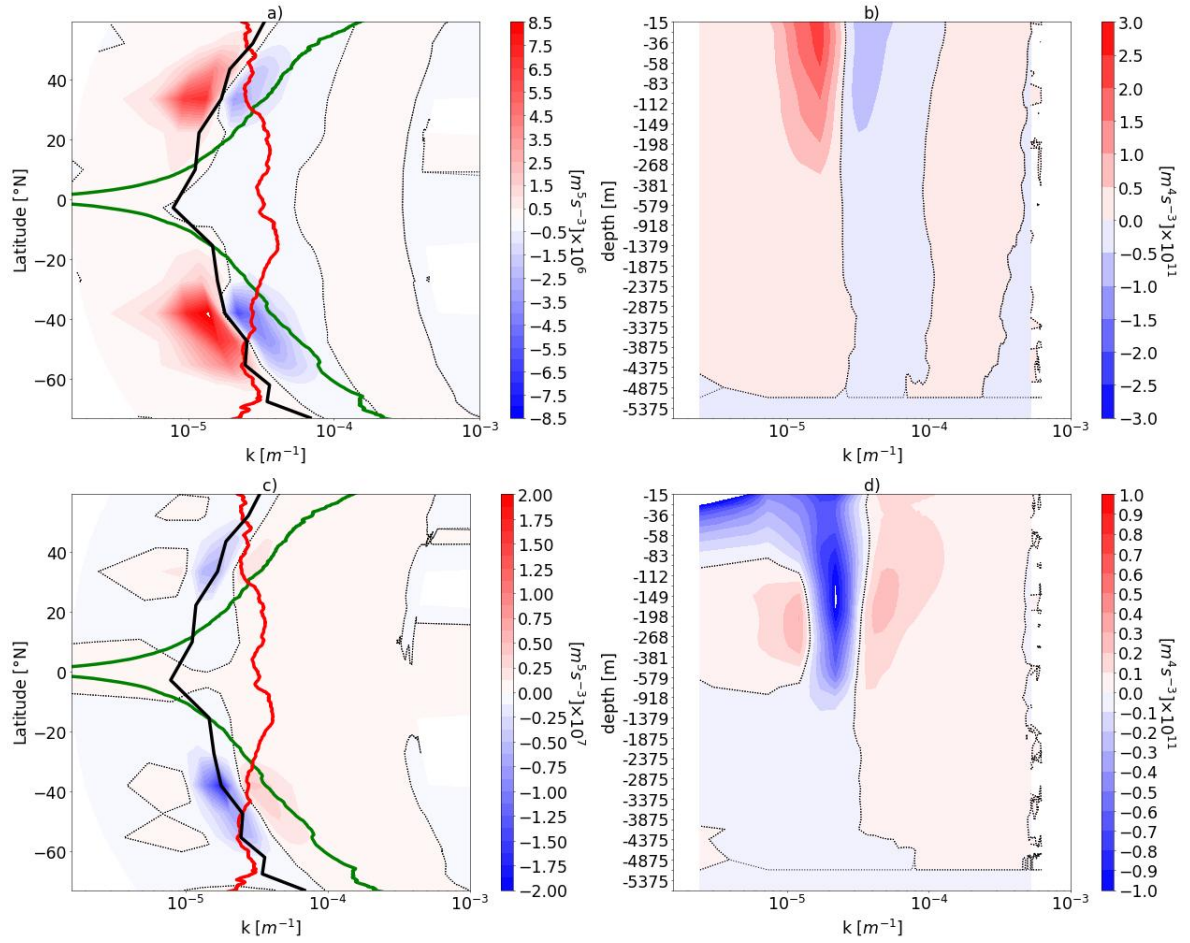


Figure 2.5: Divergence of spectral eddy kinetic energy flux $F_K(k)$ and divergence of spectral eddy potential energy flux $F_P(k)$ in REAL. (a): Vertically and zonally integrated $F_K(k)$. (b): Zonally and meridionally integrated $F_K(k)$. (c): Vertically and zonally integrated $F_P(k)$. (d): Zonally and meridionally integrated $F_P(k)$. The solid lines indicate the wavenumber k_{max} of the maximum in $K(k)$ (black), the first Rossby radius $1/L_1$ (green) and the first mode Rhines scale $1/\lambda_1$ (red) as defined in Section 2.b.3.

sign. Between 30° and 50° latitude, $F_K(k)$ is positive for $k < k_{max}$ and negative for $k > k_{max}$. This indicates an inverse cascade of K from smaller to larger scales. Like B , $F_K(k)$ is much weaker equatorward of 30° and poleward of 50° . $F_K(k)$ is maximal to the surface and decreases with depth. The maximum in $F_K(k)$ gets narrower in k with depth.

P (not shown) is similar to K . It has one maximum between 30° N and 50° N and one maximum between 20° S and 60° S. There is no maximum at the equator. The wavenumber of the maximum in P is slightly smaller than k_{max} for K , indicating that longer waves are dominating for P . As for K , the wavenumber of the maximum in P increases with latitude. In the vertical, P shows two maxima. The first one at the surface at the smallest resolved k , and the second one at about 150 m depth at slightly

smaller k than k_{max} for K .

$F_P(k)$ is shown in Fig. 2.5 c) and d). It has two maxima between 30° and 50° latitude like B and $F_K(k)$. $F_P(k)$ is negative close to k_{max} , and positive at smaller and larger scales. The source of P at $k > k_{max}$ is stronger than the source of P at $k < k_{max}$. This indicates a forward cascade of P from k_{max} to smaller scales, but there is also a weaker inverse cascade from k_{max} to larger scales. In the vertical, the sink of P at k_{max} and the source of P at larger k are found from the surface down to about 1000 m depth. The source of P at $k < k_{max}$ on the other hand is not found at the surface and peaks at about 200 m depth. At the surface the sink of P extends to the smallest resolved k . Therefore the forward cascade of P is found from the surface down to about 1000 m while the inverse cascade is only found below about 100 m.

Geographical distribution of baroclinic conversion B

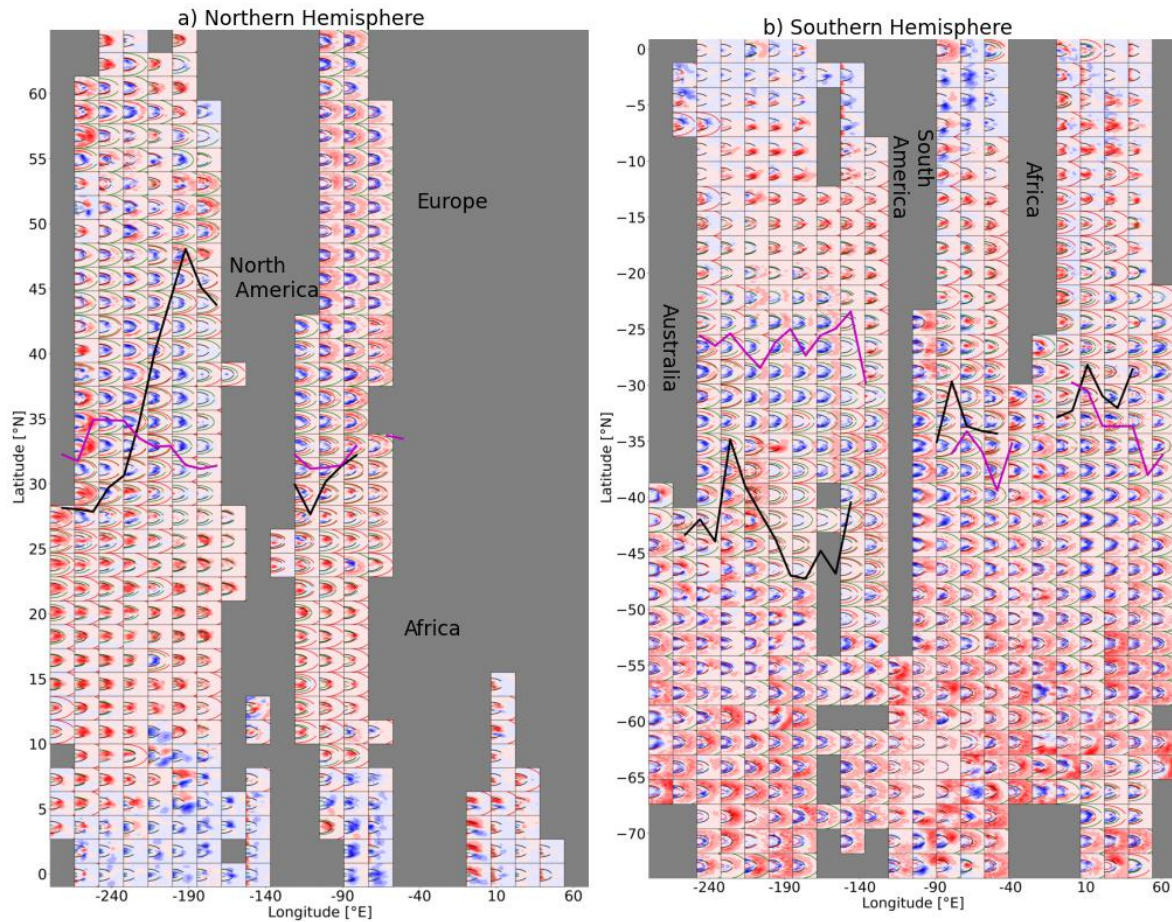


Figure 2.6: Baroclinic conversion term $B(\mathbf{k})$ normalised by its maximum for all windows in REAL at 293 m depth as a function of $k_x L_1$ and $k_y L_1$. In each panel, L_1 (green), λ_1 (red), and k_{max} (black) are also shown. Panel size is $2(L_1 k_{max}^{-1})^{-1}$ to account for variations in the spatial scale of B . The solid black line across panels indicates $L_1/\lambda_1 = 1$ and the solid magenta line across panels indicates where $\bar{u}$ at 293 m depth changes from westward equatorward to eastward poleward of the line.

Fig. 2.6 shows (the normalised) $B(\mathbf{k})$ in REAL for all windows at the secondary sub-surface peak in Fig. 2.4, but without averaging in α as before. In the North Atlantic Ocean, there is no region in $\mathbf{k}$ -space where $B < 0$ between 10° N and 24° N, but polewards of 33° N there is $B < 0$ for every window. Between 24° N and 33° N, $B < 0$ is also found, but less pronounced and more asymmetric in α compared to the western boundary currents or the ACC, where regions with $B < 0$ are clearly pronounced. In the South Atlantic Ocean, there is also no or small $B < 0$ between 10° S and 21° S, and small $B < 0$ which is also asymmetric in α between 21° S and 32° S, but polewards of 32° S we find again regions with dominant and symmetric $B < 0$. Similarly, in the North Pacific Ocean, there is most evidently no $B < 0$ between 10° N and 24° N, between 24° N and 32° N there is little $B < 0$ and asymmetric in α , and polewards of 33° N regions with $B < 0$ are found for every window. In the South Pacific Ocean, there is again no or little $B < 0$ between 10° S and 23° S, and poleward of that latitude $B < 0$ is found for every window. At the equator, B gets negative in all ocean basins, but we exclude this region from the following discussion, since equatorial dynamics differ from the rest of the ocean and can not be interpreted within the quasi-geostrophic framework.

In summary, a similar clear geographical distribution of $B < 0$ at small k can be found in each of the four major ocean basins: No, or little $B < 0$ which is also asymmetric in α between about 10° and 20° to 30° latitude. This is seen most clearly in the North Pacific Ocean. Polewards of about 30° latitude, $B < 0$ at small k is found almost everywhere, indicating two different regimes, which we aim to characterise now with different measures.

Distinguishing the two regimes of B

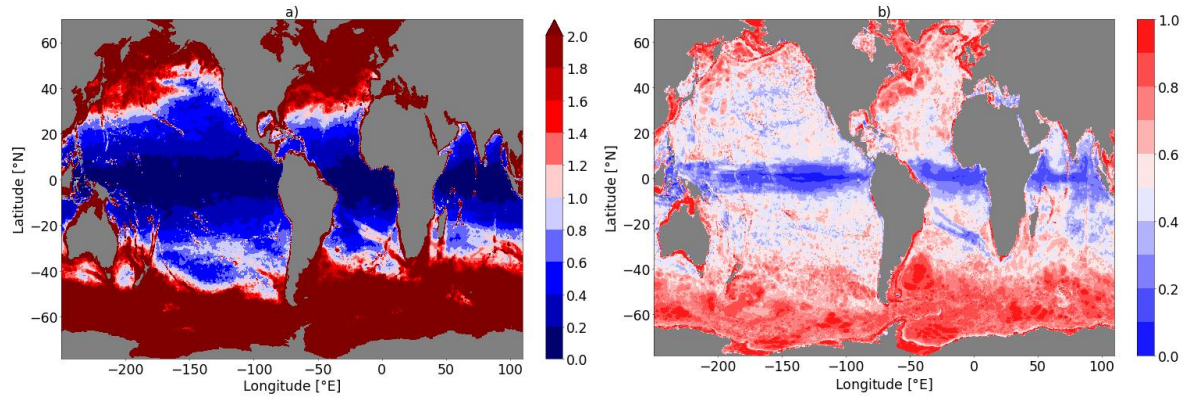


Figure 2.7: a) Ratio of Rhines scale to Rossby radius λ_1/L_1 in REAL. b) Fraction of barotropic eddy energy $R_0 = E_0/(E_0 + E_1 + E_2 + E_3)$ in REAL.

Fig. 2.7 a) shows the ratio of Rhines scale λ_1 to the Rossby radius L_1 in REAL. The Rhines scale λ_1 associated to the first vertical modal velocities and the first baroclinic Rossby radius L_1 are defined in Section 2.b.3. In REAL, AQUA_FLAT and AQUA_ROUGH most eddy kinetic energy is contained in the barotropic and first

baroclinic modes with the ratio $(E_2 + E_3)/(E_0 + E_1)$ remaining below 0.2 in most of the extratropical ocean. The modal energies E_n are also defined in Section 2.b.3. L_1 is dominated by the change of the Coriolis parameter f with latitude, while λ_1 is dominated by the variability of the modal speed U_1 . L_1 is largest close to the equator and decreases with latitude. λ_1 on the other hand, generally increases with latitude (Fig. 2.4). There is a critical latitude ϕ_c where polewards λ_1 becomes larger than L_1 . Note that ϕ_c is similar for λ_0/L_1 (not shown). In REAL, ϕ_c is at roughly 30° latitude, but appears to be shifted polewards in the eastern parts of the ocean basins.

Fig. 2.7 b) shows the fraction of eddy kinetic energy in the barotropic mode $R_0 = E_0/(E_1 + E_2 + E_3)$. In the equatorial ocean there is more energy in the higher modes. Polewards of the critical latitude ϕ_c , the fraction increases and most of the energy is in the barotropic mode. The change between the two regimes of B at the critical latitude ϕ_c coincides also with the change of sign of the near surface mean zonal current (Fig. 2.2). There is an asymmetry between the northern and southern hemisphere but the transition is found in each basin. In summary, three different important aspects of the circulation are found to coincide with the two regimes of B : the ratios λ_1/L_1 and R_0 , and the sign of $\bar{u}$.

To summarize the results thus far: We find in REAL the same dipole structure of the baroclinic conversion term B with a sink of eddy kinetic energy K at large scales as in Wang et al. (2019) and Rouillet et al. (2012). However, it is not always present and we relate the geographical distribution of $B < 0$ at large scales to the ratios λ_1/L_1 and R_0 , as well as the sign of $\bar{u}$. In the following section, we examine these relationships in detail using the less complex but still similar aquaplanet simulations, AQUA_ROUGH and AQUA_FLAT.

2.3.2. Aquaplanet models

It turns out that the circulation in AQUA_FLAT and AQUA_ROUGH is similar to REAL but less complex, and exhibits a similar spectrally resolved LEC with a similar regime change of B at about $\phi_c \approx 30^\circ$. Different to REAL, the regime change does not depend so much on longitude in AQUA_FLAT and AQUA_ROUGH and is generally more clearly defined. This makes it easier to link the regimes changes of B to different aspects of the circulation.

Mean circulation

REAL, AQUA_FLAT and AQUA_ROUGH have a similar horizontal circulation and thermocline depth as the real ocean. Observations show that the Gulf stream transport varies from about 30 Sv ($= 30 \times 10^6 \text{ m}^3 \text{ s}^{-1}$) near the Florida Strait up to 150 Sv further north (Gyory et al., 2013), the Kuroshio current transport varies from 24 Sv in the East China Sea to 65 Sv southeast of Japan (Andres et al., 2015), and the ACC transport between 100 – 150 Sv (Smith et al., 2013). Integrated over the upper 2000 m, the western boundary currents in REAL between 25° N and 45° N have a strength of about 30 Sv in the North Atlantic Ocean and 36 Sv in the North Pacific Ocean and are on the

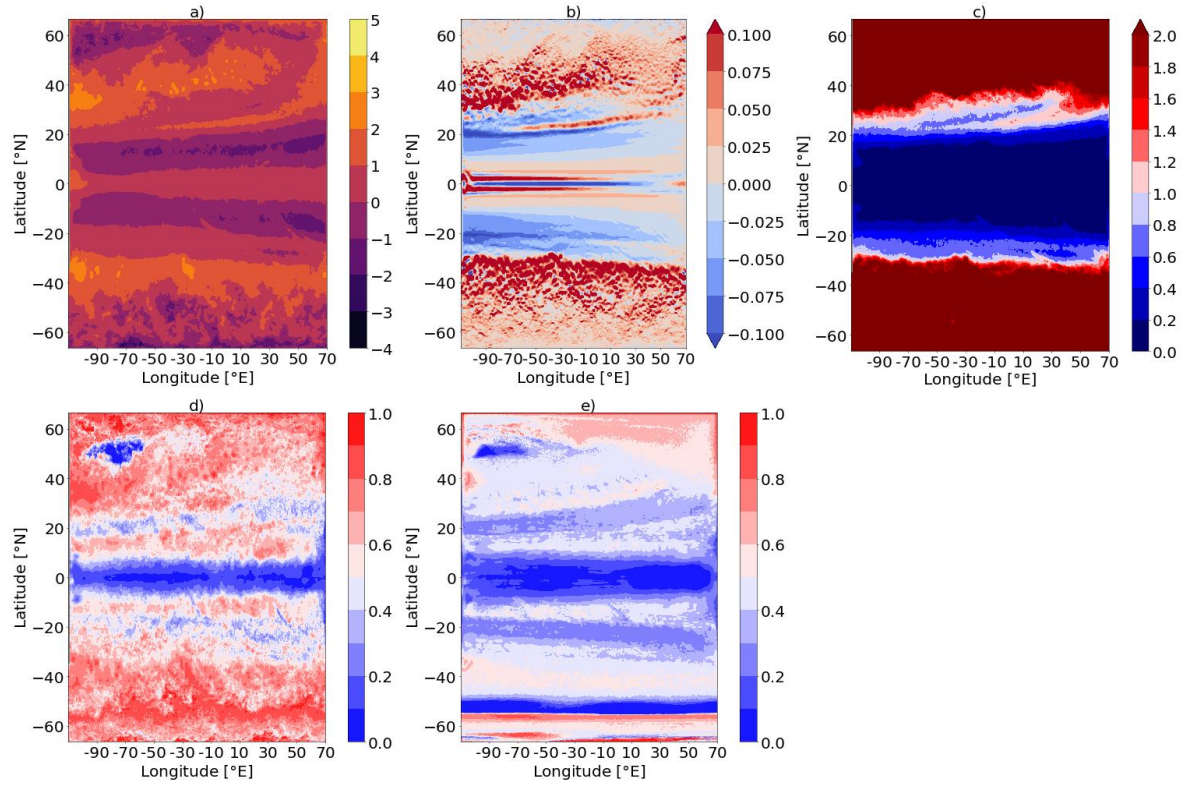


Figure 2.8: Shown are different properties of AQUA_ROUGH (a-d) and AQUA_FLAT (e). a) $\log_{10}(\int_{-h}^0 |\mathbf{u}'|^2 / 2 dz / [\text{m}^3 \text{s}^{-1}])$. b) $\bar{u}$ [m s^{-1}] at $z = -268 \text{ m}$ c) λ_1/L_1 d) $R_0 = E_0/(E_1 + E_2 + E_3)$ for AQUA_ROUGH e) R_0 for AQUA_FLAT.

lower end of the observed range. In AQUA_FLAT and AQUA_ROUGH, the western boundary currents between 25° N and 45° N are with 73 Sv and 66 Sv stronger than in REAL. The ACC transport through the Drake Passage in REAL, and between the land barrier and the southern boundary in AQUA_FLAT and AQUA_ROUGH is 100 Sv, 110 Sv and 60 Sv, respectively. The depth of the thermocline at 45° N estimated from Gouretski (2018) is approximately 1000 m in the North Atlantic and North Pacific Ocean. REAL, AQUA_FLAT and AQUA_ROUGH show a similar thermocline depth.

Eddy kinetic energy, the ratios of λ_1/L_1 , R_0 , and the sign of $\bar{u}$

Similar to REAL, both AQUA_ROUGH (Fig. 2.8 a) and AQUA_FLAT (not shown) have enhanced eddy kinetic energy in the western boundary currents and the Southern Ocean, while eddy kinetic energy is low in the eastern part of the basin and the westward return flows. Also similar to REAL, zonal jets are clearly identified. High values of $\bar{u}$ are found in the western boundary current, the southern basin and the equatorial current system in both aquaplanet models. Between 10° and 30° latitude are westward return flows, but different to REAL, they are of similar strength on both hemispheres. Similar to REAL, the ratio λ_1/L_1 changes around $\phi_c \approx 30^\circ$ from $\lambda_1/L_1 < 1$ to $\lambda_1/L_1 > 1$ in both AQUA_ROUGH and AQUA_FLAT (not shown). In

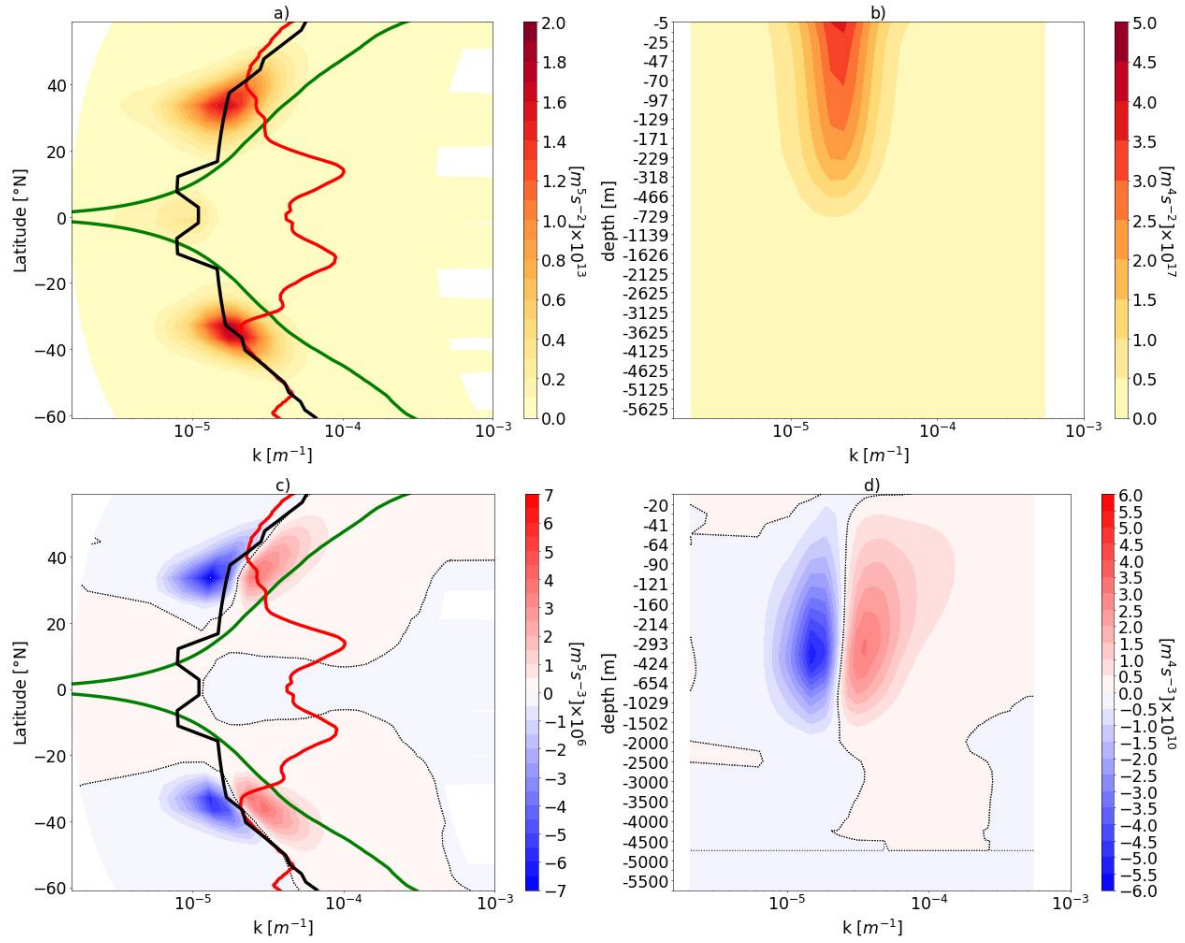


Figure 2.9: $K(k)$ and baroclinic conversion term $B(k)$ for AQUA_ROUGH. a) Vertically and zonally integrated $K(k)$. b) Zonally and meridionally integrated $K(k)$. c) Vertically and zonally integrated $B(k)$. d) Zonally and meridionally integrated $B(k)$. The solid lines indicates the wavenumber k_{max} of the maximum in $K(k)$ (black), the first Rossby radius $1/L_1$ (green), and the first mode Rhines scale $1/\lambda_1$ (red) as defined in Section 2.b.3.

AQUA_ROUGH, R_0 increases with latitude similar to REAL. AQUA_FLAT, however, shows no increase of R_0 with latitude (Fig. 2.8 e). It seems that a yet unknown interaction process of the circulation with rough topography is essential for this feature found in REAL and AQUA_ROUGH.

K and B in spectral space

Fig. 2.9 shows $K(k)$ and $B(k)$ for AQUA_ROUGH. The meridional and vertical distribution of $K(k)$ and $B(k)$ of AQUA_FLAT (not shown) is very similar to AQUA_ROUGH, and both are similar to REAL. k_{max} increases with latitude and there are two maxima in K associated to the western boundary current and the current in the southern basin. $B(k)$ in AQUA_ROUGH exhibits the same dipolar structure found in REAL, and is strongest where $K(k)$ is maximal between about 30° and 50° latitude. Different to REAL, there is only one peak in the vertical at about 300 m depth (the surface peak

in REAL is associated to the equatorial ocean, thus not discussed here). The depth of the peak in $B(k)$ in AQUA_ROUGH is also similar to REAL.

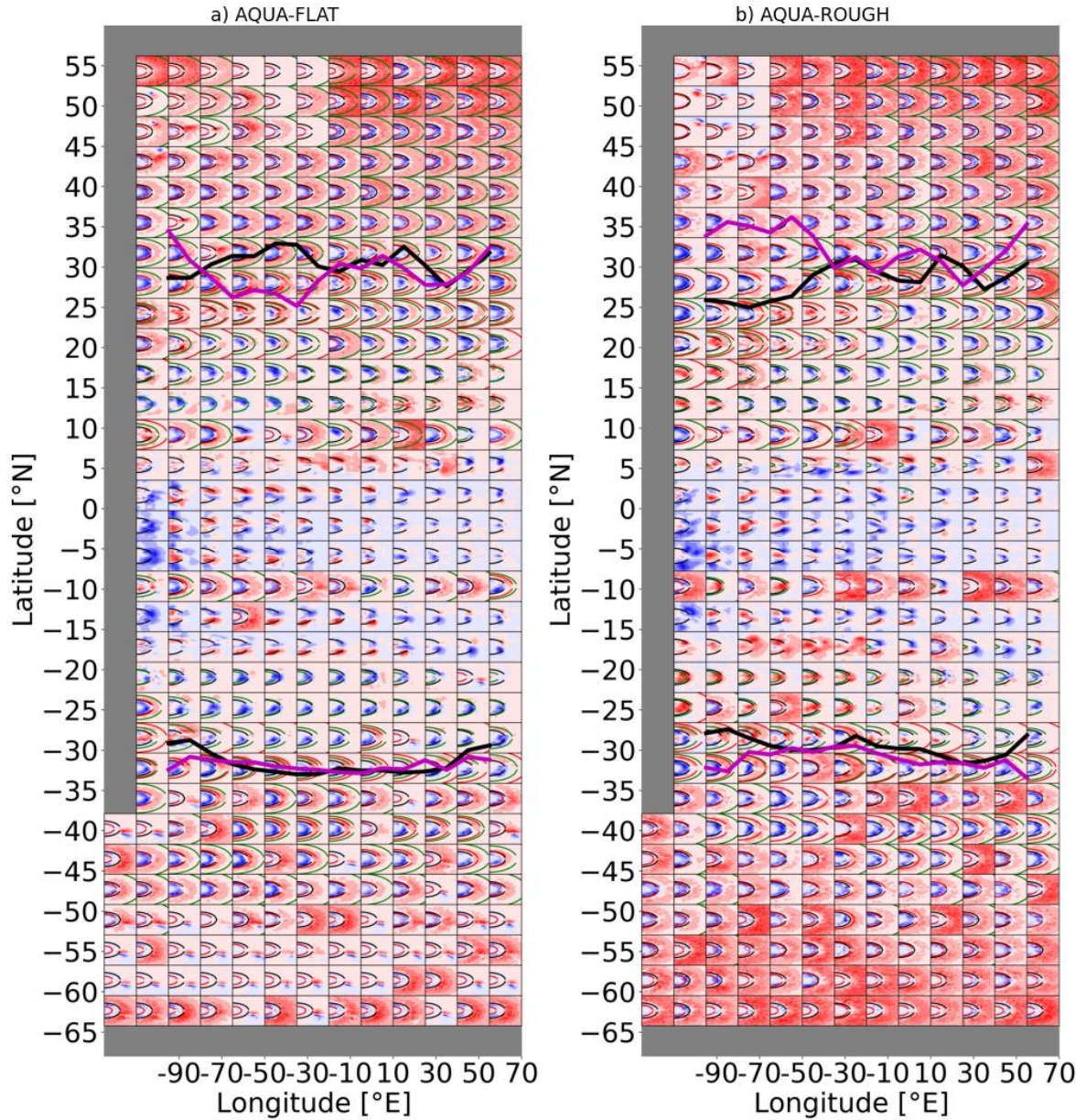


Figure 2.10: Baroclinic conversion term $B(\mathbf{k})$ normalised by its maximum for all windows in a) AQUA_FLAT and b) AQUA_ROUGH at 293 m depth as a function of $k_x L_1$ and $k_y L_1$. In each panel, L_1 (green), λ_1 (red), and k_{max} (black) are also shown. Panel size is $2(L_1 k_{max}^{-1})^{-1}$ to account for variations in the spatial scale of B . The solid black line across panels indicates $\lambda_1/L_1 = 1$ and the solid magenta line across panels indicates where $\bar{u}$ at 293 m depth changes from westward equatorward to eastward poleward of the line.

