Stability of Extra Dimensions in the Inflating Early Universe

Dissertation

zur Erlangung des Doktorgrades
an der Fakultät für Mathematik,
Informatik und Naturwissenschaften

Fachbereich Physik
der Universität Hamburg

vorgelegt von

Clemens Wieck

aus Hamburg

Hamburg

2015

Tag der Disputation: 8. Juli 2015
Folgende Gutachter empfehlen die Annahme der Dissertation:
Prof. Dr. Wilfried Buchmüller
Prof. Dr. Jan Louis

Abstract

Cosmic inflation is an attractive paradigm to explain the initial conditions of the universe. It can be conveniently described by the dynamics of a single scalar field within $\mathcal{N} = 1$ supergravity. Due to the high energy scale during the inflationary epoch, which is favored by recent observations of the cosmic microwave background radiation, and the flatness of the inflaton potential it is necessary to consider inflation in the context of a UV-complete theory like string theory. To this end, we study the effects of moduli stabilization on inflation models in supergravity, focussing on Kähler moduli in type IIB string theory which govern the size of extra dimensions. For generic models of F-term inflation we calculate back-reaction terms by integrating out the moduli at a high energy scale. When the moduli are stabilized supersymmetrically, all effects decouple in the limit of very heavy moduli. The corrections, however, may be sizeable for realistic moduli masses above the Hubble scale and affect the predicted observables of many models like chaotic inflation and hybrid inflation. If, on the other hand, moduli stabilization entails spontaneous supersymmetry breaking, there are non-decoupling effects like soft mass terms for the inflaton. By the example of chaotic inflation we show that a careful choice of parameters and initial conditions is necessary to reconcile large-field inflation with popular moduli stabilization schemes like KKLT stabilization or the Large Volume Scenario. Furthermore, we study the interplay of moduli stabilization and D-term inflation. If inflation is driven by a constant Fayet-Iliopoulos term, the back-reaction decouples but the gravitino mass in the vacuum is surprisingly constrained. For a field-dependent Fayet-Iliopoulos term associated with an anomalous $U(1)$ symmetry we discuss a number of obstructions to realizing inflation. Moreover, we propose a way to evade them using a new mechanism for supersymmetric moduli stabilization with world-sheet instantons.

Zusammenfassung

Kosmische Inflation ist ein attraktives Szenario zur Beschreibung der Anfangsbedingungen unseres Universums. Inflation kann durch die Dynamik eines skalaren Feldes beschrieben werden, beispielsweise im Rahmen von $\mathcal{N} = 1$ Supergravitation. Allerdings hängen solche Theorien, aufgrund der hohen Energieskalen und der Flachheit des Inflatonpotentials, von neuer Physik an der Planck-Skala, die beispielsweise durch Stringtheorie beschrieben werden kann, ab. Wir untersuchen die Auswirkungen von Moduli Stabilisierung auf Inflationsmodelle in Supergravitation. Dabei konzentrieren wir uns auf Kähler Moduli in Typ IIB Stringtheorie, welche die Größe zusätzlicher Dimensionen parametrisieren. Wir bestimmen die Rückkopplung solcher Moduli für beliebige Modelle von F-Term Inflation durch Ausintegrieren an einer hohen Skala. Wenn die Moduli supersymmetrisch stabilisiert sind, entkoppeln alle Effekte im Limes unendlich schwerer Moduli. Für realistische Modulimassen oberhalb der Hubble-Skala werden die Korrekturterme allerdings relevant für die vorhergesagten Observablen vieler Modelle, wie beispielweise Hybridinflation oder Chaotischer Inflation. Wenn die Stabilisierung andererseits Supersymmetrie spontan bricht, treten Effekte auf, die nicht entkoppeln. Anhand von Chaotischer Inflation zeigen wir, dass nur eine sorgfältige Wahl der Parameter und Anfangsbedinungen Moduli Stabilisierung und Inflation miteinander vereinbaren kann. Des Weiteren untersuchen wir die Effekte von schweren Moduli in D-Term Inflation. Im Falle eines konstanten Fayet-Iliopoulos Terms entkoppeln alle Korrekturen durch Moduli, während die mögliche Gravitinomasse im echten Vakuum stark eingeschränkt ist. Andererseits ist ein feldabhängiger Fayet-Iliopoulos Term nur schwer mit D-Term Inflation vereinbar. Wir diskutieren eine Reihe von Einschränkungen, und präsentieren einen Vorschlag, diese zu umgehen.

This thesis is based on the following publications:

- W. Buchmüller, V. Domcke, and C. Wieck "No-scale D-term inflation with stabilized moduli", Phys.Lett. B730 (2014) 155160
- W. Buchmüller, C. Wieck, and M. W. Winkler "Supersymmetric Moduli Stabilization and High-Scale Inflation", Phys.Lett. B736 (2014) 237240
- W. Buchmüller, E. Dudas, L. Heurtier, and C. Wieck "Large-Field Inflation and Supersymmetry Breaking", JHEP 1409 (2014) 053
- C. Wieck and M. W. Winkler "Inflation with Fayet-Iliopoulos Terms", Phys.Rev. D90 (2014) no. 10, 103507
- W. Buchmüller, E. Dudas, L. Heurtier, A. Westphal, C. Wieck, and M. W. Winkler "Challenges for Large-Field Inflation and Moduli Stabilization", JHEP 1504 (2015) 058

Contents

Chapter 1

Introduction

The study of the early universe is a difficult but rewarding task. Modern particle physics experiments like the Large Hadron Collider (LHC) can probe physics up to an energy scale of approximately 1 TeV. In the early universe, according to the well-established Hot Big Bang scenario, this corresponds to a period about 10^{-10} seconds after the Big Bang singularity. The LHC has tested the predictions of the Standard Model (SM) of particle physics with remarkable precision, in particular the structure of electroweak symmetry breaking [1–3]. It has, furthermore, put lower bounds on the masses of supersymmetric particles close to the TeV scale [4]. However, in order to illuminate processes in the very early universe, i.e., processes significantly above the TeV scale, we need different tools.

Since the discovery of the cosmic microwave background (CMB) radiation by Penzias and Wilson [5], cosmological observations have developed into such a precision tool. Measurements of the CMB temperature fluctuations by the Cosmic Background Explorer (COBE) [6], the Wilkinson Microwave Anisotropy Probe [7], and more recently the Planck satellite [8], as well as maps of the distribution of large-scale structure [9] and supernova redshift surveys [10, 11] have established a standard model of cosmology, the ΛCDM model. Furthermore, a new generation of experiments have begun to measure the polarization of the CMB with increasing accuracy [12, 13], possibly leading to a discovery of primordial gravitational waves in the near future. Among others, ground-based observatories like the BICEP telescope, the Keck Array, POLARBEAR, the Atacama Cosmology Telescope [14], and the South Pole Telescope [15] will continue to make cosmology a highly interesting field of research in the next years.

The success in relating the various observations to the Hot Big Bang theory, i.e., the thermal afterglow of processes in the first 10^{-10} seconds of the universe, motivates us to ask what the origin of the observed fluctuations in the CMB could be. An attractive paradigm to explain these primordial fluctuations, as well as the remarkable isotropy of the CMB on large scales, is a phase of cosmic inflation [16–18] in the very early universe. The principle idea of this paradigm is that a phase of exponential expansion of space, believed to have taken place approximately 10^{-34} seconds after the Big Bang singularity, can solve a series of problems of the Hot Big Bang scenario, as well as explain the origin of the CMB fluctuations leading to the formation of structure as the universe evolves. Such an accelerated expansion may have occurred during an early phase of vacuum domination with negative pressure. According to the paradigm, after inflation has ended the universe can reheat and subsequently cool down, leading to the successful predictions of the Hot Big Bang theory like, for example, Big Bang Nucleosynthesis.

In field theory, inflation is most simply driven by the vacuum energy of a single scalar field called inflaton. For sufficient expansion to occur the scalar field must roll slowly in a flat potential. However, both the particle physics origin of the inflaton field and the exact shape and origin of its potential are currently debated problems. Among successful models are chaotic inflation [19] with a monomial potential, most simply a mass term for the inflaton field, and hybrid inflation [20], involving the spontaneous breaking of a $U(1)$ symmetry. Moreover, the first release of Planck CMB data [21, 22] generated renewed interest in one of the very first inflation models developed by A. Starobinsky [23] which involves terms in the gravity action beyond the standard Einstein-Hilbert term. Alternatively, inflation may be driven by D-terms of various kinds, cf. [24, 25]. Recent measurements of the CMB temperature and polarization anisotropies strongly favor the realization of single-field slow-roll inflation in nature, resulting in nearly scale-invariant, Gaussian, and adiabatic scalar perturbations. While they severely constrain many models, cf. [13], they do not nominate a clear favorite theory of inflation.

For many reasons it is instructive to consider inflation in the context of string theory. On the one hand, string theory postulates the existence of many candidate fields for the inflaton. On the other hand, the required flatness of the inflaton potential makes it susceptible to new physics at the Planck scale. This is most pronounced in large-field inflation, i.e., in models where the inflaton traverses trans-Planckian distances during slow-roll, which would be favored by the observation of primordial tensor perturbations. It is therefore necessary to evaluate the absence or size of certain correction terms in a candidate theory of quantum gravity that can describe physics up to the Planck scale. String theory, though demanding the existence of ten space-time dimensions, is arguably the best-developed of those candidate theories. While the time-resolved analysis of the cosmological evolution in a ten-dimensional setup is, thus far, technically impossible, many relevant phenomena can be studied by means of four-dimensional effective field theories. After compactifying six dimensions on a suitable complex manifold, the supersymmetric four-dimensional macroscopic theory can be described by $\mathcal{N} = 1$ supergravity, i.e., local supersymmetry. As its name implies, supergravity contains a supersymmetrized version of gravity and is thus a convenient framework to analyse an ultraviolet (UV) completion of inflation. A particularly attractive class of compactification spaces is Calabi-Yau manifolds. In heterotic string theory [26, 27] they naturally give rise to $\mathcal{N} = 1$ supergravity in four dimensions [28], and in type IIB string theory they do so after orientifolding, cf. [29, 30] for reviews.

A generic prediction of string compactifications on Calabi-Yau manifolds is the existence of moduli superfields whose scalar components parameterize either the shape or the size of subspaces of the manifold. For multiple reasons these additional scalar fields should be heavier than the characteristic energy scale during inflation. First, for single-field slow-roll inflation to proceed successfully there must be only one light field. Second, for the size of the extra dimensions to remain small enough to escape detection, the so-called Kähler moduli must be fixed at a suitable vacuum expectation value without the possibility to run away towards infinity. This raises the critical issue of moduli stabilization whenever inflation is treated in the context of string theory. Since the fundamental works [31, 32] substantial progress has been achieved especially in type IIB string theory, but also in heterotic string theory, cf. [33, 34] and references therein.

Studying the supergravity effective theory after integrating out heavy moduli fields yields crucial information on higher-dimensional operators descending from the UV-complete theory. The interaction of such correction terms with inflation is the subject of this thesis. It is organized as follows. Chapter 2 begins with an overview of the Hot Big Bang scenario, cosmic inflation, the problems it solves, and its relation to string theory and supergravity. Subsequently, we introduce a number of successful inflation models and their descriptions in four-dimensional $\mathcal{N} = 1$ supergravity. Chapter 3 contains a brief review of moduli stabilization, focussing on Kähler moduli in type IIB string theory, as well as detailed descriptions of Minkowski or de Sitter vacua with spontaneously broken supersymmetry in a number of examples. In Chapter 4 we begin to put these two pieces together and describe the interaction of supersymmetric Kähler moduli stabilization, a rather simple case, with inflation driven by F-terms. We give general expressions for higher-dimensional operators in the effective action arising from moduli stabilization and discuss two examples. Subsequently, in Chapter 5 we give a detailed analysis of chaotic inflation combined with moduli stabilization and spontaneous supersymmetry breaking. Again, we present general expressions as well as numerical studies of useful examples. Chapter 6 is devoted to inflation driven by D-terms and how it interacts with heavy Kähler moduli. We describe cases with both a constant Fayet-Iliopoulos term and a field-dependent one. We summarize our results and conclusions in Chapter 7, and give a short outlook. Details about Fayet-Iliopoulos terms and supergravity formulae are collected in several appendices.

The results presented in this thesis have been previously published in [35–39]. Parts of it also refer to two separate publications [40, 41]. While the results of [35–41] are undisputed, their presentation in this work may be focussed on specific aspects, resulting in a slightly different treatment. These aspects are emphasized in the beginning of each chapter and reflect the author's personal contribution to the respective publication.

Chapter 2

Cosmic Inflation and its Supergravity Embedding

Motivated by shortcomings of the established Hot Big Bang scenario, cosmic inflation can explain the initial conditions of the universe. In this chapter we give a brief review of the original motivation for inflation and summarize its principal virtues. Afterwards, on the basis of simple arguments we illustrate the intimate connection between inflation and other extensions of the SM like supersymmetry and string theory. As a prelude to subsequent chapters, we introduce the standard treatment of inflation in quantum field theory, and discuss some of the most successful models of single-field slow-roll inflation and their embedding in supergravity.

For a thorough treatment of the Hot Big Bang theory we refer the reader to [42]. Useful reviews of inflationary cosmology and its embedding in supergravity and string theory can be found in [43–45]. As an introduction to four-dimensional $\mathcal{N} = 1$ supergravity we recommend the standard reference [46].

2.1 Puzzles in the early universe

The Hot Big Bang theory is a remarkably successful framework which can explain the expansion history of the universe, the existence of the CMB radiation, the relative abundances of light elements, and the formation of structure. E. Hubble's observation [47] that the universe is currently in a state of accelerated expansion brought forward the idea that it may have originated from a very hot and dense initial state, a cosmic singularity. Starting from this, the thermal history of the universe can be reconstructed by comparing the interaction rate of particle species with the expansion rate of the universe. When the former is much greater than the latter the respective particle species are in a state of thermal equilibrium largely unaffected by the expansion of the universe. As the universe cools down, interaction rates may decrease faster than the expansion rate. When both are of the same order, particle species decouple from the thermal bath and freeze out. In this way photons in the hot plasma decoupled 380 000 years after the initial singularity to form the remarkably isotropic CMB radiation. Analogously, Big Bang Nucleosynthesis successfully predicts the relative abundances of light elements in the universe, cf. [48] for a review. The consequences of small-scale temperature fluctuations observed in the CMB are another success of the Hot Big Bang theory. Such fluctuations can act as seeds of structure formation since small initial inhomogeneities grow with time due to the attractive nature of gravity.

The horizon, flatness, and monopole problems

Despite its successes a number of puzzles arise in this description of the universe, which the Hot Big Bang theory cannot explain. Most importantly the universe seems to have evolved from highly fine-tuned initial conditions, as emphasized in [49].[1]

First, according to the Big Bang model the universe must have emerged from a remarkably isotropic state. As mentioned before, inhomogeneities grow with time due to gravitational instabilities. Thus, we expect the small-scale inhomogeneities observed in the CMB to have been even smaller at earlier times. This is particularly surprising since one can show that at the time of last scattering, i.e., at the birth of the CMB radiation, the universe consisted of numerous causally disconnected patches. Why should these causally disconnected regions of space show such similar physical conditions? In more technical terms, the comoving particle horizon, a measure for the distance a light ray can travel in any given time, monotonically increases with time if the expansion of the universe is matter- or radiation-dominated as in conventional Big Bang expansion. This means that comoving scales entering the horizon today must have been far outside the horizon at the time of CMB decoupling. This is commonly referred to as the horizon problem.

Second, the initial state of the universe must have been spatially flat to incredible accuracy to explain the observed flatness of the universe today. This is because any primordial spatial curvature grows with time as long as the universe expands as postulated by the Hot Big Bang theory. Again this is due to the fact that the comoving

[1] Notice, however, that the question whether initial conditions should be treated as part of a physical theory or not is highly debatable. Some argue that physics should predict the future evolution of a system given a set of initial conditions, as in Newtonian dynamics [44]. Thus, it is far from obvious that a consistent cosmological theory should predict or explain its own initial conditions.

horizon increases with time. Quantitatively, the universe can have deviated from spatial flatness at most by a fraction of 10^{-16} during the epoch of Big Bang Nucleosynthesis, and by 10^{-55} during a possible era of Grand Unification at 10^{16} GeV [44]. This apparent shortcoming of the Big Bang model is known as the flatness problem.

Notice that these are indeed shortcomings of the predictive power of the theory, and not conceptual inconsistencies. If the universe was fine-tuned to be extremely isotropic and flat across super-Horizon distances, it would have evolved as predicted in the Hot Big Bang scenario. Before we continue to discuss how a period of cosmic inflation can dynamically generate these rather specific initial conditions let us mention a third problem, which was part of the original motivation to propose inflation [16]. As the universe cools down from a hot initial state different kinds of phase transitions can occur when symmetries are broken. During some of these phase transitions, topological defects may be produced whose presence would perturb the successful predictions of the Hot Big Bang theory. In particular, magnetic monopoles from a Grand Unified Theory (GUT) phase transition could overclose the universe by contributing large amounts of energy density [50, 51]. This is sometimes called the monopole problem.[2]

Inflation to the rescue

In order to solve or circumvent these problems the concept of inflationary cosmology was proposed in [16–18]. It was noticed that in an epoch dominated by vacuum energy, i.e., an epoch of de Sitter space-time which is dominated by energy with negative pressure, the universe would expand exponentially while the comoving horizon would shrink. From what we have discussed so far, it is quite clear how this can solve all of the aforementioned problems. On the one hand, if the comoving horizon shrinks in some early phase of the universe, patches which were causally disconnected during the time of CMB decoupling may have been in causal contact before. This naturally explains the large-scale homogeneity of the CMB. On the other hand, the inverted behavior of the comoving horizon drives the universe towards flatness. Qualitatively, any spatial curvature present in an initial state of the universe is smoothed out by the exponential expansion of space. Furthermore, inflation dilutes the number density of topological defects produced during early-stage phase transitions. In fact, all of the above shortcomings are ameliorated if inflation lasted long enough to increase the size of the universe by a factor of 10^{27}, or approximately e^{60}.

[2]Notice that overclosure of the universe is not a problem unique to magnetic monopoles and GUT symmetry breaking. Similar features are shared by the spontaneous breaking of discrete symmetries during which supermassive domain walls are produced [52, 53].

Apart from this, inflation predicts the presence of small perturbations in the CMB radiation. In a spectacular triumph for early universe cosmology and the inflationary paradigm itself these have been confirmed by the COBE satellite [6]. They are a generic consequence of the quantum mechanical treatment of the inflationary de Sitter space-time. Quantum fluctuations generated on sub-horizon scales exit the horizon, and thus "freeze in", once the Hubble horizon becomes smaller than their comoving wavelength. Once inflation has ended and the horizon expands in the subsequent Big Bang evolution they may re-enter the horizon and become classical density perturbations. Through gravitational collapse these perturbations evolve to form the presently observed large-scale structure in the universe. For a thorough treatment of the evolution of initial quantum fluctuations during inflation, we refer the reader to [54].

The study of inflationary cosmology in quantum field theory has been the subject of research for more than three decades. Quite early it has been realized that the vacuum energy of a scalar field, the inflaton, can drive inflation as long as its kinetic energy is much smaller than its potential energy. This means that the inflaton must roll slowly in a suitable potential for a sufficiently long period of time. After this phase of vacuum domination the universe reheats, for example, by coherent oscillation of the inflaton field around its vacuum expectation value.[3] After it has reached a thermal state Big Bang expansion may proceed as in the original Hot Big Bang scenario.

Although there is no unique theory of inflation with a clear origin of the inflaton field and its potential, substantial progress has been made in constructing models in supersymmetric field theories. In Section 2.3 we describe in detail how inflation can be implemented in (locally) supersymmetric theories. But first, let us comment briefly on why supersymmetry is a useful extension of quantum field theory, and why considering inflation in the context of string theory is instructive and necessary.

Why supersymmetry?

There are many reasons, related to both particle physics and cosmology, to believe that the underlying theory describing inflation and the evolution of the universe should be space-time supersymmetric. Low-energy supersymmetry can solve the gauge hierarchy problem by protecting the mass of the Higgs field from radiatively induced divergences of momentum integrals, cf. [56] for a review. The same holds for the mass of the scalar inflaton field, which can similarly be stabilized by supersymmetry. In addition, it is appealing from the perspective of GUTs since unification of the three SM gauge couplings

[3] Reheating and preheating after inflation is an interesting field of research by itself. In this thesis we cannot treat it with proper attention, and instead refer the reader to [55] and the respective chapters in [54].

is only sufficiently accurate after taking supersymmetric partner fields into account [57–59]. It also gives rise to suitable candidates for dark matter particles, like the neutralino, possibly explaining the origin of roughly 25% of the energy density of the universe.

However, since supersymmetry is broken in nature it cannot be a global symmetry. Otherwise, theories with spontaneously broken supersymmetry were to postulate an unobserved massless goldstino fermion by virtue of Goldstone's theorem. In local supersymmetry the massless goldstino is eaten by the massive superpartner of the graviton, the gravitino. This illuminates another motivation to consider supersymmetry a useful idea: it is naturally connected to gravity. The supersymmetry algebra contains the operator of space-time translations, thus local supersymmetry must contain general space-time coordinate transformations, the invariance group of general relativity. Equivalently the spin-2 graviton, belonging to a gravity supermultiplet together with the spin-3/2 gravitino, must be part of the supersymmetric spectrum to preserve Lorentz invariance.

Apart from these apparent virtues supersymmetry plays a crucial role in the structure of string theory. In fact, its first appearance as a symmetry in physics was in the attempt to extend bosonic string theory by space-time fermions [60, 61]. World-sheet supersymmetric string theories naturally lead to space-time supersymmetry in simple backgrounds, guaranteeing the absence of tachyons in the spectrum and thus providing stable vacua. For a thorough treatment, cf. [62].

2.2 Connection to string theory and extra dimensions

The emergence of four-dimensional space-time supersymmetry from string compactifications is already a convincing reason to consider string theories which consistently describe the evolution of the universe. However, a number of other reasons are worth pointing out. As discussed below, inflation is generically susceptible to physics relevant at higher energy scales, i.e., it is UV-sensitive. Therefore, inflation should be addressed in a framework which can describe Planck-scale physics, for which string theory is arguably the best-developed candidate. Since string theory is a fundamental theory containing gravity we expect it to correctly describe the cosmological evolution of the universe. Conversely, cosmological observations may provide important insights and constraints on possible string theory vacua. Superstring theory requires the existence of ten space-time dimensions, meaning that six dimensions must be compactified in order to describe inflation in the remaining four dimensions. A study of the cosmological history in full string compactifications with time-dependent four-dimensional backgrounds is technically challenging, if not, thus far, impossible in realistic setups. Many questions, however, can be addressed by means of four-dimensional effective fields theories which encode information about the underlying string theory. This is possible as long as there is a hierarchy of scales,

$$H \ll M_{\mathrm{KK}} \ll M_{\mathrm{s}} \,, \tag{2.1}$$

where H denotes the Hubble scale, parameterizing the energy scale during the inflationary epoch, M_{KK} denotes the Kaluza-Klein scale, and M_{s} denotes the string scale.

Effective field theory descriptions

Starting from the observation that new physics must enter at the Planck scale in order to render graviton-graviton scattering unitary,[4] we can ask how this new physics affects a description of inflation. Note that the following arguments hold independent of whether physics at the Planck scale is described by a finite theory of quantum gravity, such as string theory, or is in turn an effective theory of some unimagined physics at even higher scales. In four-dimensional effective field theory the effects of physics above a cut-off scale Λ are parameterized by Lagrangian operators of the form

$$\frac{\mathcal{O}_\delta}{M^{\delta-4}} \,, \tag{2.2}$$

where $M > \Lambda$ is the mass scale of fields which have been integrated out and δ denotes the mass dimension of the operator. Planck-scale processes and operators of very high dimension are usually suppressed and decouple from low-energy physics. For inflation, however, the picture is different. The flatness of the inflaton potential $V(\varphi)$ makes it susceptible to operators of the form

$$\frac{\mathcal{O}_6}{M_{\mathrm{P}}^2} = \frac{\mathcal{O}_4}{M_{\mathrm{P}}^2}\varphi^2 \,, \tag{2.3}$$

where M_{P} denotes the reduced Planck mass. Such operators are allowed if the inflaton field φ is not protected by an appropriate symmetry. If the operator $\mathcal{O}_4$ has a vacuum expectation value of the same order as the energy scale during inflation, the inflaton potential becomes too steep for inflation to be possible. This is one manifestation of the so-called η problem, discussed in more detail in Section 2.3. In more general terms, we

[4]String theory does so by introducing a characteristic length scale ℓ_{s} which cuts off the divergences in graviton scattering. This happens at the energy scale $M_{\mathrm{s}} \sim \ell_{\mathrm{s}}^{-1}$ where the extended nature of the string becomes important. This results in a finite description of quantum gravity.

can expand the effective Lagrangian for a real inflaton field φ as follows,[5]

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4}\lambda\varphi^4 - \sum_{k=1}^{\infty} c_k \mathcal{O}_{4,k}\frac{\varphi^{2k}}{M_{\mathrm{P}}^{2k}}\,. \tag{2.4}$$

Here, m denotes the mass of the inflaton field, λ is a dimensionless coupling, and c_k are real numbers. This expression illustrates that in small-field models of inflation, i.e., when the inflaton traverses distances smaller than the Planck scale, dimension-six operators are potentially dangerous and higher-order operators are suppressed by powers of φ/M_{P}. For large-field models, with $\Delta\varphi > M_{\mathrm{P}}$ during inflation, the UV sensitivity is dramatically enhanced. In this case an infinite series of higher-dimensional operators from Planck-scale physics is potentially relevant. Hence in general, if the UV theory has no additional symmetries all coefficients c_k must be fine-tuned to very small values for the operators to be negligible. From this discussion it becomes clear that knowledge of such higher-dimensional operators and symmetries is required to guarantee that a given model supports 60 e-folds of slow-roll inflation.

One possibility to protect the flatness of the inflaton potential is to impose a global shift symmetry,

$$\varphi \to \varphi + c\,, \tag{2.5}$$

where c is a real constant. This kind of symmetry was first used in the context of in inflation in [63]. Such a symmetry may, however, be broken by higher-dimensional operators. Thus, the consistency of the symmetry in the effective field theory must be checked in the UV theory. Independent of the form of additional operators, a shift symmetry like (2.5) can solve a different manifestation of the η problem in supergravity, as discussed in Section 2.3.

Additional fields from string compactifications

Dangerous operators as in Eq. (2.4) generically arise when integrating out heavy fields descending from the string compactification, such as stabilized moduli fields. The study of such operators and their effect on inflation is the main subject of this thesis. As explained in more detail in Chapter 3, moduli fields are massless at tree level. Thus, they must be stabilized by some mechanism to guarantee that single-field slow-roll inflation is not spoiled by fluctuations of other light fields, and to guarantee the stability of the extra dimensions themselves. To this end, it is desirable to achieve a hierarchy

$$H \ll M_{\mathrm{Moduli}}\,, \tag{2.6}$$

[5] For the sake of simplicity we have imposed the reflection symmetry $\varphi \to -\varphi$.

in addition to the one in (2.1). In fact, this hierarchy during the inflating phase of the universe must be achieved for any additional scalar field in the theory. As discussed above, integrating out such heavy fields, i.e., solving their equations of motion to find the Wilsonian effective action for the inflaton, generically yields operators relevant for inflation.

However, additional scalar degrees of freedom from string compactifications are not only obstructive to realizing inflation. Generically, some of the new fields postulated by string theory are viable candidates for the inflaton field. This holds most prominently for axions, which can be part of complex moduli fields. They are naturally equipped with a shift symmetry of the form (2.5) which guarantees sufficient flatness of their potentials. In recent years, a large number of inflation models has been constructed from string theory, with various choices for the inflaton field and a plethora of possible potential shapes. An attempt to list them is beyond the scope of this thesis. Instead we refer the interested reader to a number of well-written reviews, for example [45, 64–66], and references therein.

In most of this thesis we do not specify the string theory origin of the inflaton field. Instead we treat moduli and inflaton as separate fields to study their interaction in string-effective models of supergravity. Before discussing the details of moduli stabilization we proceed by introducing the embedding of slow-roll inflation in four-dimensional supergravity.

2.3 Inflation in supergravity

The physics of slow-roll inflation

In quantum field theory a real scalar field φ minimally coupled to gravity obeys the action[6]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right) , \tag{2.7}$$

where R denotes the Ricci scalar. When the four-dimensional space-time background is homogeneous and isotropic[7] it can be described by the Friedman-Lemaître-Robertson-Walker metric defined by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 . \tag{2.8}$$

[6] Notice that from now on we work in natural units, where the reduced Planck mass is set to one.
[7] This was first dignified as the "Cosmological Principle" in [67].

Here, $a(t)$ is the scale factor of the expanding universe and $\mathbf{x}$ are suitable coordinates of three-dimensional space. With $g_{\mu\nu}$ of this form, and assuming homogeneity of the scalar field, $\varphi(t, \mathbf{x}) \equiv \varphi(t)$, the dynamics of the theory are governed by the Friedman equation and the Klein-Gordon equation,

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\,, \qquad \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0\,, \tag{2.9}$$

where dots denote time derivatives and primes indicate derivatives with respect to φ. H is the aforementioned Hubble scale, defined by

$$H \equiv \frac{\dot{a}}{a}\,. \tag{2.10}$$

If the potential energy V dominates over the kinetic energy accelerated expansion of space is possible. This can be translated into a condition on the so-called first slow-roll parameter

$$\tilde{\epsilon} \equiv -\frac{\dot{H}}{H^2} < 1\,. \tag{2.11}$$

For slow-roll inflation to last long enough there is a condition on a second slow-roll parameter, defined by

$$\tilde{\eta} \equiv -\frac{\ddot{\varphi}}{H\dot{\varphi}}\,. \tag{2.12}$$

Imposing $|\tilde{\eta}| < 1$ ensures that the change of $\tilde{\epsilon}$ during inflation is small enough. More commonly, the two slow-roll conditions are expressed as conditions on the shape of the potential $V(\varphi)$,

$$\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1\,, \qquad |\eta| \equiv \left|\frac{V''}{V}\right| \ll 1\,. \tag{2.13}$$

If these are satisfied[8] Eqs. (2.9) become

$$3H^2 \approx V(\varphi) \approx \text{const.}\,, \qquad 3H\dot{\varphi} \approx -V(\varphi)\,, \tag{2.14}$$

and the scale factor increases exponentially, $a(t) \sim e^{Ht}$. Inflation ends when the slow-roll conditions (2.13) are violated, i.e., when $\epsilon(\varphi_{\text{end}}) \approx 1$ or $\eta(\varphi_{\text{end}}) \approx 1$. Before this happens, a convenient way to parameterize the duration of the inflationary phase is to specify the number of e-folds of spatial expansion. It can be computed as follows,

$$N_e \equiv \ln\frac{a_{\text{end}}}{a} \approx \int_{\varphi_{\text{end}}}^{\varphi} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon}}\,. \tag{2.15}$$

[8] In the slow-roll regime $\tilde{\epsilon} \approx \epsilon$ but $\tilde{\eta} \approx \eta - \epsilon$.

Solving the horizon and flatness problems requires a total number of e-folds $N_e \approx 60$, while the details depend on the specific mechanism of inflation and on the way the universe reheats after inflation. The fluctuations observed in the CMB were produced when the scales probed by the CMB exited the horizon. This instance corresponds to a point in field space, denoted by $\varphi_\star$, which typically obeys

$$40 \lesssim N_\star \approx \int_{\varphi_{\text{end}}}^{\varphi_\star} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon}} \lesssim 60 \,. \tag{2.16}$$

We move on to discussing suitable scalar potentials in supergravity shortly, but before let us mention constraints on the shape of $V(\varphi)$ from recent CMB measurements.

Constraints from CMB observations

There are a number of important quantities related to the CMB radiation and its fluctuations which allow us to draw inferences on the mechanism of inflation. Some of them have been measured with remarkable accuracy [68]. First, the amplitude of scalar fluctuations is measured to be

$$A_{\text{s}} = (2.207 \pm 0.076) \times 10^{-9} \,. \tag{2.17}$$

In the slow-roll approximation it is given by

$$A_{\text{s}} = \frac{H^2}{8\pi^2\epsilon} \,. \tag{2.18}$$

Thus, the measured value of the amplitude can be used to constrain the parameters of a model, as all other observables can. Second, the spectral index of scalar fluctuations, $n_{\text{s}} = 1 - 6\epsilon + 2\eta$ during slow-roll, is

$$n_{\text{s}} = 0.9645 \pm 0.0049 \,. \tag{2.19}$$

In both measurements the errors correspond to 1σ uncertainties. Finally,[9] there is an upper bound on the ratio of tensor-to-scalar fluctuations, defined by

$$r \equiv \frac{A_{\text{t}}}{A_{\text{s}}} = 16\epsilon \,, \tag{2.20}$$

[9] Let us remark that there are many more observable parameters relevant for inflation than the ones listed here. Among others the level of non-Gaussianities in the CMB fluctuations, the amplitude of isocurvature fluctuations, the level of spectral distortions of the CMB black body spectrum, the scale-dependence of tensor modes, and the running of n_{s}. However, they are of less relevance to this thesis. Cf. [44] for a more thorough discussion of additional tests of inflation.

where the last equality holds during slow-roll and $A_{\mathrm{t}} = 2H^2/\pi^2$. Notice that all slow-roll expressions are to be evaluated at the scale of CMB horizon re-entry, i.e., at $\varphi = \varphi_\star$. As of this writing r is constrained to be

$$r < 0.1\,, \tag{2.21}$$

at 95% confidence level [68]. These observations contain valuable information about the energy scale of inflation,

$$V(\varphi_\star) = 3H_\star^2 = \frac{3\pi^2 A_{\mathrm{s}}}{2}\, r = (1.88 \times 10^{16}\,\mathrm{GeV})^4\, \frac{r}{0.1}\,. \tag{2.22}$$

This is one of many reasons why a discovery of primordial gravitational waves, i.e., a detection of $r \neq 0$, would be a substantial step forward for the study of inflation.

Embedding in supergravity

Supergravity, as the low-energy effective field theory of string theory, is a useful framework to describe inflation and effects from a UV-complete theory.[10] The bosonic degrees of freedom of a general four-dimensional $\mathcal{N} = 1$ supergravity theory are the metric $g_{\mu\nu}$, gauge fields A_μ^a, and complex scalar fields z^α. We are particularly interested in the interactions of the scalars, which are encoded in the holomorphic superpotential $W(z^\alpha)$ and in the Kähler potential $K(z^\alpha, \bar{z}^{\bar{\alpha}})$, which is a real analytic function of the fields. In the absence of gauge interactions the scalar Lagrangian takes the form

$$\mathcal{L} = -K_{\alpha\bar{\alpha}} \partial_\mu z^\alpha \partial^\mu \bar{z}^{\bar{\alpha}} - V_F\,, \tag{2.23}$$

where $K_{\alpha\bar{\alpha}} = \partial_\alpha \partial_{\bar{\alpha}} K$ is the Kähler metric on the complex manifold spanned by the scalar fields. The F-term scalar potential describes the self-interactions of the fields. It is given by

$$V_F = e^K (K^{\alpha\bar{\alpha}} D_\alpha W D_{\bar{\alpha}} \overline{W} - 3|W|^2)\,, \tag{2.24}$$

where $D_\alpha W = \partial_\alpha W + K_\alpha W$ denotes the Kähler-covariant derivative of the superpotential. Moreover, in a supergravity theory with $U(1)$ gauge interactions the gauge kinetic function f and the Killing vectors k^α, which parameterize the gauge transformations of

[10] The detailed construction of the supergravity action, first performed in [69, 70], is beyond the scope of this thesis. We restrict ourselves to introducing relevant expressions for reference in subsequent chapters. More details and useful supergravity formulae concerning both F-term and D-term potentials can be found in Appendices A.1 and A.2.

the z^α, must be specified. In this case the scalar part of the Lagrangian has an additional D-term piece,

$$V_D = \frac{1}{2\mathrm{Re}\, f} D^2 \,, \tag{2.25}$$

with

$$D = -ik^\alpha K_\alpha + \xi \,, \tag{2.26}$$

with ξ denoting a Fayet-Iliopoulos (FI) term.[11]

From the previous discussions it is clear that in supergravity the real inflaton field φ must be part of a complex scalar field $\phi = \frac{1}{\sqrt{2}}(\chi + i\varphi)$. The form of the F-term potential in Eq. (2.24) suggests that sufficient flatness of the scalar potential for φ is not a generic property of supergravity [71]. This becomes evident when we expand K around some chosen origin, for example, $\phi = 0$. Then the scalar potential for the complex field containing the inflaton can be written as

$$V = V_0 \left(1 + K_{\phi\bar{\phi},0}\, \phi\bar{\phi} + \dots\right) \,, \tag{2.27}$$

where $V_0 = V_F(\phi = 0)$ and similarly for $K_{\phi\bar{\phi},0}$. Hence, once ϕ is canonically normalized, the second term in the bracket gives a model-independent contribution to the inflaton mass and hence to the slow-roll parameter η,

$$\Delta\eta = 1 \,. \tag{2.28}$$

This is the supergravity η problem briefly mentioned in Section 2.2. In addition to the steep potential contribution from expanding e^K there may be model-dependent contributions to the inflaton mass from expanding the rest of the scalar potential, i.e.,

$$K^{\phi\bar{\phi}} D_\phi W D_{\bar{\phi}} \overline{W} - 3|W|^2 \,, \tag{2.29}$$

which are generically of the same order as (2.28). Typically, this problem can be evaded if the model-independent piece in Eq. (2.27) cancels the model-dependent contribution from (2.29), or if the inflaton direction is protected by a symmetry. In most of this thesis we choose the latter path and impose a shift symmetry for ϕ. This is best illustrated in a number of examples.

