Abraham Wald

Abraham Wald – A statistician as a key figure for modern econometrics

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Abstract

Today Abraham Wald (AW) is almost forgotten. At most in probability theory he is cited by name with the term 'Wald's equation' (also known as 'Wald's identity' [Janssen, Manca (2006)] or 'Wald's lemma' [Bruss, Robertson (1991)]). The field of the Sequential Analysis, in a wide range the heritage of AW (Ch. 5), regained a resurrection during the last years, especially in medicine and clinical psychology. [Krohne, Hock (2007)] This fact at first attracted our attention to AW and was the starting point to engage ourselves with his life and work. But in the course of our thesis a new aspect became visible. AW within the history of science is much more than a footnote in statistics. His main relevance lies in another field: the econometrics. We examined the overseen relevancy of AW as a founding father of econometrics and a key figure and trigger for Nobel Price laureates like Trygve Haavelmo. This role came into daylight, when all necessary 'layers' of his personnel path of life (the historical background, the political incidents, his social context and connections, and last but not least all the scientific work he had achieved) were brought together such, that the stations he had passed through during his formative years and what he had transformed into new scientific realms got traceable and coherent.

To classifiy AW's econometric achievements we also had to work out a basic blueprint for the history of mathematical economics, its momentums and roots (Ch. 6). In the history of science we see in the last years several attempts to

draft a consistent ‘History of econometrics’, a undertaking that not yet is finished and still needs more research to be done. During the 1930s, the technology of mathematical modeling (from measurement methods to full statistical approaches) was introduced, a process having been matured with its adolescence for the next 20 years. An important part of economics step by step became reshaped into a tool-based discipline. The coincidence of the upcoming econometrics within the 1930s did not happen by accident. The world economy at that time was seriously malfunctioning. Consequently many economists saw the need to develop a complete new approach, but did not have the right methods at hand. (Ch. 4)

Economic weakness after WWI and the hyper-inflation in Austria in the 1920s not only affected everyones life but desperately asked for an alternative economic policy. The liberals around von Mises (the secretary of the Chamber of Commerce) lost their momentum and O. Morgenstern (following v. Mises) became the director of the Institute for Business Cycle Research (Ch. 4). It was AW, who trained Morgenstern (“in fact no mathematician, but for mathematicians very interesting.” [Wiener Zeitung (2008)]) and helped him to change radically the course of the Institutes research into a mathematically controlled discipline that were acceptable by the fascists as this was a nonpolitical and technocratic ansatz. Morgenstern never had spoken about AW’s role. Analysing Morgenstern’s diary and correspondence (OM Archive, Durham) we describe the unknown role of AW in that process. The later collaboration of Morgenstern and John von Neuman would never been have possible without AW. In Chs. 3 and 4 we analyse and describe Morgenstern’s role for AW’s progression and the interdependences between both.

The jumping-off point for AW was Karl Menger, his mentor and teacher. Karl Menger, son of the famous Carl Menger, the founder of the Austrian School of

Economics, not only boosted the 'mathematical investigation of economics' in his 'Colloquium', but also was the gate for AW, the poor eastern Jew, to bankers like Karl Schlesinger (who got also trainings from AW in mathematics) and O. Morgenstern (Chs. 2 and 4). AW in his desperate situation earned only little money to have a living in interwar Vienna and the life-threatening situation for a Jew like him was worsening daily. The development of the independent Austrian fascism (a fact that is accepted by most of the historians today) is shown in Ch. 4., where we especially investigate the university's tipping into the bad mind and AW's positioning (he indeed was very unpolitical, but more by fear of his life). It was the Cowles Commission that saved AW's life in the end. And especially it were AW's papers in mathematical economics that sparked Cowles interest in him. (Ch. 4)

Ch. 6 analyzes besides the classification of AW in a sketched history of econometrics, AW's different periods in econometric work and also his impact on Ragnar Frisch and Trygve Haavelmo. It can be shown, that Haavelmo was highly influenced by AW, a fact that let him develop an alternative framework for econometrics instead of his former confluence analysis. In the US AW had started his career as a professor for economics and laid with students like Milton Sobel or Jacob Wolfowitz the groundwork of a decade-ranging predominance of the US in econometric research.

The whole text shows AW's rising in mathematical economics that began in the 1930s in Vienna, blossoming up into a very productive period during the 1940s and mainly the beginning 1950s after his emigration to the US. AW's work and influence on people like Oskar Morgenstern and Trygve Haavelmo makes him a first class protagonist who laid ground and path for Nobel Prize Winners and men who produced well-known classics in mathematical economics. Following the uneasy circumstances of AW who had not a whiff of a chance to succeed in old Vienna and also got deeply into the harsh times of an tumultuous era in Europe's history between the wars, we trace his constant personal fate towards mathematical economics that turned out to be a godsend in science. A man of his talents and mathematical abilities was able to form out a new kind of science, of which the 1930s already were partially pregnant: the econometrics.

To undertake all mentioned research it was necessary to visit the US for the Karl Menger Archive (KM Archive, cp. Appendix H), the Morgenstern Archive (OM Archive) and the private collection of Prof. Robert Wald, the son of AW, in 5514 South Woodlawn Ave., Hyde park, Chicago (Appendix H).

The first archive we screened was the Karl Menger Archive in Durham in Oct. 2007. The KM Archive is dated back to 1995 when it came as a gift of the 'David M. Rubenstein Rare Book & Manuscript Library' to Duke University. It incorporates 29,500 items and makes a length of 40.7 feet. In 1996 an initial processing was undertaken by Keary Warne, but not until 2008 an update was made by Meghan Lyon [Duke Library (2012)]. On the basis of the available screening the relief was screened again by us for documents and letters that indicate any communication with or about AW (see the inventory list in Appendix H). Until today the collection is unprocessed. Materials may not have been ordered and described beyond their original condition.

The papers of Oskar Morgenstern span the years 1927-1979. There is correspondence, the diaries, subject files, printed material, audiovisual material, manuscript and printed writings and their supporting papers. The collection principally was screened for notes in diaries and other documents for AW using searchable subject entries in the online catalogue. The whole collection is well screened since 2001. Performing an online search on these subjects brought up other related research materials. This was as well done by us in Oct. 2007.

The Robert Wald Collection consists of 2 briefcases (files) as well as a separate envelope. Everything together has an extent of about 0.15 meter. It consists thereby of approximate 10 photos, official documents (a passport, a naturalization document, honors etc.), correspondences, manuscripts, newspaper cutaways as well as special editions dating of about 1934 to 1950. The material is handwritten, machine written and printed in German, Hungarian, Hebrew, French and mainly the English language. The relevant part for this work was copied, the remaining part is only outlined in Appendix H. We visited R. Wald in Oct. 2007.

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I am also pretty grateful to Mrs. Sassenberg, a great historian, who helped me in archive work to find many of AW's letters and moreover was a wonderful discussion partner for me in order not to loose the quintessential points in the subject. Finally, I owe special thanks to my partner Sandra for inspiring conversations and great support of various forms, for reading drafts and providing helpful comments. I will thank Albert, who taught me that working time is a scarce good and has to be used efficiently, and last but not least a thank-you goes to my mother for motivational words when I needed a fresh momentum to go forward. A special thought goes to my late father, to whom i want to dedicate my work. Finally I would like to thank all my friends who encouraged me throughout the last years.

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1. Introduction

1.1 It is time to write about AW

Abraham Wald (in the follwing abbreviated with AW) as a scientist, even more as person, with his individual story, is almost forgotten, although every student of statistics surely knows to explain in theory what the term 'Wald's Equation' means (cp. [Wald 1947], [Wald (1944) a,]), or surely has ever heard of the 'Wald Test' in lectures. All the rest about AW is likely unknown. I remember the day back in 2006 when I introduced my doctoral theme to my professor, who immediately knew much about the Vienna circles and of course most of the protagonists like Karl Menger or Oskar Morgenstern, but had also little knowledge about AW. That fact additionally encouraged me to digg further. The other point of interest for me was that a friendly psychologist in 2005 had mentioned AW's methods (i.e. the sequential analysis, see Ch. 5) and their current rediscorvery in disciplines like medicine and clinical psychology. [Krohne, Hock (2007)] But these achievements of AW are less the overseen marking point in AW's contributions. The more I dug in his personnel evolution, another role came into daylight: it is his catalytic influence on surrounding people not only in respect to his innovativeness but also his ability to bridge deep into other scientific fields, bringing in approaches and the exactness of mathematics. So AW, the seemingly unimportant figure in a corner of the large business of science, has to be revaluated. To disprove the undervaluation of a man like him I wrote this book.

The text highlights a series of AW's important contributions of his extensive scientific life and work. The main subjects, where he earned most of his fame, are outlined in detail: Statistical Decision Theory and Sequential Analysis. Although we intend to outline the eventful and from historical upheavals so hard affected live of AW that mainly took place in Austria and in its final culmination in the US, we lay emphasis on important parts of his work especially in econometrics and related fields what for the most part is overseen. AW's main contribution to statistics is his theory on sequential sampling. He made numerous advances in many other fields of statistics: asymptotic maximum likelihood theory, nonparametric statistics, tolerance intervals, optimal experimental designs, discriminance analysis, statistical quality control, random walks, the problem of incidental parameters, linear models with errors in the variables, and many more. "He often enough gave decisive impulses or even originating new directions of research."[Schneeweiss (2005)]

AW might have become an important geometer if he had more luck to find a position at the university. But he had the wrong background to succeeed in Vienna. So he became at first a private lecturer in mathematics to Karl Schlesinger, a "banker with great interest in the mathematical foundations of economics". [Schneeweiss (2005)] At that time AW learned about the Walrasian equilibrium in economics. In such an equilibrium, demand and supply have to match, leading to a system of equations for unknowns of the amount of the commodities traded and its prices. AW was the first to present sufficient conditions for the existence of a unique solution with nonnegative prices. This early work in the middle of the 1930s was later taken up by economists in the beginning 1950s, leading to that what is called today econometrics. Nobel laureate G. Debreu later in 1983 acknowledged AW's work in his Nobel Lecture speech.

AW worked tirelessly producing paper after paper in a wide range of different scientific fields. That alone is not the impressive part, but it is the fact that his findings were often outstanding innovative. Looking more precisely on AW's work, it shows up that besides his pure mathematical research he (not only for financial reasons) came step by step deeper into economics. We examined this variety more exactly and are at the point to say that he not only carried new mathematical results into daylight, but evenly shaped the face of its application fields like economics. He did this not only through his own work but also strongly by his role as a mathematical trainer of O. Morgenstern to shift him within a few years into the mathematical maturation to be ready for a fertile cooperation with John v. Neumann (one that culminated in the famous book 'The Theory of Games and economic Behaviour' [Neumann (1944)]). All this is simply not conceivable without the catalytic work of AW. He built Morgenstern up (due to own financial shortages), without ever getting a public acknowledgment from him. Nevertheless Morgenstern managed to win a small scholarship for AW. But here we see a not seldom occurring moment in the history of science: people, who rest on others shoulders, are celebrated, but fail to mention their inspirators. Already here we mention one thing AW was lacking pretty hard: the ability to play to the gallery. This is a consequence of his ancestry, but also due to his decent personality. Not to forget to mention Trygve Haavelmo, the Nobel Prize winner of 1989, who significantly got impacted by AW's work for own research on the probability theory foundations of econometrics.

1.2 Vienna: AW's soil and origin to start in econometrics

From the perspective of economics (and also many other fields) interwar Vienna was one of the most vibrant communities in modern European history. Besides AW it was home to Ludwig von Mises, Friedrich von Hayek, Hans Mayer, Gottfried Haberler, Fritz Machlup, Oskar Morgenstern and Karl Menger. This community began to flourish with the end of WWI, lasting productive until the early 1930s, when the suffering, a result of growing political instability and rising anti-semitism, started. With the 'Anschluss' in March 1938, the group finally collapsed completely, never having a chance to recover. Drawing on the personal papers of AW, Oskar Morgenstern and Karl Menger we try to portrait that community, to chronicle its evolution and its dramatic ending. Attention is paid to the circles around Menger and Morgenstern. The latter collaborated especially with AW in mathematics, to become years later the scientific 'partner' of John von Neumann, culminating in the 'Theory of Games and Economic Behaviour'. [Neumann, Morgenstern (1944)] Our account shows conflicts and tensions during the lifetime of that Vienna community around the discipline of economics.

It were Carl Menger's 1871 publication 'Grundsätze' [Menger (1871)] that stamped a landmark into the wide field of economic studies. The 'Grundsätze' themselves had an impact on most of the following developments in economics in the late 19th- and early 20th century, going on into the times between the wars. In addition with his later published ‘Untersuchungen’, C. Menger can rightly be regarded as a founding father of Viennese economics. The following generation, led by v. Mises, willingly carried forward his legacy. All that is known in the history of science, but less obvious is the path that economics went through C. Mengers son K. Menger, whose students, first of all AW, let O. Morgenstern more and more grew in his role as an antipode to L. v. Mises, by getting higher mathematics involved. AW as often played the loyal catalyst, but actually stayed unseen as a brilliant innovator. An also not obvious point is the choice of the key event which marks the closing of all Viennese circle activities. One could choose 1934, when the intellectual leader L. von Mises finally left Vienna for Geneva. An official version could account for 1938, when the ‘Anschluss’ came with the German troops. It is clear that the Vienna discourse and conversation moved abroad, but of course not in their hitherto well-tried circles.

AW was not integrated into the intellectual life in Vienna of the early 20th century, although it was pretty lively. Of course not as a harmonic atmosphere of cooperation (far from such a naive belief) but so, that the academics mutually effected one another for intellectual stimulus. In consequence of the curiosity of those individuals many of the groups overlapped, as the participants visited several at a time. So some of the gatherings went on in one particular discipline and others interdisciplinaryly spanned several fields. The bright side of a cultivated elite publicly being present on one hand was closely coupled with a dark side of a pervasive feeling of anxiety. Fear about Austria’s political and economic forthcoming. Especially AW was such to speak an inhabitant of the dark side, who always snatched the glimpse of the light that surrounded established friends and colleages from the old Viennese families, but never could change allegiance in any way. To give a decent support, Karl Menger in the mid-1930s twice raised a sum of money for his impecunious students and organised for them a short series of public lectures. Menger could do so knowing, that the Viennese really would pay to come and hear AW and others speak. He remembers: "*Many members of the legal, financial, and business world: publishers and journalists, physicians and engineers took intense interest in the work of scholars of various kinds. They created an intellectual atmosphere which, I have always felt, few cities enjoyed*.". [Menger (1994), p9]

We show that the climacteric poles for AW in the 1930's were the Austrian Institute for Business Cycle Research, managed by Oskar Morgenstern, and to the same degree Karl Mengers Mathematical Colloquium. The Austrian Institute for Business Cycle Research conducted applied and theoretical economic research, while Menger's Colloquium, although devoted primarily to mathematics, especially towards the mid-1930's, gave attention to economic questions and theory. To all of this might be added an Otto Neurath from the Vienna Circle of Moritz Schlick, not least because of his interests in economic organization and planning.

To put out an additional thought we came to within all the storydigging about AW and his complex circumstances, it is the necessity that historians dealing with WWI and its political turbulences should gaze away from the 'western front' view in the other direction to become aware of the eastern countries problems, its refugees, its political surveillance, bereavement and collective identities of all ethnic groups there. As a second insight we simply realize the fact that, especially exemplified through AW, a too fast classification of a scientist into one scientific field can be crucial as important pathes aside, leading over to other persons and fields of science, get easily lost.

1.3 O. Morgenstern's role to understand AW's progression

A linchpin for our story is the outstanding race of men like Oskar Morgenstern. His life and career are specific in a way from which we are able to understand the background of AW. Morgenstern started his career in economics in the mid-1920s and only 10 years later was able to monopolize the direction of the Austrian 'Institut für Konjunkturforschung'. In the end he was one of the last who started a life in exile. Morgenstern was a junction within the Viennese economists scene and moreover had excellent connections all over the world. Besides he cultivated important alliances, including K. Menger's group (and especially AW). His drive came on th ehand hand from his will to make a notable career with innovative methods in economics combined with his dissatisfaction with the Austrian School and the Austrianism itself. At the same time he was a member and also a heavy critic of the Vienna's economists (cp. [Morgenstern (1934)], where Morgenstern describes his view on the relationship between economics and politics). Morgenstern was a querulant and an outlier. But he had enough self-confidence to emphasize his differences with von Mises, Mayer and Hayek. So he tried to be an outlier in economical methodology. Without AW this would be unimaginable. AW was Morgenstern's secret teacher, his mastermind, his tutor. But this is visible only in Morgenstern's diary. He never has spoken about this. Morgenstern was vain and AW was an unimportant and a poor student of K. Menger. The later collaboration of Morgenstern and John von Neuman would never have been possible without the training with AW. AWin fact is the missing link.

AW trained Morgenstern in basic mathematics, especially in differential calculus and algebra. His impact on Morgenstern may not be underestimated, as the latter made many records about AW and his work with him in his diaries: "*Another mathematics lesson, very interesting. I feel as though I am making* *real progress. Wald told me of his new works. An amazing thing. It isn't enough, as Walras assumed, to consider only monotonically decreasing utility functions, because he [Wald] proved that with many of them, simple exchanges never lead to an equilibrium! Similar paradoxes for the addition of demand curves, which were considered before to be totally harmless! That should have far-reaching consequences*." (Oskar Morgenstern, Diary, Nov. 2, 1935, Duke Univ., OM Archive). By the end of 1935, AW assured Morgenstern that he soon would be able to figure out almost every topic in mathematical economics.

The Austrian School of Economics got disparaged after 1934. Only the remaining protagonists like Morgenstern exhibited a friendly attitude towards the fascist state. Morgenstern was not shy of addressing the idea of a 'strong state', that powerfully regulated the economy. [Morgenstern (1934), pp129-130] [1] He regarded his challenge to realize a 'technocratic' ansatz to intervene into economy. The former liberal gave up a more differentiated and critical approach and saw the advantage of a controlled economic policy that he tried to influence. Morgenstern's emancipation from von Mises' dominance and the views of the Austrian liberals went forward in form of a combination of harsh criticism and attempts to find revolutionary alternatives. But his attempts at first were poorly conceived and occasionally confusing. [2] Here the work of AW became highly important for him, as AW offered with his exactness the ideal (nonpolitical) instrument in developing an mathematical econometric.

1.4 The much to early end

After the long lasting difficult times in Austria *AW* finally succeeded after his emigration to attain a prominent position among American mathematics and statisticians total in contrary to his teacher Karl Menger, whose career definitively lost momentum in the US. Within a few years AW became a dominant figure in the US statistics. After his emigration to the US, AW set more focus on statistical and economical problems. He used the methods of his former work in statistics to solve problems in economics. He got numerous impulses from his knowledge of geometry, topology and set theory, which showed to be helpful and of an innovative influence. Not seldom probabilistic questions are treated in a geometric or set-theoretic way. Invited as a meanwhile famous teacher of statistics, AW visited India for a lecture tour in the late 1950s. He and his wife died on December 13 when their small airplane crashed through a fog bank into the Xkidri mountains (south India). [Morgenstern (1951)] With the words of L. A. Steen [Steen (1978)]: "*Despite his short career, he was a prolific contributor to statistical theory. In two cases, he created new subjects within statistics. One of these is sequential analysis, the other his general decision theory. Wald showed that the idea of seeking optimal methods for making decisions could be used to give a general framework for statistical problems and he carried out a profound study of those procedures that are acceptable for use in a particular problem.*" This last travel of AW happened on an invitation of the Indian Government, which had arranged an extensive lecture course for him. "*Thus, suddenly, in a remote and wild corner of a distant land, ended one of the most brilliant careers in the social sciences*", O. Morgenstern. [Morgenstern (1951)]

2

2. Formative Years

Wald is really intelligent. I consider these works to be very important; they throw new light on the application of mathematics to economics. One will not be able to do without these at all.

—Oskar Morgenstern, Diary, Nov. 2, 1935, Duke Univ., OM Archive

It was Abraham Wald, starting from Gustav Cassel's 1918 formulation of the Walrasian model, who eventually in Vienna in 1935-36 provided the first solution in a series of papers that attracted so little attention that the problem was not attacked again until the early fifties.

—G. Debreu, Nobel laureate in economics [debr1983]

I have no doubt that he is one of the very ablest of the men working upon problems of statistical technique as applied to business cycle analysis.

—T. Kittredge of the Rockefeller Foundation after interviewing AW in 1936 [3]

2.1 The young man's world

AW was born on October 31, 1902, in Klausenburg, Siebenbürgen (Transylvania), then belonging to the Austro-Hungarian Empire. After WWI, the district of Siebenbürgen fell to Romania and the town Klausenburg changed its name to Cluj (since 1974 Cluj-Napoca). AW was raised in a multicultural and multilingual atmosphere combining Hungarian und German Jewish tradition and culture. He spoke Hungarian, and "... *never developed any affinity for Romania, nor did he know the language of that country*." [Morgenstern (1951)]. Although his father was an orthodox Jewish baker, a small-scale businessman, there was a strong affinity for intellectualism in AW's family. This certainly leads back to his grandfather, a famous rabbi [Wolfowitz (1952)]. AW, as common, grew up in an extended familiy. As an orthodox Jew, AW could not attend school on Saturdays, the day of the Jewish Sabbath, as was required at that time by the Hungarian school system. So he was privately educated (i.e. home-schooled "*mainly under the direction of his elder brother Martin*" [Menger (1952)], p1, and by his parents, as they were themselves "... *quite knowledgeable and competent as teachers*" [Morgenstern (1951)]).

However, he went to the gymnasium to obtain the matura at the Roman Catholic Gymnasium in Cluj in 1921 [4], "*... an examination that was recognized by the University of Vienna, where he registered* [in 1927] *to study mathematics*." [Morgenstern (1951)]. Before his admittance to the Vienna University, AW had visited the University of Cluj from 1922 to 1927, where he had graduated as Licentiate in Mathematics [5]. "*After graduation from this local university he experienced considerable difficulties in entering the University of Vienna because of religious restrictions. He spent a year in the engineering school at Vienna, but finally was admitted to study mathematics at the University of Vienna*" [Wolfowitz (1952)]. AW had five siblings. His brother Martin, equally considered as intellectually gifted, became an electrical engineer and made numerous inventions [6]. AW later "*rendered mathematical help to his brother in a few of the latter's researches*." [Wolfowitz (1952)]. AW's sisters, his brother Martin, their spouses and children, his parents and other relatives, all died in the Nationalsocialist concentration camps. Only Hermann, a bit elder than AW, found a way to survive and started a new live in the United States.

It was the Treaty of Trianon (ratified on June 4, 1920) that forced Hungary to forgo its claims (dating from the Austro-Hungarian Monarchy) over Transylvania. Transylvania hence fell to the Kingdom of Romania, in consequence of Romania's reentering into war and systematic occupation of Transylvania (all in accordance to local leaders there, who themselves actively had arranged that unification over the years). The so enlarged Romania finally got renamed to 'Greater Romania'. By exactly the same treaty Bucovina and Bessarabia were incorporated into Romania, a fact that justifies its labeling with 'Greater Romania'. Without further ado AW became a Romanian citizen, resident in Cluj (the then renamed Klausenburg). All in, through these events the Kingdom of Romania was indeed a victor of WWI. With the declaration of independence by Czechoslovakia and Hungary after WWI, the breakup of the former Austro-Hungarian Empire started. The Balkan territories of Slovenia, Croatia, and Bosnia fell to Yugoslavia. Galicia became a part of the anew independent Poland.

2.1.1 The dawn of the Empire

1867 marks the final year of the independency revolution (beginning in 1848) of Hungarian and Bohemian (Czech) nationalists against Austrian rule. The Austrian military, supported by the Russians, succeeded in getting control over the Magyars. With the Settlement of 1867 [7] (the 'Ausgleich' or Austro-Hungarian Compromise of 1867), the alliance of Austria and Hungary saw the light of day. The Austro-Hungarian Empire, also called Austria-Hungary, Dual Monarchy, Dual State, the 'imperial and royal' Monarchy ('k.u.k.-Monarchie') or unofficially but popularly called Danubian Monarchy ('Donaumonarchie') ranged historically from 1867 until 1918. Geographically, the constitutional monarchic union constituted the second largest country in Europe (beaten by the Russian Empire) spanning a territory of 415.000 miles ^{2}. With a whole population of over 50 million people Austria-Hungary comes in third (concerning the number of inhabitants) after Russia and the German Empire. [Sked (1989)] The German-speaking and the Magyar [8]population formed the largest linguistic parts (each about nearly 10 million people). The rest of the 50 million inhabitants were Poles, Serbs, Italians, Czechs, Slovaks, Romanians, Ruthenians, Croats, Slovenes, and some smaller ethnic groups [Kuprian (1997)]. National ambitions of course did not stop with that conglomerate. The multi-ethnic and multi-religious (there were Roman Catholics, Protestants, eastern Orthodoxs, Jews and Muslims) Empire always battled with disputes among its national groups. Only partly a modernisation of the economic processes in the different areas took place. The state was forked into different social spheres (cp. chapter 4, the two Austria's): into urban spots, that underwent a rapid industrialisation, but also into peasant structures, that showed to be immoveable until the beginning of the 20th century. In the beginning, democratic reforms and liberal ideas found its way into politics, but towards the end of the 19th century, a rising of ideologies (e.g. socialism) and a new inflammation of nationalism brought classical liberal ideals to a halt. The shout for national autonomy got louder. Each group longed for economic privileges (at the expense of the others). This groupwise political and economic collectivism was the centrifugal force that gradually tried to crack the Empire into pieces.

The emperors of Austria-Hungary was the Habsburg dynasty. It ruled as Emperors of Austria and as Kings of Hungary. Under the governance of the Habsburgs Hungary indeed relished to a certain part self-government and self-representation, mostly in foreign relations or defence. The Empire bore as full name 'The Kingdoms and Lands Represented in the Imperial Council and the Lands of the Crown of St. Stephen'. The capital of the state was Vienna. Regarding the structure of the government we see it organized in three different parts: the Hungarian ('Transleithanian') government, the Austrian (also called 'Cisleithanian') government, and a unified administration under the monarch. That means separate parliaments, one for Hungary and one for Austria, each with an own prime minister. The February Patent of 1861 [9] constitutes the foundation of the Austrian government. It defined a bicameral system, the 'Reichsrat', consisting of an upper chamber (appointed by the Emperor) and a lower chamber (its members were elected indirectly). In Hungary the March Laws of 1848 [10] built the basis for its government. The common monarch was Franz Josef I (crowned Emperor of Austria in 1848 at age of 18 and crowned

King of Hungary in 1867). His regency lasted until his death in 1916, aging 86. Both governments needed a common coordination. For that purpose a ‘gemeinsamer Ministerrat‘ (common council of Ministers) was created. Its power was limited in practice. The common council of Ministers was responsible for the army, the navy, foreign policy, and of course for the union. The joint cabinet was controlled foremost by Austrian and Hungarian aristocrats [Okey (2001)] [11].

It is obvious that the Magyar aristocracy pursued a policy of Magyarization. They instigated minorities like Croats and Serbs to assimilate, i.e. to adopt the Hungarian language and other Hungarian manners. This policy also concerned the Jews. Their ambitions for independence provoked repression on either side the Emperor and the common government [12]. But the repressions only fired the growth of pan-Slavic terrorist groups like the ‘Black Hand’, which in 1914 was responsible for the assassination of the archduke Franz Ferdinand [Rauch 1997]. The smoldering conflict of Austria-Hungary with the Serbs and therefore Serbia finally triggered WWI. The Serbians, supported by Russia, having an old rivalry with Austria dating back to the Crimean War, mobilized in early summer of 1914. In response, Austria-Hungary declared war on July 28, 1914. On July 31 a full Russian mobilization began. The ‘*Schlieffen* Plan’ [Ehlert (2006)], which intended a quick strike against France, forced the Germans to declare war against Russia on 1 August and two days later against France. When Germany violated Belgium's neutrality by its march to Paris, the British Empire went into war. Five of the six European powers had been involved into the largest continental European conflict since Napoleon in 1914.

2.1.2 November 1918, key events

Karl Franz Josef (anglicised ‘Charles’) von Habsburg-Lothringen (August 17, 1887 - April 1, 1922) was the last monarch of the Habsburg dynasty. [13] He reigned as Karl I, last Emperor of Austria and also last King of Hungary and Bohemia (named Károly IV) from November 1916 until 1918, the year when he finally abstained from any participance in state affairs, without to abdicate. Karl became heir-presumptive in the very moment when his uncle, *Franz Ferdinand*, was assassinated. *Karl's* enthronement automatically made his eldest son *Otto von Habsburg*, who today still lives in Bavaria, to the Crown Prince of Austria, Hungary and Bohemia. But in 1917, Karl secretly negotiated with France by his brother-in-law, Prince Sixtus of Bourbon-Parma, an officer in the Belgian army. Karl foiled the negotiations of his foreign minister, *Ottokar Czernin*, when he offered a separate peace to France, ignoring Czernin who aimed at a general peace including Germany as well. In April 1918 Karls activities had been revealed [14], what led to Czernin's resignation as Austria-Hungary had lost a positive negotiating position [Okey (2001)]. On November 11, 1918, the Empire collapsed. Karl was forced to issue a proclamation [15] concerning his retreat from all state affairs. Two days later, he issued a similar proclamation as King of Hungary. Even he did not abdicate [Gombás (2002)], this sealed the end of the Habsburg dynasty. [16]

On November 12, 1918, German Austria was officially announced a republic. The preceding act was that ethnic German deputies of the ‘Cisleithanian’ parliament (including representatives from Bohemia, Moravia, and Austrian Silesia) had declared themselves ‘provisional national assembly of the independent German Austrian state’ of the new rump state ’German Austria’. *Karl Renner* was chosen the new chancellor of German Austria. In 1919 the new government enacted a law that banished the Habsburgs from Austria. Not before they would forsake the throne and accept to be common citizens, a return would be impossible. *Karl I* indeed tried several times to recuperate the throne of Hungary, until in 1921 the Hungarian government decided to enact a law that annuled *Karl's* rights. Initial *Otto von Habsburg* (still the head of the house of Habsburg today) renounced all claims to the throne. The assembly of German Austria quickly drafted a constitution that defined "*German Austria ... a democratic republic*" (Article 1) and "*German Austria ... a component of the German Republic*" (Article 2). On 22 November 1918, the national assembly claimed for all ethnic German areas of Cisleithania, but was to powerless to put through this against Italy, Czechoslovakia, and the Kingdom of Serbs. German Austria in the end encompassed the current area of the Republic of Austria, the province of Bolzano-Bozen, the town of Tarvisio, southern Carinthia, southern Styria, Sudetenland and German Bohemia. German Austria seized about 118 311 km^2, within 10.4 million people. [Neck (1968)]

On November 16, 1918, the 'Hungarian Democratic Republic' was officially proclaimed. Its president was *Mihály Károlyi*. He hoped to reform the new Hungary following *Wilson's* principles. [17] His government intented to establish ethnic autonomy and land reforms. But Karolyi was to weak to reanimate the economy, he even failed to keep Hungary's interests at the peace conference at a bearable level. Hence the Trianon Treaty (June 4, 1920), was extremely bitter for Hungary. [Kovacs-Bertrand (1997)] Károlyi resigned. In the end the Croatians took Croatia, Slavonia and Dalmatia. The Serbs took Voyvodina and the Banat became a part of Yugoslavia. Romanian troops captured Transylvania. 71% of Hungary's land and 63% of its population were lost. [Craig (1966)] Károlyi was followed by *Béla Kun,* a former journalist of the pre-war socialist party, who created a kind of Bolshevik movement in Hungary. In March 1919 Kun declared Hungary a Soviet Republic (the 'Hungarian

Soviet Republic'). [Pastor (1976)] Kun combined communism with nationalism and was accepted as long his armed forces fended the Slovaks and Romanians. In June 1919, Kun's opponents acted from exile and succeeded to make deals with the Romanian army. They initiated a rival cabinet. [Mac Cartney (1937)] Kun left the country, Romanian troops occupied Budapest and ended the Hungarian Soviet Republic after 133 days. [18] A reactionary 'White' government began its bloody work. Over 5,000 people met their death during the days of the 'White Terror' [Vardy (1997)] 70,000 (allegedly suspected) Communists, among them many Jews [19], were arrested. This was the beginning of various military leaderships in the following years. In January 1920 Admiral Miklós Horthy was installed as regent. He ruled until October 16, 1944, as the Treaty of Trianon did not touch Horty's regency. [Grenville (1974)] [20]

The allies did not pay attention to the different national movements within the Empire, in contrary they could proceed undisturbed on their own. [Valiani (1973)] Czech, Polish and Croatian National Councils founded complete new states. [Schulze (1994)] In Czechoslovakia and Poland new regimes were installed. Soon Romania and Serbia announced own claims. A National Council of Slovenes, Croats and Serbs called for the unification of all the South Slavs from the old Empire. Hence with their unification the Yugoslav state came into existence. All this happenend before the actual peace talks had started. The peace conference was a play of different forces: Leninism, Wilson's points [17], old-style diplomatic demands of the European allies and the new national regimes. [Brook-Sheperd (1997)]

2.1.3 Ethnicity and nation, the First Republic

The Paris Peace Conference started in January 1919 and held up for over one year. Each of the defeated nations got its own peace treaty. [22] The defeated states had to accept the results of the negotiations, without to be allowed to take part. Minor allies, like Greece, Serbia or Romania only participated partly in some sessions, but for main decisions Britain, France, Italy and the US were solely responsible. Only Russia was absent. [Steininger (1997)] On September 10, 1919, *Karl Renner*, chancellor of German Austria, signed the Treaty of Saint Germain. The Treaty of Versailles (from June 28, 1919), that covered Germany, and also the Treaty of Saint-Germain explicitly interdicted an unification between Austria and Germany. The allies planned to disempower Germany and categorically tried to avoid its return as a yet larger conglomerate. In such a mind the name 'Republic of German Austria' got shortened to 'Republic of Austria'. Its official name was 'Federal State of Austria'. Article 88 of the Treaty (the so-called 'Anschluss'-prohibition) says that "*the independence of Austria is inalienable otherwise than with the consent of the Council of the League of Nations*." This was the beginning of the First Republic. The Treaty also forced German Austria to give off ethnic German areas of Sudetenland to Czechoslovakia, Tyrol to Italy and southern parts to the later so-called Yugoslavia. The constitution, enacted as 'Österreichische Bundesverfassung' on October 1, 1920, had been revised in 1929. It established de-facto a republic, but did not identify the First Republic as such. [Steininger (1997)] In an answer to ethnic questions the First Republic had to organize divers plebiscites [23], although the treaty partially gave regulations for these claims. [Niederstätter (2007)] [24]

One of Wilson's principles was national self-determination and autonomy. But this implies economic viability and also military defensibility, what often stands in contrast to ethnic principles. We want to sketch this with the following example: when Romania got control of some parts of Transylvania, it won over important railroad lines for a better military and economic transfer within the Romanian territory. The price was that the newly added Magyars got subjugated and fell to the Romanian population. This not only fired Hungarian rejectionism, but also caused severe ethnic questions. [Berger (2007)] In short, the idea of ‘national self-determination’ was unable to create post-1918 Balkan borders that adequately reflected the needs of most of the ethnic groups. Some groups failed to become recognized as own ethnicity (e.g. Slavic-speaking Macedonians and the Bosnian Muslims). Other groups were recognized but faced depressing conditions and a lack of protection of their own culture like the Vlachs, the Gypsies or the Jews.

In chronological views, the First Republic stands for the period from 1919 to 1938, the year when its end came with the ‘Anschluss’ (Annexation) to Nazi Gemany. But the constitution was annihilated with the establishment of the Austro-fascist dictatorship in 1933/34 that followed the Austrian Civil War. This turning point can be seen according to most historians as the real end of the First Republic.

2.2 Jewish life in the Habsburg Empire

The Ashkenazi Jews [25] came to Eastern Europe mainly of persecution. Many of them swarmed in from north and northwest, being descendants of expelled Jews from England, France and the German states dating back to a time span between the 12th and the 13th century. [Goldberg (2001)] A small number of them [26] had lived in Balkan states since the 15th century. Then they had better conditions to live, what encouraged more to follow. Their migration found further areas within Poland and Lithuania. During the early modern period a federal state Poland-Lithuania was founded, that indeed offered almost a kind of religious tolerance. Between 1772 and 1795 Poland-Lithuania got fragmented between Russia, Prussia and Austria. The former tolerance decreased as the Jews got integrated into these states. Under the tsars, Jews only were permitted to settle in certain areas and additionally faced legal and personal chicanery. [Biale (2002)] Influenced by the ideals of Enlightenment Joseph II (March 13, 1741 - Feb. 20, 1790) decreed toleration edicts that noticeable improved the living conditions for Jews within the Habsburg Empire. They were allowed to live in their traditions, moulded by religion, family ties and close local communities. This held for orthodox as well as for liberal Jews. In spite of that the society they lived in still treated them as an alien element. [Haumann (2001)]

The areas that merged to the Kingdom of Romania had the largest Jewish population in the Balkans. Notwithstanding the Romanians never integrated the Jews, and regarded them as nonassimilable. This in combination with political and economic myths (see the section below) was exploitable by nationalist demagogues with ease. When Romania was still a Ottoman vassal state, its Jews enjoyed indeed better conditions from the millet system. [27] [Brook (2003)] As many Jews entered Romania from the south of Russia, especially the northern province of Moldavia developed a central role for Jewish life. [28] Fewer ones lived in the Wallachia. [29] In the 19th century, when Romania shook the Turkish and Phanariot Greek rule, the constitution, renewed in 1878 with Romania's independece, marginally changed the fomer passus of an exclusive natural citizenship only for christian foreigners. The requirements for a citizenship remained pretty hard. Hence in 1899 only 4,000 of Romania's 250,000 Jews had succeeded to become citizens. The government put further obligations on them: non-citizen Jews must not hold an office, they could neither vote nor own land. They were forced into a social and economic life that further accounted for their disparity among the Romanians. In cities the Jews had an easier living in respect to the restrictions by law. [30] Outside the towns Jews mainly could be seen in the merchant sector, they also were active as estate managers by order of landlords. Many Jewish merchants took over the function of the lacking banks, and started to work as credit houses, lending money to peasants. It is clear, that this role bore a huge potential of conflict. [31]

In Hungary the Jewish population rose during the 18th century. This traced back to the Turkish conquest in 1711, when Hungary was completely opened to immigrants and to the 1772 annexation of Galicia by the Empire, where many Polish Jews lived. Now residents of Austria, the Jews from Galicia easily could migrate to Hungary. [32] One century later still more Jews had arrived. About 1850 Jews made up about 4% percent of the whole population. At the turn of the century their percentage was at 8%, i.e. over 800,000 people. We find in Hungary at the beginning of the last century the second largest Jewish population compared to the other Balkan states. [Szalai (2002)] As said above, due to legal, social and economic pressures the cities were the better place to live for the Jews. In 1890 about one quarter of Budapest was Jewish. [33] In 1800 about 10% of Prague’s population (i.e. 8500 people) were Jewish. Their number increased to 11.700 in 1848.

At the end of 1915 the Austrian Ministry of the Interior had an estimation of 385,645 refugees inside the Empire, of whom 157,630 (41%) were Jewish. Vienna had 77,090 Jewish refugees, Bohemia 57,159, Moravia 18,429 and about 30,000 Jewish refugees were located in Hungary. [Rozenblit (2001)] It is not clear how many people fled or were deported from the war-torn areas of the eastern front of WWI. Some historians estimate their numbers to several million. [Gatrell (1999)] Poles, Germans, Ukrainians and Jews massively got displaced during WWI. These years brought the first refugee catastrophe of the 20th century.

2.2.1 Political Manners

It is a known fact that a great majority of Jews living in fin-de-siecle Empire adored their state and its emperorking, Franz-Joseph. The Habsburg Jewry developed an avid loyalty to its monarchy due to two reasons: at first it protected them from anti-Semitism, rampant in important parts of the society, and secondly it allowed them to develop their own kind of identity, the so-called 'tripartite identity', to use Rozenblit's terms [Rozenblit (2001), pp9-12]. This means an identity that combines political loyalty to the state, affiliation with a major culture of the region (predominantly, but not exclusively, the German) and a strong sense of an own Jewish ethnicity. The 'tripartite identity' stands in contrast to the common term of 'national identity'. It stands for a combinatoric identity, partly political, partly cultural and partly ethnic. The Austrian-half of the monarchy was receptive to this tripartite identity since "*it was a political construct not a nation,* [making] *it very easy for the Jews to adumbrate staunch Austrian loyalty without having to adopt any particular national identity*" [Rozenblit (2001), p9]. The loyalty to the Habsburgs of course was not universal within the Jewry. As liberalism appealed to many Jewish intellectuals (driven by the ideas of freedom and equality), these not seldom developed anti-cleric and anti-Habsburg attitudes.

The Hungarian half of the Empire was less receptive to the tripartite identity, and Jews were increasingly pressured to link their identity into a Magyar national identity. [Rozenblit (2001)] [34] They rapidly assimilated to ward off anti-semitic affronts and to show a patriotic mindset. Many Hungarian Jews were active in the Magyar nationalist movement (e.g. there were Jews among Kossuth's supporters in 1848). [Dawidowicz (1967)] (Cp. [Korn (1948)])

With the beginning of WWI the Jewish population of Austria-Hungary mobilized physically, economically and emotionally on behalf of their home state. They remained mobilized until the end of the Empire four years later. [Rozenblit (2001)] Therein M. Rozenblit presents in detail the activities of Viennese Jews on behalf of (Jewish) war refugees.

2.2.2 Economic myths and truth

To regard the economic conditions for Jews it is necessary to remember the year 1848. During the 1848 revolution many Jewish intellectuals entered into revolutionary groups, which mainly recruted from liberal students and nationalists. The ‘Pillersdorf constitution’, ready in April 8, 1848, granted to all inhabitants of the Empire full civil rights and furthermore religious freedom. But the revolution was blasted away and Franz Joseph I became the new Emperor. As a consequence of this much of the new governmental liberalism got abolished, including the rights for Jews: Jewish civil servants had to be inaugurated to give proof of their loyalty to the the state (1851). Additionally Jews were forbidden to own land (since 1853), and also to execute professions like teaching (1855). Hence intellectual Jews started in publishing, a field that they were admitted to. In consequence the public press was split into a part of ‘old order’, that used to be more Habsburg friendly, Catholicistic oriented and absolute monarchistic, and a second part with more progressive representants, who showed secular and republican tendencies, causing the conservative part of the press to develop anti-semitic thoughts and tendencies.

Resentments of Jews often have increased when they became outstanding or in any way prominent in their professional position. This typically goes hand in hand with the myth that most Jews were wealthy (a partial explanation for that can be seen in the fact that most Jews lived in urban areas and the wealthier of them became highly visible in such closed communities). The reality sketches an other picture: most Jews worked in low-payed jobs, like salesmen, clerks or industrial hands [Dawidowicz (1967)]. A similar myth says that most Jews were professionals, like doctors and lawyers. Because Jews were socially (and sometimes legally) banned from other careers in the civil service, the military or the school system run by Catholics, some of them indeed became doctors or lawyers. In Hungary of 1910 the numbers are the following: 55% of the merchants, 42% of the journalists, 45% of the lawyers and 49% of the physicians were Jews. This figures are misleading, as they indicate more about a 'sociology' of for instance the medical profession than about the real social status of the Jews on the whole. When we regard that 2,300 out of the Hungary's 4,800 doctors were Jewish in 1900 and know that these were only 2,300 out of 800,000 Jews, we get another picture. Census figures show large numbers of Jews in business, but those figures do not differentiate between the wealthy ones, who e.g. owned big factories, and the poorer ones, for example holder of small village stores. [Dawidowicz (1967)]

2.2.3 Viennese Jews

Viennas Jewish community was pretty small at the beginning of the 19th century. They numbered up to only 500 or 600 (i.e. a small percentage of 0.25

of the whole population of the town). The Viennese Jewish families were mostly well assimilated and wealthy. Until 1848 their number rose through immigration from within the Empire, particularly from Hungary, Galicia, and the Bukovina, to 4,000 (0.8% of the total population). [Vital (1999)] By 1923, Vienna was housing the third largest Jewish community in Europe. [Botz (2008)], [Beller (1989)] [35] Between 1867 to 1934 Vienna had evolved into a cultural and intellectual centre of the old continent. Whether in fine arts, music, literature, science, medicine, technology, in humanities, economics or philosophy, there was almost no discipline in which Vienna did not harbour outstanding and reputated people. Most of these contributors were at least partly Jewish. Intermarriage between upper-class Jews and well-situated origin Austrians generated a multiplicative effect on the number of outstanding people (who were later claimed to be Jewish). Intermarried Jews gave up on speaking Hebrew or Yiddish and stopped practicing their religion. [Botz (2002)]

But there was also a counter-movement. Based on anti-assimilative habits it helped to emerge a special Jewish culture in Vienna. The traditional Ashkenazi [36] rite was changed to a 'Wiener Ritus' ('Viennese Rite'), which spread all over the Empire. Yiddish lost its importance in Vienna, also partly in Bohemia and Moravia, and was replaced gradually by German. From this tendency can be seen how the names of Jewish children changed from the traditional Hebrew to typical Austrian ones. As a symbol of Jewish integration in 1858 the 'Stadttempel' synagogue of Vienna was built. It is one of the most elaborated synagogues in Europe. [Brugger (2006)]

A prominent example of an intermarried and assimilated Jewry is the Wittgenstein family. Only the father of the philosopher Ludwig Wittgenstein was of Jewish origin, his mother Leopoldine was Catholic (whereby her father was of Jewish descent). Ludwig was baptized according to the Catholic rite.

His burial was also Catholic, although he never in his life practiced this religion. Other parts of his family were liberal Protestants (Ludwig's grandparents had converted and changed their name into the germanized ‘Wittgenstein'). Nevertheless Ludwig was later claimed Jewish. Like other assimilated Jews of Central Europe, L. Wittgenstein may have been directly or indirectly exposed to Hebraic culture and Talmudic logic. [37] Another outstanding figure of partly Jewish origin was Wolfgang Pauli, Karl Menger’s class-mate at ‘Döblinger Gymnasium’ in Vienna. His grandparents originated from prominent Jewish families of Prague, but his father converted from Judaism to Roman Catholicism. His mother was a Catholic. W. Pauli himself was raised as a Catholic, although he left the Church in 1929. [Enz (2002)]

2.2.4 Viennese anti-Semitism, looking backwards

The ‘Heldenplatz’ (Heroes’ Place) in Vienna is a metaphor of Austrian and German history: on March, 15, 1938 Adolf Hitler proclaimed there the Anschluss [38] (annexation) of Austria to Nazi Germany. Seventy years after that, the Austrian parliament invited to a public ceremony. Other events for this historic incident are under way. [39] Among other things, pupils and Catholic youths will be lighting candles at the place of events in rememberance of the victims of the Holocaust. [40] Without doubt the ambivalent place of commemoration [41] is suitable to remind of the crimes and pose questions: e.g. the question for the traditional threads of anti-Semitism. Thomas Bernhard had already asked this question in his 1988 drama ‘Heldenplatz’ [Bernhard (1984)] and by this disunited the cultural and political Austria. At a second reading it becomes obvious: the drama has hardly lost its topicality and therefore it seems likely in these days to take a look at anti-Semitism in Vienna in the 19th and 20th century.

The antagonism against Jews has been known in Europe for two thousand years. Anti-Jewish prejudice and stereotype out of the tradition of the Christian anti-Judaism reach to the present [42]. In the 19th century antagonisms against Jews assumed the 'modern' shape of anti-Semitism, evolved as a reaction to the Jewish emancipation. The term was coined in 1879 by the Hamburg journalist Wilhelm Marr [43] and very quickly moved mainstream in large parts of Europe. In England as well as in France and Austria-Hungary the form of antagonism against Jews that was in theory and practice partly politically motivated, partly religiously, partly economically and partly by racist delusions, found supporters in all levels of the population. Like Imperialism and Colonialism the modern anti-Semitism was in accordance with a European trend, though developing a unique shape against the background of a rich past of Jewish traditions.

For the European Jews Vienna had been a centre of attraction since the High Middle Ages. [44] Sources talk of a first Jewish settlement in 1194 and of an age-long history of eviction (1421 and 1670) and resettlement. With the economic success of the Jewish 'Hoffaktor' (court Jew) in the 17th century the advancement of Austrian Jews began which was flanked by the Toleranzpatent (edict of tolerance) of Joseph II. In the course of the Jewish enlightenment Vienna continuously developed into a centre of Jewish scholarliness and Hebrew literature. In 1852 a Jewish religious community constituted itself there for the first time. The constitution of 1867 that ensured all Austrian Jews of civil equality and freedom of settlement favoured the immigration of Jews to Vienna. Many came from Bohemia and Moravia, later from other parts of Eastern Europe. Their history and culture was documented by the worldwide first Jewish museum that had been opened 1896 in Vienna. At about 1900 the community was established, the 'Stadttempel' of Vienna and the 'Leopoldstädter Tempel' were symbols of Jewish self-esteem in a democratic society (it was completely destroyed during the 'Reichskristallnacht' on November 10, 1938). Until 1938 others were added, a total of 19 synagogues and 63 smaller meeting houses. [45] Jews played a part in the development of Vienna as a centre of European culture in the modern sciences, the arts, literature and music. The census of 1910 documented 175318 Jewish inhabitants in Vienna. With this the Viennese Jews not only accounted for more than half of all Austrian Jews but also for 8.6 percent of the total population of the city. [46] In Europe, only the Jewish community in Warsaw was larger. [47] Viennese Jews predominantly belonged to the upper or lower middle classes: 69.2 percent of the Jewish men were clerks or self-employed in industry, trade and transport, 30.8 percent workers. [48] The public sector had become accessible despite some anti-Semitic barriers and a growing number of young Jews saw their best chances for advancement in this area. [49] According to the English political scientist Peter Pulzer, the specific form of Austrian anti-Semitism differs from anti-Semitism in the other parts of Europe in threefold ways: it began later, was party-politically successful like nowhere else before WWI and remained Christian-conservative throughout. [50] The term 'Wiener Antisemitismus' (Viennese anti-Semitism), a notion that was already familiar at the end of the 19th century, is also a synonym for that. [51] As origin of radical ideas, as a centre of clericalism and a treasure box of diffuse traditionalism, Vienna had created the premises that made it particularly receptive to anti-Semitic ideas. The term "Wiener Antisemitismus" reflects this ambivalence. According to Pulzer, around 1900 public life could not be imagined without anti-Semitism. [52]

Since 1895 the Christian-social Viennese anti-Semites provided a safe coalition on a broad social basis. [53] In 1897 they attained the election of Karl Lueger as mayor, a dazzling demagogue who dominated the anti-Semitic discourse of the time with his political opportunism. Lueger employed anti-Semitism as effective instrument in the political fight against the liberals. Anti-semitism was the common denominator all could agree on: German Nationalists, members of the clergy, craftsmen and immigrants from the east of the dual monarchy who had brought their local Jew-antagonistic traditions with them to Vienna. Only social democrats and the "*remains of Austrian Liberalism*" [Pulzer (1988)] remained outside the social consensus, less out of conviction though than out of political habit. [54] What made the anti-Semitism of the Christian Socialists so Viennese was, according to the English historian Steven Beller, its ambiguity: its language was radical, its action pragmatic and its entirety a matter of interpretation. Anti-Semitism, as shown by the sociologist Alphons Silbermann [55] is neither bound to professions nor to education, age, social stratum or gender. Universities had never be exempted from it, neither in Berlin nor in Vienna. In 1879/80 professors made the antagonism against Jews socially acceptable in their 'Berliner Antisemitismusstreit' (Berlin anti-Semitism dispute). [56] At about the same time the German National fraternities at the Viennese University started their anti-Semitic agitation, excluded Jewish members and declared them to be 'incapable of satisfaction'. The ideological basis for this was provided by Georg Ritter von Schönerer, German Nationalist thought leader and "*figure-head of the new anti-Semitism*" [57] whose radical racist ideas remained largely ineffective outside the university. Inside the Viennese University the antagonistic thread proceeds, also after WWI. Mainly the generation of the so-called war students held the Jews responsible for the fragile situation within the society (economic crisis, housing shortage) and university (shortage of university places, increased fees). They regarded the Jews to be the ones who took everything away from them, accommodation, laboratory and university places. The lack of logic of this argumentation - Jewish students were in fact not over-represented at the Viennese University, they never constituted more than a quarter of the total student body though [58]- seemed to be insignificant. Anti-Semitic student associations were increasingly en vogue. [59] Together with the University they funded the 'Siegfriedkopf' (Siegfried's Head) monument, erected 1923 in the auditorium of the University in rememberance of soldiers killed in action during WWI. The anti-semitic connotation of 'Deutschtum' (hokey identification with German culture) and 'Dolchstosslegende' (stab-in-the-back myth) was well understood. [60] The 1920s and early 1930s saw an advancing radicalisation of the anti-Semitic student body, acts of violence against Jewish fellow students are repeatedly documented. [61]

On the part of the professorate the institutional adjustment to national socialist structures was carried out following the same patterns as in Germany in 1938. The hasty '*self-alignment*' of the Viennese University is described as a "*dynamic, self-enforcing process*" in Austrian historical research. [62] This process was preceded by staffing measures: the whole top of the University was replaced and substituted by professors, true to party lines, from their own colleagues. The teaching authorisations of Jewish professors were withdrawn in the same year. On the Jewish side the reactions to the rise of anti-Semitism did initially hardly differ from those in the other parts of Europe: retreat into the private sphere, reconsideration of a religious Judaism, Zionism, political resistance. Many sat, as Thomas Bernhard calls it, "*in the Viennese trap*", felt as Viennese and republicans and by this in possession of constitutionally guaranteed rights while at the same time experiencing the erosion of these rights. Already in the 1920s, conversion did not provide protection against anti-Semitism any more, as shown by the Hans Kelsen case. Hans Kelsen, since

1919 professor for state and administrative law at Vienna University, was leading at the conception of the democratic constitution of the Austrian republic. After an anti-Semitic campaign against him he left the University in 1929 and changed to Cologne. [63] At that times the Viennese anti-Semitism was also very explicit and could ostracise if required comparable to the nationalsocialist anti-Semitism that had become state doctrine after the 'Anschluss' (annexation) to Nazi Germany. Most of those who emigrated after 1938 went to the USA or England like the (deceased) mathematics professor Josef Schuster in Thomas Bernhard's scandal-creating play whose reason for his former emigrating was as simple as it was the right one: "*In Oxford there is no Heldenplatz / in Oxford Hitler has never been / in Oxford there are no Viennese / in Oxford the masses don't scream*.". [64]

Most Viennese enthusiastically embraced Austria's 'Anschluss' and the majority participated in the national socialist policy of ostracism and persecution against the Jews. [65] Still in March 1938 the offices of the Jewish community and the Zionist Organisation were closed, their board members deported to Dachau concentration camp. [66] After the November-pogrom in which 49 synagogues, Hasidic meeting houses and private praying rooms had been destroyed [67], the Jewish community of Vienna intensified their support of those willing to emigrate. [68] Until December 1939 126440 Jews had left Austria. [69] In the same year National Socialists evicted 13600 Jewish families from their Viennese homes and started the publicly legitimated deprivation of Jewish assets. In several subsequent phases 5000 Jews had been deported to Lodz, 5200 to Riga, 6000 to Izbica, 20476 to Minsk and 13776 to Theresienstadt until 5th October 1942. [Botz (2008)] On 1st November 1942 the organisation of the Jewish community was suspended, the Centre for Jewish Emigration was closed on 31st March 1943. Only 300 Viennese Jews lived to see the day of liberation, in hiding places and as forced labourers in department stores and SS households.

The collage on the invitation of the parliament for the ‘Commemoration ceremony on the occasion of the 70th anniversary of the invasion of German troups into Austria’ sets the Berlin Reichstag, not the Viennese Heldenplatz into the foreground which shows that even twenty years after Thomas Bernhard’s drama there is a lot to be done. After all, the Austrian research has launched a multiplicity of publications to deal with ‘history and commemoration’, also research in anti-Semitism has progressed. For 2008 the height was the international conference organised by the ‘Institut für Zeitgeschichte der Universität Wien’ (Institute of Contemporary History of the University of Vienna) titled ‘Vienna and the Jewish Experience 1900 until 1938. Acculturation, anti-Semitism, Zionism’. [Stern, Eichinger (2009)]

2.2.5 Cluj – AW’s hometown, a Jewish metropolis

Siebenbuergen, located in today's north of Romania, became known in Western Europe particularly by Bram Stoker’s literary figure of the count Dracula, its Jewish past remained to a large extent terra incognita. Siebenbuergen’s urban center is Klausenburg (Cluj). Klausenburg, an originally German settlement was founded in 12th century, and given the Hungarian Name Kolozsvar in 16th century. After WWI, Kolozsvar fell to Romania and was called Cluj from 1940 to the end of the War [70], with exception of a Hungarian intermezzo. Only in 1974 the city was called by its current name: Cluj Napoca. Changing territorial allocations mark the general history of Klausenburg and Siebenbuergen and also affect the local and regional Jewish history. [71]

It is said, the first Jews came with the Romans, but rabbinical sources from the 11th and 12th century are to be considered as the earliest certificates of Jewish existence in Siebenbuergen. A continuous settlement story begins only in the early modern period. On trade routes to the north of Europe, Sephardic Jews from Turkey arrived in Siebenbuergen, settled themselves and in 1623 founded the first Jewish municipality in Alba Julia. [72] [Encycl. (2007)] Siebenbuergen then was known for its central sales market for goods from the more progressive Turkish economy, in particular for iron goods and textiles which were dealt at Klausenburger's trade fair. Especially Jewish business people were involved in this import business. [Patai (1996), p154] This however had no positive influence on their legal position. Since the 17th century a part of the Habsburger monarchy [73]practiced a certain tolerance in faith due to its political autonomy. However the right to religious freedom referred meanwhile to Lutherans, Calvinists, Catholics and Antitrinitats. Jews remained excluded and were further discriminated by numerous special rights. [Baleanu (2001)] This only changed after the death of empress Maria Theresia and the change in politics of her successor Joseph II.

It was Joseph's goal to make of the Jews useful citizens, faithful subjects and reliable tax payers. His means were: dismantling of economic and social barriers as well as free access to education. [74] His edict of tolerance in 1781 marked the beginning of emancipation for Siebenbuergen's Jews. Due to the edict, the right of domicile was liberalized, the free choice of trade was initiated. This made it possible for Jews to establish companies and to acquire urban real estate, but above all Jews were no more obliged to wear a stigmatizing mark which made them recognizable as Jews. [75] In Joseph's political concept of a German speaking centralized state, Jews had access to all educational facilities including universities. A further innovation was a law for the acceptance of German surnames. [76] At the same time, the use of Hebrew and Yiddish was to be limited as much as possible. The new political line was often obstructed, particularly by the cities which adhered to their restrictive Jew politics or even tightened them. In 1784, the city Klausenburg forbade the Jews free trade, Christians were neither allowed to accommodate Jews nor to let houses to them, except at fair times. [77] But also within the Jewish community it was argued about the pro and cons of the emancipatory laws: there were the supporters of the so called Haskala, the Jewish enlightment, which was based in Budapest and had an intellectual influence on the Jews' history in Eastern Europe, the innovations meant an important contribution to integration. To the orthodox Jews of Siebenbuergen these innovations were rather to a large extent an intrusion into the inner-Jewish autonomy.

Despite all resistance the whole emancipation process could not be dispelled. The new regulations prepared the ground for developing Jewish municipalities everywhere. Also Klausenburg's municipality began to constitute itself: In 1807, sources occupy a praying area, 1818 a Jewish community and a synagogue, 1837 a funeral company and 1840 a cemetery. In 1843, the first school was founded in Klausenburg where German, Hungarian and Hebrew was tought. [78] The (unsuccessful) Hungarian revolution in 1848 against the Habsburg regime had political effects on emancipation, but was not able to put them into action. [79] When Hungary received its own constitution in 1867 by political reconciliation with the Viennese court, the missed was retrieved and the Jews' emancipation regularized. The reconciliation marked a phase of upswing in the Jewish history of Hungary and also in the Hungarian Siebenbuergen. By a congressional resolution in 1869, the new rights were followed by a long prepared reform of the cult and the reorganisation of the municipalities. As in the rest of Europe, this reorganisation involved a differentiation of the Jewish community. This reorganisation was now split into first congress municipalities (also called neological communities) endorsing the full integration into the Hungarian state; second, into orthodox municipalities declining this process and third, into the so-called Status-quo (-ante) municipalities taking an intermediate position (in addition there also existed municipalities with a totally different opinion, e.g. the so-called Hasidians. [80] In Klausenburg, the situation was the following: since its formation in 1869, the orthodox municipality was facing a congress municipality, founded at the same time, as well as an emerging Hasidic movement. [81] Altogether 4 synagogues (constructed in 1850, 1867, 1881, and 1887) and several praying houses demonstrated a lively municipality. The predominant majority of the highly differentiated Hungarian Jewry was on its way of acculturation and stood a chance to integrate into state and society. At the turn of the century, the Hungarian Jews had all reasons to look optimistically into the future: They were politically equal, socially recognized, successful in industry and economics, and also increasing in politics and science. [82] Correspondingly, the population numbers grew. The majority of Klausenburg's orthodox Jewish population grew as well and between 1869 and 1910, it increased continuously from 994 to 10,633 inhibitants. This made 11% of the total population. [83] Around 1900, Klausenburg was shaped very Hungarian (in contrary to its rural environment). [84] Increasingly more people indicated Hungarian as their colloquial language – in 1890, 55.6 percent, whereas in 1910, 73.3 percent indicated it as their first language. [85] This development is also reflected in church service: Orthodox sermons were held in German, Yiddish and Hebrew, while neologists exclusively preached in Hungarian language. In some parts of Hungary with German minorities, also including Siebenbuergen, there were many Jews, though, who despite their Hungarian colloquial language regarded themselves as belonging to the German minorities. [86]

Independently of their linguistic and religious identity, most Jewish families combined advancement with their affiliation to the Hungarian state. Orthodoxy was no barrier for education and carrier advancement, e.g. Theodor Fischer, who was attorney in Klausenburg around 1900. He came from an orthodox Jewish house, went through higher education, became an officer in the K. and K. Army and became a political functionary of the Zionist movement after the end of war. [87] Due to the territorial reorganization after the First World War and the contract of Versailles, Siebenbürgen then belonged to 'greater Romania'. The Western powers had conceded the new provinces to belong to Romania, provided that all Jews would gain full citizenship. [88] This was a rather delicate point as Romania regarded the Jews from former Hungarian areas as public enemy because of their commitment during WWI. Only under the pressure of an ultimatum set by the Western powers, Romania embodied the civil equal rights of the Jews in its new condition of 1923. After one year, Romania partly withdrew the civil equal rights again: Jews from the former Hungarian areas, thereunder also Siebenbürgen, only received the Romanian nationality if they could prove that they had already lived there before 1918. [89] Furthermore, they had no access any more to the national administration, the officer corps and to a higher university career. [90] Parallel to the social exclusion of the Jews since the accession to Romania, Jewish cultural life developed in Siebenbürgen, particularly in Klausenburg. Numerous Jewish institutions were based there, among them the Jewish national federation, the rabbi association of Siebenbürgen, a Jewish credit bank, a national orphan welfare association and two Jewish schools. Since 1920, the city was the publication place of 'Uj Kelet', one of the most important Zionist newspaper.

The arguments of the Zionist movement, which had been institutionalized in the Jewish national federation for Siebenbürgen since 1918, received increasing weight. Zionism was one of several possible answers to the anti-Semitism that strongly grew in Romania. That anti-Semitic current was organized since 1923 in ‘league for national Christian defense’, later in the ‘legion of archangel Michael’. [91] Hence the 1920s experienced some bloody excesses at the universities, although in 1923 the Romanian constitution was rewritten and granted all Jewish inhabitants full citizenship and civil rights. This however was not accomplished without foreign pressure on Romania’s government. Although their rights were granted, Jews continously were regarded as foreign elements by ethnic Romanians. When Bessarabia [92] and Bukovina [93] were transferred to Romania from Russia after WWI, many new Jews added to the existent population. Until 1930, Romania counted about 800,000 Jews. There was still little assimilation. Anti-Semitism later became a major feature of Romanian fascism.

In 1927, in Oradea located in the north of Siebenbürgen, right-wing extremists students provoked a pogrom, in whose process five synagogues were devastated and Torah writings were publicly burned. Similar excesses were reported also from other cities of Siebenbürgen, among them Klausenburg [94], where eight praying houses were pillaged. Anti-Semitism became increasingly public, which made many people escape to abroad countries allegedly assumed to be safer. Most however felt bound to Klausenburg by family, tradition and occupation. [95] Less connection existed to Romania, which made it impossible at any time to accommodate to the new state, compared to Hungary. The situation of Siebenbürgen's Jews remained therefore as a double minority: the Christians were despised as Jews and the Romanians as Hungarians. [96] In the 1930s, Romania's political line orientated to a large extent at the German National Socialism. In 1938, the government Goga-Cuza declared the anti-

Semitism as state policy. [97] Shortly after the beginning of WWII, the northern part of Siebenbürgen fell to Hungary, the south remained under Romanian administration. For the Jews this meant only gradual difference of the pursuit. Following the national socialist model, extermination camps were built in the north of Siebenbürgen. Mass deportations followed, also from Klausenburg, where a ghetto existed since 1944. All inhabitants of Siebenbürgen's thirteen ghettos were deported to Auschwitz in the same year. [98] According to estimations of the Jewish community of Romania, 135,000 Jews of Siebenbürgen were murdered during Holocaust. [99]

Today, in Klausenburg there are many places of remembrance. The former "synagogue of the craftsmen" on the outskirts of Klausenburg's Old Town was reshaped into a place of cultural interest and a meeting place. Most other places of the Jewish history in Klausenburg are ruinous or forgotten. [100] Since the opening of the Eastern Bloc, people increasingly become aware of the Jewish history of Klausenburg and Siebenbürgen. The Moshe Carmilly Institute for Hebrew and Jewish History of Klausenburg university, founded in fall 1990, is a political sign for the future.

2.2.6 The worst still ahead

It was a watershed event for the East Central European history when the Habsburg monarchy got dissolved in 1918. In place of a polyethnic and more or less tolerant state appeared so-called nation-states (or 'nationalizing states' [Brubaker (1996)]), which lost the tolerance of its predecessors (except of Czechoslovakia [Brubaker (1996)]). Rozenblit [Rozenblit (2001), p67], mentiones a "*crisis of identity*" among the Empire's Jewry, engendered by the disappearance of the multi-ethnic Habsburg monarchy and its replacement by nation-states. Contrasting to pre-1918, Jews then experienced an increase in pressure to identify themselves with the leading national groups in the newly created states. Simoultaneously a legitimate place for them was forstalled by anti-semitic currents [101], questioning their right to be equal members of the new national policies. [102]

On the eve of WWI the Jewish population within the Empire's borders accounted to more than two million people. Rozenblit showed that they made the passage into the new nation-states "*extremely poorly*" [Rozenblit (2001)]. Regarding Wilson's 'national self-determination' one can immediately see a contradiction between the need of a cohesive national policy and the needs of all ethnic minorities. The common assumption, that a state would consist of one national group, let the Jews (and other ethnic minorities) face nothing else than worsening conditions. Political nationalism proved itself to be unable to solve inter-ethnic frictions or bring stability into the regions.

Within the shrinked post-war Hungary, we see about 400,000 Jews left (making up 6% of the whole population). [Sugar et al. (1994)] Their life grew worse after 1918. Before the WWI, assimilated Jews were an important political factor. This changed in post-war Hungary as they showed to be the only remaining ethnic minoriy. When Jewish socialists actively supported the Bela Kun regime of 1919, this provoked a backlash of anti-Semitism during the White Terror after Kun's fall. In 1920, new anti-Semitic laws appeared. For example, they limited the number of students in the universities. As a result, almost two-thirds of the enrolled Jewish students had to leave. Hungary had its own fascist (and apparently anti-Semitic) organizations between the wars. However, it was the Holocaust, that deeply damaged the Hungarian Jewry. [Sugar et al. (1994)]

2.3 AW and the problem of 'betweennes'

AW was a latecome. In the advanced age of 25 he entered graduate school at the University of Vienna. It was autumn 1927, when AW stood in front of Karl Menger's door (a man of the same age) in the Mathematical Institute of the Vienna University. There he became his student. AW spoke German, was Hungarian-accented and mathematically trained (from Cluj University). Menger recalls that AW had registered at the university, but was not seen there for over two years, as he did not attend classes and had to serve in the Rumanian army. [Menger (1952), p1] Early in 1930, AW reappeared. K. Menger described AW as small and lank, obviously poor, neither young nor old looking, a strange contrast to the lively first-year university students. Back in Vienna, AW became acquainted with Hans Hahn through K. Menger. [103] H. Hahn and K. Menger quickly had recognized not only the outstanding gift of this "*most promising student*" [Menger (1952)] but also his edacious ambitions for work. In these early days a lifelong friendship between AW and K. Menger should evolve as from Menger he not only learned mathematics, also a "*... close association and friendship developed between the two*." [Morgenstern (1951)]. K. Menger's economic background [104], especially laid by his father's work, made it almost inevitable that the social and economic sciences could not only occur in his but also in his students research. [Giandomenica (2009)] In his beginnings AW was particularly interested in geometrical questions. When AW told Menger about his readings of Hilbert's 'Grundlagen der Geometrie' (Foundations of Geometry) he immediately mentioned that "*improvements could be made by dropping some postulates and relaxing others*". AW indeed made some, and one of his proofs has been included in a later edition of Hilbert's book.

In 1930 Menger showed him the problem of 'betweenness'. A point q is said to be 'between' the two points p and r, if and only if $p \neq q \neq r$ and the equality $d(p, q) + d(q, r) = d(p, r)$ holds, whereby $d()$ is a distance measure. Within a month, AW had characterized the topic of 'betweenness' as a ternary relation in a metric space, yielding four publishable papers. His results appeared in 1931 as 'Axiomatik des Zwischenbegriffes in metrischen Räumen' in the Wiener Akad. Anzeiger, [Wald (1931) a,], and also in the Math. Annalen [Wald (1931) b,] and as 'Axiomatik des metrischen Zwischenbegriffes' in the EemK. [Wald (1931) c,] Later in 1933 the paper 'Zur Axiomatik des Zwischenbegriffes' appeared [Wald (1934) b,]. With these successes AW quickly got K. Menger's accreditation to participate as a full member at the Mathematical Colloquium. In 1931 AW already had attained his doctorate under K. Menger's supervision. In Vienna AW had indeed only taken three courses before getting his Ph.D. His thesis was titled 'On Hilbert's System of Axioms'. Having solved the problem of the axiomatization of 'betweeness' in metric spaces, AW "*kept asking for more*". [Menger (1952), p54] 'More' literally means that AW has developed a lively interest in many mathematical fields like axiomatics, topology, the theory of dimensions, analysis and differential geometry. His diverse activities enabled him later to do his widely recognised achievements in statistics and econometrics.

During these years AW more and more stepped away from geometry to topology. K. Menger already had proposed that the whole of geometry could be developed differently by starting from sets of points. As an elementary term without further precision K.Menger proposed the generally kept 'pieces'. Certain nested sequences of these 'pieces' could then comprise a point. AW managed to characterize these sequences, which in accordance built the point itself. In his 1935 article 'Sur la courbure des surfaces' [Wald (1935) a,] AW formulated necessary and sufficient conditions for a compact metric space to be congruent with an ordinary region with a Gaussian curvature. An additional success was AW's characterisation of metric spaces with complex coordinates as spaces, that are congruent (i.e. they are the same up to isometry) to subspaces of n-dimensional Euclidian spaces or Hilbert spaces (cp. [Popper (1934)]) He also managed the generalization of K. Menger's definition of the curvature of n-dimensional spaces. Already in this phase AW's tendency towards extension and generalization could be seen. AW had the talent to deal with and prove a given problem not only for some particular cases but in a most general form. This became even more obvious during his later work on problems of statistics and economics. It might be surprising for his later contributions to statistics and economics, but AW's geometrical-topological way of thinking (which was also influenced by the probability theory) laid the basis for a perspective on statistical problems, that was suitable for dealing with relevant economic problems. AW used in his argumentations for statistical problems very often topological methods, which opened a very general access.

AW's main problem were endouring limited financial means. He often was absent from Vienna for a visit in Cluj to see his family and earn some money with his brother Martin, who had some mathematical tasks to solve in his job as an engineer. Adding to these burdens AW became responsible for his ageing parents. [105] In late 1931, he wrote to Menger that a return to Vienna would be impossible due to financial reasons (Table 1), but he anyhow would be willing to continue his work on topology of the k-dimensional interval and at the same time also would visit a course in insurance mathematics at the University of Cluj. Further letters to K. Menger followed in 1932. In these AW presented his new results on axiomatics and the theory of convex spaces.

Table 1. AW to Menger, Cluj, 23. October 1931, (KM Archive Durham, Box 1, Folder 21).

Dear Professor!

Unfortuately, I am unable to return for the present to Vienna for financial reasons, no matter how much I would like to continue working under the direction of the Professor. Because a personal consultation is impossible, I would be very grateful, if you could communicate your remarks regarding the paper about the << relation to me by letter.
In the course of the past academic year I was unable to work intensively because I participated in a course for insurance practise which made considerable demands on my time. At the very moment I delve into the problem of the topological characterisation of the k-dimensional interval. I am taking the liberty to send the results arising from that in the attachment, briefly summed up without proof, to you, Professor. If these results are new and of interest, I will also assemble the complete proof and send it to you.

Cluj, 23rd October 1931
Respectfully yours
Your devoted
A.Wald

Address: A. Wald, Cluj Str. Tudor, Vladimiresen 17, Rumania

Sehr geehrter Herr Professor!

Leider ist mir aus materiellen Gründen vorläufig nicht möglich, nach Wien zurückzukehren, wie gerne ich auch unter der Leitung des Herrn Professors dort weiterarbeiten möchte. Da eine persönliche Rücksprache nicht möglich ist, wäre ich sehr dankbar, wenn Herr Professor Ihre Bemerkungen betreffend die Arbeit über die ≪ Relation mir brieflich mitteilen würden.

Im Laufe des vergangenen Studienjahres konnte ich nicht intensiv arbeiten, da ich einen Kurs für Versicherungstechnik absolvierte, welcher meine Zeit ziemlich in Anspruch nahm. Gegenwärtig befasse ich mich mit dem Problem der topologischen Charakterisierung des k-dimensionalen Intervalles. Die dabei erzielten Resultate erlaube ich mir anbei kurz zusammengefasst ohne Beweis Herrn Professor einzusenden. Falls diese Ergebnisse neu und von Interesse sind, werde ich auch die vollständige Beweisführung zusammenstellen und einsenden.

Cluj, den 23. Oktober 1931

Mit vorzüglicher Hochachtung

Ihr sehr ergebener

A.Wald

Adresse: A. Wald, Cluj Str. Tudor, Vladimiresen 17, Rumania

Table 2. AW to Menger, Cluj, 11. August 1932, (KM Archive Durham, Box 1, Folder 12).

Dear Professor,
Referring to the theory of convex spaces I have attained several additional results in the meantime, that I would like to convey to you.

It holds the proposition: a locally connected continuum (a set with the cardinality of $\mathbb{R}$, note from the author) is convexible, which means, it is homeomorphic with a pseudometric convex space with a continuous metric. The question is still open, whether a locally connected continuum also is homeomorphic with a convex weak metric space.
In my last letter I have posed the question, if in a compact, weak metric space, for any two points there exists a connecting curve which is congruent to the segment. This question has to be negated. It even exists a compact, convex and weak metric space in which there is no curve at all that is congruent with the segment (between any two points, note from the author). As I have communicated before, a convex, compact weak metric space is connected and locally connected. This proposition remains valid even if only completeness of the space is assumed instead of compactness.

As regards my travel to Zurich, I am still unable to convey anything definite. The decision about that was announced to me in the course of a few days.

Yours respectfully
A. Wald

Sehr geehrter Herr Professor!

Bezüglich der Theorie konvexer Räume bin ich seither zu einigen weiteren Resultaten gelangt, die ich Herrn Professor mitteilen möchte.

Es gilt der Satz: ein im kleinen zusammenhängendes Kontinuum ist konvexifizierbar, d.h. es ist homöomorph mit einem halbmetrischen konvexen Raum mit stetiger Metrik. Die Frage, ob ein im kleinen zusammenhängendes Kontinuum auch mit einem konvexen schwachmetrischen Raum homöomorph ist, ist noch offen.

In meinem vorigen Brief habe ich die Frage gestellt, ob in einem konvexen, kompakten, schwachmetrischen Raum zu je zwei Punkten ein diese Punkte verbindender, mit der Strecke kongruenter Bogen existiert. Diese Frage ist zu verneinen. Man kann sogar einen kompakten konvexen schwachmetrischen Raum angeben, in dem überhaupt kein mit der Strecke *kongruenter Bogen existiert. Wie ich bereits mitgeteilt habe, ist ein konvexer kompakter schwachmetrischer Raum zusammenhängend und zusammenhängend im kleinen. Dieser Satz bleibt auch dann gültig, wenn statt Kompaktheit blosse Vollständigkeit des Raumes vorausgesetzt wird.*

In der Angelegenheit meiner Züricher Reise kann ich noch immer nichts Endgültiges mitteilen. Die Entscheidung hierüber wurde mir in einigen Tagen in Aussicht gestellt.

Mit vorzüglicher Hochachtung

A.Wald

The following letter is written in Vienna in the end of 1932. AW was back in the town he had missed so much for month. But soon he had to return to Cluj for financial reasons. Without rest he did research on topological questions and apprised Menger of current advances.

Table 3. AW to Menger, Vienna, 27.12.1932 (KM Archive, Durham, Box 1, Folder 21)

Dear Professor!

Regarding the convexification I am unfortunately unable to convey a final result. The combinatorial proposition is false and the given example is convexible. It is indeed possible to give an remetrization for which it passes into a convex space. I initially tried to carry out certain alterations on the given example but to no success. I hardly believe that there will be a counter-example which is a sum of a countable number of arcs. The cyclic intermediate relation can be defined explicitely and also axiomatically, if it is considered, following your proposal, as a four-point-relation. Namely the following metric propositions are valid:
I, For four points $a_1, a_2, a_3, a_4$of a metric space are embeddable into a circle (with radian), it is necessary and sufficient that for any permutation i_1, i_2, i_3, i_4 of the numbers 1, 2, 3, 4 the two linear intermediate relations are valid: a_{i1} a_{i2} a_{i3} und a_{i2}, a_{i3}, a_{i4}.
II, If every four points of a pseudometric space R can be embedded in a circle (it is not required that every four points are embeddable into the same circle, it is only required that every four points of R are congruent with four points of any circle) then the space R is congruent with the subset of the circle.
Therefore it seems to be justified to define the cyclic intermediate relation as follows. For the points a,b,c and d the cyclic relation shall be valid if and only if the four points are pairwise different and following equations are valid: ab + bc = ac and bc + cd = bd.
Then proposition I, can be expressed as: for the points to be embeddable in a circle it is necessary and sufficient that the cyclic intermediate relation is valid for these points in any ordering. This would be the analogon to the proposition that three points go into a straight line if and only if the linear intermediate relation is valid for these in any ordering.
Due to proposition II, one can see that the cyclic intermediate relation cannot be

defined more precisely, because when you only add the assumption that for every four points of the space in some order the cyclic relation is valid, then the space is congruent to a subset of the circle, hence it is ordered cyclically.
The cyclic order is led back to the linear order in accordance to this explicit definiton. It is possible to adduce this relation axiomatically as a four-point-relation. It can equally be characterised topologically.

I wish you pleasant holidays, professor, a good recreation and a happy new year.

With kind regards

Yours respectfully
A. Wald

Sehr geehrter Herr Professor!

Bezüglich der Konvexifizierbarkeit kann ich leider noch kein endgültiges Ergebnis mitteilen. Der kombinatorische Satz ist falsch und das angegebene Beispiel ist konvexifizierbar. Man kann tatsächlich eine Ummetrisierung angeben, wodurch es in einen konvexen Raum übergeht. Ich versuchte zunächst, an dem angegebenen Beispiel gewisse Änderungen vorzunehmenn, aber ohne Erfolg. Ich glaube kaum, dass ein Gegenbeispiel existiere, das Summe von abzählbar vielen Bögen sei. Die zyklische Zwischenrelation lässt sich sehr gut explizit und auch axiomatisch definieren, wenn man diese, wie Sie es vorgeschlagen haben, als Vierpunktrelation betrachtet. Es gelten nämlich folgende metrische Sätze:

I, Damit vier Punkte a_1, a_2, a_3, a_4 *eines metrischen Raumes in einen Kreis (mit Bogenmass) einbettbar seien, ist notwendig und hinreichend, dass für eine Permutation* i_1, i_2, i_3, i_4 *der Zahlen 1, 2, 3, 4 die beiden linearen Zwischenrelationen gelten:* $a_{i1}\ a_{i2}\ a_{i3}$ *und* a_{i2}, a_{i3}, a_{i4}.

II, Wenn je vier Punkte eines halbmetrischen Raumes R in irgend einen Kreis einbettbar sind, (es wird nicht verlangt, dass je vier Punkte in denselben Kreis einbettbar seien, es wird bloss verlangt, dass je vier Punkte von R mit vier Punkten irgend eines Kreises kongruent seien) dann ist der Raum R kongruent mit der Teilmenge eines Kreises.

Es scheint daher vielleicht gerechtfertigt, die zyklische Zwischenrelation als Vierpunktrelation folgendermassen zu definieren. Für die Punkte a,b,c und d soll die zyklische Zwischenrelation dann und nur dann gelten, wenn die vier Punkte paarweise verschieden sind und die Gleichungen bestehen: ab + be = ac und bc + cd = bd.

Den Satz I, kann man dann so aussprechen: Damit die Punkte in einen Kreis einbettbar sind ist notwendig und hinreichend, dass für diese Punkte in irgend einer Anordnung die zyklische Zwischenrelation gilt. Dies wäre das Analogon zum Satze, dass drei Punkte dann und nur dann in eine Gerade eingehen, wenn für diese in irgend einer Anordnung die lineare Zwischenrelation gilt.

Aufgrund von Satz II, sieht man, dass die zyklische Zwischenrelation auch nicht schärfer definiert werden kann, denn wenn man bloss die Forderung hinzufügt, dass für je vier Punkte des Raumes in irgendeiner Anordnung die zyklische Relation gilt, dann ist der Raum kongruent mit einer Teilmenge eines Kreises, ist also zyklisch geordnet.

Die zyklische Anordung ist nach dieser expliziten Definition auf lineare Anordnung zurückgeführt. Man kann auch diese Relation als Vierpunktrelation axiomatisch anführen. Ebenso ist möglich, diese Relation auch topologisch zu charakterisieren.

Ich wünsche herzlich Herrn Professor angenehme Ferien, gute Erholung und ein glückliches Neujahr.

Mit den besten Grüßen

Vorzügliche Hochachtung

A.Wald

Table 4. AW to K. Menger, Cluj, written in the beginning of the winter semester 1932/1933, (KM Archive, Durham, Box 1, Folder 21)

Dear Professor

Please forgive me that I have not written for such a long time, but I had waited for something certain to convey. Unfortunately, no post arose from Temesvar. In Palestine there is no post available either at the moment, however I was assured, that if I will be there and master the language I will be able to find a position somewhere as countrywide new schools will be opened. As Prof. Fekete informed me, he does not see any possibility for an employment at the University of Jerusalem, but he also assumes that I will manage to get a position at a secondary school in a short time. I have therefore decided to travel to Palestine in spring (after Easter), if I can obtain the permit to enter the country and the required financial means.

For the time until Easter I have no definite programme for the present. I would prefer to continue working during this time, basically I would like to keep myself busy with the investigations in differential geometry and various questions in metric geometry. Unfortunately, I hardly can work here because I do not have the required quietness, until now I have also been very busy, because I am working together with my brother [Dr. Ing. Martin Wald]. I would be extremely happy if I could be in Vienna until Easter and work in your colloquium. Unfortunately, this is impossible for me for financial reasons. If I were enabled to do so from Vienna, I would be very pleased to go there. Of the new questions we have compiled in the summer [1932], the first five are referring to pseudo-tuple and quasi-congruence. I think that these questions have already been answered by my investigations in the summer.

The interesting question, wether the K_∞ and the $R_{\infty,\infty}$ are universal, is not yet decided, although this seems not to be too difficult. Anyway it is true, that a countable complex- or real-metric space is congruent with a subset of K_∞ resp. $R_{\infty,\infty}$. Is R a given separable space and the sequence of points $\{p_i\}$ dense in R, then there exists in $R_{\infty,\infty}$, what is easy to show, a sequence $\{p_i'\}$ congruent with $\{p_i\}$, that fulfills following condition: if and only if the points $\{p_{i_k}\}$, k = 1, 2,... set up a Cauchy sequence, the points $\{p_{i_k}'\}$ converge to a point p' of $R_{\infty,\infty}$, assumed convergence is understood as convergence by coordinates. Is p any point in R, than there exists a subsequence $\{p_{i_k}\}$ that converges to p. The appropriate sequence $\{p_{i_k}'\}$ converges to a point p'. If one assigns the point p to the point p', it is easy to confirm, that this mapping of R onto a subset of R_ω is topological. - It is not impossible, that the metric theory of R , $R_{n,n}$and $R_{\infty,\infty}$

will often be useful in the field of topological embedding.

The colloquia will surely have started already. I am really sorry not to participate.
With best regards and remaining respectfully yours
A. Wald

P.S. Thank you very much indeed for the separata sent to me. From the note 'Simplified Proof of Steinitz' Theorem' I have received too many copies.

Sehr geehrter Herr Professor!

Verzeihen Sie bitte, dass ich so lange nicht schrieb, ich wartete bis ich etwas Sicheres mitteilen kann. Aus der Stelle in Temesvar ist leider nichts geworden. In Palästina ist momentan auch keine Stelle frei, jedoch versichert man mir, dass falls ich dort bin und die Sprache beherrsche, so wird es mir gelingen unterzukommen, da im Lande neue Schulen eröffnet werden. Wie Prof. Fekete mitteilte, sieht er jetzt keine Möglichkeit dafür, dass die Univ. in Jerusalem mich anstelle, doch glaubt er auch, dass es mit gelingen wird in kurzer Zeit in einer Mittelschule eine Stelle zu bekommen. Ich entschloss mich daher im Frühling (nach Ostern) nach Palestina zu fahren, falls ich die Einreisebewilligung beschaffen kann und über die nötigen Mittel verfügen werde.

Für die Zeit bis Ostern habe ich vorläufig kein bestimmtes Programm. Am liebsten möchte ich ja während dieser Zeit weiterarbeiten, hauptsächlich möchte ich mich mit den begonnenen diff.geom. Untersuchungen und verschiedenen Fragen der metrischen Geometrie beschäftigen. Leider kann ich hier kaum arbeiten, da ich nicht die nötige Ruhe habe, bis jetzt war ich auch sehr beschäftigt, da ich mit meinem Bruder [Dr. ing. Martin Wald] *zusammenarbeitete. Es würde mich ausserordentlich freuen, wenn ich bis Ostern in Wien sein und in ihrem Kolloquium arbeiten könnte. Leider ist mir dies aus materiellen Gründen nicht möglich. Wenn mir dies von Wien aus ermöglicht werden könnte, so möchte ich sehr gerne hinfahren. Von den neuen Fragen, die wir im Sommer* [1932] *zusammengestellt haben, beziehen sich die ersten 5 auf pseudo-Tupel und Quasikongr. Ich glaube, dass diese Fragen durch meine Untersuchungen im Sommer bereits beantwortet sind.*

Die interessante Frage, ob der K_∞ *und der* $R_{\infty,\infty}$ *universell ist, ist noch unentschieden, zwar dürfte dies nicht allzu schwierig sein. Jedenfalls gilt aber, dass ein abzählbarer komplex- oder* *reell-metrischer Raum mit einer Teilmenge des K_∞ bzw. $R_{\infty,\infty}$ kongruent ist. Ist R ein beliebiger separabler Raum und $\{p_i\}$ eine in R dicht liegende Folge von Punkten, so gibt es im $R_{\infty,\infty}$, wie man leicht zeigen kann, eine zu $\{p_i\}$ kongruente Folge $\{p_i'\}$, wobei auch folgende Bed. erfüllt ist: dann und nur dann, wenn die Punkte $\{p_{i_k}\} k = 1, 2,...$ eine Cauchysche Folge bilden, konvergieren die Bildpunkte $\{p_{i_k}'\}$gegen einen Punkt p' des $R_{\infty,\infty}$ falls unter Konvergenz Koordinatenkonvergenz verstanden wird. Ist nun p irgendein Punkt von R, so gibt es eine gegen p konvergente Teilfolge $\{p_{i_k}\}$. Die entsprechende Folge $\{p_{i_k}'\}$konvergiert dann gegen einen Punkt p'. Ordnet man den Punkt p den Punkt p' zu, so kann man leicht bestätigen, dass diese Abbildung von R auf eine Teilmenge des R_ω eine topologische ist. - Es ist nicht ausgeschlossen, dass die metrische Theorie des R und des $R_{n,n}$und des $R_{\infty,\infty}$ noch verschiedene Anwendungen im Problemkreis der topolog. Einbettungen finden wird.*

Die Kolloquien haben sicher bereits begonnen. Es tut mir wirklich leid nicht dabei zu sein.

Mit besten Grüssen und vorzüglicher Hochachtung

Ihr A. Wald

P.S. Ich danke bestens für die eingesendeten Separata. Aus der Note "Vereinf. Bew. d. Steinitzschen Satzes" habe ich allzuviel Exemplare bekommen.

AW could never be a part of Vienna's establishement. He was one of the 'Ostjuden' (Eastern Jews) standing at the lower end of the hierarchy, even amongst Vienna's Jews. People like him flooded into 'Leopoldstadt', Vienna's by and large low-grade Jewish ghetto. Moreover AW's different accent and his appearance made him conspicuously distinguishable from his assimilated teachers and superiors like Mises or Schlesinger, who had a Jewish origin, but were integrated in Vienna's culture and habits. Menger, the gentile, always took a clear opposition against nationalism and anti-semitism, hence belonged to a minority at the University. With his opinion that AW in the Mathematical Colloquium "*had exactly the spirit which prevailed among the young mathematicians who gathered together about every other week*" [Menger (1952), p15], he also refers to AW's stamina in adverse surroundings, not only to his mathematical genius. Regarding the political climate in Vienna in the beginning of 1930, Menger's Colloquium seems like an island for collective work in mathematics, giving its members a kind of protective intimacy and stability. AW not only frequently visited this place of intellectuality and intensive interchange of mathematical work, but also became an important contributor and soon assistant editor of the regularly issued 'Ergebnisse' (EemK) of Menger's Colloquium.

An academic position for AW was unimaginable during the 1930s, as he had a background that bared him from any career in the university of Vienna. Hence the only way to continue with his mathematical work was to get any employment. With Menger's support, he came in touch with Karl Schlesinger, a well-known banker and (amateur) economist, who had enough time and leisure for extra intellectual ambitions to improve his mathematical skills. AW's position then was private tutor for Schlesinger in mathematics and mathematical economics. Years before, in 1914, Schlesinger had published a book on the Walrasian system. He also was actively involved in the Viennese Economic Society. AW's mathematical training proved to be successful: in 1935 Schlesinger's paper on the modified Cassel system [Schlesinger (1935)] appeared. It introduced inequalities to the general equilibrium problem and thus replaced Walras' simple counting of equations and unknowns. [106] Also AW published on this topic: in 1935 'Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen' [Wald (1935) b,] appeared. Typical for AW is the fact, that he quickly published in a series of several papers on systems of equations in mathematical economics, treating production and exchange variants of the Walrasian general equilibrium equation system, and also Cournot's duopoly model. [107] (Cp. Appendix B, publication list 1935, 1396)

The following letter can be dated back to March 1934. AW tutored K. Schlesinger during this time and simoultaneously developed own results on the topics of his tutorials. O. Morgenstern wanted him to pubish his results in the 'Zeitschrift für Nationalökonomie', but AW, inherently cautious, wished to discuss this with K. Schlesinger, who refused. The mentioned manuscript of G. Nöbeling (a former student of K. Menger and participant in his Colloquium, who worked on topology and had left the Institute in the Winter Semester 1933/34 for a position as assistent in the mathematical seminar of Otto Haupt in Erlangen/Germany) is also an issue in a letter of A. Flores to Menger (written on March 26, 1934, Menger Archives, Durham, Box 2, Folder 7). AW and A. Flores had to examine G. Nöbeling's work. In a following letter to K. Menger on August 22, 1934 (Menger Archives, Box 2, Folder 7) Flores writes that he meanwhile had finished the examination of G. Nöbeling's work and postally had passed this to AW, who in summer 1934 again stayed in Cluj waiting for A. Flores' transmittal of parts of G. Nöbeling's results.

Table 5. Wald to Menger, Vienna, Spring 1934 (KM Archive Durham, Box 1, Folder 21)

Dear Professor,

Thanks a lot for your nice card. I have already read a part of Nöbeling's work (ca. the half) with Mr. Flores and I hope to finish it the next days. Lecturer Morgenstern asked me right after the lecture that I should publish the work in the Viennese Econ. Magazine. I did not appeal to him for the reason that I have to discuss this with Mr. Schlesinge at first. Friday I visited Schlesinger and told him about the wish of Mr. Morgenstern. He was against it and however, he would like to publish in an English magazine. I will tell him now that I also want to discuss this with you after your return. I believe that Ms. Taussky has already mentioned to you that she proved the theorems about congr. ord. for a certain class of fields.

Currently I am busy with copying the work on the 'econ. equations'. Mr. Schlesinger even pushes already.

Yours respectfully
A. Wald

Sehr verehrter Herr Professor!

Besten Dank für Ihre l. Karte. Von der Nöbelingschen Arbeit habe ich einen Teil (ca. die Hälfte) mit Herrn Flores bereits gelesen u. hoffe in einigen Tagen beendigen zu können. Doz. Morgenstern hat mich gleich nach dem Vortrag ersucht, dass ich die Arbeit in der Wiener Ökon. Zeitschrift publiziere. Ich habe ihm nichts zugesagt mit der Begründung, dass ich dies zuerst mit Herrn Schlesinger besprechen muss. Freitag war ich bei Schlesinger u. habe ihm den Wunsch des Herrn Morgenstern mitgeteilt. Er war aber dagegen u. möchte aber in einer englischen Zeitschrift zu publizieren. Ich werde jetzt ihm sagen, dass ich das auch mit Ihnen nach Ihrer Rückkehr besprechen will. Ich glaube, dass Fräulein Taussky Ihnen bereits erwähnt hat, dass Sie die Sätze über Kongr. Ord. für eine gewisse Klasse von Körpern bewiesen hat.

Momentan beschäftige ich mich mit dem Abschreiben der Arbeit über die Ökon. Gleich. Herr Schlesinger drängt schon nämlich darauf.

Mit vorzüglicher Hochachtung

Ihr A. Wald

Table 6. AW to Menger, Cluj, Summer 1934 (KM Archive Durham, Box 2, Folder 7)

Dear Professor,

When I met Dr. Schlesinger the last time, we talked about the publication of the economic paper. I told him, that my part is such mathematical that the publication is only suitable in a mathematical magazine. He was resolute against that and suggested, if you agree, I should publish my part as well as in the economical magazine as in the Results (EemK, ann. of the author). He believes that Morgenstern is not opposed to that. I told him, I have to discuss this with you. Therefore I ask for your opinion, in order that I can inform Schlesinger.

Dr. Schlesinger will send his part to me when it is finished. I would like to write the additions of my part afterwards, if it is not too late, because I would like to add some remarks to the statements of Dr. Schlesinger to the system of Walrass. My card from Vienna and the manuscript of Flores, which I sent in at the same time, you have surely received already. Mr. Flores this time really wrote everything and wanted to hand over the manuscript before your departure, but he could not reach you any longer. Before my departure, I handed over the work about abstract fields and metrics to Ms. Taussberg. After having read, she will send it to you. Unfortunately I could not read the manuscript of Nöbeling in Vienna, because I had a lot to do before my departure. I handed the manuscript over to Mr. Flores; he will send it in hither in some days. Afterwards I will also read it.
I wish you a good recovery and a pleasant summer.

With cordial greetings
Yours, A. Wald
My address:
Cluj, Str. Minerva 7
Rumania

Sehr verehrter Herr Professor!

Als ich zuletzt bei Dr. Schlesinger war, haben wir über die Publikation der ökonomischen Arbeit gesprochen. Ich sagte ihm, dass mein Teil so mathematisch ist, dass die Veröffentlichung derselben nur in einer mathematischen Zeitschrift geeignet ist. Er war aber entschieden dagegen und hat mir vorgeschlagen, dass ich meinen Teil, falls Sie damit einverstanden sind, sowohl in der ökon. Zeitschrift als auch in den Ergebnissen publizieren möge. Er glaubt, dass

Morgenstern sicher nichts dagegen haben wird. Ich sagte ihm, dass ich dies mit Ihnen besprechen muss. Bitte daher, mir Ihre Meinung zu schreiben, damit ich dem Schlesinger Bescheid sagen kann.