Fig. 2.10 shows $B(\mathbf{k})$ in AQUA_FLAT (a) and AQUA_ROUGH (b) for all windows at the sub-surface peak in Fig. 2.9 d). AQUA_FLAT and AQUA_ROUGH show a

similar geographical distribution of $B \leq 0$ as REAL, where clear, symmetric $B < 0$ is found for almost every window poleward of 30° degree latitude and less, asymmetric (in α) $B < 0$ is found between 10° degree and 30° degree latitude. In the northern hemisphere the pattern is less distinct, but $B < 0$ is clearly reduced and less symmetric between 10° N and 30° N, especially close to the boundary. The reduction of $B < 0$ between 10° and 30° latitude is more strongly pronounced in AQUA_ROUGH than in AQUA_FLAT, especially in the southern hemisphere and at 15° and 20° N close to the boundary. There, no $B < 0$ is found for AQUA_ROUGH while some $B < 0$ shows for AQUA_FLAT. Therefore the rough bottom topography produces a distribution of $B \leq 0$ more closely to REAL. Overall the regime change from less or no $B < 0$ between 10° and 30° latitude to $B < 0$ everywhere poleward of the critical latitude $\phi_c = 30^\circ$ is less pronounced in AQUA_FLAT and AQUA_ROUGH compared to REAL but clearly visible. As for REAL, $B < 0$ is found poleward of ϕ_c , where $\lambda_1 > L_1$ and the zonal mean current is eastward, while $B < 0$ is reduced or absent equatorward of ϕ_c where $\lambda_1 < L_1$ and the zonal mean current is westward.

In summary, AQUA_FLAT and AQUA_ROUGH show the same dipole structure in B and support the relation of regions with $B < 0$ to the ratio λ_1/L_1 and the sign of $\bar{u}$. Unlike in REAL, however, $B < 0$ in AQUA_FLAT and AQUA_ROUGH occurs independently of R_0 . A link between R_0 and the different regimes of B can therefore be already excluded. Since both λ_1/L_1 and the sign of $\bar{u}$ change at ϕ_c , it is not possible to determine which of these two characteristics is related to $B < 0$. Therefore, we discuss experiments with the more idealized CHANNEL model in the following section to clarify.

2.3.3. CHANNEL models

The CHANNEL models are set up to replicate the dynamical conditions of REAL, AQUA_ROUGH and AQUA_FLAT at the latitudes 25° , 35° , 45° and 55° (CHANNEL_25, CHANNEL_35, CHANNEL_45 and CHANNEL_55) characterized by f , β , the ratio λ_1/L_1 , and the sign of $\bar{u}$. To align λ_1 and L_1 with the regimes found in the global models, the vertical and meridional buoyancy gradients N_0^2 and M^2 of the initial condition to which the models are restored (see Section 2.a.3), are adjusted as given in Table 2.2. Each of the experiments are also integrated with the opposite sign of M^2 (CHANNEL_25_R etc.).

Circulation aspects

The CHANNEL models display a similar spectrally resolved LEC and exhibit comparable spectral fluxes of K and P (not shown). Fig. 2.11 compares λ_1 and L_1 in CHANNEL with the global models REAL, AQUA_FLAT and AQUA_ROUGH at their respective latitudes. The CHANNEL models match roughly the mean L_1 and λ_1 from the global models, and capture in particular the transition from $L_1 > \lambda_1$ to $L_1 < \lambda_1$ at 30° latitude, when going from CHANNEL_25 to CHANNEL_35. The direction of the mean current has little influence on λ_1 and L_1 . λ_1 in CHANNEL_55 is slightly larger

	f [s^{-1}]	β [$(sm)^{-1}$]	M^2 [s^{-2}]	N_0^2 [s^{-2}]	L_1 [km]	λ_1 [km]
CHANNEL_25	5.1×10^{-5}	2.1×10^{-11}	2.9×10^{-5}	3.8×10^{-3}	39	30
CHANNEL_35	7.4×10^{-5}	1.9×10^{-11}	5.5×10^{-5}	3.1×10^{-3}	24	35
CHANNEL_45	9.5×10^{-5}	1.6×10^{-11}	4.0×10^{-5}	2.8×10^{-3}	17	40
CHANNEL_55	1.3×10^{-4}	1.3×10^{-11}	3.2×10^{-5}	2.3×10^{-3}	12	46

Table 2.2: Parameter in CHANNEL.

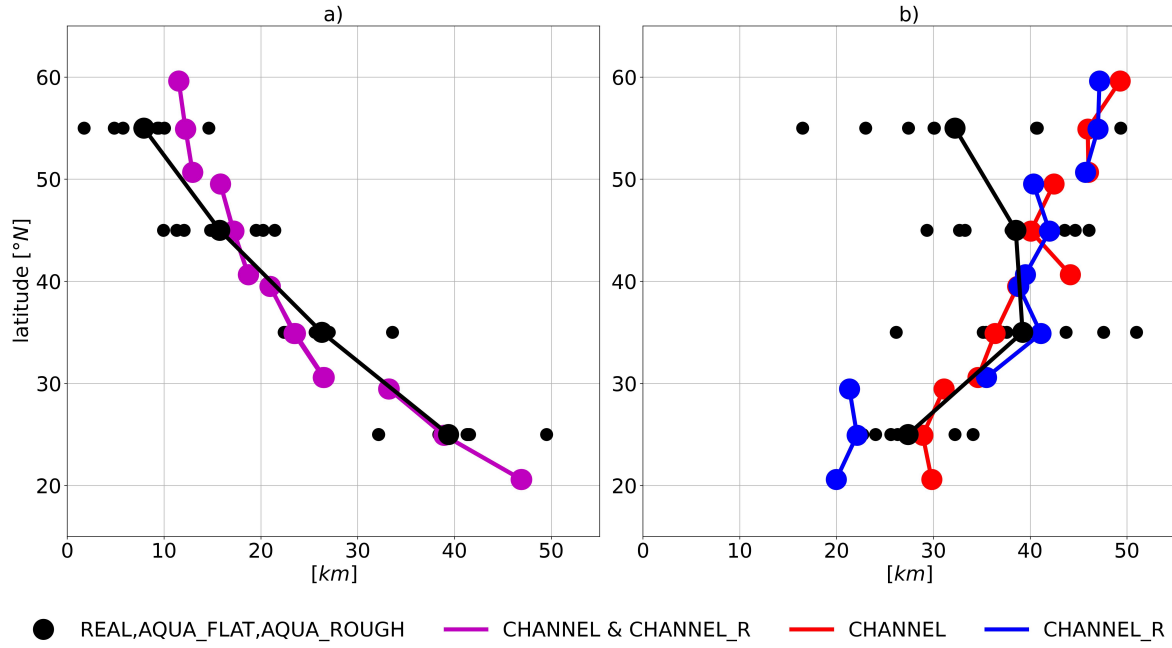


Figure 2.11: Comparison of L_1 (a) and λ_1 (b) across REAL, AQUA_ROUGH, AQUA_FLAT, and CHANNEL. For CHANNEL, the three values represent zonal averages at the southern boundary, the center, and the northern boundary of the channel. These values are plotted at the latitude where the local Coriolis parameter matches that of REAL, AQUA_ROUGH, and AQUA_FLAT. Small black dots indicate zonal averages from both hemispheres in REAL, AQUA_ROUGH, and AQUA_FLAT, while thick black dots represent the overall averages of these values. Since L_1 is nearly identical for CHANNEL and CHANNEL_R, both are shown using the same color.

compared to the global models because of larger eddy kinetic energy.

Fig. 2.12 shows the zonal and temporal average of the zonal velocity for the CHANNEL models. The amplitude of $\bar{u}$ in the different experiments follows thermal wind scaling with $\bar{u}(z = -h) = 0$ using the parameter of Tab. 2.2. In CHANNEL_25 (Fig. 2.12 a), three eastward, surface intensified jets are seen, which change in CHANNEL_35 (Fig. 2.12 b) to a broad eastward, surface intensified flow of similar magnitude, coinciding as expected with the regime change from $\lambda_1/L_1 < 1$ in CHANNEL_25 to $\lambda_1/L_1 > 1$ in CHANNEL_35. In contrast to CHANNEL_25, CHANNEL_25_R (Fig. 2.12 e) does not feature zonal jets, but weaker and broad

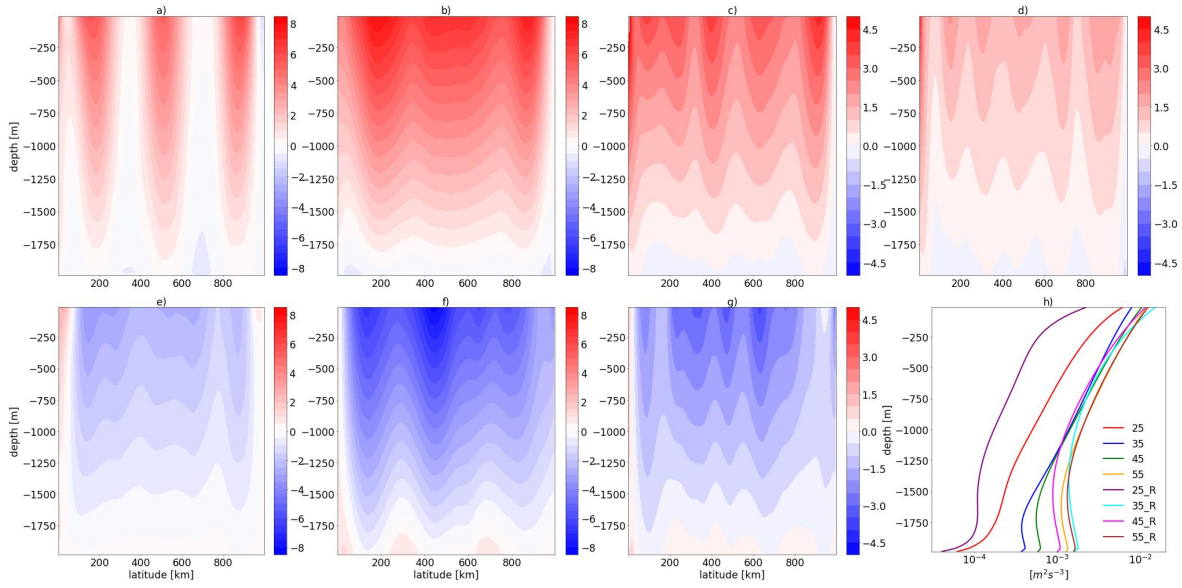


Figure 2.12: Zonal average of $\bar{u}$ [cm s^{-1}] for CHANNEL_25 - CHANNEL_55 (a-d) and CHANNEL_25_R - CHANNEL_45_R (e-g) and zonally and meridionally averaged eddy kinetic energy for CHANNEL_25 - CHANNEL_55 and CHANNEL_25_R - CHANNEL_55_R (h). Note the different color scale in panels c, d and g.

westward flow. With meridionally reversed forcing as in CHANNEL_25_R, eastward zonal jets are still generated but would be bottom intensified, compare e.g. Fig. 2 j) and Fig. 2 p) of Eden (2010). The large bottom friction in CHANNEL, however, damps the bottom-intensified jets and only weak and broad westward mean flow is left. We consider the case with bottom friction as more appropriate here, since bottom-intensified jets are not found in REAL or the aquaplanet models. A regime as in CHANNEL_25, characterized by eastward mean flow and $L_1 > \lambda_1$, is also rarely met in REAL or the aquaplanet models since often $\bar{u} < 0$ is found at latitudes equatorwards of ϕ_c . Consequently, such strong eastward, surface-intensified jets are uncommon in both REAL and aquaplanet models. Instead a regime as in CHANNEL_25_R is found in the westward return flows of the subtropical gyres.

In CHANNEL_35_R, the broad westward flow intensifies to a similar magnitude as the eastward flow in CHANNEL_35, and no strong jets are found since $\lambda_1/L_1 > 1$. Also in both CHANNEL_45 and CHANNEL_45_R, no strong zonal jets can be seen, but weaker east- or westward flow than in CHANNEL_35 and CHANNEL_35_R due to the thermal wind balance, which is further reduced in CHANNEL_55 and CHANNEL_55_R (not shown). In the absence of the strong eastward jets, signs of weaker, incoherent zonal jets in the same direction as the mean flow are emerging for both mean flow directions, which are embedded in the broad zonal flow, although $\lambda_1/L_1 > 1$. This is also seen often in REAL and observations (e.g. Maximenko et al., 2005; van Sebille et al., 2011).

The mean eddy kinetic energy in the different CHANNEL models scales roughly

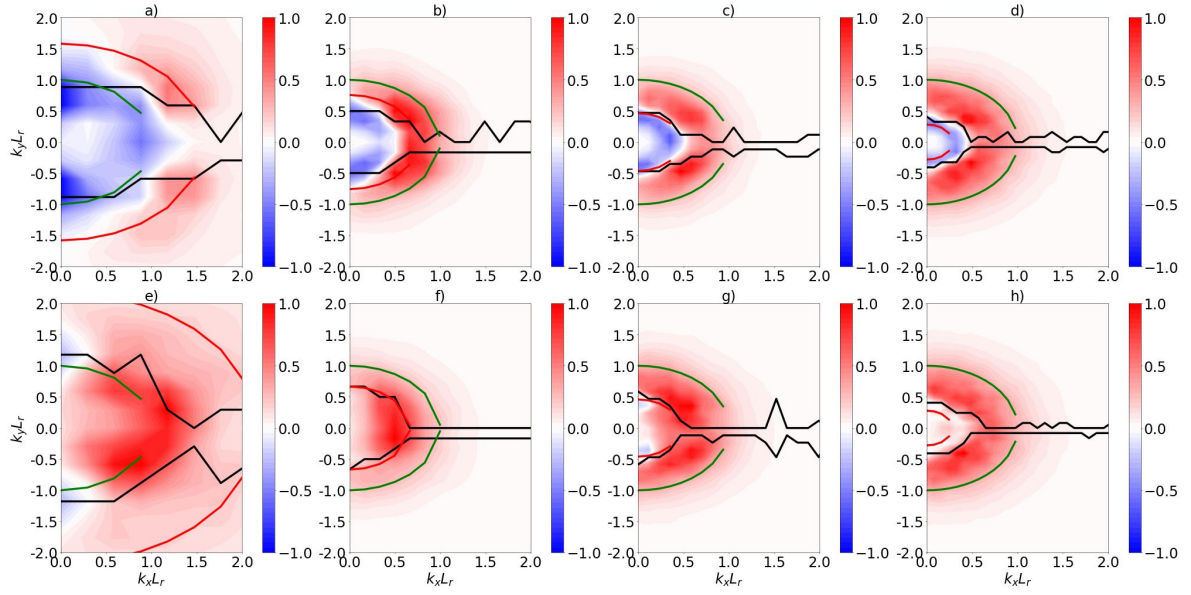


Figure 2.13: Vertical integral of baroclinic conversion term $\int B(\mathbf{k}) dz$ normalised by its maximum as a function of $k_x L_1$ and $k_y L_1$ for CHANNEL_25 (a), CHANNEL_35 (b), CHANNEL_45 (c), CHANNEL_55 (d), CHANNEL_25_R (e), CHANNEL_35_R (f), CHANNEL_45_R (g), and CHANNEL_55_R (h). L_1 (green), λ_1 (red), and k_{max} (black) are also shown.

with γ^2/k^2 , where $\gamma = 0.3M^2/N_0$ is the maximal Eady growth rate and $k = 1.6f/(N_0 h)$ the corresponding wavenumber, which is independent of the mean flow direction. However, CHANNEL_25_R shows lower eddy kinetic energy, and CHANNEL_35_R more than the scaling suggests, which transfers also to the baroclinic conversion term B , which we discuss next. Eddy kinetic energy in CHANNEL is strongly surface intensified (Fig. 2.12 h) with values approximately an order of magnitude higher at the surface than at the bottom. There is no strong dependence on latitude.

The baroclinic conversion term B in CHANNEL

Fig. 2.13 shows the vertical integral of $B(\mathbf{k})$ for the CHANNEL models. Compared to REAL, AQUA_ROUGH, and AQUA_FLAT, $B(\mathbf{k})$ shows less vertical variations in CHANNEL (not shown). While $B > 0$ is found throughout the whole water column, $B < 0$, when present, is reduced near the bottom. The magnitude of $\overline{b'w'} = \int B d\mathbf{k}$ roughly scales with $L^2 \gamma^3$, although the magnitude is smaller in CHANNEL_25_R but larger in CHANNEL_35_R than the scaling suggests. It is striking that only for the CHANNEL models with eastward background current, $B < 0$ for small k is found, while it is much reduced or not found at all for the corresponding CHANNEL models with westward mean current. Since $B < 0$ is found for all CHANNEL models with eastward background current, including CHANNEL_25 where $L_1 > \lambda_1$, there seems to be no link between $B < 0$ and the ratio λ_1/L_1 . It seems to be the sign of $\bar{u}$ which is linked to the emergence of $B < 0$ for small k .

To assess the effects of dissipation and the grid resolution, experiment CHAN-

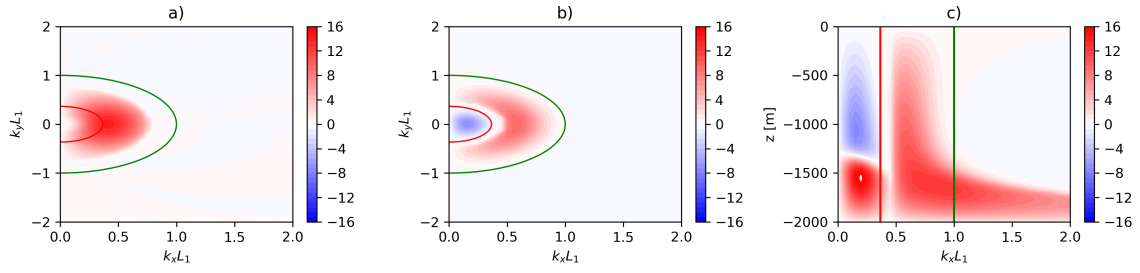


Figure 2.14: Baroclinic conversion term $B = -\text{Re}(b^*w)$ from linear stability analysis for parameter as in CHANNEL_35. The units of B are arbitrary because of the unknown amplitude of the linear stability analysis. a) with $\beta = 0$ at 1000 m depth, with energy scaling $E = (k^3 + (2Nh/f)^{-3})^{-1}$. b) Same as a) but for $\beta > 0$. c) Same as b) but for $k_y = 0$. L_1 (green) and λ_1 (red) are also shown.

NEL_45 is repeated with two times larger and two times smaller values of the biharmonic diffusivity, viscosity, and bottom friction coefficient. In another sensitivity experiment, we used a two times larger horizontal resolution, but the same dissipation as in CHANNEL_45. We found that the general characteristics of the spectrally resolved LEC and the eddy kinetic energy sink in B are unaffected by these changes. An increase in bottom friction, however, slightly enhances the magnitude of B .

In summary, we also find in the CHANNEL models – representative of particular latitudes of the aquaplanet model – the dipole structure in the baroclinic eddy conversion term B , but only for eastward background current. We find no relation to the ratio λ_1/L_1 and conclude that the sign of $\bar{u}$ is linked to the emergence of $B < 0$ at large scales.

2.3.4. Linear stability analysis

As a final step of simplification, we consider the linear stability analysis of a mean flow with constant vertical shear similar to the CHANNEL configuration. For $\beta = 0$, this becomes identical to the classical Eady case, where growth rates of instabilities are non-zero for $k < 2.4f/Nh$, with a maximum $\gamma = 0.3M^2/N$ at $k = 1.6f/(Nh)$. From the analytical solution of the Eady case it is also possible to calculate buoyancy and vertical velocity and finally the baroclinic conversion term $B = -\text{Re}(b^*w)$, up to an unknown amplitude. For $\beta > 0$ there is no analytical solution, and therefore we use here the numerical method by Thomsen et al. (2014) for the linear stability analysis, modified to include a β -plane, which is appropriate for the primitive equations used in CHANNEL.

The growth rate γ (not shown) is in general similar in the Eady case and for $\beta > 0$. The effect of β is that γ is suppressed but non-zero for small k , and finite but small for $k > 2.4f/Nh$, where γ vanishes for the Eady case. Fig. 2.14 shows the baroclinic conversion B for the Eady case with $\beta = 0$, and for the case with $\beta > 0$, for eastward and surface-intensified zonal mean flow as in CHANNEL_35. To account for the unknown amplitude of the linear stability analysis, B is scaled with

an depth-independent energy spectrum similar to the one found in the CHANNEL models. While $B > 0$ everywhere for the case $\beta = 0$ (Fig. 2.14 a), the suppressed γ at small k by the β effect comes along with $B < 0$ at small k (Fig. 2.14 b). In fact, the whole pattern of B from the linear stability analysis becomes strikingly similar to B in CHANNEL_35. The integrated conversion $\int B dk$ also remains positive as in the model with our energy scaling. Repeating experiment CHANNEL_35 but setting $\beta = 0$ (not shown), accordingly produces only incoherent or no $B < 0$ at small k . It is thus β or the presence of planetary Rossby waves, which causes $B < 0$ at small k .

Different to CHANNEL_35, however, B from the linear stability analysis becomes positive again at small k at larger depth (Fig. 2.14 c). For westward mean flow as in CHANNEL_35_R, B from the linear stability analysis stays the same for the Eady case $\beta = 0$, but for the case $\beta > 0$ the depth dependency of B reverses relative to Fig. 2.14 c), with $B < 0$ at small k towards the bottom instead of the surface, while the dependency of B to k stays the same. Note that the energy scaling for B is depth independent, while in all CHANNEL experiments, eddy energy is strongly surface intensified by the effect of the bottom friction. The bottom friction will also damp the negative eddy conversion close to the bottom in CHANNEL_35_R, but not in the region with $B < 0$ close to the surface in CHANNEL_35. Assuming that the actual perturbations in CHANNEL have a similar depth dependency as the solution of the linearised system, this damping at the bottom would explain the appearance of $B < 0$ at small k in CHANNEL for eastward mean flow, but its lack for westward flow.

2.4. Summary and discussion

In this study, the Lorenz energy cycle and especially the baroclinic conversion rate are spectrally resolved in a hierarchy of ocean models ranging from a realistic global model to idealised channel models. The baroclinic conversion term $B(\mathbf{k})$ exhibits in several geographical regions of the global models and in several of the idealised models a dipolar structure in spectral space, acting as a source of eddy kinetic energy at smaller scales and a sink at larger scales. Such an eddy kinetic energy sink at the large scales created by B is not accounted for in the classical QG turbulence theory (Charney, 1971). The feature was previously identified by Rouillet et al. (2012) in an idealised model and by Wang et al. (2019) in a global eddy-resolving model similar to REAL. Our findings confirm their results. Here we discuss the regional dependency of the large-scale sink of eddy kinetic energy to understand the process.

In regions where it is active, the sink by B partly balances the inverse cascade of eddy kinetic energy towards large scales by conversion to eddy available potential energy on large scales. Since we find a (partly) forward cascade of eddy available potential energy towards the smaller scales, where dissipation is likely, this opens up a possible new route to eddy energy dissipation in these regions. The schematic in Fig. 2.15 summarizes these findings.

It is tempting to complete the LEC by also considering the transfer from mean

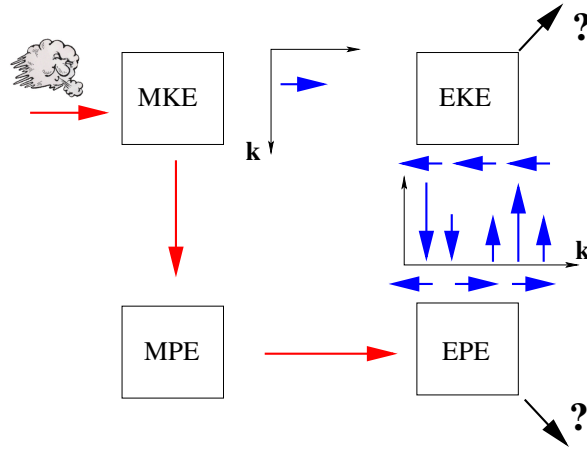


Figure 2.15: Schematic of the LEC with the energy reservoirs mean kinetic energy (MKE), eddy kinetic energy (EKE), mean available potential energy (MPE), and eddy available potential energy (EPE). The red arrows indicate the main energy pathway by baroclinic instability from the forcing of MKE by the wind to EPE, and further to the unknown eddy dissipation. For the regions with a sink of eddy kinetic energy by the baroclinic conversion term B at the large scales, the vertical blue arrows between EKE and EPE indicate the spectrally resolved B of different sign, the horizontal blue arrows the spectral energy fluxes of EKE and EPE, F_K and F_P , respectively. The horizontal blue arrow between MKE and EKE denotes the spectrally resolved transfer by horizontal shear production, which is generally of minor importance. The diagonal black arrows indicate the unknown eddy energy dissipation, which could also be at small scales fed by the forward cascade of EPE and the negative baroclinic conversion term B at small k .

to eddy potential energy, and also from mean kinetic to mean potential energy, but this is hampered by the lack of a consistent and practical definition of available potential energy. It should be noted that the diagnosis of the cascade of eddy available potential energy in our model is also affected by this problem. While this problem does not effect the baroclinic conversion term B or the energy K , it is often argued that averaging along isopycnals (or isopycnal thickness weighted averaging) is superior to averaging at constant height, which should also be done for B and K . However, practical problems with such a diagnostic in our z-level model and theoretical problems by a lack of a consistent formulation of the LEC in isopycnal averaging framework led us to stick to the simple averaging at constant height. We assume that the dipolar structure of the baroclinic conversion term in spectral space does not depend on the averaging framework.

Note that a quantitative comparison of the magnitude of the LEC terms in the realistic model with other studies (e.g. von Storch et al., 2012; Wang et al., 2019) is not done here, although they are similar. The exact magnitude of these terms, however, will depend on the model parametrizations for friction and mixing, the model domain (we exclude the Arctic Ocean), the surface forcing, and other differences. Note that globally integrated values of B , etc. are also reduced by the Hamming window we apply to the data before performing the Fourier transform. A quantitative comparison is therefore left for future studies.

We aim in this study to understand the mechanism behind the eddy kinetic energy sink by B at large scales by considering the geographical distribution of B , and its relationship to different dynamical regimes. In REAL and the aquaplanet models, $B < 0$ at small wavenumber k predominantly appears polewards of 30° latitude, whereas equatorward it is either absent or substantially reduced. Poleward of 30° latitude the Rhines scale λ_1 exceeds the Rossby radius L_1 , indicating the change from an anisotropic and an isotropic turbulence regime. In the anisotropic turbulence regime where $\lambda_1 < L_1$, eddies can transfer kinetic energy to Rossby waves and zonal jets (Theiss, 2004). When $\lambda_1 > L_1$ (polewards of 30°) the Rossby waves cannot transform their energy into zonal jets and the eddy scales remain isotropic.

The geographical distribution of $B < 0$ in REAL and the aquaplanet models also correlates with the direction change of the zonal background current from westwards equatorward of 30° to eastward poleward of 30° , and the change from more eddy kinetic energy in the baroclinic modes equatorward of 30° to more eddy kinetic energy in the barotropic mode polewards of 30° . However, the latter can be ruled out as related to the mechanism leading to $B < 0$, since for AQUA_FLAT the fraction of eddy kinetic energy in the barotropic mode R_0 does not increase with latitude, as found for REAL and AQUA_ROUGH, and most eddy kinetic energy is in the higher modes at all latitudes, yet $B < 0$ shows a similar distribution as in REAL and AQUA_ROUGH.

The idealised channel models replicate roughly the dynamical conditions of the global models at different latitudes, in particular for $\lambda_1 < L_1$ or $\lambda_1 > L_1$, and west- or eastward mean flow. $B < 0$ at small k appears independently of the ratio λ_1/L_1 indicating that $B < 0$ may not be tied to the anisotropic or isotropic turbulence regime. However, $B < 0$ at small k disappears when the mean flow changes from eastward to westward. We conclude that it is the sign of $\bar{u}$ which is linked to the emergence of $B < 0$ for small k .

Linear stability analysis shows that $\beta > 0$ and thus the presence of planetary Rossby waves is reducing the initial growth rates of perturbations at small k . $\Re(b^*w)$ calculated from the linearised solution is positive for all k for $\beta = 0$, but including $\beta > 0$ in the analysis generates the negative $\Re(b^*w)$ at small k . Repeating experiment CHANNEL_35 but setting $\beta = 0$ (not shown), accordingly produces only incoherent or no $B < 0$ at small k , demonstrating the importance of planetary Rossby waves for the effect. The pattern of (the energy-scaled) B from the linear stability analysis for $\beta > 0$ is strikingly similar to B in CHANNEL, including $B < 0$ at small k for the upper water column. But different to CHANNEL, B becomes positive again at small k at depth. Reversing the mean flow direction, reverses this depth dependency. It is argued that bottom friction damps the effect of $B < 0$ at small k at depth for westward mean flow, but not $B < 0$ at small k towards the surface for eastward mean flow.

We can thus explain with linear stability analysis the appearance (lack) of the dipole structure in B in the idealised model by the effect of β and the presence of planetary Rossby waves, and the direction of the mean flow. However, it should be noted that the reason for surface-intensified flow in CHANNEL is different from

REAL, both for east- and westward flow. In CHANNEL, we force the surface-intensified flow by bottom friction, while in REAL westward propagating long Rossby waves which dissipate at the western boundary intensify the surface circulation of the subtropical gyre (Anderson and Gill, 1975). We speculate that our results from CHANNEL transfer nevertheless to REAL and the real ocean.

Acknowledgments

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Datastatement

The model code can be found at <https://github.com/ceden/py0M2>.