[11] Consult Appendix A.1 for a more thorough discussion of FI terms in supergravity and string theory.

2.3.1 Chaotic inflation

Chaotic inflation, first proposed in [19], proposes inflation to be driven by a monomial F-term potential for a real scalar field φ. We focus here on its simplest realization, given by a free massive field theory with potential

$$V = \frac{1}{2}m^2\varphi^2 \,. \tag{2.30}$$

It predicts a scalar spectral index of $n_{\rm s} \approx 0.967$ and a tensor-to-scalar ratio of $r \approx 0.13$ for 60 e-folds of inflation beginning at $\varphi_\star \approx 15$.[12] The amplitude of the CMB scalar fluctuations constrains the inflaton mass to be $m \approx 6 \times 10^{-6}$ in Planck units.

In supergravity, one may think that this form of potential can be obtained by choosing a quadratic superpotential,[13]

$$W = \frac{1}{2}m\phi^2 \,, \tag{2.31}$$

with $\phi = \frac{1}{\sqrt{2}}(\chi + i\varphi)$ as above. However, for a generic Kähler potential the scalar potential is far too steep to allow for inflation, a manifestation of the η problem. It was proposed in [72] that this can be circumvented by imposing a global shift symmetry for the complex scalar, $\phi \to \phi + ic$, where c is a real constant. This way, the inflaton field φ does not enter the Kähler potential and the steep contribution from the factor e^K is absent. At the same time, the real part χ does receive a large mass from supergravity terms and thus does not interfere with single-field inflation. Specifically, one may choose

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 \tag{2.32}$$

However, for the large inflaton field values required by chaotic inflation the potential resulting from Eqs. (2.31) and (2.32),

$$V = \frac{1}{2}m^2\varphi^2 - \frac{3}{16}m^2\varphi^4 \,, \tag{2.33}$$

is negative and unbounded from below. The dangerous second piece in this expression may either be forbidden by an (approximate) symmetry or set to zero by the dynamics of an additional chiral multiplet. While the first possibility is explored in detail in Chapter 5, we focus here on the second one, suggested in [72]. It relies on the presence

[12]Although such a large value of r is mildly disfavored by observations we consider this a useful toy model. As discussed in subsequent chapters, different effects from other heavy scalars in the theory may flatten the potential and decrease the value of r.

[13]For convenience we denote complex scalar fields and their supermultiplets by the same symbol throughout this thesis. The difference will be clear from the context surrounding a given expression.

of a so-called stabilizer field S which couples to the inflaton. We can define the scalar Lagrangian by

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S|^2 - \frac{1}{\Lambda^2}|S|^4 + \dots\,, \qquad W = mS\phi\,. \tag{2.34}$$

The term quartic in S in the Kähler potential is necessary to make the stabilizer field heavy enough to not interfere with inflation.[14] It may be produced radiatively by couplings to heavy fields which have been integrated out at a scale $\Lambda > H$, cf. [73] and the detailed discussion in the appendix of [74].[15] The scalar potential, with S and χ stabilized at the origin, is then identical to Eq. (2.30). Furthermore, the authors of [75, 76] have demonstrated that the superpotential in Eq. (2.34) may be generalized to depend on general holomorphic functions $f(\phi)$ coupled to S. This leads to a variety of possible scalar potentials and predictions.

2.3.2 F-Term hybrid inflation

The concept of hybrid inflation was first suggested in [20] and was subsequently studied in supergravity theories in [71,77]. In its simplest form it contains three chiral superfields, one containing the inflaton and the other two containing the so-called waterfall fields, responsible for ending inflation. In contrast to chaotic inflation it is a small-field model, meaning that the inflaton field traverses sub-Planckian distances during the last 60 e-folds of slow-roll inflation. Hybrid inflation can be implemented with a canonical Kähler potential,

$$K = |\phi|^2 + |S_1|^2 + |S_2|^2\,, \tag{2.35}$$

where S_1 and S_2 denote the complex waterfall fields. The supergravity η problem can be avoided by choosing a linear superpotential for ϕ,

$$W = \lambda\phi\left(v^2 - S_1 S_2\right)\,, \tag{2.36}$$

where λ is a dimensionless coupling and v is a mass scale. As long as the inflaton field takes values larger than the critical value $\phi_{\mathrm{c}} = v$ the two waterfall fields are heavy and stabilized at $S_1 = S_2 = 0$. The superpotential on this inflationary trajectory reduces to

$$W = \lambda v^2 \phi\,, \tag{2.37}$$

[14] Nevertheless, in the presence of high-scale supersymmetry breaking after inflation the stabilizer field may give sizeable contributions to the effective inflaton potential, as discussed in Section 5.1.

[15] We come back to this discussion in Chapter 5, when evaluating the effects of high-scale supersymmetry breaking on this setup of chaotic inflation.

and we can write the inflaton potential as follows,

$$V = \lambda^2 v^4 + V_{\text{CW}} + V_{\text{Sugra}} \,. \tag{2.38}$$

During inflation the energy density of the universe is dominated by the false vacuum contribution $\lambda^2 v^4$. In the small-field regime $\phi \ll 1$ the slope of the potential is dominated by the Coleman-Weinberg one-loop potential, denoted by V_{CW}. It is generated by the Yukawa coupling between ϕ and the two heavy waterfall fields and approximately scales like

$$V_{\text{CW}} \sim \frac{\lambda^4 v^4}{16\pi^2} \ln \frac{\phi^2}{v^2} \,. \tag{2.39}$$

The supergravity terms contained in V_{Sugra} are of quartic or higher order in the inflaton field. Once the inflaton field surpasses the critical value from above, a linear combination of the waterfall fields becomes tachyonic and destabilizes the inflationary trajectory. Inflation ends in a waterfall transition and the system settles in its true vacuum state.

For Yukawa couplings $\lambda \gtrsim 10^{-5}$ comparable to SM Yukawa couplings, the mass scale v must lie close to the GUT scale, $v \approx 10^{-2}$. Hybrid inflation then predicts, for 60 e-folds of inflation, a scalar spectral index of $n_{\text{s}} \approx 0.98$ and negligible tensor-to-scalar ratio. Such a large value of n_{s} is disfavored by CMB data, cf. Eq. (2.19). A number of natural extensions of the simplest model can ameliorate this phenomenological shortcoming. On the one hand, as discussed in [78], supersymmetry breaking after inflation may introduce a linear term in the inflaton potential which can decrease the spectral index as far as $n_{\text{s}} \approx 0.96$. On the other hand, a similar effect can be induced by the back-reaction of a supersymmetrically stabilized modulus from a string compactification, as discussed in Chapter 4.

2.3.3 D-term hybrid inflation

A related model of hybrid inflation, driven by D-terms instead of F-terms, may be constructed in the presence of a $U(1)$ gauge symmetry [24,25]. In D-term hybrid inflation (DHI) an FI term ξ associated with the $U(1)$ symmetry may constitute the vacuum energy, while again quantum corrections govern the dynamics of the inflaton field. The waterfall fields are charged under the $U(1)$ symmetry and the superpotential consists of a single Yukawa coupling,

$$W = \lambda \phi S_+ S_- \,. \tag{2.40}$$

The notation indicates that the waterfall fields $S_\pm$ carry $U(1)$ charge $q_\pm$, respectively. During and after inflation one of the waterfall fields is stabilized at the origin, $S_+ = 0$.

The F- and D-term potentials read

$$V_F = \lambda^2 e^{|S_-|^2} |S_-|^2 \phi^2 + \mathcal{O}(\phi^4)\,, \tag{2.41}$$

$$V_D = \frac{g^2}{2}\left(q_- |S_-|^2 + \xi\right)^2\,, \tag{2.42}$$

where g denotes the gauge coupling of the $U(1)$ symmetry. Notice that gauge invariance requires $q_+ + q_- = \xi$. The scalar potential $V = V_F + V_D$ has a supersymmetric Minkowski minimum at

$$\langle S_- \rangle^2 = \frac{\xi}{|q_-|}\,, \qquad \phi = 0\,. \tag{2.43}$$

As in F-term hybrid inflation the potential has a plateau for large inflaton field values, $\phi > \phi_c \equiv g\sqrt{|q_-|\xi}/\lambda$. Here $S_- = 0$ and the gauge symmetry is restored. The corresponding potential energy is determined by the FI term,

$$V_0 = \frac{g^2 \xi^2}{2}\,. \tag{2.44}$$

As before, the Yukawa interaction in the superpotential lifts the potential at the one-loop level, generating a slope for the inflaton. Supergravity corrections of quartic order or higher are again suppressed since $\phi \ll 1$ in Planck units. When $\phi < \phi_c$, S_- becomes tachyonic and settles in its true vacuum state defined by Eqs. (2.43). Inflation ends in a waterfall phase transition with spontaneous breaking of the $U(1)$ gauge symmetry.

This simple implementation of DHI has a potential problem due to the generation of cosmic strings during the $U(1)$ phase transition. Furthermore, it predicts a scalar spectral index of $n_s \gtrsim 0.98$ in tension with CMB data. However, minor modifications of the Kähler potential can reconcile the model with observations. The reader can find an appropriate treatment of both issues in [79].

2.3.4 Subcritical D-term hybrid inflation

It has recently been realized that DHI does not necessarily terminate after the $U(1)$ phase transition [80]. The analysis of this curiosity was subsequently extended in [38, 81]. The treatment in this thesis is based on the account given in [38].

If the critical field value is very large, $\phi_c \gg 1$, the scalar potential in the waterfall regime may be sufficiently flat for inflation to continue. This occurs if the Yukawa coupling λ is suppressed compared to the $U(1)$ gauge coupling g. Indeed, the inflaton potential (2.41) is of the form $m^2\phi^2$ close to the supersymmetric minimum, which suggests the possibility of chaotic inflation. In order to see that, in the waterfall regime, DHI is indeed identical to the implementation of chaotic inflation in supergravity discussed in Section 2.3.1, we consider the full Kähler potential including the $U(1)$ vector superfield $\mathscr{V}$,

$$K = \bar{S}_+ e^{2q_+\mathscr{V}} S_+ + \bar{S}_- e^{2q_-\mathscr{V}} S_- + \frac{1}{2}(\phi + \bar{\phi})^2 + 2\xi\mathscr{V}\,. \tag{2.45}$$

Notice that we have introduced a shift symmetry for the inflaton field to protect the potential from corrections terms in V_{Sugra}. In contrast to the previous two scenarios of hybrid inflation, these are potentially destructive due to the large field values we consider. After a suitable field redefinition and a Kähler transformation we can write the superpotential and Kähler potential in the instructive form

$$W = \lambda\phi S_+ \langle S_- \rangle\,, \tag{2.46}$$

$$K = \bar{S}_+ e^{2q_+\mathscr{V}} S_+ + \langle S_- \rangle^2 e^{2q_-\mathscr{V}} + \frac{1}{2}(\phi + \bar{\phi})^2 + 2\xi\mathscr{V}\,. \tag{2.47}$$

The chiral superfield S_- has disappeared from the spectrum, its vacuum expectation value is given by (2.43). It has been eaten by the vector superfield $\mathscr{V}$, which became massive in turn. Integrating out $\mathscr{V}$ supersymmetrically by solving its equation of motion,

$$\partial_{\mathscr{V}} K = 0\,, \tag{2.48}$$

yields an intriguing expression for the effective Lagrangian.

$$\mathscr{V} = -\frac{q_+}{2|q_-|\xi}\,|S_+|^2 + \mathcal{O}\left(|S_+|^4\right) \tag{2.49}$$

solves Eq. (2.48) and yields the effective super- and Kähler potential,

$$W = m\phi S_+\,, \tag{2.50}$$

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S_+|^2 - \frac{|S_+|^4}{\Lambda^2}\,, \tag{2.51}$$

with $m = \lambda\langle S_- \rangle e^{\langle S_- \rangle^2/2}$ and $\Lambda^2 = 2|q_-|\xi/q_+^2$. Remarkably, Eqs. (2.50) and (2.51) define the embedding of chaotic inflation in supergravity described in Section 2.3.1. Here, S_+ plays the role of the stabilizer field. An important difference is that previously the quartic Kähler potential term for the stabilizer field was introduced by hand. In subcritical DHI it arises in the effective theory through the exchange of the heavy $U(1)$ gauge boson. This mechanism has previously been observed in [82, 83].

One more remark is in order. The supersymmetric limit considered thus far is valid only in the vicinity of the minimum $\phi = 0$. However, corrections are suppressed as long as the scale of the $U(1)$ breaking is large compared to the supersymmetry breaking scale, which coincides with the Hubble scale. This becomes evident by considering the full scalar potential. Consistently integrating out S_- in its vacuum defined by $\langle S_- \rangle$ yields the effective inflaton potential[16]

$$V = V_0 \left(1 - \frac{V_0}{2g^2\xi^2} + \dots \right) , \tag{2.52}$$

valid in the regime $\phi < \phi_\mathrm{c}$, with $V_0 = \frac{1}{2}m^2\varphi^2$ and $\varphi = \sqrt{2}\mathrm{Im}\,\phi$. This implies that, if $\sqrt{g\xi} > M_\mathrm{GUT} \sim 10^{-2}$ in Planck units, the last 60 e-folds of inflation can occur within the quadratic regime of the potential. Notice, furthermore, that the cosmic string problem of DHI is absent in this case since the $U(1)$ symmetry is already broken during inflation. Cosmic strings produced during the phase transition are diluted by subsequent e-folds of inflation.

[16]We neglect the small correction to V from the pre-factor e^K.

Chapter 3

Moduli Stabilization and Supersymmetry Breaking in Supergravity

Many aspects of the UV sensitivity of inflation can be studied in four-dimensional effective supergravity theories as they arise in string theory compactifications. Whenever inflation is described in string-effective setups, the crucial issue of moduli stabilization must be addressed. In the last decade substantial progress has been made in the search for stable string vacua. However, since many scenarios which provide moduli stability yield supersymmetric anti-de Sitter (AdS) or Minkowski vacua, a mechanism for supersymmetry breaking, or equivalently "uplifting", must be specified. In this chapter we review the origin of geometric moduli in compactifications on Calabi-Yau manifolds and discuss the most successful examples, focussing on four-dimensional effective actions from type IIB orientifold compactifications with fluxes. Furthermore, we discuss how to obtain Minkowski or de Sitter (dS) vacua with spontaneously broken supersymmetry, with particular focus on F-term supersymmetry breaking.

As in the previous chapter we introduce the most important concepts for reference in later chapters. For detailed reviews of moduli stabilization and supersymmetry breaking in string theory and supergravity we refer to [30, 45]. For details on type IIB orientifold compactifications with D-branes and fluxes, cf. the reviews [29, 84]. Many of the facts summarized here have been previously reviewed in [35,36,39,40]. The stabilization mechanism discussed in Section 3.4.3 was developed in [38]. Similarities in the discussions are intended and reflect the author's contribution to those publications.

3.1 Geometric moduli from Calabi-Yau compactifications

Consistency of critical superstring theory requires the existence of ten space-time dimensions. To establish contact with a four-dimensional description of the universe, the ten-dimensional background space X_{10} is usually decomposed as follows,

$$X_{10} = \mathbb{R}_{1,3} \times X_6 \,. \tag{3.1}$$

$\mathbb{R}_{1,3}$ denotes Minkowski space-time with four large dimensions and X_6 is a compact six-dimensional manifold. This is known as a compactification of string theory on X_6. The best-understood class of compactification manifolds is Calabi-Yau (CY) manifolds. We can define CY manifolds as compact Kähler manifolds with Ricci-flat metric, i.e., with

$$\mathrm{Ric}(g_{X_6}) = 0 \,, \tag{3.2}$$

where g_{X_6} denotes the metric on X_6.[1] CY manifolds are intimately connected to space-time supersymmetry in the four-dimensional effective theory. Compactification of heterotic string theory on CY three-folds naturally preserves $\mathcal{N} = 1$ supersymmetry [28]. In type IIB string theory $\mathcal{N} = 2$ supersymmetry remains, which can be broken to $\mathcal{N} = 1$ supersymmetry by orientifolding, cf. [29, 84] for instructive reviews.

The geometric moduli appearing as scalar fields in the four-dimensional effective theory are deformation modes of the CY metric which respect the property Eq. (3.2).[2] A number of them are associated with deformations of the complex structure of X_6, so-called complex structure moduli, and others are associated with deformations of the Kähler structure, so-called Kähler moduli. The latter parameterize the volume of submanifolds of X_6. If there is only one Kähler modulus, it parameterizes the total volume of X_6.

Moduli stabilization is particularly well understood in CY orientifold compactifications of type IIB string theory with D-branes and fluxes. The authors of [31] have demonstrated that in the presence of suitable three-form fluxes all complex structure moduli and the complex dilaton field can be stabilized supersymmetrically at a mass scale close to the string scale. Consequently, we assume that this has been achieved at a high scale and that those fields decouple from the dynamics of inflation. In particular, we assume that the Gukov-Vafa-Witten superpotential [86]

$$W_{\mathrm{GVW}} = \int_{X_6} G_3 \wedge \Omega \,, \tag{3.3}$$

[1] There are a number of equivalent definitions of CY manifolds, cf. [85].

[2] For obvious reasons only those deformations which cannot be undone by coordinate or Kähler transformations appear as physical degrees of freedom in the effective theory.

is fixed at a vacuum expectation value $\langle W_{\mathrm{GVW}}\rangle \equiv W_0$. Here Ω denotes the unique holomorphic three-form on X_6 and G_3 is the three-form flux which stabilizes the complex structure moduli and the dilaton. The Kähler moduli fields, however, do not enter the flux superpotential and their Kähler potential is of no-scale type, i.e., it satisfies $K_\alpha K^{\alpha\bar{\alpha}} K_{\bar{\alpha}} = 3$ at tree level [87]. It reads

$$K = -2\ln\mathcal{V}\,, \tag{3.4}$$

where $\mathcal{V}$ is the volume of X_6. Formally it can be expressed in terms of the Kähler form J on X_6 as follows,

$$\mathcal{V} = \int\limits_{X_6} J\wedge J\wedge J\,. \tag{3.5}$$

The volume implicitly depends on the Kähler moduli through the Kähler form. The specific form of J and $\mathcal{V}$ depends on the topology of the considered CY manifold. A thorough treatment of this issue is, however, beyond the scope of this thesis. We restrict ourselves to very simple examples with only one or two dynamical Kähler moduli which are to be stabilized. As a helpful introduction to the geometry and topology of complex manifolds we recommend [85] and the relevant chapters in [88].

A no-scale Kähler potential and the absence of a tree-level superpotential implies that all Kähler moduli are massless at this stage. However, for a number of reasons, for example, due to constraints from cosmological observations and fifth-force experiments [89–91], they should be massive. Furthermore, in metastable vacua the potential barrier must be larger than the Hubble scale H to preserve compactness of the internal manifold during inflation. Since the Kähler moduli govern the size of sub-manifolds of X_6, or, in extreme cases, the total volume of X_6, a run-away of these moduli fields towards infinity would imply decompactification to ten-dimensional space-time. As will become clear in the examples we discuss, the height of this potential barrier is generically proportional to the moduli masses. Moreover, very heavy moduli avoid the infamous cosmological moduli problem [92–95].

Fortunately, Kähler moduli can be stabilized by including quantum corrections in the effective action. These can be perturbative and non-perturbative corrections in the Kähler potential or non-perturbative corrections in the superpotential, or both. We demonstrate this in the following sections by means of successful examples. We distinguish two different classes. On the one hand, moduli stabilization mechanisms which necessarily entail spontaneous breaking of supersymmetry. Prominent examples are the mechanism of KKLT [32], Kähler Uplifting [96, 97], and the Large Volume Scenario

(LVS) [98, 99]. In all these examples the scale of supersymmetry breaking is related to the mass of the stabilized Kähler moduli. On the other hand, we present mechanisms of supersymmetric moduli stabilization, in which the scale of supersymmetry breaking is independent of the moduli masses. This is sometimes referred to as "strong moduli stabilization", cf. [100, 101]. As examples we choose racetrack stabilization as proposed in [102] and a recently developed mechanism involving world-sheet instanton couplings and D-terms of an anomalous $U(1)$ gauge symmetry [38].

3.2 KKLT mechanism and F-term uplifting

The possibly simplest setup to stabilize Kähler moduli via non-perturbative effects was proposed in [32]. The original model assumes all complex structure moduli of a compact CY manifold and the dilaton to be stabilized by fluxes, as discussed above. The remaining effective theory contains a single lightest Kähler modulus, in the following denoted by T, which parameterizes the volume of the compact manifold, i.e.,

$$\mathcal{V} = (T + \overline{T})^{3/2} \,. \tag{3.6}$$

Assuming that only one Kähler modulus is dynamical is a strong simplification, but a justified one. Once the mechanism that stabilizes the lightest modulus is specified, one may envision a setup in which all other Kähler moduli are stabilized by the same mechanism but at a higher mass scale. Although the stabilization of multiple moduli may be challenging computationally, the basic principle can be explained by considering only the lightest one.[3]

3.2.1 Finding an AdS vacuum

With Eq. (3.6) the Kähler potential in Eq. (3.4) becomes

$$K = -3 \ln (T + \overline{T}) \,. \tag{3.7}$$

To break the no-scale symmetry and generate a potential for T we can consider non-perturbative terms in the superpotential, i.e.,

$$W_{\text{np}} = \sum_i A_i e^{-a_i T} \,, \tag{3.8}$$

[3]On a more technical note, the separation between complex structure moduli stabilization and Kähler moduli stabilization is possible because the mass scale of heavy fields stabilized by fluxes is determined by the flux quantization condition, and thus is independent of W_0. As we will see below, the mass of the remaining modulus is actually proportional to W_0. Thus, by tuning W_0 to stabilize T the stability of the other moduli is not affected. We refer to the discussion in [103] for more details.

Here, A_i are constant coefficients which generically depend on the vacuum expectation values of complex structure moduli. The values of the a_i depend on the origin of the non-perturbative terms. For example, a Euclidean D3 instanton has $a = 2\pi$ [104], while $a = 2\pi/N$ for an $SU(N)$ gaugino condensate on a stack of D7 branes [105–108]. The authors of [32] have shown that a single non-perturbative term suffices to stabilize T. Thus, we consider the superpotential

$$W = W_0 + Ae^{-aT} \,. \tag{3.9}$$

W_0 and A are assumed to be real in what follows, since a relative phase between the two can be compensated by a field redefinition. The scalar potential

$$V = e^K \left(K^{T\overline{T}} D_T W \overline{D_T W} - 3|W|^2 \right) \,, \tag{3.10}$$

has two extrema defined by $\partial_T V = 0$. One of them is a global maximum at $T = \infty$ where the potential vanishes, and the other one is a global minimum which satisfies

$$D_T W = 0 \,, \tag{3.11}$$

i.e., it corresponds to a supersymmetric AdS vacuum. Its position in field space, T_{AdS}, is defined by

$$W_0 = -Ae^{-aT_{\text{AdS}}} \left(1 + \frac{2}{3} a T_{\text{AdS}} \right) \,. \tag{3.12}$$

With all parameters in the superpotential chosen to be real, T_{AdS} is real. $\operatorname{Im} T$ is stabilized at the origin at the same mass scale as $\operatorname{Re} T$. For the effective theory in this vacuum to be consistent with the supergravity approximation, i.e., the assumption that the characteristic length scale of the compact manifold is much larger than the string scale, the vacuum must satisfy $T_{\text{AdS}} \gtrsim 1$. Furthermore, to justify the single-instanton approximation it must be $aT_{\text{AdS}} \gtrsim 1$. In fact, these two requirements are not specific to the KKLT setup but must be met in any string-effective supergravity theory.

3.2.2 Uplift to de Sitter with F-terms

To uplift the AdS vacuum to a dS or near-Minkowski vacuum, we must break supersymmetry in a different sector of the theory. To this end, the authors of [32] introduced an anti-D3 brane. However, since this breaks supersymmetry explicitly[4] numerous other ways of uplifting with F-terms or D-terms have been proposed since.[5]

[4]For very recent treatments of this subtle issue, cf. [109, 110].

[5]We recommend Chapter 3 of [45] for an exhaustive list of references.

Supersymmetry breaking in the Polonyi model

We choose the arguably simplest example, which is uplifting with the F-term of a Polonyi field [111,112], as was first discussed in [113]. The Polonyi field X enters the Lagrangian as follows,

$$W = W_0 + fX \,, \qquad K = |X|^2 - \frac{|X|^4}{\Lambda^2} \,. \tag{3.13}$$

Apparently, the parameter f determines the scale of supersymmetry breaking. This setup corresponds to a version of the O'Raifeartaigh model [114] where the heavy fields coupling to the supersymmetry-breaking field have been integrated out at a high scale $\Lambda \ll 1$. Quantum corrections thus produce the quartic term in K. For a thorough discussion of the interaction of the O'Raifeartaigh model with KKLT moduli stabilization, cf. [115]. The Lagrangian defined by Eqs. (3.13) has a local minimum on the real axis at

$$X_0 = \overline{X}_0 = \frac{\sqrt{3}}{6}\Lambda^2 \,. \tag{3.14}$$

The vacuum energy is cancelled when $f = \sqrt{3}W_0$.[6] The vacuum expectation value of X lies close to the origin of field space since the cut-off scale must satisfy $\Lambda \ll 1$ in Planck units. This is important from the perspective of cosmology, since it avoids the so-called Polonyi problem first discussed in [92, 93, 116].

Since the F-term of X breaks supersymmetry spontaneously the gravitino becomes massive by eating the goldstino fermion, which is the superpartner of the scalar Polonyi field. The gravitino mass in the vacuum is

$$m_{3/2} \equiv e^{K/2} W \approx W_0 = \frac{f}{\sqrt{3}} \,, \tag{3.15}$$

neglecting terms suppressed by powers of Λ. The mass of X, on the other hand, can be much larger due to the quartic term in K. One finds

$$m_X = \frac{f}{2\Lambda} \gg m_{3/2} \,. \tag{3.16}$$

This is again favorable from the perspective of cosmology. Since during inflation we aspire a hierarchy $m_X > H$ for the Polonyi field to remain stabilized, choosing the scale Λ appropriately allows for a separation of scales between H and $m_{3/2}$.[7] As discussed

[6] We remark that there is also a global minimum at $X \approx \sqrt{3} - 1$. The necessary longevity of the metastable vacuum at X_0 has been studied and confirmed in [93, 116].

[7] Notice that the cut-off scale itself is constrained by $H < \Lambda \ll 1$ to ensure consistency of the effective field theory.

in the following, this becomes impossible when the theory is coupled to KKLT moduli stabilization. In the treatment of uplifted vacua with a Polonyi field in subsequent chapters, we usually assume that X can be made heavy enough, and X_0 can be made small enough, so that the field decouples from the dynamics of inflation and moduli stabilization. This assumption has been justified in more detail in [115].

The uplifted KKLT vacuum

With this in mind, we observe that when KKLT moduli stabilization is coupled to a Polonyi field sector, the supersymmetry-breaking piece of the theory is not significantly altered.[8] The potential of the modulus, however, changes significantly. The uplifted scalar potential, defined by

$$K = -3\ln(T+\overline{T}) + |X|^2 - \frac{|X|^4}{\Lambda^2}\,, \tag{3.17}$$

$$W = W_0 + Ae^{-aT} + fX\,, \tag{3.18}$$

evaluated at $X \approx 0$, now has three extrema. One is the run-away vacuum at $T = \infty$, one is the uplifted AdS minimum, and the third is a local maximum in between. The position of the local minimum, which is now metastable, has been slightly shifted by the supersymmetry-breaking sector,

$$T_0 = T_{\mathrm{AdS}} + \frac{f^2}{2a^2 T_0 W_0^2} + \ldots\,, \tag{3.19}$$

omitting terms suppressed by higher orders of aT_0. Here we have used that $W(T_0) \approx W_0$. The background in the uplifted minimum is Minkowski when

$$f = \sqrt{3}W_0\left(1 - \frac{3}{2aT_0} + \ldots\right)\,. \tag{3.20}$$

The modulus potential is displayed in Fig. 3.1 for a typical set of parameters.

As opposed to the original AdS vacuum, the modulus actually contributes to supersymmetry breaking,

$$\langle F_T\rangle = e^{K/2}\sqrt{K^{T\overline{T}}}D_T W\Big|_{T_0} \approx -\frac{3\sqrt{3}W_0}{a(2T_0)^{5/2}} \approx -\frac{3\langle F_X\rangle}{4aT_0}\,. \tag{3.21}$$

However, the dominant contribution to supersymmetry breaking stems from the Polonyi field. The gravitino mass in the Minkowski vacuum is

$$m_{3/2} = \frac{W_0}{(2T_0)^{3/2}}\left(1 - \frac{3}{2aT_0} + \ldots\right) \approx \frac{W_0}{(2T_0)^{3/2}}\,. \tag{3.22}$$

[8] With the exception that the scalar potential is rescaled by $e^K \approx 1/\mathcal{V}^2$.

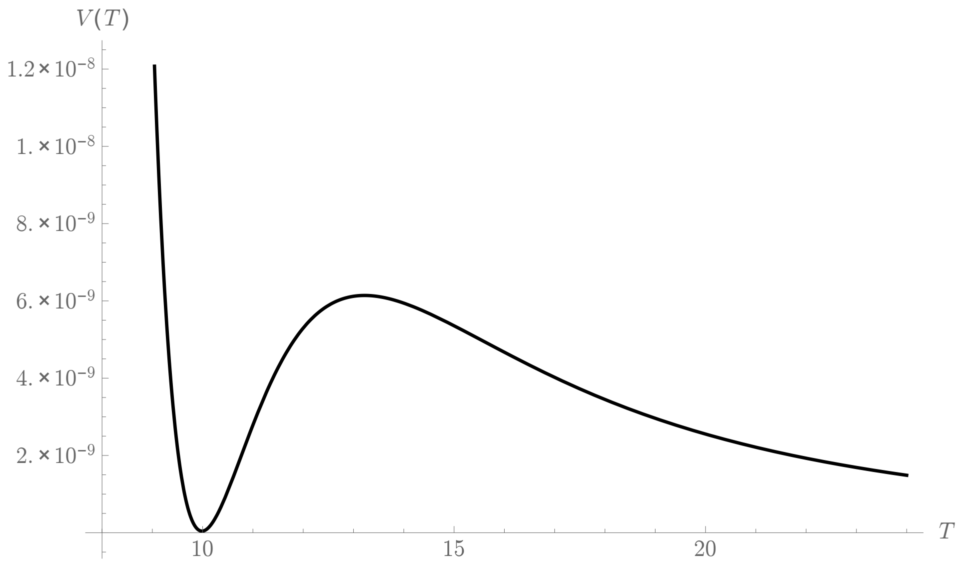

Figure 3.1: *Uplifted modulus potential in KKLT for* $W_0 = 0.009$*,* $A = -0.75$*, and* $a = 2\pi/10$*. A metastable Minkowski vacuum lies at* $T_0 \approx 10$*. Clearly visible are, furthermore, the run-away global minimum at the barrier separating the two.*

It is closely related to the mass of the canonically normalized modulus,

$$m_T \approx 2aT_0 m_{3/2}\,. \tag{3.23}$$

As mentioned above, the uplifted Minkowski vacuum is protected by a barrier from the run-away vacuum at $T = \infty$. The height of the barrier is approximately the same as the depth of the original AdS minimum. Conveniently, we can express it in terms of the gravitino mass,

$$V_{\mathrm{B}} \approx 3m_{3/2}^2\,. \tag{3.24}$$

Evidently, all relevant scales in the KKLT potential are related to the gravitino mass in the uplifted vacuum. This fact becomes important when discussing the interaction with inflation. Before we discuss moduli stabilization schemes which do not share this property in Section 3.4, let us mention two more stabilization mechanisms closely related to the KKLT scenario.

3.3 Related mechanisms: Kähler Uplifting and the Large Volume Scenario

There are two more instructive examples of moduli stabilization with spontaneous supersymmetry breaking worth pointing out. Both make use of a single non-perturbative term in the superpotential. One of them takes a perturbative correction to the Kähler potential into account, the other stabilizes the volume of a manifold with two Kähler moduli at exponentially large values.

3.3.1 Kähler Uplifting

Kähler Uplifting was first proposed in [96, 97] and further developed in [117–119]. An appealing feature of this scheme is that Kähler moduli can be stabilized in Minkowski or dS vacua without the need of an uplift sector. It is based on the observation that the interplay between the KKLT superpotential and the leading-order α'-correction in the Kähler potential can produce local minima in the scalar potential with both negative and positive cosmological constant. In particular, for a careful choice of parameters the Lagrangian defined by

$$\begin{aligned} K &= -2\ln\left[\left(T+\overline{T}\right)^{3/2}+\xi\right], \\ W &= W_0 + Ae^{-aT}, \end{aligned} \tag{3.25}$$

can stabilize T in a suitable Minkowski vacuum. Here, $\xi = -\frac{\zeta(3)}{4(2\pi)^3}\chi\langle\mathrm{Re}\, S\rangle^{3/2}$ where χ denotes the Euler number of the compactification manifold and S denotes the complex dilaton field whose real part determines the string coupling.[9] We assume the dilaton to be stabilized supersymmetrically at a high scale so that ξ can be treated as a constant. We remark that this mechanism only works if ξ is positive, hence we only consider negative Euler numbers. For more details on the α' expansion in effective theories of type IIB orientifold compactifications we refer to the original works [120, 121].

The scalar potential defined by Eqs. (3.25) has solutions to the constrained equations of motion for T, $\partial_T V = V = 0$, one of which is a local minimum. This solution can be written as two illuminating relations between the parameters. For convenience we expand the solution in powers of

$$\eta_0 \equiv \frac{\xi}{2(2T_0)^{3/2}}, \tag{3.26}$$

where T_0 denotes the position of the metastable Minkowski vacuum.[10] The latter is then defined by

$$aT_0 = \frac{5}{2} - \frac{27\eta_0}{8} + \mathcal{O}(\eta_0^2), \tag{3.27}$$

[9] ξ is not to be confused with the FI term introduced in Chapter 2.

