Quantum Computation and Natural Language Processing

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To my father — who taught me what meaning is.

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Zusammenfassung

In dieser Dissertation wird ein neuer Ansatz zum Verstehen und zur Verarbeitung natürlicher Sprache eingeführt. Der Ansatz basiert auf einer Analogie zwischen den physikalischen Objekten auf der Quantenebene und den Aktivitäten des menschlichen Geistes. Auf dieser Weise gelingt es die physikalischen und seelischen Phänomene in einem einheitlichen Rahmen zusammenzufassen. Als Konsequenz ergibt sich, daß sich die Eigenschaften vom Geist und Materie nicht grundsätzlich unterscheiden, sondern als unterschiedliche Darstellungen der makroskopischen Materie und des makroskopischen Geistes aufgrund unterschiedlicher Eigenzustände des zugrundeliegenden Quantensystems zu verstehen sind. Die scheinbaren Unterschiede sind daher eher *quantitativ* anstatt *qualitativ.*

Die in der menschlichen Kognition verwendeten Symbole kann man als Quanteneigenzustände bezüglich eines bestimmten Quantenexperiments behandeln. Darüberhinaus wird die Behauptung aufgestellt, daß es sich bei *Gedankengang* und logischer *Schlußfolgerung* um semiotische Transformationen handelt, wobei die Symbole als die Eigenzustände bezüglich eines Formulierungsoperators zu verstehen sind. Der Operator ist eine Analogie zu einem “Observable” in der Quantenmechanik. Im Allgemeinen hat ein “State-of-affairs” (eine Superposition von Eigenzuständen) keine wohldefinierten physikalischen Eigenschaften bis zu dem Zeitpunkt, wo er tatsächlich *gemessen* wird. Deswegen ist auch die klassische Semantik (als die Zuweisung klassischer Symbole zur klassischen physikalischen Realität) nicht wohldefiniert. Im Unterschied zur klassischen Semantik soll *Bedeutung* in einem quantenmechanischen Rahmen als eine *aktive Messung* von einem State-of-affairs behandelt werden.

Wenn man Kognition als Vorgänge auf einem Repräsentationssystem betrachtet, erkennt man, daß das Gedächtnis ein sprachähnliches System ist. Jedoch ist das Gedächtnis größtenteils ein klassisches Phänomen, da die chemischen Aktivitäten im Gehirn der Aggregatsgrenzfall der Quantenmechanik (also ein Phänomen einer sehr großen Menge von Quanten) sind. Daher sind Repräsentationen im Kognitionssystem im strengen Sinne auch nicht wohldefiniert.

Eigenschaften der Sprache, die eng mit dem Alltagsschließen (common sense logic) zusammenhängen, sind Gegenstand des folgenden Abschnitts. Die offenbare Tendenz, sich einer präzisen Definition zu entziehen, und die inhärente Ambiguität lassen sich gut in einem quantenmechanischen Rahmen behandeln. Es handelt sich hierbei um ein zur Quantenmechanik analoges *Unschärfeprinzip* und impliziert eine “Begriff-Symbol-Dualität”. Als Anwendung lässt sich der quantenmechanische Formalismus auf Kognitionsvorgänge übertragen. Zum Beispiel kann man *nichtmonotone Schlußfolgerungen* und *Counterfactuals* in diesem Rahmen erklären. Im Einzelnen können die zeit-asymmetrischen Eigenschaften und die genuine Unbekanntheit von nichtmonotonen Schlußfolgerungen in einem quantenmechanischen Modell einfach erklärt werden. Dies gilt auch für Potentialität und Aktualität, die für eine Erklärung von Counterfactuals sehr wichtig sind. Darüberhinaus kann Kausalität als eine Form von Counterfactuals betrachtet werden.

Der zweite Teil der Dissertation behandelt die Simulation und die technische Anwendung der obengenannten Prinzipien auf natürlichsprachliche Verarbeitungsaufgaben. Zuerst werden einfache Experimente mit Beispielen zum Alltagsschließen (exklusives Oder, nichtmonotones Schließen und Counterfactuals) dargestellt. Diese zeigen, daß das klassische Erscheinungsbild der Beispiele implementiert werden kann. Jedoch hat der quantenmechanische Ansatz zusätzliche “Feinheiten”, die man in den klassischen Ansätzen nicht finden kann.

Im Folgenden wird gezeigt, daß sich einfache natürlichsprachliche Verarbeitungsaufgaben auf unterschiedlichen Corpora simulieren lassen. Als Erstes werden die syllogistischen Schlußfiguren als quantenmechanisches System modelliert. Dabei konnten ausgezeichnete Ergebnisse erzielt werden. Als Zweites wird eine monolinguale Syntaxmanipulationsaufgabe auf quantenmechanischer Grundlage simuliert, wobei die Ergebnisse deutlich besser als die vergleichbarer konnektionistischer Ansätze sind. Zum Abschluß wird das Quantensystem auf eine deutsch-englische Übersetzungsaufgabe angewandt, in denen schwierige Eigenschaften, wie z. B. lexikalische Ambiguität, abtrennbare Verbpräfixe, Konjugationsendungen, und Umstellungen der Wortreihenfolge bei der Übersetzung vorkommen. Auch bei dieser Aufgabe konnten mit der quantenmechnischen Architektur recht gute Ergebnisse erreicht werden.

Abstract

In this thesis, a novel approach to natural language understanding inspired by quantum mechanical principle is proposed. It is based on an analogy between the physical objects at the quantum level and human's mental states. In this way, the physical and the mental phenomena are to be understood within the same framework. It is also proposed that the apparent differences between mind and matter do not lie in the fundamental differences of their properties, but in the different manifestation of macroscopic matter and macroscopic mind owing to their different composition of pure quantum eigenstates. The apparent differences are therefore *quantitative* rather than *qualitative.*

Specifically, symbols in various cognitive functions are to be treated as eigenstates with respect to a particular quantum experimental arrangement. Moreover, I claim that *reasoning* and *inference* can be treated as transformations of semiosis with symbols being the *eigenstates* of a particular formulation operator. The operator is the counterpart of an *observable* in quantum mechanics. A state of affairs (a superposition of these eigenstates) does not have well-defined physical properties until it is actually *measured.* Consequently the classical semantics (as classical symbols' referring to the classical physical reality) is also not well-defined and may be a misleading idea. Different from classical semantics, *meaning* in the quantum mechanical framework should be treated as an *active measurement* done on a state of affair.

Moreover, the ill-definedness also manifests itself in the cognition internal to a person if we regard *memory* as a language-like representational system. Nevertheless, memory, treated as a specific language system, is a largely quasi-classical phenomenon in that the

chemical activities in the brain are an *aggregate limiting case* of quantum mechanics with a very large number of quanta. The classical "objective" physical reality is therefore a limiting case of quantum reality as well.

The general language in which common sense logic is embedded is then investigated and the apparent evasiveness and ambiguity of language can be accommodated in a quantum framework. This is done by postulating an analogous *Uncertainty Principle* and observing the implication of it. An important implication is the "concept-symbol" duality. As applications, the quantum mechanical formalism is applied to cognitive processes. For instance, *non-monotonicity* and *counterfactual conditionals* can be accommodated and assimilated in this framework. Specifically, the time-asymmetric property and the genuine unknown state of non-monotonic reasoning can be easily explained in quantum mechanics. This is also the case for the *potentiality* and *actuality*, which are crucial ideas for explaining counterfactual reasoning. Furthermore, causality can be regarded as a disguise of counterfactual reasoning.

The second part of the thesis is devoted to simulations and technical applications of the aforementioned principle in natural language processing. First the preliminary experiments of common sense logic are presented. These show that the "classicization" of common sense logic can be implemented with very simple quantum mechanical systems. Moreover, the richness of the quantum framework goes well beyond what a classical system can offer. There can be "fine-structures" within seemingly simple logical arguments (XOR, for example). This is also the case for non-monotonic and counterfactual reasoning.

Simple natural language tasks are also simulated based on different natural language corpora. First the syllogistic arguments embedded in natural language are simulated with a quantum system, which delivers quite remarkable results. Secondly, a monolingual syntax manipulation is implemented with a quantum system, in which the quantum mechanical approach can achieve much better performance than connectionist one. In the last experiment, a quantum mechanical architecture is trained for bilingual translation between English and German, in which there are several thorny properties in the natural language corpus, for example lexical ambiguity, separable prefixes, complicated conjugation, and

Chapter 1

Introduction

> 道可道非常道名可名非常名
>
> *(Spoken Tao is not eternal Tao. Spoken name is not eternal name. — Translated by Jeff Rasmussen.)*
>
> *(The Tao that can be trodden is not the enduring and unchanging Tao. The name that can be named is not the enduring and unchanging name. — Translated by James Legge.)*
>
> *(The Way that can be experienced is not true; The world that can be constructed is not true. — Translated by Peter A. Merel.)*
>
> *(The experience of* flow *is surface; The world of things is drama. — "Interpolated" by Peter A. Merel.)*
>
> — 老子道德經
>
> — Laotsu (Taoteking)

1.1 A machine-translation example

Let us begin with an example of state-of-the-art machine translation. We have here a German sentence together with its English translation[1]:

---

[1] The original sentence in German is the grand conclusion of Wittgenstein's *Tractatus Logico-Philosophicus* [2]. The English counterpart is carefully translated by C.K. Ogden, presumably with the

> Wovon man nicht sprechen kann, darüber muß man schweigen.
>
> (Whereof one cannot speak, thereof one must be silent. [Translated by C.K. Ogden])

The German sentence is submitted to a popular machine translation system[2], and the following translation in English is carried out automatically:

> About which one cannot speak, over it one must be silent.

At first sight, the performance of the machine translation system seems fair. It is not a bad translation. In fact, the sense is kept almost faithfully except for somewhat bizarre wording. Heartened by this positive result, the translated English sentence is submitted to the machine translation system again, only this time the target language is set to German. We now have the following translation:

> Über welche man nicht sprechen kann, über ihm muß man leise sein.

This result is amusing! For one thing, the English word *"it"* is translated as *"ihm"*[3], so we feel almost completely lost about what this word refers to. Perhaps more strangely, *"silent"* is translated as *"leise"*, which should be considered an error as far as the meaning of the sentence is concerned. For a further test, this German sentence is submitted to the machine translation system again with English as target language. This time the translation turns out to be:

> About which one cannot speak, over it one must be quiet.

It is sometimes surprising how creative the machine translation can be! Nevertheless, it cannot be considered a bad translation, for it has kept the sense of the "bad" German translation to a certain degree. Encouraged by the result, the experiment is continued. This sentence is submitted to the machine translation system again. Now we have:

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translator's thorough *understanding* of Wittgenstein's original text.

[2] See http://www.systransoft.com/.

[3] Dative masculine or neutral pronoun in German, and indeed there is no way for a machine translation system to know which gender a pronoun should be without looking into context.

> Über welche man nicht sprechen kann, über ihm muß man ruhig sein.

Nevertheless, it is a translation which has gone a long way from the original. To see how far this procedure can go, the sentence is submitted to the machine translation again, this time we have

> About which one cannot speak, over it one must be calm.

At this point, I guess one would be convinced that computers are not only creative but also humorous! The fact is, a computer program does not have the slightest understanding what an utterance might *mean.*

For one thing, a large-scale machine translation system has a huge linguistic database, in this case perhaps even bigger than that of a competent Germanic student. For someone who is unfamiliar with natural language processing (NLP), it is hard to believe that equipped with so much information (and so heavily invested), a computer cannot deliver a decent translation of a moderately complicated sentence. But this is no surprise for someone working in NLP. They know how difficult it is to program a computer so that it can *understand* a natural language sentence. In fact, many workers in NLP even assume that a computer will *never* understand what human says and direct their attention in more productive areas (such as *computer-aided human NLP*). The fact is, there is almost no adequate account of *aboutness* of natural language. At the present time, most NLP systems simply mechanically manipulate symbolic structures.

A problem immediately arises: are current main-stream NLP systems on the right track? After decades of endeavor in symbolic artificial intelligence (AI), we can hardly believe it is so [3, 4]. For if it were the case, a state-of-the-art computer which can execute several million instructions per second (that could be millions of times *faster* than an ordinary human) would not have performed so poorly in natural language processing. The fact is, a computer can not even approximate a tiny fraction of human capability in natural language processing tasks. Indeed, it is very implausible that our own slow “computer” (the prevalent and one-sided, if not totally misleading, metaphor of the human mind) could achieve its current performance if it did not do it in a much smarter way than computers do. A revealing fact is to see how fast a computer can compile a very complex C++ program and how seldom an experienced C++ programmer can write a short program without a syntax error on the first try. A computer is a remarkable genius of Chomskyan languages [5, 6], but natural language is not something it is good at.

Indeed, a common weakness of many NLP projects today can be mostly attributed to their inability to accommodate *meaning* and their unbalanced attention to syntax. Many errors of today's NLP systems can be traced to the radical differences between their way of representing *meaning* and *context* (or absence thereof) and that of a human. When we talk about syntax, this includes different kinds of semantic formalisms as well, because according to the computer metaphor of the human mind, slot-filler and category-instance can be regarded as syntactic objects at a more abstract level and therefore deprived of any meaning — the meaning we human beings acquire in a bio-socio-cultural context. Specifically, *meaning* is something which is entangled with the experiences of individuals in a very complicated way. In this respect, meaning depends heavily on *contexts* — linguistic, socio-cultural, and ontogenetic / phylogenetic biological factors, which are *holistic* in essence. This points out the first inadequacy of a computational approach, because classical computation is serial and local.

Moreover, something can make sense only if it makes sense for *somebody*, who must be a sentient being. So meaning is derived from *subjectivity* and *intention.* But there is no place for intention in a Turing machine — a (for many, *the*) metaphor of the human mind. In this picture, at best, one has to smuggle intention into a program from without (that is, from the sentient *program designer(s)*) in order to "breath the spirit into the nostril of the robot made of earth." Without an account of holistic context or sentient beings, we cannot avoid ending up with a theory of *zombies.* This summarizes the inadequacy of a top-down or computational approach as a unified scientific view of human mind and language. This also has an unfortunate impact on NLP, for meaning is the central issue of natural language *understanding.*

It is often argued, however, that NLP is an engineering discipline, thus the question of meaning is only remotely related to NLP and should be put off. Instead, it is argued, one should pay more attention to practical issues. But this view is very limited. History has taught us all too often that a more successful engineering (this includes medicine) is always based on a "better" science. Now how can we tell which theory is "better"? An existing or an old theory backed up by authority does not make it automatically a good theory. A "better" science must *explain* Nature more *intelligibly.* Moreover, a "good" theory has to accommodate *more* facts — especially anomalies, in addition to the facts deliberately selected to fit into the theory (the practitioners in a "normal" science tend to ignore the anomalies [7]; they usually postulate *ad hoc* solutions to these anomalies). So it usually begins with the account of anomalies. (We have already encountered an important anomaly that the top-down computational approach cannot account for — holistic context and intention.)

At this moment, the reader may think I am advocating an alternative *bottom-up* or physicalist approach to mind and language. This is largely the case, but we should be careful not to fall into another questionable view — that the human mind is the activities of a classical machine, or a clockwork. In this view, we will unfortunately end up with another theory of zombies. Before we continue, let us consider the hurdles for a theory of meaning in the existing scientific frameworks — both from the top down and from the bottom up.

1.2 A scientific account of meaning

In professional as well as in lay communities, science is too often taken in a very limited (and arguably conceited, as we shall see) sense that science is a theory about naive *external* and *objective reality* in Nature.[4] In this view, Nature is passive and mechanistic. It is

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[4] This emphasis of science on natural phenomena, however, is mostly an Anglo-Saxon tradition. In German, for example, the concept of science is much broader. There are *Geisteswissenschaften* (humanities, literally sciences of mind) — *Sprachwissenschaften* (philology, linguistics), *Literaturwissenschaft* (literature, literature studies), and even *Rechtswissenschaft* (jurisprudence, law) and *Betriebswissenschaft*

therefore very often argued that the meaning-giving human beings, equipped with all their consciousness, artistic creativity, free will, and moral judgment, "naturally" can not be a part of passive and mechanistic Nature. This leads many to believe that a new scientific account of meaning is impossible right from the start[5]. But this does not have to be the case. Let us see why.

To clear the matter up a bit, let us consider what "scientific account" means anyway. We have just encountered our first question of meaning. And I hope the following discussion will shed some light on what a *scientific account of meaning* would look like. Now, as far as "an account of meaning" (call it X) is concerned, a scientist is a person who believes in and strives for *intelligible* accounts of meaning (an intelligible account is an explanation one finds persuasive and rational). Moreover, a scientist is a *naturalist*, at least when she practices her profession. A naturalist is a person who believes that in the realm of discussion there is no account other than those found in Nature[6]. Armed with these concepts, we can reformulate our target as

a naturalist intelligible account of meaning.

At this moment, an objection to the possibility of this account can be largely attributed to the belief that Nature is passive and mechanistic. For many, this position seems to be the only choice, for Nature seems to consist of matter and matter follows the *Law of Nature*

(business management). All these disciplines are seen as sciences. However, at least in the Western civilization, Nature is often taken as an antithesis of Humanity, in which the human will is transcendental to natural *laws.*

[5] There was, and perhaps still is, a substantial trend in the disciplines of *humanities* in which natural sciences, such as physics or biology, are taken as shining examples of their own discipline. A salient example is the so-called social science. The trend started with Auguste Comte (1798-1857), who coined the word "sociology" and is taken as the founder of positivism. In a sense, the modern school of *cognitive science* and various endeavors to reduce human psychology to neuronal activities (classical bio-chemistry) can be seen as microscopic versions of positivism.

Nevertheless, one should not ignore the fact that there are also significant critics of positivistic philosophy — its modern form can be traced back to Karl Marx (1818-1883). It is, with justification, termed as "negative philosophy." (In the social theory context, see, for example, [8]). In a sense, the dialog and dispute of what is positive (in Nature) and what is negative (human will and critics) comprise a centerpiece of the Western civilization.

[6] In general, a naturalist does not have to be a scientist (unless she believes there is an intelligible account of Nature) and a scientist does not have to be a naturalist (unless she believes there are no supernatural accounts).

without exception; but human beings seem to be able to *"break the law."*[7] In this sense, one could say matter is passive and objective but mind is active and subjective. If this view is correct, a naturalist has to answer this question:

> *why are mind and matter so different in that mind is active and subjective but matter is not?*

Convinced that the pre-condition of this question is correct (i.e. matter and mind are inherently different), an antagonist of the *naturalist intelligible account of meaning* has a point. This renders the question untouchable, because it does not need any further explanation (it can be taken as it *is*). Nevertheless, this question sounds quite similar to a question à la Newton: *why are earthly bodies and heavenly bodies so different in that an apple falls but the moon floats?* — remember the properties of heavenly bodies *were* an untouchable scientific question in the Middle Ages. For Newton, it turns out that the question has a simple answer: the moon *does* fall, so does the apple, and indeed so does *everything.* Asserting that, the age-old Aristotelian tenet of differentiating celestial from terrestrial body falls apart! Would the answer to the question above be the same? — that matter (indeed the physical world as a whole) *is* active too?! Or, alternatively, the mind is also passive and our subjective intuition is only delusion?! If it is the second case, we end up with another theory of zombies, and the reader should stop reading right away because *nothing* makes sense anymore. On the other hand, if it is the first case, we have to revise our conventional way of thinking of objectivity. This is a monistic view[8] of the universe relying on the refutation of Cartesian dualism. At this point, it seems to me that a "better" naturalist intelligible account of meaning must be a genuine monist theory.

The monistic approach to mind and matter is not a new idea. In fact, it can perhaps be traced all the way back to Democritus' theory of atoms and his stance as a *panpsychist.*

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[7] It is arguable whether all living beings are able to *"break the law"* in its everyday sense as well. Nevertheless, following *instincts* is, at least for most conventional natural scientists, following the law. But *knowing what instincts are and overcoming them consciously* — to sleep on a bed of nails and be hurt, for instance, poses a more profound question about what the "law" really is.

[8] Monism, for one thing, sees matter and mind to something unified.

In the era of classical physics and rationality, however, monism has given way to Cartesian dualism [9] and lost its influence Although seldom explicitly taught, Cartesian dualism is still deeply embedded in the way classical physics is presented. It remains the case even as the crucial argument of René Descartes (1596–1650) — the concept of *God* has deteriorated ever since. Ironically, an extreme form of materialism (disguised as a sort of monism, although it is not, as we shall see) has emerged from Cartesian dualism.

To see how deep-rooted Cartesian dualism is in the alleged monist materialism, let us consider the orbit of Pluto as an example. The orbit of Pluto is presented in the textbook as a movie-clip in the eye of an *external* observer — in the "God's view," so to speak, although Pluto's period of revolution is much longer than the life expectancy of today's human and it has not even completed a single revolution since its discovery. So from human's view, the observation (or the experiment) is not even finished yet. What we have is only a firm belief that Pluto will follow its course pretty much like Earth follows its course. (It is very likely the case, but it is a belief nevertheless, therefore qualitatively different from absolute objectivity.) In fact, it is only from the "God's view" — and indeed, one needs very strong faith in it — that a naive (viz. objective) materialism can emerge. Since objectivity must be established by an external observer, the observer *can not* be a part of the universe — which, by definition of monism, must include *everything.* Now it is clear that the absolute observer is the subjectivity being smuggled in. Consequently this can not be a genuine monism. In fact, this is one of most important motivations for us to shift our interest from ontology to epistemology and see the whole matter from *inside out.* A consequence of this shift is the so-called *positivism.* But a naive positivistic view of Nature cannot work either.

Thanks to the standard textbooks of sciences, today many students of science hold a naive positivistic stance that the purpose of science is to "model natural phenomena as closely as possible". That is, to offer predictions of natural phenomena as accurately as possible. This seems to be an epistemic approach. But the naivety lies *literally* in this view, because it begs for a model and an objective standard of "closeness." It is nevertheless dualism in disguise. The implicit dualist stance will become clearer if we pose the following two questions: *who* is modeling? and *to what* is the model considered close? For one thing, there must be the absolute objectivity (the matter in Nature) to which scientific theory (in the mind of scientists) can model and the numerical prediction can approach. For another, the concept of model itself tears up the universe into what is modeling and what is being modeled. In fact, this view of separability has been subject to question in modern physics and in a way has motivated the epistemic approach to science.

Let us begin with the fundamental question posed by quantum theory. Indeed, it can be argued that a sort of proto-mind must be embedded in the sub-atomic phenomena which are not separable from their physical properties (in a quite obscure and indirect way, however). For one thing, in quantum mechanics, the observer — this is extended by a set of measurement instruments that obey classical mechanics — may play a crucial role and influence the experiment outcomes dramatically. In certain experimental arrangements, for example, an electron will shy away from a particular property if it "knows" that it is being watched (see Section 3.2 for details). In these cases, the absolute objective view has to be modified, if not given up. In a sense, quantum objects have some mind-like properties which make a monistic approach to mind and matter attractive again. Observing this fact, the *qualitative* question above is not justified and should be transformed to a *quantitative* one:

> *in which situations should we talk about an object is matter-like and/or mind-like?*

This will be a crucial question addressed in this thesis. And indeed, quantum mechanics offers a handy formalism not only for physical objects but also for mental "objects." This will comprise the basis of our naturalist intelligible account of meaning.

1.3 Quantum theoretically speaking

A philosophy-prone reader may notice that this view is not without question. To clear this issue a bit, let us take a short excursion to the philosophical problem of quantum mechanics. First of all, the mathematical formalism of quantum mechanics is a language (mathematics) and its interpretation is about the *physical meaning* of the language. Indeed, the tool with which we talk about physical meaning — *language*, is such an intimate part of us that we cannot tell the difference between the meanings the utterances confer and the "empty words" used to convey it. Unfortunately, this confusion manifests itself in quantum mechanics as well. As far as the *meaning* of quantum mechanics is concerned, the interpretations of quantum mechanics are not only diverse but also obscure [10], for quantum mechanics itself is in some way inconsistent and paradoxical. More specifically, the paradox is deeply buried in the coexistence of classical objects which are not subject to uncertainties, and micro-objects, with the former measuring the latter. In a way, this paradoxical coexistence manifests itself as "a puzzle of two languages" [11]. In quantum mechanics we need an everyday language with which we can communicate with each other unambiguously — this is strengthened by the language of classical physics; and a formalism that can only predict the result stochastically — this renders the "reality" pointed to by the symbols in the formalism inherently *ambiguous.*

But knowing the inconsistency of quantum mechanics is not to refute the theory, which is the most accurate theory we have. For one thing, quantum mechanics is not a theory *out of nothing.* In fact, quantum mechanics was developed by competent *classical* physicists to solve problems that are formulated classically but cannot be solved classically. In a sense, the history of quantum theory shows that even though the quantum and classical world-views are incompatible, quantum mechanics nevertheless grew out of classical physics (and paradoxically still has a foot rooted in classical mechanics). Interestingly, the "compatibility" and "harmony" is restored by demonstrating the *correspondence* between classical physics and quantum mechanics. That is: in the limiting case when Planck's constant approaches zero and/or the number of quanta approaches infinity, the statistical behaviors of quantum theory approach the deterministic properties of classical physics. Considering the broad phenomena which quantum mechanics can explain, it is the most "consistent" theory — because the correct predictions of classical mechanics are subsumed by that of quantum mechanics.

But what will the quantum paradox tell us? Let us take a closer look from the view of scientific development. Indeed, a continuous development of our understanding of Nature is not only of pedagogical merit, it is crucial for us to understand anything in physics at all. This consists of our basic stance as naturalist scientists that Nature is a harmonious, integrated, and intelligible affair. In this sense, any sophisticated world view must have caught certain important aspects of Nature. Consider the following example: although there is inconsistency and incompatibility in quantum mechanics and classical mechanics, it is hardly imaginable that we can understand the mathematical formalism of quantum mechanics without first understanding what classical velocity, acceleration, momentum, and time are. We certainly do not think of these classical concepts in terms of the limiting cases of quantum properties. The reader should notice, therefore, that the purpose of the following discussion is *not* advocating or refuting a certain philosophical position on science. Nor is my aim to force incompatible views together. Rather, the purpose is to present a stepping stone (*boot-strapping*) to understanding the *content* of science and identifying the problem of quantum mechanics by arguing its difficulty and probing its implication *from within*.

In light of this, let us start with how science is conceived in classical physics, which, I believe, is still an often taken stance by practicing physicists and scientists of other disciplines. As Heinrich Hertz put it, in science we make ourselves "pictures" ("Bilder") of the fact in such a way that "the logically necessary consequences" ("die denknotwendigen Folgen") of the "picture" agree with "the necessary natural consequences" ("die naturnotwendigen Folgen") of the real object or facts. Being somewhat obsolete and incompatible with quantum mechanics, there is nevertheless a crucial merit of this view. In fact, it points out that scientific research is *not merely* striving steadily to improve the accuracy of the theoretic prediction of experimental results. A good scientific theory must be a theory which can *explain* and show the *connections* among phenomena.

As far as the content of this view is concerned, it works well with classical physics. But as mentioned, while it is very important to boot-strap our understanding of physics, it has to at least be modified, if not totally abandoned. As Dirac stated, perhaps for pedagogical purpose [12]:

> [I]n the case of atomic phenomena, no pictures can be expected to exist in the usual sense of the word 'picture' ... One may, however, extend the meaning of the word 'picture' to include any *way of looking at the fundamental laws which makes their self-consistency obvious.*

In this sense, a picture in quantum mechanics, if there is any, can only be conceived as a picture at a higher level (looking at the *laws* instead of objects). In any case, while an extension of picture to the higher level may help us comprehend physics, it is, so to speak, plagued by its implicit dualist stance. But as far as a boot-strapping process is concerned, it is an adequate argument (for this moment) and offers a point which is relevant to our discussion. In fact, it points out that language must play a crucial role in quantum mechanics, for it is in language (mathematics) that the laws of quantum mechanics are formulated and it is in language that the confusion, and paradox, etc. manifest themselves. Moreover, it is in the language "at the higher level" that the consistency is restored. We should note, however, that this hierarchy cannot go infinitely upwards, because we need an account from within (hierarchy is always a view seen from without). This suggests that it is unlikely to have an adequate account of quantum mechanics without an adequate account of language. Interestingly, seen from within, quantum mechanics may also offer a good formalism to *analyze* the problem of language.

Now if language and mind is to be treated as a natural phenomenon of quantum mechanics, mysticism can be kept to a minimum, if not totally eliminated. But there is a price to pay, for such an account *cannot* be consistent as far as classical logical explanation is concerned. I suspect this is a characteristic of any monistic world views that include quantum mechanics. For one thing, a consistent explanation demands that the subject matter (in this case that about quantum objects) is to be *objectified unambiguously and without uncertainty.* This is, however, forbidden according to the Principle of Uncertainty. However, I do believe an adequate account of language can be shown and this will turn out to be both a quantum mechanical account of language and a linguistic account of quantum mechanics *at the same time.* This is where an analytic boot-strapping process as shown above has its merit. If this step is taken, as in the tradition of analytic philosophy, we understand that it is not important to solve the problem, but instead to offer a *dissolution*. This is also an important motivation of this thesis.

Observing this, one should be forewarned that this thesis can inevitably capture only one aspect of the affair — both of physics and of linguistics. The other aspects, however, are guarded by the fundamental principles of quantum mechanics and have to remain literally *unspeakable* and *unthinkable* forever. In other words, these aspects are beyond our horizon and excluded from any discourse — including those of the sciences. But as in the case of approaching the horizon, there remain quite a lot of issues that can be discussed. These include the *naturalist intelligible account of meaning.* This will be argued more deliberately in the following chapters.

Now I have come to my statement of thesis.

1.4 Statement of thesis

1. There exists a strong analogy between quantum physical objects and our mental objects: thus the phenomena in the physical and those in the mental world are to be understood within the same framework. The apparent differences of mind and matter do not lie in the fundamental differences of the properties of both, but in the different manifestations of macroscopic matter and macroscopic mind owing to their different dispositions in quantum subtlety.

2. Analogous to *particle-wave* duality in quantum mechanics there is a *symbol-concept* or *word-sense* duality in language. Consequently there is an Uncertainty Principle in language, which in a sense agrees with the view of *signs* in Saussurean linguistics.

3. Natural language and common sense logic (which can be only embedded in natural language) can be described as quantum computational systems. Therefore evasiveness and ambiguity are a manifestation of the *Uncertainty Principle.* Furthermore, non-monotonicity, counterfactual conditionals and causality can be accommodated

(or assimilated) in this framework.

4. In preliminary experiments with computer simulation, it can be shown that a quantum computational framework can be applied to classical and common sense logic. Furthermore, non-monotonic and counterfactual reasoning can be demonstrated as well.
5. Simple natural language tasks (syllogistic arguments, syntactic transformations, and translation on different corpora) are also simulated with quantum computational models. It can be shown that a quantum computational framework can indeed deliver very satisfactory results.

The logical dependency of chapters in this thesis is shown in Figure 1.1.

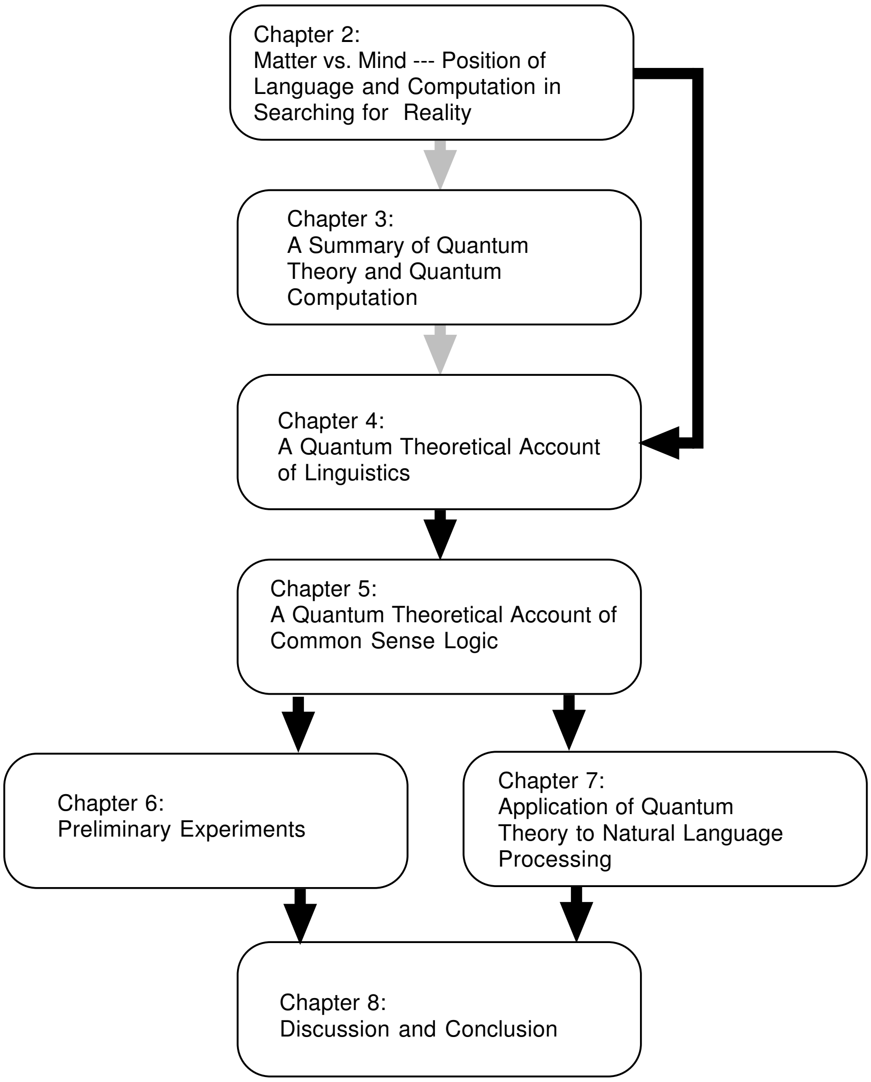

Figure 1.1: Logical dependency of chapters in this thesis.

Part I

Quantum Theory and Natural Language

Chapter 2

Matter vs. Mind — Position of Language and Computation in the Search for *Reality*

> *Wir machen uns Bilder der Tatsachen. (We make to ourselves pictures of facts.) ... Das Bild hat mit dem Abgebildeten die logische Form der Abbildung gemein. (The picture has the logical form of representation in common with what it pictures.)*
>
> — Ludwig Wittgenstein (Tractatus logico-philosophicus)

2.1 Matter vs. Mind, or Physics vs. Mathematics

In this chapter, we will first discuss the apparent close but puzzling relationship between physics and mathematics and will propose a view showing why it is the case in light of language usage. I propose that this will help to clear the so-called "hard problem" of consciousness [13] in cognitive science, in which we have to explain why subjective *qualia* (that "something it is like") can emerge from pure physical processes. The key is to treat language as a way of computation in light of quantum theory, and confer upon it a pivoting role in understanding thought (the mental reality), which in turn points to physical reality[1]. I will argue that the concept of *classical computation* is inadequate. Specifically, classical computation should be treated as a limiting case of a more subtle computation (*rheomode computation*).[2] We will begin with the argument of why physics and mathematics are relevant to our topic: a naturalist intelligible account for meaning as the activity of quantum physical objects.

2.1.1 Why physics?

There are several reasons to place physics at the center of our argument:

1. Physics is usually seen as the hardest of all hard scientific disciplines today. It has everything to say about what we call physical "reality" in the world — from galaxies to atoms to elementary particles. For many, a physical world is *the* world.

2. The whole scientific community (including psychology, cognitive science, in some respect linguistics, etc.) is dominated by an active or passive *physicalist* world view. This view is sometimes very active, such as in chemistry or astronomy. In these disciplines, physics offers a foundation for all explanations. Their explanatory frameworks are to be seen as either derivation or approximation of underlying physics. In other cases, physics plays a passive role. For instance, it is accepted that *no* scientific discipline whatsoever could ever violate the laws of physics. In all these cases, physics does *mesh* with other scientific disciplines and is regarded as more subtle. In other words, a fact established in physics is to be established as a fact in other disciplines. For example, no linguist is in the position to argue for a theory that is in any way *incompatible* with the laws of physics. Indeed, no human, as a physical entity, can utter a physically impossible sound.

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[1] In fact, the argument can be turned around with equal validity that the physical reality (whatsoever it may be) points to language. Remember a hierarchical thinking (from without) can not be genuine monistic.

[2] *Rheomode* is a concept coined by David Bohm [1] — 'rheo' comes from a Greek verb, meaning 'to flow.'

3. Physics offers good theories for many engineering disciplines “for all practical purposes” (FAPP, as John Bell calls it). This includes those which are heavily physics-oriented such as aeronautics and electronics and those which are more distant, such as architecture and information/communication technology. In the latter case, physics usually plays a supporting but *indispensable* role. Also, note that many mind-related scientific disciplines rely heavily on the help of the equipment built according to the knowledge of advanced physics — positron-emission tomography (PET) or nuclear magnetic resonance (NMR) tomography in psychology, psycholinguistics or cognitive science, to name some. In these cases, they take the results measured by physical instruments as the basis of any reliable evidence.

Although physics is indeed very successful in explaining the phenomena which we can or can not see (ranging from the Big Bang in the beginning of the universe to the stability of atoms on your finger tips), the relevance of modern physics to sciences of mind, including linguistics, in my view, is rather due to its *crisis* rather than its success in accommodating physical “reality.” In fact, the main theme of this chapter is that the naively-understood physical “reality” — an invariant objective substance — is only a *limiting case* of a more subtle reality, in which activeness has its place. I am not prone to the opinion, however, that this more subtle reality is supernatural (remember that I am advocating a naturalist account) or unintelligible. Nor do I think that there is mind or soul that can exist independently of physical objects. There is no doubt that it can be argued that way, as many students of humanities would prefer to. They ask: how can you otherwise accommodate *intention*, *free will* and *consciousness* in physics without resorting to an autonomous *mind*[3]? This conception, however, in my opinion, is largely owing to a misunderstanding of physics. It is all too easy to accept the well-established but out-dated Newtonian/Cartesian world view — let us call it ‘folk physics,’ which has penetrated so deeply in our everyday life. In

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[3] Indeed, even in quantum theory this view is often taken by physicists. There are similar but serious arguments to get one out of the difficulty of quantum mechanics as provided by the Copenhagen interpretation (for a summary of interpretations of quantum mechanics see [10]) by resorting to somewhat mysterious consciousness and by rendering the most subtle physical “reality” (in its everyday sense) as “meaningless” — this is by no means something to which physicalists might seriously subscribe.

this view the physical world is lifeless and mechanistic, in short, the universe is qualitatively identical to a clockwork. But this is not correct even if only physics is concerned, as we shall see.

Before we proceed, something must also be mentioned about the role physics plays in functionalist or emergentist approaches to mind-related sciences. In emergentism, physical objects are the substrate on which new phenomena (mind) emerge. In functionalism, physical objects are the realization of a specific function. So physics itself is often held as a macroscopically irrelevant or uninteresting topic from the view point of so-called levels of explanation. However, if any theory happens to imply a violation of existing physical laws or starts with assumptions that are refuted by physics, it is sufficient to falsify the whole theory as unscientific. In other words, newly established physical facts have the power to falsify approaches in other disciplines. Now what if the most subtle physical "reality" ceases to be "meaningful" and there is no other way except through "consciousness" or "mind" to establish physical facts, as Copenhagen Interpretation of quantum mechanics implies [10]?

It may be pointed out that all scientific disciplines can be treated as some sort of *functionalism* in that they are interested in the logical/causal relationship between the relevant entities in their corresponding disciplines. These entities are mostly defined through their corresponding functions or roles. For example, consider what role *genes* play in biology or the *Federal Reserve* in macro-economy. Although one cannot deny that there is a *realization* of the functioning unit, one is apt to think that this is irrelevant. But this view can turn out to be fruitless. To see why, consider astrology: if an astrologer can predict the solar or lunar eclipse very accurately (he can) and tell the ups and downs of Dow-Jones (alleged being influenced by these celestial events), would these facts establish astrology as a science? In fact, if the investors in Wall Street do believe in the astrologer, his prediction must be correct to a certain degree. Now we will ask: isn't it the realization of a function (the good prediction here) that makes an account of social psychological explanation of the impact of astrology on financial markets more scientific than astrology? Isn't this realization crucial in finding a more plausible causal explanation?

This motivates us to take a closer look at the foundation of physics, for according to a physicalist account physics is the ultimate realization of any function. But before delving into physics, let us take a look at the other center piece of our arguments — mathematics.

2.1.2 Why mathematics?

Mathematics is perhaps the purest of all the pure mental endeavors of humankind. During the times of Euclid and Pythagoras, mathematics was seen as a pure mental exercise that could deliver truth and nothing but truth. Today, this view is subject to a minor modification: the "truth" is related to a set of starting propositions (called *axioms*). An axiom can be, in some cases, completely lacking intuitive content and beyond intuitive or empirical verification. In most of the cases, axioms are, however, propositions which we take as self-evident. From this view, the relevance of mathematics to our topic can be seen the following two ways:

1. The (apparent?) sense of absoluteness and universality of mathematics on its own and its relationship to thoughts;
2. The efficacy of pure mathematical argument on physical reality (by way of sophisticated theoretic physics).

For one thing, mathematics is seen by many as an exact deductive science which has its own *reality.* But unlike other disciplines in natural sciences, they think, a theorem is *absolutely* and *universally* true. As long as a theorem is proved by *a* mathematician, *all* mathematicians should be able to prove (at least to verify) the theorem as well and the theorem is considered simply *proved.*[4] The strong belief that mathematics forms a

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[4] Strictly speaking, the mathematicians referred to here are those who are trained by the same logical method. For example, a mathematician trained in constructive school [14] (e.g. with intuitionist logic which accepts $p \to \neg\neg p$ but not $\neg\neg p \to p$) might refuse to accept a theorem proved by another "traditional" mathematician using an *ad absurdum* argument.

In fact, the refutation of exclusive middle is a consequence of the philosophical view of *constructive mathematics.* In short, a constructive mathematician does *not* accept that there is objective mathematical reality. Consider the following proof which is not accepted by constructive mathematicians:

consistent unity may justify our calling it *mathematical realism*. In this sense, mathematical objects (such as numbers, theorems, proofs, etc.) exist on their own and have objective existence independent of the minds of mathematicians. We may call them "mathematical reality." According to this position, the job of mathematicians, exactly as their colleagues in physics, is to *discover* the hidden reality, so that the truth can "fall into place."

It is indeed this fascinating belief that has raised an interesting question: what exactly are the rules of mathematical reasoning and why don't the outcomes contradict each other? This is a topic of mathematical logic. Many questions are answered positively in this domain — mathematically [15].[5] Interestingly, as by-products of this discipline, different "logics" have been discovered (or developed). For example, the first order intuitionist logic that turns down the law of double-negation can be still shown to be compact and complete. Nevertheless, there are also many puzzling and pessimistic results, for example Gödel's Incompleteness Theorem [15].

It turns out that the development of mathematical logic has in many ways also aided the growth of modern computer science — formal language, automata theory, proof theory, and recursion theory, to name a few area strongly influenced by mathematical logic. Moreover, it was the ambition of a branch of computer science — artificial intelligence (AI) that again brought to light profound problems about the definition of mind. This, no doubt, will have significant impact on natural language understanding and/or processing. In fact, it is because of our customary way of treating logic (indeed, classical first order logic) as a better way of reasoning (for some, it is *the* perfect way) and taking other everyday reasoning (non-monotonic, modal, context-sensitive) as *frictional* or *impure* forms thereof that has led to many difficulties in AI (see Chapter 1 for examples).

Theorem 1 *There exist two irrational numbers a and b such that a^b is rational.*

Proof: Now $(\sqrt{2})^{\sqrt{2}}$ is either rational or irrational. In the first case, we may take $a = b = \sqrt{2}$; in the second case, we may take $a = (\sqrt{2})^{\sqrt{2}}$ and $b = \sqrt{2}$, since then $a^b = 2$ is rational.

However, there is no known contradiction between the theorems proved by intuitionist mathematicians and those proved by traditional mathematicians given the same set of axioms. The controversy is rather on "acceptable" proofs. Interestingly, it is perhaps the belief in universality of mathematics that has driven constructive mathematicians to prove "existing" theorems again.

[5] For example, the Compactness Theorem and the Completeness Theorem of (classical) first order logic.

We have to see that mathematics plays a crucial role in our contemporary understanding of physical reality. In a sense, this role is active and somewhat tyrannical. For one thing, mathematics is not just a crucial tool for describing experiments or observation. Rather, the description and prediction power of mathematics is attributed to Nature's *agreeing* with mathematics. Einstein, for example, spent more than eight years of his lifetime devoted to the development of the General Theory of Relativity *without* the slightest clue from physical experiments and observations. The ultra-high agreement of the General Theory of Relativity to observed data in some areas (up to 10^{-14}) certainly suggests that it is not merely a matter of luck. There must have been something in Einstein's mind that held the key to the mystery of the universe.

Indeed, many important discoveries of today's physics are guided by mathematical theories rather than the other way around (Gedankenexperiments with pencil and paper alone are in principle mathematical exercises). The role of experiments is to *confirm* or *refute* an existing mathematical theory. The job of experiment is therefore passive in this sense. An experimentalist physicist will not be surprised to see outcomes predicted by a mathematical theory. On the contrary, she is surprised when the phenomenon predicted by the theory is *not* there.

An observation of the power of logic/mathematics renders a naive sub-symbolic [16, 17, 18] approach highly implausible. For one thing, the sub-symbolic school is an alternative view seeing frictionless reasoning as an idealized version of a more subtle classical physical activity and attacking the difficulties of AI from the bottom up. In light of the efficacy of mathematics and logic, it is hardly imaginable that a mental framework emerging from this classical substrate may give rise to a highly abstract understanding of multidimensional geometry, for example.

2.1.3 Physics and computation

Almost every serious computer scientist has some knowledge of physics. But the deeper physical background of computer science remains a seldomly addressed issue. (By "physical background" we mean an intelligible relationship between the physical properties of a piece of hardware and the computation — or mathematics — it delivers.) Nevertheless, almost every computer engineer holds an implicit working hypothesis that this connection is solid. So solid that the hardware does carry out the computation faithfully according to *anthropocentric* mathematics.

Let us first examine this issue more closely from the stand-point of a mind-matter dualist. The dualist position is a strongly held tenet in the Western tradition since René Descartes. According to the dualist position, matter is an extended and inert substance, while mind's intuition and deduction are the means for mind to understand matter.

Now, an algorithm is a set of abstract procedures devised by computer programmers (applied mathematicians) based on nothing but their knowledge of logic and mathematics. The algorithm is therefore a pure recipe of an intelligent mind. On the other hand, the hardware, although designed by competent engineers, consists of only matter and it works according to physical laws. But, according to Cartesian tenets, matter is independent of the mind of the designer. Now, how can this connection between physical hardware and mental computation be solidly established? Why is the outcome of the calculations as a physical process the same as our mathematical expectation, which is the outcome of a mental process?[6] To answer these questions, a dualist has to *postulate de facto* that it is solid. For Descartes, this is attributed to God. In fact, it is difficult for a dualist to establish a genuine solid relation between mind and matter without resorting to some sort of supernatural causes. In a sense, mind is itself supernatural in Cartesian dualism.

Nevertheless, for a naturalist dualist the connection between computation and physics has to be established *empirically* but not *deductively.* Thus this connection falls short of the expectation of most mathematicians. And it disproves the working hypothesis of

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[6] In fact, the modern digital computer works on a principle of approximation. For example, if the voltage across a junction in a CMOS memory chip is higher than a threshold value, a register is interpreted as "1," otherwise "0." The tension between computation and physics can be seen more clearly on an analog computer. Consider a scale, for example. For a scale to be balanced, the weight on the left arm *times* the length of the left arm should be equal to the weight on the right arm *times* the length of the right arm. It is hard to see any obvious and compulsory reason that an abstract multiplication operation should have a physical embodiment.

computer programmers in its strongest form.

Here a materialist or an idealist has an upper hand on this issue. For a materialist, human brain consists of matter only. So mind obeys the same laws of physics that matter does. If matter follows the laws of nature, so does its activity — and this is mind. Thus the connection between mathematics and physics has to be solid. The same argument is valid for an idealist, except that she has to see a piece of hardware as an extension of (her) mind and will argue the other way around.

For many, materialism and idealism are not good alternatives. For, it is argued, to avoid rendering oneself an idealist, in which case one is apt to collapse into solipsism, one has to take a materialist stance. This latter position is implausible for many who take matter as an inert substance that passively *obeys* the laws of physics. If it were the case, they think, in mathematics all their conscious decisions would have ceased to have any meaning. And indeed, they do not *want* this to be so. This unwillingness alone is enough for them to refute a materialist stance right from the beginning. This is a crisis of belief lying at the heart of the tension between science/technology and humanities. For a discipline of mind, it seems to me that there can not be any serious new developments without first facing this crisis. In a sense, this is the "hard-problem" in disguise. And now it is time to take a look at physics again.

2.1.4 Way out of the crisis?

Indeed, the most fundamental theory of modern physics — quantum mechanics — offers a very interesting alternative picture of physical objects. In quantum mechanics, the behavior of a physical object is related to the experimental arrangement. So the property of quantum objects depends on the observer, at least to a certain degree. It this case, an electron may "know" what the observer has decided and, strangely enough, what the observer is about to decide *before* the decision is really made. In quantum mechanics, mind can be taken as activity of matter without hurting our intuitive understanding of mind, for quantum objects seem to have some mind-like properties.

Another advantage of this account is that it *explains* why physics and mathematics mesh without resorting to supernatural effect without sacrificing our intuitive freedom of subjective mind. Indeed, since Galileo, mathematics has not only become the *lingua franca* of physicists, mathematics has also been assumed implicitly by many to be the ultimate ontology of physical reality. Indeed, a modern electrical engineer seems to have few problems in accomplishing her job dealing with, say, satellite telemetry by simply "deducing" everything from the four Maxwell's equations of electromagnetism[7] The cogent relationship between existing theories of physics of this sort, and the relationship's consistency is taken as an evidence that physics and mathematics do go hand in hand.

Interestingly enough, the nature of computation and mathematics show why classical physics *cannot* offer an adequate account of the solid relation between computation and physics[8], because classical physics is *passive* and *continuous* but mathematics is an *active* and *discrete* creative endeavor. In fact, today's computation theory is nothing but *discrete mathematics.* As far as discreteness is concerned, computation turns out to be an important quantum effect[9]. [19]

2.2 Physical reality

As far as reality is concerned, few scientists will claim themselves to be non-realists or anti-realists. In other words, few scientists admit that they are not interested in (a non-realist position) or deny (an anti-realist position) the existence of *objective* reality. Thus, if *realism* is the tenet of believing in objective reality, almost every scientist will claim herself to be a *realist.*

But what, then, is physical reality? A standard answer can be traced back to René

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[7] For the sake of pure mathematical aesthetics, even four equations are redundant. In fact, two of the four equations can be deduced from the other two with the help of the Theory of Special Relativity.

