

# ESSAYS IN FINANCIAL RISK MANAGEMENT AND DERIVATIVE PRICING

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*To my parents  
for continuous love and support*



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<b>ABM</b>	arithmetic Brownian motion.....	170
<b>ADF</b>	Augmented Dickey-Fuller .....	36
<b>BDI</b>	Baltic Dry Index .....	18
<b>BIFFEX</b>	Baltic International Freight Futures Exchange .....	87
<b>BOA</b>	Baltic Option Assessment .....	91
<b>BSPA</b>	Baltic Sale and Purchase Assessment .....	19
<b>CAPM</b>	Capital Asset Pricing Model.....	89
<b>CFS</b>	closed-form solution .....	192
<b>CI</b>	confidence interval .....	194
<b>CSL</b>	Clarkson Securities Limited.....	19
<b>DWT</b>	deadweight tons .....	6
<b>EMH</b>	Efficient Market Hypothesis .....	21
<b>EU</b>	European Union.....	82
<b>FFA</b>	Forward Freight Agreement.....	3
<b>FoSVA</b>	Forward Ship Value Agreement.....	19
<b>GBM</b>	geometric Brownian motion.....	4
<b>IAS</b>	International Accounting Standards .....	18
<b>IMO</b>	International Maritime Organization.....	33
<b>IFRS</b>	International Financial Reporting Standards.....	18
<b>LIBOR</b>	London Interbank Offered Rate.....	29
<b>LPM</b>	lower partial moment .....	7
<b>MA</b>	moving average.....	25
<b>MAR</b>	minimum acceptable return .....	105
<b>MC</b>	Monte Carlo.....	4
<b>MVCHM</b>	minimum-variance cross-hedging model .....	6
<b>NIG</b>	normal inverse Gaussian.....	167
<b>OTC</b>	over-the-counter .....	67

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**OU** Ornstein-Uhlenbeck ..... 171  
**PDE** partial differential equation ..... 5  
**PDF** probability density function ..... 104  
**PP** Phillips-Perron ..... 36  
**SBIC** Schwarz-Bayes Information Criterion ..... 111  
**SE** standard error ..... 192  
**SIN** Clarksons Shipping Intelligence Network ..... 22  
**SPAN** Standard Portfolio Analysis of Risk ..... 97  
**SPM** structural pricing model ..... 3  
**USD** United States Dollar ..... 29  
**VLOC** very large ore carrier ..... 33  
**WFR** Clarkson Research World Fleet Register ..... 108

## CHAPTER 1

# SYNOPSIS

This cumulative dissertation consists of three individual essays that are generally concerned with the following two, broader topics: financial risk management and derivative pricing. Within part I, the first two essays (i.e., chapters 2 and 3) focus on financial risk management in the shipping industry. Within part II, the third essay (i.e., chapter 4) focuses on derivative pricing for Asian options. The first two essays have a clear focus on shipping markets, whereas the third essay is generally applicable to a broad set of market settings (including shipping markets). However, the numerical example within this essay relies on model parameters estimated from shipping market data. Not only that the two parts, financial risk management and derivative pricing, are anyway intimately connected with each other, but all three essays also share at least a partial connection to shipping markets. The following sections elaborate on the general motivation for the dissertation as well as on the motivation, research questions, approach, results, and contribution to the academic literature of the three individual essays.

## 1.1 General motivation

Concerning part I of the dissertation, financial risk management refers to identifying exposure to risk factors, such as market or credit risk, and managing (i.e., hedging) this exposure using financial instruments, such as derivatives. Financial risk management and the potential associated benefit for corporations are the subject of numerous efforts in the academic literature following the fundamental work of Smith and Stulz (1985) who study the question whether risk management does add value for a company. They conclude that a value-maximizing corporation may decide to hedge for three reasons – namely, costs of financial distress, managerial risk aversion, and taxes. Smithson and Simkins (2005) provide a comprehensive overview on recent studies whether there is a relation between active risk management and firm value. They find empirical evidence for reduced risk for derivative-using financial institutions and industrial firms as well as a positive effect on firm value for industrial firms using interest rate and currency derivatives. Concerning commodity price risks, hedging is apparently only beneficial for commodity users but for not producers. Stulz (1996) adds another perspective to the classical ‘minimum-variance’ approach in risk management by rather focusing on the prevention of lower-tail outcomes that increase financial distress cost as primary goal of risk management. Active financial risk management reduces the expected cost of financial distress and thus, may help corporations to attain their preferred capital and/or ownership structure. Shleifer and Vishny (1992) find that distressed companies receive only substandard prices in liquidation or fire sales of assets to improve their liquidity situation due to the fact that their industry peers and potential asset buyers are also suffering from the same unfavorable market conditions. As a result, illiquid market conditions cause asset prices to plummet in bad times which in turn increase the cost of financial distress.

With respect to the shipping industry, it should be noted that the industry is historically characterized by comparatively high volatility of both freight rates as well as ship prices. Albertijn et al. (2011) offer a broad perspective on risk management practices within the industry, especially in the light of the recent financial crisis. For listed shipping companies, they foresee the increasing need to comply with latest accounting standards (e.g., fair value accounting) making the volatility of asset prices (i.e., the company’s ships or fleet in their balance sheet) more transparent. This might force shipping companies to recognize considerable impairment losses in bad

times that potentially threaten the financial stability of the company. Simultaneously, financial institutions providing funds for the industry are expected to hold substantially more equity for their comparatively risky shipping loans under the new Basel III regulations. Consequently, these institutions probably either tighten their loan commitment in the shipping industry or increase their demands with respect to securities or collaterals of their loans. Altogether, this urges shipping companies to practice diligent financial risk management in this new environment (Albertijn et al., 2011). Another complicating factor in this context is the typically relatively high leverage ratio of shipping companies leaving only little potential to cover extreme losses which increases financial distress cost (Drobetz et al., 2013).

The first and second essay address this issue and examine the hedge effectiveness of freight derivatives to cross-hedge dry bulk Capesize ship price risks in order to protect the shipping company's balance sheet from impairment losses through adverse ship price fluctuations. Unfortunately, no direct, liquid hedge instruments on ship prices currently exist. As a consequence, shipping companies are limited to cross-hedging ship price risks using freight derivatives. The first essay focuses on Forward Freight Agreements (FFAs) as hedge instruments, while the second essay concentrates on freight options exploiting their asymmetric payoff structure. Both the first and the second essay rely on a structural pricing model (SPM) estimated from past vessel transactions in order to determine the desired hedge exposure. For a little more detailed perspective on the individual motivation, research questions, approach, results, and contribution to the academic literature, please see the following sections 1.2 and 1.3.

Concerning part II of the dissertation, successful hedging of market or price risks using derivative instruments requires a sound understanding of fair prices for such instruments. Otherwise, the chosen hedging strategy might turn out as less effective than intended. Accordingly, the two parts of the dissertation are intimately connected with each other as initially stated. The asymmetric payoff structure of options renders them somewhat more difficult to price than forwards or futures. Additionally, options come in different styles with respect to their exercise modalities. Typically, three broad categories are distinguished: classical European or 'plain vanilla' options, American options, and so-called 'exotic' options. European options may only be exercised at the maturity date, while American options may be exercised at any point in time before the maturity date. The category of 'exotic' options comprises the set of options with

more complex payoff profiles, such as, for instance, Asian options (Hull, 2012).

Asian options are path-dependent options where the payoff is not determined by the price of the underlying at the maturity date alone but determined by the average of the underlying price within the delivery period (i.e., from a certain period of time prior to the maturity date until the maturity date) (Hull, 2012). This type of option allows companies to hedge continuous risks or exposures, such as the average interest rate or average cost across the accounting year. According to Longstaff (1995), an option on average interest rates is far more cost-effective for hedging purposes than a set of individual, standard interest rate options. Accordingly, they offer a cheaper way to hedge regular, periodic cash flows (Zhang, 1998). Additionally, these kind of options are usually preferred in less liquid markets (e.g., commodities) to prevent any price manipulations of the underlying close to maturity, while European and American option are typically preferred in classical, liquid financial markets (e.g., equities, bonds, or currencies). As a consequence of these characteristics, Asian options have gained much interest by market participants in terms of traded volume over the years (Zhang, 1998).

Due to the inherent challenges to accurately price them, Asian options have also caught some attention in the academic literature over the years. Black and Scholes (1973) provide a paradigm shift with their closed-form solution for European options under their Black and Scholes (1973) price dynamics assuming that the spot price follows a geometric Brownian motion (GBM). This liberated market participants from the need to apply complex and, at that time, computationally demanding numerical methods, such as Monte Carlo (MC) simulations. Since then, numerous studies on more complex price dynamics than the GBM describing the spot price as well as on the valuation of options of different types under such price dynamics have been published (see, for instance, Heston (1993), Schwartz (1997), or Schwartz and Smith (2000)). However, closed-form solutions remain often restricted to options of European type or to comparatively simple price dynamics assumed for the underlying spot price. For Asian options, Kemna and Vorst (1990) find a closed-form solution for geometric Asian options (i.e., the average within the delivery period is computed as geometric average) under Black and Scholes (1973) price dynamics. For the more common arithmetic Asian options (i.e., the average within the delivery period is computed as arithmetic average), they propose a MC control variate simulation approach. This variance-reduction technique for MC simulations reduces the computational effort

considerably due to the almost perfect correlation between arithmetic and geometric average. Alternatively, market participants can also apply semi-analytical solutions relying on numerical methods (see, for instance, Carverhill and Clewlow (1990) or Geman and Yor (1993)), approximate closed-form solutions (see, for instance, Turnbull and Wakeman (1991)), or partial differential equation (PDE) methods using finite differences (see, for instance, Rogers and Shi (1995), Alziary et al. (1997), or Zhang (2001)).

The third essay extends the approach of Kemna and Vorst (1990) – developing a closed-form solution for a geometric Asian option which then is applied as control variate in a MC simulation – to certain other class of price dynamics of the underlying spot price. A general pricing framework for geometric Asian options is proposed that is applicable to the entire set of affine  $n$ -factor Gaussian diffusions. The fact that the geometric average of a normally distributed variable is itself normally distributed allows to find closed-form solutions for geometric Asian options for affine Gaussian diffusions (see, for instance, Hull (2012), Kemna and Vorst (1990), or Zhang (1998)). Besides, the almost perfect correlation of the arithmetic and geometric average predestines the application of a MC control variate simulation approach to price arithmetic Asian options. For a little more detailed perspective on the motivation, research question, approach, results, and contribution to the academic literature of the third essay, please see the following section 1.4.

## 1.2 Hedging Capesize ship price risks using Forward Freight Agreements

The first essay is concerned with hedging dry bulk Capesize ship price risks using FFAs. As already mentioned, the shipping industry is historically known for the volatile nature of freight rates and also second-hand ship prices. Together with the increasing need to comply with fair value accounting principles that might cause the recognition of impairment losses on their balance sheets, shipping companies need effective strategies to hedge against such adverse ship price fluctuations. The aim of the first essay is to examine potential hedging approaches and empirically assess their hedge effectiveness.

As no direct, liquid hedge instruments for ship prices exist, shipping companies are restricted to using freight derivatives, such as FFAs, as cross-hedge instruments. With respect to deriving the desired hedge exposure, two different approaches are compared within the study. On the one hand, the idea of Alizadeh and Nomikos (2012) is translated into a minimum-variance cross-hedging model (MVCHM) basically relying on weekly time series data of panelists' estimations of second-hand prices for dry bulk Capesize reference vessels. On the other hand, a SPM based on the idea of Adland and Koekebakker (2007) is estimated from real dry bulk Capesize sale and purchase transactions for the competing hedging approach. The model includes ship-specific, deterministic factors as well as market-driven or risk factors, such as the FFA rate or slope of the FFA curve as well as interaction terms. Both approaches are empirically tested for their hedge effectiveness in two different hedging set-ups as well as in further robustness checks. Following the well-known effort of Ederington (1979), the hedge effectiveness is measured in terms of variance reduction.

The results suggest that the MVCHM achieves empirically only a hedge effectiveness of about 67 % variance reduction over a time horizon of one year, whereas Alizadeh and Nomikos (2012) claim variance-reduction levels of more than 85 %. Concerning the SPM, the second-hand price of dry bulk Capesize vessels may be well described by market-driven explanatory variables, such as the FFA+1CAL<sup>1</sup> rate as well as the slope between the FFA+2CAL and FFA+1CAL rate, and deterministic, ship-specific explanatory factors, such as the age, deadweight tons (DWT), speed, and consumption of the individual vessel. In terms of hedge effectiveness, the SPM-based approach consistently outperforms the MVCHM-based approach and achieves a variance reduction of about 77 % over a time horizon of one year.

The first essay contributes to the academic literature in various ways. This is the first empirical study testing the MVCHM-based approach suggested by Alizadeh and Nomikos (2012). Moreover, the study first considers additional explanatory variables other than simple FFA rates, vessel age, or size in DWT in a SPM. Finally, the study shows that the proposed SPM-based approach consistently outperforms the MVCHM-based approach in terms of variance reduction and thus, provides shipping companies with more a effective way to determine the desired hedge exposure and hedge their ship price risks.

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<sup>1</sup> FFA+1CAL contracts, for instance, are next calendar-year FFA contracts.

## 1.3 Hedging Capesize ship price risks using freight options

The second essay is concerned with hedging dry bulk Capesize ship price risks using freight options. The basic motivation for the study is largely similar to the one of the first essay. However, the second essay focuses more on eliminating downside risks rather than trying to completely offset ship price fluctuations via FFA-based cross-hedging. Accordingly, the aim of this paper is to empirically assess the hedge effectiveness of different freight option-based cross-hedging strategies using several risk-, downside-risk-, as well as return-based measures.

With respect to deriving the desired hedge exposure, a SPM-based approach is chosen again estimated from real dry bulk Capesize sale and purchase transactions. The hedge effectiveness of different freight option hedging strategies (i.e., long at-the-money put, long 10 % out-of-the-money put, replicated FFA, and zero-cost collar) is empirically tested in a hedging set-up over a fixed time horizon one year prior to the sale for the same dry bulk Capesize sale and purchase transactions. The simple FFA-based hedging strategy serves as reference. In terms of hedge-effectiveness measures, the pure variance-reduction perspective of Ederington (1979) fails to accurately assess the one-sided option-based hedging strategies as they only eliminate downside risk and still allow for positive variation. Accordingly, additional measures based on the risk-return perspective (i.e., the revised measure of Howard and D'Antonio (1987) which is largely based on the concept of the Sharpe (1966) ratio), downside-risk perspective (i.e., lower partial moment (LPM) measures which were brought to portfolio theory by Bawa (1975), Bawa and Lindenberg (1977), Fishburn (1977), and Bawa (1978)), and combined perspective of downside risk and return (i.e., the Sortino ratio developed by Sortino and Price (1994)) are computed. This broad set of hedge-effectiveness measures allows to reflect different risk and return preferences of shipping companies. The robustness of the presented results and findings is checked for two subsets of the data set as well as in an alternative hedging set-up.

The results suggest that freight options generally qualify quite well as cross-hedge instruments for dry bulk Capesize ship price risks. Specifically, one-sided option-based hedging strategies show no inferior performance from a pure risk or downside-risk perspective. From a risk-return perspective as well as from a combined perspective

of downside risk and return, they perform worse than the classical two-sided hedging strategies. This, however, turns out to be somewhat caused by the data set of transactions used within the study consisting of relatively few vessels that would benefit from the one-sided, option-based hedging strategies. In a robustness check, the opposite hedging position is considered and the beneficial mechanics of the one-sided, option-based hedging strategies are shown in this alternative hedging set-up. Consequently, one-sided, option-based hedging strategies prove to be beneficial compared to classical two-sided hedging strategies in case the market development does not require any downside-risk protection.

The second essay also contributes to the academic literature in various ways. This is the first effort investigating whether freight options generally qualify as hedge instruments for dry bulk Capesize ship price risks and empirically assesses the hedge effectiveness of different one- and two-sided option-based hedging strategies. Finally, the study shows that one-sided option-based hedging strategies present a viable alternative to FFA-based hedging strategies and might be even a superior choice for shipping companies with certain risk, downside-risk, or return preferences.

## **1.4 Pricing of Asian options for affine Gaussian diffusions**

The third essay is concerned with pricing of Asian options for affine Gaussian diffusions. As already outlined, Asian options have gained some popularity among both market participants and researchers. Their pricing, however, can be a challenging task at times as it requires quite some mathematical effort to either grasp the distribution of the average value at maturity in closed form or to numerically evaluate it. For geometric Asian options, closed-form solutions can luckily be found for affine Gaussian diffusions as the distribution of the geometric average of an exponential of a Gaussian random process is itself lognormal. For arithmetic Asian options, however, the distribution of the arithmetic average is even unknown for an exponential of a Gaussian random process and thus, closed-form solutions cannot be found. The aim of third essay is threefold. Firstly, a general pricing framework for continuously monitored geometric Asian call options for affine  $n$ -factor Gaussian diffusions is developed. Secondly, closed-form solutions for geometric Asian call options for three mean-reversion

commodity pricing models are practically derived. Their accuracy is examined via MC simulation in a numerical example. Thirdly, the geometric Asian call option is used as control variate in a MC simulation in order to price an arithmetic Asian call option under these pricing dynamics. Finally, an extension to forward-start Asian options is outlined as these are quite common in commodity markets.

The general pricing framework is applicable to affine  $n$ -factor Gaussian diffusions and relies on the concept of the characteristic function. The latter allows to determine the distribution of the geometric average rather easily and the required mathematical theory for treating affine processes is provided in an excellent effort by Duffie et al. (2003). The general pricing framework provides a closed-form solution for continuously monitored geometric Asian call options. Subsequently, the general pricing framework is applied to three mean-reversion pricing models (i.e., the Schwartz (1997) one-factor model, the Schwartz and Smith (2000) two-factor model, and the Korn (2005) two-factor model) and specific closed-form solutions for geometric Asian call options are derived. For the sake of completeness, a closed-form solution for the Black (1976) one-factor model is also derived which can be rather simply converted to the result of Kemna and Vorst (1990) for classical Black and Scholes (1973) price dynamics. Afterwards, the accuracy of the developed closed-form solutions is examined via MC simulation in a numerical example relying on model parameters of Prokopczuk (2011). He estimated model parameters for the four price dynamics considered within the study for four different dry bulk freight futures. Furthermore, the geometric Asian call option is applied as control variate in a MC simulation in order to price an arithmetic Asian call option.

The results show that the derived closed-form solutions are accurate and that the MC control variate simulation approach for arithmetic Asian options yields considerable variance reduction of more than 97 %. This can be translated into substantial savings in computation time and allows market participants to quickly price such derivative instruments. The developed general approach and the presented results are neither prone to changes in model selection nor prone to changes in model parameters. Therefore, the applicability of the general pricing framework is by no means limited to the mean-reversion models or commodity markets considered within this study.

The third essay contributes to the academic literature three important ways. This is the first effort providing a general pricing framework for continuously monitored geometric Asian call options for affine  $n$ -factor Gaussian diffusions as well as stating

specific closed-form solutions for the three considered mean-reversion commodity pricing models. The effort extends Kemna and Vorst (1990) to affine  $n$ -factor Gaussian diffusions but remains a special case of Hubalek et al. (2014) as they develop a pricing framework for continuously monitored geometric Asian options for general affine stochastic volatility models with jumps. However, Hubalek et al. (2014) are limited to semi-analytical solutions for this general model class. Furthermore, the geometric Asian call option is used as control variate in a MC simulation in order to price an arithmetic Asian call option. Finally, an extension of the MC simulation to forward-start Asian options is outlined. The findings for valuing geometric and arithmetic Asian options are widely applicable (i.e., to the entire set of affine Gaussian diffusions as well as in a broad set of markets).

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# PART I

## FINANCIAL RISK MANAGEMENT



## CHAPTER 2

# HEDGING CAPESIZE SHIP PRICE RISKS USING FORWARD FREIGHT AGREEMENTS

### ABSTRACT

The shipping industry is historically known for its volatile nature of freight rates and second-hand ship prices. The aim of this paper is to examine potential hedging approaches and empirically assess their hedge effectiveness in order to provide the shipping industry with guidance on effective measures to counter the recognition of threatening impairment losses on their fleet in their balance sheets. Firstly, the idea of Alizadeh and Nomikos (2012) to use Forward Freight Agreement (FFA) contracts as hedging instruments for entire dry bulk Capesize vessels is translated into a minimum variance cross-hedging model that can be applied to real Capesize sale and purchase transactions. Secondly, a structural pricing model (SPM) for dry bulk Capesize vessels following the effort of Adland and Koekebakker (2007) is developed that serves as basis of a competing hedging approach. The empirical findings suggest that the hedging approach based on the developed structural pricing model consistently outperforms the minimum variance cross-hedging approach with respect to hedge effectiveness. At the same time, the associated cost of the hedges based on the structural pricing model turn out to be lower. The presented results show robustness to different subsets of the sample size as well as to different hedging set-ups.

## 2.1 Introduction

With approximately 9.2 billion tons of goods loaded in 2012, more than 90 % of the world's merchandise trade was handled by sea and thus, seaborne transportation is an integral part and driving force of the global economy (UN, 2013; UNCTAD, 2013). Transportation of dry bulk goods accounted for roughly 69 % of the total volume loaded and therefore, represents the most important sector of the shipping industry (UNCTAD, 2013). The global financial crisis of 2008-2009 and the resulting declining demand for maritime transportation had a severe impact on the shipping market concerning the level of ship prices and freight rates. Especially, as the industry was booming in the years prior to the financial crisis, many orders for new vessels had been placed prior to the financial crisis. The resulting overcapacity in terms of number of ships and loading capacity in the years following the breakout of the financial crisis significantly worsened the situation and the shipping industry has been facing a severe recession since.

The shipping industry has always been rather volatile compared to other industries. Albertijn et al. (2011), for instance, found the annualized volatility between January 1990 and April 2011 of the Baltic Dry Index (BDI) and of the Bulker Second-Hand Price Index to be 53 % and 32 %, respectively. Furthermore, the shipping industry is, on the one hand, very capital intensive and highly leveraged compared to other industries with more than 80 % of all external funding needs being prevalingly covered by debt financing (Drobotz et al., 2013). On the other hand, the asset side of a shipping company largely consists of the carrying amounts of the company's vessels (Stopford, 2009). According to Albertijn et al. (2011), listed shipping companies will also increasingly face the need to comply with the fair value accounting principles<sup>1</sup> defined by the International Financial Reporting Standards (IFRS). Accordingly, the ship price fluctuations will become more visible. These particular characteristics of the shipping industry imply that shipping companies exhibit a higher exposure towards large or extreme losses due to negative ship price fluctuations, but only have a comparatively small portion of equity in order to potentially cover such losses. As a result, protection of the company's balance sheet against adverse ship price

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<sup>1</sup> According to International Accounting Standards (IAS) rule 36, these carrying amounts are subject to regular impairment tests. If the recoverable amount of the ship (the higher of value in use and fair value less cost to sell) is less than the carrying amount, the difference needs to be recognized as an impairment loss (Deloitte, 2009; KPMG, 2012; PwC, 2005).

fluctuations by hedging the exposure may be desirable for shipping companies.

Unfortunately, there are currently no direct liquid instruments available for hedging ship prices. In the mid-2000s, Clarkson Securities Limited (CSL) tried to launch Forward Ship Value Agreements (FoSVAs). These are cash-settled forward contracts on the Baltic Sale and Purchase Assessment (BSPA)<sup>2</sup> (Adland et al., 2004). The liquidity of these instruments, however, has been extremely limited (Alizadeh and Nomikos, 2009). Jallal (2013) even stated that ‘so far no paper trade on the BSPA has been reported.’

A first effort on hedging ship price risks was conducted by Alizadeh and Nomikos (2012) using Forward Freight Agreement (FFA) contracts. In their effort, they studied how much of the price fluctuations in the BSPAs for selected dry bulk vessel classes (i.e., Capesize, Panamax, and Supramax) may be explained by the respective price fluctuations of the corresponding FFA+2CAL contracts.<sup>3</sup> They found that up to 93 % of the second-hand ship prices may be explained by these FFA contracts using minimum variance hedge ratios and suggested that FFAs may serve as valid, alternative cross-hedging instruments to FoSVAs for second-hand ship prices (Alizadeh and Nomikos, 2012).

From a practical financial risk management perspective, shipping companies will be most of the time confronted with situations when their ship specifications, such as age, size and configuration vary significantly from the generic reference vessels used in the BSPAs. Adland and Koekebakker (2007) estimated ship values with a non-parametric multivariate pricing model using cross-sectional data on Handysize bulk carriers from actual sale and purchase transactions in the second-hand market. They identified three relevant factors in the second-hand price determination of dry bulk ships: size (measured in deadweight tons (DWT)), age and 1-year time charter freight rates. However, they also argue that this three-factor model is not fully capable of explaining the observed vessel prices in the market (Adland and Koekebakker, 2007).

The aim of this paper is to test whether the suggested hedging performance by Alizadeh and Nomikos (2012) does hold up for data from real sale and purchase transactions in the second-hand market for Capesize vessels. Furthermore, a structural

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<sup>2</sup> BSPA is a second-hand ship price estimation by panelists of certain 5-year old reference vessels provided on a weekly basis by The Baltic Exchange.

<sup>3</sup> FFA+2CAL contracts are second-next calendar-year FFA contracts.

pricing model (SPM) for dry bulk Capesize vessels is developed using various individual ship specifications of real sale and purchase transactions based on the efforts of Adland and Koekebakker (2007). This model allows to separate deterministic and risky or market-driven factors of second-hand ship prices and offers another approach to determine hedging exposures than the minimum variance hedging approach suggested by Alizadeh and Nomikos (2012). It is empirically investigated which of the two hedging approaches offers a superior hedge effectiveness in a one-year fixed time horizon prior to the sale for available real dry bulk Capesize sale and purchase transactions as well as between available real dry bulk Capesize sale and resale transactions of individual vessels.

The remainder of the paper is structured as follows. Section 2 reviews the academic literature on the formation of second-hand ship prices as well as hedging of these. Section 3 elaborates on the methodology applied within this study. Section 4 provides a thorough description of the data used within this study. Section 5 presents the empirical results, an interpretation of these results as well as further robustness checks of the presented results. Finally, section 6 concludes the findings of this study and provides an outlook on further research opportunities in this area.

## 2.2 Review of academic literature

As this paper is concerned with the hedging of second-hand vessel prices, knowledge on the formation of second-hand ship prices, on properties of the hedging instruments itself (i.e., FFAs), and on hedging is essential. There has been a considerable amount of research in the existing academic literature on the aforementioned topics.

With respect to ship price determination or formation, early efforts in the area of maritime economics focused on comprehensive econometric models of the shipping industry in order to model ship prices as well as demand for shipping or transportation services. An early effort that specifically stated equations on ship prices was by Charemza and Gronicki (1981). They developed an econometric model in which ship prices gradually adjust to freight and activity rates. Beenstock (1985) argued that classical demand and supply analysis is not sufficient to determine ship prices as vessels are to be considered as capital assets with an economic life of significant duration. He, for instance, developed an econometric model that explains ship prices

using world wealth, fleet size, interest rates, and expectations on operation income and second-hand ship prices. In the following, many general and partial equilibrium models have been developed (see, among others, Beenstock and Vergottis (1989*a*), Beenstock and Vergottis (1989*b*), Beenstock and Vergottis (1993*a*), Beenstock and Vergottis (1993*b*), Dikos and Marcus (2003), Kalouptsidi (2014), Strandenes (1984), Tsolakis et al. (2003), and Tvedt (2003). Glen (2006) provided an excellent overview on these efforts.

Another prominent line of research concerning the formation of ship prices focused on investigating whether the Efficient Market Hypothesis (EMH) introduced by Fama (1970) does hold for ship prices. Ship prices would be efficient according to the definition by Fama (1970) if they already incorporate all currently available information. Hale and Vanags (1992), for instance, studied the market efficiency of dry bulk second-hand ship prices using the cointegration technique based on monthly ship price data from October 1979 to July 1988. They found no support for the EMH which has been assumed for shipping markets in many models. These results have been further supported by Glen (1997) and Alizadeh and Kavussanos (2002). The latter attributed the failure of the EMH to existing time-varying risk premiums that connect excess returns to the investors' perception of risk. More recent efforts in this area are, for instance, Adland and Koekebakker (2004) or Sødal et al. (2009).

Volatility dynamics of dry bulk second-hand ship prices were studied by Kavussanos (1996) using ARCH/GARCH models. He found that shipping companies operating in the timecharter market face higher risks or volatilities than shipping companies operating in the spot market as well as higher risks or volatilities for larger vessels due to less flexibility in trades or loaded goods and draft restrictions limiting the number of accessible ports. In another effort, Kavussanos (1997) confirms and complements these findings. He found that volatility largely depends on the vessels classes or sizes and in particular that second-hand price of smaller vessels are characterized by lower volatility than larger vessels. Concerning price and volume dynamics of second-hand ship prices, Syriopoulos and Roumpis (2006) found that price changes have an impact on trading volume reflecting that higher possibility of capital gains cause higher activity in the sale and purchase market for ships. Furthermore, they found a negative relationship between trading volume and the volatility of ship prices in the dry bulk market. They argued that new information seems to flow sequentially to all market participants in the shipping market due to low trading volume, transparency,

and lack of official price quotes rather than simultaneously as in other classic capital markets.

A present value perspective on ship prices was suggested by Alizadeh and Nomikos (2006) and Psarftis et al. (2012). They argued that shipping companies are economically entitled to the present value of operating income generated by the respective ship as well as any capital gains or losses of the ship itself. Expectations on future freight rates for the remaining economic life as well as expectations on future second-hand ship prices allow to determine the respective second-hand price of a ship of certain age (Alizadeh and Nomikos, 2006; Psarftis et al., 2012). Deviations of dry bulk Capesize second-hand ship prices from underlying fundamentals (i.e., freight and newbuilding markets) were studied by Adland et al. (2006). They found second-hand prices to be closely cointegrated with these fundamentals and did not find any support for an asset bubble in the boom time from 2003 to 2005 in the dry bulk market.

So far, the aforementioned studies and efforts mostly relied on time series data of second-hand prices, such as the BSPA or Clarksons Shipping Intelligence Network (SIN) second-hand time series. However, these time series are panelists' estimations of second-hand ship prices for certain reference vessels of a particular age (e.g., 5-year old ships in the case of the BSPA). Pruyt et al. (2011), for instance, highly doubt whether these panelists' estimation time series accurately reflect the true market dynamics of second-hand ship prices. Besides, they provide a comprehensive overview on second-hand ship valuation related research in the past 20 years in their effort. As already stated in section 1, Adland and Koekebakker (2007) were one of the first to provide a multivariate, nonparametric analysis of second-hand ship valuation based on cross-sectional real sale and purchase transaction data. They found that a partially nonlinear function of DWT, age, and the state of the freight market (i.e., one-year timecharter rates) is able to describe second-hand ship prices quite accurately.

In terms of FFAs, research in the academic literature on their statistical properties, volatility dynamics, and predictive power has found quite some interest. Kavussanos and Visvikis (2006) provide a comprehensive summary of the development, use and market perception, and research on pricing, volatility dynamics, and forecasting performance of FFAs. Interestingly, Kavussanos et al. (2007) found that understanding of and practical hedging with FFAs has only reached an early stage of development within the Greek shipping industry despite the high volatility of freight income and ship prices. An early effort on testing the unbiasedness hypothesis for FFAs was made

by Kavussanos and Nomikos (1999). They found that futures prices with only one or two months remaining until maturity provide unbiased forecasts of realized spot prices, although forward rates are not tied to spot rates by a classical arbitrage relationship as they are nonstorable goods. Kavussanos and Visvikis (2004) identified that FFA prices are relevant in the price formation of spot prices and Kavussanos et al. (2004) argued that the validity of the unbiasedness hypothesis for FFAs is subject to the time to maturity of the contract, specific market characteristics, and the trading route.

Concerning the hedging of ship price risks, Adland et al. (2004) studied the pricing of at that time recently introduced FoSVAs as well as the term structure of second-hand vessel prices. As already pointed out in section 1, however, Alizadeh and Nomikos (2009) and Jallal (2013) emphasized the nonexistent liquidity of these instruments, so that other hedging instruments need to be considered. As outlined in section 1, Alizadeh and Nomikos (2012) applied the concept of a minimum variance hedge developed by Ederington (1979) to the dry bulk second-hand-ship prices using FFA+2CAL contracts as hedging instruments. Their approach suggested a hedging effectiveness of up to 93 % and successfully established FFAs as alternative hedging instruments to FoSVAs.

This paper contributes to the existing academic literature in three important ways. Firstly, the cross-hedging performance of the suggested reduced form model by Alizadeh and Nomikos (2012) is tested in an empirical setting using real sale and purchase transaction data for dry bulk Capesize vessels. Secondly, a structural pricing model for dry bulk Capesize vessels is developed using additional deterministic, ship-specific factors other than DWT and age as well as incorporating the forward curve as additional factor to capture the state of the freight market. Thirdly, the aforementioned structural pricing model is used as alternative approach to determine the hedging exposure and the hedging effectiveness of both approaches is tested in two different settings using real sale and purchase transaction data. Moreover, the research findings will have relevant implications for the risk management practice of shipping companies operating in the dry bulk market.

## 2.3 Empirical methodology

With respect to the empirical methodology, the following paragraphs elaborate on the classical cross-hedging approach suggested by Alizadeh and Nomikos (2012), the developed structural pricing model for second-hand dry bulk Capesize vessels, and the two settings in which the hedging effectiveness of both approaches are tested.

### 2.3.1 Minimum variance cross-hedging model

In their effort, Alizadeh and Nomikos (2012) applied the concept of minimum variance hedge ratios based on the principles of classical portfolio theory developed by Ederington (1979). Accordingly, they used a simple linear regression set-up in order to determine the minimum variance hedge ratio,  $h^*$  as shown in the following equation (2.1):

$$\Delta p_t = \alpha + \beta \Delta f_t + \varepsilon_t \quad \text{with} \quad \varepsilon_t \stackrel{iid}{\sim} (0, \sigma^2). \quad (2.1)$$

In the regression above, the log ship price return between time  $t$  and  $t - 1$  is denoted by  $\Delta p_t$ , the constant of the regression by  $\alpha$ , the log FFA rate return between time  $t$  and  $t - 1$  by  $\Delta f_t$ , and the error term at time  $t$  by  $\varepsilon_t$ . The time index,  $t$ , corresponds to weeks here.  $\beta$  is equivalent to the unconditional variance between changes in spot and futures prices over the unconditional variance of changes in futures prices which is the the minimum variance hedge ratio,  $h^*$ , as defined by Ederington (1979). The  $R^2$  of the regression reflects the hedge effectiveness (i.e., percentage of the variability in ship price returns eliminated through hedging with FFAs). Alizadeh and Nomikos (2012) used 52-week log differences within their regression set-up as they considered a hedging period of one year that corresponds to the time frame of the calendar-year FFA contracts (i.e., the FFA+2CAL contract for the second-next calendar year). Accordingly, the regression set-up changes to the following form as shown in equation (2.2) below:

$$\Delta_{52} p_t = \alpha_{52} + \beta_{52} \Delta_{52} f_t + \eta_t. \quad (2.2)$$

Within equation (2.2) above, the 52-week ship price log return,  $\Delta_{52} p_t$ , is equal to  $p_t - p_{t-52}$  and the 52-week FFA log return,  $\Delta_{52} f_t$ , is equal to  $f_t - f_{t-52}$ , respectively.

Alizadeh and Nomikos (2012) elaborated on the potential issues with the error term,  $\eta_t$ , caused by the use of overlapping observations which result in  $\eta_t$  following a moving average (MA) process of order 51. This creates inefficient standard errors as well as an incorrect  $R^2$  of the regression as  $\eta_t$  is no longer *iid*. Nevertheless, the coefficient estimates for the minimum variance hedge ratio is unbiased and consistent. Unfortunately, using nonoverlapping 52-week log difference observations is not possible as the FFA price quotes are only available roughly from 2005 onwards and this would leave only a few nonoverlapping annual return observations for the analysis.

In order to operationalize the suggested minimum variance hedging approach by Alizadeh and Nomikos (2012) for the empirical testing of the hedge effectiveness, the following adjustments were made. Alizadeh and Nomikos (2012) estimated a minimum variance hedge ratio and an associated hedge effectiveness of 5-year old reference vessels as they used BSPA time series data for ship prices within their analysis. In the real world, dry bulk vessels usually have an economic life of approximately 30 years until they are scrapped and sale and purchase transactions of second-hand dry bulk vessels occur seldom exactly when the vessel is 5 years old (Stopford, 2009). The suggested approach of Alizadeh and Nomikos (2012) was complemented by estimating minimum variance hedge ratios for all Capesize second-hand ship price time series of differently-aged reference vessels available (i.e., 5-year old ships: BSPA time series data and 10-, 15-, and 20-year old ships: SIN time series data). For new or zero-year old ships and 30-year old ships, SIN time series for Capesize newbuilding and scrap rates were used. The used data is more accurately described in section 4. This allows to estimate minimum variance hedge ratios for Capesize vessels at exactly six, specific age points of a vessels lifetime using the regression set-up as shown in equation (2.3) below:

$$\Delta_{52}p_{t, Age} = \alpha_{52, Age} + \beta_{52, Age} \Delta_{52}f_t + \eta_t \quad \text{with} \quad Age \in \{0, 5, 10, 15, 20, 30\}. \quad (2.3)$$

In order to overcome the problem of  $\eta_t$  following a MA process of order 51, Alizadeh and Nomikos (2012) generated a larger data set using the stationary bootstrap resampling technique of Politis and Romano (1994) and found only marginal differences in their results using this larger, generated data set. As only the minimum hedge ratios from the regression in equation (2.3) were used and the hedging performance empirically tested using real sale and purchase transaction data, this issue should not distort the results of the empirical analysis within this effort. Nevertheless, the sta-

tionary bootstrap procedure by Politis and Romano (1994) was also used in order to get a perspective on the accuracy of the estimated  $\beta_{52,Age}$ -coefficients. Therefore,  $n$  resampled data sets of 52-week log returns were generated and the initial experiment (i.e., the regression based on overlapping 52-week log returns of equation (2.3)) was repeated  $n$  times. Subsequently, the mean  $\beta_{52,Age}$ -coefficients,  $\bar{\beta}_{52,Age}$ , as well as a 95 % confidence interval based on the empirical distribution of the  $\beta_{52,Age}$ -coefficients of the regressions performed on the resampled data sets was derived. A detailed description of the algorithm that was applied for the stationary bootstrap can be found in the appendix A.1 on page 63. Finally, linear interpolation was used to derive the respective minimum variance hedge ratios for Capesize ship ages other than the six, specific age points from 0 to 30 years.

### 2.3.2 Structural pricing model

As ships in the dry bulk Capesize vessel class are quite heterogeneous in terms of specifications, such as size (e.g., length, DWT, breadth, or loading capacity), engine, speed, consumption, or yard built, ships in real second-hand sale and purchase transactions are rarely comparable to the reference vessels underlying the panelists' estimations of second-hand ship prices. Therefore, the analysis followed the intention of Adland and Koekebakker (2007) and a structural pricing model for dry bulk Capesize vessels was estimated from real sale and purchase transactions of the form as shown in equation (2.4) below:

$$\begin{aligned}
 p_{i,t} = & \alpha + \beta_1 x_{1,i} + \dots + \beta_n x_{n,i} + \beta_{n+1} x_{n+1,i,t} + \dots + \beta_{n+m} x_{n+m,i,t} \\
 & + \beta_{n+m+1} x_{n+m+1,t} + \dots + \beta_{n+m+k} x_{n+m+k,t} + \varepsilon_{i,t}
 \end{aligned} \tag{2.4}$$

Within the model in equation (2.4), the price or value of vessel  $i$  at time  $t$  is represented by  $p_{i,t}$  and the constant of the regression by  $\alpha$ . The ship-specific, time-invariant characteristics of vessel  $i$ , such as length, beam, DWT, loading capacity, speed, consumption, or engine are denoted by  $x_{1,i}$  to  $x_{n,i}$ , the ship-specific, time-varying characteristics of vessel  $i$ , such as age at time  $t$  by  $x_{n+1,i,t}$  to  $x_{n+m,i,t}$ , and the time-varying explanatory variables that are not ship specific, such as the FFA rate or slope of the FFA curve at time  $t$  by  $x_{n+m+1,t}$  to  $x_{n+m+k,t}$ . The respective slope coefficients for the explanatory variables are denoted by  $\beta_1$  to  $\beta_{n+m+k}$  and  $\varepsilon_{i,t}$  represents the pricing error term of the model for vessel  $i$  at time  $t$ . One benefit of the model structure is that

it allows to separate the deterministic and risky or market-driven factors of second-hand ship prices, such as FFA prices or slope of the FFA curve. Together with their estimated coefficients, these can be directly translated to the vessel-specific exposure towards these factors and used for hedging purposes. Within the empirical analysis in section 5.2, various combinations of deterministic and risky or market-driven explanatory variables of Capesize dry bulk second-hand prices were tested optimizing the explanatory power of the model subject to the constraint of explanatory variables that can be used for hedging purposes (e.g., FFA+1CAL and FFA+2CAL contracts as well as the slope of the FFA curve which is the difference between a FFA+2CAL and FFA+1CAL contract). This means that the risky or market-driven explanatory variables need to be tradeable in order to hedge the exposure towards them. Using timecharter or spot rates as explanatory variables might enhance the explanatory power of the structural pricing model. However, these cannot be used as hedging instruments without having a vessel that physically meets the contractual obligations of the desired hedging position.

### **2.3.3 Hedging set-up**

In order to assess the hedge effectiveness of the two previously described approaches (i.e., the minimum variance cross-hedging model and the developed structural pricing model), the approaches were empirically tested in two different hedging set-ups. On the one hand, a fixed hedging time horizon of one year prior to the sale of the vessel was considered and, on the other hand, the analysis specifically looked at vessels that have been sold two or more times within in the data set and considered the time period between those sales as individual hedging horizons. Both approaches as well as the methodology of measuring the hedge effectiveness are explained in detail in the following paragraphs.

#### **2.3.3.1 Fixed time horizon of one year**

Within the hedging set-up of a fixed time horizon of one year, it was assumed that a shipping company owning a particular vessel  $i$  knows ex ante that it wants to sell the vessel  $i$  at a certain date in the future,  $t_i$ . The time index,  $t_i$ , corresponds to vessel-specific sales dates here. Besides, it was assumed that the shipping company wants to

hedge against any vessel price fluctuations for a period of  $l_i$  trading days prior to the planned sales date,  $t_i$ . Accordingly, the hedge for the vessel  $i$  is initiated at  $t_i - l_i$ . As the time horizon was fixed to one year (i.e., 252 trading days or 52 weeks) within this first hedging set-up,  $l_i$  is equal to 252 for all  $i$ .<sup>4</sup> The shipping company does not know the real sales or transaction price of the vessel  $i$  until the sales or transaction date,  $t_i$ . Nevertheless, the shipping company is able to estimate a value or price of vessel  $i$  at time  $t_i - 252$ . As the shipping company already knows the sales or transaction date,  $t_i$ , the value or price of vessel  $i$  at time  $t_i - 252$  is estimated using the age of the vessel at the sales or transaction date,  $a_{t_i}$ . Hence, the aging-related loss of value of the vessel within the hedging period is factored in. The shipping company would have to account for depreciation of the vessel within this time period anyway and does only want to hedge vessel price fluctuations besides this normal aging-related loss of value. For the minimum variance cross-hedging approach, the model value or price,  $\hat{m}_{i,t_i-252,a_{i,t_i}}$ , can be derived using a linear interpolation between the different new, second-hand, and scrap ship price quotes by panelists' which is shown equation (2.5) below:

$$\hat{m}_{i,t_i-252,a_{i,t_i}} = q_{a_l,t_i-252} + \frac{a_{i,t_i} - a_l}{a_u - a_l} (q_{a_u,t_i-252} - q_{a_l,t_i-252}) \quad (2.5)$$

with  $a_l < a_{i,t_i} < a_u$   
 and  $(a_l, a_u) \in \{0, 5, 10, 15, 20, 30\}$ .

Within equation (2.5), the estimated model price of vessel  $i$  at time  $t_i - 252$  using the age of vessel  $i$  at time  $t_i$ ,  $a_{i,t_i}$ , is denoted by  $\hat{m}_{i,t_i-252,a_{i,t_i}}$ . The new, second-hand, and scrap ship price quotes by panelists' for the differently aged reference vessels at time  $t_i - 252$  are represented by  $q_a$ . For the rare occasions in which the price for 5-year-old vessels exceeds the price for new vessels or the price for scrap vessels exceeds the price for 20-year old vessels, the interpolation between these values was adjusted accordingly in order to ensure positive estimated second-hand model prices of vessel  $i$ . Concerning the structural pricing model approach, the value can be derived using equation (2.4). The resulting values reflect the physical position of the shipping company that it wants to hedge.

Concerning the corresponding hedge position, the respective hedge ratios for the minimum variance cross-hedging approach were derived based on the linear interpolation

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<sup>4</sup> Within the hedging set-ups, the perspective was changed to trading days as daily margining for the hedge positions was considered. This is explained later in this section.

between the six hedge ratios resulting from equation (2.3). The desired hedge exposure is given by multiplying the estimated physical position of vessel  $i$  at time  $t_i - 252$  by the corresponding interpolated hedge ratio,  $\beta_{i,t_i-252,a_i,t_i}$ . As the typical lot size of dry bulk Capesize FFAs is one day and these contracts are quoted in United States Dollar (USD) per day, the FFA quotes were first transformed in USD million per day and then, the number of FFA contracts or days that need to be shorted was calculated (i.e., the negative sign indicates the short position) at time  $t_i - 252$ ,  $d_{i,t_i-252}$ , by dividing the desired hedge exposure,  $\beta_{i,t_i-252} \cdot \hat{m}_{i,t_i-252,a_i,t_i}$ , by the respective FFA contract price,  $f_{t_i-252}$ , as shown in equation (2.6) below:

$$d_{i,t_i-252} = -\beta_{i,t_i-252,a_i,t_i} \frac{\hat{m}_{i,t_i-252,a_i,t_i}}{f_{t_i-252}}. \quad (2.6)$$

With respect to the hedging approach based on the structural pricing model, the desired hedge exposure is given by the respective coefficients of the time-varying, market-driven explanatory variables or risk factors, such as the FFA rate or slope of the FFA curve, in equation (2.4). Obviously, the hedge exposure needs to be aggregated across hedging instruments if the structural pricing model contains multiple time-varying, market-driven explanatory variables or cross-terms of these variables with ship-specific, time-invariant characteristics of vessel  $i$ , such as length or consumption, or ship-specific, time-varying characteristics of vessel  $i$ , such as age.

Furthermore, the following additional assumptions regarding divisibility of the hedging instruments, margining, rollover dates were made within the hedging set-up. For the purpose of the study, unlimited divisibility of the hedging instruments was assumed. The calendar-year FFA contracts used as hedging instruments are rolled over once per year, typically around December 22<sup>nd</sup> or 23<sup>rd</sup>. As FFA contracts are typically cleared, interest effects on any accumulated gains or losses on the margin account were accounted for using continuously compounded USD London Interbank Offered Rate (LIBOR) overnight rates. As a consequence to the interest effect, the initial hedge position was adjusted by applying a tailing factor,  $b_{t_i-252}$ , to the initial number of days shorted for each hedging instrument as shown in equation (2.7) below:

$$b_{t_i-252} = e^{-r_{t_i-252} \frac{(t_i - (t_i - 252))}{252}} = e^{-r_{t_i-252}}. \quad (2.7)$$

As the fixed hedging horizon was set to one year or 252 trading days, each hedge

encounters a rollover date during the hedging horizon. Therefore, the hedge is split into two consecutive hedges for practicality reasons and also to be consistent with the second hedging set-up considered. Accordingly, the first hedge covers the time period from  $t_i - 252$  until the rollover date and the second hedge from the rollover date until the transaction date,  $t_i$ . Just before the rollover date, the first hedge is closed out and just after the rollover date, the second hedge is initiated. The age effect on the physical position, hedge ratio, and tailing factors were adjusted accordingly for the two hedges. For instance, the age effect for the entire year was proportionally split across the two consecutive hedges.

The aim of the shipping company's hedging effort is minimizing the variation or fluctuation of the aggregated portfolio over the time period from  $t_i - 252$  to  $t_i$ ,  $\Delta_{252}v_i$ , consisting of the physical position in the vessel  $i$  and the hedge position as shown in equation (2.8) below:

$$\begin{aligned} \Delta_{252}v_i = & (p_{i,t_i,a_i,t_i} - \hat{m}_{i,t_i-252,a_i,t_i}) \\ & + \sum_{j=1}^{252} \left( (w_{i,t_i-j+1} - w_{i,t_i-j}) \left( \prod_{k=0}^{j-1} e^{r_{t_i-k} \cdot \frac{1}{252}} - 1 \right) \right). \end{aligned} \quad (2.8)$$

Within equation (2.8), the fluctuations of the ship price or physical position is given by the difference between the transaction price at time  $t$ ,  $p_{i,t_i,a_i,t_i}$ , and the estimated value or price at time  $t_i - 252$ ,  $\hat{m}_{i,t_i-252,a_i,t_i}$ . The initial hedge position,  $w_{i,t_i-252}$ , is determined by number of days shorted according to equation (2.6) for the minimum variance cross-hedging approach as well as according to the exposure to risk factors in equation (2.4) for the hedging approach based on the structural pricing model. Accordingly,  $w_{i,t_i-j+1} - w_{i,t_i-j}$  represents the daily difference of the aggregated or netted hedge position which is multiplied by  $\prod_{k=0}^{j-1} e^{r_{t_i-k} \cdot \frac{1}{252}} - 1$  in order to account for the interest effect on any accumulated gains or losses on the margin account.

Finally, the hedge effectiveness of both hedging approaches was measured in percentage reduction of variance between aggregated portfolios and simple physical positions across  $n$  Capesize vessels as shown in equation (2.9) below:

$$\begin{aligned} HE_{ED} = & 1 - \frac{\text{Var}[\mathbf{y}]}{\text{Var}[\mathbf{x}]} \\ \text{with } \mathbf{x} = & (p_{1,t_1,a_1,t_1} - \hat{m}_{1,t_1-252,a_1,t_1}, \dots, p_{n,t_n,a_n,t_n} - \hat{m}_{n,t_n-252,a_n,t_n}) \end{aligned} \quad (2.9)$$

and  $\mathbf{y} = (\Delta_{252}v_1, \dots, \Delta_{252}v_n)$ .

The measure for the hedge effectiveness in equation (2.9) is largely based on Ederington (1979). However, the suggested measure was slightly adjusted in order to fit the context of the effort. Several transactions of one and the same vessel take place rather seldom. As a result, time series of real second-hand prices of individual vessels are not available, not to mention in regular frequency. Hence, the variance of physical positions and portfolio positions consisting of the physical and hedge position was measured between two specific, individual points in time across different vessels, whereas Ederington (1979) measured the variance of one unhedged and one hedge position over time.

### 2.3.3.2 Between sale and resale

The measurement of fluctuations of the physical position in the hedging set-up with a fixed time horizon of one year might be biased because the value of the physical position or vessel  $i$  at the initiation of the hedge at time  $t_i - 252$  is implied by the respective model. For this reason, it was decided to test both hedging approaches in another hedging set-up between two real sale and purchase transactions of one and the same vessel  $i$ . Accordingly, the time horizon of the hedge of vessel  $i$  is no longer fixed and changes from 252 to  $l_i$  trading days. It was assumed that the shipping company does not know ex ante the exact date of the resale,  $t_i$ , if the time horizon of the hedge,  $l_i$ , is larger than 252 trading days. If  $l_i$  is less than or equal to 252 trading days, it was assumed that the date of the resale,  $t_i$ , is known to the shipping company and it directly engages in a hedge for the time period  $l_i$ . If  $l_i$  is greater than 252, it was assumed that the shipping company first engages into a hedge until the next rollover date and subsequently, engages in consecutive hedges until the next rollover date again until the remaining time until the resale date,  $t_i$ , is less than or equal to 252 trading days.

In order to factor in the usual depreciation in this hedging set-up as well, the value of the physical position or vessel  $i$  at the initiation of the hedge,  $\hat{p}_{i,t_i-l_i,a_i,t_i}$ , was set to the transaction price at  $t_i - l_i$ ,  $p_{i,t_i-l_i,a_i,t_i-l_i}$ , less the model implied aging-related loss between  $t_i - l_i$  and  $t_i$  if  $l_i$  is less than or equal to 252 trading days. This relationship

is shown in equation (2.10) below:

$$\hat{p}_{i,t_i-l_i,a_i,t_i} = p_{i,t_i-l_i,a_i,t_i-l_i} - (\hat{m}_{i,t_i-l_i,a_i,t_i-l_i} - \hat{m}_{i,t_i-l_i,a_i,t_i}). \quad (2.10)$$

In case  $l_i$  is greater than 252 trading days, the aging-related loss from  $t_i - l_i$  to  $t_i$  was proportionally spread across the consecutive hedges and adjusted the value of the physical position of vessel  $i$  at the initiation of the consecutive hedges accordingly. The assumptions and formulas on the hedge ratio, number of days shorted per hedging instrument, and tailing factor largely stayed the same, except for the change from  $l_i = 252$  to  $l_i$  being vessel-specific in this hedging set-up. So, the hedge ratio picked from the linear interpolation of equation (2.3) changes to  $\beta_{i,t_i-l_i,a_i,t_i}$  and equations (2.6) and (2.7) change to equations (2.11) and (2.12) as shown below:

$$d_{i,t_i-l_i} = -\beta_{i,t_i-l_i,a_i,t_i} \frac{\hat{p}_{i,t_i-l_i,a_i,t_i}}{f_{t_i-l_i}} \quad (2.11)$$

$$b_{t_i-l_i} = e^{-r_{t_i-l_i} \frac{(t_i - (t_i - l_i))}{252}} = e^{-r_{t_i-l_i} \frac{l_i}{252}}. \quad (2.12)$$

Again, the aim of the shipping company is to minimize the variation or fluctuation of the aggregated portfolio over the time period from  $t_i - l_i$  to  $t_i$  changes from equation (2.8) to equation (2.13) and the hedge effectiveness for both hedging approaches across  $n$  Capesize vessel resales from equation (2.9) to equation (2.14) as shown below:

$$\begin{aligned} \Delta_{l_i} v_i &= (p_{i,t_i,a_i,t_i} - \hat{p}_{i,t_i-l_i,a_i,t_i}) \\ &+ \sum_{j=1}^{l_i} \left( (w_{i,t_i-j+1} - w_{i,t_i-j}) \left( \prod_{k=0}^{j-1} e^{r_{t_i-k} \cdot \frac{1}{252}} - 1 \right) \right) \end{aligned} \quad (2.13)$$

$$HE_{ED} = 1 - \frac{\text{Var}[\mathbf{y}]}{\text{Var}[\mathbf{x}]} \quad (2.14)$$

$$\text{with } \mathbf{x} = (p_{1,t_1,a_1,t_1} - \hat{p}_{1,t_1-l_1,a_1,t_1}, \dots, p_{n,t_n,a_n,t_n} - \hat{p}_{n,t_n-l_n,a_n,t_n})$$

$$\text{and } \mathbf{y} = (\Delta_{l_1} v_1, \dots, \Delta_{l_n} v_n).$$

## 2.4 Description of the data

As already stated in section 1, the focus of this effort lies on the dry bulk Capesize vessel class which is defined as dry bulk vessels larger than 100,000 DWT in size.

Capesize vessels are almost exclusively engaged in the transport of iron ore, coal, and grain from Southern America to Northern America, and Europe as well as from Australia to Asia (Alizadeh and Nomikos, 2009; Stopford, 2009). There is no official upper size boundary and recent launches of so called ‘Valemax’ vessels by the Brazilian mining company Vale S.A. passed the 400,000 DWT mark. These very large Capesize vessels belong to the Capesize subcategory of very large ore carriers (VLOCs). On the contrary, the size of the current Capesize reference vessel as defined by The Baltic Exchange, which is the underlying vessel panelists’ estimations of the BSPA, is 172,000 DWT.

As this study focuses, on the one hand, on estimating a structural pricing model for second-hand dry bulk Capesize vessel prices and, on the other hand, on assessing the hedge effectiveness of the minimum variance hedging approach as well as the hedging approach based on the exposure derived by the structural pricing model, the considered time frame within the empirical analysis starts from January 2005 onwards as FFA time series data has only been available since then.

Real sale and purchase data were obtained from SIN consisting of 646 dry bulk Capesize vessel transactions for the time period ranging from March 1995 until October 2013. Next to the transaction date and transaction price, the gathered data set includes the following other ship specifications: vessel name, age at transaction, DWT, gross tonnage, length over all, length between perpendicular, beam, draft, speed, consumption, engine, horse power, bunker capacity, holds, hatches, grain capacity, yard, and International Maritime Organization (IMO) number as unique ship identifier. After excluding any transactions that took place prior to the time frame of consideration or before the ship building was completed<sup>5</sup>, have been part of an ‘en bloc’ sale<sup>6</sup>, or had missing entries in the above mentioned specifications, 206 dry bulk Capesize vessel transactions remain in the data set. Clearly, a larger number of transactions would have been conducive for the robustness of the developed structural pricing model. However, the Capesize vessel class is characterized by the smallest fleet size in the dry bulk sector and accordingly, the number of second-hand vessel transactions is also lower than for other dry bulk vessel classes, such as Panamax, Handymax,

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<sup>5</sup> The data set contained several transactions that took place before the ship building was completed, notably in the years 2006 and 2007. As these transactions were considered incomparable to classical second-hand transactions, these observations were excluded from the analysis.

<sup>6</sup> ‘En bloc’ sales are transactions in which two or more ships are sold for a consolidated price. Unfortunately, breakdowns of the consolidated prices or allocations to individual vessels are not available.

and Handysize. Nevertheless, studying the second-hand price dynamics of Capesize vessels and possibilities to hedge associated price risks was deemed as of particular importance for the following three reasons: firstly, Capesize vessels are economically the most important vessel class in the dry bulk sector with roughly 41 % loading capacity in deadweight tons (DWT); secondly, these ships are more capital-intensive than smaller dry bulk vessels and thus, the associated price risks are analogously larger; and finally, the vessel class shows by far more pronounced heterogeneity in terms of size than other dry bulk vessel classes and thus, deviations from available second-hand price time series, such as the BSPA, are larger lowering the transparency on fair second-hand prices for shipping companies for their vessels.

TABLE 2.1: Descriptive statistics of second-hand Capesize vessel transactions 2005-2013

Year	#	Price in USD million			Age in years			DWT in metric tonnes		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
2005	23	40.77	12.50	80.30	14.16	0.67	24.25	160,388	129,237	194,941
2006	26	38.73	11.00	73.50	13.98	1.67	24.25	164,331	141,014	275,616
2007	35	59.56	9.00	152.00	16.35	1.92	26.17	157,032	105,496	188,334
2008	13	58.57	25.00	130.00	16.46	2.83	26.00	157,885	136,999	184,403
2009	32	30.07	3.65	62.00	14.59	2.67	25.00	165,338	128,826	233,016
2010	19	29.00	10.50	84.90	16.08	0.33	25.25	174,363	148,982	275,616
2011	17	24.51	14.50	58.00	13.93	0.50	27.58	164,796	148,535	180,265
2012	21	15.09	6.65	38.00	14.67	0.25	22.08	170,351	147,048	322,457
2013	20	20.19	7.50	36.00	11.91	3.42	18.92	174,237	149,396	180,310
Total	206	36.12	3.65	152.00	14.71	0.25	27.58	164,939	105,496	322,457

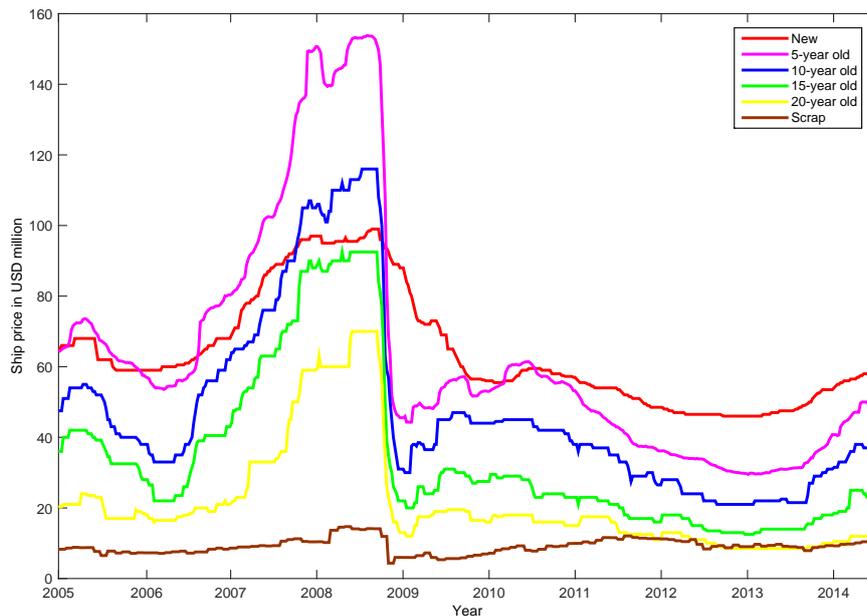
The table shows selected descriptive statistics of the remaining second-hand Capesize vessel sales for the time period ranging from January 13<sup>th</sup>, 2005 to October 30<sup>th</sup>, 2013 in the data set.

TABLE 2.1 shows selected descriptive statistics of the second-hand transactions in the considered data set. The mean sales price was USD 36.12 million, the mean vessel age at transaction 14.71 years, and the mean vessel size in DWT 164,939 metric tonnes. The least expensive sale was settled at USD 3.65 million, whereas the most expensive sale was settled at USD 152.00 million. The youngest vessel sold had an age of 0.25 years and the oldest vessel sold an age of 27.58 years. The above mentioned heterogeneity in terms of size in DWT can be also seen by means of minimum and maximum vessel size of 105,496 and 322,457 DWT, respectively.

Additionally, weekly second-hand Capesize sale and purchase assessment time series data was retrieved from The Baltic Exchange (i.e., BSPA for 5-year old Capesize vessels of 172,000 DWT size) for the time period ranging from January 4<sup>th</sup>, 2005

to June 30<sup>th</sup>, 2014 as well as from SIN (i.e., for 10-, 15-, and 20-year old Capesize vessels of 170,000, 170,000, and 150,000 DWT size) from January 7<sup>th</sup>, 2005 to June 27<sup>th</sup>, 2014. For zero- and 30-year old Capesize vessels, weekly newbuilding as well as weekly scrap price time series data was collected from SIN for the time period ranging from January 7<sup>th</sup>, 2005 to June 27<sup>th</sup>, 2014 and December 31<sup>st</sup>, 2004 to June 30<sup>th</sup>, 2014, respectively.

FIGURE 2.1: Capesize ship price time series 2005-2014



The graph shows weekly new, second-hand (i.e., for 5-, 10-, 15-, and 20-year old ships), and scrap time series based on panelists' estimations for the time period ranging from January 7<sup>th</sup>, 2005 to June 27<sup>th</sup>, 2014.

*Source:* own graph based on weekly data from The Baltic Exchange and SIN

FIGURE 2.1 shows a plot of these time series. The graph clearly illustrates the boom in the shipping industry from mid-2006 until mid-2008, the sharp decline in ship prices at the end of 2008 caused by the financial crisis, and the severe recession that the shipping industry has been facing since then. Interestingly, the newbuilding prices for dry bulk Capesize vessels exhibit less fluctuations than the second-hand prices for 5-, 10-, or 15-year old vessels and the second-hand prices for 5- and 10-year old vessels even exceeded the price for new vessels for short periods within the boom phase. At that time, the order books of yards were full and placing an additional order for a Capesize vessel would have prevented shipping companies to benefit from the extraordinary freight rates. Consequently, shipping companies were willing to pay premiums on second-hand vessels that could be delivered immediately compared to new vessels with a construction time of two or more years at that time. This effect is

less prominent in older vessels or vessels that were close to scrapping as the remaining economic life of these vessels is rather short and their prices are rather influenced by the dynamics of the world steel price.

TABLE 2.2: Descriptive statistics for Capesize ship price time series

Statistic	Type	Capesize					
		New	5 years	10 years	15 years	20 years	Scrap
Mean (in USD m)	Level	63.977	63.958	47.518	34.104	21.547	9.152
Mean (in %)	Log ret.	-1.291	-3.150	-3.520	-6.187	-7.453	2.415
Stand. dev. (in %)	Log ret.	6.035	17.202	22.691	26.586	29.430	41.887
Skewness	Log ret.	-1.490	-5.085	-1.212	-0.903	-0.868	-10.189
Kurtosis	Log ret.	13.788	57.531	28.816	19.171	16.928	189.793
Jarque-Bera	Log ret.	2,578***	63,337***	13,839***	5,450***	4,055***	726,733***
ADF	Level	-1.031	-1.603	-1.853	-1.753	-1.869	-2.498
ADF	Log ret.	-6.243***	-6.604***	-7.789***	-7.731***	-8.062***	-22.160***
PP	Level	-0.917	-1.389	-1.623	-1.524	-1.780	-2.633*
PP	Log ret.	-17.512***	-10.374***	-16.845***	-17.612***	-16.366***	-22.200***

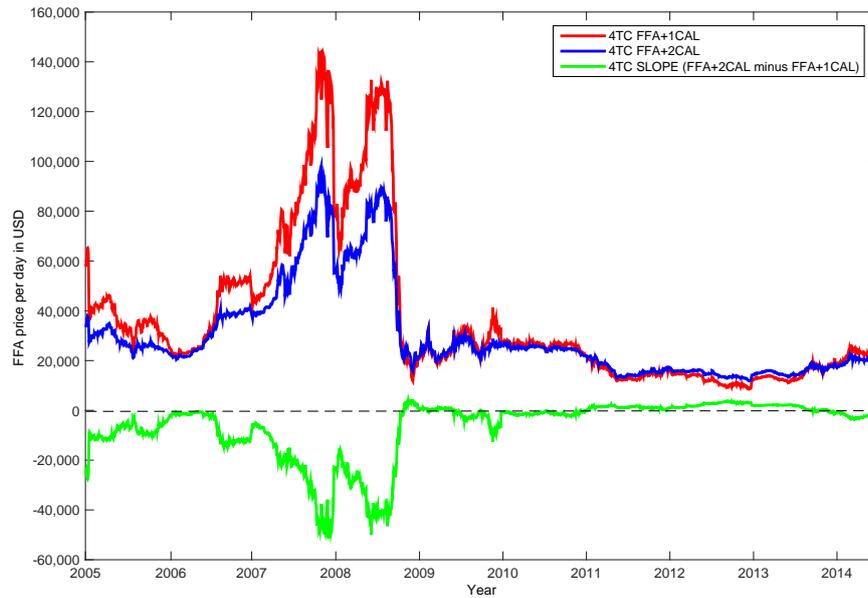
The table shows descriptive statistics for weekly Capesize ship price time series for new, 5-, 10-, 15-, 20-year old, and scrap vessels over the period from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014. This leaves 495 weekly level observations and 494 log return observations. The mean is given for level data and log returns and the mean and standard deviation of the log returns are annualized based on an average of 52 weekly observations per year in the considered time frame. The remaining statistics are based on log returns. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis. The Jarque and Bera (1980) test statistic for normality is  $\chi^2(2)$  distributed with critical values of 4.60, 5.99, and 9.21 at the 10 %, 5 %, and 1 % level, respectively. ADF refers to the Augmented Dickey-Fuller (ADF)-test developed by Dickey and Fuller (1981) and PP refers to the Phillips-Perron (PP)-test developed by Phillips and Perron (1988). The lag length was chosen by minimizing the SBIC criterion. The 10 %, 5 %, and 1 % critical values for the ADF- and PP-tests are -2.570, -2.867, and -3.443, respectively.

TABLE 2.2 shows the corresponding descriptive statistics for these ship price time series. The mean price for a new Capesize vessel in the considered time frame was USD 63.977 million. Due to the overshooting of the second-hand prices for 5-year old vessels as shown in FIGURE 2.1, the mean price of such vessels of USD 63.958 million was almost as high as the average for new vessels. With respect to log returns, 20-year old vessels showed the lowest performance with a mean annualized log return of -7.453 %, whereas the mean annualized log return for scrap vessels was even slightly positive (i.e., 2.415 %). Concerning the volatility, the annualized standard deviation of the log return time series increases from 6.035 % for new vessels to 41.887 % for scrap vessels. All six log return time series exhibit left-skewness, significant excess kurtosis, and are significantly different from a normal distribution. The unit root tests indicate that all level series are non-stationary, whereas the log differences of these series are stationary. Only the Phillips-Perron-test finds the level scrap price time series to be stationary at the 10 % significance level.

With respect to FFA data, daily time series data was collected for dry bulk Capesize

FFA+1CAL and FFA+2CAL contracts for the average of the four mostly used trip charter routes (i.e., so called 4TC-FFA+1CAL and 4TC-FFA+2CAL) from The Baltic Exchange for the time period ranging from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014. The FFAs are quoted in USD per day for the entire ship. The slope of the forward curve was derived by calculating the difference between the FFA+2CAL and FFA+1CAL contract price quote for each day.

FIGURE 2.2: Capesize FFA price time series 2005-2014



The graph shows daily 4TC FFA+1CAL, FFA+2CAL, and the slope between FFA+2CAL and FFA+1CAL price time series for the time period ranging from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014.

*Source:* own graph based on daily data from The Baltic Exchange

FIGURE 2.2 shows a plot of these time series. Similarly to the plot of the ship price time series, the graph indicates the strong increase of FFA rates in the shipping boom period from mid-2006 until mid-2008, the sharp decline in FFA rates at the end of 2008 caused by the financial crisis, and the severe recession that the shipping industry has been facing since then. Furthermore, the plot shows the development of the slope of the FFA curve or difference between the FFA+2CAL and FFA+1CAL price. The FFA+1CAL price was in 64.5% of the observations larger than the FFA+2CAL price and thus, the difference between the two prices was negative reflecting that expected future spot prices for longer term contracts (e.g., FFA+2CAL) are lower than for shorter term contracts (e.g., FFA+1CAL). This spread significantly widened during the boom period in the shipping industry from mid-2006 until mid-2008 and fluctuated closely around zero after the boom period.

TABLE 2.3: Descriptive statistics for Capesize FFA price time series

Statistic	Type	4TC FFAs	
		+1CAL	+2CAL
Mean (in USD)	Level data	36,384.905	29,878.332
Mean (in %)	Log returns	-10.942	-5.608
Standard deviation (in %)	Log returns	50.914	35.438
Skewness	Log returns	-3.155	-2.841
Kurtosis	Log returns	56.290	47.163
Jarque-Bera	Log returns	286,765.911***	197,433.943***
ADF	Level	-2.198	-2.114
ADF	Log returns	-38.973***	-36.463***
PP	Level	-1.724	-1.491
PP	Log returns	-39.628***	-36.869***

The table shows descriptive statistics for daily Capesize 4TC FFA+1CAL and FFA+2CAL price time series for the time period ranging from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014. This leaves 2,391 daily level observations and 2,390 log return observations. The mean is given for level data and log returns and the mean and standard deviation of the log returns are annualized based on an average of 252 trading days in the considered time frame. The remaining statistics are based on log returns. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis. The Jarque and Bera (1980) test statistic for normality is  $\chi^2(2)$  distributed with critical values of 4.60, 5.99, and 9.21 at the 10 %, 5 %, and 1 % level, respectively. ADF refers to the Augmented Dickey-Fuller-test developed by Dickey and Fuller (1981) and PP refers to the Phillips-Perron-test developed by Phillips and Perron (1988). The lag length was chosen by minimizing the SBIC criterion. The 10 %, 5 %, and 1 % critical values for the ADF- and PP-tests are -2.570, -2.867, and -3.443, respectively.

TABLE 2.3 shows the corresponding descriptive statistics of these FFA price time series. The mean prices per day for FFA+1CAL and FFA+2CAL contracts in the considered time frame were USD 36,385 and USD 29,878, respectively. The mean difference per day between the FFA+2CAL and FFA+1CAL contract or slope of the FFA curve was USD -6,507. Both FFA contracts showed negative annualized log returns with -10.942 % and -5.608 % for the FFA+1CAL and FFA+2CAL contract, respectively. The volatility expressed as standard deviation was 50.914 % for the FFA+1CAL contract and 35.438 % for the FFA+2CAL contract. The log return time series of the two FFA contracts also exhibit left-skewness, significant excess kurtosis, and are significantly different from a normal distribution. As it was the case for the ship price time series, the results of the unit root tests indicate that the level series are non-stationary, whereas the log differences of these series are stationary.

