Construction and Bayesian Estimation of DSGE Models for the Euro Area

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To my wife

Niniejszą pracę dedykuję mojej żonie.

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Chapter 1

Introduction

1.1 Objective of the study

Dynamic Stochastic General Equilibrium (DSGE) models have become a standard tool in various fields of economics. These models involve a fully-specified stochastic dynamic optimization, as opposed to reduced-form decision rules, and are therefore not subject to the Lucas critique. DSGE models are also attractive because they include New-Keynesian elements accounting for, paraphrasing Mankiw (1989), *the possibility of market failure on a grand scale*. Thus, contrary to their Real Business Cycle predecessors, the new generation of DSGE models can adequately replicate the high persistence which is undoubtedly present in nominal aggregates.

Few years ago, calibration, as advocated by Kydland and Prescott (1982), was by far the most common approach to examining the empirical properties of these models.[1] Although calibration is a very useful tool for understanding the dynamic properties of the system, the fully-fledged econometric estimation allows for construction of a more *realistic* economic model. The estimation procedure quantifies the uncertainty regarding model parameterization and allows for obtaining estimates of the parameters which could not be directly measured in micro-studies. Finally, estimated DSGE models can be validated by applying formal statistical methods allowing them to be compared in terms of forecast accuracy with other classes of econometric models.

Over the course of time, various formal methods have been proposed for quantifying DSGE models. The literature documents the attempts to estimate these models with Maximum Likelihood (ML), Generalized Method of Moments (GMM), Simulated Method of Moments (SMM) and a wide range of Limited Information methods. The most recent developments are marked by system-based Bayesian methods. The popularity of the Bayesian approach is particulary attributed to the fact that it allows us to incorporate extraneous information regarding model parametrization into the estimation procedure. Therefore, Bayesian estimation is, in the context of microfounded models, a reasonable compromise between the still very popular calibration and the fully-fledged estimation.

The application field of estimated DSGE models is substantially larger than that of their calibrated counterparts. These models are currently used not only for policy experiments, but also as a forecasting tool. Nominal stickiness (besides the well established real rigidities) has emerged as one of the most important factors that helps models in the New Keynesian tradition to match features of the data while maintaining a coherent micro-founded framework. The empirical properties of alternative mechanisms available in theoretical literature have been, however, only sporadically examined in the general equilibrium framework. Yet, the vast majority of DSGE models adopt the Calvo (1983) scheme for introducing both price and wage rigidities. Despite its popularity

[1]Kydland and Prescott (1982) see a much broader role for calibration. Calibration, in their opinion, might also provide signals to microeconomists about important gaps in knowledge, which when filled will improve the credibility of the quantitative analysis based on the microfounded models.

the Calvo set-up has been criticized for its several unappealing assumptions. As an alternative Mankiw and Reis (2002) propose the sticky information mechanism, which (on the theoretical ground) seems to outperform the Calvo scheme. In our study, we shed light on the problem of the empirical comparison of both schemes. We consider a state-of-the-art general equilibrium model, which is estimated under different assumptions regarding mechanisms for incorporating nominal rigidities. The comparison is done in the context of the Euro area aggregate data due to Fagan, Henry, and Mestre (2001) and the results provide guidance to researchers dealing with the estimation of Euro area DSGE models in general. We also exploit the outcome of this research while establishing a multi-country framework for modeling the Euro zone.

The construction of DSGE models for the Euro area appears to be a relatively new phenomenon. This direction of research has been initiated by Smets and Wouters (2003), who, employing the aforementioned 'synthetic' data set, have estimated a closed-economy DSGE model following the lines of Christiano, Eichenbaum, and Evans (2001). Meanwhile, the literature on estimated area-wide DSGE models is growing rapidly. Yet, DSGE models for the member states of the European Economic and Monetary Union (EMU) practically do not exist. In this thesis, we address problems related to the construction and the system-based estimation of multi-region models. Moreover, we present a coherent economic and statistical framework that approximates the structure of the EMU and explicitly accounts for the historical monetary regime change. In such a framework the disaggregate information on the Euro area can be utilized, so that one can explain the area-wide aggregates and also examine the cross-region linkages. Finally, the multi-region model can be estimated with a longer data sample without misspecifying the monetary policy regime.

1.2 Summary of the chapters

This thesis is divided into three parts. In Chapter 2 the methods of Bayesian estimation of DSGE models are presented. Chapter 3 examines the empirical validity of alternative price and wage setting schemes. The analysis is based on the estimated versions of the seminal closed-economy DSGE model for the Euro area. Chapter 4 presents a prototypical two-region DSGE model for the Euro area. We also propose and implement a coherent statistical framework for its estimation.

Bayesian estimation of DSGE models In Chapter 2 we introduce the system-based Bayesian methods for estimation and evaluation of DSGE models. Besides the use of an apparatus that is strictly associated with Bayesian analysis, the estimation of DSGE models exploits the concepts established for numerical methods of rational expectations model solutions, estimation of state space models and Monte Carlo Markov Chains. The aim of Chapter 2 is to discuss the algorithms used while estimating the models to be presented in the remainder of the thesis.

Sticky contracts or sticky information? Evidence from an estimated Euro area DSGE model In Chapter 3, we empirically evaluate two competing classes of models for introducing nominal rigidities: the Calvo model vs. the sticky information model of Mankiw and Reis (2002). We estimate variants of the Smets and Wouters (2003) DSGE model for the Euro area with Bayesian methods under different assumptions regarding mechanisms of price and wage setting. Our main finding is that the Calvo model overwhelmingly dominates the standard sticky information model in terms of the posterior odds ratio. The origin for the poor fit appears to be the inability of sticky information models to match simultaneously the autocorrelation as well as the volatility of inflation and the real wage.

In a second step, we ask whether heterogeneity in price and wage setting provides a better empirical fit. We develop a model with heterogenous agents in which one fraction follows the Calvo scheme and the other the sticky information scheme. This innovation allows us to validate the empirical relevance of a particular scheme within the same model. For the standard specification we find that the fraction of Calvo price setters is estimated at 99%. The fraction of Calvo wage setters is estimated at 93%. Thus, the data ascribes almost zero mass to sticky information considerations in price setting and only small mass for wage setting. We then allow the distribution of cohorts of information sets to follow two less restrictive patterns than in the baseline sticky information model and find slightly more support for the sticky information idea. This leads to an estimated population share of sticky information agents of roughly 15% for price setting and 30%-35% for wage setting. However, the marginal density of the nested model is lower than for the pure sticky price model a la Calvo. Summing up, our analysis turns around the view of Mankiw and Reis (2002) that the Calvo model is *hard to square with the facts* and concludes that the data strongly favors the Calvo model over the sticky information model.

Transmission of economic fluctuations in an estimated two-region DSGE model for the Euro area In Chapter 4 we propose and implement a coherent statistical framework for the estimation of a two-region DSGE model for the Euro area. The model is an augmented version of that discussed in Chapter 3 and is primarily used to study the linkages between the German economy and the rest of the area. Our contribution is twofold. First, we emphasize the use of regime-switching models in the DSGE framework (in our case the threshold is known exactly and the switch from free float to monetary union is permanent). This augmentation allows us to use a longer data sample (prior to EMU and afterwards) in the estimation without misspecifying the monetary regime. Second, we advance the growing body of the empirical literature on forecasting aggregate European economic performance. We utilize disaggregate information on the Euro area employing the (unfiltered) national accounts data along with the 'synthetic' Euro area data by Fagan, Henry, and Mestre (2001) in the model estimation. Subsequently, we quantify the effects associated with the extended information set as well as those arising from the New Open Economy Macroeconomics restrictions on the model's ability to forecast the key Euro area aggregates. Moreover, Chapter 4 offers: (i) a robustness check of the estimation results with respect to alternative priors, alternative data approaches and various restrictions imposed on the model's structure, (ii) assessments of the relative importance of various shocks and frictions, and the role of international price discrimination within the Euro area in particular, for explaining the model dynamics, (iii) an evaluation of the model's empirical properties, and (iv) a thorough analysis of international spillovers within the area.

Our results make a clear case for relying on a multi-country (disaggregate) DSGE modeling approach when analyzing and forecasting the Euro area economy, and suggest that a line of research worth pursuing is the estimation of relationships within the area and their effects on economic policies.

Chapter 2

Bayesian estimation of DSGE models

This chapter reviews Bayesian methods that have been developed in recent years to estimate and evaluate dynamic stochastic general equilibrium models. Our intention is not to give an exhaustive treatment of the topic. We rather aim to provide a selfcontained presentation of the econometric tools which will be employed in Chapters 3 and 4.

Chapter 2 proceeds as follows. Section 2.1 presents the literature review. Section 2.2 discusses the basic concepts of Bayesian inference. Section 2.3 outlines a generic procedure for estimation of DSGE models. In remaining sections more detail is provided on the particular steps of the procedure. So, Section 2.4 presents solution methods and transformation of the DSGE model to a state space model. The issues related to the determination of the prior distribution for DSGE models are discussed in Section 2.5. Section 2.6 presents the algorithms for computation of filtering and prediction densities as well as for evaluation of the data likelihood for state space models. Section 2.7 outlines the methods of Monte Carlo Markov Chain that are applied to approximate the posterior distribution. The methods for evaluation of empirical model properties as well as the issues of identification and sensitivity analysis are discussed in Section 2.8.

2.1 Literature review

Many alternative econometric procedures have been proposed to parameterize DSGE models, ranging from calibration, e.g. Kydland and Prescott (1982), over generalized method of moments and estimation of equilibrium relationships, e.g. Christiano and Eichenbaum (1992), to full-information likelihood-based estimation as in Altug (1989), McGrattan (1994), Leeper and Sims (1994), Kim (2000) and Ireland (2000). Given the complexities involved in estimating state-of-the-art DSGE models with full-information methods, a strand of the literature has considered limited information methods and focused on whether the model matches the data only along certain dimensions. For instance, Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005) propose minimum distance estimation based on the discrepancy among VAR and DSGE model impulse response functions. In turn, Canova (2002) proposes an alternative limited information approach where only a qualitative matching of responses is sought. The methodological debate surrounding the various estimation (and model evaluation) techniques has been summarized in papers by Kydland and Prescott (1996), Hansen and Heckman (1996), Sims (1996), Diebold (1998) and Ruge-Murcia (2003).

The focus of this thesis is Bayesian estimation, which marks the most recent developments in the DSGE research. The literature on likelihood-based Bayesian estimation of DSGE models began with works by Landon-Lane (1998), DeJong, Ingram, and Whiteman (2000), Schorfheide (2000), and Otrok (2001). DeJong, Ingram, and Whiteman (2000) estimate a stochastic growth model and examine its forecasting performance. Otrok (2001) fits a real business cycle with habit formation and assesses the welfare costs of business cycles. Schorfheide (2000), in turn, considers cash-in-advance monetary DSGE models. This approach has been generalized in Lubik and Schorfheide (2004) who estimate the benchmark New Keynesian DSGE model without restricting the parameters to the determinacy region of the parameter space. A similar approach has also been applied in Belaygorod, Chib, and Dueker (2005).

Growing interest in microfounded research at policy making institutions has created a demand for more complex and more realistic large-scale DSGE models that include capital accumulation and additional real and nominal frictions along the lines of Christiano, Eichenbaum, and Evans (2001). These models have been first estimated and analyzed by Smets and Wouters (2003) and Smets and Wouters (2004a), both for the US and the Euro area. Models similar to Smets and Wouters (2003) have been estimated by Laforte (2004), Onatski and Williams (2004), Ratto, Roeger, intVeld, and Girardi (2005a), Juillard, Karam, Laxton, and Pesenti (2004) and Levin, Onatski, Williams, and Williams (2005) to study monetary policy. Also our analysis of the empirical validity of alternative price and wage setting mechanisms, to be presented in Chapter 3 of this thesis, is based on variants of those New Keynesian DSGE models.

Bayesian estimation techniques have also been used in the open economy literature.[1] Lubik and Schorfheide (2003) estimate the Small Open Economy (SOE) extension of the seminal New Keynesian model to examine whether the central banks of Australia, Canada, England, and New Zealand respond to exchange rates. Justiniano and Preston (2004) estimate a SOE DSGE model in the presence of an imperfect exchange rate passthrough. Adolfson, Laseen, Linde, and Villani (2005a) analyze a multi-sector SOE model for the Euro area. Lubik and Schorfheide (2005), De Walque and Wouters (2004) and Adjemian, Paries, and Smets (2004) have estimated multi-country DSGE models to examine the interactions between the US economy and the Euro area. A multi-country model explaining the interactions within the Euro area will be presented in Chapter 4 of this thesis.

As mentioned, many central banks have pursued the process of developing DSGE models along the lines of Smets and Wouters (2003) that can be estimated with Bayesian techniques and used for policy analysis and forecasting. The first theoretically coherent attempt to estimate the DSGE model on the raw data, so that it could be used directly for forecasting, is documented in Altig, Christiano, Eichenbaum, and Linde (2003). They estimate the model under the constraints of balanced growth. As the balanced growth mechanism in its pure form is too restrictive to capture the different growth rates of real aggregates, the majority of existing DSGE models are still estimated with detrended data. In Chapter 4 we point to some stochastic extensions of the balanced growth mechanism and discuss their implementation into a two-region model.

2.2 Basic concepts of Bayesian analysis

This section provides a brief overview of Bayesian analysis. The purpose is to familiarize the reader with the concepts and notation which are subsequently used in the context of Bayesian estimation of DSGE models.[2]

In any empirical modeling exercise, there are three potential sources of uncertainty: the model itself (including the characterization of the underlying probability distribution), the parameterization conditional on the model, and the data. The stance on the issue of uncertainty is the most important difference between the classical (frequentist) and Bayesian approach. In classical analysis the probability of an event, i.e. the measure of uncertainty associated to the occurrence of the event, is the event's relative frequency. In Bayesian framework the probability of an event is

[1] One of the first attempts to develop an estimated New Open Economy Macroeconomics (NOEM) model may be found in the paper by Ghironi (2000). He develops an open economy model for the Canadian economy. The model is estimated 'equation by equation'. Nowadays, system estimation methodology is more common. One tends to use either generalized method of moments or a wide range of full information maximum likelihood methods.

[2] Comprehensive treatments of Bayesian analysis offer Jeffreys (1961), Zellner (1971) and Geweke (1999). Empirical applications are presented in Osiewalski (2001) or Canova (2005).

determined by two components, the subjective beliefs of the researcher and the frequency of that event.

Bayesian inference takes place in the context of one or more parametric econometric models.[3] Let y_t denote a vector of observable random vectors over a sequence of discrete time units $t = 1, ..., T$. The history of the sequence $\{y_t\}$ at time t is given by $Y_t = \{y_i\}_{i=1}^{t}$. A model M specifies a corresponding sequence of probability density functions $p(y_t|Y_{t-1}, \theta, M)$ in which θ is a vector of unknown parameters.

The probability density function of the data Y_T, conditional on model M and parameter vector θ, is

$$p(Y_T|\theta, M) = \prod_{t=1}^{T} p(y_t|Y_{t-1}, \theta, M). \tag{2.1}$$

Conditional on the observed Y_T the likelihood function is any function proportional to the probability density function

$$L(Y_T|\theta, M) \propto p(Y_T|\theta, M). \tag{2.2}$$

If the model specifies y_t to be independent and identically distributed then

$$p(y_t|Y_{t-1}, \theta, M) = p(y_t|\theta, M) \tag{2.3}$$

and

$$p(Y_T|\theta, M) = \prod_{t=1}^{T} p(y_t|\theta, M). \tag{2.4}$$

In classical analysis the parameters of a model are treated as fixed, unknown quantities. The data, in turn, are treated as unknown in the sense that their probability distribution (or the likelihood (2.2)) is the center of focus. Since the average value of the sample estimator converges to the true value via Law of Large Numbers, the unbiasness of the estimators is so important in classical analysis.

In the Bayesian approach the model M provides additionally the distribution of θ, which allows for determination of the joint distribution of the data Y_T and parameters θ. Properties of estimators and tests in small samples are uninteresting since beliefs are not necessarily related to the relative frequency of an event. In particular, if $p(\theta|M)$ denotes the prior density (conditional on model M) then

$$\begin{aligned} p(Y_T, \theta|M) &= p(\theta|M) \prod_{t=1}^{T} p(y_t|Y_{t-1}, \theta, M) && (2.5) \\ &= p(\theta|M)p(Y_T|\theta, M). && (2.6) \end{aligned}$$

(2.6) may also be expressed as

$$p(Y_T, \theta|M) = p(\theta|Y_T, M)p(Y_T|M), \tag{2.7}$$

where

$$p(Y_T|M) = \int p(Y_T|\theta, M)p(\theta|M)d\theta \tag{2.8}$$

[3] In this section, we take the specification of the model as given, though in principle the uncertainty about the nature of the model specification can be accommodated as well (see Section 2.8).

is the marginal likelihood of Y_T given model M and

$$p(\theta|Y_T, M) = \frac{p(Y_T|\theta, M)p(\theta|M)}{p(Y_T|M)} \propto p(Y_T|\theta, M)p(\theta|M) \tag{2.9}$$

is the posterior density of parameter vector θ in model M.[4]

Expressions (2.7), (2.8) and (2.9) are central to scientific learning. The joint density (2.7) is used to express the reduction of reality to θ inherent in the model M. The marginal likelihood (2.8) is used to learn about reality from the perspective of the particular model M and plays a crucial role in Bayesian model comparisons. The posterior density (2.9), in turn, is used to learn about the unknown parameters θ and functions of parameters $h(\theta)$ conditional on the data Y_T, which is referred to as Bayesian inference.[5] Formally the scope of the Bayesian inference is to compute the expected value for function $h(\theta)$:

$$E(h(\theta)|Y_T, M) = \int h(\theta)p(\theta|Y_T, M)d\theta, \tag{2.10}$$

where $h(\theta)$ could represent posterior moments, posterior quantiles of θ or forecasts for $\{y_t\}_{t=T+1}^{T+n}$. Yet another useful class of functions is

$$h(\theta) = L(a_1, \theta) - L(a_2, \theta), \tag{2.11}$$

in which $L(a, \theta)$ denotes the loss incurred if action a is taken and then the realization of the vector of interest is θ.

For comparison with non-Bayesian methods, sometimes a point estimate of $h(\theta)$ and an associated measure of uncertainty is reported. These measures are justified from a Bayesian point of view either as crude approximations to the peak and the curvature of the posterior, or as a summary of posterior information as $T \rightarrow \infty$. Let denote by $\widehat{h(\theta)}$ the estimator of $h(\theta)$ and let $L(\widehat{h(\theta)}, h(\theta))$ be a loss function. A Bayesian point estimator $\widetilde{h(\theta)}$ chosen to minimize expected loss conditional on the data is:

$$\begin{aligned} \widetilde{h(\theta)} &= \underset{\widehat{h(\theta)}}{\arg\min} E\left[L(\widehat{h(\theta)}, h(\theta))|Y_T, M\right] & (2.12)\\ &= \underset{\widehat{h(\theta)}}{\arg\min} \int L(\widehat{h(\theta)}, h(\theta))p(h(\theta)|Y_T, M)dh(\theta). & (2.13) \end{aligned}$$

Of particular importance for analysis in the subsequent chapters are two types of loss functions:[6]

1) Quadratic loss:

$$L(\widehat{h(\theta)}, h(\theta)) = \left(\widehat{h(\theta)} - h(\theta)\right)' W \left(\widehat{h(\theta)} - h(\theta)\right), \tag{2.14}$$

where W is a positive definite weighting matrix. Then $\widetilde{h(\theta)} = E(h(\theta)|Y_T, M)$, i.e. $\widetilde{h(\theta)}$ is simply equal to the posterior mean. Note that if $W = I$, $\widetilde{h(\theta)}$ minimizes the Mean Square Error of $h(\theta)$.

2) 'All or nothing' loss:

$$L(\widehat{h(\theta)}, h(\theta), \epsilon) = 1 - I_{\epsilon(h(\theta))}h(\theta), \tag{2.15}$$

[4]As $p(Y_T|M)$ does not depend on the unknown parameter it may be treated as a proportionality factor and may be neglected in the estimation procedure. See, e.g., Hamilton (1994).

[5]Density for a function of parameters (or other vector of interest) is given by: $p(h(\theta)|Y_T, M)$ and together with $p(Y_t|\theta, M)$ and $p(\theta|M)$ determines a so-called complete model (see Geweke (1999)).

[6]Extended list of loss functions may be find in Geweke (1999) or Canova (2005).

where $\epsilon(h(\theta))$ is an open ϵ-neighborhood of $\widehat{h(\theta)}$. Since in this case $\lim_{\epsilon \longrightarrow 0} \arg\min_{\widehat{h(\theta)}} L\left(\widehat{h(\theta)}, h(\theta), \epsilon\right) = \arg\min_{\widehat{h(\theta)}} p(h(\theta)|Y_T, M)$, then $\widetilde{h(\theta)} = \arg\min p\left(h(\theta) \in \epsilon(\widehat{h(\theta)}|Y_T, M)\right)$, i.e. $\widetilde{h(\theta)}$ is equal to the posterior mode.[7]

It should be noted that Bayesian and frequentist estimates might be equivalent under some conditions. Frequentist estimator is obtained conditional on a true parameter value $\widehat{h(\theta)}$, i.e.

$$\begin{aligned} \widetilde{h(\theta)} &= \arg\min_{\widehat{h(\theta)}} E\left(L\left(\widehat{h(\theta)}, h(\theta)\right)|h(\theta)\right) && (2.16)\\ &= \arg\min_{\widehat{h(\theta)}} \int L\left(\widehat{h(\theta)}, h(\theta)\right) p\left(Y_T|h(\theta)\right) dy. && (2.17) \end{aligned}$$

So, the solution is a function of $h(\theta)$. If we instead choose to minimize:[8]

$$\begin{aligned} \widetilde{h(\theta)} &= \arg\min_{\widehat{h(\theta)}} \int\int \mathcal{W}(h(\theta)) L\left(\widehat{h(\theta)}, h(\theta)\right) dy dh(\theta) && (2.18)\\ &= \arg\min_{\widehat{h(\theta)}} \int \left[\int L\left(\widehat{h(\theta)}, h(\theta)\right) dh(\theta)\right] \mathcal{W}(y) dy, \end{aligned}$$

where $\mathcal{W}(h(\theta))$ is a special weighting function such that $p(h(\theta)|Y_T)\mathcal{W}(y) = p(Y_T|h(\theta))\mathcal{W}(h(\theta))$. Thus, the expression $\int L\left(\widehat{h(\theta)}, h(\theta)\right) dh(\theta)$ is minimized and the estimator is Bayesian. Hence, a specification with weighting function $\mathcal{W}(h(\theta))$ also implies that Bayesian estimator is also the best from a frequentist perspective. Furthermore, despite pretty different reasoning behind frequentist and Bayesian analysis may be shown that the two procedures are equivalent in large samples and posterior mode converges to the true parameter value.[9] Finally, the posterior distribution converges to the normal distribution with the mean equal to the true parameter and the variance proportional to the Fisher's information matrix.

Before closing this section, we point to some practical issues related to the evaluation of the posterior distribution. Construction of the posterior is in many cases computationally extremely demanding. Therefore, two simplifying assumptions are made: First, usually the kernel of the posterior $p(Y_T|\theta, M)p(\theta|M)$, which preserves the moments of the posterior, is maximized or used to simulate the random numbers.[10] Second, in many applications priors are assumed to be independently distributed, i.e. $p(\theta|M) = \prod_{t=1}^{N} p(\theta_i)$, where N is the number of estimated parameters. Then, the logarithm of the posterior may be straightforward calculated as:

$$\ln p(\theta|Y_T, M) = \ln L(Y_T|\theta, M) + \sum_{i=1}^{N} \ln p(\theta_i|M), \quad (2.19)$$

where $p(\theta_i|M)$ are analytically tractable distributions.

In the subsequent sections other issues complementary for Bayesian analysis, such as prior selection and posterior simulations, this time strictly in the context of DSGE models, are discussed.

[7] For proofs see Canova (2005) and references given therein.
[8] See Canova (2005).
[9] See Canova (2005) and references given therein.
[10] Any nonnegative function proportional to a probability density is a density kernel.

2.3 Estimation procedure: an overview

As mentioned, the focus of this thesis is on the Bayesian estimation of DSGE models. An and Schorfheide (2005) list three main characteristics of this approach. First, unlike the GMM estimation based on equilibrium relationships, the Bayesian analysis is system-based and fits the solved DSGE model to a vector of aggregate time series.[11] Simultaneous estimation of all equations allows for unambiguous interpretation of structural shocks, which presents an important advantage for model-based policy analysis. Second, the estimation is based on the likelihood function generated by the DSGE model rather than, for instance, the discrepancy between DSGE model responses and VAR impulse responses (see Canova (2002) or Christiano, Eichenbaum, and Evans (2005)). Third, prior distributions can be used to incorporate additional information into the parameter estimation.[12] In this thesis we follow Schorfheide (2000) and apply a two-stage estimation procedure involving calibration and Bayesian Maximum Likelihood methods. This approach is sometimes referred to as a Bayesian calibration.

Schematically the method consists of the following steps:

Algorithm 1 (Steps typically used to evaluate the posterior of a DSGE model) .

1) Construct a log-linear representation of the DSGE model and solve/ transform it into a state space model.

2) Specify prior distributions for the structural parameters, fix the parameters which are not identifiable.

3) Compute the posterior density numerically, using draws from the prior distribution and the Kalman filter to evaluate the likelihood of the data.

4) Draw sequences from the joint posterior of the parameters using the Metropolis algorithms (to undertake a more extensive inference on the structural parameters by characterizing the shape of the posterior). Check whether the simulated distribution converge to the posterior distribution .

5) Construct statistics of interest using the draws in 4).

6) Evaluate the model and examine sensitivity of the results to the choice of priors.

In the subsequent sections we elaborate on the implementation of particular steps of the algorithm.

2.4 DSGE framework

This section presents the transformation of a DSGE model to a rational expectations system. Subsequently, the solution of the model and its state space representation are derived. The goal is to set up a suitable framework for estimation of DSGE models.

2.4.1 Model solutions

As the DSGE model may be estimated in its non-linear form, step 1) of Algorithm 1 is, in general, unnecessary. However, the methods for estimation of non-linear models are computationally

[11] For instance, Linde (2002) also argues by means of Monte Carlo simulations that the Full Information Maximum Likelihood approach improves the estimation results considerably in comparison with single-equation methods even if the model and the policy rule are misspecified.

[12] Likelihood-based inference presents a series of issues: specifically the lack of identification (multiple maxima, over-parametrization, i.e. the maximum is given by a complex multidimensional combination rather then by a single point in the parameter space). From the computational point of view, the Bayesian approach and the use of a prior makes the optimization algorithm more stable, namely because curvature is introduced in the objective function.

extremely demanding, which causes that at this point only the most basic RBC model has been estimated by non-linear likelihood methods (see Fernandez-Villaverde and Rubio-Ramirez (2004b)). Indeed, a full Bayesian non-linear approach is hardly feasible on currently available computers.[13]

The set of equilibrium conditions of a wide variety of DSGE models takes the form of a non-linear rational expectations system of variables vector s_t and innovations u_t:

$$E_t\left[G_t(s_{t+1}, s_t, u_t)\right] = 0 \tag{2.20}$$

The rational expectations system has to be solved before the DSGE model can be estimated. The solution takes the form:

$$s_t = A_t(s_{t-1}, u_t, \theta), \tag{2.21}$$

where s_t may be seen as a state vector and the equation (2.21) is a nonlinear state transition equation. The presence of parameter vector θ in the equation (2.21) indicates the dependence of the solution (and its existence, in particular) on the parameter constellation.

A variety of numerical techniques are available to solve rational expectations systems.[14] In the context of system-based DSGE model estimation linear approximation methods are very popular because they lead to a state-space representation of the DSGE model that can be analyzed with the Kalman filter (see Section 2.6). Indeed, applying linear approximation makes it also feasible to estimate large scale DSGE models. Several solution algorithms have been put forward, for instance, Blanchard and Kahn (1980), Uhlig (1999), Anderson (2000), Klein (2000) and Sims (2002).

Depending on the parameterization of the DSGE model there are three possibilities: no stable rational expectations solution exists, the stable solution is unique (determinacy), or there are multiple stable solutions (indeterminacy). In this thesis the models are estimated under the assumption of determinacy. Therefore, we restrict the parameter space accordingly. Since the linearized models presented here cannot be solved analytically (singularity problem), we decide to apply the method by Sims (2002).[15] The method involves using the QZ algorithm to solve the generalized eigenvalue problem. It produces solutions quickly, enabling us to solve the model for many different values of the underlying parameters in a reasonable amount of time. In addition, the approximate solution has a tractable form for the likelihood function.

Step 1) of the Algorithm 1 might be detailed as follows:

The model's equations are log-linearized around the non-stochastic steady state vector $\bar{s}$, where $\bar{s}$ is the solution of $(G_t(\bar{s}, \bar{s}, 0) = 0)$.[16] The log-linearized equations yield a first order linear difference equation system of the form:[17]

$$G_0 E_t s_{t+1} = G_1 s_t + F u_t, \tag{2.22}$$

[13] We have checked that estimation of the seminal non-linear DSGE model using its second order approximation around the steady state and the particle filter for evaluation of the likelihood is 200-1000 times slower than estimation based on a log-linearized version of the model with the likelihood evaluated by Kalman filter recursion.

[14] The algorithms to construct a second-order accurate solutions have been developed by Judd (1998), Collard and Juillard (2001), Schmitt-Grohé and Uribe (2004)) and Kim, Kim, Schaumburg, and Sims (2005). The so called perturbation method which includes higher order terms in the approximation, and therefore takes both curvature and risk into account are discussed in Juillard and Collard (1996) and Judd (1998).

[15] The models presented in Chapter 3 are solved with the Klein (2000) algorithm, which is included into the Matlab pre-processor Dynare. The models presented in Chapter 4 are solved with the Sims (2002) algorithm and estimated employing our own codes.

[16] Throughout the thesis the deviation of a variable s_t from its steady state, calculated as $\hat{s}_t = \ln(\frac{s_t}{\bar{s}})$, will be denoted by a circumflex.

[17] (2.22) is in the literature referred to as a Blanchard and Kahn's formulation.

We further extend the vector s_t by replacing terms of the form $E_t s_{t+1}$ with $\tilde{s}_t = E_t s_{t+1}$. The restriction linking the newly defined elements of the s_t to its old elements is then added in a form of equation: $s_t = \tilde{s}_{t-1} + \eta_t$. Finally, the system may be rewritten in the following compact form:[18]

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi u_t + \Pi \eta_t, \tag{2.23}$$

where η_t is a rational expectations error and $E_t \eta_{t+1} = 0$ for all t. This notational convention suggested by Sims (1997) implies that all variables dated t are observable at t, thus no separate list of what is predetermined is needed to augment the information that can be read off from the linearized equations themselves.

In what follows we will assume that u_t is an independent and identically distributed process with zero mean. Sims (2002) deals with the most general case where u_t can follow any distribution. However, in many applications the i.i.d. assumption is not very restrictive since one might decompose the original random variable into an endogenous component and a white noise process.

Before we detail on the solution method, it is important to understand what is meant by 'solution' of (2.23). The objective is to express the sequence of $\{s_{t+i}\}_{i=1}^{\infty}$ as a function of realizations of the exogenous random process $\{u_{t+i}\}_{i=1}^{\infty}$ and some initial conditions for the state vector. This requires solving for the endogenously determined η_t. The procedure consists of three steps:

Algorithm 2 (A generic algorithm for solving linear rational expectations system) .

1) Triangularize the system (2.23) using the QZ decomposition

2) Find the set of solutions to the transformed system (determine the η_t such that the stable solution is unique)

3) Reverse-transform this solution into the format of the original system

In the first step, using the QZ factorization we decompose Γ_0 and Γ_1 into unitary and upper triangular matrices such that:[19]

$$\Gamma_0 = Q'\Lambda Z' \tag{2.24}$$

$$\Gamma_1 = Q'\Omega Z' \tag{2.25}$$

where Q and Z are both unitary and possibly complex. The $'$ symbol indicates here both transposition and complex conjugation. The matrices Λ and Ω are possibly complex and are upper triangular. Although the QZ decomposition is not unique, the collection of values for the ratios of diagonal elements of Ω and Λ, denoted by$\left\{\psi_i = \frac{\omega_{ii}}{\lambda_{ii}}\right\}$, is unique.[20] Furthermore, we can always choose the matrices Ω, Z, Λ and Ω in a way that the generalized eigenvalues (in absolute value) are organized in exceeding order.

Defining $r_t \equiv Z's_t$ and pre-multiplying (2.23) by Q we obtain the transformed system of the form:

$$\Lambda r_t = \Omega r_{t-1} + Q\Pi\eta_t + Q\Psi u_t \tag{2.26}$$

Let $\bar{\xi}$ denote the maximum growth rate allowed for any component of s. $\bar{\xi}$ may be available from the transversality condition of the economic problem. For growth rates larger than $\bar{\xi}$ the system

[18] Models with more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated in this framework by expanding the vector s_t.

[19] Matrices Q and Z are unitary if $Q'Q = Z'Z = I$.

[20] The ratios $\psi_i = \frac{\omega_{ii}}{\lambda_{ii}}$ are called generalized eigenvalues of the matrix pencil $\psi\Gamma_0 - \Gamma_1$. If Γ_0 is the identity matrix, the ψ_i are just the eigenvalues of Γ_1. If Γ_0 is singular some of its diagonal elements are equal zero, in which case the relevant ψ_i is treated as infinite. Technically the generalized eigenvalues are $\{|\Gamma_1 - \psi_i\Gamma_0| = 0\}$.

becomes explosive. In particular we can partition the system (2.26) so that $|\psi_i| \geq \bar{\xi}$ for all $i > k$ and $|\psi_i| < \bar{\xi}$ for all $i \leq k$. Hence the system (2.26) may be expanded as:

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} r_t^1 \\ r_t^2 \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} r_{t-1}^1 \\ r_{t-1}^2 \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} \\ Q_{2\cdot} \end{bmatrix} [\Pi\eta_t + \Psi u_t] \tag{2.27}$$

Because of the way the generalized eigenvalues are grouped, the lower block of equations (2.27) is purely explosive. It has a solution that does not explode as long as we solve it forward to make r^2 a function of future u's and η's, such that the latter offset the exogenous process in a way that put r^2 on a stationary path:

$$\begin{aligned} Z'_{\cdot 2}s_t &= r_t^2 = \Omega_{22}^{-1}\Lambda_{22}r_{t+1}^2 - \Omega_{22}^{-1}Q_{2\cdot}\,[\Pi\eta_{t+1} + \Psi u_{t+1}] \\ &= -\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1}Q_{2\cdot}\,[\Pi\eta_{t+i} + \Psi u_{t+i}] \end{aligned} \tag{2.28}$$

Taking expectations conditional on information available at time t leaves the left hand side of (2.28) unchanged, i.e. $E_t r_t^2 = r_t^2$. The right hand side becomes then:

$$\begin{aligned} & -\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1}Q_{2\cdot}\,[\Pi\eta_{t+i} + \Psi u_{t+i}] \\ = \; & E_t\left[-\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1}Q_{2\cdot}\,[\Pi\eta_{t+i} + \Psi u_{t+i}]\right] \end{aligned} \tag{2.29}$$

Since $E_t\eta_{t+i} = 0$ and $E_t u_{t+i} = 0$ for $i \geq 1$, we get $Z'_{\cdot 2}s_t = r_t^2 = 0$ as a solution for the explosive block.[21] In the absence of any additional constraints, this implies that the upper block of equation (2.27) can support any solution of the form:

$$[\Lambda_{11}]\left[r_t^1\right] = [\Omega_{11}]\left[r_{t-1}^1\right] + [Q_{1\cdot}]\,[\Pi\eta_t + \Psi u_t]\,, \tag{2.30}$$

which still depends on the endogenous η_t. The equality (2.29) imposes, however, certain constraints on the left-hand side and on η_t. Knowing that $E_t\eta_{t+i} = 0$ and $E_t u_{t+i} = 0$ we obtain:

$$-\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1}Q_{2\cdot}\,[\Pi\eta_{t+i} + \Psi u_{t+i}] = 0 \tag{2.31}$$

We further notice that all future shocks can be eliminated by taking expectations conditional on information available at time $t+1$. After doing so, and shifting the equation one period backward we obtain:

$$Q_{2\cdot}\Pi\eta_t = -Q_{2\cdot}\Psi u_t \tag{2.32}$$

Sims (2002) concludes from (2.32) that a necessary and sufficient condition for existence of a solution is that the space of $Q_{2\cdot}\Psi$ is to be contained in that of $Q_{2\cdot}\Pi$.

Assuming a solution exists, we can combine (2.32) with some linear combination of equations in (2.27) to obtain a new complete system in r that is stable. What remains do be done is to free the new equation from references to the endogenous term η. Form (2.32) we see that $Q_{2\cdot}\Pi\eta_t$ depends on exogenous shock at time t. The equation (2.27) involves, however, other linear combination of

[21] The equality in (2.28) follows on the assumption that $(\Omega_{22}^{-1}\Lambda_{22})^t \longrightarrow 0$ as $t \longrightarrow \infty$. Note that if some of diagonal elements of $\Lambda = 0$, there are equations in (2.27) containing no current values of r (This corresponds to the singularity in matrix Γ_0). While this cases does not imply explosive paths, the corresponding components of (2.28) are still valid. For instance we have $0r_t^i = \psi_{ii}r_{t-1}^i + F(\eta_t, u_t)$. It can still be solved for r_{t-1}^i producing corresponding component of (2.28) .For further details see Sims (2002).

η, $Q_1.\Pi\eta_t$. In general it is possible that knowing $Q_2.\Pi\eta_t$ is not sufficient to tell the value of $Q_1.\Pi\eta_t$, in which case the solution to the model in not unique. To assure that solution is unique it is necessary and sufficient that the row space of $Q_1.$ be contained in that of $Q_2.$. Which is equivalent to:

$$Q_1.\Pi = \Phi Q_2.\Pi \tag{2.33}$$

for some matrix Φ. Premultiplying (2.27) by matrix $\begin{bmatrix} I & -\Phi \end{bmatrix}$ we obtain the system free of reference to η :

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \end{bmatrix} \begin{bmatrix} r_t^1 \\ r_t^2 \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \end{bmatrix} \begin{bmatrix} r_{t-1}^1 \\ r_{t-1}^2 \end{bmatrix} + [0]\,\eta_t + [Q_1.\Psi - \Phi Q_2.\Psi]\,u_t \tag{2.34}$$

Combining (2.34) with (2.28) we obtain:

$$\begin{aligned} \begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} r_t^1 \\ r_t^2 \end{bmatrix} \;=\; & \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_{t-1}^1 \\ r_{t-1}^2 \end{bmatrix} \\ & + E_t \begin{bmatrix} 0 \\ -\sum\limits_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1} Q_2.\Psi u_{t+i} \end{bmatrix} \\ & + \begin{bmatrix} Q_1.\Psi - \Phi Q_2.\Psi \\ 0 \end{bmatrix} u_t \end{aligned} \tag{2.35}$$

Since exogenous shocks are serially uncorrelated $E_t \left[-\sum\limits_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1} Q_2.\Psi u_{t+i} \right] = 0.$

The solution of the system in s can be recovered using that $Z's_t = r_t$:

$$s_t = As_{t-1} + Ru_t \text{ and } u_t \sim N(0, \Sigma_u), \tag{2.36}$$

where

$$A = \left(\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} Z' \right)^{-1} \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} Z'$$

and

$$R = \left(\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} Z' \right)^{-1} \begin{bmatrix} Q_1.\Psi - \Phi Q_2.\Psi \\ 0 \end{bmatrix}$$

The matrices A, R and Σ_u are functions of structural parameters stored in the vector θ.

2.4.2 Setting up a state space framework

As mentioned, estimation of the DSGE model requires to transform it into a state space form, which represents the joint dynamic evolution of an observable random vector y_t and a generally unobservable state vector s_t.[22] The state space model contains a measurement equation and a transition equation. The transition equation governs the evolution of the state vector s_t and in the DSGE context is equivalent to the model solution (2.21). The measurement equation completes the model by specifying how the state interacts with the vector of observations.

The evolution of the state vector s_t is governed by a dynamic process of the form:[23]

$$s_t = A_t(s_{t-1}) + u_t, \tag{2.37}$$

[22] More precisely, y_t stacks the time t observations that are here used to estimate the DSGE model.

[23] For a more detailed treatment see Lütkepohl (1993), Hamilton (1994) or Lemke (2005).

where A_t is a function which may depend on time. The innovation vector u_t is a serially independent process with mean zero and finite covariance matrix $\Sigma_{t,u}$, which may also depend on time.

The measurement equation determines the vector of observations y_t as possibly time dependent function of the state and of the error term u_t^m.

$$y_t = B_t(s_t) + u_t^m \tag{2.38}$$

The vector u_t^m is also a serially independent process with mean zero and finite covariance matrix $\Sigma_{t,m}$.

Some preliminary comments about the functioning of the model are in order. Having explicitly specified an initial condition, i.e. a distribution for the state vector s_t at time $t = 0$, or simply s_0, the process s_t is started by a draw from this distribution and evolves according to (2.37). The process has the Markov property.[24] That is, the distribution of s_t at time t given the entire past realizations of the process, is equal to the distribution of s_t given s_{t-1} only. The evolution of the observation vector y_t is determined by the state vector. In addition, the error u_t^m measures the deviations between the systematic component $B_t(s_t)$ and the observed vector y_t.

The general set-up above may be restricted by making the following assumptions: first, the functions $A_t(\cdot)$ and $B_t(\cdot)$ define linear transformations. Second, the distributions of s_0, u_t and u_t^m are normal. Models satisfying these assumptions will be referred to as linear Gaussian state space models and are the common structure used in the estimation of DSGE models.

The transition equation is then given by:

$$s_t = As_{t-1} + Ru_t, \tag{2.39}$$

which in the DSGE context coincides with the model solution (2.36). For the measurement equation we have:

$$y_t = Gx_t + Bs_t + Hu_t^m, \tag{2.40}$$

where x_t is the vector of predetermined variables to be discussed shortly.

The joint evolution of state innovation and measurement error are assumed to satisfy:

$$\begin{bmatrix} u_t \\ u_t^m \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_u & 0 \\ 0 & \Sigma_m \end{bmatrix} \right) \tag{2.41}$$

$\begin{bmatrix} u_t \\ u_t^m \end{bmatrix}$ and $\begin{bmatrix} u_{t-i} \\ u_{t-i}^m \end{bmatrix}$ are independent for all t and i. The initial conditions write $s_0 \sim N(\bar{s}_0, \bar{P}_0)$ and finally $E_t(u_t s_0') = 0$ and $E_t(u_t^m s_0') = 0$ for all t.

It is possible that exogenous or predetermined variables enter both the transition equation and the measurement equation. Concerning the applications of state space models in this thesis, the additional explanatory variables will show up only in the measurement equation. For instance, the vector of predetermined variables x_t stores the long-run rates of technology growth, steady state inflation and steady state nominal interest rate, which all are functions of structural parameters.

The naming of u_t^m as a measurement error stems from the use of the state space framework in the engineering or natural sciences. In the context of the DSGE model estimation it might account for the discrepancies between the variables of theoretical model and definitions of aggregates used by statistical offices. However, in many applications the issue of incorporating the measurement errors into the state space model is dictated by the need to obtain a non-singular forecast error covariance matrix resulting while predicting the vector y_t. This singularity is an obstacle to likelihood estimation (see Section 2.6). In general, the DSGE model generates a rank-deficient covariance matrix for y_t if the number of shocks stacked in the vector u_t is lower than the number

[24] See, e.g., Lemke (2005).

of time series to be matched. Adding measurement errors u_t^m reflecting the uncertainty regarding the quality of the data to equation (2.38) or augmenting the stochastics of the theoretical DSGE model solves the singularity problem. The former procedure is applied in Altug (1989) and Ireland (2004). In this thesis we pursue the latter approach, applied in Smets and Wouters (2003), by considering models in which the number of structural shocks is at least as high as the number of observables.

2.4.3 Using the model

In this subsection, we present some complementary issues related to the model solution. In particular we show how the solution may be used to study the dynamic properties of the model from a quantitative point of view. Basically, two issues are addressed here: Impulse response functions (IRF) and computation of second moments. In case the impulse responses and second moments are computed for an estimated model, they may be subsequently used to validate the model (see Section 2.8).

Impulse response functions

The impulse response function of a variable to a shock gives us the expected response of the variable to a shock at different horizons. In other words this corresponds to the best linear predictor of the variable if the economic environment remains the same in the future.

Provided the solution of the system is known, the immediate response to one of the fundamental shocks, shock k, is given by:

$$s_t = Au_{k,t}, \tag{2.42}$$

where $u_{k,t}$ is a vector with all entries equal to zero except one, which stands for the shock k.

The response at horizon j is then given by:

$$s_{t+j} = As_{t+j-1} \tag{2.43}$$

Computation of moments

Let us focus on the moments for the system (2.23). Since the system is linear the theoretical moments can be computed directly. In what follows we consider the stationary representations of the system such that the covariance of the state vector is $\Sigma_{ss} = E\left(s_{t+j}s'_{t+j}\right)$ whatever j. Hence, we have

$$\begin{aligned}\Sigma_{ss} &= A\Sigma_{ss}A' + AE\left(s_{t-1}u'_t\right)R + RE\left(u_t s'_{t-1}\right)A' + R\Sigma_u R' \qquad (2.44)\\ &= A\Sigma_{ss}A' + R\Sigma_u R' \qquad (2.45)\end{aligned}$$

Solving this equation for Σ_{ss} can be achieved remembering that $vec(ABC) = (A\otimes C')vec(B)$, hence

$$vec\left(\Sigma_{ss}\right) = \left(I - A\otimes A\right)^{-1} vec\left(R\Sigma_u R'\right) \tag{2.46}$$

The computation of covariances at leads and lags proceeds in a very similar way. From the model solution (2.36) we know that

$$s_t = A^j s_{t-j} + \sum_{i=0}^{j-1} A^i R u_{t-i} \tag{2.47}$$

Hence,

$$E\left(s_t s'_{t-j}\right) = A^j E\left(s_{t-j} s'_{t-j}\right) + \sum_{i=0}^{j-1} A^i R E\left(u_{t-i} s'_{t-j}\right) \tag{2.48}$$

Since u is the vector of innovations orthogonal to any past values such that $E\left(u_{t-i}s'_{t-j}\right) = 0$ if $i < j$ and for $i = j$ $E\left(u_{t-i}s'_{t-j}\right) = E\left(u_{t-i}(As_{t-j-1} + Ru_{t-i})'\right) = \Sigma_u R'$. Then the previous equation reduces to

$$E\left(s_t s'_{t-j}\right) = A^j \Sigma_{ss} + A^j R \Sigma_u R'. \tag{2.49}$$

2.5 Prior distributions in DSGE model estimation

Step 2) of Algorithm 1 requires specification of the prior densities. As is evident from equation (2.9) the prior re-weights the likelihood. This can bring to bear information that is not contained in the estimation sample $\{y\}_{t=1}^T$. Therefore, the Bayesian approach helps to tackle problems which are common in the pure maximum-likelihood estimation. These problems can be summarized under three items. First, the estimates of structural parameters obtained with maximum likelihood procedures based on a set of observations $\{y\}_{t=1}^T$ are often at odds with out-of-sample information. Second, due to the stylized nature of DSGE models the likelihood function often peaks in the parameter region which is at odds with the micro-evidence. Finally, priors adding curvature to a likelihood function that may be flat in some dimensions of the parameter space (some parameters are not identifiable upon the date) influence the shape of the posterior distribution and allow for its numerical optimization. The issues related to the lack of identification, particulary in the context of open economy DSGE models, are more extensively discussed in Chapter 4 of this thesis.

As mentioned, the procedure applied in the empirical parts of this thesis involves calibration and Bayesian Maximum Likelihood methods. There is also a sense in which calibration can be interpreted as a Bayesian procedure where the prior density of θ is degenerate and concentrated on a single numerical value. With such a tight prior, observations of the data series contribute nothing to our knowledge of the parameter values and the posterior density coincides with the prior one.

In the DSGE context non-degenerate priors are typically selected to be centered around standard calibrated values of the structural parameters and are often motivated by the microeconomic evidence. Since the macro theory hardly ever gives us a guidance regarding the volatility of structural shocks, in the applications to be presented in this thesis, we centered the priors for those so that the model roughly replicates the volatility of the data. The standard errors of the prior distributions generally reflect subjective prior uncertainty faced by an investigator. One could also specify standard errors so as to cover the range of existing estimates (see, e.g., Onatski and Williams (2004)). In some applications, it may be convenient to select diffuse priors over a fixed range to avoid imposing too much structure on the data. However, in the majority of applications, the form of the prior reflects computational convenience. Canova (2005) suggests to assume gamma or inverse gamma distribution for parameters bounded to be positive (e.g. standard deviation of structural shocks), beta distributions for parameters bounded between zero and one (e.g. parameters of the shocks persistence, Calvo stickiness parameters, indexation parameters, habit persistence parameters) and normal distribution for the remaining parameters.

2.6 Computation of the data likelihood

In this section we focus on the issues related to the computation of the likelihood of the state space model. We introduce the general concepts of filtering distribution and prediction distribution and subsequently present the algorithms applicable for linear Gaussian models.

2.6.1 Problem of filtering and prediction

As indicated at the beginning of this chapter the computation of the posterior (see also step 3 of Algorithm 1) requires beforehand the evaluation of the data likelihood. This, however, in a state space framework, is associated with the more general problem of estimation of the unobservable sequence of state variables $\{s_t\}_{t=1}^T$ using a set of observations $Y_T = \{y_t\}_{t=1}^T$.

For fixed t we consider the problem of estimating s_t in terms of $Y_i = \{y_j\}_{j=1}^i$. If $i = t$ the problem is called a filtering problem, $i < t$ defines a prediction problem, and the case $i > t$ is referred to as a smoothing problem. Besides predicting the unobservable state, we consider here the problem of forecasting the observation vector y_t. Applying the squared error as optimality criterion, the best estimators of the state vector are functions $\hat{s}_t$ that satisfy:[25]

$$E\left[(s_t - s_t(Y_i))\,(s_t - s_t(Y_i))'\right] \geq E\left[(s_t - \hat{s}_t(Y_i))\,(s_t - \hat{s}_t(Y_i))'\right] \tag{2.50}$$

for every function $s_t(Y_i)$. The inequality sign denotes that the difference of the right hand side and the left hand side is a negative semidefinite matrix.

The optimal estimator in terms of (2.50) for s_t conditional on Y_i is given by the conditional expectation:

$$\hat{s}_t(Y_i) = E\left(s_t|Y_i\right) = \int s_t p\left(s_t|Y_i\right) ds_t \tag{2.51}$$

This means that for finding the optimal estimators $s_t(Y_t)$ (filtered state), $s_t(Y_{t-1})$ (predicted state) and $s_t(Y_T)$ (smoothed state), one has to find the respective conditional densities (we will refer to them as filtering, prediction and smoothing densities) $p(s_t|Y_t)$, $p(s_t|Y_{t-1})$ and $p(s_t|Y_T)$, and then compute the conditional expectation given by (2.51). Similarly, for obtaining $\hat{y}_t(Y_{t-1})$, the optimal one-step predictor for the observation vector, one has to find the conditional density $p(y_t|Y_{t-1})$ in order to compute $E(y_t|Y_{t-1})$.

Required conditional densities may be constructed iteratively as follows:[26]

Algorithm 3 (A generic algorithm for computation of conditional densities) .

1) Initialize the predictive density with $p(s_0|Y_0) = p(s_0)$

2) Given the density $p(s_{t-1}|Y_{t-1})$ *compute the predictive density of the state vector*

$$p(s_t|Y_{t-1}) = \int p(s_t|s_{t-1})p(s_{t-1}|Y_{t-1})ds_{t-1} \tag{2.52}$$

3) Compute the predictive density of the observables

$$p(y_t|Y_{t-1}) = \int p(y_t|s_t)p(s_t|Y_{t-1})ds_t \tag{2.53}$$

4) Compute the filtering density of the state

$$p(s_t|Y_t) = \frac{p(y_t|s_t)p(s_t|Y_{t-1})}{p(y_t|Y_{t-1})} \tag{2.54}$$

5) Repeat steps 2)-4) until $t = T$.

[25] The circumflex is used here to denote the estimator of the state vector. The same symbol is also used to denote the deviation from the steady state. However, we think that it is always clear from the context which is referred to.

[26] For the derivation see Lemke (2005) and references given therein.

The conditional density $p(s_t|Y_T)$ required for computing the smoothing estimates $E(s_t|Y_T)$ (for $t = 1, ..., T-1$) is obtained by backward integration:

$$p(s_t|Y_T) = p(s_t|Y_t) \int \frac{p(s_{t+1}|Y_T)p(s_{t+1}|s_t)}{p(s_{t+1}|Y_t)} ds_{t+1} \tag{2.55}$$

Computation of $p(s_{t-1}|Y_T)$ is straightforward given the recursive formula (2.55).

Since the state space model contains unknown parameters, stored in the vector θ, both in transition and measurement equation, we are interested in the evaluation of the joint data density (or the data likelihood) $p(y_1, ..., y_T)$. This density may be written as a product of conditional densities using the so-called prediction error decomposition (see also Section 2.2):

$$p(y_1, ..., y_T) = \prod_{t=1}^{T} p(y_t|Y_{t-1}) \tag{2.56}$$

The conditional densities $p(y_t|Y_{t-1})$ are obtained within the iterative procedure (2.52)-(2.54). Thus, for a given parameter constellation θ the iterations above can be used to compute the log-likelihood:

$$\ln L(Y_T|\theta) = \sum_{t=1}^{T} \ln p(y_t|Y_{t-1}, \theta), \tag{2.57}$$

which is subsequently used to construct the log-posterior (2.9). Note that here we have explicitly added the argument θ, but also densities (2.52)-(2.56) are conditional on θ.

2.6.2 Prediction, filtering and likelihood of linear Gaussian models

The subsection above gives a relatively full but also general exposition of filtering and prediction problems. The integration steps required in Algorithm 3 for functions $A_t(\cdot)$ and $B_t(\cdot)$ can be performed only under two very special circumstances: First, when the support of the state variables is discrete (and finite): then the integrals are just summations. Second, when the state and the measurement equations are both linear and the disturbances are Gaussian.

In the former case Sequential Monte Carlo (SMC) methods (e.g. the particle filter) can be applied. Fernandez-Villaverde and Rubio-Ramirez (2004b) and An (2005) are the first studies in which these techniques are used for DSGE models. The idea of the method is straightforward. The filtering density is obtained in two steps: First, draw a large number of realisations from the distribution of s_{t+1} conditioned on y_t. Second, assign them a weight which is determined by their 'distance', computed via measurement equation, from y_{t+1}.

The advantage of SMC methods is that they are also applicable to non linear approximations. The disadvantage is that much more computation time, as opposed to Kalman filter, is required. Arulampalam, Maskell, Gordon, and Clapp (2002) also indicate that SMC methods are very sensitive to outliers and degeneracies frequently arise. Moreover, the researcher has to monitor carefully the numerical efficiency indicators of the SMC.

The computations simplify significantly if transition and measurement equations are linear. For the model given by equations (2.39) and (2.40) the transition density $p(s_{t+1}|s_t)$ and measurement density $p(y_t|s_t)$ are normal. It implies that also filtering and prediction densities are normal:

$$s_t|Y_{t-1} \sim N\left(s_{t|t-1}, P_{t|t-1}\right) \tag{2.58}$$

$$s_t|Y_t \sim N\left(s_{t|t}, P_{t|t}\right) \tag{2.59}$$

$$y_t|Y_{t-1} \sim N\left(y_{t|t-1}, F_{t|t-1}\right) \tag{2.60}$$

Since the multivariate normal densities are completely described by their means and covariance matrix, it is sufficient to find the sequences of conditional means, $s_{t|t-1}$, $s_{t|t}$, $y_{t|t-1}$, and the sequences of conditional covariance matrices, $P_{t|t-1}$, $P_{t|t}$ and $F_{t|t-1}$ to evaluate the likelihood of the data. These quantities can be iteratively obtained from the Kalman filter. For the observer system (2.39) and (2.40), the Kalman filter algorithm is as follows:

Algorithm 4 (Kalman Filter) .[27]

1) Select initial conditions. If all eigenvalues of A are less then one in absolute value, set $s_{1|0} = E(s_1)$ and $P_{1|0} = AP_{1|0}A' + R\Sigma_u R'$ or $vec(P_{1|0}) = (I - (A \otimes A')^{-1})vec(R\Sigma_u R')$, in which case the initial conditions are the unconditional mean and variance of the process. When some of the eigenvalues of A are greater than one, initial conditions cannot be drawn from the unconditional distribution and one needs a guess (say, $s_{1|0} = 0$, $P_{1|0} = \kappa I$, κ large) to start the iterations.[28].

2) Predict y_t and construct the mean square of the forecasts using the information from $t-1$.

$$E(y_{t|t-1}) = Bs_{t|t-1} \tag{2.61}$$

$$\begin{aligned} E(y_t - y_{t|t-1})(y_t - y_{t|t-1})' &= EB'(s_t - s_{t|t-1})(s_t - s_{t|t-1})'B + H\Sigma_m H' \\ &= B'P_{t|t-1}B + H\Sigma_m H' = F_{t|t-1} \end{aligned} \tag{2.62}$$

3) Update state equation estimates (after observing y_t):

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}BF_{t|t-1}^{-1}(y_t - Bs_{t|t-1}) \tag{2.63}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}BF_{t|t-1}^{-1}BP_{t|t-1} \tag{2.64}$$

where $F_{t|t-1}$ is defined in (2.62).

4) Predict the state equation random variables next period:

$$s_{t+1|t} = As_{t|t} = As_{t|t-1} + K_t v_t \tag{2.65}$$

$$P_{t+1|t} = AP_{t|t}A' + R\Sigma_u R' \tag{2.66}$$

where $v_t = y_t - \hat{y}_{t|t-1} = y_t - Bs_{t|t-1}$ is the one-step ahead forecast error in predicting the observed variables vector and

$$K_t = AP_{t|t-1}BF_{t|t-1}^{-1} \tag{2.67}$$

is the Kalman gain.

5) Repeat steps 2)-4) until $t = T$. Note the equations (2.63) and (2.64) provide the input for the next step of the recursion.

Under normality assumptions (2.58)-(2.60), the distribution of y_t conditional on Y_{t-1} is the n-dimensional normal distribution with mean $y_{t|t-1}$ and variance-covariance matrix $F_{t|t-1}$. Thus, the conditional density of y_t can be written as:[29]

$$p(y_t|Y_t, \theta) = \left[(2\pi)^{n/2}\sqrt{|F_{t|t-1}|}\right]^{-1} \exp\left\{-1/2\left(y_t - \hat{y}_{t|t-1}\right)' F_{t|t-1}^{-1}\left(y_t - \hat{y}_{t|t-1}\right)\right\} \tag{2.68}$$

The log-likelihood function becomes then:

$$\ln L(Y_T|\theta) = \ln p(y_1, ..., y_T|\theta) = -\frac{Tn}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|F_{t|t-1}| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_{t|t-1}^{-1} v_t \tag{2.69}$$

Note that the function (2.69) only depends on the prediction errors $v_t = y_t - \hat{y}_{t|t-1}$ and their covariance matrices $F_{t|t-1}$ which are both the output of the Kalman Filter.

[27]For simplicity of exposition, here we do not incorporate the predetermined elements into the transition and measurement equations.

[28]For the initialization of diffuse Kalman Filter see Koopman and Durbin (2003)

[29]See, e.g., Hamilton (1994).

Some complementary notes on Kalman filter

Below we present some complementary notes which are important for application of the Kalman filter.

Steady state matrices From Algorithm 4, it is evident that the sequences $\{P_{t|t-1}\}_{t=1}^{T}$, $\{P_{t|t}\}_{t=1}^{T}\}$, and $\{F_{t|t-1}\}_{t=1}^{T}\}$ of covariance matrices depend on the system matrices A, R, B, H and initial conditions but not on the observations Y_T . It may be also shown that if the eigenvalues of the matrix A are inside the unit circle, if $R\Sigma_u R'$ and $H\Sigma_m H'$ are positive semidefinite, and if at least one of them is strictly positive definite, the sequence $\{P_{t|t-1}\}_{t=1}^{T}$ will converge to a unique steady state matrix P as T goes to infinity.[30] Inserting equations (2.62) and (2.64) into (2.66) one obtains the matrix difference equation:

$$P_{t+1|t} = A\left[P_{t|t-1} - P_{t|t-1}B'\left(BP_{t|t-1}B' + H\Sigma_m H\right)^{-1} BP_{t|t-1}\right]A' + R\Sigma_u R', \tag{2.70}$$

which is to be satisfied by $\{P_{t|t-1}\}_{t=1}^{T}$. The steady state matrix P is computed from the so-called Riccati equation:

$$P = A\left[P - PB'\left(BPB' + H\Sigma_m H\right)^{-1} BP\right]A' + R\Sigma_u R', \tag{2.71}$$

which is solved for P applying numerical methods. From equations (2.62) and (2.66) it is evident that the convergence of $\{P_{t|t-1}\}_{t=1}^{T}$ leads to convergence of: $\{F_{t|t-1}\}_{t=1}^{T}$ and $\{K_t\}_{t=1}^{T}$. These properties are important because if $|P_{t+1|t} - P_{t|t-1}| \leq \varepsilon$, for small ε the computations of $P_{t|t-1}$, $F_{t|t-1}$ and K_t can be avoided for $t > t_\varepsilon$ which might significantly reduce the computation time.

Kalman filter and non-Gaussian models In what follows, we only briefly touch on the problems related with applying the Kalman filter to a model with linear measurement and transition equation for which the errors are not Gaussian.[31] If we drop the assumption of Gaussian errors, the Kalman Filter outputs $s_{t|t-1}$, $y_{t|t-1}$, and $s_{t|t}$ still preserve an optimality property: they are the linear projections of s_t and y_t on Y_{t-1} and Y_t, respectively. Hence, they are estimators which have smallest mean square errors in the restricted class of all linear estimators. However, they are not conditional expectations any more, since in the non-Gaussian case, the conditional expectations function is generally nonlinear in the conditioning variables. If in the linear model the state innovation and measurement error are not Gaussian, one can still obtain estimates of the model parameters by (falsely) assuming normality, computing the log-likelihood by means of the Kalman filter, and maximizing it with respect to θ. This approach is known as quasi-maximum likelihood estimation. Under certain conditions it will still lead to consistent estimators which are asymptotically normally distributed.

2.7 Approximations of the posterior distribution

In this section we focus on the methods for approximation of the posterior distribution. We begin by presenting the simulation methods, including Markov Chain Monte Carlo. Subsequently, the numerical optimization methods, used to locally approximate the posterior, are discussed.

[30]See, e.g., Hamilton (1994).
[31]This paragraph summarizes the extensive discussion provided by Lemke (2005) and Lütkepohl (1993).

2.7.1 Posterior simulations

Having specified the likelihood and the prior, we proceed to analyze the posterior distribution (step 4 of Algorithm 1). Knowledge of the posterior is required for implementation of the Bayesian inference, the objective of which is $E(h(\theta)|Y_T, M) = \int h(\theta)p(\theta|Y_T, M)d\theta = \frac{\int h(\theta)L(Y_T|\theta,M)p(\theta|M)d\theta}{p(Y_T|M)}$. Since only the kernel of the posterior $p^*(\theta|Y_T, M) = L(Y_T|\theta, M)p(\theta|M)$ is available but the marginal density $p(Y_T|M)$ is unknown, the above expression cannot be evaluated analytically. Only in very special situations, the integral $\int h(\theta)p(\theta|Y_T, M)d\theta$ can be approximated using the method of Monte Carlo integration. Then, producing a random sequence $\{\theta_k\}_{k=1}^{n_{sim}}$ using the kernel $p^*(\theta|Y_T, M)$ one could guarantee that $\frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} h(\theta_k) \to E(h(\theta)|Y_T, M)$ 'almost surely' as $n_{sim} \to \infty$.[32] However, as the posterior kernel is usually analytically intractable, it is likewise impossible to generate the random numbers from it directly. What may be done instead is to generate the random numbers from different analytically tractable distributions and correct these draws to better approximate the posterior distribution. Formally, a sequence of $\{\theta_k\}_{k=1}^{n_{sim}}$ together with a generic weighting function $w(\theta_k)$ with the property that

$$\frac{\sum_{k=1}^{n_{sim}} w(\theta_k)h(\theta_k)}{\sum_{k=1}^{n_{sim}} w(\theta_k)} \to E(h(\theta)|Y_T, M) \text{ 'almost surely' as } n_{sim} \to \infty \tag{2.72}$$

is the subject of interest.

There is a huge amount of literature dealing with this issue: from acceptance sampling, importance sampling, to Markov Chain Monte Carlo (MCMC) approaches (Gibbs sampler and the class of Metropolis-Hastings algorithms). The latter approach is the one which is applied in the recent literature on Bayesian analysis of DSGE models.[33] In order to present the posterior simulations algorithms used in this thesis, we first familiarize the reader with the basic concepts of Markov chains and subsequently give a general idea of the MCMC method. Finally, we detail the Random Walk Metropolis algorithm.

Introduction to Markov chains

Before introducing the MCMC methods, a few general and introductory comments on Markov chains are in order. Let X_t denote the value of a random variable at time t, and let the state space refer to the range of possible X values.[34] The random variable is a Markov process if the transition probabilities between different values in the state space depend only on the random variables current state, i.e.,

$$P(X_{t+1} = z_j|X_0 = z_k, ..., X_t = z_i) = P(X_{t+1} = z_j|X_t = z_i) \tag{2.73}$$

Thus for a Markov random variable the only information about the past needed to predict the future is the current state of the random variable. A Markov chain refers to a sequence of random variables $(X_0, ..., X_n)$ generated by a Markov process. A particular chain is defined by its transition probabilities (or the transition kernel), $P(i, j) = P(i \to j)$, which is the probability that a process at state space z_i moves to state z_j in a single step,

$$P(i, j) = P(i \to j) = P(X_{t+1} = z_j|X_t = z_i) \tag{2.74}$$

[32] The almost surely means that convergence is subject to some regularity conditions of the function $h(\theta)$ specifically absolute convergence of the integral $\int h(\theta)p(\theta|Y_T, M)d\theta$ must be satisfied. See, e.g., Geweke (1995).

[33] The development of Markov chain Monte Carlo methods has been in large part possible because of the dramatic decrease in the cost of computing in the last few years. For discussion see, e.g., Neal (1993).

[34] This subsection closely follows Walsh (2004).

Let

$$\pi_j(t) = P(X_t = z_j) \tag{2.75}$$

denote the probability that the chain is in state j at time t, and let $\pi(t)$ denote the vector of the state space probabilities at step t. We start the chain by specifying a starting vector $\pi(0)$.

The probability that the chain has state value z_i at step $t+1$ is given by the Chapman-Kolomogorov equation, which sums over the probabilities of being in a particular state at the current step and the transition probability from that state into state z_i,

$$\begin{aligned} \pi_i(t+1) &= P(X_{t+1} = z_i) \\ &= \sum_k P(X_{t+1} = z_i | X_t = z_k) P(X_t = z_k) \\ &= \sum_k P(k \to i) \pi_k(t) = \sum_k P(k,i) \pi_k(t) \end{aligned} \tag{2.76}$$

Iterations of Chapman-Kolomogorov equation describe the evolution of the Markov chain.

More compactly the Chapman-Kolomogorov equation may be written in a matrix form as follows. Define the transition matrix P as the matrix whose $P_{i,j}$ element is equivalent to the probability $P(i,j)$. This implies that $\sum_j P(i,j) = \sum_j P(i \to j) = 1$. The Chapman-Kolomogorov equation becomes

$$\pi(t+1) = \pi(t) P \tag{2.77}$$

Iterating the above equation yields

$$\pi(t) = \pi(0) P^t \tag{2.78}$$

Defining the n-step transition probability $p_{i,j}^{(n)}$ as the probability that the process is in the state j given that it started in state i n periods ago, i.e.,

$$p_{i,j}^{(n)} = P(X_{t+n} = z_j | X_t = z_i) \tag{2.79}$$

this probability is also an i,j element of P^n.

Finally, a Markov chain is said to be irreducible if for all i,j and n $p_{i,j}^{(n)} > 0$. That is, all states communicate with each other. A chain is aperiodic when the number of steps required to move between two states is not required to be multiple of some integers. A Markov chain may also reach a stationary distribution π^*, where the vector of probabilities of being in any particular state is independent on the initial condition. This distribution satisfies:

$$\pi^* = \pi^* P \tag{2.80}$$

The conditions for existence of a stationary distribution π^* are that the chain is irreducible and aperiodic. A sufficient conditions for a unique stationary distribution are detailed as follows (for all i and j):

$$P(j \to k) \pi_j^* = P(k \to j) \pi_k^* \tag{2.81}$$

If equation (2.81) holds for all *i,k,* the Markov chain is said to be reversible. This reversibility condition implies that

$$(\pi P)_j = \sum_i \pi_i P(i \to j) = \sum_i \pi_j P(j \to i) = \pi_j \sum_i P(j \to i) = \pi_j$$

The basic idea of discrete-state Markov chain can be generalized to a continuous state Markov process by having a probability kernel $P(x,y)$ that satisfies[35]

$$\int P(x,y)\, dy = 1$$

[35]See, e.g., Chib and Greenberg (1995).

and the continous extension of Chapman-Kolomogorov equation is

$$\pi_t(y) = \int \pi_{t-1}(x) P(x,y)\, dy \tag{2.82}$$

Finally, the stationary distribution satisfies

$$\pi^*(y) = \int \pi^*(x) P(x,y)\, dy \tag{2.83}$$

Markov Chain Monte Carlo methods

As mentioned, one problem with applying Monte Carlo integration is in obtaining samples from some complex probability distribution, $p(\theta|Y_T, M)$, in our case. Attempts to solve this problem are the roots of MCMC methods. In particular, they trace to attempts by mathematical physicists to integrate very complex functions by random sampling (Metropolis and Ulam (1949), Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and Hastings (1970)), and the resulting Metropolis-Hastings algorithm. A detailed review of this method is given by Neal (1993), Tierney (1994), Chib and Greenberg (1995), Geweke (1995) and Geweke (1999).

Our goal is to draw samples from the distribution $p(\theta|Y_T, M) = \frac{p^*(\theta|Y_T,M)}{p(Y_T|M)}$, where $p(Y_T|M)$ may be treated as an unknown normalizing constant, which is in fact very difficult to compute. The Metropolis algorithm (Metropolis and Ulam (1949) and Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953)), which generates a sequence of draws from this distribution, is as follows:

Algorithm 5 (The Metropolis algorithm) .

1) Start with any initial value θ_0 satisfying $p^*(\theta|Y_T, M) > 0$. Set k=0.

2) Using the current value of θ, sample a candidate point $\theta^{candidate}$ from some jumping distribution $q(\theta_1, \theta_2)$, which is the probability of returning a value of θ_2 given a previous value of θ_1. This distribution is also referred to as the proposal or candidate-generating distribution. The only restriction on the jump density is that it is symmetric, i.e. $q(\theta_1, \theta_2) = q(\theta_2, \theta_1)$.

3) Given the candidate point $\theta^{candidate}$, calculate the ratio of the density at the candidate point $\theta^{candidate}$ and the current point θ_{k-1}

$$r = \frac{p(\theta^{candidate}|Y_T, M)}{p(\theta_{k-1}|Y_T, M)} = \frac{p^*(\theta^{candidate}|Y_T, M)}{p^*(\theta_{k-1}|Y_T, M)}$$

Notice that because we are considering the ratio of $p(\theta|Y_T, M)$ under two different values of θ the constant $p(Y_T|M)$ cancels out.

4) If the jump increases the density $(r > 1)$, accept the candidate point $\theta_k = \theta^{candidate}$ and return to step 2). If the jump decreases the density $(r < 1)$ then with probability r accept the candidate point, else reject it, set $\theta_k = \theta_{k-1}$, and return to step 2. Do until k=n_{sim}.

We can summarize the Metropolis sampling as first computing

$$r = \min\left(\frac{p^*(\theta^{candidate}|Y_T, M)}{p^*(\theta_{k-1}|Y_T, M)}, 1\right)$$

and then accepting a candidate point with probability r (the probability of a move).[36] This generates a Markov chain $(\theta_0, \theta_1, ..., \theta_{n_{sim}})$, as the probability from θ_k to θ_{k+1} depends only on θ_k and not

[36] The Metropolis algorithm uses the mechanism of acceptance-rejection sampling. The idea behind this mechanism is to generate a random vector from a distribution that is 'similar' to the distribution we want to approximate, and then to accept that draws with a probability that depends on the drawn value of the vector. If this acceptance probability function is chosen correctly then the accepted values will have the desired distribution.

on the history of the chain. Following a sufficient burn-in period, the chain approaches its stationary distribution and (as we demonstrate shortly) the samples from the vector $(\theta_{n_{burn\_in}}, ..., \theta_{n_{sim}})$ are samples from the distribution of interest $p(\theta|Y_T, M)$. Hastings (1970) generalized the Metropolis algorithm by using an arbitrary probability function $q(\theta_1, \theta_2) = P(\theta_1 \rightarrow \theta_2)$, and setting the acceptance probability for a candidate point as

$$r = \min\left(\frac{p^*(\theta_k|Y_T, M)q\left(\theta^{candidate}, \theta_{k-1}\right)}{p^*(\theta_{k-1}|Y_T, M)q\left(\theta_{k-1}, \theta^{candidate}\right)}, 1\right)$$

Assuming that the proposal distribution is symmetric, i.e. $q(x, y) = q(y, x)$, the original Metropolis algorithm may be recovered.[37]

Metropolis-Hasting sampling as a Markov Chain

In what follows, we demonstrate that the Metropolis-Hasting sampling generates a Markov chain whose equilibrium density is the candidate density $p(x)$ (here $p(x)$ is a shortcut for our density of interest $p(\theta|Y_T, M)$). To show this, it is sufficient that the Metropolis-Hasting transition kernel satisfies equation (2.81).[38]

Under M-H algorithm, we sample from $q(x, y) = P(x \rightarrow y|q)$ and then accept the move probability $r(x, y)$, so that the transition probability kernel is given by

$$P(x \rightarrow y) = q(x, y)r(x, y) = q(x, y)\min\left(\frac{p(y)q(y, x)}{p(x)q(x, y)}, 1\right)$$

Thus if the M-H kernel satisfies $P(x \rightarrow y)p(x) = P(y \rightarrow x)p(y)$ or $q(x, y)r(x, y)p(x) = q(y, x)r(y, x)p(y)$ for all x, y then that stationary distribution from this kernel corresponds to draws from the target distribution.

Below we analyze possible cases:

1) Let $q(x, y)p(x) = q(y, x)p(y)$. Hence $r(x, y) = r(y, x) = 1$ implying $P(x, y)p(x) = q(x, y)p(x)$ and $P(y, x)p(y) = q(y, x)p(y)$ and hence $P(x, y)p(x) = P(y, x)p(y)$, fulfilling the reversibility condition (2.81).

2) Let $q(x, y)p(x) > q(y, x)p(y)$, in which case

$$r(x, y) = \frac{p(y)q(y, x)}{p(x)q(x, y)} \text{ and } r(y, x) = 1$$

Hence

$$\begin{aligned} P(x, y) &= q(x, y)r(x, y)p(x) \\ &= q(x, y)\frac{p(y)q(y, x)}{p(x)q(x, y)}p(x) \\ &= q(y, x)p(y) = q(y, x)r(y, x)p(y) \\ & \quad P(y, x)p(y) \end{aligned}$$

3) Let $q(x, y)p(x) < q(y, x)p(y)$. Here

$$r(x, y) = 1 \text{ and } r(y, x) = \frac{q(x, y)p(x)}{q(y, x)p(y)}$$

[37] See Chib and Greenberg (1995).

[38] This demonstration is based on the proof in Chib and Greenberg (1995).

Hence

$$\begin{aligned} P(y,x)p(y) &= q(y,x)r)y,x)p(y) \\ &= q(x,y)\frac{q(x,y)p(x)}{q(y,x)p(y)}p(y) \\ &= q(x,y)p(x) \\ &= q(x,y)r(x,y)p(x) \\ &= P(x,y)p(x). \end{aligned}$$

Choosing a Jumping (Proposal) Distribution

There are two general approaches for choosing the jumping distribution; one may decide either for random walks or independent chain sampling. While using the proposal distribution based on a random walk chain, the new value y equals the current value x plus a random variable z. In this case $q(x,y) = g(y-x) = g(z)$, the density associated with the random variable z. If $g(z) = g(-z)$, i.e., the density for the random variable z is symmetric (as occurs with a normal or multivariate normal with mean zero, or a uniform centered around zero), then we can use Metropolis sampling as $q(x,y)/q(y,x) = g(z)/g(-z) = 1$. The variance of the proposal distribution is selected to get better 'mixing'.

Under a proposal distribution using an independent chain, the probability of jumping to point y is independent of the current position (x) of the chain, i.e., $q(x,y) = g(y)$. Thus, the candidate value is simply drawn from a distribution of interest, independent of the current value. Again, any number of standard distributions can be used for $g(y)$. In this case, the proposal distribution is generally not symmetric, as $g(x)$ is generally not equal to $g(y)$, and Metropolis-Hasting sampling must be used.[39]

In this thesis the posterior simulations are performed with the Random Walk Metropolis algorithm.[40] Since the models considered here are estimated under the assumptions that they yield the unique stable solution, the following rule is introduced to the RWM algorithm. If the parameter value θ implies indeterminacy (or non-existence of a stable rational expectations solution) then the log-posterior is set to minus infinity. If a unique stable solution exists then the Kalman filter (Algorithm 4) is used to evaluate the likelihood function associated with the linear state-space system (2.39) and (2.40). Since the prior $p(\theta)$ is generated from well-known densities (uniform, beta, inverse gamma, normal), its computation is straightforward.

Algorithm 6 (Random Walk Metropolis) .

1) Use a numerical optimization routine to maximize the logarithm of the posterior kernel $\ln L(Y_T|\theta) + \ln p(\theta)$. *Denote the posterior mode by* $\tilde{\theta}$ *(see algorithm 8 below).*

2) Let $\Sigma_{\tilde{\theta}}^{-1}$ *be the inverse of the Hessian computed (numerically) at the posterior mode* $\tilde{\theta}$.

3) Draw θ_0 *from* $N(\tilde{\theta}_0, \Sigma_{\tilde{\theta}}^{-1})$ *(draw from the multivariate normal distribution centered at the posterior mode)*

4) For $k = 1, ..., n_{sim}$ *draw from the proposal distribution* $N(\theta_{k-1}, c\Sigma_{\tilde{\theta}}^{-1})$ *(centered at the last accepted draw). Note that distribution* $N(\theta_{k-1}, c\Sigma_{\tilde{\theta}}^{-1})$ *corresponds to the transition distribution* q *defined above.* c *is the scaling factor set to improve the efficiency of the algorithm.*[41] *The jump*

[39] See Walsh (2004).

[40] The RWM Algorithm was first used to generated draws from the posterior distribution of DSGE model parameters by Schorfheide (2000) and Otrok (2001).

[41] Gelman, Carlin, Stern, and Rubin (1995) consider that among this kind of jumping rules, the most efficient is scale $c^{-1} \approx 2.4\sqrt{N}$, where N is the number of parameters to estimate.

from θ_{k-1} is accepted ($\theta_k = \theta^{candidate}$) with probability $\min\left[1, r(\theta_{k-1}, \theta^{candidate}|Y_T)\right]$ *and rejected ($\theta_k = \theta_{k-1}$) otherwise (see the acceptance-rejection sampling, footnote 36). Here*

$$r(\theta_{k-1}, \theta^{candidate}|Y) = \frac{\exp\left(\ln L(Y_T|\theta^{candidate}) + \sum_{i=1}^{N} \ln p(\theta_i^{candidate})\right)}{\exp\left(\ln L(Y_T|\theta_{k-1}) + \sum_{i=1}^{N} \ln p(\theta_{i,k-1})\right)} \tag{2.84}$$

Parameter constellations not yielding the unique stable solution are rejected (by assumption $\exp\left(\ln L(Y|\theta) + \ln p(\theta)\right) = -\infty$*)*

The series of accepted draws $\{\theta_{k-1}\}_{k=1}^{n_{sim}}$ is serially correlated, therefore the number of draws n_{sim} and scaling factor c should be chosen to assure that the sequence $\{\theta_{k-1}\}_{k=n_{burn\_in}}^{n_{sim}}$ (after a burn-in period) converges to the posterior distribution. In its simplistic form the convergence check may be performed using the CUMSUM statistics (for each element θ^i of the vector θ):

$$CUMSUM_{\theta^i}(J) = \frac{1}{j}\sum\nolimits_j \frac{\theta_j^i - \bar{\theta}_j^i}{\sqrt{var(\theta_j^i)}}, \text{ where } j = 1, 2, ..., J \tag{2.85}$$

In order to avoid the so-called local optima problem, it is reasonable to start the optimization from different points in the parameter space to increase the likelihood that the global optimum is found (we discuss this issue while estimating the models in Chapter 4). Similarly, in Bayesian computation it is helpful to start MCMC from different regions of the parameter space, or simply run the parallel posterior simulations and check whether the results in all blocks converge. The convergence can be then assessed by comparing variation between and within simulated sequences until 'within' variation approximates 'between' variation, as suggested by Gelman, Carlin, Stern, and Rubin (1995). Only when the distribution of each sequence is close to that of all the sequences mixed together, they can all be used to approximate the posterior distribution. The between-chain variance and pooled within-chain variance are defined by:

$$B = \frac{n_{sim}}{m-1}\sum_{j=1}^{m}(\hat{\theta}_{.j} - \hat{\theta}_{..})^2, \text{ where } \hat{\theta}_{.j} = \frac{1}{n_{sim}}\sum_{i=1}^{n_{sim}}\theta_{ij} \text{ and } \hat{\theta}_{..} = \frac{1}{m}\sum_{j=1}^{m}\hat{\theta}_{.j} \tag{2.86}$$

$$W = \frac{1}{m}\sum_{j=1}^{m} s_{.j}^2, \text{ where } s_j^2 = \frac{1}{n_{sim}-1}\sum_{i=1}^{n_{sim}}(\theta_{ij} - \hat{\theta}_{.j})^2, \tag{2.87}$$

where m is the number of sequences and n_{sim} the number of draws in each sequence. The marginal posterior variance of each parameter will be a weighted average of W and B.

$$\widehat{var(\theta|Y)} = \frac{n_{sim}-1}{n_{sim}}W + \frac{1}{n_{sim}}B \tag{2.88}$$

To check the convergence we calculate the potential scale reduction factor (PSRF) for each parameter

$$\hat{R} = \sqrt{\frac{\widehat{var(\theta|Y)}}{W}}, \tag{2.89}$$

which declines to 1 as $n \to \infty$. If the PSRF is high, one should proceed with further simulations to improve the inference.[42]

[42] It is common practice to complement the convergence measures by visualization of the MCMC chains. Visualizations are useful especially when analyzing reasons of convergence problems. See, e.g., Brooks and Gelman (1998).

If convergence is satisfactory, the posterior expected value of a parameter function $h(\theta)$ might be approximated by $\frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} h(\theta_k)$, which is the step 5) of Algorithm 1.[43] Throughout the study we use the approximations of the posterior distribution of the following parameter functions: the posterior mean of the parameters, parameter 90% confidence intervals as well as confidence intervals for the impulse response functions and replicated second moments of the variables.

2.7.2 Numerical optimization of the posterior

As indicated above, in order to increase the efficiency of simulations, the RWM algorithm (Algorithm 6) starts at the posterior mode. The computation of the posterior mode and the matrix of second derivatives at the posterior mode may also be useful for local approximations of the posterior and subsequently for evaluating the marginal density (see Section 2.8). The maximization of the log-posterior may be performed by extending the Kalman filter algorithm as follows:

Algorithm 7 (A generic procedure for numerical optimization of the posterior) .

1) Choose some initial $\theta = \theta_0$.

2) Do steps 1)-4) of algorithm 4 (Kalman filter).

3) At each step save $v_t = y_t - y_{t|t-1}$ and $F_{t|t-1}$. Construct the log-likelihood using prediction error decomposition (2.69). Assign the prior and compute the log-posterior (2.9)

4) Update initial estimates of θ using the unconstrained optimization routine described below (Algorithm 8).

5) Repeat steps 2)-4) until a convergence criterion is met.

Algorithm 7 is here accomplished by Newton's type optimization routine.

Consider the following maximization problem $\max_{\theta} \ln p(\theta|Y_T, M)$. Suppose that $\ln p(\theta|Y_T, M)$ is twice continuously differentiable with respect to θ. The first order necessary condition means that if $\ln p(\theta|Y_T, M)$ achieves its minimum at a point $\tilde{\theta}$, then

$$\nabla \ln p(\tilde{\theta}|Y_T, M) = 0 \tag{2.90}$$

that is, $\tilde{\theta}$ is a stationary point. Since function $\ln p(\theta|Y_T, M)$ is 'complicated' and it is impossible to solve (2.90) analytically, numerical methods are required. The basic idea of Newton's method is to generate a sequence of points approximating a solution of (2.90). In particular, the Taylor approximation of

$$\nabla \ln p(\theta|Y_T, M) \approx g_0 + \Sigma_0 (\theta - \theta_0) \tag{2.91}$$

is considered, where $g_0 = \nabla \ln p(\theta_0|Y_T, M)$ and $\Sigma_0 = \nabla^2 \ln p(\theta_0|Y_T, M)$. The fundamental idea of the method is to solve the linear system of equations given by

$$g_0 + \Sigma_0 (\theta - \theta_0) = 0 \tag{2.92}$$

instead of (2.90) and take the solution of (2.92) as a new solution to (2.90). In general, one can write Newton's method as $\theta_{k+1} = \theta_k - g_k \Sigma_k^{-1}$ for $k = 0, 1, 2, ...$ Newton's method in its original form is, however, ineffective for the optimization of the posterior distributions of DSGE models. This is because the method requires evaluation of the Hessian matrix at each step, which is computationally extremely expensive. The method also does not guarantee that the sequence of $\{\ln p(\theta_k|Y_T, M)\}$ at each step is monotonically decreasing.[44]

For this reason, a quasi-Newton method with update of the estimated inverse Hessian and line search is applied in this thesis. To present the method we assume that we are able to compute the sequence of estimates of Σ_k^{-1}.

[43] Usually we discard about 50% of initial draws.

[44] See, e.g., Dennis and Schnabel (1989).

Algorithm 8 (The Quasi-Newton method with line search) .

1) Choose some initial $\theta = \theta_0$, set k=0.

2) Calculate gradient $g_k = \nabla \ln p(\theta_k|Y_T, M)$ and estimate (see the BFGS method below) the inverse Hessian Σ_k^{-1}. If $g_k = 0$, stop.[45]

3) Find the maximum of quadratic approximation of the posterior $\ln p(\theta_k|Y_T, M)$. Since the posterior is in fact not quadratic solve for the optimum iteratively setting $\theta_{k+1} = \theta_k + d_k$, where $d_k = -\Sigma_k^{-1} g_k$ is called the direction of search. The direction is a vector describing a segment of a path from the starting point to the solution, where the inverse of the Hessian, Σ_k^{-1} determines the angle of the direction and the gradient, g_k determines its size extremum. Check if under parametrization θ_{k+1} the DSGE model yields the unique stable solution and if $\theta_{k+1} \in \Theta$. If any of these conditions are not met set $\ln p(\theta_{k+1}|Y_T, M) = -\infty$.

4) When the quadratic approximation of the posterior $\ln p(\theta_k|Y_T, M)$ is 'good', the Hessian is well-conditioned and the convergence quadratic. In the case of DSGE models, the posterior, i.e. the function being optimized, can be not well behaved in the region of θ_k. To deal with this, the Newton step is redefined as $\theta_{k+1} = \theta_k + \alpha_k d_k$, where α_k is called the step length and is determined by a local optimization of the function, called a line search, that is given the direction and the starting point $\alpha_k = \arg\min_{\alpha_k} \ln p(\theta_k - \alpha_k d_k|Y_T, M)$.

5) Set $k = k + 1$ go to step 2). Repeat steps 2)-4) until the convergence criterion is met. The convergence criterion is here a relative gradient, a gradient adjusted for scaling and may be stated as $\max |\ln p(\theta_{k+1}|Y_T, M) \frac{g_{k+1}}{\theta_{k+1}}| < \varepsilon$, where ε is small.

In the implementations of the quasi-Newton algorithm, one normally requires that the steplength α_k satisfies the Wolfe conditions:

$$\ln p(\theta_k + \alpha_k d_k|Y_T, M) - \ln p(\theta_k|Y_T, M) \leq \delta_1 \alpha_k d_k' g_k \tag{2.93}$$

$$d_k' \nabla \ln p(\theta_k + \alpha_k d_k|Y_T, M) \geq \delta_2 d_k' g_k, \tag{2.94}$$

where $\delta_1 \leq \delta_2$ are constants in $(0, 1)$.

In what follows, we explain how Σ_k^{-1} can be possibly calculated, which is the key point of quasi-Newton methods.

Suppose we have calculated $g_0 = \nabla \ln p(\theta_0|Y_T, M)$, $\Sigma_0 = \nabla^2 \ln p(\theta_0|Y_T, M)$ and θ_1 (using Newton's method). Instead of calculating $\nabla^2 \ln p(\theta_1|Y_T, M)$, we would like to find a matrix Σ_1 to replace $\nabla^2 \ln p(\theta_1|Y_T, M)$. Note that

$$\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M) \approx \nabla^2 \ln p(\theta_0|Y_T, M)\,(\theta_0 - \theta_1) \tag{2.95}$$

and we want Σ_1 to satisfy

$$\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M) = \Sigma_1\,(\theta_0 - \theta_1) \tag{2.96}$$

or equivalently we want to find Σ_1^{-1} such that

$$\Sigma_1^{-1}\,(\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M)) = (\theta_0 - \theta_1) \tag{2.97}$$

This condition is called quasi-Newton condition. In general, it may be written as:

$$\Sigma_{k+1}^{-1} \gamma_k = \delta_k, \tag{2.98}$$

where $\gamma_k = \nabla \ln p(\theta_{k+1}|Y_T, M) - \nabla \ln p(\theta_k|Y_T, M)$ and $\delta_k = \theta_{k+1} - \theta_k$. If we can find matrix Σ_{k+1}^{-1}, we are able to compute the search direction $d_{k+1} = -\Sigma_{k+1}^{-1} g_{k+1}$. However, the matrix satisfying

[45] We use the 'csminwel' algorithm by C. Sims. This algorithm is also robust against certain pathologies common on likelihood functions, eg. against 'cliffs', i.e. hyperplane discontinuities.

(2.98) is not unique. The general idea to construct Σ_{k+1}^{-1} is to update it from Σ_k^{-1} using the gradient information at both θ_k and θ_{k+1}. The most important methods for estimation of the inverse Hessian matrix are Broyden (1967) method, which uses rank one correction from Σ_k^{-1} to write Σ_k^{-1} and the DFP (for Davidson (1959), Fletcher and Powell (1963)), and the BFGS (for Broyden (1969), Fletcher (1970), Goldfarb (1970), and Shanno (1970)). The BFGS is generally regarded as the best performing method (see, e.g., Dai (2002)).

Similarly to DFP method the BFGS uses rank two correction from Σ_k^{-1}, i.e.

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + a\mathrm{uu}' + b\mathrm{vv}' \tag{2.99}$$

Σ_{k+1}^{-1} must also satisfy quasi-Newton condition:

$$\Sigma_k^{-1}\gamma_k + a\mathrm{uu}'\gamma_k + b\mathrm{vv}'\gamma_k = \delta_k \tag{2.100}$$

u and v are not unique in this case. An obvious solution is: u$=\delta_k$, v$=\Sigma_k^{-1}\gamma_k$, $a = 1/\left(\mathrm{u}'\gamma_k\right)$ and $b = -1/\left(\mathrm{v}'\gamma_k\right)$. Hence

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + \frac{\delta_k\delta_k'}{\delta_k'\gamma_k} - \frac{\Sigma_k^{-1}\gamma_k\gamma_k'\Sigma_k^{-1}}{\left(\Sigma_k^{-1}\gamma_k\right)'\gamma_k} \tag{2.101}$$

Using that $\gamma_k = \Sigma_{k+1}\delta_k$ we may transform (2.101) to obtain

$$\Sigma_{k+1} = \Sigma_k + \frac{\gamma_k'\gamma_k}{\gamma_k'\delta_k} - \frac{\Sigma_k\delta_k\delta_k'\Sigma_k}{\left(\Sigma_k\delta_k\right)'\delta_k}. \tag{2.102}$$

The inverse Hessian is calculated as

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + \left(1 + \frac{\gamma_k'\Sigma_k^{-1}\gamma_k}{\delta_k'\gamma_k}\right)\frac{\delta_k\delta_k'}{\delta_k'\gamma_k} - \left(\frac{\delta_k\gamma_k'\Sigma_k^{-1} + \Sigma_k^{-1}\gamma_k\delta_k'}{\delta_k'\gamma_k}\right) \tag{2.103}$$

Since Σ_{k+1}^{-1} is the unique solution of the following problem:

$$\min_{\Sigma^{-1}}|\Sigma^{-1} - \Sigma_k^{-1}| \tag{2.104}$$

$$\begin{aligned} \text{s.t. } \Sigma^{-1} &= \left(\Sigma^{-1}\right)', \\ \Sigma^{-1}\gamma_k &= \delta_k, \end{aligned} \tag{2.105}$$

this means that, among all symmetric matrices satisfying quasi-Newton's condition, Σ_{k+1}^{-1} is the closest, in some sense, to the current matrix Σ_{k+1}^{-1}.

2.8 Model evaluation

This section studies model evaluation techniques (the last step in the Bayesian estimation algorithm). In particular, we focus on the assessment of model's relative fit, which is typically conducted by applying Bayesian inference and decision theory to the extended model space.[46] Moreover, the issues of parameter identification and the robustness of Bayesian inference are briefly discussed.

[46]The assessment of absolute fit of the model can be implemented by a sampling based model check. The model is considered as inaccurate if it is very unlikely to reproduce with it the particular feature of the data (e.g., observed inflation persistence or international spillovers). Such model checks, though they provide valuable insights about the overall quality of the estimated model, may be, however, controversial from a Bayesian perspective because the models with very diffuse predictions would be clearly favored.

2.8.1 Model-data fit

A natural method to assess the empirical validity of the DSGE model is to compare its predictive performance, measured by the integrated likelihood (or marginal likelihood), with other available models including DSGE models or perhaps an even larger class of non-structural linear reduced-form models.[47] Marginal likelihood $p(Y_T|M_i)$ measures how well model M_i predicts the observed data Y_T. The formal link between the marginal likelihood of a model $p(Y_T|M_i)$ and its predictive interpretation has been established by Geisel (1977).

First, let us consider the distribution of the sequence $y_{u+1},...,y_t$ conditional on the data Y_u and model M_i,[48]

$$p(y_{u+1},...,y_t|Y_u,M_i) = \int p(\theta|Y_u,M_i)\prod_{s=u+1}^{t} p(y_s|Y_{s-1},\theta,M_i)d\theta \tag{2.106}$$

Expression (2.106) also may be interpreted as the predictive density of $y_{u+1},...,y_t$ conditional on Y_u and model M_i, because the judgment on $y_{u+1},...,y_t$ is done based on Y_u and before observing $y_{u+1},...,y_t$. Following the observation of $y_{u+1},...,y_t$ expression (2.106) is the known number the so-called predictive likelihood of $y_{u+1},...,y_t$ conditional on Y_u and model M_i. Furthermore $p(y_1,...,y_t|Y_0,M_i) = P(Y_t|M_i)$ if $Y_0 = \{\emptyset\}$. Substituting for the posterior density in (2.106) we have

$$\begin{aligned}
p(y_{u+1},...,y_t|Y_u,M_i) &= \int \left\{ \begin{array}{c} \frac{p(\theta|M_i)\prod_{s=1}^{u} p(y_s|Y_{s-1},\theta,M_i)}{\int p(\theta|M_i)\prod_{s=1}^{u} p(y_s|Y_{s-1},\theta,M_i)d\theta} \\ \times \prod_{s=u+1}^{t} p(y_s|Y_{s-1},\theta,M_i) \end{array} \right\} d\theta & (2.107)\\
&= \frac{\int p(\theta|M_i)\prod_{s=1}^{t} p(y_s|Y_{s-1},\theta,M_i)d\theta}{\int p(\theta|M_i)\prod_{s=1}^{u} p(y_s|Y_{s-1},\theta,M_i)d\theta} & (2.108)\\
&= \frac{p(Y_t|M_i)}{p(Y_u|M_i)} & (2.109)
\end{aligned}$$

Hence for any $0 \leq u = s_0 < s_q = t$, we have

$$\begin{aligned}
p(y_{u+1},...,y_t|Y_u,M_i) &= \frac{p(Y_{s_1}|M_i)}{p(Y_{s_0}|M_i)}\frac{p(Y_{s_2}|M_i)}{p(Y_{s_1}|M_i)}\cdots\frac{p(Y_{s_q}|M_i)}{p(Y_{s_{q-1}}|M_i)} & (2.110)\\
&= \prod_{l=1}^{q} p(y_{s_{l-1}+1},...,y_{s_l}|Y_{s_{l-1}},M_i). & (2.111)
\end{aligned}$$

This decomposition shows that the marginal likelihood, i.e. if $u = 0$ and $t = T$, summarizes the out of sample model performance as expressed in predictive likelihoods $p(Y_T|M_i) = \prod_{l=1}^{q} p(y_{s_{l-1}+1},...,y_{s_l}|Y_{s_{l-1}},M_i)$.

[47] In this study we make a preliminary comparison of DSGE models with a VAR(1) and VAR(3) models, using RMSEs, MAEs and log determinant statistics. Sims (2003) provides a general discussion about pitfalls of Bayesian model comparison methods, highlighting several ways they tend to misbehave. In this view, there is no point in comparing, e.g., the marginal data density obtained here with the data density of a VAR where the priors are defined with a training set. As discussed by Sims, such kind of comparison could be totally arbitrary and meaningless. Alternative methods of comparison of DSGE models with VAR and Bayesian VAR models is proposed by Del Negro and Schorfheide (2004). Moreover, in Chapter 3 we introduce the so-called nested specfications that avoid the critique of Sims and allow for comparison of alternative specifications within the same model.

[48] See Geweke (1999).

The computation of the marginal likelihood $p(Y_T|M_i)$, and more precisely the computation of the integral (2.8) is unfeasible analytically in most cases. There have been proposed methods for estimation of the marginal likelihood using a sample from the posterior distribution. The most popular are the estimators by Geweke (1999) and Chib and Jeliazkov (2001). Alternatively, the calculation of the integral (2.8) may be based on the local approximations of the posterior.

The marginal data density of the DSGE model is in this study approximated with Geweke's modified harmonic mean estimator. Harmonic mean estimators are based on the following identity:

$$\frac{1}{p(Y_T|M_i)} = \int \frac{f(\theta)}{L(Y_T|\theta, M_i)p(\theta)} p(\theta|Y_T, M_i)d\theta, \tag{2.112}$$

where $f(\theta)$ has the property that $\int f(\theta)d\theta = 1$. Conditional on the choice of θ the estimator of $p(Y)$ is:

$$\hat{p}(Y_T|M_i) = \left[\frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} \frac{f(\theta_k)}{L(Y_T|\theta_k, M_i)p(\theta_k)}\right]^{-1}, \tag{2.113}$$

where θ_k is drawn from the posterior distribution $p(\theta|Y_T, M)$ using Algorithm 6 (RWM algorithm). To make the numerical approximation efficient, $f(\theta)$ is to be chosen so that the summands are of equal magnitude. Geweke (1999) proposed to use the density of a truncated multivariate normal distribution:

$$\begin{aligned} f(\theta) &= \tau^{-1}(2\pi)^{-d/2}|V_\theta|^{-1/2} \exp\left(-0.5(\theta-\bar{\theta})'V_\theta^{-1}(\theta-\bar{\theta})\right) \\ &\quad \times p\left\{(\theta-\bar{\theta})'V_\theta^{-1}(\theta-\bar{\theta}) \leq F^{-1}_{\chi^2_N}(\tau)\right\}, \end{aligned} \tag{2.114}$$

where $\bar{\theta}$ and V_θ are the posterior mean and posterior covariance matrix (calculated form output of Algorithm 6). N is here the dimension of parameter vector θ, $F_{\chi^2_N}$ is the cumulative density of a χ^2 random variable with N degrees of freedom and $\tau \in (0, 1)$.

When the likelihood is highly peaked around the mode and close to symmetric, the posterior density kernel can be locally approximated by the multivariate normal density (Laplace approximation):

$$\ln p(Y_T|\theta, M_i) + \ln p(\theta|M_i) \approx \ln p(Y_T|\tilde{\theta}, M_i) + \ln p(\tilde{\theta}|M_i) + \frac{1}{2}\left(\theta-\tilde{\theta}\right)' \Sigma_{\tilde{\theta}} \left(\theta-\tilde{\theta}\right), \tag{2.115}$$

where $\tilde{\theta}$ denotes the posterior mode and $\Sigma_{\tilde{\theta}}$ is the Hessian computed at the posterior mode. Integrating with respect to θ we obtain the following estimator of the marginal likelihood:

$$\hat{p}(Y_T|M_i) = (2\pi)^{\frac{N}{2}} |\Sigma_{\tilde{\theta}}|^{-\frac{1}{2}} p(\tilde{\theta}|Y_T, M_i)p(\tilde{\theta}|M_i), \tag{2.116}$$

where N is the number of estimated parameters.

Having computed the approximation of (2.8) Bayesian model selection is done pairwise comparing the models through posterior odds ratio:[49]

$$PO_{i,j} = \frac{p(Y_T|M_i)p(M_i)}{p(Y_T|M_j)p(M_j)}, \tag{2.117}$$

where the prior odds $\frac{p(M_i)}{p(M_j)}$ (researchers can place subjective probabilities on competing models) are updated by the Bayes factor: $B_{ij} = \frac{p(Y_T|M_i)}{p(Y_T|M_j)}$. Jeffreys (1961) suggested rules of thumb to interpret the Bayes factor as follows:

[49] It has been shown under various regularity conditions, see, e.g., Fernandez-Villaverde and Rubio-Ramirez (2004a), that posterior odds (or their large sample approximations) asymptotically favor the DSGE model that is closest to the 'true' data generating process in the Kullback-Leibler sense.

$B_{ij} < 1$	support for M_j
$1 \leq B_{ij} < 3$	very slight support for M_j
$3 \leq B_{ij} < 10$	slight evidence against M_j
$10 \leq B_{ij} < 100$	strong evidence against M_j
$B_{ij} \geq 100$	decisive evidence against M_j

Smets and Wouters (2004b), Adolfson, Laseen, Linde, and Villani (2005b) or Ratto, Roeger, intVeld, and Girardi (2005a) suggest to evaluate Bayesian estimated DSGE models based on the point estimates and applying standard statistical tools. For example, one can test the residuals for serial correlation and neglected autoregressive conditional heteroscedasticity, compare the root mean square errors of the DSGE model with those of another DSGE model or a Vector Autoregression or perform tests of parameter stability. All this is valuable information in the construction of more realistic economic models.

2.8.2 Robustness of the results and identification issues

In this subsection, we discuss the problem of parameter identification as well as issues related to Bayesian sensitivity analysis.

Identification problems have been extensively studied in econometric theory at least since the 1950's (see Koopmans (1950)). The more recent contributions include Rothenberg (1971) and Pasaran (1981). In the context of DSGE models, identification issues have been addressed in Onatski and Williams (2004), Lubik and Schorfheide (2005) and Canova and Sala (2005). The problem of parameter identification can be defined as the ability to draw inference about the parameters of a theoretical model from an observed sample. There have been defined several reasons for which the data might not deliver the sufficient information for an unambiguous identification of the parameters. First, the data might not distinguish between different structural forms of the model. It means that the loss function upon which the models are estimated does not account for the distinct features of alternative models, i.e.[50]

$$\min_{\theta} L(\theta, M_1) = \min_{\xi} L(\xi, M_2), \tag{2.118}$$

where $L(\cdot)$ is the loss function and θ and ξ are parameter vectors of models M_1 and M_2 respectively.

Second, some of the estimated parameters might enter the loss function proportionally. Then, partitioning the parameter vector θ to θ_1 and θ_2 and the parameter space to $\Theta = [\Theta^1, \Theta^2]$ we have

$$\min_{\theta_1,\theta_2} L(\theta_1, \theta_2, M_1) = \min_{\theta_1} L(\theta_1, \theta_2, M_1) \ \forall \ \theta_2 \in \Theta_2 \subset \Theta^2. \tag{2.119}$$

This problem is referred to as a partial identification. In practical application, the easiest way to tackle this problem is to estimate only one of the parameters entering the loss function proportionally and to fix the rest.

Third, even though all parameters enter the loss function independently and the population objective function is globally concave, its curvature may be insufficient.

$$L(\tilde{\theta}, M_1) - L(\theta, M_1) \leq \varepsilon \ \forall \theta \in \Theta^* \subset \Theta, \text{ for small } \varepsilon, \tag{2.120}$$

where $\tilde{\theta}$ is the parameter constellation yielding the minimum of the loss function. This problem is referred to as a weak identification and is particularly important from the perspective of numerical optimization.

Finally, the parameters which are one to one related to the unstable root of the system may be unidentifiable upon the observed time series, which obey the transversality condition (see Lucke and Gaggermeier (2001)).

[50] See, e.g., Canova and Sala (2005).

All types of identification problems listed above are relatively common in the estimation of DSGE models. Their source is often the discrepancy between a model's definition of economic aggregates and the available time series. Moreover, some structural parameters of DSGE model might not be identifiable due to the fact that detrended and seasonally adjusted time series may contain little information about the deterministic steady state. While in small scale models the identification issue may generally be resolved by careful inspection of single equations, we have no possibility in larger models of telling ex ante which parameters are identifiable. In addition, identification problems in DSGE models are difficult to detect because the mapping from the vector of structural parameters θ into the state-space representation (2.39) - (2.40) that determines the likelihood of Y_T is highly nonlinear. The diagnosis is also impeded by the fact that the likelihood has to be evaluated numerically.

Some numerical procedures to detect the identification problems have been proposed. For example, in the context of Maximum Likelihood estimation, the identification problem (more precisely the weak identification problem) may be detected by examination of the Hessian at the optimum or by ploting the data likelihood in the neighborhood of the optimum.

As mentioned in Section 2.5, one technical reason for the popularity of the Bayesian approach is that by incorporating even a weakly informative prior the curvature into the posterior density surface can be introduced. This, in turn, facilitates numerical maximization and the use of MCMC methods. However, the uncritical use of Bayesian methods, including employing prior distributions which do not truly reflect the existing location uncertainty, whereas the data carries no information about parameters, may - according to Canova and Sala (2005) - *hide identification problems instead of highlighting them.* There is a simple diagnostic for detecting a lack of identification in the Bayesian framework, a diagnostic unavailable in the classical setup. The identification issue may be examined by estimating the model with more and more diffuse priors, which is referred to in the literature as Bayesian sensitivity analysis. Thus, the posterior of parameters with doubtful identification features will also become more and more diffuse.

Though interrelated, the concept of Bayesian sensitivity analysis has a broader meaning than solely detecting parameter identifiability. Let us consider the case in which the data carries information on estimated parameters but the prior distribution has subjective features or the sample is small. In this case posterior and prior can have different locations. It is important, however, to check how sensitive posterior outcomes are to the choice of prior distributions. A way to assess the robustness of the posterior conclusions is to select an alternative prior density $p_2(\theta)$, with support included in $p(\theta)$, and use it to reweight posterior draws.[51]

Let $w(\theta) = \frac{p_2(\theta)}{p(\theta)}$ so that

$$E_2(h(\theta)) = \int h(\theta)p_2(\theta)d\theta = \int h(\theta)p(\theta)w(\theta)d\theta \tag{2.121}$$

so that

$$h_2(\theta) = \frac{\sum_{i=1}^{n_{sim}} h(\theta)w(\theta)}{\sum_{i=1}^{n_{sim}} w(\theta)}. \tag{2.122}$$

As a general rule, the results are assessed not robust if the means of the $h_2(\theta)$ statistics lie outside the 90% posterior interval constructed for $h(\theta)$.

The most commonly performed robustness check in the DSGE literature is based on the selection of univariate distributions for some estimated structural parameters. This approach is very similar to the constrained optimization of the likelihood (see Onatski and Williams (2004)). We discuss this issue more extensively in Chapter 4 of this thesis.

[51]See Canova and Sala (2005).

The identification issue in the context of standard closed economy DSGE models has been extensively documented in many empirical studies (see Ireland (2001), Smets and Wouters (2003) or Onatski and Williams (2004)). In fact, it seems that models such as Smets and Wouters (2003) had some success in exploiting the information contained in the aggregated macroeconomic time series. However, these problems have not yet been satisfactorily recognized in the context of estimated New Open Economy Macroeconomics DSGE models. In fact, the inference is often based on the posterior obtained under very tight priors and very flat likelihood. The only, preliminary, discussion is presented in Lubik and Schorfheide (2005). They examine the identification issues based on the estimated small-scale two-country model for the US and the Euro area. The authors underline that due to the problem with constructing the bilateral current account data, for instance, parameters standing for ineffectiveness of financial markets are in general unidentifiable in open economy models. Similar problems might be found while estimating the parameters related to trade within the Euro area (see Chapter 4 of this thesis). Moreover, when applying the available macroeconomic aggregates, it may be difficult to identify the parameters related to the multi-sector structure often incorporated into Small Open Economy models as in Adolfson, Laseen, Linde, and Villani (2005a). So, as we see it, keeping the theoretical structure of estimated open economy DSGE models relatively simple might just be a way to avoid some of the aforementioned problems.

Chapter 3

Sticky contracts or sticky information? Evidence from an estimated Euro area DSGE model

3.1 Introduction

Nominal stickiness has emerged as one of the most important factors that help DSGE models to match features of the data while maintaining a coherent micro-founded framework.

The New-Keynesian Phillips Curve - according to McCallum (1997) *the closest thing there is to a standard specification* for nominal rigidities - has recently been challenged by the proposal of Mankiw and Reis (2002) to replace it with a sticky information framework.[1] While some prices are exogenously fixed for certain periods in sticky price models such as Calvo (1983), sticky information models assume that information sets are only updated sporadically. This allows agents to change their prices in any period, but typically at a different level than in a full information environment.

The contribution of this chapter is to perform an extensive empirical comparison of these two competing frameworks for introducing nominal rigidities in wage and price setting. Our study is motivated by a discrepancy in the literature; the shortcomings of the extensively studied Calvo model are very well known, while relatively little research exists that aims at empirically evaluating sticky information models.[2] The models we estimate are identical in the specification of the real side of the economy to Smets and Wouters (2003). The main features of this closed economy model are external habit formation in consumption, capital adjustment costs specified in terms of the rate of change of investment, variable capacity utilization as well as a large number of shocks.

We empirically evaluate sticky information models and the Calvo model in a Bayesian framework by comparing posterior odds ratios using Euro area aggregate data. This comparison has two parts. In the first part, we compare the standard Calvo model to a baseline specification of a truncated Mankiw and Reis (2002) model. Agents base their prices and wages on information that is outdated by at most 12 quarters. Furthermore, the shares of agents working with information sets outdated by j periods is geometrically declining in j as in Mankiw and Reis (2002). Our striking result is that the posterior odds ratio shows overwhelming support for the Calvo model. The difference in log marginal density between the two competing models is so large that one would require a prior probability for the sticky information model that is 10^{48} times larger than the one for the Calvo model. According to Jeffreys' rule of thumb this is very strong evidence against the

[1] This chapter is based on the outcome of the joint research with Matthias Paustian.

[2] Mankiw and Reis (2002) point out that the New-Keynesian Phillips cannot explain the gradual and delayed effects of monetary shocks on inflation. They further criticize that it cannot reproduce the conventional view that announced and credible disinflations should be contractionary and fails to account for a positive correlation of the change in inflation with output.

sticky information model. Extending the maximum age of outdated information sets from 12 to 24 quarters and allowing for different parameterizations of the size of cohorts operating with the various information sets improves the performance of the sticky information model only slightly. These results are striking since Mankiw and Reis (2002, p. 1295) criticize the Calvo model as *hard to square with the facts*. Our Bayesian model comparison indicates the opposite: It appears difficult to defend the sticky information model against the Calvo model.

In a second part of the chapter we ask whether we can find partial support for the sticky information idea once we allow for heterogeneity in price and wage setting.[3] To this end we build two models of heterogeneous price and wage setters. In the first model, a fraction of agents is assumed to set Calvo contracts whereas the remaining fraction of agents operates according to the sticky information framework. Such a setup allows us to assess the importance of sticky information vs. Calvo contracts in a nested model that reduces to either specification in the extreme cases. We can thus estimate the share of sticky information agents in a nested Calvo-Mankiw population of wage and price setters. This estimated share serves as another measure for how relevant the sticky information framework is in Euro Area aggregate data. Again there is overwhelming evidence against the standard sticky information model. For the baseline specification the mode of the posterior distribution of the fraction of agents operating according to the Calvo framework is 0.99 for price setters and 0.93 for wage setters despite the fact that our priors assume shares of 50 percent each. In other words, the data ascribes almost zero mass to sticky information agents in price setting and only little more in wage setting. Consequently, there is very little support for sticky information in wage and price setting.