[8] This is not to say that a computer cannot be simulated by hardware obeying classical physics. In fact, by carefully squeezing the transient state of classical electromagnetic circuitry, clever engineers can build computers that simulate discrete computation.

[9] Perhaps the most significant computation is *evolution* in Nature. Not surprisingly, the chemical reactions and mutations on which evolution is based are all quantum effects.

Descartes (1596-1650): physical reality is *matter* and the properties thereof. Moreover, matter is an extended, inert substance. These "things" are simply there whether somebody is watching or not. In other words, physical reality is independent of observers. More specifically, these properties (such as linear momentum, angular momentum, energy, coordinates, charge, mass, etc.) are *well defined* since there are methods to retrieve them and they yield the same properties every time. Let us call this the "classical concept of physical reality."

This belief in objective reality squares well with the classical Newtonian world view, although this view has to be subject to a great but not essential revision in the Special and General Theory of Relativity. In the Theory of Relativity, mass and energy can be converted into each other, therefore substance is not inert; moreover, physical properties *are dependent* on the observer at his/her space-time vantage point. Nevertheless, the "classical concept of reality" remains sound and valid as far as its *well-definedness* is concerned, for objective properties can still be retained. Specifically, gravitation — as the curvature of space-time — is to be contemplated *from outside* of space-time and is an objective property.

Even in classical statistical mechanics, in which the exact determination of momentum and position of a particle is completely out of the question, the "classical concept of reality" still squares well with the Newtonian world view. This is because in classical statistic mechanics the position and momentum of the particle are well-defined — the position and momentum of the particle are objectively *there*, even if *I* (or anyone else) do not *know* how big they are. It is qualitatively different to say that one *cannot* know how big they are. In fact, what are relevant in classical statistical mechanics are the aggregate properties of particles (e.g. molecules) such as temperature or pressure. A *realist* position can be still retained.

When it comes to quantum mechanics, the picture of "classical concept of reality" encounters a real crisis. First of all there is the *Uncertainty Principle* stating that one *cannot* accurately measure momentum and position at the same time. Furthermore, the decision of what to observe may play a crucial role: either the position or the momentum can be measured, but not both. The *observer* has the *freedom*, so to speak, to choose which one she prefers and this will change the properties of a quantum system. Specifically, if the position of a particle is measured accurately, its momentum will turn out to be fully random. If, however, the momentum of a particle is measured accurately, its position will turn out to be fully random.

Before going into details, we have to mention a standard high-school "explanation" of the Uncertainty Principle, which is seemingly able to restore the classical view. According to this "explanation," a measurement "disturbs" the system so that the particle is either violently pushed away (when momentum is being measured and one cannot know the exact position) or confined (when the position is being measured and one cannot know the exact momentum). Objective properties such as momentum and position are "actually" there. In this way one hopes that the "classical concept of reality" can still be maintained. But this is not correct. The disturbance "interpretation" has been refuted again and again, most recently by the experimental tests of Bell's Inequality (see e.g. [10]). In fact, for many there seems to be no intuitively valid models that get away from the Uncertainty Principle without resorting to non-realist (such as the Copenhagen Interpretation) or intuitively very bizarre models (such as the Many-World Interpretation [19]).

Moreover, in quantum mechanics one talks about the *duality* of wave and particle. The behavior of a particle is described by a complex-valued wave function. The Uncertainty Principle states that coordinates alone are enough to describe the behaviors of a quantum object. These behaviors are *stochastic*, however. Specifically, if the wave function of a particle is $\psi(\vec{x}, t)$, where $\vec{x}$ is a coordinate vector, the probability of finding a particle in an infinitesimal volume S is:

$$\int_S |\psi(\vec{x}, t)|^2 \, dV. \tag{2.1}$$

A bizarre implication of wave functions is that a wave function is seldom confined to a finite space. Thus the particle can be everywhere, albeit with extremely low probability in some places. Only when a measurement is performed, can a physical property manifest itself. This is a profound challenge to well-definedness, for what happens if *nobody* does the measurement? Are the properties still there? A standard answer is that we *cannot* know so we *should not* care. In this sense, quantum mechanics demands a fundamental revision of the "classical concept of reality," if not a total abandonment.

2.3 Mental reality

When it comes to the mental world, it is an age-old controversy whether there is objective reality. For one thing, everything that deserves to be called a mental object exists only in my or your mind. Can there be *concepts* which are independent of observers? Can a sentence *mean* anything to nobody? Speaking introspectively, we seem to be able to render all mental "things" *subjective.*

However, this is not necessarily the case. For example, consider a mathematical expression $1 + 1 = 2$. This equation is a mental object. To establish this equation, one must already have the concepts 1, 2, +, and =. Almost everyone claims that she/he *understands* this equation. Would one argue that these concepts are also subjective, in the sense that my 1 is not equal to your 1? At least for mathematical realists (and it seems to me that most of us are educated as realists), there must be some mental objects, such as well-defined mathematical expressions, that deserve to be called "reality." These are the "objective" mental "things" — at least it appears so.

In fact, any serious mind-related science should be able to accommodate logic and mathematics. Better yet, a good mind-oriented science should either *explain* why logic and mathematics are the way they are; or offer alternatives, say, an alternative Pythagoras theorem in Euclidean geometry.

Let us now make our first attempt to unify physical reality and mental "things." For this purpose, it is worthwhile to notice that at the present time the prevailing scientific view of mental phenomena is *physicalist.* This includes various schools of reductionism, materialism, functionalism, and emergentism. According to these views, mental "things" are nothing but movements of physical objects, so the objectivity of mental "things" can be guaranteed by the objectivity of physics. Let us, for this moment, take quantum theory as the ultimate theory of physics. Now a mathematical wave function such as Equation 2.1 must literally point to physical properties. And according to our working hypothesis, it must be taken as a part of mental reality (because it is well-established mathematics). If this is correct, we have a unified explanation of logico-mathematical mental objects and seemingly subjective mental objects (e.g. *qualia*).

But this naive physicalist approach cannot work. This is because the physical properties pointed out by the wave function are physically *not* well-defined, therefore not objective. In fact, a quantum mechanical account of mental reality will render the complementary quantities (technically speaking, conjugate observables) totally in limbo. For the sake of argument, let us assume that an abstract object as Equation 2.1 refers to (called it the particle picture) is a classical picture of particle movement (and indeed quantum mechanics needs it, for measured results are classical mechanical objects[10]). If the particle picture is to be asserted in my mind, a complementary object of Equation 2.1 (i.e. a wave picture which uses momenta as basis) *cannot* be asserted.

Interestingly enough, even mathematical expressions that look well defined are not necessarily qualified to be called mental "reality." A notable example is *Russell's Paradox* of Naive Set Theory. In Naive Set Theory, a primary relation is *member-of* (denoted by $\in$). A *set* is then a mathematical object associated with a member statement which determines whether or not an entity is a member of the set. Now consider the following set:

$$A \equiv \{x | x \notin x\}. \tag{2.2}$$

Clearly, A is not the empty set ($\emptyset$), for at least one entity $\emptyset \notin \emptyset$, thus $\emptyset \in A$ according to the member statement. The paradox manifests itself when we consider whether

$$A \in A?$$

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[10] See [20] "[I]t is in principle impossible, however, to formulate the basic concepts of quantum mechanics without using classical mechanics...The possibility of a quantitative description of the motion of an electron requires the presence also of physical objects which obey classical mechanics to a sufficient degree of accuracy."

Now suppose it is the case (i.e. $A \in A$), we conclude that A is a member of the set A, so A must have fulfilled the member statement. Consequently, $A \notin A$. *Ad absurdum.* Therefore $A \notin A$. But if it is the case, according to the member statement, A must be a member of A, therefore $A \in A$. Again, *ad absurdum.* $A \in A$ turns out to be an *undecidable* statement.

There are several approaches that allow us to get away with this paradox, notably the Axiomatic Set Theory [21], according to which a "thing" as A is simply *not* a set. This leads to the question: is A qualified as an adequate object of discussion in the sense that a concept associated with Equation 2.2 exists? Or it is just something conjured by a naughty mathematician? Even in clear-cut mathematics, the objectivity may be subject to question. In a sense, quantum mechanics asserts at the same time "classical mechanics $\in$ quantum mechanics" and "classical mechanics $\notin$ quantum mechanics." We therefore have a similar self-referring situation as in Equation 2.2.

In everyday life, there are many mental "things" that are difficult to clear up, no matter if they are conjured subjectivity or universal objectivity. An example is *qualia.* Qualiae are introspectible and seemingly monadic properties of sense-data, the raw feelings. They are "something that it is like." A raw feeling like "redness" is a concept built around a set of sensorial data. My sensorial data are never the same as yours. Consequently "my redness" can never be "your redness," strictly speaking.

Indeed, this question of *private* qualia has a deeper philosophical root. For one thing, for a purist physicalist we have nothing but our sensorial data. Our concepts — let them be mathematical or whatsoever — are based on our eidetic experiences. These experiences, however, can not float around without physical substrate. In other words, we need *memory* of these data for later perusal. Consequently, a concept such as "redness" must be seen as a constant comparison between experience and a reconstructed environment based on memory of the past perception of "redness." This is also the case for mathematical concepts such as π or 1. The question is how these "things" are memorized. For one thing, memory is never the real thing, it is a *representation* of the real thing (if there is anything real). Thus we have reduced the problem down to *representation.* This process is illustrated in

Figure 2.1. (More details in next section.)

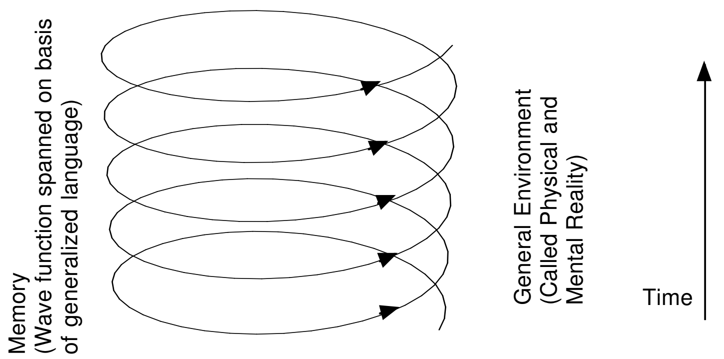

Figure 2.1: A spiral view of mental reality (cf. Bohm [1]).

2.4 Language

Now we come to our main concern — *language.* Mathematical expressions are themselves language. Moreover, it is in language (partly artificial and partly natural) that the logical relation between mathematical objects is explained. Language is also something with which Equation 2.1 is presented. And of course, physicists *talk* about electrons or quarks, their energy, momenta, charge, mass, and even colors. All these are discourses in language.

A striking insight is that our memory itself is a patterned system, so it can be seen as a language as well. This must be the case, otherwise our memory would have to consist of verbatim records of experiences, and this is very unlikely. I know how roses smell, because I have the memory of how roses smell, although I don't have access to the sensorial data *now.* And indeed, I have a memory of roses so that I know there are things which are roses. In my thought, I can see roses, experience how they smell, how they sound, and how it feels to touch them. This leads me to conclude that there is *something* which *is* a rose.

We have to use the term *language* in a very broad sense. For the purpose of discussion, any compact system capable of generating *images* — all kinds of sensorial environment — is entitled to be called a language.

In fact, when I think of roses, I think of things which are *called* “roses.” If something pops up in my mind and I cannot tell whether they are roses, I *call* them “something that I cannot identify as either roses or not roses.” Every time I think of something, it has a name. Whether the color of a rose is red, or not red, or I can not tell if it is red, has to be *called* “red,” “not red,” or “something that is undetermined if it is red.” Whether a rosebud falls, floats, or neither, has to be *called* “falls,” “floats,” or “neither.” Even the higher level categorical images such as movements or attributes have to be *called* “movements” or “attributes.” Thus we name everything, including those which can not be named.

It turns out that this habit of giving everything a name is typical in many languages, especially in Western Indo-European (WIE) languages. In German, for example, nouns are even called *Hauptwörter* (head-words or main-words). We have reason to believe that this habit of *objectification* predisposes one to think of everything as “objects,” and there is no other way to think about reality other than crystallizing. Its ultimate form may be *information theory*, in which information is reduced to well-defined objects (bits) and the *structure* thereof. There are many prejudices of this kind, for example in [22], Dretske stated:

> ... there is something *in* nature (not merely in the minds that struggle to comprehend nature), some objective, observer-independent fact or set of facts, that forms the basis of one thing’s meaning or indicating something about another.

Indeed, whenever we talk, something is spoken out. It is a *description* (Latin: write-down) of a state of affairs. A spoken or written utterance consists of sounds or words, which take the form of symbols. In this sense, symbols are objects, or rather, symbols are something being objectified. However, it is a fallacy to confuse the necessary objectification of words with the objectification of reality. It seems that many have over-generalized the subjective *naming* to the (conjured) objective information. It is even more erroneous to equate information to meaning. In fact, we should not forget that meaning is a dynamic *process* which *brings forth* the world. As Whorf stated:

> Sense or meaning does not result from words or morphemes but from patterned relations between words or morphemes. (P.67 [23])

Indeed, this observation has caused some linguists to question the adequacy of the discrete symbolic approach to language. For example, Kenneth Pike [24] has proposed a view of language as "particle, wave and field." He has also proposed the difference between *emics* and *etics* (e.g. phonemics vs. phonetics). In a sense, these insights reveal a similarity between wave-particle duality in quantum mechanics and word-sense relation in language[11]. In quantum theory, a particle is localized and exclusive — it is either there or not there. It is about static structure. On the other hand, a wave is always holistic and synergistic. In waves, what is important is the patterned relation. It is about dynamic process.

This shift from structure to process is in a way similar to David Bohm's thought experiment with language and thought in [1]. He calls it the *rheomode* of language by putting the verb at the center of language usage. The purpose is to emphasize the effect of "participation" instead of "interaction" in understanding what the world is.

If this is an adequate account of language, language usage deserves to be called *rheomode computation*. It is a sort of quantum computation, except that the activeness of the quantum system should be emphasized. In light of this, the memory in Figure 2.1 should not be taken as a classical object but a *quantum object*, which is represented by a superposition of eigenstates (for a summary of quantum mechanics see Chapter 3). Each eigenstate is a monadic entity (a *name* in a language — manifested as a *symbol*.) Following the tradition of cognitive science, this superposition is called a *state of affairs*. If a particular memory happens to be a "pure" state (an eigenstate) such as in the case of an

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[11] As in quantum mechanics, what can be observed directly are only the properties of particles, in language what can be observed directly are only words or morphemes. A wave is a patterned relation of particles and can be understood only indirectly. A wave is influenced by the experimental setup as a whole.

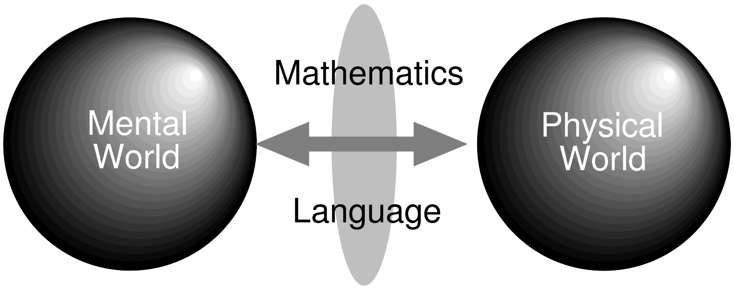
invariant mathematical symbol (e.g. π), a measurement will not distort the sensorial data generated by the memory. In day-to-day language, the state of affairs is mostly impure (i.e. with multiple components of mutual orthonormal eigenstates).

Within this framework, mathematics can be regarded as a quantum computation done on pure states (so it is always reversible); while everyday reasoning is a quantum computation done on superposed states (so it seems to be random and irreversible). Moreover, the Newtonian view is also a quantum computation on the expectation value of superpositions. Since most macroscopic objects have a huge number of quanta, the expectation values of physical properties approach that predicted by classical physics.

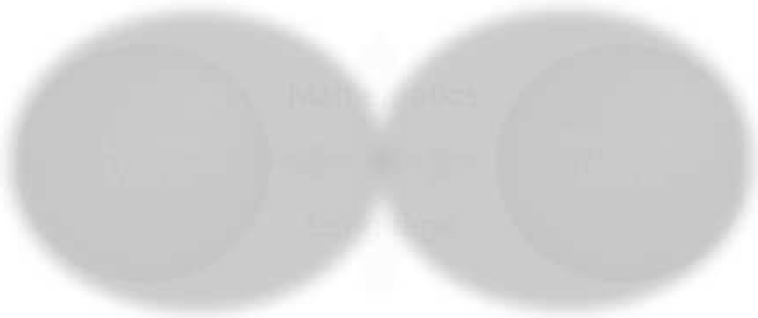
There can be a crucial impact on the interpretation of quantum mechanics. A closer look at the formalism of quantum mechanics reveals that the paradox of quantum mechanics lies in the unavoidable objectification of mathematics. When we realize this, the paradoxical question in quantum mechanics dissolves. Quantum mechanics, and indeed any science, consists of pictures or utterances that are *speakable*.

Now we can make a picture of the physical and mental world comprising all that what is speakable (according to the very broad sense of language). This is shown in Figure 2.2. In this view, language and mathematics play a pivoting role in bringing forth the physical and mental world and bridging them. Nevertheless, this is the case only if language (or mathematics) *is* being used to describe the world (labeled with *Particle-like view of world* in the figure). However, there is another way of understanding the world (labeled with *Wave-like view of world* in the figure). In this view, the subject matter cannot be spoken and everything becomes blurred. The formalism of quantum theory is speakable, this is the case only if we see it at a more subtle level (that is, if it is brought forth this way). If this picture is taken as the subtle reality, physical and mental reality can be seen as two aspects of the underlying reality.

To conclude, an intelligible naturalist account of meaning consists of a formalism based on a strong analogy between the physical and the mental world.



(A) “Particle”-like view of world



(B) “Wave”-like view of world

Figure 2.2: A wave-particle duality of relationship between the mental and physical world.

Chapter 3

A Summary of Quantum Theory and Quantum Computation

> *If you really believe in quantum mechanics, then you can't take it seriously.*
>
> — Bob Wald

3.1 Introduction

To make this thesis self-contained, a brief summary of quantum mechanics is given in this chapter. A reader who is familiar with quantum mechanics can skim or skip this chapter. For a thorough treatment of quantum mechanics, one can refer to [25] or [12]. A good introduction can be found in *The Feynman's Lectures on Physics* [26]. The notation used in this thesis is mostly due to P. A. M. Dirac [12]. A brief summary of quantum computation is also presented in Section 3.5. For more details, the reader can refer to [27]. For quantum computation in general and its applications, the reader can refer to [28] and the references therein.

To begin with, quantum mechanics is one of the greatest triumphs of modern science. Indeed, it is the most important foundation of modern physics. Perhaps more importantly, quantum mechanics provides an adequate account for atomic events which in turn offer a theoretical foundation for chemistry and molecular biology. This chain goes further and further and, as many believe, will eventually encompass all natural sciences[1].

Quantum mechanics is a theory describing the physical world of very small scale. In fact, any theory of atoms — or any other elementary building blocks of matter — intrinsically has an *absolute* concept of what is large and what is small, for otherwise the substance can be further divided into yet smaller parts, *ad infinitum*, according to the continuity of physical substance and physical laws. As Dirac stated,

> [I]n order to give an absolute meaning to size, such as is required for any theory of the ultimate structure of matter, we have to assume that *there is a limit to the fineness of our powers of observation and the smallness of the accompanying disturbance — a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer (p.3-4 [12].)*

This is the fundamental principle of quantum mechanics known as *Heisenberg's Uncertainty Principle*. Specifically, Heisenberg's Uncertainty Principle states

$$\Delta p \Delta q \geq \frac{1}{2}\hbar \tag{3.1}$$

where p and q being canonical momentum and coordinate; $\hbar = h/2\pi$ with h being the *Planck Constant* ($h = 6.62608\,10^{-34}$ Joule Second); $\Delta S = \sqrt{\langle (S - \langle S \rangle)^2 \rangle}$, for $S \in \{p, q\}$; $\langle \cdot \rangle$ is the expectation value. Before delving into the formalism of quantum mechanics, we start with two experiments which, we hope, can disclose the key properties (and indeed "strangeness") of quantum mechanics.

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[1] Strictly speaking, this cannot be correct, for at least at the present time the Theory of General Relativity is still not unified with quantum mechanics. However, there are already several candidates that might offer a unified framework for quantum mechanics and the Theory of General Relativity (e.g. String theory). In any case, quantum mechanics will probably be subject to only minor modification and the formalism will remain largely valid.

3.2 Two-slit experiment

The first experiment is the *two-slit experiment of electron interference* illustrated in Figure 3.1. In this experiment, a thermal electron gun emits high-speed electrons shooting at an electron-sensitive plate (shown at the right side of the figure). Between the plate and the electron gun there is a thin wall which has two slits. Electrons are absorbed if they hit somewhere other than these two slits on the wall, only those electrons that go through the slits can arrive at the plate and generate sparks.

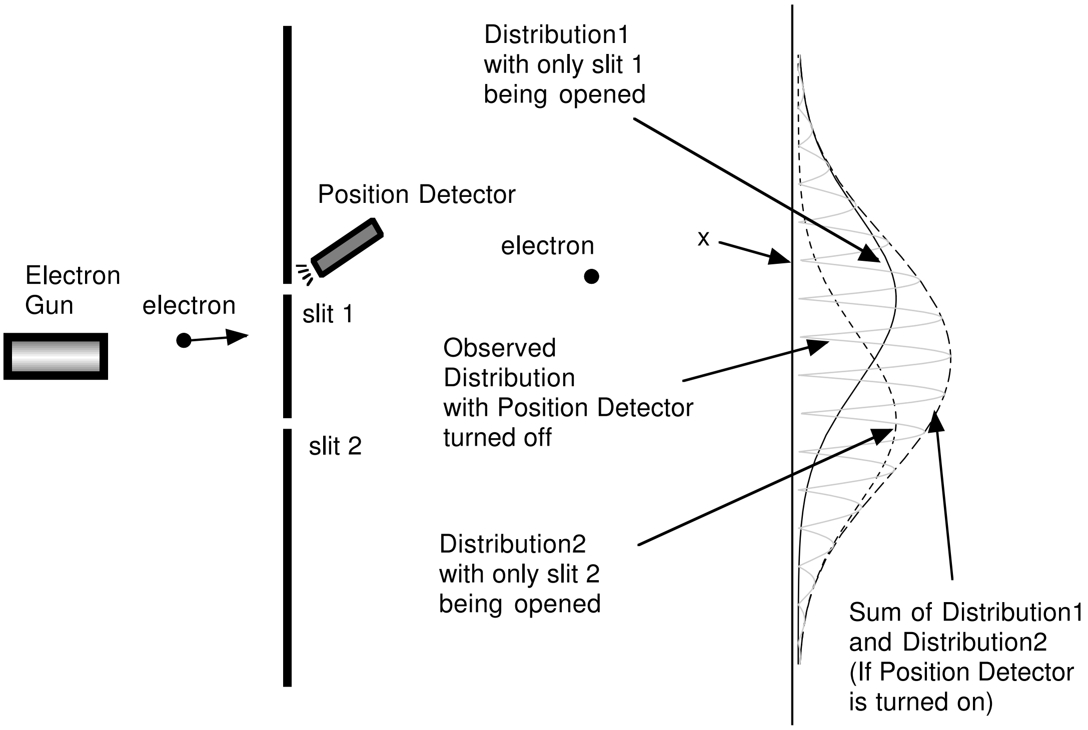

Figure 3.1: Two-slit experiment of electron interference.

The experiment goes like this: if slit 1 is opened and slit 2 is closed, the distribution of electrons which have arrived at the plate equals distribution 1, shown in the figure. On the other hand, if slit 2 is opened and slit 1 is closed, the distribution of electrons which have arrived at the plate is the curve labeled distribution 2. Now if both slits are opened, classical mechanics predicts that the joint distribution shall be the sum of distribution 1 and distribution 2, but quantum mechanics predicts differently. The classical account goes as follows. Assuming that the initial momenta of electrons at the electron gun have random distribution, a particular electron will traverse *either* through slit 1 *or* through slit 2 (but not both) on account of its initial momentum at the electron gun. Moreover, where this particular electron will hit is independent of where the other electrons will hit (assuming the electron stream is not very dense so that the collisions between electrons can be neglected). Consequently, the joint distribution should be the sum of distribution 1 and distribution 2. According to classical mechanics, the fate of an electron is determined right at the start of the electron gun, although we may not be able to know its fate technically.

The experiment outcome is *not* that which is predicted by classical mechanics! Instead, the distribution is a pattern of interference quite similar to that of light or water waves going through two slits (the undulating gray curve shown in the figure). For one thing, there are positions (e.g. the point marked with x in the figure) that are very likely to be hit with either slit 1 or slit 2 is closed but are *never* hit if both slits are opened. This phenomenon cannot be explained in classical mechanics: the fact that an electron that should have hit x when slit 2 is closed (that is, an electron that possesses the initial momenta to go through slit 1) is somehow pushed away from x simply because slit 2 is opened.

At first sight, one might argue that this particular electron could be indeed pushed away by other electrons that go through slit 2. But this is not the case. In fact, the undulating distribution remains the same even if the electron gun is throttled down so that it will emit only one electron at a time, and also when the interval between two emission is prolonged in such a way that there can never be two electrons flying at the same time. The “lonely” electron nevertheless seems to interfere with itself. Indeed, according to quantum mechanics, *a particle interferes only with itself.*

Now we encounter the first strangeness of quantum mechanics: if this electron has a particular initial momentum such that it will arrive at position x if slit 2 is closed, how come it is expelled from x if slit 2 is opened? To avoid hitting x, the electron seems to “know” that slit 2 is opened, so that it may “decide” where it should hit. Or maybe it goes through slit 1 and slit 2 at the same time?

But, according classical physics, isn't it the case that an electron can go through either slit 1 or slit 2 but not both? To corroborate or falsify this hypothesis of exclusiveness, one can introduce a position detector near slit 1 so that whenever an electron comes through slit 1, a spark is generated. In order for the electron to be able to continue its journey to the plate, the position detector has to employ some sort of nondestructive measurement technique, such as shining a light on the electron. In this way one knows whether the electron goes through slit 1 or slit 2. It turns out that it is indeed possible to check whether an electron goes through slit 1 or slit 2. But in this case, the undulating distribution disappears and the curve predicted by classical mechanics is observed. Classical mechanics becomes suddenly correct again.

A common "explanation" of this is: since one has to use photons to detect the position of passer-by electrons and to determine the position of electrons highly accurately (so that one knows with certainty that a particular electron goes through slit 1 but not slit 2), one has to use light with a shorter wave-length (and therefore higher frequency ν). According to quantum mechanics, we know that the energy of a photon is

$$E = h\nu,$$

so photons with higher frequency must have higher energy. As a consequence, collisions between photons and the electron will push the electron back to position x. Sadly, this *cannot* explain everything. For one thing, why should an electron go back to x and not somewhere else when the position detector is turned on? Moreover, it also does not explain what happens to the electrons when one is not "watching" (with position detector turned off)? Does the electron go through either slit 1 or slit 2? Indeed, a haunting question in quantum mechanics can be formulated simply: what happens to a physical system when *nobody* is watching? It shows that the presumably "objective" physical reality depends on the observer's "way of looking."

In fact, electrons have properties of both *wave* (going through slit 1 and slit 2 simultaneously) and *particles* (going through *either* slit 1 *or* slit 2). This is usually called the *wave-particle duality* in quantum mechanics. For practical purposes, it is enough to assume that an electron does somehow "know" whether both slits are opened. To determine the properties of a wave (e.g. wave length or frequency), we have to assume that the wave extends into infinity. So these properties are *holistic.* Since waves's properties are holistic, this "knowledge" must be holistic as well. In more concrete terms, this "knowledge" is described with a *wave function.* One should bear in mind, however, that the only properties a system can manifest are those of particles (in this experiment, sparks).

3.3 Elitzer-Vaidman bomb testing problem

Another strangeness and indeed power of quantum mechanics is that quantum mechanics can test something that *might* have happened but *did not* happen. A question formulated by Elitzer and Vaidman in 1993 clearly demonstrates this property (cf. Penrose [29]). The experiment goes like this: in a fictitious scenario, there is a large collection of bombs. Each bomb has an ultra-sensitive detonator on its nose connected with a mirror. The detonator is so sensitive that a single photon hitting the mirror will set off the bomb. However, there are a large number of duds in the collection whose plungers connected with the mirrors can get stuck. The problem is then: is there any way to test the bomb so that one can identify whether a particular bomb is a dud without setting it off if it happens to be a good one?

At first thought there is no solution, for any testing procedure will set off a good bomb, because according to quantum mechanics one has to observe (shooting photons at the bomb) whether the plunger is stuck. However, there is a solution and strangely enough, we need quantum mechanics to arrive at it. The solution is illustrated in Figure 3.2. In this setup, the light source emits only one photon. Now if a bomb is dud, the mirror on its nose functions as a normal mirror. In this case, the wave function describing the photon indicates that there are two separate states, one state is the photon passing through the half-silvered mirror and heading towards the dud bomb and the other state is the photon being reflected by the half-silver mirror and taking the upper path. The setup is arranged in such a way that the length of each path is exactly the same (based on the classical

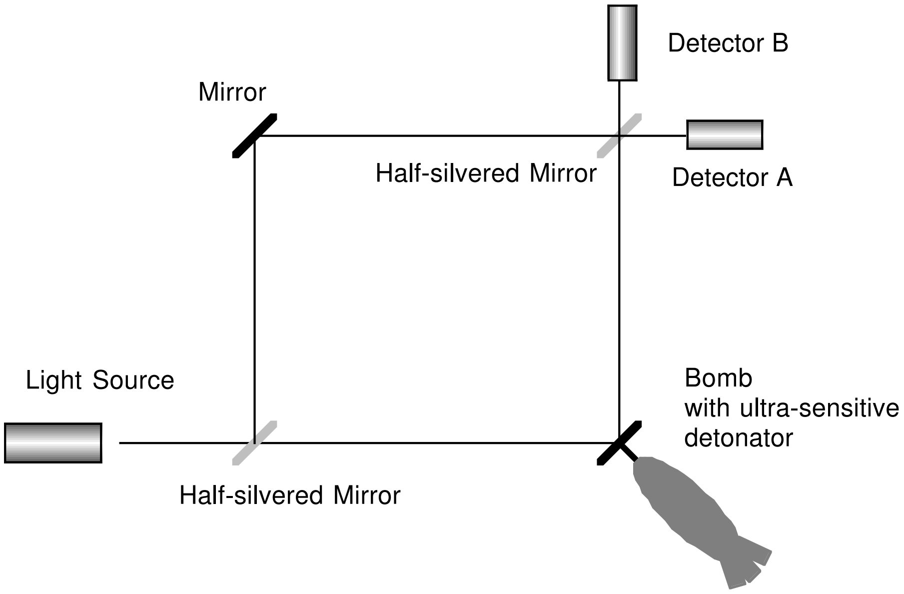

Figure 3.2: Elitzer-Vaidman bomb testing problem.

Mach-Zehnder interferometer), so the state at the detectors is a superposition of the two states. According to quantum mechanics, the wave function of the photon will be canceled out at detector B. Thus, if the bomb is a dud, the detector A is always activated, and never B.

On the other hand, if we have a good bomb, the mirror on the nose of the bomb does not function as a normal mirror, but as a *measuring device.* This is because the bomb can tell which of the alternative states the photon is in. Now if the photon takes the lower path (it has a 50% probability), the bomb will explode. In this case we know that the photon has taken the lower path, and we have lost a good bomb. However, if the photon takes the upper path, the bomb does not explode. Then we know that the photon must have taken the upper path. In other words, a good bomb measures the upper path of the photon by *not measuring* a photon. And this photon has a 50-50% chance hitting detector A or B. So only if the bomb is a good one, there is a 50% chance for detector B to receive the photon. Every now and then a photon is detected at B. The detection of a photon at B indicates that the bomb must be good and it did not explode.

In quantum mechanics, a real result can come from what has not happened. This is a significant departure from classical mechanics, in which all real effects must have real causes. On the other hand, this may also be the *power* of quantum mechanics. A similar quantum mechanical system may provide a brand new computational possibility, for all the existing computations result from what indeed happen in a real computer.

3.4 A summary of formalism of quantum mechanics

Now we have some ideas of physical characteristics in quantum mechanics. But the real power of quantum mechanics lies in its exact mathematical formalism. It is summarized in this section.

In quantum mechanics, a system's state is represented by a vector of complex numbers and is written as $|a\rangle$ (called a *ket* vector). There is another kind of state vector called *bra* vector, which is denoted by $\langle\cdot|$. The *scalar product* of a bra vector $\langle b|$ and $|a\rangle$ is a linear function that is defined as follows: for any ket $|a'\rangle$, the following conditions are fulfilled,

$$\langle b|\{|a\rangle + |a'\rangle\} = \langle b|a\rangle + \langle b|a'\rangle,$$

$$\langle b|\{c|a'\rangle\} = c\langle b|a'\rangle,$$

c being any complex number. There is a one-to-one correspondence between the bras and the kets if the conditions above are taken, with $\langle b|$ replaced with $\langle a|$, in addition to a definition that the bra corresponding to $c|a\rangle$ is $\bar{c}$ times the bra of $|a\rangle$. The bra $\langle a|$ is called the *conjugate imaginary* of the ket $|a\rangle$. Furthermore, we assume

$$\langle b|a\rangle = \overline{\langle a|b\rangle}.$$

Replacing $\langle b|$ with $\langle a|$, we find that $\langle a|a\rangle$ must be a real number. In addition, it is assumed

$$\langle a|a\rangle > 0,$$

except when $|a\rangle = 0$. Operations can be performed on a ket $|a\rangle$ and transform it to another ket $|a'\rangle$. There are operations on kets which are called *linear operators*, which satisfy the following: for a linear operator α,

$$\alpha\{|a\rangle + |a'\rangle\} = \alpha|a\rangle + \alpha|a'\rangle,$$

$$\alpha\{c|a\rangle\} = c\alpha|a\rangle,$$

with $c \in \mathbb{C}$ being a complex number. Furthermore, the sum and product of two linear operators α and β are defined as follows,

$$\{\alpha + \beta\}|a\rangle = \alpha|a\rangle + \beta|a\rangle,$$

$$\{\alpha\beta\}|a\rangle = \alpha\{\beta|a\rangle\}.$$

Generally speaking, $\alpha\beta$ is not necessarily equal to $\beta\alpha$. Together with the definition of bra, one can define the *adjoint* of an operator α by defining that the ket corresponding to $\langle a|\alpha$ is $\bar{\alpha}|a\rangle$, in which $\bar{\alpha}$ (also denoted as $\alpha^\dagger$) is called the adjoint of α. There is a special kind of operator that satisfies

$$\xi^\dagger = \xi. \tag{3.2}$$

This kind of operators is called *Hermitian.*. They are the counterparts of real numbers in operators. In quantum mechanics, all meaningful dynamical variables in quantum physical systems are represented by Hermitian operators. More specifically, every experimental arrangement in quantum mechanics is associated with a set of operators describing the dynamical variables that can be observed. These operators are usually called *observables*. For an Hermitian operator (an observable) ξ, there is a set of kets (or states) that satisfies

$$\xi|x\rangle = \lambda|x\rangle,$$

with $\lambda \in \mathbb{R}$ and $|x\rangle \neq 0$. The ket $|x\rangle$ here is called an *eigenket* or *eigenstate* of ξ and λ

is called an *eigenvalue* of ξ. Eigenvalues can be either discrete or continuous. For brevity, the discrete eigenvalues are enumerated with a subscript (e.g. ξ_i) and their corresponding eigenstates with norm equal to one (i.e. $\langle \xi_i | \xi_i \rangle = 1$) are written as $|\xi_i\rangle$. Eigenkets that have continuous eigenvalues (e.g. ξ') with norm equal to one (i.e. $\langle \xi' | \xi' \rangle = 1$) are labeled with their eigenvalues. It can be shown that

$$\langle \xi_i | \xi_j \rangle = \delta_{ij} \tag{3.3}$$

where ξ_i and ξ_j are discrete eigenvalues and δ_{ij} is Kronecker delta function

$$\left.\begin{array}{l} \delta_{ij} = 1 \ if \ i = j \\ \delta_{ij} = 0 \ if \ i \neq j \end{array}\right\}$$

and

$$\langle \xi' | \xi'' \rangle = \delta(\xi' - \xi'') \tag{3.4}$$

where ξ' and ξ'' are continuous eigenvalues and $\delta(\cdot)$ is the Dirac delta function

$$\left.\begin{array}{r} \int_{-\infty}^{\infty} \delta(x) dx = 1 \\ \\ \delta(x) = 0 \ for \ x \neq 0 \end{array}\right\}$$

In the experimental arrangement, any ket $|p\rangle$ can be expressed as

$$|p\rangle = \int |\xi'\rangle d\xi' \langle \xi' | p \rangle + \sum_r |\xi^r\rangle \langle \xi^r | p \rangle \tag{3.5}$$

where $|\xi'\rangle$ and $|\xi^r\rangle$ are all eigenkets of ξ. Moreover,

$$\int |\xi'\rangle d\xi' \langle \xi' | + \sum_r |\xi^r\rangle \langle \xi^r | = 1$$

An abstract space in which every state can be expressed as in Equation 3.5, is called a *Hilbert space.* The set of $\{|\xi'\rangle\}$ is called the *orthonormal basis* or *eigenbasis* of the Hilbert space. Given an eigenbasis, it is convenient to express a ket as a column vector of complex numbers whose components are the *projection* of the ket on the kets of the basis. This is called a *representation* of the ket. Specifically, a ket $|p\rangle$ can be represented as

$$|p\rangle = (\langle\xi_1|p\rangle, \langle\xi_2|p\rangle \cdots)^t. \tag{3.6}$$

where ^{t} denotes the transpose of a vector. The conjugate imaginary of $|p\rangle$ is then a row vector

$$\langle p| = (\langle p|\xi_1\rangle, \langle p|\xi_2\rangle \cdots). \tag{3.7}$$

It is clear that if a ket is represented by $\vec{p}$, the bra corresponding to p is $((\vec{p})^*)^t$ which is the conjugate transpose of the vector $\vec{p}$. Furthermore, for two vectors $\vec{p_1}$ and $\vec{p_2}$, $\langle p_1|p_2\rangle$ is a complex number

$$\langle p_1|p_2\rangle = (\vec{p_1})^* \cdot \vec{p_2}. \tag{3.8}$$

where $\cdot$ is the usual inner product of vectors. A linear operator α can be represented by a matrix

$$\begin{pmatrix} \langle\xi_1|\,\alpha\,|\xi_1\rangle & \langle\xi_1|\,\alpha\,|\xi_2\rangle & \cdots \\ \langle\xi_2|\,\alpha\,|\xi_1\rangle & \langle\xi_2|\,\alpha\,|\xi_2\rangle & \cdots \\ \vdots & \vdots & \cdots \end{pmatrix}. \tag{3.9}$$

With this representation, it is clear that for an operator α, the adjoint of α is

$$\alpha^\dagger = (\alpha^*)^t. \tag{3.10}$$

In this thesis, only operators with discrete eigenvalues are used. Furthermore, while the dimension of a Hilbert space can be infinite, the dimensions of bases used in this thesis are finite. In this sense, a ket is a finite-dimensional vector with complex components and an operator is a matrix with complex components.

There is a class of operators that preserve the norm of kets (i.e. $\langle p'|p'\rangle = \langle p|p\rangle$ with $|p'\rangle = U|p\rangle$). These matrices are called *unitary*. Specifically, a unitary operator is an operator with the following property

$$U^\dagger U = UU^\dagger = I. \tag{3.11}$$

where I is the identity operator (i.e. $I|x\rangle = |x\rangle$ for any $|x\rangle$).

The physical interpretation of Hermitian operators is the following. Given an Hermitian operator ξ pertaining to a particular dynamical variable (e.g. coordinate) in a particular experimental setup, each time one makes a measurement, exactly one of the eigenket (or eigenstate) will manifest itself and the eigenvalue thereof is the measured quantity. This is sometimes called the *collapse of the wave function.* Recall that the eigenvalues of an Hermitian operator are real, consequently, all the physical quantities are real. Furthermore, a state in quantum mechanics describes the experiment outcomes *stochastically.* Specifically, if a measurement is performed on a state described in Equation 3.5, the probability of getting the outcome ξ_i is

$$P(\xi_i) = |\langle \xi_i|p\rangle|^2 \tag{3.12}$$

for discrete eigenvalues. For continuous eigenvalues, the probability of measuring ξ' within an infinitesimal interval of $d\xi$ is

$$P(\xi')d\xi = |\langle \xi'|p\rangle|^2 \, d\xi. \tag{3.13}$$

where P is usually called the probability density function (PDS). In general, for any observable η, the average value of the corresponding physical quantity is

$$\langle \eta \rangle = \langle x|\eta|x\rangle.$$

We are now ready to discuss motion in quantum mechanics, starting with an analogy between quantum mechanics and classical mechanics. In classical mechanics, any two dynamical variables u and v have a Poisson Bracket (P.B.), denoted by $\{u, v\}_{P.B.}$, which is defined by

$$\{u, v\}_{P.B.} = \sum_r \left(\frac{\partial u}{\partial q_r} \frac{\partial v}{\partial p_r} - \frac{\partial u}{\partial p_r} \frac{\partial v}{\partial q_r} \right)$$

where q_r and p_r are canonical coordinates and momenta.

In quantum mechanics, the quantum P.B. of two operators u and v is defined as

$$[u, v] \equiv uv - vu = i\hbar\{u, v\}_{P.B.} \tag{3.14}$$

where $[u, v]$ is also called the *commutator* of u and v. For canonical momenta and coordinates, it can be easily confirmed that

$$\begin{aligned} q_r q_s - q_s q_r = &\quad 0, & (3.15) \\ p_r p_s - p_s p_r = &\quad 0, & (3.16) \\ q_r p_s - p_s q_r = &\quad i\hbar\delta_{rs}. & (3.17) \end{aligned}$$

which are the *fundamental quantum conditions.* These conditions also show that *classical mechanics may be regarded as the limiting case of quantum mechanics when $\hbar$ tends to zero.*

The variance of a physical quantity is defined as

$$\Delta\alpha \triangleq \sqrt{\langle(\alpha - \langle\alpha\rangle)^2\rangle}. \tag{3.18}$$

If two observables α and β do not commute (i.e. $[\alpha, \beta] \neq 0$), it can be shown by applying *Schwarz's Inequality* that

$$\Delta\alpha\Delta\beta \geq \frac{1}{2} \left|\langle[\alpha, \beta]\rangle\right|$$

where $[\alpha, \beta]$ is the commutator of α and β. Specifically, the Heisenberg's Uncertainty Principle (Equation 3.1) can be established. Moreover, q's (or p's) alone form a complete set of observables on which a state in quantum mechanics can be represented. In fact, the momentum is an operator represented by coordinate q's:

$$p_r = -i\hbar\frac{\partial}{\partial q_r}.$$

The evolution of a closed quantum system is governed by the equation of motion. It can be written as:

$$i\hbar\frac{\partial}{\partial t}\,\psi(t)\rangle = H\,\psi(t)\rangle\,, \tag{3.19}$$

where H is the *Hamiltonian* (energy), being an Hermitian operator. That is:

$$H^{\dagger} = H.$$

Equation 3.19 is known as *Schrödinger's wave equation* and its solutions $\psi(t)$ are *time-dependent wave functions.* In the literature, this is called the *Schrödinger picture.* In Schrödinger picture, the state of undisturbed motion is described by a moving ket with the state at time t represented by $|\psi(t)\rangle$. The time dependent wave function $\psi(t)$ representing a stationary state of energy H (associated with a Hamiltonian operator H) will evolve with time according to the law

$$\psi(t) = \psi_0 e^{-iHt/\hbar}, \tag{3.20}$$

where ψ_0 is the wave function at $t = 0$. Because H is Hermitian, it is clear that $e^{-iHt/\hbar}$ is a unitary operator, because according to Equation 3.11,

$$e^{-iHt/\hbar}\{e^{-iHt/\hbar}\}^{\dagger} = \{e^{-iHt/\hbar}\}^{\dagger}e^{-iHt/\hbar} = e^{iHt/\hbar}e^{-iHt/\hbar} = I,$$

where I is the identify operator.

A quantum mechanical system is linear. That is, if $|s_1\rangle$ and $|s_2\rangle$ are both physical states allowed by a particular quantum system, a *superposition* of them

$$|s'\rangle = c_1|s_1\rangle + c_2|s_2\rangle$$

with $c_1, c_2 \in \mathbb{C}$ being complex numbers, is also a physical state which is allowed by the quantum system.

In this thesis, the operators are represented by matrices with finite dimensions. For an Hermitian matrix A, e^{iA} is defined as

$$e^{iA} = \sum_{n=0}^{\infty} \frac{i^n A^n}{n!}.$$

3.5 Quantum computation

The idea of quantum computation goes back to as early as 1982, when Richard Feynman considered simulating quantum-mechanical objects with other quantum systems. However, the unusual power of quantum computation was not really appreciated until 1985 when David Deutsch published a theoretical paper [27] in which he described a universal quantum computer. Then in 1994 Peter Shor devised the first quantum algorithm that, in principle, can perform efficient factorisation [30]. In a sense, Shor's algorithm is a 'killer application,' which can do something very useful that is also, it is believed, intractable on conventional computers. In fact, the difficulty of factorising large integers is a working hypothesis on which the security of many common methods of encryption (e.g. RSA) is based. For one thing, RSA is a very popular public key encryption scheme used in many e-commerce applications today. In this section, a brief summary of the quantum computer is presented.

In [27], Deutsch laid down the foundation of quantum computation by considering the *Church-Turing conjecture*:

> *Every 'function which would naturally be regarded as computable' can be computed by the universal Turing machine.*

in physical terms. According to Deutsch, instead of considering the Church-Turing conjecture as a pure mathematical formulation, one should consider the physical version of the Church-Turing principle, which is

> *'Every finitely realizable physical system can be perfectly simulated by a universal model computing machine operating by finite means.'*

Indeed, since classical dynamics is continuous, the possible states of a classical system necessarily form a continuum. But there are only countably many ways of preparing a finite input for a Turing machine. Therefore, a Turing machine *cannot* perfectly simulate any classical dynamic system. Consequently, the Church-Turing principle does not hold in classical physics. On the other hand, a universal quantum computer is capable of perfectly simulating any finite, realizable physical system.

Specifically, a quantum computer $\mathcal{Q}$ consists of two components, a finite processor and an infinite memory. The computation proceeds in steps of fixed duration T, and during each step only the processor and a finite part of the memory interact, the rest of the memory remaining static [27].

The processor consists of M 2-state observables

$$\hat{\mathbf{n}} \equiv \{\hat{n}_i\}, (i \in \mathbb{Z}_M) \tag{3.21}$$

where $\mathbb{Z}_m$ is the set of integers from 0 to $M-1$. The memory is an infinite sequence

$$\hat{\mathbf{m}} \equiv \{\hat{m}_i\}, (i \in \mathbb{Z}) \tag{3.22}$$

of 2-state observables. This corresponds to the infinitely long memory tape in a Turing machine. One needs another observable $\hat{x}$ to specify the address number of the currently scanned tape location. Thus the state of $\mathcal{Q}$ is a unit vector of the space $\mathcal{H}$ spanned by the simultaneous eigenvectors

$$|x; \mathbf{n}; \mathbf{m}\rangle \equiv |x; n_0, n_1 \cdots n_{M-1}; \cdots m_{-1}, m_0, m_1 \cdots\rangle \tag{3.23}$$

of $\hat{x}$, $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$, labelled by the corresponding eigenvalues x, $\mathbf{n}$ and $\mathbf{m}$. Usually the spectrum of the 2-state observables is taken as $\mathbb{Z}_2$ (i.e. the set $\{0, 1\}$) and is called a *qubit*. The dynamics of $\mathcal{Q}$ is described by a unitary operator U on $\mathcal{H}$. During a single computation step, $\mathcal{Q}$ is described by

$$|\psi(nT)\rangle = U^n|\psi(0)\rangle, (n \in \mathbb{Z}^+), \tag{3.24}$$

with $|\psi(t)\rangle \in \mathcal{H}$ being the state of the quantum computer at time t; n being the "clock-step." The computation starts at $t = 0$, when the state of a finite number of $\hat{\mathbf{m}}$ is prepared as the program. In this program, the inputs of the quantum computer and the rest of qubits are set to zero. Thus

$$\left.\begin{array}{r} |\psi(0)\rangle = \sum_m \lambda_m \, |0;\mathbf{0};\mathbf{m}\rangle \\ \\ \sum_m |\lambda_m|^2 = 1 \end{array}\right\} \tag{3.25}$$

where a finite number of the λ_m are non-zero. It can be shown that in computing strict functions $\mathbb{Z} \to \mathbb{Z}$, a quantum computer generates the classical recursive functions on account of the correspondence principle between quantum mechanics and classical mechanics.

In the architectures proposed in this thesis, each eigenstate $|0;\mathbf{0};\mathbf{m}\rangle$ is associated with a symbol in a particular language. Furthermore, the preparation of the starting state as described in Equation 3.25 is referred to as a representation of a *state of affairs*. In general, a quantum computer has to have an additional state which is reserved to signal the *halt* of the computation. However, as far as our language processing applications in this thesis are concerned, we assume that the quantum computer will in any case halt after a sufficiently long sequence of execution (with sufficiently large n in Equation 3.24). Since what we are interested in is the end state of a quantum computer (that is, when the calculation is successfully carried out), n operations are absorbed into one operator with

$$\hat{U} \equiv U^n.$$

For brevity, $\hat{U}$ is denoted as U hereafter.

Chapter 4

A Quantum Theoretical Account of Linguistics

The truth is that man's capacity for symbol mongering in general and language in particular is so intimately part and parcel of his being human, of his perceiving and knowing, of his very consciousness itself, that it is all but impossible for him to focus on the magic prism through which he sees everything else.

— Walker Percy [31].

4.1 Introduction

There was a time when hominoid species other than our own – *homo sapiens sapiens* – lived simultaneously in the vicinity of our ancestors [32]. Somewhat mysteriously our species became the only species to survive, while other hominoid (Neanderthals, e.g.) became extinct.

There are different theories accounting for this paleoanthropological conundrum. A convincing hypothesis is that *homo sapiens sapiens* must have some very special and superior mental abilities. The remains of highly complex ritual paintings show our ability to use symbols, and this may be the key ability. Using symbols is definitely a relevant factor:

other hominids may also have had articulated language skills used to achieve complex cooperative tasks (highly coordinated hunting, for instance), however, as far as the ability of manipulating symbols is concerned, they were probably light-years behind our ancestors. For one thing, the language skill of Neanderthals (and perhaps other intelligent animals) may have been enough for them to achieve complicated hunting, but it was probably far too poor for them to talk about abstract issues and to accomplish more complicated mental tasks. This speculation is based on their ultra-simplistic cultural behaviors in comparison with ours. While Neanderthals did bury their dead, they probably did that simply out of worldly concerns — to keep other predators away [32]. It is humans who can give, and in fact need, an elegiac address in a funeral. It is humans who know what to say and what not to on account of respect, contempt, or cultural taboos in such situations. While other animals run away from danger, or toward food and mates, due to instincts or conditioned experiences, humans ponder which word should be used to avoid embarrassment.