As different time series and regression analyses are going to be performed jointly on weekly data for Capesize ship prices based on panelists' estimations as well as on FFA+2CAL rates for the time period ranging from January 4<sup>th</sup>, 2005 to October 28<sup>th</sup>, 2013, unit root tests were also performed for these time series. The results of these tests as shown in TABLE 2.4 confirm the findings that were already shown

TABLE 2.4: Results of further unit root tests

Statistic	Type	Ship prices						FFA
		New	5 years	10 years	15 years	20 years	Scrap	+2CAL
ADF	Level	-1.031	-1.603	-1.853	-1.753	-1.869	-2.498	-2.498
ADF	Log ret.	-6.243***	-6.604***	-7.789***	-7.731***	-8.062***	-22.160***	-22.160***
PP	Level	-0.917	-1.389	-1.623	-1.524	-1.780	-2.633*	-2.633*
PP	Log ret.	-17.512***	-10.374***	-16.845***	-17.612***	-16.366***	-22.200***	-22.200***

The table shows descriptive statistics for weekly Capesize ship price time series for new, 5-, 10-, 15-, 20-year old, and scrap vessels as well as for FFA+2CAL price time series over the period from January 4<sup>th</sup>, 2005 to October 28<sup>th</sup>, 2013. This leaves 454 weekly level observations and 453 log return observations. ADF refers to the Augmented Dickey-Fuller-test developed by Dickey and Fuller (1981) and PP refers to the Phillips-Perron-test developed by Phillips and Perron (1988). The lag length was chosen by minimizing the SBIC criterion. The 10 %, 5 %, and 1 % critical values for the ADF- and PP-tests are -2.570, -2.867, and -3.443, respectively.

for the longer time horizon for the weekly ship price time series as well as for the daily FFA+2CAL price time series. The time series are non-stationary in levels but stationary in log differences.

Besides, daily USD LIBOR overnight rates were collected from Datastream for the time period from January 3<sup>rd</sup>, 2005 to June 30<sup>th</sup>, 2014. These interest rates were used for determining appropriate tailing factors as well as the accumulated interest on the margin account in the empirical analysis of the hedging performance.

## 2.5 Estimation results

In this section, the results of the conducted empirical analyses are presented and interpreted. First, the estimation results of the underlying models for the two different hedging approaches (i.e., minimum variance cross-hedging approach and the hedging approach based on the structural pricing model) is discussed. Subsequently, the paragraphs elaborate on the performance of both hedging approaches in two different set-ups (i.e., over a fixed time horizon of one year and between sale and resale) as well some further robustness checks.

### 2.5.1 Minimum variance cross-hedging model

In order to obtain age-dependent minimum variance hedge ratios required for the minimum-variance cross-hedging model (MVCHM), 402 overlapping 52-week log dif-

ferences of the relevant time series of panelists' estimations for the differently-aged Capesize reference vessels (i.e., new (0-), 5-, 10-, 15-, 20-year old and scrap (30-year old)) as well as for the corresponding Capesize FFA+2CAL time series were first derived. Afterwards, six regression models of the form as shown in equation (2.3) were estimated, each based on 52-week log returns of one ship price time series and the FFA+2CAL time series.

TABLE 2.5: Estimates for the minimum variance cross-hedging model

$$\Delta_{52}p_{t,Age} = \alpha_{52,Age} + \beta_{52,Age}\Delta_{52}f_t + \eta_t \quad \text{with } Age \in \{0, 5, 10, 15, 20, 30\}$$

	0-year old (new)	5-year old	10-year old	15-year old	20-year old	30-year old (scrap)
$\alpha$	-0.0160 (0.0194) [-0.8233]	-0.0321 (0.0207) [-1.5542]	-0.0313 (0.0253) [-1.235]	-0.0484 (0.0292) [-1.6589]	-0.0327 (0.0380) [-0.8609]	0.0542* (0.0304) [1.7827]
$\beta_{Age}$	0.2650*** (0.0407) [6.5046]	0.8146*** (0.0464) [17.5587]	0.8223*** (0.0570) [14.4153]	0.9673*** (0.0676) [14.3097]	0.9565*** (0.0772) [12.3853]	0.4451*** (0.0681) [6.5342]
$R^2$	0.4384	0.8293	0.7764	0.7816	0.6954	0.4035

The table shows regression estimates for the  $\beta_{52,Age}$ -coefficients as well as corresponding  $R^2$ -values for the minimum variance cross-hedging model based on 402 weekly 52-week log differences of the respective underlying time series for Capesize vessels (i.e., SIN newbuilding, BSPA 5-year old, SIN 10-, 15-, and 20-year old, and SIN scrap) and FFA+2CAL time series from January 9<sup>th</sup>, 2006 to October 28<sup>th</sup>, 2013. The standard errors have been corrected for autocorrelation and heteroscedasticity using the method by Newey and West (1987). Figures in () and [] reflect the corresponding standard errors and t-statistics, respectively. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

TABLE 2.5 shows the respective regression estimates. The  $\beta_{52,Age}$ -coefficients increase from 0.2652 for new vessels to 0.9677 for 15-year old vessels and decrease to 0.4425 for scrap vessels. This implies that the desired hedge exposure in FFA+2CAL contracts increases up to almost 100 % of the vessel price from new to 15- or 20-year old vessels. Once the vessel has passed the age of 20-years, the desired hedge exposure in FFA+2CAL contracts significantly declines again as there is continuously less economic life of the vessel remaining and the price dynamics of these kind of vessels cannot be adequately mirrored using FFA+2CAL contracts. Similarly, the price dynamics of new vessels are also relatively different from the ones of FFA+2CAL contracts. The corresponding  $R^2$  is highest for 5-year old vessels with 82.82 % followed by 15-, 10-, and 20-year old vessels with 78.10 %, 77.58 %, and 69.49 %, respectively. There is again a considerable difference for new and scrap vessels as the  $R^2$ s are significantly lower with 43.87 % and 40.55 %, respectively.

As already mentioned in subsection 2.3.1, the use of overlapping observations for the 52-week log returns causes the error term,  $\eta_t$ , to follow a MA process of order 51 rather than being *iid*. Accordingly, the regression estimates contain inefficient standard errors as well as an incorrect  $R^2$ s. Alizadeh and Nomikos (2012) found that the resulting coefficients or hedge ratios of such a regression are nevertheless consistent and unbiased. The stationary bootstrap technique developed by Politis and Romano (1994) was used to generate  $n = 1,000$  resampled data sets of 52-week log returns and the initial experiment (i.e., the regression based on overlapping 52-week log returns) was repeated  $n$  times. Therefore, the stationary bootstrap algorithm was applied jointly over 402 52-week log returns of dry bulk Capesize new, 5-, 10-, 15-, 20-year old, and scrap price time series as well as the Capesize FFA+2CAL time series for the time period ranging from January 10<sup>th</sup>, 2005 to October 28<sup>th</sup>, 2013. The mean block length was selected as  $q = 0.005$  reflecting a mean block length of 200. As 52-week log returns exhibit significant autocorrelation,  $q$  was chosen based on an inspection of the autocorrelation functions of the original 52-week log returns as Politis and White (2004) suggested. Subsequently, the regressions of equation (2.3) were performed again for each resampled data set and the mean  $\beta_{52, Age}$ -coefficients determined as well as 95 % confidence intervals based on the empirical distributions of the  $\beta_{52, Age}$ -coefficients.

TABLE 2.6: Results of regressions on bootstrapped 52-week log returns

Age	Original	Stationary bootstrap			
	$\beta_{52, Age}$	Mean $\bar{\beta}_{52, Age}$	Median $\bar{\beta}_{52, Age}^{med}$	95 % confidence interval	
				Lower bound	Upper bound
0-year old (new)	0.2650	0.2540	0.2650	[ 0.0821	- 0.3764 ]
5-year old	0.8146	0.8032	0.8146	[ 0.6678	- 0.8759 ]
10-year old	0.8223	0.8140	0.8223	[ 0.6233	- 0.9177 ]
15-year old	0.9673	0.9519	0.9673	[ 0.6948	- 1.0633 ]
20-year old	0.9565	0.9220	0.9565	[ 0.5225	- 1.0916 ]
30-year old (scrap)	0.4451	0.4287	0.4451	[ 0.0279	- 0.6488 ]

The table shows aggregated regression estimates using 1,000 resamples using the stationary bootstrap technique by Politis and Romano (1994) of the original underlying, overlapping 52-week log return time series for Capesize vessel prices and FFA+2CAL prices from January 4<sup>th</sup>, 2005 to October 28<sup>th</sup>, 2013. The ‘smoothing parameter’,  $q$ , was chosen 0.005 reflecting a mean block length of 200. The lower and upper bounds of the 95 % confidence interval refer to the 2.5 %- and 97.5 %-quantiles of the empirical distributions of the  $\beta_{Age}$ -coefficients.

TABLE 2.6 shows the mean and median  $\beta_{52, Age}$ -coefficients,  $\bar{\beta}_{52, Age}$  and  $\bar{\beta}_{52, Age}^{med}$ , of the bootstrapped, resampled data sets of 52-week log returns as well as the 95 % confidence intervals based on the empirical distribution for each of the six  $\beta_{Age}$ -coefficients.

The results indicate that the 95 % confidence intervals based on the empirical distributions of estimated  $\beta_{52,Age}$ -coefficients contain the estimated  $\beta_{52,Age}$ -coefficients based on the original data and that the mean  $\beta_{52,Age}$ -coefficients,  $\bar{\beta}_{52,Age}$ , of the bootstrapped, resampled data sets are only negligibly different from the  $\beta_{52,Age}$ -coefficients from the initial regressions. The median  $\beta_{52,Age}$ -coefficients,  $\bar{\beta}_{52,Age}^{med}$ , are even similar to the  $\beta_{52,Age}$ -coefficients from the regressions using the original data set. Furthermore, the width of these bootstrapped confidence intervals is larger than the width of classical confidence intervals based on the standard errors of the initial regressions. This confirms that the standard errors from the initial regressions are in fact inefficient and tend to underestimate the true standard errors of the  $\beta_{52,Age}$ -coefficients. It was also tested for the sensitivity of these results towards a change of the ‘smoothing parameter’,  $q$ , corresponding to a mean block length of  $1/q$  in the stationary bootstrap, but found the results to be relatively robust towards changes in the choice of  $q$ . The results of the sensitivity analysis can be found in the appendix A.2 on page 64. However, as the  $\beta_{52,Age}$ -coefficients from the initial regressions are consistent estimators of the true coefficients and are contained in the bootstrapped confidence intervals, these coefficients were used as minimum variance hedge ratios for 0-, 5-, 10-, 15-, 20-, and 30-year old vessels. As indicated in subsection 2.3.1, linear interpolation was used to derive the respective minimum variance hedge ratios for Capesize ship ages other than the six, specific age points from 0 to 30 years.

## 2.5.2 Structural pricing model

Numerous combinations of explanatory variables were tested from the available ship-specific data mentioned in section 2.4, FFA rates containing freight market information data that may be used as hedging instruments, and cross-terms between pairs of these variables to estimate Capesize second-hand ship prices in a SPM of the form as shown in equation (2.4).

One model in particular and two variations of it turned out to yield a balanced trade-off between pricing accuracy and suitability as basis for hedging efforts. Concerning the freight market information or market-driven explanatory variables, two components were relied on: the FFA+1CAL rate and the slope or difference between the FFA+2CAL and FFA+1CAL rate on the respective transaction date both transformed in USD million per day. Future earnings from freight rates are the major component

of a vessel's second-hand price from a discounted cash flow perspective. Accordingly, the model incorporates information on expected freight rate levels over the next calendar year as well as information on the expected price trend from the next calendar year until the second-to-next calendar year. Further out maturity FFA contracts are less liquid and price data is only available for a shorter time horizon, so the focus was on the most liquid of these instruments which additionally nicely fit the hedging horizon of one year that was considered within the analysis of the performance of both hedging approaches.

Concerning vessel-specific or deterministic risk factors, the age at transaction as well as the consumption of the vessel were included in the model. As other efforts have already shown, the ship's age at transaction possesses considerable explanatory power because it determines the remaining economic life in which the vessel is still able to generate earnings in the future (Adland and Koekebakker, 2007). The consumption of a vessel is usually measured in metric tonnes per day. However, Capesize vessels are quite heterogeneous with respect to size measured in DWT as was shown and besides also with respect to speed measured in knots per hour. The individual consumption in metric tonnes per day value was deemed as distortive in terms of bunker cost per DWT that are comparable across differently-sized vessels. Adland and Koekebakker (2007) already identified the vessel size as another important driver of the ship's second-hand price. Alternatively, the vessel's size or DWT as well as the vessel's speed were used to derive a value for the vessel's consumption per 1,000 metric tonnes of DWT per 1,000 nautical miles<sup>7</sup> which is appropriate for fair comparisons across vessels. Bunker costs reflect a major part of the respective voyage costs for shipping companies (Stopford, 2009). Consequently, an appropriately comparable measure for the vessel's bunker or fuel efficiency is also a reasonable driver of second-hand ship prices as efficient ships operate at a lower cost base and shipping companies are willing to pay more for such vessels than for inefficient vessels.

Moreover, cross-terms between the FFA+1CAL and the vessel's age, the FFA+1CAL and the derived comparable consumption, and vessel's age and the derived comparable consumption were included in order to account for interaction effects between these variables. For the FFA+1CAL and the vessel's age, for instance, the influence of the FFA+1CAL rate on the second-hand price of the vessel decreases with increasing age of the vessel as the lower remaining economic life of the ship does only allow to

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<sup>7</sup> Voyaging at one knot per hour corresponds to a travelled distance of one nautical mile per hour.

generate future earnings from freight rates for a shorter amount of time. The resulting, considered models are shown in equations (2.15), (2.16), and (2.17) below:

$$\begin{aligned} \text{SPM 1: } p_{i,t} = & \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{Consum} \cdot Consum_i \\ & + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i \\ & + \beta_{Age \cdot Consum} \cdot Age_{i,t} \cdot Consum_i + \varepsilon_{i,t} \end{aligned} \quad (2.15)$$

$$\begin{aligned} \text{SPM 2: } p_{i,t} = & \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{Consum} \cdot Consum_i \\ & + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i + \varepsilon_{i,t} \end{aligned} \quad (2.16)$$

$$\begin{aligned} \text{SPM 3: } p_{i,t} = & \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} \\ & + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i + \varepsilon_{i,t}. \end{aligned} \quad (2.17)$$

Within equations (2.15), (2.16), and (2.17) above, the price or value of vessel  $i$  at time  $t$  in USD million is referred to by  $p_{i,t}$  and to the constant of the regression by  $\alpha$ . The price of the FFA+1CAL contract at time  $t$  in USD million is denoted by  $f_t$  and the slope of the FFA curve or difference between the FFA+2CAL and FFA+1CAL contract at time  $t$  in USD million by  $sl_t$ . The age of vessel  $i$  at time  $t$  is represented by  $Age_{i,t}$  and the derived comparable consumption per 1,000 metric tonnes of DWT per 1,000 nautical miles of vessel  $i$  by  $Consum_i$ . The coefficients of the respective corresponding explanatory variables are denoted by  $\beta_f$ ,  $\beta_{sl}$ ,  $\beta_{Age}$ ,  $\beta_{Consum}$ ,  $\beta_{f \cdot Age}$ ,  $\beta_{f \cdot Consum}$ , and  $\beta_{Age \cdot Consum}$ . The error term of the structural pricing model is represented by  $\varepsilon_{i,t}$ .

TABLE 2.7 shows the estimation results of the three structural pricing models of equations (2.15), (2.16), and (2.17) and indicates that the coefficient estimates are mostly significant at the 1 % level, some are only significant at the 5 % level. The estimates for the  $\beta_f$ -coefficient of 1,980.7053 in the SPM 1, for instance, implies to the vessel's exposure to the FFA+1CAL contract within this model. As the lot size of the FFA+1CAL contract is one day, the value 1,980.7053 can be directly interpreted as days of FFA+1CAL exposure. Moreover, the signs of the coefficients are also largely in line with what one would expect from economic theory. Accordingly, positive coefficient signs for  $\beta_f$  and  $\beta_{sl}$ , on the one hand, correspond to higher estimated vessel prices if the freight rates increase or the slope of the FFA curve or difference between the FFA+2CAL and FFA+1CAL contract widens. On the other hand, negative coefficient signs for  $\beta_{Age}$ ,  $\beta_{Consum}$ ,  $\beta_{f \cdot Age}$ , and  $\beta_{f \cdot Consum}$  correspond to lower estimated vessel prices if the vessel's age or consumption is higher or interac-

TABLE 2.7: Estimates for different Capesize structural pricing models

	SPM 1		SPM 2		SPM 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
SPM 1:	$p_{i,t} = \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{Consum} \cdot Consum_i + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i + \beta_{Age \cdot Consum} \cdot Age_{i,t} \cdot Consum_i + \varepsilon_{i,t}$					
SPM 2:	$p_{i,t} = \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{Consum} \cdot Consum_i + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i + \varepsilon_{i,t}$					
SPM 3:	$p_{i,t} = \alpha + \beta_f \cdot f_t + \beta_{sl} \cdot sl_t + \beta_{Age} \cdot Age_{i,t} + \beta_{f \cdot Age} \cdot f_t \cdot Age_{i,t} + \beta_{f \cdot Consum} \cdot f_t \cdot Consum_i + \varepsilon_{i,t}$					
$\alpha$	74.4084*** (21.2165) [3.5071]	0.0006	24.0854** (11.3193) [2.1278]	0.0346	30.0874*** (4.7346) [6.3547]	0.0000
$\beta_f$	1,980.7053*** (253.2217) [7.8220]	0.0000	1,862.9160*** (253.8745) [7.3379]	0.0000	1,782.7990*** (213.2596) [8.3598]	0.0000
$\beta_{sl}$	1,554.1709*** (437.3689) [3.5535]	0.0005	1,538.7278*** (444.7028) [3.4601]	0.0007	1,549.7201*** (443.5717) [3.4937]	0.0006
$\beta_{Age}$	-4.8208*** (1.0529) [-4.5787]	0.0000	-1.9483*** (0.2173) [-8.9659]	0.0000	-1.9470*** (0.2169) [-8.9750]	0.0000
$\beta_{Consum}$	-45.6605** (21.6886) [-2.1053]	0.0365	6.5018 (11.1338) [0.5840]	0.5599		
$\beta_{f \cdot Age}$	-12.0131*** (4.2425) [-2.8316]	0.0051	-10.8958** (4.2947) [-2.5371]	0.0119	-11.1348*** (4.2681) [-2.6089]	0.0098
$\beta_{f \cdot Consum}$	-652.9065*** (194.7444) [-3.3526]	0.0010	-552.1529*** (194.5819) [-2.8376]	0.0050	-459.7032*** (112.9465) [-4.0701]	0.0001
$\beta_{Age \cdot Consum}$	2.9481*** (1.0581) [2.7862]	0.0059				
$R^2$	0.7872		0.7788		0.7785	
Adj. $R^2$	0.7797		0.7722		0.7729	
Log likelihood	-796.7559		-800.7170		-800.8933	
SBIC criterion	7.9424		7.9550		7.9308	
Akaike criterion	7.8132		7.8419		7.8339	
Standard error	11.8061		12.0050		11.9852	

The table shows linear regression coefficient estimates for three different SPMs based on data for 206 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to October 30<sup>th</sup>, 2013.  $p_{i,t}$  refers to the price of vessel  $i$  at time  $t$  in USD million,  $f$  to the price of a FFA+1CAL contract at time  $t$  in USD million,  $sl$  to the slope of the forward curve between a FFA+2CAL and FFA+1CAL contract at time  $t$  in USD million,  $Age$  to the age of vessel  $i$  at time  $t$  in years, and  $Consum$  to the consumption per 1,000 nautical miles per 1,000 DWT of vessel  $i$  in metric tonnes. Figures in () and [] reflect the corresponding standard errors and t-statistics, respectively. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

tion term of freight rate and age or consumption is higher. The latter of the two captures the joint dynamics and is rather of corrective nature to the individual single coefficients. The only coefficient sign that is not economically intuitive is the one for  $\beta_{Age \cdot Consum}$ . One would expect that a larger value for this interaction term (i.e., an older and more inefficient vessel) would cause the estimated vessel price to fall. However, the positive sign of the coefficient indicates an increasing relationship. An outlier analysis was performed on the 10 largest observations of the interaction term  $Age \cdot Consum$  in the data set of 206 real Capesize sale and purchase transactions and found that these extreme values for the interaction term are not evenly distributed across the time period considered. Nine out of 10 of the largest observations for the interaction term  $Age \cdot Consum$  occurred before the shipping crisis and seven of these nine largest observations within the boom period from mid-2006 until mid-2008. The age of these 10 largest observations lies between 24 and 28 years and the consumption between 1.21 and 1.63 metric tonnes per 1,000 DWT per 1,000 nautical miles which both are at the higher end of all observations within the data set. It seems likely that the positive sign is caused by the relatively expensive sales of old and inefficient ships prior to the shipping crisis. A model-implied pricing surface with changing age and consumption values and fixed FFA+1CAL and slope between FFA+2CAL and FFA+1CAL values was plotted confirming this hypothesis. The 3D plot of the pricing surface can be found in FIGURE 2.4 in the appendix A.3 on page 65. As a consequence, the interaction term  $Age \cdot Consum$  was eliminated from the model and estimated the model SPM 2. As the  $\beta_{Consum}$ -coefficient is not significant and the sign of the coefficient not economically intuitive in the SPM 2, a third structural pricing model, SPM 3, was estimated in which the plain consumption value as explanatory variable was eliminated.

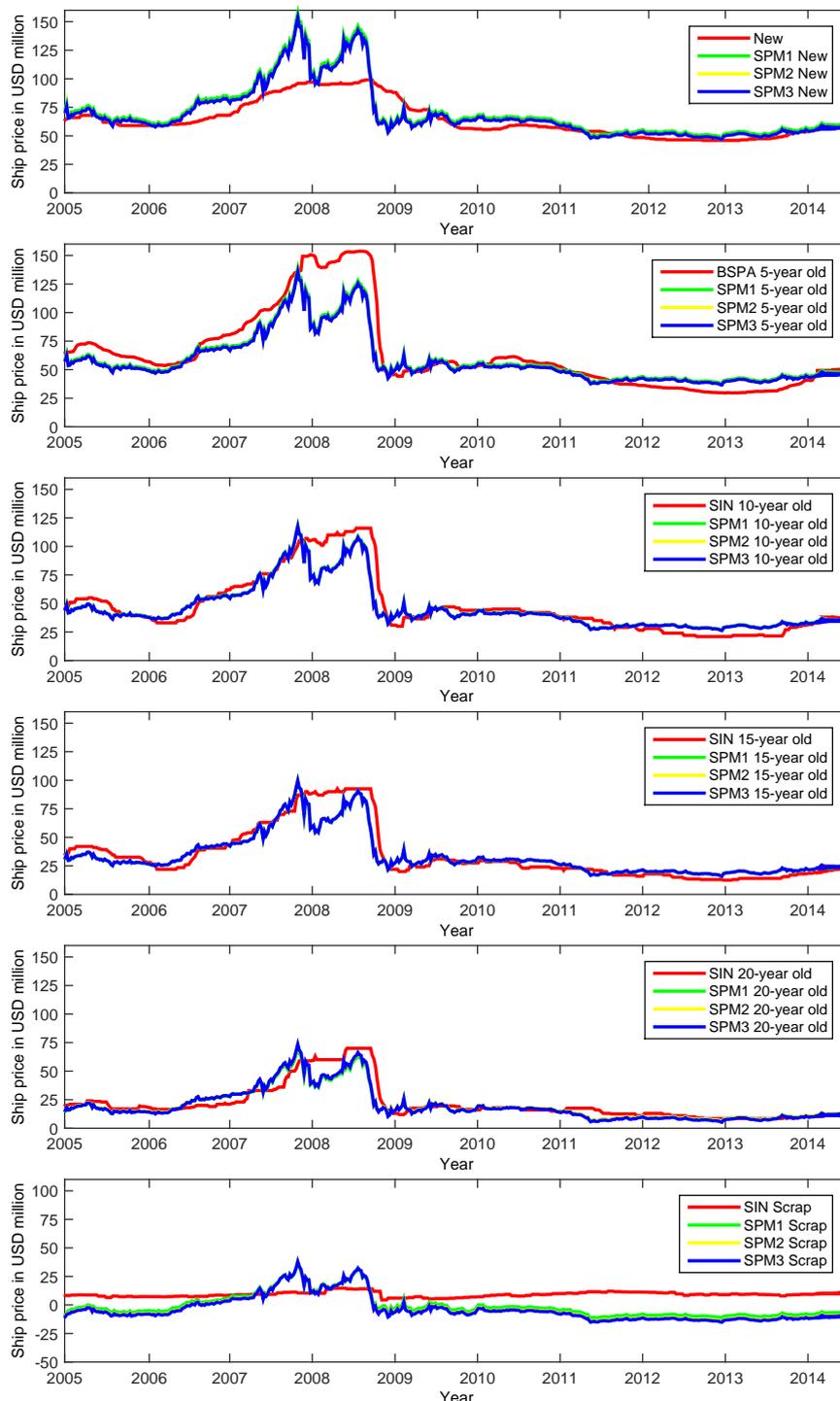
From a model selection perspective, the SPM 1 shows the highest adjusted  $R^2$  of the three models with 0.7797. The SBIC criterion prefers the SPM 3, whereas the Akaike criterion prefers SPM 1. As the models are nested, a log likelihood ratio test whether SPM 1 is significantly better than SPM 2 was performed. It found that SPM 1 is better at the 1 % level with a log likelihood test statistic of 7.9222 against the 1 % critical value of 6.6349 of the chi-square distribution with one degree of freedom. Similarly, SPM 1 was tested against SPM 3 and it was found that SPM 1 is better at the 5 % level with a log likelihood test statistic of 8.6748 against the 5 % critical value of 5.9915 of the chi-square distribution with two degrees of freedom. From a log likelihood ratio test perspective, SPM 2 and SPM 3 are not significantly different.

However, SPM 2 is strictly dominated by either SPM 1 or SPM 3 in all the other test statistics. The standard errors of the models is lowest for SPM 1 with USD 11.8061 million followed by SPM 3 with USD 11.9852 million. The standard error of SPM 1, however, is still of considerable size in light of the mean transaction price in the data set of USD 36.12 million. Consequently, SPM 1 is considered as the preferred model, although the positive sign of the  $\beta$ -coefficient of the interaction term  $Age \cdot Consum$  likely seems to be a result of overfitting the model to the particular data set.

Subsequently, all three structural pricing models were tested against the time series of panelists' estimations of second-hand prices for differently-aged reference vessels. In order to do so, the three structural pricing models were evaluated every week from January 4<sup>th</sup>, 2005 until June 30<sup>th</sup>, 2014 using the vessel specifications of the reference vessels underlying the respective panelists' estimation time series as well as the respective FFA+1CAL and FFA+2CAL rates at that time. FIGURE 2.3 shows plots of model-implied ship prices vs. the panelists' estimations for new, 5-, 10-, 15-, 20-year old and scrap vessels. The plots reveal that the structural pricing models are not able to accurately capture the price dynamics of new and scrap vessels as was already presumed. For new vessels, there are considerable deviations in the boom period. The model-implied prices show a stronger price increase within this period due to the increase in FFA rates. As prices for new vessels are rather driven by the demand for new vessels, costs of raw materials, yard-utilization, and development costs than by FFA rates, the panelists' estimations for new vessels did not react so strongly within the boom period as well as during the breakout of the crisis.

For the 5-, 10-, 15-, and 20-year old vessels, the structural pricing models capture the overall price dynamics rather accurately. Major deviations only occurred within the boom period where the model-implied prices already exhibit a considerable price decline in late 2007 when the financial crisis has not hit until later in the following year 2008 and within the breakout of the financial crisis where the model-implied prices seem to react faster to the breakout of the crisis. This is actually interesting as the panelists' estimations for the second-hand ship prices are claimed to be provided by experts of the dry bulk shipping industry. Nevertheless, the panelists' estimations did not show a noticeable reaction to the already prominent declines in freight and FFA rates in late 2007. Moreover, the delayed reaction of the panelists' estimations to the breakout of the financial crisis might be a sign for an underlying MA process in the derivation of the panelists' estimations. Besides, the suggested structural pric-

FIGURE 2.3: Model-implied prices vs. panelists' estimations for differently-aged Capesize reference vessels 2005-2014



The graphs show weekly comparisons of panelists' estimation time series for new, 5-, 10-, 15-, 20-year old, and scrap Capesize reference vessels with respective model-implied ship prices from SPM 1 to SPM 3 for the time period ranging from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014. *Source:* own graph based on weekly data from The Baltic Exchange and SIN

ing models do not even include all available information. Certain information, such as spot freight rates, newbuilding price quotes, or scrap price quotes, have been excluded as market-driven explanatory variables from the models as exposure to these factors cannot be simply created from a hedging perspective. Consequently, including these additional factors would even result in a more accurate structural pricing model outperforming the panelists' estimations even more. Additionally, the price quotes provided by the panelists' have two further significant drawbacks. Firstly, the price quotes are only available for six selected age points and secondly, each of these price quotes refers only to a certain reference vessel. As the Capesize dry bulk vessel class is characterized by considerable heterogeneity with respect to size in DWT, the information provided by these price quotes might only be of limited use for a ship owner with a vessel of an age that is between two of the six specific age points and is significantly smaller or larger in terms of size in DWT.

With respect to scrap vessels, the model-implied prices show considerable deviations from the panelists' estimations across the entire time period considered as well as even negative prices for a considerable amount of time. These negative implied prices are obviously not meaningful. Accordingly, the structural pricing models are less reliable for extremely young and old vessels because the vast majority of transactions in the data set were vessels aged between 2.5 and 25 years.

### **2.5.3 Hedging results**

With respect to the hedge effectiveness, both approaches based on the models discussed above were tested in two different hedging set-ups according to the methodology as described in subsection 2.3.3 (i.e., over a fixed time horizon of one year and between sale and resale of one and the same vessel). The following paragraphs elaborate on the results of these tests as well as on further robustness checks.

#### **2.5.3.1 Fixed time horizon of one year**

In this first hedging set-up, a fixed hedging time horizon of one year or 252 trading days prior to the transaction date requires that FFA rates have already been available at the initiation of the hedge. Accordingly, only Capesize sale and purchase transactions that occurred later than January 4<sup>th</sup>, 2006 were considered as The Baltic Exchange started

quoting FFA rates on January 4<sup>th</sup>, 2005 and 23 transactions had to be eliminated from the initial data set of 206 Capesize sale and purchase transactions. Moreover, another three transactions were eliminated because the vessel's age would have been lower than zero at the initiation of the hedge. This left 180 Capesize sale and purchase transactions for which the hedge effectiveness of the two different hedging approaches was tested over the course of the fixed hedging period of one year or 252 trading days. The analysis followed the methodology described in subsection 2.3.3.1.

In order to apply the structural pricing models in this hedging set-up, the aggregated exposure to each of the FFA contracts needs to be determined. As the slope of the FFA curve is nothing else than the difference between FFA+2CAL and FFA+1CAL contract, the resulting exposure to this explanatory variable can be replicated by taking a long position of  $\beta_{sl} = 1,554.1709$  FFA+2CAL days together with a short position of  $\beta_{sl} = 1,554.1709$  FFA+1CAL days for the SPM 1, for instance. As the analysis is concerned with hedging the respective exposure, the contrary position needs to be taken. So, within the SPM 1, a shipping company would want to short 1,554.1709 days of FFA+2CAL contracts. As the model-implied, aggregated exposure towards the FFA+1CAL contract is determined by various  $\beta$ -coefficients, the aggregated short exposure towards the FFA+1CAL contract in days can be determined in the following way for all three structural pricing models as shown in equation (2.18) below. As already indicated above, the short exposure towards the FFA+2CAL contract in days for all three structural pricing models is given by the respective negative  $\beta_{sl}$ -coefficient as shown in equation (2.19) below:

$$d_{i,t_i-252,FFA+1CAL} = -(\beta_f + \beta_{f.Age} \cdot Age_{i,t_i} + \beta_{f.Consum} \cdot Consum_i - \beta_{sl}) \quad (2.18)$$

$$d_{i,t_i-252,FFA+2CAL} = -\beta_{sl}. \quad (2.19)$$

TABLE 2.8 shows the results of both hedging approaches within this first hedging set-up over a fixed time horizon. The mean values for the model-implied physical positions including the total age effect at the hedging start date,  $t_i - 252$ , indicate that the MVCHM tends to estimate model-implied physical positions that are about USD 2.0 million higher compared to the model-implied physical positions by the three SPMs. Accordingly, the mean delta or loss from the physical position is about USD 2.0 million larger for the MVCHM. With respect to the initial hedge exposure, the results show that the desired exposure is about USD 3.0 million lower for the MVCHM

TABLE 2.8: Hedging results over fixed time horizon of one year

Physical position		in	MVCHM	SPM 1	SPM 2	SPM 3
Start (age effect incl.)	Mean	USD m	39.0540	37.0162	37.0166	36.9187
End (transaction price)	Mean	USD m	35.1450	35.1450	35.1450	35.1450
Hedge exposure		in	MVCHM	SPM 1	SPM 2	SPM 3
Start	Mean	USD m	-32.7414	-35.7461	-35.6310	-35.6992
End	Mean	USD m	-39.1678	-34.3159	-34.3539	-34.5134
Delta/change in values or profit/loss		in	MVCHM	SPM 1	SPM 2	SPM 3
Physical position	Mean	USD m	-3.9090	-1.8712	-1.8716	-1.7737
	Median	USD m	-1.8968	-2.4995	-2.3760	-2.6078
	Variance	USD m	981.3950	760.6406	749.2735	749.0990
	Stand. dev.	USD m	31.3272	27.5797	27.3729	27.3697
	Skewness		-0.5876	-0.2831	-0.2173	-0.2085
	Kurtosis		3.7132	3.9087	3.7222	3.7003
Hedge	Mean	USD m	-1.5855	0.0114	-0.1792	-0.2808
	Median	USD m	0.5553	1.4129	1.4205	1.4620
	Variance	USD m	612.7425	691.3574	687.7393	685.6391
	Stand. dev.	USD m	24.7536	26.2937	26.2248	26.1847
	Skewness		-0.0012	0.4953	0.4751	0.4693
	Kurtosis		4.1321	3.7163	3.5830	3.5432
thereof: interest effect	Mean	USD m	-0.1506	-0.1511	-0.1539	-0.1556
Hedged position (physical position + hedge)	Mean	USD m	-5.4945	-1.8598	-2.0508	-2.0545
	Median	USD m	-1.5517	-1.1274	-1.0606	-1.4934
	Variance	USD m	321.8105	171.2304	173.8485	174.8681
	Stand. dev.	USD m	17.9391	13.0855	13.1852	13.2238
	Skewness		-2.0567	-3.2952	-3.3760	-3.4043
	Kurtosis		11.4808	22.8123	22.8226	23.0600
Hedge effectiveness		in	MVCHM	SPM 1	SPM 2	SPM 3
Reduction of	Variance	%	67.2089	77.4887	76.7977	76.6562
	Stand. dev.	%	42.7365	52.5539	51.8313	51.6846

The table shows selected descriptive statistics for the start and end values as well as for the delta of the physical position, hedge exposure, and portfolio position over the fixed hedging horizon of one year prior to the individual vessel transaction. The considered sample size is 180 vessel transactions as transactions in 2005 had to be excluded due to unavailability of FFA time series data prior to 2005 as well as vessels that would have been negatively aged at the initiation of the hedge. Furthermore, the results for the hedge effectiveness for the different hedging approaches are displayed.

compared to the three SPMs. From a practical perspective, the shipping company would have to deposit higher initial margins at the clearing house if they applied the hedging approach based on the SPM. Although the mean hedge exposure is initially lower for the MVCHM, the mean loss from the hedge is about USD 1.3 to 1.6 million higher for the MVCHM. The displayed mean end values for the hedge exposures show that the exposure at the end of the hedge is about USD 4.5 to 4.7 million larger for the MVCHM than for the SPMs. The reported hedge profits, however, cannot be directly derived from subtracting the start values from the end values as the hedge is set up in two stages (i.e., from hedging start date,  $t_i - 252$ , to the next rollover date and from the rollover date to the hedging end date,  $t_i$ ). The first hedge is closed out just before the rollover date and the second hedge is set up from scratch using the model-implied physical position adjusted for the remaining age effect from rollover date to hedging end date,  $t_i$ . Accordingly, the initial hedge exposure cannot be directly compared to the final or end hedge exposure in order to derive the hedge profit or loss. The hedge profit or loss is aggregated across the two consecutive hedges. Typically, one would expect a profit from the hedge effort if the vessel prices declined and vice versa. However, this seems not to be the case here as the mean hedge profit is negative (i.e., a loss), although the physical position also incurred a loss. This picture, however, is largely caused by the aggregation across different hedges. Looking at hedge results for individual vessels reveals that the hedge profit is positive for the vast majority of the observations if the physical position incurred a loss. The interest effect on the margin account seems to play an insignificant role. Although the mean cost of the hedge or loss is considerably lower for the SPMs, the hedge effectiveness of the approaches based on these models turns out to be significantly better than for the MVCHM. The SPM 1 achieves a hedge effectiveness of 77.49 % variance reduction, whereas the MVCHM only achieves a hedge effectiveness of 67.21 % variance reduction. The SPM 2 and the SPM 3 show only slightly lower hedge effectiveness compared to the SPM 1 with 76.80 % and 76.66 %, respectively. Consequently, the empirical analysis shows consistently superior results of the SPMs in terms of hedge effectiveness at lower costs (i.e., losses from the hedge positions) in this first hedging set-up.

Concerning the number of days shorted of the individual hedging instruments in both hedging approaches, the MVCHM suggests as mean a number of about 1,025 FFA+2CAL days shorted at the initial set-up of the hedge. On the contrary, the SPMs suggest an additional mean long position of about 354 to 363 FFA+1CAL days and a mean short position of about 1,515 to 1,530 FFA+2CAL days at the initial

set-up of the hedge depending on the respective SPM.

FIGURE 2.6 in the appendix B.1 on page 68 shows histogram plots of the individual physical position as well as hedged position outcomes for the MVCHM as well as the SPM 1. From visual inspection, the shape of the hedged position histograms clearly become more leptokurtic than the shape of the physical position histograms. Moreover, the effect also seems to be larger for the SPM 1 than for the MVCHM. One detailed hedge example of the ship ‘Partagas’ can be found in the appendix B.2 on page 69. Ship-specific details as well as transaction details are provided and the cumulative development of the hedge position is shown in plot over time.

### 2.5.3.2 Between sale and resale

In this second hedging set-up, only vessels that have been sold twice in the data set were considered and the hedging horizon was changed from a fixed time period of one year to the vessel-specific time period between sale and resale. Accordingly, 134 transactions had to be eliminated from the initial data set of 206 Capesize sale and purchase transactions. 36 transaction pairs (i.e., 72 transactions in total) of corresponding sale and resale of one and the same vessel remained. Of the 36 resale pairs, six vessels have been sold twice, so that a total of 72 transactions or 36 individual resale pairs remained. The mean hedge duration (i.e., time between sale and resale,  $l_i$ ) of the these vessels was 2.51 years or 632 trading days. The shortest hedge covered a time period of 0.32 years or 80 trading days, whereas the longest hedge covered a time period of 7.79 years or 1,962 trading days. Again, the hedge effectiveness of the two different hedging approaches was tested for these transactions following the methodology as described in subsection 2.3.3.2.

TABLE 2.9 shows the results of both hedging approaches within this second hedging set-up between sale and resale of one and the same vessel  $i$ . Contrary to the first hedging set-up, the mean values for the physical position at the hedging start date,  $t_i - l_i$ , are roughly similar for the MVCHM, SPM 2, and SPM 3. Only the SPM 1 slightly differs from the other models suggesting a model-implied physical position at hedging start date which is about USD 0.4 million lower than the values for the other models. This, however, results from the fact that the basic physical position at the hedging start date,  $t_i - l_i$ , is largely driven by the observed transaction price of the first sale of vessel  $i$  in this second hedging set-up. The physical positions only differ

TABLE 2.9: Hedging results between sale and resale

Physical position		in	MVCHM	SPM 1	SPM 2	SPM 3
Start (age effect incl.)	Mean	USD m	37.4065	37.0050	37.4087	37.3806
End (transaction price)	Mean	USD m	35.4694	35.4694	35.4694	35.4694
Hedge exposure		in	MVCHM	SPM 1	SPM 2	SPM 3
Start	Mean	USD m	-34.5153	-38.6890	-38.7551	-38.9178
End	Mean	USD m	-39.6569	-38.4402	-38.7171	-38.3178
Delta/change in values or profit/loss		in	MVCHM	SPM 1	SPM 2	SPM 3
Physical position	Mean	USD m	-1.9371	-1.5355	-1.9393	-1.9111
	Median	USD m	0.4588	4.1322	3.9630	3.9711
	Variance	USD m	945.6753	1,009.11012	1,038.2353	1,036.7972
	Stand. dev.	USD m	30.7518	31.7664	32.2217	32.1993
	Skewness		-0.3062	-0.4448	-0.5051	-0.5034
	Kurtosis		4.1047	3.9733	4.0541	4.0518
Hedge	Mean	USD m	-12.5407	-7.9770	-8.1718	-8.1862
	Median	USD m	-17.8551	-9.3927	-10.6074	-11.0172
	Variance	USD m	1,380.5035	954.2877	942.3268	927.7435
	Stand. dev.	USD m	37.1551	30.8915	30.6973	30.4589
	Skewness		1.5566	0.4762	0.4921	0.5107
	Kurtosis		8.4748	4.0358	3.8837	3.7816
thereof: interest effect	Mean	USD m	-1.2746	-1.0776	-1.0744	-1.0713
Hedged position (physical position + hedge)	Mean	USD m	-14.4778	-9.5125	-10.1110	-10.0973
	Median	USD m	-2.4673	-7.7468	-7.2825	-6.1319
	Variance	USD m	856.5239	481.3571	489.3534	486.7741
	Stand. dev.	USD m	29.2664	21.9399	22.1213	22.0630
	Skewness		-0.9421	0.5642	0.5727	0.6115
	Kurtosis		3.5008	4.9665	4.7860	4.8967
Hedge effectiveness		in	MVCHM	SPM 1	SPM 2	SPM 3
Reduction of	Variance	%	9.4273	52.2984	52.8668	53.0502
	Stand. dev.	%	4.8303	30.9337	31.3464	31.4801

The table shows selected descriptive statistics for the start and end values as well as for the delta of the physical position, hedge exposure, and portfolio position between sale and resale transactions. The considered sample size is 36 individual sale and resale transaction pairs (i.e., 72 transactions in total). The remaining 134 vessel transactions in the data set occurred without corresponding resale within the time frame and therefore, had to be neglected in this second hedging set-up. Furthermore, the results for the hedge effectiveness for the different hedging approaches are displayed.

in the model-implied adjustment for the age effect. Concerning the hedge exposure, the results are pretty similar compared to the first hedging set-up. The initial mean hedge exposure in the MVCHM is about USD 4.2 to 4.4 million lower than for the SPMs. Again, the shipping company would have to deposit higher initial margins at the clearing house if they applied the hedging approach based on the SPM. The mean hedge loss, however, is again considerably higher for the MVCHM than for the SPMs with differences of about USD 4.4 to 4.6 million. The same limitations as in the first hedging set-up concerning the derivation of the hedge profit by subtracting the end hedge exposure from the initial hedge apply in this hedging set-up as multiple stage hedges (i.e., even more than two-stage hedges) are considered if the hedge duration is larger than one year or 252 trading days and thus, covers more than one rollover date. This is the case for the vast majority of the considered 36 observations. Equally to the first hedging set-up, one would expect profits from the hedge effort if vessel prices declined and vice versa. In this case, however, the mean losses incurred from hedging are even significantly larger with USD 8.0 to 8.2 million for the SPMs and about USD 12.5 million for the MVCHM. On the hand, this may be caused by the longer hedge horizons, so profits or losses from the hedge had more time to pile up. On the other hand, the shipping industry experienced a boom period of about two years in which freight rates and vessel prices significantly rose. When the financial crisis hit the shipping industry, however, there was only one severe setback of freight rates and second-hand prices that occurred in a rather short time horizon in late 2008. Accordingly, there might be some hedges that actually ended before the crisis hit the shipping industry and thus, accumulated considerable losses. Actually, 14 of the 36 considered resale pairs ended before the crisis hit the shipping industry, 16 covered the outbreak of the shipping crisis, and six only started after the outbreak of the crisis. Again, the interest effect on the margin account seems to play an insignificant role compared to the overall size of hedge losses. Accordingly, the non-representative sample of resales which disproportionally covered the boom period led to the result of the rather high mean hedge losses in this hedging set-up. Similarly to the first hedging set-up, the hedge effectiveness of the SPMs is significantly higher than for the MVCHM. In this set-up, the SPM 3 achieves the highest hedge effectiveness with 53.05 % variance reduction, whereas the MVCHM only achieves a hedge effectiveness of 9.43 % variance reduction. Generally, the level of hedge effectiveness is lower and the associated mean losses from the hedging efforts or costs of hedging are considerably larger in this second hedging set-up than in the first hedging set-up. Admittedly,

this may result from the relatively low sample size in this hedging set-up of only 36 sale and resale transaction pairs in the data set. The larger associated losses partly result from the fact that the majority hedges covered the boom period before the breakout of the shipping crisis resulting in gains from the vessel itself and losses from hedge. However, the empirical analysis again suggests consistently superior results of the SPMs in terms of hedge effectiveness at lower costs (i.e., losses from the hedge positions) in this second hedging set-up.

With respect to the number of days shorted of the individual hedging instruments in both hedging approaches, the MVCHM suggests as mean a number of about 938 FFA+2CAL days shorted at the initial set-up of the hedge. On the contrary, the SPMs suggest an additional mean long position of about 373 to 385 FFA+1CAL days and a mean short position of about 1,503 to 1,518 FFA+2CAL days at the initial set-up of the hedge depending on the respective SPM. Consequently, the overall short exposures are slightly lower for both hedging approaches compared to the first hedging set-up.

## **2.5.4 Robustness checks**

The results so far suggest that a significant part of the fluctuations of dry bulk Capesize second-hand ship prices may be hedged using either a minimum variance cross-hedging approach or a hedging approach based on a structural pricing model. The latter approach, however, shows consistently better performance with respect to hedge effectiveness in the so far considered hedging set-ups. In order to ensure robustness of these research findings presented above, several checks with different subsets of the initial sample size were performed. In particular, a subset of the data set was considered from which we excluded multiply transacted vessels as well as a subset of the data set from which vessels that were younger than five years and vessels that were older than 20 years at the transaction date were excluded.

### **2.5.4.1 Excluding multiply sold vessels**

Dry bulk Capesize vessels are typically assets that are held by shipping companies for a considerable amount of time as the economic life of such a vessel may reach up to 30 years for new vessels. Consequently, sale and resale of one and the same vessel

within a short period of time (e.g., a few years as in the data set) may indicate that the shipping company partly bought the vessel for speculative purposes. As already stated, a considerable number of resale pairs ended before the crisis hit the shipping industry and again a considerable number of resale pairs covered the outbreak of the crisis. The former might have successfully made gain from speculating on rising vessel prices in the boom period, whereas the latter might also just have bought the vessel for speculative purposes but were surprised by the shipping crisis.

In order to eliminate any effects from speculative vessel trades, a robustness check was conducted in which all vessels were excluded that have been multiply sold within the sale and purchase transactions data set. 66 transactions belong to vessels have been sold more than once in the data set. Accordingly, this left 140 transactions of vessels that have been sold only once in the data set and the SPMs were re-estimated using these transactions. The results are shown in TABLE 2.12 in the appendix C.1 on page 71. The estimations for the  $\beta$ -coefficients changed compared to the initial estimations. In particular, the resulting exposures to FFA+1CAL rates as well as to the slope between FFA+2CAL and FFA+1CAL rates are larger in all three SPMs. Furthermore, the interaction term  $f \cdot Age$  turns insignificant in all three models. The  $R^2$  and adjusted  $R^2$  values and standard errors remain fairly constant compared to the initial estimations.

Subsequently, the linearly interpolated hedge ratios in the MVCHM as well as the re-estimated SPMs were used to assess the hedge effectiveness of both approaches again in the first hedging set-up over a fixed time horizon of one year or 252 trading days. Another 17 transactions had to be eliminated from the considered sample as they occurred prior to January 4<sup>th</sup>, 2006 and FFA rates would not have been available at the initiation of the hedge one year or 252 trading days prior to the transaction as well as another three transactions because the vessel's age would have been lower than zero at the initiation of the hedge. The results shown in TABLE 2.13 in the appendix C.1 on page 72 with respect to the hedge effectiveness are even slightly higher for the SPMs with close to 80 % variance reduction than the initial results presented in subsection 2.5.3.1. On the contrary, the hedge effectiveness is slightly lower for the MVCHM with about 66 % variance reduction. With respect to the cost of hedging, the former losses from the hedge for the full sample decreased by USD 2.6 to 3.3 million when excluding the multiply sold vessels and turned into actual profits from the hedge of USD 1.0 to 3.3 million. This is consistent with subsection 2.5.3.2

which showed that the majority of resales occurred even before the breakout of the shipping crisis resulting rather in gains from the vessel itself and losses from the hedge. Consequently, the SPMs consistently outperform the MVCHM with respect to the hedge effectiveness and these results increase the robustness of the findings presented earlier.

#### 2.5.4.2 Excluding vessels younger than five and older than 20 years

It was identified that the price dynamics for new and scrap vessels are quite different from the price dynamics of typical second-hand vessels in the minimum variance cross-hedging approach as well as that the structural pricing models do not necessarily perform well for extremely young and extremely old vessels due to few extreme transactions in the data set with respect to age.

Accordingly, a robustness check was conducted in which vessels were excluded from the dry bulk Capesize sale and purchase transactions data set that were younger than five years or older than 20 years at that the transaction date. This left 138 transactions and the SPMs 1 to 3 were re-estimated using these transactions. The results are shown in TABLE 2.14 in the appendix C.2 on page 73. In general, the estimations for the  $\beta$ -coefficients changed quite a bit. The resulting exposures to FFA+1CAL rates as well as to the slope between FFA+2CAL and FFA+1CAL rates are considerably larger. This is intuitive though as vessels were eliminated from the data set whose prices were not necessarily tied to the FFA rate dynamics as mentioned above and now, the link of the second-hand price for vessels aged between five and 20 years to FFA rates becomes more apparent. The consumption only remains significant in the interaction term with the FFA+1CAL rate. Interestingly, the estimated  $\beta$ -coefficients show more consistency across the three SPMs, although the sample size underlying the regressions is considerably lower. The standard errors of the three SPMs and information criteria are also lower than for the SPMs based on the entire data set, whereas the  $R^2$  and adjusted  $R^2$  values are about 10 percentage points higher. Together, this indicates increased robustness of the SPMs.

Subsequently, the linearly interpolated hedge ratios for 5- to 20-year old vessels in the MVCHM as well as the re-estimated SPMs were used again to assess the hedge effectiveness of both approaches again in the first hedging set-up over a fixed time horizon of one year or 252 trading days. Another 14 transactions had to be eliminated

from the considered sample as they occurred prior to January 4<sup>th</sup>, 2006 and FFA rates would not have been available at the initiation of the hedge one year or 252 trading days prior to the transaction. The results shown in TABLE 2.15 in the appendix C.2 on page 74 are even more promising than the initial results presented in subsection 2.5.3.1 as the SPMs achieve a hedge effectiveness of more than 86.50 % variance reduction and also consistently outperform the MVCHM with 79.63 % variance reduction. Again, these results increase the robustness of the findings presented earlier.

## 2.6 Conclusion

The shipping industry is historically known for its volatile nature of freight rates and second-hand ship prices. The boom period from mid-2006 until mid-2008 and the following shipping crisis that lasts until today have even pronounced this characteristic of the shipping industry. Together with the increasing need to comply with the IFRS fair value accounting principles that cause ship price fluctuations to become more visible, the need for effective hedging strategies arises. Unfortunately, no direct, liquid hedging instruments on dry bulk Capesize ship values currently exist. The aim of this paper was to examine potential other hedging approaches and empirically assess their hedge effectiveness in order to provide the shipping industry with guidance on effective measures to counter the recognition of threatening impairment losses on their fleet in their balance sheets.

Within this study, the idea of Alizadeh and Nomikos (2012) to use FFA contracts as hedging instruments for entire dry bulk Capesize vessels was first translated into a minimum variance cross-hedging model that can be applied to real Capesize sale and purchase transactions. Secondly, a structural pricing model was developed for dry bulk Capesize vessels following the effort of Adland and Koekebakker (2007). The model is based on ship-specific, deterministic factors from the data set of real Capesize sale and purchase transactions as well as market-driven or risk factors, such as the FFA rate or slope of the FFA curve as well as interaction terms. Thirdly, the hedge effectiveness of the two different approaches (i.e., the minimum variance cross-hedging approach and the hedging approach based on the structural pricing model) was empirically studied in two different hedging set-ups. Finally, the robustness of the presented results was checked.

Firstly, it was found that the minimum variance cross-hedging approach suggested by Alizadeh and Nomikos (2012) does not achieve the hedge effectiveness of more than 85 % variance reduction claimed by Alizadeh and Nomikos (2012) within the empirical analysis of real dry bulk Capesize sale and purchase transactions. The estimated  $R^2$  values for the time period considered only reach close to 83 % variance reduction for 5-year old vessels. However, it was empirically found that the MVCHM only achieves a hedge effectiveness of about 67 % variance reduction over a time horizon of one year. Secondly, it was discovered that the second-hand price of dry bulk Capesize vessels may be well described by a structural pricing model containing market-driven explanatory variables, such as the FFA+1CAL rate as well as the slope between the FFA+2CAL and FFA+1CAL rate, and deterministic, ship-specific explanatory factors, such as the age, DWT, speed, and consumption of the individual vessel. The model was estimated using data from 206 real dry bulk Capesize sale and purchase transactions. Such a model overcomes the problem of relying on panelists' estimations that only exist for a few specifically aged reference vessels. As dry bulk Capesize vessels are quite heterogeneous, shipping companies often have vessels that are different from the reference vessels and, in particular, have a different age at the transaction. Furthermore, the structural pricing model shows faster reaction to changing market circumstances than the panelists' estimations even though not all available information have been included as explanatory variables. Thirdly, it was found that the SPMs consistently outperform the MVCHM with respect to the hedge effectiveness in two different hedging set-ups (i.e., over a fixed time horizon and between sale and resale of one and the same vessel). The SPMs achieve a hedge effectiveness of about 77 % variance reduction over a fixed time horizon of one year. Fourthly, the results indicate that the average cost of hedging or losses from the hedges are considerably lower for the hedging approach based on one of the SPM. The only drawback of the hedging approach based on the SPM is the apparently higher initial hedge exposures causing the clearing houses to demand higher initial margins. Finally, the robustness of the research findings stated above was confirmed by performing several checks with subsets of the initial sample size. For instance, it was found that the SPMs even achieve a hedge effectiveness of about 87 % variance reduction if vessels younger than five and older than 20 years were excluded from the analysis.

With respect to limitations of this study, the short time frame of available data on FFA rates from 2005 onwards is obviously a constraint. Furthermore, the overall number of dry bulk Capesize transactions in the data set is rather low. Because of

these constraints, the data at hand had to be used for both estimating the MVCHM and the SPMs as well as assessing their hedge effectiveness in sample. Furthermore, it was generalized to some extent by means of several assumptions on the circumstances of the two hedging set-ups. The aim here was to test the hedging approaches in set-ups that, on the one hand, reflect as much as possible the reality of shipping companies as well as, on the other hand, treat each transaction in a similar way. These circumstances, however, may be different for individual shipping companies and the determined hedge effectiveness in this study may be exceeded or not achieved in individual cases in the real world.

In terms of practicability of the discussed hedging approaches, sufficient liquidity of the applied hedging instruments is crucial factor for the practical implementation of the approaches. An average trading volume of about 11,000 days per week from July 7<sup>th</sup>, 2007 to June 30<sup>th</sup>, 2014 for the entire dry bulk Capesize FFAs clearly shows that initiating a hedge for an entire vessel might be well feasible. However, simultaneously initiating hedges for an entire fleet of a shipping company would probably need to be staged or successively built up. Moreover, insufficient liquidity for the re-initiation of multiple hedges at rollover dates clearly represents a bottleneck for a large scale practical implementation. For further details on historical FFA trading volumes, see appendix A.4 on page 67.

Accordingly, this paper contributes to the existing academic literature in various ways. Firstly, the effort is the first empirical study of the hedge effectiveness of the minimum variance cross-hedging approach for Capesize dry bulk vessels using FFA contracts as suggested by Alizadeh and Nomikos (2012). Secondly, this effort is the first to consider additional factors to simple FFA rates, age of the vessel, or size in DWT as explanatory factors for dry bulk Capesize second-hand ship prices in a structural pricing model that may well serve as basis for a competing hedging approach. Finally, the study is the first to empirically test the hedge effectiveness of both approaches and found that the effectiveness of the hedging approach based on the structural pricing model consistently exceeds the effectiveness of the minimum variance cross-hedging approach in different set-ups.

Concerning further research opportunities in this area, estimating and testing the hedge effectiveness of both hedging approaches out-of-sample represents a valid extension of the presented research within this paper once a longer time horizon of relevant data is available. Furthermore, investigating the hedge effectiveness of other

hedging instruments, such as FFAs with different maturities or freight options, might be another direction of impact for future research in this area. Especially the latter are interesting instruments from a hedging perspective because freight options allow to only eliminate the downside but to keep the upside potential of ship price fluctuations. It would be interesting to examine whether the benefit of the asymmetrical payoff structure of these instruments outweighs the cost of the hedge (i.e., the option premiums that need to be paid at the initiation of the hedge).

## A Appendix A – Miscellaneous

### A.1 The stationary bootstrap resampling technique

The stationary bootstrap developed by Politis and Romano (1994) is a time series resampling technique with randomly varying block length that produces a pseudo-time series that is stationary conditional on a strictly stationary and weakly dependent original time series. The length of these randomly varying blocks follows a geometric distribution (Politis and Romano, 1994). The description and notation of the stationary bootstrap algorithm below largely follow the efforts by Politis and Romano (1994) and Sullivan et al. (1999).

At first, a ‘smoothing parameter’,  $q$ , is chosen a priori, such that  $q = q_n$ ,  $0 < q_n \leq 1$ ,  $q_n \rightarrow 0$ ,  $nq_n \rightarrow \infty$  as  $n \rightarrow \infty$ . Subsequently, the following steps are carried out in order to resample the pseudo-time series,  $X_t^*$ , from the original time series,  $X_t$  with  $t = \{1, \dots, T\}$ :

1. The first observation of the pseudo-time series,  $X_1^*$  with  $t = 1$ , is randomly (i.e., independently and uniformly) selected from the original  $t$  observations of  $X_t$ , so that  $X_1^* = X_{I_1}$ .
2. Increment  $t$  by one. If  $t > T$ , stop the iterations. If  $t \leq T$ , pick a standard uniformly distributed random variable  $u$  which is independent from all other variables.
  - a. If  $u < q$ , let  $X_2^*$  (or  $X_t^*$  in later recursive rounds) be picked at random (i.e., independently and uniformly) from the original  $t$  observations of  $X_t$ .
  - b. If  $u \geq q$ , continue the block by setting  $X_2^* = X_{I_1+1}$  (or  $X_t^* = X_{I_{t-1}+1}$  in later recursive rounds). Accordingly,  $X_2^*$  (or  $X_t^*$  in later recursive rounds) is the next observation following  $X_{I_1}$  (or  $X_{I_{t-1}}$  in later recursive rounds) in the original time series. If  $I_{t-1} + 1 > T$ , then reset  $I_{t-1} + 1$  to 1.
3. Repeat step 2 until  $t = T$  and a resampled value to  $X_T^*$  has been assigned.
4. Repeat steps 1 to 3  $n$  times in order to get  $n$  independently resampled pseudo-time series of the original time series.

Following the algorithm above yields in a resampled pseudo-time series of varying

block length which follows a geometric distribution with mean block length  $1/q$  (Sullivan et al., 1999). Although the a priori choice of  $q$  seems to have a considerable impact on the algorithm, Sullivan et al. (1999) found that the results of their study on data snooping and technical trading rule performance are insensitive to the choice of  $q$ . They stated that a larger  $q$  is more appropriate for data with little dependence, whereas a smaller  $q$  is rather appropriate for data with more dependence. Politis and Romano (1995) and Politis and White (2004) studied the automatic block length selection for bootstrapping methods for dependent data and suggested an inspection of the autocorrelation function of the original data. They argued that looking for the smallest integer,  $\bar{m}$ , after which the correlogram turns insignificant, is a valid first indication for the selection of the mean block length.

## A.2 Sensitivity analysis with respect to the choice of $q$

A sensitivity analysis was performed with respect to the choice of the ‘smoothing parameter’,  $q$ , which corresponds to the mean block length of  $1/q$ . Besides the choice of  $q = 0.005$ , the stationary bootstrap was rerun on the 52-week log returns additionally for  $q = 0.001$ ,  $q = 0.01$ , and  $q = 0.1$  corresponding to a mean block length of 1,000, 100, and 10, respectively.

TABLE 2.10: Results of sensitivity analysis with respect to the choice of  $q$

Age	$q = 0.001$				$q = 0.01$				$q = 0.1$			
	$\beta_{52, Age}$		95 % CI		$\beta_{52, Age}$		95 % CI		$\beta_{52, Age}$		95 % CI	
	mean	med.	Low	Up	mean	med.	Low	Up	mean	med.	Low	Up
0	0.262	0.265	[0.158 - 0.324]		0.253	0.262	[0.073 - 0.394]		0.267	0.267	[0.150 - 0.375]	
5	0.810	0.815	[0.704 - 0.850]		0.797	0.814	[0.603 - 0.883]		0.809	0.813	[0.689 - 0.906]	
10	0.819	0.822	[0.729 - 0.873]		0.809	0.823	[0.574 - 0.955]		0.821	0.825	[0.678 - 0.930]	
15	0.962	0.967	[0.841 - 1.025]		0.944	0.967	[0.607 - 1.097]		0.962	0.968	[0.783 - 1.100]	
20	0.944	0.957	[0.738 - 1.051]		0.908	0.957	[0.352 - 1.101]		0.945	0.948	[0.730 - 1.130]	
30	0.438	0.445	[0.148 - 0.570]		0.415	0.452	[-0.066 - 0.681]		0.429	0.438	[0.176 - 0.614]	

The table shows aggregated regression estimates using 1,000 resamples using the stationary bootstrap technique by Politis and Romano (1994) of the original underlying, overlapping 52-week log return time series for Capesize vessel prices and FFA+2CAL prices from January 4<sup>th</sup>, 2005 to October 28<sup>th</sup>, 2013 for different ‘smoothing parameters’,  $q$ . The lower and upper bounds of the 95 % confidence interval refer to the 2.5 %- and 97.5 %-quantiles of the empirical distribution of the  $\beta_{52, Age}$ -coefficients.

The results in TABLE 2.10 show that the  $\beta_{52, Age}$ -coefficients from the initial regressions also lie within the other three confidence intervals and therefore, confirm the

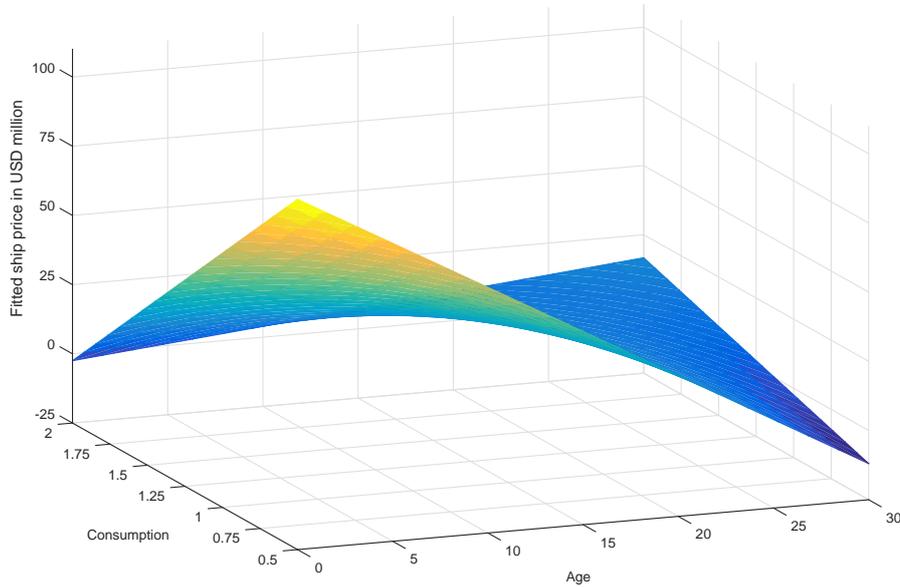
results presented earlier in subsection 2.5.1. The median  $\beta_{52,Age}$ -coefficients,  $\bar{\beta}_{52,Age}^{med}$ , are negligibly different from each other with respect to the choice of  $q$  as well as from the  $\beta_{52,Age}$ -coefficients from the regressions using the original data set. By taking a closer look, the results indicate that the width of the 95 % confidence intervals for the estimated  $\beta_{52,Age}$ -coefficients is smallest for  $q = 0.001$ . This is intuitive as the mean block length of 1,000 exceeds the length of the original data set by more than twice and thus, deviations in the resampled data sets from the original data set get less likely. Interestingly, the width of the confidence intervals is largest for  $q = 0.01$  and especially for the  $\beta_{52,Age}$ -coefficients for new and scrap vessels. In this case, the confidence interval for 30-year old or scrap vessels does contain zero and thus, the corresponding  $\beta_{52,Age}$ -coefficient is statistically not significantly different from zero. However, this is the only case in which a confidence interval does contain zero. From visual inspection, the empirical distributions of the  $\beta_{52,Age}$ -coefficients exhibit significant excess kurtosis or thick-tailedness for  $q = 0.001$ , less excess kurtosis and slightly right-skewness for  $q = 0.01$ , and approach a rather normal distribution for  $q = 0.1$ . The results of the initial choice of  $q = 0.005$  are closest to the case of  $q = 0.01$ . However, the 95 % confidence interval for the  $\beta_{52,Age}$ -coefficient for scrap vessels does not contain zero in that case. The overall differences in the 95 % confidence intervals with respect to the choice of  $q$ , however, are so small that the results and inferences presented earlier in subsection 2.5.1 are considered robust.

### A.3 Pricing surface for SPM 1

In order to graphically illustrate the interaction of the variables age and consumption in the SPM 1, a 3D plot of the corresponding pricing surface was drawn with respect to changes in age or consumption values. As a 3D plot is limited to three dimensions and the intention was to show the effect on the explained variable price, the additionally required input factors FFA+1CAL and slope between FFA+2CAL and FFA+1CAL were held constant at their mean in the considered time frame. The ranges of the age and consumption values considered in the plot were largely selected based on minimum and maximum observations in the dry bulk Capesize transactions data set.

FIGURE 2.4 shows the 3D plot containing the model-implied pricing surface of the SPM 1. The shape of the surface indicates that the model-implied vessel price declines with increasing age and consumption. For a combination of, for instance, extremely

FIGURE 2.4: Pricing surface for SPM 1 with changing age and consumption values



The graph shows a fitted pricing surface for the SPM 1 for changing age (in years) and consumption (in metric tonnes per 1,000 nautical miles per 1,000 DWT) values. For the FFA+1CAL and the slope between FFA+2CAL and FFA+1CAL values, the mean values of all daily observations in the time frame ranging from January 4<sup>th</sup>, 2005 to June 30<sup>th</sup>, 2014 were used.  
*Source:* own graph based on SPM 1 model estimations

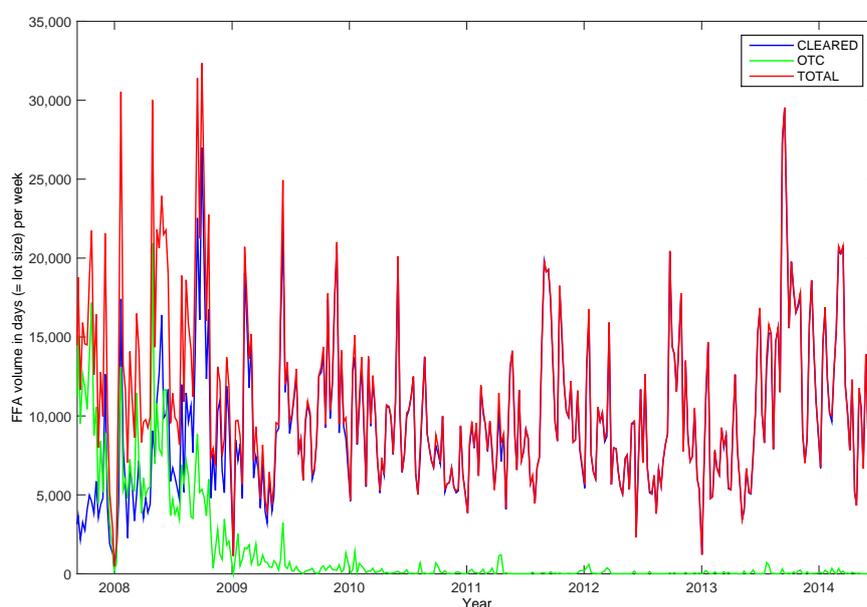
high age values and extremely low consumption values, the model even gets negative prices. This is similar for the case of extremely high consumption values and extremely low age values. However, this changes for combinations of large age and consumption values. In this case, the shape of the pricing surface is again upward sloping indicating higher prices for extremely old and inefficient vessels of about USD 17.50 million in the extreme end. As already discussed, the positive sign of the  $\beta_{Age-Consum}$ -coefficient is economically not intuitive and might be caused by overfitting to the biased sample containing disproportionately many relatively expensive sales of old and inefficient ships within the boom period prior to the shipping crisis.

In order to investigate whether the above described effect of implausibly rising prices for old and inefficient ships gets intensified or attenuated for different FFA rates, pricing surfaces for the SPM 1 were also estimated using extreme values for the FFA+1CAL rate together with the corresponding slope between FFA+2CAL and FFA+1CAL rate. Interestingly, the effect gets intensified when using the minimum of the FFA+1CAL rates with the corresponding slope between FFA+2CAL and FFA+1CAL rate and gets attenuated when using the maximum of the FFA+1CAL rates with the corresponding slope between FFA+2CAL and FFA+1CAL rate.

## A.4 Historical FFA trading volumes

As sufficient liquidity of the applied hedging instrument is a crucial factor for the practical implementation of the suggested hedging approaches, historical FFA volumes for the dry bulk Capesize sector were studied. The Baltic Exchange started to publish aggregated weekly FFA volumes for this sector on July 9<sup>th</sup>, 2007. Unfortunately, this does not allow to examine volumes for specific contracts, such as the trip-charter average FFA+1CAL or the trip-charter average FFA+2CAL contract.

FIGURE 2.5: Historical dry bulk Capesize FFA trading volumes 2007-2014



The graph shows weekly dry bulk Capesize FFA volumes from July 9<sup>th</sup>, 2007 to June 30<sup>th</sup>, 2014. As the lot size of these contracts is days, the weekly FFA volume shown here is also in days.

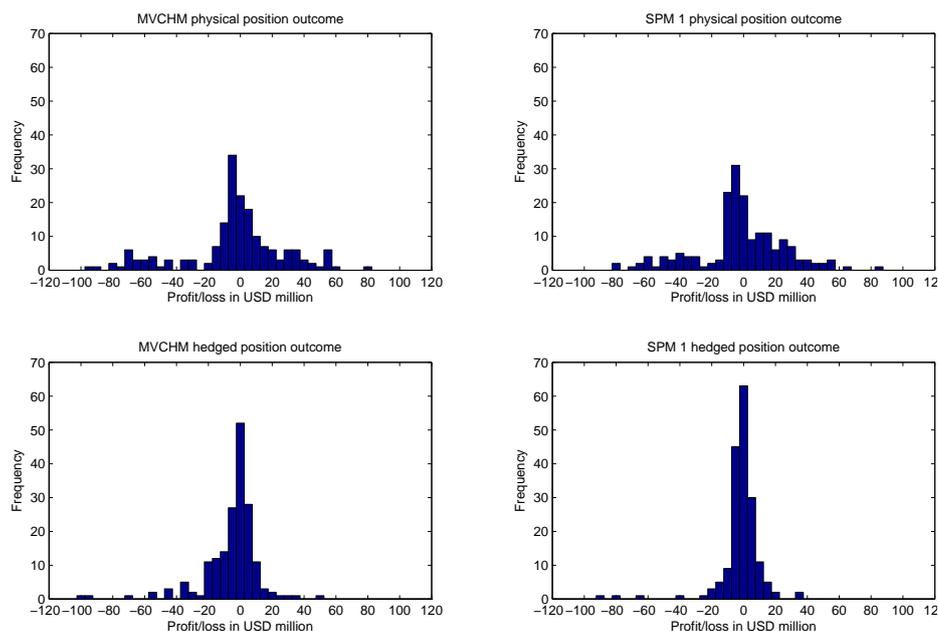
*Source:* own graph based on weekly data from The Baltic Exchange

FIGURE 2.5 shows a plot of the weekly FFA trading volumes from July 9<sup>th</sup>, 2007 to June 30<sup>th</sup>, 2014. In 2007, the majority of the FFA contracts were still traded over-the-counter (OTC) and thus, uncleared. This, however, changed in 2008 and nowadays, FFAs are almost exclusively traded via hybrid exchanges and are cleared. This significantly reduces the counterparty risk of hedging with FFAs. The mean trading volume is 10,979 lots or days per week with a standard deviation of 5,274 lots or days per week. In general, the trading volume fluctuates quite a bit and seemed to be higher before the crisis hit the shipping industry.

## B Appendix B – Selected illustrative plots

### B.1 Histogram plots

FIGURE 2.6: Histograms of MVCHM and SPM 1 physical position and hedged position outcomes



The graphs show individual physical position as well as hedged position (physical position + hedge) outcomes for the 180 vessels for MVCHM approach as well as for the hedging approach based on the SPM 1 over a fixed time horizon of one year.

