

Computational Study of Skyrmions in Magnetic Materials

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Abstract

Exploiting the potential of magnetic skyrmions for next-generation spintronic devices requires a detailed understanding of their excitation dynamics and stability in engineered magnetic systems. This thesis addresses these aspects through two computational studies. The first investigates the excitation dynamics of Néel-type skyrmions in confined synthetic antiferromagnets (SAFs), from a single skyrmion to finite skyrmion chains. The second examines the equilibrium stability and internal magnetic structure of skyrmions in ferrimagnetic DyCo_3 thin films.

In the SAF systems, ringdown and eigenvalue methods are used to determine how antiferromagnetic interlayer coupling and lateral confinement modify the skyrmion excitation spectrum. For a single SAF skyrmion in square confinement, the coupling between the two magnetic layers gives rise to a pair of gyrotropic modes and in-phase and out-of-phase breathing modes. Changing the geometry from square to rectangular confinement makes the low-frequency response predominantly translational. The investigation is then extended to chains containing two, five, and 25 skyrmions. Interactions between neighbouring skyrmions transform the isolated translational and breathing resonances into collective modes with different spatial and phase relationships. As the chain length increases, these modes become more closely spaced in frequency and form bands. Local excitation of the breathing modes in the 25-skyrmion chain produces a signal that propagates along the chain.

The DyCo_3 study investigates the equilibrium configurations and internal structure of skyrmion-like magnetic textures. Depth-resolved simulations shows two distinct bubble states can persist in the absence of Dzyaloshinskii–Moriya interaction (DMI): a continuously wound type 1 bubble and a type 2 bubble containing Bloch lines. Fitting an analytical skyrmion profile to individual layers reveals that the helicity changes through the film thickness, from Néel-like configuration near the surfaces to a more Bloch-like configuration in the interior. The inclusion of interfacial DMI shifts the internal structure towards a Néel-like configuration. A simplified model is subsequently used to examine

how the relaxed magnetic state depends on the DMI constant and effective anisotropy. Autoencoder-based classification separates the resulting states into in-plane, multidomain, and isolated-skyrmion configurations, with the isolated skyrmions further distinguished by their helicity. Comparison with experimentally observed skyrmion radii identifies the region of parameter space consistent with the experimentally observed structures.

These findings show how interlayer coupling, geometrical confinement, and interactions between neighbouring skyrmions determine the excitation modes of SAF skyrmions. They also provide a micromagnetic interpretation of the stability and internal structure of skyrmions observed in ferrimagnetic DyCo_3 thin films.

Zusammenfassung

Um das Potenzial magnetischer Skyrmionen für Spintronik-Bauelemente der nächsten Generation ausschöpfen zu können, ist ein detailliertes Verständnis ihrer Anregungsdynamik und Stabilität in künstlich hergestellten magnetischen Systemen erforderlich. Diese Dissertation behandelt diese Fragestellungen anhand zweier numerischer Studien. Im ersten Teil wird die Dynamik von Néel-Skyrmionen in mikroskopisch kleinen synthetischen Antiferromagneten (SAFs) untersucht, beginnend mit einem einzelnen Skyrmion bis hin zu endlich langen Ketten von Skyrmion. Im zweiten Teil werden die Stabilität und die innere magnetische Struktur von relaxierten Skyrmionen in ferrimagnetischen DyCo_3 -Dünnschichten analysiert.

Für die SAF-Systeme werden die Ringdown- und die Eigenwertmethode verwendet, um zu bestimmen, wie antiferromagnetische Kopplung der beiden Schichten und lateral begrenzte Probengröße das Anregungsspektrum der Skyrmionen verändern. Für ein einzelnes SAF-Skyrmion in einer quadratischen Probe ergeben sich durch die Kopplung der beiden magnetischen Schichten ein Paar gyrotropischer Moden sowie in-phase und out-of-phase breathing Moden. Der Übergang von einer quadratischen zu einer rechteckigen Geometrie bewirkt, dass die niederfrequente Anregung überwiegend translationalen Charakter annimmt. Die Untersuchung wird dann auf Ketten mit zwei, fünf und 25 Skyrmionen erweitert. Wechselwirkungen zwischen benachbarten Skyrmionen überführen die isolierten Translations- und breathing Moden in kollektive Anregungen mit unterschiedlichen räumlichen Verteilungen und Phasenbeziehungen. Zunehmende Kettenlänge führt zu verringerten Abständen der Moden im Frequenzraum und damit zur Bildung von Bändern. Eine lokale Anregung der breathing Moden in der 25-Skyrmionen-Kette erzeugt ein Signal, das sich entlang der Kette ausbreitet.

Die DyCo_3 Studie konzentriert sich auf mögliche Konfigurationen und innere Struktur skyrmionenartiger magnetischer Texturen im Gleichgewicht. Tiefenaufgelöste Simulationen zeigen, dass in Abwesenheit der Dzyaloshinskii-Moriya-Wechselwirkung (DMI) zwei unterschiedliche bubble Zustände existieren können: eine kontinuierlich gewundene Typ 1

bubble und eine Typ 2 bubble mit Blochlinien. Der Fit eines analytischen Skymionprofils für die einzelnen Schichten zeigt, dass sich die Helizität über die Filmdicke verändert, von einer Néel-artigen Konfiguration nahe der Oberflächen zu einer stärker Bloch-artigen Konfiguration im Inneren. Die Einbeziehung von interfacial DMI verschiebt die innere Struktur in Richtung einer Néel-artigen Konfiguration. Anschließend wird ein vereinfachtes Modell verwendet, um zu untersuchen, wie der relaxierte magnetische Zustand von der DMI-Konstante und der effektiven Anisotropie abhängt. Eine autoencoder-basierte Klassifikation gruppiert die resultierenden Zustände in in-plane, multidomain und isolated-skyrmion Konfigurationen, wobei die isolierten Skymionen sich zusätzlich in ihrer Helizität unterscheiden. Der Vergleich mit experimentell beobachteten Skymionradien identifiziert den Bereich des Parameterraums, der mit den experimentell beobachteten Strukturen konsistent ist.

Diese Ergebnisse zeigen, wie die Kopplung zwischen magnetischen Schichten, geometrische Begrenzung und Wechselwirkungen zwischen benachbarten Skymionen die Anregungsmoden von SAF-Skymionen bestimmen. Außerdem liefern sie eine mikromagnetische Interpretation der Stabilität und der inneren Struktur von Skymionen, die in ferromagnetischen DyCo_3 -Dünnschichten beobachtet wurden.

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Nomenclature

α	Gilbert damping parameter
δ_0	Bloch wall parameter
ℓ_{ex}	exchange length
γ	gyromagnetic ratio
λ	trajectory visualisation scaling factor
$\mathbf{H}_{\text{eff}}$	effective magnetic field
$\mathbf{M}$	magnetisation
$\mathbf{m}$	unit magnetisation vector field
σ	RKKY surface exchange coefficient
A	exchange energy constant
B	applied flux density
D	Dzyaloshinskii–Moriya interaction constant
f	frequency
J	exchange integral
K_{eff}	effective anisotropy constant
K_{u}	uniaxial anisotropy constant
L_{d}	helical period
M_{s}	saturation magnetisation
Q	topological charge
V	volume

w	energy density
AFM	antiferromagnetic
CCW	counter clockwise
CMOS	complementary metal-oxide-semiconductor
CW	clockwise
DFT	discrete Fourier transform
DMI	Dzyaloshinskii–Moriya Interaction
FM	ferromagnetic
GPU	graphics processing unit
LLG	Landau–Lifshitz–Gilbert equation
LTEM	Lorentz transmission electron microscopy
MFM	magnetic force micorscopy
OOMMF	Object Oriented MicroMagnetic Framework
PSD	power spectral density
RE-TM	Rare-earth transition-metal
RKKY	Ruderman–Kittel–Kasuya–Yosida interaction
SAF	Synthetic antiferromagnet
SAXRMS	small-angle X-ray resonant magnetic scattering
STXM	scanning transmission X-ray microscopy
UMM	unified micromagnetic modelling
XMCD	X-ray magnetic circular dichroism

Chapter 1

Introduction

Modern society relies increasingly on digital technologies, from smartphones and embedded sensors to large-scale cloud data centres, which are now integral to daily life and industrial operations. This widespread digital integration has led to a dramatic rise in computational workloads and in the energy required to sustain them, with a growing share of the energy budget of a modern chip spent not on computation itself but on maintaining volatile memory and moving data between memory and logic [1]. To continue meeting the demand for data storage, processing speed, and energy efficiency, scientific efforts have increasingly turned to the nanoscale [2, 3].

As electronic components continue to shrink, attention has shifted from conventional silicon-based complementary metal-oxide-semiconductor (CMOS) technology [4] towards magnetic nanostructures, which are central to emerging paradigms of information processing that aim to combine high speed with low power consumption [5]. Among the most promising approaches is spintronics, which exploits the intrinsic spin of electrons in addition to their charge [6–8]. Spintronics has a proven technological track record: the discovery of giant magnetoresistance, recognised by the 2007 Nobel Prize in Physics, transformed hard-disk read heads within a decade of its discovery [9, 10], and magnetoresistive random-access memories are now in commercial use [1]. A magnetic state, unlike an electric charge, is retained without power, making magnetic nanostructures natural candidates for non-volatile, energy-efficient information technologies [11].

Engineered magnetic systems, such as thin films, multilayers, and nanodots, can host different non-uniform magnetic states, including domain walls [12, 13], magnetic vortices [14], and magnetic skyrmions [15–17]. A skyrmion is a localised spin structure, illustrated in Figure 1.1, in which the magnetisation changes smoothly from one direction at the centre to the opposite direction in the surrounding region. Because of this continu-

ous winding, a skyrmion has a non-zero topological charge and cannot be transformed into a uniform magnetic state without changing the topology of the magnetisation field [18].

In magnetic thin films, skyrmions are stabilised by the competition between several energy contributions, including exchange, the Dzyaloshinskii–Moriya interaction (DMI) [19, 20], magnetic anisotropy, magnetostatic energy, and applied magnetic fields [21]. Their small size and stability make them interesting for future spintronic applications, especially where magnetic information needs to be stored or moved in confined geometries [22–24]. They have also been discussed in the context of logic [25, 26] and neuromorphic computing [27].

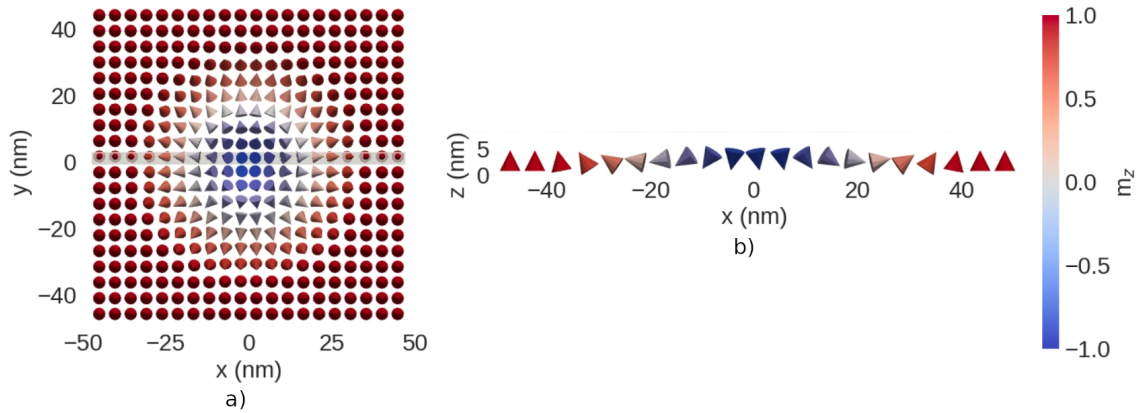


Figure 1.1: Visualisation of a magnetic skyrmion in a two-dimensional ferromagnetic lattice with an approximate radius of 40 nm. **(a)** Top view showing the full spin texture, where arrows represent the magnetisation direction and colours denote the out-of-plane component (m_z), ranging from red (positive) to blue (negative). The skyrmion exhibits a Néel-type configuration, in which the spins rotate radially from the centre to the edge. **(b)** Side view along the xz plane, revealing the variation of the magnetisation through the skyrmion core and confirming the preservation of its Néel character.

To enable the development of practical skyrmion-based devices, a detailed understanding of their behaviour under realistic operating conditions is essential. Investigating the dynamical response of skyrmions to external stimuli reveals their eigenfrequencies, internal excitation modes, and interaction mechanisms, which together determine how reliably skyrmions can be controlled, detected, and read out. Characteristic excitation modes such as breathing [28, 29] and gyration [30, 31] have been proposed as information carriers for data-processing applications [32–34], connecting skyrmion dynamics to the broader programme of magnonics and wave-based computing [35].

While these prospects are promising, the practical integration of skyrmions into device

architectures is hindered by limitations inherent to conventional ferromagnetic systems. A prominent challenge is the skyrmion Hall effect, a deflection of a moving skyrmion transverse to the driving current, which is a direct consequence of its topological charge and complicates reliable control [36, 37]. Synthetic antiferromagnets (SAFs), composed of antiferromagnetically coupled ferromagnetic layers, have emerged as promising candidates for suppressing this effect: a skyrmion in one layer is bound to a partner of opposite topological charge in the other, so that the net topological charge of the pair vanishes [38, 39]. Room-temperature SAF skyrmions have been stabilised experimentally [39], and current-driven SAF skyrmions have recently reached velocities well beyond those of their ferromagnetic counterparts [40], underlining the practical potential of this platform.

The dynamics of individual skyrmions in SAF structures have accordingly become a central research theme, from the characterisation of intrinsic modes in confined geometries [41–44] to manipulation and transport in SAF racetracks [45–47]. Future spintronic devices, however, are expected to employ interacting skyrmion ensembles arranged in structured geometries [48]. This raises a key question: when multiple skyrmions are arranged in a quasi-one-dimensional SAF nanostrip, how do their excitation modes interact and hybridise? The first part of this thesis addresses this question, examining how the interplay between interlayer coupling across the SAF layers and inter-skyrmion interactions along the chain shapes the spectrum of collective modes, and how a local excitation can propagate along a finite skyrmion chain as a travelling signal.

The second part of this thesis turns to ferrimagnets, in which two antiparallel magnetic sublattices partially compensate each other, combining antiferromagnet-like dynamics with the ferromagnet-like ability to read the magnetic state electrically [49]. Ferrimagnetic films with strong perpendicular anisotropy have been shown to host compact skyrmions: in the amorphous rare-earth transition-metal alloy DyCo_3 , isolated skyrmions with antiparallel Dy and Co moments have been observed directly using X-ray microscopy and Hall effect measurements [50], with subsequent imaging identifying their Néel-type character [51]. The origin and strength of the DMI in such alloys remain active research topics [52–55], and the experimental images alone do not reveal which interactions stabilise the observed textures, or what internal structure they hide through the thickness of the film. Motivated by these observations, this thesis develops a depth-resolved micromagnetic model of the DyCo_3 film and asks which bubble states the film supports, how their helicity and topology are organised through the film thickness, what role the DMI actually plays in their stability, and which material parameters are consistent with the experimentally observed skyrmion sizes.

Together, the two strands form a computational study of how confinement, coupling,

collective behaviour, and material-specific interactions shape magnetic textures and their excitations, carried out within a single micromagnetic framework. The remainder of this introduction outlines the computational approach, the research questions and main contributions, and the structure of the thesis.

1.1 Computational approach and scope

The work presented in this thesis is based on continuum micromagnetics, in which the magnetisation is treated as a continuous vector field and its time evolution is governed by the Landau–Lifshitz–Gilbert (LLG) equation [56]. The full theoretical framework, including the relevant energy terms and the numerical implementation, is presented in Chapter 2. At the level of the introduction, it is sufficient to note that micromagnetics provides the natural framework for studying both equilibrium spin textures and their small-amplitude dynamics in confined nanostructures.

Two different methods are used to analyse skyrmion dynamics in Chapters 4 and 5. The first is the ringdown method. In this approach, the relaxed magnetic configuration is perturbed by a broadband excitation pulse, the subsequent time-dependent magnetisation is recorded, and the resonant response is extracted using Fourier analysis and the power spectral density (PSD) [57]. This approach is straightforward and physically transparent, since it closely mirrors the way resonances are identified from time-domain dynamics.

The second method is the eigenvalue method, in which the LLG equation is linearised around the relaxed equilibrium configuration and the resulting normal-mode problem is solved directly [58, 59]. In this way, the resonance frequencies and complex mode profiles can be obtained without performing a long time-domain simulation. In the present work, this analysis is carried out using the eigensolver implementation available in `magnum.np` [60]. The eigenvalue method is especially useful when individual modes need to be identified and interpreted clearly, while the ringdown method remains important for comparing against directly excited dynamics and for assessing how a given excitation couples to the available modes.

The ferrimagnetic DyCo_3 study of Chapter 6 concerns the equilibrium textures of the film and is therefore built around energy minimisation: magnetic states are relaxed to local minima of the micromagnetic energy, and their internal structure is quantified by fitting an analytical skyrmion profile that yields the radius, domain-wall width, and helicity layer by layer through the film thickness. Large sweeps of the material parameters are then performed with a reduced model, and because such sweeps produce far more relaxed states than can be inspected manually, the resulting textures are organised using

an unsupervised, autoencoder-based classification, whose output is interpreted physically and assembled into material-parameter phase diagrams.

The simulations in this thesis were carried out using Ubermag [61–63] and `magnum.np` [60]. Both tools are used from Python and support a scriptable workflow for setting up micromagnetic simulations, running calculations, and analysing the magnetisation dynamics.

1.2 Research questions and main contributions

This thesis addresses three closely related research questions. First, how does interlayer antiferromagnetic coupling modify the eigenmodes of an isolated Néel-type skyrmion in a confined synthetic antiferromagnetic geometry, and how does the shape of the host structure affect the modes that appear? Second, how do these isolated-skyrmion modes change when several SAF skyrmions are arranged in a finite chain and begin to interact with each other? Third, how can experimentally observed ferrimagnetic skyrmion-like textures in DyCo_3 thin films be understood in terms of micromagnetic energetics, DMI, and depth-dependent internal magnetic structure?

For the isolated-skyrmion problem, the ferromagnetic system is first used to establish the relation between the skyrmion core polarity and the preferred direction of gyration. In the square SAF geometry, the interlayer RKKY coupling gives rise to two nearly degenerate gyrotropic modes at low frequencies and two in-phase and out-of-phase breathing modes at higher frequencies. In the lower-frequency gyrotropic mode, the skyrmions in both layers rotate clockwise, whereas in the higher-frequency mode they rotate counter-clockwise. Since the two skyrmions have opposite core polarities and therefore prefer opposite directions of gyration, their coupled motion is frustrated. When the square symmetry is broken by rectangular confinement, the two gyrotropic modes become translational modes. The lower-frequency mode is aligned with the longer axis of the sample, while the higher-frequency mode is aligned with the shorter axis.

The second main contribution concerns the emergence of collective modes in finite SAF skyrmion chains. Starting from a two-skyrmion strip and extending to chains of five and 25 skyrmions, the thesis shows how the isolated translational and breathing resonances evolve into collective excitations with different left–right and top–bottom phase relations. As the number of skyrmions increases, the individual modes remain identifiable but become more closely spaced in frequency and form band-like groups. Their spatial profiles also show standing-wave-like behaviour along the chain. For the longest system studied, a local excitation of the breathing band produces a propagating signal along the skyrmion chain,

linking the frequency-domain eigenmodes directly to time-domain information transport.

The ferrimagnetic part of the thesis addresses skyrmion stability in DyCo₃ thin films, motivated by the experimental observations of Ref. [50]. The central questions here concern the equilibrium state itself: which state does the film support, what internal structure they carry, and which effective material parameters reproduce the experimental observations. Depth-resolved simulations show that even without DMI the film supports two bubble states that are nearly indistinguishable in their out-of-plane profiles yet topologically distinct: a continuously wound type 1 bubble and a type 2 bubble whose domain wall is segmented by Bloch lines. For the type 1 state, fitting an analytical skyrmion profile layer by layer reveals a helicity that rotates continuously through the film thickness, driven by the demagnetising field alone, and interfacial DMI is shown to bias this internal structure towards a Néel-like configuration rather than being required for stability itself. A reduced model is then used to scan the (K_{eff}, D) parameter space. The relaxed states are organised into physically interpretable classes using an unsupervised, autoencoder-based classification, which is used to construct the phase diagram of texture type. The skyrmion radius, domain-wall width, and helicity are obtained separately by fitting an analytical skyrmion profile to the simulated states. Comparing the fitted radii with the experimentally observed skyrmion sizes constrains the plausible material parameters of the DyCo₃ film, while simulated experimental contrast indicates how the remaining ambiguity in bubble type could be resolved using in-plane-sensitive imaging. This connects the dynamical SAF part of the thesis to a broader question in computational magnetism: how reliably can experimentally observed magnetic textures be interpreted and constrained using micromagnetic modelling?

Taken together, the results show how interlayer antiferromagnetic coupling and the geometry of the system affect the excitation modes of an isolated SAF skyrmion, and how these modes develop into collective excitations when several skyrmions are arranged in a finite chain. The results further show how the micromagnetic interactions and effective material parameters influence the stability and internal magnetic structure of skyrmion-like textures in DyCo₃ thin films.

1.3 Thesis structure

The remainder of the thesis is organised as follows. Chapter 2 introduces the micromagnetic framework used throughout the thesis. It presents the relevant energy contributions, the Landau–Lifshitz–Gilbert equation, the basic magnetic structures considered in this work, and the two main methods used for the analysis of resonant dynamics, namely the

ringdown and eigenvalue approaches.

Chapter 3 documents the contributions made to the Ubermag scientific software ecosystem during this PhD, including improvements to the simulation metadata, data-access workflows, documentation work and analysis tooling. These contributions supported the reproducibility of the computational results presented in the following chapters.

Chapter 4 investigates the eigenmodes of an isolated skyrmion. The chapter begins with a ferromagnetic reference case and then moves to isolated skyrmions in synthetic antiferromagnetic geometries. Particular emphasis is placed on the effect of interlayer coupling and lateral confinement on the gyrotropic, breathing, and translational responses.

Chapter 5 extends the analysis from the isolated SAF skyrmion to finite skyrmion chains containing two, five, and 25 skyrmions. The central theme of this chapter is the evolution from isolated resonances to collective excitations. It further examines signal propagation through an extended SAF skyrmion chain and concludes with a discussion of the main physical trends and possible future directions.

Chapter 6 turns to ferrimagnetic skyrmions in DyCo_3 thin films. Motivated by the experimentally observed compact skyrmions in this material, the chapter develops the corresponding micromagnetic model and resolves the depth-dependent internal structure and topology of the stabilised bubble states. A reduced model is then used to study how DMI and the effective material parameters affect the stability, size, and helicity of the skyrmions. An unsupervised classification of the simulated textures is used to construct the phase diagram of texture type, while the skyrmion radius, domain-wall width, and helicity are determined by fitting an analytical skyrmion profile and compared with experiment. The accompanying Appendix B.1 examines which experimental measurements could distinguish the topologically distinct bubble states identified in the chapter.

Finally, Chapter 7 summarises the main findings of the thesis and brings together the two strands of the work. It highlights the common physical themes connecting isolated and collective skyrmion dynamics with ferrimagnetic texture stability, and outlines several directions in which the present work can be extended.

Chapter 2

Micromagnetics

The origin of magnetism is fundamentally quantum mechanical, rooted in the properties of electrons. Each electron possesses, intrinsic spin and orbital angular momentum¹. These quantised angular momenta intrinsically generate associated magnetic dipole moments. The net magnetic moment of an atom is the vector sum of these individual electron magnetic moments, governed by the Pauli exclusion principle [64] and Hund's rules [65] which dictate electron configurations and spin alignments within atoms.

In solids, the magnetic behaviour of a material depends on how these atomic magnetic moments interact. The dominant mechanism responsible for magnetic order is the exchange interaction (discussed in section 2.1.1.1), a quantum mechanical effect arising from the Pauli exclusion principle and Coulomb repulsion, which favours parallel or antiparallel alignment of neighbouring spins [66]. The collective behaviour of magnetic moments gives rise to different types of magnetic ordering in materials and are shown in Fig. 2.1:

1. Ferromagnetism, where magnetic moments align parallel resulting in spontaneous magnetisation.
2. Antiferromagnetism, where neighbouring moments align antiparallel, resulting in zero net magnetisation.
3. Ferrimagnetism, where unequal antiparallel moments yield a non-zero net magnetisation.

In addition to these, diamagnetism and paramagnetism describe the fundamental responses of electrons to an external magnetic field. Diamagnetism arises from the induced currents generated by changes in the orbital motion of bound electrons when exposed to

¹Arising from its quantum mechanical state and spatial distribution within the atom.



Figure 2.1: Magnetic ordering of magnetic moments in a 1D lattice. **(a)** Parallel arrangement giving rise to ferromagnetic materials. **(b)** Antiparallel arrangement which leads to antiferromagnetic materials. **(c)** Antiparallel with unequal magnitude which makes ferrimagnetic materials.

a magnetic field, which according to Lenz’s law, create magnetic moments that oppose the applied field. This response is universal and temperature-independent, though typically very weak. In contrast, paramagnetism occurs in materials with unpaired electrons that carry permanent magnetic moments. In the absence of a field, thermal agitation causes these moments to orient randomly, producing no net magnetisation. When a field is applied, the moments tend to align along the field direction, resulting in a net positive magnetisation [66–68].

The emergent macroscopic magnetic quantity that arises from the collective behaviour of atomic magnetic moments is known as magnetisation $\mathbf{M}$. It is defined as the magnetic moment per unit volume:

$$\mathbf{M} = \frac{1}{V} \sum_{i \in V} \mu_i$$

where μ_i are the magnetic moments within a representative volume V .

2.1 Micromagnetic model

Micromagnetism is a phenomenological theory based on the continuum representation of the magnetisation. This model is based on the assumption that the internal forces within the magnetic material are strong enough to align the magnetisation over a characteristic length scale, which is much larger than the lattice constant [56]. Micromagnetism addresses samples with length scales on the order of tens of nanometers to microns. In micromagnetics, the magnetisation is treated as a continuous vector field that varies smoothly with space and time. It is defined as:

$$\mathbf{M} = \mathbf{M}(\mathbf{r}, t)$$

where $\mathbf{r}$ denotes the spatial coordinate and t represents time. For simplicity, this time dependence is often omitted unless required explicitly. Micromagnetic theory typically makes the following two assumptions:

1. **Continuity:** The magnetisation field $\mathbf{M}(\mathbf{r}, t)$ is continuous and differentiable throughout the material. Discontinuities are only allowed at interfaces between distinct magnetic materials.
2. **Constant magnitude:** The magnitude of the magnetisation remains constant at all points in space and time:

$$|\mathbf{M}(\mathbf{r}, t)| = M_s$$

where M_s is the *saturation magnetisation*, a material-specific constant. The assumption of constant M_s means that thermal effects are not captured in this model. Consequently, the magnetisation can be written in terms of a unit vector field:

$$\mathbf{M}(\mathbf{r}, t) = M_s \mathbf{m}(\mathbf{r}, t), \quad (2.1)$$

where $\mathbf{m}(\mathbf{r}, t)$ is a unit vector satisfying $|\mathbf{m}| = 1$, and it describes the local direction of the magnetisation.

2.1.1 Micromagnetic energy contributions

Consider a ball placed at the rim of a curved bowl. Once released, the ball will tend to roll to the bottom, seeking a position which minimises its potential energy. The ball may oscillate for a while due to energy conservation, and gradually it rests at the bottom. This occurs because the system moves in a direction that minimises potential energy.

Similarly, magnetisation in a magnetic material develops under the effects of different energy terms. Just as the ball aims to fall to the bottom of the bowl, so too does the magnetisation rotate so that it can align in an arrangement of lowest local energy. When such equilibrium is established, the magnetic state is stable until subjected to an external perturbation.

Micromagnetics is a semi classical theory with quantum mechanical energy contributions like exchange, Dzyaloshinskii-Moriya interaction, anisotropy but the magnetisation field and its evolution with respect to time is treated classically over a continuum [69]. In the micromagnetic model, most of the energy terms are a function of local direction of magnetisation, which may vary from point to point in the material. In order to consider the overall system energy, these local energy densities are integrated over the volume of interest and summed. This yields the total micromagnetic energy, given by:

$$E[\mathbf{m}] = \int_V \sum w(\mathbf{m}) d^3r \quad (2.2)$$

Where V is the volume of the system, and summation is over all the energy terms contributing to the system. The micromagnetic energy comprises several distinct contributions, each reflecting different physical interactions:

- Isotropic exchange interaction energy density, w_a , which tends to align neighbouring magnetic moments either parallel or antiparallel.
- Dzyaloshinskii–Moriya interaction energy density, w_{dmi} , an antisymmetric exchange that favours chiral spin textures.
- Zeeman energy density, w_z , accounting for the influence of externally applied magnetic fields.
- Magnetocrystalline anisotropy energy density, w_K , which reflects the preference of magnetisation to align along specific crystallographic directions.
- Demagnetisation energy density, w_{demag} , arising from magnetostatic interactions within the material.
- Interlayer exchange coupling energy density, w_{RKKY} , relevant in multilayer structures, representing the coupling between magnetic layers mediated by conduction electrons across nonmagnetic spacers.

Together, these energy contributions govern the magnetic response and equilibrium configuration of the system.

$$E[\mathbf{m}] = \int_V [w_a(\mathbf{m}) + w_{\text{dmi}}(\mathbf{m}) + w_z(\mathbf{m}) + w_K(\mathbf{m}) + w_{\text{demag}}(\mathbf{m}) + w_{\text{RKKY}}(\mathbf{m})] d^3r \quad (2.3)$$

In addition to the energy contributions discussed above, other terms may also play significant roles in specific systems. A more comprehensive overview can be found in Refs. [56, 69–71].

2.1.1.1 Exchange energy

One of the most important contribution for ferromagnets is the exchange energy, and it arises due to the quantum mechanical interaction of neighboring atomic spins. The exchange interaction favours parallel and antiparallel spin alignment and is responsible for ferromagnetism and antiferromagnetism in many materials [72].

In magnetic materials, the exchange interaction arises from a combination of the Pauli exclusion principle [64] and Coulomb repulsion. The net effect of these quantum mechanical effect on the interaction energy between neighbouring atomic spins $\mathbf{S}_i$ and $\mathbf{S}_j$ is often described by the Heisenberg model [73] as:

$$w_a = -J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j \quad (2.4)$$

Here, $J_{i,j}$ is the exchange integral between spins, and its sign determines the preferred spin alignment:

- When $J_{i,j} > 0$, the system lowers its energy when neighbouring spins are parallel. This leads to ferromagnetic ordering as shown in Fig. 2.1(a).
- When $J_{i,j} < 0$, the system prefers antiparallel spin alignment. This results in anti-ferromagnetism, as shown in Fig. 2.1(b).

In micromagnetics, which operates at the continuum limit, this quantum mechanical exchange interaction is often approximated by an energy density term. For ferromagnetic materials (where $J_{i,j} > 0$), this term reflects that configurations where neighbouring magnetic moments point in the same direction have lower energy than those that are misaligned. The energy density is given by [68, 72]:

$$w_a(\mathbf{m}) = A(\nabla \mathbf{m})^2 \quad (2.5)$$

where A is the exchange stiffness constant, which depends on the material and quantifies the strength of the interaction, $\nabla \mathbf{m}$ is the gradient of the magnetisation vector. This energy is minimised when $(\nabla \mathbf{m})^2$ is minimised, which is when the gradient of the magnetic moment field is zero. The derivation from Eq. (2.4) to Eq. (2.5) can be found in Ref. [71].

2.1.1.2 Dzyaloshinskii-Moriya interaction energy

Dzyaloshinskii-Moriya Interaction (DMI) energy is a type of super-exchange interaction that arises in systems where inversion symmetry is broken. It is also referred as anti-symmetric exchange energy. Unlike exchange interaction, it promotes a twisted or chiral alignment between neighbouring spins. The concept was initially proposed by Dzyaloshinskii [19], and was later explained by Moriya [20, 74], who linked the effect to spin-orbit coupling. Mathematical expressions describing the DMI can be derived by expanding the magnetic free energy based on the symmetries inherent to the crystal structure. In the continuum approximation, this expansion yields energy contributions that are linear in

the first spatial derivatives of the magnetisation vector. These contributions are referred to as Lifshitz invariants. Bogdanov and Yablonskii [16, 75] initially applied Lifshitz invariants to model the formation of vortex-like magnetic structures in crystals with specific crystallographic symmetries. These invariants were later recognised as a manifestation of DMI [76, 77]. Lifshitz invariants are defined as:

$$\mathcal{L}_{ij}^{(k)} = m_i \frac{\partial m_j}{\partial x_k} - m_j \frac{\partial m_i}{\partial x_k} \quad (2.6)$$

where $i, j, k \in \{x, y, z\}$. In the continuum limit, expressions for DMI can be derived and presented using the notation of Lifshitz invariants. In noncentrosymmetric crystal² structures [19, 20, 74], the corresponding DMI terms are mostly referred as bulk type DMI³ and the energy density is given by:

$$\begin{aligned} w_{\text{dmi}}(\mathbf{m}) &= D\mathbf{m} \cdot (\nabla \times \mathbf{m}) \\ &= \mathcal{L}_{yz}^{(x)} + \mathcal{L}_{xz}^{(y)} + \mathcal{L}_{xy}^{(z)} \end{aligned} \quad (2.7)$$

DMI can also occur at interfaces between different materials, where inversion symmetry is broken along a particular axis [78]. This is mostly referred as interfacial DMI⁴. Depending on the direction along which the symmetry is broken, different forms of the interfacial DMI energy density arise. When inversion symmetry is broken along the z axis, the interfacial DMI energy density is:

$$\begin{aligned} w_{\text{dmi}}^z(\mathbf{m}) &= D[m_z \nabla \cdot \mathbf{m} - (\mathbf{m} \cdot \nabla)m_z] \\ &= \mathcal{L}_{xz}^{(x)} + \mathcal{L}_{yz}^{(y)} \end{aligned} \quad (2.8)$$

Likewise, if symmetry is broken along the x axis, the corresponding energy density becomes:

$$\begin{aligned} w_{\text{dmi}}^x(\mathbf{m}) &= D[m_x \nabla \cdot \mathbf{m} - (\mathbf{m} \cdot \nabla)m_x] \\ &= \mathcal{L}_{yx}^{(y)} + \mathcal{L}_{zx}^{(z)} \end{aligned} \quad (2.9)$$

And for symmetry breaking along the y axis, the form is:

$$\begin{aligned} w_{\text{dmi}}^y(\mathbf{m}) &= D[m_y \nabla \cdot \mathbf{m} - (\mathbf{m} \cdot \nabla)m_y] \\ &= \mathcal{L}_{zx}^{(z)} + \mathcal{L}_{xy}^{(x)} \end{aligned} \quad (2.10)$$

²Such crystals lack inversion symmetry, its atomic structure is not invariant under a spatial inversion through a central point.

³Only T and O point groups are referred as bulk type

⁴In this report, we consider interfacial DMI arising from inversion symmetry breaking along the z axis, and employ Eq. (2.8) in all simulations.

The constant D defines how strong the twisting tendency is compared to other energy contributions. The exchange and DMI act in competition: the former favours parallel alignment of neighbouring magnetic moments, while the latter promotes a chiral twist in their orientation. The ratio of their strength ($|A/D|$) controls how rapidly the twist will occur [76, 79], this relationship with A is often used to determine the value of D as:

$$D = \frac{4\pi A}{L_d} \quad (2.11)$$

Here, L_d , the helical magnetic order period, represents the distance over which the magnetic moments complete a full rotation, in the presence of only exchange and DMI interactions.

2.1.1.3 Zeeman energy

When a magnetic field is applied to a material, it influences the magnetic moments within the material, tending to align them along the direction of the applied field. This effect is described by the Zeeman energy [69], and the density is given by:

$$w_z(\mathbf{m}) = -\mu_0 M_s \mathbf{m} \cdot \mathbf{H} \quad (2.12)$$

where μ_0 is the permeability of free space, M_s is the saturation magnetisation, $\mathbf{m}$ is the unit magnetisation vector, and $\mathbf{H}$ is the applied magnetic field. This energy is lowest if the magnetisation is aligned with the external field. In strong fields, the Zeeman energy becomes the dominant term, overriding other competing energies to align the system's magnetisation with the field.

2.1.1.4 Anisotropic energy

The anisotropic energy is due to the spin-orbit interaction of the electrons. Electron orbits are inherently linked to the crystallographic structure. Through spin-orbit interaction, these orbitals influence the electron spins, causing them to preferentially align along well-defined crystallographic axes. Therefore, there are directions in space, in which a magnetic material is easier to magnetise than in others [80].

There are two main types of uniaxial anisotropy: the easy-axis anisotropy, where magnetic moments align parallel or antiparallel with the anisotropy axis, and hard-axis or easy-plane anisotropy, where the anisotropy causes the magnetic moments to align perpendicular to the anisotropy axis. Some materials have more than one preferred direction, and each of these affects the total magnetic energy [72]. When a material exhibits only

one preferred direction for magnetisation alignment, the resulting anisotropy is referred to as uniaxial anisotropy. The corresponding local energy density due to anisotropy can be expressed as:

$$w_K = -K_1 \cos^2 \theta - K_2 \cos^4 \theta \quad (2.13)$$

where K_1 and K_2 are the scalar anisotropy constant, typically determined through experimental measurements. The angle θ represents the angle between the magnetisation vector $\mathbf{m}$ and the preferred easy axis of anisotropy. In most cases, the contribution of the second-order anisotropy constant K_2 is negligible due to its relatively small magnitude. From the given Eq. (2.13), it is evident that the anisotropy energy reaches its minimum value when $\theta = 0$ or π , meaning the magnetisation aligns with the preferred direction given K is positive. Using $\cos(\theta) = \mathbf{m} \cdot \hat{\mathbf{u}}$, where $\hat{\mathbf{u}}$ is the unit vector pointing along the easy axis of anisotropy, this leads to the following expression for the uniaxial anisotropy energy:

$$w_K = -K_1 (\mathbf{m} \cdot \hat{\mathbf{u}})^2 \quad (2.14)$$

Another form of anisotropy, which involves three preferred directions, is known as cubic anisotropy. The corresponding local anisotropy energy density can be expressed as:

$$w_K = K_1 (m_x^2 m_y^2 + m_y^2 m_z^2 + m_x^2 m_z^2) + K_2 m_x^2 m_y^2 m_z^2 \quad (2.15)$$

where, similar to the case of uniaxial anisotropy, the coefficients K_1 and K_2 are determined experimentally and m_x , m_y , m_z are the components of normalised magnetisation vector $\mathbf{m}$ [81].

2.1.1.5 Magnetostatic energy

Magnetostatic energy is the energy that originates from the interaction between the magnetic fields created by different parts of a magnetic material. Inside a material, each magnetic moment creates a field which interacts with other magnetic moments.

This energy is also called demagnetising energy or stray field because it tries to reduce the total magnetisation of the system. In other words, the internal magnetic fields act against the overall magnetic order, making it harder for all the magnetic moments to point in the same direction. The demagnetisation energy tends to align the magnetic moments in a way that reduces the stray magnetic fields. [72]. In micromagnetics, this effect causes the magnetisation to form closed loops, which helps to minimise both surface and volume magnetic charges [71, 80]. The energy density of demagnetisation contribution is given as:

$$w_{\text{demag}} = -\frac{\mu_0}{2} M_s \mathbf{m} \cdot \mathbf{H}_{\text{demag}} \quad (2.16)$$

where $\mathbf{H}_{\text{demag}}$ is the demagnetising field, which is the magnetic field created by all the magnetic moments inside a material.

Most magnetic energy terms, such as anisotropy and Zeeman, depend only on the magnetisation at an exact point in the material and are referred to as local interactions in the pointwise sense. However, exchange and DMI, while still considered local, depend on the spatial variation of the magnetisation. But magnetostatic energy depends on the entire magnetisation in the material. This means that every magnetic moment affects every other, although the effect gets weaker with distance. That is why it is called a long-range interaction.

As previously discussed, the exchange interaction favours the parallel alignment of magnetic moments, whereas the magnetostatic energy promotes configurations in which the magnetisation forms closed loops to minimise magnetic surface and volume charges, even though the magnetostatic interaction is weaker, it works over larger distances. This balance between exchange (short-range) and magnetostatic (long-range) effects causes conflicting forces inside the material. As a result, the material creates small regions called magnetic domains, where the magnetisation points in different directions to keep the energy low overall [81].

The formation of magnetic domains leads to the emergence of domain walls⁵. The width of these domain walls is determined by the competition between the exchange interaction, and demagnetisation energy. This characteristic length scale is called as exchange length [69, 82] and is given by:

$$l_{\text{ex}} = \sqrt{\frac{2A}{\mu_0 M_s^2}} \quad (2.17)$$

It serves as a measure of the competition between exchange and magnetostatic energies. At sufficiently small length scales, the exchange interaction dominates, resulting in magnetic moments aligning parallel to minimise the exchange energy.