In fact, the ability to use symbols indicates a central concern of the human being — *meaning*, which we tend to think of as an inherent property of symbols. These seemingly perpetual symbols offer us something to ponder; to ask questions about; and to answer. It has become very difficult for us to think of life without symbolic manipulation, for our lives are so bound to our way of making meaning by way of symbols. Without symbols, our lives would be much more ephemeral. Indeed, our species should have been called *homo sapiens significans* — the meaning-making man instead of *homo sapiens sapiens* — the intelligent man.

4.2 Meanings, symbols, and linguistic reality

4.2.1 A net of meaning

To begin with, we live in a net of meaning. But what do we mean by *meaning?*

When someone says “Good morning!”, what does that *mean?* It *means* that the time of the utterance is in the morning (say between 7:00 and 11:00 a.m.); it *means* that the speaker observes the habitual social politeness; but if this sentence is uttered in late afternoon, it may *mean* that the speaker is laughing at you, or if she *means* it, she must be somewhat insane or at least a little absent-minded; if she says these words cheerfully, it may *mean* that she had a nice sleep; if, however, she says them cheerlessly, it *means* that she didn't sleep well or may be sick; it may *mean* that she grew up in a region with a particular English dialect, it may also mean that she is not a native English speaker — all depends on her pronunciation; it may *mean* that she is timid; it may *mean* that she is arrogant; ... and so on and so forth. As it is all too often the case, if we ask what an utterance means, we will end up with a caboodle of defining or describing sentences. If, however, we ask further what the answering sentences mean, we begin to wander aimlessly in a "net of meaning."

Strangely enough, we nevertheless seem to know what "Good morning!" means. A second look at this matter shows that our understanding of meaning is not only about the meaning of a word or an utterance. Meaning is about our "lives!" We work, play, learn, or rest, and all these activities seem to *make sense*. Furthermore, the meaning of these activities is not just about the minimum goal of life — to survive. My preparation for a better education for myself or for my child seems to go beyond the minimal purpose. Few will disagree that it certainly means something more. But then *what does it mean?*

It may be argued that this is an ill-posed question and may blur the issue of "the meaning of meaning." Indeed, when we ask what our life "means," we are actually asking the purpose, which is, roughly speaking, in the subjective (intentional) realm. As an endeavor to restore the exactness of science, one might suggest that there are in fact two kinds of *meaning*: a "ghost-free" meaning such as smoke "means" fire; and a "subjective" meaning that has to do with *intention*. In the former case, it is believed, we are talking about a substantial connection between a symbol and an object or an objective property. And this is to be distinguished from *teleological* or *intentional* meaning, the latter case mentioned, which is subjective in essence.

It is this dichotomy that has nourished in some respects the belief that semantics can be either treated as a stand-alone discipline in which *intention* has its say, or as a reduction of “exact science” in which the apparent intention is to be explained away or precluded on account of objective physical properties. In the first category, as the students of literature critics may be prone to, one seems to find a comfortable place for either a Cartesian dualist standpoint or an idealist standpoint. In the second category, as a computer scientist (or a Chomskyan linguist) may be prone to, while Cartesian dualism is still attractive, a naive physicalist stance (that there is no such thing as pure mind except physical phenomena) is perhaps taken more often. Since the workers in the first camp likely would not bother to call themselves “scientists,” let us concentrate on the view taken by the second camp.

Unfortunately, modern physical science can not back up the naive physicalist view of the second camp. On the contrary, quantum mechanics implies a position against dividing subjective minds from observed physical objects. At this moment, whenever a physicist asks herself *seriously* what physical reality is in the hope that it will offer the final support of physical meaning, she is doomed to become lost in the net of (physical) meaning. This is because physical reality suffers a crisis of meaning (albeit not admitted by most physicists), which led Heisenberg to think that the problem in the interpretation of quantum mechanics is linguistic in essence.

A view started with the consideration of physical properties does not have to lead to naive physicalist reductionism or dualism. Nor does it have to end in aimless linguistic wandering. In fact, quantum mechanics has something very profound to say about the question of meaning. According to quantum mechanics, if not actually being measured, mathematical symbols cannot take any physical manifestation. That is, mathematical symbols are physically meaningless and therefore cannot contribute to a serious reductionist account of objective linguistic meaning or logical truthfulness.

For one thing, mathematical symbols themselves do not *mean* anything concrete. Their meaning is embedded in the *context* in which they appear. (A mathematical discourse usually begins with “Let x be y...”.) Indeed, mathematics as a whole is an abstract enterprise of mathematical context. In quantum mechanics, the situation is very similar, only here *physical meaning* is concerned. That is to say, the framework of meaning (physical meaning — the physical properties pointed out by symbols in a mathematical formalism) in the quantum world is enfolded in the context of apparently "meaningless" symbols.

Like our apparently endless searching for meaning on a linguistic net of meaning, reality in quantum mechanics lies in a net of meaning as well. This view of physics suggests that we might be able to profit by looking at linguistics.

4.2.2 "Signifier" and "signified" in computation

In [33], Ferdinand de Saussure proposes the idea of "sign," which is composed of "signifier" and "signified." He stresses that the signifier and the signified are as inseparable as the two sides of a piece of paper. In this sense, "signs" are atomic, since they are not further dividable. "Sign" consists of the centerpiece of semiotics and captures a crucial characteristic of language. For the sake of argument, let us try to "force" this view into a modern computational framework.

For a computer scientist who is accustomed to an analytic way of thinking, a dichotomy of "signifier" and "signified" seems to invite further analysis. Indeed, when it comes to the question of the "meaning" of a symbol, modern computer scientists (including computational linguists) tend to go deeper into the "signified." For example, a naive *ontology* can be developed to represent the "signified" as a set of non-linguistic concepts. Specifically, in a computational implementation, the relation between "signifier" and "signified" is slot-filler or container-content.

In a computational model, these "non-linguistic" concepts of fillers or contents are entities implemented in a formal computer language engineered by human experts. In practice, a concept very often (perhaps always) turns out to be a composite concept and can be further analyzed. A computational linguist who is constrained by limited computational resources has to know when the analysis should come to a stop. In most cases, it is taken as a practical question — it depends on the capacity of computational resources, the complexity of the domain of discourse, etc. These constrains result in a limited set of primitive concepts that are at the very bottom of the representation scheme. In any case, these primitives are linguistic objects embodied in a formal programming language.

Moreover, the abstract embodiments are atomic, since they are not analyzed further. They are frames (because they are symbols) but also fillers. As far as the atomicity of these primitives is concerned, the "slot-filler" picture is remarkably similar to the original idea of the inseparability of "signifier" and "signified."

Note that the way with which we arrive at this conclusion is independent of the kind of computation used. This conclusion is equally valid in a quantum computational framework.

4.2.3 Duality of symbol and concept – a thought experiment

The genuine inseparability of the "signifier" and the "signified" of a sign invites us to suspect that there is some sort of *duality* or *complementary* property of the "signifier" – symbol and the "signified" – concept. This is rather similar to the particle-wave duality in quantum mechanics. (Duality can be seen as two aspects of an entity.)

Now, for the sake of argument, let us think of a linguistic symbol as a particle that has a well-defined position. In an observation of physics, a particle sits *stationary* at a particular reference point (a grid-point in a Cartesian-like coordinate). Similarly in language, a symbol sits squarely at a particular place of a vocabulary set.

For example, a symbol *"ouch"* is the symbol *"ouch"*, nothing less, nothing more. It sits *stationary* at the reference coordinate of one's set of English vocabulary, ordered alphabetically. To understand the *concept* represented by this symbol, one needs other symbols to define its content. For instance: "*'ouch'* is an utterance showing pain;" "*'ouch'* is a sound to express dismay," "*'ouch'* is a word usually not used in a scientific paper," etc. The more symbols (with their inseparable concepts as vehicles of definition) one employs, the better one can define "the" concept represented by this symbol. In an ideal case one should travel through all the symbols and their combinations in one's vocabulary in order to completely define the meaning of any one symbol. This said, we can understand that a *concept* is in fact a highly *dynamic* and *holistic* property. In this sense, *concept* has the properties of a wave (which is dynamic and holistic as well). In quantum theory, a wave is not a physically real object and therefore can not be grasped physically. A particle, on the other hand, is a classical object. It is well defined and stationary (in the sense that it has a well-defined trajectory in classical space-time).

The stationaryness of symbols may be better appreciated when we consider an example in science. For instance, when we say the trajectory of the moon is

$$\vec{x}(t)$$

where t is time, we have to treat the symbol "t" (*not* the referent of "t," which is time itself) as stationary. If "t" can suddenly change to another symbol, say "u," in the next lines of calculation without our noticing it, a discourse on the moon's trajectory may become completely illegible. In fact, the whole science may tumble down this way.

Now consider the following example in language. *"Love,"* (or its sound /l∧v/, for that matter) as a symbol, exists synchronically.[1] The concept the symbol *"love"* represents is, however, largely diachronic. The concept, for example, depends on the experience a speaker might have, which is, again roughly speaking, ontogenetic. The concept also depends on the socio-historical environment, which is, roughly speaking, phylogenetic. Without understanding this, one would be surprised by where English people in the 18th / 19th century "made love" in the novels of Jane Austin.

The sound /l∧v/ is after all only a symbol: its phonetics are supposed to be exactly the same no matter the word is pronounced by an English lady in 18th century; by a three-year-old girl today; or by a computer speech synthesizer; for what matters is the frequency-characteristics[2]. The concept the sound represents (or may represent) is nevertheless something in time domain. This leads us to consider the duality of symbol and concept in another way.

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[1] If an apparent scrabbling delivered to us from the late Renaissance or a noisy recording of an utterance is deciphered as "love," it *is* the symbol of "love," which, strictly speaking, has become time-independent.

[2] Strictly speaking, it also depends on the orientation of the listener. While the majority of native English speakers may agree on the ending consonant is /v/, a native Chinese speaker may identify it as /b/ in one case and /f/ in another. In fact, this example shows that an invariant inventory of symbols across the language border is not realistic. In quantum mechanical framework of natural language, the different interpretation of /v/, /b/, or /f/ can easily be understood as orthogonal eigenstates of different languages.

4.2.4 Duality in speech signal processing

In fact, duality is a well-known property in disciplines that are seemingly not related to quantum mechanics. In signal processing, for example, duality is embedded in the frequency domain analysis. It is well known that given a signal in the time domain, there is a conjugate signal in the frequency domain that is the Fourier transform of the original signal. Time domain and frequency domain are therefore complementary to each other and can be considered a special case of duality. In the time domain the dynamic aspect of a signal is manifested. In the frequency domain, on the other hand, the stationary aspect is manifested. Not very surprisingly, this frequency-time duality can also be derived from quantum mechanics by considering a signal as electromagnetic properties. In quantum mechanics, an electromagnetic wave is treated as photons. Photons have properties of both waves and particles. Specifically, we have the following relation:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

where E is the energy of a photon and t the time. Together with

$$E = h\nu = 2\pi\hbar\nu$$

with ν being the frequency of the photon, we then have

$$\Delta\nu\Delta t \geq \frac{1}{4\pi}.$$

which is a well-known relation in signal processing.

4.2.5 Physical account of linguistic reality

As far as reality is concerned, it has to begin with what can be seen and touched. In a nutshell, it should begin with *sensorial* data. In a physicalist account, these sensorial data have to be grounded in physical properties. It is a long tradition in the West that one envisages an *ontic* object that holds together all kinds of sensorial properties of this (maybe conceived) object, such as smell, taste, shape and color. For example, we think of the smell, shape, and color of a rose instead of disconnected sensorial data. In classical physics, these sensorial properties are taken as something that can be derived from more elementary properties. For example, temperature is the macroscopic manifestation of molecular velocity. Specifically, length, mass, time and charge consists of the elementary physical properties of a classical physical object (a rigid body). Other properties such as momentum, velocity, acceleration, energy, etc. can then be derived from the elementary properties. We notice that the Cartesian coordinates can also be envisaged as three mutually perpendicular rigid yardsticks of infinite length.

In a more modern form (mainly due to the Theory of Relativity), all physical properties have to rely on *operational definitions.* That is, for every physically meaningful property there must be a physically feasible operation that can be performed and will reveal this property. For example, simultaneity can be established only by synchronizing two clocks with a physically feasible medium — light flashes, from which the property of time can be defined. Combining this with the world view above, an operation has to *be performed on something*, and this something can be regarded as an ontic "physical object." Intuitively, physical reality consists of all the objects that manifest physical properties. The objectivity of physical reality guarantees that even if no observer is present, all physical properties are well defined and stable. Although not necessarily leading to materialism, a world view like this is *classical realist.* In short, in the classical view something is *real* if and only if it is *operationally* well defined. Furthermore, in the classical world view, the universe is well defined.

However, in quantum theory, this naive view has to be revised. In discussing quantum objects, all properties (represented by mathematical symbols) are *physical* in that they can indeed be "seen" and "touched" if one performs proper measurements. However, we have to abandon the naive assumption of certain ontic objects. Nevertheless, we can still hold these properties as *"real,"* if we extend our understanding of "real" a little bit. As to what a "real" stone means in everyday life — when it is "kicked," it "kicks back," and the quantum objects do “kick back” and can be therefore taken as real. The “kicking” in quantum mechanics, however, is accomplished by performing measurement. But what “kicks” back in the quantum world depends on the arrangement of measuring devices (we assume quantum mechanics is correct in this regard). The merit of this view is that *real* does not have to be a synonym of naively objective.

If we stick to this idea of reality and apply it to language, linguistic objects can be taken as *real* because when they are “kicked,” they “kick back,” as in physical reality. However this time what kicks back seems to be the “meaning” of the symbol. Moreover, the “meaning” is embedded in the concept generated by an aggregate of symbols. Since quantum theory is our best theory of physics, we have to resort to quantum measurement to account for a physicalist process of “kicking” as well.

Let us see where this position leads to. As in quantum theory, meaning can be considered neither symbol nor concept but as a measuring process. The measurement, however, is active and participatory in that a human arranges his/her instrument in a particular orientation and chooses to perform a particular measurement at a specific place and time. Therefore, a symbol that is supposed to point to an apparently “objective” concept, such as “smoke means fire,” has nevertheless intentional property. It is the “kicking-back” that answers in response to measurement. In quantum mechanical terms, the objectivity can be approximately regarded as the pureness and/or average of a superposed quantum state. In this sense, the objectivity is a *quantitative* property rather than a qualitative one.

Now we are ready to formally give a quantum theoretical account of language.

4.3 Description of natural language in quantum theoretical terms

4.3.1 Postulates of quantum linguistics

Postulate 1 *Language is the result of neural activities in the brain and nothing else.*

Remarks: This postulate has a strong *physicalist* (and therefore*anti-dualist*) position. One should note, however, the apparent physicalist reductionist stance does not have to be naive physicalist (materialist) reductionism. This is because the underlying world view of modern physics is not classical physics but quantum mechanics.

We have to refer to language in a very broad sense here. Language includes the public or pragmatic language (mostly verbal) with which individuals communicate with each other in a community to achieve various goals; it also includes the *private* language (mostly non-verbal) of an individual, with which she *represents* and *thinks* about various subject matter. In this sense, pictures of animals, spears, or fire which were drawn in a cave must also be considered as a sort of language.

Postulate 2 *The brain is a quantum mechanical system with quasi-classical memories.*

Remarks: Since classical physics can be regarded as a limiting case of quantum mechanics, the brain can be conveniently treated as a classical measuring device which is coupled with other quantum computational systems in the brain. The articulated language (remember this includes sounds as well as all kinds of signs in a patterned system) takes a classical manifestation and can be treated largely as a classical object.

Memory is crucial in this postulate. Since our sensorial data "right on the spot" is so ephemeral, we have to resort to memory in order to have access to reality in the world and to be able to reason upon the information. Because memory is aggregate quantum phenomena with a very large number of quanta, it is very stable. In fact, the probability may approach one, which is the absolute objectivity advocated in classical physics.

Postulate 3 *The "reality" in the brain is a quantum mechanical experimental setup arranged by the brain and consists of the subject matter of thought.*

Remarks: Our impression of stable and invariant "substance"and modern neurology indicate that our memory may be largely classical properties embodied in the specific orientation of nerves or synapse strengths, etc. According to modern physics, these classical properties are quantum properties in the limiting case. Nevertheless, there is also "reality" in our mind which is not stable and seems to be very evasive — for example when we reflect on aesthetic or ethic judgments in some cases. This fact indicates that the proper quantum effects in our brain are probably playing a crucial part.

According to this view, thought in the brain is largely the interaction of the brain with its own memory and occasionally external sensorial data. Moreover, there must be something *active* in our mental process which can be accounted for only in quantum theory. In quantum mechanics, measurement results generally depend on the experimental arrangement. In the mental realm this active arrangement is *participation* instead of mere *interaction*. A corollary is that mental "reality" is something actively constructed instead of something passively given.

Postulate 4 *Language understanding is a quantum measurement performed by the language user.*

Remarks: Language understanding is a process of grasping *meaning*. Only after a measurement is performed, can meaning be given to a particular physical situation. As noted in the remarks of Postulate 1, language is used here in a very broad sense. Therefore, a mental image of a flower or a person standing up, when a particular memory composition is *actively* identified as such, is a kind of understanding. An image of a bunch of red spots on a colorful background is, however, *meaningless*. So is the pronunciation of *"rose"* (/ro:z/). It does not mean anything until it is actively identified as rose (the flower) or past tense of the verb "rise." Since a quantum measurement can render only one of the eigenstates, a mental image can be either a flower or not a flower, but not both. Before the measurement is performed, on the other hand, a physical state is generally a superposition of eigenstates, so all utterances are basically *ambiguous*.

Quantum measurement is inherently participatory and active. In the same way all meaningful language understanding has to be participatory and active.

In a communication scenario, an individual grasps the meaning of the utterance of his/her partner by measuring his/her own quantum system coupling with the external physical environment. The external environment is influenced by his/her partner. For example, a speaker participating in a conversation may pronounce a series of sounds, draw pictures on a black board, pointing at something, or wink. All these activities alter the neighboring environment. The resulting physical quantity change is largely classical, the air pressure, the chalk residue, and the reflection of light are all classical properties. However, the final understanding is a quantum measurement, for the classical signals are coupled to something inside the brain, which is a quantum mechanical phenomenon according to Postulate 2.

The listener does not have to perform measurement on her/his eidetic experience resulting from his own memory and immediate external utterances. However, a not-measured eidetic experience, roughly speaking, does not enter consciousness and is not really of any value for a sophisticated linguistic activity, although it may result in coordinated behavior.

Definition 1 *A language formulation (in this thesis called* representationing*) is a series of quasi-classical physical quantities conveyed by a speaker resulting from his/her active measurement of the physical situation in his/her brain (mental states or states of affairs).*

Remarks: A language formulation (short: formulation) or *representationing* is an utterance which is supposed to convey *meaning.* Gibberish may be interpreted by the listener to mean something. However, it is very likely not what the speaker means. On the other hand, a cough from a person, if purposefully made, may mean that she needs attention. If this is correctly interpreted by the listener, it is a formulation. Usually an utterance can mean something to others only if it first means something to the speaker herself. This is why a quantum measurement has to be performed *before* a state of affair is uttered.

A formulation is a description (incidentally, the etymology of describe is Latin*de-scribe*: to write down) or representationing of the internal physical state of the speaker made by writing down some kind of orthographic notations. The process of “writing down” is generally classicizing a quantum state, where something[3] is doomed to be lost. A formulation

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[3] It is tempting to call this something information, but as information (in Shanon’s sense) is objective representation, at best we can argue that it is the representation at a “higher” level. The hierarchy is here clear but may be misleading.

can therefore be conceived as a projection from quantum state to classical state. Nevertheless, it is possible to rebuild the original quantum state to considerable accuracy by conveying more information. For instance, personal conversation may be a more complete form of reconstructing the original state of affairs than video teleconferencing; the latter, in turn, is a more complete form than a telephone conversation, etc.

Postulate 5 *Language understanding and language formulation do not commute.*

Remarks: The fact that language understanding and language formulation do not commute can be demonstrated in the following example: suppose someone is given an arbitrary utterance. He reads it aloud to himself, takes a breath, and tries to grasp its meaning without speaking a word. Then he formulates the same subject matter again. If the subject matter is moderately complicated, with great chance he will end up with a different formulation (in the narrower sense, viz. orthographically). We have to understand the subject matter before we can re-formulate it. In fact, we are not mechanistic copycats. Once we *understand* something, we can formulate the matter quite *freely* (the only constraint, so to speak, is its *meaning*).

Apparently, if we take a verbatim audio or video record of our uttering the same sentence twice, in succession, the two utterances are definitely different in some minor temporal (frequency) and spatial (amplitude) aspects. But this is not the only source of non-commuteness. Non-commuteness of language has a deeper source: the collapse of a wave function. Collapse of a wave-function puts a quantum system into an irreversible state. In other words, the superposed wave-function of a quantum system is gone forever; only the eigenstate is left out and remains accessible.

Non-commuteness of formulation and understanding does not only manifest itself in colloquial language, it does in rigorous mathematical language as well. Try proving Pythagoras Theorem yourself, very likely you will end up with different formulations every time, especially if the interval between two proofs is long enough. Why? It is because you *understand* Pythagoras Theorem. A computer automatic theorem proving system built on a deterministic algorithm, on the other hand, yields the same proof every time. Few people will believe that such a program can come anywhere close to the understanding of a mathematician.

4.3.2 State of affairs as a representation in a Hilbert space

In classical frameworks, language and logic are modeled by algebra. For instance, a *lattice* is employed by many as an efficient tool to model intuitionistic logic and certain aspects of natural language (e.g. grammar). Generally speaking, algebra largely pertains to something *qualitative*. In logic, the main concern is “truth” and “falsehood” and the soundness of an argument; in (Chomskyan) language, the main concern is the grammaticality and well-formedness of an utterance. All these are qualitative concepts. In the West, these have been the main-stream (classical) thoughts.

In a quantum mechanical approach to logic and linguistics, however, the underlying structure must instead be Hilbert space. A shift from a classical to a quantum mechanical approach to linguistic and logic studies is to be accompanied by a shift in the underlying structure from algebra to Hilbert space.

Definition 2 *A Hilbert space is vector space* $\mathfrak{H}$ *with an inner product* $\langle f, g\rangle$ *such that the norm defined by*

$$|f| \equiv \sqrt{\langle f, f\rangle}$$

turns it into a complete metric space.

In quantum mechanics, the inner product is usually denoted as $\langle f \mid g\rangle$. $|f\rangle$ is called a *ket* vector, whereas $\langle f|$ is called a *bra* vector with $\langle f| = |f\rangle^\dagger = (|f\rangle^*)^t$, where * is the complex conjugate and ^{t} is the transpose operator, respectively. Generally speaking, the dimensions of a Hilbert space can be infinite. As far as our treatment is concerned, we will first start with a Hilbert space with finite dimensions defined on $\mathbb{C}$ with $\langle f \mid g\rangle$ defined as the usual complex inner product. A state of affairs is to be represented by a complex-valued vector.

As we have discussed in Section 4.2.3, quantum mechanics suggests that symbols be considered eigenstates pertaining to an observable operator in a brain quantum system. Thus we have the following definition.

Definition 3 *A* linguistic formulation operator *(or a* representationing*)* $\mathbb{S}$ *is a quantum measurement operator:*

$$\mathbb{S} : B \to S$$

where B *is the space of quantum states (the wave function) of a brain called* mental states*;* S *is a set of symbols in a language called the* vocabulary*. Moreover, the elements in* S *are shorthands of* eigenstates *of* $\mathbb{S}$*. That is,*

$$S = \{s \mid \mathbb{S}\,|s\rangle = \lambda\,|s\rangle, \langle s \mid s\rangle = 1\}$$

where $\lambda \in \mathbb{R}$ *is an eigenvalue of* $\mathbb{S}$*.*

The restriction of $\lambda \in \mathbb{R}$ is implied by the physical consideration of $\mathbb{S}$. In quantum mechanics, physical operators are always Hermitian. An Hermitian operator has real eigenvalues.

Moreover, a brain can legitimately choose its linguistic formulation operator $\mathbb{S}$ and can therefore choose its preferred vocabularies. Generally speaking, the vocabulary of every individual is not static. Whenever we learn a new word or a new meaning, our vocabulary changes. Consequently, the same state of affair may be formulated in many different ways. Furthermore, a state of affairs can be formulated in spoken words, written language, graphics, gestures, and non-linguistic sound. All these are patterned systems of general language. In this sense, every person is *multilingual*.

To explore multilingual properties, let us take a look at the “multilingualness” in verbal languages. For example, a person may speak both English and German. In this case, very often two formulation operators do not *commute*. Thus a pure state (an eigenstate) in a language (pertaining to $\mathbb{S}_1$) is not necessarily a pure state in another language (pertaining to $\mathbb{S}_2$). For instance, the German symbol *“Taube”* has to be represented in English by a superposition of *"dove"* and *"pigeon"*. In this regard, it is clear that a German-operator and an English-operator do not commute.

In quantum mechanical terms, multilingualness manifests itself in that there are multiple ways of decomposing a Hilbert space into bases. An important implication of quantum multilingualness is that a formulation can be *active* and *holistic*, because the operator must have access to the whole universe. One should note that the capacity of the brain is not limited by representationing, only the capacity of formulation itself is.

Since $|s\rangle$ is an eigenstate pertaining to $\mathbb{S}$, we have,

Corollary 1 *For each* $s_i, s_j \in S$ *and* $s_i \neq s_j$, $\langle s_i \mid s_j \rangle = 0$.

In other words, all symbols in S are *orthogonal* to each other. Intuitively, this implies that the symbols do not *mingle* with each other. This is straight-forward: a symbol x is not any symbol which is *not* x. This manifests itself as the Principle of Excluded Middle of symbols. Furthermore, since S is an eigenbasis of B, any states in the brain, as far as a linguistic formulation is concerned, can be considered a superposed state of these symbols. So we have,

Corollary 2 *For any* $|m\rangle \in B$, $|m\rangle$ *can always be decomposed into a projection on each member of* S*. That is,*

$$|m\rangle = \sum_n c_n |s_n\rangle$$

where $c_n \in \mathbb{C}$ *and* $c_n = \langle s_n \mid m\rangle$ *is the* projection *of* $|m\rangle$ *on* $|s_n\rangle$.

An interesting fact is that any complete set of symbols can serve as a basis for a Hilbert space. These symbols are symbols *proper* — they can not have any concept attached to them. So they are quite similar to the signs in Saussurean linguistics.

As for how mental states evolve, notice that a mental state, as a physical system, must follow the law of quantum mechanics. Moreover, *thinking* or *reasoning* is very likely a closed process (i.e. energy is conserved). Consequently, the evolution of a mental state is a *unitary* operator and can be called *thinking* or *reasoning.*

Definition 4 Thinking *or* reasoning *is a unitary operator U operating on a mental state with $U^\dagger = U^{-1}$.*

A symbol can be superposed with other symbols and form a composite mental state. Indeed, most mental states are composite. A sentence, for example, can be considered as a composition of multiple symbols each of which is an eigenstate. For the purpose of reasoning, a composite mental state can then be subject to unitary operations. This is what one might call *flow of thoughts*. While the outcome of a flow of thoughts may be fruitful, it has to be *measured* in order to be stored in the physico-chemical substrate in the brain or somewhere else (in computer files, for example) for later perusal or communication. This can be considered classicizing a quantum states. A classical object is all that is stable and well defined[4].

In short, quantum mechanics has something profound to say about the quality of mental objects. If thought can only be mediated by symbols as stated in Postulate 2 and Postulate 3, *realization* (grasping a particular concept or idea) is to be understood in its literal sense. It is a process of "real"-izing a complex mental state by collapsing a superposed (impure) mental state into an eigenstate of $\mathbb{S}$. Recall that a mental state is a vector of complex numbers which has both real and imaginary parts. The collapse of a mental state suffers from, roughly speaking, loss of information. There is an exception though: if the mental state is in one of the eigenstates of $\mathbb{S}$, *realization* can reconstruct the original state. Mental states in the latter situation behave *exactly* the same as what is deemed as "true" or "well-defined" in a classical logic or linguistic framework. In other words, eigenstates, being invariant under measurement, are not to be distinguished from true and/or well defined properties in a classical framework. In this sense, the quantum computational cognitive framework *extends* the classical one.

In fact, a quantum computational framework of cognition includes also those mental situations which are "not true" or "ill-defined" according to the classical view. In the classical

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[4] Strictly speaking, a classical object is nevertheless a quantum object with a huge number of quanta. The stability is to be understood as an extremely high probability (≈ 1) of finding a particular property of the object.

view, only objects of qualitative upper-hand (i.e. what is "true" or what is "well-formed") *can* have quantitative properties (and vice versa). In quantum mechanics, however, one has the tool to discuss classically qualitatively inferior objects *quantitatively.* Specifically, qualitative is a special case of quantitative manifestation (the pureness of a mental state).

One should note a limitation of this approach: viz. what can be described is that which is *measured*, that is, anything that can be classicized. Since description depends on the arrangement of a measuring device, the true mental state remains in an area forever inaccessible to language. Any measurement (for the sake of representation of the mental state) will destroy the original state.

Nevertheless, *the state of affairs* is still something that can be discussed, for it is of practical interest, especially in aggregate behavior. Thus,

Corollary 3 *A state of affairs is a vector (among many alternatives) in a Hilbert space.*

Indeed, the representation of states of affairs is crucial in order for us to build up reality. After all, what an individual has access to is only his/her *memories* of the past and his/her ephemeral eidetic experience at present. The semi-stability of memories and our competence to access it (by continually *measuring* memory and reconstructing the corresponding eidetic experience of the past) contributes to our understanding of reality. Since memory is largely classical, we *construct a* "reality" conforming to classical physics and call it substance (matter in Cartesian sense). We go even further to substantialize many other active processes, such as thinking and mind, and treat them as something like matter, only to realize that the mind is different because it is difficult to be substantialized.

4.3.3 The Uncertainty Principle of language

In quantum mechanics, if two operators do not commute, there exists an uncertainty relation between the physical quantities represented by the two operators. Consequently, the uncertainty principle in language can be expressed formally as follows.

Corollary 4 *If S_1 and S_2 are two linguistic formulation operators which do not commute, we have*

$$[S_1, S_2] \equiv S_1 S_2 - S_2 S_1 = i\hbar I$$

where I is the identical operator. As a consequence, we have

$$\Delta S_1 \Delta S_2 \geq \frac{\hbar}{2}$$

The non-commuteness of two linguistic formulation operators can be easily checked by the eigenbasis of the operators: If S_1 and S_2 have different eigenstates, they do not commute. Thus, the non-commuteness can be established on account of *counter-examples.* For instance, *"Taube"* in German and *"dove"* or *"pigeon"* in English indicates that there is an uncertainty relation between German and English[5].

There is another aspect of Uncertainty Principle as far as language understanding and language formulation is concerned. This can be derived directly from Postulate 5.

Corollary 5 *If U and F are the two operators associated with language understanding and language formulation, we have*

$$\Delta U \Delta F \geq \frac{\hbar}{2}$$

The implication of Corollary 5 is twofold. First, it is an Uncertainty Principle of *interpersonal* communication. It undermines the possibility of perfect communication. But this is not to say that there is no effective communication[6]. In fact, the effectiveness can be understood in a stochastic sense and has to be established through engaged "tuning up" of the common language used by the participating parties. They can achieve this by

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[5] There are many cases where a specific state (and the associated symbol) is an eigenstate of both linguistic formulation operators. For instance, there is little room to dispute that an eigenstate associated with *"I"* in English is also an eigenstate associated with *"ich"* in German. Nevertheless, if there is an eigenstate which is not that of two languages at the same time, there is an uncertainty relation between the two. This is because what matters is the representation of the states of affairs, not single symbols.

[6] In a sense, a classical mistake of linguistics is to confuse what is perfect with what is effective.

negotiating the definitions in their vocabulary, for example. Secondly, Corollary 5 applies to the "private" language and thought of an individual as well. It therefore implies that the reality envisaged by an individual is intrinsically not perfect, for an individual has to memorize his/her eidetic experiences — either by keeping them in the head or writing them down on paper — for later perusal. In this sense, Corollary 5 can be interpreted as an Uncertainty Principle of memory and of the eidetic experience the memory is supposed to represent.

Understanding the Uncertainty Principle in language does not imply that the apparent logic and stability of language will fall apart. Instead, it tells us in which situation a property is stable and can be counted on. For one thing, it implies that aggregate properties of language are quite predictable, albeit stochastic. Moreover, since the Uncertainty Principle is effective only if the eigenbases of two operators commute, one can "tune" the language so that the symbols in the language remain eigenstates for a long period of time (technically speaking, that is keeping the system in *coherent*). The language used in mathematics may be an example.

Indeed, mathematics is perhaps the best example in which the symbols of the language are well tuned. The eigenstates of a mathematical discourse are kept by writing them down on a blackboard or paper constantly. This process is the "classicization of symbols." For the purpose of mathematical inference, these symbols are then constructed to form pure states and are subject to reversible logical inferences, after which the pure states remain pure. (This may be where an impression of mathematical objectivity can be built). According to quantum mechanics, the pure states of mathematical language are *coherent* states in the underlying representational language. So they can survive a long time without spreading out.

Chapter 5

A Quantum Theoretical Account of Common Sense Logic

> *The peculiar function of language consists in the symbolic expression of mental phenomena, which expression we in part need for communication of those phenomena, and in part need as a sensuous support for our own inner movements of thought.*
>
> — Edmund Husserl "Review of Ernst Schröder's Vorlesungen Über die Algebra der Logik."

5.1 Introduction

Few people doubt that common sense is very effective in helping us to "find the way." In the wilderness, even the most brilliant mathematician will not (and probably cannot) prove that he should keep away from a hungry lion. Common sense is simply crucial for his survival. However, when it comes to science, common sense seems to become a deadly enemy who one should fight against. For one thing, there is no place for common sense, it is widely believed, in pure mathematics. Everything must be rigorously *logical!* Since it is (classic) logic that is efficacious in mathematics, all "exact" sciences which talk mathematics should be strictly logical as well. But this view cannot be adequate.

It turns out that a crucial difference between logic and common sense is that logic is about *truth* but common sense (and indeed *science*) is about *relevant* information. Specifically, information about something must be capable of being *memorized* (it is representation which can be encoded and decoded in a systematic way). Therefore, information must rely on a system which is *language-like.* Thus the information about a fire does not have to be the fire itself. And the information of the danger associated with fire is not the danger itself.

In fact, if one realizes that common sense is thus mingled with language, one realizes that *information* alone may become a misleading concept. What is important is the relevance of a piece of apparently "objective" information. In fact, even a simple sentence like "hungry lions are dangerous" may not carry any objective information, strictly speaking. A hungry lion may not be dangerous for another hungry lion. It is certainly not dangerous for safari tourists who are in a well protected jeep, either. It is dangerous for an unarmed human or a sick zebra. In this regard, *relevant* information (and common sense) is always *intentional.* There must be someone or something who sees a piece of information as *relevant.* In this regard, it is hard to imagine a satisfactory framework of common sense without an adequate account of meaning[1].

This motivates us to begin our account of common sense logic with a closer inspection of language.

5.2 Information, situations and linguistic context

Before we delve into the question of how relevant information may be embedded in language, it is fruitful to elucidate what is *syntax* and what is *semantics* from the view point of both linguistics and the study of logic.

In linguistics, syntax is *the arrangement of words in sentences, clauses, and phrases, and the study of the formation of sentences and the relationship of their component parts.*

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[1] This is not to say that there cannot be an "effective" account. But that would be an engineering concern, not a scientific one.

Although whether "words" should be taken as the proper building block is arguable, any atomic entities can be replaced in the definition above. Syntax is a property taken to be intra-lingual. Classically speaking, one does not have to resort to the outside world (i.e. to any extra-lingual) factors for the study of syntax. On the other hand, semantics is the study of the meaning of linguistic constituent parts, and this has to resort to extra-lingual properties.

There are other ways of seeing syntax and semantics. The most interesting one is how most mathematicians or logicians see "syntax" (as a formal property) and "semantics" (as truthfulness). For them, syntax is *the study of the well-formed formulas of a logical system.* In the study of mathematical logic, syntax is associated with a deductive system and denotes how a formula (a sentence) can be *formally* derived from a set of axioms and very parsimonious "meta-rules" (Modus Ponens, cut, etc.). This is called a deduction. On the other hand, *semantics*[2] is the study of the truthfulness of a deductive system. In modern mathematical logic, semantics is an algebraic structure which is *outside* and *independent* of the deductive system. If all the formally (i.e. syntactically) deducible formulas of a deductive system are also true (corresponding to the semantic structure), the system is said to be *sound.* On the other hand, if all the true formulas (corresponding to the semantic structure) can be formally deduced, the system is said to be *complete.* In the literature of mathematical logic, syntax and semantics validity are expressed by $\vdash$ and $\models$ respectively.

For mathematicians, there is not much room to dispute whether such a separation of syntax and semantics is desirable or whether completeness or soundness is of any value. Given the strong Platonist (objective idealist) or intuitionist (subjective idealist) position of most mathematicians, it is rather a matter of the rules of the game. The rules (here the analytic, axiomatic, and abstract ones) are simply given by years and years of hard training.

In linguistics, because grammar is traditionally put at the center of many main-stream

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[2] Modern mathematical logicians prefer to call semantics a *model.*

linguistic studies since Chomsky's influential works [34], grammar rules are treated pretty much the same way as "syntax" is treated in mathematical logic and grammaticality in formal language. In comparison with syntax, which studies the relationship of language constituent parts, semantics has inevitably become peripheral, and is, strictly speaking, an "extra-linguistic" study.

But this dichotomy is problematic. Let us see why. A closer look at language in light of mainstream cognitive science (using the computer as a metaphor of mind) reveals that syntactic properties may not be qualitatively difference from what are conventionally called semantic. If someone knows that ravens and sparrows are similar — called birds, it is hard to imagine that he cannot know 'go' and 'give' are similar — called verbs. On the other hand, if the fact that 'ravens and sparrows are birds' is to be represented in a classical cognitive system, this fact becomes purely syntactical. This is because the *relations* between the representation of 'raven', 'sparrow' and 'bird' are nothing but the relations of *representations* in a formal language system.

In fact, even in language itself it is hardly noticed that syntactical constituents (such as 'verb', 'noun' etc.) may also have 'semantics' — in that they refer to something that is *not* in the language being used but only in the scholarly discourse of linguists. To begin with, if we say

Example 1 *The verb conjugation used in that sentence is not correct.*

we do *mean* something, don't we? In fact, the subject matter ('the verb conjugation') lies in an utterance of *someone else* in a specific language environment. It can only be *outside* of the speaker when she says 'This verb conjugation is not correct.' For example, if she says (the purposeful mistake is marked with the asterisks),

Example 2 *The following verb conjugation ∗are∗ not correct.*

the utterance is obscure if not illegible. In this sense, roughly speaking, syntax and its constituents are something that carry 'linguistic meaning' according to a specific linguistic school. As a consequence, something supposed to be syntactic (therefore without meaning) suddenly acquires its meaning at a 'higher level.'

This issue cannot bother a conventional mathematician. For him, it is a matter of hierarchical thinking (which is therefore a view from without). An honest linguist cannot afford to accept this habitual hierarchical thinking without criticism, however. For one thing, one of the greatest emancipations of modern linguistics was to get out of a *normative* paradigm and become *descriptive.* The difficulty lies in that the *describing* happens to be the *described* as well. This forces us to take a view *from within.* As far as this kind of self-reference is concerned, the situation in linguistics is similar to that of mathematical logic. But mathematical logicians are much better off, for they do not have to shy away from being 'normative.' What they are interested in is the *soundness* of their proofs. On the other hand, if language is a scheme that is *not* mathematical, we need a persuasive account that it is appropriate to use a normative mathematical "model" to describe language. Specifically, what good is it to attribute some features to semantic and some others to syntactic if they turn out to be mixed up?

In any case, we have to clear up our understanding of syntax a bit. It seems that *context* is a moderately complicated and syntactic property that we should begin with. So let us begin our endeavor by posing the following question: *"How does context influence the use of language?"* Consider the following example (cf. Barwise [35]):

Example 3 *It is 4:00 p.m.*

At first sight, the information this utterance conveys seems very clear. Unfortunately, it is not the case if we analyze it further. For one thing, it is a short-hand form of a detailed description, such as "It is *here* (e.g. $15°E$ and daylight saving time) and *now* 4:00 p.m. (with the sun light making a particular angle to the meridian, etc.)" So, though a little tedious, it is all right if one says,

Example 4 *It is 4:00 p.m here.*

In this regard, it seems that these two sentences convey the same information (so they have the same meaning)[3]. But this is not always the case. This can be seen by putting Example 3 and Example 4 in different situations (e.g. as answers to different questions). It turns out that if the question is “What time is it?”, they do mean the same. However, if the asker poses the question: “Do you know what time is it now in Tokyo?” — she may want to make a phone call to Tokyo — the second sentence becomes meaningless or irrelevant. In the latter scenario, the asker probably already knows it is 4:00 p.m. here. The information she actually wants is the time difference between here and Tokyo. This simple example shows how context may play a part in the information of everyday life. Information is, roughly speaking, context-sensitive.

One may come up with a theory in order to get rid of context sensitivity. For example, one can transform the whole scenario into “grids” in an informational framework and come up with a static context independent picture. (In this example, by extending *any* sentences to its fullest form.) But this is misleading. There is something more than naive information deeply buried in context.

First, the apparent “information” conveyed in the above dialog is, by and large, also context sensitive. For example, it is implicitly assumed that cooperation and courtesy are desirable (this can be falsified if the parties involved here are foes). If the latter assumption did not stand, the “information” conveyed could become purposeful “misinformation.”

Second, “information” is taken here as something which has well-defined properties. While it is the case in many situations[4], it is also very often not so. For the sake of argument, consider what would happen if the dialog took place in a fictitious world where watches *never* tell the correct time? Artificial as it may look, a similar scenario is common in this world. For example, we can replace the watch in the above argument with

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[3] For someone who has minimal knowledge of the General Theory of Relativity, it could be immediately pointed out that the first sentence is actually meaningless, while the second is OK. This is because any time statement is attached to a specific framework. However, this is not the point of this example. We are discussing common sense here.

[4] Strictly speaking, well-definedness is a stochastic (a quantitative rather than qualitative) property in quantum mechanics.

a "virtual oracle" which defines "degree of beauty." An informational approach has a particularly hard time to accommodate situations that are about beliefs (and misbeliefs). Unfortunately, these are *not* rare in natural language and common sense.

Before going into the third difficulty of an informational approach, we have to realize that any information is physical (information has to be grounded in "reality") and modern physics is obscure, as far as reality is concerned. (This is a recurrent theme.) This brings up the third difficulty of an informational approach, which is much more profound. The concept of physical "information" itself could turn out to be context-sensitive in the sense that it may depend on quasi-linguistic contexts (here mathematics) that are extra-physical and therefore automatically become extra-informational. The whole informational framework, aiming at eliminating context-sensitivity, suffers from the same crisis of context-sensitivity. If the postulates delineated in Section 4.3 are correct, the brain may amplify all these quantum effects into ill-defined "information." We are using an deceitful yardstick to measure the land.

The difficulty of a naive informational approach to meaning may encourage us to think over the alternatives. In this sense, it is interesting to mention a seemingly Chomskyan-opposite Whorfian view on language and thought:

> His thinking itself is in a language — in English, in Sanskrit, in Chinese. And every language is a vast pattern system, different from others, in which are culturally ordained the forms and categories by which the personality not only communicates, but also analyzes nature, notices or neglects types of relationships and phenomena, channels his reasoning, and builds the house of his consciousness. — Benjamin L. Whorf (p. 252 [23])

Despite numerous controversies and critics of the so-called "Whorfian Hypothesis,"[5] it seems to be compatible with the world view of modern physics. If symbols are to be

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[5] While the so-called "Whorfian Hypothesis," or linguistic relativity, is mostly interpreted as a language-determinism, it seems that Whorf espouses a theory in which the relation between culture and language is bidirectional.

treated as eigenstates of a formulation operator in a Hilbert space, these symbols are stripped of any semantic content once they are measured. The relationship between these symbols is purely syntactic. Consequently, syntax alone, the way of using language *in* language without resorting to outside world, is enough to weave the web of meaning. In this regard, syntax and semantics merge into a unified whole. We can call the whole the *context* or the *situation*. It is upon this unification that an active mind takes each piece of "information" into account and finds *meaning* embedded in a natural language utterance or representations.

As stated in the beginning of this chapter, "information" is in fact a misleading idea, for what matters is the active extraction of "information" out of a unity of context[6]. This is how information can become relevant. Taking the mental world as a quantum system in which the observing instrument and the observed cannot be separated, the whole experience and thoughts of a person contribute to this person as a whole. Likewise, this can not be separated from his access to linguistic context. In this sense, human reasoning, including rigorous logical reasoning and common sense, is strictly context-sensitive.

There are several implications of this view. For one thing, a quantum mechanical view of language lends itself to the belief that translation between languages can only be achieved with limited success. It should be noticed that the picture to which quantum linguistics subscribes is not only *untranslatability* but also the *intangibility* of the language usage of a person *per se*. This is the problem of *private language* à la Ludwig Wittgenstein [37]. However, quantum mechanics offers a much brighter picture. Since quantum mechanics is holistic, the parties involved in a discourse *can* form a unified system. The entanglement between the parties then makes error-free communication possible[7]. But this *cannot* be linguistic, for the Uncertainty Principle of language forbids even an error-free formulation of "private" language. It must also be noticed that since actively aligning the measuring devices of the parties plays a crucial role in establishing the quantum wholeness, so does the

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[6] David Bohm called this *active information* [36]. In my opinion, instead of *active information*, *meaning* might be a more suitable word as far as language is concerned.

[7] The two parties may understand each other. However, they may not be able to formulate their understanding.

active participation (in its everyday sense) of parties in a discourse contribute to the understanding. This seems to have been an ancient wisdom of interpersonal communication. It falls out automatically from a quantum mechanical approach to language.

There are also practical implications of this approach to linguistic context — for example in machine translation in which context is an immediate problem. Words in one language are seldom related to words in other languages in a one-to-one fashion. Translation therefore heavily depends on the correct identification of context in the source language in order to distinguish among these many-to-many mappings. Any moderate machine translation system has to take context into account. Here is where language à la quantum mechanics may provide a niche. A quantum machine translator can be designed right from the beginning based on the idea of wholeness and entanglement. One only has to employ the vocabularies of the source and target language respectively as two complete bases of a common mental state, and then train the machine to achieve corresponding *reasoning* (a unitary transformation) in translating between the two. This idea is pursued in Chapter 7.

To sum up, quantum theory can offer an account of relevant information. In the following sections, we will apply this approach to several thorny logical problems in common sense reasoning — non-monotonicity, counterfactual conditionals, and causal explanation.

5.3 Non-monotonicity

One of the "bugs" (or "features") of common sense reasoning is *non-monotonicity.* Let us begin with an example. Suppose I have an appointment at 8 a.m. on a certain Sunday and I take a look at my watch. The scenario is:

Example 5 *My watch shows 7:30 (p), so I still have enough time (q).*

However, on my way to the meeting I quickly learn that it is the Sunday when daylight-saving time takes effect, so I am certainly too late for the appointment. The scenario changes to:

Example 6 *My watch shows 7:30 (p)* and *today is the first day of DST (r), so I am too late (¬q).*

Since both reasonings seem to be sound, in symbolic logic, the two sentences above can be written as follows:

$$p \to q$$

$$p \wedge r \to \neg q$$

where p is the proposition "My watch shows 7:30;" q, "my having enough time for being punctual;" r, "it is the first day of DST." Asserting both formulas as sound is to assert that the following formula is true:

$$(p \to q) \wedge (p \wedge r \to \neg q) \tag{5.1}$$

However, it is a theorem in classical propositional logic:

$$(p \to q) \to (p \wedge r \to q)$$

which is clearly contradictory to Equation 5.1. Here we can see that classical logic is monotonic but common sense is not.

More generally speaking, non-monotonicity is a situation in which the facts which can be derived from a collection of premises $\mathfrak{C}$ is less than those that can be derived from an extension of $\mathfrak{C}$, $\mathfrak{C}'$. In plain language, non-monotonicity implies: the more one believes, the less one knows.

Apparently, if classical logic is the most reliable form of reasoning (many implicitly believe it is the case), something must have gone wrong in the above argument. Of course, unless we abandon classical logic, we have to find a way to accommodate non-monotonicity. Indeed, it is a common practice to treat the situation in Example 5 as "normal" and that in Example 6 as "pathological." More specifically, the apparent non-monotonicity, it may be argued, is a situation in which one does not have complete information. For instance, one can devise a "situation logic" to correct the "fallacy" of common sense above. This is done by treating "My watch says 7:30" as a situation (among every possible situation) in which my watch indeed agrees with an objective measurement of time. The measurement, then, is a situation in which the sunlight makes a particular angle in relation to the meridian. The premise "my watch says 7:30" is then true and Example 5 is applicable. But if some social institution offers another convention for synchronizing clocks, as the additional fact in Example 6 shows, the premise is no longer true and should be considered a different situation. Example 5 should not automatically be applied. With this approach, it is hoped, non-monotonicity can be eliminated.

In fact, any endeavor to accommodate non-monotonicity *mathematically* is an endeavor to *eliminate* non-monotonicity (this is because mathematics itself is monotonic). This is possible only if we have a *physically* sound information theory. Unfortunately, in the same vein as the arguments in Section 5.2, this is impossible.

We can look at the last statement in another, somewhat amusing, way. Notice that many instances of non-monotonicity come from false belief. For instance, I believed that my watch told the correct time in Example 5, but it turns out to be a false belief. What, then, is "genuine" belief? Shouldn't it grounded in reality? I think most physicists do believe that there is always something new to discover in physics. A newly discovered scientific fact may falsify an existing theory and therefore modify existing valid statements (by predicting the facts more precisely, for example), which makes the whole scientific endeavor non-monotonic. This indicates why the reduction of information down to physical properties is pointless.

Another interesting example is to be found in physics itself. For one thing, non-monotonicity is very common in theories of physics. Consider the following statement of the law of motion (Newton's second law):

Example 7 *The acceleration of a rigid body is proportional to its mass and the force acted upon it.*

It is a very effective tool. We know that ballistic rockets and communication satellites can be designed according to this law, but we also know it is not applicable if the body is moving at a very high speed or if the force of gravitation is very strong. In such situations, the concept of a rigid body is not even remotely correct and the whole statement in Example 7 becomes a false belief.