*Source:* own graph based on hedging results

The histogram plots of FIGURE 2.6 show histograms the physical as well as hedged position outcomes for the MVCHM and the SPM 1 over the fixed time horizon of one year. The graphs clearly show that the distribution of the outcomes narrows for the hedged position consisting of the physical position plus hedge. Furthermore, the effect is stronger for the SPM 1 than for the MVCHM from visual inspection of the graphs.

## B.2 Hedge example for one single vessel

In order to present one single, detailed hedge example, selected detailed facts on the example vessel as well as on the corresponding hedge results are presented in TABLE 2.11 below.

TABLE 2.11: Hedge example facts – ship ‘Partagas’

Fact	Value	Unit	Fact	Value	Unit
Name	Partagas		Sales date	01/26/2006	
IMO number	9272345		Transaction price	60.0000	USD m
Size	173,880	DWT	Physical pos. profit/loss	-5.8030	USD m
Age at sales date	1.67	years	Hedge pos. profit/loss	6.3490	USD m
Speed	14.00	knots	thereof: interest effect	0.0737	USD m
Consumption	54.70	mt per day	Total profit/loss	0.5460	USD m
Consumption	0.94	mt per 1,000 nm per 1,000 DWT			

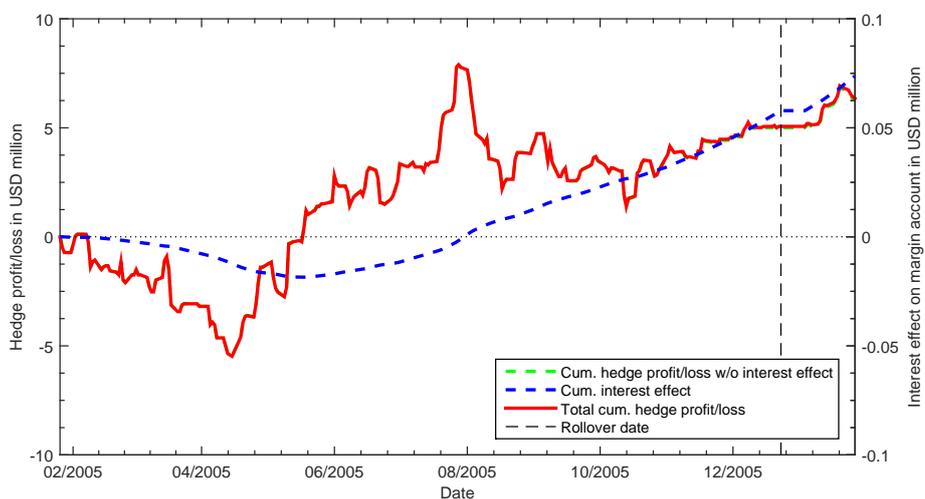
The table shows selected facts on the vessel of the example hedge as well as on the corresponding hedge results.

In order to illustrate the development of the hedge position for one single vessel based on the MVCHM, FIGURE 2.7 shows the cumulative hedge profit/loss with and without interest effect as well as the cumulative interest effect itself from hedging start (i.e., one year prior to the transaction) until hedging end (i.e., transaction date).

The plot shows that the hedge started to accumulate a loss during the first two months of the hedge. Then, the market conditions changed, FFA prices fell, the cumulative loss of the hedge reduced, and turned into a profit of about USD 6.35 million. The cumulative interest effect curve follows the cumulative hedge profit/loss curve in a lagged way. However, the overall size of the interest effect remained inconsiderable with about USD 0.074 million. This is underlined by the fact that the cumulative hedge profit/loss with interest effect curve shows hardly any deviation from the cumulative hedge profit/loss without interest effect curve.

Together with the loss of USD 5.80 million on the physical position, the shipping owner would have ended up with a profit of USD 0.55 million if he had chosen to hedge the ship price one year prior to the sale using the MVCHM approach.

FIGURE 2.7: Cumulative hedge profit/loss development for ship ‘Parta-gas’



The graph shows cumulative hedge profit/loss development of the hedge of one single vessel from the start until the end of the hedge over a fixed time horizon of one year. The cumulative profit/loss of the hedge both without and with interest effect itself is shown on the left y-axis in USD million. The cumulative interest effect is separately shown on the right y-axis in USD million as well.

Source: own graph based on hedging results

## C Appendix C – Robustness checks

### C.1 Excluding multiply sold vessels

TABLE 2.12: Estimates for different Capesize structural pricing models

	SPM 1		SPM 2		SPM 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
$\alpha$	61.6535** (23.7116) [2.6001]	0.0104	28.0677** (13.7136) [2.0467]	0.0427	31.3805*** (5.1387) [6.1066]	0.0000
$\beta_f$	2,122.7471*** (319.9196) [6.6353]	0.0000	2,002.9422*** (314.6641) [6.3653]	0.0000	1,953.8393*** (251.1887) [7.7784]	0.0000
$\beta_{sl}$	2,003.7934*** (479.3967) [4.1798]	0.0001	1,991.9051*** (482.9246) [4.1247]	0.0001	2,002.2326*** (479.6203) [4.1746]	0.0001
$\beta_{Age}$	-4.3111*** (1.2073) [-3.5707]	0.0005	-2.2727*** (0.2649) [-8.5788]	0.0000	-2.2788*** (0.2629) [-8.6669]	0.0000
$\beta_{Consum}$	-32.2696 (24.7002) [-1.3064]	0.1937	3.5393 (13.5763) [0.2607]	0.7947		
$\beta_{f \cdot Age}$	-4.4363 (6.0248) [-0.7363]	0.4628	-2.3250 (5.9439) [-0.3912]	0.6963	-2.3207 (5.9232) [-0.3918]	0.6958
$\beta_{f \cdot Consum}$	-759.8149*** (265.0323) [-2.8669]	0.0048	-673.7581** (262.2642) [-2.5690]	0.0113	-618.1367*** (151.9893) [-4.0670]	0.0001
$\beta_{Age \cdot Consum}$	2.1597** (1.2485) [1.7299]	0.0860				
$R^2$	0.7814		0.7764		0.7763	
Adj. $R^2$	0.7698		0.7663		0.7679	
Log likelihood	-533.4468		-535.0160		-535.0517	
SBIC criterion	7.9030		7.8902		7.8554	
Akaike criterion	7.7350		7.7431		7.7293	
Standard error	11.2550		11.3390		11.2995	

The table shows linear regression coefficient estimates for three different SPMs based on data for 140 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to October 30<sup>th</sup>, 2013. Vessels that have been multiply sold within the data set have been excluded from the analysis. Figures in () and [] reflect the corresponding standard errors and t-statistics, respectively. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

TABLE 2.13: Hedging results over fixed time horizon of one year

Physical position		in	MVCHM	SPM 1	SPM 2	SPM 3
Start (age effect incl.)	Mean	USD m	40.3371	38.7789	38.8807	38.8119
End (transaction price)	Mean	USD m	33.2354	33.2354	33.2354	33.2354
Hedge exposure		in	MVCHM	SPM 1	SPM 2	SPM 3
Start	Mean	USD m	-33.8736	-36.6701	-36.5106	-36.5496
End	Mean	USD m	-39.3479	-32.3155	-32.2815	-32.4159
Delta/change in values or profit/loss		in	MVCHM	SPM 1	SPM 2	SPM 3
Physical position	Mean	USD m	-7.1016	-5.5435	-5.6453	-5.5765
	Median	USD m	-4.2041	-5.5149	-6.4832	-6.6402
	Variance	USD m	921.7994	742.1537	714.8575	713.6859
	Stand. dev.	USD m	30.3611	27.2425	26.7368	26.7149
	Skewness		-0.4628	-0.2700	-0.2175	-0.2028
	Kurtosis		3.3547	3.6561	3.4220	3.4100
Hedge	Mean	USD m	0.9818	3.3273	3.1774	3.0749
	Median	USD m	1.9754	2.3996	2.3959	2.4114
	Variance	USD m	631.9555	682.3050	678.5301	677.1443
	Stand. dev.	USD m	25.1387	26.1210	26.0486	26.0220
	Skewness		0.0851	0.4938	0.4559	0.4389
	Kurtosis		3.9391	3.6219	3.4887	3.4572
thereof: interest effect	Mean	USD m	-0.1270	-0.1151	-0.1174	-0.1189
Hedged position (physical position + hedge)	Mean	USD m	-6.1199	-2.2162	-2.4678	-2.5016
	Median	USD m	-2.3854	-1.6724	-1.4946	-1.6557
	Variance	USD m	313.0890	151.4174	152.2321	154.0489
	Stand. dev.	USD m	17.6943	12.3052	12.3382	12.4116
	Skewness		-1.6901	-3.0552	-3.4335	-3.4480
	Kurtosis		11.1569	21.1281	22.7866	22.8296
Hedge effectiveness		in	MVCHM	SPM 1	SPM 2	SPM 3
Reduction of	Variance	%	66.0350	79.5976	78.7045	78.4150
	Stand. dev.	%	41.7205	54.8309	53.8530	53.5404

The table shows selected descriptive statistics for the start and end values as well as for the delta of the physical position, hedge exposure, and portfolio position over the fixed hedging horizon of one year prior to the individual vessel transaction. The considered sample size is 120 vessel transactions. Transactions in 2005 had to be excluded due to unavailability of FFA time series data prior to 2005 as well as transactions of vessels that would have been negatively aged at the initiation of the hedge. Furthermore, the results for the hedge effectiveness for the different hedging approaches are displayed.

## C.2 Excluding vessels younger than five and older than 20 years

TABLE 2.14: Estimates for different Capesize structural pricing models

	SPM 1		SPM 2		SPM 3	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
$\alpha$	48.5424 (34.7262) [1.3979]	0.1645	19.7831 (15.0461) [1.3148]	0.1909	18.8586*** (5.7831) [3.2610]	0.0014
$\beta_f$	2,874.2448*** (314.7620) [9.1315]	0.0000	2,859.7529*** (314.1803) [9.1023]	0.0000	2,872.9795*** (242.5463) [11.8451]	0.0000
$\beta_{sl}$	2,037.1615*** (412.2773) [4.9412]	0.0000	2,018.4832*** (411.5318) [4.9048]	0.0000	2,018.5660*** (409.9751) [4.9236]	0.0000
$\beta_{Age}$	-3.5695* (2.1298) [-1.6760]	0.0961	-1.6394*** (0.3535) [-4.6372]	0.0000	-1.6322*** (0.3355) [-4.8649]	0.0000
$\beta_{Consum}$	-31.7494 (36.1469) [-0.8783]	0.3814	-0.8910 (13.3778) [-0.0666]	0.9470		
$\beta_{f \cdot Age}$	-35.8552*** (7.1494) [-5.0151]	0.0000	-35.5469*** (7.1373) [-4.9804]	0.0000	-35.6027*** (7.0612) [-5.0420]	0.0000
$\beta_{f \cdot Consum}$	-986.3322*** (257.2336) [-3.8344]	0.0002	-980.7313*** (257.0088) [-3.8159]	0.0002	-993.9378*** (162.9047) [-6.1013]	0.0000
$\beta_{Age \cdot Consum}$	2.0856 (2.2693) [0.9190]	0.3598				
$R^2$	0.8761		0.8753		0.8753	
Adj. $R^2$	0.8694		0.8696		0.8705	
Log likelihood	-500.9225		-501.3694		-501.3417	
SBIC criterion	7.5454		7.5162		7.4805	
Akaike criterion	7.3757		7.3677		7.3532	
Standard error	9.4008		9.3952		9.3597	

The table shows linear regression coefficient estimates for three different SPMs based on data for 138 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to October 30<sup>th</sup>, 2013. Vessels younger than five years and older than 20 years at the transaction date have been excluded from the analysis. Figures in () and [] reflect the corresponding standard errors and t-statistics, respectively. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

TABLE 2.15: Hedging results over fixed time horizon of one year

Physical position		in	MVCHM	SPM 1	SPM 2	SPM 3
Start (age effect incl.)	Mean	USD m	41.4580	39.9703	39.9641	39.9559
End (transaction price)	Mean	USD m	37.2242	37.2242	37.2242	37.2242
Hedge exposure		in	MVCHM	SPM 1	SPM 2	SPM 3
Start	Mean	USD m	-37.2431	-36.2615	-35.9642	-35.9529
End	Mean	USD m	-44.2598	-33.6335	-33.4074	-33.4475
Delta/change in values or profit/loss		in	MVCHM	SPM 1	SPM 2	SPM 3
Physical position	Mean	USD m	-4.2338	-2.7462	-2.7399	-2.7317
	Median	USD m	-3.8083	-5.2518	-5.5722	-6.4593
	Variance	USD m	1,086.5196	778.4303	775.6993	774.2485
	Stand. dev.	USD m	32.9624	27.9004	27.8514	27.8253
	Skewness		-0.5605	0.0656	0.1194	0.1285
	Kurtosis		3.6044	3.5891	3.5755	3.5750
Hedge	Mean	USD m	-0.8162	1.2362	1.1869	1.1573
	Median	USD m	1.8828	2.2727	2.3114	2.3121
	Variance	USD m	774.6261	692.3205	682.2251	678.0224
	Stand. dev.	USD m	27.8321	26.3120	26.1194	26.0389
	Skewness		-0.0039	0.3214	0.3129	0.3114
	Kurtosis		3.5097	3.4046	3.3577	3.3407
thereof: interest effect	Mean	USD m	-0.1600	-0.1345	-0.1343	-0.1346
Hedged position (physical position + hedge)	Mean	USD m	-5.0500	-1.5100	-1.5530	-1.5744
	Median	USD m	-1.6517	-1.5108	-1.5530	-1.5744
	Variance	USD m	221.3359	99.3165	102.5720	102.4454
	Stand. dev.	USD m	14.8774	9.9658	10.1278	10.1215
	Skewness		-1.1121	-1.8619	-1.7825	-1.8012
	Kurtosis		7.4026	16.4447	16.1403	16.1032
Hedge effectiveness		in	MVCHM	SPM 1	SPM 2	SPM 3
Reduction of	Variance	%	79.6289	87.2414	86.7768	86.7684
	Stand. dev.	%	54.8656	64.2809	63.6363	63.6247

The table shows selected descriptive statistics for the start and end values as well as for the delta of the physical position, hedge exposure, and portfolio position over the fixed hedging horizon of one year prior to the individual vessel transaction. The considered sample size is 124 vessel transactions aged between five and 20 years. Transactions in 2005 had to be excluded due to unavailability of FFA time series data prior to 2005. Furthermore, the results for the hedge effectiveness for the different hedging approaches are displayed.

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## CHAPTER 3

# HEDGING CAPESIZE SHIP PRICE RISKS USING FREIGHT OPTIONS

### ABSTRACT

The nature of the shipping industry has been historically characterized by high volatility compared to other industries and downside-risk protection against adverse ship price fluctuations may be beneficial for shipping companies from a risk management perspective as well as from a cash/liquidity perspective. The aim of this paper is to empirically assess the hedge effectiveness of different freight option-based cross-hedging strategies using several risk-, downside-risk-, as well as return-based measures. The results indicate that a one-sided, option-based cross-hedging strategy presents a relevant alternative to the classical two-sided, Forward Freight Agreements (FFAs)-based cross-hedging strategy providing similar downside-risk protection. Such a strategy, however, allows to keep the upside potential in case no downside-risk protection is required. The finding that a synthetically replicated FFA using options actually outperforms the FFA-based reference strategy contradicts findings from other studies in other financial markets and actually implies redundancy of FFAs. Given the comparatively low liquidity of freight options, this result should not be over-interpreted and rather seen as a friction from a not yet fully developed freight option market.

## 3.1 Introduction

With 3.1 % compound annual growth rate from 1970 until 2013 to a total of 9.5 billion tons of goods loaded in 2013, maritime transportation continuously played a steady but crucial role in the growth of the global economy (UNCTAD, 2014). The increasing level of globalized trade flows of raw materials, semifinished, and finished goods has caused increasing demand for seaborne transportation. More than 90 % of the global merchandise trade<sup>1</sup> is nowadays handled by sea (IMO, 2012; UN, 2013).

In the past years, the maritime shipping industry has undergone remarkable highs and lows. The Baltic Dry Index (BDI)<sup>2</sup>, for instance, rose from 2006 to early 2008 to an all-time high of 11,793 index points on May 20<sup>th</sup>, 2008. With the outburst of the financial crisis, the index plummeted to a low of 663 index points on December 5<sup>th</sup>, 2008. That is an incredible decline of more than 94 % in just over six months. Ever since, the shipping industry has been facing a severe recession which was partly caused by overcapacity resulting from newbuilding orders placed prior to the outburst of the financial crisis. In early 2015, the BDI almost reached its all-time low of 554 index points from July 31<sup>st</sup>, 1986. Although the shipping industry has been historically known for its volatile nature of freight rates and second-hand ship prices, these past years presented some very challenging market circumstances (Albertijn et al., 2011).

Besides the rather high volatility of freight rates and second-hand ship prices, some further specific characteristics of the industry intensify the difficulties of being able to cope with these market circumstances. Firstly, shipping companies typically have a high asset concentration in the form of the carrying amount of the company's fleet in their balance sheets (Stopford, 2009). Secondly, shipping companies are commonly highly leveraged compared to other industry sectors (Drobetz et al., 2013). Thirdly, compliance with the fair value accounting principles<sup>3</sup> of the International Financial Reporting Standards (IFRS) will be an increasingly important topic for listed shipping companies (Albertijn et al., 2011). Accordingly, shipping companies suffer not only

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<sup>1</sup> excluding intra-European Union (EU) trade

<sup>2</sup> The BDI is an index provided by The Baltic Exchange measuring the cost for maritime transportation of dry bulk goods (i.e., iron ore, coal, and grain) aggregated across different vessel classes (i.e., Capesize, Panamax, Supramax, and Handysize) and shipping routes.

<sup>3</sup> According to International Accounting Standards (IAS) rule 36, these carrying amounts are subject to regular impairment tests. If the recoverable amount of the ship (the higher of value in use and fair value less cost to sell) is less than the carrying amount, the difference needs to be recognized as an impairment loss (Deloitte, 2009; KPMG, 2012; PwC, 2005).

from operative losses in case of low freight rates but also from potentially considerable impairment losses on the carrying amount of their fleet. Given their low share of equity, such losses might be threatening and protection of the company's balance sheet by hedging the exposure to such adverse ship price fluctuations may be beneficial for the shipping company. Finally, hedging at least the downside risk of ship prices might also be beneficial from a liquidity perspective if the vessel itself is used as a collateral for a granted loan to the shipping company with the additional covenant of increased financing costs or an additional collateral if the ship loses in value.

Unfortunately, hedging ship prices directly is not possible as no direct liquid hedge instruments on ship prices exist. Clarkson Securities Limited (CSL) tried to launch Forward Ship Value Agreements (FoSVAs) in the mid-2000s. FoSVAs are cash-settled forward contracts on the Baltic Sale and Purchase Assessment (BSPA)<sup>4</sup> (Adland et al., 2004). However, the liquidity these instruments is extremely low and Jallal (2013) even stated that FoSVA have actually never been traded so far. Accordingly, cross-hedging ship price risks using liquid hedge instruments with high correlation to ship prices is the only viable alternative. Alizadeh and Nomikos (2012) were the first ones to address the topic of hedging ship prices and suggested a minimum variance cross-hedge using Forward Freight Agreements (FFAs). They found that up to 93 % of the price fluctuations of the BSPA for selected dry bulk vessel classes (i.e., Capesize, Panamax, and Supramax) can be explained by price fluctuations of the respective FFA+2CAL contracts.<sup>5</sup>

In addition to FFAs, the freight derivatives market offers another instrument, freight options, that might potentially serve for cross-hedging purposes. Freight options have been launched in 2008 and the options are arithmetic Asian options on FFAs. The asymmetric payoff structure of freight options allows for more flexibility in the hedging strategy. For instance, using long put options rather than FFAs will only limit the downside of the underlying position, but it allows to keep upside potential in case freight rates and ship prices increase. Clearly, this benefit only comes at certain costs, the option premium. Especially in light of the rather high volatility in the industry, the feature of asymmetric protection could be of interest for shipping companies.

From a practical risk management perspective, a hedging strategy relying on panelists'

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<sup>4</sup> BSPA is a second-hand ship price panelists' estimation of certain 5-year old reference vessels provided on a weekly basis by The Baltic Exchange.

<sup>5</sup> FFA+2CAL contracts are FFA contracts for the entire second-next calendar-year.

estimations for reference vessels might be suboptimal as the company's vessel specifications might considerably differ from the assessed reference vessel by the panelists. Adland and Koekebakker (2007) estimated a non-parametric, multivariate pricing model based on real sale and purchase transaction data for second-hand Handymax vessels. In their model, they used three relevant factors of dry bulk ships for the second-hand price determination: size (measured in deadweight tons (DWT)), age and 1-year time charter freight rates (Adland and Koekebakker, 2007). Such a structural pricing model (SPM) allows to determine the exposure of the value certain ship to the freight market more accurately than the rather generic panelists' estimations.

The aim of the paper is to examine whether freight options qualify as suitable hedge instruments for dry bulk Capesize ship price risks. Firstly, a SPM is estimated using real dry bulk Capesize second-hand transactions. The model is used to determine the desired exposure to the respective hedge instruments and thus, serves as basis of the hedging approach. Secondly, the hedge effectiveness of different freight option hedging strategies (i.e., long at-the-money put, long out-of-the-money put, replicated FFA, and zero-cost collar) is empirically tested in a hedging set-up over a fixed time horizon one year prior to the sale for the same dry bulk Capesize transactions. Furthermore, it is investigated whether using freight options rather than FFAs as hedge instruments achieves superior hedge effectiveness for hedging dry bulk Capesize ship price risks. The hedge effectiveness is evaluated using several, well-established risk and downside-risk measures as well as risk- and downside-risk-adjusted return measures. It is refrained from specifying a generalized utility function for shipping companies as the risk preferences of shipping companies are assumed to be rather diverse.

The remainder of the paper is structured as follows. Section 3.2 reviews the relevant academic literature. Section 3.3 elaborates on the methodology applied within this empirical study. Section 3.4 provides a thorough description of the data used within this study. Section 3.5 presents the empirical results, an interpretation of these results as well as further robustness checks. Finally, section 3.6 concludes the findings of this study and provides an outlook on further research opportunities in this area.

## 3.2 Review of academic literature

The following review of the academic literature covers the subsequent topics: general research on ship prices, FFAs, and freight options; research on hedging of ship price risks; research on the pricing of freight options; as well as specific research on hedging using options (i.e., strategies and suitable measures to assess the effectiveness of hedging strategies involving options).

Ship prices and second-hand ship prices in particular have found quite some interest within the academic literature and the relevant studies can be largely classified in five different categories. The first category of studies focused on econometrically modeling the dynamics of the shipping industry, including demand, freight rates, and ship prices. An early, often-cited effort in this category is the work by Beenstock (1985) who modeled ship prices using expectations on income operations, fleet size, world wealth, and interest rates. Other general and partial equilibrium models have been developed based on this effort (see, among others, Beenstock and Vergottis (1989*a*), Beenstock and Vergottis (1989*b*), Beenstock and Vergottis (1993*a*), Beenstock and Vergottis (1993*b*), Dikos and Marcus (2003), Kalouptsidi (2014), Strandenes (1984), Tsolakis et al. (2003), and Tvedt (2003)) and Glen (2006) provides an almost comprehensive overview on these efforts. The second category of studies is concerned with testing the market efficiency (i.e., the Efficient Market Hypothesis (EMH) developed by Fama (1970)) for ship prices. Market efficiency is an assumption in many models, Hale and Vanags (1992), however, found no support for an efficient market for dry bulk second-hand ship prices using the cointegration technique. These results have been underlined by Glen (1997) and Alizadeh and Kavussanos (2002). On the contrary, Adland and Koekebakker (2004) support the EMH for dry bulk second-hand prices based on the absence of excess performance of technical trading rules in the market and Sødal et al. (2009) also largely consider the market for dry bulk ships to be efficient based on a real option approach indicating no excess profits of switching between the tanker and dry bulk market. The third category of studies concentrate on volatility and price-volume dynamics of second-hand ship prices. Using ARCH/-GARCH models, Kavussanos (1996) found higher volatilities for shipping companies acting in the timecharter market rather than in the spot market as well as higher volatilities for larger ships due to the operational limitations (i.e., less accessible ports due to the higher draft). Syriopoulos and Roumpis (2006) studied the impact of price

changes on the trading volume for the dry bulk and tanker market. They found a positive relationship between price changes and trading volume resulting from higher probability of capital gains causing higher trading activity. Besides, they found a negative relationship between trading volume and price volatility of second-hand ship prices for the dry bulk market. The fourth category takes the present value perspective on second-hand ship prices. Alizadeh and Nomikos (2006) as well as Psaraftis et al. (2012) argue that owning a ship entitles to the present value of future income from operating the ship as well as to any capital gains or losses from selling the ship at a later point in time. The last category of studies questions the validity of the aforementioned studies for the real market dynamics of second-hand ship prices. Previous efforts exclusively relied on second-hand ship price time series data, either from the BSPA or Clarksons Shipping Intelligence Network (SIN). Due to the relatively low trading activity, these time series are panelists' estimations for reference vessels of a certain age and are not based on actual vessel transactions. In particular, Pruyn et al. (2011) doubt whether these time series are a fair representation of the real market dynamics of second-hand ship prices. As already stated in section 3.1, Adland and Koekebakker (2007) were one of the first to work with actual dry bulk vessel transaction data. They estimated a multivariate, nonparametric model of second-hand ship prices using age, size in DWT, and one-year timecharter rates as explanatory variables.

The academic literature on FFAs mainly focuses on their statistical properties, volatility dynamics, and predictive power for future spot rates. An almost comprehensive summary of research in these areas is provided by Kavussanos and Visvikis (2006). Research on the unbiasedness hypothesis for FFAs, for instance, has been conducted by Kavussanos and Nomikos (1999), Kavussanos and Visvikis (2004), and Kavussanos et al. (2004). The latter found that the validness of the unbiasedness hypothesis depends on the specific market characteristics, trading route, and maturity of the FFA.

With respect to freight options, the academic literature mainly focuses on the pricing aspect of these instruments. The literature on pricing futures options starts with Black (1976) who adjusted the Black and Scholes (1973) option pricing model to accommodate futures options. Concerning Asian or average options, Kemna and Vorst (1990) stated that exact pricing formulas only exist for geometric average options but not for arithmetic average options as the distribution of the arithmetic average of a lognormal process is unknown. Turnbull and Wakeman (1991) presented a closed-form

solution for European geometric average options and an approximation for European arithmetic average options which is still valued by researchers as well as practitioners today. The approximation is based on the assumption of averaging in continuous time. Levy (1997) and Haug et al. (2003), for instance, provided alternatives to price Asian futures options in discrete time. For freight options in particular, Tvedt (1998) developed a model to price freight options on the Baltic International Freight Futures Exchange (BIFFEX) contract that existed at that time. The model is an adapted version of the Black (1976) model which has been adjusted for the statistical properties of the freight market. Another model was suggested by Koekebakker et al. (2007). They also provided a model based on the Black (1976) framework assuming that the price dynamics of FFAs are lognormal prior to the start of the settlement or averaging period but not lognormal within the averaging period. More recently, Nomikos et al. (2013) suggested a diffusion model overlaid with jumps of random magnitude and timing to extend the lognormal representation for risk-neutral spot freight rate dynamics as well as an option valuation framework to price the current version of freight options that have been launched in 2008.

Turning to the hedging of ship price risks, FoSVAs have been studied by Adland et al. (2004). They suggested a pricing methodology for these contracts as well as the unbiasedness of implied forward prices for second-hand ships. Unfortunately, these instruments have never been really accepted by the market participants and Alizadeh and Nomikos (2009) and Jallal (2013) raise the nonexistent liquidity of these instruments. Alternatively, Alizadeh and Nomikos (2012) suggested using FFA+2CAL contracts in a minimum-variance cross-hedging set-up. As already outlined in section 3.1, they stated that variance reductions of up to 93 % for dry bulk second-hand ship prices should be possible. A first empirical application of FFAs as cross-hedge instruments for dry bulk Capesize ship prices is provided by CHAPTER 2 of this dissertation which finds that a SPM-based approach to derive the desired hedge exposure is more effective than a minimum-variance approach. Freight options have not been considered so far in the academic literature as potential hedge instruments for ship price risks.

However, the hedging performance of options in general has found some attention in the academic literature. On the one hand, there are several efforts comparing the hedge effectiveness of futures and option hedging strategies for different underlyings (see, among others, Chang and Shanker (1986) on currencies, Cheung et al. (1990) on currencies, Sakong et al. (1993) on farmers' production risks, Whaley (1993) on

volatility risks, Garcia et al. (1994) on hog production risks, Hancock and Weise (1994), on the S&P500 index, Hsin et al. (1994) on currencies, Benet and Luft (1995) on the S&P 500 index, Lien and Tse (2001) on currencies, or Wong (2003) on currencies). On the other hand, there is an elaborate discussion in the academic literature on how to fairly assess the effectiveness of hedging strategies involving options. Ederington (1979) suggested the relative variance reduction of the hedged vs. the unhedged position for minimum-variance futures hedges.

Given the asymmetric nature of options, Bookstaber and Clarke (1981) noted the implications that options have on the return distributions of portfolios. Together with underlying and options, almost any return characteristic can be created and thus, the traditional mean-variance perspective developed by Markowitz (1952) leads to suboptimal portfolio decisions. Consequently, the hedge effectiveness measure of Ederington (1979) may also lead to wrong conclusion if options are involved. Bookstaber and Clarke (1985) elaborated more closely on this issue. Some academic researchers have subsequently turned to risk-return measures of hedge effectiveness starting with Howard and D'Antonio (1984). They developed a measure largely based on the idea of the Sharpe ratio (Sharpe, 1966) and suggested to use the ratio of the increase in excess return (i.e., expected return exceeding the risk-free rate) per unit of risk with hedge instruments additionally available and the increase in excess return per unit of risk for investing in the spot alone. Chang and Shanker (1987) suggested an improvement of the measure by Howard and D'Antonio (1984) to eliminate ambiguous results if the Sharpe ratio of the unhedged portfolio is negative and the Sharpe ratio of the hedge portfolio is positive. Their improved measure of hedge effectiveness is the increase in the Sharpe ratio from unhedged to hedged portfolio over the absolute value of the Sharpe ratio of the unhedged portfolio. Howard and D'Antonio (1987) suggested a further improvement to assess the hedge effectiveness by just simply taking the increase in the Sharpe ratio from unhedged to hedged portfolio. Kuo and Chen (1995) and Satyanarayan (1998) added to this discussion.

Another direction is the downside-risk perspective that emerged with research on the safety first principal developed by Roy (1952), semivariance and later on the lower partial moment (LPM). Markowitz (1959) already acknowledged the importance of the downside-risk perspective and defined the below-mean semivariance and below-target semivariance. However, in his considerations about optimal portfolio selection, he stayed with the simpler variance concept. The concept of the LPM emerged when

Bawa (1975) proposed the third order stochastic domination rule as an optimal selection rule for ordering uncertain prospects for agents with decreasing absolute risk averse utility functions (i.e., von Neumann and Morgenstern (1947)-type utility functions that correspond to observed economic behavior). He was the first one to define the mean-LPM with varying degrees of risk aversion (i.e., the different moments). Moreover, he introduced the mean-lower partial variance which he later uses to develop a Capital Asset Pricing Model (CAPM) based on downside risk instead of the classical mean-variance CAPM and studied optimal portfolio choice under the safety first principle and stochastic dominance (Bawa and Lindenberg, 1977; Bawa, 1978). Fishburn (1977) extended the LPM definition from Bawa (1975) to a general target-LPM and proved the equivalence of the LPM measure with results from stochastic dominance for all moments greater than zero. Bawa and Lindenberg (1977) already suggested to use the risk-free rate as target return and Nantell and Price (1979) found that equilibrium rates of return are the same for models based on variance or semivariance notion of portfolio risk if the risk-free rate is used as target in the semivariance case. They assumed the bivariate return distribution to be normal. Price et al. (1982) extended this research to skewed, lognormal distributions and question the generally accepted variance measure for systematic risk due to the violation of the restrictive underlying assumptions. Concerning portfolio selection or choice, Nawrocki (1991) developed two algorithms that can be tailored to the specific risk aversion of investors using also higher moments of the  $n$ -degree LPM risk measure. He found that the two algorithms are either equivalent or even superior to traditional covariance analysis while allowing for a broader set of utility choices for the investor at the same time. In another effort, Nawrocki (1992) studied the characteristics of portfolios selected by the  $n$ -degree LPM compared to portfolios selected by traditional mean-variance or covariance analysis. He discovered, for instance, that  $n$ -degree LPM portfolios contain fewer securities, that the portfolio skewness increases with  $n$ , and that  $n$ -degree LPM portfolios achieve superior risk-return performance. From a practitioner's side, Sortino and van der Meer (1991), Sortino and Price (1994), and Merriken (1994), for instance, concentrated on the performance measurement using the LPM downside-risk measure. Sortino and Price (1994) defined another downside-risk performance measure, the Sortino ratio. It brings together the risk-return perspective with the concept of downside risk and is a modification of the Sharpe ratio. It measures the excess return above a certain target over the square root of the second-degree target-LPM measure. Merriken (1994) particularly focused on assessing the downside risk

of different hedging strategies involving stock options and interest rate swaps. A comprehensive overview of the LPM-based research is provided by Nawrocki (1999).

This paper contributes to the existing academic literature in two important ways. Firstly, the general suitability of freight options as hedge instruments for dry bulk Capesize ship price risks is empirically assessed for real sale and purchase transactions. Secondly, the hedge effectiveness of hedging strategies involving freight options (i.e., long at-the-money put, long out-of-the-money put, replicated FFA, and zero-cost collar) is compared the hedge effectiveness of FFAs. Furthermore, the research findings will have relevant implications for the risk management practice of shipping companies operating in the dry bulk market.

### **3.3 Empirical methodology**

With respect to the empirical methodology applied within this study, the following subsections elaborate on the pricing model applied to value freight options, the developed SPM for second-hand dry bulk Capesize vessels to derive the desired hedge exposure, the general hedge set-up in which the hedging effectiveness is empirically tested, and the approach how to appropriately measure the hedge effectiveness for strategies involving instruments with asymmetric payoff structures (e.g., options).

#### **3.3.1 Freight option prices**

Freight options belong to the category of Asian options and therefore, they are somewhat more complex than typical European options. The following paragraphs focus on the specific nature of these instruments as well as on a suitable methodology to price these instruments accurately.

Freight options are European-style arithmetic average options on FFAs and exist for different dry bulk standard routes (e.g., individual routes as well as trip-charter average routes) and maturities (e.g., several months, quarters, or calendar years into the future). The averaging period is typically one month and the averaging is done discretely (i.e., once per day) (Alizadeh and Nomikos, 2009). With respect to available price quotes, The Baltic Exchange does not directly quote freight option prices

but only daily Baltic Option Assessments (BOAs) in the form of at-the-money implied volatilities. For that matter, The Baltic Exchange assumes that the Turnbull and Wakeman (1991) model holds as factual relationship between option prices and volatility. Accordingly, freight option prices have to be computed first in order to use these instruments in the empirical study of hedge effectiveness.

Currently, there is no closed-form solution for pricing European-style arithmetic average options. Turnbull and Wakeman (1991) developed a closed-form solution for European-style geometric average options that may be used as approximation for European-style arithmetic average options. Several relevant clearing houses (e.g., LCH.Clearnet or NOS Clearing) state that the industry standard formula being used for pricing freight options is the Turnbull and Wakeman (1991) approximation. Therefore, the Turnbull and Wakeman (1991) approximation is used to derive freight option prices within this study. The description and notation of the approximation below largely follow the efforts by Haug (2007) and Alizadeh and Nomikos (2009). The approximation itself is shown below in equation (3.1):

$$\begin{aligned}
 C &\approx S \cdot e^{(b_a - r)t} \cdot \mathcal{N}(d_1) - X \cdot e^{-rt} \cdot \mathcal{N}(d_2) \\
 P &\approx X \cdot e^{-rt} \cdot \mathcal{N}(-d_2) - S \cdot e^{(b_a - r)t} \cdot \mathcal{N}(-d_1)
 \end{aligned} \tag{3.1}$$

$$\begin{aligned}
 \text{with } d_1 &= \frac{\ln\left(\frac{S}{X}\right) + \left(b_a + \frac{\sigma_a^2}{2}\right) \cdot t}{\sigma_a \cdot \sqrt{t}} \\
 d_2 &= d_1 - \sigma_a \cdot \sqrt{t}.
 \end{aligned}$$

Within equation (3.1), the approximated price of a call is denoted by  $C$  and the approximated price of a corresponding put by  $P$ . The variables  $S$  and  $X$  refer to the spot price of the underlying and the strike price of the option, respectively. The risk-free interest rate is denoted by  $r$  and the time to maturity in years by  $t$ . The adjusted volatility,  $\sigma_a$ , and adjusted mean,  $b_a$ , are given by the formulas in equation (3.2) below:

$$\begin{aligned}
 \sigma_a &= \sqrt{\frac{\ln(M_2)}{t} - 2 \cdot b_a} \\
 b_a &= \frac{\ln(M_1)}{t}.
 \end{aligned} \tag{3.2}$$

The variables  $M_1$  and  $M_2$  refer to the first and second moment of the arithmetic average under the condition of risk neutrality, respectively. They are defined in equa-

tion (3.3) below:

$$\begin{aligned}
 M_1 &= \frac{e^{b \cdot t} - e^{b \cdot t_1}}{b \cdot (t - t_1)} \\
 M_2 &= \frac{2 \cdot e^{(2 \cdot b + \sigma^2) \cdot t}}{(b + \sigma^2) \cdot (2 \cdot b + \sigma^2) \cdot (t - t_1)} + \frac{2 \cdot e^{(2 \cdot b + \sigma^2) \cdot t_1}}{b \cdot (t - t_1)^2} \cdot \left( \frac{1}{2 \cdot b + \sigma^2} - \frac{e^{b \cdot (t - t_1)}}{b + \sigma^2} \right). \quad (3.3)
 \end{aligned}$$

Within equation (3.3),  $t_1$  refers to the time to the start of the averaging period in years and  $b$  to the cost of carry. As freight options are options on FFAs, the cost of carry,  $b$ , is equal to zero and  $M_1$  and  $M_2$  can be derived according to the following, simplified formulas as stated in equation (3.4) below:

$$\begin{aligned}
 M_1 &= 1 \\
 M_2 &= \frac{2 \cdot e^{\sigma^2 \cdot t} - 2 \cdot e^{\sigma^2 \cdot t_1} \cdot (1 + \sigma^2 \cdot (t - t_1))}{\sigma^4 \cdot (t - t_1)^2}. \quad (3.4)
 \end{aligned}$$

As  $M_1 = 1$  and  $b = 0$ , the adjusted volatility,  $\sigma_a$ , is the volatility of the average on the underlying futures or FFA volatility,  $\sigma$ . If the averaging period has already started, the strike price has to be replaced by  $\hat{X}$  and the option value has to be multiplied by  $t/t_2$ , where  $t_2$  refers to the length of the averaging period in years. The corresponding formula for  $\hat{X}$  can be seen in equation (3.5) below:

$$\hat{X} = \frac{t_2}{t} \cdot X - \frac{\tau}{t} \cdot S_a. \quad (3.5)$$

Within equation (3.5),  $\tau$  reflects the already realized time in the the averaging period in years and  $S_a$  the average asset price during the realized part of the averaging period so far. Now, if  $\tau > 0$  and  $\hat{X} < 0$ , a call option will be exercised for certain and the value of the call option can be derived according to the following formula as shown in equation (3.6) below:

$$\begin{aligned}
 C &\approx (\mathbb{E}(S_a) - X) \cdot e^{-r \cdot t} \\
 \text{with } \mathbb{E}(S_a) &= S_a \cdot \frac{t_2 - t}{t_2} + S \cdot M_1 \cdot \frac{t}{t_2}. \quad (3.6)
 \end{aligned}$$

In this case, the corresponding put will be out of the money and will expire worthless.

Although the averaging of freight options is done discretely and the Turnbull and Wakeman (1991) approximation assumes continuous averaging, the approximation is nevertheless used within this effort as it is considered as the industry standard option

pricing formula for freight options by the relevant clearing houses.<sup>6</sup>

Concerning volatility smiles of freight options to create moneyness (i.e., in- or out-of-the-money options), the relevant clearing houses (e.g., LCH.Clearnet or NOS Clearing) use the same implied volatility for all strike prices as the current liquidity of the freight options market does not allow to determine a correct volatility smile. Within this study, volatility smiles of freight options are equally handled and the same implied volatility is used for all strike prices to derive the option price. Commodity options usually exhibit a reverse smirk rather than the constant implied volatility assumed within Black and Scholes (1973) or Turnbull and Wakeman (1991). Accordingly, out-of-the-money put options and in-the-money call options should be cheaper in practice than the model implies and in-the-money put options and out-of-the-money call options should be more expensive in practice than the model implies (Bates, 1991).

Another specific characteristic of freight options is that after the purchase or sale of the option, quarterly and calendar year options are automatically equally split into three or 12 monthly options, respectively (Nomikos et al., 2013). The individual options are then settled as monthly options. Consequently, quarterly and calendar year options are basket or strip options consisting of three or 12 individual monthly options and need to be priced accordingly (Nomikos et al., 2013). Therefore, the most granular level of implied volatility figures available for the respective individual monthly options is used by the clearing houses to price these individual options. As dry bulk Capesize FFAs are quoted in USD per day, prices of freight options on these FFAs are similarly in USD per day. Hence, the price of a quarterly option is the trading day-weighted average price of three individual options for the respective months of the quarter and the price of a calendar year option is the trading day-weighted average price of 12 individual monthly options of the calendar year (Nomikos et al., 2013).

### 3.3.2 Structural pricing model

Dry bulk Capesize vessels are rather heterogeneous with respect to size (e.g., length, DWT, breadth, or loading capacity) and also with respect to other ship specifica-

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<sup>6</sup> Levy (1997) or Haug et al. (2003), for instance, have developed discrete time approximations for European arithmetic average options, but these approximations have apparently not yet gained ground at the relevant clearing houses for freight options.

tions (e.g., engine, speed, consumption, or yard built). Accordingly, individual vessels might be quite different from the reference vessel underlying the panelists' estimations for second-hand ship prices. Hence, transparency on a fair and adequate second-hand price for an individual vessel at a certain point in time is rather low. Adland and Koekebakker (2007) had the idea of estimating a multivariate, non-parametric model from past vessel transactions using a market indicator (i.e., the one-year timecharter rate) as well as ship specific indicators (i.e., age in years and size in DWT) as explanatory variables. This study follows the intention by Adland and Koekebakker (2007). However, a parametric approach in the form of a multiple linear regression has been chosen and a SPM is estimated based on real dry bulk Capesize sale and purchase transactions. This allows to determine the desired exposure to the selected hedge instruments (i.e., FFAs and options on these). The SPM follows the form as shown in equations (3.7) and (3.8) below:

$$\begin{aligned}
 \text{SPM 1: } p_{i,t} = & \alpha + \beta_{f,1\text{CAL}} \cdot f_{1\text{CAL},t} + \beta_{\text{Age}} \cdot \text{Age}_{i,t} + \beta_{\text{DWT}} \cdot \text{DWT}_i \\
 & + \beta_{\text{Consum}} \cdot \text{Consum}_i + \beta_{f_{1\text{CAL}} \cdot \text{Age}} \cdot f_{1\text{CAL},t} \cdot \text{Age}_{i,t} \\
 & + \beta_{f_{1\text{CAL}} \cdot \text{DWT}} \cdot f_{1\text{CAL},t} \cdot \text{DWT}_i \\
 & + \beta_{f_{1\text{CAL}} \cdot \text{Consum}} \cdot f_{1\text{CAL},t} \cdot \text{Consum}_i + \varepsilon_{i,t}
 \end{aligned} \tag{3.7}$$

$$\begin{aligned}
 \text{SPM 2: } p_{i,t} = & \alpha + \beta_{f,2\text{CAL}} \cdot f_{2\text{CAL},t} + \beta_{\text{Age}} \cdot \text{Age}_{i,t} + \beta_{\text{DWT}} \cdot \text{DWT}_i \\
 & + \beta_{\text{Consum}} \cdot \text{Consum}_i + \beta_{f_{2\text{CAL}} \cdot \text{Age}} \cdot f_{2\text{CAL},t} \cdot \text{Age}_{i,t} \\
 & + \beta_{f_{2\text{CAL}} \cdot \text{DWT}} \cdot f_{2\text{CAL},t} \cdot \text{DWT}_i \\
 & + \beta_{f_{2\text{CAL}} \cdot \text{Consum}} \cdot f_{2\text{CAL},t} \cdot \text{Consum}_i + \varepsilon_{i,t}
 \end{aligned} \tag{3.8}$$

Within the equations (3.7) and (3.8), the price of vessel  $i$  at time  $t$  is denoted by  $p_{i,t}$ . The constant of the regression is represented by  $\alpha$ . The prices of the FFA+1CAL and FFA+2CAL contract at time  $t$  in USD million are denoted by  $f_{1\text{CAL},t}$  and  $f_{2\text{CAL},t}$ , respectively. The age of vessel  $i$  at time  $t$  in years is represented by  $\text{Age}_{i,t}$ , the size of vessel  $i$  in DWT by  $\text{DWT}_i$ , and the consumption of vessel  $i$  in metric tonnes per 1,000 DWT per 1,000 nautical miles<sup>7</sup> by  $\text{Consum}_i$ . Selected interaction terms were included in the regressions to correct the exposure to the FFA contract for extremely young and old as well as for extremely efficient and inefficient vessels. These are represented by  $f_{1\text{CAL},t} \cdot \text{Age}_{i,t}$ ,  $f_{1\text{CAL},t} \cdot \text{DWT}_i$ ,  $f_{1\text{CAL},t} \cdot \text{Consum}_i$ ,  $f_{2\text{CAL},t} \cdot \text{Age}_{i,t}$ ,  $f_{1\text{CAL},t} \cdot \text{DWT}_i$ , and  $f_{2\text{CAL},t} \cdot \text{Consum}_i$ . The corresponding regression coefficients for the explanatory

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<sup>7</sup> Voyaging at one knot per hour corresponds to a traveled distance of one nautical mile per hour.

variables are denoted by  $\beta_{f,1\text{CAL}}$ ,  $\beta_{f,2\text{CAL}}$ ,  $\beta_{Age}$ ,  $\beta_{DWT}$ ,  $\beta_{Consum}$ ,  $\beta_{f_{1\text{CAL}} \cdot Age}$ ,  $\beta_{f_{2\text{CAL}} \cdot Age}$ ,  $\beta_{f_{1\text{CAL}} \cdot DWT}$ ,  $\beta_{f_{2\text{CAL}} \cdot DWT}$ ,  $\beta_{f_{1\text{CAL}} \cdot Consum}$ , and  $\beta_{f_{2\text{CAL}} \cdot Consum}$ . The error term of the SPM is represented by  $\varepsilon_{i,t}$ . So, equations (3.7) and (3.8) are actually identical except for the FFA+1CAL vs. FFA+2CAL contract being used as plain explanatory variable as well as in the interaction terms.

The consumption of dry bulk vessels is usually measured in metric tonnes per day. As dry bulk Capesize vessels are quite heterogeneous with respect to size in DWT and speed in knots per hour, the consumption measure,  $Consum_i$ , in metric tonnes per 1,000 DWT per 1,000 nautical miles has been derived in order to allow undistorted comparisons of fuel oil consumption across vessels. Bunker costs (i.e., costs for fuel oil) account for a large share of a vessel's voyage costs (Stopford, 2009). Accordingly, fuel efficiency should be an important driver of second-hand ship prices.

The model allows to determine the exposure to the market-driven explanatory variables (i.e., FFA+1CAL or FFA+2CAL contracts) at time  $t$  for ships with specific time-varying and deterministic vessel characteristics. As freight options are options on FFAs, the model above is equally suited to determine the desired exposure to FFAs or corresponding freight options as hedge instruments.

### 3.3.3 Hedging set-up

In order to fairly assess the hedge effectiveness of competing hedging strategies, the hedging set-up in which the different strategies are tested needs to be well defined. The following paragraphs elaborate on the chosen hedging set-up as well as the assumptions made to create a set-up that is, on the one hand, close to reality for shipping companies facing such hedging decisions and, on the other hand, allows adequate comparisons across different vessels.

With respect to the general hedging set-up, it is assumed that a shipping company owning a vessel  $i$  knows ex ante that it wants to sell the vessel  $i$  at a particular date in the future,  $t_i$ . Moreover, the shipping company seeks protection against ship price risk for certain time period,  $l_i$ , before the vessel transaction. Specifically, the time horizon of the hedge is set to one year for all vessels and thus,  $l_i$  corresponds to 252 trading days. The length of the hedge time horizon has been set to one year for convenience reasons as the considered hedge instruments (i.e., FFA+1CAL and

FFA+2CAL contracts as well as corresponding options on these) cover a time frame of one calendar year. Correspondingly, the hedge is initiated at  $t_i - 252$  for vessel  $i$ . The sales or transaction price,  $p_{i,t}$ , of vessel  $i$  is not known to the shipping company until the sales date,  $t_i$ . Nonetheless, the SPM of equation (3.7) or (3.8) allows the shipping company to estimate a model-implied value of vessel  $i$  at  $t_i - 252$ ,  $\hat{m}_{i,t_i-252,Age_{i,t_i}}$ , using the age of vessel  $i$  at  $t_i$ ,  $Age_{i,t_i}$ . Taking the age at the sales date,  $Age_{i,t_i}$ , to estimate the value of vessel  $i$  at  $t_i$  corrects for the expected aging-related loss (i.e., depreciation) at time  $t_i - 252$  that the shipping company would have to account for in any case. The shipping company's objective rather is to hedge any downside vessel price fluctuations besides the normal, expected depreciation. Of course, unexpected additional or reduced aging-related loss may occur throughout the hedge from time  $t_i - 252$  to  $t_i$  if the market circumstances change. Accordingly, the aging-related loss realized at the sales date,  $t_i$ , is eventually market-driven. The model-implied value,  $\hat{m}_{i,t_i-252,Age_{i,t_i}}$ , corresponds to the value of the physical or unhedged position of vessel  $i$  at the hedging start date,  $t_i - 252$ , and its computation follows from equation (3.9) below:

$$\begin{aligned}
 \hat{m}_{i,t_i-252,Age_{i,t_i}} &= \alpha_{\{\cdot\}} + \beta_{f,\{\cdot\}} \cdot f_{\{\cdot\},t_i-252} + \beta_{Age,\{\cdot\}} \cdot Age_{i,t_i} + \beta_{DWT,\{\cdot\}} \cdot DWT_i \\
 &\quad + \beta_{Consum,\{\cdot\}} \cdot Consum_i + \beta_{f_{\{\cdot\}} \cdot Age} \cdot f_{\{\cdot\},t_i-252} \cdot Age_{i,t_i} \\
 &\quad + \beta_{f_{\{\cdot\}} \cdot DWT} \cdot f_{\{\cdot\},t_i-252} \cdot DWT_i \\
 &\quad + \beta_{f_{\{\cdot\}} \cdot Consum} \cdot f_{\{\cdot\},t_i-252} \cdot Consum_i \\
 &\text{with } \{\cdot\} \in \{1\text{CAL}, 2\text{CAL}\}.
 \end{aligned} \tag{3.9}$$

Regarding the corresponding hedge position that is taken at  $t_i - 252$ , the desired hedge exposure is determined using the SPM of equation (3.7) or (3.8). Aggregating the exposure to  $f_{1\text{CAL},t}$  or  $f_{2\text{CAL},t}$  contracts yields the desired hedge exposure for vessel  $i$  in number of lots. As FFAs are quoted in USD per day and the lot size is one day, the FFA value is first transformed to USD million to match quoted vessel prices. The desired hedge exposure in number of lots is shown in equation (3.10) below:

$$\begin{aligned}
 d_{i,t_i-252,\{\cdot\}} &= \beta_{f,\{\cdot\}} + \beta_{f_{\{\cdot\}} \cdot Age} \cdot Age_{i,t_i} + \beta_{f_{\{\cdot\}} \cdot DWT} \cdot DWT_i \\
 &\quad + \beta_{f_{\{\cdot\}} \cdot Consum} \cdot Consum_i \\
 &\text{with } \{\cdot\} \in \{1\text{CAL}, 2\text{CAL}\}.
 \end{aligned} \tag{3.10}$$

For reasons of practicality, the result of equation (3.10) is rounded to two decimal

digits as the minimum number of lots per contract for FFAs and freight options is  $\frac{1}{100}$ -th of a lot according to the product specifications provided by NOS Clearing.

In addition, the following general assumptions regarding the hedging set-up have been made. Firstly, as the calendar-year FFA contracts and corresponding options are rolled over once per year, typically around December 22<sup>nd</sup> or 23<sup>rd</sup>, the hedging position is closed out on the last trading day before the rollover date and re-entered on the first trading day after the rollover date. Secondly, transaction costs are considered in the form of a fixed fee of USD 8 per lot or contract traded for both FFAs and freight options<sup>8</sup> plus half the mean estimated bid-/ask-spread for the respective hedging instrument. Given the generally higher liquidity of FFAs compared to freight options (see appendix A.2 on page 135 for details), hedging strategies involving freight options usually face relatively higher transaction costs in the form of larger bid-/ask-spreads. As no bid-/ask-spreads are available from The Baltic Exchange, the mean bid-/ask-spreads for the individual instruments were estimated from the historical weekly log return autocovariances using the method of Roll (1984). Accordingly, the differences in bid-/ask-spreads between FFAs and freight options have been accounted for in order not to distort any hedging results towards freight options. Details on the average bid-/ask-spread estimations can be found in the appendix A.2 on page 135.

Finally, as FFAs are usually cleared and ‘marked to market’ by respective clearing houses, the interest effect on any accumulated gains or losses on the margin account is considered in the form of a daily margining using the USD London Interbank Offered Rate (LIBOR) overnight rate. According to LCH.Clearnet, freight options are not ‘marked to market’ in a typical way. Nevertheless, these instruments are cleared as well by clearing houses. The option premium is exchanged up-front and the margin requirement is calculated using London Standard Portfolio Analysis of Risk (SPAN). For long option positions, the net liquidation value (i.e., the amount of money required to close out a position in case of default of one of the counterparties) is credited on the margin account and this usually results in a close-out profit. For short options, the net liquidation value is debited on the margin account besides the margin requirement. Both margin requirement and net liquidation value are reassessed on a daily basis. For reasons of simplification, the margining of option positions is done largely similarly

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<sup>8</sup> LCH.Clearnet and NASDAQ OMX reported a fixed transaction and margining fee of USD 8 per lot or contract traded for FFAs and freight options for block trades. Due to the considerable number of lots traded for each hedge, it is assumed that the trades considered within this empirical study qualify as block trades.

to FFAs by just deriving daily option prices for the margin account within this study. However, for long option positions, only positive interest effects on cumulative gains of the option position are considered as any potential losses have already been covered by the payment of the option premium at the hedge initiation and no interest effect on the cumulative losses of the position need to be considered on the margin account. The contrary happens for short option positions. Only negative interest effects on cumulative losses of the position are considered as any potential gains have already been covered by the receipt of the option premium at the hedge initiation. Besides, interest on the option premium itself is considered for both long and short option positions on the margin account. As a result of the margining, the desired exposure at the hedge initiation,  $t_i - 252$ , is corrected using a tailing factor. The concept of tailing a hedge has been developed by Figlewski et al. (1991). The tailing factor at  $t_i - 252$ ,  $b_{t_i-252}$ , is derived according to equation (3.11) as shown below:

$$b_{t_i-252} = e^{-r_{\text{LIBOR-12M},t_i-252} \frac{(t_i-(t_i-252))}{252}} = e^{-r_{\text{LIBOR-12M},t_i-252}}. \quad (3.11)$$

For the tailing factor, the USD LIBOR 12-month rate,  $r_{\text{LIBOR-12M},t_i-252}$ , is used to match the hedging time horizon of one year. The tailed number of lots is again rounded to two decimal digits for reasons of practicality as stated earlier in this subsection.

As already stated above, the considered hedge instruments are rolled over once per year and thus, each hedge encounters such a rollover date. Accordingly, the hedge is split into two consecutive hedges (i.e., from  $t_i - 252$  to the rollover date and from the rollover date to  $t_i$ ). Accordingly, the expected age effect on the physical position, the desired exposure, and the tailing factor are dynamically adjusted. The expected age effect, for instance, is proportionally split across the two consecutive hedging periods.

The change in the physical position from  $t_i - 252$  to  $t_i$ ,  $\Delta_{252}v_{\text{PHYPOS},i}$ , is the difference between the sales price of vessel  $i$  at  $t_i$ ,  $p_{i,t_i, \text{Age}_{i,t_i}}$ , and the model-implied value for vessel  $i$  at  $t_i - 252$ ,  $\hat{m}_{i,t_i-252, \text{Age}_{i,t_i}}$ , taking the age of vessel  $i$  at  $t_i$ ,  $\text{Age}_{i,t_i}$ . The formula to compute  $\Delta_{252}v_{\text{PHYPOS},i}$  is shown in equation (3.12) below:

$$\Delta_{252}v_{\text{PHYPOS},i} = p_{i,t_i, \text{Age}_{i,t_i}} - \hat{m}_{i,t_i-252, \text{Age}_{i,t_i}}. \quad (3.12)$$

The change in the hedge position from  $t_i - 252$  to  $t_i$ ,  $\Delta_{252}v_{\text{HEPOS},i}$ , consists of the cumulative profit/loss of the position itself plus the interest effect on the margin

account. The computation of  $\Delta_{252}v_{\text{HEPOS},i}$  for hedging strategies involving short FFAs, long put, or short call freight options is shown in equations (3.13), (3.14), and (3.15) below:

Hedging strategy involving short FFAs:

$$\begin{aligned} \Delta_{252}v_{\text{HEPOS},i} = & (w_{i,t_i} - w_{i,t_i-252}) + \sum_{j=1}^{251} \left( (w_{i,t_i-252+j} - w_{i,t_i-252+j-1}) \right. \\ & \left. \left( \prod_{k=0}^{j-1} e^{r_{\text{LIBOR-ON},t_i-252+k} \cdot \frac{1}{252}} - 1 \right) \right) \end{aligned} \quad (3.13)$$

Hedging strategy involving long put freight options:

$$\begin{aligned} \Delta_{252}v_{\text{HEPOS},i} = & (w_{i,t_i} - w_{i,t_i-252}) \\ & + \sum_{j=1}^{251} \left( \mathbb{1}_{\{x_{i,j}>0\}}(x_{i,j}) \cdot x_{i,j} \left( e^{r_{\text{LIBOR-ON},t_i-252+j} \cdot \frac{1}{252}} - 1 \right) \right) \\ & + \sum_{k=2}^{251} \left( \sum_{j=1}^{k-1} \left( \mathbb{1}_{\{x_{i,j}>0\}}(x_{i,j}) \cdot x_{i,j} \right. \right. \\ & \left. \left. \left( e^{r_{\text{LIBOR-ON},t_i-252+j} \cdot \frac{1}{252}} - 1 \right) \right) \left( e^{r_{\text{LIBOR-ON},t_i-252+k} \cdot \frac{1}{252}} - 1 \right) \right) \\ & - z_{i,t_i-252} \left( \prod_{h=1}^{252} e^{r_{\text{LIBOR-ON},t_i-252+h} \cdot \frac{1}{252}} - 1 \right) \end{aligned} \quad (3.14)$$

with  $x_{i,j} = w_{i,t_i-252+j} - w_{i,t_i-252}$

$$\text{and } \mathbb{1}_{\{x_{i,j}>0\}}(x_{i,j}) = \begin{cases} 1 & \text{if } x_{i,j} > 0 \\ 0 & \text{if } x_{i,j} \leq 0 \end{cases}$$

Hedging strategy involving short call freight options:

$$\begin{aligned}
 \Delta_{252}v_{\text{HEPOS},i} = & (w_{i,t_i} - w_{i,t_i-252}) \\
 & + \sum_{j=1}^{251} \left( \mathbb{1}_{\{x_{i,j} \leq 0\}}(x_{i,j}) \cdot x_{i,j} \left( e^{r_{\text{LIBOR-ON},t_i-252+j} \cdot \frac{1}{252}} - 1 \right) \right) \\
 & + \sum_{k=2}^{251} \left( \sum_{j=1}^{k-1} \left( \mathbb{1}_{\{x_{i,j} \leq 0\}}(x_{i,j}) \cdot x_{i,j} \right. \right. \\
 & \left. \left. \left( e^{r_{\text{LIBOR-ON},t_i-252+j} \cdot \frac{1}{252}} - 1 \right) \right) \left( e^{r_{\text{LIBOR-ON},t_i-252+k} \cdot \frac{1}{252}} - 1 \right) \right) \\
 & + z_{i,t_i-252} \left( \prod_{h=1}^{252} e^{r_{\text{LIBOR-ON},t_i-252+h} \cdot \frac{1}{252}} - 1 \right)
 \end{aligned} \tag{3.15}$$

with  $x_{i,j} = w_{i,t_i-252+j} - w_{i,t_i-252}$

$$\text{and } \mathbb{1}_{\{x_{i,j} \leq 0\}}(x_{i,j}) = \begin{cases} 1 & \text{if } x_{i,j} \leq 0 \\ 0 & \text{if } x_{i,j} > 0 \end{cases}.$$

Within the equations (3.13), (3.14), and (3.15) above, the initial hedge position is denoted by  $w_{i,t_i-252}$  and results from multiplying the tailed number of lots of equation (3.10) (i.e., the desired exposure to the respective hedge instrument) by the respective FFA or freight option price depending on the hedging strategy. The overall change of the hedge position excluding interest effect is given by  $w_{i,t_i} - w_{i,t_i-252}$ . The daily cumulative delta of the hedge position,  $w_{i,t_i-252+j} - w_{i,t_i-252}$ , is multiplied by  $e^{r_{\text{LIBOR-ON},t_i-252+j} \cdot \frac{1}{252}} - 1$ , where  $r_{\text{LIBOR-ON},t_i-252+j}$  denotes the USD LIBOR overnight rate at time  $t_i - 252 + j$ , to account for the daily interest effect on any accumulated gains or losses on the margin account. The cumulative daily interest effect itself is then multiplied by  $e^{r_{\text{LIBOR-ON},t_i-252+k} \cdot \frac{1}{252}} - 1$  to account for compound interest. For the case with freight options, the option premium,  $z_{i,t_i-252}$ , is paid initially for long put freight options and received initially for short call freight options. The corresponding interest effect on the option premium is computed using the USD LIBOR overnight rate and is also considered in the hedge position delta from  $t_i - 252$  to  $t_i$ ,  $\Delta_{252}v_{\text{HEPOS},i}$ .

The general goal of the shipping company is to minimize the variation of the aggregated portfolio. The variation of the aggregated portfolio from  $t_i - 252$  to  $t_i$ ,  $\Delta_{252}v_{\text{PORTF},i}$ , consists of the change in the physical position and the change in the hedge position. The computation of  $\Delta_{252}v_{\text{PORTF},i}$  for all hedging strategies is shown

in equation (3.16) below:

$$\Delta_{252}v_{\text{PORTF},i} = \Delta_{252}v_{\text{PHYPOS},i} + \Delta_{252}v_{\text{HEPOS},i}. \quad (3.16)$$

In order to measure the hedge effectiveness of any hedging strategies later on, annualized log returns of both the physical position and the aggregated portfolio need to be derived first. The annualized log return of the physical position of vessel  $i$  from time  $t_i - 252$  to  $t_i$ ,  $r_{\text{PHYPOS},i,t_i,\text{ann.}}$ , is computed according to equation (3.17) below:

$$\begin{aligned} r_{\text{PHYPOS},i,t_i,\text{ann.}} &= \ln \left( \frac{p_{i,t_i,\text{Age}_{i,t_i}}}{\hat{m}_{i,t_i-252,\text{Age}_{i,t_i}}} \right) \cdot \frac{t_i - (t_i - 252)}{252} \\ &= \ln \left( \frac{p_{i,t_i,\text{Age}_{i,t_i}}}{\hat{m}_{i,t_i-252,\text{Age}_{i,t_i}}} \right). \end{aligned} \quad (3.17)$$

Accordingly, the annualized log return of the aggregated portfolio of vessel  $i$  from time  $t_i - 252$  to  $t_i$ ,  $r_{\text{PORTF},i,t_i,\text{ann.}}$ , is computed according to equation (3.18) below:

$$r_{\text{PORTF},i,t_i,\text{ann.}} = \ln \left( \frac{v_{\text{PORTF},i,t_i}}{v_{\text{PORTF},i,t_i-252}} \right) \cdot \frac{t_i - (t_i - 252)}{252} = \ln \left( \frac{v_{\text{PORTF},i,t_i}}{v_{\text{PORTF},i,t_i-252}} \right). \quad (3.18)$$

Within equation (3.18) above,  $v_{\text{PORTF},i,t_i-252}$  and  $v_{\text{PORTF},i,t_i}$  represent the value of the aggregated portfolio at initiation of the hedge,  $t_i - 252$ , and at the end of the hedge,  $t_i$ , respectively. As it is assumed that the initial option premium is financed at the risk-free rate, the initial portfolio value,  $v_{\text{PORTF},i,t_i-252}$ , is equal to the physical position at time  $t_i - 252$ ,  $\hat{m}_{i,t_i-252,\text{Age}_{i,t_i}}$ .

As a reference, the annualized log risk-free return from time  $t_i - 252$  to  $t_i$ ,  $r_{\text{RF},i,t_i,\text{ann.}}$ , is computed by taking the log of the average daily USD LIBOR 12-month rate from  $t_i - 252$  to  $t_i$ .

The peculiarity of this empirical study is that transaction prices of one and the same vessel cannot be observed in a regular frequency like other typical financial or asset price time series. Accordingly, only one return observation per vessel of the physical position as well as the portfolio from time  $t_i - 252$  to  $t_i$  is observed. In order to draw inferences on the hedge effectiveness of different, competing hedging strategies, an aggregated perspective across vessels is taken for the purpose of this study. Aggregating across vessels yields the following mean log returns and variances for the physical

position as well as the portfolio as shown in equations (3.19) to (3.23) below:

$$\bar{r}_{\text{PHYPOS}} = \frac{1}{i} \sum_{j=1}^i r_{\text{PHYPOS},j,t_j,\text{ann.}} \quad (3.19)$$

$$\bar{r}_{\text{PORTF}} = \frac{1}{i} \sum_{j=1}^i r_{\text{PORTF},j,t_j,\text{ann.}} \quad (3.20)$$

$$\bar{r}_{\text{RF}} = \frac{1}{i} \sum_{j=1}^i r_{\text{RF},j,t_j,\text{ann.}} \quad (3.21)$$

$$\sigma_{\text{PHYPOS}}^2 = \frac{1}{i-1} \sum_{j=1}^i \left( r_{\text{PHYPOS},j,t_j,\text{ann.}} - \bar{r}_{\text{PHYPOS}} \right)^2 \quad (3.22)$$

$$\sigma_{\text{PORTF}}^2 = \frac{1}{i-1} \sum_{j=1}^i \left( r_{\text{PORTF},j,t_j,\text{ann.}} - \bar{r}_{\text{PORTF}} \right)^2. \quad (3.23)$$

### 3.3.4 Hedge effectiveness measures

Regarding the measurement of the hedge effectiveness, several measures are discussed below and used to assess the hedge effectiveness within this empirical study. These measure largely arise from the following three categories: variance perspective, risk-return perspective, and downside-risk perspective.

#### 3.3.4.1 Variance perspective

The classical variance perspective was developed by Ederington (1979). He suggested to measure the hedge effectiveness of futures hedges by the variance reduction of the hedge vs. the unhedged position's log returns. The measure as defined by Ederington (1979) is shown in equation (3.24) below:

$$HE_{\text{ED}} = \frac{\sigma_s^2 - \sigma_p^2}{\sigma_s^2} = 1 - \frac{\sigma_p^2}{\sigma_s^2}. \quad (3.24)$$

Within equation (3.24) above,  $\sigma_s^2$  and  $\sigma_p^2$  refer to the variance of the spot (unhedged) and portfolio (hedged) log returns, respectively. Ederington (1979) observed spot and portfolio positions over time in order to assess the variance of the positions.

Within the context of this empirical study, the measure needs to be slightly adjusted

and equation (3.24) is changed to equation (3.25) as shown below:

$$HE_{ED} = 1 - \frac{\sigma_{\text{PORTF}}^2}{\sigma_{\text{PHYPOS}}^2}. \quad (3.25)$$

As this measure does only take into account the reduction in variance, it is not adequately suitable for measuring the effectiveness of hedging strategies involving options because of the asymmetric payoff structure of options. Nevertheless, the measure is computed within this empirical study for comparison purposes.

### 3.3.4.2 Risk-return perspective

Several alternatives combining the risk as well as return perspective have been proposed within the academic literature. These measures are mainly based on the idea of the Sharpe ratio (Sharpe, 1966). The revised measure by Howard and D'Antonio (1987) was chosen as a representative for the risk-return perspective as it addresses some flaws identified in the original measure by Howard and D'Antonio (1984). The revised measure assesses the increase in the Sharpe ratio from unhedged to hedged portfolio which is shown in equation (3.26) below:

$$HE_{\text{HDA revised}} = \frac{r_p - r_{rf}}{\sigma_p} - \frac{r_s - r_{rf}}{\sigma_s} = \theta_p - \theta_s. \quad (3.26)$$

Within equation (3.26) above,  $r_s$  and  $r_p$  refer to the log returns of the spot (unhedged) and portfolio (hedged) position, respectively. The risk-free rate is denoted by  $r_{rf}$  and  $\sigma_s$  and  $\sigma_p$  refer to the standard deviation of the spot and portfolio log returns, respectively.  $\theta_s$  and  $\theta_p$  denote the Sharpe ratio of the spot and portfolio position, respectively.

Applied to the context of this empirical study, the measure also needs to be adjusted slightly to the following form as shown in equation (3.27) below:

$$HE_{\text{HDA revised}} = \frac{\bar{r}_{\text{PORTF}} - \bar{r}_{\text{RF}}}{\sigma_{\text{PORTF}}} - \frac{\bar{r}_{\text{PHYPOS}} - \bar{r}_{\text{RF}}}{\sigma_{\text{PHYPOS}}}. \quad (3.27)$$

Accordingly, the measure provides a risk-adjusted perspective on the generated returns. With the asymmetric payoff structure of option-based hedging strategies, however, this measure may also not be adequately suitable for measuring the hedge

effectiveness of the strategies considered within this study.

### 3.3.4.3 Downside-risk perspective

Turning to the downside-risk perspective, another approach to assess the hedge effectiveness of instruments with asymmetric payoff structures, such as options, is the concept of the LPM. The concept has been initially brought to portfolio theory by Bawa (1975), Bawa and Lindenberg (1977), Fishburn (1977), and Bawa (1978). The LPM only considers deviations that are below a certain boundary, threshold, or target,  $c$ . The LPM of order  $m$  is defined for a continuous random variable,  $X$ , with  $f(x)$  being the probability density function (PDF) of  $X$  as shown in equation (3.28) below:

$$LPM_m(c, X) = \mathbb{E}(\max(c - X, 0)^m) = \int_{-\infty}^c (c - x)^m \cdot f(x) dx. \quad (3.28)$$

The LPM is a family of risk measures specified by the boundary, threshold, or target,  $c$ , and the order of the moment,  $m$ . The boundary, threshold, or target is often set to the risk-free rate, the inflation rate, or simply to zero. By choosing the order of the moment an investor can adjust the measure to suit his risk aversion level. Intuitively, large values of  $m$  penalize larger deviations from the boundary or target more than smaller deviations.

Applying equation (3.28) to the context of this study would require to estimate empirical return distributions from the return outcomes of the physical position as well as portfolio. Making distributional assumptions might be a source of error for the results and corresponding interpretations and implications. As a consequence, sample lower partial moments across all vessels from 1 to  $i$  of the following general form are rather considered within the study as shown in equation (3.29) below:

$$LPM_m(\bar{r}_{RF}, X) = \mathbb{E}(\max(\bar{r}_{RF} - X, 0)^m) = \frac{1}{i} \sum_1^i (\max(0, \bar{r}_{RF} - x_i))^m. \quad (3.29)$$

Within equation (3.29), the mean risk-free rate,  $\bar{r}_{RF}$ , has been selected as the boundary, threshold, or target similar as in the  $HE_{HDA}$  revised-measure. Accordingly, the physical and portfolio position LPM of order  $m$  are computed according to the equa-

tions (3.30) and (3.31) as shown below:

$$LPM_m(\bar{r}_{RF}, r_{PHYPOS,i,t_i,ann.}) = \frac{1}{i} \sum_1^i (\max(\bar{r}_{RF} - r_{PHYPOS,i,t_i,ann.}, 0))^m \quad (3.30)$$

$$LPM_m(\bar{r}_{RF}, r_{PORTF,i,t_i,ann.}) = \frac{1}{i} \sum_1^i (\max(\bar{r}_{RF} - r_{PORTF,i,t_i,ann.}, 0))^m. \quad (3.31)$$

The hedge effectiveness,  $HE_{LPM_m}$ , is then measured as the reduction of the  $m^{\text{th}}$ -order LPM from the physical (unhedged) to the portfolio (hedged) position. The computation of the measure is shown in equation (3.32) below:

$$HE_{LPM_m} = 1 - \frac{LPM_m(\bar{r}_{RF}, r_{PORTF,i,t_i,ann.})}{LPM_m(\bar{r}_{RF}, r_{PHYPOS,i,t_i,ann.})}. \quad (3.32)$$

With respect to the order,  $m$ , of the LPM-based hedge effectiveness measure, values of 2, 3, and 4 have been selected for  $m$  in order to reflect moderately risk-averse shipping companies (i.e.,  $m = 2$ ) as well as strongly risk-averse shipping companies (i.e.,  $m = 3$  and  $m = 4$ ). The latter two measures punish larger negative deviations from the target return level (i.e., the risk-free rate) more than smaller negative deviations.