The second class of models with heterogeneity in price and wage setting combines a fraction of sticky information agents with a fraction of purely backward looking rule of thumb price and wage setters. Such a hybrid sticky information model is in the spirit of hybrid Calvo models as in Galí and Gertler (1999) that generate inertia in the Phillips curve by adding rule of thumb price setters. Again the posterior odds ratio slightly favors the standard Calvo model of wage and price setting over all versions of these hybrid sticky information models. The fraction of agents operating according to the sticky information framework is very small for price setting (1%) but almost all wage setters are estimated to be forwardlooking.

Next, we allow the distribution of the shares of agents working with different outdated information sets to follow two less restrictive patterns than in the baseline model. This leads to an estimated population share of sticky information agents of roughly 15% for price setting and 30-35% for wage setting. The estimated fraction of forwardlooking price setters in hybrid model increases to about 8%. It should be noted however that both nested sticky price - sticky information models and hybrid models have smaller marginal density than the standard Calvo model which is a special case of the nested model. The fact that the rich model ranks worse than the nested simple model is due to the Bayesian model comparison criteria we employ. These criteria often favor parsimonious models. Finally, the distribution of the population shares of agents working with different outdated information sets is highly irregular. In particular, the idea that the shares of agents working with older information sets should decline as in the standard model of Mankiw and Reis (2002) is not supported by the data.

What explains the overwhelmingly poor performance of sticky information models? Inspecting impulse responses and the match of models moments with the data suggests the following. First, the estimated sticky information models deliver the delayed and hump-shaped response of output and inflation to monetary shocks that were pointed out by Mankiw and Reis (2002) for their highly stylized model. Hence, the poor performance cannot be explained by a lack of robustness of the Mankiw-Reiss framework to deliver inflation inertia as is found in Coiboin (2006) or Keen (2005). These papers argue that large real rigidities are necessary to generate inflation inertia in sticky information models and that interest rate rules rather than money growth rules make inflation

[3]Heterogeneity in price setting has been emphasized by Carvalho (2005).

inertia less likely. These issues are not problematic in our model. However, it appears that the estimated sticky information model has difficulties at matching the volatility of inflation as well as the persistence of inflation and real wages. In the Smets and Wouters (2003) sticky price model, so-called markup shocks to the Phillips curve allow the model to match both the persistence and the volatility of inflation. Without such markup shocks, inflation would be too smooth as the underlying marginal cost series is smooth. In sticky information models, markup shocks induce quite different impulse responses. Here, markup shocks increase inflation volatility at the cost of reducing the autocorrelation of inflation. Hence, this trade off is one possible explanation for the poor fit of the sticky information models. It should be noted that price markup shocks are not only important in helping the Calvo model match the volatility of inflation, they are also a big factor for overall model fit. Estimated versions of the Calvo model with price markup shocks strongly dominate the restricted versions without the markup shock as indicated by a Bayes factor of roughly 50.

We now turn to how our findings compare with the literature. Much of the related literature has compared sticky information models to the data along a few selected dimensions. For instance, Collard and Dellas (2004) and Trabandt (2005) compare sticky price and sticky information models based on their ability to match stylized facts. Khan and Zhu (2002) estimate sticky information Phillips curves in a partial equilibrium framework without testing the model against a specific alternative. Carroll (2003) estimates an inflation formation process using U.S. micro-data in which households only sporadically update their inflation expectations based on professional forecasts. A similar study by Döpke, Dovern, Fritsche, and Slacalek (2005) is conducted for the Euro area.

A related paper is Korenok (2005) who compares sticky information and sticky price models based on the theoretical relations these models imply between prices and unit labor costs. The work by Korenok (2005) for the U.S. also favors the sticky price model over the Mankiw-Reis model. Andres, Lopez-Salido, and Nelson (2005) compare the Calvo model with a sticky information model by maximum likelihood estimation and find that the sticky information model attains a higher value of the likelihood function than the Calvo model. However, they consider a very stylized model without capital accumulation and wage rigidities which is driven by only three shocks. Most closely related to our approach is the recent paper by Laforte (2005) who also compares different price setting models based on Bayesian estimation using U.S. data. Our study differs from Laforte (2005) in the following aspects. We consider both wage and price setting with sticky information vs. Calvo contracts whereas Laforte (2005) considers only sticky prices. Furthermore, the structure of the economy assumed in this chapter incorporates a larger number of frictions and shocks that are often claimed to be necessary for achieving a good fit with data. Finally, our model is estimated on a total of 7 observable time series whereas Laforte (2005) estimates the model on 4 observable variables. As Canova and Sala (2005) point out problems of parameter identification are less likely to occur when the model is estimated using a large number of observed variables. These differences notwithstanding, Laforte (2005) finds that sticky price models dominate the sticky information model for USA data as we do for Euro area data. We finally note that none of the studies mentioned above include price and wage markup shocks into the model. These studies also do not consider sticky information in both wage and price setting. These differences may explain why we find far stronger evidence against the sticky information model than the previous studies.

This chapter proceeds as follows. Section 3.2 outlines the core of the model that is independent across contracting schemes in wage and price setting. Section 3.3 describes Calvo wage and price setting as well as the sticky information schemes that we use for the estimation of the models. Section 3.4 collects the equilibrium conditions and presents their log-linearized form. Section 3.5 presents the data and estimation strategy. Section 3.6 presents our estimation results for the baseline model and two extensions. The results for models with heterogenous price and wage setters are presented in Section 3.7. Section 3.8 provides an overall discussion of the dynamics of alternative models. Finally, Section 3.9 concludes. An appendix contains tables with variance decompositions and figures displaying impulse responses.

3.2 Outline of the model

In this section we derive and present the DSGE model that will serve as a basis for the analysis of price and wage setting mechanisms. The real side of the model has been described extensively in Smets and Wouters (2003) who built on the work of Christiano, Eichenbaum, and Evans (2001). In the model economy households maximize a utility function with two arguments (goods and leisure) over an infinite life horizon. The preferences incorporate external habit persistence. They rent capital services to firms and decide on capital accumulation given certain capital adjustment costs. The rental price of capital depends on capital utilization rate. There are two types of firms (intermediate good and final good producers). Firms producing differentiated goods decide on labor and capital inputs. The monopoly power of firms and households implies that they are price and wage setters in their respective markets.

3.2.1 Households

There is a continuum of households indexed by $j \in [0,1]$ supplying differentiated labor $l_t(j)$, consuming $C_t(j)$ of the final output good and accumulating capital $K_t(j)$. Households' objective is to maximize the intertemporal utility:[4]

$$E_0 \sum_{t=0}^{\infty} \beta^t [U_t \left(C_t(j), l_t(j)\right)], \tag{3.1}$$

where instantanous utility of household j is given by:

$$U_t(j) = \varepsilon_t^b \left[\frac{\left(C_t(j) - H_t(j)\right)^{1-\sigma_c}}{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(l_t(j)\right)^{1+\sigma_l} \right]. \tag{3.2}$$

Here, $H_t(j) = hC_{t-1}$ is an external habit stock. C_{t-1} is aggregate past consumption and $h \in (0,1)$ is the habit persistence parameter.[5] ε_t^b denotes a preference shock affecting the intertemporal elasticity of substitution, ε_t^l is a labor supply shock. Both shocks are assumed to follow first order autoregressive processes. $\beta \in (0,1)$ is the discount factor, σ_c is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution, σ_l represents the inverse of the elasticity of work effort with respect to the real wage. $l_t(j)$ is the labor effort (or 'hours worked'). Let $l_t(f,j)$ denote the number of hours of type f labor. There exists a continuum of labor types, indexed by $f \in [0,1]$, then the variable that appears in the utility is defined as: $l_t(j) = \int\limits_{f=0}^{1} l_t(f,j)df$.[6]

The budget constraint of household j is

$$b_t \frac{B_t(j)}{P_t} + C_t(j) + I_t(j) = \frac{B_{t-1}(j)}{P_t} + \frac{1}{P_t} W_t(j) l_t(j) + A_t(j) + \left(r_t^k z_t(j) - \Psi(u_t(j)) \right) K_{t-1}(j) + Div_t(j). \tag{3.3}$$

Households buy one period bonds B_t at price b_t. P_t is the aggregate price index, $W_t(j)$ the nominal wage, $A_t(j)$ is the real net cash inflow from participating in the market for state-contingent

[4]We abstract from the real money balances in the utility function. If money enters additively into (3.1), money market equilibrium plays no role for the dynamics when the nominal interest rate is set to be the instrument of monetary policy.

[5]External habits are relative to past aggregate consumption (catching up with the Joneses, Abel (1990)).

[6]Throughout this thesis, for a generic variable $X_t(j)$, its aggregate counterpart is defined as $X_t \equiv \int_0^1 X_t(j)dj$

securities. r_t^k is the real rental rate households obtain from renting out capital to firms. $\Psi(z_t(j))$ is a function capturing the resource cost of capital utilization when the utilization rate is $z_t(j)$. It is assumed that the cost of capital utilization is zero when capital utilization is one ($\Psi(1) = 0$). $Div_t(j)$ are dividends to household j from the intermediate good firms.

Households choose the capital stock, the utilization rate and investment subject to the following capital accumulation equation

$$K_t(j) = (1-\delta)K_{t-1}(j) + \left[1 - S\left(\varepsilon_t^I \frac{I_t(j)}{I_{t-1}(j)}\right)\right] I_t(j). \tag{3.4}$$

δ is the depreciation rate and $S(\cdot)$ is a function capturing costs of altering the rate of change of investment. In the steady state $S(\cdot)$ equals zero. Furthermore, the first derivative evaluated at the steady state is also assumed to equal zero such that the linearized dynamics only depend on the second derivative.[7] ε_t^I is a shock to the investment adjustment cost function.

The assumption of state contingent securities implies that households can fully insure against variations in their labor income arising from nominal rigidities in wage setting, allowing us to drop the subscript j. Maximizing the objective with respect to bonds and consumption subject to the budget constraints yields the familiar consumption Euler equation

$$\lambda_t = \beta E_t \frac{R_t}{\pi_{t+1}} \lambda_{t+1}. \tag{3.5}$$

Here, $R_t \equiv \frac{1}{b_t}$ is the gross nominal interest rate on the non-contingent bond, $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the aggregate CPI inflation rate and $\lambda_t \equiv \varepsilon_t^b \left(C_t - hC_{t-1}\right)^{-\sigma_c}$ is the marginal utility of consumption.

Optimizing the objective function with respect to capital, investment and the capital utilization subject to the budget constraint and the capital accumulation equation yields the following first-order conditions:

$$r_t^k = \Psi'\left(z_t\right) \tag{3.6}$$

$$\lambda_t Q_t = \beta E_t \lambda_{t+1}\left[(1-\delta)Q_{t+1} + z_{t+1} r_{t+1}^k - \Psi\left(z_t\right)\right] \tag{3.7}$$

$$1 = -Q_t\left[1 - S\left(\cdot\right)_t - S'\left(\cdot\right)_t \frac{\varepsilon_t^I I_t}{I_{t-1}}\right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} S'\left(\cdot\right)_{t+1} \varepsilon_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2 \tag{3.8}$$

Here $S'\left(\cdot\right)_t$ is shorthand for $S'\left(\varepsilon_t^I \frac{I_t}{I_{t-1}}\right)$. (3.6) equates the marginal gains from higher capacity utilization to the marginal cost. (3.7) states that the household installs capital until the cost today equals the discounted gains tomorrow.

3.2.2 Firms

The country produces a single final good and a continuum of intermediate goods indexed by z, where z is distributed over the unit interval. The final good sector is perfectly competitive. The final good is used for consumption and investment by the households. There is monopolistic competition in the markets for intermediate goods.

Final good sector

Final good producers aggregate the intermediate goods $Y_t(z)$ to final output Y_t according to the Dixit and Stiglitz (1977) technology

$$Y_t^{1/(1+\lambda_{p,t})} = \int_0^1 \left(Y_t(z)\right)^{1/(1+\lambda_{p,t})} dz, \tag{3.9}$$

[7] Functional from for $S\left(\cdot\right)$ that has the required properties is e.g. $S(x) = c_3\left\{\exp\left(c_1 x\right) + \frac{c_1}{c_2}\exp\left(-c_2 x\right) - \quad 1 + \frac{c_1}{c_2} \quad\right\}$, where c_1, c_2, c_3 are positive constants. Furthermore, we have $S(0) = 0$, $S'(0) = c_1 c_3\left\{\exp\left(c_1 0\right) - \exp\left(-c_2 0\right)\right\} = 0$, and $S''(0) \equiv S'' = c_1 c_3\left(c_1 + c_2\right) > 0$.

where $\lambda_{p,t}$ is a time varying markup parameter capturing fluctuations in the degree of market power. The production function above exhibits diminishing marginal product, which causes firms to diversify and produce using all intermediate goods available.

The cost minimization conditions in the final goods sector yield:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t, \tag{3.10}$$

where $P_t(z)$ is the price of intermediate good z.

The aggregate consumption based price index P_t that corresponds to the aggregator (3.9) is defined by the well-known relation:

$$P_t^{-1/\lambda_{p,t}} = \int_0^1 (P_t(z))^{-1/\lambda_{p,t}}\, dz. \tag{3.11}$$

Intermediate goods producers

Each intermediate good $Y_t(z)$ is produced by a firm z. Differentiated labor $l_t(f,z)$ is aggregated to composite labor $L_t(z)$ that enters as input in the firms production via the relation

$$(L_t(z))^{1/(1+\lambda_{w,t})} = \int_0^1 (l_t(f,z))^{1/(1+\lambda_{w,t})}\, df, \tag{3.12}$$

where $l_t(f,z)$ represents the quantity of f type labor used by firm z. $\lambda_{w,t}$ is time varying wage markup. The aggregate wage index is defined by $W_t^{-1/\lambda_{w,t}} = \int_0^1 (W_t(f))^{-1/\lambda_{w,t}}\, df$. The production function for intermediate output is given by

$$Y_t(z) = \varepsilon_t^a \left(\tilde{K}_t(z)\right)^{\alpha} (L_t(z))^{1-\alpha} - \Phi.$$

Here, ε_t^a denotes the exogenous total factor productivity shock, $K_t(z)$ are effective capital services defined as $\tilde{K}_t(z) \equiv z_t K_t(z)$, $L_t(z)$ denotes the aggregate labor index and Φ is a fixed cost parameter. We assume that capital is freely mobile across firms. Therefore, an economy wide rental market for capital induces all firms to operate with the same capital-labor ratio. Cost minimization implies

$$\alpha \frac{W_t}{P_t} L_t = (1-\alpha) r_t^k \tilde{K}_t(z) \tag{3.13}$$

3.2.3 Market clearing, monetary policy and exogenous processes

Clearing of the final goods market requires

$$\frac{\varepsilon_t^a}{\tilde{P}_t} K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t + G_t + \Psi(z_t) K_{t-1} + \Phi \tag{3.14}$$

Here, $\tilde{P}_t \equiv \left(\frac{P_t(z)}{P_t}\right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} dz$ is an index of price dispersion that captures the output loss due to asynchronized price setting. Since we analyze small fluctuations around a steady state of zero price dispersion this term can be ignored for a log-linear analysis. Monetary policy is assumed to follow an interest rate rule of the following form:

$$\ln\left(\frac{R_t}{\overline{R}}\right) = \rho \ln\left(\frac{R_{t-1}}{\overline{R}}\right) + r_{d\pi} \Delta \ln\left(\frac{\pi_t}{\overline{\pi}}\right) + r_{dy} \Delta \ln\left(\frac{Y_t}{Y_t^{pot}}\right) \tag{3.15}$$
$$+ (1-\rho)\left\{ r_\pi \ln\left(\frac{\pi_t}{\overline{\pi}}\right) + r_y \ln\left(\frac{Y_t}{Y_t^{pot}}\right)\right\} + u_t^R.$$

Here, Δ is the first difference operator, $\bar{\pi}$ the steady state gross inflation rate (assumed to be unity) and $\bar{R}$ denotes the steady state nominal interest rate. According to this interest rate rule, the nominal rate reacts to its own lag, to the first difference of inflation and of the output gap Y_t/Y_t^{pot}, defined as the ratio of actual output and potential output, as well as to inflation and output gap with coefficients $\rho, r_{d\pi}, r_{dy}, r_\pi$ and r_y, respectively. The natural rate of output Y_t^{pot} is defined as output with perfectly flexible wages and prices in the absence of the cost-push shocks $\{u_t^w, u_t^p, u_t^q\}$.[8]

The model is driven by 9 exogenous stochastic processes: The five shocks $\{\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^I, \varepsilon_t^a, G_t\}$ are modeled as following mutually uncorrelated AR(1) processes in logs with AR coefficients $\rho_b, \rho_l, \rho_I, \rho_a, \rho_G$. The stochastic wage and price markup parameters obey the equations $\ln(\lambda_{p,t} - \lambda_p) = u_t^p$ and $\ln(\lambda_{w,t} - \lambda_w) = u_t^w$, where λ_p, λ_w are the steady state values and u_t^p, u_t^w are stochastic i.i.d. innovations.[9] As in Smets and Wouters (2003) we include an innovation u_t^q into the log-linearized equation for the price of capital that is not derived from first principles. This 'external finance premium' shock drives a wedge between the risk free real rate and the expected return on physical capital. We do not include an inflation objective shock as in Smets and Wouters (2003). These authors only linearly detrend the observed nominal interest rate and inflation data, such that the detrended data exhibits a downward trend for which the time varying inflation objective may account. In this chapter, these time series are HP filtered as in Juillard, Karam, Laxton, and Pesenti (2004) eliminating the need for the inflation objective shock.

3.3 Staggered wage and price setting

The monopoly power of firms and households implies that they are price and wage setters in their respective markets. In this section we describe the baseline version of the Calvo and the sticky information model of staggered wage and price setting.

3.3.1 Calvo set up

In each period firms receive a random signal with constant probability $1 - \xi_p$ that allows them to change the price.[10] Is the signal not received, they update the posted price by indexing it to the last period inflation rate: $P_t(z) = \pi_{t-1}^{\gamma_p} P_{t-1}(z)$. Here, $\gamma_p \in [0,1]$ is a parameter allowing for partial indexation. Firms that are allowed to change their price maximize expected profits as valued by the households marginal utility in those states of the world where the price remains fixed. If we set the firms' stochastic discount factor equal to $\beta^i \Lambda_{t,t+i} = \beta^i \frac{U_{C,t+i}}{U_{C,t}}$, where $U_{C,t+i}$ is the household's marginal utility of consumption at time $t+i$, the optimization problem of the firm is defined as

$$\max_{P_t^{opt}} E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \left[\frac{P_t^{opt} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} - MC_{t+i}^{nom}}{P_{t+i}} Y_{t+i}(z) \right] \tag{3.16}$$

subject to the total demand it faces.

The first-order condition for the optimal nominal reset price P_t^{opt} is

$$E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} Y_{t+i}(z) \left(-\frac{1}{\lambda_{p,t+i}} \right) \left[\begin{array}{c} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} \frac{P_t^{opt}}{P_{t+i}} \\ -(1 + \lambda_{p,t+i}) MC_{t+i} \end{array} \right] = 0 \tag{3.17}$$

[8]Recent work by Benigno and Woodford (2004) has shown that the classification of shocks into cost-push shocks (shocks that imply that the flexible price and wage allocation is not fully desirable from a welfare point of view) and non-cost push shocks is non-trivial. In particular, with a distorted steady state any shock may become a cost-push shock. We stick to the cost-push shock definition in Smets and Wouters (2003), since these shocks are likely to be most relevant.

[9]The interpretation of these 'structural' shocks is somewhat ambiguous. They enter the log-linear Phillips curve as disturbances and are isomorphic to time variation in distortionary taxation of the firms revenue.

[10]Full derivation of Calvo wage and price setting is presented in Appendix C.

Here, $MC_t = \frac{1}{P_t} W_t^{1-\alpha} \left(r_t^k\right)^{\alpha} / \varepsilon_t^a \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$ is the real marginal cost. The aggregate price index evolves according to

$$P_t^{-1/\lambda_{p,t}} = (1-\xi_p)\left(P_t^{opt}\right)^{-1/\lambda_{p,t}} + \xi_p \left(P_{t-1}\pi_{t-1}^{\gamma_p}\right)^{-1/\lambda_{p,t}}. \tag{3.18}$$

Similarly for wage setting, we assume that households face a constant probability $1-\xi_w$ of receiving a signal that allows them to change their wage. Households that do not receive the signal to update their nominal wage index it to last period's price inflation rate: $W_t(j) = \pi_{t-1}^{\gamma_w} W_{t-1}(j)$.

The jth household that can re-optimize its wage faces the following optimization problem (irrelevant terms have been suppressed in the household's objective):

$$\max_{W_t^{opt}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -(1+\tau_{w,t+i})\varepsilon_t^b \varepsilon_t^L \frac{(l_{t+i}(j))^{1+\sigma_L}}{1+\sigma_L} \\ +(1+\tau_{w,t+i})\lambda_{t+i} \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} \frac{W_t^{opt}}{P_t} l_{t+i}(j) \end{array} \right\} \tag{3.19}$$

subject to the labor demand:

$$l_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t,$$

where $(1+\tau_{w,t}) = \frac{\lambda_{w,t}}{1+\lambda_{w,t}}$ is a wage subsidy chosen as to offset some steady state effects of monopolistic wage setting.

The wage setting optimization problem results in the following first-order condition:

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[\frac{W_t^{opt}}{P_t} \left(\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+i}/P_{t+i-1}} \right) \frac{L_{t+i} U_{C,t+i}}{1+\lambda_{w,t+i}} - L_{t+i} U_{L,t+i} \right] = 0. \tag{3.20}$$

Here, $U_{C,t+i}$ is the marginal utility of consumption and $U_{L,t+i}$ is the marginal utility of labor in those states of the world where the price remains fixed. The aggregate wage index evolves according to

$$W_t^{-1/\lambda_{w,t}} = (1-\xi_w)\left(W_t^{opt}\right)^{-1/\lambda_{w,t}} + \xi_w \left(W_{t-1}\pi_{t-1}^{\gamma_w}\right)^{-1/\lambda_{w,t}}. \tag{3.21}$$

For γ_p, γ_w equal to zero the specification of Calvo price and wage setting collapses to the form originally proposed by Calvo (1983). We will refer to this specification as to a standard Calvo.

3.3.2 Sticky information

We consider several versions of sticky information scheme. For computational reasons, all schemes truncate the infinite tail in the age distribution of information sets that is present in the original formulation of the sticky information Philips curve as suggested by Mankiw and Reis (2002). Agents set their prices and wages based on information outdated by no more than $J = 12$ periods, i.e. 3 years. This allows us to compute the model solution fast enough to estimate the parameters while still allowing for quite outdated information.[11] We also estimate the model with $J = 24$ to identify how the choice of the truncation point affects the fit of sticky information models.

We begin by outlining the basic sticky information price setting problem. Each period a randomly chosen fraction of agents updates their information set about the state of the world. We can think of the firms problem at the time of receiving the most recent information as choosing a whole sequence of prices. It chooses $J+1$ different prices all based on current information, one for

[11] The models with a higher truncation point imply very large state space and, thus, unfeasible long estimation time.

each of the following $J+1$ periods, including the current period, and applicable only if the firm will not receive more recent information in the meantime.

In the aggregate this will imply that at time t a fixed proportion ω_j^p of firms set their price $P_{t,t-j}^{opt}$ based on the state vector j periods ago.

The optimization problem is given by:

$$\max_{P_{t,t-j}^{opt}} \sum_{j=0}^{J} \omega_j^p E_{t-j} \left[\lambda_t \left(P_{t,t-j}^{opt} - P_t MC_t \right) Y_t(z) \right] \tag{3.22}$$

subject to the total demand.

The first-order condition for the price setting problem of the generic cohort j is

$$0 = E_{t-j} \left\{ -\lambda_t Y_t \lambda_{p,t}^{-1} \left(P_{t,t-j}^{opt} \right)^{-\frac{1+2\lambda_{p,t}}{\lambda_{p,t}}} P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \left[P_{t,t-j}^{opt} - (1+\lambda_{p,t}) MC_t P_t \right] \right\}. \tag{3.23}$$

Once log-linearized, this yields the condition that prices are a markup over the conditional expectation of current period nominal marginal cost.

In the standard version of the sticky information scheme, price setting is completely described by $J+1$ such conditions, the definition of aggregate price index and the $\omega_j^p,\ j=0,1,...,J$ denoting the shares of agents working with information sets outdated by j periods. The aggregate price index is given by:

$$P_t^{-1/\lambda_{p,t}} = \sum_{j=0}^{J} \omega_j^p \left(P_{t,t-j}^{opt} \right)^{-1/\lambda_{p,t}}. \tag{3.24}$$

Wage setting under the sticky information scheme is analogous to price setting.

The household's objective is given by:

$$\max_{W_{t,t-j}^{opt}} \sum_{j=0}^{J} \omega_j^w E_{t-j} \left\{ -\varepsilon_t^b \varepsilon_t^L \frac{(l_t(j))^{1+\sigma_L}}{1+\sigma_L} + \lambda_t \frac{W_{t,t-j}^{opt} l_t(j)}{P_t} \right\} \tag{3.25}$$

subject to the labor demand:

$$l_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t \tag{3.26}$$

The first-order condition for the generic cohort j is

$$E_{t-j} \left\{ \begin{array}{c} -\frac{1}{\lambda_{w,t}} \left(\frac{W_{t,t-j}^{opt}}{W_t} \right)^{-\frac{(1+\lambda_{w,t})}{\lambda_{w,t}}} L_t W_t \\ \times \left[-(1+\lambda_{w,t}) \varepsilon_t^b \varepsilon_t^L \left(\left(\frac{W_{t,t-j}^{opt}}{W_t} \right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t \right)^{\sigma_L} \frac{W_t}{W_{t,t-j}^{opt}} + \lambda_t \frac{W_t}{P_t} \right] \end{array} \right\} = 0 \tag{3.27}$$

or simply

$$E_{t-j} \left\{ -\frac{1}{\lambda_{w,t}} l_{t,t-j} \left(W_t \right)^2 / P_t \left[-(1+\lambda_{w,t}) \varepsilon_t^b \varepsilon_t^L \left(l_{t,t-j} \right)^{\sigma_L} \frac{P_t}{W_{t,t-j}^{opt}} + \lambda_t \right] \right\} = 0 \tag{3.28}$$

Once log-linearized, this condition states that households set the real wage $W_{t,t-j}^{opt}$ as a markup over the marginal rate of substitution between consumption and leisure based on time $t-j$ conditional expectation. There are $J+1$ such conditions characterizing wage setting together with the following definition for the wage index:

$$W_t^{-1/\lambda_{w,t}} = \sum_{j=0}^{J} \omega_j^w \left(W_{t,t-j}^{opt} \right)^{-1/\lambda_{w,t}} \tag{3.29}$$

To evaluate the empirical fit of sticky information models two important choices must be made. First which truncation point J to choose. Second whether to estimate all ω_j^p and ω_j^w in an unrestricted fashion or to impose some restrictions in order to reduce the number of parameters. It is often noted that the Bayesian model selection criterium that we employ in later sections has a built-in *Occam's razor* that punishes models with a large number of parameters (see e.g. Rabanal and Rubio-Ramirez (2005)). Therefore, we estimate three versions of the model, labeled parsimonious parameterization, intermediate parameterization I, intermediate parameterization II. For some selected models we also estimate a rich specification that imposes no restrictions on the shares.

These specifications are described in detail in Section 3.6.

3.4 Equilibrium and linearized equations

A rational expectations equilibrium in the Calvo model is a sequence of allocations $\{K_t, L_t, C_t, I_t, Z_t, Q_t\}_{t=0}^{\infty}$ as well as prices $\left\{r_t^k, R_t, P_t, P_t^{opt}, W_t^{opt}, W_t\right\}_{t=0}^{\infty}$ that satisfy equations (3.4)-(3.21) as well as transversality conditions for capital holdings given initial values for $\{K_{-1}, P_{-1}\}$ and the exogenous sequences $\{\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^I, \varepsilon_t^a, G_t, u_t^R, u_t^p, u_t^w, u_t^q\}_{t=0}^{\infty}$.

In the model with sticky information the rational expectations equilibrium is given by sequences of allocations $\{K_t, L_t, C_t, I_t, Z_t, Q_t\}_{t=0}^{\infty}$ as well prices $\left\{r_t^k, R_t, P_t, W_t, \left\{P_{t,t-j}^{opt}\right\}_{j=0}^{J}, \left\{W_{t,t-j}^{opt}\right\}_{j=0}^{J}\right\}_{t=0}^{\infty}$ that satisfy equations (3.4)-(3.15) $J+1$ conditions of the type (3.23) and $J+1$ conditions of the type (3.28), the wage and price indices (3.24) and (3.29) for given initial values of $\{K_{-1}, W_{0,-j}^{opt}, P_{0,-j}^{opt}, \text{ for } j = 1, ...J\}$.

We analyze the empirical properties of this model log-linearized around the steady state using the algorithm of Klein (2000). Below we collect the core equations that remain unchanged when we vary our assumptions about wage and price setting. These equations are log-linearized around a zero inflation deterministic steady state.

$$\begin{aligned}\hat{c}_t = &\frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}E_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma_c}\left(\hat{r}_t - E_t\hat{\pi}_{t+1}\right) \\ &+ \frac{1-h}{(1+h)\sigma_c}\left(\hat{\varepsilon}_t^b - E_t\hat{\varepsilon}_{t+1}^b\right)\end{aligned} \tag{3.30}$$

$$\hat{\imath}_t = \frac{1}{1+\beta}\hat{\imath}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} + \frac{\varphi}{1+\beta}\hat{Q}_t - \frac{\beta}{1+\beta}\left(E_t\hat{\varepsilon}_{t+1}^I - \hat{\varepsilon}_t^I\right) \tag{3.31}$$

$$\hat{Q}_t = (\hat{r}_t - \hat{\pi}_t) + \frac{1-\delta}{1-\delta+\bar{r}^k}E_t\hat{Q}_{t+1} + \frac{\bar{r}^k}{1-\delta+\bar{r}^k}E_t\hat{r}_{t+1}^k + u_t^q \tag{3.32}$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{\imath}_{t-1} \tag{3.33}$$

$$\hat{l}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{k}_{t-1} \tag{3.34}$$

$$\hat{y}_t = c_y\hat{C}_t + \delta k_y\hat{I}_t + g_y\hat{\varepsilon}_t^G + \bar{r}^k k_y\hat{z}_t \tag{3.35}$$

$$\hat{y}_t = \phi\hat{\varepsilon}_t^a + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{l}_t + \phi\alpha\hat{k}_{t-1} \tag{3.36}$$

$$\begin{aligned}\hat{r}_t = &\rho\hat{r}_{t-1} + (1-\rho)\left\{r_\pi\hat{\pi}_{t-1} + r_y\left(\hat{y}_t - \hat{y}_t^{pot}\right)\right\} \\ &+ r_{d\pi}\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + r_{dy}\left(\hat{y}_t - \hat{y}_t^P - (\hat{y}_{t-1} - \hat{y}_{t-1}^{pot})\right) + u_t^R\end{aligned} \tag{3.37}$$

Here, $\varphi \equiv 1/\bar{S}''$ is the inverse of the second derivative of the investment adjustment cost function evaluated at the deterministic steady state. $\psi \equiv \psi'(1)/\psi''(1)$ is the inverse of the elasticity of capital utilization cost function at the steady state. c_y, k_y, g_y are steady-state shares of consumption, capital and government spending in total output, respectively. ϕ is one plus the share of fixed cost in production.

The autoregressive shocks are determined as follows:

$$\hat{\varepsilon}_t^a = \rho_a \hat{\varepsilon}_{t-1}^a + u_t^a \tag{3.38}$$

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + u_t^b \tag{3.39}$$

$$\hat{\varepsilon}_t^G = \rho_g \hat{\varepsilon}_{t-1}^G + u_t^G \tag{3.40}$$

$$\hat{\varepsilon}_t^l = \rho_l \hat{\varepsilon}_{t-1}^l + u_t^l \tag{3.41}$$

$$\hat{\varepsilon}_t^I = \rho_I \hat{\varepsilon}_{t-1}^I + u_t^I \tag{3.42}$$

We next turn to the wage and price setting block of the model. In the Smets and Wouters (2003) specification, wage and price setting is described by the following two log-linear equations:

$$\begin{aligned} \hat{\pi}_t &= \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p\,(1+\beta\gamma_p)}\left(\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \frac{\lambda_p}{(1+\lambda_p)}\hat{\lambda}_{p,t}\right) \\ &\quad + \frac{\beta}{(1+\beta\gamma_p)}\hat{\pi}_{t+1} + \frac{\gamma_p}{(1+\beta\gamma_p)}\hat{\pi}_{t-1} \end{aligned} \tag{3.43}$$

$$\hat{w}_t = \frac{(1-\xi_w)}{\xi_w(1+\beta)}\left[\begin{array}{c} \frac{\beta\xi_w}{(1-\xi_w)}E_t\hat{w}_{t+1} + \frac{\xi_w}{(1-\xi_w)}\hat{w}_{t-1} + \frac{\beta\xi_w}{(1-\xi_w)}E_t\hat{\pi}_{t+1} \\ -\left(\frac{\xi_w+\gamma_w\beta\xi_w}{(1-\xi_w)}\right)\hat{\pi}_t + \frac{\gamma_w\xi_w}{(1-\xi_w)}\hat{\pi}_{t-1} \\ +\frac{(1-\xi_w\beta)}{(1+\frac{1+\lambda_w}{\lambda_w}\sigma_L)}\left[\sigma_L\hat{l}_t + \hat{\varepsilon}_t^l + \frac{\sigma_C}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) - \hat{w}_t + \frac{\lambda_w}{1+\lambda_w}\hat{\lambda}_{w,t}\right] \end{array}\right] \tag{3.44}$$

3.4.1 Log-linearized versions of sticky information price and wage equations

We define the log-linearized optimality conditions in terms of stationary variables $\hat{w}_{t,t-j}^+ \equiv \frac{d(\frac{W_{t,t-j}^{opt}}{W_{t-j}})}{(\frac{W_{t,t-j}^{opt}}{W_{t-j}})}$ and $\hat{p}_{t,t-j}^+ \equiv \frac{d(\frac{P_{t,t-j}^{opt}}{P_{t-j}})}{(\frac{P_{t,t-j}^{opt}}{P_{t-j}})}$:

$$\left(\frac{\sigma_L(1+\lambda_w)}{\lambda_w}+1\right)\hat{w}_{t,t-j}^+ = E_{t-j}\left[\begin{array}{c} \left(\frac{\sigma_L(1+\lambda_w)}{\lambda_w}+1\right)\sum_{k=0}^{J}\hat{\pi}_{t-k}^w \\ +\sigma_L\hat{l}_t - \hat{w}_t + \frac{\sigma_C}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) + \hat{\varepsilon}_t^b + \frac{\lambda_w}{(1+\lambda_w)}\hat{\lambda}_{w,t} \end{array}\right] \tag{3.45}$$

$$p_{t,t-j}^+ = E_{t-j}\left[\sum_{k=0}^{J}\hat{\pi}_{t-k} + \widehat{mc}_t + \frac{\lambda_p}{(1+\lambda_p)}\hat{\lambda}_{p,t}\right] \tag{3.46}$$

, where $\hat{\pi}_t^w = \hat{\pi}_t + (\hat{w}_t - \hat{w}_{t-1})$. The log-linearized price and wage indices are for standard sticky information setting given by[12]

$$0 = \sum_{j=0}^{J} \omega_j^w \hat{w}_{t,t-j}^{+} - \sum_{k=1}^{J} \sum_{j=k}^{J-1} \omega_j^w \hat{\pi}_{t-k}^w, \tag{3.47}$$

$$0 = \sum_{j=0}^{J} \omega_j^p \hat{p}_{t,t-j}^{+} - \sum_{k=1}^{J} \sum_{j=k}^{J-1} \omega_j^p \hat{\pi}_{t-k}. \tag{3.48}$$

3.5 Empirical analysis

We apply the system-based Bayesian approach which allows us to fit the solved DSGE model to a vector of aggregate time series. Simultaneous estimation of all equations also allows for unambiguous interpretation of structural shocks. This approach has been discussed by many authors in the literature in the last few years, e.g. Schorfheide (2000), Smets and Wouters (2003), and Lubik and Schorfheide (2004). For details see also Chapter 2 of this thesis.

3.5.1 The data

The models considered here are estimated with quarterly Euro area data for the period 1970:Q1-2003:Q4.[13] The data set we employ was first constructed by Fagan, Henry, and Mestre (2001) for the Area Wide Model database (see Appendix D for details on the sources and properties of the data). The time series from this database have been used by Smets and Wouters (2003) and Adolfson, Laseen, Linde, and Villani (2005a), to mention only a few of the wide range of estimated DSGE models for the euro area. Adolfson, Laseen, Linde, and Villani (2005a) assume a common trend in the real variables while estimating the model. This assumption might be, however, not satisfied empirically, see Del Negro, Schorfheide, Smets, and Wouters (2004). To address this issue we eliminate the trend in both real and nominal variables by applying HP filter with a high smoothing parameter ($\lambda = 10,000$) as advocated by Juillard, Karam, Laxton, and Pesenti (2004) or Detken and Smets (2004).[14] Using detrended data becomes also important as some nominal variables trend up or downward over the whole sample period, which could additionally bias the estimates. Since there is no consistent Euro area measure of hours worked, the model is estimated on employment series EM_t which is likely to respond more sluggishly to shocks. As in Smets and Wouters (2003) we work with the following auxiliary equation for employment

$$\Delta\widehat{em}_t = \beta E_t \Delta\widehat{em}_{t+1} + \frac{(1-\xi_L)(1-\beta\xi_L)}{\xi_L}(\hat{l}_t - \widehat{em}_t) \tag{3.49}$$

In each period, only a fraction ξ_L of firms can adjust employment to the desired total labor input and the difference is take up by unobserved hours worked per employer. Our models explain the same variables as the model by Smets and Wouters (2003) and the vector of observables y_t^{obs} includes: real GDP, consumption, investment, CPI inflation the real wage, employment and the nominal interest rate.[15] Hence, the measurement equation has the following form:

[12] Assuming that the weights ω_j^p add up to unity, it is possible to make the model stationary by dividing the optimal price by the price level j periods ago.

[13] The data from the 1970's are used as a training sample for initialization of the Kalman Filter.

[14] Smets and Wouters (2003) remove a linear trend from the data.

[15] Note that Smets and Wouters (2003) allow for a unit root in the inflation target which has the similar effect on model dynamics as a direct detrending of inflation series.

$$y_t^{obs} = \begin{bmatrix} \hat{y}_t \\ \hat{c}_t \\ \hat{\imath}_t \\ \hat{\pi}_t \\ \hat{w}_t \\ \widehat{em}_t \\ \hat{r}_t \end{bmatrix} \tag{3.50}$$

In this chapter we apply Bayesian estimation methods as outlined in Chapter 2. An optimization algorithm 'csminwel' is used to obtain an initial estimate of the mode of the posterior parameter distribution. To check that the optimization routine converges to the same value we start it from a number of various initial values before launching the MCMC chains. The latter is based on two chains of 50.000 - 100.000 draws. Convergence of the algorithm is checked via PSRFs and appears by and large satisfactory.[16]

3.5.2 Calibrated parameters and priors

The Bayesian approach facilitates the incorporation of prior information from other macro as well as micro studies in a formalized way. In specifying the prior density $p(\theta)$ we assume that all parameters are independently distributed of each other, i.e. $p(\theta) = \prod_{i=1}^{N} p_i(\theta_i)$, which allows for a straightforward evaluation of the posterior. The set of priors is heavily influenced by Smets and Wouters (2003). Prior to the estimation we have checked whether the models simulated with the mean of the prior distribution can roughly match the volatility and persistence of the data. As the estimation is based on HP-detrended series, which are slightly less volatile than linearly detrended series employed in Smets and Wouters (2003), the priors for the standard deviations of structural shocks have been in general scaled down.

The full set of priors can be found in Table 3.1. In particular, the Calvo parameters ξ_p and ξ_w are assumed to be beta distributed with the mean of 0.75 and standard deviation of 0.05. The parameters standing for the degree of indexation γ_p and γ_w are set to be beta distributed with the mean of 0.75 and standard deviation of 0.15.

The sticky information model is characterized by $J+1$ structural parameters, corresponding to the shares of agents ω_j using information sets outdated by j periods, where $j = 0, ..., J$.[17] Estimating all of these parameters would imply deterioration of model's out-of-sample fit and would result in penalizing the whole class of sticky information models. Therefore, we reduce the dimensionality of the parameter space by imposing some functional forms on the relation between the ω_j for $j = 0, ...J$. In the baseline model we assume that the fraction of agents using information outdated by j periods decays geometrically, i.e $\omega_j = c(1-\tilde{\xi})\tilde{\xi}^j$ for $j = 0, 1, ..., J$. Here, $\tilde{\xi} \in (0,1)$ is the single parameter to be estimated and c ensures that the shares add up to unity.[18] This specification is analogous to the Calvo wage and price setting model, where the share of agents whose price was last updated j periods ago also declines geometrically. Furthermore, Reis (2005, Proposition 7) has shown that under certain assumptions the share of agents not having planned for j periods follows the exponential distribution. For large J this parameterization approximates the distribution advocated by Reis (2005). For the prior distribution of the information structure we choose fairly loose densities defined on the interval $(0,1)$, leaving an important role for the data. We assume the same priors for the parameters describing sticky information wage as well as price setting. In particular, the parameter $\tilde{\xi}$ is assumed to be prior beta distributed with the mean

[16] As in any applied work with MCMC chains, there is always a chance that convergence appears to have occurred when in fact it has not.

[17] In the subsequent paragraphs we drop the distinction between the ω_j^p describing price setting and ω_j^w describing wage setting and use the simpler notation ω_j to refer to both.

[18] I.e. $c^{-1} = \sum_{i=0}^{J}(1-\tilde{\xi})\tilde{\xi}^j$.

of 0.8 and standard deviation of 0.15. The prior distributions of the all parameters are presented in the Table 3.1 below.

parameter	symbol	type	mean	std/df
investment adj. cost	S''	norm	4	1.5
consumption utility	σ_c	norm	1	0.375
habit persistence	h	beta	0.7	0.1
Calvo wage stickiness	ξ_w	beta	0.75	0.05
Calvo price stickiness	ξ_p	beta	0.75	0.05
information rigidity prices	$\tilde{\xi}_p$	beta	0.8	0.15
information rigidity wages	$\tilde{\xi}_w$	beta	0.8	0.15
labor utility	σ_l	norm	2	0.75
indexation wages	γ_w	beta	0.75	0.15
indexation prices	γ_p	beta	0.75	0.15
capital utilization adj. cost	ϕ	norm	0.2	0.075
fixed cost	ψ	norm	1.45	0.125
Calvo employment stickiness	ξ_L	beta	0.5	0.15
response to inflation	r_π	norm	1.7	0.1
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1
interest rate smoothing	ρ	beta	0.8	0.1
response to output gap	r_y	norm	0.125	0.05
response to diff. output gap	r_{dy}	norm	0.063	0.05
persistence tech. shock	ρ_a	beta	0.85	0.1
persistence preference shock	ρ_b	beta	0.85	0.1
persistence gov. spending. shock	ρ_g	beta	0.85	0.1
persistence labor supply shock	ρ_L	beta	0.85	0.1
persistence investment shock	ρ_I	beta	0.85	0.1
stdv. productivity shock	σ_a	invg	0.4	2
stdv. preference shock	σ_b	invg	0.02	2
stdv. gov. spending shock	σ_g	invg	0.15	2
stdv. labor supply shock	σ_L	invg	1	2
stdv. equity premium shock	σ_q	invg	0.2	2
stdv. monetary shock	σ_R	invg	0.15	2
stdv. investment shock	σ_I	invg	0.03	2
stdv. price markup shock	σ_π	invg	0.03	2
stdv. wage markup shock	σ_w	invg	0.3	2

Table 3.1: Prior parameter distribution

As in Smets and Wouters (2003) some of the structural parameters are calibrated, as they mainly influence the steady state and are not or only weakly identifiable from log-linearized equations. Following Smets and Wouters (2003) we calibrate $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.3$ and $\lambda_w = 0.5$.

3.6 Results for the baseline model

In this section we discuss the estimation results. Before turning to the comparisons of Calvo and sticky information models, we validate a version of the seminal DSGE model by Smets and Wouters (2003), which serves as a base for the analysis in this chapter. Subsequently, we report posterior odds ratio, root mean squared in sample error as well as the match of the models autocorrelations with those of the actual data, to compare the fit of the models.

3.6.1 Empirical properties of the seminal closed-economy DSGE model

In this subsection we compare stylized facts implied by the actual data to those of simulated data from the estimated seminal DSGE model. First, we compare vector autocovariance functions in the model and the data (see, e.g., Fuhrer and Moore (1995)), and second, we compare some selected second-order moments as is standard in the RBC literature. The empirical cross-covariances are based on a VAR(1) estimated on the same data sample as the DSGE model. The model-based cross-covariances are likewise calculated by estimating a VAR(1) on 1000 random samples generated from the DSGE model. Of each sample we utilize 95 observations. The empirical and model based cross-covariances are reported in Figure 3.13 on page 84 in Appendix. The unconditional second moments are calculated directly from data and compared to the theoretical moments implied by the recursive representation of the DSGE model (see Section 2.4).[19] It should be underlined that, as opposed to the calibration exercise, the likelihood-based estimation procedure applied here seeks to match *all* second order moments of the model to the data. Finally, we report the comparisons of in-sample root mean square errors calculated for the DSGE model and a VAR model, as is done in Smets and Wouters (2003).

From Tables 3.2 and 3.3 on the following page we see that the seminal DSGE model fits the data reasonably well. The fit is especially good for variables which are at the center of analysis of this chapter, i.e. the real wage and the inflation. In particular, Table 3.2 illustrates that the persistence of real wages and inflation implied by the model is very similar to the persistence found in the data. Still, a few properties are not matched satisfactorily by the model. First, the correlation between real variables and interest rate is not strong enough. Second, the model tends to overpredict the volatility of output, consumption and investment.

When it comes to comparing root mean squared errors, see Table 3.4 on page 51, the model's in-sample performance is close to that of VAR(1) model. It should be underlined that, relative to a VAR, the seminal DSGE model, which is here estimated with HP-filtered data, performs raughty the same as the (estimated with linearly detrended data) model of Smets and Wouters (2003).

Variable	std (data)	std (model)	first order autocorr. (data)	first order autocorr. (model)
$\hat{c}_t$	1.47	(2.23, 3.57, 6.04)	0.94	(0.96, 0.98, 0.99)
$\hat{y}_t$	1.36	(2.36, 3.24, 4.66)	0.95	(0.96, 0.98, 0.99)
$\hat{\imath}_t$	4.00	(5.45, 7.17, 8.90)	0.95	(0.97, 0.98, 0.99)
$\widehat{em}_t$	1.21	(0.93, 1.37 , 2.42)	0.98	(0.91, 0.98, 0.99)
$\hat{r}_t$	0.34	(0.22, 0.26, 0.31)	0.93	(0.83, 0.87, 0.90)
$\hat{w}_t$	1.09	(1.10, 1.40, 1.83)	0.92	(0.91, 0.94, 0.96)
$\hat{\pi}_t$	0.33	(0.26, 0.30, 0.36)	0.63	(0.45, 0.57, 0.69)

Table 3.2: Comparison of second moments

[19]Notwithstanding the subjective choice of the prior uncertainty, the exercise above demonstrates some of advantages of the Bayesian approach, where the small sample distributions can be obtained for all model statistics.

cross-correlations (data)							
	$\hat{c}_t$	$\hat{y}_t$	$\hat{\imath}_t$	$\widehat{em}_t$	$\hat{r}_t$	$\hat{w}_t$	$\hat{\pi}_t$
$\hat{c}_t$	1						
$\hat{y}_t$	0.92	1					
$\hat{\imath}_t$	0.92	0.95	1				
$\widehat{em}_t$	0.91	0.90	0.90	1			
$\hat{r}_t$	0.51	0.50	0.45	0.61	1		
$\hat{w}_t$	0.54	0.44	0.38	0.48	0.71	1	
$\hat{\pi}_t$	0.28	0.28	0.25	0.41	0.71	0.41	1
cross-correlations (model)							
	$\hat{c}_t$	$\hat{y}_t$	$\hat{\imath}_t$	$\widehat{em}_t$	$\hat{r}_t$	$\hat{w}_t$	$\hat{\pi}_t$
$\hat{c}_t$	1						
$\hat{y}_t$	(0.59, 0.87, 0.95)	1					
$\hat{\imath}_t$	(0.59, 0.77, 0.84)	(0.77, 0.89, 0.92)	1				
$\widehat{em}_t$	(-0.04, 0.71, 0.91)	(0.59, 0.79, 0.93)	(0.35, 0.58, 0.72)	1			
$\hat{r}_t$	(-0.40, -0.26, -0.06)	(-0.25, -0.09, 0.13)	(-0.34, -0.15, 0.05)	(-0.12, 0.01, 0.23)	1		
$\hat{w}_t$	(0.24, 0.36, 0.57)	(0.34, 0.48, 0.67)	(0.41, 0.54, 0.69)	(-0.09, 0.05, 0.16)	(-0.03, 0.20, 0.42)	1	
$\hat{\pi}_t$	(-0.18, -0.07, 0.07)	(-0.09, 0.02, 0.22)	(-0.10, 0.03, 0.18)	(-0.06, 0.03, 0.27)	(0.40, 0.52, 0.67)	(0.00, 0.17, 0.36)	1

Table 3.3: Comparison of second moments (cont.)

Root mean squared error	$\hat{c}_t$	$\hat{y}_t$	$\hat{i}_t$	$\widehat{em}_t$	$\hat{r}_t$	$\hat{w}_t$	$\hat{\pi}_t$
Calvo SW	0.52	0.51	1.26	0.25	0.11	0.41	0.25
VAR(1)	0.41	0.41	1.07	0.12	0.11	0.38	0.22

Table 3.4: In-sample accuracy of the seminal DSGE model

3.6.2 Model comparisons

In a Bayesian framework, model comparisons are based on posterior model probabilities, denoted by $p(Y_T|M_i) = \int p(Y_T|\theta, M_i)p(\theta|M_i)d\theta$ for some model M_i. It is important to note that the marginal likelihood is a predictive density based on the prior distribution as a summary of the parameter uncertainty. No data is consumed to estimate the parameters of the model when computing the marginal likelihood. This makes it possible to interpret the marginal likelihood as a measure of out-of-sample predictive performance, rather than in-sample fit. Given alternative models with prior probabilities $p(M_i)$ and $p(M_j)$, the posterior odds ratio is given by $PO_{i,j} = \frac{p(Y_T|M_i)p(M_i)}{p(Y_T|M_j)p(M_j)}$. When all models are assigned equal prior probabilities, i.e. $p(M_j) = p(M_i)$, then the posterior odds boil down to the ratios of the marginal likelihood, i.e. the Bayes factor. The Bayes factor summarizes the evidence contained in the data in favor of one model as opposed to another. Fernandez-Villaverde and Rubio-Ramirez (2004a) provide an extensive discussion of the advantages of the Bayes factor for model comparison over likelihood ratio. They also specify the sense in which the Bayes factor is a consistent selection device even when the models are non-nested or misspecified.

Table 3.5 reports marginal densities of the Calvo model and versions of the sticky information model.

Model	description	log of marginal density
Calvo	with indexation	-307.8
Calvo	no indexation	-304.1
Sticky information	J = 12	-416.5
Sticky information	J = 24	-412.0^*

Table 3.5: Log of marginal densities: baseline models. The asterisks (*) denotes that the marginal density has been calculated via the Laplace Approximation.

The model with the highest marginal density is the Calvo model without indexation. Compared with the standard sticky information model with $J = 12$ quarters of information lags, there is overwhelming evidence in favor of the Calvo model. Since the difference in log-marginal likelihood is 112.4, one would require a prior probability for the sticky information model that is $\exp(112.4) = 6.52 \times 10^{48}$ larger than the prior for the Calvo model to favor the sticky information model based on posterior odds. Fernandez-Villaverde and Rubio-Ramirez (2004a) point out that a log-difference of 7 is often used as bound for DNA testing in forensic science. Hence, there is overwhelming evidence in favor of the Calvo model.

Summary statistics for the posterior distribution of the parameters can be found in Table 3.6. The table shows that the structural parameters not related to wage and price setting are estimated at similar posterior modes and within similar 90% posterior intervals. The models examined here, contrary to those used in the literature to compare Calvo and sticky information frameworks, include besides sticky prices and wages many other frictions, so they are less likely to be misspecified in some other dimensions. This finding has also been obtained by Andres, Lopez-Salido, and Nelson (2005) in their comparison of sticky prices and sticky information models. Notable differences are found in the estimated volatility of the disturbances entering the wage and price setting equations, i.e the standard deviations of the labor supply shock, the wage markup shock and the price markup shock. This issue is interpreted later on.

		Calvo			Mankiw-Reiss J=12		
parameter		mode	90% post. interval	$\hat{R}$	mode	90% post. interval	$\hat{R}$
investment adj. cost	S''	4.054	(0.562, 2.954, 4.724)	1.041	5.615	(3.806, 5.691, 7.547)	1.051
consumption utility	σ_c	0.954	(0.750, 1.179, 1.599)	1.002	1.446	(0.987, 1.423, 1.889)	1.003
habit persistence	h	0.476	(0.358, 0.491,0.625)	1.085	0.516	(0.422, 0.545, 0.658)	1.091
labor utility	σ_l	2.386	(1.854, 2.741, 3.690)	1.004	2.635	(1.856, 2.868, 3.893)	1.009
indexation wages	γ_w	0.482	(0.201, 0.409, 0.604)	1.040			
indexation prices	γ_p	0.548	(0.257, 0.393, 0.544)	1.000			
Calvo wages	ξ_w	0.714	(0.624, 0.699, 0.771)	1.004			
Calvo prices	ξ_p	0.875	(0.878, 0.899, 0.923)	1.002			
information rigidity prices	$\tilde{\xi}_p$				0.998	(0.998, 0.998, 0.998)	1.003
information rigidity wages	$\tilde{\xi}_w$				0.856	(0.802, 0.853, 0.904)	1.006
capital util. adj. cost	ϕ	0.346	(0.252, 0.358, 0.471)	1.002	0.370	(0.269, 0.370, 0.477)	1.004
fixed cost	ψ	1.778	(1.384, 1.558, 1.723)	1.000	1.735	(1.575, 1.741, 1.897)	1.003
Calvo employment	ξ_L	0.200	(0.628, 0.689, 0.752)	1.000	0.159	(0.081, 0.208, 0.329)	1.002
response to inflation	r_π	1.683	(1.503, 1.681, 1.853)	1.014	1.660	(1.482, 1.650, 1.818)	1.023
response to diff. inflation	$r_{d\pi}$	0.150	(0.076, 0.146, 0.215)	1.010	0.145	(0.100, 0.159, 0.222)	1.025
interest rate smoothing	ρ	0.927	(0.939, 0.959, 0.981)	1.000	0.948	(0.908, 0.936, 0.965)	1.002
response to output gap	r_y	0.051	(0.040, 0.109, 0.176)	1.036	0.043	(0.013, 0.046, 0.081)	1.021
response to diff. output gap	r_{dy}	0.136	(0.164, 0.197, 0.227)	1.007	0.103	(0.075, 0.118, 0.158)	1.009
persistence techn. shock	ρ_a	0.926	(0.874, 0.910, 0.948)	1.007	0.845	(0.799, 0.843, 0.891)	1.012
persistence preference shock	ρ_b	0.745	(0.361, 0.493, 0.623)	1.042	0.712	(0.545, 0.669, 0.792)	1.051
persistence gov. spending. shock	ρ_g	0.976	(0.851, 0.898, 0.943)	1.003	0.979	(0.931, 0.965, 0.999)	1.007
persistence labor supply shock	ρ_L	0.725	(0.980, 0.988, 0.997)	1.004	0.833	(0.785, 0.864, 0.947)	1.012
persistence investment shock	ρ_I	0.267	(0.090, 0.308, 0.715)	1.072	0.325	(0.190, 0.309, 0.435)	1.091
stdv. productivity shock	σ_a	0.277	(0.339, 0.422, 0.504)	1.035	0.285	(0.250, 0.299, 0.339)	1.031
stdv. preference shock	σ_b	0.088	(0.116, 0.148, 0.185)	1.003	0.098	(0.075, 0.109, 0.143)	1.002
stdv. gov. spending shock	σ_g	0.321	(0.277, 0.321, 0.361)	1.004	0.320	(0.284, 0.321, 0.360)	1.008
stdv. labor supply shock	σ_L	2.399	(0.852, 1.229, 1.597)	1.015	4.547	(2.463, 4.242, 5.903)	1.028
stdv. equity premium shock	σ_q	0.093	(0.046, 0.279, 0.848)	1.090	0.093	(0.046, 0.192, 0.364)	1.091
stdv. monetary shock	σ_R	0.066	(0.037, 0.057, 0.077)	1.000	0.052	(0.039, 0.060, 0.080)	1.005
stdv. investment shock	σ_I	0.514	(0.173, 0.504, 0.729)	1.006	0.517	(0.441, 0.529, 0.619)	1.004
stdv. price markup shock	σ_π	0.461	(0.392, 0.692, 0.962)	1.050	0.036	(0.025, 0.037, 0.050)	1.003
stdv. wage markup shock	σ_w	0.630	(0.287, 0.713, 1.152)	1.001	0.936	(0.647, 1.034, 1.396)	1.004

Table 3.6: Posterior distribution: Calvo model with indexation and truncated Mankiw-Reiss model with $J = 12$.

We now turn to what the estimated model implies for information rigidity in price and wage setting. The estimated population shares of agents working with information outdated by j periods, denoted by ω_j, are proportional to the parameter $\tilde{\xi}$ to the power j. Here $\tilde{\xi}$ is the only estimated parameter of the truncated sticky information scheme governing the speed with which the shares decline in age. For $\tilde{\xi}$=0, all firms act with current information, for larger $\tilde{\xi}$ the shares decline more slowly over time indicating the presence of more information rigidity. Figure 3.1 plots the age distribution of information sets used by price and wage setters according to the prior of the $\tilde{\xi}$ parameter and according to posterior mode of $\tilde{\xi}$.

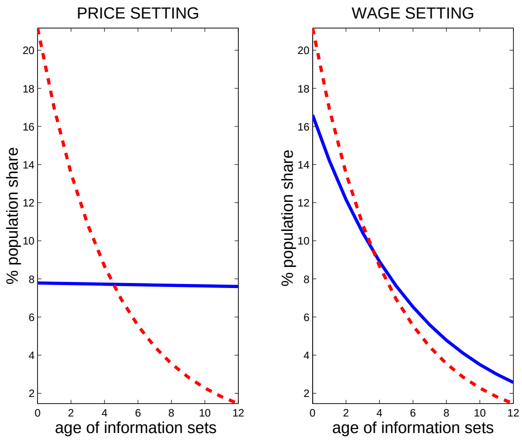

Figure 3.1: Age distributions of information sets $J = 12$. Dashed line - distribution according to prior. Solid line - distribution according to posterior.

For price setting, the posterior mode of $\tilde{\xi}_p$ is 0.998. As can be seen from the graph, this implies that the age distribution of information sets follows a Taylor type pattern with age invariant shares rather a Calvo type pattern with geometrically declining shares. For wage setting, the posterior mode of $\tilde{\xi}_w$ is 0.856, generating a Calvo type pattern. Apparently, the data favors the maximum possible amount of information rigidity for price setting.

Next, we analyze the impulse responses generated from the models with the parameters evaluated at the posterior mode. Figure 3.2 shows that the estimated sticky information model is capable of generating a delayed and hump shaped response of output and inflation to monetary shocks that is a stylized fact from identified VARs. The work of Christiano, Eichenbaum, and Evans (2005) finds that inflation takes as much as 3 years to reach its peak response to a monetary shock. As noted by Trabandt (2005), the Calvo model with indexation in price setting can also generate this basic pattern. The sticky information model generates the strongest response of inflation 12 quarters after the shock when all agents have updated their information set. Due to the endogenous propagation mechanism in the model, the effect of the monetary shock on inflation is propagated beyond the duration of information rigidities.

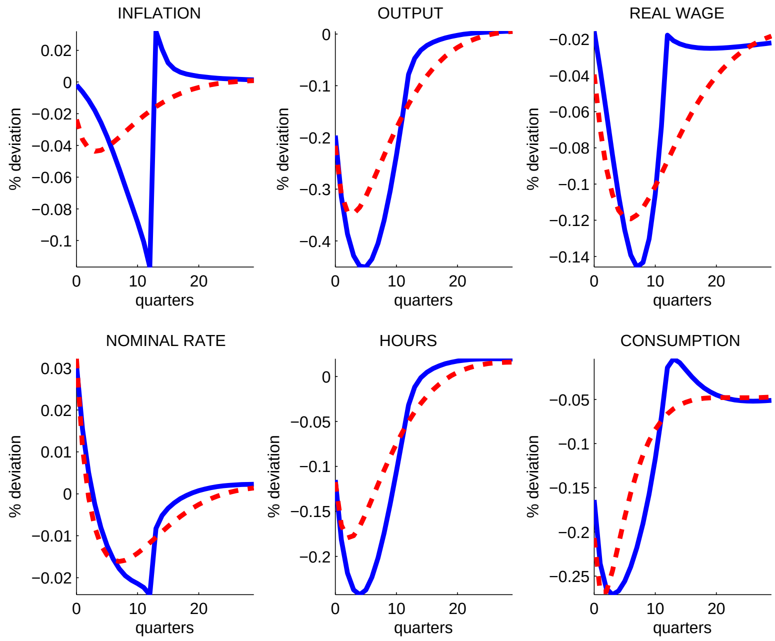

Figure 3.2: Impulse response to monetary shock. Solid line - sticky information model J=12, dashed line - Calvo model with indexation. Impulse responses are calculated at the posterior mode.

The full set of impulse responses as well as forecast error variance decomposition can be found in the appendix to this chapter. The impulse responses to most other shocks do not add much more insight into the difference between sticky information models and sticky price models except for the price and wage markup shocks. For these shocks the impulse response are drastically different. Figure 3.3 displays the response to price markup shock, i.e. a stochastic variation in the elasticity of substitution of differentiated goods in the CES aggregator.

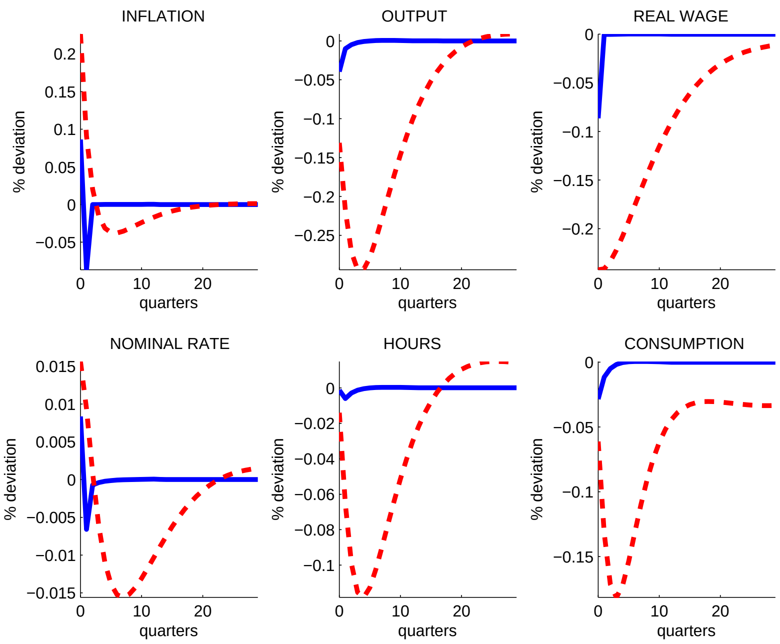

Figure 3.3: Impulse response to markup shock. Solid line - sticky information model J=12, dashed line - Calvo model. Impulse responses are calculated at the posterior mode.

Two observations emerge. First, the uncorrelated markup shock induces a persistent decline in most variables for the Calvo model. However, in the sticky information model these variables return to their steady state values very rapidly. Second, inflation and the nominal rate oscillate for two periods following the shock, inducing a negative first-order autocorrelation of inflation conditional on this shock. It is well known from Smets and Wouters (2003) that the price markup shock is the dominant shock driving inflation in their model. This is confirmed in Table 3.23 on page 81 in the appendix for our estimation of the Calvo model. At horizon 1 and 4 quarters, markup shocks account for 95 and 70 percent, respectively, of the variance in the forecast error of inflation. Note further from Table 3.5 that estimating a restricted Calvo model without the markup shock reduces the marginal likelihood drastically.

Inflation is empirically a volatile process, but responds sluggishly to marginal cost in the model. Hence, significant markup shocks are needed to match the volatility of inflation. This, however, creates problems for the sticky information model.[20] More volatile price markup shocks help to match the standard deviation of inflation in the data, but come at the cost of reducing the persistence of inflation and the real wage - as can be seen from the impulse responses in Figure 3.3. The following table summarizes the the match of the model with respect to standard deviation and autocorrelation of inflation and the real wage.

variable	data	Calvo	SI J=12	SI J=24
stdv inflation	0.33	0.31	0.3	0.4
stdv wage	1.09	1.27	1.19	1.84
AR(1) inflation	0.63	0.59	0.46	0.17
AR(1) wage	0.92	0.93	0.35	0.67

Table 3.7: Standard deviations and autocorrelation of the models and the data

The table shows that both version of the estimated sticky information models fail to match the persistence present in the inflation and real wage series, which the Calvo model matches reasonably well. The sticky information model with $J = 24$ implies also much to volatile real wage series. Furthermore, we report how well the different models replicate the autocorrelations of other times series used for estimation. Figure 3.4 displays the autocorrelations $corr(X_t, X_{t-j})$ for a generic variable X. where the abscissa denotes the displacement $j = 0, ..., 8$. The figure shows that the sticky information model replicates the autocorrelations of the data roughly as well as the Calvo model with one major exception: It fails dramatically to generate the serial correlation of the real wage series.

[20] These issues do not arise when one compares the Calvo model with the Mankiw Reis model only along a few selected dimension. Our likelihood based method considers all model implications in an environement with many shocks as opposed to say matching inflation persistence.

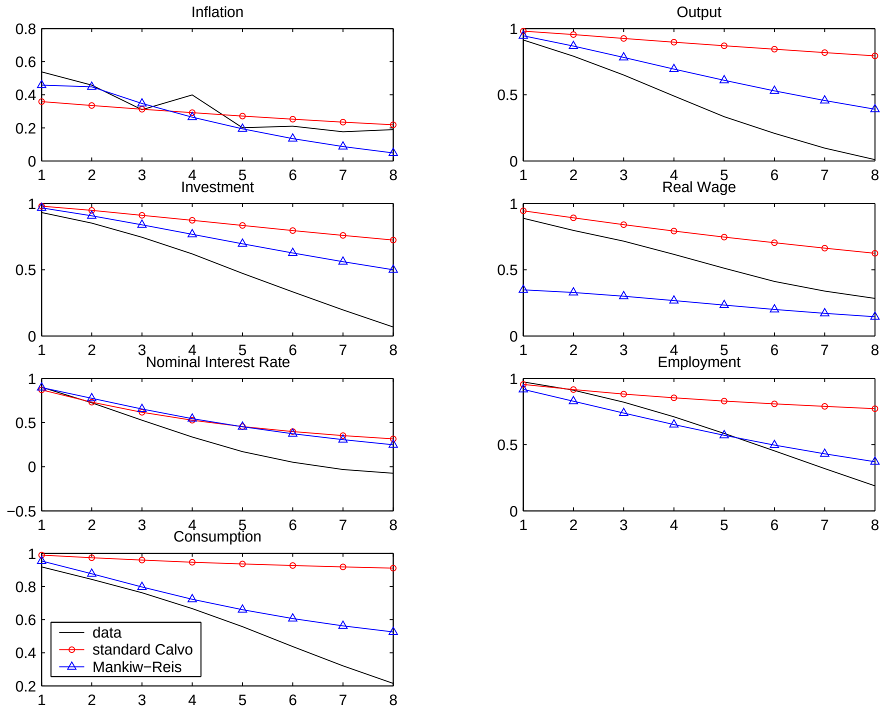

Figure 3.4: Replicated autocorrelations

This inadequate match of the real wage series is further confirmed by comparing the root mean squared in sample error that is displayed in Table 3.8.

Root mean squared error	$\hat{c}_t$	$\hat{y}_t$	$\hat{i}_t$	$\widehat{em}_t$	$\hat{r}_t$	$\hat{w}_t$	$\hat{\pi}_t$
Calvo SW	0.52	0.51	1.26	0.25	0.11	0.41	0.25
sticky information J=12	0.52	0.52	1.27	0.29	0.11	0.86	0.24
sticky information J=24	0.52	0.53	1.26	0.22	0.11	1.00	0.25

Table 3.8: Root mean squared error: Calvo, standard sticky information

The sticky information models have a similar RMSE to the Calvo model for all variables except for the real wage. The RMSE of the real wage series is roughly twice as high for the sticky information model than for the Calvo model.

Finally one may ask whether there is support for sticky information schemes in price setting while wage setting is modeled as following the Calvo scheme and vice versa. In other words, is it sticky information in wages or sticky information in prices that accounts for the bad fit? To this end, Table 3.9 presents the marginal density of so called mixed models. In these mixed models only price setters follow sticky information and wage setters Calvo or vice versa.

Model	description	log of marginal density
Calvo	without price markup shock	-359.9
Calvo	without wage markup shock	-361.9
mixed: Calvo Wages, SI prices		-348.1
mixed: Calvo Wages, SI prices	without price markup shock	-361.0
mixed: SI Wages, Calvo prices		-320.7
mixed: SI Wages, Calvo prices	without wage markup shock	-344.5

Table 3.9: Log of marginal densities: mixed models

As can be seen from that table, the mixed models per se do not attain higher marginal densities than the Calvo model. However, once one estimates the Calvo model without price or wage markup shocks and compares this model to mixed models without these markup shocks, the picture looks more favorable for sticky information. For instance, the pure Calvo model without price markup shocks attains roughly the same marginal density as the model with Calvo wage setting and sticky information price setting absent markup shocks. Hence, it appears that including or excluding markup shocks is an important decision for the relative model performance. Another important decision is whether to allow these shocks to be autocorrelated or not. We proceed by estimating models with price and wage markup shocks as they are important sources of inflation dynamics and overall model fit.

3.6.3 Extension I: Increasing the truncation point

We have further examined whether the remarkably weak performance of the standard sticky information model is caused by choosing the truncation point $J = 12$ too small. For $J = 12$ the average age of information sets for price setting implied by $\tilde{\xi}_p = 0.998$ is roughly 7 quarters.[21] This compares to an estimated average age of price contracts of 8 quarters in the Calvo model. Hence, one may want to allow more outdated information in the Mankiw-Reis type model.[22] Therefore, the model is re-estimated with $J = 24$. Trabandt (2005) has shown that the recursive equilibrium law of motion in his sticky information model does not change by more than an arbitrarily small tolerance criterion if further lags are added beyond the 20th lag. Hence, the choice of $J = 24$ is expected to be a reasonable approximation to the infinite lag inherent in Mankiw and Reis original sticky information model. Note that we include many more lags in the model than other studies that also need to decide about a truncation point. For instance, Andres, Lopez-Salido, and Nelson (2005) truncate after 3 periods. Laforte (2005) considers information sets that are outdated by at most 9 quarters. With such a large number of lags it becomes infeasible to perform a full blown Bayesian analysis on a desktop PC. Therefore, no MCMC chains are run and we restrict ourselves to characterizing the posterior mode via optimization algorithms.

Increasing the maximum lag for outdated information sets from $J = 12$ to $J = 24$ improves the fit of the sticky information model only slightly. The log of the marginal density now increases from -416 to -412 as computed via the Laplace approximation, but is still far off from the marginal density of for the Calvo model with indexation of -307.

The estimation results are presented in Table 3.10, for comparison we again provide the estimates of the sticky information model with $J = 12$. Again the estimates of the structural parameters unrelated to price and wage setting are by and large similar across models. The posterior mode of the parameters describing information rigidity in the sticky information model are now estimated at 0.90 for price setting and 0.52 for wage setting. This gives rise to the age distribution of information sets as depicted in Figure 3.5.

[21] This is easy to see: Note that $\tilde{\xi} \sim 1$ implies that the 13 age cohorts have roughly equal population share. Hence the average age is roughly similar to the average age formula for Taylor pricing: $1/n \sum_{i=1}^{N} i = (N+1)/2$. For $N = 13$ this yields 7.

[22] Another possible way to compare the models is to also truncate the Calvo sticky price and wage model at $J = 12$ thereby leveling the playing field.

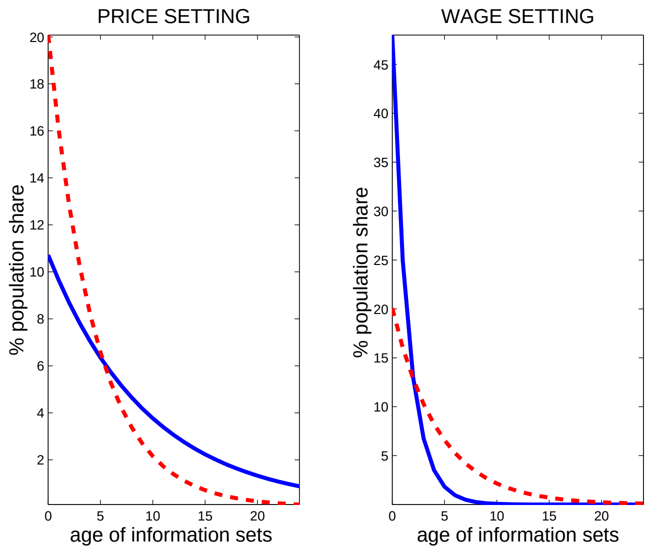

Figure 3.5: Age distributions of information sets $J = 24$. Broken line - distribution according to prior. Solid line - distribution according to posterior.

Inspecting the figure shows that the truncation point is clearly sufficient for wage setting, since the estimated age distribution converge to zero. For price setting there remains a nonzero mass at age 24, indicating that a further increase in the number of lags might provide a slightly better approximation to the infinite lag structure.

		Mankiw-Reiss J=12		Mankiw-Reiss J=24
parameter		mode	90% post. interval	mode
investment adj. cost	S''	5.615	(3.806, 5.691, 7.547)	3.983
consumption utility	σ_c	1.446	(0.987, 1.423, 1.889)	1.187
habit persistence	h	0.516	(0.422, 0.545, 0.658)	0.472
labor utility	σ_l	2.635	(1.856, 2.868, 3.893)	2.735
information rigidity prices	$\tilde{\xi}_p$	0.998	(0.998, 0.998, 0.998)	0.901
information rigidity wages	$\tilde{\xi}_w$	0.856	(0.802, 0.853, 0.904)	0.520
capital util. adj. cost	ϕ	0.370	(0.269, 0.370, 0.477)	0.345
fixed cost	ψ	1.735	(1.575, 1.741, 1.897)	1.563
Calvo employment	ξ_L	0.159	(0.081, 0.208, 0.329)	0.680
response to inflation	r_π	1.660	(1.482, 1.650, 1.818)	1.653
response to diff. inflation	$r_{d\pi}$	0.145	(0.100, 0.159, 0.222)	0.131
interest rate smoothing	ρ	0.948	(0.908, 0.936, 0.965)	0.975
response to output gap	r_y	0.043	(0.013, 0.046, 0.081)	0.107
response to diff. output gap	r_{dy}	0.103	(0.075, 0.118, 0.158)	0.219
persistence techn. shock	ρ_a	0.845	(0.799, 0.843, 0.891)	0.904
persistence preference shock	ρ_b	0.712	(0.545, 0.669, 0.792)	0.534
persistence gov. spending. shock	ρ_g	0.979	(0.931, 0.965, 0.999)	0.898
persistence labor supply shock	ρ_L	0.833	(0.785, 0.864, 0.947)	0.989
persistence investment shock	ρ_I	0.325	(0.190, 0.309, 0.435)	0.261
stdv. productivity shock	σ_a	0.285	(0.250, 0.299, 0.339)	0.397
stdv. preference shock	σ_b	0.098	(0.075, 0.109, 0.143)	0.146
stdv. gov. spending shock	σ_g	0.320	(0.284, 0.321, 0.360)	0.313
stdv. labor supply shock	σ_L	4.547	(2.463, 4.242, 5.903)	1.305
stdv. equity premium shock	σ_q	0.093	(0.046, 0.192, 0.364)	0.094
stdv. monetary shock	σ_R	0.052	(0.039, 0.060, 0.080)	0.038
stdv. investment shock	σ_I	0.517	(0.441, 0.529, 0.619)	0.557
stdv. price markup shock	σ_π	0.036	(0.025, 0.037, 0.050)	0.058
stdv. wage markup shock	σ_w	0.936	(0.647, 1.034, 1.396)	0.253

Table 3.10: Posterior distribution: Truncated Mankiw-Reiss model with $J = 12$ and $J = 24$.

We next turn to a description of impulse responses of this sticky information model. Again the parameters are evaluated at the posterior mode for plotting responses.

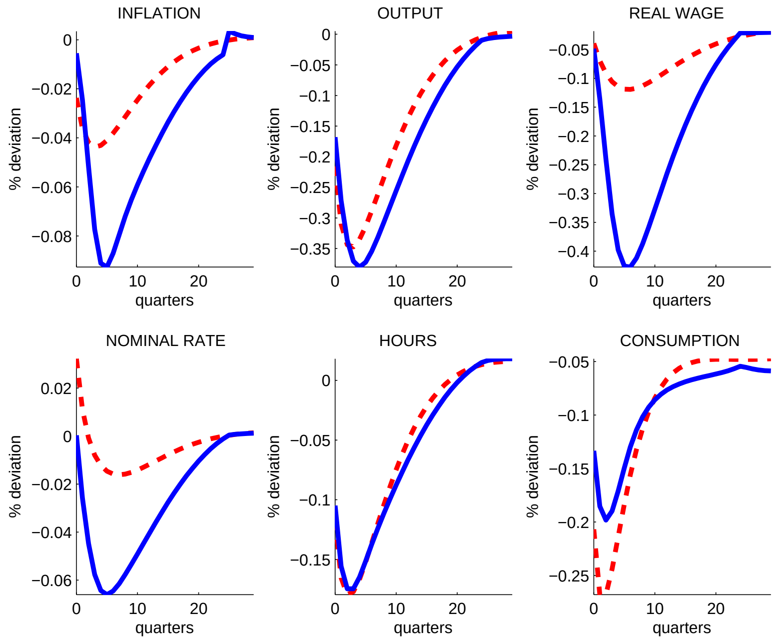

Figure 3.6: Impulse response to monetary shock. Solid line - sticky information model J=24, dashed line - Calvo model. Impulse responses are calculated at the posterior mode.

The monetary shock brings about a qualitatively very similar delayed and hump-shaped responses of the observable variables in the Calvo model as in the sticky information model, see Figure 3.6. The real wage, inflation and the nominal rate react stronger in the sticky information model. This may be due to the fact that the the estimated information rigidity $\tilde{\xi}_w = 0.52$ in wage setting is smaller than the Calvo wage setting parameter $\xi_w = 0.71$. This similarity between the models is confirmed when considering further shocks. Here, we only show some selected responses. Figure 3.7 displays the impulse responses to the technology shock.

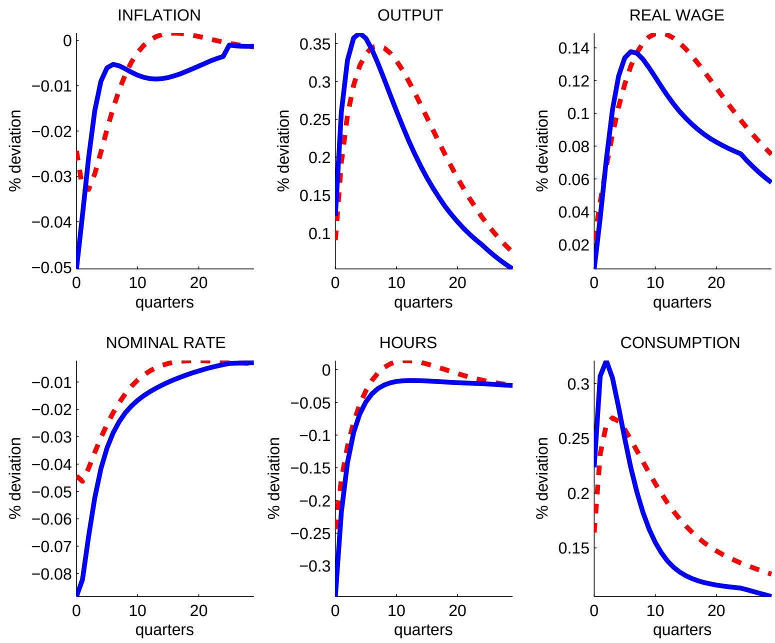

Figure 3.7: Impulse response to technology shock. Solid line - sticky information model J=24, dashed line - Calvo model. Impulse responses are calculated at the posterior mode.

The impulse response to the technology shock and most other shocks displayed in the appendix do not point to obvious differences between the models. However, sharp discrepancies arise from the impulse responses to the price (and similarly wage) markup shock displayed in Figure 3.8 and Figure 3.9.

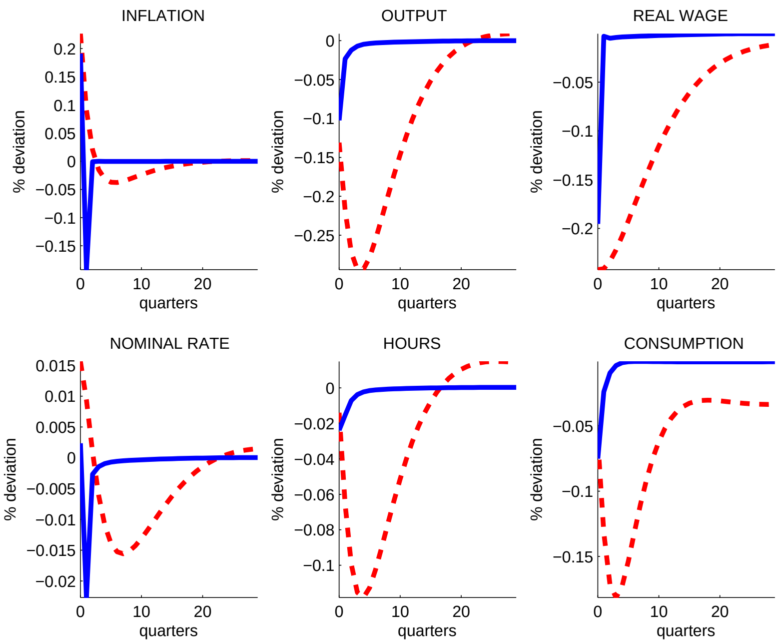

Figure 3.8: Impulse response to price markup shock. Solid line - sticky information model J=24, dashed line - Calvo model. Impulse responses are calculated at the posterior mode.

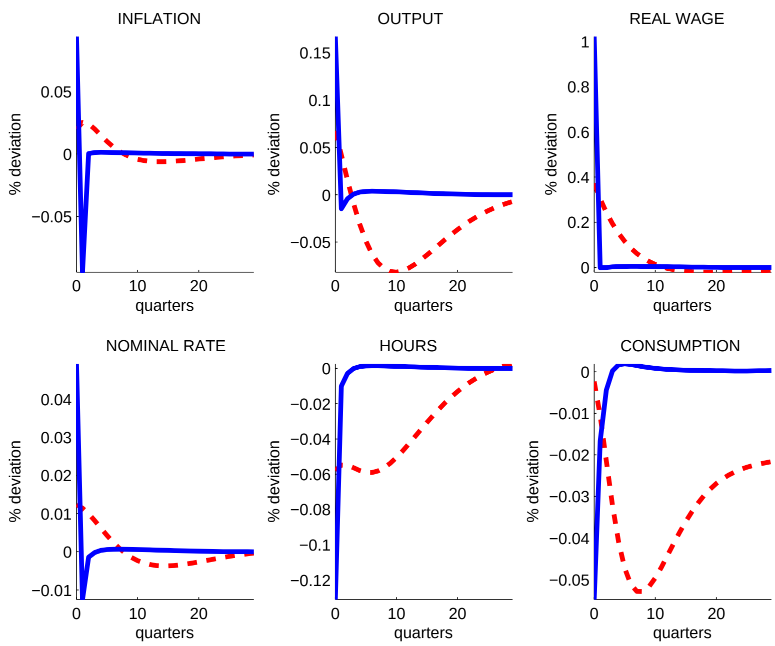

Figure 3.9: Impulse response to wage markup shock. Blue solid line - sticky information model J=24, red dashed line - Calvo model. Impulse responses are calculated at the posterior mode.

As for the case of $J = 12$, the markup shock induces persistent response in most variables for the Calvo model, but very different dynamics for the sticky information model. As indicated by the modest improvement in the marginal likelihood extending the truncation does not improve the overall fit of the sticky information model by much. This finding is confirmed by inspecting the RMSE for the extended model, displayed in the last row of table 3.8 on page 56. For most series there is not much of a change in the RMSE relative to the model with $J = 12$. The RMSE for the employment and investment improves, but the real wage series is now matched even worse.

Finally, it is worthwhile to compare our estimated degree of information rigidity to what is reported in other studies. Carroll (2003) finds that the average U.S. household updates inflation expectations roughly once per year. A study by Döpke, Dovern, Fritsche, and Slacalek (2005) has reported a similar frequency of updating information sets for 5 major European countries. In the baseline model with $J = 12$ the average age of information sets relevant for price setting is roughly seven quarters and for wage setting 5 quarters. For $J = 24$ the average age is roughly 8 quarters for price setting, but only 2 quarters for wage setting. Compared to the micro studies, a very high degree of information stickiness in price setting is needed for the DSGE models to match the data. Note though that absent the markup shocks we have obtained diametrally different estimates of the information stickiness (see Table 3.16 on page 75 and Table 3.17 on page 76). These estimates are now more plausible from theoretical point of view and indicate almost no importance of information sets outdated by more than 3-5 periods both for wages and prices, which is also consistent with the estimates in Laforte (2005). It should be, however, underlined that the overall fit of models estimated without markup shocks is not satisfactory. In particular, these models fail to simultaneously match the persistence as well as volatility of inflation and wage series. So, the sticky information model that is preferred by the data implies high information stickiness. One should bear in mind though that the estimated average age of the reset price in the Calvo model is also very high. Given the estimate of $\xi_p = 0.875$, on average prices are a roughly two years old in the Calvo model. Estimating version of this model with firm specific input factors is likely to decrease both the estimated average age of prices and of information sets. A more extensive discussion of nominal rigidities, including the international price discrimination, is provided in Chapter 4 of this thesis, where the complex two-region DSGE model for the Euro area is presented.

3.6.4 Extension II: Departing from exponential decay

Next, we explore another avenue that can potentially provide a better fit of sticky information models. That is, we relax the assumption that the age distribution of information sets follows the exponential decay. This modification is motivated by Laforte (2005), who has shown that the fit of sticky price models and sticky information models can be improved by allowing for a more general pattern of price adjustment in the spirit of Wolman (1999).

First, we estimate a rich parameterization where all J parameters ω_j are estimated. The restriction $w_0 = 1 - \sum_{j=1}^{J} \omega_j$ follows from the requirement that shares add up to unity. The purpose of estimating the rich parameterization is to obtain more insights regarding the information distribution in the data and use this knowledge for parameterizing more parsimonious models. Therefore, we do not compute the marginal density for this richly parameterized model and thus do not include it in the set of models to be compared via posterior-odds ratios.

As will be shown below, the estimates from the rich parameterization do not support the idea that population shares that are strictly declining in age as suggested by Reis (2005). Rather, the age distribution of information sets appears to be hump shaped. We therefore introduce two intermediate parameterizations, which can capture this hump shaped pattern while estimating only a small number of parameters. In the intermediate specification I, we estimate only the share of agents using the current and yesterday's information set, i.e. $\tilde{\omega}_0$ and $\tilde{\omega}_1$. This specification allows the age distribution of information sets to be hump-shaped. The current information set may now have very little weight and the peak is allowed to be at lag 1. Further lags have geometrically decaying weights. In intermediate parameterization I, the parameter $\tilde{\omega}_0$ is beta distributed with the mean of 0.2 and standard deviation of 0.15. The parameter $\tilde{\omega}_1$ which imposes geometrically decreasing relationship for the remaining information sets is prior beta distributed with mean 0.8 and standard deviation of 0.15.

The intermediate parameterization II allows for a more general hump shaped pattern. Under this specification, we estimate directly the parameters $\{\tilde{\omega}_0, \tilde{\omega}_1, \tilde{\omega}_5\}$ and linearly interpolate the values for the remaining shares assuming that $\tilde{\omega}_J = 0$.[23] This parameterization requires some transformations to obtain the structural parameters ω_j, which are bounded to be positive and sum up to unity. This may be straightforward achieved by letting $\omega_j = \tilde{\omega}_j / c$. Here, the constant $c = \sum_{j=0}^{J} \tilde{\omega}_j$. This pattern allows for at most two peaks in the age distribution or for a plateau at intermediate lags.

The priors assumed for estimated parameters $\tilde{\omega}_0$, $\tilde{\omega}_1$, $\tilde{\omega}_5$ in the intermediate parameterization II are also beta distributed with the mean of 0.2 and standard deviation of 0.15.

The different versions of the sticky information models estimated here are summarized in Table 3.11.

parameterization	estimated parameters	remaining parameters
rich	$\omega_j, \quad j = 1, ..., J$	$w_0 = 1 - \sum_{j=1}^{J} \omega_j$
parsimonious	$\tilde{\xi}$	$\omega_j = \frac{(1-\tilde{\xi})\tilde{\xi}^j}{\sum_{j=0}^{J}(1-\tilde{\xi})\tilde{\xi}^j} j = 0, ..., J$
intermediate I	$\tilde{\omega}_0$,$\tilde{\omega}_1$	$\omega_0 = \frac{\tilde{\omega}_0}{\tilde{\omega}_0 + \sum_{j=1}^{J} \tilde{\omega}_1^j (1-\tilde{\omega}_1)}$ $\omega_j = \frac{\tilde{\omega}_1^j (1-\tilde{\omega}_1)}{\tilde{\omega}_0 + \sum_{j=1}^{J} \tilde{\omega}_1^j (1-\tilde{\omega}_1)}$ j=1,...,J
intermediate II	$\tilde{\omega}_0$, $\tilde{\omega}_1$, $\tilde{\omega}_5$	linearly interpolated

Table 3.11: Different parameterizations of the sticky information model

The estimated age distributions are summarized in Figure 3.10 on page 67 and Figure 3.11 on page 68. The estimated share of agents working with current information is small. Hence, that data favor a specification where the age distribution of information sets of price and wage setter follows a hump-shaped pattern. However, these modification improve the fit of sticky information models only mildly. The marginal density of the sticky information with intermediate parameterization I and a maximum of 12 information lags is -404.7. For the intermediate parameterization II the marginal density is -409.2. Hence, the improvement over the baseline model is large enough to reject geometric decay. However, relative to the sticky price model whose marginal density is -307.8, this modified sticky information model still fares far worse.

3.7 Alternative model comparison method - models of heterogenous price and wage setters

The analysis so far has shown overwhelming dominance of the sticky price model relative to the sticky information model based on the Bayes factor as the model comparison criterion. Sims (2003) has pointed out that Bayesian model comparison methods can misbehave. He notes that results may be sensitive to the prior distributions of parameters, to seemingly minor aspects of model specification and tend to be implausibly sharp. He cites Gelman, Carlin, Stern, and Rubin (1995) who note that posterior probabilities are useful, when "each of the discrete models make scientific sense and there are no obvious scientific models in between". Sims expands on this point by noting

[23]The remaining parameters are calculated as follows: $\tilde{\omega}_j = (\tilde{\omega}_5 - \tilde{\omega}_1)(j-1)/4 + \tilde{\omega}_1$ for $j = 2, 3, 4$ and $\tilde{\omega}_j = (0 - \tilde{\omega}_5)(j-5)/7 + \tilde{\omega}_5$ for $j = 6, 7, 8, 9, 10, 11$.

that overwhelming favor for one model against an alternative model based on posterior odds may be an indication that the selection of models is simply too sparse. He advocates to expand the range of models or to vary the model specification.

We address these potential pitfalls of applying Bayesian model selection criteria in the following way. We address the requirement of Gelman, Carlin, Stern, and Rubin (1995) and form an obvious in between model: This is a nested model, where an estimated fraction of firms follows the Calvo apparatus and the remaining fraction follows the sticky information approach. Our structural interpretation of such a nested model is based on the idea that for some firms menu costs may be a more important source of nominal rigidities, whereas for other firms it may be information acquisition costs.

Moreover, we test the less restrictive version of the sticky information model - a hybrid model - where an estimated fraction of agents is backwardlooking. Such a hybrid sticky information model is in the spirit of hybrid Calvo models as in Galí and Gertler (1999) that generate inertia in the Phillips curve by adding rule of thumb price setters.

3.7.1 The nested Calvo - sticky information model

In order to validate the empirical relevance of Calvo and Sticky Information scheme within the same model we introduce a nested Calvo - Sticky Information model. In this model a fraction α of firms operates according to the Calvo Model. The remaining fraction $1-\alpha$ sets prices according to the sticky information scheme. Within the sticky information scheme, we allow the information set to be outdated by at most J periods. In the following exposition, we aim to keep the notation simple and assume that all sticky information agents set their price based on yesterdays information set. At the expense of notation, the model can easily be expanded to arbitrary information lags $j = 0, 1, ..., J$. In fact, we estimate a model with $J = 12$. For empirical analysis we use the nested models with parsimoniously as well as with rich parameterized sticky information part.

Let P_t^{opt} denote the optimal price of the Calvo price setters that re-optimize their price in period t and let $P_{t,t-1}$ denote the price that the sticky information agents chooses today based on information yesterday. ξ_p stands for the probability that Calvo agents cannot re-optimize their price.

The aggregate price index P_t is defined as[24]

$$P_t^{-1/\lambda_{p,t}} = \alpha \left\{ (1-\xi_p)\left(P_t^{opt}\right)^{-1/\lambda_{p,t}} + \sum_{j=1}^{\infty}(1-\xi_p)\xi_p^j \left(P_{t-j}^{opt}\right)^{-1/\lambda_{p,t}} \right\} + (1-\alpha) P_{t,t-1}^{-1/\lambda_{p,t}} \quad (3.51)$$

In order to eliminate the infinite sum, we define an auxiliary index X_t via the relation $X_t^{-1/\lambda_{p,t}} \equiv \sum_{j=0}^{\infty}(1-\xi_p)\xi_p^j \left(P_{t-j}^{opt}\right)^{-1/\lambda_{p,t}}$. Note that the infinite sum above is $\xi_p X_{t-1}^{-1/\lambda_{p,t}}$. X_t evolves as

$$X_t^{-1/\lambda_{p,t}} = (1-\xi_p)\left(P_t^{opt}\right)^{-1/\lambda_{p,t}} + \xi_p X_{t-1}^{-1/\lambda_{p,t}} \quad (3.52)$$

Our price index is therefore

$$P_t^{-1/\lambda_{p,t}} = \alpha \left\{ (1-\xi_p)\left(P_t^{opt}\right)^{-1/\lambda_{p,t}} + \xi_p X_{t-1}^{-1/\lambda_{p,t}} \right\} + (1-\alpha)(P_{t,t-1}^{opt})^{-1/\lambda_{p,t}} \quad (3.53)$$

Thus, price setting in the mixed model is described by equation (3.17) defining P_t^{opt} and by equation (3.23) defining the optimal $P_{t,t-1}^{opt}$ along with the equations (3.52) and (3.53) derived above.

[24]Note that the term $\sum_{j=1}^{\infty}(1-\xi_p)\xi_p^j \; P_{t-j}^{opt} \;^{-1/\lambda_{p,t}}$ does not equal $\xi_p P_{t-1}^{-1/\lambda_{p,t}}$ as in the Calvo model, due to the presence of sticky information agents. This fact requires us to introduce the auxiliary variable X_t.

In order to solve the model with standard methods, a stationary version of above two equations is needed. This is obtained as follows: Equation (3.53) is divided by P_t and the stationary variables $x_t \equiv X/P_t, p_{t,t-1} \equiv P^{opt}_{t,t-1}/P_t$ and $p^+_t \equiv P^{opt}_t/P_t$ are defined.

$$1^{-1/\lambda_{p,t}} = \alpha \left\{ (1-\xi_p) \left(p^+_t\right)^{-1/\lambda_{p,t}} + \xi_p \left(\frac{x_{t-1}}{\pi_t}\right)^{-1/\lambda_{p,t}} \right\} + (1-\alpha) \left(\frac{p_{t,t-1}}{\pi_t}\right)^{-1/\lambda_{p,t}} \tag{3.54}$$

Here, π_t is price inflation between period $t-1$ and period t. Divide (3.52) by P_t to arrive at

$$x_t^{-1/\lambda_{p,t}} = (1-\xi_p) \left(p^+_t\right)^{-1/\lambda_{p,t}} + \xi_p \left(\frac{x_{t-1}}{\pi_t}\right)^{-1/\lambda_{p,t}} \tag{3.55}$$

The same logic applied to the wage setting scheme yields:

$$1^{-1/\lambda_{p,t}} = \alpha \left\{ (1-\xi_w) \left(w^+_t\right)^{-1/\lambda_{p,t}} + \xi_w \left(\frac{x_{t-1}}{\pi^w_t}\right)^{-1/\lambda_{p,t}} \right\} + (1-\alpha) \left(\frac{w_{t,t-1}}{\pi^w_t}\right)^{-1/\lambda_{p,t}} \tag{3.56}$$

$$(x^w_t)^{-1/\lambda_{p,t}} = (1-\xi_w) \left(w^+_t\right)^{-1/\lambda_{p,t}} + \xi_w \left(\frac{x^w_{t-1}}{\pi_t}\right)^{-1/\lambda_{p,t}} \tag{3.57}$$

where $\pi^w_t = W_t/W_{t-1}$ denotes wage inflation and stationary variable $\hat{w}^+_t$ is defined as $\hat{w}^+_t \equiv W^{opt}_t/W_t$.

Linearized version of the nested model

Log-linearizing the FOC for Calvo price setters yields:

$$\hat{p}^+_t = (1-\beta\xi_p) \left(\widehat{mc}_t + u^p_t\right) + \beta\xi_p E_t \hat{p}^+_{t+1} + \beta\xi_p E_t \hat{\pi}_{t+1}, \tag{3.58}$$

where $\hat{p}^+_t \equiv \frac{d(\frac{P^{opt}_t}{P_t})}{(\frac{P^{opt}_t}{P_t})}$.

Log-linearizing the CES aggregate price index (3.54) we obtain:

$$0 = \alpha \left\{ (1-\xi_p)\hat{p}^+_t + \xi_p \left(\hat{x}_{t-1} - \hat{\pi}_t\right) \right\} + (1-\alpha) \left(\hat{p}_{t,t-1} - \hat{\pi}_t\right) \tag{3.59}$$

The linearized auxiliary index X_t has the following form:

$$\hat{x}_t = (1-\xi_p)\hat{p}^+_t + \xi_p \left(\hat{x}_{t-1} - \hat{\pi}_t\right) \tag{3.60}$$

Hence, price setting is described by these three equations and by the equation for the optimal price $\hat{p}_{t,t-1}$.

Similarly, the loglinearized FOC for Calvo wage setteres along with the loglinearized wage index (equation 3.62), auxiliary index X^w_t (equation 3.63) and conditions for optimal wage $\hat{w}_{t,t-1}$ (equation 3.4.1) describe wage setting in the nested Calvo-sticky information model:

$$\begin{aligned}\hat{w}^+_t = {} & \frac{(1-\xi_w\beta)}{(1+\frac{1+\lambda_w}{\lambda_w}\sigma_l)} \left[\sigma_l \widehat{L}_t + \hat{\varepsilon}^l_t + \frac{\sigma_c}{1-h} \left(\hat{c}_t - h\hat{c}_{t-1}\right) - \hat{w}_t + \frac{\lambda_w}{1+\lambda_w} u^w_t \right] \\ & + \beta\xi_w \left(\hat{\pi}^w_{t+1} + \hat{w}^+_{t+1}\right)\end{aligned} \tag{3.61}$$

$$0 = \alpha \left\{ (1-\xi_w)\hat{w}^+_t + \xi_w \left(\hat{x}^w_{t-1} - \hat{\pi}^w_t\right) \right\} + (1-\alpha) \left(\hat{w}_{t,t-1} - \hat{\pi}^w_t\right) \tag{3.62}$$

$$\hat{x}_t^w = (1-\theta)\hat{w}_t^+ + \xi_w \left(\hat{x}_{t-1}^w - \hat{\pi}_t^w\right), \tag{3.63}$$

where $\hat{w}_t^+ \equiv \frac{d(\frac{W_t^{opt}}{W_t})}{(\frac{W_t^{opt}}{W_t})}$, $\hat{w}_t \equiv \frac{d(\frac{W_t}{P_t})}{(\frac{W_t}{P_t})}$, and $\hat{\pi}_t^w \equiv \frac{d(\pi_t^w)}{(\pi_t^w)}$.

3.7.2 Hybrid sticky information with backward looking agents

As a last alternative model we specify the so-called hybrid sticky information models. In this class of models a fraction of agents simply follows a rule of thumb when setting the price (and wage) while the other fraction follows the standard sticky information scheme. In particular, we assume that a fraction $(1-\varkappa_p)$ of firms charges a price that is equal to the aggregate price level of the last period index to lagged inflation. In that case the aggregate price index is given by

$$P_t^{-1/\lambda_{p,t}} = \varkappa_p \sum_{j=0}^{J} \omega_j^p \left(P_{t,t-j}^{opt}\right)^{-1/\lambda_{p,t}} + (1-\varkappa_p)\left(P_{t-1}\pi_{t-1}^{\gamma_p}\right)^{-1/\lambda_{p,t}} \tag{3.64}$$

Here, γ_p is the degree of indexation similar as in hybrid Calvo models. [25]

Similarly to price setting, the aggregate wage index is in a hybrid model given by the following expression

$$W_t^{-1/\lambda_{w,t}} = \varkappa_w \sum_{j=0}^{J} \omega_j^w \left(W_{t,t-j}^{opt}\right)^{-1/\lambda_{w,t}} + (1-\varkappa_w)\left(W_{t-1}(\pi_t^w)^{\gamma_w}\right)^{-1/\lambda_{w,t}}, \tag{3.66}$$

where $(1-\varkappa_w)$ is the fraction of rule of thumb wage setters that link their wages to the last period wage rate. The indexation degree is denoted by γ_w.

3.7.3 Estimation results for heterogenous agents models

Before presenting the estimation results we mention the prior distribution of the parameters that are specific to the models with heterogenous agents. For these models one has to estimate the shares of sticky information agents: $\varkappa^P$, $\varkappa^w$ in hybrid models and α^P, α^w in the nested model. We assume that these parameters are beta distributed with the mean of 0.5 and standard deviation of 0.2.

Table 3.12 on the next page collects the log marginal densities as our summary statistics for the overall model evaluation.

[25] We also estimate the parsimoniously parametrized hybrid models, where the number of estimated parameters is the same as in Smets and Wouters' model. For this purpose we assume that the information age distribution is described by function of the singe estimable parameter $\tilde{\xi}_p^j$. So $\tilde{\omega}_j^p = (1-\tilde{\xi}_p)\tilde{\xi}_p^j$ for $j = 0,1,...,J$. The remaining mass of the distribution $\tilde{\omega_p}^{J+1} = 1 - \sum_{j=0}^{J}\tilde{\omega}_p^j$ may be now ascribed to the rule of thumb price setters (or to the fraction o Calvo price setters, see the section above). The price index is then given by

$$P_t^{-1/\lambda_{p,t}} = \sum_{j=0}^{J}\tilde{\omega}_j^p (P_{t,t-j}^{opt})^{-1/\lambda_{p,t}} + (1-\sum_{j=0}^{J}\tilde{\omega}_j^p)(P_{t-1}\pi_{t-1}^{\gamma_p})^{-1/\lambda_{p,t}}. \tag{3.65}$$

Model	parameterization	log marginal density
Calvo	with indexation	-307.8
Calvo	without indexation	-304.1
Nested Calvo-SI model	parsimonious, J=12	-322.6
Nested Calvo-SI model	intermediate I, J=12	-323.2
Nested Calvo-SI model	intermediate II, J=12	-317.4
Hybrid SI model	parsimonious, J=12	-326.2
Hybrid SI model	intermediate I, J=12	-311.3
Hybrid SI model	intermediate II, J=12	-313.1

Table 3.12: Comparison of marginal densities for heterogenous agents models

The numbers indicate that the data fit of models with heterogenous agents is much better than that of their standard sticky information counterparts whose marginal likelihood is roughly -400. However, these models do not attain a higher marginal density than the standard Calvo model. In fact, based on the Bayes factor the standard Calvo model again dominates any of the heterogenous agents models.[26]

We now turn to the estimated shares of sticky information agents. Table 3.13 summarizes these shares for the different models we estimate. For the parsimoniously parameterized nested models, we estimate the share of Calvo agents for price setting very close to unity. In other words, the data ascribes almost zero mass to sticky information price setters. For wage setting the share of agents following sticky information schemes is estimated at 7%. This is consistent with the finding that the Calvo model overwhelmingly dominates the parsimoniously parameterized sticky information model in terms of posterior odds. Hence, one can conclude that the overwhelming evidence against the sticky information model based on Bayes factor is also confirmed by the evidence in the nested model.

model		parsimonious	intermediate I	intermediate II
nested	price setting	0.01	0.15	0.15
	wage setting	0.07	0.30	0.35
hybrid	price setting	0.01*	0.08	0.08
	wage setting	0.99*	0.28	0.28

Table 3.13: Estimated shares of sticky information agents. The asterisks (*) denotes that the model is parameterized as described in the footnote 25 on the previous page

For the other estimated parameterizations, we estimate the share of sticky information price setters at roughly 15%. For wage setting the share of sticky information households is estimated between 30% and 35%. In other words, there seems to be little evidence for sticky information considerations in price setting, but more in wage setting.

Next, the estimated age distribution of information sets is discussed. Figure 3.10 on the following page and Figure 3.11 on page 68 show the posterior distribution of ω_j - the shares of agents working with information outdated by j periods- for wage and price setting. The starred line shows the age distribution under the unrestricted estimation where all ω_j are estimated freely.

[26] As is evident from Table 3.12 the Calvo model with indexation receives smaller marginal density than the model with pure Calvo specification. This is in stark contrast with the empirical New-Keynesian Phillips Curve literature (Galí and Gertler (1999), Galí, Gertler, and Lopez-Salido (2001)) that finds an evidence for backward looking agents. The potential explanation may be that incorporating a high number of nominal and real frictions into the model (i) may imply overparameterization and (ii) some frictions (especially capital adjustment costs and habit formation implying some backward-lookingness in the model) may have similar effects on the model dynamics as incorporating the backward-lookingness in wage and price settings. It should also be underlined that since the monetary policy rule is defined on the lagged inflation, inflation is in our model a state variable. So, its behavior is more persistent compared to the models with different specifications of the monetary feedback rule.

This age distribution is highly irregular. It does not follow either one of the two typically employed schemes in the sticky information literature: Geometrically declining weights as in Calvo or equal weights as in Taylor type models.[27]

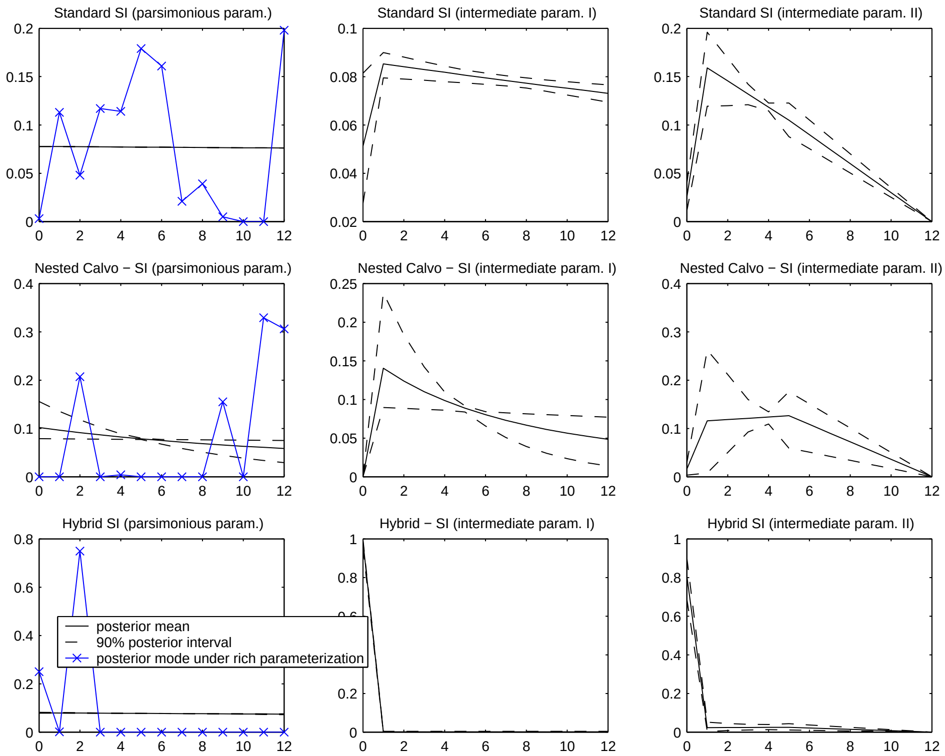

Figure 3.10: Age distribution of information sets: price setting

[27]See Coiboin (2006) and Dupor and Tsuraga (2005) for the use of sticky information models that employ these types of age distributions.

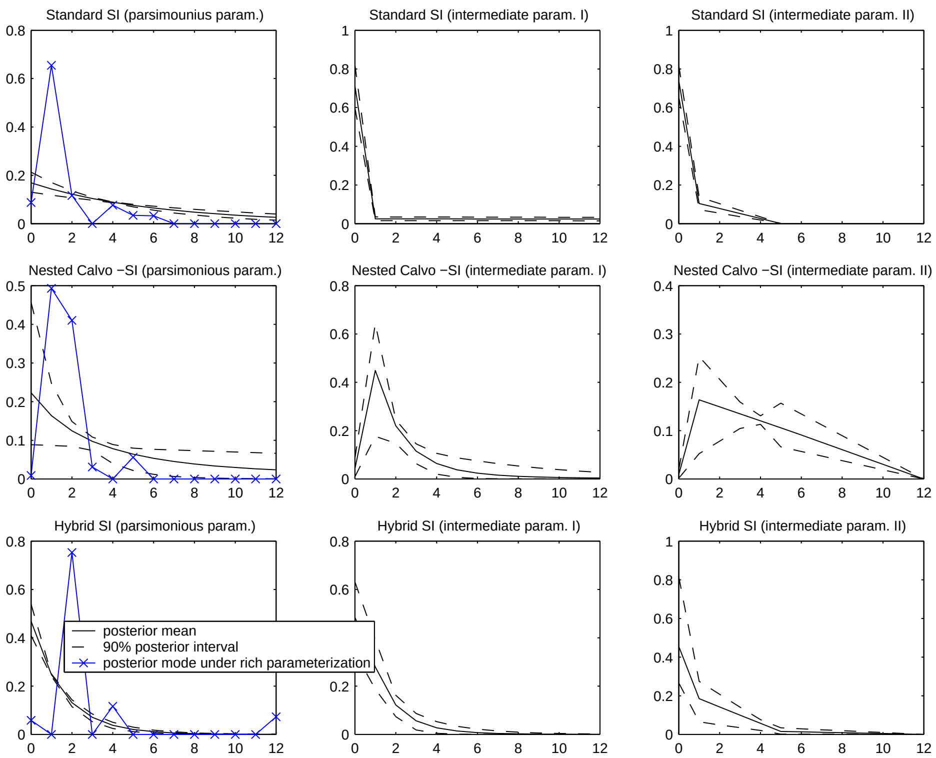

Figure 3.11: Age distribution of information sets: wage setting

For wage setting, the estimated unrestricted age distribution appears to be well approximated by hump shaped pattern with a peak at lag 1.

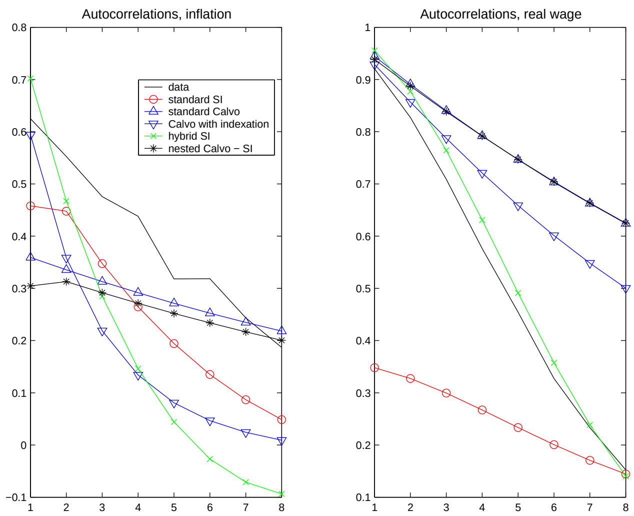

Figure 3.12: Replicated autocorrelations

Finally, Figure 3.12 shows that hybrid models cope with the problem of replicating the autocorrelation patterns of inflation and real wage much better than their standard sticky information counterparts.

Summarizing, we find only very limited empirical evidence for the finite horizon sticky information scheme in price setting. In turn, higher estimates for the fraction of sticky information wage setters both in hybrid and nested models indicate that there are features of the sticky information wage scheme which are attractive for the data.