[10] For the α' expansion to be under control η_0 must be a small parameter. In other words, we demand $\mathcal{V} \gg \xi$.

i.e., the vacuum expectation value of the modulus only depends on a and ξ.[11] Furthermore, we obtain a relation between the remaining parameters of the model in the vacuum,

$$W_0 = -\frac{4}{3\eta_0} a T_0 A e^{-aT_0} - \frac{1}{3} A e^{-aT_0}(3 + 7aT_0) + \mathcal{O}(\eta_0)\,. \tag{3.28}$$

Since $\eta_0 \ll 1$ it follows that $W_0 \gg A$, contrary to the KKLT case where W_0 typically must be small to allow for a sufficiently large vacuum expectation value of the modulus. However, similar to KKLT, the superpotential in the vacuum is dominated by the constant piece, $W(T_0) \approx W_0$. Clearly, the auxiliary field of T breaks supersymmetry and the gravitino mass is given by

$$m_{3/2} = \frac{W_0}{(2T_0)^{3/2}}\left(1 - \frac{23\eta_0}{10} + \mathcal{O}(\eta_0^2)\right) \approx \frac{W_0}{(2T_0)^{3/2}}\,. \tag{3.29}$$

The canonically normalized real and imaginary parts of T have the following masses,

$$m^2_{\mathrm{Re}\,T} \approx 5 m^2_{3/2}\eta_0\,, \qquad m^2_{\mathrm{Im}\,T} \approx \frac{25}{2} m^2_{3/2}\eta_0\,, \tag{3.30}$$

respectively. Again, we observe that the modulus mass is proportional to the gravitino mass. In fact, a few more comments about the peculiar vacuum structure of Kähler Uplifting are in order.

First, the potential has a global minimum at $T = \infty$ with $D_T W = \partial_T V = V = 0$, as in KKLT. Hence, there must exist a local maximum separating the two minima. This time, the corresponding barrier is lower than in KKLT, but again proportional to the mass of T. One finds

$$V_{\mathrm{B}} \approx \eta_0 m^2_{3/2}\,. \tag{3.31}$$

This suppression is due to the approximate no-scale cancellation between F_T and W, which is absent in KKLT because F_T is too small. This approximate no-scale symmetry becomes important in the discussions in Chapter 5.

Second, the suppression of all scales by η_0 hints towards the fact that this vacuum is actually not the uplifted KKLT vacuum. It disappears if $\xi \to 0$, in which case Eqs. (3.25) equal the setup of KKLT. In fact, the KKLT AdS vacuum is still a solution to the F-term constraint $D_T W = 0$ in Kähler Uplifting. However, it lies at small field values, $T_{\mathrm{AdS}} \ll 1$, due to the required magnitude of W_0 and is thus unphysical in our supergravity treatment.

[11] Notice that the numerical value $aT_0 \approx 2.5$ is at the border of control regarding the single-instanton approximation.

3.3.2 The simplest Large Volume Scenario

Our last example of moduli stabilization with spontaneously broken supersymmetry is the Large Volume Scenario developed in [98, 99]. It is based on the observation that, for certain types of CY compactifications with multiple Kähler moduli, the scalar potential may have a non-supersymmetric AdS minimum at exponentially large volume. A particularly simple example of this type is given by a "swiss-cheese" CY manifold with a single "hole", i.e., a manifold whose volume can be written as

$$\mathcal{V} = (T_{\rm b} + \overline{T}_{\rm b})^{3/2} - (T_{\rm s} + \overline{T}_{\rm s})^{3/2}\,, \tag{3.32}$$

where $T_{\rm b}$ is the Kähler modulus of a big four-cycle of the manifold, i.e., the "cheese", and $T_{\rm s}$ controls the volume of a small four-cycle, the "hole". The simplest setup for a Large Volume Scenario is then described by

$$\begin{aligned} K &= -2\ln(\mathcal{V} + \xi)\,, \\ W &= W_0 + Ae^{-aT_{\rm s}}\,, \end{aligned} \tag{3.33}$$

with ξ defined as in Section 3.3.1. As in the previous examples we consider real superpotential parameters, and hence restrict our attention to the real parts of the moduli, i.e., we set $T_{\rm b,s} = \overline{T}_{\rm b,s}$ in the following. In the search for possible vacua it is convenient to parameterize the scalar potential in terms of $\mathcal{V}$ and $T_{\rm s}$ instead of $T_{\rm b}$ and $T_{\rm s}$. At leading order in inverse powers of the volume one finds

$$V \approx \frac{2\sqrt{2}\,a^2A^2\sqrt{T_{\rm s}}\,e^{-2aT_{\rm s}}}{3\mathcal{V}} - \frac{4aAW_0T_{\rm s}\,e^{-aT_{\rm s}}}{\mathcal{V}^2} + \frac{3\xi W_0^2}{2\mathcal{V}^3}\,. \tag{3.34}$$

To obtain this form, the imaginary part of $T_{\rm s}$ has been fixed at its minimum given by $\langle \mathrm{Im}\, T_{\rm s}\rangle = \pi/a$. In this case W_0 and A must have the same sign for the stabilization mechanism to work. Minimizing with respect to $\mathcal{V}$ and $T_{\rm s}$ by solving $\partial_{\mathcal{V}}V = \partial_{T_{\rm s}}V = 0$ reveals a local AdS minimum at

$$\begin{aligned} \mathcal{V}_{\rm AdS} &\approx \frac{3\sqrt{T_{\rm s,AdS}}\,e^{aT_{\rm s,AdS}}\,W_0}{\sqrt{2}aA}\,, \\ T_{\rm s,AdS} &\approx \frac{\xi^{2/3}}{2} + \frac{1}{3a}\,, \end{aligned} \tag{3.35}$$

assuming $aT_{\rm s,AdS} \gg 1$. The volume of the CY manifold indeed depends exponentially on the vacuum expectation value of $T_{\rm s}$. The depth of the AdS vacuum is of the order

$$V_{\rm AdS} \sim -\frac{W_0^2}{\mathcal{V}^3}\,, \tag{3.36}$$

instead of $W_0^2/\mathcal{V}^2$ as one may naively expect. Again, this is due to an approximate no-scale cancellation between F_{T_b} and W_0. As will become clear in the following discussion of the uplift to Minkowski space-time, an important difference to KKLT is that in LVS the moduli contribute most to supersymmetry breaking. The resulting approximate no-scale symmetry is important in our discussion of inflation in an LVS background in Chapter 5.

To uplift this AdS vacuum to a Minkowski vacuum we employ, once more, a Polonyi field X as a toy example. Treating the uplift in the same way as in KKLT moduli stabilization, we assume that X is stabilized at a high scale with a nearly-vanishing vacuum expectation value. However, in LVS the quartic term in the Kähler potential is not required since X is stabilized by its soft mass term. The contribution of the Polonyi field then amounts to a term $V_\mathrm{up} = f^2/\mathcal{V}^2$ in the scalar potential. To cancel the cosmological constant in the vacuum, it must be

$$f^2 \approx \chi_0 W_0^2\,, \qquad \chi_0 = \frac{9\sqrt{2T_0}}{2a\mathcal{V}_0} \ll 1\,, \tag{3.37}$$

up to terms suppressed by higher powers of $\mathcal{V}$ or aT_s. Here, $\mathcal{V}_0$ and T_0 denote the values of the two real fields in the uplifted vacuum. Note that χ_0 plays a role analogous to the parameter η_0 in Kähler Uplifting. We find the uplifted vacuum at

$$\mathcal{V}_0 = \mathcal{V}_\mathrm{AdS}\big|_{T_\mathrm{s}=T_0}\,, \tag{3.38}$$

$$T_0 \approx \frac{\xi^{2/3}}{2} + \frac{1}{a}\,. \tag{3.39}$$

The F-terms of the fields in this vacuum are given by

$$F_{T_\mathrm{b}} \approx -\sqrt{3}\,\frac{W_0}{\mathcal{V}_0}\,, \qquad F_{T_\mathrm{s}} \approx \sqrt{6aT_0\chi_0}\,\frac{W_0}{\mathcal{V}_0}\,, \qquad F_X \approx \sqrt{\chi_0}\,\frac{W_0}{\mathcal{V}_0}\,. \tag{3.40}$$

Clearly, the dominant contribution to supersymmetry breaking comes from the volume mode. As expected, the uplift sector is important to cancel the cosmological constant but its contribution to supersymmetry breaking is suppressed in the large volume limit. The corresponding gravitino mass is again

$$m_{3/2} \approx \frac{W_0}{\mathcal{V}_0}\,, \tag{3.41}$$

up to terms suppressed by higher powers of the inverse volume or aT_0. The masses of the canonically normalized moduli are schematically[12]

$$m_{T_{\rm b}} \sim \frac{W_0}{\mathcal{V}_0^{3/2}}\,, \qquad m_{T_{\rm s}} \sim \frac{W_0}{\mathcal{V}_0}\,. \tag{3.42}$$

The uplifted vacuum is protected by a potential barrier of height

$$V_{\rm B} \approx \chi_0 m_{3/2}^2\,. \tag{3.43}$$

As in Kähler Uplifting, we observe that the barrier is suppressed by a small parameter compared to KKLT, due to the approximate no-scale symmetry in the vacuum.

3.4 Supersymmetric stabilization

Let us now turn to the second class of moduli stabilization schemes, in which the fields are stabilized in supersymmetric Minkowski vacua. Spontaneous supersymmetry breaking can then be introduced in a separate sector, completely independent of all scales in the modulus potential. This class we refer to as supersymmetric stabilization.

3.4.1 Avoiding conflicts with inflation

The motivation to consider models of this type is cosmic inflation. Shortly after the KKLT scenario was developed some of the authors recognized that a period of inflation severely constrains the parameter space of the theory. The authors of [102] emphasized that inflation corresponds to a second uplifting which can lift the modulus over its potential barrier. Schematically, a theory of inflation with energy scale H coupled to KKLT moduli stabilization can be described by

$$V \approx V_{\rm KKLT}(T) + \frac{3H^2}{T^3}\,, \tag{3.44}$$

where the inverse T-dependence stems from the overall factor e^K. Apparently, moduli stabilization in this setting is only possible for small values of the inflaton potential, or for large values of the potential barrier.[13] Since high-scale inflation is favored by observations, cf. Eq. (2.22), the only solution seems to be high-scale supersymmetry breaking, i.e., choosing

$$m_{3/2} > H\,. \tag{3.45}$$

[12]Note that the axion of $T_{\rm b}$ is exactly massless at this level and thus irrelevant during inflation. It may, however, give rise to interesting discussions of dark radiation, cf. [122]. The axion of $T_{\rm s}$ is stabilized at the same mass scale as the real part of $T_{\rm s}$.

[13]A similar observation concerning dilaton stabilization was previously made in [123].

Notice that this constraint is only valid in the KKLT setup. In Kähler Uplifting and LVS the bound on the gravitino mass is even more severe due to the suppression of the potential barrier. Concerning LVS this fact was pointed out in [124] and we discuss it more thoroughly in explicit examples in Chapter 5.

In order to avoid this restriction we may resort to moduli stabilization schemes in which the height of the potential barrier, and the mass of T, is independent of $m_{3/2}$. This allows for a large modulus mass and a stable vacuum, while the scale of supersymmetry breaking is almost arbitrary.[14]

3.4.2 Stabilization in a racetrack

The authors of [102] proposed a solution which relies on the inclusion of more parameters in the superpotential, which allows for a fine-tuning that achieves

$$D_T W = W = 0\,, \tag{3.46}$$

in the vacuum. In particular, this is possible when a second non-perturbative term for the same modulus T is taken into account,

$$W = W_0 + Ae^{-aT} + Be^{-bT}\,, \tag{3.47}$$

under the assumption that all parameters can be tuned independently.[15] The scalar potential is called a "racetrack potential" due to the appearance of another AdS minimum. We have illustrated the potential for typical parameter values in Fig. 3.2.

One of the solutions to Eqs. (3.46) is a local minimum at

$$T_0 = \frac{1}{a-b} \ln \left| \frac{aA}{bB} \right| . \tag{3.48}$$

Note that for real superpotential parameters T_0 is real, and $\mathrm{Im}\, T$ is stabilized at the origin at the same mass scale. At the same time Eqs. (3.46) imply a specific relation among the parameters,

$$W_0 = -A \left| \frac{aA}{bB} \right|^{\frac{a}{b-a}} - B \left| \frac{aA}{bB} \right|^{\frac{b}{b-a}} . \tag{3.49}$$

The mass of the complex modulus field,

$$m_T^2 = \frac{2}{9} abAB(a-b) \left| \frac{aA}{bB} \right|^{-\frac{a+b}{a-b}} \ln \left| \frac{aA}{bB} \right| , \tag{3.50}$$

[14]Different approaches to circumvent the problem in specific setups have been suggested in [125–128].

[15]Note that this assumption is non-trivial in string theory constructions. Cf. [129] for an instructive discussion in effective supergravities from M-theory.

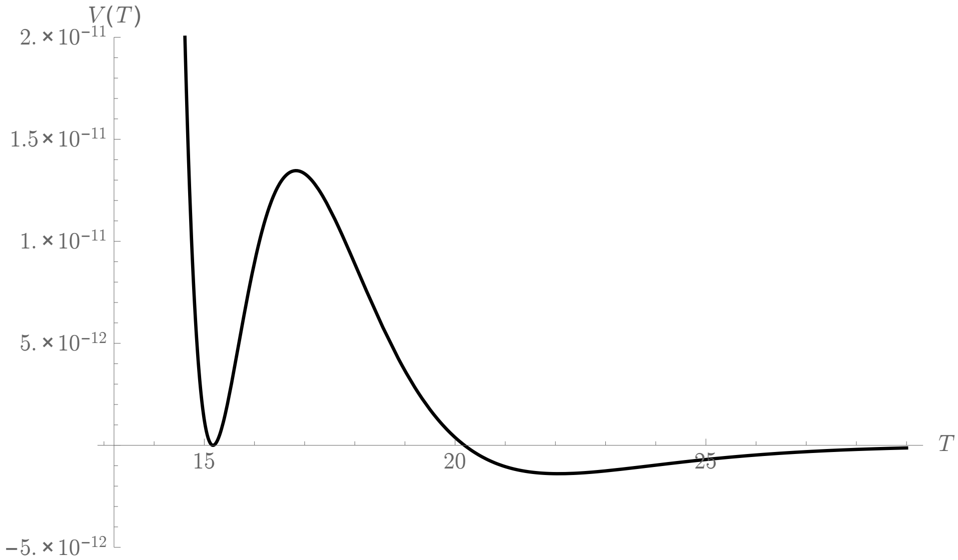

Figure 3.2: *Racetrack potential for $A = 4$, $B = -1.5$, $a = 2\pi/12$, $b = 2\pi/14$. The value of W_0 is fixed by the condition* (3.49)*. A metastable Minkowski vacuum lies at $T_0 \approx 15$. The barrier separating this vacuum from the adjacent AdS vacuum is independent of the gravitino mass, which vanishes in the absence of other fields.*

is large for typical parameter values, $m_T > 10^{-5}$, while the gravitino is massless in the vacuum. We can break supersymmetry subsequently in a separate sector, for example with the F-term of a Polonyi field. The gravitino mass is then proportional to W_0 but can be much smaller than the modulus mass. We refer the reader to [100] for a detailed discussion of supersymmetry breaking in supersymmetrically stabilized backgrounds.

3.4.3 Stabilization with world-sheet instanton couplings

Another mechanism with supersymmetric stabilization, which involves more fields but less fine-tuning of parameters than the previous example, was recently proposed in [38]. It has since found application in the context of axion inflation in heterotic string theory, cf. [41, 130].

Let us consider a supergravity theory with one modulus T and two chiral superfields χ_+ and S_- which are charged under an anomalous $U(1)$ gauge symmetry, denoted by $U(1)_\mathrm{A}$. In the context of heterotic string theory, the authors of [131] observed that a shift of T, i.e., a non-linear $U(1)_\mathrm{A}$ gauge transformation, may cancel all anomalies associated with the gauge symmetry. More details on Green-Schwarz anomaly cancellation and the subsequent generation of field-dependent FI terms are presented in Appendix A.1. These details are, however, mostly irrelevant to the mechanism of moduli stabilization.

Superpotential couplings of charged superfields which are forbidden by gauge invariance may, in such a setup, be allowed by couplings to world-sheet instantons involving the charged modulus. For example, the superpotential for the fields introduced above could be

$$W = \chi_+ S_-^2 e^{-T/\delta_{\rm GS}} - m\chi_+ S_- \,. \tag{3.51}$$

This superpotential is gauge-invariant if χ_+ and S_- carry $U(1)_{\rm A}$ charge ± 1, respectively, and the modulus transforms as

$$T \to T - i\delta_{\rm GS}\epsilon \,, \tag{3.52}$$

where $\delta_{\rm GS} \sim \mathcal{O}(1)$ is a real constant which depends on the charged spectrum of the theory, cf. Eq. (A.10), and ϵ is a chiral superfield gauge transformation parameter. Yukawa couplings like the one in Eq. (3.51) generically arise in intersecting D-brane models of type IIB string theory, cf. [30, 132], and at fixed points in heterotic orbifold compactifications, cf. [133]. Moduli stabilization via Yukawa-type interactions has been previously studied in [134] in the context of heterotic string theory, as well as in [135] with the inclusion of a constant piece in the superpotential. In generic string compactifications the mass parameter m in the second term of Eq. (3.51) is field-dependent. For the sake of simplicity we assume that the term arises from a gauge-invariant Yukawa coupling and that the scale m is fixed by the vacuum expectation value of an uncharged chiral superfield stabilized at a higher scale.

As in all other examples we assume a no-scale Kähler potential, written in a gauge-invariant way,

$$K = -3\ln\left(T + \overline{T} - 2\delta_{\rm GS}\mathscr{V} - \bar{S}_- e^{-2\mathscr{V}} S_- - \bar{\chi}_+ e^{2\mathscr{V}}\chi_+\right) \,, \tag{3.53}$$

where $\mathscr{V}$ denotes the vector multiplet of $U(1)_{\rm A}$. This choice of K is convenient and well-motivated from a string theory perspective, but the specific form of K does not affect the following discussion. Already at the level of global supersymmetry it is clear that the F-term of χ_+ stabilizes T at a non-vanishing vacuum expectation value. Moreover, the D-term potential

$$V_D = \frac{4\pi}{T + \overline{T}}\left(K_{\chi_+}\chi_+ - K_{S_-}S_- + \frac{3\delta_{\rm GS}}{T + \overline{T}}\right)^2 \,, \tag{3.54}$$

stabilizes $\langle S_- \rangle = \sqrt{\delta_{\rm GS}}$.[16] We can find the effective theory for χ_+ and T by integrating out the heavy vector multiplet, which eats the symmetry-breaking field S_-. This works

[16] Once more, details about the origin of this potential can be found in Appendix A.1 and in the original work [38].

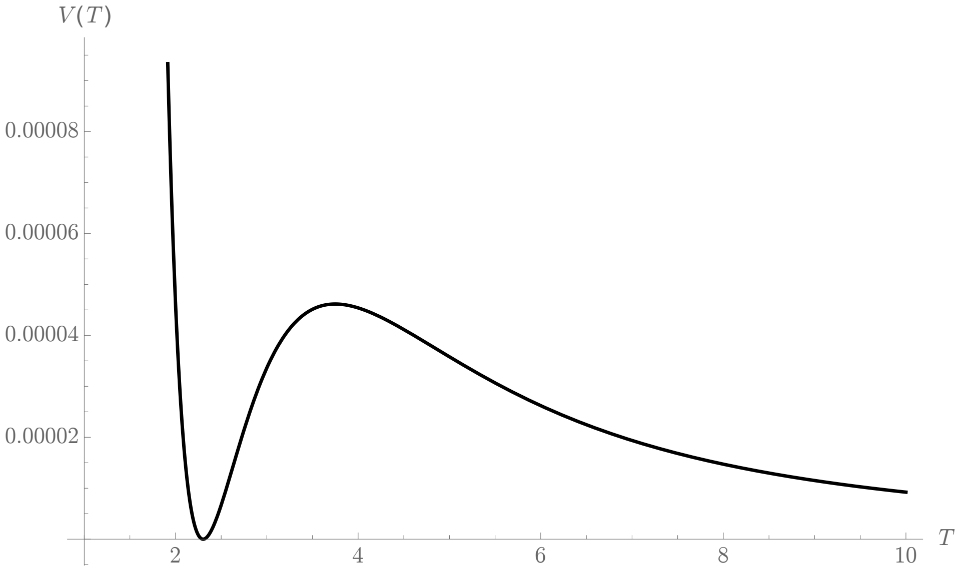

Figure 3.3: *Scalar potential $V(T)$ after integrating out S_- and χ_+, for $m = 0.1$ and $\delta_{GS} = 1$. Unsurprisingly, the shape of the potential bears resemblance to the KKLT potential. The important difference is, however, that the vacuum at $T_0 \approx 2.4$ has supersymmetric Minkowski background space-time. Therefore, the potential barrier only depends on m and δ_{GS}, but not on $m_{3/2}$.*

analogous to the discussion in Section 2.3.4, for details we refer to [38]. After a straightforward computation we find

$$\begin{aligned} K &= -3\ln\left(T + \overline{T} - \delta_{\text{GS}} - |\chi_+|^2 + \frac{|\chi_+|^4}{2\delta_{\text{GS}}}\right), \\ W &= \delta_{\text{GS}}\chi_+\left(e^{-T/\delta_{\text{GS}}} - \frac{m}{\sqrt{\delta_{\text{GS}}}}\right). \end{aligned} \tag{3.55}$$

The effective Lagrangian has a supersymmetric Minkowski minimum at

$$T_0 = \delta_{\text{GS}} \ln \frac{\sqrt{\delta_{\text{GS}}}}{m}, \qquad \chi_{+,0} = 0. \tag{3.56}$$

The mass of the canonically normalized modulus is

$$m_T = \frac{m}{3\sqrt{\delta_{\text{GS}}}}, \tag{3.57}$$

which coincides with the mass of χ_+. Since m is a free parameter in our discussion, this mechanism again allows for a separation of scales between m_T and $m_{3/2}$, after supersymmetry is spontaneously broken in a separate sector. Fig. 3.3 illustrates the scalar potential of T for fixed values of S_- and χ_+.

Chapter 4

Supersymmetric Moduli Stabilization and F-Term Inflation

Whenever inflation is treated in string-effective supergravity models, stabilization of all moduli including the dilaton, Kähler and complex structure moduli must be addressed. As outlined in Chapter 3, a standard procedure is to use gaugino condensates and fluxes [31,32], which can lead to metastable Minkowski vacua. In general, moduli stabilization in the entire cosmological history leads to upper bounds on the reheating temperature [123] and the energy scale of inflation [102]. In setups with spontaneous supersymmetry breaking the modulus mass and the barrier protecting the vacuum are proportional to the gravitino mass. This leads to a tension between high-scale inflation and low-energy supersymmetry breaking, cf. Section 3.4.1.

To avoid this tension, in this chapter we study supersymmetric moduli stabilization in combination with F-term inflation. It is instructive to study this rather simple case before moving on to including the effects of supersymmetry breaking by the moduli in Chapter 5. In general, the coupling to heavy Kähler moduli generates corrections to the inflaton potential which can be expanded in powers of H/m_T, the ratio of the Hubble scale during inflation and the modulus mass. Besides general corrections induced by the modulus back-reaction we systematically study the effective inflaton potential in two examples, F-term hybrid inflation and chaotic inflation. In hybrid inflation, the back-reaction of T produces a linear term in the inflaton at leading order. This is analogous to the effect of supersymmetry breaking which induces a linear term proportional to the gravitino mass [136]. Depending on its size such a linear term can have a significant effect on inflationary observables, in particular the spectral index of scalar fluctuations [78, 137–139]. In chaotic inflation the leading-order correction is suppressed by an additional power of H/m_T compared to hybrid inflation. Nevertheless, the modulus-induced terms can have severe consequences for CMB observables in the case of a large Hubble scale.

Detailed analyses of moduli dynamics in specific inflation and supersymmetry breaking models have been carried out in [140–142]. The results presented here constitute an extension and generalization of these works. They have been published in [36]. The treatment in this thesis focusses on the derivation of the general expression for the inflaton effective potential, Eq. (4.9), which is the central result of this chapter.

4.1 Supersymmetrically integrating out heavy moduli

Let us consider inflation in a sector comprised of chiral superfields ϕ_α in a four-dimensional effective theory with a single modulus T. As discussed in Chapter 3, T typically has the classical Kähler potential Eq. (3.7). Stabilization of the modulus is achieved by an appropriate superpotential $W_{\text{mod}}(T)$. Here we assume that W_{mod} is such that the scalar potential has a local minimum at $T_0 = \overline{T}_0$ which is supersymmetric and Minkowski, i.e.,

$$W_{\text{mod}}(T_0) = 0\,, \qquad D_T W_{\text{mod}}(T_0) = 0\,. \tag{4.1}$$

With these conditions fulfilled one finds for the mass of the canonically normalized modulus

$$|m_T| = \frac{\sqrt{2T_0}}{3}|W''_{\text{mod}}(T_0)|\,, \tag{4.2}$$

where primes denote derivatives with respect to T. For simplicity we choose $W''_{\text{mod}}(T_0)$ to be real. As discussed before, to ensure that the modulus remains stabilized in the entire cosmological evolution its mass must be larger than the scale of any other dynamics in the effective theory, for example, the scale of inflation. The same must hold for the barriers protecting metastable vacua.

Notice that the conditions (4.1) correspond to the conditions for supersymmetric moduli stabilization discussed in Section 3.4. Although examples for this kind of mechanism are known and listed in Section 3.4 we do not specify the exact form of $W_{\text{mod}}(T)$ since our analysis does not depend on it.

To study the back-reaction of the heavy modulus on the inflationary potential, we consider a class of supergravity theories defined by

$$\begin{aligned} K &= -3\ln(T+\overline{T}) + K_{\text{inf}}(\phi_\alpha, \bar{\phi}_\alpha)\,, \\ W &= W_{\text{mod}}(T) + W_{\text{inf}}(\phi_\alpha)\,. \end{aligned} \tag{4.3}$$

Note that in general the Kähler potential and superpotential of the inflaton sector can be moduli-dependent. The precise coupling of moduli to matter fields depends on the string model under consideration, cf. [100, 143] for examples. In some cases the dependence on the lightest modulus can even be absent. In the following we neglect the dependence of the inflaton sector on T, following the discussions in [100, 140–142]. The relevance of these supergravity models to string constructions has to be examined for each particular case. Even with the simplifying assumption that T couples only gravitationally to the inflaton sector we can draw crucial inferences on the structure of the effective theory after integrating out T.

The scalar potential resulting from Eqs. (4.3) can be written as

$$V = \frac{e^{K_{\rm inf}}}{(T+\overline{T})^3}\left(\frac{(T+\overline{T})^2}{3}|D_T W|^2 + K_{\rm inf}^{\alpha\bar{\alpha}} D_\alpha W D_{\bar{\alpha}}\overline{W} - 3|W|^2\right) . \tag{4.4}$$

Generically, there is a non-trivial interaction between the modulus and inflaton sectors. Due to the large positive energy density during inflation the minimum of the modulus potential is shifted by an amount $\delta T(\phi_\alpha) = T(\phi_\alpha) - T_0$. In other words, the inflationary energy density acts as an uplift on the modulus potential.

We can find this displacement by solving the equations of motion for T during inflation, i.e., by imposing $\partial_T V|_{T_0+\delta T} = 0$ in the new minimum. Thus, we expand the scalar potential as follows,

$$V = V|_{T_0} + (\partial_T V)|_{T_0}\delta T + \frac{1}{2}(\partial_T^2 V)|_{T_0}\delta T^2 + \dots \tag{4.5}$$

and then minimize with respect to δT. As before, we choose all superpotential parameters to be real. In that case only the real part of T is affected by inflation. Thus, $T = \overline{T}$ is implied in the above expression and in everything that follows. Allowing for complex parameters in the superpotential does not change our results qualitatively. We can conveniently write the resulting expression as a power series in the ratio H/m_T. The aforementioned requirement of $m_T > H$ makes this analysis self-consistent. Including all terms up to second order in H/m_T, we find[1]

$$\delta T = \frac{W_{\rm inf}}{\sqrt{2T_0}\, m_T} + \frac{1}{(2T_0)^2 m_T^2}\Bigg[K_{\rm inf}^{\alpha\bar{\alpha}} D_\alpha W_{\rm inf}\partial_{\bar{\alpha}}\overline{W}_{\rm inf} - |W_{\rm inf}|^2 \tag{4.6}$$

$$- W_{\rm inf}^2\left(\frac{1}{2} + \frac{(2T_0)^{3/2}\, W_{\rm mod}'''(T_0)}{6 m_T}\right)\Bigg] . \tag{4.7}$$

This implies

$$D_T W|_{T_0+\delta T} = \frac{3}{(2T_0)^{5/2}\, m_T} K_{\rm inf}^{\alpha\bar{\alpha}} D_\alpha W \partial_{\bar{\alpha}}\overline{W} + \mathcal{O}\left(\frac{H^2}{{m_T}^2}\right) . \tag{4.8}$$

[1] We assume that $W_{\rm mod}''$ is not hierarchically suppressed compared to higher derivatives of $W_{\rm mod}$.

We observe that T contributes to supersymmetry breaking during inflation, but its F-term is suppressed by one power of H/m_T.

After setting the modulus to its proper minimum, the effective inflaton potential reads

$$\begin{aligned} V \approx \frac{V_{\text{inf}}(\phi_\alpha)}{(2T_0)^3} &- \frac{3}{2(2T_0)^{9/2}\, m_T}\Big\{W_{\text{inf}}\Big[V_{\text{inf}}(\phi_\alpha) + e^{K_{\text{inf}}} K_{\text{inf}}^{\alpha\bar{\alpha}} \partial_\alpha W_{\text{inf}} D_{\bar{\alpha}} \overline{W}_{\text{inf}}\Big] + \text{ c.c.}\Big\} \\ &- \frac{3e^{K_{\text{inf}}}}{(2T_0)^6\, m_T^2}\Big|K_{\text{inf}}^{\alpha\bar{\alpha}} D_\alpha W_{\text{inf}} \partial_{\bar{\alpha}} \overline{W}_{\text{inf}}\Big|^2 , \end{aligned} \tag{4.9}$$

at leading order in H/m_T. Here $V_{\text{inf}}(\phi_\alpha)$ denotes the inflationary potential in the absence of a modulus sector, i.e.,

$$V_{\text{inf}}(\phi_\alpha) = e^{K_{\text{inf}}}\Big\{K_{\text{inf}}^{\alpha\bar{\alpha}} D_\alpha W_{\text{inf}} D_{\bar{\alpha}} \overline{W}_{\text{inf}} - 3|W_{\text{inf}}|^2\Big\} . \tag{4.10}$$

Notice that all powers of T_0 in Eq. (4.9) can be absorbed by a redefinition of W_{inf}. As naively expected, all corrections vanish in the limit of an infinitely heavy modulus. Since T does not break supersymmetry in the vacuum it completely decouples from the dynamics of inflation. Supersymmetry breaking after inflation can be achieved in a separate sector without affecting this discussion. In the following we study the effect of the leading-order correction operators in two representative examples of F-term inflation.

4.2 Examples

4.2.1 Hybrid inflation

As a first example we consider F-term hybrid inflation as introduced in Section (2.3.2). With K_{inf} and W_{inf} defined by Eqs. (2.35) and (2.36), respectively, we find for the leading-order potential at tree level

$$V = V_0\left[1 - \frac{3\sqrt{V_0}}{m_T}\left(\phi + \bar{\phi}\right)\right] + \dots , \tag{4.11}$$

with $V_0 = \frac{1}{4}\tilde{\lambda}^2 v^4$ and $\tilde{\lambda}^2 = \lambda^2/(2T_0)^3$. The modulus induces a linear term in the inflaton potential. In the limit $m_T \to \infty$ we recover the original potential up to a total rescaling factor which can be absorbed by a redefinition of λ.

Such a linear term in the inflaton field is also induced by soft supersymmetry breaking, cf. the detailed discussion in [136]. Depending on its size relative to the one-loop potential, it can significantly affect the dynamics of inflation [136–138]. This is particularly important for the spectral index of scalar fluctuations, which in hybrid inflation is typically $n_\text{s} \simeq 0.98$ [77]. This value can be reduced to the currently measured value

$n_s \approx 0.96$ if the linear term is taken into account [78, 138, 139]. In particular, a cancellation between the slopes generated by the linear term and the Coleman-Weinberg one-loop potential may lead to a flatter potential for certain parameter regimes. Hence, modulus-induced corrections to the inflaton potential can be used to reconcile F-term hybrid inflation with CMB observations. A consistent discussion of the effective potential (4.11) necessitates a two-field description since the modulus-induced term treats the real and imaginary parts of ϕ differently. This is, however, beyond the scope of this thesis. Instead we refer the reader to the original publication [36] and the related discussion in [78].

Based on Eq. (4.11) one may expect that corrections from the back-reaction of T become relevant if $m_T \sim H$. However, the authors of [36] have shown that the modulus induces an $\mathcal{O}(1)$ correction to the slope of the potential if

$$m_T \sim 6\pi\sqrt{2N_e}\,v^2 \sim 0.2 M_{\rm GUT}\,, \tag{4.12}$$

with $N_e \approx 60$. This implies that even if the modulus is stabilized at a scale close to $M_{\rm GUT} \sim 10^{-2}$, it can significantly affect the dynamics of hybrid inflation. Notice that m_T in Eq. (4.12) is much larger than the naive estimate $m_T \sim H \sim 10^{-7}$, where we have assumed $\tilde{\lambda} \sim 10^{-2}$, the largest coupling for which hybrid inflation works. This strong back-reaction may appear surprising, but it is a consequence of the enormous flatness of the inflaton potential.

4.2.2 Chaotic inflation

Our second example is chaotic inflation with a stabilizer field, defined by the Kähler potential in Eq. (2.34) and the superpotential

$$W_{\rm inf} = S f(\phi)\,, \tag{4.13}$$

where $f(\phi)$ is a holomorphic function. The stabilizer field S generates the inflaton potential via its F-term but decouples from the inflationary dynamics. This situation is engineered by including a sufficiently large quartic term in the Kähler potential. This lifts the mass m_S above the Hubble scale during inflation and stabilizes S at the origin of field space. The inflaton is identified with the imaginary part of ϕ which is protected against supergravity corrections by a shift symmetry. The real part of ϕ is stabilized at the origin with a Hubble-scale mass for typical choices of $f(\phi)$, in particular monomial functions.[2]

[2] This does not change when coupled to a heavy modulus. Thus, we consider $\mathrm{Re}\,\phi = 0$ in what follows.

Including the leading-order modulus correction we can write the scalar potential as follows,

$$V = m_S^2|S|^2 + |\tilde{f}(\phi)|^2 - \frac{3|\tilde{f}(\phi)|^2}{m_T}\left[\left(S\tilde{f}(\phi) + \text{ c.c.}\right) + \frac{|\tilde{f}(\phi)|^2}{m_T} + \mathcal{O}(|S|^2)\right], \tag{4.14}$$

up to higher order terms in $|S|$. Here,

$$\tilde{f}(\phi) = \frac{f(\phi)}{(2T_0)^{3/2}}, \qquad m_S^2 = \left|\tilde{f}'(\phi)\right|^2 + \frac{4}{\Lambda^2}\left|\tilde{f}(\phi)\right|^2. \tag{4.15}$$

Apparently the modulus back-reaction induces a linear term in S. The resulting displacement of S affects the potential of ϕ. At leading order in H/m_T, the new minimum of the stabilizer field lies at

$$\bar{S} = \frac{3|\tilde{f}(\phi)|^2\tilde{f}(\phi)}{m_T\, m_S^2}. \tag{4.16}$$

The effective inflaton potential after integrating out T and S becomes

$$V = V_0\left(1 - \frac{3V_0}{m_T^2} - \frac{9V_0^2}{m_T^2\, m_S^2} + \ldots\right), \tag{4.17}$$

with $V_0 = |\tilde{f}(\phi)|^2$. Again, all corrections vanish in the limit of a very heavy modulus, as naively expected in the absence of supersymmetry breaking. Notice that the leading modulus correction only appears at order m_T^{-2}. The absence of a correction of order m_T^{-1} results from the suppression of W_{inf} by one power of m_T. This is an important difference compared to hybrid inflation. A more pronounced example of this suppression arises in Chapter 6, in D-term hybrid inflation with a constant FI term coupled to supersymmetric moduli stabilization. In that case the superpotential of the inflaton sector vanishes identically due to the vacuum expectation values of two waterfall fields.

