Soft breaking terms from $N = 2$ Supergravity

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I, Luca Tripodi, hereby declare, on oath, that I have written the present dissertation titled ‘Soft breaking terms from $N = 2$ Supergravity’ by my own and have not used other than the acknowledged resources and aids.

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The effort to understand the universe is one of the very few things which lifts human life a little above the level of farce and gives it some of the grace of tragedy.

Steven Weinberg

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Abstract

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In this thesis we study the rigid limit of $N = 2$ supergravity and we perform a model-independent analysis of the supersymmetry breaking terms arising from a gauged $N = 2$ supergravity spontaneously broken to $N = 1$. We find that at least when no visible hypermultiplet is present, the global $N = 2$ theory is softly broken by $N = 1$ supersymmetric soft terms.

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Contents

Chapter 1

Introduction

Even if there is no clear experimental signature of supersymmetric theories at the present day [1], supersymmetry is still a central concept in modern theoretical physics. As the only possible loophole to the Coleman-Mandula theorem [2], it provides a unique extension of the Poincaré symmetry while being both physically and mathematically rich. The best candidate for a theory-of-everything, string theory, is supersymmetric at the core, and advances in several mathematical areas were triggered or inspired by supersymmetry. Supergravity as well, nowadays seen as a low-energy limit of a more fundamental theory such as string theory, arises from the attempt to build a supersymmetric version of general relativity, or - viewed from the supersymmetric side - to promote global supersymmetry to a local symmetry, and yields new geometrical structures in the process. But how much supersymmetry can we have and do we need? The most common choice for phenomenological purposes is $N = 1$ (four scalar supersymmetric charges), which helps realizing the grand-unification of the Standard Model gauge couplings, can offer stable candidates for the dark matter particles and alleviates the hierarchy problem. Increasing the number of supersymmetries the resulting theories become more mathematically constrained and in general less attractive from a phenomenological point of view, since they do not contain chiral matter. Starting from the maximum number of supersymmetries, one can construct $N = 8$ maximal supergravities in four dimensions, which have 32 supersymmetries (and can also exist in at most eleven dimensions with $(10, 1)$ Lorentzian signature). These theories, however, do not possess matter multiplets[1], and all the geometric properties are completely determined once the space-time dimension is fixed. On the other hand, matter multiplets become possible if the number of supersymmetries is limited to 16, e.g. N = 4 in four dimensions, and in this case the geometry is determined

[1]Here with "matter multiplets" we mean both vector and hypermultiplets.

by the number of vector multiplets coupled to supergravity. When the number of supersymmetries is eight (i.e. $N = 2$ supersymmetry), the geometry becomes more flexible and in the vector multiplet sector, for instance, it can be related to holomorphic functions called *prepotentials*. In $N = 1$ supersymmetry the set of possible geometries is even larger: e.g, in four dimensions we deal with general Kähler manifolds, but in the $N = 2$ vector multiplet sector this class is restricted to the "special Kähler manifolds", entirely determined by the prepotential. In this perspective, $N = 2$ theories hold a special place, since they can be regarded as the maximal supersymmetric theories whose geometry is not fully constrained by the number of matter superfields.

In any case, since superpartners are not observed at the current energy reach of particle collider experiments, even the minimum amount of supersymmetry - if it is realized in nature - must be broken. Thus one might be seduced by the idea that at very high energies a large amount of symmetry becomes manifest and in particular an extended supersymmetry is realized and then broken at lower energies. Nevertheless, breaking completely, for instance, $N = 2$ supersymmetry to $N = 0$ is not eligible from a phenomenological point of view since the breaking of $N = 2$ by itself does not provide chiral matter. One possibility is to consider an extended supersymmetry only in a gauge sub-sector and insist on $N = 1$ for the chiral matter sector [3]. Furthermore, a supersymmetry cascade as $N = 2 \to N = 1 \to N = 0$, where the supersymmetries are broken at two different energy scales, was proposed in [4]. With this strategy, the hope is that chirality might be restored at low energies via some mechanism (e.g. [5]). While the step from $N = 1$ to $N = 0$ is widely explored in the literature, that from $N = 2$ to $N = 1$, albeit already extensively studied, might be relevant as well and deserves further investigations. Trying to construct models of spontaneously broken extended supersymmetry in a Lorentz invariant background usually leads to theories in which the supersymmetries are completely broken. The discussion on the possibility of partial $N = 2 \to N = 1$ spontaneous supersymmetry breaking in four space-time dimensions was triggered at the beginning by string theory [6, 7], but at that time also heavily discouraged by a no-go theorem [8, 9] that forbids the partial breaking for purely electric gaugings. Therefore, counterexamples have been found only several years later: in [10] it was shown that any $N = 1$ matter can be made $N = 2$ supersymmetric with the help of the Goldstone multiplet of a partially broken $N = 2$ theory, while in [11] it was found that a magnetic Fayet-Iliopoulos term triggers a partial $N = 2 \to N = 1$ breaking. Extensions to special classes of gauged supergravity were presented in [12–14], where the no-go theorem was avoided choosing a particular basis for the vector multiplet scalars. The first attempt to draw a general picture was [15], for three dimensions, while a four-dimensional systematic analysis was finally given in [16, 17].

In general it was shown that in order to avoid the no-go theorem forbidding partial spontaneous $N = 2$ SUSY breaking one has to consider special frames (as we shall see later) or introduce magnetic charges in the theory. The most suited formalism to deal with both electric and magnetic charged fields is the embedding tensor formalism introduced in [18, 19]. The general solution for the embedding tensor inducing a consistent partial breaking in a Minkowski or AdS vacuum was found in [16], and in [17] was derived the effective $N = 1$ lagrangian integrating out all the fields becoming massive after the breaking.

The goal of this thesis is different. In the same spirit of [20], which performed a model-independent analysis of the SUSY-breaking terms (free of quadratic ultra-violet divergences, usually referred as *soft terms*) in $N = 1$ supergravity, we aim to derive the general form of the soft terms that explicitly break a generic global $N = 2$ supersymmetric theory to $N = 1$, by performing the global limit of an arbitrary spontaneously broken $N = 2$ supergravity. In fact, our starting point will be the scalar potential of a general $N = 2$ gauged supergravity in a four-dimensional $N = 1$ Minkowski vacuum provided by the solutions for the embedding tensor [16], and instead of integrating out all the massive fields as in [17], we retain all of them and evaluate the rigid limit by taking the Planck mass $M_{\rm Pl}$ to infinity. However, the most direct global limit of an arbitrary $N = 2$ supergravity (consisting of taking only $M_{\rm Pl}$ to infinity) merely leads to the global $N = 2$ supersymmetric counterpart even if we start from an $N = 1$ vacuum. In order to get a softly-broken $N = 2$ global theory, then, we need to perform the limit keeping an intermediate scale fixed, namely the mass scale of the gravitino related to the broken supersymmetry, which becomes massive in the partial breaking. In this way we are able to reach a $N = 2$ rigid theory *explicitly* broken by soft terms whose form is entirely determined by the original $N = 2$ supergravity. Another crucial ingredient in our work, inspired by string theory, is the separation of the fields in two distinct sectors: an hidden sector, which contain all the gravitational interactions and whose scalars can have a background value of the order of the Planck mass $M_{\rm Pl}$, and a visible sector, which encodes all the non-trivial gauge dynamics and whose scalar backgrounds are much smaller than the Planck mass. Even if this distinction can in general only be made locally, in the partial breaking scenario we are merely interested in a small region of the moduli space where we have an $N = 1$ vacuum and the breaking occurs. This separation in two sectors helps elucidating the rigid limit procedure in general, but is also essential in our derivation of the $N = 2$ soft terms: indeed, in order to keep the gravitino mass fixed in the limit, we assume that the gauge group of the hidden sector has a gauge coupling g' whose limit $g' \to 0$ compensates that on the Planck mass ($M_{\text{Pl}} \to \infty$).[2] The results we get for the $N = 2$ soft terms resemble those of [20], with some important distinctions due to the different geometries, the presence of the hypermultiplets and the fact that our explicitly broken scalar potential must be $N = 1$ supersymmetric and it should be possible to reorganize it in terms of a standard $N = 1$ form.

This thesis is organized as follows:

In Chapter 2 we review some general topics related to global $N = 2$ supersymmetry and $N = 2$ supergravity, with a main focus on the bosonic lagrangians and on the geometries involved in the scalar sectors: the special Kähler manifolds and the quaternionic Kähler manifolds. Then we summarize the structure of the gauged $N = 2$ supergravity and the embedding tensor formalism.

In Chapter 3 we introduce the idea of the local separation between hidden and visible fields and we explain how to perform the rigid limit of general ungauged and gauged $N = 2$ supergravities, following our paper [21]. We shall also show how the local geometric structures flow to their global counterparts and how the hidden sector becomes decoupled from the visible one.

In Chapter 4 we review the ways in which a partial supersymmetry breaking can occur in $N = 2$ supergravity, presenting the specific example given in [12] and then summarizing the partial breaking in the general case, following [16]. The results reviewed here shall be left implicit in the following chapters.

In Chapter 5 we tackle the soft $N = 2$ supersymmetry breaking problem using the knowledge from the previous chapters. We focus on the scalar potential of the gauged lagrangian and we analyse separately the contributions coming from the gravitino and the gaugino parts. Furthermore, we show at least in the simple scenario in which there is no visible hypermultiplet that in the global limit our theory is $N = 1$ supersymmetric.

In Chapter 6 we recapitulate the results of our work and we propose some future directions of research.

Appendix A we present our conventions and some useful formulas for $N = 2$ supergravity used in this thesis; in Appendix B it is shown that it is impossible to realize a partial supersymmetry breaking in a Minkowski background when an intermediate field scale between the Planck scale and the visible one is assumed;

[2]This formal procedure is useful to retain the leading order in the gravitino mass, but is not to be taken literally from a physical point of view.

finally, in Appendix C we collect several detailed computations used in the $N = 2$ soft breaking chapter.

Chapter 2

$N = 2$ supergravity: a review

In this chapter we review $N = 2$ supergravity coupled to an arbitrary number of vector multiplets and hypermultiplets. Different constructions exist in the literature: one of the first [22–24] is based on the so-called superconformal multiplet calculus, is entirely off-shell and allows to derive the $N = 2$ Poincaré supergravity by starting from the $N = 2$ superconformal gravity and gauge fixing the superconformal symmetry. A more recent on-shell approach [25–27] is the "geometric formulation" or "intrinsic formulation", which allows a description in terms of symplectic section on the scalar space without making explicit use of a prepotential. While the latter is well suited for very special cases in which a prepotential does not exist, the former helps elucidating the structures underlying the theory while relying on the holomorphic prepotential and the so called "special coordinates" (both crucial ingredients in our treatment of the rigid limit and the partial breaking). We shall start our review with global $N = 2$ supersymmetry.

2.1 Global $N = 2$ supersymmetry in four space-time dimensions

The maximal space-time dimension for theories with 8 real supercharges is six. It is possible in fact to derive many features of the four-dimensional theory starting with the six-dimensional one and then doing a dimensional reduction (see e.g. [28, 29] for a review). In four space-time dimensions the supersymmetry generators are

Majorana spinors (two for $N = 2$), but it is also possible to use Weyl spinors[1], and both descriptions can be found in the literature.

The Weyl spinors Q^A_α and $\bar{Q}_{\dot{\beta}B}$ representing the supersymmetry generators satisfy the anticommutation relation [30]

$$\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A_B, \tag{2.1}$$

where we used the usual Van der Waerden notation [31] for chiral and antichiral indices and for $N = 2$ we have $A, B = 1, 2$. In absence of central charges in the algebra, it is possible to interpret the supersymmetry generators as creation and annihilation operators acting on a vacuum state of a certain helicity, and with the additional requirement of CPT-invariance one can then derive the spectrum of the massless $N = 2$ supersymmetry, consisting of a vector multiplet and a hypermultiplet [32]. Since the on-shell components for the massless fields are representations of the little group $SO(D-2)$ ($SO(2)$ for $D = 4$) and of the $SU(2)$ related to the extended SUSY R-symmetry, the vector multiplet and the hypermultiplet can be organized as follows:

A_μ	λ^A_α	t_1	t_2	vector multiplet
(2,1)	(2,2)	(1,1)	(1,1)	
ζ_α	ζ'_α	q^A_1	q^A_2	hypermultiplet
(2,1)	(2,1)	(1,2)	(1,2)	

The numbers in parentheses indicate the representation with respect to $SO(2)$ and $SU(2)$ respectively. The index $\mu = 0, ..., 4$ is the Lorentz index for the vectors, $\alpha = 1, 2$ is the spinor index and $A = 1, 2$ runs over the two supersymmetries. Each vector multiplet has a vector, two Weyl spinors (the *gauginos*) and a complex scalar (in the table t_1 and t_2 are its real and imaginary parts). The hypermultiplet consists of two Weyl spinors (the *hyperinos*) and two complex scalars (four real scalars in total). Neglecting the fermionic parts, the lagrangian for an abelian (ungauged) $N = 2$ global supersymmetric theory with n_v vector multiplets and n_h hypermultiplets reads [26]

$$\mathcal{L} = i(\bar{\mathcal{N}}_{IJ} F^{I-}_{\mu\nu} F^{J-|\mu\nu} - \mathcal{N}_{IJ} F^{I+}_{\mu\nu} F^{J+|\mu\nu}) + \tilde{g}_{i\bar{j}} \partial_\mu t^i \partial^\mu \bar{t}^{\bar{j}} + \tilde{h}_{uv} \partial^\mu q^u \partial_\mu q^v, \tag{2.2}$$

[1]Every Majorana spinor field can be expressed in terms of a Weyl field and its complex conjugate.

where the indices $I, J = 1, ..., n_v$ run over the vectors and $i, j = 1, ..., n_v$ over the complex scalars, while $u, v = 1, ..., 4n_h$ denote the real scalars of the hypermultiplets; $F^+_{\mu\nu}$ and $F^-_{\mu\nu}$ are defined in terms of the dual and anti-dual gauge field strengths as

$$\begin{aligned} F^{\pm}_{\mu\nu} &= \frac{1}{2}(F_{\mu\nu} \pm \tilde{F}_{\mu\nu}), \\ \text{with} \quad \tilde{F}_{\mu\nu} &= -\frac{1}{2} i\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \end{aligned} \tag{2.3}$$

The scalar field space is a direct product of the vector and the hypermultiplet sectors:

$$M = M_v \times M_h. \tag{2.4}$$

Global supersymmetry requires M_v to be a *rigid special Kähler* space and its metric $\tilde{g}_{i\bar{j}}$ is defined via the Kähler potential K as

$$\tilde{g}_{i\bar{j}} = \partial_i \partial_{\bar{j}} K = \partial_i \partial_{\bar{j}} \left[i t^k \bar{F}_k(\bar{t}) - i \bar{t}^k F_k(t) \right] = 2 \, \mathrm{Im} F_{ij}. \tag{2.5}$$

With $F_{k..l}(t)$ we indicate the nth derivative of the holomorphic *prepotential* F. The metric $\tilde{h}_{uv}$ of the hypermultiplet scalar field space M_h describes instead a *hyperKähler* space. We shall give more details about these geometries in the context of supergravity and its rigid limit. The matrix $\mathcal{N}_{IJ}$ encodes the gauge couplings, and in global $N = 2$ takes the form

$$\mathcal{N}_{ij} = \bar{F}_{ij}. \tag{2.6}$$

If we now require the two supersymmetries to be local, we reach the $N = 2$ supersymmetric generalization of Einstein gravity coupled to matter, namely $N = 2$ supergravity, which is the starting point of this thesis.

2.2 $N = 2$ (ungauged) supergravity bosonic lagrangian

In addition to the n_v vector multiplets and the n_h hypermultiplets of the global $N = 2$ theory, $N = 2$ supergravity also contains a gravity multiplet. In the following table we summarize the field content:

$g_{\mu\nu}$	ψ^A_μ	A^0_μ	gravity multiplet
A_μ	λ^A	t	vector multiplet
	ζ^A	$q^{\hat{u}}$	hypermultiplet

Here we suppressed the spinor indices for clarity and we left only the Lorentz indices and the index A running over the two supersymmetries. The gravity multiplet contains a graviton, two gravitini (one for every supersymmetry) and a vector (the *graviphoton*). As in the global case, the vector multiplet has one vector, two gaugini and a complex scalar, while the hypermultiplet consists of two hyperini and four real scalars ($\hat{u} = 1, ..., 4$). The gravitons are symmetric traceless tensors and have two components (in four dimensions); the gravitini, γ-traceless vector-spinors[2], have two components as well; finally, the vectors and the spinors have also two components each (for a more detailed review see e.g. [26, 28, 29]).

The general bosonic lagrangian for an ungauged theory is given by

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2\kappa^2}R + \tfrac{1}{4}\mathrm{Im}\mathcal{N}_{IJ}(t,\bar{t})\, F^I_{\mu\nu}F^{\mu\nu\, J} - \tfrac{1}{8}\mathrm{Re}\mathcal{N}_{IJ}(t,\bar{t})\, \varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma} \\ &\quad - g_{i\bar{\jmath}}(t,\bar{t})\, \partial_\mu t^i \partial^\mu \bar{t}^{\bar{\jmath}} - h_{uv}(q)\, \partial_\mu q^u \partial^\mu q^v ,\end{aligned} \tag{2.7}$$

where R is the scalar curvature and $\kappa = \frac{1}{8\pi M_{\mathrm{Pl}}}$. The vector index I runs now from 0 to n_v, since we need to take into account the graviphoton as well. Moreover, in $N = 2$ supergravity, the scalar field space (2.4) displays different geometries with respect to the global case: M_v is required to be a *local* special Kähler space, while M_h must be a quaternionic space. $g_{i\bar{\jmath}}(t,\bar{t})$ is then the (positive definite) metric on M_v and h_{uv} is the metric on M_h. $\mathrm{Im}\mathcal{N}_{IJ}$ is a negative definite matrix dependent on the scalar fields whose vacuum expectation values give the gauge coupling constants, while the vacuum expectation values of $\mathrm{Re}\mathcal{N}_{IJ}$ are the so-called theta angles. The explicit form of $\mathcal{N}_{IJ}$ in $N = 2$ supergravity will be given later.

Despite the complexity of its details, a generic $N = 2$ supergravity theory can be uniquely defined if only three ingredients are given [26], namely:

a) the specific local special Kähler manifold describing the vector scalar space M_v;
b) the specific quaternionic Kähler manifold describing the hypermultiplet space M_h;
c) the gauge group $\mathcal{G}$, that in the non-abelian case is a subgroup of the isometry group G of the scalar field space M.

Before reviewing in some detail the two geometries of the vector and hypermultiplet scalar sectors, we need to introduce another concept related to the symmetry group of the equations of motion one can derive from (2.7), which will be crucial in the following: the symplectic transformations. Indeed, the isometry group G is embedded in the larger symplectic group of the equations of motion, and

[2] The condition for irreducible vector-spinors is $\Gamma^\mu \psi^A_\mu = 0$

the formalism we are going to use in the next chapters will make this situation manifest.

2.3 Symplectic transformations and special Kähler geometry

The symplectic transformations can be considered generalizations of the electromagnetic dualities [33]. Let us consider the kinetic terms of the vector fields for an arbitrary number n_v of vectors. If we define

$$G^{\mu\nu}_{+I} \equiv 2i\frac{\partial\mathcal{L}}{\partial F^{+I}_{\mu\nu}} = \mathcal{N}_{IJ}F^{+J}_{\mu\nu} \tag{2.8}$$

as a magnetic field strength, the Bianchi identities and the field equation can be casted respectively as

$$\begin{aligned} \partial^{\mu}\mathrm{Im}F^{+I}_{\mu\nu} &= 0, \\ \partial_{\mu}\mathrm{Im}G^{\mu\nu}_{+I} &= 0. \end{aligned} \tag{2.9}$$

These equations are left invariant under the action of $GL(2(n_v+1),\mathbb{R})$:

$$\begin{pmatrix} \tilde{F}^+ \\ \tilde{G}_+ \end{pmatrix} = \mathcal{S}\begin{pmatrix} F^+ \\ G_+ \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} F^+ \\ G_+ \end{pmatrix}, \quad \mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{2.10}$$

If we now restrict the transformations using (2.8), we have

$$\tilde{G}^+ = (C+D\mathcal{N})F^+ = (C+D\mathcal{N})(A+B\mathcal{N})^{-1}\tilde{F}^+, \tag{2.11}$$

that is

$$\tilde{\mathcal{N}} = (C+D\mathcal{N})(A+B\mathcal{N})^{-1}. \tag{2.12}$$

$\tilde{G}_{+\mu\nu}$ should be the derivative of a transformed lagrangian in order to still describe the field equations, and this in particular means that $\tilde{\mathcal{N}}$ should be symmetric. For an arbitrary $\mathcal{N}$ this implies the conditions:

$$A^TC - C^TA = 0, \quad B^TD - D^TB = 0, \quad A^TD - C^TB = \mathbb{1}; \tag{2.13}$$

in other words

$$\mathcal{S} \in \mathrm{Sp}(2(n_v+1),\mathbb{R}), \tag{2.14}$$

with

$$\mathcal{S}^T\Omega\mathcal{S} = \Omega \quad \text{where} \quad \Omega = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}. \tag{2.15}$$

Therefore, the constrained transformations are real symplectic transformations in dimension $2(n_v + 1)$. A symplectic vector is defined as the $2(n_v + 1)$-component object V transforming under the symplectic transformation as $\tilde{V} = \mathcal{S}V$. We also define the inner product of two symplectic vectors V and W as

$$V^\Lambda W_\Lambda \equiv V^T\Omega W, \tag{2.16}$$

where $\Lambda = 0, ..., 2n_v$.

It should be noted that the symplectic transformations are symmetries of the equations of motion, but *not* of the action.