3.8 Overall discussion of model dynamics

In this section, we briefly summarize the comparison of sticky information models and sticky contracts in terms of their impulse response functions and variance decomposition. Figure 3.14 on page 85 to Figure 3.18 on page 89 compare the estimated impulse responses of six selected variables: output, inflation, nominal interest rate, wage inflation, real wage and labor to key structural shocks across alternative models.[28] We plot the 90% confidence posterior band around the median of the IRF.

Figure 3.14 on page 85 presents a response to the technology shock. Due to the rise in productivity, the marginal cost falls on impact. However, contrary to the interpretation provided in Smets and Wouters (2003), we find that gradual fall of inflation is common to all models with an indexation mechanism and is not a result of a *not strong enough* response of the monetary authority. Finally, in all models, except for the hybrid sticky information model, we note that the real wage rises only gradually and not very significantly following the positive productivity shock. It also becomes evident that the standard sticky information model with $J = 24$ displays much more regular responses than the version with $J = 12$.

Figure 3.15 shows the effects of a monetary shock on the variables under alternative model specifications. This temporary shock leads to a rise in the nominal and real short-term interest rates. This leads to a hump-shaped fall in output, consumption and investment. In line with the stylized facts following a monetary policy shock, real wages fall. The maximum effect on investment is about three times as large as that on consumption. The effects of this shock are broadly similar across all models considered here.

Figure 3.16 on page 87 shows the effects of a negative labor supply shock. The qualitative effects of this supply shock on output, inflation and the interest rate are very similar to those of a negative productivity shock. In the hybrid sticky information model, where the fraction of backwardlooking agents is estimated at about 99%, the labor supply shock effects are much more delayed than in the remaining models.

A qualitatively similar impulse responses can be obtained with a negative wage markup shock (Figure 3.16 on page 87). The most important differences between Calvo and standard sticky information model regarding the effects of the markup shocks have been discussed in Section 3.6. These differences are also present in the hybrid version of the sticky information scheme. In turn, due to a very small estimated fraction of sticky information agents, the nested model displays only a few features common to the sticky information scheme. However, the immediate positive and then negative effect of wage markup shock on wage inflation, typical for sticky information models, are still visible in the nested model.

The price mark-up shock (Figure 3.16 on page 87) has, in all models except for the standard sticky information model, a strong impact on inflation (see discussion in Section 3.6). Strikingly, the response of inflation to this shock in the hybrid sticky information model is almost identical to that under the Calvo specification and displays a strong increase on impact and then a gradual decline.

We also assess the differences across the alternative settings by comparing the importance of shocks for explaining the fluctuations of model variables. Tables 3.21 on page 80 to 3.26 on page 83 report the variance of six selected variables explained by the nine shocks, as defined in Smets and Wouters (2003). In the short run, the variance of output is explained by preference, investment, government spending and technology shocks. Immediately after shock occurrence, the importance of the technology shock is significantly lower in the sticky information models. In the long run, output is explained by a highly persistent labor supply shock and technology shock. The labor supply shock explains more than 50% of the nominal interest rate volatility in the sticky information model compared to its almost negligible role in the remaining models.

[28] In the case of the sticky information models we plot the IRFs obtained under parsimonious specification.

For standard and hybrid sticky information models, the effects of monetary shocks on inflation are less important immediately after shock occurrence but more important in the long run than in Calvo models. In line with the results of Smets and Wouters (2003) we notice that the price markup shock in Calvo models explains above 90% of inflation volatility in the short run and above 60% in the long run, which also explains the importance of this shock for model fit. The role of the price markup shock in explaining inflation volatility is much lower in the sticky information model. In turn, the wage markup shock is more important in the latter model. Finally, we notice that the wage markup shock has significant impact on the price inflation only in the standard sticky information setting.

The complementary discussion of the roles of various structural innovations in explaining the volatilities of European aggregates is presented in Chapter 4.

3.9 Conclusion

In this chapter we have evaluated the empirical performance of sticky information models in wage and price setting relative the Calvo model. Our primary finding is that the baseline sticky information model is strongly dominated as measured via the Bayes factor by the Calvo model. This finding holds for both considered truncation points $J = 12$ and $J = 24$ and for all considered age distribution of information sets. One potential explanation for this finding is the inability of sticky information models to match the persistence and volatility in real wages and inflation. Thus, the standard sticky information model is *hard to square with the facts* once we let likelihood based methods decide what the facts are rather than matching a few selected stylized facts.

As a second method for comparing sticky information and sticky price models, we form a nested model. A share of agents is assumed to follow the Calvo apparatus whereas the remaining agents sets prices according to sticky information model. This method also delivers strong evidence against sticky information ideas. When the age distribution follows a truncated exponential decay, the posterior mode indicates that only 1% of agents follows sticky information schemes in price setting and only 7% in wage setting. Hence, the nested model also suggest that the data does not offer much support sticky information schemes. Finally, we show that there is little evidence in Euro area data for the arrival of information to follow a Poisson process as suggested by Reis (2005). Such a poisson process implies that the largest share of firms uses current information, which is not what we estimate in our framework. On the contrary, our estimates suggest that the share of agents using current information is typically very small.

In comparing sticky information models to the Calvo model we have made a number of choices that could be relaxed in future work. First, we have only considered sticky information in wage and price setting. This natural choice implies a form of dichotomy: Household make consumption decisions based on full information, but wage decision based on outdated information. A similar dichotomy holds for investment and pricing decisions of firms. Future work could empirically evaluate the whole sticky information paradigm by incorporating it into all decisions of economic agents.[29] In order to allow for a fair comparison with the Calvo model we have chosen not pursue this avenue in the current work. Second, future work could investigate to what extent our findings are sensitive to the specification of the real side of the economy by incorporating real rigidities in the form of firm specific factor markets. We leave these issues for future work.

Finally, a note on our methodology is in order. We view likelihood based methods such as the Bayesian model comparison employed in this paper as the appropriate way to compare models to the data. In our view, this is preferable over selecting a few stylized facts from the vast range of implications that a model has for observable variables. Essentially, we let the data decide how to weigh all the different moments that one may potentially decide to match. However, some may

[29] See Coiboin (2006) for an analysis of the interactions between sticky information in consumption and in price setting.

criticize this approach because the so-called structural shocks are not regarded as truly structural. For instance, Schmitt-Grohé and Uribe (2005) view some of the shocks employed in recent DSGE models as "a reflection of the fact that theory lags behind business cycle", i.e. as misspecification errors rather than structural shocks. They further criticize that these non-structural errors often explain a majority of observed business-cycle fluctuations. In principle, this criticism also applies to this study. For instance, we have argued that the difference in marginal likelihood between sticky information models and sticky price models may in part be attributable to how markup shocks affect the dynamics in the respective models. These markup shocks are viewed by some in the profession as non-structural shocks reflecting a host of issues neglected in the model. In our view, this is not an argument against the methodology per se. If these shocks are found to be important for model comparison purposes, this only highlights the need to further explore the interpretation and foundation of these shocks.

Tables and Figures

		prior			Calvo SW		Standard Calvo	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	4.054	(0.562, 2.954, 4.724)	2.295	(1.064, 2.794, 4.382)
consumption utility	σ_c	norm	1	0.375	0.954	(0.750, 1.179, 1.599)	1.169	(0.755, 1.187, 1.572)
habit persistence	h	beta	0.7	0.1	0.476	(0.358, 0.491, 0.625)	0.459	(0.337, 0.473, 0.608)
Calvo wages	ξ_w	beta	0.75	0.05	0.714	(0.624, 0.699, 0.771)	0.668	(0.617, 0.686, 0.764)
labor utility	σ_l	norm	2	0.75	2.386	(1.854, 2.741, 3.690)	2.485	(1.790, 2.699, 3.641)
Calvo prices	ξ_p	beta	0.75	0.05	0.875	(0.878, 0.899, 0.923)	0.887	(0.869, 0.889, 0.911)
indexation wages	γ_w	beta	0.75	0.15	0.482	(0.201, 0.409, 0.604)		
indexation prices	γ_p	beta	0.75	0.15	0.548	(0.257, 0.393, 0.544)		
capital util. adj. cost	ϕ	norm	0.2	0.075	0.346	(0.252, 0.358, 0.471)	0.351	(0.242, 0.353, 0.460)
fixed cost	ψ	norm	1.45	0.125	1.778	(1.384, 1.558, 1.723)	1.476	(1.327, 1.502, 1.671)
Calvo employment	ξ_L	beta	0.5	0.15	0.200	(0.628, 0.689, 0.752)	0.711	(0.646, 0.706, 0.760)
response to inflation	r_π	norm	1.7	0.1	1.683	(1.503, 1.681, 1.853)	1.680	(1.510, 1.674, 1.850)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.150	(0.076, 0.146, 0.215)	0.143	(0.085, 0.154, 0.222)
interest rate smoothing	ρ	beta	0.8	0.1	0.927	(0.939, 0.959, 0.981)	0.964	(0.933, 0.958, 0.982)
response to output gap	r_y	norm	0.125	0.05	0.051	(0.040, 0.109, 0.176)	0.088	(0.012, 0.084, 0.157)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.136	(0.164, 0.197, 0.227)	0.194	(0.164, 0.198, 0.232)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.926	(0.874, 0.910, 0.948)	0.900	(0.866, 0.904, 0.942)
persistence preference shock	ρ_b	beta	0.85	0.1	0.745	(0.361, 0.493, 0.623)	0.517	(0.373, 0.518, 0.666)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.976	(0.851, 0.898, 0.943)	0.903	(0.862, 0.903, 0.946)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.725	(0.980, 0.988, 0.997)	0.990	(0.978, 0.987, 0.997)
persistence investment shock	ρ_I	beta	0.85	0.1	0.267	(0.090, 0.308, 0.715)	0.246	(0.153, 0.252, 0.355)
productivity shock	σ_a	invg	0.4	2	0.277	(0.339, 0.422, 0.504)	0.439	(0.360, 0.439, 0.524)
preference shock	σ_b	invg	0.02	2	0.088	(0.116, 0.148, 0.185)	0.142	(0.106, 0.144, 0.181)
gov. spending shock	σ_g	invg	0.15	2	0.321	(0.277, 0.321, 0.361)	0.311	(0.281, 0.319, 0.356)
labor supply shock	σ_L	invg	1	2	2.399	(0.852, 1.229, 1.597)	1.112	(0.855, 1.217, 1.558)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.046, 0.279, 0.848)	0.093	(0.046, 0.177, 0.336)
monetary shock	σ_R	invg	0.15	2	0.066	(0.037, 0.057, 0.077)	0.051	(0.038, 0.057, 0.076)
investment shock	σ_I	invg	0.03	2	0.514	(0.173, 0.504, 0.729)	0.568	(0.488, 0.573, 0.661)
price markup shock	σ_π	invg	0.03	2	0.461	(0.392, 0.692, 0.962)	0.514	(0.347, 0.565, 0.764)
wage markup shock	σ_w	invg	0.3	2	0.630	(0.287, 0.713, 1.152)	0.456	(0.275, 0.607, 0.988)

Table 3.14: Posterior distribution: Calvo model with indexation (Smets and Wouters 2003), standard Calvo model without indexation

		prior			Calvo without price markup shock		Calvo without wage markup shock	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	5.495	(3.804, 5.707, 7.645)	6.143	(4.430, 6.403, 8.251)
consumption utility	σ_c	norm	1	0.375	0.986	(0.662, 1.108, 1.538)	1.180	(0.812, 1.308, 1.762)
habit persistence	h	beta	0.7	0.1	0.692	(0.484, 0.661, 0.850)	0.777	(0.604, 0.746, 0.866)
Calvo wages	ξ_w	beta	0.75	0.05	0.770	(0.725, 0.773, 0.821)	0.639	(0.512, 0.620, 0.720)
labor utility	σ_l	norm	2	0.75	1.385	(0.897, 2.030, 3.128)	1.595	(0.844, 1.929, 3.006)
Calvo prices	ξ_p	beta	0.75	0.05	0.619	(0.573, 0.618, 0.665)	0.854	(0.835, 0.857, 0.880)
indexation wages	γ_w	beta	0.75	0.15	0.946	(0.754, 0.871, 0.996)	0.418	(0.195, 0.428, 0.654)
indexation prices	γ_p	beta	0.75	0.15	0.087	(0.035, 0.109, 0.174)	0.507	(0.374, 0.515, 0.662)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.365	(0.269, 0.371, 0.473)	0.332	(0.234, 0.341, 0.445)
fixed cost	ψ	norm	1.45	0.125	1.187	(0.993, 1.183, 1.369)	1.827	(1.667, 1.824, 1.987)
Calvo employment	ξ_L	beta	0.5	0.15	0.849	(0.820, 0.846, 0.873)	0.115	(0.047, 0.132, 0.214)
response to inflation	r_π	norm	1.7	0.1	1.665	(1.477, 1.661, 1.827)	1.608	(1.436, 1.609, 1.780)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.304	(0.227, 0.305, 0.384)	0.222	(0.151, 0.227, 0.302)
interest rate smoothing	ρ	beta	0.8	0.1	0.873	(0.826, 0.867, 0.908)	0.856	(0.803, 0.856, 0.905)
response to output gap	r_y	norm	0.125	0.05	0.105	(0.041, 0.108, 0.172)	-0.012	(-0.047, 0.000, 0.043)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.006	(-0.036, 0.002, 0.035)	0.041	(0.015, 0.045, 0.074)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.898	(0.794, 0.878, 0.960)	0.900	(0.852, 0.897, 0.943)
persistence preference shock	ρ_b	beta	0.85	0.1	0.438	(0.295, 0.463, 0.624)	0.522	(0.339, 0.532, 0.730)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.886	(0.834, 0.883, 0.935)	0.953	(0.899, 0.945, 0.991)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.999	(0.998, 0.998, 0.999)	0.539	(0.344, 0.568, 0.787)
persistence investment shock	ρ_I	beta	0.85	0.1	0.349	(0.228, 0.354, 0.472)	0.456	(0.278, 0.452, 0.622)
productivity shock	σ_a	invg	0.4	2	0.449	(0.389, 0.484, 0.579)	0.268	(0.239, 0.279, 0.314)
preference shock	σ_b	invg	0.02	2	0.169	(0.126, 0.166, 0.208)	0.144	(0.094, 0.144, 0.193)
gov. spending shock	σ_g	invg	0.15	2	0.294	(0.265, 0.302, 0.339)	0.320	(0.284, 0.326, 0.364)
labor supply shock	σ_L	invg	1	2	1.036	(0.852, 1.365, 1.877)	8.595	(3.413, 9.714, 16.718)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.045, 0.293, 0.844)	0.093	(0.048, 0.169, 0.335)
monetary shock	σ_R	invg	0.15	2	0.126	(0.111, 0.129, 0.147)	0.115	(0.103, 0.119, 0.134)
investment shock	σ_I	invg	0.03	2	0.501	(0.429, 0.512, 0.597)	0.438	(0.343, 0.449, 0.547)
price markup shock	σ_π	invg	0.03	2			0.326	(0.242, 0.351, 0.453)
wage markup shock	σ_w	invg	0.3	2	0.900	(0.552, 1.327, 2.063)		

Table 3.15: Posterior distribution: Calvo model estimated without price markup shock, Calvo model estimated without wage markup shock

		prior			SI prices, Calvo wages		SI prices, Calvo wages without price markup shock	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	4.853	(2.792, 4.613, 6.500)	5.439	(3.757, 5.701, 7.547)
consumption utility	σ_c	norm	1	0.375	1.593	(1.047, 1.546, 2.003)	0.804	(0.474, 0.967, 1.422)
habit persistence	h	beta	0.7	0.1	0.407	(0.315, 0.443, 0.553)	0.760	(0.526, 0.696, 0.876)
Calvo wages	ξ_w	beta	0.75	0.05	0.655	(0.614, 0.696, 0.788)	0.830	(0.783, 0.830, 0.873)
labor utility	σ_l	norm	2	0.75	2.268	(1.806, 2.774, 3.724)	1.331	(0.734, 1.818, 2.770)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.298	(0.203, 0.308, 0.426)	0.343	(0.242, 0.348, 0.457)
fixed cost	ψ	norm	1.45	0.125	1.714	(1.589, 1.749, 1.912)	1.284	(1.043, 1.228, 1.417)
Calvo employment	ξ_L	beta	0.5	0.15	0.695	(0.521, 0.645, 0.758)	0.853	(0.823, 0.849, 0.875)
response to inflation	r_π	norm	1.7	0.1	1.634	(1.461, 1.638, 1.822)	1.642	(1.468, 1.632, 1.800)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.142	(0.086, 0.150, 0.214)	0.275	(0.201, 0.275, 0.356)
interest rate smoothing	ρ	beta	0.8	0.1	0.977	(0.962, 0.976, 0.988)	0.847	(0.798, 0.843, 0.891)
response to output gap	r_y	norm	0.125	0.05	0.124	(0.052, 0.115, 0.185)	0.085	(0.037, 0.103, 0.166)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.221	(0.183, 0.228, 0.270)	-0.025	(-0.054, -0.022, 0.008)
information rigidity prices	ω_0^p	beta	0.8	0.15	0.999	(0.999, 0.999, 0.999)	0.603	(0.527, 0.597, 0.667)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.890	(0.855, 0.890, 0.928)	0.781	(0.705, 0.789, 0.861)
persistence preference shock	ρ_b	beta	0.85	0.1	0.611	(0.445, 0.588, 0.728)	0.402	(0.282, 0.440, 0.601)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.898	(0.858, 0.904, 0.952)	0.890	(0.835, 0.886, 0.941)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.983	(0.970, 0.979, 0.991)	0.999	(0.999, 0.999, 0.999)
persistence investment shock	ρ_I	beta	0.85	0.1	0.269	(0.159, 0.269, 0.366)	0.324	(0.195, 0.322, 0.447)
productivity shock	σ_a	invg	0.4	2	0.415	(0.300, 0.399, 0.497)	0.483	(0.395, 0.493, 0.585)
preference shock	σ_b	invg	0.02	2	0.119	(0.090, 0.127, 0.163)	0.180	(0.138, 0.175, 0.211)
gov. spending shock	σ_g	invg	0.15	2	0.313	(0.279, 0.315, 0.349)	0.300	(0.268, 0.306, 0.339)
labor supply shock	σ_L	invg	1	2	1.593	(1.345, 1.854, 2.369)	0.976	(0.670, 1.216, 1.647)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.049, 0.219, 0.496)	0.093	(0.051, 0.204, 0.498)
monetary shock	σ_R	invg	0.15	2	0.038	(0.030, 0.042, 0.054)	0.121	(0.109, 0.125, 0.141)
investment shock	σ_I	invg	0.03	2	0.547	(0.476, 0.561, 0.644)	0.516	(0.442, 0.524, 0.605)
price markup shock	σ_π	invg	0.03	2	0.050	(0.041, 0.054, 0.067)		
wage markup shock	σ_w	invg	0.3	2	0.513	(0.356, 0.936, 1.753)	1.594	(0.806, 2.164, 3.496)

Table 3.16: Posterior distribution: Mixed model - sticky information prices and Calvo wages, Mixed model - sticky information prices and Calvo wages estimated without price markup shock

		prior			Calvo prices, SI wages		Calvo prices, SI wages without wage markup shock	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	3.476	(2.166, 3.808, 5.523)	4.847	(2.996, 4.963, 6.870)
consumption utility	σ_c	norm	1	0.375	1.126	(0.675, 1.130, 1.524)	1.109	(0.745, 1.118, 1.486)
habit persistence	h	beta	0.7	0.1	0.564	(0.444, 0.572, 0.692)	0.377	(0.291, 0.392, 0.509)
labor utility	σ_l	norm	2	0.75	3.823	(3.169, 3.920, 4.721)	2.417	(1.790, 2.559, 3.306)
Calvo prices	ξ_p	beta	0.75	0.05	0.874	(0.849, 0.874, 0.898)	0.838	(0.811, 0.838, 0.869)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.348	(0.249, 0.354, 0.462)	0.360	(0.248, 0.365, 0.478)
fixed cost	ψ	norm	1.45	0.125	1.637	(1.493, 1.658, 1.824)	1.869	(1.716, 1.859, 2.018)
Calvo employment	ξ_L	beta	0.5	0.15	0.671	(0.574, 0.651, 0.733)	0.213	(0.089, 0.238, 0.377)
response to inflation	r_π	norm	1.7	0.1	1.664	(1.493, 1.665, 1.824)	1.568	(1.417, 1.587, 1.750)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.150	(0.084, 0.158, 0.227)	0.311	(0.233, 0.313, 0.387)
interest rate smoothing	ρ	beta	0.8	0.1	0.929	(0.893, 0.923, 0.954)	0.806	(0.758, 0.806, 0.849)
response to output gap	r_y	norm	0.125	0.05	0.060	(0.003, 0.064, 0.126)	0.091	(0.033, 0.091, 0.142)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.188	(0.156, 0.190, 0.221)	0.175	(0.135, 0.176, 0.218)
information rigidity wages	ω_0^w	beta	0.8	0.15	0.530	(0.476, 0.535, 0.593)	0.076	(0.056, 0.080, 0.105)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.917	(0.880, 0.913, 0.946)	0.912	(0.864, 0.908, 0.949)
persistence preference shock	ρ_b	beta	0.85	0.1	0.409	(0.269, 0.419, 0.554)	0.757	(0.632, 0.732, 0.832)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.916	(0.875, 0.918, 0.958)	0.963	(0.920, 0.955, 0.989)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.985	(0.969, 0.981, 0.994)	0.988	(0.973, 0.985, 0.997)
persistence investment shock	ρ_I	beta	0.85	0.1	0.239	(0.142, 0.245, 0.352)	0.271	(0.159, 0.277, 0.396)
productivity shock	σ_a	invg	0.4	2	0.406	(0.320, 0.404, 0.479)	0.270	(0.234, 0.280, 0.323)
preference shock	σ_b	invg	0.02	2	0.161	(0.125, 0.164, 0.201)	0.076	(0.057, 0.085, 0.110)
gov. spending shock	σ_g	invg	0.15	2	0.317	(0.284, 0.323, 0.358)	0.320	(0.285, 0.325, 0.365)
labor supply shock	σ_L	invg	1	2	1.544	(1.270, 1.649, 2.025)	1.237	(1.000, 1.300, 1.607)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.055, 0.146, 0.248)	0.093	(0.051, 0.181, 0.341)
monetary shock	σ_R	invg	0.15	2	0.072	(0.058, 0.078, 0.099)	0.132	(0.117, 0.135, 0.154)
investment shock	σ_I	invg	0.03	2	0.553	(0.475, 0.559, 0.640)	0.532	(0.450, 0.539, 0.630)
price markup shock	σ_π	invg	0.03	2	0.496	(0.336, 0.525, 0.707)	0.294	(0.203, 0.305, 0.408)
wage markup shock	σ_w	invg	0.3	2	0.083	(0.063, 0.093, 0.121)		

Table 3.17: Posterior distribution: Mixed model - Calvo prices and sticky information wages, Mixed model - Calvo prices and sticky information wages estimated without wage markup shock

		prior			Standard SI (J=12)		Nested Calvo - SI		Hybrid SI	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	5.615	(3.806, 5.691, 7.547)	2.169	(1.024, 2.665, 4.178)	4.172	(2.667, 4.401, 6.088)
consumption utility	σ_c	norm	1	0.375	1.446	(0.987, 1.423, 1.889)	1.161	(0.730, 1.142, 1.562)	1.277	(0.857, 1.304, 1.730)
habit persistence	h	beta	0.7	0.1	0.516	(0.422, 0.545, 0.658)	0.457	(0.353, 0.490, 0.633)	0.539	(0.400, 0.530, 0.664)
Calvo wages	ξ_w	beta	0.75	0.05			0.664	(0.598, 0.662, 0.726)		
labor utility	σ_l	norm	2	0.75	2.635	(1.856, 2.868, 3.893)	2.553	(1.878, 2.760, 3.597)	3.798	(3.012, 3.806, 4.626)
Calvo prices	ξ_p	beta	0.75	0.05			0.885	(0.864, 0.883, 0.903)		
indexation wages	γ_w	beta	0.75	0.15					0.844	(0.515, 0.745, 0.982)
indexation prices	γ_p	beta	0.75	0.15					0.661	(0.516, 0.650, 0.787)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.370	(0.269, 0.370, 0.477)	0.349	(0.252, 0.357, 0.460)	0.373	(0.276, 0.381, 0.496)
fixed cost	ψ	norm	1.45	0.125	1.735	(1.575, 1.741, 1.897)	1.484	(1.337, 1.511, 1.673)	1.592	(1.443, 1.625, 1.818)
Calvo employment	ξ_L	beta	0.5	0.15	0.159	(0.081, 0.208, 0.329)	0.707	(0.631, 0.692, 0.752)	0.743	(0.676, 0.731, 0.787)
response to inflation	r_π	norm	1.7	0.1	1.660	(1.482, 1.650, 1.818)	1.680	(1.517, 1.677, 1.827)	1.641	(1.496, 1.647, 1.813)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.145	(0.100, 0.159, 0.222)	0.147	(0.091, 0.159, 0.224)	0.198	(0.126, 0.200, 0.272)
interest rate smoothing	ρ	beta	0.8	0.1	0.948	(0.908, 0.936, 0.965)	0.964	(0.936, 0.957, 0.979)	0.902	(0.860, 0.902, 0.945)
response to output gap	r_y	norm	0.125	0.05	0.043	(0.013, 0.046, 0.081)	0.092	(0.022, 0.096, 0.163)	0.035	(0.004, 0.040, 0.078)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.103	(0.075, 0.118, 0.158)	0.196	(0.163, 0.197, 0.229)	0.160	(0.135, 0.166, 0.198)
information rigidity prices	ω_0^p	beta	0.8	0.15	0.998	(0.998, 0.998, 0.998)	0.984	(0.564, 0.854, 1.000)	0.992	(0.990, 0.993, 0.997)
information rigidity wages	ω_0^w	beta	0.8	0.15	0.856	(0.802, 0.853, 0.904)	0.822	(0.601, 0.784, 0.994)	0.545	(0.478, 0.542, 0.612)
fraction of Calvo agents, prices	α^p	beta	0.5	0.2			0.990	(0.982, 0.991, 1.000)		
fraction of Calvo agents, wages	α^w	beta	0.5	0.2			0.929	(0.830, 0.906, 0.985)		
persistence techn. shock	ρ_a	beta	0.85	0.1	0.845	(0.799, 0.843, 0.891)	0.898	(0.859, 0.898, 0.940)	0.910	(0.876, 0.907, 0.939)
persistence preference shock	ρ_b	beta	0.85	0.1	0.712	(0.545, 0.669, 0.792)	0.511	(0.360, 0.507, 0.655)	0.458	(0.327, 0.483, 0.633)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.979	(0.931, 0.965, 0.999)	0.904	(0.864, 0.904, 0.945)	0.918	(0.878, 0.917, 0.956)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.833	(0.785, 0.864, 0.947)	0.989	(0.975, 0.986, 0.997)	0.989	(0.981, 0.988, 0.996)
persistence investment shock	ρ_I	beta	0.85	0.1	0.325	(0.190, 0.309, 0.435)	0.243	(0.153, 0.252, 0.355)	0.249	(0.154, 0.256, 0.352)
productivity shock	σ_a	invg	0.4	2	0.285	(0.250, 0.299, 0.339)	0.432	(0.361, 0.442, 0.529)	0.466	(0.371, 0.464, 0.562)
preference shock	σ_b	invg	0.02	2	0.098	(0.075, 0.109, 0.143)	0.144	(0.115, 0.149, 0.189)	0.150	(0.111, 0.150, 0.187)
gov. spending shock	σ_g	invg	0.15	2	0.320	(0.284, 0.321, 0.360)	0.312	(0.279, 0.316, 0.352)	0.319	(0.286, 0.322, 0.362)
labor supply shock	σ_L	invg	1	2	4.547	(2.463, 4.242, 5.903)	1.116	(0.859, 1.208, 1.552)	1.664	(1.293, 1.679, 2.088)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.046, 0.192, 0.364)	0.093	(0.046, 0.133, 0.220)	0.093	(0.048, 0.202, 0.421)
monetary shock	σ_R	invg	0.15	2	0.052	(0.039, 0.060, 0.080)	0.051	(0.039, 0.058, 0.077)	0.077	(0.061, 0.083, 0.105)
investment shock	σ_I	invg	0.03	2	0.517	(0.441, 0.529, 0.619)	0.573	(0.496, 0.572, 0.651)	0.531	(0.460, 0.544, 0.626)
price markup shock	σ_π	invg	0.03	2	0.036	(0.025, 0.037, 0.050)	0.485	(0.323, 0.488, 0.655)	1.018	(0.632, 1.224, 2.000)
wage markup shock	σ_w	invg	0.3	2	0.936	(0.647, 1.034, 1.396)	0.439	(0.244, 0.470, 0.688)	0.090	(0.064, 0.096, 0.125)

Table 3.18: Posterior distribution, parsimonious parameterization: standard sticky information and nested Calvo - sticky information model

parameter		prior			Standard SI (J=12)		Nested Calvo - SI		Hybrid SI	
		type	mean	std/df	mode	90% post. interval	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	4.297	(2.916, 4.750, 6.779)	2.365	(0.906, 2.043, 3.551)	1.867	(0.876, 2.456, 4.174)
consumption utility	σ_c	norm	1	0.375	1.350	(0.885, 1.307, 1.710)	1.149	(0.675, 1.110, 1.539)	0.939	(0.602, 0.998, 1.372)
habit persistence	h	beta	0.7	0.1	0.471	(0.365, 0.492, 0.613)	0.462	(0.339, 0.478, 0.602)	0.454	(0.338, 0.474, 0.607)
Calvo wages	ξ_w	beta	0.75	0.05			0.691	(0.619, 0.684, 0.743)		
labor utility	σ_l	norm	2	0.75	3.584	(2.611, 3.585, 4.470)	2.550	(1.672, 2.646, 3.563)	2.245	(1.467, 2.449, 3.401)
Calvo prices	ξ_p	beta	0.75	0.05			0.883	(0.864, 0.890, 0.918)		
indexation wages	γ_w	beta	0.75	0.15					0.294	(0.168, 0.294, 0.426)
indexation prices	γ_p	beta	0.75	0.15					0.400	(0.285, 0.399, 0.497)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.364	(0.255, 0.374, 0.476)	0.346	(0.234, 0.342, 0.453)	0.337	(0.215, 0.336, 0.446)
fixed cost	ψ	norm	1.45	0.125	1.744	(1.605, 1.784, 1.944)	1.507	(1.373, 1.542, 1.724)	1.443	(1.313, 1.478, 1.633)
Calvo employment	ξ_L	beta	0.5	0.15	0.625	(0.291, 0.509, 0.693)	0.702	(0.586, 0.664, 0.755)	0.748	(0.691, 0.743, 0.793)
response to inflation	r_π	norm	1.7	0.1	1.628	(1.454, 1.642, 1.801)	1.680	(1.523, 1.686, 1.857)	1.691	(1.518, 1.687, 1.857)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.145	(0.104, 0.157, 0.218)	0.145	(0.087, 0.161, 0.235)	0.081	(0.030, 0.089, 0.152)
interest rate smoothing	ρ	beta	0.8	0.1	0.959	(0.922, 0.947, 0.971)	0.963	(0.941, 0.962, 0.980)	0.994	(0.983, 0.991, 0.999)
response to output gap	r_y	norm	0.125	0.05	0.088	(0.026, 0.077, 0.127)	0.101	(0.035, 0.096, 0.165)	0.107	(0.000, 0.089, 0.167)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.187	(0.149, 0.182, 0.218)	0.199	(0.168, 0.202, 0.236)	0.180	(0.150, 0.183, 0.215)
information rigidity prices	ω_0^p	beta	0.2	0.15	0.004	(0.001, 0.010, 0.020)	0.000	(0.000, 0.000, 0.000)	0.102	(0.019, 0.238, 0.452)
information rigidity prices	ω_1^p	beta	0.8	0.15	0.994	(0.970, 0.982, 0.997)	0.974	(0.800, 0.910, 0.996)	1.000	(1.000, 1.000, 1.000)
information rigidity wages	ω_0^w	beta	0.2	0.15	0.107	(0.103, 0.272, 0.410)	0.016	(0.003, 0.031, 0.067)	0.381	(0.156, 0.394, 0.616)
information rigidity wages	ω_1^w	beta	0.8	0.15	0.997	(0.983, 0.990, 0.996)	0.503	(0.302, 0.568, 0.820)	0.453	(0.244, 0.453, 0.632)
fraction of Calvo agents, prices	α^p	beta	0.5	0.2			0.855	(0.696, 0.825, 0.971)		
fraction of Calvo agents, wages	α^w	beta	0.5	0.2			0.643	(0.500, 0.719, 0.967)		
fraction of forwardlooking agents, prices	χ^p	beta	1.5	1.2					0.088	(0.066, 0.085, 0.105)
fraction of forwardlooking agents, wages	χ^w	beta	2.5	2.2					0.312	(0.220, 0.291, 0.366)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.881	(0.832, 0.876, 0.918)	0.901	(0.858, 0.899, 0.949)	0.922	(0.882, 0.924, 0.962)
persistence preference shock	ρ_b	beta	0.85	0.1	0.570	(0.409, 0.580, 0.734)	0.512	(0.329, 0.485, 0.638)	0.516	(0.377, 0.509, 0.640)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.911	(0.881, 0.931, 0.991)	0.903	(0.857, 0.901, 0.945)	0.899	(0.853, 0.897, 0.938)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.975	(0.920, 0.958, 0.986)	0.989	(0.979, 0.987, 0.995)	0.845	(0.661, 0.806, 0.986)
persistence investment shock	ρ_I	beta	0.85	0.1	0.236	(0.152, 0.245, 0.343)	0.238	(0.175, 0.535, 0.933)	0.271	(0.156, 0.273, 0.386)
productivity shock	σ_a	invg	0.4	2	0.379	(0.262, 0.349, 0.432)	0.427	(0.332, 0.418, 0.495)	0.439	(0.357, 0.438, 0.522)
preference shock	σ_b	invg	0.02	2	0.134	(0.091, 0.135, 0.178)	0.144	(0.111, 0.151, 0.191)	0.150	(0.119, 0.152, 0.188)
gov. spending shock	σ_g	invg	0.15	2	0.311	(0.279, 0.317, 0.354)	0.313	(0.280, 0.318, 0.351)	0.307	(0.276, 0.315, 0.352)
labor supply shock	σ_L	invg	1	2	1.935	(1.623, 2.300, 2.938)	1.146	(0.809, 1.176, 1.610)	0.445	(0.255, 0.649, 0.982)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.046, 0.147, 0.261)	0.093	(0.056, 0.887, 1.821)	0.093	(0.045, 0.163, 0.287)
monetary shock	σ_R	invg	0.15	2	0.040	(0.033, 0.050, 0.069)	0.049	(0.038, 0.053, 0.068)	0.056	(0.044, 0.064, 0.082)
investment shock	σ_I	invg	0.03	2	0.566	(0.472, 0.568, 0.646)	0.566	(0.007, 0.294, 0.610)	0.573	(0.486, 0.572, 0.648)
price markup shock	σ_π	invg	0.03	2	0.053	(0.024, 0.063, 0.104)	0.561	(0.356, 0.744, 1.109)	0.082	(0.064, 0.088, 0.110)
wage markup shock	σ_w	invg	0.3	2	0.125	(0.099, 0.151, 0.199)	0.791	(0.336, 0.723, 1.138)	0.401	(0.228, 0.519, 0.809)

Table 3.19: Posterior distribution, intermediate parameterization I: standard sticky information, nested Calvo - sticky information and hybrid sticky information model

		prior			Standard SI (J=12)		Nested Calvo - SI		Hybrid SI	
parameter		type	mean	std/df	mode	90% post. interval	mode	90% post. interval	mode	90% post. interval
investment adj. cost	S''	norm	4	1.5	2.976	(1.660, 3.166, 4.717)	2.316	(0.693, 1.997, 3.680)	1.855	(0.964, 2.538, 4.352)
consumption utility	σ_c	norm	1	0.375	1.355	(0.951, 1.380, 1.799)	1.157	(0.689, 1.089, 1.578)	0.992	(0.613, 1.088, 1.528)
habit persistence	h	beta	0.7	0.1	0.489	(0.364, 0.487, 0.592)	0.462	(0.327, 0.463, 0.604)	0.451	(0.361, 0.476, 0.611)
Calvo wages	ξ_w	beta	0.75	0.05			0.685	(0.620, 0.684, 0.755)		
labor utility	σ_l	norm	2	0.75	3.515	(2.687, 3.475, 4.265)	2.513	(1.656, 2.581, 3.456)	2.425	(1.795, 2.680, 3.720)
Calvo prices	ξ_p	beta	0.75	0.05			0.884	(0.861, 0.885, 0.903)		
indexation wages	γ_w	beta	0.75	0.15					0.262	(0.195, 0.362, 0.512)
indexation prices	γ_p	beta	0.75	0.15					0.401	(0.320, 0.437, 0.549)
capital util. adj. cost	ϕ	norm	0.2	0.075	0.368	(0.273, 0.374, 0.487)	0.347	(0.256, 0.361, 0.464)	0.333	(0.230, 0.342, 0.442)
fixed cost	ψ	norm	1.45	0.125	1.534	(1.378, 1.562, 1.746)	1.497	(1.365, 1.522, 1.701)	1.460	(1.381, 1.537, 1.688)
Calvo employment	ξ_L	beta	0.5	0.15	0.637	(0.550, 0.625, 0.699)	0.705	(0.588, 0.672, 0.746)	0.746	(0.665, 0.726, 0.778)
response to inflation	r_π	norm	1.7	0.1	1.639	(1.453, 1.626, 1.809)	1.680	(1.524, 1.678, 1.840)	1.691	(1.530, 1.687, 1.851)
response to diff. inflation	$r_{d\pi}$	norm	0.3	0.1	0.163	(0.110, 0.179, 0.243)	0.146	(0.104, 0.168, 0.228)	0.082	(0.037, 0.102, 0.166)
interest rate smoothing	ρ	beta	0.8	0.1	0.937	(0.884, 0.924, 0.965)	0.962	(0.938, 0.961, 0.983)	0.994	(0.981, 0.989, 0.997)
response to output gap	r_y	norm	0.125	0.05	0.081	(0.011, 0.081, 0.159)	0.094	(0.031, 0.087, 0.154)	0.105	(0.023, 0.109, 0.194)
response to diff. output gap	r_{dy}	norm	0.063	0.05	0.193	(0.161, 0.193, 0.228)	0.197	(0.168, 0.204, 0.239)	0.180	(0.154, 0.186, 0.219)
information rigidity prices	ω_0^p	beta	0.2	0.15	0.020	(0.014, 0.050, 0.091)	0.001	(0.000, 0.018, 0.039)	0.144	(0.194, 0.417, 0.646)
information rigidity prices	ω_1^p	beta	0.2	0.15	0.083	(0.101, 0.289, 0.482)	0.056	(0.003, 0.208, 0.445)	0.002	(0.000, 0.019, 0.041)
information rigidity prices	ω_5^p	beta	0.2	0.15	0.057	(0.053, 0.224, 0.373)	0.100	(0.027, 0.294, 0.547)	0.001	(0.000, 0.011, 0.020)
information rigidity wages	ω_0^w	beta	0.2	0.15	0.130	(0.142, 0.396, 0.647)	0.010	(0.000, 0.029, 0.063)	0.139	(0.067, 0.319, 0.546)
information rigidity wages	ω_1^w	beta	0.2	0.15	0.020	(0.020, 0.077, 0.129)	0.137	(0.053, 0.343, 0.601)	0.008	(0.022, 0.088, 0.162)
information rigidity wages	ω_5^w	beta	0.2	0.15	0.000	(0.000, 0.000, 0.000)	0.003	(0.060, 0.129, 0.206)	0.002	(0.000, 0.013, 0.030)
fraction of Calvo agents, prices	α^p	beta	0.5	0.2			0.927	(0.860, 0.930, 0.995)		
fraction of Calvo agents, wages	α^w	beta	0.5	0.2			0.702	(0.676, 0.819, 0.975)		
fraction of forwardlooking agents, prices	χ^p	beta	1.5	1.2					0.086	(0.059, 0.080, 0.099)
fraction of forwardlooking agents, wages	χ^w	beta	2.5	2.2					0.321	(0.159, 0.288, 0.458)
persistence techn. shock	ρ_a	beta	0.85	0.1	0.863	(0.821, 0.861, 0.908)	0.898	(0.846, 0.892, 0.930)	0.924	(0.885, 0.921, 0.964)
persistence preference shock	ρ_b	beta	0.85	0.1	0.436	(0.318, 0.441, 0.559)	0.511	(0.315, 0.474, 0.616)	0.518	(0.383, 0.520, 0.653)
persistence gov. spending. shock	ρ_g	beta	0.85	0.1	0.922	(0.880, 0.920, 0.961)	0.903	(0.856, 0.897, 0.938)	0.901	(0.853, 0.896, 0.945)
persistence labor supply shock	ρ_L	beta	0.85	0.1	0.983	(0.972, 0.981, 0.991)	0.989	(0.977, 0.987, 0.996)	0.852	(0.680, 0.856, 0.999)
persistence investment shock	ρ_I	beta	0.85	0.1	0.234	(0.142, 0.235, 0.326)	0.238	(0.173, 0.505, 0.953)	0.275	(0.167, 0.284, 0.389)
productivity shock	σ_a	invg	0.4	2	0.412	(0.331, 0.416, 0.497)	0.431	(0.340, 0.416, 0.483)	0.435	(0.346, 0.428, 0.498)
preference shock	σ_b	invg	0.02	2	0.164	(0.129, 0.163, 0.197)	0.143	(0.119, 0.157, 0.206)	0.147	(0.112, 0.150, 0.189)
gov. spending shock	σ_g	invg	0.15	2	0.313	(0.285, 0.317, 0.357)	0.313	(0.281, 0.315, 0.356)	0.308	(0.279, 0.318, 0.356)
labor supply shock	σ_L	invg	1	2	1.618	(1.306, 1.669, 2.027)	1.135	(0.748, 1.130, 1.512)	0.443	(0.267, 0.726, 1.125)
equity premium shock	σ_q	invg	0.2	2	0.093	(0.048, 0.159, 0.305)	0.093	(0.053, 0.670, 1.436)	0.093	(0.041, 0.160, 0.322)
monetary shock	σ_R	invg	0.15	2	0.060	(0.046, 0.066, 0.088)	0.050	(0.035, 0.052, 0.071)	0.057	(0.048, 0.065, 0.083)
investment shock	σ_I	invg	0.03	2	0.573	(0.496, 0.582, 0.666)	0.567	(0.008, 0.347, 0.620)	0.574	(0.486, 0.568, 0.667)
price markup shock	σ_π	invg	0.03	2	0.099	(0.064, 0.166, 0.286)	0.528	(0.331, 0.555, 0.736)	0.086	(0.079, 0.120, 0.161)
wage markup shock	σ_w	invg	0.3	2	0.132	(0.090, 0.163, 0.221)	0.717	(0.344, 0.704, 1.118)	0.204	(0.118, 0.616, 1.148)

Table 3.20: Posterior distribution, intermediate parameterization II: standard sticky information, nested Calvo - sticky information and hybrid sticky information model

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(1.6, 5.5, 17.3)	(8.6, 17.8, 24.1)	(22.6, 29.9, 38.2)	(0.0, 22.1, 27.9)	(5.5, 8.9, 15.2)	(5.9, 10.3, 18.4)	(0.6, 1.6, 3.4)	(0.0, 0.0, 13.2)	(1.2, 1.8, 2.9)
	4	(13.1, 23.8, 39.3)	(2.5, 7.9, 12.7)	(7.7, 12.4, 17.7)	(0.0, 14.2, 20.0)	(12.1, 17.7, 24.4)	(11.3, 18.7, 28.3)	(1.5, 3.2, 6.9)	(0.0, 0.0, 3.7)	(0.4, 0.7, 1.3)
	8	(18.1, 31.9, 45.5)	(1.2, 3.9, 6.7)	(4.1, 7.1, 10.8)	(0.0, 8.4, 13.1)	(15.6, 22.7, 31.2)	(12.0, 20.9, 30.9)	(1.7, 3.7, 8.1)	(0.0, 0.0, 1.9)	(0.2, 0.4, 0.7)
	20	(17.0, 32.3, 48.0)	(0.7, 2.1, 3.7)	(2.3, 4.0, 6.3)	(0.0, 4.6, 7.6)	(23.0, 33.8, 45.8)	(9.8, 18.2, 29.5)	(1.4, 3.1, 6.7)	(0.0, 0.0, 1.0)	(0.2, 0.3, 0.8)
Standard Calvo	1	(2.6, 6.4, 12.7)	(12.0, 17.4, 23.7)	(22.3, 28.8, 36.0)	(18.2, 22.6, 28.5)	(5.6, 8.9, 13.7)	(6.2, 10.1, 15.2)	(1.6, 2.6, 4.6)	(0.0, 0.0, 0.4)	(0.9, 1.4, 2.1)
	4	(15.5, 24.5, 35.4)	(4.4, 8.0, 13.1)	(8.4, 12.4, 17.1)	(10.4, 15.2, 21.2)	(12.3, 17.5, 24.2)	(11.7, 17.9, 24.8)	(1.2, 2.3, 5.0)	(0.0, 0.0, 0.2)	(0.3, 0.5, 0.8)
	8	(20.6, 31.1, 43.9)	(2.1, 4.1, 7.2)	(4.7, 7.4, 10.8)	(5.8, 9.4, 14.2)	(16.2, 22.8, 31.1)	(13.5, 20.7, 29.1)	(1.0, 2.0, 4.6)	(0.0, 0.0, 0.1)	(0.2, 0.3, 0.4)
	20	(17.5, 29.1, 44.5)	(1.2, 2.2, 4.0)	(2.6, 4.3, 6.5)	(3.1, 5.3, 8.5)	(24.5, 34.2, 46.3)	(12.3, 20.5, 30.9)	(0.8, 1.6, 3.9)	(0.0, 0.0, 0.1)	(0.1, 0.2, 0.4)
Standard SI	1	(0.0, 0.0, 0.4)	(17.3, 22.4, 28.2)	(20.7, 26.1, 31.3)	(20.5, 25.4, 31.4)	(3.6, 8.2, 14.6)	(5.8, 9.0, 13.6)	(0.1, 0.4, 0.8)	(0.0, 0.0, 0.1)	(5.4, 7.2, 10.2)
	4	(0.3, 1.3, 3.6)	(12.6, 18.9, 26.7)	(11.2, 14.9, 19.4)	(16.5, 23.4, 32.0)	(5.6, 15.9, 31.5)	(14.5, 21.6, 31.8)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.0)	(1.0, 1.4, 2.2)
	8	(0.5, 2.2, 5.6)	(7.2, 12.6, 19.9)	(8.3, 12.5, 17.3)	(11.1, 18.4, 26.8)	(5.2, 18.7, 41.4)	(20.3, 31.1, 45.7)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.5, 0.8, 1.2)
	20	(0.7, 2.8, 8.2)	(4.1, 9.2, 15.0)	(5.5, 11.8, 20.8)	(6.8, 13.7, 23.0)	(5.5, 28.8, 63.8)	(13.5, 28.8, 45.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.4, 0.6, 0.9)
Hybrid SI	1	(0.0, 1.2, 5.9)	(16.8, 22.8, 29.9)	(22.8, 28.8, 35.1)	(17.8, 23.5, 29.9)	(4.2, 7.1, 12.1)	(5.7, 9.1, 13.8)	(1.2, 5.3, 8.4)	(0.0, 0.0, 0.1)	(0.5, 0.8, 1.3)
	4	(1.8, 6.7, 17.4)	(7.5, 11.7, 18.6)	(8.6, 12.6, 17.0)	(12.6, 17.7, 25.6)	(8.4, 12.5, 18.3)	(13.8, 20.6, 29.2)	(4.1, 15.4, 23.7)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.2)
	8	(2.2, 8.4, 19.4)	(3.7, 6.4, 11.8)	(5.1, 8.2, 12.6)	(8.3, 12.3, 20.5)	(11.3, 15.8, 22.1)	(18.7, 29.8, 41.1)	(6.1, 15.7, 25.7)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)
	20	(4.7, 12.6, 25.7)	(2.0, 4.0, 7.0)	(2.8, 5.2, 8.6)	(4.5, 7.7, 13.5)	(18.4, 29.5, 40.4)	(12.7, 24.8, 52.5)	(5.7, 10.9, 18.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
Calvo-SI	1	(2.3, 6.1, 13.0)	(12.4, 18.0, 23.9)	(22.1, 28.5, 35.5)	(18.3, 23.0, 28.2)	(4.8, 8.1, 13.1)	(6.0, 10.0, 15.1)	(2.0, 3.0, 5.5)	(0.0, 0.0, 0.2)	(1.0, 1.6, 2.1)
	4	(14.9, 25.8, 36.8)	(4.4, 8.3, 13.3)	(8.0, 11.9, 16.4)	(9.7, 15.0, 21.0)	(11.6, 16.4, 23.3)	(12.5, 17.8, 25.0)	(1.2, 2.4, 4.8)	(0.0, 0.0, 0.1)	(0.3, 0.5, 0.8)
	8	(19.8, 33.7, 45.5)	(2.1, 4.3, 7.4)	(4.5, 7.0, 10.3)	(5.7, 9.2, 13.3)	(15.1, 21.6, 30.0)	(13.9, 20.5, 29.5)	(0.9, 2.0, 4.3)	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.4)
	20	(16.7, 32.6, 46.7)	(1.1, 2.4, 4.0)	(2.5, 4.1, 6.3)	(3.1, 5.2, 8.0)	(23.1, 33.7, 45.7)	(12.7, 19.4, 29.6)	(0.7, 1.4, 3.5)	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.3)

Table 3.21: Variance decomposition: Output

Note: The abbreviations for the models used in the table are as follows: **SW** refers to the Calvo specification with indexation as e.g. in Smets and Wouters (2003) model. **Calvo** refers to the Calvo model without indexation. **Standard SI** refers to the parsimoniously parameterized sticky information model. **Hybrid SI** refers to the parsimonious parameterization of the sticky information model with fraction of backward-looking agents. **Calvo-SI** is the nested Calvo - sticky information model.

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(42.8, 53.9, 63.7)	(20.0, 26.9, 35.0)	(2.2, 3.6, 5.8)	(0.0, 3.3, 4.8)	(1.4, 3.1, 5.6)	(0.0, 1.6, 8.7)	(0.1, 2.3, 7.4)	(0.0, 0.0, 11.5)	(1.4, 2.1, 3.8)
	4	(43.7, 55.2, 64.6)	(16.0, 21.5, 30.4)	(2.7, 4.1, 6.5)	(0.0, 6.4, 9.8)	(1.1, 2.9, 5.8)	(1.7, 3.8, 11.9)	(0.5, 1.4, 3.5)	(0.0, 0.0, 8.0)	(1.0, 2.0, 4.0)
	8	(36.7, 51.2, 62.8)	(12.2, 16.8, 25.1)	(2.7, 3.9, 6.5)	(0.0, 6.7, 10.4)	(0.9, 2.8, 5.9)	(4.3, 10.3, 27.6)	(1.4, 2.9, 5.8)	(0.0, 0.0, 6.1)	(0.9, 1.8, 3.8)
	20	(28.6, 44.5, 57.9)	(9.7, 14.2, 21.4)	(2.3, 3.6, 6.1)	(0.0, 6.0, 9.4)	(0.8, 2.7, 6.2)	(7.9, 19.4, 41.4)	(2.3, 4.4, 7.9)	(0.0, 0.0, 4.5)	(0.8, 1.6, 3.5)
Standard Calvo	1	(45.7, 56.3, 65.8)	(19.6, 26.9, 35.3)	(2.2, 3.6, 5.8)	(2.3, 3.7, 5.7)	(1.5, 3.2, 6.3)	(0.0, 1.2, 6.5)	(0.1, 1.5, 5.1)	(0.0, 0.0, 0.3)	(1.1, 1.9, 3.0)
	4	(44.7, 55.3, 65.1)	(15.3, 22.4, 32.8)	(2.7, 4.1, 6.3)	(4.8, 7.2, 10.9)	(1.3, 3.0, 6.4)	(1.7, 3.3, 8.1)	(0.6, 1.4, 2.8)	(0.0, 0.0, 0.2)	(0.8, 1.4, 2.5)
	8	(41.1, 51.8, 62.6)	(12.3, 18.3, 28.1)	(2.8, 4.1, 6.3)	(5.2, 7.8, 11.4)	(1.1, 2.9, 6.3)	(4.3, 9.4, 20.2)	(1.0, 2.0, 3.5)	(0.0, 0.0, 0.2)	(0.7, 1.2, 2.3)
	20	(33.9, 45.5, 57.9)	(9.9, 15.5, 24.1)	(2.5, 3.8, 5.9)	(4.4, 6.9, 10.4)	(1.0, 2.8, 6.5)	(9.0, 19.1, 36.8)	(1.3, 2.4, 4.2)	(0.0, 0.0, 0.2)	(0.6, 1.0, 2.0)
Standard SI	1	(6.3, 13.9, 26.4)	(10.2, 16.9, 26.2)	(0.2, 0.7, 1.5)	(0.6, 1.4, 3.0)	(22.9, 45.4, 68.2)	(3.4, 10.4, 25.5)	(0.1, 0.6, 2.0)	(0.0, 0.0, 0.0)	(3.8, 7.1, 12.5)
	4	(6.3, 12.8, 24.9)	(11.5, 19.2, 29.3)	(0.3, 0.9, 1.9)	(1.7, 3.4, 7.0)	(32.6, 55.2, 72.5)	(1.5, 3.7, 11.2)	(0.0, 0.2, 0.9)	(0.0, 0.0, 0.0)	(1.0, 1.9, 3.8)
	8	(6.0, 12.1, 22.3)	(10.2, 16.8, 26.3)	(0.4, 1.2, 2.3)	(2.3, 4.4, 8.5)	(37.7, 57.2, 72.5)	(2.5, 4.6, 10.4)	(0.0, 0.2, 0.6)	(0.0, 0.0, 0.0)	(0.7, 1.3, 2.7)
	20	(5.7, 11.3, 19.7)	(8.9, 15.1, 23.6)	(0.7, 1.6, 2.8)	(2.5, 4.3, 8.4)	(39.2, 57.7, 70.7)	(4.5, 7.4, 13.9)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.0)	(0.6, 1.1, 2.3)
Hybrid SI	1	(36.2, 47.8, 59.3)	(13.7, 20.3, 27.5)	(1.6, 2.8, 5.0)	(2.0, 3.1, 5.0)	(1.5, 2.9, 4.8)	(8.2, 16.4, 28.1)	(0.2, 4.3, 10.5)	(0.0, 0.0, 0.1)	(0.3, 0.5, 1.0)
	4	(43.3, 53.8, 66.1)	(10.9, 18.2, 28.9)	(2.2, 3.7, 6.3)	(5.1, 8.0, 11.5)	(1.2, 2.8, 5.2)	(3.9, 7.3, 15.0)	(0.6, 2.6, 7.6)	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.4)
	8	(38.8, 53.1, 63.9)	(8.2, 13.6, 22.5)	(2.4, 4.0, 6.3)	(6.4, 9.6, 13.2)	(0.9, 2.5, 4.9)	(6.2, 10.3, 16.1)	(2.4, 5.2, 10.4)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)
	20	(30.1, 48.7, 60.4)	(6.4, 10.9, 18.3)	(2.0, 4.1, 6.4)	(5.8, 8.6, 13.0)	(0.8, 2.3, 4.9)	(8.3, 15.7, 38.5)	(3.3, 6.5, 12.7)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.2)
Calvo-SI	1	(45.3, 56.3, 65.2)	(19.5, 26.9, 36.4)	(2.1, 3.6, 5.9)	(2.2, 3.7, 5.9)	(1.3, 2.6, 5.0)	(0.0, 1.4, 6.6)	(0.0, 1.7, 5.6)	(0.0, 0.0, 0.1)	(1.2, 1.9, 2.8)
	4	(42.2, 55.1, 65.4)	(15.5, 23.5, 34.1)	(2.6, 4.1, 6.3)	(4.7, 7.3, 11.4)	(1.0, 2.5, 5.2)	(1.8, 3.0, 7.6)	(0.7, 1.5, 3.3)	(0.0, 0.0, 0.1)	(0.7, 1.2, 2.1)
	8	(38.6, 51.8, 62.9)	(13.0, 19.6, 30.1)	(2.7, 4.1, 6.1)	(5.2, 7.8, 11.8)	(0.9, 2.4, 5.1)	(4.8, 9.0, 20.6)	(1.1, 2.0, 3.9)	(0.0, 0.0, 0.1)	(0.6, 1.0, 1.9)
	20	(33.0, 46.2, 58.7)	(10.9, 16.6, 26.7)	(2.4, 3.9, 5.8)	(4.6, 7.1, 10.5)	(0.8, 2.3, 5.2)	(9.6, 17.7, 33.1)	(1.3, 2.3, 4.3)	(0.0, 0.0, 0.1)	(0.5, 0.9, 1.7)

Table 3.22: Variance decomposition: Nominal interest rate

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(0.7, 1.5, 3.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.9, 2.0, 4.1)	(92.4, 95.6, 97.4)	(0.0, 0.0, 0.0)	(0.3, 0.5, 1.4)
	4	(2.7, 5.5, 9.9)	(0.1, 0.2, 0.7)	(0.1, 0.2, 0.4)	(0.0, 0.2, 0.5)	(0.0, 0.1, 0.5)	(6.1, 11.4, 20.2)	(70.3, 79.3, 86.4)	(0.0, 0.0, 0.0)	(0.8, 1.8, 4.7)
	8	(3.1, 6.3, 11.2)	(0.1, 0.2, 0.9)	(0.1, 0.3, 0.6)	(0.0, 0.2, 0.7)	(0.0, 0.2, 0.8)	(11.5, 20.1, 33.4)	(57.5, 69.4, 79.1)	(0.0, 0.0, 0.0)	(0.8, 1.9, 5.1)
	20	(2.7, 5.8, 10.5)	(0.1, 0.3, 1.0)	(0.1, 0.3, 0.7)	(0.0, 0.2, 0.7)	(0.0, 0.4, 1.3)	(16.3, 27.5, 43.8)	(48.4, 62.1, 73.7)	(0.0, 0.0, 0.0)	(0.7, 1.7, 4.8)
Standard Calvo	1	(1.3, 2.7, 5.2)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(2.4, 4.3, 7.9)	(87.2, 91.5, 94.6)	(0.0, 0.0, 0.0)	(0.3, 0.7, 1.6)
	4	(2.7, 5.6, 10.2)	(0.1, 0.3, 0.9)	(0.1, 0.2, 0.5)	(0.1, 0.2, 0.6)	(0.0, 0.2, 0.6)	(8.0, 13.7, 23.2)	(67.9, 77.6, 84.6)	(0.0, 0.0, 0.0)	(0.6, 1.3, 2.9)
	8	(2.8, 6.1, 11.4)	(0.1, 0.4, 1.2)	(0.1, 0.3, 0.7)	(0.1, 0.3, 0.8)	(0.0, 0.3, 1.0)	(12.8, 21.4, 34.1)	(56.3, 69.0, 77.7)	(0.0, 0.0, 0.0)	(0.5, 1.3, 2.9)
	20	(2.5, 5.7, 11.2)	(0.1, 0.4, 1.4)	(0.1, 0.4, 0.9)	(0.1, 0.3, 0.9)	(0.1, 0.5, 1.5)	(18.0, 29.4, 45.1)	(45.9, 61.1, 71.7)	(0.0, 0.0, 0.0)	(0.5, 1.1, 2.7)
Standard SI	1	(3.2, 5.6, 9.3)	(0.1, 0.1, 0.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(37.7, 61.4, 75.9)	(0.0, 0.0, 0.0)	(19.9, 32.5, 53.2)
	4	(2.8, 5.4, 8.7)	(0.3, 0.8, 1.9)	(0.0, 0.1, 0.2)	(0.2, 0.3, 0.7)	(0.6, 2.1, 5.4)	(0.9, 1.6, 3.9)	(35.5, 58.1, 73.4)	(0.0, 0.0, 0.0)	(18.5, 30.5, 50.0)
	8	(2.5, 4.3, 6.8)	(0.3, 1.2, 3.5)	(0.2, 0.5, 1.0)	(0.4, 0.9, 2.0)	(3.6, 10.0, 19.4)	(9.6, 17.7, 33.5)	(20.6, 41.2, 60.0)	(0.0, 0.0, 0.0)	(13.5, 21.5, 31.7)
	20	(1.4, 2.4, 4.3)	(0.5, 1.8, 4.6)	(0.5, 1.2, 2.0)	(0.6, 1.5, 3.3)	(5.5, 17.4, 35.2)	(30.6, 48.8, 66.9)	(5.6, 16.3, 30.9)	(0.0, 0.0, 0.0)	(5.8, 8.4, 11.2)
Hybrid SI	1	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(99.9, 100.0, 100.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	4	(0.2, 1.1, 2.4)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.0)	(0.1, 0.3, 0.7)	(96.5, 98.4, 99.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	8	(0.6, 3.9, 8.3)	(0.1, 0.3, 1.1)	(0.0, 0.3, 0.8)	(0.1, 0.8, 1.9)	(0.0, 0.1, 0.4)	(1.7, 5.0, 12.1)	(79.1, 88.4, 97.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	20	(2.0, 6.4, 12.6)	(0.2, 0.6, 1.5)	(0.2, 0.7, 1.4)	(0.4, 1.4, 3.0)	(0.0, 0.3, 1.0)	(8.0, 19.0, 32.9)	(58.4, 70.8, 81.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Calvo-SI	1	(1.5, 2.7, 4.5)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(2.4, 4.1, 7.0)	(88.1, 92.0, 94.9)	(0.0, 0.0, 0.0)	(0.3, 0.6, 1.2)
	4	(3.0, 5.3, 8.4)	(0.1, 0.3, 0.8)	(0.1, 0.2, 0.5)	(0.0, 0.2, 0.6)	(0.0, 0.1, 0.4)	(8.2, 13.2, 21.1)	(70.6, 78.9, 85.7)	(0.0, 0.0, 0.0)	(0.4, 1.0, 2.0)
	8	(3.0, 5.4, 9.1)	(0.1, 0.3, 1.0)	(0.1, 0.3, 0.6)	(0.0, 0.2, 0.8)	(0.0, 0.2, 0.6)	(13.2, 20.5, 31.9)	(60.6, 70.5, 79.4)	(0.0, 0.0, 0.0)	(0.4, 1.0, 2.0)
	20	(2.7, 5.1, 8.8)	(0.1, 0.3, 1.1)	(0.1, 0.3, 0.8)	(0.1, 0.2, 0.8)	(0.0, 0.3, 1.1)	(17.4, 27.5, 41.5)	(50.4, 63.8, 74.7)	(0.0, 0.0, 0.0)	(0.4, 0.9, 1.9)

Table 3.23: Variance decomposition: Inflation

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(0.0, 0.0, 0.3)	(0.2, 0.6, 2.2)	(0.0, 0.1, 0.4)	(0.0, 0.2, 0.8)	(0.0, 0.2, 0.6)	(1.3, 3.3, 7.9)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(88.5, 95.3, 98.2)
	4	(0.0, 0.1, 1.0)	(0.2, 0.7, 2.4)	(0.1, 0.2, 0.7)	(0.0, 0.5, 1.5)	(0.1, 0.3, 1.0)	(5.4, 11.6, 22.4)	(1.1, 3.8, 8.4)	(0.0, 0.0, 0.1)	(69.6, 81.6, 89.4)
	8	(0.0, 0.2, 1.1)	(0.2, 0.6, 2.2)	(0.1, 0.3, 0.7)	(0.0, 0.5, 1.4)	(0.1, 0.3, 1.0)	(8.2, 16.9, 29.7)	(1.2, 3.8, 8.4)	(0.0, 0.0, 0.1)	(62.9, 76.2, 86.4)
	20	(0.1, 0.3, 1.3)	(0.2, 0.6, 2.2)	(0.1, 0.3, 0.7)	(0.0, 0.5, 1.4)	(0.1, 0.4, 1.1)	(9.4, 19.4, 33.5)	(1.2, 3.9, 8.3)	(0.0, 0.0, 0.1)	(59.1, 73.4, 84.9)
Standard Calvo	1	(0.0, 0.1, 0.8)	(0.3, 1.0, 2.6)	(0.1, 0.2, 0.6)	(0.1, 0.4, 1.0)	(0.1, 0.3, 0.8)	(2.4, 5.5, 10.8)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(84.8, 92.2, 96.7)
	4	(0.0, 0.1, 1.0)	(0.3, 1.1, 3.1)	(0.1, 0.4, 0.9)	(0.2, 0.7, 1.8)	(0.1, 0.5, 1.3)	(7.0, 14.3, 25.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.0)	(69.9, 82.2, 91.2)
	8	(0.0, 0.2, 1.2)	(0.3, 1.1, 2.9)	(0.1, 0.4, 0.9)	(0.2, 0.7, 1.8)	(0.1, 0.5, 1.3)	(9.8, 18.7, 31.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.0)	(64.1, 77.7, 87.9)
	20	(0.1, 0.5, 1.8)	(0.3, 1.0, 2.8)	(0.1, 0.3, 0.8)	(0.2, 0.7, 1.7)	(0.1, 0.5, 1.3)	(11.5, 21.1, 34.9)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.0)	(60.7, 75.0, 85.6)
Standard SI	1	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.1, 0.4, 1.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(98.4, 99.3, 99.8)
	4	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.1, 0.5, 1.1)	(0.1, 0.2, 0.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(98.2, 99.2, 99.7)
	8	(0.0, 0.0, 0.1)	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)	(0.2, 0.6, 1.3)	(0.3, 0.7, 1.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(96.8, 98.4, 99.2)
	20	(0.0, 0.0, 0.1)	(0.1, 0.1, 0.4)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.4, 0.8, 1.6)	(0.9, 1.6, 2.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(95.4, 97.2, 98.3)
Hybrid SI	1	(1.1, 4.4, 8.7)	(5.9, 10.8, 18.3)	(0.5, 1.3, 2.3)	(0.5, 1.2, 2.2)	(0.8, 1.7, 3.4)	(1.1, 2.3, 4.3)	(0.0, 0.0, 0.5)	(0.0, 0.0, 0.0)	(64.1, 78.0, 87.9)
	4	(0.6, 3.2, 8.3)	(3.5, 6.0, 10.8)	(0.5, 1.5, 2.8)	(2.0, 4.5, 8.3)	(0.5, 1.3, 2.7)	(9.6, 17.9, 30.0)	(0.3, 1.9, 5.7)	(0.0, 0.0, 0.0)	(44.0, 60.8, 78.4)
	8	(0.7, 3.2, 8.2)	(3.0, 5.4, 10.7)	(0.4, 1.5, 2.7)	(1.9, 4.4, 8.1)	(0.4, 1.2, 2.4)	(14.7, 26.4, 41.2)	(0.8, 2.7, 7.7)	(0.0, 0.0, 0.0)	(35.2, 52.2, 70.6)
	20	(1.3, 3.3, 8.8)	(2.8, 5.1, 10.3)	(0.4, 1.6, 2.7)	(1.9, 4.7, 8.6)	(0.4, 1.2, 2.5)	(16.3, 27.7, 42.6)	(1.0, 3.9, 11.0)	(0.0, 0.0, 0.0)	(32.3, 49.5, 67.7)
Calvo-SI	1	(0.0, 0.1, 0.8)	(0.4, 1.2, 3.2)	(0.1, 0.2, 0.6)	(0.2, 0.4, 1.2)	(0.1, 0.2, 0.7)	(2.7, 5.2, 9.7)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(84.9, 92.2, 95.9)
	4	(0.0, 0.2, 0.8)	(0.4, 1.3, 3.5)	(0.1, 0.4, 0.9)	(0.3, 0.8, 1.9)	(0.1, 0.4, 1.0)	(7.4, 13.6, 23.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.0)	(71.3, 82.8, 90.3)
	8	(0.0, 0.2, 0.9)	(0.4, 1.2, 3.4)	(0.1, 0.3, 0.9)	(0.2, 0.7, 1.9)	(0.1, 0.4, 1.0)	(10.1, 18.2, 30.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.0)	(65.6, 78.5, 87.2)
	20	(0.2, 0.5, 1.5)	(0.4, 1.2, 3.3)	(0.1, 0.3, 0.9)	(0.3, 0.7, 1.9)	(0.1, 0.4, 1.0)	(11.6, 20.2, 33.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.0)	(62.5, 75.7, 85.6)

Table 3.24: Variance decomposition: Wage inflation

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(0.0, 0.3, 0.7)	(0.1, 0.4, 1.3)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.3, 1.0, 2.6)	(16.6, 23.3, 31.6)	(0.0, 0.0, 0.0)	(65.1, 74.3, 81.6)
	4	(0.7, 2.7, 6.1)	(0.3, 0.9, 3.1)	(0.1, 0.2, 0.7)	(0.0, 0.6, 2.0)	(0.1, 0.4, 1.2)	(2.4, 6.6, 14.9)	(15.3, 24.4, 37.2)	(0.0, 0.0, 0.1)	(46.0, 62.6, 75.8)
	8	(2.6, 8.4, 17.6)	(0.3, 0.9, 3.1)	(0.1, 0.3, 1.0)	(0.0, 1.1, 3.0)	(0.2, 0.6, 1.8)	(6.6, 15.6, 30.3)	(14.1, 23.4, 36.2)	(0.0, 0.0, 0.2)	(29.4, 46.9, 63.8)
	20	(7.0, 20.0, 40.2)	(0.2, 0.6, 2.2)	(0.1, 0.2, 0.7)	(0.0, 1.2, 3.0)	(0.3, 0.7, 1.8)	(12.2, 26.4, 45.9)	(10.7, 18.6, 30.3)	(0.0, 0.0, 0.2)	(15.4, 27.9, 44.0)
Standard Calvo	1	(0.0, 0.3, 0.8)	(0.1, 0.6, 1.6)	(0.0, 0.1, 0.3)	(0.1, 0.2, 0.5)	(0.0, 0.1, 0.4)	(0.4, 1.2, 2.8)	(17.2, 23.9, 31.9)	(0.0, 0.0, 0.0)	(64.5, 73.1, 80.4)
	4	(0.8, 2.9, 6.4)	(0.4, 1.4, 4.1)	(0.1, 0.3, 0.9)	(0.3, 1.0, 2.4)	(0.2, 0.6, 1.5)	(3.0, 8.2, 17.2)	(17.3, 23.4, 31.3)	(0.0, 0.0, 0.0)	(47.8, 60.7, 72.0)
	8	(2.8, 8.6, 17.7)	(0.4, 1.5, 4.5)	(0.1, 0.4, 1.2)	(0.6, 1.7, 3.8)	(0.3, 0.9, 2.2)	(8.0, 19.3, 34.6)	(14.4, 20.3, 28.2)	(0.0, 0.0, 0.0)	(31.0, 44.9, 59.6)
	20	(6.0, 17.5, 35.4)	(0.3, 1.0, 3.2)	(0.1, 0.3, 1.0)	(0.8, 1.9, 4.3)	(0.4, 0.9, 2.1)	(17.0, 34.1, 53.4)	(9.0, 14.6, 22.5)	(0.0, 0.0, 0.0)	(15.9, 26.2, 41.1)
Standard SI	1	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.1, 0.4, 1.2)	(0.0, 0.0, 0.1)	(0.3, 0.8, 1.6)	(0.0, 0.0, 0.0)	(97.2, 98.5, 99.3)
	4	(0.1, 0.3, 0.6)	(0.5, 1.2, 3.1)	(0.0, 0.0, 0.1)	(0.1, 0.3, 0.6)	(1.6, 5.1, 11.6)	(0.5, 1.0, 2.5)	(0.3, 0.7, 1.4)	(0.0, 0.0, 0.0)	(83.2, 90.9, 95.8)
	8	(0.3, 1.0, 2.2)	(0.6, 2.0, 4.7)	(0.0, 0.1, 0.3)	(0.5, 0.9, 1.8)	(4.6, 11.6, 21.4)	(3.2, 5.7, 12.4)	(0.2, 0.6, 1.2)	(0.0, 0.0, 0.0)	(61.2, 77.2, 87.9)
	20	(0.8, 2.6, 7.4)	(0.6, 1.9, 4.7)	(0.1, 0.2, 0.4)	(1.0, 1.8, 3.0)	(5.7, 12.6, 22.6)	(5.4, 8.9, 17.1)	(0.2, 0.6, 1.1)	(0.0, 0.0, 0.0)	(54.1, 69.5, 81.0)
Hybrid SI	1	(0.7, 2.4, 5.2)	(4.0, 6.8, 12.4)	(0.4, 0.8, 1.5)	(0.4, 0.8, 1.4)	(0.5, 1.1, 2.2)	(0.7, 1.4, 2.7)	(25.3, 36.4, 45.3)	(0.0, 0.0, 0.0)	(39.2, 49.2, 64.5)
	4	(0.3, 3.1, 8.5)	(4.0, 8.0, 15.4)	(1.1, 2.2, 3.7)	(3.6, 5.9, 9.5)	(0.8, 2.0, 4.0)	(11.7, 19.3, 33.9)	(35.7, 50.7, 61.1)	(0.0, 0.0, 0.0)	(3.6, 6.1, 13.5)
	8	(0.3, 2.0, 7.6)	(1.8, 4.3, 9.4)	(0.9, 2.5, 4.2)	(3.7, 6.9, 12.3)	(0.7, 2.0, 4.1)	(25.7, 41.3, 60.7)	(20.5, 36.1, 48.8)	(0.0, 0.0, 0.0)	(1.1, 2.2, 4.0)
	20	(0.3, 6.0, 12.8)	(1.2, 3.4, 7.7)	(0.6, 2.3, 3.9)	(2.6, 6.2, 12.3)	(0.5, 2.0, 4.2)	(29.1, 47.8, 79.5)	(12.2, 29.1, 42.1)	(0.0, 0.0, 0.0)	(0.6, 1.4, 2.7)
Calvo-SI	1	(0.0, 0.3, 0.8)	(0.2, 0.7, 1.9)	(0.0, 0.1, 0.3)	(0.1, 0.2, 0.6)	(0.0, 0.1, 0.3)	(0.4, 1.1, 2.4)	(18.4, 24.7, 35.3)	(0.0, 0.0, 0.0)	(60.4, 72.5, 79.0)
	4	(1.1, 3.8, 7.8)	(0.6, 1.9, 5.2)	(0.1, 0.4, 1.0)	(0.5, 1.2, 2.6)	(0.2, 0.5, 1.3)	(3.9, 8.6, 17.5)	(18.6, 25.1, 35.2)	(0.0, 0.0, 0.0)	(40.7, 56.5, 68.9)
	8	(3.7, 11.5, 20.4)	(0.5, 1.8, 4.7)	(0.1, 0.5, 1.2)	(0.8, 2.0, 4.0)	(0.3, 0.8, 1.8)	(9.9, 20.4, 35.9)	(15.4, 21.1, 31.4)	(0.0, 0.0, 0.0)	(24.6, 39.6, 54.4)
	20	(6.8, 22.8, 39.4)	(0.3, 1.2, 3.2)	(0.1, 0.3, 0.9)	(0.8, 1.9, 3.9)	(0.4, 0.8, 1.8)	(18.8, 34.8, 51.9)	(9.4, 14.4, 21.8)	(0.0, 0.0, 0.0)	(13.0, 21.9, 36.1)

Table 3.25: Variance decomposition: Real wage

Model	H	technology	discount	government	investment	labor	monetary	price markup	risk premium	wage markup
Calvo (SW)	1	(38.2, 49.8, 62.3)	(5.5, 8.7, 12.5)	(11.9, 16.6, 22.5)	(0.0, 11.6, 15.6)	(2.8, 5.4, 11.4)	(2.3, 4.6, 11.4)	(0.0, 0.1, 0.4)	(0.0, 0.0, 9.6)	(0.6, 1.1, 2.0)
	4	(16.5, 29.5, 42.8)	(3.1, 6.9, 10.4)	(9.2, 12.2, 16.7)	(0.0, 13.1, 17.7)	(12.9, 19.9, 32.2)	(7.9, 14.2, 29.2)	(0.2, 0.6, 2.1)	(0.0, 0.0, 5.4)	(0.6, 1.2, 2.0)
	8	(9.5, 19.6, 30.5)	(1.8, 4.5, 7.2)	(6.5, 9.1, 12.9)	(0.1, 9.7, 13.8)	(23.9, 33.9, 46.0)	(10.6, 18.7, 33.5)	(0.5, 1.4, 3.6)	(0.0, 0.0, 3.2)	(0.7, 1.2, 2.0)
	20	(5.6, 11.7, 19.3)	(1.1, 2.7, 4.4)	(3.9, 5.8, 8.5)	(0.1, 5.9, 8.8)	(42.6, 54.8, 65.6)	(8.2, 15.7, 28.7)	(0.5, 1.3, 3.2)	(0.0, 0.0, 1.8)	(0.6, 1.1, 2.0)
Standard Calvo	1	(37.4, 49.7, 62.1)	(6.0, 8.9, 13.0)	(11.6, 16.4, 21.5)	(9.0, 12.5, 16.7)	(2.9, 5.7, 10.0)	(2.6, 4.7, 8.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.6, 1.1, 1.9)
	4	(17.8, 28.5, 41.3)	(4.3, 7.1, 11.2)	(9.1, 12.4, 16.1)	(10.8, 14.3, 19.5)	(13.2, 20.5, 29.7)	(8.3, 13.9, 21.8)	(0.0, 0.3, 1.2)	(0.0, 0.0, 0.2)	(0.7, 1.2, 2.0)
	8	(11.0, 18.9, 29.5)	(2.7, 4.7, 7.5)	(6.7, 9.4, 12.7)	(7.7, 10.8, 15.4)	(24.7, 34.4, 45.3)	(11.2, 18.5, 28.4)	(0.1, 0.4, 1.7)	(0.0, 0.0, 0.1)	(0.7, 1.1, 1.9)
	20	(6.4, 11.3, 18.9)	(1.6, 2.8, 4.7)	(4.1, 6.0, 8.6)	(4.4, 6.5, 9.6)	(42.8, 53.9, 63.9)	(9.8, 16.8, 27.1)	(0.1, 0.4, 1.5)	(0.0, 0.0, 0.1)	(0.5, 0.9, 1.5)
Standard SI	1	(30.0, 38.0, 47.5)	(8.6, 11.2, 14.9)	(11.0, 14.7, 18.5)	(10.7, 14.0, 18.8)	(3.4, 6.1, 10.5)	(2.9, 4.8, 7.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(6.2, 9.2, 14.7)
	4	(15.2, 21.4, 29.2)	(8.5, 12.6, 17.9)	(8.7, 11.4, 15.0)	(12.2, 17.1, 24.3)	(8.3, 17.1, 31.7)	(10.0, 15.0, 22.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(1.6, 2.6, 4.3)
	8	(9.8, 14.6, 21.5)	(5.4, 9.4, 14.5)	(7.3, 10.8, 15.1)	(8.9, 14.2, 22.2)	(8.9, 23.7, 46.6)	(13.6, 22.1, 34.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(1.0, 1.6, 2.6)
	20	(6.1, 11.5, 17.3)	(3.2, 7.3, 12.3)	(5.2, 11.5, 20.2)	(5.8, 11.3, 19.2)	(8.9, 32.5, 64.9)	(9.2, 20.8, 34.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.7, 1.2, 2.1)
Hybrid SI	1	(44.5, 58.6, 70.1)	(4.9, 8.1, 12.4)	(9.1, 12.9, 18.4)	(7.5, 10.5, 14.2)	(2.4, 4.3, 8.0)	(1.8, 3.6, 6.1)	(0.0, 0.2, 0.8)	(0.0, 0.0, 0.0)	(0.3, 0.5, 1.1)
	4	(25.1, 39.7, 52.3)	(4.0, 6.7, 11.2)	(7.3, 9.8, 13.8)	(9.4, 12.5, 16.1)	(10.6, 15.9, 25.9)	(5.5, 10.0, 16.5)	(0.3, 2.7, 5.8)	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.4)
	8	(17.0, 29.4, 41.8)	(2.8, 4.7, 8.3)	(6.0, 7.9, 11.1)	(7.2, 9.9, 13.5)	(20.9, 28.2, 40.5)	(7.0, 12.8, 21.1)	(1.2, 4.1, 7.8)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)
	20	(10.6, 19.1, 29.5)	(1.6, 3.0, 5.5)	(3.7, 5.4, 8.2)	(4.3, 6.3, 9.2)	(39.8, 50.5, 59.9)	(5.4, 10.6, 21.9)	(1.5, 3.1, 5.9)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)
Calvo-SI	1	(36.3, 51.3, 63.9)	(6.0, 9.1, 13.1)	(11.9, 15.8, 20.0)	(8.8, 12.3, 16.1)	(2.8, 4.9, 9.6)	(2.4, 4.7, 9.1)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.1)	(0.3, 0.6, 1.1)
	4	(15.9, 29.5, 43.6)	(4.4, 7.5, 10.8)	(9.3, 12.1, 15.1)	(10.7, 14.4, 18.6)	(12.6, 18.8, 28.1)	(8.8, 14.4, 24.6)	(0.1, 0.4, 1.4)	(0.0, 0.0, 0.1)	(0.3, 0.6, 1.1)
	8	(9.8, 19.7, 32.1)	(2.9, 5.0, 7.7)	(6.7, 9.3, 12.0)	(7.7, 10.8, 14.0)	(23.6, 32.0, 43.6)	(12.5, 19.4, 30.8)	(0.1, 0.5, 1.8)	(0.0, 0.0, 0.1)	(0.3, 0.6, 1.0)
	20	(6.0, 12.5, 21.1)	(1.7, 3.1, 4.9)	(4.4, 6.1, 8.1)	(4.5, 6.8, 9.1)	(41.7, 52.1, 63.4)	(10.4, 16.8, 27.7)	(0.1, 0.5, 1.5)	(0.0, 0.0, 0.0)	(0.3, 0.5, 0.8)

Table 3.26: Variance decomposition: Employment

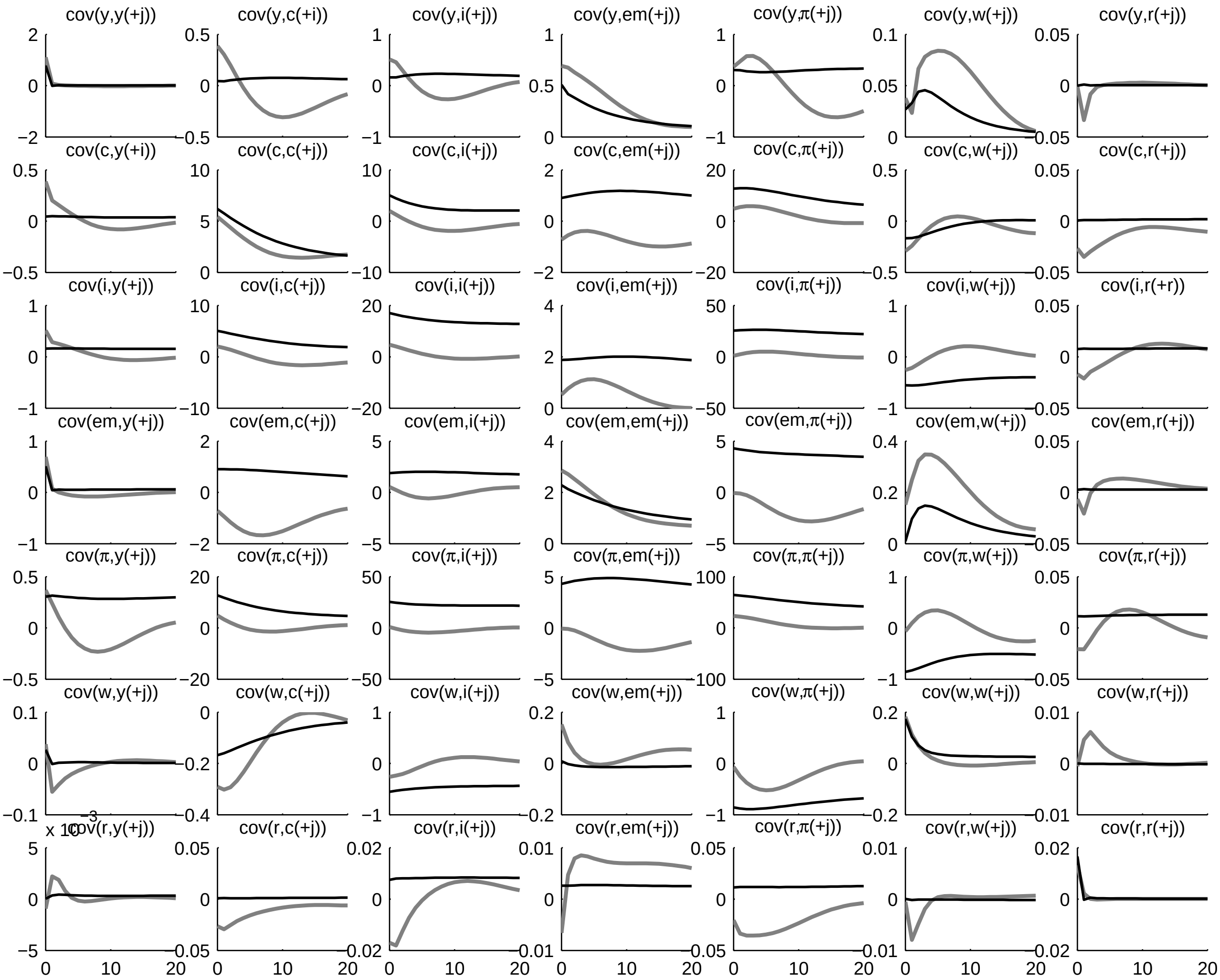

Figure 3.13: Comparison of cross-covariances of the DSGE model (thin line) and the data (thick line).

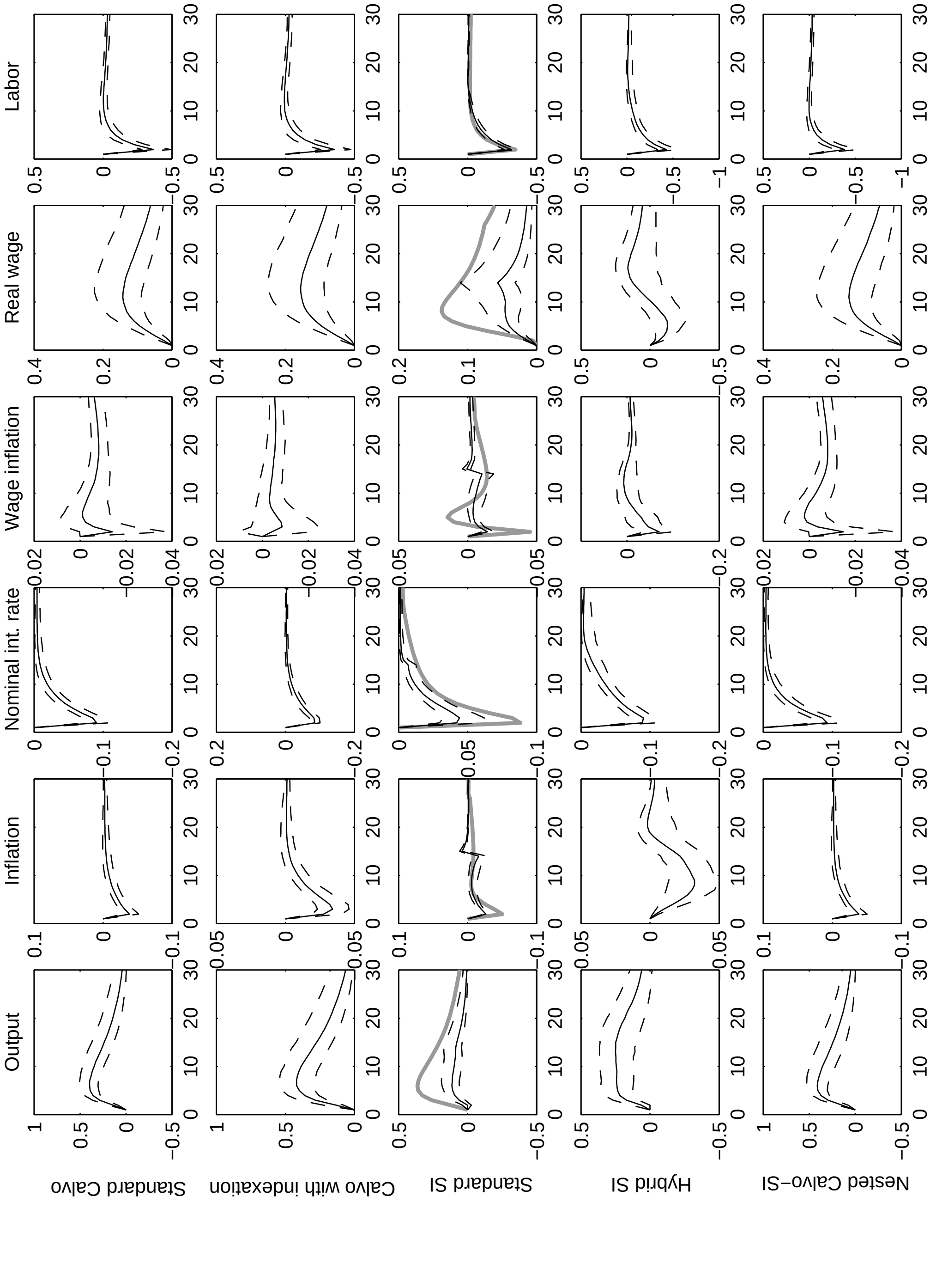

Figure 3.14: Estimated impulse responses: Technology shock. Mode of the standard Sticky Information model, J=24 (thick line), remaining models, including sticky information models, J=12 (thin solid and dashed lines).

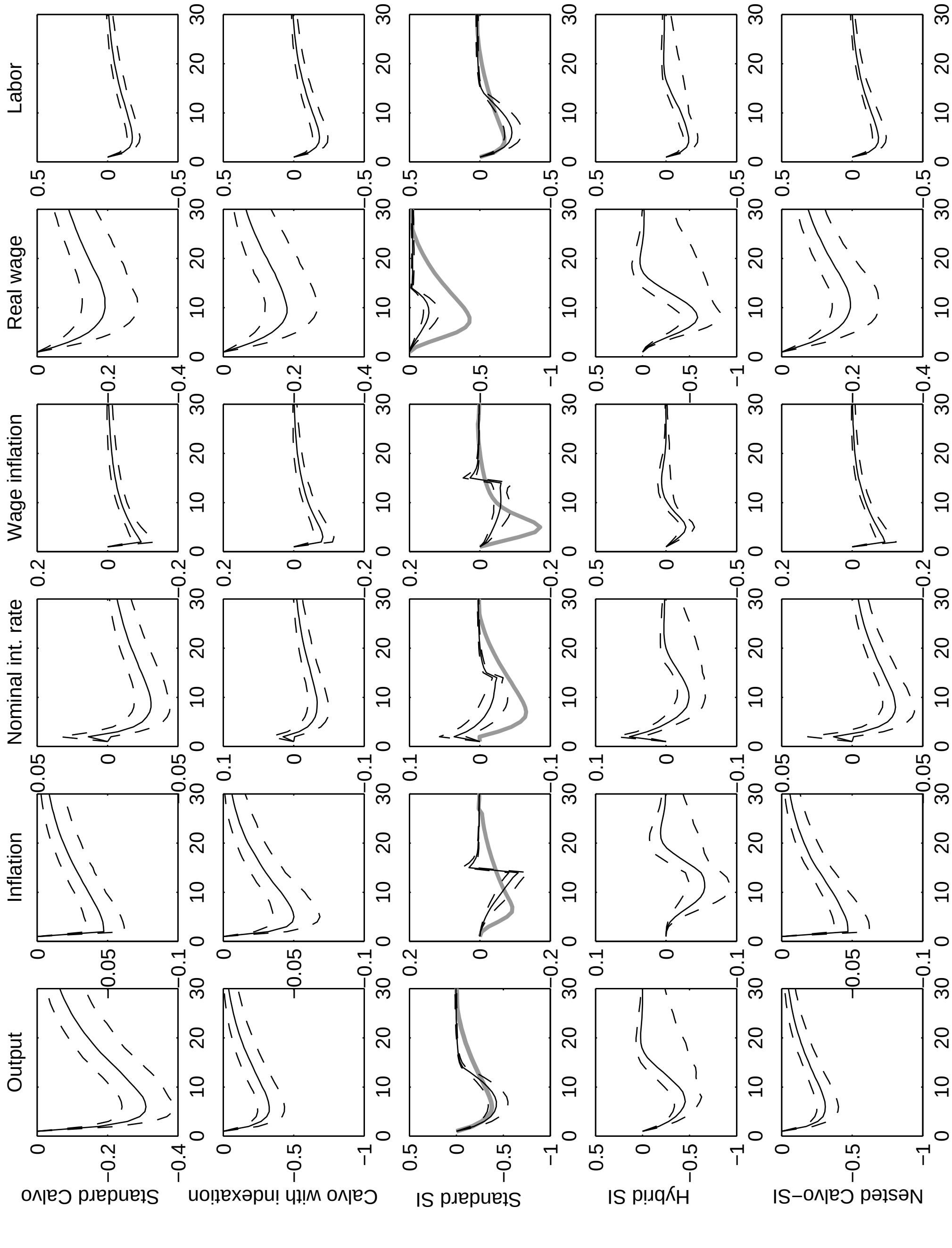

Figure 3.15: Estimated impulse responses: Monetary shock. Mode of the standard Sticky Information model, J=24 (thick line), remaining models, including sticky information models, J=12 (thin solid and dashed lines).

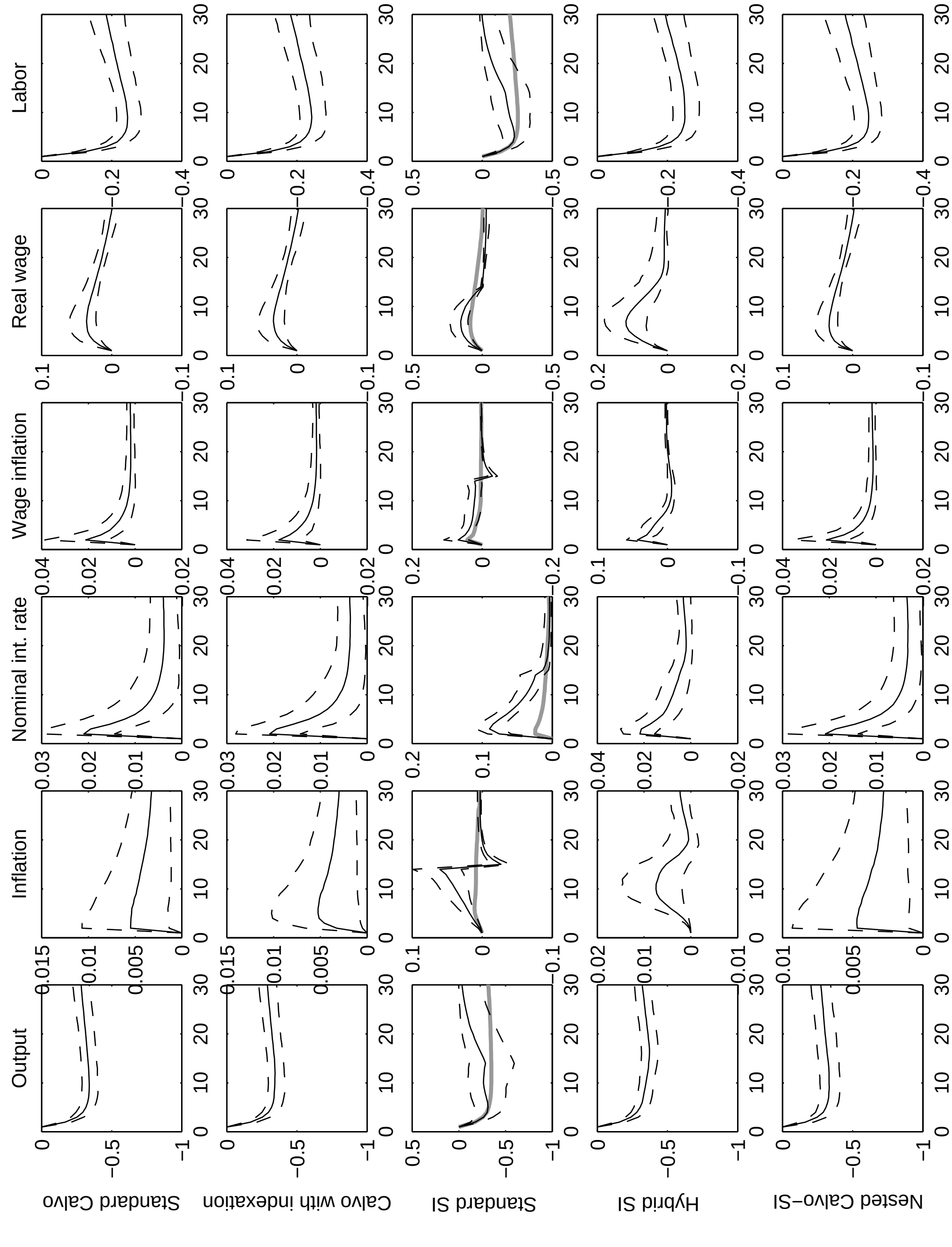

Figure 3.16: Estimated impulse responses: Labor supply shock. Mode of the standard Sticky Information model, J=24 (thick line), remaining models, including sticky information models, J=12 (thin solid and dashed lines).

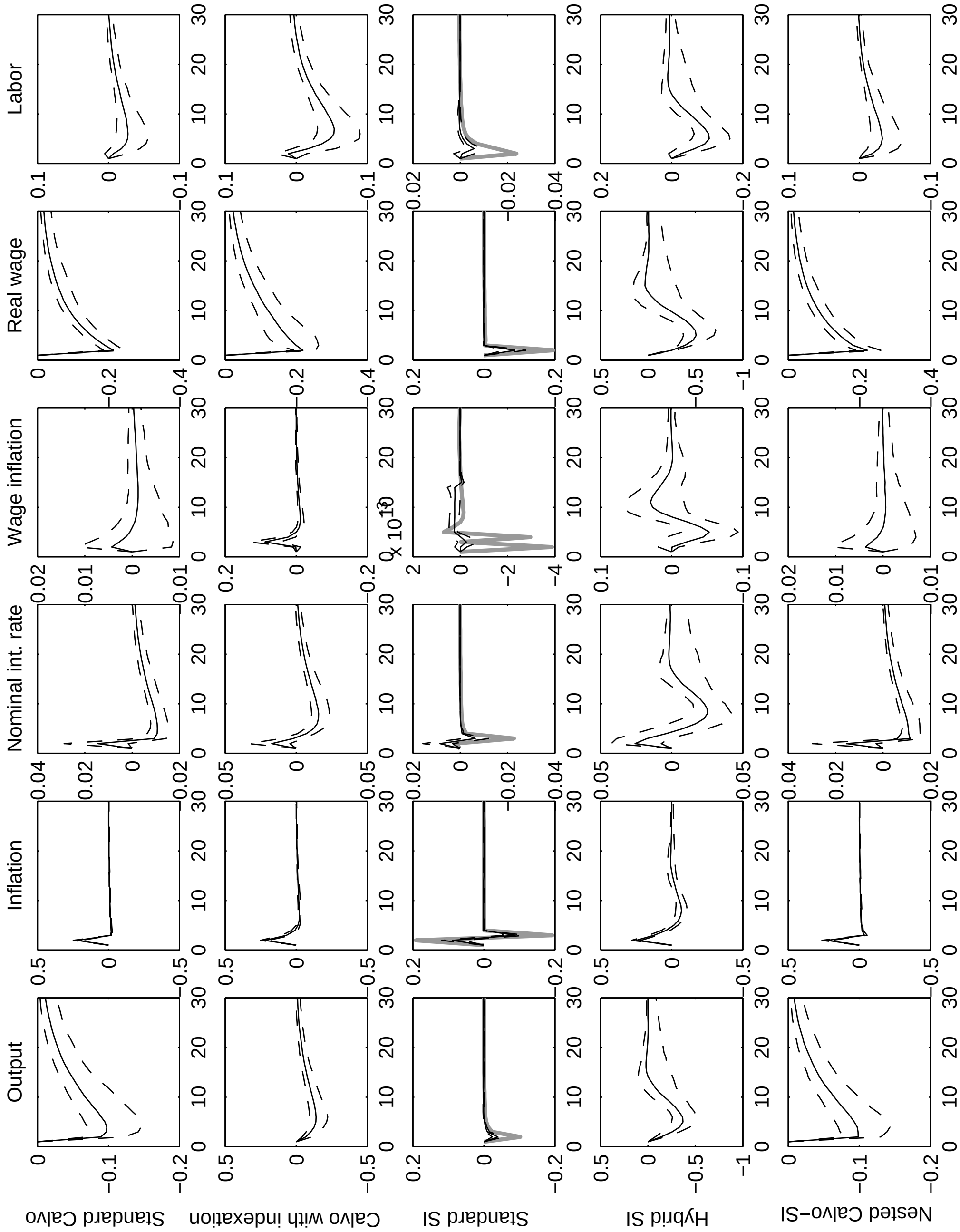

Figure 3.17: Estimated impulse responses: Price markup shock. Mode of the standard Sticky Information model, J=24 (thick line), remaining models, including sticky information models, J=12 (thin solid and dashed lines).

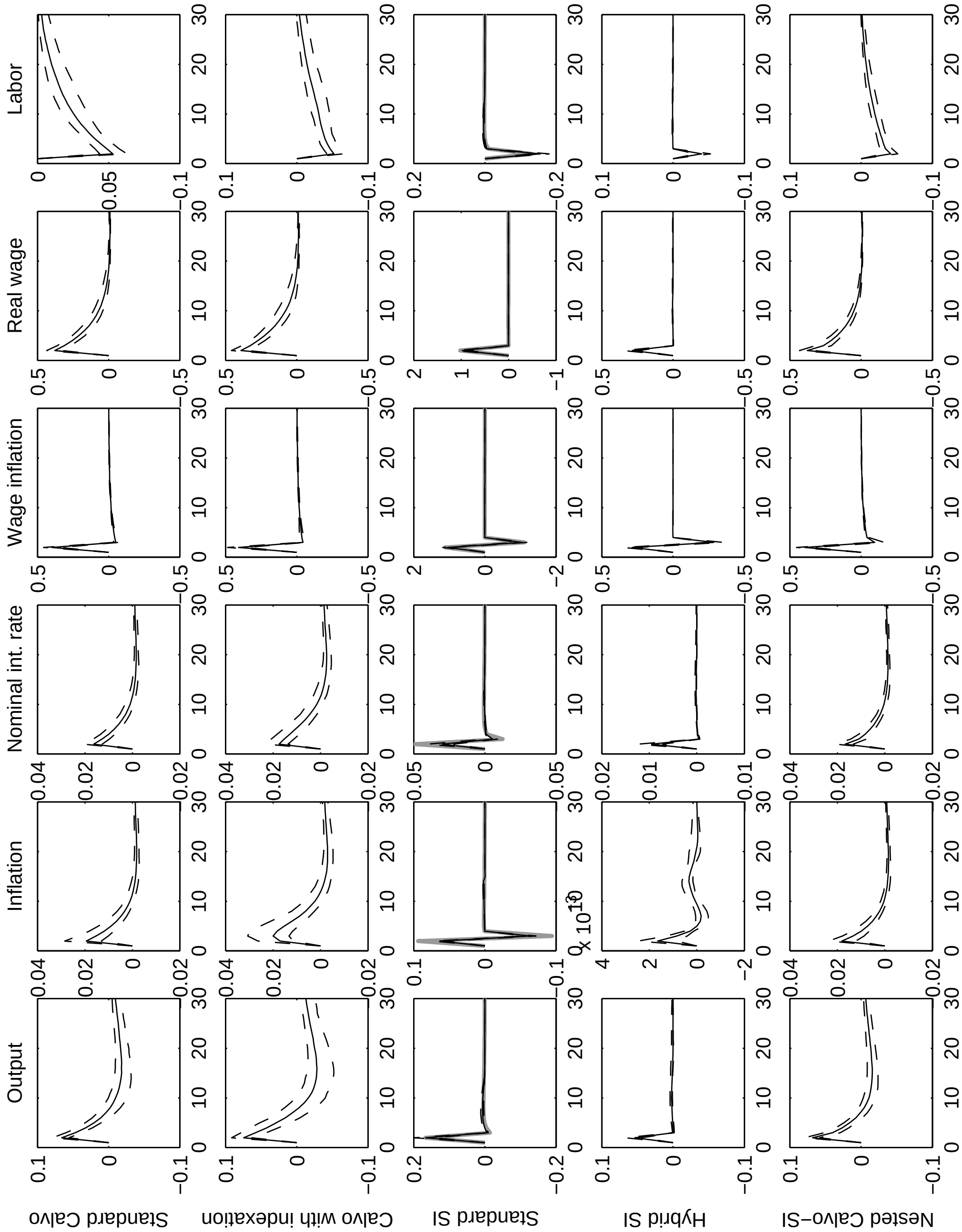

Figure 3.18: Estimated impulse responses: Wage markup shock. Mode of the standard Sticky Information model, J=24 (thick line), remaining models, including sticky information models, J=12 (thin solid and dashed lines).

Chapter 4

Transmission of economic fluctuations in an estimated two-region DSGE model for the Euro area

4.1 Introduction

The EMU is an integrated economic area with a large number of interactions between the participating countries. Yet, the available empirical evidence (mainly based on reduced form estimates) indicates that the asymmetry of economic fluctuations is still an important issue. The effects of common shocks on the core Euro area economies, particularly Germany, might be smaller relative to the effects of idiosyncratic disturbances, though meaningful.[1] This stochastic heterogeneity results mainly from differences in the institutional frameworks of participating countries and seems to be much more pronounced than in other monetary unions or federal states (the US being the most obvious comparison). More importantly, these differences are expected to persist at least for some time into the future.[2] All this evidence suggests that the area should be modeled in a disaggregate (multicountry) framework rather than in aggregate.

The contribution of this chapter is to develop an *estimated* New Open Economy Macroeconomics DSGE model to study the linkages between the regions of the Euro area. Subsequently, we investigate whether this model can accurately reflect basic features of the economies and account for transmission of economic fluctuations within the area - so that we could trust the model for policy analysis. The (Bayesian) system estimation procedure, employed here, seeks to match *all* second order moments of the data and allows for unambiguous interpretation of the sources of disturbances.

Our research approach comprises several challenges that range from the general rationality to specific problems of an appropriate model structure and the data. The first challenge we are confronted with is to determine a tractable framework for macro econometric modeling.[3] We decide to subdivide the area into two heterogenous regions, Germany and rest of the Euro area. This subdivision is only one of the possible ways and has been suggested, for instance, by Monteforte (2004). The two-region framework allows us to exploit the disaggregate information on the Euro

[1] See, e.g., Monteforte and Siviero (2003) and Stock and Watson (2005).

[2] See, e.g., Plasmans, Engwerda, van Aarle, Michalak, and Bartolomeo (2006).

[3] Tractability requires that the model is restricted to a two-region framework. Such a framework may be useful for discussing issues pertaining to the links between two large blocks (here we concentrate on explaining the links between Germany and the rest of the Euro area) but it can hardly be viewed as a realistic description of policy making in the Euro area, currently made up of twelve countries. The modification of existing two-country models may allow us to incorporate an arbitrarily large number of countries (see, e.g., Gali and Monacelli (2004)). However, extending the analysis for a larger number of countries might lead to ignoring nominal and real rigidities and to restricting the model parametrization.

area; we employ the German national accounts data along with the Euro area-wide aggregates (disaggregated via the model's measurement equation) in the estimation.

One other important aspect is the introduction of the Euro as a common currency of the European Economic and Monetary Union. This institutional change is expected to have important implications on the transmission of economic fluctuations. Since the 'genuine' time series for the EMU are relatively short, an estimated model necessarily relies, to a large extent, on a sample period before the actual establishment of the monetary union. In order to deal with the monetary regime change, we determine two separate theoretical DSGE models corresponding to both current and historical regimes. Since the model of currency area is a restricted version of the flexible exchange rate model, with possibly the same meaning of the state vector elements, the Kalman filter at the point of regime change can be initialized with the last values of the state vector in the previous regime. So, the joint likelihood of both models can be recursively evaluated on the entire sample. Crucial for our approach is the assumption that all regime-invariant parameters, including those pertaining to the model's stochastics, are constant over the period considered.[4] Thus, the model with a flexible exchange rate allows us to utilize the information available prior to EMU while computing the posterior distribution for the monetary union model.[5] Our approach is, therefore, very close to the idea of data based prior distribution for the Euro area model.

When it comes to the theoretical models, we extend the closed economy benchmark of Christiano, Eichenbaum, and Evans (2001) and Smets and Wouters (2003) by incorporating open economy mechanisms into it. Incomplete asset market structure is designed to capture the departures from the Uncovered Interest rate Parity (UIP) and deal with exchange rate indeterminacy prior to EMU. The common monetary policy, in the model for the EMU, is a part of the endogenous propagation mechanism that explains symmetric developments within the area. Asymmetric developments are, in turn, captured by defining the relative price channels. We allow for the international price discrimination within the area, which eventually results in incomplete pass-through from the foreign to the domestic economy. Finally, in order to capture the time series properties of the main macro-economic data our model includes various nominal and real frictions common in the literature.

The stochastics of the model differs from the benchmark in some important respects. We incorporate the balanced growth mechanism as in Altig, Christiano, Eichenbaum, and Linde (2003) but identify two cointegrated permanent technology processes along with the stationary process that measures the asymmetries in technological progress across the regions. Including all these mechanisms allows us to integrate aspects of growth and business cycle theory. Thus, the model can, at the same time, match both low and high frequencies of the real aggregates. Since the sample we based our estimation on is not homogenous in terms of monetary regimes in force, we also incorporate exogenous processes for the inflation targets. Finally, in order to relax the model's cross-equation restrictions, common components on structural shocks have been introduced.

The empirical analysis of this chapter has two parts. In the first part we validate the model based on comparing the model implied second moments (and spillovers in particular) to those in the data (the reduced form models being the obvious benchmark). This analysis offers several insights:

The baseline two-region DSGE model fairly well replicates the actual co-movements of the

[4]This is, in particular, motivated by the results in Castelnuovo (2004) and Canova and Ciccarelli (2006). Castelnuovo (2004) shows stability of the open economy Phillips curve estimates over the two different exchange rate regimes, which translates into the quantitative insignificance of the Lucas' critique. Canova and Ciccarelli (2006), in turn, using the BVAR with time varying coefficients detect no major changes in structural shocks over the period considered here.

[5]The assumption of autonomous central banks and flexible exchange rates in the Euro area prior to the adoption of the Euro is common in many empirical works (see, e.g., Jondeau and Sahuc (2004)). This assumption is not very restrictive for the period directly preceding EMU (the currency fluctuations had to be contained within a margin of $\mp$15%). However, this is more crude for the period before 1993. The fluctuations of local currencies were then restricted to $\mp$2.25%. In this chapter we abstract from modeling of an anticipated monetary regime change.

German and the Euro-area aggregates. Success in this dimension hinges on the inclusion of cointegrated processes for the technological progress. The model also satisfactorily mimics the volatility of the data but overpredicts the persistence in growth rates of the real variables. The version re-estimated on the detrended data seems to cope better with the persistence but lies behind in replicating the observed co-movements. We also note that the estimated persistence of stationary shocks is much higher in a balanced growth model relative to a model estimated with detrended data, which means that the endogenous propagation mechanism is much weaker in the former.

Analyzing the implied variance decompositions we find that the DSGE model, similarly to reduced form models available in the literature, identifies the idiosyncratic shocks as the main source of economic fluctuations in the Euro area (both prior to EMU and after adoption of the common currency). However, while the identified VAR (a Bayesian VAR model with a DSGE prior and estimated under orthogonality assumption) detects that as much as 25% of business cycle frequency fluctuations of the German output have been, prior to EMU, explained by spillovers from the rest of the Euro area, the DSGE model estimates the trade-related spillover effects at 4-9%. The origin for only moderate direct spillovers appears to be, besides the exchange rate disconnect, DSGE model's inability to generate symmetric open economy effects of the idiosyncratic shocks. We find that these effects can be partially attributed to and generated by including the common components on structural shocks. Hence, an estimated open economy DSGE model (with common components) might be not far away from offering explanation for co-movements detected in the data prior to EMU. We further detect that while under a free float regime spillover operates mainly through trade linkages, an additional spillover is generated after introducing a common currency working through the common interest rate. This indirect spillover multiplies the total effect of foreign shocks.

Analyzing the model implied correlations between cyclical components of output in Germany and the rest of the area, we find that, though member states are severely exposed to the idiosyncratic shocks in a monetary union, the business cycles tend to be more synchronized. We link this result to the fact that there is no scope for irrational nominal exchange rate forecasts after transition to EMU.

Finally, our estimates offer some new insights into the nature of the bilateral exchange rate fluctuations prior to EMU. Results indicate that these fluctuations can be explained by monetary and real shocks, contrary to hypothesis in some NOEM work suggesting the UIP shock and cost push shocks as an explanation.

In a second part, we ask whether the restrictions arising from the multi-country structure do not negatively affect model's ability to forecast aggregate European economic performance (that is often of particular interest for policy makers). We repeatedly estimate the model over samples of increasing lengths, forecasting out-of-sample one to eight quarters ahead at each step. The forecasts of the key Euro area-wide aggregates are then compared with those arising from the area-wide DSGE model in the spirit of Smets and Wouters (2003). The econometric literature does not univocally predict what can be the outcome of this comparison. The two-region model offers a more detailed description and exploits disaggregate information on the Euro area, but this may also be a drawback if some restrictions are too strong for the data or if the area-wide model appropriately averages the country specific uncertainty.