One more remark is in order before we move on to moduli stabilization with supersymmetry breaking. Similar to hybrid inflation, the modulus mass in chaotic inflation is forced to be very large for a four-dimensional description to be consistent. The outlined analysis is only meaningful as long as $V_0^{1/4} < m_T$, beyond which the modulus is destabilized by the uplift of the inflaton potential. In chaotic inflation typically $V_0^{1/4} \sim M_{\text{GUT}} \sim 10^{-2}$, meaning that all moduli must be stabilized around or above the GUT scale to guarantee stability of the vacuum. In that sense our analysis implies severe consequences for studies of extra dimensions during high-scale inflation. When treating the more general case of moduli stabilization with spontaneous supersymmetry breaking in the next chapter, we make these consequences more explicit.

Chapter 5

Supersymmetry-Breaking Moduli Stabilization and Chaotic Inflation

As discussed in Chapter 2, in large-field inflation the effects of high-scale physics from a UV complete theory are generically most severe. In this chapter we study these effects on chaotic inflation, in its simplest manifestation with a quadratic potential, as a representative example for large-field inflation. Having treated supersymmetric moduli stabilization in the previous two chapters, we move on to the more general case of stabilization schemes which necessarily involve supersymmetry breaking. We have observed that in most cases, with the exception of D-term inflation driven by a field-dependent FI term, all effects from the modulus sector decouple if it is made very heavy. This is not true once supersymmetry breaking is involved. There are well-known non-decoupling effects, such as soft mass terms, which can have severe effects on the sensitive inflaton potential. As will become clear, these effects may be constructive or destructive for realizing inflation. In particular, they enable us to realize chaotic inflation with a single chiral superfield, i.e., without the need of a stabilizer field. On the other hand, large correction terms in the form of higher-dimensional operators may destabilize the moduli and the inflationary trajectory. We investigate correction terms in a general way, which constitutes a generalization of the results presented in Section 4.1, and then discuss three prominent examples: KKLT stabilization, Kähler Uplifting, and the simplest Large Volume Scenario. However, before we study these non-decoupling effects in Sections 5.2 to 5.5, we begin this chapter with a systematic analysis of the effects of high-scale supersymmetry breaking in chaotic inflation with a stabilizer field. This is an instructive analysis as it illuminates the need for a different approach to chaotic inflation in supergravity whenever supersymmetry is broken at a high scale. This alternative approach is then studied in the subsequent sections.

The discussion of supersymmetry breaking in chaotic inflation with a stabilizer field, cf. Section 5.1, is based on the publication [37]. In this thesis we focus on the simplest example of supersymmetry breaking with a Polonyi field, as well as the inclusion of additional renormalizable interactions in the superpotential. The remainder of the chapter is based on the results found in [39]. Here we emphasize in particular the three examples for moduli stabilization given in that reference, and the numerical study of the back-reaction on inflation. As before, expressions and figures taken from the publications [37, 39] reflect the author's contribution to those works.

5.1 Chaotic inflation and supersymmetry breaking

In Section 2.3.1 we discussed in detail how chaotic inflation can be implemented in supergravity. The simplest way to avoid the supergravity η problem and dangerous negative terms in the potential seems to be the inclusion of a stabilizer field S and a shift symmetry for the complex inflaton field ϕ. Therefore, we consider the supergravity model defined by Eqs. (2.34) as a candidate model for inflation. In the following we study its interplay with a sector of F-term supersymmetry breaking, represented by a Polonyi model or an O'Raifeartaigh model. We analyze how different couplings between the two sectors affect the allowed supersymmetry breaking scale and derive upper bounds on the gravitino mass from the requirement of successful inflation.[1] We find that in none of our setups the gravitino mass can be larger than the Hubble scale during inflation. Among other things, this implies that chaotic inflation is challenged when combined with moduli stabilization, where typically $m_{3/2} > H$ is required, cf. the discussion in Section 3.4.1. This motivates the detailed analyses in the subsequent sections of this chapter.

5.1.1 Back-reaction of a Polonyi field

Although chaotic inflation and many of its variants have been extensively studied in the literature, its connection to supersymmetry breaking was not as closely investigated. Generic setups to achieve F-term supersymmetry breaking are the O'Raifeartaigh model and the Polonyi model, as discussed in Section 3.2.2. Coupling the inflaton sector to a supersymmetry-breaking sector turns out to be more difficult than expected. In the simplest working scenarios we find that the gravitino mass is bounded from above. To

[1] For a related analysis of the effects of supersymmetry breaking on inflation, cf. [144].

see this explicitly in a simple toy model, let us consider the effective theory defined by[2]

$$W = mS\phi + fX + W_0\,, \tag{5.1}$$

i.e., by adding a Polonyi-like sector with a chiral superfield X to the inflation model of Section 2.3.1. In the way proposed here, the two sectors decouple except for gravitational interactions. As in previous cases we choose the parameters m, f, and W_0 to be real, which can always be achieved by field redefinitions and Kähler transformations. Similar to the superpotential, a suitable Kähler potential is obtained by adding the contributions from the inflation and supersymmetry breaking sectors, i.e.,

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S|^2 + |X|^2 - \xi_1|X|^4 - \xi_2|S|^4\,, \tag{5.2}$$

where $\xi_{1,2} \gg 1$ are real parameters.[3] Note, once more, that the two quartic terms are necessary to stabilize the respective fields during and after inflation. In the absence of the term proportional to ξ_2, the stabilizing scalar S gets no Hubble-scale contribution to its mass. The quartic term for the supersymmetry-breaking field X stabilizes the corresponding scalar in the true vacuum and circumvents the Polonyi problem. Both terms may result from integrating out heavy degrees of freedom at the quantum level. Since X gets a Hubble-scale mass during inflation, we often neglect the term involving ξ_1 in our discussion of inflation.

Vacuum after inflation

In this combined model, if f is small compared to all other scales in the theory, m still corresponds to the inflaton mass, f denotes the scale of supersymmetry breaking in the true vacuum after inflation, and W_0 can be chosen such that the vacuum energy vanishes after inflation. The latter implies $W_0 \approx f/\sqrt{3}$ at leading order in f. After inflation the true vacuum of the model lies at

$$\langle\phi\rangle = \langle S\rangle = 0\,, \qquad \langle X\rangle \approx \frac{1}{2\sqrt{3}\xi_1}\,. \tag{5.3}$$

In this vacuum the gravitino mass is given by

$$m_{3/2} \approx W_0 \approx \frac{f}{\sqrt{3}}\,, \tag{5.4}$$

up to terms suppressed by powers of f or ξ_1^{-1}.

[2]Notice that this form of superpotential has been studied, in slightly different contexts, in [142, 145].

[3]Notice that, for convenience, we have slightly changed the notation of the quartic terms compared to Eqs. (2.34). As explained below, ξ_1 and ξ_2 are inversely proportional to the square of cut-off scales at which heavy degrees of freedom have been decoupled.

An important observation made in [37] is that this vacuum structure is altered if f is chosen to be larger than m. To see this, consider the full scalar potential

$$V = e^K \left(|mS + (\phi + \bar{\phi})W|^2 + K_{S\bar{S}}^{-1}|m\phi + K_S W|^2 + K_{X\bar{X}}^{-1}|f + K_X W|^2 - 3|W|^2 \right) , \quad (5.5)$$

where

$$K_X = \bar{X}(1 - 2\xi_1|X|^2) , \qquad K_{X\bar{X}} = 1 - 4\xi_1|X|^2 , \quad (5.6)$$

$$K_S = \bar{S}(1 - 2\xi_2|S|^2) , \qquad K_{S\bar{S}} = 1 - 4\xi_2|S|^2 . \quad (5.7)$$

We can expand V up to second order in all real scalars to obtain

$$\begin{aligned} V = & f^2 - 3W_0^2 - 2\sqrt{2} f W_0 \, \alpha + 2mW_0 \, \varphi\chi + \frac{1}{2} f^2 \left(2\zeta^2 + \chi^2 + \psi^2\right) \\ & - W_0^2 \left(\alpha^2 + \beta^2 + \zeta^2 + \chi^2 + \psi^2\right) + \frac{1}{2} m^2 \left(\zeta^2 + \chi^2 + \psi^2 + \varphi^2\right) \\ & + 2f^2\xi_1 \left(\alpha^2 + \beta^2\right) , \end{aligned} \quad (5.8)$$

where we have defined

$$S = \frac{\psi + i\chi}{\sqrt{2}} , \quad X = \frac{\alpha + i\beta}{\sqrt{2}} , \quad \phi = \frac{\zeta + i\varphi}{\sqrt{2}} . \quad (5.9)$$

We can study the mass matrix of this system of scalar fields to look for possible vacua. We find that $W_0 = f/\sqrt{3}$ leads to a tachyonic direction close to the origin of the potential if

$$f > m . \quad (5.10)$$

Specifically, only for $f < m$ there is a stable vacuum at $\langle\phi\rangle = \langle S\rangle = 0$ and $f^2 = 3W_0^2$ cancels the cosmological constant. For larger f a linear combination of ϕ and S obtains a non-vanishing vacuum expectation value. Cancellation of the cosmological constant is then ensured by

$$\langle V\rangle = f^2 - 3W_0^2 + \frac{m^2 \left(f^2 - 6W_0^2\right)}{256 \left(f^2 - 2W_0^2\right)^4 \left(f^2 - W_0^2 + 2\xi_2 \left(f^2 - 3W_0^2\right)\right)} = 0 , \quad (5.11)$$

at leading order in m. This effect, although small, is taken into account in our analysis of the inflaton dynamics.

Interaction during inflation

During inflation all real degrees of freedom must be stabilized at a high scale, i.e., with masses larger than the Hubble scale, so they can be integrated out. Considering the scalar potential in Eq. (5.5) and its expansion in Eq. (5.8) it is evident that all fields are stabilized at the origin with large masses, except for the inflaton $\varphi = \sqrt{2}\mathrm{Im}\,\phi$ and the imaginary part of the stabilizer field $\chi = \sqrt{2}\mathrm{Im}\,S$. Due to the presence of the additional scale f and the constant W_0, χ is displaced from its original minimum at $\langle\chi\rangle = 0$. Assuming that $\chi \ll 1$, we can expand the potential Eq. (5.5) up to second order around $\chi = 0$. The result reads

$$V = f^2 - 3W_0^2 + \frac{1}{2}m^2\varphi^2 + 2mW_0\varphi\chi + \frac{1}{2}\left(f^2 - 2W_0^2 + m^2 + 2m^2\varphi^2\xi_2\right)\chi^2\,, \tag{5.12}$$

neglecting the small vacuum expectation value of X. In particular, the linear term

$$2mW_0\varphi\chi\,, \tag{5.13}$$

causes a displacement from $\chi = 0$ whenever $\varphi \neq 0$. By minimizing with respect to χ we find

$$\chi \approx -\frac{2mW_0\varphi}{f^2 - 2W_0^2 + m^2 + 2m^2\varphi^2\xi_2}\,, \tag{5.14}$$

during inflation. Notice that Eq. (5.14) depends on f and W_0, as well as on φ, and that only the imaginary part of S receives a shift. By means of a numerical analysis of the full equations of motion we can verify that S indeed remains stabilized in its new minimum for the entire inflationary epoch. While the inflaton slowly rolls down its quadratic potential the stabilizer field trails its inflaton-dependent minimum almost instantly. This behaviour is illustrated in Fig. 5.1.

Thus, we can still treat S as a heavy degree of freedom and integrate it out at its shifted vacuum expectation value given by Eq. (5.14). This yields an effective potential for the inflaton direction which reads[4]

$$V(\varphi) = f^2 - 3W_0^2 + \frac{1}{2}m^2\varphi^2\left(1 - \frac{4W_0^2}{f^2 - 2W_0^2 + m^2 + 2m^2\varphi^2\xi_2}\right)\,. \tag{5.15}$$

Evidently, depending on the magnitude of f and hence the gravitino mass, the correction resulting from integrating out χ may severely alter the predictions of chaotic inflation.

Bounds on the gravitino mass

Considering the effective inflaton potential in Eq. (5.15) alteration of the CMB observables, in particular the scalar spectral index n_s and the tensor-to-scalar ratio r, is to

[4]Notice that this procedure is very similar to integrating out a heavy modulus field. This time, however, the back-reaction on φ is not only sourced by inflationary vacuum energy, but also by the contribution of the Polonyi field.

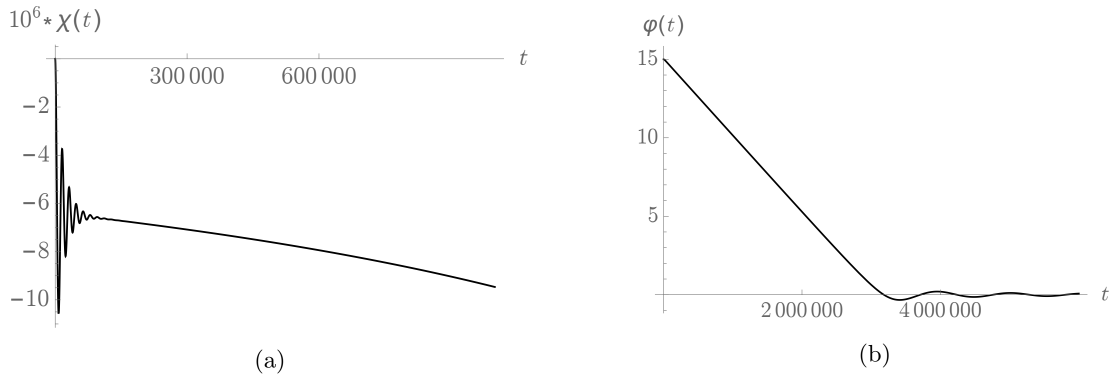

Figure 5.1: *Evolution of the canonically normalized imaginary part of S (a) and the inflaton φ (b) during inflation, for $\xi_1 = \xi_2 = 10$, $f = 10^{-8}$, and $m = 6 \cdot 10^{-6}$. In this case, since $f < m$, cancellation of the cosmological constant implies $W_0 = f/\sqrt{3}$. Depending on its initial value the stabilizer field settles in its shifted minimum very early and remains stabilized for the rest of the inflationary epoch and beyond. Due to its inflaton-dependence, the vacuum expectation value of χ evolves with time. The inflaton field rolls slowly in its potential and then oscillates around its minimum at $\varphi = 0$. Notice the different time scales in the two figures. The characteristic time until the end of inflation is m^{-1}.*

be expected at $f \gtrsim m$. We expect that increasing f even further will make inflation impossible at a value which satisfies

$$3m^2 \lesssim f^2 \lesssim 2m^2\varphi^2\xi_2\,, \tag{5.16}$$

neglecting the correction to W_0 in Eq. (5.11). Since m is fixed by observations to be $m \simeq 6 \times 10^{-6}$ in Planck units, it is necessary to specify realistic values of ξ_2 to obtain a meaningful upper bound on the gravitino mass. As stated previously, we assume that the Kähler potential terms involving ξ_1 and ξ_2 stem from couplings to heavy fields, i.e., from

$$W_{\text{heavy}} \supset \lambda_1 S\psi_1^2 + \lambda_2 X\psi_2^2 + \text{mass terms}\,, \tag{5.17}$$

where ψ_i denotes heavy superfields of mass M_i. One-loop corrections of the Coleman-Weinberg type generate a quartic term for S in K,

$$K_{\text{1-loop}} \approx |S|^2\left[1 - \frac{\lambda_1^2}{16\pi^2}\log\left(1 + \frac{\lambda_1^2|S|^2}{M_1^2}\right)\right] \approx |S|^2 - \frac{\lambda_1^4}{16\pi^2 M_1^2}|S|^4\,, \tag{5.18}$$

and similarly for X, cf. the discussion in [115]. Thus, in the generic case $\lambda_i \sim \mathcal{O}(1)$ the ξ_i are related to the mass scales M_i as follows,

$$\xi_i \sim \frac{1}{16\pi^2 M_i^2}\,. \tag{5.19}$$

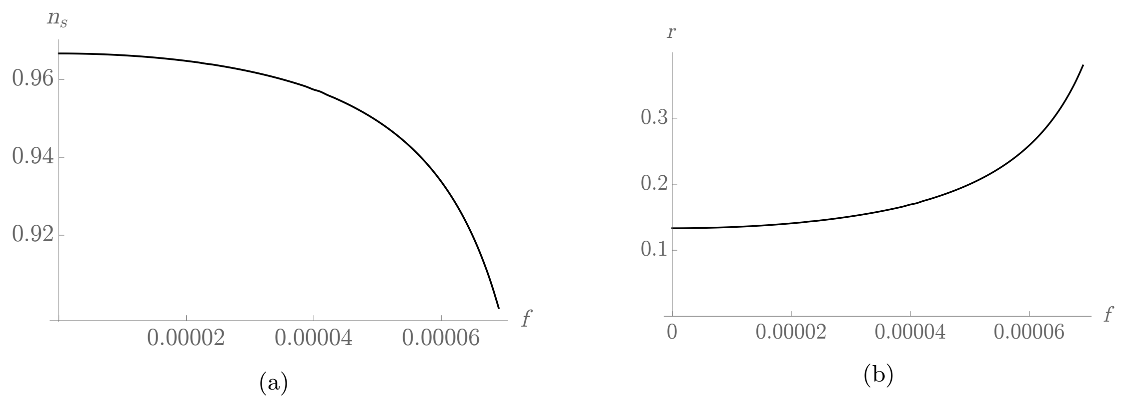

Figure 5.2: *The CMB observables n_s and r as a function of the supersymmetry breaking scale f. Clearly, the model is ruled out by observation at values of f quite below* 10^{-4}.

Given that the heavy degrees of freedom should be integrated out above the energy scale of inflation, $M_i \gtrsim \rho_{\text{inf}} \sim M_{\text{GUT}} \simeq 0.01$, but below the Planck scale, we assume

$$\xi_1 \sim \xi_2 \sim 10 \tag{5.20}$$

to be reasonable values for the coefficients. We remark that quartic terms in the Kähler potential could also arise from α' corrections in string theory. In such a setup the coefficients would rather be $\xi_i \sim 1/M_{\text{s}}^2$, where M_{s} denotes the string scale. In order for string modes to decouple, M_{s} would have to be larger than the energy scale during inflation, but smaller than the Planck scale. Due to the absence of the loop suppression factor $16\pi^2$, this could result in substantially larger coefficients.

Using these estimates, the spectral index and tensor-to-scalar ratio resulting from the corrected potential Eq. (5.15) are displayed in Fig. 5.2, as a function of f. Evidently, above a value of $f \approx 6 \times 10^{-5}$ the tensor-to-scalar ratio increases above $r \sim 0.2$ and n_{s} drops below 0.94, a point at which the model is essentially ruled out by observation. This translates into the following bound on the gravitino mass,

$$m_{3/2} \lesssim 10^{14}\,\text{GeV}\,. \tag{5.21}$$

Hence the most minimal way to achieve supersymmetry breaking in chaotic inflation excludes the possibility $m_{3/2} \gtrsim H$. This has interesting implications for setups with string-inspired supersymmetry breaking in which the supersymmetry breaking scale is usually very high, as recently investigated in [146–148]. We study this possibility in Sections 5.2 to 5.5.

5.1.2 Effects of additional interactions

In an attempt to relax the gravitino mass bound (5.21) the authors of [37] have extended the minimal model. By including a renormalizable coupling between X and ϕ in the superpotential, i.e.,

$$W = mS\phi + MX\phi + fX + W_0 \,, \tag{5.22}$$

one may hope that the situation can be improved. The new mass scale M contributes, together with m, to the mass of the inflaton, i.e.,

$$V = \frac{1}{2}m^2\varphi^2 \longrightarrow V = \frac{1}{2}(m^2 + M^2)\varphi^2 \,, \tag{5.23}$$

in the absence of supersymmetry breaking. The associated Kähler potential can be written as

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S|^2 + |X|^2 - \xi_1|X|^4 \,. \tag{5.24}$$

The quartic term in S is no longer necessary to stabilize the corresponding scalars in this setup.

Vacuum after inflation

In this framework, the fields are stabilized at different vacuum expectation values in the vacuum, and the constant W_0 consequently takes a different value to cancel the cosmological constant. Specifically, writing the complex scalars in terms of their real components

$$S = \frac{\psi + i\chi}{\sqrt{2}} \,, \quad X = \frac{\alpha + i\beta}{\sqrt{2}} \,, \quad \phi = \frac{\zeta + i\varphi}{\sqrt{2}} \,, \tag{5.25}$$

the associated vacuum expectation values after inflation are given by

$$\langle\varphi\rangle = \langle\chi\rangle = \langle\beta\rangle = 0 \,, \quad \langle\zeta\rangle \approx -\sqrt{2}\frac{Mf}{m^2 + M^2} \,, \quad \langle\alpha\rangle \approx \frac{1}{\sqrt{6}\xi_1}\frac{m}{\sqrt{m^2 + M^2}} \,, \tag{5.26}$$

and

$$\langle\psi\rangle \approx \frac{M}{(m^2 + M^2)^{3/2}}\frac{f^2\left(m^2 + 3M^2\right) - 3m^2\left(m^2 + M^2\right)}{3\sqrt{6}\xi_1 m^2} \,, \tag{5.27}$$

at leading order in f and ξ_1^{-1}. The gravitino mass in the true vacuum is given by

$$m_{3/2} \simeq W_0 \simeq \frac{m}{\sqrt{m^2 + M^2}}\frac{f}{\sqrt{3}} \,. \tag{5.28}$$

Notice that, as in the minimal model, this vacuum will be corrected for large values of f. The corrections will, however, not alter our conclusions about the allowed gravitino mass by much. Therefore, in what follows we use the value of W_0 stated in Eq. (5.28) as a leading-order approximation.

Interaction during inflation

Again, the supersymmetry breaking scale f induces a shift of the imaginary part of S during inflation. In fact, some of the other real scalars are shifted as well, but their vacuum expectation values are suppressed compared to that of χ. A numerical analysis once more confirms that all vacuum expectation values are reached quickly and that all fields, except the inflaton φ, remain stabilized during inflation. In the same manner as in the previous section, expanding up to second order in χ and integrating out the field gives a leading-order effective potential for the inflaton. The result reads

$$V(\varphi) = \frac{1}{2}(1+\delta^2)m^2\varphi^2 \left(1 - \frac{8f^2}{f^2(2+8\delta^2+6\delta^4)+3m^2(1+\delta^2)^2(2+\delta^2\varphi^2)}\right) , \tag{5.29}$$

where we have introduced the dimensionless parameter

$$\delta = \frac{M}{m} . \tag{5.30}$$

Notice that, in the limit $\delta \to 0$, Eq. (5.29) reduces to the effective potential of the minimal model. The only difference is that in the present setup $\xi_2 = 0$. Again, it appears that in this model chaotic inflation is not possible for arbitrarily large values of f.

Bounds on the gravitino mass

As shown in more detail in [37], the bound on $m_{3/2}$ actually becomes more stringent. Since $m_{3/2}$ is inversely proportional to δ there is a finite optimal value of δ. The best situation is $\delta \approx 4$. In this case, after integrating out χ, the model is ruled out by observation if

$$f \gtrsim 3 \cdot 10^{-5} , \tag{5.31}$$

a bound which is twice as stringent as in the minimal model. The situation does not improve if we re-introduce a quartic term for S in K.

In addition to extending the minimal model by renormalizable interactions, the authors of [37] explored the option of embedding chaotic inflation in the O'Raifeartaigh model. To guarantee stability of the model this requires the inclusion of unspecified α' corrections to K or the use of nilpotent superfields, as first proposed in [149]. However, even if the model is relieved from tachyonic instabilities there are upper bounds on the gravitino mass which are similar to (5.21). Hence, it appears that the bound $m_{3/2} \lesssim H$ is generic in chaotic inflation with a stabilizer field.

5.2 Integrating out supersymmetry-breaking moduli: General results

On the path to embedding chaotic inflation in string theory with consistent moduli stabilization, the results of the previous section force us to consider a different mechanism of chaotic inflation if the moduli break supersymmetry spontaneously. This is because the constraint $m_{3/2} > H$ found in Chapter 3 and the constraint $m_{3/2} \lesssim H$ from Section 5.1 are obviously at odds.

In Section 2.3.1 we elaborated on the troubles of chaotic inflation without a stabilizer field, i.e., with the simple superpotential

$$W = \frac{1}{2} m\phi^2 \,. \tag{5.32}$$

We have already hinted towards the fact that additional ingredients, apart from a stabilizer field, may remedy the model. In this section we prove that heavy moduli which break supersymmetry spontaneously are such an ingredient. This makes our setup interesting from the perspective of UV completion, since moduli are present in any string compactification and spontaneous supersymmetry breaking in the moduli sector is quite generic.

There has been substantial progress in implementing chaotic inflation without a stabilizer, and related models, in string theory. For recent discussions, cf. [147, 148, 150–156]. In particular, the authors of [153, 154] have analyzed the effects of moduli stabilization in F-term axion monodromy inflation. A general supergravity analysis comparing the scale of inflation and the gravitino mass has been performed in [157, 158]. On the other hand, it has proven difficult to implement the model proposed in [72], i.e., the one with a stabilizer field, in explicit string constructions. Both moduli stabilization and the origin of the stabilizer field are difficult questions to address. For recent treatments we refer to [150, 159].

In this part of the thesis we discuss, in general terms, how integrating out heavy moduli which break supersymmetry can have strong effects on the effective inflaton potential. We derive explicit formulae for the latter, assuming that the inflaton and moduli sectors interact only gravitationally. We then illustrate these general results with three examples – KKLT moduli stabilization, Kähler Uplifting, and the Large Volume Scenario – in Sections 5.3, 5.4, and 5.5 respectively. We derive bounds on the gravitino mass and the field value of the inflaton arising from stability of the moduli. Furthermore, each case is illustrated by means of a numerical example. In Section 5.6 we discuss the universality of the leading-order effective inflaton potential arising in all our examples and the shared universal CMB observables this predicts.

5.2.1 Non-decoupling effects from supersymmetry breaking

As before, we are interested in string-effective supergravity models in which the inflaton field φ, which is the imaginary part of a complex scalar field $\phi = \frac{1}{\sqrt{2}}(\chi + i\varphi)$, interacts with heavy moduli and supersymmetry breaking fields, collectively denoted by T_α. The effective action is defined by

$$\begin{aligned} K &= K_0(T_\alpha, \overline{T}_{\bar{\alpha}}) + \frac{1}{2} K_1(T_\alpha, \overline{T}_{\bar{\alpha}})(\phi + \bar{\phi})^2 \,, \\ W &= W_{\text{mod}}(T_\alpha) + \frac{1}{2} m\phi^2 \,. \end{aligned} \tag{5.33}$$

It can potentially reconcile chaotic inflation, moduli stabilization, and supersymmetry breaking. We are interested in the regime where the moduli and the supersymmetry breaking fields T_α are much heavier than the inflaton. Such heavy fields usually decouple from low-energy dynamics once they settle into their minima, denoted by $T_{\alpha,0}$. Remember that we studied the case without supersymmetry breaking in Chapter 4. We showed that for a single heavy modulus T with $K_0(T, \overline{T}) = -3 \ln \left(T + \overline{T}\right)$ and $K_1(T, \overline{T}) = 1$ the effects on the dynamics of inflation are given by Eq. (4.9). In particular, all corrections stemming from integrating out the heavy modulus disappear in the limit $m_T \to \infty$.

However, if any of the fields T_α break supersymmetry the picture changes. In this case, there are well-known effects that do not decouple from inflation. In the context of low-energy supersymmetric models these lead to soft-breaking terms whose size is controlled by the gravitino mass. In particular, considering spontaneous supersymmetry breaking we expect the effective inflaton potential to be of the form

$$V = V_{\text{Sugra}} + \frac{c}{2} \tilde{m} m_{3/2} \varphi^2 + \dots \,, \tag{5.34}$$

where c is a model-dependent real constant and V_{Sugra} is to be computed using

$$K = \frac{1}{2} \left(\phi + \bar{\phi}\right)^2 \,, \qquad W = \frac{1}{2} \tilde{m} \phi^2 \,, \tag{5.35}$$

with $\tilde{m} = K_1^{-1} e^{\frac{1}{2} K_0(T_0, \overline{T}_0)} m$ and the wave-function normalization $\phi \to K_1^{-1/2} \phi$ to match the notation of Eq. (5.33). Notice that in Eq. (5.34) a term proportional to $m_{3/2}^2 \varphi^2$ is absent due to the shift symmetry $\phi \to \phi + ic$, which is broken softly by the mass term in the superpotential.[5] Computing V_{Sugra} from Eqs. (5.35) while imposing cancellation

[5] We encounter this situation again in D-term hybrid inflation, cf. Section 6.1.3, where a soft mass term for the inflaton is present unless the Kähler potential is shift-symmetric.

of the cosmological constant at the end of inflation and setting the heavy real scalar χ to its minimum at $\langle\chi\rangle = 0$, we find

$$V = \frac{1}{2}\tilde{m}^2\varphi^2 + \frac{c}{2}\tilde{m}m_{3/2}\varphi^2 - \frac{3}{16}\tilde{m}^2\varphi^4 + \ldots . \tag{5.36}$$

This expression deserves our attention. Apparently, the second piece only decouples from inflation if $m_{3/2} \ll \tilde{m}$. If $m_{3/2}$ is large, however, it may potentially replace the supersymmetric mass term proportional to $\tilde{m}^2$ in driving inflation. In that case the term may even be large enough to dominate over the negative third piece for a sufficient field range.

The dots in Eqs. (5.34) and (5.36) denote sub-leading terms and higher powers in φ, for example, terms of order $\mathcal{O}(\tilde{m}m_{3/2}\varphi^4)$. Usually, such terms can be discarded easily. In large-field inflation, however, trans-Planckian excursions of φ can make corrections relevant. Therefore, in the following we systematically calculate corrections to the leading-order potential in Eq. (5.36). We are curious to find out if corrections from the modulus sector can cancel the third term in the effective potential, which makes V unbounded from below. Furthermore, if the modulus sector has an approximate no-scale symmetry we expect a cancellation of the bilinear soft mass term, i.e., $c \ll 1$. We wish to discuss if, in this situation, chaotic inflation can proceed via the supersymmetric mass term of φ without spoiling the stabilization of moduli.

5.2.2 Effective inflaton potentials

Therefore, we must attempt to find a generalization of Eq. (4.9), including the effects of supersymmetry breaking in the moduli sector. We start from the effective theory defined by Eqs. (5.33) and we assume that the moduli fields adiabatically trace the minimum of their potential during inflation. This is justified as long as their masses are larger than the Hubble scale. Specifically,

$$\nabla_\alpha V = 0 \qquad \Rightarrow \qquad G^I \nabla_\alpha G_I + G_\alpha = 0 \, . \tag{5.37}$$

Here ∇_α denotes the covariant derivative on field space, i.e., $\nabla_\alpha G_I = G_{\alpha I} - \Gamma^J_{\alpha I} G_J$ in terms of the Kähler function $G = K + \ln|W|^2$, where Γ is defined in Appendix A.2.

We can then integrate out the heavy fields T_α to obtain an effective scalar potential for the inflaton φ. Using that χ is heavy due to its soft mass and stabilized at the origin we can expand V in powers of the inflaton field,

$$V = V_0(T_\alpha, \overline{T}_{\bar{\alpha}}) + \frac{1}{2}V_1(T_\alpha, \overline{T}_{\bar{\alpha}})m\varphi^2 + \frac{1}{4}V_2(T_\alpha, \overline{T}_{\bar{\alpha}})m^2\varphi^4 \, . \tag{5.38}$$

The explicit coefficients V_0, V_1, and V_2 and other details of the computation are given in Appendix A.3. For a more thorough treatment of the derivation of the effective potential we refer to the original publication [39]. As we have seen many times before, during inflation the fields T_α are displaced from their minima,

$$T_\alpha = T_{\alpha,0} + \delta T_\alpha \,. \tag{5.39}$$

We can expand the coefficients V_i in Eq. (5.38) at leading order in δT_α as long as $|\delta T_\alpha| \ll |T_{\alpha,0}|$. Minimizing the result with respect to the displacement, and inserting back into the potential gives the effective potential for φ in its most general form,

$$\begin{aligned} V = &\frac{1}{2} V_1\left(T_{\alpha,0}, \overline{T}_{\bar\alpha,0}\right) m\varphi^2 + \frac{1}{4} V_2\left(T_{\alpha,0}, \overline{T}_{\bar\alpha,0}\right) m^2\varphi^4 \\ &- \frac{1}{2}\begin{pmatrix} \frac{\partial V_1}{\partial T_\alpha} & \frac{\partial V_1}{\partial \overline{T}_{\bar\alpha}} \end{pmatrix} \begin{pmatrix} (m^{-2})^{\alpha\bar\beta} & (m^{-2})^{\alpha\beta} \\ (m^{-2})^{\bar\alpha\bar\beta} & (m^{-2})^{\bar\alpha\beta} \end{pmatrix} \begin{pmatrix} \frac{\partial V_1}{\partial \overline{T}_{\bar\beta}} \\ \frac{\partial V_1}{\partial T_\beta} \end{pmatrix} m^2\varphi^4 + \ldots \,. \end{aligned} \tag{5.40}$$

The supergravity masses $m^2_{\alpha\beta}$ and $m^2_{\alpha\bar\beta}$ are defined in Appendix A.2. A useful observation made in [39] is that this rather unwieldy expression simplifies significantly when supersymmetry is only weakly broken, cf. the more detailed discussion in Appendix A.3. This is the case when the supersymmetric mass, i.e., the mass of the fermions associated with the scalars T_α, is much larger than the gravitino mass.[6] Specifically, when

$$\text{Eigenvalues}\left[(m_F)_{\alpha\beta}\right] = \text{Eigenvalues}\left[e^{G/2}\left(\nabla_\alpha G_\beta + \frac{1}{3} G_\alpha G_\beta\right)\right] \gg m_{3/2} \,. \tag{5.41}$$

Alternatively, one may consider the case where the supersymmetry breaking scale is large but the supersymmetry-breaking sector decouples from moduli stabilization. An example for this is supersymmetry breaking by the F-term of a very heavy Polonyi field, as discussed in the previous chapters. For both of these possibilities the effective inflaton potential becomes

$$\begin{aligned} V \approx &\frac{m\varphi^2}{2} e^{K_0}\left\{-\frac{1}{2} K_0^{\alpha\bar\beta}\left(K_{0,\bar\beta} D_\alpha W_{\text{mod}} + K_{0,\alpha}\overline{D}_{\bar\beta}\overline{W}_{\text{mod}}\right) + mK_1^{-1} + \frac{3}{2}(W_{\text{mod}} + \overline{W}_{\text{mod}})\right\} \\ &+ \frac{m^2\varphi^4}{16} e^{K_0}\left\{ -3 + e^{K_0/2}\left[K_\delta \left(m_F^{-1}\right)^{\beta\delta}\left[- K_0^{\epsilon\bar\epsilon}(K_{\beta\epsilon} + K_\beta K_\epsilon - \Gamma^\gamma_{\beta\epsilon} K_\gamma)\overline{D}_{\bar\epsilon}\overline{W}_{\text{mod}} \right.\right.\right. \\ &\left.\left.\left. + 2D_\beta W_{\text{mod}} + 3K_\beta \overline{W}_{\text{mod}} + 2mK_1^{-2}\left(K_{0,\beta}K_1 - K_{1,\beta}\right)\right] + \text{h.c.}\right]\right\}, \end{aligned} \tag{5.42}$$

which is the desired generalization of Eq. (4.9). Notice, however, that the quadratic term in the first line is independent of the small-supersymmetry breaking approximation. It

[6] With the exception of the goldstino, of course.

is simply the total mass – supersymmetric and soft mass – of the inflaton in the true vacuum, computed from the effective action defined by Eqs. (5.33). Indeed, using the definition of the supergravity scalar masses in Eqs. (A.16) we find that the inflaton mass is

$$m_\varphi^2 = m_{\phi\bar{\phi}}^2 - \frac{1}{2}\left(m_{\phi\phi}^2 + m_{\bar{\phi}\bar{\phi}}^2\right) . \tag{5.43}$$

It can be shown that Eq. (5.43) indeed equals the mass term in the first line of Eq. (5.42).