Let us turn now to the geometry of the vector scalar space M_v. Since M_v is a Kähler manifold, the metric $g_{i\bar{j}}(t, \bar{t})$ can be expressed in terms of the Kähler potential K as

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K, \tag{2.17}$$

where $\partial_i = \frac{\partial}{\partial t^i}$. The metric remains the same after the Kähler transformation

$$K \to K + f(t) + \bar{f}(\bar{t}). \tag{2.18}$$

Moreover, M_v is also (local) *special* Kähler, and this means the Kähler potential can be expressed as a logarithm of a symplectic product [23, 24] (for a review of special Kähler geometry, see e.g. [34]):

$$K(t, \bar{t}) = -\ln i(\bar{V}^\Lambda V_\Lambda) \equiv -\ln i(\bar{X}^I F_I - X^J \bar{F}_J), \tag{2.19}$$

where

$$V^\Lambda = \begin{pmatrix} X^I \\ F_I \end{pmatrix} \tag{2.20}$$

is a $2(n_v + 1)$-dimensional symplectic vector, with $I = 0, \cdots, n_v$. $X^I(t)$ are holomorphic functions of the vector scalars t and are referred to as *homogeneous coordinates*, and $F_I = \partial F / \partial X^I$ is the derivative of the prepotential $F(X)$, which supersymmetry constrains to be homogeneous of degree two in the homogeneous coordinates. The domain of the complex scalars t^i is then restricted by the condition

$$i(\bar{V}^\Lambda V_\Lambda) > 0, \tag{2.21}$$

defining the "Kähler cone", and by the requirement that the metric should be positive definite. Moreover, in special Kähler geometry the following relation holds:

$$X^I \tilde{\nabla}_i F_I - F_I \tilde{\nabla}_i X^I = 0, \tag{2.22}$$

where $\tilde{\nabla}_i = \partial_i + \frac{1}{2M_{\rm Pl}^2}\partial_i K$ is the Kähler covariant derivative.[3] Furthermore, since F_I plays the role of the magnetic component of the symplectic vector V, the matrix $\mathcal{N}_{IJ}$ satisfies

$$F_I = \mathcal{N}_{IJ} X^J, \quad \tilde{\nabla}_i F_I = \mathcal{N}_{IJ} \tilde{\nabla}_i X^J, \tag{2.23}$$

from which one can solve the explicit expression

$$\mathcal{N}_{IJ} = (\tilde{\nabla}_{\bar{j}} \bar{F}_I(\bar{X}) \quad F_I(X))(\tilde{\nabla}_{\bar{j}} \bar{X}^J \quad X^J)^{-1}, \tag{2.24}$$

where an outer product of the two vectors is intended. Applying a symplectic transformation on the symplectic vector V, one gets a new coordinate $\tilde{X}$ as a function of the old X:

$$\tilde{X}^I = A^I{}_J X^J + B^{IJ} F_J(X). \tag{2.25}$$

The full symplectic transformation (2.10) described by the matrix $\mathcal{S}$ is always invertible, but this part related to the sub-matrices A and B may not be. But if it is invertible, then $\tilde{F}_I(\tilde{X})$ becomes the derivative of a new function $\tilde{F}(\tilde{X})$ of the new coordinates $\tilde{X}$:

$$\tilde{F}_I(\tilde{X}) = \frac{\partial \tilde{F}(\tilde{X})}{\partial \tilde{X}^I}. \tag{2.26}$$

Thanks to the homogeneity of the prepotential in supergravity, $\tilde{F}$ can be obtained as

$$\tilde{F}(\tilde{X}(X)) = \frac{1}{2} V^T \begin{pmatrix} C^T A & C^T B \\ D^T A & D^T B \end{pmatrix} V. \tag{2.27}$$

What we get is a new formulation of the theory in terms of a new prepotential $\tilde{F}$. If $\tilde{F}$ is different from $F(\tilde{X})$ the two theories are only *classically* equivalent; in this case we are dealing with a "pseudo symmetry" and the transformations are called symplectic reparametrizations. On the other hand, if after the symplectic transformations the function F remains the same, we can talk about a *proper symmetry* that reflects an isometry of the target space. These symmetries are called *duality symmetries* because are connected to the duality transformations of the field equations and Bianchi identities. The isometry group of the scalar field

[3] $\tilde{\nabla}$ differs from the ∇ only for the factor $1/2$ in the second term. We shall use both notations in the following.

space M is in general a subspace of the symplectic group:

$$\text{Iso}(M) \in \text{Sp}(2(n_v+1), \mathbb{R}). \tag{2.28}$$

After a symplectic transformation, the dual formulation has generically different symmetries in perturbation theory and there are known examples in which a prepotential does not even exist in that symplectic frame. In particular, these three conditions are equivalent [34]:

1. A prepotential $F(X)$ does exist.
2. Special coordinates are possible; these are defined in terms of the homogeneous coordinates by

$$t^i = \frac{X^I}{X^0} \quad \text{or} \quad X^0 = M_{\text{Pl}}, \;\; X^I = t^i. \tag{2.29}$$

3. The matrix $\begin{pmatrix} X^I & \tilde{\nabla}_i X^I \end{pmatrix}$ is invertible.

If these conditions hold, namely we are in a certain symplectic frame in which a prepotential exists, the matrix $\mathcal{N}_{IJ}$ can be put in the form

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i \frac{\text{Im}F_{IK}X^K \text{Im}F_{JL}X^L}{X^L \text{Im}F_{LK}X^K} \tag{2.30}$$

(compare (2.6) for the global case). Moreover, it is possible to express the holomorphic prepotential F by means of an homogeneous function $\mathcal{F}$, as

$$F = (X^0)^2 \mathcal{F}\left(\frac{X^i}{X^0}\right) = \kappa^{-2}\mathcal{F}(t^i), \tag{2.31}$$

such that the Kähler potential (2.19) can be expressed as

$$K^{\text{v}} = -\kappa^{-2} \ln \mathrm{i}\, Y \;, \qquad Y = 2(\mathcal{F} - \bar{\mathcal{F}}) - (t-\bar{t})^i(\mathcal{F}+\bar{\mathcal{F}})_i \;. \tag{2.32}$$

Finally, the curvature tensor $R_{i\bar{j}k\bar{l}}$ for a local special Kähler manifold reads [35]

$$R_{i\bar{j}k\bar{l}} = g_{\bar{l}k}g_{i\bar{j}} + g_{\bar{l}i}g_{k\bar{j}} - M_{\text{Pl}}^2 e^{\frac{2K}{M_{\text{Pl}}^2}} Q_{ikm} g^{m\bar{n}} \bar{Q}_{\bar{n}\bar{l}\bar{j}}, \tag{2.33}$$

where (see also [36])

$$Q_{ikm} = (\nabla_i \nabla_k V^\Lambda)(\nabla_m V_\Lambda), \tag{2.34}$$

and $\nabla_i = \partial_i + \frac{K_i}{M_{\text{Pl}}^2}$. In special coordinates, Q_{ikm} simply becomes

$$Q_{ikm} = F_{ikm}, \tag{2.35}$$

where F_{ikm} is the third derivative of the prepotential F.

2.4 Quaternionic Kähler geometry

The metric h_{uv} in (2.55) is required by supersymmetry to describe a quaternionic-Kähler space. Despite the name, such manifolds are in general non Kähler and have holonomy

$$SU(2) \times Sp(n_h), \tag{2.36}$$

where the $SU(2)$, related to the R-symmetry, promotes the hyperkähler geometry of the hypermultiplets scalar of rigid $N = 2$ supersymmetry (holonomy $Sp(n_h)$) to the quaternionic geometry. These spaces admit a triplet of almost complex structure J^x $(x = 1, 2, 3)$ which satisfies the quaternionic algebra:

$$J^x J^y = -\delta^{xy} + \epsilon^{xyz} J^z. \tag{2.37}$$

The metric h_{uv} is hermitean with respect to each complex structure, and three Kähler 2-forms can be defined by

$$K^x_{uv} = h_{uw}(J^w_v)^x. \tag{2.38}$$

ahler manifolds case these forms are closed; in the quaternionic case of supergravity they are covariantly closed with respect to the $SU(2)$ connection ω^x:

$$\nabla K^x \equiv dK^x - \epsilon^{xyz}\omega^y \wedge K^z = 0. \tag{2.39}$$

K^x can then be viewed as a the $SU(2)$ field strength of the connection ω^x, namely:

$$K^x = \mathrm{d}\omega^x + \tfrac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z \ . \tag{2.40}$$

There is a linear relation between the K¨ $SU(2)$ curvature $\Omega^x \equiv d\omega^x - \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z$ defined by

$$\Omega^x = \lambda K^x, \tag{2.41}$$

where λ is a constant that is identically zero in the hyperkähler case and non-zero for quaternionic manifolds. The Ricci scalar curvature is always negative for quaternionic spaces and is given (in natural units) by $R = -8n_h(n_h + 2)$. Finally, we can decompose the metric in terms of vielbein on the quaternionic-Kähler manifold $\mathcal{U}^A_\alpha$ as

$$h_{uv} dq^u dq^v = \mathcal{U}^A_\alpha \epsilon_{AB} \mathcal{C}^{\alpha\beta} \mathcal{U}^B_\beta , \tag{2.42}$$

where $\mathcal{C}^{\alpha\beta}$ is the $Sp(n_h)$ invariant metric.

2.4.1 The special quaternionic Kähler manifold and the c-map

As an explicit example of quaternionic Kähler manifolds we consider a well known case in string theory, where the compactifications of type II strings at tree level lead to a $\mathbf{M}_{\rm h}$ that is the image of the so-called c-map [37, 38]:

$$\frac{SU(1,1)}{U(1)} \times \mathbf{M}^{2n-2}_{SK} \to \mathbf{M}^{4n}_{QK} . \tag{2.43}$$

$\mathbf{M}^{2n-2}_{SK}$ is a special Kähler base with coordinates $z^a, a = 1, \dots, n_{\rm h} - 1$, the $\frac{SU(1,1)}{U(1)}$ factor is spanned by the dilaton ϕ and the axion $\tilde{\phi}$, and the remaining fields are $2n_{\rm h}$ real Ramond-Ramond scalars $\xi^A, \tilde{\xi}_A, A = 0, \dots, n_{\rm h} - 1$ describing the toroidal fibration of the base that doubles its coordinates. Because $\mathbf{M}^{2n-2}_{SK}$ is a special Kähler manifold, its prepotential G fully determines the quaternionic manifold, that in this case takes the name of *special* quaternionic Kähler. An explicit form of the metric (known as Ferrara-Sabharwal metric) was given in [38] and reads:

$$\begin{aligned} \mathcal{L} \; = \; & -(\partial\phi)^2 - e^{4\kappa\phi}(\partial\tilde{\phi} + \kappa\tilde{\xi}_A \partial\xi^A - \kappa\xi^A\partial\tilde{\xi}_A)^2 + g_{a\bar{b}}\partial z^a \partial\bar{z}^{\bar{b}} \\ & - e^{2\kappa\phi} \mathrm{Im}\mathcal{M}^{AB-1}(\partial\tilde{\xi} + \mathcal{M}\partial\xi)_A(\partial\tilde{\xi} + \bar{\mathcal{M}}\partial\xi)_B \; . \end{aligned} \tag{2.44}$$

$g_{a\bar{b}}$ is the special Kähler metric on $\mathbf{M}_{\rm sk}$ and is again determined by the prepotential $G(Z)$ (compare (2.19) with F replaced by G). The kinetic matrix determining the couplings is also analogous to (2.30) with F replaced by G:

$$\mathcal{M}_{AB} = \bar{G}_{AB} + 2\mathrm{i}\, \frac{\mathrm{Im}(G_{AC})Z^C \mathrm{Im}(G_{BD})Z^D}{\mathrm{Im}(G_{AC})Z^A Z^C} \; , \tag{2.45}$$

where $Z^A = (M_{\rm Pl}, z^a)$ are the homogeneous coordinates on $\mathbf{M}_{\rm sk}$.

2.5 Gauging the action: the embedding tensor formalism

Gauging a theory means making part of its global symmetries local, and in the process some of the scalars become charged. The symmetry group we are dealing with is the isometry group G of the scalar field space M. In the ungauged case the equations of motion are invariant under the $Sp(2(n_v+1))$ electromagnetic duality group acting on the symplectic vectors $(F^I_{\mu\nu}, G_{I\mu\nu})$ and (X^I, F_I). If some of the scalars are charged (namely the theory is gauged) the symplectic group is broken to a smaller one and the final theory is determined by the magnetic and electric charges carried by the fields. In the following we shall be mainly interested in the case in which the $N=2$ supersymmetry is partially broken to $N=1$, and as it will be explained later, the scenario requires symplectic frames in which magnetic charges are present. The symplectic tensor formalism (introduced in [18, 19], for a review see also [39]) is a well suited mean to provide the theory with both electric and magnetic charges while treating them on the same footing.

The isometry group G of the $N=2$ product manifold $M = M_v \times M_h$ splits accordingly into $G_{SK} \times G_{QK}$. The isometries for the vector and hypermultiplet scalar sector respectively can be expressed as

$$\begin{aligned} \delta_{G_{SK}} t^i &= -g_v \; k^i_\iota \alpha^\iota \\ \delta_{G_{QK}} q^u &= -g_h \; k^u_\lambda \beta^\lambda, \end{aligned} \tag{2.46}$$

where g_v and g_h are coupling constants, α^ι and β^λ are local parameters and $(k^i_\iota(\mathrm{t})$, $k^u_\lambda(q))$ are the Killing vectors of the isometries that must satisfy the commutation relations of their Lie algebra, given by

$$\begin{aligned} [k_\iota, k_\kappa] &= f^\xi_{\iota\kappa} k_\xi \\ [k_\lambda, k_\rho] &= f^\sigma_{\lambda\rho} k_\sigma \end{aligned} \tag{2.47}$$

for the vector and hypermultiplet sector isometries respectively. In the following, except when explicitly stated, we will assume $f^\xi_{\iota\kappa} \neq 0$, namely a non-abelian group for the vectors. The gauging can be then encoded in the embedding tensor

$$\Theta^\delta_\Lambda = \begin{pmatrix} \Theta^\iota_\Lambda \\ \Theta^\lambda_\Lambda \end{pmatrix}, \tag{2.48}$$

where the two components are referring to the vector and hypermultiplet sector respectively. The Λ index runs over the symplectic coordinates $(\Theta^\lambda_\Lambda = (\Theta^\lambda_I, \Theta^{I\lambda})$

in such a way that the original symplectic symmetry (unbroken in the ungauged theory) is still manifest after the gauging. The spacetime derivatives appearing in the vector and hypermultiplet sigma models become covariant derivatives:

$$\begin{aligned} \partial_\mu t^i &\to D_\mu t^i \equiv \partial_\mu t^i - A^I_\mu \Theta_I{}^\iota k^i_\iota + B_{\mu I}\Theta^{I\iota} k^i_\iota, \\ \partial_\mu q^u &\to D_\mu q^u \equiv \partial_\mu q^u - A^I_\mu \Theta_I{}^\lambda k^u_\lambda + B_{\mu I}\Theta^{I\lambda} k^u_\lambda, \end{aligned} \tag{2.49}$$

where we introduced the magnetic gauge potential $B_{\mu I}$ together with the electric gauge vector A^I_μ. The embedding tensor encodes the charges and needs in general two further constraints:

$$\begin{aligned} f^\sigma_{\lambda\rho}\Theta_\Lambda{}^\lambda\Theta_\Sigma{}^\rho + (t_\lambda)_\Lambda{}^\Xi\Theta_\Lambda{}^\lambda\Theta_\Xi{}^\sigma &= 0 \\ \Theta^{[\lambda}\Theta_I^{\kappa]} &= 0. \end{aligned} \tag{2.50}$$

The first one is required by the closure of the Lie algebra generators $(t_\lambda)_\Lambda{}^\Xi$. The second one assures the mutual locality of the electric and magnetic charges.[4] Introducing both electric and magnetic couplings in the theory upsets the balance of the degrees of freedom in the lagrangian. To counterbalance one has to introduce a set of 2-form gauge potential $B^M_{\mu\nu}$ whose couplings preserve gauge symmetry and supersymmetry. For a review see again [18, 19].

Another consequence of gauging is that the supersymmetry variations get modified, and in order to keep the actions invariant a scalar potential needs to be introduced. In global $N = 2$ supersymmetry the scalar potential with both electric and magnetic charges takes the form (in the embedding tensor formalism) [26, 40]:

$$\begin{aligned} \mathcal{V}^{\text{vis (rig)}} =& V^M\bar{V}^N(\Theta_M^{\ \iota}\Theta_N^{\ \kappa} g_{m\bar{n}} k^m_\iota k^{\bar{n}}_\kappa + 4\ \Theta_M^{\ \nu}\Theta_N^{\ \rho} h_{wy} k^w_\nu k^y_\rho) \\ &+ \mathbf{P}_\nu\cdot\mathbf{P}_\rho\Theta_M^{\ \nu}\Theta_N^{\ \rho}(g^{m\bar{n}}\partial_m V^M\partial_{\bar{n}}\bar{V}^N), \end{aligned} \tag{2.51}$$

where the P's are real Killing prepotentials (more details in the next sections) and

$$\mathbf{P}_\nu\cdot\mathbf{P}_\rho = P^1_\nu P^1_\rho + P^2_\nu P^2_\rho + P^3_\nu P^3_\rho. \tag{2.52}$$

For later convenience, we also consider the situation in which there is no hypermultiplet in the theory. In this case (2.51) simply reads

$$\mathcal{V}^{\text{NoHyp (rig)}} = V^M\bar{V}^N\Theta_M^{\ \iota}\Theta_N^{\ \kappa} g_{m\bar{n}} k^m_\iota k^{\bar{n}}_\kappa. \tag{2.53}$$

[4]Doing a symplectic rotation on the equations of motion and Bianchi identities can lead to a symplectic frame with both electric and magnetic charges, but these mutually local charges cannot appear simultaneously in the local lagrangian.

In the following section we shall consider the gauging in the case of $N = 2$ supergravity.

2.6 $\mathcal{N} = 2$ gauged supergravity

After gauging, the supersymmetry variations of the fields change accordingly. In particular, for the gravitino, gaugino and hyperino we have:

$$\begin{aligned} \delta_\epsilon \Psi_{\mu A} &= D_\mu \epsilon_A - S_{AB}\gamma_\mu \epsilon^B + \cdots \\ \delta_\epsilon \lambda^{iA} &= W^{iAB}\epsilon_B + \cdots \\ \delta_\epsilon \zeta_\alpha &= N_\alpha^A \epsilon_A + \cdots \ . \end{aligned} \tag{2.54}$$

Here we are only interested in the scalar part of the variations and we also leave apart (with the dots) the terms vanishing in maximally symmetric ground states. A complete treatment of gauged $N = 2$ supergravities including both electric and magnetic charges can be found in [41–43]. These new variations must be compensated in the gauged lagrangian in order to leave the theory supersymmetric. In particular a scalar potential $\mathcal{V}$ (always present in $N = 1$ supergravities but absent in ungauged $N = 2$) is introduced. The general bosonic lagrangian for gauged supergravity is then given by

$$\begin{aligned} \mathcal{L} \ =& \ \tfrac{1}{2\kappa^2} R + \tfrac{1}{4}\mathrm{Im}\mathcal{N}_{IJ}(t,\bar{t})\, F^I_{\mu\nu} F^{\mu\nu\, J} - \tfrac{1}{8}\mathrm{Re}\mathcal{N}_{IJ}(t,\bar{t})\, \varepsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \\ & - g_{i\bar{\jmath}}(t,\bar{t})\, D_\mu t^i D^\mu \bar{t}^{\bar{\jmath}} - h_{uv}(q)\, D_\mu q^u D^\mu q^v - \mathcal{V}(t,\bar{t},q) \ . \end{aligned} \tag{2.55}$$

The derivatives of the sigma models have been promoted to the covariant derivatives (2.49) and $\mathcal{V}$ is the dimension four scalar potential, that takes the form:

$$\mathcal{V}(t,\bar{t},q) = -\frac{12}{M_{\mathrm{Pl}}^2} S_{AB} S^{AB} + g_{i\bar{\jmath}} W^{iAB} W^{\bar{\jmath}}_{AB} + 2 N^A_\alpha N^\alpha_A, \tag{2.56}$$

with S_{AB}, W^{iAB} and N^A_α given in the embedding tensor formalism by:

$$\begin{aligned} S_{AB} &= \frac{i}{2} e^{\frac{K}{2M_{\mathrm{Pl}}^2}} V^\Lambda \Theta_\Lambda{}^\lambda P^x_\lambda (\sigma^x)_{AB}, \\ W^{iAB} &= e^{\frac{K}{2M_{\mathrm{Pl}}^2}} [i g^{i\bar{\jmath}} (\nabla_{\bar{\jmath}} \bar{V}^\Lambda) \Theta_\Lambda{}^\lambda P^x_\lambda (\sigma^x)^{AB} + \epsilon^{AB} \Theta_\Lambda{}^\iota k^i_\iota \bar{V}^\Lambda], \\ N^A_\alpha &= 2 e^{\frac{K}{2M_{\mathrm{Pl}}^2}} \bar{V}^\Lambda \Theta_\Lambda{}^\lambda \mathcal{U}^A_{\alpha u} k^u_\lambda . \end{aligned} \tag{2.57}$$

V^Λ is again the symplectic vector (2.20) with $X^I(t)$ and $F_I(t)$ smooth functions of the vector scalar fields t and its Kähler covariant derivative ∇_i is $\nabla_i V^\Lambda = \partial_i V^\Lambda + \frac{1}{M_{\mathrm{Pl}}^2} K_i V^\Lambda$, with $K_i = \partial_i K$. k^i_ι and k^u_λ are as usual the Killing vectors related to the isometries of the vector and the hyper manifolds respectively, while $\mathcal{U}^{A}_{\alpha u}$ is the vielbein defined in (2.42). The matrices $(\sigma^x)_{AB}$ and their inverse are constructed applying the $SU(2)$ metric ϵ^{AB} and its inverse to the Pauli matrices. Their explicit form is (see Appendix A for more details):

$$(\sigma^1)_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\sigma^2)_{AB} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad (\sigma^3)_{AB} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \tag{2.58}$$

Finally, P^x_λ, $(x = 1, 2, 3)$ is a triplet of real Killing prepotentials (moment maps) defined by

$$-2k^u_\lambda K^x_{uv} = \nabla_v P^x_\lambda, \tag{2.59}$$

and with $P^x_\lambda(\sigma^x)_{AB}$ we indicate the sum

$$P^x_\lambda(\sigma^x)_{AB} \equiv P^1_\lambda(\sigma^1)_{AB} + P^2_\lambda(\sigma^2)_{AB} + P^3_\lambda(\sigma^3)_{AB}. \tag{2.60}$$

Chapter 3

The rigid limit of $N = 2$ supergravity

Our task will be the discussion of the explicit partial supersymmetry breaking of $N = 2$ via soft terms whose form should be determined by the supergravity theory we start with. In order to obtain a global theory together with partial-breaking soft terms, we will need to take the rigid limit ($M_{\rm Pl} \to \infty$) of supergravity, while assuming the existence of an intermediate scale that compensates the limit on the Planck mass and keeps the gravitino mass (the scale of the partial SUSY breaking) fixed. Before considering this case, it is worthwhile to thoroughly deal with the general rigid limit procedure in $N = 2$ supergravity first. This problem was originally tackled in [23, 44–48] (for reviews see, for example, [26, 28]). We shall review our recent approach [21], inspired by the common situation in string theory where we typically have two classes of light scalar fields: the so-called moduli, that are only gravitationally coupled, and the charged scalars which can have gauge interactions at low energies. While the former are generally frozen to their background value Φ_0 that can be of the order of $M_{\rm Pl}$, the latter have in general non-trivial dynamics and in gauge symmetry-breaking scenarios can take a background value φ_0 much smaller than $M_{\rm Pl}$. We call the first sector "hidden" and the second one "visible" and we assume that the two sectors are decoupled. Summarizing, we have:

$$\Phi_0 = \mathcal{O}(M_{\rm Pl}), \qquad \varphi_0 \ll \mathcal{O}(M_{\rm Pl}). \tag{3.1}$$

We should remark that this distinction can in general be made only *locally*. The rigid limit procedure is actually a zooming on a specific region of the moduli space, where a generic point is at distance $M_{\rm Pl}$ and will be sent to infinity in the limit.

As we shall see, if the only massive scale in the game is $M_{\rm Pl}$, the rigid limit is trivial and flat. We consider then a generic additional scale $\Lambda < M_{\rm Pl}$ (for instance the Seiberg-Witten scale [45]) that allows for non-trivial couplings. It is also to be noted that we shall perform the limit in a Minkowski vacua as in this context we are mainly interested in scenarios with vanishing cosmological constant.

3.1 Preliminaries

We start again with the bosonic lagrangian for general $N = 2$ supergravity (2.55) and we set canonical mass dimension one for the vector and scalar fields so that the sigma-model metrics $g_{i\bar{\jmath}}(t,\bar{t})$ and $h_{uv}(q)$, the kinetic matrix $\mathcal{N}_{IJ}(t,\bar{t})$ and the space-time metric $g_{\mu\nu}$ are dimensionless. For the moment we ignore the (four dimensional) scalar potential $\mathcal{V}$; we shall come back to it in the last section, in the context of the rigid limit of gauged supergravity. In the following sections we deal with the rigid limit of the ungauged theory.

First, we expand the space-time metric around a Minkowski background $\eta_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} + \dots \ ; \tag{3.2}$$

similarly, we expand the scalar fields around their background values t_0 and q_0 as

$$t^i = t^i_0 + \delta t^i \ , \qquad q^u = q^u_0 + \delta q^u , \tag{3.3}$$

where with δt^i and δq^u we indicate small fluctuations in the neighbourhoods of t^i_0 and q^u_0. Using (3.3), we can expand the couplings in (2.55) as well:

$$\begin{aligned} \mathcal{N}_{IJ}(t,\bar{t}) &= \mathcal{N}_{IJ}(t_0,\bar{t}_0) + \partial_i\mathcal{N}_{IJ}(t_0,\bar{t}_0)\delta t^i + \partial_{\bar{\imath}}\mathcal{N}_{IJ}(t_0,\bar{t}_0)\delta t^{\bar{\imath}} + \dots \ , \\ g_{i\bar{\jmath}}(t,\bar{t}) &= g_{i\bar{\jmath}}(t_0,\bar{t}_0) + \partial_k g_{i\bar{\jmath}}(t_0,\bar{t}_0)\delta t^k + \partial_{\bar{k}} g_{i\bar{\jmath}}(t_0,\bar{t}_0)\delta t^{\bar{k}} + \dots \ , \\ h_{uv}(q) &= h_{uv}(q_0) + \partial_w h_{uv}(q_0)\delta q^w + \dots \ . \end{aligned} \tag{3.4}$$

As we said, in our mass conventions the couplings $\mathcal{N}_{IJ}, g_{i\bar{\jmath}}, h_{uv}$ are dimensionless, therefore their derivatives must have mass dimension -1. If $M_{\rm Pl}$ is the only mass scale in the theory, all higher order terms in the expansions (3.4), namely all terms including derivatives of the couplings, have order κ and thus vanish in the rigid limit $\kappa \to 0$. Therefore, only the first terms of the expansions survive and the couplings are evaluated at the constant background values of the scalar fields, where $\mathcal{N}_{IJ}, g_{i\bar{\jmath}}$ and h_{uv} are constant and diagonalizable. Thus, in this situation the scalar field space of the rigid theory is flat and the gauge kinetic matrix can be put in a diagonal form. This is the mentioned trivial situation in which we have the $M_{\rm Pl}$ as the only scale. But if we allow for an intermediate scale Λ (like in QCD or Seiberg-Witten theories), the derivatives of the coupling do not scale necessarily like κ, but could also carry a Λ^{-1} factor. In general, insisting in the distinction between hidden and visible scalar sectors and given that the couplings (3.4) are dimensionless for us, they can in principle depend on the ratios $\frac{\Phi}{M_{\rm Pl}}, \frac{\Phi}{\Lambda}, \frac{\varphi}{M_{\rm Pl}}, \frac{\varphi}{\Lambda}$. The possibility $\frac{\Phi}{\Lambda}$ is excluded by the fact that the ratio would diverge in the limit (because Φ has a vev of order $M_{\rm Pl}$), while $\frac{\varphi}{M_{\rm Pl}}$ would vanish. Therefore we only need to keep $\frac{\Phi}{M_{\rm Pl}}$ and $\frac{\varphi}{\Lambda}$: the derivatives with respect to Φ will then scale like $M_{\rm Pl}$, namely its fluctuations will vanish in the limit and the field will be frozen to its background value; on the other hand, the derivatives with respect to φ scale like Λ^{-1} and are kept. Therefore we have

$$g(\Phi, \varphi) = g(\Phi_0, \varphi) = g(\Phi_0, \varphi_0) + \partial_\varphi g(\Phi_0, \varphi_0)\delta\varphi + \dots \ , \tag{3.5}$$

where the only dynamical field left is φ.