Dr. Schlesinger wird seinen Teil, als er damit fertig ist, mir einsenden. Ich möchte, wenn es nicht zu spät ist, die Ergänzungen zu meinem Teil erst nachher schreiben, da ich zu den Ausführungen von Dr. Schlesinger über das Walrassche System einige Bemerkungen hinzufügen möchte. Meine Karte aus Wien und das Manuskript von Flores, das ich gleichzeitig eingesendet habe, haben Sie sicher bereits erhalten. Herr Flores hat diesmal wirklich alles geschrieben und wollte Ihnen das Manuskript noch vor Ihrer Abreise übergeben, nur konnte er Sie schon nicht mehr erreichen. Die Arbeit über abstrakte Körper und Metrik habe ich vor meiner Abreise Frl. Taussberg übergeben. Sie wird es Ihnen, nachdem sie es gelesen hat, einsenden. Das Manuskript von Nöbeling konnte ich in Wien leider nicht lesen, da ich vor meiner Abreise zu viel zu tun hatte. Ich habe das Manuskript Herrn Flores übergeben; er wird es mir in einigen Tagen hierher einsenden. Nachher werde ich es auch lesen.

Ich wünsche Ihnen gute Erholung und einen angenehmen Sommer.

Mit herzlichen Grüssen

Ihr A. Wald

Meine Adresse:

Cluj, Str. Minerva 7

Rumänien

On July 2, 1934, Hans Hahn surprisingly died. His outstanding students were K. Menger, K. Gödel and Witold Hurewicz. Meanwhile AW was in Cluj, but had been informed, that he would receive a stipend of the Rockefeller Foundation, managed by O. Morgenstern. AW sedulously worked on the uniqueness of the solution of the system of equations on production (cp. the 1935 'Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen', EemK, 6, 12-20, [Wald (1935) b,]). G. Debreu mentioned AW's pathbreaking work in his Nobel speech in 1983: "*One must, however, immediately add that the mathematical tools that later made the solution of the existence problem possible did not exist when Walras wrote one of the greatest classics, if not the greatest, of our science. It was Abraham Wald, starting from Gustav Cassel's 1918 formulation of the Walrasian model, who ... in Vienna in 1935-36 provided the first solution in a series of papers that attracted so little attention that the problem was not attacked again until the early fifties*". [Debreu (1983)]

Table 7. AW to Menger, Cluj, August 1934 (KM Archive Durham, Box 2, Folder 7)

Dear Professor,

I was deeply shattered by the sad news about Professor Hahn. He was not very old. – I am very happy that I will receive the grant for the next year. –
In the attachment I am sending the proof for the necessity of the economic conditions. The condition that $\lim_{S_i=\infty} f_i(S_i) = 0$ is not necessary. I think that this condition is not used to prove the theorem, hence this condition can simply be deleted. Should it have entered into the proof still anywhere and it would be cumbersome to alter the proof at the place in question, so that the condition is not used any more, this could be done in an additional note. I am communicating the proof for the non-necessity of the condition in the enclosed draft. – I proved that the monotony is necessary in the stronger sense for the functions f_i by defining an system of equations, whereby the functions f_i are not strictly monotonous and this system of equations has continuously many solutions. Wether there are systems of equations that do not have any solution at all, I do not know. If the number of points is ≤ 2, then there is at least one solution.

My younger brother wants to enroll in Vienna for physics in the next year. He had already studied a few semesters here in Klausenburg. I do not know what difficulties a foreign person will currently have to get enrolled. Is that possible at all?

With cordial greetings
Yours, A. Wald

Sehr geehrter Herr Professor!

Die traurige Nachricht über Professor Hahn hat mich sehr erschüttert. Er war noch nicht so alt. – Es freut mich sehr, dass ich das Stipendium für das nächste Jahr bekomme. – Beiliegend sende ich den Beweis für die Notw. der ökonomischen Bedingungen ein. Die Bedingung, dass $\lim_{S_i=\infty} f_i(S_i) = 0$ *ist nicht notwendig. Ich glaube, dass diese Bedingung nirgends im Beweis des Theorems verwendet wird, so dass man diese Bedingung einfach streichen kann. Sollte aber irgendwo dies doch in den Beweis eingehen und es wäre umständlich, an der betreffenden*

Stelle den Beweis so abzuändern, dass diese Bedingung nicht verwendet wird, so könnte man dies in einer nachträglichen Bemerkung tun. Den Beweis für die Nichtnotwendigkeit der Bedingung teile ich im beiliegenden Konzept mit. – Dass die Monotonie im schärferen Sinne der Funktionen f_i notwendig ist, beweise ich so, dass ich ein Gleichungssystem angebe, wobei die Funktionen f_i nicht im schärferen Sinne monoton sind und das Gleichungssystem kontinuierlich viele Lösungen hat. Ob es auch Gleichungssysteme gibt, die gar keine Lösung besitzen, weiß ich nicht. Wenn die Anzahl der Punkte ≤ 2 ist, gibt es jedenfalls mindestens eine Lösung.

Mein jüngerer Bruder möchte sich im nächsten Jahr in Wien auf Physik einschreiben. Er hat schon hier in Klausenburg einige Semester studiert. Ich weiss nicht, welche Schwierigkeiten jetzt hier bei der Aufnahme eines Ausländers sind. Ist es überhaupt möglich?

Mit herzlichen Grüssen

Ihr A. Wald

Table 8. AW to Menger, Cluj, August/September 1934 (KM Archive, Durham, Box 2, Folder 7)

Dear esteemed Professor!

Thank you very much for your n. (nice, ann. of the author) letter. I have received the corrections from Mses. Dr. Taussky not until today. I have already finished the corrections on the galley proofs of the economic work and I am sending them concurrently with this letter. I could have finished the remaining corrections (Mrs. Taussky has sent me all papers that got published in journal 6) not until Sunday and am afraid that they wont arrive in Prague in time. Therefore, I am going to send these to your new address as soon as I came to know it.
I have examined the economic paper thereupon, if the condition $\lim_{s_i->\infty} f_i(s_i) = 0$ is dispensable. Indeed it is no part of the proof and therefore I have eliminated it in 4. The condition $\lim_{s_i->0} f_i(s_i) = \infty$ is certainly necessary. The proof for this is included in the remarks about the necessity of this condition I have sent to you some weeks ago. Accidentally, you seem to assume in your letter, that $\lim_{s_i->\infty} f_i(s_i) = 0$ is necessary and ask, if the condition $\lim_{s_i->0} f_i(s_i) = \infty$ has to go into the proof. On galley proof 6 I wrote in the proposition (+) instead of $\lambda \leq \lambda_k$: $\lambda < \lambda_k$ because when for almost all k is valid : $\lambda = \lambda_k$, (+) does not have to be valid at all.
On galley proof 7 in the first line I have included the remark that here is $\lim_{\lambda=\bar{\lambda}} \Pi(\lambda) = \lim_{\lambda=\bar{\lambda}} \overline{\Pi}(\lambda)$ because this is used on galley proof 8.
5-6 days ago I have sent to you on your Viennese address because I did not know that you are still in Strobel. Hence I am not sure wether you have received the letter and therefore am repeating the message, that I have also proved the unambiguous resolvability of the system of equations for the case that $\sigma_j = f_j(s_1, ..., s_n)$. About the function $f_j(s_1, ..., s_n)$, $j = 1, .., n$ it is only assumed that $\Delta s_1 \sigma_1 + .. + \Delta s_n \sigma_n < 0$ if for at least one i $\Delta s_i \neq 0$. Thereby mean the $\Delta s_1, .., \Delta s_n$ the alterantions of $s_1, .., s_n$ and $\Delta\sigma_1, .., \Delta\sigma_n$ the according changes of the prices. This condition is an natural generalisation of the assumed monotonicity in the case of $\sigma_j = f_j(s_j)$.
I am returning to Vienna not later than the end of October.

With kind regards
Your A. Wald

Sehr geehrter und lieber Herr Professor!

Besten Dank für Ihren l. Brief. Die Korrekturen von Frau Dr. Taussky habe ich erst heute erhalten. Die Fahnen der ökon. Arbeit habe ich bereits korrigiert und sende es gleichzeitig mit diesem Brief ein. Die übrigen Korrekturen (Frl. Taussky hat mir sämtliche in Heft 6 erscheinende Arbeiten eingesendet) könnte ich erst Sonntag fertig haben und fürchte, dass es schon nicht mehr rechtzeitig nach Prag ankommt. Ich werde diesselben daher auf Ihre neue Adresse, sobald ich sie erfahren habe, einsenden.

Ich habe die ökon. Arbeit daraufhin geprüft, ob die Voraussetzung $\lim_{S_i->\infty} f_i(s_i) = 0$ überflüssig ist. Sie geht tatsächlich nirgends in den Beweis ein und habe deswegen in 4. dies gestrichen. Die Voraussetzung $\lim_{S_i->0} f_i(s_i) = \infty$ ist sicher notwendig. Der Beweis hierfür ist in den Bemerkungen über die Notwendigkeit der Bedingung enthalten, die ich Ihnen vor einigen Wochen eingesendet habe. Sie nehmen scheinbar aus Versehen in Ihrem Briefe an, dass $\lim_{S_i->\infty} f_i(s_i) = 0$ notwendig sei und fragen, ob die Bedingung $\lim_{S_i->0} f_i(s_i) = \infty$ in den Beweis eingeht. Auf Fahne 6 habe ich im Satze (+) statt $\lambda \leq \lambda_k :\ \lambda < \lambda_k$ geschrieben, denn wenn für fast alle $k :\ \lambda = \lambda_k$ ist, muss (+) gar nicht gelten.

Auf Fahne 7 in der ersten Zeile habe ich die Bemerkung eingeschaltet, dass hier $\lim_{\lambda=\bar{\lambda}} \Pi(\lambda) = \lim_{\lambda=\bar{\lambda}} \overline{\Pi}(\lambda)$ ist, da dies auf Fahne 8 verwendet wird.

Vor 5-6 Tagen habe ich Ihnen auf Ihre Wiener Adresse geschrieben, da ich nicht wusste, dass Sie noch in Strobel sind. Ich bin jetzt nicht sicher, dass Sie diesen Brief erhalten haben und wiederhole daher die Mitteilung, dass ich die eindeutige Lösbarkeit des Gleichungssystems auch für den Fall bewiesen habe, dass $\sigma_j = f_j(s_1,\ ...,\ s_n)$. Über die Funktion $f_j(s_1,\ ...,\ s_n),\ j = 1,\ ..,\ n$ wird bloss vorausgesetzt, dass $\Delta s_1\, \sigma_1 + .. + \Delta s_n\, \sigma_n < 0$ist, falls für mindestens ein $i\ \Delta s_i \neq 0$. Dabei bedeutet $\Delta s_1,\ ..,\ \Delta s_n$ die Änderungen von $s_1,\ ..,\ s_n$ und $\Delta\sigma_1,\ ..,\ \Delta\sigma_n$die entsprechenden Änderungen der Preise. Die Bedingung ist eine natürliche Verallgemeinerung der vorausgestzten Monotonie im Falle $\sigma_j = f_j\left(s_j\right)$.

Ich fahre spätestens Ende Oktober nach Wien zurück.

Mit herzlichen Grüßen

Ihr A. Wald

In-between AW returned back to Vienna, seeking for a job position to be able to stay for longer in the city. He always tried to be close to K. Menger, his friend and mentor, to work in the Colloquium, that was his true homeland. But it was the time of a hard economic depression in Austria. Hence his quest for income and a job opportunity in mathematics did not find an end. K. Menger arranged that AW met O. Morgenstern, who was to become the director of the 'Institut für Konjunkturforschung' (Austrian Institute for Trade Cycle Research), a position that O. Morgenstern held from 1935 to 1938. O. Morgenstern quickly recognized AW's outstanding talent and more and more employed him in the Institute. AW finally became (as Franz Alt) a mathematical coach of Morgenstern. This was a vantage point in his life. AW rapidly became involved in economic questions and started to work on their mathematization. Besides he did not stop his engagement with geometrical topics, because K. Menger and the Colloquium constantly were active in this field. Despite AW's efforts O. Morgenstern never became an important mathematician, but anyhow was a goldmine for K. Menger and especially John von Neumann [Freeman (1968)] with whom O. Morgenstern later published the 'Theory of Games and economic Behaviour'. O. Morgenstern became a lifelong friend of AW. It cannot be overseen that AW was the mathematical motor (much stronger than Alt), that trimmed Morgenstern in a way that he in the end was able to coauthor with von Neumann [von Neumann (1944)]. This field of research "*in itself became one of Wald's later mathematical interests*.". [Wolfowitz (1952), p4]

We write the year 1933 when AW gradually developed a kind of professional relationship to O. Morgenstern getting a small stipend from the Rockefeller Foundation to the Institute to employ him in order to "*to undertake a methodological study of the decomposition of statistical series*". [108] For the following year AW monthly got about 300$ from the Institute. Up to 1933 this was his biggest payoff. In the beginning of 1935, Morgenstern again wrote to the Rockefeller Foundation, praising AW's work in mentioning his outstanding statistical and mathematical ability for applied problems, whereas there was "*still very much purifying to be done*". [109] AW, during his research time at the Institute, defined a method for seasonal decomposition, that varied from that of Warren M. Persons (and his method of the 'link relatives'). O. Morgenstern presented AW's paper at Louvain and Paris in 1935. [110] Therein a time series is assumed to be combined of trend, seasonal and accidental component, i.e. $f(t)$, $S(t)$ and $E(t)$. W. M. Persons assumed, that the seasonal component was multiplicative, i.e. $S(t) = f(t) * r(t)$, with $r(t)$ as a periodic (12-month) function of time. AW assumed a seasonal variation in the form of $S(t) = h(t) * r(t)$, whereby $h(t)$ is non-negative and slowly varying over time. W. M. Persons' method of deseasonalization, applied to Austrian unemployment data (for the period from 1923 to 1934) produced a residual series with a low tracking error (compared to the original data) until 1932, but showed than a pretty contradictory behaviour for the further years. In contrast to that, AW's correction fitted much better. When O. Morgenstern gave his talk about AW's paper, he included a graphic showing this phenomenon. AW's activities peaked into the 1936 book 'Berechnung und Ausschaltung von Saisonschwankungen' [Wald (1936) c,] (cp. [Morgenstern (1990), p84]).

Table 9. O. Morgenstern to AW, October 31, 1934, Private Collection Robert M. Wald, Chicago

Österreichisches Institut für Konjunkturforschung
Wien I, Stubenring 8-10
Wien, 31. Okt 1934

Herrn Dr. A. Wald
Wien, VII.,
Breitegasse 7
Dear Doctor!

For formal reasons the Institut informs you in written form, that the Rockefeller Foundation, Paris, grants to you with their letter from August 3rd, 1934, a scholarship for ca. one year with an overall amount of $3.500 for scientific work (investigations in seasonal variations and/or other math.-statistical problems) and mandates the Institut to the accounting and/or disbursement of this amount. In accordance with the verbally agreements, this scholarship will be disbursed in monthly rates of $300 to you. The first rate for this scholarship was disbursed on October 16th to you. The further payments will take place cca around the middle of each month.
Your scientific work has always to be carried out in agreement with the director of the institute, Herrn Dr. Oskar Morgenstern and he has to be informed about respective results of your analysis.

As the Institut asks you to take knowledge of this, it draws faithfully
Austrian Institut for market research
The leader;
Morgenstern

Österreichisches Institut für Konjunkturforschung

Wien I, Stubenring 8-10

Wien, 31. Okt 1934

Herrn Dr. A. Wald

Wien, VII.,

Breitegasse 7

Sehr geehrter Herr Doktor!

Das gefertigte Institut teilt Ihnen der Ordnung halber schriftlich mit, dass die Rockefeller Foundation , Paris mit Ihrem Schreiben vom 3. August 1934 Ihnen ein Stipendium für cca ein Jahr in der Gesamthöhe von $ 3.500.-- für wissenschaftliche Arbeiten (Saisonschwankungsuntersuchungen, bezw. andere mathem.-statistische Probleme) bewilligt und das Institut mit der Verrechnung, bezw. Auszahlung dieses Betrages betraut hat. Gemäss den mündlich getroffenen Vereinbarungen wird dieses Stipendium in monatlichen Raten von cca $ 300.- an Sie zur Auszahlung gebracht. Die erste Rate für dieses Stipendium wurde am 16. Oktober an Sie ausgezahlt. Die weiteren Zahlungen werden cca um die Mitte jeden Monats erfolgen.

Ihre wissenschaftlichen Arbeiten sind stest im Einvernehmen mit dem Leiter des Institutes, Hernn Dr. Oskar Morgenstern, zu führen und von den jeweiligen Ergebnissen Ihrer Untersuchungen ist ihm Mitteilung zu machen.

Indem Sie das Institut ersucht, davon Kenntnis zu nehmen zu wollen, zeichnet es

hochachtungsvoll

Österreichisches Institut für Konjunkturforschung

Der Leiter:

Morgenstern

AW was more than a simple member of the Institute's staff, who only tracked for own research tasks. O. Morgenstern engaged him (in that role succeeding F. Alt) as his personnel trainer, at first in basic mathematics like algebra or differential calculus.[111] The impact, AW had on Morgenstern was considerably high, as the numerous entries in Morgenstern's diary show.[112]And indeed Morgnstern made fine advances. Soon he developed a broad understanding of most mathematical topics in economics. But this was held private by him. In public, it is interesting to see that Morgenstern always kept AW in distance (cp. the formal letters over all the years). We could not find any evidence that O. Morgenstern ever spoke or wrote publicly of his training lessons given by the jew AW. We state a certain ambivalence in their relationship. A decade before O. Morgenstern had written about the (in his opinion) impureness of foreign culture on the German culture and shown some aversion to the Jews of the Mises Circle and the 'Geist-Kreis'. [Leonard (1998)]

AW had to bow and scrape over three years. And he was not the only one, who started to consider leaving the country. O. Morgenstern was always the key for many students to get attributions and fellowships of the Rockefeller Foundation. During the 1930s a couple of O. Morgenstern's assistants at the Institute got grants for university studies abroad (some of them went to Harvard as O. Morgenstern did himself as student). In the middle of 1935 AW had an interview with Tracy Kittredge of the Rockefeller Foundation in Vienna. O. Morgenstern had arranged this, convinced that AW would benefit from some studying time in the United States or England, especially concerning his engagement in timeseries problems.[113] But nothing came out for AW. He had to continue his search for a solid job. In the late 1935 AW, for a short time, saw a possibility in Palestine, but it smashed. His hope had been Jacob Fraenkel at

Jerusalem (cp. Table 10). AW wrote back to K. Menger to go to Palestine anyway, supposed he would get a entry permit and had enough money for a travel. His hopes got battered. So AW went back to Cluj, to work on mathematical questions of metric geometry and economics. But it was difficult as he had to do work with his brother (see letter to K. Menger below). Unflinchingly he looked forward, hoping for a return to Vienna and 'his' Colloquium.

Again on K. Menger's insistence, O. Morgenstern continued to plague the Rockefeller Foundation concerning AW and the possibilty for a fellowship. It is interesting to see that since 1936 the question of AW's origin more and more took on a central position in the correspondence between the Rockefeller Foundation and the Institute of Trade Cycle Research. As a result T. Kittredge again interviewed AW in Vienna. It was in the beginning of February 1936. T Kittredge immediately wrote back to Van Sickle (a former member of the Mises' circle and leading functionary of the Foundation) in New York. [114] Van Sickle replied to Kittredge: "*Although Wald's work is too mathematical for me to have any opinion based upon direct examination of his publications, I have no doubt that he is one of the very ablest of the men working upon problems of statistical technique as applied to business cycle analysis. It is a pity that his nationality and race combined make his future so precarious*. [...] [However, we] *have given so many fellowships to Morgenstern's group that I think we should lay our emphasis elsewhere for a while after we have made an award to Dr. John. Wald should be kept under observation, but I am not inclined to recommend any early award*". [3] Only a few months later, it was in July 1936, Kittredge yet again made an interview with AW. Morgenstern wished a study visit for AW in Princeton, either at the department of mathematics, or at the IAS (Institute for Advanced Study). Kittredge again diagnosed that for AW, with his (eastern) Jewish background, it would be very unlikely to get an university employment, or to "*ever become a permanent member of the staff of the Institute*". He advised again against funding AW. [116] In September 1936 nothing had happened in favour of AW. Van Sickle noted about AW, that he was "*obviously a man of exceptional ability but, unfortunately, a man without a country*. [...] *It is impossible to foresee what the future holds in store for him. His development should be kept under observation as he may prove in time to be one of those rare individuals whom we are justified in aiding regardless of immediate prospects. It is hard on him, but I am satisfied that we should not recommend him for a fellowship in the near future*". [117]

AW tireless set himself to work. Additionally to his papers in mathematical economics which he mainly had written in Vienna AW worked on the problem of consistency of the concept of a 'Kollektiv' (cp. in Appendix B the years 1937 and 1938). AW solved the problem, at that time a "*difficult and a noteworthy achievement*" and also "*an important step in von Mises' axiomatization of probability*". [Wolfowitz (1952)] In Wolfowitz words the "... *the problem* [of the 'Kollektiv'] *was difficult chiefly because of the way it was put. A simple consequence of the strong law of large numbers for identically distributed and independent variables already has as a consequence that almost every sequence of observations is a Kollektiv. Thus the modern measure-theoretic approach to the axiomatization of probability theory does away with the need for this pretty piece of work by Wald*". [Wolfowitz (1952)] Anyhow the work of AW eventually gave von R. v. Mises' theory a substantiated mathematical foundation, providing an explication of von Mises' intuitive notation of a gambling system in a way that the axioms could be formulated and proved to be consistent.

Table 10. AW to K. Menger, Cluj, Spring 1936 (KM Archive, Box 1, Folder 21)

Very honoured and dear Professor!

Thank You for your friendly letter. Above all I congratulate cordially to the joyful and a long time expected event. I hope that both, your wife as well as the little baby, are quite well. It makes me happy to know that in September with my return to Vienna i will meet the upcoming great mathematician.

I have sent the Separata and the book about the 'Saisonschw.' to professor Fraenkel. I am happy about the favorable prospects to get a job in Jerusalem. Of course i am pretty sorry to have to leave the Colloquium. I would be even more delighted, if I could get a post with the possibility to continue our cooperation in future. Perhaps it will turn out like this some day. I am back again since Wednesday of the previous week. During this time I have mainly studied English and i made some good progress. On Saturday I drive to Tilurta where I also have been the year before, and i will return on August, 15 to Cluj. Where will you spend the summer? I wish you a pleasant holiday and a good recreation.

With the best greetings
Faithfully A. Wald

Sehr verehrter und lieber Herr Professor!

Vielen Dank für Ihren freundlichen Brief. Vor allem gratuliere ich Ihnen herzlichst zu dem freudigen und lange erwarteten Ereignis. Ich hoffe, dass es sowohl Ihrer Frau Gemahlin wie auch dem kleinen Baby recht gut geht. Es freut mich schon im September bei meiner Rückkehr nach Wien den zukünftigen grossen Mathematiker kennenzulernen.

Die Separata und das Saisonschw.-Buch habe ich Professor Fraenkel eingesendet. Es freut mich sehr, dass so günstige Aussichten bestehen in Jerusalem eine Stelle zu bekommen. Es wird mir nur leid tun das Wiener Kolloquium zu verlassen. Noch grösser wäre natürlich meine Freude, wenn ich eine Stelle bekäme, wo ich mit Ihnen weiter zusammenarbeiten könnte. Vielleicht wird es auch einmal dazu kommen. Ich bin seit vorige Woche Mittwoch hier. Während dieser Zeit habe ich hauptsächlich Englisch gelernt und habe bereits gute Fortschritte *gemacht. Samstag fahre ich nach Tilurta wo ich auch voriges Jahr war, und komme am 15. Aug. nach Cluj zurück. Wo werden Sie den Sommer verbringen?Ich wünsche Ihnen recht angenehme Ferien und gute Erholung.*

Mit den besten Grüssen

Ihr sehr ergebner

A. Wald

2.3.1 AW's mentor and teacher

Karl Menger was born in 1902. His father was the Austrian economist Carl Menger (the founder of the Austrian School of Economics whose famous 'Principles of National Economics' ('Grundsätze der Volkswirtschaftslehre') from 1871 introduced the theory of marginality at the same time as W.S. Jevons and L. Walras. [Schumpeter (1954)]) Menger's mother Hermione was a noted novelist and a musician. His father was tutor in economics to Crown Prince Rudolph (the heir-apparent of Kaiser Franz Joseph I of Austria and his only son, who tragically sealed the fate of the Habsburg monarchie through his death in the so-called Mayerling incident, where he and his mistress, baroness Mary Vetsera, came in January 1889 to death, apparently through suicide at his Mayerling hunting lodge). His uncle Anton Menger was a renowned professor of law [Johnston (1972)], and his second uncle Max Menger a liberal deputy in the 'Reichsrat' for more than 30 years. [Menger (1998)] So without overstating, it easily can be seen that Karl junior was born into a distinguished and successful familiy. Karl Menger attended the Döblinger Gymnasium in Vienna (1913-1920), where two of his fellow students were the later Nobel Laureates Richard Kuhn and Wolfgang Pauli. Another classmate of Menger was Heinrich Schnitzler, the son of Arthur Schnitzler. Through Heinrich Karl Menger was able to seek some less positive advice of A. Schnitzler concerning his attitudes towards literture, a circumstance that finally drew Menger's interests to his real destination, the mathematics. [Menger (2002)]

In the autumn of 1920 (it was Einstein's peak time) Menger at first matriculated to theoretical physics (he attended lectures of physicist Hans Thirring), but soon began to drift towards pure mathematics. That change in his scientific focus was induced through a charismatic new professor for mathematics: Hans Hahn. Inspired by Hahn he started his first research attempts in geometry. But weakened by to much working in unheated libraries, Menger got infected by tuberculosis (also called 'Morbus Viennensis') in 1921, then a deadly illness for thousands, and was forced to leave Vienna for a one year stay in a sanatorium in Aflenz (lying in the mountains of Styria). Whilst in Aflenz, Menger lost both parents. In 1923 Menger made his first publication. Prior to 1923 he already had done a couple of notes on dimension and curves, but these were not published until 1929. [118] Menger's most productive phase was between 1928 to 1937. Besides his own research, he acted as a supervisor for the second edition of his father's 'Principles', which "... *included a wealth of revisions*.". [Menger (2002)]

K. Menger, not only interested in Luitzen E. J. Brouwer's fundamental work in topology and his philosophy of intuitionism (which he regarded as an alternative to E. Mach's positivism), supported with a Rockefeller scholarship, went in 1925 to Amsterdam. At the Amsterdam University he worked for two years as assistant professor for L. Brouwer. Their relationship was more than complicated. [Menger (1979)] Their differences finally exacerbated when Menger felt misrepresented in his contributions to dimension theory by L. Brouwer. [Menger (2002)] The salvage for K. Menger came from Vienna. In 1927, K. Menger, at the age of 25, was invited by H. Hahn, to accept the chair of geometry at the University when Kurt Reidemeister had left for Königsberg in 1925.

Figure 1. Cover of the 1994 Springer Edition of the "Reminiscences of the Vienna Circle and the Mathematical Colloquium" by Menger and Contributors, published posthumously [Menger (1994)]

In his posthumously published 'Reminiscences' [Menger (1994)] K. Menger gives a clear view of the topics that absorbed the participants of the Vienna Circle like L. Wittgenstein and Gödel. Especially with K. Gödel K. Menger held a lasting friendship. There is also a section about Menger's time at Harvard and the Rice Institute in Houston, Texas during 1930 and 1931, where he met most of the leading US mathematicians of this time. There Percy Bridgman and Emil Post made the strongest impression on him. K. Menger regarded P. Bridgman as the successor to E. Mach. And admired Post, who together with Haskell Curry, was a source for K. Menger's later work in the algebra of functions.

Elaborating his ideas on intuitionism (argueing against Brouwer) K. Menger went over to questions on the uniqueness of language and logic. This opened him the doors to the Vienna circle, where he won over Carnap to his ideas. In

1932 Menger joined the Vienna Circle. [Menger (2002)] In 1931, in tandem with the Vienna Circle, Menger ran his own circle, the Mathematical Colloquium. There Gödel announced his epoch-making incompleteness results. Menger published their advances in the 'Ergebnisse eines Mathematischen Kolloquiums' between 1931 and 1937. Even Einstein and Schrödinger were among the speakers. When Hahn died in 1934 Menger could not get his chair because it was abolished. Also his hope to follow W. Wirtinger as full professor had been destroyed.

Still shocked by the death of Schlick, Menger went in 1936 to the International Congress of Mathematicians in Oslo, where he acted as one of its vice-presidents. There arrived, he told the terrific situation in his country to friends and colleagues. Shortly thereafter Menger received a cable offering him a professorship at the University of Notre Dame. He accepted and finally arrived at south Bend (with his family) in 1937. In March 1938, the month of the annexation, Menger resigned his professorship in Vienna (and never return to Austria until 1963). K. Sigmund writes: "*After the war, the reconstruction of the bombed-out State Opera was accorded highest priority by democratic new Austria. Men like Menger, however, were politely told that the University of Vienna had no place for them*.". [Sigmund (1995)] Contrary to many European intellectuals Menger felt at home in America. In the years 1937 to 1946 he organised a mathematical colloquium at the *University of Notre Dame*, modeled after his Vienna Colloquium. He also resumed the EemK with the new title 'Reports of a Mathematical Colloquium, Second Series'. Eight issues appeared between 1938 and 1946. But the war thwarted him. Since 1941 the academic life in the US was cumbered. Hence the second Mathematical Colloquium and its proceedings failed to evolve any influence.

L. R. Ford, whom Menger knew from the time at Rice in 1931, meanwhile hold the post of the mathematics department chairman of the newly founded Illinois Institute of Technology in Chicago. In 1946 K. Menger was asked to join the IIT as professor. Menger accepted and worked there from 1946 to 1971. With his change to IIT, Menger discontiued the Series. He also taught for a time at Duke University but remained in Chicago for the rest of his life. K. Menger loved Chicago, a cosmopolitan town, where Carnap and colleagues organized their 'Chicago Circle'. In 1971 K. Menger retired from IIT. The university of Vienna never had tried to call him back, although he never officially had abrogated. In 1938 Menger had only asked for an 'extended leave' to anticipate his dismissal NSDAP leaders.

1963 was the first return of Menger to Austria. He was invited from the IHS ('Institut für höhere Studien'), founded by O. Morgenstern and P. F. Lazardsfeld in the same year. When Menger died on October 5, 1985 no obituary notice appeared, except in Austria. [Hlawka and Menger (1986)] His career had a time span of over 60 years, and in that time he had published 234 papers, 65 of them before his 30th birthday. Mengers scientific activities covered fields of logic, set theory, differential geometry, calculus of variations, graph theory, complex functions, algebra of functions, quantum theory, economics, philosophy, and pedagogy. In present-day terms Menger's geometry also includes general topology. He wrote 4 important papers entitled 'Untersuchungen über allgemeine Metrik', which not only laid the groundwork for the Vienna Colloquium, but also was the beginning of distance geometry. [Blumenthal (1970)] Above all his early work makes him outstanding in the history of mathematics. [Kass (1996)]

Figure 2. Karl Menger in front of the Vienna University around 1925 (source: www.univie.ac.at). One of his famous citations : "Nicht etwa, daß bei größerer Verbreitung des Einblickes in die Methode der Mathematik notwendigerweise viel mehr Kluges gesagt würde als heute, aber es würde sicher viel weniger Unkluges gesagt.” ("Not that, if one were to spread the insight into the methods of mathematics more widely, this would necessarily result in many more intelligent things being said than today, but certainly many fewer unintelligent things would be said.").

2.3.2 Karl Menger’s workscope

In March 1921 Hans Hahn became professor of the mathematical department in Vienna. One course of H. Hahn K. Menger attended was titled ‘What's new concerning the concept of a curve’. This was a moment of initiation for Menger. The first problem H Hahn posed was to give a precise definition of a curve, which indeed no one had been able to do before in a satisfactory way. Hahn lectured about a series of unsuccessful attempts made by Cantor, Jordan, and Peano. Hahn himself, stimulated by Peano and Hilbert, had shown (cp. the later so-called Hahn-Mazurkiewicz theorem) that every compact, connected and locally connected set is always the continuous image of an interval. The topology Hahn used was new to Menger, but he "*was completely enthralled and left the lecture room in a daze*". [Menger (1994), p41] Within a week Menger gave a new definiton of a curve, knewing recent commentaries by Hausdorff and Bieberbach to that problem [Menger (1998)], and showed it to Hahn still before the second lecture has taken place. Hahn recognized Menger's attempt as promising and encouraged him to work on it further. Neither Hahn nor Menger knew about Poincaré's advances, going in a similar direction. [Menger (1994)] Before leaving Vienna weeks later, caused by tuberculosis, he "*in a feverish haste ... wrote down his preliminary results on curves, and deposited them in a sealed envelope at the Academy of Science*". [Menger (1998)] In the sanatorium Menger elaborated his theory of curves and dimension and submitted in 1922 a paper to the 'Monatshefte für Mathematik und Physik'. His contribution contained a recursive definition of dimension in a separable metric space. P. S. Urysohn (who had died in an accident before he could publish his paper on dimension) simultaneously and independently of Menger developed an equivalent definition. The Menger-Urysohn definition has become a cornerstone in the theory of dimension. [Hurewicz (1941)] Against all odds he returned completely recovered to the Vienna University, meanwhile having "*developed a full-fledged theory of curves*". [Menger (1998)] This advance let *Menger* earn in 1924 his doctorate in mathematics with H. Hahn.

Before and during his stay in Amsterdam Menger showed a deep interest in foundational problems. One of his first papers in this field was a kind of ‘dictionary‘ for Brouwer’s often “*obscure*” mathematical definitions. [Menger (1994)] In 1927 Menger was glad to leave Amsterdam since he had had heavy disputes with *Brouwer*. He was angry that Brouwer did not mention his contributions in an adequate form in an posthumous publication of Pawel S. Urysohn’s work. Therein l. Brouwer rather stresses the link between him and P. Urysohn (referencing a little-known note from 1913, [Menger (1998)]).

In 1926 Menger got his habilitation in Vienna and was appointed ‘Extraordinarius für Geometrie’ in February 1927. [Menger (1998)] Menger also became a member of the Vienna Circle, where he shone bright with his extensive knowledge of Hahn’s logistic and Brouwer’s intuitionistic approach to foundational problems of mathematics. His ‘algebra of geometry‘ became “*a leading theme of the Colloquium, and soon played an important role in John von Neumann’s mathematical foundation of quantum mechanics*.“. [Menger (1998)] When appointed professor Menger started an extensive program for lectures in geometry: Euclidean, affine, projective, differential and general (then called set-theoretic) geometry should be covered. During 1927 and 1928, preparing material on the foundation of projective geometry, Menger constructed an axiomatic basis for principal features like joining and intersecting. In 1928 Menger already had developed the lattice-theoretic tools for the n-dimensional projective geometry. [119] In this regard Menger criticized D. Hilbert’s and Oswald Veblen’s methods: D. Hilbert required a different primitive for each dimension and Veblen gave points a distinguished role aside the fact that usually hyperplanes play the same role as points. J. v. Neumann in contrast took a position accordant to Menger’s, seeking “*to complete the elimination of the notion of point (and line and plane) from geometry*” [Neumann (1936)]. He refers to Menger as “*the first to replace distinct classes of ‘undefined entities’ by a unique class which consists of all linear subspaces of the given space, an essential part of his system being the axiomatic* *requirement of a linear dimensionality function*.". [Neumann (1936)] K. Menger subsequently defined a dual set of axioms for the system. In the same year K. Menger collected his topological results in the 'Dimensionstheorie'. [Menger (1928)] Exactly fifty years later, J. Keesling writes reviewing [Keesling (1958)]: "*This book has historical value. It reveals at one and the same time the naivete of the early investigators by modern standards and yet their remarkable perception of what the important results were and the future direction of the theory*". In [Menger (1994)] a famous theorem of him can be read: Every n-dimensional separable metric space is homeomorphic to a part of a certain 'universal' n-dimensional space, which can in turn be realized as a compact set in (2n+1)-dimensional Euclidean space. The universal 1-dimensional space appears later in Mandelbrot's 'Fractal Geometry' [Mandelbrot (1982)], there it is called 'Menger universal curve' or 'Menger sponge'.

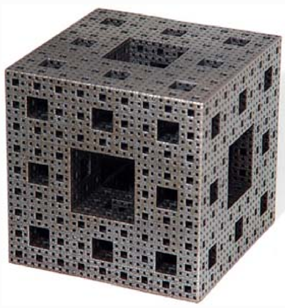


Figure 3. The 'Menger Sponge', a fractal which is the three-dimensional analog of the so-called *Sierpinski* carpet. For the first time described by *K. Menger* in 1926 [Menger (1993)]

In the early 1930s K. Menger developed a method of a general curvature of an arc in a compact and convex metric space. S. Kass [Kass (1996)] describes this topic as following: "*Consider a triple of points of an arc A, where A is an ordered continuum, not necessarily described by equations or functions. The triangle inequality implies the existence of three points in the Euclidean plane isometric to the given triple, and their Menger curvature is the reciprocal of the radius of the circumscribing circle. This curvature is zero if and only if one of the points is between the other two. Menger defined the curvature at a point of A to be the number (if it exists) from which the curvature of any three sufficiently close points in the Euclidean plane differs arbitrarily little*". Many achievements concerning K. Menger's concept were made by F. Alt and K. Gödel. [Gödel (1986)] The extension to higher-dimensional manifolds was done by AW, who obtained a fundamental new way of introducing the Gaussian curvature. K. Menger commented AW's result with the words: "*This result should make geometers realize that (contrary to the traditional view) the fundamental notion of curvature does not depend on coordinates, equations, parametrizations, or differentiability assumptions. The essence of curvature lies in the general notion of convex metric space and a quadruple of points in such a space.*". [Menger (1952)] In 1932 K. Menger's 'Kurventheorie' (in collaboration with G. Nöbeling) appeared [Menger (1932)], therein is contained K. Menger's 'n-Arc Theorem': Let G be a graph with A and B two disjoint n-tuples of vertices. Then either G contains n pairwise disjoint AB-paths (each connecting a point of A and a point of B), or there exists a set of fewer than n vertices that separates A and B. In an issue of the Journal of Graph Theory [Menger (1981)], dedicated to Menger's work, F. Harary calls this theorem "*the fundamental theorem on connectivity of graphs*" and "*one of the most important results in graph theory*". K. Menger's 'New foundations of projective and affine geometry' of 1936 [Menger (1936) a,] gave an axiomatic characterization of finite-dimensional projective spaces. This was a thematic highlight in the Vienna Colloquium. It also played an important role in J. v.

Neumanns mathematical foundation of quantum mechanics.

In 1939 K. Menger persuaded K. Gödel to visit him at *Notre Dame,* but was not able to convince him to stay there and accept a position. During this time K. Menger's interests in mathematics broadened and he started to do research on hyperbolic geometry, probabilistic geometry and the algebra of functions. Menger's work on geometry failed to have the impact that had his work on dimension theory before. This is due to the fact, that his working area simply was not asked for, particularly in the pre-war US. During the war years Menger taught navy officers-to-be in the V-12 program [120], what motivated him to try a new formulation of the foundations of mathematics. He published a monograph called 'Algebra of analysis'. [Menger (1944)] Therein he aimed at a systematization of an mathematical basis for analysis. The man from the Vienna circle with a strong emphasis on clear thoughts, gave still more transparent notations and a reformulation of calculus in 'Calculus, a modern approach'. [Menger (1953)]) Written in a rigorous style, the book radically broke with the usual approach. But this work never got a serious attention. K. Menger sent a copy to A. Einstein, who replied positively but warned him to do too much "*housecleaning*". [Keesling (1958)] The fact that his book was ignored saddenend Menger deeply. Menger also wrote on hyperbolic geometry and formulated a statistical notion of distance, which he wanted to analyse jointly with AW. [Menger (1998)] But AW died tragically in 1950 in India.

In the note 'Statistical Metrics' [Menger (1942)] K. Menger tried to resolve the interpretative issue of quantum mechanics by transferring the probabilistic notion from physics to the underlying geometry. He showed how to replace a numerical distance between points p and q by a distribution function. Studies of such spaces by numerous authors like Berthold Schweizer (a former student of Menger) and Abe Sklar (a colleague of Menger at IIT) followed. In 1951 K.

Menger formulated the idea of a 'hazy set' [Menger (1951)], in which the element-set relation gets replaced by the probability of an element belonging to a set. Hazy sets were rediscovered and renamed 'fuzzy sets' in the 1960s. [Bellman and Kalabe (1966)]. In 1979 Menger published a volume with selected papers. [Menger (1979)] During his last years he worked on a book about the Vienna Circle. He left the manuscript incomplete but evolved enough to be publishable. Almost 9 years after his death the 'Reminiscences' appeared. [Menger (1994)] An annual award sponsored by a fund mainly contributed by his family was established 1990 at Duke University. The majority of the income from this fund is spent as the annual 'AMS Karl Menger Memorial Award' at the International Science and Engineering Fair ISEF. Since 2007 at IIT yearly helds a honoring 'Karl Menger Lecture'. Parallel to the lecture at IIT the 'Karl Menger Student Award' for exceptional scholarship is granted.

His life long K. Menger was deeply interested in economics. [Menger (1979)] Already in young years he worked on economic topics but published them much later in a note called 'Bernoullische Wertlehre und Petersburger Spiel' (in the 6th volume of the EemK, [Menger (1934), EemK]). A full version was published years later as 'Das Unsicherheitsmoment in der Wertlehre'. [Menger (1934)] From O. Morgenstern we know that K. Menger's paper 'Das Unsicherherheitsmoment in der Wertlehre' [Menger (1934)] played a huge role to persuade J. v. Neumann to do a formal treatment of utility. [Kuhn and Tucker (1958)] Two essays of Menger appeared in the 'Economic Activity Analysis', a collection of essays about the Princeton research project edited by O. Morgenstern.

3

Circles and Schools

He [Haberler] *is far from understanding the fundamentals of these exact things. Mises talked pure nonsense.*

- O. Morgenstern about the econometric approach, Morgenstern diary, 31 Dec 1935, O. Morgenstern papers, Duke University, box 14, folder 6.

Topology and mathematical logic would have flourished in Vienna even without him [Menger]*, but not the mathematics of social and economic problems.*

- K. Sigmund in [Menger (1998)], p18.

[Morgenstern] *truly* [was] *no mathematician, but for mathematicians very interesting*.

- F. Alt in [Wiener Zeitung (2008)].

3.1 Vienna Circles

In 1925 Vienna was a centre for international academic discourses, with scientific topics on the agenda, and numerous connections between their protagonists. [Haller, Stadler (1993)] The most known schools were the ‘School of Theory of Law’ (founded by Hans Kelsen and his students), the ‘School of Logical Positivism’ (that operated actively between the years 1922 and 1938), with its leading figures Moritz Schlick, F. Waismann, Rudolph Carnap and O. Neurath. Alternative names for the School of Logical Positivism are generally known as ‘Vienna Circle‘ or the ’Vienna School’. [121] [Hacohen (2000)] Another famous circles were the ‘Kolloquium’ (or ’mathematical Colloquium’) of K. Menger, and the Mises group, centered around the famous ’Privatseminar’ of Ludwig von Mises), a multidisciplinal group of economists, sociologists and philosophers. Besides these there existed psycho-analytic circles of different orientations, “*a large number of social democratic-discussions groups*“, a group “*debating educational reforms*“, another one on “*art history*“ and also one on “*phenomenology*“. [Menger (1994)][Leinfellner (1993)]

In sharp contrast to the academic highlights in Vienna we see Hitler’s epigones agitating intensively since 1933 for the unification with Germany and heating up the political atmosphere inside Austria. With the beginning of the Austrian fascism under E. Dollfuss in 1934 (who indeed fitted himself with dictatorial powers), the intellectual life in Vienna became stunted. Nationalists of every shade began to fill the faculties of the Vienna University. Also other students became infected. In a row the Vienna Circle got disparaged and threatened. H. Hahn, always a progressive and leading force, died in 1934, and in June 1936 M. Schlick was shot dead by an addleheaded student. Though most of the original members of the circles had left Vienna before 1933 and started together with their numerous students new careers in Universities and research institutions around the globe. Main countries of exile were the anglo-saxon ones, where the 'Vienna' influence spread continuously. [Feichtinger (2001)] Regarding the development of philosophy of science after WWII in these countries we consider a considerable influence. [Feigl (1981)] The disappearance of the Vienna schools and circles has left a significant gap in the intellectual life of Vienna, which never became filled again, not even after WWII, when Austria faced an economic and political revival. [Dahms (1987)]

3.1.1 The Vienna Circle

The Vienna Circle was preceded by regular discussion meetings on the philosophy of science in an old Viennese coffeehouse [Uebel (2000), p65], where a couple of academics gathered from 1907 onward. This 'First Vienna Circle' (in fact [Uebel (2000)] distinguishes between a First and a Postmodern Vienna Circle) was pushed by people like *P. Frank*, *H. Hahn* and *O. Neurath*. [Dee (1996)][Uebel (1991)] Frank, the youngest of the group remembers [Uebel, (2003)]: "*After 1910 there began in Vienna a movement which* *regarded Mach's positivist philosophy of science as having great importance for general intellectual life ... [in trying] ... to supplement Mach's ideas by those of the French philosophy of science of Henri Poincaré and Pierre Duhem, and also to connect them with the investigations in logic of such authors as Couturat, Schröder, Hilbert, etc*.". Their meetings found an end in 1912, the year Frank went to Prague. [122] Hahn had left Vienna until 1921. Back in Vienna Hahn and Frank reactivated their meetings and in 1922 they convinced *M. Schlick* to enter into them. [Stadler (1997)] Four years later Schlick and Hahn persuaded R. Carnap to join, and another two years passed when their cirlce officially operated as association 'Verein Ernst Mach' (Ernst Mach Society) with Schlick as its leader. Under his direction, anew regular series of meetings began.

Quite a number of mathematicians belonged to the Vienna Circle, such as Hahn's sister Olga Hahn-Neurath, Hahn's students K. Menger and K. Gödel, as well as K. Reidemeister, G. Bergmann and T. Radakovic. [Menger (1998)] Additionally worth mentioning as members are the philosophers and historians like V. Kraft (author of 'Der Wiener Kreis' in 1950 [Kraft (1950)]) and E. Zilsel, a Jewish marxist, who was not shy to write critical words on the Circle. Further names are H. Feigl, M. Natkin and Rose Rand. Apart from them, the Vienna Circle had three dozen philosophers, logicians, mathematicians, as well as some natural and social scientists. [123] Regarding the development of the circle it is legitime to appoint Schlick, Neurath and Hahn as its founding members. [Uebel (2000)] The meetings of the Circle took place in a mean room on the ground floor of the building that housed the mathematical and physical institutes (situated in the Boltzmanngasse, Vienna). As the philosophers of the circle like F. Waismann and R. Carnap also were experienced in mathematics and interested in foundational questions, they took the opportunity to hear

Tarski's lectures on logic (1929) in Menger's Colloquium. [124] [Wolenski (1999)] This shall be one small hint showing their multiple interconnections.

The Vienna Circle entered public in 1929 with a manifesto entitled 'The Scientific World View' [125] (edited by Hahn, Neurath and Carnap). Following the thinking of *E. Mach* ("*Der Polyhistor war sowohl als Zentralfigur im Wien der Jahrhundertwende als auch als Reformer im Bereich der Naturwissenschaften bedeutsam*", "*The polyhistor was important by the central role he played in Vienna at the turn of the century and also as a reformer in the field of the natural sciences*" [Stadler and Haller (1993), p14]), they aimed to combine empiricism with modern logic to form a unified science without the need of any metaphysics. The Circle members made an effort to translate all available scientific statements into a comprehensive formal language in order to represent with it all objects of philosophy (i.e. statements, terms or theories). [Stadler (1997)], [Haller (1993)], [Kraft (1953)]

The magazine 'Erkenntnis' (since 1937 renamed in 'Journal of Unified Science') and also the series 'Schriften zur wissenschaftlichenWeltauffassung' ('The Scientific World View') und 'Einheitswissenschaft' ('Unified Science') worked as journalistic organs for the circle (cp. the disseration of Hammed Aziz Said [Said (1987), p66-69]). In 1938 in the US-exile the 'International Encyclopedia of Unified Science' started. [Geier (1998)] But only the first two volumes (of planned 26) were published between 1938 and 1969 (See Appendix D). With 'The Scientific World View' the Circle had achieved its international reputation. [Kruntorad (1991)] The series was dispersed when the Nazi party came to power in Germany and the emigration of its members began. Only Schlick remained in Austria, a deadly fault. Many years later in 1991 the institute 'Vienna Circle' was founded, devoted to the documentation, study and advancement of the philosophy of the Vienna Circle (cp. [Stöltzner and Uebel (2006)], therein contained the full program of the Vienna Circle from 1929 on as well an overview of the newest research for a new valuation of the Vienna Circle's work, see especially the introduction of the publishers, pIX-CIV).

3.1.2 Ludwig von Mises 'Privatseminar' 1920-1934

Figure 4. Ludwig Heinrich Edler von Mises (September 29, 1881 – October 10, 1973), a famous economist and exponent of the modern libertarian movement in economics. Mises did his studies at the Vienna University during the heyday of the 'Austrian School of Economics'. A student fellow of Mises was F.A. Hayek (who won the Nobel Prize in economics in 1974 partly for his work on business cycles).

Regular participants of Mises 'Privatseminar' (the 'Mises Circle') were several economists who years later in 1947 united into the 'Mont Pelerin Society'. Notable people were F.A. Hayek, Fritz Machlup (a former student of L. v.

Mises), the late Alfred Schutz and in his very early days, John van Sickle. It was regarded a great honor to be invited to the Mises' circle. Visiting scholars like Howard S. Ellis (University of California), Ragnar Nurkse (Professor of Economics in Columbia University, New York), Karl Bode (Stanford University, later Washington University), Alfred Stonier (University College in London), and many others took part. As assistent of Mises there of course was present O. Morgenstern, additionally the late Karl Schlesinger and Richard Strigl, two brilliant economists. Alike worth to be mentioned is Felix Kaufmann, philosopher of the Social Sciences, including the law and economics. [126] It was Kaufmann who brought in poetry and music to the meetings, that took place in the 'Café Künstler' in Vienna. Kaufmann wrote and performed altogether 28 songs. [127] Based on folk sound and known songs, Kaufmann did his lyrics as a mixture of High German and Austrian dialect with clever references to their debate topics and other internals of the Mises Circle. The Mises Circle of course had certain connections to the Vienna Circle, especially through Kaufmann, who himself had been a member of both circles. [The Austrian Economics Newsletter (1980)]

The Circle held each friday at 7 p.m. its meeting in Mises' office located in the building of the Chamber of Commerce on the Stubenring 5-12, Vienna. Usually Mises openend the session with an introductory paper or as alternative any other member started with a report on some problem of economics or on methods of the social sciences. Sometimes the opening lecture moved into a discussion on economic policy (Mises, as a leading head of the so-called Austro-Liberals, was a staunch opponent of any governmental interventionism). Their profound discussions lasted until 10 p.m. and got carried on in the nearby Italian Restaurant 'Ancora Verde' to have dinner. Later on they proceeded in the 'Cafe Künstler'. [Haberler (1961)]

In 1935 Mises went to Geneva and joined the Institut Universitaire des Hautes

Etudes Internationales, where he taught until 1940. [Hülsmann 2007] In that year he emigrated to the US as many students of him had done before. Those who remained in Vienna, felt lost and exposed to a insecure future. Kaufmann's song gives an expression to this situation:

Table 11. 'Elegy of the Mises Kreis' from F. Kaufmann's songs about the Mises Circle. [library.mises.org (2012)]

Klagelied des Mises-Kreises	*Elegy of the Mises Kreis*
Und der Kreis kraenkt sich sehr, *Wenn das Zentrum fort will ziehen* *Bleiben Peripherien so einsam und leer.* *Bald enstehen Krisenstadien,* *Denn es treffen sich die Radien,* *Die verbindenden Radien* *In keinem Punkte mehr.* *Unter Traenen ruft pi:* *"All ihr tranzendenten Zahlen* *Ihr ermesst nicht die Qualen der Melancholie.* *Viele meiner Naeherungswerte* *Schon die Sehnsucht fast verzehrte,* *Bis zur tausendsten Stelle* *Verwundet bin I!"* *Zentrum komm wieder her!* *In der altgewohnten Weise* *Fueg Dich ein Deinem Kreise,* *Wir warten so sehr.* *Alle Radien und Durchmesser* *Fuehlen sich dann taeglich besser* *Und der Jubel von pi,* *So was gibt es sonst nie!*	*And the circle grieves so.* *When its center starts to saunter,* *Its circumference feels empty, disheartened, and low.* *There are crises here to ponder* *Radii are left to wander* *Radii are left to wander with nowhere to go.* *Radii are left to wander with nowhere to go.* *And now pi cries in rue:* *"All you numbers transcendental,* *Melancholy and anguish are no match for you.* *Many of my estimations* *Are consumed by lamentations,* *I am mortally wounded to the thousandth place, it's true* *Mortally wounded to the thousandth place, it's true!".* *Center, come back again!* *Old familiar place assuming,* *We are bent on presuming our hopes aren't in vain.* *Radii and their fellows* *Ever gleeful, sadness mellows* *As for pi's jubilation, it cannot be restrained.* *As for pi's jubilation, it cannot be restrained!*

3.2 Menger's Colloquium and the beginning of mathematical economics

In his article 'Sull'indirizzo di idee e sulle tendenze prinzipali del colloquio matematico di Vienna' [Menger (1935)] Menger (also fluent in Italian) indicates the three main topics of his Colloquium: geometry, logic and the mathematical investigation of economics. An known alias for the 'Colloquium' is the term 'Vienna Colloquium'. as Menger's Colloquium met in Vienna from 1929 to 1936. [Stadler (1991)] It "*had a long lasting influcence on economic theory*". [Menger (1998)][Alter (1990), p14], [Menger (1998)] When we evaluate today Menger's role and the sum of his contributions, he can be seen "*as a catalyst for several essential developments in mathematical economics*". The colloquium is his "*most original contribution. Topology and mathematical logic would have flourished in Vienna even without him, but not the mathematics of social and economic problems*.". [Caldwell (1990)], see also therein Caldwell's altercation with the state of research about K. Menger in the section'What do we know about Menger?', ibid. pp313-348.

The at that time contemporary developments in geometry laid a heavy emphasis on coordinates in a way which implies its degeneration into an arithmetics of n-tuples. In consequence a distinct aim of Menger's Colloquium was the development of a geometry without coordinates, based on a set-theoretic foundation. [128] A further field of research was dimension theory, and questions on convexity and congruence. [129] [Menger (1994)] Besides, based on themes about exotic objects like the so called 'Peano continua' (showing clearly how misleading an intuitive approach can be) philosophers of the Vienna Circle got attracted to the Colloquium, for it encouraged their existing anti-intuitionistic attitudes. *H. Hahn* expresses this in drastic words: "*Intuition* *is not, as Kant thought, a synthetic a priori means of knowledge, but merely force of habit rooted in psychological inertia*“. [Menger (1994)]

After Menger's emigration to the US, his Colloquium further on was continued by AW and F. Alt. The letter below, written by AW to Menger on April 4, 1937, shows that Menger in the meantime had missed overall 4 meetings of the Colloquium. Even Gödel had participated, who occasionally lapsed into a mood to withdraw himself completely from public and even his friends. AW at that time already was on first-name terms with his former professor. While AW still was busy with the attempt of a complete axiomatisation of the theory of probability (in particular the concept of the Collectif), he anyhow gradually developed a strong interest in the price index theory to attack the problem of the measurement of the utility. In this regard AW characterizes Ragnar Frisch's ansatz as “*in certain aspects as correct. I believe, that with similar methods, as I determine the price index, one also will be able to measure the marginal utility of money*.”.

Table 12. AW to Menger, Vienna, April 4, 1937 (KM Archive, Box 1, Folder 21)

Dear Professor!
Above all, I would like to clarify a misunderstanding which possibly exists. When Alt and I visitied you lately before your departure, I completely overheard the statement you made on your farewell that the three of us shall drop all formalities in future. A few weeks ago, Alt mentioned this to me. It was therefore not in the least my intention to not accept your offer. I do hope you did not assume it the other way round. Behaving like this would be completely senseless for me. You know best how I cherish our friendship.

We gratefully received your detailed letter. I'm very happy that your lectures are of great interest to the audience there and that you also have sufficient time for your scientific work. I'm eager to learn the proof of Euler's equation. We will do our best that the colloquium book will be published as soon as possible. Of course I agree to the suggested amendment of the coversheet. Concerning the note to calculate the 2nd derivative, the situation is as follows: Dr. Schlesinger's secretary (at that time, Dr. Schlesinger made a journey and only returned a couple of days ago) called me a few days ago and told me to send you my manuscript along with the one of Dr. Schwarz on higher derivatives. I believe that except from Schwarz's note, it is not my manuscript you wish, but the manuscript of your book from Pauc. Therefore I have asked Pauc and Schwarz, that they will send the desired manuscripts as soon as possible. Pauc has done this immediately, but Schwarz has then not yet finished the manuscript. I was not thinking at all on my note, as we agreeded before your departure, that I will publish my note not till then it is more incorporated in detail. But as I understand from your letter, you want me to publish the idea in brief, therefore I will do this. The note of Schwarz will have been probably completed already. But i am not sure about this, as I have not seen it since then. Now I want to report on the meetings of the Colloquium. Until now there were held a total of four. During the first session I've talked about probability theory; I have shown some new theorems on the operation of 'division'. These results form part of my research for the purpose of a complete axiomatization of probability theory (in particular the generation of new collectivs from given ones). By the time I have everything completed, I will send the manuscript to you. In the second meeting Mostowski talked about Borel fields. It was quite interesting. He presented a number theoretical as well as a set theoretical interpretation.
[...]
Gödel takes part in the meetings on a regular basis. In the last two months I mainly dealt besides probability theory with the question of the price indices (cost of living index, income index, real income). I think having resolved the problem in a satisfying way that itself is mathematically interesting too. I will publish this in the Zeitschrift für Nationalökonomie. The manuscript already is handset and will appear in a few weeks. As soon as I get separata, I will send you one. I also think now to be able to tackle the problem of the measuring of the utility. A prerequisite for this was that the question of the price index had already been resolved.
[...]
R. von Mises has been here in Vienna two months ago. I met him and we have a lot of discussions about the probability theory. He finally begins to understand my work and is now very enthusiastic about it. He has sent me an expert opinion, of that I am allowed, as he writes to me, to make any use. I send herewith a copy of it, because you will be interested to know what he writes.

I still have sent my book to Hotelling in the summer of last year and at that time he responded with an acknowledgement of receipt. I wonder why he did not know anything about it. From

Schulz I got a letter a few weeks ago. He was very appreciative of my work 'Einige Gleichungssysteme der mathematischen Ökonomie', that I have sent him as a separatum. He sent at the same time separata of his works to me, but I have not yet found the time to read them. I will write him soon and send him a copy of my book. I got a lot of separata from Hotelling. In respect to my situation there has no substantial change occurred yet. From Palestine I have received no further message. Here at the Institute my fellowship runs until the end of this year. It is open whether it can be renewed for the next year. Not until this summer Morgenstern will be able to tell me something reliable on it. I have no choice but to wait with patience and hope that perhaps a favorable turn will come from somewhere. Do you have received my letter of January and the enclosed separatum? I think I have already reported everything in detail and hope to hear from you soon.

With cordial greetings in sincere friendship

P.S. Please also address greetings to your wife from me. How is little Karli? Is he well-behaved?

Lieber Herr Professor!

Vor allem möchte ich ein Missverständnis aufklären, das möglicherweise besteht. Als Alt und ich das letzte Mal vor deiner Abreise bei Dir waren, habe ich deine beim Abschied gemachte Bemerkung, dass wir drei von nun an zueinander per Du sein könnten, vollkommen überhört. Er vor einigen Wochen erfuhr ich es zufälligerweise von Alt. Es liegt daher nicht im geringsten eine irgendwie beabsichtigte Nichtbeachtung vor. Ich hoffe, dass Du dies gar nicht vorausgesetzt hast. Ein solches Verhalten wäre von mir vollkommen sinnlos. Du weisst ja selbst, wie hoch ich Deine Freundschaft schätze.

Deinen ausführlichen Brief haben wir mit bestem Dank erhalten. Es freut mich sehr, dass Deine Vorlesungen dort viel Interesse finden, und dass Du auch zur wissenschaftlichen Arbeit genügend Zeit hast. Ich bin schon auf den Beweis der Eulerschen Gleichung sehr gespannt. Was das Kolliquiumheft anbelangt, werden wir schauen, dass alles in bester Ordnung geht und dass es so rasch als möglich erscheint. Mit der vorgeschlagenen Änderung des Titelblattes bin ich selbstverständlich gerne einverstanden. Betreffend die Noten über zweite Ableitung ist die Sachlage die folgende: Dr. Schlesingers Sekretärin (er selbst war damals verreist und ist erst vor einigen Tagen zurück gekommen) hat mich vor einigen Wochen angerufen und sagte mir, dass ich mein Manuskript und das von Dr. Schwarz über höhere Ableitungen Dir einsenden soll. Ich glaubte, dass du ausser der Note von Schwarz nicht mein Manuskript, sondern das Manuskript Deines Buches von Pauc haben möchtest. Ich habe daher Pauc und Schwarz gebeten, dass sie die gewünschten Manuskripte Dir so rasch als möglich einsenden mögen.

Pauc hat es gleich erledigt, aber Schwarz hatte damals das Manuskript noch nicht fertig. An meine Note habe ich überhaupt nicht gedacht, umsomehr, da wir vor Deiner Abreise dabei geblieben sind, dass ich meine Note erst dann publizieren werde, wenn sie schon mehr in Details eingearbeitet sein wird. Wie ich aber aus Deinem Brief entnehme, möchtest Du doch, dass ich zumindest die Idee kurz publiziere, ich werde daher dies tun. Die Note von Schwarz wird wahrscheinlich bereits längst fertiggestellt sein. Sicher weiss ich das nicht, da ich sie seither nicht gesehen habe.

Nun möchte ich Dir über die Kolliquiumssitzungen berichten. Es waren bis jetzt insgesamt vier. Das erste Mal habe ich über Wahrscheinlichkeitsrechnung gesprochen; einige neue Sätze über die Operation der "Teilung" habe ich vorgetragen. Diese Ergebnisse bilden einen Teil meiner Untersuchungen zwecks einer vollständigen Axiomatisierung der Wahrscheinlichkeitsrechnung (insbesondere die der Bildung neuer Kollektivs aus gegebenen). Sobald ich alles fertig habe, werde ich Dir das Manuskript einsenden. In der zweiten Sitzung sprach Mostowski über Borel'sche Körper. Es war recht interessant. Er gab eine zahlentheoretische und eine mengentheoretische Deutung.

[...]

Gödel nimmt an den Sitzungen regelmässig teil. In den letzten zwei Monaten habe ich mich ausser Wahrscheinlichkeitsrechnung hauptsächlich mit der Frage der Preisindizes (Lebenskostenindex, Einkommensindex, Realeinkommen) beschäftigt. Ich glaube das Problem, das auch mathematisch interessant ist, befriedigenderweise gelöst zu haben. Ich publiziere es in der Zeitschrift für Nationalökonomie. Das Manuskript wird bereits gesetzt und erscheint in einigen Wochen. Sobald ich Separata bekomme, werde ich Dir eines schicken. Nun glaube ich auch, das Problem der Messung des Nutzens in Angriff nehmen zu können. Eine Vorbedingung hierfür war, dass die Frage des Preisindex bereits gelöst sei.

[...]

R. v. Mises war vor zwei Monaten hier in Wien. Ich bin mit ihm zusammen gekommen und wir haben viel über die Wahrscheinlichkeitsrechnung diskutiert. Endlich beginnt er meine Arbeiten zu verstehen, er ist jetzt sehr begeistert davon. Er hat mir ein Gutachten darüber eingesendet, wovon ich, wie er mir schreibt, jeden Gebrauch machen kann. Ich schicke beiliegend eine Abschrift ein, da Dich vielleicht interessieren wird, was er schreibt.

Hotelling habe ich mein Buch noch im vorigen Jahr Sommer geschickt und er hat mir den Erhalt desselben seinerzeit auch bestätigt. Es wundert mich, wieso er nichts davon wusste. Von Schulz habe ich vor einigen Wochen einen Brief bekommen. Er äussert sich sehr anerkennend über meine Arbeit "Einige Gleichungssysteme der mathematischen Ökonomie", von der ich ihm eine Separatum eingesendet habe. Er hat mir gleichzeitig Separata von seinen Arbeiten geschickt, die ich aber noch keine Zeit hatte zu lesen. Ich werde ihm demnächst schreiben und auch ein Exemplar meines Buches einsenden. Von Hotelling habe ich ebenfalls eine Menge Separata bekommen. In meiner Situation ist bis jetzt keine wesentliche Änderung eingetreten. Von Palästina habe ich seither keine weitere Nachricht bekommen. Hier beim Institut läuft mein Stipendium bis Ende dieses Jahres. Ob es für das nächste Jahr verlängert werden kann, ist noch fraglich. Erst im Sommer wird mir Morgenstern sicheres darüber sagen können. Es bleibt mir nichts anderes übrig, als mit Geduld zu warten und zu hoffen, vielleicht wird doch von irgendwo eine günstige Wendung kommen. Hast Du meinen Brief von Januar und das beigelegte Separatum erhalten? Ich glaube, dass ich über alles bereits ausführlich berichtet habe und hoffe bald wieder etwas von Dir zu hören.

Mit vielen herzlichen Grüssen und in aufrichtiger Freundschaft

P.S. Bitte richte auch Deiner Frau Gemahlin Grüsse von mir aus. Wie geht es dem kleinen Karli? Ist er brav?

The proceedings of the Colloquium were collected by K. Menger as booklets and published as ‘Ergebnisse eines mathematischen Kolloquiums’ (EemK) from 1931 onwards for the price of 2 ‘Reichsmark’ [H. Hahn (1931)], published by B. G. Teubner in Vienna. The EemK were de facto no standard reading among economists, not only because of the langague barrier and a title, that threatens the reader with more mathematics than any other economic publication does. Besides, the EemK were also not easy to receive aside from specialized libraries or directly from its originators. A further reason for its humble circulation among economists was that its authors were brilliant people in mathematics or statistics but usually not in common economics. Exceptional here is AW, who had knowledge not only in actuarial science (back in Cluj he had taken courses in that field) but also knew the writings of many economists, e.g. Max Weber (AW mentioned him in a letter to O. Morgenstern). With his expertise AW played indeed a key role in the evolvement of mathematical economics within the EemK. Hence almost “*15 years elapsed before the strength of their impact was felt*.” (Debreu in [Menger (1998)], and his “*echo was finally heard in Chicago in 1949*” (Debreu in [Menger (1998)]. When in 1949 in Chicago the famous linear programming conference, sponsored by the Cowles Commission, was hold, ”*from that time on things began to unfold rapidly*.“. [Menger (1998)]

Table 13. AW to Menger, Vienna, late June, 1937 (KM Archive, Durham, Box 1, Folder 21)

Dear Professor!

Already for a long time i wanted to write to you, because there is much to report, but i have been busy with various tasks, so I could not get around to do so earlier.
[...]
During the summer semester we had several meetings of the colloquium, which were quite interesting. ... Gödel has shown the consistency of the axiom of choice... Gödel also has shown a proof for the consistency of the the continuum hypothesis. But he neither is able to verify the independency of the axiom of choice nor the independency of the continuum hypothesis. ... Next week we will hold the final meeting of the colloquium. ... Volume 8 finally has been released. I just today have obtained 50 separata from the prob. calc.. ()
In early July I am going to Cluj to my parents and will stay there until the end of August. My address there is: Str Goga Octavian 7. During the months September and October I probably will stay in Geneva. Morgenstern even has spoken to the director of the "Geneva Research Centre", that one will give a scholarship for 2-3 months to me, therewith to study in Geneva the material about international price indices. This would be so pleasant to me, because in September the meeting of the Econometrica in Geneva takes place, and so I could attend. Prof. R. Frisch wrote a few days ago to me that he has read my paper on price indices with great interest, and that he considers the obtained results to be very important. Concluding, he writes, that he hopes to meet me at the Ecometrica conference, on which occasion he would like to discuss with me several problems in this field (also the measurement of the utility).
Two months ago R. v. Mises was in Vienna and asked me wether I would attend the meeting of the probability theorists held in October in Geneva (Frechet organizes it) and give a talk about my results on the foundations of probability. He has also offered me to arrange an officiall invitation for me and that I will get 250 Swiss francs for travel expenses. I of course told him yes. Just yesterday I received a letter from Mises, in which he writes that he will travel to Paris these days a. that he will everything discuss in person with Frechet. ...

I wish you a very pleasant summer and remain
with cordial greetings
Wald

Lieber Herr Professor!

Ich wollte Dir schon lange schreiben, da ich Vieles zu berichten habe; ich war aber so sehr mit verschiedenen Arbeiten beschäftigt, dasss ich nicht früher dazu kam.

[...]

Im Sommersemester hatten wir mehrere Kolloquiumssitzungen, die recht interessant waren. ... Gödel hat den Beweis der Widerspruchsfreihet des Auswahlaxioms vorgetragen. ... Gödel hat auch die Widerspruchsfreiheit der Kontinuumshypothese bewiesen. Er kann aber weder die

Unabhängigkeit des Auswahlaxioms noch die Unabhängigkeit der Kontinuumshypothese beweisen. ... Nächste Woche werden wir das letzte Kolloquium haben. ... Das Heft 8 ist endlich erschienen. Ich habe eben heute die 50 Separata aus der Wahrscheinl.rech. erhalten.

Anfang Juli fahre ich nach Cluj zu meinen Eltern und bleibe bis Ende August dort. Meine dortige Adresse ist: Str. Goga Octavian 7. In den Monaten September bis Oktober werde ich wahrschienlich in Genf sein. Morgenstern hat nämlich mit dem Direktor des "Geneva Research Centre" gesprochen, dass man mir für 2-3 Monate ein Stipendium gibt, damit ich in Genf die Materialien über internationale Preisindizes studieren kann. Dies wäre mir auch deswegen angenehm, weil im September die Tagung der Econometrica in Genf stattfindendet, und so könnte ich daran teilnehmen. Prof. R. Frisch schrieb mit vor einigen Tagen, dass er meinen Aufsatz über Preisindizes mit grossem Interessse gelesen hat und dass er die erzielten Resultate für sehr wichtig hält. Zum Schluss schreibt er noch, dass er hofft mich an der Econometrica Tagung zu treffen, bei welcher Gelegenheit er gerne verschiedene Probleme auf diesem Gebiet (auch die Messung des Nutzens) mit mir besprechen möchte.

Vor zwei Monaten war R. v. Mises in Wien und hat mich gefragt, ob ich nicht an der in Oktober in Genf stattfindenden Tagung der Wahrscheinlichkeitstheoretiker (Frechet organisiert es) teilnehmen und einen Vortrag über meine Ergebnisse bezüglich die Grundlegung der Wahrscheinlichkeit halten würde. Er hat mir auch angeboten, dass er veranlassen wird, dass ich offiziell eingeladen werde und dass ich 250 schweizer Franc für Reisespesen bekomme. Ich habe ihm natürlich ja gesagt. Eben gestern erhielt ich einen Brief von Mises, in welchem er schreibt, dass er in diesen Tagen nach Paris fährt u. wird alles mit Frechet persönlich besprechen. ...

Ich wünsche einen recht angenehmen Sommer und verbleibe

mit vielen herzlichen Grüssen

Wald

3.2.1 AW's compagnion: Franz Alt

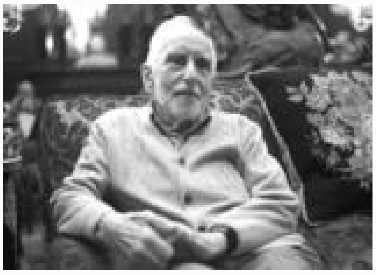


Figure 5. Franz Alt, born in Vienna in 1910, emigrated to the US in 1938. The pioneer in computer programming and fellow of K. Gödel revisited Vienna in May 2007 with the age of 96 to get honored with the ' Ehrenkreuz für Wissenschaft und Kunst I. Klasse' by the Austrian 'Bundespräsident' H. Fischer. Source: Wiener Zeitung, May 18, 2007. [Wiener Zeitung (2008)]

F. Alt wrote his doctoral thesis, titled 'Metrische Definition der Krümmung einer Kurve' ('metrical definition of the curvature of a curve'), with Menger in 1932. Exactly at that time Menger accomodated Alt to his Colloquium (where indeed only invited people were allowed to participate). Alt published many of his papers in the EemK and even became together with AW coeditor of the last volume. His most influential teachers and mentors were Menger and Hahn. [130]

It was the increasing antisemitism that closed not only for Alt all doors within the academical hierarchy. Consequently he absolved in 1933 the examination for a high school teaching post and completed 1934 his probationary year, but "*it disappointed me and convinced me about the fact that I would not have become a good teacher*.". [Wiener Zeitung (2008)] But even the high school job was blocked by political circumstances: through rationalization measures at the high schools the expensive physics laboratories were closed and former teachers of physics foremostly had the right on the freed mathematics posts. So Alt was unemployed for over 2 years and had sufficient time to take part (unpaid like AW) in the activities at the mathematical Institute of the Vienna University (it were the years between 1935 and 1938). He continued this engagement, also when starting "*of necessity at a life assurance company*". [Wiener Zeitung (2008)] With this work "*i had to make the best of a bad bargain*". [Wiener Zeitung (2008)] [Punzo (1989)]

Through Menger, Alt also knew O. Morgenstern. In Alt's own words Morgenstern (at that time director of the institute for business cycle research) was "*in fact no mathematician, but for mathematicians very interesting*.". [Wiener Zeitung (2008)] Indeed it was Morgenstern who wakened Alt's interest in mathematical economics. Thus inspired, Alt wrote several articles in economics (a field that will be later called 'econometrics'), among them 'Über die Messbarkeit des Nutzens' ('About the measurability of the utility') in 1936 [Alt (1936)], a work that meanwhile is a classic in economic literature.

3.2.2 The 'Colloquium' in brief

During 1928-29, "*... students at the 'Seminar für Mathematik' (the forerunner of the institute of mathematics) of the University of Vienna asked their professor, Karl Menger, to direct a mathematical colloquium*.". [Menger (1998)] He agreed and began to organise one in form of a discussion circle (cp. the introduction to the EemK, Vol1, 1931, [Menger (1931)]) with regular meetings every Tuesday, but only during each running semester. "*It had a flexible agenda including lectures by members or by invited guests, discussions of unsolved problems and reports on recent publications*.". [Menger (1998)] Menger's most capable doctoral students like AW, Gustav Beer or F. Alt got his allowance to participate. Menger additionally drew in O. Taussky, K. Gödel (he had completed at the age of 23 his doctoral thesis under Hahn's supervision), Hans Hornich (who wrote his dissertation with Menger, coadvised by W. Wirtinger, in 1929 titled ‘Über einen zweigradigen Zusammenhang’) and Georg Nöbeling. F. Alt remembers himself: "*Manche von ihnen* [Mengers Doktoranden] *blieben auch nach ihrer Promotion Teilnehmer. Dazu gehörten Kurt Gödel, Abraham Wald, Olga Taussky, Gustav Beer und zuletzt ich*.” (“*Some of them* [Menger’s doctoral students] *also stayed participants after their graduation. Including Kurt Gödel, Abraham Wald, Olga Taussky, Gustav Beer and at last myself*”). [Wiener Zeitung (2008)]

Beginning in autumn 1929, Menger recorded their meetings and published them (in contrast to the Viennese Circle) in a form of notes, at first with the ‘Teubner Verlag’ and then by the Austrian publishing company ‘Deuticke’ (today ’Zsolnay and Deuticke’), titled ’Ergebnisse eines mathematischen Kolloquiums’ (EemK). The EemK were written in a rigorous style at a time “*when logical rigor was not believed to be a basis for theorizing on economics*” (G. Debreu in [Menger (1998)]). All in all eight booklets appeared between Nov. 1931 and Nov. 1936. The ninth, already set booklet, could not be published in 1938 due to the complicated political circumstances. [131] “*Kurt Gödel was one of the main contributors and co-editors (besides K. Menger and G. Nöbeling) of the Ergebnisse eines mathematischen Kolloquiums which also contain important papers by A. Tarski and other logicians*.”. [Menger (1998)] In the field of topology (at that time frequently called ‘mengentheoretische Geometrie’ [Menger (1998)]) the EemK contain also important proceedings. In mathematical logic and especially in mathematical economics the EemK is a profound source. Casual visitors like *K. Borsuk*, *E. Čech* and *G. T. Whyburn* held lectures at the Colloquium. Other important visitants were *John von Neumann*, *Norbert Wiener* and *Marston Morse* (See Appendix F). [Punzo (1989)] Especially AW quickly became “*one of the major figures of the*

Colloquium, and one of the first co-editors of the [Eemk].". [Menger (1998)]

3.3 AW and the Institute for Trade Cycle Research

The 1930s were crucial years for the Austrian School of Economics. During that phase it was at the beginning on the top of its influence to loose more and more its position until it got dissolved after 1945. Vienna had always been the center of communication for all the members of the school. Outpouring from their capital city, the economists of the Austrian School ever tried to gain influence on the course of the Austrian economic policy, in particular during the Great Depression. Critics today regard the weak performance during the 1930s as a result of the damaging effect of the Austrian School. In 1934 the parliamentary democracy found its end and was replaced by the 'corporate state', a special form of fascism. The question is how the Austrian School mastered its progression until 1938, the year of the annexation. [132] [Tàlos and Neugebauer (1988)], [Senft (2002)][Bischof, Pelinka and Lassner (2003)] In the next section we try to describe the state of the Austrian School in the begining of the Austrian fascism regarding its position within an undemocratic political system. The evolvement of the school is accompagnied by the change of its leaders during this time, from Mises to Morgenstern, two pretty divergent men, whereby Mises symbolizes the 'classical' liberal ethos in economics and Morgenstern stands for a kind of technocracy in economics, a mindset that was in some aspect helpful during fascism.

We see two different strands within the Austrian School: on the one hand the

'Austroliberalism' [133] (its tradition goes back to Carl Menger, Eugen Böhm-Bawerk [134] and in the beginning of the 1930s L. v. Mises), and on the other hand a less coherent group around F. Wieser, who held a less liberal position in theory and in practice. Known epigones of Mises were F. Hayek, G. Haberler and F. Machlup. [135] Prominent economists around F. Wieser were Hans Mayer, and among the younger ones O. Morgenstern, A. Gerschenkron. There were other economic strands like that of the Austromarxists and that of the romantic tradition of 'universalism'. Aside from scientific and journalistic publications, an important part of the economic discourse was organised in cirles like Mises' private seminar, the 'Geistkreis' (the circle of spirit) or the seminar of the 'Nationalökonomische Gesellschaft' (Austrian Economic Society). [Craver (1986), pp13-18]. [Müller (1988), pp51-69] Mises official job was that of a secretary of the Chamber of Commerce, but inofficially he was the dominant figure in economic theory as well as in economic policy. At Mises' initiative the Austrian Institute for Business Cycle Research was founded in 1927, essentially financed by money of the Rockefeller foundation. The first director was Hayek (until 1931). Haberler and Machlup were members of the staff from early on. Mayer was the only member of the Austrian School with a chair at the University of Vienna [136], but his influence was not lasting. [Craver (1986), pp10-13] Morgenstern worked as Mayer's assistant, and after three years of working with a Rockefeller fellowship abroad, he succeeded Hayek as the director of the Institute and remained in this position until 1938.

Not only the rising hostality towards liberals and Jews, but also the dwindling career opportunities in 1934, let many members of the Austrian School leave the country. [137] Hayek already had left in 1931 for a professorship at the London School of Economics. [138] Haberler was at the League of Nations in

Geneva, working on business cycles. Machlup spent a two-year Rockefeller Fellowship in the United States and in Great Britain. In September 1934 Mises quit his position and entered into the Geneva Institute of International Studies, where he started a professorship. [139]

3.3.1 Mises, ambiguous leader of the Austrian School

In Mises' 'Liberalismus' which appeared in 1927 [Mises (1927)], in his 'Kritik des Interventionismus' [Mises (1929)], and also in Morgenstern [Morgenstern (1934)] one can read about both authors concerns on potential conflicts between a liberal economic policy and the fascist state. More barefaced statements on the special political situation in Austria can be found in unpublished notes and correspondence. [140] V. Mises followed the ideas of classical liberalism in a straight way and was a convinced democrat. [141] [Mises (1978) a,], [Mises (1978) b,] This picture gets strangely blurred when he writes "*that Fascism and similar movements ... are full of the best intentions and that their intervention has, for the moment, saved European civilization*" [Mises (1978) b, p51], but anyhow he warned that if democracy "*is on the road to destruction ... force is never a means of overcoming these difficulties*". [Mises (1978) b, p45] Mises oversaw the danger from the right spectrum, but identified the conflict between liberalism and the momentary democracy caused by a misdirected public opinion that followed socialist ideas. [Mises (1978) b, p173] He accused especially the Social Democrats and in particular the trade unions to be the paralyzing forces in Austria. Mises wrote about the "*terror apparatus ... of the Social Democratic party*" [Mises (1978) b, p88], and further regarded strikes organized by trade unions as illegitimate means of violence. [Mises (1931), p17] Machlup agreed with Mises breakdown of

Austria's economy and policy. [142] Mises and Machlup regarded the Austrian democracy as a failing experiment. Of course they also ascribe its failure to economic faults: inflationism, excessive taxes, social burdens and rigid wages. [Klausinger (2001)] Liberal rules (symbolized by the gold standard [143]) in their eyes require the avoidance of inflation [144] and of budget deficits, and at best no governmental intervention, in any case a reduction of social expenditures to disburden the economy of taxes and social duties, completed by wage cuts for saving costs. All this stood in clear contrast to the organized Socialists' interests. But Mises' ambiguity became more apparent when he regarded once again the fascist state as a chance to survive in independence. He even accepted a possible support of Mussolini. [Mises (1978) b, p140]

The biggest leak in Mises' views was that he never accepted the possibility of a combination of capitalist market economy with social legislation that had been enforced by the political power of the working class after the war. [70] He regarded in social justice the cause for business losses, financial weakness, and "*capital consumption*", in his view a threefolded factor for economic stagnation. Attempts to measure capital consumption in Austria were made by Morgenstern and Machlup. [Morgenstern (1931)] , [Machlup (1935)] Attached to Mises' mindset Hayek asked: "*Can capital consumption be avoided in the long run in a democratic society in which the majority of the population is anticapitalistic?*". [Hayek (1984), p155] Eventually the liberal economists regarded the new authoritarian government as a better choice to solve the problem of a stagnating economy. For Mises that only meant a replacement of one ruling clique by another one: "*The cliché, a Ständestaat, ... merely shielded the aspiration of the Christian-Social Party and its allied homeguard for complete party rule*.". [Mises (1978) b, p136] Although the liberals finally conformed to the strict course of the 'Ständestaat' (hoping for a consequent implementation of fiscal and monetary policy) they never accepted its clerical and moderate anti-semitic tendencies. [146] Max Mintz, present at Mises' seminars, noted in final words: "*Never in the last 80 years has political and economic liberalism been as dead as today*.". [147]

Haberler, Machlup and Mises made their emigration permanent. This indeed indicates how they saw their country's future. Machlup had withdrawn his habilitation thesis from the University of Vienna, liquiditated his company and went to the University of Buffalo, where he got a chair in economics. Mises enforced his activities in Geneva until his emigration in 1940 to the US. Machlup specified the Nazi problem as the reason for his decision to definitely leave Austria. [69] Mises addressed the economic situation: "*In a country where capital is consumed the whole situation is unstable*.". [149]

The remaining members of the Austrian School made the best of their nightmarish position and indeed exhibited a friendly attitude towards the fascist state. Morgenstern was not shy of addressing the idea of a 'strong state', that stood powerful above all special interests and was able to implement all necessary and hard measures to counteract the economic and social crisis. [Morgenstern (1934), pp129-130] [1] Morgenstern's mindset was less pretended than an actual common view within the conservative elites of the country. [151] Hence Morgenstern regarded his challenge not to follow any idealistic principles like that of liberalism as a base for consultancy in economic policy, but realizing his 'technocratic' ansatz to intervene into economy on the part of the new government. In the same way Mayer flexibly went his way in adapting some opportunist habits. He immediatedly gave a seminar on the 'Economics of the corporate state'. Only short time before Mayer had tried (with the support of Schumpeter and other German professors) to thwart Josef

Dobretsberger's (a convinced backer of the 'Ständestaat') appointment as professor to a chair at the University of Graz. [152] [153] The former scholar of the liberals gave up a differentiated and critical approach and accepted the end of the democracy as unavoidable. It was not only pure opportunism, but they really saw an advantage of a powerful controlled economy that they tried to influence with a nonpolitical, technocratic approach. At this point the work of AW became highly important for men like Morgenstern, as AW offers ideal instruments to develop a kind of early econometrics.

3.3.2 Morgenstern, the central figure for upcoming econometrics

The vacuum left behind by the emigrated members of the Mises' group could not be filled by people like Mayer. This was Morgenstern's chance to get the leading position within Vienna's economists. Auxiliary a couple of factors worked for Morgenstern: one was his directorship of the Institute, a second was that the funding of the Institute of Business Cycle Research by the Rockefeller Foundation with a grant until 1935 [154] gave him enough space. In the very moment when Mayer and Mises had failed to win the Rockefeller Foundation for an alternative project of social science research, Morgenstern got the funding for his Institute extended ad infinitum. We read in Morgenstern's diary: "*Mises, Mayer, etc. are not going to be asked anymore*.". [155] Morgenstern's position was steeled since the Rockefeller Foundation guaranteed him an open-ended funding. This allowed him to open up the research in the Institute towards a mathematical foundation and to more abstraction in the theory. Morgenstern managed that many of his collaborators, especially the younger ones like E. John, R. Kamitz, J. Steindl and G. Tintner got a Rockefeller Fellowship for studies and work. Only AW, the ‘difficult’ Jew from Romania, was never given any fellowship. It is noteworthy that AW never had any political affinities, neither to the (ambiguous) liberal strand nor to Morgenstern’s more pragmatic attitude. [156] AW only deeply wished to get assured a small income. Despite all aversities AW ever enjoyed to follow Morgenstern in establishing the early states of econometrics. [Leonard 2006]

The Institute was Morgenstern’s base for his activities as a policy advisor. This and also the Rockefeller funding made the Institute independent from Austria’s state bureaucracy. The Institute also provided employment for people outside the university circus like AW, who there undertook important developments in mathematical economics and so could be the booster behind Morgenstern, to aim at a mathematical, i.e. in Morgenstern’s view ‘scientific’, economic theory. Morgenstern was also very active as editor: he was responsible for the ‘Monatsberichte’ (Monthly Bulletins) and the ‘Schriftenreihe’ (Book Series), with contributions by Hayek, Machlup, himself, Nurkse, and AW. Additionally he was the managing editor of the ‘ Zeitschrift für Nationalökonomie‘. [157] Under his auspicies the journal took on a critical stance towards Mises and the liberal economists. Finally, Morgenstern was a referee for the Viennese publisher Julius Springer.

Morgenstern’s rise indicated in return the decline of the Mises group. Through the absence of many former adherer of Mises, their communication and influence had waned. The dominance of the liberals within the Austrian Economic Society was gone. [158] Besides their disappearing presence in different circles and debating groups also their journalistic activities evaporated. As an example, Machlup had written between 1932 and 1934 more than one hundred weekly columns (the ‘Zwei Minuten Volkswirtschaft’ (‘Two Minutes of Economics’)) for the ‘Neues Wiener Tagblatt‘ [Klausinger (2002)] with comments on matters of economic policy. [97] All this had been stopped. Since 1934 additionally a hard censorship escalated, inhibiting all-to liberal interpretations and commenting: "*On the most important topics I must not write, and to write on unimportant topics is not worth the while*." (Machlup to Hugo Glaser a journalist of the 'Neues Wiener Tagblatt', on April, 6, 1934 (FMP, Box 55, Folder 18). [160] The universities had to fill the vacant chairs with followers of the new government. These people, like Dobretsberger in Graz or Kerschagl at Vienna's 'Hochschule für Welthandel', propagated non-liberal views and an accordant interpretation of economic matters. Also Morgenstern, the technocrat, received the honorary title of an 'extraordinary professor'. The remaining Austrian liberals found themselves outside academia. The influence of the Austrian School kept on during the 'Ständestaat', but not longer from a liberal strand, but by Morgenstern and his followers. In 1936 Rosenstein-Rodan left Austria for the University College of London. Tintner emigrated in 1937 for the Iowa State University.

Until Morgenstern's appointment as director of the Institute in 1934 the liberals had an eminent influence on economic policy. As long as Mises was secretary of the Chamber of Commerce the door for liberal attitudes in policy always was wide open. Even Morgenstern sang from the same hymn sheet as long as he did not lead the Institute. When Mises had left for Geneva, from that time on Morgenstern became not only the most important advisor in economic policy [161], but also became the representant of the Austrian government at international conferences, e.g. at the at the League of Nations. Morgenstern's informal relations to Kienböck, the President of the Austrian central bank, and to the Ministry of Finance were of an huge importance. Due to his consultancy Morgenstern wrote divers memoranda, some of them got published later. [Morgenstern (1936) a,]

3.3.3 From apriorism to exactness

The transition from Mises to Morgenstern was not only an exchange of persons, but a break in theoretical traditions within the Austrian School. Already in the 1930s Morgenstern tried to find his own independent theoretical standpoint, critical on Mises and Hayek (implicitely questioning the leadership of Mises and the liberal mindset of the Austrian School). In 1933 he wrote: "*For the Viennese it is not unimportant that it is realized in Germany that the common impression as if Mises were the leader of the local economics community and all the other economists, especially the younger ones, look up at him, does not correspond to the facts*.". [162] Morgenstern's efforts to get an autonomous point of view on economics, independent of contemporary economic thoughts, was a combination of harsh criticism and attempts to find revolutionary alternatives. But his attempts were sketchy and sometimes confusing. [2] Compared to the liberals, Morgenstern had developed opposed views on the the foundations of economic policy. [164] Mises explains the superiority of economic liberalism as derivable from the propositions of economic theory, based on aprioristic foundations: "*[T]he science of economics proves with cold, irrefutable logic that the ideals of those who condemn making a living on the market are quite vain, that the socialist organization of society is unrealizable, that the interventionist social order is nonsensical and contrary to the ends at which it aims, and that therefore the market economy is the only feasible system of social cooperation*.". [Mises (1960), p169]

In Morgenstern's view (therein strictly contrasting Mises) policy advice should be free of judgment against the reigning system, the economist should accept the political process as given and act on that basis. [Morgenstern (1934), Ch.4

and 9] In that regard his technocratic ansatz of economic policy was highly compatible with the interventionist government of the 'Ständestaat'. Morgenstern of course faced a chilly reception among the liberals. Hayek for example characterized his book as "*miserable and yet impertinent in its tone*.". [165] Closely related to Morgenstern's general view was the question of methodology. Mises had always preferred aprioristic foundations for economics, that he regarded as a 'science of action', followed by "*praxeology*" as a corrective. In contrast to Mises Morgenstern was in search of a "*exact*" (i.e. axiomatic-mathematical) basis for economic theories, that again should be validated by empirical data (cp. [Morgenstern (1936) b,]). Morgenstern's approach is no surprise as he was shaped by the Vienna circle and also the Colloquium by K. Menger. The anti-Morgenstern position of Mayer abhorred the use of mathematics. [Weintraub (1985), pp62-80] But Morgenstern by himelf simply was unable to develop a mathematical foundation of economics. A planned book on time and economic equilibrium [166] never saw the light of day. But exactly the argument of lacking exactness was his ever present argument against his rivals. Morgenstern, for example, characterized an article of Hayek on Karl Menger's lecture on the law of returns as "*a model of confusion*". [167] Morgenstern and Menger jointly published later that topic in the 'Zeitschrift für Nationalökonomie'. [Menger (1936) b,]

A certain suspicion, fired by Tintner, arose within the Foundation, that the Institutes would become more and more dependent from the government. [168] The Rockefeller Foundation was still the main sponsor of the Institute. But Morgenstern succeeded in dispelling this suspicion. A glance at the 'Monatsberichte' quickly shows that the comments on governmental policies were indeed extremely meek and rare. This meekness explains partly that Morgenstern in his business cycle analyses in the 'Monatsberichte' had at least to argue against the Mises' group, that immovably believed in their framework of liberal rules (symbolized by the gold standard [143]) requiring the avoidance of inflation [144] and of budget deficits, the freedom of trade, and a minimum of governmental intervention (cp. Rosner [Rosner (1999), pp221-222]). Even in 1938, Morgenstern, already in the US, together with Kamitz censored a contribution to the 'Monatsberichte' in that spirit. [Steindl (1988), p400]

Summarizing, with Morgenstern's leadership a turn-away from the Austrian liberals took place and especially with regard to the Institute, a reorientation towards a mathematized research began. His critique of current economic theory needed men like AW who were able to formulate the foundations.

3.3.4 Morgenstern's pragmatic failure and his turn towards theory

For Morgenstern the year 1936 was a crucial one. After the breakdown of the 'gold bloc' (the term gold bloc then was applied to seven nations, France, Belgium, Luxembourg, the Netherlands, Italy, Poland, and Switzerland, that kept the gold standard during the world economic crisis of 1929 to 1933) and the devaluation of all currencies of its members, the country had to decide whether it should devalue the Schilling or stabilize the current parity. [Klausinger (2002)] As the control of Austrian economic policies by the League of Nations and the foreign creditors already had been eased, Austria could independently take decisions on that issue. In his role as policy advisor Morgenstern recommended a stable currency. Thereby Austrian missed its last chance to stimulate the economy. As Morgenstern's decision was not based on a specific economic theory he anyhow was pretty astonished that "*all the* *foreigners are ever asking, why we did not devalue*" (cp. Morgenstern's Diary, 24 Jul 1937, OMP, Box 13). Morgenstern pragmatically had aimed towards a stable currency that would allow a supplementary policy of lowering prices. This would be realized directly by price controls (of e.g. agricultural products), but also by opening up some industries to international competition. When his hopes to influence economic policy for the better highly got disappointed by colliding with organized interests of the Ständestaat, Morgenstern abandoned his political activities [Morgenstern (1936) b,] and started to concentrate on his scientific career.

In January 1938 Morgenstern left Austria, at first for a three-month visit in a series of US institutes. In respect to the "*limits of economic policy*" [Morgenstern (1939)] Morgenstern had finalized his career as a policy advisor to the Austrian government. It were these limits and not the 'Anschluss' that had brought him to leave the country. In 1939, retrospective, Morgenstern started to criticize the economic policies of the 'Ständestaat' with devastating words: it should be relegated to the "*great field ... which might appropriately be called the 'pathology of economic policy'*". [Morgenstern (1939), p39] Nevertheless in these years, due to Morgenstern, the economic policy had been replaced by a more pragmatic (or technocratic) approach. It is remarkable that this did not imply a policy change in the fields of monetary and financial policy, where the gold standard mentality continued. In fields like trade and industrial policy, economic reasoning also was inferior to political partisanship.

4

The Abyss

Viennese culture resembled a bed of delicate flowers to which its owner refused soil and light while a fiendish neighbour was waiting for a chance to ruin the entire garden.