So Example 7 is not true. Its “truthfulness” depends on the situation. For example, in the situation described above, the “truthfulness” of Newton’s second law can be very “low,” in contrast to the “normal” situation, where it is very “high.” In this case our crucial question becomes how to find a situation (or a class of situations) in which a particular physical law is applicable. Interestingly, as far as the applicability of physics is concerned, the last question is exactly the question that we want to solve at the outset, in the hope that the knowledge of “situation” may eliminate non-monotonicity.

Since a rigorous (pure classical logical plus physical) way of eliminating non-monotonicity is impossible, one might want to suggest a *statistical* approach (actually it is an informational approach without delving “too” deeply into how one gets the information — let us for the moment say that the *probability* is given *a priori*.). For example, the “truthfulness” of Example 5 can be transformed into a formula of conditional probability:

$$P(q|p) = \frac{P(q,p)}{P(p)} \tag{5.2}$$

and that of Example 6

$$P(q|p,r) = \frac{P(q,p,r)}{P(p,r)} \tag{5.3}$$

where $P(x)$ is the probability of proposition x being true; $P(x|y)$ is the conditional probability of x being true given the fact that y is true. So if the probability of a situation can be known *a priori*, the seeming contradiction to classical logic can be technically explained away.

Let us formalize non-monotonicity first. In formal terms, a non-monotonic reasoning is:

$$\mathfrak{C} \vdash \phi$$
$$\mathfrak{C}, \psi \vdash \neg\phi$$

where $\neg\phi$ is the negation of statement ϕ; $\mathfrak{C}$ is a collection of premises; ψ is an additional premise. In non-monotonic reasoning, additional knowledge *changes* the facts that can be derived. In a classical context, this simply suggests that $\mathfrak{C}$ and/or the newly formed collection of premises $\{\mathfrak{C}, \psi\}$ is not consistent. This inconsistency cannot propagate into a statistical meta-framework. So the meta-framework remains consistent.

This approach turns out to be just as problematic. This is because the problem of non-monotonicity in fact lies much deeper. Before discussing the problem, we introduce non-monotonicity of strong kind.

Definition 5 *Non-monotonicity of strong kind is a reasoning process in which ψ is statistically* independent *of the original collection of premises $\mathfrak{C}$ and*

$$\mathfrak{C} \vdash \phi$$
$$\mathfrak{C}, \psi \vdash \neg\phi$$

Now suppose there is indeed non-monotonicity of strong kind in Nature, with Equation 5.2 and Equation 5.3 in lieu of Equation 5.1 we have,

$$P(q|p,r) = \frac{P(q,p,r)}{P(p,r)} = \frac{P(q,p)P(r)}{P(p)P(r)} = \frac{P(q,p)}{P(p)} = P(q|p)$$

That is, if the non-monotonicity described in Equation 5.1 is of strong kind, the conclusion of classical logic should hold, (in other words, $P(q|p,r) = P(q|p)$ means that both assertions have the same "truthfulness" — if $p \rightarrow q$ is "quite true", $p \wedge r \rightarrow q$ should be "quite true," too.).

Intuitively, the statistical independence of ψ and $\mathfrak{C}$ suggests that they are compatible with each other and therefore consistent. This is a new kind of non-monotonicity which can not be accommodated in a classical framework. In short, the newly introduced knowledge seems to *actively* change the reasoning structure and can derive *novel* facts and / or falsify old facts. Of course, the crucial question becomes whether there are cases of non-monotonicity of strong kind in nature. The answer is an unequivocal *yes.*

For example, in the electron two-slit experiment (see Section 3.2): if both slits are open, there is an interference pattern on the plate. However, if a measuring device is placed near one of the slits and records whether an electron passes the slit or not, the interference pattern disappears. Treating information as well-defined "things," the knowledge of whether an electron passes a particular slit is *independent* of the knowledge of other gadgets in the experiment, so it must be consistent with the original premises collection. But the experiment results prove otherwise. The consistent and independent additional knowledge *changes* the fact!

In fact, non-monotonicity of strong kind is not remote to our everyday reasoning. Consider the following situation: one day I opened the door of my apartment, and as I was a little bit distracted I did not notice that my neighbor was just walking by the door and I bumped into him. It is an extremely rare situation, but it happened. Now I have the following description:

Example 8 *I opened the door at 3:30 p.m., thus I bumped into my neighbor.*

One can imagine that given the extremely low probability of my opening the door at exactly 3:30 p.m., and equally low probability of someone's standing in front of my door at exactly that time, the probability of my bumping into him can be extremely high. However, if I had known that my neighbor was passing by the door (if I had peeked through the key-hole, for instance), I wouldn't have bumped into him. So it seems valid to say:

Example 9 *I opened the door at 3:30 p.m. and as I knew my neighbor was there, I did not bump into him (but I did greet him).*

But how can my *knowing* of his presence change the probability distribution of bumping into him? The situation would be clearer if the "bumping event" was replaced by another event. Suppose the pipeline in front of my apartment broke at 3:20 that afternoon, and the doorway was flooded. If I open the door, my shoes will be wet. If I know that the pipeline is broken (by peeking through the key-hole again) but still open the door, my shoes will again be wet. In this situation, my knowing of the event does not seem to change the consequence, but wherein lies the difference?

We are all able to give a folk psychological explanation: through my behavior I *can* change the first situation but not the second one (at least it would be very hard to change it).

A key issue is here raised implicitly: we seem to have *free will* so that we *can* do something (but we *do not have to*). There are situations in which free will plays a crucial role and there are situations where it does not. It is on this issue that classical mechanics (including scientific frameworks built on naive "folk physical" understanding) and classical logic have a hard time. In fact, many endeavor to remove this sort of folk psychology and restore the exactness of "science." (that is, "mechanistic.") As long as the Cartesian mechanistic view is not abandoned, these endeavors are not likely to be successful.

According to quantum theory, the world at the quantum level is inherently strongly non-monotonic. Most importantly, a quantum measurement is irreversible — the world is different before the measurement versus after the measurement. It is not to say that classical logic cannot be accommodated in a quantum computational account of cognition. In fact, monotonicity can be maintained in quantum theory in two ways: either by holding the quantum state in a pure-state (eigenstate) so that it remains invariant and reversible; or by resorting to the derived classical system with a large number of quanta. Strictly speaking, the latter case is a limiting case statistically approaching monotonicity.

A final point: while classical reduction of non-monotonicity seems quite hopeless, the probability picture (which is a classical framework nevertheless) of non-monotonic reasoning is not completely out of focus. In fact, it can be treated as a derived characteristic and can be better understood in a framework of *possibility* vs. *actuality* in a quantum mechanical context. This will become clearer after we discuss counterfactual conditionals.

5.4 Counterfactual conditionals

Counterfactual reasoning is a thorny problem that interests many logicians (cf. Lewis [38], for example). Roughly speaking, counterfactual reasoning is drawing a conclusion based on antecedents which are *not* (or not yet) the case. For example, "if I paint the moon red, it is not green." The state of affairs of counterfactual reasoning is therefore not (or not yet) *actual* in this world. For a long time, it has been taken as an epiphenomenon of sound logical reasoning and should be at best tolerated and at worst totally removed. After all, as many believe, a sound argument should be based on facts but not fiction, and this is what logic is all about.

However, this view misses a very important point of counterfactual reasoning, which plays a crucial part in our life. In fact, every decision made, when carefully thought over, is based on some sort of counterfactual reasoning. Consider the following example. A university senior had to decide whether she should attend graduate school or get a job. At the moment of this decision, her attending graduate school was definitely not actuality, but neither was her taking a job. In order to make a better decision, she had to *imagine* a thread of the future in which she attended graduate school (the 'school'-thread) and another one in which she did not (the 'job'-thread). In the first thread she would have to branch further to threads in which, for example, she took either computer science or physics as her major. Likewise if she followed the 'job'-thread. Her typical reasoning about this would probably be,

> ... If I had a graduate degree, I would get a better job.
> ... But if I attend graduate school, I would have to take a loan because I don't have enough money at this time.
> ... I don't really like the life in the academic world. But if I attended graduate school, I'll have to live with it, at least for a while.

> ... But if I get a good job now, I'll actually have a better chance in my career than if I go on the job market in two years (e.g. the Internet boom in the late 1990s.)

To accommodate this kind of reasoning, one needs a competent model of, as well as and information about, the actual world, such as how much one gets paid with an undergraduate degree and how much graduate school costs, etc. In this regard, it has become an expert system problem. Classical logic and optimization are sufficient to solve this reasoning problem.

But this is not all that counterfactual is about. Consider the following example (cf. [35]). Jack and Jim are old friends. Under normal circumstances they help each other. But Jim is very proud, so he will never ask for help from someone with whom he has recently quarreled. Jack, on the other hand, is very unforgiving. So he will never help someone with whom he has just quarreled. Jack and Jim have a quarrel. Now an interesting question is this:

Example 10 *If Jim asks Jack for help, then Jack will help him.* .

First of all, this sentence seems to *mean* something. There is a state of affairs being addressed. But is this statement true? Since Jack and Jim just had a quarrel, Jim wouldn't ask Jack for help and of course Jack wouldn't help him. To answer the question, however, we have to envisage proper counterfactual conditions. We have to either ignore the fact that they just had a quarrel, or ignore the fact that Jim is very proud. Now if they haven't had a quarrel, Jim would ask Jack for help and Jack would help him. So the sentence is true. But if Jim is not proud (but they did have a quarrel), Jack wouldn't help him since Jack is unforgiving. So the sentence is false. This sentence seems to be true and false at the same time.

It turns out that counterfactual reasoning is not only common in everyday life, as well as carefully conceived logic games, it is also one of the most important activities involved in constructing scientific theories. A boy imagines that he can run at the speed of light (surely he does not and cannot), and he might ask himself what will happen. Similar thought experiments can lead to the Theory of Relativity. But if one tries to argue along the same line, à la Einstein, but assumes that one can accelerate oneself to a speed faster than light, and then asks what will happen, the question becomes physically meaningless and cannot be answered. (As long as the Theory of Relativity is correct in this regard.)

So, is there any way to accommodate all these varieties of counterfactual reasoning in classical frameworks? Let us try. To begin with, counterfactual is a problem of representation. According to a physicalist account, in any reasoning process, everything must indeed take place in the brain and the brain must have access to a similar or identical physical environment which is *represented* by the counterfactual state of affairs. In short, counterfactual is a "simulated" situation. It can be argued that, in this simulated environment the information (or probability) is taken into account in order to tell which is a sound or plausible argument and which is not. The parameters are acquired through processing all the sensorial data the brain has ever encountered. Let us call this a *bottom-up* approach to counterfactual reasoning.

For a cognitive scientist who uses computation as a model of the mind, representation plays a similar role. Moreover, the cognitive scientist may contend that a "theory" is built into the head like a program in a computer. And there is a set of rules with which he can manipulate the components of the theory. The soundness of a conclusion is evaluated, for example, according to how it violates the original theory. Let us call this a *top-down* approach.

The gap between a bottom-up and a top-down approach is very difficult, if not impossible, to bridge. Nevertheless, unless one wants to argue for *everything* based on *any arbitrary* imagination, the final arbitration of the validity of a counterfactual argument lies in the physical world. Whether for everyday or for serious scientific arguments, counterfactual is a physical problem and has to find its solution in physics. In a sense, a bottom-up approach has the upper hand, for it is inherently physical. A top-down approach has a hard time in "calibrating" a mental theory with the physical world. That is, according to a bottom-up approach, all counterfactual conditions *must* exist in the physical world, and the representation is only a pointer.

But the bottom-up approach has difficulty accommodating an argument such as that in Example 10. Nor can it account for a scientific argument such as the case of running at the speed of light. This is because all these situations are novel, non-existent, and, in a sense, *creative.* All in all, a representational account must be able to represent both *possibility* (for all that is counterfactual) and *actuality* (the factual). It also has to be efficacious in saying how likely a possibility will become actuality.

Here we come to a real strength of quantum mechanics. In quantum mechanics, a physical state can be constructed as a superposition of eigenstates. Every time a system is measured, the original physical system *collapses* into one of its eigenstates. In this sense, a pre-measurement quantum system can be conceived of as a superposition of possibilities in which the probability of each possibility (eigenstate) is the absolute square of the coefficient of the corresponding eigenstate. Therefore, every pre-measurement quantum system is counterfactual, although each of the eigenstates points to a physical world that has at the same time existed. A *novel* world can be regarded as a novel superposition of existing worlds. After measurement, however, the superposed system is *actualized* by collapsing into one of the eigenstates (the *actuality*).

In this way one can see why a quantum computational cognition model can accommodate these counterfactual arguments in a very elegant way. For one thing, a quantum brain is directly coupled to the physical world through nerves and other tissues. This makes it a physical system from the beginning (so it is bottom-up). Furthermore, quantum mechanics is a stochastic theory that can easily accommodate probabalistic reasoning but does not suffer from the weakness of most probabalistic models[8]. Moreover, since quantum mechanics can be discrete, measured results may “jump” between incompatible representations — such as truth and falsity in Example 10..

To see a quantum computational account of counterfactual reasoning, consider Exam-

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[8] Most probabalistic models are weak at explaining highly structural and abstract cognition, for probability is a real number between zero and one (thus continuous), while structural theory and symbols themselves are discrete (either zero or one).

ple 10 again. Both of the counterfactual premises can be rendered true as a superposition of two negative eigenstates of the system (viz. "Jim and Jack did not have a quarrel" E_1 and "Jim is not a proud person" E_2). The initial quantum state is then subject to a unitary evolution (a counterfactual *reasoning*). The outcome of this counterfactual conditional is a superposition of eigenstates of the system, in which both the eigenstates — "Jack helped Jim" and "Jack did not help Jim" — have a share. The answer to the question is a measured result of the end-state, which yields either "Jack helped Jim" or "Jack did not help Jim."

To elaborate this issue, notice that we are accustomed to situations in which the answer *cannot* be true and false at the same time. This discernibility (the XOR function or the law of exclusive middle) is crucial to logic — perhaps any kind of logic. In fact, a quantum mechanical framework implies that a measurement is indeed either true or false. Only if a system is *not* measured, can it be something in between. Moreover, if the question is asked multiple times, the outcome might jump back and forth between these two eigenstates but never stop in between.

Moreover, quantum mechanics has more to say than this sort of "quantum leap." Quantum mechanics offers a numerical framework within which one can predict the frequency of true or false. For example, probability may manifest itself this way: if Jim is actually not an absolutely proud person then he may still ask Jack for help (E_2) if, say, the situation is very urgent or the person with whom he just had a quarrel is a very close friend. Or, if the quarrel Jack and Jim had (E_1) is actually just a trivial squabble and it has been quite a long time since the quarrel. Under these circumstances, we have a probabilistic model with $|E_1|^2 = p_1$ and $|E_2|^2 = p_2$, where p_1 and p_2 are the respective probabilities of E_1's being true and E_2's being true. After subjecting the initial state to a unitary operator, we have a result that has a different composition of being true or false.

One might argue that all these advantages may be well accommodated in a hybrid model of probabilistic approaches and symbolic approaches to counterfactual. This is only partly true. All statistical models have *real* parameters and are therefore *accumulative.* They cannot accommodate wave-like interference where the probability may vanish at certain points.

5.5 Time and causality

In classical physics (including the General and Special Theory of Relativity), time can be conceived of as an additional dimension alongside three-dimensional space. Thus, motion in a physical system can be regarded as a *static* and *continuous* curve traversing through a four-dimensional spacetime. It is also *reversible.* If we turn all the particle's momentum backward, the motion can be completely reversed. As long as a *snapshot* (an event — made up of the momenta and coordinates) in the spacetime is known completely, the equations of motions *uniquely determine* the past and the future evolution of the physical system. According to classical physics, if one knows the positions and momenta of all particles in the universe at a certain time, the future of the whole (physical) universe is completely determined.

If one additionally takes a materialist position, the implication for our lives may become very profound. For one thing, *free will* will be completely eliminated. According to this view, the future is not really open for us, it is just that we *do not know* what the future is. No matter how eagerly we may try, our knowledge and apparent free will cannot alter it one bit. Furthermore, since one does not actually have free will, there is no such thing as *responsibility*. There is only a false belief in it. Since there is no such thing as responsibility, there is no reason to punish a wrong-doing. Since there is no free will, there is no reason to advocate democracy or human rights or even science in a society.

The classical spacetime theory plus materialism (let us call it *materialist determinism* or simply *determinism*) also has a side effect on causality. Causes and effects become totally meaningless in this framework. According to determinism, an event is nothing but an immediate and necessary follower of another event in spacetime and the causal chain can be traced all the way back to the beginning of the universe (if any), as well as forward to the end of the universe. Thus, a statement such as:

Example 11 *An atomic bomb in Hiroshima claimed hundreds of thousands of lives in 1945.*

is meaningless (has no content) according to determinism. If we want to talk about causality, we have to be able to *discern* what is *internal* to the system in question and what is *external.* Without smuggling in the idea of mind, which an honest determinist cannot allow, he has to treat all the entities in Example 11 — the human beings, the atomic bomb, Hiroshima, human lives, etc. — as intrinsic properties of the material world, therefore *internal.* In fact, if a determinist must talk about causality in this case, he should probably say "It is the physical laws that caused the loss of lives in Hiroshima in 1945."

But this is obviously at odds with our experience. We really *can*, we firmly believe, in a broad range of situations, determine something in the future. For example, one can choose to stretch out one's left or right foot, if one is conscious of it. That "explanation" stating which foot is first stretched is in fact pre-determined seems extremely implausible. This causal inefficacy is a major embarrassment to cognitive scientists who subscribe to *materialist determinism*, explicitly or implicitly.

In fact, causal efficacy is particularly notable in our use of language. It is highly implausible to regard the works of Shakespeare as simply "caused" by the laws of physics. Espousing determinism, art, science and indeed most valuable human activities have to be reduced to nothing but mechanistic trivialities. It is no wonder that this particular problem has lured many to subscribe to one or another variation of Cartesian dualism.

However, Cartesian dualism is not the only solution if one takes quantum mechanics into account. For one thing, the classical view of time is not correct in quantum mechanics. In quantum mechanics, for example, the collapse of a wave function is not deterministic. It is also irreversible. In essence, quantum mechanics has set the four-dimensional "frozen jelly" in classical spacetime "free." Now if brain really works according to quantum mechanics, it may be able to accommodate free will. This is because the experimental arrangement in the brain can be considered a result of a series of quantum measurements.

We should note, however, that the formalism of quantum mechanics alone is not enough to *explain* free will. A fatal criticism of a purely quantum physicalist account of free will is that free will of this kind is, in fact, a derived effect a quantum phenomenon, with the formalism of quantum mechanics being, mathematically speaking, still deterministic. Thus the whole enterprise simply surrenders the free-will problem to another deterministic framework (namely the formalism of quantum mechanics) and therefore eliminates causality as well[9]. Nevertheless, if language (and *thinking* in an internal language) is to be treated as a quantum system, the explanation of free will may turn out to be irrelevant, for in order to pose such a question the original mental state must have been destroyed. "Free will," as a symbol for genuine free will, is not *free* anymore once it is captured. In other words, genuine free will is an *unthinkable* issue.

In any case, physical cause(s) of a physical event can be technically regarded as the immediate and salient antecedent(s) of the event (let us call it causality stripped of free will). In this case, the inherent difficulty of classical spacetime becomes even more obvious, for in classical spacetime, the immediate antecedent (the infinitesimal deviation to the event) has to be the event itself, due the continuity of spacetime. Thus, technically speaking, the only cause of an event has to be the event itself. This does not explain anything.

It turns out that quantum mechanics can offer a much more satisfactory explanation in this regard. This is because to understand causality of this kind, we have to identify possibilities and actualities and establish an effective link between them. Therefore, a physical and/or cognitive framework capable of dealing with counterfactuals can explain "x causes y" as well. Specifically, the causality embedded in the utterance can be transformed to a counterfactual argument as "If it were not the case x, then it is necessarily not y." In quantum mechanics, we only have to examine the antecedent of the counterfactual conditional (the superposition of a state of affairs $|s\rangle$ in terms of *all* eigenbasis consisting of $|\neg x\rangle$) and look for the projection of $|s\rangle$ on $|\neg x\rangle$. If it is zero, the causal relation is established[10].

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[9] Technically speaking, a wave function in quantum mechanics is a superposition of an ensemble of *all* possible states. A statement talking about *all* possibilities (so-called God's view) inevitably introduces a sort of Cartesian dualism.

[10] In fact, there is no way to claim with certainty that a projection (the complex coefficient) is zero. So

One should note that the causality discussed here pertains not only to states of affairs of physical nature, such as "The revolution of the moon *causes* the tides on earth." but also to states of affairs which are highly cognitive, such as in Example 11. One should also note that a framework like this is often, if not always, *multi-causal.* In Example 11, we can surely assert that "the law of physics is *a* cause of the loss of lives." This is correct, because the atomic bomb indeed has followed the law of physics. If it had not, the bomb would not have exploded. However, thanks to the counterfactual aspect of quantum mechanics, we can identify the *relevant* causes of a situation. For instance, if the bomb dropped in Hiroshima had been replaced with a normal bomb, it wouldn't have claimed so many lives. In this fictitious situation, "the law of physics is *a* cause of the *negation* of the loss of many lives." Now it is clear that "the law of physics" cannot be very *relevant* in this case.

In fact, a quantum mechanical account of causality is not only qualitative but also *quantitative.* The strength of quantum mechanics comes as no surprise since causality can be regarded as disguised counterfactual conditionals with quantitative aspects. Furthermore, quantum mechanics can accommodate situations with mutually contradictory causes. It can also offer an account of probabilistic cause-effect relations, which are of particular practical interest. For instance, many causal explanations of medical science and engineering are based on statistical research.

We should notice, however, that a quantum mechanical account of a situation such as "Smoking causes lung cancer" should not be treated as the disguise of a statement such as "80% of lung cancer cases are caused by smoking." A statement like this has a real number probability instead of the complex component of the cause. Some information is bound to be lost. In fact, this may be a dangerous practice, for there can be cases where an additional contradictory cause may cancel out the cause completely (for example if a fictitious antidote against cancer were taken). In quantum mechanics, this is a kind of destructive interference that is difficult, if not impossible, to accommodate in a probabilistic framework.

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there is no way to establish absolute causality. One has to repeatedly perform actual measurements until an average probability can be obtained .

Part II

Applications of Quantum Theory on Natural Language Processing

Chapter 6

Preliminary Experiments

> *The opposite of a correct statement is a false statement. But the opposite of a profound truth may well be another profound truth.*
>
> — Niels Bohr

6.1 Introduction

In this chapter, two preliminary symbolic processing experiments inspired by quantum mechanics are presented. The underlying idea is to treat a symbolic computation (described by a function $\mathfrak{L}$ of discrete logic values — $\mathbb{F}$ for false, and $\mathbb{T}$ for true) as a physical experiment that obeys the laws of quantum mechanics. The input symbols are prepared as an initial state (an input state) for a particular quantum mechanical experimental arrangement. We assume that the input state can be represented in terms of a complete basis of eigenstates corresponding to an observable operator S that *writes down* the symbol of a state. It is helpful to think of S as an analogy of coordinate (position) observable X, as in the case of solving a Schrödinger equation (Equation 3.19). Every time a position-measurement is performed, the system yields a precise and well-defined location of each particle. In our case, the analogy of position is a well-defined symbol. An S-measurement can then yield one and only one of the eigenstates and the measured value can be one and only one of the eigenvalues (corresponding to $\mathbb{T}$ or $\mathbb{F}$).

Once the system is prepared, it is allowed to evolve without any external disturbance. The way the system evolves depends solely on the arrangement (it is helpful to think of this arrangement as different ways of putting "pegs" or "traps" in an analogous quantum physical system — some sort of quantum billiard table), with the total energy of the system constant. In this case, the system can be described by a (classical) Hamiltonian H and a unitary operator $U(t)$ associated with H. After a specific duration, the system is measured again against S, which yields an eigenstate of S ($\mathbb{T}$ or $\mathbb{F}$). The corresponding symbol is then said to be the result of the corresponding symbolic computation. Specifically, suppose an input $\vec{x}_{in}$ of symbols ($\vec{x}_{in} \in \{\mathbb{T}, \mathbb{F}\}^n$, where n is the dimension of the input symbolic vector) is prepared as an input state ϕ_0, the computation is carried out by an underlying physical system, as shown in the following diagram:

$$\begin{tikzcd}
\phi_0 \arrow[r, "U(t)"] \arrow[d, "S"'] & \phi_t \arrow[d, "S"] \\
\vec{x}_{in} \arrow[r, "\mathfrak{L}"] & \vec{x}_{out}
\end{tikzcd}$$

The evolution of the system (described by a wave function) is deterministic and continuous, both in spatial and temporal terms. However, since S is a quantum measurement involving an abrupt collapse of the wave function, the outcome of the computation is *discrete* and *irreversible.* Notice that this scheme is stochastic. In other words, we can only predict the aggregate behavior of the system according to the absolute square of the projection the end-state on each eigenstate (corresponding to either $\mathbb{T}$ or $\mathbb{F}$), or

$$\sigma : \phi_t \to (p_{\mathbb{T}}, p_{\mathbb{F}})^m,$$

where $p_{\mathbb{T},m}$ ($p_{\mathbb{F},m}$) is the probability of finding the system in $\mathbb{T}$ ($\mathbb{F}$) eigenstate for m-th output eigenstates with

$$\sum_{k\in\{\mathbb{T},\mathbb{F}\}^m} p_k = 1.$$

(In the experiments presented in this chapter, there is only one output, namely $m = 1$). In this chapter it will also be demonstrated that the quantum computational scheme "extends" the classical computation but has a much richer structure. In other words, classical computation can be regarded as a special case of its quantum computational counterpart.

6.2 Exclusive OR (XOR) Problem

In this section, a quantum computational solution of the XOR-problem is presented. The reader should bear in mind, however, that the XOR-problem is presented here in a *bottom-up* fashion. That is, the architecture is *not* designed to perform XOR operation, as they usual are performed in the quantum computation literature [39], but to *learn* the behavior of XOR from training data. As far as the underlying structure is concerned, the quantum structure is *reversible* (without being measured) while classical XOR is not.

It is well known that the common classical logic operators ($\wedge$, $\vee$, $\neg$, and $\rightarrow$), called *logical primitives*, are in fact redundant. For instance, from Negation-AND – NAND (or equivalently Negation-OR – NOR) alone all four logical operators can be derived. However, NAND and NOR have very obscure intuitive content. It is quite implausible that our cognitive process is built on NAND or NOR. On the other hand, it seems that XOR and AND are intuitively more "primitive" operations on which our cognition is based[1]. Furthermore, XOR is somewhat "intimate" to S-measurement in the sense that every quantum measurement manifests itself in an *either ... or ...* (XOR) scheme. This seems to be a good incentive for us to begin with the discussion of a quantum computational

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[1] *Exclusive OR* is a very primitive logical operation in decision making. In the early phase of development, a child has to discern what is desirable and what is not in order to learn anything at all. XOR may be the most primitive judgement. On the other hand, *AND* is a primitive logical operation to *juxtapose* two concepts and make a judgement.

realization of the XOR function.

Indeed, XOR is a classical problem that cannot be solved with a simple linear model. In a classical connectionist treatment of XOR, it is well-known that one needs at least one hidden layer as well as processing units (neurons) with a non-linear threshold function [40]. While a quantum mechanical system is linear, the problems a quantum system can successfully solve are not necessarily linear. We shall see that the dynamics and characteristics of this implementation are much richer than that of a classical logic gate or a connectionist architecture. It also suggests that the "fine-structure" is more realistic for our everyday reasoning and natural language. This will be discussed in the following sections.

6.2.1 Experimental setup

The model is based on a quantum experimental arrangement. We assume that the only possible measurement outcomes, as well as input states, of this system are either 0 or 1. They are the eigenstates of an *observable* S of the system and can be written as $|0\rangle_k$ and $|1\rangle_k$, where the subscript k represents the position of the quantum bit (qubit). We further assume that the qubit on the first input, the second input, as well as the output are represented by *different* eigenstates,[2] so we have in total 6 orthogonal eigenstates[3], which we claim forms a complete basis of a Hilbert space in which the state of affairs is to be discussed. A quantum computation is then to be understood as a trajectory in this abstract space. At the beginning of each quantum experiment, the initial state is prepared as a superposition of input eigenstates — either $|0\rangle$ or $|1\rangle$ on their corresponding positions.

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[2] It is a convenient rule that if we *can* tell the difference between two symbols at first sight, they should be represented by different eigenstates. This is the case if we draw a diagram of an XOR-gate on paper and look at the input. The examples discussed in this chapter have a parallel structure, so it is natural to use different symbols based on their spatial position. This can be different if the input is a sequence of symbols, in which case time plays a crucial role. These examples are discussed in Chapter 7

[3] Readers who are familiar with quantum computation should notice that we "flatten" the two input bits and one output bit as eigenstates pertaining to *one single* operator, not three operators each of which is oriented in different bit-positions (so the result is a direct product of three two-dimensional Hilbert spaces). In the latter case, the dimensions of the resulting Hilbert space is $2^3 = 8$.

For example, to compute $XOR(0,1) = 1$, the (initial) input state is prepared as:

$$c_{01}\left|0\right\rangle_1 + c_{12}\left|1\right\rangle_2, \tag{6.1}$$

where the subscripts of an eigenket $|x\rangle$ represent the position of the qubit and the coefficient c_{mn} (a complex number) is the component of the corresponding eigenstate $|m\rangle$ at position n. The system is then set free to evolve without external perturbation. Technically speaking, the initial state is subject to a unitary evolution U, which is determined by the specific arrangement of the experiment. U can be expressed as:

$$U = e^{-i\frac{H}{\hbar}t} \tag{6.2}$$

where H is the Hamiltonian (the energy operator) of the system and $\hbar$ is the Planck constant divided by 2π. (For simplicity and without losing generality, in the implementation we can take $\hbar$ simply as 1.) The linear superposition is straight-forward for a "parallel" configuration, where both inputs are fed into the quantum logic gate at the same time. It is also the case for a "serial" configuration, which is the norm in natural language processing/understanding. In the case of classical XOR, these two schemes are equivalent. This can be justified as follows: suppose the input qubits are fed into the system one after another, say first $\left|0\right\rangle_1$ then, after the a time delay t, $\left|1\right\rangle_2$, we should have the end state of the system as:

$$U\left(a_{01}\left|0\right\rangle_1 + U'a_{12}\left|1\right\rangle_2\right),$$

where U' is another unitary operator that can be written in the following general form:

$$U' = e^{-i\frac{H'}{\hbar}t},$$

where H' is the Hamiltonian of a preparation process. Since the classical XOR is a function that does not tell the difference between an input's temporal position (therefore is symmetric with respect to the input position), we should have the same outcome from the system if we put $|1\rangle_2$ first and, after the a time delay t, $|0\rangle_1$ afterwards. Namely

$$U\left(a_{01}\left|0\right\rangle_1 + U' a_{12}\left|1\right\rangle_1\right) = U\left(U' a'_{01}\left|0\right\rangle_1 + a'_{12}\left|1\right\rangle_2\right).$$

The best way to achieve this is to assume that the preparation process will not *mix* each eigenstate with others. That is, H' has to be a diagonal matrix. In this case, we can write the input as shown in equation 6.1. Furthermore, the absolute square of the coefficient of a particular eigenstate relative to the sum of absolute squares of all components (or the square of input vector length) is the probability of finding the system in this particular eigenstate. So, assuming symmetry, we have:

$$|c_{01}| = |c_{12}| = \sqrt{1/2}.$$

The end-state or output of the experimental setup is a superposition of eigenstates:

$$c'_{01}\left|0\right\rangle_1 + c'_{11}\left|1\right\rangle_1 + c'_{02}\left|0\right\rangle_2 + c'_{12}\left|1\right\rangle_2 + c'_{0,out}\left|0\right\rangle_{out} + c'_{1,out}\left|1\right\rangle_{out}.$$

Generally speaking, there can be "residues" of the input eigenstates in the end state. However, since we are only interested in the relative probability of the output qubit, we expect to measure the output states either as $|0\rangle_{out}$ or $|1\rangle_{out}$, with corresponding probability of

$$p_0 = \frac{\left|c'_{0,out}\right|^2}{\left|c'_{0,out}\right|^2 + \left|c'_{1,out}\right|^2} \; or \; p_1 = \frac{\left|c'_{1,out}\right|^2}{\left|c'_{0,out}\right|^2 + \left|c'_{1,out}\right|^2}.$$

In this experiment, we have a total of 36 *real* parameters (the free variables of the targeted unitary operator) which are to be found. This can be easily transferred to a standard minimization problem by defining a cost function:

$$C(\vec{v}) = \sum_{i\in\{0,1\}} (p_i - t_i)^2, \tag{6.3}$$

where $\vec{v}$ is the 36-dimensional parameter vector and t_i is 1 if the corresponding classical symbol ($\mathbb{T}$ or $\mathbb{F}$) is present, 0 otherwise. A standard minimization procedure can then be used. We use the conjugate gradient method [41] (see Appendix A), starting with a small random initial vector[4].

6.2.2 Result and analysis

In a typical experiment run, the training goal can be easily achieved. This is the case both when the input qubits are *in phase* and when they are *not in phase* (in which there is a constant phase difference between the coefficients of the two input eigenstates). If the input is prepared exactly as in the training process, the system *implements* the classical XOR operation, subject to small contingent fluctuation ($\approx 2\%$ statistical error). If a threshold is applied to the output ensemble (that is, whenever an eigenstate has a relative probability greater than the threshold, say 80%, it is taken as the output), an accuracy of 100% is achieved.

As is common in many "bottom-up" approaches to cognition at first sight, there is an annoying small error. In two out of a hundred experiments, the system gives a wrong answer. Since XOR is a very primitive logical operator, one might expect that the performance should be as crisp as in classical logic. But a closer look at human reasoning suggests that this is not the case. For one thing, humans do err. If human reasoning works according to classical logic, one will build a reasoning chain by putting one error-free logical block upon another. But where does the error come from? It seems a quantum account can accommodate this better.

Furthermore, there are many interesting cases in natural language where even very simple utterances can get us very confused. For instance, if the initial states are symbols that are not well defined (e.g. *oxymora* — "*either* cruel kindness *or* kind cruelty," etc.); or if the initial states are well defined but "interfere" with each other (e.g. "Psychology is a discipline *either* of sciences *or* humanities."), the outcome of an XOR operation may

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[4] Generally speaking, the choice of minimization method has to take both the computational resources and the quality of result into consideration. In this preliminary study, however, these issues are not addressed very sophisticatedly. Conjugate gradient method is chosen largely because it is a commonly used minimization algorithms that can deliver very accurate minima. It nevertheless requires more computational resources than, for example, the random walk algorithm.

fluctuate. The reason that classical logic treats the XOR function as an accurate operation is perhaps because it regards XOR as an *atomic* symbol. And this symbol is associated with an operation. (N.B. an operation is not a symbol). In this regard, the performance of quantum computational XOR is in fact more similar to that of a human.

If, however, the input state is not prepared exactly as in the training process, with the classical logic value maintained, the system shows phenomena that are not to be found in its classical or connectionist counterpart.

The analysis is done on a result trained with all input qubits prepared in phase, that is, when $c_{0,m} = c_{1,n} = e^{i\theta}/\sqrt{2}$. In fact, θ can be set to zero without losing generality since U is a linear operator. (See Appendix B for the numerical result used in this section.)

For example, if the input states are prepared in such a way that the arguments (phases) of the two input qubits' coefficients vary independently, while the absolute value of every qubit remains the same as in the training process (since the coefficient of one of each qubit pair must be zero, we have only two independent parameters instead of four), we then record the deviation compared to the targeted output. The result is shown in Figure 6.1. In this figure, the input state is prepared as:

$$\frac{e^{i\theta_1}}{\sqrt{2}}|x_1\rangle_1 + \frac{e^{i\theta_2}}{\sqrt{2}}|x_2\rangle_2 \tag{6.4}$$

where $x_1, x_2 \in \{0, 1\}$. There are four combinations in total. The four graphics in the figure are labeled with their classical logic truth-table counterparts at the top. The error (deviation from classical result) is defined as in Equation (6.3). In the figure we can see that if the input state consists of eigenstates that do not have the same phase-difference as in the training process, the deviation of the quantum computational result from its classical counterpart can be very large. The maximum is located at $|\theta_1 - \theta_2| = \pi$. A classical XOR can be found only within a small area where the two coefficients are almost in phase (viz. $|\theta_1 - \theta_2|$ is small). If $|x_1\rangle_1$ and $|x_2\rangle_2$ have coefficients that are not in phase, the output can be flipped. In fact, Figure 6.1 suggests that a phase difference of π has a similar effect, flipping $|1\rangle$ to $|0\rangle$ and vice versa. The meaning of phase is at this moment unclear. A hypothetical interpretation of phase angle will be presented in Section 6.4.

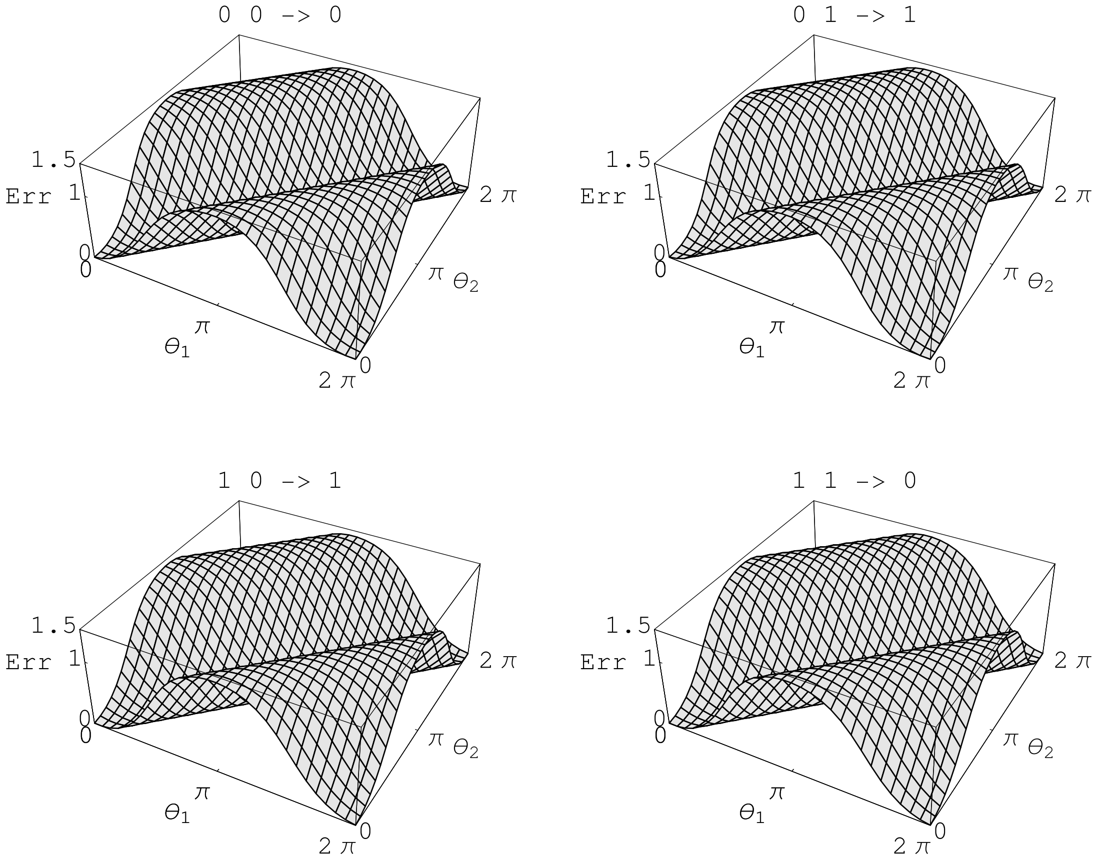

Figure 6.1: The deviation from the targets due to phase-difference in preparing the input state (θ_1, θ_2)

At first sight this seems to be a disadvantage because the result of a computation is sensitive to the phase-difference of inputs. What kind of logic is this if the result of the same input, say $XOR(1, 1)$ is sometimes 1, sometimes 0? (We are talking about the "wrong" answer, which is almost always the result when inputs are out of phase, say if $\theta_1 - \theta_2 = \pi$, not statistical errors.) A question then is, how can a system "know" how to prepare the input "correctly?" This appears puzzling because even if these symbols have difference in phases, they have the *same* symbolic interpretation. In fact, without knowing the underlying complex coefficient, one cannot tell the difference between various kinds of input states with any physically possible method. Without going to the quantum level, there can be many degenerate states that have the same "phenotype." They may heavily influence the outcomes of a computation. That is, symbols which, roughly speaking, have different "meanings" degenerate to an identical appearance. They are some sort of "homonyms." In logic study, we are in fact advocating *truth* values that have different "fine structures" — i.e. *logical homonyms* of truth and falsity. It turns out that if we restrict the preparation process so that the components with respect to each eigenstate are in phase, we have a quite stable classical system. In this sense, our quantum computational scheme *implements* the classical one. However, the fine structure allows the system to go *beyond* its classical counterpart and therefore *extends* the classical logic.

To explore this idea further, a linear preparation function is introduced to prepare the quantum input states from a given symbolic representation. More specifically, the input state is generated as follows: first the symbolic representation of the input is subject to a linear function:

$$\Phi : \{1,0\}^4 \to [0,2\pi)^4$$

to generate four phases $(\phi_1, \phi_2, \phi_3, \phi_4)$. Then the actual input state is prepared as:

$$\left\langle x_1 e^{i\phi_1}, x_2 e^{i\phi_2}, x_3 e^{i\phi_3}, x_4 e^{i\phi_4} \right\rangle,$$

where $x_1, x_2, x_3, x_4 \in \{0,1\}^4$ is the symbolic representation of the input. Note that each component of the input state prepared in this way has the same absolute value as its classical symbolic counterpart. They are therefore all "classical symbolic equivalents." The experimental arrangement is not altered, so the unitary operator remains the same in the following discussion, as in the bare XOR system.

The linear mapping function Φ can be represented by a 4 by 4 matrix resulting in 16 *real* parameters, in total, which are to be found by using a standard minimization procedure (we use a random walk algorithm [41][5]). The cost function is as defined in Equation (6.3).

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[5] A random walk algorithm can search the domain of optimization more thoroughly. It is a global minimization method. Given the relatively few parameters, it can be achieved in reasonably short time. Nevertheless, the minimization efficiency is not an important issue in this preliminary study.

But this time, instead of XOR, we now set the target function to AND. Interestingly, the system can still be trained to achieve the goal (a stochastic error of $\approx 0.01\%$, 100% if threshold is used). Now if we prepare the input state by varying the phase of each component in the "raw" symbolic representation and multiplying each component with a complex number with unit length, but independently varying phase (again, since exactly two of the relevant input qubits have non-zero component, the number of independent variables is two (θ_1, θ_2) instead of four, see Equation (6.4)), the deviation from the target is illustrated in Figure 6.2. Unlike the bare-XOR, this system delivers the correct answer only in the vicinity of the origin (or multiples of 2π).

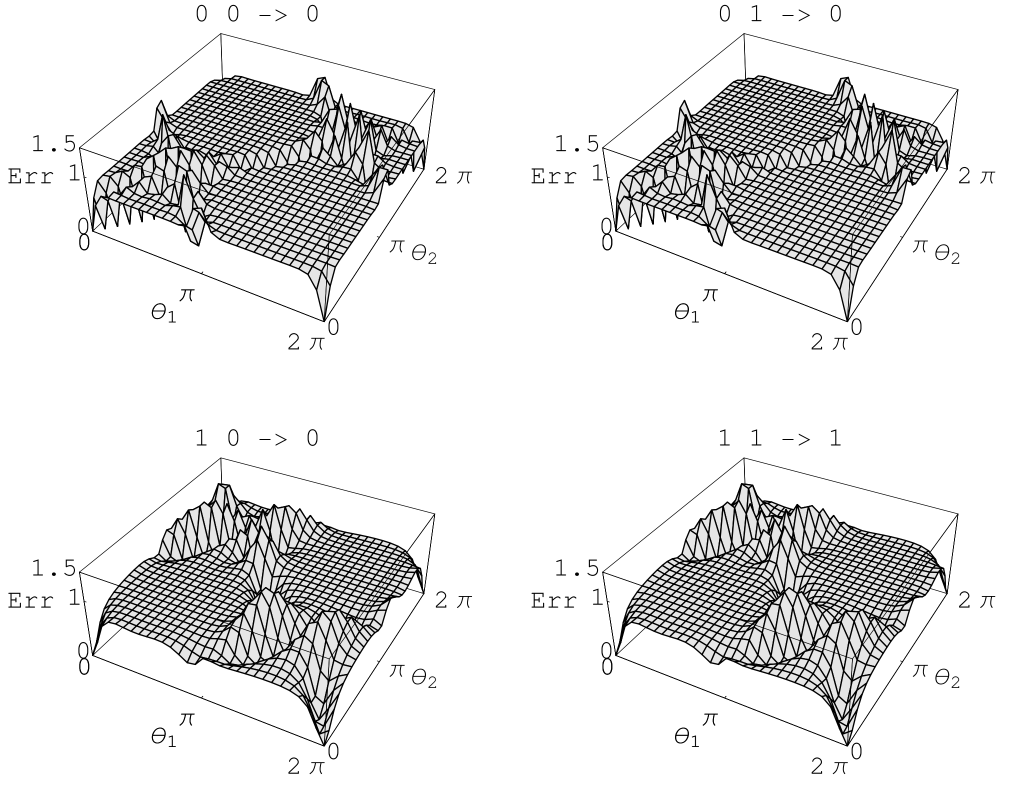

Figure 6.2: The deviation of the output of U from the target (the classical AND-function) w.r.t phase difference of inputs. The linear preparation function is trained for AND data.

In the same vein, Φ can also be prepared such that the classical OR function can be implemented (a stochastic error of $\approx 0.01\%$, 100% if threshold is used). The result is shown in Figure 6.3. As shown in these figures, the phase-"landscape" of a simple quantum computational scheme can be very complex.

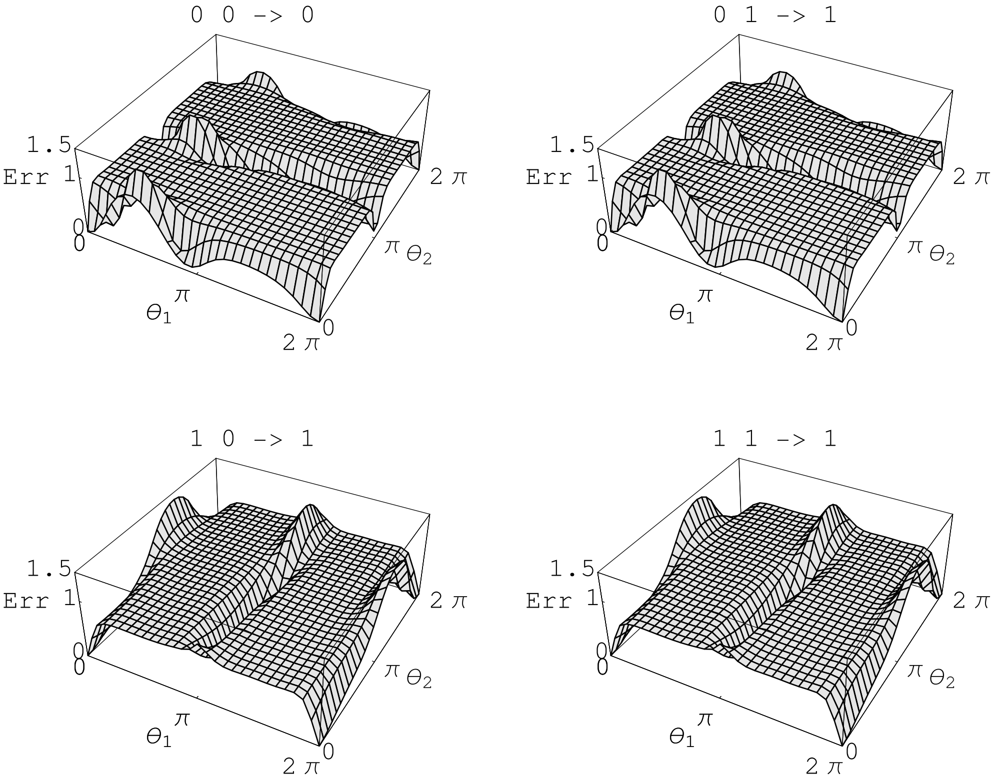

Figure 6.3: The deviation of the output of U to the target (the classical OR-function) w.r.t phase difference of inputs. The linear preparation function is trained for OR data.

In fact, it is possible to achieve different classical functions with the same quantum mechanical arrangement. There we can see a feature of quantum computation: although the input is prepared in such a way that *no* physical observation can tell the difference between two experimental preparations of inputs, the underlying structure (a complex vector) may deliver significantly different results. Moreover, the outcomes still remain *crisp* and well-defined. This is different from what is suggested by connectionist or statistical approaches. In these approaches, the output is a real number given by a smooth function of input. This is quite unnatural as far as logic is concerned.

A more interesting implication is the fine structure of symbols. A particular input (e.g. $\langle 1, 1\rangle$, read: $\langle True, True\rangle$) does *not* have to be realized identically. The difference manifests itself only at a deeper level, namely when it is described by a state vector that has complex numbers as components.

So far we have only discussed the cases in which classical truth values do not confer differences while the quantum schemes do. In these cases, the truth values of input states are well-defined in the sense that the probability of finding a particular symbol ($\mathbb{T}$ or $\mathbb{F}$) is either zero or one. There are other features in common-sense reasoning which a quantum scheme can easily accommodate — fuzziness and uncertainty, for example. In quantum mechanics, these concepts are involved in an "impure" superposition of eigenstates. For example, if the input state is prepared in a totally undetermined state such as:

$$\left\langle \frac{e^{i\theta_1}}{\sqrt{4}}, \frac{e^{i\theta_1}}{\sqrt{4}}, \frac{e^{i\theta_2}}{\sqrt{4}}, \frac{e^{i\theta_2}}{\sqrt{4}} \right\rangle,$$

the system can still draw a probabilistic conclusion. In this preparation, the first qubit (consisting of the first two components) is kept in phase with phase θ_1. Likewise, the second qubit (consisting of the last two components) is kept in phase with phase θ_2. The output is illustrated in Figure 6.4. In this figure the outputs are shown where θ_1 is set to zero and θ_2 (the horizontal axis) is allowed to vary independently. The vertical axis is the probability of the output being asserted (dotted curve — absolute square of the assertion output state) or refuted (solid curve — absolute square of the refutation output state). It is clear that under a totally undetermined situation, the system is still able to deliver results. Interestingly, when phase difference is near π, the classical output (suppose we apply a threshold to obtain a deterministic classical output) is flipped over. Such a thing happens only when the phase difference is in the vicinity of π. This suggests that under a totally undetermined situation, the phase difference plays a very important role. The intuition of this "out-of-phase" condition is at this moment unclear. The implication of phases can be more clearly conjectured when we come to counterfactual reasoning. We will come back to this issue in Section 6.4.