2.1.1.6 Indirect exchange interaction – RKKY

In metallic systems, magnetic moments can interact indirectly via conduction electrons, a mechanism commonly known as the RKKY interaction. The theoretical background of this interaction is well elaborated in standard textbooks such as [81, 83], and is therefore not discussed in detail in this report.

⁵narrow regions where the magnetisation gradually rotates from the direction in one domain to that of the neighbouring domain.

In the micromagnetic model, this interaction is modelled by the following energy term:

$$w_{\text{RKKY}} = \frac{\sigma}{t} (1 - \mathbf{m}_{\text{layer1}} \cdot \mathbf{m}_{\text{layer2}}). \quad (2.18)$$

Here, σ denotes the surface exchange coefficient which depends on the thickness t of the nonmagnetic spacer layer. Furthermore, $\mathbf{m}_{\text{layer1}}$ and $\mathbf{m}_{\text{layer2}}$ are the unit vectors of magnetisation for the two ferromagnetic layers, respectively.

2.1.2 The Landau-Lifshitz-Gilbert equation: magnetisation dynamics

In the micromagnetic model, the effective field $\mathbf{H}_{\text{eff}}$ arises from the variational derivative of the total energy with respect to the magnetisation. In the continuum limit, it is defined as:

$$\mathbf{H}_{\text{eff}}(\mathbf{m}) = -\frac{1}{\mu_0 M_s} \frac{\delta E[\mathbf{m}]}{\delta \mathbf{m}} \quad (2.19)$$

where $E[\mathbf{m}]$ is the total energy density functional. The Landau-Lifshitz-Gilbert equation (LLG) explains the dynamics of magnetisation [84, 85], it is written as:

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} \quad (2.20)$$

where t is time, and γ is the gyromagnetic ratio of an electron, and α is a dimensionless parameter, called Gilbert damping parameter, this helps the system to dissipate the energy until it reaches a minimum. Eq. (2.20) features the rate of change of magnetisation on both sides, classifying it as an implicit equation. Consequently, $\frac{d\mathbf{m}}{dt}$ cannot be explicitly isolated without further mathematical manipulation, thereby posing challenges for direct implementation in computational simulations. To overcome this issue, we can write the Eq. (2.20) as:

$$\frac{d\mathbf{m}}{dt} - \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} \quad (2.21)$$

Take cross product with $\mathbf{m}$,

$$\mathbf{m} \times \left(\frac{d\mathbf{m}}{dt} - \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} \right) = \mathbf{m} \times (-\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}}) \quad (2.22)$$

Using the vector triple product identity $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ and knowing that $\mathbf{m}^2 = 1$ and $\mathbf{m} \cdot \frac{d\mathbf{m}}{dt} = 0$, it follows that:

$$\mathbf{m} \times \left(\mathbf{m} \times \frac{d\mathbf{m}}{dt} \right) = -\frac{d\mathbf{m}}{dt}$$

Therefore Eq. (2.22) simplifies to,

$$\mathbf{m} \times \frac{d\mathbf{m}}{dt} + \alpha \frac{d\mathbf{m}}{dt} = -\gamma (\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})) \quad (2.23)$$

rewriting Eq. (2.23)

$$\mathbf{m} \times \frac{d\mathbf{m}}{dt} = -\alpha \frac{d\mathbf{m}}{dt} - \gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) \quad (2.24)$$

and substituting $\mathbf{m} \times \frac{d\mathbf{m}}{dt}$ from Eq. (2.24) in Eq. (2.20):

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha \gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) - \alpha^2 \frac{d\mathbf{m}}{dt}$$

and hence, the final explicit form becomes:

$$(1 + \alpha^2) \frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha \gamma \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) \quad (2.25)$$

$$\frac{d\mathbf{m}}{dt} = -\gamma' (\mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})) \quad (2.26)$$

where $\gamma' = \frac{\gamma}{1+\alpha^2}$. The two vector cross product terms in the Eq. (2.20) represents two types of motion of the magnetic moments. The first term describes the precession of the magnetic moment and the second term represents the slowing down of this motion (damping) as shown in Fig. 2.2.

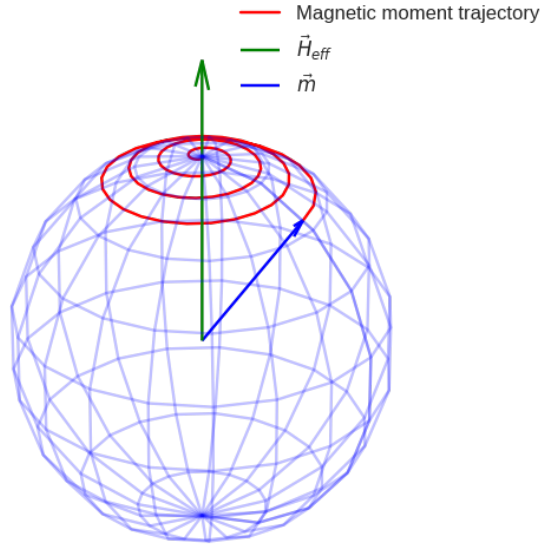


Figure 2.2: Damped precessional motion of a magnetic moment on the unit sphere under the influence of an effective magnetic field $\mathbf{H}_{\text{eff}}$. The red spiral trajectory indicates the effect of damping, where the magnetic moment $\mathbf{m}$ loses energy over time and gradually aligns with $\mathbf{H}_{\text{eff}}$. This demonstrates the gyrotropic motion described by the Eq. (2.26)

As $\alpha > 0$, the magnetic moment vector $\mathbf{m}$ gradually moves towards a state of lower energy over time. This low-energy configuration is called an equilibrium state. If the system reaches the absolute lowest possible energy, the state is called the ground state. While equilibrium states are stable in micromagnetic simulations, they may not remain stable in experiments⁶. The explicit form of the LLG equation, Eq. (2.26), allows us to compute the time development of the magnetisation by solving it as an initial value problem. Given a known initial state $\mathbf{m}(t = 0)$, one can calculate how it evolves over time.

2.1.2.1 Numerical micromagnetics

Micromagnetics aims to describe the behaviour of the magnetic vector field expressed in Eq. (2.1). This magnetisation field $\mathbf{m}(\mathbf{r}, t)$ is a continuous function of both position and time. However the exact analytical solution to the micromagnetic equations is only possible in a small number of idealised cases. In practice, these equations are too complex to be solved analytically and must therefore be approached through numerical methods.

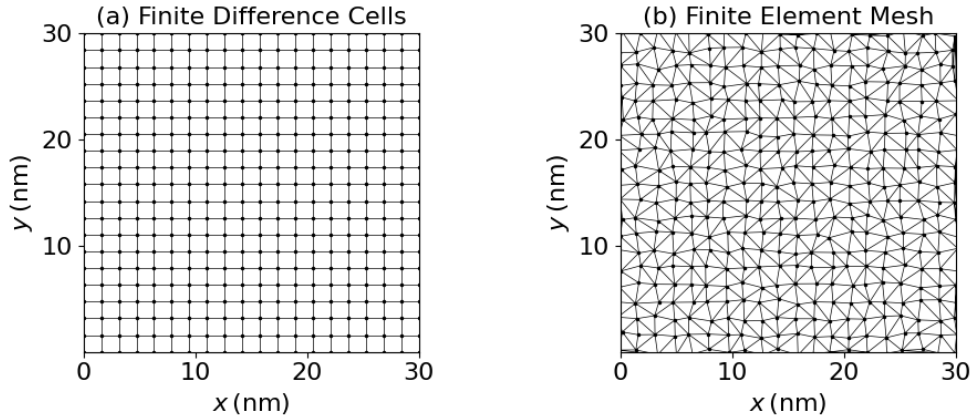


Figure 2.3: (a) Finite difference mesh of a $30 \times 30 \text{ nm}^2$ thin film geometry, representing the regular cell discretisation. (b) Finite element mesh of the same geometry, composed of an unstructured triangular mesh.

To perform numerical calculations, the continuous magnetisation field must be approximated in a way that computers can process, this involves discretising the field. There are two widely used strategies for this discretisation: finite elements and finite difference. The finite difference [80, 86] approach involves dividing the simulation space into a uniform grid of rectangular cells, with each cell assigned a single magnetic moment. In this numer-

⁶This is because external effects like temperature can push the system out of an equilibrium state into a stable state with even lower energy

ical method, spatial derivatives in the micromagnetic equations are approximated using differences between adjacent cell values rather than continuous calculus. This converts the original differential equations into algebraic ones. However, this method is not ideal for modelling structures with curved geometries, since a grid of rectangular cells cannot accurately represent non-linear boundaries.

The finite element method [80, 86] offers a more suitable framework for simulating systems with complex or irregular shapes. Instead of using regular grids, this method divides the domain into unevenly shaped elements that can adapt to intricate geometries. Within each element, a set of local basis functions is defined, and the overall solution is built by combining these functions. This transforms the partial differential equations into a set of linear equations, that can be handled computationally. Despite its flexibility, a notable disadvantage of the finite element method is the increased mathematical and programming complexity required for its implementation.

2.1.2.2 Computational tools

The numerical strategies described above form the basis of several micromagnetic simulation packages, each differing in the way the discretised equations are implemented and solved. In this work, two such packages are used, both built upon the finite difference approach: `magnum.np` [60] and `Ubermag` [61–63]. The choice of using two distinct tools was not arbitrary; rather, it was guided by the specific requirements of each physical system under study, as each package offers capabilities better suited to a particular type of investigation.

`Magnum.np` is a finite difference micromagnetic solver written in Python and built on top of the `PyTorch` [87] library. This design allows the magnetisation field and the various effective field contributions to be represented as tensors, so that the computationally demanding operations can be transparently accelerated on a graphics processing unit (GPU)⁷. A further consequence of building upon `PyTorch` is the availability of automatic differentiation, through which the derivatives required for the effective field can be obtained directly from the energy expressions. Most relevant to the present work, `magnum.np` provides a built-in implementation of the eigenvalue method described in Section 2.4.2, which makes it well suited to the study of the eigenmodes and ringdown behaviour of the skyrmion in the SAF geometry, presented in Chapter 4 and 5.

⁷A GPU is a highly parallel programmable processor with high arithmetic throughput and memory bandwidth [88]. This makes it particularly useful for the large, repetitive field evaluations encountered in micromagnetic simulations.

Ubermag [61–63] provides a Python-based workflow for micromagnetic simulations. In this workflow, the simulation system, material parameters, energy terms, and driving conditions are defined in Python. The numerical calculation is then performed using a chosen solver backend, such as Object-Oriented MicroMagnetic Framework (OOMMF) [89] or MuMax³ [90]. After the simulation has finished, the resulting magnetisation fields and table data can be read back into Python for visualisation and further analysis. This makes Ubermag particularly useful for building reproducible simulation and post-processing workflows. In this work, Ubermag is used to study the DyCo₃ system presented in Chapter 6.

2.2 Magnetic structures

In a simple ferromagnetic system governed solely by exchange interaction, where all magnetic moments point in the same direction, introducing other types of energy interactions can lead to a wide range of new magnetic states.

Beyond their energetic origin, some of these magnetic structures, such as vortices, skyrmions, and Bloch points, can also be understood through the lens of topology. When a configuration cannot be continuously deformed into a uniformly magnetised state without introducing singularities, it is termed topologically non-trivial [18], such configurations are known as topological defects, and they often exhibit enhanced stability due to their quantised nature. Their characterisation involves the concept of a topological charge or winding number⁸, which measures how the magnetisation wraps around a unit sphere. For two-dimensional textures, the winding number Q is given by [91]:

$$Q = \frac{1}{4\pi} \iint \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) dx dy \quad (2.27)$$

The prefactor of $\frac{1}{4\pi}$ normalises the total solid angle of the sphere, ensuring that the topological charge Q takes quantised values. A uniformly magnetised ferromagnetic state has $Q = 0$, indicating its topologically trivial nature. In contrast, non-trivial magnetic structures are characterised by non-zero topological charge. For example, magnetic vortices has a fractional topological charge of $Q = \pm 1/2$, whereas isolated skyrmions are characterised by an integer charge of $Q = \pm 1$, where the sign indicates the chirality of the spin structure.

⁸In the context of magnetic chiral textures, this topological charge (Q) is directly equivalent to the skyrmion number.

2.2.1 Domain walls

At small scales, ferromagnetic materials tend to exhibit uniform alignment of magnetic moments due to the dominance of the exchange interaction, however, this interaction acts only over short distances. As we move to larger length scales, magnetostatic effects become increasingly influential. These effects encourage magnetic moments to curl into closed-loop patterns, reducing magnetic charges both inside the material and at its boundaries, the result is the formation of magnetic domains.

The structure of domain walls depends on material properties such as exchange stiffness and magnetocrystalline anisotropy. These two factors determine the wall's characteristic width. The balance between exchange and anisotropy is quantitatively expressed by the Bloch wall parameter, a characteristic length scale defined as [81]:

$$\delta_0 = \sqrt{\frac{A}{K}} \quad (2.28)$$

where δ_0 is Bloch wall parameter, A is the exchange energy constant and K is the anisotropy energy constant. Strong exchange forces favour gradual changes in magnetisation across a wider region, while anisotropy promotes narrower transitions that better align with the crystal's preferred magnetic directions. The geometry of the sample also plays a role, since it affects the demagnetising field distribution. Two basic types of domain walls are commonly found: the Néel wall and the Bloch wall. In a Néel wall, the magnetisation rotates in a plane perpendicular to the domain wall, a structure more energetically favourable in thin films due to reduced surface magnetic charges, shown in Fig. 2.4(a). Conversely, in a Bloch wall, the magnetisation rotates within the plane parallel to the wall itself, typically seen in bulk materials as shown in Fig. 2.4(b).

2.2.2 Vortices

A vortex is a two-dimensional magnetisation configuration where the magnetic moments rotate around a single point, the vortex core. At the core, the magnetisation turns out of the plane—either pointing upward or downward, while remaining in-plane elsewhere. This arrangement leads to the formation of a topological defect characterised by a non-zero topological charge of $Q = \pm 1/2$.

A vortex emerges due to the competition between exchange energy and demagnetisation energy. The vortex configuration satisfies both: it rotates gently (minimising exchange energy) and avoids surface and volume magnetic charges by forming a closed in-plane flux pattern (minimising demagnetisation energy) [72]. Each vortex is uniquely defined by two

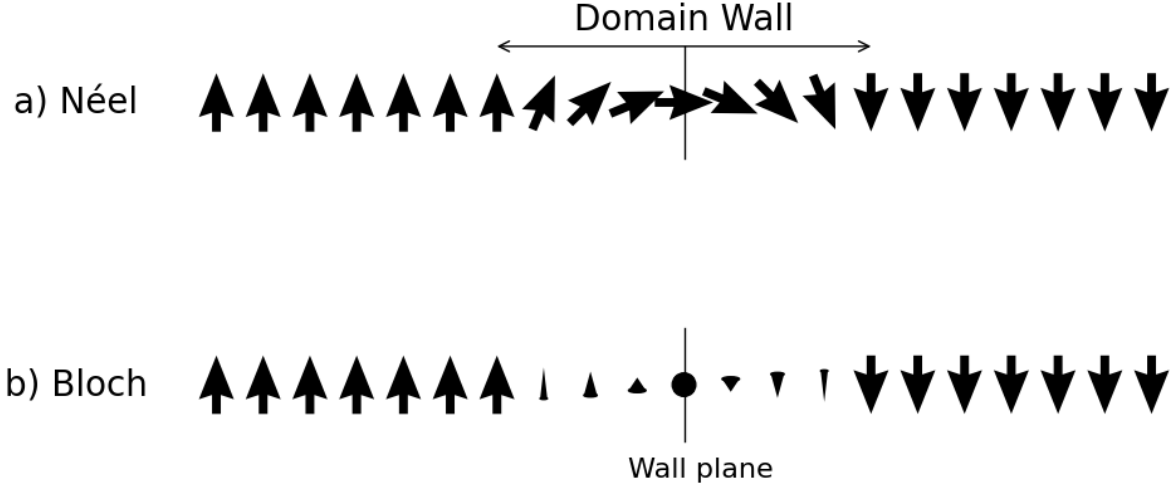


Figure 2.4: Schematic representation magnetic domain walls. (a) Néel-type domain wall, where the magnetisation rotates in the plane perpendicular to the domain wall, within the xz plane, resulting in divergence of the in-plane component along the x axis. (b) Bloch-type domain wall, where the magnetisation rotates within the plane parallel to the domain wall, the yz plane, forming a curl like structure.

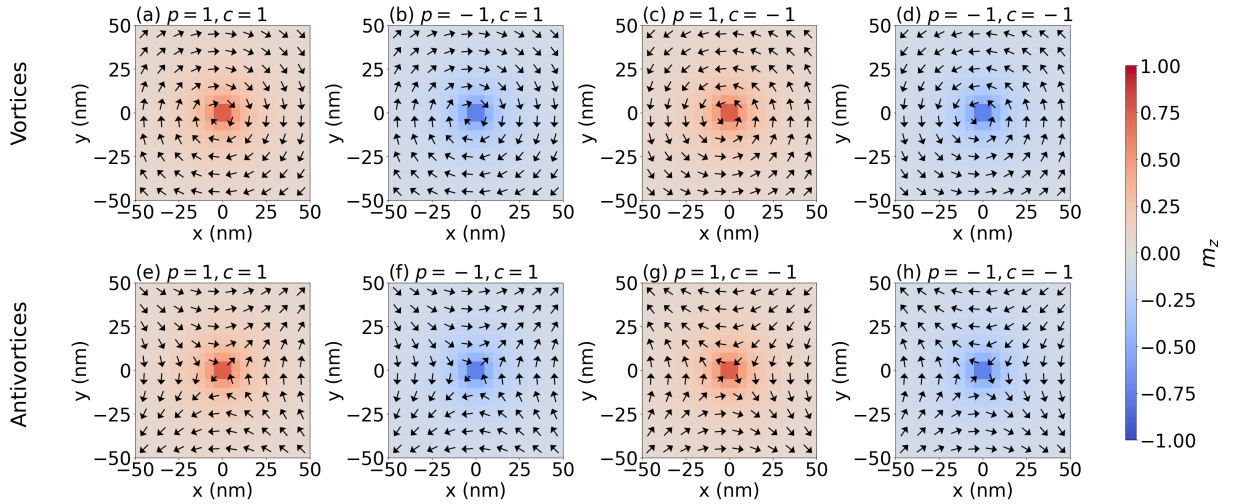


Figure 2.5: Spin textures of vortex and antivortex structures characterised by different combinations of polarity p and circularity c . The background colour indicates the out-of-plane magnetisation component m_z , while black arrows represent the in-plane components. (a–d) Vortex configurations and (e–h) Antivortex configurations.

parameters: polarity $p = \pm 1$, the direction (up/down) of the core magnetisation and circularity $c = \pm 1$, the sense of in-plane rotation (clockwise or anticlockwise) [92]. Another related topological object is the antivortex. Unlike vortices, which are rotationally sym-

metric, antivortices exhibit a two-fold rotational symmetry. This distinction arises from the opposite sense of rotation of magnetic moments around the core [93].

2.2.3 Helices

Helical magnetic states arise from the competition between symmetric exchange interaction, which favours parallel alignment of neighbouring spins, and DMI, which prefers orthogonal spin alignment with a fixed sense of rotation. This balance results in a continuous rotation of the magnetisation vector, forming a spin helix [94] with a characteristic period. The periodicity of the helical rotation is determined by the so-called helical length, given in Eq. (2.11). The periodicity and direction of the helix are governed by the propagation vector $\mathbf{q}$, which indicates the axis along which the magnetisation rotates. The magnitude of this vector is inversely related to the helical length $q = \frac{2\pi}{L_d}$ [18, 91]. Two principal types of helices are typically observed, depending on the nature of the DMI, as illustrated in Fig. 2.6:

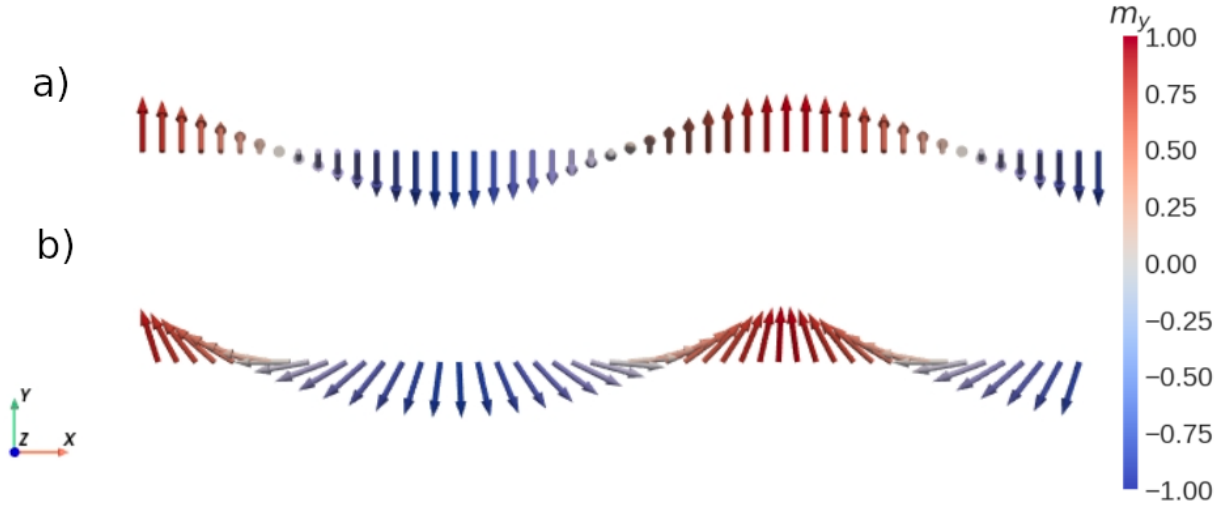


Figure 2.6: Schematic illustration of spin configurations in (a) Helical state and (b) Conical state. The colour scale represents the m_y component of the magnetisation, with red and blue indicating positive and negative values, respectively.

- Helical structure exhibits a magnetisation rotation in a plane perpendicular to the propagation vector q . If q is along the x axis, the spins might rotate in the yz plane.
- Cycloidal spiral, where the magnetisation rotates within a plane that contains the propagation vector q . If q is along x axis, the spins might rotate in the xy plane or xz plane.

Helical spin textures often serve as precursor states to more complex chiral structures, such as skyrmions, particularly under the influence of external magnetic fields or geometric confinement [18].

2.2.4 Skyrmions

A magnetic skyrmion is a two-dimensional swirling spin configuration which exhibits a smooth, continuous variation in the direction of magnetisation along any radial line. The term skyrmion originates from the work of Tony Skyrme in the context of quantum field theory, in which skyrmions were first predicted [15]. It has a non-zero topological charge, indicating that it cannot be continuously deformed into a ferromagnetic (uniformly magnetised) state without encountering a discontinuity [71]. This property holds true for an infinite system; however, in confined geometries, topological protection no longer applies, as a skyrmion can be annihilated simply by pushing it to the system's boundary or applying a very strong external magnetic field. Nevertheless, it remains possible to define a topological charge, which is a physically measurable quantity [95].

In helimagnetic materials, magnetic skyrmions can form due to the interplay between exchange interactions and DMI. In confined geometries, skyrmions may represent the ground state in the absence of an external field [96]. Since their experimental observation [91], magnetic skyrmions have attracted considerable attention due to their topological stability [18], offering immense potential for next-generation high-density, low-power spintronic devices [21, 23, 24, 97] and have been the subject of extensive research. Comprehensive overviews of their properties, dynamics, and applications are available [21, 24, 98–106]. In three dimensions, skyrmions transform into tubes or strings, the extended structure preserves the topological nature of their two-dimensional counterparts while acquiring a cylindrical form along the thickness of the material [107–110].

Magnetisation profile of skyrmions

The topological charge Q , given in Eq. (2.27) and also called the skyrmion number, quantifies how many times the magnetisation vector wraps the unit sphere. To describe this wrapping, it is convenient to express the magnetisation in polar coordinates, where the unit vector field takes the form as [76, 111, 112]:

$$\mathbf{m}(\rho, \phi) = \begin{pmatrix} \cos \Phi(\phi) \sin \theta(\rho) \\ \sin \Phi(\phi) \sin \theta(\rho) \\ \cos \theta(\rho) \end{pmatrix}. \quad (2.29)$$

where $\rho = \sqrt{x^2 + y^2}$ is the radial distance from the skyrmion centre and $\phi = \text{atan2}(y, x)$ is the azimuthal angle in the film plane. The polar angle $\theta(\rho)$ describes the radial variation of the magnetisation, while $\Phi(\phi)$ determines the orientation of its in-plane component. Under these assumptions, the topological charge simplifies to:

$$Q = \frac{1}{4\pi} \int_0^\infty d\rho \int_0^{2\pi} d\phi \sin \theta(\rho) \frac{\partial \theta(\rho)}{\partial \rho} \frac{\partial \Phi(\phi)}{\partial \phi}. \quad (2.30)$$

Using the radial symmetry of the skyrmion and separating the integrals, this expression becomes:

$$Q = -\frac{1}{2} \cos \theta(\rho) \Big|_{\rho=0}^{\rho=\infty} \cdot \frac{1}{2\pi} \Phi(\phi) \Big|_{\phi=0}^{\phi=2\pi}$$

The out-of-plane magnetisation of a skyrmion is opposite at its centre compared to its boundary. This difference is captured by the first parameter, known as the polarity.

$$p = -\frac{1}{2} \cos \theta(\rho) \Big|_{\rho=0}^{\rho=\infty} = \pm 1$$

The sign is determined by the out-of-plane magnetisation of the surrounding material. Because the magnetisation must remain continuous, the polar angle can only wind in integer multiples of 2π , which defines the second parameter, the vorticity.

$$m = \frac{1}{2\pi} \Phi(\phi) \Big|_{\phi=0}^{\phi=2\pi} = 0, \pm 1, \pm 2, \dots$$

The two-dimensional integral simplifies to a product of the polarity and the vorticity [92], permitting only integer values for the topological charge, which correspond to different types of skyrmions.

$$Q = p \cdot m = \pm 1, \pm 2, \pm 3$$

In addition to their topological charge, skyrmions can also be distinguished by their in-plane magnetisation. If the azimuthal angle Φ of the in-plane magnetisation rotates smoothly and uniformly with the polar angle of the position vector ϕ , it can be expressed as a linear function:

$$\Phi(\phi) = m\phi + \gamma, \quad (2.31)$$

where γ is the *helicity*, describing the overall rotation or twist of the in-plane spin texture. When vorticity is fixed at $m = 1$, the helicity γ determines the skyrmion type. A helicity of $\gamma = 0$ corresponds to Néel type skyrmions, in which spins rotate radially outwards. These are typically stabilised with interfacial DMI and are commonly found in materials with C_{nv} symmetry, as shown in Fig. 2.7(a). A helicity of $\gamma = \pi/2$ defines Bloch type skyrmions, where the in-plane magnetisation rotates tangentially. Such skyrmions are

typically stabilised in bulk chiral magnets with T or O crystallographic symmetries, as shown in Fig. 2.7(b). Antiskyrmions represent a distinct class of topological spin textures that lack full rotational symmetry. They generally arise in materials with anisotropic DMI, particularly those belonging to the D_{2d} crystal class. Unlike conventional skyrmions, antiskyrmions possess a negative topological charge and are often characterised by $m = -1$ and $\gamma = 0$, as depicted in Fig. 2.7(c). A comprehensive study on antiskyrmions can be found in Ref. [113].

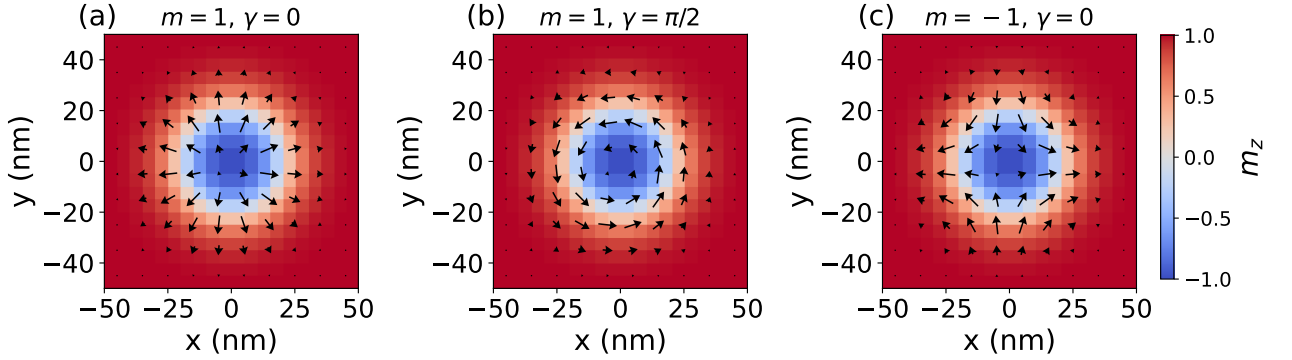


Figure 2.7: Model skyrmion configurations generated using the magnetisation profile given in Eq. (2.29), with the radial profile defined by Eq. (2.32). The colour map shows the out-of-plane magnetisation component m_z , while the arrows indicate the in-plane magnetisation components (m_x, m_y). The skyrmion radius and domain-wall width are fixed to $R = 20$ nm and $w = 10$ nm, respectively, and the polarity is $p = 1$. The panels show (a) a Néel-type skyrmion with $m = 1$ and $\gamma = 0$, (b) a Bloch-type skyrmion with $m = 1$ and $\gamma = \pi/2$, and (c) an antiskyrmion with $m = -1$ and $\gamma = 0$.

The function $\theta(\rho)$ describes the radial variation of the polar angle from the skyrmion core to the surrounding magnetic background. In a more realistic description, the radial profile of the polar angle may be written as [111]

$$\theta(\rho) = 2 \arctan \left(\frac{\sinh(R/w)}{\sinh(\rho/w)} \right), \quad (2.32)$$

where R is the skyrmion radius and w is the domain-wall width.

2.3 Synthetic antiferromagnets

Synthetic Antiferromagnets (SAFs) are multilayered magnetic structures in which two or more ferromagnetic layers are separated by a thin non-magnetic spacer layers, such that

the magnetic moments of the ferromagnetic layers couple antiferromagnetically through Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction, as shown in Fig. 2.8 effectively producing a system with near-zero net magnetisation. For applications, the SAF architecture offers several advantages over traditional ferromagnetic configurations, including reduced stray fields, enhanced magnetic stability, and compatibility with nanoscale device integration.

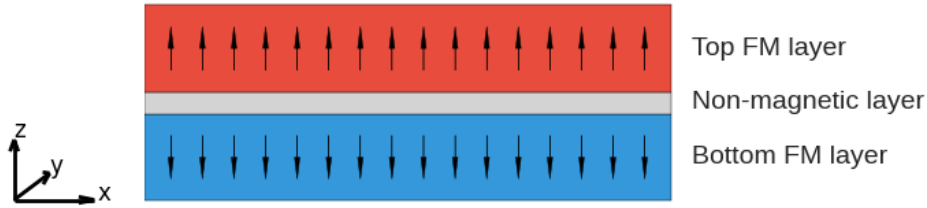


Figure 2.8: Two-dimensional schematic of a SAF structure consisting of two ferromagnetic (FM) layers separated by an insulating spacer.

The physical foundation of SAFs lies in a quantum mechanical effect known as the RKKY interaction, an indirect exchange mechanism through which conduction electrons in a metallic spacer mediate magnetic coupling between spatially separated spins (discussed in 2.1.1.6). This theoretical framework originated with Ruderman and Kittel (1954), who first proposed it in the context of nuclear magnetic moments in metals [114]. Kasuya (1956) extended the model to describe interactions between localised d -electron spins in dilute magnetic alloys, offering insight into metallic ferro- and antiferromagnetism [115]. Yosida (1957) later applied the theory to real materials such as Cu–Mn alloys, demonstrating agreement with experimental magnetic susceptibility data [116].

One of the most important physical properties of SAFs is their zero net magnetisation, arising from the antiparallel orientation of the FM layers. This significantly reduces stray fields and minimises magnetic crosstalk in high-density spintronic circuits [117]. Another key feature is the tunability of exchange coupling: the strength and nature of the interlayer interaction can be precisely adjusted by varying the spacer thickness and material, allowing control over whether the coupling is ferromagnetic or antiferromagnetic [118]. SAFs also exhibit enhanced thermal stability due to their strong interlayer coupling, which helps maintain consistent magnetic behaviour even under thermal stress—a critical factor in memory and logic device reliability [119].

Magnetic skyrmions in SAFs

As discussed in section 2.2.4, magnetic skyrmions are topologically protected structures found within ferromagnetic materials and can be stabilised by DMI energy contribution. Ferromagnetic skyrmions face several limitations that hinder their use in practical devices. Firstly, while non-local stray fields in ferromagnets contribute to the stability of skyrmions, they also restrict their minimum size to several tens of nanometres at room temperature [120], and an external magnetic field is commonly required⁹ to maintain their stability [120, 122–124]. Secondly, the motion of skyrmions in nanotracks is negatively affected by the skyrmion Hall effect, which arises from their non-zero topological charge [18]. This effect causes skyrmions to deviate from a straight path and move towards the edges of the track, where they can be annihilated, leading to data loss [25]. Thirdly, the topological charge also gives rise to what is known as topological damping [120, 125], which further slows down their movement and limits their velocity in ferromagnetic materials [126].

To overcome these limitations, it has been suggested to use antiferromagnetic (AF) systems instead of ferromagnetic systems [127, 128]. These AF systems have two main advantages. Firstly, the absence of net magnetisation in AFs significantly reduces stray fields, which could lead to smaller skyrmion sizes. Secondly, their zero net topological charge cancels the skyrmion Hall effect and simulations predict a straight trajectory along the current with enhanced velocities than their FM counterparts [128–130]. Moreover, as a result of their vanishing net magnetisation, they exhibit strong resilience to the influence of external magnetic fields [131]. To integrate AF skyrmions into devices, it is important to demonstrate their room-temperature, zero-field stability and controlled nucleation—both challenging due to their near-zero magnetic moment. Recent studies report AF spin textures in bulk materials [132] and AF half-skyrmions and bimerons at room temperature [133]. However, the ability to nucleate isolated AF skyrmions at room temperature using local excitation methods is still lacking.

SAFs [118] offer a promising platform for stabilising magnetic skyrmions. Recent experiments have also shown that skyrmions can be stabilised [39] in such materials. As discussed in section 2.3, it consists of two or more ultra-thin FM layers coupled antiferromagnetically via a non-magnetic spacer through RKKY interlayer exchange. In SAFs, skyrmions appear as paired skyrmions with opposite orientations in each FM layer. Unlike bulk antiferromagnets, the magnetic properties of SAFs—such as perpendicular magnetic anisotropy, DMI, and interlayer coupling—can be engineered by tuning the layer materials and thickness. This tunability enables better control over skyrmion size and stability.

⁹though zero-field stability has been demonstrated [96, 121].

SAF skyrmions also show robustness against moderate external magnetic fields, provided the field strength does not exceed the interlayer coupling [131].

2.4 Resonant modes in magnetic nanosystems

Magnetic systems, like many physical systems, naturally exhibit oscillations when disturbed. To understand how such systems respond to external influences, it is useful to investigate their normal modes. These modes describe a specific way in which every part of a system moves together, oscillating at the same frequency and maintaining a constant phase relationship. Analysing these modes provides insight into how a system can be driven or disrupted.

The concept of normal modes serves as a powerful and widely applicable tool across various areas of micromagnetics. One well-known example is the spin-torque nano-oscillator (STNO) [134–136], where the output power spectrum is governed by the system’s resonance frequencies. In such devices, identifying and tuning these resonant frequencies is essential for optimising performance, such as maximising power output or achieving frequency selectivity.

More broadly, normal modes play a crucial role in understanding the internal dynamics of magnetic textures, like skyrmions, which exhibit distinct eigenmodes including the gyrotropic mode¹⁰ [30, 31] and the breathing mode¹¹ [28, 29]. These excitations have been extensively studied and are well understood from both theoretical [137] and experimental perspectives [138, 139]. Analysing these modes helps reveal their stability, response to perturbations, and potential for use in next-generation memory and logic devices [32, 48, 140].

Normal mode analysis is also fundamental in magnonics, where spin-wave modes in patterned structures define the operating principles of magnonic waveguides, filters, and logic elements [141]. In SAFs, mode coupling and hybridisation can give rise to coupled dynamic behaviour [44]. Dynamic modes of skyrmion in a SAF geometry are discussed in chapters 4 and 5.

2.4.1 The ringdown method

The ringdown method [142] is a widely used approach to compute the normal modes of a micromagnetic system. The procedure usually includes the following steps:

¹⁰clockwise (CW) and counter clockwise (CCW) circular motion

¹¹where the skyrmion expands and contracts about its core

1. Obtain the equilibrium magnetisation state $\mathbf{m}_0$, by minimising the system's total energy.
2. Apply a small excitation¹² to disturb the system from equilibrium.
3. Remove the excitation and allow the system to evolve freely. Record the magnetisation over time during this relaxation phase for Fourier analysis.

The computational implementation of the procedure involves recording the magnetisation vector at regular time intervals, denoted by $t_k = k \cdot \Delta t$, where $k = 1, \dots, N$, Δt is the time between samples, and $T = N \cdot \Delta t$ is the total time over which the data is collected. This sampling rate sets the maximum frequency that can be captured in the spectrum, known as the Nyquist frequency, defined as $f_N = \frac{1}{2\Delta t}$. Additionally, the spacing between neighbouring frequencies in the spectrum, called the frequency resolution Δf , is given by $\Delta f = \frac{1}{T}$.

To make this clearer, let us consider an example. Suppose the magnetisation is recorded every $\Delta t = 0.1$ ns, for a total of $N = 1000$ time steps. This means the magnetisation is sampled at times $t_k = k \cdot \Delta t$, where $k = 1, 2, \dots, 1000$, over a total simulation time of $T = 1000 \cdot 0.1$ ns = 100 ns. From this, the Nyquist frequency is:

$$f_N = \frac{1}{2 \cdot \Delta t} = \frac{1}{2 \cdot 0.1 \text{ ns}} = 5 \text{ GHz},$$

which means that any frequency higher than 5 GHz cannot be properly resolved and may result in aliasing. The frequency resolution is:

$$\Delta f = \frac{1}{T} = \frac{1}{100 \text{ ns}} = 10 \text{ MHz}.$$

In practical terms, this setup allows us to detect oscillatory modes up to 5 GHz with a precision of 10 MHz. If two modes lie closer together than this they would appear as a single peak in the spectrum.

At each time step t_k , the magnetisation $m_\alpha(\mathbf{r}_j, t_k)$ where $\alpha \in \{x, y, z\}$, is recorded at a number of spatial sampling locations $\mathbf{r}_j$, which typically correspond to the mesh of discretised cells in a finite difference simulation. This generates a time series of magnetisation data at each spatial point for all three magnetisation components. While the mathematical formulation applies equally to all components, the physical response may differ depending on the geometry of the system and the nature of the excitation.

¹²Typical excitations include magnetic field pulses that are either uniform or non-uniform, and may vary in space or time (*e.g.*, Gaussian, sinc, or sinusoidal), as well as a static fields, and spin-polarised currents.

Certain modes may be more prominent in one component than in others, depending on how the excitation couples to the magnetisation dynamics. As a result, to capture the complete resonant behaviour of the system, it is important to compute and analyse the power spectra for all three magnetisation components: m_x , m_y , and m_z .

To extract resonant mode information from the time-dependent magnetisation data, we first compute the local power spectra for each magnetisation component. At each spatial location $\mathbf{r}_j$, we define the local power spectral density as:

$$S_\alpha(\mathbf{r}_j, f) = |\mathcal{F}_\alpha(\mathbf{r}_j, f)|^2 \quad (2.33)$$

where the discrete Fourier transform is given by:

$$\mathcal{F}_\alpha(\mathbf{r}_j, f) = \sum_{k=1}^N m_\alpha(\mathbf{r}_j, t_k) e^{-i2\pi f t_k}. \quad (2.34)$$

By averaging the local power spectra over all spatial points, we obtain the global power spectrum for the magnetisation component:

$$\text{PSD}_\alpha(f) = \langle S_\alpha(\mathbf{r}_j, f) \rangle_{\mathbf{r}} \quad (2.35)$$

For a fixed resonant frequency f , the spatial mode profile of the corresponding eigenmode can be reconstructed from the complex Fourier coefficients $\mathcal{F}_\alpha(\mathbf{r}_j, f)$. To better understand the methodology, we refer to the study by Baker et al. [57], which applied and implemented this approach. The original code and repository for reproducing the results can be found in [143].