Using this result we can, in principle, calculate the effective potential with corrections for any model of moduli stabilization described by the ansatz (5.33). In practice, however, the approximation outlined above to obtain Eq. (5.42) – more precisely, the quartic term, as explained above – is not always applicable. In that case, either a more general expression for the effective potential can be used, given by Eq. (5.40), or the calculation can be significantly simplified by expanding in small parameters while performing the above analysis. In the following we demonstrate this in three popular examples of moduli stabilization with spontaneously broken supersymmetry.

5.3 Chaotic inflation and KKLT stabilization

We have discussed KKLT moduli stabilization in some detail in Section 3.2. The results obtained there allow us to start this discussion with the interaction between KKLT stabilization and chaotic inflation without a stabilizer field.

5.3.1 Effective inflaton potential: Analytic approach

Treating the interaction between the modulus and inflaton sectors in the simplest way, we assume that their superpotentials and Kähler potentials completely decouple. Thus, the theory is defined by

$$W = W_0 + Ae^{-aT} + fX + \frac{1}{2}m\phi^2 , \tag{5.44a}$$

$$K = -3\ln\left(T + \overline{T}\right) + k\big(|X|^2\big) + \frac{1}{2}\left(\phi + \bar{\phi}\right)^2 . \tag{5.44b}$$

In particular, in the notation of Section 5.2 we choose

$$W_{\text{mod}}(T_\alpha) = W_0 + Ae^{-aT} + fX , \tag{5.45}$$

$$K_0(T_\alpha, \overline{T}_{\bar{\alpha}}) = -3\ln\left(T + \overline{T}\right) + k\big(|X|^2\big) \tag{5.46}$$

$$K_1(T_\alpha, \overline{T}_{\bar{\alpha}}) = 1 . \tag{5.47}$$

Note that the relative phase between W_0 and m is physical. In the following we choose all superpotential parameters to be real, so that only the real part of T is affected by inflation. Therefore, we set $T = \overline{T}$ in the following discussion. Our results do not change qualitatively if we allow for m and/or W_0 to be complex. Notice that we uplift the KKLT AdS vacuum with a Polonyi field X in the way discussed in Sec. 3.2.2. Hence, we assume that the function k stabilizes X close to the origin with a large mass. On the inflationary trajectory the superpotential reads

$$W = W_0 + Ae^{-aT} - \frac{1}{4} m\varphi^2 \,. \tag{5.48}$$

A natural question to ask is the following: can the effective theory of inflation defined by Eqs. (5.44) resemble chaotic inflation, after integrating out T at a high scale?

To answer this question we must again find the displacement of T during inflation by expanding

$$V = V|_{T_0} + (\partial_T V)|_{T_0} \delta T + \frac{1}{2}(\partial_T^2 V)|_{T_0} \delta T^2 + \mathcal{O}(\delta T^3) \,, \tag{5.49}$$

along the lines of the general analysis in Section 5.2, and minimizing subsequently. The result reads, at leading order,

$$\frac{\delta T}{T_0} = \frac{\tilde{m}\varphi^2}{4aT_0 m_{3/2}} + \mathcal{O}(T_0^{-2}) \,, \tag{5.50}$$

with $\tilde{m} = m/(2T_0)^{3/2}$ and $m_{3/2}$ given by Eq. (3.22). Alternatively, using Eq. (3.23) we can write δT in terms of the modulus mass as

$$\frac{\delta T}{T_0} = \frac{\tilde{m}\varphi^2}{2m_T} + \mathcal{O}(T_0^{-2}) \,. \tag{5.51}$$

With this, the effective inflaton potential including the leading-order correction becomes, at quartic order in φ and leading order in $(aT_0)^{-1}$ and $\tilde{m}/m_{3/2}$,

$$V(\varphi) = \frac{1}{2}\tilde{m}^2\varphi^2 + \frac{3}{2}\tilde{m}m_{3/2}\varphi^2 - \frac{3}{16}\tilde{m}^2\varphi^4 - \frac{3}{4aT_0}\left(3\tilde{m}m_{3/2}\varphi^2 + \frac{3}{4}\tilde{m}^2\varphi^4\right) + \dots \,. \tag{5.52}$$

To obtain higher-order corrections to the potential, the potential must be expanded at higher orders in δT, and δT must be computed up to higher powers in T_0^{-1}. Thus, it seems that after integrating out T the negative definite term proportional to $\tilde{m}^2\varphi^4$ still appears in the potential, making it unbounded from below. This is related to the fact that the modulus is only a sub-leading source of supersymmetry breaking.[7]

[7] Notice that this way of obtaining the leading-order potential, i.e., the first three pieces in Eq. (5.52), is equivalent to the naive treatment outlined in Sec. 5.2, which resulted in Eq. (5.36).

However, things are not quite as they seem by merely studying the result in Eq. (5.52). For large values of φ, i.e., when the quartic term in the effective potential dominates, the modulus can be destabilized by the potential energy of φ. In this case, the inflationary trajectory becomes tachyonic and the modulus can no longer be integrated out. This is to be expected when

$$\tilde{m}\varphi^2 \gtrsim 4m_{3/2}^2\,, \tag{5.53}$$

which corresponds to the bound $m_{3/2} > H$ discussed previously. This will become more clear in our numerical example. There, a more detailed analysis reveals that, actually, the local maximum of the effective inflaton potential Eq. (5.52) is never reached while the modulus is stabilized.[8] Hence, the negative quartic term is no obstruction to inflation, which may be driven by the soft mass term as long as T remains stabilized.

One more remark is in order before we proceed to our numerical example. In a manner similar to integrating out T, it is possible to verify that the displacement δX of the Polonyi field during inflation gives negligible contributions to the inflaton potential. For the particular choice

$$k\left(|X|^2\right) = |X|^2 - \frac{|X|^4}{\Lambda^2}\,, \tag{5.54}$$

for example, the displacement of X is at leading order

$$\delta X = \Lambda^2 \delta T\,. \tag{5.55}$$

Since $\Lambda \ll 1$ to stabilize X at a high scale with a small vacuum expectation value, the contribution of integrating out X at Eq. (5.55) is clearly negligible. Among other things, this means that the sector which dominates supersymmetry breaking can be completely decoupled from the dynamics of inflation. In this case, it is possible to obtain the effective potential Eq. (5.52) essentially by applying the approximated general expression Eq. (5.42).

5.3.2 A numerical example

Let us now study whether 60 e-folds of slow-roll inflation can be realized with the effective inflaton potential Eq. (5.52), and whether the resulting predictions for the CMB observables resemble those of chaotic inflation. It is worth noting that in the parameter regime where T is stabilized, i.e., when $m_{3/2}$ is very large, the bilinear term

[8] In fact, the full potential defined by Eqs. (5.44) is bounded from below at all points in field space.

proportional to $\tilde{m}m_{3/2}$ actually dominates in V and drives inflation. In this case, the relevant terms in the inflaton potential are

$$V(\varphi) \approx \frac{3}{2}\tilde{m}m_{3/2}\varphi^2 \left(1 - \frac{1}{8}\frac{\tilde{m}}{m_{3/2}}\varphi^2\right) . \tag{5.56}$$

Consequently, inflation is only possible if $\tilde{m}$ and $m_{3/2}$ have the same sign. With Eq. (3.24) the corrections can be interpreted as a power series in H^2/V_{B}, the squared Hubble scale divided by the barrier height of the modulus potential. This is a natural expansion parameter because the modulus is destabilized when the vacuum energy of φ lifts the modulus over the barrier, cf. (5.53). Neglecting order-one coefficients, COBE normalization imposes $\sqrt{|\tilde{m}m_{3/2}|} \sim 3 \times 10^{-6}$. This puts a lower bound on the gravitino mass, i.e.,

$$m_{3/2} > \sqrt{|\tilde{m}m_{3/2}|}\,\varphi_\star \sim 5 \times 10^{-5} \sim H\,, \tag{5.57}$$

where $\varphi_\star \approx 15$ denotes the inflaton field value at the beginning of the last 60 e-folds of inflation. This means that the gravitino must be very heavy and there must be a moderate hierarchy between the gravitino and inflaton mass for 60 e-folds of chaotic inflation to be possible. This is illustrated in Fig. 5.3 for a suitable set of parameters.

Indeed, 60 e-folds of inflation can take place starting at $\varphi_\star \approx 15$. The CMB observables in our example are

$$\begin{aligned} n_{\mathrm{s}} &= 0.966\,, \\ r &= 0.106\,, \end{aligned} \tag{5.58}$$

which are slightly below the predictions of pure quadratic inflation. This is due to the flattening of the quadratic potential by the negative quartic term. Notice that the modulus is destabilized and the inflaton trajectory becomes tachyonic at the critical value $\varphi_{\mathrm{c}} \approx 24$, corresponding to the bound in (5.53). Therefore, Eq. (5.56) and the dashed line in Fig. 5.3 are only meaningful up to this point.

Moreover, the interplay between inflaton and modulus can be illustrated by means of the full scalar potential as a function of T and φ, depicted in Fig. 5.4. The minimum in the modulus direction is uplifted as φ increases, until the point where it disappears at $\varphi_{\mathrm{c}} \approx 24$.

5.4 Chaotic inflation and Kähler Uplifting

Another instructive example for moduli stabilization with broken supersymmetry is Kähler Uplifting, as discussed in Section 3.3.1. We can analyze its interplay with chaotic inflation in a similar way as for KKLT stabilization.

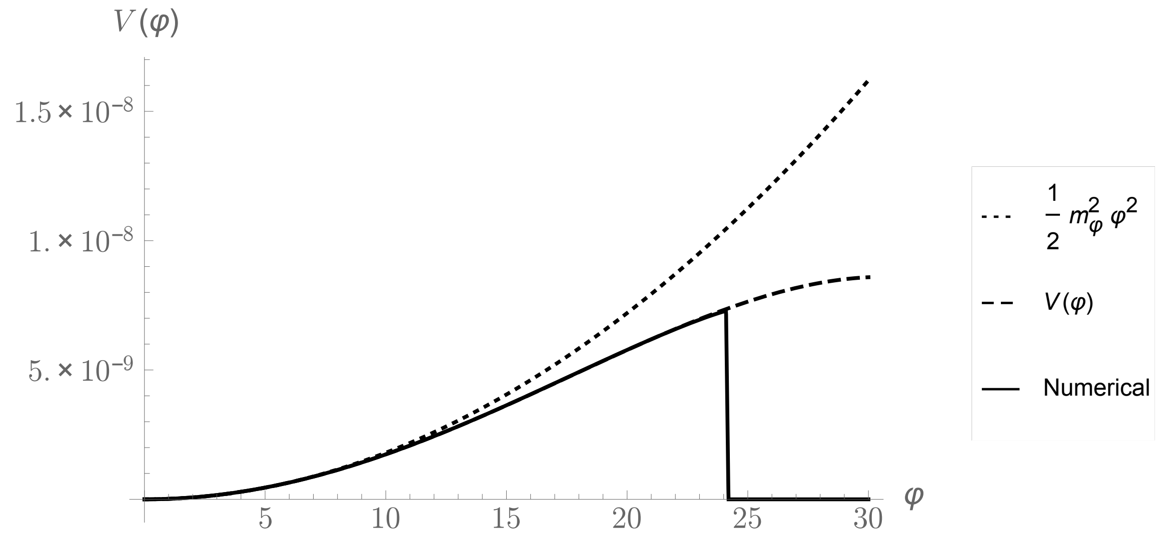

Figure 5.3: *Effective inflaton potential in KKLT for* $W_0 = 0.009$, $A = -0.75$, $a = \frac{2\pi}{10}$, *and* $m = 1.67 \times 10^{-5}$. *With these parameters we find* $T_0 = 10$ *and* $m_{3/2} = 10^{-4}$. *The dotted line denotes a purely quadratic potential with* $m_\varphi = 6 \times 10^{-6}$ *imposed by COBE normalization. The dashed line is the effective potential Eq.* (5.52) *evaluated at all orders in* $(aT_0)^{-1}$. *This potential is valid only as long as the modulus remains stabilized. The solid line is obtained numerically by setting the modulus to its minimum value at each value of* φ. *Evidently, above the critical value* $\varphi_c \approx 24$ *the modulus is destabilized towards the run-away minimum at* $T = \infty$ *and the theory can no longer be described by Eq.* (5.52).

5.4.1 Effective inflaton potential: Analytic approach

As before, to simplify the discussion we assume that the interactions between modulus and inflaton sector are purely gravitational. Hence, we study the theory defined by

$$W = W_0 + Ae^{-aT} + \frac{1}{2}m\phi^2\,, \tag{5.59a}$$

$$K = -2\ln\left[\left(T + \overline{T}\right)^{3/2} + \xi\right] + \frac{1}{2}\left(\phi + \bar{\phi}\right)^2. \tag{5.59b}$$

Again, since we choose real superpotential parameters only the real part of T is affected by inflation. Hence, we set $T = \overline{T}$ in the scalar potential. Since the modulus F-term in this case is bigger than in KKLT, at leading order it cancels the negative contribution to the inflaton potential. At leading order in δT, η_0 and T_0^{-1} it is simply

$$V = \frac{1}{2}\tilde{m}^2\varphi^2 + \mathcal{O}(\delta T, \eta_0, T_0^{-1})\,. \tag{5.60}$$

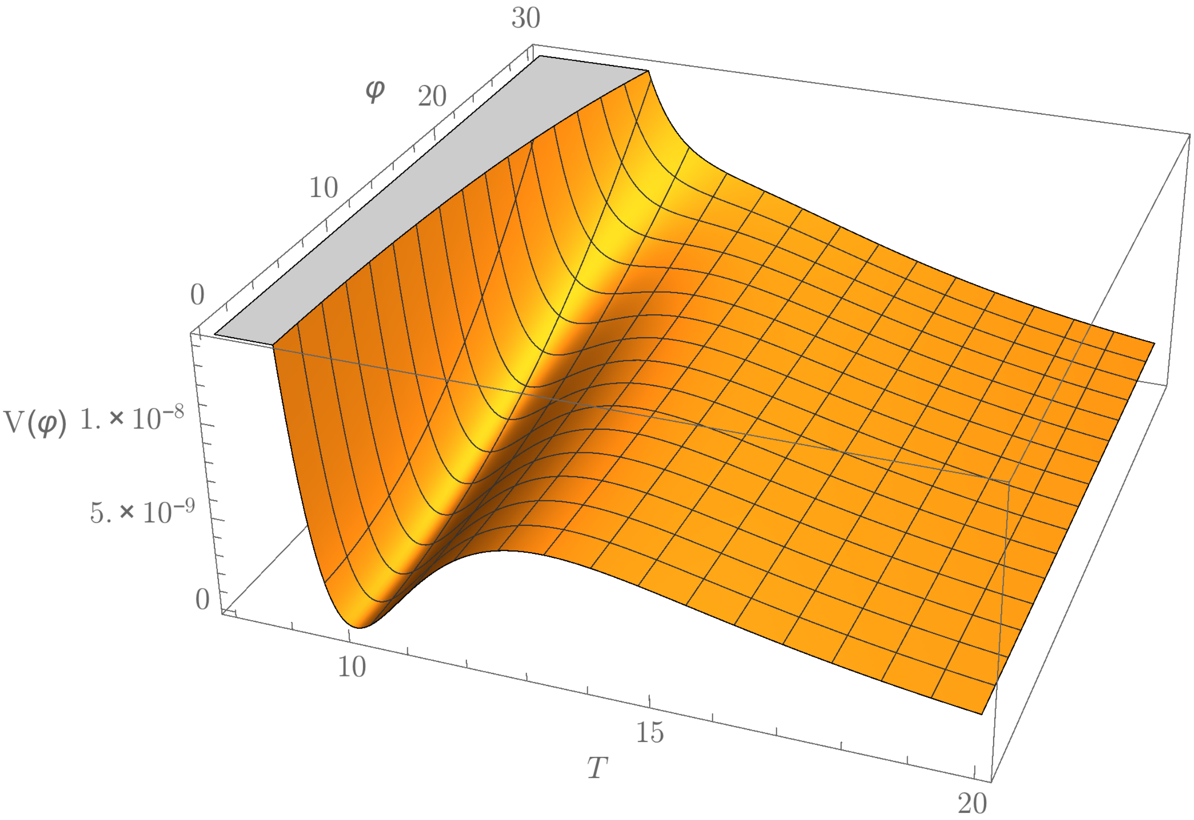

Figure 5.4: *Scalar potential as defined by Eqs. (5.44) as a function of T and φ, for the same parameter example as in Fig. 5.3. Apparently, a minimum for the modulus exists for $\varphi \lesssim \varphi_c \approx 24$. Beyond this point the modulus runs away towards $T = \infty$ and can no longer be integrated out. For $\varphi < \varphi_c$ inflation may take place in the valley of the uplifted modulus minimum.*

The displacement of T during inflation contains two pieces,

$$\frac{\delta T}{T_0} = \frac{\tilde{m}^2\varphi^2}{5\eta_0 m_{3/2}^2} - \frac{9\tilde{m}\varphi^2}{20 m_{3/2}} + \dots\,, \tag{5.61}$$

where the dots denote higher-order terms in η_0 and T_0^{-1}. Consequently, there are two upper bounds on the value of the inflaton field to guarantee stability of the modulus potential. Specifically,

$$\begin{aligned} \tilde{m}\varphi^2 &\lesssim 4 m_{3/2}\,, \\ \tilde{m}^2\varphi^2 &\lesssim \eta_0 m_{3/2}^2\,. \end{aligned} \tag{5.62}$$

Clearly, if these conditions are fulfilled the expansion in δT converges. Expanding the inflaton potential in $\delta T/T_0$ and η_0, we find at leading order

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2 - \frac{3\eta_0}{4}\tilde{m} m_{3/2}\varphi^2 - \frac{3}{20\eta_0}\frac{\tilde{m}^4\varphi^4}{m_{3/2}^2} + \frac{27}{40}\frac{\tilde{m}^3\varphi^4}{m_{3/2}} - \frac{183\eta_0}{320}\tilde{m}^2\varphi^4 + \dots\,, \tag{5.63}$$

which contains negative quartic terms in the inflaton field, analogous to the KKLT case. This time, however, they are suppressed by factors of $\delta T/T_0$ or η_0 due to the approximate no-scale symmetry of Kähler Uplifting.[9]

As in the previous section we may rewrite Eq. (5.61) in terms of m_T. We find

$$\frac{\delta T}{T_0} = \frac{4\tilde{m}^2\varphi^2 - 9\eta_0\tilde{m}m_{3/2}\varphi^2}{4m_T^2} + \dots\,. \tag{5.64}$$

From this the situation is quite clear: the first term in the numerator is the leading-order inflaton uplift of the potential and the second terms arises due to the incomplete no-scale cancellation at the shifted modulus vacuum expectation value,

$$\delta V \propto K^{T\overline{T}}|D_T W|^2 - 3|W|^2 \sim \eta_0|W|^2\,. \tag{5.65}$$

In the following we study the phenomenology of inflation resulting from this effective potential in two numerical examples. To this end, it is instructive to rewrite the effective potential as

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2\left(1 - \frac{3}{10\eta_0}\frac{\tilde{m}^2}{m_{3/2}^2}\varphi^2\right) - \frac{3\eta_0}{4}\tilde{m}m_{3/2}\varphi^2\left(1 + \frac{61}{80}\frac{\tilde{m}}{m_{3/2}}\varphi^2\right)\,. \tag{5.66}$$

At leading order $V(\varphi)$ consists of two quadratic terms and one relevant correction to each, suppressed by one power of H^2/V_{B}. The second piece in Eq. (5.66) is very similar to the leading-order potential found in the KKLT case, but is suppressed by one power of η_0. This means that the supersymmetric mass term for φ can drive inflation as well. Before discussing inflation in more detail, let us remark that to guarantee stability of T we require $H^2 < V_{\mathrm{B}}$. Using Eq. (3.31) this leads to a generic bound on the gravitino mass,

$$m_{3/2} > \frac{H}{\sqrt{\eta_0}} \sim \frac{10^{-4}}{\sqrt{\eta_0}}\,. \tag{5.67}$$

5.4.2 Numerical examples

Starting from the effective potential Eq. (5.66) we can distinguish two cases. Inflation can either be driven by the supersymmetric term proportional to $\tilde{m}^2\varphi^2$, or by the bilinear soft term proportional to $\tilde{m}m_{3/2}\varphi^2$.

[9] The procedure to find the effective potential is significantly simplified by expanding all quantities in powers of η_0. Since, in this case, T is the only field which contributes to supersymmetry breaking in the vacuum and $m_{3/2}$ is generically very large, the general formula Eq. (5.42) does not apply. However, it is possible to obtain Eq. (5.63) by applying the most general result Eq. (5.40), which does not contain assumptions about the scale of supersymmetry breaking.

The supersymmetric mass term dominates
If $\eta_0 m_{3/2} \ll \tilde{m}$ chaotic inflation may be realized in the "traditional" sense. The leading-order potential in this parameter regime is simply the first piece of Eq. (5.66), i.e.,

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2 \left(1 - \frac{3}{10\eta_0}\frac{\tilde{m}^2}{m_{3/2}^2}\varphi^2\right) . \tag{5.68}$$

The viable parameter regime in this scenario is particularly constrained. On the one hand, $\eta_0 m_{3/2}$ must be small for the soft term to be suppressed. On the other hand, $\eta_0 m_{3/2}^2$ must be large enough to guarantee a high barrier in the modulus potential. Specifically, we find

$$m_{3/2} \gg \frac{\tilde{m}^2\varphi_\star^2}{\eta_0 m_{3/2}} \gg \tilde{m}\varphi_\star^2 \gtrsim 10H \sim 10^{-3} . \tag{5.69}$$

A suitable example is illustrated in Fig. 5.5. As expected, the parameter choices are quite elaborate, especially from the perspective of string theory. Specifically, the hierarchy between W_0 and A as well as the size of η_0 are rather particular. With such a small value of ξ it is doubtful whether the string coupling can be small enough to allow for a perturbative description of the theory.

If one ignores this problem inflation can be realized and we find for the solid line

$$\begin{aligned} n_{\mathrm{s}} &= 0.966 , \\ r &= 0.116 , \end{aligned} \tag{5.70}$$

for $\varphi_\star \approx 15.2$. The modulus is destabilized at $\varphi_{\mathrm{c}} \approx 19$.

The bilinear soft term dominates
The scenario $\eta_0 m_{3/2} \gg \tilde{m}$ seems slightly more appealing since it can be realized with more realistic choices for the input parameters. The leading-order potential becomes

$$V(\varphi) \approx -\frac{3\eta_0}{4}\tilde{m} m_{3/2}\varphi^2 \left(1 + \frac{61}{80}\frac{\tilde{m}}{m_{3/2}}\varphi^2\right) . \tag{5.71}$$

Notice the sign difference of the soft term compared to KKLT. Since $\eta_0 > 0$ this means that $\tilde{m}$ and $m_{3/2}$ must have opposite signs for inflation to work in this parameter regime. COBE normalization imposes $\sqrt{|\eta_0 \tilde{m} m_{3/2}|} \sim 5 \times 10^{-6}$. Since η_0 is allowed to be larger in this case, the only bound on $m_{3/2}$ is the generic one, (5.67). An example is depicted in Fig. 5.6.

The corresponding CMB observables are

$$\begin{aligned} n_{\mathrm{s}} &= 0.965 , \\ r &= 0.107 , \end{aligned} \tag{5.72}$$

at $\varphi_\star \approx 15$. In this case the modulus is destabilized at $\varphi_{\mathrm{c}} \approx 20$.

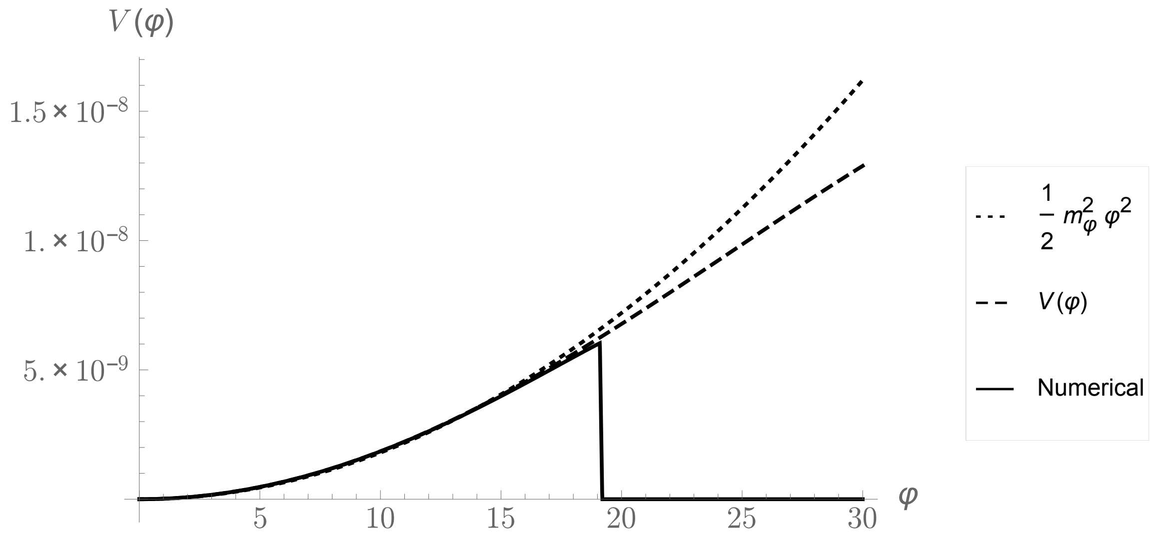

Figure 5.5: *Effective inflaton potential in Kähler Uplifting for* $W_0 = 4.67$, $A = -3.4 \times 10^{-4}$, $a = \frac{2\pi}{30}$, $m = 8 \times 10^{-4}$, *and* $\xi = 0.0047$. *With these parameters we find* $T_0 = 11.9$, $m_{3/2} = 0.04$, *and* $\eta_0 = 2 \times 10^{-5}$. *The dotted line denotes a purely quadratic potential with* $m_\varphi = 6 \times 10^{-6}$ *imposed by COBE normalization. The dashed line is the effective potential Eq.* (5.63) *evaluated at all orders in* η_0. *The solid line is obtained numerically by setting the modulus to its minimum value at each value of* φ. *In this case, modulus destabilization occurs at* $\varphi_c \approx 19$. *Again, Eq.* (5.63) *and the dashed line are only meaningful for* $\varphi < \varphi_c$.

5.5 Chaotic inflation and the Large Volume Scenario

As our last example we present moduli stabilization via the simplest Large Volume Scenario, as discussed in Section 3.3.2. Although the structure of the vacuum is more complicated than in the last two examples, the results after coupling to chaotic inflation are qualitatively similar.

5.5.1 Effective inflaton potential: Analytic approach

Our starting point for the coupled model is this time

$$W = W_0 + Ae^{-aT_{\mathrm{s}}} + fX + \frac{1}{2}m\phi^2\,, \tag{5.73}$$

$$K = -2\ln\left[\left(T_{\mathrm{b}} + \overline{T}_{\mathrm{b}}\right)^{3/2} - \left(T_{\mathrm{s}} + \overline{T}_{\mathrm{s}}\right)^{3/2} + \xi\right] + k\left(|X|^2\right) + \frac{1}{2}\left(\phi + \bar{\phi}\right)^2. \tag{5.74}$$

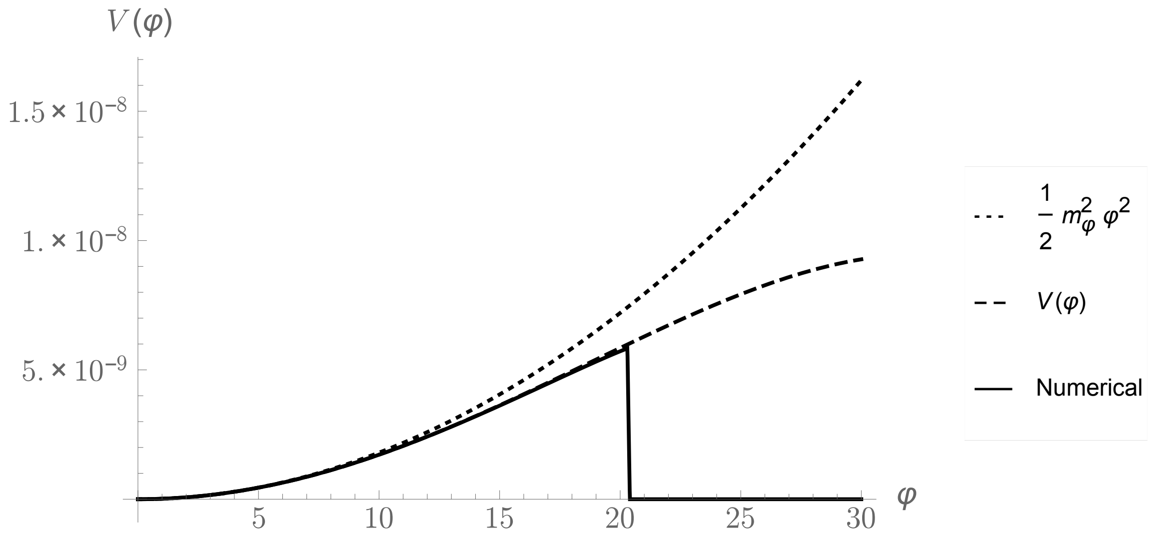

Figure 5.6: *Effective inflaton potential in Kähler Uplifting for* $W_0 = 0.23$, $A = -0.008$, $a = \frac{2\pi}{30}$, $m = -1.37 \times 10^{-4}$, *and* $\xi = 2.29$. *With these parameters we find* $T_0 = 11.8$, $m_{3/2} = 0.002$, *and* $\eta_0 = 0.01$. *The dotted line denotes a purely quadratic potential with* $m_\varphi = 6 \times 10^{-6}$ *imposed by COBE normalization. The dashed line is the effective potential Eq.* (5.63) *evaluated at all orders in* η_0. *The solid line is obtained numerically by setting the modulus to its minimum value at each value of* φ. *In this setup, modulus destabilization occurs at* $\varphi_c \approx 20$. *Again, Eq.* (5.63) *and the dashed line are only meaningful for* $\varphi < \varphi_c$.

The uplift sector is treated as described above, since it is safe to neglect its influence on inflation. The scalar potential at leading order in $\mathcal{V}^{-1}$ reads

$$\begin{aligned} V = & \frac{2\sqrt{2}\, a^2 A^2 \sqrt{T_\mathrm{s}}\, e^{-2aT_\mathrm{s}}}{3\mathcal{V}} - \frac{16 a A T_\mathrm{s}\, e^{-aT_\mathrm{s}}\, (4W_0 - m\varphi^2)}{\mathcal{V}^2} \\ & + \frac{3\xi\, (4W_0 - m\varphi^2)^2}{32\mathcal{V}^3} + \frac{(\mathcal{V} - 2\xi)\left(f^2 + \frac{1}{2}m^2\varphi^2\right)}{\mathcal{V}^3} \,. \end{aligned} \tag{5.75}$$

Comparing this expression to Eq. (3.34) we observe that, in principle, the contribution of the inflaton can be absorbed in a redefinition of W_0 and f. As before, we treat inflation as a perturbation of the true vacuum. Hence, we naively expect chaotic inflation to be successful in LVS as long as

$$m^2\varphi^2 \ll f^2 \,, \qquad m\varphi^2 \ll W_0 \,, \tag{5.76}$$

neglecting order-one coefficients. It will become clear in the following that these two conditions precisely guarantee that the inflaton energy density does not destabilize the moduli.

To compute the effective inflaton potential we have to take the back-reaction of both moduli into account. Hence, we expand the potential around

$$\delta\mathcal{V} = \mathcal{V} - \mathcal{V}_0\,, \qquad \delta T_\mathrm{s} = T_\mathrm{s} - T_0\,. \tag{5.77}$$

Minimizing the result with respect to both displacements yields

$$\frac{\delta\mathcal{V}}{\mathcal{V}_0} \approx \frac{\tilde{m}^2\varphi^2}{\chi_0 m_{3/2}^2} + \frac{\tilde{m}\varphi^2}{4m_{3/2}}\,, \tag{5.78a}$$

$$\frac{\delta T_\mathrm{s}}{T_0} \approx \frac{\tilde{m}^2\varphi^2}{aT_0\chi_0 m_{3/2}^2} + \frac{\tilde{m}\varphi^2}{2aT_0 m_{3/2}}\,, \tag{5.78b}$$

up to terms suppressed by higher powers of $\mathcal{V}^{-1}$ or $(aT_0)^{-1}$. Note that the shifts have the same form as in Kähler Uplifting, cf. Eq. (5.61). Furthermore, the displacement of T_s is relatively suppressed by one power of $\mathcal{V}_0$. This is to be expected because T_s is the heavier of the two moduli. Nonetheless, δT_s must be taken into account to find the correct leading-order result.

Integrating out the displacements of both moduli, we are left with the leading-order effective potential

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2 + \frac{\chi_0}{4}\tilde{m}m_{3/2}\varphi^2 - \frac{1}{2\chi_0}\frac{\tilde{m}^4\varphi^4}{m_{3/2}^2} - \frac{1}{4}\frac{\tilde{m}^3\varphi^4}{m_{3/2}} - \frac{\chi_0}{16aT_0}\tilde{m}^2\varphi^4\,. \tag{5.79}$$

We refrain from rewriting this unwieldy expression in terms of the moduli masses, but the idea is the same as in our previous examples. Some of the correction terms are suppressed by inverse powers of m_{T_b} and m_{T_s} and vanish in the limit of very heavy moduli. Others, like the supersymmetry-breaking second term in Eq. (5.79) grow with the moduli masses, and hence do not vanish. As in the previous examples, the region where $V(\varphi)$ is unbounded from below is never reached since the moduli are destabilized at smaller values of φ.

As in our model with Kähler Uplifting we rewrite the effective potential to study inflation. In particular,

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2\left(1 - \frac{1}{\chi_0}\frac{\tilde{m}^2}{m_{3/2}^2}\varphi^2\right) + \frac{\chi_0}{4}\tilde{m}m_{3/2}\varphi^2\left(1 - \frac{1}{4aT_0}\frac{\tilde{m}}{m_{3/2}}\varphi^2\right)\,. \tag{5.80}$$

Again, $V(\varphi)$ contains a supersymmetric mass term and a bilinear soft term – suppressed by one power of χ_0 –, both with a correction proportional to H^2/V_B. By requiring the barrier to be larger than the Hubble scale during inflation, the gravitino mass is generically constrained as follows,

$$m_{3/2} > H\sqrt{\mathcal{V}_0} \sim 10^{-4}\sqrt{\mathcal{V}_0}\,. \tag{5.81}$$

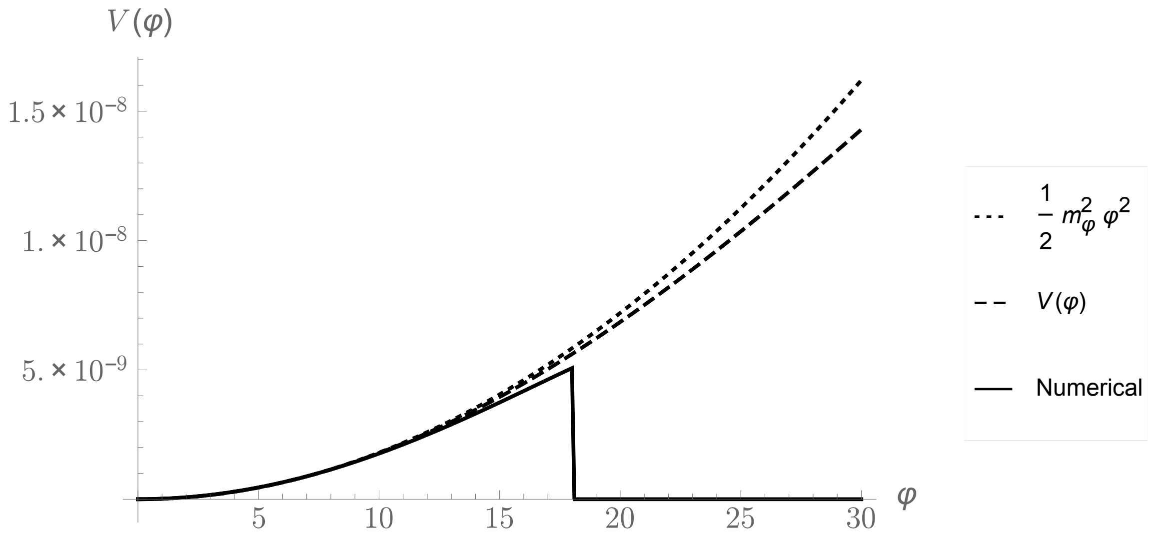

Figure 5.7: *Effective inflaton potential in LVS for $W_0 = 1$, $A = 0.13$, $a = 2\pi$, $m = 5.8 \times 10^{-4}$, and $\xi = 1.25$. With these parameters we find $T_0 = 0.75$, $\mathcal{V}_0 = 200$, and $m_{3/2} = 0.005$. The dotted line denotes a purely quadratic potential with $m_\varphi = 6 \times 10^{-6}$ imposed by COBE normalization. The dashed line is the effective potential Eq. (5.79) evaluated at all orders in aT_0. The solid line is obtained numerically by setting the modulus to its minimum value at each value of φ. Since the barrier height and Hubble scale are the same as in the previous example, modulus destabilization occurs at $\varphi_c \approx 18$. Again, Eq. (5.79) and the dashed line are only meaningful for $\varphi < \varphi_c$. Notice that the difference between the dashed and the solid line is comparably large in this example. This is because the relatively small value of $\mathcal{V}_0$ limits the precision of the expansion in $\mathcal{V}^{-1}$.*

As before, this constraint is equivalent to demanding that φ is not large enough to uplift the modulus minimum to a saddle point or beyond.