Another important difference of the quantum computational scheme used in this chapter is that the unitary operator is time-dependent. For simplicity, we harvest the results of a computation at $t = 1$ (arbitrary unit). Nevertheless, the result can be taken at a t

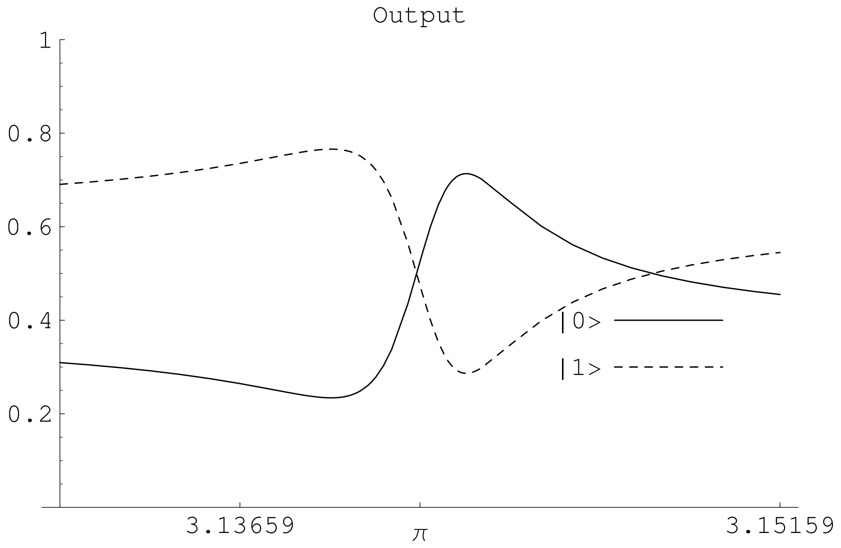

Figure 6.4: The output of a totally undetermined input state w.r.t phase difference.

other than $t = 1$. To explore this issue further, the relation of the total error of the XOR function to time is shown in Figure 6.5. In Figure 6.6, the error of the *individual* input is shown for the first 16 time-units. It can be seen in the figures that the system can deliver correct results only in the vicinity of $t = 1$. The error begins to fluctuate wildly as time goes by. The performance of the system therefore depends *critically* on when the result is measured.

At first sight, this may be a drawback. But we can still find some interesting implications. According to Equation (6.2), the unitary function is, in general, an aperiodic function of time. In fact, U is periodic only if the eigenvalues of H are multiples or rational fractions of each other. This can be verified by considering the calculation of $e^{-iH/\hbar}$, since the exponent of the unitary operator U is "pure-imaginary" (i times a Hermitian matrix H). Consequently, if the eigenvalues of H are non-rational numbers and/or not multiples or fractions of each other (I believe this is usually the case), the evolution of the system may penetrate a very large portion of the possible unitary transformations. This suggests that the same experimental arrangement may implement a wide variety of logical functions, each of which has very different characteristics (as shown in their corresponding phase-landscapes). Indeed, by varying the time t at which the results of the computation are taken, we can implement the classical AND function, with the system originally trained for XOR, to a certain accuracy. The relation of total error as AND function to time using the same data trained for XOR target is shown in Figure 6.7.

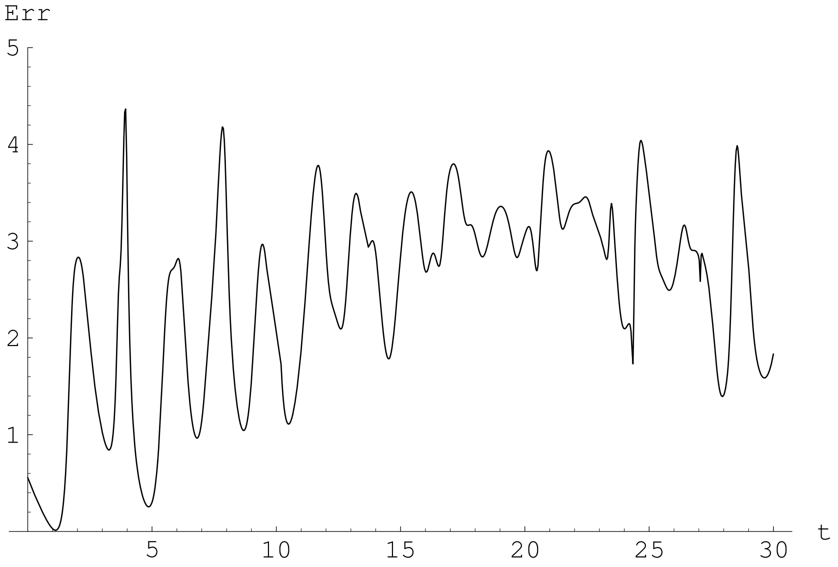

Figure 6.5: The relation of the error of the XOR function to the time at which the outcome is measured. The error is defined as in Equation 6.3.

6.3 Non-monotonic reasoning

6.3.1 Experimental setup

In this section we consider an example of non-monotonic reasoning as follows,

$$(p \rightarrow q); (p \wedge r \rightarrow \neg q).$$

where p, r are propositions functioning as “antecedents” and q is the “conclusion.” An everyday reasoning of this sort states that whenever p is true, q is true. However, if one asserts additionally that r is also true, then q can no longer be true. This scheme captures a prototypical *non-monotonic reasoning.*

In the same vein as described in the previous experiment, reasoning like this is treated

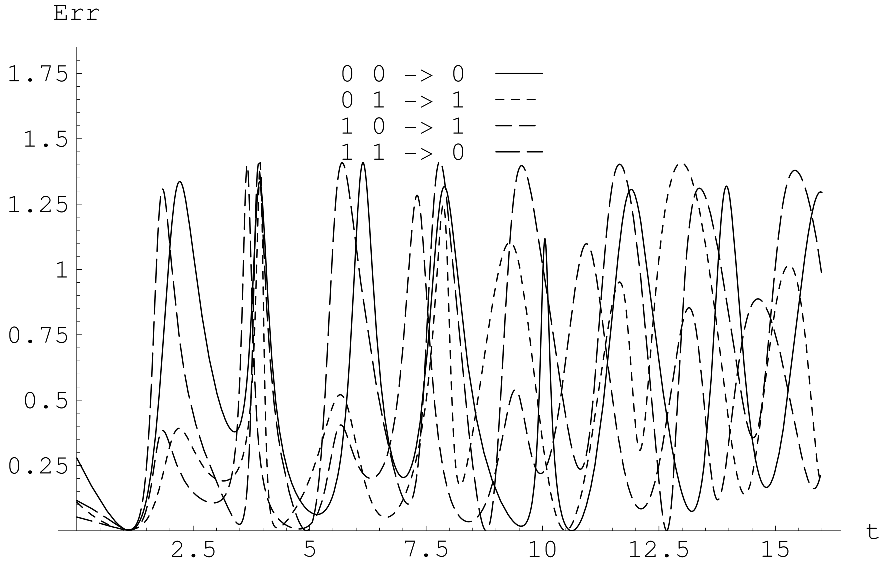

Figure 6.6: The relation of the error of the XOR function to the time at which the outcome is measured. The relation of each input in the training set is shown here separately.

as a quantum mechanical experimental setup. In this scenario, there are, in total, 6 eigenstates ($|p-\rangle$, $|p+\rangle$, $|q-\rangle$, $|q+\rangle$, $|r-\rangle$, and $|r+\rangle$), (for these, the plus sign $+$ following a proposition symbol indicates that the proposition is asserted while a minus sign $-$ indicates that it is refuted.) They are eigenstates corresponding to an operator S that asserts or refutes the state of affairs. A true proposition is therefore represented by an assertion eigenstate alone. And a false proposition is represented by a refutation eigenstate. In common non-monotonic reasoning, for each proposition, there can be a third situation in which the proposition is neither asserted nor refuted. This situation is usually called unknown and will be symbolized by X in a classical account.

In fact, an unknown status of a situation σ is nevertheless a state *known* at a higher level. By this we mean a reasoner *knows* that he does not know σ, so he *asserts* the unknown status of σ. In this case, he can consider the consequence based on the unknown status. Our training data consists of this higher level knowledge. Specifically, the training data employing the "unknown status" used in this section is from the vantage point of *the other* observer at a higher level, which should not be confused with the "knowingly unknown" status of the reasoner.

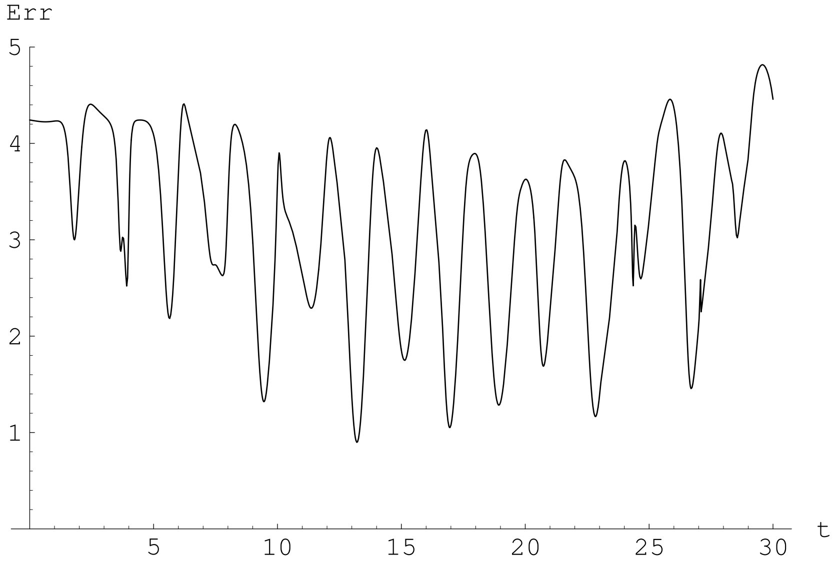

Figure 6.7: The relation of the error of the AND function to the time at which the outcome is measured.

In non-monotonic reasoning, however, if the reasoner does not even know that he does not know σ, he can not consider the consequence of the unknown status. It can be argued that this genuine unknown status is a very important source of non-monotonicity. In a quantum computational approach, an eigenstate is an unknown status *per se.* If it is not measured, it is genuinely unknown. We use this feature of quantum mechanics to account for non-monotonicity. If the "knowingly unknown" is to be included in non-monotonic reasoning, one has to introduce another qubit that asserts or refutes the *knowing status* of a situation. This is quite another question and is beyond the discussion of this section.

We start with what can be regarded as intuitively valid arguments (i.e. classically). That is, the starting point is the sound conclusions one can draw from a temporal and spatial vantage point. These arguments are listed in the following table,

p	r	q	p	r	q
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$
$\mathbb{T}$	X	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	X
$\mathbb{F}$	$\mathbb{F}$	X	$\mathbb{F}$	X	X
X	$\mathbb{T}$	X	X	$\mathbb{F}$	X

More specifically, the table can be regarded as a formulation representing a person's reflection about her previous non-monotonic reasoning. In the example given in Section 5.3, non-monotonicity appeared only after I learned that it was the first Sunday of daylight savings time and I *had not* noticed that fact. In essence, a set of rules such as the above table may eliminate non-monotonicity. But this is true only if one sees the issue from a temporal vantage point or from the spatial vantage point of a third observer. Notice that this is exactly the perspective to which the reasoner *cannot* have access, at the moment of non-monotonic reasoning. Thus the above table should be regarded as a "classicization" of non-monotonic reasoning. As I have argued in Section 5.3, a classical scheme like this cannot accommodate non-monotonicity of strong kind.

In a quantum mechanical framework, however, it is easy to express the unknown status of a proposition (an unknown status for the reasoner) *without* introducing artificial unknown status. This can be done by simply leaving out both the eigenstates pertaining to this proposition. Technically speaking, the components of the eigenstates corresponding to this proposition are set to zero. Therefore an input state is prepared as

$$c_{p-}\,|p-\rangle + c_{p+}\,|p+\rangle + c_{r-}\,|r-\rangle + c_{r+}\,|r+\rangle, \tag{6.5}$$

where $c_{xy} \in \mathbb{C}$ is the coefficient of the eigenstate corresponding to the y-state of proposition x. As a concrete example, the input state of affairs corresponding to $(p, r) = (\mathbb{T}, \mathbb{F})$ can be written as

$$\frac{e^{i\theta_1}}{\sqrt{2}}|p+\rangle + \frac{e^{i\theta_2}}{\sqrt{2}}|r-\rangle. \tag{6.6}$$

The ninth possibility $(p, r) = (X, X)$ is excluded from the table because this situation is represented by a zero vector that always is a null output (it is correct, though), and thus will not contribute to training.

In a reasoning process, an input state of affairs is subject to a unitary *reasoning* operator U. The architecture is trained with the states of affairs as shown in the table of valid arguments. The training algorithm is the same as described in the previous experiment. Specifically, these states of affairs are prepared with phases (arguments of complex components) being zero (e.g. $\theta_1 = 0$ and $\theta_2 = 0$ in Equation 6.6).

6.3.2 Result and analysis

In a typical experimental run, the training goal can be achieved with an average of 3% of contingent fluctuation. That is, in 3 out of a hundred tests, the system gives a wrong answer according to the training table. This is due to the statistical nature of quantum mechanics. If a threshold is applied to the output ensemble, an accuracy of 100% can be achieved. In this sense, a quantum mechanical architecture "implements" a prototypical everyday non-monotonic reasoning that has access to information from a temporal and spatial vantage point.

However, a quantum mechanical architecture can offer richer structures. For example, if the input states are prepared in such a way that the arguments (phases) of the two input qubits' coefficients vary independently, while the absolute value of every qubit remains the same as in the training process (since the coefficient of either the assertion eigenstate or the refutation eigenstate of a particular qubit must be zero, we have only two independent phase parameters instead of four), we find that the deviation from the targeted output can be very large. Specifically, the input state is prepared as:

$$\frac{e^{i\theta_1}}{\sqrt{2}}|x_1\rangle + \frac{e^{i\theta_2}}{\sqrt{2}}|x_2\rangle. \tag{6.7}$$

where $x_1 \in \{p+, p-\}$; $x_2 \in \{r+, r-\}$. The deviation from the targeted output is shown in Figure 6.8, in which the error (deviation to classical result) is defined as in Equation (6.3). The more interesting situations are when $(p, r) = (\mathbb{T}, \mathbb{F})$ and $(p, r) = (\mathbb{T}, \mathbb{T})$ (see the second and the third graphics of the first row in Figure 6.8). As can be seen in the figure, the intuitive valid argument can be implemented only in a narrow area along the diagonal (when $\theta_1 \approx \theta_2$). On the other hand, in a situation where $(p, r) = (\mathbb{F}, \mathbb{T})$, the output remains nevertheless largely unknown. In a situation where $(p, r) = (\mathbb{F}, \mathbb{F})$, the architecture seems to be quite stable in the output (q is unknown).

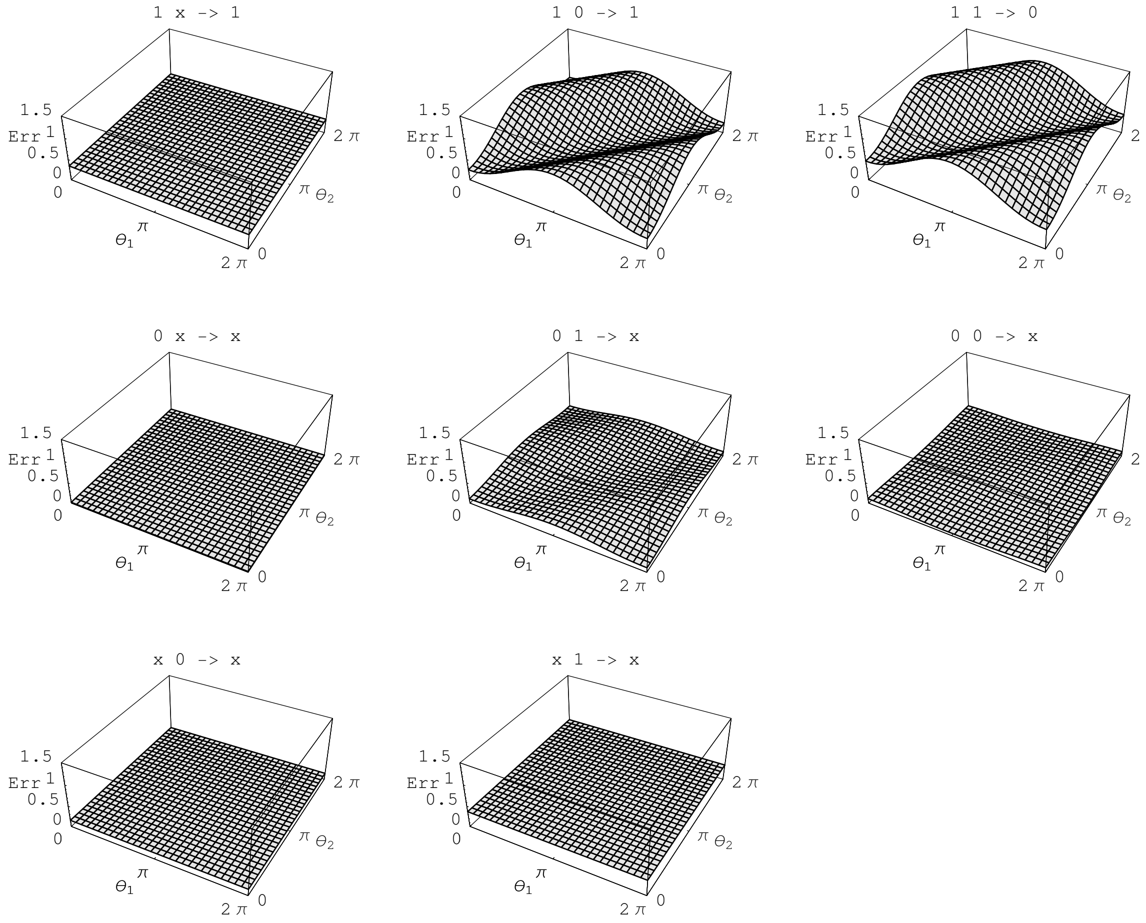

Figure 6.8: The deviation from the targets due to phase-difference in preparing the input state (θ_1, θ_2)

It is common in everyday reasoning that the input is not “well-behaved.” That is, sometimes we cannot be sure of how “true” the antecedents are. In a quantum mechanical framework, this situation can be easily represented by a mixed state of affairs. For example, if proposition p is known to be true but r is *refuted* to a certain degree, the result should become somewhat uncertain as well. Specifically, in this situation the input state can be prepared as follows

$$\frac{|p+\rangle + \rho e^{i\theta}|r-\rangle}{1+\rho^2},$$

where $\rho \in \mathbb{R}$ with $0 < \rho \leq 1$. (The denominator $1+\rho^2$ is introduce so that the input state is normalized.) The deviation to the targeted output is shown in Figure 6.9. As can be seen in the figure, proposition q remains largely asserted if the phase of p and that r are far away from π. However, if the phase difference between p and r happens to be near π and ρ is near 1, the output is switched ($r \approx \mathbb{F}$).

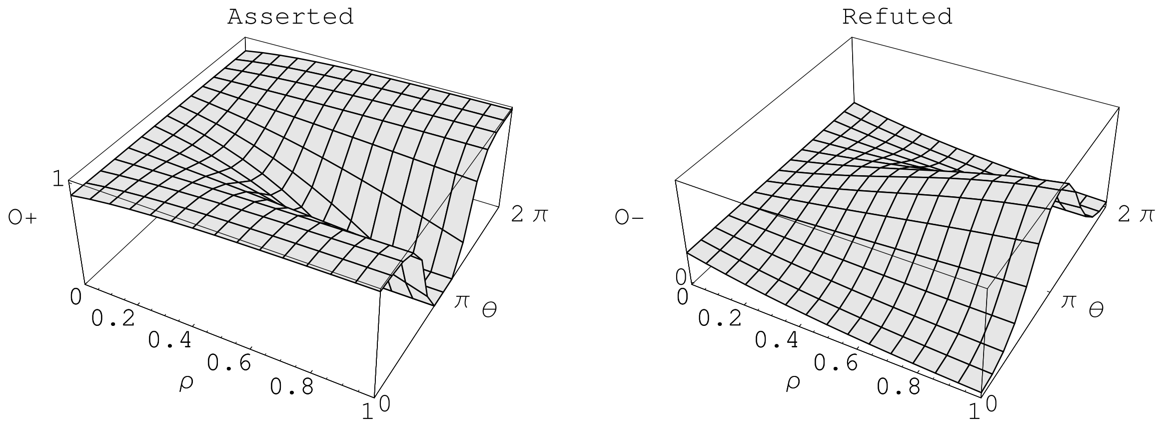

Figure 6.9: Relationship between the argument (θ) / absolute value (ρ) of the refuted second antecedent and the output

On the other hand, if proposition p is known to be true but r is *asserted* to a certain degree, we expect that q will become somewhat "fuzzy" as well. Specifically, the input state is prepared as follows

$$\frac{|p+\rangle + \rho e^{i\theta}|r+\rangle}{1+\rho^2},$$

where $\rho \in \mathbb{R}$ with $0 < \rho \leq 1$. If the situation is correspondingly prepared, the relative probability of the output state is shown in Figure 6.10. It looks somewhat like the complementary picture of Figure 6.9, but a close comparison with Figure 6.10 shows that this is *not* the case. This should come with no surprise, for in many everyday arguments, we do not treat a statement that is to a certain degree refuted as the logical complement of a statement that is to a certain degree asserted, especially when we are not sure whether the statement stands. As is shown in this example, a quantum architecture also does not treat statements this way.

Another example is when p is known to be true but r is both asserted and refuted to a

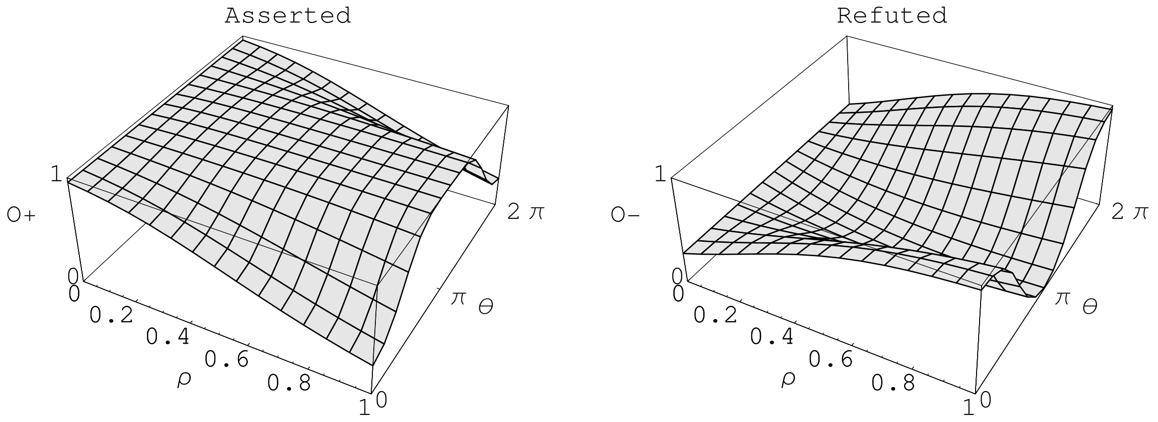

Figure 6.10: Relationship between the argument (θ) / absolute value (ρ) of the asserted second antecedent and the output

certain degree at the same time. In this situation, we, intuitively, treat the "truth-value" of r as a complementary state of refutation and assertion. To pursue this issue further, the input state of affair can be represented as

$$\frac{|p+\rangle + \rho e^{i\theta}|r+\rangle + (1-\rho)e^{i\theta}|r-\rangle}{1+\rho^2+(1-\rho)^2},$$

where $\rho \in \mathbb{R}$ with $0 < \rho \leq 1$. The deviation from the targeted output is shown in Figure 6.11. This figure shows the complexity of this situation. Specifically, there is a semi-"equipotential" contour of $\rho - \theta$ graph, on which the increase of ρ (i.e. assertion of r is counteracted by the increase of θ).

6.4 Counterfactual reasoning

6.4.1 Experimental setup

In this section, a quantum mechanical implementation of the counterfactual reasoning presented Example 10 in Section 5.4 is proposed. The example is briefly described again here (cf. [35]): Jack and Jim are old friends. Under normal circumstances they will help each other. But Jim is very proud, so he will never ask for help from someone with whom he has recently quarreled. Jack, on the other hand, is very unforgiving. So he will never

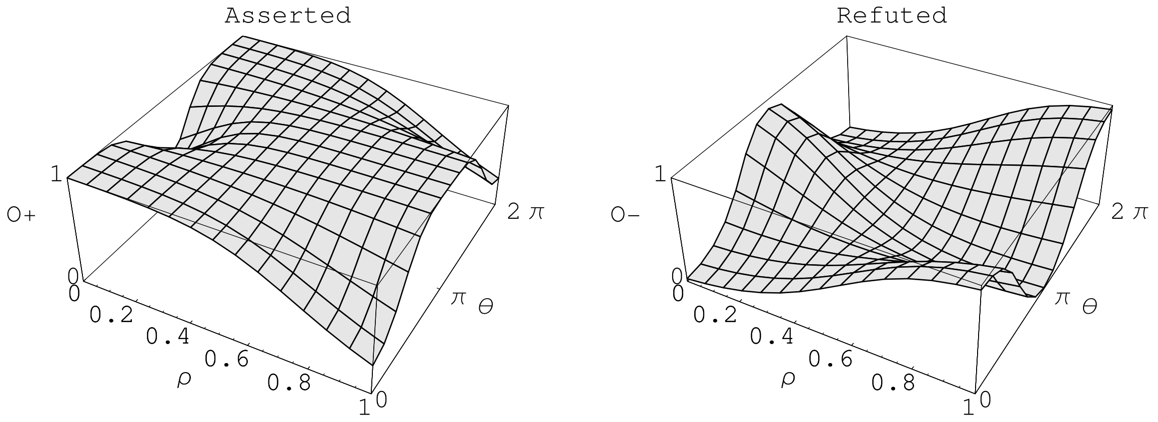

Figure 6.11: Relationship between the argument (θ) / absolute value (ρ) of the second antecedent when it is both asserted and refuted at the same time, and the output

help someone with whom he has just quarreled. Now Jack and Jim have a quarrel. Our question is:

> If Jim asked Jack for help, then Jack would help him.

This scenario is implemented as follows. First we begin with the facts and construct a quantum mechanical reasoning scheme based upon them. Let p be the proposition "Jim is very proud," q be "Jack is very unforgiving," and r be "Jim and Jack have a quarrel." p, q, r are eigenstates of an operator S that asserts the states of affairs discussed here. Let the *inference* operator be a unitary operator U which transforms an initial state of affairs to an end state. Under unambiguous circumstances U should be able to deliver a univocal answer s (whether Jack helps Jim — $\mathbb{T}$ or $\mathbb{F}$) that is another eigenstate of S. In the experiment, each proposition is associated with one *assertion* eigenstate and one *refutation* eigenstate, which are respectively denoted by a plus sign (+) or a minus sign (−) attached to the proposition symbol. Furthermore, we suppose that $\{p\pm, q\pm, r\pm, s\pm\}$ is a complete eigenbasis of the states of affairs presented here. Consequently, any input state of affairs can be represented as:

$$c_{p+}\left|p+\right\rangle + c_{p-}\left|p-\right\rangle + c_{q+}\left|q+\right\rangle + c_{q-}\left|q-\right\rangle + c_{r+}\left|r+\right\rangle + c_{r-}\left|r-\right\rangle$$

with

$$\sum_{\psi \in \{p\pm, q\pm, r\pm, s\pm\}} |c_\psi|^2 = 1,$$

where $c_\psi \in \mathbb{C}$ is the projection of a state of affairs on $|\psi\rangle$. For brevity, a state of affairs is represented by an eight-dimensional complex valued vector

$$(p_-, p_+, q_-, q_+, r_-, r_+, s_-, s_+)^t,$$

where ϕ_- (ϕ_+) is the component of the eigenstate representing proposition ϕ being false (true), $\phi \in \{p, q, r, s\}$. For example, a state of affairs in which Jim is not proud and Jack is not unforgiving and they do not have a quarrel is represented by,

$$(\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, 0, 0, 0)^t$$

The training set is constructed according to a variety of possible but coherent situations[6]. That is, situations in which the inference rule can be unquestionably applied and may lead to a coherent answer. In classical physics, one can have access to an omniscient vantage point as an external observer. This is the position we take in constructing the training set[7]. We assume that these situations are "real" in an ensemble of "possible worlds," and treat them as "factual" situations. They are thus constructed only for the sake of training the quantum system to "implement" a naive *classicization* of counterfactual reasoning. These situations are summarized in the following table:

p	q	r	s	p	q	r	s
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$

---

[6] A coherent situation is a situation in which there is no ambiguity in classical logic. It should not be confused with a coherent state in quantum mechanics, which is a pure state.

[7] It should be noticed that once a quantum system is measured, the outcome becomes *classical*. That is, the outcomes of a quantum measurement are always well defined. We assume that the relationship between classical mechanics and quantum mechanics is similar to that between classical logic and quantum computation.

In a sense, a set of these situations can be seen as the coherent experience of an individual in a real world. For example, the situation is unambiguous if Jim and Jack in fact do not have a quarrel: Jim will ask for help and Jack will help him (the first row in the table).

The questionable situations are the following two:

1. Jim is very proud and Jack is *not* unforgiving and they have a quarrel.

2. Jim is very proud and Jack is unforgiving and they have a quarrel.

The first questionable assertion is in fact not very problematic, for it leads to the same conclusion anyway: If Jim were not proud or they do not quarrel, Jim would ask Jack for help. Under both circumstances, Jack would help him, since Jack is *not* unforgiving. Thus s should be true. The reason to omit it as questionable is because the antecedent in the original statement is not true, and then one needs counterfactual reasoning (even in an imaginary "classicized" situation). The really problematic situation is the second assertion. It is a counterfactual conditional that can be both true and false even in an ensemble of imaginary "possible worlds." In fact, it *is* our original problem, which is difficult to account for in classical logic.

The training scheme is similar to that in the previous sections. The Hamiltonian has a total of 64 free parameters to be decided. An error function as defined in Equation 6.3, and a standard conjugate gradient method are used to obtain the parameters.

6.4.2 Result and analysis

In typical experiments, the training goal can be achieved (see Appendix B for the numerical result used in this section). A quantum mechanical architecture has acquired the "common sense" based on its "experience" of coherent day-to-day situations.

In the most interesting situation, as described in the second questionable situation above, the absolute square of the assertion-component of the result is 0.24. That is, in a quantum measurement evaluating the symbolic result of s ($\mathbb{T}$ or $\mathbb{F}$), about one fourth of the cases comes up as true. The outcomes jump back and forth between true and false.

Moreover, the phases of p and q may play an important part in this scenario. This can be shown by preparing the input as

$$\frac{e^{i\theta_1}|p+\rangle}{\sqrt{3}} + \frac{e^{i\theta_2}|q+\rangle}{\sqrt{3}} + \frac{|r+\rangle}{\sqrt{3}}.$$

The corresponding assertion state of s is shown in Figure 6.12. As can be seen in the figure, if the phase of $|q+\rangle$ is somewhere near π (relative to the phase of p and r), s is almost always asserted. This phenomenon seems enigmatic. However, this situation might indicate something about our intuition regarding such a state of affairs. If we take a phase difference of two, asserting eigenstates as some sort of "relevance" measure of two propositions, we may regard $\theta_2 = \pi$ as indicating that q is "irrelevant" to s. Thus we have an intuitive explanation about why s is almost always asserted in this situation, for if q is taken as irrelevant to the state of affairs under discussion, whether q is true (that is, whether Jack is unforgiving) no longer plays a crucial role in determining whether Jack helps Jim in a counterfactual situation (i.e. that Jim is *not* proud and that they do not have a quarrel). Indeed, this same hypothesis seems to offer an adequate account for the graphics presented in the previous sections. As can be seen in several examples in the previous sections, an irrelevant proposition enables a classical logic operation to deliver outcomes that are not governed by classical logic.

Another questionable argument is when $(p, q, r) = (\mathbb{T}, \mathbb{F}, \mathbb{T})$. Since the antecedent of the counterfactual conditional is not true, a counterfactual conditional cannot be applied. Specifically, an input state of affairs of this sort is

$$\frac{e^{i\theta_1}|p+\rangle}{\sqrt{3}} + \frac{e^{i\theta_2}|q-\rangle}{\sqrt{3}} + \frac{|r+\rangle}{\sqrt{3}}.$$

The corresponding assertion state of s is shown in Figure 6.13. Not very surprisingly, s is asserted in the vicinity of the origin. The troughs appear when the phase difference is roughly π, and can be explained as above.

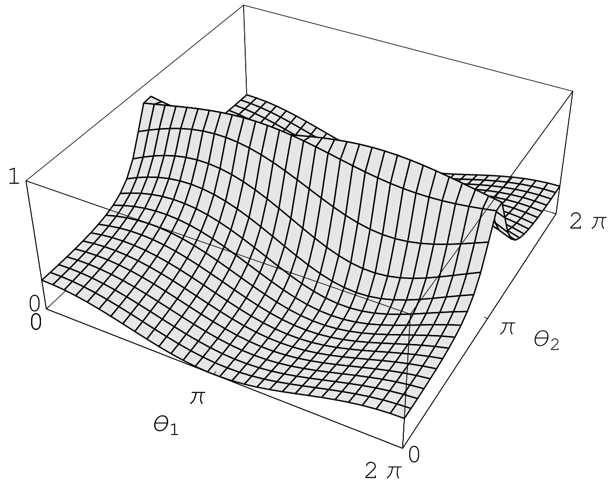

Figure 6.12: The probability of s being asserted based on counterfactual situations where $(p, q, r) = (\mathbb{T}, \mathbb{T}, \mathbb{T})$. The input states are prepared with different phase (θ_1, θ_2)

There are situations where Jim and Jack do have a quarrel, and whether Jim is proud is asserted to a certain degree, and so is the fact that Jack is unforgiving. For example, the input state of affairs can be represented as

$$\frac{\rho e^{i\theta}|p-\rangle + (1-\rho)e^{i\theta}|p+\rangle + |q+\rangle + |r+\rangle}{\sqrt{2+\rho^2+(1-\rho)^2}},$$

where $\rho \in \mathbb{R}$ with $0 < \rho \leq 1$. The corresponding assertion state of s is shown in Figure 6.14. It seems that in such situations, the "refutation-degree" of whether Jim is proud has little influence on proposition s. This agrees with our intuition about the state of affairs in these situations, for whether Jim asks for help does not influence whether Jack would help him (Jack is unforgiving, so he will not help Jim anyway).

Alternatively, the input state of affairs can be represented as

$$\frac{\rho e^{i\theta}|q-\rangle + (1-\rho)e^{i\theta}|q+\rangle + |p+\rangle + |r+\rangle}{\sqrt{2+\rho^2+(1-\rho)^2}},$$

where $\rho \in \mathbb{R}$ with $0 < \rho \leq 1$. The corresponding assertion state of s is shown in Figure 6.15. As can be seen in this figure, p depends heavily on both the "refutation-degree" (ρ) and phase (θ) of q. If the phase difference is kept small, the assertion of s is a monotonously

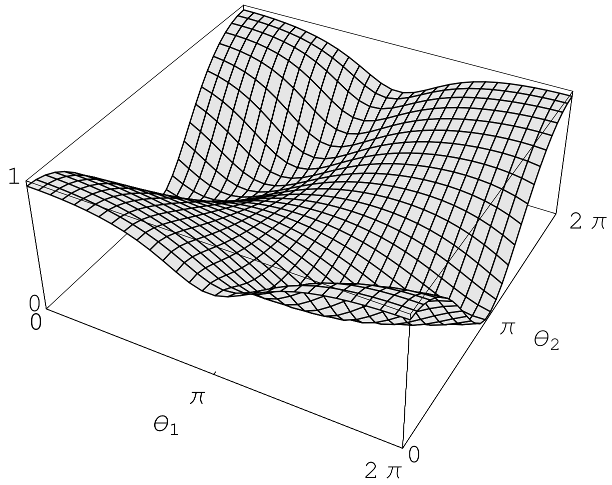

Figure 6.13: The probability of s being asserted based on counterfactual situations where $(p, q, r) = (\mathbb{T}, \mathbb{F}, \mathbb{T})$. The input states are prepared with different phase (θ_1, θ_2)

increasing function of ρ. This is not surprising. However, if the phase difference is about π, the degree of assertion behaves very strangely depending on ρ. The detailed relation between ρ and the output when $\theta = \pi$ is illustrated in Figure 6.16. When $\rho = 0.258609$ there is a minimum. When $\rho = 0.343384$ there is a maximum of 0.998. At this moment, there is no intuitive explanation for this enigmatic phenomenon.

6.5 Discussion

In this chapter, we have shown that quantum mechanical architectures can indeed implement basic classical logic functions and tackle much more thorny issues such as non-monotonic and counterfactual reasoning. In a sense, this approach to logic — as a framework for reasoning in general — is a significant departure from the conventional approach to logic. For one thing, in a conventional framework, logic is basically *normative*, while in this chapter, the architectures proposed are *descriptive* and *explanatory*. As an explanatory framework, it shows how reasoning can be “boot-strapped” with quantum computational systems.

Indeed, just like classical grammar is normative while modern linguistics is descriptive and explanatory, the science of thought should also strive to be descriptive. Just as col-

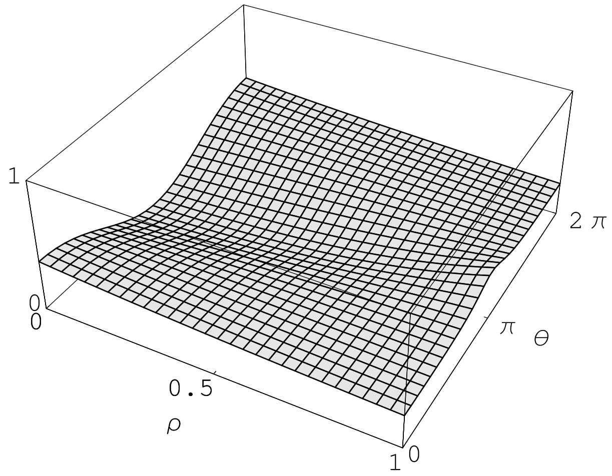

Figure 6.14: The probability of s being asserted based on counterfactual situations where p is partially asserted.

loquial language was largely unduly ignored in classical linguistics, common sense is all too often ignored or taken as a “frictional” or “impure” form of classical logical argument. Unfortunately, this latter stance is also taken by most students of artificial intelligence, in that they try to model common sense and non-monotonicity with higher level classical frameworks.

As it is shown in the XOR example, a quantum mechanical approach to a classical question shows many interesting and “non-classical” phenomena. The missing points in classical logic are that the “classical” region consists of only a fraction of the whole set of possibilities and that classical logic takes the measured outcomes in these regions incorrectly as the underlying “reasoning mechanism.” In this way we have shown an alternative computational model that includes and *extends* a classical one.

However, this is not to say that classical logic is incorrect. On the contrary, classical logic is no doubt a very powerful tool to help us draw conclusions and make decisions. What we want to point out is that even in a more serious context such as a scientific, ethical, and judicial one, we need something more than classical logic. In fact, what we want to show is how a single framework can accommodate both classical logic and common sense.

A quantum computational approach to logic shows that the “Law of Thought” as

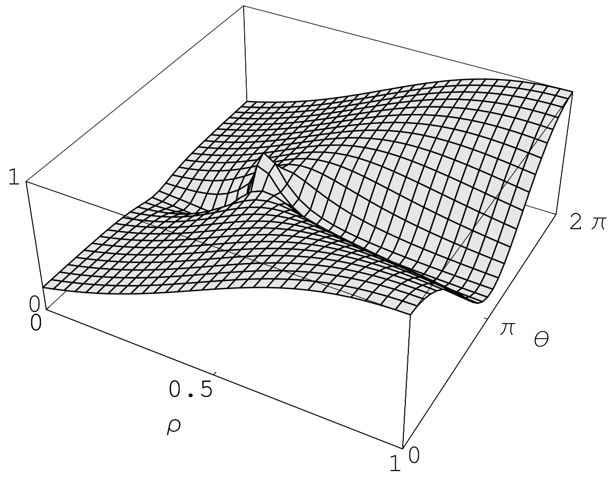

Figure 6.15: The probability of proposition s being asserted based on counterfactual situations where q is partially asserted.

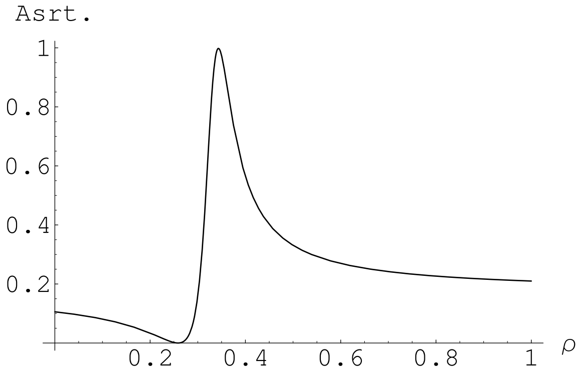

Figure 6.16: The detailed relation between ρ and proposition s's being asserted when $\theta = \pi$.

proposed by George Boole reveals only one aspect of our way of reasoning. (Remember classical logic functions can be implemented very accurately if a phase function is introduced to generate input state of affairs. See Section 6.2.) Nevertheless, this picture squares well with classical principles as far as the measurement outcomes are concerned. For one thing, the law of exclusive middle always holds, since either $|0\rangle$ or $|1\rangle$ (but not both) may manifest itself as output. There is no other thing in between. In this sense, all quantum assertions are XOR-type assertions, therefore two-valued ($\mathbb{T}$ or $\mathbb{F}$). Strictly speaking, there is no *knowingly* unknown state in quantum mechanics, only the *absence* of certain eigenstates. A multi-valued logical approach to common sense has incorrectly asserted the redundant logical value(s) which can be true or false only at a higher level (i.e. from a temporal and / or spatial vantage point)[8].

Our ability to consider the *necessity* and the *possibilities* of every situations is crucial for us to understand the world. In classical logic, however, only necessity is concerned. So perhaps most importantly, quantum mechanics provides an account of our way of knowing and seeing the world. This comes as no surprise, for in quantum mechanics one has an adequate picture of what is *possible* and what *is realized.* Complex numbers and the superposition of eigenstates in quantum mechanics offer the picture.

In the realm of possibilities, contradictory situations can peacefully coexist, as demonstrated in the examples in this chapter. In such schemes, we have a superposition with mutually contradictory states of affairs, each of which has a corresponding complex coefficient. This kind of reasoning is everywhere in our everyday life and indeed in science and any rational activities as well.

To conclude, quantum computation deserves a serious consideration as a general model for common sense logic. Although one can draw this conclusion from a postulated *analogy* between matter (physics) and mind (cognitive science/linguistics) — see Chapter 2, it is hoped that this and the following chapter may persuade the reader that this approach could be of practical interest as far as engineering (artificial intelligence or NLP) is concerned.

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[8] Few, if any, multi-valued logic endeavors have multi-valued logic as their meta-logic (the logic at a higher level). In fact, when we assert that the logic value of a certain statement is ζ (which can be other than $\mathbb{T}$ or $\mathbb{F}$), we assert at the same time that it is not not-ζ. In this sense, the meta-logic is still two-valued.

Chapter 7

Application of Quantum Theory to Natural Language Processing

A bird is a bird
Slavery means slavery
A knife is a knife
Death remains death.

— Zbigniew Herbert

7.1 Issues of natural language processing (NLP)

In this chapter we will consider the practical applications of quantum computation in natural language processing. In the conventional approach to natural language processing, a “rule-based” linguistic framework [5, 42, 43, 6] is usually employed in designing NLP systems. These approaches analyze the data and the regularities of natural languages in order to *formalize* the results as explicit statements about how to manipulate symbols. These statements are called *linguistic rules*. Accompanied and motivated by the symbol manipulating power of digital computers, many *formalisms* have been developed. Interestingly, at one time this was assumed to be the only right way for NLP and was considered a synonym for *computational linguistics* by many researchers in this field. This is referred to in the following sections as a rationalist view of computational linguistics. A schematic description of rationalist natural language processing is illustrated in Figure 7.1.

As shown in the figure, the "raw" linguistic data (either as phonetic transcripts or orthographic expressions) are subject to a rule-based analyzer and transformed into a well-defined abstract structure. All the subsequent processing is done on the abstract structures. For example, in a machine translation task the structure can be a parse-tree or some other graph. The translation is a series of rule-based symbol-manipulations done on the graph. The resulting structure is subject to another rule-based system to generate the result either as phonetic transcript or orthographic expression. All these rules are mostly hand-coded by experts who may work closely with conventional linguists.

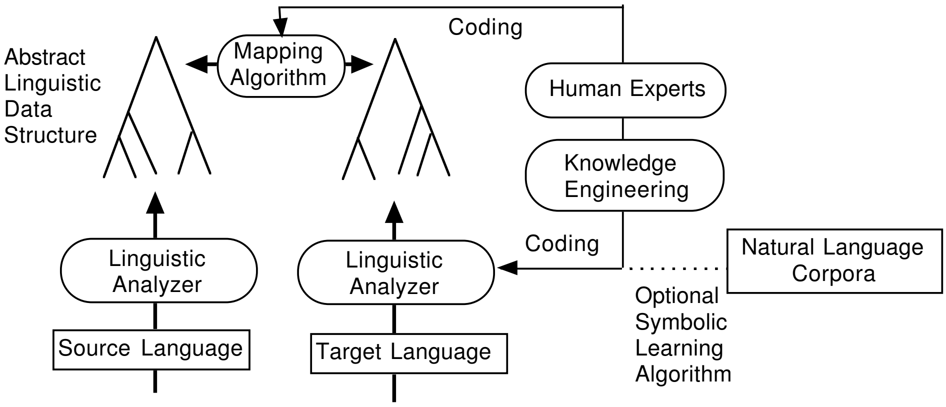

Figure 7.1: Rationalist NLP.

However, a glance at natural language data from everyday life shows that there are many *anomalies* that are difficult, if not impossible, to be accommodated in a rationalist framework[1]. These would have been regarded as "pathological" from the rationalist viewpoint and treated exceptionally. The use of language is strongly influenced by the so-

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[1] For example, the disagreement in the so-called argument structure — the verb *to give* normally requires a direct object and an indirect object, but in certain contexts these can be omitted: *Do you plan to give money to UNICEF? No, I gave last week.*

ciocultural environment of the speakers[2], as well as non-linguistic factors[3]. These turn out to be very difficult to model in a rationalist framework. Instead of blaming the speakers for not using the language correctly, a sophisticated NLP system should at least take these issues into serious account. In fact, all these "anomalies" may be just as normal as other "authentic" usages. In this regard, the rationalist linguists are incomplete at best.

Owing to these shortcomings, there is recent surge in the number of "bottom-up" approaches to NLP, most of which are motivated by speech recognition research. In these approaches one tries to shift the burden of gathering empirical data from human experts to computer programs. Moreover, one emphasizes the method of *describing* the language *per se* instead of using abstract grammar rules which are essentially *knowledge about* the language. These approaches are called *empiricist* in the following.

The philosophy of an empiricist computational linguistic NLP is to keep the linguistic formalism minimal and let a carefully designed mechanism gather *rules* by itself, although these rules might be unintelligible to a human. Particularly in a very practical application of NLP such as machine translation, the example-based [44, 45, 46, 47] and statistical approaches [48, 49, 50] both assume this thesis and have achieved certain success. Specifically, an example-based machine translation approach acknowledges the need to extract empirical rules from corpora. These rules are, however, *"shallow"* in comparison with those of a rationalist approach. Thus an example-based machine translation system does *not* inherently exclude the possibility of applying a theoretical linguistic framework. In this regard, it deserves to be called a *hybrid* approach. A *purist* statistical machine translation, on the other hand, assumes a totally empirical modeling of natural language and rejects any top-down knowledge *about* the language as a whole. From a statistical NLP's point of view, the mechanism is just a hidden Markov model (HMM) [51] or a frequency/position distribution estimator, and that is by no means *linguistic* in its conventional sense[4].

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[2] For example, consider the difference between American English and British English; or the different speaking habits of an attorney and a teenager.

[3] For example the polite form of a Japanese utterance is usually determined by the different social status of the speakers and listeners.

[4] For example, statistical machine translation is based on communication theory on a noisy channel.

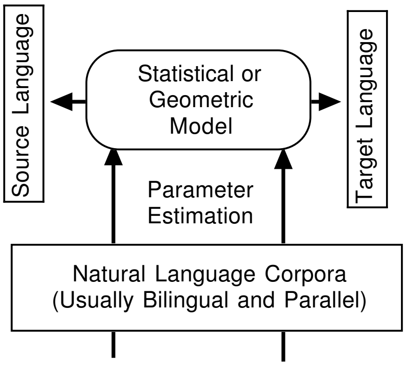

Figure 7.2: Empiricist NLP (Application in Machine Translation).

The framework of an empiricist approach to NLP (e.g. machine translation) is illustrated in Figure 7.2. As shown in the figure, the task of a NLP system is to gather reliable parameters from natural language corpora. The “raw” linguistic data are then modeled by these parameters and subject to other “parameter-manipulations.”

While the aforementioned empiricist approaches emphasize the learning capacity of an NLP system, they do not assume that the underlying *hardware* of an NLP system should resemble our brain. This is mostly due to practical considerations. In fact, research on the neurological bases of language shows that the “hardware” of human language is very complicated and very likely works according to a principle totally different from that of a Turing machine (today’s digital computer) [52, 53] or a hidden Markov model mechanism. Interestingly, while many aforementioned NLP frameworks have the ability to “learn,” the process of acquiring a first and second language reveals quite a lot properties which have been overlooked in these approaches [54, 55, 56].

Recently there have been attempts to employ connectionist techniques for modeling cognition in general and NLP in particular [57, 58, 59]. Connectionism carries naive empiricism one step further. For instance, connectionism provides an alternative and a convenient method of gathering linguistic *rules* and generating the representations of the *symbols* implicitly. Those symbols can then be manipulated by the underlying compu-

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The translation is a procedure to “recover” the original signal (sentence in the target language) given a deteriorated version (sentence in the source language.) It is hard to imagine that the parameters of such a model would have any *linguistic* meanings in the conventional sense.

tational agent. Few connectionists will dispute the strength of classical symbols. Most connectionists are convinced, however, that connectionism *implements* and *extends* the classical symbolic approach. As a result, considerable effort has been devoted to establish the correspondence between the "rules" and the "mechanism" of an artificial neural network [60, 61, 62, 63, 64, 65]. That is, how can neural mechanism *implement* symbols and rules? While promising in some limited areas, such as pattern recognition, a connectionist approach is nevertheless based on the assumption of *classical physics.* This undermines its ability to account for many interesting aspects of human language phenomena.