—Karl Menger [Menger (1998)]

It would be a great misfortune for me were I to lose this position. I would then be facing the abyss and would not even have the financial means to travel anywhere

—AW to Menger about the Cowles Commission offer, Vienna, April 18, 1938, KM Archive, Durham, Box 1, Folder 21

4.1 Economic and political crisis

The victory for the Allied after WWI implied also the victory for the Western European resp. Anglo-American model of nationbuilding and constitution of society. The post-WWI nation-states in Central and Eastern Europe inherited parliamentarism and capitalistic economy, in the false belief, that prosperity and stability soon would follow. But this translation of Western concepts brought about huge problems. The tribute of war had been much higher in the Balkans concerning landscape damage or civil victims. In Western Europe WWI happened in small strips of land (the trench warfare) and most of the losses concerned the armies involved. In the Balkans a rough rule of thumb says, that each battlefield casualty, another three men were wounded. About 150,000 South Slav soldiers found their dead serving for the Empire. Romania lost 336,000 soldiers in campaigns and ailment. Additionally 275.000 Romanian civilians lost their life on account of the war. [Bobango (1979)]

Many parts of the former Empire were split away from the successing First Republic, a fact implying the losing of important economic regions that now became own nation-states by themselves. This again implicated complications as these new states were still dependent on the Austrian banks. The fragmentation of Austria-Hungary also brought about the annulment of the old currency in an orgy of paper-money inflation. "*The initial hope that the 'Krone' could be maintained in a monetary association with the successor states quickly evaporated*". [Oesterreichische Nationalbank (2008)] Notwithstanding the coalition government established a progressive socioeconomic and labour legislation. But in defiance of all that political and economic rigour, the intellectual life in Vienna blossomed (at least until 1934, when the fascism foiled everything). The Austrian Socialists, having their voting public almost completely in Vienna [Lauridson (2007), p55], attempted a change of the cultural face of their capital in trying to bring a kind of ‘new people‘ (they used the term ‘Neue Menschen’ in their pamphlets) out of the working class into being. The ‘new people’ should face also new living standards in new architectured environments. Still today Vienna shows buildings of that ‘municipal Socialism’ with functional, affordable flats in extra quarters for the working class. [171] During the political phase of ‘Red Vienna’ for the first time a policy of real redistribution of wealth began (whereby more than half of the local rates were imposed on the wealthy segments of the population). This indeed bestowed some successes upon the government, e.g. in educational policy or health care. Also a brief economical revival had been ignited, but the backdrop soon followed and a deep depression started from 1921 on. In 1922, the unemployment rate lay at 4.3 percent, in 1923 already at 9.3 percent and then rose ceaselessly until 1934 to 25,5 percent, in a way that the impact on population was profound. [Eigner (1999), p171] The Great Depression of 1929 had an additional impact on the First Republic. The worst inflation ever hit the country. [Menger (2002)] Closely related to the economic situation was the mounting political radicalisation as the appeal of fascism [172] got boosted by the social dislocation among the unemployed (even including university graduates). The cataclysmic idea of an ‘All-German Reich‘ [173], that should be achieved by force if necessary, affected from 1933 on also Austrian NS sympathizers to threaten their state from within. In the following section we undertake a quick look at the social and political schism between the so-called bourgois [174] and socialist Austria, a fundamental condition for the development of the Austrian fascism.

The Balkan nations answered to the weaking economy with protectionism. But their attempt to hold off foreign competitors backfired. With higher tariffs and higher prices the Balkan states tapered the demand for its (agricultural) exports. [Farlow (1971)] The grain exports of Romania e.g. declined by 75% from 1929 to 1934. [Hitchins (1996)] (also cp. [Machildon (2010)] on global wheat trade) Counted across all Balkan nations the whole export levels fell down to 40% of their 1929 number. [Hitchins (1996)] Hand in hand with the economic decline the Bakan governments went on to enhance authoritarian structures. Treatises about fascism in Europe often ignore the analogous developments in Hungary or Romania. But this aspect is in our view crucial for Balkan history. Unfortunately this complicates the analysis of fascism in general.

Romania was indeed a victor in 1918. Romania saw no revolution that would have dislodged its grown authorities. The peasant masses had been before and also remained after WWI irrelevant regarding power and politics. Between 1918 and 1921 land reforms took place. They came late distributing 9 million acres to the huge number of 1.5 million peasant Romaian families. On one hand the reforms were a political success for the ruling Liberal party, but on the other hand, from an economical perpective, they were a fault as the peasant farms were simply much too small to be profitable. [Lipcsey (1994)] In an answer since the 1930s, the National Peasant Party emerged in rivalry to the Liberal Party. The Great Depression did one more thing. Romania had too few resources to act in a considerable way. Hence King Carol II, who was able to exploit splits within the parties, succeded to dominate politics in the 1930s. For all practical purposes, the King Carol ruled. [Bulei (1996)]

4.1.1 Austrias split into social blocs

A capitalist production mode in the Empire got evolved, but showed some mannerisms in comparision to Western Europe. Around 1910 about 53% of the working population was occupied with agriculture, while only 23% were engaged in industry and the trades (in comparision Germany's peasant workers made up a percentage of 35%, the Suisse 31% and Great Britain 30%). The Empire was a typical industrialized agrarian society. [Matis (1972), p423], [Brusatti (1965), p960] Looking at the timespan from 1869 to 1937 the percentage (again of the whole working population) in industry and trades steadily rose in the Austrian territories from 24.8% to 37.5%. Simultaneously the portion of workers in the agricultural sector inversely fell from 54.2% to 37.5%. [Bodzenta (1980)], [Otruba (1980)] The quota of those in commerce, transport and finance rose between 1910 and 1934 from 14.0% to 18.2%, although after 1918 Vienna had lost its position as the centre of banking and commerce to its neighbour capitals and the Austrian export numbers stagnated. [Lauridson (2007)] The industry showed a huge concentration into a few areas of the Empire: the iron industry in Styria, around the coalrich regions of Bohemia, Moravia and Silesia the heavy industry colonized. The light industry (e.g. food and clothing production, electrical and chemical businesses) was centered around Vienna, where not only the administrative apparatus but also the aristocracy in its hunger for luxurious life standards made up a huge market potential. The seemingly contradictory increase in the non-productive sector indicates with [Hertz (1970), p96], [Lauridson (2007)] that the productive sector (industry, trades, agriculture) could not provide sufficient employment. Hence many people were forced to make their living in service and distribution, in liberal professions or as small merchants. Compared to the Hungarian part of the Empire the massive weakness of the industrial sector becomes apparent, as the percentage of employees in the agricultural sector was about 69% in 1910. [Lauridson (2007)] As large and middle-range enterprises were inhibited by heavy taxes, small businesses made up a fundamental part of the Austrian industry (41.1% of all industry workers worked in businesses with 10 or less employees). [Statistisches Handbuch (1932), Vol. 13, p90] An Austrian speciality was the one-sided dependence of the industry on capital, caused by a chronic shortage of it. [Rudolph (1976), p13] As the Austrian banks were controlled by large feudal landowners and the aristocracy with a functioning connection to the Emperor, the evolution of a national bourgeoisie was impeded, also a stratum of powerful capitalists has been lacking in consequence of the majority of small businesses. [Hauptmann (1974)]

Through the 1848 emancipation all peasants had the guarantee to keep their land that they hold through feudal entailment. Additionally they got discharged from their obligations to their former feudal lords, especially to pay dues in any form. In return the former lord got a compensation by fixed payments of their peasant over a period of 40 years. [Matis (1972)], [Brusatti (1965), p410] Yet these costs combined with heavy state taxes not seldom had a crushing effect on the peasants. Hence their debts steadily increased. [175] Based on different natural conditions, the agriculture in the Austrian part of the Empire lagged behind its Hungarian counterpart in production and technical development. [Matis (1972), p122] Until 1912 the agrarian production in the Austrian part was far below the actual needs. [176] In effect, although in freedom, the economic dependencies of the peasants from the ruling class didn't change much. Depite these adversities there was no extensive migration into urban areas, also the decrease of agrarian units was very moderate. [177] Between 1902 and 1930 the number of workers on small farms (below 20 hectars), who mostly consisted of family members, increased [Statistisches Handbuch (1932), Vol 10]. Just as well the small business predominated in industry, the small farm was predominant in Austrian agriculture. [178]

Already before the start of the First Republic the social structure of Austria was divided in mainly two parts: on the one hand we see an agrarian domain and on the other hand the industrial domain, both also cut-off spatially from each other. Gulick uses the term ‘two Austria’s’ for such a status. [Gulick (1948)] Aside from Vienna, we find only a few and far between spots with industry. Besides Graz and Linz [179] an evolved urbanisation could only be found in Vienna (that harboured almost 28.5% of the Austria’s population). [Lauridson (2007)] Besides the larger cities urban development elsewhere was pretty underdeveloped. [Danneberg (1927)] The ‘petty bourgoisie’ [180], aside from Vienna, constituted only a very small part of the population. The old middle class (mainly based on the peasants) in Austria remained undisturbed in its position and maintained its prominent position, because no new (lower) middle class became predominant. (Cp. [Arendt (1958)]) As a result we see on the one hand a distinct agrarian stratum and an urban bourgoise (mainly situated in Vienna) and on the other hand a large class of industrial workers, concentrated primarily in and around Vienna. The landowning peasants organized themselves in more traditional forms: councils, chambers and corporations. Organisation that aimed on detaining commercialisation. The CSP (the Christian-Social party), with anticapitalist, antiindustrial, antiliberal, and antisemitic campaigns, got a broad acceptance among the peasants and also the petty bourgeoisie. Both groups did disapproved capitalist production, as in their belief it caused their economic dilemma. [Leichter (1964), p14]

4.1.2 The political schism

The Social Democrats campaigned for political reforms and improvements for the working class. They also called for a more democratic electoral system. This would have catapulted them into (at least) participation of power. [Hornik (1976), p347 et seqq.] The SDAP (see Appendix A for acronyms), always in a missionary mood, convinced to be on the right track within a historic inevitable process of capitalism, worked energetic on concentrating and strengthening its unity. [Lauridson (2007), p86] To that end the SDAP started a recruitment program in connection with numerous (e.g. cultural) activities for its (potential) members. This indeed had an impact and attracted primarily the young people. [181] The intensive party propaganda of the SDAP, combined with a tight organisation, implicated a deepening of the extremes between the agrarian and the industrial stratum. The simple question of the own political orientation became more and more acute. The dominant parties (CSP and SDAP), generated a strong social and political polarization. [Leichter (1964), p141, p185] Beyond that political schisma no mediating liberal party found its place. [Mommsen (1981), p427, p431] To be more precise: a liberal party quite simply lacked. In part this goes back to the strong Catholicism in Austria, that always exerted a convinced antiliberal influenc on the population, not only on the peasants, but also in the towns. Only Vienna showed tendencies of liberalism. Just this absence of a liberal center, both opposite parties, CSP and SDAP, more and more drifted away from each other. After getting into political responsibility, the CSP gradually abolished its oppositional stance. [182] Karl Lueger and his followers simply forgot their former critique of capitalism and also their emphasis on the Catholic social teachings. [Leichter (1964), p206] This turnaround into the political establishment in effect widened the gap betwen the CSP and the SDAP. As there was no liberal middle, the SDAP became the only strong opposition in pre-1918. Its relation the bourgeois parties was almost null.

The CSP never had the need to share power with the SDAP. It steadily came more and more into political power and likewise showed amost no difference to the old elite in respect to social questions. But it was confronted with the SDAP that also showed to be a growing movement. In a kind of mimicry the CSP made its own recrutional programme like the SDAP. [183] [Lauridson (2007), p88] When we go back three decades prior to 1918, the organisation of masses had a start. Then 1918, confronted with a democtratic system, these parties, especially the CSP and the SDAP were badly primed, because they never had learned to make and to work with political compromises. The new democratic republic cleared a space for smaller political parties to cluster around the big two. On the left the KPÖ (‘Communist Party of Austria’, ‘kommunistische Partei Österreichs’) was founded in November 1918. On the right side of the CSP we see the ‘Grossdeutsche Volkspartei’ GVP (‘Pan-Germanic Party'), a reservoir of converged german nationalists. The GVP drew its followers from anticleric groups among the urban middle class. Politically akin goes the ‘Landbund für Österreich’ LB (‘Agrarian League of Austria‘), with a strong constituency within the higher-income peasants, that partially developed an anti-clerical habit. [Hawlik (1971)] Not to forget another small party: the NSDAP (‘National Socialist German Workers Party’, ‘Nationalsozialistische Deutsche Arbeiterpartei’), a continuation of the DAP (‘German Workers Party’, ‘Deutsche Arbeiterpartei’) from 1904. Within the bitter conflict between the bourgeois and socialist bloc the fascist movement grew out from a political reconfiguration within the bourgeois group. After 1929, when the ‘Heimwehren’ (‘Homeguards’) heavily lost ground, the NSDAP grew at that. [Lauridson (2007), p90]

4.1.3 Economic weakness and dependency

Economic weakness after the WWI is of course related to wartime economic decisions. Wartime economies not only had been centralized from top down, but also often got under the supervision of military authorities, who controlled investments and production in all facilities (factories, mines or farms). [184] The grave handicap of this form of control is that local needs completely get neglected. In contrary, it fosters corruption and a bureaucracy that constrains market forces. The command economy in consequence not only lets newer civil and political rights erode, but also shortens the maintenance for civilians. Additionally peasants were forced to labour unpaid for the authorities. Analog official industrial cartels controlled harshly the urban areas. Besides that, captured assets by the opponent of war were expropriated and often removed. Excessive taxes, peasant obedience and inefficient monopolies returned with WWI. All that had already been removed by the revolutions of 19th century.

Economic resources were destroyed, especially livestock. [185] Livestock was a key resource for the economy: transportation, plowing, fertilizer, meat, dairy, wool and leather depended all on livestock. Also the agricultural equipment was destroyed on a large scale. Land reforms became necessary as the agricultural productivity excessively had declined. [186] These reforms implied a break-up of larger (in war expropriated) estates. But the sinking infant mortality in the Balkans combined with harsher US immigration regulations let the population unduly increase. The surplus of rural workers had to find labour in the drowned industry, that was not able to absorb them. In Romania about 50% (in 1930) of the rural population was superfluous in regard to the productivity as well in the agricultural sector as in the industry. In Bulgaria this figure was at 53%. [Lipcsey (1994)] The small farms had a low productivity. The average size of a Yugoslav farm was about 12 acres. In Macedonian, Bosnian and Dalmatia farms were even smaller. Subsistence farming was all what these small farms allowed. The farmers were too poor to afford machines or chemical fertilizer. This kept the agricultural productivity an a very low level. As most of peasants were too poor to buy consumer goods, the industry gained no profit from them.

The breakup of the Empire created new barriers between industries and resources. [187] In series factories were closed down. Producers of commodities and other raw material had lost their puchasers. The new borders additionally handicaped international trade linkages. [188] Due to the excessive costs of war and wartime borrowing, a phase of hyper-inflation began. The Hungarian currency sank to 40% in 1918 and to 15% in 1919 of its pre-war value. In Romania it was even worser. The Lei stood in 1920 at 20% of its pre-war value. [Lipcsey (1994)] As a result banks and also individuals lost their savings. The proportion of working people in the industry stuck at a low level. The highest percentage had Hungary: almost 23% of the population were employed in the industrial sectors. Romania lay far behind: the pecentage was 7% in 1920. when we compare the industrial output per capita, we see in 1938 $26 in Hungary and $12 in Romania. By way of comparision that number was at 140$ in 1938 in Great Britain. The national income numbers from 1938 paint the same gloomy picture: we see $440 per capita for Great Britain, $120 in Hungary, and between $75 and $80 in Romania. With its decline in revenue numbers, the Balkan states got pressed hard and came into huge trouble to repay foreign loans that they had incurred in the 1920s. Their debts made the Balkan states more than dependend on foreign control, especially Nazi Germany hold therein a strong position. [189] The Balkan states got into a situation like economic colonies charged with a tremendous German leverage. [190] That made it hard for them not to become political satellites of Nazi Germany.

4.1.4 Hyper-Inflation

“*When the inevitable consequences of inflation appear and prices soar,* *[people] think that commodities are becoming dearer and fail to see that money is getting cheaper. In the early stages of an inflation only a few people discern what is going on, manage their business affairs in accordance with this insight, and deliberately aim at reaping inflation gains. The overwhelming majority are too dull to grasp a correct interpretation of the situation. They go on in the routine they acquired in noninflationary periods. Filled with indignation, they attack those who are quicker to apprehend the real causes of the agitation of the market as 'profiteers' and lay the blame for their own plight on them. This ignorance of the public is the indispensable basis of the inflationary policy*." [L. von Mises (1953), Ch21] Like all other European nations, that were participants of WWI, the Austro-Hungarian government printed new money to cover the increasing military costs. In July 1914 in Austria- Hungary the currency floating around summed up to 3.4 billion crowns. By the end of 1916 there were over 11 billion crowns circulating. When WWI came to an end the currency had expanded to 33.5 billions. 3 years later in December 1921 the sum was 174.1 billions and only two years after that in December 1923 the barrier of 7.1 trillion crowns had been passing. [Sandgruber (1995), p355] [Ebeling, (2006)] With haunting words in "*The World of Yesterday*" S. Zweig puts this period of inflation in a nutshell: "*In the collapse of all values a kind of madness gained hold particularly in the bourgeois circles which until then had been unshakable in their probity. … How wild, anarchic and unreal were those years, years in which, with the dwindling value of money all other values in values in Austria and Germany began to slip! It was an epoch of high ecstasy and ugly scheming, a singular mixture of unrest and fanaticism. Every extravagant idea that was not subject to regulation reaped a golden harvest: theosophy, occultism, yoga … Anything that gave hope of newer and greater thrills, anything in the way of narcotics, morphine, cocaine, heroin found a tremendous market; on the stage, incest and parricide; in politics communism and fascism constituted the most favored*

themes; unconditionally proscribed, however, was any representation of normality and moderation.… Nothing ever embittered the German people so much - it is important to remember this - nothing made them so furious with hate and so ripe for Hitler as the inflation". [Zweig (1964)] The cost-of-living index rose from 100 in July 1914 to 1,640 by November 1918 and peaked up to 9,956 in January 1921, again one year later stood at 83,000, and a second year later in the beginning of 1923 had reached 1,183,600. [Ebeling, (2006)] But the worst of the upcoming economic disaster was not even on the horizon. The state of German-Austria (and two years later the First Republic) found itself isolated from other regions of the former empire caused by high tariff barriers and other trade restrictions. Additionally border wars broke out between the Austrian and the neighbouring Czech and Yugoslavian armies.

War costs and war loans forced the Austrian government to increase money supply excessively. [191] When the regions within Austria imposed (affected by government policy [192]) additional price and tariff barriers, prices rose anew. Of course Vienna was affected. Hence a black market spread overall for the most important goods. [193] The regions began to hoard food and combustibles. As a consequence in 1921 50% of Austria's budget deficit resulted from food subsidies for the cities and also the exuberant bureaucracy. In an answer the social democrats boosted the taxes on the business sector. That led in succession to a heavy blood-letting of the middle-class. The value of the Austrian currency mirrors the economic disaster: in January 1919 1US$ had a countervalue of 16.1 Crowns. That proportion rose to 70.800 Crowns in May 1923. [Ebeling, (2006)] At a meeting of the 'Verein für Sozialpolitik' ('Association for Social Policy') in 1925, L. von Mises spoke to the audience following: "*Three years ago a colleague from the German Reich, who is in this hall today, visited Vienna and participated in a discussion with some Viennese* *economists. ... Later, as we went home through the still of the night, we heard in the 'Herrengasse' [a main street in the center of Vienna] the heavy drone of the Austro-Hungarian Bank's printing presses that were running incessantly, day and night, to produce new bank notes*" (cp. [Boese (1939)], [Ebeling, (2006)]). Throughout the country, many of the industrial enterprises stood idle, others openened only parttime their doors. In late 1922 and early 1923 the inflation could be brought to a halt. The government got help from the League of Nations, granting Austria an international loan in the autumn of 1922 to cover a part of its high expenditures. The supervision of the loan stayed at the League of Nations. [Brook-Shepherd (1997)] It did not fail its expected effect: the state could avoid bankruptcy and stabilize its currency. That in turn accounted for a healing of the drowned economy. The negative side was that Austria became dependent on the League of nations. The League's commissioner, staying in Vienna, was not an official member of the government, but in fact took part [Ausch (1975)], as the credit was not directly paid to the Austria, but lay on an account solely disposed by him. Following the claims of the League of Nations the government started to accomplish administrative reforms and a restructuring of the national budget. That in the end was financed by a massive tax increase. [Ausch (1975), p32] The League arrogated an end to food subsidies and a cutback in the bureaucracy of 70.000 employees. At the same time, the Austrian National Bank was reorganized, with the bylaws partly written by L. v. Mises. The gold standard was reestablished in 1925. Also the Crown was deprecated and replaced by the new Austrian Shilling.

4.1.5 AW and the cost of living index

In the context of price and cost of living indexing AW contributed in 1937 a work on price index numbers, 'Zur Theorie der Preisindexziffern' [Wald (1937) b,] and 1939 another paper on the index of cost of living: 'A new formula for the index of cost of living'. [Wald (1939) c,] For the measuring of the change of prices from period 1 to period 2, one usually computes the Laspeyres or the Paasche price index. Either of these indices is defined as the ratio of the expenditures for a indexed bundle of commodities (with q as a fixed vector of quantities of this commodity bundle) under the two price vectors p_1 and p_2 prevailing in periods 1 and 2, respectively: the price change is then equal to $I_{12} = \frac{q' p_2}{q' p_1}$. That index has the disadvantage, that it neglects the fact that consumers adjust their consumption to price changes in dependence of what they prefer or not. Hence the correct cost of living index must not assume a fixed bundle of commodities, but is has to start from a fixed utility level. It is computed by the ratio of the expenditures for two optimal [194] commodity bundles q_1, q_2 with the price vectors p_1, p_2 provided that the utility derived from commodity bundles is equal: $I_{12} = \frac{q_2' p_2}{q_1' p_1}$, $u(q_1) = u(q_2)$. Typically q_1 is the quantity vector observed in period 1 and $q_2$is not observed. It is constructed such that it has an equal utility to $q_1$and minimizes expenditure under the prices $p_2$in period 2.

For an economist, the cost of living index seems quite natural, but the statistician has the problem to compute such an index. If we simply postulate the utility function of the consumer, the cost of living index is easy to compute. But usually it is not available and assumptions have to made. AW assumed that the utility function is quadratic, at least in a neighborhood of q_1: $u(q) = q' A q + a' q$, whereby A is a symmetric coefficient matrix and a is a coeffcient. He shows that, given a price p, the quantity q, that maximizes the utility under the budget constraint $q' p = e$ is a linear function of the total expenditure e: $q = b e + c$. [195] Now we are able to construct the true cost of living index from the estimated coeffcients of the Engel curves of period 1 and period 2. Assumed that b_t and c_t are these coeffcients for period t under the price vector $p_t,\ t = 1, 2,$ then we have

$$I_{12} = \sqrt{\frac{b_1' p_2}{b_2' p_1}} + \frac{c_1' p_2 - c_2' p_1 \sqrt{b_1' p_2 / b_2' p_1}}{c_2' p_1 \left(1 + \sqrt{(b_1' p_2)(b_2' p_1)}\right)}$$

If $c_1 = c_2 = 0$, i.e. the Engel curve goes through 0, I_{12} degenerates to Fisher's price index. Fisher’s price index is the quadratic mean of Laspeyres’ and Paasche’s index. In official statistics that formula never replaced Laspeyres or Paasche’s index. The reason is that on one hand many more data are needed to compute the true index (for the necessary econometric estimation of the Engel functions) and on the other one the model for the utility function is to restrictive. [196] Nevertheless, AW’s definition of the cost of living index is still a topic in reseach. [Balk (1981)] AW had the idea to link pure economics with a mathematical ansatz that does not lack an empirical background.

4.1.6 The failing of Red Vienna

During the Empire phase, Vienna, the capital town, developed during the second half of the 19th century into a global center for the modern currents in arts and psychology. In 1890, a further expansion of Vienna happenend. The whole area of the ‘Vorstädte’ (suburbias) within the ‘Linienwall’ got incorporated as the 2nd to 9th districts. The ‘Vororte’ (suburbs) beyond the old Linienwall were included as districts 11 to 19. [197] Parts of the 4th and 5th district became in 1874 the 10th district. The 21st district finally became Floridsdorf in 1904. 60 years back, during the 'Gründerzeit' [198], the population of Vienna experienced a sharp increase, whereby immigration was a main cause. Census were regularly carried out from 1869 onwards. These show a peak number of population in Vienna in 1910 with 2,031,000 townsmen. This was the last year of Karl Lueger's [199] mayorship in Vienna. He was a leading figure of city policy. Orientated with social policy he indeed succeeded on a broad base [199], but coupled his achievements for municipality with an anti-Semitism, that he showed in public. [200] Karl Lueger of course inspired some of the later right-wing leaders, such as Ignaz Seipel, Engelbert Dollfuss and Kurt Schuschnigg. Less his anti-Semitic habits were a role model for these epigones, but rather his combative stance towards ideologies and political opponents. Unfortunately such attitudes proved to be very destructive for democratic proceedings. Hence it is not surprising that the three mentioned politicians played decisive roles in the development towards Austrian fascism.

Vienna's living conditions got pretty complicated after WWI. It were less military threats or outright fightings that let Vienna suffer, but it was the lack of supplies, caused by an embargo of the Entente powers. The result was a heavy shortage in food, clothes etc. The tremendous inflation had also wiped out the savings of Vienna's middle-class. The town itself had shrunk to the capital of a small new state. It seemed like a oversized 'hydrocephalus' of the fledgling young republic.

In 1918, the Social Democrats hold the majority in Vienna. The linkage between Vienna and Lower Austria (although rural and conservative) implicated also a Social Democratic government there. In consequence the CSP massively aimed at a separation from Vienna. [Opll (2007)] They succeeded and Vienna got dissociated from Lower Austria in 1922 [201] the Social-Democratic city administration could start an independent reform policy. It was Hugo Breitner, who laid the financial basis for the creation of 'Red Vienna'. Breitner was the Finance Councilor, who installed a social graded tax system. Initially he disposed the rent tax, which charged each rent with the same tax rate, and introduced instead a new rent tax for the more pricy ones, what affected only a percentage of 20% in total. As the returns for the financing of the ambitious housing program were insufficient, Breitner developed together with Robert Danneberg in 1923 the housing tax. Even though this had an effect on tenancy on the whole, it was outstandingly progressive: 82% of all households covered 22% of the community charge, whereas the small amount of real prosperous households, 0,5%, made 45% of the entire income. This money served to finance approximately one third of the housing project. The rest was taken from the general budget. This taxation system helped to build over 64.000 new flats between 1923 and 1934 in Vienna. A big step which met the enormous need of lodge and made housing affordable to large parts of the population with low income. [Blau (1999)] Many nowadays famous 'Gemeindebauten' (communal estates) resulted from that policy. These offered (atypical to the poor flats that were built decades before) running water, toilets and adequately natural daylight. There was also enough green space between the secluded buildings. That phase of 'Red Vienna' became internationally known. It showed a flagship model for Social Democratic municipal government. [202]

Figure 1. After Hugo Breitner temporarily had the idea to found his own party, he joined in 1918 the SDAP, wherein he soon got recognized as a financial expert. In early December 1918, Breitner got into the Provisional Municipal Council, and stayed also in the regular Council. Soon afterwards he got elected as Executive City Councillor of Finance. No other Social Democrat was affronted so heavily as H. Breitner. During a campaign speech at the 'Heldenplatz' the Home Guards leader and Austrian Interior Minister Ernst Rüdiger Starhemberg 1930 exclaimed: "*To the Viennese I will give a good recipe for the election: they should run the election battle in the sign of Breitner. Only when the head of this Asian* ['Asian' is an allusion to Jewish origins; note from the author] *rolls in the sand, the victory will be ours*". [W. Fritz (2000)]

But did the 'Socialists class experiment' [Gruber (1991)] really work? Was there a real relationship between the SDAP and the workers? Against the Socialists' intentions, their paternalistic attitudes and also their administrating-from-above habit in the end was stuck as a barrier between the party and the workers. Their understanding, based on a fixed picture of how a worker had to be, hence lacked a direct aproach to its clientele. Workers were treated like human material, that could be reshaped following a predefined form. [203]

Workers daily routines were neglected, their own subculture or initiatives were ignored in favor of organised and overloaded educational and cultural programs. [204] Surely there were critics on the SDAP's policy such as Wilhelm Reich and Kathe Leichter, but these had no position of authority within Vienna's Socialists. Only the housing experiment stabilized social tensions and offered a better environment for educational programs within the working class. [Blau (1999)] But its success was limited. [Gruber (1991)] An explanation can be found in the upcoming mass movements and culture of mass media: the movies, the radio or spectator sports to a much larger extend absorbed people than traditional SDAP activities. [205] The reformist power of the SDAP was also limited due to their almost 'conservative' attitude towards women: the hardships women faced in work or in households were no part on the political agenda. The SDAP acted to much within a home-grown ideology and was not well prepared to face the upcoming political challenge imposed by the Pan-Germans, the Home Guards, the CSP and the Nazi sympathizers.

Figure 7. The Karl-Marx Hof in Viennas Döbling district, was planned by Karl Ehn in a modern style and erected between 1927- 1932. It is a very prominent example for the communal housing program financed through the 'Breitner Tax'. Only 18.5% of the 156.000 m^2 area was built over,

the rest is used for play areas or gardens. It has 1325 apartments for around 5500 residents and also a variety of community facilities from laundromats to kindergartens, health centers and a parent information center. The 'Karl-Marx-Hof' strechtes over one kilometer and is arranged in form of a closed courtyard. [S. Reppe (1993)] [Graphic from http://www.GreatBuildings.com]

4.1.7 The Austrian fascism

With the irreconciliably separated camps of the SDAP and the CSP Austrian politics got more and more ideological. That was less caused by the parties themselves but by the wide ranging power structures these meanwhile possessed. Concretely spoken, the SDAP as well the CSP had its own paramilitary forces. The Conservatives (representated by the CSP, and also German Nationalists ('Großdeutsche Volkspartei', GDVP [206]) began to organize the 'Heimwehren' (Home Guards, in a wider sense also termed with 'Heimatschutz' (Home Defense) [207] or 'Volkswehr' within the German-speaking lands of Moravia, Bohemia and Silesia (the 'Sudetenland') [208]). The different Home Guards mutually united, beginning from Tyrol, at first on a province level. As their common target, after the state borders finally had been constituted, remained the militant marxism. In their own view the Home Guards offered a protective force for the middle class. "*The Heimwehr movement can be ... categorized as the sort of organization that, like the German Fatherland Party or later the Freicorps, opposed socialism and democracy without really being given a chance politically*.". [Mommsen (1981), p182] But the Heimwehr was a "*radically right and not a fascist movement*". [Lauridson (2007), p66] The response on the part of the SDAP of course was not a long time coming. They formed their own well-organized and well-equipped paramilitary group as a counterbalance, and called it the

'Republikanische Schutzbund' ('Defense League'). [Brook-Shepherd (1997)] That happended in 1923/24. It is not surprising, that all sorts of incidents and clashes had been provoked by these paramilitary groups. Although the economic situation had improved and inflation was kept in check, a political polarisation occured. But there was no balance between the fighting groups. During the process of further radicalisation the Home Guards more and more gained ground.

In 1927 members of the 'Frontkämpfervereinigung' ('Front Combat Union' - a paramilitary association close to the conservative camp) shot a boy and a war-veteran marching with the 'Schutzbund' in Schattendorf in Burgenland. Their attorney Walter Riehl (amongst others leader of the 'Deutschsozialer Verein', a National Socialist group) defended them in the following 'Schattendorfer Prozess' and achieved their acquittal. K. Menger wrote "*a jury acquitted a group of militant right-wingers, who had fired into a socialist parade, killing two workers and a child*". [Menger (2002)] That was indeed the wrong signal for the Socialists. The 'Schattendorfer' scandal led to violent excesses in Vienna. The outraged masses caused the so-called July massacre among the workers. [Brook-Sheperd (1997)] On July 15, 1927 Vienna was confronted with a general strike and demonstrations, that got out of hand when a police station was attacked, followed by the police shooting at the demonstrators. [Menger (2002)] The angry mob set fire to the Palace of Justice. In the end, 89 people laid dead in the streets. Many hundreds suffered from injuries. Only days after the Law Court fire, the Home Guards experienced an enormous upswing. But they never were able to strike off their role as a paramilitary arm of the conservative parties and adopt an active role in politics. Later in 1930 the Home Guards formulated in the 'Korneuburger Eid' ('Korneuburger oath') [209] anti-democratic ambitions, already referring to the Austrofascism. [210] The Korneuburger program took an opposite standpoint to the Linzer program of the Social Democrats, that was in line with a democratic way to gain political power, but also was propagandistically alleged finally to aim at a dictatorship of the proletariat. [Kerekes (1966)]

Attempts to unify the Home Guards nationwide under a common leadership were undertaken several times, but regularly failed by differing aims of the individual Home Guard groups and their rivalling leaders. [211] The splitting into a monarchistic and a German Nationalists wing implied a further weakening. [Chraska (1981)], [Wiltschegg (1985)] For the elections to the National Council in 1930 the Starhemberg group positioned its own list, the ‘Heimatblock‘ (Homeland Bloc), whereas the residual group under the guidance of E. Feys (particularly in Lower Austria and Vienna) fraternized with the CSP. [Schweiger (1964)] Although the Homeland Bloc had won 8 seats in the National Council, internal differences crippled its opposition. [Carsten (1978)] Due to the pro-German, nationalist movement of the upcomig Austrian Nazis the Home Guards in the end lost their political significance. They never were as strong and ‘grass-rooted‘ as they posed to be.

Figure 8. Parade of Home Guards in Viennas ‘Ringstrasse’, about 1930. (Source: Österreichis-ches Institut für Zeitgeschichte, Wien)

The Home Guards got financial and logistical support (also with arms) from industrialists, large landowners, Italian fascists, the Hungarian Horthy-regime and also Bavarian right-wingers. [Kerekes (1966), p23] As military ’mentors’ and officials therein acted former front officers. Not few leading members of the Home Guards emanated from the old aristocracy, or additionally recruited from notabilities, who particularly stood close to the GDVP. [Tálos (1995), p260] In October 1936 the remaining Home Guard elements were incorporated by Dollfuss' successor, Kurt Schuschnigg, into the VF (‘Vaterländische Front’, ‘Fatherland‘s Front’ or ‘Patriotic Front’, a right-wing, austrofascist political party founded on May 20, 1933, by E. Dollfuss) to get control over the rightist movement in whole Austria in unifying them under one banner, the ‘Kruckenkreuz’ (‘Crutch Cross’). The coup succeeded with the help of right-wing militaries and especially Ernst Rüdiger Starhemberg, who again got governmental power as Secretary of the Interior.

In March 1933 the events overturned: on March 4, the Dollfuss government shut down parliament. Caused by a heated parliamentary debate, convened by the SDAP, over an imperial war decree from July 25, 1914, specifying the deactivation of the railways in war situations as forbidden. [Hasiba (1984), p100] This law had never been used before. Analogously to the KWEG from 1917 (‘Kriegswirtschaftliches Ermächtigungsgesetz’, ‘Wartime enabling Act’) it has been transfered unaltered into the republic. Foregone was a strike of the railroadmen on March 1, 1933, with the deploy of the constabulary, the police and the federal army at the strikebound railroad stations. Numerous strikers were arrested. During the debate, the National Assembly’s presidents Renner and Straffner had to intervene to restore the parliamentary order. Detecting irregularities in the following votes Renner finally left his chair. [cp. Stenografisches Protokoll (1933), p3389] A new vote was held under the vice-chairman, the second president of the national assembly, Rudolf Ramek. When SDAP rejected the second vote demanding the house to acclaim the first vote as legal, Ramek also left the president's chair and resigned. [Stenografisches Protokoll (1933), p3393] The third president Sepp Staffner refused his own chairmanship during a third attempt. [Stenografisches Protokoll (1933), p3393] Without a president the National Council was officially unable to work, and in particular unable to close the current vote. Its members were barred from the possibility to elect a new president, what also implied an operating presidentship. The Austrian National Council was incapable of action. On the same day, on March 1, the 'Republikanische Schutzbund' was banned, but not the SDAP. On March 5, the leading members of the CSP agreed to continue the government even without parliament. Their decision was done devoid of consulting the chancellor or the federal president. The Austrian National-Socialists (starched by the political victory of the German NSDAP) clamored for Dollfuss' resign and reelections of the National Council, that they most likely would have won. [Busshoff (1968] To prevent this, the CSP rejected the National-Socialists' demands and made every possible effort to avoid reelections (this time in consensus with Dollfuss and the president Wilhelm Miklas). [Portisch (1989), p422] On March 7, the government decided to stay in office [Stenografisches Protokoll (1933), p3393] and proclaimed at the end of the meeting of the Council of Ministers: "*Der Nationalrat, die gesetzgebende Versammlung Österreichs, ist gelähmt und handlungsunfähig ... Die Regierung wünscht nicht, dass das Land dauernd einer aktionsfähigen, dem allgemeinen Wohl dienenden Volksvertretung entbehrt. Die Führung des Staates liegt aber nicht allein bei der Gesetzgebung, sondern ebenso beim Staatsoberhaupt und der Regierung. Die vom Herrn Bundespräsidenten ernannte gesetzmäßige Regierung ist im Amte. Sie ist von der Parlamentskrise, die ohne ihr Zutun heraufbeschworen wurde, nicht berührt, es gibt daher keine Staatskrise. ... Um in dieser aufgeregten Zeit Ruhe und Ordnung zu sichern,* *hat die Bundesregierung bis auf weiteres alle Aufmärsche und Versammlungen verboten und durch eine Verordnung auf Grund des Kriegswirtschaftlichen Ermächtigungsgesetzes zum Pressegesetz die Möglichkeit geschaffen, staats- und volksschädigende Missbräuche der Pressefreiheit sowie Verstöße gegen die öffentliche Sittlichkeit zu verhindern und zu bestrafen. ... Mitbürger! Die Bundesregierung führt euch diesen Weg und wird alle Mittel der gesetzmäßigen Autorität für dieses Ziel einsetzen. Folgt uns, helft uns! ...*" (*"The national council, the legislative assembly in Austria, is paralyzed and incapable of action… the government does not wish that the country permanently missing a parliament attended to common wellfare. The guidance of the state however not only is incumbent on the legislative, but also on the head of state and the government. The legal government, appointed by the federal president, is in office. It is not affected by the crisis of the parliament, which was caused through no fault on the government, hence there is no national crisis. … To ensure law and order in these agitational times, the federal government has forbidden any deployment and meeting until further notice and created by regulation of the press law due to the wartime enabling act the possibility of preventing and punishing any the nation and its people damaging abuse of the press freedom as well as any offences against the public morals. … Fellow citizens! The federal government leads you this way and will use by all means its according to law authority for this aim. Follow us, help us! …*"). [Frass (1967), p155]

The KWEG of 24. 7. 1917 had been an ideal instrument for Dollfuss purposes. It namely gave the government a possibility to rule without parliament (the Council of Ministers could decree resolutions without the need of the Federal President's acceptance). Given the case, the Federal President would have issued an emergency decree, he would have been bound on the Council of Ministers. Indeed Miklas requested several times to issue a Emergency decree, but the government never complied with him. [Welan (1984), p87] The proclamations accrding to the KWEG meant not only an assembly and deployment ban but also a censorship of the press. Especially the ‘Wiener Arbeiterzeitung‘ (the ‘Vienna Workers’ newspaper) was affected by the press censorship, as well as the traditional deployment of the SDAP on May 1. The government meanwhile had laid the cornerstone of its authoritarian course, that was seminal in the following months up to the moment of the proclamation of the May constitution and also helped to apply pressure against the opposition exactly at that time it would let the National Council convene again. Under this pressure the Social Democrats acquiesced to accept a constitutional reform that strengthened the position of the government and brought corporatist elements into the constitution. Dollfuss not only planned minor changes to the emergency decree rights of the Federal President and to the possibilities of the National Council, but also downgraded the Federal Council to a chamber for questions on economic policy and a kind of corporatist and federal council, as he already had envisaged in 1929. The National Council should be reduced to social policy issues. Above all he aimed to prevent new elections that would have been led to a huge victory of the Social Democrats and especially the NSDAP. [Busshoff (1968] So Dollfuss took up more and more time to prevent the reactivation of the National Council. At March 9, when the third national Council’s president, Sepp Straffner, convoked a session of the National Council for March 15, 1933, to close officially its unceased session from March 4 (what finally meant a reelection of the chair and the return to a constitutional order) Dollfuss dissolved the session (with approval of the Federal President) with police forces as an illegal assembly following the regulations from March 7. Because Straffner had resigned before, the government and also the Federal President regarded Straffner’s action as not conform to the constitution. [Welan (1984), p85] The claim of the Federal Council from March 17 to the Federal President to dismiss the government and its regulations basing on the KWEG including the withdrawal of the National Council and proclaiming new elections the government rejected (justificating this with the inferior role of the Federal Council). [Busshoff (1968]

On Mai 26, 1933, the communist party got banned and dissolved. [212] On March 31, 1933, the Republican Schutzbund met the same fate. This action was a concession of the government to the Home Guards. On April 10, 1933, the so-called 'Freiwilligen Assistenzkörper' ('Voluntary Assistance Corps') were formed and subordinated to the Federal Army, for its support. This had been de facto a size increase of the army in contradiction to the Treaty of St. Germain. On April 21, 1933, a general strike prohibition was proclaimed. On May 10, 1933, the government interdicted free elections in Austria, shortly after the NSDAP had reached 41 percent in Innsbruck on April 23, 1933, on the municipal council election. This verdict had been temporary at first, but was extended until May 1, 1934. On the same day a new official oath for civil servants had been introduced. It now had to be done to the 'Federal State of Austria and the government, instructed by the Federal President' ('Bundesstaat Österreich und die vom Bundespräsidenten bestellte Regierung').

On 19 June 1933, the government illegalised the NSDAP in Austria (in response to Hitler's 1000-Reichsmark fee on each departure from Germany to Austria). Dollfuss was always rigorously against the NSDAP triggered by repeated public attacks from NSDAP fanatics. On July 7, by law, the 'Freiwillige Schutzkorps' ('Voluntary Protection Corps') got organised out of the Home Guard troops and subordinated to the Secretary of Security, Emil Fey. They had a similar function as the 'Voluntary Assistance Corps'. [Busshoff (1968] The government also created detention camp for political prisoners. Although they are not comparable to the concentration camps of the Nazis in the German Reich, they form a clear parallel to the fascist system in

Italy and Germany. In the beginnings of 1934 it was not hard to foresee Austria's political future: "*Nach einer Erklärung die [Oberst] Redulic [der österreichische Militärattaché in Paris] mir gegenüber machte, kann das Vordringen und der Sieg des Nationalsozialismus in Österreich nicht mehr verhindert werden. ... Sein Eindruck ist der, dass Kanzler Dollfuß, Minister Fey und andere wahrscheinlich als Folge ihres politischen Standpunktes das Leben verlieren werden. Seiner Meinung nach kann in der gegenwärtigen Zeit von einer offenen Einverleibung Österreichs in Deutschland keine Rede sein. Daraus schließt er, dass die Umbildung Österreichs in einen nationalsozialistischen Staat eintreten wird ...*" [213] ("*According to a statement [Colonel] Redulic [the Austrian military attaché in Paris] made to me, the encroachment and the victory of Nazism in Austria can not be prevented any longer. ... His indentation is that Chancellor Dollfuss, Minister Fey and others as a result of their political position will lose their lives. In his opinion, in the present time there is no question of an open incorporation of Austria into Germany. He concludes that a transformation of Austria in a NS state will take place ...*").

On February 12, 1934, in an answer to the government's anti-SDAP measures a civil war broke out. Between February 12, and February 16, 1934, socialist and conservative-fascist forces grimly fighted, starting from Linz [214], flashing over to greater locations as Vienna, Graz, Wiener Neustadt, Steyr and Sankt Pölten. In Vienna members of the 'Defense League' barricaded themselves into several municipal housings, such as Karl-Marx-Hof (cp. 4.1.6). [Streibel (1994)] Police and Home Guards positioned themsevles outside these complexes and started to fire back at first with small arms, but when the federal army entered the conflict, they operated with light artillery. Many flats were destroyed before the socialist fighters finally surrendered. [Reppe (1993)]

Viennese and Upper Austrian fightings ended on February 13. At February 16, 1934, the Austrian Civil War ended. Several hundred people had died in the armed conflict, more than thousand people got injuries. The government enforced arrests and reimplemented the death penalty for several offences. Nine prominent Defense League members were executed according to martial law. [Brook-Shepherd (1996)] The SDAP, all trade unions and all Socialist worker organizations were banned. Subsequently, the democratic constitution was suspended and the period (from May 1, 1934 to March 3, 1938) of a fascist state [215] (friendly labeled as ‘Ständestaat‘, ‘Corporative State’ [216]) began. The Patriotic Front, into which the Home Guards and the CSP had been merged, became the only and monopolistic party. [Kriechbaumer (2005)] Dollfuss had finally knocked off legislative and judiciary powers, undermining the former constitutionally anchored separation of powers. [Botz (1979)], [Tálos (2005)] More pro-labour laws got abolished. [217] The constitution of 1920/29 was replaced by the ‘May Constitution’ in 1934. The events were ideologically accompagnied by Pope Pius XI. in the ’Quadragesimo Anno’, an encyclica from 1931, therein he appealed to all catholics: “*... from an altercation of the classes an unanimous cooperation of corporative parts of the population*“ (“*aus einer Auseinandersetzung der Klassen zur einmütigen Zusammenarbeit der Stände*“. [Pius (1933)]

4.1.8 Menger’s attempt in social logic

“*While the political situation in Austria made it extremely difficult to concentrate on pure mathematics, socio-political problems and questions of ethics imposed themselves on everyone almost every day. In my desire for a comprehensive world view i asked myself wether some answers might not come* *through exact thought*". [Menger (1998)] Menger had the plan to sketch a general theory of ethics, that, in analogy to a mathematical approach, should describe the relations between individuals and groups basing upon their different demands on each other. His result was the 1930 published booklet 'Morality, Decision and Social Organisation' [Menger (1974)] ('Moral, Wille und Weltgestaltung. Eine Grundlegung zur Logik der Sitten'), written in parts in form of succeeding letters and also in dialogue form, like a Platonic discouurse. Despite his hopes Menger earned incomprehension ("... *the whole fomulation of the question fills me with loathing*", Nöbeling to Menger [Menger (1998)]) or polite disaffirmation ("... *doubt wether there is scope in this field for a mathematician of your prowess*.", Veblen to Menger. [Menger (1998)]) The book's concept stands within the tradition of the Vienna Circle. Menger could not argue by justifying value judgements with facts (what indeed would have been the 'optimal' approach form the Circle's view), but had to eschew them by conceptualize social norms on the investigation of relationships within cohesive social groups. He carried out a kind of typology of social collectives. In his own words, the book "... *was a transfer of the tolerance principle from logic to ethic*.". [Menger (1998)]

4.1.9 The Balkan's drift to the right

As we have sketched above, the new nation-states of the Balkans could not solve their problems with the numerous ethnic minorities they accomodated. The new parliamentary governments had no tradition and experience to make political compromises. In contrary, their answer to instability and the manifold demands of the ethnic divisions was in fact a return to authoritarianism. The traditional elites, controlling the economy as before, appreciated such a political turn-back, as they never had agreed to any democratic candidness. The authoritarian states of the Balkans did not better in the long run in regard to the economic hardships, but than it seemed to be the way out of a incapability of action concerning a liberal and parliamentary system.

The fascism in Hungary started by the end of WWI. The center of ultra-nationalism was borne by former officers, civil servants and the landed gentry [218], mainly because these groups had lost much of their former wealth since the end of war and especially through Bela Kun's ‘Soviet Republic‘. The ‘White Terror‘ in 1919 was their answer to the former leftist ‘Red Terror‘. Parts of the ‘White Terror‘ forces came from Horthy's ‘National Army‘ and hence prepared the way for Horthy to come into power. [Bela (2004)] With Admiral Horthy [219] Hungary changed into a royal authoritarian state with a military power obeying the regent. The political Left was annihilated. But with the upcoming ‘revisionist’ tendencies in Hungary's foreign policy (claiming for the lost provinces), conservatives and nationalists more and more got a predominant position. One of the leaders of the ‘White Terror‘, the fascist Gyula Gombos [220], a former army officer, founded in 1923 the so-called Race-Protecting Party. Even more radical forces emerged like the ‘Arrow Cross’ movement (led by Ferenc Szalasi [221], a former army officer like Gombos). With Hitler's rise, the influential, conservative elite of Hungary knew it acceptable to jump on this bandwagon. The strength of the fascism not only grew the more the more the depression lasted, even when it had waned this movement developed its own untenable drive.

The face of the Romanian fascism was different in some aspects. It also grew by the activities from displaced former military members or civil servants (in analogy to Hungary), but mainly got developed in the impoverished rural regions where anti-urban, anti-Semitic and anti-capitalist attitudes were a daily occurrence. The Romanian King himself tried to enforce his dictatorship with the fascist movement. In his search for allies he did not shrink from the violent mischief-makers. A third element of Romanian fascism was its fear of Russia (a century old Rumanian angst), but especially the new Russian Communism was the adverse concept. But as denoted, Romanian fascism had its roots within the misery of the peasants. [222] In their rejection of any urban and commercial, the fascists [223] had a clear focus on the Jews. The fascist 'Legion of the Archangel Michael' grew until 1937, when it achieved 16% in the 1937 elections. [Paine (2001)] After that political success King Carol II startet to suppress the potential rivals. After 1938 the 'Legion' dissapered from the political arena.

4.2 The annexation of the mind

A quintessential point of the annexation procedure was the replacement of the intellectual elite and the attainment of a strong conformity within the remaining protagonists of teachings and research. Any opponents to the NS-ideology (foremost the ideology of the Austrian and German Nazi parties) or any teaching stuff that could not be included in correspondance to the 'Nürnberger Gesetze' ('Nuremberg Laws of Race') should be eliminated by banishment and later by deportation, whereas on the other side privileges were granted to the conform Austrians against the inflow of scientific immigrants from the 'Reich' in controlling vacant positions and means and by preferring their own network. In this way the 'new scientific generation' evolved under the auspices of the established and collaborating ones. A central theme was the assimilation of existing standards or an immediate or gradual replacing of existing standards with those of the German 'Reich'. This was combined with general reorganizational measures according to Nazi structures (the 'leader principle'). This hard ideological standardizing was more prosecuted by external authorities, also the student body and lower situated scientists. [Müller (2007)] With these 'Säuberungen' ('Cleansings') traditions of scientific innovation were lastingly destroyed. Research and teachings had been realigned towards an applied, utilitaristic and ideological steeped war-oriented direction, connected with tendencies of a planned economy. [Müller (2007)] Ethnic and political 'cleansings' within the universites caused an irreversible 'Brain-Drain'. It is a known fact that the three Austrian Universities of Vienna, Graz and Innsbruck as well as the Technical University of Vienna, the University of Natural Resources and Applied Life Sciences ('Universität für Bodenkultur'), the School of Economics in Vienna and the 'Montanuniversität' in Leoben had been in various ways core zones of Nazi activities, both regarding the development of organizational structures and the transfer and formation of ideology [Müller (2007)]. A enduring concealing of the immense engagement of Austrian universities in these affairs - caused by continuing of many participants in their positions, particularly at the Viennese Academy of Sciences - caused a differentiated and earnest debate for the first time in the 1980s. [Müller (2007)]

4.2.1 The 'Anschluss' of the University

On February 12, 1938, K. Schuschnigg (Austria's chancellor) was forced by Hitler on the occasion of a meeting in Berchtesgaden (Bavaria) to relegalize the Austrian NSDAP. [224] Hitler also claimed for a release of all imprisoned NSDAP members (including the Dollfuss murderers [Shirer (1984)]). To enforce his demands, Hitler threatened with military action. Schuschnigg bent [226], but in secret planned to accomplish a plebiscite in Austria to prevent the imminent annexation. His plan failed [227], so on March 11, he announced his resignation in the radio at 19.47 o'clock, after Hitler had issued an ultimatum for noonday, and Göring did the same in the afternoon at 15.05, both demanding his resignation and replacement with Seyss-Inquart. The Austrian Nazi's were well organized and informed. Within hours they succeeded to control political institutions in Vienna, even the Ministry of Internal Affairs. Until midnight the Austrian President W. Miklas refused to appoint Seyss-Inquart chancellor, but in the end complied. In the early night hours NSDAP members hoised above the police headquarters at the 'Schottenring' the Nazi flag. The guards of the city hall capitulated. The CSP mayor immediately was arrested. [Mayerhofer (1998)] In the morning hours of March 12, 1938, the German invasion began.

The director of the institute of botany and the Vienna gardens, the Austrian *Fritz Knoll*, a man who held lectures in SS-trousers since 1933, became 'Gaudozentenführer' ('NS leader of lectures') and in 1938 on instigation of the Vienna NSDAP rector of the Vienna University after having been elected its provisional leader. He dismissed the deans of five faculties and appointed new ones. [Heiber (1994)] The former mode of election was suspended and instead the 'leader principle' was established. These concepts of a 'leader-rector' lead back to the German practice since 1933. A significant fact and an important historical factor in the context of the 'cleansings' should be seen in the circumstance that the Austrian Oswald Menghin, professor of pre- and early history at Vienna University, was appointed minister of education under Seyss-Inquart on March, 11, 1938. During 1935/36 he has been rector of the Vienna University. O. Menghin had published 1933 the racist and antisemitic book 'Geist und Blut' (Spirit and Blood) and was active in Nazi circles through the

1930s. Apparently he was the right man to oversee and force the ‘cleansing’ of the University of Vienna in his short term of office until the end of May 1938. [Mühlberger (1993)] The divorcement in political reliable and unreliable members of the university has been carried out via the formal act of swearing them in on the ’leader’ A. Hitler, to what only ‘racial’ and political acceptable parts of the past personnel were admitted (completely similar to other public mechanisms). Before these official ‘cleansings‘ in individual institues ‘wilde Arisierungen‘ (wild aryanisations) took place. [Mayerhofer (1938)]

The process of the transformation of the university was not only the effect of a centralistic penetration by Berlin offices, but above all house-made - carried out of mostly ‘old-served’ representatives of the Viennese university. [Heiß (1989)] In this way complicated communications between the involved political instances evolved. The total extent of the ’cleansings’ could not be captured completely until today. Table 14 shows all dismissed and defrocked professors, partially relieved into retirement by compulsion during the NS-period, including also a minority of Nazi-teachers who became ideologically insubordinate. The portion of this group of the teaching body was calculated with 45 per cent. [Berger (1999)] Most of all the medical faculty was affected, followed by the philosophical and the jurisprudential faculties.

Table 14. Number of the dismissed, obligated to retire, professors at the University of Vienna 1938-1945

	Total	*I*	*II*	*III*	*IV*	*V*
Full Prof.s	82	3	1	14	26	38
Assistent Prof.s	233	3	0	31	146	54
Sum	315	6	1	45	172	98

Catholic-Theological Faculty I, Evangelic-Theological faculty II, Jurisprudential faculty III, Medical Faculty IV, Philosophical Faculty V [Mühlberger (1993)]

Of course the ’cleansings’ also concerned the students. For Jewish students at first a restrictive ‘Numerus Klausus‘ (a minimum of high marks) was ordered, and after the ‘Reichspogromnacht’ (during the night from Nov. 9 until Nov. 10, 1938) it became general custom that Jewish students were refused to enter the university. [Stadler (1995)]

4.2.2 New rules for science?

A known rule in science is, simplified spoken, that every scientific insight is independent of its discoverers race or nationality. A further rule can be formulated as the ’defense of the own guild’, what means the protective function of a specialist network for its scientific members, if these keep its fundamental agreements. Both rules were deeply hurt during the ’cleansings’, nevertheless they were at no time completely abrogated. Even in May 1941 F. Knoll tried to justify the NS-course of action. In a lecture hold at the German scientific Institute in Bucharest he said “*One cannot say, the Jew is incapable* *of research. A correct determined fact remains a fact, if it were determined by a Jew*." Continuing, he mentioned that National Socialism would rather fight how "*Jews use scientific results due to their hereditary disposition and their traditional education... . " and that "Jewish science differs generally not much from Jewish journalism. Both are due to achieve primarily world dominance and a exceeding prestige*.". [Mühlberger (1993)] A further change in traditional rules constitutes the new mechanism in the appointment of lecturers. The former commission, initiated by the faculties, was confronted with ordered consultants from the NS-federation of lecturers. The decisive criterion of political compatibility was placed to the criterion of the technical suitability and the acknowledgment through specialized professors. Within the group of the former lecturers who stayed in their positions during the annexation the necessity arose to get registered as one of the 'new order' to keep the competence of teaching. [Stadler (1995)]

4.2.3 The downfall of the Vienna University

After a few years the centuries-old primacy of the Vienna University in science and research apparently was broken compared to the corresponding german university strongholds. This fact was even a great disappointment for convinced NS-adherents within the university stuff. The crisis of the Vienna University already had begun in the 1930s with decreasing numbers of students, what further continued due to the NS-restrictions. On the teaching side the vacancies could not be filled adequately. The attempt, following orders from Berlin, to concentrate on 'Südostforschung' (the south-eastern regions as object of research not only in a racingismic sense) partly failed as this brought the Vienna University into a sharp competition to Graz or Prague. [Berger

(1999)] New foundations palmed as 'applied research' from 'Fächer des Frauenschaffens' (womens work knowledge) over 'Volkskunde' (folklore) to 'Rassenbiologie' (racingismic biology) also brought no advance (albeit only three of these 'disciplines' were again disestablished after 1945: that one for 'Rechtsvereinheitlichung' (standardization of law), for 'Rassenbiologie' and for 'Geschichte des Postwesens' ('history of postal service'). The utilitaristic orientation of research and teachings connected with war-oriented tendencies not only irritated the traditional educational elite but also NS-ideologists, who felt relegated in their own interests in research. [Mühlberger (1993)] It has to be pointed out, that there was no uniform NS-science pursuing uniform goals, and so many intrigues and feuds were on the agenda. Rivaling situations analogue to the fundamental one between the NS study group 'SS Ahnenerbe' (the 'German Ancestral Heritage' founded by H. Himmler et al. on July 1, 1935, palming itself as 'study society for intellectual ancient history') and the 'Office Rosenberg' (a smaller, more professional group of archeologists (at least in their schooling) driven by A. Rosenbergs confuse theories to find the archeological evidence of the superiority of 'germanic culture') often arose. [Kater (2001)] When later in 1940 Berlin appointed a curator to control the four Viennese NS university rectors, these felt heavily provoked (also by the upcoming university reform that brought deeper changes in unversity tradition than ever before) and began to resist their ministry. [Stadler (1995)] They remained vastly unsuccessful. In 1943 F. Knoll was replaced as rector by the Austrian anatomist Eduard Pernkopf, the Austro-Fascist and dean of medicine at the Vienna university since the annexation (who was the founding editor of a major volumes on human anatomy, that are still considered as 'masterpiece' [Williams (1988)], but are also in an ongoing discussion suspected showing portrayed victims of political terror. [Israel and Seidelman (1997)]

4.3 The Circle members

The Vienna Circle and the Colloquium kept their meetings despite the hostile political climate. The University often had to close its doors, it had been become itself a hotbed of Nazism and a place for agitation. But the members of the Vienna Circle soon dispersed. Many of them emigrated to the US, and not seldom continued to teach at various universities. Their disintegration began about 1934, the year when right-oriented Austrian publicists started mean attacks agiants the Circles. Although "*the annexation had not yet been completed ... there was something in the air that one did not want to stay. there was such a feeling that Austria would become a bad place to work*.". [Wiener Zeitung (2007)] Carnap had accepted a chair in Prague. Hahn died quite unexpected at the age of 55. Neurath could not return to Austria as the fascist regime had outlawed Social Democrats and would have arrested him as leftist agitator. O. Schlick remained in Austria, a deadly fault, as in 1936 he was killed by the fascist sympathizer Dr. Nelböck on the stairs to the University of Vienna. Newspapers condemned the murder but also pointed out (in a demagogic way by mixing up pseudo-catholic critics with fascist ideas) the 'demoralising' effects of free philosophers like Schlick on youth in its religious education. In the course of such arguments the high proportion of Jewish members in the Institute of Philosophy was also addressed to. [Stadler (1997)] Schlesinger remained in Vienna. On the very day of the Annexation he commited suicide.

Table 15. From 'Sturm über Österreich' on Schlick's death, 27. 9. 1936, [Stadler (1997)]

At the end of June this year, live bullets were fired at the Viennese University. An academic student shot dead his former teacher, Dr. Moritz Schlick - a respected professor of philosophy. This disastrous deed was committed by thirty-year-old Dr. Nelböck, probably deeply tortured in his soul by his former professor's destructive teachings. The murder case was heavily covered by the the Viennese daily papers who have acquired a pro Christian heart in recent years, differing from the mainstream Austrian media. Extrinsic elements, which overpopulate most Viennese newspapers, did not however forget to celebrate Professor Schlick's famous standing among philosophers and intellectuals.

In condemnation of the heinous act of a teacher's murder, we all have the same opinion. That probably requires no further comment. However, we shall also state here our wider perspective. Just now as the school doors re-open, the most active of commentators fall silent. And yet we all understand the reasons behind the murder. Should we remain silent if we see the education of our youth endangered, what is the more important?

Who was Professor Schlick? We do not want to question a dead man who has been called to trial by God already. We have to rely on God's justice and mercy to judge him. But the fact he was an atheist and a teacher of the youth in new Austria, forces us to treat this subject publicly.
And the question is: is it possible that in new Austria, a Dollfuss Austria, our youth is being educated wrongly? Can teachers who believe in pure materialism, simply carry on and remove faith from our young people? Heroic Chancellor Dollfuss declared, "We want a Christian state!" and sacrificed his life for this holy desire. His successors work tenaciously and intensively to carry on the Dollfuss program. Using their minds and might they were, and are, ready to fight for a Christian native country with total conviction. While teachers of today's youth - if unchecked - could destroy the foundation of the state and the Christian religion in the hearts of young people, by presenting it as senseless gimmick of half-fools!
Let us be clear! We do not want to be Methodists! We do not want to force our opinion upon anybody! Terror of conscience and subjugation of the mind were weapons of the past "red era". But what Christian people and Christian parents demand is that the Austrian youth must be educated according to Christian spirit! The state schools must help to build up Christianity and not act corrosively against it.

Some may claim overanxiously: "And what of the liberty of science?" Julius Langbehn, a Rembrandt-German says, "Any science that dispossesses one of

Langbehn, a remarkable German says, "Any science that dispossesses one of God can objectively be regarded as a lie." We say "Education without God is fraud." And we ask: "What are lies compared to fraud? Does that mean we disregard it all? Cherish the liberty of science! However, anarchic liberty without any bounds, undermining the basis of the state, is not liberty! Modern science increasingly approaches the opinion of the famous chemist, Louis Pasteur: "Despite the huge research I've carried out, I was able to keep the faith of a Breton farmer. When I will have explored even more, maybe I could have the faith of a Breton farmer's wife!" World views, which are scientifically outdated, do not have the right to be further taught in new Austria! Young people are the future of the state. All our special concern should be for them! Today's battles need really strong believing Austrians. We have to make sure that our youth will be saved from the spirit of godlessness! We should all help to educate young Ostmark Germans to believe in Christ!

In den letzten Junitagen des heurigen Jahres krachten auf der Wiener Universität Schüsse. Ein Hochschüler streckte mit einigen Revolverschüssen seinen ehemaligen Lehrer, den ordentlichen Professor für Philosophie Dr. Moritz Schlick, nieder. In seiner Seele tief erschüttert und verzweifelt, wohl nicht zuletzt durch die destruktiven Lehren seines Meisters, hatte der 32jährige Dr. Nelböck diese unselige Tat begangen. Das Echo, das der Mordfall in Wiens Tagesblättern fand, war ungeheuer. Zeitungen, die seit wenigen Jahren erst, als der Wind von einer anderen Seite blies, ihr Herz ganz urplötzlich für ein christliches Österreich entdeckt hatten, behandelten in spaltenlangen Ausführungen das Mordereignis. Gewisse volksfremde Elemente, die die Redaktionsstuben so mancher Wiener Zeitungen übervölkern, vergaßen wohlweislich nicht, Professor Schlick als weltberühmte Größe unter Philosophen und Denker zu feiern.

In der Verdammung des unseligen Mordes an den persönlich höchst liebenswürdigen Lehrer sind wir uns einig. Das bedarf wohl keiner Worte. Doch müssen auch wir vom Standpunkt unserer Weltanschauung einige Worte dazu sagen. Gerade jetzt, wo die Vielrederei geschäftiger Schwätzer um diesen Fall verstummt ist und wir die Mordtat in ihren Zusammenhängen sehen. Die Tore der Schule haben sich wiederum geöffnet. Sollen wir schweigen, wenn wir die Erziehung des Kostbarsten, was wir haben, der Jugend in Gefahr sehen?

Wer war Professor Schlick? Wir stellen fest: Wir wollen hier nicht zu Gericht sitzen über einen Toten, den der ewige Gott schon vor seinen Richterstuhl gerufen hat. Gottes Gerechtigkeit und

Gottes Barmherzigkeit müssen wir den Richtspruch über diesen Menschen überlassen. Doch die Tatsache: ein Atheist - Lehrer der Jugend im neuen Österreich, zwingt uns, eine Frage ins Licht der Öffentlichkeit zu rücken. Und diese Frage lautet: Darf es möglich sein, daß im neuen Österreich, im Dollfuß-Österreich, unsere Jugend, die Zukunft Österreichs, gottlos erzogen wird? Dürfen Lehrer, deren Weltanschauung nackter Materialismus ist, weiterhin den Glauben unseren jungen Menschen entreißen? Heldenkanzler Dollfuß erklärte: "Wir wollen den christlichen Staat!" und gab für dieses sein heiliges Wollen sein Leben. Seine Nachfolger arbeiten mit Zähigkeit und Kraft, das Dollfuß-Programm zur Tat werden zu lassen. Arbeiter der Stirne und Faust sind und waren bereit, mit ihrem Herzblut für ein christliches Vaterland zu kämpfen - und Lehrer der Jugend dürfen es ungestraft wagen, das Fundament des Staates, die christliche Religion, in den Herzen junger Menschen zu vernichten, als sinnlose Spielerei von Halbnarren hinzustellen! Man verstehe uns nicht schlecht! Wir wollen keine Mucker sein! Wir wollen niemand unsere Meinung aufzwingen. Gewissensterror und Gesinnungsknechtung waren Kampfmittel der vergangenen roten Ära. Aber das verlangt das christliche Volk und die christlichen Eltern: "Österreichs Jugend, so weit sie dem christlichen Bekenntnisse angehört, muß im christlichen Geiste erzogen werden! Die Schulen des Staates müssen mithelfen, aufbauend, nicht zersetzend zu wirken."

Und die Freiheit der Wissenschaft? So meinen manche Überängstliche. Der Rembrandtdeutsche Julius Langbehn sagt: "Jede Wissenschaft, die von Gott abzieht, ist objektiv eine Lüge. Ja, Bildung ohne Gott ist Betrug!" Und wir fragen: Gibt es eine Freiheit der Lüge, dem Betruge gegenüber? Heißt das Freiheit, Lüge und Betrug in die Halme schießen zu lassen? Freiheit der Wissenschaft in Ehren! - aber schrankenlose, zügellose, zersetzende Freiheit, die die Grundlagen des Staates unterwühlt und untergräbt, ist keine Freiheit! Die moderne Wissenschaft nähert sich immer mehr und mehr der Ansicht des großen Chemikers Louis Pasteur: "Trotzdem ich soviel geforscht habe, konnte ich mir den Glauben eines bretonischen Bauern bewahren. Wenn ich noch viel mehr erforscht haben werde, könnte ich vielleicht noch einmal den Glauben einer bretonischen Bäuerin besitzen!" Weltanschauungen, die wissenschaftlich längst erledigt sind, haben nicht das Recht, weiterhin im neuen Österreich gelehrt zu werden! Die Jugend ist des Staates Zukunft. Ihr gilt, ihr muß unsere besondere Sorge gelten! Die Kämpfe der Zeit erfordern ganze, glaubensstarke Österreicher. Sorgen wir, daß unsere Jugend vor dem Geiste der Gottlosigkeit bewahrt wird! Helfen wir mit, junge, christustreue Ostmarkdeutsche zu erziehen!

Karl Schlesinger 1889 - March 12, 1938

Wien, II. Venedigerau 3, Telephon R-41-0-75.

Vier Selbstmorde.

Im Badezimmer ihrer Wohnung, 9., Währingergürtel, hat sich gestern vormittag die 40jährige Beamtin Alma Biro mit einem Rasiermesser Schnittwunden an der rechten Handbeuge beigebracht und sich sodann mit Leuchtgas vergiftet. Sie wurde von der Rettungsgesellschaft tot aufgefunden. — Gleichfalls gestern vormittag hat sich der 49jährige Schriftsteller Dr. Karl Schlesinger in seiner Wohnung, 9., Alserbachstraße, eine Kugel in die rechte Schläfe gejagt. Er wurde bei seinem Schreibtisch sitzend von der Rettungsgesellschaft schon tot aufgefunden. — Mit einem Schlafmittel hat sich gestern die 69jährige Private Helene Kuhner in ihrer Wohnung, Würthgasse, getötet. Das Motiv ist unbekannt. — Aus einem Fenster seiner Wohnung Custozzagasse hat sich gestern nachmittag aus unbekanntem Motiv der 36jährige Beamte Leopold Wien auf die Straße gestürzt.

Grillparzer-Gesellschaft.

Die für Montag den 14. d., 18.30 Uhr, anberaumte außerordentliche Jahresversammlung und die Vorlesung von Frau Lisa Bernauer-Michalek werden bis auf weiteres verschoben.

Schulverein für Beamtentöchter.

Figure 9. A Viennese newspaper article reporting the daily suicides, March, 1938, source: http://www.oemg.ac.at/

K. Schlesinger committed suicide on the day of Hitler's invasion in Vienna, the 12th of March, 1938. [Cepa (2008)] Figure 4 shows an article in a Viennese newspaper concerning the daily suicides: "*Yesterday morning the 49 year old writer Dr. Karl Schlesinger in his dwelling, 9. Alserbachstrasse, had shot a bullet into the right temple. He was already found dead by the rescue team,* *lying besides his desk.*“ (“*Gleichfalls gestern Vormittag hat sich der 49 jährige Schriftsteller Dr. Karl Schlesinger in seiner Wohnung, 9. Alserbachstrasse, eine Kugel in die rechte Schläfe gejagt. Er wurde bei seinem Schreibtisch liegend von der Rettungsgesellschaft schon tot aufgefunden.*“) Already soon thereafter one forbade to the Viennese newspapers reporting on the amassment of the suicides. [Sigmund (2001)] On the treatment of Jews following the ‘Anschluss’, see Oxaal [Oxaal, Pollak (1987)], Wistrich [Wistrich (1992)], and Pauley [Pauley (1992)]. Botz [Oxaal, Pollak (1987), pp185-204] reports that despair among the Jewish upper middle classes dramatically increased the number of suicides in the months following the Anschluss, with 220 reported in March alone.

4.4 AW’s struggle for emigration

Morgenstern, a German, who was born in Görlitz (1902), could have lived in a NS regime. But he decided to go. His passage from Vienna was much easier than that of AW. Not only did he arrive in the US with a fix fellowship from the Carnegie Endowment for International Peace, but also the Rockefeller Foundation helped him to settle in Princeton by paying half of his salary for a while. Menger had been left already in the beginning of 1937. The latter left behind a request by letter for Alt to take great care of Gödel, who meanwhile lived quite secluded, with the words: “*The heaven knows, wherein he spins himself, if he does not occasionally talk to you and the other friends in Vienna. It is therefore on my own responsibility that you are intrusive if necessary.*“. [Wiener Zeitung (2007)]

F. Alt and his wife Alice (both had married briefly before their departure) were well informed by friends from abroad and submitted betimes an application for US immigration visas short before the annexation took place. [Wiener Zeitung (2007)] On the occasion of the symposium 'mathematics and emigration' in Vienna in 2001 Alt remembers: "*Already on the 7th of May 1938 i took the night train from Vienna to Zurich. ... In Feldkirch the duty officer took one hour to look at each piece in my suitcase. I tried to retain calm outwardly but I am convinced that he looked for only one plea in order to arrest and send me into a concentration camp. When we finally left, I could not yet believe being off from Austria and in freedom.*" They had the luck to reach New York, where parents and friends waited for them. But many of Alt's family members got perished by the Nazis. Alt was again unemployed. He sends out 60 applications and receives 58 refusals: "*Three months of full despair - however nevertheless only three months*". [Wiener Zeitung (2007)] Finally he got two job offers on only one day. With the members of the Mathematical Colloquium, from which the majority meanwhile had been arrived in the US, the liaisons persisted: "*Actually over there I became acquainted with Wald much better*". [Wiener Zeitung (2007)] Nöbeling, who had abrogated his position as an assistent at the University of Vienna, was the only one who decided to pursue his career in Nazi Germany. He did this much to Menger's chagrin. [Menger (1998)]

In early 1937, AW continued to worry. Morgenstern continued to press his case. Van Sickle continued to resist: "*In spite of Morgenstern's guarantee of employment in the Institute on his return to Vienna, I doubt whether there is any real future for [Wald] there. Growing anti-semitism has closed the doors to such men throughout most of central Europe. It is a tragic situation but I don't see how we can use our fellowships to combat the trend. If an award were made to Wald to study in this country I am convinced that he would use the sojourn here to seek permanent employment*". [228] Van Sickle made the suggest to obtain the opinion of further scholars, wether AW was "*really gifted*". [229]

Meanwhile Morgenstern had sent AW's general equilibrium paper to van Sickle. But van Sickle struggeled with it, and decided to give it W. Weaver, a man at the Rockefeller offices in New York. In his enclosed letter he deprecatingly wrote the line: "*He is one of those homeless Jews whom it is very difficult to place*". [230] Weaver again forwarded AW's work to Harold Davis at the Cowles Commission, sharing van Sickle's view. [231]

AW more and more grew anxious. He expressed his fear to Menger, having no clue wether his employment at Morgenstern's Institute would continue. By necessity he sent reprints of current papers he had worked on as well to Hotelling and as to Schultz. Then out of the blue (but on mediation of Morgenstern) he got an invitation by Hans Staehle, the director of economic research at the League of Nations, to come to Geneva in the months of September and October. His challenge should be to work on price indices, within a research program of the International Labour Office, that conducted several cost-of-living analyses. Here, building on earlier work by Haberler, Leontief and Staehle, he showed how an improved approximation to the true cost of living index could be constructed: under the assumption that the utility function could be approximated by a second-degree polynomial, given certain other restrictions on the indifference mapping. By the same means, he showed how statistical data could be used to numerically estimate the underlying utility function and hence the demand functions. AW again produced quickly and inventive 3 papers. These appeared in 1937 as 'Zur Theorie der Preisindexziffern' (Zeitschrift für Nationalökonomie, vol. 8, pp179-219), in 1939 as 'A New Formula for the Index of Cost of Living' (Econometrica, Vol. 7, pp280-306) and in 1940 as 'The approximate determination of indifference surfaces by means of Engel curves' (Econometrica, Vol. 8, pp96-100).

On behalf of AW proceedings Staehle was motivated to write to Kittredge about his positive experience with AW. Therein he especially recommended a Fellowship for him. Staehle mentioned that Haberler, Menger, Frisch and Tinbergen should also be asked on that question. [232] Kittredge remained restive. [233] Also Morgenstern inconstancy became evident in AW's case. After a conversation between Morgenstern and Kittredge on the same day, Kittredge wrote in a confidential letter to van Sickle about a turn in the matter: "*OM of course shares Staehle's views as to AW's quite unusual abilities . . . [but] Morgenstern still feels however that if only one appointment from Vienna can be envisaged in 1938, he personally would give preference to the candidacy of Kamitz. K. has become Morgenstern's chief of staff and has been sharing increasing responsibility for the theoretical as well as for the practical investigations of the Institute. If an exceptional appointment could be made to Wald in addition to the ordinary fellowship appointment requested for Kamitz, Morgenstern would be delighted*". [234] It appears embarrassing that Morgenstern, at this very moment, did not advocate for AW. It is clear, that Kamitz would not face the question of being a 'homeless' Jew, he not even was a Jew. AW was rebuffed, again, what redounded to Kamitz' advantage. [235] In late 1937 the Rockefeller Foundation continued funding Morgenstern's Institute, that had "*greatly abridged the freedom of the individual*". [236] It was reassured to do so by the League of Nations officials (probably Staehle) saying that the Institute woud be "*a bright spot in a part of Europe not conspicuous for scientific detachment... The work is not subjected to political pressure and ... the Director's* [Morgenstern's] *views are scientific and detached as far as any human being's can be*". [236]

4.4.1 Salvage from the Cowles Commission

During the discussion about AW and Kamitz the Foundation asked for opinions on the scientific benefits of both men. Tintner and Haberler rated AW wide above Kamitz. Kamitz, the unanimous opinion, would not even have the ability of E. John, the previous Rockefeller Fellow. [237] Morgenstern meanwhile had emigrated to the US. Van Sickle spoke to him about a Carnegie fellowship for AW for the first few months of 1938. After their meeting van Sickle wrote: "*I am quite ready to believe that Wald is quite unusually gifted. I still do not see how we can give him a fellowship, in view of the fact that he would be almost certain to use the fellowship to secure a permanent position in this country. ... Morgenstern yesterday ... said that Wald had been offered a Cowles Commission fellowship. This offers $1,000, but nothing for travel. As Wald is responsible for his parents in Rumania, he has not been able to save anything and cannot, therefore, finance the trip to Colorado. Morgenstern expressed the hope that we might be able to make a grant-in-aid to get him over here. I told him that I did not see how we could possible do so, much as I should like to help Wald. I suggested that he attempt to interest some well-disposed American Jew in Wald with a view to getting the slight assistance that was needed.*". [105] In the end of that blue chapter in AW's life, he not even got some travel money from the Foundation.

Table 16. O. Morgenstern to AW, January 28, 1938, Private Coll. Robert M. Wald, Chicago

Cowles Commission for RESEARCH IN ECONOMICS
Colorado Springs,
Alfred Cowles, 3rd President
January 28, 1938

Dr. A. Wald
c/o Oesterreichisches Institut für Konjunkturforschung
Vienna I., Stubenring 8-10
Austria

Dear Dr. Wald:

I have your letter of January 13 with inclosures and note that the circumstances at Vienna might end your time until that fall. Whether or not we had a fellowship for you at that time would depend on whether the one now available were to be awarded to someone able to come here this spring. I cannot, herefore, promise you a fellowship for next fall. If, however, you find it possible to arrange your work so as to come here this spring, we will definitely award you a fellowship of $ 1000 for 12 months in residence at Colorado Springs and, in addition, $ 200 to apply on your travelling expenses from Vienna. As a Research Fellow of the Cowles Commission you would also receive travelling expenses to scientific meetings in the United States.

It would be quite acceptable for you to continue here investigations you began at the Geneva Research Centre.

Sincerely yours,
Alfred Cowles 3rd m.p.

It was the end of January 1938. The Nazi activities became more and more publically visible. Infringements on Jews were a daily occurrence. Anti-Semitic scribblings on walls had only been the start for more violence against Jews. In February Schuschnigg, after the famous Berchtesgaden meeting with Hitler, had capitulated, Seyss-Inquart had been admitted to the Austrian cabinet as

Minister of the Interior. Hence the Nazi sympathisant had now the power to control the police. Only days later, after Schuschniggs planned plebiscite had failed, he stepped down and Hitler's favourite Seyss-Inquart was appointed Chancellor. That happened on March 11, 1938. On the same night crowds of Nazis roamed through the streets of Vienna. Three days after the German invasion, on March 15, 1938, E. Wagemann, then director of the Berlin Institute (the German Institute for Economic Research), arrived in Vienna. He had the instruction to liquidate Morgenstern's Institute. After only one week, Wagemann had dismissed most of the staff, also AW and formally Morgenstern, who had been since several months in the US. Only the Kamitz and John were allowed to stay. Kamitz was made acting Director. He was not allowed to write to Morgenstern or any foreign institution (including the Rockefeller Foundation). But he did not obey. In early May, Kamitz held a secret meeting with Kittredege, in an old café on the outskirts of Vienna. [239] Kittredge assured Kamitz further support from the Foundation, if he could guarantee the independence of the Institute, as well in reporting on Austria's ecnomic condition as in their fundamental work. Kamitz talked to Wagemann about the offer. But Wagemann, under the whip of the German Nazis, could not accept any independent research. He threatended to skip the monograph series of the Institute. Kamitz acted pragmatically. He still informed Kittredge about the developments, telling him that he had "*no personal difficulties*" to be asked to do lectures at the 'Hochschule für Welthandel'. Kamitz substituted discharged professors and saw the possibility for an employment as dozent, so his "*prospects for an academic career . . . seemed good*.". [240]

On March 19th, as President of the National Economics Assocation, Hans Mayer wrote to all members: "*In consideration of the changed situation in the German Austria I am informing you that under the respective laws now applicable also to this state, all non-Aryan members are leaving the Economic*

Society". [Mises (1978) b, p99] (In Mises Recollections, written in 1940 when he had just arrived in the US and was bitterly upset at the turn of events, Mises condemns Mayer as a Nazi collaborator, and dismisses him as an economist. The lack of the historical interest in Mayer's economics from the outbreak of the WWII onward was shaped by his actions during the nazi phase of Austria and especially by v. Mises' 1940 condemnation). Not until 1994 some of Mayer's work has been translated into English, in a volume of Austrian readings, edited by Israel Kirzner (1994). But, by then, many of those members, Christian, Jewish, and the 'mixed group' alike, had already left or were, in one manner or another, leaving Vienna. Mises was in Geneva, and Hayek had long been in London. Menger was now at Notre Dame, Tintner in Iowa City, Haberler in Harvard, Machlup in Buffalo. Morgenstern was in the US, searching for a new university. In late 1937, he had broken with the Austrian regime over its unwillingness to face up to agrarian special interests in the matter of downward price adjustments. When the Nazis took over the Institute in March 1938, Morgenstern, who by then had left the city, was deemed persona non grata.

Within a few weeks, Menger also managed to obtain an American Immigration visa for AW. AW's plans to continue the joint work with Menger on differential geometry could not be continued.

Table 17. AW to Menger, Vienna, 17.2.1938, Menger Archive, Durham, Box 2, Folder 7

Dear professor!

I have just received the message from Mr. Cowles that he can assure me only the fellowship when I move this spring and offers to me at the same time 200 dollars for travel expenses. I cannot postpone the decision any longer. The offered fellowship is very scarce and will not be enough for a return journey which might be necessary; but I believe that I may not miss this favorable opportunity. I therefore inform Mr. Coles that I will accept his offer and will come over either at the end of April or at the beginning of May. I feel very sorry to leave Vienna and the Institute where I have worked for so many years. I always felt very comfortable there, I could mainly dedicate myself to purely scientific questions and was not very restricted concerning the selection of problems. I owe all this to you and your special understanding for my scientific tests. I will always keep this in grateful memory. Because of time scarceness, I am afraid that I cannot finish my tests on price indices and level indifferences in Vienna.

However, I will do this in Colorado Springs and will then send the manuscript to the Institute. There is here now much to do, a set of new series will be represented in the February issue. I dealt a lot with the quantum index for external commerce to maintain concrete results. With the new series, which are to be represented in the February issue, various statistical problems occurred, which I also had to solve. I am willing, even if I am in America, to take over the handling of statistical problems for the Institute. If it does not concern very urgent questions, perhaps this would be possible. It would make me quite happy to maintain the contact with the Institute. Have you spoken to Mr. v. Sickle in New York? As I will accept Mr. Cowles's offer, it would be extraordinarily favorable, if I could get a certain subsidy from the Rockefeller Foundation. You would prove a large favor to me, if you did something in this affair. How are you? I am sure that you have large success with your lectures. I would be extraordinarily pleased, if we could meet in America. How long do you intend to stay there?

With best greetings

A. Wald

Lieber Herr Professor!

Ich habe soeben von Mr. Cowles die Mitteilung erhalten, dass er mir das fellowship nur dann zusichern kann, wenn ich noch in diesem Frühling hinfahre, und biete mir zugleich für Reisespesen 200 Dollar an. Ich kann nun die Entscheidung nicht mehr hinausschieben. Es ist zwar das angebotene fellowship sehr knapp und für eine Rückreise, die unter Umständen notwendig sein kann, reicht es schon nicht; doch glaube ich, dass ich diese günstige Gelegenheit nicht verscherzen darf. Ich werde daher Mr. Cowles schreiben, dass ich sein Angebot annehme und werde Ende April oder Anfang Mai hinüber fahren. Es tut mir sehr leid, Wien und das Institut zu verlassen, wo ich so lange Jahre gearbeitet habe. Ich hatte es hier immer sehr angenehm, ich konnte mich hauptsächlich rein wissenschaftlichen Fragen widmen und war auch in der Auswahl der Probleme nicht sehr gebunden. Das alles habe ich in erster Reihe Ihnen und Ihrem besonderen Verständnis für meine wissenschaftlichen Untersuchungen zu verdanken. Ich werde dies stets in dankbarer Erinnerung aufbewahren. Wegen Knappheit der Zeit fürchte ich, dass ich meine Untersuchungen über Preisindizes und Indifferenzflächen nicht noch in Wien zum Abschluss bringen kann.

Ich werde dies aber jedenfalls in Colorado Springs tun und werde dann dem Institut das Manuskript einsenden. Es gibt hier jetzt viel zu tun, in dem Februarheft werden eine Menge neue Reihen dargestellt. Mit dem Quantumindex für den Aussenhandel habe ich mich viel abgegeben, um möglichst konkrete Ergebnisse zu erhalten. Bei den neuen Reihen, die im Februarheft dargestellt werden sollen, sind ebenfalls verschiedene statistische Probleme aufgetreten, die ich zu lösen hatte. Ich bin gerne bereit, auch wenn ich in Amerika bin, die Behandlung von statisischen Problemen für das Institut zu übernehmen. Wenn es sich nicht um sehr dringende Fragen handelt, wäre vielleicht dies ganz gut möglich. Es würde mich sehr freuen, auf diese Weise mit dem Institut in stetigem Kontakt bleiben zu können. Hast du in New York mit Mr. v. Sickle gesprochen? Da ich das Angebot des Mr. Cowles annehme, so wäre es ausserordentlich günstig, wenn ich von Rockefeller Foundation nicht einen gewissen Zuschuss bekommen könnte. Sie würden mir einen grossen Freundschaftsdienst erweisen, wenn Sie in dieser Angelegenheit etwas tun könnten. Wie geht es Ihnen? Ich bin sicher, dass Sie mit Ihren Vorträgen grosse Erfolge haben. Ich wäre ausserordentlich erfreut, wenn wir uns in Amerika treffen könnten. Wie lange beabsichtigen Sie, dort zu bleiben?

Mit besten Grüssen

Ihr sehr ergebener

A. Wald .

In Vienna the Nazis raged against the Jews who were forced into demeaning acts. They defaced their shops, destroyed Jewish property and plundered them. By April 3rd, O. Morgenstern, meanwhile in Wisconsin, wrote to van Sickle about another suicide: besides Schlesinger, Kunwald, another economist, had taken his own life. On April 11th, the Monthly Bulletin of the Institute of Business Cycle Research appeared with following foreword written by Wagemann: "*The vast historical development of these days, which has inspired and widened the life the German people in all its aspects, emphasizes also new ways for this publication. Out of the union of Austria with the Reich there has developed on the economic side two important issues. It will now be necessary, in general, to provide for the fusion of the economic and constitutional life of these two different State economies and, in particular, to overcome the economic distress of Austria. This has to be accomplished by the powerful and quickly- effective means and methods which National Socialism has developed and which were completely lacking in the former Austrian government with its remarkable lack of understanding ... The close collaboration of both* [the Berlin and Vienna] *research organizations will make possible our fruitful collaboration in the great tasks which lie before us*". [241] Excluded from this project, and fearful of the currishness of the Nazis, AW was still in Vienna. He wrote to K. Menger about the bureaucratic difficulties made by the Rumanian government that would only issue a 3-month passport, whereas the Cowles position was offered for one year. AW hoped Cowles would not make any difficulties for him: "*It would be a great misfortune for me were I to lose this position. I would then be facing the abyss and would not even have the financial means to travel anywhere*". AW could not even leave Austria to go home to Cluj because the Rumanian government had forbidden the reentry without the special permission of the Ministry of the Interior. Then, at the eleventh hour, he got out, making it to the US, and Colorado, via Cuba. With the exception of one brother, who also made it to the US, all of AW's immediate family disappeared forever in concentration camps. In Vienna, for all the expression of venom and hate of early 1938, the city was struck silent. Mises, Menger, Morgenstern, Schlesinger, Machlup, Tintner, Haberler, AW - all were gone. The Institute was not more then a shell. Of those present at the K. Menger's talk on the Petersburg Paradox a decade ago, there remained only Mayer, now presiding over the Economics Society.

Table 18. AW to Menger, Vienna, April 18, 1938, KM Archive, Durham, Box 1, Folder 21.

Dear Professor,
I hoped very much to receive some news from you. Did you get my letters in which I informed you, that I have obtained a scholarship from Cowles for one year and that I intend to travel over in the beginning of May? I do not even know wether my letters of last summer (I wrote one from Vienna and another one briefly before from Rumania), in which I included among other things notes on the Euler equations, you have achieved. Mr. Alt mentioned to me that he has received in January and now recently a few lines from you in that you neither mention the Euler equations nor my letters. Perhaps a letter has been lost. I should have to take office at Cowles already in May.
However, some difficulties occurred there, by what my journey may be possibly delayed a bit. For I have got the pass from the Romanian consulate only for 3 months. In the longer term passes currently can be issued only with the approval of the Ministry of the Interior, as in the next two months there will be held a revision of citizenships. With a pass only valid for 3 months, I can not travel over, as the US *visitor visa (I just get a visitor visa) expires two months earlier than the pass. I have therefore requested in Bucharest for a renewal of my passport and I pretty much hope that it will be done conveniently in a short time. Should this not be possible against all expectations, so I have to wait about two months until the revision of citizenships is conducted. After that I can get a passport without difficulty for the longer term. I have written everything to Cowles and hope that he will make no problems on that. It would be a great misfortune for me if I lose this job; than I am left with nothing and do not even have the financial means to travel anywhere.*
Since March 15, I am no longer at the Institute and I have no income. I wanted to travel to Cluj to visit my parents for there to await the settlement of my passport issue, but it was not possible. After a decision of the Romanian government the Romanian citizens residing in Austria are only allowed to travel home with a special authorisation of the Ministry of the Interior and it is quite difficult to get one. I hope that everything will be done advantageously and that I will still to be able to take the mid-May trip. I would love to debark on the way and visit you. I beg you to inform me if I can meet you. A reply by mail to my address in Vienna (VI, Kasernenweg 13) will probably still reach me. I would be extremely happy to see you again and to have the opportunity to discuss several mathematical questions. It would be so wonderful if it would arise the opportunity of a closer cooperation.

In the hope to see you again soon, I'm with kind regards A. Wald

Lieber Herr Professor!

Ich habe schon sehr gehofft von Dir Nachrichten zu bekommen. Hast Du meine Briefe erhalten, in denen ich mitteilte, dass ich von Cowles ein Stipendium auf ein Jahr habe und dass ich Anfang Mai hinüberzufahren beabsichtige? Ich weiss nicht einmal, ob meine Briefe vom vorigen Sommer (einen habe ich noch von Wien und einen kurz darauf aus Rumänien geschrieben), in denen ich u.a. auch über die Eulerschen Gleichungen einiges mitgeteilt habe, Dich erreicht haben. Herr Alt erwähnte mir, dass er im Januar und jetzt vor kurzem einige Zeilen von Dir erhalten hat, dass Du aber weder von den Eulerschen Gleichungen noch von meinen Briefen irgendetwas erwähnst. Es ist vielleicht doch ein Brief verloren gegangen. Meine Stelle bei Cowles müsste ich schon im Mai antreten.

Es sind jedoch gewisse Schwierigkeiten aufgetreten, wodurch meine Reise eventuell etwas verzögert werden kann. Ich habe nämlich vom rumänischen Konsulat den Pass nur für 3 Monate bekommen. Auf längere Zeit können Pässe derzeit nur mit Bewilligung des Innenministeriums ausgestellt werden, da in den nächsten zwei Monaten eine Revision der Staatsbürgerschaften stattfindet. Mit einem nur für 3 Monaten gültigen Pass kann ich aber nicht hinüberfahren, da das amerikanische Besuchsvisum (ich bekomme nur ein Besuchsvisum) mit zwei Monaten früher als der Pass abläuft. Ich habe daher in Bukarest wegen Verlängerung meines Passes angesucht und hoffe sehr, dass es innerhalb kurzer Zeit günstig erledigt wird. Sollte dies wider aller Erwarten nicht möglich sein, so müsste ich ca. zwei Monate warten, bis die Revision der Staatsbürgerschaften durchgeführt ist. Nachher kann ich ohne Schwierigkeit einen Pass auf längere Zeit bekommen. Ich habe Cowles alles geschrieben und hoffe, dass er mir deswegen keine Schwierigkeiten machen wird. Es wäre ein grosses Unglück für mich, wenn ich diese Stelle verliere; ich stehe dann vor dem Nichts und verfüge nicht einmal über die finanziellen Mittel um irgendwo hinfahren zu können.

Seit dem 15. März bin ich nicht mehr beim Institut und habe auch kein Einkommen. Ich wollte nach Cluj zu meinen Eltern fahren um dort die Erledigung meiner Passangelegenheit abzuwarten, es war aber nicht möglich. Nach einer Verfügung der rumänischen Regierung können nämlich die rumänischen Staatsbürger, die in Österreich wohnhaft sind nur mit einer Spezialbewilligung des Innenministeriums nachhause fahren und es recht schwierig eine solche *zu bekommen. Ich hoffe sehr, dass alles günstig erledigt wird und dass ich noch vor Mitte Mai die Reise antreten werde können. Ich möchte sehr gerne unterwegs aussteigen und dich besuchen. Ich bitte dich sehr, mich zu verständigen, ob ich dich antreffen kann. Ein Antwortschreiben postwendend auf meine Wiener Adresse (VI, Kasernenweg 13) wird mich wahrscheinlich noch erreichen. Es würde mich ausserordentlich freuen dich wiederzusehen und die Gelegenheit zu haben verschiedene mathematische Fragen zu diskutieren. Es wäre so schön, wenn sich wieder die Möglichkeit einer engeren Mitarbeit ergeben könnte.*

In der Hoffnung auf ein baldiges Wiedersehen, bin ich mit vielen herzlichen Grüssen A. Wald

Since March 15, 1938, AW had been no longer in the Department of Mathematics at the University of Vienna. With anxious hope he was working hard to realize his emigration to the US as soon as possible. His hopes before his departure to return to Romania for a last time got dashed because he lacked the allowance to enter Romania. He never will see his parents again.

Table 19. AW to O. Morgenstern, Colorado Springs, ca. June 1938, priv. Collection R. M. Wald, Chicago.

Dear Professor!
Many thanks for your kind letter from Chocones as well as your regards. I feel very comfortable here and I got quite used to the new situation. Cowles and also the other members of the commission are very nice. I presented a work programme about index numbers and definition of index level to the Cowles commission which has been accepted. This is very satisfactory for me as I will be working at the solution of problems which are very interesting for me. The conference is already over. It was not overwhelmingly interesting, but I had the chance to meet a lot of people. Schultz from Chicago was also there. He was briskly interested in my works about index numbers and index levels. He especially thought that the definition of index intersections is extraordinarily important.

It is planned that I will give lectures about mathematical logics at the local college. With regards of my knowledge in the English language, I will only start at the beginning of the second semester. I make good progress in learning English. I spoke English at the conference and according to public opinion my spoken language was okay. Everything would be well if I had not the sorrow about my stay permit to run out at the end of March. I already informed you that I only received a visitor visa as the Rumanian quote has been closed for years. The only way for me is to find an adequate position at the university. Because then I will be able to get an immigration visa beyond the quote. But it is very difficult to find a position within such a short period of time. I received a very nice letter from Menger some days ago. He also sent a letter to me to Vienna, but this one apparently got lost. Everything is okay between us. I hope you are well and I wish you a very pleasant summer.

With best regards
Your A. Wald

Lieber Herr Professor!

Vielen Dank für Ihren freundlichen Brief aus Chocones, und für Ihre guten Wünsche. Ich fühle mich hier ganz wohl u. habe mich in die neuen Verhältnisse bereits gut eingelebt. Cowles u. auch die übrigen Mitglieder der Commission sind sehr nett und freundlich. Ich habe ein

Arbeitsprogramm über Indexziffern u. Bestimmung von Indifferenzflächen der Cowles Commission vorgelegt, das angenommen wurde. Dies ist für mich sehr angenehm, da ich jetzt über Probleme arbeiten werde, die mich sehr interessieren. Die Konferenz ist bereits vorüber. Sie war nicht überwältigend interessant, aber ich hatte die Gelegenheit viele Leute kennen zu lernen. Schultz aus Chicago war auch hier. Er hat sich lebhaft für meine Abeiten über Indexziffern u. Indifferenzflächen interessiert. Besonders die Bestimmung der Indifferenzflächen hielt er für ausserordentlich wichtig.

Es wird geplant, dass ich an dem hiesigen College über math. Logik lesen soll. Mit Rücksicht auf meine englischen Kentnisse werde ich damit erst im zweiten Semester anfangen. Ich mache im englischen ganz gute Fortschritte. Auf der Konferenz habe ich englisch gesprochen u. nach allgemeiner Meinung war es auch sprachlich ganz in Ordnung. Es wäre alles gut, wenn ich nicht die Sorge hätte, dass meine Aufenthaltsbewilligung Ende März abläuft. Ich habe Ihnen bereits geschrieben, dass ich nur ein Besuchsvisum bekommen konnte, weil die rumänische Quote auf Jahre geschlossen ist. Der einzige Ausweg für mich ist eine Universitätsstelle. Dann kann ich nämlich ein Einwanderungsvisum ausserhalb der Quote bekommen. Es ist aber sehr schwer innerhalb so kurzer Zeit eine Stelle zu finden. Von Menger erhielt ich vor einigen Tagen einen sehr netten Brief. Er hat mir auch nach Wien geschrieben, aber dieser Brief ist scheinbar verloren gegangen. ***Es ist also alles in Ordnung zwischen uns.***

Ich hoffe dass es Ihnen gut geht u. wünsche Ihnen einen sehr angenehmen Sommer.