2.4.2 The eigenvalue method

The ringdown method described in Section 2.4.1 determines the normal modes indirectly. The system is perturbed from equilibrium, allowed to relax, and its magnetisation is recorded over time; the resonant frequencies then appear as peaks in the Fourier spectrum of this time series. While straightforward, this approach is computationally demanding. It requires a sufficiently long simulation with a small time step to resolve the spectrum, and the resulting peaks are often noisy, requiring further post-processing before the modes can be identified.

The eigenvalue method offers a direct alternative. Rather than recovering the modes from the time evolution of the system, it computes them directly from the equations of motion. The approach rests on the assumption that the oscillations about equilibrium are small, so that the dynamics can be linearised. Under this assumption the system behaves

as a set of coupled harmonic oscillators whose natural frequencies can be obtained without any time integration. The treatment presented here follows d'Aquino et al. [58] and the implementation in `magnum.np` [60].

The method begins from small fluctuations $\mathbf{v}(\mathbf{x}, t)$ about the equilibrium magnetisation $\mathbf{m}_0$, the state obtained by minimising the system's total energy. Since these fluctuations are small, the LLG equation can be linearised by retaining only terms of first order in $\mathbf{v}$ and neglecting higher-order contributions. This linearisation is the central approximation of the method.

Oscillatory solutions are then sought, in which each part of the system oscillates at a common frequency ω_k while maintaining a fixed spatial profile $\tilde{\varphi}_k(\mathbf{x})$, in keeping with the definition of a normal mode introduced at the start of this chapter. Substituting solutions of this form transforms the linearised equation of motion into an algebraic condition that the modes must satisfy:

$$A_{0\perp} [\tilde{\varphi}_k] = \omega_k B_0 [\tilde{\varphi}_k]. \quad (2.36)$$

Here $A_{0\perp}$ represents the restoring action of the effective field, projected onto the plane in which the magnetisation is free to precess, while B_0 accounts for the gyroscopic precession that characterises the magnetisation dynamics; the explicit form of both operators is given in Ref. [58]. An equation of this form, in which one seeks the spatial profiles $\tilde{\varphi}_k$ and the associated frequencies ω_k , constitutes a generalised eigenvalue problem, whose solutions are precisely the normal modes and resonant frequencies of the system. In the case of a skyrmion, for instance, these eigenmodes are exactly the gyrotropic and breathing modes introduced at the beginning of this section.

Both operators are Hermitian. This is convenient for two reasons: it guarantees that the frequencies ω_k come out real, as they must for genuine oscillations, and it ensures that the modes come out mutually orthogonal, so that each can be treated independently of the others¹³.

Effect of damping

The eigenvalue problem above is solved without damping and therefore gives real eigenfrequencies ω_k . For weak Gilbert damping, the effect of damping can be included using first-order perturbation theory. The damped eigenfrequency is written as

$$\omega'_k = \omega_k + i\delta\omega_k, \quad (2.37)$$

¹³This Hermitian structure also permits the use of the Lanczos algorithm [144], an iterative scheme that requires only the same effective-field evaluations already used in time-domain simulations, and that computes only the lowest few modes of interest rather than the full spectrum.

where $\delta\omega_k$ is the decay rate of mode k . Using the normalisation of the eigenmodes introduced above, the decay rate is given by

$$\delta\omega_k = \omega_k^2 \langle \tilde{\boldsymbol{\varphi}}_k, \alpha \tilde{\boldsymbol{\varphi}}_k \rangle. \quad (2.38)$$

This expression follows from the first-order perturbation of the damped eigenvalue problem and corresponds to Eq. (30) of Ref. [59]. For a spatially uniform damping parameter, Eq. (2.38) becomes

$$\delta\omega_k = \alpha \omega_k^2 \|\tilde{\boldsymbol{\varphi}}_k\|^2. \quad (2.39)$$

The decay rate is therefore proportional to α , but also depends on the frequency and spatial profile of the individual mode.

To first order in α , the small perturbation of the mode profile is neglected. The time-dependent perturbation is then

$$\mathbf{v}_\alpha(\mathbf{x}, t) \approx \sum_k a_k \tilde{\boldsymbol{\varphi}}_k(\mathbf{x}) e^{i\omega_k t} e^{-\delta\omega_k t}. \quad (2.40)$$

The corresponding mode lifetime is $\tau_k = 1/\delta\omega_k$. The same decay rate determines the width of the associated spectral peak, whose full width at half maximum is $2\delta\omega_k$ [59].

Response to a harmonic excitation.

Once the modes are obtained, the response of the system to an oscillating field is determined. The ringdown method would address this through a further time-domain simulation, whereas the eigenvalue method provides the answer directly from the modes already computed.

Since the eigenmodes form a complete and orthogonal set, any response of the system can be expressed as a superposition of them, in the same way that a periodic signal may be decomposed into its constituent harmonics. Driving the system with a harmonic field $\tilde{\mathbf{h}}^a$, the steady-state response is written as a sum over the modes, $\delta\tilde{\mathbf{m}}(\mathbf{x}, \omega) = \sum_k \tilde{a}_k(\omega) \tilde{\boldsymbol{\varphi}}_k(\mathbf{x})$, where each amplitude $\tilde{a}_k(\omega)$ quantifies the contribution of mode k . Two factors determine the magnitude of each amplitude. The first is the proximity of the driving frequency ω to the mode's resonant frequency ω_k : a mode is excited strongly only when it is driven close to its own resonance. The second is the degree to which the spatial profile of the driving field matches that of the mode, since even at resonance a field couples to a mode only insofar as their profiles overlap. This overlap is quantified by the modal projection $\tilde{g}_k = \langle \tilde{\boldsymbol{\varphi}}_k, M_s P_0 \tilde{\mathbf{h}}^a \rangle$, which is large when the field and the mode are well matched and vanishes when they are not.

A given mode therefore appears in the response only when both conditions are satisfied: the driving frequency must lie close to its resonance, and the driving field must overlap with its spatial profile. This is the quantitative counterpart of the observation made for the ringdown method, namely that an excitation applied along a particular direction preferentially excites the modes associated with that direction; here the selectivity is expressed precisely through the modal projection $\tilde{g}_k$.

Power spectrum.

Of particular interest is the total response of the system as a function of the driving frequency, which corresponds to the quantity measured in many experiments. Summing the contributions of the individual modes yields the volume-averaged power spectrum,

$$\tilde{p}(\omega) \approx \frac{\gamma^2}{2} \sum_k \frac{\omega_k^2 |\tilde{h}_k|^2 \|M_s \tilde{\varphi}_k\|^2}{(\omega_k - \omega)^2 + \delta\omega_k^2}, \quad (2.41)$$

in which each mode contributes a single peak [59]. The peak is centred on the resonant frequency ω_k , its height is determined by how strongly the field couples to the mode, and its width is set by the decay rate $\delta\omega_k$, so that strongly damped modes produce broad peaks and weakly damped ones sharp peaks. In contrast to the sampled and inherently noisy spectrum obtained from the ringdown method, this expression is a smooth analytic function that can be evaluated at arbitrary frequency resolution from a single computation of the modes.

The ringdown and eigenvalue methods provide two different approaches to analysing the resonant response. In the ringdown calculations used in this thesis, the excitation is chosen to be sufficiently small that the system remains close to its equilibrium state and returns towards the initial state after the excitation is removed. The modes observed in the resulting spectrum also depend on how strongly they couple to the chosen excitation. The eigenvalue method explicitly assumes small-amplitude oscillations and obtains the normal modes directly from the linearised equations of motion, independently of a particular excitation. In chapters 4 and 5, we apply both methods to study the dynamic modes of a skyrmion chain in a SAF geometry.

Chapter 3

Software contributions to the Ubermag ecosystem

3.1 Motivation and scope

Parts of the computational work presented in this thesis were carried out using the Ubermag ecosystem, which provides a unified micromagnetic modelling (UMM) framework [63]. Ubermag was used to define micromagnetic systems, run simulations with external solvers, access simulation data, and perform post-processing. During this work, I also contributed to several parts of the Ubermag code base. These contributions were not aimed at developing a new micromagnetic solver. Instead, they addressed practical aspects of the simulation workflow, such as documentation, testing, metadata handling, data access, and maintenance of existing package behaviour.

This chapter summarises those software contributions and places them in the context of the simulation work presented in the following chapters. The aim is not to describe every pull request in detail. A complete list of related pull requests is given in Appendix C. Instead, this chapter focuses on the main themes of the work and explains why these changes were useful for computational micromagnetics workflows.

The contributions discussed in this chapter can be grouped into five main categories. The first category concerns metadata handling for simulation runs, in particular the use of `info.json` files. The second category concerns documentation and tutorials, including updates to examples in `discretisedfield` and the Ubermag tutorials. The third category concerns data access in `micromagneticdata`. The fourth category contains exploratory or unmerged work, which is included for completeness but is not presented as completed package functionality. The final category concerns testing and maintenance of package

behaviour, including checks related to drivers and system definitions.

3.2 The Ubermag ecosystem in this work

Ubermag is a Python-based ecosystem for micromagnetic simulations. It allows users to define micromagnetic models in Python and to run those models using external simulation backends. In the work presented in this thesis, Ubermag was used mainly as a high-level interface for setting up systems, storing simulation outputs, and post-processing data.

The ecosystem consists of several packages with different roles. `micromagneticmodel` provides the Python representation of micromagnetic systems, including energy terms, dynamics terms, and drivers. Solver interface packages such as `oommfc` and `mumax3c` connect these model definitions to external simulation tools. The package `discretisedfield` provides data structures for fields and finite different meshes, which are used to represent magnetisation configurations and other spatially resolved quantities. The package `micromagneticdata` provides tools for accessing and organising simulation output data. Other packages, such as `ubermagutil`, the tutorial repository, and the project website, support common utilities, examples, and user-facing documentation.

The software contributions described in this chapter were distributed across several of these packages. This is important because the changes were not isolated to one part of the ecosystem. For example, metadata handling is connected to both the model definition and the solver interface packages, while data access depends on how simulation directories are written and later read. Similarly, documentation changes are useful only if they remain consistent with the current behaviour of the underlying packages.

For this reason, the chapter is organised by the type of contribution rather than by repository. This makes it easier to explain the role of each change in the overall workflow.

3.3 Metadata handling for simulation runs

The largest single theme of the software work was the handling of metadata for simulation runs. In Ubermag-based workflows, simulation output is often stored in directories together with additional information about the system, driver, backend, and run parameters. This information is needed when results are inspected later, especially in parameter sweeps where many simulations are generated with similar names.

In this work, particular attention was given to the `info.json` file. This file stores metadata associated with a simulation drive. Such metadata can include the driver type,

adapter, drive number, time of execution, and additional user-provided information. For the work in this thesis, this kind of metadata was useful because many simulations were run in series, and the post-processing often required information about the simulation parameters after the calculation had finished.

Previously, parts of the `info.json` writing logic were handled in solver-interface packages. This made the behaviour more difficult to maintain because similar logic could be repeated in more than one place. A change was therefore made to introduce an `update_info_json` method in `micromagneticmodel`. This moved the responsibility for updating the metadata closer to the common model layer. As a result, duplicated `_write_info_json` functionality could be removed from `oommfc` and `mumax3c`.

This change is useful because metadata handling is not specific to one solver backend. Although different backends may run the simulation in different ways, the need to record and update metadata is common to the workflow. Keeping this logic in a shared layer makes the behaviour easier to maintain and reduces the risk that different backends write inconsistent metadata.

A related change concerned the time format used in metadata. The time format was changed to an ISO format in `ubermagutil`. This makes the stored time information easier to read and less ambiguous. This is particularly helpful when simulation data are inspected later or when results from different runs are compared.

Metadata handling was also connected to changes in `micromagneticdata` where the display and organisation of data information were rewritten. This is important because metadata are only useful if they can be accessed and understood during later analysis. In this sense, the metadata work involved both writing information during the simulation and improving how this information is accessed afterwards.

The aim of these changes was not to add new physical modelling capability. Rather, the aim was to make simulation runs easier to identify, inspect, and reproduce. This is especially relevant for parameter sweeps and automated workflows, where the simulation directory alone is often not sufficient to understand which model and parameters produced a given result.

3.4 Documentation and tutorial maintenance

A significant part of the software work involved updating and maintaining documentation. Documentation is especially important in a package ecosystem such as Ubermag because users interact with the software through examples, notebooks, and tutorials. If these examples become outdated, it becomes harder for new users to understand the current

interface, and harder for existing users to check whether their own scripts follow the recommended usage.

Several contributions were made to the documentation notebooks in `discretisedfield`. These notebooks cover basic field definitions, field normalisation, field input and output, rotations, spatially varying fields, line objects, mesh construction, subregions, boundary conditions, mesh padding, widgets, and region visualisation. The purpose of these changes was to keep the notebooks consistent with the current behaviour of the package and to improve the clarity of examples used by users of the package.

This work did not introduce new physics into the package. Its role was more practical. Many micromagnetic workflows begin with basic operations on meshes, fields, and regions. If these basic examples are unclear or outdated, errors can propagate into later simulation scripts. Updating these notebooks therefore supported the usability of the package and reduced the chance of confusion when working with field and mesh objects.

The tutorial repository was also updated. This included corrections to existing tutorial material, changes to examples, and the addition of a tutorial notebook based on a `micromagneticdata` workflow. These tutorials are useful because they connect individual package functions to complete simulation and analysis examples. In this sense, they provide a bridge between the package API and practical micromagnetic workflows.

Further documentation-related work concerned visualisation. In particular, the documentation was updated to reflect the transition away from `k3d`-based visualisation towards `pyvista`-based visualisation. A deprecation message was also added for `k3d` visualisation. This kind of change is small at the level of code, but useful for users because it makes the expected direction of the package clearer and reduces uncertainty about which visualisation tools should be used in future work.

Overall, the documentation contributions were mainly maintenance work. They made examples more consistent with the current state of the packages and helped keep the user-facing material aligned with the code base.

3.5 Data access in `micromagneticdata`

The package `micromagneticdata` is used to access and organise data generated by Ubermag simulations. During this thesis, several changes were made to improve how simulation data can be initialised, inspected, and used in later analysis.

One practical contribution was the addition of an option to initialise a `Data` object directly from a path. Previously, data access relied more strongly on the combination of a

simulation name and a directory name. This is suitable for some workflows, but it can be inconvenient when simulation data have been moved, copied, renamed, or organised in a different directory structure. Allowing initialisation from a path makes the interface more flexible for post-processing existing data.

This change is particularly relevant for the type of work carried out in this thesis. Large simulation studies often generate many output directories, and these directories may later be copied to other locations for analysis or archiving. In such cases, it is useful to access the data by pointing directly to the relevant path rather than reconstructing the original name and parent directory convention.

The data information displayed by `micromagneticdata` was also rewritten. This change was connected to the metadata work described in Section 3.3. The goal was to make the stored information about a simulation easier to inspect from the data-access layer. This is important because post-processing often happens after the simulation itself has finished, and the user may need to recover information about the driver, backend, or run parameters from the stored output.

A further contribution added a test for checking the validity of the norm in the data. This type of test is useful in micromagnetic workflows because the magnetisation norm has physical meaning, and invalid data can lead to misleading analysis if it is not detected. The test does not replace physical interpretation by the user, but it provides an additional check at the software level.

Together, these changes made `micromagneticdata` more convenient for working with existing simulation output. They are especially relevant for workflows in which simulation data are revisited after the original calculation, moved between directories, or processed in separate analysis scripts.

3.6 Exploratory and unmerged contributions

Not all software work led directly to merged pull requests. Some pull requests remained open or were closed after discussion, replacement, or changes in the package design. These are listed in Appendix C for completeness, but they are not presented here as completed package contributions.

One example is the exploratory work on ringdown analysis in `micromagneticdata`. Ringdown analysis is central to several parts of this thesis, where time-dependent magnetisation data are transformed into the frequency domain in order to identify resonant modes. A prototype pull request explored how such analysis could be supported more

directly in `micromagneticdata`. This prototype also informed the ringdown functionality later implemented in `mag2exp` [145].

Other ongoing work concerns `discretisedfield`, including a proposed mesh resampling feature and checks related to dimensionality in `point_columns`. The mesh resampling feature is intended to support the transfer of field data between different discretisations, while the dimensionality checks improve the handling of field data during export. These pull requests remain under development and are expected to be completed after further review and testing. They are therefore included in the appendix as ongoing contributions rather than as part of the currently released package functionality.

Some closed pull requests were also part of the development process. In open source software, a closed pull request does not necessarily mean that the work was irrelevant. It may be replaced by a different implementation, superseded by another pull request, or closed because the package design changed. For this reason, the appendix includes both merged and unmerged work, whereas this chapter focuses mainly on the merged contributions and the main development themes.

Minor corrections were also made in other repositories. These include typo corrections, small example updates, and changes to outdated references. Such changes are not discussed in separate sections because they do not form a major theme of the chapter. However, they are part of the maintenance work listed in the appendix.

3.7 Testing and maintenance of package behaviour

Another part of the work concerned testing and maintenance of existing package behaviour. In simulation software, tests are important not only for checking that individual functions return the expected output, but also for making assumptions explicit. This is particularly relevant in micromagnetic simulations, where a driver may require the system to contain certain energy or dynamics terms.

One set of changes added checks for the presence of energy and dynamics terms in micromagnetic systems. These checks were introduced in `micromagneticmodel` and were reflected in the solver interface packages `oommfc` and `mumax3c`. The purpose was to make driver requirements more explicit. For example, a time-dependent simulation requires a dynamics equation, whereas an energy minimisation requires an energy expression. If such requirements are not checked early, the user may only encounter an error later, often after the simulation has already been passed to an external solver.

Related work concerned the `RelaxDriver`. A test was added to check the behaviour of this driver, and the treatment of the damping term in `mumax3c` was adjusted so that

damping was not required for the `RelaxDriver`. This is a practical correction because relaxation and time integration have different requirements. Making these requirements explicit helps prevent unnecessary or misleading input constraints.

Additional maintenance work included filtering out unsupported tests in `oommfc`. This type of change is not directly visible to users in a typical simulation script, but it is important for maintaining the package because it keeps the test suite aligned with the supported functionality of the backend. A test related to the validity of the field norm was also added in `micromagneticdata`. Such tests help catch inconsistent or invalid data early in the analysis workflow.

These contributions should be understood as maintenance and clarification of package behaviour. They do not change the physical equations solved by the external simulators. Instead, they help ensure that the Python-level interface checks the simulation setup more clearly before the calculation is run.

3.8 Summary

This chapter summarised software contributions made to the Ubermag ecosystem during this thesis. The contributions were not directed towards the development of a new micromagnetic solver. Instead, they addressed practical parts of the computational workflow: documentation, tutorials, tests, metadata handling, data access, and exploratory post-processing support.

The documentation work helped keep examples and tutorials consistent with the current package behaviour. The testing and maintenance work made some driver and system requirements more explicit. The metadata work improved how simulation-run information is updated and stored, in particular through the `info.json` workflow. The data-access work in `micromagneticdata` made it easier to inspect and load existing simulation output. Exploratory work, including ringdown-analysis support and field-related utilities, is included in the appendix but is not treated as completed package functionality unless it was merged.

The main role of these contributions in the context of this thesis was to support reproducible and inspectable simulation workflows. They do not change the physical results presented in the previous chapters, but they are part of the software environment in which those results were generated, stored, and analysed.

Chapter 4

Eigenmodes of an Isolated Skyrmion in a Synthetic Antiferromagnet

Parts of this chapter have been submitted to Scientific Reports, and a preprint is available in Ref. [146].

When studying the dynamic modes of skyrmions in synthetic SAFs, recent studies [43, 44] have shown that strong interlayer coupling leads to a splitting of both gyrotropic [43] and breathing modes [44] into in-phase and out-of-phase components. Previous studies have shown that the dynamic excitation modes of skyrmions are highly sensitive to the geometry of the system [31, 41, 43, 147, 148]. The gyration and breathing modes of a single skyrmion in a cylindrical geometry of SAF have been explored, through micromagnetic simulations [43, 44], these studies have identified both in-phase and out-of-phase breathing modes in SAF consisting of two circular ferromagnetic layers coupled via interlayer exchange. Moreover, these dynamic excitations have been shown to play a significant role in the development of skyrmion-based magnonic crystals [32–34, 149]. For instance, a strong coupling observed between gyration and breathing modes in one-dimensional skyrmion lattices formed in ferromagnetic thin-film nanostrips have been studied [150, 151]. In this chapter, we extend the previous study [44] by investigating the dynamics of a Néel-type skyrmion confined in a rectangular geometry within SAF, resembling the ferromagnetic strip configuration discussed in [32, 150].

The chapter is organised as follows. Section 4.1 first introduces the geometries and material parameters used for the ferromagnetic and SAF skyrmion systems. Section 4.2 then summarises the simulation workflow, including relaxation, excitation, frequency-domain analysis, and eigenmode visualisation. Section 4.3 presents the ferromagnetic reference case, where the basic gyrotropic, breathing, and rotational modes of an isolated skyrmion

are identified. Section 4.4 extends the analysis to a square SAF geometry and examines how interlayer antiferromagnetic coupling modifies the gyrotropic and breathing modes. The effect of rectangular confinement is then discussed in the final results section, where the lowest-frequency gyrotropic modes evolve into translational modes. Finally, Section 4.6 summarises the main findings and connects the isolated-skyrmion results to the multi-skyrmion SAF chains studied in Chapter 5.

4.1 Geometry and material parameters

Figure 4.1 shows the geometries considered in this work. All systems are modelled as rectangular thin films. In the ferromagnetic case, the film has a thickness of 2 nm. The synthetic antiferromagnetic (SAF) systems are modelled as two ferromagnetic layers, referred to as the top and bottom layers, which are coupled antiferromagnetically through the RKKY interaction [114]. The spacer layer is not included explicitly in the simulation geometry. Instead, the SAF is represented by two magnetic layers coupled across the interface. Unless stated otherwise, each ferromagnetic layer has a thickness of 1 nm, giving a total magnetic thickness of 2 nm.

The study begins with a single skyrmion in a chiral ferromagnetic film. Two core polarisations are considered: a negative core polarisation, shown in Fig. 4.1a, and a positive core polarisation, shown in Fig. 4.1b. These ferromagnetic systems provide a reference case before moving to the SAF configuration 4.3.

For the SAF systems, the same total magnetic thickness of 2 nm is divided into two antiferromagnetically coupled layers of 1 nm each. The analysis starts from a single SAF skyrmion in a square geometry, shown in Fig. 4.1c. This configuration is then modified by considering unequal layer thicknesses, shown in Fig. 4.1d, and a rectangular geometry, shown in Fig. 4.1e. Finally, the rectangular geometry is extended along the x direction to form strips containing two, five, and 25 skyrmions, as shown in Figs. 4.1f–h. The results for these multi-skyrmion systems are discussed in Chapter 5.

The material parameters are adopted from Ref. [44], where an isolated skyrmion was studied in a cylindrical SAF geometry. The parameters used in the simulations are summarised in Table 4.1.

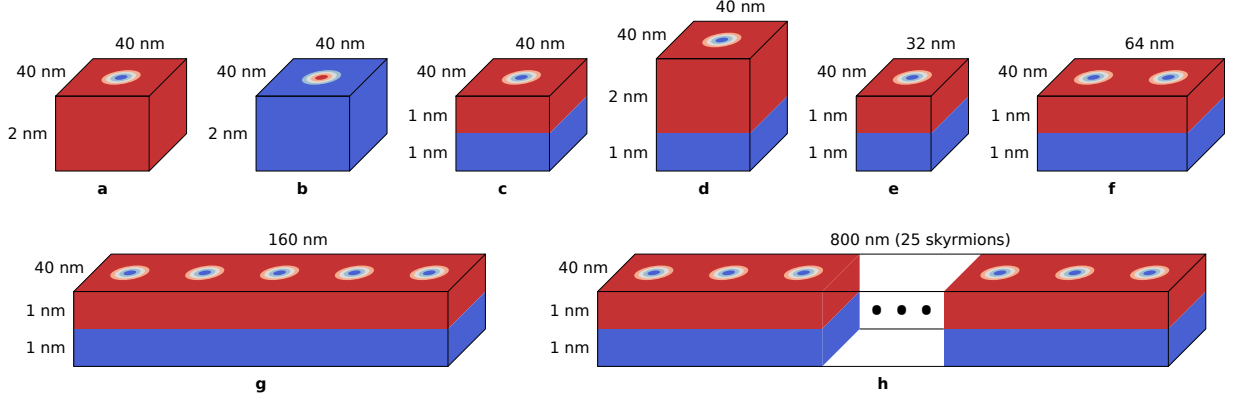


Figure 4.1: Schematic of geometries and skyrmion distribution studied in this work.

Table 4.1: Material and simulation parameters used for in the synthetic antiferromagnet.

Parameter	Symbol	Value
Exchange stiffness	A	15 pJ m^{-1}
Uniaxial anisotropy constant	K_u	1 MJ m^{-3}
RKKY coupling constant	σ	$-3 \times 10^{-4} \text{ J m}^{-2}$
Interfacial DMI constant	D	3 mJ m^{-2}
Saturation magnetisation	M_s	0.795 MA m^{-1}

4.2 Methodology

The static and dynamic behaviour of skyrmions were investigated by numerically solving the Landau-Lifshitz-Gilbert (LLG) equation (2.26) for the magnetisation. Simulations were carried out using the `magnum.np` [60].

In our model, we include the following energy terms:

$$E[\mathbf{m}] = \int_V \left[\underbrace{-A \mathbf{m} \cdot \nabla^2 \mathbf{m}}_{w_a} \underbrace{-K(\mathbf{m} \cdot \mathbf{u})^2}_{w_K} + \underbrace{D(\mathbf{m} \cdot \nabla m_z - m_z \nabla \cdot \mathbf{m})}_{w_{\text{dmi}}^z} \right. \\
 \left. - \underbrace{\frac{\sigma}{t}(1 - \mathbf{m}_{\text{top}} \cdot \mathbf{m}_{\text{bottom}})}_{w_{\text{RKKY}}} - \underbrace{\frac{1}{2} \mu_0 M_s \mathbf{m} \cdot \mathbf{H}_d}_{w_{\text{dem}}} \right] d^3r \quad (4.1)$$

In Eq (4.1), w_a is the isotropic exchange interaction, w_K is the perpendicular uniaxial anisotropy energy, w_{dmi}^z is the interfacial DMI, w_{RKKY} is the RKKY interaction between the two FM layers and w_{dem} is the magnetostatic energy of the material.

We initialise the system in a state comprising a Néel-type skyrmions in each of the ferromagnetic layers (Fig. 4.1), with a cell discretisation of $(1 \times 1 \times 1) \text{ nm}^3$. The system is

excited using the ringdown method, as discussed in section 2.4.1, using the following steps and the overall simulation sequence is summarised in Fig. 4.3.

1. The equilibrium state ($\mathbf{m}_0$) has been determined by relaxing the system for 10 ns with a relatively high damping constant of $\alpha = 0.1$. A larger damping factor increases the speed of energy dissipation, ensuring the magnetisation reaches a global or local energy minimum without getting stuck in transient or metastable states.
2. The system is excited by applying a spatially uniform time-varying magnetic field in the form of a sinc pulse, as shown in Fig. 4.2(a). For a single skyrmion, this external field is applied on x axis to excite the gyrotropic modes of the skyrmions and also along z axis to excite the breathing modes of the skyrmions.

$$\mathbf{H}_{\text{sinc}}(t) = H_0 \frac{\sin(2\pi f(t - t_0))}{2\pi f(t - t_0)} \hat{\mathbf{z}} \quad (4.2)$$

where $\mu_0 H_0 = 1$ mT (0.79 kA/m) is the amplitude, and $f = 50$ GHz of frequency. The time variable is offset with $t_0 = 1$ ns. The magnitude of the applied excitation was chosen to be small to ensure the response of the system remained within the linear regime. The use of a sinc-shaped excitation pulse offers a key advantage, as illustrated in Fig. 4.2(b), the Fourier transform of the sinc pulse yields a rectangular spectrum, providing a flat amplitude distribution up to the cutoff frequency. This ensures that all resonant modes—compatible with the direction of the applied field—are excited with approximately equal spectral weight within the range $[-f, f]$.

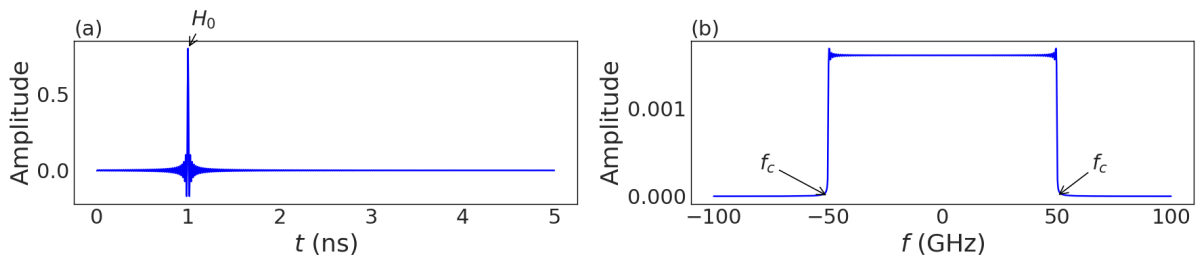


Figure 4.2: (a) Applied sinc pulse for 5 ns with maximum amplitude at 1 mT (0.79 kA/m) with time offset of 1 ns. (b) Fourier transform of the sinc excitation pulse. The resulting rectangular spectrum exhibits a flat response within $[-f, f]$ allowing uniform excitation of all frequencies up to f .

To observe the resulting magnetisation dynamics with minimal artificial damping, the damping parameter is set to a lower value of $\alpha = 0.001$ during this step. This

allows observation of natural magnetisation dynamics, skyrmion breathing or gyration modes, with minimal artificial loss of energy. This allows for more long-lived oscillations, improved spectral resolution, and more precise frequency analysis when applying Fourier transforms. Dynamics of the skyrmions has been studied using the said method in many studies some of these are [32, 41, 44, 140, 150, 152, 153].

3. After the excitation, the applied field is removed and simulate magnetisation dynamics for 20 ns, sampling the magnetisation field every 5 ps. The recorded magnetisation dynamics is analysed by using the spatially resolved method. The time-dependent magnetisation dynamics is extracted by subtracting the static equilibrium configuration $\Delta m_k(\mathbf{r}_i, t_j) = m_k(\mathbf{r}_i, t_j) - m_k(\mathbf{r}_i, t_i)$ where $k \in \{x, y, z\}$, denotes the cartesian components of the magnetisation, $\mathbf{r}_i$, indicates the mesh point, t_i is the time corresponding to the relaxed state before excitation and t_j represents a time instant during the free evolution period $t_j = j\Delta t$ ($\Delta t = 5$ ps)¹. This removes the static background, which would otherwise introduce a dominant zero-frequency peak in the Fourier spectrum and obscure higher-order dynamic modes [154]. We then apply a discrete Fourier transform (DFT) to Δm_k over time and compute the sum of squared coefficients to obtain the power spectral density (PSD) for vector component $k \in \{x, y, z\}$: [96]

$$\text{PSD}_k(f) = \frac{1}{N} \sum_{i=1}^N \left| \underbrace{\sum_{j=1}^n \Delta m_k(\mathbf{r}_i, t_j) e^{-i2\pi f t_j}}_{\mathcal{F}_k(\mathbf{r}_i, f)} \right|^2 \quad (4.3)$$

where N is the total number of mesh nodes and n is the number of time steps. The inner summation performs the DFT at each mesh point. The squared modulus gives the power at each frequency, and the outer sum averages this quantity over all mesh points. The PSD is computed for each magnetisation component (m_x, m_y, m_z), capturing the directional contributions of the system's dynamics.

The eigenvalue analysis was carried out using the `magnum.np` implementation described in Section 2.4.2. In this approach, the linearised LLG equation is solved around the relaxed equilibrium configuration, ($\mathbf{m}_0$), to obtain the resonance frequencies and their corresponding complex mode profiles.

¹The free evolution occurs between the excitation end time (t_e) and the final relaxation time (t_r), as shown in Fig. 4.3.

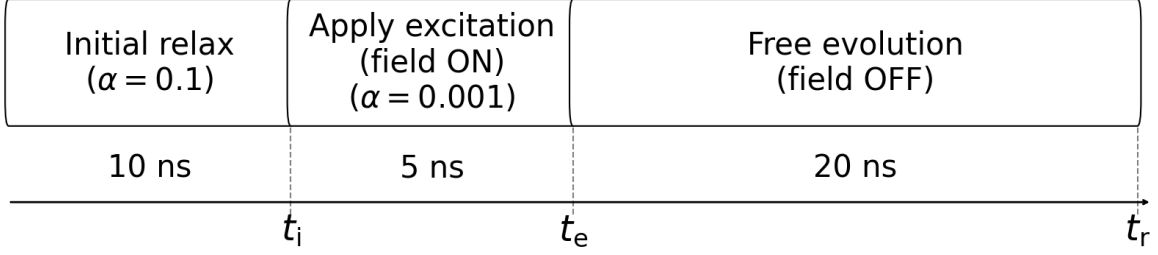


Figure 4.3: Schematic of the simulation timeline showing three stages: initial relaxation with high damping ($\alpha = 0.1$), where t_i marks the initial time (10 ns) to obtain a stable ground state. An excitation magnetic field is then applied with reduced damping ($\alpha = 0.001$) for 5 ns, with t_e denoting the excitation time. After t_e , the field is removed and the system is allowed to freely evolve for 20 ns, with t_r representing the relaxation time. Data are recorded at 5 ps intervals during this final relaxation stage.

The power spectral density (PSD) can be obtained from the calculated eigenmodes in two ways. In the first approach, the PSD is computed directly from the equilibrium configuration together with the spatial profile of the applied excitation. This is computationally efficient, as it does not require a time-dependent simulation. In the second approach, the equilibrium state is explicitly excited and the time evolution is computed under the applied pulse. The PSD is then calculated from the difference between the excited magnetisation and the equilibrium configuration.

Both approaches were implemented in `magnum.np` and give peaks at the same resonance frequencies. However, the relative mode intensities can differ because the two approaches weight the eigenmodes differently. In this work, the second approach is used, as it gives a power distribution that is more directly comparable to the full ringdown simulations.

To visualise the dynamic behaviour of the eigenmodes, we analyse the phase information contained in the complex Fourier amplitudes of the magnetisation signal. For this, we employ a combined phase–amplitude mapping approach. At each spatial point, the Fourier amplitude is computed, from which the phase is extracted as $\phi_k = \arctan(\Im[\mathcal{F}_k], \Re[\mathcal{F}_k])$ and the power spectral density as shown in equation (4.3). The transparency or the opacity of each pixel is adjusted based on the corresponding amplitude value. This method ensures that only phase data from high-confidence (high-amplitude) regions are prominently visible, reducing visual clutter from noisy or low-signal areas. Phase is displayed using a cyclic color map, where hue represents the phase angle. For these visualisations to work without artifacts it is crucial to select a colormap for the hue that has uniform lightness. The commonly used “hsv” colormap does not fulfil this requirement and intro-

duces artifacts. We instead use the “husl” colormap from [seaborn](#), more details on this are discussed in Appendix A.

Data analysis

To analyse the dynamics of the mode associated with frequency f , we can compute the dynamics of the magnetisation vector at position $\mathbf{r}$ using:

$$m_k(\mathbf{r}, t, f) = m_{0,k}(\mathbf{r}) + B \Re[\mathcal{F}_k(\mathbf{r}, f) \exp(2\pi i f t)], \quad (4.4)$$

where B is an artificial scaling factor to enhance the visibility of the eigenmode in real-space visualisations. We use $B = 0.5 / \max(\Re[\mathcal{F}_k(\mathbf{r}, f) \exp(2\pi i f t)])$ throughout this work.

To visualise the motion of the skyrmions for a given eigenmode of the system, we compute the first moment of the topological charge density to track the skyrmion’s core per z layer. From the reconstructed magnetisation vector field (4.4) we can compute the local topological charge density:

$$q(\mathbf{r}, t) = \frac{1}{4\pi} \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right), \quad (4.5)$$

where the spatial derivatives are evaluated numerically using central differences on the interior of the simulation grid. Similar to [155] we track the skyrmion’s core position (R_x, R_y) through the first moment of the topological charge density:

$$R_k(t) = \frac{\sum_i k_i q(\mathbf{r}_i, t)}{\sum_i q(\mathbf{r}_i, t)}, \quad (4.6)$$

where the summation runs over all $\mathbf{r}_i = (x_i, y_i, z)$ in the layer and $k \in \{x, y\}$. For systems containing multiple skyrmions, the computational domain is partitioned into spatial sub-regions, and Eq. (4.6) is applied independently within each region to extract individual trajectories.

The displacement from the equilibrium position $R_{k,\text{eq}}$ is very small and by just plotting $R_k(t)$ trajectories would barely be visible. Therefore, for visualisation we scale them with a factor λ :

$$R_{k,\text{vis}}(t) = R_{k,\text{eq}} + \lambda (R_k(t) - R_{k,\text{eq}}), \quad (4.7)$$

where we need to take into account that the skyrmion in the equilibrium configuration is generally not located at the origin.

In SAFs, the top and bottom magnetic layers are aligned antiparallel at equilibrium. This antiparallel alignment leads to a non-trivial interpretation of phase data in the dynamic mode analysis: physically in-phase motion—where both layers oscillate coherently

relative to their own equilibrium position—can appear as out-of-phase in the raw Fourier transformed data. To ensure that in-phase dynamics are visually represented as such, we apply a global phase shift of π to the top layer before plotting. This transforms the top layer's phase as $\phi'_{\text{top}} = \phi_{\text{top}} + \pi$, with values wrapped back into the $[-\pi, \pi]$ interval.

4.3 Skyrmion in FM square geometry

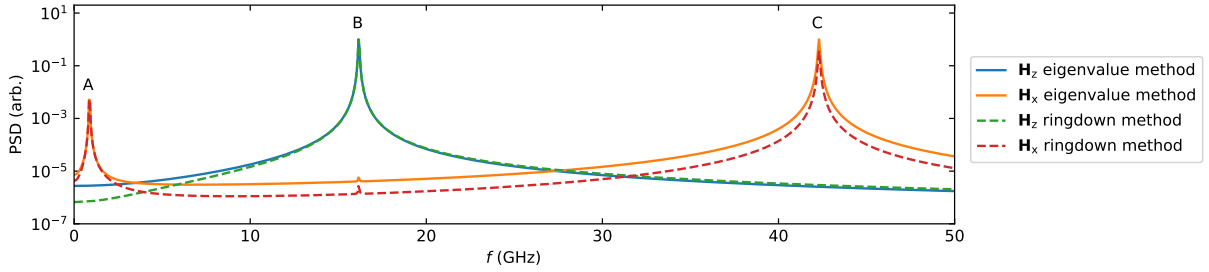


Figure 4.4: Power spectral density (PSD) of a Néel skyrmion in the xy plane for two different excitations. An in-plane excitation field acting in the x direction excites modes A and C, while an out-of-plane excitation field acting in the z direction excites mode B. The solid lines show the spectra computed from the eigenvalue method, and the dashed lines show the spectra obtained from the ringdown method.

Figure 4.4 shows the power spectral density (PSD) for a Néel skyrmion in a (chiral) ferromagnetic material (equilibrium configuration shown in Fig. 4.5). The dashed lines show the PSD obtained from the ringdown simulations. Solid lines show spectra computed from the eigenvalues obtained with the eigenvalue method.

The maxima of the PSD curves identify frequencies of normal modes, which we label with letters. The computed PSD depends on the excitation signal. By choosing a spatially uniform field (acting in the x or the z -direction), we have chosen an excitation that can be realised experimentally. Modes A (0.84 GHz) and C (43.63 GHz) are excited by the in-plane excitation field, and mode B (15.68 GHz) is excited by the out-of-plane excitation. The eigenvalue method is numerically more efficient than the ringdown method, and also guaranteed to find all eigenmodes (irrespective of the excitation), so we will use the eigenvalue method for the rest of this chapter. Figure 4.5 shows the spatial representation of the three modes (see Appendix A, Fig. A.2 for more modes at higher frequencies).

Mode A is a clockwise (CW) gyrotropic mode. Mode B is a breathing mode. Mode C is a counter clockwise (CCW) rotating mode where the skyrmion core position is fixed and

the surrounding skyrmion domain wall gyrates around the fixed core position. Furthermore the background, in particular near the sample edges, is also weakly excited.

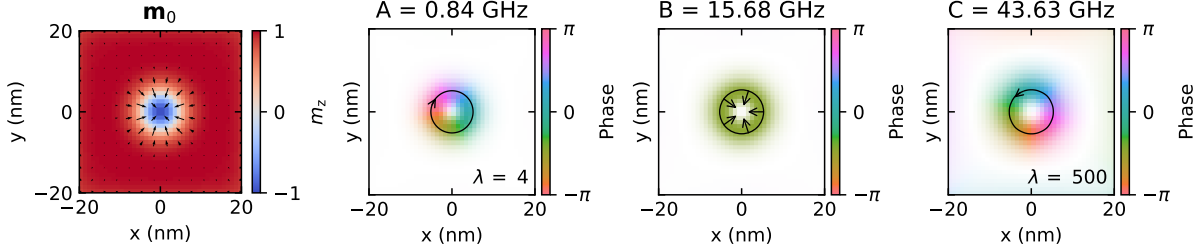


Figure 4.5: Equilibrium configuration and first three eigenmodes A, B, and C of a Néel skyrmion with core pointing in the $-z$ direction, corresponding to the geometry shown in Fig. 4.1a.