3.2 Vector multiplet sector

Now we study the details of each sector, starting from the vectors. Coming back to the distinction between hidden and visible sectors, as first step we split the homogeneous function $\mathcal{F}$ given in (2.31) in two components:

$$\mathcal{F} = \mathcal{F}^{\hat{\Phi}}(\kappa\Phi) + \mathcal{F}^t(\kappa\Phi, t^i, \kappa, \Lambda) \ , \tag{3.6}$$

where we removed the indices from the Φ for simplicity and from now on we identify the hidden parts with $\hat{\Phi}$. The first component is only dependent on the hidden fields frozen around their background values and can be treated as a constant piece, while the second one also depends on the gauge-interacting fields and the additional scale Λ, and provides all the non-trivial dynamics together with the interactions between hidden and visible fields. With $\mathcal{F}^t(\kappa\Phi, t^i = 0) = 0$ we imply that all the terms independent of the visible fields are included in the first piece.

Let us first expand $\mathcal{F}^{\hat{\Phi}}$ for small fluctuations $\Phi = \Phi_0 + \delta\Phi$:

$$\mathcal{F}^{\hat{\Phi}} = \mathcal{F}^{\hat{\Phi}}(\kappa\Phi_0) + \kappa\partial\mathcal{F}^{\hat{\Phi}}(\kappa\Phi_0)\,\delta\Phi + \tfrac{1}{2}\kappa^2\partial^2\mathcal{F}^{\hat{\Phi}}(\kappa\Phi_0)\,\delta\Phi^2 + \mathcal{O}(\kappa^3) \ , \tag{3.7}$$

with the first term being constant ($\kappa\Phi_0$ is dimensionless). Inserting this expansion into the definition of the Kähler potential (2.32) and expanding the logarithm, the $\mathcal{F}^{\hat{\Phi}}$ contribute becomes

$$K^{\mathrm{v}} = -\ln Y(\kappa\Phi_0) + Y^{-1}(\kappa\Phi_0)\, K^{\mathrm{v}}_{\mathrm{r}} + \mathcal{O}(\kappa) \ , \tag{3.8}$$

where

$$K^{\mathrm{v}}_{\mathrm{r}} = h(\delta\Phi) + \bar{h}(\delta\bar{\Phi}) + g_{\Phi\bar{\Phi}}(\kappa\Phi_0)\, \delta\Phi^{\Phi}\delta\bar{\Phi}^{\bar{\Phi}} \ . \tag{3.9}$$

$h(\delta\Phi)$ and $h(\delta\bar{\Phi})$ are holomorphic functions and do not contribute to the metric, while $Y^{-1}(\kappa\Phi_0)$ and its logarithm are constant terms and can be reabsorbed in the definitions of the metric or the fields and the function h. Moreover, we need a non-zero $Y(\kappa\Phi_0)$ in order to expand the logarithm. As expected, only the quadratic term of $K^{\mathrm{v}}_{\mathrm{r}}$ contributes in the limit, leading to a constant metric $g_{\Phi\bar{\Phi}} = g_{\Phi\bar{\Phi}}(\kappa\Phi_0)$. Furthermore, choosing the appropriate $\delta\Phi$ one can always put $Y^{-1}g_{\Phi\bar{\Phi}} = \delta_{\Phi\bar{\Phi}}$. The second term of (3.6) is the interesting one. Ignoring the constant pieces coming from combinations t_0/Λ, we can write $\mathcal{F}$ as a power series in κ:

$$\mathcal{F}^t(\kappa\Phi_0, t^i, \kappa, \Lambda) = \sum_{n=0}^{\infty} \kappa^n \mathcal{F}^{t(n)}(\kappa\Phi_0, t^i, \Lambda) \ . \tag{3.10}$$

There are two constraints to consider: $\mathcal{F}$ is dimensionless, so the first term of the expansion, $\mathcal{F}^{t(0)}$, must be dimensionless as well, and being a constant piece is absorbed by definition in $\mathcal{F}^{\hat{\Phi}}$; if we allow the $\mathcal{F}^{t(1)}$ to be non-zero, we would have terms linear in t, which would violates gauge invariance, and furthermore the prepotential F would contain mixed product like $F \sim X^0X^i$ and/or $F \sim X^{\Phi}X^i$ that violate the decoupling assumption between the hidden and visible sectors. Our requirements are then:

$$\mathcal{F}^{t(0)} = \mathcal{F}^{t(1)} = 0 \ . \tag{3.11}$$

Going to the limit $\kappa \to 0$, only $\mathcal{F}^{t(2)}(\kappa\Phi_0, t^i, \Lambda)$ survives. Inserting it into the Kähler potential (2.32) and considering this time the contributions from both the hidden and the visible fields, we get

$$K^{\mathrm{v}}_{\mathrm{r}} = h(\delta\Phi, t) + \bar{h}(\delta\bar{\Phi}, \bar{t}) - \mathrm{i}\left(\delta\bar{\Phi}^{\Phi}\mathcal{F}^{\hat{\Phi}(2)}_{\Phi} - \delta\Phi^{\Phi}\bar{\mathcal{F}}^{\hat{\Phi}(2)}_{\bar{\Phi}}\right) - \mathrm{i}\left(\bar{t}^i\mathcal{F}^{t(2)}_i - t^i\bar{\mathcal{F}}^{t(2)}_{\bar{i}}\right) \ . \tag{3.12}$$

where we have defined $\mathcal{F}^{\hat{\Phi}(2)} = \frac{i}{4}g_{\Phi\bar{\Phi}}(\kappa\Phi_0)\,\delta\Phi^2$ and again the holomorphic $h(\Phi, t)$

does not contribute to the metric. (3.12) is then the Kähler potential of a rigid special Kähler manifold with the metric (up to the renormalization factor $Y^{-1}(\kappa\Phi)$)

$$g = \begin{pmatrix} g_{\Phi\bar{\Phi}} & 0 \\ 0 & g_{i\bar{j}} \end{pmatrix} = 2 \begin{pmatrix} \mathrm{Im}\mathcal{F}^{\hat{\Phi}(2)}_{\Phi\Phi} & 0 \\ 0 & \mathrm{Im}\mathcal{F}^{t(2)}_{i\bar{j}} \end{pmatrix} , \tag{3.13}$$

so that we can identify $\mathcal{F}^{\hat{\Phi}(2)} + \mathcal{F}^{t(2)}$ as the rigid prepotential. Now we consider the kinetic matrix (2.30) that encodes all the vector couplings. Our assumption on the decoupling of the hidden scalars Φ and the visible scalars t must reflect also on the vectors, since they belong to the same multiplets. We can let the graviphoton A^0_μ to interact with the hidden vectors $A^{\hat{\Phi}}_\mu$, but we must forbid couplings between A^0_μ or $A^{\hat{\Phi}}_\mu$ and the visible vectors A^i_μ. This translates to the requirement that $\mathcal{N}$ must be block-diagonal ($\mathcal{N}_{0i} = \mathcal{N}_{\Phi i} = 0$). We can check the consistency with the constraints (3.11): first, we observe that both $F_{0i} \sim \mathcal{O}(\kappa)$ and $F_{\Phi i} \sim \mathcal{O}(\kappa)$, as the derivative with respect to t^i (or X^i) means that the only contribution is from $\mathcal{F}^{t(2)}$. For the components $\mathcal{N}_{0i}$ and $\mathcal{N}_{\Phi i}$ the non-holomorphic second term in (2.30) is suppressed because in both case it contains the Planck suppressed $\mathcal{F}^{t(2)}$ in the numerator. Therefore $\mathcal{N}_{0i} \sim \mathcal{O}(\kappa)$ and $\mathcal{N}_{\Phi i} \sim \mathcal{O}(\kappa)$, so the gravitational and visible sectors decouple and we have shown that the requirements on the prepotential $\mathcal{F}$ and the $\mathcal{N}$ matrix are compatible. On the other hand, the components $\mathcal{N}_{00}, \mathcal{N}_{0\Phi}, \mathcal{N}_{\Phi\Phi}$ are constant at leading order as they depend on $\mathcal{F}^{\hat{\Phi}}$. Finally, in $\mathcal{N}_{ij}$ the non-holomorphic second term is Planck suppressed, as the numerator only depends on $\mathcal{F}^{t(2)}$, while the denominator can be $\mathcal{O}(1)$. Thus, from

$$\mathcal{N}_{ij} = \bar{F}_{ij} = \bar{\mathcal{F}}^{t(2)}_{ij} , \tag{3.14}$$

we can also recover the correct relations for global $N = 2$ supersymmetry (2.5) and (2.6), namely:

$$\mathrm{Im}\mathcal{N}_{ij} = -\mathrm{Im}\mathcal{F}^{t(2)}_{ij} = -\tfrac{1}{2} g_{ij} . \tag{3.15}$$

3.2.1 Two examples

1. Consider as an example a theory with prepotential

$$F = \tfrac{\mathrm{i}}{4}\big((X^0)^2 - \eta_{ij} X^i X^j\big) , \qquad i = 1, \ldots, n_{\mathrm{v}} , \tag{3.16}$$

where η_{ij} is real. Using again special coordinates and the expression (2.31) we get

$$\mathcal{F} = \tfrac{\mathrm{i}}{4}(1 - \kappa^2 \eta_{ij} t^i t^j) , \qquad K = -\ln\big(1 - \kappa^2 \eta_{ij} t^i \bar{t}^j\big) + \mathrm{const.} , \tag{3.17}$$

which is the Kähler potential of the space $\frac{SU(1,n_{\rm v})}{U(1)\times SU(n_{\rm v})}$.[1] The expansion of $\mathcal{F}$ yields the rigid prepotential $\mathcal{F}^{(2)} = -\frac{\rm i}{4}\eta_{ij}t^i t^j$ and inserting this into (5.45) we obtain the flat metric $g_{i\bar{j}} = \eta_{ij}$.

2. Now we take

$$F = \frac{\rm i}{4}\frac{X^1}{X^0}(X^2X^3 - \eta_{ij}X^iX^j) \ , \qquad i,j = 4,\dots,n_{\rm v} \ , \tag{3.18}$$

with η_{ij} again real. Inserting it into (2.32) and using the special coordinates

$$S = \kappa^{-1}\frac{X^1}{X^0} \ , \quad T = \kappa^{-1}\frac{X^2}{X^0} \ , \quad U = \kappa^{-1}\frac{X^3}{X^0} \ , \quad t^i = \kappa^{-1}\frac{X^i}{X^0} \tag{3.19}$$

we have

$$\mathcal{F} = \frac{\rm i}{4}\kappa^3 S(TU - \eta_{ij}t^it^j) \ , \tag{3.20}$$

and

$$K = -\ln(S-\bar{S}) - \ln\big((T-\bar{T})(U-\bar{U}) - \eta_{ij}(t-\bar{t})^i(t-\bar{t})^j\big) + \text{const.} \ , \tag{3.21}$$

which is the Kähler potential of the space

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n_{\rm v}-1)}{SO(n_{\rm v}-1)\times SO(2)} \ . \tag{3.22}$$

S, T and U are the gravitationally coupled scalars and we then give them a background value S_0, T_0, U_0 of order $M_{\rm Pl}$, in the usual spirit of [21], while the t^i are in the visible sector. The rigid prepotential is then

$$\mathcal{F}^{t(2)} = -\tfrac{\rm i}{4}\kappa S_0\eta_{ij}t^it^j, \tag{3.23}$$

producing the flat metric $g_{i\bar{j}} = \kappa S_0\eta_{ij}$. Considering a non-abelian gauge theory with one-loop and non perturbative correction, $\mathcal{F}$ becomes [45] (see also [49] for a generalization to arbitrary gauge groups and a string theory derivation):

$$\mathcal{F}^{t(2)} = -\tfrac{\rm i}{4}\kappa S_0 t^2 + t^2\ln\frac{t^2}{\Lambda^2} + t^2\sum_{k=1}^{\infty}\mathcal{F}_k\left(\frac{\Lambda}{t}\right)^{4k} \ . \tag{3.24}$$

[1] Note that we could add terms of the form X^0X^i in F which for a purely quadratic F can always be rotated away. Furthermore, any imaginary part of η_{ij} does not contribute to K (and thus the metric) but does contribute to the θ-angle as can be seen from (3.14).

3.3 Hypermultiplet sector

Evaluating the rigid limit for the hypermultiplet sector in the most general case is a difficult problem, and only specific examples are known in the literature (e.g. [48]). We shall start with general observations and then proceed with the case of the *special* quaternionic Kähler manifolds, that are easily manageable within our approach.

As we reviewed in section (2.4), in a quaternionic space the two-forms (2.40) are *covariantly* closed with respect to the $SU(2)$ connection ω^x, but in the rigid $N=2$ supersymmetry case they need to be exactly *closed*, and the target space is an hyper-Kähler manifold with holonomy $Sp(n)$. Moreover, hyper-Kähler manifolds are Ricci-flat while quaternionic Kähler manifolds are Einstein (namely the Ricci tensor is proportional to the metric). Therefore, in the rigid limit both the second piece of the covariant derivative (2.39) and the non-Ricci-flat part of the metric must go to zero. Indeed, it has been shown in [23] that the Riemann curvature tensor can be decomposed in two parts:

$$R_{stuv} = \kappa^2 \hat{R}_{stuv} + W_{stuv} \ , \tag{3.25}$$

where $\hat{R}_{tsuv}$ is the (dimensionless) $SU(2)$ curvature that vanish for $\kappa \to 0$, while W_{stuv} is the Ricci-flat Weyl-curvature of a hyper-Kähler manifold. The $SU(2)$ part of the curvature that goes away in the limit can be traced back to the metric block related to the only-gravitationally coupled fields $h_{\Phi\Phi}$, while the second part encodes all the non trivial dynamics allowed by (for instance) an additional scale Λ, as previously discussed.

3.3.1 The rigid limit of special quaternionic Kähler manifolds and the c-map

Let us come back to the example of quaternionic Kähler manifold discussed in 3.3.1. We can make the usual distinction between gravitationally coupled scalars Φ with background values of $M_{\rm Pl}$ order and the observable scalars z^a with small background values. Since here, as in the vector multiplet sector case, we deal with a special Kähler (sub)manifold, the rigid limit works in the same way, with the Kähler potential (3.12) in which we replace $\mathcal{F}^{t(2)}$ with $\mathcal{G}^{z(2)}(\kappa\Phi_0, z, \Lambda)$

$$K^{\rm h} = h(\Phi, z) + \bar{h}(\bar{\Phi}, \bar{z}) - {\rm i}\left(\delta\bar{\Phi}^{\Phi}\mathcal{G}^{\hat{\Phi}(2)}_{\Phi} - \delta\Phi^{\bar{\Phi}}\bar{\mathcal{G}}^{\hat{\Phi}(2)}_{\bar{\Phi}}\right) - {\rm i}\left(\bar{z}^{\bar{a}}\mathcal{G}^{z(2)}_{a} - z^{a}\bar{\mathcal{G}}^{z(2)}_{\bar{a}}\right) \ , \tag{3.26}$$

where $\mathcal{G}_{\Phi}^{\hat{\Phi}(2)} = \frac{i}{4} g_{\Phi\bar{\Phi}}(\kappa\Phi_0)\,\delta\Phi^2$. Again we have allowed for another scale Λ to guarantee non-trivial dynamics also in the hypermultiplet sector. It is also clear that the matrix $\mathcal{M}_{AB}$ in (2.45) behaves as the $\mathcal{N}$ matrix of the previous section, becoming block diagonal in the rigid limit:

$$\mathcal{M}_{AB} = \begin{pmatrix} \mathcal{M}_{\Phi\Phi} & 0 \\ 0 & \bar{\mathcal{G}}^{z(2)}_{ab} \end{pmatrix} , \tag{3.27}$$

where $\mathcal{M}_{\Phi\Phi}$ is constant and includes the graviphoton direction $A = 0$ while $\bar{\mathcal{G}}^{z(2)}_{ab}$ is the second derivative of the prepotential $\bar{\mathcal{G}}^{z(2)}$.

We are left with the first two terms in (2.44). Their limit for $\kappa \to 0$ is simply $(\partial\phi)^2 + (\partial\tilde{\phi})^2$. The metric (2.44) splits in two parts: a flat part for the fields $\phi, \tilde{\phi}$ and the hidden Φ of the special Kähler manifold $\mathbf{M}_{\rm sk}$, plus the corresponding RR-scalars $\xi^{\Phi}, \tilde{\xi}_{\Phi}$ which include the $A = 0$ direction, and a non-flat part containing the visible scalars $(z^a, \bar{z}^{\bar{a}}, \xi^a, \tilde{\xi}_a)$, with rigid Kähler potential

$$K_{\rm rc} = {\rm i}(\bar{z}^a \mathcal{G}^{z(2)}_a - z^a \bar{\mathcal{G}}^{z(2)}_a) - \tfrac{1}{2}({\rm Im}\,\mathcal{G}^{z(2)})^{-1ab}(C + \bar{C})_a (C + \bar{C})_b \ , \tag{3.28}$$

where we defined [38]

$$C_a = {\rm i}(\tilde{\xi}_a + \mathcal{G}^{z(2)}_{ab}\xi^b) \ . \tag{3.29}$$

While the flat part of the metric is trivially hyper-Kähler, the other part is determined by $K_{\rm rc}$, known to be the Kähler potential of the rigid c-map and the corresponding metric is hyper-Kähler [37]. We can also add flat directions to $K_{\rm rc}$ by replacing $\mathcal{G}^{z(2)}$ in (3.28) by $\mathcal{G}^{(2)} = \frac{\rm i}{4}\tau^2 + \mathcal{G}^{\hat{\Phi}(2)}_{\Phi} + \mathcal{G}^{z(2)}$, for $\tau = \phi + {\rm i}\,\tilde{\phi}$, and defining C_{Φ} in terms of $\xi^{\Phi}, \tilde{\xi}_{\Phi}$ as in (3.29).

It should also be noted that the hyper-Kähler metric we derived as a rigid limit from a special quaternionic one has a rigid prepotential $\mathcal{G}^{(2)}$ that is different from the prepotential G describing the quaternionic metric. In the quaternionic–hyper-Kähler correspondence, instead, (see e.g. [50]) the hyper-Kähler metric and the quaternionic metric share the same prepotential.

3.4 The rigid limit of the scalar potential

Now we come back to the gauged lagrangian (2.55). The additional piece we have to consider, which will be of crucial importance in the following, is the scalar potential (2.56). In a Minkowski background and for an unbroken theory, the supersymmetry variations (2.54) must vanish in the background in order to preserve

Lorentz invariance (see e.g. [51, 52]), namely the vacuum expectation value $\langle \mathcal{V} \rangle$ of the scalar potential is identically zero. Before treating the case in which supersymmetry is partially broken, it is useful to compute the rigid limit of the scalar potential in a generic $N = 2$ background. If we introduce

$$L^\Lambda = \begin{pmatrix} L^I \\ M_I \end{pmatrix} \equiv e^{\frac{K}{2M_{\rm Pl}^2}} \begin{pmatrix} X^I \\ F_I \end{pmatrix} = e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda, \quad f_i^I \equiv \tilde{\nabla}_i L^I, \tag{3.30}$$

with $\tilde{\nabla}_i = \partial_i + \frac{1}{2M_{\rm Pl}^2} K_i$ and $K_i = \partial_i K$, the scalar potential $\mathcal{V}$ can be expressed in an explicit and compact way [26]:

$$\begin{aligned} \mathcal{V} = {} & L^\Lambda \bar{L}^\Pi (\Theta_\Lambda{}^\iota \Theta_\Pi{}^\kappa g_{i\bar{j}} k_\iota^i k_\kappa^{\bar{j}} + 4\Theta_\Lambda{}^\lambda \Theta_\Pi{}^\pi h_{uv} k_\lambda^u k_\pi^v) \\ & + \mathbf{P}_\lambda \cdot \mathbf{P}_\pi \Theta_\Lambda{}^\lambda \Theta_\Pi{}^\pi (g^{i\bar{j}} f_i^\Lambda \bar{f}_{\bar{j}}^\Pi - \frac{3}{M_{\rm Pl}^2} L^\Lambda \bar{L}^\Pi), \end{aligned} \tag{3.31}$$

where the definition (2.52) is used. As in the previous sections, we let all the fields in the hidden sector to be frozen around their background values ($\mathcal{O}(M_{\rm Pl})$) and the visible fields to get a vev $\varphi_0 \ll \mathcal{O}(M_{\rm Pl})$. We split the indices in (3.31) in hidden and observable sectors as

$$\begin{aligned} \text{Symplectic index:} \quad & \Lambda \to (\text{Hidden}) \;\; \Sigma, \Xi \ldots \; + (\text{Visible}) \;\; M, N, \ldots \\ \text{Hypermultiplet isometry index:} \quad & \lambda \to (\text{Hidden}) \;\; \gamma \ldots \; + (\text{Visible}) \;\; \nu \ldots \\ \text{Hypermultiplet scalar index:} \quad & u, v \to (\text{Hidden}) \;\; \mathcal{A}, \mathcal{B} \ldots \; + (\text{Visible}) \;\; w, y \ldots \\ \text{Vector scalar index:} \quad & i, j \to (\text{Hidden}) \;\; \Phi, \Psi \ldots \; + (\text{Visible}) \;\; m, n \ldots, \end{aligned} \tag{3.32}$$

and we identify the hidden or visible components according to their indices. In general we assume the hidden fields to be charged only under the hidden isometry group and the visible fields only under the visible one; namely we give the embedding tensor a block diagonal form. For instance, for the hypermultiplet isometries we have:

$$\Theta_\Lambda{}^\lambda = \begin{pmatrix} \Theta_\Sigma{}^\gamma & 0 \\ 0 & \Theta_M{}^\nu \end{pmatrix}, \tag{3.33}$$

where $\Theta_\Sigma{}^\gamma$ and $\Theta_{\mathbf{M}}{}^\nu$ refers to the hidden and visible sectors respectively. As a consequence, if the symplectic indices (Λ, Π) run over the visible or hidden fields, the indices (λ, ρ) are restricted to the isometries in the visible or hidden sector respectively. For the hypermultiplets we allow for isometries in both the hidden (where the partial supersymmetry breaking will take place, as it will be explained in the following chapters) and the observable sector. For the vectors, on the contrary, we only need to consider isometries in the visible sector (e.g. the non-abelian gauge groups of the Standard Model), since the hidden sector of the vectors does not play any role in the partial supersymmetry breaking, and from now on we assume for simplicity

$$\Theta_\Lambda{}^\iota = \begin{pmatrix} 0 & 0 \\ 0 & \Theta_M{}^\iota \end{pmatrix}. \tag{3.34}$$

We also rewrite the Killing vectors for each sector:

$$\begin{aligned} &\text{hypermultiplets: (hidden) } k_\gamma^{\mathcal{A}}, \quad \text{(visible) } k_\nu^w \\ &\text{vector multiplets: (visible) } k_\iota^m. \end{aligned} \tag{3.35}$$

Let us also specify the sections f_i^Λ explicitly:

$$\begin{aligned} f_i^\Lambda = \tilde{\nabla}_i L^\Lambda &= (\partial_i + \frac{K_i}{2M_{\text{Pl}}^2}) e^{\frac{K}{2M_{\text{Pl}}^2}} \begin{pmatrix} X^I \\ F_I \end{pmatrix} = \\ &= e^{\frac{K}{2M_{\text{Pl}}^2}} \frac{K_i}{2M_{\text{Pl}}^2} \begin{pmatrix} X^I \\ F_I \end{pmatrix} + e^{\frac{K}{2M_{\text{Pl}}^2}} \begin{pmatrix} \partial_i X^I \\ \partial_i F_I \end{pmatrix} + e^{\frac{K}{2M_{\text{Pl}}^2}} \frac{K_i}{2M_{\text{Pl}}^2} \begin{pmatrix} X^I \\ F_I \end{pmatrix} \\ &= e^{\frac{K}{2M_{\text{Pl}}^2}} \left[\begin{pmatrix} \partial_i X^I \\ \partial_i F_I \end{pmatrix} + \frac{K_i}{M_{\text{Pl}}^2} \begin{pmatrix} X^I \\ F_I \end{pmatrix} \right] = e^{\frac{K}{2M_{\text{Pl}}^2}} \nabla_i V^\Lambda. \end{aligned} \tag{3.36}$$

In the first line we used the definitions (3.30), in the second line we evaluated the derivative of the product and in the third line we put $\nabla_i = \partial_i + \frac{1}{M_{\text{Pl}}} K_i$ (see the footnote in section (2.3)). Let us now focus for a moment on the mixed covariant derivatives, namely the covariant derivative of a visible symplectic vector V^Λ with respect to hidden fields and viceversa ($\nabla_m V^\Sigma$ and $\nabla_\Phi V^M$): in accord with the assumptions of the previous sections, in order to preserve gauge invariance and forbid the coupling between the hidden and the visible sectors, we put the mixed derivatives of the prepotential to zero, namely

$$\partial_\Phi F_m = \partial_m F_\Phi = 0; \tag{3.37}$$

moreover, the terms $\frac{K_\Phi}{M_{\text{Pl}}^2} \begin{pmatrix} X^M \\ F_M \end{pmatrix}$ and $\frac{K_m}{M_{\text{Pl}}^2} \begin{pmatrix} X^\Sigma \\ F_\Sigma \end{pmatrix}$ are of order $\sim \frac{1}{M_{\text{Pl}}}$, hence they are subleading in the limit $M_{\text{Pl}} \to \infty$; together with the assumption (3.37) this means we can safely forbid the mixed sections f_m^Σ and f_Φ^M.