Mit besten Grüssen

Ihr sehr ergebener A. Wald

4.4.2 The Cowles Commission

The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. In 1955 the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation. (cited from [Cowles Foundation (2007)])

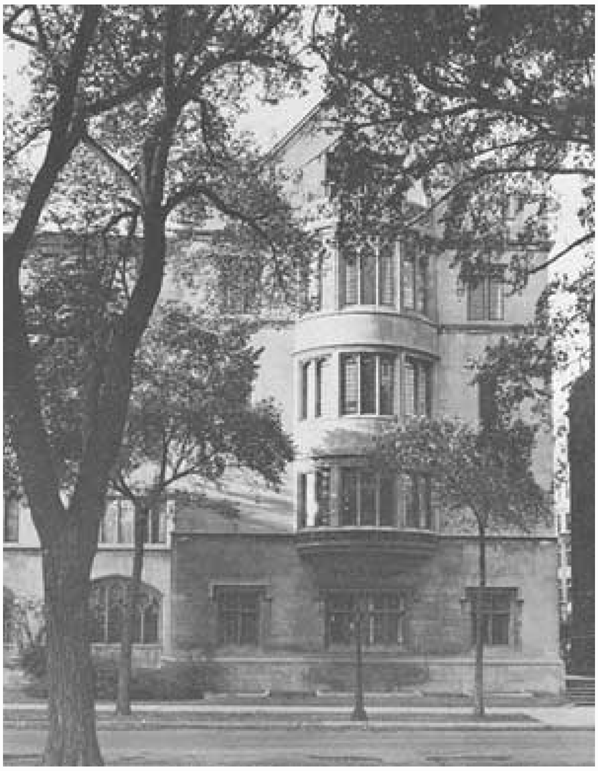


Figure 1: The Cowles Commission, University of Chicago, Social Science Building, 139-1955, source: http://cowles.econ.yale.edu/archive/gallery/cf_chicago.html.

During the years 1937-1939 in Colorado (the move to Chicago happened in 1939) the Cowles Commision held a number of summer conferences. The proceedings after 1936 were published by the Cowles Commission itself. Among the participants in 1937–1939 who were not previously connected with the Cowles Commission was AW. [Cowles Foundation (2007)]. G. Debreu remembers: "*The 1930s papers by von Neumann and Wald were well known at Cowles but that small group at Cowles was not representative of the profession. It was exceptional in many ways. His group lived in a universe of its own which made it possible for them to live very comfortably with ideas that were not orthodox. There were much fewer colloquia in those days than there are now, so there were fewer opportunities to be confronted with other viewpoints and when the colloquium was on linear programming there was no reason to disagree. Incidentally, von Neumann played an important role in the development of linear programming. You will find in his collected papers the paper he wrote on the duality theorem which was never published*.". [Weintraub (2002)]

5

Milestones

They [AW, J. Wolfowitz] *have shown the way to better economic inventions*

—J. Marschak, at the Boston Meeting of the Econometric Society, December 1951 [Marshak 1952].

The book makes effective use of the modern theory of measure and integration, and operates at a high level of rigor and abstraction. For this reason few statisticians will be prepared to read it, yet its ultimate liberating effect on statistical theory will be great.

—H. Robbins, Review of the ‘Statistical decision functions‘ 1951, [Robbins 1951]

5.1 The Sequential Analysis

5.1.1 Developments before and after the 'Sequential Analysis'

In July 1942 (for the years between 1939 and 1942 see Ch. 6) the SRG ('Statistical Reseach Group') was founded at Columbia University. The SRG was headed by W. A. Wallis. The SRG directly collaborated with the US Defense Department. Amongst other members of the SRG were AW, H.A. Freeman, M. Friedman, M.A. Girshik, H. Hotelling and J. Wolfowitz. In early 1943 M. Friedman and W. A. Wallis formulated the problem of carrying out a test of a hypothesis in sequences and proposed their sketch to AW. [Wallis (1980)] AW's followed the methodology to take the simplest case at first. This often shows to be an effective approach as the simplest case occasionally shows surpring results. He started with the most basic test alias the hypothesis H_0 (the distribution of independent observations is $f_1(x)$) that had to be evaluated against hypothesis H_2 (the distribution is $f_2(x)$ with a prescribed maximum probability of error). Taking that line AW showed on one hand that a test procedure based on constant limits $S_n := \sum_{i=1}^{n} \log f_1(x_i) - log\, f_2(x_i)$ minimizes the expected number of observations (the so called 'optimum property of the Sequential Probability Ratio Test', abbreviated as 'SPRT', what we will define below), and on the other hand he obtained pretty good approximations for these limits (in terms of the given error probability). AW presented an elaborated proof one year later in the 1948 paper 'Optimum character of the sequential probability ratio test‘ [Wald, Wolfowitz (1948)] after "*a number of attempts, and by the methods of Wald's decision theory and the manipulation of Bayes solutions. This was the paper Wald himself liked best*". [Wolfowitz (1952), p3] The simplicity of AW doing approximations to limits basing on cumulative sums describes Wolfowitz as "*typical of Wald.. . His intuition about approximations was uncanny and the rudest of methods in his hands struck gold*". [Wolfowitz (1952)] It was indeed important that it was possible to formulate good approximations to ensure the further progress on this topic. The next step was to go over to composite hypotheses.

AW had caught fire and started an intensive work on sequential analysis. Within a few months (until summer 1943) he had discovered most of the results that later appeared in the 'Sequential Analysis'. [Wald (1947) c,] They were first published in September of the same year in the volume 'Sequential Analysis of Statistical Data', where AW devises the SPRT and drafts a basic theory. This was one of the reports for the Applied Mathematics Panel of National Defense Research Committee. [242] Hence it was categorized 'restricted' [243] (following the so-called 'Espionage Act‘), it was only available for some authorized people (to the regret of AW as "*Wald chafed greatly under this restriction*" [Wolf 1950]). In 1944 a couple of advances regarding the SPRT were made independently from AW. The operating characteristic (**OC**) curve of the **SPRT** in the case of a binomial distribution was formulated by M. Friedman and unknown to him by *G. W. Brown*. Only a short time before *C. M. Stockman* had presented an OC curve in a similar way. [Stockman (1944)] However again it was AW who gave a general OC for the SPRT. [Wald (1944) a,] A few months later AW developed the general theory of cumulative sums, which not only provides the OC curve of any SPRT, but also the characteristic function of the number of observations required by the test and various more. These results were published by AW in September, 1944, in 'On Cumulative

Sums of Random Variables‘ [Wald (1944) a,] and in June, 1945 in the ‘Annals of Mathematical Statistics’. [Wald (1945) c,] For the Statistical Research Group he issued a revised edition. [Wald (1945) c,] [244]

To facilitate the use of the sequential analysis for the Army and the Navy, the *Statistical Research Group* edited a second report in July 1944. That report was elementary and non-mathematical. It described the use of the SPRT and illustrated it with tables and charts. [Freeman (1944)] The ‘restriction‘ on the first report [Wald (1943) a,] lasted until the end of war, i.e. first in May 1945 the classification was removed. After that AW published his achievements. The first paper in that regard had already appeared in 1944. It bears the inconspicious title ‘On cumulative sums of random variables‘. Therein he expresses the close connection between the random walk problem and the sequential analysis. In 1945 a whole bunch of papers by AW appeared (cp. Appendix B, the paper ‘Sequential tests of statistical hypotheses‘ [Wald (1945) b,] does indeed not differ much from the 1947 book ’Sequential Analysis’ [Wald (1947) c,], the paper ‘Sequential method of sampling for deciding between two courses of action‘ [Wald (1945) c,] from the same year gives an elementary exposition of the subject), all concerned with the sequential analysis and especially with the optimality of the SPRT. One of the most interesting results is the following: if one neglects the excess of the cumulative sums over the prescribed limits then the SPRT minimizes the expected number of observations under each distribution. But AW’s interest in the sequential analysis did by no means end with his book ‘Sequential Analysis‘, a book ”*written ... clear, lucid, most of the researches on the subject his own. It was put together hurriedly without too much thought of elegance or of reference to related fields*”. [Wolfowitz (1952)] AW continued his research with the paper ‘Some improvements in setting limit, for the expected number of observations required by a sequential probability ratio test‘ and ‘Differentiation under the expectation sign in the fundamental identity of sequential analysis‘, both published in 1946. In 1947 the paper ‘Limit distribution of the maximum and minimum of successive cumulative sums of random variables‘ and in 1948 the ‘Asymptotic properties of the maximum likelihood estimate of an unknown parameter of a discrete stochastic process‘ followed. Both papers (basing on [Erdös (1946)]) cover questions on the theory of random walk. Through the sequential analysis AW arrived at that topic. From the very beginning of his research on the sequential analysis AW always asked for a ‘cost’ saving [245] effect of his method in the process of estimation. “*Several memoranda written for the Statistical Research Group contained a proof of the fact that no saving could be effected, under restrictions which AW found obnoxious*”. [Wolfowitz (1952)] AW in 1946 formulated an estimation process without these restrictions and published in 1947 (together with his student C.M. Stein) a revised proof in the paper ‘Sequential confidence intervals for the mean of a normal distribution with known variance‘.

The remaining question is wether AW was the sole inventor of the sequential analysis or not. Bristish scientists claim to have it formulated in principle as first. But the SPRT is inarguable the merit of AW: “*The notion of taking observations sequentially was not Wald's. It is probably an old idea, although a number of people lay claim to it, no doubt having discovered it independently. The brilliant and difficult part is the invention of the sequential probability ratio test. This is solely the work of Wald, and no trace of this idea exists in the literature prior to Wald's work*”. [Wolfowitz (1952)]

The 1949 ‘A sequential decision procedure for choosing one of three hypotheses concerning the unknown mean of a normal distribution‘ [Wald, Sobel (1949)] is also based onto a sequential process. AW targeted (together with his student Milton Sobel) the problem of “*deciding sequentially in which* *of three intervals the mean of a normal distribution with known variance lies"*. [Wolfowitz (1952)] Their solution is an adaptation of AW's solution on the problem of two hypotheses.

5.1.2 A survey of the 'Sequential Analysis'

The first what catches the reader's eye on sequential analysis is that it does not need a fixed number of observations in comparision to the classical procedures in testing statistical hypotheses. As a matter of fact the final number of observations is not defined in advance of a procedure but is determined stepwise, i.e. at each step of the experiment a decision, based on all previous results, is made wether the experiment is terminated or will be carried forward. Knowing that character of decision, one is able to construct the procedure in a way that "*a substantially smaller number of observations compared to equally reliable test procedures based on a predetermined number of observations*". [Wald (1947) c,] [246]

The book 'Sequential Analysis' describes the theory of the sequential analysis method. It presents especially the sequential probability ratio test (SPRT). A basic forerunner of a sequential test procedure (i.e. a test for which the number of observations is not determined in advance) goes back to *H. F. Dodge* and *H. G. Romig*. [Dodge and Romig (1929)] A more general application presents *W. Bartky* in '*Multiple Sampling with Constant Probability*'. [Bartky (1943)] [247] Dodge, Roming and Bartkey recognized immediately the fact, that they required a smaller number of observations. The occasional practice of designing a large scale experiment in successive stages may be regarded as a forerunner of sequential analysis. A further "pre-sequential" idea of designing a large scale experiment in succesive stages has been described 1941 by *H. Hotelling* in '*Experimental Determination of the Maximum of a Function*'. [Hotelling (1941)] A notable example of this type was formulated by *P. C. Mahalanobis* in 1940. [Mahalanobis (1940)] [248] We conclude this paragraph with an annotation of J. Wolfowitz: "*Wald's sequential probability ratio test was a great statistical achievement*". [Wolfowitz (1952)]

5.1.2.1 Quick Repetitions

The outcome of an experiment is usually a variable, since generally it can take different values. Concerning this outcome, it is often possible to make probability statements. A variable x is called a **random variable** if for any given value v a definite probability can be ascribed to the event that x will take a value less than v. An experiment where the outcome is a random variable can be realized through a (finite) set of objects (population, universe) with a varying measurable characteristic. We suppose that each object has an equal chance to be chosen and should be selected at random. Assuming that our objects are produced goods, a proper (categorial) two-valued characteristic is incidental if we classifiy the products in one of two categories: **defective** (value **1**) or **non-defective** (value **0**).

Let x be a random variable and as usual F(a) the probability that x will take a value less than a given value a (generally known as cumulative distribution function (c.f.d) of x with following known properties : **[1]** $0 \le F(a) \leqslant 1$, **[2]** $a < b \Rightarrow F(a) \le F(b)$, and **[3]** $\mathbb{P}(\{x \mid a \le x < b\}) = F(b) - F(a), \ \forall \ a, b \ \in \mathbb{R}$). The c.f.d of a finite population is always a step function, but can be regarded as continuous if we treat the population as if it were infinite (meaning an ordered infinite sequence of objects). So we regard any c.f.d as a limiting form of a c.f.d arising from a finite population and are so able to interpret every! probability as limiting value of the proportion of objects satisfying given certain measurable charactistics.

The derivative of a c.d.f. F(x) (if it exists and fulfills a normalization condition) is called the **probability density function** f(x) (p.d.f) of the variable x. It follows from the definition of the p.d.f that for small positive values Δ the product f(a)Δ is a good approximation for $\mathbb{P}\left(\left\{x \mid x \in \left[a - \frac{\Delta}{2}, a + \frac{\Delta}{2}\right]\right\}\right)$. A discrete random variable x (i.e x can take only a countable number of values) has no p.d.f, because the c.f.d is a step function. In that case f(a) denotes the value $\mathbb{P}(\{x \mid x = a\})$. As usual the expected value (center-mean) of x is defined as $\mathbb{E}(x) = \sum_t t * f(t)$, respectively $\mathbb{E}(x) = \int_t t\, f(t)\, dt$. Accordingly the r-th central-moment (referred to the mean) is defined as $\mathbb{E}((x - \mathbb{E}(x))^r)$. (The second moment of this kind is called variance, and its square root is named standard deviation).

5.1.2.2 Notions of a statistical hypothesis

If x is a random variable with unknown distribution, the statistical challenge is to draw some inference concerning the unknown distribution of x on the basis of a limited number of observations on x. Frequently, the distribution of x is not entirely unknown, i.e., some partial knowledge of the distribution of x is available a priori : f.e. the distribution function is known except for the values of a finite number of parameters $\theta_1, .., \theta_k$. A **statistical hypothesis** is called **simple**, if it determines **uniquely values for the parameters** $\theta_i, i = 1, .., k$, otherwise it is called **composite** (f.e. if we only have $\theta_1 = \theta_2$). The decision to accept or reject a hypothesis is always made on the basis of a finite number ($n \in \mathbb{N}$) of sucessive and independent observations on x (this set is called a sample of x of size n). Independence means here that the conditional probability distribution of the i-th sample is not affected by its preceeding values (what is never strictly but approximately fulfilled in the case of a finite population; cp. [Wald (1943) a, p14]) So the joint probability repectively the joint probability distribution to obtian a special sample x_1, ..., $x_n$is for discrete distributions simply the product $\prod_{i=1}^{n} f(x_i)$.

5.1.2.3 The general nature of a test procedure

The test procedure leading to acceptance or rejection of a hypothesis can be interpreted as a subdividing operation of the totality of all possible samples of size n into two mutually exclusive parts (say part l and part 2) together with the application of the rule that the **hypothesis be rejected if the observed sample is contained in** part l and that the hypothesis be accepted if the observed sample is contained in part 2. Part l is also called the **critical region W**. Part 2 is uniquely determined by part 1. Thus, choosing a test procedure is equivalent to determining a critical region. In general one has infinitely many possibilities for choosing a critical region, that are naturally not! equally good. So a fundamental problem in testing hypotheses is to set up principles for a proper choice of the critical region (cp. [Neyman, Pearson (1936) b, pp1-37]).

5.1.2.4 Principles for choosing a critical region

The basic idea of the Neyman-Pearson theory in the simple case of a single parameter θ which can take only two values, θ_0 and θ_1, can be drafted in the following way : is $f(x, \theta)$ the distribution of x and denote $f_i(x) := f(x, \theta_i)$, $i = 0, 1$, then the null hypothesis is formulated as $H_0 : \theta = \theta_0$.The alternative hypothesis is $H_1 : \theta = \theta_1$.The test procedure is to decide between H_0 and H_1on the basis of n independent samples x_1, .., x_n of x . As an **error of the first kind** we consider the decision of **rejecting *H_0* although it is true**, and analogous an **error of second kind** is the **acceptance of *H_0* although it is false**. After a particular critical region W has been chosen, the probability of committing an error of the first kind, as well as the probability of committing an error of the second kind, is uniquely determined. The **probability *α* to commit an error of the first kind** is equal to the probability that the sample x_1, .., $x_n$is contained in the critical region (meaning $H_0$is rejected) under the assumption that H_0 is true. The **probability *β* of committing an error of the second kind** is equal to the probability, determined on the assumption that H_1 is true, that the observed sample will fall outside the critical region (meaning H_0 is accepted). In the long run of generating many samples the proportion of the samples that wrongly result in a rejection of H_0 approaches α . The same is true for β : if $H_1$is true, the proportion of all samples that cause wrongly a rejection of H_1 approximates β. Hence ***α* is called the size of W**, and **1-*β* is called the power of W**. Clearly a critical region is desirable that has minimal values of α and β. Although either α or β can be made arbitrarily small by a proper choice of W, it is **impossible to make both α and *β* arbitiarily small for a fixed value of n** (i.e. a fixed sample size). **The principle of Neyman-Pearson is as follows : if we restrict ourselves to critical regions W of a fixed size α, the task is to determine W in a way that *β* is minimized, i.e we seek for the most powerful W**.

If the sample size n is fixed, the functional dependency between α and β is simply α=$\alpha(\beta)$, respectively β=$\beta(\alpha)$, i.e. one of these two parameters can be choosen arbitrarily. The Neyman-Pearson theory leaves the question of this choice open. Indeed this choice will be influenced by the relative importance of the errors of the first and second kinds in each particular application. Neyman-Pearson show that given α, a critical region W, consisting of all samples $(x_1, .., x_n)$ which satisfy

$$\frac{f_1(x_1) \cdot ... \cdot f_1(x_n)}{f_0(x_1) \cdot ... \cdot f_0(x_n)} \geq k(\alpha),\ k = k(\alpha) \in \mathbb{N} \text{ is choosen so, that } W \text{ will have size } \alpha.$$

is a most powerful critical region for testing the hypothesis H_0 against the alternative hypothesis H_1. [Wald (1947) c, p18]

5.1.3 Sequential test of a statistical hypothesis

5.1.3.1 Determining the number of observations if α and β have preassigned values

Lets now alternatively assume that both parameters α and β are given and the task is to determine the minimum value of n for which the power of the most powerful region of size α is greater than or equal to $1 - \beta$. Denotes β_n the probability of an error of the second kind associated with a most powerful W of size α, when the test is based on n observations, then β_n decreases, or at least does not increase, with increasing n. [Wald (1947) c, p20] (Generally, β_n will approach 0 as n increases indefinitely). Is $n(\alpha, \beta)$ the smallest value of n for which $\beta_n \leq \beta$, than a test procedure t with $n_t \geq n(\alpha, \beta)$ features $\alpha_t = \alpha$ and $\beta_t \leq \beta$. Using $n_t = n(\alpha, \beta)$ we get a most powerful W (because n_t is minimal in the above sense).

5.1.3.2 Notion of a sequential test

Usually the number of observations, i.e. the sample size n is treated as a constant for a test procedure. An essential feature of **sequential test** is, that **the number of observations depends on their successive outcome** and is, therefore, a random variable. Is H_0 the hypothesis to be tested, the **sequential method** is carried out stepwise. We have a rule for making one of the following three decisions at each singular stage of the experiment, i.e. at the m-th trial, $m = 1, 2, 3, \ ...$, we settle for 1, accepting H_0, 2, rejecting H_0, or 3, continuing with the experiment by making one additional observation. If decision 1, or 2, is made, the process is terminated. If 3, is made, a further trial is performed. This process is continued until either decision 1, or 2, is made. Clearly the number n of observations required by such a test procedure is a random variable. (We shall consider only sequential tests for which the probability is one that the process will eventually terminate.)

Is M_m the m-dimensional space containing the totality of all possible samples $(x_1, \ .., x_m)$ of size m, then for each integer value m M_m is split into three mutually exclusive subsets, R_m^0, R_m^1, and R_m. Now the testing procedure of H_0 can be formally described as follows :

$$(x_1, \ ..., x_i) \in \begin{cases} R_i^0 & \Rightarrow H_0 \text{ accepted}, \ n = i \\ R_i^1 & \Rightarrow H_0 \text{ rejected}, \ n = i \\ R_i & \Rightarrow \text{next trial } (i \to i+1) \end{cases} \qquad i = 1, 2, 3, \ ...$$

As the mentioned subsets are mutually exclusive and add up to the whole sample space, the sequential test is completely defined by the R_m^0, R_m^1, and R_m, $m = 1, 2, 3,$ ....A sample $(x_1, .., x_m)$ is called **ineffective**, if it contains an initial segment $(x_1, .., x_{m'})$, where $m' < m$, such that $(x_1, .., x_{m'}) \in R_{m'}^0 \bigcup R_{m'}^1$. Otherwise a sample is called **effective**. During a sequential test procedure we clearly handle only effective samples, i.e the sets R_m^0, R_m^1, and R_m cannot contain ineffective ones. These sets defining a sequential test can be chosen in many ways, and a fundamental problem is that of a proper choice of these sets.

5.1.3.3 Defining the operating characteristic (OC) function

After a particular choice of the R_m^0, R_m^1, and R_m , $m = 1, 2,$ ...has been made, the probability that the process will terminate with the acceptance of H_0 under test depends only on the distribution of the considered random variable x. Is the distribution of x known except for the values of a finite number of parameters $\underline{\theta} = (\theta_1, .., \theta_k)'$, it can be noted as a function $f(x, \theta_1, .., \theta_k)$, where the functional form f is known, but the true values of the parameters θ_i , $i = 1, .., k$ are unknown. Accordingly the **probability of accepting** H_0 will be a function of $\underline{\theta}$. This function is called **operating characteristic** (**OC**) function $L(\underline{\theta})$. So, if $\underline{\theta}$ is consistent with H_0 to be tested, $L(\underline{\theta})$ is the probability of making a correct decision. Otherwise if the true parameter point $\underline{\theta}$ is not consistent with H_0, the probability of making a correct decision is equal to 1-$L(\underline{\theta})$. Clearly an OC function is considered more favorable the higher the value of $L(\underline{\theta})$ for $\underline{\theta}$ consistent with H_0 and the lower the value of L(θ) is for θ that are not consistent with H_0. Since we assume that all our tests will terminate with the probability of 1, the probability of rejecting H_0 is equal to $1 - L(\underline{\theta})$ and this again is equal to $1 - \beta$ (β from above).

5.1.3.4 The ASN function of a sequential test

As pointed out, the number n of observations required by a sequential test is not predetermined, but is a random variable. Of particular interest is the expected value $\mathbb{E}(n)$ (the average value of n in the long run), that depends only on the distribution of x. Since the distribution of x is determined by $\underline{\theta}$, we write $\mathbb{E}_{\underline{\theta}}(n)$ and call this the **ASN** (average sample number) function. The OC function describes how well the test procedure achieves its objective of making correct decisions, and the ASN function represents the price we have to pay. Thus, in judging the relative merits of two different test procedures, we shall compare their OC and ASN function. [Wald (1947) c, p26,27]

5.1.3.5 Degree of preference for acceptance or rejection

Denote by ω **the set of all those $\underline{\theta}$ which are consistent with H_0** (i.e. H_0 is precisely the statement that the true parameter point is included in the set ω). Since a correct decision is preferred to a wrong decision, we can say that acceptance of H_0 is preferred whenever $\underline{\theta}$ is in ω, and rejection of $H_0$is preferred whenever $\underline{\theta}$ is outside ω. It would be desireable to have some knowledge about the degree of preference for acceptance or rejection as a function of $\underline{\theta}$. Depending how near $\underline{\theta}$ lies to the (topological) boundary of ω, we get a measure for the accuracy of the decision we make : f.e. if the true parameter point $\underline{\theta}$ lies in ω but is near the boundary, the preference for acceptance of H_0 will only be slight, or if $\underline{\theta}$ is directly a boundary point, it is indifferent wether we accept or reject H_0, and so on. In this manner we are able to subdivide the parameter space into three mutually exclusive zones: a zone (subset of ω) consisting of all $\underline{\theta} \in \omega$ for which acceptance of H_0 is strongly preferred (zone of acceptance), a second zone (subset of the complement $\overline{\omega}$ of ω) consisting of points $\underline{\theta}$ for which rejection of H_0 is strongly preferred (zone of rejection), and thirdly a zone (consting of points near the boundary of ω) consisting of all points $\underline{\theta}$ for which neither acceptance nor rejection of $H_0$is strongly preferred (zone of indifference). This subdivision of the parameter space is not a statistical problem as made on the basis practical considerations.

A refined description of the degree of preference for one or the other decision can be given by two functions $w_0(\underline{\theta})$ and $w_1(\underline{\theta})$, where $\boldsymbol{w_0(\underline{\theta})}$ expresses the relative importance of the **loss caused by the error of accepting $\boldsymbol{H_0}$ when $\boldsymbol{\underline{\theta}}$ is true**, and **$\boldsymbol{w_1(\underline{\theta})}$ expresses the relative importance of the error of rejecting $\boldsymbol{H_0}$when $\boldsymbol{\underline{\theta}}$ is true**.

$$\underline{\theta} \in \begin{cases} \omega \\ \overline{\omega} \end{cases} \Rightarrow$$

resp. $w_1(\underline{\theta}) > 0$ (increasing with increasing dist($\underline{\theta}$, $\partial\omega$), $\partial\omega$ is the topological bo
$w_0(\underline{\theta}) > 0$ (increasing with increasing dist($\underline{\theta}$, $\partial\omega$)) , resp. $w_1(\underline{\theta}) = 0$

Contrarily the subdivision of the parameter space into three zones can also be done by the (continuous) functions $w_0(\underline{\theta})$ resp. $w_1(\underline{\theta})$, if we define appropriate constants $c_0 > 0$ resp. $c_1 > 0$for the zone of indifference

$$\left.\begin{array}{l} w_0(\underline{\theta}) = 0\,,\ \text{resp. } w_1(\underline{\theta}) > c_1 > 0 \\ w_0(\underline{\theta}) > c_0 > 0,\ \text{resp. } w_1(\underline{\theta}) = 0 \end{array}\right\} \Rightarrow \begin{cases} \underline{\theta} \in \omega \\ \underline{\theta} \in \overline{\omega} \end{cases}$$

For most practical purposes the use of step functions $w_i(\underline{\theta})$, $i = 1, 2$, implied by the subdivision of the parameter space into three zones will give a sufficiently good approximation for most practical purposes. [Wald (1947) c,]

5.1.3.6 Requirements on the OC and the ASN function

The acceptance of H_0depends on $\underline{\theta}$ lying in ω. Hence we wish to make the probability of accepting H_0 as high as possible when $\underline{\theta}$ lies in ω, and as low as possible when when $\underline{\theta}$ is outside of ω. This probability is by definition equal to the OC function $L(\underline{\theta})$. The ideal $L(\underline{\theta})$ would be equal 1 for any $\underline{\theta}$ in ω, and equal 0 for any $\underline{\theta}$ outside ω. Achieving an ideal form of the OC function is done at the expense of the needed sample size, why the sequential test is desireable more and more. Since the closer we approach the ideal form of the OC function, the larger, in general, will be the number of observations required by the test. To achieve a compromise between these two conflicting desiderata, we may proceed as follows. First we formulate requirements concerning the closeness of the OC function to the ideal function and then consider only tests which satisfy these requirements. From these tests we try to select one for which the expected number of observations required by the test is as small as possible. Concerning the subdivision of the parameter space into the three mutually exclusive zones, we dont impose any conditions on the behavior of $L(\underline{\theta})$ within the zone of indifference, but in the **zone of acceptance** of H_0 this probability should be less than or equal to a preassigned value α (i.e. $\mathbf{1 - L(\underline{\theta}) \leq \alpha < 1}$, (*)), respectively for the zone of rejecting of H_0 the accordant probability should be less than or equal to a preassigned value β (i.e. $\mathbf{L(\underline{\theta}) \leq \beta < 1}$, (**)). Clearly the values α, β < 1 are preselected (after the parameter space has been divided) on the basis of practical considerations in each particular case. **A sequential test S is called admissible if it satisfies the requirements (*) and (**)**. As we wish to have tests with minimal $\mathbb{E}_{\underline{\theta}}(n)$ (see above), our aim is to have an admissible test S obeing following condition : $\mathbb{E}_{\underline{\theta}}(n\,|\,S) \approx \min_{S' \text{ admissible sequential test}} \mathbb{E}_{\underline{\theta}}(n\,|\,S')$ .The ideal case of " = "does not exist, i.e. it is not possible to minimize $\mathbb{E}_{\underline{\theta}}(n\,|\,S)$ simultaneously for all $\underline{\theta}$ ("*...the* *various possibilities have not yet been fully investigated.*", AW in [Wald (1945) b, p34]).

5.1.3.7 Defining example for the "efficiency" of a sequential test

Lets say $\underline{\theta}$ takes only two values $\underline{\theta}_0$ and $\underline{\theta}_1$, and let H_i be the hypotheses that $\underline{\theta} = \underline{\theta}_i$, $i = 1, 2$ (null hypothesis and alternative hypothesis). With any sequential test of testing H_0 against H_1 we commit an error of the first kind (assumed $H_0$is true) with a probability α, and an error of the second kind (assumed $H_1$is true) with the probability of β. Two tests S and S' will be said to be of **equal strength** if the values (a, β) associated with S are equal to the corresponding values (a', β') associated with S'. If $\alpha < \alpha'$ and $\beta \leq \beta'$ or if $\alpha \leq \alpha'$ and $\beta < \beta'$ S is called **stronger** than S' (or S' **weaker** as S). If $\alpha < \alpha'$ and $\beta > \beta'$ or if $\alpha > \alpha'$ and $\beta < \beta'$ we say that the strength of S ist **not comparable** with that of S'. Are S and S' of the same strength, that one with the smaller expected number of observations will be preferred. If S and S' are of equal strength such that $\mathbb{E}_{\underline{\theta}_0}(n \mid S) \leq \mathbb{E}_{\underline{\theta}_0}(n \mid S')$ and $\mathbb{E}_{\underline{\theta}_1}(n \mid S) < \mathbb{E}_{\underline{\theta}_1}(n \mid S')$, or $\mathbb{E}_{\underline{\theta}_0}(n \mid S) < \mathbb{E}_{\underline{\theta}_0}(n \mid S')$ and $\mathbb{E}_{\underline{\theta}_1}(n \mid S) \leq \mathbb{E}_{\underline{\theta}_1}(n \mid S')$, S is considered preferable to S'. We call S an **optimum test**, if $\mathbb{E}_{\underline{\theta}_0}(n \mid S) \leq \mathbb{E}_{\underline{\theta}_0}(n \mid S')$ and $\mathbb{E}_{\underline{\theta}_1}(n \mid S) < \mathbb{E}_{\underline{\theta}_1}(n \mid S')$, $\forall$ S' of strength equal to that of S.

Is S a sequential test of strength (a, β), we define $n_i(\alpha, \beta \mid S) := \inf_{\underline{\theta}_i} \mathbb{E}_{\underline{\theta}_i}(n \mid S)$, $i = 1, 2$. S is an optimum test, if $n_i(\alpha, \beta \mid S) = \inf_{\underline{\theta}_i} \mathbb{E}_{\underline{\theta}_i}(n \mid S)$, $i = 1, 2$ (whereas the existence of a optimum test has not been proved when AW wrote the book [Wald (1947) c,]; ann. of the author). AW conjectured in 1947 "*...that the sequential probability ratio test is* *exactly an optimum test, but ... [i] ... did not succeed in proving this.”*. [Wald (1947) c, p35] The **efficiency** of a test S ist defined by the ratio $\frac{n_0(\alpha,\beta|S)}{\mathbb{E}_{\underline{\theta}_0}(n|S)}$ when $H_0$is supposed to be true, otherwise it is the ratio $\frac{n_1(\alpha,\beta|S)}{\mathbb{E}_{\underline{\theta}_1}(n|S)}$. Clearly the efficiency lies always between 0 and 1. **The greater the efficiency of a sequential test of a given strength the more desirable it is.** An optimum test has the efficiency 1 under both hypotheses (as the SPRT almost nearly has. [Wald (1947) c,])

5.1.4 The sequential probability ratio test (SPRT) for testing a simple hypothesis against a single alternative

$f(x, \underline{\theta})$ should denote the distribution of a random variable x (meaning in the continuous case the probability density function, if it exists, and in the discrete case the probability, that the random variable under consideration takes the value x), and are $H_0 : \underline{\theta} = \underline{\theta}_0$, resp. $H_1 : \underline{\theta} = \underline{\theta}_1$ the null- resp. alternative hypothesis, then the distribution of x is given by $f\left(x, \underline{\theta}_0\right)$ when H_0 is true, and by $f\left(x, \underline{\theta}_1\right)$ when H_1 is true. **It is our intention to cover both cases simultaneously**, excepting some statements that have to be formulated slightly different. Are successive observations on x denoted by $x_1, x_2,$ .., we get for any positive integer m the probability that a sample x_1, .., x_m is obtained as product $p_{0m} = f(x_1, \theta_0)$, ..., $f(x_m, \theta_0)$if $H_0$is true, and as $p_{1m} = f(x_1, \theta_1)$, ..., $f(x_m, \theta_1)$ if $H_1$is true. Here we assume that the samples appear independently from each other. At each m-th stage of the experiment the **probability ratio** p_{1m} / p_{0m} is computed (for purposes of practical computation, it is much more convenient to compute the logarithm of the probability ratio). If for two positive constants $A, B\,(B < A)$, $B < p_{1\,j} / p_{0\,j} < A$, $j = 1, .., m-1$, we will terminate the test if $p_{1\,m} / p_{0\,m} \geq A$ (this is considered to be fulfilled also in the case $p_{1\,m} > 0$, $p_{0\,m} = 0$) with the rejection of H_0(acceptance of H_1), and if $p_{1\,m} / p_{0\,m} \leq B$, we stop the trial with the rejection of H_1(acceptance of H_0). The constants A and B are to be determined, so that the test will have the prescribed strength (α, β). In this context we define the probaility ratio as equal to 1, if $p_{1\,m} = 0$ and $p_{0\,m} = 0$.

5.1.4.1 Fundamental relations between the parameters α, β, A and B

We derive now inequalities satisfied by the quantities (α, β, A, B) to provide a basis for determining the constants A and B in the SPRT. A **sample** (x_1, .., x_m) is called to be of **type 0** resp. **type 1**, if $B < p_{1\,j} / p_{0\,j} < A$ for $j = 1, ..., m-1$ and $p_{1\,m} / p_{0\,m} \leq B$ resp. $p_{1\,m} / p_{0\,m} \geq A$. Thus, a sample of type 0 leads to the acceptance of H_0 and a sample of type l leads to the acceptance of H_1 (rejection of H_0). The probability of obtaining a sample of type 1 is at least A times as large under H_1 as under H_0. So the probabilty measure of the totality of all samples of type 1 is A times larger under H_1 as under H_0, and is per definition equal to α when H_0 is true and equal to $1-\beta$, when H_1 is true (assumed that the sequential process will always terminate). This leads to the following upper limit for A (and in an analogue argumentation to a lower limit for B) :

$$\frac{1-\beta}{\alpha} \geq A \text{ , and } \frac{\beta}{1-\alpha} \leq B$$

From these inequalities we derive $\alpha \leq \frac{1}{A}$, and $\beta \leq B$.

In the case the successive samples $x_1, x_2,$ ..are not longer independent, we have only to replace the products $p_{\mathrm{jm}} = f(x_1, \theta_j), \ldots, f(x_m, \theta_j)$, $j = 1, 2$ by more general functions $p_{\mathrm{jm}} = f(x_1 \ldots, x_m)$, $j = 1, 2$, and see immediately that above inequalities remain valid.

5.1.4.2 Determination of the parameters A and B in practice

Suppose that we wish to have a test procedure of strength (α, β). Our task is to determine the constants $A = A(\alpha, \beta)$ and $B = B(\alpha, \beta)$, such that the resulting test will have the desired strength (α, β). "*The exact determination of the values $A(\alpha, \beta)$ and $B(\alpha, \beta)$ is usually very laborious*.". [Wald (1947) c, p44] When we define $a(\alpha, \beta) := \frac{1-\beta}{\alpha}$ and $b(\alpha, \beta) := \frac{\beta}{1-\alpha}$ we have following unequalities : $A(\alpha, \beta) \le a(\alpha, \beta)$ and $B(\alpha, \beta) \ge b(\alpha, \beta)$. If we set simplifying $A = a$ and $B = b$, we have to investigate how the errors of the first and the second kind are altered (we denote the resulting probabilities in this case by α' and β'), and derive immediately

$$\frac{\alpha}{1-\beta} = \frac{1}{a} \equiv \frac{1}{A} \ge \frac{\alpha'}{1-\beta'}, \text{ and } \frac{\beta}{1-\alpha} = b \equiv B \ge \frac{\beta'}{1-\alpha'},$$

$$i.e.\ \alpha' \le \frac{\alpha}{1-\beta} \text{ and } \beta' \le \frac{\beta}{1-\alpha}$$

and with appropriate factors we get $\alpha' + \beta' \le \alpha + \beta$. This shows that at least one of the probabilities α or β may be increased. Since the "... *values α and β* [are] *usually ... small in practical applications*", and "*most frequently they ... lie in the range from 0.01 to 0.05*." [Wald (1947) c, p46], it is clear that the amount by which α' may exceed α, resp. β' may exceed β is "*very small and can be neglected for all pratical purposes*.". [Wald (1947) c, p46] AW recapitulates : "*The use of $a(\alpha, \beta)$ and $b(\alpha, \beta)$ instead of $A(\alpha, \beta)$ and $B(\alpha, \beta)$,* *respectively, cannot result in any appreciable increase in the value of either* α *or* β*. In other words, for all practical purposes the least corresponding to* $A = a(\alpha, \beta)$ *and* $B = b(\alpha, \beta)$ *provides at least the same protection against wrong decisions as the test corresponding to* $A = A(\alpha, \beta)$ *and* $B = B(\alpha, \beta)$*.*“. [Wald (1947) c, p46] (The possibility of a decrease of α or β will not be discussed as this would improve the tests protection against wrong decisions). The only real disadvantage may result in an appreciable increase in the number of observations n'required by the modified test. Since $[B, A] \subset [b, a]$ the number n' can never be smaller than n. Since $A \le a$ and $B \ge b$ the test will mostly terminate with $p_{1m} / p_{0m} > A$ resp. $p_{1m} / p_{0m} < B$ (in the case of "=" it follows $A = a$ resp. $B = b$ and subsequently $n' = n$). " ... *a possible excess of* p_{1m} / p_{0m} *over the boundaries A and B at the termination of the test procedure is caused only by the discontinuity of the number of observations, i.e., by the fact that the number of observations can take only integral values. Thus, if fractional observations were possible, i.e., if the number m of observations were a continuous variable,* p_{1m} / p_{0m} *would also be a continuous function of m and consequently* $A(\alpha, \beta)$ *and* $B(\alpha, \beta)$ *would be exactly equal to* $a(\alpha, \beta)$ *and* $b(\alpha, \beta)$*, respectively*.". [Wald (1947) c, p47] AW concludes that "... *the increase in the necessary number of trials caused by the use of* $a(\alpha, \beta)$ *and* $b(\alpha, \beta)$ *will generally be slight is strongly indicated by the fact that the discrepancy between* $A(\alpha, \beta)$ *and* $a(\alpha, \beta)$*, as well as that between* $B(\alpha, \beta)$ *and* $b(\alpha, \beta)$*, arises only from the discontinuity of the number of observations*.". [Wald (1947) c, p47] An interesting fact is that contrary to the usual test the sequential test can be carried out without finding the probability distribution of the statistic on which the test is based (since the ratio p_{1j} / p_{0j} "*can be calculated from the data of the problem without solving any distribution problems*." [Wald (1947) c, p48]).

5.1.4.3 The OC function of the SPRT

Regarding the SPRT for tests with a (single or multidimensional) parameter θ, that can take values $\neq \theta_0, \theta_1$, it is of interest to derive the whole OC function $L(\theta)$ of the test. [249] (L(θ) has been defined as the probability that the sequential process will terminate with the acceptance of H_0 when θ is the true value of the parameter). Here we try to derive an approximation for L(θ), neglecting the excess of p_{1m} / p_{0m} over the boundaries A and B at the termination of the process. A rigorous derivation can be found in [Wald (1947) c, Appendix, Section A.2.3]. We determine for each θ a value $h(\theta)$, so that we have in the continuous resp. discrete case

$$\int_{-\infty}^{\infty}\left(\frac{f(x,\theta_1)}{f(x,\theta_0)}\right)^{h(\theta)} f(x,\theta)\,dx = 1\,, \quad \text{resp.} \sum_x \left(\frac{f(x,\theta_1)}{f(x,\theta_0)}\right)^{h(\theta)} f(x,\theta)\,dx = 1\,,\ \forall\,\theta$$

AW shows in [Wald (1947) c, Appendix, A.2.1], that under "*some slight restrictions*" [Wald (1947) c, p49], there exists **exactly one value $h(\theta)$** such that the above Equation is fulfilled. With these assumptions it is clear that

$$f^*(x,\theta) = \left(\frac{f(x,\theta_1)}{f(x,\theta_0)}\right)^{h(\theta)} f(x,\theta)$$

is a distribution function. Besides that we have to investigate the two possibilities for $h(\theta)$: $h(\theta) > 0$ and $h(\theta) < 0$. AW shows quickly that the SPRT S* (testing $H: f(x,\theta)$ is the true distribution, against $H^*: f^*(x,\theta)$ is the true distribution, with $A^{h(\theta)}$ and $B^{h(\theta)}$) is equivalent to the SPRT S ($H_0: \underline{\theta} = \underline{\theta}_0$, against $H_1: \underline{\theta} = \underline{\theta}_1$, with A and B) [Wald (1947) c, p49], i.e. the value of $L(\theta)$ is exactly equal to that of the probability that S* leads to the acceptance of H, i.e H_0 due to the equivalence. Is (α', β') the strength of S*, and is '≥' resp. '≤'

replaced by '≈' in the equation from 5.1.4.2 we obtain quickly

$$\alpha' \approx \frac{1-B^{h(\theta)}}{A^{h(\theta)}-B^{h(\theta)}}, \text{ and hence with } L(\theta) \equiv 1-\alpha', \text{ we get } L(\theta) \approx \frac{A^{h(\theta)}-1}{A^{h(\theta)}-B^{h(\theta)}}$$

The last equation also holds in the case of $h(\theta) < 0$. [Wald (1947) c, p50]

5.1.4.4 The ASN function of the SPRT

Let $\mathbb{E}_\theta(n)$ denote the expected value of n (the so called ASN function) when ***θ* is the true value** of the parameter and n is the number of observations. In [Wald (1947) c, pp52-53] AW derives an approximation formula for the ASN function, "*neglecting the excess of p_{1m} / p_{0m} over the boundaries A and B at the termination of the sequential process*" (cp. [Wald (1947) c, App A.3]). With $z := \ln \frac{f(x,\theta_1)}{f(x,\theta_0)}$ and $\mathbb{E}_\theta(z)$ as expected value of z when θ is the true value, AW gets following

$$\mathbb{E}_\theta(n) \approx \frac{L(\theta)\ \mathrm{lnB} + (1-L(\theta))\ \mathrm{lnA}}{\mathbb{E}_\theta(z)}$$

5.1.4.5 Saving the number of observations by the SPRT compared to the usual test procedure

Assuming a test with strength (α, β) by taking hypothesis H_0: x is normally distributed with mean θ_0 and variance 1, against hypothesis H_1: x is normally distributed with mean θ_1 and variance 1, we compare the expected number of observations ($\mathbb{E}_\theta(n)$, θ is the true value) required by the SPRT for testing H_0

against H_1 with the fixed number of observations (denoted by $n(\alpha, \beta)$) needed for the usual most powerful test with the same strength (α, β). We assume without loss of generality that $\theta_0 < \theta_1$. Regarding a test with (fixed) $n(\alpha, \beta)$ we accept H_0 if the arithmetic mean $\overline{x} = \frac{1}{n(\alpha,\beta)} \sum_{i=1}^{n(\alpha,\beta)} x_i$ is less or equal a constant d. The probability of $\overline{x} \leq d$ (if H_0 is true) is equal to the probability that $\sqrt{n(\alpha, \beta)}\ (\overline{x} - \theta_0) \leq \sqrt{n(\alpha, \beta)}\ (d - \theta_0)$ (if H_0 is true). [Wald (1947), p54] So the probability to accept H_0 (when H_0 is true) is equal to $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{n(\alpha,\beta)}\ (d-\theta_0)} e^{-\frac{t^2}{2}} dt = : G\left(\sqrt{n(\alpha, \beta)}\ (d - \theta_0)\right)$ (since $\sqrt{n(\alpha, \beta)}\ (\overline{x} - \theta_0)$ has mean 0 and variance 1), and this again is per definitionem equal to $1 - \alpha$. Analogously one obtains for the probability to accept H_0 although it is false the equation $G\left(\sqrt{n(\alpha, \beta)}\ (d - \theta_1)\right) = \beta$. [Wald (1947), p55] Finally we get for the fixed number of observations [Wald (1947) c, p55]:

$$n(\alpha, \beta) = \left(\frac{\lambda_1 - \lambda_0}{\theta_1 - \theta_0}\right)^2 \text{ with } \lambda_i := G^{-1}\left(\sqrt{n(\alpha, \beta)}\ (d - \theta_i)\right),\ i = 1, 2\,.$$

We still have to derive the value of $\mathbb{E}_\theta(n)$ required by the SPRT of strength (α, β). Approximatively we use the equalities $A = a(\alpha, \beta)$ and $B = b(\alpha, \beta)$, what at most increases the value of $\mathbb{E}_\theta(n)$ and does not afflict our demonstration of the saving effect by the SPRT. Supposing that $|\theta_1 - \theta_0|$ is small (i.e. we can neglect the excess of p_{1m} / p_{0m} over the boundaries A and B), we use the equation from 5.1.4.4 and get

$$\mathbb{E}_{\theta=\theta_0}(n) \overset{L(\theta_0)=1-\alpha}{\approx} \frac{(1-\alpha)\ln \mathrm{B} + \alpha \ln \mathrm{A}}{\mathbb{E}_{\theta_0}(z)}\ ,\ \text{when } \theta_0 \text{ is true}\ ,$$

$$\text{resp.}\ \mathbb{E}_{\theta=\theta_1}(n) \overset{L(\theta_1)=\beta}{\approx} \frac{\beta \ln \mathrm{B} + (1-\beta)\ln \mathrm{A}}{\mathbb{E}_{\theta_1}(z)}\ ,\ \text{when } \theta_1 \text{ is true}$$

From that AW ([Wald (1947) c, p56]) derives easily the ratios

$$\frac{\mathbb{E}_{\theta=\theta_0}(n)}{n(\alpha,\,\beta)} = \frac{2\,(-(1-\alpha)\,\mathrm{lnB} - \alpha \mathrm{lnA})}{(\lambda_1 - \lambda_0)^2},$$

$$\text{resp. } \frac{\mathbb{E}_{\theta=\theta_1}(n)}{n(\alpha,\,\beta)} = \frac{2\,(\beta \mathrm{lnB} + (1-\beta)\,\mathrm{lnA})}{(\lambda_1 - \lambda_0)^2}$$

Interestingly these ratios are independent of the parameter values θ_i , $i = 0, 1$. As the table in [Wald (1947) c, p57, Table 1] shows, the SPRT results "*for the range of α and β from .01 to .05 ... in an average saving of at least 47 per cent in the necessary number of observations as compared with the current test*.". [Wald (1947) c, p57]

5.1.4.6 Increase in the expected number of observations when we replace the exact values A and B

Lets describe the changes $\Delta\mathbb{E}_{\theta=\theta_i}(n)$, $i = 0, 1$, when A resp. B is replaced by $a(\alpha,\,\beta)$ resp. $b(\alpha,\,\beta)$. From the equation in 5.1.4.4 we derive with $L(\theta_0) = 1 - \alpha$ resp. $L(\theta_1) = \beta$ following (approximate) equations [Wald (1947) c, p66] :

$$\Delta\mathbb{E}_{\theta_0}(n) \approx \frac{(1-\alpha)\ln\frac{b(\alpha,\beta)}{B} + \alpha \ln\frac{a(\alpha,\beta)}{A}}{\mathbb{E}_{\theta_0}(z)}\;,\;\; \Delta\mathbb{E}_{\theta_1}(n) \approx \frac{\beta \ln\frac{b(\alpha,\beta)}{B} + (1-\alpha)\ln\frac{a(\alpha,\beta)}{A}}{\mathbb{E}_{\theta_1}(z)}$$

With [Wald (1947) c, Appendix 2.1], $\ln\frac{b(\alpha,\beta)}{B} < 0$ and $\ln\frac{a(\alpha,\beta)}{A} > 0$ AW concludes from the last equation [Wald (1947) c, p67] :

$$\Delta\mathbb{E}_{\theta_0}(n) < \frac{\ln\frac{b(\alpha,\beta)}{B}}{\mathbb{E}_{\theta_0}(z)} \leq \frac{\ln\eta_{\theta_0}}{\mathbb{E}_{\theta_0}(z)},\ \Delta\mathbb{E}_{\theta_1}(n) < \frac{\ln\frac{a(\alpha,\beta)}{A}}{\mathbb{E}_{\theta_1}(z)} \leq \frac{\ln\delta_{\theta_0}}{\mathbb{E}_{\theta_1}(z)}$$

Since the exact values of $A(\alpha,\beta)$ and $B(\alpha,\beta)$ are not known, we use in the last equation the substituting quantities $\delta_{\theta_0}$and η_{θ_0}, fulfilling $\frac{a(\alpha,\beta)}{A} \leq \delta_{\theta_0}$and $\frac{b(\alpha,\beta)}{B} \geq \eta_{\theta_0}$. [Wald (1947) c, Appendix 2.3] (Bear in mind, that $\ln\frac{b(\alpha,\beta)}{B} < 0$ from definiton).

We derive now the lower limit of the probability that the SPRT will terminate with a number of trials less than or equal a given number n_0. In [Wald (1947) c, Appendix A.6] AW presents an approximate formula for the probability distribution of the number of observations required by the SPRT in the case in which z is normally distributed. He further states : "... *the same distribution function of n can be regarded as an approximation to the exact distribution even when z is not normally distributed* .. [if] *the absolute value of* $\mathbb{E}_\theta(z)$ *and the standard deviation of z are sufficiently small as compared with log A and log B*". [Wald (1947) c, p58]

In [Wald (1947) c, p58] AW presents a simpler method with almost the same accuracy : let p_i denote the probability that $n \leq n_0$when $H_i : \theta = \theta_i$, $i = 0, 1$is true. We assume that $n_0$is sufficiently large, so that the sum $\sum_{i=1}^{n_0} z_i$ is normally distributed. [250] Because $\sum_{i=1}^{n_0} z_i \geq \ln A$ resp. $\sum_{i=1}^{n_0} z_i \leq \ln B$ implies directly $n_0 \geq n$, we have $p_1(\sum_{i=1}^{n_0} z_i \geq \ln\mathrm{A}) \leq p_1(n \leq n_0)$ resp. $p_0(\sum_{i=1}^{n_0} z_i \leq \ln\mathrm{A}) \leq p_0(n \leq n_0)$. Is $G(\lambda)$ the probability that a normally distributed variable with variance unity will take a value less than λ, than AW formulates easily a lower bounds for $p_i(n \leq n_0)$, $i = 0, 1$ [Wald (1947) c, pp59-60] :

$$1 - G\left(\frac{\ln A - n_0\, \mathbb{E}_{\theta=\theta_1}(z)}{\sqrt{n_0}\ \sigma_1(z)}\right) \leq p_1(n \leq n_0)\ ,$$

$\sigma_1(z)$ standard deviation of z when H_1 is true

$$G\left(\frac{\ln B - n_0\, \mathbb{E}_{\theta=\theta_0}(z)}{\sqrt{n_0}\ \sigma_0(z)}\right) \leq p_0(n \leq n_0)\ ,$$

$\sigma_0(z)$ standard deviation of z when H_0 is true

5.1.4.7 Truncation of the sequential test procedure

"Although it is shown ... that the probability is 1 that the sequential test procedure will eventually terminate, it is occasionally desirable to set a definite upper limit, say n_0, for the number of observations.". [Wald (1947) c, p61] We attain this by adding a simple rule for acceptance or rejection of H_0 at the n_0th trial : if the SPRT does not lead to a decision for $n \leq n_0$, we accept H_0, if $\ln B \leq \sum_{i=1}^{n_0} z_i \leq 0$, and reject H_0, if $0 < \sum_{i=1}^{n_0} z_i < \ln A$. This however changes the probabilites of the errors of the first and the second kind. Lets denote these changed values by $\alpha(n_0)$ and $\beta(n_0)$. Is $p_0(n_0)$ the probability under H_0that the truncated process leads to the rejection of H_0, while the "normal" SPRT leads to the acceptance of H_0, then we have : $\alpha(n_0) \leq \alpha + p_0(n_0)$. The inequality results from the fact that there may be samples for which the truncated process leads to the acceptance of H_0, while the non-truncated process leads to the rejection of H_0. Taking $G(\lambda)$ from above AW [Wald (1947) c, p61-64] derives quickly upper bounds (" *... considerably above the true $\alpha(n_0)$ and $\beta(n_0)$...*" [Wald (1947) c, p64]) for $\alpha(n_0)$ and $\beta(n_0)$:

$$\alpha(n_0) \leq \alpha + G\left(\frac{\ln A - n_0\, \mathbb{E}_{\theta=\theta_0}(z)}{\sqrt{n_0}\ \sigma_0(z)}\right) - G\left(\frac{-n_0\, \mathbb{E}_{\theta=\theta_0}(z)}{\sqrt{n_0}\ \sigma_0(z)}\right),$$

$$\text{and } \beta(n_0) \leq \beta + G\left(\frac{-n_0\,\mathbb{E}_{\theta=\theta_1}(z)}{\sqrt{n_0}\;\sigma_1(z)}\right) - G\left(\frac{\ln B - n_0\,\mathbb{E}_{\theta=\theta_1}(z)}{\sqrt{n_0}\;\sigma_1(z)}\right)$$

5.1.5 A theory of sequential tests of simple and composite hypotheses against a set of alternatives

In this paragraph we discuss "... *sequential tests of simple and composite hypotheses against infinitely many alternatives*.". [Wald (1947) c, p70] In applications we often decide between two alternative hypotheses H_0 and H_1. The preference for one or the other depends on the value of a parameter θ. Is ω the set of all θ for which $H_0$is preferred, than the decision can be formulated as problem of testing the hypothesis H_ω: θ lies in ω. H_ω can be understood as a simple hypothesis (if ω consists of one value and induces a discontinuity in the decision making) or a composite hypothesis (if the degree of preference for one or the other action varies continuously with θ). "*Nevertheless, frequently it will be expedient to approximate the composite hypothesis with a simple one, since the latter is usually a simpler problem to treat*.". [Wald (1947) c, p71] "*In terms of the zones of preference for acceptance, of preference for rejection, and of indifference, the simple hypothesis may be characterized by the condition that the zone of preference for acceptance consists of a single point*.". [Wald (1947) c, p72]

A simple hypothesis against one-sided alternatives

Lets consider the simple case in which there is only one unknown parameter θ and the hypothesis $H_0 : \theta = \theta_0$ is tested against the one-sided alternative hypothesis that $\theta > \theta_0$. The zone of preference simply consists of the point θ_0. The degree of preference for rejection of the hypothesis generally increases with increasing value of $\theta > \theta_0$.The acceptance of θ is considered as an error of practical importance if $\theta \geq \theta_1 > \theta_0$ for an appropriate value $\theta_1 > \theta_0$. Accordingly the domain $\theta_0 < \theta < \theta_1$ is considered as zone of indifference. The requirements on the OC function of this test are the usual one : the probability of a wrongly rejection of $H_0(\theta = \theta_0)$ should be equal to α and the probability of wrongly accepting H_0 $(\theta \geq \theta_1)$ should not exceed a preassigned value β.

Test of a simple hypothesis on an unknown parameter vector

A bit more general is the case when $\underline{\theta}$ is a vector, meaning that we deal with a couple of parameters $\underline{\theta} = (\theta_1, .., \theta_k)$. So the hypothesis to be tested is $H_0 : \underline{\theta} = \underline{\theta}_0$ .As above the zone of preference for acceptance consists of the single parameter point $\underline{\theta}_0$. AW denotes the zone of preference for rejection by ω_r. The requirements on the OC function are analogously formulated as in the last section. Is therefore $\beta(\underline{\theta})$ for each $\underline{\theta} \in \omega_r$ the probability of wrongly accepting H_0, it follows that $\beta(\underline{\theta}) < \beta$, $\forall\ \underline{\theta} \in \omega_r$, when β is the error of the second kind. A modified requirement on the error of the second kind could be, that the weighted average of $\beta(\underline{\theta})$ (weighted with a given weight function $w(\underline{\theta})$, should be equal to β, i.e. :

$$\int_{\omega_r} \beta(\underline{\theta})\, w(\underline{\theta})\, d\underline{\theta} = \beta \;,\ \text{ where } w_{\omega_r} \geq 0 \text{ with } \int_{\omega_r} w(\underline{\theta})\, d\underline{\theta} = 1$$

The requirements on α are maintained as before. A proper SPRT for these requirements can be constructed in following way : let $p_{0\,n} = \prod_{i=1}^{n} f(x_i, \underline{\theta}_0)$ be the probability distribution of the sample $(x_1, .., x_n)$ and let $p_{1n} = \int_{\omega_r} \prod_{i=1}^{n} f(x_i, \underline{\theta}_1)\, w(\underline{\theta})\, d\,\underline{\theta}$ a weighted average of the probability distribution functions (what itself is a probability distribution [Wald (1947) c, p75]). Is H_1 the hypothesis, that distribution of the sample $(x_1, .., x_n)$ is given by p_{1n}, the SPRT of testing H_0against H_1 is given as usual : We choose constants A and B and reject H_0if $\frac{p_{1n}}{p_{0n}} \geq A$, accept H_0if $\frac{p_{1n}}{p_{0n}} \leq B$ and add a new observation if $B < \frac{p_{1n}}{p_{0n}} < A$. For "*most practical purposes*" AW recommends $A = \frac{1-\beta}{\alpha}$and $B = \frac{\beta}{1-\alpha}$. [Wald (1947) c, p75] This SPRT "*can be shown to satisfy*" the equation above. [Wald (1947) c, pp76-77] "*In practical problems however, it seems more reasonable to maintain the original requirements*.". [Wald (1947) c, p75] "*There are, in general, infinitely many sequential tests which satisfy these requirements, and we want to select one for which the expected number of observations is as small as possible*.". [Wald (1947) c, p75] 251

A still open question is how to choose the weight function $w(\underline{\theta})$ for a SPRT of class C (a class C SPRT is one that is based on the ratio $\frac{p_{1n}}{p_{0n}}$ with p_{0n} and p_{1n} from above). If the quantities A and B are already determined, its clear that the choice of $w(\underline{\theta})$ only affects $\beta(\underline{\theta})$ but not α. [Wald (1947) c, p76] "*Thus, the following choice of $w(\underline{\theta})$ seems reasonable : For given values of A and B the weight function $w(\underline{\theta})$ is chosen for which the maximum of $\beta(\underline{\theta})$ with respect to $\underline{\theta}$ ($\underline{\theta}$ is, of course, restricted to points in ω_r) takes its smallest value*". [Wald (1947) c, p76] This principle leads to the desired probabilities α and β that themselves depend only on the quantities A and B (cp. [Wald (1947) c, pp76-77]). For some special cases AW has concretely determined the weight function $w(\underline{\theta})$. [Wald (1947) c, Appendix 8]

Example for a composite hypothesis: Testing the mean of a normal distribution with known variance

Lets formulate the simple hypothesis H_0 as follows: the mean θ of a normal distribution with known standard deviation σ is equal to a value θ_0. The acceptance of H_0 is not considered as a serious error if $\theta \neq \theta_0$ lies in the near of θ_0. So the region of preference for rejection (ω_r) can be defined as the set of all values θ for which $\frac{|\theta-\theta_0|}{\sigma} \geq \delta$is fulfilled. Accordingly the region of indifference consists of all values θ in $0 < \frac{|\theta-\theta_0|}{\sigma} < \delta$. The probability density of the sample $(x_1, .., x_n)$ under H_0 is given by $p_{0\,n} = \frac{1}{(2\,\pi)^{\frac{n}{2}}\,\sigma^n}\,e^{-\frac{1}{2\,\sigma^2}\sum_{i=1}^n (x_i-\theta_0)^2}$and $p_{1\,n}$ as weighted average of the probability distributions can be expressed in following form : $p_{1\,n} = \frac{1}{(2\,\pi)^{\frac{n}{2}}\,\sigma^n}\,e^{-\frac{1}{2\,\sigma^2}\sum_{i=1}^n (x_i-\theta_0)^2}$. [Wald (1947) c, Appendix 8.2] Then the test is carried out as usual : if $B < \frac{p_{1\,n}}{p_{0\,n}} < A$ the observations continue, if $\frac{p_{1\,n}}{p_{0\,n}} \geq A$ we reject H_0, and finally in the case $\frac{p_{1\,n}}{p_{0\,n}} \leq B$ we accept H_0.

Composite hypotheses : discussion of a special case

A problem of particular importance is the quality control of manufactured products. The task to be performed is about testing of the hypothesis H_0: the unknown parameter θ does not exceed a specified value θ'. The importance of an error of the first (second) kind will increase steadily with decreasing (increasing) value of θ in the domain $\theta \leq \theta'(\theta > \theta')$. For the desired test procedure the probability for an error of the first kind should be less than or equal to a preassigned α whenever $\theta \leq \theta_0 < \theta'$ (zone of preference) and the probability of an error of the second kind is less than or equal to a preassigned value β whenever $\theta \geq \theta_1 > \theta'$(zone of rejection). "*In most of the important cases ... in practice ... the sequential probability test of strength* (α, β) *for testing the hypothesis* H_0': $\theta = \theta_0$*against the single alternative* $\theta = \theta_1$ *will have the desired properties and provides a satisfactory solution to the problem*. ". [Wald (1947) c, p79] If this SPRT leads to acceptance of H_0' , AW advises to accept H_0. Otherwise from the rejection of H_0' AW advises the rejection of H_0. [Wald (1947) c, p79]

Composite hypotheses : the general case

In the general case of a **composite** hypothesis H_ω (when the unknown parameter θ lies in a subset ω of the parameter space), the parameter space is subdivided into three mutually exclusive zones : the zone of acceptance ω_a , the zone of rejection ω_r and the zone of indifference. For any θ in ω AW denotes with $\alpha(\theta)$ the probability that $H_\omega$will wrongly be rejected (α is varying in general with θ), and analogously the probability of wrongly accepting H_ω AW varies with θ and denotes it by $\beta(\theta)$. As usual $\alpha(\theta) < \alpha$ in ω_a and $\beta(\theta) < \beta$ in ω_r for preassigned values α and β. A modification of the desired test procedure can be formulated with non-negative weight functions $w_a(\theta)|_{\omega_a}$ and $w_r(\underline{\theta})|_{\omega_r}$ as above :

$$\int_{\omega_a} w_a(\underline{\theta})\, d\underline{\theta} = 1 \ , \ \int_{\omega_r} w_r(\underline{\theta})\, d\underline{\theta} = 1 \ ,$$ [252]

$$\text{with } \int_{\omega_a} \alpha(\underline{\theta})\, w_a(\underline{\theta})\, d\underline{\theta} = \alpha \ , \ \int_{\omega_r} \beta(\underline{\theta})\, w_r(\underline{\theta})\, d\underline{\theta} = \beta \ \ (*) \ \text{ and}$$

$$p_{0\,n} = \int_{\omega_a} \prod_{k=1}^{n} f(x_k,\, \underline{\theta})\, w_a(\underline{\theta})\, d\underline{\theta}\,,\;\; p_{1\,n} = \int_{\omega_r} \prod_{k=1}^{n} f(x_k,\, \underline{\theta})\, w_r(\underline{\theta})\, d\underline{\theta}\;\; (**)$$

where $f(x_k, \underline{\theta})$denotes the probability distribution of x provided that $\underline{\theta}$ is the true parameter value. H_0^* ought to denote the hypothesis that the distribution of $(x_1, ..., x_n)$ is given by $p_{0\,n}$, and H_1^* should name the hypothesis that this distribution is given by $p_{1\,n}$. The SPRT of strength (α, β) for testing H_0^* against H_1^*(where as usual $\alpha = \int_{\omega_a} \alpha(\theta)\, w_a(\underline{\theta})\, d\underline{\theta}$ is the probability that we wrongly reject H_0^*, and $\beta = \int_{\omega_r} \beta(\theta)\, w_r(\underline{\theta})\, d\underline{\theta}$ is the probability that we wrongly accept H_1^*). The constants A and B have to be choosen adequately. For practical purposes AW suggests again $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$. [Wald (1947) c, p82] Possible weight functions have to fulfill

$$\alpha(\underline{\theta})\,|_{\omega_a} \leq \alpha \;\;,\; \beta(\underline{\theta})\,|_{\omega_r} \leq \beta$$

SPRTs satisfying both equations (*) and (**) are also called to be of “class C”. Hence a SPRT of class C is uniquely determined by A,B,w_a and w_r. Thus $\alpha(A, B, w_a, w_r) := \max_{\omega_a} w_a(\underline{\theta})$ and $\beta(A, B, w_a, w_r) := \max_{\omega_r} w_r(\underline{\theta})$ are determined uniquely by A,B,w_a and w_r. For given A and B, the weight functions are more desirable the smaller they make $\alpha(A, B, w_a, w_r)$ and $\beta(A, B, w_a, w_r)$. In the case of finding w_a and w_r that simultaneously minimize $\alpha(A, B, w_a, w_r)$ and $\beta(A, B, w_a, w_r)$, AW calls them **optimum weight functions** [Wald (1947) c, p82]. (AW shows that in special cases optimum weight functions do exist. [Wald (1947) c, Appendix 9.2] ”*However, it is not known whether they generally exist*.”. [Wald (1947) c, p83]

5.1.6 Application of the general theory to special cases

Testing the mean of a binomial distribution: formulation of the problem and tolerated risks

Here we deal with the problem of an acceptance inspection of a (huge) lot of particularly classified units. Each unit represents f.e. a manufactured product, that is either defective (x=1) or not (x=0), where x is a random variable which can take only the values 0 and 1. p denotes the (unknown) probability that x takes the vaIue 1 (p is also the proportion of the defectives in the lot). Thus the probability of a unit to be non-defective is $1-p$. The task is now to "... *deal here with the problem of testing the hypothesis that p does* ***not*** *exceed some specified value* p' ".[Wald (1947) c, p89] The lot will be rejected if $p > p'$(this is the alternative hypothesis). [253] In AW's own words : "*A proper sampling plan (test procedure) is to be devised for deciding whether the lot submitted for inspection should be accepted or rejected*." [Wald (1947) c, p89]. Because every incomplete inspection may lead to a wrong decision one has to state a sampling plan in combination with an indication of the maximum risks of wrong decisions. If $p = p'$ a decision cannot be reached. $p > p'$ is an indicator for rejection and contrarily leads $p < p'$ to acceptance, whereas the equality $p = p'$ denotes a status of indifference. If the difference between p and p' is only slight a won decision won't count as a serious error. A serious error of practical consequence has been made if p lies outside a predefined intervall (p_0, p_1) with $p' \in (p_0, p_1)$. After this intervall has been chosen, the risks can be formulated in following way [Wald (1947) c, p89] : "*The probability of rejecting the lot should not exceed some small preassigned value α whenever* $p \leq p_0$, *and the probability of accepting the lot should not exceed some* *preassigned value β whenever $p \geq p_1$*". Hence the characterisation of the tolerated risks can be done by 4 numbers : p_0, p_1,α and β. Their choice is **not** a statistical task but done "*on the basis of practical considerations in each particular case*". [Wald (1947) c, p89]

Testing the mean of a binomial distribution: Formulation of the SPRT corresponding to p_0, p_1,α and β

A sampling plan is given by the SPRT of strength (α, β) for testing the hypothesis $H_0: p = p_0$ against $H_1: p = p_1$. We define the i-th inspection result as $x_i = \begin{cases} 0 \text{ unit is ok} \\ 1 \text{ unit is defective} \end{cases}$. The probability for a sample $(x_1, \ ..., x_m)$ is given by $p^{d_m}(1-p)^{m-d_m}$, where d_m is the number of defectives in the sample. Given H_i , $i = 0$ resp. $i = 1$ as correct, we have as probabilities for obtaining this sample $p_{\text{im}} = p_i{}^{d_m}(1-p_i)^{m-d_m}$, $i = 0$ resp. $i = 1$. Now the SPRT is carried out as follows : at the m-th inspection (for each successive m) we compute $\ln \frac{p_{1m}}{p_{0m}}$and coninue the inspection as long as $\ln \frac{\beta}{1-\alpha} < \ln \frac{p_{1m}}{p_{0m}} < \ln \frac{1-\beta}{\alpha}$. For the first time this unequality does not hold, the lot is rejected (accepted) if $\ln \frac{\beta}{1-\alpha} \leq \ln \frac{p_{1m}}{p_{0m}}$ $(\ln \frac{1-\beta}{\alpha} \geq \ln \frac{p_{1m}}{p_{0m}})$. AW [Wald (1945) a, p91] easily derives :

$$d_m \geq \frac{\ln \frac{1-\beta}{\alpha}}{\ln \frac{p_1}{p_0} - \ln \frac{1-p_1}{1-p_0}} + m \frac{\ln \frac{1-p_0}{1-p_1}}{\ln \frac{p_1}{p_0} - \ln \frac{1-p_1}{1-p_0}} =: h_1 + \text{ms} =: r_m \ (*) \text{ and}$$

$$d_m \leq \frac{\ln \frac{\beta}{1-\alpha}}{\ln \frac{p_1}{p_0} - \ln \frac{1-p_1}{1-p_0}} + m \frac{\ln \frac{1-p_0}{1-p_1}}{\ln \frac{p_1}{p_0} - \ln \frac{1-p_1}{1-p_0}} =: h_0 + \text{ms} =: a_m \ (**),$$

whereas a_m is called the acceptance number, and r_m denotes the so-called rejection number. These two numbers are computed at each stage. Now the SPRT is carried out as follows : inspection continues as long as we have $a_m < d_m < r_m$. If $d_m \geq r_m$ ($d_m \leq a_m$) the lot is rejected (accepted). Because a_m and r_m depend only on the four numbers α, β, p_0 and p_1, they can be computed before the inspection starts. (If a_m resp. r_m is not an integer, we replace them by $[a_m]$ resp $[r_m + 1]$, whereby [c] denotes the integer part of any $c \in \mathbb{R}$). As all points (m, r_m) resp. (m, a_m), $m = 1, 2, ..$, lie on straight, parallel lines L_1 and L_0 with a slope s. As long as (m, d_m) is located between these lines, the inspection is continued. The SPRT regarded as graphical procedure, we first draw the lines L_i , $i = 0, 1$ before inspection starts and continue as long as the successive points (m, d_m) lie between them (without touching).

The OC function $L(p)$ (specifying the probability that the lot will be accepted when p is the true proportion of defectives (by definition of p)) clearly takes following values: $L(0) = 1$, $L(1) = 0$, $L(p_0) = 1 - \alpha$ and $L(p_1) = \beta$. For $p = s$ AW derives with the rule of de l'Hôpital (and values α, β such that $\ln B < 0$) $L(s) = \frac{h_1}{h_1 + |h_0|}$. [Wald (1947) c, p95] From the last equation in 5.1.4.3 with arbitrarily chosen values for $h \in (-\infty, +\infty)$ we can compute more points $(p, L(p))$ of the OC curve. [Wald (1947) c, p96] Interestingly for a negative value -h the corresponding pair $(p', L(p'))$ can be computed from $(p, L(p))$ (belonging to h) with $p' = \frac{p_1}{p_0}$ and $L(p') = \left(\frac{\beta}{1-\alpha}\right)^h L(p)$. [Wald (1947) c, p94]

The probability ratio for a single observation can take only the values $\ln \frac{p_0}{p_1}$ and $\ln \frac{1-p_0}{1-p_1}$. With the equation for the slope s we have $\ln \frac{p_0}{p_1} = \left(\frac{1}{s} - 1\right) \ln \frac{1-p_0}{1-p_1}$. In [Wald (1947) c, p98], AW derives an exact formula for $L(p)$ in the case that $\frac{1}{s}$ is an integer. A different method for deriving an exact formula for $L(p)$ was given by M. A. Girshick. [Girshik (1946)]

The ASN (average sample number) is a random variable, because it depends on the outcome of the experiment. The expectation value $E_p(n)$ can be obtained with the approximation formula i.e. the equation in 5.1.4.4. This applied to the binomial case gives

$$E_p(n) = \frac{L(p)\ln B + (1 - L(p))\ln A}{p \ln \frac{p_1}{p_0} + (1-p)\ln \frac{1-p_1}{1-p_0}}$$

With that equation we are able to compute $L(p)$ for the five values $0,\ p_0,\ p_1\ ,\ 1,\ s$. Especially for the derivation of $E_s(n)$ resp. $L(s)$ we need the approximation formula A:99 in [Wald (1947) c,] . Then these five points "*give a fairly good idea of the shape of the whole curve*". [Wald (1947) c,] If it is desired to get the curve of $E_p(n)$ for all p, it is necessary to compute first the values of $L(p)$.

Grouping (i.e. observations taken in groups)

An advanced examination must be undertaken when the **observations are taken in groups** (what is sometimes useful). Then the test is carried out as follows : a group g_1 consisting of v units is drawn from the lot. If the number of defectives d_v in this group is less than or equal to the acceptance number a_v (greater than or equal to the rejection number r_v), the inspection terminates with the acceptance of the lot (rejection of the lot). Only if $a_v < d_v < r_v$ a further group g_2 of observations is drawn. g_2 is accepted (rejected) if the **total** number of defectives d_{2v} is less than or equal to a_{2v} $(d_{2v} \geq r_{2v})$, otherwise g_{3v} is drawn.

The question is how grouping increases (or doesn't) the number of observations (denoted by n in the non-grouping i.e single case) : clearly, if accidentally

$n = k \cdot v$, $k \in \mathbb{N}$, the grouped experiment terminates with the group g_{kv} and we dont need more data as the single case needs. Is however kv $< n < (k+1)\,v$, we have to inspect at least $k+1$ groups (it can be more than $k+1$, if d_n lies outside the intervall (a_n, r_n), but $a_{(k+1)\,v} < d_{(k+1)\,v} < r_{(k+1)\,v}$). In other words the increase over n can exceed v in some cases.

The inequalities $\alpha' \le \frac{\alpha}{1-\beta}$ and $\beta' \le \frac{\beta}{1-\alpha}$ (cp. the equation from 5.1.4.2) "*also remain valid when the observations are taken in groups*.". [Wald (1947) c, p102] This means that the effect of grouping on the OC curve is neglectable. As the values α and β are usually very small, α' and β' at the most exceed α resp. β by a pretty small quantity, "*which can be neglected for all practical purposes*". [Wald (1947) c, p102] **Hence grouping does not affect the protection against wrong decisions provided by the test**. Otherwise α' and β' can be substantially smaller than α and β with increasing number of observations. This smallness compensates for the data increase.

If the number v of units in a group is equal to the reciprocal of the common slope s of the acceptance and rejection lines and if the intercepts of these lines are integers, the OC curve is not affected at all by grouping (cp. [Wald (1947) c, p103]).

Limits for the OC and ASN curves in the case of grouping

[Wald (1947) c, p103] uses three auxiliary sequential sampling plans to obtain these limits : let h_0, h_1, s the parameters from Equation 5.1.6 (*), resp. 5.1.6 (**) be altered to $h_0^* := h_0 - \mathrm{vs}$, h_1, s (first auxiliary plan), h_0, $h_1^* := h_1 + \mathrm{vs}$, s (second auxiliary plan) and h_0^*, h_1^*, s (third auxiliary plan), let $L_i(p)$ the OC and $E_{\mathrm{pi}}(n)$ the ASN function of all three plans (when item-by-item inspection i used, i = 1, 2, 3), and finally $\overline{L}(p)$ and $\overline{E}_p(n)$ the OC resp. ASN function of the grouped procedure, than it easily follows from the acceptance of the lot by the first auxiliary plan (using item-by-item inspection) the acceptance of the lot by the original plan (taking observations in groups). This can be written as $L_1(p) \leq \overline{L}(p)$. Similarly we derive that from the rejection of the lot according to the second auxiliary plan (item-by-item inspection) we maintain the rejection of the lot by the original plan. Hence $\overline{L}(p) \leq L_2(p)$.

To get a upper limit for $\overline{E}_p(n)$ we make use of the third auxiliary plan. If this plan (item-by-itern inspection) terminates at the inspection of the n-th unit, the original plan (using grouping) must terminate at the latest with the inspection of the group in which the n-th item is included (or with an earlier group). As the number of inspected units of the original plan cannot exceed $n+v$, we get $\overline{E}_p(n) \leq E_{\mathrm{p}3}\left(n\right)+v$, and with $E_p(n) \leq \overline{E}_p(n)$ finally the term $E_p(n) \leq \overline{E}_p(n) \leq E_{\mathrm{p}3}(n)+v$. In [Wald (1947) c, Appendices 2.3, 3.1] AW derives limits with alternative methods, which are relative close to the just derived limits when $\frac{p_1}{p_0}$ and $\frac{1-p_1}{1-p_0}$ are near 1 and vs does not exceed 1. [Wald (1947) c, p104]

The corresponding OC function

As known the OC function $L(\sigma)$ denotes for any value σ the probability that the test terminates with the acceptance of the product. For the general case we have $L(\sigma) = \frac{\left(\frac{1-\beta}{\alpha}\right)^h - 1}{\left(\frac{1-\beta}{\alpha}\right)^h - \left(\frac{\beta}{1-\alpha}\right)^h}$, whereas the parameter h is the root of the integral equation. To get a finite value for this integral we need the inequality

$\frac{h}{\sigma_1^2} - \frac{h}{\sigma_0^2} + \frac{1}{\sigma^2} > 0$ to be fulfilled. [Wald (1947) c, p130] Using this in the last mentioned integral equation, AW solves it with respect to σ and obtains finally $\sigma = \sqrt{\frac{\left(\frac{\sigma_0}{\sigma_1}\right)^{2h} - 1}{\frac{h}{\sigma_1^2} - \frac{h}{\sigma_0^2}}}$. [Wald (1947) c, p130] Having this we are able to compute the pair $(\sigma, L(\sigma))$ for any given value of h.

Principles for the selection of a sequential sampling plan

At this stage it is necessary to investigate the heaviness of a wrong decision depending on the parameter θ. Let ω_i be the set of parameters θ consistent with the hypothesis H_i (i.e. H_i is the statement that θ is contained in ω_i). If θ is in ω_i but not far from H_j, the acceptance of H_i is not regarded as a serious error. It is often possible to express the importance of wrong decisions by k non-negative error-weights $w_i(\theta),\ i = 1,\ ..k$, where $w_i(\theta)$ is a measure for the importance of the error when H_i is accepted and θ is true. Of course is $w_i(\theta) = 0\ \forall\ \ \theta \in \omega_i$. The choice of a sampling plan will be influenced by the $w_i(\theta)$. Their determination is not a statistical problem. They will be choosen on practical considerations.

Example: When we withhold the product from the market and θ is only slightly above p' we regard this decision not as serious error. Otherwise if θ is significantly above p', the error of the decision is getting more massive.

The risk function of a given sampling plan

We define as risk function $r(\theta)$ the expected value of the loss. This value is equal to $\sum_i L_i(\theta)\, w_i(\theta)$ (separate from the costs for each sample step that would make an extra addend like $c\,\mathbb{E}_\theta(n)$ with a constant c). So we judge the quality of a sampling plan by $r(\theta)$ and the ASN function $\mathbb{E}_\theta(n)$. This means that a sampling plan is better the smaller the risk and the smaller the ASN function are. But we have to find a compromise between the two functions as they are conflicting in following way: the smaller we get $r(\theta)$ the larger n will become. Our first condition should be: $r(\theta) < r_0 \ \forall\ \theta$. This proposed we choose the sampling plan with the smallest $\mathbb{E}_\theta(n)$. The choice of r_0 is not a statistical problem but based on practical considerations. Now we have to simplify the $w_i(\theta)$ in a non-specific way (since constructing specific error-weights is practically awkward). A very rough method is following simplification: $\varpi_j(\theta) = \begin{cases} 0 & \omega_j(\theta) \le c_j \\ 1 & \omega_j(\theta) > c_j \end{cases}$. Following this we define as zone of preference for hypothesis H_j the set $\left\{\theta \mid \varpi_i(\theta) = 1 \ \forall\ i \neq j \text{ and } \varpi_j(\theta) = 0\right\}$ and as zone of indifference between H_i and H_j the set $\left\{\theta \mid \varpi_k(\theta) = 1 \ \forall\ k \neq i,\ j \text{ and } \varpi_j(\theta) = \varpi_i(\theta) = 0\right\}$. In this way we define in an analogue way as zone of indifference between H_i, H_j and H_k the set $\left\{\theta \mid \varpi_l(\theta) = 1 \ \forall\ k \neq i,\ j,\ k \text{ and } \varpi_i(\theta) = \varpi_j(\theta) = \varpi_k(\theta) = 0\right\}$ and so on.

Example: Lets consider again the last example: H_1 is the hypothesis that $\theta \le a$, H_2 is the hypothesis that $a < \theta < b$ and H_3 is the hypothesis that $b \le \theta$. Reasonably we define as simplified error-weights $\varpi_1(\theta) = \begin{cases} 0 & \theta < a + \delta \\ 1 & \theta \ge a + \delta \end{cases}$ $\varpi_2(\theta) = \begin{cases} 0 & a - \delta < \theta < b + \delta \\ 1 & \text{elsewhere} \end{cases}$ and $\varpi_3(\theta) = \begin{cases} 0 & \theta \ge b - \delta \\ 1 & \text{elsewhere} \end{cases}$ with a positive constant δ. Then the zone of indifference between H_1 and H_3 is the empty set $\emptyset$, the zone of indifference between H_2 and H_3 is the set $\{\theta \mid b - \delta \le \theta < b + \delta\}$ and the zone of indifference between H_1, H_2 and H_3 is again empty.

With that we say a wrong decision is made if we accept H_i despite $\varpi_i(\theta) = 1$.

The risk $r(\theta)$ as probability for making a wrong decision is the summation over all error-weights with $\varpi_i(\theta) = 1$: $r(\theta) = \sum_{j|\varpi_j(\theta)=1} L_j(\theta)$. Now our aim is to develop sampling plans for which the risk of making wrong decisions is less than r_0. From this class of sampling plans the best is that with the smallest number of observations n. AW could not give a common solution to this problem but could construct a special class of sampling plans based on following lemma [Wald (1947) c, p146]:

Lemma: Let $x_1, x_2, ..$ be a sequence of variates, let $p_{\mathrm{im}}(x_1, x_2, .., x_m)$ be the joint probability under hypothesis H_i, $i = 1, 2$. Is the constant $A > 1$, then under H_0 the probaility that $\frac{p_{1m}(x_1, x_2, .., x_m)}{p_{0m}(x_1, x_2, .., x_m)} < A$ holds for all $m = 1, 2, ...$ is $\geq 1 - \frac{1}{A}$.

Knowing this we are able to construct a sampling plan with a probability for a wrong decision not exceedig a given value r_0. Denotes $p_m(x_1, x_2, .., x_m, \theta) = f(x_1, \theta)\, f(x_2, \theta) \ldots f(x_m, \theta)$ where $f(x, \theta)$ is the probability distribution of x when θ is true and is $p^*(x_1, x_2, .., x_m, \theta)$ an given arbitrary distribution, than the probability that

$$\frac{p_{*m}(x_1, x_2, .., x_m, \theta)}{p_m(x_1, x_2, .., x_m, \theta)} < A \ \forall\ m$$

is $\geq 1 - \frac{1}{A}$ when θ is true. Is $\omega_n(E_n)$ the set of all θ which fulfill Equation 1 for all $m \leq n$, than the probability that the true value θ is included in $\omega_n(E_n)$ $\forall\ n$ is exactly $\geq 1 - \frac{1}{A}$. Knowing this we define the sampling plan as follows:

we continue taking additional observations as long as each of the $\varpi_1(\theta), ..., \varpi_k(\theta)$ is > 0 in $\omega_n(E_n)$. At the first time when one of the weight functions $\varpi_1(\theta)$ is identically 0 in $\omega_n(E_n)$ we stop the process accepting the hypothesis that correspondends to the weight function which is zero for all parameters in $\omega_n(E_n)$. The probability of this sequential sampling plan to make a wrong decision does not exceed $1/A$. Defining $A = 1/r_0$, the probability of making a wrong decision will not exceed r_0, as required.

Since the distribution function $p_{*m}(x_1, x_2, .., x_m, \theta)$ can be chosen arbitrarily, we get a wide class C of sequential sampling plans with the required property. Yet "it is doubtful whether this class C of sampling plans contains an optimum plan in the sense of the definition of the [ϖ_j]". [Wald (1947) c, p147] If we accept sampling plans of class C, we "still have the problem of choosing $p_{*m}(x_1, x_2, .., x_m, \theta)$ as to make the expected number of observations required by the plan as small as possible.". [Wald (1947) c, p147] "This problem, has not yet been solved." [Wald (1947) c, p147] Another waste happens probably in letting $A = 1/r_0$, since this may result in a probability of making a wrong decision that is considerably less than the value r_0. A possible choice of $p_{*m}(x_1, x_2, .., x_m, \theta)$ which may give "reasonably good results" [Wald (1947) c, p147], may be obtained by setting $p_{*m}(x_1, x_2, .., x_m, \theta)$ equal a chosen weighted average of $p_m(x_1, x_2, .., x_m, \zeta)$ where ζ is a variable parameter, i.e. we set $p_{*m}(x_1, x_2, .., x_m, \theta) = \int_\Omega \rho_\theta(\zeta)\, p_m(x_1, x_2, .., x_m, \zeta)\, d\zeta$, where the integration is taken over the whole parameter space Ω and a non-negative averaging function $\rho_\theta(\zeta)$ that satisfies $\int_\Omega \rho_\theta(\zeta)\, d\zeta = 1$. The choice of $\rho_\theta(\zeta)$ depends on the weight functions $\varpi_i(\theta)$, $i = 1, ..k$. (If $\varpi_j(\theta) = 0$ for the considered parameter θ, we set $\rho_\theta(\zeta) = 0$ for all ζ with $\varpi_j(\zeta) = 0$, since we do not discriminate between parameter points for which the same decision is correct).

5.2 Statistical Decision Functions

AW's most mentioned and most praised work in statistics [254] is his last big take in 1950: the 'Statistical Decision Functions' [Wald (1950) a,] [255] (it has to be mentioned that he already had prepared that subject in early 1939. [Wald (1939) b,] The book, published shortly before his death is outgrown from AW's working period from 1939 to 1949 on the general theory of statistical decision. "*The book marks the end of the first chapter of work on decision functions. It contains many of Wald's results to date, and in some respects the most general results". *[Wolfowitz (1952)] "*However,* [the] *papers* [Statistical Decision Functions [Wald (1949)], and 'Bayes solutions of sequential decision problems' from 1950 [Wald, Wolfowitz (1950)]] *contain results not in the book. The papers* ('Elimination of randomization in certain problems of statistics and of the theory of games' [Wald, Dvoretzky, Wolfowitz (1950)], 'Elimination of randomization in certain statistical decision procedures and zero-sum two-person games' [Wald, Wolfowitz (1951) b,], 'Two methods of randomization in statistics and the theory of games' [Wald, Wolfowitz (1951) a,] and 'Characterization of the minimal complete class of decision functions when the number of distributions and decisions is finite' [Wald, Wolfowitz (1951) d,]) *date after the book and contain entirely new results. The results of are basic for many purposes in decision theory and deserve some mention. They were obtained in January, 1948, in connection with work on the optimum character of the sequential probability ratio test* ['Optimum character of the sequential probability ratio test' [256], [Wald, Wolfowitz (1948)]*, but because of various delays were not published until much later.".* [Wolfowitz (1952)] In the book AW formulated statistics on the basis of Decision Theory, concretely spoken he generalized and unified the theory of estimation and the theory of testing hypotheses. In [Wald (1939) a,] AW outlined a class of decision problems and gave a notation of a loss function. AW not only introduced the minimax principle but also highlights the fact that it is possible to achieve minimax solutions under special restrictions by using Bayes solutions. The use of Bayes solutions, provides a method for solving specific decision problems. But it remains as main problem to determine a Bayes solutions explicitely. In the 1947 paper 'Sequential confidence intervals for the mean of a normal distribution with known variance' AW attacks that problem in the case of estimating the mean of a normal distribution with known variance and shows (jointly with Charles Stein) that the usual nonsequential procedure provides confidence intervals of fixed length and confidence coefficients that minimize the maximum expected number of observations. In the 1948 published 'Optimum character of the sequential probability ratio test' AW shows that the Bayes solutions for testing a simple hypothesis against a simple alternative are, when the cost per observation is constant, SPRT's. In the 'Asymptotic minimax solutions of sequential point estimation problems' [Wald (1951)] (posthumously published in 1951) AW takes the maximum likelihood method and obtains sequential solutions for estimation problems, which are asymptotically minimax.

Not all of the later definitions get noted by AW in their final form, also elements like the complete class concept and so forth are missed out in 1939. But the paper shows a first sketch of a unified general theory of decision along with the mathematical tools. AW stood at the beginning of a deep statistical understanding to construct such a theory, but had to get familiarized more itensive. He still had a longer way to go and returned to that not until 1946. Since that very year AW published a bunch of papers on the subject and bundled these in his last book 'Statistical Decision Functions' [Wald (1950) a,], one year before his death. His last paper concerning the decision theory appeared in 1952.[257] Titled ‘Basic ideas of a general theory of statistical decision rules‘ it outlines the topic exemplified through a finite number of decisions. One year before AW examined in three papers[258] the common disturbing problem that at each step of the decision process one has to choose between various decisions based on probabilities, i.e. the final decision is also made at random. He tried to find out when randomization could be omitted. The last of the mentioned papers shows a method how to get an identic result as with randomization when at the start of an experiment one single randomization is made over a class of nonrandomized strategies. In ‘Relations among certain ranges of vector measure‘ and ‘Elimination of randomization in certain statistical decision procedures and zerosum two-person games‘ AW covers a finite parameter space and atomeless distributions dispensing with randomization.

Already in 1949 AW had published the paper (sponsored by the Naval Research) ‘Statistical Decision Functions’ in ‘The Annals of Mathematical Statistics’. There he refers to the 1947 work ‘Foundations of a General Theory of Sequential Decision Functions’ [Wald (1947) d,] where he yet had discussed “*the foundations of a general theory of statistical decision functions, including the classical non-sequential case as well as the sequential case*“. [Wald (1949), p165] A couple of the assumptions in [Wald (1947) d,] is not always the case in conventional statistical problems. In the 1949 paper AW weakens considerably several of the former assumptions in order to reach most “*of the statistical problems treated in literature*“. [Wald (1949), p165]

5.2.1 A quick survey of decision theory

To sketch the ‘Statistical Decision Functions‘ [Wald (1950) a,] we assume a family of distributions $f(x,\ \theta)$ to be given on a sample space X. The unknown parameter is denoted by $\theta \in \Theta$. The decision d we are allowed to take comes from a (measurable) decision space D. Each decision d is associated with a nonnegative loss function $W(\theta, d)$ that measures the loss we put up with in our decision d, given θ as actual parameter.

We consider the term of a '(randomized) decision function' as a function $\delta : X \rightarrow \Pi(D)$, where Π(D) is the set of all probability measures on D such that for each sample $x \in X$, $\delta(x)$ is a probability distribution on D, i.e. for any subset $E \subset D$ $\delta(x)\,(E)$ is the probability that our taken decision d is a element of E. We derive from the loss function $W\,(\theta, d)$ the risk function $r(\theta, \delta)$. It is defined as the expected loss when we assume a decision function δ when θ is the actual parameter value:

$$r(\theta, \delta) = E_\theta(E_{\delta(x)}\,(W(\theta, d)|\, x)) = \int\int W(\theta, d)\, \mathrm{d}\delta(x)\,(d)\, f(x, \theta)\, dx\,.$$

The risk function is the expected value of the total loss. By nature it depends upon the statistical decision function adopted.

On the basis of the risk function we make decisions. Different decision functions in general have different risk functions. The next quesion is how to compare two different decision functions δ_1 and δ_2. If $r\,(\theta, \delta_1) \leq r(\theta, \delta_2)$ for all θ, δ_1 is said to be at least as good as δ_2. δ_1 is defined as to be uniformly 'better' than δ_2, if $r(\theta, \delta_1) < r(\theta, \delta_2)$ for all θ (whereby the strict inequality must hold for at least one θ). A decision function δ is called 'admissible' if there is no better decision function. A class C of decision functions is called 'complete' (essentially complete) if for any decision function δ‘ that doesn't lay in the class C there exists a decision function in C which is uniformly better than (at least as good as) δ'. From that definiton follows that the search for a 'best‘

decision function can be restricted to a complete class. Hence that concept is of a pretty importance. However, the quest for the 'best' decision function out of a complete class needs a further criterion.

Now we assume a distribution (a probability measure) π on Θ as given. A Bayes solution with respect to π is defined as a decision function δ_B such that the expected risk $\int r(\theta, \delta)\, \mathrm{d}\pi(\theta)$ will be minimized. The importance of Bayes' solutions for AW laid in the fact that the totality of all Bayes solutions (or a class derived from this totality) constitutes (under certain circumstances) a complete or essentially complete class. A minimax solution is a decision function δ_M for which the maximum risk $\sup\limits_{\theta} r(\theta, \delta)$ is a minimum. A least favorable a priori distribution π_0 is one for which the term $\inf\limits_{\delta} \int r(\theta, \delta)\, \mathrm{d}\pi(\theta)$ is a maximum. The decision theory tries to find minimax solutions. To that end different criteria for the existence of a minimax solution have been given. Under special conditions (that are mostly of a topological nature), the class of Bayes solutions is complete and a minimax solution is a Bayes solution with respect to a least favorable (prior) distribution. That shows the importantance of Bayes solutions in the decision theory, even if a prior distribution in the statistical sense does not exist (because μ is not random) or is unknown to the statistician.

We state one of AW's results in that concern [Wald, Wolfowitz (1949)]:

$W(\theta, d)$ should be bounded. Then the space D can be endowed with a metric (whereby 'distance' is defined as follows: $r(d_1, d_2) = \sup\limits_{\theta} \left| W(\theta, d_1) - W(\theta, d_2) \right|$). If D is compact, then there exists a minimax solution δ_0 and to each prior distribution there exists a corresponding

Bayes solution. Furtheremore if $\pi_0$is a least favorable prior distribution, then δ_0 is a Bayes solution with respect to π_0 .

AW also covers the case when it is possible to dispense with randomized decision functions. That is possible fo example, when Θ and D are finite and the distribution of x is absolutely continuous.

5.2.1.1 Example for a discontinuous weight function

We try to test the hypothesis H:the mean θ of a normally distributed chance variable X with unit variance is equal to 0. Thereby should $d_1$denote the decision to accept H, and $d_2$the decision to reject H. Let $W(\theta, d_i)$ be the weight function equaling 0 whenever a correct decision is made, resp. 1 whenever a wrong decision is made: $W(\theta, d_1) = 0$ if $\theta = 0$ and $= 1$ for $\theta \neq 0$, resp. $W(\theta, d_2) = 0$ if $\theta \neq 0$ and $= 1$ for $\theta = 0$. W is obviously discontinuous.