5.5.2 Numerical examples

Based on the effective potential Eq. (5.80) we can distinguish two cases in which 60 e-folds of inflation may be realized.

The supersymmetric mass term dominates

If $\tilde{m} \gg \chi_0 m_{3/2} \sim m_{3/2}/\mathcal{V}_0$, in principle the supersymmetric quadratic term in Eq. (5.80) could dominate, yielding the leading-order potential

$$V(\varphi) \approx \frac{1}{2}\tilde{m}^2\varphi^2 \left(1 - \frac{1}{\chi_0}\frac{\tilde{m}^2}{m_{3/2}^2}\varphi^2\right) . \tag{5.82}$$

However, this scenario is excluded by a consistency requirement of the LVS scheme. Specifically, the gravitino mass must not exceed the Kaluza-Klein scale which, as discussed in [160], means that $W_0 \ll \mathcal{V}_0^{1/3}$. Requiring the supersymmetric term to be larger than the soft term while both moduli are stabilized always violates this bound.

The bilinear soft term dominates

If, on the other hand, $\tilde{m} \ll \chi_0 m_{3/2} \sim m_{3/2}/\mathcal{V}_0$, the term proportional to $\tilde{m} m_{3/2}$ may drive inflation. In this case, the leading-order inflaton potential reads

$$V(\varphi) \approx \frac{\chi_0}{4} \tilde{m} m_{3/2} \varphi^2 \left(1 - \frac{1}{4aT_0} \frac{\tilde{m}}{m_{3/2}} \varphi^2 \right) . \tag{5.83}$$

The gravitino mass is constrained by the generic requirement (5.81). Interestingly, by requiring $m_{3/2} < M_{\mathrm{KK}}$ for consistency, the volume of the compactification manifold is bounded from above,

$$\mathcal{V}_0 \lesssim 10^3 . \tag{5.84}$$

A numerical example for this scenario is depicted in Fig. 5.7. The CMB observables in this case are

$$\begin{aligned} n_{\mathrm{s}} &= 0.964 , \\ r &= 0.116 , \end{aligned} \tag{5.85}$$

at $\varphi_\star \approx 15.2$. Modulus destabilization towards the run-away minimum occurs at $\varphi_{\mathrm{c}} \approx 18$.

5.6 Universality and CMB observables

We can make a number of intriguing observations regarding all effective potentials found in our three examples. We observe that a simple expression captures all models and their flattening of the inflaton potential by moduli back-reaction,

$$V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 , \qquad \lambda > 0 . \tag{5.86}$$

This expression is valid at leading order in the modulus shift, and thus holds for a certain range $\varphi < \varphi_{\mathrm{c}}$ until the moduli are destabilized.

Due to the negative quartic term the potential has a local maximum at $\varphi_{\mathrm{M}} = m/\sqrt{\lambda}$. All three scenarios share the property that the moduli destabilization point occurs to the left of the maximum of the leading-order inflaton potential,

$$\varphi_{\mathrm{c}} < \varphi_{\mathrm{M}} . \tag{5.87}$$

Hence, $V(\varphi)$ is a good approximation for $\varphi < \varphi_c$. Two parameters determine the effective potential, $m/\sqrt{\lambda}$ gives the position of the maximum and m fixes the overall normalization of $V(\varphi)$. Thus, we can write the potential in terms of m and $\varphi_{\rm M}$,

$$V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2 \left(1 - \frac{\varphi^2}{2\varphi_{\rm M}^2}\right) . \tag{5.88}$$

As long as $\varphi_{\rm M}, \varphi_{\rm c} \gg 1$ inflation can occur to the left of the local maximum. For $\varphi_{\rm M} \to \infty$ the potential asymptotes to the pure quadratic form. In this limit, the field value $\varphi_\star$ corresponding to $N_e(\varphi_\star)$ e-folds of slow-roll before the end of inflation takes the limiting value $\varphi_\star = 2\sqrt{N_e}$, which for $N_e = 50 - 60$ is about 15.

For smaller $\varphi_{\rm M}$ the 60 e-fold point lies increasingly close to the local maximum and the destabilization point. Thus, for $\varphi_{\rm c} \to \varphi_\star$ the inflationary dynamics changes continuously from the quadratic large-field behaviour to a nearly hill-top small-field model. Correspondingly, the scalar spectral index and r are decreased compared to pure quadratic inflation. For more details and an illuminating parameter plot we refer to the original publication [39].

Inflaton potentials of this type arise in the context of non-minimally coupled quadratic inflation [161] and more recently in subcritical models of D-term hybrid inflation [38, 80, 81]. As the leading-order scalar potential is the same in all our models, the CMB observables agree as well. A detailed account is given in [80,81]. There it was found that imposing the most recent observational constraints on $n_{\rm s}$ and r leads to a lower bound on the tensor-to-scalar ratio,

$$r \gtrsim 0.05 , \tag{5.89}$$

for 60 e-folds of inflation. For the reasons given above, this bound applies to all of our example models. It is particularly interesting in the light of the most recent CMB data, which favor a tensor-to-scalar ratio of this magnitude [13].

Chapter 6

D-Term Inflation and Moduli Stabilization

Many of the interesting back-reaction effects we studied in the previous chapters also arise when inflation is driven by D-terms instead of F-terms. In D-term hybrid inflation (DHI) the vacuum energy sourcing the accelerated expansion of the universe is determined by a Fayet-Iliopoulos term associated with a $U(1)$ gauge symmetry, as outlined in Section 2.3.3. As a step towards a UV embedding of this mechanism, we study the consistency of DHI with supersymmetric moduli stabilization. String theory embeddings of hybrid inflation have been discussed thoroughly in the literature, and in all of the proposed scenarios moduli stabilization is a critical issue [162–167]. Along the lines of Chapter 4 we analyze D-term inflation and its coupling to a single supersymmetric Kähler modulus in string-effective supergravity models. Depending on the source of the FI term the interplay between inflation and moduli stabilization can be highly non-trivial. We study two different cases, inflation with a constant FI term and with a modulus-dependent one.

In the first case, treated in Section 6.1, the back-reaction of the modulus simplifies significantly because the inflaton superpotential vanishes exactly during inflation. This becomes clear from the results obtained in Chapter 4. The back-reaction of a heavy Kähler modulus on the dynamics of DHI has been previously studied in [168]. Furthermore, we point out an intriguing relation between our setup of DHI with stabilized modulus and superconformal D-term inflation, cf. [79, 169]. After inflation we break supersymmetry spontaneously with a Polonyi field without spoiling moduli stabilization or inflation, and derive stringent bounds on the allowed gravitino mass.

With regard to a possible UV embedding of DHI, it was noted in [170–172] that constant FI terms in supergravity are potentially inconsistent. However, supergravity models in which the arguments of these references do not apply have been studied in [173–175]. Whatever the outcome of this discussion, so far there are no known four-dimensional effective theories derived from string theory which contain constant FI terms. String theory, however, provides an elegant mechanism which generates "field-dependent FI terms"[1] which, from the perspective of cosmology, may play a similar role as their constant counterparts. We study inflation driven by these field-dependent FI terms in Section 6.2. In this case, even a very heavy modulus never fully decouples from the dynamics of inflation. After listing the potential problems of known setups, we propose a mechanism which achieves moduli stabilization and a phase of subcritical DHI, as discussed in Section 2.3.4.

The results summarized here have been published in [35, 38]. Regarding inflation with a constant FI term, in this thesis we focus on the modulus back-reaction and the inclusion of spontaneous supersymmetry breaking via a Polonyi field. The emphasis of Section 6.2 lies on the challenges of stabilizing a charged modulus, and the interplay between inflation and the stabilization mechanism described in Section 3.4.3.

6.1 Inflating with a constant FI term

6.1.1 Moduli corrections

To study the interaction of DHI, as introduced in Section 2.3.3, with a supersymmetrically stabilized modulus we start with a no-scale Kähler potential of the form

$$K = -3\ln\left[T + \overline{T} - \frac{1}{3}\left(|\phi|^2 + |S_+|^2 + |S_-|^2\right) - \frac{\chi}{6}\left(\phi^2 + \bar{\phi}^2\right)\right], \tag{6.1}$$

and the superpotential

$$W = W_{\text{inf}} + W_{\text{mod}}(T)\,, \tag{6.2}$$

where W_{inf} is given by Eq. (2.40). The particular form of W_{mod} is irrelevant for our discussion, as long as the conditions (3.46) are fulfilled in the true vacuum after inflation. Our choice of Kähler potential, although it has only little impact on our results, deserves a few remarks. The first two pieces in the logarithm, i.e., the case $\chi = 0$, arise in many string compactifications and are thus well-motivated from the perspective of the UV-complete theory. The last piece proportional to χ, a real constant, is necessary because

[1]Notice that this terminology is somewhat misleading. The "field-dependent FI term" is the D-term of a modulus field which transforms non-linearly under a $U(1)$ symmetry. Thus, it is quite different in nature from the constant gauge-invariant term introduced by Fayet and Iliopoulos in [176].

the inflaton contained in ϕ must be protected by an appropriate symmetry to avoid the η problem. Our choice corresponds to a version of the underlying superconformal symmetry of supergravity. A detailed treatment of this symmetry is beyond the scope of this thesis. For a detailed introduction to superconformal supergravity we recommend [177], as well as [169] for the connection to D-term inflation. Notice that for $\chi = 1$ the Kähler potential reduces to the standard no-scale form with a shift symmetry for ϕ. Therefore, the reader may consider the additional piece and χ as a parameterization of a generalized shift symmetry.

An important ingredient in addition to the F-term potential defined by Eqs. (6.1) and (6.2) is the D-term potential, which reads

$$V_D = \frac{g^2}{2}\left[\frac{1}{\Xi}(|S_+|^2 - |S_-|^2) + \xi\right]^2 , \tag{6.3}$$

where Ξ denotes the argument of the logarithm in Eq. (6.1) and ξ is an FI term of unspecified origin, treated here as a constant.[2] As before, $S_\pm$ are chosen to carry $U(1)$-charge ± 1.

Integrating out T, S_+, and S_- to study the effective potential of ϕ works analogously to the procedure discussed in Section (4.1). Although the form of the Kähler potential is different, the results are similar. S_+ and S_- remain stabilized at the origin until the waterfall transition. In particular, they are not displaced by the back-reaction of T. Integrating out T yields correction terms proportional to $W_{\rm inf}$, which vanishes identically during inflation. Therefore, all leading-order modulus corrections vanish in this case. This is, however, only true as long as T is stabilized supersymmetrically, and thus $W_{\rm mod}(T_0) = W'_{\rm mod}(T_0) = 0$. In this sense, the interaction between DHI and a supersymmetrically stabilized modulus is trivial and inflation proceeds unperturbed as described in Section 2.3.3.

6.1.2 Relation to superconformal symmetry and the Starobinsky model

We mentioned before that our choice of Kähler potential is motivated by the superconformal symmetry of supergravity. By illuminating this connection we can make contact with another successful supergravity model of inflation, the Starobinsky model proposed in [23].

Specifically, the authors of [79] developed a model of D-term inflation with the same

[2] As explained in more detail in App. A.1.1 this can actually not be a constant FI term since the superpotential we choose is gauge-invariant. It may, for example, be an FI term which depends on the vacuum expectation values of mesonic fields [178].

superpotential as ours, and the following Kähler potential,

$$K = -3\ln\left(-\frac{1}{3}\Phi\right), \tag{6.4}$$

where

$$\Phi = -3 + |\phi|^2 + |S_+|^2 + |S_-|^2 + \frac{\chi}{6}\left(\phi^2 + \bar{\phi}^2\right), \tag{6.5}$$

is the so-called frame function. This type of frame function characterizes a large class of models, cf. [179] for details. The superconformal symmetry, which is the starting point in constructing these models, is explicitly broken by the term proportional to χ and by gauge fixing the so-called compensator field, resulting in the appearance of the Planck scale in Eq. (6.5). This particular symmetry-breaking structure allows to keep the attractive features implied by the superconformal symmetry.

In [79] the D-term scalar potential is found to be

$$V_{\rm D} = \frac{g^2}{2}\left[\Omega^2\left(|S_+|^2 - |S_-|^2\right) + \xi\right]^2, \tag{6.6}$$

with $\Omega^2 = -\frac{3}{\Phi}$. This is identical to Eq. (6.3) after a suitable field redefinition,

$$\phi = \sqrt{T+\overline{T}}\,\phi', \qquad S_\pm = \sqrt{T+\overline{T}}\,S'_\pm. \tag{6.7}$$

The F-term scalar potential is invariant under these redefinitions since the Kähler function $K + \ln|W|^2$ is invariant. In particular,

$$\begin{aligned} K(\phi, S_\pm) &= -\frac{3}{2}\ln\left(T+\overline{T}\right) + \ln\Omega^{-2}(\phi', S'_\pm), \\ \ln|W(\phi, S_\pm)|^2 &= +\frac{3}{2}\ln\left(T+\overline{T}\right) + \ln|W(\phi', S'_\pm)|^2. \end{aligned} \tag{6.8}$$

Hence, even after rescaling inflation proceeds as discussed in Section 2.3.3 and the F-term potential vanishes along the inflationary trajectory, as it does in in the model of [79].

Another intriguing connection was pointed out in [169]. In the large-field regime, i.e., if inflation proceeds in the regime $\phi \gg 1$, superconformal D-term inflation is asymptotically equivalent to the Starobinsky model. In particular, at leading order in N_e^{-1} the predictions for the scalar spectral index and the tensor-to-scalar ratio coincide,

$$n_{\rm s} \approx 1 - \frac{2}{N_e}, \qquad r \approx \frac{12}{N_e^2}. \tag{6.9}$$

This makes D-term hybrid inflation even more appealing, since these predictions match CMB observations very well for $N_e \approx 55 - 60$.

6.1.3 Bounds on the scale of supersymmetry breaking

During inflation the D-term determined by ξ and g breaks supersymmetry. After inflation, however, the model defined by Eqs. (6.1)-(6.3) has a supersymmetric vacuum. Therefore, it is necessary to check whether it can be combined with a separate sector of supersymmetry breaking without spoiling either inflation or moduli stabilization.

As in our discussion of moduli stabilization in Chapter 3 we can break supersymmetry with a Polonyi field X in the way outlined in Section 3.2.2. Again, we choose

$$W = W_0 + fX\,, \qquad K = |X|^2 - \frac{|X|^4}{\Lambda^2}\,, \tag{6.10}$$

so that $X_0 \approx \frac{\sqrt{3}}{6}\Lambda^2$ and

$$m_X^2 = \frac{f^2}{2T_0^3\Lambda^2}\,. \tag{6.11}$$

By an appropriate choice of parameters we can realize a hierarchy of scales,

$$m_T > m_X > H \gg m_{3/2}\,, \tag{6.12}$$

and allow for low-energy supersymmetry breaking. Remember that $m_{3/2}$ is determined by f, cf. Eq. (3.15). Requiring that $m_X \gtrsim H$ and demanding $\Lambda^2 \gtrsim f$ in the effective theory defined by Eqs. (6.10) leads to the lower bounds

$$f,\, \Lambda^2 \gtrsim 10^{-10}\,, \tag{6.13}$$

where we have used that $H \sim 0.1 M_{\mathrm{GUT}}^2 \approx 10^{-5}$ in DHI.

This appears surprising because, starting from supersymmetric moduli stabilization, one may have expected that an arbitrarily small value of the gravitino mass is possible. However, since both m_X and the mass scale Λ are constrained by the scale of inflation, one is driven to a regime of intermediate-scale supersymmetry with

$$m_{3/2} \gtrsim 10^5\,\mathrm{GeV}\,. \tag{6.14}$$

Even if the Polonyi field is allowed to be lighter than H but heavier than the inflaton, thus taking part in the dynamics of inflation, this bound is not significantly relaxed. Furthermore, it is independent of the mechanism of inflation. It only depends on the energy scale H and is therefore generic in all models of high-scale inflation.

Notice that the choice of parameters in the Polonyi sector only slightly influences the modulus sector and vice versa. Therefore, in a large portion of parameter space the proposed mechanism of supersymmetry breaking does not interfere with moduli stabilization. Quantifying the impact of the Polonyi field on the dynamics of inflation is slightly more involved. During inflation, X is displaced by the inflationary vacuum energy. Integrating it out consistently leads to a mass term for the inflaton of the form

$$\Delta m_\phi^2 = m_{3/2}^2 \, (1+\chi)^2 \ , \tag{6.15}$$

at leading order in f.[3] Note that this term is present even before integrating out T, which yields small corrections of higher order in ϕ^2. Eq. (6.15) implies that successful inflation also puts an upper bound on the gravitino mass, unless $\chi = -1$, which corresponds to a shift-symmetric Kähler potential.[4] For $\chi \neq -1$, we can demand that the correction to the slow-roll parameter η induced by (6.15) does not alter the prediction for n_s by more than 1σ. This leads to the upper bound

$$m_{3/2} \lesssim \frac{10^{10}\,\mathrm{GeV}}{|1+\chi|} \, . \tag{6.16}$$

The bound resulting from the correction to the slow-roll parameter ϵ is less severe. We conclude that our model can be extended by a simple supersymmetry breaking sector without spoiling any of its features. In this setup, the gravitino mass has to satisfy lower and upper bounds,

$$10^5\,\mathrm{GeV} \lesssim m_{3/2} \lesssim 10^{10}\,\mathrm{GeV} \, , \tag{6.17}$$

which are due to the high scale of inflation and an inflaton mass term induced by supersymmetry breaking in the vacuum, respectively.

6.2 Inflating with a field-dependent FI term

The interaction between T and D-term inflation becomes much more interesting if we drop the notion of a constant FI term and instead attempt to drive inflation with a T-dependent term. Such terms arise when the $U(1)$ symmetry under consideration has anomalies which are cancelled by the transformation of the axion associated with T. This was first observed in context of the Green-Schwarz mechanism [131] in heterotic string theory [180]. A similar situation may arise in type IIB string theory, cf. [181] for an instructive discussion. For a more detailed treatment of D-terms associated with anomalous $U(1)$ symmetries we refer the reader to Appendix A.1. There it is shown that

[3] We have assumed that $2T_0 \gg |\phi|^2$ towards the end of inflation, which can be satisfied even in the large-field regime discussed in [169].

[4] As naively expected, a shift symmetry protects the inflaton from soft mass terms like the one in Eq. (6.15). Remember that we previously encountered this important fact in Chapter 5.

the D-term potential of an anomalous $U(1)_{\mathrm{A}}$ for a Kähler modulus T and a number of chiral superfields ϕ_α can be written as,

$$V_D = \frac{4\pi}{T+\overline{T}} \left(\sum_\alpha q_\alpha K_\alpha \phi_\alpha + \xi_{\mathrm{GS}} \right)^2 , \tag{6.18}$$

with

$$\xi_{\mathrm{GS}} \equiv -\delta_{\mathrm{GS}}\, \partial_T K \simeq \frac{3\delta_{\mathrm{GS}}}{T+\overline{T}} . \tag{6.19}$$

In particular, we assume the absence of a constant FI term. It seems that, if T is stabilized at a high scale with a suitable vacuum expectation value, the last piece in V_D can play the role of a constant FI term and thus drive D-term inflation. In fact, this was already proposed in the original reference [180] and later in [182].

However, it was realized later that moduli stabilization is a subtle issue in the presence of the field-dependent FI term [183,184]. Gauge invariance of the modulus superpotential poses severe restrictions on possible setups [181, 185–187]. In particular, it has been shown that invoking non-perturbative superpotential terms for T requires the inclusion of additional fields charged under $U(1)_{\mathrm{A}}$. Otherwise, T can not be stabilized in a gauge-invariant way. This can be achieved, for example, by including a non-Abelian gauge sector with chiral matter which undergoes gaugino condensation [188, 189].

The authors of [38] have attempted to clarify whether a field-dependent FI term can play the role of an effective constant which drives inflation. Following that account we discuss a series of obstacles which prevent possible setups from resembling the simple controllable model reviewed in Section 2.3.3. Taking stabilization of all additional fields into account, it turns out that in all feasible setups of modulus stabilization with non-perturbative superpotentials the modulus never decouples from the dynamics of inflation, leading to much more complicated multi-field inflation models. We remark, however, that in cases where the modulus that generates the FI term does not appear in the superpotential some of our arguments may be avoided. This can be realized, for example, in the Large Volume Scenario, cf. the related discussion in [190].

In subcritical DHI, on the other hand, a separation of the modulus from the inflaton dynamics seems possible in many moduli stabilization schemes. We provide an example in which the effective theory, after integrating out the modulus and the heavy $U(1)$ vector supermultiplet supersymmetrically, is identical to single-field chaotic inflation. This proceeds along the lines of the discussion of the inflation model in Section 2.3.4.

6.2.1 Stabilizing a charged modulus field

As explained above, the field-dependent FI-term of a modulus T scales as $(T+\overline{T})^{-1}$ and the corresponding Lagrangian scales as $(T+\overline{T})^{-3}$. Thus, to prevent T from running away to infinity we must consider an appropriate mechanism to stabilize it.

Naively, we could assume that T obtains a large supersymmetric mass $m_T \gg \xi_{\mathrm{GS}}$ by some unspecified mechanism so that the field-dependent FI term becomes an effective constant. However, it was argued in [184] that this assumption is inconsistent. The reason is that the vector superfield $\mathscr{V}$ of $U(1)_{\mathrm{A}}$ would receive the same large mass m_T via the Stückelberg mechanism. This, however, immediately implies that one can integrate out $\mathcal{V}$ supersymmetrically at the scale m_T which excludes the existence of an FI term in the effective theory. Hence, a more careful treatment of modulus stabilization is required in the presence of the field-dependent FI term.

The standard procedure, as discussed in Chapter 3, is to stabilize T by employing instantonic contributions to the superpotential of the form

$$W = W_0 + \sum_j A_j e^{-a_j T} \,. \tag{6.20}$$

The interplay of one or several such terms with a constant W_0 or with corrections to the Kähler potential can lead to stable minima for T.

The coefficients A_j are typically assumed to be constant in the effective theory and may arise from integrating out other heavy moduli. However, if T contains the Green-Schwarz axion, constant coefficients A_j would result in a violation of $U(1)_{\mathrm{A}}$ gauge invariance. In order to remedy the theory, the A_j must be promoted to functions $A_j(\phi_\alpha)$ of chiral superfields ϕ_α which carry charge under $U(1)_{\mathrm{A}}$. Writing each piece of the superpotential in the form

$$W \supset A(\phi_\alpha)\, e^{-q_0 T/\delta_{\mathrm{GS}}} \,, \tag{6.21}$$

gauge invariance implies $q\,[A(\phi_\alpha)] = -q_0$ for the charge of the function A, cf. the transformation (A.9). Superpotential terms as in Eq. (6.21) arise, for example, in intersecting D-brane models where the couplings between matter fields are suppressed by the world-sheet instanton action. Generation of Yukawa couplings of this type has first been treated in [191, 192], for a review cf. [132]. Similar couplings are well-known in heterotic string theory [133].

Alternatively, the ϕ_α can be associated with mesonic states of a strongly coupled non-Abelian gauge theory. Let us consider an $SU(N_{\mathrm{c}})$ gauge theory with one pair of quarks $\{Q, \tilde{Q}\}$ transforming as (N_{c}, q) and $(\bar{N}_{\mathrm{c}}, \tilde{q})$ under $SU(N_{\mathrm{c}}) \times U(1)_{\mathrm{A}}$, respectively. In other words, they are matter fields in the fundamental and antifundamental representations of

$SU(N_c)$. Since they are also charged under $U(1)_A$, they enter the D-term potential. To ensure that their D-terms do not cancel ξ_{GS}, which must be non-zero to drive inflation, we assume $q + \tilde{q} > 0$. The gauge theory undergoes gaugino condensation at a scale

$$\Lambda = e^{-2\pi T/(3N_c - 1)} . \tag{6.22}$$

At energy scales below Λ the effective theory can be described by canonically normalized mesonic degrees of freedom

$$M \equiv \sqrt{2Q\bar{Q}} . \tag{6.23}$$

The gauge-invariant superpotential, first computed in [193], reads[5]

$$W = (N_c - 1) \left(\frac{2\, e^{-2(q+\tilde{q})T/\delta_{GS}}}{M^2} \right)^{\frac{1}{N_c - 1}} , \tag{6.24}$$

after inserting the expression for δ_{GS} in Eq. (A.10). Thus, in the case of gaugino condensation the function A in Eq. (6.21) is generically non-analytic. This is important since any field with negative $U(1)_A$ charge[6] entering A can potentially cancel the FI term through its vacuum expectation value. Only for non-analytic A the inclusion of negatively charged fields is unnecessary.[7]

From the perspective of modulus stabilization the dependence of A on other chiral fields is undesirable. In particular, the non-perturbative superpotential of Eq. (6.21) induces couplings of the modulus to other light degrees of freedom, rather than generating a mass term. Only if the fields ϕ_α themselves are stabilized appropriately an effective modulus mass term may arise. As will become clear in the following discussion, this is difficult to achieve in combination with successful hybrid inflation.

6.2.2 The challenge of realizing inflation

Having discussed modulus stabilization, let us analyze whether DHI can proceed with a field-dependent FI term. We are interested in situations where the modulus T is stabilized during inflation and does not perturb the dynamics of DHI, i.e., a situation similar to the one found in Section 6.1. As a starting point, assuming that the superpotential explicitly depends on the charged modulus, we consider

$$W = W_{\text{inf}} + W_{\text{mod}}(T) , \tag{6.25}$$

[5] Note that the superpotential for a gaugino condensate without an anomalous $U(1)$ was proposed much earlier in [188, 189].

[6] Notice that exchanging 'negative charge' with 'positive charge' is merely a choice of convention. Only the sign relative to ξ_{GS} is of importance.

[7] This fact was used in [181] to construct consistent string models with KKLT stabilization and D-term uplift.

where $W_{\rm inf}$ is again given by Eq. (2.40). Following the previous discussion, we choose

$$W_{\rm mod} = A(\phi_\alpha) e^{-q_0 T/\delta_{\rm GS}} + \dots\,, \tag{6.26}$$

responsible for modulus stabilization. In order to promote the instanton contribution to a mass term, stabilization of the fields ϕ_α must be achieved by one of the following mechanisms.

Vector-like mass terms

The presence of gauge anomalies implies charged chiral states in the spectrum. However, the ϕ_α which enter $A(\phi_\alpha)$ may be only a subset of the charged spectrum. Hence they might not contribute to the anomaly, i.e., they could still have large vector-like mass terms of the form

$$\mathcal{L}_{\rm vector} = m_{\rm v}^2 \phi_\alpha \bar{\phi}_\alpha\,. \tag{6.27}$$

In this case we can integrate out all fields ϕ_α and $\bar{\phi}_\alpha$ supersymmetrically. This, in turn, yields $A(\phi_\alpha) = 0$. This would imply that the instanton term disappears in the effective theory below the scale $m_{\rm v}$. Note, however, that this is not necessarily true if $A(\phi_\alpha)$ is a non-analytic function as in gaugino condensation. In that case vector-like mass terms do not appear because the effective degrees of freedom, the mesons in Eq. (6.23), are already two-particle states.

Soft mass terms

Soft mass terms for the ϕ_α may be generated by non-vanishing F- and D-terms, i.e., by supersymmetry breaking. If the field-dependent FI term is not canceled, gauge-mediated soft masses of the form

$$\mathcal{L}^D_{\rm soft} = g^2 q_\alpha\, \xi_{\rm GS} |\phi_\alpha|^2\,, \tag{6.28}$$

arise. In addition, depending on the mechanism of modulus stabilization and supersymmetry breaking, gravity-mediated soft terms may appear.[8] For a minimal choice of the Kähler potential, these are expected to be of the form

$$\mathcal{L}^F_{\rm soft} = m_{3/2}^2 |\phi_\alpha|^2\,. \tag{6.29}$$

[8]Notice that the inflaton, when protected by a shift symmetry of the Kähler potential, does not receive a soft mass term at tree level.

Mass terms from spontaneous symmetry breaking
Finally, if $U(1)_{\mathrm{A}}$ is broken spontaneously, Yukawa couplings can become effective mass terms. Consider, for example, a mesonic state M which couples to the waterfall field S_- as follows,

$$\mathcal{L}_{\mathrm{Yuk}} = \lambda S_- M^2 \,. \tag{6.30}$$

In the true vacuum of the theory S_- cancels the FI term and the meson receives the mass $\lambda\langle S_-\rangle$.

From this discussion it is clear that modulus stabilization either requires the spontaneous breaking of $U(1)_{\mathrm{A}}$ or the breaking of supersymmetry. In principle, all ingredients exist within the simple DHI setup of Section 2.3.3. During inflation supersymmetry is broken by the inflaton sector while the $U(1)$ symmetry is intact. After inflation supersymmetry is restored but the $U(1)$ is spontaneously broken by the vacuum expectation value of S_-. While this may lead to successful stabilization of all fields, the responsible mechanism is clearly different during and after inflation. Therefore, the modulus sector does not decouple from the inflaton dynamics and we are left with an inflation model with several dynamical degrees of freedom. This may happen, for example, in the supersymmetric racetrack scheme studied in [102]. Moreover, depending on the shape of the potential, the motion of the modulus at the end of inflation may lead to a manifestation of the Polonyi problem.

To obtain the simple controllable DHI setup, the same mechanism of modulus stabilization must operate in the entire cosmological history. This requires the inclusion of additional sources of supersymmetry breaking which fix the modulus during and after inflation. A similar conclusion has previously been drawn in [184]. There it was noted that, in a field-dependent realization of DHI, F-terms and D-terms must split their roles in a way that F-terms provide modulus stabilization while D-terms drive inflation. Assuming that the modulus mass is comparable to the gravitino mass, $m_T \sim m_{3/2}$, as in many of the mechanisms reviewed in Chapter 3, results in the constraint

$$m_{3/2} > g\xi_{\mathrm{GS}} \,, \tag{6.31}$$

which ensures that the modulus decouples from inflation. In the following, we wish to point out that a series of problems arises even if this constraint is satisfied.

First, no negatively charged fields beyond S_- should be present in the spectrum. Such fields receive large tachyonic masses during inflation, analogous to Eq. (6.28), and tend to cancel ξ_{GS}. Therefore, we consider the case of gaugino condensation, where the function A in the instanton term contains only the positively charged mesonic fields, cf. the discussion in Section 6.2.1. It turns out that in this setup, condition (6.31) is insufficient to decouple the modulus sector from inflation. This is because during inflation the FI term induces a soft mass term proportional to $m_M \sim g\sqrt{\xi_{\rm GS}}$ for the meson fields.[9] These soft masses are enhanced compared to the Hubble scale since they originate from gauge mediation. In order to avoid that the mesons, and as a consequence also the modulus, are shifted by large amounts at the end of inflation, the gauge-mediated masses should be sub-dominant. This can be achieved by introducing even larger gravity-mediated soft masses $m_M \sim m_{3/2} > g\sqrt{\xi_{\rm GS}}$. At the same time the waterfall fields must be protected against such large gravity-mediated masses by a specific choice of Kähler potential, otherwise inflation would never end. The origin of this sequestering could lie in a higher-dimensional theory where the dominant source of supersymmetry breaking is localized on a different brane than the waterfall fields, cf. the discussion in [194].

Second, even in this case, another type of problem occurs related to the size of the instanton term. Given that modulus stabilization must proceed via supersymmetry breaking, we expect that

$$F_T \sim A(\phi_\alpha)\, e^{-aT} \sim m_{3/2}\,. \tag{6.32}$$

For the case of a condensing $SU(N_{\rm c})$ gauge theory with a single meson M, one finds

$$A(M) = (N_{\rm c}-1)M^{-2/(N_{\rm c}-1)}\,. \tag{6.33}$$

The meson is then stabilized by the interplay of this instanton term and its soft mass term, as explained in detail in [185]. Evidently, the instanton term is responsible for a large vev of M, which we can schematically write as

$$\langle M\rangle \sim \frac{F_T}{m_M}\,. \tag{6.34}$$

Given a meson mass $m_M \sim m_{3/2}$, the minimum lies at $\langle M\rangle \sim 1$ in Planck units. Therefore, if the constraint (6.31) holds, the D-term contribution of the meson exceeds the size of the field-dependent FI term. This is inconsistent with having an effective realization of standard DHI.[10]

In order to find a possible way out of this apparent predicament, one may invoke schemes of modulus stabilization with $m_T \gg m_{3/2}$. An example of this kind may be stabilization via additional Kähler potential terms as proposed, for example, in [82]. But

[9] Without loss of generality we assume $q_M \sim \mathcal{O}(1)$ for the $U(1)_{\rm A}$ charge of the mesons.

[10] While it may be possible to obtain an approximate version of DHI with an FI term generated by stabilized mesons, the analysis of such schemes is beyond the scope of this work. For a detailed discussion of this option we refer the reader to [178].

even this is not a full solution to the problem, as stabilization of the mesons still requires a very large gravitino mass and DHI is spoiled by the large displacement of the meson, cf. Eq. (6.34).

To summarize, in models where a field-dependent FI term drives inflation there is always an intimate connection between modulus stabilization and inflation. Generically, the modulus does not decouple from the dynamics of inflation. When trying to obtain the simple controllable scheme of DHI as an effective theory, a series of problems arises. These problems are related to the fact that inflation back-reacts on the modulus stabilization and vice versa. While we have shown that there is no straight-forward realization of DHI with a field-dependent FI-term, we can not exclude that it arises to some approximation by a very delicate engineering of the Kähler potential and the mechanism of modulus stabilization.

These considerations lead us to consider D-term inflation in a regime where the D-term has actually been cancelled. This is the mechanism of subcritical DHI discussed in Section 2.3.4. In this scheme, T can be stabilized conveniently via the breaking of $U(1)_{\mathrm{A}}$, without the need for gaugino condensates. The obstacles mentioned above are absent in this case.

6.2.3 Inflating in the chaotic regime of D-term inflation

From the previous discussions it is clear that to circumvent the above problems, inflation may proceed in the subcritical regime of DHI, while the Kähler modulus which cancels the anomalies of $U(1)_{\mathrm{A}}$ is stabilized by the mechanism of Section 3.4.3. This is, of course, different from the standard mechanism of DHI. In fact, it is not even D-term inflation since the vacuum energy during inflation is determined by the F-term of one of the waterfall fields. Nevertheless, we consider it an instructive example.