Appendix: Spectral Lorenz energy cycle equations

Our derivation of the LEC is standard (e.g. Olbers et al., 2012), except for a modification regarding the reference density and the spectral resolution, which we detail here. We start with the momentum and continuity equations in hydrostatic and Boussinesq approximation, given by

$$D_t \mathbf{u}_h + \underline{f} \mathbf{u}_h + \nabla_h p = 0, \quad \partial_z p = -g \rho_{full} / \rho_0, \quad \nabla_h \cdot \mathbf{e} c \mathbf{u}_h + \partial_z w = 0 \quad (2.7)$$

with $D_t = \partial_t + \mathbf{u}_h \cdot \nabla_h + w \partial_z$ and the pressure p (scaled by the reference density ρ_0 of the Boussinesq approximation), horizontal and vertical velocity $\mathbf{u}_h$ and w , respectively. The hook in $\underline{f} \mathbf{u}_h$ denotes anticlockwise rotation of $\mathbf{u}_h$ by 90° . Friction has been ignored in Eq. 2.7. We define a reference density $\rho_{ref}(\mathbf{x}_h, z)$ and split full density ρ_{full} into reference and perturbation $\rho_{full} = \rho_{ref}(\mathbf{x}_h, z) + \tilde{\rho}(\mathbf{x}_h, z, t)$. We will assume below that the horizontal variations of ρ_{ref} are much smaller than the ones of $\tilde{\rho}$ in a WKB-sense, i.e. we assume that ρ_{ref} only varies in $\mathbf{x}_h$ on the large scales, while $\tilde{\rho}$ also varies on the smaller meso-scale. On the other hand, most of the vertical variation of ρ_{full} is as usual accounted for by ρ_{ref} . Note that in the standard derivation of the Lorenz energy cycle $\rho_{ref} = \rho_{ref}(z)$ is assumed, but we aim to allow for large-scale horizontal variations in ρ_{ref} . In consequence, the non-available potential energy, which is related to ρ_{ref} , will also be allowed to vary on large-scales, which would otherwise be not the case.

Taking the material derivative of $\tilde{\rho}$ yields

$$D_t \tilde{\rho} = D_t \rho_{full} - D_t \rho_{ref} = \rho_0 (\beta D_t S - \alpha D_t T) + c_{full}^{-2} D_t p_0 - \mathbf{u}_h \cdot \nabla_h \rho_{ref} - w \partial_z \rho_{ref} \quad (2.8)$$

using the equation of state appropriate for the primitive equations (e.g. Eden, 2015), where $p_0 = p_0(z)$ is the reference pressure of the Boussinesq approximation, and with the expansion coefficients $\alpha = -\rho_0^{-1} \partial \rho / \partial T$ and $\beta = \rho_0^{-1} \partial \rho / \partial S$, and the speed of sound c with $1/c^2 = \partial \rho / \partial p_0$. Using now the adiabatic versions of the conservative temperature and salinity equation, $D_t T = 0$ and $D_t S = 0$, and neglecting the term $\mathbf{u}_h \cdot \nabla_h \rho_{ref}$ in a WKB-sense as mentioned above yields

$$D_t \tilde{\rho} = (c_{full}^{-2} - c_{ref}^{-2}) w d_z p_0 + w \rho_0 N^2 / g, \quad N^2 = -g / \rho_0 \left(\partial_z \rho_{ref} - c_{ref}^{-2} d_z p_0 \right) \quad (2.9)$$

with the stability frequency N^2 . We now define buoyancy $\tilde{b} = -g \tilde{\rho} / \rho_0$, and neglect the difference between the actual speed of sound c_{full} and the one of the reference state c_{ref} . This yields the familiar buoyancy equation

$$D_t \tilde{b} + w N^2 = 0 \quad (2.10)$$

Note that except for the treatment of the large-scale variation of ρ_{ref} , all our assumptions are standard to derive the LEC (e.g. Olbers et al., 2012; von Storch et al., 2012).

In practice, we use $\rho_{ref} = \bar{\rho}$, where the overbar denotes the time mean. Introducing now Reynolds averaging $u = \bar{u} + u'$ and similar for v, w , etc., it becomes clear that $b' = \tilde{b}'$ and $\bar{b}' = 0$, while N^2 is calculated from $\bar{\rho}$. Total eddy available potential energy is given by $\overline{b'^2} / N^2$, while mean available potential energy is combined in a single reservoir with non-available potential energy by the choice $\rho_{ref} = \bar{\rho}$. The latter is no problem since we focus here on eddy available potential energy. In a more general study, one may modify our choice for ρ_{ref} towards more general approaches as in e.g. Hieronymus and Nycander (2015); Saenz et al. (2015).

We combine the Reynolds averaging with a Fourier transform of Eq. 2.7 and Eq. 2.10. We use the wave ansatz $\mathbf{u} = \int d\mathbf{k} \hat{\mathbf{u}}(\mathbf{k}, z, t) \exp(i\mathbf{k} \cdot \mathbf{x}_h)$ with the horizontal wavenumber vector $\mathbf{k}$ and the Fourier amplitudes $e\hat{u}$, and similar for p, b etc., multiply by $\exp(-i\mathbf{k}' \cdot \mathbf{x}_h)$, and integrate over $\mathbf{x}_h$, which yields

$$\partial_t \hat{\mathbf{u}} + \widehat{e\mathbf{c}\mathbf{u} \cdot \nabla \mathbf{u}} + f \hat{\mathbf{u}} + i\mathbf{k} \hat{p} = 0, \quad \partial_t \hat{b} + \widehat{\mathbf{u} \cdot \nabla b} + \hat{w} N = 0 \quad (2.11)$$

with the Fourier transforms of the non-linear terms

$$\widehat{\mathbf{u} \cdot \nabla \mathbf{u}} = (2\pi)^{-2} \int d\mathbf{x}_h \exp(-i\mathbf{k} \cdot \mathbf{x}_h) \mathbf{u} \cdot \nabla \mathbf{u} \quad (2.12)$$

$$\widehat{\mathbf{u} \cdot \nabla b} = (2\pi)^{-2} \int d\mathbf{x}_h \exp(-i\mathbf{k} \cdot \mathbf{x}_h) \mathbf{u} \cdot \nabla b \quad (2.13)$$

where $\mathbf{u}$ is the three-dimensional velocity, and ∇ the three-dimensional gradient. Here we have ignored lateral variations of N in the Fourier transform, which is only necessary for the buoyancy equation and thus available potential energy, and assumed a double periodic domain in x_h . The treatment of topography is discussed in Section 2 b). Next we use the Reynolds decomposition for $\hat{\mathbf{u}}$, etc., multiply the momentum equation with $\hat{\mathbf{u}}_h^*$ and average, and the complex conjugate of the momentum equation with $\hat{\mathbf{u}}_h'$ and average, and similar for the buoyancy equation but multiplied with $\hat{b}'^*$ and $\hat{b}'$, respectively. Using also the hydrostatic relation $\partial_z \hat{p} = \hat{b}$ and the continuity equation $i\mathbf{k} \cdot \mathbf{u}_h + \partial_z w = 0$, Eq. 2.2 is obtained.

Vertical structure of eddy kinetic energy transfers

Jan Dettmer and Carsten Eden (2026): “Vertical Structure of Eddy Kinetic Energy Transfers” submitted to Journal of Physical Oceanography

Abstract

The vertical structure of oceanic eddy kinetic energy is analyzed in realistic global and idealized models using vertical Sturm-Liouville eigenmodes. Independent of the model configuration, and given sufficient resolution, most eddy kinetic energy resides in the barotropic and first baroclinic mode and the fundamental energy pathway remains the same: eddy kinetic energy is generated via baroclinic instability in the baroclinic modes, while barotropic production by horizontal shear remains small. Subsequently eddy kinetic energy is transferred to the barotropic mode by non-linear transfers. This transfer is non-local in modal space with energy transferred directly from the higher modes to the barotropic mode. The presence of a pycnocline shifts baroclinic generation and non-linear transfers to higher modes and increases the complexity of these transfers, introducing an energy loop between the first and higher baroclinic modes that operates alongside the direct transfer to the barotropic mode. In model configurations with a rough bottom, topographic scattering of eddy kinetic energy can locally be significant, but remains small in the global integral, and is of secondary importance next to baroclinic generation and non-linear transfers. A spectral decomposition of the modal eddy kinetic energy conversion and transfers reveals that while the baroclinic modes facilitate a large-scale eddy kinetic energy sink via negative baroclinic conversion, such a sink is absent in the barotropic mode. Since non-linear transfers in both modal and spectral space accumulate barotropic large scale eddy kinetic energy, the removal of large-scale barotropic eddy kinetic energy must rely on alternative routes to dissipation.

3.1. Introduction

The ocean can be described as a sea of eddies with spatial scales of approximately 5–150 km and time scales ranging from days to months. The imprint of these mesoscale eddies can be seen in satellite observations and they emerge in numerical ocean simulations with small enough resolution. Mesoscale eddies modulate the stratification, mixing, tracer transport, and dissipation pathways of oceanic flows over a wide range of spatiotemporal scales. Many aspects of the generation and propagation of mesoscale eddies are well understood, however, despite their importance, several key aspects remain uncertain. These include the production of eddy energy and its distribution among vertical modes, as well as the respective roles of advection and bottom topography. The objective of this study is to advance the understanding of eddy energy pathways in the ocean through projecting eddy kinetic energy reservoirs and transfers on vertical modes. The vertical modes employed in this study are the Sturm-Liouville eigenmodes derived from the linearized primitive equations under the assumptions of a rigid lid and a flat bottom (Olbers and Eden, 2013). In the presence of non-linear flow dynamics and realistic bottom topography, the latter modifying the bottom boundary condition, these modes are no longer the natural eigenmodes of the system. Nevertheless, they still provide a complete and useful basis for expanding the dynamical variables and for diagnosing the vertical structure of eddy energy and its transfers. More details are given below.

A comparison between the observed spatial scales of mesoscale eddies and the scales predicted by classical analytical theories (Eady, 1949; Phillips, 1954; Charney, 1947) suggests that these eddies are primarily generated by baroclinic instability (Smith, 2007; von Storch et al., 2012; Böning and Budich, 1992), which can be understood as the conversion of eddy available potential energy to eddy kinetic energy. It has been shown that their horizontal kinetic energy is mostly contained by the barotropic and first baroclinic vertical mode (Wunsch, 1997). Mesoscale eddies are to leading order in geostrophic balance and can therefore be described within the framework of quasigeostrophic (QG) theory. QG dynamics predict a forward cascade of (potential) enstrophy and an inverse cascade of kinetic energy to smaller horizontal wavenumbers. Observations of sea surface height as well as eddy-resolving ocean models provide clear evidence for such an inverse kinetic energy cascade over spatial scales of roughly 150–1000 km (Scott and Wang, 2005; Wang et al., 2015; Schlösser and Eden, 2007). Vallis (2017) argue that in quasigeostrophic case it is the three-dimensional wavenumber that is important and as a consequence, the energy cascade to larger horizontal scales is generally accompanied by a cascade to larger vertical scales — a barotropization of the flow. This inverse cascade in the vertical and the effect of non-constant stratification is studied by Smith and Vallis (2001), who analyze numerical quasigeostrophic simulations of freely decaying turbulence and later forced-dissipative turbulence (Smith and Vallis, 2002). They find that there is a transfer of kinetic energy to the lower vertical modes, which is fast and efficient in the case of constant stratification, while it is less efficient in the case of a strong

pycnocline. In the case of constant stratification they find that in each vertical mode energy cascades to the respective Rossby radius, where it is directly transferred to the barotropic mode. In the case of non-constant, surface intensified stratification however, energy of the higher modes is first transferred to the first baroclinic mode at the respective Rossby radius before it is transferred to the barotropic mode at the first baroclinic Rossby radius. It is noted that this is an idealized picture and in fact the transfers are not that localized in physical space and there is significant transfer of energy from the higher to the lower modes at scales larger than the respective Rossby radius. Using multi-layer quasi-geostrophic models, Shevchenko and Berloff (2017) partially confirm this modal pathway, by demonstrating that energy in high baroclinic modes can also be transferred directly to the barotropic mode without passing through the first baroclinic mode. Furthermore, while an inverse energy transfer prevails, high-resolution vertical configurations reveal a simultaneous forward energy transfer from lower to higher modes.

Mesoscale eddy energetics and transfers between vertical modes have also been investigated in a regional model of the Agulhas region by Tedesco et al. (2022) using a eddy kinetic energy budget decomposed into vertical modes. They focus on decomposing the barotropic and first baroclinic from the higher baroclinic modes and show that mesoscale variability is dominated by the barotropic and first baroclinic mode along the path of the intense boundary currents, while higher baroclinic modes become important in regions of strong eddy–topography interaction. Their modal eddy kinetic energy budget reveals multiple energy pathways, including transfers toward bottom-intensified mean flows, dissipation by bottom friction, and non-linear transfers to higher baroclinic modes. Away from the shelf, they identify an inverse modal cascade, similar to Smith and Vallis (2001). Due to strong local eddy generation, the Agulhas Current region remains a net source of mesoscale eddy contrary to the paradigm of mesoscale eddy decay upon western boundaries.

The vertical structure of mesoscale eddy energetics have also been analyzed in a model hierarchy by Yankovsky et al. (2022). It shown that as horizontal resolution increases from $1/4^\circ$ to $1/32^\circ$ and eddies become increasingly resolved, kinetic energy shifts toward the barotropic mode reflecting more efficient barotropization of the flow. At coarse resolution, unresolved eddy dynamics lead to suppressed vertical energy fluxes and trapping of energy in higher baroclinic modes. The sensitivity of this partitioning is especially pronounced for the eddy part of kinetic energy and depends strongly on the underlying dynamical regime: regions characterized by weak, baroclinic mean flows exhibit pronounced resolution dependence, whereas strongly barotropic regimes, are comparatively insensitive.

While kinetic energy production by baroclinic instability and inverse transfers in modal space with the tendency toward barotropization have been documented, significant gaps remain in understanding the vertical structure of eddy energy pathways. The underlying interactions of the inter-modal transfers, topographic scattering and sensitivity to stratification, as well as global relevance remain insufficiently understood. It is the focus of this study to address these uncertainties by formulating

and diagnosing a vertically resolved eddy kinetic energy equation in a hierarchy of ocean models ranging from idealized high resolution channel models to realistic global models at different horizontal and vertical resolution. Section 2 details the energy diagnostic and gives a description of the ocean models. Section 3 presents the results derived from these models and Section 4 gives a discussion and summary.

3.2. Methods

The first part of this section describes and discusses the vertical mode decomposition and our modal eddy kinetic energy diagnostic. The second part details the numerical models, to which the diagnostic is applied.

3.2.1. Linear modal energy equations for flat bottom

Starting point of the discussion are the linearized primitive equations with rigid lid and flat bottom

$$\partial_t \mathbf{u} + f \underline{\mathbf{u}} + \nabla p = 0 \quad , \quad \partial_z p = b \quad , \quad \partial_t b + w N^2 = 0 \quad , \quad \nabla \cdot \mathbf{u} + \partial_z w = 0 \quad (3.1)$$

All vectors are two dimensional, $\mathbf{u}$ is horizontal velocity, p is pressure scaled by the Boussinesq reference density, w is vertical velocity, b is buoyancy and $N(z)$ is the background buoyancy frequency. The symbol $\underline{\mathbf{u}} = (-v, u)$ denotes a 90° anticlockwise rotation of $\mathbf{u} = (u, v)$. The assumptions made are standard (Olbers and Eden, 2013), and frictional and diabatic processes are neglected. The following vertical mode ansatz is used

$$\mathbf{u} = \sum_m \mathbf{u}_m \phi_m(z) \quad , \quad p = \sum_m p_m \phi_m(z) \quad , \quad w = \sum_m w_m d_z \phi_m(z) / N^2 \quad , \quad b = \sum_m b_m d_z \phi_m(z) \quad (3.2)$$

with the vertical structure function $\phi_m(z)$ and modal amplitudes $\mathbf{u}_m(\mathbf{x}, t)$, $p_m(\mathbf{x}, t)$ etc. This leads by separation of dimensions in the continuity equation to the following standard eigenvalue equation (Olbers et al., 2012)

$$\phi_m + c_m^2 d_z (d_z \phi_m / N^2) = 0 \quad \text{with} \quad d_z \phi_m|_{-h}^0 = 0 \quad (3.3)$$

with the eigenfunctions ϕ_m and eigenvalues c_m , and to the modal continuity equation $\nabla \cdot \mathbf{u}_m = c_m^{-2} w_m$. In the numerical models the modes are solved numerically for which details are given in the Appendix. For now, we assume a constant water depth h and rigid lid, for which the kinematic boundary condition transfers to the boundary conditions given in Eq. (3.3), i.e. $w|_{-h}^0 = 0$. From the linear buoyancy equation it also follows that $b|_{-h}^0 = 0$.

Since Eq. (3.3) is of Sturm-Liouville type, the following orthogonality conditions apply

$$\int_{-h}^0 \phi_m \phi_n dz = \delta_{m,n} , \quad c_m^2 \int_{-h}^0 d_z \phi_m (d_z \phi_n) / N^2 dz = \delta_{m,n} \quad (3.4)$$

The second condition is derived by partial integration of Eq. (3.3). Using the orthogonality conditions it follows that

$$\begin{aligned} \mathbf{u}_n &= \int \mathbf{u} \phi_n dz , & p_n &= \int p \phi_n dz \\ w_n &= c_n^2 \int w (d_z \phi_n) dz , & b_n &= c_n^2 \int b (d_z \phi_n) / N^2 dz \end{aligned} \quad (3.5)$$

Using Eq. (3.2) in Eq. (3.1), multiplying the horizontal and vertical momentum equation with ϕ_n and the buoyancy equation with $(d_z \phi_m) / N^2$, and integrating over depth we obtain the modal equations

$$\partial_t \mathbf{u}_n + f \mathbf{u}_n + \nabla p_n = 0 , \quad p_n = b_n , \quad \partial_t b_n + w_n = 0 , \quad \nabla \cdot \mathbf{u}_n = w_n / c_n^2 \quad (3.6)$$

The modal momentum (buoyancy) equation is multiplied with $\mathbf{u}_n$ (with b_n / c_n^2) to arrive at the modal kinetic (potential) energy equation

$$\partial_t \mathbf{u}_n^2 / 2 + \nabla \cdot \mathbf{u}_n b_n - b_n w_n / c_n^2 = 0 , \quad \partial_t b_n^2 / 2 c_n^2 + w_n b_n / c_n^2 = 0 \quad (3.7)$$

Each mode is separated from the others, i.e. there is no coupling between the modes for the flat bottom and linear case. Only by topographic variations or non-linearities the modes get coupled, as discussed now.

3.2.2. Effect of topography and variable N^2

Now we allow for $h = h(x)$ and also small, large-scale horizontal variations in $N^2 = N^2(x, z)$ in a WKB sense as in Dettmer and Eden (2025). With varying topography h , the boundary conditions $w|_{0,-h} = 0$ is replaced with $\mathbf{u}|_{-h} \cdot \nabla h + w|_{-h} = 0$, but we keep the rigid lid $w|_0 = 0$ for simplicity. It is still possible to define $\phi_n(x, z)$ locally as solution of Eq. (3.3) with the same orthogonality conditions Eq. (3.4) and boundary conditions $d_z \phi|_{0,-h} = 0$ as before, but the ϕ_n are not the natural eigenmodes of the system anymore. The following consequences arise: while it is still possible to use the expansion $\mathbf{u} = \sum_n \mathbf{u}_n \phi_n$, the expansion $w_{flat} = \sum w_n (d_z \phi_n) / N^2$ cannot be used anymore, because w is in general non-zero at the bottom, but $d_z \phi|_{-h} = 0$. We can still define the coefficients $w_n = c_n^2 \int w d_z \phi_n dz$, but they take now a different meaning than in the flat bottom case. In the same way, we also define $p_n = \int p \phi_n dz$ but cannot use $p_{flat} = \sum_n p_n \phi_n$ anymore to allow for buoyancy anomalies at the bottom. They would otherwise be zero since then $\partial_z p_{flat}|_{-h} = \sum_n p_n d_z \phi_n|_{-h} = 0 = b_{flat}|_{-h}$. We can

also still define $b_n = c_n^2 \int b(d_z \phi_n)/N^2 dz = p_n$, but both expansion coefficients p_n and b_n take a different meaning than in the flat bottom case.

Using $\mathbf{u} = \sum_n \mathbf{u}_n \phi_n$, the linear momentum equation multiplied with ϕ_n and integrated over depth becomes now

$$\partial_t \mathbf{u}_n + f \underline{\mathbf{u}}_n + \int_{-h}^0 \phi_n \nabla p dz = 0 \quad (3.8)$$

The pressure force in Eq. (3.8) is not written in terms of p_n as before, since the p_n are not expansion coefficients for p anymore. However, the kinetic energy $\mathbf{u}^2/2$ decomposes as for the flat bottom case into modal components $\int_{-h}^0 \mathbf{u}^2 dz = \sum_n \mathbf{u}_n^2$. The evolution equation for the modal kinetic $\mathbf{u}_n^2/2$ follows by multiplication of Eq. (3.8) with $\mathbf{u}_n$, which we complement with the total potential energy equation

$$\partial_t \mathbf{u}_n^2/2 + \mathbf{u}_n \cdot \int_{-h}^0 \phi_n \nabla p dz = 0, \quad \partial_t \int_{-h}^0 b^2/(2N^2) dz + \int_{-h}^0 b w dz = 0 \quad (3.9)$$

Note that a modal potential energy equation as for the flat bottom case is missing now, since the b_n are not expansion coefficients for b anymore. Summing the term $\mathbf{u}_n \cdot \int_{-h}^0 \phi_n \nabla p dz$ over all modes n equals $-\int_{-h}^0 b w dz$ plus a flux divergence term, thus total energy is conserved. The individual terms $\mathbf{u}_n \cdot \int_{-h}^0 \phi_n \nabla p dz$ also contain a flux divergence, the actual exchange term with potential energy, but also the scattering to other kinetic energy modes. They are worked out explicitly in the following.

The pressure gradient term in Eq. (3.8) can be written as

$$\int_{-h}^0 \phi_n \nabla p dz = \int_{-h}^0 \nabla \phi_n p dz - \int_{-h}^0 p \nabla \phi_n dz = \nabla p_n - \phi_n p|_{-h} \nabla h - \int_{-h}^0 p \nabla \phi_n dz \quad (3.10)$$

with $\nabla p_n = \nabla \int_{-h}^0 \phi_n p dz = \int_{-h}^0 \nabla \phi_n p dz + \phi_n p|_{-h} \nabla h$. The exchange term between kinetic and potential modal energy in Eq. (3.9) then becomes

$$\begin{aligned} \mathbf{u}_n \cdot \int_{-h}^0 \phi_n \nabla p dz &= \nabla \cdot (\mathbf{u}_n p_n) - p_n \nabla \cdot \mathbf{u}_n - \phi_n p|_{-h} \mathbf{u}_n \cdot \nabla h \\ &\quad - \int_{-h}^0 p \mathbf{u}_n \cdot \nabla \phi_n dz \\ &= \nabla \cdot (\mathbf{u}_n p_n) - p_n w_n c_n^{-2} - \phi_n p|_{-h} \mathbf{u}_n \cdot \nabla h \\ &\quad - \int_{-h}^0 (p \mathbf{u}_n + p_n \mathbf{u}) \cdot \nabla \phi_n dz - \phi_n p|_{-h} \mathbf{u}_n \cdot \nabla h \end{aligned} \quad (3.11)$$

For the second step the continuity equation multiplied with ϕ_n and integrated over depth, and the bottom boundary condition $w|_{-h} + \mathbf{u}|_{-h} \cdot \nabla h = 0$ was used. The last two

terms in Eq. (3.11) are the scattering between the vertical modes, while the first is a flux divergence which cancels in a global integral. The second term is identical to the exchange between potential and kinetic energy in the flat bottom case, but note that the expansion coefficients $p_n = b_n$ have different meaning for varying topography. Note also that

$$\begin{aligned} \int_{-h}^0 \mathbf{u} \cdot \nabla p \, dz &= \sum_n \mathbf{u}_n \cdot \int_{-h}^0 \phi_n \nabla p \, dz \\ &= \int_{-h}^0 \nabla \cdot (\mathbf{u} p) \, dz - p w \Big|_{-h}^0 - \int_{-h}^0 w b \, dz \\ &= \sum_n \nabla \cdot (\mathbf{u}_n p_n) - \int_{-h}^0 w b \, dz \end{aligned}$$

with $\sum_n \nabla \cdot \mathbf{u}_n p_n = \nabla \cdot \int_{-h}^0 \mathbf{u} p \, dz = \int_{-h}^0 \nabla \cdot \mathbf{u} p \, dz + \mathbf{u} p \cdot \nabla h$ and using the bottom boundary condition. Combining with Eq. (3.11) summed over all n , it follows that

$$\int_{-h}^0 b w \, dz = \sum_n p_n w_n / c_n^2 + \sum_n \int_{-h}^0 (p_n \mathbf{u} + \mathbf{u}_n p) \cdot \nabla \phi_n \, dz + \sum_n \phi_n p \Big|_{-h} \mathbf{u}_n \cdot \nabla h \quad (3.12)$$

and the modal energy cycle for varying topography $h = h(\mathbf{x})$ as

$$\begin{aligned} \partial_t \mathbf{u}_n^2 / 2 + \nabla \cdot \mathbf{u}_n p_n &= p_n w_n c_n^{-2} + T_n, \\ \partial_t \int_{-h}^0 b^2 / 2 N^2 \, dz &= - \sum_n (p_n w_n / c_n^2 + \tilde{T}_n) \end{aligned} \quad (3.13)$$

with the topographic scattering term

$$\tilde{T}_n = \int_{-h}^0 (p \mathbf{u}_n + p_n \mathbf{u}) \cdot \nabla \phi_n \, dz + \phi_n p \Big|_{-h} \mathbf{u}_n \cdot \nabla h \quad (3.14)$$

$\tilde{T}_n$ vanishes for the flat bottom (and $N^2 = N^2(z)$) case and we recover Eq. (3.7). Note that we were unable to show that in general $\sum_n \tilde{T}_n = 0$, thus $\tilde{T}_n$ may not be purely a topographic scattering between the modes, but may contain also a net transfer of kinetic to potential energy, but we expect this net transfer to be small and evaluate it below. Note also that $\nabla \phi_n$ in $\tilde{T}_n$ contains in principle the variation due to changes in $h(\mathbf{x})$ but also of changes in $N^2(\mathbf{x})$, which is evaluated below.

3.2.3. Effect of non-linear terms and eddy kinetic energy

We continue with the non-linear momentum equation

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + f \underline{\mathbf{u}} + \nabla p = 0 \quad (3.15)$$

The term $w\partial_z \mathbf{u}$ has been neglected since it is small in quasi-geostrophic scaling. Even for flat bottom, the ϕ_n are not the eigenmodes of the non-linear system anymore, but we still use them. Expressing with the vertical modes yields

$$\partial_t \mathbf{u}_n + \sum_{ij} B_{ijn} \mathbf{u}_i \cdot \nabla \mathbf{u}_j + f \mathbf{u}_n + \int_{-h}^0 \phi_n \nabla p dz = - \sum_{ij} \mathbf{u}_j (\mathbf{u}_i \cdot \int_{-h}^0 \phi_n \phi_i \nabla \phi_j dz) \quad (3.16)$$

introducing the coupling coefficients $B_{ijn} = \int_{-h}^0 \phi_i \phi_j \phi_n dz$. The dependency of ϕ_n on x by $h(x)$ or $N^2(x, z)$ generates the term on the right hand side of Eq. (3.16), which vanishes for $h = \text{const}$ and $N^2(z)$.

A standard Reynolds decomposition $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$ and similar for the other variables including the coefficients $\mathbf{u}_n, p_n$, etc is applied. The Reynolds average is defined as a time average over the diagnosis period and denoted by an overbar. Eq. (3.16) is multiply with $\mathbf{u}'_n$, and the Reynolds average is applied, which yields

$$\partial_t E_n + \sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}_i : \nabla \mathbf{u}'_j} + \sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}'_i : \nabla \bar{\mathbf{u}}_j} = -\nabla \cdot \overline{\mathbf{u}'_n p'_n} + \overline{p'_n w'_n c_n^{-2}} + T_n \quad (3.17)$$

with the modal eddy kinetic energy $E_n = \overline{(\mathbf{u}'_n)^2}/2$, with the inner tensor product $\mathbf{A} : \mathbf{B} = A_{ij} B_{ij}$, and with the topographic scattering term

$$\begin{aligned} T_n = & \int (\overline{p' \mathbf{u}'_n} + \overline{p'_n \mathbf{u}'}) \cdot \nabla \phi_n dz \\ & + \phi_n \overline{p' |_{-h} \mathbf{u}'_n} \cdot \nabla h \\ & - \sum_{ij} \overline{\mathbf{u}'_n \cdot \mathbf{u}_j} (\mathbf{u}_i \cdot \int_{-h}^0 \phi_n \phi_i \nabla \phi_j dz) \end{aligned} \quad (3.18)$$

The first two terms in T_n are related to the pressure work term, the last term to the advective terms.

The term $\sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}_i : \nabla \mathbf{u}'_j} = \sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}'_i : \nabla \mathbf{u}'_j} + \sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \bar{\mathbf{u}}_i : \nabla \mathbf{u}'_j}$ denotes exchange between modal eddy kinetic energy E_n of different vertical modes due to non-linear eddy-mean and eddy-eddy interactions. When summed over all modes n , the term can be written as a flux divergence and therefore does not generate total eddy energy, but only redistributes energy between the modes. The term $\sum_{ij} B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}'_i : \nabla \bar{\mathbf{u}}_j}$ on the other hand can not be written as a flux divergence and is interpreted as modal eddy energy production related to lateral shear instability of the mean flow. Summed over all n it becomes the familiar transfer term of total eddy kinetic energy by lateral shear instability from mean kinetic energy. The term $\overline{p'_n w'_n c_n^{-2}}$ is a production term related to vertical shear instability, that converts eddy available potential energy to eddy kinetic energy.