”*The main results of* [the book ‘Statistical Decision Functions‘ [Wald (1950) a,]] *are existence theorems and complete class theorems”*. [Wolfowitz (1952)]

The main theorem for existence proofs is Theorem 2.15 in [Wald (1950) a,]. [260] Under weaker restrictions AW proved the existence of a minimax solution and of Bayes’ solutions, and under stronger conditions he showed the existence of a least favorable a priori distribution. In both cases, AW delivers complete and essentially complete classes (in terms of Bayes’ solutions). A relatively short and easy-to-read exposition to all this can be found in the posthumous published ‘Basic ideas of a general theory of statistical decision rules‘. “*The statistician who wants to apply the results of* [the ‘Statistical Decision Functions‘ [Wald (1950) a,]] *to specific problems is likely to be disappointed. Except for special problems, the complete classes are difficult to characterize in a simple manner and have not yet been characterized.*

Satisfactory general methods are not yet known for obtaining minimax solutions". Wolfowitz continues: "*If one is not always going to use a minimax solution (to which serious objections have been raised) or a solution satisfying some given criterion, then the statistician should have the opportunity to choose from among 'representative' decision functions on the basis of their risk functions. These are not available except for the simplest cases. It is clear that much remains to be done… . The theory provides a rational basis for attacking almost any statistical problem, and, when some computational help is available and one makes some reasonable compromises in the interest of computational feasibility, one can obtain a practical answer to many problems which the classical theory is unable to answer or answers in an unsatisfactory manner. However, for this purpose a relatively simple exposition would suffice to instruct the reader in the rationale of such procedures and it is unnecessary for him to tackle the mathematical details of the theory. The principal value of Wald's book must therefore be for research workers, and the practicing statistician can probably content himself with a reading of the first, and perhaps parts of the last, chapters*.". [Wolfowitz (1952)] AW had formulated Theorem 2.15 already in 1947. In the 1950 book [Wald (1950) a,] he mentiones the connection of his theorem to one of Krylov and Bogolyubov [Krylow and Bogolyubov (1937)]. That is best described with following theorem of Helly-Bray [Billingsley (1999)]: If the probability measures ξ_i converges weakly against the probability measure ξ_0, and if $\phi(x)$ is any bounded, continuous function, then (*) $\int \phi(x)\, d\xi_i(x) \to \int \phi(x)\, d\xi_0(x)$ (weak convergence). Krylov and Bogolyubov have proved an existence theorem, that states that for each given sequence of probability measures ξ_k on a compact metric space, there exists a probability measure ξ_0 and a subsequence ξ_i such that (*) holds for any bounded continuous ϕ.

In the 1951 paper 'Characterization of the minimal complete class of decision functions when the number of distributions and decisions is finite‘, written jointly with Wolfowitz, the weight function W is allowed to depend additionally on the sequence of observations. The total loss is the sum of the loss due to the decision made and the cost of the observations. A statistical decision function says the statistician at the i-th stage ($i = 1, 2, \ldots$) of the experiment whether or not to step forward. At each step the decision function is a probability distribution of the forewent sample values. Governed by this distribution the actual decision is made by an independent chance mechanism.

5.2.2 The three lines of research

In outlining the development of AW's general decision theory we find three main pathes of research: at first the construction of usable sequential procedures, at second an evaluation of their performance characteristics, and finally the determination of minimax or other optimum solutions of specific decision problems. AW was initiator of these topics. Until today the discourse has never been stopped and shows the significance of AW's primings. In regard to the first topic AW in his attempts to generalize his theory dealt with the a priori problem for the statistician to decide which observations he favors and upon that selection how to formulate the experiment. The class of feasible experiments consists of those which serve the same purpose (in certain sense). AW showed indeed in the 1943 paper ‘On the efficient design of statistical investigations‘ how to choose at various levels of generality the most efficient one. He proved that within a certain class of experimental designs the Latin square is most efficient.

Regarding the general theory, several investigations have been carried out on a possible weakening of the conditions. [Gosh (1952)] [261] AW himself had formulated in the 1949 'Statistical decision functions' a weaker condition for the case of discrete variables than he did later in his book. In 1953/54 A. Birnbaum and M. Sobel worked out complete classes for certain decision processes. [Birnbaum (1954)], [Sobel (1953)] The defect of their complete classes was that they lacked of being really finite. To that regard Wolfowitz introduced in 1951 his concept of a ε-complete class τ of decision procedures. This means that to any procedure $\delta \in \tau$ there exists $\delta' \in \tau$, such that the risk function of δ' never exceeds that of δ by more than ε. [Wolfowitz (1951)] Wolfowitz had proved that finite ε-complete classes exist without any complex conditions.

As indicated in the last section the minimax principle plays a central role. During the last decades several investigations were made on these. In many cases the minimax solutions turn out to be choosen on intuitive grounds [262], in others the resulting procedures are quite unsatisfactory. [263] That led to various modifications of the principle.

5.2.3 Jacob Wolfowitz (1910 - 1981)

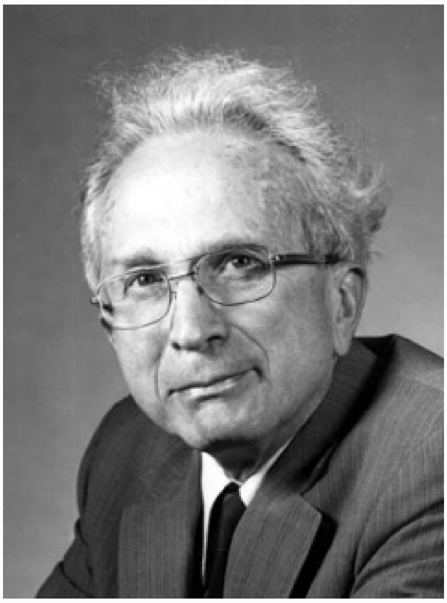


Figure 11. Wolfowitz around 1978. One of Wolfowitz famous sayings was: "*Let us look at what happens in Euclidean n-space. This was good enough for my grandfather and therefore also for me*.". Photo courtesy of http://www.nap.edu/.

As one of the leading US statisticians, Jacob Wolfowitz died on July 16, 1981, in Tampa, Florida, after suffering a heart attack. He was born on March 19, 1910, in Warsaw. In 1933 he received his diploma in mathematics from Columbia University, New York, and the Ph.D. degree from New York University in 1942. At the Columbia University Wolfowitz met AW in 1938. This was the beginning of a lifelong friendship and also a fruitful and unique collaboration. Both produced some fundamental results in theoretical statistics, particularly in the area of decision theory and sequential analysis. The phase with AW was the most decisive in Wolfowitz's research. It ended tragically in 1950 with AW's death. Shortly after that incident, in 1951, Wolfowitz went to Cornell University, NY, where he started his second and also pretty productive period. There he also collaborated, like AW before, with some of his students (like J. Kiefer). He succeeded to extend the frontiers opened up by AW in the areas of decision theory and sequential analysis. Kiefer and Wolfowitz made important contributions in new fields such as stochastic approximation, or the design of experiments. A discussion of his contributions can be found in the 'Selected Papers'. [Wolfowitz (1980)]

J. Wolfowitz was one of the first who realized Shannon's foundational power in information theory. In 1957 he presented in 'The coding of messages subject to chance errors' his own approach to coding theory. He authored over 20 papers and articles on coding theory (one coauthored with J. Kiefer and four with R. Ahlswede). Compare to that his book 'Coding Theorems of Information Theory'. [Wolfowitz (1978)] This meanwhile is a true classic in information theory. Ahlswede writes "*... the progress in multi-user information theory during the last decade is hard to imagine without the use of* [Wolfowitz] *'typical sequences'...*". [Ahlswede (1981)] With the age of 60 Wolfowitz went to the University of Illinois and retired there in 1978. The final decade of his life he applied himself to the method of maximum probability estimators and to the asymptotic theory of estimation. Among his numerous honors (e.g. a fellowships of the Econometric Society and the International Statistics Institute) he was elected as Wald Lecturer of the IMS (Institute of Mathematical Statistics, Ohio)

5.2.4 The link to the Game Theory

There is a strong link between Statistical Decision Theory and Game Theory. The statistical decision problem is a two-person zero-sum game between the statistician and the nature. The strategies of the statistician are the decision functions, the strategies of nature are all parameters $\theta \in \Theta$. The risk function

$r(\theta, \delta)$ finally is the pay-off function of nature. Mixed strategies of the statistician are the decision functions, and mixed strategies of nature are the prior probability distributions. While game theory is in a strict sense not statistical it is yet motivated partly by AW's theory of decision functions. AW became interested in the theory of games on reading [Neumann (1944)] and recognized its connection with his own work. The main ideas of his own theory were already present 5 years before in his 1939 paper on decision theory. Specific statistical elements of AW's theory go far beyond Game Theory.

AW was the first to prove in 'Generalization of a Theorem by v. Neumann Concerning Zero Sum Two Person Games' [Wald (1945) a,] that, if one player of a zero-sum two-person game possesses only a finite number of pure strategies, the game is determined. This result has long since been generalized by AW in 'Foundations of a general theory of sequential decision functions' [Wald (1947) d,] in 1947, where he needed a similar theorem for his theory of decision functions. AW's most general result on the determinateness of a game is Theorem 2.23 in his book. [Wald (1950) a,]

In his book 'Statistical Decision Functions' AW used decision functions as outcome at random at each stage of the experiment. That means, after each observation a random experiment is made to determine the next decision. Another method of randomization is to randomize once for all at the beginning of the sequence of observations and then to proceed in a nonrandom manner. In 'Two methods of randomization in statistics and the theory of games' [Wald, Wolfowitz (1951) a,] the equivalence of the two methods under rather general conditions is shown. In 'Elimination of randomization in certain statistical decision procedures and zerosum two-person games' [Wald, Wolfowitz (1951) b,] it is proved, inter alia, that when the number of decisions and possible distribution functions is finite, randomization can be eliminated if the distributions are continuous. At least in this case the role of randomization is to break up “*atoms*“ of probability. The proof rests on a general measure theoretic result proved in ‘Relations among certain ranges of vector measure‘ [Wald, Wolfowitz (1951) c,], an extension of Lyapunov's theorem. In ‘Characterization of the minimal complete class of decision functions when the number of distributions and decisions is finite‘ [Wald, Wolfowitz (1951) d,] from 1951 one can see clearly the intuitive basis of AW’s theorem that the totality of Bayes solutions is complete. Characterizations of admissible solutions are given there.

6

How Economics became a Science

Being only since a few years in this country, I do not feel competent in recommending foreign scientists in this country. But I do believe that if any young man would be a desirable addition to the imposing scientific ... , then it would certainly be Dr. Wald.

—K. Menger in his testimonial for AW, R. Wald collection, Chicago, Appendix I. 1.1

6.1 Columbia, New York

AW's econometric work meanwhile became known in the US, and we write the year 1937 when he was invited by Alfred Cowles to become a fellow of the Cowles Commission for Research in Economics. The events in Austria made it impossible to stay any longer. AW was dismissed from the Business Cycle Research Institute, went to Romania and from there into the US This was a great luck, as almost all his relatives in Romania got murdered by the Nazis. Only AW's brother Hermann reached the U.S after the war (cp. Table 29). During the summer of 1938, AW had arrived. Although in the US, AW had still problems to be able to stay permanently. He continued to hope for a fixed employment. Hotelling turns the situation a bit and offers him a fellowship. However AW conceals from Hotelling his problem with the Rumanian visa:

Table 20. Letter to Morgenstern: Princeton, 20. 09. 1938, KM Archive, Durham, Box 10

Dear Professor!

Many thanks for your friendly telegram. I told Hotelling in a letter last Wednesday that I am more than pleased to accept the position. I did not mention the question regarding the visa to him. I prefer to wait for your letter you announced in your telegram. It's also better to discuss this question with Hotelling personally. Maybe Hotelling can make it possible that I will teach there to get the visa. In the meantime, the solution of this question has become more urgent. An order hast been released by the Rumanian government that each citizen who stays abroad has to have the pass checked every 3 months by the consulate. Under certain circumstances, it can happen that the consulate turns down to issue the visa and you have to return to Rumania. I will most likely arrive in New York on 1st October. Will you also be in New York soon? I would be more than happy to meet you there. I hope to receive the letter you announced to me soon.
Thank you and best regards. Your A. Wald

Lieber Herr Professor!

vielen Dank für Ihr freundliches Telegramm. Ich habe noch vorige Woche Mittwoch Hotelling geschrieben, dass ich die Stelle gerne annehme. Über die Frage des Visums habe ich ihm vorläufig nichts erwähnt. Ich möchte erst Ihren Brief, den Sie in Ihrem Telegramm angekündigt haben, abwarten. Es ist auch besser, diese Frage mit Hotelling persönlich zu besprechen. Vielleicht wird Hotelling ermöglichen können, dass ich auch unterrichte dort, um das Visum bekommen zu können. Inzwischen ist die Lösung dieser Frage noch dringender geworden. Es ist nämlich eine Verordnung der rumänischen Regierung erschienen, dass jeder Bürger der im Ausland sich aufhält, muss sich 3-monatlich beim Konsulat melden und seinen Pass Visieren lassen. Das Konsulat kann unter Umständen das Visum verweigern und dann muss man nach Rumänien zurück. Ich werde voraussichtlich am 1. Oktober in New York ankommen. Werden Sie nicht demnächst in New York sein? Es würde mich riesig freuen, wenn wir uns treffen könnten. Ich hoffe, Ihren angekündigten Brief bald zu bekommen und bin mit bestem Dank und herzlichen Grüssen. Ihr sehr ergebener A. Wald.

Table 21. Cable from Morgenstern to AW: Princeton, 20. 09. 1938, KM Archive, Durham, Box 1(

Dear Dr. Wald:

I would have written to you earlier, if I hadn't have to go to hospital suddenly. I've been released today and just wanted to let you know that I am very pleased to receive Hotellings offer. In New York, you have a great diversity of different options and I will do my best to find an easy way to get your visa. I do think it is possible indeed. It's better to talk things through when we actually meet instead of saying it in a letter.

I hope you will visit me soon and would like to suggest that you come to Princeton on 1st October and be my guest over Sunday. The best is to take the train from Pennsylvania Station at 4p.m. (Daylight Saving time!) and you'll arrive 57 min. later. We will then have the chance to talk thoroughly about your work you've done so far; I myself have an interesting problem for you. Let's discuss this verbally.
With best regards.

Lieber Herr Doktor Wald:

Ich hätte Ihnen schon längst geschrieben, wenn ich nicht ganz plözlich in ein Spital hätte gehen müssen. Heute bin ich den ersten Tag wieder frei und will Ihnen nur schnell sagen, dass ich mich über das Angebot Hotellings sehr freue. In New York haben sie ganz andere Möglichkeiten und ich werde mich sehr bemühen, dass sich Ihr Visum günstig erledigen lässt. Ich glaube schon, dass es möglich sein wird. Wir können das alles mündlich besser ergrüden, als durch Briefe jetzt.

Ich hoffe, Sie werden mich gleich besuchen und schlage vor, dass Sie Samstag den 1. Oktober nach Princeton kommen und hier über Sonntag als mein Gast bleiben. Sie nehmen am besten den Zug ab Pennsylvania Station um 4p.m. (Daylight Saving time!) und sind 57 Min. später hier. Dann werden wir auch auf die vielen schönen Arbeiten eigehen können, die Sie unternommen haben; ich selbst habe Ihnen ein nettes Problem zu stellen. Alles dies mündlich.

Mit besten Grüssen

It was during fall of 1938, when AW accepted the fellowship of the Carnegie Corporation and went to the Columbia University. Both, invitation and fellowship, have been organised by Harold Hotelling. Hotelling was the senior statistician at Columbia, who forcefully campaigned for AW and his work since yonder had put his name on the map. AW at that time only had little knowledge in contemporary statistics, and had to spent the following months in learning this field, especially by attending Hotelling's lectures. This was not as self-evident as it might sound: "*The great difficulty in learning statistics then was due to the obscure manner in which much of the statistical literature was written*.". [Wolfowitz (1952)] But AW "*brought to statistics a very high degree of mathematical ability and knowledge*.". [Wolfowitz (1952)] In spite of his abstract and theoretical bent, AW in the end became, a full-blooded statistician combining mathematical thinking with practical intuition. [Schneeweiss (2005), p19] [264] AW still was unsure about his future employment at Columbia. Hence he did not hesitate to follow Morgenstern's invitation to Princeton and already hit the road 10 days later. AW had a concrete reason to visit Morgenstern. He still was in need of a permanent job and expressed his wish that Morgenstern (who knew the president of Columbia University) would talk to the president about his case as long as Hotelling would not object to that (see Table 23).

Table 22. AW to Morgenstern: 26. 09. 1938, KM Archive, Durham, Box 9, Folder 5.

Dear Professor,

I just received your letter dated the 20th of September. I will arrive in New York on 30th September and will then come to Princeton on Sunday afternoon (1st October). I will take the train you suggested (4 p.m. from Pennsylvania station). I will leave from here on 27th September and will interrupt my journey for one day to visit Menger. If you want to tell me something before I come to Princeton please send your letter to the following address: c/o Dr. Karol Klein, 1518 First Avenue, New York City.

I look forward very much to seeing you again and remain with best regards
Yours very truly A. Wald

P.S.: Please let me know in any case where I can meet you in Princeton.

Lieber Herr Professor,

ich habe Ihren freundlichen Brief vom 20. September soeben erhalten. Ich werde am 30. September in New York ankommen und komme dann Sonntag (1. Oktober) Nachmittag nach Princeton hinüber. Ich werden den Zug nehmen, den Sie vorgeschlagen haben (4 p.m. ab Pennsylvania station). Ich fahre von hier am 27. September weg und unterbreche die Reise auf einen Tag, um Menger zu besuchen. Falls Sie etwas mir mitteilen wollen, bevor ich nach Princeton komme, so bitte an die Adresse zu schreiben: c/o Dr. Karol Klein, 1518 First Avenue, New York City.

Ich freue mich ausserordentlich, dass wir uns bald wiedersehen werden und bin mit den besten Grüssen

Ihr sehr ergebener A. Wald

P.S.: Bitte schreiben Sie mir auf jeden Fall, wo ich Sie in Princeton treffen kann.

Table 23. AW to Morgenstern: 10. 10. 1938, KM Archive, Durham, Box 9, Folder 6.

Dear Professor
I especially would like to thank you again for your nice welcome. I am feeling very well here. Hotelling is a very nice person and an important scientist. I look forward very much to working with him. I already discussed the matter with my visa with him. He tries very hard to get a teaching position for me at Columbia. He already talked to them in this issue. One dean (of whom I did not understand the name) thinks positively about it, but the other dean from Graduate School creates problems. Hotelling will do everything to get this through. It would be favourable if you could write to Hotelling telling him that you know the president at Columbia and that you would be willing to talk to him if Hotelling wouldn't mind.
I was invited to Montain Labes at Hotelling's place on Sunday where I also met Willes. How are you? You are surely fully occupied with your lectures. When do you come to New York? Menger will be here at the end of October. It would be very nice if you could also be here again.

With best regards
Your very truly A. Wald

My address: Fayerweather Hall (room 509), Columbia University

Please let Hotelling know how the confirmation letter to the University must be written. He asked me whether it is allowed for a temporary visitor to accept a University position? I reckon that the University offers me the position first, I get the Non-Quota-Visa and only then I can accept the position. I would be very grateful if you could soon write to me as this is a very urgent matter to me.

Lieber Herr Professor,

vor allem möchte ich nochmals herzlichst danken für die liebe Gastfreundschaft. Hier geht es mir sehr gut. Hotelling ist ein besonders netter Mensch und ein sehr bedeutender Wissenschaftler. Ich freue mich sehr, dass ich mit ihm zusammenarbeiten kann. Meine Visumsangelegenheit habe ich mit ihm bereits erwähnt. Er bemüht sich sehr, mir eine Teaching position am Columbia zu verschaffen. Er hat bereits in dieser Angelegenheit vorgesprochen. Der eine dean (den Namen habe ich nicht verstanden), ist günstig eingestellt, aber der dean der

Graduate School macht gewisse Schwierigkeiten. Hotelling wird alles tun, um die Sache durchzusetzen. Es wäre vielleicht günstig, wenn Sie Hotelling schreiben würden, dass Sie den Präsidenten am Columbia kennen und dass Sie bereit wären, mit ihm zu sprechen, falls Hotelling dies für geeignet hält.

Sonntag war ich nach Montain Labes zu Hotelling eingeladen, wo ich auch Willes angetroffen habe. Wie geht es Ihnen? Sie werden sicherlich mit Ihren Vorlesungen sehr beschäftigt sein. Wann kommen Sie nach New York? Menger wird Ende Oktober hier sein. Es wäre sehr schön, wenn Sie auch abermals hier sein könnten.

Mit herzlichen Grüssen

Ihr sehr ergebener A. Wald

Meine Adresse: Fayerweather Hall (room 509), Columbia University

Bitte schreiben Sie dem Hotelling, wie die Berufung von der University abgefasst sein muss. Er hat mich nämlich gefragt, ob es gestattet ist, dass ein temporary visitor eine Universitätsstelle hier annimmt? Ich stelle mir die Sache so vor, dass die Universität bloss ein Angebot macht, woraufhin ich das Non-Quota-Visum bekomme und erst dann die Stelle annehmen kann. Ich wäre Ihnen dankbar, wenn Sie sobald als möglich schreiben würden, da die Angelegenheit mir ziemlich dringend ist.

Already from 1939 on, AW started with his own work on statistics and soon developed first ideas on decision theory (in his opinion an extension of Neyman-Pearson test theory) and a steady stream of papers came from his pens for the following years. It "... *was in that academic year* [1938-39] ... [when] *probably his most important paper* [the 'Contributions to the theory of statistical estimation and testing hypotheses'] *was written at a time when his knowledge of statistics was rather limited*.". [Wolfowitz (1952)] This paper was written before AW had indeed any detailed knowledge of modern statistics. Already here he designed a common approach to the two main problems of statistics: estimation and hypothesis testing. His approach was principially a decision theoretic one, although his 'Decision Theory' still laid in future. But AW had integrated the main concepts of decision theory, loss and risk function, Bayes solution, minimax solution, admissibility [265] and so on. But he not always named these like we do today. Astonishing enough AW operated in his paper with Bayes solutions for a priori distributions, what was quite unusual at that time. [266] AW used Bayes solutions "*purely as a mathematical tool and without invoking any objectionable statistical connotations*". [Wolfowitz (1952)] But the paper went almost completely unnoticed, because only few statisticians had the mathematical competence to read AW's paper. In a review in the 'Zentralblatt für Mathematik und ihre Grenzgebiete' in 1941, the following text can be found: "*Es ist gerade der grosse Fortschritt gewesen, dass J. Neyman und E. S. Pearson im Gegensatz zu Th. Bayes ohne zusätzliche Annahmen ausgekommen sind. Die Einführung der beiden Hilfsfunktionen* [i.e. the weight function and the a priori distribution function] *wirkt demgegenuber wie ein Rückschritt. Die Sätze des Verfassers bleiben leere Theorie, die kurzen Beispiele sprechen keineswegs für die praktische Bedeutung seiner Ansätze"* ("*It was the great advance that J. Neyman and E. S. Pearson managed without additional assumptions, compared to Th. Bayes. Against that, the introduction of the two auxiliary functions seems like a setback. The author's theorems remain empty theory, the short examples indicate in no way the practical meaning of his ansatz.*").

But AW's main problem intractably stayed. Only laborious AW came forward with his visa problem. In December 1938 he somewhat eased told Morgenstern the prolongation of his passport until May 1940. But his visa issue remained open:

Table 24. AW to Morgenstern: 17. 12. 1938, KM Archive, Durham, Box 9, Folder 6.

Dear Professor
It's been along time since I've heard something from you. I don't even know if you received my letter of October in which I asked you to write to Hotelling. I hope you are well and that you have a pleasant and interesting working field there.
I'm very happy here. Hotelling is a very nice and convenient person whom it is a pleasure to work with. I have already written quite a few working papers which I will soon send in for publication.
In Detroit, I will give two lectures, the first at the Economic Society and the other one at the Institute for Mathematical Statistics. As I learn from the programme, you will also be in Detroit. I look forward very much to seeing you again.
The matter with my visa is as follows: My passport has been prolonged until 1 May 1940 and I hope that also the stay permit will be prolonged accordingly. Prof. Hotelling tried to get an instructorship for me, but it seems not to be possible these days. I hope to find something by autumn.

I wish you some nice and pleasant bank holidays and hope to hear from you soon.

Yours very truly A. Wald

P.S.: I just noticed that I have not returned the fomulars regarding the regulations for imigration. Please excuse the delay and find the documents enclosed.

Lieber Herr Professor,

es ist schon sehr lange her, dass ich von Ihnen nichts gehört habe. Ich weiss auch nicht, ob Sie meinen Brief in Oktober, in dem ich sie bat, Hotelling zu schreiben, erhalten haben. Ich hoffe, dass es Ihnen gut geht und dass Sie dort einen angenehmen und interessanten Arbeitskreis haben.

Ich bin hier sehr zufrieden. Hotelling ist ein sehr netter und angenehmer Mensch, mit dem man gut zusammenarbeiten kann. Ich habe bereits eine ganze Reihe von neuen Arbeiten geschrieben, die ich demnächst für Publikation einreichen werde.

In Detroit werde ich zwei Vorträge halten, einen beim Econom. Society und einen beim Institute for Math. Statistics. Aus dem Programm ersehe ich, dass Sie auch in Detroit sein werden. Ich *freue mich sehr, dass wir uns dort wiedersehen werden.*

Meine Visumsangelegenheit ist jetzt die folgende: mein Pass wurde verlaengert bis zum 1. Mai 1940 und hoffe, dass auch die Aufenthaltsbewilligung wird entsprechend verlaengert. Prof. Hotelling versuchte, ein Instructorship fuer mich hier durchzusetzen, aber scheinbar geht es derzeit nicht. Ich hoffe, dass ich bis Herbst etwas finden werde.

Ich wünsche Ihnen recht angenehme Feiertage und hoffe, bald von Ihnen etwas zu hören.

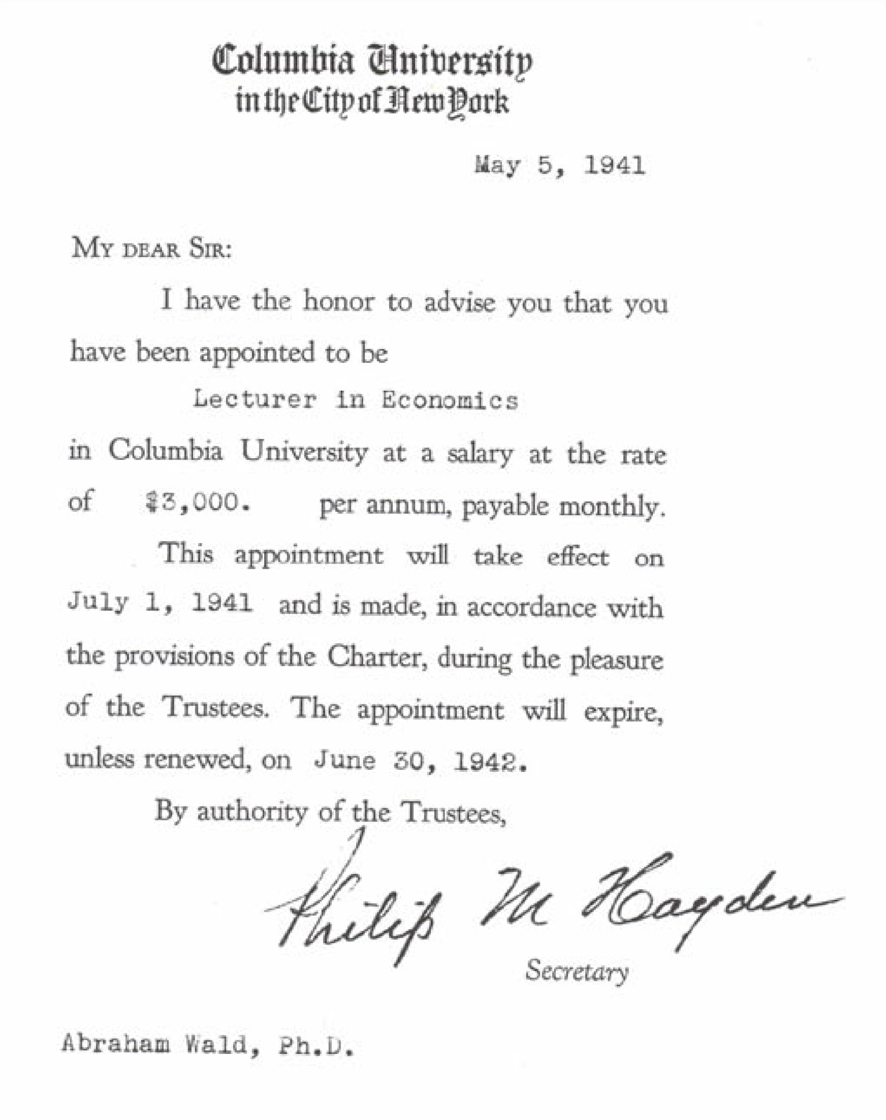
Mit den besten Gruessen

Ihr sehr ergebener A. Wald

P.S.: Ich merke soeben, dass ich die Drucksachen über Einwanderungsvorschriften aus Versehen nocht nicht zurueckgeschickt habe. Bitte entschuldigen Sie dafür, ich lege sie hiermit bei.

6.1.1 Professor of Economics

The presence of Hotelling and AW made Columbia a famous place for mathematical statistics. Already as fellow AW started with own lectures and mastered the new challenge. Wolfowitz, at this time his student, writes : "... *his lectures at Columbia in 1939-40 were noted for their lucidity and mathematical rigor*". [Wolfowitz (1952)] R. J. Brookner, later obtaining his Ph.D with AW, took notes of the lectures (they can be found in the Menger archive, Duke University, but since 2008 also at books.google.com) which were reproduced and circulated among the students. The lectures 1939-40 mark the beginning of AW's career as teacher. So it was no big surprise that his fellowship ended in 1941 and he was made at first Lecturer in Economics (July 1941) and exact one year later Assistant Professor of Economics. That finally ended his long endouring search for a fixed employment and made it possible for him to get a permanent residence authorisation for the US.



Columbia University
in the City of New York

May 5, 1941

My dear Sir:

I have the honor to advise you that you have been appointed to be

Lecturer in Economics

in Columbia University at a salary at the rate of $3,000. per annum, payable monthly.

This appointment will take effect on July 1, 1941 and is made, in accordance with the provisions of the Charter, during the pleasure of the Trustees. The appointment will expire, unless renewed, on June 30, 1942.

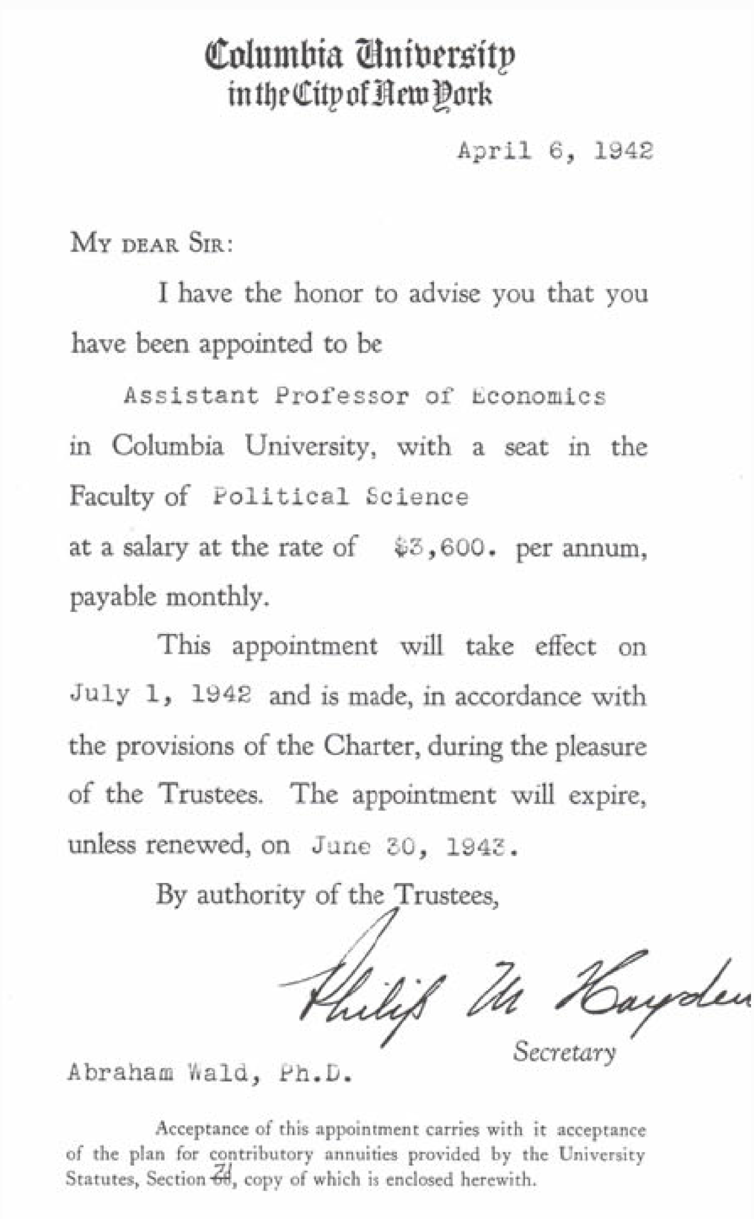
By authority of the Trustees,

Philip M Hayden

Secretary

Abraham Wald, Ph.D.

Figure 12. AW's appointment to Lecturer in Economics at Columbia, Robert Wald, private Collection, Chicago



Columbia University
in the City of New York

April 6, 1942

MY DEAR SIR:

I have the honor to advise you that you have been appointed to be

Assistant Professor of Economics

in Columbia University, with a seat in the Faculty of Political Science at a salary at the rate of $3,600. per annum, payable monthly.

This appointment will take effect on July 1, 1942 and is made, in accordance with the provisions of the Charter, during the pleasure of the Trustees. The appointment will expire, unless renewed, on June 30, 1943.

By authority of the Trustees,

Philip M Hayden
Secretary

Abraham Wald, Ph.D.

Acceptance of this appointment carries with it acceptance of the plan for contributory annuities provided by the University Statutes, Section ~~68~~ 24, copy of which is enclosed herewith.

Figure 13. AW's appointment to Assistant Professor of Economics at Columbia, Robert Wald, private Collection, Chicago

6.1.2 AW and Columbias rise to a center of statistics

Although AW meanwhile had developed prolific ideas in statistics (stepwise originating the Decision Theory), he still was professor for economics. In this regard he immediately picked out a classical economic question which circulated for quite a while. It is the problem of estimating a linear equation $\eta = \alpha + \beta\xi$, when the variables x and y are measured with errors δ and ε : $x = \xi + \delta,\ y = \eta + \varepsilon$. The expectation of the errors should be equal to 0. The errors should also be independent of the error free variables ξ and η. AW advanced in the 1940 paper 'The fitting of straight lines if both variables are subject to error' as follows: he subdivided a sample $(x_i, y_i),\ i = 1,\ ...,n$ into two groups and joined the two centers of gravity by a straight line. The subdivision must be independent of the errors and the x-coordinates of the two centers have to differ by a positive (> 0) amount whenever $n \to \infty$. AW not only proved that α and β can be consistently estimated, but also specified a confidence region for α and β when the errors δ and ε are normally distributed. A simple solution for a long-standing problem, solved in a typical AW style, yet often misunderstood. [267]

As the outside recognition increased, AW's career quickly developed. He was promoted to an associate professorship in 1943 and then to a professorship in 1944. With growing fame AW's involuntary lectures notes began to circulate outside of Columbia. Hence "*many of the new generation of American statisticians learned the theory of the analysis of variance from the notes of his course. They are rigorous, accurate, and clear, but some of the proofs are clumsy, and the organization of the notes could be improved. The original lectures were given under great time pressure, and there was no time to search for the most elegant proofs, or to plan the organization of the course long in* *advance. Wald seldom bothered to rework his writings for mathematical elegance or clarity - only new results interested him. Thus the original notes were allowed to stand without alteration, although he would depart from them in his class lectures*". [Wolfowitz (1952)] When Hotelling left Columbia in 1946 (he moved to Chapel Hill as head of the newly founded statistics department), the Institute of Mathematical Statistics meanwhile had also been formed with AW, professor of mathematical statistics, as its president (See Table 25). "*His fame was at its height and students came from all over the world to hear him*". [Wolfowitz (1952)]

Nevertheless AW's econometric work continued. In collaboration with H. B. Mann he published in 1943 the paper 'On the statistical treatment of linear stochastic difference equations'. Therein they proved consistency and asymptotic normality of the Quasi-ML estimator, a result "*which was fundamental to the theory of dynamic simultaneous equation models of econometrics*". [Schneeweiss (2005), p10]

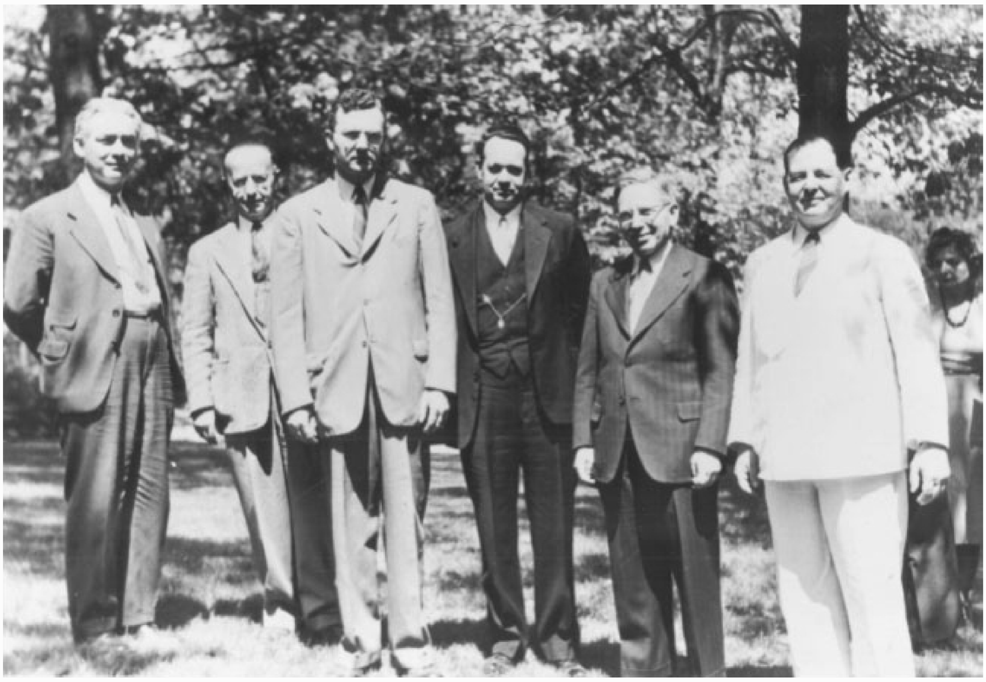
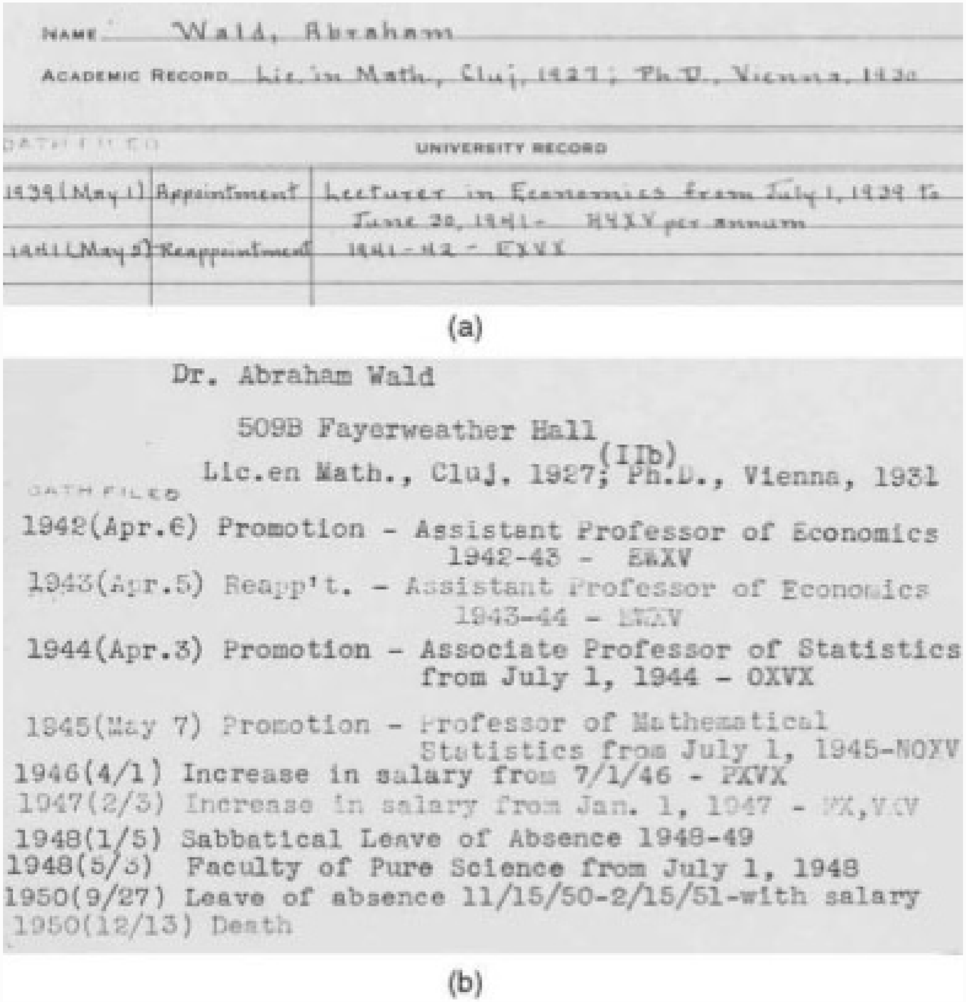


Figure 14. Around 1945. The founding fathers of the Columbia Statistics Institute: Willy Feller, Walter Sheward, Sam Wilks, Paul Dwyer, AW, Harold Hotelling (from left to right). Photo courtesy of Milton Sobel.

Soon after the ‘Contributions to the Theory of Statistical Estimation and Testing Hypotheses’ paper [Wald (1940)], AW gave in the 1940 work ‘A note on the analysis of variance with unequal class frequencies‘ confidence limits for the intraclass correlation coefficient. His method was later labeled ‘Model II’ of the analysis of variance. Wolfowitz describes his ”... *method is forthright and simple*”. [Wolfowitz (1952)] H. Hotelling had confronted AW with the problem. AW presented a generalization of this result in ‘On the analysis of variance in case of multiple classifications with unequal class frequencies‘ one year later. AW’s work on decision functions rested until 1946. [Wolfowitz (1952)] [268] When he started to rework his former approaches he was spurred on by the connection between his theoretical blueprints and the ground-breaking results of 'the Theory of Games and Economic Behaviour'. Especially this book caused a new arousal of a general interest among economists in mathematical economics.



NAME Wald, Abraham

ACADEMIC RECORD Lic. in Math., Cluj, 1927; Ph.D., Vienna, 1930

OATH FILED

UNIVERSITY RECORD

1939 (May 1)	Appointment	Lecturer in Economics from July 1, 1939 to June 30, 1941 - HXXY per annum
1941 (May 5)	Reappointment	1941-42 - EXVX

(a)

Dr. Abraham Wald

509B Fayerweather Hall

(IIb)

Lic.en Math., Cluj, 1927; Ph.D., Vienna, 1931

OATH FILED

1942(Apr.6) Promotion - Assistant Professor of Economics 1942-43 - EXXV

1943(Apr.5) Reapp't. - Assistant Professor of Economics 1943-44 - EXXV

1944(Apr.3) Promotion - Associate Professor of Statistics from July 1, 1944 - OXVX

1945(May 7) Promotion - Professor of Mathematical Statistics from July 1, 1945-NOXV

1946(4/1) Increase in salary from 7/1/46 - PXVX

1947(2/3) Increase in salary from Jan. 1, 1947 - MX,VXV

1948(1/5) Sabbatical Leave of Absence 1948-49

1948(5/3) Faculty of Pure Science from July 1, 1948

1950(9/27) Leave of absence 11/15/50-2/15/51-with salary

1950(12/13) Death

(b)

Figure 15. Columbia University, 1950. AW's personnel card at Columbia

Table 25. The 1948 report of the president of the IMS [Wald (1948) a,]

The last few years have seen a considerable growth of the Institute. The upward trend has continued throughout 1948. The Institute has acquired 126 new members during the year, but this gain is to be balanced against losses due to resignation and suspension for non-payment of dues. The Institute starts the year 1949 with a membership of about 1,100 as against the membership of 1,037 at the beginning of 1948. While the net gain is still substantial, it is not quite as much as hoped for, and this may serve as an incentive for an increased membership drive in 1949. The constantly increasing interest and research activities in statistical theory and methodology are well reflected in our meetings and the publications appearing in the Annals.

The growth of the Institute in the past few years has brought about a considerable increase in its various activities. This manifested itself particularly in the extensive and rich programs of the meetings held during the year 1948. In addition to the usual invited addresses and contributed papers, the programs included a considerable number of symposia on various important subjects such as the theory of games (Berkeley, June; Madison, September), stochastic difference equations (Madison, September), scales of measurement (New York, April), sampling for industrial use (Berkeley, June), etc. The eleventh summer meeting was held in conjunction with the meetings of the American Mathematical Society and the Econometric Society (Madison, September). The eleventh annual meeting (Cleveland, December) was held in conjunction with the American Statistical Association, Econometric Society and Biometric Society. There were also three regional meetings: New York (April), Berkeley (June) and Seattle (November). The Berkeley meeting was held in conjunction with the Pacific Division of the American Association for the Advancement of Science and some of the sessions of the Seattle meeting were sponsored jointly with the Biometric Society.

To facilitate the organization of meetings and arrangements of programs, instead of a single program committee there were three program committees appointed, one for Eastern, one for Mid-Western and one for Far-Western meetings. These committees consisted of the following members. Eastern Committee: W. G. Cochran, C. Eisenhart (Chairman), F. Mosteller, and J. Wolfowitz; Mid-Western Committee: C. C. Craig, H. B. Mann, and A. M. Mood (Chairman); Far Western Committee: Z. W. Birnbaum, M. A. Girshick, P. G. Heel, and J. Neyman (Chairman). To coordinate the work of these three program committees, a coordinating committee was appointed consisting of J. W. Tukey (Chairman) and the three chairman of the three program committees.

This committee was also charged with the responsibility of making recommendations to the Board of Directors as to times and places for future meetings. Another innovation introduced during the past year was the appointment of assistant secretaries in connection with the meetings. S. B. Littauer acted as assistant secretary for the New York meeting, K. J. Arnold for the summer meeting in Madison, Z. W. Birnbaum for the Seattle meeting and W. R. Van Voorhis for the Cleveland meeting. The assistant secretaries were charged with the task of looking after the local arrangements that had to be made in connection with the meetings. The appointment of assistant secretaries proved to be a great success not only in facilitating the necessary local arrangements for meetings but also in relieving the burden on the secretary's office. On the basis of this year's experience, it seems very desirable to continue with this practice in the future.

No Rietz Memorial lecture was given in 1948 in accordance with a decision of the Board of Directors that these lectures should not be given every year. It is planned, however, to have a Rietz lecture for 1949 and the Board of Directors invited J. Neyman to deliver it.

The New Constitution. One of the major events of the year was the adoption of the new constitution at the meeting in Madison. The growth of the Institute in recent years made parts of the old constitution obsolete and the need for a revision was apparent. Our thanks are due to the Committee on Planning and Development which has devoted much time and consideration to the study of the problem and prepared a draft of a revised constitution. M. H. Hansen was chairman of this Committee. Other members were: J. H. Curtiss, W. G. Cochran, W. Feller, J. Neyman, H. W. Norton, F. F. Stephan, J. W. Tukey, and W. A. Wallis. A draft of the new By-Laws was prepared by J. W. Tukey, who acted as a subcommittee of the Committee on Planning and Development.

The growth of the Institute during the past few years has manifested itself also in a constantly increasing number of manuscripts submitted for publication in the Annals. While it is very gratifying to see this upward trend, it rais's some problems of financial nature. At the rate manuscripts are coming in, an expansion of the publication facilities of the Institute would seem very desirable. Increase of the volume of the Annals would, however, mean increased cost and the present financial situation of the Institute could not allow such an additional burden unless some new sources of income can be found. Apart from a possible increase in the cost of printing the Annals, it seems that additional expenditures will be necessary for secretarial help in 1949. It was decided at the membership meeting in Madison that additional funds be raised through the contributions of universities and other organizations with strong interest in mathematical statistics and through the contributions of the members. Appeals for such contributions were sent out and it is hoped that there will be a generous response. The new constitution permits the appointment of responsible associete Editors. This brings up the whole qucstion of editorial set-up and policies. A committee with S. S. Wilks as *chairman was appointed to make a thorough study of the Institute's publication experience and to make recommendations as to publication policies and editorial set-up. Other members of this committee are: W. G. Cochran, W. Feller, M. A. Girshick, J. Neyman, P. S. Olmstead, W. A. Wallis and J. Wolfowitz. The committee gave much thought and consideration to the problems involved and will report to the newly elected officers and Council. The Annals has developed under the leadership of the Editor, S. S. Wilks, to one of the outstanding professional journals. I am sure that I can speak for all our members in expressing the Institute's indebtedness to S. S. Wilks for his untiring and most successful work.*

The problem of classification of statisticians in the Government service is naturally of considerable importance to the statistical profession. A committee consisting of W. E. Deming (chairman) and C. Eisenhart was appointed to make a thorough study of this question with a view to advising the Civil Service Commission. The committee prepared a report in which three main categories of statisticians in Government Service are distinguished: mathematical statisticians, statistical analysts and, data-collecting statisticians. The report was transmitted to the Civil Service Commission with the approval of the Board of Directors. The members of this committee are to be commended for the excellent work they have done in spite of the severe limitation of time allotted by the Civil Service Commission. The work on the problem of classification of statisticians still goes on and a committee of experts consisting of members of the Washington Statistical Society, the Institute of Mathematical Statistics, and the American Statistical Association has been set up to advise the Civil Service Commission on this problem. Our representatives on this committee of experts are: W. E. Deming, C. Eisenhart, M. H. Hansen and S. Weiss.

The advances in numerical computations in recent years has made an enlargement and reorganization of the Committee on Tabulation necessary. Its present members are: R. L. Anderson, C. Eisenhart (Chairman), A. M. Mood, F. Mosteller, H. G. Romig, L. E. Simon, and J. W. Tukey. The objectives of this committee, as outlined by the chairman are: (1) to prepare a comprehensive list of new mathematical tables that would be of value in statistical theory and applications, (2) to assemble an American Collection of "Tables for Statisticians", (3) to prepare a list of mathematical tables of importance in statistical theory and applications to be recommended for inclusion in the proposed National Bureau of Standards volume of "Tables for the Occasional Computer". To implement the program of the committee, the following sub-committees have been constituted: (1) "On Computing Centers" with L. E. Simon as Chairman, (2) "On Ranks and Runs" with A. M. Mood as Chairman, (3) "On Serial Correlations" with II. L. Anderson as Chairman, (4) "On 2 x 2 Tables" with C. Eisenhart as Chairman, (5) "On Order Statistics" with F. Mosteller as Chairman, (6) "On Binomial, Poisson, and Hypergeometric Distributions" with H. G. Romig as Chairman, (7) "On Miscellaneous Tables" with J. W. Tukey as Chairman.

On the recommendation of the membership committee, consisting of H. Scheffé (chairman), C. C. Craig, P. G. Hoel and F. F. Stephan, the following members have been elected as Fellows: J. Berkson, E. L. Lehmann, E. J. G. Pitman, H. E. Robbins and C. M. Stein. The members of the finance committee for 1948 were P. S. Dwyer (chairman), C. F. Roos, L. A. Knowles and T. N. E. Greville. The Nominating Committee for 1948 consisted of W. Bartky (chairman), C. C. Craig, J. F. Daly, H. A. Freeman, E. L. Lehmann and W. G. Madosv. The committee nominated J. Neyman for President, J. L. Dobb for President-Elect and 24 Council members for the 12 positions to be filled. In accordance with the provisions of the new constitution, the Nominating Committee for 1949 has also been appointed. The members of this Committee are: W. G. Cochran (Chairman), M. H. Hansen, H. B. Mann, A. M. Mood and H. G. Romig. The Board of Directors has been exploring the possibilities for a closer cooperation with our colleagues abroad and for making foreign statistical publications more easily accessible to our members. In particular, there has been correspondance with Professor E. S. Pearson, ManagingEditor of Biometrika, on the question of a possible reduction of the subscription rate of Biometrika for our members. As a result of these discussions, Professor Pearson offered certain reductions, provided that a sufficient number of subscribers can be secured. Detailed information on this was contained in a memorandum of the Secretary, P. S. Dwyer, in the November mailing to the membership. It is hoped that many of our members will make use of this opportunity. With the new constitutions of the American Statistical Association and the Institute of Mathematical Statistics adopted, the way is cleared for the consideration of possible federation plans of the various statistical organizations by the Inter-Society Committee on Federation. J. H. Curtiss and P. S. Olmstead continued to serve as our representatives on the aforementioned committee during 1948. W. Feller was our representative on the Policy Committee for Mathematics, and F. C. Mosteller and S. S. Wilks represented the Institute on the Joint Committee for the Development of Statistical Application in Engineering and Manufacturing. W. Bartky was reappointed for a three-year term as our representative to the Division of the Physical Sciences of the National Research Council, and H. Hotelling was our representative to the American Association for the Advancement of Science.

In conclusion, I wish to thank all committee members and others who participated in the work of the Institute during the past year. The heaviest burden falls, of course, on the Secretary and it is hard to express adequately our appreciation for his unselfish efforts and devotion. The smooth and efficient conduct of the affairs of the Institute is largely due to his work.

ABRAHAM WALD

President, 1948
December 31, 1948

6.1.3 The 1941 lecture cycle: a pleading for exactness and abstraction

In 1942 AW's teaching cyclus (started in February 1941) had been printed at the University of Notre Dame titled 'On the principles of statistical inference'. Already in his introductory words AW clarifies the validity of axiomatisation and abstraction, i.e. he emphasizes the step from real phenomena to the model, which itself may not have a direct reference towards a real context, but however is able to describe natural phenomema. It is remarkable that AW indicates points, lines etc. as examples for such abstract elements, on which the geometry is actually constructed. It is highly visible that AW's former occupation with geometry formed his addicition to clear and abstract foundational concepts. He quasi imports that approach into statistics. In that mind he remarks that the term of probability, as a fundamental one in statistics, lacks also clarity. He shows its interpretations with different schools (Richard von Mises, Andrei Nikolaevich Kolmogorov and so on) and criticizes these. Among other things AW points out that expressions like 'approximately' and so on are used without to specify a precise definition. To demonstrate his critics AW cites the sentence: 'In the long run the Event e happens approximately with the probability p', whereby nothing is stated about the meaning of 'long' or 'approximately'. It is evident that such 'intuitive' definitions are inadequate for AW. He calls for an axiomatisation of the statistics.

In the further progress of his lecture AW presents the terms of probability density and probability distribution. He stresses that the usually used distributions represent only a small subset of all possible distributions, which correspond to natural phenomena. In the following he discusses the term of a statistical hypothesis. He points out that there is a priori no evidence which

which distribution should be taken. In the end of his introduction AW presents the most general and most strict formulation of the statistical inference. AW asks a set-theoretic question towards a definiton: which of the elements h of a set of hypotheses H is to accept on the basis of a sample, if each h corresponds to a distribution d from an appropriate subset of distributions D (of the class of allpossible distributions)? At this point AW's upcoming decision theoretical ansatz becomes pretty visible. In the most general formulation this problem deals with the selection of a certain element out of a possible infinite set with the help of a finite number of observations (measurements, data and so on). The decision has been done such that the 'real content' is approximated as good as possible. One question hereby is the already mentioned selection of h out of H. Another question deals with the selection of the best strategy (in terms of the game theory) on the basis of a finite number of observations of the opponent's strategy.

The second part of AW's lecture cycle treats the theory of statistical hypotheses set up by Neyman and Pearson. [Neyman, Pearson (1933), (1936), (1938)] AW stresses that this theory does not constitute the most general formulation of inference, as mentioned above, but is a special case of it. AW describes that special case in detail and ends up with the question how to test a specific hypothesis. He analyzes which subset of his most general formulation of inference the method of Neyman and Pearson constitutes. AW depicts the connection of this statistic procedure with operations on sets, describing the inference problem as a problem of the selection of a hypothesis from a possible infinite set of hypotheses assisted by a finite number of samples. He demonstrates that the difficulty often consists in the fact that one does not have any a priori information about the true distribution, but the accepted hypothesis already implicates a certain distribution, that should be approved by samples. The problem is that parameters like the mean, the variance and so on, not only depend in their value from the unknown distribution, but also are determined in their existence on it. As an example we mention here the typical case of the measuring a population. Typically the mean and the variance are calculated by a sample. For that we use usually equations, which are only valid if the population would be normally distributed. However the distribution of the sample and that of the population can deviate from each other pretty much. Furthermore it is possible that the distribution of the sample and the population are alike, but both are no normal distribution. Here closes the circle. In estimating the parameters a model is already accepted that actually should be determined. Even a successful test of a hypothesis is no mathematical evidence that the accepted model is the right choice.

In the third part of his lecture AW describes the problem of the statistically supported estimation according to the theory of R.A.Fisher (see [R.A. Fischer (1921), (1925), (1935)]). Again he clarifies that the concept is vague, and presents a sharp definition, which is based on the minimum variance of the value that has to be estimated. His definiton is valuable for practical applications, since AW avoids an 'intuitive' weighing out concerning the quality of the estimation. Nevertheless he also mentions that although an estimation with small variance has a high probability to lie near to the true value the opposite of that may not be true. There are estimations, which are close to the true value but nevertheless possess a high variance. At that point AW's set-theoretical view becomes visible again, because he considers the estimated value as a random variable, with its own distribution, variance and so on, whereby, under certain conditions, the distribution of the sample is mapped on the distribution of estimate. Beyond AW gives a sharp definition of an efficiently estimated value, which already shows some characteristics of his upcoming decision theory.

The fourth part of AW's lecture covers the estimation on the basis of confidence intervals and clarifies terms like the shortest confidence interval, the most efficient confidence interval and the evenly most efficient confidence interval, including the appropriate test. The proposed treatment of the topic by AW again had a strong set-theoretical character, and continued in the fifth part with the introduction of the asymptotic smallest confidence intervals. These are used mostly in cases, if the determination of a critical range is (almost) impossible.

In part six AW discusses the restrictions, which are present in the contemporary approaches, and prevent on the one hand the construction of a complete theory and on the other their full applicability. So the theories of Fisher, Neyman and Pearson ([Neyman, Pearson (1933), (1936), (1938)], [R.A. Fisher (1921), (1925), (1935)]) were limited already by the fact that they only treat a test of a single hypothesis, and that this test was made exclusively over point intervals. An additional restriction was the fact that solely the family of the k-parametric distributions got examined, what brought about that e.g. two normal distributions with different means were comparable, but not a normal distribution with a binomial one. There is additionaly a multitude of problems in practice, which neither are caused by estimation nor by the testing of a hypothesis. For example the mentioned question how to classify an individual in one out of three specified classes is such a problem. AW outlined the general framework (at that time not yet existing) of a theory of statistic inference, and all the requirements, which it will cover on the pure theoretical side but also related to applicability. It is characteristical again that AW develops his ideas on fundamental set-theoretical considerations, from what a high generality would follow. In his lecture cycle it is well recognizable, how important for AW the completeness of the future statistical theory was. He never lost sight of the possibilities for applications apart from the pure mathematical considerations.

6.1.4 AW's quest for von Neumann

AW had never been satisfied with the minimax criterion. The concrete problem was to find a criterion for choosing a decision function from a complete class Ω of decision functions. AW asked himself how it would be possible to have a criterion for choosing a member of the complete class Ω despite no information would be available about which distribution of Ω is the true one. One solution he supposed could be found in an admissible minimax decision function as the right choice. The advantage would be to have a constant risk function (under certain conditions) and to be independent of any a priori distribution on Ω. But he was not really sure about his idea.

In his 'Statistical Decision Functions' AW states: "*Nevertheless, since Nature's choice is unknown to the experimenter, it is perhaps not unreasonable for the experimenter to behave as if Nature wanted to maximize the risk.*". [Wald (1950) a, p27] AW however tempers that statement by mentioning: "*But, even if one is not willing to take this attitude, the theory of games remains of fundamental importance ... .*" AW was searching for other criteria (his last joint work with J. Wolfowitz concerns this problem). He was discontent with the results he had found on the problem. AW also had no faith in the benefit of the minimax criterion. He hoped to find a way out with the help of the ingenious J.v. Neumann. As so often he tried to reach Neumann with Morgenstern's help, who gave lectures in Princeton like v. Neumann.

Table 26. AW to Morgenstern, New York, April 6, 1948, KM Archive Durham, Box 9, Folder 6

Columbia University
in the City of New York
Department of Mathematical Statistics

Professor Oscar Morgenstern
Department of Economics
Princeton University
Princeton, New Jersey

Dear Morgenstern,
It has been a long time since I heard from you. I trust that everything is all right with you. I plan to leave for California at the end of May and as you can imagine I am quite busy with preparations for the trip since I shall go with my family and intend to stay there for a whole year. I would very much like to see you before I leave. Could you give me a ring next time when you will be in New York?
I am sorry that my visit to Princeton in February did not materialize. von Neumann sent me a telegram at that time that he could not be in Princeton on the day I planned to come out. He also indicated in the telegram that he will write a letter proposing some other dates, but I haven't heard anything from him so far.

Hoping to see you soon, I am,

Cordially yours,
A. Wald

Table 27. AW to Morgenstern, May 4, 1948, KM Archive Durham, Box 21, Folder 6

Columbia University
in the City of New York
Department of Mathematical Statistics
May 4, 1948

Professor Oscar Morgenstern
Department of Economics
Princeton University
Princeton, New Jersey

Dear Morgenstern,

I wrote you a letter some weeks ago and I wonder whether you received it. We will leave for California at the end of this month and I would like to see you before.
I just wrote to von Neumann in some matter and at the same time I asked him whether I could come out to see him on Tuesday, May 18. I happened to meet him in New York about two weeks ago, and he told me that he would like to discuss some problems with me relating to the minimax theorem in the infinite case. It would be very nice if all three of us could get together. Is Tuesday, May 18, a good time for you? If not, would you get in touch with von Neumann and find out what date would be mutually convenient?

With best regards,

Cordially yours,
A. Wald

Table 28. Morgenstern to AW, May 10, 1948, Private Collection, Robert Wald

Professor A. Wald
Department of Mathematical Statistics
Columbia University
New York 27, N.Y.

Dear Wald:

Thank you for your nice letter of May 4th and please accept my apologies for not having answered your earlier letter, but I knew that von Neumann was in touch with you and I thought, therefore, that the matter had been arranged. If you come here on Tuesday, the 18th, I shall certainly be available and indeed be only to glad to see you. The three of us could have a meeting and there are certainly very many questions I would like to talk over with you. I shall be able then to tell you of a new research project on which I am about to start and which may be of some interest to you.
I am leaving the middle of June myself for Europe and shall be back by September. If you plan to spend some time in Stelton before you go west, please let me know because I would like to drive over so as to continue our talks.

Kind regards,

Cordially yours,
Oskar Morgenstern

Table 29. AW to Morgenstern, New York, June 27, 1947, KM Archive Durham, Box 21, Folder 7

Columbia University
in the City of New York
Department of Mathematical Statistics

June 27, 1947

Dear Morgenstern:

I hope that you will be in Vienna by the time this letter arrives there and that you will find your parents and sister in good health. How does it feel to be back there after having been away for 9 years? Vienna now must be quite different from what it was 10 years ago.
How was your lecture tour? There is no particular news here. My book came out a few days after you left. I asked Wiley to send you a complimentary copy. My brother Hermann is still in Prague waiting for his visa. According to the latest word I had from him, the Consul is willing to give him the visa after he receives some information he requested from The American Legation in Bucarest.
Before you left you kindly told me that you may be able to visit my brother in Prague. I would be very grateful to you could do it, but please make sure first that he is still there. My brother's address: Hotel Maceka, Hastalska Namesti, Prague. Of course, I shall be glad to reimburse you for all expenses you may have.

Please, given my very best regards to your parents and your sister. I still remember well the many pleasant hours I have spent in their house.
I would be glad to hear from you if you can have the time to drop me a line. When do you expect to be back here?

Cordially yours,
A. Wald

P.S.: I shall review the second edition of your book for the Mathem. Reviews.

Figure 16. California, 1949. AW seated left, his wife Lucille, their children Robert and Betty (on the floor), unknown, Lucille's mother and Milton Sobel (from left to right). Photo courtesy of Milton Sobel. Milton Sobel (1919-2002), son of Hungarian immigrants in New York City, was a former student of AW. There he became one of the first Ph.D'.s in Mathematical Statistics in 1951 with AW. Sobel is well known for his contributions (among other things) in decision theory, sequential analysis, reliability analysis well as (statistical) computing. Sobel was later Professor for Statistics at the University of California at Santa Barbara [Sobel (2008)].

6.2 How economics became a science

6.2.1 Introduction

A theme of vital importance shall be focused in following provocative question: **did economics become a science during the 20th century and did it produce useful knowledge?** First of all it is important to define the attributes of a science‘: a science consists of formulated theoretical models and observable facts that confront the models with reality. Websters Dictionary defines ‘science‘ as “*systemized knowledge derived from observation, study, experimentation carried on in order to determine the nature or principles of what is being studied*.”[Gove (1997)] In the case of economics, it’s knowledge (as far as it can be produced) should be systematizable into an observational part and into an hypothetical part which should be refutable through observations. [Bearn (1999)]

The foundational part of a science, in particular its models and theoretical parts, which not only permit insight into phenomena and also contain the potential of forecast, must offer at least the possibility to their examination or refutation. This secures the gained ground and permits at the same time further developments, refinements and extensions, which thereby expand the possibilities for the interpretation of its objects of study. K. Popper’s definition is hereby pretty useful: A theory must be falsifiable and may not contain any metaphysical element. [Popper (1959 (orig. 1934))] If economics is to be considered to be a science, it thus must fulfill Popper’s criterion and abstain from any ‘intuitive judgement’. [Popper (1959 (orig. 1934))] At that point we have arrived at the specific question on when to regard economics as a science: Are there any observable facts (data) adequate to examine an economic theory? It is obvious that economical phenomena cannot be treated with the same precision as it is the case e.g. in physics. It cannot be expected that the forecast of the price of a stock coincides with the observed ones to the same extent as we can do this with similar data forecasts in natural science. The reasons for that ‘incognizability’ are indeed manifold.

A simple example shall advance our sensitivity on the problem. Regarding the model of Black Scholes in order to estimate the prices of options as functions over time [Black and Scholes (1973)] (for their work Merton and Scholes were honoured 1997 with the Nobel Prize, while Black had already died in 1995) we see a mechanism that really gives pretty good forecasts, with an accuracy that takes on a privileged position in economics. [Shah (1997)] [269] However, Shah also mentions in his article that there is always an element "*of wisdom*" [270] in economics. In extreme cases the model can collapse, as newer a-posteriori verifications of the stock market crash on October 19, 1987 have showed. [Lewis (1980)] Anyhow the validity of the model has never been second-guessed. [271] [272] The ambiguous situation is the following: in order to develop a theory as a logical framework for an understanding of the phenomena, the phenomena themselves must be observed and the results of the observations have to be summarized. But the observations are not only accompanied by a certain level of vagueness and uncertainty but in principle not even 'real' prices (meaning the value) as they are manipulated by chains of market forces (the traders). So how can the model credibly predict such 'wrong' prices or even be derived from a multitude of them?. This is a situation that resembles the situation in quantum physics. The 'object' (an entity that sometimes behaves as a wave and sometimes as a particle) is not accessible and the data about it (the 'observables') are influenced by the observer. Additionally the observables themselves are a result of several interacting 'elementary' rules which hardly cannot be examined seperatly.

Differently to statistical thermodynamics, macro-economic phenomena, regarded as averages of individual processes, never have thermodynamic dimensions (concerning the number of the involved 'particles') nor are a free of individuality (i.e. they dont obey to physical laws but have a free will). There is no theoretical 'rationality' that dictates the behaviour of all protagonists. Hence macro-economic phenomena only work on the ground of statistical probabilities within certain limits of confidence. Lets consider an example from risk management to show the practical consequence of such statistical description of economic situations: given is a manager who has the choice between two decisions about his tactics on a running project. The first possibility causes fixed costs of 10.000 € and fails in 20% of all cases. The second one causes costs of 7.000 € but fails in 40% of all cases. In case of a failure consequential costs of 20.000 € are generated with both methods. So the manager looks at the probably most favorable option. It can be determined by weighted addition of all costs. In the first case the probable costs are: 10.000 + 0.2 * 20.000 = 14.000 €. In second case we summarize: 7.000 + 0.4 * 20.000 = 15.000 €. Thus the first option will be taken by the manager. Following the above case in AW's terms, the probabilities of a failure are simplified risk functions, which depend in a realistic case on multiple external factors. The summation of all losses is represented by a simple decision function, which is aimed at the minimisation of the maximal possible loss. It is obvious that an economic science should be able to deliver the required data in assistance of statistical and other mathematical methods. Mathematical statistics supplies the necessary equipment by treating the economic uncertainties. I.e. the development of scientific economics is strictly accompagnied by the evolvement of statistical methods. These methods are not only sufficient but absolutely necessary in order to replace the already mentioned 'armchair-strategies' by examinable principles.

The huge need to adopt and develop appropriate mathematical methods (both to express economic theories and also to formalize arguments) is not only a mathematical one, but has in general a wider generic qualitiy. A good example for this is the question on how to develop an appropriate index-number formula ([Fisher (1922)]) as the measured inflation and growth of the US economy got wiped out during the 1990s on the change of the measurement formula (cp. on that discussion point the Boskin report in ‘Symposium on Measuring the CPI‘, Journal of Economic Perpectives, 1998, 12:1, 3-87)). Arguments also arose on the conditions for the measurability of unobservable elements such as the so-called ‘utilities‘ and about the appropriateness of measurement formulas of economic concepts like the ‘capital‘ and of the above mentioned business cycles.

6.2.2 Preeconometrics

6.2.2.1 Times before 1900

Prominent economic ideas before 1900 are part of our common knowledge as for example the ‘invisible hand’ by Adam Smith [273], ‘the comparative advantage’ by David Ricardo [274] and the with that connected idea of ‘gains from specialization of work’ (what describes the situation when team production increases if comparative advantages go into account). As we see below not only *Adam Smith's* brilliant ideas in no way got turned into mathematics at least the 1930s. Afore regarding Smith in the 18th century and John Stuart Mill in the 19th, we see two sides of the one medal in making political economics: the first one inheres in the positive view (i.e. the inherent working of economics, trying to find its lawlike doctrines) whereby the latter holds the normative one (Mill, as a member of the English parliament, contrasts Smith in eying economic policy as fashionable following economic insights). Mill is a representative of economic governance. [Rosen 2003], [Buchan 2006] These two opposite aspects simply show the main difference in the way economics was formerly both constituted and practiced. The twentieth century (mauled by heavy economic ideologies) finally tried to reach not only a kind of consensus but a self-disenthrallment in the attempt to develop technologies, that would be free from ideologies, to practice economics in the scientific domain, that also should cover the policy domain. Traditionally economists had always laid claims to their expertise in public policy (not only throughout the 19th century, but also in newer times, exemplified especially with O. Morgenstern above), although the range of economic policy that called for the responsibility of the state varied by nation and governments. For example the late nineteenth century economists view was that the gold standard [275] should be the ultimate benchmark to maintain the health of the national and international economy and making monetary/exchange rate policy automatic and self-stabilizing. This was widely adopted in the Western world.

The modern precision in economic investigation bundled with mathematical techniques is fully alien to the 19th century. Its ‘engineering’ workflow embodies not only a design aspect of an economic theory but also the operation and governmental control of the economy itself (cp. [Craufurd 1975] for a discussion of different interventionist models). An extant body of economic knowledge was published in the ‘Principles of Economics’ in 1890 by Alfred Marshall (1842 - 1924) [Marshall (1890)] that shows minimal data and not a single test of one of the hypotheses. Especially A. Marshall railed against the use of mathematics in economics and stressed the miserable notion of economics as a ‘moral’ science. It is common sense to see Marshall’s way at least as important as that of Léon Walras and Vilfredo Pareto for the neoclassical [277] thinking (since he formulated insights on the nature of production to explore the partial equilibrium markets, over time and by goods). Scattered mathematical works in economics were authored by Antoine A. Cournot and Francis Edgeworth. Additionally only a "*few economists were able to read their papers*.". [Bearn (1999)] History has reputed the english statistician and social scientist George Udny Yule having been the first who did a multiple regression analysis in 1899 for the purpose to mark the determinants of the phenomenon that different poor law authorities gave out different amounts of relief payments. Walras demonstrated the existence of an equilibrium set of prices and quantities by constructing a set of simultaneous equations (representing the demand for each good and arbitrage condition). The general equilibrium approach of Walras focused on the combination of individual sellers and buyers not only inspired the Irving Fisher (a student of the American physicist Willard Gibbs) and Pareto (the Italian economist who succeeded Walras in Lausanne) but especially AW who headmost found a first field of research in his quest for meaningful solutions of such systems of equations. [Tintner (1952), p21-28]

Sometimes lacking mathematization could also be a balk: e.g. comparing himself with Walras, Carl Menger felt a less impact of his ideas as a result of the absence of mathematics. [277] [278] [279] Despite this insight the neoclassical economist Carl Menger principially stood against the use of mathematics in economics, but could not circumvent that especially in the marginal economics the following generation started a mathematitzing along the joint mathematical trajectories set by Jevons and Walras. Jevon's account of individual feelings got formulated with the differential calculus. Other variants of the marginalists' utility economics, seemingly appropriate for mathematization, did not adopt mathematics. [281] Jevon's work traditionally understood as being concerned with decisions on the marginal utilities of the individual, or of individuals in exchange situations, was taken up by the Irish economist Edgeworth, an excellent mathematician and statistician. Questions of equity and distribution (such as those raised by Henry George's single tax movement or by Fabian socialists) were treated with the new marginal and neoclassical tools. John B. Clark replaced his earlier analyses of fair exchange with a mathematical account of the return paid to each factor of production in equilibrium. Arthur

Cecil Pigou used marginal analysis to understand the divergence between private and social interests and Marshall's neoclassical concepts provided the basis for later tool-based analyses of equity and distributive questions arising from governmenral actions. These forms of 'social engineering' had been developed by French engineers [282] during the 19th century, but became publicly visible for the first time in the economic decision theory in the course of the 20th century.

6.2.2.2 The turn into the 20th century

Despite some mathematization around 1900 most of the great economic ideas could not constitute a serious science, because economics had been based on some deductive logic, historical description and verbal tradition. At that time we see separate societies and journals in the field of economics, and its subject had become more separate from its older ancestors, moral philosophy and politics, and from newer siblings such as sociology. *A. Bearns* affirms that "*economics circa 1900 was not a science until the 1960s* ... [when it] ... *has been completely transformed into a technical discipline*.". [Bearn (1999)] The pioneering *Trygve Haavelmo* sings from the same hymn sheet in his nobel lecture: "*There were lots of deep thoughts but a lack of quantitative results*.". [Haavelmo (1989)] Economics was a vital pluralism of beliefs, ideas, theories and rudimental methods. But considered as an 'official' field of science, economics already had gathered sufficient academic respectability to possess a couple of chairs in universities by the mid 19th century, although none of these schools of economics (even such delineated in national terms, like the Austrianism or the American Institutionalism) can be regarded as being dominant over another, as there were clear national differences (cp. [Beckhouse (1985)]). The creation of university departments of economics, the increase of professional jobs inside and outside academia, and the becoming of a student education were absolutely subject to national variations in timing and outcomes. [John Maloney (1985)] With its academic autonomy, economics (nationwise coloured) evolved specialized subfields (e.g. labor economics, international trade) and suffered more and more on local demarcation disputes within particular fields like business management and industrial relations that developed independent positions. [Dorothy Ross (1991)] In the very beginning economists used suchlike statistical methods that provided the measurement of parameters in simple relations. E.g. to get an understanding of the law of demand, the statistical analysis observes the relations between data on the prices and quantities of goods. [283]

6.2.2.3 Upcoming Modeling during the first third of the 20th century

The drive to measure economic phenomena can be seen as a movement that had started from the late 19th century running to the mid 20th century. [284] A history regarding that point should be written as there is no overall work on this. [Klein (2001)], [Studenski (1958)] The hiherto little mathematics (or modelling) inherent in most of the economic works was followed by a massive growth of collected economic data in the first third of the 20th century (driven primarily by economic interwar troubles), accompagnied by associated empirical investigations. [MacKenzie (1981)] This built up a profound and detailed knowledge base, that loudly called for specialized methods and tools. Of course academic economists often initiated and collected their own data on specific questions they followed and also showed a dependecy on institutionalist or historical influences (e.g. the research in the US strongly stood within the ‘Progressive Era’, whereas the European colleagues found themselves morely in a period of liberalist and welfarist movement). [Flanagan (2007)] The parallel movement in other social sciences as psychometrics or sociometrics developed their own particular statistical methods. Nevertheless, these parallel movements did not take on board the commitment to mathematical models or mathematical methods as was done by the econometricians. For a short review of some origins of econometrics in the US before WWII and its development see [Klein (1971)]. There he gives an insightful account of the pioneering works of H.L.Moore [Moore (1914)] on economic cycles, Working [Working (1927)] on demand curves, or Schultz [Schultz (1938)] on the theory and measurement of demand, and of course Tinbergen [Tinbergen (1939)] on business cycles.

The entities that often appear in economic theories are not seldom represented by the task to aggregate and combine representative numbers. To exemplify that: to measure the so-called ‘price level’ we need appropriate ways to combine the data collected on prices and quantities of many different goods into a consistent set of numbers from which a price-Ievel series could be calculated. Especially the upcoming diversification of doing measurements was naturally in a strong connection with a gradual adoption and use of mathematics. [286] A main moment in economic research of that period were the investigations on the business cycle [286] phenomenon. There was no general concept of it, neither on its shape, its regularity or its length. Its cause was as unclear as the methods or measurement procedures to cover it. In follows different business cycle institutes (from Cambridge, Massachuserts, Berlin, Vienna to Moscow) evolved following a high demand from politics, that tried to find the turning points in their economies especially during the interwar periods between 1910 to 1930. Measurement was not an end in itself, but understood as the necessary prerequisite for the prediction of turning points within the the cycles.

During the 1930s the technology of mathematical modeling (from measurement methods to full statistical approaches) was introduced, a process that went on with its adolescence for the next 20 years. An important part of economics step by step became reshaped into a tool-based discipline. The coincidence of the upcoming econometrics with the 1930s did not happen by accident. The world economy at that time was seriously malfunctioning. Consequently many economists saw the need to develop a complete new approach, but did not have the right methods at hand. This was an ideal combination of circumstances for AW to start shaping new methods and finally being the accoucheur to a new kind of economics, namely econometrics. It is clear that a totalitarist belief in a mathematized economics itself would be not more than ideological, but the econometrics' strong ability to generate new economic knowledge based on statistical methods disburdens the new discipline from the esteem of being only a shallow modernism.

Mathematization in neoclassical economics meant less the replacement of words by equations but especially since Jan Tinbergen more the appearing of 'models' for the economic context (cp. Robert M. Solow, 'How Did Economics Get That Way and What Way Did It Get?', [Solow (2005)]). Tinbergen himself had a background in physics and hence was able to formulate mathematical representations (i.e. 'models') for complex economic procedures what in itself showed to be an adequate basis for a statistical analysis of the inherent data. [Bouman (1993)] Tinbergen was one of the leaders of the international econometrics movement during the interwar phase. He worked on both statistical and also pure mathematical methods. These years showed to be those when 'econometrics' more and more went towards a statistical approach on models of economics. A second prominent name for the 1940s was that of Trygve Haavelmo who developed his own branch of theoretical statistics. The range of models reached from time patterns to behavioral mechanisms (also cp. hereby the work of Lawrence R. Klein. Modeling became a distinctive element of both inductive and deductive economics in scientific and also policy domains. [287] Models were the basis for advice to governments and firms and forced the emerging of its own ‘theoretical’ and ‘applied’ sub-fields. To come again to the example of the business cycles, models such as Tinbergen’s gave both mathematical formulation of former non-mathematical theories and provided the basis for data to be seen as reasonable measurements for the involved parameters. Business cycle work suddenly gained a high degree of exactness in its claims. Ragnar K. Frisch, a Norwegian econometrician of that period, tried to develop a planning model based on consumption requests. [Jonung (1991)] It was the adoption of the modeling style that made large parts of economics primarily a mathematized discipline.

Not by accident we see economics partly shape into econometrics since the 1930s. Apart from methods of measurement, the demands from the policy domain for economic expertise and especially for a ‘usable’ economics during the 1930s until the 1950s, were responsible for such developments. E.g., the League of Nations supported Tinbergen's econometric research activities in end of the 1930s as part of their attempts to stem international and domestic problems brought by the Great Depression. The timing and also the nature of such policy demands had an impact on the character and of further developments in the economic science. In that context of increasing mathematisation of economics Morgenstern in 1935 especially emphasized in his institutes report for the Rockefeller foundation [108] “*purely scientific work*”, citing the publication of several monographs, mentioning Hayek's 1931 ‘Preise und Produktion’, and his own 1934 publication ‘Die Grenzen der Wirtschaftspolitik’. Morgenstern stressed the established links to the University of Vienna, having been realized through lectures and seminars, e.g. in the

Mathematical Colloquium. He noted several outstanding people from Menger's Colloquium who had worked on questions in pure theory, especially AW. Morgenstern underlined the theoretical work (ibd. p11): “*On the basis of the experiences of the last years I have worked out a program for research which I beg to outline briefly. This program provides for purely theoretical work as well as for empirical studies. These assume even relatively more importance than before; they are necessitated in order to examine theories of the Trade Cycle and procure a basis for new abstract thinking. It is my particular desire to harmonize more than has been done before both ways of research. I am absolutely convinced that abstract theoretical work, even making use of mathematical analysis or of the modern methods of Logic that have not yet been applied to Economics, are just as necessary as the systematic collection of facts”* and continues on page 14: "*Economists have so far entirely neglected the progress of mathematics and notably of logic during the last 30 years, so that it seems indispensable to subject economic theories of various kinds to the more rigorous test of these new ways of thinking and research*“.

6.2.2.4 A bridge to the Sequential Analysis

As we have seen in earlier sections, statistics in the late nineteenth and early twentieth century was mainly engaged with agriculture and industry phenomena, and their analysis usually was carried out on the base of already collected data. The focus therefore is targeted on a single examination of the data. In the same way probability estimates got based on a single evaluation of the data in stock. Hence an accurate course of action is given, suitable for a fixed sample size to make statistical decisions during the planning stage, whereby an examination is never done before the collection of the sample is completed. The fixed sample has known disadvantages: its implementation implicates unnecessary costs and risks to the observed objects and its size incurs considerable ecconomic and ethical disadvantages. On many occasions, the effects of treatments are much more obvious than envisaged during planning. E.g. in clinical research, subjects are recruited and data collected in a sequential manner, and a research project may extend over a long period. It is evident that the nature of the data would be more suitable for sequential examination than a single analysis at the end of the long period.

Sequential Analysis, until WWII quite an strange field of research, starts as a mathematical model to analyze the behaviour of a set of probabilities when applied repeatedly over time. In the late 1930s and early 1940s with its massive increase in industrial production of war materials, there was a huge need to ensure reliability of such products especially ammunition to its users. The usual testing of these products was not only pretty expensive, but at time destructive (as bullets get destroyed by usual tests). So the need to develop a method of minimal testing costs was enormous. AW and his group developed the theoretical basis and methodologies of the Sequential Probability Ratio Test (SPRT). Instead of defining a sample size, a pair of thresholds are computed, one to decide the rejection of the null hypothesis and the other to accept it when crossed by the actual test statistic. Data are obtained sequentially, and plotted against these borders. After each single measurement, one of 3 decisions can be made. These are to stop further testing and reject the null hypothesis, to stop further testing and accept the null hypothesis, or to continue testing. The success of this methodology greatly reduced the costs and increased the reliability of supplies.

A couple of years later, a group under Barnard and Armitage [289] [Armitage (1957)] (Armitage was on the staff of the Statistical Research Unit of the Medical Research Council at London School of Hygiene and Tropical

Medicine) also developed its own concept of Sequential Analysis focussing not only industrial production processes, but also testing the efficacy of the products itself. Their results can be found in the much quoted book [Armitage (1975)] on sequential medical trials. They introduced so called paired comparisons, where a paired observation is made, and the difference between the pair members is tested sequentially against the null hypothesis.

6.2.2.5 Until 1950

In the beginning of the mathematization process its proponents saw in their way the most truthful one to achieve the 'reality' of economics, but as the century proceeded the common commitment to the strength of mathematics in economic reasoning was accompanied with the weakening claim that the mathematical representation should be meaningful when understood as accurate descriptive language for economics. [Ingrao, Israel (1990)] This loose of 'realism' went hand in hand with the insight that the mathematical form not only preceeds over the economic content and primarily expresses an abstract model of the concrete case. It was the process of 'modeling' that had started in the 1930s and was around the 1950s in full bloom in the US as many of the emigrants like AW there had the freedom and also the scientific network to establish their body of thought. Although Jan Tinbergen is a candidate of having brought modeling into economics, we regard especially AW in front of this development, as he not only worked together with Tinbergen in Geneva (see below the section 'Haavelmo, AW and the probabilistic approach') but also started his late career in the US at the Cowles Commission to become soon afterwards professor of economics at Columbia University.

The whole half of the century before 1950 is full of developments in basic methods, in evolving practice and also conceptual frameworks. All these developments were undertaken to come to a feasible theory of applied economics and statistics. In terms of its appliance it was the conometrics that evolved as a new field of research out of economics and statistics. An allocateable time span for the process of formation of econometrics can be the years between 1861 and 1952. Econometrics is closely related to the work and the lifetime of AW. It is also important to mention the difference in the perception of meanwhile prominent papers in early econometrics and those which were important in the process of developing ideas. [290] Prominent pioneers immediately come to mind as there are AW, Irving Fisher, Ragnar Frisch, *Trygve Haavelmo*, Tjalling Koopmans, Henry Moore, Henry Schultz, and Jan Tinbergen. Less known authors with nonetheless important contributions are *R. H. Hooker*, *Robert Lehfeldt*, *Marcel Lenoir*, *Albert Marget*, *Warren Persons* and *Philip Wright*. In the phase before 1940 the scientific progress happened less in using mathematical statistics, but in new theoretical treatments of known econometric problems, i.e. primarily in innovative applications. Since the 1940s "*theoretical work took over from applied work*". [Hendry and Morgan (1997), p2] Basic knowledge for econometrics comes from the development of time-series analysis. "*The statistical techniques available to economists were either those developed in nineteenth-century social science to deal with masses of largely crosssection data or those developed in physical science in which data might relate to different points in time, but for which data observations at different time periods were usually independent of each other (as in astronomers' observations)*". [Hendry and Morgan (1997), p9] Economists and statisticians such as *Yule* and *AW*, both heavily engaged in the treatment of time-series problems, began to develop their own special form of statistical analysis to fit the time-related data of economics. The occasionally spread assumption that econometrics is primarily a post-1951 development is not correct. Concerning its foundational elements this assumption is simply false. It is sufficient to regard AW's 'case' to see quickly the contrary.

After WWII, economists had access to a bewildering variety of 'official' data. Rarely since then economists have set out to take their own measurements. For the main part macroeconomic data came out from national sources against what microecomonic data resulted mainly from investigations of privately held institutions or research. Their ambitions in the realm of measurement soon led them (along with other social scientists) develop mathematical methods (i.e. statistical ansätze). Measurements that had been valued earlier for their own sake, as sufficient evidence in tables and graphs, were now asked to be the source of causal explanations. The methods of correlation and regression, originally designed for biometric data, were immediately adapted and developed by statisticians operating in the social science community. [291] During the post 1950 period, the goal was more evolutionary. Pupils and followers of the founding fathers like AW started to buttress the newly won insights in econometrics. Not at least the emigrant AW played an important role in the upcoming dominance of the US' ideas within the Western economies during the second half of the 20th century.

In [Christ (1985)] the story of early progresses in estimating economic relationships in the US is sketched. The start of modern econometrics, as we know it until today, can be put into the 1940s. Following Klein [Klein (1971)] the usage of statistical inference in econometric problem is initiated by Haavelmo [Haavelmo (1944)], Mann and AW [Wald, Mann (1943)]. This ansatz got elaborated during the last years of the 1940s and the beginning of the 1950s by J. Marschak, T.C. Koopmans, and L. Hurwicz etc. at the Cowles Commission. Klein writes: "*At this time econometrics and mathematical economics had to fight for academic recognition. In retrospect, it is evident* *that they were growing disciplines and becoming increasingly attractive to the new generation of economic students after World War II, but only a few of the largest and most advanced universities offered formal work in these subjects. The mathematization of economics was strongly resisted.”* [Klein (1971), p416]

6.2.2.6 Now what finally is econometrics?

To follow the definition of Haavelmo, it is “*the method of econometric research aims, essentially, at a conjunction of economic theory and actual measurements, using the theory and technique of statistical inference as a bridge pier*“ [Haavelmo (1944)], or with Samuelson it “... *may be defined as the quantitative analysis of actual economic phenomena based on the concurrent development of theory and observation, related by appropriate methods of inference*.“ [Samuelson, Koopmans, Stone (1954)]. As Ragnar Frisch [292] writes in the first issue of Econometrica, econometrics can be seen as the unification of statistics, economics and mathematics, whereby each of these disciplines by itself is necessary but never sufficient to come to a real understanding of quantitative relations in economics. Econometrics aims at giving empirical content to economic relationships. Lawrence Klein [Klein (1971)] (1980 winner of the Nobel Prize in economics) has always emphasized economic theory, statistical methods and practical economics (i.e. also economic data) as key ingredients for econometrics to work. To work in a sense that econometrics is able to verify or refute economic laws (such as purchasing power parity, the quantity theory of money, etc.). Econometrics bridges the gap between economic theorizing and actual measurements to make economic phenomena understandable in a stricter sense. It also provides quantitative estimates of e.g. price and income elasticities of demand or estimates of returns to scale in production, technical efficiency, the velocity of money, etc, and finally enables the possibility to predict future economic data like interest rates, unemployment, or GNP growth. Lawrence Klein emphasized this last function of econometrics:"*Econometrics had its origin in the recognition of empirical regularities and the systematic attempt to generalize these regularities into "laws" of economics. In a broad sense, the use of such 'laws' is to make predictions - about what might have or what will come to pass. Econometrics should give a base for economic prediction beyond experience if it is to be useful. In this broad sense it may be called the science of economic prediction"*. [Klein (1971)] It should always be clear that data in economics are not generated under ideal conditions and cannot be replicated. They are afflicted with error and are in some cases only proxies for variables that are either not observed or cannot be measured.

6.2.3 AW's first period: econometric foundations

AW's turn into statistical-economical problems was the result of its own diversified interests but predominantly a result of the worsening conditions in Austria, which made impossible for a young Jewish scientist to come into an academic employment. With Menger's recommendation AW was able to meet Schlesinger in 1931 and start to work on economical questions. Also by Menger's initiative AW got a recommendation to visit Morgenstern, who quickly gave him a position in his institute for market research. In 1935 AW published the article 'Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen' [Wald (1935) b,] and in 1936 'Über einige Gleichungssysteme der mathematischen Ökonomie', two articles that unfortunately get noticed hardly due to their abstract and mathematical buildup. AW's relentless interpretation of the contemporary economists therein surely played a role. In AW's view in economics a kind of wishful thinking was predominant, the thinking that one could describe economy and economic behavior with vague 'rules' (thereby getting a method for profit maximizing on the ease). Such an 'approach' AW regarded as naive and hasty, a fact that he made clear with a couple of examples. Already in his early work AW's persuasion became apparent, that the development of a substantiated economic theory so far had been prevented by the lack of mathematical models. After the introduction AW analyzes the meaning of different variables in the Walras-Cassel set of equations modified by Schlesinger. [293]

Schlesingers modification as a matter of fact consisted in the use of inequations instead of equations. [Schlesinger (1935)] See [Wald (1936) a,]: "*Schlesinger have suggested a modification of the above system on considering the following objection: in Walras, only the 'scarce' factors of production are included in the factors R_1, R_2,... R_m, which appear in the equations, that is, those factors for which the whole available amount is used in production and which cannot be obtained without cost. Just which factors of production are 'scarce' is considered by Walras to be a datum of the economy. But this is not so, for the scarcity, or abundance, of a factor depends upon the demand functions $f_j(s_1, \dots s_n)$ for the products, upon the technical coefficients a_{i_j}, etc. In other words, whether factors are free or scarce cannot be considered, a priori, a datum of the economy; it can only be determined on the basis of the production equations. Therefore the following modification of the Walras-Cassel equations is proposed: R_1, R_2,... R_m, are all available factors, both free and scarce. It is not assumed that for each factor R_i the total available quantity r_i will be used in production, but that for some R_i there will be an unused excess. The first m equations of Walras are then modified as follows:*

$$r_1 = a_{11}\, s_1 + a_{12}\, s_2 + \,..+ a_{1\,n}\, S_n + u_1$$
$$r_2 = a_{21}\, s_1 + a_{22}\, s_2 + \ldots + a_{2\,n}\, S_n + u_2 \ldots$$
$$r_m = a_{\mathrm{m}1}\, s_1 + a_{\mathrm{m}2}\, s_2 + \ldots + a_{\mathrm{mn}}\, S_n + u_m$$

where for each i, $u_i \geq 0$. Those factors for which $u_i > 0$ are the free factors and have a price equal to 0. Therefore, the following m side conditions are formulated: If $u_i > 0$, then $\rho_i = 0$, $i = 1, 2, \ldots, m$, which is equivalent to the following m equations: $u_i * \rho_i = 0$, $i = 1, \ldots, m$. All other equations of the system remain unchanged.".

At that time it had simply been assumed that this system of equations must have a solution, such that it would represent the market equilibrium. This was infered from the fact that the number of equations was equal to that of the unknowns. "*As a rule, economists have contented themselves with equating the number of equations and unknowns and have assumed, without further investigation, that the system of equations had a meaningful solution from an economic viewpoint, and that this solution was unique. But the equality of the number of equations and unknowns does not prove that a solution exists, much less the uniqueness of a solution, as can already be seen from quite simple examples such as the equations: $x_2 + y_2 = 0$, $x_2 - y_2 = 1$, which have no solutions for the unknowns x and y. On the other hand, there are systems of equations where, despite the agreement in the number of equations and unknowns, there are many, even an infinite number, of solutions. For example, the equations: $x_2 - 2 = 0$, $x + y - z = 0$, $x * z = 0$ have an infinite number of solutions for x, y, and z*.". [Wald (1934) a,], [Wald (1936) a,] From a mathematical view this is neither necessary nor sufficient. AW gave a formulation of the constraints and restrictions necessary to guarantee a solution. These became a stringent part of the solution away from any subjective matters of faith and taste, wrongly alleged as economical objective. AW formulated six sufficient conditions, which guarantee a solution. In a further step he loosened some restrictions, e.g. that one of the possibility of an unused surplus from certain goods. AW finally develops the first proof of the existence of the equilibrium. [Wald (1934) a,], [Wald (1936) a,] Regarding AW's six necessary conditions we quickly see the scope of validity of the existence theorem. The quantities of the factors of production are positive. The quantities for the production of a unit of a product likewise are positive, and at least one factor of production is necessary for a product to be produced. There are no price gaps, they evolve continuously as functions of the quantities of the products. A restriction is that the demand for a product only totally disappears if its price becomes infinite. AW characterizes this as a idealized, but nevertheless regards it as a substantial condition. [Wald (1934) a,] Karl Menger later loosened this condition by setting the price, for which the demand converges against 0, on a finite, calculable level. AW's last condition describes a lowering value of a combination of goods when the demand for that combination is increasing. [Wald (1934) a,]

AW's entire paper was an important step forward, and this not solely because of his (the first ever) existence proof of an equilibrium, but also on the conditions he gave, necessary for such an equilibrium. The implementation of the first three conditions made it impossible to get an equilibrium. Using the first four conditions, he got as result of infinite many possible equilibria. Only in combination with the last two conditions the existence of one unique equilibrium was guaranteed. From an epistemological view AW's article was a huge step making economical systems understandable. It opened the way to find general forms of systems of conditions, necessary for an equilibrium. AW had pushed the door open to search after imbalancing conditions within an economic system to find ways for its rebalancing. Introducing strict mathematical conditions, he broke traditions in various ways but finally made clear how to rely on examinable facts. AW discussed that extensively thru the final part of his text. He pleads for a pure mathematical foundation of the elementary principles in economics. AW's article was an independent achievement from v. Neumann's work on equilibrium. [295]

In 1936 AW wrote the book 'Berechnung und Ausschaltung von Saisonschwankungen. Beiträge zur Konjunkturforschung', published by J. Springer. [Wald (1936) c,] That work was appraised by Wolfowitz as an example showing clearly AW's mind to achieve applicability without leaving any mathematical consistency. [Wolfowitz (1952)] Examples for such kind of work were AW's 1936/37 papers 'Über die Produktionsgleichungen der ökonomischen Wertlehre', 'Grundsätzliches zur Berechnung des Produktionsindex', 'Zur Theorie der Preisindexziffern' and 'Extrapolation des gleitenden 12-Monatdurchschnitts'. [Wald (1935 c,)] Therein the author's priority on deriving statistical methods for the treatment of economic problems became more and more vital. In the same year (1936) AW published 'Sur la notion de collectif dans le calcul de probabilités'. Therein AW shows a method to specify the term of the choice regarding 'collectives' (introduced by R. v. Mises as a fundamental term for random sequences). [Wald (1936) b,] Another of AW's papers giving a consistency proof of the collective was 'Die Widerspruchsfreiheit des Kollektivbegriffs der Wahrscheinlichkeitsrechnung' in 1937 and also the 1938 paper 'Die Widerspruchsfreiheit des Kollektivbegriffs'. Karl Popper had already tried to specify the concept of a random sequence and had presented his ideas in Menger's Colloquium. [Popper (1935)] AW exhaustively examined the collective, proved its consistency and reduced the irregularity requirements of Copeland, Popper and Reichenbach to special cases of his general treatment. [Wald (1937) a,] Wolfowitz mentions AW's proofs as remarkable and an important step towards the axiomatization of the probability theory. [Wolfowitz (1952)] By working on time series of seasonal variations AW interest in a strict mathematical treatment of statistical-economic problems strengthened. For AW the term of randomness was equal with 'incalculability' or the 'absence of recognizable patterns'. A view that was criticized first (e.g. by Ville in 1939 [Shafer and Vovk (2005)]), but later got recognized as a fundamental one. Shannon's information theory is based on the same concept on randomness. [Shannon (1948)] The modern view in quantum mechanics as 'causeless coincidence' confirms AW's point of view.