Let us investigate the theory defined by the superpotential

$$W = \chi_+ \left(S_-^2 e^{-T/\delta_{\mathrm{GS}}} - m S_- \right) + \lambda \phi S_+ S_- \,, \tag{6.35}$$

which is obtained by adding the superpotential of DHI to the superpotential in Eq. (3.51) and identifying the waterfall field S_- with the field which renders the instantonic term gauge-invariant. As in previous examples, the inflaton field $\varphi = \sqrt{2}\mathrm{Im}\,\phi$ is protected by a shift symmetry in the Kähler potential. The Kähler potential reads

$$K = -3 \ln \left[T + \overline{T} - |S_-|^2 - |S_+|^2 - |\chi_+|^2 + (\phi + \bar{\phi})^2 \right] \,. \tag{6.36}$$

Here, as in our previous examples, we implicitly assume that the inflaton is part of the matter sector of a possible string theory embedding. In this particular case it could be associated with a Wilson line scalar with the shift symmetry being a consequence of higher-dimensional gauge invariance. For a recent discussion of large-field inflation with such Wilson lines, cf. [150].

Along the lines of Section 2.3.4 we can absorb S_- into the vector superfield, which we integrate out supersymmetrically to obtain the effective theory for the remaining degrees of freedom. For more details we refer to the original publication [38]. We find the following effective superpotential and Kähler potential,

$$W = \delta_{\rm GS}\,\chi_+\left(e^{-T/\delta_{\rm GS}} - 3m_T\right) + \lambda\sqrt{\delta_{\rm GS}}\,\phi S_+\,, \tag{6.37}$$

$$K = -3\ln\left[T + \overline{T} - \delta_{\rm GS} - |S_+|^2 - |\chi_+|^2 + \frac{(|S_+|^2 + |\chi_+|^2)^2}{2\delta_{\rm GS}} + (\phi + \bar{\phi})^2\right]\,, \tag{6.38}$$

where we have used Eq. (3.57) to express m in terms of the modulus mass. If $m_T > H$ the modulus decouples from the dynamics and can be integrated out together with χ_+. The resulting effective theory for ϕ and S_+ is defined by

$$W = \hat{m}\hat{\phi}\hat{S}_+\,, \qquad K = |\hat{S}_+|^2 - \frac{|\hat{S}_+|^4}{2\xi_{\rm GS}} - \frac{1}{2}(\hat{\phi} + \bar{\hat{\phi}})^2\,, \tag{6.39}$$

where we have introduced the mass parameter $\hat{m} = \lambda\sqrt{\xi_{\rm GS}}/3\sqrt{6}$ and the canonically normalized superfields

$$\hat{S}_+ = \sqrt{\frac{3}{2T_0 - \delta_{\rm GS}}}\,S_+\,, \qquad \hat{\phi} = \sqrt{\frac{6}{2T_0 - \delta_{\rm GS}}}\,\phi\,, \tag{6.40}$$

with T_0 given by Eq. (3.56). Evidently, by integrating out all heavy degrees of freedom we have obtained the standard realization of chaotic inflation, cf. Section 2.3.1, as an effective theory.

A few more comments are in order. For finite m_T a small correction to the predictions of chaotic inflation arises due to a displacement of the modulus during inflation, similar to the one derived in Chapter 4. Integrating out T at its shifted vacuum expectation value induces an inflaton-dependent correction to the scalar potential. The leading-order correction can be found by expanding V around T_0, along the lines of Section 4.1. Specifically,

$$V = \frac{1}{(T + \overline{T} - \delta_{\rm GS})^2}\left[\frac{1}{2}\hat{m}^2\hat{\varphi}^2(2T_0 - \delta_{\rm GS})^2 + 3m_T^2\,|T - T_0|^2\right]\,, \tag{6.41}$$

where $\hat{\varphi} = \sqrt{\rm Im}\,\hat{\phi}$ is the canonically normalized inflaton field. Minimizing this expression with respect to T gives

$$T - T_0 = \frac{2T_0 - \delta_{\rm GS}}{3m_T^2}\,\hat{m}^2\hat{\varphi}^2\,. \tag{6.42}$$

For the leading-order effective inflaton potential we obtain

$$V = V_0 \left(1 - \frac{4}{3} \frac{V_0}{m_T^2} \right) , \tag{6.43}$$

with $V_0 = \frac{1}{2}\hat{m}^2\hat{\varphi}^2$. Notice that the numerical coefficient of the correction term differs from the result obtained in Section 4.2.2 due to the different choice of Kähler potential. As naively expected in supersymmetric stabilization, the correction induced by the shift of the modulus disappears in the limit where T is infinitely heavy.

Similar to our discussion in Chapter 5, modulus stabilization in the effective theory implies constraints on the initial conditions of the system. In particular, inflation can not begin at arbitrarily large field values of $\hat{\varphi}$ because, to ensure that T remains stabilized in the entire cosmological history, the energy density of the universe must never exceed the modulus mass. This is a conceptual shortcoming which remains to be addressed in many effective theories of inflation with moduli stabilization. It is a common problem of all four-dimensional descriptions of string-effective supergravity.

Chapter 7

Conclusion and Outlook

Embeddings of cosmic inflation in supergravity and string theory give rise to a number of interesting phenomena. In the scenarios considered here, inflation takes place at an energy scale close to the GUT scale and the Planck scale. This, as well as the flatness of the inflaton potential, makes supergravity descriptions of inflation sensitive to Planck-scale physics, as, for example, described by string theory. Whenever inflation is treated in the context of string theory, the issue of moduli stabilization must be addressed. We have demonstrated that the requirement of stability of all moduli in the entire cosmological history leads to a number of constraints on both moduli stabilization and inflation schemes.

In F-term inflation we have formulated these constraints in quite general terms. For supersymmetric moduli stabilization, i.e., for setups which produce supersymmetric Minkowski vacua after inflation, we have calculated back-reaction terms from heavy Kähler moduli for arbitrary superpotentials. All of these correction terms vanish if the moduli are infinitely heavy. For realistic moduli masses above the Hubble scale, the corrections are sizeable and affect the predicted CMB observables in many models. We have demonstrated this in the examples of hybrid inflation and chaotic inflation with a stabilizer field. On the other hand, inflation affects the potential for the moduli. The vacuum energy sourced by the inflaton acts as an uplift of the moduli, so that the moduli masses must lie close to the GUT scale to ensure stability of the extra dimensions.

Furthermore, we have generalized this analysis by including the effects of spontaneous supersymmetry breaking in the moduli sector. The latter occurs in many successful moduli stabilization schemes, such as KKLT stabilization or the Large Volume Scenario. In these cases, there are a number of non-decoupling effects like soft mass terms for the inflaton. The strength of the back-reaction of these terms increases with the mass of the moduli. By the example of chaotic inflation without a stabilizer field we have shown that this leads to challenges for large-field inflation in string theory. In particular, the parameters and initial conditions must be chosen carefully to ensure stability of the extra dimensions and to allow for inflation in accordance with observations. Moreover, chaotic inflation with a stabilizer field is incompatible with such moduli stabilization schemes due to mutually exclusive constraints on the gravitino mass. On the one hand, moduli stabilization with spontaneous supersymmetry breaking requires at least $m_{3/2} \gtrsim H$ to ensure stability of all moduli. On the other hand, supersymmetry breaking induces a back-reaction of the stabilizer field which makes inflation unfeasible unless $m_{3/2} \lesssim H$.

Lastly, we have studied the back-reaction of heavy Kähler moduli on D-term inflation. If inflation is driven by a constant FI term, all moduli corrections decouple due to a suppression of the inflaton superpotential. There are, however, constraints on the gravitino mass in the vacuum after inflation, forcing supersymmetry to be broken at an intermediate scale $10^5\,\mathrm{GeV} \lesssim m_{3/2} \lesssim 10^{10}\,\mathrm{GeV}$. Driving inflation with the field-dependent FI term of an anomalous $U(1)$ symmetry, on the other hand, is non-trivial. Gauge invariance of the superpotential forces the introduction of additional fields in the modulus sector, making it difficult to decouple the latter from inflation. We have discussed a series of obstacles which prevent the realization of the standard DHI scheme. To evade these obstacles we have proposed a version of subcritical DHI in which the moduli are stabilized by Yukawa couplings and world-sheet instantons.

The phenomena studied in this thesis may have a variety of consequences for string theory models of inflation. The alleged discovery of primordial gravitational waves by the BICEP2 collaboration [12], by now ascribed to foreground contaminations, has sparked renewed interest in such models. Explicit or semi-realistic string theory constructions like the ones in [41,130,148,150,151,154,195,196] have demonstrated that the back-reaction of moduli on the inflationary trajectory is important. In particular, in all of them the back-reaction can be studied by means of four-dimensional effective supergravity Lagrangians as described here. Constraints from inflation, like the necessity of moduli masses close to the GUT scale, hold in all setups available so far. They teach us valuable lessons about possible compactification manifolds and the selection of string theory vacua. At the same time constraints from moduli stability may leave testable string theory imprints in the CMB fluctuations which are in reach of experiments in the near future.

Let us conclude this thesis with a few biased remarks. The concepts and models studied in this work are connected to a number of open discussions, and raise a number of questions which deserve to be commented on.

First, assuming that the very early universe developed in a phase of single-field inflation, it is presently unknown whether small-field or large-field inflation is realized in nature. From a conceptual point of view small-field inflation entails a number of advantages. These are, for example, control over the effective field theory and Planck-suppressed operators, as well as its well-understood relation to post-inflationary thermal evolution like reheating and the production of dark matter, cf. [197] for an enlightening discussion and a comprehensive list of references. Furthermore, it has been shown in many examples that small-field inflation can be realized in string theory, cf. [45]. Large-field inflation seems to require the existence of appropriate symmetries which protect the flatness of the inflaton potential on super-Planckian distances. While such symmetries – approximate ones, at least – do exist in string theory, realistic models of large-field high-scale inflation are difficult to reconcile with moduli stabilization, and potentially with other aspects of cosmology and particle physics. Without resorting to full string theory compactifications we have analyzed several of these difficulties in this thesis. Moreover, the effectiveness of axionic shift symmetries in string theory has recently been questioned by the authors of [198–200]. They argue that gravitational instantons generically obstruct large-field inflation driven by axions. Given that we have made use of shift symmetries in our examples of large-field inflation, these arguments may apply to string theory implementations of some of the models presented in this thesis. Hence, on purely conceptual grounds it seems that small-field inflation is simply easier to treat. From the perspective of CMB observations as guidance for model building, on the other hand, large-field inflation is clearly more appealing. The discovery of a non-vanishing tensor-to-scalar ratio from primordial gravitational waves would be constructive proof for large-field inflation, while the confirmation of $r \approx 0$ would merely hint towards the realization of small-field inflation in nature. In the latter case one has to use other observables to distinguish between different models of inflation and to find definite proof for an inflationary epoch itself. Therefore, as emphasized before, the CMB data expected in the next few years is vital to achieving progress in inflationary model building. Until then it seems more instructive to constrain existing models than to invent new ones with a plethora of predictions.

Second, in this thesis we have not taken Standard Model fields into account. But, as mentioned before, during inflation all fields must be heavier than the Hubble scale for single-field inflation to work. Since supersymmetry is always broken during inflation all relevant matter fields are expected to receive soft mass terms of the same order as the Hubble scale. In many cases this implies a mass hierarchy between the matter fields and the inflaton. But fields with Hubble-scale masses may in some cases still contribute to the fluctuations generated by the inflaton field. As pointed out in [201–206], and most recently in [207], a very interesting situation arises when many fields are stabilized close to the Hubble scale. On the path to a consistent theory of inflation and particle physics, the models discussed in this thesis certainly deserve a careful investigation of this issue.

Our third and last remark is related to the scale of supersymmetry breaking and the mass of the lightest supersymmetric partners. As stated in the beginning, the LHC has set lower bounds on these scales close to the TeV scale. However, high-scale inflation with consistent moduli stabilization seems to favor supersymmetry at a much higher scale. If supersymmetry is broken by Kähler moduli as discussed in this thesis, or by fluxes in the internal manifold, one typically has $m_{3/2} \lesssim M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$. In this case many of the virtues of supersymmetry are futile. High-scale supersymmetry does not solve the Standard Model hierarchy problem, and may even be unfit to protect the inflaton mass from quantum corrections. The only way to avoid this predicament seems to be supersymmetric moduli stabilization. However, this either requires a substantial amount of fine-tuning or the presence of additional stabilizer fields which interact with the moduli in a specific way. Hence it is doubtful whether supersymmetric stabilization is implemented in nature to reconcile high-scale inflation with low-energy supersymmetry. This implies that a discovery of low-energy supersymmetry during the next run of the LHC would put immense pressure on many available setups. One may view this as a shortcoming of high-scale inflation or of the available mechanisms for moduli stabilization. A few critical arguments regarding the latter are worth pointing out: As explained before, most moduli stabilization schemes require substantial fine-tuning for self-consistency and for compatibility with inflation and phenomenology. Thus, many examples which are viable as toy models are unnatural in some way. Furthermore, since these schemes are mostly toy models, usually a large portion of the full string theory superpotential is discarded. Generically, there is a plethora of non-perturbative terms which can depend on various moduli in intricate ways. It is computationally and conceptually challenging to prove that the mechanisms discussed in this thesis are viable in full string theory compactifications. Connected to this is the question whether reheating can be implemented successfully. The reheating mechanism crucially depends on couplings between the inflaton and other matter fields which are encoded in the superpotential. Therefore, neglecting parts of the latter can have severe implications. Moreover, it may be troubling that in high-scale inflation the moduli masses are necessarily close to the GUT scale. This means that in generic string compactifications with $\mathcal{O}(100)$ moduli fields all of them must have masses between the GUT scale and the string scale, i.e., in an interval of typically one or two orders of magnitude. It is far from obvious that this can be achieved. Of course, these shortcomings of moduli stabilization schemes do not imply that high-scale inflation describes the evolution of the early universe correctly.

But, hopefully, this question can be settled by precision CMB data in the near future.

Acknowledgements

I would like to thank Wilfried Buchmüller for agreeing to supervise my work as a PhD student. As a great teacher, mentor, and collaborator he has given me invaluable insights into the field of cosmology, into physics, and beyond. I am deeply indebted to him for the patience and thoughtfulness with which he encouraged my work. I am grateful to Jan Louis, Daniela Pfannkuche, Christian Sander, and Alexander Westphal for agreeing to act as referees of my dissertation and disputation.

The collaboration with Ido Ben-Dayan, Valerie Domcke, Emilian Dudas, Lucien Heurtier, Shenglin Jing, Fabian Rühle, Alexander Westphal, and Martin Winkler was very enjoyable and fruitful. Furthermore, there are a number of colleagues at DESY and other institutions to whom I owe gratitude for various discussions and comments on my work. In particular Jean-Pierre Derendinger, Sergio Ferrara, Daniel Z. Freedman, Thomas Grimm, Arthur Hebecker, Kwang Sik Jeong, Renata Kallosh, Rolf Kappl, Andrei Linde, Jan Louis, Hans Peter Nilles, Francisco Pedro, Robert Richter, and Lukas Witkowski. Markus Dierigl and Fabian Rühle gave helpful comments on the manuscript of this thesis. I am especially grateful to Emilian Dudas and Fabian Rühle for their relentless patience in answering my questions and for sharing their remarkable knowledge of mathematics and physics.

Throughout my time at DESY I have been financially supported by a generous scholarship of the Joachim Herz Foundation. Specifically, I wish to thank Eva Ackermann and Jörg Maxton-Küchenmeister for their continuous support, which made focussing on physics so easy. Furthermore, I am grateful to the team of the PIER Helmholtz Graduate School, the PhD initiative DOIT, and the PIER Running Group for many hours spent together. Without my friends and fellow PhD students at DESY the past years would have been much less stimulating and fun, so thank you! This goes especially to my marvellous office mates Laura Sagunski and Václav Tlapák, who always made me carry a smile on my face. Last but not least, I wish to thank my family for their unconditional love and support. No matter the hour, they always encourage me to achieve whatever I want to achieve.

Appendix A

A.1 Fayet-Iliopoulos terms in supergravity and string theory

A.1.1 Constant FI terms

In a supergravity theory with $U(1)$ gauge interactions the Lagrangian is determined by the choice of superpotential W, Kähler potential K, gauge kinetic function f, and Killing vectors k^α specifying the gauge transformation properties of chiral superfields ϕ_α. The superpotential and Kähler potential enter the Lagrangian in the combination

$$G = K + \ln |W|^2 \,, \tag{A.1}$$

which must be gauge-invariant. The gauge kinetic function transforms trivially under the $U(1)$ up to a possible shift required for anomaly cancellation. It determines the gauge coupling as $g^2 = (\mathrm{Re}\, f)^{-1}$. In case the $U(1)$ symmetry is linearly realized, chiral superfields ϕ_α transform as

$$\phi_\alpha \to e^{iq_\alpha \epsilon} \phi_\alpha \,, \tag{A.2}$$

where ϵ is a chiral superfield gauge transformation parameter and q_α denotes the charge of ϕ_α. This corresponds to the choice of Killing vector $k^\alpha = iq_\alpha \phi_\alpha$. The transformation of the $U(1)$ vector superfield $\mathscr{V}$ can be written as

$$\mathscr{V} \to \mathscr{V} - \frac{i}{2}(\epsilon - \bar{\epsilon}) \,. \tag{A.3}$$

The scalar potential may contain an F-term and a D-term piece, i.e., $V = V_F + V_D$ with

$$V_F = e^K \left(K^{\alpha\bar{\alpha}} D_\alpha W D_{\bar{\alpha}} \overline{W} - 3|W|^2 \right) \,, \tag{A.4}$$

$$V_D = \frac{1}{2\mathrm{Re}\, f} D^2 \,. \tag{A.5}$$

We can write D-terms associated with the $U(1)$ as [175]

$$D = -ik^\alpha G_\alpha = -ik^\alpha K_\alpha \underbrace{-i \frac{W_\alpha}{W} k^\alpha}_{\equiv \xi} \,. \tag{A.6}$$

Notice that, by gauge invariance of the supergravity Lagrangian, W may transform with a constant phase denoted by ξ. This is the local variant of the constant FI term introduced in [176] in the context of globally supersymmetric theories. This means, in particular, that in supergravity a constant FI term can only be present when the superpotential is not gauge-invariant.

A.1.2 Field-dependent FI terms

Depending on the full gauge group and chiral spectrum of the theory under consideration, a $U(1)$ symmetry like the one in Section A.1.1 can have anomalies, in which case we denote it by $U(1)_{\mathrm{A}}$. This is actually the generic situation in many string compactifications. The anomalies manifest as divergences of the gauge current J, i.e.,

$$\partial_\mu J^\mu \propto c_1 A_{G^2-U(1)_{\mathrm{A}}} \operatorname{tr} \mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu} + c_2 A_{U(1)_{\mathrm{A}}^3} F_{\mu\nu}\tilde{F}^{\mu\nu} + c_3 A_{\mathrm{grav}^2-U(1)_{\mathrm{A}}} \operatorname{tr} R_{\mu\nu}\tilde{R}^{\mu\nu} , \qquad \text{(A.7)}$$

where $\mathcal{F}$, F, and R denote the field strengths of a non-Abelian gauge group piece G, $U(1)_{\mathrm{A}}$, and the Riemann tensor, respectively. The pre-factors c_i are model-dependent and the anomaly coefficients A are given by

$$A_{G^2-U(1)_{\mathrm{A}}} = \sum_f q_f \ell(\mathbf{R}_f) , \quad A_{U(1)_{\mathrm{A}}^3} = \sum_\alpha q_\alpha^3 , \quad A_{\mathrm{grav}^2-U(1)_{\mathrm{A}}} = \sum_\alpha q_\alpha . \qquad \text{(A.8)}$$

The first sum runs over all chiral fermions transforming in the representation $\mathbf{R}$ of G and $\ell(\mathbf{R})$ denotes the quadratic index of $\mathbf{R}$. The sums in the second and third expression run over all chiral fermions.

For the theory to be consistent, all anomalies must be canceled by the four-dimensional version of the Green-Schwarz mechanism [131]. This means there must be at least one axion which shifts under $U(1)_{\mathrm{A}}$, and this shift cancels all anomalies via its coupling to the field strengths. Motivated by compactifications of type IIB string theory, we take the axion to be the imaginary part of a Kähler modulus T and assume all other moduli to be stabilized by fluxes [31]. Note that the discussion proceeds analogously in heterotic string theory with the dilaton playing the role of the Kähler modulus. The transformation of T under $U(1)_{\mathrm{A}}$ reads

$$T \to T - i\delta_{\mathrm{GS}}\epsilon , \qquad \text{(A.9)}$$

which corresponds to the Killing vector $k^T = -i\delta_{\mathrm{GS}}$. In what follows we consider the case $G = SU(N_{\mathrm{c}})$ and N_{f} quark pairs transforming as (N_{c}, q) and $(\bar{N}_{\mathrm{c}}, \tilde{q})$ under $SU(N_{\mathrm{c}}) \times U(1)_{\mathrm{A}}$, respectively. Cancellation of the pure $U(1)_{\mathrm{A}}^3$ and the mixed $SU(N_{\mathrm{c}}) \times U(1)_{\mathrm{A}}^2$

anomaly then implies [181]

$$\delta_{\rm GS} = \frac{1}{6\pi\kappa}\sum_\alpha q_\alpha^3 = \frac{1}{4\pi\tilde{\kappa}} N_{\rm f}(q+\tilde{q})\,, \tag{A.10}$$

where the sum again runs over all chiral fermions. We do not impose additional constraints on $\delta_{\rm GS}$ related to the cancellation of the gauge-gravity anomaly, as in type IIB orientifold compactifications the coupling of the axion to the Riemann tensor is model-dependent [208]. The coefficients κ and $\tilde{\kappa}$ which enter the above equation are $\mathcal{O}(1)$ constants which appear in the gauge kinetic functions, i.e.,

$$f = \frac{\kappa}{2\pi} T\,, \qquad \tilde{f} = \frac{\tilde{\kappa}}{2\pi} T\,, \tag{A.11}$$

for $U(1)_{\rm A}$ and $SU(N_{\rm c})$, respectively. The $U(1)_{\rm A}$ gauge coupling is given by

$$g^2 = \frac{1}{{\rm Re}\, f} = \frac{4\pi}{\kappa(T+\overline{T})}\,, \tag{A.12}$$

and similarly for the gauge coupling of the $SU(N_{\rm c})$. In the following we choose a normalization which coincides with the one used in the work of KKLT [32], i.e., $\kappa = \tilde{\kappa} = \frac{1}{2}$.

Since T transforms non-trivially under $U(1)_{\rm A}$, the familiar no-scale Kähler potential must be modified accordingly,

$$K = -3\ln\left(T+\overline{T}\right) \quad \longrightarrow \quad K = -3\ln\left(T+\overline{T} - 2\delta_{\rm GS}\mathscr{V}\right). \tag{A.13}$$

Allowing for the presence of additional chiral fields ϕ_α which transform linearly under $U(1)_{\rm A}$, the D-term potential reads

$$V_D = \frac{4\pi}{T+\overline{T}}\left(\sum_\alpha q_\alpha K_\alpha \phi_\alpha + \xi_{\rm GS}\right)^2\,, \tag{A.14}$$

where we have assumed gauge invariance of W, i.e., the absence of a constant FI term in V_D. The piece

$$\xi_{\rm GS} \equiv -\delta_{\rm GS}\,\partial_T K \simeq \frac{3\delta_{\rm GS}}{T+\overline{T}}\,, \tag{A.15}$$

is usually called a field-dependent FI term in the literature [180]. Notice that a D-term like this can only arise in theories with an anomalous $U(1)$ symmetry. In a theory without anomalies, a non-trivial shift of an axion associated with a modulus or the dilaton field introduces an anomaly itself.

A.2 Useful supergravity formulae

Scalar masses in supergravity with Minkowski background are given by [185, 209–211]

$$\begin{aligned} m^2_{\alpha\bar{\beta}} &= e^G\left(G_{\alpha\bar{\beta}} - R_{\alpha\bar{\beta}\gamma\bar{\delta}}G^\gamma G^{\bar{\delta}} + \nabla_\alpha G^{\bar{\gamma}}\nabla_{\bar{\beta}}G_{\bar{\gamma}}\right) , \\ m^2_{\alpha\beta} &= e^G\left(2\nabla_\alpha G_\beta + G^\gamma \nabla_\alpha\nabla_\beta G_\gamma\right) , \end{aligned} \tag{A.16}$$

without taking canonical normalization into account. Here, $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$ is the Riemann curvature of the Kähler manifold and $\Gamma^\alpha_{\beta\gamma} = G^{\alpha\bar{\alpha}}\partial_\beta G_{\gamma\bar{\alpha}}$. Notice that these expressions can be used to compute physical masses in the ground state of the theory, but not during inflation. The fermionic mass matrix, on the other hand, is given by

$$(\tilde{m}_F)_{\alpha\beta} = e^{G/2}(\nabla_\alpha G_\beta + G_\alpha G_\beta) . \tag{A.17}$$

After extracting the goldstino-gravitino mass mixing, the fermionic mass matrix becomes

$$(m_F)_{\alpha\beta} = e^{G/2}\left(\nabla_\alpha G_\beta + \frac{1}{3}G_\alpha G_\beta\right) = e^{K/2}\left(D_\alpha D_\beta W - \frac{2}{3W}D_\alpha W D_\beta W\right) . \tag{A.18}$$

The fermionic masses also define the supersymmetric contribution to the scalar masses. Hence, we can define the soft scalar mass matrix m_0 by subtracting the fermionic mass contribution,

$$\begin{aligned} m^2_{\alpha\bar{\beta}} &= (m_F m_F^\dagger)_{\alpha\bar{\beta}} + e^G\left(G_{\alpha\bar{\beta}} - R_{\alpha\bar{\beta}\gamma\bar{\delta}}G^\gamma G^{\bar{\delta}} + \frac{1}{3}G_\alpha G_{\bar{\beta}}\right) \equiv \left(m_F m_F^\dagger\right)_{\alpha\bar{\beta}} + \left(m_0^2\right)_{\alpha\bar{\beta}} \ , \\ m^2_{\alpha\beta} &= 2e^{G/2}(m_F)_{\alpha\beta} + e^G\left(-\frac{2}{3}G_\alpha G_\beta + G^\gamma\nabla_\alpha\nabla_\beta G_\gamma\right) , \end{aligned} \tag{A.19}$$

where $\left(m_F m_F^\dagger\right)_{\alpha\bar{\beta}} = G^{\gamma\bar{\gamma}}(m_F)_{\alpha\gamma}(\overline{m}_F)_{\bar{\beta}\bar{\gamma}} \equiv (m_S^2)_{\alpha\bar{\beta}}$. Furthermore, it is useful to define the inverse supersymmetric mass matrix,

$$\left(m_S^{-2}\right)^{\bar{\alpha}\delta} = G_{\beta\bar{\gamma}}\left({m_F}^{-1}\right)^{\beta\delta}\left(\overline{m}_F^{-1}\right)^{\bar{\alpha}\bar{\gamma}} , \tag{A.20}$$

which satisfies the relations

$$(m_F)_{\alpha\beta}\left(m_S^{-2}\right)^{\alpha\bar{\beta}} \;=\; G_{\beta\bar{\gamma}}(\overline{m}_F^{-1})^{\bar{\beta}\bar{\gamma}} , \qquad \left(m_S^{-2}\right)^{\alpha\bar{\beta}}(\overline{m}_F)_{\bar{\beta}\bar{\gamma}} = G_{\beta\bar{\gamma}}\left({m_F}^{-1}\right)^{\beta\alpha} . \tag{A.21}$$

A.3 Integrating out supersymmetry-breaking moduli

A.3.1 Obtaining the general result

The coefficients of the Taylor series in Eq. (5.38) are given by

$$V_0 = e^{K_0}\left\{K_0^{\alpha\bar\beta}\left[W_\alpha + K_{0,\alpha}W_{\rm mod}\right]\left[\overline{W}_{\bar\beta} + K_{0,\bar\beta}\overline{W}_{\rm mod}\right] - 3|W_{\rm mod}|^2\right\}, \tag{A.22a}$$

$$V_1 = e^{K_0}\left\{-\frac{1}{2}K_0^{\alpha\bar\beta}\left(K_{0,\bar\beta}D_\alpha W_{\rm mod} + K_{0,\alpha}\overline{D}_{\bar\beta}\overline{W}_{\rm mod}\right) + mK_1^{-1} + \frac{3}{2}(W_{\rm mod} + \overline{W}_{\rm mod})\right\}, \tag{A.22b}$$

$$V_2 = \frac{1}{4}e^{K_0}\left\{K_0^{\alpha\bar\beta}K_{0,\alpha}K_{0,\bar\beta} - 3\right\}, \tag{A.22c}$$

where $D_\alpha W_{\rm mod} = W_{{\rm mod},\alpha} + K_{0,\alpha}W_{\rm mod}$. Expanding these coefficients at leading order in $\delta T_\alpha \ll T_{\alpha,0}$ leads to

$$V_0(T_\alpha, \overline{T}_{\bar\alpha}) = \frac{1}{2}\begin{pmatrix}\delta T_\alpha & \delta\overline{T}_{\bar\alpha}\end{pmatrix}\begin{pmatrix} m^2_{\alpha\bar\beta} & m^2_{\alpha\beta} \\ m^2_{\bar\alpha\bar\beta} & m^2_{\bar\alpha\beta}\end{pmatrix}\begin{pmatrix}\delta\overline{T}_{\bar\beta} \\ \delta T_\beta\end{pmatrix} + \dots, \tag{A.23a}$$

$$V_1(T_\alpha, \overline{T}_{\bar\alpha}) = V_1(T_{\alpha,0}, \overline{T}_{\bar\alpha,0}) + \frac{\partial V_1}{\partial T_\alpha}\delta T_\alpha + \frac{\partial V_1}{\partial \overline{T}_{\bar\alpha}}\delta\overline{T}_{\bar\alpha} + \dots, \tag{A.23b}$$

$$V_2(T_\alpha, \overline{T}_\alpha) = V_2(T_{\alpha,0}, \overline{T}_{\bar\alpha,0}) + \dots, \tag{A.23c}$$

keeping only the leading-order terms up to fourth order in φ. $m^2_{\alpha\bar\beta}$ and $m^2_{\alpha\beta}$ denote the mass matrices of the moduli fields in the true vacuum, defined by Eqs. (A.16). In the expansion of V_0 we have used that the cosmological constant vanishes in the vacuum and that the moduli trace their minima adiabatically. In particular,

$$V_0(T_{\alpha,0}, \overline{T}_{\bar\alpha,0}) = \partial_\alpha V_0|_{T=T_0} = 0\,. \tag{A.24}$$

Plugging the results in Eqs. (A.23) back into V and inserting the correct moduli displacement in the potential leads to the general expression in Eq. (5.40). By a straight-forward computation we find

$$\frac{\partial V_1}{\partial T_\alpha}|_{T=T_0} = e^{K_0}\Big\{ -\frac{1}{2}K_0^{\beta\bar\gamma}\left[K_{\bar\gamma}D_\alpha D_\beta W_{\rm mod} + (K_{\alpha\beta} + K_\alpha K_\beta - \Gamma^\gamma_{\alpha\beta}K_\gamma)\overline{D}_{\bar\gamma}\overline{W}_{\rm mod}\right]$$
$$+ D_\alpha W_{\rm mod} + K_\alpha\overline{W}_{\rm mod} + mK_1^{-2}(K_\alpha K_1 - K_{1,\alpha})\Big\}, \tag{A.25}$$

where $D_\alpha D_\beta W = \nabla_\alpha D_\beta W + K_\alpha D_\beta W$.

Using the mass formulae of Appendix A.2, we can further simplify the effective potential. In particular, using the approximation that the supersymmetric mass scale is much larger than $m_{3/2}$ we find

$$
\begin{aligned}
&\begin{pmatrix} \frac{\partial V_1}{\partial T_\alpha} & \frac{\partial V_1}{\partial \overline{T}_{\bar\alpha}} \end{pmatrix} \begin{pmatrix} (m^{-2})^{\alpha\bar\beta} & (m^{-2})^{\alpha\beta} \\ (m^{-2})^{\bar\alpha\bar\beta} & (m^{-2})^{\bar\alpha\beta} \end{pmatrix} \begin{pmatrix} \frac{\partial V_1}{\partial \overline{T}_{\bar\beta}} \\ \frac{\partial V_1}{\partial T_\beta} \end{pmatrix} \\
&\approx \frac{\partial V_1}{\partial T_\alpha}(m^{-2})^{\alpha\bar\beta}\left[\frac{\partial V_1}{\partial \overline{T}_{\bar\beta}} - m^2_{\bar\beta\bar\gamma}(m^{-2})^{\bar\gamma\beta}\frac{\partial V_1}{\partial T_\beta}\right] + \text{h.c.} \\
&\approx \frac{1}{2}e^{K_0}K_0^{\alpha\bar\beta}K_{0,\alpha}K_{0,\bar\beta} - \frac{1}{2}e^{3K_0/2}\Big\{K_\delta(m_F^{-1})^{\beta\delta}\Big[-K_0^{\epsilon\bar\epsilon}(K_{\beta\epsilon} + K_\beta K_\epsilon - \Gamma^\gamma_{\beta\epsilon}K_\gamma)\overline{D}_{\bar\epsilon}\overline{W}_{\text{mod}} \\
&\qquad\qquad + 2D_\beta W_{\text{mod}} + 3K_\beta\overline{W}_{\text{mod}} + 2mK_1^{-2}\left(K_{0,\beta}K_1 - K_{1,\beta}\right)\Big] + \text{h.c.}\Big\}.
\end{aligned}
\tag{A.26}
$$

After inserting this into V we find the approximate effective potential Eq. (5.42). We remark that there are subtleties involved: when supersymmetry is broken, the fermion mass matrix has a zero eigenvalue, corresponding to the goldstino direction. Therefore, it is necessary to make the scalar partner of the goldstino very heavy so that its entry in the inverse scalar mass matrix can be neglected and Eq. (5.42) indeed can be used to obtain the leading-order result. However, it would be interesting to find an analogous expression to Eq. (5.42) in cases where this is not possible.

A.3.2 Nearly-supersymmetric stabilization

If the supersymmetric masses are much larger than the supersymmetry breaking scale, $m_F \gg m_{3/2}$, we can expand the inverse mass matrix,

$$m^2_{\alpha\bar\beta} = \left(m_S^2\right)_{\alpha\bar\gamma}\left[\delta^{\bar\gamma}_{\bar\beta} + \left(m_S^{-2}\right)^{\bar\gamma\delta}\left(m_0^2\right)_{\delta\bar\beta}\right] \tag{A.27}$$

to obtain

$$\left(m^{-2}\right)^{\bar\alpha\beta} \approx \left(m_S^{-2}\right)^{\bar\beta\beta}\left[\delta^{\bar\alpha}_{\bar\beta} - \left(m_S^{-2}\right)^{\bar\alpha\delta}\left(m_0^2\right)_{\delta\bar\beta}\right]. \tag{A.28}$$

In this limit the holomorphic terms $m^2_{\alpha\beta}$ are small, so that for the inverse of the mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{\alpha\bar\beta} & m^2_{\alpha\beta} \\ m^2_{\bar\alpha\bar\beta} & m^2_{\bar\alpha\beta} \end{pmatrix}, \tag{A.29}$$

we find

$$\mathcal{M}^{-2} \approx \begin{pmatrix} (m^{-2})^{\bar\beta\gamma} & -(m^{-2})^{\bar\beta\gamma}m^2_{\gamma\beta}(m^{-2})^{\beta\bar\gamma} \\ -(m^{-2})^{\beta\bar\alpha}m^2_{\bar\alpha\bar\beta}(m^{-2})^{\bar\beta\gamma} & (m^{-2})^{\beta\bar\gamma} \end{pmatrix}. \tag{A.30}$$

This result allows us to find the approximate scalar potential in Eq. (5.42).