With all these considerations in mind we can effectively split the scalar potential into a hidden, an observable and a mixed part:

$$\mathcal{V} = \mathcal{V}^{\text{hid}} + \mathcal{V}^{\text{vis}} + \mathcal{V}^{\text{mix}}, \tag{3.38}$$

with

$$
\begin{aligned}
\mathcal{V}^{\text{hid}} &= L^{\Sigma}\bar{L}^{\Xi}(4\Theta_{\Sigma}{}^{\gamma}\Theta_{\Xi}{}^{\delta}h_{\mathcal{A}\mathcal{B}}k_{\gamma}^{\mathcal{A}}k_{\delta}^{\mathcal{B}}) \\
&\quad + \mathbf{P}_{\gamma}\cdot\mathbf{P}_{\delta}\Theta_{\Sigma}{}^{\gamma}\Theta_{\Xi}{}^{\delta}(g^{\Phi\bar{\Psi}}f_{\Phi}^{\Sigma}\bar{f}_{\bar{\Psi}}^{\Xi} - \frac{3}{M_{\text{Pl}}^2}L^{\Sigma}\bar{L}^{\Xi}), \\
\\
\mathcal{V}^{\text{vis}} &= L^{M}\bar{L}^{N}(\Theta_{M}{}^{\iota}\Theta_{N}{}^{\kappa}g_{m\bar{n}}k_{\iota}^{m}k_{\kappa}^{\bar{n}} + 4\ \Theta_{M}{}^{\nu}\Theta_{N}{}^{\rho}h_{wy}k_{\nu}^{w}k_{\rho}^{y}) \\
&\quad + \mathbf{P}_{\nu}\cdot\mathbf{P}_{\rho}\Theta_{M}{}^{\nu}\Theta_{N}{}^{\rho}(g^{m\bar{n}}f_{m}^{M}\bar{f}_{\bar{n}}^{N} - \frac{3}{M_{\text{Pl}}^2}L^{M}\bar{L}^{N}), \\
\\
\mathcal{V}^{\text{mix}} &= L^{\Sigma}\bar{L}^{M}(4\Theta_{\Sigma}{}^{\gamma}\Theta_{M}{}^{\nu}h_{\mathcal{A}w}k_{\gamma}^{\mathcal{A}}k_{\nu}^{w}) \\
&\quad + \mathbf{P}_{\gamma}\cdot\mathbf{P}_{\nu}\Theta_{\Sigma}{}^{\gamma}\Theta_{M}{}^{\nu}(g^{\Phi\bar{n}}f_{\Phi}^{\Sigma}\bar{f}_{\bar{n}}^{M} - \frac{3}{M_{\text{Pl}}^2}L^{\Sigma}\bar{L}^{M}) + \text{c.c.}.
\end{aligned}
\tag{3.39}
$$

Now we are ready to compute the limit. First of all, we freeze as usual the fields in the hidden sector on their background values and $\mathcal{V}^{\text{hid}}$ becomes a cosmological constant term, which we can assume vanishing in a Minkowski background. Then, we take the limit $M_{\text{Pl}} \to \infty$ in $\mathcal{V}^{\text{vis}}$. The factors $e^{K/2M_{\text{Pl}}^2}$ go to a constant we can reabsorb in the definition of the vector, namely

$$
L^{M} \to V^{M}. \tag{3.40}
$$

Moreover, since $L^{M} \sim O(1)$, the term $-\frac{3}{M_{\text{Pl}}^2}L^{M}\bar{L}^{N}$, related to the $-\frac{12}{M_{\text{Pl}}^2}S_{AB}S^{AB}$ contribution, goes to zero and the covariant derivative $\nabla_{\bar{j}}$ reduces to the standard derivative $\partial_{\bar{j}}$. Turning to the mixed term $\mathcal{V}^{mix}$, we use the fact that the metrics become block-diagonal in the rigid limit (as shown in the previous sections), while the piece $-\frac{3}{M_{\text{Pl}}^2}L^{\Sigma}\bar{L}^{M}$ is of order $\frac{1}{M_{\text{Pl}}}$ and is subleading. Thus (3.38) correctly reduces to (2.51), which is the scalar potential for global $N = 2$.

Chapter 4

Partial $N = 2 \to N = 1$ supersymmetry breaking

As we have seen in the last chapter, taking the ($M_{\mathrm{Pl}} \to \infty$) limit of general $N = 2$ supergravities lead as expected to global $N = 2$ supersymmetric theories. What we want to reach, though, is a global $N = 2$ theory partially broken to $N = 1$ by soft terms. In order to achieve this goal, we need to derive the rigid limit in an $N = 1$ vacuum of a spontaneously partially broken $N = 2$ supergravity (while keeping the gravitino mass fixed). Now that we have some control on the general rigid limit procedure, in this chapter we shall proceed summarizing some aspects of $N = 2 \to N = 1$ partial supersymmetry breaking.

4.1 How to partially break SUSY

In the case of interest for us, asking for a partially broken $N = 2$ supersymmetry means requiring that the supersymmetry transformations (2.54) get a vanishing background value for the $N = 1$ unbroken supersymmetry together with a non-zero value for the broken one. Namely, if we call the parameters for the unbroken and broken supersymmetries ϵ_1 and ϵ_2 respectively, we must have

$$\langle \delta \Psi_{\mu A} \rangle = \langle \delta \lambda^{iA} \rangle = \langle \delta \zeta_\alpha \rangle = 0 \tag{4.1}$$

for a vacuum with $\epsilon_1 \neq 0$ and $\epsilon_2 = 0$, and

$$\langle \delta \Psi_{\mu A} \rangle \neq 0 \qquad \langle \delta \lambda^{iA} \rangle \neq 0 \qquad \langle \delta \zeta_\alpha \rangle \neq 0 \tag{4.2}$$

for $\epsilon_1 = 0$ and $\epsilon_2 \neq 0$. The problem consists in finding the properties of the couplings S_{AB}, W^{iAB} and N^A_α such that (4.2) are satisfied. These properties are strictly related to the isometries of the scalar manifold $\mathbf{M}$, as it should be clear looking at the explicit forms (2.57).

4.1.1 Physical requirements

Before giving the solution found in [16], we review some physical statements about the general ingredients of partial $N = 2 \to N = 1$ supersymmetry breaking [51, 53]. A partial breaking of $N = 2$ supersymmetry implies giving a mass to one of the gravitinos, let us say m_{ψ_2}, leaving the other one massless. Moreover, since the vacuum is $N = 1$ supersymmetric, the massive gravitino must be part of a massive spin-3/2 multiplet $s = (3/2, 1, 1, 1/2)$. Then also two massive vectors A^0_μ, A^1_μ and a massive fermion χ should be part of the spectrum. Similarly, besides the would-be Goldston fermion (the goldstino) eaten by the gravitino, two would-be Goldstone bosons (sGoldstinos) must be eaten by the vectors that become massive [54]. The minimal content of $N = 1$ *massless* multiplets that can realize a *massive* 3/2-multiplet is then

a) one $N = 1$ spin-3/2 multiplet (3/2, 1)
b) one $N = 1$ vector multiplet (1, 1/2)
c) one $N = 1$ chiral multiplet (1/2, 0, 0).

This matter content should now be expressed in terms of $N = 2$ multiplets. The vector and the chiral multiplets seem to fit well in a non-abelian $N = 2$ vector multiplet, but vector multiplet scalars are singlet under the $SU(2)$ R-symmetry of $N = 2$ supergravity, therefore they cannot yield the splitting of the gravitino multiplet masses. For an abelian theory, one must then take the sGoldstinos scalars from an $N = 2$ charged hypermultiplet, while the other gauge boson besides the one in the gravitational multiplet requires at least one vector multiplet. In summary, in terms of massless $N = 2$ multiplets we need at least

a) one $N = 2$ gravitational multiplet (2, 3/2, 3/2, 1)
b) one $N = 2$ vector multiplet (1, 1/2, 1/2, 0, 0)
c) one $N = 2$ hypermultiplet (1/2, 1/2, 0, 0, 0, 0).

From a geometric perspective, the main point is that the two eaten sGoldistinos in the hypermultiplet correspond to *two gauged commuting isometries on the* *hypermultiplet moduli space* M_h. If we define the Killing vectors of these two isometries as k_1^u, k_2^u, from the Killing prepotential equation (2.59) it is also implied the existence of two non-zero Killing prepotentials (momentum maps)

$$P_1^x, \quad P_2^x \tag{4.3}$$

in the ground state. It should also be required for the two momentum maps not to be proportional to each other, otherwise a linear combination of the two Killing vectors could give a vanishing P^x.

These minimum requirements are only necessary conditions for the partial supersymmetry breaking to take place, but are not a proof that the breaking is possible. Indeed, the no-go theorem of [8] even forbid it, under certain assumptions. In the following sections we shall assume that everything takes place in a hidden sector. Since there is no visible sector to distinguish from, with a slight abuse of notation we will make use of the general indices (check Appendix A).

4.2 Two into one will go, after all

4.2.1 The no-go theorem and its loophole

In the original paper "Two into one won't go" [8], using the conformal tensor calculus it was shown that for gauged $N = 2$ supergravities containing only electric charges, a partial supersymmetry breaking is impossible. In particular, in this case the gravitino mass matrix is degenerate, meaning that the supersymmetries can be both broken or both preserved, with no chances for $N = 1$ vacua. Let us briefly sketch the argument (for a full review see [16]). Denoting ϵ_1 the parameter of the unbroken supersymmetry and ϵ_2 that of the broken one and expressing them as vectors in the supersymmetry parameters space, in a Minkowski background we have the conditions

$$W_{iAB}\epsilon_1^B = N_{\alpha A}\epsilon_1^A = S_{AB}\epsilon_1^B = 0 \tag{4.4}$$

for the unbroken SUSY and

$$W_{iAB}\epsilon_2^B \neq 0 \quad \text{or} \quad N_{\alpha A}\epsilon_2^A \neq 0, \quad \text{and} \quad S_{AB}\epsilon_2^B \neq 0 \tag{4.5}$$

for the broken one. In a point of the moduli space $(X_0^I,\, q_0^u)$ in which (4.4) and (4.5) hold, supersymmetry is partially broken to $N = 1$. For pure electric gaugings the embedding tensor selects only the first components of the symplectic vector

V^Λ (namely, F_I does not appear) and the gravitino variation (2.57a) reads (in a Minkowski background)

$$S_{AB}\epsilon_1^B = \frac{1}{2}e^{K/2}X^I\Theta_I{}^\lambda P_\lambda^x\sigma^x_{AB}\epsilon_1^B = 0, \tag{4.6}$$

while the complex conjugate of the gaugino variation (see (2.54)) is[1]

$$\begin{aligned} W_{iAB}\epsilon_1^B &= ie^{K/2}(\nabla_i X^I)\Theta_I{}^\lambda P_\lambda^x\sigma^x_{AB}\epsilon_1^B \\ &= ie^{K/2}(\partial_i X^I)\Theta_I{}^\lambda P_\lambda^x\sigma^x_{AB}\epsilon_1^B = 0, \end{aligned} \tag{4.7}$$

where in both cases we suppressed the index 0 from the scalar fields for simplicity. In the second line of (4.7) the usual expression for the Kähler covariant derivative was used and then (4.6) was inserted. This is a system of $2(n_v+1)$ equations, that can be further simplified using special coordinates to yield

$$\Theta_I{}^\lambda P_\lambda^x\sigma^x_{AB}\epsilon_1^B = 0. \tag{4.8}$$

Because the momentum maps P^x are real, the only complex quantity in (4.8) is $\sigma^x_{AB}\epsilon_1^B$, and from (4.8) and its complex conjugate follows

$$\Theta_I{}^\lambda P_\lambda^x = 0. \tag{4.9}$$

Substituting this expression back in the fermion variations (2.54), we would get $S_{AB} = W_{iAB} = 0$. Since we need a non vanishing background value for the parameter of the broken supersymmetry, this means that a partial breaking is impossible. A crucial ingredient in the proof given in [8] was the possibility of choosing special coordinates, related to the existence of a prepotential (see (2.3)). Indeed, the first counterexamples to the no-go theorem made use of symplectic frames in which the prepotential does not exist. But from such frames it is always possible to reach a new frame where a prepotential does exist via a symplectic transformation, at the price of rotating electric and magnetic charges into each other. Starting from a frame with only electric charges, as in the no-go theorem case, this means considering frames in which magnetic charges are present as well. Hence, the no-go argument has been reconsidered (e.g. [16]) with both electric and magnetic charges. The expression (4.7) for the gaugino variation now becomes

$$e^{K/2}(\partial_i X^I - \partial_i F_I\Theta^{I\lambda})P_\lambda^x\sigma^x_{AB}\epsilon_1^B = 0, \tag{4.10}$$

[1] Note that since we are in the hidden sector and we assume the isometries of the vectors to be visible, the Killing vectors for the vector moduli space are not present.

where $\Theta^{I\lambda}$ are now the magnetic charges and F_I, the derivative of the prepotential, is the component of the symplectic section being absent before. After some manipulations (see [16]), the analogous of (4.8) reads:

$$\begin{aligned}(\Theta_I{}^{\lambda} - F_{IJ}\Theta^{J\lambda})P^x_\lambda \sigma^x_{AB}\epsilon_1^B &= 0,\\ (\Theta_I{}^{\lambda} - \bar{F}_{IJ}\Theta^{J\lambda})P^x_\lambda \sigma^x_{AB}\epsilon_1^B &\neq 0.\end{aligned} \tag{4.11}$$

It is apparent that the presence of the magnetic charges avoids the final step of the no-go theorem (which implied $\Theta^{J\lambda}P^x_\lambda = 0$). The solutions for the embedding tensor such that both (4.11) are fulfilled were found in [16], and we shall give them in the last section of this chapter. In the next section we shall review instead the explicit example of partial supersymmetry breaking given in [12], derived in a symplectic frame in which the prepotential does not exist.

4.2.2 Partial breaking in a no-prepotential frame: an example

The spectrum of this $N = 2$ supergravity model consists of the minimum content necessary for the partial breaking, as we discussed before: the gravitational multiplet, a charged hypermultiplet and a vector multiplet. The hypermultiplet scalars span the quaternionic manifold $SO(4,1)/SO(4)$ and the vector scalars the K¨ $SU(1,1)/U(1)$, described in a symplectic frame with no prepotential. The defining elements are then the geometries of the quaternionic manifold ahler manifolds, together with the prepotentials P^x_Λ [27]. We denote as usual the coordinates on the quaternionic manifold by q^u, with in this case $u = 0, 1, 2, 3$. The $SU(2)$ connections and related curvatures (field strengths) of the quaternionic geometry read

$$\omega^x_u = \frac{1}{q^0}\delta^x_u, \quad \Omega^x_{0u} = -\frac{1}{2q^{0\,2}}\delta^x_u, \quad \Omega^x_{yz} = -\frac{1}{2q^{0\,2}}\epsilon^{xyz}, \quad x, y, z = 1, 2, 3. \tag{4.12}$$

From the $SU(2)$ field strengths we can derive the metric h_{uv} using the relation $h^{st}\Omega^x_{us}\Omega^y_{tv} = -\delta^{xy}h_{uv} - \epsilon^{xyz}\Omega^z_{uv}$:

$$h_{uv} = \frac{1}{2q^{0\,2}}\delta_{uv}, \tag{4.13}$$

and the vielbein $\mathcal{U}^{\alpha A}$ (with $\alpha, A = 1, 2$)

$$\mathcal{U}^{\alpha A} = \frac{1}{2q^0}\epsilon^{\alpha\beta}(dq^0 - i\sigma^x db^x)_\beta^A. \tag{4.14}$$

The Kähler potential (2.19) on the manifold $SU(1,1)/U(1)$ (with $I = 0, 1$) is determined by the symplectic components

$$X^0(t) = -\frac{1}{2}, \quad X^1(t) = \frac{i}{2} \quad F_0 = it, \quad F_1 = t \tag{4.15}$$

which yield Kähler potential and related metric

$$\begin{aligned} K &= -\ln(t + \bar{t}), \\ g_{t\bar{t}} &= \frac{1}{(t + \bar{t})^2}. \end{aligned} \tag{4.16}$$

Note again that in this frame the symplectic component F_I are not derivative of a function F because the prepotential does not exist.

The hypermultiplet manifold has the global isometry $U(1)^2$, where the first $U(1)$ comes from the $N = 2$ graviphoton and the second one from the matter vector. Thus, we have two commuting isometries that must be gauged, in accord with the prescriptions of the previous section. Since the metric (4.13) is invariant under translation of the coordinates q^1, q^2, q^3, we can choose to gauge for instance the translations along q^1 and q^2, with related Killing vectors

$$k_0^u = g\delta^{u1}, \quad k_1^u = g'\delta^{u2}, \tag{4.17}$$

where g and g' are the gauge couplings of the two $U(1)'s$. From equation (2.59) we can derive the respective Killing prepotentials

$$P_0^x = g\frac{1}{q^0}\delta^{x1}, \quad P_1^x = g'\frac{1}{q^0}\delta^{x2}. \tag{4.18}$$

Finally we can compute the explicit expressions for the gaugino, hyperino and gravitino variations (2.54) in terms of the matrices

$$\begin{aligned} W_{AB}^{\bar{t}} = -i(t + \bar{t})^{1/2}\frac{1}{q^0}X_{AB}, \quad N_A^\alpha &= -i(t + \bar{t})^{-1/2}\frac{1}{q^0}\epsilon^{\alpha\beta}X_{\beta A}, \\ S_{AB} &= -\frac{1}{2}(t + \bar{t})^{-1/2}\frac{1}{q^0}X_{AB}, \end{aligned} \tag{4.19}$$

with X_{AB} given by

$$X_{AB} = -\frac{g}{2}(\sigma^1)_A^C\epsilon_{CB} + i\frac{g'}{2}(\sigma^2)_A^C\epsilon_{CB} = \begin{pmatrix} \frac{g'-g}{2} & 0 \\ 0 & \frac{g'+g}{2} \end{pmatrix}. \tag{4.20}$$

If we insert these definitions into the scalar potential (2.56), we find $\mathcal{V} = 0$, meaning that this model has vanishing potential (hence cosmological constant) for arbitrary g and g'. Putting now $g = g' \neq 0$ and considering the gravitino mass matrix $2S_{AB}$, whose eigenvalues are the masses of the gravitinos, we see that the first gravitino is massless, corresponding to the unbroken supersymmetry, while the second one is massive, corresponding to the broken one. Thus, in this model $N = 2$ supersymmetry is partially broken to $N = 1$, at the price of a symplectic frame in which no prepotential exists.

4.2.3 Spontaneous partial supersymmetry breaking: embedding tensor solutions

Gravitino and gaugino variations

We shall now review the solutions of (4.11), valid in a Minkowski background. As we anticipated in the first section, for a partial breaking to occur we need at least two commuting isometries. In fact, if we only consider one isometry k_1, (4.11a) splits in two factors, and in order to satisfy the equation either $({\Theta_I}^1 - F_{IJ}\Theta^{J1})$ or $P_1^x \sigma^x_{AB} \epsilon_1^B$ must be zero, while (4.11b) requires both factors to be non-zero. Thus, having only one isometry is compatible with full $N = 2$ or with a completely broken supersymmetry; a partial breaking cannot occur. Going now to two isometries and remembering that the two momentum maps must be linearly independent, the analysis is simplified choosing an $SU(2)$ frame in which P_1^x and P_2^x lie on the $x = 1, 2$ plane and defining the complex combinations

$$P^+_\lambda = P^1_\lambda + iP^2_\lambda, \qquad P^-_\lambda = P^1_\lambda - iP^2_\lambda. \tag{4.21}$$

Equations (4.11) become

$$\begin{aligned} &({\Theta_I}^1 - F_{IJ}\Theta^{J1})P_1^- + ({\Theta_I}^2 - F_{IJ}\Theta^{J2})P_2^- = 0, \quad \text{for all I,} \\ &({\Theta_I}^1 - \bar{F}_{IJ}\Theta^{J1})P_1^- + ({\Theta_I}^2 - \bar{F}_{IJ}\Theta^{J2})P_2^- \neq 0 \quad \text{for some I.} \end{aligned} \tag{4.22}$$

The solutions for the embedding tensor in a $N = 1$ Minkowski vacuum are then [16]

$$\begin{aligned} \Theta_I^1 &= -\mathrm{Im}(P_2^+ F_{IJ} C^J), \quad & \Theta^{I1} &= -\mathrm{Im}(P_2^+ C^I), \\ \Theta_I^2 &= \mathrm{Im}(P_1^+ F_{IJ} C^J), \quad & \Theta^{I2} &= \mathrm{Im}(P_1^+ C^I), \end{aligned} \tag{4.23}$$

where C^I is an arbitrary complex vector. The requirement that the P's should not be proportional is essential here in assuring that (4.11b) does not vanish for an arbitrary C. Imposing also the mutual locality condition (2.50b), which for two isometries become

$$\Theta^{I1}\Theta_I^2 - \Theta^{I2}\Theta_I^1 = 0, \tag{4.24}$$

the complex vector C is found to obey the constraint

$$\bar{C}^I(\mathrm{Im}F)_{IJ}C^J = 0, \tag{4.25}$$

derived inserting (4.23) into (4.24). $(\mathrm{Im}F)_{IJ}$ has signature $(n_v, 1)$ (see e.g. [16]), hence this constraint can be always fulfilled. The existence of a solution for the embedding tensor shows that the partial supersymmetry breaking is indeed possible. Note again that this result was obtained in a symplectic frame in which both magnetic and electric charges exist. Nevertheless, as stated before, it is always possible to rotate the charges via a symplectic transformation in a way that only electric charges are present. The partial breaking must still occur in this frame, but according to the no-go theorem, in a pure electric frame in which a prepotential exists, the partial breaking should be impossible. This means that in this new frame a prepotential cannot exist (for a proof see again [16]).

In the next chapter we want to consider a general vacuum in which $N = 2$ is broken to $N = 1$, and since until now we specialized to the case with a minimum amount of commuting isometries (two) needed to realize the partial breaking, it is worth it to ask what happens in the case of an arbitrary number of isometries (namely larger than two). If we have n gauged commuting isometries, it is always possible to find a basis of Killing vectors in which $P^x_\lambda \neq 0$ for only three vectors k^λ in the vacuum. Moreover, the three non-vanishing P^x_λ are subject to the constraint (4.11a), telling us that at least one combination of the P's has to be zero. At the end, we are again in the situation in which only two Killing vectors (with their prepotentials) participate in the breaking, while the others could give rise at most to additional masses (since the derivatives of the $P's$ are not obviously zero). It should be noted that fulfilling the partial breaking conditions on the gravitino and the gaugino variations does not require to be in a specific point of the moduli space. The result is in fact background independent as long as the hypermultiplet space supports two commuting isometries and the two Killing prepotentials are not proportional to each other in the vacuum.

Let us now revisit the minimal example with $\mathbf{M}_h = SO(1,4)/SO(4)$ [12] reviewed in section (4.2.2). Inserting the Killing prepotentials $P^x_\lambda = \frac{g}{q^0}\delta^x_\lambda$ (with $g = g'$) into the embedding tensor solution (4.23), it becomes

$$\begin{aligned} \Theta_I{}^1 &= -\mathrm{Re}(F_{IJ}C^J), \quad \Theta^{I1} = -\mathrm{Re}C^I, \\ \Theta_I{}^2 &= \mathrm{Im}(F_{IJ}C^J), \quad \Theta^{I2} = \mathrm{Im}C^I. \end{aligned} \tag{4.26}$$

Using now (4.26), it is possible to show that $N_{\alpha A}\epsilon_1^A = 0$ holds automatically and we retrieve the same $N = 1$ partially broken vacuum. The important difference is that while in [12] a precise vector multiplet moduli space $\mathbf{M}_v$ was specified, in this case the result holds for a general $\mathbf{M}_v$: the two commuting isometries on $\mathbf{M}_h$ are the only geometric requirement. Studying the general case for every possible $\mathbf{M}_h$ admitting two commuting isometries is quite difficult. In [16] it is considered the situation in which $\mathbf{M}_h$ is a special quaternionic manifold. There, the embedding tensor solution (4.23), with the added constraints $N_{\alpha A}\epsilon_1^A = 0$ and the specific Killing prepotentials for a special quaternionic space, becomes identical to (4.26).