6.2.4 The decline of the time-series approach

AW's contribution to time-series analysis 'Berechnung und Ausschaltung von Saisonschwankungen' [Wald (1936) c,] from 1936 was done by him on request of O. Morgenstern. (An english version can be found in [Hendry and Morgan (1997), p178]. AW defines in the beginning components in terms of causal categories imposed externally prior to investigation. The difficulties lie in providing complete, consistent and unique definitions of the categories. His problem is that individual components cannot always be uniquely determined. AW's answer is to turn to 'internal' definitions, components hypothesized to exist in the original series, which are expected to have some correspondence with externally conceived components. This requires a form of test for these internal components - if the a correspondence appears 'improbable' - if not, the hypotheses about the internal components need to be modified (cp. [Hendry and Morgan (1997), p16]. The tests depend on finding empirical regularities between components of the same or different series. These again could be identified with externally categorized notions from economics. As the difficulties of interpretation of harmonic components are immense AW concludes that the role of external categories can never be more than heuristic (cp. 'Berechnung und Ausschaltung von Saisonschwankungen', 1936 [Wald (1936) c,]). AW recognized the problem of identifying (the hidden) mathematical time-series components in the economic data and proving their correspondence to the economic categories. This is the reason why the time series methods lost their importance since the 1930s. The decomposition method of economic data into mathematical components was basically empirist without an underlying theoretical method. Since 1910 the growing economic movement started to aim at the introduction of more theoretical models. It was Jakob Marschak (he founded, together with Roy Radner, the economic theory of teams and organisations) who generally organized the informal meetings of econometricians which took place in New York during the 1930's and 1940's, to discuss with AW, Haavelmo and others. [Christ (1993), pp71-94] Not until the 1960s the time-series programme was reactivated, partly due to Morgenstern's role in the Princeton time-series project.

Parallel to the time series approach (from about 1920 to 1940) all kind of social scientists developed their own appropriate statistical techniques. Fields like sociometrics or psychometrics established their own approaches to statistical questions. This was due to the fact that statistics is not immediately transferable to social science problems. Using the example of Hotelling that split is pretty apparent. Hotelling's contributions to mathematical economics during the 1930s can be differentiated against his statistical work in the reception history. Where the first mentioned publications were highly regarded, his papers in statistics did not resonate so strongly at that time.

6.2.5 AW's second period: Statistics and applicability

In the 1939 paper 'Contributions to the theory of statistical estimation and testing hypotheses' [Wald (1939) b,] AW tied in with the work of Neyman and Pearson, that provided a lesser effective method of estimating and testing hypotheses, as was initially assumed of the statisticians at first. AW generalized these methods for the multivariate and multi-parametric case, and uses weight functions, risk functions and the concept of parameters as distributions, in order to get parameters based on given observations. His work was partly ahead of the times, even von Schelling [Schelling (1955)] sees therein, because of the "*subjective*" use of weight functions, rather a step back, and additionally no relevance for practical applications. Yet AW's weight functions are nevertheless no 'subjective' implementation but a systematization on subjectivity which never can be completely eliminated from an application. AW never makes any assumptions on the structure of the weight functions to avoid any restriction in a general ansatz. Also the frequentistic view can be hold. AW's proof is set-theoretic, by what a variety of apparently different cases can be subsumed under a generalized model. AW also examined different kinds of errors more differentiated than Neyman and Pearson. Thereby the work becomes more relevant for the praxis. AW gives an existence theorem for a distribution, which minimizes the risk. He exhaustively analyzes the necessary and sufficient conditions for several special cases. In the following discussion he mentions that further investigations with less and weaker restrictions have to be made, in order to extend the theory.

A typical example for practical use is shown by AW in the article 'Contributions to the theory of statistical estimation and testing hypotheses'. [Wald (1939) b,] It is the determination of the unknown mean value θ of a normal distribution on the basis of a finite sample $E = (x_1, x_2, \dots, x_n)$. Without restriction of the generality it is assumed that the variance is equal to 1. The task is to find with a finite number of observations the center of gravity of the normal distribution with known 'width'. In that case the probability for θ

the true mean value is: $p(E\,|\,\theta) = (2\,\pi)^{-1/2}\,e^{-1/2\,\Sigma(x_i-\theta)^2}$. Is the weight function defined as follows: $W(\theta,\,\theta') = 2\,(\theta' - \theta)$ if $\theta' \leq \theta$, and $W(\theta,\,\theta') = \theta - \theta'$ if $\theta' > \theta$ (whereby θ' is an supposed value for the mean), than the best estimation for the mean value θ is that that minimizes the integral: $\int_{-\infty}^{\infty} W(\theta,\,\theta')\,e^{-1/2\,\Sigma(x_i-\theta)^2}\,d\theta =$. The calculation of

$$\int_{-\infty}^{\theta'} 2\,(\theta - \theta')\,e^{-1/2\,\Sigma(x_i-\theta)^2}\,d\theta + \int_{\theta'}^{\infty} (\theta - \theta')\,e^{-1/2\,\Sigma(x_i-\theta)^2}\,d\theta$$

that integral delivers a mean that is different from the normal average value!.

Also in 1939 AW published, in cooperation with Wolfowitz, the paper 'Confidence limits for continuous distribution functions', a continuation and extension of the work of Fisher and Neyman. They show, on what conditions and by what means one is able to specify a lower and upper confidence limit. The treatment of exemplary cases shows, how flexible and general valid their theoretical model is 22. In the same year the article 'Limits of a distribution function determined by absolute moments and inequalities satisfied by absolute moments' appears. [Wald (1939) a,] It covers necessary and sufficient conditions for a random variable having a given lower and a given upper limit in its distribution. These limits get defined by moments and a characteristic random variable. The practical meaning of the work lies therein, that such limits get calculable for a variety of distributions. The results of this work found their application in psychology or medicinepractice to classify individuals into two or more groups. [Solomon (1956)]

In 1940 the article 'The fitting of straight lines if both variables are subject to error' appears, which represents an additional generalization of the theory of regression. The variable, afflicted with errors, is not only the dependent one. [296] AW mentioned the fact that earlier work on that topic contained to many a priori assumptions and also did not accomplish the regression solely on the basis of the data. Additionally the determined equation of the regression line was not invariant regarding a change of the coordinates. AW questioned also the justification of the principle of least squares, which seems to be more ‘intuitive’ than logically compelling. He treats the topic by renouncing additional a priori assumptions. His results showed to be pretty useful for practical applications in economics.

In 1941 AW releases the ‘Asymptotically most powerful tests of statistical hypotheses’. Herein he connects the method of the optimal estimation with the method of the asymptotic most effective test, defined by himself, a conection that mathematicians up to this time believed to be impossible. The connection shows AW’s steady effort for generalizing his theories. The article ‘Some examples of asymptotically most powerful tests’ from 1941 continues the preceding one, and shows further examples for such tests, which are able to stand in a practical context, as the estimation, in which certain set out of predefined sets an individual belongs, on which different measurements were made.

In 1942 AW published the article ‘Setting of tolerance limits when the sample is large’. [Wald (1942) a,] Therein he attends to the special problem, which emerges from huge samples. This is a situation that often occurs in industrial production. Thus its treatment is of a special practical importance. If the appropriate distribution is known as element of the family of the k-parametric functions, AW’s method exceeds that of Wilks. [Wilks (1941)] AW actually generalizes his ansatz later for the multivariate case. He also gives in the paper a practical example for an a priori known normal distribution and the proper tolerance limits. In his 1944 paper ‘On a statistical problem arising in the classification of an idividual into one of two groups‘. [Wald (1944) b,] AW also treated a practical question: it is the procedure of a binary classification in industry and production (the ok-defect selection). AW succeeds to solve the statistical problems linked with it and also specifies the associated risk functions. This work is in its nature set-theoretic and even for the results appropriate geometrical interpretations are given. In the same year AW issued the paper 'Asymptotically shortest confidence intervals'. [Wald (1942) b,] Here he completes Neymans results [Neyman (1937)], that are valid so far only for small samples. In particular he examines the case, when the sample size becomes infinite. So he completes on the one hand the theory, and on the other he prepares it for applicability, e.g. for industrial production. Thereby the existing equipment to get samples out of a production can be used further on, because it is only the method that changes. The article 'On the power function of the analysis test' is a further generalization. It derives a theorem, which contains Hsus [Hsu (1941)] earlier result as a special case. AW finds that the size of a critical test region is smaller than all other test regions with the same effect, if their size equals the size of the canonical form. [297]

1943 AW published the on practice oriented article 'On the efficient design of statistical investigations'. Therin he generalizes the up to this time developed methods for statistical investigations of agricultural production and expands them to other areas. AW indicates measures to characterize the efficiency and the quality of a statistical investigation and defines critical regions for the appropriate tests. Thereby he admits a great importance to the practicability of the tests and describes mathematical procedures for their implementation. In the same year AW extended the method of Wilks (cp. [Wilks (1941)]) for tolerance limits, published them in 'An extension of Wilk's method for setting tolerance limits' [Wald (1943) b,]. In that paper he systematically broadened Wilks' method to the multivariate case. The method remains independent of the underling distribution. The proof contains geometrical arguments, and also an inductive argumentation for an arbitrary number of random variables. It is characteristical for AW to indicate a pure geometrical procedure for the definition of the tolerance limits.

In 1944 the article ‘On cumulative sums of random variables‘ appeared. [Wald (1944) a,] AW kept himself busy with the probabilities, under which these cumulative sums cut across or fall below determined upper and lower limits. His approach was to interprete cumulative sums and all other combinations of distributions as distributions themselves. In this way AW could reduce the number of fundamental terms, what showed to be important for a new axiomatization of statistics. Up to that time statistical parameters were regarded as ‘fixed‘ and of course not as distributions. The examined characteristics of the cumulative sums and in particular ‘Wald’s equation’ $E\left(\sum_{i=1}^{T} X_i\right) = E(T)\,E(X)$(whereby T is a integer random variable, X a random variable and E the expectation) showed up to be pretty important in many areas of research, e.g. [Miller (1961)], [Kryukov (1976)] or the model of the ‘random walks’ in physics (the Brownian movement), where the distance of a walking particle is the cumulative sum of all its basic random movements (thus the traveled distance can be calculated on the basis of appropriate probabilities). In the same year AW published the text ‘Note on a lemma’ [Wald (1944) c,], where he eliminated the restriction to bounded functions of his earlier lemma in ‘On the power function of the analysis test‘ for variance tests.

In 1945 AW published the article ‘Sampling inspection plans for continuous production which insure a prescribed limit on the outgoing quality‘ [Wald and Wolfowitz (1945)] in cooperation with Wolfowitz. They present procedures for the testing of products in a continuous production, which guarantee that the number of defect products will stay below an upper limit, what holds the quality of production on a desired level. The presented methods are held completely general, which keep the desired quality with a minimum of control. The 1945 article ‘Some generalizations of the theory of cumulative sums of random variables' [Wald (1945) c,], generalizes the hitherto existing achievements. It points out the probabilities, with which the cumulative sums exceed certain upper and lower bounds. AW finds the characteristic functions and distributions of the cumulative sums themselves. That standardizing view in regarding calculated values as random variables themselves, showed to be beneficial for further developments of the sequential analysis. In 1946 AW enhanced the previous results in 'Some improvements on setting limits for the expected number of observations required by a sequential probability ratio test'. He especially aimed at making them more useable in praxis but without to abstain from mathematical exactness. In particular AW presents methods to forecast upper and lower limits for the number of necessary samples in the case of a SPRT. These limits are defined as functions of cumulative sums and may not afreshly be determined at each stage of the experiment, but base on the outcomes of the preceding steps.

The relevance, that AW attached to the elimination of ad hoc assumptions for building strict proofs, gets again visible in his 1946 article 'Differentiation under the expectation sign in the fundamental identity of sequential analysis'. Here he formulated general conditions, on which a differentiation under the expectation sign is feasible. With that an important part of the sequential analysis is freed from a possibly doubtful proof. In 1947 AW continued his regular contributions to the sequential analysis. The article 'Sequential confidence intervals for the mean of a normal distribution with known variance' [Wald and Stein (1947)], written in collaboration with Charles Stein, presents a solution for a recurrent problem in practical applications, which results from the need to determine an average value by sequential sampling within given confidence intervals, assumed the variance is known. It is characteristic that AW does not lose his practically oriented sight despite the abstract problem. He again aimed at a solution, that guarantees a feasible procedure in a production cycle. The mathematical treatment remains nevertheless stringent. In the same year AW published ’A note on regression analysis’. [Wald (1947) e,] In this article he regards the regression coefficients themselves as a distribution around zero with a certain variance, and specifies confidence intervals by using a F-distribution. The idea to integrate statistical parameters into the same family of basic objects (distributions) and to apply statistical theorems on them, completed the theory in a very elegant way. Statistical parameters are specified to a certain extent by statistical characteristics, which describe them.

AW continued with the systematization of the sequential analysis. Thereto he wrote ‘An essentially complete class of admissible decision functions’ [Wald (1947) f,] that appeared in 1947. Beginning with the special Bayes solution for a decision function in the case of an a priori distribution of a single parameter, he extended this ansatz to the class of all possible Bayes solutions for all possible a priori distributions. In turn, AW used this generalisation for the practical situation when one has to decide between two hypotheses. The same fundamental approach AW showed in ‘Foundation of a general theory of sequential decision functions’ [Wald (1947) d,], where a general theory for the foundation of decision functions in the sequential analysis is presented. Unlike to the ‘Theory of games and economic behavior’ AW formed an expanded theory, containing the problem of the decision between two hypotheses only as a special case. His paper is built on set-theoretical considerations and that way a typical approach for AW. Before he attacked economic problems, he at first tried to provide a comprehensive theory as a consistent equipment for actual problems. [299] In the 1947 paper 'Limit distribution of the maximum and minimum of successive sums of random variables' [Wald (1947) a,] AW defined a range, where the sequence of the cumulative sums can be located. In particular AW not only examined the case, when the sequences converge, but also when they reach infinity. For the case of convergence he proved that the probabilities of the maximum and minimum values are independent of the distribution.

In 1947 AW's work cumulated in his book 'Sequential Analysis' [Wald (1947) c,], that summarizes all to that time won findings. The sequential analysis had a great potential for the treatment of a variety of practical problems, which determine everyday's life in companies in the range of finance, insurance or production. It however seemed, that AW's strict mathematical treatment of these phenomena only had been hesitantly noticed. Although his methods were generally formulated and precisely developed, these required precise studies before they could be applied. In the field of economics the dominant expectation was to have 'mechanical' procedures which automatically lead to the right results. Nevertheless the economist and mathematician Leonid Hurwicz [Hurwicz (1947)] creates in his appraisal of the 'The Theory of Games and economic Behavior' [Morgenstern, v. Neumann (1944)] in 1947 the outward impression that the decision-theoretical treatment of economic questions can be attributed in equal parts to v. Neumann and AW, because the half of his double sided article is devoted to AW. Hurwicz writes in [Hurwicz (1947)]: ' *The readers of the Annals will be particularly interested in the connection between the 'Theory of Games' and the theory of statistical inference. As has been pointed out by Abraham Wald the problem faced by the statistician is somewhat similar to that of a player in a game of strategy. The theory of statistical inference may be viewed as a theory of rational behavior of the Statistician. His 'strategy' consists in adopting an optimal test or estimate, more generally an optimal decision function. This optimal decision function must be chosen without the knowledge of the 'a priori' distribution of the population parameters. Wald's basic postulate of minimization of maximum risk is equivalent, to regarding the statistician as a player in a game of* *strategy, with 'Nature' as the other player. The optimal decision function is chosen in a way which (as shown by Wald) is equivalent to assuming the 'least, favorable' a priori distribution of the parametens. As Wald says, "we cannot say that Nature wants to maximize the statistician's risk. However, if the statistician is completely ignorant as to Nature's choice, it is perhaps not unreasonable to base the theory of a proper choice of the decision function on the assumption that Nature wants to maximize the statistician's risk"".*

The year 1948 was also a fruitful one for AW. He published the article 'Asymptotic properties of the maximum likelihood estimate of an unknown parameter of a discrete stochastic process'. Herein, contrary to previous consideratons, it is not assumed that the observations should be independent from each other. After a detailed analysis of the so connected stochastic convergence AW succeeded to set up a proof showing that the maximum-likelihood procedure (i.e. if $\{X_i\}$ is a sequence of random variables and $p_n(X_1, \dots X_n)$ the appropriate distribution with a parameter Θ. Then the (minimizing) maximum-likelihood equation is formulated as $\frac{\partial \ln p_n}{\partial \Theta} = 0$) under the given terms has always a solution, which itself is a consistent estimation. Beyond that, the proof of the equality of the distributions of the observations and also of the derived means in the limiting case of "*infinite many observations*" is a fundamental insight, because the equivalent view of data and parameters gets a theoretical reasoning. From the same year originates the paper 'Estimation of a parameter when the number of unknown parameters increases indefinitely with the number of observations'. [Wald (1948) b,] It's method is pretty practicably oriented. AW examined the problem of the estimation of a statistical parameter, given an increasing number of parameters by a growing sample size. He determines the necessary and sufficient condition for the existence (or the non-existence) of a consistent estimation and defines the information content of the first n observations in analogy to Fisher's information function. He also extends Neymans definition of structural parameters. [299] The 1948 article 'On the distribution of the maximum of successive cumulative sums of independently but not identically distributed chance variables' treats cumulative sums of variables as distributions. The former condition of the identity of the distributions is not longer implied, by what the results are of a higher generality. In particular AW examined the special case, when for the random variables only the values ± 1 are allowed (they are thus subject to a classification in the way of 'wrong-false' or 'ok-defect'). AW's tendency, to connect any mathematical achievement with usability, becomes again visible in the context of the article. The paper 'A sequential decision procedure for choosing one of three hypotheses concerning the unknown mean of a normal distribution' [Wald, Sobel (1949)] of 1949, written in cooperation with Milton Sobel [300], provides a solution for a multiple decision problem. Here more than two competing hypotheses on the value of a parameter of the distribution of the sample are possible. In the cited paper actually three different hypotheses on the mean value θ of a distribution are given. Practically said, the three hypotheses concern the cases $\theta < a$, $a < \theta < b$, and $b < \theta$. The authors describe in detail a sequential procedure, that enables them to select one of three hypotheses. They compare their procedure with the optimum non-sequential one. Thereby they use parameter spaces, containing the decisive parameters as elements. Despite the abstract set-theoretical and topological methods, the suggested procedure remains astonishing simple in its feasibility. The enormous output of AW continues in 1949 in 'On distinct hypotheses', a paper he wrote together with A. Berger. The article treats a problem suggested by Neyman, concerning the conditions under which two compound hypotheses are distinct from each other. Two hypotheses are said to be mutually distinct, if under these hypotheses the probabilities of a sample falling in given region are unequal. The set-theoretical analysis of these compound hypotheses results in a necessary and sufficient condition, guaranteeing the mutual distinctness of the hypotheses. 1949 was also the year of AW's publication 'Statistical decision functions'. [Wald (1949)] That paper is mathematically seen of great value, but unsuitable for a practical use.

Afreshly in 1949 AW developed better proofs of some theorems (e.g. [Wald (1948) b,]), but did not lose sight of the practical side of his research. For example he does not only accomplish his investigations for non-sequential but also for sequential cases. Thus also cases are covered, that don't allow a sequential sampling due to the kind of the production process without a huge expenditure. AW introduces new methods of research, that broaden the framework for statistical investigations. There are no 'intuitive proposals' on conditions and restrictions, but only such, that stand real requirements. These are formulated in an most abstract way, even if the proofs become very difficult and time-consuming. This is a characteristic of AW's mode of operation. He produces not always elegant but also pretty lumbering proofs, if it is necessary. In the mentioned article he uses a variety of game-theoretical, topological and set-theoretical considerations, which he interpretes in the context of making decisions. A very important result of the 'Statistical decision functions' was the proof on the conditions, which determine strictly a zero-sum, two-person game. AW describes complete classes of such decision functions, which are valid even in case of non-countable infinite sets of distributions and decision functions. In the same year AW's article 'Note on the consistency of maximum likelihood estimate' appeared, which provided a simpler proof for the consistency of the estimation procedure. In particular the proof no longer needed differentiability, and hence even permitted discrete distributions. Thus it is shown that the procedure is consistent in its independence from 'smooth' distributions. The article contains explicit definitions of a kind of distance between the parameters, so that the regarded spaces become metric. AW characteristically aims for a topological generalisation of his research.

6.2.6 Haavelmo, AW and the probabilistic approach

During the 1940s econometricians started to search for alternatives to the time-series approach. The key role in development of the probabilistic approach holds Trygve Haavelmo. [Morgan (1990), Ch8] Haavelmo had contact with the central protagonists between the contemporary economists and statisticians who published in modern econometrics like AW, R. Frisch, Jan Tinbergen, G. Tintner, J. A. Schumpeter, H. Hotelling, Leonid Hurwicz, P. A. Samuelson, J. Neyman, T. Koopmans, Jakob Marschak et al. In mid-January 1938 Haavelmo travelled to Geneva, primarily to work with Tinbergen at the League of Nations' Financial Section. [301] Geneva was in the mid-1930s a center for economists. AW himself worked from September to December 1937 at the Institute des Hautes Etudes (supported by Rockefeller Foundation) but had left Geneva before Haavelmo's arrival. During that time AW had collaborated with Tinbergen in establishing a system of equations reflecting the main forces acting in business cycles. From Geneva AW went back to the 'Institut für Konjunkturforschung' in Vienna, but soon left for the US in the spring of 1938 to start a new life in the US beginning at the Cowles Commission. [302]

In the summers from 1935 to 1940 altogether six Cowles Commission research conferences took place at Colorado Springs [Cowles Foundation (2007)]. The first conference was held in June 1935. In that year first publications on econometrics (from the Cowles Commission members Harold T. Davis and William C. F. Nelson (cp. [Davis, Nelson (1935)]) were done. Davis and

Nelson's book was full of examples but contained almost no theory. AW, freshly arrived from Europe, started with his position as a research fellow of the Commission, and took part in the July 1938 conference where he already was scheduled to speak on indifference surfaces. The fifth conference in July 1939 had a pretty high attendance. Among the audience and speakers were Harold T. Davis, Jakob Marschak (Oxford), Gerhard Tintner (Iowa State College), AW (already at the Columbia University) and many more. [303] Haavelmo presented a paper on the statistical test of a dynamic system. [Haavelmo (1939)] Therein he presented the 'inversion problem' that Frisch and Haavelmo were working on: how should one derive a (damped) solution of a dynamic system from the observed series. AW's contribution was entitled 'The fitting of straight lines if both variables are subject to error'. He showed that consistent estimates could be made with a procedure that was close to a method of confluence analysis. Haavelmo sent Frisch a letter on the conference. He was very satisfied as his own contribution had been received very well. Of the present scientists he ranked AW and Marschak on top. After the Cowles event Haavelmo stayed in Colorado Springs for a couple of weeks and talked further with AW, also Marschak and Tintner. Haavelmo and AW attended also the sixth Cowles Conference in July 1940. In that year the Commission had moved to Chicago, but the conference remained in Colorado Springs. AW meanwhile had dived deeper into statistics and spoke on a new foundation of the maximum likelihood method. (cp. the Report of the Conference, pp33-35). To Haavelmo's pleasance AW had succeeded to show under general assumptions that all maximum-likelihood estimators tended towards a normal distribution. After the conference Haavelmo and AW went hiking in Colorado.

Years before Haavelmo made his pioneering work public [Haavelmo (1941)], [Haavelmo (1944)] [304], R. Frisch in 1934 had written a work on confluence analysis [Frisch (1934)] and also G. Tintner had issued a paper on the variate difference method. [Tintner (1940)] The first generation of monographs on a pristine mathematical treatment of statistical theory (e.g. like the workings of Samuel Wilks (1941) or Harald Cramer (1946)) had not yet appeared. Following the Cowles Commission Conference in 1939 Haavelmo went to the University of California in Berkeley at the end of August or the beginning of September. There he stayed for around two months until the beginning of November. From handwritten notes of a seminar lecture given by him on 2 Nov. 1939 at the Statistical Lab (filed at the Haavelmo Archive, University of California) it is conveyed that in his view any statistical modelling and testing of economic questions is closely connected with the explicit definiton of 'irregular' errors, i.e. errors that have to be given as random variations. From Berkeley Haavelmo travelled back to the Cowles Commission. On his way he stopped for a couple of days in Ames, Iowa, to hold a seminar on invitation of G. Tintner, proceeding on November, 9th, titled 'Problems in the Statistical Testing of Economic Relations'. [305] Finally in Chicago he gave lectures on confluence analysis, reporting to Frisch: "*I have been quite used to being a missionary for Confluence Analysis*". Haavelmo's missionary activities had barely started. [306] He also got his Rockefeller fellowship for 1940 confirmed. On the way to his next location, New York, in December 1939, he primarily was in quest for AW to continue their intellectual exchange. Apparently both liked themselves very much from the first meeting in Colorado Springs and Haavelmo instinctively knew that there was much that could be learned from AW. Both started their work on the determination of coefficients in simultaneous structural relations (the 'reducibility' problems in R. Frisch's 1938 memorandum [307]). Haavelmo wrote to Frisch in March 1940 about AW's conviction that there were no theorems in general mathematical literature that would be of any help. Their problem was to ascertain the rank of matrices where the unknown coefficients themselves entered. The rank could thus be insufficient for ‘critical’ values of the coefficients. It was, according to Haavelmo, “*not always possible to give rules – in the form of a finite number of steps – wherewith one can verify if just these critical values of the coefficients are possible values with regard to a set of observations constrained by the simultaneously fulfilled relations*.”. [308]

Jakob Marschak (head of the Cowles group, was installed for the development of statistical theories on structural estimation from 1943 to 1948) was in the US on a Rockefeller fellowship. It was the time of the outbreak of WWII. Marshak had decided to stay, when an job offer reached him for a position at the New School for Social Research in New York. There he organized a seminar on mathematical economics under the auspices of the NBER (the National Bureau of Economic Research). Haavelmo took part and also presented his ideas. Koopmans regularly attended the seminar in 1940-41. Haavelmo wrote Frisch of him continuing the work on the idea he had set out in a letter from November 1939. He now nicknamed it ‘Distribution-Dynamics’. Haavelmo tried its application in a dynamic analysis of consumption and saving behaviour [308]. From autumn 1940 Haavelmo was at Harvard, where he more or less remained until 1942. There he had a frequent contact with J. A. Schumpeter who read much of his work and advised him on publication. Beeing at Harvard Haavelmo wrote a short review for ‘Social Research’ of a new book by Gerhard Tintner on his ‘variate difference method’ (Cowles Commission Monograph No. 5, Bloomington, Indiana, 1940) on the analysis of economic time series. Thinking deeper about the issues involved, he wrote a longer and more technical note which was published in ‘Econometrica’ [Haavelmo (1941)]. Haavelmo took issue with Tintner on an important point. Tintner assumed that a time series could be modelled as $w_i = m_i + x_i$, with m_i deterministic and all x_i independently and identically distributed. Haavelmo in [Haavelmo (1941), p75], rates this unsatisfactory: “*In modern economic dynamics a simple scheme* *of additive random elements, like the x's above, takes a secondary place as compared with the schemes where the random elements form an integrating part of the fundamental system of dynamic relations. Random events, whether they be "from outside" or resulting from characteristic random spreads in the behavior of different individuals, firms, or groups, usually strike deep into the very structure of economic movements , they change velocities, accelerations, and so forth; they create new initial conditions. Only in very particular schemes would the result be additive – independent random errors "pasted" on the top of some "true" smooth curve.*"

Haavelmo's Rockefeller fellowship got an extension for the year 1941, also in view of the war situation. Haavelmo continued as a missionary for his Confluence Analysis. In springtime 1941 he visited the University of Michigan, in Ann Arbor. There he lectured on Confluence Analysis in February 1941, gave seminars at Harvard and a mimeographed booklet titled 'The Elements of Frisch's Confluence Analysis', appearing at Harvard in the same year. Haavelmo told Frisch that he had refined a technique for presenting the idea and methods of Confluence Analysis and thus been better able to kill misunderstandings of what it was about. [309] Haavelmo added in the letter that he had nearly completed a manuscript on "*statistical verification of economic relations*". Schumpeter was reading it and Haavelmo intended to let AW look at the more technical sections. The manuscript was 'On the Theory and Measurement of Economic Relations', an early version of 'Probability Approach'. [310] The preface is dated April 1941, but it was not completed and mimeographed until after hiking with AW in Maine in August 1941. Haavelmo was very keen to have another summer hike with AW. The two earlier summers they had spent weeks together in Colorado Springs. Those had clearly been very fruitful periods for Haavelmo. As he told AW: "*I have learned more during our previous summers together than during all the time I have spent* *elsewhere in this country*". [313] Haavelmo's manuscript was mimeographed at Harvard at the end of August 1941 and distributed to a number of economists and statisticians. In a letter to Frisch Haavelmo gave a brief summary of the main ideas: "*The more I have worked on the question of statistical verification of economic laws, the question of structural vs. confluent relationships etc., the more convinced I have become about the necessity and fruitfulness of studying these problems from the viewpoint of probability and random variables. In the above mentioned treatise I have thus gone the whole step and tried to put all problems on a probabilistic basis. The problems of statistical verification, measurement of structural coefficients etc. can then be formulated such that they fall under the Neyman-Pearson scheme for testing statistical hypotheses. I have tried to give the general principles for such a formulation. From the beginning I thought that this would cover only a very special group of econometric research problems, and that other kinds of apparatus would be needed for a whole group of problems "where probability considerations do not apply". But this way of looking at it I have come to abandon. There might of course arise problems for which another technique is better, but with regard to the problems that are currently discussed in econometric research, they can all, as far as I can see, be formulated – and that pleasantly precisely and also enormously generally – as questions about testing statistical hypotheses in the Neyman-Pearson sense. Something about this I try to set out and argue for in the Ms. I send you.*" [312]

It can not be overseen that Haavelmo was highly influenced by Neyman and AW, a fact that lead him away from Frisch when he developed an alternative framework for econometrics instead of his confluence analysis. Haavelmo 's objections concerning Neyman's and Pearson's disregard of Fisher's maximum-likelihood method [313] as having no foundation with regard to most powerful tests, shortest confidence intervals etc., could be cleared out by AW. AW had shown in a series of articles that the maximum-likelihood method could be solidly founded on the basis of Neyman-Pearson's scheme and thus Fisher was right after all.

In 1943 Haavelmo reworked the 1941 treatise. The revised version eventually became the 'Probability Approach'. Haavelmo reshuffled the sections in the early part of the thesis and created a new chapter (III) on 'Stochastical Schemes as Basis for Econometrics'. He also added a whole new chapter (VI) on prediction and a short conclusion. The preface and acknowledgements were practically unchanged in the revised version, but clearly the impact of AW had increased. Haavelmo refers in the acknowledgement part of the preface to recent work by AW having given "*a more explicit statistical treatment of problems that in the present study have only been mentioned or dealt with in general terms*". [Haavelmo (1941)] Not long after Haavelmo moved to Washington the 'Probability Approach' was published. An editorial note by the acting editor in the April issue announced that "*in view of the deficiency of material, it has been decided to publish, instead of the July issue, a special supplement containing an extensive monograph by Trygve Haavelmo, "The Probability Approach in Econometrics". A preliminary version of this was privately circulated by the author*." (Econometrica 12, p142). The note also impressed upon the reader that the Probability Approach together with AW 1943 'On the Statistical Treatment of Linear Stochastic Difference Equations' "*gives the foundations for a rigorous statistical testing of economic theory applied to time series and should be very helpful to those working in the field*."

In the 'On the Statistical Treatment of Linear Stochastic Difference Equations' AW discusses several cases. G Tintner decribes this paper [Tintner (1952), p26]: "*He* [AW] *discussed several cases. Suppose first we have a single linear stochastic difference equation. This is a linear difference equation which* *includes a random term. We assume that the random terms are independently (but not necessarily normally) distributed. The stochastic process is stationary. This excludes a trend. Then it can be shown that the application of the classical method of least squares leads to consistent estimates of the constants in the difference equation. In the limit, that is for large samples, the estimates are jointly normally distributed. The covariance matrix of the estimates is ascertained for the large-sample case. This permits the testing of hypotheses. A more complicated case is a system of simultaneous linear stochastic difference equations. The random terms are not autocorrelated and the total system is stationary. If the matrix of the coefficients which involve no time lag is the unit matrix, then the maximum likelihood estimates are shown to follow from an application of the method of least squares to each single equation in the system. The large-sample limiting distribution of the estimates is derived. They are again jointly normally distributed and their covariance matrix is estimated. It is also shown that in the general case there are no unique estimates. We need a priori restrictions on the coefficients of the system or on the covariance matrix of the random elements. These contributions are of great importance in practical econometric work. It is to be hoped that they can be amplified and extended and that they will stimulate work leading to the derivation of small-sample distributions in this field. These would be more useful than the large-sample distributions derived by Wald but are much more difficult to obtain*."

6.2.7 Last papers in econometrics and statistics

At the hight of his career AW became president of the Columbia Institute of Mathematical Statistics in 1948 and also vice president of the American Statistical Association in the same year. After a fruitful 1949 in the US he was invited in November 1950 by the Indian government for a lecture tour through several Indian universities and research centers, what he immediately accepted to do. AW was accompanied by his wife. As his book on statistical decision functions had recently been published, it was his firm intention to teach the new theory to Indian statisticians. During the tour, on December 13, 1950, the Air India plane, therein AW and his wife Lucille as passengers, crashed into a peak of the Nilgiris mountains, sightless of heavy fog. All aboard got killed. Cut off in the prime of his activity AW's work had already changed the course and emphasis of statistics until today. "*The personal loss will be felt by his numerous friends, but all must mourn for the statistical discoveries yet unmade which were buried in the flaming wreckage on a mountain side in South India and which will slowly and painfully have to be made by others.*" [Wolfowitz (1952)].

In 1950 AW's 'Note on zero sum two person games' [Wald (1950) b,] appeared. Here he starts from v. Neumann's game theory [Neumann (1967)], and indicates the restrictions for mixed strategies in a zero-sum two-person game, under which the game stays strictly determined. Again AW's tendency for completion of a theory and also his set-theoretic approach became visible. In that paper his condition for determinateness links the classes of for the two players applicable strategies with the appropriate distributions and density functions, which are tied to their part with the result function, crediting the players with profit or loss. [Wald (1950) b, p740]

In a paper published in the famous No.10 Cowles Commission Monograph from 1950 AW gave "*a new, somewhat unusual, characterization of the identifcation of a linear simultaneous equation system*" [Schneeweiss (2005)]: "*Such a system is given by the matrix equation* $Ax = u$, *wehre* x *is an observable stochastic p-vector and u an unobservable stochastic q-vector with*

$E(u) = 0$, $V(u) = \Sigma$ and A a $q \times q$-matrix of unknown coeffcients. Any linear transformation $A^* = C A$, $u^* = C u$ will lead to a similar system with the same empirical content. Thus A and Σ are not identifiable from given data x_t, $t = 1, 2, \dots$. However economic theory usually provides lots of restrictions on A (and sometimes also on Σ) and if these are rich enough, A (and Σ) will be identifiable. Wald states necessary and suffcient conditions for the unknown parameters of A and Σ to be identifiable."(ibid.)

In 1951 posthumously 'On the fundamental lemma of Neyman and Pearson' was published, a paper that AW wrote in cooperation with G. Danzig. [Wald, Danzig (1951)] AW therein succeeded with a set-theoretical proof in generalizing the Lemma of Neyman and Pearson [Neyman 1936] using much weaker conditions. In the same year the article 'Two methods of randomization in statistics and the theory of games' [Wald, Wolfowitz (1951) a,] in cooperation with J. Wolfowitz appeared. It is shown that the (in AW's book 'Statistical decision functions' already implemented) randomized sequential decision functions are regarded as a special 'mixture' of non-randomized decision functions. This article only experienced little notice despite its theoretical importance of having a basis of (non-randomized) decision functions, with which each randomized decision function can be built as a combination of them. For v. Neumann's game theory this article implies that it is not relevant wether the pure strategies, as statistical decisions, are regarded as randomized or non-randomized functions!

With 'On a relation between changes in demand and price changes', appeared in 1952, AW treats formerly introduced necessary conditions for the existence and uniqueness of an equilibrium of the Walras-Cassel set of equations. AW showed that these conditions are fulfilled, if the matrix of the partial derivatives of the prices is negative definite and the second partial derivatives of the product prices after their quantities are continuous.

7

Summary and Conclusion

7.1 Conclusion

AW contributed his whole life with vital energy to the advancement of mathematical statistics through his broad and fundamental research, that long time after his death continues to influence the development of statistical theory and practice. We have elaborated, that AW, although always bent on a certain mathematical pureness, was eminently important in the mutation of economics into econometrics (at least of parts of economics). All this seemingly happens during his short period in the US. But contentually everything is already put on during his Viennese time, that was formative not only for him, but also for Morgenstern. But AW's bloom time only starts after his emigration and his own accrueing from Menger's and Morgenstern's influence. Morgenstern after the annexation was appointed to Princeton University, where his collaboration with John von Neumann marked the climax of his scientific work and his career, well prepared to this through the long time collaboration and training with AW. In the first chapter of the jointly authored work on game theory Morgenstern finally succeeded in summing up his critique of the Austrian economic school – and it may be considered an irony that after half a century gone, game theory presently finds itself as an indispensable part of mainstream economic theory. AW's roots in the circle(s) of Vienna are undeniable present. In a certain mind he stands also for the idea of the unified science (although he never formulated such philosophical ideas as far as the author has knowledge) as he takes mathematical methods into other fields of interest, employing mathematics as a common base of language for more than one single discipline.

AW, as many other European immigrants, of course engaged himself ardously in war research. As we have shown, one of his achievements for the military was the development of sequential analysis. It was its baffling simplicity of the test procedure made it a favorite one among practitioners. Still, its use is not so wide spread as one might have thought. The main reason seems to be that the sequential sampling procedure as such is not always the cheapest way to improve the quality of the objects of the examination. Although a gain in efficiency (as made by the reduction of average sample size) it is critical if the proportion of the defectives in the lot is either very low or very high. On the other hand, for ethical reasons, sequential sampling has gained a new and bright importance in clinical trials. And just this endouring fact was the starting point for our engagement with AW. It shows that the chapter ' Abraham Wald' is not at all completed.

7.1.1 The man behind

AW never made any effort to popularize his ideas or to make them accessible to a less mathematically agile public. He was deeply interested in refining the mathematical basis of his theorems i.e. achieving generalizations of their assumptions. So minded he arrogated his students to work on abstract questions or to weaken restrictions of some of his theorems. [Wolfowitz (1952)] AW always was ready to do mathematics, but steadily uninterested to popularize his ideas or to work on special applications. This mental attitude surely influenced the reception of his research (a "*program of research* [that] *was not calculated to popularize his theory*" [Wolfowitz (1952)]). Anyhow AW was practical-minded in "*that he always kept statistical ends in sight when working in statistical theory". When the latter was finished to his satisfaction he was not interested in its special application to practical problems*." [Wolfowitz (1952)].

AW himself was a man completely immersed in his work and in no way a public figure like J. v. Neumann. Especially the famous 'Theory of Games and Economic Behavior' never came into reality without AW who in the end made Morgenstern to an real econometrician. AW in fact backed down to his family, his wife Lucille Lang (who met her untimely end with him), and preferably went hiking as it was his chief diversion. AW also was an indefatigable walker and some of his joint papers were worked out on long hikes. He was shy of controversies and avoided all forms of it. He was much absorbed in his family life and very devoted to the children. AW loved his privacy and also his institutes circle of persons he worked with in his pacific way: "*As a gentle and kindly friend and collaborator Abraham Wald was unsurpassed*". [Hotelling (1951), p19]

Years after his death AW for the first time was attacked by Ronald A. Fisher for his design of experiments, his ignoring of basic ideas as set out by Fisher and F. Yates. [Fisher (1955)] This first public altercation found its defendor with Jerzy Neyman in 1956. [Neyman (1956)] Years later in 1986 AW gets credits from Lucien Le Cam, that in statistical decision theory “*the ideas and techniques used, reflect first and foremost the influence of Abraham Wald's writings*.“. [Lucien Le Cam (1986)]

7.1.2 The man between

During the 20th century there were three remarkable changes in the discipline of economics. The mathematisation of economics and in follows the foundation of econometrics, and in a contrast to econometrics the emergence of macro-economics by J. M. Keynes from 1936 on. T. Haavelmo, J. Tinbergen and R. Frisch formed the body of the new economic knowledge, that was modeled predominantly on parameters that themselves were determined statistically. This enthusiastic opening towards a mathematisation had its start in the US. What we we have shown througout the text is the step behind that common view, finding AW as a key in many of the fundamental developments in the early years of econometrics. Among others, but at the front edge, AW formed economics into a science (as we gave a definition of ‘science‘ in chapter 6 following Webster). There happened also a feedback into the field of economics: it became more dataled, as numerous data sources and the econometric methods for the analysis of these data became available, including statistical tests of theories. As we have shown, economic modeling played a main role, statements about probability distributions, and techniques, which give a statistical control came to the forefront. The way how unexpected economic events or demands of a huge magnitude exerted a strong discipline on mathematical pattern of economics to be a least partially tradeable.

To come to a conclusion about AW’s achievements, we will have a short gaze on two opposed economists and their views, so see how valuable AW's approach is. Heilbroner saw in some economists a kind of 'worldly philosophers': "*The worldly philosophers thought their task was to model all the complexities of an economic system - the political, the sociological, the psychological, the moral, the historical*" and in others "*... au contraire, do not want so complex a vision. They favor two-dimensional models that in trying to be scientific leave out too much and leave modern economists without a true understanding of how the system works*.". [Heilbroner (1953)] Following Heilbroner, both groups lack a certain correspondence to reality. The remedy of that deficit would be a regaining 'worldliness' by taking out to much prosaic over-complexity and also becoming aware what is really modeled within the mathematical formulas used. In both cases AW was exemplary. He never loosed sight of what he modeled and always took the line of an efficient simplicity avoiding complex and error-prone approaches. Another witness for the casa AW could be Leonard Savage, whose book 'The Foundations of statistics' [Savage (1954)] is an important component in the development of modern economics. Savage version of the statistical decision theory was a subjective expected utility theory with Bayes rules for the changes of decision. It was Savage, who established the 'state act consequence' model. And the remarkable hereby is, that he used the minimax-ansatz of AW to show the trueness of his subjectivistic view, by showing that all statistic problems can be represented as AW's minimax problem, and that each minimax problem has a subjectivistic interpretation. In Savage interpretation uncertainty is a kind of probability-driven personal faith, but with strict rational restrictions (the rationality is reached by means of consistency). Although boisterously attacked, Savage theory (on the shoulders of AW) found its way to become a prominent part into economics.

The opposite occurred to AW's own contributions. As said above he lacked a broader reception. While AW did not lose sight of his aim to treat the economic problems as ‘scientific’ as possible (following the paradigm of mathematics) and spur it into eonometrics, his results mainly got studied by mathematicians and protagonists of the mathematically oriented research. AW’s methods are as tools for a accurate consideration highly important, but bear a formal barrier that makes it necessary to study and adapt his work. His work opposes a simple transmission into arbitrary economic questions, what prevents at first a quick and appropriate appliance in economics. The circumstance, that a wide audiance in economics at the times of AW were ‘worldly philophers‘ (to use Heilbroner’s phrase), of course played the role that his papers were unrecognized in economics, but also the fact that the other fraction of ‘modelists‘ stays within its formalist modus operandi. But AW was between, an always ingrate position. Finally we hope to have worked on AW’s revalidation in the history of (neoclassical) economics, that is mainly dominated by the US. Perhaps he is the one who has shown a way to avoid to much tiling at windmills, the ones on the abstract side and the ones on the prosaic side. AW stands for a new kind of engineering intervention in economics.

7.2 In honour of AW

7.2.1 Abraham Wald Prize in Sequential Analysis

With the permission and a monetary support from the AW’s family, Taylor & Francis and Associate Editors of the ‘Sequential Analysis‘ journal, the so-called ‘Abraham Wald Prize in Sequential Analysis’ has been established in 2004. It is awarded for the best judged published article in the ‘Sequential Analysis’. This prize will be given during spring time of each year from 2005 on, taking into account all articles of an author that he has published in the 'Sequential Analysis' journal during the preceeding full year. Eventually the prize is combined with a special 'Sequential Analysis Lecture' during an annual statistical event. An account dedicated to the 'Abraham Wald Prize in Sequential Analysis' has been established within the Department of Statistics at the University of Connecticut.

7.2.2 Sequential Analysis Journal: Abraham Wald Centennial Celebration

The Sequential Analysis Journal celebrated in 2004 the centenary of AW's birth with a call for papers that were appearing in all four issues for the year 2004, titled as 'Abraham Wald Centennial Celebration: Invited Papers'. The journal, in its own wording, has the 'aims and scopes to embrace new and emerging areas in statistics'. [SQA (2008)]

7.2.3 Abraham Wald PhD programme at the Vienna University

A Ph.D program at the department of statistics of the Vienna University (valid from 2008 on) has been titled 'Abraham Wald PhD in Statistics and Operations Research'. The accompanying text on the institutes website includes following line: "*The eponym Abraham Wald studied and taught during 1927 - 1938 at the University of Vienna. His works in Statistical Decision Theory as well as Linear Optimization and Equilibrium Theory were pioneering*". [Department of Statistics, Vienna University (2008)]

7.2.4 Abraham Wald memorial lectures at the IMS

"The Wald Memorial Lectures honor Professor Abraham Wald. The Wald Lecturer gives two, three or four one-hour talks on one subject. This gives sufficient time to develop material in some detail and make it accessible to nonspecialists." [IMS (2013)]

7.2.5 AMS Abraham Wald memorial edition (1952)

The 'Annals of Mathematical Statistics' (AMS) was published between 1930 and 1972 by the Institute of Mathematical Statistics. The 1952 edition is dedicated to the memory of AW. [Wolfowitz (1952)]

8 Appendix

A: Acronyms and Abbreviations

- AW: Abraham Wald
- CSP: Christian Social Party / Christlich Soziale Partei
- DAP: German Worker's Party / Deutsche Arbeiterpartei
- EemK: Ergebnisse eines mathematischen Kolloquiums
- GE: General Equilibrium
- GJE: Abbreviation for the '*Handbuch zur Geschichte der Juden in Europa*', [Baleanu(2001)]
- GDVP: Pan-German Party / Großdeutsche Volkspartei
- SPRT: Sequential Probability Ratio Test
- KM: Karl Menger, e.g. KM Archive
- KPÖ: Communist Party of Austria / Kommunistische Partei Österreichs
- KWEG: Wartime Economy Enabling Act /

Kriegswirtschaftliches Ermächtigungsgesetz

□ NSDAP: Nationalsozialistische Deutsche Arbeiterpartei

□ OM: Oskar Morgenstern, e.g. OM Archive

□ SDAP: Social Democratic Workers' Party /

Sozialdemokratische Arbeiterpartei

□ SRG: Statistical Research Group

B: Bibliography of the original works of AW

(citation scheme: [coauthor,] (book)title, [magazine, volume, number,] page(s) [, publishers, cities].)

1931

□ On Hilbert's System of Axioms, Dissertation of AW, 1931.

□ Axiomatik des Zwischenbegriffes in metrischen Räumen, Wiener Akadademischcr Anzeiger, No. 16, pp. 1-3.

□ Axiomatik des Zwischenbegriffes in metrischen Räumen, Math. Ann., Vol. 104, pp476-484.

□ Axiomatik des metrischen Zwischenbegriffes, Ergebnisse eines Math. Kolloquiums, Vol. 2, p17.

□ Über den allgemeinen Raumbegriff, Ergebnisse eines Math. Kolloquiums, Vol. 3, pp. 6-11.

□ Über das Hilbertsche Axiomensystem der Geometrie, Ergebnisse eines Math. Kollioquiums, Vol. 3, pp23-24.

1933

□ Zur Axiomatik der Verknüpfungsbeziehungen, Ergebnisse eines Math. Kolloquiums, Vol. 4, p6.

□ Zur Axiomatik des Zwischenbegriffes, Ergebnisse eines Math. Kolloquiums, Vol. 4; pp23-24.

□ Über die Volumsdeterminante, Ergebnisse eines Math. Kolloquiums, Vol. 4, pp25-28.

□ Über das Volumen der Euklidischen Simplexe, Ergebnisse eines Math. Kolloquiums, Vol. 4, pp32-33.

□ Halbmetrische Räume und Konvexifizierbarkeit, Ergebnisse eines Math. Kolloquiums, Vol. 5, p. 7.

□ Vereinfachter Beweis des Steinitzschen Satzes über Vektorenreihen im $\mathbb{R}_n$, Ergebnisse eines Math. Kolloquiums, Vol. 5, pp10-13.

□ Bedingt konvergente Reihen von Vektoren im $\mathbb{R}_n$, Ergebnisse eines Math. Kolloquiums, Vol. 5, pp13-14.

□ Reihen in topologischen Gruppen, Ergebnisse eines Math. Kolloquiums, Vol. 5, pp14-16.

□ Komplexe und indefinite Räume, Ergebnisse eines Math. Kolloquium., Vol. 5, pp32-42.

1934

□ Über die eindeutige Lösbarkeit der neuen Produktionsgleichungen, Ergebnisse eines Mathematischen Kolloquiums, Vol I, Num. 6, pp12–18, 1933–34.

1935

□ Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen (Mitteilung 1), Ergebnisse eines Math. Kolloquiums, Vol. 6, pp. 12-18.

□ Über die Anzahl der abgerundeten Mengen, Ergebnisse eines Math. Kolloquiums, Vol. 6, p. 26.

□ Eine Charakterisierung des Lebesgueschen Masses, Ergebnisse eines Math. Kolloquiums, Vol. 6, pp. 27-29.

□ Zur Differentialgeometrie der Flächen: Eine neue Definition der Flächenkrümmung, Ergebnisse eines Math. Kolloquiums, Vol. 6, pp. 29-39.

□ Sur la courbure des surfaces, C. R. Acad. Sci. Paris, Vol. 201, pp. 918-920.

□ La nature et le calcul des variations saisonnières. Un expose preliminaire par

A. Wald, Memorandum par Dr. A. Wald, distribué à l'occasion de la conférence de Dr. Oskar Morgenstern, prononcée le 6 mai 1935 à l'Institute Scientifiques des Recherches Econmiques et Sociales, Paris. (Menger archive, Durham, Box 9, Folder 2)

□ Über die Produktionsgleichungen der ökonomischen Wertlehre (Mitteilung II), Ergebnisse eines Mathematischen Kolloquiums, Vol 7, pp1–6.

1936

□ Über einige Gleichungssysteme der mathematischen Ökonomie, Zeitschrift für Nationalökonomie, 7, pp637-670. English translation by Otto Eckstein.

□ Sur la notion de collectif dans le calcul des probabilités, C. R. Acad. Sci. Paris, Vol. 202, pp. 180-183.

□ Begründung einer koordinatenlosen Differentialgeometrie der Flächen, Ergebnisse eines Math. Kolloquiums, Vol. 7, pp24-46.

□ On some systems of equations of mathematical economics, Econometrica, Vol. 19 (1951), pp368-403.

□ Berechnung und Ausschaltung von Saisonschwankungen, Beiträge zur Konjunkturforschung, Vol. 9, Verlag Julius Springer, Vienna.

1937

□ Zur Theorie der Preisindexziffern, Zeitschrift für Nationalökonomie 8, 179-219.

□ Extrapolation des gleitenden 12-Monatsmittels, Monatsberichte des Österreichischen Institutes für Konjunkturforschung, Vol. 11, Appendix 8 of November issue, pp. i-vii.

□ Ein Streckenbild, für welches λ_φ nicht existiert, obwohl λ_φ für jeden Anfangsabschnitt existiert, Ergebnisse eines Math. Kolloquiums, Vol. 8, pp34-35.

□ Die ϕ-Länge im Hilbertschen Raum, Ergebnisse eines Math. Kolloquiums, Vol. 8, pp. 36-37.

□ Grundssätz1iches zur Berechnung des Produktionsindex, Monatsberichte des Österreichischen Institutes für Konjunkturforschung, Vol. 11, Appendix 6 of February issue, pp. i-vu.

□ Die Widerspruchsfreiheit des Kollektivbegriffes der Wahrscheinlichkeitsrechnung, Ergebnisse eines Math. Kolloquiums, Vol. 8, pp 38-72.

1938

□ Die Widerspruchsfreiheit des Kollektivbegriffes der Wahrscheinlichkeitsrechnung, Actualités Scientifiques et Industrielles 755, Colloque Consacré à la Théorie des Probabilités, Hermann et Cie., Paris, pp79-99.

□ Generalization of the Inequality of Markoff, Annals of Math. Stat., Vol.9, No.

4, pp. 244-255.

1939

□ Long cycles as a result of repeated integration, Am. Math. Monthly, Vol. 46, pp136-141.

□ Confidence limits for continuous distribution functions (with J. Wolfowitz), Annals of Math. Stat., Vol. 10, pp305-118.

□ Limits of a distribution function determined by absolute moments and inequalities satisfied by absolute moments, Trans. Am. Math. Soc., Vol. 46, pp280-306.

□ A new formula for the index of cost of living, Econometrica, Vol. 7, pp319-331.

□ Contributions to the theory of statistical estimation and testing hypotheses, Annals of Math. Stat., Vol. 10. pp. 299-326.

1940

□ A note on the analysis of variance with unequal class frequencies, Annals of Math. Stat., Vol. 11, pp96-100.

□ The approximate determination of indifference surfaces by means of Engel curves, Econometrica, Vol. 8, pp144-175.

□ The fitting of straight lines if both variables are subject to error, Annals of Math. Stat., Vol. 11, pp284-300.

□ On a test wether two samples are from the same population (with J. Wolfowitz), Annals of Math. Stat., Vol. 11, pp147-162.

□ Approximate determination of indifference surfaces by means of Engel curves, Econometrica 8, April, S144–73, 1940.

□ Contributions to the Theory of Statistical Estimation and Testing Hypotheses, Annals of Mathematical Statistics, 10, 1940, pp299-326.

□ Note on the Analysis of Variance with Unequal Class Frequencies, The Annals of Mathematical Statistics, 1940, pp69-100.

1941

□ Note on confidence limits for continuous distribution functions (with J. Wolfowitz), Annals of Math. Stat., Vol. 12, pp118-119.

□ Asymptotically most powerful tests of statistical hypotheses, Annals of Math. Stat., Vol. 12, pp1-19.

□ On the distribution of Wilks' statistic for testing the independence of several groups of variates (with R. J. Brookner), Annals of Math. Stat., Vol. 12, pp 137-152.

□ On the analvsie of variance in case of multiple classifications with unequal

class frequencies, Annals of Math. Stat., Vol. 12, pp346-350.

□ Some examples of asymptotically most powerful tests, Annals of Math. Stat.,Vol. 12, pp396-400.

1942

□ 0n the Principles of Statistical Inference, Notre Dame Mathematical Lectures, No. 1, University of Notre Dame.

□ Asymptotically shortest confidence intervals, Annals of Math. Stat., Vol. 13, pp127-137.

□ On the choice of the number of class intervals in the application of the chi-square teat (with H. B. Mann), Annals of Math. Stat., Vol. 13, pp306-317.

□ Setting of tolerance limits when the sample is large, Annals of Math. Stat., Vol 13, pp389-399.

□ On the power function of the analysis of variance test, Annals of Math. Stat., Vol. 13, pp. 434-439.

1943

□ Tests of statistical hypotheses concerning several parameters when the number of observations is large, Trans. Am. Math. Soc., Vol. 54, pp426-482.

□ Sequential Analysis of Statistical Data, a report submitted by the Statistical

Research Group, Columbia University, to the Applied Mathematics Panel, National Defense Research Committee.

□ An extension of Wilks' method for setting tolerance limits. Annals of Math. Stat., Vol. 14, pp45-55.

□ On a statistical generalization of metric spaces. Proceedings of the National Academy of Sciences, Vol. 29, pp196-197.

□ On the efficient design of statistical investigations. The Annals of Mathematical Statistics, Vol. 14, pp134-140.

□ with H. B. Mann, On stochastic limit and order relationships, Annals of Math. Stat., Vol. 14, pp. 217-226. Also in Econometrica, Vol. 11, pp173-229.

□ with J. Wolfowitz, An exact test for randomness in the nonparametric case based on serial correlation, Annals of Math. Stat., Vol. 14, pp. 376-388.

1944

□ A General Method of Deriving the Operating Characteristics of any Sequential Probability Ratio Test, unpublished memorandum submitted to the Statistical Research Group, Columbia University, April, 1944.

□ On Cumulative Sums of Random Variables. The Annals of Mathematical Statistics, Vol 15, pp. 283-296.

□ On a Statistical Problem Arising in the Classification of an Individual into One of Two Groups. The Annals of Mathematical Statistics, Vol 15, pp145-162.

□ Note on a Lemma, The Annals of Mathematical Statistics, Vol. 15, No. 3, Sep., pp330-333.

□ with J. Wolfowitz, Statistical tests based on permutations of the observations, Annals of Math. Stat., Vol. 15, pp. 358-372.

1945

□ Sequential Tests of Statistical Hypotheses, in The Annals of Mathematical Statistics, 16, p117-186, June, 1945.

□ Sequential Tests of Statistical Hypotheses, revised edition for the Statistical Research Group.

□ Statistical Decision Functions Which Minimize the Maximum Risk. The Annals of Mathematics, Vol 46, No. 2, pp265-280.

□ Sequential Method of Sampling for Deciding Between Two Courses of Action. Journal of the American Statistical Association, Vol. 40, pp. 277-306.

□ Generalization of a Theorem by v. Neumann Concerning Zero Sum Two Person Games, The Annals of Mathematics, Vol.46, April 1945.

□ Some Generalizations of the Theory of Cumulative Sums of Random Variables, The Annals of Mathematical Statistics, 1945.

□ with J. Wolfowitz, Sampling inspection plans for continuous production which insure a prescribed limit on the outgoing quality, Annals of Math. Stat., Vol. 16, pp30-49.

□ Statistical decision functions which minimize the maximum risk, Annals of Math. Stat., Vol. 46, pp. 265-280.

□ Generalization of a theorem by v. Neumann concerning zero-sum two-person games, Annals of Mathematics, Vol. 46, No. 2, pp. 281-286.

□ Sequential tests of statistical hypotheses, Annals of Math. Stat., Vol. 16, pp. 117-186.

□ Some generalizations of the theory of cumulative sums of random variables, Annals of Math. Stat., Vol. 16, pp. 287-293.

1946

□ with J. Wolfowitz, Tolerance limit for a normal distribution, (with J. Wolfowits), Annals of Math. Stat., Vol. 17, pp. 208-215.

□ Some improvements in setting limit, for the expected number of observations required by a sequential probability ratio test, Annals of Math. Stat., Vol. 17, pp. 466-474.

□ Differentiation Under the Expectation Sign in the Fundamental Identity of Sequential Analysis. The Annals of Mathematical Statistics, Vol 17,pp 493-497 1946.

□ The Efficient Design of experimental Investigations, (a group of AW's students), Lecture notes of an one semester course of AW given at the Columbia University, NY, 1943. Published in script form in 1946 with special permission of AW.

1947

□ Limit distribution of the maximum and minimum of successive cumulative sums of random variables, Bull. Am. Math. Soc., Vol. 63, pp. 142-153.

□ with C. Stein, Sequential confidence intervals for the mean of a normal distribution with known variance, Annals of Math. Stat., Vol. 18, pp. 427-433.

□ Sequential Analysis, John Wiley & Sons Inc., New York, Chapman & Hall, Ltd., London, 1947

□ An Essentially Complete Class of Admissible Decision Functions, Annals of Mathematical Statistics, Vol. 18, pp549-555.

□ Limit distribution of the maximum and minimum of successive cumulative sums of random variables. Bull. Amer. Math. Soc.

□ Foundations of a general theory of sequential decision functions, Econometrica, Vol 15, pp. 279-313, October, 1947.

□ with C. Stein, Sequential Confidence Intervals for the Mean of a Normal Distribution with Known Variance. The Annals of Mathematical Statistics.

□ An Essentially Complete Class of Admissible Decision Functions. The Annals of Mathematical Statistics.

□ A Note on Regression Analysis, The Annals of Mathematical Statistics, Vol. 18, No. 4, Dec., pp586-589.

□ The Theory of Games and Economic Behaviour by J. v. Neumann and O. Morgenstern, Book Review by AW, The Review of economic Statistics, Vol

XXIX, No 1, pp47-61, Febr, 1947.

1948

□ with Jacob Wolfowitz, Optimum character of the sequential probability ratio test. Annals of Mathematical Statistics, Vol. 19, pp326-339, 1948.

□ Note on the Consistency of the Maximum Likelihood Estimate. The Annals of Mathematical Statistics, 1948.

□ Asymptotic Properties of the Maximum Likelihood Estimate of an Unknown Parameter of a Discrete Stochastic Process, The Annals of Mathematical Statistics, 1948.

□ Estimation of a Parameter When the Number of Unknown Parameters Increases Indefinitely with the Number of Observations, The Annals of Math. Stat., Vol. 19, No. 2 Jun., pp220-227, 1948.

□ Asymptotic propertie. of the maximum likelihood estimate of an unknown parameter of a discrete stochastic process, Annal. of Math. Stat., Vol. 19, pp40-46.

□ On the distribution of the maximum of successive cumulative sums of independently but not identically distributed chance variables, Bull. Am. Math. Soc., Vol. 54, pp. 422-430.

□ Estimation of a parameter when the number of unknown parameters increases indefinitely with the number of observations, Annals of Math. Stat., Vol. 19, pp. 220-227.

□ with J. Wolfowitz, Optimum character of the sequential probability ratio test, Annals of Math. Stat., Vol. 19, pp. 326-339.

1949

□ with Agnes Berger, On Distinct Hypotheses, The Annals of Mathematical Statistics, Vol. 20, No. 1, Mar., pp104-109, 1949.

□ Statistical Decision Functions, Ann. Math. Statist. [314], Vol. 20, No. 2, pp165-205.

□ with J. Wolfowitz, Bayes solutions of sequential decision problems, Proc. Nat. Acad. Sc., Vol. 35, pp99-102.

□ with Agnes Berger, On distinct hypotheses, Annals of Math. Stat., Vol. 20, pp104-109.

□ Statistical Decision Functions, The Annals of Mathematical Statistics. Volume 20, Number 2, pp165-205.

□ Note on the consistency of the maximum likelihood estimate, Annals of Math. Stat., Vol. 20, pp. 506-601.

□ with Milton Sobel, A sequential decision procedure for choosing one of three hypotheses concerning the unknown mean of a normal distribution, Annals of Math. Stat., Vol. 20, pp502-622.

1950

□ Statistical Decision Functions, John Wiley & Sons Inc., New York, Chapman & Hall Ltd., London.

□ Note on Zero Sum Two Person Games, The Annals of Mathematics, Vol 52(3), p739

□ with J. Wolfowitz, Bayes solutions of sequential decision problems, Annals of Math. Stat., Vol. 21, pp82-89.

□ with A. Dvoretzky and J. Wolfowitz, Elimination of randomization in certain problems of statistics and of the theory of games, Proc. Nat. Acad. Sci., Vol. 36, pp256-200.

□ Note on the identification of economic relations: Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph No. 10, John Wiley and Sons, pp238-244.

□ Remarks on the estimation of unknown parameters in incomplete systems of equation, Statistical Inference in Dynamic Economic Models, Cowles Commission, Monograph No. 10, John Wiley and Sons, pp306-310.

1951

□ Note on zero-sum two-person games, Annals of Mathematics, Vol. 52, pp. 739-742.

C: Reprints, posthumous publications and reviews

1951

□ On some systems of Equations of Mathematical Economics, Econometrica, Vol. 19, Num. 4, pp368-403, 1951. [Translation of the 1936 paper]

□ O. Morgenstern, Abraham Wald, 1902-1950. Econometrica, 1951.

□ with G. Dantzig, On the Fundamental Lemma of Neyman and Pearson, The Annals of Mathematical Statistics, Vol. 22, No. 1, Mar., 1951, pp. 87-93, 1951.

□ Harold Hotelling, Abraham Wald, The American Statistician, Vol. 5, No. 1, Feb., 1951, pp. 18-19, 1951.

□ L. J. Savage, The Theory of Statistical Decision, Journal of the American Statistical Association, Vol. 46, No. 253, Mar., pp55-67, 1951.

□ B de Finetti, L'opera di Abraham Wald e l'assestamento concettuale della statistica matematica moderna, Statistica, Milano 11 (1951), 185-192.

□ S. N. Roy, Obituary: Abraham Wald, Calcutta Statist. Assoc. Bull. 3 (1951), 133-138.

□ L. Schmetterer, Obituary: Abraham Wald, Statist. Vierteljschr. 4 (1951), 69-74.

□ with A. Dvoretzky and J.Wolfowitz, Relations among certain ranges of vector measure, Pacific Jour. Math., Vol. 1, pp59-74.

□ with A. Dvoretzky and J. Wolfowitz, Elimination of randomization in certain statistical decision procedures and zerosum two-person games, Annals of Math. Stat., Vol. 22, pp1-21.

□ with O.B. Dantzig, On the fundamental lemma of Neyman and Pearson, Annals of Math. Stat., Vol. 22, pp87-93.

□ with J. Wolfowitz, Two methods of randomization in statistics and the theory of games, Annals of Mathematics, Vol. 53, pp581-586.

□ Asymptotic minimax solutions of sequential point estimation problems, Proceedings of Me Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press.

□ with J. Wolfowitz, Characterization of the minimal complete class of decision functions when the number of distributions and decisions is finite, Proc. Second Berkeley Symposium on Mathematical Statistics and Probability, Univ. of California Press, pp149-157.

1952

□ J. Wolfowitz, Abraham Wald, 1902-1950. The Annals of Mathematical Statistics, Vol 23, pp1-13, 1952.

□ K. Menger, The Formative Years of Abraham Wald and His Work in Geometry, The Annals of Mathematical Statistics, 1952.

□ On a relation between changes in demand and price changes, Econometrica, Vol. 20, No. 2, pp304-305.

□ G. Tintner, Abraham Wald's Contributions to Econometrics, The Annals of Mathematical Statistics, Vol. 23, No. 1, Mar., pp21-28, 1952.

□ The publications of Abraham Wald, Ann. Math. Statistics 23 (1952), 29-33.

□ Basic ideas of a general theory of statistical decision rules, Proceedings of the International Congress of Mathematicians, Harvard University Press.

1955

□ Tenting the difference between the means of two normal populations with unknown standard deviations, in Selected Papers in Statistics and Probability, McGraw-Hill Book Company, New York-Toronto-London, 1955.

□ Abraham Wald, Selected papers in statistics and probability, McGraw-Hill Book Company, New York-Toronto-London, 1955. (2nd Editon 1957)

1968

□ H. Freeman, Abraham Wald, in D. L. Sills (Ed.), International Encyclopedia of Social Sciences 16, 1968, pp435-438.

1980

□ A Method of Estimating Plane Vulnerability Based on Damage of Survivors, Reprint, Center of Naval Analyses, 1980.

1997

□ Sequential Tests of Statistical Hypotheses and Statistical Decision Functions, in: Breakthroughs in Statistics, Vol 1, Springer, 1997.

□ L. Weiss, Abraham Wald, in N. L. Johnson and S. Kotz (eds.), Leading personalities in statistical sciences, New York, pp164-167, 1997.

D: International Encyclopedia of Unified Science

In 1938 the 'International Encyclopedia of Unified Science' as a new series of publications started in the US. Its volumes are titled 'Fundamentals of Unified Science' (FUS). It was an ambitious project, but never completed. The series was an output of former members of the Vienna Circle. Only the first area on the 'Foundations of the Unity of Sciences' was published. It contains two volumes, consitsing of twenty monographs in total, that were published between 1938 and 1969.

Volume I

□ Encyclopedia and Unified Science (FUS I-1), Otto Neurath, Niels Bohr, John Dewey, Bertrand Russell, Rudolph Carnap, and Charles Morris

□ Foundations of the Theory of Signs (FUS I-2), Charles Morris

□ Foundations of Logic and Mathematics (FUS I-3), Rudolph Carnap

□ Linguistic Aspects of Science (FUS I-4), Leonard Bloomfield

- Procedures of Empirical Science (FUS I-5), Victor F. Lenzen
- Principles of the theory of probability (FUS I-6), Ernest Nagel
- Foundations of Physics (FUS I-7), Philipp Frank
- Cosmology (FUS I-8), E. Finlay-Freundlich
- Foundations of Biology (FUS I-9), Felix Mainx
- The Conceptual Framework of Psychology (FUS I-10), Egon Brunswik

Volume II

- Foundations of the social sciences (FUSII-1), Otto Neurath
- Structure of scientific revolutions (FUSII-2), Thomas S. Kuhn
- Science and the structure of ethics (FUSII-3), Abraham Edel, 1961.
- Theory of valuation (FUSII-4), John Dewey, 1939.
- Technique of theory construction (FUSII-5), J. H. Woodger, 1939.
- Methodology of mathematical economics (FUSII-6), Gerhard Tintner, 1968.
- Fundamentals of concept formation in empirical science (FUSII-1) / Hempel.
- Development of rationalism and empiricism (FUSII-1) / Santillana and Zilsel.
- Development of logical empiricism (FUSII-1) / Joergensen.

□ Bibliography and index (FUSII-10), Herbert Feigl and Charles Morris.

E: List of works edited by the Vienna Circle

The following lists the **Schriften zur wissenschaftlichen Weltauffassung** (papers on scientific world-view), edit by Schlick and Frank.

□ R. von Mises, Wahrscheinlichkeit, Statistik und Wahrheit, 1928, (Probability, statistics, and truth, New York : Macmillan company, 1939)

□ R. Carnap, Abriss der Logik, 1929

□ M. Schlick, Fragen der Ethik, 1930, (Problems of ethics, New York : Prentice-Hall, 1939)

□ O. Neurath, Empirische Soziologie, 1931

□ P. Frank, Das Kausalgesetz und seine Grenzen, 1932, (The law of causality and its limits, Dordrecth ; Boston, Kluwer, 1997)

□ O. Kant, Zur Biologie der Ethik, 1932

□ R. Carnap, Logische Syntax der Sprache, 1934, (The logical syntax of language, New York, Humanities, 1937)

□ K. R. Popper, Logik der Forschung, 1934, (The logic of scientific discovery, New York, Basic Books, 1959)

□ J. Schächeter, Prolegomena zu einer kritischen Grammatik, 1935,

(Prolegomena to a critical grammar, Dordrecth, Boston, D. Reidel Pub. Co., 1973)

- V. Kraft, Die Grundlagen einer wissenschaftliche Wertlehre, 1937, (Foundations for a scientific analysis of value, Dordrecth, Boston, D. Reidel Pub. Co., 1981)
- Einheitswissenschaft (Unified science), edited by R. Carnap, Frank, H. Hahn, Neurath and Joergensen (after Hahn's death), also Morris (from 1938 on)
- H. Hahn, Logik, Mathematik und Naturerkennen, 1933
- O. Neurath, Einheitswissenschaft und Psychologie, 1933
- R. Carnap, Die Aufgabe der Wissenschaftlogik, 1934
- P. Frank, Das Ende der mechanistichen Physik, 1935
- O. Neurath, Was bedeutet rationale Wirtschaftsbetrachtung, 1935
- O. Neurath, E. Brunswik, C. Hull, G. Mannoury, J. Woodger, Zur Enzyclopädie der Einheitswissenschaft. Vorträge, 1938
- R. von Mises, Ernst Mach und die empiritische Wissenschaftauffasung, 1939

These works are translated in 'Unified science - The Vienna Circle monograph series originally edited by Otto Neurath' in [Neurath (1987)].

F: Original members of the Vienna Circle

□ Rudolf Carnap (Ronsdorf, 1891 - Santa Monica, CA, 1970)

Rudolf Carnap, a German-born philosopher of logical positivism, was very influential in Europe until 1935 and later in the US. Carnap was a leading figure of the Vienna Circle and a prominent exponent of logical positivism. Carnap visited from 1910 to 1914 the University of Jena, intending to study physics. But instead, he attended a course about Kant taught by Bruno Bauch, and was one of the few to participate Frege's courses in mathematical logic. After WWI (fighting for Germany) he continued with physics at the University of Berlin, where Albert Einstein was a newly appointed professor. Back in Jena Carnap finally in 1922 wrote a thesis under Bauch's supervision in philosophy.

In 1923 Carnap met Hans Reichenbach and was introduced by H. Reichenbach to Moritz Schlick, who in turn gave him (from 1926 on) a position in his department and brought Carnap into the Vienna Circle. In 1929 Carnap (with H. Hahn and O. Neurath) wrote the 1929 manifesto of the Circle, and founded with H. Reichenbach the journal ‘Erkenntnis’. In 1928 he published his famous ‘Der logische Aufbau der Welt’, in which he developed a formal version of empiricism, defining scientific terms in phenomenalistic correspondents. [Carnap (1928)] This book was highly visible influenced by the ‘Principia Mathematica’ by B. Russel and A.N. Whitehead. [Russel (1913)] In 1930 Carnap met Tarski, who lectured in Vienna. Hence Carnap learned much about Tarski's model theoretic approach to semantics. In 1931, Carnap, meanwhile Professor of the German language at the University of Prague, wrote the ‘Logische Syntax der Sprache’ [Carnap (1934)], a book, that made him the most famous logical positivist and member of the Vienna Circle. [Schilpp (1936)]

Carnap, a socialist and pacifist, had no illusions about the Third Reich, hence emigrated to the US in 1935 and became there a naturalized citizen in 1941. From 1936 to 1952, Carnap was a professor of philosophy at the University of Chicago. There he productively wrote a series of books on semantics and modal logic. After a period at the Institute for Advanced Study in Princeton, Carnap joined the philosophy department at UCLA in 1954, where Hans Reichenbach had died one year before. At UCLA, Carnap taught until his retirement in 1961. He died in Santa Monica (California) on 14 September 1970. Carnaps writings on thermodynamics and on the foundations of probability and induction, were published posthumously. [Schilpp (1936)]

□ Herbert Feigl (Reichenberg, 1902 - Minneapolis, 1988)

In 1922 H. Feigl enrolled at the LMU in Munich for physics, mathematics and philosophy in Munich and continued his studies in 1922 in Vienna under Moritz Schlick. In that year he joined the Vienna Circle. In 1927, also at the Vienna University, he received his doctorate in philosophy. In 1930 H. Feigl had to emigrate to the US, where he received a Rockefeller Fellowship at Harvard University. In 1933 Feigl was appointed Assistant professor at the University of Iowa and 1937 Associate Professor, ibid. In 1940 he got for the second time a Rockefeller Fellowship at Harvard University and Columbia University and accepted a position as professor of philosophy at the University of Minnesota, where he remained for 31 years. In 1953 he founded the Minnesota Center for Philosophy of Science. In 1971 he resigned as its director, but worked until his death in 1988 as philosopher. [Savage (1988)]

□ Philipp Frank (Vienna, 1884 - Cambridge, MA, 1966)

Philipp Frank, a born Austrian, worked as a physicist and mathematician but was also an influential philosopher in logical positivism during the first half of the 20th century. [Holton (1968)] He graduated from the University of Vienna in 1907 with a thesis in theoretical physics under Ludwig Boltzmann. It was no less than Albert Einstein who gave his recommendation for Frank (impressed by Frank's 1907 paper on causality) to be the right one for a professorship at the Charles University of Prague, a position which Frank held in effect from 1912 until 1938, the year when he emigrated to the US to became a lecturer of physics and mathematics at Harvard University. Frank himself not only was a huge admirer of Mach and Einstein but also made both to topics to work on. At Harvard Frank lectured on Mach. [MacTutor (2011)]

Frank, Hahn and R. von Mises were the most active parts of the Vienna Circle from 1920 onwards. They were influenced in their philosophical thinking by Frege, Russell and Whitehead. [Horton (1968)] Frank worked in mathematics on the calculus of variations, Fourier series, function spaces, and in physics on Hamiltonian geometrical optics, Schrödinger wave mechanics, and relativity. [Connor, Robertson (2011)]

□ Kurt Gödel (Brno, Moravia, 1906 - Princeton, NJ, in 1978)

Kurt Gödel, born in Austria-Hungary, is until today one of the most famous logicians. His impact on logic and set theory to understand the foundations of mathematics and also philosophy was huge and absolutely pioneering. His most known results are the two incompleteness theorems, which he published in 1931 at the age of 25, already a doctor in mathematics at the University of Vienna. To prove one his theorems Gödel used the so called 'Gödel numbering', a function that assigns to each symbol and well-formed formula of some formal language a unique natural number.

Gödel was a ethnic German. He never had financial shortages like AW, as his father Rudolf was the manager of a textile factory in Brno. At the end of WWI and the break-up of Austria -Hungary Gödel became a Czechoslovak citizen at age of 12. That changed automatically after the annexation when he got the German citizenship. Aged 18, Gödel entered the University of Vienna, intending to study theoretical physics. Visiting seminars of the Vienna Circle run by Schlick, Hahn and Carnap, Gödel came at first to number theory and soon later focused on mathematical logic, "*a science prior to all others, which contains the ideas and principles underlying all sciences*." [Gleick (2011)] 1928 was the crucial year for Gödel when Hilbert and Wilhelm Ackermann posed in their 'Grundzüge der theoretischen Logik' [Hilbert (1928)] the problem of completeness. This was Gödels topic for his doctoral work under Hahn, that he completed in 1929 at the Vienna University. Two years later he published in Vienna his ground-breaking incompleteness theorems.

Gödel habilitated at Vienna University in 1932, and in 1933 there he became a 'Privatdozent'. Triggered by the Schlick incident in 1936 his 'nervous crisis' attacks began, accompanied by paranoid symptoms and his always imminent fear of being poisoned. Subsequently he spent several months in a sanitarium for nervous diseases. [Dawson (2005)] From 1933 on Gödel visited Princeton, lso giving lectures there on computability and recursive functions. From 1934 he gave a series of lectures on the IAS (Institute for Advanced Study) in Princeton. In autumn 1935 Gödel visited the again the IAS. He did not return until 1937 as in 1936 he had to recover from a depression. But he was able to work during that phase on the proof of consistency of the axiom of choice and on the continuum hypothesis. After his marriage in 1938 Gödel left again for the US, spending the late 1938 at the IAS and following the early months of 1939 at the University of Notre Dame. Despite the annexation, Gödel tried to stay in Austria. Not until 1940, caused by the forlorn circumstances, his need for an income, and the lost lectureship (Gödel was regarded to be a contributor to 'Jewish mathematics') and a conscription by the German army, he quickly changed his mind and escaped Nazi Germany through the Soviet Union and Japan to the US. In the early 1940's, arrived in Princeton, Gödel became a position at IAS.

At the IAS Gödel quickly resumed to mathematical research. In 1940, he published the work 'Consistency of the axiom of choice and of the generalized continuum-hypothesis' [Gödel (1938)] which is a classic in mathematics today. In Princeton he developed a strong friendship with Einstein. On December 5, 1947, Gödel finally got the US citizenship not without heavy irritaitons, caused by himself with some paranoid ideas regarding the US Constitution. [Dawson (2005)] But Einstein and Morgenstern turned the situation around and Gödel got an US passport. Gödel in series became a permanent member of the IAS in 1946. At the IAS he became a full professor in 1953 and an emeritus professor in 1976. In his late phase of life Gödel suffered heavily from his mental desease. He refused to eat and in 1978 died in the Princeton hospital of 'malnutrition and inanition caused by personality disturbance'. [Toates (2002)]

□ Hans Hahn (1879, Vienna - 1934, Vienna)

In 1899, after a try in the discipline of law a the Vienna University, Hahn changed over to mathematics. He continued these studies at the universities of Strasbourg, Munich and Göttingen. In 1902 he got his doctoral degree in Vienna. H.Hahn in 1905 habilitated in Vienna and after some years as a private lecturer (‘Privatdozent’), he became extraordinary professor of mathematics in Czernowitz in 1909. After being seriously wounded as soldier of the Austro-Hungarian army in WWI, he was releaved and soon got the chance to become in 1916 Professor extraordinarius and in 1917 regular Professor for mathematics in Bonn. In 1921 Hahn returned to Vienna accepting a chair at the Vienna University. There he stayed until his rather early death in 1934 at the age of 55, following complications related to cancer surgery. [Lense (1966)]

Hahn’s most famous student was Kurt Gödel. He also was K. Menger's Ph.D. thesis advisor. Cp sections 3.1 and 3.1.1 concerning Hahn’s importance for the Vienna Circles. Hahn himself is known in the history of mathematics for the proof of the Hahn-Banach theorem (1927) or the Hahn-Mazurkiewicz theorem. [Lense (1966)]

□ Victor Kraft (1880, Vienna - 1975, Vienna)

Kraft studied philosophy, history and geography at the University of Vienna. In 1903 he received his Ph.D. with a dissertation titled ‘Die Erkenntnis der Außenwelt‘. Further studies in Berlin followed, among others with George Simmel, Wilhelm Dilthey and Carl Stumpf. In 1912, back in Vienna, he accepted a job at the at the philosophical library of the University. Until 1914 Kraft had written his habilitation under Adolf Stöhr. In 1924 he got associate professor for philosophy at the Vienna University. In 1925 he was promoted scientific official with the function of an librarian assistant. Kraft attended regularly the Vienna Circle and at the same time was an active member of the

Gomperz Circle. He also had contacts to peripheral people of the Vienna Circle like Karl Popper.

After the annexation Kraft was forced to leave his job at the library because of his wife's Jewish background. He also lost his venia legendi. Not until 1945 he regained the post at the library. In 1947 in the course of the restauration he became 'Generalstaatsbibliothekar' (national librarian). In the same year he was reappointed associate professor and three years later he became a full professor and additionally co-director of the faculty of philosophy. A philosophical circle, chaired by Kraft and named after him, developed between 1949 and 1953. Among students of Kraft are known philosophers like Paul Feyerabend and Ernst Topitsch as well as the author Ingeborg Bachmann, who attained her doctorate with a critical work on Martin Heidegger. [Kainz (1975)]

□ Karl Menger (1902, Vienna - 1985, Chicago)

Cp. Chapter 2 for the biographical background of K. Menger.

□ Otto Neurath (Vienna, December 10, 1882 - Oxford, December 22, 1945)

Otto was the son of Wilhelm Neurath, a known economist in Austria. O. Neurath studied mathematics in Vienna and made his Ph.D. at the University of Berlin at the department of Political Science and Statistics. In his second marriage it was Hans Hahn's sister Olga Hahn who gave him her hand.

Until WWI O. Neurath taught political economy at the 'Neue Wiener Handelsakademie', and became later in Leipzig the director of the 'Deutsches

Kriegwirtschaftsmuseum' (German Museum of War Economy). After that position he ran an office for central economic planning in Munich. As a convinced socialist he joined the German SPD in 1918.

Back in Vienna, after a post there as secretary for the 'Verband für Siedlungs- und Kleingartenwesen' (Austrian Association for Housing and Small Gardens) he became the director of the 'Siedlungsmuseum' (museum for housing and city planning). Neurath was active in the development of Red Vienna (cp. Chapter 4). In 1925 he opened the 'Gesellschafts-und Wirtschaftsmuseum' (Social and Economic Museum) which led him to work on graphic design and visual education. With the illustrator G. Arntz and Marie Reidemeister (his third wife after his flight to England), Neurath created the method of 'isotype' (what later led to the 'Isotype Institute' in Oxford, GB).

In the 1920s, Neurath joined the logical positivists, and was the main author of the Vienna Circle manifesto. He also was the driving man behind the 'Unity of Science' movement and the 'International Encyclopedia of Unified Science' (cp. for this Section 3.1.1). In England Neurath should advise the redevelopment of the slums of Bilston. But he suddenly died in 1945. Marie Neurath continued the work of the Isotype Institute and publishing Neurath's work, thereunder many childrens books basing on the isotype method, posthumously. [Cartwright (1996)]

□ Theodor Radakovic (1895, Graz - 1938, Vienna)

Th. Radakovic was the son of Michael Radaković, a known Austrian physicist. Th.Radakovic was a student of Hahn and later assistant professor of mathematics at the Polytechnical Institute in Vienna and later professor for mathematics at the University of Graz. He wrote in 1921 his disseration at the ‘Friedrich-Wilhelms-Universität’ in Bonn advised by Hahn titled ’Über singuläre Integrale und Interpolationsformeln’. [Dt. Mathematiker Vereinigung (2011)]

□ Kurt Reidemeister (1893-1971)

Kurt Reidemeister was examined by Edmund Landau and wrote his doctoral thesis on algebraic number theory as assistant of Erich Hecke. Astonishingly K. Reidemeister never (except for one paper) returned to the field of his doctoral thesis. [Chandler (1982)] On H. Hahn's recommendation, K. Reidemeister was appointed associate professor of geometry at the Vienna University in 1923. There he came in contact with Wilhelm Wirtinger who brought him into knot theory. In Vienna, K. Reidemeister encountered the ‘Tractatus’ by L. Wittgenstein and actively started to study and discuss the deep ideas on logic and mathematics within the Vienna circle in his work.

In 1927 Reidemeister was offered a chair in Königsberg. In 1930 the German Mathematical Congress met in Königsberg and Reidemeister organised the first international conference on the philosophy of mathematics. Althoug not Jewish, he was forced to leave his chair in Königsberg in 1933 by the Nazis, who he strongly opposed. After being temporarily suspended from his chair he later was appointed to Hensel's chair in Marburg (what was considered a smaller and less prestigious university).

Reidemeister worked on the foundations of geometry and he wrote an important book on knot theory [Reidemeister (1932)]. He established a geometry and topology based on group theory without the concept of a limit.

Reidemeister had a huge influence on group theory, partly through his work on knots and groups, partly through his influence on Schreier. [Reichel (1994)]

□ Moritz Schlick (1882, Berlin - 1936, Vienna)

Moritz Schlick studied after his 'Abitur' (university-entrance diploma) natural sciences and mathematics in Heidelberg, Lausanne and Berlin. In 1904, back in Berlin, he wrote his dissertation thesis with Max Planck titled 'Über die Reflexion des Lichtes in einer inhomogenen Schicht.' (about the reflection of light in an inhomogenous layer). Between 1907 and 1909 Schlick studied psychology in Zurich, mainly the the work of Gustav Störrings and Wilhelm Wundt, as well as the writings of Henri Poincaré. In 1910 he removed to Rostock where he habilitated to a private lecturer at the University of Rostock with the writing 'Das Wesen der Wahrheit nach der modernen Logik' [Schlick (1910)] (The nature of truth following modern logic). For a time-span of 10 years Schlick lectured and did his research work in Rostock. In these years he met Albert Einstein and started a close friednship with him. Schlick was one of the first to work on Einstein's relativity theory (in 1917 Schlick published the book 'Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie' [Schlick (1917)]).

Schlick was released from army service due to a heart and a lung suffering. But in 1917 in the course of the worsening war situation for the German Reich Schlick is drafted by the 'Vaterländischen Hilfsdienstgesetz' (patriotic emergency law) for two years into war service. He works in a physical laboratory on the area of the 'Königliche Flugzeugmeisterei Adlershof-Johannisthal' near Berlin

In 1917 Schlick received the title of a lecturer of the University of Rostock. At the end of 1918 Schlick returned to Rostock and resumed from 1919 on with his teachings at the university. In 1918 his ‘Allgemeinen Erkenntnislehre‘ [Schlick (1918)] appeared. In 1921 Schlick becomes an extraordinary professor for ethics and nature philosophy in Rostock. In October of the same year he was appointed to full professorship at the University of Kiel, before taking the position of Professor of the Philosophy of Inductive Science at the University of Vienna in 1922.

Schlick, similar to E. Mach and L. Boltzmann (who both predecessed him in Vienna), was strictly against any metaphysical ideas and focused on empiricism in philosophy. In his believe only pure observation would be able to verify factual knowledge and help to renounce a priori knowledge. An aim should be to create a theory of knowledge based on symbolic logic. With other scientists and philosophers such as Rudolf Carnap and Otto Neurath he tried to give philosophical research a new direction. Influenced by L. Wittgenstein and also B. Russell, Schlick helped spread the idea of logic of science and ethics. [Engler (2008)] Schlick was a gifted essayist, who published many writings out of the Vienna Circle. That gave Schlick the opportunity to be a visiting professor at Stanford University in 1929. In the United States he further opened up the ideas of logical positivism.

The last years of his life Schlick spent in Vienna, where he continued to advance the philosophy of science. On June 22, 1936, Schlick was shot by a mentally insane student. The death of Schlick sealed the aprupt end of the Vienna Circle. [Engler (2008)]

□ Friedrich Waismann (1896, Vienna - 1959,)

Waismann, a born Viennese, studied mathematics and physics at the University of Vienna. From 1922 on, he studied philosophy under Moritz Schlick. From

1927 until 1936, Waismann had an extensive contact to L. Wittgenstein. Their conversations were recorded by Waismann and published in 1979. [Waismann (1979)]

Waismann emigrated to Cambridge, UK, to become a reader in philosophy of science at the University. He held this post until 1939 and then changed to Oxford University as lecturer in philosophy of mathematics. This was the position he held until his death. [Buchholz (2008)]

Some of the less regular guests

□ Alfred Jules Ayer (1910, London – 1989, London)

Ayer was a british philosopher known for his writings on logical positivism. [Ayer (1936)] Ayer worked during WWII for the MI6 [Scott-Smith (2002), p109] Ayer held the Grote Chair of the Philosophy of Mind and Logic at University College London from 1946 until 1959. Then he became 'Wykeham Professor of Logic' at the University of Oxford. He presided the Aristotelian Society from 1951 to 1952.

□ Gustav Bergmann (1906, Vienna - 1987 Iowa City, USA)

Bergmann studied mathematics, law, and philosophy in Vienna. In 1938 he emigrated to the US to teach in the Departments of Philosophy and Psychology at the University of Iowa. He did this for 40 years. Bergmann's contributions to philosophy are explained in a collection of essays by his former students and colleagues. [Moltke (1974)]

□ Carl G. Hempel (1905, Oranienburg, Germany - 1997, Princeton, New Jersey)

Carl Hempel was a leading figure in 20th-century logical empiricism. He had studied mathematics, physics, and philosophy at the University of Göttingen, Heidelberg and Berlin. He is known for the deductive-nomological model of scientific explanation. His name is also connected to the so-called 'Raven paradox', which exemplifies the problem of induction. [IEP (2011)]

□ Felix Kaufmann (1895, Vienna - 1949, New York))

F. Kaufmann studied the law and philosophy at the Vienna Univeristy. From 1922 until 1938 he worked there as lecturer. Kaufmann wrote on the foundations of mathematics where he attempted to apply the phenomenology of Edmund Husserl into mathematics. In 1938 he emigrated to the US to teach at the Graduate Faculty of the New York School for Social Research as professor for law. Kaufmann aided numerous Austrian emigres during the pre-war years. [ÖBL (1965)]

□ Richard von Mises (1883, Lwów – 1953 Boston, Massachusetts))

Richard Edler von Mises (the younger brother of Ludwig v. Mises) worked not only as a mathematician in probability theory and statistics but also as scientist in fields like fluid and solid mechanics, aerodynamics and aeronautics. The position he held at last was the Gordon-McKay professorship of Aerodynamics and Applied Mathematics at Harvard University. Richard v. Mises also worked in the theory of science, whereby his views were shaped substantially by the

Viennese circle (cp. his book ‘Kleines Lehrbuch des Positivismus – Einführung in die empiristische Wissenschaftsauffassung‘ [Richard v.Mises (1990)]). V. Mises contributions were neo-positivistic, going in line with the thougts of Ernst Mach. His first philosophical phase went from 1907 through 1914 accompagnied by important men like Philipp Frank, Hans Hahn, and Otto Neurath.[Frank (1954)] During his time in Istanbul, v. Mises maintained a close relation to Philipp Frank, who was professor of physics in Prague until 1938. His older brother, Ludwig von Mises, held an opposite point of view with respect to positivism and epistemology. [Machlup (2004)]

During WWI he served in the Austria-Hungarian army. Due to his scientific knowledge in aircraft construction and his training as a pilot v. Mises was detached to work as as test pilot and flight trainer. He also was the lead scientist of a new airplane project (the ‘Mises Flugzeug‘), which got finished in 1916, but never took action. After the war he held the chair for hydrodynamics and aerodynamics at the technical university in Dresden. In 1919 he changed to the University of Berlin and became there a director of the institute for applied mathematics. After coming into power, the National Socialists did not remove him from his post as for participants of WWI a special arrangement prevailed. He also was well-known for his German national statements. Because of the uncertainty and intolerability of the situation v. Mises notwithstanding emigrated in 1933 to istanbul, where he got a chair for pure and applied mathematics. [Kaznelson (1962)]

□ Ernest Nagel (1901, Nové Mesto nad Váhom, Slovakia – 1985, New York)

E. Nagel was born in the Austria-Hungarian Empire. Nagel was not a ‘classic’ emigrant like man yothers fleeing from the Nazi Regime as he already at the age of 10 had emigrated to the US with his family. Yet in 1919 he became an US citizen. He passed through a BSc at the City College of New York in 1923, and obtained a PhD from Columbia University in 1931. After a lasting scientific career at Columbia he there became a professor for philosophy in 1967 and held this post until his retirement in 1970.

Nagel worked on the philosophy of mathematical fields like geometry and probability. He also was engaged in the philosophy of reductive and inductive theories of science. He was an advocate for the social sciences to be scientific, demanding that these should come to the same standards as natural sciences. His 1961 book ‘The Structure of Science’ [Nagel (1961)] had an initial impact in inaugurating the field of analytic philosophy of science. Nagel is not an unimportant figure in the logical positivist movement (besides men like R. Carnap, H. Reichenbach, and Carl Hempel). [Suppes (2006)] He published in 1958 on Gödels incompleteness theorem, publishing together with James R. Newman a booklet for non-mathematicians. Between 1939 and 1956 he edited the ‘Journal of Philosophy’ and from 1940 to 1946 also the ‘Journal of Symbolic Logic’. [Suppes (2006)]

□ John von Neumann (1903, Budapest - 1953, Washington)

John von Neumann of a surety was the mathematician of the 20th century with the largest influence beyond mathematics. Born 1903 in Budapest, he already formulated the mathematical base to quantum mechanics just 24 years old in Göttingen. He was a honorary member of the Eötvös society. He became a professor at the IAS in Princeton, where he projected the programmable electronic computer. He is the founder of the modern game theory. V. Neumann received the Fermi medal (of the AEC for his contributions to the development of the electronic computer) and 1956 the Medal OF Freedom as the first from president Eisenhower. [Marx (1994)] V. Neumann also got two US Presidential Awards.