Bibliography

[1] **ATLAS** Collaboration G. Aad *et al.* "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Phys.Lett.* **B716** (2012) 1–29 [1207.7214].

[2] **CMS** Collaboration S. Chatrchyan *et al.* "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Phys.Lett.* **B716** (2012) 30–61 [1207.7235].

[3] **Particle Data Group** Collaboration K. Olive *et al.* "Review of Particle Physics," *Chin.Phys.* **C38** (2014) 090001.

[4] **ATLAS, CMS** Collaboration T. Yamanaka "Third Generation SUSY Searches at the LHC,".

[5] A. A. Penzias and R. W. Wilson "A Measurement of excess antenna temperature at 4080-Mc/s," *Astrophys.J.* **142** (1965) 419–421.

[6] C. Bennett, A. Banday, K. Gorski, G. Hinshaw, P. Jackson, *et al.* "Four year COBE DMR cosmic microwave background observations: Maps and basic results," *Astrophys.J.* **464** (1996) L1–L4 [astro-ph/9601067].

[7] **WMAP** Collaboration G. Hinshaw *et al.* "Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," *Astrophys.J.Suppl.* **208** (2013) 19 [1212.5226].

[8] **Planck** Collaboration P. Ade *et al.* "Planck 2015 results. XIII. Cosmological parameters," [1502.01589].

[9] **SDSS** Collaboration K. N. Abazajian *et al.* "The Seventh Data Release of the Sloan Digital Sky Survey," *Astrophys.J.Suppl.* **182** (2009) 543–558 [0812.0649].

[10] **Supernova Cosmology Project** Collaboration S. Perlmutter *et al.* "Measurements of Omega and Lambda from 42 high redshift supernovae," *Astrophys.J.* **517** (1999) 565–586 [astro-ph/9812133].

[11] **Supernova Search Team** Collaboration A. G. Riess *et al.* "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astron.J.* **116** (1998) 1009–1038 [astro-ph/9805201].

[12] **BICEP2** Collaboration P. Ade *et al.* "Detection of B-Mode Polarization at Degree Angular Scales by BICEP2," *Phys.Rev.Lett.* **112** (2014) no. 24, 241101 [1403.3985].

[13] **BICEP2, Planck** Collaboration P. Ade *et al.* "Joint Analysis of BICEP2/*KeckArray* and *Planck* Data," *Phys.Rev.Lett.* **114** (2015) no. 10, 101301 [1502.00612].

[14] S. Das, T. Louis, M. R. Nolta, G. E. Addison, E. S. Battistelli, *et al.* "The Atacama Cosmology Telescope: temperature and gravitational lensing power spectrum measurements from three seasons of data," *JCAP* **1404** (2014) 014 [1301.1037].

[15] Z. Hou, C. Reichardt, K. Story, B. Follin, R. Keisler, *et al.* "Constraints on Cosmology from the Cosmic Microwave Background Power Spectrum of the 2500 deg^2 SPT-SZ Survey," *Astrophys.J.* **782** (2014) no. 2, 74 [1212.6267].

[16] A. H. Guth "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Phys.Rev.* **D23** (1981) 347–356.

[17] A. D. Linde "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," *Phys.Lett.* **B108** (1982) 389–393.

[18] A. Albrecht and P. J. Steinhardt "Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking," *Phys.Rev.Lett.* **48** (1982) 1220–1223.

[19] A. D. Linde "Chaotic Inflation," *Phys.Lett.* **B129** (1983) 177–181.

[20] A. D. Linde "Hybrid inflation," *Phys.Rev.* **D49** (1994) 748–754 [astro-ph/9307002].

[21] **Planck** Collaboration P. Ade *et al.* "Planck 2013 results. XVI. Cosmological parameters," *Astron.Astrophys.* **571** (2014) A16 [1303.5076].

[22] **Planck** Collaboration P. Ade *et al.* "Planck 2013 results. XXII. Constraints on inflation," *Astron.Astrophys.* **571** (2014) A22 [1303.5082].

[23] A. A. Starobinsky "A New Type of Isotropic Cosmological Models Without Singularity," *Phys.Lett.* **B91** (1980) 99–102.

[24] P. Binetruy and G. Dvali "D term inflation," *Phys.Lett.* **B388** (1996) 241–246 [hep-ph/9606342].

[25] E. Halyo "Hybrid inflation from supergravity D terms," *Phys.Lett.* **B387** (1996) 43–47 [hep-ph/9606423].

[26] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm "The Heterotic String," *Phys.Rev.Lett.* **54** (1985) 502–505.

[27] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm "Heterotic String Theory. 1. The Free Heterotic String," *Nucl.Phys.* **B256** (1985) 253.

[28] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten "Vacuum Configurations for Superstrings," *Nucl.Phys.* **B258** (1985) 46–74.

[29] T. W. Grimm "The Effective action of type II Calabi-Yau orientifolds," *Fortsch.Phys.* **53** (2005) 1179–1271 [hep-th/0507153].

[30] L. E. Ibanez and A. M. Uranga "String theory and particle physics: An introduction to string phenomenology," (2012).

[31] S. B. Giddings, S. Kachru, and J. Polchinski "Hierarchies from fluxes in string compactifications," *Phys.Rev.* **D66** (2002) 106006 [hep-th/0105097].

[32] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi "De Sitter vacua in string theory," *Phys.Rev.* **D68** (2003) 046005 [hep-th/0301240].

[33] L. B. Anderson, J. Gray, A. Lukas, and B. Ovrut "Stabilizing All Geometric Moduli in Heterotic Calabi-Yau Vacua," *Phys.Rev.* **D83** (2011) 106011 [1102.0011].

[34] M. Cicoli, S. de Alwis, and A. Westphal "Heterotic Moduli Stabilisation," *JHEP* **1310** (2013) 199 [1304.1809].

[35] W. Buchmuller, V. Domcke, and C. Wieck "No-scale D-term inflation with stabilized moduli," *Phys.Lett.* **B730** (2014) 155–160 [1309.3122].

[36] W. Buchmuller, C. Wieck, and M. W. Winkler "Supersymmetric Moduli Stabilization and High-Scale Inflation," *Phys.Lett.* **B736** (2014) 237–240 [1404.2275].

[37] W. Buchmuller, E. Dudas, L. Heurtier, and C. Wieck "Large-Field Inflation and Supersymmetry Breaking," *JHEP* **1409** (2014) 053 [`1407.0253`].

[38] C. Wieck and M. W. Winkler "Inflation with Fayet-Iliopoulos Terms," *Phys.Rev.* **D90** (2014) no. 10, 103507 [`1408.2826`].

[39] W. Buchmuller, E. Dudas, L. Heurtier, A. Westphal, C. Wieck, *et al.* "Challenges for Large-Field Inflation and Moduli Stabilization," [`1501.05812`].

[40] I. Ben-Dayan, S. Jing, A. Westphal, and C. Wieck "Accidental inflation from Khler uplifting," *JCAP* **1403** (2014) 054 [`1309.0529`].

[41] F. Ruehle and C. Wieck "Natural inflation and moduli stabilization in heterotic orbifolds," [`1503.07183`].

[42] E. W. Kolb and M. S. Turner "The Early Universe," *Front.Phys.* **69** (1990) 1–547.

[43] A. D. Linde "Particle physics and inflationary cosmology," *Contemp.Concepts Phys.* **5** (1990) 1–362 [`hep-th/0503203`].

[44] D. Baumann "TASI Lectures on Inflation," (2009) [`0907.5424`].

[45] D. Baumann and L. McAllister "Inflation and String Theory," (2014) [`1404.2601`].

[46] J. Wess and J. Bagger "Supersymmetry and supergravity," (1992).

[47] E. Hubble "A relation between distance and radial velocity among extra-galactic nebulae," *Proc.Nat.Acad.Sci.* **15** (1929) 168–173.

[48] S. Sarkar "Big bang nucleosynthesis and physics beyond the standard model," *Rept.Prog.Phys.* **59** (1996) 1493–1610 [`hep-ph/9602260`].

[49] R. Dicke and P. Peebles "The big bang cosmology: Enigmas and nostrums," (1979).

[50] Y. Zeldovich and M. Y. Khlopov "On the Concentration of Relic Magnetic Monopoles in the Universe," *Phys.Lett.* **B79** (1978) 239–241.

[51] J. Preskill "Cosmological Production of Superheavy Magnetic Monopoles," *Phys.Rev.Lett.* **43** (1979) 1365.

[52] Y. Zeldovich, I. Y. Kobzarev, and L. Okun "Cosmological Consequences of the Spontaneous Breakdown of Discrete Symmetry," *Zh.Eksp.Teor.Fiz.* **67** (1974) 3–11.

[53] **ADMX** Collaboration P. Sikivie "Of Axions, Domain Walls and the Early Universe," *Phys.Rev.Lett.* **48** (1982) 1156–1159.

[54] V. Mukhanov "Physical foundations of cosmology," (2005).

[55] L. Kofman, A. D. Linde, and A. A. Starobinsky "Towards the theory of reheating after inflation," *Phys.Rev.* **D56** (1997) 3258–3295 [`hep-ph/9704452`].

[56] S. P. Martin "A Supersymmetry primer," *Adv.Ser.Direct.High Energy Phys.* **21** (2010) 1–153 [`hep-ph/9709356`].

[57] S. Dimopoulos, S. Raby, and F. Wilczek "Supersymmetry and the Scale of Unification," *Phys.Rev.* **D24** (1981) 1681–1683.

[58] S. Dimopoulos and H. Georgi "Softly Broken Supersymmetry and SU(5)," *Nucl.Phys.* **B193** (1981) 150.

[59] L. E. Ibanez and G. G. Ross "Low-Energy Predictions in Supersymmetric Grand Unified Theories," *Phys.Lett.* **B105** (1981) 439.

[60] P. Ramond "Dual Theory for Free Fermions," *Phys.Rev.* **D3** (1971) 2415–2418.

[61] A. Neveu and J. Schwarz "Factorizable dual model of pions," *Nucl.Phys.* **B31** (1971) 86–112.

[62] M. B. Green, J. Schwarz, and E. Witten "Superstring Theory. Vol. 1: Introduction," *Cambridge Monogr.Math.Phys.* (1987).

[63] K. Freese, J. A. Frieman, and A. V. Olinto "Natural inflation with pseudo - Nambu-Goldstone bosons," *Phys.Rev.Lett.* **65** (1990) 3233–3236.

[64] R. Kallosh "On inflation in string theory," *Lect.Notes Phys.* **738** (2008) 119–156 [`hep-th/0702059`].

[65] A. Westphal "String Cosmology - Large-Field Inflation in String Theory," [`1409.5350`].

[66] D. F. Chernoff and S.-H. H. Tye "Inflation, String Theory and Cosmology," *Int.J.Mod.Phys.* **D24** (2015) 0010 [`1412.0579`].

[67] E. A. Milne "Relativity, Gravitation and World Structure," *Oxford Clarendon Press* (1935).

[68] **Planck** Collaboration P. Ade *et al.* “Planck 2015 results. XX. Constraints on inflation,” [1502.02114].

[69] E. Cremmer and B. Julia “The SO(8) Supergravity,” *Nucl.Phys.* **B159** (1979) 141.

[70] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen “Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and SuperHiggs Effect,” *Nucl.Phys.* **B212** (1983) 413.

[71] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands “False vacuum inflation with Einstein gravity,” *Phys.Rev.* **D49** (1994) 6410–6433 [astro-ph/9401011].

[72] M. Kawasaki, M. Yamaguchi, and T. Yanagida “Natural chaotic inflation in supergravity,” *Phys.Rev.Lett.* **85** (2000) 3572–3575 [hep-ph/0004243].

[73] M. T. Grisaru, M. Rocek, and R. von Unge “Effective Kahler potentials,” *Phys.Lett.* **B383** (1996) 415–421 [hep-th/9605149].

[74] K. A. Intriligator, N. Seiberg, and D. Shih “Dynamical SUSY breaking in meta-stable vacua,” *JHEP* **0604** (2006) 021 [hep-th/0602239].

[75] R. Kallosh and A. Linde “New models of chaotic inflation in supergravity,” *JCAP* **1011** (2010) 011 [1008.3375].

[76] R. Kallosh, A. Linde, and T. Rube “General inflaton potentials in supergravity,” *Phys.Rev.* **D83** (2011) 043507 [1011.5945].

[77] G. Dvali, Q. Shafi, and R. K. Schaefer “Large scale structure and supersymmetric inflation without fine tuning,” *Phys.Rev.Lett.* **73** (1994) 1886–1889 [hep-ph/9406319].

[78] W. Buchmuller, V. Domcke, K. Kamada, and K. Schmitz “Hybrid Inflation in the Complex Plane,” *JCAP* **1407** (2014) 054 [1404.1832].

[79] W. Buchmuller, V. Domcke, and K. Schmitz “Superconformal D-Term Inflation,” *JCAP* **1304** (2013) 019 [1210.4105].

[80] W. Buchmuller, V. Domcke, and K. Schmitz “The Chaotic Regime of D-Term Inflation,” *JCAP* **1411** (2014) no. 11, 006 [1406.6300].

[81] W. Buchmuller and K. Ishiwata “Grand Unification and Subcritical Hybrid Inflation,” [1412.3764].

[82] N. Arkani-Hamed, M. Dine, and S. P. Martin "Dynamical supersymmetry breaking in models with a Green-Schwarz mechanism," *Phys.Lett.* **B431** (1998) 329–338 [hep-ph/9803432].

[83] D. Gallego and M. Serone "Moduli Stabilization in non-Supersymmetric Minkowski Vacua with Anomalous U(1) Symmetry," *JHEP* **0808** (2008) 025 [0807.0190].

[84] M. Grana "Flux compactifications in string theory: A Comprehensive review," *Phys.Rept.* **423** (2006) 91–158 [hep-th/0509003].

[85] M. Nakahara "Geometry, topology and physics," (2003).

[86] S. Gukov, C. Vafa, and E. Witten "CFT's from Calabi-Yau four folds," *Nucl.Phys.* **B584** (2000) 69–108 [hep-th/9906070].

[87] E. Witten "Dimensional Reduction of Superstring Models," *Phys.Lett.* **B155** (1985) 151.

[88] M. B. Green, J. Schwarz, and E. Witten "Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies And Phenomenology," *Cambridge Monogr.Math.Phys.* (1987).

[89] E. Fischbach and C. Talmadge "Ten years of the fifth force," (1996) [hep-ph/9606249].

[90] E. Adelberger, B. R. Heckel, and A. Nelson "Tests of the gravitational inverse square law," *Ann.Rev.Nucl.Part.Sci.* **53** (2003) 77–121 [hep-ph/0307284].

[91] O. Bertolami, J. Paramos, and S. G. Turyshev "General theory of relativity: Will it survive the next decade?," (2006) [gr-qc/0602016].

[92] G. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross "Cosmological Problems for the Polonyi Potential," *Phys.Lett.* **B131** (1983) 59.

[93] M. Dine, W. Fischler, and D. Nemeschansky "Solution of the Entropy Crisis of Supersymmetric Theories," *Phys.Lett.* **B136** (1984) 169.

[94] T. Banks, D. B. Kaplan, and A. E. Nelson "Cosmological implications of dynamical supersymmetry breaking," *Phys.Rev.* **D49** (1994) 779–787 [hep-ph/9308292].

[95] B. de Carlos, J. Casas, F. Quevedo, and E. Roulet "Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings," *Phys.Lett.* **B318** (1993) 447–456 [hep-ph/9308325].

[96] V. Balasubramanian and P. Berglund "Stringy corrections to Kahler potentials, SUSY breaking, and the cosmological constant problem," *JHEP* **0411** (2004) 085 [hep-th/0408054].

[97] A. Westphal "de Sitter string vacua from Kahler uplifting," *JHEP* **0703** (2007) 102 [hep-th/0611332].

[98] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **0503** (2005) 007 [hep-th/0502058].

[99] J. P. Conlon, F. Quevedo, and K. Suruliz "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking," *JHEP* **0508** (2005) 007 [hep-th/0505076].

[100] E. Dudas, A. Linde, Y. Mambrini, A. Mustafayev, and K. A. Olive "Strong moduli stabilization and phenomenology," *Eur.Phys.J.* **C73** (2013) no. 1, 2268 [1209.0499].

[101] J. L. Evans, M. A. Garcia, and K. A. Olive "The Moduli and Gravitino (non)-Problems in Models with Strongly Stabilized Moduli," *JCAP* **1403** (2014) 022 [1311.0052].

[102] R. Kallosh and A. D. Linde "Landscape, the scale of SUSY breaking, and inflation," *JHEP* **0412** (2004) 004 [hep-th/0411011].

[103] H. Abe, T. Higaki, and T. Kobayashi "Remark on integrating out heavy moduli in flux compactification," *Phys.Rev.* **D74** (2006) 045012 [hep-th/0606095].

[104] E. Witten "Nonperturbative superpotentials in string theory," *Nucl.Phys.* **B474** (1996) 343–360 [hep-th/9604030].

[105] V. Novikov, M. A. Shifman, A. Vainshtein, M. Voloshin, and V. I. Zakharov "Supersymmetry Transformations of Instantons," *Nucl.Phys.* **B229** (1983) 394.

[106] V. Novikov, M. A. Shifman, A. Vainshtein, and V. I. Zakharov "Instanton Effects in Supersymmetric Theories," *Nucl.Phys.* **B229** (1983) 407.

[107] M. Dine, R. Rohm, N. Seiberg, and E. Witten "Gluino Condensation in Superstring Models," *Phys.Lett.* **B156** (1985) 55.

[108] S. Ferrara, L. Girardello, and H. P. Nilles "Breakdown of Local Supersymmetry Through Gauge Fermion Condensates," *Phys.Lett.* **B125** (1983) 457.

[109] R. Kallosh and T. Wrase "Emergence of Spontaneously Broken Supersymmetry on an Anti-D3-Brane in KKLT dS Vacua," *JHEP* **1412** (2014) 117 [`1411.1121`].

[110] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen, and T. Wrase "$\overline{\mathrm{D3}}$ and dS," [`1502.07627`].

[111] J. Polonyi "Generalization of the Massive Scalar Multiplet Coupling to the Supergravity," (1977).

[112] H. P. Nilles "Supersymmetry, Supergravity and Particle Physics," *Phys.Rept.* **110** (1984) 1–162.

[113] O. Lebedev, H. P. Nilles, and M. Ratz "De Sitter vacua from matter superpotentials," *Phys.Lett.* **B636** (2006) 126–131 [`hep-th/0603047`].

[114] L. O'Raifeartaigh "Spontaneous Symmetry Breaking for Chiral Scalar Superfields," *Nucl.Phys.* **B96** (1975) 331.

[115] R. Kallosh and A. D. Linde "O'KKLT," *JHEP* **0702** (2007) 002 [`hep-th/0611183`].

[116] G. Coughlan, R. Holman, P. Ramond, and G. G. Ross "Supersymmetry and the Entropy Crisis," *Phys.Lett.* **B140** (1984) 44.

[117] S. de Alwis and K. Givens "Physical Vacua in IIB Compactifications with a Single Kaehler Modulus," *JHEP* **1110** (2011) 109 [`1106.0759`].

[118] M. Rummel and A. Westphal "A sufficient condition for de Sitter vacua in type IIB string theory," *JHEP* **1201** (2012) 020 [`1107.2115`].

[119] J. Louis, M. Rummel, R. Valandro, and A. Westphal "Building an explicit de Sitter," *JHEP* **1210** (2012) 163 [`1208.3208`].

[120] K. Becker, M. Becker, M. Haack, and J. Louis "Supersymmetry breaking and alpha-prime corrections to flux induced potentials," *JHEP* **0206** (2002) 060 [`hep-th/0204254`].

[121] T. W. Grimm and J. Louis "The Effective action of N = 1 Calabi-Yau orientifolds," *Nucl.Phys.* **B699** (2004) 387–426 [`hep-th/0403067`].

[122] M. Cicoli, J. P. Conlon, and F. Quevedo "Dark radiation in LARGE volume models," *Phys.Rev.* **D87** (2013) no. 4, 043520 [`1208.3562`].

[123] W. Buchmuller, K. Hamaguchi, O. Lebedev, and M. Ratz "Dilaton destabilization at high temperature," *Nucl.Phys.* **B699** (2004) 292–308 [`hep-th/0404168`].

[124] J. P. Conlon, R. Kallosh, A. D. Linde, and F. Quevedo "Volume Modulus Inflation and the Gravitino Mass Problem," *JCAP* **0809** (2008) 011 [0806.0809].

[125] S. Mooij and M. Postma "Hybrid inflation with moduli stabilization and low scale supersymmetry breaking," *JCAP* **1006** (2010) 012 [1001.0664].

[126] T. He, S. Kachru, and A. Westphal "Gravity waves and the LHC: Towards high-scale inflation with low-energy SUSY," *JHEP* **1006** (2010) 065 [1003.4265].

[127] T. Kobayashi and M. Sakai "Inflation, moduli (de)stabilization and supersymmetry breaking," *JHEP* **1104** (2011) 121 [1012.2187].

[128] S. Antusch, K. Dutta, and S. Halter "Combining High-scale Inflation with Low-energy SUSY," *JHEP* **1203** (2012) 105 [1112.4488].

[129] L. Carlevaro and J.-P. Derendinger "Five-brane thresholds and membrane instantons in four-dimensional heterotic M-theory," *Nucl.Phys.* **B736** (2006) 1–33 [hep-th/0502225].

[130] R. Kappl, H. P. Nilles, and M. W. Winkler "Natural Inflation and Low Energy Supersymmetry," [1503.01777].

[131] M. B. Green and J. H. Schwarz "Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory," *Phys.Lett.* **B149** (1984) 117–122.

[132] R. Blumenhagen, M. Cvetic, S. Kachru, and T. Weigand "D-Brane Instantons in Type II Orientifolds," *Ann.Rev.Nucl.Part.Sci.* **59** (2009) 269–296 [0902.3251].

[133] S. Hamidi and C. Vafa "Interactions on Orbifolds," *Nucl.Phys.* **B279** (1987) 465.

[134] F. Brummer, R. Kappl, M. Ratz, and K. Schmidt-Hoberg "Approximate R-symmetries and the mu term," *JHEP* **1004** (2010) 006 [1003.0084].

[135] E. Dudas, Y. Mambrini, S. Pokorski, and A. Romagnoni "Moduli stabilization with Fayet-Iliopoulos uplift," *JHEP* **0804** (2008) 015 [0711.4934].

[136] W. Buchmuller, L. Covi, and D. Delepine "Inflation and supersymmetry breaking," *Phys.Lett.* **B491** (2000) 183–189 [hep-ph/0006168].

[137] K. Nakayama, F. Takahashi, and T. T. Yanagida "Constraint on the gravitino mass in hybrid inflation," *JCAP* **1012** (2010) 010 [1007.5152].

[138] C. Pallis and Q. Shafi "Update on Minimal Supersymmetric Hybrid Inflation in Light of PLANCK," *Phys.Lett.* **B725** (2013) 327–333 [1304.5202].

[139] W. Buchmuller, V. Domcke, K. Kamada, and K. Schmitz "A Minimal Supersymmetric Model of Particle Physics and the Early Universe," [1309.7788].

[140] S. C. Davis and M. Postma "Successfully combining SUGRA hybrid inflation and moduli stabilisation," *JCAP* **0804** (2008) 022 [0801.2116].

[141] S. C. Davis and M. Postma "SUGRA chaotic inflation and moduli stabilisation," *JCAP* **0803** (2008) 015 [0801.4696].

[142] R. Kallosh, A. Linde, K. A. Olive, and T. Rube "Chaotic inflation and supersymmetry breaking," *Phys.Rev.* **D84** (2011) 083519 [1106.6025].

[143] L. Aparicio, D. Cerdeno, and L. Ibanez "Modulus-dominated SUSY-breaking soft terms in F-theory and their test at LHC," *JHEP* **0807** (2008) 099 [0805.2943].

[144] H. Abe, S. Aoki, F. Hasegawa, and Y. Yamada "Illustrating SUSY breaking effects on various inflation mechanisms," *JHEP* **1501** (2015) 026 [1408.4875].

[145] K. Nakayama, F. Takahashi, and T. T. Yanagida "Gravitino Problem in Supergravity Chaotic Inflation and SUSY Breaking Scale after BICEP2," *Phys.Lett.* **B734** (2014) 358–363 [1404.2472].

[146] L. E. Ibanez and I. Valenzuela "BICEP2, the Higgs Mass and the SUSY-breaking Scale," *Phys.Lett.* **B734** (2014) 354–357 [1403.6081].

[147] E. Palti and T. Weigand "Towards large r from [p, q]-inflation," *JHEP* **1404** (2014) 155 [1403.7507].

[148] A. Hebecker, S. C. Kraus, and L. T. Witkowski "D7-Brane Chaotic Inflation," *Phys.Lett.* **B737** (2014) 16–22 [1404.3711].

[149] V. Akulov and D. Volkov "Goldstone fields with spin 1/2," *Theor.Math.Phys.* **18** (1974) 28.

[150] F. Marchesano, G. Shiu, and A. M. Uranga "F-term Axion Monodromy Inflation," *JHEP* **1409** (2014) 184 [1404.3040].

[151] T. W. Grimm "Axion Inflation in F-theory," *Phys.Lett.* **B739** (2014) 201–208 [1404.4268].

[152] L. E. Ibanez and I. Valenzuela "The inflaton as an MSSM Higgs and open string modulus monodromy inflation," *Phys.Lett.* **B736** (2014) 226–230 [1404.5235].

[153] R. Blumenhagen, D. Herschmann, and E. Plauschinn "The Challenge of Realizing F-term Axion Monodromy Inflation in String Theory," *JHEP* **1501** (2015) 007 [1409.7075].

[154] A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski "Tuning and Back-reaction in F-term Axion Monodromy Inflation," [1411.2032].

[155] L. E. Ibanez, F. Marchesano, and I. Valenzuela "Higgs-otic Inflation and String Theory," *JHEP* **1501** (2015) 128 [1411.5380].

[156] I. Garca-Etxebarria, T. W. Grimm, and I. Valenzuela "Special Points of Inflation in Flux Compactifications," [1412.5537].

[157] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma, *et al.* "de Sitter vacua in no-scale supergravities and Calabi-Yau string models," *JHEP* **0806** (2008) 057 [0804.1073].

[158] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma, *et al.* "Constraints on modular inflation in supergravity and string theory," *JHEP* **0808** (2008) 055 [0805.3290].

[159] E. Dudas "Three-form multiplet and Inflation," *JHEP* **1412** (2014) 014 [1407.5688].

[160] M. Cicoli, J. P. Conlon, A. Maharana, and F. Quevedo "A Note on the Magnitude of the Flux Superpotential," *JHEP* **1401** (2014) 027 [1310.6694].

[161] A. Linde, M. Noorbala, and A. Westphal "Observational consequences of chaotic inflation with nonminimal coupling to gravity," *JCAP* **1103** (2011) 013 [1101.2652].

[162] K. Dasgupta, C. Herdeiro, S. Hirano, and R. Kallosh "D3 / D7 inflationary model and M theory," *Phys.Rev.* **D65** (2002) 126002 [hep-th/0203019].

[163] R. Kallosh and A. D. Linde "P term, D term and F term inflation," *JCAP* **0310** (2003) 008 [hep-th/0306058].

[164] J. P. Hsu, R. Kallosh, and S. Prokushkin "On brane inflation with volume stabilization," *JCAP* **0312** (2003) 009 [hep-th/0311077].

[165] F. Koyama, Y. Tachikawa, and T. Watari "Supergravity analysis of hybrid inflation model from D3 - D7 system," *Phys.Rev.* **D69** (2004) 106001 [hep-th/0311191].

[166] K. Dasgupta, J. P. Hsu, R. Kallosh, A. D. Linde, and M. Zagermann "D3/D7 brane inflation and semilocal strings," *JHEP* **0408** (2004) 030 [`hep-th/0405247`].

[167] P. Chen, K. Dasgupta, K. Narayan, M. Shmakova, and M. Zagermann "Brane inflation, solitons and cosmological solutions: 1.," *JHEP* **0509** (2005) 009 [`hep-th/0501185`].

[168] P. Brax, C. van de Bruck, A. Davis, S. C. Davis, R. Jeannerot, *et al.* "Moduli corrections to D-term inflation," *JCAP* **0701** (2007) 026 [`hep-th/0610195`].

[169] W. Buchmuller, V. Domcke, and K. Kamada "The Starobinsky Model from Superconformal D-Term Inflation," *Phys.Lett.* **B726** (2013) 467–470 [`1306.3471`].

[170] Z. Komargodski and N. Seiberg "Comments on the Fayet-Iliopoulos Term in Field Theory and Supergravity," *JHEP* **0906** (2009) 007 [`0904.1159`].

[171] K. R. Dienes and B. Thomas "On the Inconsistency of Fayet-Iliopoulos Terms in Supergravity Theories," *Phys.Rev.* **D81** (2010) 065023 [`0911.0677`].

[172] Z. Komargodski and N. Seiberg "Comments on Supercurrent Multiplets, Supersymmetric Field Theories and Supergravity," *JHEP* **1007** (2010) 017 [`1002.2228`].

[173] N. Seiberg "Modifying the Sum Over Topological Sectors and Constraints on Supergravity," *JHEP* **1007** (2010) 070 [`1005.0002`].

[174] J. Distler and E. Sharpe "Quantization of Fayet-Iliopoulos Parameters in Supergravity," *Phys.Rev.* **D83** (2011) 085010 [`1008.0419`].

[175] F. Catino, G. Villadoro, and F. Zwirner "On Fayet-Iliopoulos terms and de Sitter vacua in supergravity: Some easy pieces," *JHEP* **1201** (2012) 002 [`1110.2174`].

[176] P. Fayet and J. Iliopoulos "Spontaneously Broken Supergauge Symmetries and Goldstone Spinors," *Phys.Lett.* **B51** (1974) 461–464.

[177] D. Z. Freedman and A. Van Proeyen "Supergravity," (2012).

[178] V. Domcke, K. Schmitz, and T. T. Yanagida "Dynamical D-Terms in Supergravity," *Nucl.Phys.* **B891** (2015) 230–258 [`1410.4641`].

[179] S. Ferrara, R. Kallosh, A. Linde, A. Marrani, and A. Van Proeyen "Superconformal Symmetry, NMSSM, and Inflation," *Phys.Rev.* **D83** (2011) 025008 [`1008.2942`].

[180] M. Dine, N. Seiberg, and E. Witten "Fayet-Iliopoulos Terms in String Theory," *Nucl.Phys.* **B289** (1987) 589.

[181] A. Achucarro, B. de Carlos, J. Casas, and L. Doplicher “De Sitter vacua from uplifting D-terms in effective supergravities from realistic strings,” *JHEP* **0606** (2006) 014 [hep-th/0601190].

[182] J. Casas, J. Moreno, C. Munoz, and M. Quiros “Cosmological Implications of an Anomalous U(1): Inflation, Cosmic Strings and Constraints on Superstring Parameters,” *Nucl.Phys.* **B328** (1989) 272.

[183] D. H. Lyth and A. Riotto “Particle physics models of inflation and the cosmological density perturbation,” *Phys.Rept.* **314** (1999) 1–146 [hep-ph/9807278].

[184] P. Binetruy, G. Dvali, R. Kallosh, and A. Van Proeyen “Fayet-Iliopoulos terms in supergravity and cosmology,” *Class.Quant.Grav.* **21** (2004) 3137–3170 [hep-th/0402046].

[185] E. Dudas and S. K. Vempati “Large D-terms, hierarchical soft spectra and moduli stabilisation,” *Nucl.Phys.* **B727** (2005) 139–162 [hep-th/0506172].

[186] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski “Soft supersymmetry breaking in KKLT flux compactification,” *Nucl.Phys.* **B718** (2005) 113–133 [hep-th/0503216].

[187] G. Villadoro and F. Zwirner “De-Sitter vacua via consistent D-terms,” *Phys.Rev.Lett.* **95** (2005) 231602 [hep-th/0508167].

[188] T. Taylor, G. Veneziano, and S. Yankielowicz “Supersymmetric QCD and Its Massless Limit: An Effective Lagrangian Analysis,” *Nucl.Phys.* **B218** (1983) 493.

[189] I. Affleck, M. Dine, and N. Seiberg “Dynamical Supersymmetry Breaking in Four-Dimensions and Its Phenomenological Implications,” *Nucl.Phys.* **B256** (1985) 557.

[190] A. Hebecker, S. C. Kraus, M. Kuntzler, D. Lust, and T. Weigand “Fluxbranes: Moduli Stabilisation and Inflation,” *JHEP* **1301** (2013) 095 [1207.2766].

[191] R. Blumenhagen, M. Cvetic, and T. Weigand “Spacetime instanton corrections in 4D string vacua: The Seesaw mechanism for D-Brane models,” *Nucl.Phys.* **B771** (2007) 113–142 [hep-th/0609191].

[192] L. Ibanez and A. Uranga “Neutrino Majorana Masses from String Theory Instanton Effects,” *JHEP* **0703** (2007) 052 [hep-th/0609213].

[193] P. Binetruy and E. Dudas “Gaugino condensation and the anomalous U(1),” *Phys.Lett.* **B389** (1996) 503–509 [hep-th/9607172].

[194] L. Randall and R. Sundrum "Out of this world supersymmetry breaking," *Nucl.Phys.* **B557** (1999) 79–118 [hep-th/9810155].

[195] R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, E. Plauschinn, *et al.* "A Flux-Scaling Scenario for High-Scale Moduli Stabilization in String Theory," [1503.07634].

[196] A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski "Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?," [1503.07912].

[197] V. Domcke "Matter, Dark Matter and Gravitational Waves from a GUT-Scale U(1) Phase Transition," (2013).

[198] T. Rudelius "Constraints on Axion Inflation from the Weak Gravity Conjecture," [1503.00795].

[199] M. Montero, A. M. Uranga, and I. Valenzuela "Transplanckian axions !?," [1503.03886].

[200] J. Brown, W. Cottrell, G. Shiu, and P. Soler "Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation," [1503.04783].

[201] X. Chen and Y. Wang "Large non-Gaussianities with Intermediate Shapes from Quasi-Single Field Inflation," *Phys.Rev.* **D81** (2010) 063511 [0909.0496].

[202] X. Chen and Y. Wang "Quasi-Single Field Inflation and Non-Gaussianities," *JCAP* **1004** (2010) 027 [0911.3380].

[203] C. T. Byrnes, K. Enqvist, and T. Takahashi "Scale-dependence of Non-Gaussianity in the Curvaton Model," *JCAP* **1009** (2010) 026 [1007.5148].

[204] D. Baumann and D. Green "Signatures of Supersymmetry from the Early Universe," *Phys.Rev.* **D85** (2012) 103520 [1109.0292].

[205] V. Assassi, D. Baumann, and D. Green "On Soft Limits of Inflationary Correlation Functions," *JCAP* **1211** (2012) 047 [1204.4207].

[206] T. Noumi, M. Yamaguchi, and D. Yokoyama "Effective field theory approach to quasi-single field inflation and effects of heavy fields," *JHEP* **1306** (2013) 051 [1211.1624].

[207] N. Arkani-Hamed and J. Maldacena "Cosmological Collider Physics," [1503.08043].

[208] G. Aldazabal, D. Badagnani, L. E. Ibanez, and A. Uranga "Tadpole versus anomaly cancellation in D = 4, D = 6 compact IIB orientifolds," *JHEP* **9906** (1999) 031 [hep-th/9904071].

[209] S. K. Soni and H. A. Weldon "Analysis of the Supersymmetry Breaking Induced by N=1 Supergravity Theories," *Phys.Lett.* **B126** (1983) 215.

[210] V. S. Kaplunovsky and J. Louis "Model independent analysis of soft terms in effective supergravity and in string theory," *Phys.Lett.* **B306** (1993) 269–275 [hep-th/9303040].

[211] S. Ferrara, C. Kounnas, and F. Zwirner "Mass formulae and natural hierarchy in string effective supergravities," *Nucl.Phys.* **B429** (1994) 589–625 [hep-th/9405188].

Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Hamburg, den 4. Mai 2015 Unterschrift