We assign the following names to the different terms in Eq. (3.17) which we discuss below

- E_n scattering by eddy-eddy interaction $N_n^e = \sum_{ij} B_{ijn} \overline{u'_n u'_i} : \nabla u'_j$
- E_n scattering by eddy-mean flow interaction $N_n^m = \sum_{ij} B_{ijn} \overline{u'_n \bar{u}_i} : \nabla u'_j$
- E_n production by mean lateral shear $S_n = \sum_{ij} B_{ijn} \overline{u'_n u'_i} : \nabla u_j$
- E_n production by mean vertical shear $P_n = \overline{p'_n w'_n} / c_n^2$

with the properties $\sum_n N_n^e = 0$ and $\sum_n N_n^m = 0$. The topographic scattering is decomposed as

$$T_n = \underbrace{\int (\overline{p' u'_n} + \overline{p'_n u'}) \cdot \nabla \phi_n dz}_{T_n^\phi} + \underbrace{\overline{\phi_n p'|_{-h}} u'_n \cdot \nabla h}_{T_n^t} - \underbrace{\sum_{ij} \overline{u'_n \cdot u_j} (u_i \cdot \int_{-h}^0 \phi_n \phi_i \nabla \phi_j dz)}_{T_n^a} \quad (3.19)$$

into contributions by the pressure work term T_n^ϕ due to variations in ϕ_n and T_n^t due to variations in h , and by advection terms T_n^a . Note that all terms in Eq. (3.17) and Eq. (3.19) are depth independent and depend only on the mode n and the horizontal dimensions.

3.2.4. The coupling coefficient B_{nij}

The terms N_n^m , N_n^e and S_n contain the coupling coefficient B_{nij} . For $N^2 = \text{const}$ the eigenmodes are given by cosine functions, which exhibit a symmetry that can be used to show for which n, i, j triplets B_{nij} is non-zero and allows for non-linear interaction. B_{nij} can be expressed as the sum of four integrands of which at least one is non-zero only if $n = |i \pm j|$ for $n, i, j > 0$. Consequently, non-linear modal interactions occur only for these triads.

Fig. 3.1 shows the vertical modes ϕ_n from Eq. (3.3) and the corresponding B_{nij} for constant and exponential $N^2(z)$. For $N^2 = \text{const}$ the modes are symmetric in the vertical about $z = -h/2$ and B_{nij} is strictly confined to the combinations satisfying $n = |i \pm j|$. For $n = 0$ the $i = j$ diagonal is non-zero, while for $n = 1$ the off-diagonal elements are non-zero. As n increases the non-zero coefficients move further away from the diagonal. The largest coefficients occur for $j, i = 0$, where multiple conditions of $n = |i \pm j|$ are met, indicating strong coupling between the baroclinic modes and the barotropic mode. For exponential $N^2(z)$, the modes are surface intensified and the vertical symmetry is broken. While the overall structure of B_{nij} remains similar, neighboring combinations become weakly non-zero. In particular, the first baroclinic mode exhibits strong diagonal self interaction, consistent with the findings of Smith and Vallis (2001).

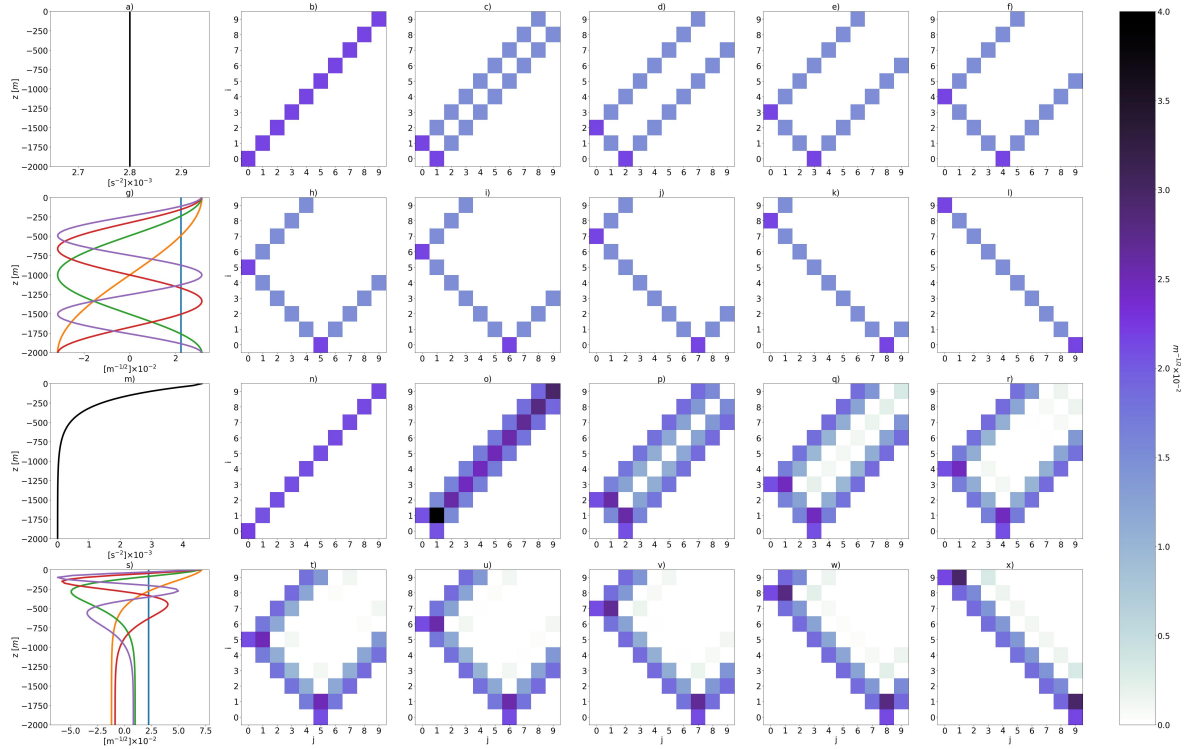


Figure 3.1: The vertical modes ϕ_n from Eq. (3.3) and the modal coupling coefficient B_{nij} for constant $N^2 = 2.8 \times 10^{-3} \text{ s}^{-2}$ in a) and exponential $N^2(z)$ with a vertical average of $2.8 \times 10^{-3} \text{ s}^{-2}$ in m), and for 2000 m depth. The modes for $N^2 = \text{const}$ and $N^2(z)$ are shown in g) and s) for $n = 0, \dots, 4$ in blue, orange, green, red and purple. B_{nij} for $N^2 = \text{const}$ and $N^2(z)$ is shown in b) to l) and n) to x) for $n = 0, \dots, 9$.

REAL	eddy permitting global ocean model with horizontal resolution of $1/10^\circ$, 42 vertical levels, realistic topography.
AQUA_FLAT	global aqua planet model, same resolution as REAL, flat bottom, single meridional barrier, open Southern Ocean.
AQUA_FLAT_HR	same as AQUA_FLAT but horizontal resolution of $1/10^\circ$ and 128 vertical levels
AQUA_ROUGH	same as AQUA_FLAT but rough bottom.
CHANNEL	set of idealized forced baroclinically unstable periodic zonal channel models

Table 3.1: Name and short description of the different ocean model experiments discussed below.

3.2.5. Numerical models

The different terms in the modal eddy kinetic energy equation Eq. (3.17) are evaluated in several numerical ocean models: a set of idealized channel models (CHANNEL) at different resolutions, three aquaplanet models (AQUA_FLAT_HR, AQUA_FLAT and AQUA_ROUGH) and an eddy-resolving global ocean model (REAL). A short description is given in Table 3.1 and more details are given below.

Experiment CHANNEL

The experiment CHANNEL is an idealized channel with zonally periodic and meridionally closed boundary conditions. The horizontal domain spans $1000 \text{ km} \times 1000 \text{ km}$ with a flat bottom at 2000 m depth. Dissipation is implemented as biharmonic mixing and friction, Rayleigh damping and linear bottom friction. Lateral biharmonic viscosity and diffusivity are scaled with Δx^{-4} and vertical viscosity and diffusivity are scaled with Δz^{-2} , with horizontal and vertical resolution Δx and Δz . The Rayleigh damping coefficient is set to $0.5 \times 10^{-7} \text{ s}^{-1}$. More parameter values for the different configurations can also be found in Table 3.2. CHANNEL uses the β -plane approximation with $f = 7.4 \times 10^{-5} \text{ s}^{-1}$ and $\beta = 1.9 \times 10^{-11} (\text{ms})^{-1}$, corresponding to 35°N and a linear equation of state. The model is initialized with a meridional buoyancy gradient of $5.5 \times 10^{-5} \text{ s}^{-2}$ and a vertical buoyancy gradient of $3.1 \times 10^{-3} \text{ s}^{-2}$. The forcing is given by restoring of the zonal mean temperature gradient towards the initial conditions with a restoring timescale of $t_{res} = 50$ days, representing a large scale forcing. By only restoring the zonal mean, damping of perturbations like mesoscale eddies is reduced. The model is integrated for 1200 days and the diagnosis is carried out with the last 365 days, using 3 daily averages.

	Δx	Δz	η_{hbi}	κ_{hbi}	η_v	κ_v	r_{bot}	$N^2(z)$
CHANNEL_HR	488	12.	3.05×10^6	1.53×10^6	1.56×10^{-6}	1.56×10^{-5}	6.17×10^{-5}	linear
CHANNEL_HR.P	488	12.	3.05×10^6	1.53×10^6	1.56×10^{-6}	1.56×10^{-5}	6.17×10^{-5}	exponential
CHANNEL_HR.LV	488	50	3.05×10^6	1.53×10^6	2.5×10^{-5}	2.5×10^{-4}	1.54×10^{-5}	linear
CHANNEL_MR	1953	50	7.81×10^8	3.91×10^8	2.5×10^{-5}	2.5×10^{-4}	1.54×10^{-5}	linear
CHANNEL_MR.P	1953	50	7.81×10^8	3.91×10^8	2.5×10^{-5}	2.5×10^{-4}	1.54×10^{-5}	exponential
CHANNEL_LR	6993	50	1.28×10^{11}	6.42×10^{10}	1.0×10^{-4}	1.0×10^{-3}	7.72×10^{-6}	linear
CHANNEL_LR.P	6993	50	1.28×10^{11}	6.42×10^{10}	1.0×10^{-4}	1.0×10^{-3}	7.72×10^{-6}	exponential
CHANNEL_LR.LV	6993	100	1.28×10^{11}	6.42×10^{10}	1.0×10^{-4}	1.0×10^{-3}	7.72×10^{-6}	linear

Table 3.2: Horizontal resolution $\Delta x [m]$, vertical resolution $\Delta z [m]$, lateral biharmonic viscosity $\eta_{hbi} \text{ m}^4\text{s}^{-1}$, lateral biharmonic diffusivity $\kappa_{hbi} \text{ m}^4\text{s}^{-1}$, vertical viscosity $\eta_v \text{ m}^2\text{s}^{-1}$ and vertical diffusivity $\kappa_v \text{ m}^2\text{s}^{-1}$ for the different configurations of experiment CHANNEL.

CHANNEL is run in four different resolution settings which are given in Table 3.2. The horizontal resolution ranges from $\Delta x = 488 \text{ m}$ for CHANNEL_HR and CHANNEL_HR.LV over $\Delta x = 1953 \text{ m}$ in CHANNEL_MR to $\Delta x = 6993 \text{ m}$ in CHANNEL_LR and the vertical resolution ranges between $\Delta z = 12.5 \text{ m}$ and $\Delta z = 100 \text{ m}$.

These different configurations are used to evaluate the sensitivity of the modal eddy energy pathway to different horizontal and vertical resolutions. The different resolutions are tested for $N^2 = \text{const}$ and an exponential $N^2(z)$ with a e-folding depth of $h/10$, but with the same vertically averaged N . These models are called CHANNEL_P etc.. Additionally one experiment with $\beta = 0$ is conducted.

Experiment AQUA_FLAT_HR

AQUA_FLAT_HR is an aquaplanet model of global scale with a flat bottom at 4750 m depth, a single narrow land part that extends from 35°S to the northern boundary at 66°N. The southern ocean between 35°S and 66°S is open. It has zonally periodic boundary conditions and is closed at the meridional boundaries. The model has $5776 \times 5776 \times 128$ gridpoints with a horizontal resolution of $1/32^\circ \times 1/32^\circ \cos(\phi)$ with latitude ϕ , resulting in a grid spacing of $\Delta x = 1.38$ km at the meridional boundaries and $\Delta x = 3.47$ km at the equator. The vertical resolution decreases from $\Delta z = 2$ m at the surface over $\Delta z = 36$ m at 1000 m depth to $\Delta z = 120 - 197$ m in the deep ocean. AQUA_FLAT_HR uses a linear equation of state and is forced by an idealized wind stress vector $[\tau_x(y), 0]$ that is strongly positive eastward in the mid latitudes and negative in the subtropics and the equator. Heating at the equator and cooling near the poles is supplied by restoring the sea surface temperature in the uppermost layer towards a target temperature with a timescale of 15 days. For more details on the forcing see Fig. 1a in Dettmer and Eden (2025). The model uses biharmonic lateral friction, linear bottom friction and a second order central difference advection scheme with superbee flux limiter for temperature and salinity, and no additional lateral diffusion is used. Instead of using an explicit sea ice model the surface heat flux from ocean to atmosphere is set to zero when the surface temperature falls below the freezing point. The model is initialized with interpolated fields from the lower resolution model AQUA_FLAT (see below) and integrated for two years. The diagnosis is carried out using 3-daily snapshots of the second year. The combined spinup period is insufficient for the large-scale circulation to reach a steady state, however, the mesoscale eddy field can be expected to have reached a quasi-stationary state.

Experiments AQUA_FLAT and AQUA_ROUGH

AQUA_FLAT and AQUA_ROUGH are also used by Dettmer and Eden (2025) and are aquaplanet simulations with the same domain size and depth as AQUA_FLAT_HR but the resolution is reduced to $1/10^\circ \times 1/10^\circ \cos(\phi)$ with 1800×1800 horizontal gridpoints and 42 vertical levels, resulting in a horizontal resolution of 4.44 km at the meridional boundaries and 11.13 km at the equator. The vertical resolution decreases from about 10 m in the upper 100 m to 250 m in the deep ocean below 1250 m.

AQUA_ROUGH is identical to AQUA_FLAT, except that it includes a randomized rough bottom topography, that is constructed to match the spectral characteristics of observed oceanic topography. For details see Dettmer and Eden (2025). AQUA_ROUGH uses no linear bottom friction. AQUA_FLAT and AQUA_ROUGH

are integrated for 20 years and diagnosis is carried out using 3-daily snapshots of the last 5 years of integration time.

Experiment REAL

Experiment REAL is a global model with realistic topography, resembling the real ocean. The same model is used in Eden (2016), Eden et al. (2021) and Dettmer and Eden (2025). REAL has the same horizontal and vertical resolution and uses identical settings as AQUA_FLAT and AQUA_ROUGH if not stated otherwise, but uses 3600×2163 grid points in the horizontal to achieve this. To avoid a singularity at the north pole, the Arctic Ocean north of 70°N is cut out, enabling the use of a regular latitude-longitude grid everywhere. The same advection scheme, biharmonic friction and handling of sea ice as AQUA_FLAT and AQUA_ROUGH is used, but REAL uses the TEOS-10 equation of state and different to AQUA_FLAT and AQUA_ROUGH vertical mixing and friction induced by small scale turbulence in the surface layer and the ocean interior are parameterised using the scheme by Gaspar et al. (1990). The IDEMIX closure Olbers and Eden (2013) is used to parameterise the effects of breaking internal gravity waves. REAL is forced by a monthly climatology from Barnier et al. (1995) applied by a restoring boundary condition with a timescale of 90 days for surface salinity. The forcing for the boundary condition is provided by the World Ocean Atlas by Zweng et al. (2019). The model is initialized with a state of rest, where the temperature and salinity are taken from the combined WOCE/ARGO dataset by Gouretski (2018). The model is integrated for 20 years and the diagnosis is conducted with 3-daily snapshots of the last 5 years of data.

3.2.6. Modal-spectral eddy kinetic energy

As a further diagnostic, the modal eddy kinetic energy, baroclinic conversion, and the advection terms are analyzed in terms of the two-dimensional horizontal wavenumber spectrum. Topographic variations are neglected here, as it will be shown that topographic scattering is small in the global integral. The modal-spectral decomposition is achieved by applying a Fourier ansatz to the modally decomposed variables: $u_n = \int dk u_n(k) \exp(ik \cdot x)$ and similar for w and p . k is the horizontal wavenumber vector, and $\hat{}$ indicates a two-dimensional Fourier transform.

The steps to derive the modal-spectral eddy kinetic energy equation follow the same logic as the separate modal derivations presented here and spectral derivations presented in Dettmer and Eden (2025), since the integral over k in the Fourier ansatz and the sum over n in the modal ansatz commute. The resulting modal-spectral eddy kinetic energy equation is given by:

$$\partial_t E_n(k) + N_n^m(k) + N_n^e(k) + S_n(k) = P_n(k) \quad (3.20)$$

With modal-spectral eddy kinetic energy $E_n(\mathbf{k}) = \overline{\hat{\mathbf{u}}_n'^* \cdot \hat{\mathbf{u}}_n'}/2$ and

$$\begin{aligned} N_n^m(\mathbf{k}) &= \text{Re} \left[\overline{\hat{\mathbf{u}}_n'^* \cdot \sum_{ij} B_{ijn}(\overline{(\mathbf{u}_i \cdot \nabla) \mathbf{u}_j'})} \right], & N_n^e(\mathbf{k}) &= \text{Re} \left[\overline{\hat{\mathbf{u}}_n'^* \cdot \sum_{ij} B_{ijn}(\overline{(\mathbf{u}_i' \cdot \nabla) \mathbf{u}_j'})} \right] \\ S_n(\mathbf{k}) &= \text{Re} \left[\overline{\hat{\mathbf{u}}_n'^* \cdot \sum_{ij} B_{ijn}(\overline{(\mathbf{u}_i' \cdot \nabla) \bar{\mathbf{u}}_j})} \right], & P_n(\mathbf{k}) &= \text{Re} \left[\overline{w_n' p_n'} \right] / c_n^2 \end{aligned}$$

where Re is the real part operator. The term $N_n^m(\mathbf{k})$ denotes the divergence of the spectral flux of $E_n(\mathbf{k})$ and inter-modal scattering by eddy-mean flow interaction. Similarly $N_n^e(\mathbf{k})$ denotes the divergence of the spectral flux of $E_n(\mathbf{k})$ and inter-modal scattering by eddy-eddy interaction. The term $S_n(\mathbf{k})$ denotes conversion of mean kinetic to eddy kinetic energy by lateral shear instability and $P_n(\mathbf{k})$ denotes conversion of eddy available potential energy to eddy kinetic energy by vertical shear instability.

While the purely spectral and the purely modal eddy kinetic energy equations contain pressure-induced flux divergence terms, such a term is absent in the combined modal-spectral formulation. This is because the operator $i\mathbf{k} \cdot \mathbf{u}$ is commutable, a property already exploited in the derivation of the spectral eddy kinetic energy equation, where it leads to the elimination of the horizontal pressure-flux divergence and leaves only a vertical divergence term. Since the vertical dependence is removed by the modal ansatz, this remaining vertical pressure-induced flux divergence vanishes identically, and the pressure-induced flux term drops out entirely of the spectral-modal eddy kinetic energy equation.

To diagnose the modal-spectral equation in AQUA_FLAT_HR, the horizontal domain is divided into windows of 512 gridpoints, providing a window size ranging from 805 km at 60° latitude to 1780 km at the equator. In CHANNEL, the diagnosis is performed over the full horizontal domain. The window size is chosen to be large enough to resolve the spectral maximum of eddy kinetic energy, yet small enough to maintain geographical resolution. Note that varying the window size slightly modifies the absolute magnitudes of the individual terms in Eq. (3.20), but not their characteristics. Other methodological details follow Dettmer and Eden (2025), including the averaging of terms in Eq. (3.20) over the wavenumber angle α (where $\mathbf{k} = k(\cos \alpha, \sin \alpha)$) to express results as a function of wavenumber magnitude k .

Note that, consistent with a WKB approximation, where the background state varies on much larger scales than the modal-spectral energy, horizontal variations of ϕ_n are neglected, within each subdomain. A spatially dependent coupling coefficient $B_{ijn}(\mathbf{x})$ would appear as a convolution with the advection terms in the spectral domain, thereby incorporating wave-background interactions alongside wave-wave interactions. This would prevent the integral of the spectral flux from vanishing and complicates the physical interpretation.

3.3. Results

In this section, the modal eddy kinetic energy equation Eq. (3.17) is evaluated in the different ocean models described in Section 2.3.2.5). We focus first on the idealized CHANNEL experiments and later the analysis is extended to the global and realistic models.

3.3.1. Idealized CHANNEL experiments

First we discuss results from two different CHANNEL model configurations with high vertical and horizontal resolution, where the 9.th baroclinic Rossby radius L_9 is resolved by five grid points, and the lower Rossby radii by more grid points. The Rossby radius is defined as $L_n = c_n/f$ with Coriolis frequency f . CHANNEL_HR has $N^2 = \text{const}$ for which the vertical modes ϕ_n and the interaction coefficients B_{nij} are shown in Fig. 3.1 a)-l), while CHANNEL_HR_P has an exponential $N^2(z)$, for which the modes and B_{nij} are given in Fig. 3.1 m)-x).

CHANNEL_HR

Fig. 3.2 shows a snapshot of relative vorticity ζ at $z = -106$ m, the modal eddy kinetic energy E_n and the terms of the modal eddy kinetic energy equation Eq. (3.17) for CHANNEL_HR. The relative vorticity ζ shows fine horizontal structures, indicating mesoscale eddy activity. The horizontal structure of E_n and the modal production and transfer terms is slightly smoother. P_1 and S_1 are predominantly positive but show some negative values, too. N_n^m and N_n^e show positive and negative values and a clear transfer direction is difficult to see. Therefore the terms will be analyzed as a global integral in the following.

Fig. 3.3 shows $\int E_n dA$ and the correspondingly integrated individual terms of the modal eddy kinetic energy equation Eq. (3.17) in CHANNEL_HR. Barotropic eddy kinetic energy ($n = 0$) is dominating, the first baroclinic eddy kinetic energy ($n = 1$) is less than half of that, the higher modes are of minor importance. The baroclinic production term P_n produces the largest amount of eddy kinetic energy for $n = 1$, vanishes for $n = 0$, and remains small for $n > 1$. The barotropic production term S_n is smaller but also positive for $n = 1$ and $n = 0$, where the former dominates. Most eddy kinetic energy is contained in $n = 0$ and only directly produced by a small S_0 contribution, but it is transferred by the terms N_1^m to N_0^m and to a lesser from N_1^e to N_0^e , almost balancing the direct production of E_1 by P_1 and S_1 . This can be interpreted as an inverse modal transfer of kinetic energy towards the barotropic mode as described by Smith and Vallis (2001). Fig. 3.3 shows that T_n – which is only non-zero by the horizontal variation of $N^2(x)$ – remains very small.

To isolate the mechanics of the non-linear transfers, Fig. 3.4 shows horizontal integrals of the non-linear scattering terms, but here the summation over mode indices i and j is omitted, such that the individual triad interactions involving mode n and modes i and j are resolved explicitly. The interaction of mode n with modes i

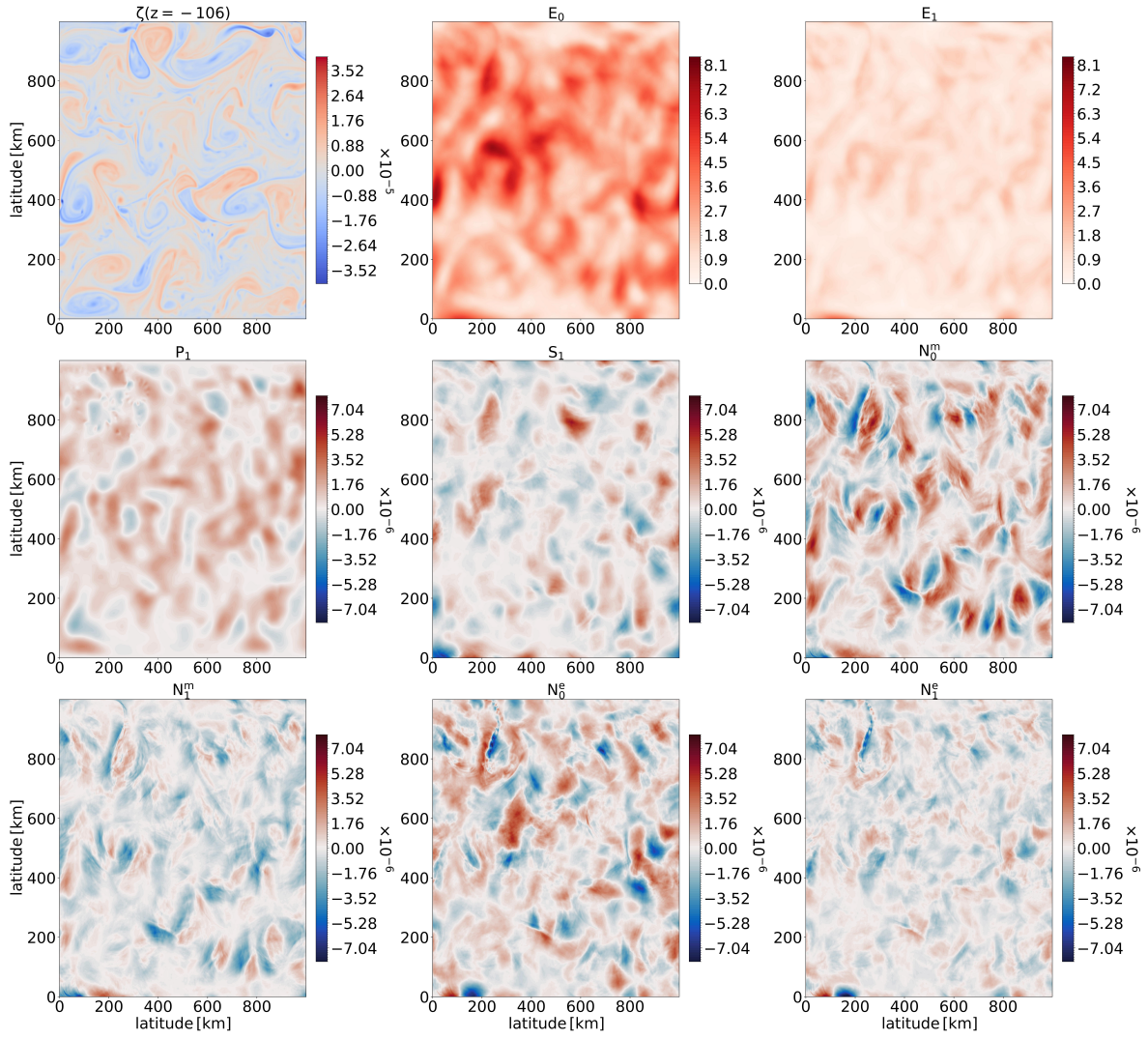


Figure 3.2: Results derived from CHANNEL_HR. Snapshot of relative vorticity $\zeta(z = -25 \text{ m}) [\text{s}^{-1}]$, E_0 and $E_1 [\text{m}^3 \text{s}^{-2}]$ and $P_1, S_1, N_0^m, N_1^m, N_0^e$ and $N_1^e [\text{m}^3 \text{s}^{-3}]$.

and j is given by

$$\mathcal{N}_n^e(i, j) = B_{ijn} \overline{\mathbf{u}'_n \mathbf{u}'_i : \nabla \mathbf{u}'_j} \quad , \quad \mathcal{N}_n^m(i, j) = B_{ijn} \overline{\mathbf{u}'_n \bar{\mathbf{u}}_i : \nabla \mathbf{u}'_j} \quad (3.21)$$

For divergence-free flow with periodic or closed boundary conditions, and neglecting the horizontal dependence of $\phi_n(\mathbf{x})$, it can be shown that the following relation holds

$$\int \mathcal{N}_n^e(i, j) dA = \int \mathcal{N}_j^e(i, n) dA \quad (3.22)$$

and likewise for $\mathcal{N}_n^m(i, j)$. The divergence of the modal perturbation velocity is given by $\nabla \cdot \mathbf{u}'_m = w'_m c_m^2$. For quasigeostrophic dynamics the vertical velocity and

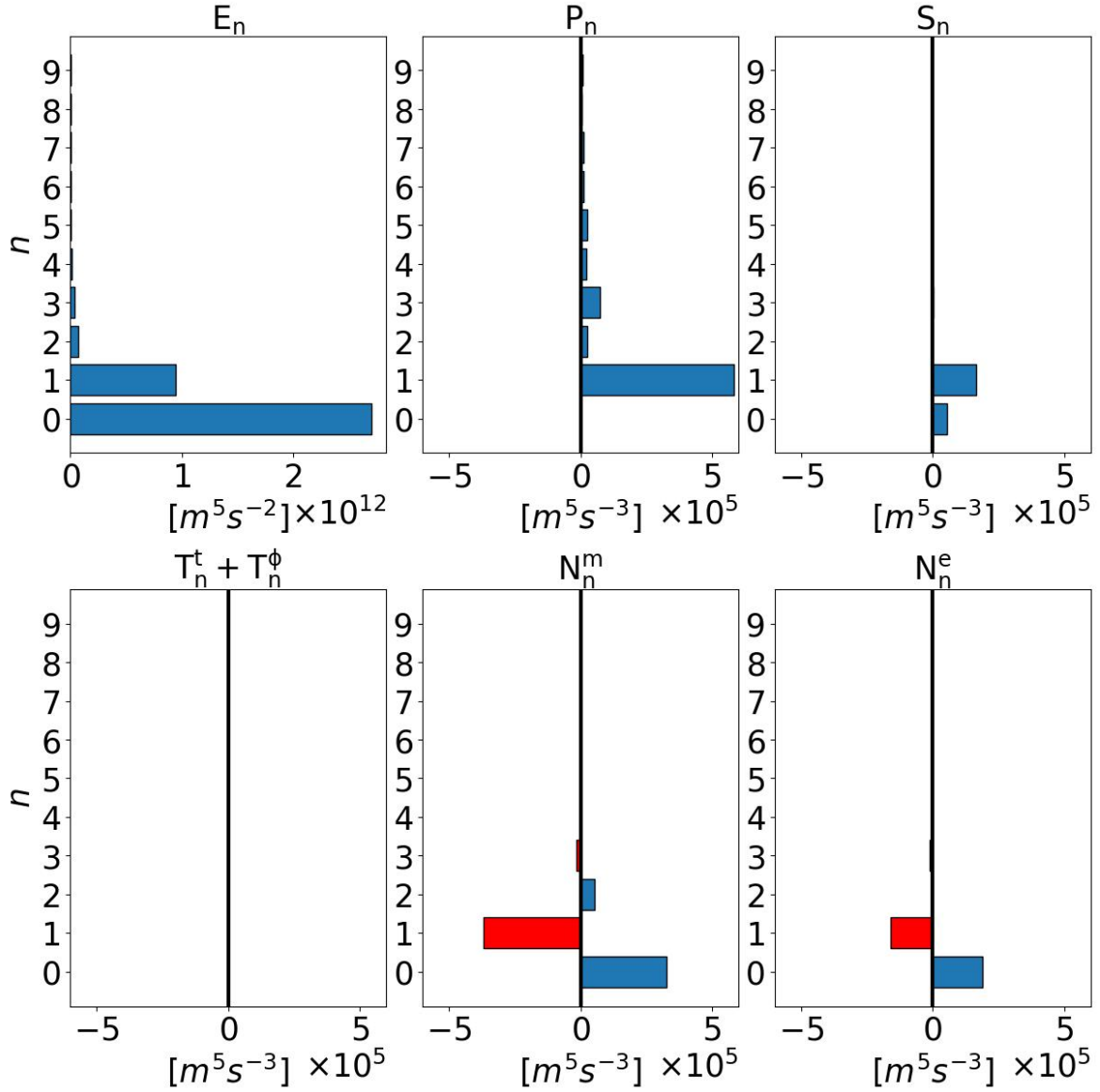


Figure 3.3: Horizontally integrated modal eddy kinetic energy $\int E_n dA$ and the transfer terms $\int P_n dA$, $\int S_n dA$, $\int T_n dA$, $\int N_n^m dV$ and $\int N_n^e dA$ in CHANNEL_HR.