A combined perspective on downside risk and return was provided by Sortino and Price (1994). They suggested to look at the excess return above the minimum acceptable return (MAR) over the downside deviation (i.e., the deviations below the MAR), which is also known as Sortino ratio. With respect to the context of this empirical study, the MAR is set to the risk-free rate and the ratio is computed for the physical and portfolio position according to equations (3.33) and (3.34) as shown below:

$$SR_{PHYPOS} = \frac{\bar{r}_{PHYPOS} - \bar{r}_{RF}}{\sqrt{LPM_2(\bar{r}_{RF}, r_{PHYPOS,i,t_i,ann.})}} \quad (3.33)$$

$$SR_{PORTF} = \frac{\bar{r}_{PORTF} - \bar{r}_{RF}}{\sqrt{LPM_2(\bar{r}_{RF}, r_{PORTF,i,t_i,ann.})}}. \quad (3.34)$$

The Sortino ratio is also used to derive a hedge effectiveness measure. Similarly to the  $HE_{HDA \text{ revised}}$ -measure, the Sortino measure for hedge effectiveness proposed within this paper looks at the simple increase in the Sortino ratio from physical (unhedged) to portfolio (hedged) position. The measure is computed according to equation (3.35)

as shown below:

$$HE_{SR} = SR_{\text{PORTF}} - SR_{\text{PHYPOS}}. \quad (3.35)$$

The set of different hedge effectiveness measures introduced above allows to assess the performance of the different hedging strategies tested within this empirical study from different angles and different investors' or shipping companies' risk aversion levels.

### 3.3.5 Hedging strategies

The different, competing hedging strategies tested within this empirical study are explained in more detail in the following paragraphs. The tested hedging strategies are the five ones below:

- A) short FFA strategy (as reference case),
- B) long at-the-money put option strategy,
- C) long 10 % out-of-the-money put option strategy,
- D) replicated short FFA strategy using options, and
- E) zero-cost collar strategy using options.

In order to have a reference strategy for the performance of option-based hedging strategies, strategy A is a simple short FFA cross-hedge using either FFA+1CAL or FFA+2CAL contracts and requires no up-front investment. The short FFA position is intended to offset positive and negative ship price fluctuations.

Strategy B is a simple long at-the-money cross-protective put hedge using options on either FFA+1CAL or FFA+2CAL contracts and requires an up-front investment of the option premium. This strategy limits the downside risk of ship price fluctuations at the cost of the option premium.

Strategy C is very similar to strategy B. However, the long put contracts are bought 10 % out-of-the-money. Accordingly, this reduces the initial investment but allows for some downside variation up to the strike price at the same time.

Strategy D is intended to replicate the payoff of strategy A of either FFA+1CAL or FFA+2CAL contracts by combining a long at-the-money put with a short at-the-

money call. As a call option is usually slightly more expensive than the corresponding put option, the proceedings from this strategy are invested at the risk-free rate (i.e., the USD LIBOR overnight rate on the margin account) to replicate the FFA payoff.

Strategy E is pretty similar to strategy D. Here, however, the put option is bought at-the-money and the call option sold slightly out-of-the-money, such that the initial investment is zero (i.e., the option premium received for the written call exactly covers the amount required to buy the long put). The strategy is also called zero-cost collar. The set-up of the strategy requires to solve the Turnbull and Wakeman (1991) approximation on the individual monthly option level for the strike prices of the individual monthly call options given the prices of the at-the-money monthly put options at the hedge initiation for any subsequent valuation of the short call options. As no closed-form solution exists for this problem, the individual call strike prices are solved numerically using the strike prices of the put option as an initial guess. The price for the basket or strip call option is then the trading day-weighted average of the 12 individual monthly call options.

Consequently, strategies B and C only limit the downside risk, whereas strategies A, D, and E limit both the risk of positive and negative ship price fluctuations. Strategies D and E involve two different hedge instruments simultaneously. Accordingly, these two hedge instruments always have to be aggregated with respect to current position, accumulated gains or losses, and associated interest effects.

In terms of hypotheses on the performance of the different hedging strategies, one would typically expect the two one-sided option strategies B and C to perform worse from a pure variance perspective as they allow for upside variation. Moreover, as options require the initial payment of the premium which leads to an interest effect burden for these one-sided strategies, one would expect these strategies also to perform worse from a risk-return perspective. However, all five strategies are expected to provide equally well downside-risk protection. From a combined perspective of downside risk and return, the benefit of keeping the upside potential should lead to superior results for the one-sided, option-based strategies B and C. For strategy D, results from studies in other financial markets suggest that synthetic futures replicated using options do not outperform the futures themselves, so that strategy A is expected to be superior to strategy D (see, for instance, Chang and Shanker (1986) and Hsin et al. (1994) on currencies or Benet and Luft (1995) on the S&P 500 index). For strategy E, one would expect that the performance is very close to the perfor-

mance of strategy D but might be slightly worse as the payoff of the strategy A is not exactly replicated.

### 3.4 Description of the data

Concerning the data used within this empirical study, the following paragraphs provide detailed descriptions of the different data sources as well as plots and descriptive statistics of the data (i.e., where applicable and adequate). As already stated in section 3.1, the research project focuses on the dry bulk Capesize vessel class. The Capesize vessel class comprises vessels of 100,000 DWT in size and larger. The recent, largest newbuildings in this class have exceeded the 400,000 DWT mark. Given their size, Capesize vessels are almost exclusively engaged in the transport of dry bulk goods that are traded in vast quantities, such as iron ore, coal, and partly grain. Accordingly, the main trading routes of these vessels follow certain trade patterns (i.e., from Southern America to Northern America, and Europe as well as from Australia to Asia) and seasonality is certainly an aspect for agricultural products, such as grain (Alizadeh and Nomikos, 2009; Stopford, 2009). The current reference vessel, as defined by The Baltic Exchange, has a size of 172,000 DWT and serves as underlying for Capesize vessel price estimations by panelists (i.e., BSPA and SIN second-hand ship price assessments).

Regarding the time horizon considered within this empirical study, the available data suggest to concentrate on hedges from January 2008 until December 2014 as freight options in their current version have only been launched in January 2008. Dry bulk Capesize vessel transactions as well as FFA data is already available from January 2005 onwards. This already allows to calibrate the SPMs from January 2005 onwards.

Dry bulk Capesize sale and purchase transaction data were obtained from Clarkson Research World Fleet Register (WFR) for the time period ranging from January 13<sup>th</sup>, 2005 until December 2<sup>nd</sup>, 2014. This data set includes 277 vessel transactions and contains the following information on the individual sales transactions: transaction date, transaction price, vessel name, age at transaction, DWT, speed, consumption, and International Maritime Organization (IMO) number as unique ship identifier. Transactions with negative age at the time of the sale were excluded from the analysis<sup>9</sup>

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<sup>9</sup> The data set contained several vessels that were sold before the ship building was completed,

just as vessels that were sold as part of an ‘en bloc’ transaction.<sup>10</sup>

TABLE 3.1: Descriptive statistics of second-hand Capesize vessel transactions 2005-2014

Year	#	Price in USD million			Age in years			DWT in metric tonnes		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
2005	26	39.80	12.50	80.30	14.37	0.70	24.27	162,783	129,237	243,850
2006	32	42.12	11.00	81.00	12.88	0.32	23.99	171,548	141,014	275,616
2007	51	54.17	9.00	152.00	15.91	0.05	25.82	177,112	105,496	285,933
2008	18	51.80	18.00	130.00	16.71	2.85	25.56	172,453	100,314	290,160
2009	37	28.83	3.65	62.00	14.73	2.69	25.02	176,790	128,826	284,480
2010	20	29.02	10.50	84.90	16.05	0.41	25.35	174,066	148,982	275,616
2011	19	24.31	12.50	58.00	13.94	0.53	27.75	165,375	148,535	180,265
2012	25	14.48	6.65	38.00	15.40	0.32	22.14	176,146	147,048	322,457
2013	29	21.26	7.50	52.00	12.71	0.75	20.12	179,518	149,210	280,537
2014	20	32.75	8.30	55.00	9.85	0.02	22.53	173,912	150,966	184,744
Total	277	35.45	3.65	152.00	14.37	0.02	27.75	173,687	100,314	322,457

The table shows selected descriptive statistics of the remaining second-hand Capesize vessel sales for the time period ranging from January 13<sup>th</sup>, 2005 to December 2<sup>nd</sup>, 2014 in the data set.

TABLE 3.1 shows selected descriptive statistics of the second-hand transactions in the considered data set. The mean sales price was USD 35.45 million, the mean vessel age at transaction 14.37 years, and the mean vessel size in DWT 173,687 metric tonnes. The transaction prices ranged from a minimum of USD 3.65 million in 2009 to a maximum of USD 152.00 million in 2008. The age of the vessels sold ranged from 0.02 years in 2014 for the youngest vessel to 27.75 years in 2011 for the oldest vessel. The heterogeneity in terms of vessel size can be seen by means of the DWT statistics. The smallest vessel sold had a size of 100,314 DWT, whereas the largest vessel sold had a size of 322,457 DWT. This is more than three times the size of the smallest vessel.

Additionally, daily dry bulk Capesize FFA price quote time series data were collected from The Baltic Exchange for the time period ranging from January 4<sup>th</sup>, 2005 until December 31<sup>st</sup>, 2014. The data set includes daily dry bulk Capesize FFA price quotes for all available maturities up to FFA+7CAL contracts<sup>11</sup> on the average of the four

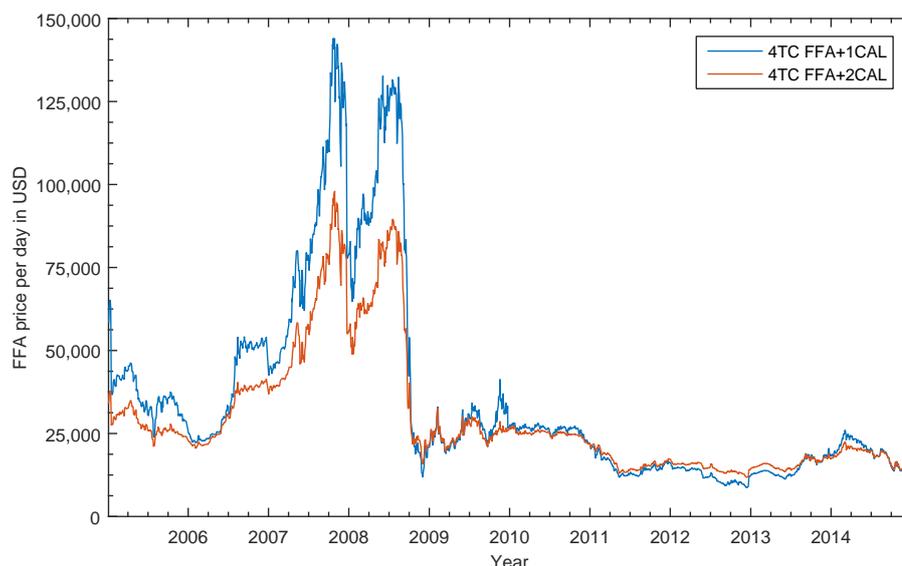
especially in the years 2006 and 2007. These transactions were considered incomparable to classical second-hand transactions and thus, were excluded from the analysis.

<sup>10</sup> ‘En bloc’ sales are transactions in which two or more ships are sold for a consolidated price. Unfortunately, breakdowns of the consolidated prices or allocations to individual vessels are not available.

<sup>11</sup> This means dry bulk Capesize 4TC FFA contracts with the following maturities: CURMON, CURQ, +1MON, +2MON, +3MON, +1Q, +2Q, +3Q, +4Q, +1CAL, +2CAL, +3CAL, +4CAL, +5CAL, +6CAL, and +7CAL.

mostly-used trip charter routes (i.e., 4TC contracts). The granularity of FFA price quotes is required in order to accurately price calendar-year basket or strip freight options.

FIGURE 3.1: Dry bulk Capesize FFA price time series 2005-2014



The graph shows daily dry bulk Capesize 4TC FFA+1CAL and FFA+2CAL price time series for the time period ranging from January 4<sup>th</sup>, 2005 to December 31<sup>st</sup>, 2014.

Source: own graph based on daily data from The Baltic Exchange

FIGURE 3.1 shows a plot of the dry bulk Capesize FFA+1CAL and FFA+2CAL time series. The plot clearly depicts the rising freight rates and FFA prices from 2006 until mid-2008, the prominent decline in late 2008 with the outburst of the financial crisis, and the subsequent recession that the shipping industry has been facing ever since. During the boom phase, the FFA+1CAL rate peaked at almost USD 145,000 per day in late 2007 and the spread between FFA+1CAL and FFA+2CAL contracts considerably widened reflecting a market in backwardation. During the ongoing recession, the FFA rates fluctuated around USD 25,000 per day from 2009 to 2011 and the price levels even fell after that.

TABLE 3.2 shows the descriptive statistics for daily FFA+1CAL and FFA+2CAL time series. Within the time frame considered, the mean prices for the FFA+1CAL and FFA+2CAL contracts were USD 35,354 and USD 29,189, respectively. Both time series are characterized considerable negative annualized log returns of -15.18 % and -9.00 %, respectively, as well as considerable volatility expressed as annualized standard deviation of the daily log returns of 50.33 % and 35.28 %, respectively. Furthermore, the two time series exhibit considerably left-skewness, excess kurtosis,

TABLE 3.2: Descriptive statistics for dry bulk Capesize FFA price time series 2005-2014

Statistic	Type	4TC FFAs	
		+1CAL	+2CAL
Mean (in USD)	Level data	35,353.607	29,188.647
Mean (in %)	Log returns	-15.175	-8.999
Standard deviation (in %)	Log returns	50.331	35.280
Skewness	Log returns	-3.079	-2.692
Kurtosis	Log returns	55.927	45.717
Jarque-Bera	Log returns	297,995.835***	194,565.451***
ADF	Level	-2.201	-2.099
ADF	Log returns	-40.071***	-37.560***
PP	Level	-1.718	-1.454
PP	Log returns	-40.720***	-37.932***

The table shows descriptive statistics for daily Capesize 4TC FFA+1CAL and FFA+2CAL price time series for the time period ranging from January 4<sup>th</sup>, 2005 to December 31<sup>st</sup>, 2014. This leaves 2,520 daily level observations and 2,519 log return observations. The mean is given for level data and log returns and the mean and standard deviation of the log returns are annualized based on an average of 252 trading days in the considered time frame. The remaining statistics are based on log returns. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis. The Jarque and Bera (1980) test statistic for normality is  $\chi^2(2)$  distributed with critical values of 4.60, 5.99, and 9.21 at the 10 %, 5 %, and 1 % level, respectively. ADF refers to the Augmented Dickey-Fuller-test developed by Dickey and Fuller (1981) and PP refers to the Phillips-Perron-test developed by Phillips and Perron (1988). The lag length was chosen by minimizing the Schwarz-Bayes Information Criterion (SBIC) criterion. The 10 %, 5 %, and 1 % critical values for the ADF- and PP-tests are -2.570, -2.867, and -3.443, respectively.

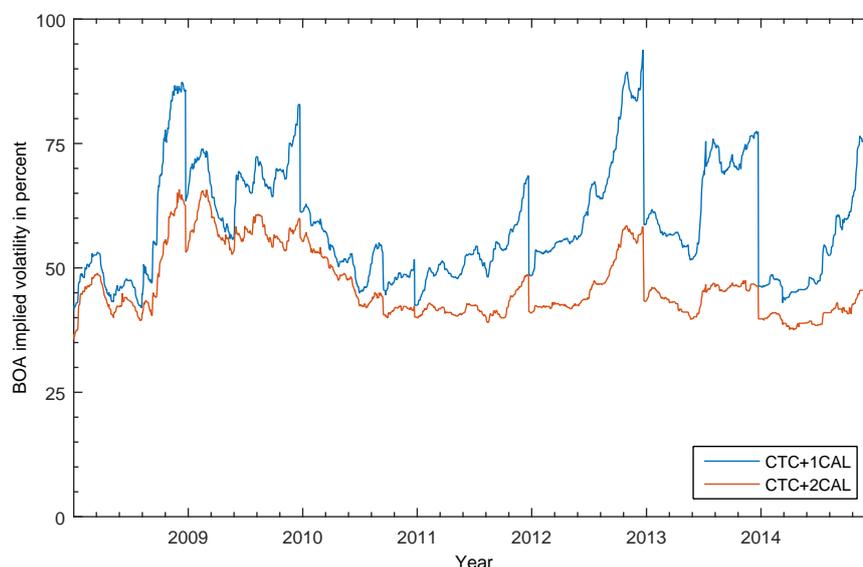
and are significantly different from a normal distribution as the Jarque and Bera (1980) test statistic for normality indicates. Unit root tests (i.e., ADF and Phillips-Perron tests) of the time series indicate that the level time series are non-stationary, whereas the log return time series are stationary.

Moreover, daily dry bulk Capesize BOA time series data in the form of at-the-money implied volatilities were obtained from The Baltic Exchange for the time period ranging from January 2<sup>nd</sup>, 2008 until December 31<sup>st</sup>, 2014. Freight options on 4TC FFAs are called CTC option contracts. As for the FFA price quote time series, the data set also includes similar maturities as for FFAs but only up to CTC+4CAL contracts.<sup>12</sup> As already mentioned in subsection 3.3.1, these BOA quotes reflect implied volatility assessments for at-the-money options. As the low liquidity within the freight option market does not allow for accurate volatility smiles, the same implied volatilities are used to compute prices for in-the-money and out-of-the-money options.

FIGURE 3.2 shows a plot of CTC+1CAL and CTC+2CAL BOA implied volatilities.

<sup>12</sup> Further out maturities have not yet been launched for dry bulk Capesize freight option contracts.

FIGURE 3.2: Dry bulk Capesize freight option BOA implied volatilities 2005-2014



The graph shows daily dry bulk Capesize BOA implied volatilities for CTC+1CAL and CTC+2CAL freight options for the time period ranging from January 4<sup>th</sup>, 2005 to December 31<sup>st</sup>, 2014.

*Source:* own graph based on daily data from The Baltic Exchange

The plot indicates that shorter maturity options exhibit higher implied volatility figures as expected. Moreover, implied volatilities seem to rise throughout the year as the time to maturity of the option decreases. At the rollover dates at the end of the year, setbacks of the implied volatility levels can be seen reflecting the increased time to maturity when the calendar-year options refer to the respective subsequent calendar year. As already indicated in subsection 3.3.1, the implied volatility figures plotted here are not used to price entire calendar-year freight options. The calendar-year freight options are basket or strip options consisting of 12 individual monthly options which are individually priced using the most granular implied volatility figure available.

Besides, daily USD LIBOR 12-month and overnight rates were collected from Datastream for the time period ranging from January 3<sup>rd</sup>, 2005 to December 31<sup>st</sup>, 2014. These interest rates are used to determine appropriate tailing factors for the hedges as well as for the margining to determine the interest effect in the empirical analysis.

## 3.5 Estimation results

Within this section, the results of the empirical analyses are presented and interpreted. Firstly, the model estimations for the SPMs that serve as basis for the subsequent hedging strategies are presented and discussed. Secondly, the performance of the five competing hedging strategies is presented and examined. Finally, several robustness checks are performed in order to ensure the reliability of the presented results as well as the drawn implications.

### 3.5.1 Structural pricing model

With respect to the SPMs, several combinations of explanatory variables have been tested. However, the two model forms as presented in equations (3.7) and (3.8) in subsection 3.3.2 yielded the best trade-off between explanatory power and suitability for the subsequent hedging purposes. Including also spot freight market information or non-linear FFA terms, for instance, would have improved the explanatory power of the models. However, trading or replicating these positions for hedging purposes is hardly possible and thus, not constructive for the intended study at hand.

Before estimating the coefficients or parameters of the two models, the FFA price quotes corresponding to the respective vessel sales date have been first transformed into USD million per day in order to match the unit of the vessel price quotes.

TABLE 3.3 shows the model estimations for the SPM 1 and SPM 2 of equations (3.7) and (3.8). The estimation results indicate that the coefficients of the explanatory variables are mostly significant at the 1 % level. Only the  $\beta_{DWT}$ -coefficient is not significant at the 10 % level in the SPM 1. However, the coefficient is significant at the 5 % level in the SPM 2. The  $\beta_{Consum}$ -coefficient is significant only at the 5 % level in the SPM 1. The coefficient signs are generally in line with economically intuitive expectations of the influence of the explanatory variable on the ship price. Only the positive sign estimated for the  $\beta_{Consum}$ -coefficient is economically not intuitive at first glance as fuel inefficient vessels should be less worth than fuel efficient vessels. However, the effect is offset on an aggregated level together with the negative sign of the rather large  $\beta_{f.Consum}$ -coefficient of the interaction term between FFA and consumption. For the respective means of the FFA+1CAL and FFA+2CAL price

TABLE 3.3: Estimates for different Capesize structural pricing models

	$p_{i,t} = \alpha + \beta_{f,1CAL} \cdot f_{1CAL,t} + \beta_{Age} \cdot Age_{i,t} + \beta_{DWT} \cdot DWT_i + \beta_{Consum} \cdot Consum_i$					
SPM 1:	$+ \beta_{f_{1CAL} \cdot Age} \cdot f_{1CAL,t} \cdot Age_{i,t} + \beta_{f_{1CAL} \cdot DWT} \cdot f_{1CAL,t} \cdot DWT_i$ $+ \beta_{f_{1CAL} \cdot Consum} \cdot f_{1CAL,t} \cdot Consum_i + \varepsilon_{i,t}$					
	$p_{i,t} = \alpha + \beta_{f,2CAL} \cdot f_{2CAL,t} + \beta_{Age} \cdot Age_{i,t} + \beta_{DWT} \cdot DWT_i + \beta_{Consum} \cdot Consum_i$					
SPM 2:	$+ \beta_{f_{2CAL} \cdot Age} \cdot f_{2CAL,t} \cdot Age_{i,t} + \beta_{f_{2CAL} \cdot DWT} \cdot f_{2CAL,t} \cdot DWT_i$ $+ \beta_{f_{2CAL} \cdot Consum} \cdot f_{2CAL,t} \cdot Consum_i + \varepsilon_{i,t}$					
	SPM 1 (FFA+1CAL)			SPM 2 (FFA+2CAL)		
	Coefficient	t-stat.	p-value	Coefficient	t-stat.	p-value
$\alpha$	13.4143 (14.6408)	0.9162	0.3604	-11.4413 (17.0311)	-0.6718	0.5023
$\beta_f$	1,862.0948*** (245.1389)	7.5961	0.0000	3,120.4766*** (397.4180)	7.8519	0.0000
$\beta_{Age}$	-1.8420*** (0.1905)	-9.6673	0.0000	-1.7095*** (0.2239)	-7.6353	0.0000
$\beta_{DWT}$	$5.925 \cdot 10^{-05}$ ( $4.002 \cdot 10^{-05}$ )	1.4802	0.1400	$9.547 \cdot 10^{-05}$ ** ( $4.644 \cdot 10^{-05}$ )	2.0557	0.0408
$\beta_{Consum}$	20.2928** (10.1484)	1.9996	0.0465	31.2894*** (11.8145)	2.6484	0.0086
$\beta_{f \cdot Age}$	-11.2887*** (3.7315)	-3.0253	0.0027	-18.7948*** (5.9147)	-3.1776	0.0017
$\beta_{f \cdot DWT}$	-0.0022*** (0.0007)	-3.2090	0.0015	-0.0040*** (0.0011)	-3.6225	0.0003
$\beta_{f \cdot Consum}$	-871.5250*** (163.8518)	-5.3190	0.0000	-1,432.5039*** (266.6249)	-5.3727	0.0000
$R^2$	0.7341			0.7425		
Adj. $R^2$	0.7272			0.7358		
Log likelihood	-1,085.3001			-1,080.8666		
SBIC criterion	7.9985			7.9665		
Akaike criterion	7.8938			7.8619		
Standard error	12.3514			12.1552		

The table shows linear regression coefficient estimates for two different SPMs based on data for 277 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to December 2<sup>nd</sup>, 2014.  $p_{i,t}$  refers to the price of vessel  $i$  at time  $t$  in USD million,  $f_{1CAL}$  to the price of a FFA+1CAL contract at time  $t$  in USD million,  $f_{2CAL}$  to the price of a FFA+2CAL contract at time  $t$  in USD million,  $Age_{i,t}$  to the age of vessel  $i$  at time  $t$  in years,  $DWT_i$  to the size of vessel  $i$  in DWT, and  $Consum_i$  to the consumption per 1,000 nautical miles per 1,000 DWT of vessel  $i$  in metric tonnes. Figures in () reflect the corresponding standard errors.

\* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

time series between 2005 and 2014, the mean aggregated influence of an increase of one unit of *Consum* is USD -10.5592 million for the SPM 1 and USD -10.5397 million for the SPM 2. This is again in line with the economic expectation of fuel inefficient vessels being worth less than fuel efficient vessels.

The coefficient estimates for  $\beta_{f,1CAL}$  and  $\beta_{f,2CAL}$  indicate that a base level exposure of 1,862.1 or 3,120.5 days is desired for the SPM 1 or SPM 2, respectively. The aggregated desired exposure level is, of course, corrected downwards by the negative coefficients of the included interaction terms (i.e.,  $\beta_{f,Age}$ ,  $\beta_{f,DWT}$ , and  $\beta_{f,Consum}$ ) and the corresponding Age, DWT, and consumption values of vessel  $i$ .

Overall, the two models show an explanatory power in form of adjusted  $R^2$  values of 72.72 % and 73.58 % for SPM 1 and SPM 2, respectively. The Akaike criterion, the SBIC criterion, and the log likelihood tend to also slightly prefer the SPM 2. Both models will be applied in the subsequent hedging efforts as basis to determine physical positions as well as to determine the desired exposure to FFAs and freight options as hedge instruments.

### 3.5.2 Hedging results

The five different hedging strategies described in subsection 3.3.5 (i.e., from A to E) are empirically tested in the hedge set-up described in subsection 3.3.3 using the two estimated SPMs. The hedge effectiveness has been measured according to the methodology described in subsection 3.3.4. The following paragraphs elaborate on the results of these empirical analyses.

Dry bulk Capesize freight options were launched and quoted by The Baltic Exchange on January 2<sup>nd</sup>, 2008. As the initiation of the individual hedges in the hedging set-up considered is one year or 252 trading days prior to the transaction date,  $t_i$ , only sale and purchase transactions that took place from January 2<sup>nd</sup>, 2009 onwards were considered in the empirical analysis of hedge effectiveness. Accordingly, 127 transactions had to be eliminated from the initial sample of 277 sale and purchase transactions in the data set. However, these transactions were used in the estimation of the SPMs as the underlying of the freight options, FFAs, were already launched on January 4<sup>th</sup>, 2005. This left 150 transactions for which the hedge effectiveness of the different hedging strategies over a fixed time horizon of one year is examined.

TABLE 3.4: Hedging results over fixed time horizon of one year (1/2)

			SPM 1				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	36.3449		-		
End	USD m	Mean	24.9513		-		
Change in values	USD m	Mean	-11.3936		-		
		Median	-6.6571		-		
		Std. dev.	18.5330		-		
		Skewness	-1.2586		-		
		Kurtosis	4.2808		-		
	Log return in %	Mean	-31.6528		1.2894		
		Median	-27.8532		0.9774		
		Std. dev.	65.9781		-		
		Skewness	1.9390		-		
		Kurtosis	18.8491		-		
			SPM 1				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	8.2981	6.9183	6.5341	8.6442	8.6477
		Median	1.3180	0.7850	0.6954	1.4926	1.4929
		Std. dev.	18.5292	14.9708	14.0682	18.3994	18.4104
		Skewness	1.6846	1.8739	1.8901	1.6898	1.6911
		Kurtosis	4.6613	5.2970	5.3495	4.6574	4.6633
thereof: interest		Mean	0.0263	-0.0012	0.0045	0.0233	0.0268
Portfolio (PHYPOS+HEDGE)							
Start		Mean	36.3449	36.3449	36.3449	36.3449	36.3449
End		Mean	33.2495	31.8697	31.4855	33.5955	33.5990
Init. opt. prem.	USD m	Mean	0.0000	-3.9188	-2.8499	0.0000	0.0000
Transaction costs		Mean	-1.1539	-0.9160	-0.7490	-1.3939	-1.3939
Change in values	USD m	Mean	-3.0954	-4.4752	-4.8595	-2.7494	-2.7459
		Median	-3.6538	-5.2552	-5.5852	-3.3134	-3.3127
		Std. dev.	6.1646	6.8556	7.2779	6.0969	6.0992
		Skewness	1.3977	1.1246	0.9472	1.3613	1.3602
		Kurtosis	8.3480	7.6525	6.9570	8.4347	8.4194
	Log return in %	Mean	-12.4228	-12.8767	-13.4122	-10.4835	-10.4803
		Median	-11.4680	-19.8232	-19.7812	-10.5282	-10.5246
		Std. dev.	45.7878	46.3742	46.7910	44.3388	44.3381
		Skewness	5.5805	6.1152	6.0486	6.3006	6.3004
		Kurtosis	54.4037	52.2319	51.0203	59.8193	59.8211

The table shows selected descriptive statistics of the SPM 1 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 150 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLE 3.5: Hedging results over fixed time horizon of one year (2/2)

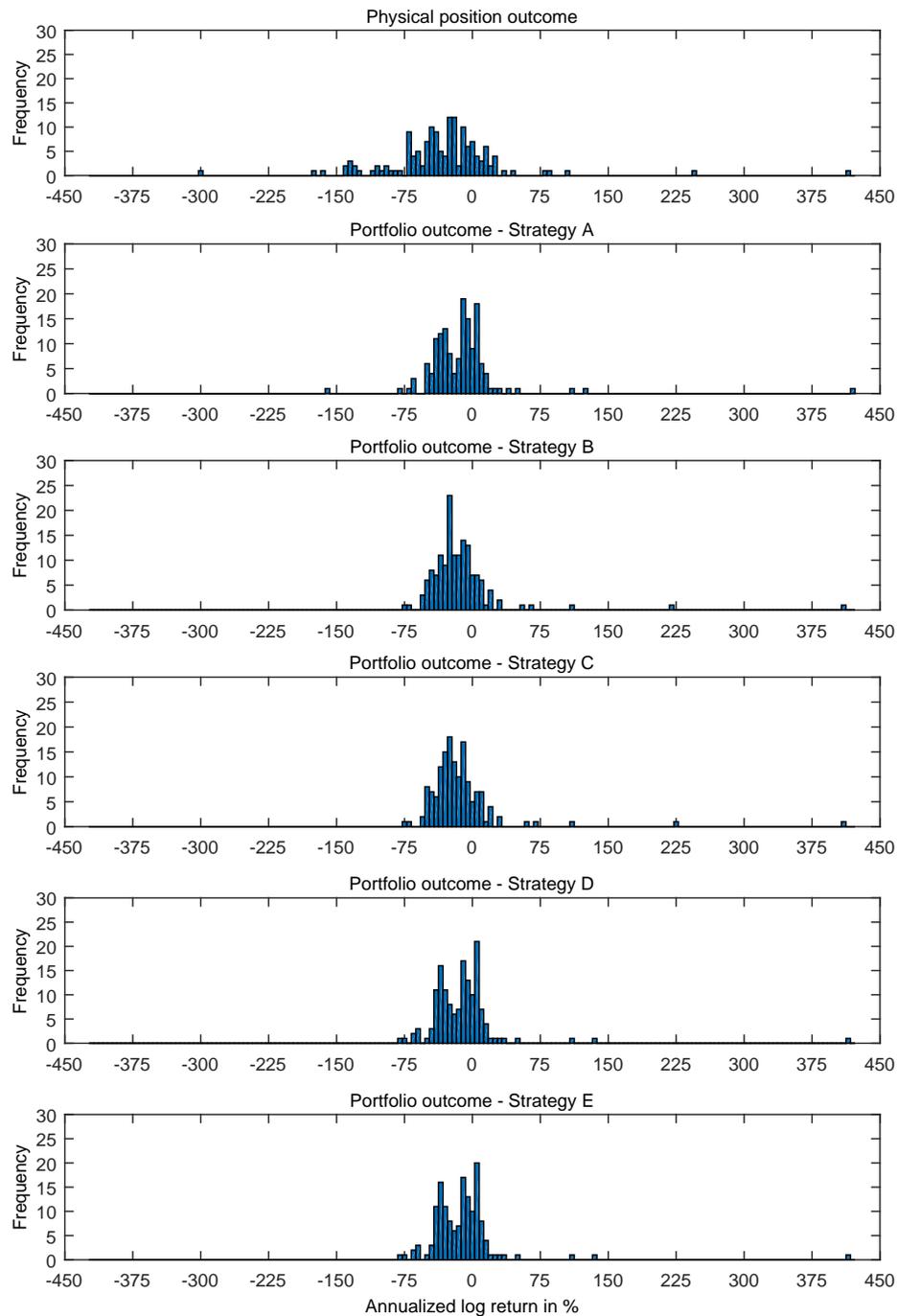
			SPM 2				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	36.2745		-		
End	USD m	Mean	24.9513		-		
Change in values	USD m	Mean	-11.3232		-		
		Median	-6.2950		-		
		Std. dev.	18.4552		-		
		Skewness	-1.2711		-		
		Kurtosis	4.2796		-		
	Log return in %	Mean	-31.7173		1.2894		
		Median	-30.4117		0.9774		
		Std. dev.	61.2647		-		
		Skewness	1.1682		-		
		Kurtosis	14.8195		-		
			SPM 2				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	8.4245	5.3908	5.0092	8.1381	8.1442
		Median	1.3923	0.6200	0.5540	1.5471	1.5481
		Std. dev.	17.3606	12.0706	11.1981	16.3302	16.3457
		Skewness	1.7157	1.8869	1.9050	1.7144	1.7160
		Kurtosis	4.8777	5.4985	5.5648	4.8813	4.8883
		thereof: interest	Mean	0.0371	-0.0183	-0.0109	0.0188
Portfolio (PHYPOS+HEDGE)							
Start		Mean	36.2745	36.2745	36.2745	36.2745	36.2745
End	USD m	Mean	33.3758	30.3422	29.9605	33.0895	33.0955
Init. opt. prem.		Mean	0.0000	-6.1242	-4.6836	0.0000	0.0000
Transaction costs		Mean	-1.2902	-0.8267	-0.6709	-1.3766	-1.3766
Change in values		USD m	Mean	-2.8987	-5.9323	-6.3140	-3.1850
	Median		-3.2356	-6.2549	-6.4248	-3.2911	-3.2905
	Std. dev.		6.2220	8.1659	8.7858	6.2286	6.2273
	Skewness		1.3618	0.3518	0.1333	1.3614	1.3618
	Kurtosis		8.7015	5.6757	5.1134	9.1237	9.1211
	Log return in %	Mean	-10.8451	-16.0939	-16.7525	-10.1067	-10.0983
		Median	-10.7754	-22.5215	-23.6922	-12.2047	-12.2071
		Std. dev.	43.5827	40.7877	41.3425	40.2679	40.2682
		Skewness	3.3827	5.5275	5.3778	5.3383	5.3375
		Kurtosis	40.4720	46.5273	44.6122	48.4148	48.4114

The table shows selected descriptive statistics of the SPM 2 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 150 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLES 3.4 and 3.5 show aggregated mean and median outcome, return, and log return statistics on the individual hedges using SPM 1 and SPM 2, respectively. For the SPM 1, the results indicate a mean loss of USD 11.39 million and a median loss of USD 6.66 million on the vessel or physical position. The losses are left-skewed and show some excess kurtosis. These losses correspond to a low mean log return figure of the physical position of -31.65 %. The log returns of the physical position are right-skewed and show considerable excess kurtosis. The mean reference log return or risk-free rate,  $r_{RF}$ , across all transactions considered was about 1.29 %. The different hedging strategies yield a mean profit of USD 6.53 to 8.65 million for strategy C (long 10 % out-of-the money put) and E (zero-cost collar), respectively. The hedge profits are right-skewed and show excess kurtosis for all strategies. The interest effect is lowest for strategy B (long at-the money put) with around USD 0.00 million and largest for strategy E (zero-cost collar) with about USD 0.03 million. For strategies B (long at-the money put) and C (long out-of-the money put), USD 3.92 million and USD 2.85 million have been paid as initial option premium for the long put options. The transaction costs range from about USD 0.75 million for strategy C (long 10 % out-of-the money put) to about USD 1.15 million for strategy A (short FFA) due to the higher transaction volume of FFAs. From a portfolio perspective, the hedging strategies reduce the mean loss of the physical position to values ranging from USD -4.86 million to USD -2.75 million for strategy C (long 10 % out-of-the money put) and E (zero-cost collar), respectively. The associated mean log return values are -13.41 % and -10.48 %, respectively. The portfolio losses and log returns are right-skewed and show excess kurtosis for all strategies. For the SPM 2, the mean loss of the physical position is slightly smaller with USD 11.32 million. However, the mean log return is slightly lower with -31.72 % resulting from the slightly lower mean model-implied value at the hedging start date. The mean profit for the different hedging strategies is slightly lower for strategies B (long at-the money put), C (long 10 % out-of-the money put), D (replicated short FFA), and E (zero-cost collar) and slightly higher for strategy A (short FFA) compared to the results of SPM 1. The interest effect is again lowest for strategy B (long at-the money put). However, it is highest for strategy A (short FFA) for SPM 2. The transaction costs are slightly higher for strategy A (short FFA) and slightly lower for all other strategies compared to SPM 1. From a portfolio perspective, strategy A (short FFA) performs best with a mean loss of USD 2.90 million and an associated log return of -10.85 %. For the other hedging strategies, the mean losses are slightly higher for SPM 2 compared to SPM 1.

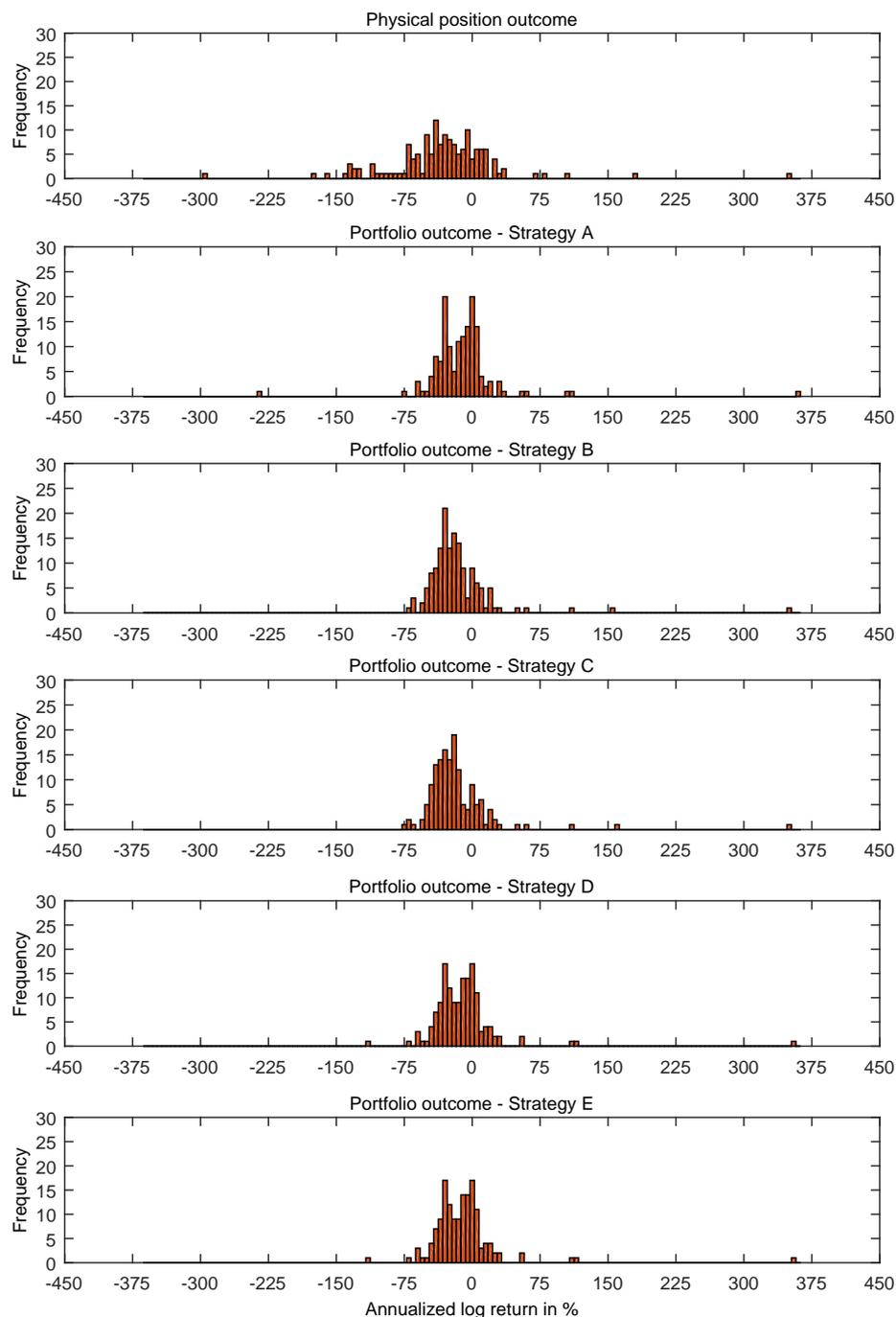
FIGURE 3.3: Histograms of physical position and portfolio outcomes for SPM 1



The graph shows histogram plots of the physical position and portfolio outcomes for SPM 1 as annualized log returns in % for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014. Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

Source: own graph based on hedging results

FIGURE 3.4: Histograms of physical position and portfolio outcomes for SPM 2



The graph shows histogram plots of the physical position and portfolio outcomes for SPM 2 as annualized log returns in % for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014. Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

Source: own graph based on hedging results

FIGURES 3.3 and 3.4 show histogram plots of the physical position log returns as well as of the portfolio log returns for the five different hedging strategies for both SPMs. From visual inspection, the distributions of the portfolio log returns for the five different strategies are clearly narrower than the distribution of the physical position log returns and extremely negative outcomes seem to be mitigated apart from one outlier for strategy A (short FFA) for both SPMs.

TABLE 3.6: Results for different hedge effectiveness measures

Model	Measure	Unit	Hedging strategy				
			A	B	C	D	E
SPM 1	$HE_{ED}$	%	51.84	50.60	49.71	54.84	54.84
	$HE_{HDA}$ revised	pp	19.98	19.38	18.51	23.38	23.38
	$HE_{LPM_2}$	%	75.59	79.54	78.72	81.03	81.03
	$HE_{LPM_3}$	%	87.61	93.66	93.41	93.48	93.48
	$HE_{LPM_4}$	%	93.11	98.39	98.33	98.12	98.12
	$HE_{SR}$	pp	8.57	2.68	1.77	9.77	9.78
SPM 2	$HE_{ED}$	%	49.39	55.68	54.46	56.80	56.80
	$HE_{HDA}$ revised	pp	26.03	11.26	10.24	25.57	25.60
	$HE_{LPM_2}$	%	73.16	76.17	74.60	81.68	81.68
	$HE_{LPM_3}$	%	76.17	92.51	91.81	93.14	93.14
	$HE_{LPM_4}$	%	74.21	98.07	97.83	97.50	97.50
	$HE_{SR}$	pp	15.99	-4.34	-4.66	10.66	10.68

The table shows the results of the different hedge effectiveness measures for the five different hedging strategies applied (i.e., from A to E) to both models SPM 1 and SPM 2 over a fixed time horizon of one year. The included sample size is 150 vessels for all strategies. For  $HE_{ED}$ ,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ , and  $HE_{LPM_4}$ , the result is given in percent. For  $HE_{HDA}$  revised and  $HE_{SR}$ , the result is given in percentage points. Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

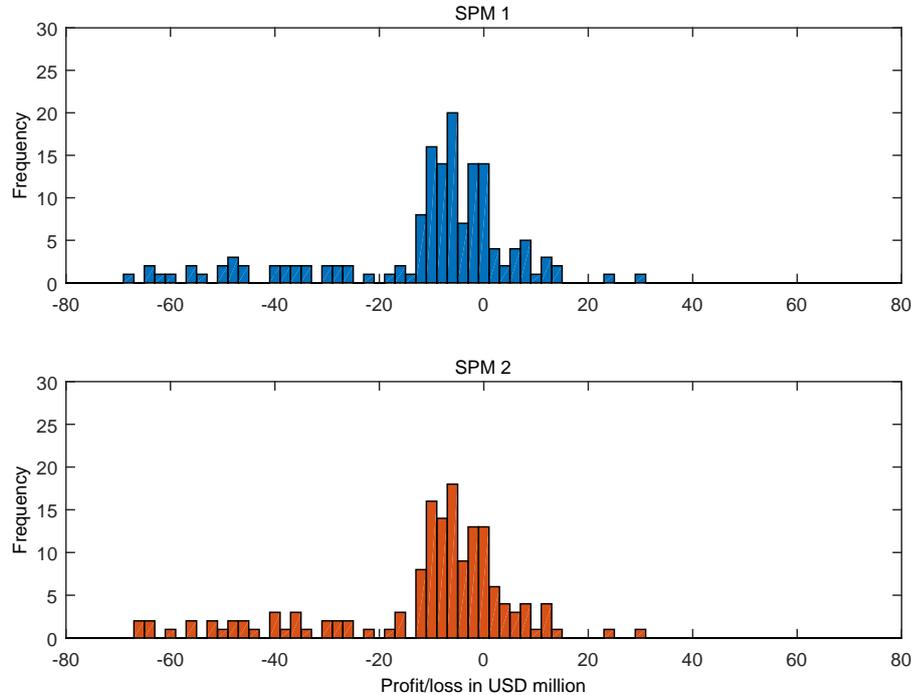
With respect to the hedge effectiveness of the different hedging strategies, TABLE 3.6 shows the results of the different hedge effectiveness measures defined in subsection 3.3.4 for SPM 1 and SPM 2. From a variance perspective, the results for  $HE_{ED}$  suggest that strategies D (replicated short FFA) and E (zero-cost collar) achieve the highest variance reduction with 54.84 % and 56.80 % for SPM 1 and SPM 2, respectively. Interestingly, strategy A (short FFA) achieves the lowest variance reduction for SPM 2. From a risk-return perspective, the results for  $HE_{HDA}$  revised indicate that strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) perform worst for SPM 1 with an increase of only 19.38 pp and 18.51 pp, respectively. Strategies D (replicated short FFA) and E (zero-cost collar) show the best performance with both an increase of 23.38 pp. From a downside-risk perspective, the results for  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ , and  $HE_{LPM_4}$  show again that strategy A (short FFA)

performs worst of the different hedging strategies. For  $HE_{LPM_2}$ , strategies D (replicated short FFA) and E (zero-cost collar) perform best with 81.03 % downside-risk reduction for SPM 1 and 81.68 % downside-risk reduction for SPM 2. For  $HE_{LPM_3}$  and  $HE_{LPM_4}$ , strategy B (long at-the-money put option) performs best for SPM 1. For SPM 2, strategies D (replicated short FFA) and E (zero-cost collar) perform best for  $HE_{LPM_3}$  and strategy B (long at-the-money put option) for  $HE_{LPM_4}$ . From a combined perspective of downside risk and return, strategies D (replicated short FFA) and E (zero-cost collar) perform best for SPM 1 and strategy A (short FFA) for SPM 2. Strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show considerably lower and even negative performance.

Referring to the initial hypotheses on the strategies' performance stated in subsection 3.3.5, the results show that strategies D (replicated FFA) and E (zero-cost collar) provide the best performance in terms of variance reduction. Strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show the lowest performance in this category for SPM 1. This is consistent with the initially established hypotheses. For SPM 2, however, strategy A (short FFA) shows the lowest performance in terms of variance reduction. This somewhat contradicts the initially established hypotheses. Nevertheless, strategies A (short FFA), D (replicated short FFA), and E (zero-cost collar) show superior results from a risk-return perspective as expected. Besides, all five strategies more or less provide equally well downside-risk protection as expected. The one-sided option strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) turn out to offer the highest protection with the higher order LPM moments,  $HE_{LPM_3}$  and  $HE_{LPM_4}$ . From a combined perspective of downside risk and return, however, the benefit of keeping the upside potential does not seem to have materialized in the results. This fact is quite striking and needs some further investigation on the reasons why that is the case later in this section. Concerning the performance of strategies D (replicated FFA) and E (zero-cost collar), the results contradict the initially established hypotheses based on findings from other financial markets. The results indicate that the two strategies outperform the reference strategy A (short FFA) in almost all hedge effectiveness measures. Strategy A (short FFA) only outperforms strategies D (replicated short FFA) and E (zero-cost collar) in the  $HE_{HDA \text{ revised}}$  and  $HE_{SR}$ -measures for the SPM 2. This finding is again quite striking given the fact that these two strategies incur higher transaction costs. Finally, contrary to the initial hypotheses, strategy E (zero-cost collar) seems to be marginally better than strategy D (replicated short FFA) from a

risk-return perspective as well as from a combined perspective of downside risk and return.

FIGURE 3.5: Histograms of physical position outcomes



The graph shows histogram plots of the physical position outcomes for SPM 1 and SPM 2 in USD million for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014.

Source: own graph based on hedging results

These findings might be caused by peculiarities of the data set used within this empirical study. Therefore, the representativeness of the transactions considered in the hedging analysis needs to be reviewed. Only vessels that were sold starting from January 8<sup>th</sup>, 2009 were included in the analysis and thus, the time period of the hedges covers the years from 2008 until 2014. Accordingly, the considered time frame mostly covers the shipping crisis period starting in September 2008. From visual inspection of FIGURE 3.1 on page 110, one can already infer that most of the individual one year hedge time frames are confronted with a falling market and therefore, seek downside-risk protection. Actually, 122 of the 150 transactions considered incurred a loss and only 28 transactions realized a gain on the physical position for SPM 1. For SPM 2, 117 incurred a loss and only 33 realized a gain on the physical position. This can also be visually seen in FIGURE 3.5 and in the negative skewness figures for the physical position outcomes in TABLES 3.4 and 3.5 on pages 116 and 117. So, for a vast majority of the transactions, the benefit of the keeping the upside potential

of strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) cannot materialize. As a consequence, the data set was divided into two groups based on the outcome of the physical position (i.e., one group with observations that incurred a loss or were equal to zero and one group that realized a gain on the physical position). The hedging results were computed again separately for the two groups and the hypothesis is that strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show a considerably increased performance from a risk-return perspective (i.e., the  $HE_{\text{HDA revised}}$ -measure) for the group of positive physical position outcomes. Unfortunately, the hedge effectiveness measures based on downside risk (i.e.,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ ,  $HE_{LPM_4}$ , and  $HE_{\text{SR}}$ ) are not able to provide any meaningful results for this group as the downside risk of the physical position is zero and a computation of the mentioned hedge effectiveness measures is not possible. Nevertheless, the  $HE_{\text{HDA revised}}$ -measure should provide meaningful insights on whether the option-based strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show an increased performance for this group of transactions. TABLE 3.7 shows the results for the  $HE_{\text{HDA revised}}$ -measure for this

TABLE 3.7: Results for  $HE_{\text{HDA revised}}$ -measure for group of positive physical position outcomes

Measure	Model	Unit	Hedging strategy				
			A	B	C	D	E
$HE_{\text{HDA revised}}$	SPM 1	pp	-21.38	-8.30	-6.92	-18.26	-18.27
	SPM 2	pp	-29.93	-11.11	-9.39	-19.71	-19.72

The table shows results for the  $HE_{\text{HDA revised}}$ -measure in percentage points for the group of positive physical position outcomes for both models SPM 1 and SPM 2. The included sample size is 28 transactions for SPM 1 and 33 transactions for SPM 2. Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

group for both SPMs. For the strategies A (short FFA), D (replicated short FFA), and E (zero-cost collar), the results show a decline of the Sharpe ratio. The results, however, indicate that there is a considerably smaller decline in the Sharpe ratio for the strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) from the physical position alone to the portfolio of physical position plus hedge. Of course, not hedging at all would have been the best scenario for this group of vessels. This is consistent with the initial hypothesis that the one-sided, option-based strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) allow to keep the upside potential in case of favorable market circumstances. However, these strategies do not come costless. The interest burden on the initial

option premium cause the decline in the Sharpe ratio. The other strategies eliminate both negative and positive fluctuations resulting in a stronger decrease of the Sharpe ratio when implementing the hedge for the group consisting of only positive physical position outcomes. This feature of the one-sided, option-based strategies seems to dissolve in the aggregated results given the higher number of negative physical position outcomes in the entire data set as the higher initial cost of these two strategies at similar downside-risk protection lead to lower performance from a risk-return perspective as well as from a combined perspective of downside risk and return.

### **3.5.3 Robustness checks**

The results and findings presented are obviously directly linked to the data set of this empirical study and their representativeness strongly depend on the representativeness of the empirical data set. Accordingly, further investigation of the data set as well as the presented results is required in order to ensure the representativeness of both the data set as well as the results and associated findings. Accordingly, two robustness checks on subsets of the initial full sample of data set were performed as well as one robustness check in an alternative hedging set-up. Firstly, vessels that were multiply sold within the data set were excluded. Secondly, vessels that were younger than five years and vessels that were older than 20 years at the transaction date were excluded. Thirdly, an alternative hedging set-up was considered in which the underlying assumption is not that a vessel shall be sold after a hedging time horizon of one year but rather bought after one year and the purchase price shall be hedged over the time horizon of one year prior to the purchase. The results of these robustness checks are presented and discussed in the following subsections.

#### **3.5.3.1 Excluding multiply sold vessels**

Dry bulk Capesize vessels are typically assets of a considerable longevity and thus, are usually also held for a considerable amount of time by shipping companies. As the data set only covers transactions from 2005 until 2014, sale and resale of one and the same vessel within this rather short period of time compared to the economic life of a dry bulk Capesize vessel might indicate that the shipping company bought and subsequently sold the vessel for speculative purposes. In order to correct for

transactions done for speculative purposes, any vessels that were multiply sold within the data set were excluded for this robustness check. Actually, 87 transactions pertain to vessels that were sold more than once in the data set. Accordingly, this left 190 transactions in this subset of the data set and the SPMs were re-estimated using these transactions. The results are shown in TABLE 3.10 in the appendix B.1 on page 139. The estimations for the  $\beta$ -coefficients only marginally changed compared to the initial estimations for both SPMs and especially the coefficient signs remained the same. Only the  $\beta_{Consum}$ -coefficient turned insignificant at the 10 % level for the SPM 1. The  $R^2$  and adjusted  $R^2$  values marginally decreased.

Subsequently, the re-estimated SPMs were used to assess again the hedge effectiveness of the five different hedging strategies. Another 78 transactions had to be eliminated from the considered sample as these transactions occurred prior to January 8<sup>th</sup>, 2009. The corresponding hedging results are shown in TABLES 3.11 and 3.12 in the appendix B.1 on pages 140 and 141. The corresponding results for the different hedge effectiveness measures are shown in TABLE 3.13 in the appendix B.1 on page 142. The results largely show the same pattern as the results presented earlier in subsection 3.5.2. From a variance perspective, the  $HE_{ED}$ -measure suggests the highest performance for strategies D (replicated short FFA) and E (zero-cost collar) for SPM 1 and SPM 2, respectively. From a risk-return perspective, strategies D (replicated short FFA) and E (zero-cost collar) show the highest performance for SPM 1, whereas strategy A (short FFA) performs best for SPM 2. Strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show considerably lower and partly even negative performance. From a downside-risk perspective, all five different strategies achieve largely similar performances. From a combined perspective of downside risk and return, strategies D (replicated short FFA) and E (zero-cost collar) perform again best for SPM 1 and strategy A (short FFA) for SPM 2. Strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) show once more considerably lower and even negative performance. Consequently, these results confirm the initial results on the entire data set and increase the robustness of the findings presented earlier.

### 3.5.3.2 Excluding vessels younger than five and older than 20 years

Second-hand ship price dynamics are largely influenced by the development of the underlying spot freight rates and FFA representing the current and future earnings potential of such vessels. The price dynamics of new and extremely old vessels, however, are not so much driven by spot freight rates or FFA rates. For extremely old vessels, for instance, the share of the scrap value (i.e., the metal price) in the remaining vessel value increases with increasing age while the share of the spot freight rates or FFA rates in the remaining vessel value declines. Consequently, a SPM with FFA rates as explanatory variable might not be able to accurately capture these price dynamics. Furthermore, the data set contains only few transactions of extremely young and extremely old vessels. Accordingly, vessels younger than five years and older than 20 years at the transaction date,  $t_i$ , were excluded from the data set for this robustness check. Actually, 92 transactions pertain to this category. This left 185 transactions in this subset of the data set and the SPMs were re-estimated using these transactions. The results are shown in TABLE 3.14 in the appendix B.2 on page 143. The estimations for the  $\beta$ -coefficients seem to change quite a bit and especially the FFA-related coefficients increased for both SPMs by about 70 %. As the aggregated exposure to the FFA contract needs to be netted across all FFA-related coefficients, the desired exposure does not necessarily increase that much. Nevertheless, an increase of the desired exposure to the FFA contracts is intuitive though because vessels were eliminated from the data set which prices were not necessarily tied to the FFA rate dynamics as mentioned above. Now, the link of the second-hand price for the remaining vessels to FFA rates becomes more apparent. Moreover, the  $\beta_{Consum}$ -coefficient also turned insignificant at the 10 % level for the SPM 1. The  $R^2$  and adjusted  $R^2$  values, however, increased by about 10 pp reflecting the above mentioned dynamics.

Similarly to the first robustness check, the re-estimated SPMs were used to assess again the hedge effectiveness of the five different hedging strategies. Again, another 82 transactions had to be eliminated from the considered sample as these transactions occurred prior to January 8<sup>th</sup>, 2009. The corresponding hedging results are shown in TABLES 3.15 and 3.16 in the appendix B.2 on pages 144 and 145. The corresponding results for the different hedge effectiveness measures are shown in TABLE 3.17 in the appendix B.2 on page 146. Once more, the results largely show the same pattern as the results presented earlier in subsection 3.5.2 and as the results presented in the first robustness check. Only the variance reduction measure,  $HE_{ED}$ , is now lowest

for strategy A (short FFA) and the  $HE_{LPM}$ -measures show also relatively low values for strategy A (short FFA) (with the  $HE_{LPM_A}$ -measure being even negative for the SPM 2). These results are, however, caused by one outlying observation for which the physical position incurs a small profit, the hedge, however, incurs a comparatively large loss. It is refrained from repeating any further detailed descriptions of the results at this point. Consequently, these results further increase the robustness of the findings presented earlier.

### 3.5.3.3 Alternative hedging set-up

As the data set of vessel transactions and the associated aggregated results within the hedging set-up considered so far conceal the benefit of one-sided, option-based hedging strategies in positive market circumstances, the considered hedging set-up was changed in order to show the underlying mechanics using the same full data set. It is rather assumed that a shipping company intends to buy a certain vessel and wants to hedge the ship price over a fixed time horizon of one year prior to the transaction. The transaction price for vessel  $i$  is again unknown to the shipping company at the hedge initiation at time  $t_i - 252$ , but the shipping company is able to estimate a model-implied value of vessel  $i$  at  $t_i - 252$ ,  $\hat{m}_{i,t_i-252,Age_{i,t_i}}$ , using the age of vessel  $i$  at  $t_i$ ,  $Age_{i,t_i}$ . Accordingly, this changes the physical position from a long to a short exposure and the hedging strategies are revised to the following five ones below:

- A) long FFA strategy (as reference case),
- B) long at-the-money call option strategy,
- C) long 10 % out-of-the-money call option strategy,
- D) replicated long FFA strategy using options, and
- E) zero-cost collar strategy using options.

Beyond that, the remaining assumptions undertaken in subsection 3.3.3 also apply in this alternative hedging set-up. The changes in the physical position, hedge position, and portfolio as well as the corresponding log returns are computed reflecting the physical exposure change as well as the change in the hedging strategies.

With respect to the SPM, the initial estimations based on the full data set of 277 transactions are still valid despite the change in the hedging perspective. Accordingly, the hedging results were computed for the 150 transactions starting from January 8<sup>th</sup>,

2009 onwards and are shown in the TABLES 3.18 and 3.19 in the appendix B.3 on pages 147 and 148. The corresponding results for the different hedge effectiveness measures are shown in TABLE 3.20 in the appendix B.3 on page 149. The results clearly show that strategies B (long at-the-money call option) and C (long 10 % out-of-the money call option) outperform the other strategies from a risk-return perspective as well as from a combined perspective of downside risk and return. The negative signs for the  $HE_{HDA \text{ revised-}}$  and  $HE_{SR}$ -measures indicate that not hedging at all would have resulted in a better outcome on an aggregated level for this data set. This, however, is intuitive as the average physical position incurred a considerable profit. For strategies B (long at-the-money call option) and C (long 10 % out-of-the money call option), the implication is that even for the one-sided, option-based strategies the benefit of keeping the upside potential did not pay off for the average vessel in the considered data set. The gains in risk and downside-risk reduction were offset at the same time by lower returns compared to the physical position alone. This is largely caused by the non-negligible transaction costs of about USD 0.50 million and the interest burden on the initial option premium for these strategies. From the variance perspective, the  $HE_{ED}$ -measure shows that strategies A (long FFA), D (replicated long FFA), and E (zero-cost collar) reduced the variance by about 55 %. Strategies B (long at-the-money call option) and C (long 10 % out-of-the money call option) show considerably lower variance reduction. This is caused by the fact that the one-sided option-based strategies still allow for upside variation. For the  $HE_{LPM}$ -measures, all strategies show a small downside-risk reduction for SPM 1 apart from strategies B (long at-the-money call option) and C (long 10 % out-of-the money call option) for the  $HE_{LPM_4}$ -measure. For SPM 2, however, all strategies show a small increase of downside risk. This is potentially caused by the left shift of the return distribution from the transaction cost burden applied to all portfolios as well as from the interest effect burden on the initial option premium for strategies B (long at-the-money call option) and C (long 10 % out-of-the money call option). Nonetheless, the one-sided, option-based strategies show hedge effectiveness gains compared to the other strategies in this alternative hedging set-up, but insurance of any type (i.e., whether it is two-sided or only one-sided) always comes at certain costs causing the  $HE_{HDA \text{ revised-}}$  and  $HE_{SR}$ -measures to be negative for all strategies.

## 3.6 Conclusion

The nature of the shipping industry has been historically characterized by high volatility compared to other industries. The boom period prior to the outburst of the financial crisis and the severe recession that the shipping industry has been facing since is a prime example for the challenging market circumstances that shipping companies operate in. Along with the increasing need to comply with the IFRS fair value accounting principles that cause large ship price fluctuations to become more visible through impairment tests or from a cash/liquidity perspective, the need for effective hedging strategies for the shipping companies arises. Unfortunately, no direct, liquid hedge instruments on dry bulk Capesize ship values, such as FoSVAs currently exist. The aim of this paper was to examine whether freight options qualify as suitable cross-hedge instruments for dry bulk Capesize ship price risks and to empirically assess whether option-based cross-hedging strategies may achieve superior hedge effectiveness than a simple FFA-based cross-hedging strategy.

Within this study, a SPM was first estimated using actual dry bulk Capesize second-hand transactions following the effort of Adland and Koekebakker (2007). The model is based on ship-specific, deterministic factors from the data set of real Capesize sale and purchase transactions as well as market-driven or risk factors, such as the FFA rate and interaction terms. It serves as basis of the hedging approach and allows to determine the desired exposure to the respective hedge instruments. Secondly, the hedge effectiveness of different freight option hedging strategies (i.e., long at-the-money put, long 10 % out-of-the-money put, replicated FFA, and zero-cost collar) was empirically tested in a hedging set-up over a fixed time horizon one year prior to the sale for the same dry bulk Capesize sale and purchase transactions. The performance of these hedging strategies was compared against the reference case of a simple FFA-based hedging strategy. The hedge effectiveness was assessed using several measures (i.e.,  $HE_{ED}$ ,  $HE_{HDA \text{ revised}}$ ,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ ,  $HE_{LPM_4}$ , and  $HE_{SR}$ ) reflecting different risk and return preferences of shipping companies. Finally, the robustness of the presented results and findings has been checked for two subsets of the data set as well as in an alternative hedging set-up.

Firstly, it was found that dry bulk Capesize second-hand prices may be rather well described by a SPM containing market-driven explanatory variables, such as the FFA+1CAL or FFA+2CAL rate, and deterministic, ship-specific explanatory factors,

such as the age, DWT, and fuel efficiency of the individual vessel. The SPM was estimated from 277 actual dry bulk Capesize sale and purchase transactions. Given the heterogeneity among dry bulk Capesize vessels, such a tailored model is beneficial for shipping companies with respect to valuation and hedging purposes. Secondly, the results indicate that all five tested hedging strategies achieved a relatively similar hedge effectiveness from a downside-risk perspective. Thirdly, the two-sided hedging strategies (i.e., strategies A (short FFA), D (replicated short FFA), and E (zero-cost collar)) achieved superior results from a risk-return perspective as well as from a combined perspective of downside risk and return. This, however, is somewhat caused by the data set used within this empirical study consisting of relatively few vessels that would benefit from the one-sided, option-based hedging strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option). Fourthly, the replicated FFA using options strategy as well as the zero-cost collar strategy using options outperform the FFA-based reference strategy. This contradicts findings from other studies in other financial markets and actually implies redundancy of FFAs. Given the comparatively low liquidity of freight options, this result should not be over-interpreted and rather seen as a friction from a not yet fully developed freight option market. Fifthly, the robustness of the results was confirmed for two subsets of the data set (i.e., excluding multiply sold vessels as well as excluding vessels younger than five years and older than 20 years). Finally, the beneficial mechanics of the one-sided, option-based strategies B (long at-the-money put option) and C (long 10 % out-of-the-money put option) were shown in an alternative hedging set-up. Consequently, one-sided, option-based hedging proved to be beneficial compared to the classical two-sided hedging in case the market development does not require any downside-risk protection. These are relevant findings for the risk management practice of shipping companies.

With respect to limitations of this study, the short time frame covered caused by the availability of freight options only from 2008 onwards obviously presents a constraint. Besides, the number of dry bulk Capesize vessel transactions is generally quite low. Taken together, the two constraints made solid testing of the presented results out-of-sample rather difficult. Moreover, the considered hedging set-up generalizes to some extent in form of certain assumptions in order to create a comparable environment for different vessel transactions. Individual shipping companies may of course face different set-ups, hedge horizons, and potentially hedging goals. Furthermore, the computed freight option prices derived from the implied volatilities quoted by The Baltic Exchange present another constraint. The absence of volatility smiling as well

as the fact that these prices were synthetically computed rather than being actually observed in the market are limitations that were difficult to overcome. Last but not least, the historically estimated bid-/ask-spreads present only a rough indication for the real transaction costs that a shipping company may face. The bid-/ask-spreads may, of course, considerably deviate from the estimations at certain times. Nonetheless, the empirical study has been conducted to the best of one's knowledge given these rather complicated market circumstances.

In terms of practicability of the discussed hedging approaches, sufficient liquidity of the hedging instruments considered is essential for a successful implementation of the suggested hedging approaches. Historical figures on dry bulk Capesize FFA and freight option trading volumes indicate that FFAs are clearly the more liquid instrument of the two. For FFAs, an average of 11,085 lots per week was traded in the time frame from July 9<sup>th</sup>, 2007 to December 22<sup>nd</sup>, 2014. On the contrary, an average of only 2,875 lots per week was traded for freight options in the time frame February 24<sup>th</sup>, 2011 to December 22<sup>nd</sup>, 2014. These numbers clearly show that initiating a hedge for an entire vessel might be well feasible for a shipping company. Only the simultaneous initiation of hedges for an entire fleet or the rollover dates present currently a bottleneck in terms of liquidity. Nevertheless, such large scale hedging efforts would probably need to be staged or successively built up anyways. For further details on historical trading volumes, see a plot of weekly trading volumes in FIGURE 3.6 in the appendix A.1 on page 134 as well as selected descriptive statistics of these weekly volume figure in FIGURE 3.8 in the appendix A.1 on page 135.

Accordingly, this paper contributes to the existing academic literature in several ways. Firstly, this is the first effort investigating whether freight options generally qualify as hedge instruments for dry bulk Capesize ship price risks. Secondly, this paper is also the first empirical study assessing the hedge effectiveness of different option-based hedging strategies (i.e., long at-the-money put, long 10 % out-of-the-money put, replicated FFA, and zero-cost collar). Finally, the study compares the performance of the option-based hedging strategies with the performance of a classical FFA-based hedging strategy and found that hedging strategies involving freight options present a viable alternative to FFAs and might be even a superior choice for shipping companies with certain risk, downside-risk, or return preferences.

Concerning further research opportunities in this area, out-of-sample tests of the hedge effectiveness of the five different hedging strategies obviously present a valuable exten-

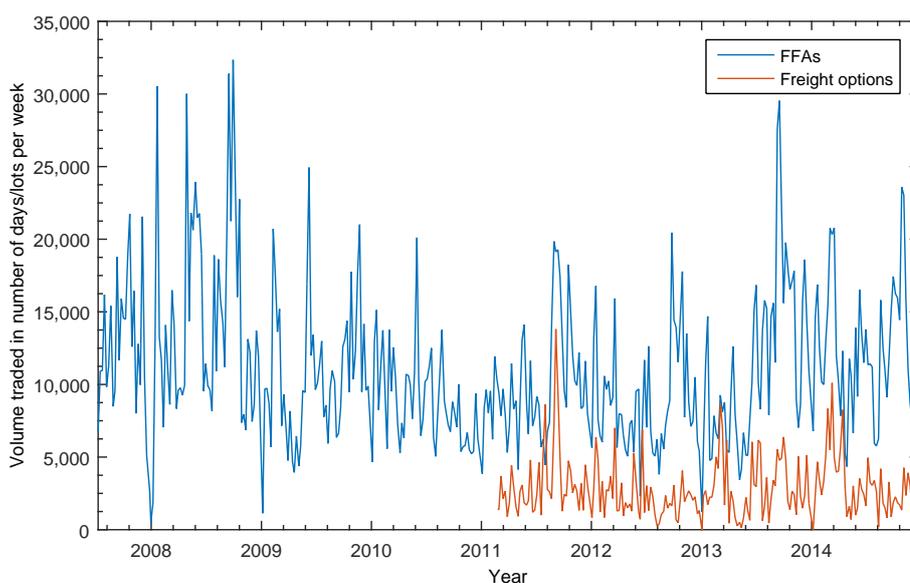
sion of the presented research within this paper once a longer time horizon of relevant data is available. Besides, the presented research could be tested for other vessel classes of the dry bulk sector with a larger number of sale and purchase transactions (e.g., Panamax, Handymax, or Handysize) or even for vessel classes of the tanker sector (e.g., VLCC, Suezmax, Aframax, or Handysize). Furthermore, investigating the hedge effectiveness of the presented hedging instruments with different maturities or the hedge effectiveness of other option-based strategies (e.g., strip, strap, straddle, strangle, bull spread, bear spread, butterfly spread, or calendar spread) might be another direction of impact for future research in this area. Alternatively, testing the hedge effectiveness of the considered hedge instruments with altered parameters of the hedging set-up (e.g., a different hedge time horizon, dynamic rebalancing of the hedge, etc.) presents another valid extension of research in this area.

## A Appendix A – Liquidity and bid-/ask-spreads

### A.1 Historical FFA and freight option trading volumes

With respect to the practical application of the suggested hedging strategies, sufficient liquidity of the hedge instruments is essential. The Baltic Exchange started to publish weekly dry bulk Capesize FFA volumes from July 9<sup>th</sup>, 2007 and dry bulk Capesize freight option volumes from February 24<sup>th</sup>, 2011. Unfortunately, these volume figures represent aggregated numbers across all available contracts (i.e., across different maturities) and a breakdown on individual contract volumes of the hedging instruments used within this study, such as the FFA+1CAL, FFA+1CAL, CTC+1CAL, or CTC+2CAL contract, is not available.

FIGURE 3.6: Historical dry bulk Capesize FFA and freight option trading volumes 2007-2014



The graph shows weekly dry bulk Capesize FFA and freight option volumes from July 9<sup>th</sup>, 2007 to December 22<sup>nd</sup>, 2014. Freight option volume data is only available from February 24<sup>th</sup>, 2011. As the lot size of these contracts is days, the weekly volume shown here is also in days.

*Source:* own graph based on weekly data from The Baltic Exchange

FIGURE 3.6 shows a plot of these weekly volume figures. From visual inspection, the plot indicates considerable volatility on the trading volume for both FFAs and freight options. Moreover, FFAs seem to be clearly more liquid instruments than freight options. Selected descriptive statistics of these weekly volume figures are presented in TABLE 3.8. The numbers confirm the lower liquidity of freight options with a mean

of only 2,875 lots traded per week compared to a mean of 11,805 lots traded per week for FFAs.