Apart from his outstanding achievements in mathematics and computer architecture v. Neumann has made important contributions to physics (quantum mechanics), to the meteorology (flow dynamics) and to the developing of econometrics (game theory [v. Neumann (1944)]). During WWII he was the leading figure within the 1944 'Manhattan Project'. The complex computations, especially in the field of hydrodynamics, were realized by the first large computers, then built with relays and tubes and only programmable by a laborious plugging-work with uncountable cables. V. Neumann's substantial contribution in this area was the development of a universal computer architecture (the so-called 'von-Neumann-architecture'), that can be found until today in personal computers. In his last years v. Neumann was active as advisor in important political and scientific committees. He died on February 8th, 1957, in Washington D.C. with 53 years of cancer, which he probably got by own imprudence through radioactive contamination during nuclear bomb tests. [Easle (1986)]

□ Olga Hahn-Neurath (1882, Vienna - 1937, Den Haag)

Olga Hahn, later Hahn-Neurath, the sister of Hans Hahn, was an Austrian mathematic. She is particularly known for having been a member of the Vienna Circle. In 1902 she started her studies in mathematics and philosophy at the University of Vienna. In 1911 she made her first degree in philosophy, being the third female graduate of all times at the Vienna University. Her thesis was highly praised by Adolf Stöhr, her supervisor, successor on the chair of Ludwig

Boltzmann. In 1912 she married Otto Neurath, whom she had met during her studies. Olga, who went blind at the age of 22 due to an inflammation of her visual nerve, became in 1924 a regular participant of the Viennese Circle. During the events of February 1934 (the Schlick murder case) she was on a visit in Moscow and together with her husband she immediateley emigrated by Poland and Denmark into the Netherlands. There she died three years later. [Korotin (2002)]

□ Rose Rand (1903, Lemberg, today Ukraine - 1980, Princeton, NJ)

Rose Rand was a logician and a philosopher. After her family had moved to Austria she visited the Polish Gymnasium in Vienna and there enrolled 1924 in the Vienna University, studying with Heinrich Gomperz, Moritz Schlick, and Rudolf Carnap. She graduated in 1928, and as a PhD candidate (working on T. Kotarbiski's Philosophy), Rand regularly participated in the Vienna Circle discussions. Between 1930 and 1937 she worked, also as a researcher, at the psychiatric-neurological Clinic of the Vienna university. [Hamacher-Hermes (2003)] Rand made her living tutoring students and also giving education to adults. In 1938 she received her PhD. Rand, like AW, unemployed and of Jewish descent, suffered great difficulties in pre-WWII Vienna. Finally in 1939 she decided to emigrate to London. She was a Jew without nationality.

In GB she at first earned money working as a nurse. After that she was admitted as 'distinguished foreigner' at the faculty of Moral Science at Cambridge University. In 1943 she lost all her privileges and had to work at a metal factory, and teach at night classes in German and psychology in the Luton Technical College and Tottenham Technical College. With the help of Karl Popper she got a small research grant. Between 1943 and 1950 she also worked in practical engineering. [Hamacher-Hermes (2003)]

In 1954 Rand moved to the US. Between 1955 and 1959 she taught elementary math, ancient philosophy and logic, and also was a research associate in the University of Chicago, Indiana University and Notre Dame University. In 1959 she returned to Cambridge, Massachusetts and after that to Princeton, New Jersey. In the following years she earned her living from grants and fellowships which were given to her mostly for her work on translations. Rand's materials (research texts, her records of the discussions in the Vienna Circle, over 1,600 letters to O. Neurath, L. Wittgenstein, A. Tarski and others) were purchased by the University of Pittsburgh. [Hamacher-Hermes (2003)]

□ Hans Reichenbach (1891, Hamburg – 1953, Los Angeles)

Hans Reichenbach, was a leading figure in the philosophy of science and an known proponent of the logical empiricism. Reichenbach founded in 1928 the 'Berlin Circle' ((also known as the 'Die Gesellschaft für empirische Philosophie', 'the Society for Empirical Philosophy'). As famous members of the Berlin Circle we notice C. G. Hempel, R. von Mises and D. Hilbert. His most famous book is the 'The Rise of Scientific Philosophy'. [Reichenbach (1951)] He studied engineering at the TH Stuttgart, also physics, mathematics and philosophy in Berlin, Erlangen, Göttingen and Munich. Reichenbach had famous teachers like E. Cassirer, D. Hilbert, M. Planck and Arnold Sommerfeld. In 1915 Reichenbach received a degree in philosophy from the University of Erlangen. In 1916 he published his dissertation on the theory of probability, supervised by Paul Hensel and Emmy Noether.

After serving in WWI he could leave the army in 1917, thanks to an illness. In series returned to Berlin. There he visited Einstein's lectures on the theory of relativity from 1917 to 1920. In 1920 Reichenbach could start his own teaching career as a private lecturer at the Technische Hochschule at Stuttgart. In the same year, he started to publish on the theory of relativity. He originated overall 4 books on this topic between 1920 and 1928. [Grünbaum (1963)] From 1930 on Reichenbach and R. Carnap started as editors the journal ‘Erkenntnis’ (‘Insight‘).

In 1933 Reichenbach, of Jewish anchestry, immediately lost his job. He emigrated to Turkey, and got the post of head of the Department of Philosophy at the University of Istanbul. In 1938, he changed to the US, taking up a professorship at the Philosophy Department of the University of California, Los Angeles. There he worked on the philosophical foundations of quantum mechanics and on symbolic logic. [Salmon (1977)] With Reichenbach the UCLA developed into a leading philosophy department in the US after WWII. Hilary Putnam, e.g. is one of Reichenbach’s outstanding students.

□ Olga Taussky-Todd (August 1906, Olmütz, (today Olomouc in the Czech Republic) - October 1995, Pasadena, California)

Her first field of research was the algebraic number theory, in what she receives a PhD in 1930 at the Vienna University. Her supervisor then was the remarkable mathematician Phillip Furtwängler. During her time in Vienna she took part in the meetings of the Vienna Circle, but left after her thesis for Göttingen, where she spent a year to edit David Hilbert's Collected Works. In Götingen she also made her first steps in teaching. From 1934-1937, she worked as a fellow at Girton College, Cambridge University. After that period, she left for the London University, where she met the british John Todd, whom she married in 1938. During WWII (working for the UK National Physical Laboratory) she revolutionized the numercis of matrix computations in order to damp the vibrations of airplanes. . In 1945 she and her husband emigrated to the US, working for the National Bureau of Standards. In 1957 both joined the CALTECH in Pasadena. In 1971 she got a full professorship at CALTECH to become there the second female professor ever. [Luchins and McLoughlin (1996)]

□ Alfred Tarski (January, 1901, Warsaw – October, 1983, Berkeley, USA)

Alfred Tarski was born (then with the name Teitelbaum) into a wealthy Jewish family in Warsaw. He grew up in Warsaw where he also enrolled in the local university. In 1923 he wrote his doctoral thesis under Lesniewski and got his PhD in 1926. In 1923 he changed his name to 'Tarski' and converted to the catholicism (although he was convinced atheist), as he hoped to have better chances with his efforts to win a professorship. But several applications were declined. From 1926 on he worked at the Warsaw University as a lecturer in logic. In 1930 he came in contact with the Viennese circle and especially Kurt Gödel. With WWII the Polish logic lost many of its protagonists and got almost destroyed. Tarski reacted very late on the serious situation. In August 1939 he got an invitation to the Unity-of-Science Conference in the US and could leave the country with the last legal ship. His woman and his children could no longer follow him, but survived the war in Poland. After WWIII Tarski, united with his family, remained in California to develop in Berkeley a school for logic. [Burdman (2004)]

□ Willard Van Orman Quine (June, 1908, Akron, Ohio – December, 2000, Boston, MA)

Van Quine worked in philosophy and logic. Quine represents a analytic philosophy, but viewing in philosophy more than only a conceptual analysis. From 1926 to 1930 he studied in Harvard mathematics and philosophy and subsequently made there in 1932 his PhD in philosophy with A. N. Whitehead. His thesis was titled 'The Logic of Sequences: A Generalization of Principia Mathematica'. After Quine had met Herbert Feigl in 1932 during Feigl's stay in Harvard and so getting a introduction into the Vienna circle and its philosophy, Quine was deeply attracted by Vienna and already traveled in 1932 for one year as 'Sheldon Traveling Fellow' to Europe. In Vienna he in particular encountered Kurt Gödel and Moritz Schlick. In Prague he visited lectures of R. Carnap and in Warsaw he attended readings of A. Tarski and of the logician Stanisław Lesniewski. [Lauener (1982)]

From 1933 to 1936 Quine was a member of the Junior Fellows in the 'Harvard Society of Fellows' and could so exclusively for three years do own research work. In 1936 he started his activity as a lecturer in Harvard and became there in 1938 a full professor of philosophy. He continously held this position in Havard until his death, filling the Edgar Pierce Chair of Philosophy from 1956 to 1978. [Lauener (1982)]

□ Edgar Zilsel (August, 1891, Vienna – March, 1944, Oakland, CA)

Edgar Zilsel was an Austrian marxist philospher, who connected Marxist views with the positivism of the Viennese Circle. But despite his link to the Vienna Circle, Zilsel also criticized the views of Circle members.

Zilsel attended from 1910 on the University of Vienna where he studied philosophy, physics, and mathematics. In 1915 he received his PhD in under the supervision of Heinrich Gomperz. [Nemeth (2011)] As a Jewish Marxist he in follows had no chance on a academic career at the Vienna University. After working at an insurance company for a few months, Zilsel worked as a teacher of mathematics and physics at a Vienna secondary school from 1917 onwards. He also taught philosophy and physics at the Vienna People's University. After the annexation Zilsel could at first escape to England and in 1939 to the US where he received a Rockefeller Fellowship. He published a great many of papers during these years of exile, the most famous of which is his ‘Die sozialen Ursprünge der neuzeitlichen Wissenschaft’ (‘Sociological Roots of Modern Science’). [Zilsel (1976)] In 1943, Zilsel was invited by Lynn White to teach physics at Mills College in California. Because of personal isolation and private disappointments he shortly thereafter committed suicide with sleeping pills. He is buried at the Viennese Central Cemetery. [Raven (2000)]

□ Tscha Hung (1909, Anhwei, China – 1992, Beijing)

Tscha Hung was encouraged by his teacher to travel in 1927 to Jena to start with an education in philosophy with Rudolf Eucken. But Eucken died shortly before his arrival. Hung at first studied mathematics, physics and philosophy with Bruno Bauch, but soon turned to Berlin to Hans Reichenbach, disappointed by Bauch’s Neokantianism. It is known that Hung got to Vienna during the winter semester in 1928/29, where he without break studied with M. Schlick. Later in 1934 he made his PhD with Schlick. His thesis was titled ‘Das Kausalproblem in der heutigen Physik’ (‘The problem of causality in nowadays physics’). [Haller (1999)] From 1931 on Hung took part in the meetings of the Vienna Circle. It is a huge merit of Hung that he not only brought the Austrian positivism to China, but also was its protagonist and apologist in his home country, what gets confirmed from his publication list on these topics. In 1945 he published the book 'The philosophy of the Vienna Circle' in Chinese and during 1982-1984 an anthology on logical empirism, also in Chinese. [Dainian (1996)]

Back in China, he was appointed in 1937 lecturer at the national Beijing University, and then got a position as lecturer from 1940 to 1945 at the national South West University. From 1945 to 1947 Hung was Research Fellow at the New College, Oxford University. From 1948 to 1951 he held the position of a professor and dean of the Department of Philosophy at the National Wuhan University. In 1951/1952 he was active as a professor at the Yenching University. Since 1965 he finally lectured at the Beijing University. [Haller (1999)]

□ Marcel Natkin (1904, Lodz - 1963, Paris)

Marcel Natkin, a fellow student and besides H. Feigl a good friend of Gödel, became later one of the most famous photographers of Paris. The friendship between the both emerges from Gödel's correspondence. Natkin studied mathematics and philosophy and made his PhD with Schlick in 1928. [IMN (2006)] His thesis was titled 'Einfachheit, Kausaliät und Induktion' ('Simplicity, Causality and Induction'), a widely circulating paper, as Natkin was counted the Circle's 'Wunderkind' [Hacohen (2000)] "For personal and practical reasons he decided on a career in business, and left Vienna after his doctorate and settled in Paris. There he became one of the world's outstanding experts on photography." [Cohen (1981), p65]

The Circle also benefitted from frequent contacts with

□ Ludwig Wittgenstein (1889, Vienna – 1951, Cambridge, UK)

L. Wittgenstein is surely the most known Austrian philosopher until today. He was born into a very wealthy family, but in young years gave away all his inheritance. His influence is still strong, as he inspired two main philosophical movements of the 20th century, the logical positivism and the philosophy of language. In his lifetime he only published a few titles, whereby of course a special accent lies on his outstanding ‘Tractatus Logico-Philosophicus’ from 1921. [Monk, 2005] [Wittgenstein (1921)] Wittgenstein himself in his young years was convinced having resolved all philosophical problems with his ‘Tractatus’ (what he later relatived). With that work he indeed caught the attention of R. Carnap and M. Schlick. The Vienna Circle spent many months working through the text, and it was Schlick, who convinced Wittgenstein to visit the Circle (at a time Wittgenstein was working as an architect for his sister Margaret building the today famous ‘Haus Wittgenstein’). [Monk, 2005]

Wittgenstein teached only from 1939 until 1947 at the University of Cambridge, as he tried to escape the philosophy several times in his life: he served during WWI on the front lines with the Austrian Army, taught from 1922 to 1926 in Austrian village schools until his resignation after an incident when Wittgenstein hit brutally a pupil. He also worked during WWII in Guy's Hospital, London, because he found it unbearable to teach philosophy when a war was going on. [Monk, 2005]

His later work (the ‘Philosophical Investigations‘ [Wittgenstein (1953)]) was published posthumously in 1953, two years after his death. In the ‘Investigations‘ he rejects many of the theses of the ‘Tractatus’. Despite the differences between his early and later work, the similarities can be understand as a the concept of therapeutic philosophy concerning also ethical and religious issues (in the so-called ‘New Wittgenstein’ family of interpretations). [Klagge (2001)] [Read (2000)]

□ Karl Popper (1902, Vienna – 1994, Croydon, UK)

Sir Karl Popper, one of the most famous philosophers of the 20th century, was of Austrian ancestry, but later got the British citizenship including a knightship by Queen Elisabeth II in 1965. Popper was a professor at the London School of Economics from 1946 to 1949, and afterwards got appointed professor of logic and scientific method at the University of London until his retirement in 1969. [Edmonds (2002)]

The fact that Karl Popper started to write down his philosophical thoughts is due to his contacts o the Viennese Circle, especially M. Schlick, R.Carnap and O. Neurath. Popper harshly criticized the neopositivistim, especially Wittgenstein for his forbiddance of “questions, on which he does not know an answer“ [Edmonds (2002)], so that M. Schlick started to dissociate himself of Popper. After an eclat at Poppers Rigorosum in 1928, that M. Schlick assessed, Popper nevermore received any invitations to the Circle’s meetings.

Popper defeated the classical inductivist method and favoured instead the empirical falsification. He also opposed to the classical justificationist account of knowledge and replaced it with critical rationalism. Popper additionally wrote extensively on social and political philosophy and advovated the liberal democracy. [Bartley (1964)]

□ Heinrich Gomperz

In 1900 Gomperz habilitated at the University of Bern in philosophy and worked there unitl 1903 as a private lecturer. In 1905 he returned to Vienna, where he was appointed in 1920 extraordinary and in 1924 ordinary professor of philosophy at the Vienna University. In 1934 he got his dismissal as Social Democrat. Thereupon he emigrated in 1935 to the US, where he got in 1942 a guest professorship at the University of Southern California. H. Gomperz is considered as an important opponent to the logical positivism, developing his own ideology, the Pathempirismus. [Stadler (1994)]

G: Participants of Menger's Colloquium

□ Franz Alt (see above)

□ Nachman Aronszajn (1907, Warsaw – 1980, Corvallis, Oregon)

N. Aronszajn was born in 1907 in Warsaw, where he went to school and university. Aronszajn received two Ph.D's. One from the University of Warsaw in 1930 under Stefan Mazurkiewicz and another from Paris University in 1935 with Maurice Fréchet. He then worked in Paris and Cambridge until 1948, when he emigrated to US. There he spent most of his career at the University of

Kansas in Lawrence. He died in Corvallis, Oregon in 1980. [James (1999)]

□ Leonhard Mascot Blumenthal (1901, ? – 1984, Columbia, Missouri?)

Blumenthal received his Ph.D. in 1927 from the Johns Hopkins University in Baltimore. He taught for the majority of his professional career at the University of Missouri. [Menger (1994)] K. Menger and Blumenthal published in 1971 the 'Studies inGeometry'. [Blumenthal (1971)]

□ Karol Borsuk (1905, Warsaw – 1982, Warsaw)

Borsuk received his diploma and also his Ph.D. from Warsaw University in 1927 resp. 1930. His Ph.D. thesis advisor was Stefan Mazurkiewicz. Borsuk is famous for his contributions to topology. His conjectures stimulated the topological research for half of the 20th century. [O'Connor (2000) a,]

□ Rudolf Carnap (see above)

□ Eduard Cech (1893, Stracov, Bohemia (now Czech Republic) – 1960, Prague, Czechoslovakia (now Czech Republic))

In 1922, Cech wrote his habilitation thesis and got a docent at the Charles University of Prague. In 1923 he successed Mathias Lerch as an extraordinary professor at the Masaryk university in Brünn. There he was appointed professor in 1928. Čech made Brünn a center of combinatorial topology during the 1930s. After WWII he started at the University of Prague, heading the mathematical institute of the Czech academy of sciences. In 1956 he was appointed director of the mathematical institute of the University of Prague. [O'Connor (2000) b,]

□ Kurt Gödel (see above)

□ Hans Hornich (1906, Vienna – 1979 ibid.)

H. Hornich studied from 1905 on mathematics and physics at the Vienna University. His academic teachers were Wirtinger, Furtwängler, Hanhn, K. Menger and Mayer. In 1929 he attained his doctorate with Menger. Hornich at first worked as librarian at the Mathematical Institute of Vienna University. Later in 1933 he habilitated there, but could work however only as an assistant there since 1936. From 1945 on he taught as assistent lecturer at the 'Universität für Bodenkultur' (Univeristy of agriculture) in Vienna, and then in 1949 was appointed a professor at the Technical University in Graz. In 1958 he got a position as professor for mathematics at the Technical University of Vienna. There he retired in 1976. as occupied with infinite rows and infinite products and set up later an important inequation for curves. [Hlawka (1980)]

□ Marston Morse (1892, Waterville, Maine – 1977, Princeton, New Jersey)

M. Morse studied first at Colby College and then at Harvard University, where he received a master's degree and a Ph.D. in 1917 in mathematics. He taught at Harvard, Brown, and Cornell University before he finally accepted a position as professor in 1935 at the IAS in Princeton. There he worked until his retirement in 1962.

□ John von Neumann (see above)

□ Georg Nöbeling (1907, Lüdenscheid – 2008, Rosenheim)

G. Nöbeling studied mathematics in Göttingen from 1927 until 1929, then went to Vienna to finish his studies with K. Menger. His main topics of research were geometry, analysis and especially topology. In 1931 he received his Ph.D with K. Menger for a generalisation of Menger's universal curve in the three-dimensional space. From the Vienna Circle, where topology was a key issue, he got numerous impulses. During his Vienna period he met talented topologists like Leopold Vietoris and Witold Hurewicz. Nöbeling worked for Menger until 1933 as an auxiliary assistant. In that time he cooperated jointly with K. Gödel, AW, Franz Alt and Olga Taussky-Todd in Mengers Colloquium. [Fasshauer, (2002)] His results from this period made Nöbeling quickly known in the mathematical community. To Menger's dismay, Nöbeling went back to Germany, becoming an assistent at the University of Erlangen in 1933. [Sigmund (2001)] There he habilitated in 1935, was appointed in 1940 adjunct professor and in 1942 he became a full professor. In 1950 Nöbeling was appointed president of the German Mathematical Society ('Deutsche Mathematiker-Vereinigung'). He celebrated his 100th birthday in 2007. [Univ. of Erlangen (2007)]

□ R. G. Putnam (?)

In the 73. Colloquium in 1933/34 (printed in the 1. Volume of the 'Ergebnisse') the mathematican R. G. Putnam showed a gap in a proof of H. Reschovsky on rational curves (Cp. [Menger (1998)]). R. G. Putnam was professor of mathematics at the New York University. In summer 1934 he took part at the Colloqium. [Menger (1998)]

□ Karl Schlesinger (see above)

□ Alfred Tarski (see above)

□ Olga Taussky-Todd (see above)

□ Abraham Wald (see whole text)

□ Gordon Thomas Whyburn (1904, Lewisville, Texas – 1969, Charlottesville, Virginia)

Whyburn at first studied chemistry at the University of Texas, but was soon attracted to mathematics by Robert Moore, a newly appointed associate professor, in 1920. It was Moore who saw the mathematical talent in Whyburn.

Moore's deep interest in topology gave also Whyburn the direction of research throughout his life.

In 1929 Whyburn, together with his wife Lucille, got financed an academic year in Europe by a Guggenheim Fellowship. He worked most time in Vienna with H. Hahn (interrupted by visiting Kuratowski and Sierpinski in Warsaw). [O'Connor (2000) c,] Soon after his studies abroad Whyburn was appointed associate professor of mathematics at Johns Hopkins University. In 1934 he was appointed full professor by the University of Virginia, where he also became the chairman of the Department of Mathematics. Whyburn spent his academic life in Virginia, although he often taught at other universities like Stanford, the UCLA or the University of Colorado. [O'Connor (2000) c,]

□ Norbert Wiener (1894, Columbia, Missouri – 1964, Stockholm)

Wiener was an US mathematician, famous for his contributions to electronic engineering, electronic communication, and control systems. He is regarded as the originator of cybernetics, what had many implications for a wide range of scientific fields.

In 1914 Wiener for the first time traveled to Europe, visiting Bertrand Russell and G. H. Hardy at Cambridge University, to study wih them. Afterwards he journeyd to D. Hilbert and E. Landau at the University of Göttingen to deepen his knowledge. Back in the US Wiener taught philosophy at Harvard, then worked as an engineer for General Electric, and also as a journalist for the Boston Herald. He never was able to get a post as a professor on a university, but became an instructor of mathematics at MIT, where he spent the rest of his professional career.

1931/32 was an important year for Wiener when he again visited Hardy at

Cambridge. There he gave a lecture course on his own contributions to the Fourier integral theory. Cambridge also worked also for him as a bridge into old Europe, where he was able to meet many colleagues like K. Menger, Ph. Frank, H. Hahn, E. Artin and Kurt Gödel. In this way he participated Menger's Colloquium in Vienna. [O'Connor (2000) d,]

□ Karl Menger (see above)

□ Gerhard Tintner (1907, Nürnberg – 1983, Vienna)

Although G. Tintner was born in Germany, he got his education in Vienna, where he studied economics, statistics and law at the University of Vienna. The influence of the the Vienna Circle on him cannot be underestimated. Especially R. Carnap, but also hsi philosophical opponent, K. Popper had a lasting impact on Tintner as he annexed the idea the idea of the 'unity of science', the unideological access to theory. Concerning his studies in economics, Tintner used the logical-deductive method and also statistical tests of economic hypotheses, what brought him step by step into the mathematization of economics (cp. 'Methodology of Mathematical Economics and Econometrics' [Tintner (1968)]). [Deistler (1984)]

In 1936 Tintner became a member of the Austrian Institute of Trade Cycle Research in Vienna, headed by O. Morgenstern., but left Austria in 1936/37 for a post as Research Fellow in Economics and Statistics at the Cowles Commission. After the Cowles interlude, he went to the Iowa State University, where he later became Professor of Economics, Mathematics and Statistics. In 1963 he joined the University of Southern California, Los Angeles, as

Distinguished Professor of Economics and Mathematics. Since 1973 he was Professor of Econometrics at the University of Technology, Vienna, from where he retired in 1978. [Deistler (1984)]

In Econometrics Tintner's most influential book is the 'The Variate Difference Method'. [Tintner (1940)] His interest in the reliability of data led him into the theory of errors. He went analogically driven by his interest in economic processes into the field of stochastic processes and the stochastic programming. [Tintner, Sengupta (1972)]

H: Report on the archival work

1. Visit of AW's son, Robert M. Wald, in Chicago, October, 2007. Evaluation and selection of relevant material

A partial relief of AW is in the private property of his son, Prof. Dr. Robert M. Wald, 5514 South Woodlawn Ave., Hyde park, Chicago. The collection consists of 2 briefcases (files) as well as a separate envelope. Everything together has an extent of approximate 0.15 meters. It consists thereby of approximate 10 photos, official documents (a passport, a naturalization document, honors etc.), correspondences, manuscripts, newspaper cutaways as well as special editions dating of about 1934 to 1950. The material is handwritten, machine written and printed in German, Hungarian, Hebrew, French and mainly the English language. The relevant part for this work was copied, the remaining part is only outlined as follows:

From the lecture held by Robert M. Wald during the 2001 conference 'Kühler Abschied von Europa' in Vienna there exists neither a manuscript nor an audio recording. Also notes in addition are missing according to his statement on this. For the additional exhibition of this conference a part of his photo material was used. Robert M. Wald has little background knowledge of the relief of his father. So our demands of contextualizing individual documents remained unsolved.

□ ‘The commercial Expanding File‘, alphabetical register, titled by hand:

A: Austrian Institute.

B: Bergmann, biography, bibliography.

C: Carnegie, Columbia Appointments, Cowles Commission.

D (not titled)

E/F: Fraenkel, Frechet, Frisch, Finetti.

G: Geneva.

HU: Hausdorff.

K/L: License/Lease.

M: Mises, Menger.

N/O: Neumann, Naturalization.

P/Q: Policies, Firelns.

R: Recommendations.

S: Scholz/Schultz.

T/U/V (not titled)

W: Herman Wald, clippings.

XYZ (not titled)

□ 'Kraftall. Visible index file', alphabetical register. In fact without assortment, material partly strongly damaged and/or in precarious condition. Correspondence (among other things) with graduate students, the administration of Columbia University, the Econometric Society, the International Statistical Institute and some memoranda of various commissions.

□ Additionally to picture and text material we found a bronze medal with the following text: 'Columbia University World War II: for Participation in the Work of the division of War Research'.

2. Visit at Duke University, Durham NC, October, 2007, Evaluation and Triage of K. Menger's Relief

The whole relief was screened (see the attached inventory list). The all in all 28 boxes have a circumference of about 45 cm x 45 cm, except of the the boxes 25 to 28 that are oversized with circa 50 cm x 70 cm. The screening was done regarding letters from/to AW and documents, manuscripts with his authorship. We also screened documents and letters for any indication of AW. We made multiple copies of every single finding to have a better readability.

(Abbreviations: **D** = document is a copy, **F** = nothing found, **./.**= not relevant)

Box 1

□ Correspondence, (partially sorted in chronological order).

□ R. v. Mises, writing about AW, a Transcript, Istanbul-Beyoglu, dated March, 23th, 1937 (there is also a copy in the private collection of R. Wald), (**D**).

□ Franz Alt to Karl Menger, Vienna, July 13, 1937, (**D**).

□ File N: 1929 May 5 - 1938 April 18, letters and manuscripts of AW (**D**), a certain part of the documents is damaged by water and hence hard to decipher, (**D**).

Box 2

□ Folder 6 Correspondence (partially sorted in chronological order)

□ Folder 7: Antonin Flores to Karl Menger, Vienna, March 26, 1934 (**D**).

□ Folder 7: Antonin Flores to Karl Menger, Vienna, 22.08.1934, (**D**).

□ Folder 7: 6 letters from AW to Karl Menger, without date.

□ Folder 13: Illinois Institute of Technology, Research Proposal tothe office of Naval Research, Feb. 25,1949, (**D**).

□ Folder 16: Leonard Blumenthal to Karl Menger, June 11,1935, (**D**).

Box 3 (./.)

Box 4 (./.)

Box 5

□ Folder 5, File 11: Harald Freeman, Abraham Wald, a biographical sketch, without date, (**D**).

Box 6 (F)

Box 7

□ Folder 7, File 14: 'Über die Grundlagen der Wahrscheinlichkeitsrechnung' after AW, undated, unauthorized, (**D**).

Box 8 (F)

Box 9

□ Folder 5: 'Abstrakte Algebra und Metrik', manuscript of AW, undated, 8 pages.

□ Folder 6: 'Memorandum par Dr. A. Wald, Wien, distribue a l'occasion de la Conference de Dr. Oskar Morgenstern, Wien, prononcee le 6 mai 1935, Institut Scientifique des Recherches Economiques et Sociales, Paris', 19 pages, (**D**).

Box 10

□ Folder 5: 'Generalization of the Inequality of Markoff, by A. Wald', special print, (**D**).

Box 11

□ Folder 15, File 1: Interview with K. Popper

□ Folder 15, File 3: Interview with Karl Menger, August 17, 1978, by Werner Schimanovich und Peter Weibel.

Box 12

□ Folder 1: 'Notes on the efficient design of experimental investigations, New York, 1946', a students record of a lecture of AW at the Columbia University, 1943, (**D**).

□ Folder 2: 'The Analysis of Variance and Covariance', a one semester course lecture of AW given at the Columbia University in 1941. The notes are prepared by Ralph J. Brookner, Fellow of the Carnegie Corporation of New York, (**D**).

□ Folder 3: 'Notes on the Theory of Statistical Estimation and Testing Hypotheses. Lectures by Dr. A. Wald', given in a one semester course at the Columbia Univ., Notes prepared by Ralph J. Brookner, Fellow of the Carnegie Corporation of New York.

Boxes 13-16 (Writings and speeches)

□ Predominantely typescripts und manuscripts, to a large extend undated. The screening for references to AW was done at random.

Box 17 (./.)

Box 18 (./.)

Box 19

□ Notebooks, that contain in essence records on mathematical questions, a considerable part is wrtitten in shorthand.

Box 20

□ Three diaries from 1918, 1919, 1920 to 1922. Not relevant in respect to AW.

Box 21

□ Rustbrown ring binder with undated notes of K. Menger. Page 1 to 3 contain some reference to AW. A palmful of photographs, none with AW.

Box 22 (F)

Boxes 23 and 24 (./.)

Box 25 (F)

□ Photographs of the moon landing, page proofs, sketch sheets from the 60s and 70s.

Box 26 (F)

□ Page proofs, sketch sheets and computer prints from the 60s and 70s.

Box 27 (F)

□ Typescripts corrected by hand, unsorted, undated.

Box 28 (F)

□ Typescripts corrected by hand, unsorted, undated.

I: Selected letters on AW's sudden death

Table 30. Karl Menger to Morgenstern, 1951. KM Archive Durham.

Dear Morgenstern:

Thanks for the beautiful article on Wald which I read with great interest.

A few minor points are, I think, not correctly stated. p. 1, line -5. Hahn and Wald knew each other hardly. To my regret, due to an idiosyncrasy on the part of Hahn, they never got well acquainted. That Wald never took any course under Hahn follows from the more general statement which he made repeatedly: that I was the only person under whom he ever took any course. I do not think, one can say that Wald got under the influence of Hahn.

p. 1, line -5. Furthwängler, like Wirtinger, Wald never even met (Er hat sich ihnen nie vorgestellt). He had not any relation with F. whatever.

The only people under whose infl. Wald came, in his formative years, were those gathered at that time round me under the name of members of my colloquium: Gödel, Nöbeling, Alt, and the numerous visitors from outside.

p. 2, line 4. as well as his studies on the concept of collective

p. 3, line -6. What you call the second development, preceded what you call the first by two years. Wald presented his solution of the equations of production in March 1934 (cf. Ergebnisse 6, p. 10)

p. 4, lines 7./8. The idea of the Überschüsse (which, in my opinion, is the main economical contribution to t he new solution), is entirely due to Zeuthen and Schlesinger. (cf. Schlesinger as the above Erg. p. 10 sq.)

Best regards to you and your family

Cordially

Karl Menger

Table 31. Morgenstern to the Editor of the Econometrica, Dr. Simpson, July 31, 1951. KM Archi
Durham.

31 July 1951

Dr. William B. Simpson
Editor
Econometrica

The University of Chicago
Chicago 37, Illinois

Dear Dr. Simpson:
Enclosed, as promised you in an earlier letter, is the obituary for Abraham Wald for publication in Econometrica. I hope you will like the note I have written and that together with photograph and the translation, which is already in your hands, a nice memorial is gotten together.
I can be reached here until 17 August, then I can be reached again in Princeton until 2 September. I am sailing on 4 September (leaving Princeton the day before) for Sweden and shall be abroad for several months. In Sweden my address for the month of September will be:
Care of Professor J. Akerman
University of Lund
Lund, Sweden

If any difficulties are encountered in trying to send the proof to me to read you are entitled to print the paper by making sure that a good copy is made of the manuscript which I am transmitting to you. In this manner the publication will not be held up; it is not imperative that I see proofs at all.

Very sincerely yours,

Oskar Morgenstern

Table 32. AW's last letter, addressed to Morgenstern, Dez. 1950. KM Archive Durham.

Columbia University
in the City of New York
Department of Mathematical Statistics

Dear Morgenstern,

Thanks for your letter. We were pleased to learn that you have a son and we want to congratulate you on this occasion.
I have been invited by the Indian Government for a lecture time in India. My wife will accompany me, but we leave the children home with their grandmother. We leave on Nov. 7 and plan to make short stops in London, Paris and Rome (I shall give a lecture at the Inst. .... ??? in Paris).
I shall be back by Feb 15, but my wife will return much earlier (early in January).
We hope to have an opportunity to visit you after our return from India.

With best wishes,

Cordially yours,

A. Wald

Table 33. Morgenstern to Mrs Lang, mother of Lucille Lang, AW's wife, May 1952. Private Collection M. Wald

May 21, 1952

Mrs. Lang
80 Arden
New York

Dear Mrs. Lang:

I am sending you enclosed four copies of an obituary I have written for Abraham Wald and his wife. I would have sent these copies earlier had it not been for my prolonged absence from this country. I hope you will keep one each for the two children so that they may be added to the many other obituaries which have appeared.
I hope I have conveyed in these pages my great admiration for the scientific work of Abraham Wald and the sense of personal friendship which I felt for him.
If less was said about Mrs. Wald, this is due to the fact that my note appeared in a scientific periodical and had to deal with Abraham Wald essentially, but I do recall so well how much they were devoted to each other and how happy she made him during their common lifetime.
Believe me,

Very sincerely yours,

Oskar Morgenstern

J: Students of AW

Students of AW listed in chronological order:

□ Herman Chernoff, Brown University, 1948, Dissertation: Studentization in Testing of Hypotheses

□ Meyer A. Girshick, Columbia University, 1947, Dissertation: Contributions to the Theory of Sequential Analysis

□ Milton Sobel, Columbia University, 1951, Dissertation: Essentially Complete Classes of Decision Functions for Certain Standard Sequential and Non-Sequential Problems

□ Charles Stein, Columbia University, 1953, Dissertation: A Two-Sample Test for a Linear hypothesis Having Power Independent of the Variance

□ Allan Birnbaum, Columbia University, 1953, started his doctorate with AW and asked after AW's death Erich Leo Lehmann (1917-2009), who was visiting Columbia at that time. Birnbaum's thesis and his early work was very much in the spirit of E. Lehmann's text 'Testing Statistical Hypotheses'. [Lehmann

(1959)]

H: Register of Persons

9 Notes

1 It is noteworthy that in the revised English version of this book [Morgenstern (1937)], pp141-143 this statement is considerably weakened

2 Haberler in a letter to Morgenstern warned: "*Your sweeping ondemnations of all possible people are very amusing. However, they put a heavy burden of responsibility on your forthcoming works.*" Haberler to Morgenstern, 15 Aug 1934. O. Morgenstern papers, box 5.

3 Memo, Van Sickle to Kittredge, 27 March 1936, Ernest John was one of Morgenstern's economic researchers at the Institute, Austrian Institute for Trade Cycle Research.

4 Biographic Memorandum of Abraham Wald. Written by himself. Abatement AW, owner Robert M. Wald, Chicago

5 Biographic Memorandum of Abraham Wald. Written by himself. Abatement AW, owner Robert M. Wald, Chicago

6 See Karl Menger, 1952, "The Formative Years of Abraham Wald and his Work in Geometry", Annals of Mathematical Statistics, 23, pp14-20, or also O. Morgenstern, "Abraham Wald, 1902-1950", Econometrica, Vol. 19, No. 4, October, pp. 361-367. Other sources are E., R., Weintraub, 1985, General Equilibrium Analysis, Studies in

Appraisal, Cambridge: Cambridge University Press.

[7] Also known as the Compromise of 1867

[8] The word Magyar in the Hungarian language refers both to the ethnicity and the language. In English and many other languages, however, "Magyar" is used to refer to the ethnicity to distinguishing ethnic Hungarians (i.e. the Magyars) from the other nationalities living in the former Kingdom of Hungary.

[9] In February 1861, the February Patent, a letters patent issued by Austrian Emperor Franz Joseph I, was adopted as the “Imperial Constitution of 1861.” It was proclaimed as a revision of the previous October Diploma, the “Irrevocable Fundamental Law of the State.”

[10] The March Laws measures enacted by the Hungarian Diet at Pozsony (modern Bratislava) during the Revolution of 1848 that created a modern national Magyar state. After revolutions had broken out in Paris (Feb. 24, 1848) and in Vienna (March 13, 1848), liberal Hungarians, who dominated the lower house of the Diet, sought to avoid radical social revolution by emphasizing reform and national liberation.

[11] For ministary cabinet resp, to structur of government see: Èva Somogyi, “Der gemeinsame Ministerrat der österreich-ungarischen Monarchie 1867-1869”, Böhlau 1996

[12] In 1903 Hungary sought Franz Joseph I to suspend the Hungarian constitution in 1903

[13] For life and creation: “Peter Broucek, Karl I.(IV). “Der politische Weg des letzten Herrschers der Donaumonarchie”, Böhlau,Berlin/Köl-

n/Weimar 1997

14 The French prime minister *Georges Clemenceau* published letters signed by Karl.

15 Emperor Karl's Abdication Proclamation, 11 November 1918: Since my accession I have incessantly tried to rescue my peoples from this tremendous war. I have not delayed the reestablishment of constitutional rights or the opening of a way for the people to substantial national development. Filled with an unalterable love for my peoples I will not, with my person, be a hindrance to their free development. I acknowledge the decision taken by German Austria to form a separate State. The people has by its deputies taken charge of the Government. I relinquish every participation in the administration of the State. Likewise I have released the members of the Austrian Government from their offices. May the German Austrian people realize harmony from the new adjustment. The happiness of my peoples was my aim from the beginning. My warmest wishes are that an internal peace will be able to heal the wounds of this war. (signed) Karl I, Source: Source Records of the Great War, Vol. VI, ed. Charles F. Horne, National Alumni 1923

16 Compare: at the end of the habsburg dynasty, for example: Richard Georg Plaschka/Karlheinz Mack, “Die Auflösung des Habsburgerreiches. Zusammenbruch und Neuorientierung im Donauraum”, München 1970

17 Wilson’s program conceptualized a new world order on nationalistic basic; several points written by: Elisabeth Kovács, Untergang oder Rettung der Donaumonarchie? Die österreichische Frage, Böhlau 2004, Bd. 1, S. 351

18 The hungarian “Räterepublik” lasts 133 days, according to Anikó Kovács-Bertrand, 1997, S 75

19 A high proportion of the Communist leadership was Jewish.

20 For life and creation, art and Karólyis: see: Steven Béloa Várdy, Historical Dictionary of Hungary, The Scarecrow Press, Inc., Lanham, Md., & London 1997 and: I. Barta/T. Berend, P. Hanák, M. Lackó, L. Makkai, Zs. L Nagy und Gy. Ránki, Die Geschichte Ungarns, Corvina Verlag, 1971, S. 675 und 677

17 Wilson’s program conceptualized a new world order on nationalistic basic; several points written by: Elisabeth Kovács, Untergang oder Rettung der Donaumonarchie? Die österreichische Frage, Böhlau 2004, Bd. 1, S. 351

22 Each treaty was separately formulated and called after its venue.

23 One concerned the south-eastern part of Carinthia, which was inhabited mostly by Slovenians. A claim of the new SHS-state was shot down through a plebiscite on October 20, 1920, in which the population chose to remain with Austria.

24 A land-claim that was banned , was Hungary's claim for Burgenland, which, under the name ‘Western Hungary‘ , had been part of the Hungarian kingdom since 1647. Through the Treaty of St. Germain it became part of the Austrian Republic in 1921.

25 The Sephardic Jews, expelled from Spain and Portugal in the end of the 15th century (after Ferdinand and Isabella established Spain as a militantly Catholic country, intolerant of religious minorities) found their

religious freedom and some economic and political success also in the Ottoman Empire, especially in Constantinople, Salonika and Dubrovnik. When the Turks completed their conquest of the Balkans in the 1500s, Sephardic Jews followed them into the interior, settling in the larger towns. Jews eventually became the majority in Salonika (in today's northern Greece). The Greek Jewish community survived there until World War II, speaking Ladino, a Spanish dialect.

26 During the Middle Ages, the European Jewish community became divided, physically and into two different cultural traditions: the Spanish (or Sephardic) Jews and the German (or Ashkenazic) Jews.

27 Millet is an Ottoman Turkish term for a confessional community in the Ottoman Empire. In the 19th century, with the Tanzimat reforms, the term started to refer to legally protected religious minority groups, other than the ruling Sunni. Millet comes from the Arabic word millah. The Millet system of Ottoman Islamic law is considered an early example of pre-modern democratic religious pluralism.

28 For that reason in 1803 there were only 15,000 Jews in Moldavia, but by 1859 they already counted to 118,000. In 1899 there already lived 197,000 Jews.

29 There were 4,000 in 1831, in 1859 they totalled up to 9,000 and in 1899 they reached almost 61,000.

30 In 1899 40% of urban dwellers were Jewish

31 During the peasant revolt of 1907, rural rioters attacked Jews as symbol of economic repression

32 From only 12.000 Jews in 1720 it were 83.000 1787. This made up 1% of the whole population

33 Jews made up 60% of the city's merchants, 51% of its lawyers and 63% of its medical doctors. But only 4% of municipal and government employees were Jewish.

34 In his early years, Leon (or Lev) Pinsker (1821-91), then later Zionist pioneer, was another former advocate of assimilation.

35 Historic steps in: Encyclopedia Judaica, Second Edition (Keter Publishing House) 2007, Bd. 20, S. 518-523. Historiic and sociological aspects in: Gerhard Botz/Ivar Oxaal/Michael Pollak (Hg.), Eine zerstörte Kultur. Jüdisches Leben und Antisemitismus in Wien seit dem 19. Jahrhundert, Buchloe 1990, S. 34-43.

36 The use of piyyuhim was introduced into northern Europe probably from Italy. There, again, the Mahzor underwent many changes, and a German ("Ashkenazi") ritual was established which is contained in the "Mahzor Ashkenaz," the "Mahzor Pehm [Bohemia] we-Polin [Poland]," and the "Minhag Zarfat" (French ritual). Of these the first two only are now in use. The French ritual was never published; it is extant partly in manuscript and partly in the ritual of the three Italian communities of Asti, Fassano, and Moncalvo, where many French Jews settled after their expulsion from France in 1306 and 1394. The several Mahzorim included in the Ashkenazic ritual vary in some details, but agree in essentials. They are distinguished from those of other rituals in containing numerous piyyuhim based upon the Halakah and Haggadah. The German ritual is used by the Jews in Germany, Bohemia, Moravia, or Silesia, Prussian Poland, Russia, Austria, Hungary, France, and

England. The Ashkenazic Mahzor was first published about 1521, the Polish in 1522. Among the commentaries on the German and Polish Mahzor, which have often been published together with the text, are those of: Benjamin ben Meïr ha-Levi of Nuremberg (Tanhausen, 1540), Isaac ben Jacob Jozebel (entitled "Hadrat Kodesh," Venice, 1554), an anonymous writer (entitled "Ma'agle Zedek," Venice, 1568), Zebi Hirsch Zundels (Lublin, 1579), Nathan Shapira (Cracow, 1604), Joseph Bezaleel Kaz Mehokek (Prague, 1616), a second anonymous writer (with additions entitled "Sefer ha-Masbir," by Joseph , and with glosses and notes entitled "Masbir he-Hadash," by Moses Kosmann, Amsterdam, 1667), Hirz Shatz (Wilhelmsdorf, 1673), Benjamin Wolf Heidenheim (Rödelheim, 1800), Uri Feibus ben Aryeh Löb (entitled “Keri'e Mo'ed," Breslau, 1805), Moses Israel Büdinger (Metz, 1817), and Jehiel Michael ha-Levi (entitled "Matteh Lewi," Slobuta, 1827). Source: Jewish Encyclopedia, www.jewishencyclopedia.com

[37] An understanding of Wittgenstein's Jewish heritage provides an important perspective on his work [Richter (2006)].

[38] The term used in 19th century Europe in the sense of “political union” signifies the unification of Austria with Nazi Germany since 1938. With pejorative connotation it hints at the forced loss of state independence which is often emphasised by the use of inverted commas in historical research, even though in broad inconsequence: in the political spectrum from conservative right wing to left-liberal, historians use the term with or without inverted commas or in connection with “so called”. Here and in the following it is used in dissociation to national socialist interpretation patterns to label a historic date and for this reason without inverted commas. Referring to the history of the term s.

Cornelia Schmitz-Berning, Vokabular des Nationalsozialismus, Berlin 2000, p. 32-33.

39 According to Dr Brigitte Bailer, Documentation Centre of Austrian Resistance, Vienna, 20th February 2008 and to the organiser of the Austrian People's Party ÖVP Brienek, Press Office of the Parliament of the Austrian Republic, 20th February 2008.

40 See mainly Judith Lechner, Riesiges Zeitzeugenprojekt startet, 10th September 2007; in: DiePresse.com; nachtdesschweigens.at and http://wien.ORF.at/stories/247417.

41 See Ernst Hanisch, Wien, Heldenplatz, in: transit., 15/1998, pp. 122-140. Referring to the concept of commemorative places: Pierre Nora, Zwischen Geschichte und Gedächtnis, Berlin 1990, and Deutsche Erinnerungsorte, ed. Etienne Francois and Hagen Schulze, 3 Vols, Munich 2003, here: Vol. 1, Introduction of the Editors, pp. 9-26.

42 See Julius H. Schoeps/Joachim Schlör (eds.), Bilder der Judenfeindschaft, München 1995.

43 Referring to the origins see Peter Pulzer, Die Entstehung des politischen Antisemitismus in Deutschland und Österreich 1867-1914, Göttingen 2004. Still worth reading: Alex Bein, Die Judenfrage. Biographie eines Weltproblems, Vol. 1, pp. 217-247 and Christina von Braun/Ludger Heid (eds): Der ewige Judenhaß, Berlin/Wien 2000.

44 Historic stations in: Encyclopedia Judaica, Second Edition (Keter Publishing House) 2007, Vol. 20, pp. 518-523. Referring to earlier history also: Ivar Oxaal, Die Juden im Wien des jungen Hitler. Historische und soziale Aspekte, in: Gerhard Botz/ Ivar Oxaal/ Michael Pollack (Eds),

Eine zerstörte Kultur. Jüdisches Leben und Antisemitismus in Wien seit dem 19. Jahrhundert, Buchloe 1990, pp. 34-43.

45 Encyclopedia Judaica, Vol. 20, p.520.

46 Demographic data according to Marsha L. Rozenblit, Die Juden Wiens, Assimilation und Identität, Wien/Köln/Graz 1988, p.27.

47 Barbara Beusy, Heimat und Hölle, Reinbek 1996, p.693.

48 See Ivar Oxaal, Die Juden im Wien des jungen Hitler, pp. 58-60.

49 See Marsha L. Rozenblit, Die Juden Wiens, p. 55-79.

50 See Peter Pulzer, Spezifische Momente und Spielarten des österreichischen und Wiener Antisemitismus, in: Botz/Oxaal/Pollak, Eine zerstörte Kultur, pp-121-140.

51 See Wilhelm Ellenbogen, Der Wiener Antisemitismus, in: Sozialistische Monatshefte 3-5 (1899). H. 9189909, pp. 418-425.

52 See Peter Pulzer, Die Wiederkehr des Hasses, in: Deutsch-Jüdische Geschichte der Neuzeit, ed. Michael Meyer and Michael Brenner, Vol III: Umstrittene Integration 1871-1918 by Steven M. Lowenstein, Paul Mendes-Flohr, Peter Pulzer and Monika Richarz, Munich 1997, pp. 193-248, here p.228.

53 See Peter Pulzer, Die Wiederkehr des Hasses, pp. 222-228.

54 Ibid, p.227.

55 In many of his writings, above all Der ungeliebte Jude. Zur Soziologie

des Antisemitismus, Zürich 1981 and Antisemitismus nach dem Holocaust. Bestandsaufnahme und Erscheinungsformen in deutschsprachigen Ländern, Köln 1986.

56 See Walter Boehlich, Der Berliner Antisemitismusstreit,Frankfurt /Main 1965 and Der "Berliner Antisemitismusstreit" 1879-1881. Eine Kontroverse um die Zugehörigkeit der deutschen Juden zur Nation. Kommentierte Quellenedition (edition of a primary source text) . By order of the Centre for Anti-Semitism Research edited by Karsten Krieger, Munich 2003.

57 Albert Lichtblau, Die Geburtsstätten des "horror judaei". Der "moderne Antisemitismus" in Berlin und Wien, www.sbg.ac.at/ges/people/lichtblau/horror.htm. About Schönerer, see also Peter Pulzer, Die Entstehung des politischen Antisemitismus in Deutschland und Österreich 1867-1914, (new edition of the volume of 1964 with extensive research report) Göttingen 2004, pp.181-192.

58 Monika Richarz, Berufliche und soziale Struktur, in: Deutsch-jüdische Geschichte in der Neuzeit, Vol.III, Umstrittene Integration 1871-1918, p.58.

59 Helge Zoitl lists ten alone at University and Technical College. Zoitl, Student kommt von Studieren, Geschichte der sozialdemokratischen Studentenbewegung in Wien, Wien/Zürich 1992, pp. 338ff. According to: Wolfgang Lamsa, Der Siegfriedskopf. Antisemitismus und Universität, in: Context XXI 7-8/2001-1/2002.

60 The "Siegfriedskopf" is still standing today in the University, however, it was transferred from the auditorium to the arcade yard and integrated

into an explanatory historical context. This does not prevent rightist extremist members of student fraternities from meeting annually at the identity creating place. See Judith Lecher, Universität Wien: Und täglich grüsst der Siegfriedskopf, Die Presse, 15th January 2008.

61 Albert Müller, Dynamische Adaptierung und “Selbstbehauptung”, in: Geschichte und Gesellschaft, 23/1997, 1, pp. 592-617, here p.605.

62 Albert Müller, Dynamische Adaptierung und “Selbstbehauptung”.

63 Elfi Pracht, Portrait Hans Kelsen, in : Ludger Heidi/Elfi Pracht/Werner Ripkens/Marina Sassenberg, Juden im Rheinland (additional booklet to the slide series), Düsseldorf 1994, pp. 41-43.

64 Thomas Bernhard, Heldenplatz, Frankfurt/Main 1988, quote p13.

65 See Eberhard Jäckel/Peter Longerich/Julius H.Schoeps (Eds), Enzyklopädie des Holocaust, 4 Vols, Munich 1998.

66 See Enzyklopädie des Holocaust, p.1591.

67 Later most of the others also were demolished or diverted from their intended use. Synagogen in Wien, Website of the Israelite Religious Community, www.ikg-wien.at and Enzyklopädie des Holocaust, p. 1594.

68 See Enzyklopädie des Holocaust, p.1592.

69 See Gerhard Botz, Die Ausgliederung der Juden aus der Gesellschaft, in: Gerhard Botz et al (ed), Eine zerstörte Kultur, Buchloe 1990, pp.285-311, tab. P. 308.

70 Common and specialized encyclopedia show differing data, e.g. the dtv-encyclopedia: 1940-1947, Encyclopaedia Judaica, 2nd edition, Keter Publishing House 2007, Vol. 4, p762: 1940-19.

71 The topic can be outlined only in general terms. The comparative European-Jewish historiography did not start until the 1990s to integrate Eastern Europe into its discourse. The study of Jewish history in Klausenburg and Siebenbürgen remains limited thereby to a large extent to the Eastern European linguistic area. Only isolated English and German-language research papers are published. The best historical investigation to the topic offers at present the bilingual monograph of Ladislau Gyémánt, The Jews of Transylvania, Cluj 2004.

72 Compare Encyclopaedia Judaica, Second Edition, Vol 20, p108.

73 Avram Andrei Baleanu, Rumänien, in: Elke-Vera Kotowski/Julius H. Schoeps und Hiltrud Wallenborn (Ed.), Handbuch zur Geschichte der Juden in Europa (called GJE-handbook in the following context), Darmstadt 2001, p281.

74 Please especially compare: András Kovács, Hungary, in: GJE-Handbook, pp151-163.

75 Compare András Kovács, Hungary, in: GJE-Handbook, p155.

76 Until then, only the first name was used according to Jewish tradition, provided by the father's name, e.g. Abraham ben Jacob.

77 Compare Ladislau Gyémánt, The Jews of Transylvania, p175.

78 Compare Ladislau Gyémánt, The Jews of Transylvania, p214.

79 Ambrus Miskolczy: How did the hungarian Jewish emancipation law arise? , in: Holger Fischer, Die ungarische Revolution von 1848/49, Krämer, Hamburg 1999, p -164

80 Compare Hiltrud Glass, Zerbrochene Nachbarschaft, Munich 1996, p37.

81 Compare Encyclopaedia Judaica, second edition, vol. 4, pp762-763. The service in the neological Judaism took frequently place following the German model, often also in German language. Also the German science of the Judaism had numerous supporters among Hungarian neologists. See Lowenstein/Mendes-Flohr/Pulzer/Richarz (Ed.), Deutsch-jüdische Geschichte in der Neuzeit, vol. 3, Munich 1997, p121. Comprehensive international research has been done regarding Chassidismus; for fast classification, compare Heiko Haumann, Die Volksfrömmigkeit des Chassidismus, in: Ders., Die Geschichte der Ostjuden, Munich 1998, p 53-57.

82 Compare Raphael Patai, Jews in Hungary, esp. chapters The Fin de Siècle and its Aftermath I, II and III, p358-428. Citation p364.

83 Compare Encyclopaedia Judaica, Second Edition, vol. 4, p762.

84 Compare Holly Case, The Jews of Kolozsvár and the Hungarian Administration 1940-1944, in: Wolfgang Müller (Ed.), Osteuropa vom Weltkrieg zur Wende, Vienna 2007, p39-53, here: p42.

85 Compare Hiltrud Glass, Zerbrochene Nachbarschaft, p37.

86 Compare Hiltrud Glass, Zerbrochene Nachbarschaft, p151-152.

87 Compare Hiltrud Glass, Zerbrochene Nachbarschaft, p86.

88 Compare Heiko Haumann, Geschichte der Ostjuden, p188 et sqq.

89 Compare Avram Andrei Baleanu, Rumania, in: GJE-Handbook.

90 Compare Heiko Haumann, Geschichte der Ostjuden, p188 et sqq.

91 Compare Avram Andrei Baleanu, Romania, in: GJE-Handbook, p282.

92 In 1918, slightly before the end of World War I, Bessarabia declared its independence from Russia and after three months united with the Kingdom of Romania.

93 Romanian control of the province was recognized internationally in the Treaty of St. Germain in 1919.

94 Compare Anda Dumitru, Romania, in: Jewish Virtual Library 2004/2008 (jewishvirtuallibrary.org/jsource/vjw/romania.html)

95 The Federal Statistical Office identifies 13% of Klausenburg's total population as "Jewish". Compare Hiltrud Glass, Zerbrochene Nachbarschaft, p39.

96 Compare Holly Case, The Jews of Kolozsvár, p42.

97 Compare Baleanu, Romania, in: GJE-Handbook, p282.

98 Compare Baleanu, Romania, in: GJE-Handbook, p284-285.

99 Website of the Romanian Jewish Community (romanianjewish.org/en/introducere.html) of 28.04.2008. Appropriate literature about Siebenbürgen and/or Klausenburg neither records estimations nor exact figures.

100 Compare Carsten Dippel, "Komm wie Du willst, aber komm!" Integration statt Auswanderung: das jüdische Rumänien, in: Jüdische Zeitung von November 2007.

101 The easiest transition was in the democratic, tolerant and relatively affluent Czechoslovakia, whose paternalistic president T.G.Masaryk had been venerated as a committed foe of anti-Semitism.

102 The Jewish population in Poland and the Ukraine experienced a series of pogroms on a massive scale between 1918-1920, which claimed the lives of a staggering number of 100,000 Jews [Rozenblit (2001)].

103 Menger notes that amongst his University colleagues, his "friend Hahn was the only mathematician who knew Wald personally. No one else showed the slightest interest in his work" (Menger 1952, p18).

104 After the death of his father, in 1921, K. Menger started to revise the second edition of the'Grundsätze', which was published two years later. In the same year he wrote his first paper on economics theory

105 Memo J. van Sickle to T.B.Kittregde, Jan. 21, 1938, Austrian Institute for Trade Cycle Research, Archive. Therein Van Sickle writes "*As Wald is responsible for his parents in Rumania, he has not been able to save anything and cannot, therefore, finance the trip to Colorado...*".

106 The Colloquium's developing interest in economics and social science appears in the fifth volume, which concerns the meetings of 1933-34, with reference to two notes by Menger, on 'Bernoullian economics and the Petersburg game' and on the relationship between finite sets and the formalization of ethics, and to the papers of March 1934 on general

equilibrium by Schlesinger and AW.

107 Cp. AW's ‘‘Berechnung und Ausschaltung von Saisonschwankungen’ and ‘Über die Produktionsgleichungen der ökonomischen Wertlehre’. [Wald (1935) c,], [Wald (1936) c,] The duopoly paper shows how the model's equilibrium depends on the form of the demand function for the commodity faced by the two firms. E.g., the existence of a unique, stable, equilibrium point requires that the demand function cut the price and quantity axes and has a negative first derivative and nonpositive second derivative. A third general equilibrium paper by Wald got lost in the riots of 1938.

108 July 24, 1933, Research Aid Grants, Paris, Rockefeller Foundation, box 4, folder 36, Austrian Institute of Business Cycle Resarch.

109 February 13, 1935, ‘Report on the Activities of the Austrian Institute for Trade Cycle Research 1931-1935', Box 4, Folder 37, AITCR, Vienna, 1935-1936

110 See "La nature et le calcul des variations saisonnières", Memorandum per Dr. A. Wald, distribué à l'occasion de la conférence de Dr. Oskar Morgenstern, Wien, 6 mai, 1935, à l'Institut Scientifique des Recherches Économiques et Sociales, Paris.

111 Franz Alt (b. 1910) entered the University of Vienna as a student of mathematics in 1928, and was a participant in the Menger Colloquium and Hahn's seminar. Upon graduation, he recalled in a 1997 interview, Menger felt guilty that he could not provide him with some employment and recommended him to Morgenstern, who appointed him as private tutor in mathematics at 20 Schillings an afternoon.

"*Morgenstern . . . very interesting, very intelligent. . . . He was convinced that mathematics was important . . . He told me once that he had wanted to study physics, but right after World War I all the interest was in the social sciences, and so he felt he should go into that . . . He had me help him read books on mathematical economics. It helped that I knew languages. We read English mostly. There was a man named Bowley who wrote a book here on mathematical economics. It was just as interesting for me as for him. I had to prepare each meeting, read a chapter in the book, and then we discussed it. He knew as much about it as I did, but perhaps once in a while I could explain something*". (From a May 1997 interview with Alt, at his New York home, conducted by Seymour Kass, Bert Schweitzer, Abe Sklar, and Mrs. Annice Alt.). Through Morgenstern, Alt met various figures, including Oskar Lange, and Paul and Alan Sweezy, and was led to publish an article on utility theory in the Zeitschrift für Nationalökonomie. In 1938, Alt moved to the US, where he was introduced by Morgenstern to Harold Hotelling. The latter, in turn, introduced him to Charles Roos, formerly of the Cowles Commission, whose book, 'Dynamic Economics', Alt had reviewed for the Zeitschrift, and who had by then left Cowles to set up a private economic forecasting consultancy, the Econometric Institute, in New York. Alt later left economics and made his career in computing. I am grateful to Professor Seymour Kass for permission to quote from this interview, the manuscript of which has been deposited in the Vienna Circle collections both at the University of Pittsburgh and in Vienna.

112 "*Yesterday more mathematics. I am beginning to see deeper and deeper, and through the ongoing implicit repetition it all seems to settle down*" (Diary, Oct. 19, 1935, OM archive, Duke Univ.). Some days

later: "*Another mathematics lesson, very interesting. I feel as though I am making real progress. Wald told me of his new works. An amazing thing. It isn't enough, as Walras assumed, to consider only monotonically decreasing utility functions, because he [Wald] proved that with many of them, simple exchanges never lead to an equilibrium! Similar paradoxes for the addition of demand curves, which were considered before to be totally harmless! That should have far-reaching consequences . . . Wald is really intelligent. I consider these works to be very important; they throw new light on the application of mathematics to economics. One will not be able to do without these at all*" (Diary, Nov. 2, 1935, OM archive, Duke Univ.).

113 See Note, undated, concerning Kittredge interview with AW on July 9, 1935, Austrian Institute for Trade Cycle Research.

114 Letter, Kittredge to Van Sickle, February 23, 1936, Austrian Institute for Trade Cycle Research

115 Memo, Van Sickle to Kittredge, 27 March 1936, Ernest John was one of Morgenstern's economic researchers at the Institute, Austrian Institute for Trade Cycle Research.

116 But, at least, in Kittredge's words, AW recently did a invention of a new device for improving radio apparatus and so assumed a minimal income for him. (Source: Kittredge's note on an interview with AW, July 11, 1936, Austrian Institute for Trade Cycle Research).

117 Letter, Van Sickle to Kittredge, 16 Sept. 1936, Austrian Institute for Trade Cycle Research.

118 Menger published them altogether in 1929.

119 *Menger* can be seen a forerunner of *G. Birkhoff*, who in 1935 introduced complemented modular lattices

120 The Navy V-12 program allowed students to complete baccalaureate degrees at civilian universities and earn commissions in the Navy and Marine Corps during World War II.

121 L. Wittgenstein, often noted as member of the Vienna Circle, only had a close relationship to the group. Anyhow the Tractatus Logico-Philosophicus (published 1918) was a central topic of discussion.

122 Frank followed A. Einstein's on his chair of theoretical physics

123 For the original members, guests and loose contacts to the Vienna Circle see Appendix D.

124 These "*were also attended by the philosophers of the Vienna Circle*" [Menger (1998)].

125 It states the scientific world-conception of the Vienna Circle, that is characterized by two features. At first an empiricist and positivist one: knowledge comes from experience. And at second, the scientific conception of the world is solely marked by logical analysis." (Source: The Scientific Conception of the World. The Vienna Circle in Sarkar, Sahotra, 1996, p. 331)

126 *F. Kaufmann* emigrated in 1938 and joined the Faculty of the New School for Social Research in New York where he taught pretty successful until his premature death [Haberler (1961)].

127 Kaufmann delivered in 1934 Gottfried von Haberler copies of all the

songs that were published in 1990

128 In contrary to that, the analytic approach defines objects by equations.

129 Congruence implicates for instance the existence of a distance-preserving function that maps a given metric space onto an euclidean n-space.

130 Compare the document "A Conversation with Franz Alt," May 19, 1997, with Annice Alt, Seymour Kass, Berthold Schweizer, Abe Sklar (recollections of Karl Menger, the Menger Colloquium, Kurt Godel, Abraham Wald, and the University of Vienna prior to 1938). Archives of Scientific Philosophy, Doc. num. ASP MC 01-02.

131 With contributions of *J. Dawson*, *R. Engelking* and *W. Hildenbrand*, the statistician *K. Sigmund* from Vienna University republished the EemK in 1998, that offer "*a revealing insight into an intellectually fascinating period*" [Menger (1998)].

132 On the politics and economics of the Ständestaat see Tàlos and Neugebauer (1988), Senft (2002), and Bischof, Pelinka and Lassner (2003).

133 The term 'Austroliberalism' follows K. H. Müller 1987, Die Idealwelten der österreichischen Nationalökonomen, in F. Stadler, ed. , 1987, 238-75.

134 On the history of the Austrian school and the distinction of two strands see Streissler (1988) and similarly Hayek (1992, 42-60). The term 'Austroliberalism' follows Müller (1987).

135 Joseph Schumpeter is not included because he is typically not considered as a member of the Austrian (or any) school

136 The two other chairs were held by Ferdinand Degenfeld-Schönburg (an economic historian) and Othmar Spann (the founder of 'universalism').

137 On the emigration of the Austrian economists see [Craver (1986)], [Müller (1987)], [Fleck (1987)]and [Rathkolb (1987)]or biographical information see [Hagemann and Krohn (1999)]).

138 From London Hayek propagated his international reputation as a leading representative of classic liberalism.

139 Of the Austroliberals only the lesser known stayed in Austria, for example, Richard Strigl, Martha Stephanie Braun (one of the very few female members), and Erich Schiff.

140 E.g. much material can be found in the F. Machlup papers, Hoover Institution archives, Stanford University, and the Morgenstern papers, Special Collections Library, Duke University.

141 In Mises' view only democracy can be consistent with liberalism

142 cp. Political Situation of Austria', F. Machlup papers, box 86, folder 43

143 On the 'gold standard mentality' see B. Eichengreen and P. Temin, 2000, The gold standard and the Great Depression. Contemporary European History 9, pp183–207.

144 As contemporary terminology equated inflation with an increase in the (effective) quantity of money, this requirement was, in the case of growth, more stringent than a stable price level.

145 See, for example, retrospectively Hayek, 1945, The future of Austria, in

The Spectator, 6 Apr.

146 Max Mintz to Machlup, 9 Dec 1934, Fritz Machlup papers, box 53, folder 25

147 Mintz to Machlup, 9 Dec 1934. Fritz Machlup papers, box 53, folder 25

148 Machlup to Mises, 25 Sept 1934. Fritz Machlup papers, box 53, folder 27.

149 Mises to Machlup, 23 Nov 1934. Fritz Machlup papers, box 53, folder 27

150 It is noteworthy that in the revised English version of this book [Morgenstern (1937)], pp141-143 this statement is considerably weakened

151 Similar pronouncements are documented for Dollfuß, Kurt Schuschnigg or Viktor Kienböck (the President of the Austrian central bank). Also Rost van Tonningen (the representative of the League of Nations in Austria) sang form the same hymnsheet. See also the 1932 correspondence between Morgenstern and Viktor Brauneis (also a director at the Austrian central bank), source: O. Morgenstern archive, Duke University, box 4.

152 Haberler to Morgenstern, undated [Nov 1935?]. O. Morgenstern papers, Duke Univ., box 5.

153 Later Mayer hailed Dobretsberger on the occassion of becoming minister of social affairs. Mayer compared him with his predecessor Böhm-Bawerk. A comparison that was ill-received among the other members

of the Austrian school. Morgenstern to Haberler, 5 Nov 1935, O. Morgenstern papers, Duke Univ., box 5.

154 On the activities of the Rockefeller Foundation see [Craver (1986)] and Leonard [Leonard (2002)]

155 Morgenstern diary, 26 Dec 1934. O. Morgenstern papers, box 13

156 Contemporarily in 1934, Braun expresses that Morgenstern could "*much misunderstood, for he will be reproached for favoring political fascism for the sake of sound economic policies*" [Braun (1934)]

157 The only German-language journal after 1933 with an international reputation

158 The loss of this center of communication caused the divergence of the theoretical concepts within the Austrian school in the 1930s. See Klausinger [Klausinger (2002)].

159 See Neues Wiener Tagblatt, undated (1931?). O. Morgenstern papers, box 6. Also Morgenstern to Grätz, letter from April, 4, 1932. O. Morgenstern papers, box 5.

160 On the problems with censorship see Morgenstern letters to Machlup on March 19 and 26, 1934. Machlup papers, Hoover Instititution archives, Stanford University, box 54, folder 15.

161 Morgenstern held some formal positions within the Austrian bureaucracy. He was advisor to the Ministry of Commerce, where he was responsible for topics on traffic infrastructure and partly for the Austrian railroads. He was also a member to the price commision that

regulated commodity prices, mainly of agricultural products.

162 O. Morgenstern in a letter to Vleugels, 6 May 1933. O. Morgenstern papers, box 7

163 Haberler in a letter to Morgenstern warned: “*Your sweeping ondemnations of all possible people are very amusing. However, they put a heavy burden of responsibility on your forthcoming works*.” Haberler to Morgenstern, 15 Aug 1934. O. Morgenstern papers, box 5.

164 Morgenstern considered liberalism (and socialism) as examples of “*rigid systems of economic policy*” that cannot be justified by the insights of economic theory [Morgenstern (1934)], ch 3

165 Hayek to Machlup, 14 Jan 1934. F. Machlup papers, Hoover Instititution archives, Stanford University, box 42, folder 2

166 cp. Morgenstern’s diary, 4 Nov 1934. O. Morgenstern papaers, Duke University, box 13.

167 O. Morgenstern in his diary, 3 Sep 1935, O. Morgenstern papers, Duke University, box 13

168 Tintner, who had worked for the Institute before and had left for the US, where he reported to the Rockefeller Foundation that Morgenstern tried to avoid any critical review of government policies (especially in the Monatsberichte) in exchange for having a free hand in his internal role as an advisor [Leonard (2002)] , p38-39

169 On the ‘gold standard mentality’ see B. Eichengreen and P. Temin, 2000, The gold standard and the Great Depression. Contemporary

European History 9, pp183–207.

170 As contemporary terminology equated inflation with an increase in the (effective) quantity of money, this requirement was, in the case of growth, more stringent than a stable price level.

171 Helmut Gruber's Red Vienna analyis from 1993 (Experiment in Working-Class Culture, 1919-1934, Gruber 1991) is critical with Socialist Vienna. He provides profound details of the political and philosophical shortcomings of party functionaries and their programs.

172 Most definitions of Fascism include these elements: Fascism is a mass movement, it is nationalist and exaggerated patriotic and subordinates the needs of the individual to the needs of the national community. Despite its character as a movement of mass participation, control flows from the top down in Fascism, usually from one charismatic figure. Fascism is anti-parliamentary, anti-democratic and in priniple even anti-political, because it depicts a world of absolute values in which debate and compromise lose their validity. Fascism replaces compromise with violence and handles difficult issues with a kind of 'final solution'. In addition, fascism is generally described as a movement that has specific enemies: one is Communism, a response to perceived threats from working-class socialism. Fascism is also anti-capitalist, a response to fear of big banks, department stores and factory-produced goods. For both of these reasons, fascism is rooted in the lower middle class, which suffers threats from both socialism and capitalism. Fascism is racist and often anti-Semitic. Ethnic nationalism rejects minority groups. The Jews in particular also suffer from their mythical identification with both Bolshevism and capitalism. Balkan fascism forces serious reexamination of one common assertion: the

middle-class nature of fascism. ('Fascism', Encyclopædia Britannica ,8 January 2008 / Passmore, Kevin. Fascism: A Very Short Introduction. Oxford University Press. ISBN 0192801554 / Laqueuer, Walter. Fascism: Past, Present, Future. Oxford University Press. ISBN 019511793X / Paxton, Robert. The Anatomy of Fascism. Vintage Books. ISBN 1400033918 / Griffin, Roger. The Nature of Fascism. Palgrave Macmillan. ISBN 0312071329 / Eatwell, Roger. Fascism: A History. University of Michigan. ISBN 071399147X / Payne, Stanley. A History of Fascism, 1914-45. University of Wisconsin Press. ISBN 0299148742 / Nolte, Ernst. Three Faces of Fascism: Action Française, Italian Fascism and National Socialism. Holt, Rinehart and Winston / Fritzsche, Peter. Rehearsals for Fascism: Populism and Political Mobilization in Weimar Germany. Oxford University Press. ISBN 0195057805 / "Collectivism", Encyclopædia Britannica, 8 January 2008).

[173] Promoted by Hitler since the beginnings of his leadership in the NSDAP and formerly stated in 1924 in his pamphlet ‘Mein Kampf‘.

[174] Bourgeoisie is a classification term used to describe a social class of people who are in the so-called upper or the merchant class, whose status or power comes from employment, education, and wealth as opposed to aristocratic origin. Petite bourgeoisie (also termed with petty bourgeoisie) is used to describe the class below the bourgeoisie but above the proletariat (usually independent operators with a small number of employees). Source: A Dictionary of Sociology, 1998, http://www.encyclopedia.com.

[175] In 1914 about 80% of peasant holdings were mortgaged [Matis (1972)], [Brusatti (1965)]

176 Before 1918 a third of the consumption had to be transferred from the Hungarian part [Klenner (1953)], p253.

177 Their number only shrank by 4.2% between 1902 and 1934. ÖWS II in [Lauridson (2007)])

178 About 50% of all farms were under 5 hectars and approximately 85% were under 20 hectars [Lauridson (2007)]

179 The only towns with more than 100.000 inhabitants

180 Anglizised form ‘petite bourgeoisie', a class between the workingmen and the capitalists. In modern usage it is used derisively, to refer to the consumption habits and tastes of the middle class and the lower middle class in particular.

181 See also Hans Mommsen’s considerations on the SPD and SDAP in Mommsen, 1981, p438

182 The CSP got the leadership of the Vienna City Council with Karl Lueger as mayor in 1897. This was forced upon the Emperor, who previously had rejected Lueger three times.

183 The CSP even initiated a Catholic worker movement to win over the working class.

184 A system that is called ‘military state socialism . D. Mitrany [Forbes and Mitrany (2006)]

185 Romania had lost a third of its livestock after WWI.

186 The grain exports of Romania in 1919 were half of prewar levels

187 Factory cities like Budapest lost access to mines and agricultural products

188 In 1921, Hungary exported flour at 30% of the pre-war level, livestock at 20%, industrial goods at only 50%. Romanian postwar imports were a third of old levels, and exports at 3%. In Bulgaria in 1920, imports had fallen by 65%, exports by 50%.

189 In 1934 Germany agreed to buy Hungarian agricultural goods at an attractive subsidized price. In return Hungary had to open its border to German industrial goods. Similiar treaties were made with other Balkan states.

190 In 1929 Germany took in 30 percent of Bulgaria's exports and accounted for 22 percent of Bulgarian imports. By 1939 those figures were 71 percent and 69 percent.

191 Between 1919 and 1923, Austria’s money supply increased by 14.250%.

192 The SDAP imposed artificially low prices on agricultural products and tried to require food by force for the cities.

193 Also thousands of Viennese regularly ransack the woods around to chop down the trees, and carry cords of firewood back into the city to keep their homes and apartments warm in the winters of 1919, 1920, and 1921

194 A commodity bundle q is optimal under a price system p and a total

expenditure e if it maximizes utility under the budget constraint $q' p = e$

195 This is the system of the so-called Engel functions, which form combined the Engel curve. They can be estimated from a survey of family budgets.

196 A simpler approximation to the cost of living index for more than two periods is the construction of chain index numbers, as e.g. proposed by Eurostat with its ‘harmonized consumer price index’.

197 The 10th district had been created in 1874 by the division of the Leopoldstadt district in 1900, whereby the northern part became the 20th district (Brigittenau).

198 Literally translated as ‘the founder’s epoch’ is stands for the economic phase in Germany and Austria before the great stock market crash of 1873. In central Europe it was the phase of the awakening industrialisation, reaching back to the 1840s. Althoug there is no precise timeframe defined for the founder’s epoch, it is accepted in Austria’s case to regard the March Revolution as its beginning. In Germany as a consequence from the war of 1870/71 a large influx of french capital fired the enduring economic boom to an excess that finally culminated in the 1873 crash. The prevailing classical liberalism of this epoch accompanies the evolvement of the middle class’ cultural leadership but fails in the realization of its immanent political demands.

199 Karl Lueger is responsible for the “Wiener Hochquellwasserleitung”, bringing fresh water from the mountains to Vienna, the creation of a belt of meadows and forests around the city and the integrated public transport system owned by the municipality.

200 This probably was more a political acumen than a true conviction [Elon

(2002)]. Years later, Adolf Hitler saw himself inspired by Lueger for his own virulent hatred of Jews, "*... however the enthusiastic tribute that Hitler paid him in 'Mein Kampf' does not seem justified, for the Jews did not suffer under his* [Lueger's] *administration*." [Poliakov (2003)].

201 The so-called 'Separation Act' erased the old link between Vienna and Lower Austria and elevated the city to the rank of a province in its own right.

202 Critics see the Socialists building program in Vienna as very conventional in selecting architectural styles and considering possible living arrangements [Gruber (1991)]. More advanced municipal apartments for example have been built in Hamburg, Frankfurt, and Berlin.

203 The Socialists building project exemplifies this critic. Vienna's municipal buildings should have been an outstanding triumph in that regard, offering functional, affordable living quarters for the working class. But socialist party administrators and planners never consulted workers when designing new buildings.

204 At its extreme, such attitudes could lead Socialists such as David Joseph Bach, the dictatorial director of the Social Democratic Art Organization (Sozialdemokratische Kunststelle), to insist that in the high arts such as music and the theatre the task was to provide the workers with the "finest classical and therefore most revolutionary art". Bach's Art Organization never really confronted the question of what "socialist" high arts might be; instead it acted almost exclusively as a ticket bureau which gave free passes to employees, managers, and intellectuals and not primarily to workers.

205 The Socialists often were harsh in their criticism of mass culture They did not recognize its possibilities and hence lost an important opportunity to use it as promoter for their program [Gruber (1991)].

206 The GDVP originates from the ‘Großdeutschen Vereinigung‘ (founded by Franz Dinghofer on August 8, 1920, as union of national and liberal minded delegates). It consolidated about 17 political groups. Foremost higher officials and high-school teachers represented a large part of its membership. The GDVP was straightened to a Pan-German movement (supported by a majority of the Austrians) without being strongly antisemitic (although such tendencies partly came to light). The GDVP was a decided opponent of the regulations of the peace treaty of Saint-Germain-en-Laye. In 1920 the GDVP was governing party, from 1921 to 1932 it was involved in government (together with the Christian-Social party) and placed between 1922-27 the vice chancellor. After the losing of governmental responsibilty many of its members withdrew and switched to the Home Guards or the National Socialists. In 1933 the GDVP made a deal with the Austrian NSDAP in founding a combat companionship with the Austrian NSDAP. During the period of Austrian fascism the GDVP officially was forbidden and after the annexation its former members run over to the NSDAP. After 1945 former GDVP members reunited and later passed into today’s FPÖ.

207 In 1916, Max Dvorak's ‘Katechismus der Denkmalpflege’ appeared. Therein he builds upon the ideas of Austrian Alois Riegl and extends the idea of ’homeland protection’ beyond a simple protection of geographical borders to encapsulate a society's culture.

208 These had been formed after the end of WWI (mainly during

1921-1923) from local guards and similar self-protecting combat units. Their beginning goes back to Nov., Dec. 1918, when border disputes with the newly founded Czechoslovakian army, Hungarian troops and military units of the SHS state (the later Yugoslavia) occurred [Chraska (1981)], p33.

209 The ’Korneuburger Eid’ was taken by home guards leader Richard Steidle on May, 18, 1930 in Korneuburg. It was a public commitment of parts of the Austrian home guards to fascism. (Wiltschegg, Walter, Die Heimwehr. Eine unwiderstehliche Volksbewegung?, in: Studien und Quellen zur österreichischen Zeitgeschichte, Bd. 7, Verlag für Geschichte und Politik, Wien 1985, ISBN 3-7028-0221-5)

210 The term ‘Austrofascism’, although used by the proponents of the regime itself, it is still disputed until today. It is predominantly used by left-wing historians, while others speak of the Ständestaat’. The ÖVP (the successor of the CS), in parts does not distance itself from the authoritarian Austrian regime. Usually they stress out that the Austro-fascists aimed at Austria's fight for independence from the Nazis. It is undisputed that the regime was an authoritarian dictatorship. It locked away members of the opposition, mostly nazis, communists and social-democrats, in concentration camps called ‘Anhaltelager’ or imprison-ment centers. Although the VF used fascist-like symbols (such as the‘Kruckenkreuz’) and was meant to be a party of the masses, it lacked a solid basis in the population, especially among the workers who tended to support the Communists or the Nazis. According to some historians the Austrian government also did not target minorities or engage in any sort of expansionism. Austrofascism was a contrived and desperate attempt to ’out-Hitler’ (‘überhitlern’) the Nazis (a term

used by Dollfuß himself). They argue that Dollfuß was interested in a renaissance of Catholicism rather than in a totalitarian state. The historian Ernst Hanisch speaks of semi-fascism. Austrofascism is sometimes also called imitation fascism. Source: Stephan Neuhäuser: 'Wir werden ganze Arbeit leisten' - Der austrofaschistische Staatsstreich 1934, ISBN 3-8334-0873-1 / Emmerich Tálos, Wolfgang Neugebauer: Austrofaschismus. Politik, Ökonomie, Kultur. 1933-1938. 5th Edition, Münster, Austria, 2005, ISBN 3-8258-7712-4 / Hans Schafranek: Sommerfest mit Preisschießen. Die unbekannte Geschichte des NS-Putsches im Juli 1934. Czernin Publishers, Vienna 2006. / Hans Schafranek: Hakenkreuz und rote Fahne. Die verdrängte Kooperation von Nationalsozialisten und Linken im illegalen Kampf gegen die Diktatur des 'Austrofaschismus'. In: Bochumer Archiv für die Geschichte des Widerstandes und der Arbeit, No.9 (1988), pp.7 - 45. / Jill Lewis: Austria: Heimwehr, NSDAP and the Christian Social State, in: Kalis, Aristotle A.: The Fascism Reader. London/New York / Lucian O. Meysels: Der Austrofaschismus - Das Ende der ersten Republik und ihr letzter Kanzler. Amalthea, Vienna and Munich, 1992 / Erika Weinzierl: Der Februar 1934 und die Folgen für Österreich. Picus Publishers, Vienna 1994 / Manfred Scheuch: Der Weg zum Heldenplatz. Eine Geschichte der österreichischen Diktatur 1933-1938. Publishing House Kremayr & Scheriau, Vienna 2005, ISBN 978-3-218-00734-4.

211 Among others known names as Walter Pfrimer (a regional head in Styria who, without any support from other home guard leaders, abortively attempted a coup in 1931), Ernst Rüdiger Starhemberg,, who was during the short time government of C. Vaugoin Minister of the Interior, and Richard Steidle, member of the CS and first federal leader

('Bundesführer') of the united regional Home Guards after 1927 [Perz, B., Safrian, H., Stuhlpfarrer, K., Safrian and Stuhlpfarrer (1982)]. As such he substantially promoted the 'Korneuburger Eid'.

212 Erich Scheithauer, Grete Woratschek, Werner Tscherne: Geschichte Österreichs in Stichworten, Teil V: Die Zeit der demokratischen Republik Österreich von 1918 bis 1934, Wien, Ferdinand Hirt GmbH & Co. KG. 1983, pp220

213 From a letter of the hungarian military atttachee in Paris, from January 24, 1934, to the hungarian General Staff in Budapest. Source: 1000 Jahre Österreich. Folienatlas und Materialsammlung zur Geschichte Österreichs. Typografik Verlag + Vertrieb, Salzburg, 1996, Schriften/-Dokumente 38.2.

214 In Linz a Home Guards force, led by the vice-chancellor Emil Fey, devastated 'Hotel Schiff', a dependance of the Social Democrats, alleged to search for weapons. But despite its illegalisation the outlawed Defense League actively resisted, sparking off an armed conflict with a conglomeration of the Home Guards, the gendarmerie, and the regular Federal Army

215 It were the the socialists that coined the term Austrofascism' as the underlying ideology essentially was that of most conservative elements of the Austrian Catholic clergy, a feature showing inconsistency with Nazism

216 Within the one-party system with the Patriotic Front the usual spectrum of a many party system should be replaced by professional organisations (a corporative system)

217 The law of working-hours got disposed, also the ‚money for the unemployed was abridged.

218 The lower middle-class was not the core of Hungarian ultra-nationalism as many of the middle-class were not ethnic Magyars.

219 The former Austro-Hungarian Admiral Horthy became regent, a convenient solution that left him free of further authority as neither the Allies nor the Magyars would have accepted a return of the Habsburgs.

220 Gombos was supported of racist secret societies, unemployed youths, ex-officers and revisionists. He even accepted donations from Jewish industrialists if they were Magyarized and patriotic. Despite his fascist rhetoric, his actual political course was more that of an opportunist.

221 He espoused 'Hungarism', a plan for a national socialist welfare state and a Balkan federation under Magyar leadership. Arrow Cross rhetoric involved revisionism and anti-Semitism.

222 70% of Romanias population around 1930 still lived from agricultural work. Land reforms after WWI had left 75% of the farms too small for a living for their owners. Romania had Europe's highest death rate, highest infant mortality. 50% of the population was illiterate. Peasants had a exceeding mistrust in banks, bureaucrats, and townsfolk.

223 The largest fascist group after 1927 was the Legion of the Archangel Michael or Iron Guard. It was founded in 1927 in Jassy (Moldavia), a town within Jewish districts. The founder, Codreanu, began his political career in the nationalist, anti-Semitic Association of Christian Students. In 1923 he founded the League of National Christian

Defence, then was briefly jailed for the murder of a fellow-student. In jail he had a vision of the Archangel Michael, an important element in the Iron Guard's messianic tone.

224 Afore called DNSAP, that split in 1923 into the ‘Deutschsozialen Verein’ and the ‘Schulz-Gruppe’, and reestablished as NSDAP (Hitler movement) on May 4, 1926 in Vienna.

225 A minister without portfolio is either a government minister with no specific responsibilities or a minister that does not head a particular ministry.

226 Schuschnigg appointed *Arthur Seyss-Inquart*, an Austrian pro-Nazi lawyer (who was later executed at the Nuremberg Trials for crimes against humanity), as Interior Minister and another Austrian Nazi, *Edmund Glaise-Horstenau* (committed later suicide at *Langwasser* military camp near *Nuremberg* in 1946), as minister, even without a portfolio [225]

227 Schuschnigg had planned a plebiscite on March 13, 1938. But he was not able to stand against German propaganda and its political pressures. Any help from France, Italy or the United Kindom, he had hoped for, did not came in.

228 Memo, J.van Sickle to T.B.Kittredge, Feb. 9, 1937, Austrian Institute for Trade Cycle Research.

229 Memo, J.van Sickle to T.B.Kittredge, Feb. 9, 1937, Austrian Institute for Trade Cycle Research.

230 Memo, J. van Sickle to Weaver, June 16, 1937, Austrian Institute for

Trade Cycle Research.

231 Letter, Weaver to H.T. Davis, June 18, 1937, Austrian Institute for Trade Cycle Research.

232 Letter, Staehle to TBK, Nov. 26, 1937, Austrian Institute for Trade Cycle Research.

233 Kittredeg reiterating van Sickle's argument about the risk of having AW enter the US in a letter to Staehle, Dec. 1, 1937, Austrian Institute for Trade Cycle Research.

234 Memo, T.B. Kittredge to J. van Sickle, Dec. 1, 1937, Austrian Institute for Trade Cycle Research.

235 Howard Ellis, at Berkeley, endorsed Kamitz, who, he appraised as "*convincing and businesslike appearance and address* [ensuring] *no lost motion in awkwardness or vagueness concerning objectives*". Ellis to van Sickle, Feb. 21, 1938, Series 705E Austria, Folder 1214.

236 See Comment on Foundation appropriation to Institute of 30,000 Austrian Schillings per year for 1938-1940, without date, late 1937, Austrian Institute for Trade Cycle Research.

237 See memo J. van Sickle to T.B. Kittredge, Jan. 6, 1938, Austrian Institute for Trade Cycle Research.

239 Memo J. van Sickle to T.B.Kittredge, Jan. 21, 1938, Austrian Institute for Trade Cycle Research.

239 See Memo, T.B.Kittredgde to Sydnor Walker, May 19, 1938, Austrian Institute for Trade Cycle Research.

240 Memo, T.B.Kittredge to Sydnor Walker, May 19, 1938, Austrian Institute for Trade Cycle Research.

241 Austrian Institute for Trade Cycle Research, Austrian Monthly Bulletin, April 11, 1938, p12.

242 The Statistical Research Group of Columbia University operated under a contract with the Office of Scientific Research and Development (directed by the Applied Mathematics Panel of the National Defense Research Committee).

243 In conjunction with "*some comments*" [Wald (1947) c,] made by Captain G. Schuyler of the Bureau of Ordnance, Navy Department, the usefulness of the SPRT in developments on military and naval equipment was brought out. This led to the 'restriction'.

244 The authorship of the revised edition, which was published by the Columbia University Press, Sept., 1945, is ascribed to the group as a whole.

245 AW considered only procedures for which the infimum of the confidence coefficient was the same, and measure the efficiency by the supremum of the expected number of observations.

246 The SPRT procedure is able (depending on the model and chance) to save as far as 50% in the number of observations over the most efficient test procedure based on a fixed number of observations.

247 Bartky's multiple sampling scheme for testing the mean of a binomial distribution provides an example of a sequential test. Without knowing

Bartkys results, Friedman and Wallis conjectured in 1943 that a sequential test procedure might be constructed, which would control possible errors committed by wrong decisions exactly to the same extent as the best current procedure based on a predetermined number of observations with a substantially smaller number of needed observations.

248 Mahalanobis treats the series of sample censuses of area of jute in Bengal. Sample censuses, steadily increasing in size, were taken primarily for the purpose of obtaining preliminary information about the parameters to be estimated. This information was then used for designing the final sampling of the whole immense jute area in Bengal.

249 The OC function for the binomial distribution was found independently by Milton Friedman and George W. Brown, and slightly earlier by C. M. Stockman in England. The derivation of the OC function in the general case is due to AW.

250 According to theorems in the theory of probability, the sum of a large number of independent random variables is nearly normally distributed under very general conditions.

251 Instead of defining p_{1n} by some weighted average, it would seem equally reasonable to define it as the maximum of $f(x_i, \underline{\theta}_1)$, $i = 1, .., n$ with respect to ω_r. The ratio $\frac{p_{1n}}{p_{0n}}$ would coincide with the so-called likelihood ratio introduced by J. Neyman and E. Pearson. The reason for preferring weighted averages is that the theory of such tests seems to be considerably simpler. If p_{1n} is defined by the maximum, it would no longer be a probability distribution.

252 The weight functions may also be discrete. Formulas valid for both

continuous and discrete weight functions could be given by using Stieltje's integrals.

253 A more detailed and non-mathematical discussion of these applications, together with a number of tables, charts, and computational simplifications, is contained in "Sequential Analysis of Statistical Data: Applications," a report prepared by the Statistical Research Group of Columbia University and published by Columbia University Press, Sept., 1945. (See references)

254 Even if his former student and later colleague Wolfowitz stroke a little critic note: “*The book is in places not easy to read. Some of the longer arguments could, with some effort, be made more accessible. A number of very minor errors has crept in*.“ [Wolfowitz (1952)].

255 “*Wald's greatest achievement was the theory of statistical decision functions, which includes almost all problems which are the raison d'être of statistics*.” [Wolfowitz (1952)]

256 The paper ‘Optimum character...‘ from 1949 united two currents of AW's approaches. The result therein was obtained by studying Bayes solutions of sequential decision problems involving two decisions. After that was done it was clear to attack the same problem for k decisions.

257 It was his contribution to the International Congress of Mathematicians at Harvard in 1950.

258 1, With A. Dvoretzky and J.Wolfowitz, Relations among certain ranges of vector measure, Pacific Jour. Math., Vol. 1, pp59-74. 2, with A. Dvoretzky and J. Wolfowitz, Elimination of randomization in certain

statistical decision procedures and zerosum two-person games, Annals of Math. Stat., Vol. 22, pp1-21. 3, With J. Wolfowitz, Two methods of randomization in statistics and the theory of games, Annals of Mathematics, Vol. 53, pp581-586.

259 The topologies are typically defined intrinsic, i.e. they are derived from the given decision model

260 This theorem says that any sequence of probability measures on a compact metric space contains a subsequence which converges to a probability measure.

261 Also: S. Karlin, 'Operator treatment of minimax principle', Contributions to the Theory of Games, Princeton University Press, 1950, pp133-154, S. Karlin 'The theory of infinite pines', Annals of Mathematics, 2d ser., Vol. 58 1953, pp371-401, and J. Kiefer, 'On Wald's complete class theorems', Annals of Math. Stat, Vol. 24, 1963, pp70-75.

262 Compare J. Wolfowitz, 'Minimax estimates of the mean of a normal distribution with known variance', Annals of Math. Stat, Vol. 21, 1950, pp218-230, M. A. Girshick and L. J. Savage, 'Bayes and minimax estimates for quadratic low functions', Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, 1951, pp53-74.

263 Compare for example J. L. Hodges, Jr., and E. L. Lehmann, 'Same problems in minimax point estimation', Annals of Math. SW., Vol. 21, 1950, pp182-197, H. Bobbins, 'Asymptotically subminimax solutions of compound statistical decision problems', Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,

University of California Press, 1951, pp131-148, P. Frank and J. Kiefer, 'Almost subminimax and biased minimiax procedures', Annals of Math Stat., Vol. 22, 1951, pp465-468.

264 AW never *"in any statistical investigation, lost sight of the fad that there was a question to be answered and a decision to be made. ... Wald not only posed his statistical problems clearly and precisely, but he also posed them to fit the practical problem and to accord with the decisions the statistician was called on to make"* [Wolfowitz (1952)].

265 At this time he already "*had the notions of an admissible test and of a least favorable a priori distribution*" [Wolfowitz (1952)]

266 Due to the "*earlier misuse*" of Bayes [Wolfowitz (1952)].

267 "*It was thought that dividing the sample such that the x_i of the first group were all smaller than those of the second group would provide a subdivision of the required kind. But this subdivision is not independent of the errors, as it depends on the observable variables x_i, which contain the errors δ_i. Consequently the estimator derived from this subdivision is not consistent. On the other hand, taking the first $n/2$ sample points for the first group and the rest for the second does, in fact, provide an independent subdivision. But now the centers of gravity converge to each other as $n \to \infty$ and again the estimator is not consistent"* [Schneeweiss (2005)].

268 AW did not "*resume serious work on decision functions until 1946*" [Wolfowitz (1952)].

269 "... the empirical accuracy of the prices forecasted by the Black–Scholes formula is unrivalled in all economics" (in Black, Merton and

Scholes: Their work and its consequences. Economic and Political Weekly, XXXII(52):3337–3342, Dec. 1997)

270 “In civil engineering, it does not take a researcher to build a bridge: the building of bridges is a routine technique which can be taught to undergraduate students, who can then go out and reliably build bridges. In contrast, a good macroeconomist cannot teach students the techniques which will enable them to understand and predict the macroeconomy; there is a large element of wisdom and experience that goes into the the making of a good macroeconomist.“ (in Black, Merton and Scholes: Their work and its consequences. Economic and Political Weekly, XXXII(52):3337–3342, December 1997)

271 “If too many investors are trying to unload stocks as a market falls, they create the very disaster they are seeking to avoid. Their desire to sell drives the market lower, triggering an even greater desire to sell and, ultimately, sending the market into a bottomless free fall. That's what happened on October 19, 1987, when the sweet logic of Black-Scholes was shown to be irrelevant in the real world of crashes and panics. Even the biggest portfolio insurance firm, Leland O'Brien Rubinstein Associates (co-founded and run by the same finance professors who invented portfolio insurance), tried to sell as the market crashed and couldn't. Oddly, this failure of financial theory didn't lead Wall Street to question Black-Scholes in general. "If you try to attack it," says one longtime trader of abstruse financial options, "you're making a case for your own unintelligence.” (in Michael Lewis, Inside Wall Street's Black Hole, Condé Nast Portfolio, March 2008)

272 “The very theory underlying all insurance against financial panic falls apart in the face of an actual panic. A few smart traders may have

abandoned the theory, but the market itself hasn't; in fact, its influence has mushroomed in the most fantastic ways. At the end of 2006, according to the Bank for International Settlements, there were $415 trillion in derivatives—that is, $415 trillion in securities for which there is no completely satisfactory pricing model. Added to this are trillions more in exchange-traded options, employee stock options, mortgage bonds, and God knows what else—most of which, presumably, are still priced using some version of Black-Scholes. Investors need to believe that there's a rational price for what they buy, even if it requires a leap of faith. "The model created markets," Seo says. "Markets follow models. So these markets spring up, and the people in them figure out that, at least for some of it, Black-Scholes doesn't work. For certain kinds of risk—the risk of rare, extreme events—the model is not just wrong. It's very wrong. But the only reason these markets sprang up in the first place was the supposition that Black-Scholes could price these things fairly." (in Michael Lewis, Inside Wall Street's Black Hole, Condé Nast Portfolio, March 2008)

273 This principle was formulated by the scottish moral philosopher and political economist Adam Smith (June 16 1723 – July 17, 1790). In his book "The Wealth of Nations" he claims that, in a free market, an individual pursuing his own interests tends to also to promote the good of his community as a whole. He argues that each individual maximizing revenue for himself maximizes the total revenue of the society as a whole.