the divergence is small, so $\nabla \cdot \mathbf{u}'_m$ is expected to be small, especially for the lower modes where c_n is large. The letters in Fig. 3.4 indicate the transfer pairs given by Eq. (3.22). Modes $n \geq 3$ are not shown since there N_n^m and N_n^e are very weak. The inverse modal transfer seen in Fig. 3.3 is dominated by the two triad interactions: $\mathcal{N}_1^e(1, 0)$ and $\mathcal{N}_1^m(1, 0)$, which can be interpreted as first baroclinic eddy-eddy and eddy-mean scattering at the lateral shear at the barotropic mode, respectively, and form by Eq. (3.22) a transfer pair with $\mathcal{N}_0^e(1, 1)$ and $\mathcal{N}_0^m(1, 1)$, which can be interpreted as barotropic eddy-mean and eddy-eddy scattering at the lateral shear of the first

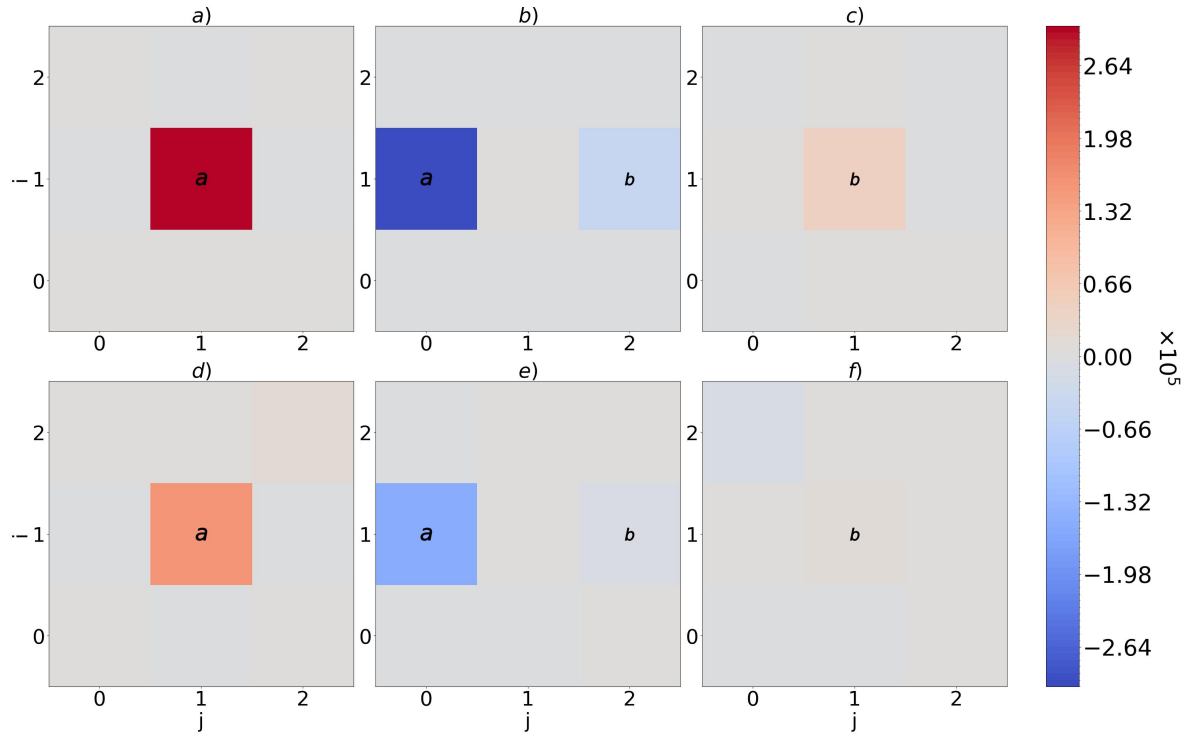


Figure 3.4: Horizontal integrals in CHANNEL_HR of $\int \mathcal{N}_n^m(i, j) dA$ (a-c) and $\int \mathcal{N}_n^e(i, j) dA$ (d-f) in $[\text{m}^5\text{s}^{-3}]$ for $n = 0, 1, 2$. The letters indicate the transfer pairs given by Eq. (3.22).

baroclinic mode, respectively.

There is also a weaker transfer of E_n to higher modes, so a forward transfer, by $\mathcal{N}_1^m(1, 2)$ to $\mathcal{N}_2^m(1, 1)$. For $\mathcal{N}_n^e$ this transfer is absent. These findings support the interpretation that relation Eq. (3.22) indeed holds and $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ form a transfer pair. Additionally, the structure of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ is shaped by B_{nij} and therefore the stratification (Fig. 3.1). For $n = 0$, B_{nij} is large for the diagonal elements where $i = j$, therefore damping all off-diagonal interactions. For $n = 1$, B_{nij} is large for the off-diagonal interactions, therefore damping all interactions along the diagonal.

CHANNEL_HR_P

CHANNEL_HR_P features a pycnocline in the form of exponential stratification with an e-folding scale of $h/10$. The horizontal structures of the modal eddy kinetic energy equation terms (not shown) are finer and smaller compared to CHANNEL_HR, which is consistent with baroclinic instability energizing smaller scales more strongly in presence of non-constant stratification (not shown). In general the terms look similar to CHANNEL_HR with P_1 and S_1 being mainly positive and the other terms showing positive and negative values in the horizontal.

Fig. 3.5 shows horizontal integrals of E_n , P_n , S_n , T_n , $\mathcal{N}_n^m$ and $\mathcal{N}_n^e$ for CHANNEL_HR_P. In presence of a strongly surface intensified stratification, E_n production

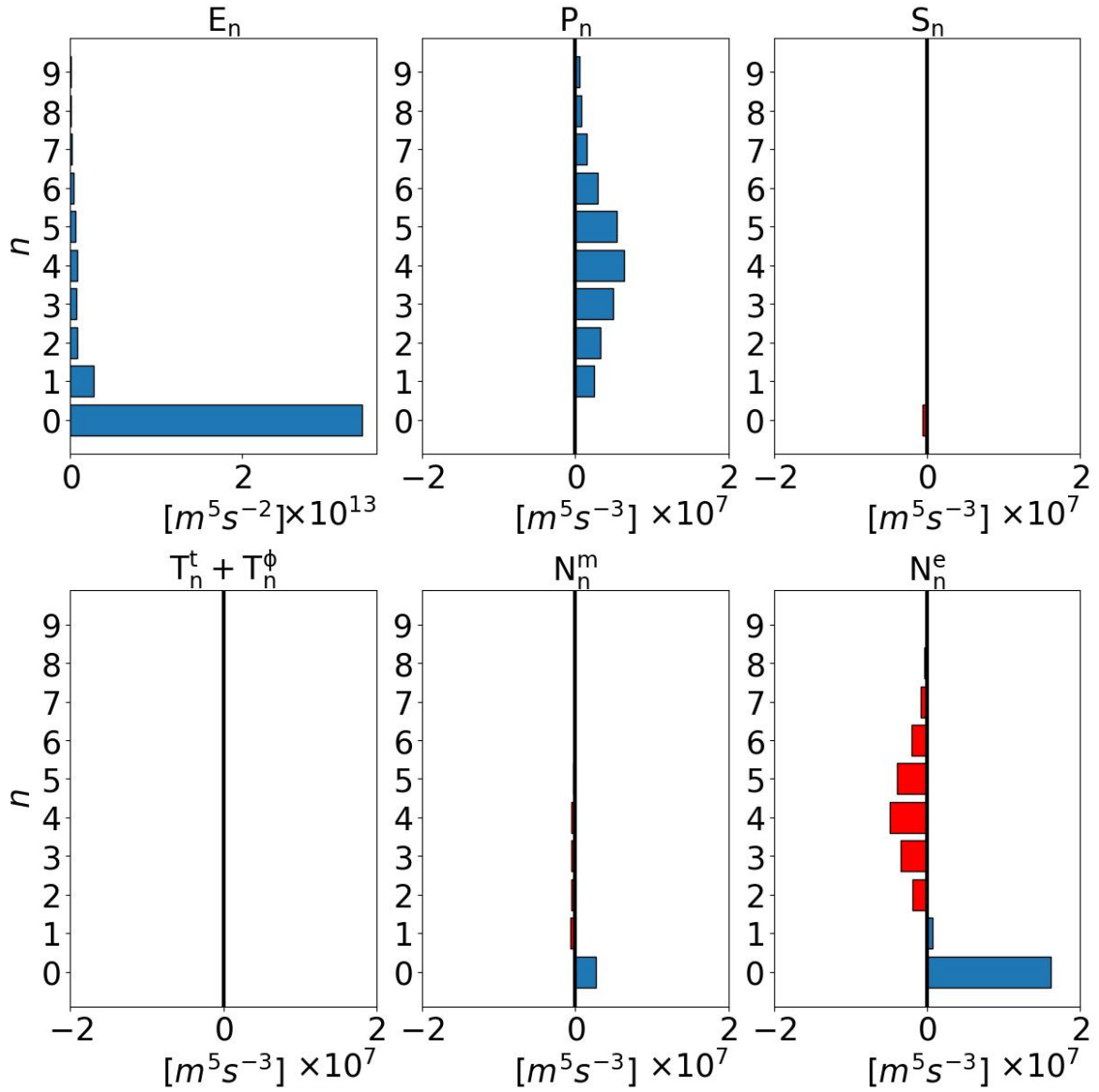


Figure 3.5: Results derived from CHANNEL_HR_P. Horizontal integrals of the modal eddy kinetic energy $\int E_n dA$ in $[m^5 s^{-2}]$ and $\int P_n dA$, $\int S_n dA$, $\int T_n^t + T_n^\phi dA$, $\int N_n^m dA$ and $\int N_n^e dA$ in $[m^5 s^{-3}]$.

by P_n is shifted toward higher modes and by approximately 1.5 order of magnitudes stronger than for constant N^2 . When experiment CHANNEL_HR_P is repeated with an e-folding depth of $h/20$ (not shown), representing a slightly less surface intensified stratification profile, it becomes apparent that the degree to which the energy pathway shifts to higher baroclinic modes is dictated by the intensity of surface stratification,. This intensified production across a broader modal range drives equally enhanced non-linear transfers and results in one order of magnitude larger E_n . Similar to CHANNEL_HR, almost all eddy kinetic energy is in the barotropic mode, and the

baroclinic E_n production is balanced by inter-modal transfer to lower modes through N_n^m and N_n^e . However, unlike CHANNEL_HR, N_n^e is stronger than N_n^m and specifically N_n^e is negative for $n > 1$ and slightly positive for $n = 1$ and strongly positive for $n = 0$. Barotropic production BT_n and topographic scattering $T_n^\phi + T_n^t$ remain negligible. In summary, while the energy cycle remains a balance between P_n production and an inverse modal transfer, the presence of the pycnocline shifts E_n generation and transfers toward higher modal indices ($n > 1$), whereas in CHANNEL_HR it is constrained to the barotropic and first baroclinic modes.

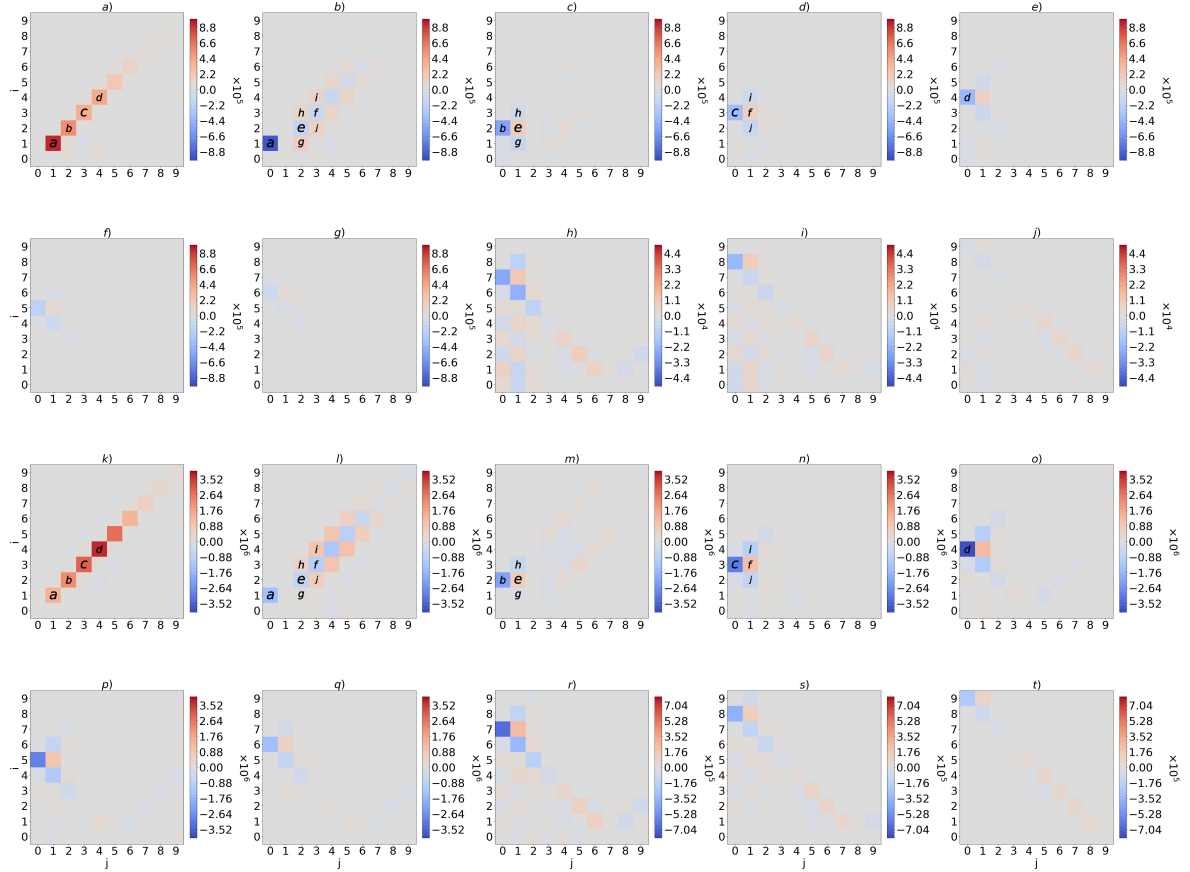


Figure 3.6: Horizontal integrals in CHANNEL_HR_P of $\int \mathcal{N}_n^m(i, j) dA$ (a-j) and $\int \mathcal{N}_n^e(i, j) dA$ (k-t) in $[m^5 s^{-3}]$ for $n = 0, \dots, 9$. The letters indicate the transfer pairs given by Eq. (3.22).

Fig. 3.6 shows the horizontal integrals of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ for CHANNEL_HR_P, analogous to Fig. 3.4 for CHANNEL_HR, but here modes up to $n = 9$ are included. Compared to the case with constant N^2 (CHANNEL_HR), a substantially larger number of (n, i, j) interactions is active for exponential $N^2(z)$. The overall structure of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ is similar, but the amplitudes of $\mathcal{N}_n^e(i, j)$ are generally stronger. For the barotropic mode, both $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ exhibit positive diagonal elements. While $\mathcal{N}_n^m(i, j)$ gains most of its energy from the first baroclinic mode through $\mathcal{N}_1^m(1, 0)$, the dominant contribution to $\mathcal{N}_n^e(i, j)$ originates from the fourth

baroclinic mode via $\mathcal{N}_4^e(4, 0)$. In both terms, the first baroclinic mode shows negative diagonal elements, representing a non-local forward transfer of energy toward higher modes (transfer pairs e and f). Simultaneously, positive off-diagonal elements indicate a non-local inverse transfer from higher modes back to the first baroclinic mode (transfer pairs h, g and i, j). The higher modes ($n > 1$) lose energy to the barotropic mode through interactions between baroclinic self-interaction ($n = i$) and horizontal barotropic shear ($j = 0$). Their interaction with the first-mode baroclinic shear ($j = 1$) is more complex: they transfer energy to the first baroclinic mode when interacting with neighboring baroclinic modes ($i = k \pm 1$), yet conversely gain energy from the first mode through interactions with baroclinic self-interaction ($i = k$).

To summarize, eddy kinetic energy is transferred directly from all baroclinic modes to the barotropic mode. In addition, the first baroclinic mode both receives energy from and loses energy to the higher baroclinic modes, establishing a closed exchange loop between the first baroclinic mode and the higher modes. This loop is absent in the case of constant N^2 , where B_{nij} vanishes along the diagonal for the first baroclinic mode, preventing energy transfer from the first baroclinic mode to higher modes. Furthermore, for constant stratification B_{nij} vanishes for the interaction $(n, i, j) = (n, n, 1)$, thereby suppressing energy transfer from the higher baroclinic modes to the first baroclinic mode. Even in the constant stratification case, B_{nij} would in principle permit direct energy transfer from higher baroclinic modes to the barotropic mode, which, however, is not observed. Compared to CHANNEL_HR, where neighbouring modes interact predominantly locally, the inter-modal transfers in the presence of non-constant stratification are primarily non-local.

3.3.2. Resolution dependency of modal eddy kinetic energy equation

As a next step, the diagnostic of the modal eddy kinetic energy equation is extended to the global models AQUA_FLAT_HR, AQUA_FLAT, AQUA_ROUGH, and REAL to estimate the vertical eddy kinetic energy budget in a realistic framework. Because these configurations have lower horizontal resolution than CHANNEL_HR, additional experiments (CHANNEL_MR and CHANNEL_LR) are performed to assess whether the resolution in the global models is sufficient to resolve the dynamically relevant scales.

Fig. 3.7 compares the ratio of the Rossby radius ($L_n = c_n/f$) to the horizontal grid spacing (Δ) for the models listed in Table 3.1. The global models REAL, AQUA_FLAT, and AQUA_ROUGH exhibit comparable resolution, where L_1 is resolved by 3 to 4 grid points, L_2 is resolved by roughly two grid points, and the ratio drops to one or below for higher modes. CHANNEL_LR closely matches this resolution. In contrast, CHANNEL_MR resolves L_1 with 12 grid points, L_2 with 6, decreasing to roughly 1.4 for L_9 . AQUA_FLAT_HR is slightly coarser than CHANNEL_MR, resolving L_1 with 10 grid points and L_3 with 3. Finally, CHANNEL_HR provides the highest resolution, with 48 grid points for L_1 (not shown), 10 for L_5 , and nearly 9 for L_9 .

In summary, CHANNEL_HR adequately resolves the Rossby radii of all modes,

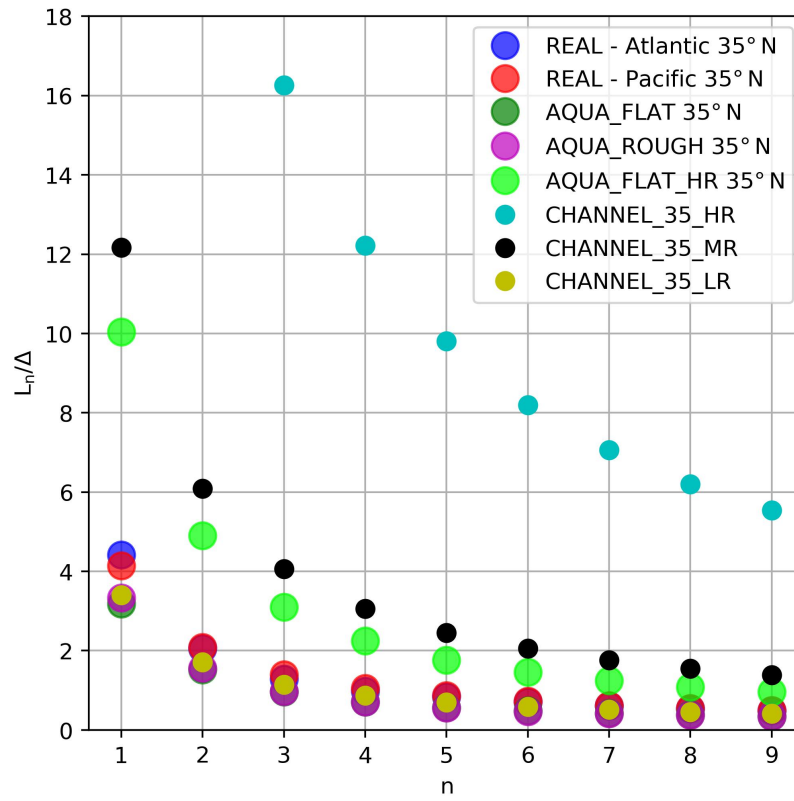


Figure 3.7: Ratio of Rossby radius L_n and horizontal resolution Δ per vertical mode n for REAL, AQUA_FLAT, AQUA_ROUGH, CHANNEL_HR, CHANNEL_MR and CHANNEL_LR. For REAL, AQUA_FLAT and AQUA_ROUGH Δ is the maximum of the zonal and meridional resolution. For CHANNEL the zonal and meridional resolution are the same. REAL is evaluated at 35°N in the Atlantic and Pacific ocean and AQUA_FLAT and AQUA_ROUGH are evaluated at 35°N in the boundary current.

whereas CHANNEL_LR and the global models (REAL, AQUA_ROUGH, AQUA_FLAT) only resolve the first baroclinic mode well. AQUA_FLAT_HR and CHANNEL_MR resolve the barotropic and the first three baroclinic modes with sufficient accuracy. Because higher modes also exhibit fine vertical structures, vertical resolution is equally critical. To isolate its impact, additional configurations with identical horizontal but reduced vertical resolution (e.g., CHANNEL_LV) are employed.

Comparing CHANNEL_HR to CHANNEL_MR and CHANNEL_LR reveals that while reduced resolution decreases the magnitude of energy reservoirs and transfers, the qualitative structure of the vertical budget remains consistent. In particular, in CHANNEL_HR, where all diagnosed modes are properly resolved, the energy content and associated transfers for modes higher than $n = 2$ are negligible. However, introducing a pycnocline at low resolution (CHANNEL_LR_P) produces unrealistically large P_n in the highest diagnosed modes whereas CHANNEL_MR_P adequately resolves the pycnocline, exhibiting a budget qualitatively similar to CHANNEL_HR_P, with only a reduction in magnitude. For constant stratification, a

reduction in vertical resolution, does not alter the budget qualitatively, but the energy conversions and transfers are reduced by a factor of 2 to 5, as indicated by comparing CHANNEL_HR and CHANNEL_LR to CHANNEL_HR_LV and CHANNEL_LR_LV. The influence of a spatially varying Coriolis parameter was assessed by repeating CHANNEL_MR with $\beta = 0$. While the budget remains qualitatively unchanged, BC_n , N_n^e and N_n^m increase by an order of magnitude, suggesting that β exerts a stabilizing influence (Thompson and Young, 2006). As the fundamental energy pathway remains consistent in the various CHANNEL configurations, the diagnostic can be extended to AQUA_FLAT_HR, AQUA_FLAT, AQUA_ROUGH and REAL, although the results have to be interpreted with care, keeping the resolution limitations in mind. Regional differences will be examined and the results will be compared to the CHANNEL models.

3.3.3. Global models

The global models REAL, AQUA_FLAT, and AQUA_ROUGH exhibit horizontal circulations and thermocline depths comparable to the real ocean (Dettmer and Eden, 2025) and AQUA_FLAT_HR likewise falls within the observed range. The CHANNEL models are designed to approximate the dynamical conditions of REAL, AQUA_ROUGH, AQUA_FLAT, and AQUA_FLAT_HR at 35° N, characterized by corresponding f and β . To align the zonal mean current strength and direction with the global models, the vertical and meridional buoyancy gradients of the initial state, toward which the models are restored are adjusted.

Experiment AQUA_FLAT_HR

The analysis first focuses on AQUA_FLAT_HR, which possesses sufficient horizontal and vertical resolution to adequately resolve the first three baroclinic modes. The stratification profile is broadly consistent with the exponential profile prescribed in CHANNEL_HR_P, with the difference of weaker values toward the surface, related to a mixed layer. Consequently, the resulting modes and B_{nij} exhibit a structure (Fig. 3.1 n)-x)).

Fig. 3.8 shows $\log_{10}(E_m)$ for $m = 0, 1$ together with P_1 , S_1 , N_0^m , N_1^m , N_0^e and N_1^e for AQUA_FLAT_HR. A snapshot of relative vorticity demonstrates that the flow is rich in mesoscale eddies, particularly within the boundary current regime between 20°N and 50°N and in the open Southern Ocean between 35°S and 50°S. In these regions, both E_0 and E_1 have large values. Reduced levels of E_n occur between 20°S and 20°N, indicating comparatively weak mesoscale activity; there E_1 exceeds E_0 .

The transfer terms P_n , S_n , N_n^m and N_n^e are strongest in regions of enhanced mesoscale variability, namely the boundary current system and the Southern Ocean. P_1 exhibits a very fine horizontal structure with predominantly positive values, but also localized negative contributions. In contrast, the horizontal patterns of S_1 , N_n^m and N_n^e are spatially broader and display both positive and negative regions. For S_1 no clear net transfer direction is immediately apparent. However, N_n^m and N_n^e are

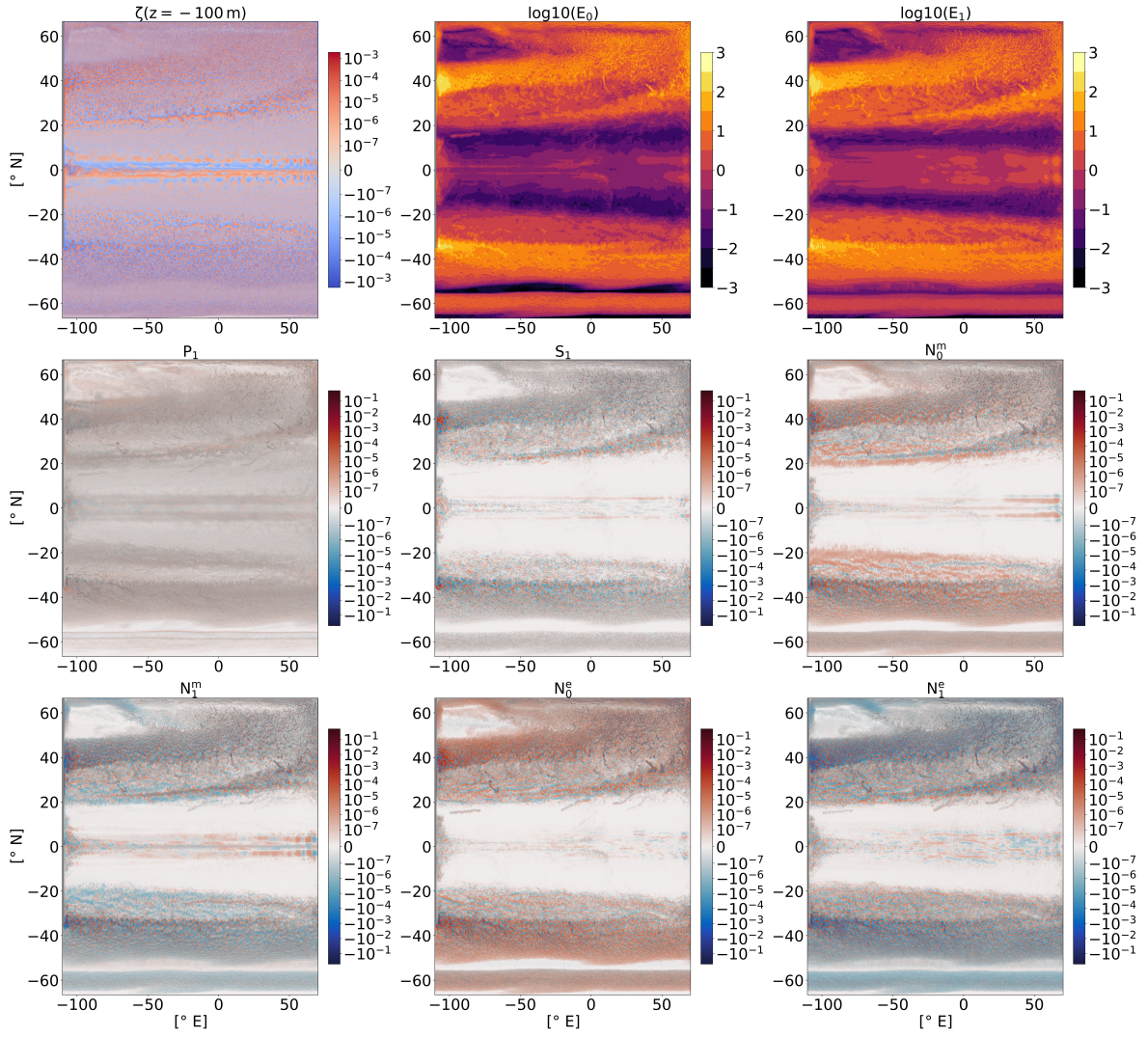


Figure 3.8: Results derived from AQUA_FLAT_HR. Snapshot of relative vorticity $\zeta(z = -97 \text{ m}) [\text{s}^{-1}]$, $\log_{10}(E_0)$ and $\log_{10}(E_1)$ and P_1 , S_1 , N_0^m , N_1^m , N_0^e and N_1^e in $[\text{m}^3\text{s}^{-3}]$.

predominantly negative for $m = 1$ and predominantly positive for $m = 0$, indicating a systematic transfer of eddy kinetic energy from the first baroclinic mode to the barotropic mode.