TABLE 3.8: Selected descriptive statistics of trading volume figures

Instrument	Mean	Median	Variance	Stand. dev.	Min.	Max.
FFAs	11,085	9,939	27,508,749	5,245	480	32,348
Freight options	2,875	2,360	4,464,089	2,113	0	13,805

The table shows selected descriptive statistics of weekly trading volume figures from July 9<sup>th</sup>, 2007 to December 22<sup>nd</sup>, 2014 for dry bulk Capesize FFAs and from February 24<sup>th</sup>, 2011 to December 22<sup>nd</sup>, 2014 for dry bulk Capesize freight options. As the lot size of these contracts is days, the weekly volume shown here is also in days.

Accordingly, it seems to be currently easier for shipping companies to set up a hedge for an entire ship using FFAs due to the generally higher liquidity of these instruments. Nevertheless, setting up a hedge for an entire ship using freight options should also be possible given the historical liquidity levels. Only the rollover dates present a bottleneck for the suggested hedging approach if entire fleets should be hedged as this would involve closing out and re-initiating all positions across vessels simultaneously around the rollover date.

## A.2 Historical bid-/ask-spread estimation

Unfortunately, The Baltic Exchange does not provide data on daily, weekly, or average bid-/ask-spreads for FFAs or freight options. Nevertheless, taking into account the differences in liquidity of the two different hedging instruments and thus, the differences in bid-/ask-spreads is ultimately important in order not to distort any hedging results towards freight options. Consequently, the method of Roll (1984) was applied to historically estimate effective bid-/ask-spreads from daily and weekly log returns for each of the hedging instruments.

In case a market maker is involved, transactions are usually not costless and the market maker requires compensation in form of a bid-/ask-spread as lower liquidity, higher volatility, or both for a financial asset causes the market maker to bear a price risk. Furthermore, Niederhoffer and Osborne (1966) stated that in presence of a market maker, negative serial dependence of price changes should be anticipated. Under the assumptions of an informationally efficient market and of stationarity of the probability distribution of observed price changes, Roll (1984) stated that the effective

bid-/ask-spread faced by the dollar-weighted average investor trading at observed prices can be determined according to equation (A.1) below:

$$\text{Bid-/ask-spread} = 2 \cdot \sqrt{-\text{Cov}(\Delta p_t, \Delta p_{t-1})}. \quad (\text{A.1})$$

Within equation (A.1) above, the bid-/ask-spread is given as a percentage of the price level if  $\Delta p_t$  and  $\Delta p_{t-1}$  are log returns and  $\text{Cov}(\Delta p_t, \Delta p_{t-1})$  is the first-order autocovariance of these log returns. Roll (1984) computed the bid-/ask-spread measure yearly from daily and weekly log returns for stocks listed on the New York and American Exchanges and found that the estimated spreads were strongly negatively related to firm size (i.e., smaller firms having less liquid stocks and thus, larger associated bid-/ask-spreads).

Among others, Harris (1990) noted the poor empirical performance of the Roll (1984) serial covariance estimator when estimated yearly from daily or weekly log returns. A major problem is that empirical first-order autocovariances are positive for many financial assets and thus, the square root in equation (A.1) is not properly defined. There are several approaches taken by researchers to deal with this problem. Roll (1984), for instance, took the square root of the absolute value of the autocovariance and preserved the sign of the autocovariance afterwards resulting in negative spreads for these cases. Another common approach in the literature is setting the bid-/ask-spread to zero. On the contrary, Lesmond (2005) and Kim and Lee (2014) simply took the square root of the absolute value of the autocovariance and did not preserve the sign of the autocovariance afterwards.

The question whether the underlying assumptions of the Roll (1984) measure are fulfilled in the FFA or freight option market is, of course, highly doubtful. Nevertheless, Harris (1990) stated that the Roll (1984) method is ‘very nearly the best serial covariance spread estimator available’. Therefore, the historical mean bid-/ask-spreads for the different hedging instruments were estimated based on the Roll (1984) measure and used to reflect differences in transaction costs for the different hedging instruments. In order to be conservative with respect to transaction costs, the approach applied by Lesmond (2005) and Kim and Lee (2014) regarding the treatment of positive autocovariances was followed. The bid-/ask-spreads were estimated yearly for each of the different hedging instruments based on daily and weekly log return autocovariances. Specifically, daily and weekly log returns were derived for FFA+1CAL,

FFA+2CAL, CTC+1CAL, and CTC+2CAL contracts from the first date after the rollover date until the first date before the subsequent rollover date. For the freight options, an at-the-money put option was considered on the first date after the rollover date and observed over the year.

FIGURE 3.7 shows plots of these yearly estimated bid-/ask-spreads both based on daily and weekly log returns for FFA+1CAL, FFA+2CAL, CTC+1CAL, and CTC+2CAL contracts from 2005 until 2014 for FFAs and from 2008 until 2014 for freight options. The plots indicate that the bid-/ask-spreads for freight options are generally higher than for FFAs and that the bid-/ask-spreads based on weekly log return autocovariances tend to be higher than the bid-/ask-spreads based on daily log return autocovariances.

TABLE 3.9: Mean estimated bid-/ask-spreads for FFAs and freight options

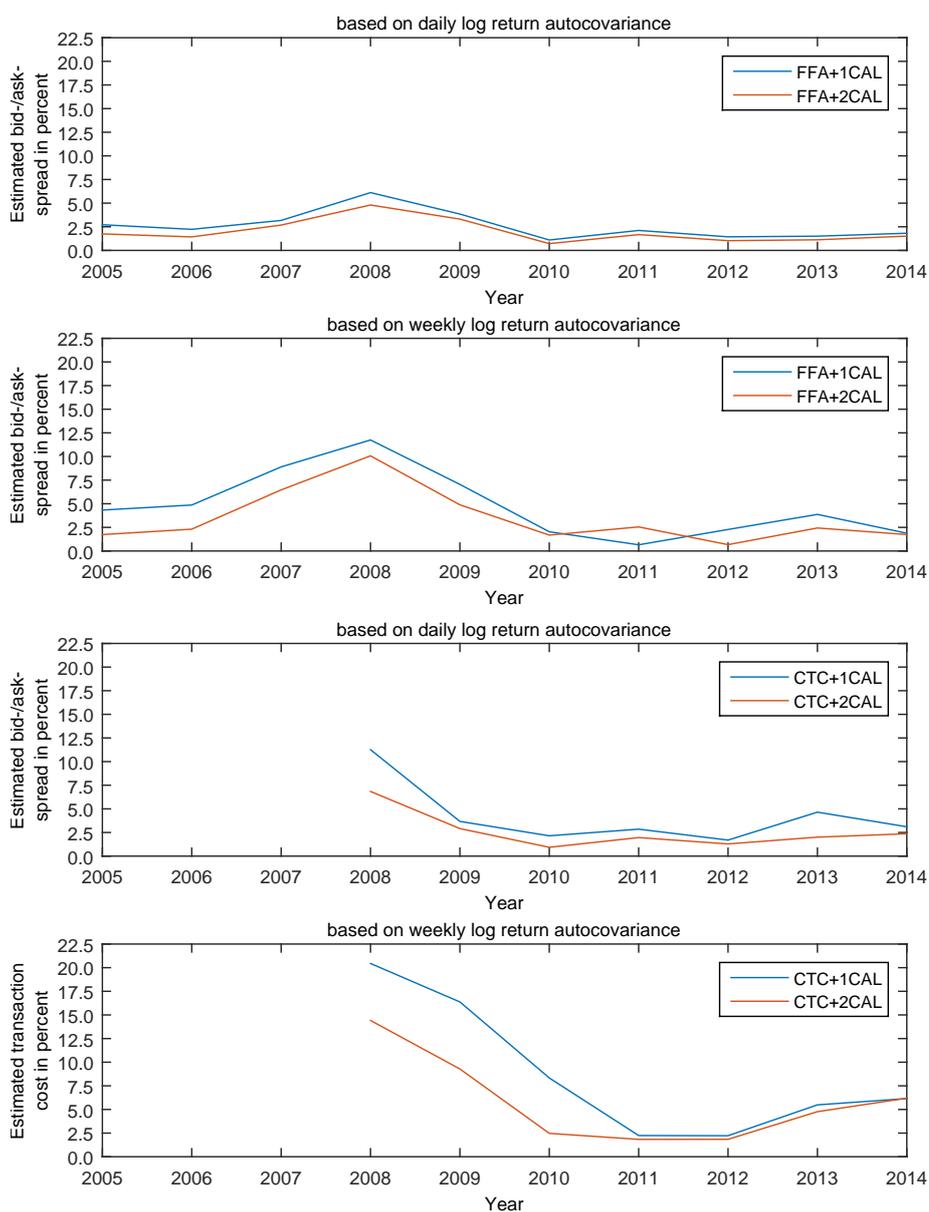
Mean	Unit	Frequency	FFAs		Freight options	
			+1CAL	+2CAL	+1CAL	+2CAL
Bid-/ask-spread	%	daily	2.60	2.00	4.20	2.62
		weekly	4.76	3.45	8.75	5.83

The table shows the mean of the yearly estimated bid-/ask-spreads both based on daily and weekly log returns for FFA+1CAL, FFA+2CAL, CTC+1CAL, and CTC+2CAL contracts from 2005 until 2014 for FFAs and from 2008 until 2014 for freight options.

TABLE 3.9 shows the mean of the yearly estimated bid-/ask-spreads. These results largely confirm the initial implications from the plot (i.e., freight options have larger bid-/ask-spreads and the bid-/ask-spreads based on weekly log return autocovariances are larger). The fact that the mean estimated bid-/ask-spreads are lower for the +1CAL contracts compared to the +2CAL contracts although their liquidity should be theoretically lower may result from the considerably higher volatility of the +1CAL contracts in the considered time frame.

As Harris (1990) concluded that the Roll (1984) measure is very noisy if estimated from daily log return data as well as for the purpose of being conservative, the mean of the yearly estimated bid-/ask-spreads based on weekly log return autocovariances was used in order to determine transaction costs within this empirical study (see TABLE 3.9, last row).

FIGURE 3.7: Estimated bid-/ask-spreads for FFAs and freight options 2005-2014



The graph shows yearly estimated bid-/ask-spreads in percent for FFA+1CAL, FFA+2CAL, CTC+1CAL, and CTC+2CAL contracts from 2005 until 2014 for FFAs and from 2008 until 2014 for freight options.

Source: own graph based on daily data from The Baltic Exchange

## B Appendix B – Robustness checks

### B.1 Excluding multiply sold vessels

TABLE 3.10: Estimates for different Capesize structural pricing models

	SPM 1 (FFA+1CAL)			SPM 2 (FFA+2CAL)		
	Coefficient	t-stat.	p-value	Coefficient	t-stat.	p-value
$\alpha$	15.1798 (17.4640)	0.8692	0.3859	-6.6668 (20.4599)	-0.3258	0.7449
$\beta_f$	1,825.5345*** (305.2212)	5.9810	0.0000	3,000.9018*** (495.8789)	6.0517	0.0000
$\beta_{Age}$	-1.8763*** (0.2251)	-8.3346	0.0000	-1.7910*** (0.2674)	-6.6977	0.0000
$\beta_{DWT}$	$6.445 \cdot 10^{-05}$ ( $4.663 \cdot 10^{-05}$ )	1.3823	0.1686	$9.215 \cdot 10^{-05}$ * ( $5.392 \cdot 10^{-05}$ )	1.7089	0.0892
$\beta_{Consum}$	18.4936 (12.2162)	1.5139	0.1318	29.0419** (14.3902)	2.0182	0.0450
$\beta_{f \cdot Age}$	-11.0260*** (4.8231)	-2.2861	0.0234	-16.8034** (7.5231)	-2.2336	0.0267
$\beta_{f \cdot DWT}$	-0.0021*** (0.0008)	-2.7218	0.0071	-0.0037*** (0.0012)	-2.9418	0.0037
$\beta_{f \cdot Consum}$	-881.3063*** (204.7747)	-4.3038	0.0000	-1,441.4829*** (334.6122)	-4.3079	0.0000
$R^2$	0.7263			0.7351		
Adj. $R^2$	0.7157			0.7249		
Log likelihood	-736.8557			-733.7519		
SBIC criterion	7.9773			7.9446		
Akaike criterion	7.8406			7.8079		
Standard error	11.9503			11.7567		

The table shows linear regression coefficient estimates for two different SPMs based on data for 190 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to December 2<sup>nd</sup>, 2014. Vessels that were multiply sold have been excluded from the analysis. Figures in () reflect the corresponding standard errors. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

TABLE 3.11: Hedging results over fixed time horizon of one year (1/2)

			SPM 1				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	37.4375		-		
End	USD m	Mean	26.4446		-		
Change in values	USD m	Mean	-10.9929		-		
		Median	-7.2386		-		
		Std. dev.	17.6589		-		
		Skewness	-1.1721		-		
		Kurtosis	4.4225		-		
	Log return in %	Mean	-35.6268		1.2816		
		Median	-31.3665		0.9722		
		Std. dev.	51.6712		-		
		Skewness	-1.2356		-		
		Kurtosis	8.2042		-		
			SPM 1				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	7.7942	6.4887	6.1253	8.1288	8.1313
		Median	1.3521	0.7365	0.6526	1.4309	1.4314
		Std. dev.	17.2910	13.9822	13.1343	17.1959	17.2049
		Skewness	1.6970	1.9013	1.9199	1.6982	1.6997
		Kurtosis	4.8259	5.5709	5.6349	4.8120	4.8189
thereof: interest		Mean	0.0218	-0.0021	0.0035	0.0198	0.0224
Portfolio (PHYPOS+HEDGE)							
Start		Mean	37.4375	37.4375	37.4375	37.4375	37.4375
End		Mean	34.2388	32.9334	32.5700	34.5734	34.5759
Init. opt. prem.	USD m	Mean	0.0000	-3.6662	-2.6632	0.0000	0.0000
Transaction costs		Mean	-1.0876	-0.8598	-0.7025	-1.3076	-1.3076
Change in values	USD m	Mean	-3.1987	-4.5042	-4.8676	-2.8641	-2.8616
		Median	-3.5496	-5.5259	-5.9299	-3.3961	-3.3980
		Std. dev.	6.4822	7.2082	7.5768	6.4828	6.4846
		Skewness	1.4022	1.1114	0.9674	1.3356	1.3353
		Kurtosis	8.2216	7.4585	6.9862	8.0120	8.0016
	Log return in %	Mean	-13.5610	-15.9599	-16.6157	-12.4468	-12.4441
		Median	-11.3477	-21.0249	-21.3702	-10.6583	-10.6520
		Std. dev.	23.7128	24.6156	25.0441	23.4702	23.4714
		Skewness	1.1472	1.4941	1.5489	1.1364	1.1360
		Kurtosis	7.3884	7.8637	7.8556	7.3908	7.3886

The table shows selected descriptive statistics of the SPM 1 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 112 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLE 3.12: Hedging results over fixed time horizon of one year (2/2)

		SPM 2					
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	37.4772		–		
End	USD m	Mean	26.4446		–		
Change in values	USD m	Mean	-11.0325		–		
		Median	-7.3899		–		
		Std. dev.	17.4765		–		
		Skewness	-1.1515		–		
		Kurtosis	4.3521		–		
	Log return in %	Mean	-35.5788		1.2816		
		Median	-32.3172		0.9722		
		Std. dev.	51.3223		–		
		Skewness	-1.2177		–		
		Kurtosis	8.0678		–		
		SPM 2					
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	8.0094	5.0707	4.7057	7.7207	7.7256
		Median	1.3550	0.7716	0.7082	1.4747	1.4755
		Std. dev.	16.1160	11.1706	10.3605	15.1231	15.1363
		Skewness	1.7081	1.8924	1.9123	1.6991	1.7009
		Kurtosis	4.9393	5.6640	5.7413	4.9277	4.9357
		thereof: interest	Mean	0.0304	-0.0182	-0.0110	0.0160
Portfolio (PHYPOS+HEDGE)							
Start		Mean	37.4772	37.4772	37.4772	37.4772	37.4772
End	USD m	Mean	34.4541	31.5154	31.1504	34.1653	34.1703
Init. opt. prem.		Mean	0.0000	-5.7407	-4.3879	0.0000	0.0000
Transaction costs		Mean	-1.2138	-0.7761	-0.6294	-1.2900	-1.2900
Change in values	USD m	Mean	-3.0231	-5.9618	-6.3268	-3.3119	-3.3069
		Median	-3.3488	-6.8045	-6.6284	-3.3696	-3.3405
		Std. dev.	6.6340	8.3038	8.8429	6.6679	6.6670
		Skewness	1.3842	0.5385	0.3339	1.3641	1.3646
		Kurtosis	8.6437	6.2584	5.6905	8.9262	8.9220
	Log return in %	Mean	-11.3313	-18.6730	-19.4968	-11.6374	-11.6295
		Median	-10.3856	-24.1879	-25.6976	-11.5264	-11.5484
		Std. dev.	23.6673	25.0062	25.7469	23.4059	23.4071
		Skewness	1.4528	1.6445	1.5851	1.5225	1.5216
		Kurtosis	8.3758	8.4558	8.0296	8.6136	8.6096

The table shows selected descriptive statistics of the SPM 2 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 112 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLE 3.13: Results for different hedge effectiveness measures

Model	Measure	Unit	Hedging strategy				
			A	B	C	D	E
SPM 1	$HE_{ED}$	%	78.94	77.31	76.51	79.37	79.37
	$HE_{HDA}$ revised	pp	8.84	1.39	-0.03	12.94	12.95
	$HE_{LPM_2}$	%	83.40	80.98	80.06	84.60	84.60
	$HE_{LPM_3}$	%	95.32	94.68	94.38	95.77	95.77
	$HE_{LPM_4}$	%	98.95	98.81	98.74	99.08	99.08
	$HE_{SR}$	pp	0.79	-4.28	-5.18	3.14	3.15
SPM 2	$HE_{ED}$	%	78.73	76.26	74.83	79.20	79.20
	$HE_{HDA}$ revised	pp	18.53	-7.98	-8.88	16.63	16.66
	$HE_{LPM_2}$	%	86.03	77.51	75.78	86.06	86.06
	$HE_{LPM_3}$	%	96.32	93.48	92.72	96.40	96.40
	$HE_{LPM_4}$	%	99.22	98.48	98.23	99.26	99.26
	$HE_{SR}$	pp	5.11	-8.55	-8.78	3.71	3.74

The table shows the results of the different hedge effectiveness measures for the five different hedging strategies applied (i.e., from A to E) to both models SPM 1 and SPM 2 over a fixed time horizon of one year. The included sample size is 112 vessels for all strategies. For  $HE_{ED}$ ,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ , and  $HE_{LPM_4}$ , the result is given in percent. For  $HE_{HDA}$  revised and  $HE_{SR}$ , the result is given in percentage points. Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

## B.2 Excluding vessels younger than five and older than 20 years

TABLE 3.14: Estimates for different Capesize structural pricing models

	SPM 1 (FFA+1CAL)			SPM 2 (FFA+2CAL)		
	Coefficient	t-stat.	p-value	Coefficient	t-stat.	p-value
$\alpha$	10.7337 (17.2727)	0.6214	0.5351	-34.4233* (19.4634)	-1.7686	0.0787
$\beta_f$	3,185.9469*** (304.0483)	10.4784	0.0000	5,371.9995*** (476.4368)	11.2754	0.0000
$\beta_{Age}$	-1.4020*** (0.3296)	-4.2532	0.0000	-0.6584* (0.3822)	-1.7225	0.0867
$\beta_{DWT}$	$5.592 \cdot 10^{-05}$ ( $3.764 \cdot 10^{-05}$ )	1.4856	0.1392	$1.078 \cdot 10^{-04}$ ** ( $4.262 \cdot 10^{-05}$ )	2.5286	0.0123
$\beta_{Consum}$	13.4123 (12.2533)	1.0946	0.2752	33.2725** (13.7492)	2.4200	0.0165
$\beta_{f \cdot Age}$	-49.0855*** (6.6105)	-7.4254	0.0000	-82.9450*** (10.3855)	-7.9866	0.0000
$\beta_{f \cdot DWT}$	-0.0032*** (0.0007)	-4.7624	0.0000	-0.0057*** (0.0010)	-5.5128	0.0000
$\beta_{f \cdot Consum}$	-1,480.9109*** (213.7153)	-6.9294	0.0000	-2,460.8855*** (334.8115)	-7.3501	0.0000
$R^2$	0.8359			0.8495		
Adj. $R^2$	0.8229			0.8436		
Log likelihood	-682.7414			-674.7063		
SBIC criterion	7.6067			7.5199		
Akaike criterion	7.4675			7.3806		
Standard error	9.9111			9.4899		

The table shows linear regression coefficient estimates for two different SPMs based on data for 185 Capesize vessel transactions and corresponding FFA time series data from January 13<sup>th</sup>, 2005 to December 2<sup>nd</sup>, 2014. Vessels that were multiply sold have been excluded from the analysis. Figures in () reflect the corresponding standard errors. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\* at the 1 % level.

TABLE 3.15: Hedging results over fixed time horizon of one year (1/2)

			SPM 1				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	35.2000		-		
End	USD m	Mean	22.4398		-		
Change in values	USD m	Mean	-12.7602		-		
		Median	-5.8700		-		
		Std. dev.	22.3791		-		
		Skewness	-1.8989		-		
		Kurtosis	6.0302		-		
	Log return in %	Mean	-32.5590		1.3090		
		Median	-23.2083		0.9792		
		Std. dev.	41.4044		-		
		Skewness	-0.6615		-		
		Kurtosis	3.0008		-		
			SPM 1				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	10.7297	8.7821	8.2703	11.0499	11.0536
		Median	1.5481	1.0348	0.9108	1.8845	1.8855
		Std. dev.	23.6812	18.9351	17.8020	23.3394	23.3530
		Skewness	1.9856	2.1160	2.1306	1.9489	1.9505
		Kurtosis	6.1834	6.6048	6.6589	5.9808	5.9889
thereof: interest		Mean	0.0301	-0.0027	0.0047	0.0268	0.0304
Portfolio (PHYPOS+HEDGE)							
Start		Mean	35.2000	35.2000	35.2000	35.2000	35.2000
End		Mean	33.1695	31.2219	30.7101	33.4897	33.4934
Init. opt. prem.	USD m	Mean	0.0000	-4.6296	-3.3635	0.0000	0.0000
Transaction costs		Mean	-1.3047	-1.0891	-0.8936	-1.6248	-1.6248
Change in values	USD m	Mean	-2.0305	-3.9780	-4.4899	-1.7102	-1.7066
		Median	-2.4812	-3.6700	-4.2930	-2.1724	-2.1769
		Std. dev.	4.4210	5.4208	6.1735	4.2260	4.2305
		Skewness	0.3115	-0.1158	-0.3600	0.3257	0.3254
		Kurtosis	3.0543	3.7152	3.8543	3.1941	3.1938
	Log return in %	Mean	-13.0291	-13.5844	-14.2848	-10.5199	-10.5174
		Median	-11.7086	-14.6289	-15.8237	-10.0723	-10.0655
		Std. dev.	24.1916	18.2330	18.5813	19.0976	19.0998
		Skewness	-2.7653	0.3176	0.4446	-0.6467	-0.6468
		Kurtosis	19.2948	3.1415	3.1637	4.4828	4.4829

The table shows selected descriptive statistics of the SPM 1 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 103 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLE 3.16: Hedging results over fixed time horizon of one year (2/2)

		SPM 2					
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean	35.1960		–		
End	USD m	Mean	22.4398		–		
Change in values	USD m	Mean	-12.7562		–		
		Median	-5.6381		–		
		Std. dev.	22.3451		–		
		Skewness	-1.8861		–		
		Kurtosis	5.9436		–		
	Log return in %	Mean	-32.1977		1.3090		
		Median	-24.2915		0.9792		
		Std. dev.	41.3014		–		
		Skewness	-0.7052		–		
		Kurtosis	3.0344		–		
		SPM 2					
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	10.8472	6.9895	6.4835	10.4086	10.4155
		Median	1.6698	0.9486	0.7999	1.9104	1.9113
		Std. dev.	22.3554	15.3010	14.1961	20.9203	20.9400
		Skewness	2.0043	2.1434	2.1604	1.9850	1.9868
		Kurtosis	6.2846	6.8239	6.8928	6.2243	6.2332
		thereof: interest	Mean	0.0423	-0.0237	-0.0142	0.0218
Portfolio (PHYPOS+HEDGE)							
Start		Mean	35.1960	35.1960	35.1960	35.1960	35.1960
End	USD m	Mean	33.2870	29.4293	28.9233	32.8484	32.8553
Init. opt. prem.		Mean	0.0000	-7.2320	-5.5267	0.0000	0.0000
Transaction costs		Mean	-1.4704	-0.9711	-0.7899	-1.5929	-1.5929
Change in values		USD m	Mean	-1.9091	-5.7667	-6.2727	-2.3476
	Median		-2.0064	-4.2452	-4.3546	-1.9031	-1.8999
	Std. dev.		4.2563	7.9375	8.9106	4.2338	4.2306
	Skewness		-0.0102	-1.0979	-1.2072	-0.3086	-0.3079
	Kurtosis		3.6762	4.2847	4.4370	3.9865	3.9914
	Log return in %	Mean	-11.7255	-16.2872	-17.0563	-10.3608	-10.3535
		Median	-8.9815	-16.8165	-18.8154	-9.8509	-9.8522
		Std. dev.	29.8848	18.4550	19.2532	19.7494	19.7535
		Skewness	-5.6649	0.3602	0.4163	-1.8637	-1.8652
		Kurtosis	48.0466	3.2482	3.1452	12.4269	12.4349

The table shows selected descriptive statistics of the SPM 2 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 103 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

TABLE 3.17: Results for different hedge effectiveness measures

Model	Measure	Unit	Hedging strategy				
			A	B	C	D	E
SPM 1	$HE_{ED}$	%	65.86	80.61	79.86	78.73	78.72
	$HE_{HDA \text{ revised}}$	pp	22.53	0.11	-2.12	19.86	19.88
	$HE_{LPM_2}$	%	72.90	81.85	80.77	83.39	83.39
	$HE_{LPM_3}$	%	74.15	93.32	92.94	92.38	92.38
	$HE_{LPM_4}$	%	66.12	97.56	97.44	96.07	96.07
	$HE_{SR}$	pp	12.04	-2.08	-3.23	9.23	9.24
SPM 2	$HE_{ED}$	%	47.64	80.03	78.27	77.13	77.13
	$HE_{HDA \text{ revised}}$	pp	37.51	-14.22	-14.26	22.04	22.09
	$HE_{LPM_2}$	%	62.86	77.87	75.92	82.33	82.33
	$HE_{LPM_3}$	%	27.83	91.49	90.49	88.36	88.36
	$HE_{LPM_4}$	%	-66.44	96.73	96.27	89.82	89.81
	$HE_{SR}$	pp	23.18	-7.46	-7.50	10.99	11.03

The table shows the results of the different hedge effectiveness measures for the five different hedging strategies applied (i.e., from A to E) to both models SPM 1 and SPM 2 over a fixed time horizon of one year. The included sample size is 103 vessels for all strategies. For  $HE_{ED}$ ,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ , and  $HE_{LPM_4}$ , the result is given in percent. For  $HE_{HDA \text{ revised}}$  and  $HE_{SR}$ , the result is given in percentage points.

Strategies explained: A (short FFA), B (long at-the-money put option), C (long 10 % out-of-the-money put option), D (replicated short FFA), and E (zero-cost collar).

### B.3 Alternative hedging set-up

TABLE 3.18: Hedging results over fixed time horizon of one year (1/2)

			SPM 1				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start	USD m	Mean	-36.3449		–		
End		Mean	-24.9513		–		
Change in values	USD m	Mean	11.3936		–		
		Median	6.6571		–		
		Std. dev.	18.5330		–		
		Skewness	1.2586		–		
		Kurtosis	4.2808		–		
	Log return in %	Mean	31.6528		1.2894		
		Median	27.8532		0.9774		
		Std. dev.	65.9781		–		
		Skewness	-1.9390		–		
		Kurtosis	18.8491		–		
			SPM 1				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	-10.6073	-2.6811	-2.1836	-11.4319	-11.4373
		Median	-2.7042	-1.3815	-1.2406	-2.8110	-2.8117
		Std. dev.	19.6267	4.2117	3.4004	20.8982	20.9119
		Skewness	-1.7198	-0.8924	-0.7449	-1.7341	-1.7354
		Kurtosis	4.7593	3.0261	2.9412	4.7638	4.7696
thereof: interest		Mean	-0.0276	-0.0241	-0.0194	-0.0233	-0.0287
Portfolio (PHYPOS+HEDGE)							
Start	USD m	Mean	-36.3449	-36.3449	-36.3449	-36.3449	-36.3449
End		Mean	-35.5586	-27.6324	-27.1349	-36.3832	-36.3886
Init. opt. prem.		Mean	0.0000	-3.9188	-3.2345	0.0000	0.0000
Transaction costs		Mean	-1.1539	-0.4779	-0.4086	-1.3939	-1.3939
Change in values	USD m	Mean	0.7863	8.7125	9.2100	-0.0383	-0.0437
		Median	1.5872	5.1620	5.5464	1.3231	1.3219
		Std. dev.	6.5596	14.9706	15.7517	7.0420	7.0495
		Skewness	-1.1628	1.1813	1.2282	-1.0437	-1.0446
		Kurtosis	6.1759	4.6477	4.6238	4.7822	4.7746
	Log return in %	Mean	3.2134	20.1156	21.8398	1.5712	1.5636
		Median	4.7494	22.0953	23.9843	3.6934	3.6904
		Std. dev.	44.6691	53.7997	55.5360	44.2123	44.2125
		Skewness	-6.1882	-4.0951	-3.7162	-6.3031	-6.3025
		Kurtosis	57.1677	33.3267	30.1299	58.0997	58.0950

The table shows selected descriptive statistics of the SPM 1 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 150 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

Strategies explained: A (long FFA), B (long at-the-money call option), C (long 10 % out-of-the-money call option), D (replicated long FFA), and E (zero-cost collar).

TABLE 3.19: Hedging results over fixed time horizon of one year (2/2)

			SPM 2				
	Unit	Statistic	Physical position		Reference return ( $r_{RF}$ )		
Start		Mean		-36.2745			-
End	USD m	Mean		-24.9513			-
Change in values	USD m	Mean		11.3232			-
		Median		6.2950			-
		Std. dev.		18.4552			-
		Skewness		1.2711			-
		Kurtosis		4.2796			-
	Log return in %	Mean		31.7173			1.2894
		Median		30.4117			0.9774
		Std. dev.		61.2647			-
		Skewness		-1.1682			-
		Kurtosis		14.8195			-
			SPM 2				
Hedge	Unit	Statistic	A	B	C	D	E
Change in values	USD m	Mean	-11.0063	-3.8469	-3.3587	-10.8914	-10.8991
		Median	-3.3474	-1.7838	-1.6638	-3.2217	-3.2221
		Std. dev.	18.2755	4.9672	4.2177	17.9822	17.9994
		Skewness	-1.7398	-1.0889	-0.9970	-1.7473	-1.7489
		Kurtosis	4.9608	3.2586	3.0663	4.9755	4.9824
thereof: interest		Mean	-0.0386	-0.0369	-0.0317	-0.0188	-0.0265
Portfolio (PHYPOS+HEDGE)							
Start		Mean	-36.2745	-36.2745	-36.2745	-36.2745	-36.2745
End		Mean	-35.9576	-28.7982	-28.3100	-35.8427	-35.8504
Init. opt. prem.	USD m	Mean	0.0000	-6.1242	-5.2888	0.0000	0.0000
Transaction costs		Mean	-1.2902	-0.5499	-0.4834	-1.3766	-1.3766
Change in values	USD m	Mean	0.3169	7.4763	7.9645	0.4318	0.4241
		Median	0.8777	4.4692	4.6251	1.1114	1.1112
		Std. dev.	6.3714	14.3410	15.0433	6.2667	6.2699
		Skewness	-1.2189	1.0434	1.1014	-1.2767	-1.2749
		Kurtosis	7.2342	4.4193	4.4098	7.5990	7.5794
	Log return in %	Mean	0.0623	15.8439	17.4629	0.1758	0.1632
		Median	2.5996	17.9851	18.7830	3.6064	3.6006
		Std. dev.	39.8165	48.5054	49.9098	39.4041	39.4046
		Skewness	-5.4528	-3.4902	-3.1870	-5.6860	-5.6853
		Kurtosis	48.2844	27.1848	24.8708	49.8488	49.8425

The table shows selected descriptive statistics of the SPM 2 hedging results for the different hedging strategies (i.e., from A to E) over a fixed time horizon of one year prior to the vessel transaction. The included sample size is 150 vessels for all strategies with transaction dates from January 8<sup>th</sup>, 2009 to December 2<sup>nd</sup>, 2014. The kurtosis measure states the estimated centralized fourth moment, not the excess kurtosis.

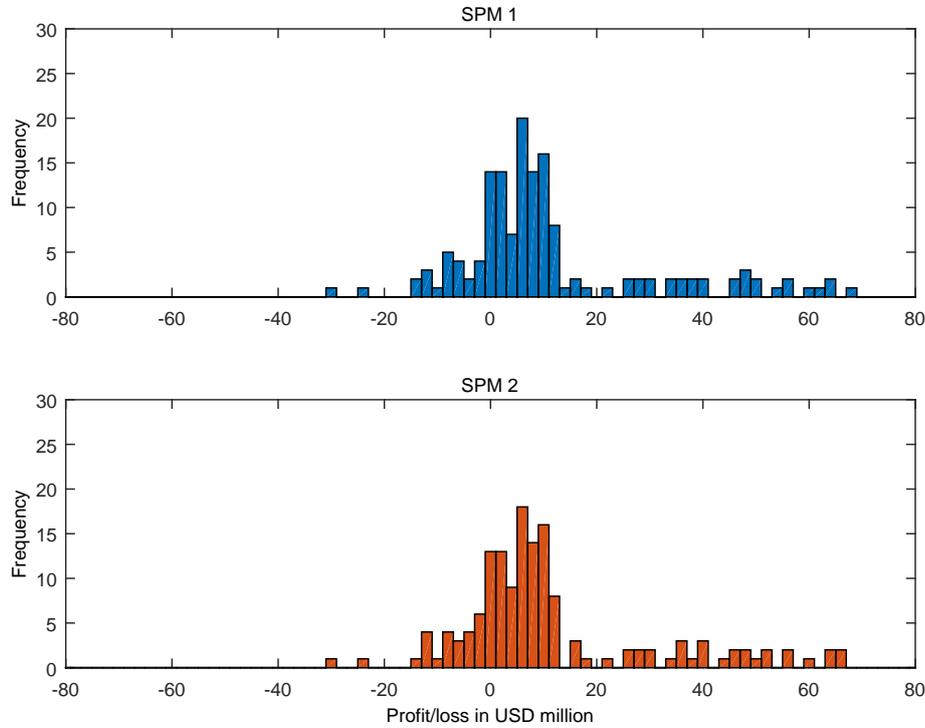
Strategies explained: A (long FFA), B (long at-the-money call option), C (long 10 % out-of-the-money call option), D (replicated long FFA), and E (zero-cost collar).

TABLE 3.20: Results for different hedge effectiveness measures

Model	Measure	Unit	Hedging strategy				
			A	B	C	D	E
SPM 1	$HE_{ED}$	%	54.16	33.51	29.15	55.10	55.10
	$HE_{HDA}$ revised	pp	-41.71	-11.03	-9.02	-45.38	-45.40
	$HE_{LPM_2}$	%	10.53	5.33	4.53	9.72	9.71
	$HE_{LPM_3}$	%	7.51	3.27	2.80	7.70	7.70
	$HE_{LPM_4}$	%	1.86	-0.03	-0.11	2.60	2.60
	$HE_{SR}$	pp	-66.96	-26.03	-22.05	-71.06	-71.08
SPM 2	$HE_{ED}$	%	57.76	37.32	33.63	58.63	58.63
	$HE_{HDA}$ revised	pp	-52.75	-19.66	-17.26	-52.49	-52.52
	$HE_{LPM_2}$	%	-2.57	-5.59	-5.69	-2.50	-2.51
	$HE_{LPM_3}$	%	-2.30	-5.50	-5.53	-2.18	-2.18
	$HE_{LPM_4}$	%	-7.36	-8.52	-8.29	-6.49	-6.50
	$HE_{SR}$	pp	-88.84	-45.67	-41.27	-88.53	-88.56

The table shows the results of the different hedge effectiveness measures for the five different hedging strategies applied (i.e., from A to E) to both models SPM 1 and SPM 2 over a fixed time horizon of one year. The included sample size is 150 vessels for all strategies. For  $HE_{ED}$ ,  $HE_{LPM_2}$ ,  $HE_{LPM_3}$ , and  $HE_{LPM_4}$ , the result is given in percent. For  $HE_{HDA}$  revised and  $HE_{SR}$ , the result is given in percentage points. Strategies explained: A (long FFA), B (long at-the-money call option), C (long 10 % out-of-the-money call option), D (replicated long FFA), and E (zero-cost collar).

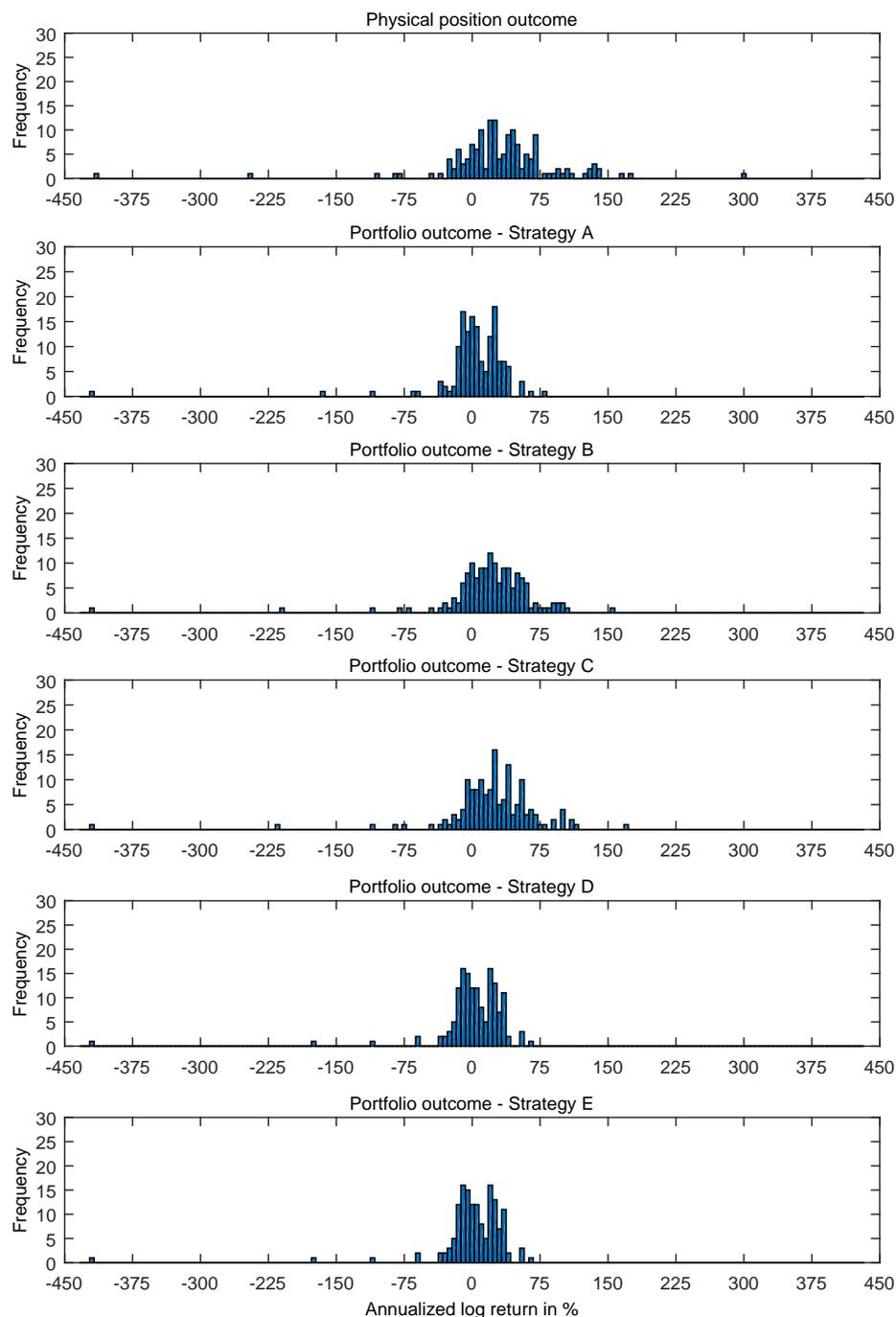
FIGURE 3.8: Histograms of physical position outcomes



The graph shows histogram plots of the physical position outcomes for SPM 1 and SPM 2 in USD million for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014.

Source: own graph based on hedging results

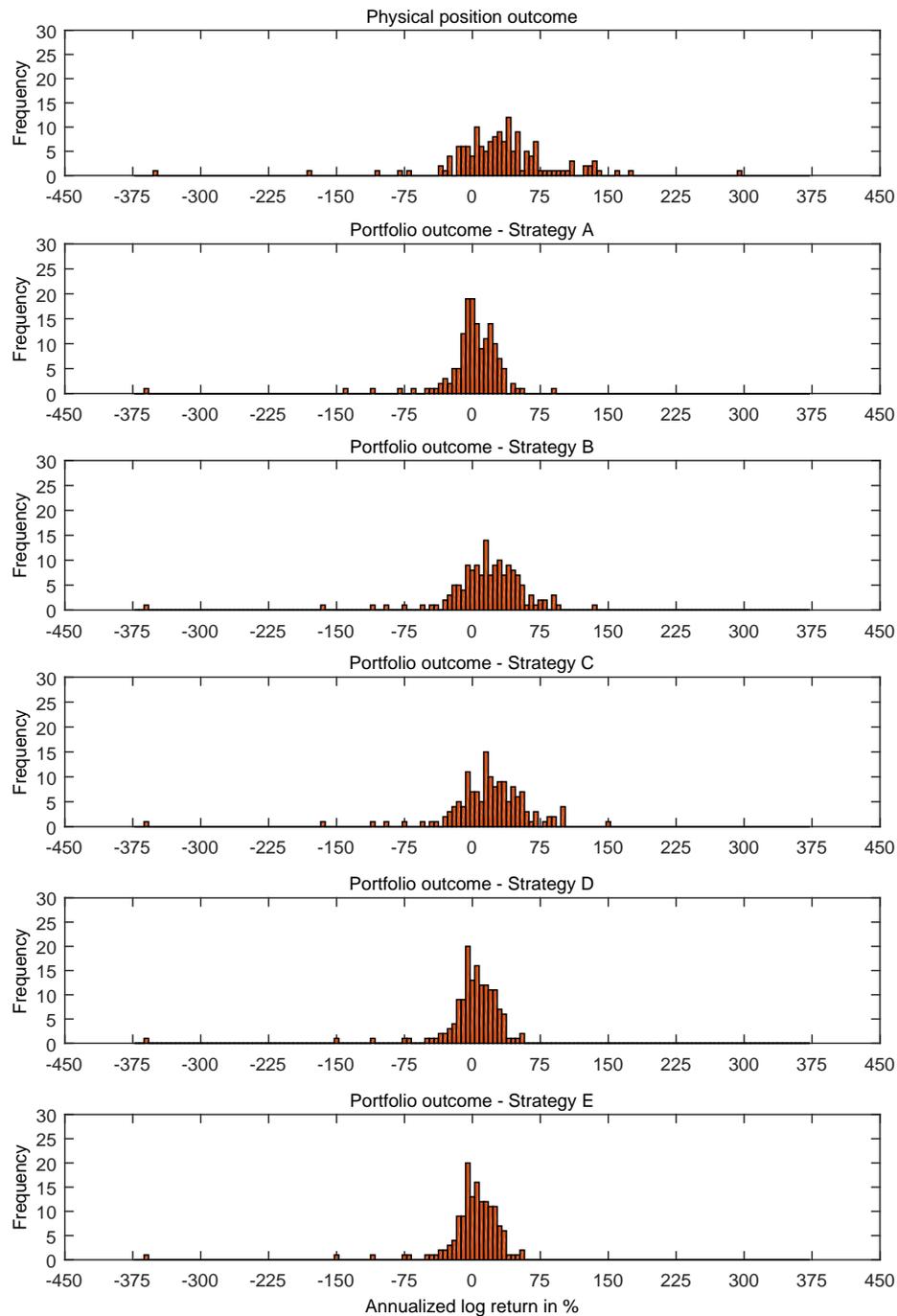
FIGURE 3.9: Histograms of physical position and portfolio outcomes for SPM 1



The graph shows histogram plots of the physical position and portfolio outcomes for SPM 1 as annualized log returns in % for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014. Strategies explained: A (long FFA), B (long at-the-money call option), C (long 10 % out-of-the-money call option), D (replicated long FFA), and E (zero-cost collar).

Source: own graph based on hedging results

FIGURE 3.10: Histograms of physical position and portfolio outcomes for SPM 2



The graph shows histogram plots of the physical position and portfolio outcomes for SPM 2 as annualized log returns in % for transactions from January 8<sup>th</sup>, 2009 until December 2<sup>nd</sup>, 2014. Strategies explained: A (long FFA), B (long at-the-money call option), C (long 10 % out-of-the-money call option), D (replicated long FFA), and E (zero-cost collar).

Source: own graph based on hedging results

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## PART II

# DERIVATIVE PRICING



## CHAPTER 4

# PRICING OF ASIAN OPTIONS FOR AFFINE GAUSSIAN DIFFUSIONS

WITH ALEXANDER SZIMAYER

### ABSTRACT

We develop a general pricing framework for continuously monitored geometric Asian call options for affine  $n$ -factor Gaussian diffusions and practically derive closed-form solutions for geometric Asian call options for three prominent mean-reversion commodity pricing models. In a numerical example, we examine the accuracy of our closed-form solutions via Monte Carlo (MC) simulation and use the geometric Asian call option as control variate in order to price an arithmetic Asian call option. The results confirm our closed-form solutions to be accurate and show that the MC control variate simulation approach provides a considerable variance reduction. This can be translated into substantial computation-time savings. Finally, we outline an extension to forward-start Asian options which are quite common in commodity markets. Our general approach and the presented results are neither prone to changes in model selection nor prone to changes in model parameters. Therefore, the applicability of our general pricing framework is by no means limited to the mean-reversion models or commodity markets considered within this study.

## 4.1 Introduction

Asian options are path-dependent options where the payoff is determined by the average of the underlying price within the period  $[T_0, T_1]$ . Such options allow market participants to hedge continuous risks or exposures (e.g., the average interest rate or average cost across the accounting year) (Fusai and Meucci, 2008; Hull, 2012; Kemna and Vorst, 1990). Longstaff (1995) states that an option on average interest rates is far more cost-effective for hedging purposes than a set of individual, standard interest rate options. Consequently, they provide a cheaper way to hedge regular, periodic cash flows (Zhang, 1998). Furthermore, Asian options protect against potential market manipulations of the underlying asset close to maturity and thus, are preferred in markets with lower trading volume than classical financial markets (Fusai and Meucci, 2008; Hubalek et al., 2014; Kemna and Vorst, 1990). This option form is predominantly used in commodity and currency markets (Geman, 2005; Zhang, 1998). Asian options typically come as arithmetic average or geometric average options and their type can either be ‘fixed strike’ or ‘floating strike’<sup>1</sup> (Hubalek et al., 2014). ‘Fixed strike’ Asian options are probably the most common from a trading volume perspective and thus, we focus on this type of Asian options within this study. From a monitoring basis of the average, Asian options can either be continuously or discretely monitored (Hubalek et al., 2014). The former is often preferred from a pricing perspective as the continuous average is, to a certain extent, mathematically easier to handle, while discretely monitoring is what practically happens in derivative markets as continuous prices rarely exist.

The pricing of Asian options can be a challenging task at times as it requires quite some mathematical effort to either grasp the distribution of the average value at maturity in closed form or to numerically evaluate it. For geometric Asian options, closed-form solutions can luckily be found for affine Gaussian diffusions as the distribution of the geometric average of an exponential of a Gaussian random process is itself lognormal. For arithmetic Asian options, however, the distribution of the arithmetic average is even unknown for an exponential of a Gaussian random process and thus, closed-form solutions cannot be found (Kemna and Vorst, 1990; Zhang, 1998). For these kind of options as well as for non-Gaussian diffusions, numerical methods remain as the only measure to price Asian options.

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<sup>1</sup> ‘Fixed strike’ Asian call options have the payoff profile  $(A_T - K)^+$ , whereas ‘floating strike’ Asian call options have the payoff profile  $(S_T - A_T)^+$ .

The aim of the paper is threefold. Firstly, we develop a general pricing framework for continuously monitored geometric Asian call options for affine  $n$ -factor Gaussian diffusions. Secondly, we practically derive closed-form solutions for geometric Asian call options for three prominent mean-reversion commodity pricing models. Finally, we use the geometric Asian call option as control variate in a Monte Carlo (MC) simulation in order to price an arithmetic Asian call option under these price dynamics and outline an extension to forward-start Asian options. The developed general pricing framework uses the characteristic function of the joint stochastic process of the underlying price dynamics and the geometric average to find the distribution of the geometric average. The three mean-reversion commodity pricing models for which we derive specific closed-form solutions for geometric Asian call options are the Schwartz (1997) one-factor model, the Schwartz and Smith (2000) two-factor model, and the Korn (2005) two-factor model. For the sake of completeness as well as to underline the validity of our chosen approach, we also derive a closed-form solution for the Black (1976) one-factor model. The obtained result can be rather simply converted to the result for classical Black and Scholes (1973) price dynamics developed by Kemna and Vorst (1990). Within the MC simulation, we examine the accuracy of the derived closed-form solutions as well as apply the geometric Asian call option as control variate to price an arithmetic Asian call option. Concerning model parameters, we rely on Prokopczuk (2011) as he estimated model parameters for the four price dynamics mentioned above for four different dry bulk freight futures.<sup>2</sup> Moreover, we outline an extension of the MC simulation to forward-start Asian options. Note that neither is our approach nor are our results prone to changes in model selection (as long as it is an affine  $n$ -factor Gaussian diffusion) or model parameters. Consequently, the applicability of our general pricing framework is by no means limited to mean-reversion models or commodity markets.

The remainder of the paper is structured as follows. Section 2 reviews the relevant academic literature. Section 3 presents the general as well as specific model price dynamics considered within the study. Section 4 develops the general pricing framework for geometric Asian call options for affine Gaussian diffusions and states the closed-form solutions (i.e., for the specific, considered model price dynamics). Section 5 elaborates on a numerical example in which we examine the accuracy of the

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<sup>2</sup> We simply choose the Prokopczuk (2011) model parameters for pure convenience reasons as he estimated a joint parameter set for the four price dynamics that we consider within this study. See subsection 4.5.2 for details.

developed closed-form solutions for geometric Asian call options in a MC simulation. Furthermore, we price arithmetic Asian call options in a MC simulation using the geometric Asian call option as control variate within the section. Finally, section 6 concludes the findings of this study and provides an outlook on further research opportunities in this area.

## 4.2 Review of academic literature

Asian options and especially the pricing of these derivative instruments have found quite some attention in the academic literature. The chosen pricing approaches can be mainly classified in the following four broad categories: closed-form and semi-analytical solutions,<sup>3</sup> approximate closed-form solutions, partial differential equation (PDE) methods and MC simulation. Note that the following literature review mainly concentrates on ‘fixed strike’-type Asian options as this is the focus of our effort.

Concerning closed-form solutions, Kemna and Vorst (1990) develop a closed-form solution for continuously monitored ‘fixed strike’ geometric Asian call options under Black and Scholes (1973) price dynamics. Angus (1999) extends the solution to evaluation within the averaging period. With respect to semi-analytical solutions under geometric Brownian motion (GBM) price dynamics, numerous efforts have been published over the years in the academic literature (see, for instance, Carr and Schröder (2004), Dufresne (2001), Dufresne (2005), Fu et al. (1999), Geman and Yor (1993), Schröder (2008), and Yor (1992) for continuously monitored ‘fixed strike’ arithmetic Asian options and Carverhill and Clewlow (1990) for discretely monitored arithmetic Asian options). All of these efforts, however, rely on numerical methods, such as the Laplace or Fourier transform, in order to compute option prices.

Leaving the world of Black and Scholes (1973) price dynamics but staying within the field of semi-analytical solutions, Kim and Wee (2014) provide a semi-analytical solution for continuously monitored geometric Asian options under the stochastic volatility price dynamics of Heston (1993). Wong and Cheung (2004) study the valuation of continuously monitored geometric Asian options for an underlying where the constant

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<sup>3</sup> We define closed-form solution as mathematical expressions involving everything up to the evaluation of the normal cumulative distribution function. We consider mathematical expressions requiring more complex numerical operations, such as the Laplace or Fourier transform, to be evaluated as semi-analytical solutions.

volatility of a GBM is replaced by stochastic volatility driven by a mean-reverting process. Turning to Lévy-type models, Albrecher (2004) and Albrecher and Predota (2004) study the pricing of discretely monitored arithmetic Asian options for exponential Lévy models and exponential Lévy models with normal inverse Gaussian (NIG) log returns. Fusai and Meucci (2008) study discretely monitored Asian options when the underlying follows a generic Lévy process. They find a semi-analytical solution for geometric Asian options involving a Fourier transform and a recursive algorithm for arithmetic Asian options. Concerning models incorporating stochastic volatility and jumps, Hubalek and Sgarra (2011) find a semi-analytical solution for continuously monitored geometric Asian options for Barndorff-Nielsen and Shephard (2001) price dynamics.

With respect to the use of the characteristic function within option pricing, Carr and Madan (1999) show how the Fourier transform can be used to value options if the characteristic function is known analytically. Bakshi and Madan (2000) provide a broader discussion on how the characteristic function is able to grasp the payoff range of derivatives. Concerning concrete application to specific price dynamics, Heston (1993) derives a semi-analytical solution for an European option under the stochastic volatility price dynamics he proposes within his study. Turning again to Asian options, Hubalek et al. (2014) develop a general pricing framework that yields a semi-analytical solution for continuously monitored geometric Asian options for general affine stochastic volatility models with jumps. Our effort can be considered as special case of Hubalek et al. (2014) as we focus only on affine Gaussian diffusions. This allows us to state closed-form solutions, whereas Hubalek et al. (2014) are limited to semi-analytical solutions due to the more general class of affine stochastic volatility models with jumps. Kyriakou et al. (2014) consider a mean-reversion model with seasonality jumps and Heston-type stochastic volatility as well as three nested models for energy commodities. They find a semi-analytical solutions for discretely monitored geometric Asian options. These efforts, however, rely again on numerical methods and thus, are also considered as semi-analytical solutions. Duffie et al. (2003) provide an excellent discussion of the required mathematical theory to treat the general class of affine stochastic processes including the derivation of the characteristic function for single or joint processes.

In terms of approximate closed-form solutions for arithmetic Asian options, the most well-known is the moment-matching approximation of Turnbull and Wakeman (1991)

under Black and Scholes (1973) price dynamics for continuously monitored arithmetic Asian options. Popular approximation for discretely monitored arithmetic Asian options include, for instance, Curran (1992) and Curran (1994) both conditioning on the geometric average as well as Levy (1997) extending the Turnbull and Wakeman (1991) approximation to discrete monitoring. Milevsky and Posner (1998) approximate the distribution of the log arithmetic average by the reciprocal gamma distribution.

Concerning PDE methods, Kemna and Vorst (1990) explicitly state a PDE describing the value of an Asian option. Rogers and Shi (1995), Alziary et al. (1997), and Zhang (2001), among others, study numerical solutions of this PDE using finite differences.

With respect to MC simulation, Boyle (1977) is the first one to apply it to option valuation. He also already applies the control variate as variance reduction technique. Kemna and Vorst (1990) are the first ones to apply the geometric Asian call option as control variate to price an arithmetic Asian option using a MC simulation. They achieve considerable variance reduction using the control variate due to the almost perfect correlation of geometric and arithmetic average. A MC simulation is nowadays one of the standard methods to price options and is also often used as benchmark for any closed-form or semi-analytical solutions, closed-form approximations, or PDE methods (see, for instance, Albrecher and Predota (2004), Curran (1994), Fu et al. (1999), Fusai and Meucci (2008), Kim and Wee (2014), Kyriakou et al. (2014), Milevsky and Posner (1998), Rogers and Shi (1995), and Turnbull and Wakeman (1991)).

With respect to the specific price dynamics we touch within this study, mean-reversion is a characteristic often attributed to commodity markets and numerous studies on the theoretical link as well as the empirical evidence have been performed. Mean-reversion is theoretically induced by either the convenience yield or time-varying risk premium existing in commodity markets, finds some empirical support, and several corresponding one-, two-, or multi-factor pricing models incorporating mean-reversion have been specified for commodity markets (see, among others, Bessembinder et al. (1995), Brennan (1991), Casassus and Collin-Dufresne (2005), Cortazar and Schwartz (1994), Cortazar and Naranjo (2006), Gibson and Schwartz (1990), Korn (2005), Lutz (2010), Schwartz (1997), or Schwartz and Smith (2000)).

Our paper contributes to the existing academic literature in three important ways. Firstly, we develop a general pricing framework for continuously monitored geometric

Asian call options for affine  $n$ -factor Gaussian diffusions. Secondly, we practically derive closed-form solutions for geometric Asian call options for three prominent mean-reversion commodity pricing models. Finally, we use the geometric Asian call option as control variate in a MC simulation in order to price an arithmetic Asian call option under these price dynamics and outline an extension to forward-start Asian options. Consequently, our findings for valuing geometric and arithmetic Asian call options are applicable to the set of affine Gaussian diffusions in a broad set of markets.

## 4.3 Price dynamics

On the one hand, we consider the general class of affine  $n$ -factor Gaussian diffusions within this effort in order to develop a general pricing framework for continuously monitored geometric Asian call options. On the other hand, we specifically apply the general pricing framework to four model price dynamics. The following subsections introduce the general price dynamics as well as the specific model price dynamics.

### 4.3.1 General price dynamics

The general price dynamics considered within this study are affine  $n$ -factor Gaussian diffusions. Consequently, we limit the general pricing framework of the subsequent section to these kind of Gaussian diffusions. Firstly, we start off with a basic definition.

**DEFINITION 1 (AFFINE  $n$ -FACTOR GAUSSIAN DIFFUSION):** *The underlying  $S$  with starting value  $S(0) = s$  is given by*

$$\ln(S(t)) = \sum_{i=1}^n w_i \cdot X_i(t) = \langle \mathbf{w}, \mathbf{X}(t) \rangle, \text{ for } t \geq 0, \quad (4.1)$$

where  $\mathbf{X}$  is an  $\mathbb{R}^n$ -valued affine Gaussian diffusion with dynamics

$$d\mathbf{X}(t) = (\mathbf{b} + \boldsymbol{\beta} \cdot \mathbf{X}(t)) dt + \boldsymbol{\Sigma} d\mathbf{W}(t), \text{ for } t \geq 0, \quad (4.2)$$

and starting value  $\mathbf{X}(0) = \mathbf{x} \in \mathbb{R}^n$  (satisfying  $\ln(s) = \langle \mathbf{w}, \mathbf{x} \rangle$ ), where  $\mathbf{w}$  is the vector containing the weights for each of the  $n$  factors,  $\mathbf{b} \in \mathbb{R}^n$  is the constant component of the drift, the matrix  $\boldsymbol{\beta} \in \mathbb{R}^{n \times n}$  is the drift component that is linear in  $\mathbf{X}$ ,  $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times d}$  is

the volatility matrix, and  $\mathbf{W}$  is a  $d$ -dimensional Wiener process under the risk-neutral measure  $\mathbb{Q}$ . Denote by  $\mathbf{a} = \frac{1}{2}\boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma}^\top$  the covariance matrix times  $\frac{1}{2}$ .

### 4.3.2 Specific model price dynamics

With respect to the specific model price dynamics considered within this study, we focus on the same set of price dynamics as in Prokopczuk (2011). This includes two one-factor and two two-factor models which are typically used for commodity pricing. The one-factor models considered are the Black (1976) model and the Schwartz (1997) one-factor model, whereas the two-factor models considered are the Schwartz and Smith (2000) two-factor model and the Korn (2005) two-factor model. With respect to notation, we largely follow Prokopczuk (2011). The price dynamics of the respective models are explained in more depth in the following two subsections.

#### 4.3.2.1 One-factor models

The first one-factor commodity pricing model is the model by Black (1976) which is a variation of the well-known Black and Scholes (1973) option pricing model. Black (1976) assumes that the log underlying price,  $S_t$ , follows an arithmetic Brownian motion (ABM). Under the real probability measure,  $\mathbb{P}$ , the price dynamics are the following:

$$\ln(S_t) \equiv \xi_t, \tag{4.3}$$

$$d\xi_t = a \, dt + \sigma_\xi \, dW_{\xi,t}^{\mathbb{P}}. \tag{4.4}$$

Within equation (4.4) above, the stochastic process is governed by the drift parameter,  $a$ , the volatility parameter,  $\sigma_\xi$ , and is driven by a standard Brownian motion or Wiener process,  $W_{\xi,t}^{\mathbb{P}}$ , with zero mean and unit variance rate. Under the risk-neutral probability measure,  $\mathbb{Q}$ , the price dynamics of equation (4.4) change to the following:

$$d\xi_t = a^* \, dt + \sigma_\xi \, dW_{\xi,t}^{\mathbb{Q}}, \tag{4.5}$$

$$\text{with } a^* = a - \lambda_\xi. \tag{4.6}$$

Under risk-neutral valuation, the drift parameter,  $a$ , is reduced by the market price of risk,  $\lambda_\xi$ , yielding the risk-neutral drift parameter,  $a^*$ .

The second one-factor commodity pricing model is the Schwartz (1997) one-factor model. Contrary to Black (1976), Schwartz (1997) assumes that commodity prices show signs of mean-reversion and models the log spot price by means of an Ornstein-Uhlenbeck (OU) process reverting to the long-term equilibrium log price level. Under the real probability measure,  $\mathbb{P}$ , the price dynamics are the following:

$$\ln(S_t) \equiv \xi_t, \quad (4.7)$$

$$d\xi_t = \kappa_\xi (a - \xi_t) dt + \sigma_\xi dW_{\xi,t}^{\mathbb{P}}. \quad (4.8)$$

Within equation (4.8) above, the stochastic process is governed by the mean-reversion parameter,  $\kappa_\xi$ , the long-term equilibrium log price level,  $a$ , the volatility parameter,  $\sigma_\xi$ , and is again driven by a standard Brownian motion or Wiener process,  $W_{\xi,t}^{\mathbb{P}}$ , with zero mean and unit variance rate. Under the risk-neutral probability measure,  $\mathbb{Q}$ , the price dynamics of equation (4.8) change to the following:

$$d\xi_t = \kappa_\xi (a^* - \xi_t) dt + \sigma_\xi dW_{\xi,t}^{\mathbb{Q}}, \quad (4.9)$$

$$\text{with } a^* = a - \lambda_\xi. \quad (4.10)$$

Under risk-neutral valuation, the long-term equilibrium log price level,  $a$ , is reduced by the market price of risk,  $\lambda_\xi$ , yielding the risk-neutral long-term equilibrium log price level,  $a^*$ .

#### 4.3.2.2 Two-factor models

Empirical research has shown that a second stochastic factor enhances the pricing accuracy in several commodity markets (e.g., Schwartz (1997), Schwartz and Smith (2000), Korn (2005)). Consequently, we also include two two-factor models in the study. The first one considered is the Schwartz and Smith (2000) two-factor model.<sup>4</sup> The model assumes that the log spot price follows a linear combination of two stochastic factors. The first stochastic factor,  $\xi_t$ , models the long-term equilibrium log price

<sup>4</sup> Note that this model is equivalent to the Gibson and Schwartz (1990) two-factor model or the Schwartz (1997) two-factor model because the stochastic factors of the model considered can be represented by a linear combination of the stochastic factor of the respective other model.

level by means of an ABM and the second stochastic factor,  $\chi_t$ , models short-term deviation by means of an OU process reverting to zero. Accordingly, the price dynamics under the real probability measure,  $\mathbb{P}$ , are the following:

$$\ln(S_t) \equiv \xi_t + \chi_t, \quad (4.11)$$

$$d\xi_t = a dt + \sigma_\xi dW_{\xi,t}^{\mathbb{P}}, \quad (4.12)$$

$$d\chi_t = -\kappa_\chi \cdot \chi_t dt + \sigma_\chi dW_{\chi,t}^{\mathbb{P}}. \quad (4.13)$$

Within equations (4.12) and (4.13) above, the long-term equilibrium log price level,  $\xi_t$ , is governed by the drift parameter,  $a$ , the volatility parameter,  $\sigma_\xi$ , and is driven by a standard Brownian motion or Wiener process,  $W_{\xi,t}^{\mathbb{P}}$ , with zero mean and unit variance rate. The short-term deviations,  $\chi_t$ , are governed by the mean-reversion parameter,  $\kappa_\chi$ , the volatility parameter,  $\sigma_\chi$ , and are driven as well by a standard Brownian motion or Wiener process,  $W_{\chi,t}^{\mathbb{P}}$ . The two Wiener processes,  $W_{\xi,t}^{\mathbb{P}}$  and  $W_{\chi,t}^{\mathbb{P}}$ , are correlated with  $dW_{\xi,t}^{\mathbb{P}} dW_{\chi,t}^{\mathbb{P}} = \rho_{\xi,\chi} dt$  where  $\rho_{\xi,\chi}$  represents the instantaneous correlation. Under the risk-neutral probability measure,  $\mathbb{Q}$ , the price dynamics of equations (4.12) and (4.13) change to the following:

$$d\xi_t = a^* dt + \sigma_\xi dW_{\xi,t}^{\mathbb{Q}}, \quad (4.14)$$

$$d\chi_t = (-\kappa_\chi \cdot \chi_t - \lambda_\chi) dt + \sigma_\chi dW_{\chi,t}^{\mathbb{Q}}, \quad (4.15)$$

$$\text{with } a^* = a - \lambda_\xi. \quad (4.16)$$

Under risk-neutral valuation, the drift parameter of the long-term equilibrium,  $a$ , is reduced by the market price of risk,  $\lambda_\xi$ , yielding the risk-neutral drift of the long-term equilibrium,  $a^*$ . The short-term deviations revert under risk-neutrality to  $-\frac{\lambda_\chi}{\kappa_\chi}$  rather than to zero. The two Wiener processes,  $W_{\xi,t}^{\mathbb{Q}}$  and  $W_{\chi,t}^{\mathbb{Q}}$ , are again correlated with  $dW_{\xi,t}^{\mathbb{Q}} dW_{\chi,t}^{\mathbb{Q}} = \rho_{\xi,\chi} dt$ .

The second two-factor model considered is the Korn (2005) two-factor model.<sup>5</sup> The model modifies the Schwartz and Smith (2000) two-factor model. Korn (2005) replaced the non-stationary long-term equilibrium log price level (i.e., which is driven by an ABM in Schwartz and Smith (2000)) by an OU process reverting to the long-

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<sup>5</sup> Note that the version of the Korn (2005) two-factor model is slightly different as in the original source. Korn (2005) rather assumes that the log spot price follows the following definition:  $\ln(S_t) \equiv \frac{\kappa_\chi}{\kappa_\xi + \kappa_\chi} \xi_t + \chi_t - \frac{\kappa_\xi \cdot a}{\kappa_\xi + \kappa_\chi}$ . We use the simplified definition as in Prokopczuk (2011) mainly for convenience reasons for the later numerical example.

term equilibrium log price level leading to stationary spot prices. Accordingly, the price dynamics under the real probability measure,  $\mathbb{P}$ , for this model are the following:

$$\ln(S_t) \equiv \xi_t + \chi_t, \quad (4.17)$$

$$d\xi_t = \kappa_\xi (a - \xi_t) dt + \sigma_\xi dW_{\xi,t}^{\mathbb{P}}, \quad (4.18)$$

$$d\chi_t = -\kappa_\chi \cdot \chi_t dt + \sigma_\chi dW_{\chi,t}^{\mathbb{P}}. \quad (4.19)$$

Within equations (4.18) and (4.19) above, the long-term equilibrium log price level,  $\xi_t$ , is governed by the mean-reversion parameter,  $\kappa_\xi$ , the long-term equilibrium log price level,  $a$ , the volatility parameter,  $\sigma_\xi$ , and is driven by a standard Brownian motion or Wiener process,  $W_{\xi,t}^{\mathbb{P}}$ , with zero mean and unit variance rate. The short-term deviations,  $\chi_t$ , are governed by the mean-reversion parameter,  $\kappa_\chi$ , the volatility parameter,  $\sigma_\chi$ , and are driven as well by a standard Brownian motion or Wiener process,  $W_{\chi,t}^{\mathbb{P}}$ . The two Wiener processes,  $W_{\xi,t}^{\mathbb{P}}$  and  $W_{\chi,t}^{\mathbb{P}}$ , are correlated with  $dW_{\xi,t}^{\mathbb{P}} dW_{\chi,t}^{\mathbb{P}} = \rho_{\xi,\chi} dt$  where  $\rho_{\xi,\chi}$  represents the instantaneous correlation. Under the risk-neutral probability measure,  $\mathbb{Q}$ , the price dynamics of equations (4.18) and (4.19) change to the following:

$$d\xi_t = \kappa_\xi (a^* - \xi_t) dt + \sigma_\xi dW_{\xi,t}^{\mathbb{Q}}, \quad (4.20)$$

$$d\chi_t = (-\kappa_\chi \cdot \chi_t - \lambda_\chi) dt + \sigma_\chi dW_{\chi,t}^{\mathbb{Q}}, \quad (4.21)$$

$$\text{with } a^* = a - \lambda_\xi. \quad (4.22)$$

Under risk-neutral valuation, the long-term equilibrium log price level,  $a$ , is reduced by the market price of risk,  $\lambda_\xi$ , yielding the risk-neutral long-term equilibrium log price level,  $a^*$ . The short-term deviations revert under risk-neutrality to  $-\frac{\lambda_\chi}{\kappa_\chi}$  rather than to zero. The two Wiener processes,  $W_{\xi,t}^{\mathbb{Q}}$  and  $W_{\chi,t}^{\mathbb{Q}}$ , are again correlated with  $dW_{\xi,t}^{\mathbb{Q}} dW_{\chi,t}^{\mathbb{Q}} = \rho_{\xi,\chi} dt$ .

## 4.4 Geometric Asian options

In order to price an arithmetic Asian call option in a MC setting using the geometric Asian call option as control variate, closed-form solutions for geometric Asian call options under the different pricing dynamics need to be derived. Firstly, we provide a general pricing framework applicable to affine  $n$ -factor Gaussian diffusions including

functional integrals of these diffusions, such as the geometric average. Secondly, we apply this general pricing framework to the specific model price dynamics considered within this study and provide individual closed-form solutions for geometric Asian call options.

#### 4.4.1 General pricing framework for Gaussian diffusions

As we set out to develop closed-form solutions for geometric Asian call options for affine  $n$ -factor Gaussian diffusions, the distribution of the geometric average needs to be determined. As already stated in subsection 4.1, the distribution of the geometric average of a lognormally distributed variable is itself lognormal. As all price dynamics considered within this study model log spot price, the log spot price as well as the log of the geometric average is normally distributed. Accordingly, the mean and variance parameters of the distribution of the log geometric average at the end of the averaging period need to be derived. Once these two distribution parameters,  $\mu_G$  and  $\sigma_G^2$ , are known, a specific, closed-form option pricing formula can be specified for a geometric Asian call option.

The following paragraphs provide the required mathematical theory and explain the derivation of the distribution parameters of the log geometric average using the concept of the characteristic function. We start off with the payoff of the geometric Asian call option on the underlying  $S$  which is described by the running average of the log price of the underlying

$$G(t; T_0, T_1) = \mathbb{1}_{t \geq T_0} \frac{1}{T_1 - T_0} \int_{T_0}^t \ln(S(u)) \, du = \mathbb{1}_{t \geq T_0} \int_{T_0}^t \langle \mathbf{v}, \mathbf{X}(s) \rangle \, ds, \quad (4.23)$$

for  $T_0 = 0 \leq t \leq T_1$ , with  $\mathbf{v} = \frac{\mathbf{w}}{T_1 - T_0}$ .

Then,  $\exp(G(T_1; T_0, T_1))$  is the geometric average of  $S$  over the interval  $[T_0, T_1]$ .

The price of a geometric Asian call option on the underlying  $S$  with averaging period  $[T_0, T_1]$ , settlement date  $T$ , and strike price  $K$  is the risk-neutral expected payoff discounted at the risk free rate,  $r$ . This is, for  $t \leq T$ ,

$$C_G(t, \mathbf{x}, g; T_0, T_1, T) = e^{-r \cdot (T-t)} \cdot \mathbb{E}_{t, \mathbf{x}, g} \left[ (\exp(G(T_1; T_0, T_1)) - K)^+ \right], \quad (4.24)$$

where  $T_0 < T_1 \leq T$  and  $\mathbb{E}_{t,\mathbf{x},g} = \mathbb{E}[\cdot | \mathbf{X}(t) = \mathbf{x}, G(t; T_0, T_1) = g]$  is the corresponding conditional risk-neutral expectation. For  $t \leq T_0$ , it follows that  $G(t; T_0, T_1) = 0$  and thus, conditioning on  $G$  becomes redundant and we write  $\mathbb{E}_{t,\mathbf{x}}[\cdot] = \mathbb{E}[\cdot | \mathbf{X}(t) = \mathbf{x}]$  instead. For  $t = 0$ , we omit the time variable and write  $\mathbb{E}_{\mathbf{x}}[\cdot] = \mathbb{E}[\cdot | \mathbf{X}(0) = \mathbf{x}]$  and  $\mathbb{E}_{\mathbf{x},g} = \mathbb{E}[\cdot | \mathbf{X}(0) = \mathbf{x}, G(0; T_0, T_1) = g]$ , respectively.