274 The principle of the comparative advantage (attributed to the economist David Ricardo) explains how trade is beneficial for all parties involved (countries or, individuals and so on) as long as they produce goods with

different relative costs. Ricardo(April, 18 1772– September, 11 1823) was a influential political economist, who systematized economics.

275 The gold standard is a monetary system in which the standard economic unit of account is a fixed weight of gold.

276 The neoclassical economics is usually dated from William Stanley Jevons's “Theory of Political Economy”, 1871, Carl Menger's “Principles of Economics”, 1871, and Leon Walras's “Elements of Pure Economics”, 1874 – 1877. These three economists laid the groundstone for the “the Marginal Revolution"

277 “Nevertheless [Karl] Menger wondered why Austrians never used mathematics. He found an answer in the correspondence between Leon Walras and Carl Menger: while for the Lausanne economist, mathematics is the unique means of research, for the Viennese it is only a method of description.“ (in ‘The economic theory of Wiener Kreis and Mathematische Kolloquium. The complex role of Karl Menger’, Giandomenica Becchio, Department of Economics ‘S. Cognetti de Martiis’, University of Turin, Paper presented at the thirty-second annual meeting of the History of Economics Society, 24-27 June 2005, University of Puget Sound, Tacoma, WA, (2005))

278 “The Austrian economists surely shared the second epistemological and methodological model (from‘Carl Menger’s struggle against Schmoller’s historical school until Schumpeter’s theory and Hayek’s studies’ in ‘The economic theory of Wiener Kreis and Mathematische Kolloquium. The complex role of Karl Menger’, Giandomenica Becchio, Department of Economics)

279 “The relation between the Kreis and the Kolloquium was very complex.

Otto Neurath introduced economics into the Kreis during the 1920s. At the end of the 1920s, the Manifesto gave economic theory a specific role in the neoempiricist program: it was a social science that could be treated by a new epistemological model, and Carl Menger was regarded as a forerunner of this new approach." (in 'The economic theory of Wiener Kreis and the Mathematische Kolloquium. The complex role of Karl Menger', Giandomenica Becchio, Department of Economics)

280 The neoclassical economics is usually dated from William Stanley Jevons's "Theory of Political Economy", 1871, Carl Menger's "Principles of Economics", 1871, and Leon Walras's "Elements of Pure Economics", 1874 – 1877. These three economists laid the groundstone for the "the Marginal Revolution"

281 On the attitudes toward mathematics held by late-nineteenth-century economists can be read in Theodore Porter's 'Rigor and Practicality: Rival Ideals of Quantification in Nineteenth-Century Economcs", in Natural Images in Economic Thought, Edt. by Ph. Mirowski, Cambridge: Cambridge Univ. Press, 1994, pp123-70.

282 Cp. Ekelund and Hebert, "The Secret Origins", 1999, Princeton Univ. Press, for the importance of french engineers as economists

283 Cp. the monograph 'History of economic Ideas' of Rosner to get a deeper insight in early developments in economics

284 As there is no overall history of this evolvement, see e.g. Paul Studenski, "The Income of Nations. Theory, Measurement, and Analysis: Past and Present", New York, New York University Press, 1958.

285 On the attitudes toward mathematics held by late-nineteenth-century

economists can be read in Theodore Porter's 'Rigor and Practicality: Rival Ideals of Quantification in Nineteenth-Century Economcs", in Natural Images in Economic Thought, Edt. by Ph. Mirowski, Cambridge: Cambridge Univ. Press, 1994, pp123-70.

286 Another term for 'business cycle' is 'economic cycle'. It tries to describe fluctuations in production or other economic activity over several months or years. These fluctuations occur around a long-term trends, and typically see sharp shifts between phases of expansion and phases of decline. Cp. Arthur Sullivan, Steven M. Sheffrin, "Economics: Principles in action", Upper Saddle River, New Jersey, 2003, pp. 57,310

287 For the pre-1950 era see Morgan, The History of Econometric Ideas with chapters on Tinbergen and Haavelmo. For the post 1940s cp. Duo Qin, 'The Formation of Econometrics', Oxford, Clarendon Press, 1993 and also Roy J. Epstein, 'A History of Econometrics', Amsterdam, North-Holland, 1987.

288 Morgenstern "Report on the Activities of the Austrian Institute for Trade Cycle Research 1931-1935", Feb. 13, 1935, AIRC, Folder 37, Austrian Institute for Trade Cycle Research, Vienna 1935-1936

289 P. Armitage was on the staff of the Statistical Research Unit of the Medical Research Council, London School of Hygiene and Tropical Medicine, London. He was a visiting scientist at the National Institutes of Health, Bethesda, Maryland, 1957-1958.

290 For example, many economists are familiar with Trygve Haavelmo's simultaneous equations paper of 1943 [Haavelmo (1943)], but fewer

people know (particularly before ltaavelmo won the Nobel Prize in 1989) of his more important contributions in 'The Probability Approach in Econometrics' of 1944.

291 For a good overview on the statistical advancements of social scientists around the beginning of the 20th century cp. Donald Mc Kenzie, 'Statistics in Britain, 1865-1930', Edinburgh Univ. Press ,1981 or Stephen Stigler 'The History of Statistics: the Measurement of Uncertainty before 1900', Cambridge, Harvard Univ. Press, 1986.

292 Ragnar Frisch is credited with coining the term 'econometrics' and he is one of the founders of the Econometrics Society, see C.F. Christ, 'The Founding of the Econometric Society' , Econometrica, 51, 1983

293 "*Walras has set up a system of equations for the determination of prices of factors and quantities of products, which was popularized by Cassel in a simplified form. Let* $r_1, r_2, .., r_m$ *be factors of production, which, in different combinations, can be used to produce n different products,* $S_1, S_2, \ldots, S_n$*, specifically in such manner that, for the production of one unit of* S_j $(j = 1, \ldots, n)$, a_{1j} *units of* R_1, a_{2j} *units of* $R_2, .., a_{mj}$ *units of* R_m *must be used. If the producer has available* r_1 *units of* R_1 , r_2 *units of* $R_2, .., r_m$ *units of* R_m*, if we also know that the price of a unit of* S_j $(j = 1, \ldots, n)$ *is* $f_j(s_1, \ldots s_n)$ *provided that* s_1 *units of* S_1, s_2 *units of* $S_2, \ldots, s_n$ *units of* S_n *are produced, then the* $m + 2n$ *Walras-Cassel equations for the m unknown prices* ρ_i *for a unit of the factor* R_i $(i = 1, m .., i)$*, for the n unknown quantities* s_j $(j = 1, \ldots, n)$ *of product* S_j*, and for the n unknown prices* σ_j*, for a unit of the product* S_j $(j = 1, \ldots, n)$*, are the following:*

$r_1 = a_{11} * s_1 + a_{12} * s_2 + \ldots + a_{1n} * s_n,$

$r_2 = a_{21} * s_1 + a_{22} * s_2 + \ldots + a_{2n} * s_n, \ldots,$

$r_m = a_{m1} * s_1 + a_{m2} * s_2 + \ldots + a_{mn} * s_n,$

$\sigma_1 = a_{11} * \rho_1 + a_{21} * \rho_2 + \ldots + a_{m1} * \rho_m, \ldots,$

$\sigma_n = a_{1n} * \rho_1 + a_{2n} * \rho_2 + \ldots + a_{mn} * \rho_m, \sigma_1 = f_1(s_1, \ldots s_n),$

$\sigma_2 = f_2(s_1, \ldots s_n), \ldots, \sigma_n = f_n(s_1, \ldots s_n)$" AW in Über einige Gleichungssysteme der mathematischen Ökonomie, Zeitschrift für Nationalökonomie, 7, 637-670, (1936). [Wald (1936) c,]

294 That means to build a proof that shows, that there is a system of quantities of goods and prices fulfilling the modified Walras-Cassel set of equations. The appropriate conditions are: 1) There are positive finite quantities of goods, which are needed for production. 2) The quantity of the property R_i, necessary for the production of the product S_i is not negative. 3) At least one the producer goods $R_1, R_2, \ldots, R_m$ are necessary for the production of the product S_i. 4) The relationship between demand and price is constant 5) The demand converges against 0, if the price becomes infinitely high. 6) A combination of goods loses in value (not necessarily in price!), if with changing prices the related demand rises.

295 In 1964, Kenneth Arrow wrote to Karl Menger asking him about “the intellectual relationship between Wald’s first paper on existence of equilibrium and von Neumann’s article on an expanding economy”. He wrote: “As von Neumann understands the replacement of equations by inequalities along Zeuthen’s lines, whereas clearly the problem he raises, that of a uniformly progressive economy, is derived from another section of Cassel’s book… did von Neumann arrive at this formulation independently of everyone else? Or did he derive it from Schlesinger or Wald? Or is it too late ever to find out?” The meaning of

Arrow's questions was that according to him: "the really interesting point is that von Neumann's mathematical methods, introduced in that paper, actually enable one to improve greatly on Wald's proofs, giving both more general and more elegant results". Source: The economic theory of Wiener Kreis and Mathematische Kolloquium. The complex role of Karl Menger, Giandomenica Becchio, Department of Economics 'S. Cognetti de Martiis', University of Turin, Paper presented at the thirty-second annual meeting of the History of Economics Society, 24-27 June 2005, University of Puget Sound, Tacoma, WA, 2005.

296 In a regression the values of he independent variable are assumed to be fix, whereas the values of the appropriate dependent variable are assumed to be afflicted with errors. E.g. in a linear regression of the measured values of the pressure of a gas in dependence of the temperature the values of the temperature are taken as error-free. The values of the pressure are treated with measurement errors. This a priori assumption is only allowed, when it is possible to guarantee that the adjustment of the temperature can be done very exactly. This is not realistic. This asymmetric view causes that the inverse regression shows a complete different regression line. There is no invariance when the variables are changed. AW eliminated that in The fitting of straight lines if both variables are subject to error' .

297 The variables $y_1,\ ..,\ y_p,\ z_1,\ ...,\ z_n$ should be normally, mutually independent, distributed, all with the a variance σ^2. Furthermore the mean values of the $z_1,\ ...,\ z_n$ should be equal 0 and the mean values $\eta_1,\ ...,\ \eta_p$of $y_1,\ ...,\ y_p$are unknown. Then the canonical form of the variance test is the test of the hypothesis: $\eta_1 = .. = \eta_r = 0$, whereby $r \le p$, basing on the region $\left(y_1{}^2 + .. + y_r{}^2\right) / \left(z_1{}^2 + .. + z_n{}^2\right) > c$, and c is a

constant, which has to be selected such that the size of the region is equal to the significance α.

298 Cp. H. von Schelling in his review [von Schelling (1950)]: " ... *während. der Verf. in der früheren Arbeit nur den Fall betrachtete, dass die Zahl n der Beobachtungen von vornherein feststeht, schließt er nun die von ihm besonders studierte Möglichkeit ein, dass diese Zahl n der Beobachtungen in irgendeiner, vor Versuchsbeginn bestimmten Weise von den Ergebnissen der aufeinanderfolgenden Beobachtungen abhängt (sequential analysis). Die auftauchenden. mathematischen Probleme haben z.T. in anderem Zusammenhang eine Lösung gefunden durch J. Von Neumann und O. Morgenstern: The Theory of Games and economic Behavior, Princeton University Press, Princeton 1944. Aber diese Autoren haben ihre Existenzsätze nur für abgeschlossene Bereiche benötigt, so dass Verf. die Untersuchung erheblich verallgemeinern muss, um sie seinen Bedürfnissen anzupassen. Das Ergebnis dieser Bemühungen ist eine nun völlig allgemeine Theorie der Entscheidungsfunktionen, die als ganz speziellen Fall die Entscheidung zwischen zwei rivalisierenden Hypothesen umfasst*."

299 Let X_1, ..., X_n, .. be a sequence of random variables with distributions dependent on unknown parameters. A parameter θ is called a structural one, when an infinite subsequence exists such, that the distribution of the subsequence is dependent on θ. If θ is no structural parameter it is called incidence parameter.

300 Milton Sobel worked for his Ph.D. with AW. He received the title after AW's death in 1951.

301 The League of Nation's Economic Intelligence Service had for several

years, supported by grant from the Rockefeller Foundation, been engaged in an inquiry into the causes of the recurrence of depressions. The outcome of the first phase in this inquiry was Gottfried Haberler (1937)'s Prosperity and Depression: A Theoretical Analysis of Cyclical Movements. The second phase was the statistical verification and mathematical testing of the alternative explanations. The investigation was led by Tinbergen since 1936. Alexander Loveday was in charge of the Financial Section and the Economic Intelligence Service (see M. Hill: The Economic and Financial Organization of the League of Nations, Washington: Carnegie Endowment for International Peace, 1946).

302 AW's sojourn in Geneva may have been mentioned to Haavelmo by Tinbergen, but AW had not yet done much work on statistical problems. Frisch had had scientific contact with AW and in connection with price index problems and encouraged him to write two articles on that topic for the Econometrica. He had probably also been instrumental in getting AW the offer as a research associate of the Cowles Commission, that gave him the opportunity to leave for the US 1938. Frisch had persuaded AW to visit him in Oslo and leave for the US from a Scandinavian port, but AW was in too much of a hurry to make the detour.

303 Mendershausen and AW had until shortly before the conference been research fellows of the Commission, Hotelling had been instrumental in offering AW a position at Columbia where he eventually would succeed Hotelling. Yntema would take over as research director in September 1939 after the position had been vacant since Charles Roos left in January 1937.

304 Haavelmo's monograph covered the whole held of econometrics in a

theoretical and conceptual way, and was the first of its kind in econometrics. "*Haavelmo was influenced both by Jerzy Neyman and by AW and this led him to reform his ideas away from Frisch to produce an alternative general framework for econometrics instead of confluence analysis*" [Hendry and Morgan (1997)], p66.

305 A handwritten note for the seminar, Haavelmo Archive, University of California, Berkeley.

306 Haavelmo/Frisch 17 Nov. 1939, Haavelmo archive, Berkely Univ.

307 'Statistical versus theoretical relations in economic macrodynamics', Memorandum prepared the Business Cycle Conference at Cambridge, England, July 1938, to discuss Professor J. Tinbergen's publications for the League of Nations.

308 Haavelmo/Frisch 27 March 1940, Haavelmo Archive, Berkely Univ.

309 Haavelmo/Frisch (undated) from spring 1941. Haavelmo Archive, Berkely.

310 A handwritten and two successive typewritten versions exist, Haavelmo archive, Berkely

311 Wald was more inclined to have another hike in Colorado Springs, and Haavelmo put persuasive pressure on him, arguing that "Maine has got everything that Colo. Spr. has, and it has got more … to choose Colo. Spr. instead of Maine would definitely be to give up something for something inferior"! (Havelmo/Wald 5 Aug. 1941, Haavelmo archive, Berkely).

312 Haavelmo/Frisch 31 Aug. 1941, Haavelmo archive, Berkely.

313 Haavelmo/Reiersøl 8 Aug. 1941, Haavelmo archive, Berkely

314 The Annals of Mathematical Statistics was published between 1930 and 1972 by the Institute of Mathematical Statistics. The journal was superseded in 1973 by The Annals of Probability and The Annals of Statistics.

315 A minister without portfolio is either a government minister with no specific responsibilities or a minister that does not head a particular ministry.

10 References

- Ahlswede, R.(1981). *Jacob Wolfowitz (1910-1981).* Retrieved 2008 from http://people.cornell.edu.

- Arendt, H. (1958). *The origins of totalitarianism* , Cleveland : World Pub. Co.

- Ayer A. J. (1936). *Language, Truth, and Logic*, London: Gollancz. (2nd edition, 1946.)

- Alt, F. (1936). *Über die Meßbarkeit des Nutzens*. *Journal of Economics*, **7**(2), 161–169.

- Alt, F. (2008), in 'Wiener Zeitung' , *Frieden mit Wien gemacht*. Retrieved May 2008 from http://www.wienerzeitung.at/DesktopDefault-.aspx?TabID=3946&Alias=wzo&cob=284626

- Alter, M. (1990). *Carl Menger and the origin of Austrian economics*. Westview Press Inc., Boulder, Colorado.

- Armitage, P. (1975). *Sequential Medial Trials*. Melbourne : Blackwell Scientific Publications.

- Armitage, P. (1958). *Sequential methods in clinical trials*. *Am. J. Public Health*, **48**, 1395–1402.

- Ausch, K. (1975). *Genfer Sanierung und der 12. Februar 1934. In: Vom Justizpalast zum Heldenplatz. Studien und Dokumentationen 1927 bis 1938.*

Wien: Verlag der Österreichischen Staatsdruckerei.

- Baleanu, A. (2001). *Handbuch zur Geschichte der Juden in Europa, im folgenden GJE*. Darmstadt : Elke-Vera Kotowski/Julius H. Schoeps und Hiltud Wallenborn (Hg).
- Balk, B., M. (1981). *Second Thoughts on Wald's Cost-of-Living Index and Frisch's Double Expenditure Method*. Econometrica Vol. 49, No. 6 (Nov., 1981), pp. 1553-1558
- Bartky, W. (1943). *Multiple Sampling with Constant Probability*. *The Annals of Mathematical Statistics*, **14**, 363–377.
- Bartley, William. (1964). *Rationality versus the Theory of Rationality*, In Mario Bunge: The Critical Approach to Science and Philosophy, The Free Press of Glencoe, 1964, section IX.
- Bearn, A. (1999). *Useful Knowledge*. In A. Bearn (Ed.), *The American Philosophical Society Millennium Program* (pp. 140–154). Philadelphia: Diane.
- Beckhouse, R. (1985). *A History of Modern Economic Analysis*. Oxford: Blackwell.
- Beller, S. (1989). *Vienna and the Jews: 1867-1938,* . Cambridge: Cambridge University Press.
- Bellman, R., Kalabe, R. (1966). *Abstraction and pattern design*. *J. Math. Anal. Appl.* , **13**, 1–7.
- Berger, P. (1999). *Die Wiener Hochschule für Welthandel und ihre Professoren 1938-1945*. *Österreichische Zeitschrift für Geschichtswissenschaften*, **10**, 9–49.

◂ Berger, P. (2007). *Kurze Geschichte Österreichs im 20. Jahrhundert, .* Wien: Facultas Universitätsverlag.

◂ Bernhard, T. (1984). *Heldenplatz.* Suhrkamp, Frankfurt/M. (1988 Edition), ISBN 3-518-01997-X.

◂ Biale, D. (2002). *Biale, David (2002): Cultures of the Jews: A New History. .* New York: Schoken Books.

◂ Billingsley, P. (1999). *Convergence of Probability Measures.* New York, John Wiley & Sons.

◂ Birnbaum, A. (1954). *Admissible tests for the mean of a rectangular distribution . Annals of Math. Stat.*, **25**, 157–161.

◂ Bischof, G., Pelinka, A., Lassner, A. (2003). *Forthcoming. The Dollfuss/Schuschnigg Era in Austria. A Reassessment.* New Brunswick-London: Transaction Publ..

◂ Black, F., Scholes, M. (1973). *The Pricing of Options and Corporate Liabilities. Journal of Political Economy*, **81**, 637–654.

◂ Blau, E. (1999). *The Architecture of Red Vienna 1919-1934.* Cambridge: MIT Press.

◂ Blumenthal, L. M. (1970). *Theory and Applications of Distance Geometry.* New York: Chelsea House Pub.

◂ Blumenthal, L. M., Menger, Karl (1971). *Studies in Geometry.* W.H.Freeman & Co Ltd (21. September 1970).

◂ Bobango, Gerald J (1979). *The emergence of the Romanian national State*, New York: Boulder.

◂ Bodzenta, E. (1980). *Änderungen der österreichischen Sozialstruktur in der*

Ersten und Zweiten Republik. In E. Zöllner (Ed.), *Österreichs Sozialstrukturen in historischer Sicht* (pp. 156–170). Wien: Schriften des Inst. f. Österreichkunde.

- Boese, F. (1939). *Geschichte des Vereins für Sozialpolitik, 1872-1932*. Berlin: Duncker Humblot.
- Botz, G. (1979). *Formen und Intensität politisch-sozialer Konflikte in der Ersten und Zweiten Republik*. *Austriaca*, **Sondernummer 3**, 427–464.
- Botz, G., Oxaal, I., Pollak , M., Scholz, N. (Edts.) (2002). *Eine zerstörte Kultur. Jüdisches Leben und Antisemitismus in Wien seit dem 19. Jahrhundert*. Wien: Czernin Verlags GmbH.
- Botz, G. (2008). *Nationalsozialismus in Wien. Machtübernahme, Herrschaftssicherung, Radikalisierung 1938/39, Wien, überarbeitete und erweiterte Neuauflage 2008, 734 S.*. Buchloe: Druck und Verlag Obermayer.
- Boumana, M. (1993). *Paul Ehrenfest and Jan Tinbergen: A Case of limited Physics Transfer*. In N. de Marchi (Ed.), *Non-Natural Social Science: Refelctions on the Enterprise of more Heat than Light*. Durham: Duke Univ. Press.
- Braun, S. M. (1934). *Review of Oskar Morgenstern, Die Grenzen der Wirtschaftspolitik*. *Mitteilungen des Verbandes der österreichischen Banken und Bankiers*, **16**, 21–23.
- Brook, K. A. (2003). *The Origins of East European Jews*. *Russian History/Histoire Russe*, **30**(1-2), 1–22.
- Brook-Shepherd, G. (1996). *The Austrians : a thousand-year odyssey*. New

York: HarperCollins.

- Brook-Shepherd, G. (1997). *The Austrians: A thousand-year odyssey*. New York: Carroll & Graf Publishers.

- Brubaker, R. (1996). *Nationalism Reframed: Nationalism and the National Question in the New Europe*. Cambridge: Cambridge University Press.

- Brugger, E., et al. (2006). *Geschichte der Juden in Österreich*. Wien: Ueberreuther.

- Brusatti, A.(1965). *Österreichische Wirtschaftspolitik vom Josephinismus zum Ständestaat* . Wien: Jupiter Verlag.

- Bruss , F., Robertson, J. B. (1991).*Wald's Lemma' for Sums of Order Statistics of i.i.d. Random Variables*. Advances in Applied Probability **23**(3): 612–623.

- Buchan, J. (2006). *The Authentic Adam Smith: His Life and Ideas*. New York: W. W. Norton & Company.

- Buchholz K. (2008) (Edt). *Friedrich Waismann*: *Was ist logische Analyse?* Europäische Verlagsanstalt, Hamburg 2008.

- Bulei, Ion (1996). Short History of the Romanians, Meronia Publishing House, Bucharest.

- Burdman Feferman, A., Feferman, S. (2004). *Alfred Tarski. Life and Logic*. Cambridge University Press, Cambridge.

- Busshoff, Heinrich (1968). *Das Dollfuß-Regime in Österreich*. Duncker & Humblot, Berlin 1968.

- Caldwell, B. (1990). *Carl Menger and His Legacy in Economics*. Durham:

Duke University Press.

- Carnap, Rudolf (1928). *Der logische Aufbau der Welt*. Leipzig. Felix Meiner Verlag.

- Carnap, Rudolf (1934). *Logische Syntax der Sprache*. Wien. J. Springer Verlag.

- Carsten, F. L. (1978). *Faschismus in Österreich: von Schönerer zu Hitler*. München: W. Fink.

- Cartwright Nancy, Cat J., Uebel T. (1996). *Otto Neurath: philosophy between science and politics*. Cambridge University Press

- Cepa (2008) *Karl Menger's Vienna Colloquium*. Retrieved 2008 from http://cepa.newschool.edu/het/schools/vienna.htm

- Chandler, B., Magnus, W. (1982). *The History of Combinatorial Group Theory: A Case Study in the History of Ideas* . New York - Heidelberg - Berlin: Springer.

- Chraska, W. (1981). *Die Heimwehr und die Erste Republik Österreich. Über-legungen zur österreichischen Staatswerdung nach dem Zusammenbruch der Monarchie*. Kiel: Selbstverlag.

- Christ, C. H. (1993). *Assessing Applied Econometric. Results Federal Reserve Bank of St. Louis Review*, **75** (2), 71–94.

- Christ, C. F. (1985). *Early Progress in Estimating Quantiative Economic Relations in America. American Economic Review*, **12**, 39–52.

- Cohen, Sonne, Robert (Edt.) (1981). *Inquiries and provocations: selected writings*, 1929-1974, Volume 1974 by Herbert Feigl, Reidel Publishing, Vol. 81

- Cowles Foundation. (2007), *History of the Cowles Commission 1932–1952*. Retrieved Oct 2007 from http://cowles.econ.yale.edu/P/reports/1932-52b.htm
- Cowles Foundation. (2007), retrieved Oct. 2007 from http://cowles.econ.yale.edu/
- Craig, G. A. (1966). *Europe since 1914*. New York: Holt, Rinehart and Winston.
- Craufurd, G. (1975). *Exhortation and Controls: The search for a Wage-Price Policy* . Washington D.C.: Brookings Institute.
- Craver, E. (1986). *The emigration of the Austrian economists*. *History of Political Economy*, **18**, 1–32.
- Craver, E. (1986, June). *Patronage and the directions of research in economics: The Rockefeller Foundation in Europe, 1924-1938*. *Minerva*, **24** (2/3), 205–222.
- Dahms, H. J. (1987). *Die Emigration des Wiener Kreises*. In F. Stadler (Ed.), *Vertriebene Vernunft I. Emigration und Exil österreichischer Wissenschaft* (pp. 66–122). Wien: Jugend und Volk.
- Dainian, F. (1996). *Hong Qian (Tscha Hung) and the Vienna Circle*, in: R.S. Cohen & R. Hilpinen & Q. Renzong (Edts.), *Realism and Anti-Realism in the Philosophy of Science*, Dordrecht (Boston Studies in the Philosophy of Science, 169).
- Danneberg, R. (1927). *Die politischen Parteien in Deutsch-Österreich* . Wien .
- Dawson, John W., Jr. (2005) *Logical Dilemmas: The Life and Work of Kurt Gödel*. A K Peters Series, A K Peters Ltd.

- Davis, Nelson., *Annual Report for the Year ending Dez. 1935*. Retrieved 2008 from http://cowles.econ.yale.edu/P/reports/1935.htm
- Dawidowicz, L. (1967). *The Golden Tradition: Jewish Life and Thought in Eastern Europe*. New York: Holt, Rinehart, and Winston.
- Debreu, G., *Economic theory in the mathematical mode*. Retrieved 2008 from http://nobelprize.org/nobel_prizes/economics/laureates/1983/debreu-lecture.pdf
- Dee, E. (1996). *Wittgenstein's Vienna*. Noosaville: Elephant PBK Edition.
- Deistler, M. (1984). *Gerhard Tintner 1907-1983,* in Nouvelles Mathematiques Intern., June 1984, Vol 38, No. 136, pp1.
- Department of Statistics, Vienna University, *Abraham Wald PhD programme*. Retrieved 2008 from http://www.isds.univie.ac.at/index.php?id=28245
- Deutsche Mathematiker Vereinigung (2011). *Theodor Radakovic*. https://www.dmv.mathematik.de/die-dmv/105-kurzbiographien/384-kurzbiographien-r.html.
- Dodge, J. F., Romig, H. G. (1929). *A Method of Sampling Inspection. The Bell System Technical Journal*, **8**, 613–631.
- Duke University, Archive Online (2012). *http://library.duke.edu/rubenstein/findingaids/mengerkarl.pdf*
- Easle, B. (1986). *Väter der Vernichtung. Männlichkeit, Naturwissenschaftler und der nukleare Rüstungswettlauf*. Reinbek, Rowohlt Taschenbuchverlag, 1986.
- Ebeling, Richard (2006). *The Great Austrian Inflation* www.fee.org.

- Edmonds, David, Eidinow, John (2002). *Wie Ludwig Wittgenstein Karl Popper mit dem Feuerhaken drohte: eine Ermittlung*. München: DVA, 2002.
- Ehlert, H. (2006). *Der Schlieffenplan. Analysen und Dokumente* . Paderborn: Gerhard P. Groß (Hg), Schöningh Verlag.
- Eigner, P. (1999). *Österreichische Wirtschafts- und Sozialgeschichte im 19. und 20. Jahrhundert*. Wien, Brandstätter Verlag.
- Elon, A. (2002). *The Pity of It All: A Portrait of the German-Jewish Epoch, 1743-1933*. New York: Picador.
- Encyclopedia Judaica. (2007). Second Edition Vol 20, p. 108. Jerusalem: Keter Publishing House .
- Engler Fynn Ole, Iven Mathias (Edts.) (2008). *Moritz Schlick. Leben, Werk und Wirkung*. Berlin: Parerga
- Enz, C. P. (2002). *No Time to be Brief. A scientific biography of Wolfgang Pauli.* . Oxford: Oxford Univ. Press. .
- Erdös, P., Kac, M. (1946). *On certain limit theorems of the theory of probability. Bull. Am. Math. Soc.*, **52**, 292–302.
- Farlow, Robert, L. (1971). Romanian foreign policy: A case of partial alignment, in Problems of Communism XX/6 (November-December 1971), pp. 54-63.
- Fasshauer, Greg (2002). Online Source: http://www.iit.edu/csl/am/about/-menger/about.shtml. *About Karl Menger*.
- Feichtinger, J. (2001). *Wissenschaft zwischen den Kulturen. Österreichische Hochschullehrer in der Emigration 1933-1945*. Frankfurt a. Main: Campus.

- Feigl, H. (1981). *The Wiener Kreis in America*. In R. S. Cohen (Ed.), *Inquiries and Provocations* (pp. 21–37). Harvard: Harvard University Press.
- Fisher, I. (1922). *The making of Index Numbers*. Boston: Houghton Mifflin.
- Fisher, Ronald, A. (1921). *On the mathematical foundation of Statistics*, Philosophical Transactions of the Royal Society of London, Series A, Vol. 22, 309-368.
- Fisher, Ronald, A. (1925). *The theory of statistical estimation*, Proceedings of the Cambridge Philosophical Society, Vol. 22, 700-725.
- Fisher, Ronald, A. (1935). *The fiducial argument in statistical inference*, Annals of Eugenics, Vol. 6, 391-398.
- Fisher, Ronald, A. (1955). *Statistical methods and scientific induction*. Journal of the Royal Statistical Society. Series B 17 (1): 69–78.
- Flanagan, M. (2007). *America Reformed: Progressives and Progressivisms, 1890s-1920s*. New York: Oxford University Press.
- Fleck, C. (1987). *Rückkehr unerwünscht. Der Weg der österreichischen Sozialforschung ins Exil*. In F. Stadler (Ed.), *Vertriebene Vernunft I: Emigration und Exil österreichischer Wissenschaft 1930-1940* (pp. 182–213). Wien: Jugend und Volk.
- Forbes, N., Mitrany, D. (2006). *The Balkans*. La Vergne: Lightning Source Inc.
- Frank, P. (1964). *The Work of Richard von Mises: 1883-1953*. Science 119 (3102), pp823–824.
- Frass, O. (1967). *Quellenbuch zur Österreichischen Geschichte*. Wien: Birken

Verlag.

- Freeman, H. (1944). *Sequential Analysis of Statistical Data: Applications*. Statistical Research Group, Columbia University, New York. B [52-260], Harold Freeman.

- Freeman, H. (1968). *Abraham Wald. International Encyclopedia of Social Sciences*, **16**, 435–438.

- Frisch, R. (1934). *Statistical Confluence Analysis by Means of Complete Regression Systems*. Oslo: Universitetets Okonomiske.

- Fritz, W. (2000). *Der Kopf des Asiaten Breitne*r. Politik und Ökonomie im Roten Wien. Hugo Breitner. Leben und Werk. Löcker Verlag, Wien.

- Gatrell, P. (1999). *A Whole Empire Walking: Refugees in Russia during World War I*. Bloomington: Indiana University Press.

- Geier, M. (1998). *Der Wiener Kreis*. Reinbek bei Hamburg : Rowohlt Taschenbuch Verlag.

- Giandomenica, B. (2009). *Ethics and economics in Karl Menger: how did social sciences cope with Hilbertism. CESMEP Working Papers*.

- Girshik, M. A. (1946). Ann. Math. Statist. Vol. 17, No. 2, pp123-143.

- Glass, H. (1996). *Zerbrochene Nachbarschaft*, Munich 1996, p37.

- Gleick, J. (2011) The Information: a History, a Theory, a Flood, London, Fourth Estate, p181.

- Gödel, K. (1986). *Collected Works*. Oxford: Oxford Univ. Press.

- Gödel, K. (1939). *Consistency-Proof for the Generalized Continuum-Hypothe-*

sis. Proc Natl Acad Sci USA. 1939, Apr. 25 (4), 220-4.

- Gödel, K. (1938). *The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis*. Proc. Natl. Acad. Sci. USA. 1938, Dec, 24 (12), 556-7.
- Goldberg, H. (2001). *The Life of Judaism*.. Berkely: University of California Press.
- Gombás, I. (2002). *Kings and Queens of Hungary & Princes of Transylvania*. Budapest: Corvina.
- Gosh, M. N. (1952). *An extension of Wald's decision theory to unbounded weight functions*. *Sankhya*, **12**, 8–26.
- Gove, P. B. (1997). *Webster's New Twentieth Century Dictionary of English Language* . Springfield: Merriam-Webster Inc..
- Grenville, J. A. (1974). *The Major International Treaties 1914-1973. A history and guides with texts*. . London: Methnen.
- Gruber, H. (1991). *Red Vienna: Experiment in Working-Class Culture, 1919-1934*. Oxford: Oxford University Press.
- Grünbaum, A., (1963). *Philosophical Problems of Space and Time*. Chpt. 3.
- Gulick, C. (1948). *Austria: From Habsburg to Hitler*. Berkeley : University of California Press.
- Haavelmo, T. (1939, July Cowles Commission for Research in Economics). *Statistical Testing of Dynamic Systems If the Series Observed Are Shock Cumulants*. *Report of Fifth Annual Research Conference on Economics and Statistics at Colorado Springs*, 45–47.

Haavelmo, T. (1941). *A Note on the Variate Difference Method*. *Econometrica*, **9**, 74–79.

Haavelmo, T. (1943). *The statistical implications of a system of simultaneous equations*. Econometrica 11, 1-12.

Haavelmo, T. (1944). *The Probability Approach in Econometrics*. Cowles foundation Paper No. 4., also New York: Econometric Society.

Haavelmo, T. (1989). *Nobel lecture*. Retrieved 1989 from http://nobelprize.org/

Haberler, G. (1961, October). *Mises's Private Seminar*. *The Mont Pelerin Quarterly*, **3**( ), 20.

Hacohen, M. (2000). *Karl Popper -The Formative Years, 1902-1945. Politics and Philosophy in Interwar Vienna*. Cambridge: Univ. Press.

Hagemann, H., Krohn, C. D. (1999). *Biographisches Handbuch der deutschsprachigen wirtschaftswissenschaftlichen Emigration nach 1933*. München: Saur.

Hahn, H. (1932). *Ergebnisse eines mathematischen Kolloquiums*. *Monatshefte für Mathematik*, **39**, 8–8. http://dx.doi.org/10.1007/BF01699094

Haller, R. (1982). *New Light on the Vienna Circle*, in: The Monist, 65/1.

Haller, R. (1991). *The First Vienna Circle*, in: Th.E. Uebel (Edts.), *Rediscovering the Forgotten Vienna Circle. Austrian Studies on Otto Neurath and the Vienna Circle*, Dordrecht.

Haller, R. (1993). *Neopositivismus. Eine kritische Einführung in die Philosophie des Wiener Kreises*. Darmstadt: Wissenschaftliche Buchgesellschaft.

- Haller, R., Stadler, F. (1993). *Der Aufstieg der wissenschaftlichen Philosophie*. Wien, Berlin, Prag: Hölder-Pichler-Tempsky.
- Haller R., Bindeawlik, Th., J. (1999). *Zufall und Gesetz. Drei Dissertationen unter Schlick: H. Feigl – M. Natkin –Tscha Hung*. Amsterdam/Atlanta. Studien zur österreichischen Philosophie Bd. XXV.
- Hamacher-Hermes, Adelheid (2003). *Rose Rand: a Woman in Logic*. In Stadler, Friedrich, (Ed.), *The Vienna Circle and Logical Empiricism: Reevaluation and Future Perspectives*. Springer. ISBN 0306482142, pp. 375-6.
- Hasiba, G. (1984). *Die „rechtliche Zeitgeschichte" – Ein anderer Weg zur Bewältigung der Vergangenheit*. In J. Desput (Ed.), *Österreich 1934-1984. Erfahrungen, Erkenntnisse, Besinnung,* Graz, Wien, Köln: Styria.
- Haumann, H. (2001). *A History of East European Jews*. Budapest: Central European University Press.
- Hauptmann, H. (1974). *Die österreichische Arbeiterbewegung vom Vormärz bis 1945*. Wien/München: Europaverlag.
- Hawlik, J. (1971). *Die politischen Parteien Deutsch-Österreichs bei der Wahl der konstituierenden Nationalversammlung 1919* (Doctoral Dissertation, Univ. of Vienna, Vienna) , **1**, 5–180.
- Hayek, F. (1984). *Money, Capital, and Fluctuations. Early Essays*. London: Routledge and Kegan Paul.
- Heiber, H. (1992 (Part 1), 1994 (Part 2)). *Universität unterm Hakenkreuz*. Munich.
- Heilbroner, Robert, L. (1953). *The Worldly Philosophers*. Simon & Schuster, 7th edition, 1999, ISBN 0-684-86214-X.

Heiß, G. (1989). *Willfährige Wissenschaft. Die Universität Wien 1938-1945*. Wien.

Hendry, D. F., Morgan, M. S. (1997). *The Foundations of Econometric Analysis*. Cambridge: Cambridge University Press.

Hertz, F. (1970). *The Economic Problem of the Danubian States. A Study in Economic Nationalism*. London: Howerd Fertig.

Hilbert, David, Ackermann, Wilhelm (1928). *Grundzüge der theoretischen Logik* (Principles of Mathematical Logic). Springer-Verlag.

Hitchins, Keith (1996). *Romania, 1866-1947, 'Society and Economy'* (Chapt. 8), Humanitas Publishing House, Bucharest.

Hlawka, E. (1980). *Nachruf auf E. Hornich. Monatshefte für Mahematik und Physik* , **89**, 1–8.

Hlawka, E., Menger, K. (1986). *Sonderabdruck. Almanach der Österreichischen Akademie der Wissenschaften*, **136**.

Holton Gerhard, Kemble E. C., Quine W. V. (1968), *Philosophy of Science*, Vol. 35, No. 1, pp. 1-5

Hornik, L. (1976). *Otte Bauer über die Ursachen und Lehren des Februar 1934. ITH Tagungsberichte*, **9**, 346–354.

Hotelling, H. (1941). *Experimental Determination of the Maximum of a Function. The Annals of Mathematical Statistics*, **12**, 20–45.

Hotelling, H. (1951). *Abraham Wald. American Statistician*, **5**, 18–19.

Hsu, P. L. (1941). *Analysis of variance from the power function standpoint,* *Biometrica*, **32**, 62–69.

- Hülsmann, J. G., *Mises. The Last Knight of Liberalism*. Retrieved 2009 from http://mises.org/story/2696
- Hurewicz, W. (1941). *Dimension Theory*. Princeton: Princeton Univ. .
- Hurwicz, L. (1947). *Book review of: The theory of games and economic behavior by John von Neumann & Oskar Morgenstern*. *American Economic Review*, **35**, 909–925.
- IEP (2011). *Internet Enzyklopedia of Philosophy*. http://www.iep.utm.edu/.
- IMN, Internationale Mathematische Nachrichten (2006), Österrreichische math. Gesellschaft, Gödelausstellung April 2006, http://www.oemg.ac.at/IMN/imn201.pdf.
- IMS (2013). Wald Memorial Lectures, http://www.imstat.org/awards/lectures.htm.
- Ingrao, B., Israel, G. (1990). *The invisible Hand : Economic Theory in the History of Science* . Cambridge: MIT Press.
- Israel, H., Seidelman, W. (1997, April 19). *Nazi Origins of an Anatomy Text: The Pernkopf Atlas*. *JAMA* , **276**(20), 1633.
- James, I., M., (Edt.) (1999). *Absolute Neighborhood Retracts and Shape Theory*, by Sibe Mardesic, in: History of Topology, Elsevier Science B.V.
- Janssen, Jacques, Manca, Raimondo (2006). Applied semi-Markov processes. Birkhäuser. p67.
- Johnston, W. M. (1972). *The Austrian Mind*. Berkeley: Univ. of California Press.

◂ Jonung, L. (1991). *The Stockholm School of Economics Revisited*. Cambridge: Cambridge University Press.

◂ Kainz Friedrich (1975). *Viktor Kraft*. in: Almanach für das Jahr 1975 der Österreichischen Akademie der Wissenschaften, Wien, pp519-557.

◂ Kass, S., *Karl Menger*. Retrieved 1996 from

◂ Kater, M. H. (2001). *Das'Ahnenerbe' der SS 1935 - 1945. Ein Beitrag zur Kulturpolitik des Dritten ReichesPublisher*. München: Oldenbourg.

◂ Kaznelson, Siegmund, Edt.. *Juden im deutschen Kulturbereich. Ein Sammelwerk*. Jüdischer Vlg. Frankfurt, 2. Ed. 1959, 1962.

◂ Keesling, J. (1958). *Review of K. Menger's Dimensionstheorie*. *Bull. Amer. Math. Soc.*, **83**, 203.

◂ Kerekes, L. (1966). *Abenddämmerung einer Demokratie. Mussolini, Gömbös und die Heimwehr*. . Wien-Frankfurt-Zürich : Europa Verlag.

◂ Klagge, James Carl. (2001). *Wittgenstein: Biography and Philosophy*. Cambridge University Press, 2001

◂ Klausinger, H. (2001). *Austrian economics in two minutes. Fritz Machlup as a journalist*. . Paper presented at the History of Economics Society Meeting, Winston Salem, North Carolina

◂ Klausinger, H. (2002). *The Austrian School of economics and the gold standard mentality in Austrian economic policy in the 1930s*. Working Paper, Center for Austrian Studies at the University of Minnesota

◂ Klein, L. R. (1971). *Whither Econometrics?*. *Journal of the American Statistical Association*, **66**, 415–421.

- Klein, J. (2001). *The Age of Economic Measurement. History of Political Economy*, **33**.
- Klenner, F. (1953). *Die österreichischen Gewerkschaften*. Wien: Verl. des österreichischen Gewerkschaftsbundes.
- Korn, Bertram. (1948). *Jewish 48'ers in America*. Delivered at the annual meeting of the American Historical Association in Washington on December 30, 1948. American Jewish Archives. *URL:* http://americanjewisharchives.org/publications/journal/PDF/1949_02_01_00_korn.pdf.
- Koopmans, T.C., MArschak, J. (1950). *Statistical Inference in Dynamic Economic Models*. Wien: Springer.
- Korotin, Ilse (2002). *Olga Hahn-Neurath*. In Keintzel, Brigitta, Korotin, Ilse (Edts.): *Wissenschafterinnen in und aus Österreich. Leben, Werk, Wirken*. Verlag Böhlau, Wien.
- Kovács-Bertrand, A. (1997). *Der ungarische Revisionismus nach dem ersten Weltkrieg. Der publizistische Kampf gegen den Friedensvertrag von Trianon, S. 9*. München: Oldenbourg Verlag.
- Kraft, V. (1950). *Der Wiener Kreis*. Wien: Springer Verlag.
- Kraft, V. (1953). *The Vienna Circle: The Origin of Neo-positivism, a Chapter in the History of Recent Philosophy*. New York: Greenwood Press .
- Kriechbaumer, R. (2005). *Österreich und Front Heil! Aus den Akten des Generalsekretariats der Vaterländischen Front. Innenansichten eines Regimes*. Wien - Köln - Weimar: Böhlau Verlag.
- Krohne, Heinz Walter, Hock, Michael, Edts. (2007). *Psychologische Diagnostik: Grundlagen und Anwendungsfelder*. Stuttgart: Kohlhammer Verlag.

◂ Kruntorad, P. (1991). *Jour Fixe der Vernunft. Der Wiener Kreis und die Folgen*. Wien: Hölder-Pichler-Tempsky.

◂ Krylow, N., Bogolyubov, N. (1937). *La théorie générale de la mesure dans son application a l'étude des systèmes dynamiques de la mécanique non-linéaire*. *Annals of Mathematics*, **38**, 65–113.

◂ Kryukov, V. I. (1976). *Wald's Identity and Random Walk Models for Neuron Firing*. *Advances in Applied Probability*, **8**(2), 257–277.

◂ Kuhn, H., Tucker, W. (1958). *Contributions to the Theory of Games*. Princeton: Princeton University Press.

◂ Kuprian, H. J. (1997). *An der Schwelle zum 20. Jahrhundert - Staat und Gesellschaft vor dem Ersten Weltkrieg*. Wien: Rolf Steininger / Michael Gehler (Hrsg.), Österreich im 20. Jahrhundert. Ein Studienbuch in zwei Bänden. Von der Monarchie bis zum Zweiten Weltkrieg (Böhlau-Studien-Bücher. Grundlagen des Studiums), Wien - Köln - Weimar 1997, S. 9-64.

◂ Lauener, Henri (1982). *Willard Van Orman Quine*. Beck, München 1982

◂ Lauridson, J. (2007). *Nazism And the Radical Right in Austria 1918-1934*. Kopenhagen: Mus. Tusculanum Press.

◂ Le Cam, Lucien (1986). Asymptotic Methods in Statistical Decision Theory. pp. xiii. Springer Series in Statistics.

◂ Lehmann, E. (1959). *Testing statistical hypotheses*. New York: Wiley.

◂ Leichter, O. (1964). *Glanz und Ende der ersten Republik*. Wien: Europa Verlag.

◂ Leinfellner, W. (1993). *Der Wiener Kreis und sein Einfluß auf die Sozialwis-*

senschaften. In F. Stadler & R. Haller (Ed.), *Zum Aufstieg der wissenschaftlichen Philosophie* (pp. 593–618). Wien: Springer.

- Lense, Josef (1966). *Hahn, Hans, Mathematiker*. In: Neue Deutsche Biographie (NDB). Band 7, Duncker & Humbolt, Berlin, S. 506.
- Leonard, R. (1998). *Ethics and the Excluded Middle : Karl Menger and Social Science in Interwar Vienna*. *Isis*, **89**(1), 1–26.
- Leonard, R. (2002). *Risk, Foresight – Flight. Morgenstern's Vienna in the 1930s*. Working Paper, University of Quebec at Montreal
- Leonard, R. (2006, March). *From Austroliberalism to Anschluss: Oskar Morgenstern and the Viennese Economist in the 1930's*. *Centre interuniversitaire de recherche sur la science et la technologie, Note de recherche 2006*, 1.
- Lewis, M. (1980). *Inside Wall Street's Black Hole*. London: Condé Nast .
- Lipcsey, Indilko (1994). *Romania and Transylvania in the 20th century*. Prepublished, Budapest, Possum Kiado.
- Library.mises.org (2012). *Elegy of the Mises Kreis*. http://library.mises.org/books/Mises%20Institute/Songs%20of%20the%20Mises-Kreis%20(Complete%20Collection)-Info.txt
- Luchins, E., McLoughlin, M. A. (1996). *In memoriam: Olga Taussky-Todd*. *Notices of the AMS*, **43**, 838–847.
- Mac Cartney, C. A. (1937). *Hungary and her successors - The Treaty of Trianon and Its Consequences 1919-1937*. Oxford: Oxford University Press.
- Machildon, G. P. (2010, July 17). *The Impact of the Great Depression on the Global Wheat Trade*. *Draft Prepared for workshop Unpeaceable*

Exchange: Trade and Conflict in the Global Economy. *University of Lisbon, Draft*.

Machlup, F. (1935). *The consumption of capital in Austria*. *Review of Economic Statistics*, **17**, 13–19.

Machlup, F. (2004). *Ludwig von Mises: A scholar who would not compromise*. Published by the Ludwig von Mises Institute.

Mac Kenzie, D. (1981). *Statistics in Britain*. Edingburgh: Edingburgh University Press.

Macmillan, M. *Paris 1919: Six Months that Changed the World*. New York: Random House.

Mahalanobis, P. C. (1940). *A Sample Survey of the Acreage under Jute in Bengal, with Discussion on Planning of Experiments*. *Proceedings of the 2nd Indian Statistical Conference, Statistical Publishing Society*.

Maloney, J. (1985). *Orthodoxy and Professionalisation of Economics* . Cambridge: Cambridge University Press.

Mandelbrot, B. (1982). *The Fractal Geometry of Nature*. San Francisco: W. H. Freeman.

Marschak, J. (1952). *Some aspects of inventory control.*. *Econometrica*, **20**, 499.

Marshall, A. (1890). *Principles of Economics*. London: Macmillan and Co. Ltd.

Marx, G: (1994). *Remembering the student years*. Középiskolai Matematikai Lapok, Sonderausgabe.

- Matis, H. (1972). *Österreichs Wirtschaft 1848-1913. Konjunkturelle Dynamik und gesellschaftlicher Wandel im Zeitalter Franz Josephs I.* Berlin: Duncker & Humbolt.
- Mayerhofer (1998), *Österreichs Weg zum Anschluss im März 1938*. Retrieved 25.05.1998 from http://www.wienerzeitung.at/linkmap/personen/miklaspopup.htm
- Menger, C. (1871). *Grundsätze der Volkswirtschaftslehre*. Aalen, Scientia-Verl., 1968, Neudr. d. 2. Aufl. Wien 1923
- Menger, K. (1928). *Dimensionstheorie*. Leipzig: Teubner.
- Menger, K. (1931). *Ergebnisse eines mathematischen Kolloquiums*. Vienna: B.G. Teubner.
- Menger, K. (1932). *Kurventheorie*. Berlin, Leipzig: B. G. Teubner.
- Menger, K. (1934) (August). *Das Unsicherherheitsmoment in der Wertlehre*. *Journal of Economics* , **5**(4), 459–485.
- Menger, K. (1934). *Bernoullische Wertlehre und Petersburger Spiel, ibid., pp26-27.*. *Ergebnisse eines mathematischen Kolloquiums*, **6**, 26–27.
- Menger, K. (1935). *Sull'indirizzo di idee e sulle tendenze prinzipali del colloquio matematico di Vienna*. *Annali di Pisa*, **4**, 1–13.
- Menger, K. (1936). *New foundations of projective and affine geometry*. *Algebra of geomerty*. *Ann. Math* , **37**, 456–482.
- Menger, K. (1936). *Bemerkungen zu den Ertragsgesetzen*. *Zeitschrift für Nationalökonomie*, **7**, 25–56.
- Menger, K. (1942). *Statistical metrics*. *Proc. Nat. Acad. Sci.*, **28**.

- Menger, K. (1944). *Algebra of analysis*. Notre Dame Mathematical Lectures, Number 3. Notre Dame, Indiana, *Notre Dame Math. Lectures,* Univ. Notre Dame .
- Menger, K. (1951). *Ensembles flous et fonctions aleatoires. Comptes Rendus Académie des. sciences*, **37**, 2001–2003.
- Menger, K. (1952). *The formative years of Abraham Wald and his work on Geometry. Annals of Math. Statistics*, **23**, 14–20.
- Menger, K. (1953). *Calculus, a modern Approach*. Chicago: Illinois Institute of Technology.
- Menger, K. (1974). *Morality, Decision and Social Organisation*. Dordrecht: D. Reidel Pub. Co..
- Menger, K. (1979). *Selected Papers in Logic and Foundations, Didactics, Economics* . Dordrecht: Kluwer.
- Menger, K. (1981). *On the origin of the n-arc theorem. J. Grapg Theory*, **5**, 341–350.
- Menger, K. (1993). *General Spaces and Cartesian Spaces*. In G. A. Edgar (Ed.), *Classics on Fractals* (pp. 285–298). London: Addison-Wesley.
- Menger, K. (1994). *Reminiscences of ihe Vienna Circle and ihe Mathematical Colloquium, Kluwer. Dordrecht. Vienna Circle Collection*, **20**.
- Menger, K. (1994). *Reminiscences of the Vienna Circle and the Mathematical Colloquium (Vienna Circle Collection)*. New York: Springer.
- Menger, K. (1998). *Ergebnisse Eines Mathematischen Kolloquiums*. Wien: Springer.

- Menger, K. (2002). *Selecta Mathematica*. Vienna, Springer.
- Miller, H. D. (1961). *A Generalization of Wald's Identity with Applications to Random Walks*. *The Annals of Mathematical Statistics*, **32**(2), 549–560.
- Mises, L. v. (1927). *Liberalismus*. Jena: Gustav Fischer.
- Mises, L. v. (1929). *Kritik des Interventionismus*. Jena: Gustav Fischer.
- Mises, L. v. (1931). *Die Ursachen der Wirtschaftskrise*. Tübingen: Mohr.
- Mises, L. v. (1953). *The Theory of Money and Credit*. New Haven: Yale University Press Edition.
- Mises, L. v. (1960). *Epistemological Problems of Economics*. Princeton: Van Nostrand.
- Mises, L. v. (1978). *On the Manipulation of Money and Credit*. New York: Free Market Books.
- Mises, L. v. (1978). *Liberalism: A Socio-Economic Exposition, Translation of Mises*. Kansas City, Sheed Andrews & McMeel.
- Mises, L. v. (1978). *Notes and Recollections*. Spring Mills: Libertarian Press.
- Mises, R. v. (1990). *Kleines Lehrbuch des Positivismus. Einführung in die empiristische Wissenschaftsauffassung*. Frankfurt, Suhrkamp.
- Moltke S. Gram, Klemke E. D. (Edts) (1974). *The Ontological Turn: Studies in the Philosophy of Gustav Bergmann*, Iowa.
- Mommsen, H. (1981). *Politik und Gesellschaft im alten und neuen Österreich: Festschrift für Rodolf Neck zum 60. Geburtstag*. Wien: Verlag für Politik und Geschichte.

- Monk, Ray (2005). *How to read Wittgenstein*. W.W. Norton & Company.
- Moore, H. L. (1914). *Economic Cycles: Their Law and Cause*. New York: McMillan.
- Morgan, M. S. (1990). *The History of Econometric Ideas*. Cambridge: Cambridge University Press.
- Morgenstern, O. (1931). *Kapital- und Kurswertänderungen der an der Wiener Börse notierten Aktiengesellschaften 1913 bis 1930*. *Zeitschrift fu¨r Nationalökonomie*, **3**, 251–255.
- Morgenstern, O. (1934). *Die Grenzen der Wirtschaftspolitik*. Wien: Julius Springer.
- Morgenstern, O. (1936). *Logistik und Sozialwissenschaft*. *Zeitschrift fu¨r Nationalökonomie*, **7**, 1–24.
- Morgenstern, O. (1936). *Die Einordnung der Verkehrspolitik in die allgemeine Wirtschaftspolitik*. *Mitteilungen des Verbandes der österreichischen Banken und Bankiers*, **18**, 11–23.
- Morgenstern, O. (1936). *Währung und Preise*. *Österreichische Zeitschrift für Bankwesen*, **1**, 166–171.
- Morgenstern, O. (1937). *The Limits of Economics*. London: William Hodge & Co.
- Morgenstern, O. (1939). *The experience with public regulation and public monopoly abroad*. *Proceedings of the Academy of Political Science*, **18**(2), 34–39.
- Morgenstern, O. (1951, Oktober). *Abraham Wald, 1902-1950*, **19** (4)

Mühlberger, K. (1993). *Vertriebene Intelligenz 1938. Der Verlust geistiger und menschlicher Potenz an der Universität Wien von 1938 bis 1945.* Wien: Dokumentation.

Müller, K. H. (1987). *Die Idealwelten der österreichischen Nationalökonomen*. In F. Stadler (Ed.), *Vertriebene Vernunft I: Emigration und Exil österreichischer Wissenschaft 1930-1940* (pp. 238–275). Wien: Jugend und Volk.

Müller, K. H. (1988). *Hochzeit der Sozialwissenschaften 1871–1938*. In J. Langer (Ed.), *Geschichte der österreichischen Soziologie* (pp. 51–69). Wien: Verlag für Gesellschaftskritik.

Müller, A., *Universitäten in der NS-Zeit - mit Focus Wien*. Retrieved 2007 from http://www.vknn.at/texte/mueller_nsuniv.html

Nagel, Ernest (1961). *The Structure of Science: Problems in the Logic of Scientific Explanation*, Hackett Publishing Co, Inc. 2nd Ed. (Dezember 1987)

Neck, Rudolf (Edt.) (1968). *Österreich im Jahre 1918. Berichte und Dokumente*. Verlag Oldenbourg, München.

Nemeth, Elisabeth (2011). *Logical Empiricism and the History and Sociology of Science,* in Richardson, Alan W. and Thomas Uebel (eds.). The Cambridge Companion to Logical Empiricism. CambridgeUniversity Press. p. 293.

Neumann, J.v. (1936, January 8). *Continuous Geometry*. *Proc. Nat. Acad. Sci.*, **22**, 92–100.

Neumann, J.v., Morgenstern, O. (1944). *The Theory of Games and Economic*

Behavior. Princeton University Press.

- Neurath, O. (1987). *Unified science - The Vienna Circle monograph series originally edited by Otto Neurath*. Dordrecht, London and Boston: Kluwer Acad. Pub..
- Neyman, J. (1937). *Outline of a theory of statistical estimation based on the classical theory of probability. Phil. Trans. Roy. Soc. London*, **236**, 333–380.
- Neyman, J. (1938). Statistical research memoires, Vol. II, University College, London, (1938)
- Neyman, J. (1956). *Note on an Article by Sir Ronald Fisher,* Journal of the Royal Statistical Society. Series B, 18 (2): 288–294.
- Neyman, J., Pearson, E. S. (1933). *On the problem of the most efficient tests of statistical hypotheses*, Philosophical Transactions of. the Royal Society of London, Series A, Vol. 231.
- Neyman, J., Pearson, E. S. (1936). In J. Neyman & E. Pearson (Ed.), *Statistical Research Memoirs* (pp. 1–37). London: University College .
- Neyman, J., Pearson, E. S. (1936). *Contributions to the theory of testing statistical Hypotheses, Stat. Res. Memoirs*, **1**, 1–37.
- Niederstätter, A. (2007). *Geschichte Österreichs*. Stuttgart: Kohlhammer Verlag.
- Noble, Holocomb (2005). *Robert Heilbroner, Writer and Economist, Dies at 85*. The New York Times, Obituaries.
- ÖBL (1965). *Kaufmann Felix. In: Österreichisches Biographisches Lexikon*

1815–1950. Band 3, Verlag der Österreichischen Akademie der Wissenschaften, Wien, p 270

- O'Connor, John, Robertson, Edmund F., (2000). *Karol Borsuk*, MacTutor History of Mathematics Archive, University of St Andrews.

- O'Connor, John, Robertson, Edmund F., (2000). *Eduard Cech*, MacTutor History of Mathematics Archive, University of St Andrews.

- O'Connor, John, Robertson, Edmund F., (2000). *G. T. Whyburn*, MacTutor History of Mathematics Archive, University of St Andrews.

- O'Connor, John, Robertson, Edmund F., (2000). *Norbert Wiener*, MacTutor History of Mathematics Archive, University of St Andrews.

- O'Connor, John, Robertson, Edmund F., (2011). *Philipp Frank. 1884 - 1966*. MacTutor History of Mathematics Archive, University of St Andrews. http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Frank.html.

- Oesterreichische Nationalbank., *Oesterreichische Nationalbank*. Retrieved 2008 from

http://www.oenb.at/en/ueber_die_oenb/geldmuseum/oesterr_geldgeschichte/history_of_austrian_money.jsp

- Okey, R. (2001). *The Habsburg Monarchy, c. 1765–1918: from enlightenment to eclipse*. New York: St Martin's Press.

- Opll, F., *From the capital of the Austro-Hungarian Empire to the capital of the Republic*. Retrieved 10/2007 from

http://www.wien.gv.at/english/history/history/vhisrv.htm

- Otruba, G. (1980). *Entstehung und soziale Entwicklung der Arbeiterschaft und der Angestellten bis zum Ersten Weltkrieg*. In E. Zöllner (Ed.), *Österre-*

ichs Sozialstrukturen in historischer Sicht (pp. 123–154). Wien: Schriften des Inst. f. Österreichkunde.

Oxaal, I., & Pollak, M. (1987). *Jews, Antisemitism and Culture in Vienna*. London and New York: Routledge & Kegan.

Pastor, P. (1976). *Hungary between Wilson and Lenin. The Hungarian Revolution of 1918-1919 and the Big Three*. New York: Columbia University Press.

Papst Pius XI, Enzyklika, *Quadragesimo Anno*. Retrieved 2008 from

http://www.vatican.va/holy_father/pius_xi/encyclicals/documents/hf_p-xi_enc_19310515_quadragesimo-anno_en.html

Patai, R. (1996). *The Jews of Hungary*. Detroit: University Press.

Pauley, B. F. (1992). *From Prejudice to Perdition: A History of Austrian Anti-Semitism*. Chapel Hill & London: University of North Carolina Press.

Payne, Stanley (2001). *A History of Fascism 1914-45*, London: Routledge, 2001.

Perz, B., Safrian, H., Stuhlpfarrer, K., B., Safrian, H., & Stuhlpfarrer, K. (1982). *Faschismus in O¨sterreich und International*. Wien: Lo¨cker.

Peter F., Hanák, P. , Tibor, F. (1994). A history of Hungary, Indiana University Press.

Poliakov, L. (2003). *The History of Anti-Semitism*. Philadelphia: University of Pennsylvania Press.

Popper, K. (1935). *Über nachwirkungsfreie Folgen. Ergebnisse eines mathematischen Kolloquiums*, **7**(12).

- Popper, K. (1959 (orig. 1934)). *The logic of scientific discovery (orig. Die Logik der Forschung)*. New York: Basic Books.
- Portisch, H. (1989). *Österreich I. Die unterschätzte Republik*. Wien: Kremayr & Scheriau.
- Pulzer, P. G. (1988). *The rise of political anti-semitism in Germany and Austria*. New York: Jon Wiley & Sons.
- Punzo, L. F. (1989). *John von Neumann and Modern Economics*. In M. Dore & S. Chakravarty (Ed.), *Von Neumann and Karl Menger's Mathematical Colloquium* (pp. 29–65). London, Clarendon Press.
- Rathkolb, O. (1987). *Überlegungen zum Exodus der Jurisprudenz* . In F. Stadler (Ed.), *Vertriebene Vernunft I: Emigration und Exil österreichischer Wissenschaft 1930-1940* (pp. 276–303). Wien: Jugend und Volk.
- Rauchensteiner, M. (1998). *Österreich-Ungarn und der Erste Weltkrieg*. Graz: Steirische Verlagsgesellschaft.
- Raven Diederick, Krohn Wolfgang (2000). *Edgar Zilsel: His Life and Work* (*1891-1944*), In Edgar Zilsel, *The Social Origins of Modern Science*, ed. Diederick Raven, Wolfgang Krohn and Robert S. Cohen (Dordrecht: Kuwer Academic Publishers, xix-lix.
- Read, Rupert, Crary, A. (2000). Edts., *The New Wittgenstein*, Routledge.
- Reichel, H.C. (1994): *Kurt Reidemeister (1893 bis 1971) als Mathematiker und Philosoph. Ein 'Meilenstein' in der Entwicklung der Topologie, der Geometrie und der Philosophie dieses Jahrhunderts*, Österreichische Akademie der Wiss., Math.-Naturwiss. Klasse, Sitzungsberichte II 203, pp117-135.

- Reichenbach, Hans. (1951). *The Rise of Scientific Philosophy*. University of California Press.
- Reidemeister, K. (1932). *Knotentheorie, in Ergebnisse der Mathematik und ihrer Grenzgebiete*, Berlin, J. Springer
- Reppe, S. (1993). *Der Karl-Marx-Hof. Geschichte eines Gemeindebaus und seiner Bewohner*. Wien: Picus Verlag.
- Richter, D. J., *Ludwig Wittgenstein (1889-1951)*. Retrieved 2006 from http://www.iep.utm.edu/w/wittgens.htm
- Robbins, H. (1951). *Statistical decision functions. By A. Wald.. Bulletin of the American Math. Soc.*, **57**, 383–384.
- Ross, D. (1991). *The Origins of American Social Science*. Cambridge: Cambridge Univ. Press.
- Rosen, F. (2003). *Classical Utilitarianism from Hume to Mill*. London: Routledge Studies in Ethics & Moral Theory.
- Rosner, P. (1999). *The Austrian research on business cycles. History of Economic Ideas*, **7**, 195–226.
- Rozenblit, M. L. (2001). *Reconstructing A National Identity: The Jews of Habsburg Austria during WorldWar I.* . New York: Oxford University Press.
- Rudolph, R. L. (1976). *Banking and Industrialization in Austria-Hungary*. Cambridge: Cambridge University Press.
- Russel, B., Whitehead, A. N. (1913). *Principia mathematica,* 1. Auflage 1910-1913, Online at the University of Michigan, http://quod.lib.umich.e-

du/cgi/t/text/text-idx?c=umhistmath;idno=AAT3201.0001.001

- Said, H. A. (1987). *Moritz Schlick, der Wiener Kreis und ihr Verhältnis zur Naturwissenschaft* (Doctoral Dissertation, Univ. of Münster). pp65–70.
- Salmon, Wesley (1977). *The philosophy of Hans Reichenbach*, Synthese 34,. 5-88.
- Samuelson, P., Koopmans, T., Stone, J. (1954). *Report of the Evaluative Committee for Econometrica*. *Econometrica*, **22**, 141–146.
- Sandgruber, R. (1995). *Ökonomie und Politik. Österreichische Wirtschaftsgeschichte vom Mittelalter zur Gegenwart*. Wien: Carl Überreuter .
- Savage, C. W. (1988). *Herbert Feigl*, in PSA, Proceedings of the Biennial Meeting of the Philosophy of Science Association, Volume Two: Symposia and Invited Papers, 15-22,Wien, Carl Überreuter.
- Savage, L. (1954). *The foundations of statistics*, John Wiley and Sons, New York.
- Schelling von, Hermann (1931).*Die wirtschaftlichen Zeitreihen als Problem der Korrelationsrechnung: mit besonderer Berücksichtigung der "lag" correlation*. Veröffentlichungen der Frankfurter Gesellschaft für Konjunkturforschung. K. Schröder.
- Schelling von, Hermann (1950).*Review on the* 'Foundation of a general theory of sequential decision functions. A. Wald, Econometrica, Menasha 15, 279-313', Zentralblatt MATH: 0029.30701.
- Schelling von, Hermann (1955).*Statistische Modelle als Hilfsmittel der Naturbeschreibung*. Mitt.-Bl. Math. Statistik 7, 173-192 (1955). Physica-Verlag, Würzburg.

- Schilpp, P. A. (1963). *The Philosophy of Rudolf Carnap*. LaSalle IL, Open Court.
- Schlesinger, K.(1935). *Über die Produktionsgleichungen der okonomischen Wertlehre*. In *Ergebnisse eines mathematischen Kolloquiums*. Vienna: F. Deuticke.
- Schlick , Moritz (1910). *Das Wesen der Wahrheit nach der modernen Logik*, in: Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie, Jg. 34, pp386-477.
- Schlick , Moritz (1917). *Raum und Zeit in der gegenwärtigen Physik*. Berlin, Verlag Julius Springer 1917 (4. Auflage 1922).
- Schlick , Moritz (1918). *Allgemeine Erkenntnislehre*. Berlin, Verlag Julius Springer, (2. Auflage 1925).
- Schmitz, N. (1972). *A note on the optimality of k-stage tests*. Metrika 19 (1972) 72-75. Retrieved 2009 from http://www.springerlink.com/content/n4n3n6uu8183gjpw/
- Schneeweiss, H.(2005). *Abraham Wald*. Retrieved 2005 from http://epub.ub.uni-muenchen.de/1808/1/paper_439.pdf
- Schultz, H. (1938). *The Theory and Measurement of Demand*. Chicago: University of Chicago Press.
- Schulze, Hagen. (1994). *States, Nations and Nationalism*. Malden: Blackwell Publishers.
- Schumpeter, J. (1954). *History of Economic Analysis* . Oxford: Oxford Univ. Press.
- Schweiger, F. (1964). *Geschichte der niederösterreichischen Heimwehr*.

Vienna: self-published .

Scott-Smith, Giles (2002). *The politics of apolitical culture: the Congress for Cultural Freedom, the CIA, and post-war American hegemony*. London: Routledge. p109

Senft, G. (2002). *Im Vorfeld der Katastrophe. Die Wirtschaftspolitik des Ständestaates, Österreich 1934–1938*. Wien: Braunmüller.

Shafer, G., & Vovk, V. (2005). *The origins and legacy of Kolmogorov's Grundbegriffe. Game-Theoretic Probability and Finance Project, Working Papers*, **4**, 1–109.

Shah, A. (1997, Dec). *Black, Merton and Scholes: Their work and its consequences.. Economic and Political Weekly*, **XXXII**(52), 3337–3342.

Shannon, C. E. (1948). *A mathematical theory of communication. Bell System Tech. J.*, **27**, 623–656.

Shirer, W. L. (1984). *20th Century Journey: A Memoir of a Life and the Times. The Nightmare Years.*. NY: Little, Brown & Company.

Siegmund-Schultze, R. (2001). Rockefeller and the Internationalization of Mathematics Between the Two World Wars. Historical Studies, Science Networks, Birkhäuser Basel, 1.Ed. (February 28, 2001)

Sigmund, K. (1995). *A philosopher's mathematician: Hans Hahn and the Vienna Circle. The Mathematical Intelligencer*, **17**, 6–29.

Sigmund, K.(2001), *Kühler Abschied von Europa – Wien 1938 und der Exodus der Mathematik*. Retrieved 2001 from http://www.oemg.ac.at/Tagungen/2001/KuehlerAbschied.pdf.

◂ Sked, A. (1989). *The Decline And Fall of the Habsburg Empire,1815–1918*. London: Longman.

◂ Sobel, M. (1953). *An essentially complete class of decision functions for certain standard sequential problems. Annals of Math. Stat*., **24**, 319–337.

◂ Sobel, M. (2008), *Milton Sobel, Professor of Statistics*. Retrieved 2008 from http://www.universityofcalifornia.edu .

◂ Solomon, H. (1956). *Probability and statistics in psychometric research - item analysis and classification techniques*. California, Berkeley: University of California Press.

◂ Solow, Robert. (2005). *How did economics get that way & what way did it get?*. Daedalus, Fall 2005, Vol. 134, No. 4, Pages 87-100.

◂ SQA, Online Source , *Abraham Wald Centennial Celebration*. Retrieved Sept. 2008 from http://www.dekker.com/servlet/product/productid/SQA.

◂ Stadler, F. (1991). In Kruntorad (Ed.), *Der Wiener Kreis – Versuch einer Typologie* (pp. 23–41). Wien: Hölder-Pichler-Tempsky.

◂ Stadler, F., Haller, R. (1993). *Zum Aufstieg der wissenschaftlichen Philosophie*. Wien: Springer.

◂ Stadler, F., Seiler, M. (1994). (Edts.) *Heinrich Gomperz, Karl Popper und die 'österreichische Philosophie'*. Beiträge zum internationalen Forschungsgespräch aus Anlaß des 50. Todestages von Heinrich Gomperz (1873-1942) und des 90. Geburtstages von Karl R. Popper (1902-). Amsterdam/Atlanta, GA, 1994, XI, 227 pp.

◂ Stadler, F. (1995). *Vertreibung der Vernunft. The Cultural Exodus from Austria*. Springer, Wien, New York.

- Stadler, F. (1997). *Studien zum Wiener Kreis*. Frankfurt a. Main, Suhrkamp.
- Stadler, F. (1997). *Friedrich Stadler, Die andere Kulturgeschichte am Beispiel von Emigration und Exil der österreichischen Intellektuellen 1930 - 1940*. In M. Gehler & R. Steininger (Ed.), *Österreich im 20. Jahrhundert. Ein Studienbuch in zwei Bänden. Von der Monarchie bis zum Zweiten Weltkrieg* (pp. 535–555). Wien, Köln, Weimar. Böhlau-Studien-Bücher.
- Statistisches Handbuch für die Rep. Österreich (1932). Bundesamt für Statistik, Wien.
- Staudinger, A. (1995) (Edt.), *Handbuch des politischen Systems Österreichs. Erste Republik 1918-1933*, Wien.
- Steen, L. A. (1978). *Mathematics Today. Twelve Informal Essays*. NY, Springer.
- Steindl, J. (1988). *Zeitzeuge*. In F. Stadler (Ed.), *Vertriebene Vernunft II: Emigration und Exil österreichischer Wissenschaft* (pp. 399–401). Wien: Jugend und Volk.
- Steininger, R., Gehler, M. (Eds.) (1997). *Österreich im 20 Jahrhundert. Ein Studienbuch in zwei Bänden. Von der Monarchie bis zum zweiten Weltkrieg*. Wien, Köln, Weimar. Böhlau Verlag.
- Stenografisches Protokoll. (1933). *125. Sitzung des Nationalrates der Republik Österreich* . Wien: Nationalrat Österreich, IV Gesetzgebungsperiode.
- Stern, F., Eichinger, B. (2009). *Wien und die jüdische Erfahrung*. 1900-1938. Böhlau, Wien.
- Stockman, C. M. (1944). *A Method of Obtaining an Approximation for the*

Operating Characteristic of a Wald Sequential Probability Ratio Test Applied to a Binomial Distribution. Series R, Q.C./R/19, Stockman

- Stöltzner, M., Uebel, T. (2006). *Wiener Kreis*. Hamburg: Felix Meiner.

- Streibel, R. (1994). *Februar in der Provinz. Eine Spurensicherung zum 12. Februar 1934 in Niederösterreich*. Grünbach: Grünbach Edition Geschichte der Heimat.

- Studenski, P. (1958). *The Income of Nations: Theory, Measurement, Analysis*. NY: NY Univ. Press.

- Suppes, P. (2006). In Sarkar S. and Pfeifer J. (Eds.), *The Philosophy of Science: An Encyclopedia,* Volume 2, N-Z Indexed, pp.491-496. New York: Routledge.

- Szalai, A. (2002). *In the Land of Hagar: The Jews of Hungary, History, Society and Culture*. Haifa: MOD Publishing House.

- Tálos, E., Neugebauer, W. (1988). *'Austrofaschismus'. Beiträge über Politik, Ökonomie und Kultur 1934-1938*. . Wien: Verlag für Gesellschaftskritik.

- Tálos E. (Edt.) (1995). *Handbuch des politischen Systems Österreichs. Erste Republik 1918-1933*. Wien: Manz.

- Tálos, E. (2005). *Zum Konstituierungsprozess des Austrofaschismus*. In . Tálos & . Neugebauer (Ed.), *Austrofaschismus* (pp. 31–52). Wien: Lit Verlag.

- The Austrian Economics Newsletter., *An Interview with Fritz Machlup (1902-1983)*. Retrieved 1980 from http://www.mises.org/journal-s/aen/aen3_1_1.asp

- Tinbergen, J. (1939). *Statistical Testing of Business Cycle Theories: Business Cycles in the USA, 1919-1932*. Geneva: League of Nations.
- Tintner, G. (1940). *The Variate Difference Method*. Bloomington, Indiana, Principia Press.
- Tintner, G. (1952). *Abraham Wald's Contributions to Econometrics*. *Ann. Math. Statist.*, **23**(1), 21–28.
- Tintner, G. (1968). *Methodology of Mathematical Economics and Econometrics*. University of Chicago Press.
- Tintner, G., Sengupta J. K. (1972). *Stochastic Economics*. Academic Press, NY.
- Toates, F. (2002). *Obsessive Compulsive Disorder: Practical Tried-and-Tested Strategies to Overcome OCD*. Class Publishing. p221
- Uebel, T. (1991). *Rediscovering the Forgotten Vienna Circle*. *Austrian Studies on Otto Neurath and the Vienna Circle*. London: Kluwer Academic Publishers .
- Uebel, T. (2000). *Vernunftkritik und Wissenschaft*. Wien, NY: Springer.
- Uebel, T. (2003). *On the Austrian Roots of Logical Empiricism,* . In P. Parrini & W. C. Salmon (Ed.), *Logical Empiricism - Historical and contemporary Perspectives* (pp. 76–93). Pittsburgh : University of Pittsburgh Press .
- Univ. of Erlangen (2007). *Zum Geburtstag des Mathematikers Prof. Dr. Georg Nöbeling*, Press release, Universität Erlangen-Nürnberg, Retrieved 2008-02-24.

- Valiani, L. (1973). *The End of Austria-Hungary, London*. London: Secker & Warburg.
- Várdy, S. (1997). *Historical Dictionary of Hungary, p. 732*. London: The Scarecrow Press Inc.Lanham, Md& London.
- Vital, D. (1999). *A People Apart: A History of the Jews in Europe*. Oxford: Oxford University Press.
- Waismann, Friedrich. (1979). *Ludwig Wittgenstein und der Wiener Kreis. Gespräche, aufgezeichnet von Friedrich Waismann*. Suhrkamp Verlag. Auflage 1
- Wald, A. (1931). *Axiomatik des Zwischenbegriffes in metrischen Räumen. Wiener Akad. Anzeiger*, (16), 1–3.
- Wald, A. (1931). *Axiomatik des Zwischenbegriffes in metrischen Räumen. Math. Annalen*, **104**, 476–484.
- Wald, A. (1931). *Axiomatik des metrischen Zwischenbegriffes. Ergebnisse eines mathematischen Kolloquiums*, **2**, 17.
- Wald, A. (1934). *Über die eindeutige Lösbarkeit der neuen Produktionsgleichungen. Ergebnisse eines Mathematischen Kolloquiums*, **1**(6), 12–18.
- Wald, A. (1934). *Zur Axiomatik des Zwischenbegriffes. Ergebnisse eines Math. Kolloquiums*, **4**, 23–24.
- Wald, A. (1935). *Sur la courbure des surfaces. C. R. Acad. Sci. Paris*, **201**, 918–920.
- Wald, A. (1935). *Über die eindeutige positive Lösbarkeit der neuen Produktionsgleichungen. Ergebnisse eines Math. Kolloquiums*, **6**, 12–18.

- Wald, A. (1935). *Über die Produktionsgleichungen der ökonomischen Wertlehre*. *Ergebnisse eines mathematischen Kolloquiums*, **6**, 10–11.
- Wald, A. (1936). *Über einige Gleichungssysteme der mathematischen Ökonomie*. *Zeitschrift für Nationalokonomie*, **7**, 637–670.
- Wald, A. (1936). *Sur la notion de collectif dans le calcul de probabilités*. *Comptes Rendus des Séances de l'Académie des Sciences*, **202**, 180–183.
- Wald, A. (1936). Berechnung und Ausschaltung von Saisonschwankungen, Beiträge zur Konjunkturforschung, Vol. 9, Verlag Julius Springer, Vienna.
- Wald, A. (1937). *Die Widerspruchsfreiheit des Kollektivbegriffes der Wahrscheinlichkeitsrechnung*. *Ergebnisse eines mathematischen Kolloquiums*, **8**, 38–72.
- Wald, A. (1937). *Zur Theorie der Preisindexziffern*, Zeitschrift für Nationalökonomie 8, 179-219.
- Wald, A. (1939). *Limits of a distribution function determined by absolute moments and inequalities satisfied by absolute moments*. *Trans. Amer. Math. Soc.*, **46**, 280–306.
- Wald, A. (1939). *Contributions to the theory of statistical estimation and testing hypotheses*. *Annals of Math. Stat.*, **10**, 299–326.
- Wald, A. (1939). *A new formula for the index of cost of living*, Econometrica, Vol. 7, pp319-331.
- Wald, A. (1940). *Contributions to the Theory of Statistical Estimation and Testing Hypotheses*, Annals of Mathematical Statistics, 10.
- Wald, A. (1942). *Setting of tolerance limits when the sample is large*. *Ann.*

math. Statistics, **13**, 389–399.

- Wald, A. (1942). *Asymptotically shortest confidence intervals. Ann. math. Statistics*, **13**, 127–137.
- Wald, A. (1943). *Sequential Analysis of Statistical Data : Theory*. Statistical Research Group, Columbia University, New York. [18], A. Wald
- Wald, A. (1943). *An extension of Wilks' method for setting tolerance limits*. Annals of Math. Stat., Vol. 14, pp45-55.
- Wald, A, H. B. Mann (1943). *On stochastic limit and order relationships*. Annals of Math. Stat., Vol. 14, pp. 217-226.
- Wald, A. (1944) (September). *On cumulative sums of random variables. The Annals of Mathematical Statistics*.
- Wald, A. (1944). *On a statistical problem arising in the classification of an idividual into one of two groups. Ann. Math. Statist.*, **15**(2), 145–162.
- Wald, A. (1944). *A General Method of Deriving the Operating Characteristics of any Sequential Probability Ratio Test*. Columbia University.
- Wald, A. (1944). *Note on a lemma. Ann. Math. Statist.*, **15**(3), 330–333.
- Wald, A. (1944). *On Cumulative Sums of Random Variables. The Annals of Mathematical Statistics*, **15**.
- Wald, A., J. Wolfowitz (1945). *Sampling inspection plans for continuous production which insure a prescribed limit on the outgoing quality. Ann. math. Statistics*, **16**, 30–49.
- Wald, A. (1945) *Generalization of a theorem by v. Neumann concerning zero-sum two-person games*, Annals of Mathematics, **46**(2), pp. 281-286.

- Wald, A. (1945) *Sequential Tests of Statistical Hypotheses*, revised edition for the Statistical Research Group.
- Wald, A. (1945). *Some generalizations of the theory of cumulative sums of random variables*. *Ann. Math. Statist.*, **16**(3), 287–293.
- Wald, A. (1945). *Sequential Method of Sampling for Deciding Between Two Courses of Action*. Journal of the American Statistical Association, Vol. 40, pp. 277-306.
- Wald, A. (1947). *Limit distribution of the maximum and minimum of successive cumulative sums of random variables*. Bull. Amer. Math. Soc. Vol. 53, No. 2, pp142-153.
- Wald, A. (1947). *Theory of games and economic behavior by John Von Neumann and Oskar Morgenstern*. *Review of Economic Statistics*, **39**, 47–52.
- Wald, A. (1947). *Sequential Analysis*. New York, London: John Wiley & Sons.
- Wald, A. (1947). *Foundations of a general theory of sequential decision functions*. *Econometrica*, **15**, 279–313.
- Wald, A. (1947). *A note on regression analysis*. *Ann. math. Statist.*, **18**, 586–589.
- Wald, A. (1947). *An Essentially Complete Class of Admissible Decision Functions*, Annals of Mathematical Statistics, Vol. 18, pp549-555.
- Wald, A., C. Stein (1947). *Sequential Confidence Intervals for the Mean of a Normal Distribution with Known Variance*. *Ann. Math. Statist.*, **18**(3),

427–433.

- Wald, A. (1948). *Report of the president of the institute for 1948*. Retrieved 2009 from projecteuclid.org/.
- Wald, A. (1948). *Estimation of a parameter when the number of unknown parameters increases indefinitely with the number of observations*. *Ann. math. Statist.*, **19**, 220–227.
- Wald, A., J. Wolfowitz (1948). *Optimum character of the sequential probability ratio test*. Annals of Mathematical Statistics, Vol. 19, pp326-339, 1948.
- Wald, A. (1949). *Statistical Decision Functions*. *Ann. Math. Statist.*, **20**(2), 165–205.
- Wald, A., M. Sobel (1949). *A sequential decision procedure for choosing one of three hypotheses concerning the unknown mean of a normal distribution*, Annals of Math. Stat., Vol. 20, pp502-622.
- Wald, A., J. Wolfowitz (1949). *Bayes solutions of sequential decision problems*, Proc. Nat. Acad. Sc., Vol. 35, pp99-102.
- Wald, A. (1950). *Statistical Decision Functions*. New York: John Wiley & Sons.
- Wald, A. (1950). *Note on Zero Sum Two Person Games*, The Annals of Mathematics, Vol 52(3), p739.
- Wald, A., Dvoretzky, A., Wolfowitz, J. (1950). *Elimination of randomization in certain problems of statistics and of the theory of games*, Proc. Nat. Acad. Sci., Vol. 36, pp256-200.
- Wald, A., Wolfowitz, J. (1950). *Bayes solutions of sequential decision prob-*

lems. Annals of Math. Stat., Vol. 21, pp82-89.

- Wald, A. (1951). *Asymptotic minimax solutions of sequential point estimation problems*, Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press.
- Wald, A., Wolfowitz, J. (1951). *Two methods of randomization in statistics and the theory of games*, Annals of Mathematics, Vol. 53, pp581-586.
- Wald, A., Wolfowitz, J. (1951). *Elimination of randomization in certain statistical decision procedures and zerosum two-person games*, Annals of Math. Stat., Vol. 22, pp1-21.
- Wald, A., Wolfowitz, J. (1951). *Relations among certain ranges of vector measure*, Pacific Jour. Math., Vol. 1, pp59-74.
- Wald, A., Wolfowitz, J. (1951). *Characterization of the minimal complete class of decision functions when the number of distributions and decisions is finite*, Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press.
- Wald, A., Dantzig G. (1951). *On the Fundamental Lemma of Neyman and Pearson*, The Annals of Mathematical Statistics, Vol. 22, No. 1, Mar., 1951, pp. 87-93.
- Wallis, W. A. (1980). *The statistical research group. J. Amer. Stat. Assoc.*, **75**, 320–334.
- Weintraub, E. R. (1985). *General Equilibrium Analysis*. Cambridge: Cambridge University Press.
- Weintraub, E. R. (2002). *How Economics Became a Mathematical Science*. Duke University Press.

- Welan, M. (1984). *Die Verfassungsentwicklung in der Ersten Republik*. In J. Desput (Ed.), *Österreich 1934-1984. Erfahrungen, Erkenntnisse, Besinnung*. Graz, Wien, Köln: Styria.
- Wilks, S. S. (1941). *Determination of sample sizes for setting tolerance limits*. *Annals of math. Statistics*, **12**.
- Williams, D. J. (1988, Spring). *The History of Eduard Pernkopf's Topographische Anatomie des Menschen*. *J. Biomed Commun*, **1**, 2–12.
- Wiltschegg, W. (1985). *Die Heimwehr: Eine unwiderstehliche Volksbewegung?* . Wien: Verlag für Geschichte und Politik.
- Wistrich, R. S. (1992). *Austrians and Jews in the Twentieth Century: from Franz Joseph to Waldheim*. New York: St. Martin's Press .
- Wittgenstein, L. (1921). *Tractatus Logico-Philosophicus*.
- Wittgenstein, L. (1953). *Philosophical Investigations*. Blackwell Publishing
- Wolenski, J. (1999). *Alfred Tarski and the Vienna Circle*. Kluwer Acad. Publishing.
- Wolfowitz, J. (1951). *On ε-complete classes of decision functions*. *Annals of Math. Stat.*, **22**, 461–46.
- Wolfowitz, J. (1952). *Abraham Wald, 1902-1950*. *Annals of Mathematical Statistics*, **23**, 1–13.
- Wolfowitz, J. (1978 (3rd. Ed.)). *Coding Theorems of Information Theory*. New York: Springer Verlag.
- Wolfowitz, J. (1980). *Selected Papers*. New York: Springer Verlag.

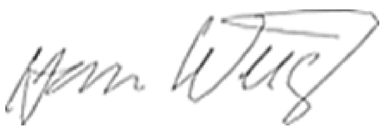
- Working, E. J. (1927). *What Do Statistical 'Demand Curves' Show?*. *Quarterly Journal of Economics*, **41**, 212–235.
- Zilsel Edgar. (1976). *Die sozialen Ursprünge der neuzeitlichen Wissenschaft* .Edt. Wolfgang Krohn, Frankfurt, Suhrkamp Verlag.
- Zweig, S. (1964). *The World of Yesterday: An Autobiography by Stefan Zweig*. Omaha: Univ. of Nebraska Press.

11 Erklärung / Declaration

Eidesstattliche Erklärung.
Hiermit erkläre ich an Eides statt, die vorliegende Dissertation selbst verfasst und keine anderen als die von mir im Literaturverzeichnis angegebenen Werke / Hilfsmittel benutzt zu haben.

Declaration on oath.
I hereby declare, on oath, that I have written the present dissertation by my own and have not used other than the acknowledged resources and aids.

Hamburg, 21.01.2013



Hans Weigl

12 Kurzzusammenfassung / Short Abstract

Kurzzusammenfassung

Obwohl AW als Wissenschaftler praktisch vergessen ist, hat seine Sequenzanalyse in letzter Zeit eine regelrechte Wiederauferstehung gefeiert, besonders in der Medizin und der klinischen Psychologie. Das war unser Ausgangspunkt, aber im Verlauf der Forschung zu AW sind wir auf einen neuen Aspekt gestoßen: AW's Bedeutung liegt gerade im Feld der Ökonometrie. Wir zeigen, dass er nicht nur ein Gründungsvater der Ökonometrie ist, sondern eine Schlüsselfigur für Nobelpreisträger wie Trygve Haavelmo war. Diese Rolle tritt zutage, wenn man alle Ebenen seiner wissenschaftlichen Karriere und seines persönlichen Lebens in Betracht zieht.

Um AW's ökonometrische Errungenschaften klassifizieren zu können haben wir eine Art Blaupause für eine Geschichte der mathematischen Ökonomie erarbeitet. Dies ist noch unerschlossenes Gebiet, das einiges an Forschungsarbeit weiterhin benötigt. In den 1930er Jahren wurde in die Ökonomie die Technik der mathematischen Modellierung eingeführt, eine Vorgehensweise, deren Entwicklung die darauf folgenden 20 Jahre bestimmt.

Die Ökonomie wurde in eine Disziplin umgeformt, die mit mathematischen Vorgehensweisen und Methoden arbeitet. Das Aufkommen der Ökonometrie in den 1930er Jahren war nicht zufällig. Die Weltwirtschaft war damals in derart ernsthaften Schwierigkeiten, dass die Ökonomen auf der Suche nach besseren Methoden zu deren Beschreibung waren.

Wirtschaftliche Schwächen nach dem ersten Weltkrieg und die Hyper-Inflation in Österreich in den 1920er Jahren liessen den Ruf laut werden nach einer neuen Wirtschaftspolitik. Die Liberalen um von Mises (Leiter der Finanzabteilung der Gewerbe- und Handelskammer) verloren an Einfluss und O. Morgenstern (als Nachfolger von von Mises) wurde zum Direktor des Instituts für Konjunkturforschung. Es war AW, der Morgenstern mathematisch ausgebildet hat und ihm dazu verhalf die Forschungsrichtung des Instituts radikal zu verändern und einen mathematischen Weg einzuschlagen. Morgenstern hat nie über die AWs Rolle gesprochen. Anhand Morgensterns' Tagebücher und seiner Korrespondenz beschreiben wir diese unbekannte Rolle von AW. Die spätere Zusammenarbeit von Morgenstern und John von Neuman wäre ohne AW nie möglich gewesen.

Das Sprungbrett für AW war Karl Menger, der Sohn des berühmten Carl Menger, dem Begründer der Österreichischen Schule der Nationalökonomie. Er machte AW, den armen Ostjuden, mit Bankiers wie Karl Schlesinger bekannt (der selber auch Schulungen von AW in Mathematik bekam) und O. Morgenstern (Kapitel 2 und 4). Die Entwicklung des unabhängigen österreichischen Faschismus (eine Tatsache, die von den meisten Historikern heute akzeptiert wird) wird in Kapitel 4 nachgezeichnet. Dort untersuchen wir vor allem, wie die Universitäten dem Ungeist anheimgefallen sind und AWs politische Positionierung. Es war am Ende die Cowles Commission, der AW sein Leben verdankt. Und vor allem waren es die Arbeiten von AW in mathematischer Ökonomie, die Cowles Interesse an ihm geweckt haben (Kap. 4).

Kapitel 6 analysiert neben der Klassifizierung von AW innerhalb der skizzierten Geschichte der Ökonometrie, die verschiedenen Perioden seiner ökonometrischen Arbeit und auch seine Auswirkungen auf Ragnar Frisch und Trygve Haavelmo. Es kann gezeigt werden, dass Haavelmo stark von AW beeinflusst wurde, eine Tatsache, die ihm bei die Entwicklung einer Alternative für die Ökonometrie an Stelle seiner Konfluenzanalyse geholfen hat. In den USA hat AW seine Karriere als Professor für Ökonomie begonnen und legte mit Studenten wie Milton Sobel oder Jacob Wolfowitz die Grundlage einer jahrzehntelangen Vorherrschaft der USA in der ökonometrischen Forschung.

Die Arbeit zeigt AW's Aufstieg in der mathematischen Ökonomie, der in den 1930er Jahren in Wien begann und in eine sehr produktive Zeit in den 1940er und zu Beginn der 1950er mündete nach seiner Emigraton in die USA. AW's Werk und Wirken auf Menschen wie Oskar Morgenstern und Trygve Haavelmo macht ihn zu einem erstklassigen Protagonisten, der den Boden für Nobelpreisträger bereitet hat. Wir verfolgen sein persönliches Schicksal hin zur mathematischen Ökonomie, in der er sich als ein Glücksfall für die Wissenschaft erwiesen hat. Ein Mann mit solchen Talenten und mathematischen Fähigkeiten war in der Lage eine neue Art von Wissenschaft mitzubegründen, mit der die 1930er Jahre bereits schwanger gingen: die Ökonometrie.

Short Abstract

Although AW is almost forgotten his Sequential Analysis regained a resurrection during the last years, especially in medicine and clinical

psychology. [Krohne (2007)] In the course of our thesis a new aspect became visible. AW's main relevance lies in the field of econometrics. We found that he indeed is a founding father of econometrics and a key figure and trigger for Nobel Price laureates like Trygve Haavelmo. This role came into daylight, when all necessary 'layers' of his career and life were brought together.

To classifiy AW's econometric achievements we had to work out a basic blueprint for the history of mathematical economics. This is still an open field that not yet is finished and still needs more research to be done. During the 1930s, the technology of mathematical modeling was introduced, a process that spanned over the following 20 years. Economics became reshaped into a tool-based discipline. The coincidence of the upcoming econometrics within the 1930s did not happen by accident. The world economy at that time was seriously malfunctioning so that economists were in search for the right methods.

Economic weakness after WWI and the hyper-inflation in Austria in the 1920s asked for a new economic policy. The liberals around von Mises (the secretary of the Chamber of Commerce) lost their momentum and O. Morgenstern (following v. Mises) became the director of the Institute for Business Cycle Research. It was AW, who trained Morgenstern and helped him to change radically the course of the Institutes research into a mathematically controlled discipline. Morgenstern never had spoken about AW's role. Analysing Morgenstern's diary and correspondence we describe the unknown role of AW in that process. The later collaboration of Morgenstern and John von Neuman would never been have possible without AW.

The jumping-off point for AW was Karl Menger, the son of the famous Carl Menger, the founder of the Austrian School of Economics. He was the gate for AW, the poor eastern Jew, to bankers like Karl Schlesinger (who got also trainings from AW in mathematics) and O. Morgenstern (Chs. 2 and 4). The development of the independent Austrian fascism (a fact that is accepted by most of the historians today) is shown in Ch. 4., where we especially investigate the university's tipping into the bad mind and AW's positioning. It was the Cowles Commission that saved AW's life in the end. And especially it were AW's papers in mathematical economics that sparked Cowles interest in him. (Ch. 4)

Ch. 6 analyzes besides the classification of AW in a sketched history of econometrics, AW's different periods in econometric work and also his impact on Ragnar Frisch and Trygve Haavelmo. It can be shown, that Haavelmo was highly influenced by AW, a fact that let him develop an alternative framework for econometrics instead of his former confluence analysis. In the US AW had started his career as a professor for economics and laid with students like Milton Sobel or Jacob Wolfowitz the groundwork of a decade-ranging predominance of the US in econometric research.

The whole text shows AW's rising in mathematical economics that began in the 1930s in Vienna, blossoming up into a very productive period during the 1940s and mainly the beginning 1950s after his emigration to the US. AW's work and influence on people like Oskar Morgenstern and Trygve Haavelmo makes him a first class protagonist who laid ground and path for Nobel Prize Winners. We trace his constant personal fate towards mathematical economics that turned out to be a godsend in science. A man of his talents and mathematical abilities was able to form out a new kind of science, of which the 1930s already were partially pregnant: the econometrics.

13 Curriculum Vitae

Persönliche Daten / Personal Data

Hans Weigl

*10.3.1968, Mering (nearby Augsburg)

Deutsch / German

Schulausbildung / Education

Grundschule / Primary School Walleshausen, Landsberg a. Lech, 1978

Gymnasium / Secondary School St. Ottilien, Landsberg a. Lech, 1987

Diplom in Mathematik / Diploma in Mathematics, LMU Munich, 1994

Berufserfahrung / Work Experience

Technical Analyst, Hornblower Fisher, Munich — 1997-1999
Technical Analyst, Allfonds BKG, Munich — 1999 - 2001
Technical Analyst, Activest Munich (Pioneer), Munich — 2001 - 2003
Portfolio Manager Total Return, Bayr. Landesbank, Munich — 2004 - 2005
Manager, VA Engineering GmbH, Walleshausen — since 2006