The general finding in this part of the analysis is that a disaggregate DSGE model has a slight edge over the area-wide counterpart in terms of predictions on some of the variables, e.g. output, consumption, investment and inflation. Also the multivariate accuracy measures seem to propose the two-region model as the best forecasting tool.

Our results make a clear case for relying on a multi-country modeling approach when analyzing and forecasting the Euro area economy, and suggest that a line of research worth pursuing is the estimation of relationships within the area and their effects on economic policies.

The remainder of this chapter is organized as follows. In the next section we review the relevant literature and the main stylized facts. Section 4.3 presents the theoretical models for both flexible exchange rate regime and currency union. We recapitulate the model on stationary variables and present the solution method in Section 4.4. In Section 4.5 we discuss the data used in the estimation. Section 4.6 briefly shows the estimation methodology. In sections 4.7 and 4.8 the empirical results are presented. We analyze the parameter estimates along with the model implied second moments. Then, the importance of common, domestic and foreign shocks is evaluated through variance decompositions. We also extensively comment on the robustness of the results and critically compare alternative approaches. Section 4.9 documents the out-of-sample accuracy of the two-region DSGE model. Section 4.10 summarizes the main findings and concludes. An appendix contains tables with model implied second moments, variance decompositions and figures displaying prior and posterior distributions as well as impulse responses.

4.2 Related literature

The empirical analysis of co-movements in economic activity across countries dates as far back as the works of Mitchell (1927), Moore and Zarnowitz (1986), Gerlach (1988) and Backus and Kehoe (1992). More recent empirical works examining the interactions between the Euro area countries are: Agresti and Mojon (2001), Schirwitz and Wälde (2004), Stock and Watson (2005), Candelon, Piplack, and Straetmans (2005), Crespo Cuaresma and Amador (2006) and Canova and Ciccarelli (2006). These works particularly focus on the aspects of business cycles synchronization.

Canova and Ciccarelli (2006) examine the properties of G7 cycles using a multicountry Bayesian panel VAR model with time variations, unit specific dynamics and cross country interdependencies. They found, however, no evidence of the existence of a Euro area specific cycle or of its emergence in the 1990's. Different conclusions have been reached, for instance, by Schirwitz and Wälde (2004), who based on correlations of cyclical components of GDP in EU14 and G7 countries found increase in synchronization in the former area starting in the 1990's. Furthermore, Artis and Zhang (1999) argue that the synchronization of business cycles in Europe might hinge on the exchange rate stability.

A preliminary attempt to disentangle common shocks and spillover effects within the Euro area, based on the reduced form multi-country setting, is provided by Monteforte and Siviero (2002) and Monteforte (2004). They found that, for the output gap, the share of the variance explained by the idiosyncratic component is considerably higher in the German economy than for the other Euro area economies. So, the German data *are the least tailored to the area-wide model*.

The factor-structural vector autoregression (FSVAR) analysis of G7 countries provided in Stock and Watson (2005) leads to the conclusion that spillovers within the Euro area are at most moderate. Also moderate are the effects of international shocks in determining the business cycles in Europe when compared to the effects they have, for instance, on the Canadian economy - a subject of the vast majority of open economy research (see Kose, Otrok, and Whiteman (2003) or more recently Justiniano and Preston (2006)).

These stylized facts have motivated a strand of microfounded research that seeks to replicate the observed results of interactions within the Euro area. However, these models are mainly compared to the data along a few *selected* dimensions, as in the contributions of Andres, Ortega, and Vallés (2003) and Benigno and Thoenissen (2003), who constructed micro-founded two-country models to study currency unions among asymmetric countries, and therefore are unsuitable for a (complex) quantitative analysis or forecasting.

This study in particular relates to the works which apply (Bayesian) estimated New Open Economy Macroeconomics DSGE models. These models have been used to study interactions between the Euro area and the US economy either in a two-country framework, see Adjemian, Paries, and Smets (2004), De Walque and Wouters (2004), Lubik and Schorfheide (2005) and Rabanal and Tuesta (2005), or in a Small Open Economy setting, see Adolfson, Laseen, Linde, and

Villani (2005a). Moreover, similar models have been used for open economy analyses of the New Zealand, Canadian, Danish and Swedish economies, see Justiniano and Preston (2004), Justiniano and Preston (2006) and Dam and Linaa (2004). The Bayesian estimation of a (dissagregate) multi-country DSGE model for the Euro area is presented in Jondeau and Sahuc (2004). Their fairly simple model has been constructed to assess the problem of heterogeneity regarding the optimal monetary policy. However, the authors use only the data from before the adoption of the Euro to estimate the model and do not account for the current monetary regime in the area. Moreover, the issue of co-movements or spillovers within the area is not addressed explicitly.

Methodologically most closely related to our approach is the aforementioned paper by Justiniano and Preston (2006) who examine the ability of an estimated DSGE model to induce international spillovers from the US to the Canadian economy. Our study differs from Justiniano and Preston (2006) in the following aspects. We consider a complex two-region DSGE model whereas Justiniano and Preston (2006) consider a Small Open Economy framework. Furthermore, the structure of the economy assumed in this chapter incorporates a much larger number of frictions, including capital adjustment costs, and shocks that are often claimed to be necessary for achieving a good fit with data. Apart from examining the trade-related spillovers, we also assess the role of common components on structural shocks in explaining the co-movements in economic activity. Finally, our model is estimated on a total of 17 observable time series, whereas Justiniano and Preston (2006) estimate the model on 10 observable variables. These differences notwithstanding, Justiniano and Preston (2006) find discrepancy between the DSGE model implied trade-related spillovers and those detected in the data. Yet, we emphasize that the contrast between model and data, as well as its apparent origin, the exchange rate disconnect, is much less pronounced while employing the European data.

This chapter is, to our knowledge, the first attempt to estimate the DSGE model for the German economy within the monetary union. We are also unaware of any microfounded studies examining the role of foreign sourced shocks within the Euro area.

4.3 Theoretical Models

In this section, we derive prototypical DSGE models for a free float regime and a currency union to approximate the structure of the Euro area.[6]

From now on, we will consider the world economy to be made up of two asymmetric countries denoted Home (Germany) and Foreign (the rest of the Euro area).[7] [8] Before forming the monetary union these two economies used to conduct 'quasi independent' monetary policies.[9]

A following normalization is assumed throughout the chapter. The world population is a continuum of agents distributed on the interval $[0, 1]$. The population of the Home country is distributed on the interval $[0, n)$ and that of the rest of the Euro area on the interval $[n, 1]$. In each country modeled here there are households, a government, two types of firms (intermediate

[6] We refer here to a currency area, which is defined as a group of regions that share the same currency. One currency means there is one central bank that is entitled to conduct monetary policy within this area. Relevant for our analysis is the simplest form of a currency area - a two-region area.

[7] Indeed, the German economy intrinsically depends on the development of exports to the Euro area, which is by far the biggest market for Germany. It should be, however, underlined that the exports to non-EU countries still account for about 43-47% of the total (source: Bundesbank), which might explain significant international spillovers from these economies found, for instance, by Stock and Watson (2005).

[8] Modeling the Euro area as a closed economy is the standard assumption made in DSGE studies. Its origin can be traced back, for instance, to the empirical works of Masson and Taylor (1993) who indicate that all the EU-countries have a high degree of mutual openness and therefore it is necessary to model them in an open-economy framework. These authors also find that the European Union as a whole is a relatively closed economy, which presents the possibility of neglecting the links between European economies and the rest of the world.

[9] 'Quasi independent' in this context means that we introduce some (exogenous) mechanisms of monetary coordination prior to EMU.

good and final good producers) and distributors which transform the final good into differentiated consumption and investment goods. These goods are sold both to households domestically and abroad. We allow for endogenous deviations from purchasing power parity and the law of one price in the short-run. Specifically, the same good can have different prices depending on where it is sold (even after adjusting for exchange rate movements as it is the case under a free float regime). Producers set prices monopolistically for the domestic as well as the foreign market in their own currency. Imported goods, however, are subject to price discrimination as monopolistic retailers charge a mark-up to consumers at the border. Households own domestic firms and receive dividends paid by these firms. Labor and physical capital are assumed to be immobile internationally.

Unless the general setting for the Foreign economy differs from the ones for the Home country, we present only equations for the latter. In order to lighten the notation, if equations for both economies are presented, the Foreign economy variables are indicated with an asterisks (*). Reviewing the model, we point to the differences between the setting for a currency area and the model with a flexible exchange rate arrangement.

4.3.1 Firms

The economy of each country produces a single final good and a continuum of intermediate goods indexed by z, where z is distributed over the unit interval; respectively over $[0, n)$ in the Home economy and $[n, 1]$ in the Foreign economy. The final good sector is perfectly competitive. In contrast, there is monopolistic competition in the markets for intermediate goods. Each intermediate good is produced by a single firm. Producers of intermediate goods use domestic physical capital and domestic labor as inputs. The final good is transformed into a consumption good, a capital good and a public good. After differentiation, e.g. brand naming, it is the subject of trade between the two countries.

The final good sector

Final good firms produce a homogenous good Y_t (Y_t^* in the Foreign country) according to the Dixit and Stiglitz (1977) production function, using differentiated intermediate goods $Y_t(z)$:

$$Y_t = \left[\left(\frac{1}{n} \right)^{\frac{\lambda_p - 1}{\lambda_p}} \int_0^n Y_t(z)^{\frac{1}{\lambda_p}} dz \right]^{\lambda_p}, \tag{4.1}$$

where $\lambda_p > 1$ is a gross markup in the goods market.[10]

The final good producer minimizes its cost choosing the input $Y_t(z)$ given the input price $P_t(z)$ subject to the production technology (4.1). The cost minimization conditions yield:

$$Y_t(z) = \frac{1}{n} Y_t \left(\frac{P_t(z)}{P_t} \right)^{\frac{\lambda_p}{\lambda_p - 1}}, \tag{4.2}$$

which may be seen as an equation of total demand for intermediate good z.

The Lagrangian multiplier P_t from the above minimization problem is the cost-minimizing price of a unit of a final good basket. Solving for P_t we obtain:

$$P_t = \left[\frac{1}{n} \int_0^n \left[P_t(z) \right]^{1/(1-\lambda_p)} dz \right]^{1-\lambda_p}. \tag{4.3}$$

As the final good sector is perfectly competitive, each firm takes the price of the final good P_t as given and equates its marginal cost to the price.

[10] Note that, contrary to the model presented in Chapter 3, the markup is here time invariant.

The output index Y_t may be either sold to the domestic households, being used in the production of the final consumption or investment good, or it may be exported.

Intermediate good producers

Intermediate good producers choose a price for their products based on the production technology and the total demand (4.2). Firms use both labor and capital bundles to produce according to the following Cobb-Douglas production function:

$$Y_t(z) = A_t^{1-\alpha}\varepsilon_t^Y \left(K_t(z)\right)^{\alpha} \left(L_t(z)\right)^{1-\alpha}, \tag{4.4}$$

where α denotes the share of capital in production, A_t is a unit root technology shock, ε_t^Y is a Kydland-Prescott type of covariance stationary technology shock, $K_t(z)$ is a bundle of physical capital used by the firm at time t. The amount of labor service $L_t(z)$ utilized by firm z is given by the following Dixit-Stiglitz aggregate:

$$L_t(z) = \left\{ \left(\frac{1}{n}\right)^{\frac{\lambda_w-1}{\lambda_w}} \int\limits_{f=0}^{n} l_t(f,z)^{1/\lambda_w} df \right\}^{\lambda_w}, \tag{4.5}$$

where $l_t(f,z)$ denotes the number of hours of type f labor used by the firm z and $\lambda_w > 1$ is a gross wage markup.

The so-called stationary technology shock is assumed to follow an AR(1) process in logs:[11]

$$\ln(\varepsilon_t^Y) = (1-\rho_Y)\ln(\bar{\varepsilon}^Y) + \rho_Y \ln(\varepsilon_{t-1}^Y) + u_t^Y + u_t^{Y^{com}}. \tag{4.6}$$

$\bar{\varepsilon}^Y$ stands here for the level of technology. Further, the growth rate of technological progress $A_t/A_{t-1} \equiv \varepsilon_t^A$ and

$$\ln(\varepsilon_t^A) = (1-\rho_A)\ln(\bar{\varepsilon}^A) + \rho_A \ln(\varepsilon_{t-1}^A) + u_t^A. \tag{4.7}$$

$\bar{\varepsilon}^A$ denotes the long run growth rate of technological progress. Because of the unit root in the technology process A_t, variables in the model evolve along the stochastic growth path. In order to calculate the steady state and solve the model in the log-linearized form, we make the model stationary by dividing the real variables by the level of technology A_t, in the Home economy and A_t^* in the Foreign economy.[12] Recapitulating the model's structure in Section 4.4, we present the equations in stationary variables.

Firm z chooses production factors: a sequence of different types of labor $l_t(f,z)$ and capital bundle $K_t(z)$ to minimize the total cost of production, given by:

$$\min_{K_t(z),l_t(f,z)} P_t r_t^k K_t(z) + \int\limits_{f=0}^{n} W_t(f) l_t(f,z) df \tag{4.8}$$

s.t.

$$Y_t(z) = A_t^{1-\alpha}\varepsilon_t^Y K_t^{\alpha}(z) \left[\left\{ \left(\frac{1}{n}\right)^{\frac{\lambda_w-1}{\lambda_w}} \int\limits_{f=0}^{n} l_t(f,z)^{1/\lambda_w)} df \right\}^{\lambda_w} \right]^{1-\alpha}, \tag{4.9}$$

[11]We allow for common components on the structural shocks, indicated with a superscript (^{com}).

[12]The growth rate of the observed variable may be calculated from the stationary variables as follows: $\ln(\frac{X_t}{X_{t-1}}) = \ln(\frac{\tilde{X}_t}{\tilde{X}_{t-1}}\frac{A_t}{A_{t-1}}) \Rightarrow \Delta \ln X_t \approx \Delta\hat{x}_t + \hat{\varepsilon}_t^A + \ln(\bar{\varepsilon}^A)$.

where $P_t r_t^k$ denotes a common nominal rental cost of capital faced by the firms in the Home country.

Cost minimization implies the following equation of the demand for labor of type $l_t(f,z)$:

$$l_t(f,z) = \frac{1}{n} L_t(z) \left[W_t(f)/W_t\right]^{\frac{\lambda_w}{1-\lambda_w}}, \tag{4.10}$$

where $W_t(f)$ represents the cost of hiring labor of type f. The aggregate labor demand is given by:

$$L_t(z) = \frac{1-\alpha}{\alpha} r_t^k P_t K_t(z)/W_t, \tag{4.11}$$

Further, an aggregate wage index, which minimizes expenditures needed to purchase one unit of labor L_t, is given by:

$$W_t = \left\{ \frac{1}{n} \int_{f=0}^{n} W_t(f)^{\frac{1}{1-\lambda_w}} df \right\}^{1-\lambda_w}. \tag{4.12}$$

Since all firms face the same prices for labor and capital inputs, the cost minimization implies that also the capital-labor ratio is the same for all firms:

$$\frac{W_t L_t}{K_t r_t^k P_t} = \frac{(1-\alpha)}{\alpha}. \tag{4.13}$$

The firms' nominal marginal cost is then given by:

$$MC_t^{nom} = MC_t P_t = \frac{1}{A_t^{(1-\alpha)} \varepsilon_t^Y} \left(\frac{r_t^k P_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{(1-\alpha)}. \tag{4.14}$$

Note that our specification of the intermediate goods producers sector is much more parsimonious than that of Smets and Wouters (2003). For instance, we ignore variable capacity utilization as well as fixed costs of production in our setting. The reason for this is that both mechanisms increase the number of estimable parameters not improving the predictive properties of the model. There is also no indication that these mechanisms have any open economy implications.

Optimal price setting

Deriving the New-Keynesian Phillips Curve we follow the methodology of Calvo (1983) as augmented in Smets and Wouters (2003) and Adolfson, Laseen, Linde, and Villani (2005a). Intermediate good producing firms are assumed to set the prices in a staggered fashion. In each period with probability $(1-\xi_p)$ a firm may adjust its price $P_t(z)$ to the level at which it maximizes the discounted future profits. Since all firms adjusting prices choose the same optimum (see, e.g., Woodford (2003)), we suppress the firm-specific indexation (z) and simply denote the price chosen by P_t^{opt}. The difference compared to the mechanism described in Chapter 3 is that firms that here are not allowed to re-optimize their price update the price indexing it not only to the last period inflation π_t but also to the inflation target for the next period $\bar{\pi}_{t+1}$:[13]

$$P_{t+1} = P_t \left(\pi_t\right)^{\gamma_p} \left(\bar{\pi}_{t+1}\right)^{1-\gamma_p}, \tag{4.15}$$

[13] We allow for the time-varying inflation targets to facilitate the estimation on raw data for inflation and the nominal interest rate (both have a downward trend starting in the 1980's). Technically, incorporating an autoregressive process for the inflation target is almost equivalent to detrending the inflation and the nominal interest rate series 'outside' the model.

where γ_p is an indexation degree.

If we define the nominal marginal cost as $MC_t^{nom} = P_t MC_t$ and set the firms' stochastic discount factor equal to $\beta^i \Lambda_{t,t+i} = \beta^i \frac{U_{C,t+i}}{U_{C,t}}$, where $U_{C,t+i}$ is the household's marginal utility of consumption at time $t+i$, the profit maximization problem is given by:

$$\max_{P_t^{opt}} E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \left[\frac{P_t^{opt} \left(\pi_t \pi_{t+1} ... \pi_{t+i-1}\right)^{\gamma_p} \left(\bar{\pi}_{t+1} \bar{\pi}_{t+2} ... \bar{\pi}_{t+i}\right)^{1-\gamma_p} - MC_{t+i}^{nom}}{P_{t+i}} Y_{t+i}(z) \right] \tag{4.16}$$

subject to the demand function (4.2).

The first order condition may be written as follows:

$$E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \left[\begin{array}{c} \frac{-\frac{1}{\lambda_p - 1} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} (\bar{\pi}_{t+1} \bar{\pi}_{t+2} ... \bar{\pi}_{t+i})^{1-\gamma_p} \frac{P_t^{opt}}{P_{t+i}}^{-\frac{\lambda_p}{\lambda_p - 1}}}{P_{t+i}} \\ + \frac{\frac{\lambda_p}{\lambda_p - 1} MC_{t+i}^{nom} \frac{P_t^{opt}}{P_{t+i}}^{-\frac{\lambda_p}{\lambda_p - 1} - 1}}{P_{t+i}} \end{array} \right] \frac{1}{n} Y_{t+i} = 0. \tag{4.17}$$

Using the definition (4.3), the aggregate price index is given by:

$$P_t = \left[(1 - \xi_p) \left(P_t^{opt} \right)^{1/(1-\lambda_p)} + \xi_p \left(\left(\pi_{t-1}\right)^{\gamma_p} \left(\bar{\pi}_t\right)^{1-\gamma_p} P_{t-1} \right)^{1/(1-\lambda_p)} \right]^{1-\lambda_p} \tag{4.18}$$

The price determined in this way may be seen as an output index deflator. In the observed data, we link it to the GDP deflator.

Production of consumption and investment good

A final consumption good is produced by a representative consumption good distributor. This firm combines part of the domestic output index with imported goods to produce a final consumption basket according to the Cobb-Douglas technology:

$$C_t(j) = \frac{C_{D,t}^{\omega_C}(j) M_{C,t}^{1-\omega_C}(j)}{\omega_C^{\omega_C} (1 - \omega_C)^{(1-\omega_C)}}, \tag{4.19}$$

where ω_C (ω_C^* in the Foreign economy) is the share of domestically produced goods in the consumer basket.[14] $C_{D,t}(j)$ and $M_{C,t}(j)$ denote the composites of a continuum of differentiated consumption goods, each supplied by a different firm located in the Home (Foreign) economy which follows the CES function:

$$C_{D,t} = \left[\left(\frac{1}{n} \right)^{\frac{\lambda_C - 1}{\lambda_C}} \int_0^n C_{D,t}(z)^{\frac{1}{\lambda_C}} dz \right]^{\lambda_C}, \tag{4.20}$$

$$M_{C,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{\lambda_C - 1}{\lambda_C}} \int_n^1 M_{C,t}(z)^{\frac{1}{\lambda_C}} dz \right]^{\lambda_C}, \tag{4.21}$$

where $C_{D,t}(z)$ and $M_{C,t}(z)$ denote the consumption of generic good z and $\frac{\lambda_C}{\lambda_C - 1}$ is the elasticity of substitution between types of differentiated domestic or foreign goods.

[14] See Armington (1969).

Given the decision on consumption $C_t(j)$ (see Subsection 4.3.2 below), household j will optimally allocate the expenditure on $C_{D,t}$ and $M_{C,t}$ by minimizing the total expenditure $P_t^C C_t(j)$ under the constraint given by (4.19):[15]

$$\min_{C_{D,t}M_{C,t}} E_0 \sum_{t=0}^{\infty} \beta^{-t}(\frac{U_{C,t}}{U_{C,0}})(P_t C_{D,t} + P_t^{C,imp} M_{C,t}) + P_t^C \left[C_t - \frac{C_{D,t}^{\omega_C} M_{C,t}^{(1-\omega_C)}}{\omega_C^{\omega_C}(1-\omega_C)^{(1-\omega_C)}} \right], \tag{4.22}$$

where P_t, $P_t^{C,imp}$ are the price sub-indices for domestic and imported goods expressed in the domestic currency. For determination of $P_t^{C,imp}$ see subsection below. P_t^C may be interpreted as a consumption price index - a shadow cost of producing an additional unit of a consumption good.

The optimal allocation of expenditures across domestic and foreign goods implies the following demand functions:

$$C_{D,t}(z) = \left[\frac{1}{n} \left(\frac{P_t(z)}{P_t} \right)^{\frac{\lambda_C}{\lambda_C - 1}} \right] \frac{P_t^C}{P_t} \omega_C C_t, \tag{4.23}$$

$$M_{C,t}(z) = \left[\frac{1}{1-n} \left(\frac{P_t^{C,imp}(z)}{P_t^{C,imp}} \right)^{\frac{\lambda_C}{\lambda_C - 1}} \right] \frac{P_t^C}{P_t^{C,imp}} (1-\omega_C) C_t \tag{4.24}$$

The overall price index P_t^C, defined as the minimum expenditure required to purchase goods resulting in the index C_t, is then given by:

$$P_t^C = (P_t)^{\omega_C} \left(P_t^{C,imp} \right)^{(1-\omega_C)}. \tag{4.25}$$

Symmetric results hold for the foreign economy. Investment good production is modeled in a very similar manner assuming preference parameters to be ω_I and ω_I^*, respectively.

Retail firms

Following Betts and Devereux (2000) we now introduce retail firms. These firms import foreign differentiated goods for which the law of one price holds at the border. However, in determining the domestic currency price of the imported good, importers are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run and is modeled through the same type of Calvo setup as in the subsection above. Hence, in any period t, a fraction $1-\xi_p^{imp}$ of firms sets prices optimally, while a fraction ξ_p^{imp} of goods prices are adjusted according to an indexation rule analogous to that for intermediate goods producers. This scheme allows for the possibility that importing firms update to the domestic inflation target $\bar{\pi}_t$.

Hence, the consumption good importing firms face the following optimization problem:[16]

$$\max_{P_t^{C,imp,opt}} E_t \sum_{i=0}^{\infty} (\xi_p^{imp}\beta)^i \Lambda_{t,t+i} \left[\begin{array}{c} \left(\pi_t^{C,imp} \pi_{t+1}^{C,imp} ... \pi_{t+i-1}^{C,imp} \right)^{\gamma_p^{imp}} (\bar{\pi}_{t+1}\bar{\pi}_{t+2}...\bar{\pi}_{t+i})^{1-\gamma_p^{imp}} P_t^{C,imp,opt} M_{C,t+i}(z) \\ -S_{t+i} P_{t+i}^* M_{C,t+i}(z) \end{array} \right], \tag{4.26}$$

[15]Note that we suppress the household-specific indexation due to the fact that households are homogeneous in each region.

[16]Since all importing firms that re-optimize their price will set the same price, so the price chosen is denoted by $P_t^{C,imp,opt}$.

where $S_{t+i}P^*_{t+i}$ is the marginal cost of the importing firm or a price paid to the foreign producer. S_t denotes the nominal exchange rate and is fixed in the model for the currency area.[17] Inserting the demand schedules (4.21) in the firms optimization problem, the first order condition with respect to the consumption good is:

$$\max_{P_t^{C,imp,opt}} E_t \sum_{i=0}^{\infty} (\xi_p^{imp}\beta)^i \Lambda_{t,t+i} \left[\begin{array}{c} \frac{2\lambda_C-1}{\lambda_C-1}\left(\pi_t^{C,imp}\pi_{t+1}^{C,imp}...\pi_{t+i-1}^{C,imp}\right)^{\gamma_p^{imp}} \left(\bar{\pi}_{t+1}\bar{\pi}_{t+2}...\bar{\pi}_{t+i}\right)^{1-\gamma_p^{imp}} \\ \times P_t^{C,imp,opt}\frac{1}{1-n}\left(\frac{P_t^{C,imp,opt}}{P_{t+i}^{C,imp}}\right)^{\frac{\lambda_C}{\lambda_C-1}} \frac{P_{t+i}^C}{P_{t+i}^{C,imp}}\left(1-\omega_C\right)C_{t+i} \\ -\frac{\lambda_C}{\lambda_C-1}S_{t+i}P^*_{t+i}\frac{1}{1-n}\left(P_t^{C,imp,opt}\right)^{\frac{1}{\lambda_C-1}} \\ \times\left(\frac{1}{P_{t+i}^{C,imp}}\right)^{\frac{\lambda_C}{\lambda_C-1}} \frac{P_{t+i}^C}{P_{t+i}^{C,imp}}\left(1-\omega_C\right)C_{t+i} \end{array} \right] = 0, \tag{4.27}$$

The aggregate import price index is given by:

$$P_t^{C,imp} = \left[(1-\xi_p^{imp})\left(P_t^{C,imp,opt}\right)^{1/\left(1-\lambda_p^C\right)} + \xi_p\left((\pi_{t-1}^{C,imp})^{\gamma_p^{imp}}\left(\bar{\pi}_t\right)^{1-\gamma_p^{imp}}P_{t-1}^{C,imp}\right)^{1/\left(1-\lambda_p^C\right)}\right]^{1-\lambda_p^C} \tag{4.28}$$

As we do not use the investment deflator series in estimation, parameters determining the price of imported investment good would be only weakly identifiable. For this reason we assume that the law of one price holds for this type of goods.[18] Hence, $P_t^{I,imp} = S_t P_t^*$.

4.3.2 Households and preferences

A typical open economy is inhabited by a representative household who owns capital, which it rents to domestic firms and provides labor in exchange for wage income. It derives satisfaction from leisure and consuming commodities which are composites of domestically produced and imported goods (see subsection above). Each period, the representative household decides what part of the income to spend on consumption and what to save, in effect to maximize its discounted utility given the budget constraint.

We consider a cashless limit of money in the utility function framework a la Woodford (2003). Each household j maximizes an intertemporal utility function given by:

$$E_0\sum_{t=0}^{\infty}\beta^t[U_t\left(C_t(j),l_t(j)\right)], \tag{4.31}$$

[17] We postulate that even after adoption of the Euro the so-called border effects, estimated by Engel and Rogers (1999) to be important within the Europe, result in the international price discrimination.

[18] By assuming that the law of one price holds, so that there is no stickiness in importer price setting, we arrive at the model similar to Obstfeld and Rogoff (2000) and Monacelli (2003). The consumer price index is then simply defined as:

$$P_t^C = (P_t)^{\omega_C}\left(S_tP_t^*\right)^{(1-\omega_C)}. \tag{4.29}$$

Time differencing the terms of trade definition implies $\mathcal{T}_t/\mathcal{T}_{t-1} = \frac{S_t\pi_t^*}{S_{t-1}\pi_t}$. The consumer price inflation reads then as follows:

$$\pi_t^C = \pi_t\left(\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}}\right)^{(1-\omega_C)}, \tag{4.30}$$

where $\pi_t = P_t/P_{t-1}$, $\pi_t^C = P_t^C/P_{t-1}^C$. We will use this result while examining the evidence of price discrimination within the Euro area. As all the deviations from the LOOP are unimportant in the long run, the result above is also useful for the derivation of the steady state of the baseline model.

Furthermore, assuming the limiting value of $\omega_C = 1$, so that foreign goods are no longer part of the domestic consumption bundle, delivers the familiar closed-economy New Keynesian model as in Chapter 3.

where $\beta \in (0, 1)$ is the discount factor, $C_t(j)$ is period t per capita consumption of the commodity bundle defined in Section 4.3.1. $l_t(j)$ is the labor effort. Let $l_t(f, j)$ denote the number of hours of type f labor. There exists a continuum of labor types, indexed for the Home country by $f \in [0, n]$, then the variable that appears in the utility is defined as: $l_t(j) = \int\limits_{f=0}^{n} l_t(f, j)df$.

The explicit form of the household's instantaneous utility assumed here is given by:

$$U_t(j) = \left[\varepsilon_t^C \ln(C_t(j) - H_t(j)) - \varepsilon_t^L A_L \frac{1}{1+\sigma_L} (l_t(j))^{1+\sigma_L}\right], \tag{4.32}$$

where $H_t(j) = hC_{t-1}$ is an external habit stock and $h \in (0, 1)$. The inverse of the elasticity of work effort with respect to the real wage is denoted by σ_L. We follow, e.g., Ireland (2002) and restrict the elasticity of intertemporal substitution of consumption to unity, which is also in line with the most recent estimates for the Euro area. This restriction is necessary to ensure balanced growth in the model.[19] ε_t^C denotes an exogenous preference shock common to all households of a given country and ε_t^L is a labor supply shock.

Given the well-documented departures from the Uncovered Interest rate Parity (UIP) condition between the major currencies in Europe (prior to EMU), we introduce the incomplete asset market into the model. It is assumed that households in the Home country are able to trade in two nominal riskless bonds B_t and B_t^* denominated in domestic and foreign currency, respectively. These bonds are issued by Home country residents in the domestic and foreign currency to finance their consumption. Households face a cost of undertaking positions in the foreign market. For simplicity, we assume that foreign residents can only allocate their wealth in bonds denominated in foreign currency.

Hence, the budget constraint of the consumer in the Home country is given by:

$$0 = \left[\begin{array}{c} P_t^C C_t(j) + P_t^I I_t(j) + B_{t+1}(j) + S_t B_{t+1}^*(j) \\ -R_{t-1}B_t(j) - W_t(j)_t(j) - P_t r_t^k K_t(j) \\ -Div_t(j) - TAX_t(j) + TR_t(j) - R_{t-1}^* \Phi\left(\tilde{B}_{t-1}^*\right) S_t B_t^*(j) \end{array}\right], \tag{4.33}$$

where $P_t r_t^k K_t(j)$ denotes household's income from renting capital, $W_t(j)l_t(j)$ represents its total wage income, $Div_t(j)$ are dividends derived from the imperfect competitive intermediate firms, $TAX_t(j)$ is a lump sum tax paid by the household and $TR_t(j)$ are transfers to the household j. Further, R_{t-1} is a gross nominal interest rate and $R_{t-1}^* \Phi\left(\tilde{B}_t^*\right)$ is the risk adjusted gross nominal interest rate on foreign currency denominated bonds. Function $\Phi(\cdot)$ depends on the real holdings of the foreign assets ($\tilde{B}_t^* \equiv \frac{S_t B_{t+1}^*}{P_t A_t}$) in the entire economy.[20]

In each country, households accumulate capital $K_t(j)$ and bear the costs of capital adjustment. The law of motion for capital owned by a household in the Home country is given by:

$$K_{t+1}(j) = (1-\delta)K_t(j) + \varepsilon_t^I F(I_t(j), I(j)_{t-1}), \tag{4.34}$$

where $0 < \delta < 1$ is the physical depreciation rate of capital. We follow Christiano, Eichenbaum, and Evans (2001) and assume that the function turning investment into physical capital has a form:

$$F(I_t(j), I_{t-1}(j)) = (1 - \tilde{S}(\frac{I_t(j)}{I_{t-1}(j)}))I_t(j), \tag{4.35}$$

[19] This restriction applies for models with an instantaneous utility function that is additively separable in consumption and leisure.

[20] This cost allows us to achieve the stationarity in the net foreign asset position. See Benigno (2001) Kollmann (2004) and Rabanal and Tuesta (2005) for applications in two-country model. Schmitt-Grohé and Uribe (2003) present four further alternatives: two versions of an endogenous discount factor, a debt-contingent interest rate premium and complete financial markets. They argue that once all five models are made to share the same calibration, the quantitative predictions are virtually identical.

where $\tilde{S}\left(\bar{\varepsilon}^A\right) = \tilde{S}'\left(\bar{\varepsilon}^A\right) = 0$. $\tilde{S}''\left(\bar{\varepsilon}^A\right) \equiv S'' > 0$ is the only identifiable parameter upon the linearized version of the model. Expression (4.35) implies that[21]

$$F_1(I_t(j), I_{t-1}(j)) = \frac{\partial F(I_t(j), I_{t-1}(j))}{\partial I_t(j)} = -\tilde{S}'(\frac{I_t(j)}{I_{t-1}(j)})(\frac{I_t(j)}{I_{t-1}(j)}) + (1 - \tilde{S}(\frac{I_t(j)}{I_{t-1}(j)})) \tag{4.36}$$

and

$$F_2(I_t(j), I_{t-1}(j)) = \frac{\partial F(I_t(j), I_{t-1}(j))}{\partial I_{t-1}(j)} = \tilde{S}'(\frac{I_t(j)}{I_{t-1}(j)})(\frac{I_t(j)}{I_{t-1}(j)})^2 \tag{4.37}$$

On the balanced growth path the following holds:[22]

$$F_1(I, I) = -\tilde{S}'(\bar{\varepsilon}^A)\bar{\varepsilon}^A + (1 - S(\bar{\varepsilon}^A)) = 1 \tag{4.38}$$

and

$$F_2(I, I) = \tilde{S}'(\bar{\varepsilon}^A)\left(\bar{\varepsilon}^A\right)^2 = 0 \tag{4.39}$$

Consumer's program

Summarizing, the intertemporal optimization problem of the representative household in the Home economy is given by:

$$\max_{C_t(j), I_t(j), l_t(j), B_{t+1}(j), B^*_{t+1}(j), K_{t+1}(j), Q_t(j), \lambda_t(j)} E_0 \sum_{t=0}^{\infty} \mathcal{L}_t(j),$$

where

$$\begin{aligned} \mathcal{L}_t(j) &= \varepsilon_t^C \ln(C_t(j) - hC_{t-1}) - \varepsilon_t^L A_L \frac{1}{1+\sigma_L}\left(l_t(j)\right)^{1+\sigma_L} \\ &+ \lambda_t(j) \left[\begin{array}{c} W_t(j)l_t(j) + P_t r_t^k K_t(j) + TR_t(j) + Div_t(j) - TAX_t(j) \\ -P_t^C C_t(j) - P_t^C I_t(j) - B_{t+1}(j) + R_{t-1}B_t(j) \\ -S_t B^*_{t+1}(j) + R^*_{t-1}\Phi\left(\tilde{B}^*_{t-1}\right) S_t B^*_t(j) \\ +P_t Q_t(j)\left[-K_{t+1}(j) + (1-\delta)K_t(j) + \varepsilon_t^I F(I_t(j), I(j)_{t-1})\right] \end{array} \right], \end{aligned}$$

and where λ_t, Q_t are multipliers associated with the budget constraint and the law of motion for capital, respectively. Q_t may be seen as the price of installed capital. We further assume that the initial level of wealth is the same across households and they work for firms sharing the profits in equal proportion, which implies that within a country all households face the same budget constraint choosing the same path of consumption, investment and capital.

The conditions characterizing the allocations in the Home economy are:

$$\frac{1}{R_t} = \beta E_t \left[\frac{\varepsilon_{t+1}^C (C_t - hC_{t-1})}{\varepsilon_t^C (C_{t+1} - hC_t)\pi_{t+1}^C}\right] \tag{4.40}$$

[21] See Adolfson, Laseen, Linde, and Villani (2005a).

[22] We have also experimented with the alternative setting by adopting an explicit functional form for capital adjustment costs which is more common in the US literature (Erceg, Guerrieri, and Gust (2003)). For the purpose of our analysis the original setting has been transformed to allow for balanced growth: $K_{t+1}(j) = (1-\delta)K_t(j) + \Psi_t K_t(j)$ and $\Psi_t = I_t(j)/K_t(j) - \frac{\phi_{I_1}}{2}(\frac{I_t(j)}{K_t(j)} - \varepsilon_t^I(\bar{\varepsilon}^A + \delta - 1))^2 - \frac{\phi_{I_2}}{2}(\frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}})^2$where $\phi_{I_1}, \phi_{I_2} \geq 0$ and $\bar{\varepsilon}^A$ is the long-term productivity growth, ε_t^I is a temporary investment shock (an unexpected increase in the demand for investment is equivalent to the increase in the capital depreciation rate.

Contrary to Juillard, Karam, Laxton, and Pesenti (2004), we find that the specification of capital adjustment *à la Christiano, Eichenbaum and Evans* implies a much higher marginal likelihood of the model (not reported in the thesis). Thus, the choice of a proper functional form is of great importance for model-based predictions.

$$\frac{S_t}{R_t^* \Phi\left(\tilde{B}_t^*\right) E_t S_{t+1}} = \beta E_t \left[\frac{\varepsilon_{t+1}^C (C_t - hC_{t-1})}{\varepsilon_t^C (C_{t+1} - hC_t)\pi_{t+1}^C} \right] \tag{4.41}$$

$$-\frac{P_t^I}{P_t} + Q_t \varepsilon_t^I F_1(I_t, I_{t-1}) + E_t(\frac{\pi_{t+1}}{R_t} Q_{t+1} \varepsilon_{t+1}^I F_2(I_{t+1}, I_t)) = 0 \tag{4.42}$$

$$-E_t(\frac{R_t}{\pi_{t+1}} Q_t) + E_t(r_{t+1}^k) + E_t(Q_{t+1}(1-\delta)) = 0 \tag{4.43}$$

Equations (4.40) and (4.41) are Euler equations and eventually imply the modified uncovered interest parity condition. Equation (4.42) may be interpreted as investment demand, (4.43) determines the price of installed capital. The first order condition for the household's optimal choice of wage rate and labor effort is provided in subsection below.

Wage determination

Following Kollmann (2001), we assume that nominal wage setting is modeled in a fashion *à la Calvo,* with random duration of contracts. The wage rate can be optimized in any particular period with the probability $1 - \xi_w$. The household after having re-optimized the nominal wage at W_t^{opt} takes into account that with the probability ξ_w it will not be able to re-optimize in the future. However, this household will be able to adjust its wage to the past inflation and, as opposed to the model considered in Chapter 3, to the announced inflation target and the predicted growth path:

$$W(j)_{t+1} = \left(\pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+1}^A\right) W_t^{opt}, \tag{4.44}$$

where γ_w dentes the indexation degree.

The jth household that can re-optimize its wage faces the following optimization problem (irrelevant terms have been suppressed in the household's objective):

$$\max_{W_t^{opt}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \frac{(l_{t+i}(j))^{1+\sigma_L}}{1+\sigma_L} \\ +\lambda_{t+i} \left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+2} \bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt} l_{t+i}(j) \end{array} \right\} \tag{4.45}$$

subject to the following demand function for time $t+i$ (households always meet the demand for labor at its chosen wage level) :

$$l_{t+i}(j) = \frac{1}{n} \left(\frac{\left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+2} \bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i}, \tag{4.46}$$

where $P_t^C \lambda_{t+i} = \frac{\varepsilon_t^C}{C_t - hC_{t-1}}$.

The resulting first order condition is given by:[23]

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{\left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} (\bar{\pi}_{t+i}...\bar{\pi}_{t+2}\bar{\pi}_{t+1})^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \\ \times \frac{\lambda_w}{1-\lambda_w} \left(\frac{\left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} (\bar{\pi}_{t+i}...\bar{\pi}_{t+2}\bar{\pi}_{t+1})^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} L_{t+i} \frac{1}{W_{t+i}} \\ \times \left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} (\bar{\pi}_{t+i}...\bar{\pi}_{t+2}\bar{\pi}_{t+1})^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) \\ + \frac{1}{1-\lambda_w} \lambda_{t+i} \left(\frac{\left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} (\bar{\pi}_{t+i}...\bar{\pi}_{t+2}\bar{\pi}_{t+1})^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \frac{1}{W_{t+i}} \\ \times \left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} (\bar{\pi}_{t+i}...\bar{\pi}_{t+2}\bar{\pi}_{t+1})^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) \end{array} \right\} \tag{4.47}$$

The aggregate wage index is given by:

$$W_t = \left[(1-\xi_w) \left(W_t^{opt}\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\left(\pi_{t-1}^C\right)^{\gamma_w} (\bar{\pi}_t)^{1-\gamma_w} \varepsilon_t^A W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{4.48}$$

By introducing wage stickiness we dampen the response of marginal cost to structural shocks and thus allow for a greater degree of endogenous price stickiness. For the full derivation see Appendix C.

Real and nominal exchange rate

The real exchange rate S_t^{real} is (using stationary variables) defined as $S_t^{real}/S_{t-1}^{real} = \left(S_t \pi_t^{C*}\right) / \left(S_{t-1} \pi_t^C\right)$. Equations (4.40) and (4.41) imply an arbitrage condition which corresponds to a modified uncovered interest rate parity:

$$\frac{E_t S_{t+1}}{S_t} = \frac{R_t}{R_t^* \Phi\left(\tilde{B}_t^*\right)} \varepsilon_t^{UIP} \tag{4.49}$$

As the departures from the UIP caused by bond holding costs $\Phi\left(\tilde{B}_t^*\right)$ are relatively small in the incomplete asset market models, we assume that the Home country's Euler equation is additionally disturbed by a stationary exogenous variable ε_t^{UIP}. This shock can be interpreted as reflecting transitory biases in households' exchange rate forecasts (see Kollmann (2004)). It also enables the model to generate highly volatile nominal and real exchange rates, under a free float regime. The consequence of which is the so-called exchange rate disconnect or the property that nominal (and real) exchange rate movements are unrelated to fundamentals (see discussion in Obstfeld and Rogoff (2000)). As under a monetary union there is no scope for irrational exchange rate forecasts, the UIP shock is after the transition to EMU set to zero.

4.3.3 Relative prices

In this subsection we briefly discuss various stationary relative price ratios which enter the model. The terms of trade ratio $\mathcal{T}_t \equiv \frac{S_t P_t^*}{P_t}$ measures the overall competitiveness of local producers. As the LOOP is assumed to hold for investment goods, $\mathcal{T}_t$ directly affects the international trade in these goods. In turn, the price of imported goods faced by local consumers, due to the international price discrimination, differs from that faced by foreign counterparts. Then, the ratios: $\mathcal{T}_t^{P^*/P^{C,imp}} \equiv \frac{S_t P_t^*}{P_t^{C,imp}}$ and $\mathcal{T}_t^{P/P^{C,imp^*}} \equiv \frac{P_t}{S_t P_t^{C,imp^*}}$ figure the relative profitability of foreign sales as compared with the local ones. Moreover, the agents make use of the following price ratios in determining the

[23]Note that when the wage is fully flexible, the wage equation is the standard $\varepsilon_t^C W_t / P_t^C = \varepsilon_t^L L_t \sigma_L (C_t - hC_{t-1})$.

composition of their consumption basket: $\mathcal{T}_t^{P/P^C} \equiv \frac{P_t}{P_t^C}$, $\mathcal{T}_t^{P^*/P^{C*}} \equiv \frac{P_t^*}{P_t^{C*}}$, $\mathcal{T}_t^{P^C/P^{C,imp}} \equiv \frac{P_t^C}{P_t^{C,imp}}$ and $\mathcal{T}_t^{P^{C^*}/P^{C,imp^*}} \equiv \frac{P_t^{C^*}}{P_t^{C,imp^*}}$.

4.3.4 Fiscal authority

The role of a fiscal authority is highly simplified in the model.[24] The fiscal authority co-creates demand but its spending rule is assumed to follow an autoregressive process. We assume that spending, financed by lump-sum taxes, falls solely on the final good G, which is entirely domestically produced, and that the price of government consumption coincides with the output deflator. Since Ricardian equivalence holds in this model, we can assume without loss of generality that there is no public debt. The government budget constraint is hence given by:

$$P_t G_t + TR_t = TAX_t \tag{4.50}$$

The budget spending represents on average a constant part of output and evolves according to the following rule:

$$G_t = \varepsilon_t^G Y_t, \tag{4.51}$$

where

$$\ln(\varepsilon_t^G) = (1-\rho_G)\ln(\bar{\varepsilon}^G) + \rho_G \ln(\varepsilon_{t-1}^G) + u_t^G + u_t^{Gcom}. \tag{4.52}$$

4.3.5 Market clearing conditions

Equilibrium requires that all markets clear. In particular, the final goods market (for consumption and investment goods) clears when the demand from the households ($C_{D,t}, I_{D,t}$), the government (G_t) and the Foreign economy ($M_{C,t}^*, M_{I,t}^*$) can be met by the production of the intermediate domestic firms.

$$Y_t = C_{D,t} + \frac{(1-n)}{n} M_{C,t}^* + I_{D,t} + \frac{(1-n)}{n} M_{I,t}^* + G_t \tag{4.53}$$

Using the information about domestically consumed goods and exports (see the optimization problem of the final good distributors), we may derive the following equation for aggregate output:[25]

$$\begin{aligned} Y_t \; &= \; \omega_C \frac{P_t^C}{P_t} C_t + \omega_I \mathcal{T}_t^{(1-\omega_I)} I_t + G_t \\ &\quad + \frac{(1-n)}{n}\left[(1-\omega_C^*)\frac{P_t^{C*}}{P_t^{C,imp^*}} C_t^* + (1-\omega_I^*)\mathcal{T}_t^{\omega_I^*} I_t^*\right]. \end{aligned} \tag{4.56}$$

A similar expression is derived for the Foreign country:

[24] Note that incorporating explicit taxation into our model could not improve its out-of-sample accuracy. Indeed, defining new exogenous processes, which would stand for tax rates, might create some problems in the identification of the remaining shocks.

[25] If the LOOP also holds for the consumption goods, the aggregate output equations reduce to:

$$Y_t = \omega_C C_t \mathcal{T}_t^{(1-\omega_C)} + \omega_I I_t \mathcal{T}_t^{(1-\omega_I)} + G_t + \frac{(1-n)}{n}\left[(1-\omega_C^*)C_t^* \mathcal{T}_t^{\omega_C^*} + (1-\omega_I^*) I_t^* \mathcal{T}_t^{\omega_I^*}\right] \tag{4.54}$$

and

$$Y_t^* = \frac{n}{n-1}\left[(1-\omega_C)C_t \mathcal{T}_t^{-\omega_C} + (1-\omega_I) I_t \mathcal{T}_t^{-\omega_I}\right] + G_t^* + \omega_C^* C_t^* \mathcal{T}_t^{(\omega_C^*-1)} + \omega_I^* I_t^* \mathcal{T}_t^{(\omega_I^*-1)}. \tag{4.55}$$

$$\begin{aligned} Y_t^* &= \frac{n}{n-1}\left[(1-\omega_C)\frac{P_t^C}{P_t^{C,imp}}C_t + (1-\omega_I)\mathcal{T}_t^{-\omega_I}I_t\right] + \\ &+G_t^* + \omega_C^*\frac{P_t^{C*}}{P_t^*}C_t^* + \omega_I^*\mathcal{T}_t^{(\omega_I^*-1)}I_t^*. \end{aligned} \tag{4.57}$$

The asset market equilibrium becomes:

$$S_tB_{t+1}^* - R_{t-1}^*\Phi\left(\tilde{B}_{t-1}^*\right)S_tB_t^* = P_tY_t - \left(P_tG_t + P_t^CC_t + P_t^II_t\right) \tag{4.58}$$

Equilibria in the factor markets require that

$$L_t = \int_{z=0}^{n} L_t(z)dz \text{ and } L_t^* = \int_{z=n}^{1} L_t^*(z)dz, \tag{4.59}$$

$$K_t = \int_{z=0}^{n} K_t(z)dz \text{ and } K_t^* = \int_{z=n}^{1} K_t^*(z)dz. \tag{4.60}$$

4.3.6 Monetary authority

The model accounts for the monetary regime change in the Euro area occurring in 1999. The data set considered in this chapter begins in 1980. Monetary policy between that year and the end of 1998 is better described as a free float regime with autonomous monetary authorities. After the fourth quarter of 1998 the monetary union was established in the area.

In the currency union monetary policy is conducted by a supranational central bank. It fixes the bilateral nominal exchange rates and sets the common nominal interest rate according to a modified Taylor rule:

$$\begin{aligned} \ln\left(\frac{R_t^{EMU}}{\bar{R}_t}\right) &= \rho_R^{EMU}\ln\left(\frac{R_{t-1}^{EMU}}{\bar{R}_t}\right) + (1-\rho_R^{EMU}) \\ &\times\left\{r_\pi^{EMU}\ln\left(\frac{\pi_{t-1}^{C_{EMU}}}{\bar{\pi}_t}\right) + r_y^{EMU}\ln\left(\frac{Y_{t-1}^{EMU}}{A_t^n\left(A_t^*\right)^{(1-n)}\bar{Y}^{EMU}}\right)\right\} + u_t^{R\_EMU} \end{aligned} \tag{4.61}$$

Hence, the deviation of the short-term gross nominal interest rate R_t^{EMU} from its target value $\bar{R}_t$ are adjusted in response to deviations of area-wide output Y_t^{EMU} from its long run growth path and to the deviations of the area-wide inflation $\pi_t^{C_{EMU}}$ from the time-varying inflation target $\bar{\pi}_t$. The inflation target is driven by the following exogenous process:

$$\ln(\bar{\pi}_t) = (1-\rho_\pi)\ln(\bar{\pi}) + \rho_\pi\ln(\bar{\pi}_{t-1}) + u_t^\pi, \tag{4.62}$$

where $\bar{\pi}$ is the steady state gross rate of inflation. There are two monetary shocks in the model, one is a persistent shock to the inflation objective $\bar{\pi}_t$, the other is a temporary Euro area-wide interest rate shock $u_t^{R\_EMU}$. Parameter ρ_R^{EMU} captures the degree of interest rate smoothing. The nominal interest rate target is given by:

$$\bar{R}_t = \frac{\bar{\varepsilon}^A}{\beta}\bar{\pi}_t \tag{4.63}$$

By definition, the consumer price inflation and aggregate output in the EMU are as follows

$$\ln\left(\pi_t^{C_{EMU}}\right) \equiv n\ln\left(\pi_t^C\right) + (1-n)\ln\left(\pi_t^{C^*}\right) \tag{4.64}$$

and

$$\ln\left(Y_t^{EMU}\right) \equiv n\ln\left(Y_t\right) + (1-n)\ln\left(Y_t^*\right). \tag{4.65}$$

Prior to EMU, we account for the autonomous monetary policies in the area:

$$\ln\left(\frac{R_t}{\bar{R}_t}\right) = \rho_R\ln\left(\frac{R_{t-1}}{\bar{R}_t}\right) + (1-\rho)\left\{r_\pi\ln\left(\frac{\pi_{t-1}^C}{\bar{\pi}_t}\right) + r_y\ln\left(\frac{Y_{t-1}}{A_t\bar{Y}}\right)\right\} + u_t^R + u_t^{R^{com}} \tag{4.66}$$

and

$$\ln\left(\frac{R_t^*}{\bar{R}_t}\right) = \rho_R^*\ln\left(\frac{R_{t-1}^*}{\bar{R}_t}\right) + (1-\rho_R^*)\left\{r_\pi^*\ln\left(\frac{\pi_{t-1}^{C^*}}{\bar{\pi}_t}\right) + r_y^*\ln\left(\frac{Y_{t-1}^*}{A_t^*\bar{Y}^*}\right)\right\} + u_t^{R^*} + u_t^{R^{com}}. \tag{4.67}$$

Similar monetary rules are used to close the closed-economy models, which are constructed by a straightforward reduction of the two-country model.

4.4 Macroeconomic equilibrium and model solution

A rational expectations equilibrium in each of the two-region models derived above is a sequence of endogenous variables:

$$\left\{\begin{array}{c} K_t,\ K_t^*,\ L_t,\ L_t^*,\ C_t,\ C_t^*,\ I_t,\ I_t^*,\ Y_t,\ Y_t^*,\ Y_t^{EMU},\ Q_t,\ Q_t^*,\ r_t^k,\ r_t^{k^*},\ MC_t,\ MC_t^*,\ W_t,\ W_t^*, \\ W_t^{opt},\ W_t^{*opt},\ P_t,\ P_t^*,\ P_t^{opt},\ P_t^{*opt}, P_t^{C,imp},\ P_t^{C,imp^*}, P_t^{C,imp,opt},\ P_t^{C,imp,opt^*},\ \pi_t,\ \pi_t^*,\ \pi_t^{C,imp}, \\ \pi_t^{C,imp^*},\ \pi_t^C,\ \pi_t^{C^*}, \pi_t^{C_{EMU}},\ R_t,\ R_t^*,\ R_t^{EMU}, S_t^{real},\ S_t,\ B_t^* \end{array}\right\}_{t=0}^{\infty}$$

that (i) satisfy the Home and Foreign consumers' optimality conditions, (ii) maximize firms' profits or minimize their costs, (iii) satisfy the market clearing conditions for each asset and each good, in all markets where it is traded, and (iv) satisfy the resource constraints.

The endogenous variables in the model are driven by exogenous shocks. The shocks arising from technology and preferences $\left\{\ A_t,\ A_t^*,\ \varepsilon_t^\pi,\ \varepsilon_t^Y,\ \varepsilon_t^{Y^*},\ \varepsilon_t^C,\ \varepsilon_t^{C^*},\ \varepsilon_t^I,\ \varepsilon_t^{I^*},\ \varepsilon_t^L,\ \varepsilon_t^{L^*},\ \varepsilon_t^G,\ \varepsilon_t^{G^*}\right\}_{t=0}^{\infty}$ are assumed to follow first-order autoregressive processes. The shocks in the linearized Phillips Curve and CPI equation $\left\{\varepsilon_t^{mc}, \varepsilon_t^{mc^*}, \varepsilon_t^{CPI}, \varepsilon_t^{CPI^*}\right\}_{t=0}^{\infty}$ are assumed to be serially uncorrelated.[26] These shocks are defined for both free float regime and monetary union models. In turn, local $\left\{u_t^R,\ u_t^{R^*}\right\}$ and common $\left\{\ u_t^{R\_EMU},\ u_t^{R^{com}}\right\}$ temporary interest rate shocks, as well as persistent shocks to the inflation target and the UIP shock are conditional on the monetary regime in force.

Since the equilibrium above does not have a closed-form solution, one of the possibilities is to approximate solutions constructed numerically from a log-linearization of the model around its steady state. However, the system above implies that in equilibrium real variables inherit a unit root from the processes for the technology progress A_t and A_t^*. In such conditions the log-linearization procedure is not accurate. The alternative procedure is to consider the system in stationary variables obtained using the following transformations: $\tilde{K}_{t+1} \equiv \frac{K_{t+1}}{A_t}$, $\tilde{K}_{t+1}^* \equiv \frac{K_{t+1}^*}{A_t^*}$,

[26] In general, the microeconomic interpretation of the shock in the Phillips curve is quite doubtful. As in the case of non-zero steady state inflation it is impossible to write the definition of the aggregate price P_t when price markup is time varying, we introduce an alternative 'fiscal' shock on the marginal cost. This shock is observationally equivalent to the one in the linearized Phillips Curve of Smets and Wouters (2003). Similarly, we introduce the inefficient shock to the linearized CPI equation. This shock is observationally equivalent to a shock to the elasticity of substitution among imported consumption goods.

$\tilde{C}_t \equiv \frac{C_t}{A_t}, \tilde{C}_t^* \equiv \frac{C_t^*}{A_t^*}$, $\tilde{I}_t \equiv \frac{I_t}{A_t}$, $\tilde{I}_t^* \equiv \frac{I_t^*}{A_t^*}$, $\tilde{G}_t \equiv \frac{G_t}{A_t}$, $\tilde{G}_t^* \equiv \frac{G_t^*}{A_t^*}$, $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$, $\tilde{Y}_t^* \equiv \frac{Y_t^*}{A_t}$, $\tilde{W}_t \equiv \frac{W_t}{A_t P_t}$, $\tilde{W}_t^* \equiv \frac{W_t^*}{A_t^* P_t^*}$, $W_t^+ \equiv \frac{W_t^{opt}}{W_t}$, $W_t^{+*} \equiv \frac{W_t^{*opt}}{W_t^*}$ and $\tilde{B}_t^* \equiv \frac{S_t B_{t+1}^*}{P_t A_t}$.[27]

The system in stationary variables is reported in the subsequent section.

4.4.1 Model in stationary variables

Recapitulating, the model we have formed may be presented in terms of stationary variables.

By multiplying equation (4.58) with the factor $\frac{1}{P_t A_t}$ and using the final good market equilibrium we obtain:

$$\tilde{B}_t^* - R_{t-1}^* \Phi\left(\tilde{B}_{t-1}^*\right) \frac{S_t \tilde{B}_{t-1}^*}{\pi_t \varepsilon_t^A S_{t-1}} = \left[\begin{array}{c} \frac{(1-n)}{n} \left((1-\omega_C^*) \frac{P_t^{C^*}}{P_t^{C,imp^*}} \frac{C_t^*}{\varepsilon_t^Z} + (1-\omega_I^*) \mathcal{T}_t^{\omega_I^*} \frac{I_t^*}{\varepsilon_t^Z} \right) \\ (1-\omega_C) \frac{P_t^C}{P_t} C_t + (1-\omega_I) \mathcal{T}_t^{(1-\omega_I)} I_t \end{array} \right] \tag{4.68}$$

The above equation states that the net foreign assets position (NFA) - an additional state variable in the incomplete asset market model - changes with accruing interest and net exports.

Note that foreign trending variables have been scaled with the technical progress factor A_t^*. As the processes for technology are assumed to be cointegrated, the ratio $\frac{A_t}{A_t^*}$ is stationary. We further define the variable $\varepsilon_t^Z = \frac{A_t}{A_t^*}$ to measure the degree of asymmetry across the regions. This asymmetric technology innovation follows an autoregressive process. Thus, the growth in the rest of the Euro area can be determined endogenously, given the exogenously driven technology progress in the German economy and the law of motion for the asymmetric shock:[28]

$$\varepsilon_t^{A^*} = \varepsilon_t^A \frac{\varepsilon_{t-1}^Z}{\varepsilon_t^Z}. \tag{4.69}$$

Here, the long run growth rates of technology are, in line with the database methodology, set to be equal across the Euro area blocks (see Section 4.5). It should also be noted that due to the included asymmetric technology shock, the model is able to capture observed differences in growth rates of real variables across the regions that apparently result from a real convergence process.

The conditions characterizing the allocations of domestic consumption in the Home and Foreign countries are:

$$\frac{1}{R_t} = \beta E_t \left[\frac{\varepsilon_{t+1}^C (\tilde{C}_t - h\tilde{C}_{t-1}/\varepsilon_t^A)}{\varepsilon_t^C (\varepsilon_{t+1}^A \tilde{C}_{t+1} - h\tilde{C}_t) \pi_{t+1}^C} \right], \tag{4.70}$$

$$\frac{S_t}{R_t^* \Phi\left(\tilde{B}_t^*\right) E_t S_{t+1}} = \beta E_t \left[\frac{\varepsilon_{t+1}^C (\tilde{C}_t - h\tilde{C}_{t-1}/\varepsilon_t^A)}{\varepsilon_t^C (\varepsilon_{t+1}^A \tilde{C}_{t+1} - h\tilde{C}_t) \pi_{t+1}^C} \right], \tag{4.71}$$

and

$$\frac{1}{R_t^*} = \beta E_t \left[\frac{\varepsilon_{t+1}^{C^*} (\tilde{C}_t^* - h^* \tilde{C}_{t-1}^*/\varepsilon_t^{A^*})}{\varepsilon_t^{C^*} (\varepsilon_{t+1}^{A^*} \tilde{C}_{t+1}^* - h^* \tilde{C}_t^*) \pi_{t+1}^{C^*}} \right] \tag{4.72}$$

As the remaining equations are symmetric for the Home and Foreign countries, we present only the conditions for the former below.

[27] Note that $\tilde{K}_{t+1} = \frac{K_{t+1}}{A_t}$ because the capital stock at the beginning of the period $t+1$ is determined in period t.

[28] The meaning of the asymmetric shock may be a little bit confusing. This shock does not accelerate the technological progress in the Home economy, it implies only the decrease (increase) in the technological progress in the Foreign economy relative to the Home economy. Therefore, the impact of this shock on output growth may be negative (positive) in both economies (e.g. lower demand in the rest of the Euro area may adversely affect the Home country's exports). A negative asymmetric technology shock also results in a temporary lower level of technology in the Euro area as a whole.

From the household's decision problem we use a capital accumulation equation

$$\varepsilon_t^A \tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \varepsilon_t^I F(\tilde{I}_t, \tilde{I}_{t-1}, \varepsilon_t^A), \tag{4.73}$$

the equation defining price of installed capital

$$-E_t(\frac{R_t}{\pi_{t+1}} Q_t) + E_t(r_{t+1}^k) + E_t(Q_{t+1}(1-\delta)) = 0, \tag{4.74}$$

and the equation for investment demand (we use that $\frac{P_t^I}{P_t} = \mathcal{T}_t^{(1-\omega^I)}$)

$$-\mathcal{T}_t^{(1-\omega^I)} + Q_t \varepsilon_t^I F_1(\tilde{I}_t, \tilde{I}_{t-1}, \varepsilon_t^A) + E_t(\frac{\pi_{t+1}}{R_t} Q_{t+1} \varepsilon_{t+1}^I F_2(\tilde{I}_{t+1}, \tilde{I}_t, \varepsilon_{t+1}^A)) = 0. \tag{4.75}$$

Production technology in the intermediate sector is given by:

$$\tilde{Y}_t = \left(\varepsilon_t^A\right)^{-\alpha} \varepsilon_t^Y \tilde{K}_t^\alpha L_t{}^{1-\alpha} \tag{4.76}$$

Cost optimization in the intermediate sector implies the following labor demand equation

$$L_t = \frac{1-\alpha}{\alpha} r_t^k \tilde{K}_t / (\tilde{W}_t \varepsilon_t^A) \tag{4.77}$$

and marginal cost equation:

$$MC_t = \frac{1}{\varepsilon_t^Y} \left[\frac{\tilde{W}_t}{1-\alpha}\right]^{1-\alpha} \left[\frac{r_t^k}{\alpha}\right]^\alpha . \tag{4.78}$$

The aggregate demand in the Home country is as follows:

$$\begin{aligned} \varepsilon_t^Z \tilde{Y}_t \quad &= \quad \varepsilon_t^Z \omega_C \frac{P_t^C}{P_t} \tilde{C}_t + \varepsilon_t^Z \omega_I \mathcal{T}_t^{(1-\omega_I)} \tilde{I}_t + \varepsilon_t^Z \tilde{G}_t \\ &\quad + \frac{(1-n)}{n} \left\{ (1-\omega_C^*) \frac{P_t^{C^*}}{P_t^{C,imp^*}} \tilde{C}_t^* + (1-\omega_I^*) \mathcal{T}_t^{\omega_I^*} \tilde{I}_t^* \right\} \end{aligned} \tag{4.79}$$

We obtain a similar expression for the Foreign country:

$$\begin{aligned} \left(\varepsilon_t^Z\right)^{-1} \tilde{Y}_t^* \quad &= \quad \frac{n}{n-1} \left\{ (1-\omega_C) \frac{P_t^C}{P_t^{C,imp}} \tilde{C}_t + (1-\omega_I) \mathcal{T}_t^{-\omega_I} \tilde{I}_t \right\} \\ &\quad + \left(\varepsilon_t^Z\right)^{-1} \tilde{G}_t^* + \left(\varepsilon_t^Z\right)^{-1} \omega_C^* \frac{P_t^{C^*}}{P_t^*} \tilde{C}_t^* + \left(\varepsilon_t^Z\right)^{-1} \omega_I^* \mathcal{T}_t^{(\omega_I^*-1)} \tilde{I}_t^* \end{aligned} \tag{4.80}$$

Derivation of stationary equations for real wage and producer price is reported in Appendix C.

Under free float regime, the two-country model is closed with two separate Taylor rules. A monetary union is achieved by fixing the nominal exchange rate $\frac{S_t}{S_{t-1}} = 1$ and by restricting the cost of foreign bond holdings (and the UIP disturbances) to zero. We further assume that the supranational monetary authority follows the Taylor-like feedback rule constructed on the aggregate Euro area variables.

Finally, the stochastic behavior of the stationary system is governed by the following exogenous processes: (ε_t^π, $\varepsilon_t^{\pi^*}$, $\varepsilon_t^{\pi\_EMU}$, ε_t^A, ε_t^Z, ε_t^Y, $\varepsilon_t^{Y^*}$, ε_t^C, $\varepsilon_t^{C^*}$, ε_t^I, $\varepsilon_t^{I^*}$, ε_t^L, $\varepsilon_t^{L^*}$, ε_t^G, $\varepsilon_t^{G^*}$, ε_t^{mc}, $\varepsilon_t^{mc^*}$, ε_t^{CPI}, $\varepsilon_t^{CPI^*}$, ε_t^{UIP}, u_t^R, $u_t^{R^*}$, $u_t^{R^{com}}$, $u_t^{R\_EMU}$).

4.4.2 Solution method

Having derived the first-order conditions and combined them with market-clearing conditions, we log-linearize both DSGE models, for flexible and fixed exchange rates respectively, around the non-stochastic steady state (see Appendix B). These linearized models can be presented as a single nested model, the structure of which is conditional on some regime driving parameters. Subsequently, this nested model is transformed to the Linear Rational Expectations (LRE) system along the lines of Sims (2002):

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)u_t + \Pi(\theta)\eta_t, \tag{4.81}$$

where s_t is a vector of model variables, u_t is a vector of structural shocks, and $\eta_t = s_t - E_{t-1}(s_t)$ is a vector of rational expectations errors. The matrices Γ_0, Γ_1, Ψ and Π which contain reduced-form parameters are functions of deep (structural) parameters stored in the vector θ. As we mentioned, the regime driving part of the vector θ is threshold dependent. Here, the threshold (i.e. the date of regime change) is known to the researcher.

We do not attempt to model the transition process from one regime to another. From the general equilibrium perspective the regime change is treated here as an unexpected event.[29] So the data generating process is determined completely by solving the two models separately for infinite horizon.[30] For $t < T^*$ the solution of the LRE system coincides with that of the flexible exchange rate model and for $t \geq T^*$ the solution for the common currency area is used. To solve the models it is necessary to determine η_t as a function of u_t such that s_t is stable. To do this we use the algorithm by Sims (2002), which is applicable also to large scale models in which matrix Γ_0 is often non-invertible. The resulting canonical multidimensional linear Gaussian state-space

[29]The bubble free model solution as well as model dynamics for the interim phase, i.e. the period in which agents anticipate the introduction of the common currency, might be determined by iterating the system backwards given the infinite horizon solution for the period after transition to monetary union. Below, we consider a special case in which the model solution can be obtained analytically. In practice, the majority of DSGE models might be reformulated to facilitate the analytical solution. This can be achieved, for instance, by allowing for a special form of the utility function, which implies that the number of dynamic equations matches the number of backward and forward looking variables.

Let

$$s_t^B = As_{t-1}^B \tag{4.82}$$

$$s_t^F = Ds_{t-1}^B \tag{4.83}$$

denote the infinite horizon solution of the DSGE model for monetary union. s_t^B and s_t^F are , respectively, the backward looking and forward looking parts of the state vector respectively. The rational expectation system for the period prior to the adoption of the common currency, i.e. for $t-1 < T^*$, is given by:

$$s_t^B = Gs_{t-1}^B + Hs_{t+1}^F \tag{4.84}$$

$$s_t^F = Ms_{t-1}^B + Ks_{t+1}^F \tag{4.85}$$

The solution for the interim period is obtained by iterating the system backwards. Plugging (4.83) into (4.84) and (4.85) we obtain the following solution for the period t conditional on the information from $t-1 = T^* - 1$:

$$s_t^B = Gs_{t-1}^B + HDs_t^B$$

$$s_t^F = Ms_{t-1}^B + KDs_t^B$$

Finally, we have

$$s_t^B = (I - HD)^{-1}Gs_{t-1}^B \equiv \Omega^{(1)}s_{t-1}^B \tag{4.86}$$

$$s_t^F = (M + KD(I - HD)^{-1}G)s_{t-1}^B \equiv \Lambda^{(1)}s_{t-1}^B \tag{4.87}$$

Plugging (4.86) into (4.84) we iteratively obtain the solution conditional on the information from $t-1 = T^* - 2, T^* - 3, ..., T^* - i$. The model dynamics for the entire interim phase are determined as follows:

$$s_t^B = \Omega^{(i)}s_{t-1}^B \tag{4.88}$$

$$s_t^F = \Lambda^{(i)}s_{t-1}^B, \tag{4.89}$$

where

$$\Omega^{(i)} = \left(I - H\Lambda^{(i-1)}\right)^{-1} \tag{4.90}$$

$$\Lambda^{(i)} = (M + K\Lambda^{(i-1)}\Omega^{(i)}) \tag{4.91}$$

Note that the above solution is not time invariant and leads to non-linearities in the transition equation of the state space representation of the DSGE model. For this reason, likelihood functions cannot be constructed with the Kalman filter that is readily available for linear models. The recursive evaluation of the likelihood is possible by applying, e.g., the particle filter. This, however, is related with the unfeasible long computation time.

[30]Our approach to the DSGE model estimation may be easily augmented. It is possible to incorporate further regimes into the estimation procedure. In order to account for the fact that before 1993 the European currencies were pegged to the Deutsche Mark (the fluctuations were restricted to $\mp$ 2.25%) one may construct an additional DSGE model for that period. Thus, the monetary policy would be determined by the imperfect peg against the DM. Since the interest rate is assumed to be the instrument which is used to keep the nominal exchange constant up to an exogenous policy shock, the monetary policy rule in the rest of the euro area countries would be given by $\hat{r}_t^* = \hat{r}_t + u_t^{EMS}$. Thus, the interest rate would respond one-to-one to changes in the Bundesbank's monetary policy but it would also be affected by an exogenous shock.

model is conditional on parametrization and has the following form:

$$s_t = A_1(\theta)s_{t-1} + R_1(\theta)u_t \quad (4.92)$$
$$y_t^{obs1} = G_1(\theta)x_t(\theta) + B_1(\theta)s_t \quad (4.93)$$

and

$$s_t = A_2(\theta)s_{t-1} + R_2(\theta)u_t \quad (4.94)$$
$$y_t^{obs2} = G_2(\theta)x_t(\theta) + B_2(\theta)s_t, \quad (4.95)$$

where equations (4.92) and (4.94) are transition equations, which coincide with the infinite horizon solutions of the free float regime and monetary union model, respectively. The measurement equations (4.93) and (4.95) complete the model by specifying how the state interacts with the vector of observations. The structural shocks u_t are assumed to be multivariate normal distributed: $u_t \sim N(0, \Sigma_1)$ for $t < T^*$ and $u_t \sim N(0, \Sigma_2)$ for $t \geq T^*$. y_t^{obs} is a vector of observable variables and its size is threshold dependent.[31] x_t is a vector of predetermined variables - the common rate of technology growth, the steady state inflation and the steady state nominal interest rate - in our model.

4.5 Data consideration

In the estimation, we mainly use the seasonally adjusted 'synthetic' quarterly data from the AWM Database by Fagan, Henry, and Mestre (2001) (update containing data from 1970:Q1 to 2003:Q4) and the German VGR data (see Appendix D).[32] A few comments are in order here. The area-wide series have been obtained by fixed-weight aggregation of the single country data:[33]

$$\ln(X) = \sum w_i \ln(x_i), \quad (4.96)$$

where X is the area-wide aggregate, x_i are the national variables and w_i are the weights employed. This method is referred to as the 'index method'. It implies that the area-wide real and nominal variables as well as corresponding prices are all exactly equal to the weighted average of national variables, which is not the case with the alternative 'current exchange rate method'. The advantages of the 'index method', as pointed out in Fagan and Henry (1998), are as follows: first, it facilitates comparison of the area-wide and national equations (or models, as is the case in this chapter) and second, the measure of price inflation implied by this method - i.e. a weighted average of national inflation rates - corresponds to the definition employed by international organisations. Finally, the fixed weights attached to national variables (see Table 4.1 on the next page) allow for a straightforward disaggregation of the area-wide series. Here, we employ the German national accounts data to disaggregate the AWM series and subsequently infer on the German economy and the rest of the Euro area.[34] It should be noted that our approach is consistent with the AWM database methodology also in other respects. Namely, deriving the DSGE model we account for the fact that the fixed weights applied in the the AWM data-set restrict the long-run growth rates of real variables to be the same across the Euro area countries.

[31] Extending the vector of observables in the DSGE model for the monetary union with a local interest rate would imply singularities in the forecast-error-matrix in the Kalman filter (see Section 4.6).

[32] The model is estimated on the sample 1980:Q1-2003:Q4. The data from the 1970's is used for initialization of the Kalman filter.

[33] For the purpose of aggregation all single country series have been re-based to the same year (i.e. 1995).

[34] We also follow the AWM methodology constructing the time series for the unified Germany by re-scaling data available only for West Germany by the ratio of the two series at the starting date of the post-unification series.

EU 12	
Belgium	0.036
Germany	0.283
Spain	0.111
France	0.201
Ireland	0.019
Italy	0.195
Luxembourg	0.003
Netherlands	0.060
Austria	0.030
Portugal	0.024
Finland	0.017
Greece	0.025

Table 4.1: Weights used in aggregation

Confronting the model with more data exploits more model implied cross-equation restrictions and delivers a more stringent test of the model. To estimate the baseline model we decide to match the following set of variables: GDP, consumption, investment, annualized GDP deflator, annualized consumption deflator, annualized nominal interest rate, real wage rate and total employment for Germany and the respective Euro area wide aggregates.[35] Moreover, the data based prior distribution for the monetary union model is constructed using the information contained in the nominal exchange rate prior to EMU (see Section 4.6 below). The nominal exchange rate we refer to is the synthetic bilateral Germany - rest of the Euro area exchange rate, computed using the corresponding exchange rates of the Deutsche Mark against European currencies and weighted with the average share in exports. For details see Appendix D.

In the estimation we follow two alternative data approaches. The model is estimated by (i) employing the HP-filtered data (a model absent long-run restrictions) and (ii) on the log-differences calculated from the raw data (a model with a balanced growth mechanism). The former approach has been followed in Chapter 3 of this thesis and is emphasized, for instance, by Juillard, Karam, Laxton, and Pesenti (2004). We follow their suggestion and eliminate the trend also in nominal variables by applying the HP filter. A DSGE model in the presence of the balanced growth mechanism is estimated on the raw data, for instance, in Adolfson, Laseen, Linde, and Villani (2005a).[36]

As the model is estimated simultaneously with both single country data and the aggregate data for the whole Euro area, the original AWM time series are disaggregated, using the index method,

[35] Note that the set of area-wide aggregates employed here contains that used by Smets and Wouters (2003). We exploit this fact while comparing the forecast accuracy of our two-region model with the one-country area-wide model in the spirit of Smets and Wouters.

[36] Note that the wage rate series included in the AWM database are calculated as a total labor compensation divided by total employment. This means, however, that the AWM definition of the wage rate does not correspond to that in the theoretical model, where the wage rate is defined as a wage per hour worked. For the sake of consistency between the theoretical model and the data, we correct the wage series, more precisely the log-differences, for the change in average hours worked per employed (see Appendix D). For this purpose the annual OECD data (see Figure 4.6 on page 179) have been employed. We intrapolate these data to quarterly using the Spline technique (in line with the AWM database methodology). Strikingly, as there has been a systematic decline in hours worked per employed over the last decades, correcting the wage series for this effect offsets the discrepancy between the balanced growth model, where all real variables are assumed to grow on average like productivity, and the AWM data, where the (uncorrected) growth rates of the real wage are on average much lower than those of the other real variables. It should be noted that the measurement issue discussed here refers to all models estimated with the original AWM data (Fagan, Henry, and Mestre (2001), Smets and Wouters (2003), Adolfson, Laseen, Linde, and Villani (2005a) and many others do not account for the discrepancy between the model and the data). Especially in the case of balanced growth models, which are constructed to explain low frequency movements of the variables and are estimated with the system methods (see e.g. Adolfson, Laseen, Linde, and Villani (2005a)), correcting for the effects related to the evolution of average hours worked is of particular importance.

via the model's measurement equation. Below we present the measurement equation for the free float model[37]

$$\begin{bmatrix} \Delta \ln Y_t^{GER} \\ \Delta \ln Y_t^{EMU} \\ \Delta \ln C_t^{GER} \\ \Delta \ln C_t^{EMU} \\ \Delta \ln I_t^{GER} \\ \Delta \ln I_t^{EMU} \\ \Delta \ln(W_t^{GER}/P_t^{GER}) \\ \Delta \ln(W_t^{EMU}/P_t^{EMU}) \\ EM_t^{GER} \\ EM_t^{EMU} \\ 4\times\Delta \ln P_t^{GER} \\ 4\times\Delta \ln P_t^{EMU} \\ \ln R_t^{GER} \\ \ln R_t^{EMU} \\ \Delta \ln S_t \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{c}_t - \hat{c}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{c}_t^* - \hat{c}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{c}_t - \hat{c}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{\imath}_t^* - \hat{\imath}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{w}_t - \hat{w}_{t-1} + \ln \bar{\varepsilon}^A + \hat{\varepsilon}_t^A \\ (1-n)(\hat{w}_t^* - \hat{w}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{w}_t - \hat{w}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \widehat{em}_t \\ n \times \widehat{em}_t + (1-n)\widehat{em}_t^* \\ 4 \times (\hat{\pi}_t + \ln \bar{\pi}) \\ 4 \times (n\hat{\pi}_t + (1-n)\hat{\pi}_t^* + \ln \bar{\pi}) \\ 4 \times (\hat{r}_t + \ln \bar{R}) \\ 4 \times (n\hat{r}_t + (1-n)\hat{r}_t^* + \ln \bar{R}) \\ \hat{S}_t - \hat{S}_{t-1} \end{bmatrix} \tag{4.97}$$

The measurement equation completing the state space representation of the DSGE model for the monetary union is

$$\begin{bmatrix} \Delta \ln Y_t^{GER} \\ \Delta \ln Y_t^{EMU} \\ \Delta \ln C_t^{GER} \\ \Delta \ln C_t^{EMU} \\ \Delta \ln I_t^{GER} \\ \Delta \ln I_t^{EMU} \\ \Delta \ln(W_t^{GER}/P_t^{GER}) \\ \Delta \ln(W_t^{EMU}/P_t^{EMU}) \\ EM_t^{GER} \\ EM_t^{EMU} \\ 4\times\Delta \ln P_t^{GER} \\ 4\times\Delta \ln P_t^{EMU} \\ \ln R_t^{EMU} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{y}_t - \hat{y}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{c}_t - \hat{c}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{c}_t^* - \hat{c}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{c}_t - \hat{c}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{\imath}_t - \hat{\imath}_{t-1} + \ln(\bar{\varepsilon}^A) + \hat{\varepsilon}_t^A \\ (1-n)(\hat{\imath}_t^* - \hat{\imath}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \hat{w}_t - \hat{w}_{t-1} + \ln \bar{\varepsilon}^A + \hat{\varepsilon}_t^A \\ (1-n)(\hat{w}_t^* - \hat{w}_{t-1}^* + \hat{\varepsilon}_t^{A^*}) + n(\hat{w}_t - \hat{w}_{t-1} + \hat{\varepsilon}_t^A) + \ln \bar{\varepsilon}^A \\ \widehat{em}_t \\ n \times \widehat{em}_t + (1-n)\widehat{em}_t^* \\ 4 \times (\hat{\pi}_t + \ln \bar{\pi}) \\ 4 \times (n\hat{\pi}_t + (1-n)\hat{\pi}_t^* + \ln \bar{\pi}) \\ 4 \times (\hat{r}_t^{EMU} + \ln \bar{R}) \end{bmatrix} \tag{4.98}$$

Since there is no official Euro area-wide data for the 'hours worked', denoted in our model by L_t ($\hat{l}_t$ in log-deviations), we use available data on employment. Similarly as in Chapter 3, the aggregate employment equation is stated as follows:

$$\Delta\widehat{em}_t = \beta E_t \Delta\widehat{em}_{t+1} + \frac{(1-\xi_L)(1-\beta\xi_L)}{\xi_L}(\hat{l}_t - \widehat{em}_t) \tag{4.99}$$

[37]Since we estimate the model with GDP, consumption and investment series, which are not transformed to per capita terms, the growth rate of technology ε_t^A is in fact a mixture of the population growth and technological progress. In turn, the growth rates of real wage rate we use in the estimation (see Appendix D) have been separated out of the reference to the population growth. Therefore, we correct the series containing the growth rates of the real wage for the change in total population in Germany and in the Euro area, respectively.

4.6 Estimation

Recent advances in estimation methods, especially applying Bayesian techniques, have made it feasible to estimate even large-scale multi-country DSGE models. In this chapter we follow Schorfheide (2000) and apply a two-step estimation procedure involving calibration and Bayesian Maximum Likelihood methods.[38]

4.6.1 Evaluation of the posterior

With the prior specified (for details see the subsequent section) we turn to the estimation of the model. The principle of the approach is straightforward. We look for a parameter vector which maximizes the posterior, given the prior and the likelihood based on the data. Subsequently, we use an approximation around the posterior mode to generate a sample of MCMC draws to undertake a more extensive inference on the structural parameters, by characterizing the shape of the posterior distribution.

Let M denote a model that accounts for the permanent monetary regime change in the Euro area. From the perspective of general equilibrium there should be uncertainty about the date of the switch and possibility that agents may anticipate the regime change. However, as mentioned above, we do not attempt to model that transition mechanism here. The change in the monetary regime is taken as a given exogenous event that creates a structural break in the economy at time T^*. Conditional on such exogenous structural break occurring, we are back to the full general equilibrium paradigm. Hence, prior to time point T^* the economy might be described by the model M_1 and by M_2 afterwards.

The probability density function of the data Y_T, conditional on the threshold dependent model M and parametrization θ is

$$p(Y_T|\theta, M) = \prod_{t=1}^{T} p(y_t|Y_{t-1}, \theta, M). \tag{4.100}$$

Taking into account the permanent regime change at T^*, the data density may be rewritten as

$$\begin{aligned} p(Y_T|\theta, M) &= p(Y_{T^*-1}|\theta, M) \prod_{t=T^*}^{T} p(y_t|Y_{t-1}, \theta, M) && (4.101)\\ &= p(Y_{T^*-1}|\theta, M_1) p(Y_T|Y_{T^*-1}, \theta, M). && (4.102) \end{aligned}$$

The joint likelihood function is, thus, any function proportional to the probability density of the first subsample Y_{T^*-1} and the conditional density of the second subsample.

$$L(Y_T|\theta, M) \propto p(Y_{T^*-1}|\theta, M_1) p(Y_T|Y_{T^*-1}, \theta, M) \tag{4.103}$$

As data distributions on both subsamples are assumed to be independent, we arrive at

$$L(Y_T|\theta, M) \propto p(Y_{T^*-1}|\theta, M_1) p(Y_{T-T^*}|\theta, M_2), \tag{4.104}$$

where $p(Y_{T-T^*}|\theta, M_2) = \prod_{t=T^*}^{T} p(y_t|y_{T^*-1}, y_{T^*}, y_{T^*+1}, ..., y_{t-1}, \theta, M_2)$ and $p(y_{T^*-1}|\theta, M_2)$ is the initial prediction density for the regime M_2.

The goal is to evaluate the posterior density $p(\theta|Y_T, M)$ conditional on the entire sample and the threshold dependent model so that we could utilize the information prior to EMU while estimating

[38] In addition to our own codes we use modified codes by Michael Juillard, Frank Schorfheide and Cristopher Sims.

the parameters of the monetary union model. Since the structural break between regimes affects the so-called regime-driving parameters, as well as the structure of the model, some preliminary comments about the prior distributions are in order. As mentioned, the models for free float regime and monetary union may be written as a single nested model M. By restricting some structural parameters to zero this nested model can be transformed either to the free float regime model or to the monetary union model. For this purpose, the parameter vector θ may be partitioned as follows: $\theta = [\theta_0, \theta_1, \theta_2]'$, where θ_0 denotes the regime invariant parameters, θ_1 and θ_2 denote the regime-driving parameters. The prior distributions for regime driving parameters are defined as follows: $p(\theta_1|M_1)$, $p(\theta_1 = 0|M_2) = 1$ and $p(\theta_2 = 0|M_1) = 1$, $p(\theta_2|M_2)$. Hence, the estimation procedure will, in fact, provide the joint densities of $\theta_0|Y_T, M$, $\theta_1|Y_T, M_1$ and $\theta_2|Y_T, M_2$, which, for the notational simplicity, will be referred to as $p(\theta|Y_T, M)$.

By Bayes theorem, the posterior density $p(\theta|Y_T, M)$ is related to the prior and the likelihood as follows:

$$p(\theta|Y_T, M) = \frac{p(Y_T|\theta, M)p(\theta|M)}{p(Y_T|M)} \propto L(Y_T|\theta, M)p(\theta|M) \tag{4.105}$$

Using the knowledge about the regime change, the posterior is rewritten as

$$p(\theta|Y_T, M) \propto p(Y_{T^*-1}|\theta, M_1)p(Y_{T-T^*}|\theta, M_2)p(\theta|M). \tag{4.106}$$

Expression (4.106) has an interesting interpretation in terms of utilizing the information from the pre-EMU subsample to obtain the posterior distribution for regime invariant parameters.

Let us consider the estimation of a model M_2 on the subsample $\{y_t\}_{t=T^*}^{T}$. In addition, we define the prior which combines the subjective beliefs of the researcher with the information based on the data set $\{y_t\}_{t=1}^{T^*-1}$, i.e. the prior which is given by

$$p^d(\theta) = \frac{p(Y_{T^*-1}|\theta, M_1)p(\theta|M)}{p(Y_{T^*-1}|M_1)} \tag{4.107}$$

By Bayes theorem we also have

$$\begin{aligned} p(\theta|Y_{T-T^*}, M_2) &= \frac{p^d(\theta)p(Y_{T-T^*}|\theta, M_2)}{p(Y_{T-T^*}|M_2)} \\ &= \frac{p(Y_{T^*-1}|\theta, M_1)p(\theta|M)p(Y_{T-T^*}|\theta, M_2)}{p(Y_{T^*-1}|M_1)p(Y_{T-T^*}|M_2)}. \end{aligned} \tag{4.108}$$

and

$$p(\theta|Y_{T-T^*}, M_2) \propto L(Y_{T-T^*}|\theta, M_2)p(\theta|M)L(Y_{T^*-1}|\theta, M_1). \tag{4.109}$$

Finally, we notice that

$$p(\theta|Y_{T-T^*}, M_2) \propto p(\theta|Y_T, M). \tag{4.110}$$

Hence, our approach, detailed as evaluation of the joint likelihood along with the posterior distribution conditional on the entire sample, is equivalent to the approach, where the prior based on the pre-EMU data is assigned and the EMU data likelihood is evaluated.

Assuming that priors are independently distributed, the logarithm of the posterior may be calculated as

$$\ln p(\theta|Y_T, M) = \ln L(Y_T|\theta, M) + \sum_{i=1}^{N} \ln p(\theta_i|M), \tag{4.111}$$

where N is the number of estimated parameters.

Exploiting the implications of (4.106) we have

$$\ln p(\theta|Y_T, M) = \ln L(Y_{T^*-1}|\theta, M_1) + \ln L(Y_{T-T^*}|\theta, M_2) + \sum_{i=1}^{N} \ln p(\theta_i|M). \tag{4.112}$$

Due to the unobserved variables in the vector s_t of the model solution and the scale of the maximization problem the log-likelihood has to be evaluated recursively by applying the Kalman filter. The Kalman filter is, in this case, applied to a state space model (4.92)-(4.95) conditional on the monetary regime in force. Note that the likelihood function will differ across the regimes M_1 and M_2, although it will still be multivariate normal. Hence, the joint likelihood function is

$$\begin{aligned}\ln(L) \quad = \quad & -\frac{n(T^*-1)+(n-2)\,(T-T^*)}{2}\ln(2\pi) - \frac{1}{2}\left(\sum_{t=1}^{T^*-1}\ln|F_{1,t}| + \sum_{t=T^*}^{T}\ln|F_{2,t}|\right) + \\ & \frac{1}{2}\left(\sum_{t=1}^{T^*-1}\ln(v_{1,t}'F_{1,t}v_{1,t}) + \sum_{t=T^*}^{T}\ln(v_{2,t}'F_{2,t}v_{2,t})\right). \end{aligned} \tag{4.113}$$

Here, n is the dimension of y^{obs}.[39] $F_{1,t}$ and $F_{2,t}$ are the regime dependent forecast-error covariance matrices:

$$F_{1,t} = B_1 P_{1,t|t-1} B_1' \text{ for } t < T^* \tag{4.114}$$

and

$$F_{2,t} = B_2 P_{2,t|t-1} B_2' \text{ for } t \geq T^* \tag{4.115}$$

$v_{1,t}$ and $v_{2,t}$ are one-step forecast errors:

$$v_{1,t} = y_t^{obs1} - y_{t|t-1}^{obs1} \text{ for } t < T^* \tag{4.116}$$

and

$$v_{2,t} = y_t^{obs2} - y_{t|t-1}^{obs2} \text{ for } t \geq T^* \tag{4.117}$$

$y_{t|t-1}^{obs1}$ and $y_{t|t-1}^{obs2}$ are one-step forecasts of observables.[40]

The state covariance matrices $P_{1,t|t-1}$ and $P_{2,t|t-1}$for the respective regimes are calculated as follows[41]:

$$P_{1,t|t-1} = A_1\left[P_{1,t-1|t-2} - P_{1,t-1|t-2}B_1'(B_1P_{1,t-1|t-2}B_1')^{-1}B_1P_{1,t-1|t-2}\right]A_1' + R_1\Sigma_1R_1' \text{ for } t < T^* \tag{4.118}$$

and

$$P_{2,t|t-1} = A_2\left[P_{2,t-1|t-2} - P_{2,t-1|t-2}B_2'(B_2P_{2,t-1|t-2}B_2')^{-1}B_2P_{2,t-1|t-2}\right]A_2' + R_2\Sigma_2R_2' \text{ for } t \geq T^* \tag{4.119}$$

The breakpoint between the regimes requires some special treatment. After the switch to the new regime, the state vector s_t is initialized with the last values from the previous regime, i.e. one-step conditional forecast $s_{T^*|T^*-1}$ is based on the realized state s_{T^*-1} and initial predictive density is $s_{T^*}|s_{T^*-1} \sim N\left(A_2 s_{T^*-1}, P_{2,T^*|T^*-1}\right)$. Likewise, the initial predictive density of the observables vector is $y_{T^*}|y_{T^*-1} \sim N\left(B_2 s_{T^*-1}, F_{2,T^*|T^*-1}\right)$, where matrices $P_{2,T^*|T^*-1}$ and $F_{2,T^*|T^*-1}$ are replaced with their steady state analogs. To allow for Kalman filtering on the entire sample the size

[39] The dimension of y_t^{obs} is $(n-2)$ after the switch to the monetary union regime.

[40] Note that the sizes of vectors $v_{1,t}$ and $v_{2,t}$ as well as $y_{t|t-1}^{obs1}$, $y_{t|t-1}^{obs2}$ are threshold dependent.

[41] To be more precise, we calculate the steady state values of $P_{1,t|t-1}$, $P_{2,t|t-1}$ for each regime outside the estimation procedure, using equations (4.118)-(4.119).

of the state-variable vector s_t as well as the meaning of its elements, has to be set the same in both regimes.

Having computed the likelihood of the data, the prior distributions are assigned to all parameters of the model. The posterior distribution (4.112) is maximized numerically. In order to tackle the problem of local optima, often faced in numerical optimization; we first apply a random search algorithm over the bounded parameter space.[42] Further, starting with the parameter values obtained in the former step (initial values for which the posterior takes the highest values) we use the 'csminwel' numerical optimization routine developed by Christopher Sims, to locally maximize the posterior. In respect to this, our approach is similar to that described in Onatski and Williams (2004).[43] The indeterminate models are ruled out during the estimation procedure (see Blanchard and Kahn (1980) or Klein (2000)).[44]

In the second stage of the Bayesian estimation procedure, the posterior mode is used as a starting point for the Random Walk Metropolis algorithm. The jumping distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian evaluated numerically at the posterior mode (see Chapter 2 for details).

To this end, in order to check how sensitive our results are with respect to the shape of the prior, we limit the prior information for the DSGE model by assuming the uniform distribution for all the parameters. This procedure is equivalent with the maximization of the likelihood over the bounded space and has been applied by Onatski and Williams (2004) and Bergin (2004). A potential drawback of the method, as we see it, is related to the corner solutions of the optimization problem.[45] [46]

4.6.2 Calibrated parameters and specification of the priors

As pointed out in Chapter 2, some of structural parameters of a DSGE model might not be identifiable upon the data. For this reason, we calibrate the whole set of parameters, which are either related to the omitted time series (like capital stock) or strictly pertain to the model's steady state, and could not be estimated upon the first order Taylor approximation. The former parameters including the physical depreciation rate of capital and the share of capital are pinned down at values common in the Real Business Cycle literature. The latter are calibrated using the simple arithmetic means of the raw data as a reference.[47]

[42] This applies especially to the model estimated with ML approach.

[43] Onatski and Williams (2004) used a genetic algorithm initialized on the draws from the prior distribution.

[44] The likelihood function is maximized here under the constraint that the parameter vector θ does not yield multiple stable solutions. In this case the numeric maximization of the likelihood function might be corrupted by the impossibility to 'wander' through the indeterminacy region even when the maximum is beyond that region. For alternative method see Kremer, Lombardo, and Werner (2004).

[45] The results in Onatski and Williams (2004) indicate that the estimates of consumption habit, fixed cost of production, Calvo parameter in the employment equation, response to a lagged inflation in the monetary feedback rule, inverse of elasticity of labor supply, and capital utilization cost may be seen as corner solutions of the optimization problem.

[46] Similarly to Onatski and Williams (2004), we restrict ourselves to the computation of the posterior mode. However, the parameters have been first transformed as in, e.g., Ireland (2003). The transformation ensures that the structural parameters are bounded within the bounds suggested by theory. For example, we transform the autoregressive parameter as follows: $\rho = \frac{\rho_{trans}^2}{1+\rho_{trans}^2}$. Undertaking the above transformation allows the numerical optimization procedure to search for any value of ρ_{trans} between $-\infty$ and $+\infty$ but restricts the estimate of ρ to be within 0 and 1. This approach does not lead to corner solutions and the point estimates of structural parameters should be very close to those obtained by applying the method of Onatski and Williams (2004).

[47] Note that in few cases the values of parameters calibrated here differ from those in Chapter 3. This is because in Chapter 3, as well as in the original model of Smets and Wouters (2003), the steady state has not been calculated explicitly and the shares of consumption, investment and government consumption in the GDP have been treated as reduced-form parameters. In contrast, the steady state GDP decomposition is linked here to 'deep' parameters.

The physical depreciation rate of capital is calibrated to 0.01. The share of capital input in the production function is fixed at 0.29, in line with the EU estimate (see European Economy (1994)). The steady state gross wage markup λ_w is fixed at 1.5. The relative country size parameter n is set to 0.283, which corresponds to the weight assigned to the German economy in the AWM database (see Table 4.1 on page 113). The levels of technology in Germany and the rest of the Euro area are calibrated to yield the cross country ratio of per capita output of 1.2. All this results in the following GDP decomposition: $\frac{C}{Y} = 0.6, \frac{C^*}{Y^*} = 0.6, \frac{I}{Y} = 0.2, \frac{I^*}{Y^*} = 0.2, \frac{G}{Y} = 0.20, \frac{G^*}{Y^*} = 0.2$. The numbers roughly reflect the mean in the 1980:Q1-2003:Q4 sample. It should be noted, however, that as the steady state, derived in Appendix A, depends on some estimated parameters, the ratios above are likewise updated during the estimation.

Finally, the discount factor β is fixed at 0.995 in our model. Along with the estimated steady-state inflation and long-run growth rate of technology it implies the annualized steady-state nominal interest rate ($\bar{R} = \frac{\bar{\pi}\bar{\varepsilon}^A}{\beta}$) equal to about 6%.[48]

The vast majority of structural parameters in our model are only bounded by the prior distribution and are the subject of estimation.

The locations of the prior distributions are the same across the regions and to a large extent correspond to those in Smets and Wouters (2003), Altig, Christiano, Eichenbaum, and Linde (2003), Adolfson, Laseen, Linde, and Villani (2005a) and Jondeau and Sahuc (2004). Note that in the models considered in this chapter, as opposed to those of Chapter 3, we do not normalize structural shocks, which explains the very different priors and estimates of the stochastics.

We set 1.004 as a mean of the prior for the technology growth rate $\bar{\varepsilon}^A$, which implies an annual growth rate of about 1.6%. Note that the parameter $\bar{\varepsilon}^A$, which we refer to in the model as a steady state growth rate of technological progress, is more or less a mixture of population growth and technological progress because we work with the data in levels, and not in per capita terms. The standard deviation of the asymmetric technology shock ε_t^Z is set to 0.6. This number is estimated from the AR(1) on the cumulated differences in GDP growth rates in the rest of the Euro area and Germany (see Adolfson, Laseen, Linde, and Villani (2005a)). The mean of the prior for the steady state rate of inflation is set at 2.4% (annualized). The persistence parameter for the inflation target is fixed at 0.99, in line with the literature. The mean of the prior for the volatility of this process is set to 0.125, which closely corresponds to the volatility of residuals obtained from regressing the HP-trend of inflation on its own lag.

The degree of openness, which in our case can be deducted from the shares of domestic goods in consumption and investment basket, could be calibrated at the mean of the trade shares in respective economies, as has been done, e.g., in Jondeau and Sahuc (2004). Given the importance of these parameters for the possible influence of foreign shocks and the model fit, we prefer to specify a prior distribution, although rather tight, around the values reported in Jondeau and Sahuc (2004). So, the means of the priors are set respectively to: $\omega_C = 0.65$, $\omega_C^* = 0.8$, $\omega_I = 0.55$, $\omega_I^* = 0.7$.[49]

The standard errors of the innovations are assumed to follow uniform distributions. Ruge-Murcia (2003) note that in DSGE models data are often very informative about the structural disturbances so those very loose priors seem well suited.[50] The distribution of the persistence parameters in the efficient shocks is assumed to follow a beta distribution with mean 0.85 and

[48]Note that the sample used in estimation is not homogenous in terms of e.g. monetary regimes in force. Moreover, some of the countries experienced a long transition phase before adopting the common currency, which might lead to a new steady state. Nevertheless, we decide to set the steady state inflation and nominal interest rates the same for both regions even prior to EMU. This assumption results in more persistent estimates of structural shocks. In addition, smoothed estimates of these shocks, not only the inflation target, might be trending in the past, which indicates presence of a strong exogenous transition mechanism in our model.

[49]Since we do not treat the trade within the rest of the Euro area in terms of exports and imports, the share of domestically consumed goods in the rest of the EMU is much higher compared to the German economy.

[50]We have chosen the mean of the priors so that the model simulated at the mean of the prior roughly replicates the volatility and persistence of the actual data.

standard error 0.1.

As the low frequency movements of the data are roughly explained by the permanent changes in productivity as well as the persistent changes in inflation target, we use the same set of priors for both the model with imposed long run restrictions (and estimated on log-differences) and a model without the balanced growth mechanism (and estimated on detrended data).

Structural parameters, except for regime-driving parameters, are assumed here to be constant over the entire estimation sample. A detailed description of the prior distribution for all estimated models can be found in Table 4.2 on page 124 and Table 4.3 on page 125. While conducting the Bayesian sensitivity analysis we maximize the likelihood of the baseline model over the bounded space. The boundaries are provided in Table 4.20 on page 152.

4.7 Results

This section presents the estimation results. The literature dealing with estimated DSGE models has paid relatively little attention to the issue of result robustness. Besides the parallel work of Lubik and Schorfheide (2005) this study is the only to extensively document the (non-Bayesian) robustness check with respect to the data.[51] Moreover, the comparison of the baseline results with those from the model estimated with the ML approach (over the bounded parameter space) is reported.[52] To this end, we examine the implications of utilizing the synthetic nominal exchange rate series in the estimation.

The thorough discussion of posterior estimates is followed by a comparison of data versus model implied unconditional second moments. Our analysis seeks then a deeper understanding of the role of a balanced growth mechanism in generating the co-movements of German and Euro area-wide aggregates. As we consider in this chapter alternative model specifications, the formal, based on the marginal likelihood, model comparisons are also reported. The analysis on international spillovers within the Euro area is postponed until Section 4.8.

4.7.1 Posterior estimates of the parameters

Tables 4.2 on page 124 and 4.3 on page 125 report parameter estimates - the posterior mode along with the 5th and 95th percentiles of the posterior distribution (obtained after 200,000 draws) - for the benchmark model, determined after extensive investigation of alternative specifications. These included various combinations of nominal and real rigidities - price and wage indexation, capital adjustment costs - as well as variants of exogenous sources of persistence. We have found (although not reported) that our main results do not depend on these alternative specifications.

The posterior distributions of the estimated parameters, plotted along with the imposed priors (see Figure 4.2 on page 177 and Figure 4.3 on page 177), indicate that a vast majority of the estimates are data, not solely prior, driven. By and large, point estimates obtained with the ML approach are close to those obtained for the baseline specification of the priors, with some exceptions to be discussed below.[53] The robustness check with respect to the data is postponed until the next subsection. As a preview of our findings, the data transformations (detrended vs. raw data) matter.

[51] Our preliminary results have been documented in Pytlarczyk (2005).
[52] For alternative methods of sensitivity analysis see Ratto, Roeger, intVeld, and Girardi (2005b).
[53] The ML estimates lie within the 90% posterior interval obtained under 'standard' specification of the prior.

Parameters of the deterministic part and frictions

We begin our analysis with a comparison of parameter estimates obtained from the area wide model, see Table 4.19 on page 151, and the two-region specification.[54]

Although the same priors have been selected for both regions, we find evidence of heterogeneity, in terms of structural parameters, between Germany and the rest of the Euro area. Employment stickiness ξ_L and wage stickiness parameters ξ_w are higher in the rest of the Euro area. For Germany the later parameter approaches the value of 0.73, implying on average wage contract duration of about 4 quarters, which fits well with intuition. The area-wide estimates of the deterministic part and frictions are, in general, located between those obtained for the two blocks by applying the disaggregate model.

Perhaps the most difficult parameters to interpret are the degrees of price stickiness. In all models estimated with the log-differences, including the area-wide model, Calvo domestic price stickiness parameters ξ_p and ξ_p^* are estimated at above 0.92. Interestingly, the posterior distribution of parameter ξ_p^* is bimodal with peaks at 0.91 and 0.95 (see Figure 4.2 on page 177). The Calvo parameter estimates imply that the average duration of intermediates price contracts is above 12 quarters which is at odds with microeconomic surveys. For instance, DeWalque, Smets, and Wouters (2004), indicate that the price contracts in Europe are no longer than 2-4 quarters. We emphasize, however, that the somewhat high degree of price rigidity is one characteristic of model estimates that is robust to alternative specifications. With the ML approach values close to the upper prior boundary have been obtained. Turning to the cross-study comparisons we see that our estimates are only slightly higher than those of Adolfson, Laseen, Linde, and Villani (2005a) and close to those in Onatski and Williams (2004), 0.89 and 0.91 respectively. For the German economy, the evidence of implausibly high Calvo parameter estimates has also been documented in Kremer, Lombardo, and Werner (2004). Those authors eventually decide to fix the parameter at a level consistent with the microevidence, which, however, results in a significant deterioration of the model fit.

One possible explanation for implausibly high estimates of the Calvo parameters may be related to the assumption of a white-noise Phillips Curve equation shock. For instance, DeWalque, Smets, and Wouters (2004) indicate that including a persistent markup shock into their model reduces the estimated degree of price stickiness to 0.73. Nevertheless, the assumption that markup shocks (or shocks to marginal cost, as in our model) follow an autoregressive process may create identification problems for other structural shocks, e.g., for the inflation target.[55] Likewise, the introduction of firm specific capital into a DSGE model may reduce the degree of price stickiness. Technically, the reduced-form parameter at the marginal cost is then multiplied by an additional term standing for real rigidities.[56] All these findings should apply to our model.

The disaggregate modeling approach offers the possibility to augment the nominal rigidities analysis of Chapter 3 by accounting for international aspects of price setting within the Euro area.

Parameters ξ_p^{imp} and $\xi_p^{imp^*}$ which determine the international price discrimination are inferred at about 0.48 and 0.51 in Germany and the rest of the Euro area respectively. In both cases, the posterior distribution appears to be strongly influenced by our choice of the prior.

Applying the ML approach we obtain a slightly lower value (0.31) for import price stickiness in the German economy, which implies that the price of imported goods is changed every 4-5 months. The estimate for the rest of the Euro area is now close to zero. This result compares well with reduced form estimates of Warmedinger (2004), who underlines that import stickiness tends to be higher for Germany. As the estimated deviations from the law of one price are fairly short lived

[54] Although Bayesian estimates for the one-country version of the Euro area model have been reported in the literature, we re-estimate the model based on our particular model specification and data set.

[55] The interpretation provided in the literature is that the data offer two explanations behind the high inflation inertia that we observe; either a rather high degree of price stickiness or highly correlated markup shocks.

[56] See Eichenbaum and Fischer (2004).

in our model, we cannot link our findings to results in Engel and Rogers (1999), who explored failures of the law of one price in Europe when using consumer price data from European cities over the period 1981-97.

Finally, it should be noted that the values of Calvo parameter for importing firms can be influenced, similarly as those for the domestic good producers, by allowing for auto-correlated markup shocks. However, the resulting interactions between Calvo pricing, price indexation and persistent shocks in the CPI equation are, then, likely to induce identification problems.

The estimates of price indexation parameters γ_p and γ_p^* indicate that German firms stronger link their prices to past inflation. The implied weights of lagged inflation in the New-Keynesian Phillips Curve are equal to about 0.23 and 0.16, respectively. In the area-wide model the indexation parameter is estimated to be closer to that of the rest of the Euro area. The estimates obtained for the pooling of closed economies and those from the models without capital (not reported in the thesis) roughly correspond to those for the baseline model. Note that in all cases the posterior is, relative to the prior, shifted to the left. Furthermore, the estimates obtained under the diffuse prior indicate no indexation to the past price or wage inflation in the rest of the Euro area. All this contrasts with the empirical New-Keynesian Phillips Curve literature that finds evidence for backward looking agents, yet confirms the findings of Chapter 3. The indexation mechanism appears to be more relevant for describing the behavior of retail firms in the rest of the area (the ML estimate approaches, then, 0.96).

The shares of imported goods in the domestic consumption and investment bundle, regardless of the local currency pricing or LOOP assumption, are remarkably stable across the models and are estimated to be around 0.20 and 0.45 in Germany and 0.1 and 0.17 in the rest of the Euro area. These shares, together with the habit persistence parameters, determine the steady state openness ratios in our model (see Appendix A). The exports to output ratio is inferred at 0.21 and 0.10 for Germany and the rest of the Euro area, respectively. For the German economy the estimate of the openness lies between the bilateral openness of around 16% and the overall openness that approaches 30%. The ML estimates of the shares of imported goods (0.22, 0.09 and 0.3, 0.15) lie within the prior boundaries, implying exports to output ratios of around 0.2 and 0.08 in Germany and the rest of the Euro area, respectively. This suggests that the data are indeed informative about the openness parameters and support the assumptions of the linkages between the regions.

On the demand side, habit formation seems to play a more important role in the German economy. The habit persistence estimates (the posterior mean is about 0.65 for Germany and 0.60 for the rest of the Euro area) differ only slightly from those obtained by Jondeau and Sahuc (2004) in the multi-country Euro area model estimated with the German, Italian and French data. The area-wide estimate is 0.76 and is much higher than those for the regions. In turn, the estimate for steady state technology growth is much higher in the disaggregate model.

Turning to the estimates of the monetary rule, we see that the long run response of the nominal interest rate to inflation is estimated in all models to be greater than one, in line with the Taylor rule.[57] The estimate from the area-wide model is somewhat lower than the estimates for the two blocks. We note, however, that for the rest of the Euro area the long run response of nominal interest rate to inflation might be influenced by our choice of the prior. Strikingly, the estimate obtained under the uniform priors is much lower, only slightly above unity.[58]

[57] Since the Taylor rule for the EMU is effectively estimated with the information contained in the twenty most recent observations (see the form of the log-likelihood function above), we assign much looser prior to it.

[58] The evidence of the low values of inflation feedback parameter and high values of estimated output gap response parameters may be found, for example, in Welz (2004). It should be underlined that the prior information has been also used in other DSGE studies to pull the Taylor rule estimates towards the values that have been obtained while estimating the Taylor rule 'separately' and are regarded as more plausible.

Persistence and volatility of structural shocks

As is usually the case in DSGE models, the data are quite informative about the persistence and volatility of shocks. Indeed, the patterns of prior and posterior distributions are relatively distinct. In several cases, however, the bimodal posterior distributions indicate that there are possible alternative explanations for the detected persistence. Applying the ML approach (over the bounded parameter space), we find that the estimates are in general similar to those in the baseline model. The more pronounced differences we find for investment and preference shocks for which persistence is then estimated at zero, i.e., at the lower boundary of the prior range.

Commenting on the estimates of the model's stochastic, we should emphasize that there is no consensus in the literature. As we shortly show, these estimates critically depend on data transformations. Moreover, many authors normalize structural shocks, which poses further problems in cross-study comparisons and implies the necessity to redefine the priors.[59] The lack of a consensus regarding the estimates of model stochastics may also be attributed to the fact that the estimates seem to be strongly affected by the choice of measurement errors (if one decides to estimate the volatility of these errors) and even by slight changes in the model structure.[60] Further differences result from the identification problems. For instance, in this chapter we estimate the DSGE model without a time-varying wage markup, contrary to Smets and Wouters (2003). Having allowed for this shock, along with the labor supply shock and inflation target shock, we find that the corresponding estimates are not stable numerically.

Notwithstanding this observation, we find some regularity in our results: First, the majority of structural shocks have a very high persistence (above 0.85), which can often be attributed to the absence of some important shocks in the specification, or to the general weakness of the endogenous persistence mechanism in the model. Second, we find that the persistence of shocks is higher if we close the international linkages, which may also be seen as an indication that the data support the open economy specification (see Tables 4.21 on page 153 and 4.22 on page 154). Third, some of the shocks in the estimated area-wide model seem to be of lower persistence and volatility than those from the disaggregate two-region model. This can probably be attributed to the statistical averaging effect. Fourth, one can expect that all shocks measuring the asymmetry between the regions, here it is the asymmetric technology shock and UIP shock, might be due to long lasting differences in economic growth and nominal interest rates estimated to be highly persistent.

Since the importance of shocks in driving economic fluctuations cannot be directly assessed from the magnitude of the associated standard deviation, we decompose the variance of key macro variables in Section 4.8.

Cross-country correlation of structural shocks

Our benchmark model assumes common components on all efficient shocks. The relevant common factors appear for preference, labor supply and interest rate shocks.

As the baseline specification de facto restricts the cross-country correlation of structural shocks to be positive, we also examine the less restrictive version of the model with a non-diagonal error covariance matrix (see Table 4.23 on page 155 and Table 4.24 on page 156).[61] Correlations between preference and government spending shocks are then estimated at 0.35 and 0.02, respectively. This model, however, does not capture the presumed correlation of monetary shocks prior to the establishment of the EMU. The correlation coefficients for supply shocks are estimated to be negative. So, these results may be seen only to a limited extent as an evidence of the European

[59]Normalization may have an impact on the numerical optimization procedure, making the model less nonlinear. In a two-country framework however not all shocks could be normalized.

[60]The evidence is presented, e.g., in Kremer, Lombardo, and Werner (2004) or Boivin and Giannoni (2005).

[61]We do not refer to this specification as to the baseline because, as shown for instance, in Bergin (2004), different orthogonalization schemes might change the variance decompositions considerably. So, it would be not a priori clear whether the influence attributed to foreign shocks is the artefact of an orthogonalization scheme.

integration or common European business cycles. The model also does not improve significantly on the baseline, at least as far as the marginal density is concerned (see section below).