To better visualise the type of motion we add overlays to the phase maps. For the rotating modes we overlay the skyrmion trajectory (computed as first moment of the topological charge density, details in Methods Sec. 4.2). For better visibility, the trajectory is scaled by an arbitrary factor λ . An arrow marks the type of motion. The arrow position indicates the skyrmion core position at a reference time, the arrow direction provides a visual indication of the instantaneous direction of motion. The different scaling values of $\lambda = 4$ for modes A and $\lambda = 500$ for mode C show that the displacement of the skyrmion in mode C is much smaller than for mode A. For the breathing mode, the overlaid circle contains the skyrmion core and the arrows show expansion or contraction of the core at the reference time. The reference time is arbitrary and only useful to see phase shifts in different layers, but not suitable to directly compare different eigenfrequencies or different systems.

In previous studies of eigenmodes of confined skyrmions both breathing modes coupling to out-of-plane excitation [41, 96, 150, 156] and gyrotropic modes coupling to in-plane excitation [96, 150, 156] have been reported, in agreement with the results here.

The low frequency gyrotropic mode A can be described by the Thiele model, which approximates the skyrmion magnetisation configuration as a rigid particle. Within that model, which describes the skyrmion only through its core position and its gyrovector $\mathbf{G}$, one low frequency gyrotropic mode is expected [31]. The sign of the Gyrovector, which is proportional to skyrmion topological charge [157], determines the rotation direction (CW or CCW) of that mode.

We repeat the study for the FM skyrmion but change the polarity of the skyrmion so that the skyrmion core points in the opposite direction (now $+z$), which reverses the sign of the topological charge. The results are shown in Fig. 4.6. Indeed the rotation direction

of the gyrotropic mode A has changed from CW to CCW and mode C has changed from CCW to CW.

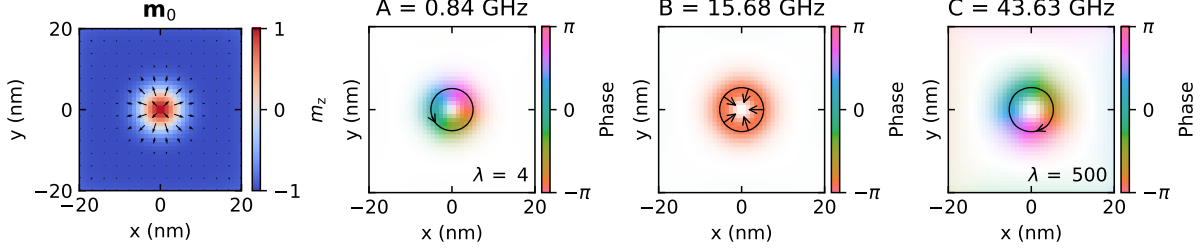


Figure 4.6: Equilibrium configuration and first three eigenmodes A, B, and C of a Néel skyrmion with core pointing in the $+z$ direction, corresponding to the geometry shown in Fig. 4.1b.

4.4 Skyrmion in SAF square geometry

We now move to the skyrmions in a synthetic antiferromagnetic (SAF) bilayer. The geometry is shown in Fig. 4.1c, and hosts two antiferromagnetically coupled skyrmions: one in the top layer with the core pointing in $-z$ direction and another in the bottom layer with the core pointing in $+z$ direction.

Figure 4.7 shows the power spectral densities for the same in-plane and out-of-plane excitations as we use in Sec. 4.3, and the obtained normal modes up to 50 GHz. Modes A and B are gyrotropic modes, modes C and D are breathing modes. Higher frequency modes start to hybridise with the sample, as we have already seen for the ferromagnetic skyrmion (mode C in Figs. 4.5 and 4.6), and are shown in Fig. A.5.

4.4.1 Gyrotropic modes

The first two modes A and B in Fig. 4.8 are gyrotropic modes with very similar frequencies $f_A = 4.68$ GHz and $f_B = 4.75$ GHz and are nearly indistinguishable in the spectrum.

As for the corresponding ferromagnetic single layer study, the gyrotropic modes are excited by the in-plane field. The two skyrmions, one in the top and one in the bottom layer of the SAF, gyrate in the same orientation: clockwise for mode A and counter clockwise for mode B.

From the study of the ferromagnetic system (Sec. 4.3) we know that given a skyrmion core polarisation, one rotation sense is preferred. In the SAF setup, the skyrmion in the top layer has core polarity -1 and, due to the AFM coupling, the skyrmion in the bottom

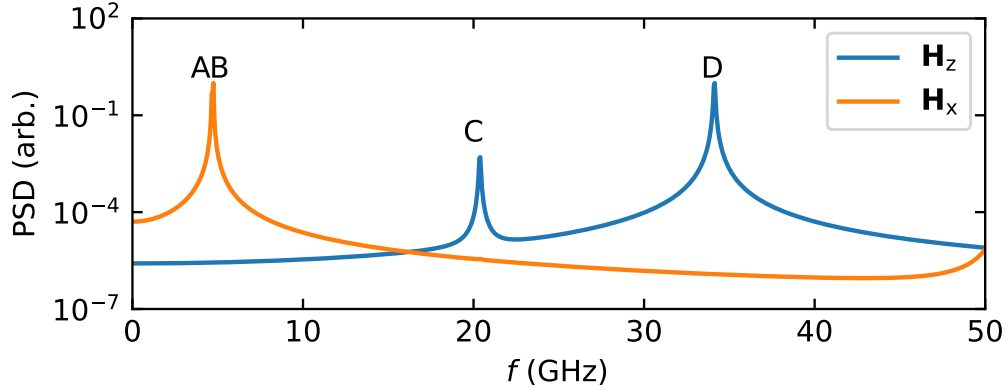


Figure 4.7: Power spectral density for a SAF skyrmion in the square geometry shown in Fig. 4.1c. The Néel skyrmion in the top layer has the vortex core pointing in the $-z$ direction, and the skyrmion in the bottom layer has the vortex core pointing in the $+z$ direction. The layers are antiferromagnetically coupled. The blue line shows the power spectral density for out-of-plane excitation acting in the z direction, and the orange line shows the power spectral density for excitation acting in the in-plane x direction.

layer has core polarity $+1$. Due to the interlayer coupling forces, top and bottom layer rotate (mostly) coupled. The system of the two skyrmions in the SAF sample is frustrated: the preferred rotation direction for the skyrmion in the top layer is the opposite of the preferred direction for the skyrmion in the bottom layer.

The frustration also manifests in the skyrmion trajectories in Fig. 4.8. The trajectory of the top layer skyrmion for mode A shows a larger diameter than the trajectory for the bottom layer skyrmion. For mode B the trajectory of the bottom layer skyrmion has the larger diameter. This is in line with the core down polarisation of the top layer skyrmion preferring the CW rotation, and the core up polarisation in the bottom layer skyrmion preferring the CCW rotation (Sec. 4.3). Due to the interlayer coupling, both layers show the same rotation but the non-preferred rotation direction has reduced amplitude (i.e. reduced trajectory diameter).

Figure 4.9 shows the gyrotropic normal modes for a modified SAF geometry with non-identical layer thicknesses: the top layer has an increased thickness of 2 nm, while the bottom layer is 1 nm thick, as shown in Figure 4.1d.

In the symmetric case (both layers 1 nm), the eigenmodes at 4.68 and 4.75 GHz are nearly degenerate. In the asymmetric geometry, this near-degeneracy is lifted, and the modes split into mode A at 2.1 GHz and mode B at 8.28 GHz. Mode A corresponds to a CW gyrotropic motion, which is the preferred rotation direction in the top layer. Since

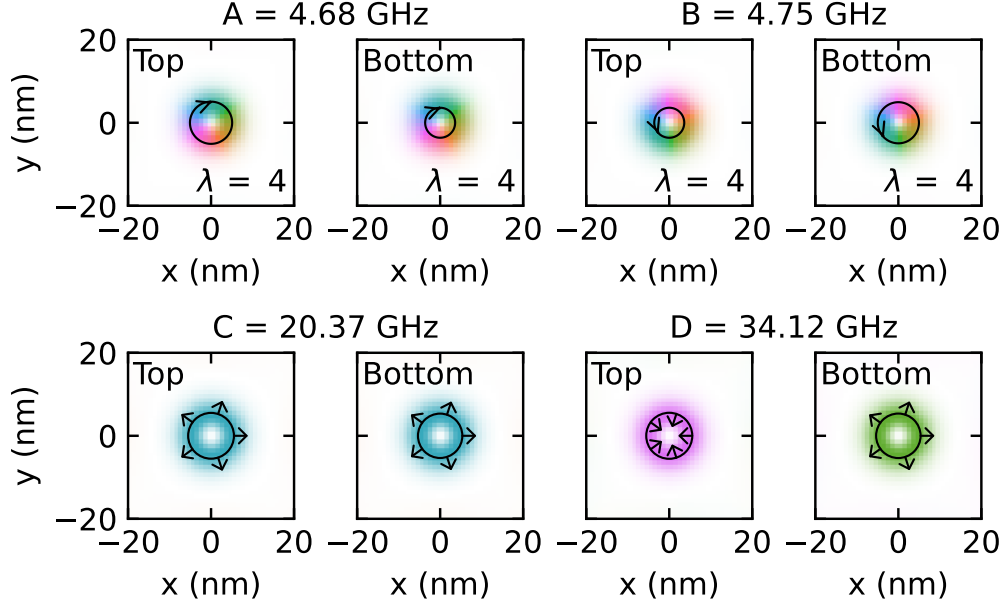


Figure 4.8: Normal modes for a SAF skyrmion in the square geometry shown in Fig. 4.1c. The observed normal modes are gyrotropic modes A and B, and breathing modes C and D.

the top layer accounts for approximately two-thirds of the total magnetic volume, this lowers the overall mode frequency. Conversely, the CCW mode is unfavourable in the thicker top layer, leading to an increase in its energy and thus a higher frequency. This behaviour is consistent with our interpretation of the coupled modes (decreased amplitude for the non-preferred rotation sense). Finally, we note that combining layers of different thicknesses can be advantageous in applications where a finite stray field is required.

A similar splitting of frequencies for clockwise and anticlockwise gyrotropic modes, where the skyrmions in the top and bottom layers rotate in the same direction, has been reported for a skyrmion in a synthetic antiferromagnet (SAF) confined to a disk geometry [43] (Fig. 4, 61 GHz and 66 GHz). However, the behaviour reported for the other modes is not consistent with our findings, and the overall interpretation of the remaining modes differs from what we observe. To the best of our knowledge, no other studies have reported a comparable splitting of these modes in SAF skyrmions.

4.4.2 Breathing modes

Modes C and D in Fig. 4.8 are breathing modes, in which the skyrmion radius grows and shrinks periodically [44, 150]. The visualisations show that in mode C at $f_C = 20.37$ GHz the top and bottom layer breath in phase, whereas in mode D at $f_D = 34.12$ GHz the top

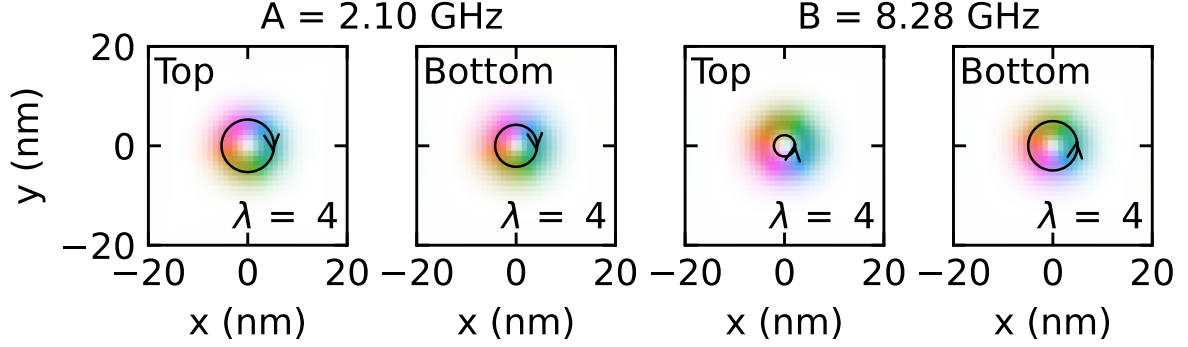


Figure 4.9: Gyrotropic normal modes and frequencies for a SAF skyrmion in square geometry with top layer thickness of 2 nm and bottom layer thickness of 1 nm, with $L_x = L_y = 40$ nm, as shown in Fig. 4.1d.

and bottom layer breath out of phase. The energetic impact of out-of-phase behaviour between top and bottom layer is high due to the large interface area over which the RKKY coupling is active. These in-phase and out-of-phase breathing modes are consistent with the results [44] reported for a skyrmion in a SAF that is confined by the disk-like geometry.

4.4.3 Effect of saturation magnetisation on the eigenmode frequencies

To examine the role of the saturation magnetisation M_s in the isolated skyrmion system, the eigenfrequencies of the two low-frequency modes were calculated for a single skyrmion in the square synthetic antiferromagnetic geometry, with M_s varied over a range spanning values below and above the reference $\mu_0 M_s = 1$ T used in the main simulations. The results are shown in Fig. 4.10. Both modes increase monotonically in frequency with increasing M_s , which indicates that the saturation magnetisation reshapes the effective field landscape experienced by the skyrmion.

This dependence can be understood from the way M_s enters the individual contributions to the total micromagnetic energy. The relevant terms here are the exchange energy, the DMI, the demagnetisation energy, the perpendicular magnetic anisotropy, and the interlayer RKKY coupling, as given in Eq. (4.1). These terms do not scale with M_s in the same way, so varying it changes their relative balance rather than simply rescaling the energy as a whole.

The most direct effect is through the magnetostatic energy, which scales as M_s^2 . Increasing M_s therefore strengthens the dipolar contribution and, with it, the internal effec-

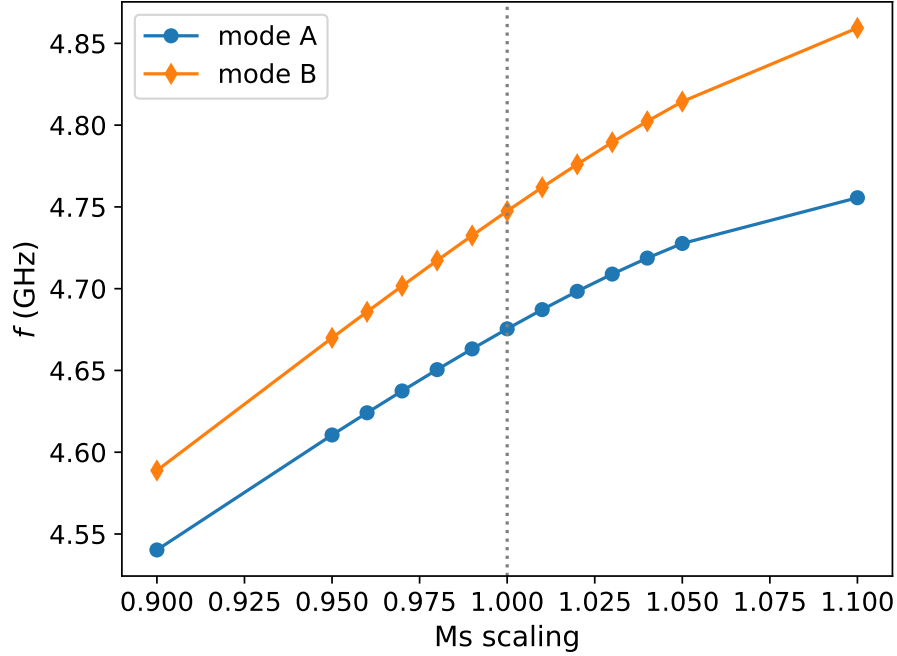


Figure 4.10: Frequencies of the two lowest gyrotropic eigenmodes of an isolated skyrmion in the 40×40 nm synthetic antiferromagnetic geometry, as the saturation magnetisation M_s is varied above and below the reference value $\mu_0 M_s = 1$ T. The dashed vertical line marks this reference value, adopted for the remaining simulations. Mode A (CW) and mode B (CCW) both rise monotonically with M_s , and their separation widens across the scanned range.

tive field of the skyrmion. The same quantity also enters the balance between crystalline and shape anisotropy, often written as an effective anisotropy $K_{\text{eff}} = K_u - \mu_0 M_s^2/2$, so a larger M_s reduces the effective perpendicular anisotropy and lowers the energy cost of out-of-plane magnetisation. The exchange and DMI terms, by contrast, set the spatial stiffness and the chiral character of the texture; with A , D , and K_u held fixed, their associated effective fields carry an inverse dependence on M_s and are thus weakened as it grows.

The combined result is a shift in the equilibrium skyrmion profile, and the coupling between the two ferromagnetic layers, which in turn alters the curvature of the energy landscape about the equilibrium state. Since the eigenfrequencies are governed by this curvature, they move accordingly. Within the scanned range the monotonic increase of both modes suggests that the net effect of raising M_s is to stiffen the dynamical response of the isolated SAF skyrmion.

Throughout the scanned range, mode B (CCW gyration) remains slightly higher in frequency than mode A (CW gyration), so the ordering and character of the two modes are

preserved while their frequencies shift with the energetic balance. The splitting between them does, however, widen as M_s increases. These results show that even a moderate change in M_s can shift the resonance frequencies of the SAF skyrmion appreciably, while leaving the underlying skyrmion state intact.

4.5 Skyrmion in SAF rectangular geometry: translational modes

Figures 4.11 and 4.12 show the PSD and the normal modes of the SAF skyrmion in a sample with $L_x = 32$ nm and $L_y = 40$ nm (geometry in Fig. 4.1e). Apart from the dimension in the x-direction, this is the same system as shown in Fig. 4.8 where $L_x = L_y = 40$. The breathing modes C and D are very similar in Fig. 4.12 and Fig. 4.8, and the small change in frequency is probably related to the changed confinement potential by going from $L_x = 40$ nm to $L_x = 32$ nm.

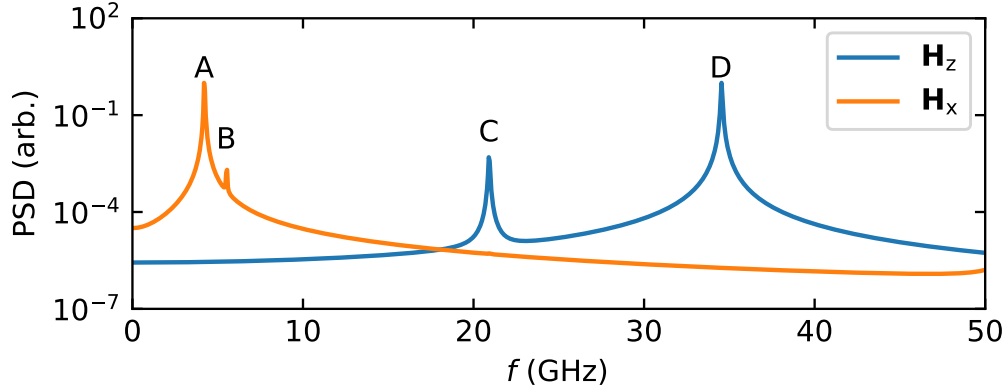


Figure 4.11: Power spectral density of a skyrmion in a SAF with sample dimensions $L_x = 32$ nm and $L_y = 40$ nm. The blue line shows the power spectral density for out-of-plane excitation acting in the z direction, and the orange line shows the power spectral density for excitation acting in the in-plane x direction.

However, modes A and B differ significantly. For the square geometry (Fig. 4.8), we find two gyrotropic modes A and B, where for mode A top and bottom layer both show CW gyration. For the rectangular geometry (Fig. 4.12) we find that mode A exhibits a CW rotation in the top layer and a CCW rotation in the bottom layer, i.e. the skyrmion in each layer gyrates in its preferred direction. The skyrmion core positions describe elliptic trajectories in each layers (although with opposite gyration sense). The SAF skyrmion

core position (the average of the top and bottom layer skyrmion position) describes a (nearly) linear motion along the y-direction as the x-components of the elliptic motion in the two layers cancel each other due to the opposite rotation sense. For the lower frequency mode A, this translational motion occurs in the y-direction, along the long axis of the sample, and for the higher frequency mode B a corresponding motion occurs in the x-direction, along the short axis of the sample.

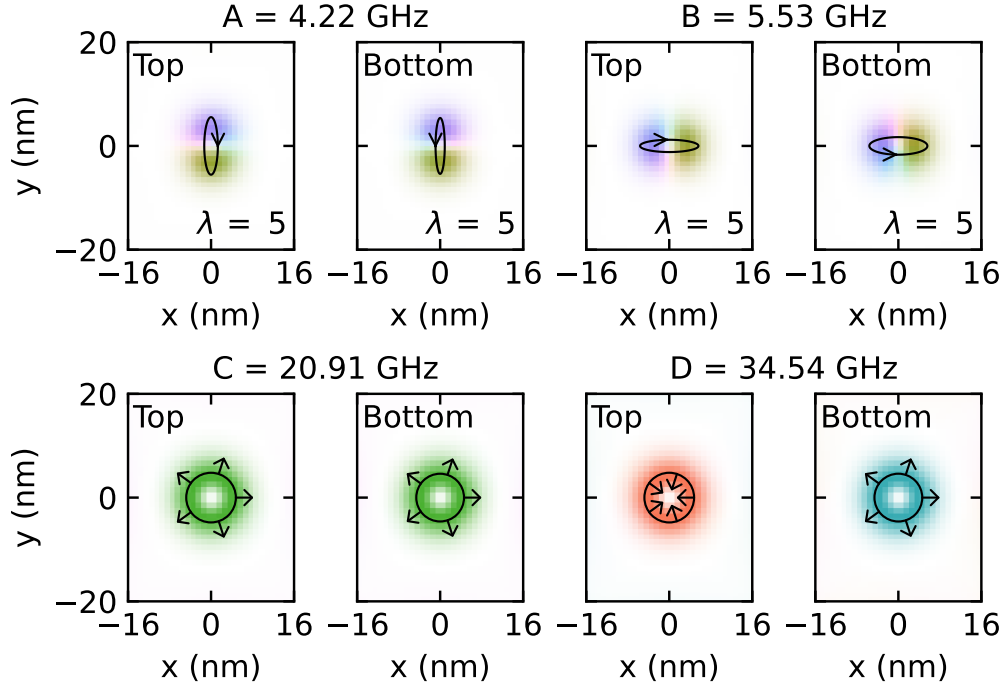


Figure 4.12: Lowest-frequency normal modes of a skyrmion in a SAF with sample dimensions $L_x = 32$ nm and $L_y = 40$ nm. Modes A and B are classified as translational modes, while modes C and D are classified as breathing modes.

Figure 4.13 shows the trajectory of the skyrmion in the top and bottom layer for the translational mode A for a number of different interlayer coupling strengths σ . The weaker coupling values show that the motion is CW in the top layer and CCW in the bottom layer. As the coupling strength σ increases, the elliptical trajectory becomes increasingly more elongated and narrower. For the strongest values studied $\sigma = 0.1$ Jm⁻² the trajectory (in both layers) appears as a line (within the resolution of our figure). We interpret this as the interlayer coupling strength enforcing increasingly more identical spin rotations in the top and bottom layer as σ increases. The x-components of the top layer skyrmion core position grows positively when the bottom layer skyrmion core position grows negatively

within one rotation cycle, whereas the y-positions of both layers change in-phase. As the coupling increases towards infinity, the out-of-phase motion is suppressed and only translational motion in the y-direction remains.

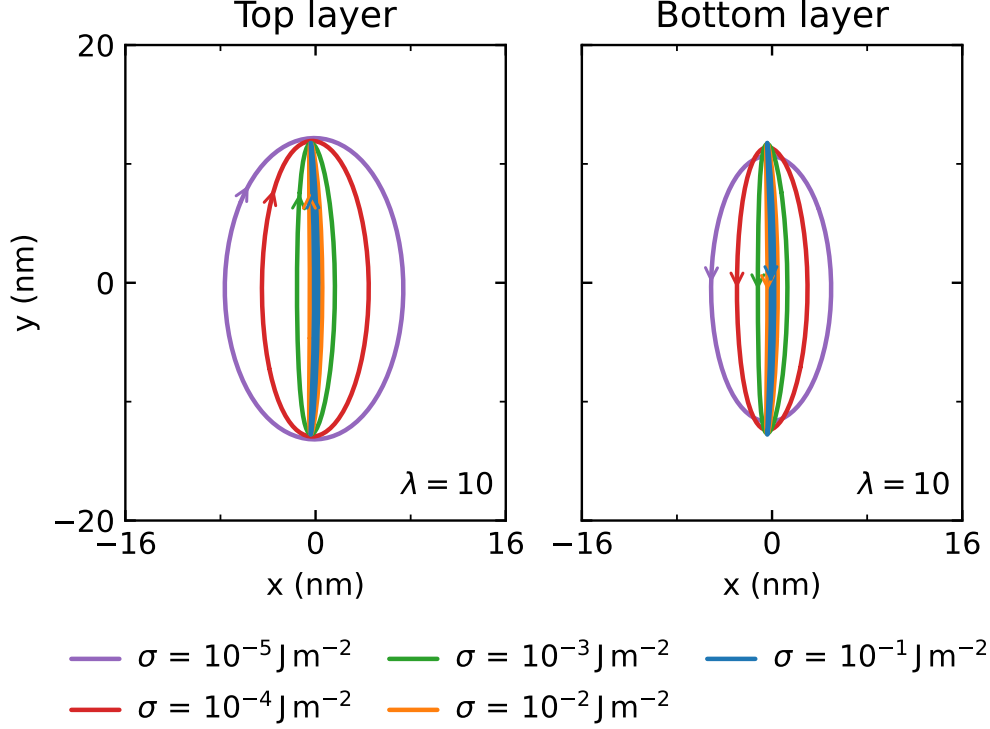


Figure 4.13: Skyrmion core trajectories for mode A of a SAF skyrmion in the $32 \text{ nm} \times 40 \text{ nm}$ geometry for different interlayer coupling strengths σ .

We have studied translational modes A and B systematically with L_x changing from 32 nm to 45 nm in steps of 1 nm (the mesh discretisation is 1 nm) and keeping $L_y = 40 \text{ nm}$ fixed. Figure 4.14 shows the frequency of the vertical translational mode A (blue squares) and the horizontal translational mode B (orange diamonds) as a function of L_x . The dashed line is a 4-th order polynomial fit through the data points. The mode aligned with the longer sample axis has the lower frequency, i.e. mode A for $L_x < L_y$ and mode B for $L_x > L_y$. Increasing the sample aspect ratio increases the mode splitting. The two green dots at $L_x = 40 \text{ nm}$ are the two gyrotropic modes (mode A and B in Fig. 4.8) observed in the square geometry.

From the eccentricity of the SAF skyrmion core trajectory (inset in Figure 4.14), we find a circular trajectory only for $L_x = L_y = 40 \text{ nm}$. For this square geometry, we call modes A and B gyrotropic modes. For aspect ratios $L_x/L_y \neq 1$, we call modes A and B

translational modes. The names *gyrotropic* and *translational* reference the trajectory of the SAF skyrmion core.

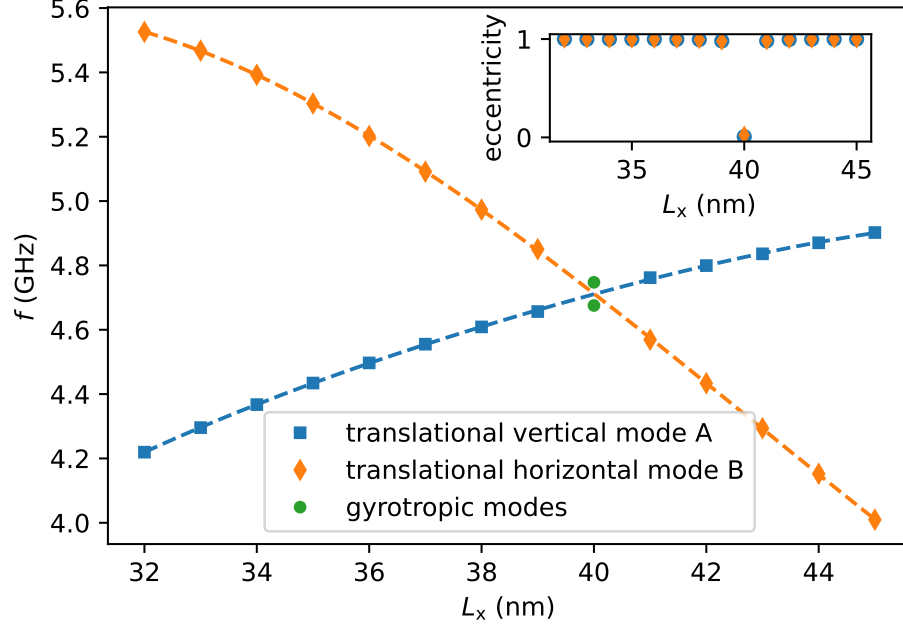


Figure 4.14: Frequency of the translational modes A and B, shown in Fig. 4.12, as a function of geometry L_x . All studied systems have $L_y = 40$ nm. For the quadratic sample with $L_x = 40$ nm and $L_y = 40$ nm, two gyrotropic modes are observed, where the skyrmion cores move on a circular trajectory, as shown in Fig. 4.8. The inset shows the eccentricity of the ellipse of the SAF skyrmion trajectories, obtained from the summed trajectories of the top and bottom layer skyrmions. A sharp transition is observed from just below 1 for $L_x \neq L_y$ to 0 for $L_x = L_y$.

4.6 Summary

This chapter discussed the dynamics of an isolated skyrmion in confined ferromagnetic and synthetic antiferromagnetic geometries. The ferromagnetic system was first used as a reference case, because it allows the basic modes of a single skyrmion to be identified before studying the coupled SAF system. For the square geometry with the skyrmion core pointing along the negative z direction, the main modes are identified as clockwise gyrotropic motion, breathing motion, and counter-clockwise rotational motion. The direction of the gyrotropic motion is related to the polarity of the skyrmion core, which shows that the internal structure of the skyrmion has an important role in its dynamical response.

The SAF system adds another level of complexity because the skyrmions in the top and bottom ferromagnetic layers are coupled through the interlayer RKKY interaction. In the square SAF geometry, this coupling gives rise to paired gyrotropic modes at lower frequencies and breathing modes at higher frequencies. The breathing modes can be further separated into in-phase and out-of-phase modes, depending on whether the oscillations in the two layers occur together or opposite to each other. When the square symmetry is broken by changing the lateral geometry, the low-frequency gyrotropic motion changes into a predominantly translational motion along the long axis of the sample. This shows that the geometry of the system strongly influences which modes are supported.

These isolated skyrmion systems provide the reference point for the next chapter. In Chapter 5, the discussion is extended to more than one SAF skyrmion, where the interaction between neighbouring skyrmions leads to collective translational and breathing modes.

Chapter 5

Collective Dynamic Modes of Skyrmion Chains in a Synthetic Antiferromagnet

Parts of this chapter have been submitted to Scientific Reports, and a preprint is available in Ref. [146].

In Chapter 4, the dynamics of an isolated skyrmion in a ferromagnetic chiral system and synthetic antiferromagnetic (SAF) system were analysed. In this chapter, the study is extended to finite SAF skyrmion chains containing two, five, and 25 skyrmions. This provides the next step towards understanding how the resonant response of an isolated SAF skyrmion changes when neighbouring skyrmions are introduced.

The main focus of this chapter is on the evolution of translational and breathing modes from single-skyrmion resonances into collective excitations of coupled skyrmion systems. By comparing chains of different lengths, we examine how the mode frequencies, spatial profiles, and relative phase relationships are modified as the number of skyrmions increases.

The simulations presented in this chapter use the material parameters listed in Table 4.1 and follow the general workflow described in the previous chapter. The relaxed SAF skyrmion chains are excited using a sinc-shaped magnetic-field pulse, and the resulting magnetisation dynamics are analysed in the frequency domain. The excitation is applied to the left most skyrmion of the strip. This local excitation makes it possible to excite both translational and breathing collective modes in the finite chains.

The chapter is organised as follows. Section 5.1 first considers the two-skyrmion system, which provides the simplest coupled SAF skyrmion geometry beyond the isolated case. In this system, the translational and breathing modes can still be identified individually,

allowing the role of left–right and top–bottom phase relationships to be analysed in detail. Section 5.2 then extends the analysis to a five-skyrmion chain, where the discrete modes begin to form mode groups with band-like features. The standing-wave character of the collective excitations is also examined in this geometry. Section 5.3 considers a longer chain of 25 SAF skyrmions and uses this system to study how a locally excited breathing-mode signal propagates through the chain. Finally, Section 5.4 summarises the main findings of the chapter and discusses possible directions for future work.

5.1 Two-skyrmion system from the eigenmode analysis

Next, we study a strip of length $L_x = 64 \text{ nm}$, which can hold two skyrmions in each layer. Figure 5.1 illustrates the geometry. We relax the system into two pairs of Néel type skyrmions, which are antiferromagnetically coupled between top and bottom layer with a core-to-core spacing of 32 nm within a given layer. For the spectrum calculation, the

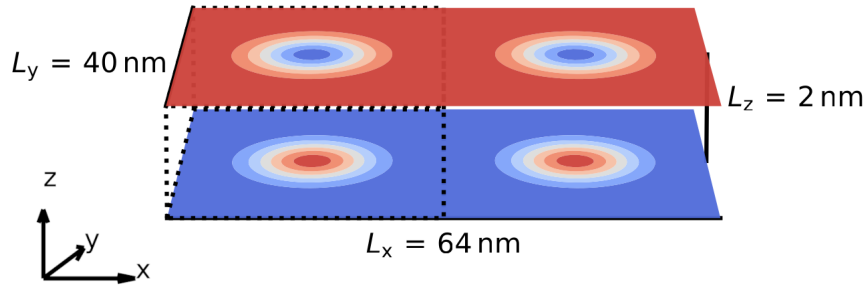


Figure 5.1: Schematic of the two skyrmion system confined in a $(64 \times 40 \times 2) \text{ nm}^3$ geometry.

excitation field is applied to the left pair of top-bottom skyrmions only (i.e. for $x < 32 \text{ nm}$, in Fig. 5.1 indicated by the dotted lines) and applied in the out-of-plane direction. For subsequent studies with more than two skyrmions, we will also excite only the left-most SAF skyrmion.

The PSD shown in Fig. 5.2, shows eight normal modes. We will identify below that the first four modes A to D are translational modes, and modes E to H are breathing modes.

Figure 5.3 shows phase-amplitude plots of the normal modes A to D. We see elliptical trajectories for all 4 modes, with opposite gyration sense in top and bottom layer for each mode. This corresponds to the translation modes we identified for the single SAF skyrmion (Fig. 4.12) but here extended to two SAF skyrmions. The lowest frequency

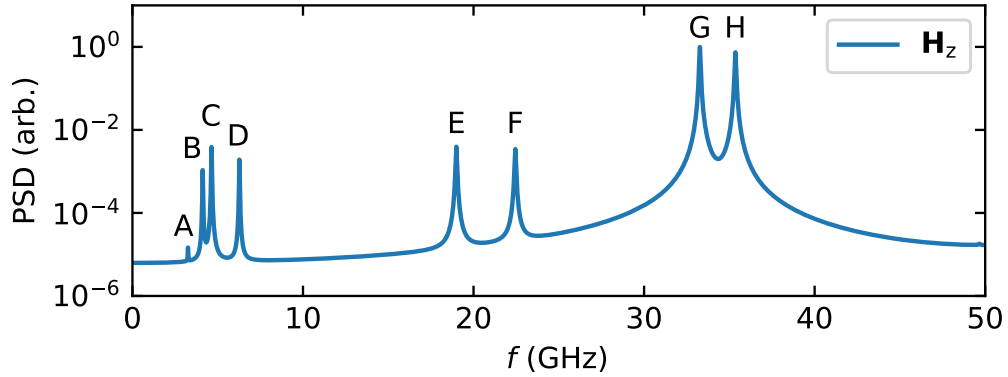


Figure 5.2: PSD for the two skyrmion system. The excitation field is acting in the out-of-plane direction and only excites the left SAF skyrmion, indicated by the dotted lines in Fig. 5.1.

mode A is now showing horizontal translation presumably because the shape anisotropy of the sample make the x-direction an easy axis for skyrmion displacement.

The same geometry ($L_x = 64$ nm and $L_y = 40$ nm) with two skyrmions has been studied as a single-layer ferromagnetic system [150] and the lowest frequency mode found shows elliptical trajectories with the long axis along the x-direction (resembling qualitatively the orientation of the elliptical trajectories of the skyrmion cores we find in the top layer in mode A). This supports the interpretation that an elliptical shape of the trajectory is independent from the interlayer coupling and driven by the shape anisotropy of the sample (see also A.4). However, the interlayer coupling makes the elliptical trajectories more anisotropic as the two layers favour opposite gyration directions. The skyrmion pairs forming a SAF skyrmion are always in-phase along the long axis of the ellipse between the RKKY coupled layers.

As we have two SAF skyrmions next to each other, they can show in-phase or out-of-phase motion relative to the other SAF skyrmion: modes A and C are in-phase between the neighbouring skyrmions and B and D are out-of-phase.

The amplitude phase maps for modes E, F, G and H are shown in Fig. 5.4. These are all breathing modes. The lowest frequency breathing mode E shows in-phase dynamics for the top and bottom skyrmions, but out-of-phase coupling between the left and the right skyrmion. While the left SAF skyrmion contracts, the right SAF skyrmion undergoes radial expansion, corresponding to a phase shift of π between the left and right SAF skyrmion. The out-of-phase dynamics of skyrmions in a given layer provides an opportunity for one skyrmion to grow and fill the space when the other shrinks, thus reducing the energetic cost of this mode. At the same time, the interlayer coupling is strong enough to

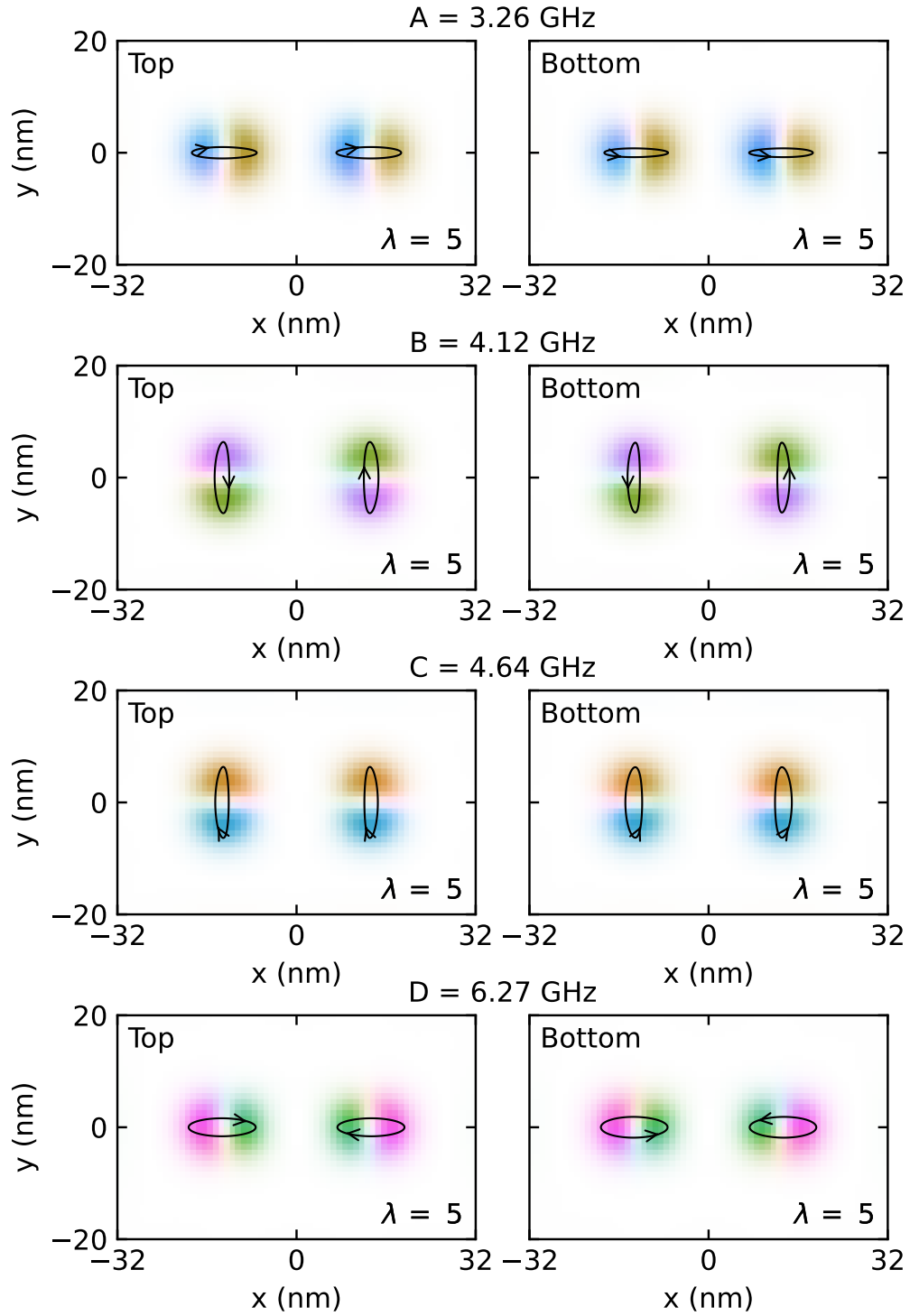


Figure 5.3: Normal modes A to D of the two skyrmion system. All four modes have translational character.