From a pure engineering point of view, all these approaches are in some way productive. In fact, one might argue that this is what NLP as engineering is about. However, owing to the theoretic weakness of these approaches (see the discussion in Chapter 4), they are not very plausible scientific accounts. History teaches us that a correct scientific theory usually leads to a more fruitful engineering application. In this regard, it is the aim of this chapter to show that a quantum mechanical approach to natural language processing is also efficacious in engineering.

7.2 Quantum mechanical NLP

In quantum mechanical terms, the state of affairs that is associated with a natural language utterance is a superposition of eigenstates of an eigenbasis pertaining to a specific *vocabulary* V. The vocabulary is a set consisting of all symbols found in a language. Moreover, all these symbols are eigenstates corresponding to a language formulation operator F. That is,

$$V = \{s_i \mid F\,|s_i\rangle = \gamma_i\,|s_i\rangle\}\,,$$

where $\gamma_i \in \mathbb{R}$ is an eigenvalue of F. According to Corollary 2, any state of affairs $|\phi\rangle$ can be treated as a superposition of components in V,

$$|\phi\rangle = \sum_n c_n\,|s_n\rangle,$$

where $c_n \in \mathbb{C}$ and $c_n = \langle s_n \mid \phi \rangle$ is the *projection* of $|\phi\rangle$ on $|s_n\rangle$.

In practice, a natural language utterance is usually written as an orthographic string. Generally speaking, this can be a string of phonetic transcriptions. In a sense, we are free to choose our "atomic" symbol set (alphabets, phonetic symbols, or ideographs). In the problems tackled in this chapter, however, orthographic *words* are used as the building blocks (symbols or eigenstates) of the string. For example, the eigenstate corresponding to the word *loves* can be denoted by

$$|loves\rangle.$$

Our first question is then: *how can we put together a string of symbols to refer to a state of affairs?* Since we are taking a physicalist account, the answer is to be found in physics. We need a particular unitary operator (called the *preparation operator* $P(t)$, which is a function of time t) to place a particular symbol in its particular position in an utterance. In general, the unitary operator P can be written as,

$$P(t) = e^{i\frac{H'}{\hbar}t},$$

where H' is an Hermitian operator. Suppose the string is constructed incrementally, we have,

$$|\phi\rangle = \sum_{k=1}^{m} P(t_k) e^{i\theta_k} \, |s_k\rangle,$$

where $s_1, s_2, ...s_m$ is a string of symbols in the orthographic natural language utterance; m is the length of the string; t_k is the time of utterance of the k-th symbol; θ_k is the phase (argument of a complex number) of the k-th symbol. Generally speaking, the preparation operator P may "mix" up one symbol with others if H' is not a diagonal matrix. Indeed, this could occur quite often in natural language[5]. However, for simplicity, we assume that the symbols in the miniature languages discussed in this chapter do not mix with each

---

[5] For example in the use of idioms or collocations, etc. E.g. *sore throat* or *pain in the neck.*

other. That is, H' is a diagonal matrix. In this case, we have

$$P(t) = \begin{pmatrix} e^{i\lambda_1 t/\hbar} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{i\lambda_n t/\hbar} \end{pmatrix},$$

where $\lambda_k \in \mathbb{R}$ is the k-th diagonal component of H'; n is the size of the vocabulary. To make the model even simpler, we assume that all λ_k are equal. Furthermore, we assume that the symbols in a string are uttered at uniform intervals ($\theta_0 = 2\pi/m + 2$) and the argument θ_k of each eigenstate $|s_k\rangle$ is zero. Thus we have, after all these simplifications,

$$|\phi\rangle = \sum_{k=1}^{m} e^{i(k-1)\theta_0} \, |s_k\rangle. \tag{7.1}$$

A state of affairs $|\phi\rangle$ thus prepared is subject to a unitary operator U (the *reasoning* operator). That is,

$$|\phi'\rangle = U|\phi\rangle = e^{\frac{-iHt}{\hbar}}|\phi\rangle,$$

where $|\phi'\rangle$ is the end state of affairs and H is the Hamiltonian of the reasoning process. The training is done by optimizing an error function. Specifically, the error function is defined as

$$err(H) = \sum_{(\phi_t, \phi_j) \in T} \left| \langle \phi_t^k \mid \phi_o^k \rangle \right|^2,$$

where H is an Hermitian matrix that is the target of the training process; T is a set of training pairs $((\phi_t, \phi_j))$; $|\phi_t\rangle$ and $|\phi_j\rangle$ are the target and input state of affairs respectively. Moreover, $|\phi_o\rangle$ is related to $|\phi_j\rangle$ as follows

$$|\phi_o\rangle = U|\phi_j\rangle = e^{\frac{-iHt}{\hbar}}|\phi_j\rangle.$$

We can then use the conjugate gradient method [41], starting with a small random initial vector to calculate H.

Once H is calculated, an unseen state of affairs can be subject to the same reasoning operator U. The end state of affairs should be then *measured* to generate the result of the natural language processing task. One should note that since the input state of affairs is not normalized, the end state of affairs is not normalized either. But this is not relevant because what we are interested in is an orthographic result; only the relative probability is crucial. Here, one needs another operator to generate the orthographic string. This should be a time-varying quantum state associated with the resulting utterance. This can be quite tricky and is very time-consuming to train[6]. Therefore, in this preliminary study a classical combinatorial optimizer is employed. Specifically, this is done by backward superposing possible orthographic strings and comparing them with the end state of affairs. Each candidate is given a score, which is calculated by preparing a candidate state according to Equation 7.1 and by calculating the absolute value of the complex inner product of the normalized state with the normalized end state of affairs. That is,

$$score(\psi) = |\langle \psi \mid \varphi \rangle|$$

where ψ is a candidate state of affairs and φ is the end state. In the ideal case, the inner product should be unity (1) for a perfect candidate. Since the vocabulary can be quite large, we suffer a combinatorial explosion if one employs a "brute force" (complete search) method. We therefore need heuristics to avoid such a disaster. This is done according to the following algorithm,

```
0. Normalize the end state of affairs; set the initial threshold
     Theta=0.01;
1. Build a set S of all symbols with absolute value greater or equal
     to Theta;
2. Calculate the score of each permutation in S; notice the one with
```

---

[6] The number of free parameters of a generating operator is proportional to the square of the size of the vocabulary. The computational complexity is proportional to the length of the sentence to be generated. Thus the total complexity of a generating operator may be 5-10 times of that in the *reasoning* process of the following section. Furthermore, there must be an additional status to indicate the end of an utterance. Since the main purpose of this preliminary study is the transformation of states of affairs, a full-scale generating scheme is dropped and reserved for further study.

```
    best score;
3. Theta := Theta+0.01;
4. If Theta <= 0.4 goto step 1;
5. Output the permutation with best score.
```

The string that yields the best score is taken as the orthographic result. The scheme described above is illustrated in Figure 7.3. We are now ready to apply this framework to NLP tasks.

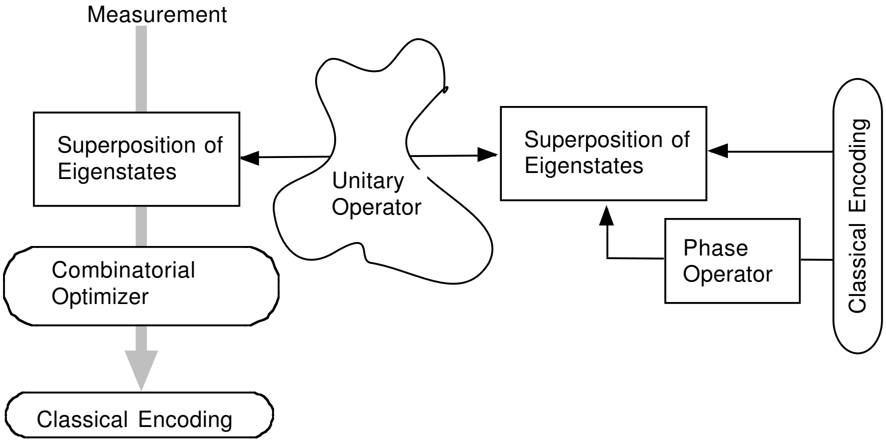

Figure 7.3: Quantum theoretical NLP.

7.3 Syllogism in natural language

Conventionally, valid deductive arguments with two premises and one conclusion are called *syllogisms* in classical logic. These arguments are among the most discussed and studied logical forms since Aristotle. Specifically, a *categorical syllogism* is an argument consisting of exactly three categorical propositions (two premises and one conclusion) containing exactly three categorical terms, each of which is used exactly twice.

One of the terms is used as the subject term of the conclusion of the syllogism. It is called the *minor term* of the syllogism . The *major term* of the syllogism is a term that is used as the predicate term of its conclusion. The third term in the syllogism does not occur in the conclusion, but must be used in each of its premises. It is called the *middle term.* An example is an argument such as:

`all men are animals` (first premise)
`all animals are mortal` (second premise)
∴ `all men are mortal` (conclusion)

where “man” is the minor term, “are-mortal” is the major term, and “animals” is the middle term.

7.3.1 “Barbara” corpus

Our first experiment is based on the argument form, traditionally called “Barbara:”

```
all m are p and all s are m -> all s are p
```

The corpus is built by replacing the term *m,p,s* with three general symbols *a,b,c* and permuting the order of the three symbols. An additional term *d* is reserved for testing purposes. Thus, there are 6 sentences in the corpus and the vocabulary of the corpus is (`all, are, and, a, b, c, d`). The corpus is then subject to the quantum architecture proposed above and trained using the conjugate gradient method starting with a small random parameter vector. In 20 experiments, run with different random start parameters, the architecture seems to have difficulty learning all these sentences. That is, the system cannot manage to generate all of the six sentences without error. There are always two sentences that cannot be learned, although we do not know in advance which two. This depends on the random initial vector in the optimization method. A closer look at the result reveals an interesting pattern. In fact, all the results agree with the following pattern:

```
all a are b and all c are a -> all c are b
all a are c and all b are a -> all b are c
all b are a and all c are b -> all are* ++
all b are c and all a are b -> all are* ++
```

```
all c are a and all b are c -> all b are a

all c are b and all a are c -> all a are b
```

in which the incorrect and missing words are marked with `*` and `+`, respectively. The absolute squares and phases are shown in Figure 7.4 and Figure 7.5. In Figure 7.4, the absolute square of each component is represented by the area of its corresponding black square. In Figure 7.5, the phase of each component is represented by the angle of the line in the circle (as the hand of a clock). The upper rows are the target sentences and the lower the actual outputs of the system.

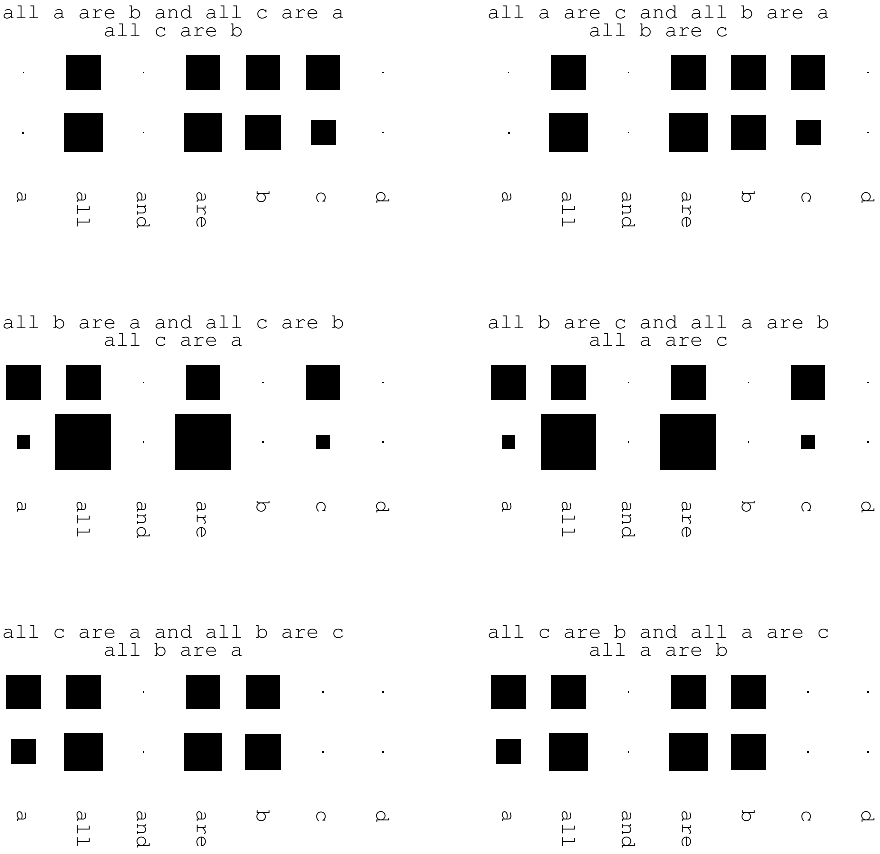

Figure 7.4: The absolute squares of the output trained with the “Barbara” corpus. The absolute square of each component is represented by the area of its corresponding black square.

A closer look at the result reveals a not very surprising fact. If one replaces `a,b,c` above with concrete categories, one will notice that the only “meaningful” solution (one

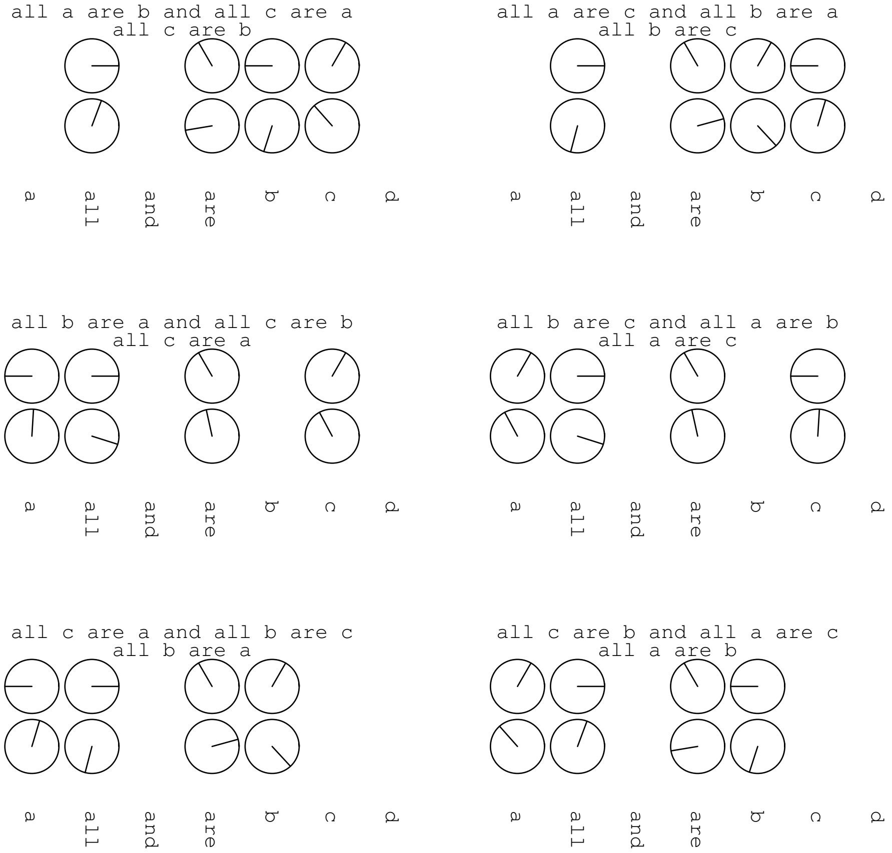

Figure 7.5: The phases of the output trained with "Barbara" corpus. The phase of each component is represented by the angle of the line in the circle.

that makes sense to our intuitive common sense) is that `a,b,c` must be exact synonyms, for otherwise the following four conclusions:

`all c are b`

`all b are c`

`all b are a`

`all a are b`

cannot simultaneously be true. In this case, both "`all a are c`" and "`all c are a`" must be true. Indeed, if the threshold of the combinatorial optimizer is fixed to 0.05 (instead of subject to combinatorial optimization), the third and the fourth sentences in the corpus will be decoded as

```
all b are a and all c are b -> all a* are c*
all b are c and all a are b -> all c* are a*
```

where both confer meaningful state of affairs, although syntactically incorrect.

Observing this fact, another corpus is constructed where the symbols `a,b,c,d` are replaced by concrete categories (`whales, dolphins, mammals, animals`). Moreover, the sentences are arranged in such a way that they reflect the “meaningful” state of affairs, as far as our knowledge about the world is concerned. The corpus is shown below:

```
all whales are mammals and all mammals are animals
 -> all whales are animals
all mammals are animals and all whales are mammals
 -> all whales are animals
all dolphins are mammals and all mammals are animals
 -> all dolphins are animals
all mammals are animals and all dolphins are mammals
 -> all dolphins are animals
all dolphins are whales and all whales are animals
 -> all dolphins are animals
all whales are animals and all dolphins are whales
 -> all dolphins are animals
all dolphins are whales and all whales are mammals
 -> all dolphins are mammals
all whales are mammals and all dolphins are whales
 -> all dolphins are mammals
```

Not surprisingly, this time the architecture can learn all the sentences. The absolute squares and phases are shown in Figure 7.6 and 7.7 respectively.

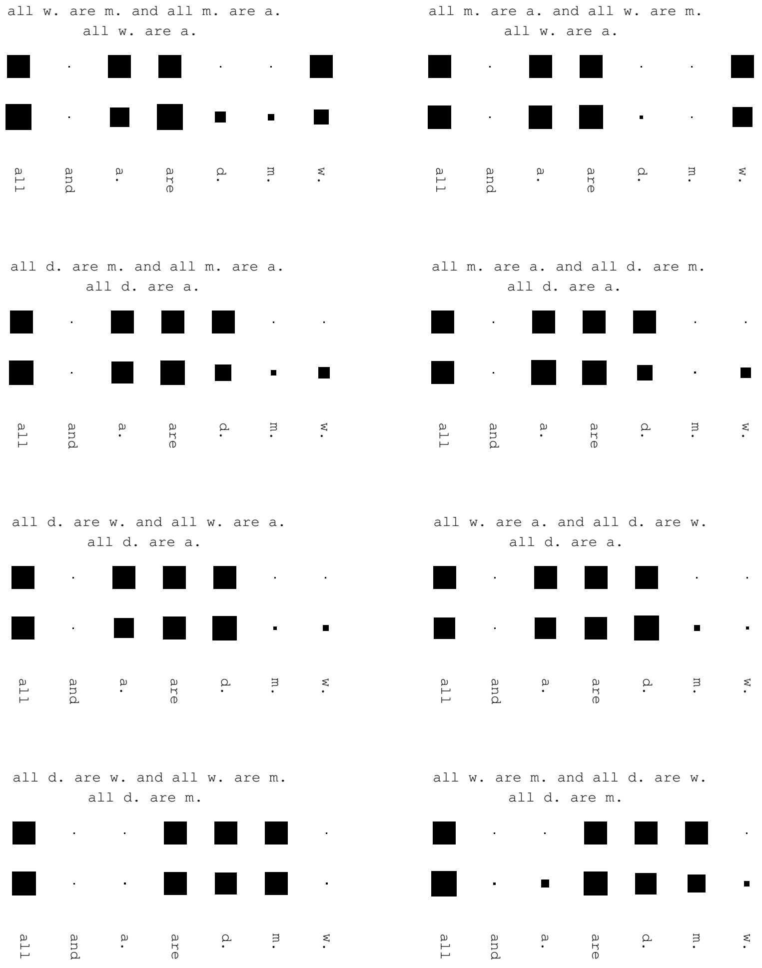

Figure 7.6: The absolute squares of the output trained with concrete “Barbara” corpus.

7.3.2 Full categorical syllogistic corpus

An argument in the form as discussed in the last section is only one example of a valid syllogism. In fact, there are 15 valid forms of arguments that have categorical predicates, negation, and two quantifiers (“all” and “some”). The corpus used in this section consists of these 15 forms of arguments and is listed below.

```
all m are p and all s are m -> all s are p
all p are m and some s are not m -> some s are not p
some m are not p and all m are s -> some s are not p
all p are m and no m are s -> no s are p
all p are m and no s are m -> no s are p
```

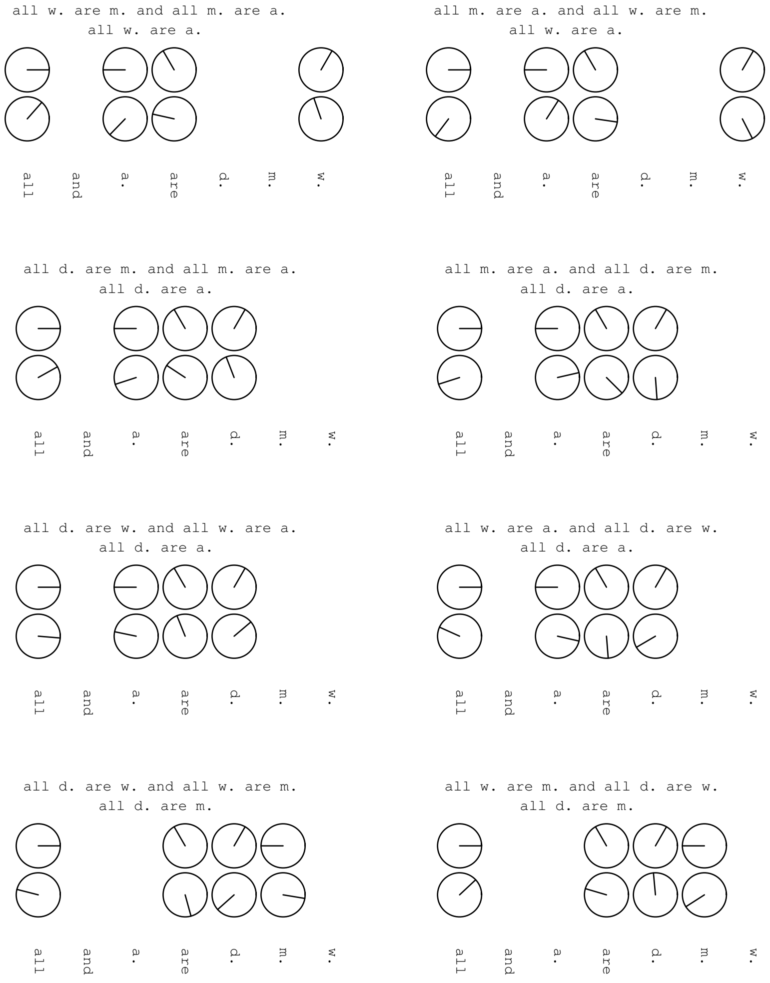

Figure 7.7: The phases of the output trained with concrete“Barbara” corpus.

```
no m are p and all s are m -> no s are p

no p are m and all s are m -> no s are p

all m are p and some s are m -> some s are p

all m are p and some m are s -> some s are p

some m are p and all m are s -> some s are p

some p are m and all m are s -> some s are p

no m are p and some s are m -> some s are not p

no p are m and some s are m -> some s are not p

no p are m and some m are s -> some s are not p

no m are p and some m are s -> some s are not p
```

If we use this corpus to train the aforementioned quantum architecture, it has no difficulty in learning all these sentences. The absolute squares and phases are shown in Figure 7.8 and 7.9 respectively. In Figure 7.8, the absolute square of each component is represented by the area of its corresponding black square. In Figure 7.9, the phase of each component is represented by the angle of the line in the circle (as the hand of a clock). The upper rows are the target sentences and the lower the actual outputs of the system.

As all students learning elementary logic know, these forms of arguments, especially when they are thus abstracted, are difficult to follow without resorting to sophisticated reasoning (with the help of Venn diagrams, for example). The ability of the quantum computational framework to learn all these arguments is remarkable and therefore very encouraging.

7.4 Syntax manipulation

7.4.1 Chalmers' syntax corpus

The experiment data used in this section is based on a corpus proposed by David Chalmers [66]. In his paper, David Chalmers claimed that a Recursive Auto-Associative Memory, (RAAM) originally proposed by Pollack [61], is a connectionist architecture that is capable of processing "compositional structure." He demonstrated that two RAAMs (as the encoder / decoder of symbolic sentences), plus a feedforward network [40] between the internal layers of the RAAM, can achieve the syntactic task of transforming an active sentence to a passive sentence. An initial experiment with 80 sentences (40 of each form) was used to train the connectionist architecture (both the RAAM encoder / decoder and the feedforward transformation network). A 65% generalization rate was reported on the rest of the 40 unseen sentences. That is, the error rate on the unseen test corpus was 35%. He then modified the experimental setup by training RAAMs with all possible sentences, and the transformation feedforward network with 75 out of the 125 possible active/passive pairs. A 100% generalization rate on the remaining 50 active/passive pairs was achieved.

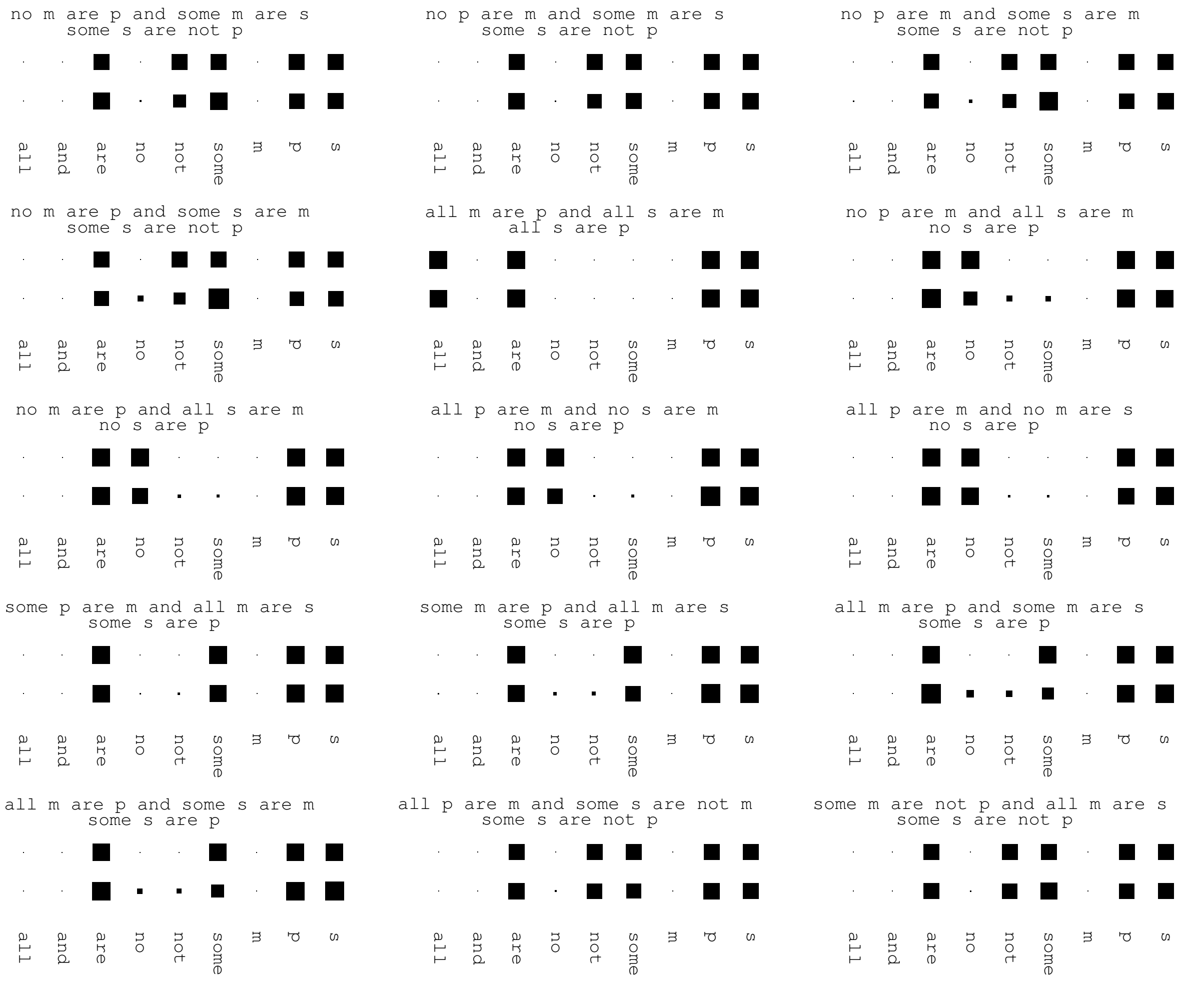

Figure 7.8: The absolute squares of the output trained with full syllogistic corpus. The absolute square of each component is represented by the area of its corresponding black square.

Specifically, the corpus used in his study consists of 5 nouns, 5 transitive verbs, one auxiliary verb (`is`), and a preposition (`by`). There are 125 sentences in active form and 125 sentences in passive. The vocabulary used in this corpus is summarized in Table 7.1. The original corpus is *not* conjugated. For example, the following sentence,

```
diane kill helen -> helen is kill by diane
```

is used in the corpus although it is incorrect as far as common English grammar is concerned. As a starting point and to establish a more accurate comparison, we begin with

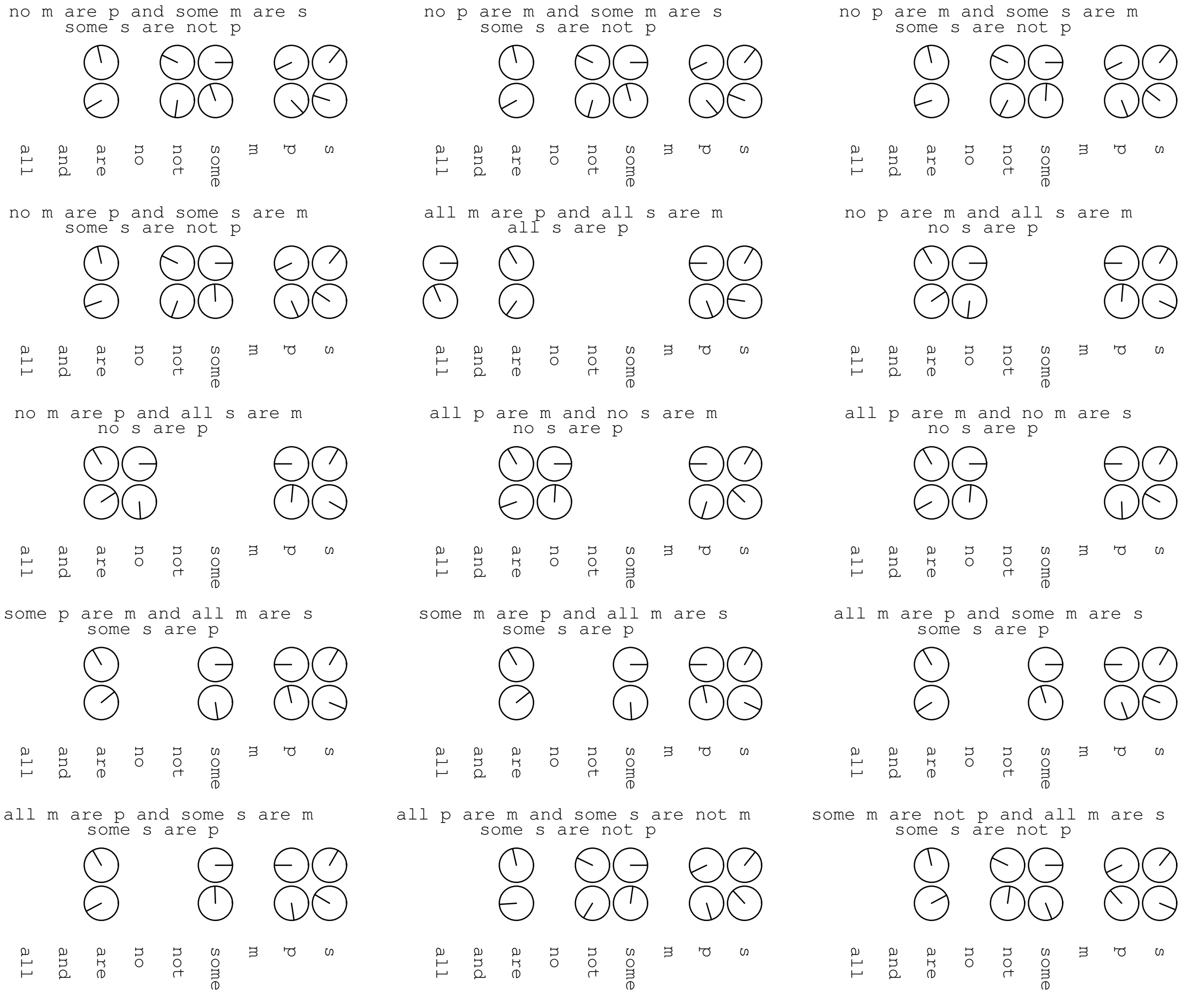

Figure 7.9: The phases of the output trained with full syllogistic corpus. The phase of each component is represented by the angle of the line in the circle.

the "initial" experimental setup in [66] by using 40 random sentences as the training set and the rest of 85 as the test set. The architecture is trained with the conjugate gradient method starting with a small ($[-0.1, 0.1]$) random parameters set. The architecture can learn all the sentences in the training set without difficulty. The generalization accuracy of the architecture on the test set is 100%.

To view the result in more detail, the output of an example sentence in the training set

```
diane kill helen -> helen is kill by diane
```

is shown in Figure 7.10. In this figure, the complex components are drawn as an array of complex planes. This is the representation of the state as a superposition of the symbol-eigenstates. For a better comparison between the end state of affairs and the target state of affairs, the output, together with the target, is shown again in Figure 7.11. In the figure, the first two rows are the absolute values of components of the target (upper) and output (lower) state of affairs. The third and the fourth rows in the figures are the phases (arguments of complex numbers) of components of the target and output respectively. As can be seen in the figures, five eigenstates ($|helen\rangle$, $|is\rangle$, $|kill\rangle$, $|by\rangle$, $|diane\rangle$) have the most significant absolute values. The permutation thereof that is most similar to the state of affairs (i.e. that has the maximal complex inner product with the state of affairs vector) is taken as the orthographic form of the result of the syntax manipulation.

The generalization ability of the architecture is very good. For example, in the test set, an unseen sentence

```
chris kill john -> john is kill by chris
```

is visualized in Figure 7.12. As can be seen in the figure, there are hardly any differences between the absolute values of the output of the unseen sentence and that of the target. There is significant variation of phases, however. Nevertheless, the target is still the best candidate for the orthographic output.

Category	Instances
Person	john michael helen diane chris
Action	love hit betray kill hug
Misc.	is by

Table 7.1: Simple Syntax Corpus

7.4.2 More complicated corpus

While Chalmers' corpus is useful to demonstrate that a connectionist architecture has interesting cognitive aspects, it misses many interesting points of natural language. For one thing, natural language utterances are well-formed and may refer to state of affairs[7]. Thus,

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[7] This is a weakness of most connectionist approaches to NLP in which the referred state of affairs is remote to its claimed grounding. In fact, it is the implementation of symbolic tasks by a connectionist

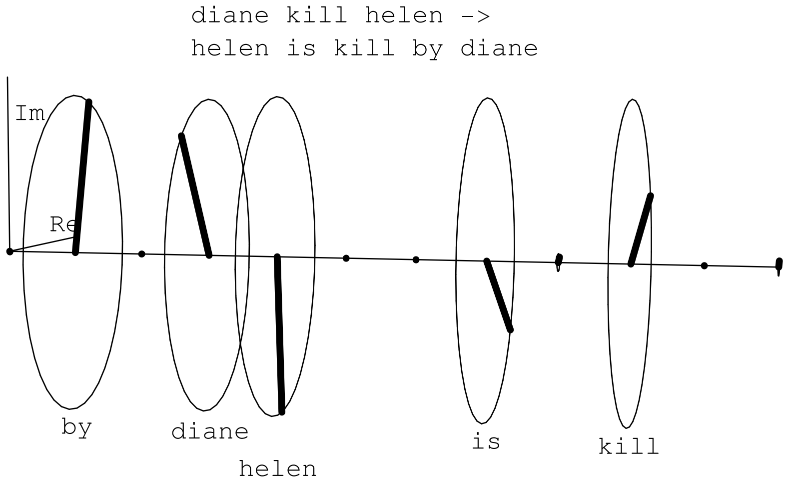

Figure 7.10: An example of the training set shown as a series of vectors on complex plane.

the corpus used in the experiment discussed in this section was modified and extended, as described in the following paragraphs.

In this more complex corpus, an additional conjunction `and` is introduced and full verb conjugations are taken into account. There are 240 sentences in total. The vocabulary is summarized in Table 7.2. There is considerable variation added to the corpus as compared to that used in [66]. For instance, the past participle of `hit` is `hit`, but that of `love` is `loved`. In addition, the conjunction `and` introduces the plurality of actor and recipient. A strategy that manipulates symbols simply on account of position will not work in this case. Moreover, `hit` (as past participle) and `hit` (as verb present plural) have the same eigenstate. In other words, they are indistinguishable according to the formulation operator F of common English. As for the verb `love`, there are three eigenstates associated with it (`loves, love, loved`). Some examples are listed below.

```
helen hits john <-> john is hit by helen
helen loves john <-> john is loved by helen
john kills diane and michael <-> diane and michael are killed by john
```

---

architecture that is mostly being studied. In other words, symbols are already abstracted by the designer of the architecture and the actual forming of symbols from the bottom up is seldom addressed.

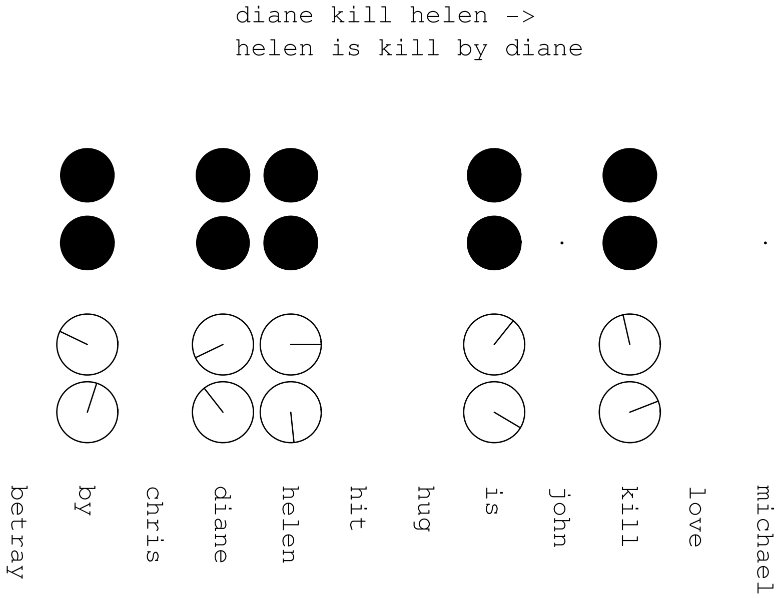

Figure 7.11: An example of the training set.

Category	Instances
Person	john michael helen diane
Action	kill love betray hit
Action Conjugated	kills loves betrays hits
Past Participle	killed loved betrayed hit*
Conjunction	and
Misc.	is are by

Table 7.2: Vocabulary used in the more complex syntax corpus. Words marked with * are homonyms that are represented by identical eigenstate in the vocabulary.

Fifty-six sentences (23% of the corpus) have been randomly chosen as the training set. The other 184 sentences are reserved as test. Using the same optimization algorithm as in the previous section, the quantum mechanical architecture can learn all the utterances in the training set. The architecture can generalize the task on all sentences in the test set (generalization rate is 100%). Given the complexity of the corpus (in comparison with that used in Chalmers' study) and the small size of the training set, this is a very encouraging result.

A typical training curve is shown in Figure 7.13. Using the conjugate gradient method, for around 100 epochs the architecture can learn all the instances in the training set. To

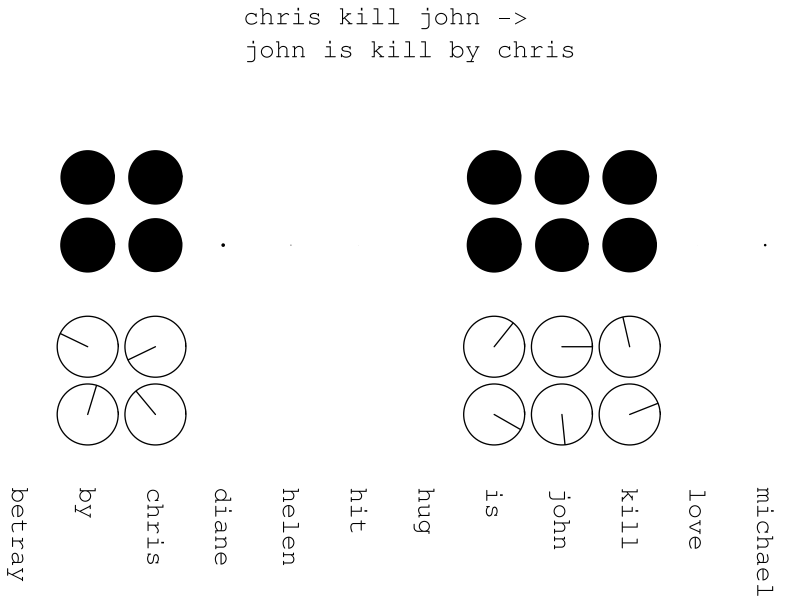

Figure 7.12: An example of the test set.

visualize more details, the output of an utterance in the training set,

`helen and diane hit john -> john is hit by helen and diane`

is shown in Figure 7.14, in which each component is illustrated as a vector on the complex plane. A comparison of the output to the target utterance is shown in Figure 7.15. The first two rows are the absolute squares (represented by the area of the black disks) of the targets and outputs respectively. As can be seen in the figure, seven eigenstates have the most significant coefficients. The lower two rows are the phases of target and output vector, respectively.

An output of an utterance in the test set,

`john kills diane and michael <-> diane and michael are killed by john`

is shown in Figure 7.16. The quantum architecture has never seen the utterance, it is remarkable that the differences in absolute squares and phases are hardly noticeable.

Theoretically, a quantum mechanical architecture can perform the reverse computation if time is reversed. In this case, if the output state of affairs is subject to the inverse of the

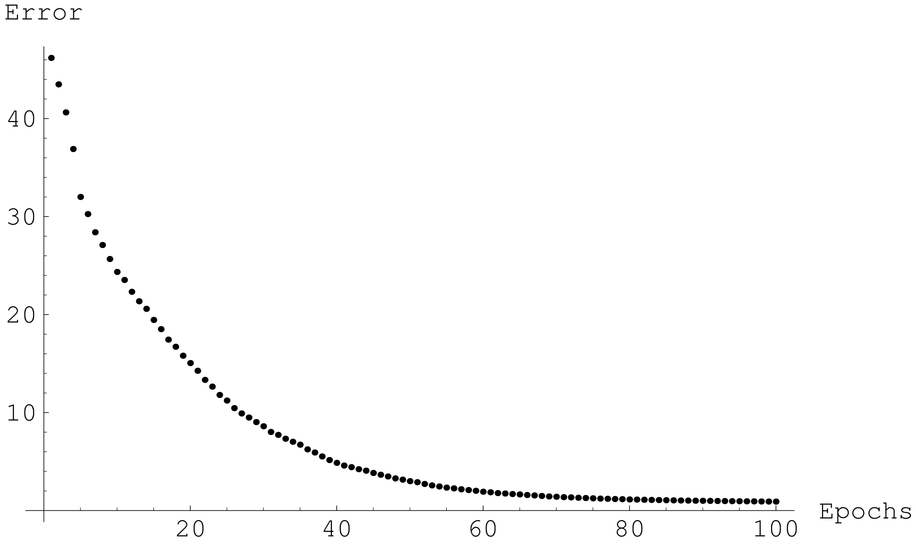

Figure 7.13: A typical training curve for the more complex syntax corpus.

unitary operator using

$$U^{-1} = e^{\frac{iHt}{\hbar}}$$

one should have the original input utterance at the input side. This is, however, the ideal case only if the output state of affairs is *not* formulated. If an orthographic output utterance is prepared according to the same procedure, there must be some minor difference between it and the genuine output state of affairs. This is shown in Figure 7.17. In this figure, the output utterance of same example of the training set above is prepared according to the standard procedure and then subject to the inverse of the unitary operator. The absolute squares and phases of the processed input are shown in the second and the fourth rows; that of the original input state of affairs is shown in the first and the third rows.

If, however, the input is a well-formed sentence but cannot be transformed to the passive form in the language, such as,

```
john kills
```

the system can still arrive at some reasonable solution. This is shown in Figure 7.18 and Figure 7.19. As can be seen in Figure 7.18, four eigenstates (`is`, `killed`, `by`, `john`) have the most significant components in the end state of affairs vector. In a sense, it suggests a

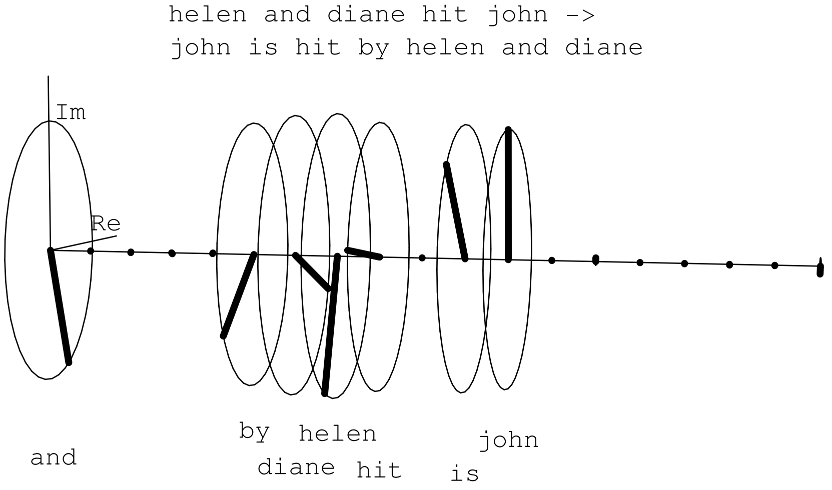

Figure 7.14: An example of the training set shown as a series of vectors on a complex plane.

well-formed utterance:

```
somebody is killed by john
```

However, in the miniature language we use here, it is not possible to identify who is killed.

7.5 Machine translation

The corpus used in this experiment is based on the more complex corpus used in the previous section. Both the active and passive form are used and translated to German. There are 480 English-German bilingual sentence pairs in total. All the verbs are correctly conjugated. As in the previous section, the atomic symbols are orthographic words. There is a separable German verb (*umbringen* — to kill) in the corpus[8] which introduces some additional complexity. For example, in the following bilingual sentence pairs,

```
helen is killed by john -> helen wird von john umgebracht
diane kills michael -> diane bringt michael um
```

---

[8] The prefix of a German separable verb is placed at the end of the sentence and represented by two independent symbols in conjugated form.

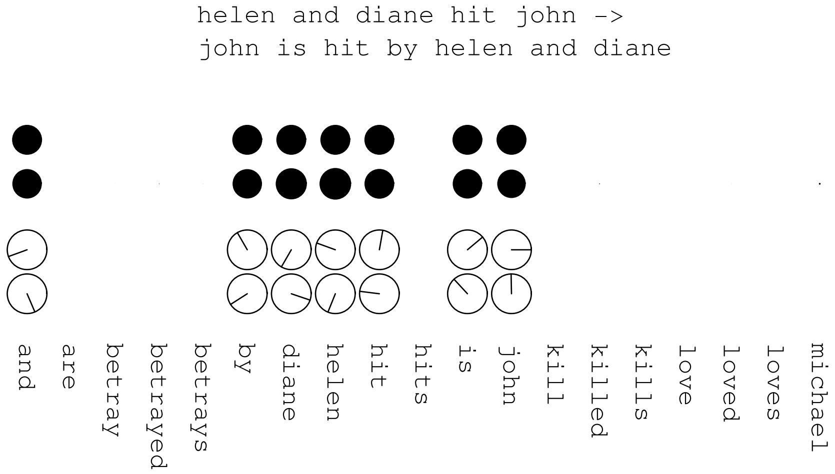

Figure 7.15: An example of the training set (the first and the second rows: absolute squares of the target and the output, respectively; the third and the fourth: the phases of the target and the output).

`umgebracht`, **`bringt`** and **`um`** are treated as mutually exclusive symbols (eigenstates, according to the common German formulation operator). The vocabulary is summarized in Table 7.3. Specifically, there are a total of 19 (20) symbols in the English (German) vocabularies. The total number of free parameters is therefore $39^2 = 1521$.

Vocabulary Category	English Instances	German Instances
Person	john michael helen diane	john michael helen diane
Action	kill love betray hit*	umbringen† lieben verraten* schlagen
Action Conjugated	kills loves betrays hits	bringt† liebt verraetet schlaegt
Past Participle	killed loved betrayed hit*	umgebracht geliebt verraten* geschlagen
Conjunction	and	und
Misc.	is are by	wird werden von um†

Table 7.3: Vocabulary used in the bilingual corpus. Words marked with * are homonyms and all are represented by identical eigenstate in the vocabulary. † German verb *umbringen* is a separable verb.

Seventy-eight sentences pairs are chosen randomly as the training set (16% of the corpus) and the remaining 408 sentences pairs are reserved as the test set. In a typical experiment, the correctness on the training set is 93.58% and the generalization rate on the test is 88.81%. We count an incorrectly decoded *sentence* as an error. On the other

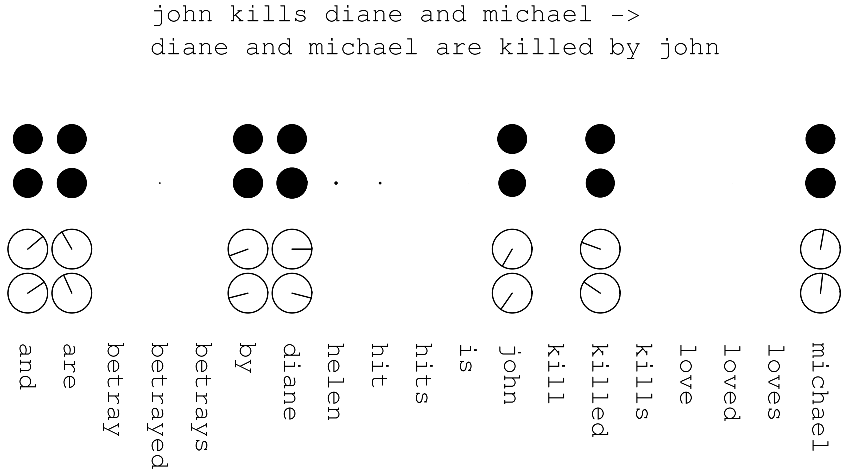
john kills diane and michael ->
diane and michael are killed by john

Figure 7.16: An example of the test set (the first and the second rows: absolute squares of the target and the output, respectively; the third and the fourth: the phases of the target and the output).

hand, if the correctness of words instead of that of sentences is counted, the correctness of the training set rises to 97.82% and the generalization accuracy of the test set rises to 95.82%. Given the small size of the training set, the generalization rate is not bad.