To summarize, we recalled that a spontaneous partial breaking $N = 2 \rightarrow N = 1$ is in general possible if we consider symplectic frames with both magnetic and electric charges, and we presented the solution (4.23) for the embedding tensor with generic Killing prepotentials P^x. We state again that these solutions are valid at any point of the moduli space $\mathbf{M}_v \times \mathbf{M}_h$. The reader should also be aware that all the results presented here have been derived for a Minkowski background; for the AdS case see [16]. In the next section we shall assume an $N = 1$ spontaneously broken vacuum as a starting point, and the solution (4.23) implicit.

Chapter 5

Soft partial breaking

Let us summarize the story so far. In Chapter 3 we learned how to compute the limit $M_{\rm Pl} \to \infty$ of a general $N = 2$ supergravity. Then, in Chapter 4, we have reviewed the possibility of partial supersymmetry breaking and recalled the embedding tensor solutions for an $N = 1$ vacuum of a spontaneously broken $N = 2$ (gauged) supergravity. Now it is time to merge the two ideas: we shall start from a spontaneously broken $N = 2$ gauged supergravity and derive from the scalar potential the mass terms which softly break a global $N = 2$ theory to $N = 1$. This is achieved by studying the rigid limit of the generic $N = 1$ vacua discussed in Chapter 4. But if we let the mass of the gravitino corresponding to the broken supersymmetry to be of order $M_{\rm Pl}$, in the limit $M_{\rm Pl} \to \infty$ we would get again the $N = 2$ global limit obtained in Chapter 3. The idea is then to perform the rigid limit while keeping the gravitino mass fixed at an intermediate scale $\Lambda << M_{\rm Pl}$, a procedure introduced in [55] and also performed in [20] for the $N = 1$ case, whose results we are going to summarize in the following section.

5.1 Soft terms from $N = 1$ supergravity

[20] carried out a model-independent analysis of the soft terms arising from a general $N = 1$ supergravity, where the fields are separated in two sectors: a hidden sector (at an intermediate scale M_{hid} much above the electroweak scale but much smaller than the Planck scale), and a visible sector which contains the Standard Model and its possible extensions. The scalar potential of such a theory has in general flat directions at the perturbative level, which can be parametrized by 'moduli' fields Φ (with jargon inspired by string theory). The spontaneous supersymmetry breaking is assumed to occur in the moduli sector, while the hidden

sector is integrated out, yielding an effective scalar potential for the moduli fields (assumed to be zero at the minimum) which reads

$$V^{eff}(\Phi,\bar{\Phi}) = \kappa^{-2}\hat{K}_{\Phi\bar{\Psi}}F^{\Phi}\bar{F}^{\bar{\Psi}} - 3\kappa^2 e^{\hat{K}}|\hat{W}|^2, \tag{5.1}$$

where all the hatted quantities are functions of the moduli only and

$$\bar{F}^{\bar{\Psi}} = \kappa^2 e^{\hat{K}/2}\hat{K}^{\bar{\Psi}\Phi}(\partial_\Phi \hat{W} + \hat{W}\partial_\Phi \hat{K}), \qquad \hat{K}^{\bar{\Psi}\Phi} = (\hat{K}_{\Phi\bar{\Psi}})^{-1}. \tag{5.2}$$

The order parameter F^{Φ} is a function of the Kähler potential $\hat{K}$, the superpotential $\hat{W}$ and their derivatives, and the spontaneous supersymmetry breaking occurs since for some Φ, $\langle F^{\Phi}\rangle \neq 0$. As a result, the gravitino acquires a mass given by

$$m_{3/2} = \kappa^2 e^{\langle\hat{K}\rangle/2}|\hat{W}(\langle\Phi\rangle)| = \langle\frac{1}{3}\hat{K}_{\Phi\bar{\Psi}}F^{\Phi}\bar{F}^{\bar{\Psi}}\rangle^{1/2} \sim \frac{M^3_{hid}}{M^2_{Pl}}. \tag{5.3}$$

The unbroken $N=1$ effective theory for the visible fields is described by an effective superpotential $W^{(eff)}$ and a Kähler potential K^v. $W^{(eff)}$ is given by

$$W^{(eff)}(Q) = \frac{1}{2}\mu_{IJ}Q^I Q^J + \frac{1}{3}Y_{IJL}Q^I Q^J Q^L, \tag{5.4}$$

where the (non-holomorphic functions of the moduli) μ_{IJ} and Y_{IJL} are the unnormalized masses of the visible fermions and their Yukawa coupling respectively. Similarly, expanding the Kähler potential in terms of the visible fields one has

$$K^v \sim M^2_{\rm Pl}K^0(\Phi,\bar{\Phi}) + Z_{I\bar{J}}(\Phi,\bar{\Phi})Q^I\bar{Q}^{\bar{J}} + \frac{1}{2}\left[H_{IJ}(\Phi,\bar{\Phi})Q^I Q^J + \bar{H}_{\bar{I}\bar{J}}(\Phi,\bar{\Phi})\bar{Q}^{\bar{I}}\bar{Q}^{\bar{J}}\right] + \ldots, \tag{5.5}$$

with $Z_{I\bar{J}}(\Phi,\bar{\Phi})$, $H_{IJ}(\Phi,\bar{\Phi})$ and $\bar{H}_{\bar{I}\bar{J}}(\Phi,\bar{\Phi})$ dimensionless functions of the moduli. In order to derive the broken effective theory for the observable fields, the moduli fields are frozen around their background value and the rigid limit $M_{\rm Pl}\to\infty$ is taken while keeping the gravitino mass $m_{3/2}$ fixed. The effective potential for the observable scalars takes then the form

$$\begin{aligned} V^{(eff)}(Q,\bar{Q}) = &\sum_{a\in G^{(obs)}}\frac{g_a^2}{4}(\bar{Q}^{\bar{I}}Z_{\bar{I}J}T_a Q^J)^2 + \partial_I W^{(eff)}Z^{I\bar{J}}\partial_{\bar{J}}\bar{W}^{(eff)} \\ &+ m^2_{I\bar{J}}Q^I\bar{Q}^{\bar{J}} + (\frac{1}{3}A_{IJL}Q^I Q^J Q^L + \frac{1}{2}B_{IJ}Q^I Q^J + \text{h.c.}), \end{aligned} \tag{5.6}$$

In the first line of (5.6) we observe the scalar potential of a global $N=1$ supersymmetric theory, while in the second line the supersymmetry breaking soft terms appear, with coefficient matrices given by

$$\begin{aligned} m^2_{I\bar{J}} &= m^2_{3/2} Z_{I\bar{J}} - F^{\Phi} \bar{F}^{\bar{\Psi}} R_{\Phi\bar{\Psi}I\bar{J}}, \\ A_{IJL} &= F^{\Phi} D_{\Phi} Y_{IJL}, \\ B_{IJ} &= F^{\Phi} D_{\Phi} \mu_{IJ} - m_{3/2} \mu_{IJ}, \end{aligned} \tag{5.7}$$

where

$$\begin{aligned} R_{\Phi\bar{\Psi}I\bar{J}} &= \partial_{\Phi} \partial_{\bar{\Psi}} Z_{I\bar{J}} - \Gamma^{N}_{\Phi I} Z_{N\bar{L}} \bar{\Gamma}^{\bar{L}}_{\bar{\Psi}\bar{J}}, \quad \Gamma^{N}_{\Phi I} = Z^{N\bar{J}} \partial_{\Phi} Z_{\bar{J}I}, \\ D_{\Phi} Y_{IJL} &= \partial_{\Phi} Y_{IJL} + \frac{1}{2} \hat{K}_{\Phi} Y_{IJL} - \Gamma^{N}_{\Phi(I} Y_{JL)N}, \\ D_{\Phi} \mu_{IJ} &= \partial_{\Phi} \mu_{IJ} + \frac{1}{2} \hat{K}_{\Phi} \mu_{IJ} - \Gamma^{N}_{\Phi(I} \mu_{J)N}. \end{aligned} \tag{5.8}$$

$R_{\Phi\bar{\Psi}I\bar{J}}$ is a sub-tensor of the curvature tensor of the Kähler manifold describing the scalar field space in $N = 1$, and D_{Φ} indicates the Kähler covariant derivative.

We shall keep in mind these results in the following.

5.2 Preliminaries and assumptions

Our starting point is again the gauged lagrangian (2.55) with scalar potential (2.56). The general strategy is the following. In the same spirit of [20, 21] and Chapter 3, we assume the existence of two field sectors: the "hidden" one, consisting of the gravitationally coupled fields and scalars with background values of order $M_{\rm Pl}$, and the "visible" one, containing all the low energy non-trivial dynamics and whose scalars can have a background value much smaller than $M_{\rm Pl}$. We assume the partial supersymmetry breaking to take place in the hidden sector, hence we consider a moduli space $\mathbf{M}_v \times \mathbf{M}_h$ where the hypermultiplet moduli space $\mathbf{M}_h$ admits at least two commuting isometries (see Chapter 4) in the hidden sector. We split accordingly the isometry group $G_{SK} \times G_{QK}$ of the moduli space into a hidden and a visible parts. As we already mentioned in section (3.4), where we studied the rigid limit for the full $N = 2$ gauged supergravity, we can assume without significant loss that the isometry group G_{SK} of the vectors is entirely visible, while for G_{QK} we have

$$G_{QK} = G^{hid}_{QK} \times G^{vis}_{QK}, \tag{5.9}$$

with gauge couplings g' and g respectively. The two commuting isometries are then in G^{hid}_{QK}. This group can in principle have more than two generators, but as we discussed in (4.2.3), it is always possible to go to a basis of Killing vectors in which only two Killing prepotentials are non-vanishing. As we did in (3.4), we assume the hidden fields to be charged only under the hidden isometry group and the visible fields only under the visible one, so that the embedding tensor takes the block diagonal form (3.33).

When $N = 2$ supersymmetry is partially broken to $N = 1$, the second gravitino acquires a mass which we want to keep fixed in the limit $M_{\rm Pl} \to \infty$. One may think that a way to achieve this is to assume the existence of a third intermediate sector between the hidden and the visible ones, with scalar background values $\phi_0 << \Lambda << M_{\rm Pl}$ (where ϕ_0 is the possible vacuum expectation value of the visible scalars) and define the order of the gravitino mass as $m_{3/2} \sim \Lambda^3/M_{\rm Pl}^2$ with a compensating limit on Λ such that $m_{3/2}$ remains finite in the double limit, in the same spirit of [20] for the $N = 1$ case. But as we show in Appendix B, in this way there is no chance to achieve a Minkowski vacuum in the partially broken $N = 1$ theory. Instead, here we stick to the existence of only two sectors, hidden and visible, with vacuum expectation values of order $\Phi_0 \sim M_{\rm Pl}$ and $\phi_0 << M_{\rm Pl}$ respectively, and we define the gravitino mass scale (which we want to keep fixed in the limit) as

$$m_{3/2} \sim g' M_{\rm Pl}, \tag{5.10}$$

where g' is again the gauge coupling constant of the hidden sector. Hence, in order to keep $m_{3/2}$ fixed when $M_{\rm Pl} \to \infty$, we also perform the formal limit $g' \to 0$.

According to the notation introduced in (3.4), the Killing vectors of the theory are

$$\begin{array}{lcc} \text{hypermultiplets:} & \text{(hidden)} & \text{(visible)} \\ & k_\gamma^a & k_\nu^w \\ \text{vector multiplets:} & & \text{(visible)} \\ & & k_\iota^m . \end{array} \tag{5.11}$$

We can also write

$$k_\gamma^a = g' \hat{k}_\gamma^a \qquad k_\nu^w = g \hat{k}_\nu^w , \tag{5.12}$$

where we made explicit the dependence of the prepotential on the hidden and visible gauge couplings g' and g.[1] Among the k_γ^a's, we denote the two Killing vectors (or combination of Killing vectors) related to the two commuting isometries as k_1^a and k_2^a. Similarly, we define the hidden and visible Killing prepotentials by

$$P_\gamma^x = g' \hat{P}_\gamma^x , \qquad P_\nu^x = g \hat{P}_\nu^x , \tag{5.13}$$

[1] The role of g' and g here is different from the one in the example (4.2.2): there the two couplings were both in the hidden sector.

and the two prepotentials related to the two commuting Killing vectors as P_1^x and P_2^x. We also use the convenient complex combinations (4.21),

$$P_\lambda^+ = P_\lambda^1 + iP_\lambda^2, \qquad P_\lambda^- = P_\lambda^1 - iP_\lambda^2,$$

where λ runs here over both hidden and visible isometries. However, it should be noted that in Chapter 4 everything was defined in the hidden sector: there we can always choose a basis of Killing vectors where only two P_σ^x are non-zero and an $SU(2)$ frame such that P_1^x and P_2^x lie on the $x = 1, 2$ plane. In this case we need to consider also the visible sector, and while we can continue using the combinations (4.21), we must take into account the contribution from P_ν^3 as well. In this thesis we focus on a Minkowski background, thus we assume a vanishing cosmological constant. In other words,

$$\langle \mathcal{V}(\Phi, \bar{\Phi}) \rangle = 0, \tag{5.14}$$

where Φ is the set of fields in the hidden sector.

We shall proceed with the study of the soft terms for each piece of the scalar potential $\mathcal{V}$ separately. After the limit we expect to find the $N = 2$ global limit plus the soft terms generated by the partial breaking.

5.3 Gravitino variation term

We consider now the contribution from the $-\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB}$ part of the scalar potential. As we have seen in Section 3.4, this term is subleading and vanishes for $M_{\rm Pl} \to \infty$, but if we now perform the limit keeping the gravitino scale fixed (namely we also take $g' \to 0$) this piece becomes relevant. Let us start studying the matrix S_{AB} explicitly. We can write

$$\begin{aligned}
S_{AB} &= \\
&= \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda \Theta_\Lambda{}^\lambda P_\lambda^x (\sigma^x)_{AB} \\
&= \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda \Theta_\Lambda{}^\lambda [P_\lambda^1 (\sigma^1)_{AB} + P_\lambda^2 (\sigma^2)_{AB} + P_\lambda^3 (\sigma^3)_{AB}] \\
&= \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda \Theta_\Lambda{}^\lambda \left[P_\lambda^1 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + P_\lambda^2 \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} + P_\lambda^3 \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \right] \\
&= \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda \Theta_\Lambda{}^\lambda \left[\begin{pmatrix} P_\lambda^- & 0 \\ 0 & -P_\lambda^+ \end{pmatrix} - \begin{pmatrix} 0 & P_\lambda^3 \\ P_\lambda^3 & 0 \end{pmatrix} \right] \\
&= \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Lambda \Theta_\Lambda{}^\lambda \begin{pmatrix} P_\lambda^- & -P_\lambda^3 \\ -P_\lambda^3 & -P_\lambda^+ \end{pmatrix} .
\end{aligned} \tag{5.15}$$

In the second step we used (2.60), in the third line we inserted in the definitions (2.58) for the σ-matrices and in the last steps we made use of the complex combinations (4.21). Here we let again the λ index to run over both the hidden sector and the visible sector isometries, but as we discussed before, we can always consider P_λ^3 vanishing in the hidden sector, namely

$$P_\gamma^3 = 0, \qquad P_\nu^3 \neq 0. \tag{5.16}$$

If we now take the vacuum expectation value of S_{AB} and neglect the possible background values of the visible fields ($\phi_0 << M_{\rm Pl}$), the only relevant contribution comes from the hidden sector and reads

$$\langle S_{AB} \rangle = \begin{pmatrix} \langle \frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Sigma \Theta_\Sigma{}^\sigma P_\gamma^- \rangle & 0 \\ 0 & \langle -\frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Sigma \Theta_\Sigma{}^\gamma P_\gamma^+ \rangle \end{pmatrix} . \tag{5.17}$$

The eigenvalues of $\langle S_{AB} \rangle$ are the masses of the two gravitini [26]. Leaving implicit the partial breaking solutions for the embedding tensor [16] given in (4.23), the first supersymmetry remains unbroken, the second one is broken and we are left with a non-zero mass for the second gravitino:

$$|\langle S_{AB} \rangle| = \begin{pmatrix} 0 & 0 \\ 0 & m_{3/2} M_{\rm Pl}^2 \end{pmatrix} , \tag{5.18}$$

where the gravitino mass is given by[2]

$$m_{3/2} \equiv |\langle -\frac{i}{2M_{\rm Pl}^2} e^{\frac{K}{2M_{\rm Pl}^2}} V^\Sigma \Theta_\Sigma{}^\gamma P_\gamma^+ \rangle|. \tag{5.19}$$

The background value of the full scalar potential term $-\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB}$ is then

$$\langle -\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB} \rangle = -12 m_{3/2}^2 M_{\rm Pl}^4. \tag{5.20}$$

The solution (4.23) for a Minkowski background guarantees the vacuum expectation value of the positive definite part of the scalar potential (2.56) to be

$$\langle g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB} + 2 N_\alpha^A N_A^\alpha \rangle = 12 m_{3/2}^2 M_{\rm Pl}^4, \tag{5.21}$$

so that $\langle \mathcal{V} \rangle = -12 m_{3/2}^2 M_{\rm Pl}^4 + 12 m_{3/2}^2 M_{\rm Pl}^4 = 0$.

From now on it will be useful to simplify the notation defining

$$\begin{aligned} &\Theta_\Sigma{}^\gamma P_\gamma^\pm \equiv P_\Sigma^\pm \qquad \Theta_M{}^\nu P_\nu^\pm \equiv P_M^\pm \\ &\Theta_M{}^\nu P_\nu^3 \equiv P_M^3 \end{aligned} \tag{5.22}$$

for the hidden and visible P's respectively. In the background V^Σ and P_Σ^+ get a vacuum expectation value of order

$$\langle V^\Sigma \rangle \sim O(M_{\rm Pl}), \qquad \langle P_\Sigma^+ \rangle \sim O(g' M_{\rm Pl}^2), \tag{5.23}$$

according to their mass dimensions. Therefore, the order of the gravitino mass is

$$m_{3/2} \sim O(g' M_{\rm Pl}). \tag{5.24}$$

Let us now consider again the $-\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB}$ term of the potential with the full S_{AB} (5.15) comprised of the hidden and visible sectors. Splitting the contributions from the two different sectors and considering again a partially broken supersymmetry we have

$$S_{AB} = \left(\frac{i}{2} e^{\frac{K}{2M_{\rm Pl}^2}} \right) \left[V^\Sigma \begin{pmatrix} 0 & 0 \\ 0 & -P_\Sigma^+ \end{pmatrix} + V^M \begin{pmatrix} P_M^- & -P_M^3 \\ -P_M^3 & -P_M^+ \end{pmatrix} \right]. \tag{5.25}$$

[2]The numeric coefficient may vary in the literature according to different conventions for the gravitino mass term $S_{AB} \bar{\psi}_\mu^A \gamma^{\mu\nu} \psi_\nu^B$.

Thus we can compute the full scalar potential term:

$$\begin{aligned}
&-\frac{12}{M_{\text{Pl}}^2} S_{AB} S^{AB} = \\
&= -\frac{3}{M_{\text{Pl}}^2} e^{\frac{K}{M_{\text{Pl}}^2}} \left[V^\Sigma \bar{V}^\Xi P_\Sigma^+ P_\Xi^- + V^\Sigma \bar{V}^N P_\Sigma^+ P_N^- + V^M \bar{V}^\Xi P_M^+ P_\Xi^- \right. \\
&\quad \left. + 2V^M \bar{V}^N (P_M^+ P_N^- + P_M^3 P_N^3) \right] \\
&= -\frac{3}{M_{\text{Pl}}^2} e^{\frac{K}{M_{\text{Pl}}^2}} \left[S^{(h)} \bar{S}^{(h)} + S^{(h)} \bar{S}^{(v)} + S^{(v)} \bar{S}^{(h)} + 2(S^{(v)} \bar{S}^{(v)} + S^{(v3)} \bar{S}^{(v3)}) \right] .
\end{aligned} \tag{5.26}$$

In Appendix C we performed the detailed calculation yielding the first step, while in the second step we made use of the definitions

$$\begin{aligned}
S^{(h)} &\equiv V^\Sigma P_\Sigma^+ & \bar{S}^{(h)} &\equiv \bar{V}^\Sigma P_\Sigma^- \\
S^{(v)} &\equiv V^M P_M^+ & \bar{S}^{(v)} &\equiv \bar{V}^M P_M^- \\
S^{(v3)} &\equiv V^M P_M^3 & \bar{S}^{(v3)} &\equiv \bar{V}^M P_M^3 .
\end{aligned} \tag{5.27}$$

Let us make some preliminary observations. The first term, $S^{(h)} \bar{S}^{(h)}$, receives contributions only from the hidden sector and belongs to $\mathcal{V}(\Phi, \bar{\Phi})$, which is vanishing in the background according to our assumption (5.14). The last term, $2(S^{(v)} \bar{S}^{(v)} + S^{(v3)} \bar{S}^{(v3)})$, contains only visible fields and vanishes in the rigid limit computed in section (3.4). Thus, we would expect the mixed terms to play a role now that we want to keep the gravitino fixed.

5.3.1 Kähler potential expansion

Before proceeding with the computation of the limit, let us expand the exponential factor $e^{\frac{K}{M_{\text{Pl}}^2}}$, where $K \equiv K^v$ is the Kähler potential (2.19). Separating the contribution from the hidden and visible fields, (2.19) becomes (in special coordinates)

$$K^v = -\ln i[C(\Phi, \bar{\Phi}) + \bar{t}^m F_m - t^n \bar{F}_n], \tag{5.28}$$

where $C(\Phi, \bar{\Phi})$ encodes the contribution of the hidden fields. Now we shall start expanding the prepotential F in terms of the visible scalars, and since the dimension-1 scalars Φ and t^m scale as

$$\Phi \sim O(M_{\text{Pl}}), \qquad t^m \sim O(1), \tag{5.29}$$

we have:

$$F(\Phi,t) \sim M_{\rm Pl}^2 F^0(\Phi) + F_{mn}t^m t^n + \dots. \tag{5.30}$$

$F^0(\Phi)$ is a function of the hidden fields only, and in order to fulfil our assumption (3.37) on the decoupling between hidden and visible sectors, F_{mn} must be a constant matrix.[3] Inserting (5.30) into (5.28) and expanding again, the Kähler potential reads

$$K^v \sim -M_{\rm Pl}^2 \ln(iC) - \frac{2}{C}(F_{mn} - \bar{F}_{\bar{m}\bar{n}})t^m \bar{t}^{\bar{n}} + \dots. \tag{5.31}$$

If we now define $K^0(\Phi,\bar{\Phi}) \equiv -\ln(iC)$, we can put $C^{-1} = ie^{K^0}$ and write

$$K^v \sim M_{\rm Pl}^2 K^0(\Phi,\bar{\Phi}) + Z_{m\bar{n}}(\Phi,\bar{\Phi})t^m \bar{t}^{\bar{n}} + \dots, \tag{5.32}$$

with

$$Z_{m\bar{n}} = -\frac{2}{C}(F_{mn} - \bar{F}_{\bar{m}\bar{n}}) = -2ie^{K^0}(F_{mn} - \bar{F}_{\bar{m}\bar{n}}) = 4e^{K^0(\Phi,\bar{\Phi})}{\rm Im}F_{mn}. \tag{5.33}$$

For the dimensionless matrix $Z_{m\bar{n}}$, which depend only on the hidden fields, we used the same convention as [20] (compare also [56], which adopts another convention for the fields). Note that the matrices H_{IJ} and $\bar{H}_{\bar{I}\bar{J}}$ appearing in the expansion (5.5) for the $N = 1$ case, are absent in the $N = 2$ case because of the structure of the Kähler potential (2.19) which describes a *special* Kähler manifold (only Kähler in $N = 1$). Inserting (5.32) into $e^{\frac{K}{M_{\rm Pl}^2}}$ and expanding we finally get

$$e^{\frac{K}{M_{\rm Pl}^2}} \sim e^{K^0}\left(1 + \frac{Z_{m\bar{n}}(\Phi,\bar{\Phi})t^m \bar{t}^{\bar{n}}}{M_{\rm Pl}^2} + \dots\right). \tag{5.34}$$

It should be also noted that the expansions (5.30) and (5.32) of the prepotential and the Kähler potential are different from (3.10) and (3.12). In Chapter 3 we expanded in terms of $M_{\rm Pl}$ in order to get the rigid expressions for the prepotential, the Kähler potential and the metric. Here we expand in terms of the visible fields without taking the proper rigid limit. Nevertheless, computing for example the visible part of the metric from (5.32), we would have

$$\partial_m \partial_{\bar{n}} K^v = Z_{m\bar{n}} + O\left(\frac{1}{M_{\rm Pl}^2}\right) + \dots. \tag{5.35}$$

Taking $M_{\rm Pl} \to \infty$ and freezing the hidden fields around their expectation values, $Z_{m\bar{n}}$ can be identified with the rigid metric. An important difference with the approach in Chapter 3 is that for simplicity we are ignoring additional scales (like

[3] Since the prepotential F is homogeneous of degree two, F^0 and F_{mn} are of degree two and zero respectively.

the Seiberg-Witten Λ), hence we do not consider combinations like (t/Λ) in the expansion (5.32) and non-flat metrics with respect to the visible fields are excluded.