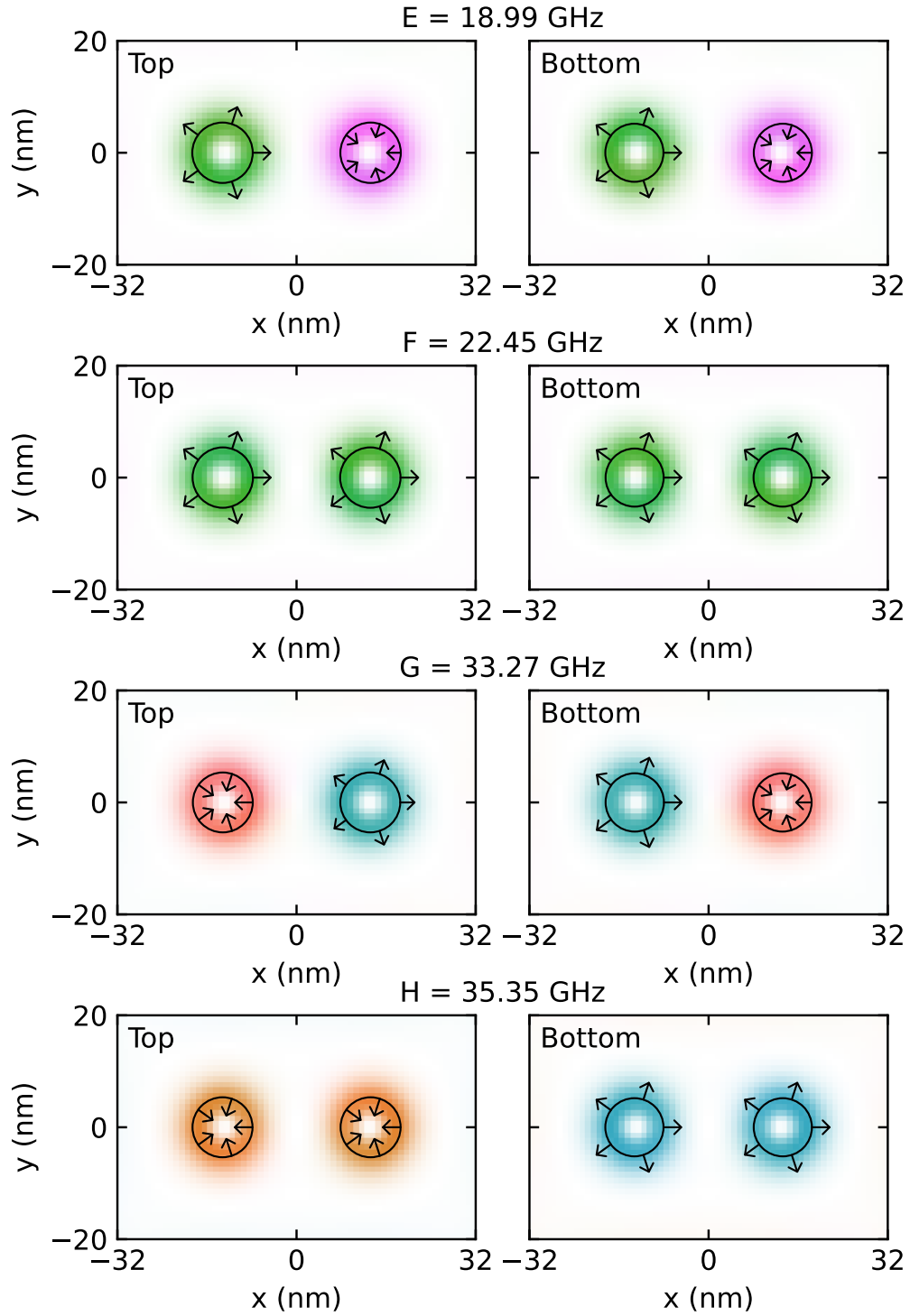


Figure 5.4: Normal modes E to H of the two skyrmion system. All four modes are breathing modes.

reward in-phase dynamics between the top and bottom layer.

Mode F shows in-phase behaviour across the layers, and between left and right skyrmion. The radial expansion and contraction occur simultaneously for the left and right skyrmions, and in both layers. We hypothesize that mode F has a higher energy than mode E because the left and right skyrmion compete for space when expanding simultaneously.

Mode G decouples motion of top and bottom layer: when the skyrmion in the top layer grows, the corresponding one in the bottom layer shrinks. This is combined with an out-of-phase behaviour between left and right skyrmion.

The most energetically expensive combination is seen for mode H: out-of-phase across the layer interface, and in-phase between left and right skyrmion.

The dependence of the resonance frequencies of the first eight modes on the interlayer coupling strength $|\sigma|$ is shown in Fig. 5.5.

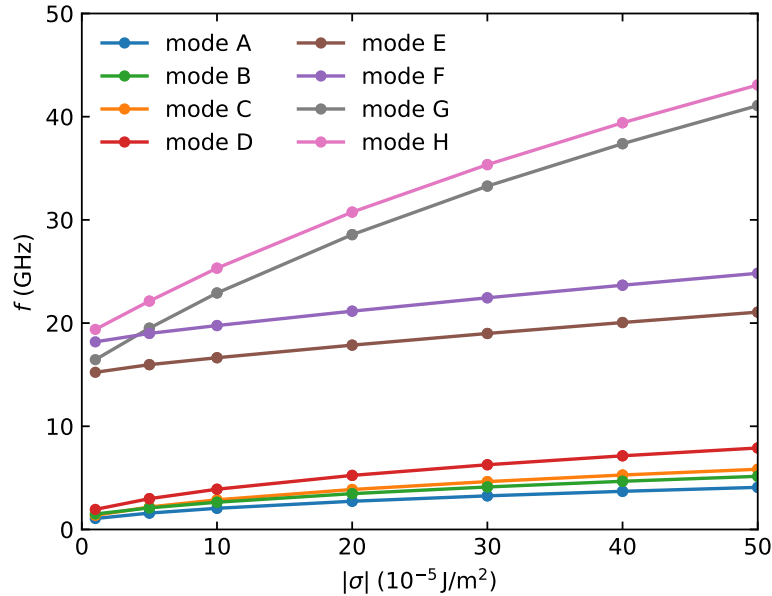


Figure 5.5: Frequencies of the first eight eigenmodes for varying interlayer RKKY coupling strength $|\sigma|$. For weak coupling we observe a different order of the modes.

Overall, we can see a decrease in frequency with decreasing interlayer coupling. This is expected as the total energy of the system decreases with decreasing $|\sigma|$. For the two modes G and H we can observe a stronger frequency decrease with decreasing $|\sigma|$ than for the other modes, and for $|\sigma| \rightarrow 0$ they approach modes E and F, respectively. In the limit $\sigma = 0$ the two layers are uncoupled (ignoring demagnetisation effects for a moment),

and the four eigenfrequencies collapse to just two, with the modes E and G (left right out of phase) at the lower frequency, and modes F and H (left right in phase) at the higher frequency. This mode reordering (swapping of modes F and G) can be seen just below $|\sigma| = 5 \cdot 10^{-5} \text{ J m}^{-2}$, where modes F and G cross. In reality the two layers are for $|\sigma| = 0$ still weakly coupled via the demagnetisation field and the frequency degeneracy at $\sigma = 0$ is lifted (instead further mode re-ordering would occur for $|\sigma| > 0$). For better clarity we exclude results for $|\sigma| < 1 \cdot 10^{-5} \text{ J m}^{-2}$ in Fig. 5.5, so that RKKY is dominating the interlayer coupling for everything shown in the figure. In most parts of this study we use the value $|\sigma| = 30 \cdot 10^{-5} \text{ J m}^{-2}$. In summary, for the two SAF skyrmion system, we have established translational modes in the frequency range 3 to 6 GHz (Fig. 5.3) and breathing modes in the frequency range 18 to 35 GHz (Fig. 5.4). For different values of σ , the frequencies adjust: stronger coupling increases the frequency of the modes.

5.2 Five-skyrmion chain

We extend the analysis of the magnetisation modes to a five-skyrmion strip geometry of length $L_x = 160 \text{ nm}$ as shown in Fig. 5.6.

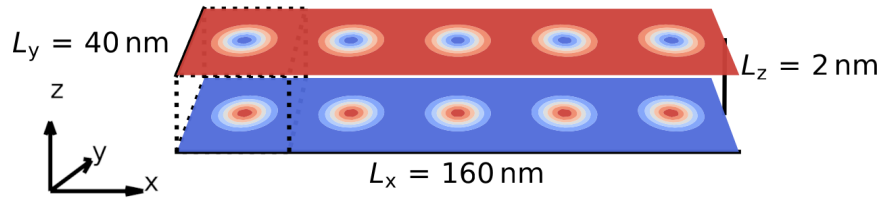


Figure 5.6: System geometry for the five skyrmion study. The dotted lines indicate the region where the excitation pulse is applied.

Figure 5.7 shows the PSD obtained for the given excitation pulse (out-of-plane on the area occupied by the left-most SAF skyrmion). For extra clarity the eigenfrequencies are also marked with lines. We note that mode B couples only very weakly to the chosen excitation and modes G and H have very nearby frequencies leading to overlapping peaks for the standard damping $\alpha = 10^{-3}$ used throughout the work.

The inset in Fig. 5.7 shows a smaller frequency range containing modes D to H for two different damping values. Here we can more clearly see that modes G and H cannot fully be resolved with $\alpha = 10^{-3}$, but split into two distinct peaks for $\alpha = 10^{-4}$.

Modes A to J are translation modes (in Fig. 5.7 horizontal and vertical translation is indicated by dashed and solid lines, respectively), modes K to O are breathing modes

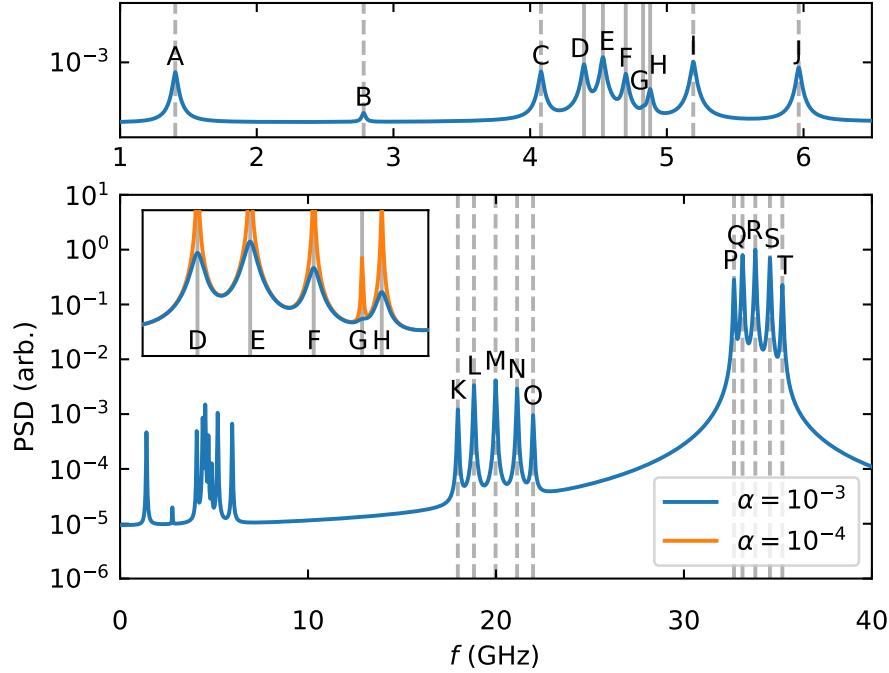


Figure 5.7: PSD in response to an out-of-plane excitation. The low-frequency range up to 6 GHz is shown in more detail in the top panel. The vertical lines indicate the eigen frequencies. As a guide to the eye, dashed and solid lines in the top panel indicate translational horizontal and vertical modes, respectively. The inset shows the translational vertical modes for two different damping values.

where top and bottom layer are in phase and modes P to T are breathing modes where top and bottom layer are out of phase.

For five skyrmions we see that the different types of modes start to form bands. For the breathing modes all eigenmodes in a band have relatively similar intensity. These modes all couple well to the out-of-plane excitation. Coupling efficiency to the translational modes depends stronger on the specifics of the mode. Furthermore, horizontal and vertical translational modes have overlapping frequencies, making the spectrum appear less clean.

The phase amplitude maps for the translational horizontal modes are shown on the left in Fig. 5.8(a), those for the translational vertical modes on the left in Fig. 5.8(b). The lowest frequency mode A for the 5 skyrmion system (Fig. 5.8(a)) exhibits horizontal translation of the skyrmion cores. All 5 skyrmions move in-phase. The amplitude of the excitations varies from skyrmion to skyrmion. It is smallest for skyrmions 1 (on the left) and 5 (on the right) and largest for skyrmion 3 (in the centre). The lower amplitude can be seen through the lower opacity of the colours in the amplitude phase map. Additionally, the

plot to the right in Fig. 5.8(b), which shows the amplitude of skyrmion centre displacement for individual skyrmions, emphasises this difference. The second lowest frequency mode B also shows variation of excitation amplitude: the centre skyrmion 3 is not significantly excited and thus appears (virtually) transparent in the phase-amplitude plot. The two skyrmions to the left (1 and 2) move together (i.e. in-phase) but are out-of-phase relative to the two skyrmions on the right (4 and 5). Similar inhomogeneities in the intensity distribution can also be observed for most of the other translational modes shown in Figs. 5.8.

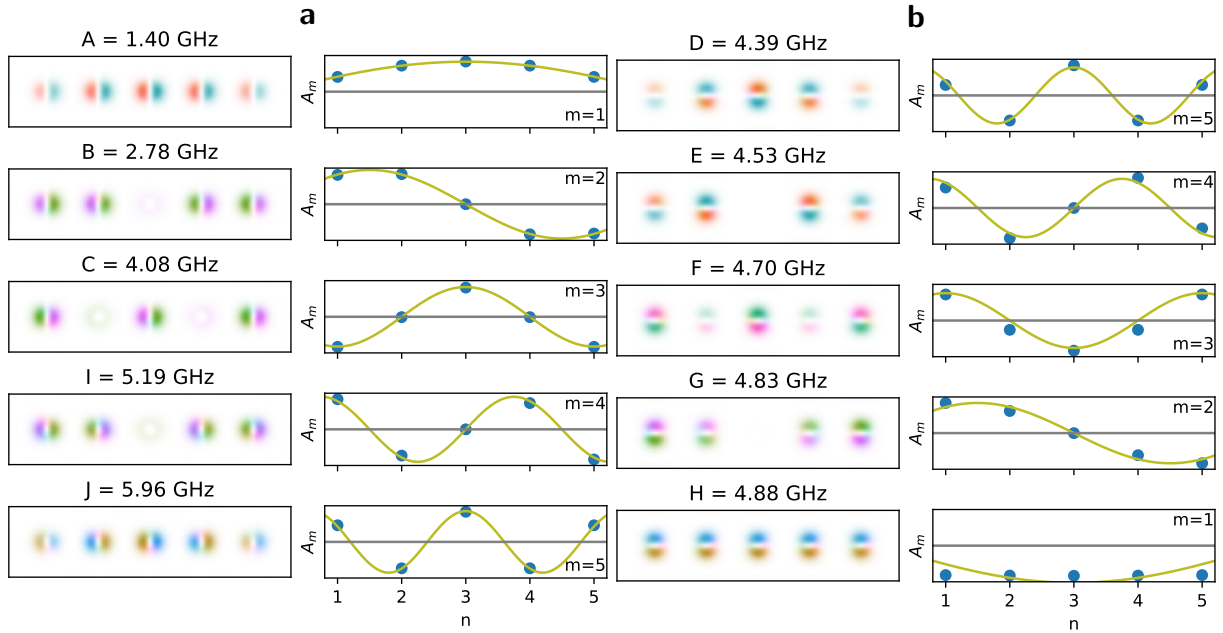


Figure 5.8: (a) Left: Phase amplitude maps for the translation horizontal modes. Right: Amplitudes $A_m(n)$ of collective skyrmion coordinates from simulation (blue points) and amplitudes $\tilde{A}_m(n)$ (5.1) of the associated standing mode m (green line). (b) Phase amplitude maps and amplitudes of the collective skyrmion coordinates for the translational vertical modes.

A model has been proposed for gyrotropic modes of skyrmions in a ferromagnet [150] and for breathing modes of skyrmions in a ferromagnet [151] where the amplitude A of the skyrmion excitation follows a standing wave

$$\tilde{A}_m(n) = a \sin(k_m n), \quad (5.1)$$

with $k_m = m\pi/(N + 1)$, where $n = 1, 2, \dots, N$ is the index of the skyrmion ranging from 1 (left-most skyrmion) to $N = 5$ (right-most skyrmion), $m = 1, 2, \dots, N$ denotes the standing wave mode index, and a is a fitting constant. Through (5.1) the boundary

conditions are that (virtual) skyrmions at positions $n = 0$ and $n = N+1$ have an amplitude of 0.

Can the same model be applied for the more complex two-layer SAF system under investigation here? Figure 5.8(a) shows good fits for the translational horizontal modes. We find that the model also describes well the breathing modes K to T as shown in Fig. 5.9

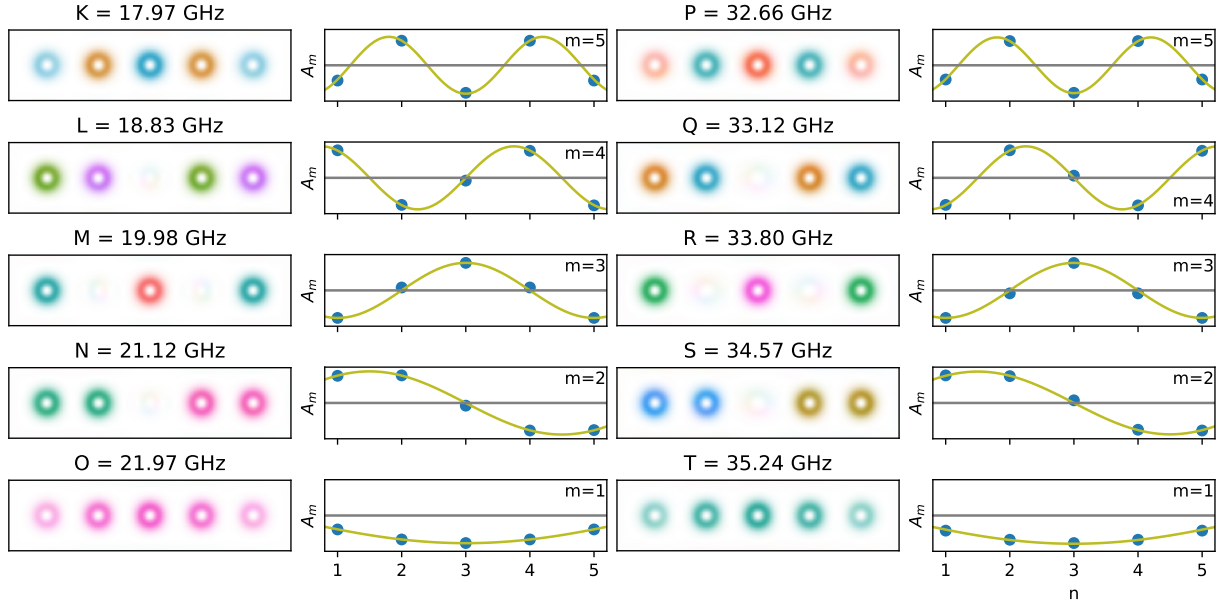


Figure 5.9: Breathing modes K to T of the five-skyrmion system. The phase amplitude maps show the top layer. For modes K to O both layers breathe in phase, for modes P to T they breathe out of phase (see Fig. A.8 for phase amplitude maps showing both layers). The plots in columns two and four show amplitudes $A_m(n)$ of collective skyrmion coordinates from simulation (blue points) and amplitudes $\tilde{A}_m(n)$ of the associated standing mode m (green line).

However, the model seems not appropriate for the translational vertical modes as shown in Fig. 5.8(b). In particular mode H at 4.88 GHz shows uniform translational motion of all 5 skyrmion cores: it is impossible to fit a sin function as in (5.1) with amplitude $\tilde{A}_m(0) = 0$ and $\tilde{A}_m(N+1) = 0$.

In summary, for five skyrmions, we find the same translational and breathing modes as for one and two skyrmions. The different types of modes start to form bands, where for a given mode type (e.g. breathing top bottom in-phase) the extrema are out-of-phase and in-phase excitation of neighbouring SAF skyrmions, respectively.

5.3 Signal propagation in an extended skyrmion chain

Finally, we show results for a longer chain of 25 SAF skyrmions, in the spirit of similar studies done for a ferromagnetic chain of skyrmions [150, 151]. In this system we can study signal propagation along the skyrmion chain.

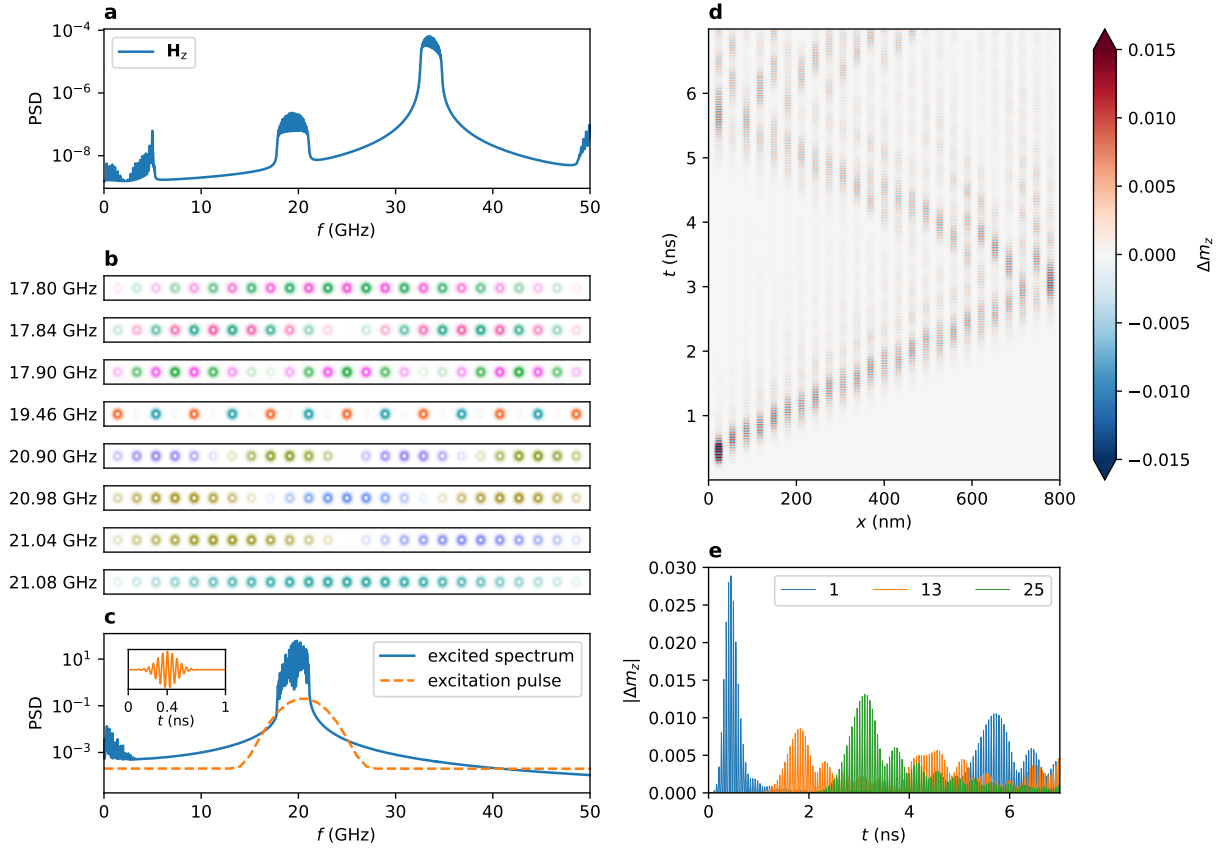


Figure 5.10: (a) PSD for 25 skyrmions. (b) Subset of the breathing modes in the 20 GHz band. (c) Spectrum after exciting the 20 GHz band with a gaussian pulse. Only the left-most skyrmion is excited by the pulse. The pulse power distribution in frequency space is shown with the dashed line. Inset: shape of the excitation pulse in time domain. (d) Propagation of excitation through the strip. (e) Excitation of selected skyrmions in the strip (1: left-most skyrmion, 13: centre skyrmion, 25: right-most skyrmion). The applied excitation has its maximum at $t = 0.4$ ns.

Figure 5.10a shows the power spectrum for out-of-plane excitation of the left-most SAF skyrmion. The different types of eigenmodes (translation, breathing) now have many possible realisations, resulting in the formation of distinct bands in the PSD. At low frequencies, we can see the translational modes, horizontal and vertical translation are

intermixed as we have already seen for five skyrmions. At around 20 GHz we see a band of top bottom in-phase breathing modes, at 35 GHz top bottom out-of-phase breathing modes. Near 50 GHz we see the onset of a higher rotational mode, similar to mode C in the ferromagnetic study (Fig. 4.5).

Figure 5.10b shows a subset of the breathing modes in the 20 GHz band: the three lowest frequency modes, the mode in the centre of the band, and the four highest frequency modes. The lowest mode at 17.80 GHz shows alternating out-of-phase excitation of neighbouring skyrmions, at the highest mode at 21.08 GHz all skyrmions breathe in-phase. At the centre mode (19.49 GHz) only every second skyrmion is excited and neighbouring excited skyrmions are out of phase. The intensity modulation seen over the whole the strip follows model (5.1).

To demonstrate signal propagation along the SAF skyrmion chain we now switch to a different type of excitation and focus on the top bottom in-phase breathing band around 20 GHz. Instead of a sinc-pulse with equal power for all frequencies up to 50 GHz, we excite the left-most skyrmion with a Gaussian pulse centred at $f_0 = 20.5$ GHz:

$$\mathbf{H}(t) = H_0 \sin(2\pi f_0 t) \exp\left(-\frac{(t - t_0)^2}{2\sigma_f^2}\right) \hat{\mathbf{z}}, \quad (5.2)$$

where $t_0 = 0.4$ ns is time of maximum excitation, $\sigma_f = 0.1$ ns controls the width of the excitation in frequency space and H_0 is the excitation power, which we choose as $\mu_0 H_0 = 1$ mT. The central frequency $f_0 = 20.5$ GHz is chosen somewhat arbitrarily and not central in the band. The exact location is irrelevant provided the overall pulse overlaps with the breathing band. The dashed line in Fig. 5.10c shows the pulse profile in frequency space (scaled by an arbitrary factor; the unusual shape of the gaussian profile is caused by the logarithmic y-axis). We evolve the LLG for 1 ns with applied excitation, i.e. $\pm 4\sigma_f$ plus an extra 0.2 ns at the end of the excitation. The excitation strength at the beginning and end of this time frame is very close to zero. After the excitation we record data of the free LLG relaxation for further 20 ns.

Figure 5.10c shows a power spectrum for the free relaxation, i.e. the last 20 ns. The whole top bottom in-phase breathing band has been excited. Furthermore, we can observe weak excitation of parts of the low-frequency translational modes. The higher-frequency top bottom out-of-phase breathing band is not excited.

Figure 5.10d shows the signal propagation through the strip during the first 7 ns (including the excitation). We plot the deviation Δm_z from the equilibrium configuration in the bottom layer averaged over the strip width. The vertical stripe-like features correspond to the 25 skyrmions. We can see the strong excitation of the left-most skyrmion

during the first ~ 0.8 ns. After that the signal propagates through the strip (to the right in the figure) until it reaches the right strip edge at around 3 ns and is reflected. Shortly before 6 ns the reflected signal arrives back at the left strip edge. We can see that the excited modes seem to have slightly different velocities and the excitation pulse disperses over time (sub-dominant features in the figure after the first excitation). Reflection at the sample edges seems to contribute significantly to the peak broadening.

Figure 5.10e shows the excitation of skyrmions 1 (left strip edge, excited by the pulse), 13 (strip centre), and 25 (right strip edge) in more detail. We can see the high-frequency oscillation of the carrier frequency of the excitation pulse. The low-frequency envelope corresponds to the signal propagating through the strip. By tracking the first maximum, we can compute the average speed of the signal, and obtain approximately 300 m/s. This is slightly slower than the value (340 m/s) reported for the ferromagnetic system [151].

5.4 Discussion and outlook

In this chapter, the dynamics of isolated SAF skyrmions discussed in Chapter 4 were extended to finite skyrmion chains. By increasing the number of skyrmions from two to five and finally to 25, we have shown how the resonant response evolves from individual skyrmion modes into collective excitations of a coupled system.

The two-skyrmion system provides the clearest starting point for understanding this coupling. In this geometry, the low-frequency modes are translational, whereas the higher-frequency modes have breathing character. Compared with the isolated SAF skyrmion, each mode is split according to the relative phase between neighbouring skyrmions and between the two antiferromagnetically coupled layers. The translational modes show elliptical core motion, with the top and bottom layers favouring opposite gyration senses. This indicates that the observed dynamics are shaped by both the lateral confinement of the strip and the interlayer RKKY coupling.

For the five-skyrmion chain, the same types of modes are retained, but the spectrum becomes more structured. Instead of isolated resonances only, groups of modes begin to form. The translational modes are more difficult to separate because horizontal and vertical displacements occur in overlapping frequency ranges. Nevertheless, the mode profiles show clear collective behaviour across the chain. In particular, the horizontal translational modes and the breathing modes can be interpreted using a standing-wave-like amplitude profile along the skyrmion index. This suggests that, despite the additional complexity introduced by the two-layer SAF structure, some features of the collective dynamics can still be understood using a coupled-oscillator picture.

The 25-skyrmion chain further demonstrates the emergence of band-like mode structures. In this longer system, the top-bottom in-phase breathing modes form a band around 20,GHz, while the top-bottom out-of-phase breathing modes appear at higher frequencies. By exciting the left-most SAF skyrmion with a Gaussian pulse centred on the lower breathing band, the propagation of a breathing-mode signal along the chain was observed. The signal travels through the strip, reflects at the boundary, and returns towards the excitation region. This shows that the collective breathing modes are not only visible in the frequency-domain spectra, but can also mediate time-dependent signal transport through the SAF skyrmion chain.

Overall, these results show that the transition from one SAF skyrmion to a finite chain is not simply a repetition of the isolated-skyrmion response. Instead, the interaction between neighbouring skyrmions introduces additional phase relationships, mode splitting, and eventually band-like collective behaviour. The RKKY interaction plays a central role in determining how the two magnetic layers move relative to one another, while the lateral coupling between skyrmions controls the formation of collective modes along the strip. This makes SAF skyrmion chains a useful platform for studying coupled magnetic excitations in confined geometries.

Outlook

Although the simulations in Chapters 4 and 5 give direct access to the full magnetisation dynamics, an experimental measurement would only detect the part of the response that is visible to a given technique. This means that the identification of the modes may not be as direct as in the simulations. A useful next step would therefore be to calculate the expected magnetic contrast or microwave response of the modes found here. This would help to determine which of the in-phase and out-of-phase skyrmion modes could be observed experimentally.

This direction is also motivated by earlier work on coupled magnetic vortices. Vogel et al. showed that two spatially separated vortex cores in permalloy squares can interact through their stray fields and behave like coupled harmonic oscillators [158]. When one vortex core was excited, the second vortex core also started to gyrate, and the system showed in-phase and antiphase normal modes. A similar physical picture appears in the SAF skyrmion systems studied in this thesis, where the coupling between the layers and between neighbouring skyrmions leads to different collective modes.

Another important point is the effect of sample imperfections. The systems studied in this thesis are idealised, with well-defined geometry and material parameters. In real

samples, defects, edge roughness, thermal fluctuations, and local variations in material parameters can change both the relaxed skyrmion state and its dynamics. Including these effects would show whether the same modes remain visible when the system is less ideal.

The excitation of selected modes can also be studied further using more realistic driving mechanisms. In this work, different excitation profiles were used to excite selected parts of the mode spectrum and to study signal propagation. A possible extension would be to use microwave antennas, spin-orbit torques, or current-induced torques as excitation sources. This would make it possible to test whether particular modes can be excited efficiently in a geometry closer to experiment.

The results in Chapters 4 and 5 show that the dynamics of synthetic antiferromagnetic skyrmions are strongly influenced by the coupling between layers and between neighbouring skyrmions. A natural extension of this work is to study how these modes would appear in experimentally measurable signals, and how stable they remain when more realistic sample conditions are included.

Chapter 6

Computational Modelling of Ferrimagnetic Skyrmions in DyCo₃ Thin Films

The theoretical prediction [159, 160] and subsequent experimental observation [91, 161] of skyrmion lattices in non-centrosymmetric bulk crystals marked a significant milestone in magnetism. In these materials, an intrinsic DMI of bulk origin provides the chiral interaction necessary for stabilising Bloch-type skyrmions [18, 21]. Despite this fundamental understanding, the practical application of these bulk skyrmion systems is often hindered by their stability conditions, typically confined to narrow low-temperature ranges and specific applied magnetic fields [111, 162]. To overcome this low-temperature limitation, researchers have explored ways to achieve room-temperature skyrmions. Strategies include reducing the material’s dimensionality using thin films [163, 164] or using interfacial DMI found in ultrathin films and multilayers, which leads to the formation of Néel-type skyrmions [97, 124, 126, 165, 166]. Additionally, skyrmions have been reported in systems without intrinsic DMI, where an *effective* DMI can arise from curvature-induced magnetic frustration [167–170], non-linear magnetic interactions [171], or stray fields in systems with strong perpendicular magnetic anisotropy [120].

In Chapter 2, it was discussed in Section 2.3 how skyrmions offer notable advantages in antiferromagnetic systems over ferromagnetic ones. As an alternative route, ferrimagnetic materials¹ have recently attracted attention for developing skyrmion-based systems suitable for technological applications [52, 172, 173]. These materials often exhibit perpendicular magnetic anisotropy [173–175] and are considered promising for stabilising skyrmion

¹particularly amorphous thin films composed of 4f rare-earth and 3d transition-metal elements.

states due to the presence of DMI. The DMI may originate from asymmetric elemental distribution in the bulk [53] or weak interfacial effects from capping layers [54]. The lack of inversion symmetry in amorphous materials further supports a non-vanishing DMI [55]. The first experimental demonstrations of skyrmions in ferrimagnetic RE–TM alloys have been reported only recently, with evidence of reduced or fully suppressed skyrmion Hall effect (SkHE) near the angular momentum compensation point [126, 176]. Specifically, in DyCo_x alloys, DMI has been observed in both single-crystal and amorphous forms [52, 177, 178].

6.1 Experimental results

In recent experiments [50], skyrmions were observed in ferrimagnetic RE–TM alloy films with strong perpendicular magnetic anisotropy. This was demonstrated by investigating a 50 nm thick ferrimagnetic DyCo₃ film capped with a 3 nm layer of tantalum (Ta) to prevent oxidation as shown in Fig. 6.1. The study employed X-ray magnetic circular dichroism (XMCD), small-angle X-ray resonant magnetic scattering (SAXRMS), high-resolution scanning transmission X-ray microscopy (STXM) with circularly polarised X-rays, and Hall transport measurements.

The results showed that under the application of a perpendicular magnetic field, the DyCo₃ thin films exhibited distinct magnetic phase transitions. Starting from negative magnetic saturation, the nucleation of isolated skyrmions was observed at 90 K, occurring at a magnetic field of -0.110 T. These skyrmions were characterised by a compact structure, featuring a core radius of approximately $35 \text{ nm} \pm 5 \text{ nm}$ and $42 \text{ nm} \pm 5 \text{ nm}$, and a domain wall width of $50 \text{ nm} \pm 5 \text{ nm}$. As the magnetic field was subsequently swept towards less negative or into positive values, these isolated skyrmions tended to merge, transitioning into a maze-like domain pattern, which persisted up to approximately $+0.24$ T. A further increase in the positive field strength initiated the annihilation of this maze-like structure, a regime where a second phase of isolated skyrmions was indicated before the film reached complete magnetic saturation in the positive direction. Unlike bubble-like skyrmions typically observed in ferromagnetic systems, the compact skyrmions² in DyCo₃ were found to have a narrow core and a broad shell, consistent with theoretical predictions for ferrimagnetic systems [21].

²Compact skyrmions, typically stabilised by DMI, are small (5–100 nm) and exhibit continuous spin textures, whereas bubble-like skyrmions emerge from dipolar interactions, are considerably larger (100 nm–1 μ m), and often form through the field-driven transformation of stripe domains, as discussed in Refs. [18, 120], a detailed analysis of the differences can be found in Ref [179].

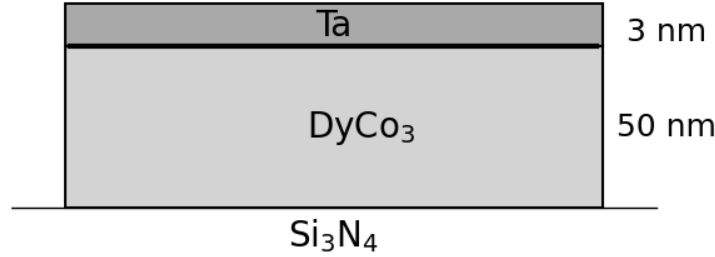


Figure 6.1: Schematic cross-section of the simulated ferrimagnetic thin-film structure. The system consists of a 50 nm thick DyCo_3 layer deposited on a Si_3N_4 substrate and capped with a 3 nm thick Ta layer. The Ta capping layer stabilises the interface and can contribute to interfacial effects such as the Dzyaloshinskii–Moriya interaction, while the Si_3N_4 substrate provides structural support.

To gain deeper insight into the stabilising interactions responsible for these experimentally observed skyrmions, we performed micromagnetic simulations in close collaboration with the experimental team. This work was initiated following their request to model the system and investigate the role of DMI in stabilising the observed skyrmion configurations.

6.2 Micromagnetic model and simulation framework

6.2.1 Geometry and discretisation

All micromagnetic simulations in this chapter were performed using the Object-Oriented MicroMagnetic Framework (OOMMF) [180] and the GPU-accelerated MuMax³ code [90], driven through the Ubermag Python interface [61–63]. The simulated film is a $1\text{ }\mu\text{m} \times 1\text{ }\mu\text{m}$ square with a thickness of 50 nm. Two complementary discretisation schemes are employed in this work:

- For the depth-resolved analysis of Sections 6.3–6.5, the film is discretised into $1000 \times 1000 \times 10$ cells of size $1\text{ nm} \times 1\text{ nm} \times 5\text{ nm}$. The ten layers in the thickness direction allow the depth-dependent magnetisation profile—and in particular the helicity $\gamma(z)$ —to be resolved while keeping the simulation tractable.
- For the two-dimensional (K_{eff} , D) parameter sweeps of Section 6.6, where the question is the dependence of the skyrmion stability on the bulk parameters rather than the depth-resolved structure, the film is discretised with a single cell of $1\text{ nm} \times 1\text{ nm} \times 50\text{ nm}$ through the thickness. This reduction is computationally

essential, as each sweep involves several hundred independent simulations, and is appropriate once the depth dependence has been characterised separately.

The relevant length scale that constrains the in-plane discretisation is the exchange length $l_{\text{ex}} = \sqrt{2A/(\mu_0 M_s^2)}$. For the parameters of Table 6.1, $l_{\text{ex}} \approx 7.3$ nm, and the chosen in-plane cell size of 1 nm safely satisfies $\Delta x, \Delta y \ll l_{\text{ex}}$.