The following THEOREM 1, COROLLARY 1, and COROLLARY 2 provide closed-form solutions for affine  $n$ -factor Gaussian diffusions for different points in time (i.e., at inception of, within, and before the averaging period, respectively).

**THEOREM 1 (PRICE OF A GEOMETRIC ASIAN CALL OPTION AT INCEPTION OF THE AVERAGING PERIOD):** *The price of a geometric Asian call option on the underlying  $S = e^{\langle \mathbf{w}, \mathbf{X} \rangle}$  with averaging period  $[0, T_1]$ , settlement date  $T$ , and strike price  $K$  is at time  $t = T_0 = 0$  given by*

$$C_G(0, \mathbf{x}, 0; 0, T_1, T) = e^{-r \cdot T} \left( e^{\mu_G + \frac{1}{2} \sigma_G^2} \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G} \right) - K \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K)}{\sigma_G} \right) \right), \quad (4.25)$$

$$\text{where } \mu_G = \frac{1}{T_1} \left\langle \mathbf{w}, \int_0^{T_1} e^{\beta \cdot t} dt \cdot \mathbf{x} \right\rangle + \frac{1}{T_1} \left\langle \mathbf{w}, \int_0^{T_1} \int_0^t e^{\beta \cdot s} ds dt \cdot \mathbf{b} \right\rangle \quad (4.26)$$

$$\sigma_G^2 = \frac{1}{T_1^2} \int_0^{T_1} \left\| \boldsymbol{\Sigma}^\top \int_0^t e^{\beta \tau \cdot s} ds \cdot \mathbf{w} \right\|_2^2 dt. \quad (4.27)$$

For the proof of THEOREM 1, see appendix A.1.1 from page 199 onwards.

**COROLLARY 1 (PRICE OF A GEOMETRIC ASIAN CALL OPTION WITHIN THE AVERAGING PERIOD):** *The price of a geometric Asian call option on the underlying  $S = e^{\langle \mathbf{w}, \mathbf{X} \rangle}$  with averaging period  $[T_0, T_1]$ , settlement date  $T$ , and strike price  $K$  is at time  $t \in [T_0, T_1]$  given by*

$$C_G(t, \mathbf{x}, g; T_0, T_1, T) = e^{-r \cdot (T-t)} \left( e^{\mu_G + \frac{1}{2} \sigma_G^2} \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G} \right) - K \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K)}{\sigma_G} \right) \right), \quad (4.28)$$

$$\begin{aligned} \text{where } \mu_G &= g + \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1-t} e^{\beta \cdot u} du \cdot \mathbf{x} \right\rangle \\ &+ \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1-t} \int_0^u e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle, \end{aligned} \quad (4.29)$$

$$\sigma_G^2 = \frac{1}{(T_1 - T_0)^2} \int_0^{T_1-t} \left\| \Sigma^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du, \quad (4.30)$$

$$\text{and } g = G(t, T_0, T_1) = \frac{1}{T_1 - T_0} \int_{T_0}^t \ln(S(u)) du = \int_{T_0}^t \langle \mathbf{v}, \mathbf{X}(s) \rangle ds. \quad (4.31)$$

For the proof of COROLLARY 1, see appendix A.1.2 from page 201 onwards.

**COROLLARY 2 (PRICE OF A GEOMETRIC ASIAN CALL OPTION BEFORE THE AVERAGING PERIOD):** *The price of a geometric Asian call option on the underlying  $S = e^{\langle \mathbf{w}, \mathbf{X} \rangle}$  with averaging period  $[0, T_1]$ , settlement date  $T$ , and strike price  $K$  is at time  $t \leq T_0$  given by*

$$\begin{aligned} C_G(t, \mathbf{x}, g; T_0, T_1, T) &= e^{-r \cdot (T-t)} \left( e^{\mu_G + \frac{1}{2} \sigma_G^2} \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G} \right) \right. \\ &\quad \left. - K \cdot \mathcal{N} \left( \frac{\mu_G - \ln(K)}{\sigma_G} \right) \right), \end{aligned} \quad (4.32)$$

$$\begin{aligned} \text{where } \mu_G &= \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1-T_0} e^{\beta \cdot (T_0-t+u)} du \cdot \mathbf{x} \right\rangle \\ &+ \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1-T_0} \int_0^{T_0-t+u} e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle, \end{aligned} \quad (4.33)$$

$$\begin{aligned} \sigma_G^2 &= \frac{1}{(T_1 - T_0)^2} \int_0^{T_1-T_0} \left\| \Sigma^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du \\ &+ \frac{1}{(T_1 - T_0)^2} \int_0^{T_0-t} \left\| \Sigma^\top \int_0^{T_1-T_0} e^{\beta^\top \cdot (u+s)} ds \cdot \mathbf{w} \right\|_2^2 du. \end{aligned} \quad (4.34)$$

For the proof of COROLLARY 2, see appendix A.1.3 from page 201 onwards.

### 4.4.2 Closed-form solutions for considered price dynamics

Now, we apply the developed general pricing framework for geometric Asian call options presented above to the specific model price dynamics introduced in subsection 4.3.2. We first derive closed-form solutions using THEOREM 1 for the case at the inception of the averaging period (i.e.,  $t = T_0 = 0$ ). For brevity of the notation, we assume that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ).<sup>6</sup> Note that we only provide the resulting closed-form solutions in the main body of the paper due to the lengthiness of the required calculus. The structure of the resulting closed-form solutions is basically equation (4.25) with varying terms for  $\mu_G$  and  $\sigma_G^2$ .

For the case within the averaging period (i.e.,  $t \in [T_0, T_1) = [T_0, T)$ ), the resulting closed-form solutions using COROLLARY 1 are provided in the respective appendix sections. For the case before the averaging period (i.e.,  $t \leq T_0$ ), we leave the application of COROLLARY 2 to the reader.

#### 4.4.2.1 Black (1976) one-factor model

The closed-form solution price of a geometric Asian call option for the Black (1976) one-factor model on the underlying  $S$ , at time  $t = T_0 = 0$  with averaging period  $[T_0, T_1] = [T_0, T]$ , settlement date  $T$ , and strike price  $K$  is given by

$$C_G = e^{\mu_G + \frac{1}{2}\sigma_G^2 - r \cdot T} \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G}\right) - e^{-r \cdot T} \cdot K \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K)}{\sigma_G}\right), \quad (4.35)$$

$$\text{with } \mu_G = \xi_0 + \frac{1}{2}a^* \cdot T,$$

$$\text{and } \sigma_G^2 = \frac{1}{3}\sigma_\xi^2 \cdot T.$$

Note that equation (4.35) is equal to the one provided by Kemna and Vorst (1990) if we set  $a^* = r - \frac{1}{2}\sigma_\xi^2$ .

For details on the derivation of equation (4.35) above as well as the closed-form solution for the case within the averaging period (i.e.,  $t \in [T_0, T_1) = [T_0, T)$ ), see appendix A.2.1 from page 204 onwards.

<sup>6</sup> This assumption can easily be relaxed by replacing  $T$  by  $T_1$  in the  $\mu_G$ - and  $\sigma_G^2$ -terms of the closed-form solutions, respectively.

#### 4.4.2.2 Schwartz (1997) one-factor model

The closed-form solution price of a geometric Asian call option for the Schwartz (1997) one-factor model on the underlying  $S$ , at time  $t = T_0 = 0$  with averaging period  $[T_0, T_1] = [T_0, T]$ , settlement date  $T$ , and strike price  $K$  is given by

$$C_G = e^{\mu_G + \frac{1}{2}\sigma_G^2 - r \cdot T} \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G}\right) - e^{-r \cdot T} \cdot K \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K)}{\sigma_G}\right), \quad (4.36)$$

$$\text{with } \mu_G = \frac{\xi_0}{\kappa_\xi \cdot T} \left(1 - e^{-\kappa_\xi \cdot T}\right) + a^* - \frac{a^*}{\kappa_\xi \cdot T} \left(1 - e^{-\kappa_\xi \cdot T}\right),$$

$$\text{and } \sigma_G^2 = \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} \left(2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi \cdot T} - e^{-2 \cdot \kappa_\xi \cdot T} - 3\right).$$

For details on the derivation of equation (4.36) above as well as the closed-form solution for the case within the averaging period (i.e.,  $t \in [T_0, T_1) = [T_0, T)$ ), see appendix A.2.2 from page 206 onwards.

#### 4.4.2.3 Schwartz and Smith (2000) two-factor model

The closed-form solution price of a geometric Asian call option for the Schwartz and Smith (2000) two-factor model on the underlying  $S$ , at time  $t = T_0 = 0$  with averaging period  $[T_0, T_1] = [T_0, T]$ , settlement date  $T$ , and strike price  $K$  is given by

$$C_G = e^{\mu_G + \frac{1}{2}\sigma_G^2 - r \cdot T} \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G}\right) - e^{-r \cdot T} \cdot K \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K)}{\sigma_G}\right), \quad (4.37)$$

$$\text{with } \mu_G = \xi_0 + \frac{\lambda_0}{\kappa_\chi \cdot T} \left(1 - e^{-\kappa_\chi \cdot T}\right) + \frac{1}{2}a^* \cdot T + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} \left(1 - \kappa_\chi \cdot T - e^{-\kappa_\chi \cdot T}\right),$$

$$\text{and } \sigma_G^2 = \frac{1}{3}\sigma_\xi^2 \cdot T + \frac{\rho_{\xi,\chi}\sigma_\xi\sigma_\chi}{\kappa_\chi^3 \cdot T^2} \left(\kappa_\chi^2 \cdot T^2 + 2 \cdot \kappa_\chi \cdot T \cdot e^{-\kappa_\chi \cdot T} + 2 \cdot e^{-\kappa_\chi \cdot T} - 2\right) \\ + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} \left(2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi \cdot T} - e^{-2 \cdot \kappa_\chi \cdot T} - 3\right).$$

For details on the derivation of equation (4.37) above as well as the closed-form solution for the case within the averaging period (i.e.,  $t \in [T_0, T_1) = [T_0, T)$ ), see appendix A.2.3 from page 210 onwards.

#### 4.4.2.4 Korn (2005) two-factor model

The closed-form solution price of a geometric Asian call option for the Korn (2005) two-factor model on the underlying  $S$ , at time  $t = T_0 = 0$  with averaging period  $[T_0, T_1] = [T_0, T]$ , settlement date  $T$ , and strike price  $K$  is given by

$$C_G = e^{\mu_G + \frac{1}{2}\sigma_G^2 - r \cdot T} \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K) + \sigma_G^2}{\sigma_G}\right) - e^{-r \cdot T} \cdot K \cdot \mathcal{N}\left(\frac{\mu_G - \ln(K)}{\sigma_G}\right), \quad (4.38)$$

$$\begin{aligned} \text{with } \mu_G &= \frac{\xi_0}{\kappa_\xi \cdot T} \left(1 - e^{-\kappa_\xi \cdot T}\right) + \frac{\chi_0}{\kappa_\chi \cdot T} \left(1 - e^{-\kappa_\chi \cdot T}\right) \\ &+ \frac{a^*}{\kappa_\xi \cdot T} \left(e^{-\kappa_\xi \cdot T} + \kappa_\xi \cdot T - 1\right) + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} \left(1 - e^{-\kappa_\chi \cdot T} - \kappa_\chi \cdot T\right), \end{aligned}$$

$$\begin{aligned} \text{and } \sigma_G^2 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} \left(2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi \cdot T} - e^{-2 \cdot \kappa_\xi \cdot T} - 3\right) \\ &+ \frac{2 \cdot \rho_{\xi, \chi} \sigma_\xi \sigma_\chi}{\kappa_\xi^2 \cdot \kappa_\chi^2 (\kappa_\xi + \kappa_\chi) T^2} \left(\kappa_\xi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\chi \cdot T} + \kappa_\xi \cdot \kappa_\chi \cdot T (\kappa_\xi + \kappa_\chi)\right. \\ &\left. + \kappa_\chi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\xi \cdot T} - \kappa_\xi \cdot \kappa_\chi \cdot e^{-(\kappa_\xi + \kappa_\chi) \cdot T} - (\kappa_\xi + \kappa_\chi)^2 + \kappa_\xi \cdot \kappa_\chi\right) \\ &+ \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} \left(2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi \cdot T} - e^{-2 \cdot \kappa_\chi \cdot T} - 3\right). \end{aligned}$$

For details on the derivation of equation (4.38) above as well as the closed-form solution for the case within the averaging period (i.e.,  $t \in [T_0, T_1] = [T_0, T)$ ), see appendix A.2.4 from page 216 onwards.

## 4.5 Numerical example

In order to test the accuracy of the developed closed-form solutions for geometric Asian call options as well as to apply them in a MC control variate setting to price arithmetic Asian call options, we conduct the following numerical example using model parameters from Prokopczuk (2011).

### 4.5.1 Freight options

For the numerical example, we consider an arithmetic Asian call option that is written on spot freight rates. Freight options are Asian options or, more specifically, European fixed-strike, arithmetic average options on the underlying spot freight rate. Alternatively, they can also be seen as plain European options on fixed-strike, arithmetic average Forward Freight Agreements (FFAs) (as their maturity date is exactly the same). FFAs are settled also against the arithmetic average of the spot freight rate within the delivery period, which is typically one month for the relevant dry bulk Capesize or Panamax contracts. As these contracts are cleared via clearing houses, traded on hybrid exchanges, and also standardized to a certain extent, they can actually be seen as futures-like contracts. From a pricing perspective for freight options, the model of Turnbull and Wakeman (1991) is quite heavily used among practitioners and, for instance, The Baltic Exchange quotes at-the-money implied volatilities for freight options based on the Turnbull and Wakeman (1991) model. The time to maturity available of these kind of products reaches up to the seven next calendar-years. For more information on FFAs or freight options in general, see, for instance, the comprehensive effort of Alizadeh and Nomikos (2009). For detailed product descriptions of traded dry bulk freight options, see, for instance, Appendix 5 to the Rulebook of NOS Clearing ASA<sup>7</sup> (NOS, 2014).

### 4.5.2 Model parameters

Within the numerical example, we rely on model parameters estimated by Prokopczuk (2011). As already indicated in section 4.1, we simply choose the Prokopczuk (2011) model parameters for pure convenience reasons as he estimated a joint parameter set for the four specific model price dynamics that we consider within this study. This allows to compare the results across the four price dynamics. Obviously, we could have estimated a joint parameter set for the four price dynamics for any other commodity data set. However, we deemed this out of scope of our study as any joint parameter set would serve equally well for the purpose of our numerical example and the estimation procedure for these models has been extensively described, for instance, in the respective original sources of the models.

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<sup>7</sup> NOS Clearing ASA (part of NASDAQ OMX) is a clearing house for over-the-counter (OTC)-traded derivatives, such as freight options.

Prokopczuk (2011) estimated model parameters for four dry bulk freight futures contracts (i.e., Capesize routes C4 and C7 as well as Panamax routes P2A and P3A) based on weekly data from January 2005 until December 2008. For a more detailed discussion including summary statistics of the raw data, see the ‘data’ section in Prokopczuk (2011). In order to estimate the model parameters, Prokopczuk (2011) applied a Kalman filter maximum likelihood approach. The estimated model parameters are shown in TABLE 4.1.<sup>8</sup> With respect to significance of the estimated parameters, it should be noticed that only the estimations of the Schwartz (1997) one-factor model are significant for all parameters for all routes. Each other model has at least one parameter that is not significant for each route. This may either result from the short time frame that has been used to estimate the models or from the fact that the freight market experienced quite some turbulences in 2008. The latter also becomes apparent in the rather high volatilities for the different models. Nonetheless, this should not affect the validity of the conclusions we draw from our numerical computations.

### 4.5.3 Monte Carlo simulation set-up

For the MC simulation, we have two objectives. Firstly, we want to test the accuracy of the developed closed-form solutions from subsection 4.4.2 by comparing them against a MC price of a geometric Asian call option. Secondly, we want to price an arithmetic Asian call option using a MC simulation and show the benefit of using the geometric Asian call option as control variate.

We assume that we are at the beginning of the averaging period,  $t = T_0 = 0$ . The option has a remaining time to maturity of  $T = T_1 = \frac{1}{12} = 0.08\bar{3}$  years or 1 month. We simulate  $k = 100,000$  paths with  $n = 21$  time steps for each path. We choose  $n = 21$  in order to reflect one price per trading day as it is common for discretely sampled Asian options in practice. Concerning our first objective, this induces, on the one hand, a bias in form of a discretization error as our closed-form solution assumes continuous monitoring. In this case, we define bias or discretization error as the deviation of the MC price from the closed-form solution.<sup>9</sup> This bias can be reduced

<sup>8</sup> Note that we changed the sign of the parameter  $a^*$  for the Black (1976) one-factor model for route P2A as this is consistent with the other routes. We assume this to be a misprint in Prokopczuk (2011).

<sup>9</sup> Note that we anchor the bias at the continuously monitored closed-form solution although Asian

TABLE 4.1: Model parameter estimates from Prokopczuk (2011)

Parameter	Capesize route C4				Panamax route P2A			
	Black (1976)	Schwartz (1997)	Smith (2000)	Korn (2005)	Black (1976)	Schwartz (1997)	Smith (2000)	Korn (2005)
$\xi_0$	3.0459****	3.0432****	2.8243****	2.9810****	10.7056****	10.7048****	10.5035****	10.9141****
$k\zeta$	—	0.2830****	—	0.8017****	—	0.1882****	—	0.4641****
$a^*$	-0.2748	1.9958****	-0.2777	4.0011****	-0.3501	8.5831****	-0.5056	9.7090****
$\sigma\zeta$	0.5109****	0.5497****	0.4446****	1.2091****	0.6167****	0.6371****	0.6159****	1.3443****
$X_0$	—	—	0.3531****	0.6098	—	—	0.2115	0.0403
$k\chi$	—	—	3.0194****	1.6660****	—	—	2.1339****	1.2128****
$\lambda\chi$	—	—	-0.2661	0.0768	—	—	-0.6127**	0.5982
$\sigma\chi$	—	—	0.5637****	1.2247****	—	—	0.5997****	1.3028****
$\rho_{\zeta,\chi}$	—	—	-0.0367	-0.8543****	—	—	-0.3450****	-0.8691****
Log likelihood	2,992.1	3,072.5	3,675.2	3,679.8	2,989.5	3,068.0	3,658.7	3,656.6
	Capesize route C7				Panamax route P3A			
Parameter	Schwartz and Korn (2005)				Schwartz and Korn (2005)			
	Black (1976)	Schwartz (1997)	Smith (2000)	Korn (2005)	Black (1976)	Schwartz (1997)	Smith (2000)	Korn (2005)
$\xi_0$	3.0220****	3.0205****	2.7744****	3.1708****	10.5389****	10.5500****	10.3743****	10.8388****
$k\zeta$	—	0.2638****	—	0.7470****	—	0.2140****	—	0.1018****
$a^*$	-0.2854	1.8796****	-0.2976	4.2374****	-0.3717	8.4965****	-0.4192	8.1302****
$\sigma\zeta$	0.4679****	0.5064****	0.4179****	1.1736****	0.6691****	0.7146****	0.6204****	0.6733****
$X_0$	—	—	0.3972****	0.8612*	—	—	0.2231*	-0.8681****
$k\chi$	—	—	2.7654****	1.5203****	—	—	3.9719****	2.9352****
$\lambda\chi$	—	—	-0.2866	-0.5794	—	—	-0.4656	1.7878
$\sigma\chi$	—	—	0.5187****	1.2032****	—	—	0.7458****	0.9781****
$\rho_{\zeta,\chi}$	—	—	-0.0821	-0.8714****	—	—	0.0011	-0.2375*
Log likelihood	3,071.0	3,152.6	3,874.2	3,878.2	2,780.4	2,832.5	3,476.3	3,441.3

The table shows estimated model parameters from Prokopczuk (2011) for four freight futures based on weekly data from January 2005 until December 2008. The estimation was done using a Kalman filter maximum likelihood approach. Note that we changed the sign of the parameter  $a^*$  for the Black (1976) one-factor model for route P2A. \* indicates significance at the 10 % level, \*\* at the 5 % level, and \*\*\*\* at the 1 % level.

by increasing the number of simulated time steps,  $n$  (Seydel, 2012). On the other hand, our first objective also suffers from a simulation error as every MC simulation does. This error can be measured by means of the MC standard deviation and can be reduced by increasing the number of simulated price paths,  $k$  (Seydel, 2012). For our first objective, the bias or discretization error is of utmost importance in judging our results if  $k$  is selected sufficiently large. With respect to our second objective, bias or discretization error is negligible as the continuously monitored closed-form solution plays only a minor role in the MC control variate approach. Both normal MC and MC control variate approach are subject to  $n$  time steps. The simulation error, however, is a key criteria when comparing both approaches and judging the benefit of the MC control variate approach.

The simulation is conducted in MATLAB R2015a using seed 1 of the ‘Mersenne twister’ as random number generator. The standard normal random numbers are drawn in a  $n \times k$  matrix. For the two-factor models, the second, correlated standard normal random number matrix is obtained using another independent standard normal random number  $n \times k$  matrix which is subsequently adjusted using the Cholesky factorization (Glasserman, 2003). This gives us

$$\mathbf{Z}_1 = \mathbf{z}_1, \quad (4.39)$$

$$\mathbf{Z}_2 = \rho_{\xi, \chi}(\Delta t) \cdot \mathbf{z}_1 + \sqrt{1 - \rho_{\xi, \chi}^2(\Delta t)} \cdot \mathbf{z}_2, \quad (4.40)$$

with  $\mathbf{z}_1$  and  $\mathbf{z}_2$  being two independent standard normal random number  $n \times k$  matrices.

As we discretely simulate the price paths, the instantaneous correlation,  $\rho_{\xi, \chi}$ , between the two Wiener processes in the two-factor models needs to be adjusted to the chosen length of the time step,  $\Delta t = \frac{T}{n}$ . The time step-dependent correlation,  $\rho_{\xi, \chi}(\Delta t)$ , is obtained for the

Schwartz and Smith (2000) two-factor model by

$$\rho_{\xi, \chi}(\Delta t) = \frac{\text{Cov}_{\xi, \chi}}{\sigma_{\xi} \cdot \sigma_{\chi}} = \frac{\rho_{\xi, \chi}}{\sqrt{\Delta t}} \sqrt{\frac{2(1 - e^{-\kappa_{\chi} \cdot \Delta t})^2}{\kappa_{\chi}(1 - e^{-2 \cdot \kappa_{\chi} \cdot \Delta t})}} \quad (4.41)$$

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options are usually discretely monitored in practice. Our goal here is to examine the accuracy of the developed closed-form solutions knowing that these have an inherent bias with respect to the monitoring frequency of Asian options in practice. The closed-form solutions, however, focus on a continuously monitored Asian options because these are mathematically easier to handle.

and for the Korn (2005) two-factor model by

$$\rho_{\xi,\chi}(\Delta t) = \frac{\text{Cov}_{\xi,\chi}}{\sigma_{\xi} \cdot \sigma_{\chi}} = \rho_{\xi,\chi} \cdot \sqrt{\frac{4 \cdot \kappa_{\xi} \cdot \kappa_{\chi} \left(1 - e^{-(\kappa_{\xi} + \kappa_{\chi}) \cdot \Delta t}\right)^2}{(\kappa_{\xi} + \kappa_{\chi})^2 (1 - e^{-2 \cdot \kappa_{\xi} \cdot \Delta t}) (1 - e^{-2 \cdot \kappa_{\chi} \cdot \Delta t})}}. \quad (4.42)$$

For each model, we discretize the price dynamics using the Euler scheme for time points  $T_0 < T_{0+i} < T_{0+n} = T$  with  $i = (1, \dots, n)$ . This yields the following for the  $k^{\text{th}}$  price path for the four different models:

Black (1976) one-factor model:

$$S_{t_i, (i, k)} = \exp \left( \xi_{t_{i-1}} + a^* \cdot \Delta t + \sigma_{\xi} \cdot \sqrt{\Delta t} \cdot Z_{1, (i, k)} \right) \quad (4.43)$$

Schwartz (1997) one-factor model:

$$S_{t_i, (i, k)} = \exp \left( \xi_{t_{i-1}} \cdot e^{-\kappa_{\xi} \cdot \Delta t} + a^* \left(1 - e^{-\kappa_{\xi} \cdot \Delta t}\right) + \sigma_{\xi} \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa_{\xi} \cdot \Delta t}}{2 \cdot \kappa_{\xi}}} \cdot Z_{1, (i, k)} \right) \quad (4.44)$$

Schwartz and Smith (2000) two-factor model:

$$\begin{aligned} S_{t_i, (i, k)} = \exp \left( \xi_{t_{i-1}} + a^* \cdot \Delta t + \sigma_{\xi} \cdot \sqrt{\Delta t} \cdot Z_{1, (i, k)} \right. \\ \left. + \chi_{t_{i-1}} \cdot e^{-\kappa_{\chi} \cdot \Delta t} - \frac{\lambda_{\chi}}{\kappa_{\chi}} \left(1 - e^{-\kappa_{\chi} \cdot \Delta t}\right) + \sigma_{\chi} \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa_{\chi} \cdot \Delta t}}{2 \cdot \kappa_{\chi}}} \cdot Z_{2, (i, k)} \right) \end{aligned} \quad (4.45)$$

Korn (2005) two-factor model:

$$\begin{aligned} S_{t_i, (i, k)} = \exp \left( \xi_{t_{i-1}} \cdot e^{-\kappa_{\xi} \cdot \Delta t} + a^* \left(1 - e^{-\kappa_{\xi} \cdot \Delta t}\right) + \sigma_{\xi} \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa_{\xi} \cdot \Delta t}}{2 \cdot \kappa_{\xi}}} \cdot Z_{1, (i, k)} \right. \\ \left. + \chi_{t_{i-1}} \cdot e^{-\kappa_{\chi} \cdot \Delta t} - \frac{\lambda_{\chi}}{\kappa_{\chi}} \left(1 - e^{-\kappa_{\chi} \cdot \Delta t}\right) + \sigma_{\chi} \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa_{\chi} \cdot \Delta t}}{2 \cdot \kappa_{\chi}}} \cdot Z_{2, (i, k)} \right). \end{aligned} \quad (4.46)$$

For details on the exact updating formula for an OU process that we apply above, see Gillespie (1996). For computational efficiency, we use vectorized versions of the above discretizations as well as parallel computing in MATLAB R2015a to simulate the price

paths. The corresponding MATLAB R2015a code including the main .m-file, the MC functions, the simulation functions, as well as the closed-form solution functions are provided in appendix C from page 237 onwards.

With respect to the initial value,  $S_0$ , we choose  $S_0 = 21$  for the Capesize routes C4 and C7 and  $S_0 = 45,000$  for the Panamax routes P2A and P3A. For the two-factor models, we split the initial value,  $\ln(S_0) = \xi_0 + \chi_0$ , proportionally between the two initial values of the two factors,  $\xi_0$  and  $\chi_0$ , based on their respective share of the estimated initial value,  $\ln(S_0)$ , from Prokopczuk (2011). Concerning the strike price,  $K$ , we price at-the-money options (i.e.,  $S_0 = K$ ) as well as 10 % and 20 % in-the-money and out-of-the-money options. We assume an interest rate of 5 % in continuous compounding.

In order to examine the accuracy of the developed closed-form solutions for geometric Asian call options, we calculate the MC price of a geometric Asian call option with payoff  $\Phi_{\text{MC G}} = \max(G(0, T) - K, 0)$  for each model, route, and strike price combination. We determine the MC price by computing the discounted arithmetic mean of the payoff for each of the  $k$  price paths. This is given by

$$C_{\text{MC G}} = \frac{e^{-r \cdot T}}{k} \sum_{j=1}^k (\max(G_j(0, T) - K, 0)), \quad \text{with } G_j(0, T) = \left( \prod_{h=1}^n S_{h,j} \right)^{-n}. \quad (4.47)$$

With respect to arithmetic Asian call options, we also determine the MC price for each model, route, and strike price combination. We compute the MC price of an arithmetic Asian call option with payoff  $\Phi_{\text{MC A}} = \max(A(0, T) - K, 0)$  as well by computing the discounted arithmetic mean of the payoff for each of the  $k$  price paths. This is given by

$$C_{\text{MC A}} = \frac{e^{-r \cdot T}}{k} \sum_{j=1}^k (\max(A_j(0, T) - K, 0)), \quad \text{with } A_j(0, T) = \frac{1}{n} \sum_{h=1}^n S_{h,j}. \quad (4.48)$$

Regarding the accuracy of the MC price, we determine the MC standard deviation for each of the MC prices according to

$$s_{\text{MC}} = e^{-r \cdot T} \cdot \sqrt{\frac{1}{k} \text{Var}[\Phi_{\text{MC}}]}. \quad (4.49)$$

Finally, we exploit our developed closed-form solutions for geometric Asian call options as control variate in the MC simulation in order to reduce the variance of the obtained

MC prices for arithmetic Asian call options. The MC control variate approach is a rather simple but effective method for variance reduction in MC simulations if the correlation between the MC estimator,  $C_{MC A}$ , and the control variate,  $C_{MC G}$ , is sufficiently high and if we know the expected value of the control variate in closed-form (i.e.,  $C_G$  in this case). Generally, the correlation between the geometric and arithmetic mean is almost perfect and Kemna and Vorst (1990) show the effectiveness of this method under Black and Scholes (1973) price dynamics. The following technical description of the MC control variate approach largely follows Glasserman (2003).

For each path  $j$  with  $j = (1, \dots, k)$ , we compute the individual MC payoff for a geometric Asian call option,  $\Phi_{MC G,j}$ , and an arithmetic Asian call option,  $\Phi_{MC A,j}$ , and the individual MC prices,  $C_{MC G,j}$  and  $C_{MC A,j}$ , respectively. For the purpose of simplified notation, we henceforth label

$$X = (C_{MC G,1}, \dots, C_{MC G,k}) \quad (4.50)$$

$$Y = (C_{MC A,1}, \dots, C_{MC A,k}) \quad (4.51)$$

$$\bar{X} = \frac{1}{k} \sum_{j=1}^k X_j \quad (4.52)$$

$$\bar{Y} = \frac{1}{k} \sum_{j=1}^k Y_j. \quad (4.53)$$

Subsequently, we determine the control variate-adjusted value of each MC price for the arithmetic Asian call option which is given by

$$Z_j(\beta_{CV}) = Y_j + \beta_{CV} (X_j - C_G). \quad (4.54)$$

The MC control variate estimator,  $C_{MC AA CV}(\beta_{CV}) = \bar{Z}(\beta_{CV})$ , for the arithmetic Asian call option is then given by

$$\begin{aligned} \bar{Z}(\beta_{CV}) &= \bar{Y} + \beta_{CV} (\bar{X} - C_G) \\ &= \frac{1}{k} \sum_{j=1}^k \left( e^{-r \cdot T} \cdot \Phi_{MC A,j} + \beta_{CV} \left( e^{-r \cdot T} \cdot \Phi_{MC G,j} - C_G \right) \right). \end{aligned} \quad (4.55)$$

According to Glasserman (2003), the resulting estimator,  $C_{MC AA CV}$ , is unbiased and

consistent. The variance of each  $Z_j$  can be computed by

$$\begin{aligned}\sigma_{Z_j}^2(\beta_{CV}) &= \text{Var}[Y_j + \beta_{CV}(X_j - C_G)] \\ &= \sigma_Y^2 + 2 \cdot \beta_{CV} \cdot \sigma_X \sigma_Y \rho_{X,Y} + \beta_{CV}^2 \cdot \sigma_X^2.\end{aligned}\tag{4.56}$$

Hence, the simple MC estimator,  $\bar{Y}$ , with  $\beta_{CV} = 0$  has variance  $\sigma_{\bar{Y}}^2 = \text{Var}[\bar{Y}] = \sigma_Y^2 \cdot k^{-1}$  and the MC control variate estimator,  $\bar{Z}(\beta_{CV})$ , has variance  $\sigma_{\bar{Z}}^2(\beta_{CV}) = \text{Var}[\bar{Z}(\beta_{CV})] = \sigma_Z^2(\beta_{CV}) \cdot k^{-1}$ . The condition for a variance reduction through applying the control variate is then  $\beta_{CV} \cdot \sigma_X > 2 \cdot \beta_{CV} \cdot \sigma_Y \rho_{X,Y}$ . The variance of the MC control variate estimator is minimized by optimally selecting  $\beta_{CV}^*$  which is given by

$$\beta_{CV}^* = -\frac{\sigma_Y \rho_{X,Y}}{\sigma_X} = -\frac{\text{Cov}[X, Y]}{\text{Var}[X]}.\tag{4.57}$$

The resulting variance reduction is eventually determined by

$$VR = 1 - \frac{\text{Var}[\bar{Z}(\beta_{CV}^*)]}{\text{Var}[\bar{Y}]} = 1 - (1 - \rho_{X,Y}^2) = \rho_{X,Y}^2.\tag{4.58}$$

As the true parameters for  $\sigma_X$ ,  $\sigma_Y$ , and  $\rho_{X,Y}$  are unknown, we use the corresponding sample counterparts. This introduces some bias. However, the resulting optimal  $\hat{\beta}_{CV}^*$  from the sample counterparts tends to the true  $\beta_{CV}^*$  and thus,  $\bar{Z}(\hat{\beta}_{CV}^*)$  is asymptotically as precise as  $\bar{Z}(\beta_{CV}^*)$ . This holds because they satisfy the same central limit theorem (see Glasserman (2003) for details).

The above described MC simulation approach can of course be easily adjusted to price an arithmetic Asian call option within the delivery period (i.e.,  $T_0 < t < T$ ). This simply requires to run an MC simulation from time  $t$  to  $T$  using the adjusted closed-form solution as control variate as well as accounting for the deterministic part of the average that already has materialized from time  $T_0$  to  $t$ .

#### 4.5.4 Numerical results

The numerical results of the MC simulation for the Capesize routes C4 and C7 as well as the Panamax routes P2A and P3A are shown in TABLES 4.2, 4.3, 4.4, and 4.5, respectively.

TABLE 4.2: Numerical results of Capesize route C4 with  $T = \frac{1}{12}$  years and  $n = 21$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.0163	(0.0055)	4.0224	-0.1522	4.0552	(0.0055)	4.0614	(0.0001)	99.9571
18.9	2.0258	(0.0050)	2.0337	-0.3886	2.0565	(0.0051)	2.0644	(0.0001)	99.9694
21	0.6197	(0.0032)	0.6289	-1.4671	0.6380	(0.0033)	0.6473	(0.0001)	99.9370
23.1	0.0976	(0.0013)	0.1019	-4.2062	0.1059	(0.0013)	0.1102	(0.0001)	99.5996
25.2	0.0081	(0.0003)	0.0086	-6.7184	0.0099	(0.0004)	0.0105	(0.0001)	97.8496
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.0113	(0.0059)	4.0179	-0.1661	4.0553	(0.0059)	4.0621	(0.0001)	99.9524
18.9	2.0431	(0.0053)	2.0520	-0.4343	2.0775	(0.0054)	2.0865	(0.0001)	99.9652
21	0.6625	(0.0034)	0.6725	-1.4946	0.6835	(0.0035)	0.6937	(0.0001)	99.9296
23.1	0.1208	(0.0015)	0.1258	-3.9868	0.1309	(0.0016)	0.1360	(0.0001)	99.6054
25.2	0.0126	(0.0004)	0.0135	-6.4875	0.0153	(0.0005)	0.0162	(0.0001)	98.0615
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.5058	(0.0068)	3.5133	-0.2133	3.5695	(0.0068)	3.5771	(0.0002)	99.9264
18.9	1.7138	(0.0057)	1.7277	-0.8037	1.7567	(0.0058)	1.7708	(0.0001)	99.9473
21	0.5870	(0.0036)	0.6013	-2.3774	0.6112	(0.0037)	0.6257	(0.0001)	99.9033
23.1	0.1365	(0.0017)	0.1439	-5.1150	0.1484	(0.0018)	0.1559	(0.0001)	99.6066
25.2	0.0221	(0.0007)	0.0243	-9.2942	0.0263	(0.0007)	0.0286	(0.0001)	98.4076
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.5314	(0.0070)	4.5354	-0.0872	4.5947	(0.0071)	4.5987	(0.0002)	99.9379
18.9	2.5566	(0.0064)	2.5659	-0.3600	2.6110	(0.0065)	2.6204	(0.0002)	99.9450
21	1.0571	(0.0047)	1.0695	-1.1617	1.0976	(0.0049)	1.1103	(0.0002)	99.9018
23.1	0.2981	(0.0026)	0.3061	-2.6105	0.3235	(0.0027)	0.3316	(0.0002)	99.6153
25.2	0.0571	(0.0011)	0.0601	-4.9877	0.0678	(0.0012)	0.0708	(0.0002)	98.4606

The table shows the numerical results for the Capesize route C4 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.3: Numerical results of Capesize route C7 with  $T = \frac{1}{12}$  years and  $n = 21$ 

Black (1976) one-factor model									
K	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.9941	(0.0051)	3.9996	-0.1363	4.0271	(0.0051)	4.0326	(0.0001)	99.9626
18.9	1.9800	(0.0047)	1.9869	-0.3459	2.0064	(0.0047)	2.0133	(0.0001)	99.9741
21	0.5513	(0.0029)	0.5596	-1.4864	0.5661	(0.0030)	0.5745	(0.0001)	99.9459
23.1	0.0686	(0.0010)	0.0718	-4.5107	0.0744	(0.0011)	0.0776	(0.0001)	99.5886
25.2	0.0038	(0.0002)	0.0041	-7.3401	0.0048	(0.0003)	0.0051	(0.0000)	97.5807
Schwartz (1997) one-factor model									
K	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.9875	(0.0054)	3.9935	-0.1494	4.0254	(0.0054)	4.0314	(0.0001)	99.9579
18.9	1.9947	(0.0049)	2.0025	-0.3882	2.0245	(0.0050)	2.0323	(0.0001)	99.9703
21	0.5942	(0.0031)	0.6033	-1.5107	0.6116	(0.0032)	0.6208	(0.0001)	99.9388
23.1	0.0886	(0.0012)	0.0927	-4.3822	0.0961	(0.0013)	0.1002	(0.0001)	99.5984
25.2	0.0067	(0.0003)	0.0072	-7.0774	0.0083	(0.0003)	0.0088	(0.0001)	97.8076
Schwartz and Smith (2000) two-factor model									
K	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.4422	(0.0062)	3.4481	-0.1706	3.4972	(0.0062)	3.5031	(0.0002)	99.9321
18.9	1.6141	(0.0052)	1.6261	-0.7396	1.6501	(0.0053)	1.6623	(0.0001)	99.9556
21	0.4920	(0.0031)	0.5044	-2.4631	0.5103	(0.0032)	0.5229	(0.0001)	99.9181
23.1	0.0925	(0.0013)	0.0979	-5.4828	0.1004	(0.0014)	0.1059	(0.0001)	99.6070
25.2	0.0109	(0.0004)	0.0123	-10.9817	0.0131	(0.0005)	0.0145	(0.0001)	98.2260
Korn (2005) two-factor model									
K	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	5.1336	(0.0066)	5.1368	-0.0628	5.1971	(0.0067)	5.2004	(0.0002)	99.9342
18.9	3.0894	(0.0064)	3.0958	-0.2069	3.1490	(0.0065)	3.1555	(0.0002)	99.9382
21	1.3775	(0.0051)	1.3892	-0.8479	1.4274	(0.0052)	1.4394	(0.0002)	99.9019
23.1	0.4060	(0.0029)	0.4145	-2.0370	0.4399	(0.0031)	0.4485	(0.0002)	99.6279
25.2	0.0760	(0.0012)	0.0795	-4.4443	0.0907	(0.0014)	0.0942	(0.0002)	98.3976

The table shows the numerical results for the Capesize route C7 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.4: Numerical results of Panamax route P2A with  $T = \frac{1}{12}$  years and  $n = 21$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,558.51	(14.15)	8,575.47	-0.20	8,675.69	(14.23)	8,692.85	(0.34)	99.94
40,500	4,452.48	(12.52)	4,475.73	-0.52	4,541.99	(12.72)	4,565.54	(0.27)	99.96
45,000	1,603.92	(8.40)	1,628.17	-1.49	1,661.66	(8.67)	1,686.24	(0.25)	99.91
49,500	373.17	(4.06)	386.87	-3.54	404.64	(4.33)	418.53	(0.27)	99.61
54,000	57.10	(1.52)	60.81	-6.09	68.29	(1.72)	72.04	(0.22)	98.33
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,487.06	(14.47)	8,504.75	-0.21	8,610.09	(14.56)	8,627.99	(0.36)	99.94
40,500	4,417.53	(12.73)	4,441.94	-0.55	4,510.58	(12.94)	4,535.32	(0.28)	99.95
45,000	1,614.85	(8.56)	1,639.90	-1.53	1,674.88	(8.84)	1,700.28	(0.26)	99.91
49,500	390.42	(4.22)	404.80	-3.55	423.49	(4.50)	438.08	(0.28)	99.61
54,000	63.71	(1.63)	67.80	-6.03	75.94	(1.84)	80.09	(0.23)	98.38
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,619.77	(15.36)	8,636.85	-0.20	8,760.46	(15.48)	8,777.77	(0.40)	99.93
40,500	4,590.70	(13.49)	4,619.76	-0.63	4,698.62	(13.74)	4,728.12	(0.32)	99.95
45,000	1,787.86	(9.31)	1,821.25	-1.83	1,859.89	(9.64)	1,893.81	(0.30)	99.90
49,500	487.02	(4.90)	508.28	-4.18	528.65	(5.23)	550.26	(0.32)	99.61
54,000	94.65	(2.06)	101.91	-7.13	112.01	(2.32)	119.40	(0.28)	98.49
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	7,348.27	(14.47)	7,361.36	-0.18	7,480.78	(14.51)	7,494.04	(0.39)	99.93
40,500	3,538.64	(12.02)	3,564.74	-0.73	3,624.74	(12.20)	3,651.20	(0.27)	99.95
45,000	1,188.79	(7.50)	1,210.97	-1.83	1,235.93	(7.74)	1,258.42	(0.23)	99.91
49,500	268.97	(3.52)	280.68	-4.17	291.96	(3.75)	303.83	(0.23)	99.61
54,000	42.04	(1.33)	45.70	-8.02	50.07	(1.49)	53.78	(0.19)	98.41

The table shows the numerical results for the Panamax route P2A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.5: Numerical results of Panamax route P3A with  $T = \frac{1}{12}$  years and  $n = 21$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,576.79	(15.27)	8,595.99	-0.22	8,712.05	(15.38)	8,731.51	(0.39)	99.93
40,500	4,549.34	(13.39)	4,575.50	-0.57	4,652.39	(13.64)	4,678.93	(0.31)	99.95
45,000	1,756.95	(9.21)	1,783.55	-1.49	1,825.92	(9.53)	1,852.94	(0.30)	99.90
49,500	472.06	(4.81)	488.17	-3.30	512.30	(5.13)	528.67	(0.32)	99.61
54,000	89.89	(2.02)	95.24	-5.62	106.57	(2.28)	112.01	(0.28)	98.48
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,448.57	(16.04)	8,469.57	-0.25	8,597.94	(16.18)	8,619.25	(0.43)	99.93
40,500	4,506.64	(13.92)	4,535.25	-0.63	4,619.00	(14.2)	4,648.08	(0.34)	99.94
45,000	1,802.26	(9.64)	1,830.64	-1.55	1,877.90	(10.00)	1,906.79	(0.33)	99.89
49,500	523.86	(5.24)	541.61	-3.28	569.15	(5.60)	587.22	(0.35)	99.61
54,000	112.93	(2.36)	119.49	-5.48	133.29	(2.65)	139.96	(0.32)	98.57
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,203.34	(19.54)	8,236.99	-0.41	8,435.21	(19.87)	8,469.65	(0.66)	99.89
40,500	4,612.43	(16.58)	4,659.59	-1.01	4,787.17	(17.08)	4,835.54	(0.55)	99.90
45,000	2,160.13	(12.11)	2,208.78	-2.20	2,283.57	(12.69)	2,333.55	(0.52)	99.83
49,500	842.63	(7.67)	878.42	-4.07	924.16	(8.26)	960.97	(0.54)	99.57
54,000	279.49	(4.35)	298.12	-6.25	326.34	(4.88)	345.50	(0.53)	98.82
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	10,690.35	(23.02)	10,726.03	-0.33	11,016.02	(23.71)	11,052.77	(0.91)	99.85
40,500	6,798.03	(20.79)	6,847.98	-0.73	7,082.42	(21.64)	7,134.05	(0.83)	99.85
45,000	3,787.32	(16.86)	3,843.95	-1.47	4,023.47	(17.83)	4,082.15	(0.81)	99.79
49,500	1,843.83	(12.20)	1,889.94	-2.44	2,028.36	(13.23)	2,076.22	(0.88)	99.56
54,000	787.31	(8.02)	820.87	-4.09	916.89	(9.02)	951.74	(0.93)	98.93

The table shows the numerical results for the Panamax route P3A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

The pattern of findings in the results is largely similar across routes. Firstly, the accuracy of the closed-form solutions (CFSs) for geometric Asian call options seems to be quite promising across routes and models. The relative bias between MC price and closed-form solution,  $\Delta$  %, is always below 2 % for at-the-money options and even considerably smaller for in-the-money options. Only for out-of-the-money options,  $\Delta$  % reaches values of up to 11 %. These cases, however, coincide with comparatively low options prices. Accordingly, the relative bias seems to be of larger magnitude, although the bias is consistently even smaller than for at-the-money or in-the-money options. Expressed in terms of number of standard errors (SEs), the bias ranges from -0.5 to -4.7 times the respective standard error. This seems somewhat high and results largely from the discretization error due to the different monitoring frequencies. The bias, however, is economically insignificant in relation to the option prices for the Capesize routes C4 and C7 or at least the level of strike prices for the Panamax routes P2A and P3A. Furthermore, the bias for the Schwartz (1997) one-factor model is similar to the Black (1976) one-factor model, which is similar to the approved closed-form solution of Kemna and Vorst (1990). For the two-factor models, the bias is negligibly higher in some cases, but this is consistent with the expectation for two-factor models. Hence, the closed-form solutions turn out to be accurate in our numerical example apart from a small discretization error which results from the inherent difference in monitoring frequencies (i.e., continuously monitored closed-form solution vs. discretely monitored MC price).

Secondly, the application of the closed-form solution for the geometric Asian call option as control variate for the arithmetic Asian call option results in considerable and consistent variance reduction levels of more than 97 % across routes and models. This finding is consistent with the variance reduction levels of Kemna and Vorst (1990) and originates from the almost perfect correlation between geometric and arithmetic average. Consequently, the computation effort can be considerably reduced if the MC control variate approach is chosen to price an arithmetic Asian call option. In order to achieve the same pricing accuracy for the standard MC approach, the number of generated MC paths,  $k$ , would need to be approximately increased by factor 1,050.<sup>10</sup>

Thirdly, the numerical results suggest that arithmetic Asian call option prices (i.e.,

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<sup>10</sup> The factor is approximated by the mean of  $\left(\frac{SE(MC)}{SE(MC\ CV)}\right)^2$  across all computed prices presented in TABLES 4.2, 4.3, 4.4, and 4.5. The minimum and maximum of the individual factors are approximately 40 and 3,850, respectively.

MC or MC CV) are always slightly larger than geometric Asian call option prices (i.e., MC or CFS). This is consistent with mathematical theory stating that the arithmetic average is greater than or equal to the geometric average of a list of non-negative numbers (Beckenbach and Bellman, 1961). Accordingly, the closed-form solution for the geometric Asian call option serves as lower boundary in the MC control variate approach for the arithmetic Asian call option.

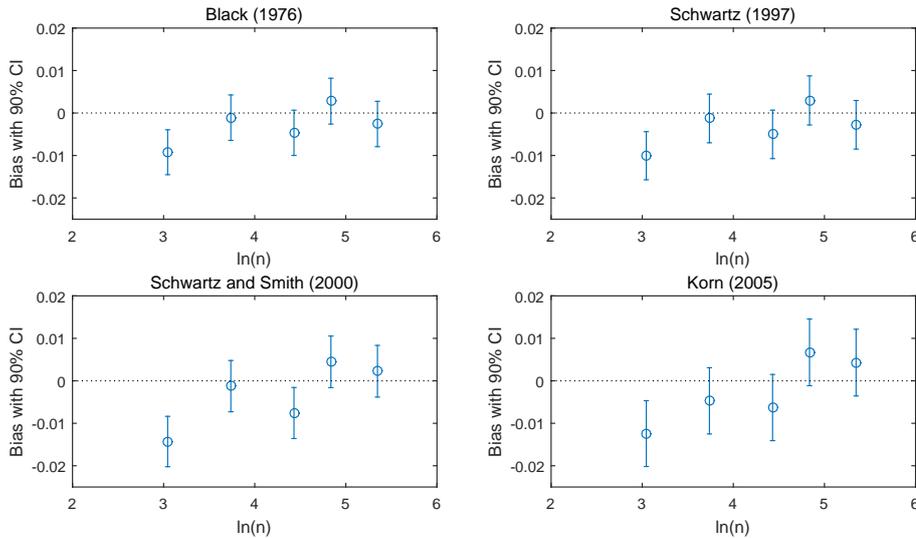
Finally, the general level of options prices varies quite significantly across models for the individual routes. This, however, results most likely from the different parameter estimates for the respective models and their partly low significance. Although a comparison and interpretation of resulting option prices across the four different models would have been interesting, the parameter circumstances do not allow to do so in a meaningful way. As already stated, our primary objectives are examining the accuracy of the developed closed-form-solutions for a geometric Asian call option as well as pricing an arithmetic Asian call option using a MC simulation and showing the benefit of using the geometric Asian call option as control variate. Thus, the lack of comparability of the resulting option prices across the four different models should not concern us any further.

Note that the computation for the all values of TABLES 4.2, 4.3, 4.4, and 4.5 takes about 23 seconds or 0.072 seconds on average for each of the four prices and standard errors or deviations of the 80 chosen combinations of route, model, and strike price. The computations are conducted in MATLAB R2015a on a system with an INTEL Core i5-3570 CPU (4 cores with 3.40 GHz clock rate) and 8 GB RAM.

In order to be robust in the conclusions we draw from our numerical computations, we repeat the numerical example in two alternative versions: increase the number of time steps to  $n = 210$  reflecting 10 prices per trading day and increase the time to maturity of the option to  $T = 1$  year with  $n = 252$  time steps reflecting again one price per trading day. The corresponding results are shown in appendices B.1 from page 226 onwards and B.2 from page 230 onwards. For the first alternative, the closed-form solution prices are obviously the same. The MC price and standard errors are largely similar as we did not increase the number of MC paths,  $k$ . The relative bias from the closed-form solution,  $\Delta$  %, however, is considerably smaller as the discretization is finer. Expressed in terms of number of standard errors, the bias consistently stays clearly within the two-standard error barrier. For the second alternative, the level of prices is obviously different and the MC standard errors are higher due the increased

time to maturity. Interestingly, the relative bias from the closed-form solution,  $\Delta$  %, is also considerably smaller, although the discretization is not finer. However, the absolute number of elements comprising the discrete average increased considerably. Expressed in terms of number of standard errors, the bias again consistently stays clearly within the two-standard error barrier.

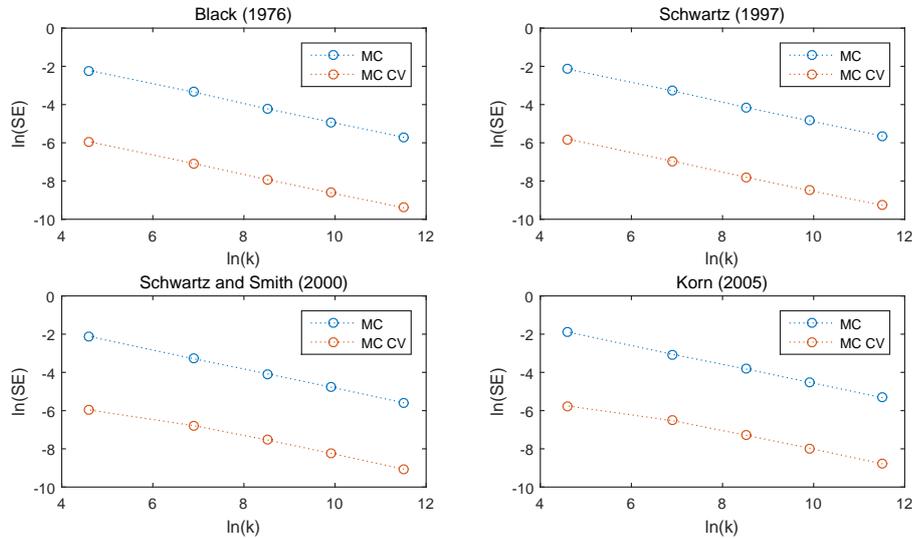
FIGURE 4.1: Impact of  $n$  on the discretization error for Capesize route C4



The graph shows the bias as well as the 90 % CI of an at-the-money geometric Asian call option for different log numbers of discretization time steps (i.e.,  $n = 21, 42, 84, 126, 210$ ) for the Capesize route C4 for all four models – with  $S_0 = 21$ ,  $r = 5$  %,  $T = \frac{1}{12}$  years, and  $k = 100,000$ .

Moreover, we examine the impact of increasing the number of discretization time steps,  $n$ , on the bias or discretization error. FIGURE 4.1 shows the bias between the MC price and closed-form solution as well as the 90 % confidence interval (CI) of an at-the-money geometric Asian call option for different log numbers of discretization time steps (i.e.,  $n = 21, 42, 84, 126, 210$ ) for the Capesize route C4 for all four models. With a finer discretization, the bias get smaller for all models and the 90 % CIs start to contain zero bias as well. As already stated above, the bias, however, is economically insignificant, so that using a discretization of one price per trading day should be sufficiently accurate. Similar plots with similar findings for the routes C7, P2A, and P3A are shown in appendix B.3 from page 234 onwards.

Furthermore, we analyze the impact of increasing the number of generated MC paths,  $k$ , on the simulation error. For the Capesize route C4 for all four models, FIGURE 4.2 shows the log standard error (SE) for the MC price as well as MC control variate price

FIGURE 4.2: Impact of  $k$  on the simulation error for Capesize route C4

The graph shows the log standard error (SE) for the MC price as well as MC control variate price of an at-the-money arithmetic Asian call option for different log numbers of generated MC paths (i.e.,  $k = 100, 1,000, 5,000, 20,000, 100,000$ ) for the Capesize route C4 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $n = 21$ .

of an at-the-money arithmetic Asian call option for different log numbers of generated MC paths (i.e.,  $k = 100, 1,000, 5,000, 20,000, 100,000$ ). The benefit in variance reduction due to applying the control variate is consistent across different numbers of generated MC paths. In particular, the plot shows that the level of SE for the MC control variate simulation approach is even lower with  $k = 100$  than for the standard MC simulation approach with  $k = 100,000$ . This reflects the above mentioned generated path increase factor of more than 1,000. Hence, MC control variate simulation approach is very powerful for pricing arithmetic Asian call options. Similar plots with similar findings for the routes C7, P2A, and P3A are shown in appendix B.4 from page 235 onwards.

#### 4.5.5 Extension to forward-start Asian options

In commodity or freight markets, Asian options with an averaging period limited to a certain amount of time before maturity are quite common. Accordingly, the averaging period does not cover the entire life of the option. These options are called forward-start Asian options and the length of the averaging period typically ranges from one week to several months prior to maturity. The life time of the option may, however, extend to several years. The MC control variate simulation set-up can be

rather simply adjusted in order to price these kind of Asian options when  $t < T_0 < T$ . We briefly outline the approach for forward-start Asian options below.

Firstly, we simulate again  $k$  price paths from time  $t$  to  $T$  according to equations (4.43), (4.44), (4.45), or (4.46), respectively. It is possible to simulate from time  $t$  to  $T_0$  in one step. From time  $T_0$  to  $T$ , however, we apply again an discretization scheme with  $n$  steps,  $T_0 < T_{0+i} < T_{0+n} = T$  with  $i = (1, \dots, n)$ . We choose  $n$ , such that we have again at least one price per day.

Accordingly, the MC price for an arithmetic Asian call option at time  $t$  changes to

$$\bar{Y} = \frac{e^{-r \cdot T}}{k} \sum_{j=1}^k (\max(A_j(T_0, T) - K, 0)), \quad \text{with } A_j(T_0, T) = \frac{1}{n} \sum_{h=1}^n S_{h,j}. \quad (4.59)$$

With respect to the control variate, we cannot simply apply our develop closed-form solution as it only provides valid prices at the inception of or within the averaging period. Thus, we do not know the expected value of an geometric Asian call option at time  $t$ . However, we can still leverage our closed-form solution at time  $T_0$ ,  $C_G(S_{T_0})$ , and apply the following as control variate:

$$\bar{X} = \frac{e^{-r \cdot (T-T_0)}}{k} \sum_{j=1}^k (\max(G_j(T_0, T) - K, 0)) - C_G(S_{T_0}), \quad (4.60)$$

with  $G_j(T_0, T) = \left( \prod_{h=1}^n S_{h,j} \right)^{-n}$ , and  $S_{T_0} = \exp(\xi_{T_0} + \chi_{T_0})$ .

The expected value of  $\bar{X}$  is zero and  $\bar{X}$  is still highly correlated with  $\bar{Y}$ . The MC control variate estimator,  $\bar{Z}(\beta_{CV})$ , for the price of an arithmetic Asian call option at time  $t$  is then given by

$$\bar{Z}(\beta_{CV}) = \bar{Y} + \beta_{CV}(\bar{X}). \quad (4.61)$$

The variance minimizing  $\beta_{CV}^*$ , the MC standard errors, and the corresponding variance reduction can be computed similarly as described in subsection 4.5.3.

## 4.6 Conclusion

To summarize, we develop a general pricing framework for continuously monitored geometric Asian call options applicable to affine  $n$ -factor Gaussian diffusions. The pricing framework relies on the concept of the characteristic function which allows to determine the distribution of the log geometric average rather easily. Furthermore, we practically apply the developed general pricing framework to three mean-reversion pricing models (i.e., the Schwartz (1997) one-factor model, the Schwartz and Smith (2000) two-factor model, and the Korn (2005) two-factor model) and derive specific closed-form solutions for geometric Asian call options. For the sake of completeness as well as to underline the validity of our chosen approach, we also derive a closed-form solution for the Black (1976) one-factor model. The obtained result can be rather simply converted to the result for classical Black and Scholes (1973) price dynamics developed by Kemna and Vorst (1990).

Moreover, we examine the accuracy of the derived closed-form solutions in a MC simulation as well as apply the geometric Asian call option as control variate to price an arithmetic Asian call option. Concerning model parameters in the numerical example, we rely on Prokopczuk (2011) as he estimated model parameters for the four price dynamics considered within the study for four different dry bulk freight futures. The results show that our derived closed-form solutions for geometric Asian call options are accurate as well as that the MC control variate simulation approach to price arithmetic Asian call options allows for a considerable variance reduction of more than 97 %. This can be translated into substantial savings in computation time. Additionally, we outline an extension of the MC simulation to forward-start Asian options as these are quite common in commodity markets. Our general approach and the presented results are neither prone to changes in model selection (as long as it is an affine  $n$ -factor Gaussian diffusion) nor prone to changes in model parameters. Therefore, the applicability of our general pricing framework is by no means limited to the mean-reversion models or commodity markets considered within this study.

With respect to further research opportunities in this area, a potential extension of the presented research is to develop a general pricing framework for discretely monitored geometric Asian call options. This allows to eliminate the inherent discretization error of the closed-form solution in our developed general pricing framework compared to the discrete monitoring of Asian options in practice. This would extend research

along the lines of, for instance, Fusai and Meucci (2008) for the specific case of affine Gaussian diffusions. Moreover, the general pricing framework can be rather simply adjusted for ‘fixed strike’ geometric Asian put options. Another potential extension of the presented research is to adjust the general pricing framework for ‘floating strike’ geometric Asian options. Perhaps, a closed-form solution would still be possible for these kind of options. Finally, applying the developed general pricing framework to other affine Gaussian diffusions to state specific closed-form solutions for these models is another potential extension of research in this area.

## A Appendix A – Geometric Asian options

### A.1 Proofs

#### A.1.1 Theorem 1

*Proof.* Include the process  $G(\cdot; 0, T_1)$  in the state vector to obtain the  $\mathbb{R}^{n+1}$ -valued enlarged state vector  $\bar{\mathbf{X}} = (\mathbf{X}^\top, G(\cdot; 0, T_1))^\top$ . Then,

$$d\bar{\mathbf{X}}(t) = (\bar{\mathbf{b}} + \bar{\boldsymbol{\beta}} \cdot \bar{\mathbf{X}}(t)) dt + \bar{\boldsymbol{\Sigma}} d\mathbf{W}(t), \quad \text{for } 0 \leq t \leq T_1, \quad (\text{A.1})$$

with starting value  $\bar{\mathbf{X}}(0) = \bar{\mathbf{x}} = (\mathbf{x}^\top, 0)^\top$ , where  $\bar{\mathbf{b}} = (\mathbf{b}^\top, 0)^\top$ ,

$$\bar{\boldsymbol{\beta}} = \begin{pmatrix} \boldsymbol{\beta} & 0 \\ \mathbf{v}^\top & 0 \end{pmatrix}, \quad \text{and } \bar{\boldsymbol{\Sigma}} = \begin{pmatrix} \boldsymbol{\Sigma} \\ 0 \end{pmatrix}. \quad (\text{A.2})$$

The characteristic function of  $\bar{\mathbf{X}}$  follows from THEOREM 2.7 of Duffie et al. (2003)

$$\mathbb{E}_{\bar{\mathbf{x}}} \left[ e^{\langle \mathbf{u}, \bar{\mathbf{X}}(t) \rangle} \right] = \exp \left( \bar{\phi}(t, \mathbf{u}) + \langle \bar{\boldsymbol{\psi}}(t, \mathbf{u}), \bar{\mathbf{x}} \rangle \right), \quad \text{for } \mathbf{u} \in i \cdot \mathbb{R}^{n+1}, \quad (\text{A.3})$$

$$\text{where } \bar{\boldsymbol{\psi}}(t, \mathbf{u}) = e^{\bar{\boldsymbol{\beta}}^\top \cdot t} \cdot \mathbf{u}, \quad (\text{A.4})$$

$$\bar{\phi}(t, \mathbf{u}) = \int_0^t \left\langle \bar{\mathbf{b}}, e^{\bar{\boldsymbol{\beta}}^\top \cdot s} \cdot \mathbf{u} \right\rangle + \frac{1}{2} \left\| \bar{\boldsymbol{\Sigma}}^\top \cdot e^{\bar{\boldsymbol{\beta}}^\top \cdot s} \cdot \mathbf{u} \right\|_2^2 ds. \quad (\text{A.5})$$

To calculate the expectation of the option payoff, the last component of  $\bar{\mathbf{X}}$ ,  $G(\cdot; 0, T_1)$ , is of interest. The characteristic function of  $G(T_1; 0, T_1)$  is obtained by setting  $t = T_1$  and  $\mathbf{u} = i \cdot q \cdot \mathbf{e}_{n+1}$ , where  $q \in \mathbb{R}$  and  $\mathbf{e}_{n+1} = (0, \dots, 0, 1)^\top \in \mathbb{R}^{n+1}$ . Then,

$$\mathbb{E}_{\bar{\mathbf{x}}} \left[ e^{i \cdot q \cdot G(T_1; 0, T_1)} \right] = \exp \left( \phi(T_1, q) + \langle \boldsymbol{\psi}(T_1, q), \bar{\mathbf{x}} \rangle \right), \quad (\text{A.6})$$

$$\text{where } \boldsymbol{\psi}(t, q) = i \cdot q \cdot e^{\bar{\boldsymbol{\beta}}^\top \cdot t} \cdot \mathbf{e}_{n+1}, \quad (\text{A.7})$$

$$\begin{aligned} \phi(t, q) &= i \cdot q \cdot \int_0^t \left\langle \bar{\mathbf{b}}, e^{\bar{\boldsymbol{\beta}}^\top \cdot s} \cdot \mathbf{e}_{n+1} \right\rangle ds \\ &\quad - \frac{1}{2} q^2 \cdot \int_0^t \left\| \bar{\boldsymbol{\Sigma}}^\top \cdot e^{\bar{\boldsymbol{\beta}}^\top \cdot s} \cdot \mathbf{e}_{n+1} \right\|_2^2 ds. \end{aligned} \quad (\text{A.8})$$

We can simplify the expression  $e^{\bar{\boldsymbol{\beta}}^\top \cdot t} \cdot \mathbf{e}_{n+1}$  by using the power series representation

$$\begin{aligned}
 e^{\bar{\boldsymbol{\beta}}^\top \cdot t} \cdot \mathbf{e}_{n+1} &= \sum_{n=0}^{\infty} \frac{t^n}{n!} \cdot \begin{pmatrix} \boldsymbol{\beta}^\top & \mathbf{v} \\ 0 & 0 \end{pmatrix}^n \cdot \mathbf{e}_{n+1} \\
 &= \mathbf{e}_{n+1} + \sum_{n=1}^{\infty} \frac{t^n}{n!} \cdot \begin{pmatrix} [\boldsymbol{\beta}^\top]^n & [\boldsymbol{\beta}^\top]^{n-1} \cdot \mathbf{v} \\ 0 & 0 \end{pmatrix} \cdot \mathbf{e}_{n+1} \\
 &= \begin{pmatrix} \sum_{n=1}^{\infty} \frac{t^n}{n!} \cdot [\boldsymbol{\beta}^\top]^{n-1} \cdot \mathbf{v} \\ 1 \end{pmatrix}.
 \end{aligned} \tag{A.9}$$

Denote  $F(t) = \sum_{n=1}^{\infty} \frac{t^n}{n!} \cdot [\boldsymbol{\beta}^\top]^{n-1}$ , then  $F(0) = 0$  and  $F'(t) = e^{\boldsymbol{\beta}^\top \cdot t}$ , for  $t \geq 0$ . Thus,

$$\sum_{n=1}^{\infty} \frac{t^n}{n!} \cdot [\boldsymbol{\beta}^\top]^{n-1} = F(t) = F(0) + \int_0^t F'(s) ds = \int_0^t e^{\boldsymbol{\beta}^\top \cdot s} ds, \text{ for } t \geq 0. \tag{A.10}$$

Finally, the simplification becomes

$$\boldsymbol{\psi}(t, q) = i \cdot q \cdot e^{\bar{\boldsymbol{\beta}}^\top \cdot t} \cdot \mathbf{e}_{n+1} = i \cdot q \cdot \begin{pmatrix} \int_0^t e^{\boldsymbol{\beta}^\top \cdot s} ds \cdot \mathbf{v} \\ 1 \end{pmatrix}. \tag{A.11}$$

Noting that the last component of  $\bar{\mathbf{x}}$  and  $\bar{\mathbf{b}}$  is zero, and the last column of  $\bar{\boldsymbol{\Sigma}}^\top$  is zero as well, we obtain

$$\begin{aligned}
 \mathbb{E}_{\bar{\mathbf{x}}} \left[ e^{i \cdot q \cdot G(T_1; 0, T_1)} \right] &= \exp \left( \phi(T_1, q) + \langle \boldsymbol{\psi}(T_1, q), \bar{\mathbf{x}} \rangle \right) \\
 &= \exp \left( i \cdot q \cdot \int_0^{T_1} \left\langle \mathbf{b}, \int_0^t e^{\boldsymbol{\beta}^\top \cdot s} ds \cdot \mathbf{v} \right\rangle dt \right. \\
 &\quad \left. - \frac{1}{2} q^2 \cdot \int_0^{T_1} \left\| \bar{\boldsymbol{\Sigma}}^\top \int_0^t e^{\boldsymbol{\beta}^\top \cdot s} ds \cdot \mathbf{v} \right\|_2^2 dt \right) \\
 &\quad \cdot \exp \left( i \cdot q \cdot \left\langle \int_0^{T_1} e^{\boldsymbol{\beta}^\top \cdot t} dt \cdot \mathbf{v}, \mathbf{x} \right\rangle \right) \\
 &= \exp \left( i \cdot q \cdot \mu_G - \frac{1}{2} q^2 \cdot \sigma_G^2 \right),
 \end{aligned} \tag{A.12}$$

where the last step follows from  $e^{\boldsymbol{\beta}^\top \cdot s} = (e^{\boldsymbol{\beta} \cdot s})^\top$  and  $\mathbf{v} = \frac{\mathbf{w}}{T_1}$ . From the form of the characteristic function, we see that  $G(T_1; 0, T_1)$  is normally distributed with mean

$\mu_G$  and variance  $\sigma_G^2$ . The option pricing formula follows by standard arguments for pricing calls on a lognormal underlying.  $\square$

### A.1.2 Corollary 1

*Proof.* For  $t \in [T_0, T_1]$ , the characteristic function of  $G(T_1; T_0, T_1)$  conditioned on  $\mathbf{X}(t) = \mathbf{x}$  and  $G(t; T_0, T_1) = g$  is given by

$$\mathbb{E}_{t, \mathbf{x}, g} \left[ e^{i \cdot q \cdot G(T_1; T_0, T_1)} \right] = \mathbb{E}_{\mathbf{x}} \left[ e^{i \cdot q \cdot (g + G(T_1 - t; 0, T_1 - T_0))} \right]. \quad (\text{A.13})$$

The result follows then from the proof of THEOREM 1.  $\square$

### A.1.3 Corollary 2

*Proof.* For  $t \leq T_0$ , the characteristic function of  $G(T_1; T_0, T_1)$  given  $\mathbf{X}(t) = \mathbf{x}$  is

$$\begin{aligned} \mathbb{E}_{t, \mathbf{x}} \left[ e^{i \cdot q \cdot G(T_1; T_0, T_1)} \right] &= \mathbb{E}_{t, \mathbf{x}} \left[ \mathbb{E}_{T_0, \mathbf{X}(T_0)} \left[ e^{i \cdot q \cdot G(T_1; T_0, T_1)} \right] \right] \\ &= \mathbb{E}_{t, \mathbf{x}} \left[ \mathbb{E}_{\mathbf{X}(T_0)} \left[ e^{i \cdot q \cdot G(T_1 - T_0; 0, T_1 - T_0)} \right] \right] \\ &= \mathbb{E}_{t, \mathbf{x}} \left[ \exp \left( i \cdot q \cdot \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1 - T_0} e^{\beta \cdot u} du \cdot \mathbf{X}(T_0) \right\rangle \right. \right. \\ &\quad \left. \left. + i \cdot q \cdot \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1 - T_0} \int_0^u e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle \right. \right. \\ &\quad \left. \left. - \frac{1}{2} q^2 \frac{1}{(T_1 - T_0)^2} \int_0^{T_1 - T_0} \left\| \boldsymbol{\Sigma}^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du \right) \right] \\ &= \mathbb{E}_{\mathbf{x}} \left[ \exp \left( \left\langle i \cdot q \cdot \frac{1}{T_1 - T_0} \int_0^{T_1 - T_0} e^{\beta^\top \cdot u} du \cdot \mathbf{w}, \mathbf{X}(T_0 - t) \right\rangle \right) \right. \\ &\quad \cdot \exp \left( i \cdot q \cdot \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1 - T_0} \int_0^u e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle \right) \\ &\quad \left. \cdot \exp \left( -\frac{1}{2} q^2 \frac{1}{(T_1 - T_0)^2} \int_0^{T_1 - T_0} \left\| \boldsymbol{\Sigma}^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du \right), \right] \end{aligned} \quad (\text{A.14})$$

where we have used the original THEOREM 1 in the third step.