5.3.2 Limit result and soft terms

Now we turn back to the expression (5.26) and we study the limit $M_{\rm Pl} \to \infty$ with $m_{3/2}$ fixed. According to (5.23) and the definitions (5.27) we can determine the orders of the various pieces, namely:

$$\begin{aligned} &S^{(h)} = g' M_{\rm Pl}^3 \hat{S}^{(h)} \qquad \bar{S}^{(h)} = g' M_{\rm Pl}^3 \bar{\hat{S}}^{(h)} \\ &S^{(v)} \sim O(1) \qquad \bar{S}^{(v)} \sim O(1) \qquad S^{(v3)} \sim O(1) \qquad \bar{S}^{(v3)} \sim O(1), \end{aligned} \tag{5.36}$$

where the hatted objects are dimensionless.

Truncating the expansion (5.34) before the dots and inserting it into (5.26), we get

$$\begin{aligned} &-\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB} = \\ &= -\frac{3}{M_{\rm Pl}^2} e^{\frac{K}{M_{\rm Pl}^2}} \left[S^{(h)} \bar{S}^{(h)} + S^{(h)} \bar{S}^{(v)} + S^{(v)} \bar{S}^{(h)} + 2(S^{(v)} \bar{S}^{(v)} + S^{(v3)} \bar{S}^{(v3)}) \right] \\ &= -\frac{3}{M_{\rm Pl}^2} e^{K^0} \left(1 + \frac{Z_{m\bar{n}} t^m \bar{t}^{\bar{n}}}{M_{\rm Pl}^2} \right) \Big[(g')^2 M_{\rm Pl}^6 (\hat{S}^{(h)} \bar{\hat{S}}^{(h)}) \\ &\quad + g' M_{\rm Pl}^3 (\hat{S}^{(h)} \bar{\hat{S}}^{(v)}) + g' M_{\rm Pl}^3 (\hat{S}^{(v)} \bar{\hat{S}}^{(h)}) + 2(S^{(v)} \bar{S}^{(v)} + S^{(v3)} \bar{S}^{(v3)}) \Big] . \end{aligned} \tag{5.37}$$

In the last step we made the relative orders of the various pieces explicit. Remember that keeping the gravitino fixed in the limit $M_{\rm Pl} \to \infty$ means to keep the combination $g' M_{\rm Pl}$ finite. We absorb the purely hidden part in $\mathcal{V}(\Phi, \bar{\Phi}) = 0$, and we see that the only terms surviving in the limit are the two mixed pieces and the combination between the subleading term of the exponential expansion and $\hat{S}^{(h)} \bar{\hat{S}}^{(h)}$:

$$\begin{aligned} &\lim_{\substack{M_{\rm Pl} \to \infty \\ g' \to 0}} \; -\frac{12}{M_{\rm Pl}^2} S_{AB} S^{AB} \\ &\qquad = -3 e^{K^0} \hat{S}^{(h)} \bar{\hat{S}}^{(h)} \left(Z_{m\bar{n}} t^m \bar{t}^{\bar{n}} \right) - 3 e^{K^0} (\bar{\hat{S}}^{(h)} S^{(v)} + \hat{S}^{(h)} \bar{S}^{(v)}) \\ &\qquad = -12 m_{3/2}^2 \left(Z_{m\bar{n}} t^m \bar{t}^{\bar{n}} \right) - 6 e^{\frac{K^0}{2}} (m_{3/2} \; S^{(v)} + \text{c.c.}), \end{aligned} \tag{5.38}$$

where in the first line we kept only the visible leading terms in (5.37) and in the second step we substituted the gravitino mass using the definition (5.19). Thus, as expected, the purely visible part vanishes and the mixed pieces are the soft terms of the gravitino variation contribution to the scalar potential. Let us now turn to the gaugino variation term.

5.4 Gaugino variation term

Now we focus on the $g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB}$ term. The matrix W^{iAB} reads:

$$\begin{aligned} W^{iAB} &= \\ &= e^{\frac{K}{2M_{\rm Pl}^2}} [i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Lambda}) \Theta_{\Lambda}{}^{\lambda} P^x_{\lambda} (\sigma^x)^{AB} + \epsilon^{AB} \Theta_{\Lambda}{}^{\iota} k^i_{\iota} \bar{V}^{\Lambda}] \\ &= e^{\frac{K}{2M_{\rm Pl}^2}} \left[i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Lambda}) \begin{pmatrix} -P^+_{\Lambda} & P^3_{\Lambda} \\ P^3_{\Lambda} & P^-_{\Lambda} \end{pmatrix} + k^i_{\Lambda} \bar{V}^{\Lambda} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right], \end{aligned} \tag{5.39}$$

where in the last line the explicit expression for $P^x_{\lambda}(\sigma^x)^{AB}$ is evaluated substituting (A.11), and we also used (5.22) (together with $\Theta_{\Lambda}{}^{\iota} k^i_{\iota} \equiv k^i_{\Lambda}$). Taking the vacuum expectation value we get

$$\begin{aligned} \langle W^{iAB} \rangle &= \\ &= \begin{pmatrix} \langle -e^{\frac{K}{2M_{\rm Pl}^2}} i g^{i\bar{j}} (\nabla_{\bar{g}} \bar{V}^{\Sigma}) P^+_{\Sigma} \rangle & 0 \\ 0 & \langle e^{\frac{K}{2M_{\rm Pl}^2}} i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Sigma}) P^-_{\Sigma} \rangle \end{pmatrix} = \\ &= \begin{pmatrix} 0 & 0 \\ 0 & \langle e^{\frac{K}{2M_{\rm Pl}^2}} i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Sigma}) P^-_{\Sigma} \rangle \end{pmatrix}. \end{aligned} \tag{5.40}$$

In the last step we implicitly used the solution (4.23) for the embedding tensor which induces the partial breaking.[4] The Killing vectors of the vector multiplet sector (and the P^3, as before) are missing, since they belong to the visible sector and the background value is relevant only for the hidden sector. Note also that while S^{AB} was the mass matrix for the gravitini and we could identify the non-zero expression with the gravitino mass of the broken supersymmetry, W^{iAB} is the coupling matrix between the gravitini and the gaugini. The mass matrices of the gauginos and hyperinos are not part of the scalar potential (for a review, see [57]).

[4] Here we let the indices $(i, \bar{j})$ to run over both the hidden and the visible fields, but this cannot affect the validity of the solutions for the embedding tensor as long as we assume it to be block diagonal.

Proceeding as in the previous section, we can split W^{iAB} in a hidden and visible part:

$$\begin{aligned} W^{iAB} = &\begin{pmatrix} 0 & 0 \\ 0 & e^{\frac{K}{2M_{\rm Pl}^2}} i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Sigma}) P_{\Sigma}^{-} \end{pmatrix} \\ &+ e^{\frac{K}{2M_{\rm Pl}^2}} \left[i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{M}) \begin{pmatrix} -P_M^{+} & P_M^3 \\ P_M^3 & P_M^{-} \end{pmatrix} + k_M^i \bar{V}^M \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right], \end{aligned} \tag{5.41}$$

where in both the hidden and the visible parts we leave for now (i, j) to run over both hidden and visible fields. Using this expression, the scalar potential terms becomes (see Appendix C for the details of the derivation)

$$g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB} = \mathcal{V}_W^{hid} + \mathcal{V}_W^{mix} + \mathcal{V}_W^{vis}, \tag{5.42}$$

with

$$\begin{aligned} \mathcal{V}_W^{hid} &\equiv e^{\frac{K}{M_{\rm Pl}^2}} g^{k\bar{k}} [(\nabla_{\bar{k}} \bar{V}^{\Sigma})(\nabla_k V^{\Xi}) P_{\Sigma}^{-} P_{\Xi}^{+}] \\ \mathcal{V}_W^{mix} &\equiv e^{\frac{K}{M_{\rm Pl}^2}} g^{k\bar{k}} [(\nabla_{\bar{k}} \bar{V}^{\Sigma})(\nabla_k V^{M}) P_{\Sigma}^{-} P_M^{+} + (\nabla_{\bar{k}} \bar{V}^{M})(\nabla_k V^{\Xi}) P_M^{-} P_{\Xi}^{+}] \\ \mathcal{V}_W^{vis} &\equiv e^{\frac{K}{M_{\rm Pl}^2}} g^{k\bar{k}} [2(\nabla_{\bar{k}} \bar{V}^{M})(\nabla_k V^{N})(P_M^{+} P_N^{-} + P_M^3 P_N^3)] + 2 e^{\frac{K}{M_{\rm Pl}^2}} g_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M. \end{aligned} \tag{5.43}$$

The term $\mathcal{V}_W^{hid}$ is comprised of the hidden sector contribution, $\mathcal{V}_W^{mix}$ contains the mixed terms and $\mathcal{V}_W^{vis}$ all the visible contributions. Since for now we left the indices $(k, \bar{k})$ and $(i, \bar{j})$ to run over both visible and hidden fields, we cannot decide a priori the order and relevance of the various pieces and we need to consider the metric $g_{i\bar{j}}$ in detail. In Chapter 3 we have seen how in the limit $M_{\rm Pl} \to \infty$ the metrics on the moduli spaces for the vectors and the hypers become block-diagonal, since with our assumptions on the two different field sectors the off diagonal terms are subleading. In this case, though, that conclusion is not valid anymore since in order to keep the gravitino scale fixed we also take the effective limit $g' \to 0$ and terms of order $1/M_{\rm Pl}^2$ in the metric might be compensated, thus in the following we need to take into account the off-diagonal blocks as well. Keeping our conventions close to [20], we can put

$$g_{i\bar{j}} = \partial_i \partial_{\bar{i}} K^v \equiv K_{i\bar{j}}. \tag{5.44}$$

Separating in hidden and visible blocks, the metric reads

$$g_{i\bar{j}} = \begin{pmatrix} K_{\Phi\bar{\Psi}} & K_{\Phi\bar{n}} \\ K_{m\bar{\Psi}} & K_{m\bar{n}} \end{pmatrix}. \tag{5.45}$$

Imposing the relationship $g^{a\bar{c}}g_{\bar{c}b} = \delta^a_b$ we can derive also the inverse metric (Appendix C for the details):

$$g^{\bar{j}i} = \begin{pmatrix} \tilde{K}^{\bar{\Psi}\Phi} & \tilde{K}^{\bar{\Psi}m} \\ \tilde{K}^{\bar{n}\Phi} & \tilde{K}^{\bar{n}m} \end{pmatrix} = \begin{pmatrix} K^{\bar{\Psi}\Phi} + K^{\bar{\Psi}\Upsilon}K_{\Upsilon\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Phi} & -K^{\bar{\Psi}\Phi}K_{\Phi\bar{p}}K^{\bar{p}m} \\ -K^{\bar{n}m}K_{m\bar{\Psi}}K^{\bar{\Psi}\Phi} & K^{\bar{n}m} \end{pmatrix}. \tag{5.46}$$

The explicit expression for the sub-metrics of $K_{i\bar{j}}$ and their inverses are (see C.2.1)

$$\begin{aligned}
K_{m\bar{n}} &= Z_{m\bar{n}}, \\
K^{\bar{n}m} &= Z^{\bar{n}m}, \\
K_{\Phi\bar{\Psi}} &= K^0_{\Phi\bar{\Psi}} + \frac{Z_{m\bar{n}\Phi\bar{\Psi}}t^m\overline{t}^{\bar{n}}}{M_{\rm Pl}^2}, \\
K^{\bar{\Psi}\Phi} &= K^{0\bar{\Psi}\Phi} - \frac{K^{0\bar{\Psi}\Upsilon}K^{0\bar{\Upsilon}\Phi}Z_{m\bar{n}\Upsilon\bar{\Upsilon}}t^m\overline{t}^{\bar{n}}}{M_{\rm Pl}^2}, \\
K_{\Phi\bar{n}} &= \frac{Z_{m\bar{n}\Phi}t^m}{M_{\rm Pl}}, \\
K_{m\bar{\Psi}} &= \frac{Z_{m\bar{n}\bar{\Psi}}\overline{t}^{\bar{n}}}{M_{\rm Pl}}, \\
K_m &= Z_{m\bar{n}}\overline{t}^{\bar{n}}, \\
K_{\bar{n}} &= Z_{m\bar{n}}t^m, \\
K_{\bar{\Psi}} &= M_{\rm Pl}K^0_{\bar{\Psi}} + \frac{Z_{m\bar{n}\bar{\Psi}}t^m\overline{t}^{\bar{n}}}{M_{\rm Pl}}, \\
K_{\Phi} &= M_{\rm Pl}K^0_{\Phi} + \frac{Z_{m\bar{n}\Phi}t^m\overline{t}^{\bar{n}}}{M_{\rm Pl}},
\end{aligned} \tag{5.47}$$

where we have defined

$$\begin{aligned}
&Z_{m\bar{n}\Phi} = \partial_\Phi Z_{m\bar{n}} \qquad Z_{m\bar{n}\bar{\Psi}} = \partial_{\bar{\Psi}} Z_{m\bar{n}} \\
&Z_{m\bar{n}\Phi\bar{\Psi}} = \partial_\Phi\partial_{\bar{\Psi}} Z_{m\bar{n}}.
\end{aligned} \tag{5.48}$$

5.4.1 Limit summary and gaugino variation soft terms

Now we consider the limit of (5.42). The full computation has been performed in (C.3) and here we only give the final result. Splitting the covariant derivative $\nabla_i = \partial_i + \frac{1}{M_{\rm Pl}^2}K_i$ and defining for convenience

$$\begin{aligned}
W_i^{(h)} &\equiv W_i^{(h1)} + W_i^{(h2)}, \\
W_i^{(v)} &\equiv W_i^{(v1)} + W_i^{(v2)},
\end{aligned} \tag{5.49}$$

with

$$\begin{aligned} W_i^{(h1)} &\equiv \partial_i V^\Sigma P_\Sigma^+, \\ W_i^{(h2)} &\equiv \frac{1}{M_{\rm Pl}^2} K_i V^\Sigma P_\Sigma^+, \\ W_i^{(v1)} &\equiv \partial_i V^M P_M^+, \\ W_i^{(v2)} &\equiv \frac{1}{M_{\rm Pl}^2} K_i V^M P_M^+, \end{aligned} \tag{5.50}$$

and

$$\begin{aligned} W_i^{(v31)} &\equiv \partial_i V^M P_M^3 \\ W_i^{(v32)} &\equiv \frac{1}{M_{\rm Pl}^2} K_i V^M P_M^3. \end{aligned} \tag{5.51}$$

where the index i can run over the hidden *or* the visible fields, we can proceed to evaluate the limit

$$\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB}|_{vis} = \lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} (\mathcal{V}_W^{hid} + \mathcal{V}_W^{mix} + \mathcal{V}_W^{vis}). \tag{5.52}$$

Considering each piece separately, we have

$$\begin{aligned} \lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{hid} =& e^{K^0}[(K^{0\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)} W_\Phi^{(h)})|_0 Z_{m\bar{n}} t^m \bar{t}^{\bar{n}} + (\tilde{K}^{\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)} W_\Phi^{(h)})|_2 \\ & + (\tilde{K}^{\bar{\Psi}m}\bar{W}_{\bar{\Psi}}^{(h)} W_m^{(h2)} + \text{c.c.}) + \tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(h2)} W_m^{(h2)}], \end{aligned} \tag{5.53}$$

$$\begin{aligned} \lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{mix} =& e^{K^0}[\tilde{K}^{\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)} W_\Phi^{(v2)} + (\tilde{K}^{\bar{\Psi}m}\bar{W}_{\bar{\Psi}}^{(h)} W_m^{(v1)} + \text{c.c.}) + \tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(h2)} W_m^{(v1)}] \\ & + \text{c.c.}, \end{aligned} \tag{5.54}$$

$$\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{vis} = 2e^{K^0}(\tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(v1)} W_m^{(v1)} + \tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(v31)} W_m^{(v31)} + K_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M), \tag{5.55}$$

where for all the quantities we consider in general the leading terms and the indexes $'0'$ and $'2'$ in (5.53) indicate that we are considering the $\sim (g'^2)M_{\rm Pl}^4$ or the $\sim (g'^2)M_{\rm Pl}^2$ part of the expansion respectively. Notice that the three pieces (5.53), (5.54) and (5.55) are of order $O(m_{3/2}^2)$, $O(m_{3/2})$ and $O(1)$ respectively. Now we want to find a more useful expression, and we make explicit all the elements containing visible fields, starting from the $\mathcal{V}_W^{hid}$ contribution. Using the definitions

(5.46) and the explicit formulas (5.47), we can rewrite it as

$$
\begin{aligned}
\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{hid} = & \\
&= e^{K^0}[K^{0\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)}W_{\Phi}^{(h)}Z_{m\bar{n}}t^m\bar{t}^{\bar{n}} \\
&\quad + K^{0\bar{\Psi}\Phi}K^{0\bar{\Upsilon}\Upsilon}W_{\Phi}^{(h)}\bar{W}_{\bar{\Upsilon}}^{(h)}(-Z_{m\bar{n}\Upsilon\bar{\Psi}} + Z_{m\bar{q}\Upsilon}Z^{\bar{q}p}Z_{p\bar{n}\bar{\Psi}})t^m t^{\bar{n}} \\
&\quad + S^{(h)}\bar{S}^{(h)}Z_{m\bar{n}}t^m\bar{t}^{\bar{n}}],
\end{aligned} \tag{5.56}
$$

where we also used $W_m^{(h2)} = \frac{1}{M_{\rm Pl}^2}K_m S^h$. The expression $(-Z_{m\bar{n}\Upsilon\bar{\Psi}} + Z_{m\bar{q}\Upsilon}Z^{\bar{q}p}Z_{p\bar{n}\bar{\Psi}})$ is a sub-tensor of the curvature tensor of the vector scalar space, and it holds for a general Kähler manifold. In fact, using $\Gamma^p_{lm} = Z^{p\bar{q}}Z_{l\bar{q}m}$, we can put

$$
(-Z_{m\bar{n}\Upsilon\bar{\Psi}} + Z_{m\bar{q}\Upsilon}Z^{\bar{q}p}Z_{p\bar{n}\bar{\Psi}}) = -(Z_{m\bar{n}\Upsilon\bar{\Psi}} - \Gamma^p_{\Upsilon m}Z_{p\bar{q}}\bar{\Gamma}^{\bar{q}}_{\bar{\Psi}\bar{n}}) = -R_{m\bar{n}\Upsilon\bar{\Psi}}, \tag{5.57}
$$

which is identical to the expression (5.8a) for the $N=1$ case. Since in $N=2$ we are dealing with a *special* Kähler manifold, (5.57) can be further specified, as we shall see in the following. Putting now $|W^{(h)}|^2 = K^{0\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)}W_{\Phi}^{(h)}$ and $K^{0\bar{\Psi}\Phi}W_{\Phi}^{(h)} = \bar{W}^{\bar{\Psi}(h)}$, and substituting the gravitino mass expression (5.19) with the help of the definitions (5.27), we can write (5.56) in a cleaner way as

$$
\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{hid} = (e^{K^0}|W^{(h)}|^2 Z_{m\bar{n}} + 4m_{3/2}^2 Z_{m\bar{n}} - e^{K^0}W^{\Phi(h)}\bar{W}^{\bar{\Psi}(h)}R_{m\bar{n}\Phi\bar{\Psi}})\, t^m\bar{t}^{\bar{n}}. \tag{5.58}
$$

This term is the analogous of the $m^2_{I\bar{J}}$-soft term in (5.6). Let us proceed with the terms of order $O(m_{3/2})$. With (5.46) and (5.47) it reads

$$
\begin{aligned}
\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} \mathcal{V}_W^{mix} = & \\
= e^{K^0}[&K^{0\bar{\Psi}\Phi}\bar{W}_{\bar{\Psi}}^{(h)}K^0_{\Phi}S^{(v)} + (K^{0\bar{\Psi}\Phi}W_{\bar{\Psi}}^{(h)}Z^{p\bar{q}}Z_{m\bar{q}\Phi}W_p^{(v1)}t^m + \text{c.c.}) + \bar{S}^{(h)}W_m^{(v1)}t^m] \\
&+ \text{c.c.} \\
= e^{K^0}[&W^{(h)\Phi}K^0_{\Phi}S^{(v)} + (W^{(h)\Phi}\Gamma^p_{m\Phi}W_p^{(v1)}t^m + \text{c.c.}) + \bar{S}^{(h)}W_m^{(v1)}t^m] + \text{c.c.}.
\end{aligned} \tag{5.59}
$$

Notice the resemblance of the first two pieces with the A_{IJL}-soft term which appears in (5.6), with the differences that here we have a Yukawa coupling between scalars belonging to both vectors and hypermultiplets; moreover according to our assumptions (3.37), $W_{\Phi}^{(v1)} = \partial_{\Phi}V^M P^{\pm}_M = 0$ (namely F_{IJ} is block diagonal), hence we do not have the $\partial_i Y_{IJL}$ part.

Adding the global piece (5.55) and collecting all the simplified results, (5.52) reads

$$\lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB}|_{vis} = \lim_{\substack{M_{\rm Pl}\to\infty \\ g'\to 0}} (\mathcal{V}_W^{hid} + \mathcal{V}_W^{mix} + \mathcal{V}_W^{vis}) =$$
$$\begin{aligned} &= (e^{K^0}|W^{(h)}|^2 Z_{m\bar{n}} + 4m_{3/2}^2 Z_{m\bar{n}} - e^{K^0} W^{\Phi(h)} \bar{W}^{\bar{\Psi}(h)} R_{m\bar{n}\Phi\bar{\Psi}})\ t^m \bar{t}^{\bar{n}}, \\ &+ e^{K^0}[W^{(h)\Phi} K_\Phi^{(0)} S^{(v)} + (W^{(h)\Phi}\Gamma^p_{m\Phi} W_p^{(v1)} t^m + \text{c.c.}) + \bar{S}^{(h)} W_m^{(v1)} t^m] + \text{c.c.} \\ &+ 2e^{K^0}[\tilde{K}^{\bar{n}m} \bar{W}_{\bar{n}}^{(v1)} W_m^{(v1)} + \tilde{K}^{\bar{n}m} \bar{W}_{\bar{n}}^{(v31)} W_m^{(v31)} + K_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M]. \end{aligned} \tag{5.60}$$

5.5 $N = 1$ scalar potential without visible hypermultiplets

If the theory we reach after the limit is $N = 1$ supersymmetric, we should be able to rewrite the global limit plus the soft terms derived from the partially broken $N = 2$ theory in terms of the standard $N = 1$ scalar potential (given in the first line of (5.6)). Let us consider a simple situation, in which we have hypermultiplets in the hidden sector (needed for the partial breaking) but not in the visible sector.