6.2.2 Energy functional

The total magnetic energy is the sum of five contributions,

$$E[\mathbf{m}] = \int_V \left[\underbrace{-A \mathbf{m} \cdot \nabla^2 \mathbf{m}}_{w_a} - \underbrace{K_u (\mathbf{m} \cdot \mathbf{u})^2}_{w_K} + \underbrace{D (\mathbf{m} \cdot \nabla m_z - m_z \nabla \cdot \mathbf{m})}_{w_{\text{dmi}}^z} \right. \\ \left. - \underbrace{\mu_0 M_s \mathbf{m} \cdot \mathbf{H}}_{w_z} - \underbrace{\frac{1}{2} \mu_0 M_s \mathbf{m} \cdot \mathbf{H}_d}_{w_{\text{dem}}} \right] d^3 r, \quad (6.1)$$

where $\mathbf{m} = \mathbf{M}/M_s$ is the unit magnetisation vector. The five terms are, respectively, the isotropic exchange interaction characterised by exchange stiffness A ; the uniaxial magnetocrystalline anisotropy with easy axis $\mathbf{u} = \hat{\mathbf{z}}$ and intrinsic anisotropy constant K_u ; the interfacial DMI of C_{nv} symmetry with constant D ; the Zeeman coupling to the externally applied field $\mathbf{H}$; and the magnetostatic (demagnetising) energy with self-induced field $\mathbf{H}_d$. The exchange term favours a spatially uniform magnetisation, $K_u > 0$ stabilises an out-of-plane background, $D \neq 0$ favours canting of neighbouring spins and selects a preferred helicity, the Zeeman term aligns the magnetisation with the applied field, and the demagnetising term penalises stray fields, particularly near the film surfaces.

6.2.3 Material and simulation parameters

The material parameters adopted in this work are summarised in Table 6.1; they are inherited from Ref. [50] where available. The saturation magnetisation per DyCo₃ cluster was experimentally found to range between $2.5 \mu_B$ and $3.5 \mu_B$ below 150 K, decreasing to $1.0 \mu_B$ above 200 K. As the experiments of Ref. [50] were conducted at 90 K, the corresponding value $M_s = 4.2 \times 10^5$ A/m is used throughout Sections 6.3–6.6.

A note on anisotropy conventions: the constant K_u in Eq. (6.1) is the *intrinsic* (magnetocrystalline) anisotropy and is the quantity provided as input to the simulation. In thin-film systems, however, the experimentally accessible anisotropy is the *effective* anisotropy [72, 181, 182], which arises from the competition between the intrinsic

Table 6.1: Material and simulation parameters used for the DyCo₃ thin film. The DMI constant D and the intrinsic uniaxial anisotropy K_u are treated as control parameters in the sweeps of Sections 6.5.2 and 6.6; the remaining parameters are held fixed.

Parameter	Symbol	Value
Exchange stiffness	A	$6 \times 10^{-12} \text{ J m}^{-1}$
Intrinsic uniaxial anisotropy	K_u	varied (see text)
DMI constant	D	varied (see text)
Saturation magnetisation	M_s	$4.2 \times 10^5 \text{ A m}^{-1}$
Applied flux density	B	-0.110 T
Gilbert damping	α	0.01

anisotropy and the demagnetising (shape) anisotropy:

$$K_{\text{eff}} = K_u - \frac{1}{2}\mu_0 M_s^2, \quad (6.2)$$

where the subtracted term represents the shape anisotropy of a thin film with out-of-plane magnetisation. Throughout this chapter, simulations are parameterised by K_u , but results are analysed and compared with experiment in terms of K_{eff} . Positive K_{eff} corresponds to perpendicular magnetic anisotropy, while negative K_{eff} describes an easy-plane regime in which demagnetisation dominates.

6.3 Depth-resolved structure of magnetic bubbles at zero DMI

Before investigating the effect of DMI, we first consider the simpler case $D = 0$, where no chirality-selection mechanism is present. In this limit, DMI does not contribute to the energy landscape and therefore does not bias the helicity of the domain wall.

In this section, we use the layered $1000 \times 1000 \times 10$ mesh introduced in Section 6.2.1, with $K_u = 1.68 \times 10^5 \text{ J m}^{-3}$. This corresponds to $K_{\text{eff}} \approx 57 \text{ kJ m}^{-3}$ via Eq. (6.2), placing the system well within the perpendicular-anisotropy regime. This choice is methodological: it provides a stable and well-resolved $|Q| = 1$ texture, as defined in Eq. (2.27), while allowing the depth dependence of the magnetisation profile to be characterised in detail.

The effect of varying K_u and D is explored separately in Section 6.6. The initial condition is a cylindrical bubble: $\mathbf{m} = +\hat{\mathbf{z}}$ inside a radius of 50 nm and $\mathbf{m} = -\hat{\mathbf{z}}$ everywhere

else. The system is relaxed to its energy minimum using the **MinDriver** in OOMMF, which iteratively minimises the total energy in Eq. (6.1).

Depending on the random perturbation added to this seed, the relaxation converges to one of two topologically distinct metastable states. Following the classification introduced by Malozemoff and Slonczewski [183], we label these *type 1* and *type 2*.

6.3.1 Type 1 and type 2 bubbles

Figure 6.2 shows top-view maps of the type 1 bubble at three representative depths: near the bottom surface ($z \approx 2.5$ nm), near the mid-plane ($z = 22.5$ nm), and near the top surface ($z \approx 47.5$ nm).³ Panels (a)–(c) display the out-of-plane component m_z . At every depth, a well-defined core with $m_z = +1$ is surrounded by a uniformly magnetised background with $m_z = -1$. The radius remains approximately unchanged through the thickness, suggesting that the texture forms an almost cylindrical tube.

Panels (d)–(f) display the in-plane magnetisation angle, $\phi_{\text{ip}} = \arctan 2(m_y, m_x)$. The angle is represented using a "husl" colour palette, while regions with a small in-plane component are faded towards white according to $|\mathbf{m}_{\parallel}| = \sqrt{m_x^2 + m_y^2}$. This makes the domain wall region visible, where the in-plane magnetisation is strongest, while suppressing the nearly uniform core and background regions. In this representation, a uniform colour around the domain wall indicates a nearly constant helicity, whereas a systematic colour change through the film thickness indicates a depth-dependent rotation of the wall magnetisation.

The type 1 bubble shows a smooth and radially symmetric helicity pattern at each depth. The colour around the domain wall remains nearly uniform within each horizontal slice, indicating that the in-plane spins rotate with the same sense around the circumference. However, the overall colour changes from the bottom to the top surface, showing that the helicity rotates continuously through the film thickness. This depth-dependent helicity variation is quantified in Section 6.5.1.

The type 2 bubble, shown in Fig. 6.3, has an m_z profile [panels (a)–(c)] that is very similar to the type 1 state. The core radius and the overall shape of the domain wall remain almost unchanged through the film thickness. The difference becomes clear in the in-plane angle maps [panels (d)–(f)]. Unlike the type 1 bubble, where the colour around the domain wall is nearly uniform within each horizontal slice, the type 2 bubble shows local interruptions in the angular pattern at discrete positions along the wall. These

³The mid-plane sample is taken at $z = 22.5$ nm rather than $z = 25$ nm because the latter lies on the boundary between two cells; we therefore choose the centre of one of the neighbouring cells.

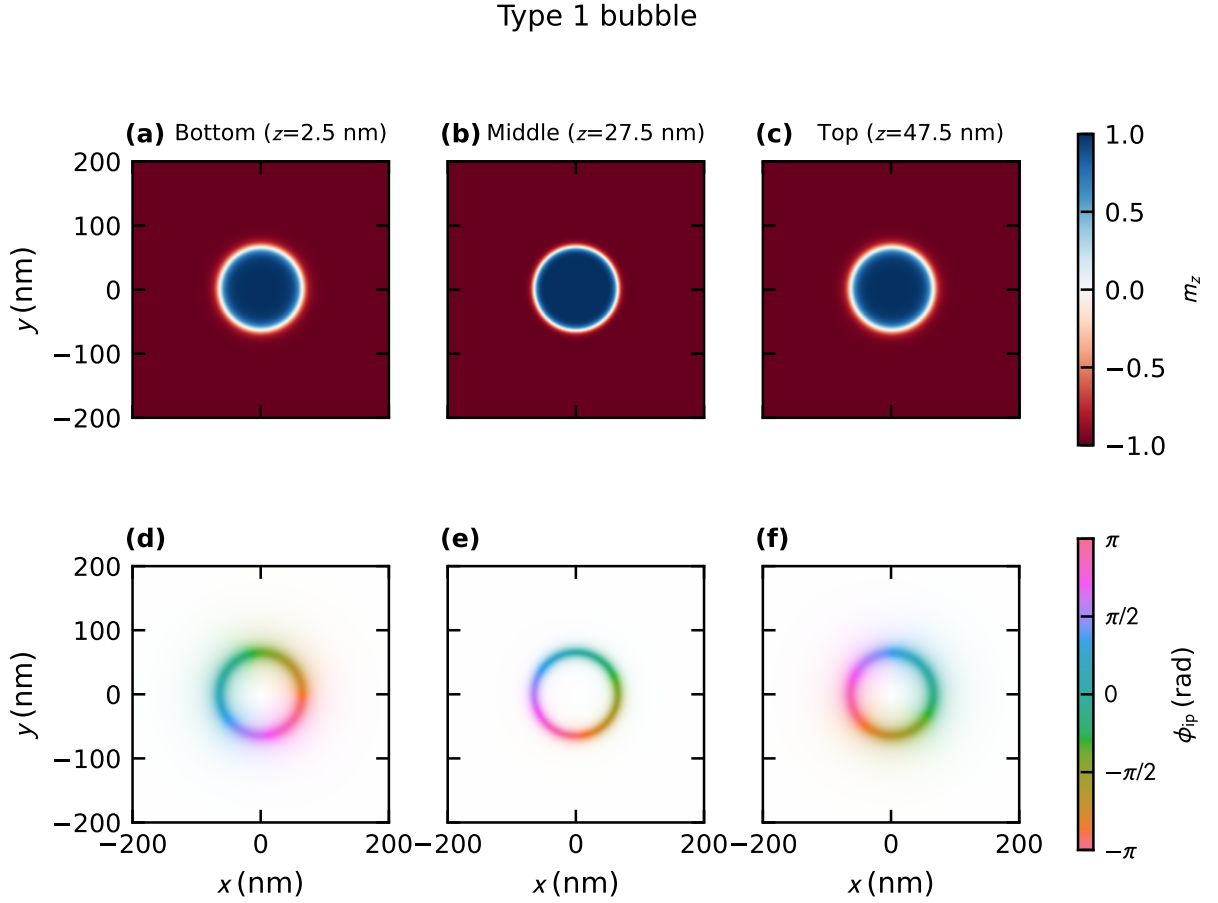


Figure 6.2: Top-view magnetisation maps of the *type 1* bubble at $D = 0$ at three depths through the DyCo₃ film. (a)–(c) Out-of-plane component m_z (red: $m_z = +1$; blue: $m_z = -1$) at $z \approx 2.5$ nm, 22.5 nm and ≈ 47.5 nm respectively. (d)–(f) In-plane magnetisation angle ϕ_{ip} shown as HUSL colour wheel. The domain-wall hue is uniform around the circumference at each depth, confirming the radial symmetry of the type 1 state. The shift in hue between panels (d) and (f) indicates a depth-dependent helicity rotation.

interruptions indicate abrupt changes in the direction of the in-plane magnetisation and mark regions where the local wall character switches between different helicities.

Such features can be interpreted as Bloch-line-like defects in the domain wall. Bloch lines are line defects that separate wall segments with different in-plane chirality [183]. Their presence breaks the continuous rotational symmetry of the wall and divides the domain wall into segments with different local helicity. As a result, the topological contributions from different parts of the texture partly cancel each other, as analysed below.

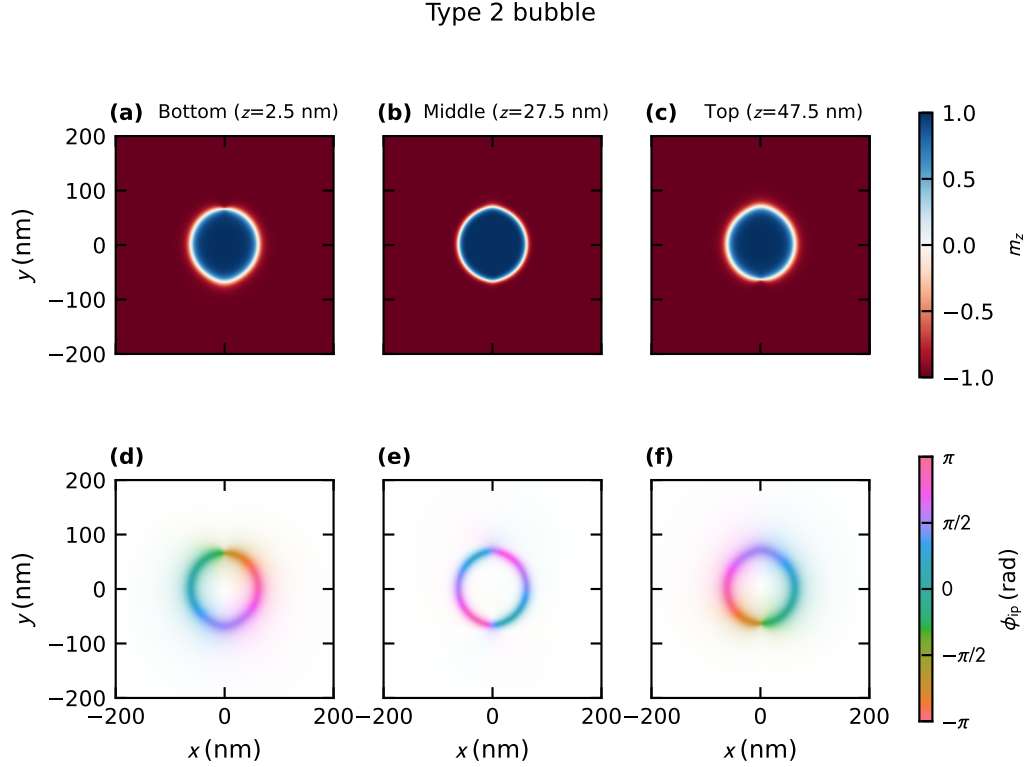


Figure 6.3: Top-view magnetisation maps of the *type 2* bubble at $D = 0$, displayed in the same format as Fig. 6.2. (a)–(c) m_z maps are nearly identical to the type 1 state. (d)–(f) The helicity maps reveal sharp discontinuities (Bloch lines) in the in-plane magnetisation around the domain wall, breaking the azimuthal symmetry of the type 1 state.

6.3.2 Topological distinction between type 1 and type 2 bubbles

The visual difference between the two bubble types can be quantified by computing the topological charge, as shown in Eq. (2.27), evaluated independently in every z -layer. For a continuously wound, axially symmetric domain wall, $|Q| = 1$: the in-plane spins wrap a full 2π around the skyrmion centre, mapping the film plane to the unit sphere exactly once. If, however, the wall contains Bloch lines that break the azimuthal winding into segments of opposite chirality, the contributions from neighbouring segments partially cancel and $|Q|$ is reduced.

Fig. 6.4 shows $Q(z)$ for both states. The type 1 bubble has $|Q| \approx 1$ at every depth, confirming that the in-plane magnetisation wraps smoothly around the core. In contrast, the type 2 bubble has $|Q| \approx 0$ throughout the bulk of the film: the cancelling sectors created by the Bloch lines suppress the net winding almost completely. The topological charge therefore provides a useful quantitative distinction between the two states: while

the m_z profiles are nearly identical, the azimuthal structure is qualitatively different.

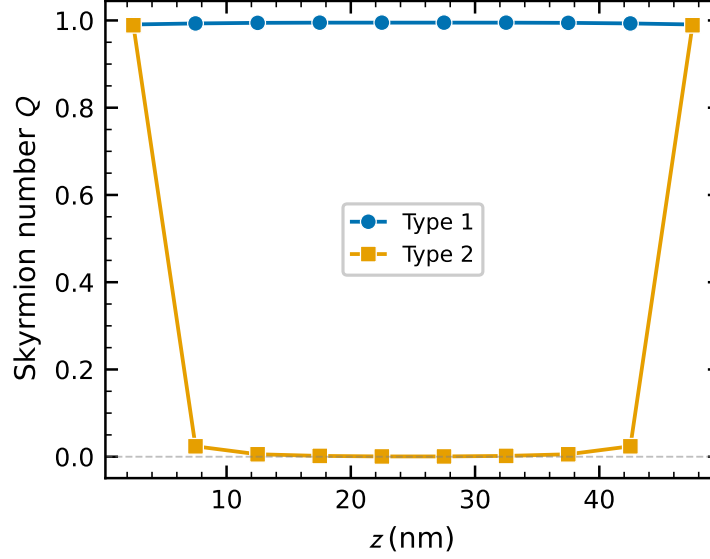


Figure 6.4: Skyrmion number Q as a function of depth for the type 1 (blue) and type 2 (orange) bubbles at $D = 0$. The type 1 state carries $|Q| \approx 1$ at every layer, while the type 2 state has $|Q| \approx 0$ throughout the bulk, reflecting the cancellation of opposite-chirality wall segments separated by Bloch lines.

6.4 Skyrmion profile ansatz

Having established that the type 1 bubble possesses the radial symmetry expected of a magnetic skyrmion, we now describe how its structural parameters are extracted from the simulated magnetisation. The analysis is based on the general skyrmion profile defined in Eq. (2.29), where the radial variation of the magnetisation is described through the polar angle profile introduced in Eq. (2.32).

For the type 1 bubble, the vorticity is fixed at $m = 1$. Since Eq. (2.32) enforces $\theta(0) = \pi$ at the core and $\theta \rightarrow 0$ far from the center, choosing the polarity $p = -1$ yields a core magnetisation along $+\hat{z}$ and a surrounding background along $-\hat{z}$, consistent with the simulated initial state. The skyrmion profile used for the fit is written as:

$$\mathbf{m}(\rho, \phi) = \begin{pmatrix} -\cos(\phi + \gamma) \sin \theta(\rho) \\ -\sin(\phi + \gamma) \sin \theta(\rho) \\ -\cos \theta(\rho) \end{pmatrix}, \quad (6.3)$$

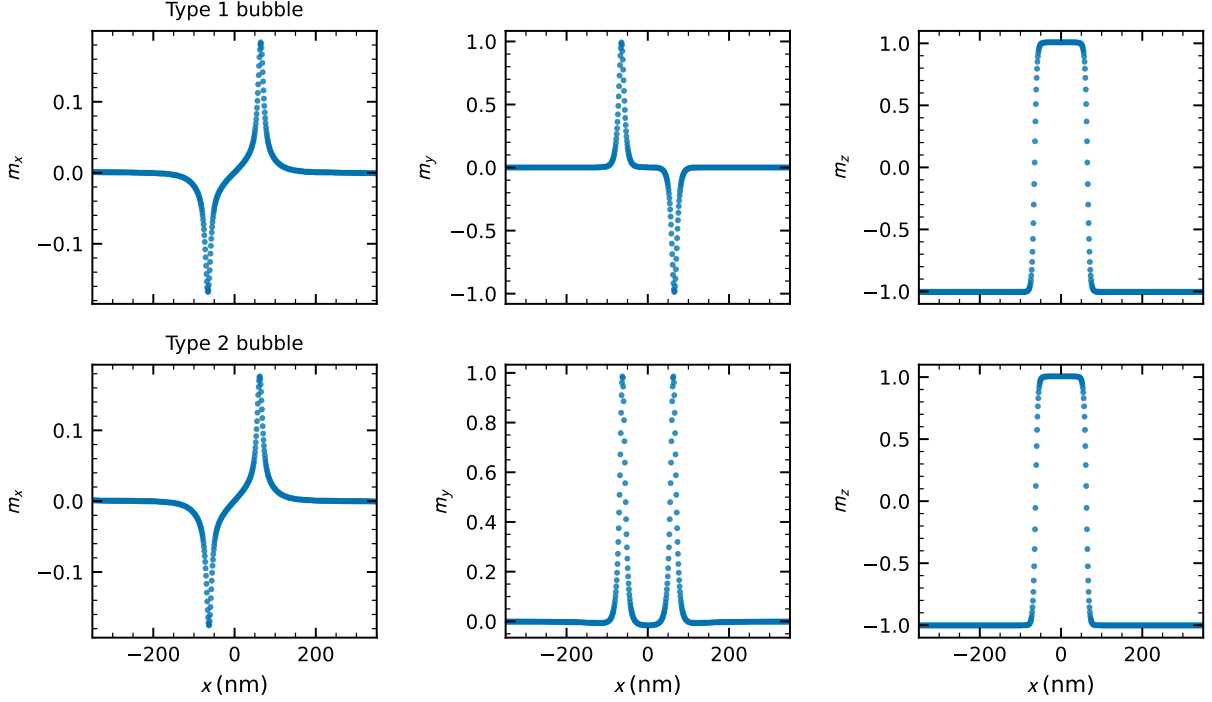


Figure 6.5: Magnetisation components along the x -axis at the mid-plane ($z = 25$ nm) for the type 1 and type 2 bubbles at $D = 0$. Panels (a)–(c) show m_x , m_y , and m_z for the type 1 bubble, while panels (d)–(f) show the corresponding components for the type 2 bubble. The m_x and m_z profiles are similar for the two states, whereas a clear difference is observed in m_y . The topological distinction between the two states arises from the azimuthal variation of the in-plane magnetisation around the domain wall.

where ρ and ϕ are measured from the skyrmion centre. The minus sign in the in-plane components fixes the reference direction used for the helicity in this chapter. With this convention, $\gamma = 0$ and $\gamma = \pi$ correspond to spins at the domain wall pointing radially inwards and outwards respectively, the two Néel chiralities, while $\gamma = \pm\pi/2$ give the two Bloch chiralities and intermediate values describe hybrid textures with mixed Néel–Bloch character. The remaining free parameters are the skyrmion radius R , defined as the radial position at which $\theta(R) = \pi/2$ and the magnetisation lies entirely in the film plane, and the domain-wall width w , which sets the length scale of the rotation from the core to the background orientation.

In contrast to the idealised textures of Section 2.2.4, where the helicity takes a single value throughout the structure, in the DyCo₃ film we allow γ to vary with depth. The helicity at each depth z characterises the local rotational sense of the spins within the domain wall, and as we demonstrate in Section 6.5.1, it varies continuously through the

thickness of the film. The measurement of this variation is the primary objective of the fitting procedure.

Reduction to a line scan

The fit is performed on line scans of the magnetisation taken along the x -axis at $y = 0$ for each depth z . Along this line, the radial coordinate simplifies to $\rho = |x|$ and the azimuthal angle takes only two values, $\phi = 0$ for $x > 0$ and $\phi = \pi$ for $x < 0$. Inserting these into Eq. (6.3) gives explicit expressions for the three components along the scan:

$$\begin{aligned} m_x(x) &= \mp \sin \theta(|x|) \cos \gamma, \\ m_y(x) &= -\sin \theta(|x|) \sin \gamma, \\ m_z(x) &= -\cos \theta(|x|), \end{aligned} \tag{6.4}$$

where the upper (lower) signs apply for $x > 0$ ($x < 0$). The two in-plane components provide complementary information about the helicity: m_x depends on $\cos \gamma$, while m_y depends on $\sin \gamma$. Fitting both components together therefore helps to constrain the value of γ .

6.5 Helicity at zero DMI and with DMI

6.5.1 Type 1 bubble at zero DMI

We first apply the fitting procedure of Section 6.4 to the type 1 bubble relaxed at $D = 0$. Figure 6.6 shows the magnetisation line scan at the mid-plane ($z = 22.5$ nm), together with the best-fit analytical profile. Panels (a)–(c) show the simulated components m_x , m_y , and m_z extracted from OOMMF (blue markers), along with the analytical profiles obtained from Eqs. (6.3)–(6.4) using the best-fit parameters (R, w, γ) (orange solid lines). The analytical profile reproduces the main features of the simulated texture, although a small deviation is visible in m_x . At zero DMI, the type 1 bubble has a three-dimensional magnetostatic structure. The domain wall is predominantly Bloch-like in the film interior, while the stray field introduces a radial Néel component that varies through the film thickness [184]. This depth-dependent structure is not fully captured by the analytical profile, which assumes a constant helicity. The difference is particularly visible in m_x because this component is small near the mid-plane. Nevertheless, the good agreement obtained for m_y and m_z , together with the main features of m_x , shows that the ansatz provides an adequate description of the simulated texture. In particular, the nonzero m_y

component in Fig. 6.6(b) is reproduced well, supporting the extraction of the helicity γ from the fit.

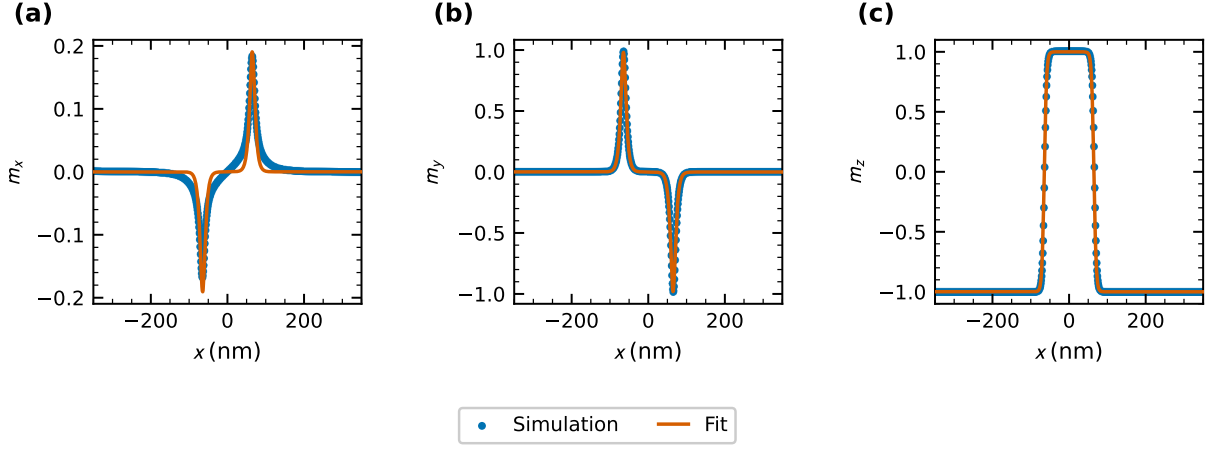


Figure 6.6: Magnetisation line scan of the type 1 bubble at $D = 0$ and $z = 22.5$ nm (mid-plane): (a) m_x , (b) m_y , and (c) m_z as functions of position along the x -axis. The blue markers represent the micromagnetic simulation data, while the orange solid lines show the best-fit analytical skyrmion profile obtained using Eqs. (6.3)–(6.4). The agreement between the simulation and fitted profiles validates the fitting procedure at zero DMI.

Having validated the fit at the mid-plane, we apply it independently to each of the ten computational layers. Figure 6.7 shows the three fitted skyrmion parameters as a function of depth z . The skyrmion radius R [Fig. 6.7(a)] is nearly constant across all layers, varying by less than 1 nm from top to bottom, which is comparable to the in-plane micromagnetic cell size. This indicates that the bubble forms an almost cylindrical tube through the film, with no clear radial tapering at the surfaces. The domain-wall width w [Fig. 6.7(b)] also varies only slightly with depth. The small modulation arises because the demagnetising field is depth-dependent: near the film surfaces the perpendicular magnetisation generates magnetic surface charges whose stray field slightly modifies the effective anisotropy and hence the wall width, while deep in the film interior this surface effect is screened.

The main result at $D = 0$ is the depth dependence of the helicity γ , shown in Fig. 6.7(c). Although no DMI is included in this simulation, γ is not constant through the film thickness. Instead, it changes smoothly from the bottom surface to the top surface, showing that the in-plane orientation of the domain wall is modified by the demagnetising field.

This behaviour can be understood from the role of magnetic surface charges. Near the film surfaces, the out-of-plane magnetisation produces surface charges, and the associated stray field favours a more Néel-like wall character. In the interior of the film, the influence

of these surface charges is reduced, allowing the wall to move towards a more Bloch-like configuration. The resulting competition produces the smooth variation of γ with depth.

This zero-DMI result provides an important baseline: a depth-varying helicity can already appear without DMI, and the role of DMI can then be understood by comparing how this helicity profile changes when the chiral interaction is introduced.

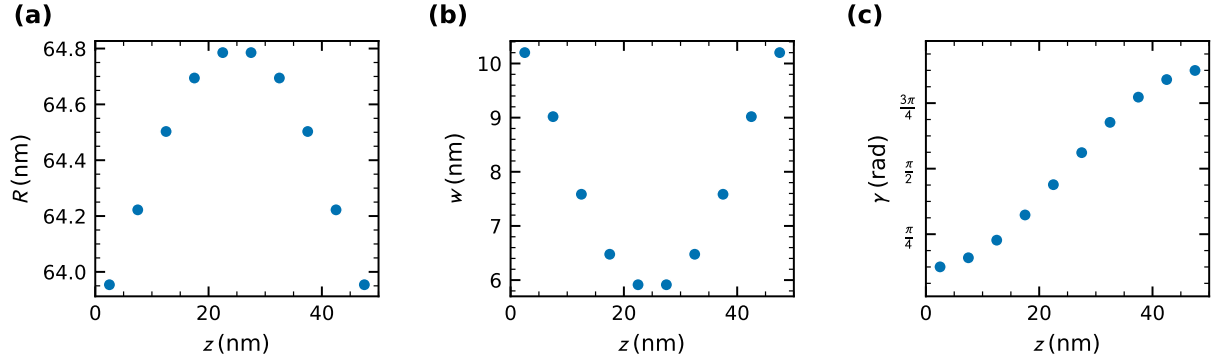


Figure 6.7: Depth dependence of the skyrmion fit parameters for the type 1 bubble at $D = 0$. (a) Skyrmion radius R . (b) Domain-wall width w . (c) Helicity γ (labelled in multiples of π). Even without DMI, γ varies smoothly through the film thickness due to the depth-dependent demagnetising field, rotating by approximately $\pi/2$ from bottom to top.

6.5.2 Helicity evolution with DMI

We now systematically switch on the interfacial DMI by sweeping the DMI constant D from 0 to $1 \times 10^{-3} \text{ J m}^{-2}$ in 21 equally spaced steps, holding K_u fixed at $1.68 \times 10^5 \text{ J m}^{-3}$. At each value of D , we initialise the simulation with an analytical Bloch-skyrmion profile ($\gamma = \pi/2$, $R = 50 \text{ nm}$, $w = 10 \text{ nm}$) and relax to the energy minimum; the use of the same well-defined initial condition at every D ensures that the relaxed state is comparable across the sweep. The skyrmion ansatz of Section 6.4 is then fitted independently to each layer, yielding the depth-resolved parameters $R(z)$, $w(z)$ and $\gamma(z)$ for every D .

Figure 6.8(a) shows the helicity profile $\gamma(z)$ at seven representative values of D spanning the sweep. At $D = 0$, the relaxed state remains predominantly Bloch-like, although a weak depth dependence of γ is still present due to the demagnetising field, as discussed in Section 6.5.1. As D increases, the interfacial DMI progressively rotates the helicity away from the Bloch configuration: the system is driven toward $\gamma \approx \pi$ (one of the Néel chiralities), with the span of γ across the film also increasing. Figures 6.8(b) and (c) show the layer-averaged radius $\langle R \rangle$ and domain-wall width $\langle w \rangle$ as functions of D . The

fitted radius increases with increasing DMI strength, showing that the DMI modifies the balance between domain-wall energy, anisotropy, and magnetostatic effects. In this fixed K_u sweep, the wall width decreases as the DMI is increased.

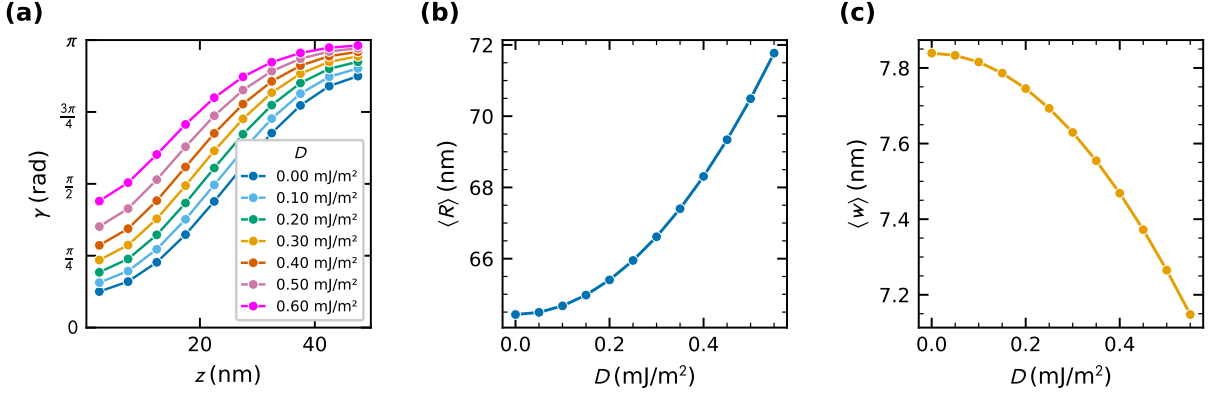


Figure 6.8: Evolution of skyrmion parameters with DMI strength. (a) Helicity profile $\gamma(z)$ at selected values of D ; as D increases, the helicity span across the film grows from near zero to $\approx \pi$, producing a hybrid skyrmion tube. (b) Layer-averaged radius $\langle R \rangle$ versus D ; the fitted radius changes systematically as DMI is increased. (c) Layer-averaged domain-wall width $\langle w \rangle$ versus D ; the wall narrows as DMI is increased.

6.6 Material-parameter phase diagrams

The depth-resolved simulations discussed in the previous sections show that the DyCo_3 film can support a topologically non-trivial bubble state even when the DMI is set to zero. For the type 1 state, the skyrmion radius remains almost constant through the film thickness, whereas the helicity changes with depth. When interfacial DMI is introduced, it mainly changes the helicity of the domain wall and drives the texture towards a more Néel-like configuration.

This result is important for the next part of the study. The depth-resolved model is useful for understanding the internal structure of the skyrmion tube, especially the variation of helicity through the film thickness. However, the experimental comparison is mainly based on the size of the skyrmion observed in real-space images. Therefore, to study a larger range of material parameters, we use a reduced model in which the film thickness is represented by a single computational cell. This model does not describe the full three-dimensional helicity profile, but it keeps the main in-plane energy competition that controls the relaxed magnetic state and the skyrmion radius.

In this section, the reduced model is used to scan the parameter space spanned by the DMI constant D and the effective anisotropy K_{eff} . The aim is to identify the range of parameters where isolated skyrmions are stabilised, and to compare the fitted skyrmion radius with the experimentally observed values.

6.6.1 Reduced single-cell model for large parameter sweeps

For the parameter sweep, the saturation magnetisation M_s , exchange stiffness A , applied flux density B , and damping constant α are kept fixed at the values listed in Table 6.1. The two varied parameters are the DMI constant D and the intrinsic uniaxial anisotropy K_u . These parameters are not directly known with high precision for the experimental DyCo₃ film, and therefore they are treated as control parameters in the simulations.

As discussed in Section 6.2.3, the simulation input is the intrinsic anisotropy K_u , while the results are analysed in terms of the effective anisotropy K_{eff} defined in Eq. 6.2. A positive value of K_{eff} corresponds to a net perpendicular easy axis, which supports an out-of-plane magnetic background. A negative value indicates that the shape anisotropy dominates and the system tends towards an in-plane magnetisation.

At each point in the (K_{eff}, D) plane, the system is initialised with an isolated Bloch-type skyrmion with $\gamma = \pi/2$, $R = 50$ nm, and $w = 10$ nm. The system is then relaxed to a local energy minimum by minimising the same total energy functional given in Eq. 6.1. The simulations are performed using the Ubermag! [61–63] interface with the GPU-accelerated MuMax³ solver [90], which makes this large two-dimensional parameter sweep computationally feasible.

This reduced model should be interpreted as a practical approximation. It cannot reproduce the depth-dependent helicity profile obtained in the ten-layer simulations, but it allows us to map the relaxed magnetic states over a much wider parameter range. It is therefore suitable for studying which textures are stabilised for different values of D and K_{eff} , and for extracting approximate skyrmion radii for comparison with experiment.

6.6.2 Classification of relaxed magnetic states

The parameter sweep produces a large number of relaxed magnetisation states, and it is not practical to classify all of them manually. To organise these states, we use an autoencoder-based unsupervised classification approach, similar in spirit to the machine-learning method demonstrated for micromagnetic vector-field configurations by Pathak et al. [185]. In this work, the classification is carried out using the (m_x) component of each

relaxed magnetisation state. The autoencoder compresses this information into a twelve-dimensional latent space, and the resulting latent features are then clustered across the (K_{eff}, D) parameter sweep.

The cluster labels are used only as a way of grouping states with similar features in the learned latent space. To assign a physical meaning to these groups, representative states from each cluster are then inspected using all three magnetisation components, (m_x) , (m_y) , and (m_z) . Based on this inspection, the relaxed states are grouped into five physically interpreted classes. Figure 6.9 shows the resulting classification in the (K_{eff}, D) plane.

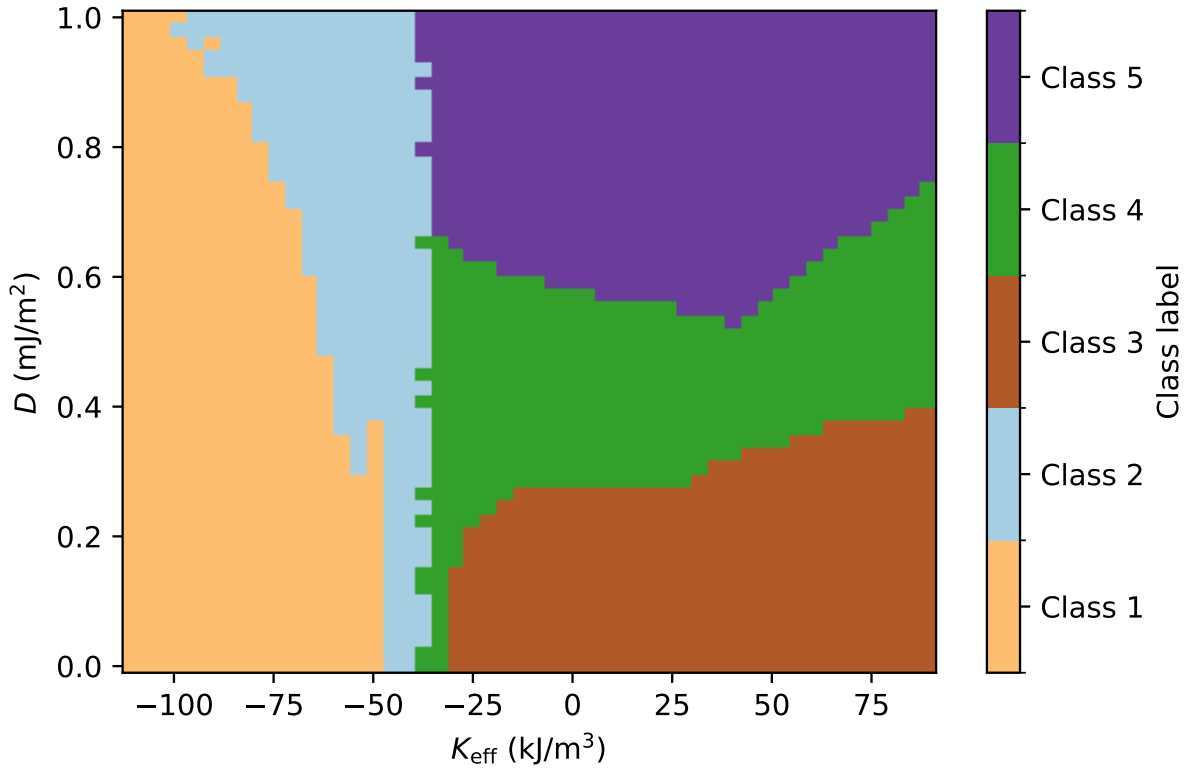


Figure 6.9: Unsupervised classification of the relaxed magnetic states in the (K_{eff}, D) plane, obtained from the reduced single-cell model. The colours denote groups of magnetisation states that are similar in the autoencoder feature space. The physical interpretation of each group is assigned by inspecting representative magnetisation states.

6.6.3 Magnetic phase diagram and the Bloch-to-Néel transition

The five classes identified by the unsupervised procedure correspond to distinct representative magnetic states. Their distribution in the (K_{eff}, D) plane is consistent with the expected role of effective anisotropy and DMI, as shown in Fig. 6.9.

Class 1 appears in the region of strongly negative K_{eff} . In this region, the effective anisotropy favours an in-plane magnetisation direction, so a perpendicular magnetic background is not established. As a result, isolated skyrmion-like textures are not stabilised, even when D is increased. The representative state in Fig. 6.10 shows broad, smooth magnetic domains with only a weak out-of-plane component. This behaviour is also reflected in Fig. 6.9, where Class 1 extends across the full range of D at the most negative values of K_{eff} . Additional representative m_x states from this class are shown in Fig. B.1.