Turning to the cross study comparison, our results roughly correspond to the estimates in Jondeau and Sahuc (2004) (positive correlation of preference shocks). They are also close to those obtained with reduced-form models by Monteforte and Siviero (2002) (negative coefficients for supply shocks and positive for demand shocks). The issue of the symmetry of shocks to the European economies, or lack thereof, has also been tackled by Eichengreen (1997) and Demertzis and Hughes-Hallett (1998). Their empirical evidence confirms and complements our results. These authors also conclude that, although the European economies have followed rather similar policies in recent years, there is little evidence of a strengthening of the degree of symmetry of the disturbances affecting the various economies.

		prior			baseline model (with common components)		
parameter		type	mean	std/df	mode	90% posterior interval	$\hat{R}$
growth rate of technology	$\bar{\varepsilon}^A$	norm	1.004	0.001	1.0051	(1.0049, 1.0058, 1.0067)	1.003
steady state inflation	$\bar{\pi}$	norm	1.006	0.001	1.0055	(1.0038, 1.0053, 1.0068)	1.001
int. rate smoothing	ρ_R^{EMU}	beta	0.8	0.2	0.848	(0.839, 0.890, 0.941)	1.004
int. rate smoothing	ρ_R	beta	0.8	0.1	0.834	(0.817, 0.857, 0.897)	1.003
int. rate smoothing	ρ_R^*	beta	0.8	0.1	0.902	(0.856, 0.886, 0.918)	1.005
inflation response	r_π^{EMU}	norm	1.5	0.2	1.647	(1.258, 1.539, 1.824)	1.001
inflation response	r_π	norm	1.5	0.1	1.498	(1.304, 1.473, 1.638)	1.013
inflation response	r_π^*	norm	1.5	0.1	1.506	(1.398, 1.531, 1.668)	1.000
output response	r_y^{EMU}	beta	0.2	0.15	0.012	(0.031, 0.100, 0.172)	1.006
output response	r_y	beta	0.2	0.15	0.209	(0.191, 0.315, 0.439)	1.033
output response	r_y^*	beta	0.2	0.15	0.076	(0.059, 0.105, 0.151)	1.010
habit formation	h	beta	0.7	0.1	0.655	(0.668, 0.741, 0.812)	1.258
habit formation	h^*	beta	0.7	0.1	0.603	(0.564, 0.618, 0.672)	1.013
capital adj. cost	S''	norm	4	2	6.362	(5.508, 6.896, 8.281)	1.062
capital adj. cost	$S^{*\prime\prime}$	norm	4	2	3.779	(3.625, 4.775, 5.871)	1.068
Calvo employment	ξ_L	beta	0.7	0.15	0.785	(0.748, 0.784, 0.821)	1.011
Calvo employment	ξ_L^*	beta	0.7	0.15	0.919	(0.881, 0.896, 0.911)	1.001
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.930	(0.922, 0.936, 0.949)	1.015
Calvo domestic prices	ξ_p^*	beta	0.7	0.05	0.954	(0.906, 0.927, 0.948)	1.003
Calvo import	ξ_p^{imp}	beta	0.5	0.05	0.488	(0.414, 0.491, 0.572)	1.001
Calvo import	$\xi_p^{imp^*}$	beta	0.5	0.05	0.513	(0.422, 0.506, 0.594)	1.001
Calvo wages	ξ_w	beta	0.7	0.05	0.725	(0.704, 0.731, 0.757)	1.048
Calvo wages	ξ_w^*	beta	0.7	0.05	0.897	(0.799, 0.844, 0.890)	1.004
indexation domestic prices	γ_p	beta	0.5	0.15	0.302	(0.134, 0.309, 0.468)	1.009
indexation domestic prices	γ_p^*	beta	0.5	0.15	0.188	(0.082, 0.211, 0.328)	1.004
indexation import	γ_p^{imp}	beta	0.5	0.15	0.462	(0.211, 0.457, 0.683)	1.000
indexation import	$\gamma_p^{imp^*}$	beta	0.5	0.15	0.442	(0.232, 0.480, 0.736)	1.000
indexation wages	γ_w	beta	0.5	0.15	0.446	(0.247, 0.476, 0.716)	1.001
indexation wages	γ_w^*	beta	0.5	0.15	0.186	(0.120, 0.261, 0.412)	1.001
share of dom. consum.	ω_C	beta	0.65	0.05	0.802	(0.757, 0.793, 0.831)	1.007
share of dom. consum.	ω_C^*	beta	0.8	0.05	0.896	(0.884, 0.916, 0.946)	1.002
share of dom. invest.	ω_I	beta	0.55	0.05	0.563	(0.525, 0.586, 0.646)	1.004
share of dom. invest.	ω_I^*	beta	0.7	0.05	0.826	(0.729, 0.781, 0.834)	1.017
risk premium	ϕ	invg	0.01	2	0.002	(0.002, 0.003, 0.003)	1.004

Table 4.2: Prior and posterior distributions, baseline model

		prior			baseline model (with common components)		
parameter		type	mean	std/df	mode	90% posterior interval	$\hat{R}$
unit root tech. shock	ρ_A	beta	0.85	0.1	0.913	(0.883, 0.917, 0.955)	1.004
asymmetric technology shock	ρ_Z	beta	0.85	0.1	0.991	(0.969, 0.982, 0.995)	1.009
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.864	(0.796, 0.856, 0.919)	1.001
stationary tech. shock	ρ_Y^*	beta	0.85	0.1	0.946	(0.719, 0.804, 0.884)	1.026
preference shock	ρ_C	beta	0.85	0.1	0.633	(0.370, 0.521, 0.673)	1.088
preference shock	ρ_C^*	beta	0.85	0.1	0.996	(0.983, 0.990, 0.997)	1.074
labor supply shock	ρ_L	beta	0.95	0.05	0.838	(0.760, 0.816, 0.875)	1.000
labor supply shock	ρ_L^*	beta	0.95	0.05	0.994	(0.961, 0.973, 0.985)	1.026
investment shock	ρ_I	beta	0.85	0.1	0.263	(0.167, 0.267, 0.357)	1.014
investment shock	ρ_I^*	beta	0.85	0.1	0.191	(0.154, 0.305, 0.445)	1.000
government spending shock	ρ_G	beta	0.85	0.1	0.976	(0.957, 0.969, 0.982)	1.000
government spending shock	ρ_G^*	beta	0.85	0.1	0.992	(0.980, 0.989, 0.998)	1.013
UIP shock	ρ_{UIP}	beta	0.85	0.1	0.995	(0.987, 0.993, 1.000)	1.004
stdv. stationary technology shock	σ_Y	unif	2	1.2	1.471	(1.065, 1.507, 1.961)	1.026
stdv. stationary technology shock	σ_Y^*	unif	2	1.2	2.315	(2.151, 2.694, 3.158)	1.313
stdv. preference shock	σ_C	unif	2	1.2	1.704	(1.944, 2.761, 3.697)	1.150
stdv. preference shock	σ_C^*	unif	2	1.2	0.000	(0.000, 0.720, 1.440)	1.007
stdv. labor supply shock	σ_L	unif	10	5.9	13.441	(13.142, 14.792, 16.500)	1.336
stdv. labor supply shock	σ_L^*	unif	10	5.9	3.085	(1.073, 3.375, 6.843)	1.141
stdv. monetary shock	σ_R	unif	0.2	0.12	0.072	(0.024, 0.074, 0.115)	1.000
stdv. monetary shock	σ_R^*	unif	0.2	0.12	0.158	(0.143, 0.176, 0.202)	1.007
stdv. monetary shock EMU	σ_R^{EMU}	unif	0.2	0.12	0.180	(0.128, 0.212, 0.298)	1.007
stdv. gov. spending shock	σ_G	unif	2	1.16	2.566	(2.265, 2.671, 3.062)	1.000
stdv. gov. spending shock	σ_G^*	unif	2	1.16	1.734	(1.545, 1.879, 2.240)	1.005
stdv. investment shock	σ_I	unif	10	5.9	15.471	(14.717, 17.324, 20.207)	1.162
stdv. investment shock	σ_I^*	unif	10	5.9	5.407	(5.103, 6.378, 7.578)	1.189
stdv. unit root technology shock	σ_A	unif	0.25	0.15	0.158	(0.119, 0.173, 0.226)	1.030
stdv. asymmetric technology shock	σ_Z	unif	0.5	0.29	0.466	(0.387, 0.459, 0.547)	1.013
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.165	(0.141, 0.168, 0.192)	1.000
stdv. Phillips Curve shock	σ_{mc}^*	unif	0.25	0.15	0.213	(0.185, 0.216, 0.248)	1.007
stdv. infl. target shock	σ_π^{EMU}	unif	0.125	0.1	0.000	(0.000, 0.060, 0.108)	1.002
stdv. infl. target shock	σ_π	unif	0.125	0.1	0.186	(0.130, 0.180, 0.229)	1.001
stdv. infl. target shock	σ_π^*	unif	0.125	0.1	0.000	(0.000, 0.065, 0.145)	1.001
stdv. CPI shock	σ_{CPI}	unif	0.25	0.15	0.334	(0.309, 0.348, 0.389)	1.053
stdv. CPI shock	σ_{CPI}^*	unif	0.25	0.15	0.350	(0.312, 0.352, 0.392)	1.000
stdv. UIP shock	σ_{UIP}	unif	0.25	0.15	0.182	(0.159, 0.214, 0.268)	1.000
stdv. com. comp. tech. shock	σ_Y^{com}	unif	2	1.2	0.004	(0.000, 0.243, 0.496)	1.008
stdv. com. comp. preference shock	σ_C^{com}	unif	2	1.2	1.892	(1.480, 2.066, 2.612)	1.009
stdv. com. comp. labor supply shock	σ_L^{com}	unif	10	5.9	2.516	(1.137, 3.179, 4.849)	1.214
stdv. com. comp. monetary shock	σ_R^{com}	unif	0.2	0.12	0.082	(0.034, 0.073, 0.115)	1.028
stdv. com. comp. gov. spending shock	σ_G^{com}	unif	2	1.2	0.374	(0.000, 0.464, 0.840)	1.010
stdv. com. comp. investment shock	σ_I^{com}	unif	10	5.9	0.000	(0.010, 1.507, 2.728)	1.000
stdv. com. comp. infl. target shock	σ_π^{com}	unif	0.125	0.1	0.094	(0.018, 0.082, 0.145)	1.017

Table 4.3: Prior and posterior distributions, baseline model (cont.)

4.7.2 Modification I: Estimation based on detrended data

So far, we imposed all the long-run restrictions implied by the baseline model, in particular, a common technology growth rate $\bar{\varepsilon}^A$, a common steady state inflation rate $\bar{\pi}$, and a common steady state nominal interest rate $\bar{R}$. In order to assess the role of these restrictions we re-estimate the (appropriately transformed) model on detrended data. The estimates are reported in Tables 4.4 on the next page and Table 4.5 on page 127.

Several broad conclusions emerge. First, the estimates of the frictions prove to be significantly lower while employing the filtered data. For instance, Calvo domestic price stickiness reduces to 0.86 in both regions. We observed a similar pattern for wage stickiness parameters. Yet, the nominal rigidities in the imported good sector seem to be immune to the data transformations. Second, it is likely that the assumption of the common steady state inflation might induce higher estimates of interest rate smoothing parameters. In the model estimated with detrended data, these parameters drop to 0.78 and 0.82 in Germany and the rest of the Euro area, respectively. The fact that the two-region model imposes common steady states for Germany and the rest of the Euro area, might also imply that the estimated fluctuations around the steady states have potentially changed. This may explain some discrepancies in the estimates of inflation and output-gap feedback parameters prior to EMU. Turning to the open economy aspects, the shares of foreign goods in the consumption basket approach now the values of 0.34 and 0.20 in Germany and the rest of the Euro area, respectively. This translates into the openness ratios of 26% and 16%.

Finally, some of the long-lasting deviations from the balanced growth path are, in a model estimated on log-differences, to be explained by other than permanent technology exogenous processes. Hence, the persistence of stationary shocks increases compared to the model estimated with detrended data. This also indicates that an endogenous propagation mechanism is weaker in the balanced growth model.

		prior			baseline model (detrended data)		
parameter		type	mean	std/df	mode	90% posterior interval	$\hat{R}$
int. rate smoothing	ρ_R^{EMU}	beta	0.8	0.2	0.797	(0.687, 0.783, 0.889)	1.004
int. rate smoothing	ρ_R	beta	0.8	0.1	0.779	(0.734, 0.782, 0.826)	1.003
int. rate smoothing	ρ_R^*	beta	0.8	0.1	0.824	(0.786, 0.831, 0.880)	1.004
inflation response	r_π^{EMU}	norm	1.5	0.2	1.447	(1.134, 1.449, 1.754)	1.002
inflation response	r_π	norm	1.5	0.1	1.458	(1.327, 1.471, 1.621)	1.001
inflation response	r_π^*	norm	1.5	0.1	1.322	(1.185, 1.341, 1.509)	1.009
output response	r_y^{EMU}	beta	0.2	0.15	0.075	(0.002, 0.102, 0.186)	1.003
output response	r_y	beta	0.2	0.15	0.071	(0.021, 0.079, 0.133)	1.012
output response	r_y^*	beta	0.2	0.15	0.144	(0.073, 0.166, 0.259)	1.013
habit formation	h	beta	0.7	0.1	0.652	(0.569, 0.691, 0.814)	1.112
habit formation	h^*	beta	0.7	0.1	0.866	(0.806, 0.867, 0.937)	1.031
capital adj. cost	S''	norm	4	2	6.107	(4.147, 6.840, 9.568)	1.023
capital adj. cost	$S^{*''}$	norm	4	2	6.858	(5.082, 7.874, 10.852)	1.009
Calvo employment	ξ_L	beta	0.7	0.15	0.675	(0.645, 0.686, 0.735)	1.011
Calvo employment	ξ_L^*	beta	0.7	0.15	0.810	(0.784, 0.809, 0.836)	1.015
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.864	(0.838, 0.863, 0.889)	1.004
Calvo domestic prices	ξ_p^*	beta	0.7	0.05	0.864	(0.839, 0.864, 0.889)	1.008
Calvo import	ξ_p^{imp}	beta	0.5	0.05	0.509	(0.435, 0.509, 0.581)	1.005
Calvo import	$\xi_p^{imp^*}$	beta	0.5	0.05	0.538	(0.441, 0.536, 0.624)	1.023
Calvo wages	ξ_w	beta	0.7	0.05	0.689	(0.662, 0.696, 0.730)	1.003
Calvo wages	ξ_w^*	beta	0.7	0.05	0.698	(0.691, 0.737, 0.789)	1.026
indexation domestic prices	γ_p	beta	0.5	0.15	0.451	(0.295, 0.472, 0.643)	1.008
indexation domestic prices	γ_p^*	beta	0.5	0.15	0.300	(0.191, 0.317, 0.456)	1.007
indexation import	γ_p^{imp}	beta	0.5	0.15	0.367	(0.167, 0.408, 0.638)	1.003
indexation import	$\gamma_p^{imp^*}$	beta	0.5	0.15	0.361	(0.172, 0.401, 0.632)	1.001
indexation wages	γ_w	beta	0.5	0.15	0.469	(0.252, 0.484, 0.727)	1.005
indexation wages	γ_w^*	beta	0.5	0.15	0.199	(0.078, 0.229, 0.367)	1.009
share of dom. consum.	ω_C	beta	0.65	0.05	0.662	(0.614, 0.668, 0.717)	1.002
share of dom. consum.	ω_C^*	beta	0.8	0.05	0.806	(0.764, 0.805, 0.849)	1.001
share of dom. invest.	ω_I	beta	0.55	0.05	0.562	(0.493, 0.562, 0.644)	1.003
share of dom. invest.	ω_I^*	beta	0.7	0.05	0.774	(0.696, 0.766, 0.829)	1.003
risk premium	ϕ	invg	0.01	2	0.003	(0.002, 0.003, 0.004)	1.004

Table 4.4: Estimation based on detrended data, prior and posterior distributions

parameter		prior			baseline model (detrended data)		
		type	mean	std/df	mode	90% posterior interval	$\hat{R}$
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.732	(0.612, 0.704, 0.800)	1.005
stationary tech. shock	ρ_Y^*	beta	0.85	0.1	0.631	(0.479, 0.626, 0.772)	1.034
preference shock	ρ_C	beta	0.85	0.1	0.681	(0.474, 0.631, 0.793)	1.003
preference shock	ρ_C^*	beta	0.85	0.1	0.273	(0.136, 0.297, 0.441)	1.005
labor supply shock	ρ_L	beta	0.95	0.05	0.556	(0.407, 0.538, 0.679)	1.005
labor supply shock	ρ_L^*	beta	0.95	0.05	0.733	(0.520, 0.651, 0.777)	1.023
investment shock	ρ_I	beta	0.85	0.1	0.509	(0.351, 0.485, 0.614)	1.009
investment shock	ρ_I^*	beta	0.85	0.1	0.329	(0.200, 0.322, 0.429)	1.003
government spending shock	ρ_G	beta	0.85	0.1	0.895	(0.809, 0.883, 0.964)	1.004
government spending shock	ρ_G^*	beta	0.85	0.1	0.743	(0.630, 0.741, 0.852)	1.000
UIP shock	ρ_{UIP}	beta	0.85	0.1	0.915	(0.845, 0.907, 0.971)	1.009
stdv. stationary technology shock	σ_Y	unif	2	1.2	0.890	(0.725, 1.007, 1.233)	1.029
stdv. stationary technology shock	σ_Y^*	unif	2	1.2	1.407	(1.027, 1.440, 2.013)	1.012
stdv. preference shock	σ_C	unif	10	5.9	3.101	(2.360, 3.691, 5.324)	1.011
stdv. preference shock	σ_C^*	unif	10	5.9	5.350	(3.337, 6.146, 9.946)	1.031
stdv. labor supply shock	σ_L	unif	10	5.9	16.264	(14.451, 17.508, 20.213)	1.056
stdv. labor supply shock	σ_L^*	unif	10	5.9	7.811	(6.898, 12.546, 18.599)	1.120
stdv. monetary shock	σ_R	unif	0.2	0.12	0.116	(0.097, 0.125, 0.150)	1.005
stdv. monetary shock	σ_R^*	unif	0.2	0.12	0.137	(0.115, 0.141, 0.166)	1.045
stdv. monetary shock EMU	σ_R^{EMU}	unif	0.2	0.12	0.085	(0.068, 0.102, 0.134)	1.031
stdv. gov. spending shock	σ_G	unif	2	1.16	2.287	(2.025, 2.319, 2.659)	1.009
stdv. gov. spending shock	σ_G^*	unif	2	1.16	1.784	(1.522, 1.770, 2.059)	1.008
stdv. investment shock	σ_I	unif	10	5.9	6.474	(3.998, 7.440, 12.082)	1.067
stdv. investment shock	σ_I^*	unif	10	5.9	10.572	(8.563, 12.562, 20.012)	1.041
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.170	(0.145, 0.178, 0.210)	1.004
stdv. Phillips Curve shock	σ_{mc}^*	unif	0.25	0.15	0.235	(0.205, 0.241, 0.273)	1.011
stdv. CPI shock	σ_{CPI}	unif	0.25	0.15	0.311	(0.277, 0.318, 0.359)	1.021
stdv. CPI shock	σ_{CPI}^*	unif	0.25	0.15	0.336	(0.298, 0.345, 0.386)	1.024
stdv. UIP shock	σ_{UIP}	unif	0.25	0.15	0.271	(0.213, 0.306, 0.385)	1.051
stdv. com. comp. preference shock	σ_Y^{com}	unif	2	1.2	0.000	(0.000, 0.181, 0.368)	1.068
stdv. com. comp. tech. shock	σ_C^{com}	unif	2	1.2	0.000	(0.000, 0.652, 1.259)	1.076
stdv. com. comp. labor supply shock	σ_L^{com}	unif	10	5.9	2.880	(0.002, 3.609, 6.889)	1.009
stdv. com. comp. monetary shock	σ_R^{com}	unif	0.2	0.12	0.074	(0.044, 0.067, 0.101)	1.005
stdv. com. comp. gov. spending shock	σ_G^{com}	unif	2	1.2	0.000	(0.000, 0.266, 0.539)	1.113
stdv. com. comp. investment shock	σ_I^{com}	unif	10	5.9	0.000	(0.001, 1.956, 3.707)	1.250

Table 4.5: Estimation based on detrended data, prior and posterior distributions (cont.)

4.7.3 Modification II: Excluding the nominal exchange rate from the set of observables

Exchange rate movements have proved so problematic that some recent research has recommended abandoning the attempt to explain them in terms of macroeconomic models.[62] In order to assess the potential misspecification of the Euro area model, we present the estimation results based on the data set excluding the nominal exchange rate series prior to EMU.

As parameters pertaining to the incomplete asset market structure would then be, due to omitted nominal exchange rate series, only weekly identifiable, we slightly modify this dimension of the model. Here, the asset structure is represented by state contingent one-period nominal bonds denominated in the domestic currency and traded both domestically and internationally. Hence, the budget constraints of the consumer in the Home economy reduces to

$$0 = \begin{bmatrix} P_t^C C_t(j) + P_t^I I_t(j) + B_{t+1}(j) + S_t B_{t+1}^*(j) \\ -R_{t-1} B_t(j) - W_t(j) l_t(j) - P_t r_t^k K_t(j) \\ -Div_t(j) - TAX_t(j) + TR_t(j) - R_{t-1}^* \Phi\left(\tilde{B}_{t-1}^*\right) S_t B_t^*(j) \end{bmatrix}. \tag{4.120}$$

Complete international asset markets imply perfect risk-sharing between households in the two countries. Hence, stochastic discount factors in equilibrium have to be equalized across the regions,

[62]See Bergin (2004) and references given therein.

which leads to the following condition:

$$\frac{S_t P_t^{C*}}{P_t^C} = \kappa \frac{\varepsilon_t^{C*} U'(C_t^*)}{\varepsilon_t^C U'(C_t)} = \kappa \frac{\varepsilon_t^{C*} (C_t^* - h^* C_{t-1}^*)^{-1}}{\varepsilon_t^C (C_t - hC_{t-1})^{-1}}, \tag{4.121}$$

where $\kappa = P_0^* U_C(C_0) / (P_0 U_{C^*}(C_0^*))$ is a constant that depicts the initial condition. Equation (4.121) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among state contingent securities (the full derivation can be found in Chari, Kehoe, and McGrattan (2002)). Combining (4.121) with the Euler equations yields the uncovered interest rate parity condition.

The estimation results in Table 4.6 on the next page and Table 4.7 on page 130 indicate that omitting the nominal exchange rate series has, in general, only minor effects on the parameter estimates. The more pronounced differences, compared to the baseline estimation, are noted in the domestic price indexation parameters and import price stickiness. For Germany ξ_p^{imp} rises to 0.57. The share of imported goods in the consumer basket are up to 0.28 and 0.18 and the openness ratios are now inferred at 0.26 and 0.14.

We do not notice, however, any regular changes in the persistence and volatility of the shocks, when the model tries to match the behavior of the synthetic nominal exchange rate.

One other aspect that is interesting to compare is the time series of the nominal exchange rate implied by the baseline model and the perfect risk sharing model. Note that for the latter model the nominal exchange rate $\hat{S}_t$ is completely unobserved and is calculated as a function of model variables. For the former model $\hat{S}_t$ matches exactly the cumulated log-differences of the observed nominal exchange rate. From Figure 4.1 it is evident that the qualitative features of both models are very similar. Our estimates of the nominal exchange rate in the perfect-risk sharing model, which satisfies the UIP condition, are consistent with the observed series and indicate a strong appreciation of the German currency against the rest of the Euro area. The above observation suggests that the development of the nominal exchange rate in Europe might reflect the fundamentals to high degree. Interestingly, models predict different levels (more precisely deviations from the steady state) at which the exchange rate has been fixed. This, however, might be attributed to slightly different implications of both models for the steady state.

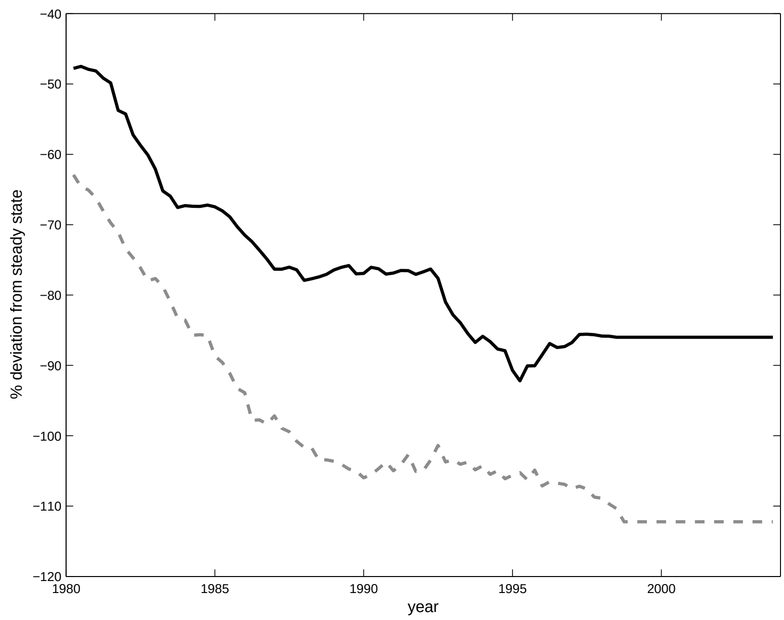

Figure 4.1: Estimates of the nominal exchange rate, baseline model (solid) vs. perfect risk sharing model (dashed)

parameter		prior			perfect risk sharing model		
		type	mean	std/df	mode	90% posterior interval	$\hat{R}$
growth rate of technology	$\bar{\varepsilon}^A$	norm	1.004	0.001	1.0057	(1.0048, 1.0059, 1.0069)	1.003
steady state inflation	$\bar{\pi}$	norm	1.006	0.001	1.0054	(1.0039, 1.0054, 1.0072)	1.001
int. rate smoothing	ρ_R^{EMU}	beta	0.8	0.2	0.919	(0.858, 0.904, 0.958)	1.005
int. rate smoothing	ρ_R	beta	0.8	0.1	0.854	(0.811, 0.851, 0.888)	1.005
int. rate smoothing	ρ_R^*	beta	0.8	0.1	0.875	(0.809, 0.852, 0.892)	1.003
inflation response	r_π^{EMU}	norm	1.5	0.2	1.494	(1.278, 1.600, 1.890)	1.002
inflation response	r_π	norm	1.5	0.1	1.342	(1.209, 1.387, 1.574)	1.001
inflation response	r_π^*	norm	1.5	0.1	1.463	(1.191, 1.369, 1.541)	1.008
output response	r_y^{EMU}	beta	0.2	0.15	0.086	(0.024, 0.107, 0.197)	1.007
output response	r_y	beta	0.2	0.15	0.214	(0.026, 0.070, 0.113)	1.001
output response	r_y^*	beta	0.2	0.15	0.053	(0.022, 0.060, 0.098)	1.005
habit formation	h	beta	0.7	0.1	0.768	(0.682, 0.748, 0.811)	1.009
habit formation	h^*	beta	0.7	0.1	0.679	(0.607, 0.670, 0.728)	1.002
capital adj. cost	S''	norm	4	2	5.151	(3.047, 4.527, 6.182)	1.023
capital adj. cost	$S^{*\prime\prime}$	norm	4	2	4.023	(2.494, 3.921, 5.432)	1.002
Calvo employment	ξ_L	beta	0.7	0.15	0.788	(0.806, 0.833, 0.860)	1.031
Calvo employment	ξ_L^*	beta	0.7	0.15	0.903	(0.876, 0.892, 0.908)	1.023
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.923	(0.927, 0.938, 0.950)	1.002
Calvo domestic prices	ξ_p^*	beta	0.7	0.05	0.926	(0.910, 0.924, 0.940)	1.000
Calvo import	ξ_p^{imp}	beta	0.5	0.05	0.570	(0.489, 0.568, 0.647)	1.005
Calvo import	$\xi_p^{imp^*}$	beta	0.5	0.05	0.531	(0.442, 0.530, 0.618)	1.010
Calvo wages	ξ_w	beta	0.7	0.05	0.714	(0.665, 0.705, 0.743)	1.042
Calvo wages	ξ_w^*	beta	0.7	0.05	0.847	(0.786, 0.834, 0.880)	1.012
indexation domestic prices	γ_p	beta	0.5	0.15	0.427	(0.750, 0.841, 0.937)	1.006
indexation domestic prices	γ_p^*	beta	0.5	0.15	0.198	(0.109, 0.232, 0.355)	1.008
indexation import	γ_p^{imp}	beta	0.5	0.15	0.590	(0.361, 0.582, 0.795)	1.003
indexation import	$\gamma_p^{imp^*}$	beta	0.5	0.15	0.464	(0.214, 0.476, 0.724)	1.003
indexation wages	γ_w	beta	0.5	0.15	0.496	(0.295, 0.523, 0.757)	1.021
indexation wages	γ_w^*	beta	0.5	0.15	0.224	(0.082, 0.207, 0.328)	1.013
share of dom. consum.	ω_C	beta	0.65	0.05	0.723	(0.628, 0.703, 0.774)	1.007
share of dom. consum.	ω_C^*	beta	0.8	0.05	0.896	(0.782, 0.850, 0.911)	1.003
share of dom. invest.	ω_I	beta	0.55	0.05	0.518	(0.459, 0.532, 0.606)	1.002
share of dom. invest.	ω_I^*	beta	0.7	0.05	0.675	(0.559, 0.630, 0.691)	1.003

Table 4.6: Prior and posterior distributions, perfect risk sharing model

parameter		prior type	prior mean	prior std/df	perfect risk sharing model mode	90% posterior interval	$\hat{R}$
unit root tech. shock	ρ_A	beta	0.85	0.1	0.930	(0.879, 0.915, 0.956)	1.003
asymmetric technology shock	ρ_Z	beta	0.85	0.1	0.992	(0.485, 0.731, 0.859)	1.014
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.832	(0.921, 0.943, 0.969)	1.021
stationary tech. shock	ρ_Y^*	beta	0.85	0.1	0.740	(0.601, 0.710, 0.816)	1.002
preference shock	ρ_C	beta	0.85	0.1	0.379	(0.217, 0.425, 0.627)	1.003
preference shock	ρ_C^*	beta	0.85	0.1	0.989	(0.955, 0.974, 0.993)	1.003
labor supply shock	ρ_L	beta	0.95	0.05	0.886	(0.883, 0.927, 0.970)	1.031
labor supply shock	ρ_L^*	beta	0.95	0.05	0.974	(0.890, 0.951, 0.993)	1.005
investment shock	ρ_I	beta	0.85	0.1	0.272	(0.204, 0.339, 0.475)	1.009
investment shock	ρ_I^*	beta	0.85	0.1	0.329	(0.190, 0.350, 0.525)	1.000
government spending shock	ρ_G	beta	0.85	0.1	0.958	(0.915, 0.953, 0.996)	1.001
government spending shock	ρ_G^*	beta	0.85	0.1	0.998	(0.998, 0.998, 0.998)	1.000
stdv. stationary technology shock	σ_Y	unif	2	1.2	1.575	(1.455, 1.839, 2.207)	1.019
stdv. stationary technology shock	σ_Y^*	unif	2	1.2	3.560	(2.409, 3.152, 4.038)	1.281
stdv. preference shock	σ_C	unif	2	1.2	2.827	(1.891, 2.565, 3.245)	1.021
stdv. preference shock	σ_C^*	unif	2	1.2	0.624	(0.059, 0.836, 1.449)	1.056
stdv. labor supply shock	σ_L	unif	10	5.9	11.426	(7.549, 9.919, 11.891)	1.025
stdv. labor supply shock	σ_L^*	unif	10	5.9	3.779	(2.360, 4.153, 6.193)	1.156
stdv. monetary shock	σ_R	unif	0.2	0.12	0.081	(0.083, 0.108, 0.133)	1.003
stdv. monetary shock	σ_R^*	unif	0.2	0.12	0.144	(0.112, 0.134, 0.153)	1.006
stdv. monetary shock EMU	σ_R^{EMU}	unif	0.2	0.12	0.145	(0.110, 0.169, 0.225)	1.004
stdv. gov. spending shock	σ_G	unif	2	1.16	2.112	(1.656, 2.193, 2.693)	1.041
stdv. gov. spending shock	σ_G^*	unif	2	1.16	1.476	(1.097, 1.551, 1.861)	1.024
stdv. investment shock	σ_I	unif	10	5.9	12.239	(7.160, 9.802, 13.329)	1.007
stdv. investment shock	σ_I^*	unif	10	5.9	5.767	(2.963, 5.155, 7.702)	1.004
stdv. unit root technology shock	σ_A	unif	0.25	0.15	0.166	(0.113, 0.170, 0.231)	1.003
stdv. asymmetric technology shock	σ_Z	unif	0.5	0.29	0.506	(0.273, 0.369, 0.473)	1.005
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.164	(0.136, 0.160, 0.178)	1.003
stdv. Phillips Curve shock	σ_{mc}^*	unif	0.25	0.15	0.217	(0.203, 0.234, 0.268)	1.008
stdv. infl. target shock	σ_π^{EMU}	unif	0.125	0.1	0.052	(0.289, 0.294, 0.298)	1.121
stdv. infl. target shock	σ_π	unif	0.125	0.1	0.216	(0.109, 0.190, 0.283)	1.034
stdv. infl. target shock	σ_π^*	unif	0.125	0.1	0.000	(0.000, 0.046, 0.089)	1.005
stdv. CPI shock	σ_{CPI}	unif	0.25	0.15	0.317	(0.260, 0.324, 0.378)	1.007
stdv. CPI shock	σ_{CPI}^*	unif	0.25	0.15	0.356	(0.305, 0.360, 0.413)	1.009
stdv. com. comp. tech. shock	σ_Y^{com}	unif	2	1.2	0.000	(0.001, 0.351, 0.656)	1.021
stdv. com. comp. preference shock	σ_C^{com}	unif	2	1.2	1.703	(1.016, 1.634, 2.299)	1.022
stdv. com. comp. monetary shock	σ_R^{com}	unif	0.2	0.12	0.084	(0.077, 0.096, 0.111)	1.007
stdv. com. comp. gov. spending shock	σ_G^{com}	unif	2	1.2	0.897	(0.140, 0.654, 1.187)	1.017
stdv. com. comp. investment shock	σ_I^{com}	unif	10	5.9	0.000	(0.000, 0.785, 1.666)	1.248
stdv. com. comp. infl. target shock	σ_π^{com}	unif	0.125	0.1	0.001	(0.000, 0.086, 0.163)	1.008

Table 4.7: Prior and posterior distributions, perfect risk sharing model (cont.)

4.7.4 Empirical performance of the model

In this section we briefly examine the empirical performance of the baseline model. First, we visually analyze the in-sample fit of the model. Subsequently, we conduct a posterior predictive analysis by comparing the statistics calculated for the actual data to those calculated for the artificial time series generated from the DSGE model. The fullblown analysis of the model's out-of-sample accuracy is postponed until Section 4.9.

In Figure 4.4 on page 178 and Figure 4.5 on page 178 we report the Kalman filtered one-side estimates of the observed variables, computed using the posterior mode of θ. These one-step in-sample forecasts are presented along with the actual variables. As is evident the in-sample fit is satisfactory for both the model estimated on log-differences (see Figure 4.4) and the one fited to the detrended data (see Figure 4.5).

Tables 4.25 on page 157 through 4.27 on page 159 report the comparison of the unconditional second moments replicated by the baseline DSGE model to those calculated from the data. For this purpose we simulate the baseline model (100 runs for a selection of 100 parameter draws from the posterior distribution), accounting for the monetary regime change.[63]

[63] The procedure is similar to that applied in Chapter 3. After discarding some initial simulated data points we utilize 95 observations for inference.

Although the model slightly overpredicts the volatility and autocorrelation of real variables, which is common for estimated balanced growth models in general (see, e.g., Adolfson, Laseen, Linde, and Villani (2005a)), it replicates very precisely the statistics for the nominal variables. Furthermore, the model does particulary well in replicating the co-movements of variables. Indeed, the medians of the model-implied statistics almost coincide with their empirical analogs. The model captures high correlations between consumption and GDP growth rates and low correlations between real variables and nominal interest rates, so the latter problematic disconnect is not a puzzle according to our model. Finally, as a result of the incorporation of the cointegrated permanent technology shocks, the model almost perfectly matches correlations between area-wide aggregates and the German variables.[64] This property might be important while applying the two region model to analyze interactions within the Euro-area.

In order to assess the effects the adoption of a common currency had on the synchronization of business cycles in the area, we simulate the free float and monetary union models separately and calculate the correlations between the cyclical components of output in Germany and the rest of the area. The 90% posterior interval is inferred at (-0.27, 0.83) under free float. In the monetary union correlation coefficient lies in the interval (-0.07, 0.94). Hence, our estimated model replicates the result of Artis and Zhang (1999), which suggests that exchange rate stability might induce more synchronized business cycles in Europe. Note that our result is not determined by inherent properties of monetary union and free float DSGE models. Indeed, in the presence of idiosyncratic shocks, the monetary union model rather tends to amplify the asymmetric effects of such shocks (see Section 4.8.3). The result above hinges on common components on structural shocks, which are freely estimated here. Moreover they may be related to the absence of irrational nominal exchange rate forecasts in the currency area.

Before closing this section, we report the comparison of empirical- and model-implied second moments for the baseline model estimated with detrended data in Tables 4.28 on page 160 to 4.30 on page 161. In short, the model seems on average to match all the second moments satisfactorily. However, the model's success in replicating the co-movements of real variables is not as spectacular as in the version with the balanced growth, though we allowed for common components on structural shocks.

4.7.5 Role of frictions and open economy mechanisms

After having validated a good fit of the baseline DSGE model, we can proceed with establishing the role of the various frictions and open economy mechanisms that are included in the model. This relative model comparison is based on the pseudo out-of-sample accuracy as measured by the marginal posterior density:[65]

$$p(Y_T|M_i) = \int p(Y_T|\theta, M_i)p(\theta|M_i)d\theta, \tag{4.122}$$

where $p(\theta|M_i)$ denotes the prior density for the model M_i, $p(Y|\theta, M_i)$ stands for data density of the model given the parameters θ. Bayesian model selection is done pairwise comparing the models through posterior odds ratio:

$$PO_{i,j} = \frac{p(Y_T|M_i)p(M_i)}{p(Y_T|M_j)p(M_j)}$$

[64] This is consistent with the results in Dellas (1986) and Canova and Marrinan (1998), who also found that to simulate realistic output fluctuations in an international business cycle model, transmission mechanism alone might be not sufficient and the presence of common technology shocks is needed.

[65] In general, the formal comparison can encompass other available linear models including alternative DSGE models or perhaps an even larger class of non-structural linear reduced-form models. Note, however, that the number of observed variables used to estimate our baseline model is threshold-dependent. Thus, it is problematic to compare its predictive density with standard BVAR models.

where the prior odds $\frac{p(M_i)}{p(M_j)}$are updated by the Bayes factor: $\frac{p(Y_T|M_i)}{p(Y_T|M_j)}$. The approach has been discussed in detail in Chapter 2 of this thesis.

Aside from the baseline specification, Table 4.8 and Table 4.9 collect the log marginal densities when i) the regions are homogenous, ii) the area is modeled as a pooling of closed economies, iii) the LOOP holds, iv) the model is estimated absent common components, and v) the error covariance matrix is non-diagonal.

It is important to note that the marginal data density penalizes the likelihood fit by a measure of model complexity. Allowing for heterogenous regions (the baseline specification) is equivalent to the removal of restrictions on parameters and increases the model complexity (approximately measured by the log-determinant of the posterior covariance matrix of the parameters). We conclude, however, that for the baseline model, the improvement in model fit dominates the penalty for increased model complexity and the marginal data density improves by 134.1 points on a log scale compared to the more parsimonious specification with the symmetric regions.[66] This means that we would need a prior that favors the second model over the first by a factor of 1.73×10^{58} in order to accept it after observing the data.

Furthermore, the two-country model with international trade and integrated financial markets is compared with the model estimated on the same data set but with countries closed to international trade and without integrated financial markets. Thus, the latter collapses to a pooling of closed economies. In this exercise, both models are estimated on the sample 1980:Q1-1998:Q4. Although the open-economy mechanisms in our models are very stylized, the model with open-economy features beats the closed economy models, in terms of marginal density (see Table 4.9).

Our baseline specification includes common components on structural shocks. This model, however, does not improve significantly on the version with orthogonal shocks and that with less restrictive error covariance matrix as far as marginal density is concerned (see Table 4.8).

Table 4.8 also indicates that the LOOP restriction is associated with a limited drop in log marginal likelihood (the Bayes factor is 4.2 in favor of pricing to market specification). Thus, the international price discrimination does not give a particulary better explanation for price setting within the Euro area.

Model	Marginal likelihood (Estimation sample 1980:Q1-2003:Q4)
baseline (with common components)	-1814.9
Model without common components	-1814.8
Model with non-diagonal error covariance matrix	-1815.4
LOOP model	-1819.1
Homogeneous regions	-1949.1

Table 4.8: Model comparisons

Model	Marginal likelihood (Estimation sample 1980:Q1-1998:Q4)
Two-country model with international trade	-1330.0
Pooling of closed economies	-1365.1

Table 4.9: Model comparisons, closed- vs. open economy frameworks

4.8 International spillover effects within the Euro area

In the previous sections we have documented the first important result of this chapter. Namely, it is possible to construct and estimate the DSGE model that can account for heterogeneity within

[66] In the model with symmetric regions we allow for stochastic heterogeneity.

the Euro area, as well as for the historical monetary regime change, and still satisfactorily match features of the actual data. The second challenge with which we confront the two-region model is its ability to induce international spillovers. To our knowledge, these effects have neither been previously documented or analyzed with an estimated microfounded model in the context of the Euro area. Methodologically, most closely related to our approach is the recent paper by Justiniano and Preston (2006), who apply an estimated small open economy DSGE model to analyze the spillovers from the US to the Canadian economy. So far, the only attempts to analyze these effects within the Euro area have been based on calibrated DSGE models (see, e.g., Fichtner (2003)) or various reduced-form models (see, e.g., Monteforte and Siviero (2003) and Monteforte (2004)). Most of the studies suggest limited convergence of the European business cycles and signal that structural and stochastic heterogeneity among Euro area economies is significant. The effects attributed to common shocks are meaningful, but idiosyncratic shocks, especially in the German economy, are still by far the most important determinants of economic fluctuations (see Monteforte (2004)).

In what follows we show that the idiosyncratic shocks are predominant in explaining the variance of macro aggregates also in an estimated New Open Economy Macroeconomics DSGE model. The trade related spillovers generated by this model are, however, moderate at most, and the significant portion of the spillover originates from the common components on structural shocks. Furthermore, we provide an extensive robustness check of our results and compare the effects of foreign shocks in a DSGE model to those arising from reduced form models.

4.8.1 Variance decomposition

To infer the role of the system's shocks in driving the fluctuations in the endogenous variables, we use variance decompositions.

Contribution of all structural shocks to the variability in real GDP, consumption, investment, real wage, employment, CPI inflation, and nominal interest rate in Germany and the rest of the Euro area, as well as variance shares of real and nominal exchange rates, are reported in Tables 4.31 on page 162 to 4.45 on page 176. The statistics are calculated for the baseline model (with common components), both under free float and monetary union regimes. Note that for the real variables effects of the shocks refer to the levels, defined as $\hat{y}_t + \ln(A_t)$, where $\hat{y}_t$ is a detrended real variable and A_t is a level of technology evolving according to the following unit root process: $\ln(A_t) = \ln(A_{t-1}) + \ln\left(\bar{\varepsilon}^A\right) + \hat{\varepsilon}_t^A$.

A comparison across tables reveals that under both free float and monetary union regimes the volatility in the levels of real variables is explained by demand shocks in the short run and supply shocks in the long run. The most important sources of GDP variability are productivity, preference, labor supply and investment shocks, followed by government spending shocks.

Variability of the nominal interest rate is mostly caused by the demand shocks and innovations to the inflation target. We also find a significant contribution of the unit root technology shock and the labor supply shock. In addition, under a free float regime the risk premium shock has been estimated to explain about 5.1% of the nominal interest rate in the German economy in the long run. The variance of inflation is mainly explained by the inefficient shocks to the Phillips Curve and CPI equations (in the short run) and investment shocks, as well as innovations to the inflation target, in the long run. Yet, in the monetary union the role of the inflation target is negligible. Under both regimes, the role of the interest rate shock is insignificant in the long run. These findings are similar to those in the literature (see, e.g., Altig, Christiano, Eichenbaum, and Linde (2003)) and are complementary to the results presented in Chapter 3.

Tables 4.31 on page 162 to 4.45 on page 176 also report the percentage of variability attributed to the combined common shocks and all domestic and foreign shocks at several forecast horizons. The first group selects, besides the common factors, a unit root technology shock, asymmetric technology shock and the uncovered interest rate parity shock (prior to EMU). In the model for a monetary union inflation target and the common interest rate shock are also included.[67]

Our main finding is that the estimated model generates *meaningful* international spillover effects within the area both under a free float regime and the monetary union. The spillovers are not only attributable to the common components on structural shocks. Under free float, the direct spillovers resulting from trade-related transmission account for about 6.6% of the volatility in the German output level (in the short run). While under a free float regime, spillover operates mainly through trade linkages, after introducing a common currency an additional spillover is generated working through the common interest rate. This indirect spillover multiplies the total effect of foreign shocks on consumption and investment.

In the monetary union spillovers from the shocks originating in the rest of the Euro area range from about 1% for real wages to 7.3% for investment in the medium run. It is worth emphasizing that the effects of foreign sourced shocks on output are larger than on consumption. This is an immediate consequence of the substitution effect and the fact that exports strongly react to foreign shocks. It seems that foreign shocks affect domestic consumption mainly through the income, and not relative price channel. As the unit root technology shock, by construction, determines the economic growth in the long run, contribution of stationary foreign shocks to the variability in the levels of real variables is the highest over the short to medium term. Despite that, after 20 quarters, foreign shocks still account for over 1% of employment fluctuations and from 1 to 10% of the variability in the levels of investment, consumption and output.

In Table 4.10, we report the forecast errors decomposition for the detrended output $\hat{y}_t$. This table also presents the results obtained under several alternative specifications. It is immediate that the role of the common technology in explaining business cycle frequency movements of real variables is only moderate (the unit root technology shock accounts then only for 1-4% of fluctuations). The long-run spillovers from the rest of the Euro area to Germany are now estimated at about 6.1% under a free float regime and at above 24% in monetary union. The model estimated with the ML approach implies even larger spillovers (9.1% and 24.6% respectively). To check robustness of the results we also report variance decompositions for a model estimated with HP-filtered data (in the absence of both unit root technology shock and the time varying inflation targets) and the perfect risk sharing model. Foreign shocks account then for about 4.5% and 8.5% of the business cycle frequency fluctuations in the German output respectively. The contribution of the German shocks to the output fluctuations in the rest of the Euro area is respectively lower.

Forecast errors at 20 quarter horizon			
Model/Shocks	common	domestic	spillovers
baseline (free float)	23.2	70.7	6.1
baseline (monetary union)	27.8	47.9	24.2
baseline ML estimation (free float)	37.9	53.1	9.1
baseline ML estimation (monetary union)	37.8	37.7	24.5
baseline estimated with HP-filtered series (free float)	9.8	85.7	4.5
baseline estimated with HP-filtered series (monetary union)	7.0	87.6	5.5
model with perfect risk sharing (free float)	12.3	79.3	8.5
model with perfect risk sharing (monetary union)	28.4	66.9	4.9

Table 4.10: Variance shares of German output attributed to domestic, foreign and common shocks. Variance shares are calculated at the posterior mode.

Turning to the nominal variables under a common currency regime, closed-economy shocks originating in the German economy account for about 2% of the variability in nominal interest

[67] The shocks to the CPI equations in both Germany and the rest of the Euro area are classified as domestic because they are unlikely to proxy for the transmission of foreign shocks. They may, however, be driven by some fluctuations in commodity markets which may well be exogenous to each economy (and the whole area) considered in this study.

rate compared to about 17% caused by the Foreign economy shocks. The rest is due to common shocks.

Table 4.11 reveals that, both under free float and monetary union, the CPI inflation is significantly affected by foreign shocks. This finding is robust to data transformations, as well as to restrictions imposed on the theoretical model. We find, however, that the result is sensitive to the choice of the prior.

Forecast errors at 20 quarter horizon			
Model/Shocks	common	domestic	spillovers
baseline (free float)	35.0	58.2	6.8
baseline (monetary union)	17.2	75.6	7.3
baseline ML estimation (free float)	60.1	33.5	6.4
baseline ML estimation (monetary union)	12.5	60.9	26.6
baseline estimated with HP-filtered series (free float)	9.8	72.6	6.3
baseline estimated with HP-filtered series (monetary union)	3.3	81.9	13.8
model with perfect risk sharing (free float)	18.6	74.5	6.9
model with perfect risk sharing (monetary union)	29.2	50.3	20.4

Table 4.11: Variance shares of German CPI inflation attributed to domestic, foreign and common shocks. Variance shares are calculated at the posterior mode.

Overall, these results indicate that an estimated New Open Economy Macroeconomics DSGE model for the Euro area can generate, say, moderate trade-related spillovers. The spillovers induced by common components on structural shocks are dominant both a under free float and in a monetary union. The majority of variance is, however, explained by domestic shocks. At this point, we should emphasize that the presence (or absence) of contribution from foreign shocks to the volatility of the German series is not an inherent feature of the structural model and has not been imposed by our choice of the priors. Indeed, we have estimated the parameters that determine spillovers, i.e., household's preferences and common components on structural shocks. Moreover, we limit the prior information by assuming the uniform distribution for the volatility of both idiosyncratic and common shocks. Strikingly, with uniformly distributed priors for all the parameters, estimates of 'economy openness' are even higher than those under the baseline prior specification (see also Section 4.7.1) which indicates that spillovers are rather a property of the data. As a preview of the results in Section 4.9, by accounting for the linkages within the area we do not adversely affect model's accuracy.

Our results are not immediately comparable to other contributions in the NOEM literature, mainly because there are no similar studies for the Euro area. Methodologically comparable results might be found in the parallel work of Justiniano and Preston (2006). While we are able to identify moderate spillovers within the Euro area, either in the form of a trade related spillover or an indirect spillover, Justiniano and Preston (2006) indicate that it is not possible to identify realistic spillovers from the US to Canadian economy employing macroeconomic series.[68] They further argue that the model's inability to account for the linkages between the two economies might be related to the real exchange rate disconnect.[69] As we have introduced staggered import price setting and inefficient shocks to the CPI equation as well as the autocorrelated UIP shock prior to EMU, the disconnect is also present in our baseline model. However, here it does not completely switch off

[68] Justiniano and Preston (2006) estimate the spillover effects in the DSGE model at below 1%. In turn, using reduced form models they have estimated the variance shares of Canadian output attributed to the U.S. sourced shocks at about 70%. The estimates of foreign spillovers to the Canadian economy are even higher in Stock and Watson (2005).

[69] The results in Dam and Linaa (2004), Adolfson, Laseen, Linde, and Villani (2005a) and Justiniano and Preston (2006) highlight that a persistent cost-push shock in the retail sector and a risk premium disturbance are essential in order to match the volatility of the exchange rate series. This is because without these disturbances the cross-equation restrictions embodied in the interest parity condition prove too stringent when confronted with the data.

the fundamentals in determining the nominal and the real exchange rates. Table 4.12 indicates that in the baseline model above 69% and in the model estimated under assumption of complete financial markets above 58% of real exchange rate fluctuations in the monetary union are due to technology, investment, preference and labor supply shocks. Under a free float regime above 25% of real exchange variance is explained by monetary shocks and the share explained by real shocks is above 28%. For comparison, portion of the real exchange variance explained by combined real and monetary shocks is estimated at below 20% in Justiniano and Preston (2006).

Forecast errors at 20 quarter horizon				
	free float		monetary union	
Shocks/Variable	nominal exch. rate	real exch. rate	real exch. rate	real exch. rate (perfect risk sharing)
real	34.3	28.0	69.4	58.0
monetary	56.0	25.9	1.1	0.5
cost-push	1.2	27.1	29.5	41.6
UIP	8.5	19.0	-	-

Table 4.12: Variance shares of nominal and real exchange rate attributed to real, monetary and a-theoretical shocks. Variance shares are calculated at the posterior mode.

Our results are particulary interesting in the context of estimated multi-country DSGE models constructed to explain the co-movements of the US and aggregate Euro area series. The nominal exchange rate fluctuations in the Euro area prior to EMU are estimated in our model to be up to 90% driven by real and monetary shocks. These results contrast with the estimates of Lubik and Schorfheide (2005) or Adjemian, Paries, and Smets (2004), who find no more than 10-15% of the US-Euro nominal exchange rate fluctuation being explained by these shocks. This suggests that the UIP shock is not being used in our model to drive the exchange rate and artificially boost the model fit. Results in Adjemian, Paries, and Smets (2004) and De Walque and Wouters (2004) also indicate that international effects of the US originated shocks on the Euro area series are, at the business-cycle frequency, insignificant.[70] This suggests that the Euro area as a whole can be modeled in the closed economy framework. In light of our results, one should rather account for the linkages within the area.

4.8.2 Evidence from reduced-form models

In order to assess the international linkages and to validate assumptions regarding the way we model the Euro area, this subsection compares the results from the DSGE models with those arising from reduced form models.

We refer here to the three reduced form models constructed to explain the business cycle frequency fluctuations in the macro aggregates. The first model is the Disaggregate Euro Area Multi-Country Model (DEAM) by Monteforte and Siviero (2002) and includes core Euro area countries (Germany, France and Italy). The second one is the FSVAR model of Stock and Watson (2005), which is estimated for G7 countries. Finally, we examine the Bayesian VAR with priors coming from a two-region DSGE model.

Disaggregate Euro Area Multi-Country Model We begin with the disaggregate Multi-Country Model by Monteforte (2004) that, similarly to our two-region DSGE model, neglects the linkages between the Euro area and the rest of the world. The DEAM model has been estimated on the data for the output gap (calculated as deviations from the trend), inflation and nominal

[70] Adjemian, Paries, and Smets (2004) estimate the trade related spillovers from the US economy to the Euro area at below 0.5% for output and below 1% for consumption.

interest rates. The model specifies the aggregate supply and the aggregate demand equations and allows for cross-country linkages. Specifically, inflation in any given country depends not only on its own lagged values and on the output gap, but also on inflation 'imported' from the other two countries. In turn, the output gap in any of the three countries depends on its own lagged values, the corresponding real interest rate and it may react to the output gap in the other two countries.

The analysis of spillovers is performed here with the Factor-Analysis-based approach, developed by Monteforte and Siviero (2002), which rests on identifying the idiosyncratic components of the single country equations. The results in Table 13 on page 36 in Monteforte (2004) can be interpreted that the DEAM model identifies 57.3% of German output gap fluctuations as being explained by idiosyncratic components (the rest are due to common components and spillovers), which is slightly less than our results obtained from the baseline free float model. Monteforte (2004) further concludes that, for the output gap, the share of the variance explained by the idiosyncratic component is considerably higher in the German economy than for the other two economies considered in the study. Some similar indications can also be found in Tables 4.31 on page 162 to 4.45 on page 176 in Appendix. The common components (but not the trade related spillovers) are more relevant in explaining the variance of the rest of the Euro area's output fluctuation.

Note that as the estimation sample in Monteforte and Siviero (2003) runs from 1978:Q1 to 1998:Q4, the model does not account for the institutional change in the area. Therefore, we see the results of Monteforte and Siviero (2003) as a valuable reference for the results we obtained under the free float regime but not for the monetary union model.

FSVAR We next turn to the FSVAR model of Stock and Watson (2005). This model, specified in terms of the growth rates of quarterly GDP in the G7 countries, identifies common international shocks, the domestic effects of spillovers from foreign idiosyncratic shocks, and the effects of domestic idiosyncratic shocks. As it has originally been constructed to analyze synchronization of business cycles in the G7 countries, the authors report only the sum of all spillovers, which are referred to as international spillovers in their work. Here, we re-estimate the model and recalculate the variance decompositions to provide more refined information regarding the spillovers from Italy and France to Germany. This extension of the scope of analysis is possible because the individual shocks are identified as contemporaneously uncorrelated.[71]

Table 4.13 reports median shares corresponding to the common shocks, domestic shocks and shocks originating from the rest of the Euro area in the FSVAR model. The spillovers from the rest of the Euro area are calculated as the sum of spillovers from Italy and France. In addition, the sum of spillovers from the US, Japan, UK and Canada to the German economy is reported. As the German imports from Italy and France account only for about 30-35% of the total imports from the Euro area, the spillovers within the Euro area might be underestimated in the FSVAR.

Forecast errors at 20 quarter horizon	
Shocks	
international	25.3
domestic	65.5
spillovers from the rest of the Euro area	1.4
spillovers from the rest of the world	7.7

Table 4.13: FSVAR model, Variance shares of German output attributed to domestic, foreign and international shocks

It is evident that the contribution of domestic shocks (and consequently, the combined effect of common, international and foreign shocks) to the variability of German output is rather similar

[71] Thanks to Mark W. Watson for making his codes available to me.

across all models considered here. The idiosyncratic shocks are identified as the major source of economic fluctuations in Germany under a free float regime. However, the effects of common and international shocks are also meaningful both in a DSGE model and FSVAR. In turn, the fraction of output variance explained by disturbances originating in the rest of the Euro area is substantially smaller in the FSVAR model compared to the DSGE model. Strikingly, effects of foreign sourced shocks in the DSGE model are rather closer to the sum of all spillovers in the FSVAR model. Spillovers from the rest of the Euro area explain only 1.4% of output volatility in the FSVAR model and from 5.1% to 9.1% in the variants of the DSGE model. As above 7% of the variation in output is attributable to shocks originating in the US, Canada, UK and Japan, in the FSVAR model its implications also contrast those of NOEM models by Adjemian, Paries, and Smets (2004) and De Walque and Wouters (2004).

VAR-DSGE Finally, we report the forecast errors decomposition obtained from a VAR-DSGE model estimated with 17 detrended time series. In order to identify structural innovations we follow Del Negro and Schorfheide (2004) and utilize a theoretical DSGE model as a prior for the VAR impulse responses.[72] The procedure details are as follows.

Let y_t be the $n \times 1$ vector of observed variables. Then, the VAR(p) model is of the form:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \epsilon_t,\ \epsilon_t \sim N\left(0, \Sigma_\epsilon\right), \tag{4.123}$$

where ϵ_t is the vector of reduced-form disturbances. If we denote by Y the $T \times n$ matrix with rows y_t' and let $k = 1 + np$, X be the $T \times k$ matrix with rows $x_t' = \left[\begin{array}{cccc} 1 & y_{t-1}' & ... & y_{t-p}' \end{array}\right]$, $\mathcal{E}$ be the $T \times n$ matrix with rows ϵ_t', and $\Phi = \left[\begin{array}{cccc} \Phi_0 & \Phi_1 & ... & \Phi_p \end{array}\right]'$, the VAR(p) can be expressed as

$$Y = X\Phi + \mathcal{E} \tag{4.124}$$

The likelihood function conditional on the past observations can be then written as

$$p\left(Y|\Phi, \Sigma_\epsilon\right) \propto |\Sigma_\epsilon|^{-T/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma_\epsilon \left(Y - X\Phi\right)' \left(Y - X\Phi\right)\right]\right\} \tag{4.125}$$

Subsequently, we specify a hierarchical prior of the form

$$p\left(\Phi, \Sigma_\epsilon, \theta\right) = p\left(\Phi, \Sigma_\epsilon|\theta\right) p\left(\theta\right),$$

where θ is the vector of DSGE model parameters. Using this prior might be thought of as augmenting the actual data with $T^* = \lambda T$ artificial observations generated from a DSGE model (as originally proposed by Theil and Goldberger (1961)), where parameter λ can be seen as a prior weight assigned to a DSGE model. For computational convenience, rather than generating random observations from an equation from the DSGE model and augmenting the actual data, we pre-multiply the likelihood function (4.125) with

$$\tilde{p}\left(\Phi, \Sigma_\epsilon|\theta\right) \propto |\Sigma_\epsilon|^{-(\lambda T + n + 1)/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma_\epsilon^{-1} \left(Y^* - X^*\Phi\right)' \left(Y^* - X^*\Phi\right)\right]\right\} \tag{4.126}$$

and solve the DSGE model to replace the sample moments $Y^{*\prime}Y^*$, $Y^{*\prime}X^*$ and $X^{*\prime}X^*$ by theoretical analogs $\Gamma^*_{yy}(\theta)$, $\Gamma^*_{yx}(\theta)$, $\Gamma^*_{xx}(\theta)$.

[72] Identification of structural shocks in an open economy reduced-form model poses some additional problems. The general idea is to impose some restrictions on the variance-covariance matrix to obtain impulse responses that conform, in one or more dimensions, with a theoretical model. Blanchard and Quah (1989) have proposed an identification strategy based on long-run properties of the shocks. Canova and DeNicolo (2001) and Uhlig (2001) focus on sign restrictions. In the open economy context a Cholesky decomposition is usually used to orthogonalize the shocks. The ordering follows the assumption that a larger economy tends to influence the block of smaller countries and not vice versa. The strategy proposed by Del Negro and Schorfheide (2004) avoids arbitrariness and is applicable both to closed and open economy VARs.

Using the scaled theoretical moments equation (4.126) can be transformed as follows:

$$\begin{aligned}\tilde{p}\left(\Phi,\Sigma_{\epsilon}|\theta\right) \quad &\propto \quad c^{-1}\left(\theta\right)\left|\Sigma_{\epsilon}\right|^{-(\lambda T+n+1)/2} \\ &\times \exp\left\{-\frac{1}{2}tr\left[\lambda T\Sigma_{\epsilon}\left(\Gamma^{*}_{yy}(\theta)-\Phi'\Gamma^{*}_{xy}(\theta)-\Gamma^{*}_{yx}(\theta)\Phi+\Phi'\Gamma^{*}_{xx}(\theta)\Phi\right)\right]\right\}\end{aligned} \tag{4.127}$$

The prior is proper provided that $\lambda T > k + n$. $c^{-1}(\theta)$ is the proportionality factor (see technical Appendix to Del Negro and Schorfheide (2004)).[73]

As shown in Del Negro and Schorfheide (2004), conditional on θ the prior distribution of the VAR model parameters is of the Inverted-Wishart (IW) and Normal form[74]

$$\Sigma_{\epsilon}|\theta \sim IW\left(\lambda T\Sigma^{*}_{\epsilon}(\theta),\lambda T-k,n\right) \tag{4.128}$$

$$\Phi|\Sigma_{\epsilon},\theta \sim N(\Phi^{*}\left(\theta\right),\Sigma_{\epsilon}\otimes\left(\lambda T\Gamma^{*}_{xx}(\theta)\right)^{-1}), \tag{4.129}$$

where

$$\Sigma^{*}_{\epsilon}(\theta)=\Gamma^{*}_{yy}(\theta)-\Gamma^{*}_{yx}(\theta)\Gamma^{*-1}_{xx}(\theta)\Gamma^{*}_{xy}(\theta) \tag{4.130}$$

and

$$\Phi^{*}\left(\theta\right)=\Gamma^{*-1}_{xx}(\theta)\Gamma^{*}_{xy}(\theta) \tag{4.131}$$

are the functions that trace out a subspace of the VAR parameter space and can be interpreted as coefficient matrix and corresponding error covariance matrix that minimize the one-step-ahead forecast errors of the VAR model provided that data has been generated from a DSGE model. Since the likelihood and prior are conjugate, the posterior $p\left(\Phi,\Sigma_{\epsilon},\theta|Y\right)$ is also of the $IW-N$ form.[75]

The empirical performance of and inference based on a VAR with DSGE prior will crucially depend on the choice of λ. For $\lambda=0$ estimates of $\Phi\left(\theta\right)$ and $\Sigma_{\epsilon}\left(\theta\right)$ equal the OLS estimates. As $\lambda\rightarrow\infty$, $\Phi\left(\theta\right)$ and $\Sigma_{\epsilon}\left(\theta\right)$ approach the restriction functions $\Phi^{*}\left(\theta\right)$ and $\Sigma^{*}_{\epsilon}\left(\theta\right)$ derived from the DSGE model. The analysis of international spillovers that will follow is conditioned on the value of λ that yields the highest marginal density:[76]

$$p_{\lambda}\left(Y\right)=\int p\left(Y|\Phi,\Sigma_{\epsilon}\right)p_{\lambda}\left(\Phi,\Sigma_{\epsilon}|\theta\right)p(\theta)d\left(\Phi,\Sigma_{\epsilon},\theta\right). \tag{4.132}$$

In general, the mapping from the one-step-ahead forecasts errors ϵ_t into structural shocks u_t is based on the relation:

$$\epsilon_t=\Sigma^{C}_{\epsilon}\Omega u_t,\ u_t\sim N(0,I_n), \tag{4.133}$$

where Σ^{C}_{ϵ} is the Cholesky decomposition of Σ_{ϵ} and Ω is the rotation matrix (an orthonormal matrix with the property $\Sigma^{C}_{\epsilon}\Omega\Omega'\Sigma^{C\prime}_{\epsilon}=\Sigma^{C}_{\epsilon}\Sigma^{C\prime}_{\epsilon}$, so that the likelihood function is invariant to Ω) with some theoretical restrictions imposed. In the case of a DSGE model, the rotation matrix $\Omega^{*}\left(\theta\right)$ is obtained in a straightforward way; if $A_0\left(\theta\right)=B\left(\theta\right)R\left(\theta\right)$ (see equations (2.39) and (2.40)) is the matrix of DSGE impulse responses on impact obtained from the model solution, the QR decomposition of $A_0\left(\theta\right)$ yields a lower triangular matrix $\Sigma^{C*}_{DSGE}(\theta)$ and a unitary matrix $\Omega^{*}\left(\theta\right)$ such that $A_0\left(\theta\right)=\Sigma^{C*}_{DSGE}(\theta)\Omega^{*}\left(\theta\right)$.

[73] As the estimation sample runs here from 1980 to 1999 (75 observations) and there are 17 observed variables, for the VAR(1) model the prior is proper only if $\lambda > 0.467$.

[74] In this application we use the stationary version of the two-region DSGE model estimated on detrended data prior to EMU. As the number of shocks equals, in this model, the number of observed variables to which the model is fitted, matrix $\Gamma^{*}_{xx}(\theta)$ is invertible. This property is important (i) for construction of the prior and (ii) for mapping from the structural innovations of the DSGE model to the VAR model.

[75] For the proof see Del Negro and Schorfheide (2004) and references given therein.

[76] The 'optimal' value of λ has been obtained by grid-search.

Turning to the identified VAR, the impulse responses and variance decomposition can be computed knowing matrices Φ, Σ_ϵ and Ω. We further note that the identified VAR approximation of the DSGE model, given by $\Phi^*(\theta), \Sigma_\epsilon^*(\theta)$ and $\Omega^*(\theta)$ defined above, induces a prior distribution for the parameters of the identified VAR. The posterior distribution is obtained by updating the distribution of Φ, Σ_ϵ and θ (as described above) and mapping θ into $\Omega = \Omega^*(\theta)$. Since the likelihood is invariant with respect to Ω, so that it is conditional on θ a priori and a posteriori the same, we learn from the data indirectly which rotation matrix to choose via learning about the DSGE model parameters θ. Posterior VAR impulse response (and variance decomposition) will differ from the prior responses also because the distributions of Φ and Σ_ϵ are being updated.

The advantage of the approach is twofold. First, there is very little room for arbitrariness and the data along with the DSGE model prior distribution will determine in which dimensions the posterior VAR responses will conform with the theoretical model. Second, the tightness parameter λ that determines to which extent the VAR-DSGE model will look like a DSGE model is chosen endogenously based on the overall model fit.

From Table 4.14 we see that the marginal density is the highest for λ close to 0.75. This result corresponds to that of Del Negro and Schorfheide (2004). Interestingly, Del Negro and Schorfheide (2004) also find that the ex post performance improves for $\lambda > 1$, i.e., if the VAR-DSGE model becomes closer to a DSGE model. We obtain similar results in Section 4.9, where the out-of-sample comparison of VARs with DSGE models is documented.

Prior weight for DSGE model (λ)	Marginal density
0.5	-1622.6
0.6	-1608.6
0.75	-1604.8
1	-1615.0
1.5	-1631.6
2	-1647.9
3	-1670.9
5	-1699.4
10	-1714.5

Table 4.14: Pseudo out-of-sample performance of VAR-DSGE models

Forecast errors decomposition for the selected variables is reported in Table 4.15. The portion of consumption volatility explained in a VAR-DSGE model by foreign shocks is significantly lower than for the output (as it is the case in the DSGE model). We see, however, that the share of the German output and CPI inflation variance explained by foreign shocks is above 24% and 26%, respectively, which contrasts with the evidence from all the DSGE models considered in this chapter.[77] Part of this difference can be attributed to the assumptions we made in the estimation of the VAR-DSGE model: In order to assign structural meaning to the VAR innovations, we set the number of structural shocks in a VAR and a DSGE model (that has been subsequently used as a prior for a VAR) the same and eliminate the common components on the shocks. We have also imposed that shocks within the area are uncorrelated to unambiguously interpret the spillovers.

[77] In addition, we have estimated the DSGE model used as a prior for the VAR (separately, using the same data and sample as in the case of VAR-DSGE model). The spillover from the rest of the Euro area to Germany is in this model estimated at below 5.5%.

Forecast errors at 20 quarter horizon				
Shocks/Variable	Output (H)	Output (F)	CPI (H)	CPI (F)
common (UIP)	5.1	2.5	10.0	9.1
Home	70.6	19.5	63.6	17.0
Foreign	24.3	78.0	26.4	73.9
Forecast errors at 20 quarter horizon				
Shocks/Variable	Consumption (H)	Consumption (F)	Investment (H)	Investment (F)
common (UIP)	0.3	2.2	1.4	1.3
Home	84.7	10.9	79.0	11.5
Foreign	14.9	86.9	19.5	87.3

Table 4.15: Variance decomposition VAR-DSGE model (prior to EMU)

The conclusion we derive from the analysis above is that the trade-related spillover effects, though meaningful, are lower in the DSGE model relative to the spillovers obtained from an identified VAR model estimated using the same data and sample. As a preview of our results to be presented in the next section, the origin for only moderate direct spillovers appears to be, besides the exchange rate disconnect, the DSGE model's inability to generate symmetric open economy effects of the idiosyncratic innovations. These symmetric effects can be, however, partially attributed to, and generated with, the common components on structural shocks (see also the evidence from FSVAR and DEAM models). Therefore, an estimated open economy DSGE model with common components might not be far away from offering an explanation for co-movements detected in the data prior to EMU. Our finding of the increased spillovers after adoption of the common currency requires, however, some confirmation from the research which employs alternative methods.

4.8.3 Impulse responses

The estimated parameters can also be converted into impulse response functions. Figures 4.10 on page 181 to 4.38 on page 195 report the sequence of IRFs for each estimated shock in the baseline model both under a free float and a common currency regime. The graphs plot the mean response together with the 90% confidence interval.[78]

Findings in this section can be summarized under three items: First, our results are complementary to those of Welz (2004) or Smets and Wouters (2003) obtained with much more restrictive DSGE models.[79] All major features of those models are preserved in our model. Second, our contribution is to analyze the open economy aspects of domestic and foreign shocks both under flexible exchange rate regime and monetary union. The responses to foreign shocks are generally stronger (and more asymmetric) in the monetary union than under a free float, where the nominal exchange rate acts as an automatic stabilizer for the real exchange rate. As the role of idiosyncratic shocks is still dominant within the area, the question of potential welfare gains or losses related to the introduction of the common currency naturally arises. Finally, we compare (along the selected dimensions) the implications of the baseline model to the fragmentary evidence from the large scale structural models and SVAR models available in the literature. We emphasize that though some mechanisms of market imperfections have been implemented, the relative price channel still determines the open economy properties of the DSGE model. So, an idiosyncratic shock has, in general, opposite effects on Home and Foreign aggregates, which might contrast with the evidence from reduced-form models. This also appears to be the origin for the limited trade-related spillovers in estimated NOEM DSGE models (and significant spillovers induced by common components).

Below we describe the main characteristics of transmission mechanisms at work in the baseline model. One should note that such reasoning entails a considerable simplification because, given

[78]The bootstrapped confidence intervals for the baseline model are calculated for a selection of 1000 parameters from the posterior sample of 200,000.

[79]See also the IRFs reported in Chapter 3.

the general equilibrium nature of the model, described events happen simultaneously.

Stationary technology shock (Figures 4.10, 4.11 and 4.12) Responses to the stationary technology shock, the shock causing the changes in the level of technology, differ in our model slightly from those obtained from the standard RBC model. These differences may be particularly attributed to the high degree of price stickiness. Nominal rigidities cause the immediate supply effect to be very limited. Furthermore, a given level of productivity can now be reached using fewer resources due to a higher level of technology. It causes labor as well as capital demand to fall, supporting the results of Galí and Gertler (1999). Also investment and capital supply drop below the balanced growth path. In the short run, lower demand may outweigh the positive supply effect, additionally limiting the increase of aggregated output. However, after some time, this shock expands production and lowers marginal costs implying a fall in prices and real wages. Investment rises after a possible initial drop due to higher expected returns from capital. Due to the habit formation, consumption rises more slowly compared to the standard RBC models. The impact of the stationary technology shock on the real exchange rate is positive. The movement in relative prices favors the goods produced in the Home economy and causes that the effect of this idiosyncratic shock on the foreign output can be negative.

Although all we stated above mostly applies under both the free float and monetary union settings, differences, mainly in the behavior of prices and the real exchange rate, arise between the two regimes. Indeed, under the flexible exchange rate regime, the nominal exchange rate acts as an automatic stabilizer for the real exchange rate. Therefore, the response of the real exchange rate is magnified in the monetary union thus yielding, for instance, a stronger purchasing power effect and subsequently a competitiveness effect.

Unit root technology shock (Figure 4.13) The reaction of the economies to the shock to the growth rate of technology is likewise in line with the theoretical literature. Higher expected future growth stimulates the demand side of the economy resulting in higher prices and interest rate hikes. Our quite high estimate of the persistence of the permanent technology shock implies that it is more profitable for individuals to adjust their investment and work effort more gradually, because it takes some time before the labor and capital input become more productive. In the case of imposed no persistence, this shock causes an immediate rise in employment, because the production input is at its highest productivity immediately after the shock (see Linde (2004)). Note that employment in both economies rises after a positive shock to the growth rate of technology, supporting the results reported, e.g., in Altig, Christiano, Eichenbaum, and Linde (2003) for the US economy.

Asymmetric technology shock (Figure 4.14) The asymmetric technology shock measures the asymmetry in technological progress within the area. This shock is estimated to be very persistent. Hence, its effects for the Foreign economy are very similar to those associated with the negative unit root technology shock. The prospects of slower economic growth in the Foreign country adversely affect the current demand. Likewise, the lower demand from abroad causes some economic slowdown in the Home economy in the mid run. However, this effect is only transitory because the Home economy takes advantage of its superior access to the technology (and improved terms of trades). As the monetary policy, by construction, does not offset asymmetric developments within the area, the effects of this shock are, in the EMU, much stronger compared to those prevailing under the free float regime.

Preference shock (Figures 4.15, 4.16 and 4.17) A higher (consumption) demand increases production and the demand for labor. The boost to demand raises inflation and nominal interest rates rise, but the presence of adjustment costs limits the extent of the price increase. Interestingly, the preference shock brings about a fall of real wages under a free float regime. The mechanism is subtle. Although the consumption preference shock raises the demand for labor, it also has an effect on the expectations of future wage inflation. In this case, workers expect future wage inflation to fall (due to, e.g., a contractionary monetary policy). The total effect on wages, then, depends on both the increase in labor and the expectations of future wage inflation. As inflation expectations are affected by a common monetary policy, a Home preference shock has, in general, positive effects on Home real wages in the EMU. The preference shock also leads to crowding out of investment and a decumulation of capital. The effects of idiosyncratic preference shocks are symmetric for output in both regions but strongly asymmetric for consumption demand. As in the EMU the exchange rate is constant, these shocks pass on to the Foreign economy at full force.

Investment-specific shock (Figures 4.18, 4.19 and 4.20) The investment-specific shock affects the cost of investment. Thus, firms invest more and increase their capital stock. The effect of this shock is positive on output and, after an initial drop, it has also a positive impact on consumption. This shock temporarily improves the competitiveness of the Home economy and, in the monetary union, negatively affects the investment in the Foreign economy.

Negative labor supply shock (Figures 4.21, 4.22 and 4.23) the labor supply shock is estimated to be highly autocorrelated. Thus, its effects are very persistent. Employment falls, and the reduction in labor supply has a negative impact on investment and output. As consumers anticipate lower incomes, consumption also falls. This shock also leads to deterioration of terms of trade and thus to positive but very limited effects on the employment in the Foreign economy. The effects of an observationally equivalent shock (an idiosyncratic labor market shock originating in the German economy) have been analyzed, based on the EUROMON model, by Demertzis and de Haan (2002). The wages in the rest of the Euro area are, then, affected by about 10% of the size of the wage shock in Germany to start with, reducing only to about 5% in the long run. This contrasts with the evidence from the NOEM DSGE model, where the effect of the German labor market shock on the real wage in the rest of the area has the opposite sign and is generally very limited. This may suggest the need to augment the labor market structure in a DSGE model by the mechanisms magnifying the pass trough.

Government spending shock (Figures 4.24, 4.25 and 4.26) There is an increase in the demand for final goods, which is satisfied by the increase in the amount of inputs. So labor demand, investments and the demand for domestic and imported intermediate goods increase. The central bank reacts by increasing the nominal interest rate as a way to return production to its steady state level.

This shock also generates an important budgetary spillover to the Foreign economy the size of which is similar to that obtained in SVAR models examined in European Commission (2006).

Shock to the domestic Phillips Curve equation (Figures 4.27 and 4.28) Following a positive Phillips Curve shock, there is a jump increase in inflation and investment, output and consumption decline. As output and investment fall, labor demand is also lower and employment falls. As the shock is transitory, all effects on real variables disappear after 3-4 years.

Shock to the CPI equation (Figures 4.29 and 4.30) Following an 'a-theoretical' shock to the CPI equation we observe that Home CPI increases on impact and almost immediately returns to the steady state. This shock, however, implies a permanent change in the relative prices (and the real exchange rate as well) within the area, which affects the competitiveness of the economies.

Two types of monetary policy shocks are considered in the model, a shock to the inflation objective and a transitory nominal interest rate shock.

Shock to the inflation target (Figures 4.31, 4.32, 4.33 and 4.34) A persistent change in the inflation objective has a permanent effect on inflation rate. Also nominal interest rates increase immediately as inflation expectations rise. Quantitative effects of this shock differ, however, from those obtained under an area-wide model (see, e.g., Ratto, Roeger, intVeld, and Girardi (2005a). With inflation up by about 0.4 percentage points (in annual terms), nominal interest rates are also up by the same magnitude. Consumption, investment and output are all higher, with the peak response reached after four quarters. This shock is estimated to have important implications for the nominal interest rate prior to the EMU.

Monetary policy shock (Figures 4.35, 4.36, 4.37 and 4.38) Turning to the dynamic effects of an unanticipated temporary increase in the nominal interest rate, we see that after 12-16 quarters all variables return to their steady state, which is in line with the estimates from VAR studies. Consumption declines because of the monetary contraction. Lower consumption implies lower output and lower employment. Decreased labor demand brings about the reduction in real wages. Note that the common interest rate shock has a non-zero effect on the relative prices and real exchange rate which may be explained by the home bias in households' preferences.

Uncovered Interest Rate Parity shock (Figure 4.39) The UIP shock permits the Home interest rate to rise relative to the foreign rate, even though the value of the domestic currency is appreciating over time, as has often been observed empirically. Although this shock has been introduced to primarily generate the volatility in the nominal exchange rate prior to the EMU, it is interesting to note that it also implies movements in a variety of real and nominal variables. In particular, the interest rate, investment and output move substantially.

4.9 Out-of-sample forecasting experiment

This section documents the out-of-sample forecasting accuracy of the two-region DSGE model for the Euro area. We repeatedly estimate the model over samples of increasing lengths, forecasting out-of-sample one to eight quarters ahead at each step. The forecasts of the key Euro area-wide aggregates are then compared with those arising from unrestricted VAR models as well as to those from the one-country area-wide DSGE model in the spirit of Smets and Wouters (2003).

Our contribution to the empirical literature on forecasting aggregate European economic performance is to verify whether the restrictions arising from the multi-country (disaggregate) structure do not negatively affect DSGE model's ability to forecast the area-wide aggregates.

The econometric literature does not univocally predict that using disaggregate data is preferable to relying on the corresponding aggregates (see, e.g., Barker and Peseran (1990) for extensive reviews of all related issues). A number of presumably relevant issues cannot be taken into account or are not well identified when applying the area-wide model. Yet, the aggregate modeling approach might be a preferable one if either the structural parameters of the disaggregate model are all the same, or if compositional stability holds (see Monteforte and Siviero (2003)). In practice, these conditions are usually found not to hold. So, aggregation bias is a very frequent occurrence. Grunfeld and Griliches (1960) show, however, that if one considers the possibility of measurement errors or misspecification of the disaggregate relationships (which may be the case in stylized NOEM models), a disaggregate approach is not necessarily better. It should also be noted that while the statistical criteria underlying aggregation of primary data to national data are well established, the choice of the most appropriate aggregating functions to be used in constructing Euro area figures is an issue still under debate (see, e.g., Fagan and Henry (1998)).

4.9.1 Design of the experiment

Estimated DSGE models have relatively seldom been used to generate out-of-sample forecasts. Usually the forecasting performance of these models is evaluated based on pseudo out-of-sample measures such as marginal likelihood (see Section 4.7.5 above). Exceptions are works by Dib, Gammoudi, and Moran (2005), Smets and Wouters (2004b) and Adolfson, Laseen, Linde, and Villani (2005b), where the authors compare the forecasting performance of linearized one-country DSGE models with a range of a-theoretical linear models.[80] To contribute to this evidence, we document the out-of-sample forecasting properties of an open economy, two-region DSGE model constructed for the Euro area and estimated using aggregated Euro area data (data set by Fagan, Henry, and Mestre (2001)) along with the national accounts data for Germany. Forecasts from this model are subsequently compared with those arising from a one-country DSGE model for the Euro area and with those from unrestricted VARs. Here, the one-country DSGE model and VAR models are estimated on the set of area-wide aggregates corresponding exactly to that in Smets and Wouters (2003). Note, however, that contrary to Smets and Wouters, all models analyzed in this section have been estimated with the unfiltered data. Equations of the one-country DSGE model can be recovered from those presented in Appendix B.

The forecasting exercise details as follows: We estimate the competing models on samples of increasing lengths and compute forecasts one-through-eight quarters out at each step. All examined models are re-estimated at quarterly frequency (sequential posterior mode estimates are presented in Figures 4.7 on page 179 to 4.9 on page 180).[81] We start this rolling forecasting in 1993Q4 with the first out of sample forecast produced for 1994Q1. The final estimation sample ends in 2003Q3. This gives us forty one-step out-of sample forecasts and thirty two eight-step forecasts of the vector of observables. These forecasts can then be compared to realized data over the same period.

Since the Bayesian predictive criteria are conditional on the data set, and here we compare models estimated on different information sets, frequentist measures of the out-of-sample model's accuracy are applied.[82]

Let $\hat{y}_{t+h|t}$ denote the h-step-ahead point forecast of y_{t+h}, standing at time t, and define $e_t(h) = y_{t+h} - \hat{y}_{t+h|t}$ as the corresponding forecast error. The measures of univariate point forecasts accuracy applied here are the mean absolute forecast error (MAE) and the root mean squared forecast error (RMSE):

$$MAE_i(h) = \frac{\sum_{t=T}^{T+N_h-1} |e_{i,t}(h)|}{N_h}, \tag{4.134}$$

$$RMSE_i(h) = \sqrt{\frac{\sum_{t=T}^{T+N_h-1} (e_{i,t}(h))^2}{N_h}}, \tag{4.135}$$

where $e_{i,t}(h)$ is the ith element of the vector of forecasts errors $e_t(h)$, N_h is the number of evaluated h-step-ahead forecasts and T denotes here the end of the estimation sample.

[80] A major difference between the analysis in Adolfson, Laseen, Linde, and Villani (2005b) and Smets and Wouters (2004b) or Dib, Gammoudi, and Moran (2005) is that the former include a unit-root stochastic technology shock. This induces a common stochastic trend in the variables and makes it possible to jointly model economic growth and business cycle fluctuations. In this case estimation and forecasting does not require to detrend the data.

[81] Note that for this rolling forecasting exercise we fixed the parameters of common monetary policy rule for the EMU at the values estimated on the sample from 1980:Q1 to 2003:Q4. The remaining parameters are re-estimated at quarterly frequency.

[82] The two region DSGE model is estimated with 17 time series data prior to monetary regime change, and with 15 time series data afterwards. One country DSGE model and VAR models are estimated with 7 time series data. The information set employed to estimate the two-region model contains that used in the estimation of remaining models.

Furthermore, we consider the log determinant statistic $\ln |\Omega_{W_{N_h}}(h)|$ as a scalar value of the following multivariate measure of point forecast accuracy:

$$\Omega_{W_{N_h}}(h) = \frac{1}{N_h} \sum_{i=T}^{T-N_h-1} W_{N_h}^{-\frac{1}{2}} e_t(h) \left[W_{N_h}^{-\frac{1}{2}} e_t(h) \right]', \tag{4.136}$$

where W_{N_h} acts as a scaling matrix and accounts for intrinsic predictability of particular series. Here, we set W_{N_h} equal to a diagonal matrix with the sample variances of the time series based on data from T to $T + N_h$ as diagonal elements.

For a class of linear models a Bayesian approach implies that the parameter vector for which the model yields the best data fit corresponds to the mean of the posterior. This may, however, not be true in the case of DSGE models which are highly non-linear with respect to parameters. Furthermore, the mean of the posterior must not necessarily belong to the parameter space for which the particular DSGE model yields a unique stable solution. Therefore, in order to produce point forecasts, we are mapping from the posterior distribution using as a criterion the maximization of the in-sample data fit of the model but discarding parameter constellations yielding the indeterminacy. We also limit the impact of subjective priors, using DSGE models with uniformly distributed priors in this exercise (for the prior range see Table 4.20 on page 152). The posterior mode is, in this case, equivalent to the point estimates obtained applying the constrained maximum likelihood estimation.

4.9.2 Empirical results

Table 4.16 on the next page shows the root mean squared forecast errors at the one to eight quarters horizon from the baseline two-region DSGE model, area-wide DSGE model and two unrestricted VAR systems, VAR(1) and VAR(3). The mean absolute forecast errors are presented in Table 4.17 on page 148. We see from these tables that the DSGE models clearly outperform VAR models. The DSGE models do particularly well in terms of forecasts on output, consumption, investment, real wage growth and GDP deflator inflation. We do not, however, notice that the theoretical structure of DSGE models matters more in the long run and DSGE models have particular advantage over VARs in the forecasting ata longer horizon. The performance of a two-region DSGE model is very similar to its area-wide counterpart, with a slight edge for the former one. The disaggregate model does slightly worse in terms of the nominal interest forecasts in the medium to long-term horizons, but surpasses the area-wide model in forecasts on output, consumption, investment and inflation at all horizons.

The multivariate statistics reported in Table 4.18 on page 148 indicate that both the NOEM model, as well as the area-wide DSGE model, outperform the VARs at all forecasts horizons, when the projections of all seven variables are jointly taken into account. The disaggregate DSGE model does slightly better than area-wide model at six out of eight forecast horizons considered here.

The above results suggest that the two-region DSGE model satisfactorily exploits the disaggregate information on the Euro area so that it can be seen as a valid alternative to the currently existing area-wide DSGE models. It also appears that the theoretical restrictions arising from the New Open Economy Macroeconomics structures are not too strong for the data and do not distort the behavior of the area-wide aggregates.[83] Furthermore, the model's accuracy is generally not affected by the assumptions regarding the monetary regime change.

[83]It should be noted that model misspecification may be particulary important in the system estimation, when misspecification of any dimension of the model adversely affects its overall performance.

Variable	Output growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.34**	**0.31**	**0.30**	**0.32**	**0.34**	**0.35**	**0.36**	**0.35**
area-wide DSGE	0.35	0.34	0.35	0.35	0.37	0.36	**0.36**	**0.35**
VAR(1)	0.38	0.39	0.40	0.40	0.40	0.40	0.39	0.38
VAR(3)	0.45	0.43	0.35	0.40	0.38	0.41	0.41	0.40
Variable	Consumption growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.36**	**0.35**	0.35	**0.37**	**0.37**	**0.38**	**0.39**	0.40
area-wide DSGE	0.38	0.37	**0.34**	0.38	0.38	**0.38**	**0.39**	**0.39**
VAR(1)	0.45	0.43	0.44	0.44	0.45	0.46	0.47	0.46
VAR(3)	0.53	0.47	0.43	0.46	0.49	0.47	0.47	0.44
Variable	Investment growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**1.28**	**1.27**	1.25	**1.28**	**1.29**	**1.24**	**1.22**	**1.19**
area-wide DSGE	1.29	1.29	1.37	1.32	1.36	1.31	1.31	1.31
VAR(1)	**1.28**	1.41	1.39	1.43	1.42	1.36	1.37	1.35
VAR(3)	1.37	1.48	**1.21**	1.35	1.39	1.38	1.39	1.32
Variable	Real wage growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.36**	0.39	0.42	0.43	0.45	0.48	0.50	0.52
area-wide DSGE	**0.36**	**0.38**	**0.41**	**0.42**	**0.44**	**0.46**	**0.46**	**0.47**
VAR(1)	0.43	0.43	0.46	0.48	0.50	0.52	0.53	0.55
VAR(3)	0.49	0.47	0.48	0.48	0.48	0.53	0.54	0.54
Variable	Employment (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.11**	**0.23**	**0.38**	**0.55**	0.73	0.91	1.10	1.31
area-wide DSGE	**0.11**	**0.23**	**0.38**	**0.55**	0.73	0.91	1.11	1.32
VAR(1)	0.19	0.38	0.57	0.75	0.93	1.08	1.21	1.30
VAR(3)	0.13	0.25	0.58	**0.55**	**0.72**	**0.88**	**1.03**	**1.14**
Variable	GDP deflator (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.75**	**0.71**	0.91	**1.04**	**1.07**	1.07	1.12	**1.12**
area-wide DSGE	0.85	0.84	**0.88**	1.05	1.08	**1.00**	**1.00**	1.15
VAR(1)	0.91	1.02	1.00	1.16	1.22	1.23	1.32	1.36
VAR(3)	0.92	1.00	1.02	1.23	1.22	1.23	1.44	1.45
Variable	Nominal interest rate (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	0.45	0.80	1.10	1.35	1.52	1.66	1.76	1.85
area-wide DSGE	0.44	0.76	**1.03**	1.20	**1.33**	**1.40**	**1.45**	**1.44**
VAR(1)	**0.41**	**0.74**	1.08	**1.40**	1.70	1.99	2.28	2.60
VAR(3)	0.51	0.95	1.38	1.82	2.25	2.65	2.95	3.24

Table 4.16: Univariate statistics, root mean squared forecast errors. Bold numbers indicate the best forecasting model.

Variable	Output growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.28**	**0.25**	**0.24**	**0.26**	0.28	**0.30**	0.31	**0.29**
area-wide DSGE	0.29	0.29	0.28	0.28	0.29	**0.30**	**0.30**	**0.29**
VAR(1)	0.32	0.33	0.33	0.33	0.34	0.33	0.32	0.31
VAR(3)	0.36	0.34	0.29	0.32	0.31	0.34	0.34	0.33
Variable	Consumption growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.30**	**0.30**	0.29	**0.30**	**0.30**	0.31	**0.32**	0.33
area-wide DSGE	0.32	0.31	**0.28**	0.31	0.31	**0.30**	**0.32**	**0.31**
VAR(1)	0.36	0.36	0.37	0.37	0.38	0.37	0.38	0.38
VAR(3)	0.41	0.36	0.33	0.35	0.38	0.35	0.35	0.34
Variable	Investment growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	1.01	**0.95**	**0.93**	**0.99**	**0.98**	**0.93**	**0.91**	**0.86**
area-wide DSGE	**0.97**	1.01	1.06	1.04	1.05	1.03	1.01	0.98
VAR(1)	0.99	1.05	1.07	1.08	1.08	1.04	1.04	1.01
VAR(3)	1.05	1.05	0.96	1.07	1.05	1.05	1.07	1.00
Variable	Real wage growth (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.28**	**0.29**	**0.31**	**0.31**	**0.33**	**0.35**	**0.37**	0.39
area-wide DSGE	**0.28**	0.30	0.32	0.34	0.36	0.38	**0.37**	**0.38**
VAR(1)	0.32	0.32	0.36	0.37	0.40	0.41	0.43	0.45
VAR(3)	0.37	0.35	0.32	0.32	0.34	0.39	0.39	0.39
Variable	Employment (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.08**	**0.18**	**0.29**	**0.40**	**0.52**	**0.69**	0.88	1.09
area-wide DSGE	0.09	0.19	0.31	0.43	0.56	0.70	0.86	1.06
VAR(1)	0.16	0.31	0.47	0.62	0.77	0.90	1.02	1.10
VAR(3)	0.10	0.19	0.33	0.47	0.59	0.72	**0.85**	**0.94**
Variable	GDP deflator (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**0.61**	0.71	0.73	**0.82**	**0.83**	0.87	0.95	0.95
area-wide DSGE	0.70	**0.69**	**0.69**	0.85	**0.83**	**0.77**	**0.76**	**0.80**
VAR(1)	0.71	0.82	0.82	0.96	0.94	0.96	1.02	1.06
VAR(3)	0.71	0.86	0.84	1.04	1.05	1.02	1.17	1.23
Variable	Nominal interest rate (Euro area)							
Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	0.40	0.69	0.93	1.12	1.24	1.33	1.38	1.44
area-wide DSGE	0.38	0.65	**0.85**	**1.02**	**1.15**	**1.21**	**1.22**	**1.20**
VAR(1)	**0.33**	**0.61**	0.90	1.17	1.42	1.69	1.98	2.27
VAR(3)	0.38	0.78	1.17	1.54	1.95	2.34	2.65	2.97

Table 4.17: Univariate statistics, mean absolute forecast errors. Bold numbers indicate the best forecasting model.

Model/Horizon	1	2	3	4	5	6	7	8
two-region (disaggregate) DSGE	**-10.55**	**-7.77**	**-6.29**	**-4.69**	**-3.49**	-2.62	-2.22	**-1.87**
area-wide DSGE	-10.05	-7.38	-6.01	-4.54	-3.44	**-2.96**	**-2.61**	-1.83
VAR(1)	-8.92	-6.30	-4.60	-3.28	-2.33	-1.81	-1.28	-1.00
VAR(3)	-8.78	-5.92	-5.08	-3.34	-2.36	-1.67	-0.99	-0.31

Table 4.18: Multivariate statistics, log determinant statistic. Bold numbers indicate the best forecasting model.

Finally, we link our results to those of Marcellino, Stock, and Watson (2001) and Monteforte and Siviero (2003), who investigated reduced form time series models (autoregressions; vector autoregressions; a model in which the Euro-area aggregate is used at the country level as a predictor for the country-specific variable; and a large-model in which forecasts are based on estimates of common dynamic factors) for forecasting Euro area-wide aggregates. They also conclude that structural macroeconometric modeling of the Euro area is appropriately done at the country-specific level, rather than directly at the aggregate level.

4.10 Conclusions and direction of further research

In this chapter, we have presented an estimated two-region DSGE model for the Euro area with a particular focus on its implications for the transmission of economic fluctuations. The novel innovation is to account for the monetary regime shift from the free float regime to the monetary union while estimating the model. This allowed us to use a longer data sample without misspecifying the monetary policy regime. A natural implication is an improved accuracy of regime-invariant parameter estimates.

The model has been estimated simultaneously with unfiltered data for Germany and the Euro area aggregates using Bayesian methods. Due to the use of a multi-country framework, as well as disaggregate information the model provides additional insights for forecasting the aggregate Euro area series and analyzing the interactions within the area. When we subject the estimated model to alternative validation methods, we find that the empirical performance is satisfactory. In particular, the model can reproduce the actual co-movements of the German and the Euro-area aggregates, as well as statistics for the nominal variables. Furthermore, we find that model implied fluctuations of cyclical components of output in Germany and the rest of the area are more synchronized in the monetary union than under a free float, which is consistent with the view of Artis and Zhang (1999).

Analyzing the implied variance decompositions we find that the DSGE model, similarly to reduced form models available in the literature, identifies the idiosyncratic shocks as the main source of economic fluctuations in the Euro area (both prior to the EMU and after adoption of the common currency). However, while the identified VAR (a Bayesian VAR model with a DSGE prior and estimated under orthogonality assumption) detects that as much as 25% of business cycle frequency fluctuations of the German output have been prior to EMU explained by spillovers from the rest of the Euro area, the DSGE model estimates the trade-related spillover effects at 4-9%. The origin for only moderate direct spillovers appears to be, besides the exchange rate disconnect, DSGE model's inability to generate symmetric open economy effects of the idiosyncratic shocks. We find that these effects can be partially attributed to the common components on structural shocks. Hence, an estimated open economy DSGE model, once we allow for common shocks, might be not far away from offering explanation for co-movements detected in the data prior to the EMU. We further detect that while under a free float regime the spillover operates mainly through trade linkages, after introducing a common currency an additional spillover, working through the common interest rate, is generated. This indirect spillover multiplies the total effect of foreign shocks.

The open-economy model that we have constructed is favored by the data over a pooling of closed-economy models, as far as marginal likelihood is concerned. It also appears that the theoretical restrictions arising from the New Open Economy Macroeconomics structures do not distort the behavior of the area-wide aggregates so that the two-region model is a valid alternative to the existing area-wide models.

There are some interesting avenues for future research, some of which we are exploring in ongoing work. The model can be augmented by incorporating the mechanism that allows the agents to anticipate the transition to monetary union. Then, the model's structure would also be suitable for analyzing the accession of the 'new' countries to the EMU. A promising line of research also consists in incorporating tradable and nontradable goods, which may help to explain spillover anomalies. Finally, by allowing for sector specific technological progress and, thus, permanent divergence of sectoral output, one could better explain the differences in growth rates of real variables at the country level.