The terms we should take into account are then only

$$-12m_{3/2}^2 Z_{m\bar{n}} t^m \bar{t}^{\bar{m}} \tag{5.61}$$

from the gravitino variation term (5.38), and

$$\begin{aligned} &(e^{K^0}|W^{(h)}|^2 Z_{m\bar{n}} + 4m_{3/2}^2 Z_{m\bar{n}} - e^{K^0} W^{\Phi(h)} \bar{W}^{\bar{\Psi}(h)} R_{m\bar{n}\Phi\bar{\Psi}})\ t^m \bar{t}^{\bar{n}} \\ &+ 2e^{K^0} K_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M \end{aligned} \tag{5.62}$$

from the gaugino term (5.60). We are excluding the visible hypermultiplets, nevertheless we should take into account the only vector contribution from the hyperino variation term $2N_\alpha^A N_A^\alpha$, which comes from the expansion of the exponential factor $e^{K/M_{\rm Pl}^2}$. Together with (5.61) and the first term in (5.62) $e^{K^0}|W^{(h)}|^2 Z_{m\bar{n}} t^m \bar{t}^{\bar{n}}$, this term belongs to an overall term which reads

$$\mathcal{V}(\Phi, \bar{\Phi}) Z_{m\bar{n}} t^m \bar{t}^{\bar{n}}. \tag{5.63}$$

As we are assuming a vanishing cosmological constant, namely $\langle \mathcal{V}(\Phi, \bar{\Phi}) \rangle = 0$, we can consider this term identically zero. Therefore, the scalar potential without visible hypermultiplets reads

$$\begin{aligned}\mathcal{V}^{NoHyp} =& e^{K^0}[(4m_{3/2}^2 Z_{m\bar{n}} - W^{\Phi(h)}\bar{W}^{\bar{\Psi}(h)}R_{m\bar{n}\Phi\bar{\Psi}})\ t^m\bar{t}^{\bar{n}} \\ &+ 2K_{m\bar{n}}k_M^m k_N^{\bar{n}} V^N \bar{V}^M].\end{aligned} \tag{5.64}$$

Let us study this expression further. The curvature sub-tensor (2.33) for a local special Kähler manifold in this case reads

$$R_{m\bar{n}\Phi\bar{\Psi}} = K_{\bar{\Psi}\Phi}K_{m\bar{n}} + K_{\bar{\Psi}m}K_{\Phi\bar{n}} - M_{\rm Pl}^2 e^{\frac{2K}{M_{\rm Pl}^2}} Q_{m\Phi i}K^{i\bar{j}}\bar{Q}_{\bar{j}\bar{\Psi}\bar{n}}, \tag{5.65}$$

where (i, j) run over both the hidden and the visible fields and

$$Q_{m\Phi i} = (\nabla_m \nabla_\Phi V^\Lambda)(\nabla_i V_\Lambda). \tag{5.66}$$

In special coordinates, we have

$$Q_{m\Phi i} = F_{m\Phi i}. \tag{5.67}$$

Because of the assumptions (3.37), in our case $Q_{m\Phi i}$ is identically zero and we are left with the first two terms of (5.65). Since in the previous section we derived $R_{m\bar{n}\Phi\bar{\Psi}}$ from the dimensionless expression $(-Z_{m\bar{n}\Phi\bar{\Psi}} + Z_{m\bar{q}\Phi}Z^{\bar{q}p}Z_{p\bar{n}\bar{\Psi}})$ obtained after the rigid limit, we have to take only the leading part of (5.65) as well. According to the definitions (5.47), $K_{\bar{\Psi}m}$ is subleading and the first two terms reduce to

$$K_{\bar{\Psi}\Phi}K_{m\bar{n}} + K_{\bar{\Psi}m}K_{\Phi\bar{n}} \to K^0_{\bar{\Psi}\Phi}Z_{m\bar{n}}. \tag{5.68}$$

Then $R_{m\bar{n}\Phi\bar{\Psi}} = K^0_{\bar{\Psi}\Phi}Z_{m\bar{n}}$ and the scalar potential (5.64) becomes

$$\begin{aligned}\mathcal{V}^{NoHyp} =& e^{K^0}m_{1/2}^2 Z_{m\bar{n}}\ t^m\bar{t}^{\bar{n}} \\ &+ 2e^{K^0}K_{m\bar{n}}k_M^m k_N^{\bar{n}} V^N \bar{V}^M,\end{aligned} \tag{5.69}$$

where we defined

$$m_{1/2}^2 \equiv 4m_{3/2}^2 - |W^{(h)}|^2. \tag{5.70}$$

We can now recast the expression (5.69) in the $N = 1$ form defining an holomorphic $N = 1$ superpotential $\mathcal{W}$ as

$$\begin{aligned}\mathcal{W} &\equiv e^{K^0/2}m_{1/2}Z_{mn}\ t^m t^n, \\ \bar{\mathcal{W}} &\equiv e^{K^0/2}m_{1/2}Z_{\bar{m}\bar{n}}\ \bar{t}^{\bar{m}}\bar{t}^{\bar{n}}.\end{aligned} \tag{5.71}$$

We see that the $N = 1$ expression

$$\partial_m \mathcal{W} Z^{m\bar{n}} \partial_{\bar{n}} \bar{\mathcal{W}}, \tag{5.72}$$

with $Z_{mn} Z^{n\bar{p}} = \delta_m^{\bar{p}}$, reproduces the first line of (5.69). The second line of (5.69) preserves the full $N = 2$ supersymmetry, since it coincides (modulus a constant factor) with the global limit of the $N = 2$ scalar potential (2.56) without hyper-multiplets (see (2.53)). We can conclude that the scalar potential

$$\mathcal{V}^{N=1} = \partial_m \mathcal{W} Z^{m\bar{n}} \partial_{\bar{n}} \bar{\mathcal{W}} + 2e^{K^0} K_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M \tag{5.73}$$

is $N = 1$ supersymmetric.

We have thus shown, at least in this simple scenario, that the global limit of the spontaneously broken $N = 2$ supergravity with $m_{3/2}$ fixed yields a rigid $N = 2$ theory explicitly broken by $N = 1$ supersymmetric soft terms.

Chapter 6

Conclusions and outlook

The aim of this thesis was to perform a model-independent analysis of the soft terms arising in the rigid limit $M_{\mathrm{Pl}} \to \infty$ from a general gauged $N = 2$ supergravity spontaneously broken to $N = 1$. In particular, we evaluated two kinds of rigid limit. First we studied the limit $M_{\mathrm{Pl}} \to \infty$ of both the ungauged and gauged theory, with a focus on the geometric structures involved. Namely, assuming the possibility of a local distinction between hidden and visible fields, we have shown that the metrics and the couplings become block-diagonal and the local manifolds describing the target spaces of the sigma models flow to their global counterparts. Specifically, the local special Kähler manifold of the vector multiplet scalar sector becomes a rigid special Kähler space and the quaternionic manifold spanned by the hypermultiplet scalars is reduced to an HyperKähler manifold. Afterwards, we considered generic $N = 2$ gauged supergravities spontaneously broken to $N = 1$, we focused on the gravitino and gaugino variation terms of the scalar potential and we performed again the rigid limit while keeping this time the gravitino mass scale related to the broken supersymmetry fixed for $M_{\mathrm{Pl}} \to \infty$. In this way we obtained the global $N = 2$ limit of the $N = 2$ supergravity together with $N = 1$ supersymmetric soft terms. Finally, we considered the simple setup in which there is no hypermultiplet in the visible sector, and we showed that the $N = 2$ global limit of the scalar potential and the soft terms can be arranged in a global $N = 1$ scalar potential defined in terms of an $N = 1$ superpotential.

Our analysis should be extended to the hyperino variation term of the scalar potential and to the general case in which also visible hypermultiplets are present. We also remind the reader that in this work only a Minkowski vacuum was considered and we leave the Anti-de Sitter case for future investigations. One might also think to relax our assumption that forbids the mixing between hidden and visible fields and to allow for more general prepotentials, considering possible additional scales like the QCD or the Seiberg-Witten scales. Finally, it would be interesting to re-do the analysis in the case of a supersymmetry breaking cascade $N = 2 \to N = 1 \to N = 0$, where the two supersymmetries are broken at two different energy scales.

Appendix A

$N = 2$ supergravity conventions and formulas

A.1 indices conventions

Symplectic indices:

$$\begin{aligned} &\text{General: } \Lambda, \Pi, \dots \\ &\text{Hidden sector: } \Sigma, \Xi, \dots \\ &\text{Visible sector: } M, N, \dots \end{aligned} \tag{A.1}$$

Isometry indices:
(Hypermultiplets)

$$\begin{aligned} &\text{General: } \lambda, \pi, \dots \\ &\text{Hidden sector: } \gamma, \delta, \dots \\ &\text{Visible sector: } \nu, \rho, \dots \end{aligned} \tag{A.2}$$

(Vectors)

$$\text{Visible sector: } \iota. \tag{A.3}$$

Scalar fields indices:
(Hypermultiplets)

$$\begin{aligned} &\text{General: } u, v \dots \\ &\text{Hidden sector: } \mathcal{A}, \mathcal{B} \dots \\ &\text{Visible sector: } w, y, \dots \end{aligned} \tag{A.4}$$

(Vectors)

$$
\begin{aligned}
&\text{General - (homogeneous coordinates): } I, J \dots \quad (\text{in } N=2)\\
&\text{General - (special coordinates): } i, j \dots\\
&\text{Hidden sector (special coordinates): } \Phi, \Upsilon, \Psi, \dots\\
&\text{Visible sector (special coordinates): } m, n, p, q \dots
\end{aligned}
\tag{A.5}
$$

A.2 $SU(2)$ and $Sp(2n_h)$ structures

The $SU(2)$ and $Sp(2n_h)$ metrics are defined as

$$
\begin{aligned}
\epsilon^{AB}\epsilon_{BC} &= -\delta^A_C, \quad \epsilon^{AB} = -\epsilon^{BA},\\
\mathcal{C}^{\alpha\beta}\mathcal{C}_{\beta\gamma} &= -\delta^\alpha_\gamma, \quad \mathcal{C}^{\alpha\beta} = -\mathcal{C}^{\beta\alpha}.
\end{aligned}
\tag{A.6}
$$

These act on the $SU(2)$ and $Sp(2n_h)$ vectors V_A as

$$
\begin{aligned}
\epsilon_{AB}V^B &= V_A, \quad \epsilon^{AB}V_B = -V^A,\\
\mathcal{C}_{\alpha\beta}V^\beta &= V_\alpha, \quad \mathcal{C}^{\alpha\beta}V_\beta = -V^\alpha.
\end{aligned}
\tag{A.7}
$$

A.2.1 Pauli and $SU(2)$ matrices

The standard Pauli matrices $(\sigma^x)_A^{\ C}$ are

$$
\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.8}
$$

The indices of the Pauli matrix can be lowered using ϵ_{AB}, namely

$$
(\sigma^x)_{AB} \equiv (\sigma^x)_A^{\ C}\epsilon_{BC}. \tag{A.9}
$$

These are the matrices which appears in the scalar potential of the $N=2$ gauged supergravity and their explicit expression is

$$
(\sigma^1)_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\sigma^2)_{AB} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \quad (\sigma^3)_{AB} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \tag{A.10}
$$

Analogously, for both upper indices we have

$$
(\sigma^1)^{AB} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\sigma^2)^{AB} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \quad (\sigma^3)^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{A.11}
$$

A.3 Dimensions summary

$[\mathcal{V}(t,\bar{t},q)] = 4$
$[S_{AB}] = 3$
$[W_i] = 2$
$[N_\alpha^A] = 2$
$[V^\Lambda] = 1$
$[k_\lambda^i] = [k_\lambda^u] = 1$
$[P_\lambda^x] = 2$
$[F] = 2$
$[K] = 2$
$[K_i] = 1$
$[g_{i\bar{j}}] = [h_{uv}] = [\mathcal{N}_{IJ}] = 0$
$[\Theta_\Lambda{}^\lambda] = 0$

Appendix B

Partial breaking with an intermediate sector

If one tries to define a third intermediate sector between the hidden (background values of order $M_{\rm Pl}$) and the visible one (background values $\phi_0 << M_{\rm Pl}$) at the scale Λ such that

$$\phi_0 << \Lambda << M_{\rm Pl}, \tag{B.1}$$

and then define the gravitino mass scale as a combination of the $M_{\rm Pl}$ scale and the Λ scale

$$m_{3/2} \equiv \frac{\Lambda^3}{M_{\rm Pl}^2}, \tag{B.2}$$

so that in the limit the gravitino mass remains fixed, namely

$$\lim_{M_{\rm Pl}\to\infty,\ \Lambda\to\infty} \quad \frac{\Lambda^3}{M_{\rm Pl}^2} \to \text{finite}, \tag{B.3}$$

it becomes impossible to construct a Minkowski vacuum. The reason is the following. We consider again the scalar potential (2.56). According to the definitions (2.57), that we rewrite here for the hidden intermediate sector only,

$$\begin{aligned}
S_{AB} &= \frac{1}{2} e^{\frac{K}{2M_{\rm Pl}^2}} V^{\Sigma} \Theta_{\Sigma}{}^{\gamma} P^x_{\gamma} (\sigma^x)_{AB}, \\
W^{iAB} &= e^{\frac{K}{2M_{\rm Pl}^2}} [i g^{i\bar{j}} (\nabla_{\bar{j}} \bar{V}^{\Sigma}) \Theta_{\Sigma}{}^{\gamma} P^x_{\gamma} (\sigma^x)^{AB}], \\
N^A_{\alpha} &= 2 e^{\frac{K}{2M_{\rm Pl}^2}} \bar{V}^{\Sigma} \Theta_{\Sigma}{}^{\gamma} \mathcal{U}^A_{\alpha u} k^u_{\gamma},
\end{aligned}$$

we can estimate the orders of the various pieces in the background. Taking the vacuum expectation values in the intermediate sector and using the canonical mass dimensions in section A.3 we have

$$\begin{aligned}
&V^{\Sigma} \sim O(\Lambda) \\
&\partial_f V^{\Sigma} \sim O(1) \\
&P^x_{\gamma} \sim O(\Lambda^2) \\
&K_f \sim O(\Lambda) \quad (\text{or } O(M_{\text{Pl}})) \\
&g_{f,\bar{g}},\ h_{ab} \sim O(1).
\end{aligned} \tag{B.4}$$

Hence, for the gravitino variation piece we can write

$$\begin{aligned}
-\frac{12}{M_{\text{Pl}}^2} S_{AB} S^{AB} \sim \frac{1}{M_{\text{Pl}}^2} |\langle V^{\Sigma} P_{\Sigma} \rangle|^2 \sim O\left(\frac{\Lambda^6}{M_{\text{Pl}}^2}\right) \\
\sim m_{3/2}^2 M_{\text{Pl}}^2,
\end{aligned} \tag{B.5}$$

where, in order to simplify the notation, we put $P_{\Sigma} \equiv \Theta_{\Sigma}{}^{\lambda} P^x_{\lambda}$.

For the gaugino piece we have instead

$$\begin{aligned}
g_{f\bar{g}} W^{fAB} W^{\bar{g}}_{AB} &\sim |\langle \nabla_f \bar{V}^{\Sigma} P_{\Sigma} \rangle|^2 \sim |\langle (\partial_f \bar{V}^{\Sigma}) P_{\Sigma} + \frac{1}{M_{\text{Pl}}^2} K_f \bar{V}^{\Sigma} P_{\Sigma} \rangle|^2 \\
&\sim \left(O(\Lambda^2) + O\left(\frac{\Lambda^4}{M_{\text{Pl}}^2}\right)\right)^2 \sim O(\Lambda^4) + O\left(\frac{\Lambda^6}{M_{\text{Pl}}^2}\right) + O\left(\frac{\Lambda^8}{M_{\text{Pl}}^4}\right) \\
&\sim O\left(m_{3/2}^2 \frac{M_{\text{Pl}}^4}{\Lambda^2}\right) + O(m_{3/2}\Lambda^3) + O(m_{3/2}^2 \Lambda^2),
\end{aligned} \tag{B.6}$$

where we omitted the metric for simplicity. Finally, for the hyperino variation part of the scalar potential we have

$$\begin{aligned}
2 N^A_{\alpha} N^{\alpha}_A \sim \langle h_{ab} V^{\Sigma} \bar{V}^T k^a_{\Sigma} k^b_T \rangle \sim O(\Lambda^4) \\
\sim O\left(m_{3/2}^2 \frac{M_{\text{Pl}}^4}{\Lambda^2}\right),
\end{aligned} \tag{B.7}$$

with $k^a_{\Sigma} \equiv \Theta_{\Sigma}{}^{\gamma} k^a_{\gamma}$.

Therefore, we have shown that the three part of the scalar potential $\mathcal{V}$ have different orders and cannot cancel between themselves, leaving a non-zero cosmological constant. We have to conclude that if the scale Λ of the partial supersymmetry breaking is given by the vacuum expectation values of the fields belonging to an intermediate sector there is no way to achieve a Minkowski vacuum.

Appendix C

Soft partial breaking: detailed calculations

C.1 Scalar potential with partial breaking

C.1.1 Gravitino variation term

If we separate the S_{AB} matrix into a hidden and a visible part and we consider the expression (5.15) for the S_{AB} matrix, the term $-\frac{12}{M_{\mathrm{Pl}}^2}S_{AB}S^{AB}$ becomes:

$$
\begin{aligned}
&-\frac{12}{M_{\mathrm{Pl}}^2}S_{AB}S^{AB} = \\
&= -\frac{12}{M_{\mathrm{Pl}}^2}\left(-\frac{1}{4}e^{\frac{K}{M_{\mathrm{Pl}}^2}}\right)\mathrm{Tr}\left\{\left[V^{\Sigma}\begin{pmatrix}0 & 0\\ 0 & -P^+_{\Sigma}\end{pmatrix} + V^M\begin{pmatrix}P^-_M & -P^3_M\\ -P^3_M & -P^+_M\end{pmatrix}\right]\cdot\right.\\
&\qquad\qquad\left.\cdot\left[\bar{V}^{\Xi}\begin{pmatrix}0 & 0\\ 0 & P^-_{\Xi}\end{pmatrix} + \bar{V}^N\begin{pmatrix}-P^+_N & P^3_N\\ P^3_N & P^-_N\end{pmatrix}\right]\right\}\\
&= -\frac{3}{M_{\mathrm{Pl}}^2}e^{\frac{K}{M_{\mathrm{Pl}}^2}}\mathrm{Tr}\left[V^{\Sigma}\bar{V}^{\Xi}\begin{pmatrix}0 & 0\\ 0 & P^+_{\Sigma}P^-_{\Xi}\end{pmatrix}\right.\\
&\qquad + V^{\Sigma}\bar{V}^N\begin{pmatrix}0 & 0\\ P^+_{\Sigma}P^3_N & P^+_{\Sigma}P^-_N\end{pmatrix} + V^M\bar{V}^{\Xi}\begin{pmatrix}0 & P^3_M P^-_{\Xi}\\ 0 & P^+_M P^-_{\Xi}\end{pmatrix}\\
&\qquad \left.+V^M\bar{V}^N\begin{pmatrix}P^-_M P^+_N + P^3_M P^3_N & -P^-_M P^3_N + P^3_M P^-_N\\ -P^3_M P^+_N + P^+_M P^3_N & P^+_M P^-_N + P^3_M P^3_N\end{pmatrix}\right].
\end{aligned}
\tag{C.1}
$$

In the first step we used (5.15) and the definitions (A.11) for S^{AB}, and in the second one we computed the explicit matrix product. Taking the trace, the off-diagonal terms do not contribute and we get

$$
\begin{aligned}
&= -\frac{3}{M_{\rm Pl}^2} e^{\frac{K}{M_{\rm Pl}^2}} \left[V^\Sigma \bar{V}^\Xi P^+_\Sigma P^-_\Xi + V^\Sigma \bar{V}^N P^+_\Sigma P^-_N + V^M \bar{V}^\Xi P^+_M P^-_\Xi \right. \\
&\quad \left. + 2 V^M \bar{V}^N (P^+_M P^-_N + P^3_M P^3_N) \right] \\
&= -\frac{3}{M_{\rm Pl}^2} e^{\frac{K}{M_{\rm Pl}^2}} \left[S^{(h)} \bar{S}^{(h)} + S^{(h)} \bar{S}^{(v)} + S^{(v)} \bar{S}^{(h)} + 2 (S^{(v)} \bar{S}^{(v)} + S^{(v3)} \bar{S}^{(v3)}) \right] ,
\end{aligned}
\tag{C.2}
$$

where we put

$$
\begin{aligned}
S^{(h)} &\equiv V^\Sigma P^+_\Sigma \qquad & \bar{S}^{(h)} &\equiv \bar{V}^\Sigma P^-_\Sigma , \\
S^{(v)} &\equiv V^M P^+_M \qquad & \bar{S}^{(v)} &\equiv \bar{V}^M P^-_M , \\
S^{(v3)} &\equiv V^M P^3_M \qquad & \bar{S}^{(v3)} &\equiv \bar{V}^M P^3_M .
\end{aligned}
\tag{C.3}
$$

C.1.2 Gaugino variation term

Separating again between hidden and visible sectors and using (5.39), for the gaugino variation term we have:

$$
\begin{aligned}
& g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB} = \\
& = e^{\frac{K}{M_{\rm Pl}^2}} g_{i\bar{j}} \mathrm{Tr} \left\{ i g^{i\bar{k}} (\nabla_{\bar{k}} \bar{V}^\Sigma) \begin{pmatrix} 0 & 0 \\ 0 & P^-_\Sigma \end{pmatrix} \right. \\
& \qquad \left. + \left[i g^{i\bar{k}} (\nabla_{\bar{k}} \bar{V}^M) \begin{pmatrix} -P^+_M & P^3_M \\ P^3_M & P^-_M \end{pmatrix} + k^i_M \bar{V}^M \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \right\} \cdot \\
& \qquad \cdot \left\{ i g^{k\bar{j}} (\nabla_k V^\Xi) \begin{pmatrix} 0 & 0 \\ 0 & -P^+_\Xi \end{pmatrix} \right. \\
& \qquad \left. + \left[i g^{k\bar{j}} (\nabla_k V^N) \begin{pmatrix} P^-_N & -P^3_N \\ -P^3_N & -P^+_N \end{pmatrix} + k^{\bar{j}}_N V^N \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right] \right\} ,
\end{aligned}
\tag{C.4}
$$

where again we used (5.39) and the definitions (A.11). Simplifying the metrics with

$$
\begin{aligned}
g_{i\bar{j}} g^{i\bar{k}} &= \delta^{\bar{k}}_{\bar{j}} \\
\delta^{\bar{k}}_{\bar{j}} g^{k\bar{j}} &= g^{k\bar{k}}
\end{aligned}
\tag{C.5}
$$

and performing the products, we have for the terms not involving the Killing vectors

$$
\begin{aligned}
&e^{\frac{K}{M_{\rm Pl}^2}} g^{k\bar{k}} \mathrm{Tr}\left[(\nabla_{\bar{k}}\bar{V}^{\Sigma})(\nabla_k V^{\Xi})\begin{pmatrix}0 & 0\\ 0 & P_{\Sigma}^- P_{\Xi}^+\end{pmatrix}\right.\\
&+(\nabla_{\bar{k}}\bar{V}^{\Sigma})(\nabla_k V^{M})\begin{pmatrix}0 & 0\\ P_{\Sigma}^- P_M^3 & P_{\Sigma}^- P_M^+\end{pmatrix}+(\nabla_{\bar{k}}\bar{V}^{M})(\nabla_k V^{\Xi})\begin{pmatrix}0 & P_M^3 P_{\Xi}^+\\ 0 & P_M^- P_{\Xi}^+\end{pmatrix}\\
&\left.+(\nabla_{\bar{k}}\bar{V}^{M})(\nabla_k V^{N})\begin{pmatrix}P_M^+ P_N^- + P_M^3 P_N^3 & -P_M^+ P_N^3 + P_M^3 P_N^-\\ -P_M^3 P_N^- + P_M^- P_N^3 & P_M^- P_N^+ + P_M^3 P_N^3\end{pmatrix}\right],
\end{aligned}
\tag{C.6}
$$

while the terms involving the Killing vector contribution are

$$
\begin{aligned}
&e^{\frac{K}{M_{\rm Pl}^2}} \mathrm{Tr}\left[i(\nabla_i V^{\Xi})(k_M^i \bar{V}^M)\begin{pmatrix}0 & -P_{\Xi}^+\\ 0 & 0\end{pmatrix}+i(\nabla_{\bar{j}}\bar{V}^{\Sigma})(k_N^{\bar{j}} V^N)\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\right.\\
&+i(\nabla_{\bar{j}}\bar{V}^{M})(k_N^{\bar{j}} V^N)\begin{pmatrix}P_M^3 & P_M^+\\ P_M^- & -P_M^3\end{pmatrix}+i(k_M^i \bar{V}^M)(\nabla_i V^N)\begin{pmatrix}-P_N^3 & -P_N^+\\ -P_N^- & P_N^3\end{pmatrix}\\
&\left.+g_{i\bar{j}} k_M^i k_N^{\bar{j}} V^N \bar{V}^M\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right].
\end{aligned}
\tag{C.7}
$$

Taking the trace from both (C.6) and (C.7) (in (C.7) only the last line contributes) we are left with

$$
\begin{aligned}
g_{i\bar{j}} W^{iAB} W^{\bar{j}}_{AB} =&\\
&= e^{\frac{K}{M_{\rm Pl}^2}} g^{k\bar{k}}[(\nabla_{\bar{k}}\bar{V}^{\Sigma})(\nabla_k V^{\Xi})P_{\Sigma}^- P_{\Xi}^+\\
&+(\nabla_{\bar{k}}\bar{V}^{\Sigma})(\nabla_k V^{M})P_{\Sigma}^- P_M^+ + (\nabla_{\bar{k}}\bar{V}^{M})(\nabla_k V^{\Xi})P_M^- P_{\Xi}^+\\
&+2(\nabla_{\bar{k}}\bar{V}^{M})(\nabla_k V^{N})(P_M^+ P_N^- + P_M^3 P_N^3)] + 2e^{\frac{K}{M_{\rm Pl}^2}} g_{i\bar{j}} k_M^i k_N^{\bar{j}} V^N \bar{V}^M,
\end{aligned}
\tag{C.8}
$$

where in the first line we have the hidden sector contribution, in the second line the mixed terms and in the last line the visible part.