Class 2 occupies an intermediate region between the in-plane state and the isolated-skyrmion region. It appears for weakly negative to small values of K_{eff} , where the anisotropy is no longer strong enough to maintain a simple in-plane state, but is also not sufficient to establish a uniform perpendicular background. As a result, the relaxed state forms an extended multidomain texture with stripe-like features, as shown in Fig. 6.10 by Class 2. In this region, domain walls are present throughout the film, but the magnetic background is not sufficiently uniform for isolated skyrmions to be clearly stabilised.

The remaining three classes correspond to isolated skyrmion-like states. Their m_z components show a localised core surrounded by an opposite background, while the main difference between the classes appears in the in-plane magnetisation. This is expected from the ansatz discussed in Section 6.4, where the in-plane components along a radial cut depend on the helicity through $m_x \propto \cos \gamma$ and $m_y \propto \sin \gamma$.

Since the clustering was performed using the m_x component, the separation between these three classes mainly reflects differences in the m_x contrast of the domain wall. Class 3 has a weak m_x signature and is therefore close to a Bloch-type skyrmion ($\gamma \approx \pi/2$), where most of the in-plane magnetisation is carried by m_y . Class 5 shows the strongest m_x contrast and is close to a Néel-type skyrmion ($\gamma \approx \pi$), where the in-plane magnetisation is mainly radial in the convention used here. Class 4 lies between these two limits and represents a hybrid Bloch–Néel state, with both m_x and m_y components present.

The progression from Class 3 to Class 5 therefore reflects a gradual change in the wall character from Bloch-like to Néel-like. This is consistent with the helicity evolution discussed in Section 6.5.2, where increasing interfacial DMI rotates the in-plane wall magnetisation towards the Néel chirality favoured by the chosen sign of D .

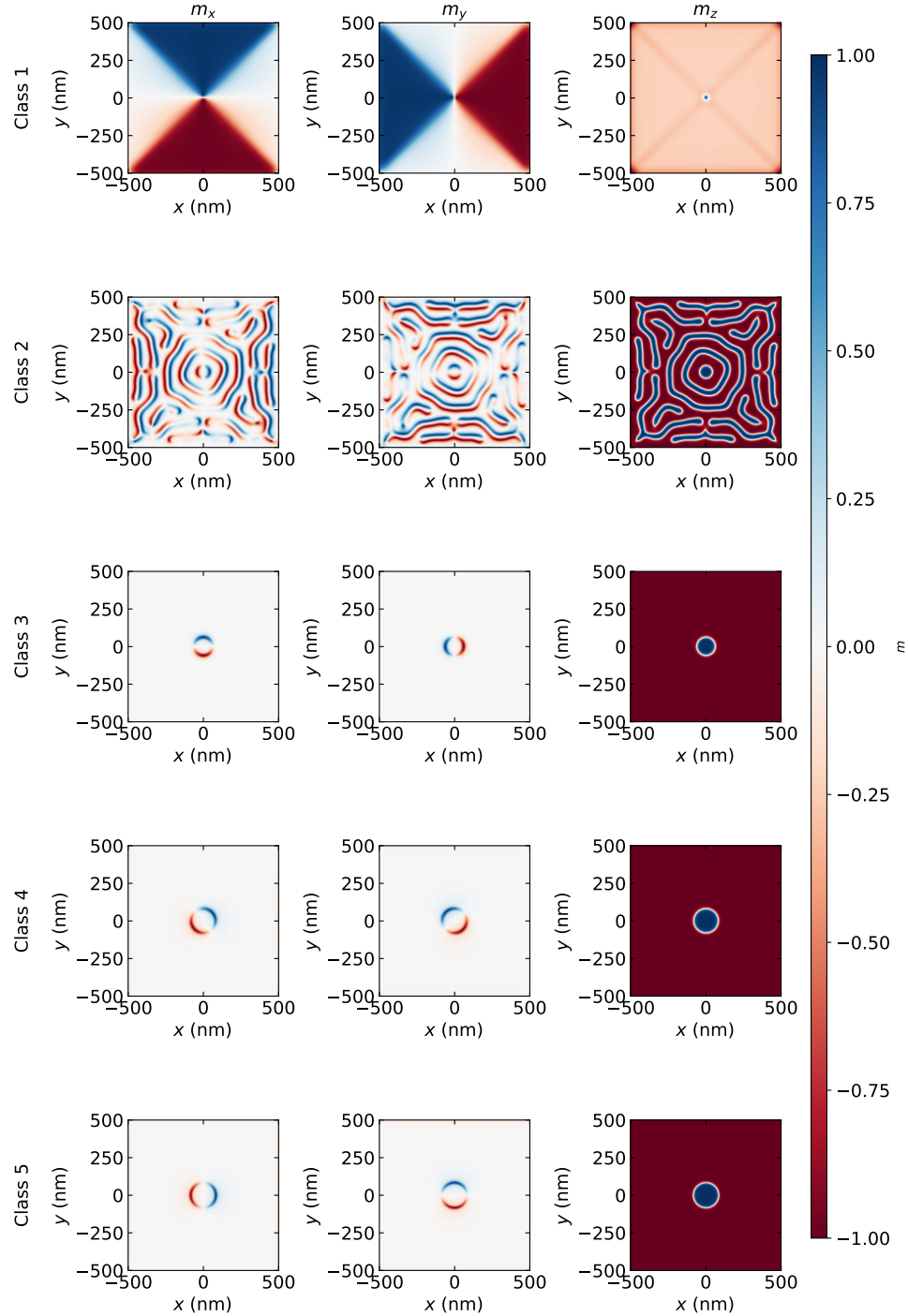


Figure 6.10: Representative relaxed magnetisation states selected from the clusters shown in Fig. 6.9. These examples illustrate the main texture types obtained in the parameter sweep, including isolated skyrmions, target-like states, stripe or helical textures, and states dominated by in-plane magnetisation.

The relation between this classification and the depth-resolved analysis of Section 6.3 should be made explicit, because the two parts of this chapter use different labels for the magnetic states. The type 1 and type 2 labels refer to the depth-resolved bubble structures discussed in Section 6.3.2. In contrast, Classes 1–5 are obtained from the reduced unsupervised classification across the (K_{eff}, D) plane.

Within this classification, Classes 3–5 correspond to isolated skyrmion-like states with continuously wound domain walls. They are therefore closest to the type 1 bubble, but are further separated according to their helicity and wall character. The type 2 bubble does not appear as a separate class in the reduced classification. This is expected because the reduced model does not resolve the full depth-dependent wall structure, and the segmented wall associated with Bloch lines is not observed in the clustered states.

In the following analysis, we focus only on the isolated-skyrmion region within the positive K_{eff} range. In this region, the analytical ansatz introduced in Section 6.4 remains appropriate, and the fitted quantities R , w , and γ have a clear physical interpretation.

6.6.4 Skyrmion radius across the phase diagram

Within the isolated-skyrmion region, the analytical profile of Eqs. (6.3)–(6.4) is fitted to each relaxed state. From these fits, the skyrmion radius (R), domain-wall width (w), and helicity (γ) are extracted at each point in the (K_{eff}, D) plane. Figure 6.11 shows the fitted radius R as a function of D and (K_{eff}) .

The radius varies smoothly across the skyrmion region, showing that the skyrmion size is controlled by the combined effect of DMI and effective anisotropy. The largest radii are obtained for high values of D and low values of K_{eff} , while the smallest radii occur for low D and high K_{eff} . This trend is consistent with the role of DMI in lowering the effective domain-wall energy, which allows the skyrmion wall to expand. In contrast, increasing K_{eff} increases the energy cost associated with the tilted spins in the wall and the reversed core, and therefore favours a smaller skyrmion.

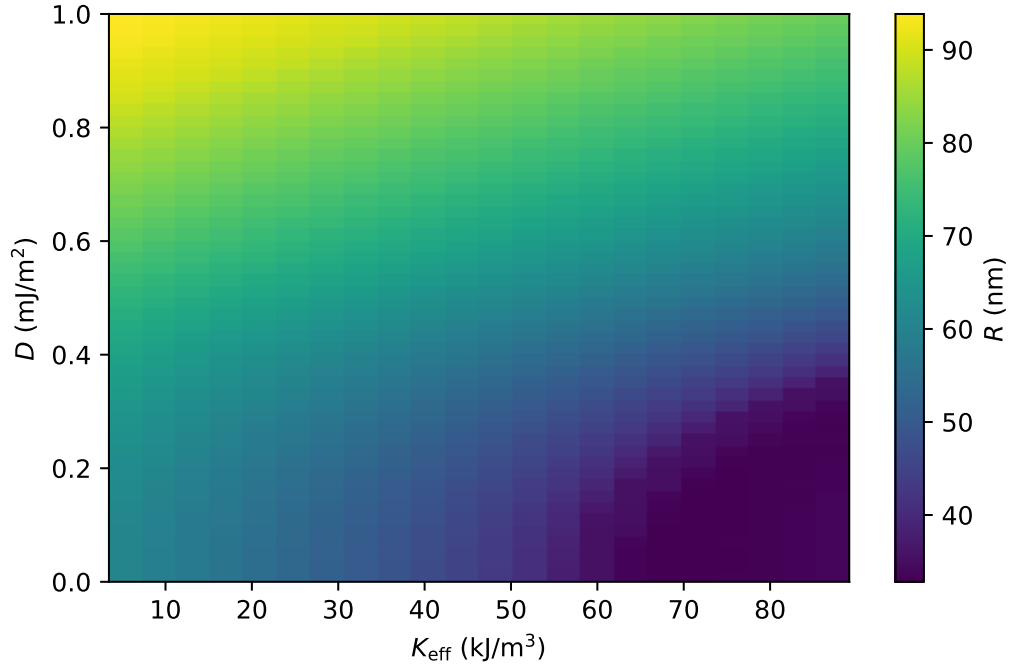


Figure 6.11: Fitted skyrmion radius R across the isolated-skyrmion region of the phase diagram. The radius increases mainly with increasing DMI strength and decreases as K_{eff} becomes larger.

6.6.5 Domain-wall width across the phase diagram

Figure 6.12 shows the fitted domain-wall width w over the same region. Unlike the radius, the wall width is smallest at high D and high K_{eff} . The wall width is set mainly by the balance between exchange, which favours a wide wall, and anisotropy, which favours a narrow wall; a larger K_{eff} stiffens the wall and reduces w . The DMI adds a further energy scale that also sharpens the wall, so increasing D narrows it as well. High D together with high K_{eff} therefore gives the narrowest wall.

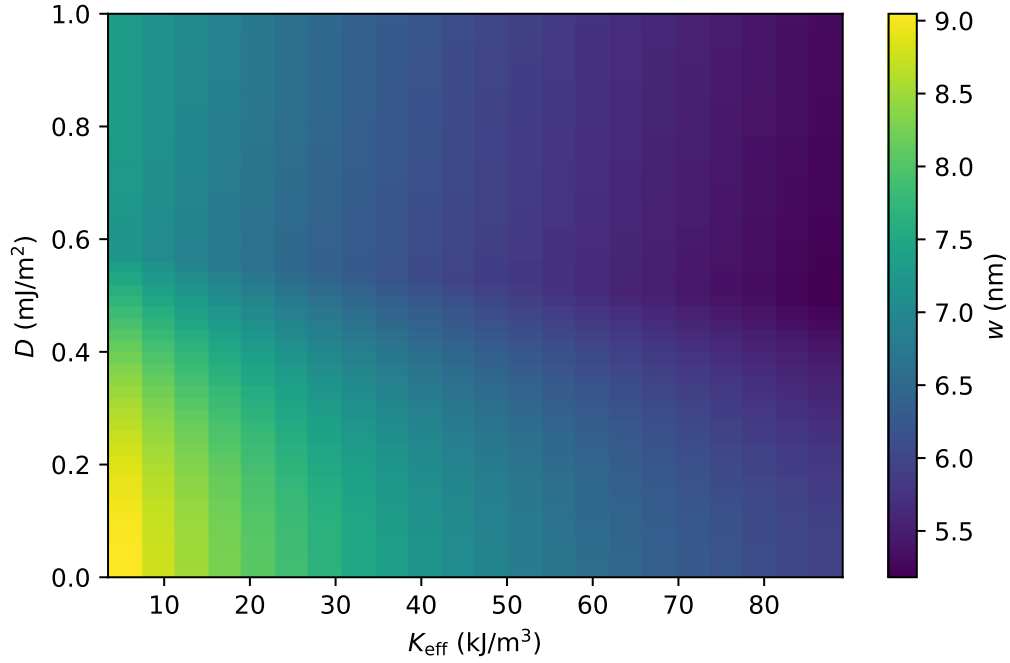


Figure 6.12: Fitted domain-wall width w across the isolated-skyrmion region of the phase diagram.

6.6.6 Helicity and the Bloch–Néel transition

The clearest variation across the phase diagram is in the helicity γ , shown in Figure 6.13(a). At low D the relaxed skyrmion stays Bloch-like ($\gamma \approx \pi/2$), close to the Bloch initial condition, in the absence of DMI, the demagnetising field favours a predominantly Bloch-like wall.

As D is increased the interfacial DMI rotates the helicity away from the Bloch configuration, through a range of mixed Bloch–Néel states, until at high D the skyrmion is fully Néel ($\gamma \rightarrow \pi$), with the in-plane spins pointing radially as preferred by the C_{nv} interfacial DMI. The change is continuous in γ , indicating a gradual rotation set by the relative strength of the DMI and the demagnetising field. To show this more clearly, Fig. 6.13(b) plots a line cut of γ against D at fixed K_{eff} , on which the continuous rotation from Bloch to Néel is easy to see.

This is the same competition seen in Section 6.5.2, viewed in a different way. The demagnetising field favours a Bloch-like wall, and adding the interfacial DMI drives a smooth crossover towards a Néel-like wall. In the depth-resolved film this competition plays out across the thickness, giving a hybrid tube whose helicity rotates from one surface

to the other; in the reduced model it plays out as a function of D , giving a continuous Bloch–Néel crossover in the single representative helicity.

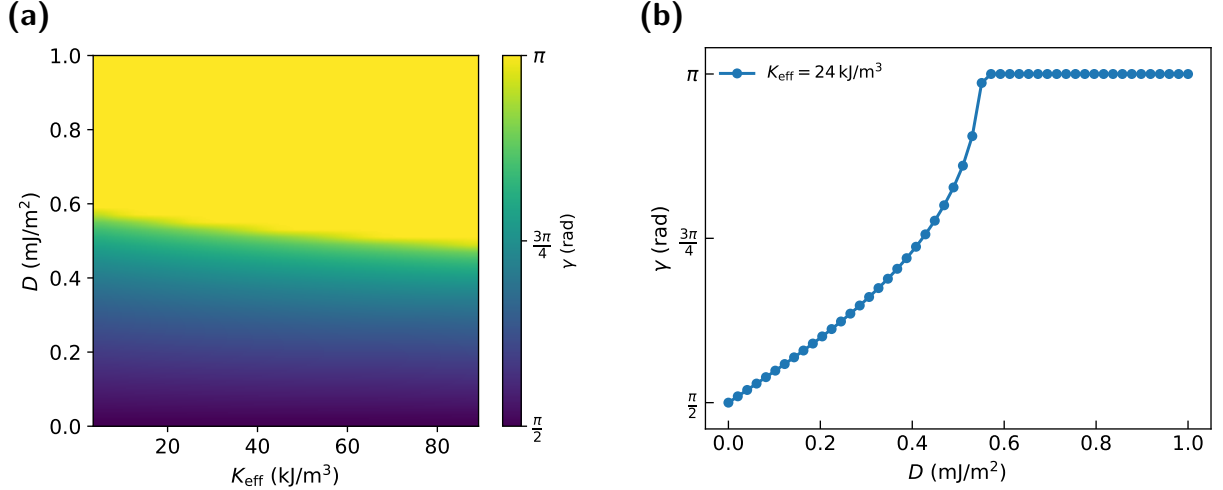


Figure 6.13: Fitted helicity γ across the isolated-skyrmion region. (a) Helicity map across the phase diagram, showing Bloch-like states ($\gamma \approx \pi/2$) at low D , mixed Bloch–Néel states at intermediate D , and Néel-like states at high D . (b) Line cut of the fitted helicity γ against D at fixed K_{eff} , taken from panel (a). The continuous rotation from a Bloch-like wall to a Néel-like wall is resolved without an abrupt transition.

6.6.7 Comparison to experiment and summary

The phase diagrams above map how the stabilised magnetic state depends on the DMI constant D and the effective anisotropy K_{eff} . This is useful because the experimental DyCo₃ film provides only limited direct access to the internal structure of the skyrmion. In particular, the skyrmion radius can be estimated from real-space images, whereas the domain-wall width and helicity are much more difficult to determine directly. The fitted radius is therefore the main quantity used to compare the simulations with the experimental observations.

The experimental skyrmions reported in Ref. [50] have radii of approximately 35 ± 5 nm and 42 ± 5 nm. The radius map in Fig. 6.11 can therefore be used to identify the region of the (K_{eff}, D) plane where the simulated skyrmion size is closest to these values. This comparison does not give a unique material parameter set, because several combinations of D and K_{eff} can produce similar skyrmion radii. However, it does constrain the plausible range of parameters and shows how the experimentally observed length scale can emerge from the micromagnetic model.

A further limitation of this comparison is that the experimental images do not directly identify the internal bubble type. The phase diagrams discussed above are based on the reduced model, where the isolated skyrmion classes correspond to continuously wound type 1-like states. The STXM images reported in Ref. [50], however, are mainly sensitive to the thickness-integrated out-of-plane magnetisation. As shown in Section 6.3.2, this projected (m_z) profile is very similar for the type 1 and type 2 bubbles. The radius comparison therefore constrains the material parameters, but it does not by itself determine whether the experimental bubbles have a continuously wound wall or a segmented wall with Bloch lines.

This ambiguity is examined in Appendix B.1, where simulated experimental contrasts are calculated from the relaxed type 1 and type 2 states using `mag2exp` [145]. These calculations show that projected out-of-plane contrast cannot reliably distinguish the two states, whereas LTEM is sensitive to the in-plane magnetisation of the domain wall and separates them more clearly. LTEM imaging would therefore provide a more direct experimental test of the internal wall structure of the DyCo₃ bubbles.

Overall, the depth-resolved simulations and the reduced parameter sweep provide complementary information. The depth-resolved model shows that a topologically non-trivial bubble can exist even at $D = 0$, and that the helicity varies through the film thickness due to demagnetising effects. Adding interfacial DMI mainly biases the domain-wall helicity towards a Néel-like configuration. The reduced single-cell model then allows a wider scan of material parameters, showing where in-plane domains, multidomain textures with stripe-like features, and isolated skyrmions are obtained.

Within the isolated-skyrmion region, the fitted radius increases mainly with increasing D and decreasing K_{eff} , while the wall width and helicity also vary systematically across the same parameter space. The helicity rotates from Bloch-like to Néel-like as D is increased. These results suggest that the experimentally observed DyCo₃ skyrmions can be understood as compact ferrimagnetic skyrmions whose size is mainly controlled by the balance of anisotropy, DMI, exchange, and demagnetising effects, while the DMI plays an important role in setting the chirality of the domain wall.

Chapter 7

Summary and outlook

This thesis presents a computational study of magnetic skyrmions in engineered magnetic materials, with a focus on two related questions. The first concerns the dynamics of skyrmions in synthetic antiferromagnets, where antiferromagnetic interlayer coupling and lateral skyrmion–skyrmion interactions shape the resonant response of the system. The second concerns the stability and internal structure of skyrmion-like textures in ferromagnetic DyCo_3 thin films, where the Dzyaloshinskii–Moriya interaction, anisotropy, demagnetising effects, and film thickness have to be considered together. Although these two parts of the thesis address different physical systems, they are connected by the same central theme: micromagnetic modelling can resolve details of magnetic textures that are difficult to access directly from experiment alone.

The theoretical and computational basis of the thesis is presented in Chapter 2. This includes the continuum representation of the magnetisation and the main energy contributions that determine the equilibrium and dynamic behaviour of magnetic systems: exchange, DMI, anisotropy, Zeeman, magnetostatic, and interlayer RKKY coupling terms. The LLG equation is introduced as the equation governing the time evolution of the magnetisation and provides the foundation for the dynamic simulations presented later in the thesis. The magnetic structures relevant to this work, including domain walls, vortices, helices, skyrmions, and synthetic antiferromagnets, are also described. Finally, the ring-down and eigenvalue methods are introduced as two different approaches for analysing resonant magnetisation dynamics.

The software contributions made to the Ubermag ecosystem during this PhD are summarised in Chapter 3. These contributions were not aimed at developing a new micromagnetic solver, but addressed practical aspects of simulation setup, documentation, testing, metadata handling, and data access. Documentation and tutorial updates kept exam-

ples consistent with the package behaviour, while testing and maintenance contributions made the requirements of different drivers more explicit. The metadata-related work made simulation runs easier to track and interpret after completion, and the work on data access in `micromagneticdata` supported the post-processing of simulation results. These contributions form an important practical part of the thesis and support reproducible computational micromagnetics workflows.

In Chapter 4, we investigate the eigenmodes of an isolated skyrmion in ferromagnetic and SAF geometries. The ferromagnetic system establishes the relation between the skyrmion core polarity and its preferred direction of gyration. In the square SAF geometry, interlayer RKKY coupling gives rise to two nearly degenerate gyrotropic modes at low frequencies and to in-phase and out-of-phase breathing modes at higher frequencies. Under rectangular confinement, the two gyrotropic modes become predominantly translational.

In the SAF system, the antiferromagnetic RKKY interaction couples the skyrmions in the top and bottom magnetic layers. In the square SAF geometry, the coupling gives rise to two nearly degenerate gyrotropic modes at low frequencies and to in-phase and out-of-phase breathing modes at higher frequencies. When the lateral symmetry is broken by rectangular confinement, the two gyrotropic modes become predominantly translational. The lower-frequency mode describes translational motion along the longer dimension of the sample, whereas the higher-frequency mode describes translational motion along the shorter dimension. These results show that the excitation modes are determined by both the interlayer coupling and the geometry of the confining structure.

In Chapter 5, we extend the analysis from an isolated SAF skyrmion to finite chains containing two, five, and 25 skyrmions. This allows the development of isolated-skyrmion modes into collective excitations to be followed as neighbouring skyrmions are introduced. The two-skyrmion system provides the simplest case in which lateral skyrmion–skyrmion coupling acts together with the coupling between the two SAF layers. The low-frequency modes have translational character, whereas the higher-frequency modes are breathing modes. Each mode is further distinguished by the relative phase between neighbouring skyrmions and between the two antiferromagnetically coupled layers, giving rise to different left–right and top–bottom phase relationships.

For the five-skyrmion chain, the same basic mode characters are retained, but the individual modes become more closely spaced in frequency and begin to form band-like groups. The mode profiles show collective behaviour across the chain, with the horizontal translational and breathing modes exhibiting standing-wave-like amplitude profiles. In the 25-skyrmion chain, a local excitation of the top–bottom in-phase breathing band produces

a signal that propagates along the chain. The signal reaches the end of the strip, reflects from the boundary, and returns towards the excitation region. Tracking the first maximum gives an average signal speed of approximately 300 m s^{-1} . This connects the frequency-domain description of the collective modes with the time-dependent propagation of an excitation through the skyrmion chain.

In Chapter 6, we study ferrimagnetic skyrmion-like textures in DyCo_3 thin films, motivated by experimental observations of compact skyrmions in this material. In contrast to the preceding chapters, which focus on resonant dynamics, this part of the thesis examines equilibrium magnetic states and their internal structure. A depth-resolved micromagnetic model is used to study the bubble states through the film thickness. At zero DMI, two bubble states are identified with similar out-of-plane magnetisation profiles but different topological structures. The type 1 bubble is a continuously wound skyrmion-like state, whereas the type 2 bubble contains Bloch lines in the domain wall. This distinction is important because projected out-of-plane contrast alone may not be sufficient to distinguish the two states experimentally.

The helicity of the type 1 bubble is analysed by fitting an analytical skyrmion profile layer by layer through the film thickness. The helicity rotates continuously through the thickness even in the absence of DMI, showing that demagnetising effects alone can produce a depth-dependent hybrid domain-wall structure. When interfacial DMI is included, it biases the domain-wall helicity towards a Néel-like configuration rather than acting only as an interaction required for the stability of the bubble. The role of DMI in DyCo_3 therefore includes controlling the chirality and internal structure of the domain wall.

A reduced single-cell model is used to scan the material-parameter space defined by the DMI constant D and the effective anisotropy K_{eff} . An unsupervised autoencoder-based classification organises the relaxed states into physically interpretable classes and is used to construct the phase diagram of texture type. The skyrmion radius, domain-wall width, and helicity are obtained separately by fitting the analytical skyrmion profile. Within the isolated-skyrmion region, the radius increases with increasing D and decreasing K_{eff} , while the wall width and helicity also vary systematically across the parameter space. The helicity changes continuously from Bloch-like to Néel-like as D increases. Comparison of the fitted radii with the experimentally observed DyCo_3 skyrmion sizes constrains the plausible range of material parameters for the film.

The simulated experimental contrast further supports the interpretation of the DyCo_3 results. Thickness-integrated out-of-plane contrast, such as that obtained from X-ray holography or STXM, cannot reliably distinguish type 1 and type 2 bubbles because both states can produce similar projected m_z contrast. In contrast, imaging methods sensitive

to the in-plane magnetisation, such as LTEM, can distinguish their internal domain-wall structures more clearly. This provides a possible route for experimentally testing the bubble classification proposed in this work.

The results show that the resonant response of SAF skyrmion systems is shaped by the combined effects of interlayer coupling, lateral confinement, and skyrmion–skyrmion interactions. In DyCo₃ thin films, the skyrmion-like textures are determined by the balance between exchange, anisotropy, demagnetising effects, and DMI, while their internal structure varies through the film thickness. In both systems, micromagnetic simulations provide access to mode profiles, phase relationships, helicity, topology, and internal domain-wall structures that are difficult to obtain directly from standard experimental images.

Several directions can be pursued from this work. For the SAF systems, a useful next step would be to connect the calculated eigenmodes and signal-propagation results with experimentally measurable signals. This is motivated by earlier work on coupled magnetic vortices, where spatially separated vortex cores were shown to behave as coupled oscillators with in-phase and antiphase normal modes [158]. It would also be important to test the stability of the predicted modes under more realistic conditions, including defects, edge roughness, thermal fluctuations, and experimentally relevant excitation mechanisms.

For the DyCo₃ system, the main future direction is a closer connection to experimental imaging. The results show that different internal structures can produce similar projected out-of-plane contrast, making a direct comparison with in-plane-sensitive imaging particularly valuable. More realistic modelling could include temperature-dependent material parameters, a detailed ferrimagnetic two-sublattice description, and a more refined treatment of the depth dependence of the DMI. The autoencoder-based classification could also be extended to include all magnetisation components and larger parameter spaces. This would allow magnetic-state classes to be identified more systematically and reduce the manual inspection required for large simulation datasets.

This thesis contributes to the computational understanding of both the dynamic and static properties of magnetic skyrmions. It shows how skyrmion modes develop into collective excitations in SAF chains and how skyrmion-like textures in DyCo₃ can be interpreted through their depth-dependent internal structure. More broadly, the work shows how micromagnetic modelling and scientific Python tools can connect microscopic magnetic interactions with experimentally relevant behaviour in complex magnetic materials.

Appendix A

Supplementary analysis of skyrmion dynamics in synthetic antiferromagnets

A.1 Selecting a suitable colormap for visualising phase amplitude maps

Here, we provide an additional example to show why the choice of colourmap is important for this type of phase visualisation. Since the hue represents the phase, the colourmap should have approximately uniform lightness. If the lightness varies strongly across the colourmap, changes in brightness can be misinterpreted as changes in mode amplitude or spatial structure. This is the case for the commonly used “hsv” colourmap, which can introduce visual artefacts. Therefore, in the main analysis we use the “husl” colourmap from [seaborn](#), as discussed in Section 4.2.

Figure A.1 illustrates this effect for the vertical translational mode of five skyrmions, corresponding to Fig. 5.8a in Chapter 5. The same mode is shown using the “hsv” colourmap in panels a and c, and the “husl” colourmap in panels b and d. In panels c and d, the global phase is shifted by (-0.6π) . Since the absolute phase has no physical meaning, only relative phase differences should affect the interpretation of the mode.

For the “husl” colourmap, the visual appearance of the mode remains consistent after the global phase shift, as shown in panels b and d. The vertical translational motion is clearly visible, while the motion in the horizontal direction is strongly suppressed. A similar interpretation can be obtained from panel c, where the “hsv” colourmap is used

with a suitable phase shift. However, panel a shows that the same “hsv” colourmap without this phase shift can make the mode appear more gyrotropic, which is misleading. This example shows why a colourmap with uniform lightness is important when phase information is encoded through colour.

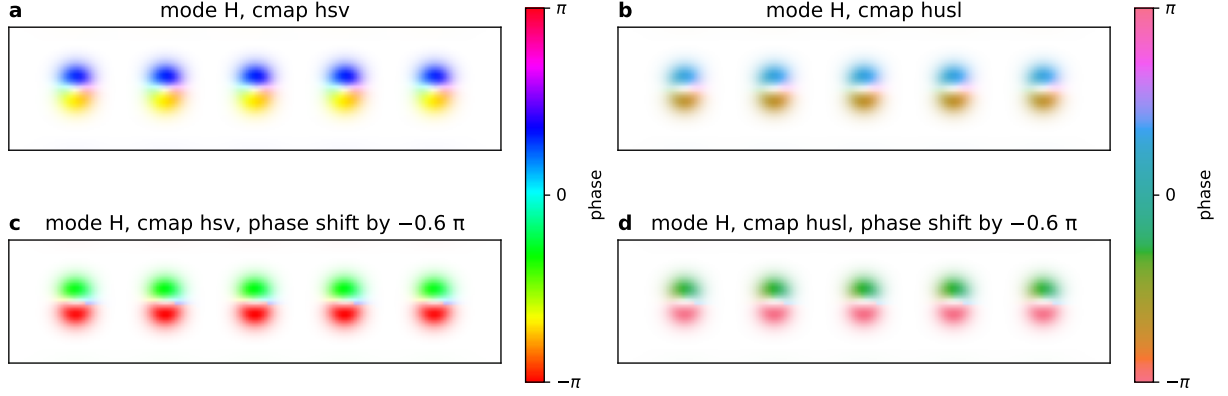


Figure A.1: Phase amplitude map for the same translational mode of five skyrmions with four different visualisations. (a) hsv to visualise the phase, (b) husl to visualise the phase, (c) hsv with all colours shifted by -0.6π , (d) husl with all colours shifted by -0.6π .

This example illustrates that homogeneous hue lightness is crucial to draw valid conclusions from the phase amplitude plots. Using colourmaps such as hsv that ignore this constraint can lead to artifacts in the visualisation.

A.2 Eigenmodes of FM and SAF skyrmions

The following Figs. A.2 to A.7 show phase amplitude maps for the first 40 eigenmodes of FM and SAF skyrmions in different system geometries.

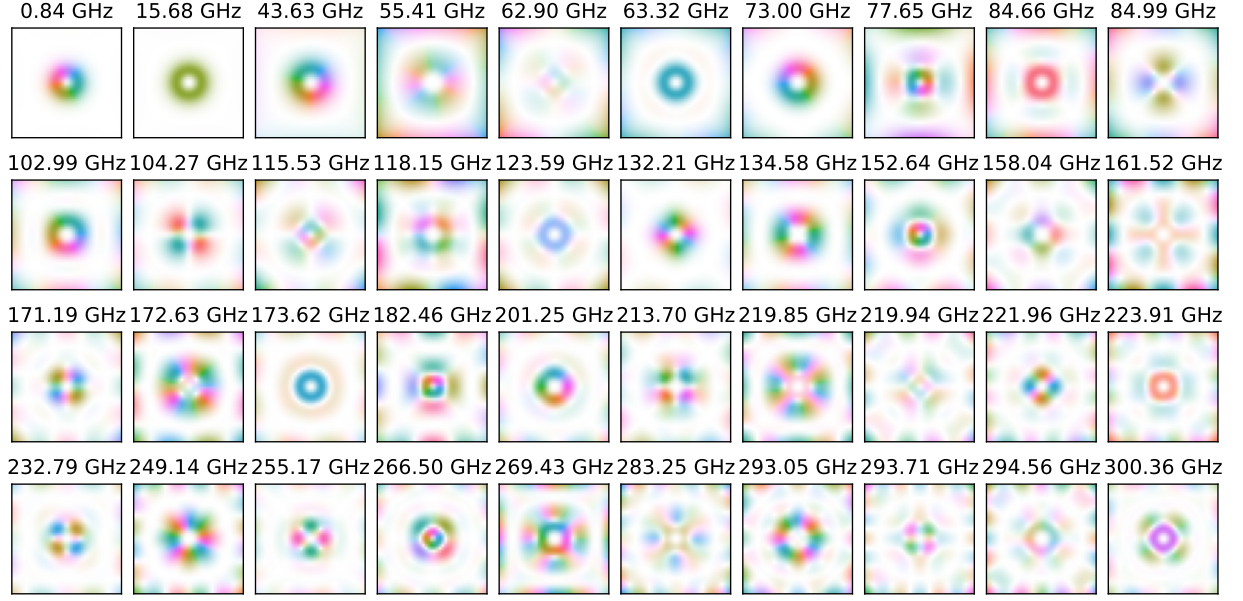


Figure A.2: Eigenmodes of a single FM skyrmion in a $(40 \times 40 \times 2) \text{ nm}^3$ geometry. The first three modes are discussed in detail in Fig. 4.5 in Chapter 4.

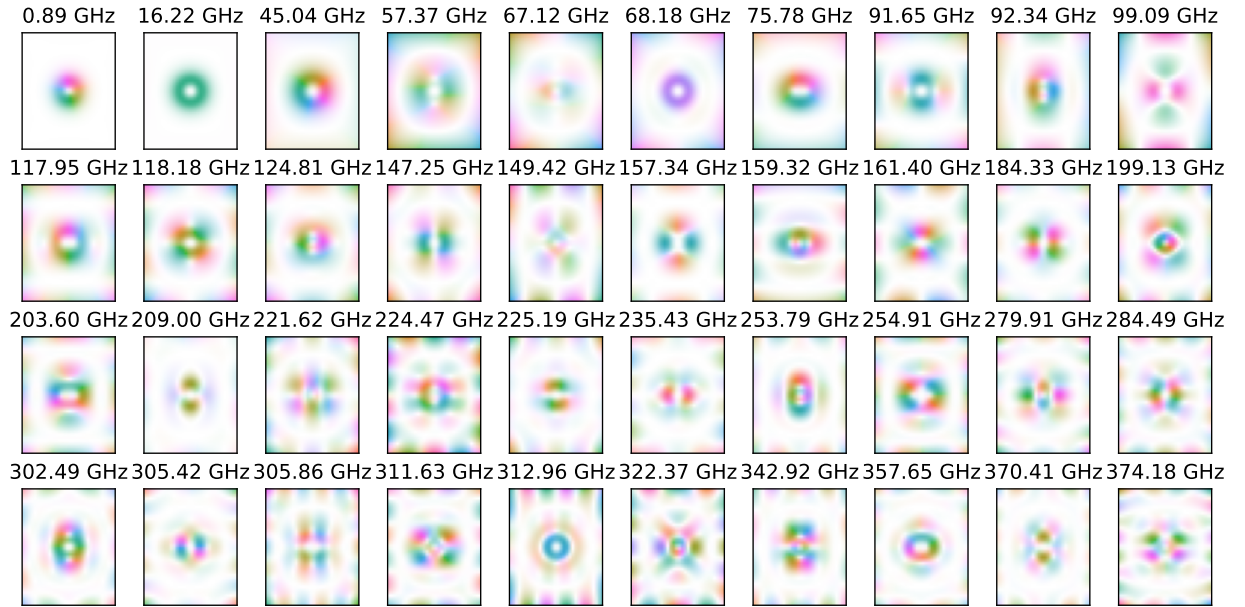


Figure A.3: Eigenmodes of a single FM skyrmion in a $(32 \times 40 \times 2) \text{ nm}^3$ geometry. Compared to the square geometry, many of the modes show elliptical distortion, see Fig. A.4 for a more detailed comparison of the gyrotropic and first rotational mode.

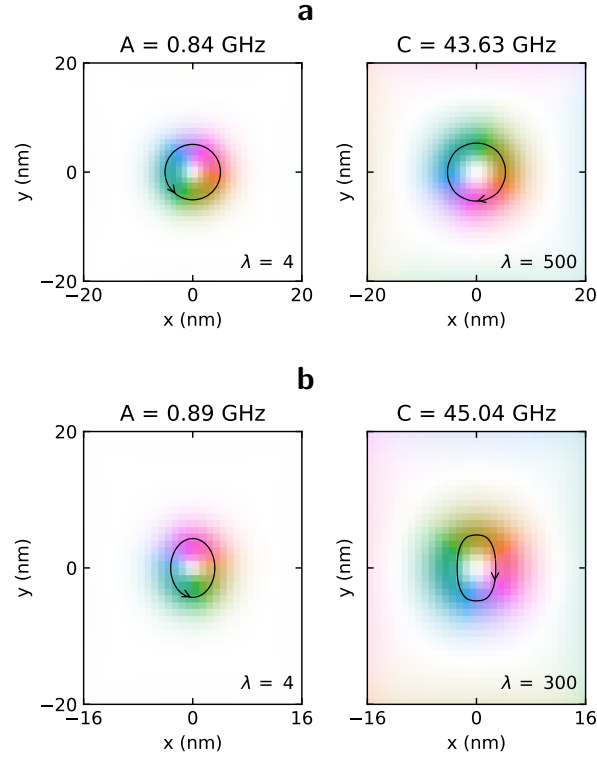


Figure A.4: Comparison of the low-frequency gyrotropic and rotational modes for a FM skyrmion in (a) square geometry of size $(40 \times 40 \times 2) \text{ nm}^3$ and (b) rectangular geometry of size $(32 \times 40 \times 2) \text{ nm}^3$. The trajectories show that in the square geometry the skyrmion moves in a circle, while in the rectangular geometry the motion is elliptical.



Figure A.5: Eigenmodes of a single SAF skyrmion in a $(40 \times 40 \times 2) \text{ nm}^3$ geometry. The two layers are marked with letters t (top) and b (bottom). The first four modes are discussed in detail in Fig. 4.8 in Chapter 4.

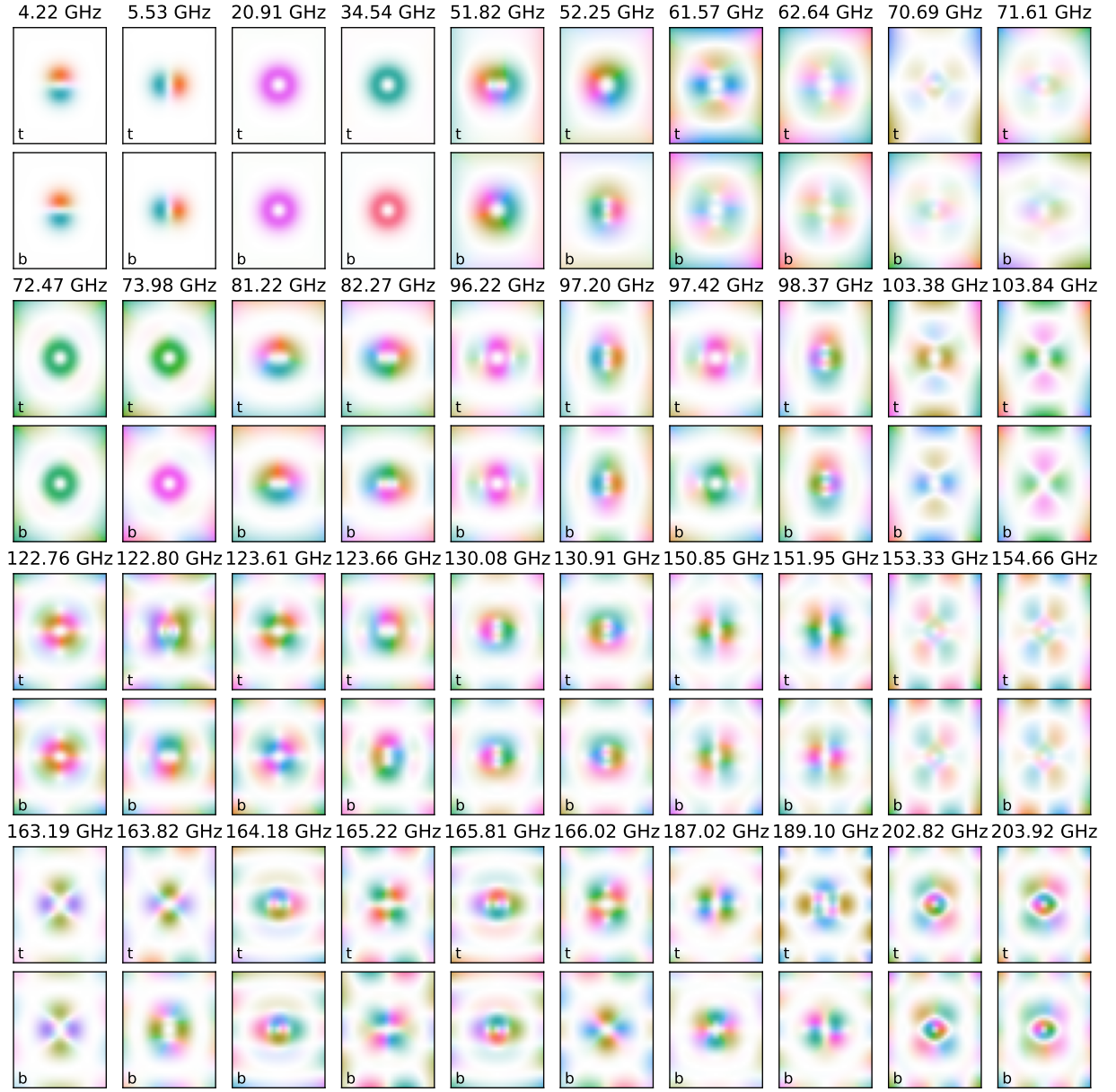


Figure A.6: Eigenmodes of a single SAF skyrmion in a $(32 \times 40 \times 2) \text{ nm}^3$ geometry. The two layers are marked with letters t (top) and b (bottom). The first four modes are discussed in detail in Fig. 4.12 in Chapter 4.