Fig. 3.9 displays the globally integrated modal energy reservoirs E_n and transfer terms P_n , S_n , $T_n^\phi + T_n^t$, N_n^m , and N_n^e for AQUA_FLAT_HR. Higher modes are omitted because their amplitudes are weak and they are not sufficiently resolved. Overall, the modal energy pathway in AQUA_FLAT_HR is qualitatively similar to that of CHANNEL_HR. Eddy kinetic energy is generated primarily by baroclinic instability in the first baroclinic mode, whereas barotropic generation remains weak. The non-linear terms N_n^m and N_n^e subsequently transfer energy toward the barotropic mode. In contrast to CHANNEL_HR, N_n^e is stronger than N_n^m and E_1 slightly exceeds

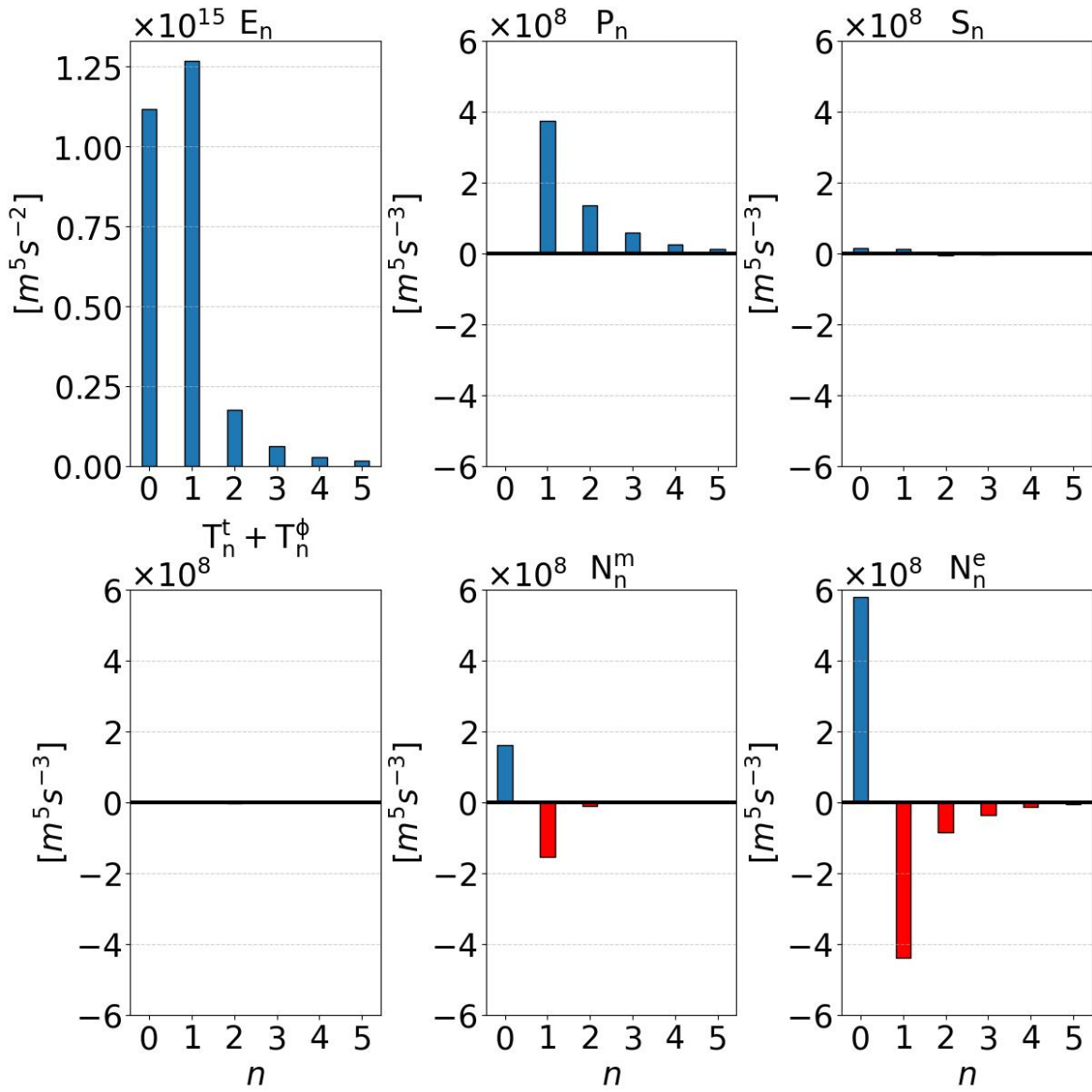


Figure 3.9: Results derived from AQUA_FLAT_HR. Horizontal integrals of the modal eddy kinetic energy $\int E_n dA$ in $[m^5 s^{-2}]$ and $\int P_n dA$, $\int S_n dA$, $\int T_n^t + T_n^\phi dA$, $\int N_n^m dA$ and $\int N_n^e dA$ in $[m^5 s^{-3}]$.

E_0 . Notably, the vertical energy cycle in AQUA_FLAT_HR more closely resembles the case with constant N^2 in CHANNEL_HR than the configuration with a pycnocline, despite the stratification and vertical modes being more similar to the latter. Simultaneously, P_n and N_n^e in AQUA_FLAT_HR exhibit relatively stronger contributions in the second and third baroclinic modes than in CHANNEL_HR, reminiscent of CHANNEL_HR_P, where the energy pathway shifts toward higher modes. In this regard, AQUA_FLAT_HR shares characteristics with CHANNEL_HR_P. It should be emphasized, that the pycnocline in CHANNEL_HR_P is considerably more pro-

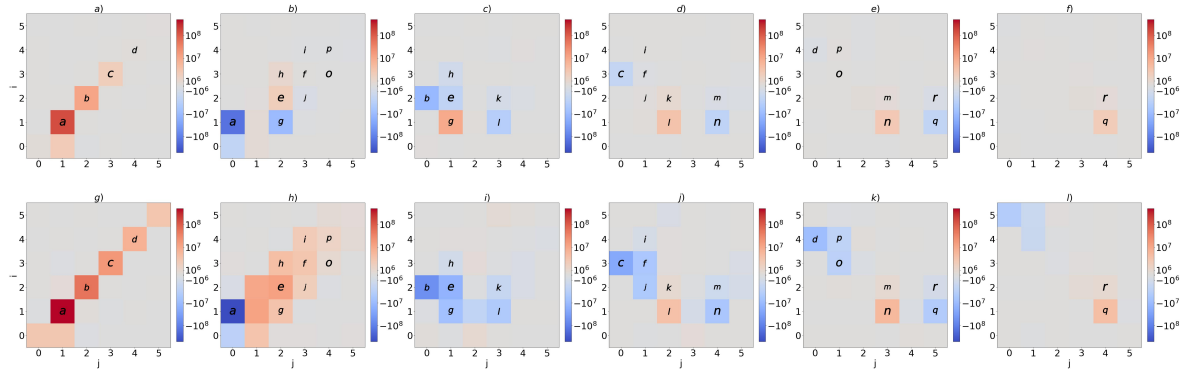


Figure 3.10: Horizontal integrals in AQUA_FLAT_HR of $\int \mathcal{N}_n^m(i, j) dA$ (a-f) and $\int \mathcal{N}_n^e(i, j) dA$ (g-l) in $[m^5s^{-3}]$ for $n = 0, \dots, 5$. The letters indicate the transfer pairs given by Eq. (3.22).

nounced than in most regions of AQUA_FLAT_HR and therefore represents an idealized, amplified configuration designed to isolate the effects of non-constant stratification rather than to replicate the exact conditions of AQUA_FLAT_HR.

Fig. 3.10 shows global integrals of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ for $n \geq 5$. In the barotropic mode, both $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ are positive along the diagonal. As in CHANNEL_HR and in $\mathcal{N}_n^m(i, j)$ for CHANNEL_HR_P, the dominant interaction occurs for $(i, j) = (1, 1)$. Similar to CHANNEL_HR_P, but in contrast to CHANNEL_HR, the higher diagonal elements are also positive. The source of this inverse modal energy transfer is again provided by all baroclinic modes (transfer pairs a, b, c and d). The closed energy loop between the first and higher baroclinic modes that was clearly identified in CHANNEL_HR_P is less pronounced here. For $\mathcal{N}_n^m(i, j)$, energy is transferred from the first baroclinic mode to higher modes via transfer pairs g, l, n and q, while transfer pairs e and h indicate a transfer from the second baroclinic mode to the first. For $\mathcal{N}_n^e(i, j)$ the pattern differs: the first baroclinic mode gains energy from all higher baroclinic modes, whereas only weak transfers from the second baroclinic mode to higher modes are found (transfer pairs k and l, m and n, and r and q). Since $\mathcal{N}_n^e(i, j)$ is approximately three times stronger than $\mathcal{N}_n^m(i, j)$ (see Fig. 3.9), the overall transfer structure is dominated by $\mathcal{N}_n^e(i, j)$. Note that while the comparable magnitudes of the transfer pairs confirm that Eq. (3.22) remains a valid interpretation for AQUA_FLAT_HR, the non-zero values in the $j = n$ columns reflect a slight departure from the divergence-free assumption.

The global integrals shown in Fig. 3.9 and Fig. 3.10 are dominated by the energetic eastward currents poleward of $30^\circ N$ and in the open Southern Ocean poleward of $37^\circ S$. Regionally, however, some differences emerge. In the subtropical ocean equatorward of approximately 30° latitude, N_n^m exceeds N_n^e , in contrast to the global integral and a larger fraction of E_n and P_n is found in the higher baroclinic modes. There, S_n is comparable to P_n in the subtropics and exceeds P_n near the equator.

Compared to the lower-resolution configuration AQUA_FLAT, AQUA_FLAT_HR has 50% more globally integrated eddy kinetic energy $\int |\overline{u'^2}|/2dx$ while the inte-

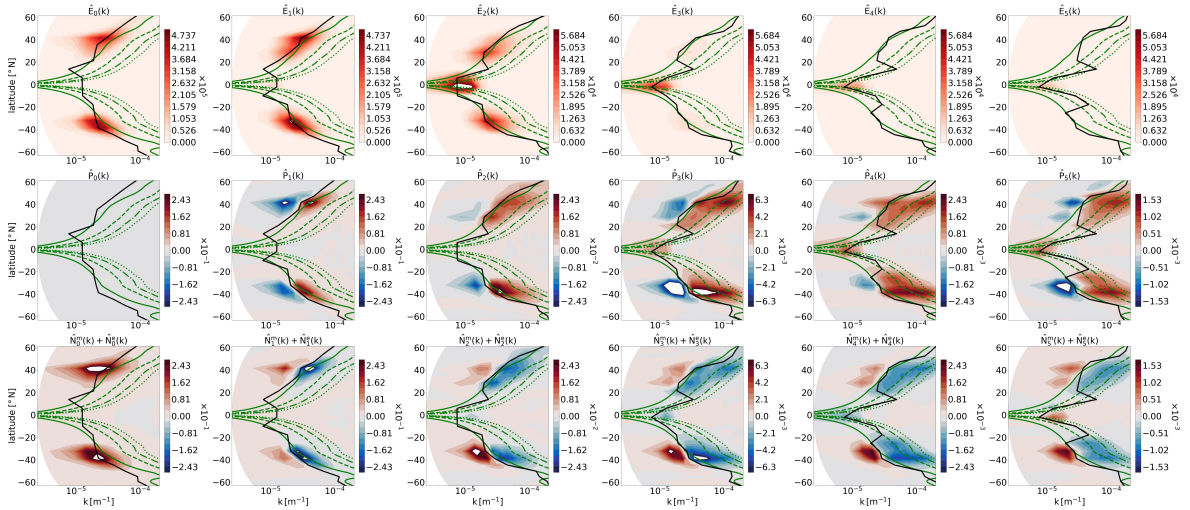


Figure 3.11: Results derived from AQUA_FLAT_HR. Zonally averaged $E_n(k)$ in $[m^4s^{-2}]$ zonally averaged $P_n(k)$ and zonally averaged $N_n^m(k)$ and $N_n^e(k)$ in $[m^4s^{-3}]$ for $n = 0, \dots, 5$. Also shown are Rossby radii L_n in green for modes 1 (solid), 2 (dashed), 3 (dash-dotted) and 4 (dotted). Additionally the wavenumber of the $E_n(k)$ maximum is shown per mode in black. Note the different color scales between modes.

grated baroclinic conversion rate $\int \overline{b'w'}dx$ is almost identical. The modal sum $\sum_{n=0}^9 E_n$ captures 93% and 96% of the total eddy kinetic energy in AQUA_FLAT and AQUA_FLAT_HR, respectively. In contrast, $\sum_{n=0}^9 P_n$ captures 73% of the baroclinic conversion rate in AQUA_FLAT while it captures 91% in AQUA_FLAT_HR. This improvement likely stems from the better horizontal and vertical resolution of baroclinic modes (particularly for $n > 1$) in the HR configuration. Nevertheless, the vertical energy pathway remains qualitatively similar across both resolutions (not shown), with AQUA_FLAT_HR exhibiting a slightly stronger signal in higher modes. In summary, AQUA_FLAT_HR exhibits a robust inverse transfer of eddy kinetic energy toward the barotropic mode, with secondary but dynamically relevant exchanges among the first and higher baroclinic modes, and a modal energy structure that is broadly consistent with the idealized channel configurations, while reflecting the increased complexity of variable stratification and different dynamical regimes.

Fig. 3.11 shows zonal averages of $E_n(k)$, $P_n(k)$, $N_n^m(k)$ and $N_n^e(k)$ for $n = 0, \dots, 5$ in AQUA_FLAT_HR. The relative magnitudes of these terms across modes are consistent with the integrated energy budget shown in Fig. 3.9 and the general spectral structure aligns with the purely spectral analysis of Dettmer and Eden (2025). In the barotropic mode, $E_n(k)$ maxima are primarily associated with western boundary currents, the Agulhas Current, and the Antarctic Circumpolar Current. In contrast, for modes $n \geq 2$, the energy maxima are concentrated in equatorial regions, though these are weaker than the barotropic contributions when integrated. The wavenumber $k_n^{\max}$ at which $E_n(k)$ is maximal generally increases with latitude, following the tendency of the Rossby radii L_n^{-1} . Poleward of 10° latitude, $k_n^{\max}$ is marginally smaller than

L_1^{-1} for the barotropic mode and slightly shifts to larger k as n increases, aligning approximately with L_n^{-1} for $n \geq 3$.

Baroclinic conversion $P_n(k)$ mainly acts in the first baroclinic mode, where it exhibits dipolar structure identified and discussed by Dettmer and Eden (2025), with positive values indicating $E_n(k)$ production at larger wavenumbers and negative values indicating $E_n(k)$ loss to eddy available potential energy at small wavenumbers. While $P_n(k)$ is significantly weaker for higher modes $n \geq 2$, the dipolar structure persists. While in the first baroclinic mode positive $P_1(k)$ peaks at approximately L_1^{-1} , in the higher modes the wavenumber range of positive $P_n(k)$ increases and shifts to higher k . Positive and negative $P_n(k)$ are approximately separated by $k_n^{\max}$ in each mode. Although $P_n(k)$ weakens significantly for higher modes ($n \geq 2$), the dipolar structure persists. While positive $P_1(k)$ in the first baroclinic mode peaks at approximately L_1^{-1} , the wavenumber range of positive $P_n(k)$ in higher modes expands and shifts toward higher k , between L_1^{-1} and L_4^{-1} . In each mode, the transition between positive and negative $P_n(k)$ is approximately separated by $k_n^{\max}$. For modes $n \geq 3$, BC produces $E_n(k)$ in the equatorial ocean at low wavenumbers, coinciding with the local $E_n(k)$ maximum. Consistent with Fig. 3.9, barotropic production is also significantly weaker than baroclinic production in spectral space and is therefore omitted.

The modal-spectral flux divergence $N_n^m(k) + N_n^e(k)$ mainly acts in the barotropic and the first baroclinic mode, dropping by one to two orders of magnitude for $n \geq 2$. When summed over all modes, the spectral flux divergence recovers the results of Dettmer and Eden (2025), with negative values at large wavenumbers and positive values at small wavenumbers, indicating an inverse spectral cascade. When integrated over wavenumber the picture presented by Fig. 3.9 is recovered, corresponding to a modal transfer toward $n = 0$. Similarly to $E_n(k)$ and $P_n(k)$, the modal-spectral flux divergence $N_n^m(k) + N_n^e(k)$ shifts to higher k as the mode index increases and the wavenumber range of negative values broadens. The modal-spectral flux divergence $N_n^m(k) + N_n^e(k)$ For baroclinic modes, this flux divergence is negative at the scales of $P_n(k)$ production and positive at larger scales, where $P_n(k)$ is negative, with positive and negative values again separated by $k_n^{\max}$. In contrast, in the barotropic mode $N_0^m(k) + N_0^e(k)$ is exclusively positive, but an barotropic inverse spectral cascade is expected to cause negative values at high wavenumbers. However, their absence here suggests that the barotropic cascade is masked by the inter-modal transfer into the barotropic mode. In summary the findings demonstrate a unified inverse transfer in both spectral and modal space, confirming that $E_n(k)$ production by $P_n(k)$ is balanced by the non-linear transfer across modal and spectral space.

While the individual contributions are not shown for brevity, $N_n^e(k)$ is found to be stronger than $N_n^m(k)$, consistent with Fig. 3.9. For $n \geq 2$ $N_n^m(k)$ is weakly positive at large wavenumbers between 30° south and 40° south. Since inter-modal transfers ($\mathcal{N}_n^m(i, j)$) are weak for modes $n \geq 2$, this E_n source suggests a spectral forward cascade driven by eddy-mean flow interactions. On the other hand the positive values at large scales of $N_0^m(k) + N_0^e(k)$ are related to eddy-eddy interaction $N_n^e(k)$

and suggest an inverse cascade confined to the baroclinic modes. Since through purely inter-modal energy transfers $\mathcal{N}_n^m(i, j)$ the first baroclinic mode gains energy only from the higher modes but not from the barotropic mode, the positive $N_n^e(k)$ at small wavenumber must come from higher baroclinic modes at larger wavenumbers, or self interaction, e.g a spectral energy flux from larger wavenumbers in the first baroclinic mode. Similarly the higher baroclinic modes $n \geq 2$ almost only loose eddy kinetic energy through purely inter-modal transfers, and consequently the source for positive $N_n^e(k)$ at small wavenumbers in these modes must be in the same mode and is found as negative $N_n^e(k)$ at higher wavenumbers. Therefore there is a inverse spectral cascade confined to the baroclinic modes.

As a next step the modal diagnostic is applied to REAL, which has the same resolution as AQUA_FLAT but includes realistic topography and more realistic surface forcing. This step allows the modal eddy kinetic energy pathway to be analyzed within a more realistic circulation and enables a quantitative assessment of the role of topographic scattering in the presence of realistic bottom topography.

Influence of topography

To evaluate the impact of topographic scattering $T_n = T_n^\phi + T_n^t + T_n^a$ the configuration AQUA_ROUGH (not shown) is considered. T_n exhibits adjacent positive and negative values indicative of localized modal scattering. Globally integrated T_n is weak compared to P_n and $N_n^m + N_n^e$ and only acts in the barotropic and first baroclinic mode and is negligible for $n \geq 2$. Topographic scattering by pressure work $T_n^\phi + T_n^t$ is negative in the barotropic and positive in the first baroclinic mode whereas advection induced topographic scattering T_n^a is weaker and always negative. Globally integrated $T_0^\phi + T_0^t$ accounts for 20% of the $N_n^m + N_n^e$ magnitude for modes 0 and 1 while for T_n^a these values reduce to 9% and 11% of $N_n^m + N_n^e$. To isolate scattering induced by horizontal $N^2(z)$ gradients, $T_n^\phi + T_n^t$ is recomputed using a globally averaged stratification profile. The resulting contribution is negligible, in agreement with AQUA_FLAT_HR, where this term can only arise from horizontal $N^2(z)$ gradients and is likewise small. Analysis of T_n^ϕ and T_n^t separately reveals that T_n^t dominates due to the constructive addition of its two components. In contrast, the four contributions forming T_n^ϕ cancel. While the specific canceling terms vary by mode, the balance always consists of two positive and two negative contributions. Consequently, the fundamental energy pathway in AQUA_ROUGH is similar to AQUA_FLAT_HR.

The horizontal integral $\int \sum_n T_n^\phi + T_n^t dA$, representing potential net conversion between kinetic and potential energy, is evaluated be 5% of $\int T_0^\phi + T_0^t dA$. These values are comparable in magnitude to $\int \sum_n N_n^m dA$ and $\int \sum_n N_n^e dA$, even though these terms are theoretically expected to vanish when summed over all modes. As a consequence, the residual is probably partly attributed to the discretization of the vertical modes and the accumulation of numerical summation errors, which is indicated by testing the orthogonality conditions of the discrete modes.

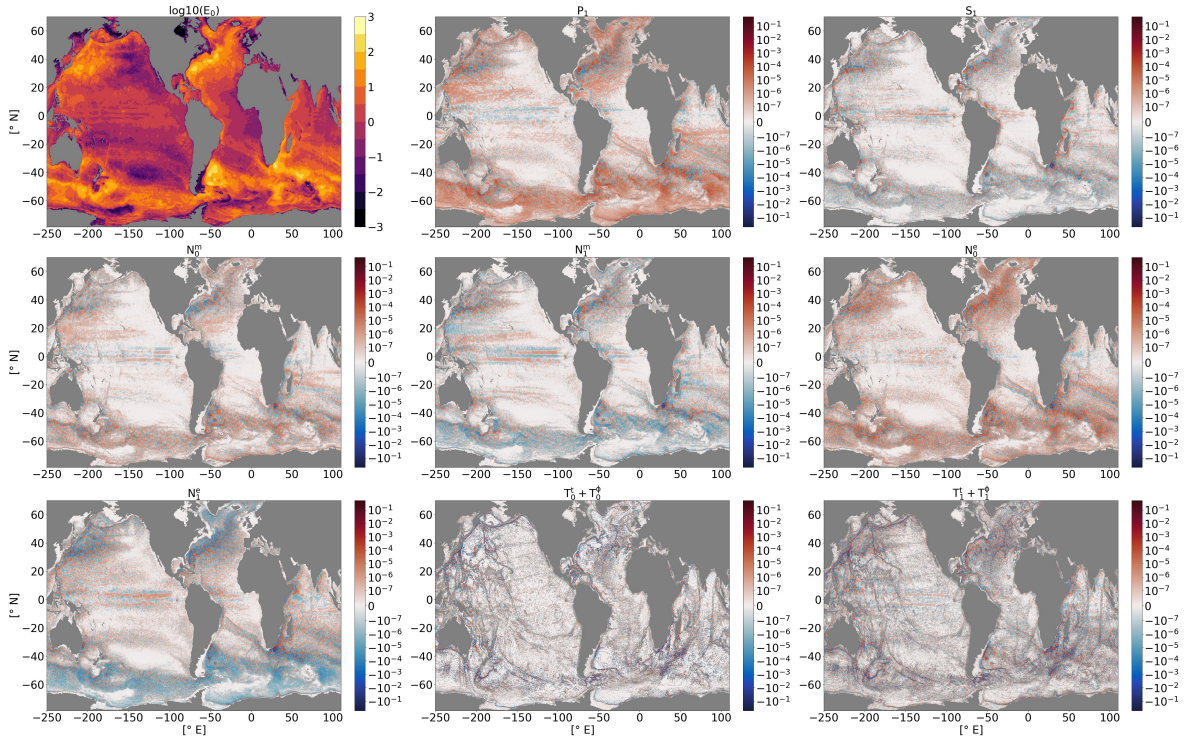


Figure 3.12: Results derived from REAL. Shown are the $\log_{10}(E_0)$, and P_1 , S_1 , N_0^m , N_1^m , N_0^e and N_1^e in $[\text{m}^3\text{s}^{-3}]$.

Experiment REAL

The circulation in REAL is realistic in the sense that it is dominated by zonal jets associated with the western boundary currents of the major ocean basins, the Antarctic Circumpolar Current, and westward return flows in the subtropical gyres, in broad agreement with observations (Dettmer and Eden, 2025). The stratification, vertical modes and interaction coefficients B_{nij} (not shown) closely resemble those found in AQUA_FLAT_HR. In most regions, $N(z)$ is large in the upper half of the water column and exhibits a subsurface maximum within the upper 1000 m, below which it decreases toward weak values at depth. The corresponding vertical modes are surface intensified, and the structure of B_{nij} is similar to AQUA_FLAT_HR.

Fig. 3.12 shows $\log_{10}(E_0)$ together with P_1 , S_1 , N_0^m , N_1^m , N_0^e and N_1^e for REAL. Enhanced E_0 values are found in the western boundary currents and along the Antarctic Circumpolar Current, whereas low values occur in the eastern parts of the basins and equatorward of 30° latitude. The horizontal distribution of E_1 is qualitatively similar (not shown). As in AQUA_FLAT_HR, the conversion and transfer terms P_1 , S_1 , N_0^m , N_1^m , N_0^e and N_1^e are strongest in regions of elevated eddy kinetic energy. The baroclinic conversion term P_1 is predominantly positive, but also exhibits localized negative values, particularly in regions with enhanced E_n . Consistent with the previous configurations, the non-linear scattering terms N_n^m and N_n^e are predominantly negative for the first baroclinic mode and positive for

the barotropic mode. The spatial structure of N_n^m is slightly smoother than that of N_n^e . Topographic scattering by pressure work $T_n^\phi + T_n^t$ reflects major topographic features such as mid-ocean ridges and continental slopes, yet as in AQUA_ROUGH, neither positive nor negative transfers clearly dominate. At western boundaries T_n is enhanced, likely due to stronger topographic slopes and increased E_n , with especially the spatial distribution of T_n^a reflecting that of E_n .

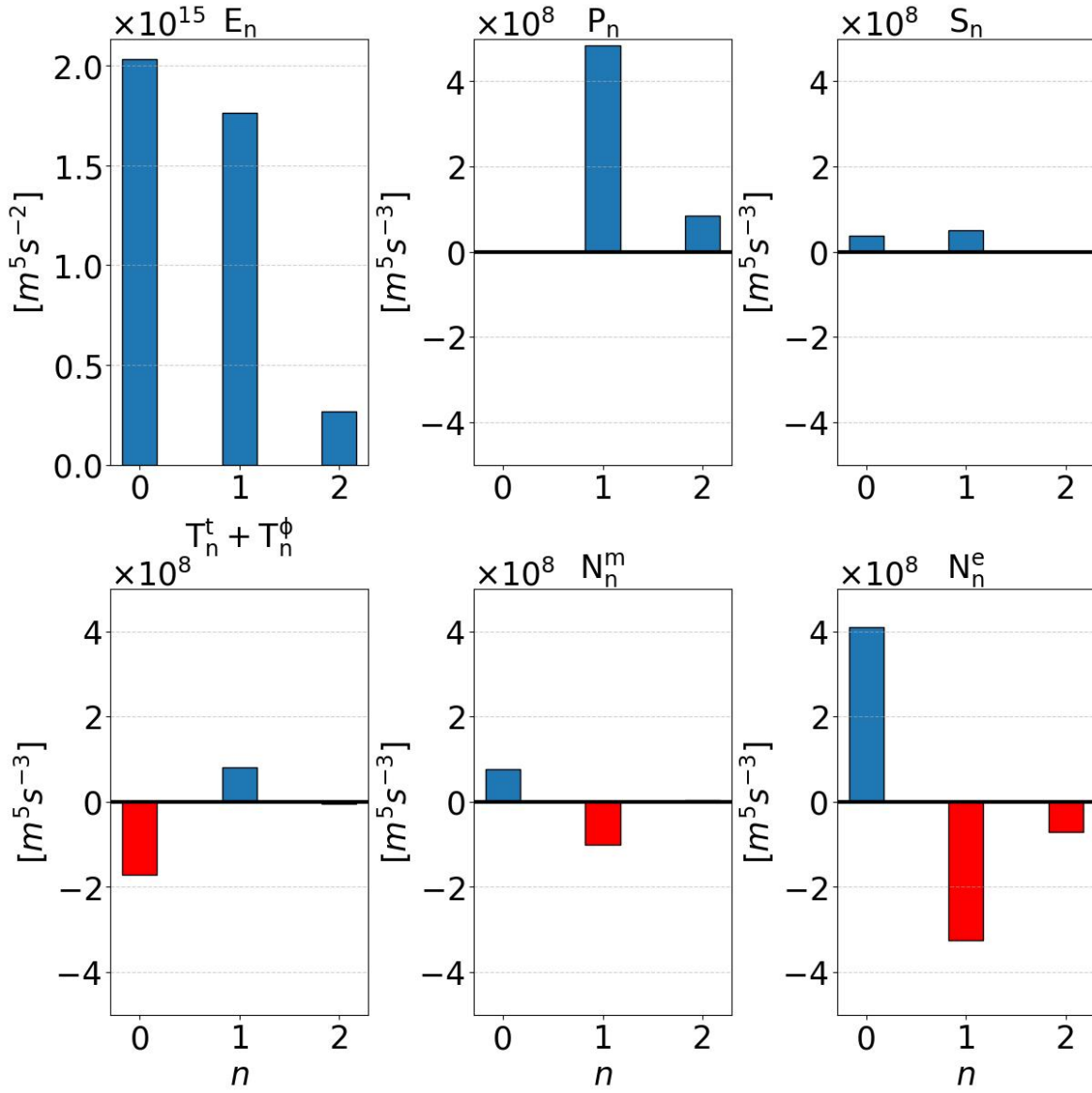


Figure 3.13: Results derived from REAL. Horizontal integrals of the modal eddy kinetic energy $\int E_n dA$ in [$m^5 s^{-2}$] and $\int P_n dA$, $\int S_n dA$, $\int T_n^\phi + T_n^t dA$, $\int N_n^m dA$ and $\int N_n^e dA$ in [$m^5 s^{-3}$].