Tables and Figures

parameter		prior type	prior mean	prior std/df	area-wide model mode	area-wide model 90% posterior interval
growth rate of technology	$\bar{\varepsilon}^A$	norm	1.004	0.001	1.0032	(1.0019, 1.0032, 1.0045)
steady state inflation	$\bar{\pi}$	norm	1.006	0.001	1.0059	(1.0041, 1.0058, 1.0075)
int. rate smoothing	ρ_R	beta	0.8	0.1	0.840	(0.803, 0.844, 0.885)
inflation response	r_π	norm	1.5	0.1	1.402	(1.277, 1.440, 1.604)
output response	r_y	beta	0.2	0.15	0.044	(0.017, 0.051, 0.084)
habit formation	h	beta	0.7	0.1	0.766	(0.683, 0.778, 0.883)
capital adj. cost	S''	norm	4	2	5.335	(3.847, 6.195, 8.476)
Calvo employment	ξ_L	beta	0.7	0.15	0.895	(0.872, 0.895, 0.918)
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.922	(0.904, 0.919, 0.935)
Calvo wages	ξ_w	beta	0.7	0.05	0.686	(0.642, 0.708, 0.775)
indexation domestic prices	γ_p	beta	0.5	0.15	0.279	(0.158, 0.289, 0.420)
indexation wages	γ_w	beta	0.5	0.15	0.349	(0.185, 0.382, 0.585)
unit root tech. shock	ρ_A	beta	0.85	0.1	0.911	(0.796, 0.874, 0.951)
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.907	(0.818, 0.879, 0.941)
preference shock	ρ_C	beta	0.85	0.1	0.381	(0.225, 0.436, 0.629)
labor supply shock	ρ_L	beta	0.95	0.05	0.975	(0.939, 0.965, 0.991)
investment shock	ρ_I	beta	0.85	0.1	0.356	(0.219, 0.372, 0.518)
government spending shock	ρ_G	beta	0.85	0.1	0.992	(0.974, 0.985, 0.999)
stdv. stationary technology shock	σ_Y	unif	2	1.2	1.487	(1.062, 1.755, 2.412)
stdv. preference shock	σ_C	unif	2	1.2	1.813	(1.082, 2.237, 3.564)
stdv. labor supply shock	σ_L	unif	10	5.9	3.738	(2.932, 4.346, 5.681)
stdv. monetary shock	σ_R	unif	0.2	0.12	0.140	(0.126, 0.144, 0.164)
stdv. gov. spending shock	σ_G	unif	2	1.16	1.549	(1.391, 1.577, 1.767)
stdv. investment shock	σ_I	unif	10	5.9	5.017	(3.341, 5.900, 8.397)
stdv. unit root technology shock	σ_A	unif	0.25	0.15	0.199	(0.120, 0.243, 0.370)
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.156	(0.134, 0.159, 0.183)
stdv. infl. target shock	σ_π	unif	0.125	0.1	0.123	(0.085, 0.132, 0.180)

Table 4.19: Prior and posterior distributions, area-wide model

parameter		prior range	ML estimate	parameter		prior range	ML estimate
growth rate of technology	$\bar{\varepsilon}^A$	(1.002, 1.006)	1.008	labor supply shock	ρ_L	(0, 1)	0.623
steady state inflation	$\bar{\pi}$	(1.002, 1.008)	1.002	labor supply shock	ρ_L^*	(0, 1)	0.928
int. rate smoothing	ρ_R^{EMU}	(0, 1)	0.975	investment shock	ρ_I	(0, 1)	0.000
int. rate smoothing	ρ_R	(0, 1)	0.751	investment shock	ρ_I^*	(0, 1)	0.000
int. rate smoothing	ρ_R^*	(0, 1)	0.936	government spending shock	ρ_G	(0, 1)	0.980
inflation response	r_π^{EMU}	(1, 2)	1.983	government spending shock	ρ_G^*	(0, 1)	0.994
inflation response	r_π	(1, 2)	1.252	UIP shock	ρ_{UIP}	(0, 1)	0.993
inflation response	r_π^*	(1, 2)	1.003	stdv. stationary technology shock	σ_Y	(0, 4)	1.127
output response	r_y^{EMU}	(0, 1)	0.978	stdv. stationary technology shock	σ_Y^*	(0, 4)	1.329
output response	r_y	(0, 1)	0.141	stdv. preference shock	σ_C	(0, 4)	3.982
output response	r_y^*	(0, 1)	0.210	stdv. preference shock	σ_C^*	(0, 4)	3.989
habit formation	h	(0, 1)	0.884	stdv. labor supply shock	σ_L	(0, 20)	19.980
habit formation	h^*	(0, 1)	0.930	stdv. labor supply shock	σ_L^*	(0, 20)	19.990
capital adj. cost	S''	(0, 10)	7.170	stdv. monetary shock	σ_R	(0, 0.4)	0.069
capital adj. cost	$S^{*''}$	(0, 10)	9.828	stdv. monetary shock	σ_R^*	(0, 0.4)	0.149
Calvo employment	ξ_L	(0, 1)	0.760	stdv. monetary shock EMU	σ_R^{EMU}	(0, 0.4)	0.142
Calvo employment	ξ_L^*	(0, 1)	0.893	stdv. gov. spending shock	σ_G	(0, 4)	2.571
Calvo domestic prices	ξ_p	(0.5, 0.95)	0.950	stdv. gov. spending shock	σ_G^*	(0, 4)	1.837
Calvo domestic prices	ξ_p^*	(0.5, 0.95)	0.950	stdv. investment shock	σ_I	(0, 20)	19.971
Calvo import	ξ_p^{imp}	(0, 0.9)	0.247	stdv. investment shock	σ_I^*	(0, 20)	16.077
Calvo import	$\xi_p^{imp^*}$	(0, 0.9)	0.129	stdv. unit root technology shock	σ_A	(0, 0.5)	0.106
Calvo wages	ξ_w	(0.5, 0.95)	0.724	stdv. asymmetric technology shock	σ_Z	(0, 1)	0.495
Calvo wages	ξ_w^*	(0.5, 0.95)	0.935	stdv. Phillips Curve shock	σ_{mc}	(0, 0.5)	0.170
indexation domestic prices	γ_p	(0, 1)	0.000	stdv. Phillips Curve shock	σ_{mc}^*	(0, 0.5)	0.241
indexation domestic prices	γ_p^*	(0, 1)	0.000	stdv. infl. target shock	σ_π^{EMU}	(0, 0.25)	0.081
indexation import	γ_p^{imp}	(0, 1)	0.000	stdv. infl. target shock	σ_π	(0, 0.25)	0.051
indexation import	$\gamma_p^{imp^*}$	(0, 1)	0.963	stdv. infl. target shock	σ_π^*	(0, 0.25)	0.000
indexation wages	γ_w	(0, 1)	0.867	stdv. CPI shock	σ_{CPI}	(0, 0.5)	0.344
indexation wages	γ_w^*	(0, 1)	0.000	stdv. CPI shock	σ_{CPI}^*	(0, 0.5)	0.355
share of dom. consum.	ω_C	(0.5, 0.8)	0.780	stdv. UIP shock	σ_{UIP}	(0, 0.5)	0.047
share of dom. consum.	ω_C^*	(0.55, 0.95)	0.915	stdv. com. comp. preference shock	σ_Y^{com}	(0, 4)	0.000
share of dom. invest.	ω_I	(0.5, 0.8)	0.698	stdv. com. comp. tech. shock	σ_C^{com}	(0, 4)	3.895
share of dom. invest.	ω_I^*	0.55, 0.85)	0.850	stdv. com. comp. labor supply shock	σ_L^{com}	(0, 20)	0.580
risk premium	ϕ	(0, 0.2)	0.000	stdv. com. comp. monetary shock	σ_R^{com}	(0, 0.4)	0.079
unit root tech. shock	ρ_A	(0, 1)	0.956	stdv. com. comp. gov. spending shock	σ_G^{com}	(0, 4)	0.003
asymmetric technology shock	ρ_Z	(0, 1)	1.000	stdv. com. comp. investment shock	σ_I^{com}	(0, 20)	0.002
stationary tech. shock	ρ_Y	(0, 1)	0.929	stdv. com. comp. infl. target shock	σ_π^{com}	(0, 0.25	0.121
stationary tech. shock	ρ_Y^*	(0, 1)	0.939				
preference shock	ρ_C	(0, 1)	0.000				
preference shock	ρ_C^*	(0, 1)	0.028				

Table 4.20: Results from ML estimation

parameter		prior			model with int. linkages		pooling of closed economies	
		type	mean	std/df	mode	90% posterior interval	mode	90% posterior interval
growth rate of technology	$\bar{\varepsilon}^A$	norm	1.004	0.001	1.0048	(1.0044, 1.0053, 1.0062)	1.0037	(1.0042 1.005 1.0058)
steady state inflation	$\bar{\pi}$	norm	1.006	0.001	1.0058	(1.0042, 1.0058, 1.0074)	1.0059	(1.0043 1.006 1.0075)
int. rate smoothing	ρ_R	beta	0.8	0.1	0.795	(0.749, 0.798, 0.847)	0.840	(0.791 0.832 0.876)
int. rate smoothing	ρ_R^*	beta	0.8	0.1	0.865	(0.838, 0.885, 0.930)	0.862	(0.839 0.880 0.925)
inflation response	r_π	norm	1.5	0.1	1.462	(1.364, 1.503, 1.647)	1.562	(1.432 1.571 1.707)
inflation response	r_π^*	norm	1.5	0.1	1.425	(1.285, 1.456, 1.625)	1.430	(1.293 1.462 1.629)
output response	r_y	beta	0.2	0.15	0.029	(0.009, 0.033, 0.058)	0.002	(0.000 0.007 0.015)
output response	r_y^*	beta	0.2	0.15	0.101	(0.052, 0.161, 0.266)	0.091	(0.081 0.198 0.307)
habit formation	h	beta	0.7	0.1	0.743	(0.663, 0.747, 0.824)	0.696	(0.588 0.685 0.782)
habit formation	h^*	beta	0.7	0.1	0.749	(0.700, 0.779, 0.862)	0.503	(0.573 0.706 0.841)
capital adj. cost	S''	norm	4	2	1.673	(1.174, 3.722, 6.149)	2.040	(1.134 1.991 2.843)
capital adj. cost	$S^{*\prime\prime}$	norm	4	2	2.530	(1.699, 4.090, 6.295)	1.972	(2.326 4.611 6.947)
Calvo employment	ξ_L	beta	0.7	0.15	0.826	(0.777, 0.815, 0.852)	0.804	(0.781 0.810 0.841)
Calvo employment	ξ_L^*	beta	0.7	0.15	0.886	(0.858, 0.880, 0.903)	0.895	(0.857 0.880 0.902)
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.918	(0.902, 0.919, 0.936)	0.916	(0.895 0.913 0.931)
Calvo domestic prices	ξ_p^*	beta	0.7	0.05	0.925	(0.907, 0.925, 0.941)	0.928	(0.910 0.925 0.940)
Calvo import	ξ_p^{imp}	beta	0.5	0.05	0.537	(0.455, 0.539, 0.625)		
Calvo import	$\xi_p^{imp^*}$	beta	0.5	0.05	0.549	(0.470, 0.552, 0.642)		
Calvo wages	ξ_w	beta	0.7	0.05	0.640	(0.619, 0.675, 0.734)	0.668	(0.602 0.658 0.713)
Calvo wages	ξ_w^*	beta	0.7	0.05	0.834	(0.796, 0.843, 0.887)	0.736	(0.705 0.770 0.838)
indexation domestic prices	γ_p	beta	0.5	0.15	0.763	(0.534, 0.721, 0.930)	0.642	(0.449 0.640 0.841)
indexation domestic prices	γ_p^*	beta	0.5	0.15	0.200	(0.115, 0.252, 0.392)	0.236	(0.146 0.297 0.437)
indexation import	γ_p^{imp}	beta	0.5	0.15	0.436	(0.219, 0.470, 0.707)		
indexation import	$\gamma_p^{imp^*}$	beta	0.5	0.15	0.526	(0.265, 0.509, 0.763)		
indexation wages	γ_w	beta	0.5	0.15	0.501	(0.293, 0.518, 0.765)	0.527	(0.273 0.519 0.750)
indexation wages	γ_w^*	beta	0.5	0.15	0.221	(0.108, 0.269, 0.420)	0.248	(0.091 0.240 0.380)
share of dom. consum.	ω_C	beta	0.65	0.05	0.688	(0.612, 0.681, 0.755)		
share of dom. consum.	ω_C^*	beta	0.8	0.05	0.820	(0.762, 0.821, 0.880)		
share of dom. invest.	ω_I	beta	0.55	0.05	0.544	(0.466, 0.538, 0.613)		
share of dom. invest.	ω_I^*	beta	0.7	0.05	0.674	(0.576, 0.655, 0.729)		

Table 4.21: Model with international linkages vs. pooling of closed economies, prior and posterior distributions

parameter		prior			model with int. linkages		pooling of closed economies	
		type	mean	std/df	mode	90% posterior interval	mode	90% posterior interval
unit root tech. shock	ρ_A	beta	0.85	0.1	0.885	(0.801, 0.867, 0.940)	0.900	(0.721 0.828 0.944)
unit root tech. shock	ρ_A^*	beta	0.85	0.1			0.907	(0.640 0.795 0.948)
asymmetric technology shock	ρ_Z	beta	0.85	0.1	0.962	(0.821, 0.919, 0.996)		
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.880	(0.805, 0.863, 0.925)	0.957	(0.866 0.925 0.982)
stationary tech. shock	ρ_Y^*	beta	0.85	0.1	0.794	(0.693, 0.795, 0.903)	0.600	(0.586 0.712 0.848)
preference shock	ρ_C	beta	0.85	0.1	0.361	(0.246, 0.468, 0.687)	0.921	(0.801 0.891 0.989)
preference shock	ρ_C^*	beta	0.85	0.1	0.394	(0.277, 0.507, 0.722)	0.965	(0.666 0.803 0.970)
labor supply shock	ρ_L	beta	0.85	0.1	0.862	(0.559, 0.729, 0.895)	0.599	(0.441 0.621 0.792)
labor supply shock	ρ_L^*	beta	0.85	0.1	0.944	(0.765, 0.867, 0.970)	0.950	(0.915 0.940 0.967)
investment shock	ρ_I	beta	0.85	0.1	0.993	(0.940, 0.967, 0.995)	0.991	(0.979 0.989 0.999)
investment shock	ρ_I^*	beta	0.85	0.1	0.321	(0.221, 0.371, 0.530)	0.245	(0.188 0.328 0.475)
government spending shock	ρ_G	beta	0.85	0.1	0.934	(0.868, 0.927, 0.988)	0.864	(0.761 0.857 0.962)
government spending shock	ρ_G^*	beta	0.85	0.1	0.899	(0.811, 0.887, 0.972)	0.971	(0.921 0.957 0.996)
stdv. stationary technology shock	σ_Y	unif	2	1.2	2.618	(1.831, 2.639, 3.431)	2.794	(2.339 2.993 3.678)
stdv. stationary technology shock	σ_Y^*	unif	2	1.2	2.171	(1.192, 2.205, 3.224)	3.817	(1.583 2.656 3.747)
stdv. preference shock	σ_C	unif	2	1.2	2.728	(2.320, 3.159, 4.078)	3.005	(2.265 3.051 4.063)
stdv. preference shock	σ_C^*	unif	2	1.2	1.782	(1.308, 2.492, 3.594)	0.000	(1.170 2.355 3.792)
stdv. labor supply shock	σ_L	unif	10	5.9	9.071	(9.504, 14.253, 20.211)	15.442	(9.810 14.523 20.217)
stdv. labor supply shock	σ_L^*	unif	10	5.9	4.611	(3.394, 9.510, 16.918)	6.145	(4.143 7.678 11.643)
stdv. monetary shock	σ_R	unif	0.2	0.12	0.146	(0.128, 0.152, 0.176)	0.142	(0.124 0.147 0.170)
stdv. monetary shock	σ_R^*	unif	0.2	0.12	0.168	(0.144, 0.170, 0.195)	0.170	(0.141 0.166 0.189)
stdv. gov. spending shock	σ_G	unif	2	1.16	2.392	(2.007, 2.471, 2.946)	2.966	(2.636 3.058 3.465)
stdv. gov. spending shock	σ_G^*	unif	2	1.16	1.656	(1.413, 1.682, 1.948)	2.037	(1.780 2.061 2.355)
stdv. investment shock	σ_I	unif	10	5.9	4.568	(3.274, 5.463, 7.396)	3.451	(2.478 3.690 4.791)
stdv. investment shock	σ_I^*	unif	10	5.9	3.495	(2.218, 5.736, 8.954)	2.973	(3.709 6.928 10.837)
stdv. unit root technology shock	σ_A	unif	0.25	0.15	0.187	(0.092, 0.189, 0.281)	0.205	(0.147 0.303 0.491)
stdv. unit root technology shock	σ_A^*	unif	0.25	0.15			0.184	(0.066 0.191 0.311)
stdv. asymmetric technology shock	σ_Z	unif	0.5	0.29	0.450	(0.325, 0.437, 0.553)		
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.158	(0.136, 0.165, 0.194)	0.160	(0.141 0.169 0.198)
stdv. Phillips Curve shock	σ_{mc}^*	unif	0.25	0.15	0.218	(0.185, 0.223, 0.259)	0.210	(0.182 0.217 0.253)
stdv. infl. target shock	σ_π^{EMU}	unif	0.125	0.1	0.153	(0.099, 0.162, 0.218)	0.106	(0.066 0.126 0.180)
stdv. infl. target shock	σ_π	unif	0.125	0.1	0.157	(0.104, 0.165, 0.226)	0.183	(0.103 0.177 0.246)
stdv. CPI shock	σ_{CPI}	unif	0.25	0.15	0.367	(0.304, 0.367, 0.428)	0.380	(0.332 0.382 0.434)
stdv. CPI shock	σ_{CPI}^*	unif	0.25	0.15	0.352	(0.309, 0.360, 0.409)	0.344	(0.300 0.350 0.399)
corr. preference shocks	σ_{C,C^*}	norm	0.2	0.4	0.406	(0.271, 0.450, 0.616)		
corr. gov. spending shocks	σ_{G,G^*}	norm	0.2	0.4	0.203	(-0.032, 0.192, 0.440)		
corr. labor supply shocks	σ_{L,L^*}	norm	0.2	0.4	-0.052	(-0.371, -0.058, 0.272)		
corr. monetary shocks	σ_{R,R^*}	norm	0.2	0.4	0.342	(0.173, 0.339, 0.515)		
corr. tech. shocks	σ_{Y,Y^*}	norm	0.2	0.4	-0.185	(-0.325, -0.124, 0.078)		
corr. investment shocks	σ_{I,I^*}	norm	0.2	0.4	-0.121	(-0.282, -0.064, 0.170)		

Table 4.22: Model with international linkages vs. pooling of closed economies, prior and posterior distributions (cont.)

parameter		prior type	prior mean	prior std/df	model with correlated shocks mode	model with correlated shocks 90% posterior interval	model without common components mode	model without common components 90% posterior interval	LOOP mode	LOOP 90% posterior interval
growth rate of technology	$\bar{\varepsilon}^A$	norm	1.004	0.001	1.0059	(1.0049, 1.006, 1.0071)	1.006	(1.0049, 1.0059, 1.0068)	1.0059	(1.005, 1.006, 1.0069)
steady state inflation	$\bar{\pi}$	norm	1.006	0.001	1.0057	(1.0041, 1.0058, 1.0074)	1.0057	(1.004, 1.0056, 1.0072)	1.0057	(1.0038, 1.0056, 1.0071)
int. rate smoothing	ρ_R^{EMU}	beta	0.8	0.2	0.909	(0.844, 0.905, 0.964)	0.915	(0.844, 0.902, 0.960)	0.913	(0.850, 0.904, 0.963)
int. rate smoothing	ρ_R	beta	0.8	0.1	0.800	(0.750, 0.797, 0.850)	0.809	(0.759, 0.802, 0.844)	0.803	(0.757, 0.800, 0.845)
int. rate smoothing	ρ_R^*	beta	0.8	0.1	0.779	(0.768, 0.847, 0.935)	0.778	(0.731, 0.796, 0.860)	0.771	(0.728, 0.783, 0.839)
inflation response	r_π^{EMU}	norm	1.5	0.2	1.413	(1.161, 1.505, 1.833)	1.439	(1.191, 1.498, 1.785)	1.439	(1.205, 1.489, 1.777)
inflation response	r_π	norm	1.5	0.1	1.538	(1.383, 1.527, 1.665)	1.547	(1.397, 1.540, 1.670)	1.559	(1.406, 1.557, 1.685)
inflation response	r_π^*	norm	1.5	0.1	1.415	(1.269, 1.425, 1.594)	1.413	(1.300, 1.441, 1.581)	1.419	(1.312, 1.444, 1.573)
output response	r_y^{EMU}	beta	0.2	0.15	0.172	(0.049, 0.249, 0.482)	0.177	(0.054, 0.195, 0.340)	0.181	(0.074, 0.226, 0.381)
output response	r_y	beta	0.2	0.15	0.154	(0.066, 0.148, 0.215)	0.149	(0.070, 0.144, 0.214)	0.151	(0.081, 0.168, 0.253)
output response	r_y^*	beta	0.2	0.15	0.142	(0.106, 0.191, 0.286)	0.142	(0.091, 0.161, 0.234)	0.143	(0.096, 0.162, 0.223)
habit formation	h	beta	0.7	0.1	0.714	(0.475, 0.644, 0.788)	0.706	(0.496, 0.626, 0.756)	0.687	(0.479, 0.654, 0.806)
habit formation	h^*	beta	0.7	0.1	0.693	(0.670, 0.753, 0.833)	0.692	(0.642, 0.724, 0.817)	0.684	(0.617, 0.707, 0.800)
capital adj. cost	S''	norm	4	2	6.536	(4.296, 6.361, 8.188)	6.522	(4.898, 6.744, 8.728)	6.389	(5.012, 6.640, 8.412)
capital adj. cost	$S^{*\prime\prime}$	norm	4	2	1.300	(1.317, 2.435, 3.477)	1.232	(0.947, 1.734, 2.501)	1.162	(0.815, 1.389, 1.910)
Calvo employment	ξ_L	beta	0.7	0.15	0.794	(0.760, 0.796, 0.845)	0.790	(0.763, 0.801, 0.841)	0.789	(0.751, 0.787, 0.826)
Calvo employment	ξ_L^*	beta	0.7	0.15	0.876	(0.858, 0.881, 0.904)	0.870	(0.851, 0.874, 0.896)	0.870	(0.848, 0.872, 0.896)
Calvo domestic prices	ξ_p	beta	0.7	0.05	0.927	(0.911, 0.925, 0.938)	0.927	(0.912, 0.925, 0.938)	0.927	(0.911, 0.925, 0.938)
Calvo domestic prices	ξ_p^*	beta	0.7	0.05	0.927	(0.922, 0.939, 0.957)	0.919	(0.907, 0.925, 0.943)	0.920	(0.908, 0.925, 0.943)
Calvo import	ξ_p^{imp}	beta	0.5	0.05	0.490	(0.413, 0.489, 0.561)	0.488	(0.410, 0.485, 0.560)		
Calvo import	$\xi_p^{imp^*}$	beta	0.5	0.05	0.506	(0.421, 0.509, 0.600)	0.506	(0.428, 0.511, 0.597)		
Calvo wages	ξ_w	beta	0.7	0.05	0.699	(0.615, 0.667, 0.717)	0.695	(0.619, 0.677, 0.733)	0.691	(0.628, 0.680, 0.734)
Calvo wages	ξ_w^*	beta	0.7	0.05	0.799	(0.794, 0.850, 0.915)	0.783	(0.743, 0.796, 0.856)	0.785	(0.745, 0.800, 0.858)
indexation domestic prices	γ_p	beta	0.5	0.15	0.380	(0.183, 0.364, 0.543)	0.373	(0.190, 0.361, 0.535)	0.361	(0.177, 0.347, 0.499)
indexation domestic prices	γ_p^*	beta	0.5	0.15	0.242	(0.108, 0.249, 0.372)	0.256	(0.131, 0.269, 0.404)	0.257	(0.120, 0.268, 0.405)
indexation import	γ_p^{imp}	beta	0.5	0.15	0.462	(0.246, 0.474, 0.727)	0.461	(0.234, 0.472, 0.721)		
indexation import	$\gamma_p^{imp^*}$	beta	0.5	0.15	0.464	(0.230, 0.466, 0.713)	0.461	(0.217, 0.467, 0.689)		
indexation wages	γ_w	beta	0.5	0.15	0.471	(0.226, 0.464, 0.696)	0.476	(0.246, 0.487, 0.715)	0.481	(0.263, 0.489, 0.723)
indexation wages	γ_w^*	beta	0.5	0.15	0.287	(0.121, 0.263, 0.413)	0.279	(0.142, 0.298, 0.454)	0.276	(0.144, 0.306, 0.481)
share of dom. consum.	ω_C	beta	0.65	0.05	0.767	(0.731, 0.770, 0.809)	0.764	(0.723, 0.763, 0.805)	0.792	(0.759, 0.793, 0.826)
share of dom. consum.	ω_C^*	beta	0.8	0.05	0.934	(0.883, 0.917, 0.952)	0.933	(0.902, 0.928, 0.958)	0.943	(0.916, 0.941, 0.967)
share of dom. invest.	ω_I	beta	0.55	0.05	0.619	(0.489, 0.582, 0.656)	0.602	(0.521, 0.586, 0.650)	0.601	(0.530, 0.593, 0.658)
share of dom. invest.	ω_I^*	beta	0.7	0.05	0.661	(0.613, 0.691, 0.776)	0.663	(0.603, 0.667, 0.727)	0.664	(0.595, 0.661, 0.722)
risk premium	ϕ	invg	0.01	2	0.002	(0.002, 0.003, 0.003)	0.002	(0.002, 0.010, 0.004)	0.002	(0.002, 0.003, 0.003)

Table 4.23: Prior and posterior distributions, model with cross-country correlated shocks, model without common components and LOOP model

parameter		prior type	prior mean	prior std/df	model with correlated shocks mode	model with correlated shocks 90% posterior interval	model without common components mode	model without common components 90% posterior interval	LOOP mode	LOOP 90% posterior interval
unit root tech. shock	ρ_A	beta	0.85	0.1	0.933	(0.881, 0.915, 0.952)	0.935	(0.893, 0.926, 0.962)	0.933	(0.890, 0.922, 0.956)
asymmetric technology shock	ρ_Z	beta	0.85	0.1	1.000	(1.000, 1.000, 1.000)	1.000	(1.000, 1.000, 1.000)	1.000	(1.000, 1.000, 1.000)
stationary tech. shock	ρ_Y	beta	0.85	0.1	0.879	(0.780, 0.848, 0.911)	0.880	(0.803, 0.856, 0.911)	0.877	(0.793, 0.851, 0.909)
stationary tech. shock	ρ_Y^*	beta	0.85	0.1	0.835	(0.800, 0.869, 0.938)	0.802	(0.725, 0.813, 0.912)	0.807	(0.737, 0.818, 0.906)
preference shock	ρ_C	beta	0.85	0.1	0.451	(0.407, 0.678, 0.991)	0.498	(0.436, 0.672, 0.956)	0.517	(0.353, 0.622, 0.963)
preference shock	ρ_C^*	beta	0.85	0.1	0.935	(0.838, 0.901, 0.958)	0.935	(0.879, 0.918, 0.959)	0.935	(0.890, 0.923, 0.960)
labor supply shock	ρ_L	beta	0.95	0.05	0.881	(0.826, 0.886, 0.950)	0.880	(0.816, 0.878, 0.950)	0.881	(0.803, 0.870, 0.933)
labor supply shock	ρ_L^*	beta	0.95	0.05	0.963	(0.861, 0.933, 0.979)	0.963	(0.942, 0.960, 0.979)	0.961	(0.932, 0.954, 0.975)
investment shock	ρ_I	beta	0.85	0.1	0.272	(0.192, 0.327, 0.460)	0.269	(0.181, 0.289, 0.395)	0.269	(0.187, 0.297, 0.396)
investment shock	ρ_I^*	beta	0.85	0.1	0.398	(0.219, 0.360, 0.482)	0.432	(0.235, 0.391, 0.542)	0.424	(0.238, 0.386, 0.531)
government spending shock	ρ_G	beta	0.85	0.1	0.970	(0.951, 0.970, 0.990)	0.973	(0.953, 0.970, 0.989)	0.971	(0.942, 0.964, 0.987)
government spending shock	ρ_G^*	beta	0.85	0.1	0.997	(0.993, 0.996, 1.000)	0.997	(0.992, 0.996, 0.999)	0.996	(0.992, 0.995, 0.999)
UIP shock	ρ_{UIP}	beta	0.85	0.1	0.997	(0.991, 0.995, 1.000)	0.997	(0.988, 0.994, 1.000)	0.996	(0.990, 0.994, 0.999)
stdv. stationary technology shock	σ_Y	unif	2	1.2	1.528	(1.200, 1.769, 2.644)	1.504	(1.257, 1.836, 2.474)	1.498	(1.200, 1.625, 2.093)
stdv. stationary technology shock	σ_Y^*	unif	2	1.2	1.834	(1.250, 1.778, 2.262)	1.892	(1.341, 1.986, 2.616)	1.858	(1.279, 1.930, 2.539)
stdv. preference shock	σ_C	unif	2	1.2	2.639	(1.947, 2.740, 3.554)	2.564	(1.565, 2.230, 2.902)	2.398	(1.610, 2.607, 3.594)
stdv. preference shock	σ_C^*	unif	2	1.2	2.309	(2.096, 3.090, 4.078)	2.255	(1.799, 2.664, 3.498)	2.183	(1.715, 2.526, 3.359)
stdv. labor supply shock	σ_L	unif	10	5.9	10.632	(6.788, 9.126, 11.545)	10.463	(6.646, 9.907, 12.808)	10.207	(7.438, 10.178, 14.662)
stdv. labor supply shock	σ_L^*	unif	10	5.9	4.107	(3.418, 6.766, 11.024)	3.957	(2.718, 4.327, 5.897)	4.044	(3.186, 4.996, 7.378)
stdv. monetary shock	σ_R	unif	0.2	0.12	0.119	(0.105, 0.122, 0.140)	0.120	(0.107, 0.125, 0.143)	0.121	(0.104, 0.124, 0.144)
stdv. monetary shock	σ_R^*	unif	0.2	0.12	0.143	(0.127, 0.152, 0.172)	0.142	(0.130, 0.151, 0.173)	0.141	(0.125, 0.143, 0.162)
stdv. monetary shock EMU	σ_R^{EMU}	unif	0.2	0.12	0.167	(0.117, 0.197, 0.267)	0.162	(0.126, 0.188, 0.255)	0.166	(0.113, 0.196, 0.265)
stdv. gov. spending shock	σ_G	unif	2	1.16	2.539	(2.206, 2.664, 2.987)	2.559	(2.373, 2.690, 3.008)	2.602	(2.293, 2.635, 2.965)
stdv. gov. spending shock	σ_G^*	unif	2	1.16	1.784	(1.535, 1.775, 2.033)	1.782	(1.583, 1.804, 2.006)	1.850	(1.659, 1.873, 2.061)
stdv. investment shock	σ_I	unif	10	5.9	15.592	(9.694, 14.969, 19.236)	15.565	(11.507, 15.881, 19.671)	15.235	(11.847, 15.500, 19.467)
stdv. investment shock	σ_I^*	unif	10	5.9	1.886	(2.227, 3.587, 4.885)	1.745	(1.408, 2.490, 3.465)	1.669	(1.231, 2.062, 2.842)
stdv. unit root technology shock	σ_A	unif	0.25	0.15	0.142	(0.105, 0.158, 0.213)	0.153	(0.115, 0.163, 0.214)	0.158	(0.115, 0.170, 0.229)
stdv. asymmetric technology shock	σ_Z	unif	0.5	0.29	0.419	(0.348, 0.449, 0.549)	0.403	(0.333, 0.422, 0.499)	0.410	(0.329, 0.421, 0.524)
stdv. Phillips Curve shock	σ_{mc}	unif	0.25	0.15	0.165	(0.147, 0.171, 0.200)	0.166	(0.143, 0.174, 0.202)	0.166	(0.143, 0.169, 0.191)
stdv. Phillips Curve shock	σ_{mc}^*	unif	0.25	0.15	0.217	(0.191, 0.221, 0.246)	0.217	(0.189, 0.220, 0.253)	0.218	(0.188, 0.222, 0.253)
stdv. infl. target shock	σ_π^{EMU}	unif	0.125	0.1	0.094	(0.051, 0.097, 0.145)	0.090	(0.059, 0.100, 0.142)	0.087	(0.047, 0.089, 0.128)
stdv. infl. target shock	σ_π	unif	0.125	0.1	0.179	(0.064, 0.141, 0.200)	0.167	(0.105, 0.170, 0.240)	0.161	(0.101, 0.162, 0.220)
stdv. infl. target shock	σ_π^*	unif	0.125	0.1	0.000	(0.000, 0.072, 0.129)	0.000	(0.000, 0.048, 0.096)	0.000	(0.000, 0.055, 0.103)
stdv. CPI shock	σ_{CPI}	unif	0.25	0.15	0.340	(0.305, 0.341, 0.379)	0.341	(0.315, 0.351, 0.391)	0.351	(0.311, 0.352, 0.397)
stdv. CPI shock	σ_{CPI}^*	unif	0.25	0.15	0.347	(0.311, 0.349, 0.388)	0.347	(0.304, 0.348, 0.389)	0.342	(0.305, 0.346, 0.387)
stdv. UIP shock	σ_{UIP}	unif	0.25	0.15	0.192	(0.145, 0.199, 0.250)	0.195	(0.151, 0.203, 0.254)	0.172	(0.135, 0.188, 0.239)
corr. preference shock	σ_{C,C^*}	norm	0.2	0.4	0.347	(0.050, 0.268, 0.504)				
corr. tech. shock	σ_{Y,Y^*}	norm	0.2	0.4	-0.250	(-0.419, -0.242, -0.070)				
corr. labor supply shock	σ_{L,L^*}	norm	0.2	0.4	-0.025	(-0.505, -0.121, 0.243)				
corr. monetary shock	σ_{R,R^*}	norm	0.2	0.4	-0.056	(-0.151, 0.093, 0.364)				
corr. gov. spending shock	σ_{G,G^*}	norm	0.2	0.4	0.027	(-0.145, 0.020, 0.170)				
corr. investment shock	σ_{I,I^*}	norm	0.2	0.4	-0.214	(-0.416, -0.210, -0.029)				
corr. infl. target shock	σ_{π,π^*}	norm	0.2	0.4	0.090	(-0.446, 0.035, 0.535)				

Table 4.24: Prior and posterior distributions, model with cross-country correlated shocks, model without common components and LOOP model (cont.)

Variable	std (data)	std (model)	first order autocorr. (data)	first order autocorr. (model)
$\Delta \ln Y_t^{GER}$	0.73	(0.80, 0.95, 1.15)	0.09	(0.04, 0.29, 0.52)
$\Delta \ln Y_t^{EMU}$	0.51	(0.61, 0.73, 0.91)	0.25	(0.18, 0.42, 0.63)
$\Delta \ln C_t^{GER}$	0.76	(0.83, 1.01, 1.22)	0.04	(0.15, 0.38, 0.56)
$\Delta \ln C_t^{EMU}$	0.53	(0.65, 0.79, 0.94)	0.12	(0.27, 0.49, 0.66)
$\Delta \ln I_t^{GER}$	2.73	(2.54, 3.08, 3.67)	0.01	(0.05, 0.29, 0.51)
$\Delta \ln I_t^{EMU}$	1.40	(1.64, 1.97, 2.44)	0.19	(0.18, 0.42, 0.63)
$\Delta \ln(W_t^{GER}/P_t^{GER})$	0.77	(1.09, 1.36, 1.72)	-0.11	(0.19, 0.40, 0.56)
$\Delta \ln(W_t^{EMU}/P_t^{EMU})$	0.45	(0.58, 0.73, 0.92)	0.17	(0.24, 0.51, 0.72)
$\mathrm{EM}_t^{GER\_HP}$	2.56	(1.52, 2.64, 4.51)	0.97	(0.87, 0.95, 0.98)
$\mathrm{EM}_t^{EMU\_HP}$	2.17	(1.01, 1.92, 3.66)	0.98	(0.88, 0.95, 0.98)
$4\mathrm{x}\Delta \ln P_t^{GER}$	1.38	(1.79, 2.89, 4.77)	0.69	(0.62, 0.84, 0.94)
$4\mathrm{x}\Delta \ln P_t^{EMU}$	2.78	(1.50, 2.40, 4.19)	0.89	(0.53, 0.79, 0.92)
$4\mathrm{x}\Delta \ln P_t^{C,GER}$	1.76	(2.22, 3.16, 4.92)	0.83	(0.30, 0.66, 0.87)
$4\mathrm{x}\Delta \ln P_t^{C,EMU}$	2.90	(1.82, 2.63, 4.33)	0.88	(0.26, 0.62, 0.85)
$\ln R_t^{GER}$	2.36	(1.68, 2.77, 4.73)	0.95	(0.76, 0.90, 0.96)
$\ln R_t^{EMU}$	3.35	(1.40, 2.37, 4.26)	0.96	(0.76, 0.90, 0.96)
$\Delta \ln S_t$	1.00	(1.67, 2.05, 2.51)	0.40	(-0.26, 0.04, 0.39)

Table 4.25: Comparison of unconditional second moments, log-differences

cross-correlations (data)							
	$\Delta\ln Y_t^{GER}$	$\Delta\ln Y_t^{EMU}$	$\Delta\ln C_t^{GER}$	$\Delta\ln C_t^{EMU}$	$\Delta\ln I_t^{GER}$	$\Delta\ln I_t^{EMU}$	$\Delta\ln(W_t^{GER}/P_t^{GER})$
$\Delta\ln Y_t^{GER}$	1						
$\Delta\ln Y_t^{EMU}$	0.80	1					
$\Delta\ln C_t^{GER}$	0.60	0.50	1				
$\Delta\ln C_t^{EMU}$	0.52	0.67	0.81	1			
$\Delta\ln I_t^{GER}$	0.58	0.52	0.32	0.28	1		
$\Delta\ln I_t^{EMU}$	0.64	0.79	0.40	0.51	0.51	1	
$\Delta\ln(W_t^{GER}/P_t^{GER})$	0.27	0.19	0.27	0.21	0.03	0.14	1
$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	0.20	0.22	0.22	0.21	-0.03	0.21	0.58
$\text{EM}_t^{GER\_HP}$	0.28	0.08	0.27	0.14	0.07	0.04	0.35
$\text{EM}_t^{EMU\_HP}$	-0.05	-0.14	-0.08	-0.08	-0.11	-0.14	0.14
$4\text{x}\Delta\ln P_t^{GER}$	-0.17	-0.27	-0.23	-0.28	-0.12	-0.31	0.09
$4\text{x}\Delta\ln P_t^{EMU}$	-0.10	-0.19	-0.12	-0.17	-0.11	-0.26	-0.08
$4\text{x}\Delta\ln P_t^{C,GER}$	-0.14	-0.21	-0.24	-0.31	-0.12	-0.31	0.08
$4\text{x}\Delta\ln P_t^{C,EMU}$	-0.15	-0.24	-0.26	-0.30	-0.14	-0.33	-0.07
$\ln R_t^{GER}$	-0.04	-0.24	-0.05	-0.19	-0.14	-0.32	0.13
$\ln R_t^{EMU}$	0.02	-0.14	0.01	-0.13	-0.06	-0.23	0.05
$\Delta\ln S_t$	0.17	0.20	0.05	0.14	0.01	0.26	0.18
	$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	$\text{EM}_t^{GER\_HP}$	$\text{EM}_t^{EMU\_HP}$	$4\text{x}\Delta\ln P_t^{GER}$	$4\text{x}\Delta\ln P_t^{EMU}$	$4\text{x}\Delta\ln P_t^{C,GER}$	$4\text{x}\Delta\ln P_t^{C,EMU}$
$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	1						
$\text{EM}_t^{GER\_HP}$	0.34	1					
$\text{EM}_t^{EMU\_HP}$	0.26	0.19	1				
$4\text{x}\Delta\ln P_t^{GER}$	0.06	0.50	-0.08	1			
$4\text{x}\Delta\ln P_t^{EMU}$	0.06	0.17	-0.24	0.70	1		
$4\text{x}\Delta\ln P_t^{C,GER}$	0.16	0.39	0.08	0.64	0.59	1	
$4\text{x}\Delta\ln P_t^{C,EMU}$	0.11	0.15	-0.20	0.69	0.92	0.72	1
$\ln R_t^{GER}$	0.23	0.62	0.02	0.81	0.72	0.78	0.74
$\ln R_t^{EMU}$	0.17	0.46	-0.33	0.75	0.88	0.66	0.86
$\Delta\ln S_t$	0.13	0.02	0.23	-0.34	-0.41	-0.12	-0.38
	$\ln R_t^{GER}$	$\ln R_t^{EMU}$	$\Delta\ln S_t$				
$\ln R_t^{GER}$	1						
$\ln R_t^{EMU}$	0.87	1					
$\Delta\ln S_t$	-0.31	-0.41	1				

Table 4.26: Comparison of unconditional second moments, log-differences (cont.)

	cross-correlations (model)						
	$\Delta\ln Y_t^{GER}$	$\Delta\ln Y_t^{EMU}$	$\Delta\ln C_t^{GER}$	$\Delta\ln C_t^{EMU}$	$\Delta\ln I_t^{GER}$	$\Delta\ln I_t^{EMU}$	$\Delta\ln(W_t^{GER}/P_t^{GER})$
$\Delta\ln Y_t^{GER}$	1						
$\Delta\ln Y_t^{EMU}$	(0.65, 0.76, 0.85)	1					
$\Delta\ln C_t^{GER}$	(0.33, 0.50, 0.65)	(0.26, 0.45, 0.61)	1				
$\Delta\ln C_t^{EMU}$	(0.23, 0.43, 0.60)	(0.52, 0.67, 0.79)	(0.49, 0.66, 0.78)	1			
$\Delta\ln I_t^{GER}$	(0.32, 0.50, 0.64)	(0.19, 0.38, 0.55)	(-0.13, 0.08, 0.29)	(-0.15, 0.07, 0.28)	1		
$\Delta\ln I_t^{EMU}$	(0.46, 0.62, 0.76)	(0.56, 0.71, 0.82)	(-0.11, 0.12, 0.35)	(-0.12, 0.15, 0.42)	(0.34, 0.52, 0.67)	1	
$\Delta\ln(W_t^{GER}/P_t^{GER})$	(-0.13, 0.10, 0.32)	(-0.08, 0.16, 0.39)	(-0.18, 0.05, 0.27)	(-0.13, 0.12, 0.35)	(-0.10, 0.12, 0.32)	(-0.12, 0.12, 0.36)	1
$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	(-0.05, 0.17, 0.39)	(0.11, 0.34, 0.55)	(-0.11, 0.11, 0.32)	(-0.08, 0.17, 0.41)	(-0.08, 0.14, 0.34)	(0.05, 0.28, 0.49)	(0.45, 0.65, 0.78)
$\mathrm{EM}_t^{GER\_HP}$	(-0.19, 0.03, 0.28)	(-0.20, 0.07, 0.36)	(-0.20, 0.02, 0.24)	(-0.22, 0.05, 0.32)	(-0.15, 0.06, 0.26)	(-0.21, 0.04, 0.29)	(-0.10, 0.19, 0.45)
$\mathrm{EM}_t^{EMU\_HP}$	(-0.18, 0.05, 0.29)	(-0.22, 0.05, 0.34)	(-0.19, 0.02, 0.24)	(-0.25, 0.01, 0.28)	(-0.17, 0.03, 0.24)	(-0.19, 0.05, 0.30)	(-0.18, 0.11, 0.38)
$4\mathrm{x}\Delta\ln P_t^{GER}$	(-0.27, -0.01, 0.27)	(-0.21, 0.07, 0.37)	(-0.25, -0.03, 0.21)	(-0.23, 0.04, 0.31)	(-0.18, 0.04, 0.27)	(-0.22, 0.04, 0.31)	(-0.02, 0.23, 0.46)
$4\mathrm{x}\Delta\ln P_t^{EMU}$	(-0.18, 0.07, 0.33)	(-0.24, 0.05, 0.36)	(-0.19, 0.03, 0.24)	(-0.27, 0.01, 0.28)	(-0.18, 0.04, 0.26)	(-0.22, 0.04, 0.31)	(-0.14, 0.14, 0.39)
$4\mathrm{x}\Delta\ln P_t^{C,GER}$	(-0.25, -0.01, 0.25)	(-0.26, 0.01, 0.30)	(-0.29, -0.08, 0.15)	(-0.27, -0.01, 0.24)	(-0.23, -0.01, 0.21)	(-0.26, -0.01, 0.25)	(-0.06, 0.17, 0.40)
$4\mathrm{x}\Delta\ln P_t^{C,EMU}$	(-0.18, 0.06, 0.31)	(-0.25, 0.02, 0.33)	(-0.20, 0.01, 0.22)	(-0.29, -0.03, 0.23)	(-0.18, 0.03, 0.24)	(-0.23, 0.02, 0.28)	(-0.14, 0.12, 0.37)
$\ln R_t^{GER}$	(-0.38, -0.13, 0.17)	(-0.37, -0.09, 0.26)	(-0.34, -0.12, 0.12)	(-0.34, -0.07, 0.22)	(-0.29, -0.06, 0.18)	(-0.36, -0.09, 0.20)	(-0.32, -0.06, 0.21)
$\ln R_t^{EMU}$	(-0.35, -0.08, 0.22)	(-0.47, -0.15, 0.24)	(-0.30, -0.07, 0.17)	(-0.42, -0.14, 0.18)	(-0.26, -0.02, 0.21)	(-0.42, -0.14, 0.18)	(-0.29, -0.01, 0.28)
$\Delta\ln S_t$	(-0.03, 0.17, 0.36)	(-0.32, -0.12, 0.09)	(-0.18, 0.00, 0.17)	(-0.28, -0.07, 0.13)	(-0.24, -0.07, 0.11)	(-0.27, -0.09, 0.10)	(-0.18, 0.00, 0.18)
	$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	$\mathrm{EM}_t^{GER\_HP}$	$\mathrm{EM}_t^{EMU\_HP}$	$4\mathrm{x}\Delta\ln P_t^{GER}$	$4\mathrm{x}\Delta\ln P_t^{EMU}$	$4\mathrm{x}\Delta\ln P_t^{C,GER}$	$4\mathrm{x}\Delta\ln P_t^{C,EMU}$
$\Delta\ln(W_t^{EMU}/P_t^{EMU})$	1						
$\mathrm{EM}_t^{GER\_HP}$	(-0.18, 0.18, 0.50)	1					
$\mathrm{EM}_t^{EMU\_HP}$	(-0.20, 0.17, 0.51)	(-0.02, 0.67, 0.93)	1				
$4\mathrm{x}\Delta\ln P_t^{GER}$	(-0.10, 0.23, 0.53)	(-0.55, 0.10, 0.68)	(-0.58, 0.09, 0.68)	1			
$4\mathrm{x}\Delta\ln P_t^{EMU}$	(-0.13, 0.20, 0.49)	(-0.54, 0.12, 0.69)	(-0.58, 0.14, 0.75)	(-0.03, 0.53, 0.84)	1		
$4\mathrm{x}\Delta\ln P_t^{C,GER}$	(-0.16, 0.14, 0.44)	(-0.52, 0.08, 0.62)	(-0.53, 0.08, 0.63)	(0.71, 0.87, 0.95)	(-0.03, 0.48, 0.79)	1	
$4\mathrm{x}\Delta\ln P_t^{C,EMU}$	(-0.13, 0.18, 0.47)	(-0.50, 0.11, 0.65)	(-0.54, 0.13, 0.70)	(-0.04, 0.47, 0.80)	(0.80, 0.91, 0.97)	(0.04, 0.50, 0.80)	1
$\ln R_t^{GER}$	(-0.32, 0.01, 0.36)	(-0.63, 0.03, 0.67)	(-0.60, 0.10, 0.72)	(0.35, 0.72, 0.90)	(-0.20, 0.41, 0.79)	(0.28, 0.65, 0.86)	(-0.18, 0.36, 0.75)
$\ln R_t^{EMU}$	(-0.27, 0.05, 0.39)	(-0.59, 0.09, 0.71)	(-0.61, 0.15, 0.77)	(-0.20, 0.40, 0.79)	(0.36, 0.73, 0.91)	(-0.15, 0.39, 0.76)	(0.28, 0.66, 0.88)
$\Delta\ln S_t$	(-0.34, -0.15, 0.06)	(-0.33, -0.03, 0.26)	(-0.33, -0.02, 0.30)	(-0.08, 0.19, 0.47)	(-0.41, -0.14, 0.14)	(0.04, 0.30, 0.53)	(-0.39, -0.12, 0.15)
	$\ln R_t^{GER}$	$\ln R_t^{EMU}$	$\Delta\ln S_t$				
$\ln R_t^{GER}$	1						
$\ln R_t^{EMU}$	(0.05, 0.64, 0.90)	1					
$\Delta\ln S_t$	(-0.16, 0.12, 0.43)	(-0.36, -0.06, 0.23)	1				

Table 4.27: Comparison of unconditional second moments, log-differences (cont.)

Variable	std (data)	std (model)	first order autocorr. (data)	first order autocorr. (model)
$\hat{y}_t^{GER}$	1.56	(1.25, 1.72, 2.40)	0.92	(0.74, 0.86, 0.93)
$\hat{y}_t^{EMU}$	1.36	(0.88, 1.20, 1.65)	0.92	(0.71, 0.85, 0.92)
$\hat{c}_t^{GER}$	2.04	(2.16, 3.07, 4.43)	0.87	(0.78, 0.88, 0.94)
$\hat{c}_t^{EMU}$	1.47	(1.20, 1.79, 2.76)	0.93	(0.77, 0.89, 0.95)
$\hat{i}_t^{GER}$	4.40	(4.24, 6.42, 9.63)	0.89	(0.83, 0.92, 0.97)
$\hat{i}_t^{EMU}$	4.00	(3.10, 4.57, 6.75)	0.93	(0.79, 0.90, 0.96)
$\hat{w}_t^{GER}$	1.21	(1.39, 1.99, 2.87)	0.80	(0.74, 0.86, 0.93)
$\hat{w}_t^{EMU}$	1.09	(0.97, 1.42, 2.10)	0.92	(0.78, 0.89, 0.95)
$\widehat{em}_t^{GER}$	1.54	(1.03, 1.52, 2.24)	0.96	(0.84, 0.92, 0.96)
$\widehat{em}_t^{EMU}$	1.21	(0.55, 0.83, 1.29)	0.97	(0.86, 0.93, 0.97)
4x$\hat{\pi}_t^{GER}$	0.99	(1.17, 1.52, 2.03)	0.45	(0.40, 0.63, 0.79)
4x$\hat{\pi}_t^{EMU}$	1.06	(1.03, 1.30, 1.67)	0.45	(0.26, 0.53, 0.73)
4x$\hat{\pi}_t^{C,GER}$	1.33	(1.52, 1.82, 2.21)	0.71	(0.07, 0.33, 0.56)
4x$\hat{\pi}_t^{C,EMU}$	1.22	(1.35, 1.61, 1.96)	0.49	(0.05, 0.31, 0.56)
$\hat{r}_t^{GER}$	1.65	(1.03, 1.37, 1.81)	0.94	(0.60, 0.78, 0.88)
$\hat{r}_t^{EMU}$	1.36	(0.87, 1.15, 1.54)	0.93	(0.63, 0.79, 0.89)
$\Delta\hat{S}_t$	0.91	(1.24, 1.50 1.79)	0.27	(-0.36, -0.12, 0.14)

Table 4.28: Comparison of unconditional second moments, detrended data

	cross-correlations (data)								
	$\hat{y}_t^{GER}$	$\hat{y}_t^{EMU}$	$\hat{c}_t^{GER}$	$\hat{c}_t^{EMU}$	$\hat{i}_t^{GER}$	$\hat{i}_t^{EMU}$	$\hat{w}_t^{GER}$	$\hat{w}_t^{EMU}$	
$\hat{y}_t^{GER}$	1								
$\hat{y}_t^{EMU}$	0.81	1							
$\hat{c}_t^{GER}$	0.82	0.55	1						
$\hat{c}_t^{EMU}$	0.77	0.92	0.69	1					
$\hat{i}_t^{GER}$	0.72	0.63	0.57	0.56	1				
$\hat{i}_t^{EMU}$	0.69	0.95	0.48	0.92	0.60	1			
$\hat{w}_t^{GER}$	0.38	0.08	0.56	0.21	0.15	0.01	1		
$\hat{w}_t^{EMU}$	0.68	0.45	0.70	0.54	0.33	0.38	0.75	1	
$\widehat{em}_t^{GER}$	0.80	0.89	0.64	0.90	0.50	0.86	0.10	0.51	
$\widehat{em}_t^{EMU}$	0.74	0.90	0.56	0.91	0.48	0.90	0.04	0.49	
4x$\hat{\pi}_t^{GER}$	0.48	0.35	0.40	0.34	0.18	0.29	0.32	0.51	
4x$\hat{\pi}_t^{EMU}$	0.39	0.35	0.31	0.34	0.17	0.31	0.19	0.40	
4x$\hat{\pi}_t^{C,GER}$	0.46	0.40	0.43	0.42	0.29	0.39	0.35	0.54	
4x$\hat{\pi}_t^{C,EMU}$	0.42	0.36	0.32	0.34	0.27	0.34	0.32	0.55	
$\hat{r}_t^{GER}$	0.70	0.59	0.58	0.60	0.39	0.53	0.47	0.66	
$\hat{r}_t^{EMU}$	0.59	0.50	0.48	0.51	0.30	0.45	0.47	0.71	
$\Delta\hat{S}_t$	0.08	0.18	-0.01	0.09	0.08	0.16	-0.12	-0.06	
	$\widehat{em}_t^{GER}$	$\widehat{em}_t^{EMU}$	4x$\hat{\pi}_t^{GER}$	4x$\hat{\pi}_t^{EMU}$	4x$\hat{\pi}_t^{C,GER}$	4x$\hat{\pi}_t^{C,EMU}$	$\hat{r}_t^{GER}$	$\hat{r}_t^{EMU}$	$\Delta\hat{S}_t$
$\widehat{em}_t^{GER}$	1								
$\widehat{em}_t^{EMU}$	0.96	1							
4x$\hat{\pi}_t^{GER}$	0.44	0.42	1						
4x$\hat{\pi}_t^{EMU}$	0.41	0.43	0.60	1					
4x$\hat{\pi}_t^{C,GER}$	0.44	0.47	0.41	0.42	1				
4x$\hat{\pi}_t^{C,EMU}$	0.41	0.43	0.54	0.57	0.74	1			
$\hat{r}_t^{GER}$	0.69	0.68	0.60	0.57	0.65	0.63	1		
$\hat{r}_t^{EMU}$	0.58	0.61	0.54	0.63	0.67	0.69	0.91	1	
$\Delta\hat{S}_t$	0.12	0.09	-0.15	-0.16	0.13	-0.08	-0.05	-0.14	1

Table 4.29: Comparison of unconditional second moments, detrended data (cont.)

	cross-correlations (model)						
	$\hat{y}_t^{GER}$	$\hat{y}_t^{EMU}$	$\hat{c}_t^{GER}$	$\hat{c}_t^{EMU}$	$\hat{i}_t^{GER}$	$\hat{i}_t^{EMU}$	$\hat{w}_t^{GER}$
$\hat{y}_t^{GER}$	1						
$\hat{y}_t^{EMU}$	(0.30, 0.63, 0.82)	1					
$\hat{c}_t^{GER}$	(0.11, 0.54, 0.80)	(-0.09, 0.36, 0.68)	1				
$\hat{c}_t^{EMU}$	(-0.10, 0.38, 0.72)	(0.09, 0.54, 0.81)	(0.06, 0.54, 0.81)	1			
$\hat{i}_t^{GER}$	(-0.20, 0.27, 0.65)	(-0.17, 0.30, 0.66)	(-0.58, -0.17, 0.32)	(-0.56, -0.09, 0.43)	1		
$\hat{i}_t^{EMU}$	(-0.16, 0.31, 0.66)	(0.18, 0.57, 0.80)	(-0.51, -0.07, 0.40)	(-0.60, -0.16, 0.37)	(-0.03, 0.47, 0.78)	1	
$\hat{w}_t^{GER}$	(-0.48, -0.09, 0.32)	(-0.43, -0.06, 0.33)	(-0.40, 0.00, 0.40)	(-0.43, 0.00, 0.42)	(-0.48, -0.08, 0.34)	(-0.48, -0.07, 0.35)	1
$\hat{w}_t^{EMU}$	(-0.49, -0.10, 0.34)	(-0.53, -0.15, 0.27)	(-0.45, -0.02, 0.42)	(-0.46, -0.01, 0.46)	(-0.55, -0.11, 0.36)	(-0.58, -0.17, 0.29)	(0.05, 0.47, 0.75)
$\widehat{em}_t^{GER}$	(0.52, 0.77, 0.90)	(0.09, 0.50, 0.76)	(-0.09, 0.41, 0.75)	(-0.24, 0.29, 0.69)	(-0.32, 0.20, 0.64)	(-0.25, 0.26, 0.66)	(-0.59, -0.22, 0.22)
$\widehat{em}_t^{EMU}$	(0.10, 0.54, 0.80)	(0.42, 0.72, 0.87)	(-0.23, 0.29, 0.67)	(-0.20, 0.37, 0.75)	(-0.28, 0.26, 0.68)	(-0.06, 0.46, 0.78)	(-0.55, -0.16, 0.29)
4x$\hat{\pi}_t^{GER}$	(-0.30, 0.03, 0.35)	(-0.33, -0.01, 0.31)	(-0.26, 0.07, 0.38)	(-0.30, 0.03, 0.35)	(-0.40, -0.09, 0.24)	(-0.37, -0.05, 0.28)	(0.18, 0.49, 0.72)
4x$\hat{\pi}_t^{EMU}$	(-0.34, -0.02, 0.29)	(-0.43, -0.10, 0.21)	(-0.31, 0.01, 0.33)	(-0.32, 0.03, 0.35)	(-0.41, -0.08, 0.25)	(-0.47, -0.16, 0.16)	(-0.12, 0.21, 0.50)
4x$\hat{\pi}_t^{C,GER}$	(-0.23, 0.07, 0.35)	(-0.28, -0.02, 0.25)	(-0.21, 0.06, 0.32)	(-0.25, 0.02, 0.28)	(-0.31, -0.04, 0.22)	(-0.33, -0.06, 0.21)	(0.00, 0.28, 0.53)
4x$\hat{\pi}_t^{C,EMU}$	(-0.29, -0.01, 0.26)	(-0.34, -0.04, 0.23)	(-0.28, 0.00, 0.27)	(-0.26, 0.03, 0.30)	(-0.37, -0.09, 0.19)	(-0.42, -0.14, 0.13)	(-0.10, 0.18, 0.43)
$\hat{r}_t^{GER}$	(-0.49, -0.13, 0.25)	(-0.51, -0.19, 0.19)	(-0.49, -0.12, 0.29)	(-0.49, -0.11, 0.29)	(-0.60, -0.25, 0.18)	(-0.53, -0.16, 0.26)	(-0.21, 0.17, 0.51)
$\hat{r}_t^{EMU}$	(-0.48, -0.13, 0.26)	(-0.61, -0.30, 0.08)	(-0.48, -0.11, 0.31)	(-0.53, -0.17, 0.26)	(-0.55, -0.18, 0.25)	(-0.63, -0.30, 0.12)	(-0.26, 0.11, 0.47)
$\Delta\hat{S}_t$	(-0.08, 0.06, 0.19)	(-0.12, 0.02, 0.16)	(-0.11, 0.03, 0.17)	(-0.13, 0.01, 0.14)	(-0.09, 0.04, 0.18)	(-0.12 , 0.02, 0.16)	(-0.16, -0.02, 0.13)
	$\hat{w}_t^{EMU}$	$\widehat{em}_t^{GER}$	$\widehat{em}_t^{EMU}$	4x$\hat{\pi}_t^{GER}$	4x$\hat{\pi}_t^{EMU}$	4x$\hat{\pi}_t^{C,GER}$	4x$\hat{\pi}_t^{C,EMU}$
$\hat{w}_t^{EMU}$	1						
$\widehat{em}_t^{GER}$	(-0.57, -0.17, 0.30)	1					
$\widehat{em}_t^{EMU}$	(-0.68, -0.34, 0.11)	(0.25, 0.68, 0.89)	1				
4x$\hat{\pi}_t^{GER}$	(-0.09, 0.25, 0.54)	(-0.22, 0.17, 0.49)	(-0.31, 0.04, 0.38)	1			
4x$\hat{\pi}_t^{EMU}$	(0.14, 0.45, 0.71)	(-0.33, 0.00, 0.34)	(-0.38, -0.01, 0.34)	(0.16, 0.41, 0.61)	1		
4x$\hat{\pi}_t^{C,GER}$	(-0.07, 0.20, 0.45)	(-0.23, 0.09, 0.37)	(-0.26, 0.03, 0.31)	(0.44, 0.62, 0.75)	(0.14, 0.36, 0.55)	1	
4x$\hat{\pi}_t^{C,EMU}$	(0.07, 0.35, 0.61)	(-0.28, 0.01, 0.28)	(-0.33, -0.01, 0.28)	(0.10, 0.33, 0.53)	(0.63, 0.75, 0.85)	(0.25, 0.44, 0.60)	1
$\hat{r}_t^{GER}$	(-0.26, 0.14, 0.50)	(-0.42, 0.00, 0.41)	(-0.47, -0.08, 0.34)	(-0.09, 0.20, 0.47)	(-0.20, 0.10, 0.39)	(-0.12, 0.12, 0.36)	(-0.16, 0.09, 0.34)
$\hat{r}_t^{EMU}$	(-0.18, 0.24, 0.60)	(-0.47, -0.07, 0.35)	(-0.54, -0.15, 0.29)	(-0.18, 0.12 0.41)	(-0.13, 0.16, 0.47)	(-0.10, 0.15, 0.38)	(-0.12, 0.13, 0.40)
$\Delta\hat{S}_t$	(-0.19, -0.04, 0.10)	(-0.15, -0.01, 0.13)	(-0.11, 0.02, 0.16)	(-0.14, 0.01, 0.17)	(-0.25, -0.09, 0.08)	(0.04 , 0.22, 0.39)	(-0.24, -0.06, 0.13)
	$\hat{r}_t^{GER}$	$\hat{r}_t^{EMU}$	$\Delta\hat{S}_t$				
$\hat{r}_t^{GER}$	1						
$\hat{r}_t^{EMU}$	(0.36, 0.64, 0.82)	1					
$\Delta\hat{S}_t$	(-0.28, -0.12, 0.05)	(-0.12, 0.04, 0.19)	1				

Table 4.30: Comparison of unconditional second moments, detrended data (cont.)

	Output (Home)				Output (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(7.0, 14.0, 22.0)	(30.0, 47.3, 61.5)	(53.3, 71.6, 83.6)	(80.4, 91.2, 96.1)	(10.9, 19.4, 30.2)	(19.9, 33.3, 47.2)	(29.6, 44.9, 60.1)	(45.0, 62.4, 76.4)
asymmetric technology	(0.1, 0.3, 0.6)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(3.8, 5.6, 8.9)	(3.3, 5.1, 8.8)	(3.4, 5.5, 9.7)	(2.7, 4.5, 7.8)
stationary technology	(0.0, 0.1, 0.6)	(0.1, 0.2, 0.7)	(0.1, 0.7, 2.3)	(0.1, 0.4, 1.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.6)	(0.1, 0.9, 3.3)	(0.6, 2.1, 6.0)	(0.5, 1.6, 5.1)
preference	(6.9, 10.5, 14.5)	(4.5, 7.9, 12.7)	(1.7, 3.3, 6.3)	(0.3, 0.7, 1.6)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.1, 1.0)	(0.0, 0.1, 1.7)	(0.0, 0.2, 2.1)	(0.0, 0.4, 2.2)
labor supply	(0.1, 1.0, 3.1)	(0.2, 0.6, 1.7)	(0.5, 2.2, 6.1)	(0.4, 1.9, 6.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
labor supply*	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.5)	(0.0, 0.2, 1.3)	(0.0, 0.3, 2.4)	(0.1, 1.8, 7.8)	(0.7, 6.3, 18.2)
investment	(14.0, 18.7, 24.3)	(7.3, 11.8, 17.9)	(3.0, 5.3, 9.2)	(0.6, 1.2, 2.3)	(0.8, 1.4, 2.4)	(0.3, 0.5, 1.0)	(0.1, 0.2, 0.5)	(0.0, 0.1, 0.1)
investment*	(1.1, 2.0, 3.4)	(0.5, 1.1, 2.2)	(0.2, 0.5, 1.0)	(0.0, 0.1, 0.2)	(7.0, 9.8, 13.3)	(2.7, 4.3, 7.7)	(1.2, 2.1, 4.1)	(0.3, 0.6, 1.3)
gov. spending	(14.0, 19.3, 27.3)	(4.3, 7.3, 11.6)	(1.7, 3.2, 5.7)	(0.4, 0.8, 1.6)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
gov. spending*	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(4.5, 7.9, 12.4)	(0.9, 2.0, 3.6)	(0.5, 1.0, 2.0)	(0.3, 0.6, 1.1)
Phillips Curve	(0.3, 0.6, 1.1)	(0.5, 1.2, 2.4)	(0.3, 0.7, 1.7)	(0.1, 0.2, 0.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.0)	(0.8, 1.4, 2.3)	(1.0, 1.7, 3.5)	(0.8, 1.2, 2.4)	(0.2, 0.4, 0.8)
CPI equation	(0.0, 0.0, 0.2)	(0.4, 0.8, 1.3)	(0.3, 0.6, 1.1)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.4, 0.5, 0.7)	(0.1, 0.1, 0.2)	(0.1, 0.1, 0.2)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.4)	(1.1, 1.7, 2.6)	(1.1, 1.6, 2.4)	(0.4, 0.6, 1.0)
inflation target	(0.0, 1.0, 4.0)	(0.0, 0.9, 4.1)	(0.0, 0.5, 2.3)	(0.0, 0.1, 0.6)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(0.7, 1.4, 2.8)	(0.2, 0.4, 0.8)	(0.2, 0.3, 0.8)	(0.0, 0.1, 0.2)	(10.7, 17.3, 26.7)	(9.4, 15.1, 25.4)	(6.0, 10.0, 19.2)	(1.6, 3.2, 7.0)
monetary	(0.1, 1.8, 3.2)	(0.1, 1.6, 0.9)	(0.0, 0.8, 0.9)	(0.0, 0.2, 0.3)	(0.0, 0.0, 27.9)	(0.0, 0.0, 26.4)	(0.0, 0.0, 20.7)	(0.0, 0.0, 8.5)
monetary*	(1.1, 2.0, 3.2)	(0.3, 0.5, 0.9)	(0.2, 0.5, 0.9)	(0.1, 0.1, 0.3)	(15.2, 20.9, 27.9)	(12.9, 18.9, 26.4)	(8.3, 13.2, 20.7)	(2.3, 4.4, 8.5)
UIP	(7.1, 10.9, 16.5)	(2.9, 5.1, 8.6)	(1.2, 2.0, 4.1)	(0.2, 0.5, 1.0)	(0.7, 1.5, 2.6)	(0.2, 0.4, 0.7)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.1)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. preference	(4.4, 8.7, 14.2)	(3.0, 5.9, 10.8)	(1.1, 2.4, 5.0)	(0.2, 0.5, 1.3)	(0.0, 1.5, 3.8)	(0.2, 2.3, 5.1)	(0.5, 2.8, 5.6)	(1.5, 3.3, 7.1)
com. comp. labor supply	(0.0, 0.0, 0.6)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.8)	(0.0, 0.3, 1.2)	(0.0, 0.1, 1.2)	(0.0, 0.3, 2.1)	(0.1, 1.5, 7.1)	(0.3, 5.2, 16.1)
com. comp. investment	(0.0, 0.3, 2.7)	(0.0, 0.2, 2.0)	(0.0, 0.1, 1.0)	(0.0, 0.0, 0.2)	(0.0, 0.4, 3.7)	(0.0, 0.2, 1.7)	(0.0, 0.1, 0.9)	(0.0, 0.0, 0.3)
com. comp. gov. spending	(0.0, 0.7, 3.2)	(0.0, 0.2, 1.2)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.1)	(0.0, 0.5, 2.4)	(0.0, 0.1, 0.6)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)
com. comp. inflation target	(0.1, 0.8, 2.6)	(0.1, 1.8, 5.8)	(0.1, 1.1, 4.1)	(0.0, 0.3, 1.1)	(0.2, 3.2, 11.6)	(0.2, 3.3, 12.3)	(0.1, 2.3, 9.4)	(0.0, 0.7, 3.4)
com. comp. monetary	(0.1, 0.5, 1.8)	(0.2, 1.4, 4.0)	(0.1, 1.1, 3.1)	(0.0, 0.2, 0.9)	(0.4, 3.7, 7.5)	(0.5, 3.9, 8.2)	(0.3, 2.8, 6.6)	(0.1, 0.9, 2.5)
common shocks	(31.1, 38.4, 46.6)	(50.0, 63.6, 74.7)	(65.2, 80.0, 89.1)	(84.7, 93.3, 97.1)	(28.5, 39.1, 47.5)	(39.0, 51.8, 61.6)	(50.3, 62.8, 73.4)	(64.1, 79.9, 89.5)
Home economy shocks	(46.0, 54.8, 62.8)	(22.7, 33.6, 46.9)	(9.6, 18.3, 32.5)	(2.4, 6.0, 14.2)	(1.1, 1.8, 2.9)	(0.3, 0.6, 1.1)	(0.2, 0.3, 0.6)	(0.1, 0.1, 0.2)
Foreign economy shocks	(4.2, 6.6, 9.6)	(1.5, 2.6, 3.9)	(1.0, 1.6, 2.8)	(0.2, 0.5, 1.2)	(50.5, 59.2, 69.5)	(37.7, 47.5, 59.9)	(26.2, 36.9, 49.3)	(10.3, 20.0, 35.4)

Table 4.31: Variance decomposition, 5th median and 95th percentiles, free float regime

	Consumption (Home)				Consumption (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(4.7, 9.5, 16.5)	(10.3, 20.4, 31.7)	(17.3, 32.2, 46.1)	(36.7, 55.9, 68.3)	(14.2, 25.2, 38.8)	(16.0, 27.7, 41.6)	(16.5, 29.2, 42.5)	(19.8, 34.1, 48.9)
asymmetric technology	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.2)	(0.1, 0.1, 0.4)	(0.1, 0.2, 0.4)	(0.7, 1.2, 2.0)	(0.7, 1.3, 2.2)	(0.8, 1.4, 2.4)	(0.8, 1.3, 2.4)
stationary technology	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.2, 0.5, 1.3)	(0.2, 0.7, 2.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.5)	(0.3, 0.8, 2.6)	(0.5, 1.6, 5.0)	(0.3, 1.1, 5.3)
preference	(32.6, 48.1, 67.7)	(21.3, 32.4, 49.5)	(11.5, 20.1, 32.7)	(4.2, 8.0, 15.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.7, 8.2)	(0.0, 1.2, 10.7)	(0.0, 1.6, 13.1)	(0.0, 2.1, 13.6)
labor supply	(0.0, 0.1, 0.4)	(0.1, 0.3, 0.7)	(0.8, 2.0, 4.3)	(1.1, 3.3, 7.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
labor supply*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.4)	(0.0, 0.3, 1.2)	(0.0, 0.2, 2.3)	(0.1, 0.9, 7.4)	(0.4, 3.5, 15.2)	(1.0, 8.0, 27.6)
investment	(0.3, 0.7, 1.4)	(0.6, 1.3, 2.7)	(0.6, 1.3, 2.8)	(0.5, 1.0, 1.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.0, 0.3)	(0.1, 0.1, 0.3)
gov. spending	(1.2, 2.6, 4.2)	(2.6, 4.9, 7.8)	(3.8, 7.1, 11.3)	(3.5, 6.8, 11.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
gov. spending*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(2.1, 4.2, 7.7)	(2.5, 4.7, 8.5)	(2.8, 4.9, 8.7)	(2.5, 4.2, 7.4)
Phillips Curve	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.5)	(0.1, 0.2, 0.5)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.5, 0.8, 1.3)	(0.6, 1.0, 1.8)	(0.4, 0.7, 1.1)	(0.1, 0.2, 0.4)
CPI equation	(2.0, 3.1, 4.6)	(4.6, 6.5, 9.1)	(6.6, 9.1, 12.6)	(6.9, 9.7, 14.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)	(7.0, 9.6, 12.6)	(7.9, 10.8, 14.2)	(7.6, 10.1, 13.3)	(5.1, 7.4, 10.0)
inflation target	(0.0, 0.3, 1.3)	(0.0, 0.3, 1.5)	(0.0, 0.3, 1.2)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(0.0, 0.1, 0.2)	(0.1, 0.3, 0.6)	(0.1, 0.3, 0.6)	(0.0, 0.1, 0.3)	(9.6, 16.6, 26.9)	(6.5, 11.8, 20.1)	(3.7, 7.2, 13.3)	(1.3, 2.6, 5.1)
monetary	(0.0, 0.6, 0.2)	(0.0, 0.5, 0.8)	(0.0, 0.4, 0.8)	(0.0, 0.2, 0.4)	(0.0, 0.0, 26.3)	(0.0, 0.0, 19.8)	(0.0, 0.0, 13.3)	(0.0, 0.0, 5.2)
monetary*	(0.0, 0.1, 0.2)	(0.2, 0.4, 0.8)	(0.2, 0.4, 0.8)	(0.1, 0.2, 0.4)	(13.2, 19.1, 26.3)	(8.7, 13.4, 19.8)	(5.2, 8.2, 13.3)	(1.8, 3.1, 5.2)
UIP	(0.7, 1.4, 2.7)	(2.6, 4.7, 8.0)	(3.9, 6.6, 11.0)	(2.0, 3.6, 6.6)	(0.1, 0.3, 0.6)	(0.2, 0.6, 1.2)	(0.2, 0.6, 1.2)	(0.1, 0.3, 0.6)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. preference	(15.7, 29.0, 48.1)	(10.4, 19.7, 36.5)	(5.8, 12.2, 25.2)	(2.0, 5.0, 11.7)	(2.3, 8.4, 15.7)	(4.8, 12.2, 20.9)	(8.0, 16.6, 25.7)	(10.5, 20.1, 28.8)
com. comp. labor supply	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.4, 1.2)	(0.1, 0.7, 2.2)	(0.0, 0.1, 1.3)	(0.1, 0.7, 3.8)	(0.2, 3.0, 8.7)	(0.5, 6.9, 16.3)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. gov. spending	(0.0, 0.1, 0.4)	(0.0, 0.2, 0.9)	(0.0, 0.3, 1.2)	(0.0, 0.2, 1.2)	(0.0, 0.2, 1.4)	(0.0, 0.3, 1.6)	(0.0, 0.3, 1.6)	(0.0, 0.3, 1.3)
com. comp. inflation target	(0.1, 0.9, 2.9)	(0.1, 1.2, 4.1)	(0.1, 1.0, 3.3)	(0.0, 0.4, 1.4)	(0.2, 3.9, 13.9)	(0.2, 2.9, 10.6)	(0.1, 1.7, 6.8)	(0.0, 0.6, 2.5)
com. comp. monetary	(0.1, 0.8, 2.0)	(0.1, 1.1, 2.5)	(0.1, 0.9, 2.1)	(0.0, 0.4, 1.0)	(0.5, 4.0, 8.5)	(0.3, 2.9, 6.9)	(0.2, 1.8, 4.6)	(0.1, 0.7, 1.8)
common shocks	(26.7, 43.1, 60.4)	(36.7, 50.6, 64.2)	(43.4, 56.2, 67.6)	(53.6, 68.4, 77.7)	(34.2, 46.0, 56.2)	(40.1, 50.9, 61.4)	(43.4, 56.8, 67.9)	(47.9, 66.3, 79.0)
Home economy shocks	(39.4, 56.5, 73.1)	(34.9, 48.5, 62.7)	(31.1, 42.6, 55.7)	(21.2, 30.6, 45.6)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)
Foreign economy shocks	(0.1, 0.2, 0.4)	(0.4, 0.9, 1.7)	(0.5, 1.1, 2.0)	(0.3, 0.8, 2.1)	(43.6, 54.0, 65.2)	(38.5, 48.9, 59.7)	(32.0, 43.1, 56.5)	(20.9, 33.6, 52.0)

Table 4.32: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Investment (Home)				Investment (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(1.0, 2.1, 3.6)	(4.3, 9.1, 16.3)	(11.3, 22.1, 35.6)	(37.5, 55.6, 70.5)	(2.7, 5.7, 10.0)	(7.1, 15.0, 25.1)	(13.1, 26.5, 41.3)	(34.4, 54.6, 70.3)
asymmetric technology	(0.0, 0.1, 0.1)	(0.2, 0.3, 0.5)	(0.4, 0.7, 1.1)	(0.5, 1.0, 1.7)	(0.4, 0.7, 1.0)	(1.0, 1.8, 2.9)	(1.9, 3.2, 5.2)	(2.2, 3.8, 6.3)
stationary technology	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.3)
stationary technology*	(0.0, 0.0, 0.1)	(0.1, 0.3, 0.8)	(0.2, 0.7, 1.9)	(0.2, 0.5, 2.3)	(0.0, 0.1, 1.3)	(0.1, 0.5, 2.7)	(0.2, 1.3, 5.3)	(0.5, 1.8, 5.3)
preference	(0.1, 0.2, 0.4)	(0.4, 0.8, 1.7)	(0.7, 1.4, 3.1)	(0.4, 0.9, 2.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.9)	(0.0, 0.3, 2.2)	(0.0, 0.4, 3.5)	(0.0, 0.3, 3.7)
labor supply	(0.1, 0.3, 0.9)	(0.1, 0.6, 2.0)	(0.1, 0.5, 2.3)	(0.1, 0.5, 1.7)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.3)	(0.0, 0.2, 1.1)	(0.0, 0.2, 1.1)
labor supply*	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.5)	(0.0, 0.4, 2.3)	(0.2, 1.9, 6.8)	(0.0, 0.5, 4.7)	(0.0, 0.5, 7.2)	(0.1, 0.8, 5.7)	(0.4, 3.1, 12.0)
investment	(85.9, 90.4, 93.1)	(56.7, 67.2, 77.2)	(29.2, 40.8, 53.7)	(10.3, 16.7, 26.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.3)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.1, 0.1, 0.4)	(52.1, 63.5, 75.2)	(19.1, 32.2, 49.7)	(8.2, 15.6, 30.0)	(3.2, 5.7, 11.8)
gov. spending	(0.1, 0.1, 0.3)	(0.3, 0.6, 1.1)	(0.5, 1.2, 2.1)	(0.5, 1.2, 2.1)	(0.1, 0.2, 0.3)	(0.2, 0.5, 0.9)	(0.3, 0.7, 1.3)	(0.3, 0.6, 1.1)
gov. spending*	(0.0, 0.0, 0.1)	(0.1, 0.2, 0.3)	(0.2, 0.4, 0.7)	(0.2, 0.4, 0.7)	(0.0, 0.1, 0.2)	(0.0, 0.2, 0.4)	(0.0, 0.2, 0.6)	(0.0, 0.1, 0.6)
Phillips Curve	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.1)	(0.1, 0.1, 0.3)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.2)	(0.4, 0.7, 1.3)	(0.8, 1.4, 2.7)	(0.8, 1.3, 2.5)	(0.3, 0.6, 1.1)
CPI equation	(0.1, 0.1, 0.2)	(0.1, 0.3, 0.5)	(0.2, 0.3, 0.7)	(0.1, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.4)	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.3)	(0.6, 1.0, 1.6)	(1.3, 2.0, 3.2)	(1.4, 2.1, 3.3)	(0.6, 1.0, 1.6)
inflation target	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
inflation target*	(0.2, 0.4, 1.0)	(0.5, 1.1, 2.7)	(0.5, 1.3, 3.4)	(0.2, 0.6, 1.8)	(2.7, 5.5, 9.5)	(4.4, 9.4, 15.8)	(4.4, 8.5, 15.6)	(1.6, 3.3, 7.5)
monetary	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
monetary*	(0.2, 0.5, 1.1)	(0.7, 1.4, 3.3)	(0.8, 1.9, 4.5)	(0.4, 0.9, 2.6)	(4.0, 6.6, 10.8)	(7.1, 11.9, 19.2)	(6.7, 11.8, 20.0)	(2.5, 4.9, 10.9)
UIP	(2.0, 3.3, 4.7)	(7.7, 12.0, 17.4)	(13.6, 19.9, 27.7)	(7.3, 12.5, 19.4)	(1.2, 2.2, 4.7)	(2.6, 5.3, 11.5)	(3.2, 7.2, 15.4)	(1.7, 4.2, 8.9)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
com. comp. preference	(0.1, 0.2, 0.6)	(0.3, 0.9, 1.9)	(0.4, 1.3, 2.9)	(0.2, 0.7, 1.7)	(0.5, 1.4, 3.5)	(1.1, 3.3, 6.9)	(1.2, 4.6, 9.6)	(0.8, 3.3, 9.1)
com. comp. labor supply	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.4)	(0.0, 0.3, 2.0)	(0.1, 1.6, 5.7)	(0.0, 0.4, 2.3)	(0.0, 0.3, 3.4)	(0.0, 0.7, 3.0)	(0.1, 2.6, 11.2)
com. comp. investment	(0.0, 0.5, 3.4)	(0.0, 0.4, 2.8)	(0.0, 0.2, 1.9)	(0.0, 0.1, 0.7)	(0.1, 1.9, 17.3)	(0.0, 0.9, 10.1)	(0.0, 0.4, 5.6)	(0.0, 0.2, 2.2)
com. comp. gov. spending	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.2)
com. comp. inflation target	(0.0, 0.3, 1.1)	(0.1, 0.7, 2.8)	(0.0, 0.7, 3.0)	(0.0, 0.3, 1.5)	(0.1, 1.5, 5.9)	(0.2, 2.5, 9.6)	(0.1, 2.1, 8.9)	(0.1, 0.9, 4.2)
com. comp. monetary	(0.0, 0.3, 0.9)	(0.1, 0.8, 2.4)	(0.1, 1.0, 2.9)	(0.0, 0.4, 1.5)	(0.2, 1.8, 4.0)	(0.4, 3.1, 6.8)	(0.3, 3.0, 6.9)	(0.1, 1.2, 3.5)
common shocks	(5.2, 7.3, 11.0)	(17.7, 25.4, 34.4)	(36.8, 48.1, 59.7)	(62.3, 73.9, 83.4)	(12.0, 19.3, 32.3)	(26.9, 37.6, 46.0)	(41.3, 52.1, 61.7)	(60.9, 74.6, 84.6)
Home economy shocks	(87.3, 91.4, 93.9)	(59.4, 70.3, 79.4)	(33.3, 45.4, 58.2)	(12.7, 20.0, 31.3)	(0.2, 0.3, 0.6)	(0.4, 0.8, 1.7)	(0.6, 1.3, 3.0)	(0.5, 1.1, 2.6)
Foreign economy shocks	(0.7, 1.2, 2.3)	(2.1, 3.8, 7.1)	(3.1, 6.0, 10.8)	(2.0, 5.4, 12.1)	(67.1, 80.4, 87.6)	(52.8, 61.5, 72.3)	(37.2, 46.2, 57.4)	(14.6, 24.0, 36.5)

Table 4.33: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Real Wage (Home)				Real Wage (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(2.9, 5.2, 8.4)	(4.7, 8.4, 13.8)	(8.0, 14.2, 23.6)	(23.8, 39.3, 56.3)	(5.0, 10.4, 16.8)	(18.2, 31.6, 45.3)	(27.0, 45.3, 60.0)	(41.4, 62.4, 76.4)
asymmetric technology	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(41.9, 60.6, 70.7)	(15.4, 28.5, 41.1)	(7.6, 14.5, 22.9)	(3.2, 5.8, 9.9)
stationary technology	(0.3, 0.9, 1.9)	(0.3, 0.9, 1.8)	(0.2, 0.8, 1.7)	(0.1, 0.5, 1.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.1, 1.1)	(0.0, 0.2, 2.2)	(0.0, 0.3, 2.6)	(0.0, 0.3, 2.0)
preference	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.5)	(0.0, 0.3, 1.4)	(0.0, 0.3, 2.0)	(0.0, 0.4, 2.3)
labor supply	(79.5, 84.9, 89.6)	(78.2, 84.6, 90.5)	(69.9, 79.2, 87.1)	(39.2, 55.8, 70.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
labor supply*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.4, 4.2, 15.8)	(1.2, 11.8, 35.5)	(1.5, 14.0, 41.5)	(1.3, 11.9, 39.0)
investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
gov. spending	(0.0, 0.1, 0.2)	(0.1, 0.3, 0.6)	(0.2, 0.5, 1.0)	(0.3, 0.8, 1.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
gov. spending*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.1, 0.2, 0.5)	(0.1, 0.3, 0.6)	(0.1, 0.3, 0.6)
Phillips Curve	(2.6, 4.3, 6.2)	(0.5, 0.9, 1.8)	(0.2, 0.5, 1.1)	(0.1, 0.3, 0.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(12.0, 17.4, 24.4)	(4.0, 7.5, 12.9)	(1.6, 3.3, 6.4)	(0.4, 0.9, 1.9)
CPI equation	(0.1, 0.2, 0.4)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.4)	(0.0, 0.1, 0.6)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)
inflation target	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.2, 1.2)	(0.2, 0.8, 2.0)	(0.2, 0.7, 1.7)	(0.1, 0.2, 0.6)
monetary	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
monetary*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.1, 0.4, 1.3)	(0.5, 1.3, 2.9)	(0.4, 1.2, 2.6)	(0.2, 0.5, 1.2)
UIP	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. stationary tech.	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. preference	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.3, 0.8, 2.3)	(1.0, 2.2, 5.5)	(1.4, 3.0, 6.7)	(1.6, 3.3, 7.5)
com. comp. labor supply	(0.3, 3.7, 7.9)	(0.3, 3.6, 8.0)	(0.3, 3.4, 7.9)	(0.2, 2.4, 6.1)	(0.2, 3.4, 10.2)	(0.6, 10.3, 23.3)	(0.8, 12.5, 25.9)	(0.7, 10.3, 21.5)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. inflation target	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.7)	(0.0, 0.1, 0.7)	(0.0, 0.0, 0.3)
com. comp. monetary	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.4)	(0.0, 0.2, 0.8)	(0.0, 0.2, 0.7)	(0.0, 0.1, 0.4)
common shocks	(5.0, 9.1, 14.4)	(7.1, 12.6, 19.0)	(10.8, 18.6, 27.6)	(26.9, 42.2, 58.6)	(62.0, 76.1, 83.2)	(52.9, 76.0, 89.0)	(52.4, 78.3, 92.3)	(57.9, 84.7, 95.8)
Home economy shocks	(85.4, 90.9, 95.0)	(80.8, 87.2, 92.6)	(72.2, 81.3, 89.1)	(41.0, 57.5, 72.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Foreign economy shocks	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(16.8, 23.9, 37.9)	(10.9, 23.9, 46.6)	(7.7, 21.7, 47.5)	(4.1, 15.3, 42.0)

Table 4.34: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Employment (Home)				Employment (Foreign)			
Shocks	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(2.8, 5.7, 9.8)	(5.0, 10.0, 17.1)	(7.8, 15.7, 26.6)	(15.9, 30.5, 48.3)	(3.7, 7.7, 13.3)	(5.0, 10.0, 17.9)	(5.8, 11.5, 21.5)	(5.2, 11.4, 22.9)
asymmetric technology	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.1, 0.2, 0.5)	(0.1, 0.3, 0.7)
stationary technology	(66.7, 77.4, 85.5)	(60.0, 72.3, 81.8)	(49.4, 63.4, 75.6)	(27.1, 40.1, 55.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(47.8, 58.0, 67.7)	(31.9, 42.9, 59.6)	(16.9, 27.7, 50.4)	(3.9, 8.5, 27.0)
preference	(1.0, 2.2, 4.2)	(0.8, 2.0, 4.2)	(0.5, 1.2, 3.0)	(0.2, 0.6, 1.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.5, 2.9)	(0.0, 0.7, 4.6)	(0.0, 1.1, 6.4)	(0.0, 1.9, 9.3)
labor supply	(0.0, 0.2, 1.8)	(0.1, 1.3, 5.4)	(0.8, 4.8, 14.5)	(2.5, 11.1, 28.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
labor supply*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.0, 0.2, 1.0)	(0.3, 4.1, 13.5)	(0.7, 7.4, 20.8)	(1.3, 13.1, 33.2)	(3.3, 25.0, 52.8)
investment	(0.4, 1.1, 2.6)	(0.1, 0.4, 1.2)	(0.2, 0.5, 1.0)	(1.4, 2.8, 5.0)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
investment*	(0.1, 0.3, 0.7)	(0.1, 0.2, 0.6)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.2)	(0.1, 0.4, 0.9)	(0.0, 0.2, 0.7)	(0.0, 0.1, 0.3)	(0.1, 0.1, 0.3)
gov. spending	(1.5, 2.6, 4.6)	(1.1, 2.2, 3.9)	(1.0, 2.2, 4.1)	(1.1, 2.7, 5.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
gov. spending*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.5, 1.1, 2.0)	(0.5, 1.2, 2.3)	(0.5, 1.4, 2.8)	(0.6, 1.7, 3.8)
Phillips Curve	(0.2, 0.4, 0.9)	(0.2, 0.6, 1.3)	(0.2, 0.5, 1.4)	(0.1, 0.3, 0.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.3, 0.5, 1.1)	(0.3, 0.6, 1.3)	(0.2, 0.5, 1.0)	(0.1, 0.2, 0.3)
CPI equation	(0.1, 0.2, 0.4)	(0.2, 0.5, 0.8)	(0.3, 0.6, 1.0)	(0.2, 0.4, 0.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.5, 0.7, 1.1)	(0.6, 1.0, 1.5)	(0.6, 1.0, 1.5)	(0.2, 0.4, 0.7)
inflation target	(0.0, 0.3, 1.5)	(0.0, 0.4, 1.7)	(0.0, 0.3, 1.5)	(0.0, 0.2, 0.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(2.4, 4.5, 9.2)	(2.3, 4.9, 10.6)	(1.7, 3.9, 8.8)	(0.4, 1.1, 2.9)
monetary	(0.0, 0.5, 1.4)	(0.0, 0.6, 1.7)	(0.0, 0.5, 1.5)	(0.0, 0.3, 0.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
monetary*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(4.0, 6.7, 10.5)	(4.4, 7.4, 12.4)	(3.4, 6.3, 11.6)	(1.1, 2.3, 4.9)
UIP	(0.9, 1.6, 3.1)	(0.7, 1.3, 2.4)	(0.6, 1.0, 1.9)	(0.6, 1.0, 1.8)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. stationary tech.	(0.0, 1.5, 10.1)	(0.0, 1.4, 9.4)	(0.0, 1.2, 8.3)	(0.0, 0.8, 5.2)	(0.0, 0.4, 2.4)	(0.0, 0.2, 1.8)	(0.0, 0.2, 1.3)	(0.0, 0.1, 0.5)
com. comp. preference	(0.7, 1.7, 3.7)	(0.5, 1.5, 3.4)	(0.3, 0.9, 2.3)	(0.2, 0.5, 1.5)	(1.9, 4.9, 8.5)	(3.0, 7.4, 12.3)	(4.8, 10.6, 16.6)	(9.0, 17.2, 26.3)
com. comp. labor supply	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.5)	(0.0, 0.2, 1.7)	(0.1, 1.1, 4.7)	(0.2, 3.2, 12.0)	(0.3, 5.7, 20.2)	(0.5, 10.4, 32.6)	(1.0, 21.0, 51.5)
com. comp. investment	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.5)
com. comp. inflation target	(0.0, 0.6, 2.2)	(0.1, 0.8, 3.4)	(0.0, 0.7, 3.4)	(0.0, 0.4, 1.9)	(0.1, 1.0, 3.4)	(0.1, 1.1, 3.7)	(0.1, 0.9, 3.2)	(0.0, 0.2, 1.0)
com. comp. monetary	(0.1, 0.6, 1.6)	(0.1, 0.9, 2.4)	(0.1, 0.9, 2.7)	(0.0, 0.5, 1.6)	(0.1, 1.4, 3.0)	(0.2, 1.5, 3.6)	(0.2, 1.3, 3.4)	(0.1, 0.4, 1.4)
common shocks	(8.4, 13.3, 21.8)	(11.2, 17.9, 27.4)	(14.4, 22.9, 34.5)	(22.9, 36.7, 52.8)	(13.2, 20.5, 30.1)	(17.6, 28.4, 43.5)	(22.4, 37.9, 58.9)	(28.6, 52.8, 79.9)
Home economy shocks	(77.6, 86.2, 91.3)	(71.8, 81.7, 88.4)	(64.8, 76.6, 85.2)	(46.2, 62.3, 76.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)
Foreign economy shocks	(0.1, 0.3, 0.7)	(0.1, 0.3, 0.7)	(0.2, 0.5, 1.0)	(0.2, 0.7, 1.9)	(69.7, 79.4, 86.6)	(56.1, 71.4, 82.3)	(40.9, 61.9, 77.5)	(20.0, 47.0, 70.5)

Table 4.35: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	CPI Inflation (Home)				CPI Inflation (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.3, 1.1, 3.0)	(2.0, 4.9, 11.0)	(3.4, 8.1, 16.5)	(4.6, 11.9, 24.2)	(0.5, 1.5, 3.6)	(1.3, 3.7, 9.2)	(1.4, 4.4, 11.7)	(1.3, 4.4, 13.5)
asymmetric technology	(0.4, 0.7, 1.3)	(0.5, 0.9, 1.7)	(0.4, 0.7, 1.5)	(0.3, 0.8, 1.6)	(0.1, 0.1, 0.3)	(0.1, 0.1, 0.3)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.2)
stationary technology	(1.5, 2.9, 5.5)	(2.9, 6.1, 11.3)	(2.8, 6.3, 12.0)	(2.1, 5.0, 10.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.1, 0.4)	(0.1, 0.3, 0.9)	(0.1, 0.3, 0.9)	(0.1, 0.2, 0.9)	(1.9, 4.0, 8.2)	(2.7, 6.2, 12.4)	(2.1, 5.1, 11.0)	(1.2, 3.0, 7.9)
preference	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.7)	(0.0, 0.2, 1.5)	(0.0, 0.3, 1.9)	(0.0, 0.4, 2.7)
labor supply	(3.6, 8.3, 15.7)	(9.0, 20.8, 35.6)	(10.5, 24.6, 42.0)	(8.5, 21.9, 41.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
labor supply*	(0.0, 0.1, 0.8)	(0.1, 0.6, 2.4)	(0.1, 1.0, 3.5)	(0.2, 1.5, 5.7)	(0.4, 3.1, 9.9)	(1.1, 7.9, 22.2)	(1.5, 11.2, 27.8)	(1.9, 13.9, 35.0)
investment	(0.0, 0.1, 0.3)	(0.1, 0.3, 0.7)	(0.2, 0.5, 1.1)	(0.4, 0.7, 1.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.1)
gov. spending	(0.9, 1.9, 3.4)	(1.1, 2.5, 4.7)	(0.8, 2.2, 4.3)	(0.7, 2.1, 4.7)	(0.1, 0.2, 0.4)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)
gov. spending*	(0.2, 0.4, 0.7)	(0.2, 0.4, 0.8)	(0.2, 0.3, 0.6)	(0.1, 0.3, 0.6)	(0.0, 0.2, 0.6)	(0.1, 0.3, 1.1)	(0.1, 0.3, 1.3)	(0.1, 0.4, 1.7)
Phillips Curve	(9.2, 13.8, 18.8)	(5.3, 8.0, 11.6)	(3.7, 5.8, 8.4)	(2.5, 4.0, 6.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.1, 0.3, 0.5)	(0.1, 0.2, 0.5)	(0.1, 0.2, 0.4)	(0.1, 0.1, 0.3)	(13.8, 20.3, 27.8)	(6.3, 10.3, 14.8)	(4.0, 6.8, 10.1)	(2.2, 3.9, 6.0)
CPI equation	(38.3, 47.6, 57.2)	(18.3, 25.3, 35.3)	(11.7, 18.0, 26.7)	(7.5, 12.6, 19.4)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.3, 0.5, 0.8)	(0.2, 0.4, 0.7)	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.4)	(38.5, 47.7, 53.9)	(17.1, 23.5, 29.7)	(10.8, 15.4, 20.4)	(5.8, 8.7, 11.9)
inflation target	(0.0, 1.3, 5.2)	(0.0, 2.8, 10.8)	(0.0, 3.6, 13.2)	(0.1, 5.5, 19.9)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(1.4, 2.9, 5.9)	(1.0, 2.2, 4.4)	(0.7, 1.7, 3.4)	(0.5, 1.2, 2.5)	(6.3, 12.3, 19.1)	(14.4, 24.7, 36.3)	(16.9, 28.6, 42.9)	(18.5, 32.6, 49.3)
monetary	(0.0, 0.2, 0.7)	(0.0, 0.2, 0.5)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
monetary*	(2.0, 3.8, 7.4)	(1.4, 3.0, 5.6)	(0.9, 2.2, 4.2)	(0.6, 1.5, 3.0)	(0.6, 1.1, 2.0)	(0.6, 1.1, 1.9)	(0.4, 0.7, 1.4)	(0.2, 0.4, 0.8)
UIP	(5.4, 8.9, 14.2)	(4.8, 7.7, 12.4)	(3.1, 5.6, 9.2)	(2.2, 4.0, 6.8)	(0.5, 1.2, 2.6)	(0.4, 1.0, 1.9)	(0.3, 0.7, 1.3)	(0.2, 0.4, 0.8)
com. comp. stationary tech.	(0.0, 0.1, 0.5)	(0.0, 0.2, 1.1)	(0.0, 0.2, 1.1)	(0.0, 0.1, 0.9)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)
com. comp. preference	(0.0, 0.0, 0.4)	(0.0, 0.1, 0.7)	(0.0, 0.1, 0.9)	(0.0, 0.1, 1.6)	(0.1, 0.7, 3.1)	(0.4, 1.9, 7.2)	(0.6, 2.7, 9.7)	(0.9, 3.8, 13.5)
com. comp. labor supply	(0.0, 0.6, 2.6)	(0.2, 2.6, 7.2)	(0.3, 3.9, 9.7)	(0.4, 4.3, 10.9)	(0.2, 2.5, 6.6)	(0.4, 7.0, 17.3)	(0.5, 9.5, 22.7)	(0.7, 11.9, 26.9)
com. comp. investment	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)
com. comp. inflation target	(0.1, 0.7, 2.5)	(0.2, 4.2, 12.0)	(0.4, 7.4, 20.0)	(0.8, 12.4, 31.8)	(0.1, 2.2, 7.1)	(0.3, 5.5, 15.9)	(0.4, 6.7, 19.5)	(0.4, 7.6, 21.7)
com. comp. monetary	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.2)
common shocks	(9.2, 13.2, 18.4)	(15.9, 22.6, 30.5)	(18.0, 28.4, 39.8)	(22.5, 37.2, 52.7)	(5.7, 9.6, 15.1)	(12.4, 22.0, 33.9)	(15.1, 27.9, 42.1)	(17.8, 32.5, 48.7)
Home economy shocks	(68.3, 77.9, 85.2)	(59.3, 69.3, 77.4)	(52.0, 64.7, 74.8)	(39.8, 56.8, 70.5)	(0.1, 0.3, 0.6)	(0.1, 0.3, 0.5)	(0.1, 0.2, 0.4)	(0.1, 0.1, 0.2)
Foreign economy shocks	(4.7, 8.5, 14.4)	(4.2, 8.0, 13.1)	(3.3, 6.7, 11.6)	(2.5, 5.7, 11.4)	(84.5, 90.1, 94.1)	(65.9, 77.7, 87.4)	(57.6, 71.9, 84.4)	(51.0, 67.4, 81.9)

Table 4.36: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Nominal Interest Rate (Home)				Nominal Interest Rate (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.0, 0.0, 0.0)	(1.8, 4.7, 10.0)	(5.7, 13.7, 26.2)	(10.0, 24.3, 43.9)	(0.0, 0.0, 0.0)	(0.6, 1.9, 5.1)	(2.1, 6.3, 15.5)	(3.2, 9.2, 23.4)
asymmetric technology	(0.0, 0.0, 0.0)	(0.4, 0.9, 1.6)	(0.4, 0.9, 1.7)	(0.3, 0.7, 1.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
stationary technology	(0.0, 0.0, 0.0)	(1.7, 4.0, 8.0)	(2.4, 5.8, 11.8)	(1.4, 4.0, 10.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.0)	(0.1, 0.3, 0.7)	(0.1, 0.4, 1.1)	(0.1, 0.2, 0.9)	(0.0, 0.0, 0.0)	(0.9, 2.3, 5.3)	(1.5, 4.3, 9.8)	(0.7, 2.2, 7.4)
preference	(0.0, 0.0, 0.0)	(0.6, 1.2, 2.1)	(0.8, 1.8, 3.1)	(0.4, 1.0, 1.9)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.4)	(0.0, 0.1, 1.3)	(0.0, 0.2, 2.4)
labor supply	(0.0, 0.0, 0.0)	(5.5, 13.7, 25.4)	(10.2, 23.7, 40.4)	(7.7, 20.1, 39.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
labor supply*	(0.0, 0.0, 0.0)	(0.0, 0.2, 1.6)	(0.1, 1.0, 3.8)	(0.2, 1.7, 6.7)	(0.0, 0.0, 0.0)	(0.3, 2.5, 8.2)	(1.1, 8.4, 23.2)	(2.1, 14.7, 35.3)
investment	(0.0, 0.0, 0.0)	(0.6, 1.3, 2.7)	(0.5, 1.3, 3.0)	(0.3, 0.7, 1.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.1)
gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.8)	(0.0, 0.3, 1.9)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)
gov. spending*	(0.0, 0.0, 0.0)	(0.2, 0.4, 0.8)	(0.2, 0.4, 0.7)	(0.1, 0.2, 0.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.7)	(0.0, 0.2, 1.3)
Phillips Curve	(0.0, 0.0, 0.0)	(2.0, 3.6, 6.4)	(0.9, 1.7, 3.2)	(0.3, 0.7, 1.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.2, 0.3, 0.5)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.0)	(1.5, 2.8, 5.4)	(0.9, 1.7, 3.0)	(0.2, 0.6, 1.0)
CPI equation	(0.0, 0.0, 0.0)	(6.8, 10.3, 13.9)	(2.8, 4.5, 6.6)	(1.1, 1.9, 3.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.0, 0.0)	(0.3, 0.5, 0.9)	(0.2, 0.3, 0.5)	(0.1, 0.1, 0.2)	(0.0, 0.0, 0.0)	(3.6, 5.8, 8.5)	(2.3, 3.5, 5.2)	(0.7, 1.1, 1.8)
inflation target	(0.0, 0.1, 0.6)	(0.0, 1.3, 5.0)	(0.0, 2.9, 11.1)	(0.1, 5.3, 19.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
inflation target*	(0.0, 0.0, 0.0)	(1.0, 2.3, 4.4)	(0.4, 1.1, 2.4)	(0.2, 0.6, 1.3)	(0.1, 0.3, 0.6)	(2.2, 4.6, 8.0)	(9.0, 16.5, 26.2)	(15.8, 27.8, 42.8)
monetary	(2.3, 46.7, 91.4)	(0.6, 11.9, 25.7)	(0.2, 4.3, 10.2)	(0.1, 1.8, 4.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
monetary*	(0.0, 0.0, 0.0)	(1.7, 3.2, 6.0)	(0.7, 1.6, 3.5)	(0.3, 0.7, 1.6)	(69.7, 82.9, 96.9)	(48.0, 60.4, 71.8)	(21.7, 32.1, 43.8)	(6.5, 10.0, 15.5)
UIP	(0.0, 0.0, 0.0)	(8.0, 12.3, 18.5)	(5.8, 9.8, 16.0)	(2.9, 5.1, 9.4)	(0.0, 0.0, 0.0)	(0.3, 0.6, 1.3)	(0.3, 0.6, 1.2)	(0.1, 0.2, 0.5)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.8)	(0.0, 0.1, 1.1)	(0.0, 0.1, 0.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
com. comp. preference	(0.0, 0.0, 0.0)	(0.2, 1.0, 2.6)	(0.2, 1.0, 2.6)	(0.2, 0.7, 1.5)	(0.0, 0.0, 0.0)	(0.0, 0.4, 1.8)	(0.1, 1.3, 6.2)	(0.2, 2.7, 12.4)
com. comp. labor supply	(0.0, 0.0, 0.0)	(0.1, 1.4, 5.3)	(0.3, 3.7, 10.5)	(0.4, 4.6, 12.1)	(0.0, 0.0, 0.0)	(0.1, 2.3, 6.7)	(0.4, 7.5, 19.4)	(0.7, 12.8, 28.5)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
com. comp. inflation target	(0.0, 0.2, 1.1)	(0.1, 1.1, 3.7)	(0.3, 5.3, 15.4)	(0.8, 12.5, 32.4)	(0.0, 0.1, 0.2)	(0.0, 0.9, 2.8)	(0.2, 3.7, 11.2)	(0.4, 6.6, 18.9)
com. comp. monetary	(7.5, 52.6, 97.3)	(2.3, 17.1, 38.7)	(0.9, 6.6, 17.6)	(0.4, 2.7, 7.1)	(2.3, 16.5, 29.9)	(1.7, 12.5, 23.3)	(0.8, 6.8, 13.6)	(0.2, 2.2, 4.7)
common shocks	(8.5, 53.2, 97.6)	(27.6, 41.0, 60.0)	(32.6, 44.8, 58.5)	(36.8, 55.4, 71.1)	(2.6, 16.5, 30.1)	(9.6, 20.1, 30.4)	(19.7, 28.9, 42.5)	(22.9, 37.9, 56.2)
Home economy shocks	(2.3, 46.8, 91.5)	(31.5, 50.6, 65.4)	(34.5, 49.4, 61.8)	(22.9, 40.6, 58.7)	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.3)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.1)
Foreign economy shocks	(0.0, 0.0, 0.0)	(4.6, 7.7, 12.5)	(2.8, 5.5, 10.2)	(1.6, 4.0, 10.0)	(69.9, 83.5, 97.3)	(69.4, 79.7, 90.2)	(57.2, 70.9, 80.1)	(43.5, 61.9, 76.9)

Table 4.37: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Real Exchange Rate				Nominal Exchange Rate			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.0, 0.2, 1.1)	(0.0, 0.4, 1.6)	(0.0, 0.5, 2.1)	(0.0, 0.5, 2.2)	(0.0, 0.2, 1.2)	(0.0, 0.2, 1.5)	(0.0, 0.2, 1.5)	(0.0, 0.4, 2.3)
asymmetric technology	(1.1, 1.9, 3.3)	(1.4, 2.3, 3.9)	(1.8, 2.9, 5.0)	(2.5, 4.4, 7.5)	(1.3, 2.2, 3.9)	(1.6, 2.7, 4.8)	(1.8, 3.2, 5.6)	(1.4, 2.9, 5.5)
stationary technology	(0.0, 0.2, 0.5)	(0.1, 0.2, 0.4)	(0.1, 0.3, 0.9)	(0.1, 0.3, 1.1)	(0.2, 0.5, 1.1)	(0.3, 0.7, 1.6)	(0.6, 1.4, 3.0)	(1.1, 2.8, 6.5)
stationary technology*	(0.0, 0.1, 0.5)	(0.1, 0.7, 2.3)	(0.4, 1.6, 4.2)	(0.6, 2.0, 5.5)	(0.0, 0.0, 0.4)	(0.0, 0.1, 0.5)	(0.0, 0.2, 0.9)	(0.2, 0.7, 2.7)
preference	(0.0, 0.2, 0.4)	(0.1, 0.2, 0.5)	(0.0, 0.2, 0.5)	(0.0, 0.1, 0.3)	(0.0, 0.2, 0.4)	(0.0, 0.2, 0.5)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)
preference*	(0.0, 0.0, 0.5)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.4)	(0.0, 0.1, 0.9)	(0.0, 0.0, 0.7)	(0.0, 0.1, 0.9)	(0.0, 0.1, 1.2)	(0.0, 0.2, 1.9)
labor supply	(0.2, 0.9, 2.8)	(0.2, 0.6, 1.6)	(0.4, 1.2, 2.9)	(0.3, 1.3, 4.1)	(0.5, 2.0, 4.9)	(0.8, 2.8, 7.0)	(2.1, 5.6, 13.0)	(4.8, 12.5, 27.7)
labor supply*	(0.0, 0.2, 2.0)	(0.0, 0.4, 1.8)	(0.2, 1.3, 5.1)	(0.7, 6.7, 18.0)	(0.0, 0.4, 3.2)	(0.0, 0.6, 4.1)	(0.0, 0.9, 5.7)	(0.1, 2.9, 12.3)
investment	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.5)	(0.1, 0.3, 0.8)	(0.2, 0.5, 1.1)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.1, 0.2, 0.5)
investment*	(0.2, 0.5, 1.5)	(0.2, 0.7, 2.0)	(0.3, 0.7, 2.2)	(0.2, 0.6, 1.7)	(0.2, 0.5, 1.6)	(0.2, 0.6, 1.8)	(0.1, 0.4, 1.5)	(0.0, 0.1, 0.4)
gov. spending	(2.3, 4.0, 6.5)	(2.7, 4.6, 7.5)	(3.0, 5.0, 7.7)	(2.9, 5.2, 7.9)	(2.6, 4.5, 7.4)	(3.2, 5.5, 9.0)	(3.2, 5.8, 9.6)	(2.2, 4.5, 8.0)
gov. spending*	(0.7, 1.3, 2.1)	(0.8, 1.4, 2.2)	(0.9, 1.6, 2.5)	(0.9, 1.8, 2.8)	(0.8, 1.5, 2.6)	(1.0, 1.9, 3.3)	(1.2, 2.2, 4.0)	(1.1, 2.3, 4.4)
Phillips Curve	(0.0, 0.1, 0.3)	(0.2, 0.3, 0.8)	(0.1, 0.3, 0.6)	(0.1, 0.2, 0.4)	(0.0, 0.1, 0.2)	(0.1, 0.2, 0.4)	(0.3, 0.5, 0.8)	(0.3, 0.4, 0.8)
Phillips Curve*	(0.5, 0.8, 1.2)	(0.6, 1.1, 2.0)	(0.6, 1.0, 1.8)	(0.4, 0.7, 1.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.1, 0.1, 0.3)
CPI equation	(4.2, 6.2, 8.8)	(6.0, 8.4, 11.6)	(6.5, 9.3, 12.6)	(7.8, 11.0, 15.1)	(0.6, 1.1, 1.7)	(0.5, 0.9, 1.5)	(0.3, 0.5, 0.9)	(0.1, 0.1, 0.2)
CPI equation*	(6.1, 8.4, 11.6)	(8.6, 11.6, 15.3)	(10.2, 13.4, 17.0)	(11.8, 15.6, 19.0)	(1.3, 2.1, 3.1)	(1.3, 2.1, 3.1)	(1.0, 1.5, 2.3)	(0.3, 0.5, 0.9)
inflation target	(0.0, 0.7, 2.8)	(0.0, 0.5, 2.1)	(0.0, 0.3, 1.6)	(0.0, 0.2, 1.0)	(0.0, 0.9, 4.0)	(0.0, 1.2, 5.0)	(0.0, 1.8, 7.3)	(0.1, 3.6, 14.1)
inflation target*	(8.5, 14.7, 22.4)	(7.0, 12.4, 19.8)	(5.5, 10.3, 17.4)	(3.5, 6.9, 11.8)	(11.7, 20.0, 28.6)	(15.1, 25.0, 34.7)	(19.5, 31.8, 42.4)	(25.0, 42.2, 57.7)
monetary	(0.1, 1.1, 2.6)	(0.0, 0.8, 1.8)	(0.0, 0.6, 1.3)	(0.0, 0.4, 0.8)	(0.1, 1.2, 2.8)	(0.0, 0.7, 1.7)	(0.0, 0.4, 1.0)	(0.0, 0.1, 0.3)
monetary*	(13.1, 18.0, 24.6)	(10.7, 15.4, 21.9)	(8.7, 13.3, 19.8)	(5.5, 9.1, 14.8)	(13.9, 19.1, 26.4)	(10.7, 15.4, 22.6)	(7.1, 11.1, 17.7)	(1.9, 3.5, 7.3)
UIP	(27.6, 35.3, 44.4)	(25.2, 32.4, 40.9)	(22.5, 29.1, 36.9)	(14.6, 20.5, 26.6)	(29.0, 37.4, 48.3)	(24.0, 32.0, 42.3)	(17.3, 23.8, 32.9)	(5.3, 8.2, 12.7)
com. comp. stationary tech.	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)
com. comp. preference	(0.0, 0.1, 1.2)	(0.0, 0.2, 1.2)	(0.0, 0.3, 1.8)	(0.0, 0.7, 5.2)	(0.0, 0.2, 1.9)	(0.0, 0.3, 2.4)	(0.0, 0.7, 3.6)	(0.2, 2.3, 7.1)
com. comp. labor supply	(0.0, 0.1, 1.2)	(0.0, 0.2, 1.5)	(0.0, 0.7, 3.7)	(0.2, 4.3, 14.9)	(0.0, 0.1, 1.5)	(0.0, 0.2, 1.9)	(0.0, 0.2, 2.6)	(0.0, 0.7, 5.1)
com. comp. investment	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
com. comp. inflation target	(0.0, 0.3, 2.1)	(0.0, 0.4, 2.6)	(0.0, 0.5, 2.9)	(0.0, 0.4, 2.2)	(0.0, 0.4, 2.5)	(0.0, 0.5, 3.1)	(0.0, 0.5, 2.8)	(0.0, 0.1, 0.9)
com. comp. monetary	(0.0, 0.5, 2.0)	(0.1, 0.6, 2.4)	(0.1, 0.7, 2.6)	(0.0, 0.6, 2.1)	(0.0, 0.6, 2.1)	(0.1, 0.7, 2.6)	(0.1, 0.7, 2.4)	(0.0, 0.2, 1.1)
common shocks	(32.2, 39.8, 47.9)	(31.0, 38.0, 45.7)	(29.6, 36.5, 44.2)	(25.3, 33.5, 44.2)	(34.6, 42.6, 52.4)	(30.5, 38.4, 47.9)	(24.0, 31.4, 40.3)	(11.8, 16.9, 23.9)
Home economy shocks	(10.1, 14.1, 19.4)	(11.9, 16.3, 21.8)	(13.2, 17.9, 23.7)	(14.4, 19.6, 26.5)	(7.3, 11.2, 16.7)	(8.5, 13.3, 19.7)	(11.0, 17.4, 26.9)	(16.1, 26.6, 43.2)
Foreign economy shocks	(37.4, 45.5, 53.9)	(37.5, 45.6, 53.1)	(37.3, 45.1, 52.9)	(35.2, 46.1, 56.5)	(35.7, 45.1, 54.9)	(37.1, 47.1, 57.1)	(38.6, 50.2, 60.0)	(39.4, 55.8, 68.1)

Table 4.38: Variance decomposition, 5th median and 95th percentiles, free float regime (cont.)