To see more details, the output of a correctly decoded example in the training set

```
diane and michael are betrayed by helen ->
diane und michael werden von helen verraten
```

is shown in Figure 7.20. The first two rows are the absolute values of the output components in which the absolute squares are represented by the area of the disks. The first is that of the target state of affairs (`diane und michael werden von helen verraten`) and the second is that of the output. The third and the fourth rows in the figure are the phases of the target and the output respectively. As can be seen in the figure, the architecture can generate the target state of affairs quite faithfully.

Nevertheless, there are some sentence-pairs that are not correctly learned both in the training and the test set. As an example, the output of an example in the test set

```
helen is killed by michael and diane ->
helen wird von michael und umgebracht* diane*
```

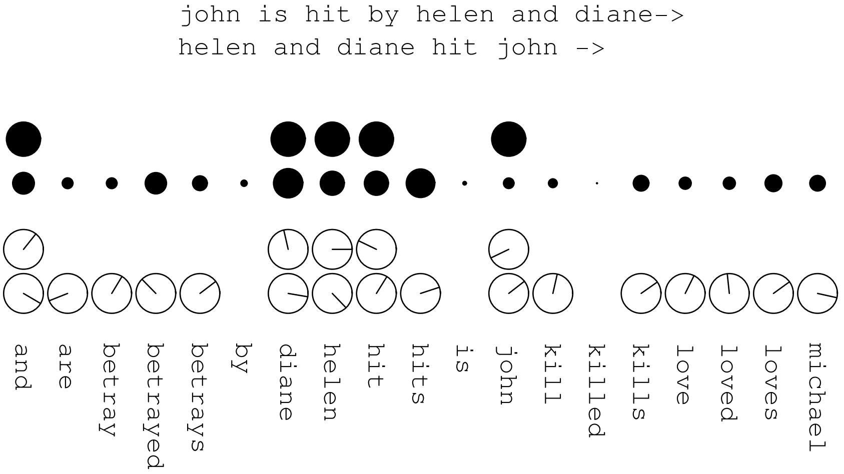

Figure 7.17: An example of the training set reverse in time (the first and the second rows: absolute squares of target and output, respectively; the third and the fourth: phases of target and output).

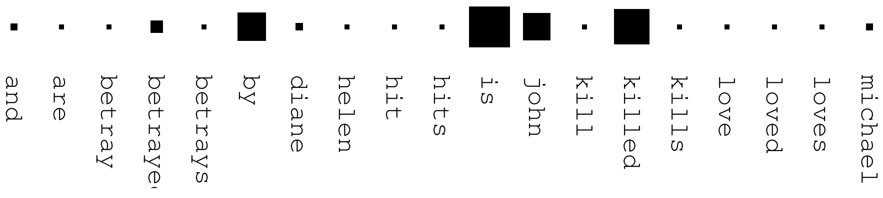

Figure 7.18: An example of an utterance which can not be transformed to passive form in the limited vocabulary of the language (absolute squares).

that is not correctly learned, is shown in Figure 7.21. Incorrectly decoded words are marked with *. As can be seen in the figures, the output state of affairs is largely similar to that of the target. The error is mostly due to shift of phases, therefore the word order is incorrect. If the order is swapped, the sentences would be correct. This suggests that another accuracy criterion may reveal more information about the performance. For one thing, we would like to know how many incorrectly decoded sentence-pairs are due to the error of phases. In fact, if the decoded sequences are permuted only once (by swapping the positions of exactly two of the symbols), we achieve an accuracy of 100% on the training set. On the test set, however, the accuracy is 98.5%. A glance at the remaining errors (6 sentences), we notice that they are all of the form as shown in the following:

```
diane kills helen ->
```

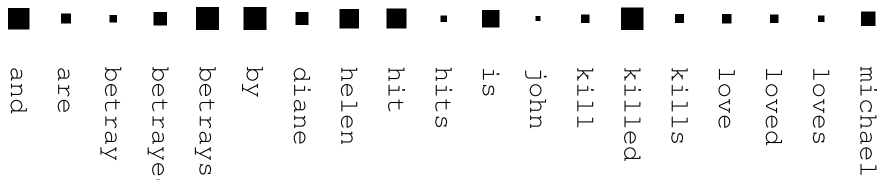

Figure 7.19: An example of an utterance which can not be transformed to passive form in the limited vocabulary of the language (arguments).

`diane bringt john* helen* um-`

This state of affairs of the above example is shown in Figure 7.22. The error of this example is due to unwanted residues of eigenstates. This kind of error can be removed by raising the threshold in the combinatorial decoding process. For example, if the threshold is set to 0.1, all error of this kind can be avoided.

This example also shows an interesting "bias" of the system to convict `john` as the killer. In fact, this comes as no surprise if we take a closer look at the training set. In the training set there are 20 sentences about "killing." In these scenarios, `john` kills 9 times and is killed 6 times, he is the most frequent killer (`michael` kills 7 times and is killed 9 times. `helen` kills 7 times and is killed 8 times. `diane` kills 7 times and is killed 3 times.) Poor `john` seems to have become the natural "black sheep" of the system, owing to the unbalanced training set.

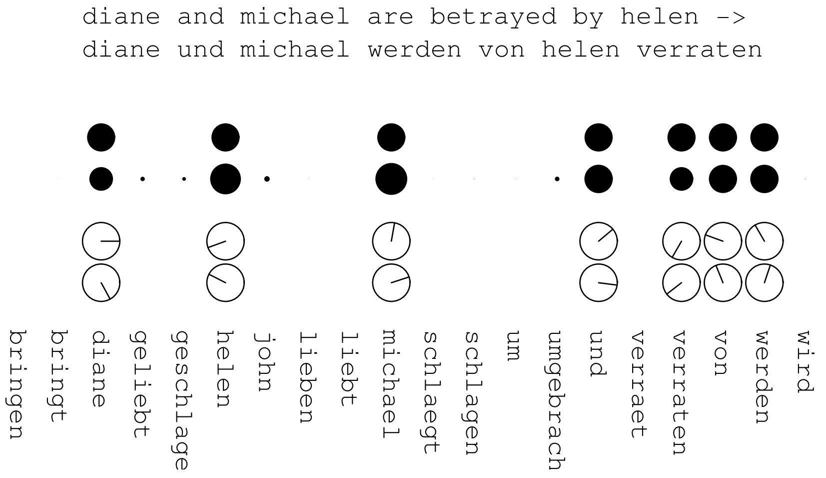
diane and michael are betrayed by helen ->
diane und michael werden von helen verraten

Figure 7.20: An example of the training set in the bilingual corpus that is correctly learned.

```
helen is killed by michael and diane ->
helen wird von michael und umgebracht*diane*
```

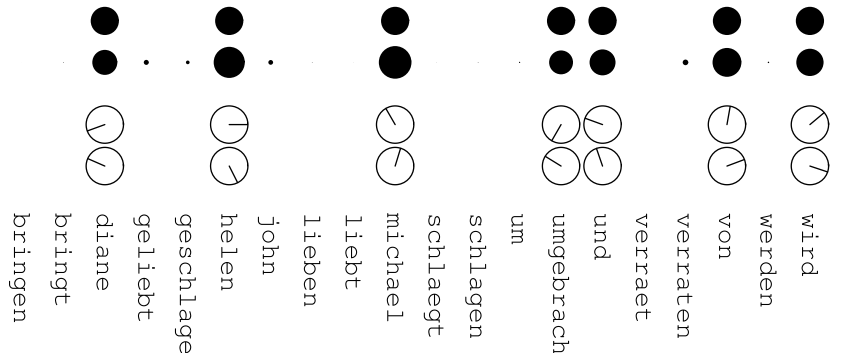

Figure 7.21: An example of the testing set in the bilingual corpus which is not correctly decoded.

In the ideal case, a quantum mechanical translator can be reversed in time in order to translate a sentence from the target language back to the source language. However, this can be done only if the end state of affairs is *not* measured (that is, no symbolic sequence in the target language is formulated). If a state of affairs formulated in the target language is subject to the time reverse version of the *reasoning* operator U, there can be a significant amount of noise in the "starting" state of affairs. As an example, the target state of affairs of the first example in the training set above (`diane und michael werden von helen verraten`) is prepared and subject to U^{-1}. The output (in fact, the reversed input) and the original state of affairs is shown in Figure 7.23. The first two rows are the absolute squares of the original state of affairs and the reversed input respectively. The last two rows are the phases of the original state of affairs and the reversed input. As can be seen in the figure, there are unwanted mixed states in the source language that are generated by the *pure* states in the target language. These mixtures are not effectively cancelled as they are in the case of forward translation.

An interesting but non-trivial by-product of this experiment is that one can use the architecture to compile a bilingual dictionary of the miniature languages. This can be done using the lexical list of English as input and looking at the end state of affairs in German. The result is shown in Figure 7.24. In the figure, the absolute square of each

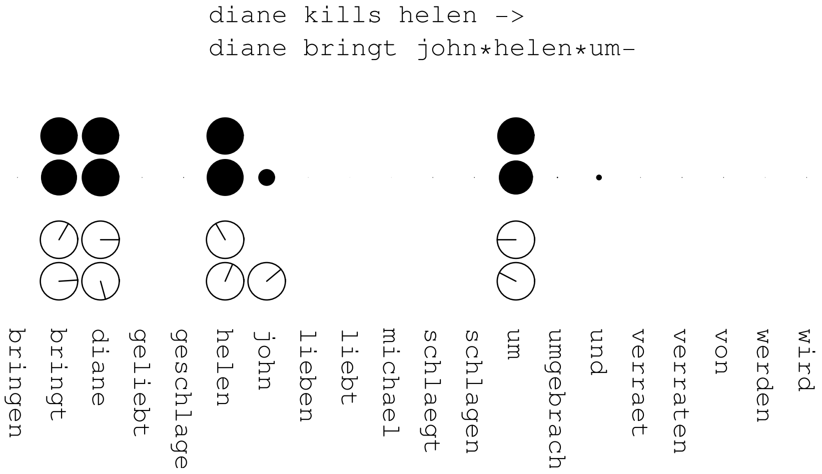

Figure 7.22: Yet another example of the test set in the bilingual corpus which is not correctly decoded. The error is mainly due to residue of irrelevant eigenstates.

component is represented by the area of the little square. As can be seen in the map, the English words are largely associated with their German translations. Interestingly, personal names and auxiliary words (`is, are, by`) are mapped to somewhat distributed German words. The German counterparts of the English personal names are nevertheless the most activated. The auxiliary words, on the other hand, show a sort of "template" relationship, which is basically a *many-to-many* mapping. These are the desirable results that correspond well to our intuitive understanding of language usage. What is puzzling is the relationship among past participles. English past participles show a tendency to associate with German past participles as a *category.* That is, the mapping shows a kind of "generalization" based on syntax (in the sense of conventional linguistics) in addition to natural associations based on content. However, this generalization proves to be incorrect for the homonym `hit` (as in plural present tense and as past participle). These errors of mappings among past participles (as far as category is concerned) seem to be due to the ambiguity of `hit` in English, which may have connected `geschlagen` to `schlagen` (`hit` as a past participle is almost totally neglected in the map). The other interesting phenomenon is the "bias" against `john`, discussed above which can be seen in the last row of the figure.

In the same vein, a reverse dictionary map using $U^{-1} = U^{\dagger}$ is shown in Figure 7.25. One should note, however, that these two maps are *not* simply transpositions of each other.

```
diane and michael are betrayed by helen ->
diane und michael werden von helen verraten
```

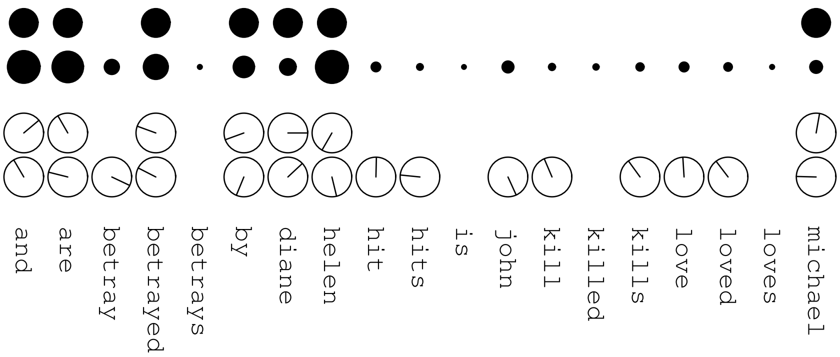

Figure 7.23: A reverse translation task.

7.6 Discussion

The experiments in this chapter show that a quantum mechanical architecture can achieve miniature natural language processing tasks quite successfully. One should bear in mind, however, that the models and problems proposed in this chapter are highly simplified. This immediately raises the question of the scalability of a quantum mechanical framework. As far as the simulation of a quantum process on a conventional computer is concerned, the efficiency is probably not feasible for NLP of very large scale. This can be seen from the complexity of the optimization procedure. For one thing, the number of free parameters in a Hamiltonian operator is proportional to the square of the number of eigenstates (or the size of the vocabulary). Therefore the complexity will grow very fast as the vocabulary grows. Secondly, in order to calculate the unitary *reasoning* operator

$$U = e^{iH}$$

one has to calculate the eigenvectors of H (using the Jacobi algorithm [67], for example). This is a tedious task, and very resource-hungry. But we should bear in mind that for these simulations we used a *conventional* computer, so the criticism on efficiency is not fair. If we can build a quantum computer in the near future (this is not unlikely) [28, 68, 69], the

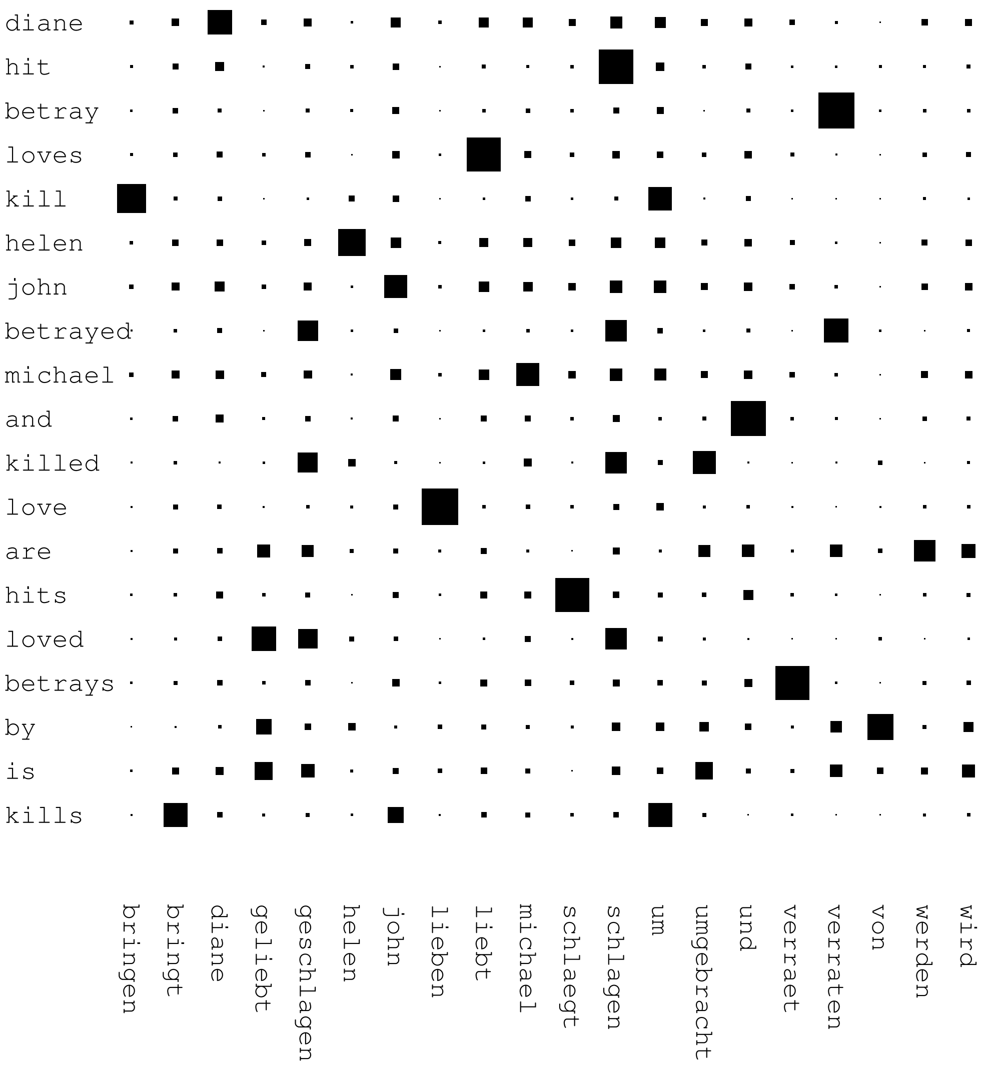

Figure 7.24: An English-German dictionary map.

lack-of-efficiency argument may not stand.

This is not to say that a quantum mechanical framework cannot have practical value at the present time. For instance, one may build a hybrid model in which classical symbolic computation and/or statistic-connectionist modules can be implemented to work with a quantum mechanical "arbitration" module that takes care of crucial decisions. This is similar to a scenario in which a human is assisted by computer programs (such as database or number-crunching programs) to make decisions. After all, if the picture described in Chapter 4 and 5 is correct, the human brain must be working in a classical-quantum hybrid

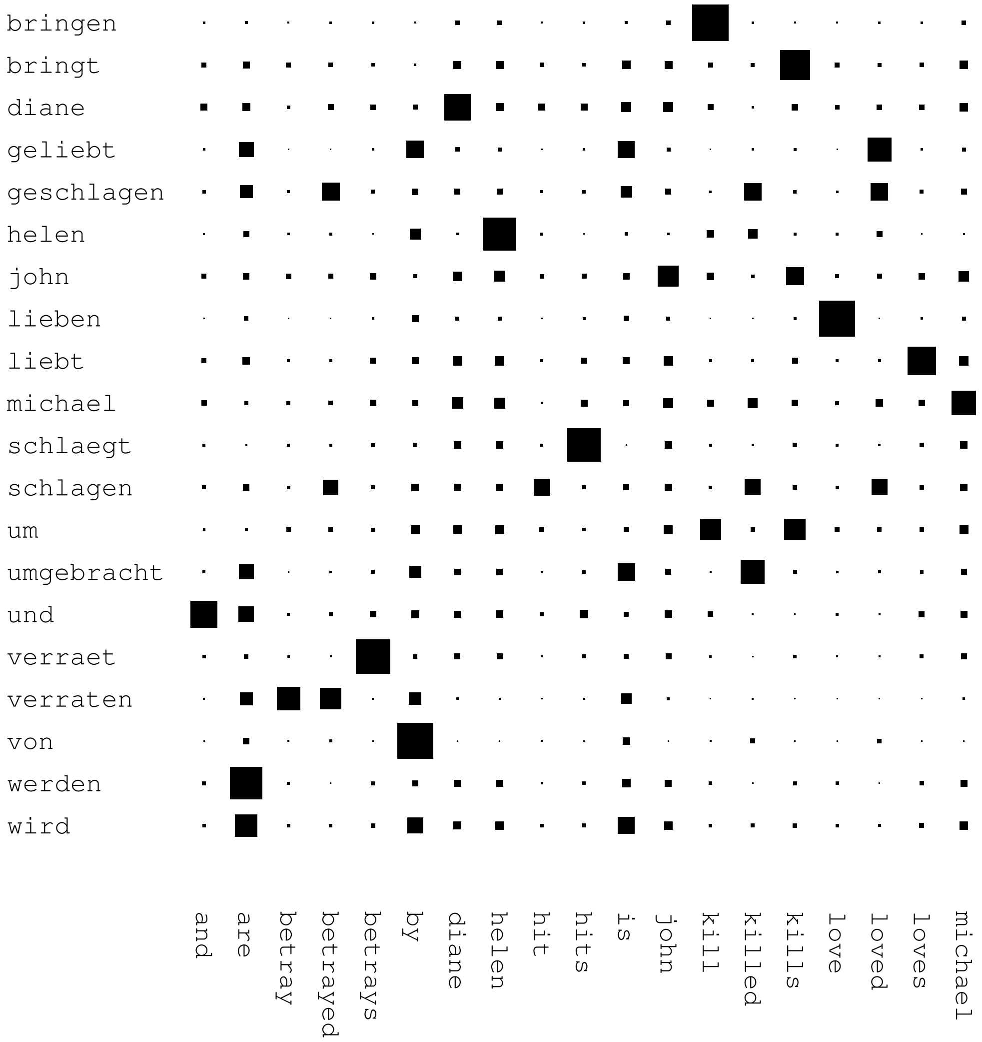

Figure 7.25: A German-English dictionary map using the reasoning operator in time reversal mode.

mode.

A closer look at the errors in this chapter shows that most of them result from the last combinatorial procedure, for the pre-measurement states of affairs are indeed quite "well-behaved." (This can be seen from the absolute squares/phases graphics of the erroneous instances.) A full scale time-dependent quantum encoder/decoder might be able to solve this problem.

A question (aside from lack-of-efficiency) arises when we notice that the experiments done in this chapter are all simulations on conventional computers (this is because we have a powerful mathematical formalism). Can simulations on conventional computers tackle all the problems a quantum mechanical NLP system can solve? This is probably not the case. In fact, we only simulate the formalism, *not* the experiment itself. There are many experimental problems that cannot be dealt with in classical computation. Therefore they also cannot be solved with simulations. For instance, genuine random numbers and counterfactual computation in quantum mechanics *cannot* be carried out on a conventional computer.

A further remark is that the models proposed here are all "first-order," by which we mean that the Hamiltonian is fixed and given. In fact, the Hamiltonian used in this chapter are calculated using classical optimization algorithms, but the brain may work very differently. In a genuine quantum computational environment, the Hamiltonian must itself be the result of a chain of quantum computations. It may be the source of *active* thinking and creativity in the brain.

The activeness is twofold: for one thing, the brain actively compiles a string of symbols (eigenstates) into a state of affairs; for the other, the brain should be able to actively arrange the quantum experiment. In general, Hamiltonians are set up by way of other quantum measurement, in which the experimental arrangement is setup by yet another quantum mechanical arrangement. This may go from one level to another *ad infinitum.* In this regard, thought can be seen as a continuous process of "preparing" and "measuring" — a constant "enfolding" and "unfolding," as David Bohm put it [1, 36].

Generally speaking, if the brain is indeed a quantum computer, an adequate simulation of it has to be a higher-order quantum computer, in order that it may exhibit similar behavior of our brains. Indeed, our superior mental ability may be closely related to these higher order quantum computations, especially for creative tasks in language and mathematics[9]. Today, we know that there are algorithms believed *intractable* on a classical computer

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[9] Other good examples are perhaps games such as *go.* Although the rules of go are very simple and the playing algorithm is well-defined, there is no computer program that can beat a moderate human go-player. The search space is simply too large even for today's best super computer. This may change, of course. But what is really fascinating is that human beings do not seem to search the problem space as a computer does, and so are able to beat a sophisticated computer.

that can be solved elegantly and quickly by a quantum computer [30]. Although this is only in theory, it is encouraging to see quantum computation able to solve some seemingly intractable human cognition problems. For a full scale application of this approach, of course, we need a genuine quantum computer.

Part III

Discussion and Conclusion

Chapter 8

Discussion and Conclusion

> *The purpose of all prayer is to uplift the words,*
> *To return them to their Source above.*
> *The world was created by the downward flow of letters:*
> *Our task is to form those letters into words*
> *And take them back to God.*
> *If you come to know this dual process,*
> *Your prayer may be joined*
> *To the constant flow of Creation–*
> *Word to word, voice to voice,*
> *Breath to breath, thought to thought.*
>
> — Liqutim Yekarim

8.1 A brief comparison with other approaches

The approach proposed in this thesis is not the only endeavor that is motivated, in a sense, by discontent with classical approaches to language and logic. Although the motivation of this thesis cannot be wholly attributed to such discontent, it may be helpful to briefly compare the merits/shortcomings of other “revolting” theories.

For one thing, classical approaches are symbolic and top-down, and use the classical Turing machine (computer) as a metaphor of human mind. More importantly, these accounts, explicitly or implicitly, assume a Cartesian dualist stance. In this case one is obliged to take an *anti-naturalist* philosophical position regarding mind. Partially driven by the need for a descriptive and naturalist account, the alternatives (the so-called *soft-computing* models) address the shortcomings of the classical approaches and try to build theories which are more or less bottom-up.

For instance, observing that human thinking is very often “fuzzy,” fuzzy logic [70] replaces the set theory underlying classical logic with a “fuzzy set theory.” In this regard, fuzzy logic offers certain insights into the fundamental characteristic of human reasoning. However, in fuzzy logic theory, while the fuzziness of human reasoning is correctly identified, it lacks a theoretic framework explaining why the member function is a monotonic *real* mapping with the range of $[0, 1]$. Furthermore, the continuity of the real-numbered member function is obviously at odds with sophisticated human reasoning — at least when a verbal or mathematical argument is *expressed.* In this sense, fuzzy logic can be at best regarded as an extension of (classical) physiological reflection (as in the example of arms, which can be treated as an automatic control system with feedbacks) to some mental tasks. A more severe problem lies perhaps in its implicit dualist stance, in which the “member-of” and other symbolic concepts are given from without. In fact, even if one can think of a thing being “0.51 big” (call it *big*) the thing *cannot* be “less than 0.51 big” (call it *medium*) anymore. So being “0.51 big” and being “less than 0.51 big” is a crisp distribution. For students of fuzzy logic, it seems that symbols, as particular categories to which an entity (or a state of affairs) belongs, must be something innate to the human mind. In this sense, fuzzy logic remains a top-down theory and does not offer a bottom-up account of why the brain functions the way it does.

In a sense, artificial neural network (ANN) or connectionist models, based on simplified neurological findings, may come to the rescue. In ANN models, the fuzziness is attributed to the continuity of neuronal activations. In fact, connectionist approaches seem to be the only class of soft-computing models that may have a deep root in physics (of biology) and deserve the name bottom-up. However, the models used in most artificial neural networks are highly simplified. They are hardly similar to real neurons in the brain. Moreover, at least at the present time, connectionist models cannot accommodate symbols in a satisfactory way [71], although it remains an open question whether a connectionist model may one day achieve this *implementation* account. Most severely, the physical objects on which the current neural network models are based are *classical.* Thus a connectionist model must carry with it all the weakness of classical physics. It is hardly convincing that out of passive, mechanistic classical physics something we call *intention* can emerge without resorting to mysterious (therefore non-naturalist) accounts. Nevertheless, there are interesting studies in neurology that may one day reveal how some perceptions can be accounted for classically (as long as classical physics is regarded as a limiting case of quantum physics). Perhaps the most interesting of recent studies is that on the neurological basis of consciousness [72]. Not surprisingly, quantum mechanics is promising in some of these studies [73, 74, 75].

Acknowledging the complexity of the physical neuronal substrate, there are simplistic connectionist approaches to language [76, 77, 78, 79, 18] which, with some success, have demonstrated how symbols and other symbolic structures may emerge from or be implemented with so-called "sub-symbolic features." Strictly speaking, these approaches have to be called *hybrid* because they still start with an abstraction of characteristic *sub-symbols.* Enumeration is a common technique of implementing these sub-symbolic features. These sub-symbols have at least the most important symbolic features, that is, *coordinates.* They are nevertheless *given* from without and remain symbols in disguise (for example, *Wickelfeatures* employed in [76]). In fact, a "neuron" in these approaches has a coordinate as its label and an activation as its content, which render it a classical variable (a classical slot-filler). These endeavors may prove successful as far as engineering applications are concerned, but their theoretic implications, aside from being a limiting case of underlying quantum computation, is rather problematic. As far as the artifact design of sub-symbols is concerned, these approaches are top-down.

Statistical approaches are perhaps the most engineering-oriented among these models for language and logic. In language research, in particular, there are a growing number of projects [80, 81, 82, 83] that employ large natural language corpora as the source of bottom-up knowledge. In reasoning, a similar motivation has spurred statistical modeling of decision making (Bayesian/Markov approach, etc.). The main merit of these approaches is that they emphasize quantitative aspects and are able to "learn." Nevertheless, the constitutions of these models are more or less arbitrarily postulated. It is more a matter of engineering. For most statistical approaches, an intelligible explanation that can be traced to the brain is rather an unimportant issue. In fact, while a statistical approach can accommodate many intuitive stochastic processes, parameters of the model and the model itself (how many hidden states, for example), it is still far from the genuine physical implementation in the brain. At best, such approaches can be regarded as an engineering-oriented abstraction of mental tasks. Moreover, since the parameters involved are real numbers (therefore additive), they cannot account for wave-like interferences.

In general, all these soft-computing models suffer an explanatory gap between primitive behaviors and the highly sophisticated structural and logical reasoning of a human being. It is like trying to explain how an earthworm might comprehend the Pythagorean Theorem and articulate a proof. (A postulate of *mechanistic* evolution does not help, since all arguments against classical physics apply to it as well[1].) I believe this difficulty lies in the heterogeneity within the theory — that discrete classical computation has to be reconciled with continuous classical physics. This is impossible without resorting to Cartesian/Newtonian dualism, which may render the theories incoherent. In fact, this may oblige many researchers to take an obscure and radical "unscientific" step in that, as Jack Copeland (who claims to be a physicalist himself) [84] put it, "the physicalism is supported by nothing but faith."

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[1] On the other hand, evolution may offer an account of why we are as we are now. But an adequate theory of evolution probably also has to include quantum mechanics [19].

8.2 Conclusion and future works

As analyzed in various parts of this thesis, there are fundamental disadvantages and failures in the classical approaches to language and logic. While very effective, one of their most grievous difficulties, for many perhaps the most fundamental one, is that they cannot provide a satisfactory explanation for why the brain, following the law of physics, can display activeness and creativity. The fact that the brain is a physical object is difficult to refute if we acknowledge the discovery of modern neurology and biochemistry. A reduction of mind down to simple physical phenomena, on the other hand, will eliminate even the most fundamental belief in individual responsibility. Ironically, this denies the responsibility of the advocators of the theories as well. These are taken as the main difficulties of anti-physicalist and physicalists, respectively.

The fatal fallacy of these arguments is, of course, that the law of physics is incorrectly conceived as to how classical physical objects *mechanistically* follow classical physics. It is also all but impossible to escape from Cartesian dualism in classical physics, and this renders the whole enterprise incoherent. On the other hand, once we are freed from a clockwork world view, a serious monistic approach to mind in general and language/logic in particular becomes conceivable again.

Specifically, the strategy of monism is an *analogy* — whatever the case is in A, that is the case in B; and indeed that is the case in *everything.* But the paradoxical coexistence of classical objects (as measuring instruments) and quantum objects forbids a naive analogy free of (classic) logical inconsistency. In this regard, one has to trace the paradox all the way to an account of meaning (especially that of *inconsistency*) and ponder the genuine roles of symbol and language. In fact, one can arrive at this account of meaning by arguing along the line of Cartesian *Meditations* and, additionally, by taking language-like memory into consideration. Thus, it is symbol that has brought us to the idea of *reality* and it is the invariance of symbol brought us to the idea of *objectiveness.* One can see not only that there is a niche for a quantum mechanical account of language, but that in addition it must be a linguistic account of quantum mechanics. In this account, the invariance of symbols is equivalent to the invariance of eigenstates under the operation of an *observable*. This is the language *formulation* or *representationing* operator mentioned in Chapter 4.

Classical qualities such as truth and grammaticality have to give way to quantities[2]. Specifically, a state of affairs is to be regarded as a *superposition* of eigenstates. In this sense, the apparent well-definedness of macroscopic classical properties is a limiting case of quantum properties. It can only be addressed in aggregate. On the other hand, qualities *per se* have to be regarded as eigenstates (symbols) corresponding to a particular quantum experimental arrangement (associated with a *formulation* or *representationing* operator). As discussed in Chapter 4, these eigenstates are *references without referent* — or *symbols per se*. In this sense, folk psychological terms such as freedom, intention, consciousness, emotion, etc. must have their own merits and can be consequently retained. The discreteness and abruptness of symbols can be satisfactorily accommodated. In fact, even classical consistency can be saved. This is because all eigenstates are *orthogonal* to each other. A symbol is *either* the case *or* not the case. Based on these observations, non-monotonic reasoning, counterfactual conditionals, and causal arguments (as counterfactual reasoning in disguise) can also be accommodated in this framework. In a sense, these approaches are a quantum computational simulation of others' subjective mind. It is therefore a framework for *intersubjectivity.*

In technical terms, the strength of quantum mechanics lies in its sophisticated mathematical formalism. Although the formalism is, mathematically speaking, well-formed and well-defined, one should not be lured to the idea that language and/or logic — both classical and common sense, can be treated as well-defined at a higher level. It is the case only if we *choose* to *represent* them at the higher level. In fact, one should be careful here that well-definedness all too often goes hand-in-hand with conventional understanding of objective reality. Taking this position of "higher level," one is apt to assert that at least the formalism of quantum mechanics is "real" (in its conventional sense). But this cannot

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[2] Modern physics is overwhelmingly *quantitative.* An extreme case is *String Theory* which promises a unified account for all four fundamental forces and elementary particles. In String Theory, fundamental physical objects are "notes" that are played on tiny strings of the Planck scale. They are all just *quantitative* aspects of the genuine "atoms" — strings [85]

be the case, for a coherent monistic theory *cannot* afford the traditional Cartesian hierarchic thinking. Rather, a quantum mechanical approach to language and logic has to be an *epistemological* instead of an *ontological* account. That is to say, the conventional way of separating *res extensa* and *res cogitans* must be put away.

Specifically, the Uncertainty Principle of language wards off the collapse of the approach as a whole. Furthermore, it also advocates the need for *narrative* as a complement of *proof*[3]. In fact, a narrative is *holistic* and content-rich; but a proof (or a classical logical argument) is *local* and content-free. So the complementarity of narrative and proof is implied in the symbol-concept duality which is associated with the Uncertainty Principle (technically speaking, it is based on the *non-commuteness* of formulation or representationing operators). In fact, the Uncertainty Principle sets the horizon of the intelligible discourse. A horizon not only indicates the limits, but also implies that there must be something beyond, but what is beyond the horizon is not describable (literally *unspeakable*).

Nevertheless, if the account proposed in this thesis is correct, there are a wide range of phenomena to be taken into account in a quantum mechanical theory of mind. In the future, a very important issue of quantum mechanical approaches to language/logic will be to *re-apply* the new theory to old linguistic/logical problems. For instance, it remains to be shown how conventional linguistic syntax (grammar, for example) and conventional semantics can be explained as an aggregate phenomenon in a language community. Also of interest is the acquisition of first and second (verbal) languages. In the case of syntax and semantics, we have to show how quantum computation can build a bridge between bottom-up non-verbal memory of the environment and structured verbal expression. In the case of language acquisition, we have to show how verbal expressions in a foreign language can first be simulated in the native language (through translation) and then become automatic (achieving fluency). Equally interesting is how situations and/or objects that are found in only one community can be expressed in another community — using the same or a different

---

[3] Gödel's Incomplete Theorem also implies that there is truth that is unaccessible through any formal proof system.

language. Of course, it still remains to be shown how a verbal expression corresponds to its logical form.

In logic study, for example, it is interesting to see how expertise can be stored as a "database" of quantum states of affairs. There are many arguments — legal, moral, and aesthetic, to name a few — which are very difficult, if not impossible to be accounted for in a classical framework. These are interesting as well as practical scientific issues.

The second part of this thesis, it is hoped, has engendered a little optimism for the practical applications of a quantum computational approach. It is a matter of turning a theory into engineering. Indeed, quantum mechanics may have profound influence on natural language understanding/processing — both in scientific and engineering terms. For one thing, classical symbolic as well as bottom-up statistical, template, or connectionist approaches to NLP do *not* offer any adequate account for subjectivity and intention. At best, the "intention" of a computer program (such as in a dialog system of diagnosis) is only a programmed function, created so that the user can make believe that the computer program is willing to help or is user-friendly.[4] Indeed, it is hardly imaginable that a program without free will or emotion can in any way be called "friendly." Moreover, it is as unlikely that an NLP system can help us in sophisticated works without understanding what utterances or texts actually *mean.* All these difficulties, it seems to me, have to be traced back to the absence of an adequate account for intention in both the classical symbolic and the classical physicalist theories of language.

The implication due to intention can be profound. For example, we come to an anticipated application of quantum mechanical NLP/AI in automatic agents, to which the intention of the host (a human user in this case) is to be understood. Without intention, it is hardly possible to talk about understanding. This is a crucial difference between a handy tool and a competent agent. For example, in an application consisting of an internet agent who is supposed to recommend to users which web pages may be relevant or of interest, the application has to have adequate access to the states of affairs of the web pages as well

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[4] In fact, we can only say that the engineers who design the NLP-system are willing to help or friendly, but not the program or hardware itself.

as that of the users adequately. In fact, for us as humans, a state of affairs is seldom a fixed and mechanistic “representation.” It is rather a dynamic and living whole that *makes sense*, most of the time, only to us. Evidently, the most suitable implementation of states of affairs of an agent are those that are genuinely similar to that of a human.

Part IV

Appendix

Appendix A

Learning Algorithm

This area is usually called *unconstrained optimization.* It is a very useful technique in many natural sciences, social sciences, and engineering disciplines. In this framework, the goal is to minimize a real scalar objective function of n-dimensional vector $\vec{x}$ on the parameter space, where n is the total number of free parameters.

A typical unconstrained optimization method can be better understood by imaging walking on an n-dimensional terrain described by the objective function and to look for the deepest valley from where one starts the exploration. One chooses every step in order to descend to a lower level. In general, one can eventually find a point at which the gradient is zero. Note that there is no guarantee to find the global minimum [86, 41].

More precisely, given a real function $f(\vec{x})$ of n variables (i.e. $\vec{x} \in \mathbb{R}^n$), we want to find a particular $\vec{x}_{min} \in \mathbb{R}^n$ such that

$$f(\vec{x}_{min}) < f(\vec{y})$$

for all $\vec{y}$ in the neighborhood of $\vec{x}_{min}$. $f(\vec{x}_{min})$ is called a *local minimum* of function f. Specifically, if f is a continuous differentiable function, at $\vec{x}_{min}$ we must have,

$$\nabla f(\vec{x})|_{\vec{x}=\vec{x}_{min}} = 0.$$

A.1 Conjugated gradient method

The conjugation gradient method [41] is a very efficient method to find a local minimum near an arbitrary initial point $\vec{x}_0$. It begins with the calculation of the gradient:

$$\vec{h}_0 = \vec{g}_0 = -\nabla f(\vec{x}_0).$$

We then minimize the function in its conjugate gradient direction $\vec{h}_{k+1}$, which is given as follows:

$$\vec{g}_{k+1} = -\nabla f(\vec{x}_{k+1})$$

and

$$\vec{h}_{k+1} = \vec{g}_{k+1} + \gamma_k \vec{h}_k$$

where γ is updated with the Fletcher-Reeves formula, by

$$\gamma_k = \frac{\vec{g}_{k+1} \cdot \vec{g}_{k+1}}{\vec{g}_k \cdot \vec{g}_k}$$

or with the Polak-Ribiere formula, by

$$\gamma_k = \frac{(\vec{g}_{k+1} - \vec{g}_k) \cdot \vec{g}_{k+1}}{\vec{g}_k \cdot \vec{g}_k}$$

where $\vec{x}_{k+1}$ is the minimal point along the direction $\vec{h}_k$. A one-dimensional minimization routine (e.g. the exact line-search algorithm) can be used to find the minimum $\vec{x}_{k+1}$.

A.2 Random walk method

The random walk method is illustrated in the following pseudo-codes:

```
for (i=1 to n)
    x0[i]=random(-SEARCHRANGE,SEARCHRANGE);
```

```
f0=f(x0);

for (counter=1 to MAXCOUNT) {
    for (i=1 to n)
        delta_x[i]=random(-STEPSIZE,STEPSIZE);

    f1=f(x0+delta_x);
    if (f1 < f0) {
        x0 = x0 + delta;
        f0 = f1;
    }
}
```

Appendix B

Simulation Data used in Chapter 6

B.1 Data used in XOR experiment

The input state can be represented by a 6-dimensional complex vector

$$\psi_{in} = \langle c_{01}, c_{11}, c_{02}, c_{12}, c_{0,out}, c_{1,out} \rangle^t.$$

In this case, the unitary operator U can be expressed in a matrix form:

$$U = e^{\frac{-iHt}{\hbar}}$$

where H is an Hermitian matrix. After the system is let go for free evolution, the actual time point t at which the measurement is performed can be absorbed into H. This is also the case for the minus sign and $\hbar$. We then write the exponent of e still as H. The reader should bear in mind that H is a short-hand form of $(-Hamiltonian \cdot t/\hbar)$. The end state of the system ψ_{out} can be then expressed as a matrix-vector multiplication:

$$\psi_{out} = U\psi_{in}$$

The simulation data used in Section 6.2 are summarized as follows.

$$Re[H] = \begin{pmatrix} -0.1400 & -0.03995 & 0.008367 & -0.2734 & -0.6253 & 0.2534 \\ -0.03995 & 0.07190 & 0.2339 & -0.1937 & 0.5120 & -0.8315 \\ 0.008367 & 0.2339 & 0.2348 & -0.06109 & -0.5260 & -0.8313 \\ -0.2734 & -0.1937 & -0.06109 & -0.2293 & 0.6233 & 0.3009 \\ -0.6253 & 0.5120 & -0.5260 & 0.6233 & -0.008325 & 0.03478 \\ 0.2534 & -0.8315 & -0.8313 & 0.3009 & 0.03478 & 0.06618 \end{pmatrix}$$

$$Im[H] = \begin{pmatrix} 0 & -0.2129 & -0.1333 & -0.1391 & 0.3124 & -0.6552 \\ 0.2129 & 0 & 0.1083 & 0.1731 & -0.7011 & 0.4363 \\ 0.1333 & -0.1083 & 0 & -0.03910 & 0.3401 & 0.3461 \\ 0.1391 & -0.1731 & 0.03910 & 0 & -0.6871 & -0.5109 \\ -0.3124 & 0.7011 & -0.3401 & 0.6871 & 0 & 0.01860 \\ 0.6552 & -0.4363 & -0.3461 & 0.5109 & -0.01860 & 0 \end{pmatrix}$$

$$Re[U] = \begin{pmatrix} 0.5320 & 0.5283 & 0.06375 & 0.09863 & -0.06494 & 0.4119 \\ 0.2825 & 0.2838 & -0.2371 & -0.2534 & 0.4904 & -0.2145 \\ -0.09931 & -0.09407 & 0.4684 & 0.4256 & -0.1432 & -0.06713 \\ -0.05263 & -0.06068 & 0.3667 & 0.4319 & 0.5171 & 0.1842 \\ 0.3032 & -0.3502 & 0.3017 & -0.3487 & -0.0001100 & 0.0006705 \\ -0.3881 & 0.3321 & 0.3306 & -0.3866 & 0.0002425 & -0.0001584 \end{pmatrix}$$

$$Im[U] = \begin{pmatrix} -0.1011 & -0.1126 & -0.1866 & -0.09938 & -0.4268 & -0.02156 \\ 0.04828 & 0.04775 & 0.06286 & 0.06781 & 0.2898 & -0.5793 \\ 0.1706 & 0.1643 & 0.1038 & 0.1530 & -0.2475 & -0.6357 \\ -0.2830 & -0.2893 & -0.1986 & -0.1493 & 0.3741 & 0.07583 \\ -0.3991 & 0.3589 & -0.3970 & 0.3567 & -0.00007909 & -5.664\,10^{-6} \\ 0.3179 & -0.3747 & -0.3728 & 0.3161 & 0.0002185 & -0.0001794 \end{pmatrix}$$

The following can be easily checked:

$$U = e^{iH}$$

$$U^\dagger = \left(e^{iH}\right)^\dagger = e^{-iH^\dagger} = e^{-iH}$$

since H is Hermitian ($H^\dagger = H$). So we have:

$$U^\dagger U = e^{-iH} e^{iH} = e^0 = I$$

and

$$U U^\dagger = e^{iH} e^{-iH} = e^0 = I$$

where I is an identity matrix of dimension 6. So $U^\dagger = U^{-1}$.

When a phase-mapping function is applied to the AND-training data (or the OR-training data) before submitting it to the unitary transformation (trained for XOR *without* phase-mapping), the system can perform AND-operation (OR-operation). The phase-mapping function can be written in a matrix form (see Section 6.2) and calculated using standard minimization procedure (we use the random walk method). The phase-mapping matrice used in Section 6.2 for the AND (OR) training data are:

$$\Phi_{\wedge} = \begin{pmatrix} -10.16 & 3.557 & 6.005 & -8.747 \\ 6.512 & 0.3859 & -7.283 & -6.283 \\ 2.802 & -4.607 & -6.734 & 3.088 \\ -2.400 & -7.193 & -3.493 & -3.156 \end{pmatrix}$$

$$\Phi_{\vee} = \begin{pmatrix} 1.289 & -7.746 & 8.088 & 1.335 \\ 7.421 & 7.618 & 7.896 & -1.015 \\ 4.014 & -7.635 & -3.313 & 3.588 \\ -8.610 & -0.02628 & -5.716 & 2.173 \end{pmatrix}$$

B.2 Data used in the non-monotonic reasoning experiment of Section 6.3

$$Re[H] = \begin{pmatrix} 0.09009 & -0.05759 & 0.03099 & -0.09969 & -0.4535 & -0.3245 \\ -0.05759 & -0.01911 & 0.04090 & -0.05086 & 0.05262 & 0.06099 \\ 0.03099 & 0.04090 & 0.02451 & 0.01931 & 0.01398 & -0.4031 \\ -0.09969 & -0.05086 & 0.01931 & -0.01469 & -0.1143 & 0.07480 \\ -0.4535 & 0.05262 & 0.01398 & -0.1143 & 0.01324 & 0.04712 \\ -0.3245 & 0.06099 & -0.4031 & 0.07480 & 0.04712 & 0.01458 \end{pmatrix}$$

$$Im[H] = \begin{pmatrix} 0 & 0.04270 & 0.2003 & 0.1403 & 1.216 & -0.6488 \\ -0.04270 & 0 & -0.06095 & -0.01343 & 0.06835 & 0.1112 \\ -0.2003 & 0.06095 & 0 & -0.04234 & -0.1879 & -0.6590 \\ -0.1403 & 0.01343 & 0.04234 & 0 & 0.6434 & 0.1664 \\ -1.216 & -0.06835 & 0.1879 & -0.6434 & 0 & 0.08628 \\ 0.6488 & -0.1112 & 0.6590 & -0.1664 & -0.08628 & 0 \end{pmatrix}$$

$$Re[u] = \begin{pmatrix} 0.08081 & -0.006018 & -0.2595 & -0.3644 & -0.6909 & 0.4428 \\ 0.04187 & 0.9836 & 0.09947 & -0.02304 & -0.09392 & -0.05617 \\ -0.0003197 & -0.003632 & 0.6875 & 0.1470 & 0.1291 & 0.6051 \\ -0.2013 & -0.04264 & 0.08906 & 0.7881 & -0.5444 & -0.1388 \\ 0.8145 & 0.02060 & -0.2714 & 0.3516 & 0.1212 & 0.1107 \\ -0.3902 & 0.1210 & -0.5011 & 0.2204 & 0.1932 & 0.4842 \end{pmatrix}$$

$$Im[u] = \begin{pmatrix} 0.03117 & -0.06811 & -0.003469 & -0.1321 & -0.2778 & -0.1402 \\ 0.006895 & -0.01870 & 0.03725 & -0.02225 & 0.06701 & 0.04102 \\ 0.003838 & 0.03267 & 0.01980 & -0.007732 & -0.02021 & -0.3478 \\ -0.001612 & -0.06294 & 0.02963 & 0.003620 & -0.06176 & 0.06548 \\ -0.2637 & -0.008408 & 0.08760 & -0.1133 & 0.003739 & -0.1477 \\ -0.2548 & 0.07469 & -0.3272 & 0.1447 & 0.2507 & -0.01640 \end{pmatrix}$$

B.3 Data used in the counterfactual reasoning experiment of Section 6.4

$$Re[H] = \begin{pmatrix} -0.1355 & -0.02028 & -0.2116 & -0.1489 & 0.01512 & 0.05923 & 0.6437 & 0.3818 \\ -0.02028 & 0.05760 & 0.1220 & 0.003952 & 0.08527 & 0.01495 & -0.01935 & 0.3882 \\ -0.2116 & 0.1220 & -0.1490 & -0.1322 & 0.1674 & 0.001046 & 0.01993 & 0.8742 \\ -0.1489 & 0.003952 & -0.1322 & -0.006610 & -0.07957 & -0.01279 & 0.6076 & -0.08570 \\ 0.01512 & 0.08527 & 0.1674 & -0.07957 & 0.03112 & 0.1469 & -0.1895 & 0.5474 \\ 0.05923 & 0.01495 & 0.001046 & -0.01279 & 0.1469 & 0.1071 & 0.7280 & 0.3969 \\ 0.6437 & -0.01935 & 0.01993 & 0.6076 & -0.1895 & 0.7280 & 0.07284 & -0.3952 \\ 0.3818 & 0.3882 & 0.8742 & -0.08570 & 0.5474 & 0.3969 & -0.3952 & 0.004064 \end{pmatrix}$$

$$Im[H] = \begin{pmatrix} 0 & -0.4598 & -0.1507 & -0.4147 & -0.8259 & 0.3757 & 0.9729 & -3.962 \\ 0.4598 & 0 & 0.4762 & 0.01666 & 0.1250 & 0.3775 & 0.05517 & -1.466 \\ 0.1507 & -0.4762 & 0 & -0.4852 & -0.7704 & 0.5942 & 0.1655 & -4.587 \\ 0.4147 & -0.01666 & 0.4852 & 0 & 0.09558 & 0.3713 & 0.8165 & -0.9131 \\ 0.8259 & -0.1250 & 0.7704 & -0.09558 & 0 & 0.7103 & -0.002297 & -3.192 \\ -0.3757 & -0.3775 & -0.5942 & -0.3713 & -0.7103 & 0 & 1.137 & -2.297 \\ -0.9729 & -0.05517 & -0.1655 & -0.8165 & 0.002297 & -1.137 & 0 & 0.4831 \\ 3.962 & 1.466 & 4.587 & 0.9131 & 3.192 & 2.297 & -0.4831 & 0 \end{pmatrix}$$

$$Re[u] = \begin{pmatrix} 0.6102 & -0.03973 & -0.2606 & -0.1692 & -0.02417 & -0.3702 & -0.2968 & 0.5051 \\ -0.1149 & 0.9358 & -0.2182 & -0.02327 & -0.1831 & -0.005787 & 0.06145 & 0.1285 \\ -0.2058 & -0.09091 & 0.4349 & 0.2786 & -0.2845 & -0.08145 & 0.2921 & 0.6892 \\ -0.2998 & 0.05458 & 0.01790 & 0.6286 & 0.1139 & -0.4093 & -0.4768 & -0.09621 \\ -0.2364 & -0.08590 & -0.4931 & 0.1332 & 0.6643 & 0.04689 & 0.1914 & 0.3405 \\ -0.3159 & -0.01102 & 0.09086 & -0.2761 & 0.07063 & 0.5739 & -0.5056 & 0.3078 \\ 0.2872 & -0.04728 & -0.2368 & 0.4766 & -0.2076 & 0.4469 & -0.0009598 & 0.007248 \\ -0.4253 & -0.3016 & -0.5689 & -0.1583 & -0.5705 & -0.1564 & 0.001117 & 0.001767 \end{pmatrix}$$

$$Im[u] = \begin{pmatrix} 0.005924 & -0.006723 & -0.03321 & -0.04426 & 0.03501 & 0.03600 & 0.2047 & 0.03310 \\ -0.02145 & -0.01176 & 0.04365 & 0.04219 & -0.02780 & -0.02083 & -0.01722 & 0.05773 \\ -0.05558 & 0.07093 & -0.04413 & -0.006187 & 0.03929 & 0.03806 & -0.1116 & 0.09590 \\ -0.03663 & 0.03160 & -0.009506 & 0.04859 & 0.01444 & -0.02391 & 0.2902 & -0.06956 \\ 0.009067 & -0.007534 & 0.1090 & -0.02379 & -0.1122 & 0.01984 & -0.2158 & 0.08749 \\ -0.01137 & -0.02112 & -0.03700 & -0.01602 & 0.05525 & 0.02411 & 0.3432 & 0.08527 \\ 0.2292 & -0.03586 & -0.1894 & 0.3796 & -0.1668 & 0.3571 & 0.001059 & -0.0005084 \\ 0.07413 & 0.05276 & 0.09832 & 0.02756 & 0.09996 & 0.02771 & 0.0006075 & 0.0001970 \end{pmatrix}$$

B.4 Data used in the "Barbara" corpus of Section 7.3.1

Since the dimension of the Hamiltonian is rather large, the data are represented in a graphic format. The *areas* of each component of the matrix represent the value. The real and imaginary part of the Hamiltonian is shown in Figure B.1 and Figure B.2, respectively.