Fig. 3.13 presents the global integrals of modal eddy kinetic energy and the associated transfer terms for REAL. Only modes up to $n = 2$ are shown, as higher

modes contain negligible energy and their Rossby radii are poorly resolved. In general, the modal energy pathway mirrors that of the previous models: eddy kinetic energy is primarily generated by baroclinic instability in the first baroclinic mode, while barotropic generation remains weaker. Subsequently, the non-linear terms N_n^m and N_n^e transfer this energy to the barotropic mode. Globally, slightly more eddy kinetic energy is found in the barotropic mode than in the first baroclinic mode, with negligible energy in higher modes. As in AQUA_FLAT_HR, N_n^e exceeds N_n^m in magnitude. Topographic scattering by pressure work $T_n^\phi + T_n^t$ is negative for the barotropic mode and positive for the baroclinic mode and T_n^a (not shown) is always negative, consistent with AQUA_ROUGH. Globally integrated these contributions remain small, with $T_n^\phi + T_n^t$ accounting for 35% and 18% of the $N_n^e + N_n^m$ magnitude in the barotropic and first baroclinic mode, and T_n^a accounting for 10% and 27%. As in AQUA_ROUGH there is internal cancellation of T_n^ϕ relative to the constructive addition of T_n^t . However, at the western boundaries, T_n becomes comparable in magnitude to $N_n^m + N_n^e$ with the transfer directions as in the global integral: Next to enhanced E_n and steeper topographic slopes this arises from an altered internal cancellation: the two components of T_n^t exhibit opposing signs, causing T_n^ϕ to dominate. Although the four contributions to T_n^ϕ oppose each other as in the global integral, their magnitudes are significantly increased, resulting in stronger net T_n^ϕ . The horizontal integral $\int \sum_n T_n^\phi + T_n^t dA$ yields a net conversion representing 24% of the barotropic scattering, which is as in AQUA_HROUGH comparable to the residuals of $\int \sum_n N_n^m dA$ and $\int \sum_n N_n^e dA$.

At western boundaries, T_n reaches magnitudes comparable to $N_n^m + N_n^e$ while maintaining transfer directions consistent with the global integral. This localized strength arises from a shift in internal cancellation: the two components of T_n^t roughly offset one another, allowing T_n^ϕ to dominate. Although the four contributions to T_n^ϕ still exhibit opposing signs as in the global integral, their individual magnitudes are significantly increased, resulting in a stronger net T_n^ϕ signal in these regions.

Fig. 3.14 shows the global integrals of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ for $n \geq 3$. Consistent with previous configurations, the positive diagonal elements associated with the barotropic mode indicate an inverse transfer into $n = 0$, dominated in REAL by the transfer from the first baroclinic mode. Beyond this, the transfer pattern is less distinct, likely due to the low energy content and marginal resolution of the higher baroclinic modes. A weak forward modal transfer mediated by $\mathcal{N}_n^m(i, j)$ is visible through transfer pairs d and e, though its magnitude remains small. The structures of $\mathcal{N}_n^m(i, j)$ and $\mathcal{N}_n^e(i, j)$ in AQUA_ROUGH and AQUA_FLAT are qualitatively similar (not shown).

As in AQUA_FLAT_HR, the global integrals in Fig. 3.13 and Fig. 3.14 are dominated by the energetic western boundary currents and the Antarctic Circumpolar Current. While higher modes exhibit enhanced E_n and P_n in the subtropical and equatorial ocean, this signal is less pronounced than in AQUA_FLAT_HR, likely

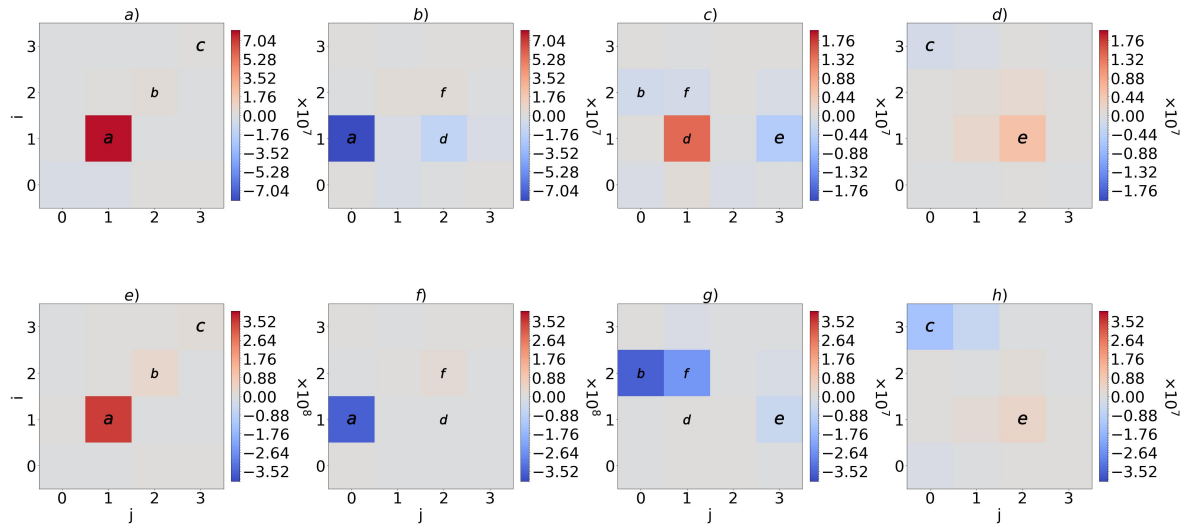


Figure 3.14: Horizontal integrals in REAL of $\int \mathcal{N}_n^m(i, j) dA$ (a-d) and $\int \mathcal{N}_n^e(i, j) dA$ (e-h) in $[m^5 s^{-3}]$ for $n = 0, 1, 2, 3$. The letters indicate the transfer pairs given by Eq. (3.22).

due to the comparatively coarser resolution of higher baroclinic modes in REAL. As in AQUA_FLAT_HR, N_n^e becomes positive for $n = 1$ in parts of the subtropics, and S_n locally exceeds P_n near the equator. In certain subtropical and equatorial regions, $T_n^\phi + T_n^t$ displays signs for $n = 0$ and $n = 1$ that are opposite to those of the global integral. However, no coherent large-scale pattern emerges, suggesting that these sign reversals are controlled by local topographic interactions. Modal-spectral transforms as in Fig. 3.11 for the REAL are not shown, as the results are qualitatively consistent with AQUA_FLAT_HR.

3.4. Discussion

In this study, a modal eddy kinetic energy equation is derived and evaluated in a hierarchy of ocean models, ranging from idealized channel configurations to global simulations with realistic topography. Despite substantial differences in forcing, stratification, and bottom boundary conditions, a robust modal energy pathway emerges: eddy kinetic energy is generated by baroclinic instability (P_n) in the baroclinic modes, while barotropic production is weak, and transferred to the barotropic mode via a non-linear inverse modal transfer. The identification of such an inverse modal transfer is consistent with Smith and Vallis (2001) and Tedesco et al. (2022).

Across all configurations, the majority of eddy kinetic energy is contained within the barotropic and first baroclinic modes, consistent with Wunsch (1997). However, the partitioning between these modes varies: AQUA_FLAT_HR exhibits higher energy in the first baroclinic mode, whereas the CHANNEL configurations show $E_0 > E_1$. This difference likely stems from the greater depth and stronger surface

intensification of mean isopycnal gradients in AQUA_FLAT_HR, which favors a more baroclinic circulation. While surface intensification and vertical structure in CHANNEL is related to the frictional parameterization, which are tuned to resolution and varies across configurations, the impact of resolution itself is evident in the E_0/E_1 ratio. Specifically, the increased E_0/E_1 ratio in AQUA_FLAT_HR relative to AQUA_FLAT suggests that higher resolution facilitates more efficient barotropization, consistent with Yankovsky et al. (2022).

Note that while the definition of E_n in principle encompasses both mesoscale eddies and internal gravity waves, their contribution is expected to be negligible. In REAL and AQUA_ROGH, the resolution is too coarse to resolve internal gravity waves, except potentially the longest ones, and in the higher resolution configurations CHANNEL and AQUA_FLAT_HR, the flat bottom and smooth forcing prohibits their excitation.

The non-linear inter-modal transfers are governed by the coupling coefficient B_{nij} . Under constant stratification, B_{nij} is nonzero only for $n = |i \pm j|$, thereby restricting triadic interactions. For vertically varying stratification, the overall structure of B_{nij} remains similar, but an important difference arises: the diagonal elements for $n = 1$ become nonzero. Furthermore, it is shown that N_n^m and N_n^e form transfer pairs (cf. Eq. (3.22)), which makes it possible to disentangle the various (n, i, j) triads of the non-linear interactions. The introduction of a pycnocline drives the transition from the simple energy cycle in CHANNEL_HR to more complex inter-modal transfers. For constant stratification the inter-modal transfers are confined to the barotropic and first baroclinic mode, which is different to Smith and Vallis (2001, 2002), where also the higher modes contribute to the inverse modal transfer. CHANNEL_HR exhibits a strong transfer from the first baroclinic to the barotropic mode and a much weaker transfer from the first baroclinic to the second baroclinic mode. In contrast, for CHANNEL_HR_P and AQUA_FLAT_HR, the higher baroclinic modes emerge as active components and transfer E_n directly to the barotropic mode while simultaneously exhibiting a bi-directional energy loop between the first baroclinic and the higher baroclinic modes. This finding differs from Smith and Vallis (2001, 2002), who, instead of diagnosing inter-modal transfers directly analyze the temporal evolution of modal-spectral kinetic energy of decaying simulations and suggest that in presence of a strong pycnocline, energy is first transferred to the first baroclinic mode and subsequently to the barotropic mode. The present results however suggest a persistent direct energy transfer from the baroclinic modes to the barotropic mode, consistent with the findings of Shevchenko and Berloff (2017), who diagnose inter-modal energy transfers. The detailed insight into the inter-modal transfers in this study is achieved by resolving individual triads and further specifying the energy transfer direction and magnitude using the transfer pair relation (cf. Eq. (3.22)). Note that both studies find an increased complexity of the barotropization pathway in presence of a pycnocline. The modal transfer in AQUA_FLAT_HR is largely similar to CHANNEL_HR_P, with higher baroclinic modes transferring E_n directly to both the barotropic and first baroclinic modes. However, a distinct shift in the forward

transfer occurs: the transfer from the first baroclinic to higher modes is non-local in CHANNEL_HR_P, whereas it is realized by local transfers in AQUA_FLAT_HR. Although the pattern in REAL is qualitatively consistent with AQUA_FLAT_HR, REAL should be interpreted with caution, as the higher baroclinic modes remain under resolved.

The relative importance of the non-linear scattering terms N_n^m and N_n^e varies both across model configurations and geographic regions. In the global integral of AQUA_FLAT_HR, N_n^e exceeds N_n^m , while the opposite is observed in CHANNEL_HR. In AQUA_FLAT_HR and REAL, N_n^e dominates in the Southern Ocean and boundary current regimes, whereas N_n^m dominates in the subtropical oceans. This contrast likely reflects a dynamical regime shift near 30° latitude: poleward the circulation is dominated by isotropic turbulent eddies, while equatorward eddies transfer energy to Rossby waves and zonal jets Theiss (2004); Rhines (1975); Eden (2007); Tulloch et al. (2009).

Consistent with Tedesco et al. (2022), topographic scattering $T_n^\phi + T_n^t$ scatters energy toward higher modes, partially opposing the inverse modal transfer by $N_n^m + N_n^e$. While T_n is locally significant, particularly at western boundaries, where steep slopes and increased E_n enhance topographic scattering, globally integrated T_n remains small due to internal cancellation. Additionally, scattering induced by horizontal $N^2(z)$ gradients is found to be negligible. Consequently, the impact of T_n on modal energy pathways is secondary, and the fundamental energy cycle remains robust across both flat and rough bottom configurations.

As demonstrated by the modal-spectral budget in Fig. 3.11, the dipolar structure of $P_n(k)$ persists across all baroclinic modes, facilitating a closed energy loop where baroclinic conversion acts as an eddy kinetic energy source at small scales and a sink at large scales. Additional evidence for such negative large-scale baroclinic conversion has been identified by Roulet et al. (2012); Wang et al. (2019); Dettmer and Eden (2025); Lobo (2025); Tro et al. (2025). Crucially, this large-scale sink is absent in the barotropic mode ($n = 0$). Consequently, the simultaneous eddy kinetic energy transfer toward the barotropic mode and the lack of barotropic energy removal via negative baroclinic conversion necessitate strong dissipation pathways for large-scale barotropic eddy kinetic energy, which are yet to be identified.

3.5. Summary

- Across all model configurations - independent of forcing, dissipation and topography - most eddy kinetic energy is found in the barotropic and first baroclinic mode.
- Given sufficient horizontal and vertical resolution, the fundamental modal energy pathway is qualitatively consistent across a wide hierarchy of models, regardless of variations in forcing, dissipation, or topography and can be described as generation of eddy kinetic energy via baroclinic instability,

primarily in the first baroclinic mode for constant (CHANNEL_HR) and more realistic (AQUA_FLAT_HR) stratification and in higher modes when stratification is strongly surface-intensified (CHANNEL_HR_P). Subsequently energy is transferred to the barotropic mode via eddy-eddy and eddy-mean flow interactions.

- The transfer of eddy kinetic energy from the baroclinic modes to the barotropic mode is non-local in modal space with energy transferred directly from the higher modes to the barotropic mode. The presence of a pycnocline increases the complexity of these transfers, introducing a bi-directional energy loop between the first and higher baroclinic modes that operates alongside the direct transfer to the barotropic mode.
- Although topographic scattering ($T_n^\phi + T_n^t$) and advection over topography (T_n^a) can be locally significant, their effects largely cancel in the global integral and $N^2(z)$ gradient induced scattering is insignificant. Consequently, topographic effects remain secondary to the primary inverse modal transfer in the global energy budget.
- While baroclinic modes possess a large-scale eddy kinetic energy sink via negative baroclinic conversion, no such sink exists for the barotropic mode. Consequently, the removal of large-scale barotropic eddy kinetic energy must rely on alternative routes to dissipation whose nature and efficiency remain to be determined.

Acknowledgments

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Datastatement

The model code can be found at <https://github.com/ceden/py0M2>.

Appendix: Diagnosis of vertical modes

For the diagnosis of the numerical models the vertical modes $\phi_n(\bar{N}^2)$ from the eigenvalue equation Eq. (3.3) are discretized at each grid point using a second-order finite-difference scheme and solved numerically for n from 0 to 9 with $\bar{N}(x, z)$ as time average of the local stratification, allowing for small, large scale horizontal variations of $\phi_n(x, z)$ induced by variations of $\bar{N}(x, z)$ and by variations of water depth.

The discretization of Eq. (3.3) on a non-equidistant grid z_m , $m = 1, \dots, N$ with spacing $\Delta z_m = z_{m+1} - z_m$ is given by:

$$c^{-2}\phi_m + \left(-\frac{\phi_m - \phi_{m-1}}{N_{m-1}^2 \Delta z_{m-1}} \right) \frac{2}{\Delta z_m + \Delta z_{m-1}} = 0, \quad m = N \quad (3.23)$$

$$c^{-2}\phi_m + \left(\frac{\phi_{m+1} - \phi_m}{N_m^2 \Delta z_m} - \frac{\phi_m - \phi_{m-1}}{N_{m-1}^2 \Delta z_{m-1}} \right) \frac{2}{\Delta z_m + \Delta z_{m-1}} = 0, \quad m = 2, \dots, N-1 \quad (3.24)$$

$$c^{-2}\phi_m + \left(\frac{\phi_{m+1} - \phi_m}{N_m^2 \Delta z_m} \right) \frac{2}{\Delta z_m + \Delta z_{m-1}} = 0, \quad m = 1 \quad (3.25)$$

The sign of the eigenmodes ϕ_n is not uniquely determined by the eigenvalue problem. This ambiguity is resolved by requiring the modes to be positive at the upper boundary. While this choice formally affects the sign of the projected dynamical variables, all diagnosed energy reservoirs and transfer terms involve the sign of ϕ_n in an even number of times. Consequently, the diagnostics presented in this study are independent of the particular sign convention adopted for ϕ_n .

4

Modal-Spectral eddy energy characteristics and transfers

In chapter 2 a spectral eddy kinetic energy equation and in chapter 3 a vertical mode eddy kinetic energy equation and a modal-spectral eddy kinetic energy equation are derived and diagnosed in a hierarchy of ocean models, ranging from a realistic eddy resolving global simulation, over three aquaplanet simulations to idealized simulations of a baroclinically unstable channel. This chapter extends the modal-spectra analysis and consists of two parts: First, spectra of eddy kinetic energy, eddy available potential energy and their sum — total eddy energy — are analyzed and second the diagnosis of the modal-spectral eddy kinetic energy equation Eq. (3.20) is extended to the CHANNEL configurations. Eddy available potential energy is given by: $\hat{A}_n(k) = \overline{\check{b}'^* \cdot \check{b}' / 2c_n^2}$. As in chapter 3 the modal Rossby radius $L_n = c_n/f$ with eigenvalue c_n of mode n and Coriolis frequency f and modal Rhines scale $\lambda_n = \sqrt{U_n/\beta}$ with modal turbulent velocity scale $U_n = \sqrt{E_n/h}$ defined from modal eddy kinetic energy E_n and water depth h are used. Unless otherwise stated, notation, methodological details and numerical models remain consistent with chapters 2 and 3.

4.1. Results

4.1.1. Eddy energy spectra

In the following, the horizontal wavenumber spectra of eddy kinetic energy $E_n(k)$ and eddy available potential energy $\hat{A}_n(k)$ will be discussed. First in AQUA_FLAT_HR and then in CHANNEL_HR and CHANNEL_HR_P.

Fig. 4.1 shows total eddy energy $E_n(k) + \hat{A}_n(k)$ at different locations in AQUA_FLAT_HR for modes $n = 0, \dots, 5$ and summed over all modes up to $n = 9$. Additionally vertically integrated spectral eddy energy $\int \overline{\check{b}'^* \cdot \check{b}' / 2N^2} + \overline{\check{u}'^* \cdot \check{u}' / 2} dz$ is shown. As expected, the modal sum and the vertical integral of eddy energy are very similar.

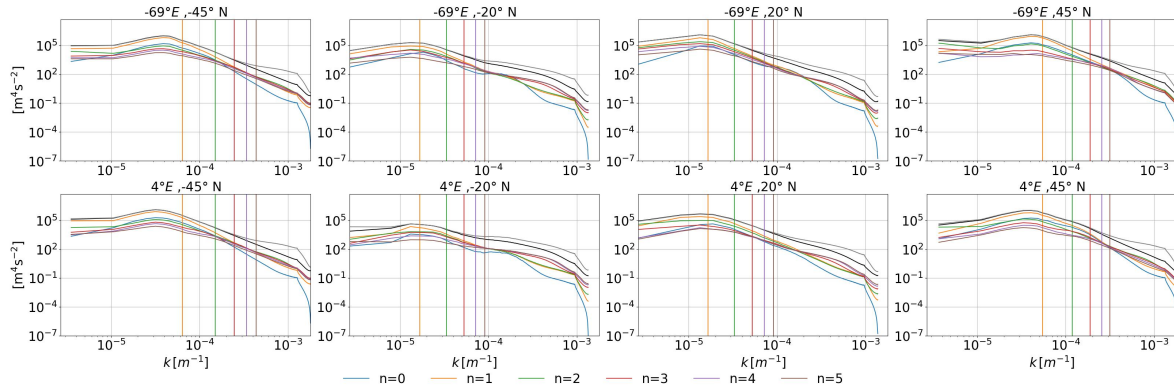


Figure 4.1: Total eddy energy $\hat{A}_n(k) + E_n(k)$ at different locations in AQUA_FLAT_HR for modes $n = 0, \dots, 5$. Additionally $\sum_{n=0}^9 \hat{A}_n(k) + E_n(k)$ is shown in black and vertically integrated total spectral eddy energy $\int \check{b}'^* \cdot \check{b}' / 2N^2 + \check{u}'^* \cdot \check{u}' / 2 dz$ is shown in gray. The vertical lines indicates the inverse Rossby radius L_n^{-1} .

At low wavenumbers there is almost no difference between the two spectra, but at high wavenumbers the modal sum of eddy energy is slightly smaller than the vertical integral of eddy energy, indicating that at small scales eddy energy is not fully but still almost resolved by the first 10 modes.

As seen before, most eddy kinetic energy is in the barotropic and first baroclinic mode. The spectra at 20° latitude are shallower than at 46° latitude, but overall the geographical, especially the zonal variability of the spectra is small. The maxima of the spectra are well resolved, as there is a drop in energy toward smaller wavenumbers than the wavenumber of the eddy energy maximum. Barotropic eddy energy drops off more steeply toward larger wavenumbers than baroclinic eddy energy, indicating that barotropic eddy energy resides mainly at large scales. As the mode index n increases, the spectra become shallower and the maximum shifts to higher wavenumbers. However, this shift is less pronounced than the corresponding increase in the deformation wavenumber L_n^{-1} . In the following the spectral slopes are analyzed further by fitting a slope to the spectra in logarithmic space. This fitting is performed between the eddy energy maximum wavenumber and the wavenumber where eddy energy drops off due to dissipation. As discussed in the introduction, two inertial ranges separated by an intermediate forcing wavenumber are expected to emerge: For small wavenumbers an energy range with a $-5/3$ slope and for larger wavenumbers an enstrophy inertial range with a -3 slope. Since in the spectra presented here, there is no pronounced dip, separating the two inertial ranges, only one slope is fitted to each spectrum.

Fig. 4.2 shows the spectral slopes of total eddy energy $E_n(k) + A_n(k)$ for modes $n = 0, \dots, 5$ in each horizontal window of AQUA_FLAT_HR. Additionally the slopes for the modal sum of eddy energy $\sum_n E_n(k) + A_n(k)$ and vertically integrated spectral eddy energy $\int \check{b}'^* \cdot \check{b}' / 2N^2 + \check{u}'^* \cdot \check{u}' / 2 dz$ are shown. The spectral slopes remain

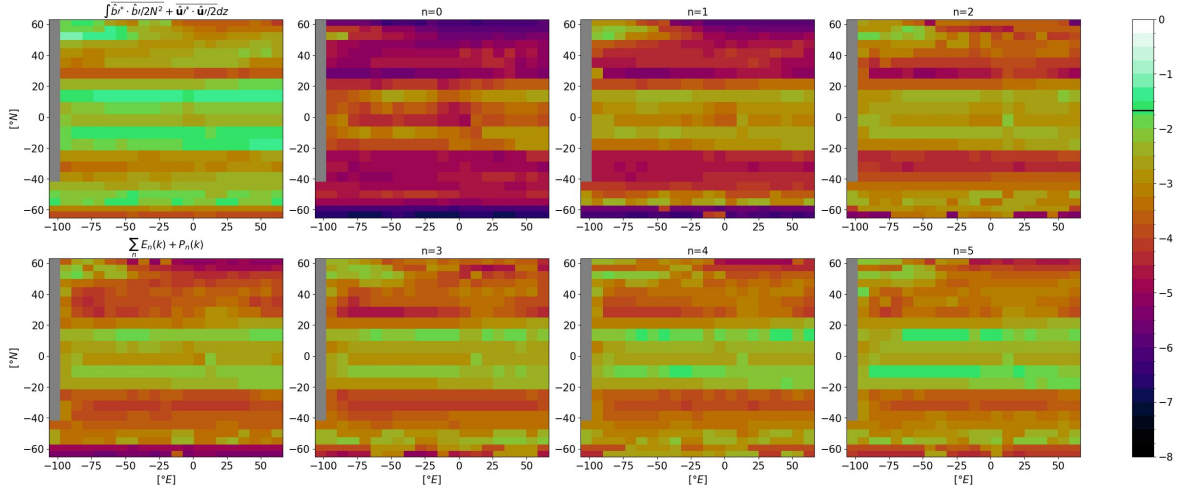


Figure 4.2: Results derived from AQUA_FLAT_HR. Spectral slopes of total eddy energy $E_n(k) + A_n(k)$ for modes $n = 0, \dots, 5$ in each horizontal window. Additionally the slopes of the modal sum

$\sum_{n=0}^9 \hat{A}_n(k) + E_n(k)$ and vertically integrated spectral eddy energy $\int \tilde{b}'^* \cdot \tilde{b}' / 2N^2 + \tilde{u}'^* \cdot \tilde{u}' / 2 dz$ are shown. One slope is fitted to each spectrum between the wavenumber of the eddy energy maximum and the wavenumber where eddy energy drops due to dissipation. The black line in the colorbar indicates a $-5/3$ slope.

nearly identical when calculated for kinetic energy instead of eddy kinetic energy. For $n = 0$ the slopes are mainly between -4 and -5 poleward of 30° latitude and between around -3 equatorward of 30° latitude. With increasing n , the spectra become progressively shallower, with a stronger reduction poleward of 30° latitude. At 28° latitude, the spectra are steeper than at adjacent latitudes, which can be attributed to a dip at intermediate wavenumbers. When the fit is restricted to wavenumbers larger than this intermediate scale, the slopes at 28° latitude become comparable to those at neighboring latitudes (not shown).

The spectral slopes of the modal sum of eddy energy are around -4 poleward of 30° latitude and around -2 equatorward of 30° latitude. The slopes of the vertically integrated eddy energy spectra are generally shallower, ranging between -1 and -4 . This behavior is likely related to the relative reduction of modal eddy energy compared to vertically integrated eddy energy at high wavenumbers. Both diagnostics exhibit a similar meridional structure, with steeper (shallower) slopes poleward (equatorward) of 30° latitude.

Fig. 4.3 shows the spectral slopes of eddy energy in CHANNEL_HR and CHANNEL_HR_P for modes $n = 0, \dots, 9$, as well as eddy energy summed over all modes and vertically integrated spectral eddy energy. As in AQUA_FLAT_HR, spectral slopes calculated for kinetic energy instead of eddy kinetic energy are nearly identical. The spectral slopes in CHANNEL_HR_P are slightly shallower than in CHANNEL_HR, but overall the values are comparable between the two configurations. As in AQUA_FLAT_HR, poleward of 30° latitude the spectra exhibit slopes of approximately -4 for $n = 0$, while the higher baroclinic modes are progressively shallower,

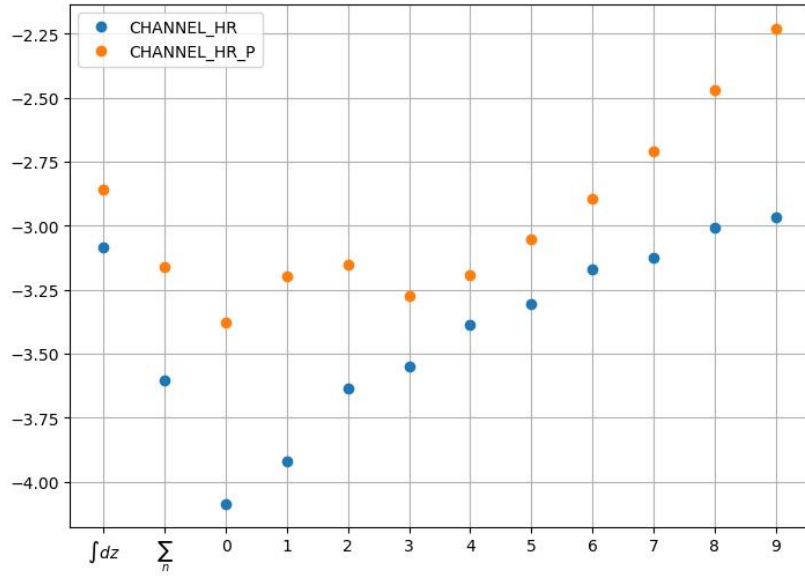


Figure 4.3: Results derived from CHANNEL_HR and CHANNEL_HR_P. Spectral slopes of total eddy energy $E_n(k) + A_n(k)$ for modes $n = 0, \dots, 5$ in each horizontal window. Additionally the slopes of the modal sum $\sum_{n=0}^9 A_n(k) + E_n(k)$ and vertically integrated spectral eddy energy $\int \tilde{b}'^* \cdot \tilde{b}' / 2N^2 + \tilde{u}'^* \cdot \tilde{u}' / 2 dz$ are shown. One slope is fitted to each spectrum between the wavenumber of the eddy energy maximum and the wavenumber where eddy energy drops due to dissipation.

reaching about -2.5 for $n = 9$. Again consistent with AQUA_FLAT_HR, the slope of the vertically integrated eddy energy is shallower than that of eddy energy summed over all modes, reflecting a similar reduction in modal eddy energy relative to vertically integrated eddy energy at high wavenumbers (not shown).

4.1.2. Modal-spectral eddy kinetic energy

In the following the modal-spectral eddy kinetic energy equation Eq. (3.20) is diagnosed in CHANNEL_HR and CHANNEL_HR_P. The modal-spectral energy pathway in AQUA_FLAT_HR is discussed in chapter 3 and is characterized by a unified inverse transfer in both spectral and modal space, where eddy kinetic energy produced by baroclinic instability in the baroclinic modes is redistributed toward lower wavenumbers and the barotropic mode. While eddy-eddy interactions $N_n^e(k)$ drive an inverse spectral cascade, partly confined to the baroclinic modes, eddy-mean flow interactions $N_n^m(k)$ exhibit a weak forward spectral cascade for higher baroclinic modes between 30° and 40° South. Next, the modal-spectral energy pathway in CHANNEL_HR is discussed and compared to AQUA_FLAT_HR.