C.2 Kähler potential and inverse metric

C.2.1 Kähler potential explicit derivatives

From the Kähler potential expansion (5.32) we can derive the following explicit expressions for the derivatives

$$
\begin{aligned}
K_{m\bar{n}} &= Z_{m\bar{n}} \\
K_{\Phi\bar{\Psi}} &= K^0_{\Phi\bar{\Psi}} + (Z_{m\bar{n}\Phi\bar{\Psi}} t^m \bar{t}^{\bar{n}})/M^2_{\rm Pl}; \\
K_{\Phi\bar{n}} &= (Z_{m\bar{n}\Phi} t^m)/M_{\rm Pl}; \\
K_{m\bar{\Psi}} &= (Z_{m\bar{n}\bar{\Psi}} \bar{t}^{\bar{m}})/M_{\rm Pl}; \\
K_m &= (Z_{m\bar{n}} \bar{t}^{\bar{m}}); \\
K_{\bar{n}} &= (Z_{m\bar{n}} t^m); \\
K_{\bar{\Psi}} &= M_{\rm Pl} K^0_{\bar{\Psi}} + (Z_{m\bar{n}\bar{\Psi}} t^m \bar{t}^{\bar{n}})/M_{\rm Pl}; \\
K_{\Phi} &= M_{\rm Pl} K^0_{\Phi} + (Z_{m\bar{n}\Phi} t^m \bar{t}^{\bar{n}})/M_{\rm Pl}.
\end{aligned}
\tag{C.9}
$$

The inverses are:

$$K^{\bar{n}m} = Z^{\bar{n}m}, \tag{C.10}$$

while for $K^{\bar{\Psi}\Phi}$ we can write

$$
\begin{aligned}
K^{\bar{\Psi}\Phi} &= \left[K^0_{\Phi\bar{\Psi}} + (Z_{m\bar{n}\Phi\bar{\Psi}} t^m \bar{t}^{\bar{n}})/M^2_{\rm Pl}\right]^{-1} \\
&= \left[K^0_{\Phi\bar{\Upsilon}}(\delta^{\bar{\Upsilon}}_{\bar{\Psi}} + K^{0\bar{\Upsilon}\Upsilon}(Z_{m\bar{n}\Upsilon\bar{\Psi}} + \dots))/M^2_{\rm Pl}\right]^{-1};
\end{aligned}
\tag{C.11}
$$

Taylor-expanding the second factor, considering only the terms up to order $O(1/M^2_{\rm Pl})$ and remembering that for two matrices A and B holds $(A \cdot B)^{-1} = (B)^{-1} \cdot (A)^{-1}$, we finally have

$$K^{\bar{\Psi}\Phi} = \left[\delta^{\bar{\Psi}}_{\bar{\Upsilon}} - K^{0\bar{\Psi}\Upsilon}(Z_{m\bar{n}\Upsilon\bar{\Upsilon}} t^m t^n + \dots)/M^2_{\rm Pl}\right] K^{0\bar{\Upsilon}\Phi}. \tag{C.12}$$

C.2.2 Inverse special Kähler metric with two different sectors

Imposing the relationship $g_{i\bar{j}} g^{\bar{j}k} = \delta^k_i$ we can derive the explicit form for the inverse full metric. Since we have:

$$g_{i\bar{j}} = \begin{pmatrix} K_{\Phi\bar{\Upsilon}} & K_{\Phi\bar{p}} \\ K_{m\bar{\Upsilon}} & K_{m\bar{p}} \end{pmatrix}, \qquad g^{\bar{j}k} = \begin{pmatrix} \tilde{K}^{\bar{\Upsilon}\Psi} & \tilde{K}^{\bar{\Upsilon}n} \\ \tilde{K}^{\bar{p}\Psi} & \tilde{K}^{\bar{p}n} \end{pmatrix}, \tag{C.13}$$

we need to find the expressions for the matrices $\tilde{K}^{\bar{j}k}$ in terms of $K_{i\bar{j}}$ and their inverses. The constraints can then be derived from

$$\begin{pmatrix} K_{\Phi\bar{\Upsilon}} & K_{\Phi\bar{p}} \\ K_{m\bar{\Upsilon}} & K_{m\bar{p}} \end{pmatrix} \begin{pmatrix} \tilde{K}^{\bar{\Upsilon}\Psi} & \tilde{K}^{\bar{\Upsilon}n} \\ \tilde{K}^{\bar{p}\Psi} & \tilde{K}^{\bar{p}n} \end{pmatrix} = \begin{pmatrix} K_{\Phi\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi} + K_{\Phi\bar{p}}\tilde{K}^{\bar{p}\Psi} & K_{\Phi\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}n} + K_{\Phi\bar{p}}\tilde{K}^{\bar{p}n} \\ K_{m\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi} + K_{m\bar{p}}\tilde{K}^{\bar{p}\Psi} & K_{m\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}n} + K_{m\bar{p}}\tilde{K}^{\bar{p}n} \end{pmatrix}$$
$$= \begin{pmatrix} \delta^{\Psi}_{\Phi} & 0 \\ 0 & \delta^{n}_{m} \end{pmatrix} \tag{C.14}$$

and read

$$K_{\Phi\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi} + K_{\Phi\bar{p}}\tilde{K}^{\bar{p}\Psi} = \delta^{\Psi}_{\Phi}, \tag{C.15}$$

$$K_{\Phi\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}n} + K_{\Phi\bar{p}}\tilde{K}^{\bar{p}n} = 0, \tag{C.16}$$

$$K_{m\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi} + K_{m\bar{p}}\tilde{K}^{\bar{p}\Psi} = 0, \tag{C.17}$$

$$K_{m\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}n} + K_{m\bar{p}}\tilde{K}^{\bar{p}n} = \delta^{n}_{m}. \tag{C.18}$$

Acting now with $K^{\bar{\Psi}\Phi}$ on (C.16) we obtain

$$\tilde{K}^{\bar{\Psi}n} = -K^{\bar{\Psi}\Phi}K_{\Phi\bar{p}}\tilde{K}^{\bar{p}n}, \tag{C.19}$$

and similarly, with $K^{\bar{n}m}$ on (C.17),

$$\tilde{K}^{\bar{n}\Psi} = -K^{\bar{n}m}K_{m\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi}. \tag{C.20}$$

Now we insert these expressions back into (C.15) and (C.18), yielding

$$\begin{aligned} K_{\Phi\bar{\Upsilon}}\tilde{K}^{\bar{\Upsilon}\Psi} + K_{\Phi\bar{p}}(-K^{\bar{p}m}K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Psi}) = \delta^{\Psi}_{\Phi} \\ \Longrightarrow \quad (K_{\Phi\bar{\Upsilon}} - K_{\Phi\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}})\tilde{K}^{\bar{\Upsilon}\Psi} = \delta^{\Psi}_{\Phi} \end{aligned} \tag{C.21}$$

and

$$\begin{aligned} K_{m\bar{\Upsilon}}(-K^{\bar{\Upsilon}\Phi}K_{\Phi\bar{p}}\tilde{K}^{\bar{p}n}) + K_{m\bar{p}}\tilde{K}^{\bar{p}n} = \delta^{n}_{m}. \\ \Longrightarrow \quad (-K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Phi}K_{\Phi\bar{p}} + K_{m\bar{p}})\tilde{K}^{\bar{p}n} = \delta^{n}_{m}. \end{aligned} \tag{C.22}$$

We have then

$$\tilde{K}^{\bar{\Upsilon}\Psi} = (K_{\Phi\bar{\Upsilon}} - K_{\Phi\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}})^{-1}\delta^{\Psi}_{\Phi}, \tag{C.23}$$

$$\tilde{K}^{\bar{p}n} = (K_{m\bar{p}} - K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Phi}K_{\Phi\bar{p}})^{-1}\delta^{n}_{m}. \tag{C.24}$$

In the same way as (C.12), we take the Taylor expansion of (C.23), and we get

$$\begin{aligned} \tilde{K}^{\bar{\Upsilon}\Psi} &= \left[K_{\Phi\bar{\Upsilon}}(\delta^{\bar{\Upsilon}}_{\bar{\Upsilon}} - K^{\bar{\Upsilon}\Psi}K_{\Psi\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}})\right]^{-1}\delta^{\Psi}_{\Phi} \\ &= (\delta^{\bar{\Upsilon}}_{\bar{\Upsilon}} + K^{\bar{\Upsilon}\Upsilon}K_{\Upsilon\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}})K^{\bar{\Upsilon}\Psi}. \end{aligned} \tag{C.25}$$

Since the term $-K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Phi}K_{\Phi\bar{p}}$ in (C.24) is of order $\frac{1}{M_{\rm Pl}^2}$ and $\tilde{K}^{\bar{p}n}$ acts on the visible fields only, we can directly put

$$\tilde{K}^{\bar{p}n} = K^{\bar{p}n}. \tag{C.26}$$

Finally, substituting (C.25) and (C.26) in (C.19) and (C.20) and truncating to $1/M_{\rm Pl}^2$ order, the inverse matrix $g^{\bar{j}k}$ is

$$\begin{pmatrix} K^{\bar{\Upsilon}\Psi} + K^{\bar{\Upsilon}\Upsilon}K_{\Upsilon\bar{p}}K^{\bar{p}m}K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Psi} & -K^{\bar{\Psi}\Phi}K_{\Phi\bar{p}}K^{\bar{p}n} \\ -K^{\bar{n}m}K_{m\bar{\Upsilon}}K^{\bar{\Upsilon}\Psi} & K^{\bar{p}n} \end{pmatrix}. \tag{C.27}$$

C.3 Gaugino variation part limit computation

Hidden piece

Let us consider the three lines of (5.42) separately. For convenience, we split the covariant derivative $\nabla_i = \partial_i + \frac{1}{M_{\rm Pl}^2}K_i$ and define

$$\begin{aligned} W_i^{(h1)} &\equiv \partial_i V^\Sigma P_\Sigma^+ \\ W_i^{(h2)} &\equiv \frac{1}{M_{\rm Pl}^2}K_i V^\Sigma P_\Sigma^+ \\ W_i^{(v1)} &\equiv \partial_i V^M P_M^+ \\ W_i^{(v2)} &\equiv \frac{1}{M_{\rm Pl}^2}K_i V^M P_M^+ \end{aligned} \tag{C.28}$$

The hidden sector piece in (5.42) reads:

$$\begin{aligned} \mathcal{V}_W^{hid} &\equiv e^{\frac{K}{M_{\rm Pl}^2}} g^{\bar{k}k}(\nabla_{\bar{k}}\bar{V}^\Sigma)(\nabla_k V^\Xi)P_\Sigma^- P_\Xi^+ = \\ &= e^{\frac{K}{M_{\rm Pl}^2}} g^{\bar{k}k}(\bar{W}_{\bar{k}}^{(h1)} + \bar{W}_{\bar{k}}^{(h2)})(W_k^{(h1)} + W_k^{(h2)}) \\ &= e^{\frac{K}{M_{\rm Pl}^2}}\{\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_\Phi^{(h1)} + W_\Phi^{(h2)}) \\ &\quad + [\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_m^{(h1)} + W_m^{(h2)}) + \text{c.c.}] \\ &\quad + \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h1)} + \bar{W}_{\bar{n}}^{(h2)})(W_m^{(h1)} + W_m^{(h2)})\}. \end{aligned} \tag{C.29}$$

Now we need to evaluate the orders of the various terms. Using (5.23) and (5.47) we have

$$
\begin{aligned}
\tilde{K}^{\bar{\Psi}\Phi} &= K^{\bar{\Psi}\Phi} + K^{\bar{\Psi}\Upsilon} K_{\Upsilon\bar{p}} K^{\bar{p}m} K_{m\bar{\Upsilon}} K^{\bar{\Upsilon}\Phi} \;\sim \hat{\tilde{K}}_0^{\bar{\Psi}\Phi} + \frac{\hat{\tilde{K}}_2^{\bar{\Psi}\Phi}}{M_{\rm Pl}^2} + \frac{\hat{\tilde{K}}_4^{\bar{\Psi}\Phi}}{M_{\rm Pl}^4} + \dots \\
\tilde{K}^{\bar{\Psi}m} &= -K^{\bar{\Psi}\Phi} K_{\Phi\bar{p}} K^{\bar{p}m} \;\sim \frac{\hat{\tilde{K}}_1^{\bar{\Psi}m}}{M_{\rm Pl}} + \frac{\hat{\tilde{K}}_3^{\bar{\Psi}m}}{M_{\rm Pl}^3}, \\
\bar{W}_{\bar{\Psi}}^{(h1)} &= \partial_{\bar{\Psi}} V^\Sigma P_\Sigma^+ \;\sim g' M_{\rm Pl}^2 \hat{\bar{W}}_{\bar{\Psi}}^{(h1)} \\
\bar{W}_{\bar{\Psi}}^{(h2)} &= \frac{1}{M_{\rm Pl}^2} K_{\bar{\Psi}} V^\Sigma P_\Sigma^+ \;\sim g'(M_{\rm Pl}^2 \hat{\bar{W}}_{(0)\bar{\Psi}}^{(h2)} + \hat{\bar{W}}_{(2)\bar{\Psi}}^{(h2)}) \\
W_m^{(h1)} &= \partial_m V^\Sigma P_\Sigma^+ \;\sim g' M_{\rm Pl}^2 \hat{W}_m^{(h1)} \\
W_m^{(h2)} &= \frac{1}{M_{\rm Pl}^2} K_m V^\Sigma P_\Sigma^+ \;\sim g' M_{\rm Pl} \hat{W}_{(1)m}^{(h2)},
\end{aligned}
\tag{C.30}
$$

where the hatted quantities have dimensions indicated in the bottom index and are dimensionless otherwise.
The orders of the various combinations are then

$$
\begin{aligned}
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi} (\bar{W}_{\bar{\Psi}}^{(h1)} W_\Phi^{(h1)}) \sim [(g')^2 M_{\rm Pl}^4]_h + [(g')^2 M^2]_v + \dots \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi} (\bar{W}_{\bar{\Psi}}^{(h1)} W_\Phi^{(h2)}) \sim [(g')^2 M_{\rm Pl}^4]_h + [(g')^2 M^2]_v + \dots, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi} (\bar{W}_{\bar{\Psi}}^{(h2)} W_\Phi^{(h2)}) \sim [(g')^2 M_{\rm Pl}^4]_h + [(g')^2 M^2]_v + \dots, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m} (\bar{W}_{\bar{\Psi}}^{(h1)} W_m^{(h1)}) \sim (g')^2 M_{\rm Pl}^3, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m} (\bar{W}_{\bar{\Psi}}^{(h1)} W_m^{(h2)}) \sim (g')^2 M_{\rm Pl}^2 + \dots, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m} (\bar{W}_{\bar{\Psi}}^{(h2)} W_m^{(h2)}) \sim (g')^2 M_{\rm Pl}^2 + \dots, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m} (\bar{W}_{\bar{n}}^{(h1)} W_m^{(h1)}) \sim (g')^2 M_{\rm Pl}^4, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m} (\bar{W}_{\bar{n}}^{(h1)} W_m^{(h2)}) \sim (g')^2 M_{\rm Pl}^3, \\
& e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m} (\bar{W}_{\bar{n}}^{(h2)} W_m^{(h2)}) \sim (g')^2 M_{\rm Pl}^2.
\end{aligned}
\tag{C.31}
$$

Remember that the combination $(g' M_{\rm Pl})$ is finite in the limit. The first three terms are comprised of the fully-hidden part, that we absorb in $\mathcal{V}(\Phi, \bar{\Phi}) = 0$, but also give rise to terms involving visible fields from the second orders of the $\tilde{K}^{\bar{\Psi}\Phi}$, W_{Ψ}^{h2} and the exponential $e^{\frac{K}{M_{\rm Pl}^2}}$ expansions. Notice also that all the pieces involving $W_m^{(h1)}$ diverge. In order to erase them, we impose the same requirement as in Chapter 3, namely

$$\partial_m V^\Sigma = 0, \qquad \partial_\Phi V^M = 0. \tag{C.32}$$

Thus, we are again saying that the off-diagonal components of the prepotential second derivative matrix F_{IJ} must be zero in order to preserve gauge invariance

and the decoupling between the hidden and the visible sector.
With $W_m^{(h1)} = 0$, (C.29) (without the term belonging to $\mathcal{V}(\Phi, \bar{\Phi})$) becomes

$$
\begin{aligned}
\lim_{M_{\rm Pl}\to\infty\ g'\to 0} \mathcal{V}_W^{hid} = \\
& e^{K^0}\Big\{K^{0\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_{\Phi}^{(h1)} + W_{\Phi}^{(h2)})|_0(Z_{m\bar{n}}t^m\bar{t}^{\bar{n}} + \dots) \\
& + \tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_{\Phi}^{(h1)} + W_{\Phi}^{(h2)})|_2 \\
& +[\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})W_m^{(h2)} + \text{c.c.}] + \tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(h2)}W_m^{(h2)}\Big\}.
\end{aligned}
\tag{C.33}
$$

All the pieces in (C.33) go like $m_{3/2}^2$ at first order. The indices $'0'$ and $'2'$ indicate that we are considering the $\sim (g'^2)M_{\rm Pl}^4$ or the $\sim (g'^2)M_{\rm Pl}^2$ part of the expansion respectively.

Mixed piece

Now we turn to $\mathcal{V}_W^{mix}$. With the definitions (C.28), we can write

$$
\begin{aligned}
\mathcal{V}_W^{mix} &= \\
&= e^{\frac{K}{M_{\rm Pl}^2}} g^{\bar{k}k}[(\nabla_{\bar{k}}\bar{V}^{\Sigma})(\nabla_k V^M)P_{\Sigma}^- P_M^+ + (\nabla_{\bar{k}}\bar{V}^M)(\nabla_k V^{\Xi})P_M^- P_{\Xi}^+] \\
&= e^{\frac{K}{M_{\rm Pl}^2}}\Big\{\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_{\Phi}^{(v1)} + W_{\Phi}^{(v2)}) \\
&\quad + [\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_m^{(v1)} + W_m^{(v2)}) + \text{c.c.}] \\
&\quad + \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h1)} + \bar{W}_{\bar{n}}^{(h2)})(W_m^{(v1)} + W_m^{(v2)})\Big\} + \text{c.c.}.
\end{aligned}
\tag{C.34}
$$

In addition to (C.30), we also have

$$
\begin{aligned}
\bar{W}_{\bar{\Psi}}^{(v1)} &= \partial_{\bar{\Psi}}V^M P_M^+ \ \sim \hat{\bar{W}}_{(2)\bar{\Psi}}^{(v1)}, \\
\bar{W}_{\bar{\Psi}}^{(v2)} &= \frac{1}{M_{\rm Pl}^2}K_{\bar{\Psi}}V^M P_M^+ \ \sim \frac{\hat{\bar{W}}_{(3)\bar{\Psi}}^{(v2)}}{M_{\rm Pl}} + \frac{\hat{\bar{W}}_{(5)\bar{\Psi}}^{(v2)}}{M_{\rm Pl}^3}, \\
W_m^{(v1)} &= \partial_m V^M P_M^+ \ \sim \hat{W}_{(2)m}^{(v1)}, \\
W_m^{(v2)} &= \frac{1}{M_{\rm Pl}^2}K_m V^M P_M^+ \ \sim \frac{\hat{W}_{(4)m}^{(v2)}}{M_{\rm Pl}^2}.
\end{aligned}
\tag{C.35}
$$

The orders of the various pieces are then

$$
\begin{aligned}
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)}W_{\Phi}^{(v1)}) \sim g' M_{\mathrm{Pl}}^2\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)}W_{\Phi}^{(v2)}) \sim g' M_{\mathrm{Pl}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h2)}W_{\Phi}^{(v1)}) \sim g' M_{\mathrm{Pl}}^2 + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h2)}W_{\Phi}^{(v2)}) \sim g' M_{\mathrm{Pl}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)}W_{m}^{(v1)}) \sim g' M_{\mathrm{Pl}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)}W_{m}^{(v2)}) \sim \frac{g'}{M_{\mathrm{Pl}}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h2)}W_{m}^{(v1)}) \sim g' M_{\mathrm{Pl}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h2)}W_{m}^{(v2)}) \sim \frac{g'}{M_{\mathrm{Pl}}} + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h1)}W_{m}^{(v1)}) \sim g' M_{\mathrm{Pl}}^2 + \dots,\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h1)}W_{m}^{(v2)}) \sim g',\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h2)}W_{m}^{(v1)}) \sim g' M_{\mathrm{Pl}}\\
&e^{\frac{K}{M_{\mathrm{Pl}}^2}}\tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h2)}W_{m}^{(v2)}) \sim \frac{g'}{M_{\mathrm{Pl}}}.
\end{aligned}
\tag{C.36}
$$

The divergent pieces of order $g' M_{\mathrm{Pl}}^2$ are again removed by the assumption (C.32) that implies $W_m^{h1} = W_\Phi^{v1} = 0$, and the only terms surviving the limit are of order $g' M_{\mathrm{Pl}}$.

Therefore, the limit on $\mathcal{V}_W^{mix}$ becomes

$$
\begin{aligned}
\lim_{M_{\mathrm{Pl}}\to\infty \; g'\to 0} \mathcal{V}_W^{mix} = \\
& e^{K^0}\Big\{\tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_{\Phi}^{(v2)})\\
&\quad + [\tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(h1)} + \bar{W}_{\bar{\Psi}}^{(h2)})(W_{m}^{(v1)}) + \text{c.c.}]\\
&\quad + \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(h2)})(W_{m}^{(v1)})\Big\} + \text{c.c.}.
\end{aligned}
\tag{C.37}
$$

Thus, in this sector, all the surviving terms are of order $\sim m_{3/2}$.

Visible piece

Let us finally proceed with $\mathcal{V}_W^{vis}$. We have

$$\begin{aligned}
&\mathcal{V}_W^{vis} = \\
&\quad 2e^{\frac{K}{M_{\rm Pl}^2}}[g^{\bar{k}k}(\nabla_{\bar{k}}\bar{V}^M)(\nabla_k V^N)(P_M^+ P_N^- + P_M^3 P_N^3) + g_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M] \\
&\quad = 2[e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(v1)} + \bar{W}_{\bar{\Psi}}^{(v2)})(W_\Phi^{(v1)} + W_\Phi^{(v2)}) \\
&\qquad + \tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(v1)} + \bar{W}_{\bar{\Psi}}^{(v2)})(W_m^{(v1)} + W_m^{(v2)}) \\
&\qquad + \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(v1)} + \bar{W}_{\bar{n}}^{(v2)})(W_m^{(v1)} + W_m^{(v2)}) \\
&\qquad + \text{similar terms involving } P^3 \\
&\qquad + g_{m\bar{n}} k_M^m k_N^{\bar{n}} V^N \bar{V}^M].
\end{aligned} \tag{C.38}$$

Using (C.30) and (C.35) we can estimate the orders of the combinations appearing in (C.38):

$$\begin{aligned}
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(v1)} W_\Phi^{(v1)}) \sim 1 \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(v1)} W_\Phi^{(v2)}) \sim \frac{1}{M_{\rm Pl}} + \dots, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}\Phi}(\bar{W}_{\bar{\Psi}}^{(v2)} W_\Phi^{(v2)}) \sim \frac{1}{M_{\rm Pl}^2} + \dots, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(v1)} W_m^{(v1)}) \sim 1, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(v1)} W_m^{(v2)}) \sim \frac{1}{M_{\rm Pl}^2}, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{\Psi}m}(\bar{W}_{\bar{\Psi}}^{(v2)} W_m^{(v2)}) \sim \frac{1}{M_{\rm Pl}^3} + \dots, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(v1)} W_m^{(v1)}) \sim 1, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(v1)} W_m^{(v2)}) \sim \frac{1}{M_{\rm Pl}^2}, \\
&e^{\frac{K}{M_{\rm Pl}^2}} \tilde{K}^{\bar{n}m}(\bar{W}_{\bar{n}}^{(v2)} W_m^{(v2)}) \sim \frac{1}{M_{\rm Pl}^4}.
\end{aligned} \tag{C.39}$$

We immediately see that all terms except three are subleading. The first two finite terms would imply a mixing between hidden and visible sectors and are canceled by our assumption (C.32) (implying in this case $\bar{W}_{\bar{\Psi}}^{(v1)} = 0$), while the last finite term is part of the global limit we have seen in the last section of Chapter 3. The terms involving P^3 behave in an analogous way, while the last piece containing the Killing vectors for the vector moduli space is already purely visible. Overall, from $\mathcal{V}_W^{vis}$ we get precisely the full global limit of the gaugino variation part of the scalar potential. We have then .

$$\lim_{M_{\rm Pl}\to\infty\ g'\to 0} \mathcal{V}_W^{vis} = 2e^{K^0}(\tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(v1)}W_m^{(v1)} + \tilde{K}^{\bar{n}m}\bar{W}_{\bar{n}}^{(v31)}W_m^{(v31)} + K_{m\bar{n}}k_M^m k_N^{\bar{n}} V^N \bar{V}^M), \tag{C.40}$$

where we also defined

$$W_i^{(v31)} \equiv \partial_i V^M P_M^3$$
$$W_i^{(v32)} \equiv \frac{1}{M_{\rm Pl}^2} K_i V^M P_M^3. \tag{C.41}$$

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