Figure A.7: Eigenmodes of two SAF skyrmions in a $(64 \times 40 \times 2) \text{ nm}^3$ geometry. The two layers are marked with letters t (top) and b (bottom). The first eight modes are discussed in detail in Fig. 5.3 and Fig. 5.4 in Chapter 5.



Figure A.8: Eigenmodes of five SAF skyrmions in a $(160 \times 40 \times 2) \text{ nm}^3$ geometry. The two layers are marked with letters t (top) and b (bottom). The first 20 modes are discussed in detail in Fig. 5.8 and Fig. 5.9 in the Chapter 5.

A.3 Signal propagation in a FM strip

Figures A.9 and A.10 show signal propagation in a ferromagnetic strip containing 25 skyrmions. These results are used as a comparison with the corresponding SAF strip discussed in Section 5.3. In both cases, only the left-most skyrmion is excited locally, while the response of the full skyrmion chain is monitored as a function of time.

The power spectral density of the ferromagnetic strip is shown in Fig. A.9(a), and selected breathing modes from the band around 15 GHz are shown in Fig. A.9(b). Compared with the SAF system, this breathing-mode band is shifted towards lower frequencies. In Fig. A.9(c), the same Gaussian pulse as used for the SAF system is applied. This pulse is centred at 20.5 GHz, so its overlap with the 15 GHz breathing-mode band is limited. As a result, the excitation of the breathing modes is weak and inhomogeneous across the band. The time-resolved signal in Fig. A.9(d) shows that no strong propagation through the strip is observed. The response is mainly localised at the first skyrmion shortly after the pulse reaches its maximum at approximately 0.4 ns. This is also visible in Fig. A.9(e), where the first skyrmion shows the strongest response, while the centre and right-most skyrmions are only weakly excited.

To check whether this weak propagation is caused by the mismatch between the excitation pulse and the breathing-mode band, the excitation was repeated with a Gaussian pulse centred at 15 GHz. The corresponding result is shown in Fig. A.10. The PSD and the selected breathing modes in Figs. A.10(a) and A.10(b) are the same as in Fig. A.9. However, the excitation spectrum in Fig. A.10(c) shows a much better overlap between the pulse and the breathing-mode band. In this case, the breathing modes dominate the excited spectrum.

This improved overlap leads to a clearer propagation of the excitation along the skyrmion chain, as shown in Fig. A.10(d). The response of selected skyrmions in Fig. A.10(e) also shows that the excitation reaches the centre and right-most skyrmions more clearly than in the 20.5 GHz excitation case. By tracking the first maxima of the signal, the propagation velocity is estimated to be approximately 255 m s^{-1} . These results show that efficient signal propagation in the ferromagnetic strip requires the excitation pulse to overlap well with the relevant breathing-mode band.

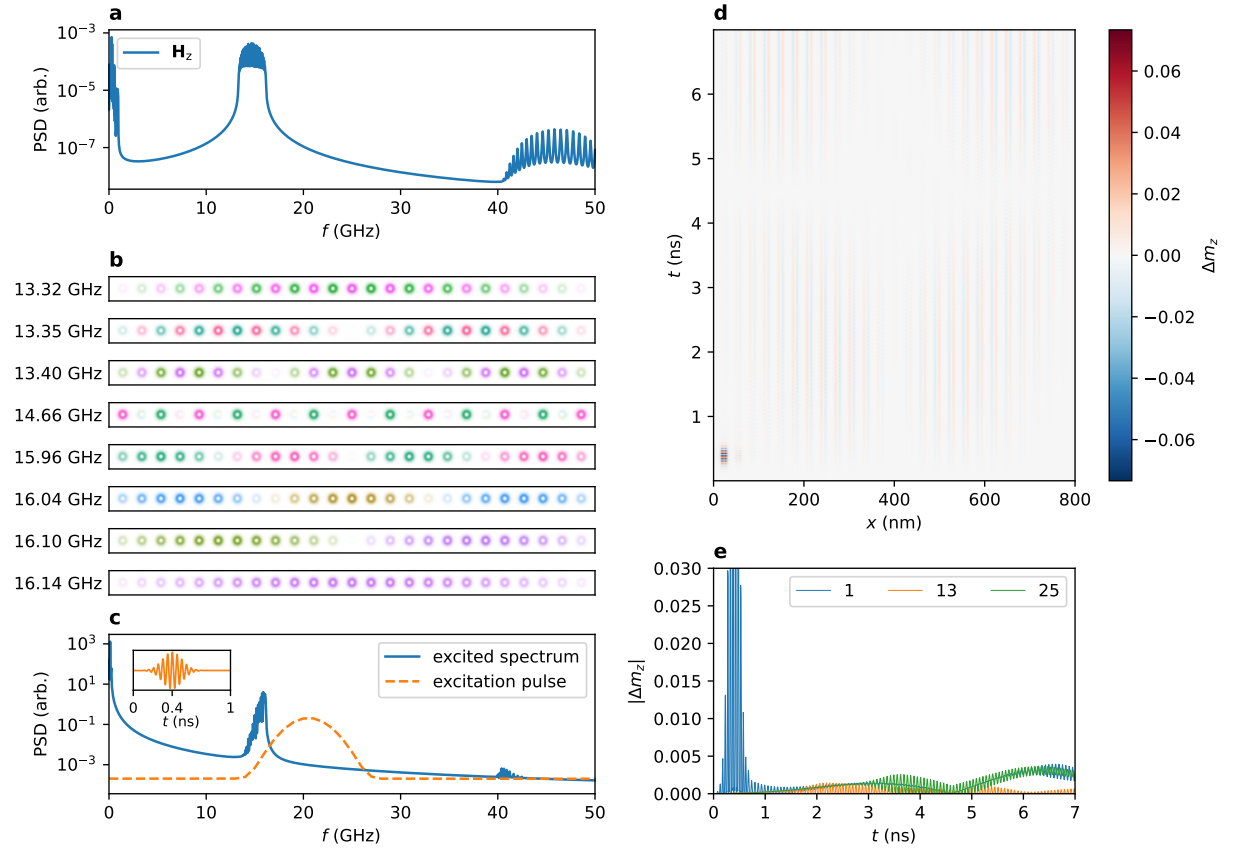


Figure A.9: Signal propagation in a ferromagnetic strip containing 25 skyrmions after local excitation with a Gaussian pulse centred at 20.5 GHz.

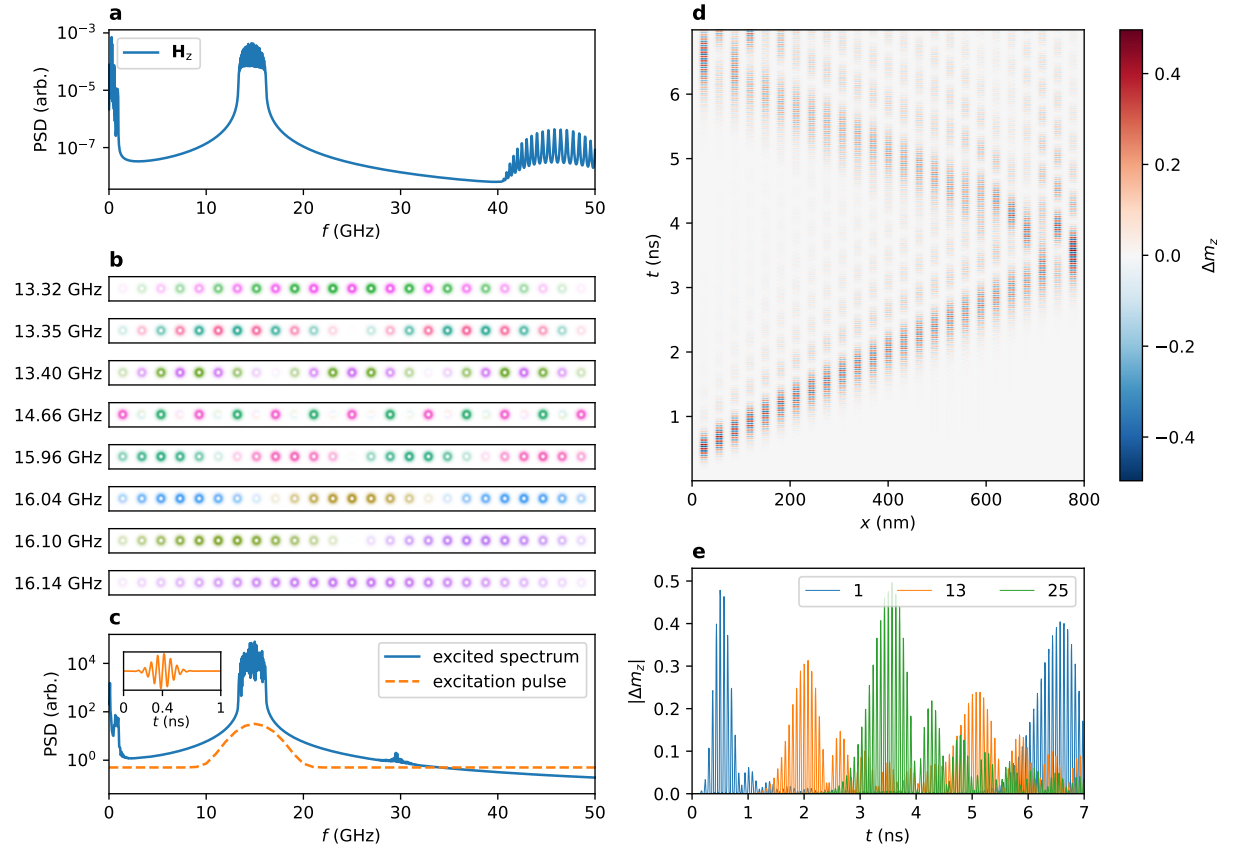


Figure A.10: Signal propagation in a ferromagnetic strip containing 25 skyrmions after local excitation with a Gaussian pulse centred at 15 GHz.

Appendix B

Supplementary classification results for DyCo₃ thin films

This appendix shows additional representative relaxed magnetic states obtained from the classification of the DyCo thin-film simulations. The classification method and the physical interpretation of these classes are discussed in Section 6.6.2. Here, the aim is not to repeat the detailed discussion, but to provide further examples of the states belonging to each class.

For each class, 40 configurations were selected across the corresponding region of the parameter space and plotted using the in-plane magnetisation component m_x . The values of K_{eff} and D are given above each configuration to show how the magnetic textures within each class vary across the sampled parameter range.

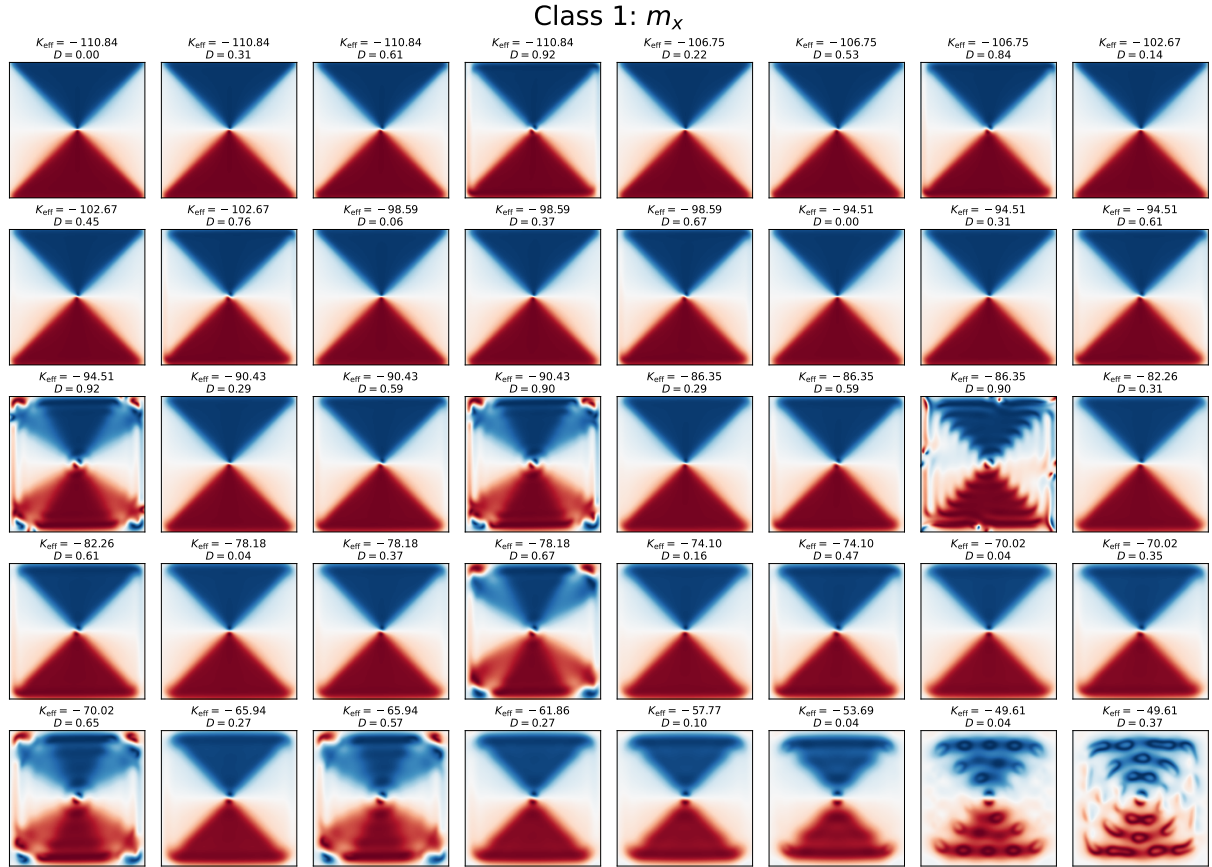


Figure B.1: Representative m_x configurations belonging to Class 1.

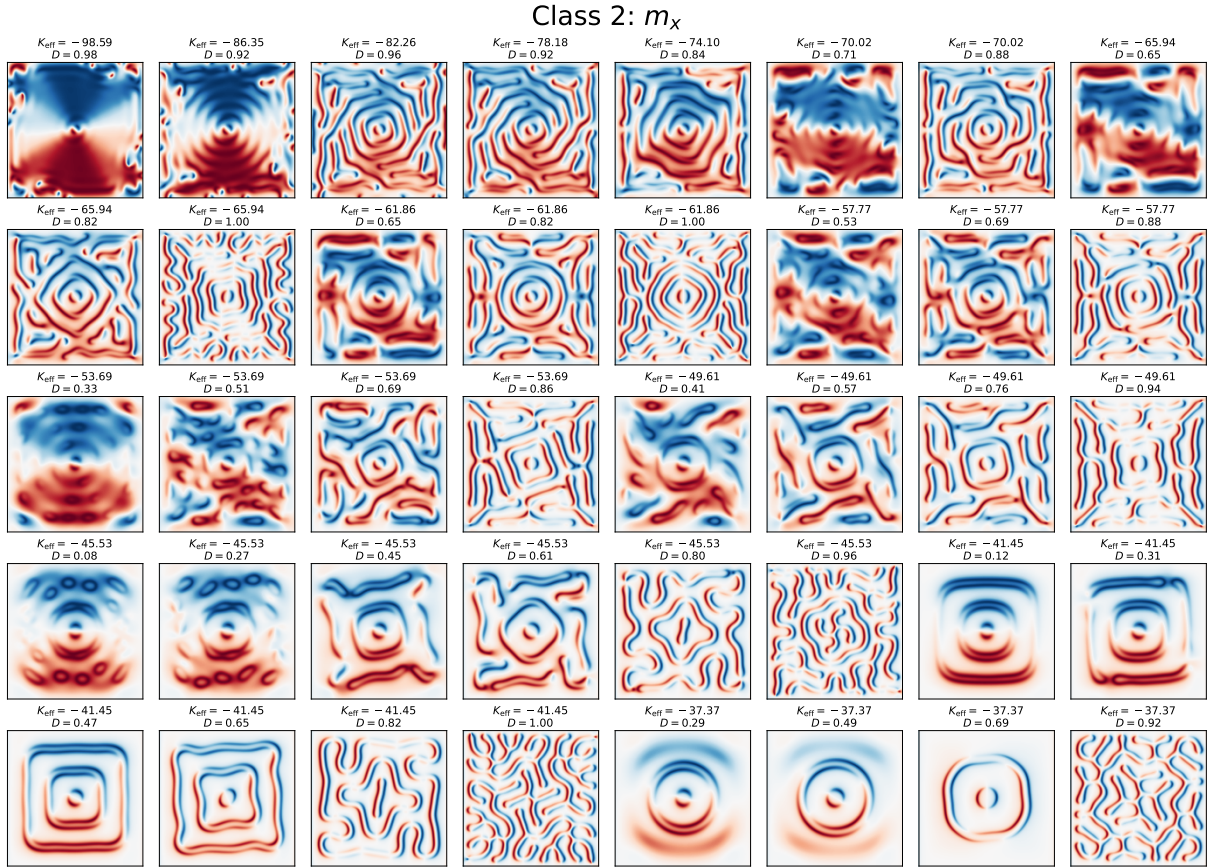


Figure B.2: Representative m_x configurations belonging to Class 2.

In the isolated skyrmion-like classes, Classes 3–5, the skyrmion radius decreases across the selected configurations. This is mainly because these states correspond to larger values of K_{eff} , which favours smaller skyrmions. Although increasing D tends to increase the skyrmion radius, the increase in K_{eff} has the dominant effect in the selected states. This behaviour is therefore consistent with the radius map discussed in Section 6.6.4.

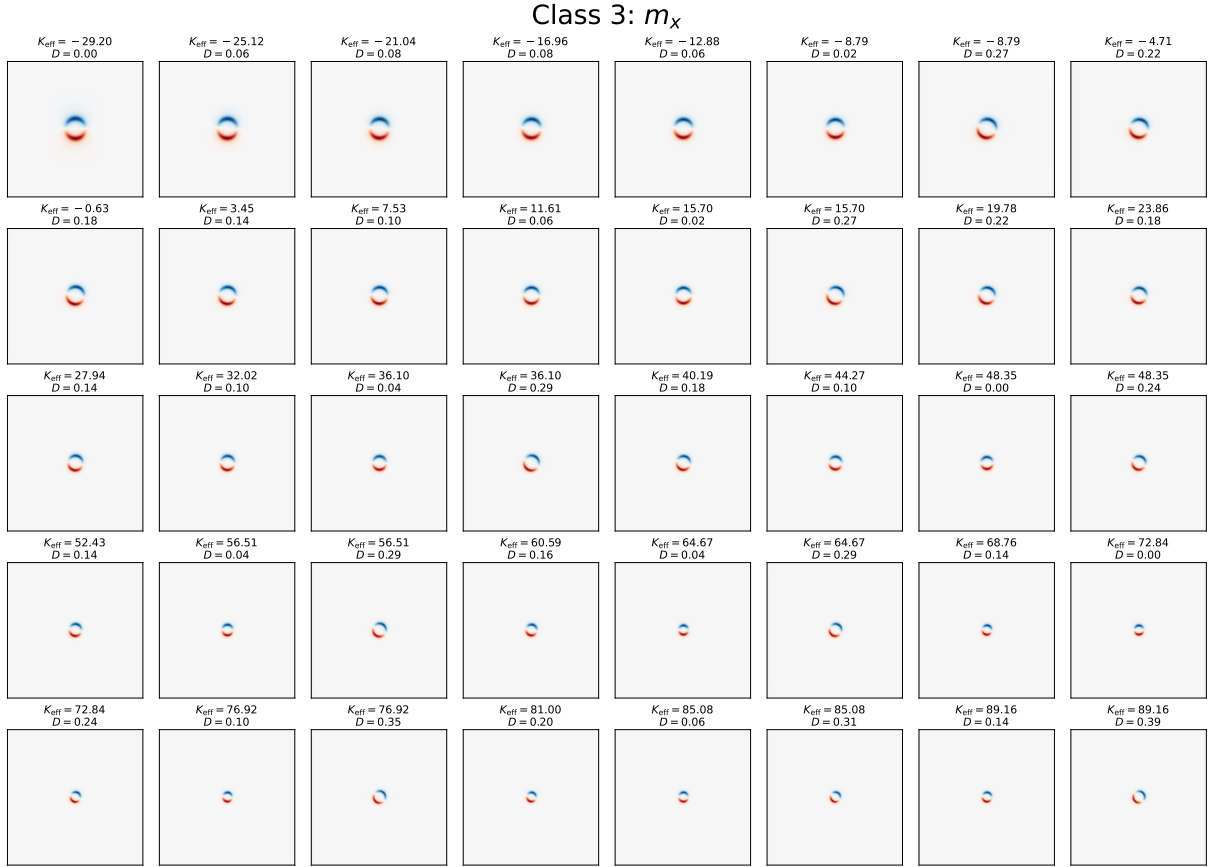


Figure B.3: Representative m_x configurations belonging to Class 3.

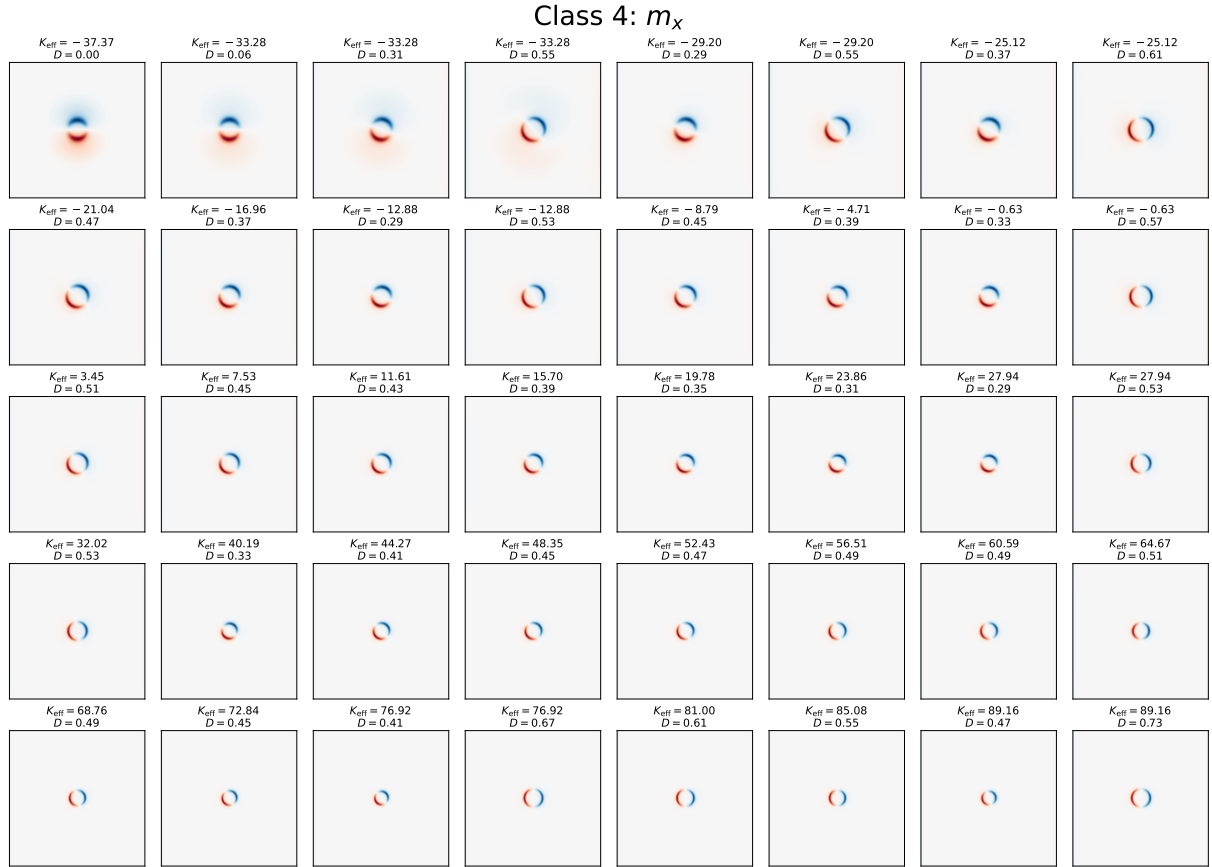


Figure B.4: Representative m_x configurations belonging to Class 4.



Figure B.5: Representative m_x configurations belonging to Class 5.

B.1 Simulated experimental contrast of type 1 and type 2 bubbles

This section examines whether the type 1 and type 2 bubbles discussed in Section 6.3 could be distinguished using experimental imaging techniques. In the simulations, the two states can be separated directly because the full vector magnetisation field is available. In experiment, however, the measured contrast depends on the imaging method and may not contain all components of the magnetisation.

The main difficulty is that the two bubble types have very similar out-of-plane magnetisation profiles. Therefore, imaging methods that are mainly sensitive to the projected (m_z) component are expected to show nearly the same contrast for both states. The difference between the two states is instead contained in the in-plane structure of the domain wall: in a type 1 bubble the in-plane magnetisation winds continuously around the wall, whereas in a type 2 bubble this winding is interrupted by Bloch lines.

To test which imaging methods are sensitive to this difference, the expected experimental contrasts were calculated from the relaxed type 1 and type 2 states using `mag2exp` [145]. The magnetic states were kept fixed, and only the imaging method was changed. This allows the resulting contrast to be compared directly for the same two underlying magnetic textures.

B.1.1 X-ray holography

X-ray holography is the natural starting point, since X-ray imaging is the technique used in the experiments of Ref. [50] and gives a reliable estimate of the position, radius and polarity of the bubbles. For a normally incident beam, however, the measured contrast is essentially the thickness-integrated out-of-plane magnetisation, which is precisely the part of the texture that differs least between the two bubble types.

Figure B.6 shows the integrated out-of-plane magnetisation for the two states, plotted with a 50 nm spatial resolution to represent the effective resolution of the holography measurement. The two maps are almost indistinguishable: both show a positive core, a negative background and nearly the same radius. X-ray holography can therefore confirm that a bubble is present and measure its size, but it cannot by itself decide whether the bubble is type 1 or type 2. The topological difference is hidden because it is encoded in how the magnetisation turns inside the wall, not in the size or position of the reversed core. This is the basis of the ambiguity discussed in Section 6.6.7: the experimental images of Ref. [50] establish the presence and size of the bubbles, but not their topological character.

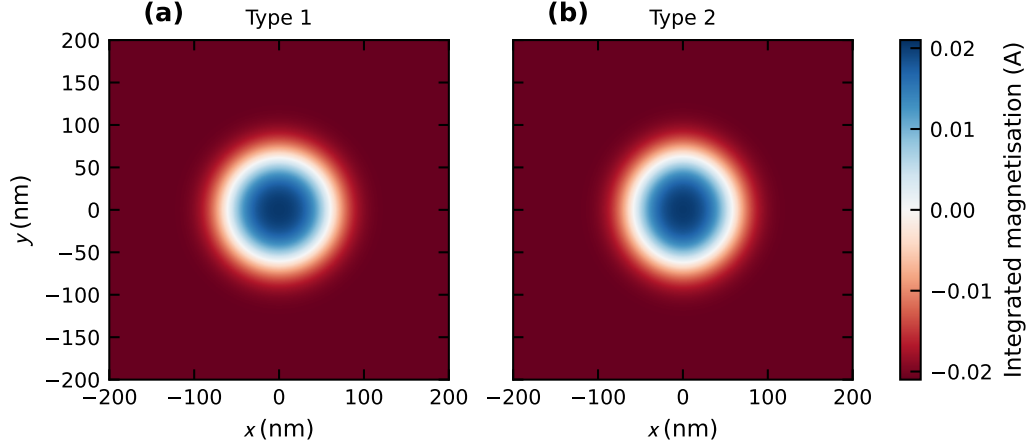


Figure B.6: Integrated out-of-plane magnetisation for the type 1 and type 2 bubbles, plotted with a 50 nm spatial resolution to represent the effective resolution of the X-ray holography measurement. **(a)** The type 1 bubble shows a positive core, a negative background and a well-defined circular domain wall. **(b)** The type 2 bubble gives an almost identical projected map. The similarity of the two images shows why projected m_z contrast alone cannot identify the bubble type.

B.1.2 Lorentz transmission electron microscopy

Lorentz transmission electron microscopy (LTEM) is better matched to this question. In LTEM the electron beam is deflected by the magnetic induction inside the sample, so the defocused image is strongly affected by the in-plane magnetisation in the domain wall, which is exactly where the two states differ. For a type 1 bubble, the smooth winding of the wall should produce a contrast pattern that is close to rotationally symmetric. For a type 2 bubble, the Bloch lines interrupt this winding, and the smooth ring should break up into angular sectors.

This is what Figure B.7 shows. The images were calculated at 300 kV with a defocus of -0.4 mm, using the same microscope parameters for both states. The type 1 bubble produces a circular ring pattern with concentric bright and dark features around the centre, while for the type 2 bubble the ring is broken into two halves. These sector-like features are the practical experimental fingerprint of the Bloch lines: a smooth circular pattern is consistent with a type 1 bubble, whereas a broken sector-like pattern is evidence for a type 2 bubble.

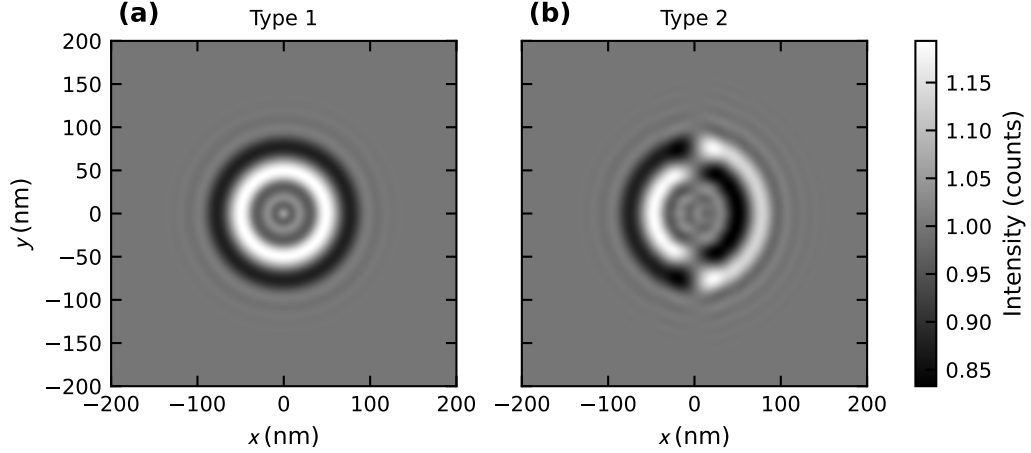


Figure B.7: Simulated LTEM defocus images for the type 1 and type 2 bubbles. **(a)** The type 1 bubble gives a nearly circularly symmetric ring pattern, reflecting the smooth winding of the in-plane magnetisation around the domain wall. **(b)** For the type 2 bubble the ring breaks into sector-like contrast because the Bloch lines interrupt the continuous winding of the wall. LTEM is therefore far more sensitive to the difference between the two bubble types than projected m_z imaging.

B.1.3 Magnetic force microscopy

Finally, the magnetic force microscopy (MFM) phase shift expected from the two states is calculated. MFM is a less direct probe than either X-ray holography or LTEM: it does not image the magnetisation itself, but the interaction between a magnetic tip and the stray field above the sample. The image therefore carries information about the internal structure, but it also depends on the lift height, the tip shape and magnetisation, and the details of the stray field.

Figure B.8 shows the simulated phase shift for the two bubbles. The type 1 bubble produces a smooth, nearly circularly symmetric pattern. The type 2 bubble is visibly asymmetric, with an additional feature appearing at the lower part of the domain wall that is absent in the type 1 state. This feature is consistent with the Bloch-line structure breaking the rotational symmetry of the wall.

The distinction is, however, more subtle than in LTEM. The type 2 MFM signal does not produce as clean a sector-like fingerprint, and the result will be sensitive to the tip and scan conditions. MFM can therefore provide supporting evidence for the bubble type, but LTEM remains the cleaner experimental test because it probes the in-plane domain-wall magnetisation directly.

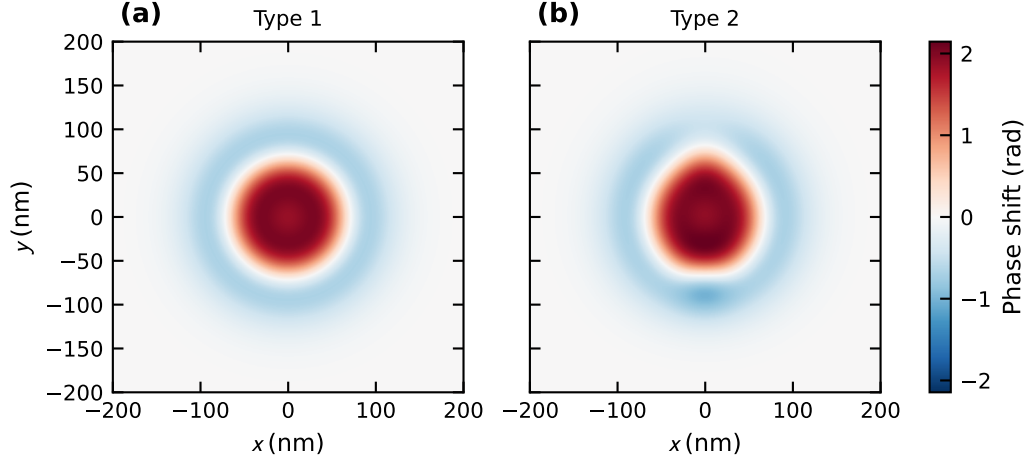


Figure B.8: Simulated MFM phase shift for the type 1 and type 2 bubbles. **(a)** The type 1 bubble produces a smooth phase-shift pattern with no strong angular feature at the domain wall. **(b)** The type 2 bubble produces an additional feature at the lower part of the wall, consistent with the Bloch-line structure. MFM can provide supporting evidence for the bubble type, although its interpretation is less direct than LTEM because it measures the stray-field response above the sample.

B.1.4 Summary

Projected out-of-plane imaging is not sufficient to distinguish type 1 and type 2 bubbles. X-ray holography identifies the bubble and measures its radius and polarity, but the two states give almost identical contrast because the difference between them lives in the in-plane structure of the wall. LTEM is the most useful probe of this structure, distinguishing clearly between the smooth circular type 1 pattern and the broken sector-like type 2 pattern. MFM also shows the symmetry breaking caused by the Bloch lines, but the signal is less direct and is best treated as supporting evidence rather than the main identifier. LTEM imaging of the DyCo_3 bubbles would therefore provide a direct experimental test of whether the bubbles observed in Ref. [50] carry the topological charge of the type 1 state, resolving the ambiguity identified in Section 6.6.7.

Appendix C

Software contributions: pull requests and tools

This appendix lists the principal pull requests (PRs) associated with the contributions described in Chapter 3, grouped by theme. Smaller corrections (typographical fixes and minor cleanups) are combined into single entries. Each PR number links to the corresponding page on GitHub; all repositories belong to the **ubermag** organisation (<https://github.com/ubermag>).

Repository	PR	Contribution
<i>Simulation metadata workflow (Section 3.3)</i>		
micromagneticdata	#84	Rewrite of the <code>info.json</code> data information workflow.
micromagneticmodel	#92	<code>update_info_json</code> method to update and extend the simulation metadata.
oommfc	#161	Removal of the superseded <code>_write_info_json</code> from the OOMMF calculator.
mumax3c	#93	Removal of the superseded <code>_write_info_json</code> from the MuMax ³ calculator.
ubermagutil	#55	Time stamps in metadata changed to ISO format.
micromagnetictests	#47	Test function for <code>RelaxDriver</code> behaviour.
<i>Data access and robustness (Section 3.5)</i>		

Repository	PR	Contribution
micromagneticdata	#75	Initialisation of <code>Data</code> objects from a path instead of name and directory name.
micromagneticdata	#69	Test for the validity of the field norm.
micromagneticdata	#62	Corrections in the data-handling code.
<i>Model and driver validation</i>		
micromagneticmodel	#86	Validation that a system defines dynamics and energy terms.
oommfc	#151	Corresponding dynamics and energy checks in the OOMMF calculator.
mumax3c	#79	Corresponding dynamics and energy checks in the MuMax ³ calculator.
mumax3c	#82	Fix: damping term not required for <code>RelaxDriver</code> .
<i>Testing and maintenance (Section 3.7)</i>		
discretisedfield	#535	Removal of the legacy type system.
discretisedfield	#534	Deprecation message for the <code>k3d</code> visualisation.
discretisedfield	#518	Test for deprecation warnings (open).
discretisedfield	#524 , #529 , #536	Smaller fixes: n -dimensional field check in <code>point_columns</code> , wildcard-point correction, mesh resampling (partly open).
oommfc	#152	Filtering of unsupported tests.
tutorials	#72	Addition of the pre-commit configuration.
<i>Ringdown analysis prototype (Section 3.6)</i>		
micromagneticdata	#76	<code>RingdownAnalysis</code> class: relaxed-state subtraction, Fourier analysis, spatially resolved and averaged power spectral density, peak detection, and mode reconstruction (open feature pull request).

Documentation and examples (Section 3.4)

Repository	PR	Contribution
discretisedfield	#390–#407	Systematic review and update of the documentation notebooks (field definition, visualisation, normalisation, input/output, rotations, tools, mesh, regions; 18 merged PRs).
tutorials	#70	New tutorial notebook for <code>micromagneticdata</code> based on a domain-wall pair conversion example.
tutorials	#63 , #67 , #68 , #71	Vortex trajectory example and corrections across the tutorial notebooks.
ubermag.github.io	#40	PyVista visualisation documentation added to the website, replacing <code>k3d</code> .
micromagneticmodel	#83 , #84 , #91	Documentation cleanups, including removal of outdated website references and examples.
mag2exp	#62 , #63	Corrections in the package documentation.

The FMR contribution to `mag2exp` described in Section 3.6 was developed in collaboration with the package authors and is documented in the author contribution statement of the associated publication [145].

Appendix D

Publications

The following publications were produced during the course of this doctoral work.

1. H. Fangohr, M. Lang, S. J. R. Holt, S. A. Pathak, **K. Zulfiqar**, and M. Beg, “Vision for unified micromagnetic modeling (UMM) with Ubermag,” *AIP Advances*, **14**, 015138 (2024).

DOI: <https://doi.org/10.1063/9.0000661>. [63]

This publication is related to the computational framework used in this thesis. It discusses the unified micromagnetic modelling approach implemented in **Ubermag**.

2. S. J. R. Holt, M. Lang, J. C. Loudon, T. J. Hicken, D. Suess, D. Cortés-Ortuño, S. A. Pathak, M. Beg, **K. Zulfiqar**, and H. Fangohr, “Virtual experiments in computational magnetism with **mag2exp**,” *npj Computational Materials*, **11**, 201 (2025).

DOI: <https://doi.org/10.1038/s41524-025-01686-3>. [145]

This publication is related to the computational magnetism tools used in this thesis. It presents the **mag2exp** package, which enables virtual experimental measurements to be computed from spatially resolved magnetisation fields.

3. **K. Zulfiqar**, M. Lang, S. J. R. Holt, S. A. Pathak, F. Bruckner, and H. Fangohr, “Eigenmodes of synthetic antiferromagnetic skyrmions,” arXiv:2606.06304 (2026).

<https://arxiv.org/abs/2606.06304>. [146]

Submitted to *Scientific Reports*.

This preprint forms the basis of the thesis Chapters 4 and 5 on skyrmion eigenmodes in ferromagnetic and synthetic antiferromagnets.

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