	Output (Home)				Output (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(6.5, 11.9, 22.2)	(15.4, 28.0, 44.7)	(25.2, 42.3, 60.0)	(49.3, 67.8, 81.0)	(11.9, 23.1, 37.5)	(19.5, 36.2, 55.7)	(26.2, 45.4, 67.1)	(41.0, 59.6, 79.3)
asymmetric technology	(0.2, 0.4, 0.7)	(0.4, 0.8, 1.5)	(0.5, 1.1, 1.9)	(0.4, 0.7, 1.4)	(2.6, 4.2, 6.6)	(2.0, 3.6, 6.3)	(2.2, 3.8, 6.6)	(2.0, 3.3, 6.0)
stationary technology	(0.0, 0.1, 0.2)	(0.1, 0.5, 1.3)	(0.5, 1.3, 3.1)	(0.7, 1.7, 3.9)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
stationary technology*	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.7)	(0.0, 0.1, 1.1)	(0.0, 0.1, 0.8)	(0.0, 0.1, 1.5)	(0.2, 1.2, 5.3)	(0.7, 2.6, 8.7)	(0.6, 2.0, 8.7)
preference	(7.1, 10.3, 15.2)	(4.1, 7.1, 11.9)	(1.9, 3.5, 6.8)	(0.6, 1.1, 2.3)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.5)	(0.0, 0.1, 1.1)	(0.0, 0.1, 1.3)	(0.0, 0.0, 0.8)	(0.0, 0.2, 2.7)	(0.0, 0.4, 3.4)	(0.0, 0.5, 3.7)	(0.0, 0.6, 3.4)
labor supply	(0.0, 0.1, 0.5)	(0.2, 1.2, 3.3)	(1.2, 4.0, 9.5)	(2.5, 7.0, 14.9)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.5)	(0.0, 0.0, 0.5)	(0.0, 0.1, 0.3)
labor supply*	(0.0, 0.1, 1.7)	(0.0, 0.3, 3.3)	(0.0, 0.5, 4.2)	(0.0, 0.5, 3.7)	(0.0, 0.2, 2.8)	(0.0, 0.6, 5.0)	(0.1, 2.1, 9.4)	(0.7, 6.8, 19.3)
investment	(10.9, 16.0, 21.7)	(5.6, 9.5, 14.1)	(2.9, 5.2, 8.5)	(0.9, 1.8, 3.4)	(0.9, 1.8, 3.7)	(0.2, 0.6, 1.6)	(0.1, 0.3, 0.7)	(0.0, 0.1, 0.2)
investment*	(1.2, 2.5, 4.6)	(0.5, 1.2, 3.0)	(0.2, 0.5, 1.5)	(0.1, 0.2, 0.6)	(6.1, 11.5, 16.2)	(2.2, 4.7, 8.6)	(1.1, 2.1, 4.7)	(0.3, 0.6, 1.5)
gov. spending	(21.4, 30.2, 41.1)	(8.6, 14.3, 21.8)	(5.3, 9.0, 14.8)	(2.2, 4.0, 7.0)	(0.1, 0.2, 0.4)	(0.1, 0.3, 0.6)	(0.1, 0.3, 0.7)	(0.1, 0.2, 0.4)
gov. spending*	(0.2, 0.5, 1.1)	(0.4, 1.0, 2.1)	(0.4, 1.1, 2.2)	(0.3, 0.6, 1.3)	(7.1, 13.2, 21.7)	(1.4, 3.6, 6.6)	(0.8, 2.0, 3.7)	(0.5, 1.1, 1.8)
Phillips Curve	(0.6, 1.0, 1.4)	(0.8, 1.4, 2.5)	(0.7, 1.3, 2.4)	(0.3, 0.6, 1.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(1.1, 1.8, 3.2)	(1.2, 2.1, 3.9)	(0.8, 1.5, 3.0)	(0.3, 0.5, 1.1)
CPI equation	(0.4, 0.8, 1.3)	(0.1, 0.2, 0.3)	(0.1, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.1, 0.2, 0.4)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.1)
CPI equation*	(0.1, 0.3, 0.7)	(0.4, 1.1, 1.9)	(0.4, 1.0, 1.8)	(0.2, 0.4, 0.8)	(0.0, 0.1, 0.4)	(0.3, 1.0, 1.9)	(0.4, 1.1, 1.8)	(0.2, 0.4, 0.8)
inflation target	(0.0, 0.5, 9.0)	(0.0, 0.8, 13.8)	(0.0, 0.6, 11.6)	(0.0, 0.2, 4.5)	(0.0, 0.9, 17.2)	(0.0, 1.0, 19.1)	(0.0, 0.7, 14.3)	(0.0, 0.2, 5.0)
monetary	(7.5, 14.8, 25.8)	(11.9, 23.2, 37.5)	(8.5, 19.4, 33.7)	(2.7, 7.2, 15.9)	(17.2, 30.5, 43.6)	(16.6, 31.6, 47.3)	(10.7, 23.0, 38.0)	(3.1, 7.8, 15.8)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
com. comp. preference	(0.5, 3.9, 9.8)	(0.3, 1.8, 7.8)	(0.5, 1.1, 4.4)	(0.3, 0.7, 2.6)	(0.5, 3.8, 8.5)	(0.6, 4.5, 10.1)	(0.9, 4.9, 10.4)	(1.9, 4.7, 10.5)
com. comp. labor supply	(0.0, 0.1, 1.3)	(0.0, 0.3, 2.6)	(0.1, 0.7, 4.5)	(0.1, 1.0, 5.6)	(0.0, 0.2, 2.3)	(0.0, 0.5, 3.5)	(0.1, 1.4, 8.1)	(0.3, 4.7, 17.4)
com. comp. investment	(0.0, 0.3, 3.1)	(0.0, 0.2, 1.8)	(0.0, 0.1, 0.9)	(0.0, 0.0, 0.3)	(0.0, 0.5, 3.8)	(0.0, 0.2, 1.8)	(0.0, 0.1, 0.9)	(0.0, 0.0, 0.3)
com. comp. gov. spending	(0.0, 0.6, 2.7)	(0.0, 0.2, 0.9)	(0.0, 0.1, 0.4)	(0.0, 0.0, 0.2)	(0.0, 0.6, 3.0)	(0.0, 0.1, 0.7)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.2)
common shocks	(26.2, 36.2, 47.3)	(47.6, 59.8, 72.3)	(56.6, 69.6, 81.2)	(67.3, 80.2, 89.0)	(56.6, 68.5, 78.8)	(71.5, 83.4, 90.7)	(71.3, 85.3, 93.3)	(68.8, 86.1, 95.3)
Home economy shocks	(47.1, 59.7, 69.9)	(22.9, 35.4, 46.8)	(14.8, 26.2, 37.7)	(9.3, 17.4, 28.1)	(1.3, 2.3, 4.3)	(0.8, 1.3, 2.5)	(0.6, 1.0, 1.8)	(0.3, 0.5, 0.9)
Foreign economy shocks	(2.5, 4.0, 7.2)	(2.8, 4.3, 8.5)	(2.3, 3.9, 8.9)	(1.2, 2.3, 6.7)	(19.2, 28.8, 40.0)	(8.3, 15.1, 26.5)	(6.0, 13.5, 27.1)	(4.2, 13.3, 30.5)

Table 4.39: Variance decomposition, 5th median and 95th percentiles, monetary union

	Consumption (Home)				Consumption (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(4.6, 9.4, 19.9)	(9.4, 20.3, 35.0)	(16.3, 31.9, 48.3)	(31.8, 51.3, 65.6)	(13.8, 27.9, 45.9)	(15.0, 28.9, 46.6)	(15.6, 29.2, 47.8)	(18.9, 32.8, 49.9)
asymmetric technology	(0.1, 0.2, 0.5)	(0.2, 0.5, 0.9)	(0.3, 0.6, 1.3)	(0.2, 0.5, 1.0)	(0.3, 0.7, 1.4)	(0.4, 0.8, 1.5)	(0.5, 0.9, 1.7)	(0.5, 1.0, 1.8)
stationary technology	(0.0, 0.0, 0.0)	(0.0, 0.2, 0.4)	(0.2, 0.6, 1.5)	(0.5, 1.2, 2.8)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.8)	(0.0, 0.1, 1.5)	(0.0, 0.1, 1.1)	(0.0, 0.1, 1.4)	(0.2, 0.9, 3.6)	(0.5, 1.7, 6.2)	(0.4, 1.3, 6.9)
preference	(35.2, 50.6, 71.5)	(24.3, 36.9, 55.0)	(13.9, 23.6, 39.5)	(5.3, 10.0, 19.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.1, 1.3)	(0.0, 0.2, 2.4)	(0.0, 0.3, 3.3)	(0.0, 0.2, 3.5)	(0.0, 1.5, 12.2)	(0.0, 1.9, 14.0)	(0.0, 2.3, 15.7)	(0.0, 2.6, 16.7)
labor supply	(0.0, 0.0, 0.1)	(0.1, 0.4, 1.0)	(0.7, 2.0, 4.4)	(1.9, 5.3, 10.9)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)
labor supply*	(0.0, 0.1, 1.3)	(0.0, 0.3, 2.7)	(0.0, 0.8, 5.4)	(0.1, 1.1, 7.2)	(0.0, 0.4, 5.5)	(0.1, 1.2, 9.3)	(0.2, 2.8, 16.4)	(0.9, 6.7, 26.1)
investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.3, 0.5, 0.9)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.1, 0.1, 0.2)
gov. spending	(0.1, 0.2, 0.3)	(0.2, 0.4, 0.8)	(0.5, 1.0, 1.8)	(0.8, 1.7, 3.5)	(0.1, 0.2, 0.5)	(0.1, 0.3, 0.5)	(0.1, 0.2, 0.5)	(0.1, 0.1, 0.3)
gov. spending*	(0.2, 0.5, 1.0)	(0.4, 0.9, 1.8)	(0.6, 1.2, 2.2)	(0.5, 1.0, 1.7)	(1.0, 2.9, 5.3)	(1.3, 3.3, 5.9)	(1.5, 3.5, 6.2)	(1.5, 3.0, 5.4)
Phillips Curve	(0.1, 0.2, 0.3)	(0.3, 0.5, 0.9)	(0.4, 0.6, 1.2)	(0.2, 0.4, 0.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.5, 0.9, 1.7)	(0.7, 1.1, 2.1)	(0.5, 0.8, 1.5)	(0.2, 0.3, 0.6)
CPI equation	(1.0, 1.5, 2.3)	(2.3, 3.4, 5.1)	(3.7, 5.6, 8.3)	(5.2, 7.4, 10.8)	(0.1, 0.2, 0.4)	(0.1, 0.2, 0.3)	(0.1, 0.1, 0.2)	(0.0, 0.0, 0.1)
CPI equation*	(0.2, 0.5, 0.9)	(0.3, 0.9, 1.5)	(0.3, 0.8, 1.4)	(0.2, 0.4, 0.7)	(4.3, 8.2, 12.6)	(5.4, 9.0, 13.3)	(5.6, 8.4, 11.7)	(4.6, 6.4, 8.7)
inflation target	(0.0, 0.4, 6.9)	(0.0, 0.6, 9.5)	(0.0, 0.5, 9.3)	(0.0, 0.3, 4.6)	(0.0, 1.2, 17.9)	(0.0, 0.8, 14.5)	(0.0, 0.5, 9.5)	(0.0, 0.2, 3.7)
monetary	(5.4, 11.0, 21.4)	(7.8, 15.7, 28.0)	(7.1, 14.5, 26.3)	(3.0, 7.0, 13.6)	(18.2, 34.0, 50.4)	(12.6, 24.4, 38.3)	(7.1, 15.1, 25.7)	(2.4, 5.5, 10.7)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. preference	(8.3, 19.1, 43.6)	(3.2, 10.3, 31.4)	(2.2, 5.8, 20.8)	(2.3, 4.1, 9.4)	(5.2, 14.3, 25.3)	(8.1, 18.2, 30.1)	(11.5, 23.0, 34.1)	(14.1, 24.8, 36.0)
com. comp. labor supply	(0.0, 0.1, 1.0)	(0.0, 0.3, 2.5)	(0.1, 0.9, 4.9)	(0.2, 1.9, 6.7)	(0.0, 0.3, 3.2)	(0.1, 0.9, 5.1)	(0.3, 2.5, 8.6)	(0.5, 6.2, 14.7)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.6)	(0.0, 0.2, 1.0)	(0.0, 0.2, 1.1)	(0.0, 0.2, 1.4)	(0.0, 0.3, 1.6)	(0.0, 0.3, 1.6)	(0.0, 0.2, 1.2)
common shocks	(26.0, 45.5, 61.4)	(36.0, 54.2, 67.3)	(44.0, 60.2, 72.3)	(52.7, 67.3, 78.4)	(70.6, 83.2, 88.9)	(65.1, 78.8, 86.7)	(58.9, 76.1, 84.9)	(54.9, 75.1, 86.1)
Home economy shocks	(37.1, 52.7, 72.8)	(29.3, 42.4, 59.9)	(23.7, 34.2, 51.2)	(18.3, 27.6, 41.2)	(0.3, 0.6, 1.0)	(0.4, 0.6, 1.1)	(0.3, 0.5, 0.9)	(0.2, 0.3, 0.5)
Foreign economy shocks	(0.8, 1.5, 3.6)	(1.7, 3.1, 7.1)	(2.3, 4.3, 9.6)	(1.8, 3.8, 10.8)	(10.4, 16.1, 27.9)	(12.7, 20.3, 33.7)	(14.6, 23.3, 40.6)	(13.6, 24.6, 44.8)

Table 4.40: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

	Investment (Home)				Investment (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.7, 2.0, 4.5)	(2.9, 7.3, 16.8)	(6.8, 16.1, 32.5)	(21.2, 37.8, 57.9)	(2.3, 6.1, 12.0)	(6.1, 15.8, 28.9)	(11.4, 27.3, 46.4)	(27.5, 53.2, 76.6)
asymmetric technology	(0.1, 0.2, 0.4)	(0.4, 0.8, 1.4)	(0.8, 1.6, 2.8)	(0.8, 1.7, 3.0)	(0.2, 0.5, 0.8)	(0.6, 1.3, 2.4)	(1.1, 2.2, 4.3)	(1.8, 3.1, 5.2)
stationary technology	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.4)	(0.1, 0.4, 1.4)	(0.5, 1.3, 3.4)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.6)	(0.0, 0.1, 0.4)
stationary technology*	(0.0, 0.0, 0.5)	(0.0, 0.1, 2.3)	(0.0, 0.2, 4.6)	(0.0, 0.2, 4.3)	(0.0, 0.1, 1.2)	(0.1, 0.5, 4.0)	(0.3, 1.9, 8.6)	(0.8, 3.1, 11.3)
preference	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.2)
preference*	(0.0, 0.0, 0.7)	(0.0, 0.2, 2.8)	(0.0, 0.3, 5.2)	(0.0, 0.3, 5.6)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.9)	(0.0, 0.0, 1.0)	(0.0, 0.0, 0.7)
labor supply	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.7)	(0.2, 0.9, 3.5)	(1.7, 5.4, 12.5)	(0.0, 0.1, 0.5)	(0.0, 0.3, 1.6)	(0.1, 0.6, 2.8)	(0.1, 0.5, 2.3)
labor supply*	(0.0, 0.1, 1.2)	(0.0, 0.7, 5.5)	(0.1, 2.0, 13.0)	(0.1, 2.9, 20.3)	(0.0, 0.6, 6.1)	(0.0, 0.8, 10.2)	(0.1, 1.0, 9.0)	(0.4, 4.1, 14.9)
investment	(83.6, 89.9, 93.9)	(56.0, 68.5, 80.7)	(31.1, 43.3, 60.4)	(11.3, 18.3, 29.6)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.6)	(0.0, 0.1, 0.3)
investment*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.2)	(44.7, 64.2, 76.9)	(17.8, 33.5, 54.7)	(8.7, 16.9, 34.5)	(3.4, 6.3, 14.2)
gov. spending	(0.1, 0.1, 0.2)	(0.1, 0.3, 0.9)	(0.2, 0.6, 1.5)	(0.1, 0.5, 1.5)	(0.1, 0.3, 0.5)	(0.4, 0.8, 1.7)	(0.6, 1.4, 2.7)	(0.7, 1.3, 2.4)
gov. spending*	(0.2, 0.3, 0.7)	(0.6, 1.3, 2.3)	(1.1, 2.4, 4.4)	(1.4, 2.7, 4.8)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.4)	(0.0, 0.1, 0.6)	(0.0, 0.1, 0.9)
Phillips Curve	(0.1, 0.2, 0.3)	(0.3, 0.5, 1.1)	(0.5, 0.9, 1.8)	(0.3, 0.6, 1.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.5, 0.9, 2.0)	(0.9, 1.8, 4.2)	(0.9, 1.9, 4.8)	(0.5, 0.9, 2.2)
CPI equation	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.1, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.1, 0.1, 0.3)	(0.1, 0.3, 0.5)	(0.1, 0.3, 0.5)	(0.1, 0.1, 0.3)
CPI equation*	(0.1, 0.2, 0.4)	(0.2, 0.6, 1.1)	(0.3, 0.8, 1.4)	(0.1, 0.4, 0.8)	(0.4, 0.9, 1.6)	(0.7, 1.7, 3.1)	(0.7, 1.9, 3.4)	(0.4, 0.9, 1.7)
inflation target	(0.0, 0.1, 2.0)	(0.0, 0.3, 4.7)	(0.0, 0.3, 5.1)	(0.0, 0.2, 2.9)	(0.0, 0.5, 11.1)	(0.0, 0.8, 16.7)	(0.0, 0.8, 16.2)	(0.0, 0.3, 9.2)
monetary	(1.5, 3.9, 7.8)	(3.9, 9.8, 19.7)	(4.7, 12.0, 25.0)	(2.3, 6.0, 14.5)	(7.6, 16.2, 27.9)	(13.7, 29.2, 46.7)	(13.3, 29.7, 48.4)	(5.3, 13.8, 27.7)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.3)
com. comp. preference	(0.1, 0.7, 2.2)	(0.1, 2.8, 7.9)	(0.2, 5.7, 15.2)	(0.2, 6.9, 17.0)	(0.0, 0.3, 2.5)	(0.0, 0.6, 4.5)	(0.0, 0.6, 5.2)	(0.0, 0.6, 3.4)
com. comp. labor supply	(0.0, 0.1, 0.8)	(0.0, 0.5, 3.5)	(0.1, 1.6, 9.6)	(0.2, 3.6, 15.8)	(0.0, 0.3, 3.7)	(0.0, 0.5, 5.2)	(0.0, 0.8, 5.4)	(0.1, 3.1, 15.8)
com. comp. investment	(0.0, 0.4, 2.7)	(0.0, 0.3, 1.8)	(0.0, 0.2, 1.2)	(0.0, 0.1, 0.5)	(0.1, 2.0, 16.9)	(0.0, 1.0, 10.4)	(0.0, 0.5, 6.4)	(0.0, 0.2, 2.7)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.4)	(0.0, 0.1, 0.4)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)
common shocks	(4.6, 8.5, 14.4)	(13.7, 25.3, 38.3)	(25.2, 42.9, 59.6)	(40.8, 61.2, 76.1)	(19.5, 31.2, 48.8)	(37.8, 56.6, 71.2)	(49.8, 69.6, 81.8)	(60.9, 79.9, 90.0)
Home economy shocks	(84.2, 90.4, 94.2)	(57.4, 69.9, 82.1)	(33.3, 46.9, 64.2)	(17.4, 27.7, 40.0)	(0.4, 0.7, 1.3)	(1.0, 1.9, 3.5)	(1.5, 3.0, 5.8)	(1.2, 2.6, 5.0)
Foreign economy shocks	(0.5, 1.0, 2.6)	(1.9, 3.7, 9.8)	(3.6, 7.3, 21.1)	(4.1, 8.6, 27.3)	(49.8, 67.9, 79.4)	(27.4, 41.2, 60.4)	(16.4, 27.1, 46.0)	(8.2, 17.4, 35.0)

Table 4.41: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

	Real Wage (Home)				Real Wage (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(3.2, 6.2, 10.4)	(5.6, 10.3, 17.5)	(9.6, 17.4, 28.8)	(28.2, 43.6, 59.5)	(5.4, 10.6, 17.3)	(19.4, 32.8, 46.0)	(28.2, 46.3, 60.3)	(44.1, 64.0, 77.1)
asymmetric technology	(0.1, 0.1, 0.2)	(0.1, 0.2, 0.4)	(0.1, 0.3, 0.6)	(0.2, 0.4, 0.8)	(40.9, 59.9, 70.7)	(14.0, 27.6, 41.3)	(6.8, 13.4, 22.4)	(2.8, 5.3, 9.2)
stationary technology	(0.2, 0.6, 1.5)	(0.1, 0.5, 1.3)	(0.1, 0.3, 0.9)	(0.2, 0.4, 0.8)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology*	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.5)	(0.0, 0.1, 1.1)	(0.0, 0.3, 2.5)	(0.0, 0.4, 3.3)	(0.1, 0.5, 3.0)
preference	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.5)	(0.0, 0.1, 0.7)	(0.0, 0.1, 1.1)	(0.0, 0.0, 0.4)	(0.0, 0.1, 1.1)	(0.0, 0.2, 1.4)	(0.0, 0.2, 1.5)
labor supply	(74.2, 80.9, 86.1)	(69.5, 78.7, 85.8)	(58.6, 71.8, 81.1)	(31.1, 46.1, 60.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
labor supply*	(0.0, 0.1, 0.8)	(0.0, 0.1, 1.2)	(0.0, 0.2, 1.9)	(0.0, 0.3, 2.9)	(0.4, 4.0, 16.5)	(1.2, 11.6, 36.8)	(1.5, 13.9, 42.6)	(1.3, 11.1, 36.4)
investment	(0.1, 0.1, 0.2)	(0.1, 0.1, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.2)
gov. spending	(0.1, 0.1, 0.2)	(0.1, 0.2, 0.3)	(0.1, 0.2, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.1)
gov. spending*	(0.1, 0.1, 0.3)	(0.1, 0.2, 0.5)	(0.2, 0.4, 0.7)	(0.2, 0.5, 0.9)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)
Phillips Curve	(3.3, 5.1, 7.3)	(0.9, 1.5, 2.8)	(0.6, 1.1, 2.2)	(0.4, 0.8, 1.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(12.5, 18.0, 24.9)	(4.4, 8.0, 13.9)	(1.8, 3.7, 7.3)	(0.5, 1.0, 2.4)
CPI equation	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
CPI equation*	(0.1, 0.1, 0.2)	(0.1, 0.2, 0.3)	(0.1, 0.2, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.2, 0.8)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)
inflation target	(0.0, 0.0, 1.4)	(0.0, 0.1, 1.6)	(0.0, 0.1, 1.8)	(0.0, 0.1, 1.3)	(0.0, 0.0, 0.8)	(0.0, 0.0, 1.2)	(0.0, 0.0, 0.9)	(0.0, 0.0, 0.4)
monetary	(0.9, 2.2, 4.3)	(1.1, 2.9, 6.1)	(1.2, 3.2, 7.6)	(0.8, 2.5, 6.6)	(0.1, 0.8, 2.4)	(0.4, 1.9, 5.9)	(0.4, 1.8, 6.1)	(0.2, 0.9, 3.5)
com. comp. stationary tech.	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. preference	(0.0, 0.1, 0.6)	(0.0, 0.2, 1.0)	(0.0, 0.5, 1.7)	(0.0, 1.0, 2.8)	(0.1, 0.4, 1.8)	(0.5, 1.3, 4.5)	(0.7, 1.8, 5.5)	(0.8, 2.0, 5.9)
com. comp. labor supply	(0.2, 2.7, 7.2)	(0.1, 2.5, 7.1)	(0.1, 2.0, 6.5)	(0.1, 1.2, 4.8)	(0.2, 3.5, 10.5)	(0.6, 10.6, 22.6)	(0.8, 12.9, 26.0)	(0.7, 10.2, 21.8)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
common shocks	(7.0, 12.2, 18.4)	(10.3, 17.2, 26.4)	(15.4, 24.8, 38.0)	(34.7, 50.1, 65.3)	(62.6, 76.3, 83.5)	(54.1, 77.9, 90.4)	(52.3, 80.6, 93.6)	(59.2, 85.9, 96.1)
Home economy shocks	(80.8, 87.3, 92.1)	(72.0, 81.9, 88.3)	(60.9, 73.9, 82.9)	(32.6, 48.0, 62.5)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.1, 0.1, 0.3)	(0.1, 0.1, 0.3)
Foreign economy shocks	(0.3, 0.5, 1.4)	(0.4, 0.7, 2.1)	(0.6, 1.1, 3.2)	(0.7, 1.4, 4.7)	(16.4, 23.5, 37.1)	(9.4, 21.9, 45.1)	(6.2, 19.2, 47.7)	(3.8, 13.9, 39.9)

Table 4.42: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

	Employment (Home)				Employment (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(2.4, 5.9, 15.0)	(3.5, 8.7, 19.8)	(4.2, 10.9, 25.6)	(5.6, 14.8, 32.9)	(3.0, 7.8, 17.4)	(3.9, 10.0, 23.2)	(4.2, 11.2, 27.1)	(3.6, 9.9, 25.6)
asymmetric technology	(0.4, 0.7, 1.4)	(0.6, 1.2, 2.3)	(0.9, 1.7, 3.2)	(1.0, 1.9, 3.8)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)
stationary technology	(33.3, 47.0, 61.0)	(22.1, 34.6, 48.9)	(12.7, 23.8, 36.9)	(6.3, 11.8, 20.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)
stationary technology*	(0.0, 0.0, 0.7)	(0.0, 0.1, 1.0)	(0.0, 0.1, 1.6)	(0.0, 0.3, 1.6)	(40.4, 53.2, 66.5)	(23.9, 37.9, 55.6)	(11.4, 22.8, 42.9)	(2.3, 6.3, 18.8)
preference	(1.9, 3.6, 6.3)	(1.6, 3.1, 6.5)	(1.0, 2.1, 4.9)	(0.5, 1.0, 2.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.0, 0.8)	(0.0, 0.1, 1.3)	(0.0, 0.1, 1.7)	(0.0, 0.1, 1.7)	(0.0, 0.7, 4.4)	(0.0, 1.0, 6.4)	(0.0, 1.4, 8.1)	(0.0, 2.1, 10.3)
labor supply	(0.6, 2.4, 6.0)	(1.4, 5.2, 12.0)	(3.4, 10.9, 24.5)	(10.4, 26.6, 46.2)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.5)	(0.0, 0.1, 0.5)
labor supply*	(0.0, 0.2, 3.3)	(0.0, 0.4, 4.6)	(0.0, 0.7, 6.5)	(0.0, 1.0, 8.3)	(0.2, 4.8, 16.8)	(0.4, 8.0, 24.3)	(1.0, 13.0, 34.4)	(3.3, 26.3, 53.3)
investment	(1.0, 2.3, 4.3)	(0.4, 1.4, 3.1)	(0.2, 0.6, 1.6)	(0.5, 1.0, 1.7)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)
investment*	(0.2, 0.5, 1.6)	(0.1, 0.4, 1.4)	(0.0, 0.2, 1.0)	(0.0, 0.1, 0.6)	(0.1, 0.3, 1.1)	(0.0, 0.2, 0.9)	(0.0, 0.1, 0.5)	(0.1, 0.1, 0.3)
gov. spending	(6.2, 9.9, 15.0)	(5.9, 10.2, 15.1)	(5.8, 10.4, 16.3)	(5.3, 10.3, 17.8)	(0.1, 0.2, 0.4)	(0.1, 0.3, 0.6)	(0.1, 0.3, 0.7)	(0.1, 0.2, 0.5)
gov. spending*	(0.3, 0.7, 1.6)	(0.4, 1.1, 2.4)	(0.6, 1.4, 3.2)	(0.5, 1.4, 3.0)	(0.9, 1.9, 3.1)	(0.9, 2.0, 3.4)	(1.1, 2.3, 3.8)	(1.1, 2.4, 4.3)
Phillips Curve	(0.6, 1.0, 1.8)	(0.8, 1.4, 2.6)	(0.9, 1.6, 3.0)	(0.8, 1.2, 2.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
Phillips Curve*	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.3)	(0.4, 0.7, 1.4)	(0.4, 0.8, 1.7)	(0.3, 0.7, 1.5)	(0.1, 0.3, 0.5)
CPI equation	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)
CPI equation*	(0.3, 0.7, 1.3)	(0.4, 1.0, 1.8)	(0.4, 1.1, 2.0)	(0.2, 0.7, 1.4)	(0.1, 0.4, 0.8)	(0.3, 0.6, 1.1)	(0.3, 0.6, 1.1)	(0.1, 0.3, 0.5)
inflation target	(0.0, 0.5, 8.3)	(0.0, 0.6, 10.2)	(0.0, 0.6, 10.0)	(0.0, 0.4, 6.3)	(0.0, 0.3, 8.0)	(0.0, 0.3, 9.8)	(0.0, 0.2, 8.5)	(0.0, 0.1, 2.6)
monetary	(6.7, 14.7, 27.7)	(9.1, 19.5, 35.4)	(8.6, 20.4, 37.5)	(4.7, 12.9, 26.9)	(4.3, 11.0, 21.7)	(4.6, 12.6, 25.8)	(3.6, 11.1, 24.7)	(1.1, 3.8, 10.6)
com. comp. stationary tech.	(0.0, 0.9, 5.9)	(0.0, 0.7, 4.4)	(0.0, 0.4, 3.1)	(0.0, 0.2, 1.6)	(0.0, 0.3, 2.2)	(0.0, 0.2, 1.7)	(0.0, 0.1, 1.2)	(0.0, 0.0, 0.5)
com. comp. preference	(0.0, 0.7, 4.1)	(0.0, 0.6, 4.2)	(0.1, 0.7, 4.2)	(0.1, 0.7, 4.2)	(2.6, 7.1, 13.0)	(3.9, 10.2, 17.2)	(6.0, 13.6, 21.7)	(10.2, 19.5, 28.9)
com. comp. labor supply	(0.0, 0.3, 3.0)	(0.0, 0.6, 5.1)	(0.1, 1.2, 9.1)	(0.2, 2.4, 14.3)	(0.1, 3.2, 13.6)	(0.1, 5.7, 21.0)	(0.3, 10.0, 31.1)	(1.0, 21.6, 49.5)
com. comp. investment	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
com. comp. gov. spending	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.4)	(0.0, 0.1, 0.5)
common shocks	(17.4, 28.5, 42.3)	(23.4, 37.5, 52.5)	(26.1, 42.0, 57.7)	(24.2, 39.7, 57.8)	(23.1, 34.4, 49.1)	(28.0, 45.1, 63.5)	(31.9, 53.4, 74.9)	(33.1, 58.0, 85.0)
Home economy shocks	(53.0, 68.2, 79.6)	(41.6, 57.9, 72.5)	(35.6, 52.5, 67.3)	(36.4, 54.7, 69.7)	(0.3, 0.5, 0.9)	(0.3, 0.6, 1.2)	(0.3, 0.6, 1.5)	(0.2, 0.4, 1.2)
Foreign economy shocks	(1.8, 2.9, 7.1)	(2.3, 3.9, 9.7)	(2.6, 4.6, 12.2)	(2.4, 4.6, 13.4)	(50.3, 64.8, 76.4)	(35.5, 54.0, 71.2)	(24.4, 45.8, 67.7)	(14.7, 41.5, 66.5)

Table 4.43: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

	CPI Inflation (Home)				CPI Inflation (Foreign)			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.7, 1.7, 3.9)	(2.9, 6.9, 14.1)	(4.3, 10.1, 21.0)	(4.6, 11.9, 25.7)	(0.5, 1.4, 3.6)	(2.0, 5.1, 11.7)	(3.0, 7.3, 16.5)	(3.1, 8.8, 21.0)
asymmetric technology	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.5)	(0.1, 0.3, 0.7)	(0.0, 0.3, 0.7)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
stationary technology	(0.6, 1.3, 2.4)	(1.6, 3.1, 5.9)	(1.3, 2.8, 5.7)	(0.8, 1.9, 4.2)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)
stationary technology*	(0.1, 0.4, 0.8)	(0.5, 1.3, 3.3)	(0.5, 1.5, 4.7)	(0.5, 1.5, 6.9)	(2.0, 4.2, 8.4)	(3.8, 8.5, 16.5)	(2.9, 7.5, 16.8)	(1.9, 5.3, 14.6)
preference	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
preference*	(0.0, 0.1, 0.5)	(0.0, 0.2, 1.7)	(0.0, 0.3, 2.6)	(0.0, 0.4, 3.6)	(0.0, 0.0, 0.4)	(0.0, 0.1, 1.5)	(0.0, 0.2, 2.2)	(0.0, 0.3, 3.6)
labor supply	(1.1, 2.6, 4.6)	(3.9, 8.7, 15.1)	(3.9, 9.5, 17.4)	(2.7, 6.9, 13.7)	(0.0, 0.1, 0.2)	(0.1, 0.3, 0.8)	(0.2, 0.5, 1.1)	(0.2, 0.5, 1.2)
labor supply*	(0.0, 0.5, 3.6)	(0.1, 2.5, 12.5)	(0.1, 4.3, 19.4)	(0.7, 9.1, 33.0)	(0.2, 2.1, 9.3)	(0.8, 7.7, 26.9)	(1.2, 11.7, 35.7)	(2.0, 15.3, 44.3)
investment	(0.0, 0.0, 0.0)	(0.1, 0.1, 0.2)	(0.1, 0.2, 0.3)	(0.1, 0.3, 0.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.2)
gov. spending	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.4)	(0.0, 0.2, 0.5)	(0.0, 0.2, 0.4)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)
gov. spending*	(0.0, 0.1, 0.4)	(0.1, 0.5, 1.3)	(0.2, 0.8, 1.8)	(0.2, 0.8, 2.2)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.8)	(0.0, 0.2, 1.4)
Phillips Curve	(9.5, 15.2, 20.8)	(5.4, 10.2, 15.2)	(3.4, 7.8, 12.8)	(2.0, 5.4, 10.5)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.1, 0.1)	(0.0, 0.0, 0.1)
Phillips Curve*	(0.2, 0.4, 0.8)	(0.3, 0.6, 1.1)	(0.2, 0.5, 0.9)	(0.2, 0.4, 0.7)	(17.2, 24.1, 31.7)	(9.5, 16.5, 23.8)	(6.2, 12.3, 20.0)	(3.6, 8.2, 15.9)
CPI equation	(62.0, 72.6, 81.6)	(25.0, 47.7, 60.0)	(14.5, 35.9, 50.1)	(8.6, 24.8, 40.5)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
CPI equation*	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.3)	(48.5, 62.6, 69.5)	(22.2, 41.9, 53.0)	(14.0, 31.9, 44.5)	(8.0, 21.5, 36.2)
inflation target	(0.0, 0.6, 8.7)	(0.0, 2.5, 25.1)	(0.0, 3.8, 32.3)	(0.0, 5.6, 40.3)	(0.0, 0.5, 7.6)	(0.0, 2.0, 22.0)	(0.0, 3.0, 28.5)	(0.0, 4.4, 34.9)
monetary	(0.1, 0.4, 1.4)	(0.4, 1.3, 5.0)	(0.3, 1.4, 6.3)	(0.2, 1.0, 5.5)	(0.0, 0.2, 1.1)	(0.1, 0.6, 3.8)	(0.1, 0.6, 4.6)	(0.1, 0.5, 3.8)
com. comp. stationary tech.	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.8)	(0.0, 0.1, 0.7)	(0.0, 0.1, 0.6)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.5)	(0.0, 0.1, 0.5)	(0.0, 0.0, 0.4)
com. comp. preference	(0.0, 0.4, 1.8)	(0.2, 1.7, 6.8)	(0.4, 2.7, 10.6)	(0.7, 4.0, 16.7)	(0.0, 0.3, 2.6)	(0.1, 1.0, 7.8)	(0.1, 1.7, 11.4)	(0.4, 3.0, 17.2)
com. comp. labor supply	(0.0, 0.9, 3.7)	(0.1, 3.8, 13.4)	(0.1, 5.6, 19.0)	(0.3, 8.9, 26.8)	(0.1, 1.8, 5.7)	(0.3, 7.1, 19.1)	(0.5, 10.7, 28.2)	(0.7, 14.5, 37.5)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)
common shocks	(2.4, 5.3, 14.7)	(11.4, 21.5, 43.5)	(17.5, 31.8, 55.1)	(24.3, 42.3, 66.2)	(2.1, 5.7, 14.6)	(8.7, 20.6, 42.0)	(13.4, 30.5, 54.9)	(19.6, 42.2, 65.8)
Home economy shocks	(79.2, 92.8, 96.4)	(40.5, 71.5, 84.2)	(27.2, 58.1, 76.0)	(16.2, 40.2, 62.9)	(0.1, 0.2, 0.4)	(0.2, 0.6, 1.1)	(0.3, 0.7, 1.4)	(0.3, 0.7, 1.7)
Foreign economy shocks	(0.8, 1.8, 4.8)	(2.1, 5.9, 17.0)	(2.5, 8.6, 25.1)	(3.2, 13.6, 39.1)	(84.9, 94.1, 97.7)	(56.3, 78.8, 90.8)	(43.9, 68.5, 86.1)	(32.7, 56.9, 79.5)

Table 4.44: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

	Nominal Interest Rate				Real Exchange Rate			
Shocks/Horizon	1Q	4Q	8Q	20Q	1Q	4Q	8Q	20Q
unit root technology	(0.0, 0.0, 0.0)	(0.6, 1.4, 4.1)	(3.0, 7.2, 16.4)	(7.1, 17.0, 35.4)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.3)	(0.0, 0.1, 0.9)	(0.0, 0.2, 1.7)
asymmetric technology	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.3)	(0.3, 0.5, 0.9)	(0.7, 1.4, 2.6)
stationary technology	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.2)	(0.1, 0.3, 0.7)	(0.1, 0.2, 0.6)	(0.2, 0.4, 0.9)	(1.3, 2.6, 4.5)	(2.2, 4.3, 7.5)	(1.6, 3.6, 7.5)
stationary technology*	(0.0, 0.0, 0.0)	(0.4, 1.2, 3.1)	(0.9, 3.1, 8.6)	(0.5, 2.5, 11.3)	(0.6, 1.4, 3.0)	(2.5, 5.6, 11.8)	(3.3, 7.8, 15.5)	(2.2, 5.5, 15.3)
preference	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
preference*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.3)	(0.0, 0.0, 1.0)	(0.0, 0.1, 2.6)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.2)	(0.0, 0.0, 0.6)	(0.0, 0.1, 1.0)
labor supply	(0.0, 0.0, 0.0)	(0.1, 0.3, 0.7)	(0.3, 1.0, 2.2)	(0.3, 1.0, 2.4)	(0.4, 0.8, 1.5)	(2.9, 6.3, 10.9)	(6.1, 13.2, 23.0)	(6.0, 14.9, 28.6)
labor supply*	(0.0, 0.0, 0.0)	(0.1, 1.1, 4.8)	(0.3, 5.0, 18.6)	(1.2, 12.1, 36.6)	(0.0, 0.4, 1.4)	(0.3, 2.7, 7.7)	(0.8, 6.8, 16.9)	(2.1, 16.3, 36.3)
investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.4)	(0.4, 0.8, 1.4)
investment*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.2)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.1, 0.1)	(0.1, 0.2, 0.4)
gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.2, 0.3, 0.6)	(0.4, 0.8, 1.3)	(0.6, 1.3, 2.0)
gov. spending*	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.4)	(0.0, 0.2, 1.2)	(0.0, 0.0, 0.1)	(0.1, 0.2, 0.4)	(0.2, 0.5, 0.8)	(0.4, 0.9, 1.6)
Phillips Curve	(0.0, 0.0, 0.0)	(0.1, 0.2, 0.4)	(0.1, 0.2, 0.4)	(0.0, 0.1, 0.2)	(4.6, 6.9, 9.1)	(4.1, 6.3, 10.0)	(2.4, 3.8, 6.4)	(0.8, 1.4, 2.4)
Phillips Curve*	(0.0, 0.0, 0.0)	(0.8, 1.6, 3.7)	(0.6, 1.3, 3.0)	(0.2, 0.6, 1.8)	(8.1, 11.5, 15.9)	(5.2, 8.2, 12.8)	(2.7, 4.6, 7.6)	(0.9, 1.6, 2.8)
CPI equation	(0.0, 0.0, 0.0)	(0.2, 0.5, 1.0)	(0.2, 0.4, 0.8)	(0.1, 0.2, 0.4)	(30.4, 37.7, 45.5)	(23.5, 30.6, 39.7)	(17.3, 23.9, 33.7)	(11.5, 17.2, 26.4)
CPI equation*	(0.0, 0.0, 0.0)	(1.5, 3.1, 6.5)	(1.1, 2.7, 5.4)	(0.3, 1.3, 2.9)	(33.0, 40.1, 46.0)	(26.1, 32.9, 40.0)	(19.5, 26.3, 34.1)	(13.2, 19.7, 26.7)
inflation target	(0.0, 0.0, 0.2)	(0.0, 0.1, 3.8)	(0.0, 1.0, 18.7)	(0.0, 3.2, 34.8)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)
monetary	(99.8, 100.0, 100.0)	(76.5, 87.3, 93.3)	(40.2, 65.9, 81.0)	(12.4, 31.1, 55.7)	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.5)	(0.0, 0.2, 1.2)	(0.0, 0.2, 1.2)
com. comp. stationary tech.	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.4)	(0.0, 0.0, 0.3)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.2)
com. comp. preference	(0.0, 0.0, 0.0)	(0.0, 0.1, 1.1)	(0.0, 0.4, 4.8)	(0.0, 1.5, 12.8)	(0.0, 0.0, 0.2)	(0.0, 0.1, 0.9)	(0.0, 0.3, 1.9)	(0.0, 0.6, 4.4)
com. comp. labor supply	(0.0, 0.0, 0.0)	(0.0, 1.0, 3.7)	(0.2, 4.7, 14.2)	(0.5, 11.3, 30.6)	(0.0, 0.1, 0.5)	(0.0, 0.8, 2.4)	(0.1, 2.3, 6.7)	(0.3, 7.9, 21.0)
com. comp. investment	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
com. comp. gov. spending	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
common shocks	(100.0, 100.0, 100.0)	(83.0, 91.0, 95.5)	(68.1, 83.8, 92.5)	(51.9, 79.0, 91.8)	(0.1, 0.3, 0.6)	(0.7, 1.6, 3.2)	(1.7, 4.1, 8.0)	(4.2, 11.5, 24.1)
Home economy shocks	(0.0, 0.0, 0.0)	(0.6, 1.1, 2.2)	(1.0, 2.0, 3.8)	(0.8, 1.7, 3.5)	(39.1, 46.1, 52.7)	(39.7, 47.2, 54.1)	(38.1, 47.6, 57.1)	(27.6, 40.9, 54.2)
Foreign economy shocks	(0.0, 0.0, 0.0)	(3.8, 7.8, 15.0)	(5.9, 13.7, 29.6)	(6.4, 19.1, 46.5)	(47.1, 53.6, 60.4)	(43.8, 51.1, 58.7)	(38.1, 48.1, 57.4)	(30.6, 46.2, 64.1)

Table 4.45: Variance decomposition, 5th median and 95th percentiles, monetary union (cont.)

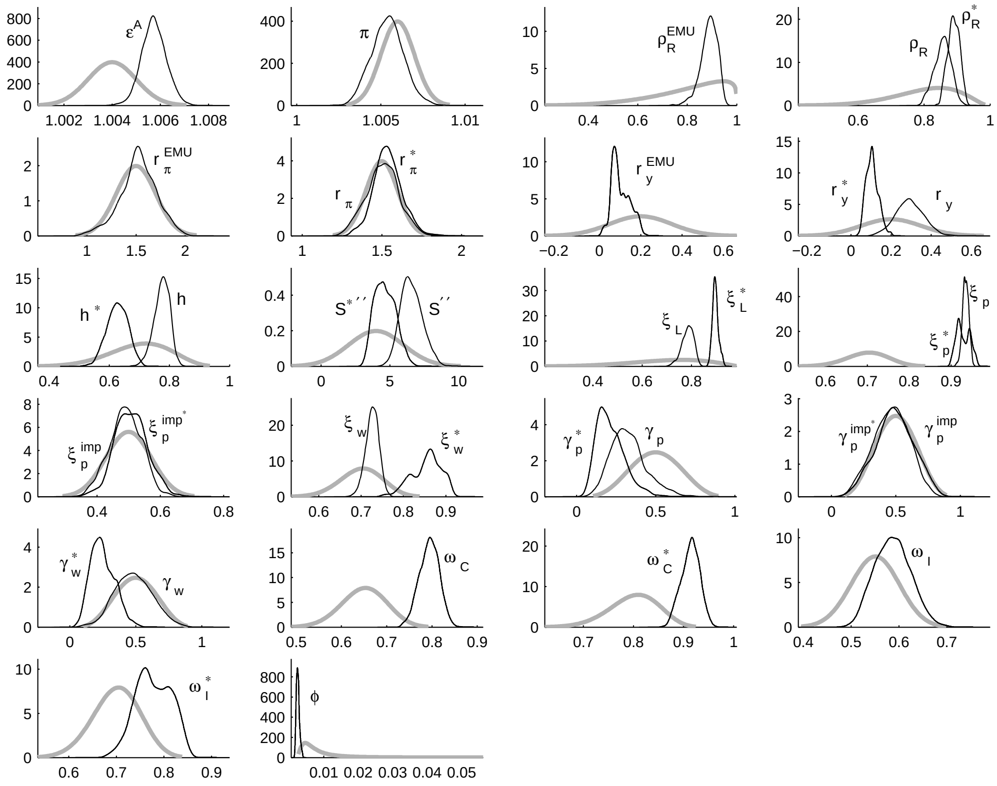

Figure 4.2: Posterior and prior distributions, structural parameters

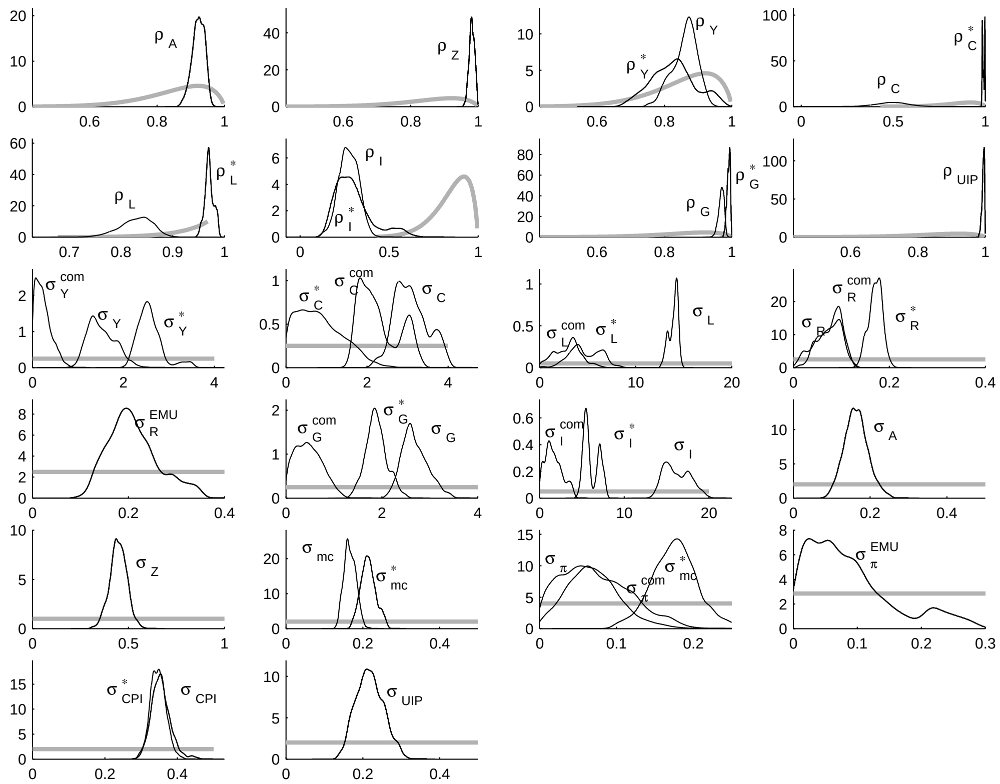

Figure 4.3: Posterior and prior distributions, structural shocks

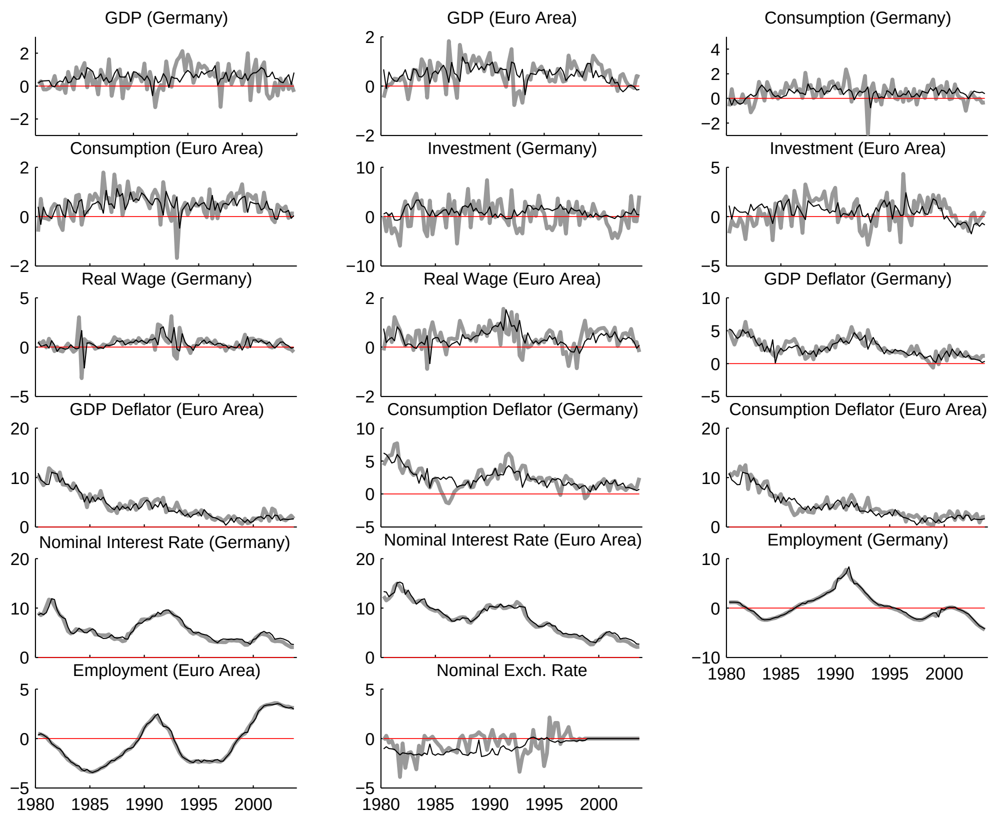

Figure 4.4: Observed data (thick) and one-step forecasts (thin), model estimated with log-differences

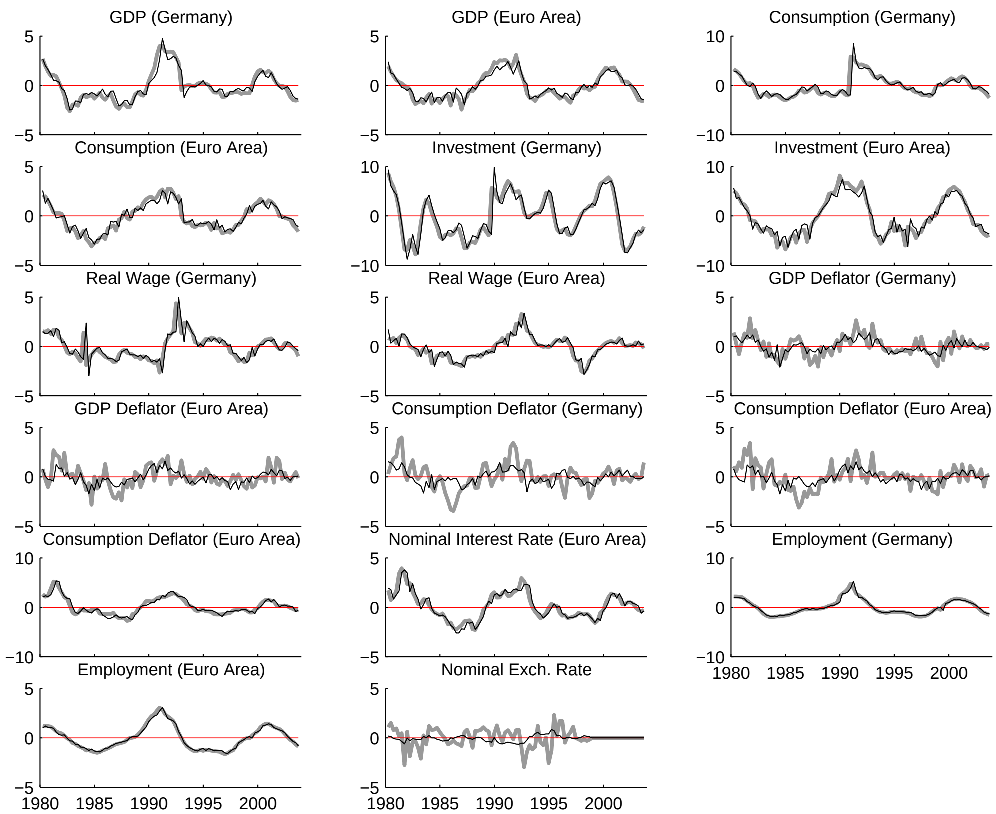

Figure 4.5: Observed data (thick) and one-step forecasts (thin), model estimated with detrended data

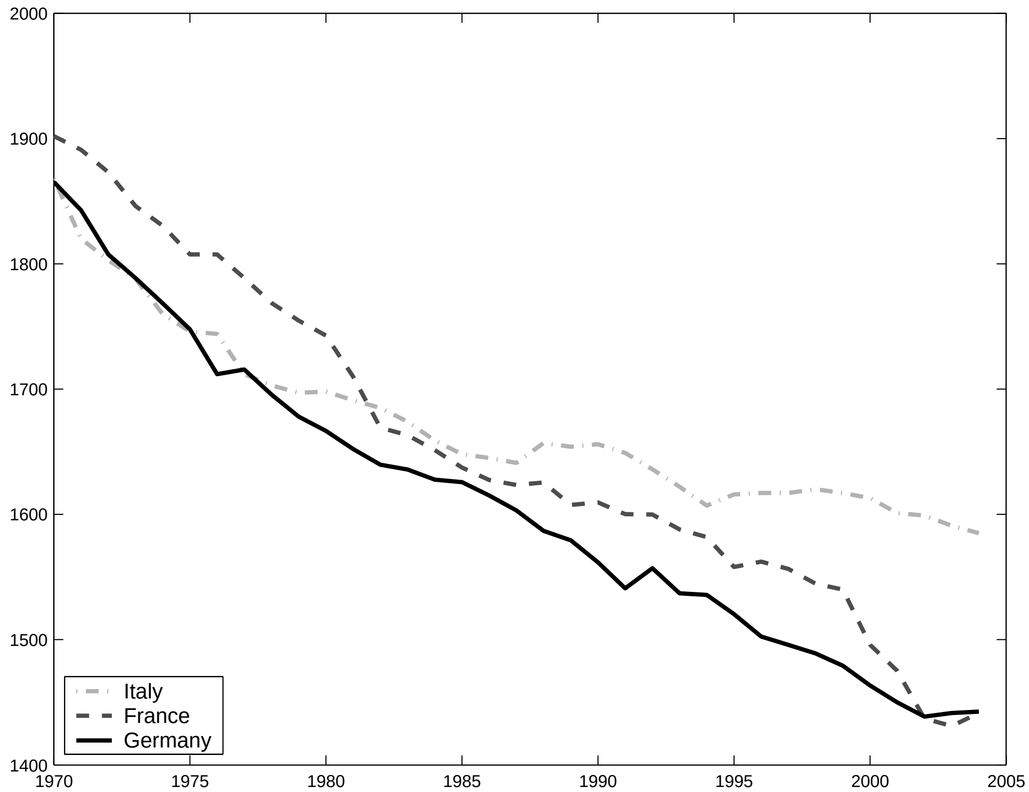

Figure 4.6: Annual hours worked per full time employed

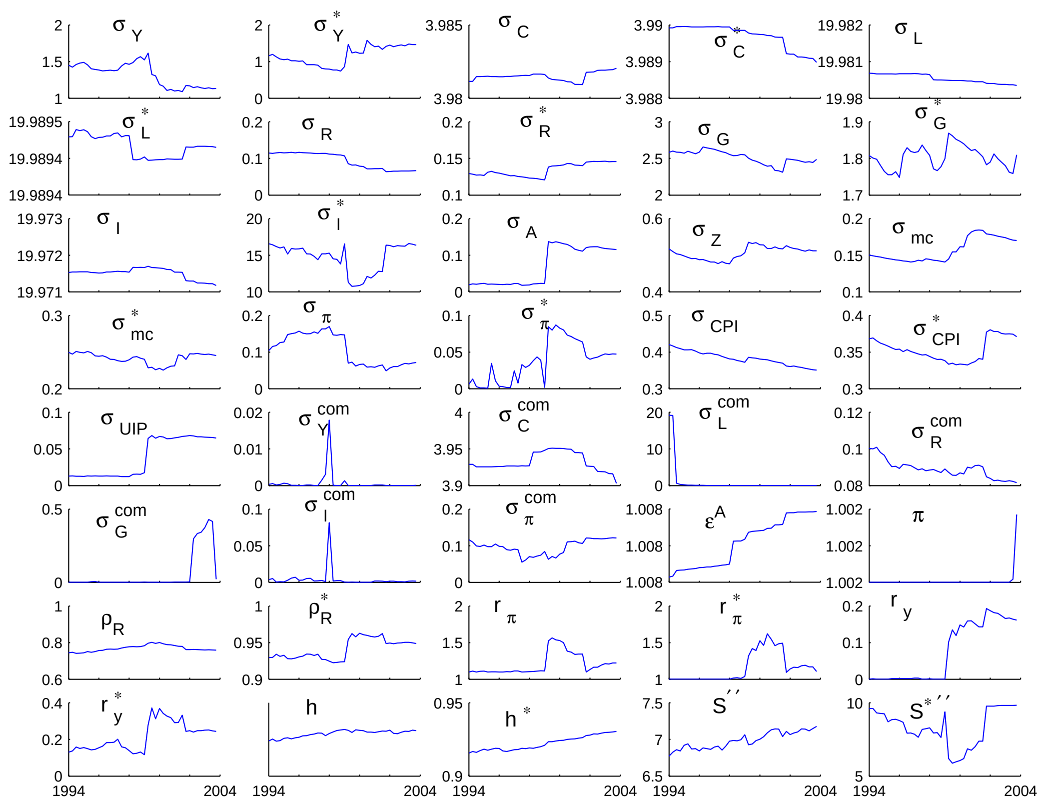

Figure 4.7: Sequential posterior mode estimates, the two-region DSGE model

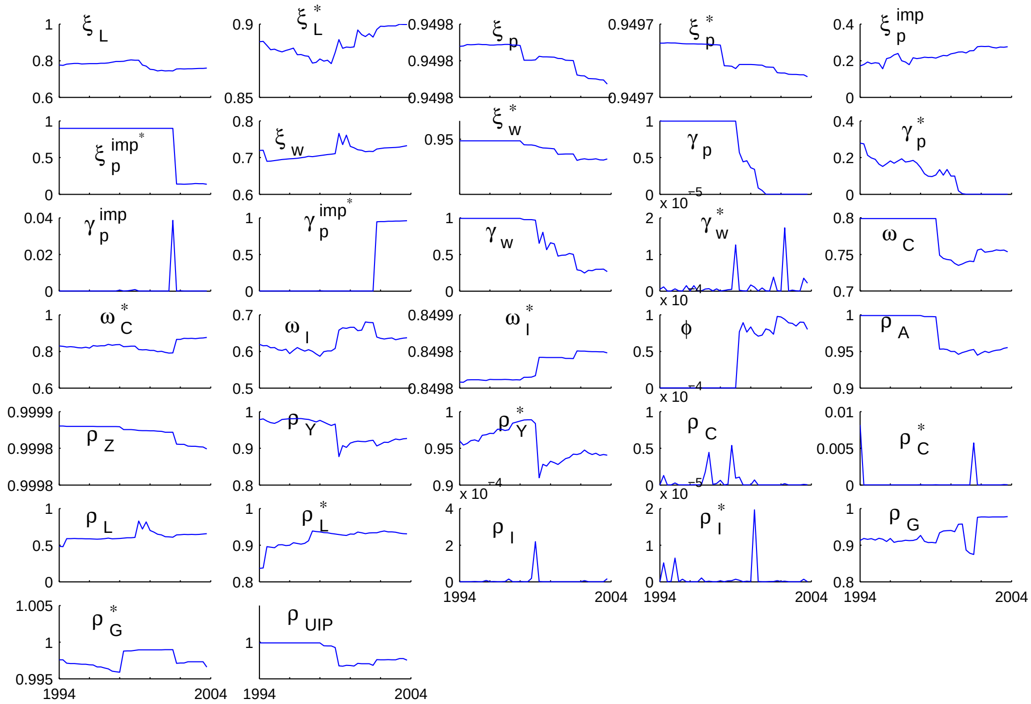

Figure 4.8: Sequential posterior mode estimates, the two-region DSGE model (cont.)

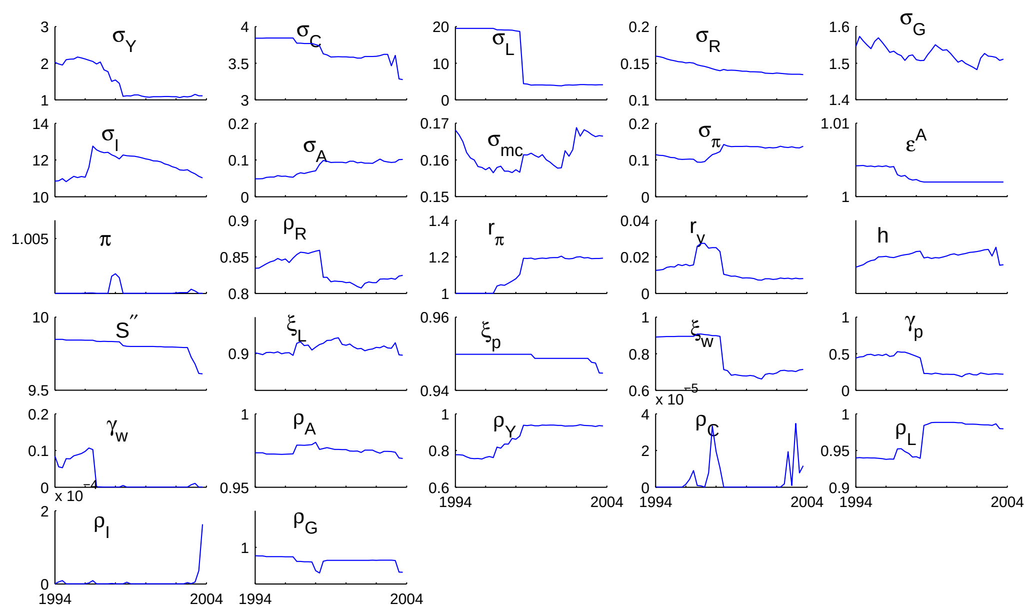

Figure 4.9: Sequential posterior mode estimates, the area-wide DSGE model

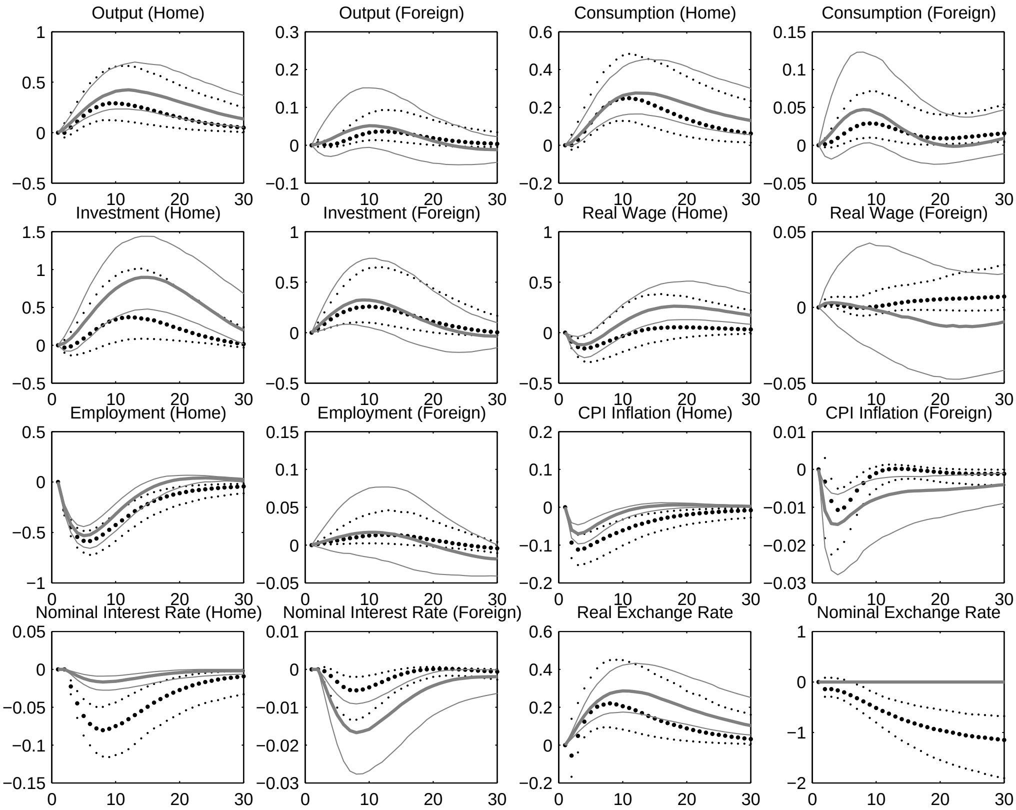

Figure 4.10: Response to Home stationary technology shock. Solid line - monetary union, dotted line - free float regime.

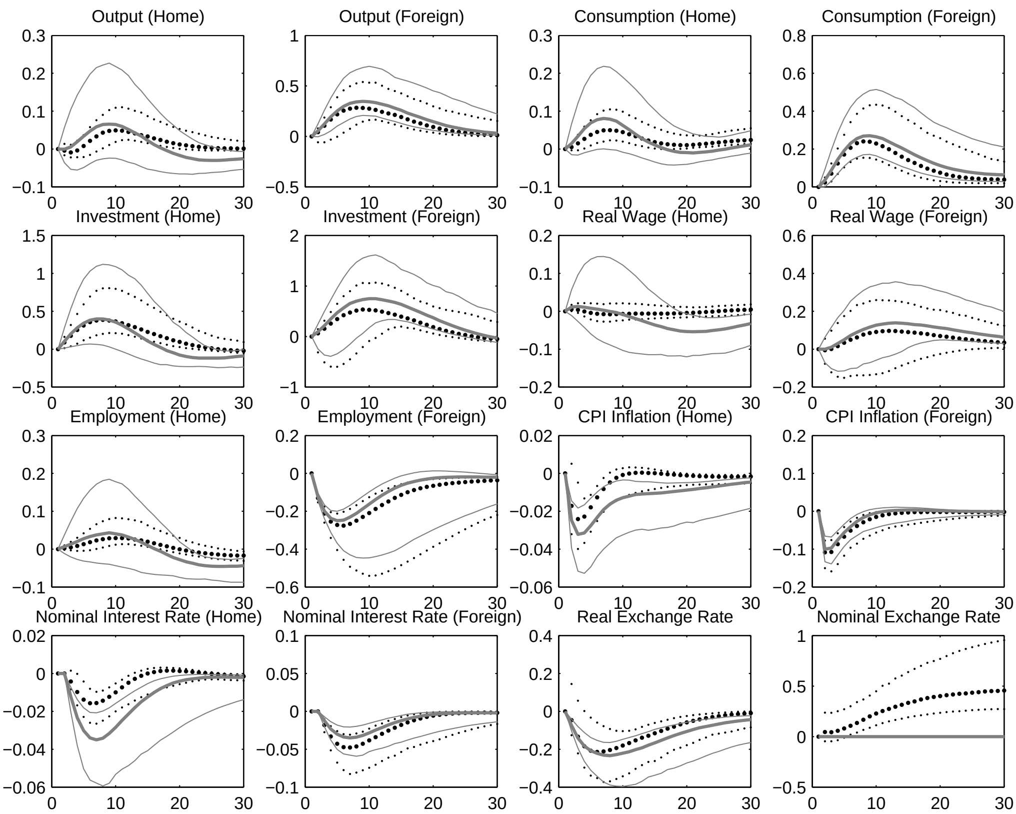

Figure 4.11: Response to Foreign stationary technology shock. Solid line - monetary union, dotted line - free float regime.

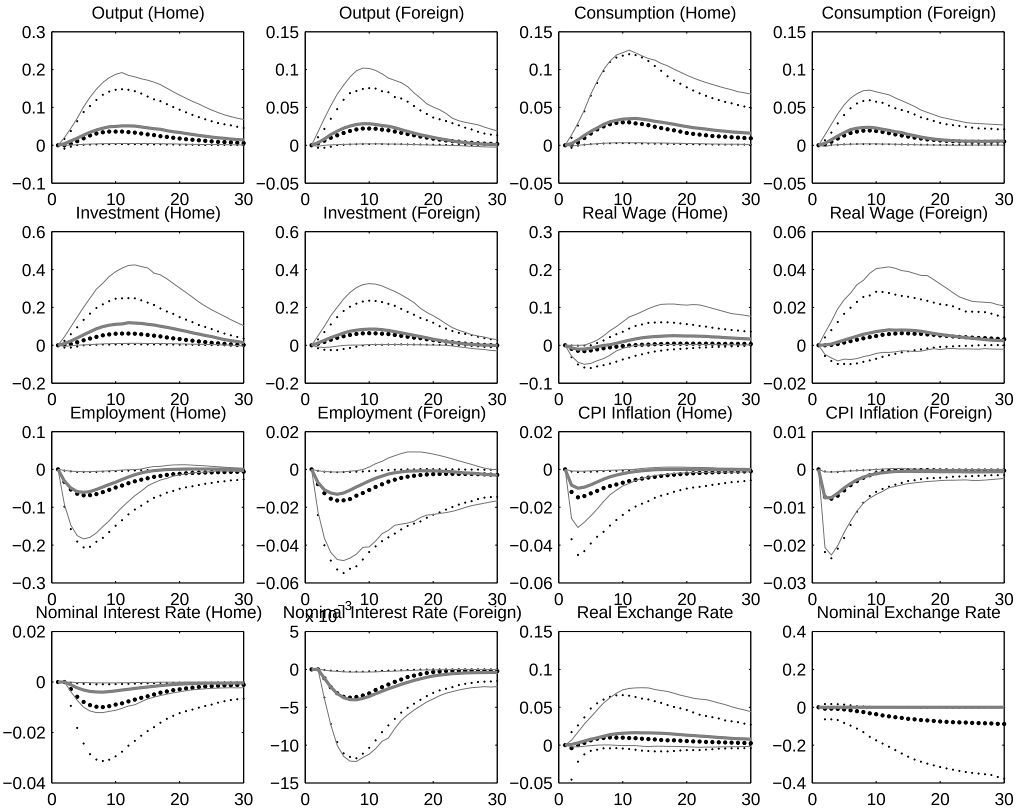

Figure 4.12: Response to a common component on the stationary technology shock. Solid line - monetary union, dotted line - free float regime.

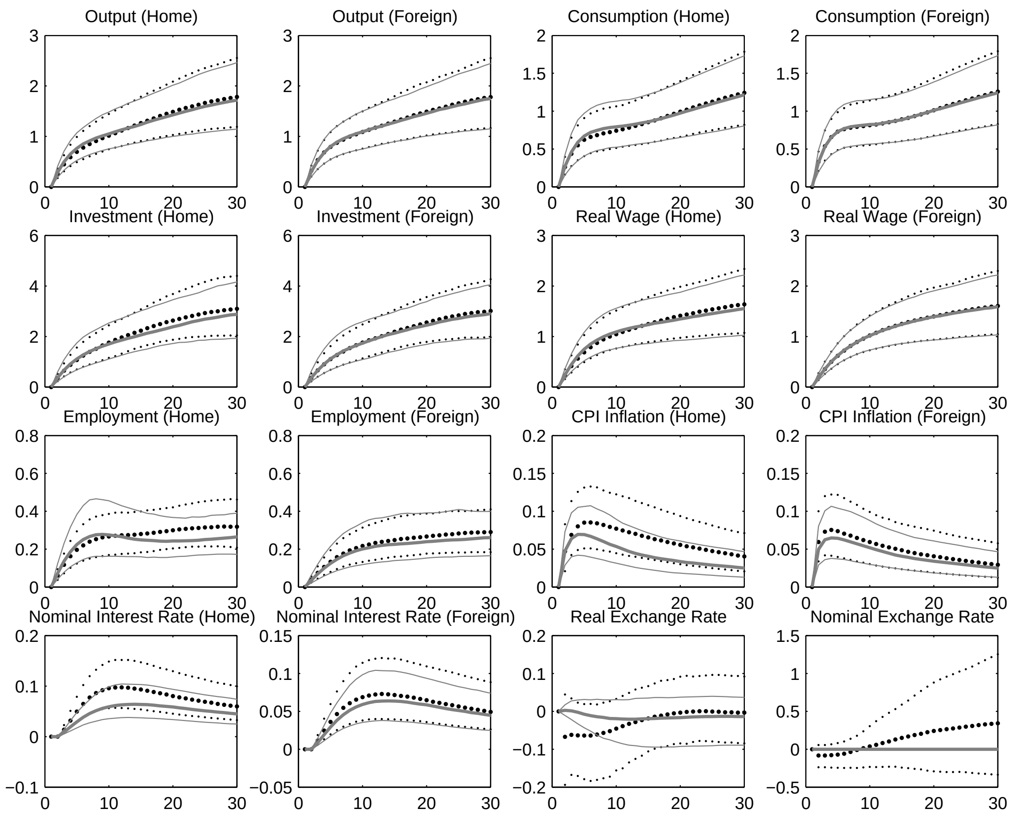

Figure 4.13: Response to a unit-root technology shock. Solid line - monetary union, dotted line - free float regime.

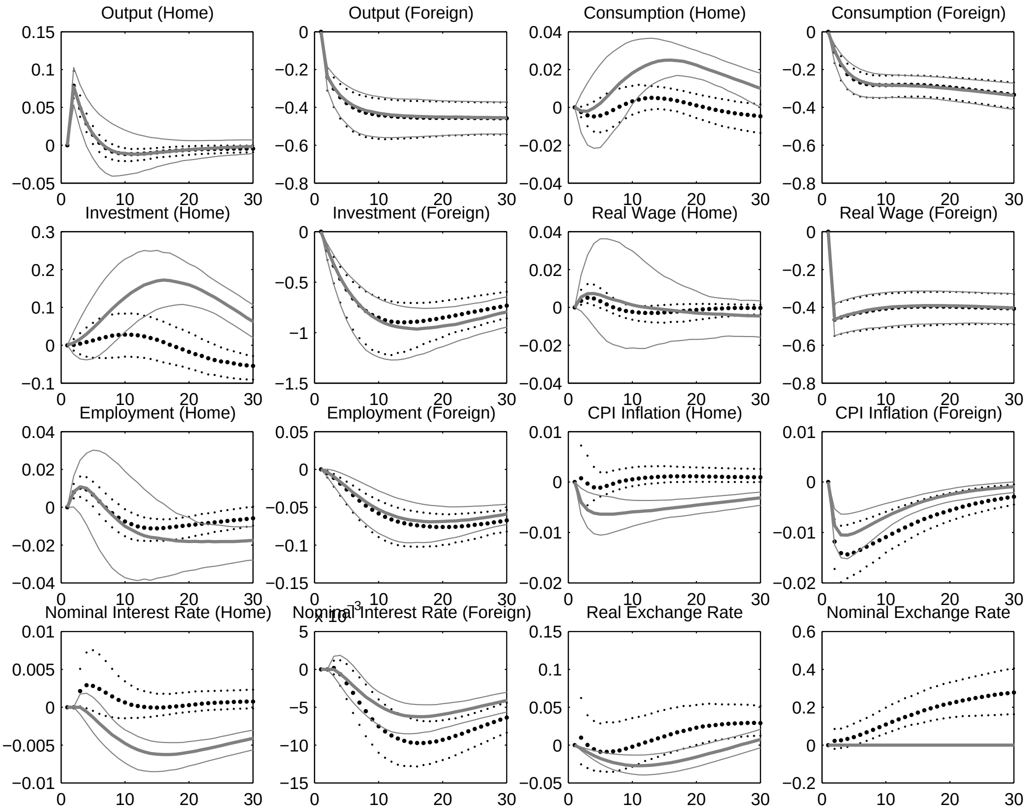

Figure 4.14: Response to an asymmetric technology shock. Solid line - monetary union, dotted line - free float regime.

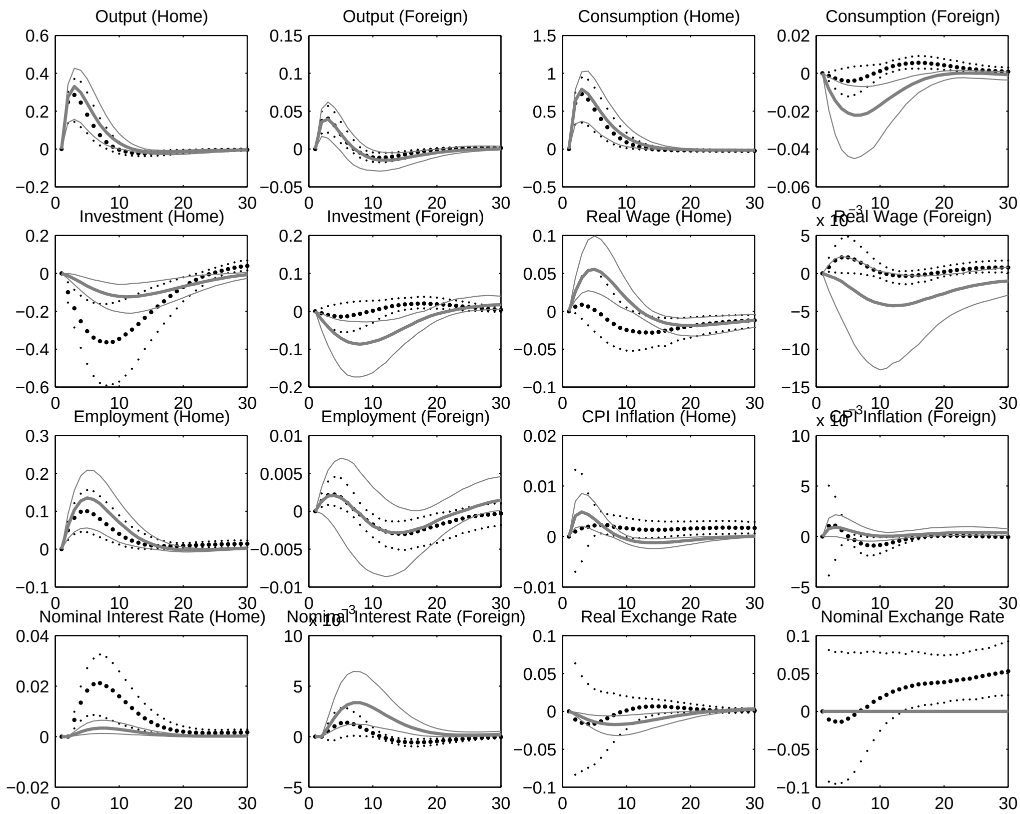

Figure 4.15: Response to Home preference shock. Solid line - monetary union, dotted line - free float regime.

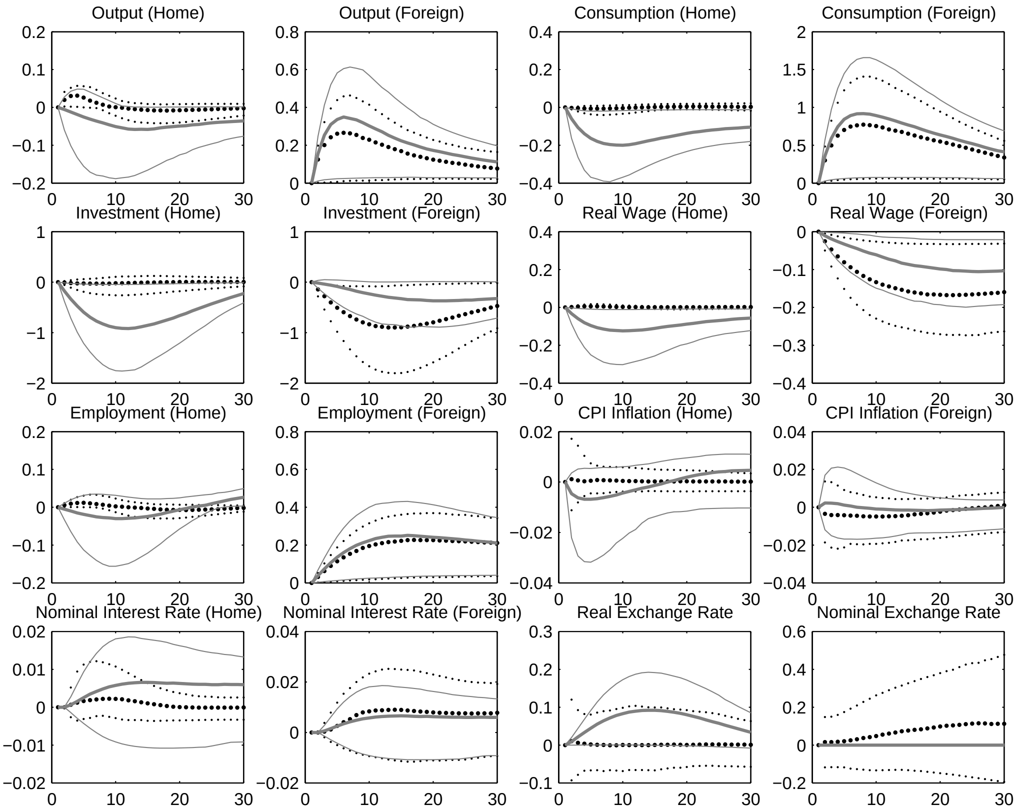

Figure 4.16: Response to Foreign preference shock. Solid line - monetary union, dotted line - free float regime.

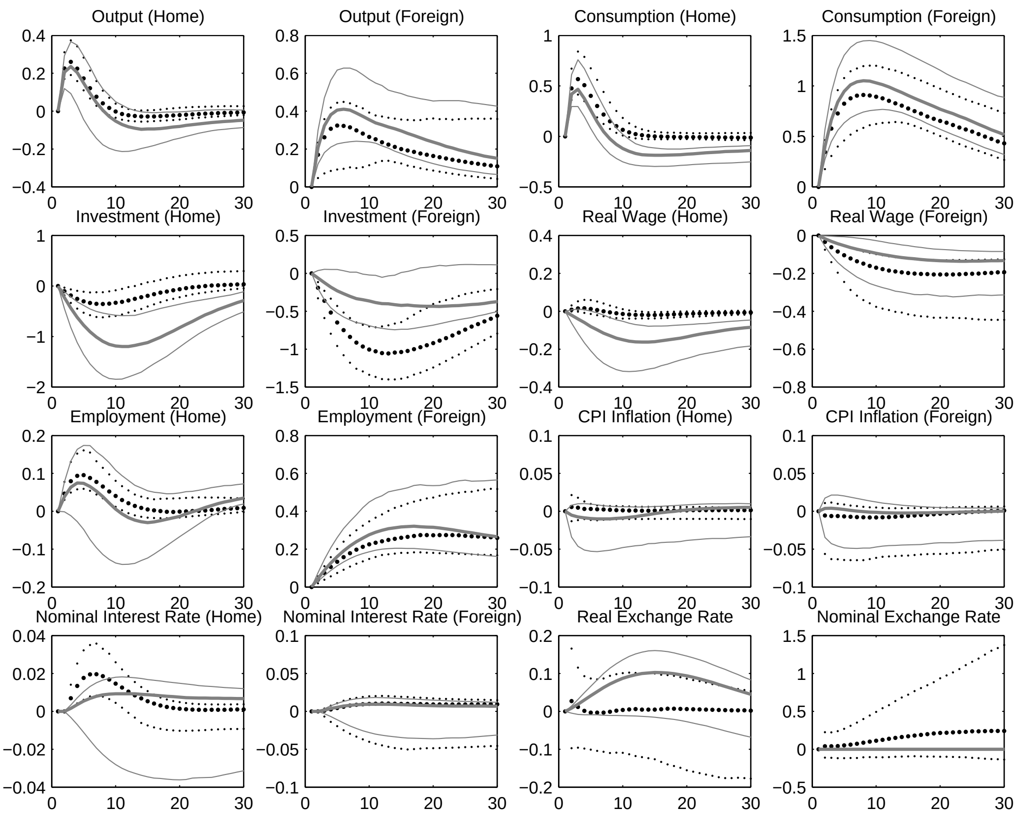

Figure 4.17: Response to a common component on the preference shock. Solid line - monetary union, dotted line - free float regime.

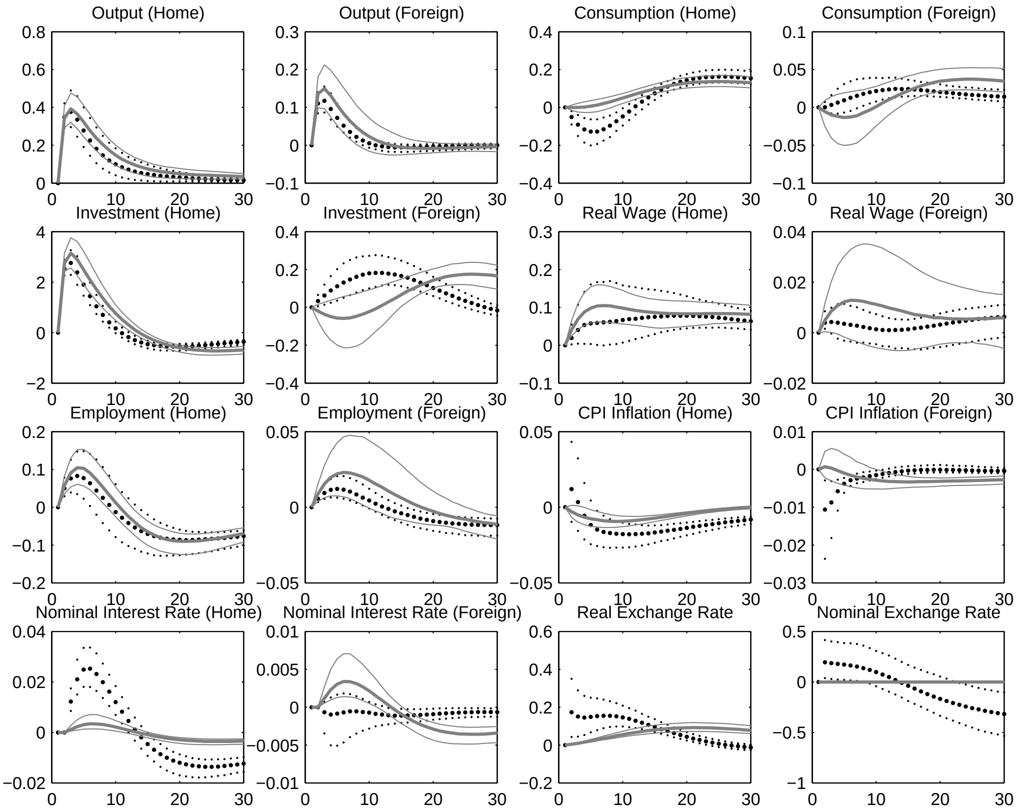

Figure 4.18: Response to Home investment shock. Solid line - monetary union, dotted line - free float regime.

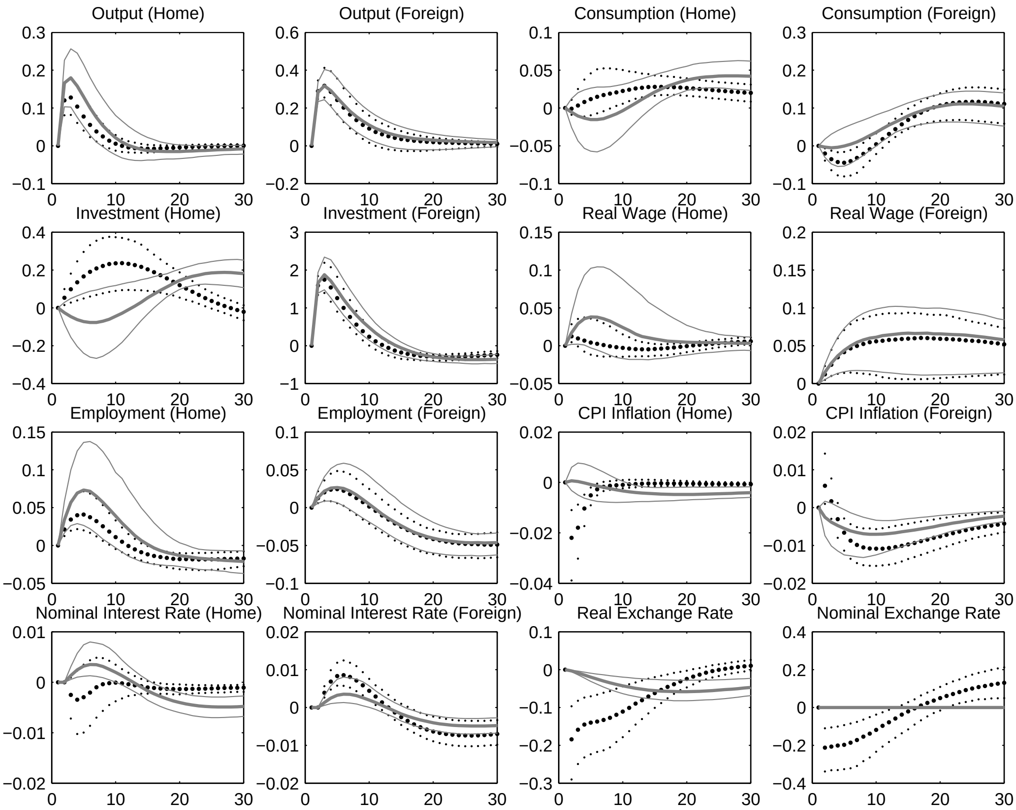

Figure 4.19: Response to Foreign investment shock. Solid line - monetary union, dotted line - free float regime.

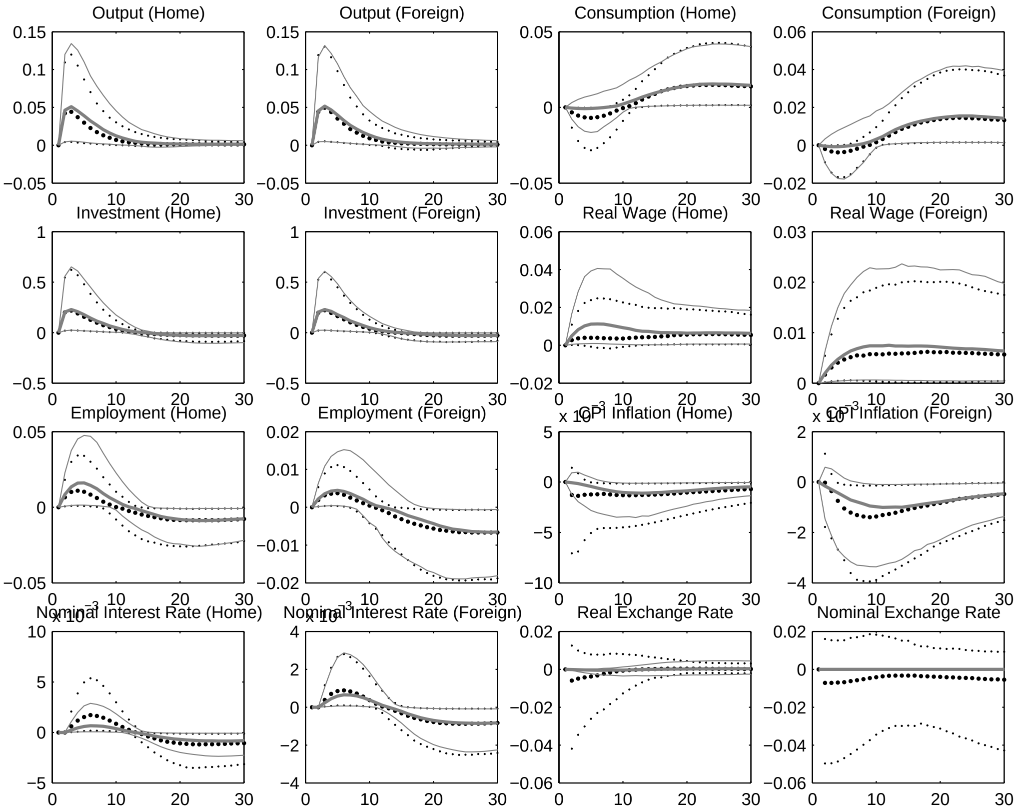

Figure 4.20: Response to a common component on the investment shock. Solid line - monetary union, dotted line - free float regime.

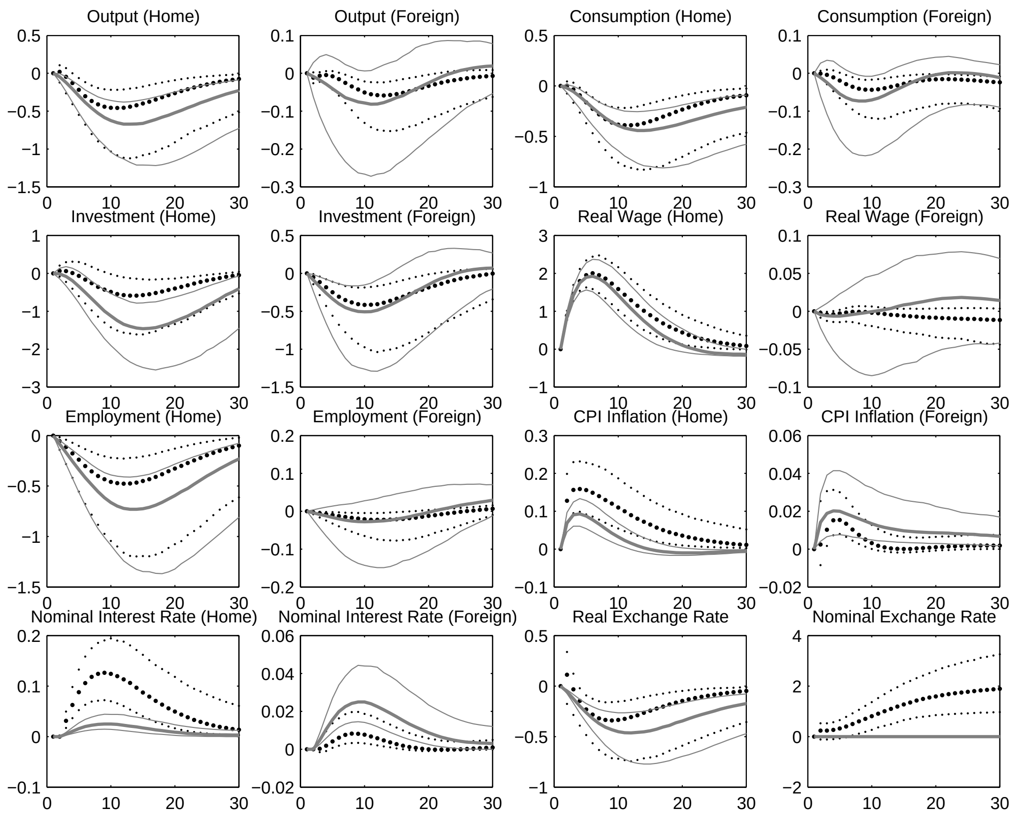

Figure 4.21: Response to Foreign negative labor supply shock. Solid line - monetary union, dotted line - free float regime.

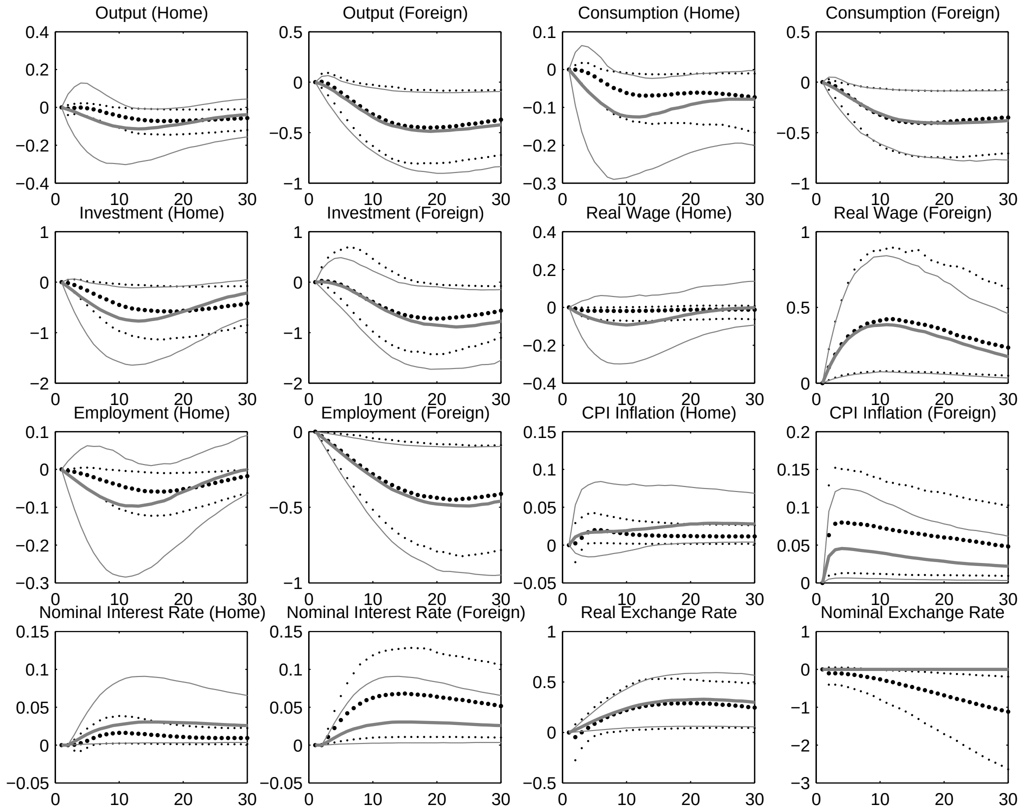

Figure 4.22: Response to Foreign negative labor supply shock. Solid line - monetary union, dotted line - free float regime.

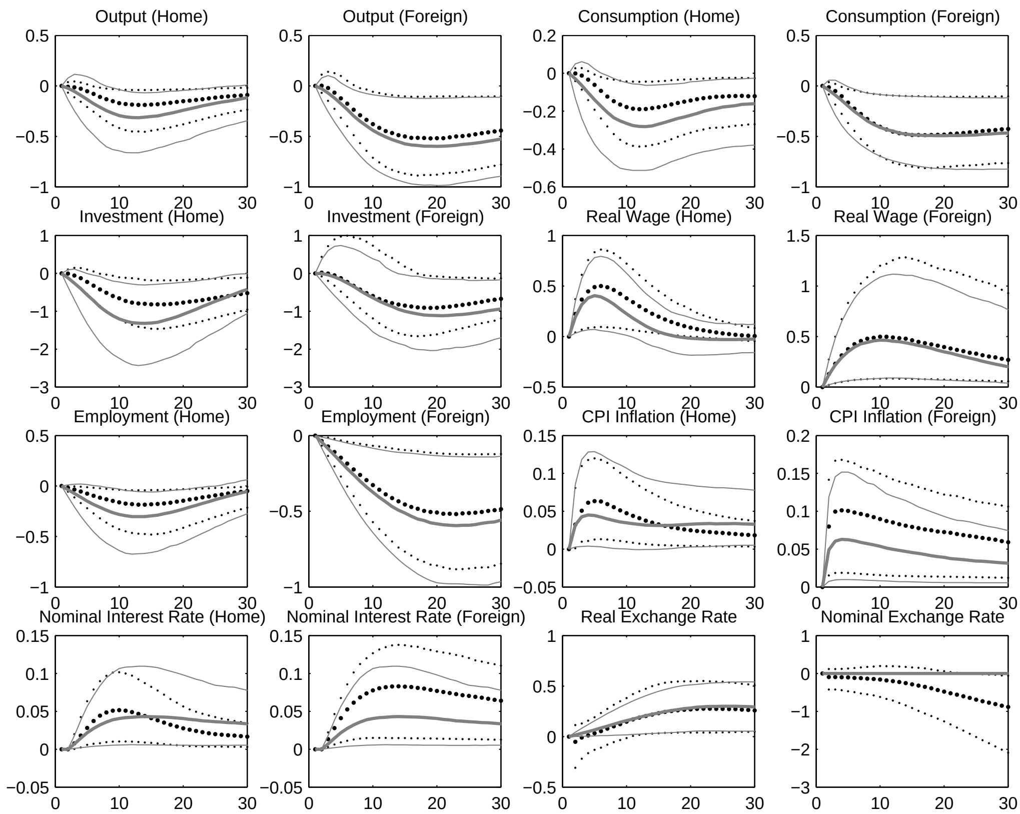

Figure 4.23: Response to a common component on the negative labor supply shock. Solid line - monetary union, dotted line - free float regime.

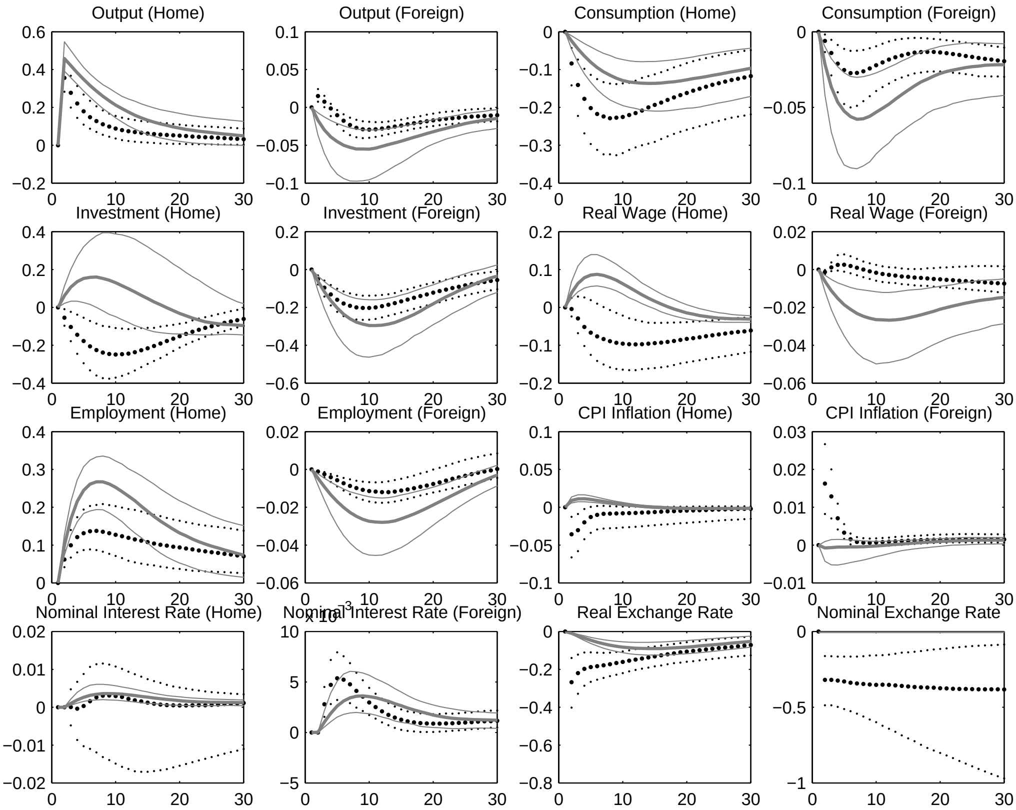

Figure 4.24: Response to Home government spending shock. Solid line - monetary union, dotted line - free float regime.

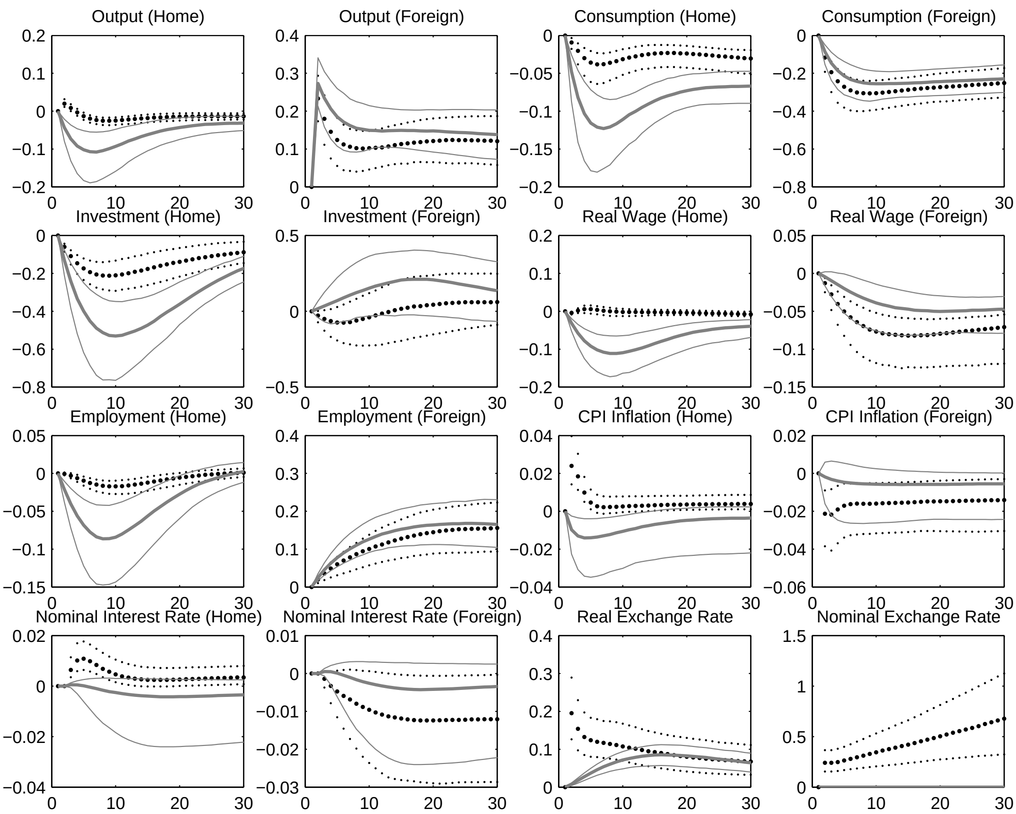

Figure 4.25: Response to Foreign government spending shock. Solid line - monetary union, dotted line - free float regime.

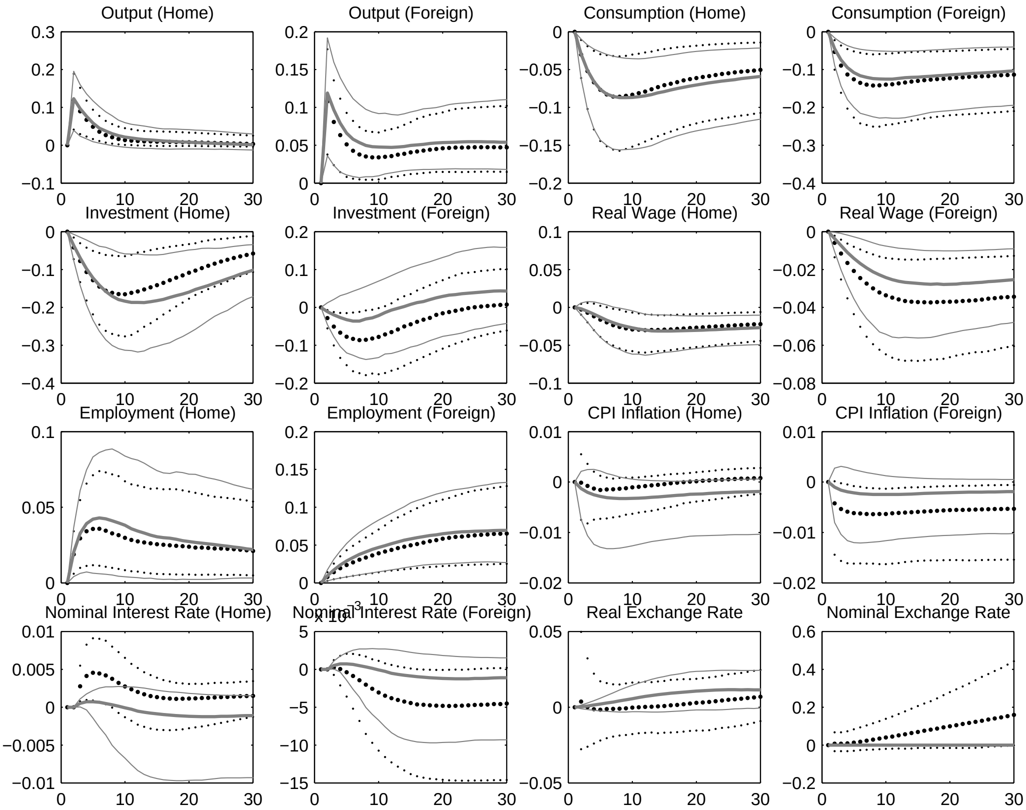

Figure 4.26: Response to a common component on the government spending shock. Solid line - monetary union, dotted line - free float regime.

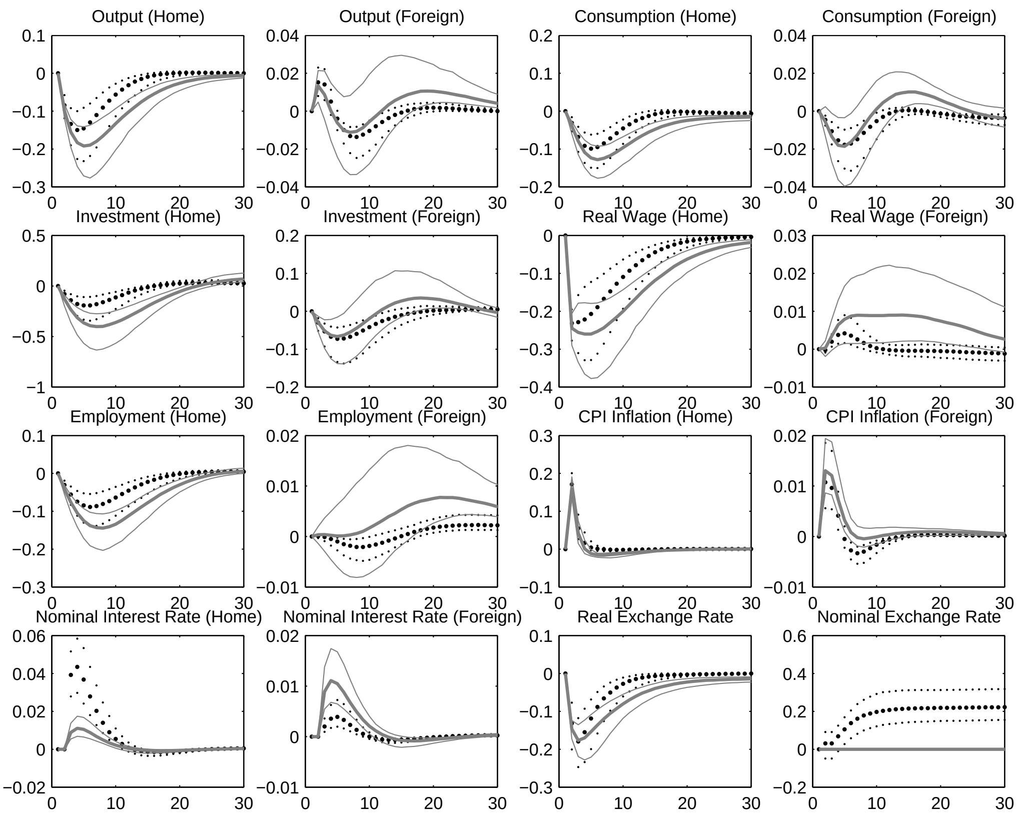

Figure 4.27: Response to Home Phillips curve shock. Solid line - monetary union, dotted line - free float regime.

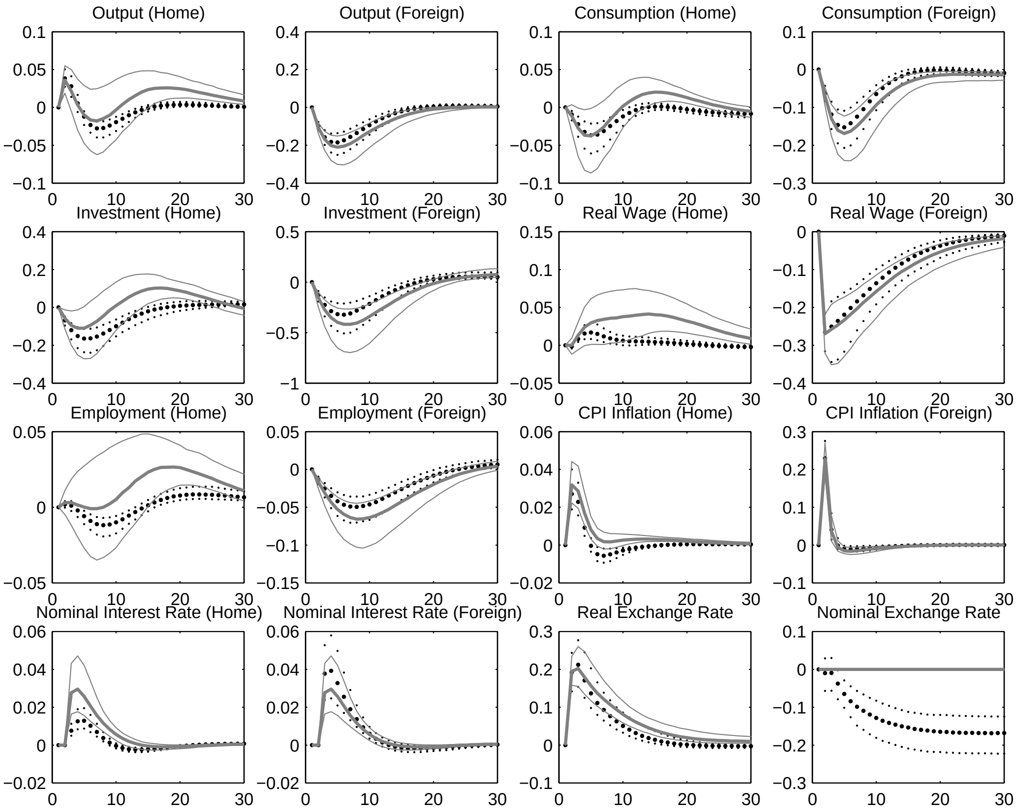

Figure 4.28: Response to Home Phillips curve shock. Solid line - monetary union, dotted line - free float regime.

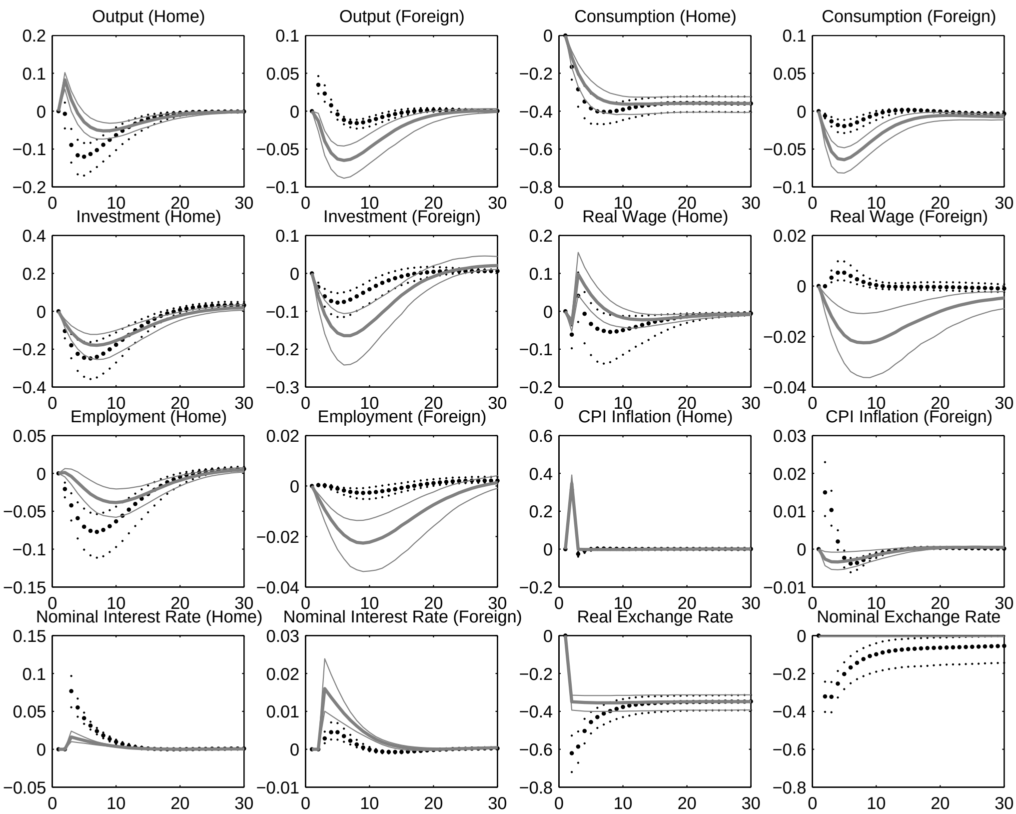

Figure 4.29: Response to Home CPI equation shock. Solid line - monetary union, dotted line - free float regime.

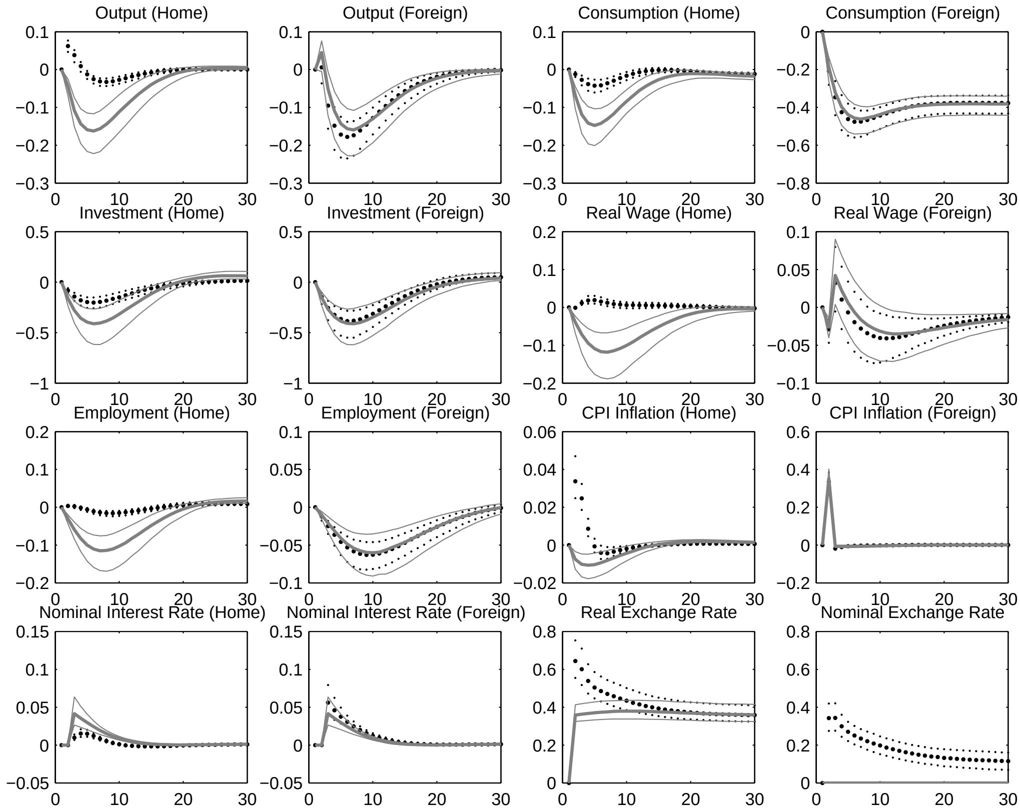

Figure 4.30: Response to Foreign CPI equation shock. Solid line - monetary union, dotted line - free float regime.