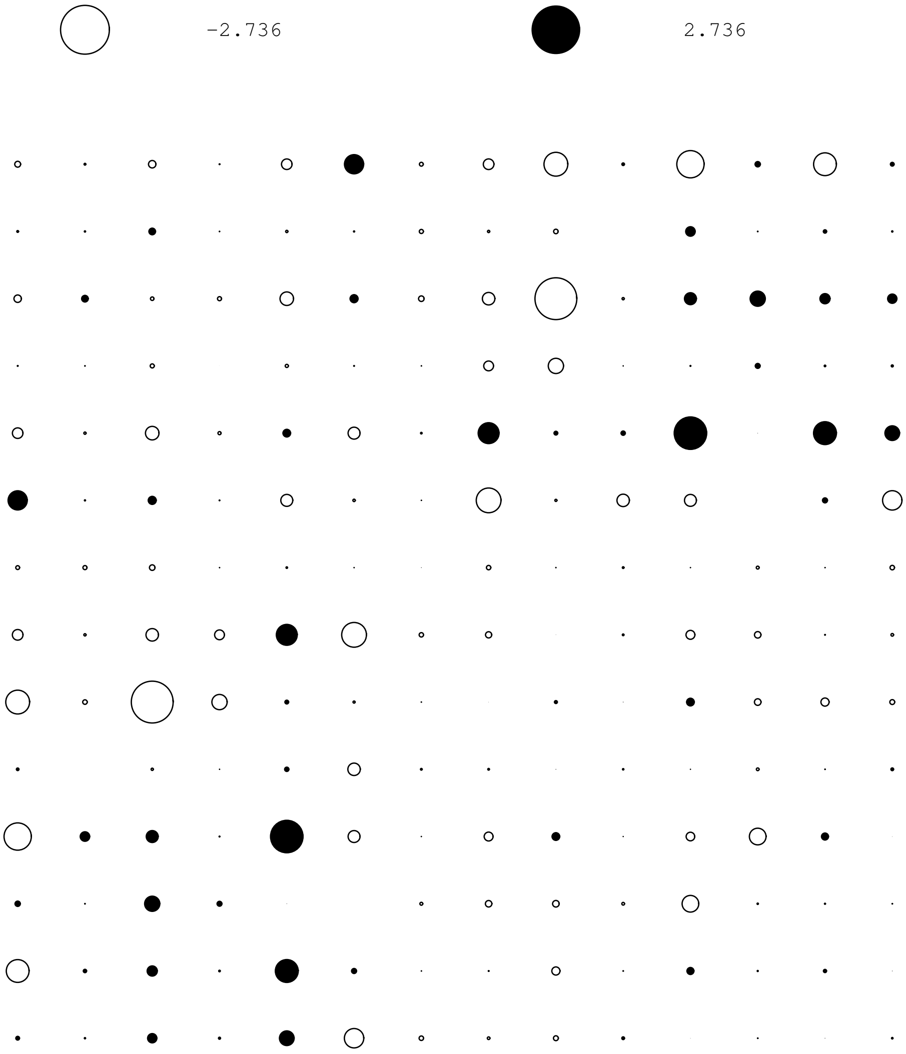

Figure B.1: The Hamiltonian (real part).

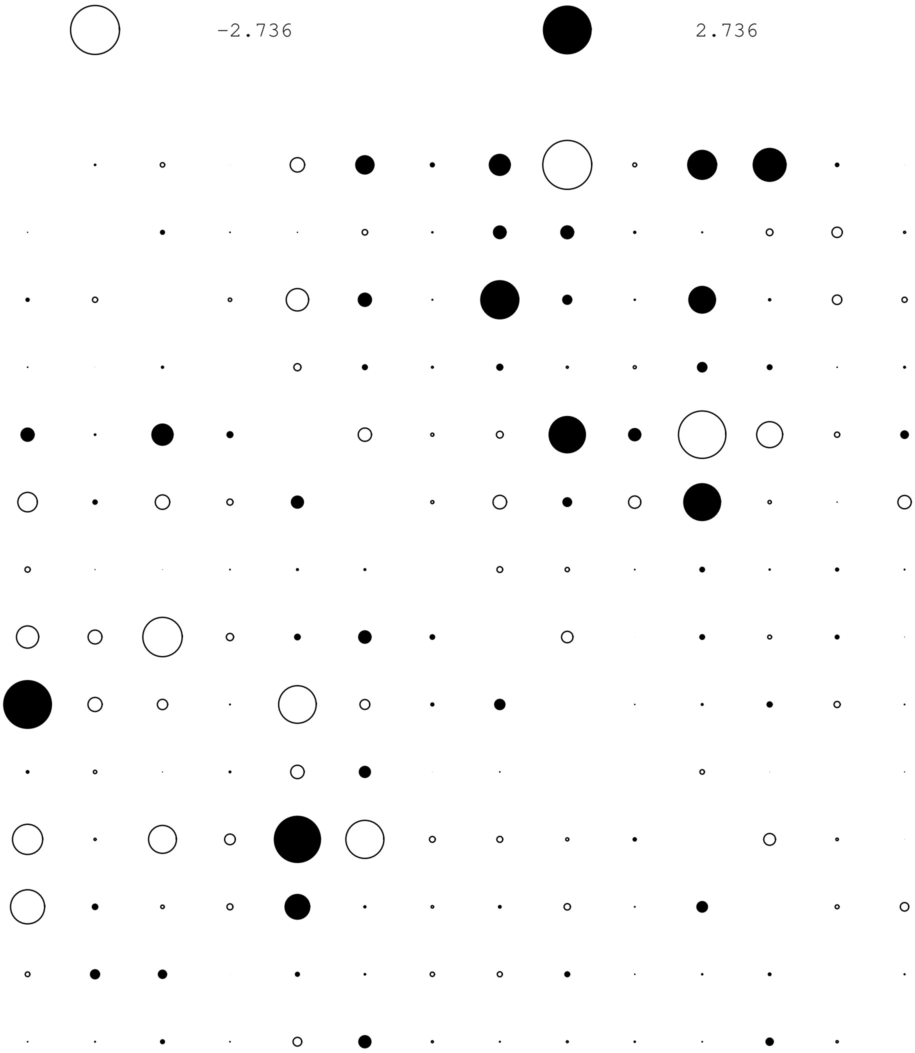

Figure B.2: The Hamiltonian (imaginary part).

B.5 Data used in the Full categorical syllogistic corpus of Section 7.3.2

The dimension of the Hamiltonian is 38. The real and imaginary part of the Hamiltonian is shown in Figure B.3 and Figure B.4, respectively.

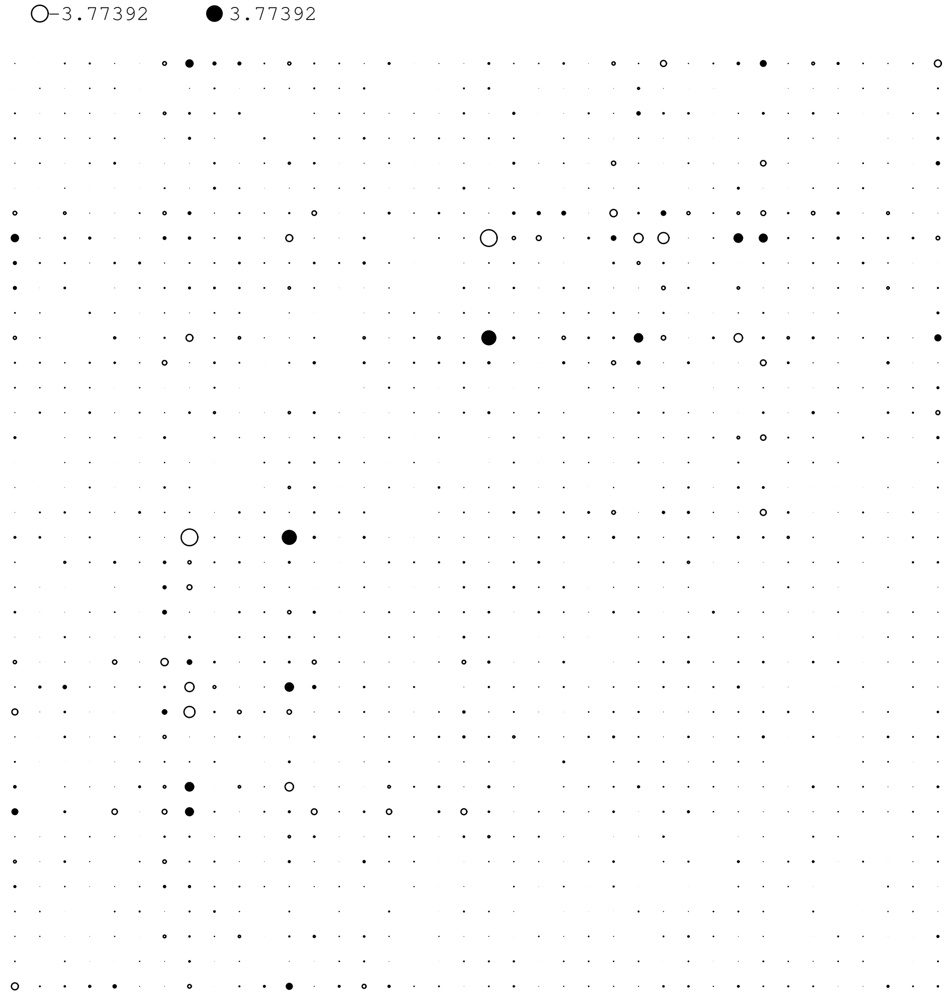

Figure B.3: The Hamiltonian (real part).

B.6 Data used in the bilingual machine translation corpus of Section 7.5

The dimension of the Hamiltonian is $20 + 19 = 39$. The real and imaginary part of the Hamiltonian is shown in Figure B.5 and Figure B.6, respectively.

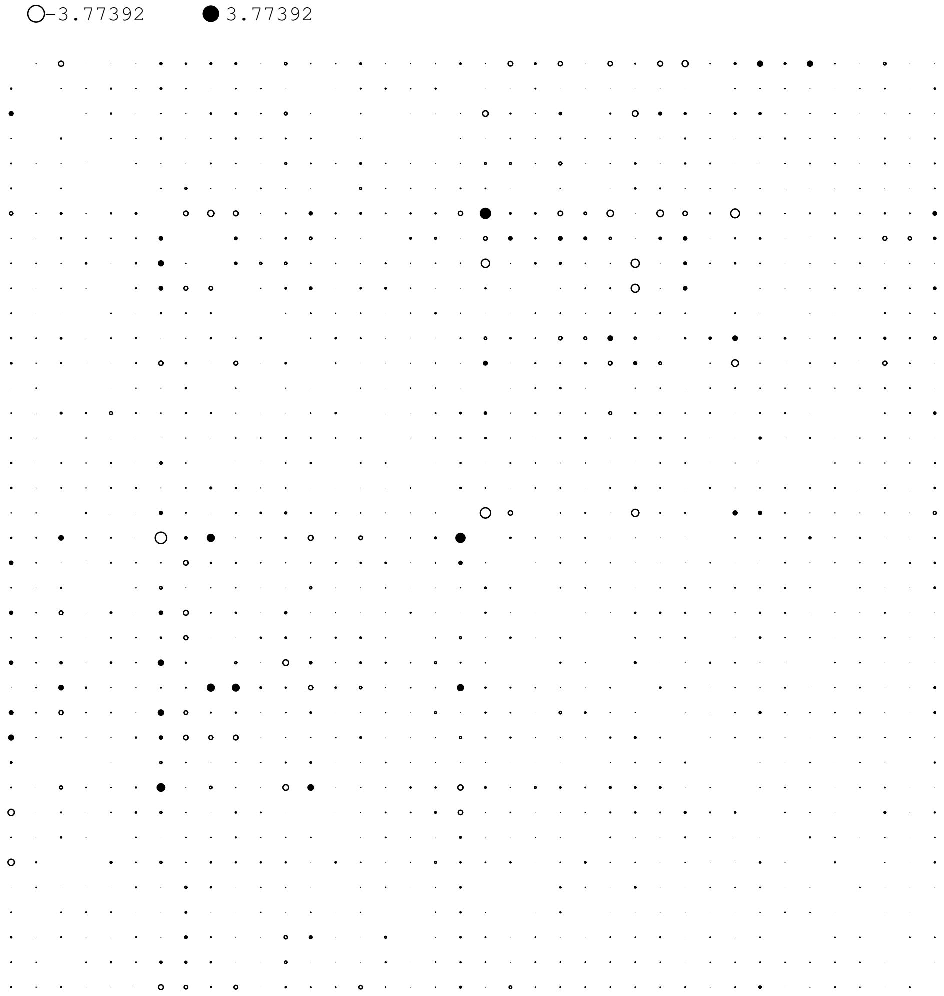

Figure B.4: The Hamiltonian (imaginary part).

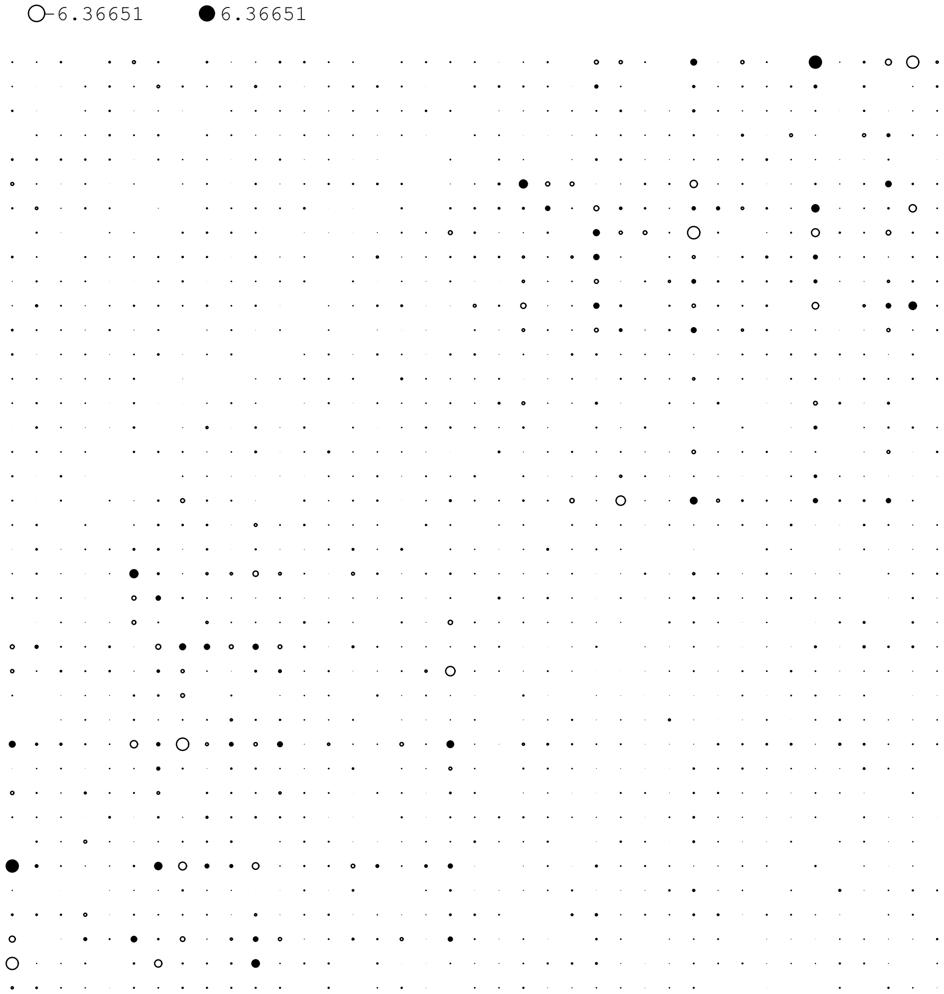

Figure B.5: The Hamiltonian (real part).

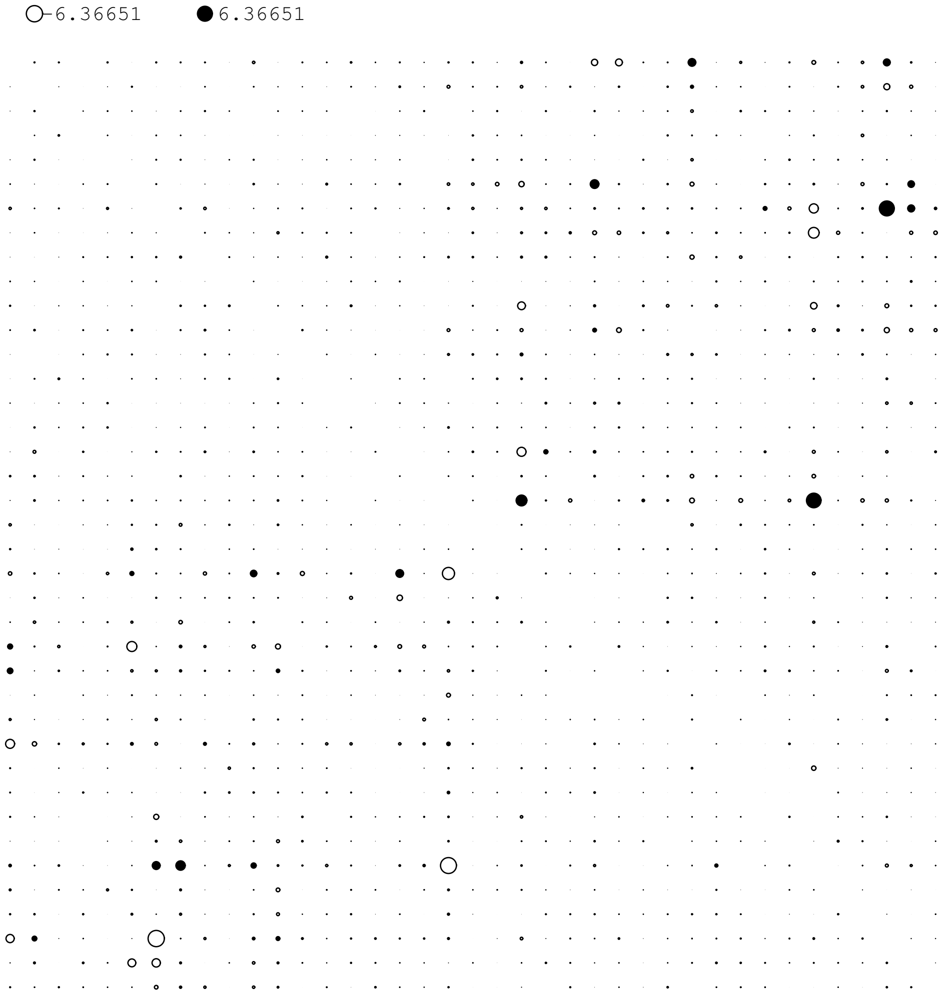

Figure B.6: The Hamiltonian (imaginary part).

Appendix C

Natural Language Corpora

In this appendix, the corpora used in Chapter 7 are summarized. The vocabulary used in Chalmers' original corpus (Section 7.4.1) is presented in Table 7.1 and repeated below:

Category	Instances
Person	john michael helen diane chris
Action	love hit betray kill hug
Misc.	is by

The vocabulary used in the augmented Chalmers' corpus (Section 7.4.2) is presented in Table 7.2 and repeated below:

Category	Instances
Person	john michael helen diane
Action	kill love betray hit
Action Conjugated	kills loves betrays hits
Past Participle	killed loved betrayed hit*
Conjunction	and
Misc.	is are by

The vocabulary used in the English-German bin lingual corpus (Section 7.5) is presented in Table 7.3 and repeated below:

Vocabulary Category	English Instances	German Instances
Person	john michael helen diane	john michael helen diane
Action	kill love betray hit*	umbringen† lieben verraten* schlagen
Action Conjugated	kills loves betrays hits	bringt† liebt verraetet schlaegt
Past Participle	killed loved betrayed hit*	umgebracht geliebt verraten* geschlagen
Conjunction	and	und
Misc.	is are by	wird werden von um†

Trainig set used in Chalmers' syntax corpus

40 sentences in total:

john hit john ⇔ john is hit by john
diane kill helen ⇔ helen is kill by diane
diane betray diane ⇔ diane is betray by diane
michael kill helen ⇔ helen is kill by michael
michael betray chris ⇔ chris is betray by michael
john kill michael ⇔ michael is kill by john
diane hug michael ⇔ michael is hug by diane
chris hit diane ⇔ diane is hit by chris
diane love diane ⇔ diane is love by diane
helen betray michael ⇔ michael is betray by helen
michael betray helen ⇔ helen is betray by michael
chris kill diane ⇔ diane is kill by chris
john hit michael ⇔ michael is hit by john
michael kill john ⇔ john is kill by michael
helen hug chris ⇔ chris is hug by helen
michael hit chris ⇔ chris is hit by michael
diane hug diane ⇔ diane is hug by diane
diane betray john ⇔ john is betray by diane
john betray michael ⇔ michael is betray by john
michael hug michael ⇔ michael is hug by michael
john hug diane ⇔ diane is hug by john
john hug michael ⇔ michael is hug by john
diane love john ⇔ john is love by diane
helen love john ⇔ john is love by helen
john love michael ⇔ michael is love by john
chris betray diane ⇔ diane is betray by chris
helen hit chris ⇔ chris is hit by helen
john betray helen ⇔ helen is betray by john
helen love chris ⇔ chris is love by helen
michael hit john ⇔ john is hit by michael
chris love chris ⇔ chris is love by chris
chris betray john ⇔ john is betray by chris
diane betray helen ⇔ helen is betray by diane
michael kill diane ⇔ diane is kill by michael
john hit helen ⇔ helen is hit by john
john kill helen ⇔ helen is kill by john
diane betray chris ⇔ chris is betray by diane
john love diane ⇔ diane is love by john
michael hit helen ⇔ helen is hit by michael
helen betray diane ⇔ diane is betray by helen

Test set used in Chalmers' syntax corpus

85 sentences in total:

john hit chris ⇔ chris is hit by john
chris hit helen ⇔ helen is hit by chris
michael betray john ⇔ john is betray by michael
diane betray michael ⇔ michael is betray by diane
chris kill john ⇔ john is kill by chris
michael love diane ⇔ diane is love by michael
helen kill helen ⇔ helen is kill by helen
helen hug helen ⇔ helen is hug by helen
chris hit michael ⇔ michael is hit by chris
john kill diane ⇔ diane is kill by john
helen betray helen ⇔ helen is betray by helen
diane hit helen ⇔ helen is hit by diane
helen love helen ⇔ helen is love by helen
chris hit chris ⇔ chris is hit by chris
michael hit michael ⇔ michael is hit by michael
michael hug helen ⇔ helen is hug by michael
diane hit chris ⇔ chris is hit by diane
chris love helen ⇔ helen is love by chris
chris love diane ⇔ diane is love by chris
helen hit michael ⇔ michael is hit by helen
john love john ⇔ john is love by john
helen hug diane ⇔ diane is hug by helen
michael betray michael ⇔ michael is betray by michael
diane hug chris ⇔ chris is hug by diane
john hug john ⇔ john is hug by john
chris kill michael ⇔ michael is kill by chris
john kill chris ⇔ chris is kill by john
diane kill chris ⇔ chris is kill by diane
john hug chris ⇔ chris is hug by john
helen betray john ⇔ john is betray by helen
michael kill michael ⇔ michael is kill by michael
michael love chris ⇔ chris is love by michael
john hug helen ⇔ helen is hug by john
helen love michael ⇔ michael is love by helen
john kill john ⇔ john is kill by john
diane hit john ⇔ john is hit by diane
diane kill michael ⇔ michael is kill by diane
michael hug diane ⇔ diane is hug by michael
chris hug john ⇔ john is hug by chris
john betray chris ⇔ chris is betray by john
helen hit diane ⇔ diane is hit by helen
john love chris ⇔ chris is love by john
chris hug diane ⇔ diane is hug by chris
diane love helen ⇔ helen is love by diane
diane love chris ⇔ chris is love by diane
michael betray diane ⇔ diane is betray by michael
chris hit john ⇔ john is hit by chris
michael kill chris ⇔ chris is kill by michael
michael love john ⇔ john is love by michael
helen betray chris ⇔ chris is betray by helen
chris betray helen ⇔ helen is betray by chris
chris betray michael ⇔ michael is betray by chris
john love helen ⇔ helen is love by john
chris hug michael ⇔ michael is hug by chris
helen kill diane ⇔ diane is kill by helen
michael hug john ⇔ john is hug by michael
diane hit michael ⇔ michael is hit by diane
michael love helen ⇔ helen is love by michael
john betray diane ⇔ diane is betray by john
michael love michael ⇔ michael is love by michael
diane love michael ⇔ michael is love by diane
helen kill john ⇔ john is kill by helen
chris kill chris ⇔ chris is kill by chris
chris betray chris ⇔ chris is betray by chris
michael hit diane ⇔ diane is hit by michael
chris hug helen ⇔ helen is hug by chris
john betray john ⇔ john is betray by john
michael hug chris ⇔ chris is hug by michael
diane kill john ⇔ john is kill by diane

helen hit helen ⇔ helen is hit by helen
helen hug john ⇔ john is hug by helen
helen hit john ⇔ john is hit by helen
diane hug helen ⇔ helen is hug by diane
helen love diane ⇔ diane is love by helen
john hit diane ⇔ diane is hit by john
helen kill chris ⇔ chris is kill by helen
helen kill michael ⇔ michael is kill by helen
chris love john ⇔ john is love by chris
diane hug john ⇔ john is hug by diane
helen hug michael ⇔ michael is hug by helen
diane kill diane ⇔ diane is kill by diane
chris hug chris ⇔ chris is hug by chris
chris kill helen ⇔ helen is kill by chris
chris love michael ⇔ michael is love by chris
diane hit diane ⇔ diane is hit by diane

Trainig set used in the fully conjugated syntax corpus

56 sentences in total:

diane loves john and michael ⇔ john and michael are loved by diane
diane and helen hit michael ⇔ michael is hit by diane and helen
john kills helen ⇔ helen is killed by john
john kills michael ⇔ michael is killed by john
michael hits helen and diane ⇔ helen and diane are hit by michael
helen and diane hit john ⇔ john is hit by helen and diane
john betrays michael and diane ⇔ michael and diane are betrayed by john
john and diane kill michael ⇔ michael is killed by john and diane
john hits helen ⇔ helen is hit by john
michael loves helen and john ⇔ helen and john are loved by michael
diane betrays helen and michael ⇔ helen and michael are betrayed by diane
helen and michael love john ⇔ john is loved by helen and michael
michael and john kill helen ⇔ helen is killed by michael and john
john and michael kill diane ⇔ diane is killed by john and michael
helen betrays michael and diane ⇔ michael and diane are betrayed by helen
diane and michael hit helen ⇔ helen is hit by diane and michael
michael and john hit diane ⇔ diane is hit by michael and john
helen and john love diane ⇔ diane is loved by helen and john
diane and john hit helen ⇔ helen is hit by diane and john
michael and john kill diane ⇔ diane is killed by michael and john
john betrays diane and helen ⇔ diane and helen are betrayed by john
helen and john kill michael ⇔ michael is killed by helen and john
john betrays helen ⇔ helen is betrayed by john
michael kills helen and john ⇔ helen and john are killed by michael
diane and john betray helen ⇔ helen is betrayed by diane and john
john loves michael ⇔ michael is loved by john
john kills helen and michael ⇔ helen and michael are killed by john
diane and michael betray john ⇔ john is betrayed by diane and michael
helen kills john ⇔ john is killed by helen
helen betrays michael and john ⇔ michael and john are betrayed by helen
john and diane kill helen ⇔ helen is killed by john and diane
diane and john betray michael ⇔ michael is betrayed by diane and john
john kills diane and helen ⇔ diane and helen are killed by john
helen hits michael and diane ⇔ michael and diane are hit by helen
helen and diane betray john ⇔ john is betrayed by helen and diane
john and michael betray helen ⇔ helen is betrayed by john and michael
helen hits john and michael ⇔ john and michael are hit by helen
diane and helen betray michael ⇔ michael is betrayed by diane and helen
michael betrays diane and john ⇔ diane and john are betrayed by michael
john and diane love helen ⇔ helen is loved by john and diane
diane hits john and michael ⇔ john and michael are hit by diane
john and michael betray diane ⇔ diane is betrayed by john and michael
helen and michael hit diane ⇔ diane is hit by helen and michael
helen hits michael ⇔ michael is hit by helen
diane hits michael ⇔ michael is hit by diane
john kills michael and helen ⇔ michael and helen are killed by john
michael hits diane ⇔ diane is hit by michael
diane betrays john and helen ⇔ john and helen are betrayed by diane
john kills helen and diane ⇔ helen and diane are killed by john
diane kills john and michael ⇔ john and michael are killed by diane
helen betrays diane ⇔ diane is betrayed by helen
john betrays diane and michael ⇔ diane and michael are betrayed by john
diane and john kill helen ⇔ helen is killed by diane and john
michael and helen love john ⇔ john is loved by michael and helen
helen hits diane and john ⇔ diane and john are hit by helen
diane and michael betray helen ⇔ helen is betrayed by diane and michael

Test set used in the fully conjugated syntax corpus

184 sentences in total:

john kills diane and michael ⇔ diane and michael are killed by john
diane and helen kill john ⇔ john is killed by diane and helen
john and helen betray michael ⇔ michael is betrayed by john and helen
john and michael hit helen ⇔ helen is hit by john and michael
helen and michael betray john ⇔ john is betrayed by helen and michael
john and helen kill diane ⇔ diane is killed by john and helen
michael betrays diane and helen ⇔ diane and helen are betrayed by michael
helen and diane betray michael ⇔ michael is betrayed by helen and diane
helen and michael kill john ⇔ john is killed by helen and michael
diane kills helen and michael ⇔ helen and michael are killed by diane
john loves michael and helen ⇔ michael and helen are loved by john
diane kills helen and john ⇔ helen and john are killed by diane
john hits helen and michael ⇔ helen and michael are hit by john
diane loves john and helen ⇔ john and helen are loved by diane
helen kills john and michael ⇔ john and michael are killed by helen
john hits diane ⇔ diane is hit by john
john and helen hit diane ⇔ diane is hit by john and helen
diane kills michael and helen ⇔ michael and helen are killed by diane
helen loves diane ⇔ diane is loved by helen
john loves diane and michael ⇔ diane and michael are loved by john
diane betrays john ⇔ john is betrayed by diane

diane kills john and helen ⇔ john and helen are killed by diane
john and michael hit diane ⇔ diane is hit by john and michael
diane kills michael and john ⇔ michael and john are killed by diane
helen betrays michael ⇔ michael is betrayed by helen
john and michael love helen ⇔ helen is loved by john and michael
michael and john hit helen ⇔ helen is hit by michael and john
diane hits michael and helen ⇔ michael and helen are hit by diane
michael betrays john and diane ⇔ john and diane are betrayed by michael
helen and michael kill diane ⇔ diane is killed by helen and michael
michael and john love helen ⇔ helen is loved by michael and john
michael hits diane and helen ⇔ diane and helen are hit by michael
john and helen kill michael ⇔ michael is killed by john and helen
john and diane hit michael ⇔ michael is hit by john and diane
diane loves helen ⇔ helen is loved by diane
helen hits diane ⇔ diane is hit by helen
diane hits helen ⇔ helen is hit by diane
john hits michael and diane ⇔ michael and diane are hit by john
john loves helen ⇔ helen is loved by john
helen betrays john ⇔ john is betrayed by helen
helen hits diane and michael ⇔ diane and michael are hit by helen
helen kills diane ⇔ diane is killed by helen
helen betrays diane and john ⇔ diane and john are betrayed by helen
michael betrays john ⇔ john is betrayed by michael
diane betrays michael and helen ⇔ michael and helen are betrayed by diane
helen and john love michael ⇔ michael is loved by helen and john
diane betrays helen and john ⇔ helen and john are betrayed by diane
john and michael kill helen ⇔ helen is killed by john and michael
michael kills diane ⇔ diane is killed by michael
helen kills john and diane ⇔ john and diane are killed by helen
helen kills michael and diane ⇔ michael and diane are killed by helen
john kills michael and diane ⇔ michael and diane are killed by john
diane and helen betray john ⇔ john is betrayed by diane and helen
michael betrays diane ⇔ diane is betrayed by michael
michael kills helen and diane ⇔ helen and diane are killed by michael
michael and john betray helen ⇔ helen is betrayed by michael and john
helen loves michael and diane ⇔ michael and diane are loved by helen
helen and diane kill michael ⇔ michael is killed by helen and diane
diane hits john ⇔ john is hit by diane
michael and helen betray john ⇔ john is betrayed by michael and helen
diane and john hit michael ⇔ michael is hit by diane and john
john loves diane and helen ⇔ diane and helen are loved by john
john and diane hit helen ⇔ helen is hit by john and diane
helen hits john and diane ⇔ john and diane are hit by helen
helen and michael betray diane ⇔ diane is betrayed by helen and michael
diane loves michael and john ⇔ michael and john are loved by diane
helen betrays diane and michael ⇔ diane and michael are betrayed by helen
john and diane love michael ⇔ michael is loved by john and diane
michael and helen hit john ⇔ john is hit by michael and helen
michael loves john ⇔ john is loved by michael
michael hits john ⇔ john is hit by michael
michael kills john ⇔ john is killed by michael
john and helen love diane ⇔ diane is loved by john and helen
michael kills helen ⇔ helen is killed by michael
diane kills helen ⇔ helen is killed by diane
helen loves john and michael ⇔ john and michael are loved by helen
michael loves helen and diane ⇔ helen and diane are loved by michael
helen betrays john and diane ⇔ john and diane are betrayed by helen
diane loves michael ⇔ michael is loved by diane
michael kills john and helen ⇔ john and helen are killed by michael
helen betrays john and michael ⇔ john and michael are betrayed by helen
diane and michael kill john ⇔ john is killed by diane and michael
diane hits helen and michael ⇔ helen and michael are hit by diane
diane and john love helen ⇔ helen is loved by diane and john
michael loves diane ⇔ diane is loved by michael
john and helen love michael ⇔ michael is loved by john and helen
diane and john kill michael ⇔ michael is killed by diane and john
diane loves john ⇔ john is loved by diane
diane and helen love john ⇔ john is loved by diane and helen
diane and john love michael ⇔ michael is loved by diane and john
diane betrays michael and john ⇔ michael and john are betrayed by diane
helen loves michael ⇔ michael is loved by helen
helen hits john ⇔ john is hit by helen
diane and helen hit john ⇔ john is hit by diane and helen
michael hits helen and john ⇔ helen and john are hit by michael
michael betrays helen and diane ⇔ helen and diane are betrayed by michael
michael and diane betray helen ⇔ helen is betrayed by michael and diane
diane hits michael and john ⇔ michael and john are hit by diane
diane and michael love john ⇔ john is loved by diane and michael
helen loves michael and john ⇔ michael and john are loved by helen
diane betrays michael ⇔ michael is betrayed by diane
helen hits michael and john ⇔ michael and john are hit by helen
john kills diane ⇔ diane is killed by john
michael betrays john and helen ⇔ john and helen are betrayed by michael
michael loves helen ⇔ helen is loved by michael
michael and john betray diane ⇔ diane is betrayed by michael and john
john betrays michael ⇔ michael is betrayed by john
michael hits john and diane ⇔ john and diane are hit by michael
michael and diane kill john ⇔ john is killed by michael and diane
diane and michael kill helen ⇔ helen is killed by diane and michael
helen and john kill diane ⇔ diane is killed by helen and john
helen loves diane and michael ⇔ diane and michael are loved by helen
helen loves diane and john ⇔ diane and john are loved by helen
michael and diane kill helen ⇔ helen is killed by michael and diane
diane betrays helen ⇔ helen is betrayed by diane
john betrays michael and helen ⇔ michael and helen are betrayed by john
helen kills diane and michael ⇔ diane and michael are killed by helen
michael and diane hit helen ⇔ helen is hit by michael and diane
michael and helen love diane ⇔ diane is loved by michael and helen
michael and diane love helen ⇔ helen is loved by michael and diane
john and diane betray michael ⇔ michael is betrayed by john and diane
john hits diane and helen ⇔ diane and helen are hit by john
michael loves diane and helen ⇔ diane and helen are loved by michael
helen and john hit michael ⇔ michael is hit by helen and john
john betrays helen and diane ⇔ helen and diane are betrayed by john

john hits diane and michael ⇔ diane and michael are hit by john
helen and john betray michael ⇔ michael is betrayed by helen and john
john loves helen and diane ⇔ helen and diane are loved by john
john and diane betray helen ⇔ helen is betrayed by john and diane
michael and diane hit john ⇔ john is hit by michael and diane
diane loves helen and john ⇔ helen and john are loved by diane
michael hits helen ⇔ helen is hit by michael
diane hits helen and john ⇔ helen and john are hit by diane
helen and diane kill john ⇔ john is killed by helen and diane
michael and helen betray diane ⇔ diane is betrayed by michael and helen
john loves helen and michael ⇔ helen and michael are loved by john
john betrays diane ⇔ diane is betrayed by john
michael and john love diane ⇔ diane is loved by michael and john
michael kills john and diane ⇔ john and diane are killed by michael
helen loves john ⇔ john is loved by helen
michael hits diane and john ⇔ diane and john are hit by michael
helen and michael love diane ⇔ diane is loved by helen and michael
michael hits john and helen ⇔ john and helen are hit by michael
michael and helen kill john ⇔ john is killed by michael and helen
helen kills diane and john ⇔ diane and john are killed by helen
michael and diane betray john ⇔ john is betrayed by michael and diane
helen kills michael ⇔ michael is killed by helen
helen and diane love michael ⇔ michael is loved by helen and diane
john hits michael and helen ⇔ michael and helen are hit by john
john betrays helen and michael ⇔ helen and michael are betrayed by john
john loves diane ⇔ diane is loved by john
diane and helen love michael ⇔ michael is loved by diane and helen
michael loves diane and john ⇔ diane and john are loved by michael
diane kills michael ⇔ michael is killed by diane
john hits helen and diane ⇔ helen and diane are hit by john
diane and michael hit john ⇔ john is hit by diane and michael
helen and diane love john ⇔ john is loved by helen and diane
helen and michael hit john ⇔ john is hit by helen and michael
michael betrays helen and john ⇔ helen and john are betrayed by michael
michael loves john and diane ⇔ john and diane are loved by michael
diane and helen kill michael ⇔ michael is killed by diane and helen
diane hits john and helen ⇔ john and helen are hit by diane
michael and helen hit diane ⇔ diane is hit by michael and helen
helen kills michael and john ⇔ michael and john are killed by helen
diane betrays john and michael ⇔ john and michael are betrayed by diane
michael kills diane and john ⇔ diane and john are killed by michael
diane loves michael and helen ⇔ michael and helen are loved by diane
john and helen hit michael ⇔ michael is hit by john and helen
diane kills john ⇔ john is killed by diane
john and helen betray diane ⇔ diane is betrayed by john and helen
diane and michael love helen ⇔ helen is loved by diane and michael
michael and helen kill diane ⇔ diane is killed by michael and helen
michael loves john and helen ⇔ john and helen are loved by michael
michael kills diane and helen ⇔ diane and helen are killed by michael
helen and john betray diane ⇔ diane is betrayed by helen and john
helen and john hit diane ⇔ diane is hit by helen and john
diane loves helen and michael ⇔ helen and michael are loved by diane
michael betrays helen ⇔ helen is betrayed by michael
helen loves john and diane ⇔ john and diane are loved by helen
john loves michael and diane ⇔ michael and diane are loved by john
michael and diane love john ⇔ john is loved by michael and diane
helen and diane hit michael ⇔ michael is hit by helen and diane
john and michael love diane ⇔ diane is loved by john and michael
john hits michael ⇔ michael is hit by john

Trainig set used in the English-German corpus

78 sentences in total:

helen and michael are killed by john ⇔ helen und michael werden von john umgebracht
michael and john are loved by helen ⇔ michael und john werden von helen geliebt
helen loves michael and john ⇔ helen liebt michael und john
michael is betrayed by john and helen ⇔ michael wird von john und helen verraten
helen and john kill michael ⇔ helen und john bringen michael um
helen and john hit michael ⇔ helen und john schlagen michael
michael loves john and diane ⇔ michael liebt john und diane
diane and michael are betrayed by helen ⇔ diane und michael werden von helen verraten
john is loved by diane ⇔ john wird von diane geliebt
john and michael kill helen ⇔ john und michael bringen helen um
john and helen are hit by diane ⇔ john und helen werden von diane geschlagen
michael is killed by helen and diane ⇔ michael wird von helen und diane umgebracht
john and diane are hit by michael ⇔ john und diane werden von michael geschlagen
michael and diane love helen ⇔ michael und diane lieben helen
helen kills diane and john ⇔ helen bringt diane und john um
diane and john love michael ⇔ diane und john lieben michael
helen betrays john and diane ⇔ helen verraet john und diane
michael and john are betrayed by diane ⇔ michael und john werden von diane verraten
john and michael are killed by helen ⇔ john und michael werden von helen umgebracht
michael is hit by john and diane ⇔ michael wird von john und diane geschlagen
john hits michael and diane ⇔ john schlaegt michael und diane
diane loves john and michael ⇔ diane liebt john und michael
john is hit by helen and diane ⇔ john wird von helen und diane geschlagen
diane and michael are killed by john ⇔ diane und michael werden von john umgebracht
michael is loved by john ⇔ michael wird von john geliebt
michael and john kill helen ⇔ michael und john bringen helen um
michael is loved by john and helen ⇔ michael wird von john und helen geliebt
diane is hit by john and helen ⇔ diane wird von john und helen geschlagen
diane is loved by helen and michael ⇔ diane wird von helen und michael geliebt
helen is hit by diane and john ⇔ helen wird von diane und john geschlagen
diane kills helen and john ⇔ diane bringt helen und john um

john and diane are hit by helen ⇔ john und diane werden von helen geschlagen
john hits michael and helen ⇔ john schlaegt michael und helen
michael loves john ⇔ michael liebt john
john is betrayed by michael and helen ⇔ john wird von michael und helen verraten
diane is loved by michael and john ⇔ diane wird von michael und john geliebt
helen is betrayed by diane and john ⇔ helen wird von diane und john verraten
john kills michael ⇔ john bringt michael um
helen and diane are killed by michael ⇔ helen und diane werden von michael umgebracht
michael loves helen ⇔ michael liebt helen
diane hits helen and michael ⇔ diane schlaegt helen und michael
diane betrays john and helen ⇔ diane verraet john und helen
helen and diane hit michael ⇔ helen und diane schlagen michael
michael is killed by diane ⇔ michael wird von diane umgebracht
helen and diane kill michael ⇔ helen und diane bringen michael um
diane and helen are loved by michael ⇔ diane und helen werden von michael geliebt
michael and helen are betrayed by diane ⇔ michael und helen werden von diane verraten
john and diane are betrayed by michael ⇔ john und diane werden von michael verraten
john and michael hit helen ⇔ john und michael schlagen helen
john and michael are hit by diane ⇔ john und michael werden von diane geschlagen
diane betrays john ⇔ diane verraet john
diane and helen kill john ⇔ diane und helen bringen john um
john loves helen ⇔ john liebt helen
michael hits helen and john ⇔ michael schlaegt helen und john
john and diane betray helen ⇔ john und diane verraten helen
helen betrays michael and diane ⇔ helen verraet michael und diane
helen is killed by john and michael ⇔ helen wird von john und michael umgebracht
diane hits michael and helen ⇔ diane schlaegt michael und helen
michael is loved by diane ⇔ michael wird von diane geliebt
helen is killed by michael ⇔ helen wird von michael umgebracht
helen betrays diane and john ⇔ helen verraet diane und john
michael is betrayed by helen and diane ⇔ michael wird von helen und diane verraten
helen is betrayed by john and michael ⇔ helen wird von john und michael verraten
john and helen love diane ⇔ john und helen lieben diane
michael is loved by john and diane ⇔ michael wird von john und diane geliebt
michael and john love helen ⇔ michael und john lieben helen
michael and john hit diane ⇔ michael und john schlagen diane
john and diane kill michael ⇔ john und diane bringen michael um
helen hits john and diane ⇔ helen schlaegt john und diane
diane and michael betray helen ⇔ diane und michael verraten helen
helen and michael kill john ⇔ helen und michael bringen john um
diane and helen are betrayed by john ⇔ diane und helen werden von john verraten
helen is killed by diane and john ⇔ helen wird von diane und john umgebracht
diane and michael are loved by helen ⇔ diane und michael werden von helen geliebt
john is killed by michael ⇔ john wird von michael umgebracht
john is betrayed by helen and diane ⇔ john wird von helen und diane verraten
john betrays diane and michael ⇔ john verraet diane und michael
michael is betrayed by helen and john ⇔ michael wird von helen und john verraten

Test set used in the English-German corpus 402 sentences in total:

michael loves john and helen ⇔ michael liebt john und helen
michael is killed by helen ⇔ michael wird von helen umgebracht
michael and diane hit helen ⇔ michael und diane schlagen helen
diane loves michael and john ⇔ diane liebt michael und john
helen and john are killed by diane ⇔ helen und john werden von diane umgebracht
helen and michael kill diane ⇔ helen und michael bringen diane um
john and helen are betrayed by michael ⇔ john und helen werden von michael verraten
michael betrays helen and john ⇔ michael verraet helen und john
michael and diane kill john ⇔ michael und diane bringen john um
john and michael love helen ⇔ john und michael lieben helen
diane kills john and helen ⇔ diane bringt john und helen um
michael betrays diane and john ⇔ michael verraet diane und john
helen betrays john and michael ⇔ helen verraet john und michael
diane is betrayed by helen and john ⇔ diane wird von helen und john verraten
helen and diane hit john ⇔ helen und diane schlagen john
diane is loved by john ⇔ diane wird von john geliebt
diane and michael kill john ⇔ diane und michael bringen john um
diane hits john ⇔ diane schlaegt john
john and diane betray michael ⇔ john und diane verraten michael
michael and diane betray john ⇔ michael und diane verraten john
diane and michael betray john ⇔ diane und michael verraten john
john loves michael ⇔ john liebt michael
michael and john betray diane ⇔ michael und john verraten diane
michael and helen are betrayed by john ⇔ michael und helen werden von john verraten
helen is loved by diane and john ⇔ helen wird von diane und john geliebt
michael and diane are killed by john ⇔ michael und diane werden von john umgebracht
john and michael love diane ⇔ john und michael lieben diane
diane and michael are loved by john ⇔ diane und michael werden von john geliebt
diane and john love helen ⇔ diane und john lieben helen
diane is betrayed by helen ⇔ diane wird von helen verraten
diane and michael love helen ⇔ diane und michael lieben helen
diane is killed by john ⇔ diane wird von john umgebracht
michael is loved by helen ⇔ michael wird von helen geliebt
john and helen love michael ⇔ john und helen lieben michael
helen is killed by michael and diane ⇔ helen wird von michael und diane umgebracht
helen and diane are hit by michael ⇔ helen und diane werden von michael geschlagen
michael and diane hit john ⇔ michael und diane schlagen john
john is killed by diane ⇔ john wird von diane umgebracht
diane betrays helen and john ⇔ diane verraet helen und john

michael kills helen ⇔ michael bringt helen um
michael kills helen and john ⇔ michael bringt helen und john um
michael betrays helen and diane ⇔ michael verraet helen und diane
helen is betrayed by michael and john ⇔ helen wird von michael und john verraten
john kills michael and helen ⇔ john bringt michael und helen um
michael is killed by john and diane ⇔ michael wird von john und diane umgebracht
helen is loved by diane ⇔ helen wird von diane geliebt
helen and john are loved by michael ⇔ helen und john werden von michael geliebt
michael and helen love john ⇔ michael und helen lieben john
helen is killed by john ⇔ helen wird von john umgebracht
helen and diane are betrayed by john ⇔ helen und diane werden von john verraten
diane and helen are hit by michael ⇔ diane und helen werden von michael geschlagen
michael is hit by helen and john ⇔ michael wird von helen und john geschlagen
michael kills john ⇔ michael bringt john um
helen hits john and michael ⇔ helen schlaegt john und michael
john betrays michael ⇔ john verraet michael
michael betrays john and helen ⇔ michael verraet john und helen
diane hits michael ⇔ diane schlaegt michael
diane and helen betray john ⇔ diane und helen verraten john
michael loves helen and john ⇔ michael liebt helen und john
diane and john are loved by michael ⇔ diane und john werden von michael geliebt
helen hits diane ⇔ helen schlaegt diane
michael and diane are betrayed by john ⇔ michael und diane werden von john verraten
helen hits michael ⇔ helen schlaegt michael
diane is loved by michael and helen ⇔ diane wird von michael und helen geliebt
michael is hit by john ⇔ michael wird von john geschlagen
michael kills helen and diane ⇔ michael bringt helen und diane um
helen kills michael ⇔ helen bringt michael um
michael and john are hit by diane ⇔ michael und john werden von diane geschlagen
john hits helen and diane ⇔ john schlaegt helen und diane
diane is killed by michael and john ⇔ diane wird von michael und john umgebracht
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