We have to compute the characteristic function of  $\mathbf{X}$  and evaluate it at  $\mathbf{u}_0 = i \cdot q \cdot \frac{1}{T_1 - T_0} \int_0^{T_1 - T_0} e^{\beta^\top \cdot u} du \cdot \mathbf{w}$ . Using once more THEOREM 2.7 of Duffie et al. (2003) gives

$$\mathbb{E}_{\mathbf{x}} \left[ e^{\langle \mathbf{u}, \mathbf{X}(t) \rangle} \right] = \exp \left( \phi(t, \mathbf{u}) + \langle \boldsymbol{\psi}(t, \mathbf{u}), \mathbf{x} \rangle \right), \text{ for } \mathbf{u} \in i \cdot \mathbb{R}^n, \quad (\text{A.15})$$

$$\text{where } \boldsymbol{\psi}(t, \mathbf{u}) = e^{\beta^\top \cdot t} \cdot \mathbf{u}, \quad (\text{A.16})$$

$$\phi(t, \mathbf{u}) = \int_0^t \left\langle \mathbf{b}, e^{\beta^\top \cdot s} \cdot \mathbf{u} \right\rangle + \frac{1}{2} \left\| \boldsymbol{\Sigma}^\top \cdot e^{\beta^\top \cdot s} \cdot \mathbf{u} \right\|_2^2 ds. \quad (\text{A.17})$$

Using the equations above, we see that

$$\begin{aligned} \mathbb{E}_{\mathbf{x}} \left[ \exp \left( \left\langle i \cdot q \cdot \frac{1}{T_1 - T_0} \int_0^{T_1 - T_0} e^{\beta^\top \cdot u} du \cdot \mathbf{w}, \mathbf{X}(T_0 - t) \right\rangle \right) \right] \\ = \exp \left( \int_0^{T_0 - t} \left\langle \mathbf{b}, e^{\beta^\top \cdot s} \cdot \mathbf{u}_0 \right\rangle + \frac{1}{2} \left\| \boldsymbol{\Sigma}^\top \cdot e^{\beta^\top \cdot s} \cdot \mathbf{u}_0 \right\|_2^2 ds \right) \\ \cdot \exp \left( \left\langle e^{\beta^\top \cdot (T_0 - t)} \cdot \mathbf{u}_0, \mathbf{x} \right\rangle \right). \end{aligned} \quad (\text{A.18})$$

Plugging in  $\mathbf{u}_0 = i \cdot q \cdot \frac{1}{T_1 - T_0} \int_0^{T_1 - T_0} e^{\beta^\top \cdot u} du \cdot \mathbf{w}$  gives

$$\begin{aligned} \mathbb{E}_{\mathbf{x}} \left[ \exp \left( \left\langle i \cdot q \cdot \frac{1}{T_1 - T_0} \int_0^{T_1 - T_0} e^{\beta^\top \cdot u} du \cdot \mathbf{w}, \mathbf{X}(T_0 - t) \right\rangle \right) \right] \\ = \exp \left( i \cdot q \cdot \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_0 - t} \int_0^{T_1 - T_0} e^{\beta \cdot (u+s)} du ds \cdot \mathbf{b} \right\rangle \right) \\ \cdot \exp \left( -\frac{1}{2} q^2 \cdot \frac{1}{(T_1 - T_0)^2} \int_0^{T_0 - t} \left\| \boldsymbol{\Sigma}^\top \int_0^{T_1 - T_0} e^{\beta^\top \cdot (u+s)} du \cdot \mathbf{w} \right\|_2^2 ds \right) \\ \cdot \exp \left( i \cdot q \cdot \frac{1}{T_1 - T_0} \left\langle \mathbf{w}, \int_0^{T_1 - T_0} e^{\beta \cdot (T_0 - t + u)} du \cdot \mathbf{x} \right\rangle \right). \end{aligned} \quad (\text{A.19})$$

Checking that

$$\begin{aligned}
 & \int_0^{T_1-T_0} \int_0^u e^{\beta \cdot s} \, ds \, du + \int_0^{T_1-T_0} \int_0^{T_0-t} e^{\beta \cdot (u+s)} \, ds \, du \\
 &= \int_0^{T_1-T_0} \int_0^u e^{\beta \cdot s} \, ds + \int_u^{T_0-t+u} e^{\beta \cdot s} \, ds \, du = \int_0^{T_1-T_0} \int_0^{T_0-t+u} e^{\beta \cdot s} \, ds \, du
 \end{aligned} \tag{A.20}$$

gives the claimed result. □

## A.2 Closed-form solutions

### A.2.1 Black (1976) one-factor model

For the Black (1976) one-factor model, we consider the stochastic process  $X = \xi_t$  with  $n = d = 1$ ,  $w = 1$ , and starting value  $X(0) = x = \xi_0 = \ln(S_0)$ . The corresponding diffusion parameters are

$$b = (a^*), \quad (\text{A.21})$$

$$\beta = (0), \quad (\text{A.22})$$

$$\Sigma = (\sigma_\xi), \quad (\text{A.23})$$

$$a = \left(\frac{1}{2}\sigma_\xi^2\right). \quad (\text{A.24})$$

First, we consider the case at inception of the averaging period (i.e.,  $t = T_0 = 0$ ) and assume that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply THEOREM 1 and determine the distribution parameters of  $G(T; 0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ . This gives us

$$\begin{aligned} \mu_G &= \frac{1}{T} \int_0^T e^{\beta \cdot t} dt \cdot x + \frac{1}{T} \int_0^T \int_0^t e^{\beta \cdot s} ds dt \cdot b \\ &= \frac{1}{T} \int_0^T 1 dt \cdot \xi_0 + \frac{1}{T} \int_0^T \int_0^t 1 ds dt \cdot a^* \end{aligned} \quad (\text{A.25})$$

$$= \xi_0 + \frac{1}{T} \int_0^T t dt \cdot a^* = \xi_0 + \frac{1}{T} \left[ \frac{1}{2} a^* \cdot t^2 \right]_0^T = \xi_0 + \frac{1}{2} a^* \cdot T,$$

$$\begin{aligned} \sigma_G^2 &= \frac{1}{T^2} \int_0^T \left\| \Sigma^\top \int_0^t e^{\beta \cdot s} ds \right\|_2^2 dt = \frac{1}{T^2} \int_0^T \|\sigma_\xi \cdot t\|_2^2 dt \\ &= \frac{1}{T^2} \int_0^T \sigma_\xi^2 \cdot t^2 dt = \frac{1}{T^2} \left[ \frac{1}{3} \sigma_\xi^2 \cdot t^3 \right]_0^T = \frac{1}{3} \sigma_\xi^2 \cdot T. \end{aligned} \quad (\text{A.26})$$

Accordingly, the log geometric average,  $G(T; 0, T)$ , is normally distributed with the following mean and variance

$$G \sim \mathcal{N} \left( \xi_0 + \frac{1}{2} a^* \cdot T, \frac{1}{3} \sigma_\xi^2 \cdot T \right). \quad (\text{A.27})$$

Inserting the two distribution parameters into equation (4.35) yields the desired closed-form solution for a geometric Asian call option.

Secondly, we consider the case within the averaging period (i.e.,  $t \in [T_0, T_1]$ ) and assume again that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply COROLLARY 1 and determine the distribution parameters of  $G(T; T_0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ , conditioned on  $X(t) = x = \xi_t = \ln(S_t)$  and  $G(t; T_0, T) = g$ . This gives us

$$\begin{aligned}
 \mu_G &= g + \frac{1}{T - T_0} \int_0^{T-t} e^{\beta \cdot u} du \cdot x + \frac{1}{T - T_0} \int_0^{T-t} \int_0^u e^{\beta \cdot s} ds du \cdot b \\
 &= g + \frac{1}{T - T_0} \int_0^{T-t} 1 du \cdot \xi_t + \frac{1}{T - T_0} \int_0^{T-t} \int_0^u 1 ds du \cdot a^* \\
 &= g + \frac{\xi_t \cdot (T - t)}{T - T_0} + \frac{1}{T - T_0} \int_0^{T-t} u \cdot a^* du \tag{A.28}
 \end{aligned}$$

$$\begin{aligned}
 &= g + \frac{\xi_t \cdot (T - t)}{T - T_0} + \frac{1}{T - T_0} \left[ \frac{1}{2} u^2 \cdot a^* \right]_0^{T-t} \\
 &= g + \frac{\xi_t \cdot (T - t)}{T - T_0} + \frac{a^* \cdot (T - t)}{2(T - T_0)}, \\
 \sigma_G^2 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \Sigma^\top \int_0^u e^{\beta \tau \cdot s} ds \right\|_2^2 du = \frac{1}{(T - T_0)^2} \int_0^{T-t} \|u \cdot \sigma_\xi\|_2^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} u^2 \cdot \sigma_\xi^2 du = \frac{1}{(T - T_0)^2} \left[ \frac{1}{3} u^3 \cdot \sigma_\xi^2 \right]_0^{T-t} \tag{A.29} \\
 &= \frac{\sigma_\xi^2 \cdot (T - t)^3}{3 \cdot (T - T_0)^2}.
 \end{aligned}$$

Accordingly, the log geometric average,  $G(T; T_0, T)$ , is normally distributed with the following mean and variance

$$G \sim \mathcal{N} \left( g + \frac{\xi_t \cdot (T - t)}{T - T_0} + \frac{a^* \cdot (T - t)}{2 \cdot (T - T_0)}, \frac{\sigma_\xi^2 \cdot (T - t)^3}{3 \cdot (T - T_0)^2} \right). \tag{A.30}$$

Inserting the two distribution parameters into equation (4.28) yields the desired closed-form solution for a geometric Asian call option within the averaging period.

### A.2.2 Schwartz (1997) one-factor model

For the Schwartz (1997) one-factor model, we consider the stochastic process  $X = \xi_t$  with  $n = d = 1$ ,  $w = 1$ , and starting value  $X(0) = x = \xi_0 = \ln(S_0)$ . The corresponding diffusion parameters are

$$b = (\kappa_\xi \cdot a^*), \quad (\text{A.31})$$

$$\beta = (-\kappa_\xi), \quad (\text{A.32})$$

$$\Sigma = (\sigma_\xi), \quad (\text{A.33})$$

$$a = \left(\frac{1}{2}\sigma_\xi^2\right). \quad (\text{A.34})$$

First, we consider the case at inception of the averaging period (i.e.,  $t = T_0 = 0$ ) and assume that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply THEOREM 1 and determine the distribution parameters of  $G(T; 0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ . This gives us

$$\begin{aligned} \mu_G &= \frac{1}{T} \int_0^T e^{\beta \cdot t} dt \cdot x + \frac{1}{T} \int_0^T \int_0^t e^{\beta \cdot s} ds dt \cdot b \\ &= \frac{1}{T} \int_0^T e^{-\kappa_\xi \cdot t} dt \cdot \xi_0 + \frac{1}{T} \int_0^T \int_0^t e^{-\kappa_\xi \cdot s} ds dt \cdot \kappa_\xi \cdot a^* \\ &= \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + \frac{1}{T} \int_0^T \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot t}) dt \cdot \kappa_\xi \cdot a^* \\ &= \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + \frac{a^*}{T} \int_0^T (1 - e^{-\kappa_\xi \cdot t}) dt \\ &= \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + \frac{a^*}{\kappa_\xi \cdot T} [\kappa_\xi \cdot t + e^{-\kappa_\xi \cdot t}]_0^T \\ &= \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + a^* - \frac{a^*}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}), \end{aligned} \quad (\text{A.35})$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{T^2} \int_0^T \left\| \Sigma^\top \int_0^t e^{\beta^\top \cdot s} ds \right\|_2^2 dt = \frac{\sigma_\xi^2}{T^2} \int_0^T \left\| \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot t}) \right\|_2^2 dt \\
 &= \frac{\sigma_\xi^2}{\kappa_\xi^2 \cdot T^2} \int_0^T (1 - e^{-\kappa_\xi \cdot t})^2 dt \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} \left[ 2 \cdot \kappa_\xi \cdot t + 4 \cdot e^{-\kappa_\xi \cdot t} - e^{-2 \cdot \kappa_\xi \cdot t} \right]_0^T \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} (2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi \cdot T} - e^{-2 \cdot \kappa_\xi \cdot T} - 3).
 \end{aligned} \tag{A.36}$$

Accordingly, the log geometric average,  $G(T; 0, T)$ , is normally distributed with the following mean and variance

$$\begin{aligned}
 G \sim \mathcal{N} \left( \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + a^* - \frac{a^*}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}), \right. \\
 \left. \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} (2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi \cdot T} - e^{-2 \cdot \kappa_\xi \cdot T} - 3) \right).
 \end{aligned} \tag{A.37}$$

Inserting the two distribution parameters into equation (4.36) yields the desired closed-form solution for a geometric Asian call option.

Secondly, we consider the case within the averaging period (i.e.,  $t \in [T_0, T_1)$ ) and assume again that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply COROLLARY 1 and determine the distribution parameters of  $G(T; T_0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ , conditioned on  $X(t) = x = \xi_t = \ln(S_t)$  and

$G(t; T_0, T) = g$ . This gives us

$$\begin{aligned}
 \mu_G &= g + \frac{1}{T - T_0} \int_0^{T-t} e^{\beta \cdot u} du \cdot x + \frac{1}{T - T_0} \int_0^{T-t} \int_0^u e^{\beta \cdot s} ds du \cdot b \\
 &= g + \frac{1}{T - T_0} \int_0^{T-t} e^{-\kappa_\xi \cdot t} dt \cdot \xi_t + \frac{1}{T - T_0} \int_0^{T-t} \int_0^t e^{-\kappa_\xi \cdot s} ds dt \cdot \kappa_\xi \cdot a^* \\
 &= g + \frac{\xi_t}{\kappa_\xi \cdot (T - T_0)} \left(1 - e^{-\kappa_\xi \cdot (T-t)}\right) \\
 &\quad + \frac{1}{T - T_0} \int_0^{T-t} \frac{1}{\kappa_\xi} \left(1 - e^{-\kappa_\xi \cdot t}\right) dt \cdot \kappa_\xi \cdot a^* \\
 &= g + \frac{\xi_t}{\kappa_\xi \cdot (T - T_0)} \left(1 - e^{-\kappa_\xi \cdot (T-t)}\right) \\
 &\quad + \frac{a^*}{\kappa_\xi \cdot (T - T_0)} \left[\kappa_\xi \cdot u + e^{-\kappa_\xi \cdot u}\right]_0^{T-t} \\
 &= g + \frac{\xi_t}{\kappa_\xi \cdot (T - T_0)} \left(1 - e^{-\kappa_\xi \cdot (T-t)}\right) \\
 &\quad + \frac{a^*}{\kappa_\xi \cdot (T - T_0)} \left(\kappa_\xi \cdot (T - t) + e^{-\kappa_\xi \cdot (T-t)} - 1\right),
 \end{aligned} \tag{A.38}$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \Sigma^\top \int_0^u e^{\beta \cdot s} ds \right\|_2^2 du \\
 &= \frac{\sigma_\xi^2}{(T - T_0)^2} \int_0^{T-t} \left\| \frac{1}{\kappa_\xi} \left(1 - e^{-\kappa_\xi \cdot u}\right) \right\|_2^2 du \\
 &= \frac{\sigma_\xi^2}{\kappa_\xi^2 \cdot (T - T_0)^2} \int_0^{T-t} \left(1 - e^{-\kappa_\xi \cdot u}\right)^2 du \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot (T - T_0)^2} \left[2 \cdot \kappa_\xi \cdot u + 4 \cdot e^{-\kappa_\xi \cdot u} - e^{-2 \cdot \kappa_\xi \cdot u}\right]_0^{T-t} \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot (T - T_0)^2} \left(2 \cdot \kappa_\xi \cdot (T - t) + 4 \cdot e^{-\kappa_\xi \cdot (T-t)} - e^{-2 \cdot \kappa_\xi \cdot (T-t)} - 3\right).
 \end{aligned} \tag{A.39}$$

Accordingly, the log geometric average,  $G(T; T_0, T)$ , is normally distributed with the

following mean and variance

$$\begin{aligned}
 G \sim \mathcal{N} \left( g + \frac{\xi_t}{\kappa_\xi \cdot (T - T_0)} \left( 1 - e^{-\kappa_\xi \cdot (T-t)} \right) + \frac{a^*}{\kappa_\xi \cdot (T - T_0)} \left( \kappa_\xi \cdot (T - t) \right. \right. \\
 \left. \left. + e^{-\kappa_\xi \cdot (T-t)} - 1 \right), \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot (T - T_0)^2} \left( 2 \cdot \kappa_\xi \cdot (T - t) \right. \right. \\
 \left. \left. + 4 \cdot e^{-\kappa_\xi \cdot (T-t)} - e^{-2 \cdot \kappa_\xi \cdot (T-t)} - 3 \right) \right). \tag{A.40}
 \end{aligned}$$

Inserting the two distribution parameters into equation (4.28) yields the desired closed-form solution for a geometric Asian call option within the averaging period.

### A.2.3 Schwartz and Smith (2000) two-factor model

For the Schwartz and Smith (2000) two-factor model, we consider the stochastic process  $\mathbf{X} = (\xi, \chi)^\top$  with  $n = d = 2$ ,  $\mathbf{w} = (1, 1)^\top$ , and starting value  $\mathbf{X}(0) = \mathbf{x} = (\xi_0, \chi_0)^\top$  with  $\langle \mathbf{w}, \mathbf{X}(0) \rangle = \ln(S_0)$ . The corresponding diffusion parameters are

$$\mathbf{b} = \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix}, \quad (\text{A.41})$$

$$\boldsymbol{\beta} = \begin{pmatrix} 0 & 0 \\ 0 & -\kappa_\chi \end{pmatrix}, \quad (\text{A.42})$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_\xi & 0 \\ \rho_{\xi,\chi}\sigma_\chi & \sqrt{1 - \rho_{\xi,\chi}^2}\sigma_\chi \end{pmatrix}, \quad (\text{A.43})$$

$$\mathbf{a} = \begin{pmatrix} \frac{1}{2}\sigma_\xi^2 & \frac{1}{2}\rho_{\xi,\chi}\sigma_\xi\sigma_\chi \\ \frac{1}{2}\rho_{\xi,\chi}\sigma_\xi\sigma_\chi & \frac{1}{2}\sigma_\chi^2 \end{pmatrix}. \quad (\text{A.44})$$

First, we consider the case at inception of the averaging period (i.e.,  $t = T_0 = 0$ ) and assume that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply THEOREM 1 and determine the distribution parameters of

$G(T; 0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ . This gives us

$$\begin{aligned}
\mu_G &= \frac{1}{T} \left\langle \mathbf{w}, \int_0^T e^{\beta \cdot t} dt \cdot \mathbf{x} \right\rangle + \frac{1}{T} \left\langle \mathbf{w}, \int_0^T \int_0^t e^{\beta \cdot s} ds dt \cdot \mathbf{b} \right\rangle \\
&= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^T e^{0 \cdot t} dt & 0 \\ 0 & \int_0^T e^{-\kappa_\chi \cdot t} dt \end{pmatrix} \cdot \begin{pmatrix} \xi_0 \\ \chi_0 \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^T \begin{pmatrix} \int_0^t e^{0 \cdot s} ds & 0 \\ 0 & \int_0^t e^{-\kappa_\chi \cdot s} ds \end{pmatrix} dt \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} T & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot T}) \end{pmatrix} \cdot \begin{pmatrix} \xi_0 \\ \chi_0 \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^T \begin{pmatrix} t & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot t}) \end{pmatrix} dt \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \xi_0 \cdot T \\ \frac{\chi_0}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot T}) \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^T t dt & 0 \\ 0 & \int_0^T \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot t}) dt \end{pmatrix} \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= \xi_0 + \frac{\chi_0}{\kappa_\chi \cdot T} (1 - e^{-\kappa_\chi \cdot T}) + \frac{1}{2} a^* \cdot T + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} (1 - \kappa_\chi \cdot T - e^{-\kappa_\chi \cdot T}),
\end{aligned} \tag{A.45}$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{T^2} \int_0^T \left\| \Sigma^\top \int_0^t e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} t & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} t \\ \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \sigma_\xi \cdot t + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \\ \frac{\sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left( \sigma_\xi \cdot t + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \right)^2 + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \quad (\text{A.46}) \\
 &= \frac{1}{T^2} \int_0^T \sigma_\xi^2 \cdot t^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi \cdot t}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) + \frac{\rho_{\xi,\chi}^2 \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 \\
 &\quad + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \\
 &= \frac{1}{T^2} \int_0^T \sigma_\xi^2 \cdot t^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi \cdot t}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) + \frac{\sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \\
 &= \frac{1}{3} \sigma_\xi^2 \cdot T + \frac{\rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\chi^3 \cdot T^2} \left( \kappa_\chi^2 \cdot T^2 + 2 \cdot \kappa_\chi \cdot T \cdot e^{-\kappa_\chi T} + 2 \cdot e^{-\kappa_\chi T} - 2 \right) \\
 &\quad + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} \left( 2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi T} - e^{-2 \cdot \kappa_\chi T} - 3 \right).
 \end{aligned}$$

Accordingly, the log geometric average,  $G(T; 0, T)$ , is normally distributed with the following mean and variance

$$\begin{aligned}
 G &\sim \mathcal{N} \left( \xi_0 + \frac{\chi_0}{\kappa_\chi \cdot T} (1 - e^{-\kappa_\chi T}) + \frac{1}{2} a^* \cdot T + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} (1 - \kappa_\chi \cdot T - e^{-\kappa_\chi T}), \right. \\
 &\quad \frac{1}{3} \sigma_\xi^2 \cdot T + \frac{\rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\chi^3 \cdot T^2} \left( \kappa_\chi^2 \cdot T^2 + 2 \cdot \kappa_\chi \cdot T \cdot e^{-\kappa_\chi T} + 2 \cdot e^{-\kappa_\chi T} - 2 \right) \quad (\text{A.47}) \\
 &\quad \left. + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} \left( 2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi T} - e^{-2 \cdot \kappa_\chi T} - 3 \right) \right).
 \end{aligned}$$

Inserting the two distribution parameters into equation (4.37) yields the desired closed-form solution for a geometric Asian call option.

Secondly, we consider the case within the averaging period (i.e.,  $t \in [T_0, T_1)$ ) and assume again that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply COROLLARY 1 and determine the distribution parameters of  $G(T; T_0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ , conditioned on  $\mathbf{X}(t) = \mathbf{x} = (\xi_t, \chi_t)^\top$ ,  $\langle \mathbf{w}, \mathbf{X}(t) \rangle = \ln(S_t)$ , and  $G(t; T_0, T) = g$ . This gives us

$$\begin{aligned}
\mu_G &= g + \frac{1}{T - T_0} \left\langle \mathbf{w}, \int_0^{T-t} e^{\beta \cdot u} du \cdot \mathbf{x} \right\rangle + \frac{1}{T - T_0} \left\langle \mathbf{w}, \int_0^{T-t} \int_0^u e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle \\
&= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^{T-t} e^{0 \cdot u} du & 0 \\ 0 & \int_0^{T-t} e^{-\kappa_\chi \cdot u} du \end{pmatrix} \cdot \begin{pmatrix} \xi_t \\ \chi_t \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^{T-t} \begin{pmatrix} \int_0^u e^{0 \cdot s} ds & 0 \\ 0 & \int_0^u e^{-\kappa_\chi \cdot s} ds \end{pmatrix} du \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} T - t & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot (T-t)}) \end{pmatrix} \cdot \begin{pmatrix} \xi_t \\ \chi_t \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^{T-t} \begin{pmatrix} u & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \end{pmatrix} du \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \xi_t \cdot (T - t) \\ \frac{\chi_t}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot (T-t)}) \end{pmatrix} \right\rangle \\
&\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^{T-t} u du & 0 \\ 0 & \int_0^{T-t} \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) du \end{pmatrix} \cdot \begin{pmatrix} a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
&= g + \frac{\xi_t \cdot (T - t)}{T - T_0} + \frac{\chi_t}{\kappa_\chi \cdot (T - T_0)} (1 - e^{-\kappa_\chi \cdot (T-t)}) \\
&\quad + \frac{a^* \cdot (T - t)}{T - T_0} + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot (T - T_0)} (1 - \kappa_\chi \cdot (T - t) - e^{-\kappa_\chi \cdot (T-t)}),
\end{aligned} \tag{A.48}$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \left\| \Sigma^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1-\rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} u & 0 \\ 0 & \frac{1}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1-\rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} u \\ \frac{1}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \sigma_\xi \cdot u + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \\ \frac{\sqrt{1-\rho_{\xi,\chi}^2} \sigma_\chi}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \left( \sigma_\xi \cdot u + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \right)^2 \\
 &\quad + \frac{(1-\rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1-e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \sigma_\xi^2 \cdot u^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi \cdot u}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \\
 &\quad + \frac{\rho_{\xi,\chi}^2 \sigma_\chi^2}{\kappa_\chi^2} (1-e^{-\kappa_\chi \cdot u})^2 + \frac{(1-\rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1-e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{1}{(T-T_0)^2} \int_0^{T-t} \sigma_\xi^2 \cdot u^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi \cdot u}{\kappa_\chi} (1-e^{-\kappa_\chi \cdot u}) \\
 &\quad + \frac{\sigma_\chi^2}{\kappa_\chi^2} (1-e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{\sigma_\xi^2 \cdot (T-t)^3}{3 \cdot (T-T_0)^2} + \frac{\rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\chi^3 \cdot (T-T_0)^2} (\kappa_\chi^2 \cdot (T-t)^2 \\
 &\quad + 2 \cdot \kappa_\chi \cdot (T-t) \cdot e^{-\kappa_\chi \cdot (T-t)} + 2 \cdot e^{-\kappa_\chi \cdot (T-t)} - 2) \\
 &\quad + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot (T-T_0)^2} (2 \cdot \kappa_\chi \cdot (T-t) + 4 \cdot e^{-\kappa_\chi \cdot (T-t)} - e^{-2 \cdot \kappa_\chi \cdot (T-t)} - 3).
 \end{aligned} \tag{A.49}$$

Accordingly, the log geometric average,  $G(T; T_0, T)$ , is normally distributed with the

following mean and variance

$$\begin{aligned}
G \sim \mathcal{N} & \left( g + \frac{\xi_t \cdot (T-t)}{T-T_0} + \frac{\lambda_t}{\kappa_\chi \cdot (T-T_0)} \left( 1 - e^{-\kappa_\chi \cdot (T-t)} \right) \right. \\
& + \frac{a^* \cdot (T-t)}{T-T_0} + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot (T-T_0)} \left( 1 - \kappa_\chi \cdot (T-t) - e^{-\kappa_\chi \cdot (T-t)} \right), \\
& \frac{\sigma_\xi^2 \cdot (T-t)^3}{3 \cdot (T-T_0)^2} + \frac{\rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\chi^3 \cdot (T-T_0)^2} \left( \kappa_\chi^2 \cdot (T-t)^2 \right. \\
& \left. + 2 \cdot \kappa_\chi \cdot (T-t) \cdot e^{-\kappa_\chi \cdot (T-t)} + 2 \cdot e^{-\kappa_\chi \cdot (T-t)} - 2 \right) \\
& \left. + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot (T-T_0)^2} \left( 2 \cdot \kappa_\chi \cdot (T-t) + 4 \cdot e^{-\kappa_\chi \cdot (T-t)} \right. \right. \\
& \left. \left. - e^{-2 \cdot \kappa_\chi \cdot (T-t)} - 3 \right) \right). \tag{A.50}
\end{aligned}$$

Inserting the two distribution parameters into equation (4.28) yields the desired closed-form solution for a geometric Asian call option within the averaging period.

### A.2.4 Korn (2005) two-factor model

For the Korn (2005) two-factor model, we consider the stochastic process  $\mathbf{X} = (\xi, \chi)^\top$  with  $n = d = 2$ ,  $\mathbf{w} = (1, 1)^\top$ , and starting value  $\mathbf{X}(0) = \mathbf{x} = (\xi_0, \chi_0)^\top$  with  $\langle \mathbf{w}, \mathbf{X}(0) \rangle = \ln(S_0)$ . The corresponding diffusion parameters are

$$\mathbf{b} = \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix}, \quad (\text{A.51})$$

$$\boldsymbol{\beta} = \begin{pmatrix} -\kappa_\xi & 0 \\ 0 & -\kappa_\chi \end{pmatrix}, \quad (\text{A.52})$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_\xi & 0 \\ \rho_{\xi,\chi}\sigma_\chi & \sqrt{1 - \rho_{\xi,\chi}^2}\sigma_\chi \end{pmatrix}, \quad (\text{A.53})$$

$$\mathbf{a} = \begin{pmatrix} \frac{1}{2}\sigma_\xi^2 & \frac{1}{2}\rho_{\xi,\chi}\sigma_\xi\sigma_\chi \\ \frac{1}{2}\rho_{\xi,\chi}\sigma_\xi\sigma_\chi & \frac{1}{2}\sigma_\chi^2 \end{pmatrix}. \quad (\text{A.54})$$

First, we consider the case at inception of the averaging period (i.e.,  $t = T_0 = 0$ ) and assume that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply THEOREM 1 and determine the distribution parameters of

$G(T; 0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ . This gives us

$$\begin{aligned}
 \mu_G &= \frac{1}{T} \left\langle \mathbf{w}, \int_0^T e^{\beta \cdot t} dt \cdot \mathbf{x} \right\rangle + \frac{1}{T} \left\langle \mathbf{w}, \int_0^T \int_0^t e^{\beta \cdot s} ds dt \cdot \mathbf{b} \right\rangle \\
 &= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^T e^{-\kappa_\xi \cdot t} dt & 0 \\ 0 & \int_0^T e^{-\kappa_\chi \cdot t} dt \end{pmatrix} \cdot \begin{pmatrix} \xi_0 \\ \chi_0 \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^T \begin{pmatrix} \int_0^t e^{-\kappa_\xi \cdot s} ds & 0 \\ 0 & \int_0^t e^{-\kappa_\chi \cdot s} ds \end{pmatrix} dt \cdot \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
 &= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot T}) & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot T}) \end{pmatrix} \cdot \begin{pmatrix} \xi_0 \\ \chi_0 \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^T \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot t}) & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot t}) \end{pmatrix} dt \cdot \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
 &= \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{\xi_0}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot T}) \\ \frac{\chi_0}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot T}) \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^T \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot t}) dt & 0 \\ 0 & \int_0^T \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot t}) dt \end{pmatrix} \cdot \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
 &= \frac{\xi_0}{\kappa_\chi \cdot T} (1 - e^{-\kappa_\chi \cdot T}) + \frac{\chi_0}{\kappa_\chi \cdot T} (1 - e^{-\kappa_\chi \cdot T}) \\
 &\quad + \frac{a^*}{\kappa_\xi \cdot T} (e^{-\kappa_\xi \cdot T} + \kappa_\xi \cdot T - 1) + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} (1 - e^{-\kappa_\chi \cdot T} - \kappa_\chi \cdot T),
 \end{aligned} \tag{A.55}$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{T^2} \int_0^T \left\| \boldsymbol{\Sigma}^\top \int_0^t e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} \frac{1 - e^{-\kappa_\xi t}}{\kappa_\xi} & 0 \\ 0 & \frac{1 - e^{-\kappa_\chi t}}{\kappa_\chi} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi t}) \\ \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left\| \begin{pmatrix} \frac{\sigma_\xi}{\kappa_\xi} (1 - e^{-\kappa_\xi t}) + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \\ \frac{\sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \end{pmatrix} \right\|_2^2 dt \\
 &= \frac{1}{T^2} \int_0^T \left( \frac{\sigma_\xi}{\kappa_\xi} (1 - e^{-\kappa_\xi t}) + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi t}) \right)^2 \\
 &\quad + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \\
 &= \frac{1}{T^2} \int_0^T \frac{\sigma_\xi^2}{\kappa_\xi^2} (1 - e^{-\kappa_\xi t})^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi \cdot \kappa_\chi} (1 - e^{-\kappa_\xi t}) (1 - e^{-\kappa_\chi t}) \\
 &\quad + \frac{\rho_{\xi,\chi}^2 \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \tag{A.56} \\
 &= \frac{1}{T^2} \int_0^T \frac{\sigma_\xi^2}{\kappa_\xi^2} (1 - e^{-\kappa_\xi t})^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi \cdot \kappa_\chi} (1 - e^{-\kappa_\xi t}) (1 - e^{-\kappa_\chi t}) \\
 &\quad + \frac{\sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi t})^2 dt \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} (2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi T} - e^{-2 \cdot \kappa_\xi T} - 3) \\
 &\quad + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi^2 \cdot \kappa_\chi^2 (\kappa_\xi + \kappa_\chi) T^2} \left( \kappa_\xi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\chi T} + \kappa_\xi \cdot \kappa_\chi \cdot T (\kappa_\xi + \kappa_\chi) \right. \\
 &\quad \left. + \kappa_\chi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\xi T} - \kappa_\xi \cdot \kappa_\chi \cdot e^{-(\kappa_\xi + \kappa_\chi) T} - (\kappa_\xi + \kappa_\chi)^2 + \kappa_\xi \cdot \kappa_\chi \right) \\
 &\quad + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} (2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi T} - e^{-2 \cdot \kappa_\chi T} - 3).
 \end{aligned}$$

Accordingly, the log geometric average,  $G(T; 0, T)$ , is normally distributed with mean and variance as specified below:

$$\begin{aligned}
G \sim \mathcal{N} & \left( \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + \frac{\chi_0}{\kappa_\chi \cdot T} (1 - e^{-\kappa_\chi \cdot T}) \right. \\
& + \frac{a^*}{\kappa_\xi \cdot T} (e^{-\kappa_\xi \cdot T} + \kappa_\xi \cdot T - 1) + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot T} (1 - e^{-\kappa_\chi \cdot T} - \kappa_\chi \cdot T), \\
& \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} (2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi \cdot T} - e^{-2 \cdot \kappa_\xi \cdot T} - 3) \\
& + \frac{2 \cdot \rho_{\xi, \chi} \sigma_\xi \sigma_\chi}{\kappa_\xi^2 \cdot \kappa_\chi^2 (\kappa_\xi + \kappa_\chi) T^2} \left( \kappa_\xi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\chi \cdot T} + \kappa_\xi \cdot \kappa_\chi \cdot T (\kappa_\xi + \kappa_\chi) \right. \\
& + \kappa_\chi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\xi \cdot T} - \kappa_\xi \cdot \kappa_\chi \cdot e^{-(\kappa_\xi + \kappa_\chi) \cdot T} - (\kappa_\xi + \kappa_\chi)^2 + \kappa_\xi \cdot \kappa_\chi \\
& \left. \left. + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot T^2} (2 \cdot \kappa_\chi \cdot T + 4 \cdot e^{-\kappa_\chi \cdot T} - e^{-2 \cdot \kappa_\chi \cdot T} - 3) \right) \right). \tag{A.57}
\end{aligned}$$

Inserting the two distribution parameters into equation (4.38) yields the desired closed-form solution for a geometric Asian call option.

Secondly, we consider the case within the averaging period (i.e.,  $t \in [T_0, T_1)$ ) and assume again that the end of the averaging period coincides with the settlement date (i.e.,  $T_1 = T$ ). Now, we apply COROLLARY 1 and determine the distribution parameters of  $G(T; T_0, T)$ ,  $\mu_G$  and  $\sigma_G^2$ , conditioned on  $\mathbf{X}(t) = \mathbf{x} = (\xi_t, \chi_t)^\top$ ,  $\langle \mathbf{w}, \mathbf{X}(t) \rangle = \ln(S_t)$ ,

and  $G(t; T_0, T) = g$ . This gives us

$$\begin{aligned}
 \mu_G &= g + \frac{1}{T - T_0} \left\langle \mathbf{w}, \int_0^{T-t} e^{\beta \cdot u} du \cdot \mathbf{x} \right\rangle + \frac{1}{T - T_0} \left\langle \mathbf{w}, \int_0^{T-t} \int_0^u e^{\beta \cdot s} ds du \cdot \mathbf{b} \right\rangle \\
 &= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^{T-t} e^{-\kappa_\xi \cdot u} du & 0 \\ 0 & \int_0^{T-t} e^{-\kappa_\chi \cdot u} du \end{pmatrix} \cdot \begin{pmatrix} \xi_t \\ \chi_t \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^{T-t} \begin{pmatrix} \int_0^u e^{-\kappa_\xi \cdot s} ds & 0 \\ 0 & \int_0^u e^{-\kappa_\chi \cdot s} ds \end{pmatrix} du \cdot \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \\
 &= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot (T-t)}) & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot (T-t)}) \end{pmatrix} \cdot \begin{pmatrix} \xi_t \\ \chi_t \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \int_0^{T-t} \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot u}) & 0 \\ 0 & \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \end{pmatrix} du \cdot \begin{pmatrix} \kappa_\xi \cdot a^* \\ -\lambda_\chi \end{pmatrix} \right\rangle \quad (\text{A.58}) \\
 &= g + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \frac{\xi_t}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot (T-t)}) \\ \frac{\chi_t}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot (T-t)}) \end{pmatrix} \right\rangle \\
 &\quad + \frac{1}{T - T_0} \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \int_0^{T-t} a^* (1 - e^{-\kappa_\xi \cdot u}) du & 0 \\ 0 & \int_0^{T-t} -\frac{\lambda_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) du \end{pmatrix} \right\rangle \\
 &= g + \frac{\xi_t}{\kappa_\chi \cdot (T - T_0)} (1 - e^{-\kappa_\chi \cdot (T-t)}) + \frac{\lambda_\chi}{\kappa_\chi \cdot (T - T_0)} (1 - e^{-\kappa_\chi \cdot (T-t)}) \\
 &\quad + \frac{a^*}{\kappa_\xi \cdot (T - T_0)} (e^{-\kappa_\xi \cdot (T-t)} + \kappa_\xi \cdot (T - t) - 1) \\
 &\quad + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot (T - T_0)} (1 - e^{-\kappa_\chi \cdot (T-t)} - \kappa_\chi \cdot (T - t)),
 \end{aligned}$$

$$\begin{aligned}
 \sigma_G^2 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \Sigma^\top \int_0^u e^{\beta^\top \cdot s} ds \cdot \mathbf{w} \right\|_2^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} \frac{1 - e^{-\kappa_\xi \cdot u}}{\kappa_\xi} & 0 \\ 0 & \frac{1 - e^{-\kappa_\chi \cdot u}}{\kappa_\chi} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \sigma_\xi & \rho_{\xi,\chi} \sigma_\chi \\ 0 & \sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot u}) \\ \frac{1}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left\| \begin{pmatrix} \frac{\sigma_\xi}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot u}) + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \\ \frac{\sqrt{1 - \rho_{\xi,\chi}^2} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \end{pmatrix} \right\|_2^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \left( \frac{\sigma_\xi}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot u}) + \frac{\rho_{\xi,\chi} \sigma_\chi}{\kappa_\chi} (1 - e^{-\kappa_\chi \cdot u}) \right)^2 \\
 &\quad + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \frac{\sigma_\xi^2}{\kappa_\xi^2} (1 - e^{-\kappa_\xi \cdot u})^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi \cdot \kappa_\chi} (1 - e^{-\kappa_\xi \cdot u}) (1 - e^{-\kappa_\chi \cdot u}) \\
 &\quad + \frac{\rho_{\xi,\chi}^2 \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi \cdot u})^2 + \frac{(1 - \rho_{\xi,\chi}^2) \sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{1}{(T - T_0)^2} \int_0^{T-t} \frac{\sigma_\xi^2}{\kappa_\xi^2} (1 - e^{-\kappa_\xi \cdot u})^2 + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi \cdot \kappa_\chi} (1 - e^{-\kappa_\xi \cdot u}) (1 - e^{-\kappa_\chi \cdot u}) \\
 &\quad + \frac{\sigma_\chi^2}{\kappa_\chi^2} (1 - e^{-\kappa_\chi \cdot u})^2 du \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot (T - T_0)^2} \left( 2 \cdot \kappa_\xi \cdot (T - t) + 4 \cdot e^{-\kappa_\xi \cdot (T-t)} - e^{-2 \cdot \kappa_\xi \cdot (T-t)} - 3 \right) \\
 &\quad + \frac{2 \cdot \rho_{\xi,\chi} \sigma_\xi \sigma_\chi}{\kappa_\xi^2 \cdot \kappa_\chi^2 (\kappa_\xi + \kappa_\chi) (T - T_0)^2} \left( \kappa_\xi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\chi \cdot (T-t)} \right. \\
 &\quad + \kappa_\xi \cdot \kappa_\chi \cdot (T - t) (\kappa_\xi + \kappa_\chi) + \kappa_\chi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\xi \cdot (T-t)} \\
 &\quad \left. - \kappa_\xi \cdot \kappa_\chi \cdot e^{-(\kappa_\xi + \kappa_\chi) \cdot (T-t)} - (\kappa_\xi + \kappa_\chi)^2 + \kappa_\xi \cdot \kappa_\chi \right) \\
 &\quad + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot (T - T_0)^2} \left( 2 \cdot \kappa_\chi \cdot (T - t) + 4 \cdot e^{-\kappa_\chi \cdot (T-t)} - e^{-2 \cdot \kappa_\chi \cdot (T-t)} - 3 \right).
 \end{aligned} \tag{A.59}$$

Accordingly, the log geometric average,  $G(T; T_0, T)$ , is normally distributed with the

following mean and variance

$$\begin{aligned}
 G \sim \mathcal{N} & \left( g + \frac{\xi_t}{\kappa_\chi \cdot (T - T_0)} \left( 1 - e^{-\kappa_\chi \cdot (T-t)} \right) + \frac{\chi_t}{\kappa_\chi \cdot (T - T_0)} \left( 1 - e^{-\kappa_\chi \cdot (T-t)} \right) \right. \\
 & + \frac{a^*}{\kappa_\xi \cdot (T - T_0)} \left( e^{-\kappa_\xi \cdot (T-t)} + \kappa_\xi \cdot (T - t) - 1 \right) \\
 & + \frac{\lambda_\chi}{\kappa_\chi^2 \cdot (T - T_0)} \left( 1 - e^{-\kappa_\chi \cdot (T-t)} - \kappa_\chi \cdot (T - t) \right), \\
 & \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot (T - T_0)^2} \left( 2 \cdot \kappa_\xi \cdot (T - t) + 4 \cdot e^{-\kappa_\xi \cdot (T-t)} - e^{-2 \cdot \kappa_\xi \cdot (T-t)} - 3 \right) \\
 & + \frac{2 \cdot \rho_{\xi, \chi} \sigma_\xi \sigma_\chi}{\kappa_\xi^2 \cdot \kappa_\chi^2 (\kappa_\xi + \kappa_\chi) (T - T_0)^2} \left( \kappa_\xi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\chi \cdot (T-t)} \right. \\
 & + \kappa_\xi \cdot \kappa_\chi \cdot (T - t) (\kappa_\xi + \kappa_\chi) + \kappa_\chi (\kappa_\xi + \kappa_\chi) e^{-\kappa_\xi \cdot (T-t)} \\
 & \left. - \kappa_\xi \cdot \kappa_\chi \cdot e^{-(\kappa_\xi + \kappa_\chi) \cdot (T-t)} - (\kappa_\xi + \kappa_\chi)^2 + \kappa_\xi \cdot \kappa_\chi \right) \\
 & + \frac{\sigma_\chi^2}{2 \cdot \kappa_\chi^3 \cdot (T - T_0)^2} \left( 2 \cdot \kappa_\chi \cdot (T - t) + 4 \cdot e^{-\kappa_\chi \cdot (T-t)} \right. \\
 & \left. - e^{-2 \cdot \kappa_\chi \cdot (T-t)} - 3 \right) \Bigg). \tag{A.60}
 \end{aligned}$$

Inserting the two distribution parameters into equation (4.28) yields the desired closed-form solution for a geometric Asian call option within the averaging period.

### A.2.5 Alternative derivation for the Schwartz (1997) one-factor model

For the Schwartz (1997) one-factor model, the mean of the log geometric average distribution can also be determined manually or directly according to equation (A.61) as shown below:

$$\begin{aligned}
 \mu_G &= \mathbb{E} \left[ \frac{1}{T} \int_0^T \ln S(t) dt \right] = \frac{1}{T} \int_0^T \mathbb{E} [\ln S(t)] dt = \frac{1}{T} \int_0^T \mathbb{E} [\xi_t] dt \\
 &= \frac{1}{T} \int_0^T \mathbb{E} \left[ e^{-\kappa_\xi \cdot t} \cdot \xi_0 + a^* (1 - e^{-\kappa_\xi \cdot t}) + \sigma_\xi \int_0^t e^{-\kappa_\xi \cdot (t-s)} dW_s \right] dt \\
 &= \frac{1}{T} \int_0^T \left( \mathbb{E} [e^{-\kappa_\xi \cdot t} \cdot \xi_0 + a^* (1 - e^{-\kappa_\xi \cdot t})] + \mathbb{E} \left[ \sigma_\xi \int_0^t e^{-\kappa_\xi \cdot (t-s)} dW_s \right] \right) dt \\
 &= \frac{1}{T} \int_0^T \mathbb{E} [e^{-\kappa_\xi \cdot t} \cdot \xi_0 + a^* (1 - e^{-\kappa_\xi \cdot t})] dt \\
 &= \frac{1}{T} \left[ \frac{e^{-\kappa_\xi \cdot t} \cdot (a^* - \xi_0) + a^* \cdot \kappa_\xi \cdot t}{\kappa_\xi} + c \right]_0^T \\
 &= a^* + \frac{e^{-\kappa_\xi \cdot T} \cdot (a^* - \xi_0) - a^* + \xi_0}{\kappa_\xi \cdot T} \\
 &= a^* - \frac{a^*}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T}) + \frac{\xi_0}{\kappa_\xi \cdot T} (1 - e^{-\kappa_\xi \cdot T})
 \end{aligned} \tag{A.61}$$

with  $a^* = a - \lambda_\xi$

$$\text{and } \mathbb{E} \left[ \sigma_\xi \int_0^t e^{-\kappa_\xi \cdot (t-s)} dW_s \right] = 0$$

because  $\int_0^T (e^{-\kappa_\xi \cdot (t-s)})^2 ds < \infty$  with  $\kappa_\xi > 0$ .

The variance of the mean of the log geometric average distribution can be determined according to equation (A.62) as shown below:

$$\begin{aligned}
 \sigma_G^2 &= \text{Var} \left[ \frac{1}{T} \int_0^T \ln S(t) dt \right] = \frac{1}{T^2} \text{Var} \left[ \int_0^T \ln S(t) dt \right] = \frac{1}{T^2} \text{Var} \left[ \int_0^T \xi_t dt \right] \\
 &= \frac{1}{T^2} \text{Var} \left[ \int_0^T \left( e^{-\kappa_\xi t} \cdot \xi_0 + a^* (1 - e^{-\kappa_\xi t}) + \sigma_\xi \int_0^t e^{-\kappa_\xi (t-s)} dW_s \right) dt \right] \\
 &= \frac{1}{T^2} \text{Var} \left[ \int_0^T \left( e^{-\kappa_\xi t} \cdot \xi_0 + a^* (1 - e^{-\kappa_\xi t}) \right) dt + \sigma_\xi \int_0^T \int_0^t e^{-\kappa_\xi (t-s)} dW_s dt \right] \\
 &= \frac{1}{T^2} \text{Var} \left[ a^* + \frac{e^{-\kappa_\xi T} \cdot (a^* - \xi_0) - a^* + \xi_0}{\kappa_\xi \cdot T} + \sigma_\xi \int_0^T \int_0^t e^{-\kappa_\xi (t-s)} dW_s dt \right] \\
 &= \frac{\sigma_\xi^2}{T^2} \text{Var} \left[ \int_0^T \int_0^t e^{-\kappa_\xi (t-s)} dW_s dt \right] = \frac{\sigma_\xi^2}{T^2} \text{Var} \left[ \int_0^T \int_s^T e^{-\kappa_\xi (t-s)} dt dW_s \right] \\
 &= \frac{\sigma_\xi^2}{T^2} \text{Var} \left[ \int_0^T \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi (T-s)}) dW_s \right] \\
 &= \frac{\sigma_\xi^2}{T^2} \left( \mathbb{E} \left[ \left( \int_0^T \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi (T-s)}) dW_s \right)^2 \right] \right. \\
 &\quad \left. - \left( \mathbb{E} \left[ \int_0^T \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi (T-s)}) dW_s \right] \right)^2 \right) \\
 &= \frac{\sigma_\xi^2}{T^2} \left( \mathbb{E} \left[ \int_0^T \left( \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi (T-s)}) \right)^2 ds \right] - 0 \right) \\
 &= \frac{\sigma_\xi^2}{T^2} \left( \mathbb{E} \left[ \frac{1}{\kappa_\xi^2} \int_0^T (1 - 2 \cdot e^{-\kappa_\xi (T-s)} + e^{-2 \cdot \kappa_\xi (T-s)}) ds \right] \right) \\
 &= \frac{\sigma_\xi^2}{T^2} \left( \frac{1}{\kappa_\xi^2} \left[ \frac{2 \cdot \kappa_\xi \cdot s - 4 \cdot e^{-\kappa_\xi (T-s)} + e^{-2 \cdot \kappa_\xi (T-s)}}{2 \cdot \kappa_\xi} + c \right]_0^T \right) \\
 &= \frac{\sigma_\xi^2}{2 \cdot \kappa_\xi^3 \cdot T^2} (2 \cdot \kappa_\xi \cdot T + 4 \cdot e^{-\kappa_\xi T} - e^{-2 \cdot \kappa_\xi T} - 3) \\
 &\text{with } \text{Var} \left[ a^* + \frac{e^{-\kappa_\xi T} \cdot (a^* - \xi_0) - a^* + \xi_0}{\kappa_\xi \cdot T} \right] = 0 \\
 &\text{and } \mathbb{E} \left[ \frac{1}{\kappa_\xi} \int_0^T (1 - e^{-\kappa_\xi (T-s)}) dW_s \right] = 0
 \end{aligned} \tag{A.62}$$

because  $\int_0^T \left( \frac{1}{\kappa_\xi} (1 - e^{-\kappa_\xi \cdot (T-s)}) \right)^2 ds < \infty$  with  $\kappa_\xi > 0$ .

Note that the manually derived  $\mu_G$  and  $\sigma_G^2$  shown above are the exactly equal to the results derived via the characteristic function shown in equations (A.35) and (A.36).

We refrain from providing alternative, manual or direct derivations for mean and variance of the two-factor models as this would get disproportionately cumbersome for these kind of models. The derivation via the characteristic function provides a considerably more efficient way for these models.

## B Appendix B – Numerical example

### B.1 Numerical results for $T = \frac{1}{12}$ and $n = 210$

TABLE 4.6: Numerical results of Capesize route C4 with  $T = \frac{1}{12}$  years and  $n = 210$

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.0244	(0.0056)	4.0224	0.0483	4.0616	(0.0056)	4.0597	(0.0001)	99.9610
18.9	2.0353	(0.0051)	2.0337	0.0789	2.0649	(0.0051)	2.0633	(0.0001)	99.9713
21.0	0.6263	(0.0032)	0.6289	-0.4111	0.6440	(0.0033)	0.6466	(0.0001)	99.9413
23.1	0.1006	(0.0013)	0.1019	-1.3065	0.1083	(0.0014)	0.1097	(0.0001)	99.6485
25.2	0.0085	(0.0004)	0.0086	-1.1251	0.0103	(0.0004)	0.0104	(0.0001)	98.2326
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.0200	(0.0059)	4.0179	0.0513	4.0622	(0.0059)	4.0602	(0.0001)	99.9564
18.9	2.0535	(0.0053)	2.0520	0.0744	2.0867	(0.0054)	2.0852	(0.0001)	99.9672
21.0	0.6697	(0.0035)	0.6725	-0.4122	0.6901	(0.0036)	0.6929	(0.0001)	99.9342
23.1	0.1242	(0.0015)	0.1258	-1.3074	0.1337	(0.0016)	0.1354	(0.0001)	99.6521
25.2	0.0133	(0.0005)	0.0135	-1.2505	0.0158	(0.0005)	0.0160	(0.0001)	98.3917
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.5193	(0.0069)	3.5133	0.1723	3.5808	(0.0069)	3.5747	(0.0002)	99.9328
18.9	1.7305	(0.0058)	1.7277	0.1626	1.7726	(0.0059)	1.7698	(0.0001)	99.9495
21.0	0.6036	(0.0037)	0.6013	0.3754	0.6273	(0.0038)	0.6250	(0.0001)	99.9087
23.1	0.1451	(0.0018)	0.1439	0.8454	0.1568	(0.0019)	0.1555	(0.0001)	99.6493
25.2	0.0249	(0.0007)	0.0243	2.4290	0.0291	(0.0008)	0.0285	(0.0001)	98.7202
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.5417	(0.0071)	4.5354	0.1390	4.6028	(0.0071)	4.5964	(0.0002)	99.9429
18.9	2.5715	(0.0065)	2.5659	0.2184	2.6241	(0.0066)	2.6184	(0.0002)	99.9483
21.0	1.0738	(0.0048)	1.0695	0.4013	1.1130	(0.0049)	1.1087	(0.0001)	99.9079
23.1	0.3077	(0.0026)	0.3061	0.5254	0.3319	(0.0028)	0.3303	(0.0002)	99.6582
25.2	0.0607	(0.0011)	0.0601	0.9851	0.0709	(0.0013)	0.0703	(0.0001)	98.7083

The table shows the numerical results for the Capesize route C4 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.7: Numerical results of Capesize route C7 with  $T = \frac{1}{12}$  years and  $n = 210$

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	4.0014	(0.0051)	3.9996	0.0452	4.0329	(0.0051)	4.0311	(0.0001)	99.9661
18.9	1.9885	(0.0047)	1.9869	0.0813	2.0139	(0.0047)	2.0123	(0.0001)	99.9759
21.0	0.5572	(0.0029)	0.5596	-0.4215	0.5716	(0.0030)	0.5740	(0.0001)	99.9497
23.1	0.0709	(0.0010)	0.0718	-1.2653	0.0764	(0.0011)	0.0773	(0.0001)	99.6412
25.2	0.0041	(0.0002)	0.0041	0.0258	0.0050	(0.0003)	0.0050	(0.0000)	98.0065
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.9954	(0.0055)	3.9935	0.0483	4.0317	(0.0055)	4.0297	(0.0001)	99.9617
18.9	2.0041	(0.0050)	2.0025	0.0800	2.0328	(0.0050)	2.0312	(0.0001)	99.9722
21.0	0.6008	(0.0031)	0.6033	-0.4214	0.6175	(0.0032)	0.6201	(0.0001)	99.9430
23.1	0.0915	(0.0012)	0.0927	-1.3084	0.0985	(0.0013)	0.0997	(0.0001)	99.6466
25.2	0.0072	(0.0003)	0.0072	-0.9428	0.0086	(0.0004)	0.0087	(0.0000)	98.2068
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.4538	(0.0063)	3.4481	0.1656	3.5067	(0.0063)	3.5009	(0.0002)	99.9388
18.9	1.6288	(0.0053)	1.6261	0.1654	1.6641	(0.0054)	1.6614	(0.0001)	99.9575
21.0	0.5066	(0.0032)	0.5044	0.4395	0.5247	(0.0033)	0.5224	(0.0001)	99.9228
23.1	0.0988	(0.0014)	0.0979	0.9550	0.1066	(0.0015)	0.1057	(0.0001)	99.6542
25.2	0.0128	(0.0005)	0.0123	4.2046	0.0150	(0.0005)	0.0145	(0.0001)	98.6018
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	5.1425	(0.0067)	5.1368	0.1112	5.2035	(0.0068)	5.1977	(0.0002)	99.9398
18.9	3.1018	(0.0064)	3.0958	0.1957	3.1591	(0.0066)	3.1530	(0.0002)	99.9427
21.0	1.3937	(0.0051)	1.3892	0.3236	1.4416	(0.0053)	1.4370	(0.0002)	99.9088
23.1	0.4164	(0.0030)	0.4145	0.4638	0.4485	(0.0031)	0.4466	(0.0002)	99.6692
25.2	0.0801	(0.0013)	0.0795	0.6525	0.0940	(0.0014)	0.0934	(0.0002)	98.6418

The table shows the numerical results for the Capesize route C7 for all four models – with  $S_0 = 21$ ,  $r = 5$  %, and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.8: Numerical results of Panamax route P2A with  $T = \frac{1}{12}$  years and  $n = 210$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,580.46	(14.23)	8,575.47	0.06	8,692.78	(14.31)	8,687.74	(0.33)	99.95
40,500	4,478.38	(12.59)	4,475.73	0.06	4,564.92	(12.77)	4,562.24	(0.26)	99.96
45,000	1,621.34	(8.48)	1,628.17	-0.42	1,677.03	(8.73)	1,683.95	(0.25)	99.92
49,500	382.11	(4.14)	386.87	-1.23	411.87	(4.39)	416.70	(0.26)	99.65
54,000	59.69	(1.57)	60.81	-1.84	70.20	(1.76)	71.33	(0.21)	98.58
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,509.90	(14.56)	8,504.75	0.06	8,627.82	(14.64)	8,622.61	(0.35)	99.94
40,500	4,444.47	(12.80)	4,441.94	0.06	4,534.52	(12.99)	4,531.97	(0.27)	99.96
45,000	1,632.85	(8.64)	1,639.90	-0.43	1,690.79	(8.90)	1,697.93	(0.26)	99.92
49,500	399.86	(4.30)	404.80	-1.22	431.17	(4.56)	436.18	(0.27)	99.65
54,000	66.56	(1.69)	67.80	-1.83	78.06	(1.89)	79.32	(0.22)	98.61
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,650.71	(15.56)	8,636.85	0.16	8,786.64	(15.68)	8,772.59	(0.39)	99.94
40,500	4,628.21	(13.68)	4,619.76	0.18	4,733.76	(13.92)	4,725.18	(0.32)	99.95
45,000	1,826.04	(9.51)	1,821.25	0.26	1,896.17	(9.82)	1,891.30	(0.30)	99.91
49,500	512.14	(5.09)	508.28	0.76	552.69	(5.40)	548.77	(0.32)	99.66
54,000	103.44	(2.22)	101.91	1.50	120.36	(2.47)	118.81	(0.27)	98.77
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	7,375.57	(14.64)	7,361.36	0.19	7,502.98	(14.68)	7,488.60	(0.38)	99.93
40,500	3,573.96	(12.16)	3,564.74	0.26	3,658.14	(12.34)	3,648.80	(0.27)	99.95
45,000	1,217.35	(7.65)	1,210.97	0.53	1,263.57	(7.88)	1,257.10	(0.23)	99.91
49,500	282.98	(3.66)	280.68	0.82	305.39	(3.88)	303.05	(0.23)	99.66
54,000	47.28	(1.45)	45.70	3.45	55.20	(1.61)	53.60	(0.18)	98.72

The table shows the numerical results for the Panamax route P2A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.9: Numerical results of Panamax route P3A with  $T = \frac{1}{12}$  years and  $n = 210$

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,601.38	(15.35)	8,595.99	0.06	8,731.07	(15.46)	8,725.62	(0.38)	99.94
40,500	4,577.89	(13.46)	4,575.50	0.05	4,677.66	(13.70)	4,675.22	(0.31)	99.95
45,000	1,776.06	(9.29)	1,783.55	-0.42	1,842.57	(9.59)	1,850.16	(0.29)	99.91
49,500	482.69	(4.89)	488.17	-1.12	520.80	(5.20)	526.37	(0.31)	99.65
54,000	93.56	(2.08)	95.24	-1.76	109.33	(2.33)	111.03	(0.27)	98.68
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,475.22	(16.13)	8,469.57	0.07	8,618.65	(16.26)	8,612.91	(0.42)	99.93
40,500	4,537.18	(13.99)	4,535.25	0.04	4,646.14	(14.26)	4,644.18	(0.34)	99.94
45,000	1,822.63	(9.73)	1,830.64	-0.44	1,895.63	(10.07)	1,903.77	(0.32)	99.90
49,500	535.64	(5.33)	541.61	-1.10	578.65	(5.67)	584.71	(0.34)	99.65
54,000	117.50	(2.43)	119.49	-1.66	136.79	(2.71)	138.81	(0.30)	98.75
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	8,251.97	(19.82)	8,236.99	0.18	8,477.31	(20.14)	8,462.00	(0.65)	99.90
40,500	4,666.84	(16.86)	4,659.59	0.16	4,838.06	(17.34)	4,830.63	(0.55)	99.90
45,000	2,213.62	(12.4)	2,208.78	0.22	2,334.54	(12.96)	2,329.58	(0.52)	99.84
49,500	884.32	(7.96)	878.42	0.67	964.29	(8.52)	958.24	(0.53)	99.61
54,000	300.29	(4.60)	298.12	0.73	346.16	(5.11)	343.93	(0.52)	98.97
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	10,746.83	(23.35)	10,726.03	0.19	11,062.36	(24.01)	11,040.95	(0.89)	99.86
40,500	6,862.21	(21.10)	6,847.98	0.21	7,138.53	(21.91)	7,123.84	(0.82)	99.86
45,000	3,854.96	(17.17)	3,843.95	0.29	4,084.05	(18.10)	4,072.65	(0.81)	99.80
49,500	1,900.22	(12.53)	1,889.94	0.54	2,078.04	(13.51)	2,067.39	(0.86)	99.59
54,000	827.07	(8.34)	820.87	0.75	951.25	(9.29)	944.82	(0.91)	99.05

The table shows the numerical results for the Panamax route P3A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

**B.2 Numerical results for  $T = 1$  and  $n = 252$** 

 TABLE 4.10: Numerical results of Capesize route C4 with  $T = 1$  year and  $n = 252$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.3014	(0.0139)	3.2930	0.2552	3.5449	(0.0148)	3.5360	(0.0009)	99.6259
18.9	2.2312	(0.0120)	2.2253	0.2615	2.4268	(0.0129)	2.4205	(0.0008)	99.6021
21.0	1.4572	(0.0100)	1.4535	0.2483	1.6155	(0.0109)	1.6116	(0.0008)	99.4976
23.1	0.9234	(0.0081)	0.9237	-0.0280	1.0516	(0.0091)	1.0519	(0.0008)	99.2750
25.2	0.5718	(0.0064)	0.5744	-0.4461	0.6748	(0.0074)	0.6776	(0.0008)	98.8716
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	3.2414	(0.0135)	3.2331	0.2573	3.4998	(0.0143)	3.4909	(0.0009)	99.6022
18.9	2.1635	(0.0116)	2.1577	0.2672	2.3690	(0.0124)	2.3629	(0.0008)	99.5745
21.0	1.3901	(0.0096)	1.3871	0.2141	1.5533	(0.0105)	1.5501	(0.0008)	99.4569
23.1	0.8642	(0.0077)	0.8650	-0.0943	0.9923	(0.0086)	0.9931	(0.0008)	99.2174
25.2	0.5236	(0.0060)	0.5266	-0.5520	0.6230	(0.0069)	0.6261	(0.0008)	98.7917
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	1.4414	(0.0088)	1.4418	-0.0270	1.6391	(0.0095)	1.6395	(0.0008)	99.2069
18.9	0.8292	(0.0069)	0.8326	-0.4052	0.9615	(0.0076)	0.9652	(0.0007)	99.1162
21.0	0.4586	(0.0052)	0.4635	-1.0560	0.5464	(0.0058)	0.5518	(0.0006)	98.8629
23.1	0.2460	(0.0038)	0.2512	-2.0705	0.3039	(0.0044)	0.3096	(0.0006)	98.3581
25.2	0.1296	(0.0028)	0.1335	-2.9226	0.1675	(0.0033)	0.1718	(0.0005)	97.5647
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	8.7951	(0.0214)	8.8002	-0.0582	9.6920	(0.0239)	9.6976	(0.0025)	98.8655
18.9	7.0305	(0.0205)	7.0362	-0.0800	7.8973	(0.0231)	7.9036	(0.0025)	98.8285
21.0	5.4577	(0.0192)	5.4647	-0.1267	6.2823	(0.0219)	6.2900	(0.0025)	98.7241
23.1	4.1189	(0.0175)	4.1276	-0.2125	4.8873	(0.0204)	4.8971	(0.0025)	98.5066
25.2	3.0281	(0.0156)	3.0391	-0.3604	3.7279	(0.0185)	3.7402	(0.0025)	98.1131

The table shows the numerical results for the Capesize route C4 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.11: Numerical results of Capesize route C7 with  $T = 1$  year and  $n = 252$

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	2.9894	(0.0124)	2.9816	0.2607	3.1872	(0.0131)	3.1790	(0.0007)	99.6941
18.9	1.9284	(0.0105)	1.9232	0.2700	2.0799	(0.0112)	2.0744	(0.0006)	99.6815
21.0	1.1888	(0.0085)	1.1861	0.2293	1.3063	(0.0092)	1.3034	(0.0006)	99.5831
23.1	0.7046	(0.0066)	0.7051	-0.0826	0.7955	(0.0073)	0.7961	(0.0006)	99.3573
25.2	0.4040	(0.0051)	0.4071	-0.7671	0.4738	(0.0058)	0.4771	(0.0006)	98.9224
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	2.9668	(0.0122)	2.9590	0.2628	3.1813	(0.0129)	3.1731	(0.0007)	99.6627
18.9	1.8996	(0.0103)	1.8945	0.2729	2.0637	(0.0110)	2.0582	(0.0007)	99.6447
21.0	1.1592	(0.0083)	1.1569	0.1980	1.2844	(0.0090)	1.2819	(0.0006)	99.5347
23.1	0.6786	(0.0064)	0.6796	-0.1425	0.7728	(0.0071)	0.7739	(0.0006)	99.2930
25.2	0.3837	(0.0049)	0.3870	-0.8606	0.4538	(0.0055)	0.4574	(0.0006)	98.8356
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	1.1225	(0.0074)	1.1237	-0.1061	1.2699	(0.0079)	1.2712	(0.0007)	99.2411
18.9	0.5929	(0.0055)	0.5964	-0.6022	0.6825	(0.0060)	0.6864	(0.0005)	99.1778
21.0	0.2975	(0.0039)	0.3023	-1.5811	0.3516	(0.0044)	0.3568	(0.0005)	98.9394
23.1	0.1442	(0.0027)	0.1480	-2.6100	0.1767	(0.0031)	0.1809	(0.0004)	98.4360
25.2	0.0685	(0.0019)	0.0707	-3.2035	0.0879	(0.0022)	0.0903	(0.0003)	97.6117
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
16.8	15.2674	(0.0259)	15.2798	-0.0810	16.9823	(0.0304)	16.9963	(0.0041)	98.1821
18.9	13.2977	(0.0258)	13.3095	-0.0886	15.0077	(0.0302)	15.0211	(0.0040)	98.2158
21.0	11.3746	(0.0254)	11.3862	-0.1024	13.0732	(0.0299)	13.0866	(0.0040)	98.2251
23.1	9.5378	(0.0247)	9.5494	-0.1217	11.2137	(0.0294)	11.2271	(0.0040)	98.1918
25.2	7.8314	(0.0237)	7.8434	-0.1531	9.4688	(0.0285)	9.4827	(0.0039)	98.0791

The table shows the numerical results for the Capesize route C7 for all four models – with  $S_0 = 21$ ,  $r = 5$  %, and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

TABLE 4.12: Numerical results of Panamax route P2A with  $T = 1$  year  
and  $n = 252$ 

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	7,407.23	(35.05)	7,389.36	0.24	8,128.14	(38.03)	8,108.74	(2.90)	99.42
40,500	5,318.99	(30.88)	5,306.16	0.24	5,921.62	(34.04)	5,907.62	(2.70)	99.37
45,000	3,751.38	(26.67)	3,743.61	0.21	4,259.92	(29.93)	4,251.40	(2.59)	99.25
49,500	2,605.36	(22.68)	2,606.10	-0.03	3,035.97	(25.99)	3,036.78	(2.56)	99.03
54,000	1,791.00	(19.08)	1,796.51	-0.31	2,154.19	(22.39)	2,160.29	(2.58)	98.67
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	6,766.94	(32.51)	6,750.40	0.25	7,452.36	(35.10)	7,434.44	(2.65)	99.43
40,500	4,747.42	(28.30)	4,736.05	0.24	5,306.48	(31.05)	5,294.11	(2.43)	99.39
45,000	3,261.59	(24.09)	3,256.41	0.16	3,720.00	(26.92)	3,714.34	(2.32)	99.26
49,500	2,202.23	(20.18)	2,204.93	-0.12	2,577.25	(23.02)	2,580.21	(2.28)	99.02
54,000	1,468.90	(16.70)	1,476.19	-0.49	1,775.46	(19.51)	1,783.48	(2.27)	98.64
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	6,179.23	(30.70)	6,179.11	0.00	6,921.01	(33.31)	6,920.87	(2.84)	99.28
40,500	4,258.02	(26.46)	4,265.05	-0.16	4,848.41	(29.22)	4,856.11	(2.57)	99.23
45,000	2,870.61	(22.30)	2,883.06	-0.43	3,338.77	(25.12)	3,352.46	(2.41)	99.08
49,500	1,902.60	(18.49)	1,918.41	-0.82	2,274.20	(21.28)	2,291.65	(2.33)	98.80
54,000	1,243.72	(15.15)	1,261.94	-1.44	1,538.91	(17.88)	1,559.10	(2.30)	98.35
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	2,380.26	(18.06)	2,394.77	-0.61	2,745.83	(19.72)	2,761.72	(1.96)	99.01
40,500	1,416.56	(14.26)	1,433.10	-1.15	1,660.00	(15.87)	1,678.20	(1.62)	98.96
45,000	827.82	(11.07)	842.17	-1.70	993.09	(12.58)	1,008.94	(1.41)	98.74
49,500	479.26	(8.52)	489.17	-2.03	592.11	(9.91)	603.10	(1.26)	98.39
54,000	277.14	(6.52)	282.23	-1.80	355.81	(7.80)	361.46	(1.16)	97.78

The table shows the numerical results for the Panamax route P2A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

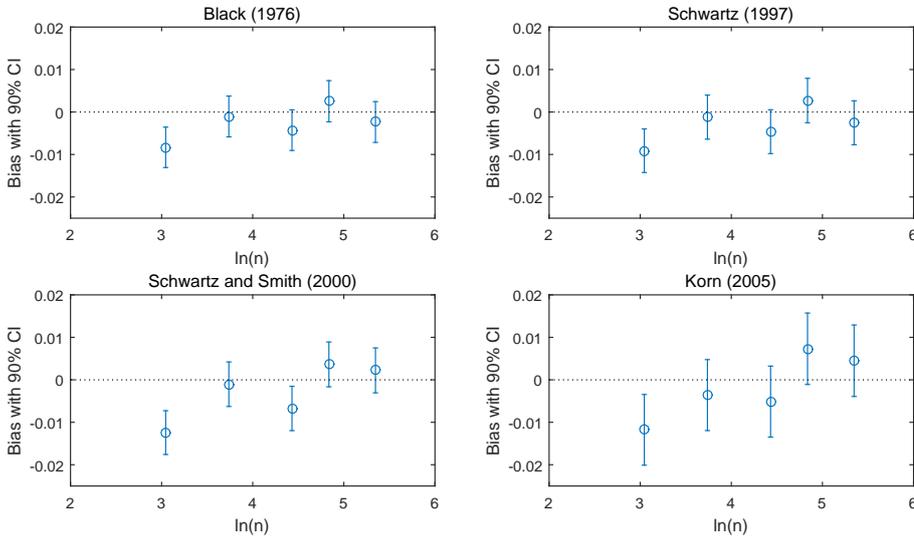
TABLE 4.13: Numerical results of Panamax route P3A with  $T = 1$  year and  $n = 252$

Black (1976) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	7,829.36	(38.35)	7,811.18	0.23	8,686.47	(42.17)	8,666.48	(3.59)	99.28
40,500	5,784.34	(34.21)	5,771.06	0.23	6,516.21	(38.23)	6,501.53	(3.37)	99.22
45,000	4,218.54	(30.03)	4,210.16	0.20	4,848.61	(34.17)	4,839.30	(3.25)	99.09
49,500	3,042.95	(26.06)	3,043.16	-0.01	3,588.00	(30.26)	3,588.24	(3.21)	98.87
54,000	2,180.31	(22.41)	2,185.47	-0.24	2,649.39	(26.63)	2,655.19	(3.23)	98.53
Schwartz (1997) one-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	6,822.53	(35.22)	6,806.49	0.24	7,641.46	(38.63)	7,623.83	(3.31)	99.26
40,500	4,935.60	(31.03)	4,924.65	0.22	5,616.03	(34.60)	5,603.93	(3.09)	99.20
45,000	3,521.10	(26.90)	3,516.96	0.12	4,089.90	(30.55)	4,085.31	(2.96)	99.06
49,500	2,484.59	(23.05)	2,488.30	-0.15	2,959.44	(26.71)	2,963.58	(2.90)	98.82
54,000	1,741.16	(19.58)	1,749.34	-0.47	2,138.38	(23.22)	2,147.53	(2.88)	98.46
Schwartz and Smith (2000) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	5,418.59	(31.39)	5,415.95	0.05	6,403.61	(35.34)	6,400.62	(3.90)	98.78
40,500	3,821.47	(27.22)	3,827.04	-0.15	4,616.17	(31.28)	4,622.50	(3.63)	98.65
45,000	2,662.04	(23.26)	2,674.23	-0.46	3,300.90	(27.32)	3,314.83	(3.45)	98.41
49,500	1,839.22	(19.69)	1,854.79	-0.84	2,352.09	(23.67)	2,369.95	(3.33)	98.02
54,000	1,263.11	(16.56)	1,280.58	-1.36	1,674.21	(20.42)	1,694.32	(3.27)	97.43
Korn (2005) two-factor model									
$K$	Geometric Asian option				Arithmetic Asian option				
	MC	SE	CFS	$\Delta$ %	MC	SE	MC CV	SE	VR %
36,000	15,874.48	(57.24)	15,894.19	-0.12	18,533.53	(65.69)	18,556.19	(8.14)	98.46
40,500	12,872.11	(53.97)	12,893.42	-0.17	15,343.16	(62.84)	15,367.77	(7.95)	98.40
45,000	10,314.13	(50.16)	10,335.57	-0.21	12,585.06	(59.38)	12,609.92	(7.83)	98.26
49,500	8,180.63	(46.07)	8,204.74	-0.29	10,249.24	(55.54)	10,277.28	(7.80)	98.03
54,000	6,432.34	(41.89)	6,462.55	-0.47	8,298.22	(51.53)	8,333.45	(7.84)	97.68

The table shows the numerical results for the Panamax route P3A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ , and  $k = 100,000$ . MC refers to the Monte Carlo price, SE to the standard error, CFS to the closed-form solution,  $\Delta$  % to the relative bias between MC price and closed-form solution, MC CV to the Monte Carlo control variate price, and VR % to the variance reduction due to the control variate.