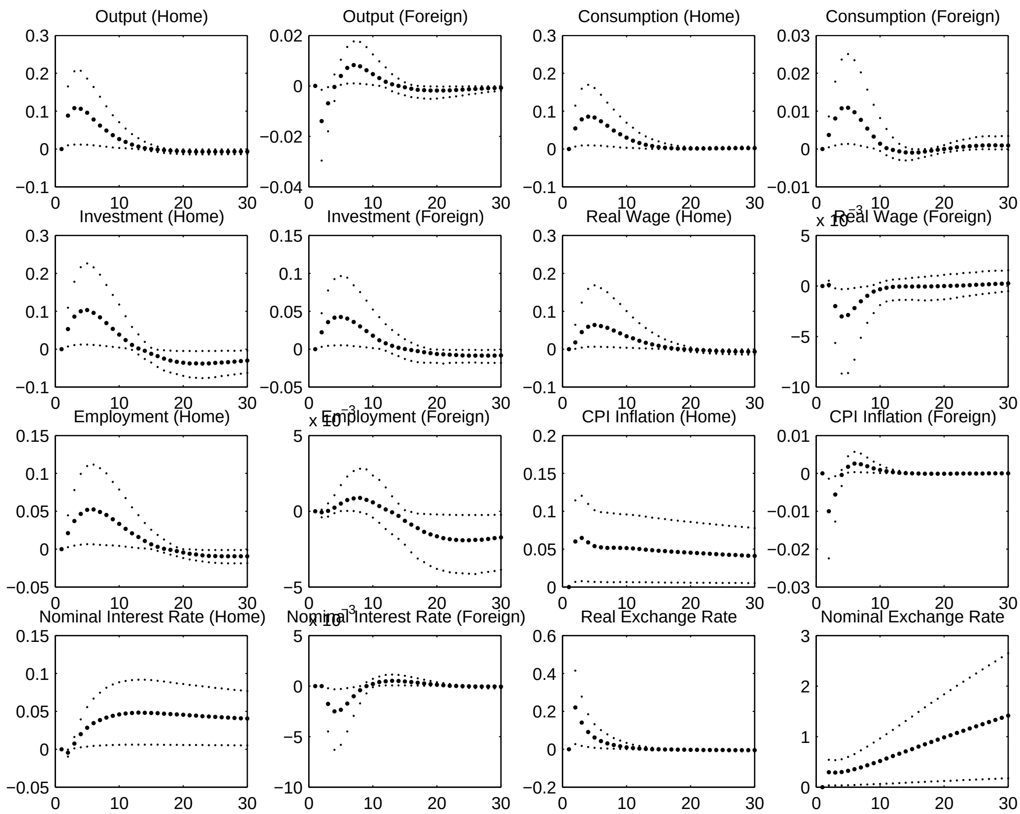

Figure 4.31: Response to Home inflation target shock under flexible exchange rate regime

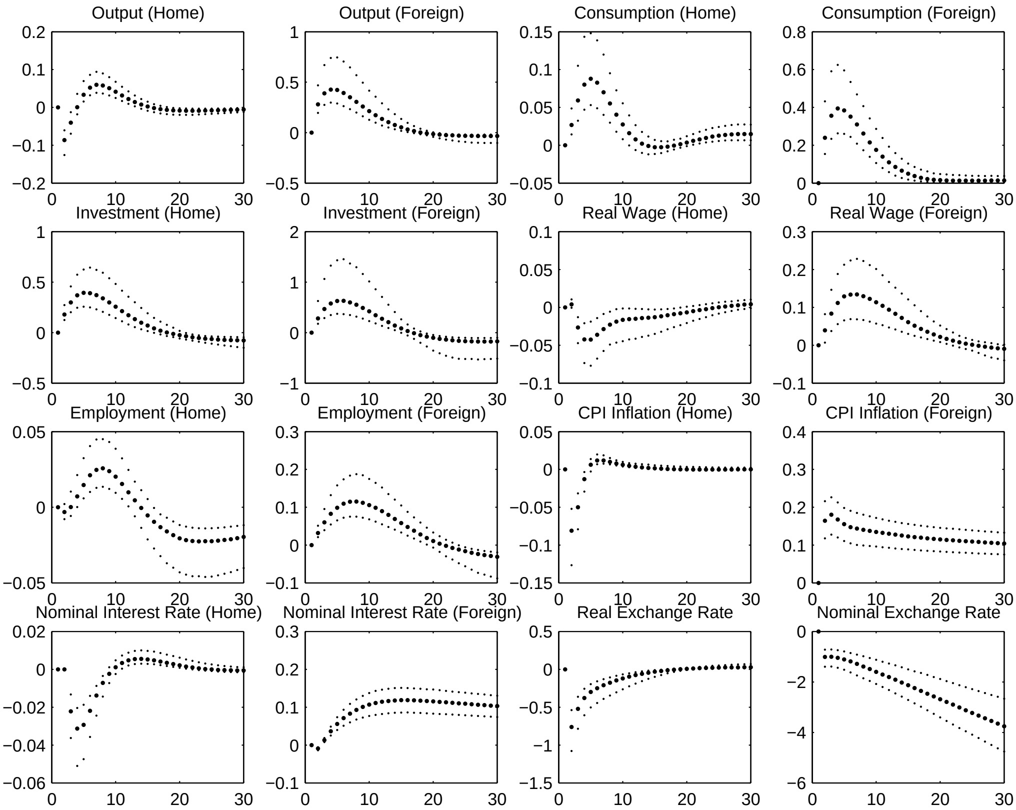

Figure 4.32: Response to Foreign inflation target shock under flexible exchange rate regime

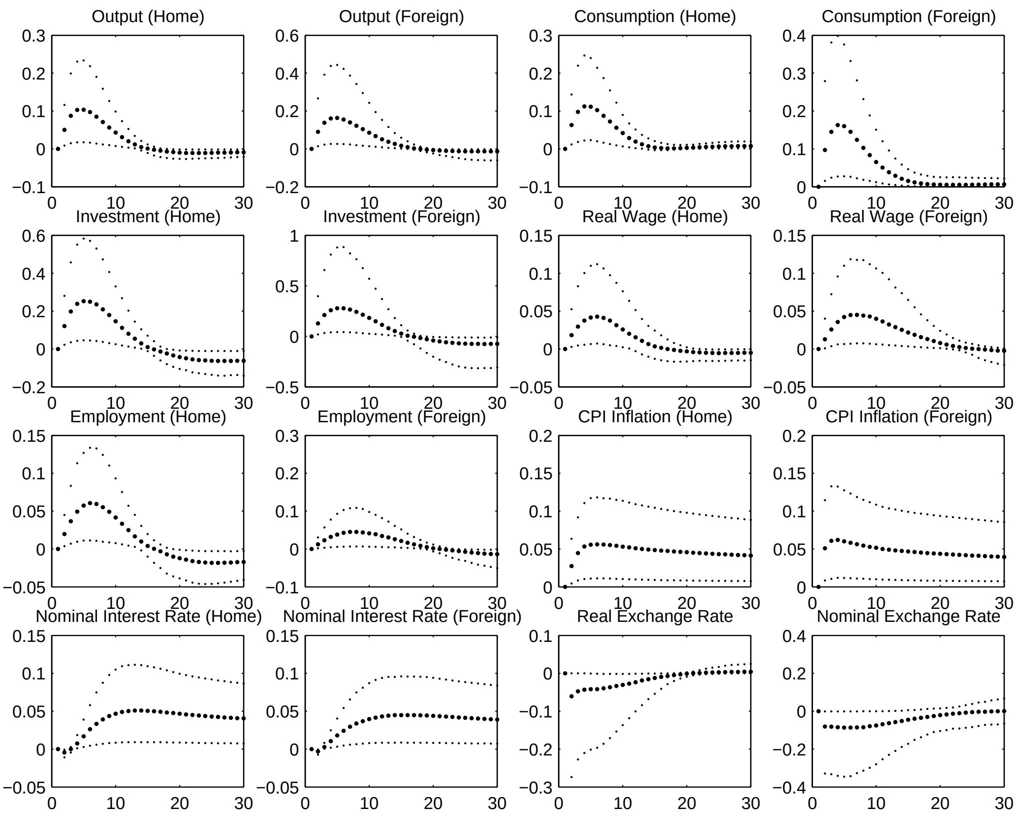

Figure 4.33: Response to a common component on the inflation target shock under flexible exchange rate regime

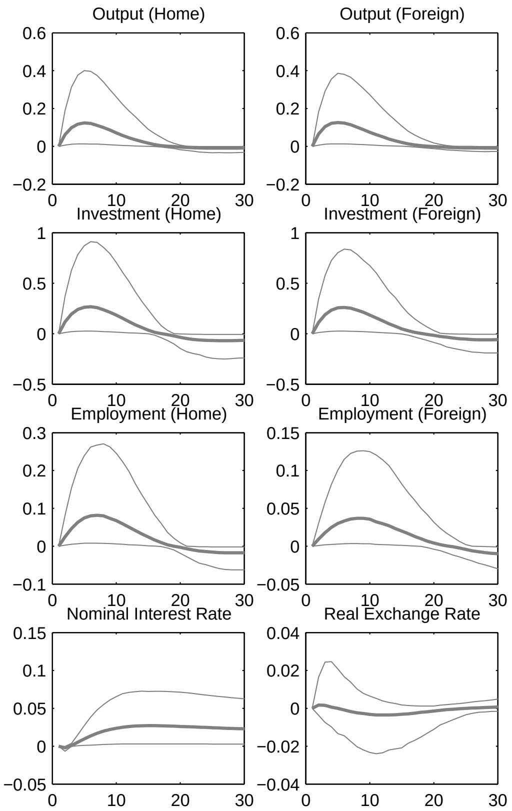

Figure 4.34: Response to a common inflation target shock in EMU

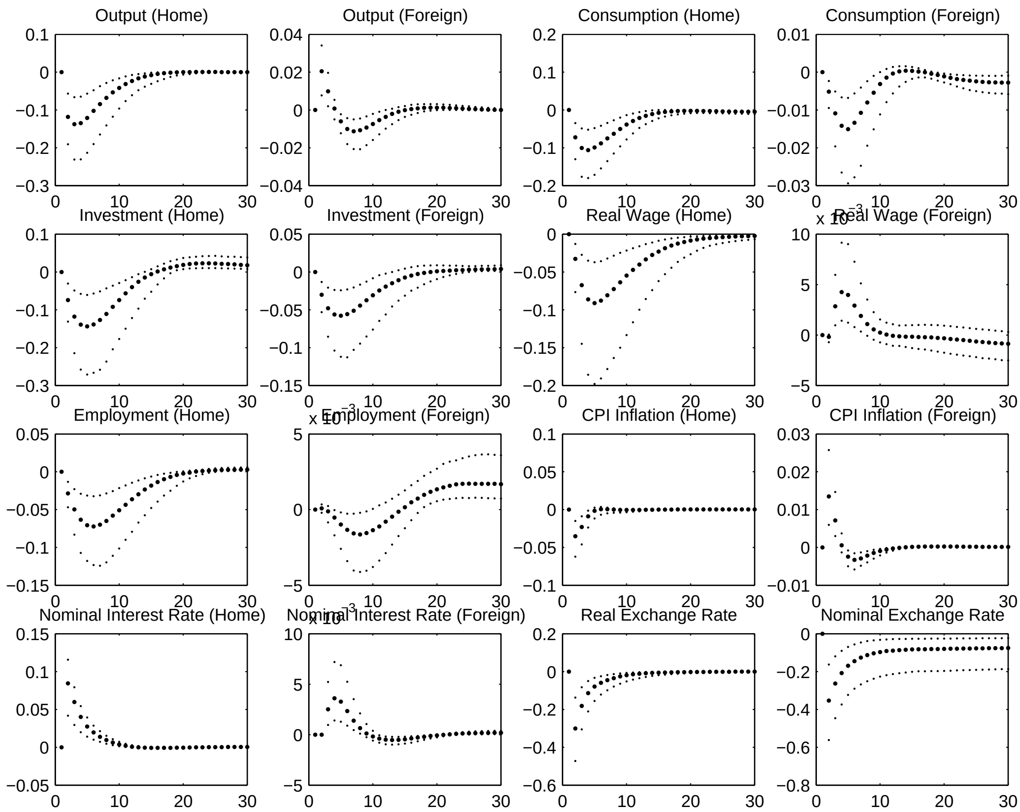

Figure 4.35: Response to Home interest rate shock under flexible exchange rate regime

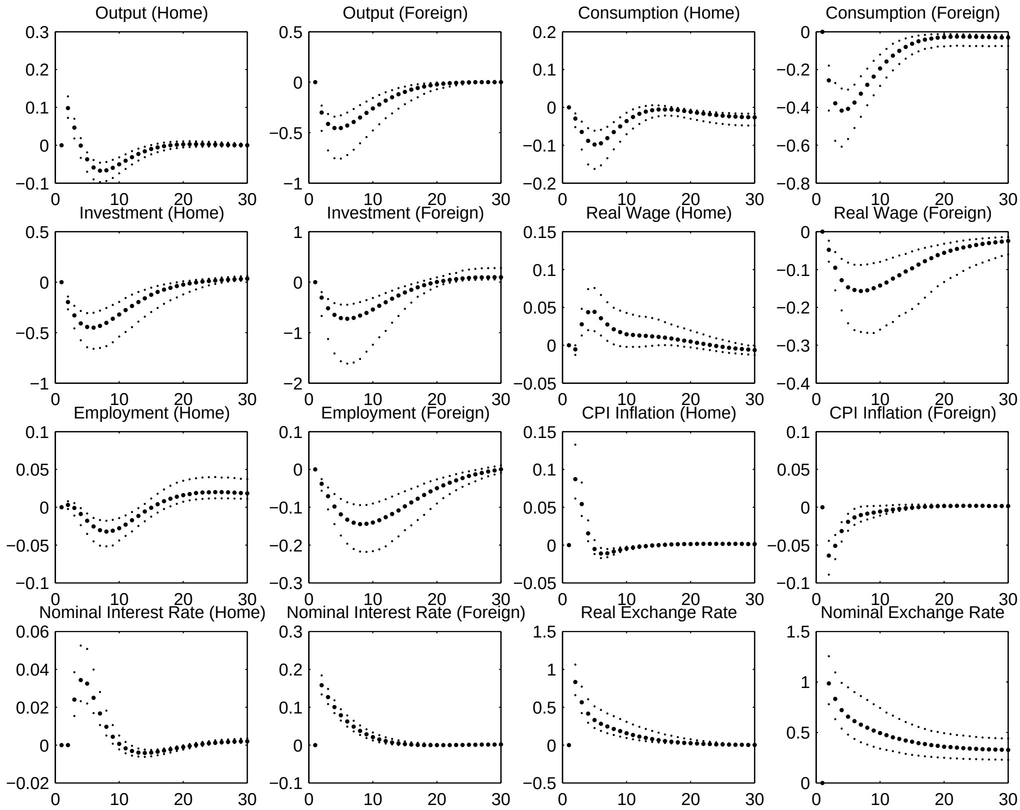

Figure 4.36: Response to Foreign interest rate shock under flexible exchange rate regime

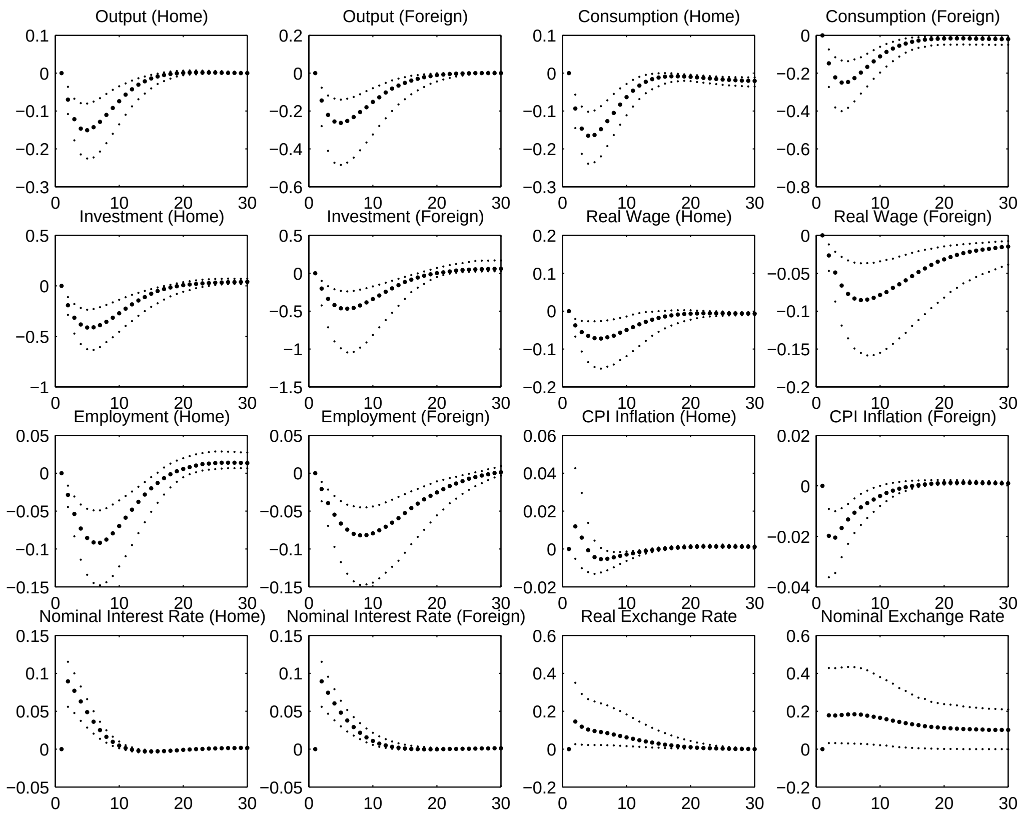

Figure 4.37: Response to a common component on the interest rate shock under flexible exchange rate regime

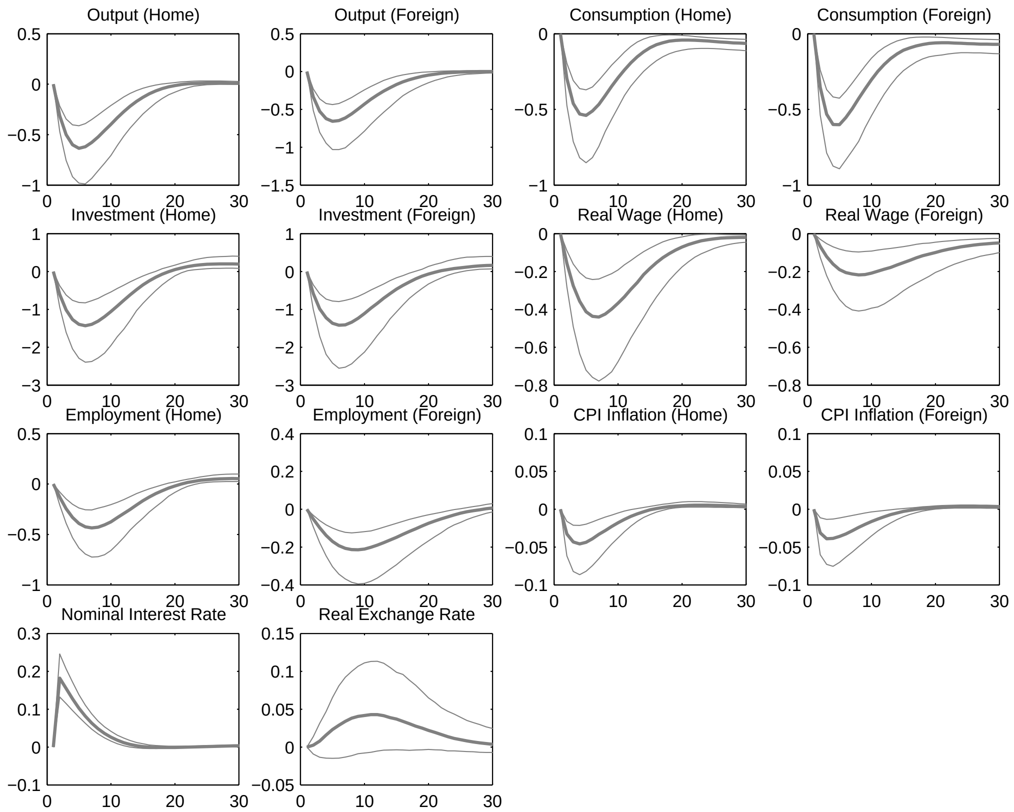

Figure 4.38: Response to a common interest rate shock in EMU

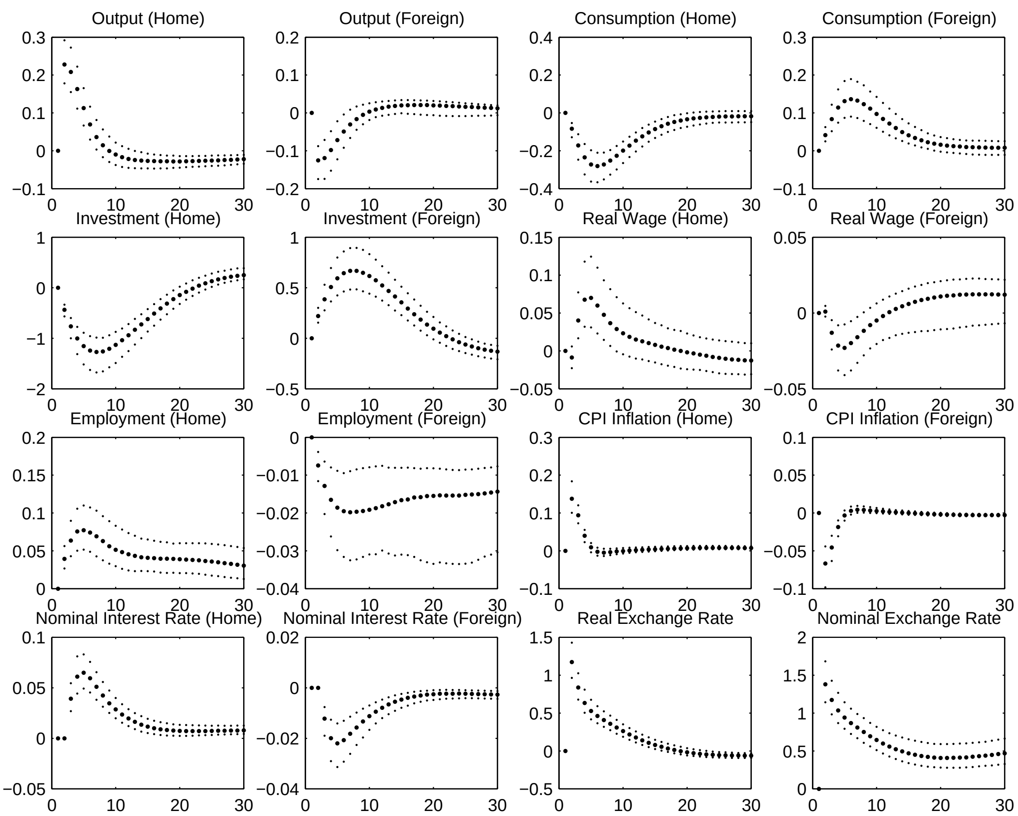

Figure 4.39: Response to UIP shock under flexible exchange rate regime

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Appendix A

Steady state

A.1 The two-region model

In this section we compute the steady state of the two-region DSGE model.[1]

The steady state gross rate of inflation is assumed to be in both regions equal to $\bar{\pi}$.[2] We further assume that the net foreign asset position $\bar{\tilde{B}}^*$, as well as a depreciation rate of the nominal exchange rate are zero in the steady state. Then, the stationary Euler equations reduce to:

$$\bar{R} = \bar{R}^* = \frac{\bar{\pi}\bar{\varepsilon}^A}{\beta} \tag{A.1}$$

As there are no long run deviations from the law of one price, the real exchange rate and various price ratios reduce to:[3]

$\bar{S}^{real} = \bar{\mathcal{T}}^{(\omega_C+\omega_C^*-1)}$, $\bar{\mathcal{T}}^{P^*/P^{C,imp}} = 1$, $\bar{\mathcal{T}}^{P/P^{C,imp^*}} = 1$, $\bar{\mathcal{T}}_t^{P/P^C} = \bar{\mathcal{T}}^{\omega_C-1}$, $\bar{\mathcal{T}}_t^{P^*/P^{C^*}} = \bar{\mathcal{T}}^{1-\omega_C^*}$, $\mathcal{T}^{P^C/P^{C,imp}} = \bar{\mathcal{T}}^{-\omega_C}$ and $\bar{\mathcal{T}}^{P^{C^*}/P^{C,imp^*}} = \bar{\mathcal{T}}^{\omega_C^*}$.

Thus, from the NFA equation we obtain:

$$(1-g)\,\bar{Y} = \bar{\mathcal{T}}^{(1-\omega_C)}\bar{C} + \bar{\mathcal{T}}^{(1-\omega_I)}\bar{I} \tag{A.2}$$

The stationary version of the first order condition for I_t and I_t^* simplifies to:

$$\bar{Q} = \bar{\mathcal{T}}^{(1-\omega_I)}, \tag{A.3}$$

$$\bar{Q}^* = \bar{\mathcal{T}}^{-\left(1-\omega_I^*\right)} \tag{A.4}$$

The first order condition for the price of installed capital reduces in the steady state to:

$$\bar{r}^k = \bar{Q}\left(\frac{\bar{\varepsilon}^A}{\beta} - (1-\delta)\right), \tag{A.5}$$

$$\bar{r}^{k^*} = \bar{Q}^*\left(\frac{\bar{\varepsilon}^A}{\beta} - (1-\delta)\right) \tag{A.6}$$

The steady state labor demand is given by:

[1] Steady state levels will be indicated by overbars.

[2] The inflation rate (and the steady state inflation) can, in general, be computed from the definition equation of the money growth, where the steady state rate of money growth is determined by the monetary authority. Since the nominal interest rate is used in our model as a monetary instrument and the money balances are not explicitly considered, the steady state inflation rate is regarded as exogenously determined. Note that the model also imposes: $\bar{\pi} = \bar{\pi}^* = \bar{\pi}^C = \bar{\pi}^{C^*} = \bar{\pi}^{C_{EMU}}$.

[3] We assume that the monopolistic power of retailers is very limited ($\lambda_C \approx 1$).

$$\bar{L}\bar{W}\bar{\varepsilon}^A = \frac{1-\alpha}{\alpha}\bar{r}^k\bar{K}, \tag{A.7}$$
$$\bar{L}^*\bar{W}^*\bar{\varepsilon}^A = \frac{1-\alpha}{\alpha}\bar{r}^{k^*}\bar{K}^* \tag{A.8}$$

The cost minimization of intermediate goods producers results in the following steady state relations:

$$\bar{\varepsilon}^Y\overline{MC} = \left(\frac{\bar{W}}{1-\alpha}\right)^{1-\alpha}\left(\frac{\bar{r}^k}{\alpha}\right)^{\alpha}, \tag{A.9}$$
$$\bar{\varepsilon}^{Y^*}\overline{MC}^* = \left(\frac{\bar{W}^*}{1-\alpha}\right)^{1-\alpha}\left(\frac{\bar{r}^{k^*}}{\alpha}\right)^{\alpha} \tag{A.10}$$

From the profit maximization of intermediate firms we obtain:

$$\overline{MC} = \frac{2\lambda_p - 1}{\lambda_p}, \tag{A.11}$$
$$\overline{MC}^* = \frac{2\lambda_p^* - 1}{\lambda_p^*} \tag{A.12}$$

The wage setting schemes yield the following relations:

$$\bar{W} = \lambda_w\bar{L}^{\sigma_L}\left(\frac{\bar{C}(\bar{\varepsilon}^A - h)}{\bar{\varepsilon}^A\bar{\mathcal{T}}^{(\omega_C - 1)}}\right), \tag{A.13}$$
$$\bar{W}^* = \lambda_w^*\bar{L}^{*\sigma_L^*}\left(\frac{\bar{C}^*(\bar{\varepsilon}^A - h^*)}{\bar{\varepsilon}^A\bar{\mathcal{T}}^{-(\omega_C^* - 1)}}\right) \tag{A.14}$$

From capital accumulation equation we obtain:

$$\bar{I} = \left(\frac{\bar{\varepsilon}^A - (1-\delta)}{\bar{\varepsilon}^A}\right)\bar{K} \tag{A.15}$$
$$\bar{I}^* = \left(\frac{\bar{\varepsilon}^A - (1-\delta)}{\bar{\varepsilon}^A}\right)\bar{K}^* \tag{A.16}$$

The production function reduces in steady state to:

$$\bar{Y} = \bar{\varepsilon}^Y\left(\bar{\varepsilon}^A\right)^{-\alpha}\bar{K}^{\alpha}\bar{L}^{(1-\alpha)}, \tag{A.17}$$
$$\bar{Y}^* = \bar{\varepsilon}^{Y^*}\left(\bar{\varepsilon}^A\right)^{-\alpha}\left(\bar{K}^*\right)^{\alpha}\left(\bar{L}^*\right)^{(1-\alpha)} \tag{A.18}$$

Finally, the aggregate demands in Home and Foreign economy are in the steady state given by:

$$\bar{Y} = \left[\begin{array}{c}\omega_C\bar{C}\bar{\mathcal{T}}^{(1-\omega_C)} + \omega_I\bar{I}\bar{\mathcal{T}}^{(1-\omega_I)} + g\bar{Y} \\ +\frac{(1-n)}{n}(1-\omega_C^*)\bar{C}^*\bar{\mathcal{T}}^{\omega_C^*} + \frac{(1-n)}{n}(1-\omega_I^*)\bar{I}^*\bar{\mathcal{T}}^{\omega_I^*}\end{array}\right], \tag{A.19}$$
$$\bar{Y}^* = \left[\begin{array}{c}\frac{n}{n-1}(1-\omega_C)\bar{C}\bar{\mathcal{T}}^{-\omega_C} + \frac{n}{n-1}(1-\omega_I)\bar{I}\bar{\mathcal{T}}^{-\omega_I} \\ +g^*\bar{Y}^* + \omega_C^*\bar{C}^*\bar{\mathcal{T}}^{(\omega_C^*-1)} + \omega_I^*\bar{I}^*\bar{\mathcal{T}}^{(\omega_I^*-1)}\end{array}\right] \tag{A.20}$$

Below we solve for $\bar{Y}$, $\bar{Y}^*$, $\bar{C}$, $\bar{C}^*$, $\bar{I}$, $\bar{I}^*$, $\bar{K}$, $\bar{K}^*$, $\bar{L}$, $\bar{L}^*$, $\bar{W}$, $\bar{W}^*$, $\bar{Q}$, $\bar{Q}^*$, $\bar{r}^k$, $\bar{r}^{k^*}$, $\overline{MC}$, $\overline{MC}^*$ and $\bar{\mathcal{T}}$. Using equations (A.3) - (A.12) we subsequently substitute for $\overline{MC}$, $\overline{MC}^*$, $\bar{Q}$, $\bar{Q}^*$, $\bar{r}^k$, $\bar{r}^{k^*}$, $\bar{W}$, $\bar{W}^*$, $\bar{K}$ and $\bar{K}^*$ in equations (A.13) - (A.18) to arrive at:

$$\bar{I} = \left(\bar{\varepsilon}^A - (1-\delta)\right) \frac{\alpha}{1-\alpha} \frac{\bar{L}}{\bar{r}^k} \left(\bar{\varepsilon}^Y \frac{(2\lambda_p - 1)}{\lambda_p}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{\mathcal{T}}^{(1-\omega_I)} \left(\frac{\bar{\varepsilon}^A}{\beta} - (1-\delta)\right)}\right)^{\frac{1}{1-\alpha}}, \quad \text{(A.21)}$$

$$\bar{I}^* = \left(\bar{\varepsilon}^A - (1-\delta)\right) \bar{L}^* \left(\bar{\varepsilon}^{Y^*} \frac{(2\lambda_p^* - 1)}{\lambda_p^*}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{\mathcal{T}}^{-(1-\omega_I^*)} \left(\frac{\bar{\varepsilon}^A}{\beta} - (1-\delta)\right)}\right)^{\frac{1}{1-\alpha}}, \quad \text{(A.22)}$$

$$\bar{C} = \bar{\mathcal{T}}^{(\omega_C - 1)} \frac{\bar{\varepsilon}^A}{\lambda_w(\bar{\varepsilon}^A - h)} \bar{L}^{-\sigma_L} \left(\bar{\varepsilon}^Y \frac{(2\lambda_p - 1)}{\lambda_p}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{r}^k}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha), \quad \text{(A.23)}$$

$$\bar{C}^* = \bar{\mathcal{T}}^{(1-\omega_C^*)} \frac{\bar{\varepsilon}^A}{\lambda_w^*(\bar{\varepsilon}^A - h^*)} \left(\bar{L}^*\right)^{-\sigma_L^*} \left(\bar{\varepsilon}^{Y^*} \frac{(2\lambda_p^* - 1)}{\lambda_p^*}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\bar{r}^{k^*}}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha), \quad \text{(A.24)}$$

$$\bar{Y} = \bar{\varepsilon}^Y \left(\bar{\varepsilon}^Y \frac{(2\lambda_p - 1)}{\lambda_p} \frac{\alpha}{\bar{r}^k}\right)^{\frac{\alpha}{1-\alpha}} \bar{L}, \quad \text{(A.25)}$$

$$\bar{Y}^* = \bar{\varepsilon}^{Y^*} \left(\bar{\varepsilon}^{Y^*} \frac{(2\lambda_p^* - 1)}{\lambda_p^*} \frac{\alpha}{\bar{r}^{k^*}}\right)^{\frac{\alpha}{1-\alpha}} \bar{L}^* \quad \text{(A.26)}$$

Finally, substituting for $\bar{Y}$, $\bar{Y}^*$, $\bar{C}$, $\bar{C}^*$, $\bar{I}$ and $\bar{I}^*$ in equations (A.2), (A.19) and (A.20) we obtain a system of three equations with three unknowns $\bar{L}$, $\bar{L}^*$ and $\bar{\mathcal{T}}$, which we solve numerically. Once equipped with the solution for $\bar{L}$, $\bar{L}^*$ and $\bar{\mathcal{T}}$ we can recursively find the steady state for the remaining variables. The derivation of the model's steady state is accomplished by computing the weighted area-wide output:

$$\bar{Y}^{EMU} = \bar{Y}^n \bar{Y}^{(1-n)} \quad \text{(A.27)}$$

In order to derive the steady state equilibrium under a complete asset market, it is sufficient to replace equation (A.2) with the perfect risk sharing condition:

$$\bar{\mathcal{T}}^{(\omega_C + \omega_C^* - 1)} = \frac{\bar{C}(\bar{\varepsilon}^A - h)}{\bar{C}^*(\bar{\varepsilon}^A - h^*)} \quad \text{(A.28)}$$

Appendix B

The stationary log-linearized system

B.1 The two-region model

For the purpose of the empirical analysis in Chapter 4, a log-linear approximation to the model's optimality conditions around a non-stochastic steady state is employed. For any stationary variable X_t define $\hat{x}_t = \ln(X_t/\bar{X})$ as the respective log deviations from steady state.

Capital equations:

$$\hat{k}_t = \frac{(1-\delta)}{\bar{\varepsilon}^A}(\hat{k}_{t-1} - \hat{\varepsilon}_t^A) + (1 - \frac{(1-\delta)}{\bar{\varepsilon}^A})(\hat{\imath}_t + \hat{\varepsilon}_t^I) \tag{B.1}$$

$$\hat{k}_t^* = \frac{(1-\delta)}{\bar{\varepsilon}^A}(\hat{k}_{t-1}^* - \hat{\varepsilon}_t^{A^*}) + (1 - \frac{(1-\delta)}{\bar{\varepsilon}^A})(\hat{\imath}_t^* + \hat{\varepsilon}_t^{I^*}) \tag{B.2}$$

Investment equations:

$$0 = \hat{Q}_t + \hat{\varepsilon}_t^I - S'' \left(\bar{\varepsilon}^A\right)^2 (\hat{\imath}_t - \hat{\imath}_{t-1}) - \beta(E_t\hat{\imath}_{t+1} - \hat{\imath}_t) + \hat{\varepsilon}_t^A - \beta E_t\hat{\varepsilon}_{t+1}^A - (1-\omega_I)\hat{t}_t \tag{B.3}$$

$$0 = \hat{Q}_t^* + \hat{\varepsilon}_t^{I^*} - S^{*\prime\prime} \left(\bar{\varepsilon}^A\right)^2 (\hat{\imath}_t^* - \hat{\imath}_{t-1}^*) - \beta(E_t\hat{\imath}_{t+1}^* - \hat{\imath}_t^*) + \hat{\varepsilon}_t^{A^*} - \beta E_t\hat{\varepsilon}_{t+1}^{A^*} + (1-\omega_I^*)\hat{t}_t \tag{B.4}$$

Price of installed capital equations:

$$0 = \hat{r}_t - E_t\hat{\pi}_{t+1} - \beta\frac{(1-\delta)}{\bar{\varepsilon}^A}E_t\hat{Q}_{t+1} + \hat{Q}_t - \frac{(\bar{\varepsilon}^A - \beta(1-\delta))}{\bar{\varepsilon}^A}\hat{r}_{t+1}^k \tag{B.5}$$

$$0 = \hat{r}_t^* - E_t\hat{\pi}_{t+1}^* - \beta\frac{(1-\delta)}{\bar{\varepsilon}^A}E_t\hat{Q}_{t+1}^* + \hat{Q}_t^* - \frac{(\bar{\varepsilon}^A - \beta(1-\delta))}{\bar{\varepsilon}^A}\hat{r}_{t+1}^{k^*} \tag{B.6}$$

Euler equations :

$$0 = \left[\begin{array}{c} \bar{\varepsilon}^A E_t\hat{c}_{t+1} - (\bar{\varepsilon}^A + h)\hat{c}_t + h\hat{c}_{t-1} + (\bar{\varepsilon}^A - h)E_t\hat{\pi}_{t+1}^C - (\bar{\varepsilon}^A - h)\hat{r}_t \\ +\bar{\varepsilon}^A E_t\hat{\varepsilon}_{t+1}^A - h\hat{\varepsilon}_t^A + (\bar{\varepsilon}^A - h)\hat{\varepsilon}_t^C - (\bar{\varepsilon}^A - h)E_t\hat{\varepsilon}_{t+1}^C \end{array}\right] \tag{B.7}$$

$$0 = \left[\begin{array}{c} \bar{\varepsilon}^A E_t\hat{c}_{t+1}^* - (\bar{\varepsilon}^A + h^*)\hat{c}_t^* + h^*\hat{c}_{t-1}^* + (\bar{\varepsilon}^A - h^*)E_t\hat{\pi}_{t+1}^{C^*} - (\bar{\varepsilon}^A - h^*)\hat{r}_t^* \\ +\bar{\varepsilon}^A E_t\hat{\varepsilon}_{t+1}^{A^*} - h\hat{\varepsilon}_t^{A^*} + (\bar{\varepsilon}^A - h^*)\hat{\varepsilon}_t^{C^*} - (\bar{\varepsilon}^A - h^*)E_t\hat{\varepsilon}_{t+1}^{C^*} \end{array}\right] \tag{B.8}$$

Rental price of capital:

$$\hat{r}_t^k = \hat{w}_t + \hat{l}_t - \hat{k}_t + \hat{\varepsilon}_t^A \tag{B.9}$$

$$\hat{r}_t^{k^*} = \hat{w}_t^* + \hat{l}_t^* - \hat{k}_t^* + \hat{\varepsilon}_t^{A^*} \tag{B.10}$$

Wage setting equations:

$$0=\begin{bmatrix} -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w E_t\hat{w}_{t+1} \\ +\left((1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{1+\beta(\xi_w)^2}{1-\xi_w}+\sigma_L\frac{\lambda_w}{1-\lambda_w}\right)\hat{w}_t \\ -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\hat{w}_{t-1} \\ +(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w\frac{\gamma_w}{(1-\xi_w)}\hat{\pi}^C_t \\ -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}\gamma_w\hat{\pi}^C_{t-1} \\ -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\beta\xi_w}{(1-\xi_w)}E_t\hat{\pi}_{t+1} \\ +(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}\hat{\pi}_t \\ -\sigma_L\hat{l}_t-\sigma_L\hat{\varepsilon}^L_t-\frac{\hat{c}_t+\hat{\varepsilon}^A_t}{\bar{\varepsilon}^A-h}+\frac{h\hat{c}_{t-1}}{\bar{\varepsilon}^A-h}+\hat{t}^{P/P^C}_t+\hat{\varepsilon}^C_t+\hat{\varepsilon}^A_t \\ +(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w\frac{(1-\gamma_w)}{(1-\xi_w)}E_t\widehat{\bar{\pi}}_{t+1} \\ -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}(1-\gamma_w)\widehat{\bar{\pi}}_t \end{bmatrix} \tag{B.11}$$

$$0=\begin{bmatrix} -(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\beta\xi^*_w E_t\hat{w}^*_{t+1} \\ +\left((1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\frac{1+\beta(\xi^*_w)^2}{1-\xi^*_w}+\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w}\right)\hat{w}^*_t \\ -(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi^*_w}\hat{w}^*_{t-1} \\ +(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\beta\xi^*_w\frac{\gamma^*_w}{(1-\xi^*_w)}\hat{\pi}^{C^*}_t \\ -(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\frac{\xi^*_w}{(1-\xi^*_w)}\gamma^*_w\hat{\pi}^{C^*}_{t-1} \\ -(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\frac{\beta\xi^*_w}{(1-\xi^*_w)}E_t\hat{\pi}^*_{t+1} \\ +(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda_w})\frac{1}{1-\beta\xi^*_w}\frac{\xi^*_w}{(1-\xi^*_w)}\hat{\pi}^*_t \\ -\sigma^*_L\hat{l}_t-\sigma^*_L\hat{\varepsilon}^{L^*}_t-\frac{\hat{c}^*_t+\hat{\varepsilon}^{A^*}_t}{\bar{\varepsilon}^A-h^*}+\frac{h^*\hat{c}^*_{t-1}}{\bar{\varepsilon}^A-h^*}+\hat{t}^{P^*/P^{C^*}}_t+\hat{\varepsilon}^{C^*}_t+\hat{\varepsilon}^{A^*}_t \\ +(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\beta\xi^*_w\frac{(1-\gamma^*_w)}{(1-\xi^*_w)}E_t\widehat{\bar{\pi}}^*_{t+1} \\ -(1-\sigma^*_L\frac{\lambda^*_w}{1-\lambda^*_w})\frac{1}{1-\beta\xi^*_w}\frac{\xi^*_w}{(1-\xi^*_w)}(1-\gamma^*_w)\widehat{\bar{\pi}}^*_t \end{bmatrix} \tag{B.12}$$

Labor demand:

$$\hat{l}_t=\frac{1}{\alpha}(\hat{y}_t-\hat{\varepsilon}^Y_t)-\frac{(1-\alpha)}{\alpha}(\hat{k}_{t-1}-\hat{\varepsilon}^A_t) \tag{B.13}$$

$$\hat{l}^*_t=\frac{1}{\alpha}(\hat{y}^*_t-\hat{\varepsilon}^{Y^*}_t)-\frac{(1-\alpha)}{\alpha}(\hat{k}^*_{t-1}-\hat{\varepsilon}^{A^*}_t) \tag{B.14}$$

Employment:

$$\widehat{em}_t-\widehat{em}_{t-1}=\beta E_t\widehat{em}_{t+1}-\beta\widehat{em}_t+\frac{(1-\xi_L)(1-\xi_L\beta)}{\xi_L}(\hat{l}_t-\widehat{em}_t) \tag{B.15}$$

$$\widehat{em}^*_t-\widehat{em}^*_{t-1}=\beta E_t\widehat{em}^*_{t+1}-\beta\widehat{em}^*_t+\frac{(1-\xi^*_L)(1-\xi^*_L\beta)}{\xi^*_L}(\hat{l}^*_t-\widehat{em}^*_t) \tag{B.16}$$

GDP deflator equation (the New Keynesian Phillips Curve):

$$(\hat{\pi}_t-\widehat{\bar{\pi}}_t)=\begin{bmatrix} \frac{\beta}{1+\beta\gamma_p}(E_t\hat{\pi}_{t+1}-E_t\widehat{\bar{\pi}}_{t+1})+\frac{\gamma_p}{1+\beta\gamma_p}(\hat{\pi}_{t-1}-\widehat{\bar{\pi}}_t) \\ +\frac{\beta\gamma_p}{1+\beta\gamma_p}(E_t\widehat{\bar{\pi}}_{t+1}-\widehat{\bar{\pi}}_t)+\frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}((1-\alpha)\hat{w}_t+\alpha\hat{r}^k_t-\hat{\varepsilon}^Y_t+\hat{\varepsilon}^{mc}_t) \end{bmatrix} \tag{B.17}$$

$$(\hat{\pi}^*_t-\widehat{\bar{\pi}}^*_t)=\begin{bmatrix} \frac{\beta}{1+\beta\gamma^*_p}(E_t\hat{\pi}^*_{t+1}-E_t\widehat{\bar{\pi}}^*_{t+1})+\frac{\gamma^*_p}{1+\beta\gamma^*_p}(\hat{\pi}^*_{t-1}-\widehat{\bar{\pi}}^*_t) \\ +\frac{\beta\gamma^*_p}{1+\beta\gamma^*_p}(E_t\widehat{\bar{\pi}}^*_{t+1}-\widehat{\bar{\pi}}^*_t)+\frac{(1-\beta\xi^*_p)(1-\xi^*_p)}{(1+\beta\gamma^*_p)\xi^*_p}((1-\alpha)\hat{w}^*_t+\alpha\hat{r}^{k^*}_t-\hat{\varepsilon}^{Y^*}_t+\hat{\varepsilon}^{mc^*}_t) \end{bmatrix} \tag{B.18}$$

The Phillips curve for the imported consumption goods:

$$(\hat{\pi}^{C,imp}_t-\widehat{\bar{\pi}}_t)=\begin{bmatrix} \frac{\beta}{1+\beta\gamma^{imp}_p}(E_t\hat{\pi}^{C,imp}_{t+1}-E_t\widehat{\bar{\pi}}_{t+1})+\frac{\gamma^{imp}_p}{1+\beta\gamma_p}(\hat{\pi}^{C,imp}_{t-1}-\widehat{\bar{\pi}}_t) \\ +\frac{\beta\gamma^{imp}_p}{1+\beta\gamma^{imp}_p}(E_t\widehat{\bar{\pi}}_{t+1}-\widehat{\bar{\pi}}_t)+\frac{(1-\beta\xi^{imp}_p)(1-\xi^{imp}_p)}{(1+\beta\gamma^{imp}_p)\xi^{imp}_p}(\hat{t}^{P^*/P^{C,imp}}_t) \end{bmatrix} \tag{B.19}$$

$$(\hat{\pi}_t^{C,imp^*} - \widehat{\overline{\pi}}_t^*) = \left[\begin{array}{c} \frac{\beta}{1+\beta\gamma_p^{imp^*}}(E_t\hat{\pi}_{t+1}^{C,imp^*} - E_t\widehat{\overline{\pi}}_{t+1}^*) + \frac{\gamma_p^{imp^*}}{1+\beta\gamma_p^{imp^*}}(\hat{\pi}_{t-1}^{C,imp^*} - \widehat{\overline{\pi}}_t^*) \\ +\frac{\beta\gamma_p^{imp^*}}{1+\beta\gamma_p^{imp^*}}(E_t\widehat{\overline{\pi}}_{t+1}^* - \widehat{\overline{\pi}}_t^*) + \frac{(1-\beta\xi_p^{imp^*})(1-\xi_p^{imp^*})}{(1+\beta\gamma_p^{imp^*})\xi_p^{imp^*}}(\hat{t}_t^{P/P^{C,imp^*}}) \end{array} \right] \tag{B.20}$$

Consumer Price Inflation:

$$\hat{\pi}_t^C = \omega_C\hat{\pi}_t + (1-\omega_C)\pi_t^{C,imp} + \hat{\varepsilon}_t^{CPI} \tag{B.21}$$

$$\hat{\pi}_t^{C^*} = \omega_C^*\hat{\pi}_t^* - (1-\omega_C^*)\pi_t^{C,imp^*} + \hat{\varepsilon}_t^{CPI^*} \tag{B.22}$$

Aggregate demand equations:

$$\hat{y}_t = \left[\begin{array}{c} \omega_C\frac{\bar{C}}{\bar{Y}}\bar{\mathcal{T}}^{(1-\omega_C)}(-\hat{t}_t^{P/P^C} + \hat{c}_t) + \omega_I\frac{\bar{I}}{\bar{Y}}\bar{\mathcal{T}}^{(1-\omega_I)}((1-\omega_I)\hat{t}_t + \hat{\imath}_t) + \frac{\bar{G}}{\bar{Y}}\hat{g}_t \\ +\frac{(1-n)}{n}(1-\omega_C^*)\frac{\bar{C}^*}{\bar{Y}}\bar{\mathcal{T}}^{\omega_C^*}(\hat{t}_t^{P^{C^*}/P^{C,imp^*}} + \hat{c}_t^* - \hat{\varepsilon}_t^Z) \\ +\frac{(1-n)}{n}(1-\omega_I^*)\frac{\bar{I}^*}{\bar{Y}}\bar{\mathcal{T}}^{\omega_I^*}(\omega_I^*\hat{t}_t + \hat{\imath}_t^* - \hat{\varepsilon}_t^Z) \end{array} \right] \tag{B.23}$$

$$\hat{y}_t^* = \left[\begin{array}{c} \frac{n}{n-1}(1-\omega_C)\frac{\bar{C}}{\bar{Y}^*}\bar{\mathcal{T}}^{-\omega_C}(\hat{t}_t^{P^C/P^{C,imp}} + \hat{c}_t + \hat{\varepsilon}_t^Z) \\ +\frac{n}{n-1}(1-\omega_I)\frac{\bar{I}}{\bar{Y}^*}\bar{\mathcal{T}}^{-\omega_I}(\hat{\imath}_t - \omega_I\hat{t}_t + \hat{\varepsilon}_t^Z) + \frac{\bar{G}^*}{\bar{Y}^*}\hat{g}_t^* \\ +\omega_C^*\frac{\bar{C}^*}{\bar{Y}^*}\bar{\mathcal{T}}^{(\omega_C^*-1)}(-\hat{t}_t^{P^*/P^{C^*}} + \hat{c}_t^*) + \omega_I^*\frac{\bar{I}^*}{\bar{Y}^*}\bar{\mathcal{T}}^{(\omega_I^*-1)}((\omega_I^*-1)\hat{t}_t + \hat{\imath}_t^*) \end{array} \right] \tag{B.24}$$

Under complete asset market the perfect risk sharing condition is given by:

$$\hat{S}_t^{real} = \left[\begin{array}{c} -\frac{\varepsilon^A}{(\varepsilon^A-h^*)}\hat{c}_t^* + \frac{h^*}{(\varepsilon^A-h^*)}(\hat{c}_{t-1}^* - \hat{\varepsilon}_t^{A^*}) + \\ +\frac{\varepsilon^A}{(\varepsilon^A-h)}\hat{c}_t - \frac{h}{(\varepsilon^A-h)}(\hat{c}_{t-1} - \varepsilon_t^A) + \hat{\varepsilon}_t^{C^*} - \hat{\varepsilon}_t^C + \hat{\varepsilon}_t^Z \end{array} \right] \tag{B.25}$$

Under incomplete markets the risk sharing condition changes. It can be now read off the NFA equation and the uncovered interest rate parity. The linearized version of the NFA equation is obtained by totally differentiating of expression (4.68), and evaluating it in the steady state, where $\overline{\tilde{B}}^* = 0$, $\Phi(0) = 1$:

$$\hat{b}_t^* = \left[\begin{array}{c} \frac{(1-n)}{n}\left(\begin{array}{c} (1-\omega_C^*)\bar{C}^*\bar{\mathcal{T}}^{\omega_C^*}\left(c_t^* - \hat{\varepsilon}_t^Z + \hat{t}_t^{P^{C^*}/P^C,imp^*}\right) \\ +(1-\omega_I^*)\bar{I}^*\bar{\mathcal{T}}^{\omega_I^*}\left(\hat{\imath}_t^* - \hat{\varepsilon}_t^Z + \omega_I^*\hat{t}\right) \end{array} \right) \\ +(\omega_C-1)\bar{C}\bar{\mathcal{T}}^{(1-\omega_C)}\left(\hat{c}_t - \hat{t}_t^{P/P^C}\right) \\ +(\omega_I-1)\bar{I}\bar{\mathcal{T}}^{(1-\omega_I)}\left(\hat{\imath}_t + (1-\omega_I)\hat{t}_t\right) + \frac{1}{\beta}\hat{b}_{t-1}^* \end{array} \right], \tag{B.26}$$

where $\hat{b}_t^* = \tilde{B}_t^* - \overline{\tilde{B}}^*$. Taking a (log-) linear approximation of (4.40) and (4.41) (around $\overline{\tilde{B}}^* = 0$) yields the modified uncovered interest rate parity condition:

$$\hat{r}_t - \hat{r}_t^* = E_t\Delta\hat{S}_{t+1} - \Phi\hat{b}_t^* + \hat{\varepsilon}_t^{UIP} \tag{B.27}$$

The real exchange rate is given by:

$$\Delta\hat{S}_t^{real} = \Delta\hat{S}_t + \hat{\pi}_t^{C^*} - \hat{\pi}_t^C \tag{B.28}$$

Definitions of relative prices:

$$\Delta\hat{t}_t = \Delta\hat{S}_t + \hat{\pi}_t^* - \hat{\pi}_t \tag{B.29}$$

$$\hat{t}_t = \hat{t}_t^* \tag{B.30}$$

$$\Delta\hat{t}_t^{P/P^C} = \hat{\pi}_t - \hat{\pi}_t^C \tag{B.31}$$

$$\Delta\hat{t}_t^{P^*/P^{C^*}} = \hat{\pi}_t^* - \hat{\pi}_t^{C^*} \tag{B.32}$$

$$\Delta\hat{t}_t^{P^*/P^{C,imp}} = \Delta\hat{S}_t + \hat{\pi}_t^* - \hat{\pi}_t^{C,imp} \tag{B.33}$$

$$\Delta\hat{t}_t^{P/P^{C,imp^*}} = -\Delta\hat{S}_t + \hat{\pi}_t - \hat{\pi}_t^{C,imp^*} \tag{B.34}$$

$$\Delta\hat{t}_t^{P^C/P^{C,imp}} = \hat{\pi}_t^C - \hat{\pi}_t^{C,imp} \tag{B.35}$$

$$\Delta\hat{t}_t^{P^{C^*}/P^{C,imp^*}} = \hat{\pi}_t^{C^*} - \hat{\pi}_t^{C,imp^*} \tag{B.36}$$

The model for flexible exchange rate regime is closed with two separate monetary feedback rules:

$$\hat{r}_t = \rho_R \hat{r}_{t-1} + (1-\rho_R)\left[\widehat{\overline{\pi}}_t + r_\pi(\hat{\pi}_{t-1}^C - \widehat{\overline{\pi}}_t) + r_y(\hat{y}_{t-1})\right] + u_t^R + u_t^{R^{com}} \tag{B.37}$$

$$\hat{r}_t^* = \rho_R^* \hat{r}_{t-1}^* + (1-\rho_R^*)\left[\widehat{\overline{\pi}}_t^* + r_\pi^*\left(\hat{\pi}_{t-1}^{C^*} - \widehat{\overline{\pi}}_t^*\right) + r_y^*(\hat{y}_{t-1}^*)\right] + u_t^{R^*} + u_t^{R^{com}} \tag{B.38}$$

The DSGE model for the monetary union is closed with the common monetary policy rule:

$$\hat{r}_t^{EMU} = \rho_R^{EMU}\hat{r}_{t-1}^{EMU} + (1-\rho_R^{EMU})\left[\widehat{\overline{\pi}}_t^{EMU} + r_\pi^{EMU}(\hat{\pi}_{t-1}^{EMU} - \widehat{\overline{\pi}}_t^{EMU}) + r_y^{EMU}(\hat{y}_{t-1}^{EMU})\right] + u_t^{R\_EMU} \tag{B.39}$$

where the aggregated Euro area output and inflation are given by:

$$\hat{y}_t^{EMU} = n\hat{y}_t + (1-n)\hat{y}_t^* \tag{B.40}$$

$$\hat{\pi}_t^{EMU} = n\hat{\pi}_t^{C^*} + (1-n)\hat{\pi}_t^{C^*} \tag{B.41}$$

In addition we impose that

$$\Delta\hat{S}_t = 0 \tag{B.42}$$

The stochastic of the model is driven by the following autoregressive processes:

Unit root technology shock in Home country:

$$\hat{\varepsilon}_t^A = \rho_A \hat{\varepsilon}_{t-1}^A + u_t^A \tag{B.43}$$

Asymmetric technology shock:

$$\hat{\varepsilon}_t^Z = \rho_Z \hat{\varepsilon}_{t-1}^Z + u_t^Z \tag{B.44}$$

Changes in the growth rate of technology in the Foreign country are determined as follows:

$$\hat{\varepsilon}_t^{A^*} = \hat{\varepsilon}_t^A - \hat{\varepsilon}_t^Z + \hat{\varepsilon}_{t-1}^Z \tag{B.45}$$

Stationary technology shocks:

$$\hat{\varepsilon}_t^Y = \rho_Y \hat{\varepsilon}_{t-1}^Y + u_t^Y + u_t^{Y^{com}} \tag{B.46}$$

$$\hat{\varepsilon}_t^{Y^*} = \rho_Y^* \hat{\varepsilon}_{t-1}^{Y^*} + u_t^{Y^*} + u_t^{Y^{com}} \tag{B.47}$$

Preference shocks:

$$\hat{\varepsilon}_t^C = \rho_C \hat{\varepsilon}_{t-1}^C + u_t^C + u_t^{C^{com}} \tag{B.48}$$

$$\hat{\varepsilon}_t^{C^*} = \rho_C^* \hat{\varepsilon}_{t-1}^{C^*} + u_t^{C^*} + u_t^{C^{com}} \tag{B.49}$$

Labor supply shocks:

$$\hat{\varepsilon}_t^L = \rho_L \hat{\varepsilon}_{t-1}^L + u_t^L + u_t^{L^{com}} \tag{B.50}$$

$$\hat{\varepsilon}_t^{L^*} = \rho_L^* \hat{\varepsilon}_{t-1}^{L^*} + u_t^{L^*} + u_t^{L^{com}} \tag{B.51}$$

Investment shocks:

$$\hat{\varepsilon}_t^I = \rho_I \hat{\varepsilon}_{t-1}^I + u_t^I + u_t^{I^{com}} \tag{B.52}$$

$$\hat{\varepsilon}_t^{I^*} = \rho_I^* \hat{\varepsilon}_{t-1}^{I^*} + u_t^{I^*} + u_t^{I^{com}} \tag{B.53}$$

Government spending shocks:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} + u_t^G + u_t^{G^{com}} \tag{B.54}$$

$$\hat{g}_t^* = \rho_G^* \hat{g}_{t-1}^* + u_t^{G^*} + u_t^{G^{com}} \tag{B.55}$$

The law of motion for the UIP shock is given by:

$$\hat{\varepsilon}_t^{UIP} = \rho_{UIP} \hat{\varepsilon}_t^{UIP} + u_t^{UIP} \tag{B.56}$$

In order to estimate the model on the unfiltered inflation and interest rate series, we define the following shocks to the respective inflation targets:

$$\widehat{\overline{\pi}}_t = \rho_\pi \widehat{\overline{\pi}}_{t-1} + u_t^\pi + u_t^{\pi^{com}} \tag{B.57}$$

$$\widehat{\overline{\pi}}_t^* = \rho_\pi \widehat{\overline{\pi}}_{t-1}^* + u_t^{\pi^*} + u_t^{\pi^{com}} \tag{B.58}$$

$$\widehat{\overline{\pi}}_t^{EMU} = \rho_\pi \widehat{\overline{\pi}}_{t-1}^{EMU} + u_t^{\pi\_EMU} \tag{B.59}$$

Closed economy models are obtained in a straightforward way by setting the share of imported consumption and investment goods to zero.

Appendix C

Derivation of wage and price equations

C.1 The New-Keynesian Phillips Curve with a time varying markup

The strategy for deriving the New-Keynesian Phillips curve involves derivation of the first order condition for the Calvo optimizing firms and determination of the aggregate price index. In order to facilitate the identification of the time varying markup, we assume that the steady state inflation rate $(\bar{\pi}-1)$ is zero.

In each period with probability $1-\xi_p$ a firm may adjust its price $P_t(z)$ to the level at which it maximizes the discounted future profits. Since all firms adjusting prices choose the same optimum, we suppress the firm-specific indexation (z) and simply denote the price chosen by P_t^{opt}. Firms not allowed to re-optimize their price update the price indexing it to the last period inflation:

$$P_{t+1} = P_t\left(\pi_t\right)^{\gamma_p}, \tag{C.1}$$

where γ_p is an indexation degree.

In period $t+i$ the price is then determined as follows:

$$P_{t+i} = \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_p} P_t^{opt}$$

If we define the nominal marginal cost as $MC_t^{nom} = P_t MC_t$ and set the firms' stochastic discount factor equal to $\beta^i\Lambda_{t,t+i} = \beta^i\frac{U_{C,t+i}}{U_{C,t}}$, where $U_{C,t+i}$ is the household's marginal utility of consumption at time $t+i$, the profit maximization problem is given by:

$$\max_{P_t^{opt}} E_t\sum_{i=0}^{\infty}(\xi_p\beta)^i\Lambda_{t,t+i}\left[\frac{P_t^{opt}\left(\pi_t\pi_{t+1}...\pi_{t+i-1}\right)^{\gamma_p} - MC_{t+i}^{nom}}{P_{t+i}}Y_{t+i}(z)\right] \tag{C.2}$$

subject to the demand function:

$$Y_t(z) = Y_t\left(\frac{P_t(z)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \tag{C.3}$$

derived from the cost minimization of the final good producer.

The first order condition can be written as follows:

$$E_t\sum_{i=0}^{\infty}(\xi_p\beta)^i\Lambda_{t,t+i}\left(P_t^{opt}\right)^{-\frac{1+2\lambda_{p,t+i}}{\lambda_{p,t+i}}}\left[\begin{array}{c} -\frac{1}{\lambda_{p,t+i}}\frac{\left(\pi_t\pi_{t+1}...\pi_{t+i-1}\right)^{\gamma_p}}{P_{t+i}}Y_{t+i}\frac{P_t^{opt}}{P_{t+i}}\left(P_{t+i}\right)^{\frac{1+2\lambda_{p,t+i}}{\lambda_{p,t+i}}} \\ +\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}MC_{t+i}Y_{t+i}\left(P_{t+i}\right)^{\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} \end{array}\right] = 0 \tag{C.4}$$

Note that by substituting back for $Y_t(z) = Y_t\left(\frac{P_t(z)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$ we arrive at expression (3.17):

$$E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} Y_{t+i}(z) \left(-\frac{1}{\lambda_{p,t+i}}\right) \left[\begin{array}{c} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} \frac{P_t^{opt}}{P_{t+i}} \\ -(1+\lambda_{p,t+i}) MC_{t+i} \end{array} \right] = 0 \tag{C.5}$$

This equation sets expected present value of future marginal revenue equal to the expected present value of future marginal cost. Rearranging terms yields the following expression for optimal price P_t^{opt} :

$$P_t^{opt} = \frac{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}} MC_{t+i} Y_t (P_{t+i})^{\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}}}{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1}{\lambda_{p,t+i}} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} Y_t (P_{t+i})^{\frac{1}{\lambda_{p,t+i}}}} \tag{C.6}$$

In order to write the FOC in terms of stationary variables we multiply (C.6) by $\frac{1}{P_t}$ to arrive at:

$$P_t^{+} = \frac{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}} MC_{t+i} Y_{t+i} (P_{t+i}/P_t) (P_{t+i}/P_t)^{\frac{1}{\lambda_{p,t+i}}}}{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1}{\lambda_{p,t+i}} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} Y_{t+i} (P_{t+i}/P_t)^{\frac{1}{\lambda_{p,t+i}}}}, \tag{C.7}$$

where $P_t^{+} = \frac{P_t^{opt}}{P_t}$.

Note that since the markup is stochastic, we make here a systematic error. However, as our final goal is to linearize the first order condition for the price setting and we know that for the linear system Certainty Equivalence holds, this error will not affect our final linearized equation.

Rewrite the expression (C.7) defining the following two auxiliary variables.

$$P_t^{+} = \frac{S_t}{V_t} \tag{C.8}$$

$$S_t \equiv E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}} MC_{t+i} Y_{t+i} (P_{t+i}/P_t)^{\frac{1+\lambda_{p,t+i}}{\lambda_{p,t+i}}} \tag{C.9}$$

$$V_t \equiv E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{1}{\lambda_{p,t+i}} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} Y_{t+i} (P_{t+i}/P_t)^{\frac{1}{\lambda_{p,t+i}}} \tag{C.10}$$

The infinite discount sums S_t and V_t have a recursive representation (we substitute for $\Lambda_{t,t+i} = \frac{\varepsilon_{t+i}^b (C_{t+i} - hC_{t+i-1})^{-\sigma_c}}{\varepsilon_t^b (C_t - hC_{t-1})^{-\sigma_c}}$):

$$S_t = \frac{1+\lambda_{p,t}}{\lambda_{p,t}} MC_t Y_t + \xi_p \beta E_t \left[\frac{\varepsilon_{t+1}^b (C_{t+1} - hC_t)^{-\sigma_c}}{\varepsilon_t^b (C_t - hC_{t-1})^{-\sigma_c}} (\pi_{t+1})^{\frac{1+\lambda_{p,t+1}}{\lambda_{p,t+1}}} S_{t+1} \right] \tag{C.11}$$

$$V_t = \frac{1}{\lambda_{p,t}} Y_t + \xi_p \beta E_t \left[\frac{\varepsilon_{t+1}^b (C_{t+1} - hC_t)^{-\sigma_c}}{\varepsilon_t^b (C_t - hC_{t-1})^{-\sigma_c}} (\pi_t)^{\gamma_p} (\pi_{t+1})^{\frac{1}{\lambda_{p,t+1}}} V_{t+1} \right] \tag{C.12}$$

Log-linearizing the auxiliary equations around a steady state with zero inflation yields:

$$\hat{p}_t^{+} = \hat{S}_t - \hat{V}_t \tag{C.13}$$

$$\begin{aligned}\hat{S}_t &= (1-\xi_p\beta)\hat{y}_t + (1-\xi_p\beta)\widehat{mc}_t - \frac{(1-\xi_p\beta)}{(1+\lambda_p)}\hat{\lambda}_{p,t} \\ &+ \xi_p\beta\left[\begin{array}{c}\left(E_t\hat{\varepsilon}^b_{t+1} - \hat{\varepsilon}^b_t + \frac{\sigma_c}{(1-h)}\left(-E_t\hat{c}_{t+1} + (1+h)\,\hat{c}_t + h\hat{c}_{t-1}\right)\right) \\ +\frac{1+\lambda_p}{\lambda_p}E_t\hat{\pi}_{t+1} + E_t\hat{S}_{t+1}\end{array}\right]\end{aligned} \tag{C.14}$$

$$\begin{aligned}\hat{V}_t &= (1-\xi_p\beta)\,\hat{y}_t - (1-\xi_p\beta)\,\hat{\lambda}_{p,t} \\ &+\xi_p\beta\left[\begin{array}{c}\left(E_t\hat{\varepsilon}^b_{t+1} - \hat{\varepsilon}^b_t + \frac{\sigma_c}{(1-h)}\left(-E_t\hat{c}_{t+1} + (1+h)\,\hat{c}_t + h\hat{c}_{t-1}\right)\right) \\ +\gamma_p\hat{\pi}_t + \frac{1}{\lambda_p}E_t\hat{\pi}_{t+1} + E_t\hat{V}_{t+1}\end{array}\right]\end{aligned} \tag{C.15}$$

Substituting expressions (C.14) and (C.15) into (C.13) one obtains:

$$\begin{aligned}\hat{p}^+_t &= (1-\xi_p\beta)\widehat{mc}_t + (1-\xi_p\beta)\left(-\frac{1}{(1+\lambda_p)}+1\right)\hat{\lambda}_{p,t} \\ &+\xi_p\beta E_t\left(E_t\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t + E_t\hat{p}^+_{t+1}\right)\end{aligned} \tag{C.16}$$

By rendering the aggregate price index stationary

$$P_t^{-1/\lambda_{p,t}} = (1-\xi_p)\left(P_t^{opt}\right)^{-1/\lambda_{p,t}} + \xi_p\left(P_{t-1}\pi_{t-1}^{\gamma_p}\right)^{-1/\lambda_{p,t}} \tag{C.17}$$

we arrive at

$$1^{-1/\lambda_{p,t}} = (1-\xi_p)\left(P_t^+\right)^{-1/\lambda_{p,t}} + \xi_p\left(\frac{1}{\pi_t}\pi_{t-1}^{\gamma_p}\right)^{-1/\lambda_{p,t}}. \tag{C.18}$$

The log-linearized aggregate price index is then

$$\hat{p}^+_t = \frac{1}{(1-\xi_p)}\left(\xi_p\hat{\pi}_t - \gamma_p\xi_p\hat{\pi}_{t-1}\right) \tag{C.19}$$

Subsituting the above expression into equation (C.16) and using the linearized equation of the marginal cost ($\widehat{mc}_t = \alpha\hat{r}^k_t + (1-\alpha)\hat{w}_t - \hat{\varepsilon}^a_t$) we arrive at:

$$\begin{aligned}\hat{\pi}_t &= \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p\,(1+\beta\gamma_p)}\left(\alpha\hat{r}^k_t + (1-\alpha)\hat{w}_t - \hat{\varepsilon}^a_t + \frac{\lambda_p}{(1+\lambda_p)}\hat{\lambda}_{p,t}\right) \\ &+\frac{\beta}{(1+\beta\gamma_p)}E_t\hat{\pi}_{t+1} + \frac{\gamma_w}{(1+\beta\gamma_p)}\hat{\pi}_{t-1}\end{aligned} \tag{C.20}$$

C.2 Wage equation with a time varying markup

We assume that households face a constant probability $1-\xi_w$ of receiving a signal that allows them to change their wage. Households that do not receive the signal to update their nominal wage index it to last period's price inflation rate: $W_t(j) = \pi_{t-1}^{\gamma_w}W_{t-1}(j)$. Let W_t^{opt} be the nominal reset wage in period t of those agents that receive a signal to change their price. The optimization problem is then given by:

$$\max_{W_t^{opt}} E_t\sum_{i=0}^{\infty}(\beta\xi_w)^i\left\{\begin{array}{c}-(1+\tau_{w,t+i})\varepsilon^b_t\varepsilon^L_t\frac{(l_{t+i}(j))^{1+\sigma_L}}{1+\sigma_L} \\ +(1+\tau_{w,t+i})\lambda_{t+i}\left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w}\frac{W_t^{opt}}{P_t}l_{t+i}(j)\end{array}\right\} \tag{C.21}$$

subject to the labor demand:

$$l_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t. \tag{C.22}$$

The optimization problem can be rewritten as follows:

$$\max_{W_t^{opt}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -(1+\tau_{w,t+i})\varepsilon_t^b \varepsilon_t^L \dfrac{\left(\frac{W_t^{opt}}{W_{t+i}}^{-(1+\lambda_{w,t+i})/\lambda_{w,t+i}} L_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\ +(1+\tau_{w,t+i})\lambda_{t+i}\left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} \frac{W_t^{opt}}{P_t}\left(\frac{W_t^{opt}}{W_{t+i}}\right)^{-(1+\lambda_{w,t+i})/\lambda_{w,t+i}} L_{t+i} \end{array} \right\} \tag{C.23}$$

The first order condition with respect to W_t^{opt} is then given by:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^b \varepsilon_t^L \left(\left(\frac{W_t^{opt}}{W_{t+i}}\right)^{-(1+\lambda_{w,t+i})/\lambda_{w,t+i}} L_{t+i}\right)^{\sigma_L} \left(-\left(\frac{W_t^{opt}}{W_{t+i}}\right)^{-\frac{1+2\lambda_{w,t+i}}{\lambda_{w,t+i}}} \frac{1}{W_{t+i}} \right) L_{t+i} \\ +\frac{1}{1+\lambda_{w,t+i}}\lambda_{t+i}\left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} \frac{1}{P_t}\left(W_t^{opt}\right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} \left(\frac{1}{W_{t+i}}\right)^{-\frac{1+\lambda_{w,t+i}}{\lambda_{w,t+i}}} L_{t+i} \end{array} \right\} = 0,$$

which we can write as:

$$\begin{aligned} 0 \;=\; & E_t \sum_{i=0}^{\infty} (\xi_w\beta)^i \left(\frac{W_t^{opt}}{W_{t+i}}\right)^{-\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}} L_{t+i}\lambda_{t+i} \qquad \text{(C.24)} \\ & \times \left[\frac{W_t^{opt}}{(1+\lambda_{w,t+i})P_{t+i}} \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} - \frac{\varepsilon_{t+i}^b \varepsilon_{t+i}^l \left(\frac{W_t^{opt}}{W_{t+i}}\right)^{-\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}\sigma_L} L_{t+i}^{\sigma_L}}{\lambda_{t+i}} \right] \end{aligned}$$

Note that by substituing for $-\varepsilon_t^l \varepsilon_t^b \left(l_t(j)\right)^{\sigma_l} = U_{L,t}(j)$ and $\lambda_t = U_{C,t}(j)$, where $U_{C,t}(j)$ is the marginal utility of consumption and $U_{L,t}(j)$ is the marginal utility of labor in those states of the world where the price remains fixed, we arrive at expression (3.20).

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[\frac{W_t^{opt}}{P_{t+i}} \frac{l_{t+i}(j) U_{C,t+i}(j)}{1+\lambda_{w,t+i}} \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} - l_{t+i}(j) U_{L,t+i}(j) \right] = 0 \tag{C.25}$$

Using equation (C.24) we can solve for the nominal wage W_t^{opt}

$$W_t^{opt} = \left(\frac{\sum_{i=0}^{\infty} (\xi_w\beta)^i \varepsilon_{t+i}^b \varepsilon_{t+i}^l W_{t+i}^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}(1+\sigma_L)} L_{t+i}^{1+\sigma_L}}{\sum_{i=0}^{\infty} (\xi_w\beta)^i W_{t+i}^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}} L_{t+i}\lambda_{t+i}[P_{t+i}(1+\lambda_{w,t+i})]^{-1} \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w}} \right)^{\frac{1}{1+\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}\sigma_L}} \tag{C.26}$$

Note that we use here the properties of the Certainty Equivalence.

Define $W_t^+ \equiv \frac{W_t^{opt}}{W_t}$, $\tilde{W}_t \equiv \frac{W_t}{P_t}$ and $\pi_{t,t+i}^w \equiv \frac{W_{t+i}}{W_t}$. We can write the above condition as

$$W_t^+ = \left(\frac{\sum_{i=0}^{\infty} (\xi_w\beta)^i \varepsilon_{t+i}^b \varepsilon_{t+i}^l (\pi_{t,t+i}^w)^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}(1+\sigma_l)} L_{t+i}^{1+\sigma_l}}{\sum_{i=0}^{\infty} (\xi_w\beta)^i (\pi_{t,t+i}^w)^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}} L_{t+i}\lambda_{t+i}\tilde{W}_{t+i}[\pi_{t+i}^w(1+\lambda_{w,t+i})]^{-1} \left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w}} \right)^{\frac{1}{1+\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}\sigma_l}}$$

Rewrite this expression defining the following two auxiliary variables.

$$G_t \equiv E_t \sum_{i=0}^{\infty} (\xi_w \beta)^i (\pi^w_{t,t+i})^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}} L_{t+i}\lambda_{t+i}\tilde{W}_{t+i}[\pi^w_{t+i}(1+\lambda_{w,t+i})]^{-1}\left(\pi_{t+i-1}...\pi_{t+1}\pi_t\right)^{\gamma_w} \quad (C.27)$$

$$D_t \equiv E_t \sum_{i=0}^{\infty} (\xi_w \beta)^i \varepsilon^b_{t+i}\varepsilon^l_{t+i}(\pi^w_{t,t+i})^{\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}(1+\sigma_L)} L^{1+\sigma_L}_{t+i} \quad (C.28)$$

These infinite sums have a recursive representation. The behavior of optimizing firms is fully described by the following three equations

$$w^+_t = \left(\frac{D_t}{G_t}\right)^{\frac{1}{1+\frac{(1+\lambda_{w,t+i})}{\lambda_{w,t+i}}\sigma_l}} \quad (C.29)$$

$$D_t = \varepsilon^b_t \varepsilon^l_t L^{1+\sigma_l}_t + \beta\xi_w E_t(\pi^w_{t+1})^{\frac{(1+\lambda_{w,t+1})}{\lambda_{w,t+1}}(1+\sigma_l)} D_{t+1} \quad (C.30)$$

$$G_t = \frac{L_t\lambda_t\tilde{W}_t}{1+\lambda_{w,t}} + \beta\xi_w E_t\left(\pi^w_{t+1}\right)^{\frac{(1+\lambda_{w,t+1})}{\lambda_{w,t+1}}-1} G_{t+1}\left(\pi_t\right)^{\gamma_w} \quad (C.31)$$

Log-linearizing the auxiliary equations defining recursively the condition for optimal wage setting around a steady state with zero wage inflation yields

$$\hat{D}_t = (1-\beta\xi_w)\left[(1+\sigma_L)\hat{l}_t + \hat{\varepsilon}^b_t + \hat{\varepsilon}^l_t\right] + \beta\xi_w\left[\frac{(1+\lambda_w)}{\lambda_w}(1+\sigma_L)E_t\hat{\pi}^w{}_{t+1} + E_t\hat{D}_{t+1}\right] \quad (C.32)$$

$$\hat{G}_t = (1-\beta\xi_w)\left[\hat{w}_t + \hat{l}_t + \hat{\lambda}_t - \frac{\lambda_w}{1+\lambda_w}\hat{\lambda}_{w,t}\right] + \frac{\beta\xi_w}{\lambda_w}\left[E_t\hat{\pi}^w{}_{t+1} + E_t\hat{G}_{t+1} + \gamma_w\hat{\pi}_t\right] \quad (C.33)$$

Substituting these two equations into the log-linearized first order condition for wage setting (C.29) yields

$$\begin{aligned}\hat{w}^+_t \quad = \quad & \frac{(1-\xi_w\beta)}{(1+\frac{1+\lambda_w}{\lambda_w}\sigma_L)}\left[\sigma_L\hat{l}_t + \hat{\varepsilon}^b_t + \hat{\varepsilon}^l_t - \hat{\lambda}_t - \hat{w}_t + \frac{\lambda_w}{1+\lambda_w}\hat{\lambda}_{w,t}\right] \\ & +\beta\xi_w\left(E_t\widehat{\pi^w}_{t+1} + E_t\hat{w}^+_{t+1} - \gamma_w\hat{\pi}_t\right)\end{aligned}$$

Here, $\hat{\lambda} = \hat{\varepsilon}^b_t - \frac{\sigma_C}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right)$. The condition then reads

$$\begin{aligned}\hat{w}^+_t \quad = \quad & \frac{(1-\xi_w\beta)}{(1+\frac{1+\lambda_w}{\lambda_w}\sigma_L)}\left[\sigma_L\hat{l}_t + \hat{\varepsilon}^l_t + \frac{\sigma_C}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) - \hat{w}_t + \frac{\lambda_w}{1+\lambda_w}\hat{\lambda}_{w,t}\right] \\ & +\beta\xi_w\left(E_t\hat{\pi}^w_{t+1} + E_t\hat{w}^+_{t+1} - \gamma_w\hat{\pi}_t\right)\end{aligned} \quad (C.34)$$

Rendering the aggregate wage index stationary

$$W_t^{-1/\lambda_{w,t}} \;=\; (1-\xi_w)\left(W^{opt}_t\right)^{-1/\lambda_{w,t}} + \xi_w\left(W_{t-1}\pi^{\gamma_w}_{t-1}\right)^{-1/\lambda_{w,t}} \quad (C.35)$$

we obtain:

$$1^{-1/\lambda_{w,t}} \;=\; (1-\xi_w)\left(W^+_t\right)^{-1/\lambda_{w,t}} + \xi_w\left(\frac{1}{\pi^w_t}\pi^{\gamma_w}_{t-1}\right)^{-1/\lambda_{w,t}} \quad (C.36)$$

The log-linearization yields:

$$\hat{w}^+_t = \frac{1}{(1-\xi_w)}\left(\xi_w\hat{\pi}^w_t - \gamma_w\xi_w\hat{\pi}_{t-1}\right) \quad (C.37)$$

Subsituting the above expression into equation (C.34) we arrive at:

$$\hat{w}_t = \frac{(1-\xi_w)}{\xi_w(1+\beta)} \left[\begin{array}{c} \frac{\beta\xi_w}{(1-\xi_w)} E_t \hat{w}_{t+1} + \frac{\xi_w}{(1-\xi_w)} \hat{w}_{t-1} + \frac{\beta\xi_w}{(1-\xi_w)} E_t \hat{\pi}_{t+1} \\ -\left(\frac{\xi_w + \gamma_w \beta \xi_w}{(1-\xi_w)}\right) \hat{\pi}_t + \frac{\gamma_w \xi_w}{(1-\xi_w)} \hat{\pi}_{t-1} \\ + \frac{(1-\xi_w\beta)}{(1+\frac{1+\lambda_w}{\lambda_w}\sigma_L)} \left[\sigma_L \hat{l}_t + \hat{\varepsilon}_t^l + \frac{\sigma_C}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) - \hat{w}_t + \frac{\lambda_w}{1+\lambda_w} \hat{\lambda}_{w,t} \right] \end{array} \right] \tag{C.38}$$

C.3 The New-Keynesian Phillips curve in the presence of non-zero steady state inflation

The derivation of the New-Keynesian Phillips curve presented here, as opposed to that of Section C.1, deals with the non-zero steady state inflation. It should be, however, noted that then it is not possible to obtain the linearized version of the Phillips Curve with an explicit time-varying markup.

In each period with probability $1-\xi_p$ a firm may adjust its price $P_t(z)$ to the level at which it maximizes the discounted future profits. Since all firms adjusting prices choose the same optimum, we suppress the firm-specific indexation (z) and simply denote the price chosen by P_t^{opt}. Firms not allowed to re-optimize their price update the price indexing it to the last period inflation π_t and to the inflation target for the next period $\bar{\pi}_{t+1}$:

$$P_{t+1} = P_t \left(\pi_t\right)^{\gamma_p} \left(\bar{\pi}_{t+1}\right)^{1-\gamma_p}, \tag{C.39}$$

where γ_p is an indexation degree.

Hence, in period $t+i$ the price is determined as follows:

$$\begin{aligned} P_{t+1} &= \left(\pi_t^C\right)^{\gamma_p} \left(\bar{\pi}_{t+1}\right)^{1-\gamma_p} P_t^{opt} \\ P_{t+2} &= \left(\pi_{t+1}^C \pi_t^C\right)^{\gamma_p} \left(\bar{\pi}_{t+2}\bar{\pi}_{t+1}\right)^{1-\gamma_p} P_t^{opt} \\ &\ldots \\ P_{t+i} &= \left(\pi_{t+i-1}^C \ldots \pi_{t+1}^C \pi_t\right)^{\gamma_p} \left(\bar{\pi}_{t+i}\ldots\bar{\pi}_{t+2}\bar{\pi}_{t+1}\right)^{1-\gamma_p} P_t^{opt} \end{aligned}$$

If we define the nominal marginal cost as $MC_t^{nom} = P_t MC_t$ and set the firms' stochastic discount factor equal to $\beta^i \Lambda_{t,t+i} = \beta^i \frac{U_{C,t+i}}{U_{C,t}}$, where $U_{C,t+i}$ is the household's marginal utility of consumption at time $t+i$, the profit maximization problem is given by:

$$\max_{P_t^{opt}} E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \left[\frac{P_t^{opt} \left(\pi_t \pi_{t+1} \ldots \pi_{t+i-1}\right)^{\gamma_p} \left(\bar{\pi}_{t+1}\bar{\pi}_{t+2}\ldots\bar{\pi}_{t+i}\right)^{1-\gamma_p} - \varepsilon_t^{mc} MC_{t+i}^{nom}}{P_{t+i}} Y_{t+i}(z) \right] \tag{C.40}$$

subject to the demand function

$$Y_t(z) = \frac{1}{n} Y_t \left(\frac{P_t(z)}{P_t}\right)^{\frac{\lambda_p}{\lambda_p - 1}} \tag{C.41}$$

derived from the cost minimization of the final good producer. Here, ε_t^{mc} is an i.i.d. shock, which can be interpreted as a fiscal shock to marginal cost.

The first order condition may be written as follows:

$$\frac{1}{n} E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \left[\begin{array}{c} \frac{\lambda_p}{\lambda_p - 1} + 1 \quad \frac{\left(P_t^{opt}\right)^{\frac{\lambda_p}{\lambda_p-1}} \frac{1}{P_{t+i}}^{\frac{\lambda_p}{\lambda_p-1}} \left(\pi_t \pi_{t+1}\ldots\pi_{t+i-1}\right)^{\gamma_p} \left(\bar{\pi}_{t+1}\bar{\pi}_{t+2}\ldots\bar{\pi}_{t+i}\right)^{1-\gamma_p}}{P_{t+i}} Y_{t+i} \\ -\frac{\lambda_p}{\lambda_p - 1} \frac{\varepsilon_t^{mc} MC_{t+i}^{nom} \frac{1}{P_{t+i}}^{\frac{\lambda_p}{\lambda_p-1}} \left(P_t^{opt}\right)^{\frac{\lambda_p}{\lambda_p-1}-1}}{P_{t+i}} Y_{t+i} \end{array} \right] = 0 \tag{C.42}$$

This equation sets expected present value of future marginal revenue equal to the expected present value of future marginal cost. Rearranging terms yields the following expression for optimal price P_t^{opt} :

$$P_t^{opt} = \left(\frac{\lambda_p}{2\lambda_p - 1}\right) \frac{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i}^{mc} \varepsilon_t^{mc} MC_{t+i} \left(\frac{P_t^{opt}}{P_{t+i}}\right)^{\frac{\lambda_p}{\lambda_p - 1}} Y_{t+i}}{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \Lambda_{t,t+i} \frac{\frac{P_t^{opt}}{P_{t+i}}^{\frac{\lambda_p}{\lambda_p-1}} (\pi_t \pi_{t+1} ... \pi_{t+i-1})^{\gamma_p} (\bar{\pi}_{t+1} \bar{\pi}_{t+2} ... \bar{\pi}_{t+i})^{1-\gamma_p}}{P_{t+i}} Y_{t+i}} \tag{C.43}$$

In order to write the FOC in terms of stationary variables we multiply (C.43) by $\frac{1}{P_t}$and utilize that $A_{t+i}/A_t \equiv \varepsilon_{t+1}^A ... \varepsilon_{t+i}^A$ to arrive at:

$$P_t^+ = \left(\frac{\lambda_p}{2\lambda_p - 1}\right) \frac{E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \tilde{\Lambda}_{t,t+i} \varepsilon_t^{mc} MC_{t+i} \left(P_{t+i}^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} \tilde{Y}_{t+i}}{E_t \left[\begin{array}{c} \sum_{i=0}^{\infty} (\xi_p \beta)^i \tilde{\Lambda}_{t,t+i} \left(P_{t+i}^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} (\pi_{t+1} ... \pi_{t+i})^{-1} \\ \times (\pi_t ... \pi_{t+i-1})^{\gamma_p} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+i})^{1-\gamma_p} \tilde{Y}_{t+i} \end{array}\right]}, \tag{C.44}$$

where $P_t^+ \equiv \frac{P_t^{opt}}{P_t}$, $P_{t+i}^A \equiv \frac{P_t^{opt}}{P_{t+i}}$, $\tilde{\Lambda}_{t,t+i} \equiv \frac{1}{\varepsilon_t^A ... \varepsilon_{t+i-1}^A} \frac{\varepsilon_{t+i}^C \left(\varepsilon_t^A \tilde{C}_t - h\tilde{C}_{t-1}\right)}{\varepsilon_t^C \left(\varepsilon_{t+i}^A \tilde{C}_{t+i} - h\tilde{C}_{t+i-1}\right)}$ and $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$ are stationary variables. We can rewrite the above using the auxiliary variables S_t and V_t.

$$\begin{aligned} P_t^+ &\equiv \left(\frac{\lambda_p}{2\lambda_p - 1}\right) \frac{S_t}{V_t} \\ S_t &\equiv E_t \sum_{i=0}^{\infty} (\xi_p \beta)^i \tilde{\Lambda}_{t,t+i} \varepsilon_t^{mc} MC_{t+i} \left(P_{t+i}^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} \tilde{Y}_{t+i} \\ V_t &\equiv E_t \left[\begin{array}{c} \sum_{i=0}^{\infty} (\xi_p \beta)^i \tilde{\Lambda}_{t,t+i} \left(P_{t+i}^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} (\pi_{t+1} ... \pi_{t+i})^{-1} \\ \times (\pi_t ... \pi_{t+i-1})^{\gamma_p} (\bar{\pi}_{t+1} ... \bar{\pi}_{t+i})^{1-\gamma_p} \tilde{Y}_{t+i} \end{array}\right] \end{aligned}$$

Fortunately, both S_t and V_t admit the recursive representation:

$$S_t = \varepsilon_t^{mc} MC_t \tilde{Y}_t \left(P_t^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} + \xi_p \beta E_t \left[\tilde{\Lambda}_{t,t+1} S_{t+1}\right] \tag{C.45}$$

$$V_t = \tilde{Y}_t \left(P_t^A\right)^{\frac{\lambda_p}{\lambda_p - 1}} + \xi_p \beta E_t \left[\tilde{\Lambda}_{t,t+1} (\pi_{t+1})^{-1} (\pi_t)^{\gamma_p} (\bar{\pi}_{t+1})^{1-\gamma_p} V_{t+1}\right] \tag{C.46}$$

Log-linearizing the auxiliary equations yields:

$$\hat{p}_t^+ = \hat{S}_t - \hat{V}_t \tag{C.47}$$

$$\begin{aligned} \hat{S}_t = {} & \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \hat{\varepsilon}_t^{mc} + \frac{\lambda_p}{\lambda_p - 1} \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \hat{p}_t^A + \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \hat{y}_t + \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \widehat{mc}_t \\ & + \xi_p \beta E_t \left[\frac{1}{\bar{\varepsilon}^A} \hat{\Lambda}_{t,t+1} + \frac{1}{\bar{\varepsilon}^A} \hat{S}_{t+1}\right] \end{aligned} \tag{C.48}$$

$$\begin{aligned} \hat{V}_t = {} & \frac{\lambda_p}{\lambda_p - 1} \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \hat{p}_t^A + \left(1 - \frac{\xi_p \beta}{\bar{\varepsilon}^A}\right) \hat{y}_t \\ & + \xi_p \beta E_t \left[\frac{1}{\bar{\varepsilon}^A} \hat{\Lambda}_{t,t+1} - \frac{1}{\bar{\varepsilon}^A} \hat{\pi}_{t+1} + \frac{\gamma_p}{\bar{\varepsilon}^A} \hat{\pi}_t + \frac{1 - \gamma_p}{\bar{\varepsilon}^A} \hat{\bar{\pi}}_{t+1} + \frac{1}{\bar{\varepsilon}^A} \hat{V}_{t+1}\right] \end{aligned} \tag{C.49}$$

Substituting expressions (C.48) and (C.49) into (C.47) one obtains:

$$\begin{aligned}\hat{p}_t^+ &= \left(1-\frac{\xi_p\beta}{\bar{\varepsilon}^A}\right)\hat{\varepsilon}_t^{mc}+\left(1-\frac{\xi_p\beta}{\bar{\varepsilon}^A}\right)\widehat{mc}_t \\ &\quad +\frac{\xi_p\beta}{\bar{\varepsilon}^A}E_t\left[\hat{\pi}_{t+1}-\gamma_p\hat{\pi}_t-(1-\gamma_p)\hat{\bar{\pi}}_{t+1}+\hat{p}_{t+1}^+\right]\end{aligned} \tag{C.50}$$

Using the definition (4.3) the aggregate price index is given by:

$$P_t=[(1-\xi_p)\left(P_t^{opt}\right)^{1/(1-\lambda_p)}+\xi_p\left((\pi_{t-1})^{\gamma_p}(\bar{\pi}_t)^{1-\gamma_p}P_{t-1}\right)^{1/(1-\lambda_p)}]^{1-\lambda_p} \tag{C.51}$$

Dividing by P_t we obtain:

$$1=[(1-\xi_p)\left(P_t^+\right)^{1/(1-\lambda_p)}+\xi_p\left((\pi_{t-1})^{\gamma_p}(\bar{\pi}_t)^{1-\gamma_p}\frac{1}{\pi_t}\right)^{1/(1-\lambda_p)}]^{1-\lambda_p} \tag{C.52}$$

The log-linearization of (C.52) yields:

$$\hat{p}_t^+=\frac{\xi_p}{(\xi_p-1)}\left(\gamma_p\hat{\pi}_{t-1}+(1-\gamma_p)\hat{\bar{\pi}}_t-\hat{\pi}_t\right) \tag{C.53}$$

If we substitute out the optimal price, equations (C.53) and (C.50) imply the Phillips Curve of the form:

$$\begin{aligned}\left(\hat{\pi}_t-\hat{\bar{\pi}}_t\right) &= \frac{\beta}{(\bar{\varepsilon}^A+\beta\gamma_p)}\left(\hat{\pi}_{t+1}-\hat{\bar{\pi}}_{t+1}\right)+\frac{\bar{\varepsilon}^A\gamma_p}{(\bar{\varepsilon}^A+\beta\gamma_p)}\left(\hat{\pi}_{t-1}-\hat{\bar{\pi}}_t\right) \\ &\quad +\frac{\beta\gamma_p}{(\bar{\varepsilon}^A+\beta\gamma_p)}\left(\hat{\bar{\pi}}_{t+1}-\hat{\bar{\pi}}_t\right)+\frac{(1-\xi_p)\left(\bar{\varepsilon}^A-\xi_p\beta\right)}{\xi_p\left(\bar{\varepsilon}^A+\beta\gamma_p\right)}\left(\widehat{mc}_t+\hat{\varepsilon}_t^{mc}\right)\end{aligned} \tag{C.54}$$

C.4 Wage equation in the presence of non-zero steady state inflation and the balanced growth

Deriving the wage equation we follow the logic of Altig, Christiano, Eichenbaum, and Linde (2005). Note though that we incorporate inflation targeting into our model, which changes the wage indexation mechanism. Moreover, the wage equation is derived for the open economy framework, where the consumer price does not coincide with the price of output.

The wage rate of a given labor type can be changed (optimized) in any particular period with the probability $1-\xi_w$. The household takes into account that if it does not get to reoptimize next period, it's wage rate then is

$$W_{t+1}=\left(\pi_t^C\right)^{\gamma_w}(\bar{\pi}_{t+1})^{1-\gamma_w}\left(\varepsilon_{t+1}^A\right)W_t^{opt}, \tag{C.55}$$

where ε_{t+1}^A is the expected growth rate of technology, $\bar{\pi}_{t+1}$ is the inflation target, W_t^{opt} denotes the nominal wage set by households that reoptimize in period t, W_t denotes the nominal wage rate associated with aggregate, homogeneous labor L_t.

In period $t+i$ the wage is determined as follows:

$$
\begin{aligned}
W_{t+1} &= \left(\pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+1}^A\right) W_t^{opt} \\
W_{t+2} &= \left(\pi_{t+1}^C \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+2}\bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt} \\
&\ldots \\
W_{t+i} &= \left(\pi_{t+i-1}^C ... \pi_{t+1}^C \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+2}\bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+2}^A ... \varepsilon_{t+2}^A \varepsilon_{t+1}^A\right) W_t^{opt}
\end{aligned}
$$

The demand curve of the individual household is given by:

$$
\begin{aligned}
l_{t+i}(W_{t+i}^{opt}(j)) &= \frac{1}{n}\left[\frac{W_{t+i}^{opt}(j)}{W_{t+i}}\right]^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \qquad (C.56) \\
&= \frac{1}{n}\left(\frac{\left(\pi_{t+i-1}^C ... \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+j}^A ... \varepsilon_{t+1}^A\right) W_t^{opt}(j)_t}{\tilde{W}_{t+i} A_{t+i} P_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i}
\end{aligned}
$$

where $P_{t+i}^C = \pi_{t+j}^C P_{t+i-1}^C = \pi_{t+i}^C ... \pi_{t+1}^C P_t^C$, $A_{t+i} = \varepsilon_{t+i}^A ... \varepsilon_{t+1}^A A_t$, $\tilde{W}_t$ is the aggregate wage scalled by $P_{t+i} A_{t+i}$.

Since all households are assumed to chose the same optimal wage, we suppress the household specific indexation. Then the stationary demand curve is given by:

$$
\begin{aligned}
l_{t+i} &= \frac{1}{n}\left(\frac{\left(\pi_{t+i-1}^C ... \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+i}^A ... \varepsilon_{t+1}^A\right) W_t^{opt}}{\tilde{W}_{t+i} \pi_{t+i} ... \pi_{t+1} P_t \varepsilon_{t+i}^A ... \varepsilon_{t+1}^A A_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \qquad (C.57) \\
&= \frac{1}{n}\left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,j}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} = \frac{1}{n}\left(\frac{W_t^{+} \tilde{W}_t}{\tilde{W}_{t+i} P_t A_t} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i},
\end{aligned}
$$

where $W_t^{+} = W_t^{opt}/W_t$ is a stationary variable.

$$
\begin{aligned}
X_{t,i} &= \frac{\left(\pi_{t+i-1}^C ... \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+1}\right)^{1-\gamma_w} \left(\varepsilon_{t+i}^A ... \varepsilon_{t+1}^A\right)}{\pi_{t+i} ... \pi_{t+1} \varepsilon_{t+i}^A ... \varepsilon_{t+1}^A} \\
&= \frac{\left(\pi_{t+i-1}^C ... \pi_t^C\right)^{\gamma_w} \left(\bar{\pi}_{t+i} ... \bar{\pi}_{t+1}\right)^{1-\gamma_w}}{\pi_{t+i} ... \pi_{t+1}}
\end{aligned}
$$

After log-linearizing we obtain:

$$
\hat{X}_{t,i} = (1-\gamma_w)\left(\hat{\bar{\pi}}_{t+i} + ... + \hat{\bar{\pi}}_{t+1}\right) + \gamma_w\left(\hat{\pi}_{t+i-1}^C + ... + \hat{\pi}_t^C\right) - \left(\hat{\pi}_{t+i} + ... + \hat{\pi}_{t+1}\right)
$$

The homogenous labor is given by:

$$
L_t = \left[\int_0^n l(j)^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}, 1 \leq \lambda_w \leq \infty
$$

The jth household optimize wage W_t^{opt} as follows (we neglect the irrelevant terms in the household objective):

$$
E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{-\varepsilon_t^L \frac{(l_{t+i}(j))^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+i} W(j)_{t+i} l_{t+i}(j)\right\},
$$

where $P_t^C \lambda_{t+i} = \frac{\varepsilon_t^C}{C_t - hC_{t-1}}$. In terms of stationary variables we may write this as follows:

$$\tilde{\lambda}_t = \frac{\varepsilon_t^C \varepsilon_t^A}{\varepsilon_t^A \tilde{C}_t - h\tilde{C}_{t-1}} \frac{P_t}{P_t^C}$$

After loglinearizing we obtain:

$$\hat{\tilde{\lambda}}_t = -\frac{\bar{\varepsilon}^A \hat{c}_t}{\bar{\varepsilon}^A - h} + \frac{h\hat{c}_{t-1}}{\bar{\varepsilon}^A - h} + \hat{t}_t^{P/P^C} - \frac{h\hat{\varepsilon}_t^A}{\bar{\varepsilon}^A - h} + \hat{\varepsilon}_t^C$$

We now derive the first order condition for W_t^{opt}. Substituting for $l_{t+i}(j)$ and making use of the definition: $\tilde{\lambda}_t = \lambda_t P_t A_t$ we obtain:

$$\max_{W_t^{opt}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \frac{\left(\frac{1}{n} \frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i}^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \\ + \frac{\tilde{\lambda}_{t+i} W_{t+i}^{opt}}{P_{t+i} A_{t+i}} \frac{1}{n} \left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} \tag{C.58}$$

Using the fact that $W_{t+i}^{opt} / (A_{t+i} P_{t+i}) = \left[W_t^{opt} / (A_t P_t) \right] X_{t,i}$ and rearranging we obtain:

$$\max_{W_t^{opt}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \frac{\left(\frac{1}{n} \frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i}^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \\ + \tilde{\lambda}_{t+i} \left(\frac{W_t^{opt}}{A_t P_t} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} \tag{C.59}$$

After differentiating with respect to W_t^{opt} we obtain the following first order condition:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \frac{\lambda_w}{1-\lambda_w} \left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t^C A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} L_{t+i} \frac{X_{t,i}}{\tilde{W}_{t+i} P_t A_t} \\ + \frac{1}{1-\lambda_w} \tilde{\lambda}_{t+i} \left(\frac{W_t^{opt}}{A_t P_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{1}{A_t P_t} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.60}$$

In order to write the FOC in terms of stationary variables we multiply by $\left(W_t^{opt} \right)^{-\frac{\lambda_w}{1-\lambda_w}} (1-\lambda_w) / \lambda_w$

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i} P_t A_t} \right)^{\frac{\lambda_w}{1-\lambda_w}-1} L_{t+i} \frac{X_{t,i}}{\tilde{W}_{t+i} P_t A_t} \\ + \frac{1}{\lambda_w} W_t^{opt} \tilde{\lambda}_{t+i} \left(\frac{1}{A_t P_t} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.61}$$

Multiply by $(P_t)^{\frac{\lambda_w}{1-\lambda_w}}$:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^{opt}}{\tilde{W}_{t+i} P_t A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i} A_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \\ + \frac{1}{\lambda_w} \frac{W_t^{opt}}{P_t} \tilde{\lambda}_{t+i} \left(\frac{1}{A_t P_t} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.62}$$

and use $W_t^+ \equiv \frac{W_t^{opt}}{W_t}$ to render all variables stationary:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ W_t}{\tilde{W}_{t+i} P_t A_t} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i} A_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \\ + \frac{1}{\lambda_w} \frac{W_t^+ W_t}{P_t} \tilde{\lambda}_{t+i} \left(\frac{1}{A_t P_t} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.63}$$

Further, we take into account that $\tilde{W}_t = \frac{W_t}{A_t P_t}$:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i} A_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \\ + \frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\lambda}_{t+i} \left(\frac{1}{A_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.64}$$

Multiply by $A_t^{\frac{\lambda_w}{1-\lambda_w}}$:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \begin{array}{c} -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \\ + \frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\lambda}_{t+i} X_{t,i} \frac{1}{n} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \end{array} \right\} = 0 \tag{C.65}$$

and arrive at:

$$E_t \sum_{i=0}^{\infty} L_{t+i} \left(\frac{X_{t,i}}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left\{ \begin{array}{c} \frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\lambda}_{t+i} X_{t,i} \frac{1}{n} \\ -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \frac{1}{n} \end{array} \right\} = 0 \tag{C.66}$$

Writing this out we obtain:

$$\begin{aligned} & L_t \left(\frac{1}{\tilde{W}_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left\{ \frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\lambda}_t \frac{1}{n} - \varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_t \right)^{\sigma_L} \frac{1}{n} \right\} + \\ & (\beta\xi_w) L_{t+1} \left(\frac{X_{t,1}}{\tilde{W}_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left\{ \begin{array}{c} \frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\lambda}_{t+1} X_{t,1} \frac{1}{n} \\ -\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_{t+1}} X_{t,1} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+1} \right)^{\sigma_L} \frac{1}{n} \end{array} \right\} + \ldots \end{aligned} \tag{C.67}$$

We now log-linearize the term $\left(\varepsilon_t^L \left(\frac{1}{n} \left(\frac{W_t^+ \tilde{W}_t}{\tilde{W}_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} L_{t+i} \right)^{\sigma_L} \right)$ obtaining:

$\sigma_L \hat{\varepsilon}_t^L + \sigma_L \frac{\lambda_w}{1-\lambda_w} \left(\hat{w}_t^+ + \hat{w}_t - \hat{w}_{t+i} + \hat{X}_{t,i} \right) + \sigma_L \hat{l}_{t+i}$

Further, we consider the term $\frac{1}{\lambda_w} W_t^+ \tilde{W}_t \tilde{\Lambda}_{t+1} X_{t,1} \frac{1}{n}$. The first term can be written as follows:

$$\begin{aligned} & \frac{1}{\lambda_w} \tilde{W} \tilde{\lambda} \left\{ (1 - \sigma_L \frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^+ + \hat{w}_t + \hat{\tilde{\lambda}}_t + \hat{X}_{t,0}) \right\} \\ & -\sigma_L \left\{ \hat{\varepsilon}_t^L + \frac{\lambda_w}{1-\lambda_w} \left(\hat{w}_t^+ + \hat{w}_t - \hat{w}_t + \hat{X}_{t,0} \right) + \hat{l}_t \right\} \\ = \; & \frac{1}{\lambda_w} \tilde{W} \tilde{\lambda} \left\{ (1 - \sigma_L \frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^+ + \hat{w}_t + \hat{X}_{t,0}) \right\} \\ & + \hat{\tilde{\lambda}}_t - \sigma_L \hat{\varepsilon}_t^L + \sigma_L \frac{\lambda_w}{1-\lambda_w} \hat{w}_t - \sigma_L \hat{l}_t \end{aligned} \tag{C.68}$$

The second term may be written as:

$$\begin{aligned} & \frac{1}{\lambda_w} \tilde{W} \tilde{\lambda} \left\{ (1 - \sigma_L \frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^+ + \hat{w}_t + \hat{\tilde{\lambda}}_{t+1} + \hat{X}_{t,1}) \right\} \\ & -\sigma_L \left\{ \hat{\varepsilon}_t^L + \frac{\lambda_w}{1-\lambda_w} \left(\hat{w}_t^+ + \hat{w}_t - \hat{w}_{t+1} + \hat{X}_{t,0} \right) + \hat{l}_t \right\} \\ = \; & \frac{1}{\lambda_w} \tilde{W} \tilde{\lambda} \left\{ (1 - \sigma_L \frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^+ + \hat{w}_t + \hat{X}_{t,1}) \right\} \\ & + \hat{\tilde{\lambda}}_{t+1} - \sigma_L \hat{\varepsilon}_t^L + \sigma_L \frac{\lambda_w}{1-\lambda_w} \hat{w}_{t+1} - \sigma_L \hat{l}_{t+1} \end{aligned} \tag{C.69}$$

The ith term would be given by:

$$\frac{1}{\lambda_w}\tilde{W}\tilde{\lambda}\left\{(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^{+}+\hat{w}_t+\hat{X}_{t,i})\right\}+\widehat{\tilde{\lambda}}_{t+i}-\sigma_L\hat{\varepsilon}_t^L+\sigma_L\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t+i}-\sigma_L\hat{l}_{t+i}$$

The FOC is given by:

$$\begin{aligned}
&\left\{(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^{+}+\hat{w}_t+\hat{X}_{t,0})+\widehat{\tilde{\lambda}}_t-\sigma_L\hat{\varepsilon}_t^L+\sigma_L\frac{\lambda_w}{1-\lambda_w}\hat{w}_t-\sigma_L\hat{l}_t\right\} \\
&+(\beta\xi_w)\left\{(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^{+}+\hat{w}_t+\hat{X}_{t,1})+\widehat{\tilde{\lambda}}_{t+1}-\sigma_L\hat{\varepsilon}_t^L+\sigma_L\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t+1}-\sigma_L\hat{l}_{t+1}\right\} \\
&+... \\
&+(\beta\xi_w)^i\left\{(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})(\hat{w}_t^{+}+\hat{w}_t+\hat{X}_{t,i})+\widehat{\tilde{\lambda}}_{t+i}-\sigma_L\hat{\varepsilon}_t^L+\sigma_L\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t+i}-\sigma_L\hat{l}_{t+i}\right\} \\
= \;& 0
\end{aligned} \tag{C.70}$$

Above condition may be expressed as:

$$\begin{aligned}
&(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}(\hat{w}_t^{+}+\hat{w}_t)+(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\sum_{i=1}^{\infty}(\beta\xi_w)^i\hat{X}_{t,i}+ \\
&\sum_{i=0}^{\infty}(\beta\xi_w)^i\widehat{\tilde{\lambda}}_{t+i}+\sigma_L\frac{\lambda_w}{1-\lambda_w}\sum_{i=0}^{\infty}(\beta\xi_w)^i\hat{w}_{t+i}-\sigma_L\sum_{i=0}^{\infty}(\beta\xi_w)^i\left(\hat{l}_{t+i}+\hat{\varepsilon}_t^L\right) \\
= \;& 0
\end{aligned} \tag{C.71}$$

Writing $\sum_{j=1}^{\infty}(\beta\xi_w)^i\hat{X}_{t,i}$ out we obtain:

$$\begin{aligned}
&\sum_{j=1}^{\infty}(\beta\xi_w)^i\hat{X}_{t,i} \\
= \;&\beta\xi_w\left[(1-\gamma_w)\left(\hat{\varepsilon}_{t+1}^{\pi}\right)+\gamma_w\left(\hat{\pi}_t^C\right)-(\hat{\pi}_{t+1})\right] \\
&+... \\
&+(\beta\xi_w)^i\left[\begin{array}{c}(1-\gamma_w)\left(\widehat{\bar{\pi}}_t+...+\widehat{\bar{\pi}}_{t+i}\right) \\ +\gamma_w\left(\hat{\pi}_{t-1}^C+...+\hat{\pi}_{t+i-1}^C\right)-(\hat{\pi}_t+...+\hat{\pi}_{t+i})\end{array}\right] \\
&(1-\gamma_w)\frac{1}{1-\beta\xi_w}\sum_{i=0}^{\infty}\widehat{\bar{\pi}}_{t+i}+\gamma_w\frac{1}{1-\beta\xi_w}\sum_{i=0}^{\infty}\hat{\pi}_{t-1+i}^C-\frac{1}{1-\beta\xi_w}\sum_{i=0}^{\infty}\hat{\pi}_{t+i}
\end{aligned} \tag{C.72}$$

After substituting into the linearized FOC:

$$\begin{aligned}
0 \;=\; &(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}(\hat{w}_t^{+}+\hat{w}_t)+(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w} \\
&\times\sum_{i=1}^{\infty}\left((1-\gamma_w)\widehat{\bar{\pi}}_{t+i}+\gamma_w\hat{\pi}_{t-1+i}^C-\hat{\pi}_{t+i}\right) \\
&+\sum_{i=0}^{\infty}(\beta\xi_w)^i\widehat{\tilde{\lambda}}_{t+i}+\sigma_L\frac{\lambda_w}{1-\lambda_w}\sum_{i=0}^{\infty}(\beta\xi_w)^i\hat{w}_{t+i}-\sigma_L\sum_{i=0}^{\infty}(\beta\xi_w)^i\left(\hat{l}_{t+i}+\hat{\varepsilon}_{t+i}^L\right)
\end{aligned} \tag{C.73}$$

Using the lag operators the above equation may be expressed as follows:

$$\begin{aligned}
0 \quad = \quad & (1-\sigma_L \frac{\lambda_w}{1-\lambda_w}) \frac{1}{1-\beta\xi_w} (\hat{w}_t^+ + \hat{w}_t) + (1-\sigma_L \frac{\lambda_w}{1-\lambda_w}) \frac{1}{1-\beta\xi_w} \frac{1}{1-\beta\xi_w L^{-1}} \\
& \times (\beta\xi_w (1-\gamma_w) \widehat{\bar{\pi}}_{t+1} + \beta\xi_w \gamma_w \hat{\pi}_t^C - \beta\xi_w \hat{\pi}_{t+1}) \\
& + \frac{1}{1-\beta\xi_w L^{-1}} \widehat{\bar{\lambda}}_t + \sigma_L \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta\xi_w L^{-1}} \hat{w}_t - \sigma_L \frac{1}{1-\beta\xi_w L^{-1}} \left(\hat{l}_t + \hat{\varepsilon}_t^L \right)
\end{aligned} \tag{C.74}$$

The aggregated wage equation is:

$$W_t = \left[(1-\xi_w) \left(W_t^{opt} \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\left(\pi_{t-1}^C \right)^{\gamma_w} (\bar{\pi}_t)^{1-\gamma_w} \varepsilon_t^A W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{C.75}$$

Dividing by $P_t A_t$ we obtain:

$$\tilde{W}_t = \left[(1-\xi_w) \left(W_t^+ \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\left(\pi_{t-1}^C \right)^{\gamma_w} \bar{\pi}_t^{1-\gamma_w} \tilde{W}_{t-1}}{\pi_t} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{C.76}$$

After differentiation we obtain:

$$\hat{w}_t = (1-\xi_w) \left(\hat{w}_t^+ + \hat{w}_t \right) + \xi_w \left(\gamma_w \widehat{\pi}_{t-1}^C + (1-\gamma_w) \widehat{\bar{\pi}}_t + \hat{w}_{t-1} - \hat{\pi}_t \right)$$

$$\left(\hat{w}_t^+ + \hat{w}_t \right) = \frac{1}{(1-\xi_w)} \hat{w}_t - \frac{\xi_w}{(1-\xi_w)} \left(\gamma_w \widehat{\pi}_{t-1}^C + (1-\gamma_w) \widehat{\bar{\pi}}_t + \hat{w}_{t-1} - \hat{\pi}_t \right) \tag{C.77}$$

Substituting to the FOC we obtain:

$$\begin{aligned}
& (1-\sigma_L \frac{\lambda_w}{1-\lambda_w}) \frac{1}{1-\beta\xi_w} \left[\frac{1}{(1-\xi_w)} \hat{w}_t - \frac{\xi_w}{(1-\xi_w)} \left(\gamma_w \widehat{\pi}_{t-1}^C + (1-\gamma_w) \widehat{\bar{\pi}}_t + \hat{w}_{t-1} - \hat{\pi}_t \right) \right] \\
& + (1-\sigma_L \frac{\lambda_w}{1-\lambda_w}) \frac{1}{1-\beta\xi_w} \frac{1}{1-\beta\xi_w L^{-1}} (\beta\xi_w (1-\gamma_w) \widehat{\bar{\pi}}_{t+1} + \beta\xi_w \gamma_w \hat{\pi}_t^C - \beta\xi_w \hat{\pi}_{t+1}) \\
& + \frac{1}{1-\beta\xi_w L^{-1}} \widehat{\bar{\lambda}}_t + \sigma_L \frac{\lambda_w}{1-\lambda_w} \frac{1}{1-\beta\xi_w L^{-1}} \hat{w}_t - \sigma_L \frac{1}{1-\beta\xi_w L^{-1}} \left(\hat{l}_t + \hat{\varepsilon}_t^L \right) \\
= \quad & 0
\end{aligned} \tag{C.78}$$

Multiplying by $1-\beta\xi_w L^{-1}$

$$\begin{aligned}
& (1-\sigma_L \frac{\lambda_w}{1-\lambda_w}) \frac{1}{1-\beta\xi_w} \\
& \times \left[\begin{array}{c} \frac{1}{(1-\xi_w)} (\hat{w}_t - \beta\xi_w \hat{w}_{t+1}) + ((1-\gamma_w) \beta\xi_w \widehat{\bar{\pi}}_{t+1} + \gamma_w \beta\xi_w \hat{\pi}_t^C - \beta\xi_w \hat{\pi}_{t+1}) \\ -\frac{\xi_w}{(1-\xi_w)} \left(\begin{array}{c} \gamma_w (\widehat{\pi}_{t-1}^C - \beta\xi_w \widehat{\pi}_t^C) + (1-\gamma_w) (\widehat{\bar{\pi}}_t - \beta\xi_w \widehat{\bar{\pi}}_{t+1}) \\ + (\hat{w}_{t-1} - \beta\xi_w \hat{w}_t) - (\hat{\pi}_t - \beta\xi_w \hat{\pi}_{t+1}) \end{array} \right) \end{array} \right] \\
& + \widehat{\bar{\lambda}}_t + \sigma_L \frac{\lambda_w}{1-\lambda_w} \hat{w}_t - \sigma_L \left(\hat{l}_t + \hat{\varepsilon}_t^L \right) \\
= \quad & 0
\end{aligned} \tag{C.79}$$

Finally, the real wage equation may be written as follows:

$$0=\left[\begin{array}{c}
-(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w E_t\hat{w}_{t+1}\\
+\left((1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{1+\beta\xi_w^2}{1-\xi_w}+\sigma_L\frac{\lambda_w}{1-\lambda_w}\right)\hat{w}_t\\
-(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\hat{w}_{t-1}\\
+(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w\frac{\gamma_w}{(1-\xi_w)}\hat{\pi}_t^C\\
-(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}\gamma_w\hat{\pi}_{t-1}^C\\
-(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\beta\xi_w}{(1-\xi_w)}E_t\hat{\pi}_{t+1}\\
+(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}\hat{\pi}_t\\
-\sigma_L\hat{l}_t-\sigma_L\hat{\varepsilon}_t^L-\frac{\bar{\varepsilon}^A\hat{c}_t}{\bar{\varepsilon}^A-h}+\frac{h\hat{c}_{t-1}}{\bar{\varepsilon}^A-h}-\frac{h\hat{\varepsilon}_t^A}{\bar{\varepsilon}^A-h}+\hat{\varepsilon}_t^C+\hat{t}_t^{P/P^C}\\
+(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\beta\xi_w\frac{(1-\gamma_w)}{(1-\xi_w)}E_t\widehat{\bar{\pi}}_{t+1}\\
-(1-\sigma_L\frac{\lambda_w}{1-\lambda_w})\frac{1}{1-\beta\xi_w}\frac{\xi_w}{(1-\xi_w)}(1-\gamma_w)\widehat{\bar{\pi}}_t
\end{array}\right] \tag{C.80}$$

Appendix D

Data description

D.1 The area-wide model

The following raw series were extracted (Series descriptions are indicated in brackets and correspond to AWM databank mnemonics):
Euro area-wide aggregates:
AWM databank of the European Central Bank

- Gross domestic product (YER)
- Private consumption (PCR)
- Investment (ITR)
- Employment (LNN)
- Nominal wage rate (WRN)
- GDP deflator (YED)
- Nominal interest rate (STN)

D.2 The two-region model

The following raw series were extracted (Series descriptions are indicated in brackets and correspond to AWM, VGR and OECD databank mnemonics):
Euro area-wide aggregates:
AWM databank of the European Central Bank

- Gross domestic product (YER)
- Private consumption (PCR)
- Investment (ITR)
- Employment (LNN)
- Nominal wage rate (WRN). In order to assure that the data on wages corresponds to the definition in the model (wage per hour worked), we correct the growth rates of nominal wage rate (WRN), calculated as nominal total labor compensation (WIN) divided by total employment (LNN), for the change in hours worked per employed, (see OECD data below).
- GDP deflator (YED)

- Nominal interest rate (STN)

German time series:
VGR databank of the Deutsches Institut für Wirtschaftsforschung

- Gross domestic product (Bruttoinlandsprodukt)
- Private consumption (Private Konsumausgaben;zu konst.Preisen)
- Investment (Bruttoinvestitionen; zu konst.Preisen)
- Employment (Erwerbstaetige im Inland)
- The nominal wage rate index is calculated as the sum of the wage bills (Bruttolohn-und-gehaltsumme, Inlandskonzept) divided by the number of employed (Arbeitnehmer im Inland). In the model estimated with log-differences we correct the growth rates of the wage rate for the change in hours worked per employed (see OECD data below).
- GDP deflator (Bruttoinlandsprodukt; Preisindex)

Databank of the Deutsche Bundesbank

- Nominal interest rate (Zeitreihe su0107: Geldmarktsätze am Frankfurter Bankplatz, Dreimonatsgeld)

Synthetic bilateral Germany - rest of the Euro area exchange rate is computed using the corresponding exchange rates of the Deutsche Mark against European currencies and weighted with the average share in exports.

- DM/NLG (Zeitreihe wu5000: Devisenkurse der Frankfurter Börse / 100 NLG = ... DM / Niederlande)
- DM/PTE (Zeitreihe wu5004: Devisenkurse der Frankfurter Börse / 100 PTE = ... DM / Portugal)
- DM/ESP (Zeitreihe wu5006: Devisenkurse der Frankfurter Börse / 100 ESP = ... DM / Spanien)
- DM/ITL (Zeitreihe wu5007: Devisenkurse der Frankfurter Börse / 1000 ITL = ... DM / Italien)
- DM/FRF (Zeitreihe wu5012: Devisenkurse der Frankfurter Börse / 100 FRF = ... DM / Frankreich)
- DM/ATS (Zeitreihe wu5015: Devisenkurse der Frankfurter Börse / 100 ATS = ... DM / Östereich)

DM/GRD series (Deutsche Mark against Greek Drahma) has been retrieved from the Global Insight database.

OECD data. Consists of weekly hours worked, annualized and then adjusted for holidays, vacations and sick leave.

- Annual hours worked per full time employed in Germany[1]
- Annual hours worked per full time employed in France
- Annual hours worked per full time employed in Italy[2]

[1]For the pre-unification period the growth rates of hours worked per employed in Germany have been calculated using the data on average hours worked per employed per day (Tägliche Arbeitszeit) provided by the DIW.

[2]The hours worked per full time employed in the Euro area have been calculated as the weighting average of those in Germany, France and Italy.