Fig. 4.4 shows $E_n(k)$, $P(k)$, $N_n^m(k)$ and $N_n^e(k)$ in CHANNEL_HR for the barotropic and first baroclinic mode. For modes $n \geq 2$, the magnitudes of these terms are significantly weaker. Overall the modal-spectral eddy kinetic energy characteristics are similar to AQUA_FLAT_HR. Eddy kinetic energy $E_n(k)$ is maximal at scales

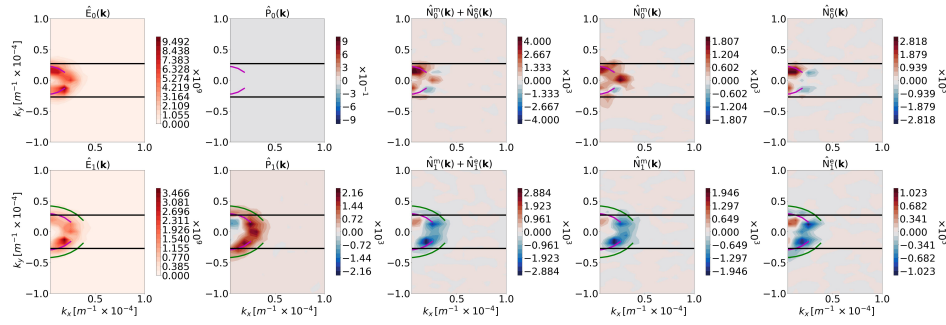


Figure 4.4: Results derived from CHANNEL_HR. $E_n(\mathbf{k})$ in $[\text{m}^5\text{s}^{-2}]$ and $P(\mathbf{k})$, $N^m(\mathbf{k})$ and $N^e(\mathbf{k})$ in $[\text{m}^5\text{s}^{-3}]$ for $n = 0, 1$. Also shown is L_n in green and λ_n in purple and the jet wavenumber given by the inverse of the distance between the jets in black

slightly larger than λ_n . Similarly to AQUA_FLAT_HR, $P(\mathbf{k})$ produces eddy kinetic energy at scales slightly larger than L_1 in the first baroclinic mode and is negative at large scales and the divergence of the spectral flux $N^m(\mathbf{k}) + N^e(\mathbf{k})$ shows the inverse cascade of eddy kinetic energy to larger scales and the barotropic mode. In contrast to AQUA_FLAT_HR no forward cascade by $N^m(\mathbf{k})$ is observed. Negative $N^m(\mathbf{k}) + N^e(\mathbf{k})$ occurs in a dipole pattern in k_y in good agreement with λ_n^{-1} and positive $N^m(\mathbf{k}) + N^e(\mathbf{k})$ are found at small k_x , likely indicating jet formation driven by Rossby wave dynamics.

Fig. 4.5 shows $E_n(\mathbf{k})$, $P(\mathbf{k})$, $N^m(\mathbf{k})$ and $N^e(\mathbf{k})$ in CHANNEL_HR_P for $n = 0 \dots 9$. The general modal-spectral eddy kinetic energy pathway is similar to CHANNEL_HR, except that eddy kinetic energy and the production and transfer rates are stronger and $E_n(\mathbf{k})$ production by baroclinic instability and the transfer to the barotropic mode by $N^m(\mathbf{k})$ and $N^e(\mathbf{k})$ are shifted toward higher baroclinic mode, as already discussed in chapter 3, where baroclinic conversion and nonlinear inter-modal transfer by eddy-eddy interaction is found to be strongest for $n = 4$. Another difference to CHANNEL_HR is that eddy kinetic energy and its associated conversion and transfer extend to higher wavenumbers, meaning that smaller scales are directly energized by baroclinic instability. The eddy kinetic energy sink at large scales by $P(\mathbf{k})$ is not present for all baroclinic modes, but is clearly visible for modes $n = 4, 6$ and 8 .

4.2. Discussion

In this chapter the modal-spectral eddy kinetic energy and its sources, sinks and transfers are analyzed and compared between AQUA_FLAT_HR, CHANNEL_HR and CHANNEL_HR_P. The modal-spectral eddy kinetic energy pathway can be summarized as eddy kinetic energy being produced by baroclinic instability in the baroclinic modes and on horizontal scales slightly larger than the respective Rossby radius. From there eddy kinetic energy is transferred toward larger horizontal scales and lower vertical modes through a combination of spectral and inter-modal

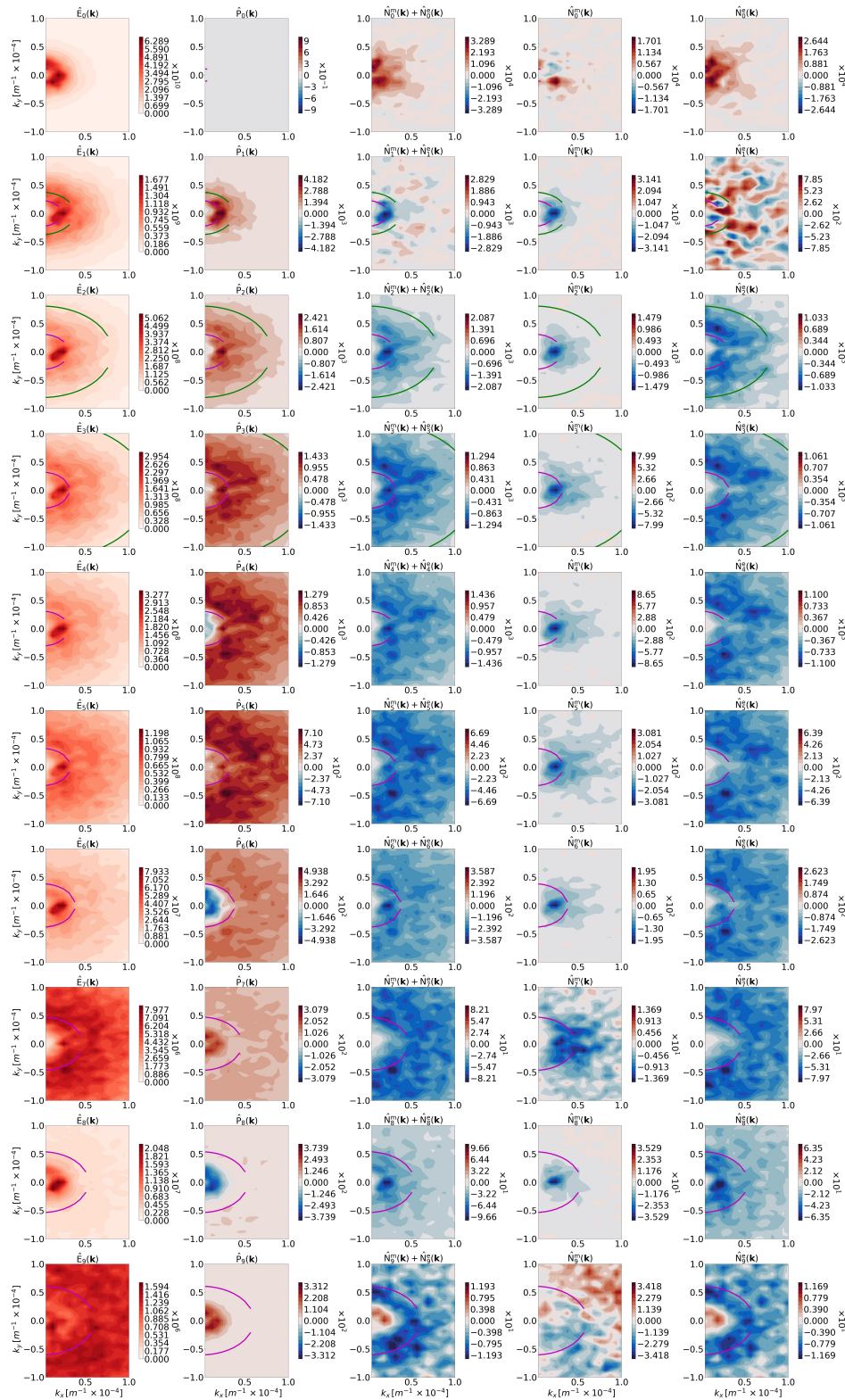


Figure 4.5: Results derived from CHANNEL_HR_P. $E_n(k)$ in $[m^5 s^{-2}]$ and $P(k)$, $N^m(k)$ and $N^e(k)$ in $[m^5 s^{-3}]$ for $n = 0 \dots 9$. Also shown is L_n in green and λ_n in purple.

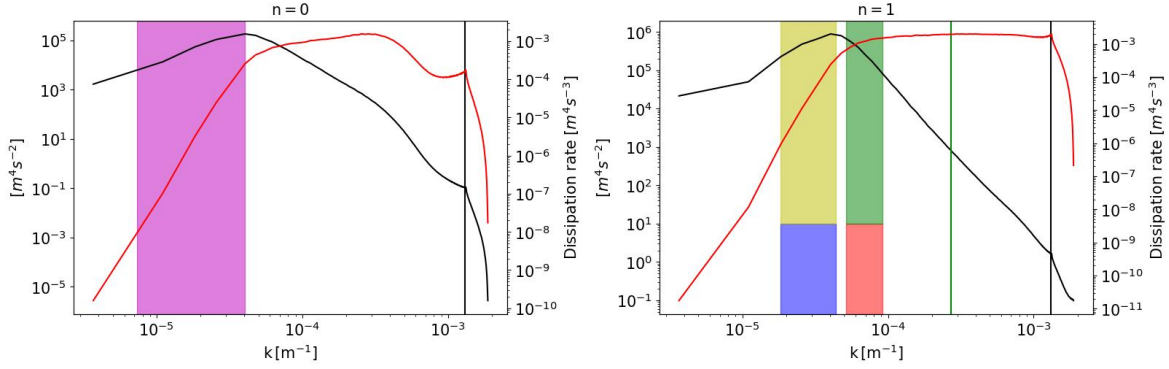


Figure 4.6: Spectra of eddy energy $E_n(k) + A_n(k)$ at -60°E and 45°N in AQUA_FLAT_HR for modes $n = 0, 1$ in black and the biharmonic dissipation rate $| -2 \eta_{hbi} k_h^4 E_n(k) |$ in red. Magenta indicates the wavenumber range where $N_0^m(k)$ and $N_0^e(k)$ are positive in the barotropic mode ($n = 0$). For the first baroclinic mode ($n = 1$), red (blue) indicate the wavenumber range where $BCK_1(k)$ is positive (negative) and yellow (green) indicates where $N_1^m(k)$ and $N_1^e(k)$ are weakly positive (negative). The vertical green line indicates L_1^{-1} and the vertical black line indicates the largest wavenumber used for fitting the spectral slope.

nonlinear transfers. As in AQUA_FLAT_HR, the inverse spectral cascade of barotropic eddy kinetic energy is masked by the inter-modal transfer into the barotropic mode. In AQUA_FLAT_HR, the higher baroclinic modes exhibit a weak forward spectral cascade driven by $N^m(k)$ and an inverse spectral cascade via $N^e(k)$, both of which remain confined to their respective vertical modes. While an inverse spectral cascade in the first baroclinic mode is identified in CHANNEL_HR, it is absent in CHANNEL_HR_P. The forward cascade by $N^m(k)$ observed in the aquaplanet simulation is not identified in either of the CHANNEL configurations.

This inverse modal-spectral transfer is likely related to the organization of the flow into zonal jets, most notably in CHANNEL_HR. The flow in CHANNEL_HR shows a tendency to form zonal barotropic jets. Although their instantaneous positions vary, the time-averaged zonal velocity reveals three equally spaced jets approximately 230 km apart. This corresponds to a jet wavenumber of $2.7 \times 10^{-5} \text{ m}^{-1}$, which aligns closely with the modal Rhines scale λ_1^{-1} . Since $\lambda_n > L_n$ in CHANNEL_HR, freely propagating Rossby waves are not expected to dominate the dynamics, which is supported by $E_n(k)$ not concentrating purely at $k_x = 0$, indicating characteristics closer to isotropic turbulence. However weak Rossby wave turbulence interactions are still sufficient to organize zonal jets near the Rhines scale. Non constant stratification, however, reduces the tendency of the flow to self organize into zonal jets and increases the energy input by baroclinic instability at smaller scales, which explains the finer horizontal structure of the modal eddy kinetic energy production and transfer terms observed in chapter 3.

The spectral slopes of total energy are between -1 and -8 in AQUA_FLAT_HR and between -4 and -2 in CHANNEL_HR and CHANNEL_HR_P. In QG theory, spectral slopes are predicted for total energy, defined as the sum of available potential

and kinetic energy. While this study focuses on the eddy components, the spectra are almost identical to those of the full kinetic energy. Moreover, given that the model resolution effectively captures the meso-scale, the total energy is expected to be dominated by the eddy field (Yankovsky et al., 2022).

A pronounced dip in the spectra separating the energy from the enstrophy inertial range is not consistently found. In general the spectra are steeper for low modes and poleward of 20° latitude and shallower for higher modes and equatorward of 20° latitude. The expected slopes are -3 in the enstrophy inertial range for larger wavenumbers, and $-5/3$ in the energy inertial range for smaller wavenumbers and -5 in the presence of β and weak friction. The observed spectral slopes are in that range, but it is not possible to distinguish the enstrophy from the energy inertial range. Furthermore in AQUA_FLAT_HR the slopes are between -3 and -5 poleward of 20° latitude and between -2 and -3 equatorward of 20° latitude, even though the influence of Rossby waves on the turbulent eddies is expected to be stronger there, which should lead to spectral slopes around -5 .

The mismatch between observed and expected spectral slopes can likely be explained by the assumptions behind the theory being violated. Fig. 4.6 shows the spectra of modal eddy energy $E_n(k) + A_n(k)$ at -60°E and 45°N in AQUA_FLAT_HR for modes $n = 0, 1$ with the wavenumber ranges where eddy energy is added or removed as well as the spectrum of the eddy kinetic energy dissipation rate by biharmonic friction. Clearly there is no inertial range with clear scale separation between forcing and dissipation scale. In the barotropic mode $E_0(k)$ is produced at large scales by $N^m(k) + N^e(k)$ and removed by biharmonic friction over a wide range of scales, mainly at wavenumbers larger than the peak wavenumber and linear bottom friction, which does not depend on k and removes $E_0(k)$ mainly at small wavenumbers, where $E_0(k)$ is strong. In the first baroclinic mode the sources of $E_1(k)$ are $BCk(k)$ in the red and green coloured wavenumber range and to a lesser extent $N^m(k) + N^e(k)$ in the blue and yellow coloured wavenumber range. The sinks of $E_1(k)$ are $P(k)$ in the blue and yellow coloured wavenumber range and $N^m(k) + N^e(k)$ in the red and green wavenumber range as well as biharmonic friction and bottom friction acting on the same wavenumbers as in the barotropic mode.

In summary eddy energy is generated and removed over a wide range of scales, which violates the assumption of inertial ranges. Additionally Danilov and Gurarie (2001) show that the spectral fluxes of energy are partly driven by non-local interactions and strongly influenced by coherent vortices — features identified in all models presented here — interacting with all scales. For these reasons it is difficult to predict or explain the observed spectral slopes. The tendency of shallower spectra equatorward of 20° and for higher vertical modes could be explained by the forcing through baroclinic instability being weaker in this region and in the higher modes. But there are multiple other possible reasons. It could be that in the higher modes the ratio of local to non-local spectral interactions is changed, the influence of Rossby waves is stronger, or that the presence and interactions of strong coherent vortices is changed in the higher modes. Disentangling these processes is not trivial and beyond

the scope of this work. In summary, the complexity of spectral interactions and the lack of clear scale separation between forcing and dissipation lead to a violation of classical inertial range assumptions explaining the non-universality of the observed spectral slopes.

5

Conclusions

This thesis derives and diagnoses spectral, modal, and combined modal-spectral energy equations across a hierarchy of numerical ocean models. These diagnostics provide a scale- and vertical structure-dependent quantification of meso-scale eddy energy production, transfer, and conversion, resolving the meso-scale energy pathways in unparalleled detail. This chapter is organized as follows: Section 5.1 presents a synthesis of the principal conclusions and provides an integrated picture of the oceanic eddy energy cycle. Section 5.2 discusses these findings in the context of recent research and outlines possibilities for future research.

5.1. Synthesis

This thesis formulates an integrated modal-spectral perspective on eddy energy reservoirs, transfers, and conversions, summarized in Fig. 5.1. The primary energy pathway originates with eddy kinetic energy generation primarily via baroclinic conversion from eddy available potential energy in the baroclinic modes at scales close to the Rossby radius, while generation by barotropic instability is much weaker. This energy is subsequently redistributed firstly via an inverse horizontal spectral cascade to larger scales, where baroclinic eddy kinetic energy is partly converted back to eddy available potential energy by negative baroclinic conversion, and secondly a direct inter-modal transfer to the barotropic mode.

5.1.1. Spectral characteristics of baroclinic conversion

A key result is the detailed spectral analysis of baroclinic conversion, which reveals a dipolar structure in spectral space, where it acts as a source of eddy kinetic energy at smaller scales and as a sink of eddy kinetic energy on larger scales, confirming the findings of Wang et al. (2019) and Rouillet et al. (2012). This large-scale sink partly balances the inverse cascade of eddy kinetic energy, facilitates a closed energy loop between kinetic and potential energy, and opens a potential pathway to small-scale energy dissipation via the forward cascade of potential energy. Going beyond

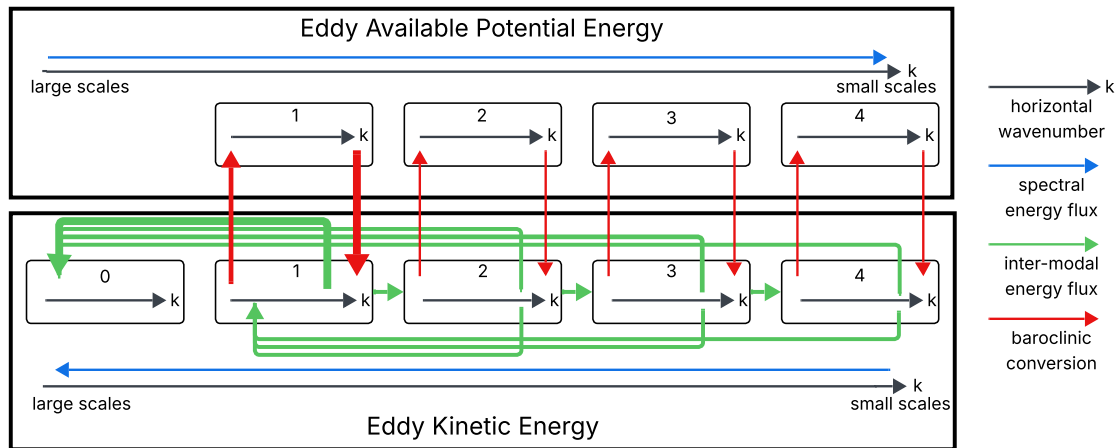


Figure 5.1: Schematic of the modal-spectral eddy energy conversions and transfers. Numbered boxes represent vertical modes 0–4 (not scaled to energy content), while the black arrow denotes the horizontal wavenumber axis. Blue and green arrows indicate spectral and inter-modal energy fluxes, respectively. Red arrows represent baroclinic conversion. Arrow thickness corresponds to the relative strength of the energy transfer. The primary eddy energy pathway involves the baroclinic conversion of eddy available potential energy to eddy kinetic energy at small scales. Subsequently eddy kinetic energy is redistributed via a direct inter-modal transfer to the barotropic mode (mode 0) and an inverse spectral cascade toward larger scales. At these larger scales, eddy kinetic energy is partly converted back into available potential energy, which undergoes a forward spectral cascade. Additionally, there is a forward inter-modal transfer between neighboring modes from the first baroclinic mode toward higher modes, as well as a direct inverse inter-modal transfer from higher baroclinic modes back to the first baroclinic mode.

the analysis of previous studies the geographical distribution, dynamical regime dependence and underlying mechanism of this energy sink is examined. In global models, the large-scale eddy kinetic energy sink is confined to regions poleward of approximately 30° latitude, where the mean flow is eastward. Idealized models and linear stability analysis confirm that the large-scale eddy kinetic energy sink by baroclinic conversion exists only in eastward mean flow and further show that finite β is necessary, suggesting that planetary Rossby waves are essential for its existence. The linear stability analysis further reveals how its vertical structure depends on mean flow direction and explains the findings: In eastward mean flow, negative baroclinic conversion appears in the upper half of the water column, whereas for westward mean flow it appears near the bottom boundary. Because eddy energy and also baroclinic conversion are surface-intensified, negative baroclinic conversion near the bottom in westward flows is damped, whereas in eastward flows it is intensified. Crucially, the modal decomposition reveals that baroclinic conversion is limited to the baroclinic modes. Consequently the negative baroclinic conversion at large scales is likewise confined to the baroclinic modes. Simultaneously, next to the inverse spectral cascade a robust inverse modal eddy kinetic energy transfer to the barotropic mode is identified (green arrows in Fig. 5.1), Consequently, while the

need for a baroclinic large-scale eddy kinetic energy sink is alleviated, the removal of large-scale barotropic eddy kinetic energy must rely on alternative processes to negative baroclinic conversion.

5.1.2. Vertical structure of eddy kinetic energy transfers

The modal decomposition further shows that EKE is primarily concentrated in the barotropic and first baroclinic modes. A key result is the detailed characterization of non-linear inter-modal transfers via individual triad interactions, which demonstrates that these transfers are fundamentally direct and non-local in modal space. Energy is transferred directly from higher baroclinic modes to the barotropic mode, contrasting with previous characterizations of these transfers as a local inverse modal cascade. The specific structure of the modal eddy kinetic energy pathway depends on the stratification profile: For a constant stratification profile, baroclinic instability acts primarily in the first baroclinic mode, limiting also the non-linear transfer to interactions between the first baroclinic and barotropic modes. In the presence of non-constant stratification the energy pathway shifts toward higher modes. Baroclinic instability energizes a broader range of baroclinic modes, and the non-linear transfer to the barotropic mode shifts accordingly. Additionally a strong pycnocline increases the complexity of the inter-modal transfers, as a bi-directional energy loop between the first and higher baroclinic modes that operates alongside the direct transfer to the barotropic mode appears. Finally, analysis of energy transfers by topographic scattering and advection over topography reveals that while these effects can be locally significant, they largely cancel in the global integral and thus do not fundamentally alter the energy pathway.

In summary, the modal-spectral energy pathway is characterized as follows: the generation of eddy kinetic energy via baroclinic conversion is balanced by a unified inverse modal-spectral transfer to larger scales and lower modes. In the baroclinic modes this transfer is in turn partly balanced by negative baroclinic conversion, provided finite β and eastward mean flows are present. However in the barotropic mode, which acts as the destination for the inter-modal transfers, this energy sink is absent, leading to an accumulation of barotropic large-scale eddy kinetic energy.

5.2. Discussion and Outlook

5.2.1. Comparison to recent studies

Recently, a similar spectral dipole in the conversion between available potential and kinetic energy was identified by Tro et al. (2025) using freely decaying, continuously stratified quasigeostrophic (QG) simulations, and by Lobo (2025) in a two layer QG model. Tro et al. (2025) attribute the large-scale negative conversion to vertical buoyancy fluxes induced by a bottom Ekman layer, where a scale-dependent relationship

between Ekman pumping and buoyancy causes a negative vertical buoyancy flux at large scales. Lobo (2025) decompose vertical velocity contributions to the baroclinic conversion and identify two primary drivers for negative conversion: bottom friction and the baroclinic advection of barotropic vorticity along jet flanks.

The combined evidence from Tro et al. (2025), Lobo (2025), and the present study indicates that the spectral dipole in baroclinic conversion is a robust dynamical feature of QG turbulence. Its identification across z-coordinate and layered models confirms that the large-scale sink is not an artifact of specific numerical formulations or ambiguities in the definition of available potential energy. Both Lobo (2025) and Tro et al. (2025) associate negative baroclinic conversion with bottom friction, which is also implemented in the flat-bottom models used in the present study. Notably, Tro et al. (2025) find that this sink is not localized to the boundary layer but extends throughout the water column. However, only this study and Lobo (2025) associate it additionally with finite β , whereas negative baroclinic conversion occurs for $\beta = 0$ in Tro et al. (2025). Neither Lobo (2025) nor Tro et al. (2025) evaluate their proposed mechanisms for negative baroclinic conversion in different mean flow directions. The analysis by Lobo (2025) is limited to eastward mean flow and in Tro et al. (2025), no mean flow develops, since the simulations are freely decaying. However, the link between mean flow direction and negative baroclinic conversion is a key result of the present work. Such an evaluation through the use of a QG model presents a clear opportunity for future research.

5.2.2. Negative baroclinic conversion and Rossby wave asymmetry

While the linear stability analysis successfully explains the existence and lack of negative baroclinic conversion in eastward versus westward mean flows, a complementary and more mechanistic explanation providing further physical insight may lie in the asymmetric propagation characteristics and dynamical interactions of planetary Rossby waves with eastward and westward mean flow. To investigate this asymmetry, consider the dispersion relation for barotropic Rossby waves in a purely zonal mean flow:

$$\omega = Uk - \frac{k\beta}{k^2 + l^2} = Uk + \omega_i$$

With frequency ω , zonal mean flow U , intrinsic frequency ω_i and zonal and meridional wavenumbers k and l . The phase speed is $c = U - \beta/(k^2 + l^2) = U + c_i$. Because the intrinsic phase speed c_i is strictly westward, it opposes eastward flow but aligns with westward flow.

The propagation characteristics are governed by the refractive index l^2 . Wave propagation is possible only where $l^2 > 0$, corresponding to the range $0 < U - c < \beta/k^2$. As illustrated in Fig. 5.2, two limits define this range: the critical layer ($U - c = 0$), where $l^2 \rightarrow \infty$ and waves likely break and nonlinear or dissipative effects dominate (Vallis, 2017), and the turning point ($U - c = \beta/k^2$), where $l^2 \rightarrow 0$ and the wave becomes purely zonal. Fig. 5.2b illustrates the asymmetric Rossby wave characteristics:

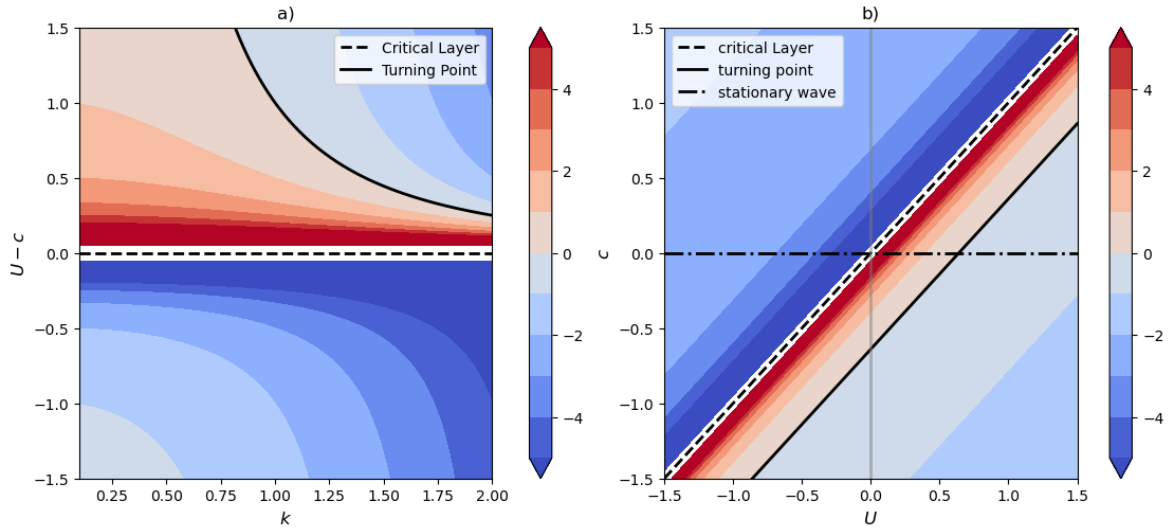


Figure 5.2: Dispersion relation expressed as the refractive index $l^2 = \beta/(U - c) - k^2$ as a function of (a) k and intrinsic phase speed $U - c$, and (b) U and c for $k = 1.25$. The dashed line indicates the critical layer ($U = c$), the solid line denotes the turning point ($l^2 = 0$), the dashed-dotted line indicates stationary waves ($c = 0$) and the gray line separates westward from eastward mean flow ($U = 0$).

In westward flow, Rossby waves are confined to westward phase speeds ($c < 0$ for $U < 0$) and encounter critical layers if their intrinsic propagation is slower than the mean flow. No stationary waves ($c = 0$) are possible. In eastward flow, however, Rossby waves can propagate both westward and eastward (relative to the ground), provided they maintain westward propagation relative to the mean flow. Critically, this allows for stationary waves ($c = 0$) where the intrinsic westward propagation exactly balances the eastward advection. For a purely zonal wave ($l = 0$), this occurs at the Rhines scale $k = \sqrt{\beta/U}$. This asymmetry suggests that while waves in both regimes may encounter critical layers, their dynamical impact should differ. We speculate that in westward flow critical layer effects lead to stronger Rossby wave breaking than in eastward flow, thereby providing a sink for kinetic Rossby wave energy, that is reduced in eastward flow. Instead, eastward flows favor negative baroclinic conversion, potentially driven by stationary wave effects at the Rhines scale, where negative baroclinic conversion is observed.

To evaluate the first part of this hypothesis and whether Rossby wave breaking is stronger in westward than in eastward flow, the Eliassen-Palm flux divergence is used as a diagnostic for Rossby wave breaking and calculated in the idealized models CHANNEL_45 and CHANNEL_45_R (see chapter 2) with eastward and westward mean flow respectively (not shown). The Eliassen-Palm flux divergence is indeed found to be negative in both models, but stronger for westward mean flow, signifying more intense wave breaking in that regime. The dynamical difference between Rossby wave propagation in eastward versus westward flow is further indicated by the time evolution of potential vorticity anomalies (not shown), which propagate rapidly

in westward flow, but in eastward flow are nearly stationary or propagate slowly eastward, providing empirical evidence for the predicted stationary Rossby waves.

The theoretically distinct propagation characteristics and mean flow interactions in eastward versus westward flow are thus apparent in both the Eliassen-Palm flux and PV anomaly evolution. This provides the first evidence for the speculated mechanism driving negative baroclinic conversion outlined above: intensified critical layer breaking in westward flow serves as a kinetic energy sink that is largely absent in the eastward flow, where instead waves become stationary and facilitate the conversion of kinetic energy back to available potential energy. However, the global significance of the difference in wave breaking strength between eastward and westward mean flow and the precise connection between stationary Rossby waves in eastward mean flow remain to be clarified and presents an opportunity for future research. The former can be addressed by a thorough global analysis of Rossby wave breaking using for example ray tracing techniques and the latter can be addressed by decomposing baroclinic conversion in frequency-wavenumber space to isolate the contribution of stationary waves with zero frequency. Alternatively, the impact of stationary waves could be tested in a pseudo-spectral QG model by applying selective damping to stationary waves. If the large-scale negative baroclinic conversion disappears under such a filter, its origin in stationary wave effects would be confirmed. Finally, alternative routes to dissipation for the large-scale barotropic kinetic energy that accumulates in the absence of this sink remain to be identified.

In conclusion, the integrated modal-spectral perspective developed in this thesis describes the conversion and transfer mechanisms of meso-scale eddy energy in unprecedented detail. By providing a scale-aware quantification of the energy cycle, this work represents a significant step forward in understanding the oceanic circulation.

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