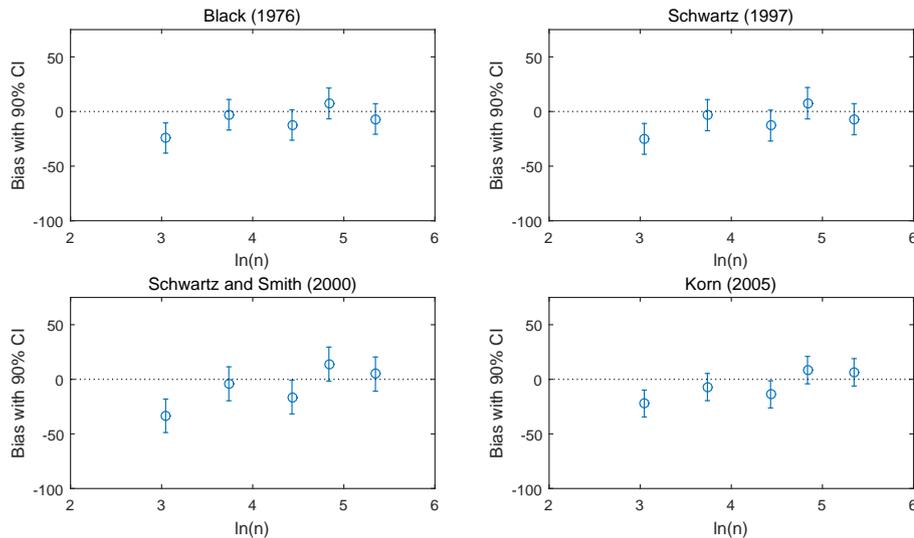
### B.3 Discretization error plots

FIGURE 4.3: Impact of  $n$  on the discretization error for Capesize route C7



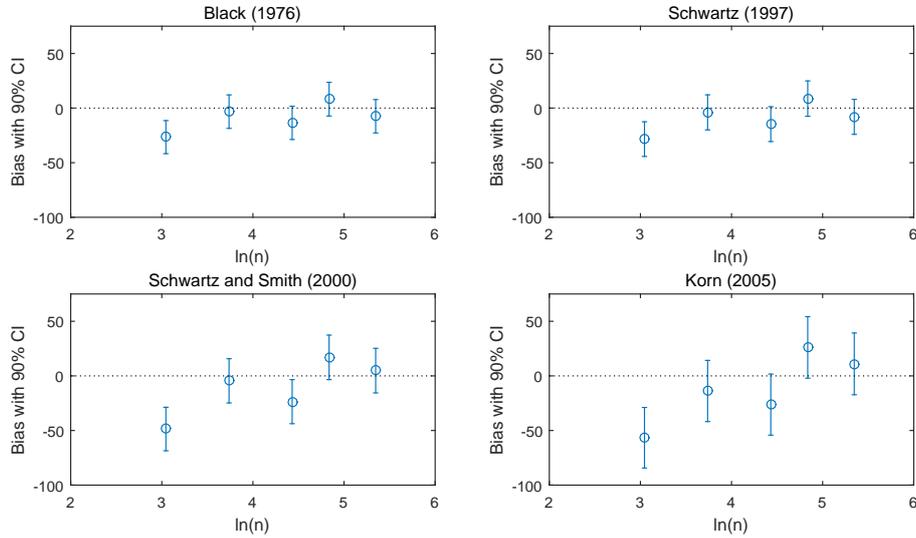
The graph shows the bias as well as the 90 % CI of an at-the-money geometric Asian call option for different log numbers of discretization time steps (i.e.,  $n = 21, 42, 84, 126, 210$ ) for the Capesize route C7 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $k = 100,000$ .

FIGURE 4.4: Impact of  $n$  on the discretization error for Panamax route P2A



The graph shows the bias as well as the 90 % CI of an at-the-money geometric Asian call option for different log numbers of discretization time steps (i.e.,  $n = 21, 42, 84, 126, 210$ ) for the Panamax route P2A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $k = 100,000$ .

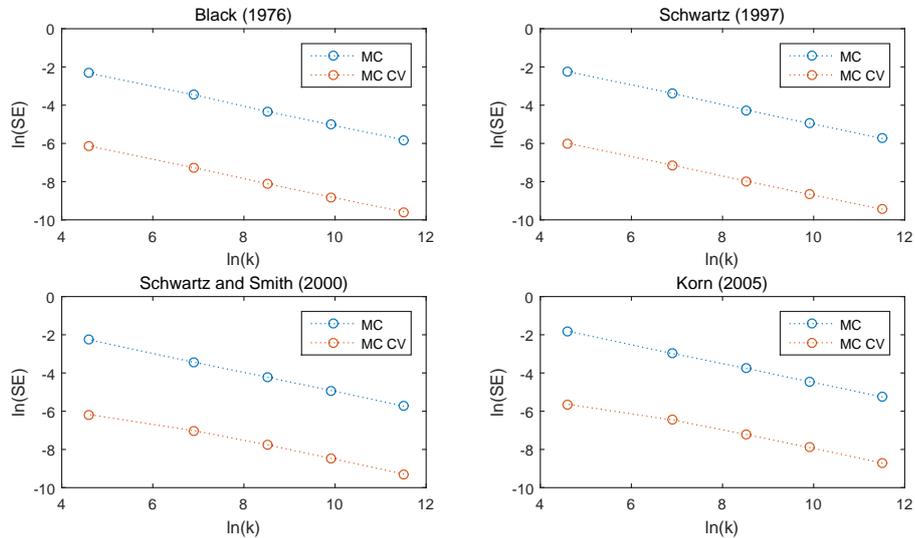
FIGURE 4.5: Impact of  $n$  on the discretization error for Panamax route P3A



The graph shows the bias as well as the 90 % CI of an at-the-money geometric Asian call option for different log numbers of discretization time steps (i.e.,  $n = 21, 42, 84, 126, 210$ ) for the Panamax route P3A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $k = 100,000$ .

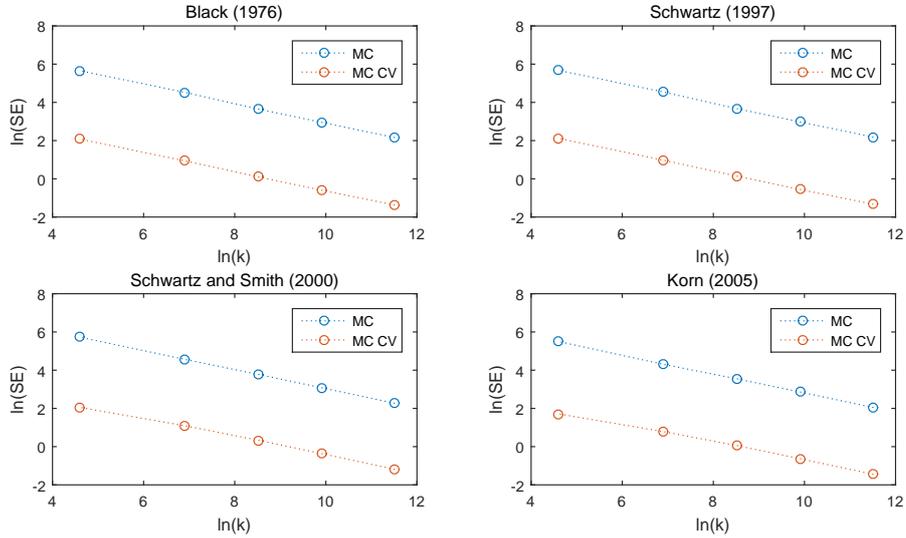
## B.4 Simulation error plots

FIGURE 4.6: Impact of  $k$  on the simulation error for Capesize route C7



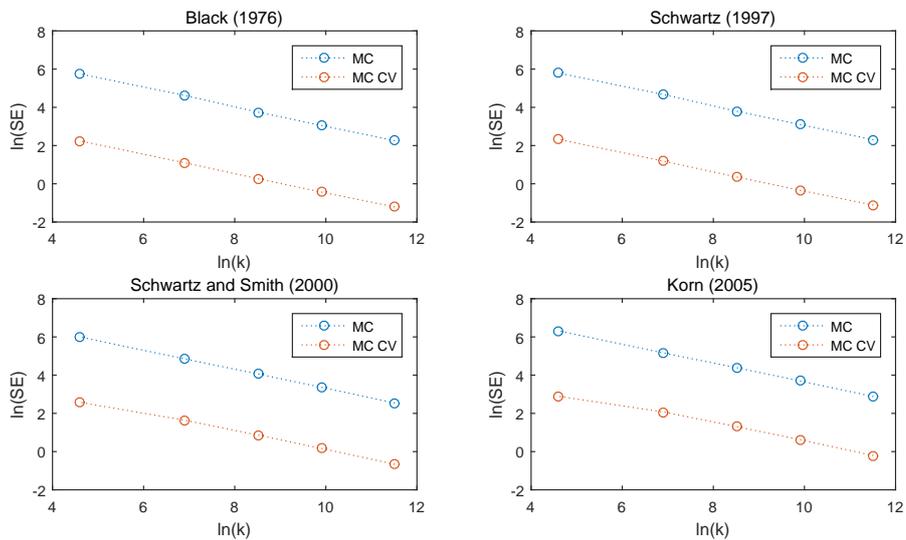
The graph shows the log standard error (SE) for the MC price as well as MC control variate price of an at-the-money arithmetic Asian call option for different log numbers of generated MC paths (i.e.,  $k = 100, 1,000, 5,000, 20,000, 100,000$ ) for the Capesize route C7 for all four models – with  $S_0 = 21$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $n = 21$ .

FIGURE 4.7: Impact of  $k$  on the simulation error for Panamax route P2A



The graph shows the log standard error (SE) for the MC price as well as MC control variate price of an at-the-money arithmetic Asian call option for different log numbers of generated MC paths (i.e.,  $k = 100, 1,000, 5,000, 20,000, 100,000$ ) for the Panamax route P2A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $n = 21$ .

FIGURE 4.8: Impact of  $k$  on the simulation error for Panamax route P3A



The graph shows the log standard error (SE) for the MC price as well as MC control variate price of an at-the-money arithmetic Asian call option for different log numbers of generated MC paths (i.e.,  $k = 100, 1,000, 5,000, 20,000, 100,000$ ) for the Panamax route P3A for all four models – with  $S_0 = 45,000$ ,  $r = 5\%$ ,  $T = \frac{1}{12}$  years, and  $n = 21$ .

## C Appendix C – MATLAB R2015a code

This section of the appendix contains the most important MATLAB R2015a .m-files and functions that have been programmed for the numerical example.

### C.1 Main .m-file

```
1 %% Description of file: rp3_mainfile
2
3 % date created: 11/09/2015
4 % date last edited: 01/25/2016
5 % author: Michael Herbener
6 % research project 3 of dissertation
7 % current folder and file locations refer to:
8 % Z:\-drive on UHH workstation or
9 % C:\drive on McK notebook
10
11 %% Clear the existing workspace
12 clear
13
14 %% Define paths
15 path = 'Z:\Dissertation\data\analyses\MATLAB';
16 % path = 'C:\Users\Michael Herbener\Documents\Private\Dissertation\data\analyses\MATLAB';
17 exportfile_path = 'Z:\Dissertation\data\analyses\Excel\rp3_results.xlsx';
18 % exportfile_path = 'C:\Users\Michael ...
    Herbener\Documents\Private\Dissertation\data\analyses\Excel\rp3_results.xlsx';
19
20 %% Set the MATLAB current folder
21 cd(path)
22
23 %% Set format to long
24 format long
25
26 %% Start parallel computing session
27 if strcmp(version('-release'),'2013a')
28     if matlabpool('size') == 0 %#ok<DPOOL>
29         matlabpool %#ok<DPOOL>
30     end
31 elseif strcmp(version('-release'),'2013b')
32     if matlabpool('size') == 0 %#ok<DPOOL>
33         matlabpool %#ok<DPOOL>
34     end
35 elseif strcmp(version('-release'),'2014a')
36     pool = gcp('ncreate');
37     if isempty(pool)
38         poolsize = 0;
39     else
40         poolsize = pool.NumWorkers;
41     end
```

```
42     if poolsize == 0
43         parpool;
44     end
45 elseif strcmp(version('-release'),'2014b')
46     pool = gcp('nocreate');
47     if isempty(pool)
48         poolsize = 0;
49     else
50         poolsize = pool.NumWorkers;
51     end
52     if poolsize == 0
53         parpool;
54     end
55 elseif strcmp(version('-release'),'2015a')
56     pool = gcp('nocreate');
57     if isempty(pool)
58         poolsize = 0;
59     else
60         poolsize = pool.NumWorkers;
61     end
62     if poolsize == 0
63         parpool;
64     end
65 elseif strcmp(version('-release'),'2015b')
66     pool = gcp('nocreate');
67     if isempty(pool)
68         poolsize = 0;
69     else
70         poolsize = pool.NumWorkers;
71     end
72     if poolsize == 0
73         parpool;
74     end
75 end
76 clearvars pool poolsize
77
78 %% Start stopwatch
79 tstart = tic;
80
81 %% Prokopczuk (2010) model parameters
82
83 fprintf('Define model parameters, intial values, and strike prices...')
84
85 % Route C4
86 % Black (1976)
87 % PARAMETERS.C4.Black1976.xi_0 = 3.0459;
88 PARAMETERS.C4.Black1976.a = -0.2062;
89 PARAMETERS.C4.Black1976.a_star = -0.2748;
90 PARAMETERS.C4.Black1976.lambda_xi = PARAMETERS.C4.Black1976.a - ...
    PARAMETERS.C4.Black1976.a_star;
91 PARAMETERS.C4.Black1976.sigma_xi = 0.5109;
92
93 % Schwartz (1997) one-factor model
94 % PARAMETERS.C4.Schw1997.xi_0 = 3.0432;
```

```
95 PARAMETERS.C4.Schw1997.kappa_xi = 0.2830;
96 PARAMETERS.C4.Schw1997.a = 2.2596;
97 PARAMETERS.C4.Schw1997.a_star = 1.9958;
98 PARAMETERS.C4.Schw1997.lambda_xi = PARAMETERS.C4.Schw1997.a - ...
    PARAMETERS.C4.Schw1997.a_star;
99 PARAMETERS.C4.Schw1997.sigma_xi = 0.5497;
100
101 % Schwartz and Smith (2000) two-factor model
102 % PARAMETERS.C4.SchwSm2000.xi_0 = 2.8243;
103 PARAMETERS.C4.SchwSm2000.a = -0.1015;
104 PARAMETERS.C4.SchwSm2000.a_star = -0.2777;
105 PARAMETERS.C4.SchwSm2000.lambda_xi = PARAMETERS.C4.SchwSm2000.a - ...
    PARAMETERS.C4.SchwSm2000.a_star;
106 PARAMETERS.C4.SchwSm2000.sigma_xi = 0.4446;
107 % PARAMETERS.C4.SchwSm2000.chi_0 = 0.3531;
108 PARAMETERS.C4.SchwSm2000.kappa_chi = 3.0194;
109 PARAMETERS.C4.SchwSm2000.lambda_chi = -0.2661;
110 PARAMETERS.C4.SchwSm2000.sigma_chi = 0.5637;
111 PARAMETERS.C4.SchwSm2000.rho = -0.0367;
112
113 % Korn (2005) two-factor model
114 % PARAMETERS.C4.Korn2005.xi_0 = 2.9810;
115 PARAMETERS.C4.Korn2005.kappa_xi = 0.8017;
116 PARAMETERS.C4.Korn2005.a = 3.0518;
117 PARAMETERS.C4.Korn2005.a_star = 4.0011;
118 PARAMETERS.C4.Korn2005.lambda_xi = PARAMETERS.C4.Korn2005.a - ...
    PARAMETERS.C4.Korn2005.a_star;
119 PARAMETERS.C4.Korn2005.sigma_xi = 1.2091;
120 % PARAMETERS.C4.Korn2005.chi_0 = 0.6098;
121 PARAMETERS.C4.Korn2005.kappa_chi = 1.6660;
122 PARAMETERS.C4.Korn2005.lambda_chi = 0.0768;
123 PARAMETERS.C4.Korn2005.sigma_chi = 1.2247;
124 PARAMETERS.C4.Korn2005.rho = -0.8543;
125
126 % Route C7
127 % Black (1976)
128 % PARAMETERS.C7.Black1976.xi_0 = 3.0220;
129 PARAMETERS.C7.Black1976.a = -0.2010;
130 PARAMETERS.C7.Black1976.a_star = -0.2854;
131 PARAMETERS.C7.Black1976.lambda_xi = PARAMETERS.C7.Black1976.a - ...
    PARAMETERS.C7.Black1976.a_star;
132 PARAMETERS.C7.Black1976.sigma_xi = 0.4679;
133
134 % Schwartz (1997) one-factor model
135 % PARAMETERS.C7.Schw1997.xi_0 = 3.0205;
136 PARAMETERS.C7.Schw1997.kappa_xi = 0.2638;
137 PARAMETERS.C7.Schw1997.a = 2.2328;
138 PARAMETERS.C7.Schw1997.a_star = 1.8796;
139 PARAMETERS.C7.Schw1997.lambda_xi = PARAMETERS.C7.Schw1997.a - ...
    PARAMETERS.C7.Schw1997.a_star;
140 PARAMETERS.C7.Schw1997.sigma_xi = 0.5064;
141
142 % Schwartz and Smith (2000) two-factor model
143 % PARAMETERS.C7.SchwSm2000.xi_0 = 2.7744;
```

```

144 PARAMETERS.C7.SchwSm2000.a = -0.0803;
145 PARAMETERS.C7.SchwSm2000.a_star = -0.2976;
146 PARAMETERS.C7.SchwSm2000.lambda_xi = PARAMETERS.C7.SchwSm2000.a - ...
    PARAMETERS.C7.SchwSm2000.a_star;
147 PARAMETERS.C7.SchwSm2000.sigma_xi = 0.4179;
148 % PARAMETERS.C7.SchwSm2000.chi_0 = 0.3972;
149 PARAMETERS.C7.SchwSm2000.kappa_chi = 2.7654;
150 PARAMETERS.C7.SchwSm2000.lambda_chi = -0.2866;
151 PARAMETERS.C7.SchwSm2000.sigma_chi = 0.5187;
152 PARAMETERS.C7.SchwSm2000.rho = -0.0821;
153
154 % Korn (2005) two-factor model
155 % PARAMETERS.C7.Korn2005.xi_0 = 3.1708;
156 PARAMETERS.C7.Korn2005.kappa_xi = 0.7470;
157 PARAMETERS.C7.Korn2005.a = 3.5702;
158 PARAMETERS.C7.Korn2005.a_star = 4.2374;
159 PARAMETERS.C7.Korn2005.lambda_xi = PARAMETERS.C7.Korn2005.a - ...
    PARAMETERS.C7.Korn2005.a_star;
160 PARAMETERS.C7.Korn2005.sigma_xi = 1.1736;
161 % PARAMETERS.C7.Korn2005.chi_0 = 0.8612;
162 PARAMETERS.C7.Korn2005.kappa_chi = 1.5203;
163 PARAMETERS.C7.Korn2005.lambda_chi = -0.5794;
164 PARAMETERS.C7.Korn2005.sigma_chi = 1.2032;
165 PARAMETERS.C7.Korn2005.rho = -0.8714;
166
167 % Route P2A
168 % Black (1976)
169 % PARAMETERS.P2A.Black1976.xi_0 = 10.7056;
170 PARAMETERS.P2A.Black1976.a = -0.3331;
171 PARAMETERS.P2A.Black1976.a_star = -0.3501; % Prokoczek (2010): +
172 PARAMETERS.P2A.Black1976.lambda_xi = PARAMETERS.P2A.Black1976.a - ...
    PARAMETERS.P2A.Black1976.a_star;
173 PARAMETERS.P2A.Black1976.sigma_xi = 0.6167;
174
175 % Schwartz (1997) one-factor model
176 % PARAMETERS.P2A.Schw1997.xi_0 = 10.7048;
177 PARAMETERS.P2A.Schw1997.kappa_xi = 0.1882;
178 PARAMETERS.P2A.Schw1997.a = 8.9127;
179 PARAMETERS.P2A.Schw1997.a_star = 8.5831;
180 PARAMETERS.P2A.Schw1997.lambda_xi = PARAMETERS.P2A.Schw1997.a - ...
    PARAMETERS.P2A.Schw1997.a_star;
181 PARAMETERS.P2A.Schw1997.sigma_xi = 0.6371;
182
183 % Schwartz and Smith (2000) two-factor model
184 % PARAMETERS.P2A.SchwSm2000.xi_0 = 10.5035;
185 PARAMETERS.P2A.SchwSm2000.a = -0.1302;
186 PARAMETERS.P2A.SchwSm2000.a_star = -0.5056;
187 PARAMETERS.P2A.SchwSm2000.lambda_xi = PARAMETERS.P2A.SchwSm2000.a - ...
    PARAMETERS.P2A.SchwSm2000.a_star;
188 PARAMETERS.P2A.SchwSm2000.sigma_xi = 0.6159;
189 % PARAMETERS.P2A.SchwSm2000.chi_0 = 0.2115;
190 PARAMETERS.P2A.SchwSm2000.kappa_chi = 2.1339;
191 PARAMETERS.P2A.SchwSm2000.lambda_chi = -0.6127;
192 PARAMETERS.P2A.SchwSm2000.sigma_chi = 0.5997;

```

```
193 PARAMETERS.P2A.SchwSm2000.rho = -0.3450;
194
195 % Korn (2005) two-factor model
196 % PARAMETERS.P2A.Korn2005.xi_0 = 10.9141;
197 PARAMETERS.P2A.Korn2005.kappa_xi = 0.4641;
198 PARAMETERS.P2A.Korn2005.a = 9.9977;
199 PARAMETERS.P2A.Korn2005.a_star = 9.7090;
200 PARAMETERS.P2A.Korn2005.lambda_xi = PARAMETERS.P2A.Korn2005.a - ...
    PARAMETERS.P2A.Korn2005.a_star;
201 PARAMETERS.P2A.Korn2005.sigma_xi = 1.3443;
202 % PARAMETERS.P2A.Korn2005.chi_0 = 0.0403;
203 PARAMETERS.P2A.Korn2005.kappa_chi = 1.2128;
204 PARAMETERS.P2A.Korn2005.lambda_chi = 0.5982;
205 PARAMETERS.P2A.Korn2005.sigma_chi = 1.3028;
206 PARAMETERS.P2A.Korn2005.rho = -0.8691;
207
208 % Route P3A
209 % Black (1976)
210 % PARAMETERS.P3A.Black1976.xi_0 = 10.5389;
211 PARAMETERS.P3A.Black1976.a = -0.3921;
212 PARAMETERS.P3A.Black1976.a_star = -0.3717;
213 PARAMETERS.P3A.Black1976.lambda_xi = PARAMETERS.P3A.Black1976.a - ...
    PARAMETERS.P3A.Black1976.a_star;
214 PARAMETERS.P3A.Black1976.sigma_xi = 0.6691;
215
216 % Schwartz (1997) one-factor model
217 % PARAMETERS.P3A.Schw1997.xi_0 = 10.5500;
218 PARAMETERS.P3A.Schw1997.kappa_xi = 0.2140;
219 PARAMETERS.P3A.Schw1997.a = 8.4040;
220 PARAMETERS.P3A.Schw1997.a_star = 8.4965;
221 PARAMETERS.P3A.Schw1997.lambda_xi = PARAMETERS.P3A.Schw1997.a - ...
    PARAMETERS.P3A.Schw1997.a_star;
222 PARAMETERS.P3A.Schw1997.sigma_xi = 0.7146;
223
224 % Schwartz and Smith (2000) two-factor model
225 % PARAMETERS.P3A.SchwSm2000.xi_0 = 10.3743;
226 PARAMETERS.P3A.SchwSm2000.a = -0.2876;
227 PARAMETERS.P3A.SchwSm2000.a_star = -0.4192;
228 PARAMETERS.P3A.SchwSm2000.lambda_xi = PARAMETERS.P3A.SchwSm2000.a - ...
    PARAMETERS.P3A.SchwSm2000.a_star;
229 PARAMETERS.P3A.SchwSm2000.sigma_xi = 0.6204;
230 % PARAMETERS.P3A.SchwSm2000.chi_0 = 0.2231;
231 PARAMETERS.P3A.SchwSm2000.kappa_chi = 3.9719;
232 PARAMETERS.P3A.SchwSm2000.lambda_chi = -0.4656;
233 PARAMETERS.P3A.SchwSm2000.sigma_chi = 0.7458;
234 PARAMETERS.P3A.SchwSm2000.rho = 0.0011;
235
236 % Korn (2005) two-factor model
237 % PARAMETERS.P3A.Korn2005.xi_0 = 10.8388;
238 PARAMETERS.P3A.Korn2005.kappa_xi = 0.1018;
239 PARAMETERS.P3A.Korn2005.a = 6.1302;
240 PARAMETERS.P3A.Korn2005.a_star = 8.1302;
241 PARAMETERS.P3A.Korn2005.lambda_xi = PARAMETERS.P3A.Korn2005.a - ...
    PARAMETERS.P3A.Korn2005.a_star;
```

```

242 PARAMETERS.P3A.Korn2005.sigma_xi = 0.6733;
243 % PARAMETERS.P3A.Korn2005.chi_0 = -0.8681;
244 PARAMETERS.P3A.Korn2005.kappa_chi = 2.9352;
245 PARAMETERS.P3A.Korn2005.lambda_chi = 1.7878;
246 PARAMETERS.P3A.Korn2005.sigma_chi = 0.9781;
247 PARAMETERS.P3A.Korn2005.rho = -0.2375;
248
249 %% Set initial values
250
251 % Route C4
252 % define S_0
253 S_0 = 21;
254
255 % determine two-factor model split
256 PARAMETERS.C4.SchwSm2000.xi_0_share = 2.8243 / (2.8243 + 0.3531);
257 PARAMETERS.C4.SchwSm2000.chi_0_share = 0.3531 / (2.8243 + 0.3531);
258 PARAMETERS.C4.Korn2005.xi_0_share = 2.9810 / (2.9810 + 0.6098);
259 PARAMETERS.C4.Korn2005.chi_0_share = 0.6098 / (2.9810 + 0.6098);
260
261 % define model initial values
262 PARAMETERS.C4.Black1976.xi_0 = log(S_0);
263 PARAMETERS.C4.Schw1997.xi_0 = log(S_0);
264 PARAMETERS.C4.SchwSm2000.xi_0 = log(S_0) * PARAMETERS.C4.SchwSm2000.xi_0_share;
265 PARAMETERS.C4.SchwSm2000.chi_0 = log(S_0) * PARAMETERS.C4.SchwSm2000.chi_0_share;
266 PARAMETERS.C4.Korn2005.xi_0 = log(S_0) * PARAMETERS.C4.Korn2005.xi_0_share;
267 PARAMETERS.C4.Korn2005.chi_0 = log(S_0) * PARAMETERS.C4.Korn2005.chi_0_share;
268
269 % Route C7
270 % define S_0
271 S_0 = 21;
272
273 % determine two-factor model split
274 PARAMETERS.C7.SchwSm2000.xi_0_share = 2.7744 / (2.7744 + 0.3972);
275 PARAMETERS.C7.SchwSm2000.chi_0_share = 0.3972 / (2.7744 + 0.3972);
276 PARAMETERS.C7.Korn2005.xi_0_share = 3.1708 / (3.1708 + 0.8612);
277 PARAMETERS.C7.Korn2005.chi_0_share = 0.8612 / (3.1708 + 0.8612);
278
279 % define model initial values
280 PARAMETERS.C7.Black1976.xi_0 = log(S_0);
281 PARAMETERS.C7.Schw1997.xi_0 = log(S_0);
282 PARAMETERS.C7.SchwSm2000.xi_0 = log(S_0) * PARAMETERS.C7.SchwSm2000.xi_0_share;
283 PARAMETERS.C7.SchwSm2000.chi_0 = log(S_0) * PARAMETERS.C7.SchwSm2000.chi_0_share;
284 PARAMETERS.C7.Korn2005.xi_0 = log(S_0) * PARAMETERS.C7.Korn2005.xi_0_share;
285 PARAMETERS.C7.Korn2005.chi_0 = log(S_0) * PARAMETERS.C7.Korn2005.chi_0_share;
286
287 % Route P2A
288 % define S_0
289 S_0 = 45000;
290
291 % determine two-factor model split
292 PARAMETERS.P2A.SchwSm2000.xi_0_share = 10.5035 / (10.5035 + 0.2115);
293 PARAMETERS.P2A.SchwSm2000.chi_0_share = 0.2115 / (10.5035 + 0.2115);
294 PARAMETERS.P2A.Korn2005.xi_0_share = 10.9141 / (10.9141 + 0.0403);
295 PARAMETERS.P2A.Korn2005.chi_0_share = 0.0403 / (10.9141 + 0.0403);

```

```

296
297 % define model initial values
298 PARAMETERS.P2A.Black1976.xi_0 = log(S_0);
299 PARAMETERS.P2A.Schw1997.xi_0 = log(S_0);
300 PARAMETERS.P2A.SchwSm2000.xi_0 = log(S_0) * PARAMETERS.P2A.SchwSm2000.xi_0_share;
301 PARAMETERS.P2A.SchwSm2000.chi_0 = log(S_0) * PARAMETERS.P2A.SchwSm2000.chi_0_share;
302 PARAMETERS.P2A.Korn2005.xi_0 = log(S_0) * PARAMETERS.P2A.Korn2005.xi_0_share;
303 PARAMETERS.P2A.Korn2005.chi_0 = log(S_0) * PARAMETERS.P2A.Korn2005.chi_0_share;
304
305 % Route P3A
306 % define S_0
307 S_0 = 45000;
308
309 % determine two-factor model split
310 PARAMETERS.P3A.SchwSm2000.xi_0_share = 10.3743 / (10.3743 + 0.2231);
311 PARAMETERS.P3A.SchwSm2000.chi_0_share = 0.2231 / (10.3743 + 0.2231);
312 PARAMETERS.P3A.Korn2005.xi_0_share = 10.8388 / (10.8388 - 0.8681); % Prokoczuk (2010): -
313 PARAMETERS.P3A.Korn2005.chi_0_share = -0.8681 / (10.8388 - 0.8681); % Prokoczuk ...
    (2010): -
314
315 % define model initial values
316 PARAMETERS.P3A.Black1976.xi_0 = log(S_0);
317 PARAMETERS.P3A.Schw1997.xi_0 = log(S_0);
318 PARAMETERS.P3A.SchwSm2000.xi_0 = log(S_0) * PARAMETERS.P3A.SchwSm2000.xi_0_share;
319 PARAMETERS.P3A.SchwSm2000.chi_0 = log(S_0) * PARAMETERS.P3A.SchwSm2000.chi_0_share;
320 PARAMETERS.P3A.Korn2005.xi_0 = log(S_0) * PARAMETERS.P3A.Korn2005.xi_0_share;
321 PARAMETERS.P3A.Korn2005.chi_0 = log(S_0) * PARAMETERS.P3A.Korn2005.chi_0_share;
322
323 % clear helper variables
324 clearvars S_0
325
326 %% Define strike price
327
328 % define strike adjustment factors
329 adj_factor_1 = 0.1;
330 adj_factor_2 = 0.2;
331
332 % route C4
333 % Black (1976)
334 PARAMETERS.C4.Black1976.K(3,1) = exp(PARAMETERS.C4.Black1976.xi_0);
335 PARAMETERS.C4.Black1976.K(1,1) = PARAMETERS.C4.Black1976.K(3,1) * (1 - adj_factor_2);
336 PARAMETERS.C4.Black1976.K(2,1) = PARAMETERS.C4.Black1976.K(3,1) * (1 - adj_factor_1);
337 PARAMETERS.C4.Black1976.K(4,1) = PARAMETERS.C4.Black1976.K(3,1) * (1 + adj_factor_1);
338 PARAMETERS.C4.Black1976.K(5,1) = PARAMETERS.C4.Black1976.K(3,1) * (1 + adj_factor_2);
339
340 % Schwartz (1997)
341 PARAMETERS.C4.Schw1997.K(3,1) = exp(PARAMETERS.C4.Schw1997.xi_0);
342 PARAMETERS.C4.Schw1997.K(1,1) = PARAMETERS.C4.Schw1997.K(3,1) * (1 - adj_factor_2);
343 PARAMETERS.C4.Schw1997.K(2,1) = PARAMETERS.C4.Schw1997.K(3,1) * (1 - adj_factor_1);
344 PARAMETERS.C4.Schw1997.K(4,1) = PARAMETERS.C4.Schw1997.K(3,1) * (1 + adj_factor_1);
345 PARAMETERS.C4.Schw1997.K(5,1) = PARAMETERS.C4.Schw1997.K(3,1) * (1 + adj_factor_2);
346
347 % Schwartz and Smith (2000)

```

```

348 PARAMETERS.C4.SchwSm2000.K(3,1) = exp(PARAMETERS.C4.SchwSm2000.xi_0 + ...
    PARAMETERS.C4.SchwSm2000.chi_0);
349 PARAMETERS.C4.SchwSm2000.K(1,1) = PARAMETERS.C4.SchwSm2000.K(3,1) * (1 - adj_factor_2);
350 PARAMETERS.C4.SchwSm2000.K(2,1) = PARAMETERS.C4.SchwSm2000.K(3,1) * (1 - adj_factor_1);
351 PARAMETERS.C4.SchwSm2000.K(4,1) = PARAMETERS.C4.SchwSm2000.K(3,1) * (1 + adj_factor_1);
352 PARAMETERS.C4.SchwSm2000.K(5,1) = PARAMETERS.C4.SchwSm2000.K(3,1) * (1 + adj_factor_2);
353
354 % Korn (2005)
355 PARAMETERS.C4.Korn2005.K(3,1) = exp(PARAMETERS.C4.Korn2005.xi_0 + ...
    PARAMETERS.C4.Korn2005.chi_0);
356 PARAMETERS.C4.Korn2005.K(1,1) = PARAMETERS.C4.Korn2005.K(3,1) * (1 - adj_factor_2);
357 PARAMETERS.C4.Korn2005.K(2,1) = PARAMETERS.C4.Korn2005.K(3,1) * (1 - adj_factor_1);
358 PARAMETERS.C4.Korn2005.K(4,1) = PARAMETERS.C4.Korn2005.K(3,1) * (1 + adj_factor_1);
359 PARAMETERS.C4.Korn2005.K(5,1) = PARAMETERS.C4.Korn2005.K(3,1) * (1 + adj_factor_2);
360
361 % route C7
362 % Black (1976)
363 PARAMETERS.C7.Black1976.K(3,1) = exp(PARAMETERS.C7.Black1976.xi_0);
364 PARAMETERS.C7.Black1976.K(1,1) = PARAMETERS.C7.Black1976.K(3,1) * (1 - adj_factor_2);
365 PARAMETERS.C7.Black1976.K(2,1) = PARAMETERS.C7.Black1976.K(3,1) * (1 - adj_factor_1);
366 PARAMETERS.C7.Black1976.K(4,1) = PARAMETERS.C7.Black1976.K(3,1) * (1 + adj_factor_1);
367 PARAMETERS.C7.Black1976.K(5,1) = PARAMETERS.C7.Black1976.K(3,1) * (1 + adj_factor_2);
368
369 % Schwartz (1997)
370 PARAMETERS.C7.Schw1997.K(3,1) = exp(PARAMETERS.C7.Schw1997.xi_0);
371 PARAMETERS.C7.Schw1997.K(1,1) = PARAMETERS.C7.Schw1997.K(3,1) * (1 - adj_factor_2);
372 PARAMETERS.C7.Schw1997.K(2,1) = PARAMETERS.C7.Schw1997.K(3,1) * (1 - adj_factor_1);
373 PARAMETERS.C7.Schw1997.K(4,1) = PARAMETERS.C7.Schw1997.K(3,1) * (1 + adj_factor_1);
374 PARAMETERS.C7.Schw1997.K(5,1) = PARAMETERS.C7.Schw1997.K(3,1) * (1 + adj_factor_2);
375
376 % Schwartz and Smith (2000)
377 PARAMETERS.C7.SchwSm2000.K(3,1) = exp(PARAMETERS.C7.SchwSm2000.xi_0 + ...
    PARAMETERS.C7.SchwSm2000.chi_0);
378 PARAMETERS.C7.SchwSm2000.K(1,1) = PARAMETERS.C7.SchwSm2000.K(3,1) * (1 - adj_factor_2);
379 PARAMETERS.C7.SchwSm2000.K(2,1) = PARAMETERS.C7.SchwSm2000.K(3,1) * (1 - adj_factor_1);
380 PARAMETERS.C7.SchwSm2000.K(4,1) = PARAMETERS.C7.SchwSm2000.K(3,1) * (1 + adj_factor_1);
381 PARAMETERS.C7.SchwSm2000.K(5,1) = PARAMETERS.C7.SchwSm2000.K(3,1) * (1 + adj_factor_2);
382
383 % Korn (2005)
384 PARAMETERS.C7.Korn2005.K(3,1) = exp(PARAMETERS.C7.Korn2005.xi_0 + ...
    PARAMETERS.C7.Korn2005.chi_0);
385 PARAMETERS.C7.Korn2005.K(1,1) = PARAMETERS.C7.Korn2005.K(3,1) * (1 - adj_factor_2);
386 PARAMETERS.C7.Korn2005.K(2,1) = PARAMETERS.C7.Korn2005.K(3,1) * (1 - adj_factor_1);
387 PARAMETERS.C7.Korn2005.K(4,1) = PARAMETERS.C7.Korn2005.K(3,1) * (1 + adj_factor_1);
388 PARAMETERS.C7.Korn2005.K(5,1) = PARAMETERS.C7.Korn2005.K(3,1) * (1 + adj_factor_2);
389
390 % route P2A
391 % Black (1976)
392 PARAMETERS.P2A.Black1976.K(3,1) = exp(PARAMETERS.P2A.Black1976.xi_0);
393 PARAMETERS.P2A.Black1976.K(1,1) = PARAMETERS.P2A.Black1976.K(3,1) * (1 - adj_factor_2);
394 PARAMETERS.P2A.Black1976.K(2,1) = PARAMETERS.P2A.Black1976.K(3,1) * (1 - adj_factor_1);
395 PARAMETERS.P2A.Black1976.K(4,1) = PARAMETERS.P2A.Black1976.K(3,1) * (1 + adj_factor_1);
396 PARAMETERS.P2A.Black1976.K(5,1) = PARAMETERS.P2A.Black1976.K(3,1) * (1 + adj_factor_2);
397

```

```
398 % Schwartz (1997)
399 PARAMETERS.P2A.Schw1997.K(3,1) = exp(PARAMETERS.P2A.Schw1997.xi_0);
400 PARAMETERS.P2A.Schw1997.K(1,1) = PARAMETERS.P2A.Schw1997.K(3,1) * (1 - adj_factor_2);
401 PARAMETERS.P2A.Schw1997.K(2,1) = PARAMETERS.P2A.Schw1997.K(3,1) * (1 - adj_factor_1);
402 PARAMETERS.P2A.Schw1997.K(4,1) = PARAMETERS.P2A.Schw1997.K(3,1) * (1 + adj_factor_1);
403 PARAMETERS.P2A.Schw1997.K(5,1) = PARAMETERS.P2A.Schw1997.K(3,1) * (1 + adj_factor_2);
404
405 % Schwartz and Smith (2000)
406 PARAMETERS.P2A.SchwSm2000.K(3,1) = exp(PARAMETERS.P2A.SchwSm2000.xi_0 + ...
    PARAMETERS.P2A.SchwSm2000.chi_0);
407 PARAMETERS.P2A.SchwSm2000.K(1,1) = PARAMETERS.P2A.SchwSm2000.K(3,1) * (1 - adj_factor_2);
408 PARAMETERS.P2A.SchwSm2000.K(2,1) = PARAMETERS.P2A.SchwSm2000.K(3,1) * (1 - adj_factor_1);
409 PARAMETERS.P2A.SchwSm2000.K(4,1) = PARAMETERS.P2A.SchwSm2000.K(3,1) * (1 + adj_factor_1);
410 PARAMETERS.P2A.SchwSm2000.K(5,1) = PARAMETERS.P2A.SchwSm2000.K(3,1) * (1 + adj_factor_2);
411
412 % Korn (2005)
413 PARAMETERS.P2A.Korn2005.K(3,1) = exp(PARAMETERS.P2A.Korn2005.xi_0 + ...
    PARAMETERS.P2A.Korn2005.chi_0);
414 PARAMETERS.P2A.Korn2005.K(1,1) = PARAMETERS.P2A.Korn2005.K(3,1) * (1 - adj_factor_2);
415 PARAMETERS.P2A.Korn2005.K(2,1) = PARAMETERS.P2A.Korn2005.K(3,1) * (1 - adj_factor_1);
416 PARAMETERS.P2A.Korn2005.K(4,1) = PARAMETERS.P2A.Korn2005.K(3,1) * (1 + adj_factor_1);
417 PARAMETERS.P2A.Korn2005.K(5,1) = PARAMETERS.P2A.Korn2005.K(3,1) * (1 + adj_factor_2);
418
419 % route P3A
420 % Black (1976)
421 PARAMETERS.P3A.Black1976.K(3,1) = exp(PARAMETERS.P3A.Black1976.xi_0);
422 PARAMETERS.P3A.Black1976.K(1,1) = PARAMETERS.P3A.Black1976.K(3,1) * (1 - adj_factor_2);
423 PARAMETERS.P3A.Black1976.K(2,1) = PARAMETERS.P3A.Black1976.K(3,1) * (1 - adj_factor_1);
424 PARAMETERS.P3A.Black1976.K(4,1) = PARAMETERS.P3A.Black1976.K(3,1) * (1 + adj_factor_1);
425 PARAMETERS.P3A.Black1976.K(5,1) = PARAMETERS.P3A.Black1976.K(3,1) * (1 + adj_factor_2);
426
427 % Schwartz (1997)
428 PARAMETERS.P3A.Schw1997.K(3,1) = exp(PARAMETERS.P3A.Schw1997.xi_0);
429 PARAMETERS.P3A.Schw1997.K(1,1) = PARAMETERS.P3A.Schw1997.K(3,1) * (1 - adj_factor_2);
430 PARAMETERS.P3A.Schw1997.K(2,1) = PARAMETERS.P3A.Schw1997.K(3,1) * (1 - adj_factor_1);
431 PARAMETERS.P3A.Schw1997.K(4,1) = PARAMETERS.P3A.Schw1997.K(3,1) * (1 + adj_factor_1);
432 PARAMETERS.P3A.Schw1997.K(5,1) = PARAMETERS.P3A.Schw1997.K(3,1) * (1 + adj_factor_2);
433
434 % Schwartz and Smith (2000)
435 PARAMETERS.P3A.SchwSm2000.K(3,1) = exp(PARAMETERS.P3A.SchwSm2000.xi_0 + ...
    PARAMETERS.P3A.SchwSm2000.chi_0);
436 PARAMETERS.P3A.SchwSm2000.K(1,1) = PARAMETERS.P3A.SchwSm2000.K(3,1) * (1 - adj_factor_2);
437 PARAMETERS.P3A.SchwSm2000.K(2,1) = PARAMETERS.P3A.SchwSm2000.K(3,1) * (1 - adj_factor_1);
438 PARAMETERS.P3A.SchwSm2000.K(4,1) = PARAMETERS.P3A.SchwSm2000.K(3,1) * (1 + adj_factor_1);
439 PARAMETERS.P3A.SchwSm2000.K(5,1) = PARAMETERS.P3A.SchwSm2000.K(3,1) * (1 + adj_factor_2);
440
441 % Korn (2005)
442 PARAMETERS.P3A.Korn2005.K(3,1) = exp(PARAMETERS.P3A.Korn2005.xi_0 + ...
    PARAMETERS.P3A.Korn2005.chi_0);
443 PARAMETERS.P3A.Korn2005.K(1,1) = PARAMETERS.P3A.Korn2005.K(3,1) * (1 - adj_factor_2);
444 PARAMETERS.P3A.Korn2005.K(2,1) = PARAMETERS.P3A.Korn2005.K(3,1) * (1 - adj_factor_1);
445 PARAMETERS.P3A.Korn2005.K(4,1) = PARAMETERS.P3A.Korn2005.K(3,1) * (1 + adj_factor_1);
446 PARAMETERS.P3A.Korn2005.K(5,1) = PARAMETERS.P3A.Korn2005.K(3,1) * (1 + adj_factor_2);
447
```

```
448 % clear helper variables
449 clearvars adj_factor_1 adj_factor_2
450
451 fprintf('done.\n')
452
453 %% Define MC simulation parameters
454
455 fprintf('Define MC simulation parameters...')
456
457 % define length of time interval considered in years
458 t = 1/12;
459 % t = 1;
460
461 % define number of discretization steps
462 n = zeros(5,1);
463 n(1) = 21;
464 n(2) = 42;
465 n(3) = 84;
466 n(4) = 126;
467 n(5) = 210;
468 % n(5) = 252;
469
470 % determine length of time step
471 dt = t./n;
472
473 % define number of MC runs
474 k = zeros(5,1);
475 k(1) = 100000;
476 k(2) = 20000;
477 k(3) = 5000;
478 k(4) = 1000;
479 k(5) = 100;
480
481 % define interest rate
482 r = 0.05;
483
484 % specify time vector
485 time.N1 = (0:dt(1):n*dt(1))';
486 time.N2 = (0:dt(2):n*dt(2))';
487 time.N3 = (0:dt(3):n*dt(3))';
488 time.N4 = (0:dt(4):n*dt(4))';
489 time.N5 = (0:dt(5):n*dt(5))';
490
491 fprintf('done.\n')
492
493 %% Random number generation
494
495 fprintf('Generate random numbers...')
496
497 % loop across discretization steps
498 for c = 1:length(n)
499
500     % differentiate discretization labels
501     if c == 1
```

```
502     d = 'N1';
503 elseif c == 2
504     d = 'N2';
505 elseif c == 3
506     d = 'N3';
507 elseif c == 4
508     d = 'N4';
509 elseif c == 5
510     d = 'N5';
511 end
512
513 % loop across MC runs
514 for a = 1:length(k)
515
516     % differentiate run labels
517     if a == 1
518         b = 'K1';
519     elseif a == 2
520         b = 'K2';
521     elseif a == 3
522         b = 'K3';
523     elseif a == 4
524         b = 'K4';
525     elseif a == 5
526         b = 'K5';
527     end
528
529     % define seed
530     seed = 1;
531
532     % define random number stream
533     stream = RandStream('mt19937ar','Seed',seed);
534     RandStream.setGlobalStream(stream);
535
536     % draw independent random numbers
537     z1 = randn(n(c),k(a));
538     z2 = randn(n(c),k(a));
539
540     % define correlated random number vectors
541     % first matrix remains unchanged
542     Z.(b).(d).Z1 = z1;
543
544     % adjust second matrix for correlation structure for two-factor models for
545     % each route
546     for i = 1:4
547
548         % differentiate routes
549         if i == 1
550             j = 'C4';
551         elseif i == 2
552             j = 'C7';
553         elseif i == 3
554             j = 'P2A';
555         elseif i == 4
```

```

556         j = 'P3A';
557     end
558
559     % determine time-dependent rho_dt for each model
560     PARAMETERS.(j).SchwSm2000.rho_dt.(b) = PARAMETERS.(j).SchwSm2000.rho * ...
        sqrt((1 - exp(-PARAMETERS.(j).SchwSm2000.kappa_chi * dt(c)))^2 * 2 / ...
        ((1 - exp(-2 * PARAMETERS.(j).SchwSm2000.kappa_chi * dt(c))) * ...
561     PARAMETERS.(j).SchwSm2000.kappa_chi * dt(c)));
562     PARAMETERS.(j).Korn2005.rho_dt.(b) = PARAMETERS.(j).Korn2005.rho * sqrt(4 ...
        * PARAMETERS.(j).Korn2005.kappa_xi * PARAMETERS.(j).Korn2005.kappa_chi ...
        * ...
563     (1 - exp(-(PARAMETERS.(j).Korn2005.kappa_xi + ...
        PARAMETERS.(j).Korn2005.kappa_chi) * dt(c)))^2 / ...
        ((PARAMETERS.(j).Korn2005.kappa_xi + ...
        PARAMETERS.(j).Korn2005.kappa_chi)^2 * ...
564     (1 - exp(-2 * PARAMETERS.(j).Korn2005.kappa_xi * dt(c))) * (1 - exp(-2 ...
        * PARAMETERS.(j).Korn2005.kappa_chi * dt(c)))));
565
566     % determine second, correlated random number matrix for each model and
567     % route
568     Z.(b).(d).(j).SchwSm2000.Z2 = PARAMETERS.(j).SchwSm2000.rho_dt.(b) * z1 + ...
        sqrt(1 - PARAMETERS.(j).SchwSm2000.rho_dt.(b)^2) * z2;
569     Z.(b).(d).(j).Korn2005.Z2 = PARAMETERS.(j).Korn2005.rho_dt.(b) * z1 + ...
        sqrt(1 - PARAMETERS.(j).Korn2005.rho_dt.(b)^2) * z2;
570
571     end
572     clearvars i j
573
574     % clear helper variables
575     clearvars stream seed z1 z2
576
577     end
578     clearvars a b
579
580     fprintf('done.\n')
581
582     %% Run MC simulation
583
584     fprintf('Run MC simulation...\n')
585
586     % loop across discretization steps
587     for c = 1:length(n)
588
589         % differentiate discretization labels
590         if c == 1
591             d = 'N1';
592         elseif c == 2
593             d = 'N2';
594         elseif c == 3
595             d = 'N3';
596         elseif c == 4
597             d = 'N4';
598         elseif c == 5
599             d = 'N5';

```

```
600     end
601
602     fprintf('\tDiscretization: %s...\n',d)
603
604     % loop across MC runs
605     for a = 1:length(k)
606
607         % differentiate run labels
608         if a == 1
609             b = 'K1';
610         elseif a == 2
611             b = 'K2';
612         elseif a == 3
613             b = 'K3';
614         elseif a == 4
615             b = 'K4';
616         elseif a == 5
617             b = 'K5';
618         end
619
620         fprintf('\t\tRun: %s...\n',b)
621
622         % loop across routes
623         for i = 1:4
624
625             % differentiate routes
626             if i == 1
627                 j = 'C4';
628             elseif i == 2
629                 j = 'C7';
630             elseif i == 3
631                 j = 'P2A';
632             elseif i == 4
633                 j = 'P3A';
634             end
635
636             fprintf('\t\t\tRoute: %s...\n',j)
637
638             % initialize dataset
639             RESULTS.(b).(d).(j).Black1976 = table;
640             RESULTS.(b).(d).(j).Schw1997 = table;
641             RESULTS.(b).(d).(j).SchwSm2000 = table;
642             RESULTS.(b).(d).(j).Korn2005 = table;
643
644             % fill S_0, K, r
645             RESULTS.(b).(d).(j).Black1976.S_0 = zeros(5,1);
646             RESULTS.(b).(d).(j).Black1976.K = zeros(5,1);
647             RESULTS.(b).(d).(j).Black1976.r = zeros(5,1);
648             RESULTS.(b).(d).(j).Schw1997.S_0 = zeros(5,1);
649             RESULTS.(b).(d).(j).Schw1997.K = zeros(5,1);
650             RESULTS.(b).(d).(j).Schw1997.r = zeros(5,1);
651             RESULTS.(b).(d).(j).SchwSm2000.S_0 = zeros(5,1);
652             RESULTS.(b).(d).(j).SchwSm2000.K = zeros(5,1);
653             RESULTS.(b).(d).(j).SchwSm2000.r = zeros(5,1);
```

```

654     RESULTS.(b).(d).(j).Korn2005.S_0 = zeros(5,1);
655     RESULTS.(b).(d).(j).Korn2005.K = zeros(5,1);
656     RESULTS.(b).(d).(j).Korn2005.r = zeros(5,1);
657
658     for p = 1:5
659         RESULTS.(b).(d).(j).Black1976.S_0(p,1) = ...
            exp(PARAMETERS.(j).Black1976.xi_0);
660         RESULTS.(b).(d).(j).Black1976.K(p,1) = PARAMETERS.(j).Black1976.K(p);
661         RESULTS.(b).(d).(j).Black1976.r(p,1) = r;
662         RESULTS.(b).(d).(j).Schw1997.S_0(p,1) = exp(PARAMETERS.(j).Schw1997.xi_0);
663         RESULTS.(b).(d).(j).Schw1997.K(p,1) = PARAMETERS.(j).Schw1997.K(p);
664         RESULTS.(b).(d).(j).Schw1997.r(p,1) = r;
665         RESULTS.(b).(d).(j).SchwSm2000.S_0(p,1) = ...
            exp(PARAMETERS.(j).SchwSm2000.xi_0 + PARAMETERS.(j).SchwSm2000.chi_0);
666         RESULTS.(b).(d).(j).SchwSm2000.K(p,1) = PARAMETERS.(j).SchwSm2000.K(p);
667         RESULTS.(b).(d).(j).SchwSm2000.r(p,1) = r;
668         RESULTS.(b).(d).(j).Korn2005.S_0(p,1) = ...
            exp(PARAMETERS.(j).Korn2005.xi_0 + PARAMETERS.(j).Korn2005.chi_0);
669         RESULTS.(b).(d).(j).Korn2005.K(p,1) = PARAMETERS.(j).Korn2005.K(p);
670         RESULTS.(b).(d).(j).Korn2005.r(p,1) = r;
671     end
672     clearvars p
673
674     % run MC simulation
675     [RESULTS.(b).(d).(j).Black1976.MC_GA, ...
676         RESULTS.(b).(d).(j).Black1976.MC_GA_SE, ...
677         RESULTS.(b).(d).(j).Black1976.CFS, ...
678         RESULTS.(b).(d).(j).Black1976.MC_AA, ...
679         RESULTS.(b).(d).(j).Black1976.MC_AA_SE, ...
680         RESULTS.(b).(d).(j).Black1976.MC_AA_CV, ...
681         RESULTS.(b).(d).(j).Black1976.MC_AA_CV_SE, ...
682         RESULTS.(b).(d).(j).Black1976.VR] ...
683     = Black1976_MC(PARAMETERS.(j).Black1976.xi_0, ...
684         PARAMETERS.(j).Black1976.a_star,PARAMETERS.(j).Black1976.sigma_xi, ...
685         dt(c),Z.(b).(d).Z1,PARAMETERS.(j).Black1976.K,r,t);
686     [RESULTS.(b).(d).(j).Schw1997.MC_GA, ...
687         RESULTS.(b).(d).(j).Schw1997.MC_GA_SE, ...
688         RESULTS.(b).(d).(j).Schw1997.CFS, ...
689         RESULTS.(b).(d).(j).Schw1997.MC_AA, ...
690         RESULTS.(b).(d).(j).Schw1997.MC_AA_SE, ...
691         RESULTS.(b).(d).(j).Schw1997.MC_AA_CV, ...
692         RESULTS.(b).(d).(j).Schw1997.MC_AA_CV_SE, ...
693         RESULTS.(b).(d).(j).Schw1997.VR] = ...
694         Schw1997_MC(PARAMETERS.(j).Schw1997.xi_0, ...
695         PARAMETERS.(j).Schw1997.kappa_xi,PARAMETERS.(j).Schw1997.a_star, ...
696         PARAMETERS.(j).Schw1997.sigma_xi,dt(c),Z.(b).(d).Z1, ...
697         PARAMETERS.(j).Schw1997.K,r,t);
698     [RESULTS.(b).(d).(j).SchwSm2000.MC_GA, ...
699         RESULTS.(b).(d).(j).SchwSm2000.MC_GA_SE, ...
700         RESULTS.(b).(d).(j).SchwSm2000.CFS, ...
701         RESULTS.(b).(d).(j).SchwSm2000.MC_AA, ...
702         RESULTS.(b).(d).(j).SchwSm2000.MC_AA_SE, ...
703         RESULTS.(b).(d).(j).SchwSm2000.MC_AA_CV, ...
704         RESULTS.(b).(d).(j).SchwSm2000.MC_AA_CV_SE, ...

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705     RESULTS.(b).(d).(j).SchwSm2000.VR] = ...
706     SchwSm2000_MC(PARAMETERS.(j).SchwSm2000.xi_0, ...
707     PARAMETERS.(j).SchwSm2000.a_star,PARAMETERS.(j).SchwSm2000.sigma_xi, ...
708     PARAMETERS.(j).SchwSm2000.chi_0,PARAMETERS.(j).SchwSm2000.kappa_chi, ...
709     PARAMETERS.(j).SchwSm2000.lambda_chi, ...
710     PARAMETERS.(j).SchwSm2000.sigma_chi,PARAMETERS.(j).SchwSm2000.rho, ...
711     dt(c),Z.(b).(d).Z1,Z.(b).(d).(j).SchwSm2000.Z2, ...
712     PARAMETERS.(j).SchwSm2000.K,r,t);
713     [RESULTS.(b).(d).(j).Korn2005.MC_GA, ...
714     RESULTS.(b).(d).(j).Korn2005.MC_GA_SE, ...
715     RESULTS.(b).(d).(j).Korn2005.CFS, ...
716     RESULTS.(b).(d).(j).Korn2005.MC_AA, ...
717     RESULTS.(b).(d).(j).Korn2005.MC_AA_SE, ...
718     RESULTS.(b).(d).(j).Korn2005.MC_AA_CV, ...
719     RESULTS.(b).(d).(j).Korn2005.MC_AA_CV_SE, ...
720     RESULTS.(b).(d).(j).Korn2005.VR] = ...
721     Korn2005_MC(PARAMETERS.(j).Korn2005.xi_0, ...
722     PARAMETERS.(j).Korn2005.kappa_xi, ...
723     PARAMETERS.(j).Korn2005.a_star,PARAMETERS.(j).Korn2005.sigma_xi, ...
724     PARAMETERS.(j).Korn2005.chi_0,PARAMETERS.(j).Korn2005.kappa_chi, ...
725     PARAMETERS.(j).Korn2005.lambda_chi,PARAMETERS.(j).Korn2005.sigma_chi, ...
726     PARAMETERS.(j).Korn2005.rho,dt(c),Z.(b).(d).Z1, ...
727     Z.(b).(d).(j).Korn2005.Z2,PARAMETERS.(j).Korn2005.K,r,t);
728
729     fprintf('\t\t\tRoute: #s...done.\n',j)
730     end
731     clearvars i j
732
733     fprintf('\t\t\tRun: #s...done.\n',b)
734     end
735     clearvars a b
736
737     fprintf('\t\t\tDiscretization: #s...done.\n',d)
738     end
739     clearvars c d
740
741     fprintf('Run MC simulation...done.\n')
742
743     %% Determine pricing error
744
745     fprintf('Determine pricing error...')
746
747     % loop across discretization steps
748     for c = 1:length(n)
749
750         % differentiate discretization labels
751         if c == 1
752             d = 'N1';
753         elseif c == 2
754             d = 'N2';
755         elseif c == 3
756             d = 'N3';
757         elseif c == 4
758             d = 'N4';

```

```

759     elseif c == 5
760         d = 'N5';
761     end
762
763     % loop across MC runs
764     for a = 1:length(k)
765
766         % differentiate run labels
767         if a == 1
768             b = 'K1';
769         elseif a == 2
770             b = 'K2';
771         elseif a == 3
772             b = 'K3';
773         elseif a == 4
774             b = 'K4';
775         elseif a == 5
776             b = 'K5';
777         end
778
779         % loop across routes
780         for i = 1:4
781
782             % differentiate routes
783             if i == 1
784                 j = 'C4';
785             elseif i == 2
786                 j = 'C7';
787             elseif i == 3
788                 j = 'P2A';
789             elseif i == 4
790                 j = 'P3A';
791             end
792
793             % determine bias
794             RESULTS.(b).(d).(j).Black1976.BIAS = RESULTS.(b).(d).(j).Black1976.MC_GA - ...
                RESULTS.(b).(d).(j).Black1976.CFS;
795             RESULTS.(b).(d).(j).Schw1997.BIAS = RESULTS.(b).(d).(j).Schw1997.MC_GA - ...
                RESULTS.(b).(d).(j).Schw1997.CFS;
796             RESULTS.(b).(d).(j).SchwSm2000.BIAS = RESULTS.(b).(d).(j).SchwSm2000.MC_GA ...
                - RESULTS.(b).(d).(j).SchwSm2000.CFS;
797             RESULTS.(b).(d).(j).Korn2005.BIAS = RESULTS.(b).(d).(j).Korn2005.MC_GA - ...
                RESULTS.(b).(d).(j).Korn2005.CFS;
798
799             % determine absolute error
800             RESULTS.(b).(d).(j).Black1976.ERROR_ABS = ...
                abs(RESULTS.(b).(d).(j).Black1976.MC_GA - ...
                RESULTS.(b).(d).(j).Black1976.CFS);
801             RESULTS.(b).(d).(j).Schw1997.ERROR_ABS = ...
                abs(RESULTS.(b).(d).(j).Schw1997.MC_GA - ...
                RESULTS.(b).(d).(j).Schw1997.CFS);
802             RESULTS.(b).(d).(j).SchwSm2000.ERROR_ABS = ...
                abs(RESULTS.(b).(d).(j).SchwSm2000.MC_GA - ...
                RESULTS.(b).(d).(j).SchwSm2000.CFS);

```

```
803     RESULTS.(b).(d).(j).Korn2005.ERROR_ABS = ...
        abs(RESULTS.(b).(d).(j).Korn2005.MC_GA - ...
            RESULTS.(b).(d).(j).Korn2005.CFS);
804
805     % determine relative error
806     RESULTS.(b).(d).(j).Black1976.ERROR_REL = ...
        RESULTS.(b).(d).(j).Black1976.BIAS ./ ...
        RESULTS.(b).(d).(j).Black1976.CFS * 100;
807     RESULTS.(b).(d).(j).Schw1997.ERROR_REL = RESULTS.(b).(d).(j).Schw1997.BIAS ...
        ./ RESULTS.(b).(d).(j).Schw1997.CFS * 100;
808     RESULTS.(b).(d).(j).SchwSm2000.ERROR_REL = ...
        RESULTS.(b).(d).(j).SchwSm2000.BIAS ./ ...
        RESULTS.(b).(d).(j).SchwSm2000.CFS * 100;
809     RESULTS.(b).(d).(j).Korn2005.ERROR_REL = RESULTS.(b).(d).(j).Korn2005.BIAS ...
        ./ RESULTS.(b).(d).(j).Korn2005.CFS * 100;
810
811     % rearrange order of table columns
812     RESULTS.(b).(d).(j).Black1976 = RESULTS.(b).(d).(j).Black1976(:, [1:6 12:14 ...
        7:11]);
813     RESULTS.(b).(d).(j).Schw1997 = RESULTS.(b).(d).(j).Schw1997(:, [1:6 12:14 ...
        7:11]);
814     RESULTS.(b).(d).(j).SchwSm2000 = RESULTS.(b).(d).(j).SchwSm2000(:, [1:6 ...
        12:14 7:11]);
815     RESULTS.(b).(d).(j).Korn2005 = RESULTS.(b).(d).(j).Korn2005(:, [1:6 12:14 ...
        7:11]);
816     end
817     clearvars i j
818     end
819     clearvars a b
820 end
821 clearvars c d
822
823 fprintf('done.\n')
824
825 %% Determine run-increase factor
826
827 fprintf('Determine run-increase factor...')
828
829 % loop across discretization steps
830 for c = 1:length(n)
831
832     % differentiate discretization labels
833     if c == 1
834         d = 'N1';
835     elseif c == 2
836         d = 'N2';
837     elseif c == 3
838         d = 'N3';
839     elseif c == 4
840         d = 'N4';
841     elseif c == 5
842         d = 'N5';
843     end
844
```

```
845 % loop across MC runs
846 for a = 1:length(k)
847
848     % differentiate run labels
849     if a == 1
850         b = 'K1';
851     elseif a == 2
852         b = 'K2';
853     elseif a == 3
854         b = 'K3';
855     elseif a == 4
856         b = 'K4';
857     elseif a == 5
858         b = 'K5';
859     end
860
861     % initialize variable
862     RUN_IN.(b).(d) = zeros(80,1);
863
864     % loop across routes
865     for i = 1:4
866
867         % differentiate routes
868         if i == 1
869             j = 'C4';
870         elseif i == 2
871             j = 'C7';
872         elseif i == 3
873             j = 'P2A';
874         elseif i == 4
875             j = 'P3A';
876         end
877
878         % determine individual run-increase factor
879         RESULTS.(b).(d).(j).Black1976.RUN_IN = ...
            (RESULTS.(b).(d).(j).Black1976.MC_AA_SE ./ ...
            RESULTS.(b).(d).(j).Black1976.MC_AA_CV_SE).^2;
880         RESULTS.(b).(d).(j).Schw1997.RUN_IN = ...
            (RESULTS.(b).(d).(j).Schw1997.MC_AA_SE ./ ...
            RESULTS.(b).(d).(j).Schw1997.MC_AA_CV_SE).^2;
881         RESULTS.(b).(d).(j).SchwSm2000.RUN_IN = ...
            (RESULTS.(b).(d).(j).SchwSm2000.MC_AA_SE ./ ...
            RESULTS.(b).(d).(j).SchwSm2000.MC_AA_CV_SE).^2;
882         RESULTS.(b).(d).(j).Korn2005.RUN_IN = ...
            (RESULTS.(b).(d).(j).Korn2005.MC_AA_SE ./ ...
            RESULTS.(b).(d).(j).Korn2005.MC_AA_CV_SE).^2;
883
884         % get all individual run-increase factors
885         RUN_IN.(b).(d)(i*20-(4*5-1):i*20-(3*5),1) = ...
            RESULTS.(b).(d).(j).Black1976.RUN_IN;
886         RUN_IN.(b).(d)(i*20-(3*5-1):i*20-(2*5),1) = ...
            RESULTS.(b).(d).(j).Schw1997.RUN_IN;
887         RUN_IN.(b).(d)(i*20-(2*5-1):i*20-(1*5),1) = ...
            RESULTS.(b).(d).(j).SchwSm2000.RUN_IN;
```

```
888         RUN_IN.(b).(d)(i*20-(1*5-1):i*20,1) = RESULTS.(b).(d).(j).Korn2005.RUN_IN;
889     end
890     clearvars i j
891
892     % determine min, max, and mean run-increase factor
893     RUN_IN_MIN.(b).(d) = min(RUN_IN.(b).(d));
894     RUN_IN_MAX.(b).(d) = max(RUN_IN.(b).(d));
895     RUN_IN_MEAN.(b).(d) = mean(RUN_IN.(b).(d));
896 end
897 clearvars a b
898 end
899 clearvars c d
900
901 fprintf('done.\n')
902
903 %% Export results to Excel file
904
905 fprintf('Export results to Excel file...')
906
907 % loop across discretization steps
908 for c = 1:length(n)
909
910     % differentiate discretization labels
911     if c == 1
912         d = 'N1';
913     elseif c == 2
914         d = 'N2';
915     elseif c == 3
916         d = 'N3';
917     elseif c == 4
918         d = 'N4';
919     elseif c == 5
920         d = 'N5';
921     end
922
923     % loop across MC runs
924     for a = 1:length(k)
925
926         % differentiate run labels
927         if a == 1
928             b = 'K1';
929         elseif a == 2
930             b = 'K2';
931         elseif a == 3
932             b = 'K3';
933         elseif a == 4
934             b = 'K4';
935         elseif a == 5
936             b = 'K5';
937         end
938
939         % loop across routes
940         for i = 1:4
941
```

```
942         % differentiate routes
943         if i == 1
944             j = 'C4';
945         elseif i == 2
946             j = 'C7';
947         elseif i == 3
948             j = 'P2A';
949         elseif i == 4
950             j = 'P3A';
951         end
952
953         % define sheet names
954         sheet.Black1976 = strcat(d, '-', b, '-', j, '-', 'Black1976');
955         sheet.Schw1997 = strcat(d, '-', b, '-', j, '-', 'Schw1997');
956         sheet.SchwSm2000 = strcat(d, '-', b, '-', j, '-', 'SchwSm2000');
957         sheet.Korn2005 = strcat(d, '-', b, '-', j, '-', 'Korn2005');
958
959         % turn off new worksheet warning
960         warning('OFF', 'MATLAB:xlswrite:AddSheet');
961
962         % export to specified sheet in Excel file
963         writetable(RESULTS.(b).(d).(j).Black1976, exportfile_path, 'Sheet', ...
964             sheet.Black1976, 'Range', 'A1');
965         writetable(RESULTS.(b).(d).(j).Schw1997, exportfile_path, 'Sheet', ...
966             sheet.Schw1997, 'Range', 'A1');
967         writetable(RESULTS.(b).(d).(j).SchwSm2000, exportfile_path, 'Sheet', ...
968             sheet.SchwSm2000, 'Range', 'A1');
969         writetable(RESULTS.(b).(d).(j).Korn2005, exportfile_path, 'Sheet', ...
970             sheet.Korn2005, 'Range', 'A1');
971     end
972     clearvars i j sheet
973 end
974 clearvars a b
975 end
976 clearvars c d
977
978 fprintf('done.\n')
979
980 %% Plots
981
982 SE_FACTOR = norminv(0.95, 0, 1);
983
984 % plot 1: discretization error
985 for i = 1:4
986
987     % differentiate routes
988     if i == 1
989         j = 'C4';
990     elseif i == 2
991         j = 'C7';
992     elseif i == 3
993         j = 'P2A';
994     elseif i == 4
995         j = 'P3A';
```

```

996     end
997
998     % create figure
999     figure
1000
1001     % Black (1976)
1002     spl1_1 = subplot(2,2,1);
1003     splot1_1 = errorbar(log([n(1),n(2),n(3),n(4),n(5)]), ...
1004         [RESULTS.K1.N1.(j).Black1976.BIAS(3), ...
1005         RESULTS.K1.N2.(j).Black1976.BIAS(3),RESULTS.K1.N3.(j).Black1976.BIAS(3), ...
1006         RESULTS.K1.N4.(j).Black1976.BIAS(3),RESULTS.K1.N5.(j).Black1976.BIAS(3)], ...
1007         SE_FACTOR .* [RESULTS.K1.N1.(j).Black1976.MC_GA_SE(3), ...
1008         RESULTS.K1.N2.(j).Black1976.MC_GA_SE(3), ...
1009         RESULTS.K1.N3.(j).Black1976.MC_GA_SE(3), ...
1010         RESULTS.K1.N4.(j).Black1976.MC_GA_SE(3), ...
1011         RESULTS.K1.N5.(j).Black1976.MC_GA_SE(3)]);
1012     xlim([2,6]);
1013     if i <= 2
1014         ylim([-0.025,0.02]);
1015     elseif i > 2
1016         ylim([-100,75]);
1017     end
1018     set(splot1_1,'LineStyle','none');
1019     set(splot1_1,'Marker','o');
1020     title('Black (1976)', 'FontSize',11,'FontWeight','normal')
1021     xlabel('ln(n)', 'FontSize',11);
1022     ylabel('Bias with 90% CI', 'FontSize',11);
1023     hline1 = reline([0 0]);
1024     set(hline1,'LineStyle',':');
1025     set(hline1,'LineWidth',0.5);
1026     set(hline1,'Color','k');
1027
1028     % Schwartz (1997)
1029     spl1_2 = subplot(2,2,2);
1030     splot1_2 = errorbar(log([n(1),n(2),n(3),n(4),n(5)]), ...
1031         [RESULTS.K1.N1.(j).Schw1997.BIAS(3), ...
1032         RESULTS.K1.N2.(j).Schw1997.BIAS(3),RESULTS.K1.N3.(j).Schw1997.BIAS(3), ...
1033         RESULTS.K1.N4.(j).Schw1997.BIAS(3),RESULTS.K1.N5.(j).Schw1997.BIAS(3)], ...
1034         SE_FACTOR .* [RESULTS.K1.N1.(j).Schw1997.MC_GA_SE(3), ...
1035         RESULTS.K1.N2.(j).Schw1997.MC_GA_SE(3), ...
1036         RESULTS.K1.N3.(j).Schw1997.MC_GA_SE(3), ...
1037         RESULTS.K1.N4.(j).Schw1997.MC_GA_SE(3), ...
1038         RESULTS.K1.N5.(j).Schw1997.MC_GA_SE(3)]);
1039     xlim([2,6]);
1040     if i <= 2
1041         ylim([-0.025,0.02]);
1042     elseif i > 2
1043         ylim([-100,75]);
1044     end
1045     set(splot1_2,'LineStyle','none');
1046     set(splot1_2,'Marker','o');
1047     title('Schwartz (1997)', 'FontSize',11,'FontWeight','normal')
1048     xlabel('ln(n)', 'FontSize',11);
1049     ylabel('Bias with 90% CI', 'FontSize',11);

```

```
1050 hline2 = reffline([0 0]);
1051 set(hline2,'LineStyle',':');
1052 set(hline2,'LineWidth',0.5);
1053 set(hline2,'Color','k');
1054
1055 % Schwartz and Smith (2000)
1056 spl1_3 = subplot(2,2,3);
1057 splot1_3 = errorbar(log([n(1),n(2),n(3),n(4),n(5)]), ...
1058     [RESULTS.K1.N1.(j).SchwSm2000.BIAS(3), ...
1059     RESULTS.K1.N2.(j).SchwSm2000.BIAS(3),RESULTS.K1.N3.(j).SchwSm2000.BIAS(3), ...
1060     RESULTS.K1.N4.(j).SchwSm2000.BIAS(3),RESULTS.K1.N5.(j).SchwSm2000.BIAS(3)], ...
1061     SE_FACTOR .* [RESULTS.K1.N1.(j).SchwSm2000.MC_GA_SE(3), ...
1062     RESULTS.K1.N2.(j).SchwSm2000.MC_GA_SE(3), ...
1063     RESULTS.K1.N3.(j).SchwSm2000.MC_GA_SE(3), ...
1064     RESULTS.K1.N4.(j).SchwSm2000.MC_GA_SE(3), ...
1065     RESULTS.K1.N5.(j).SchwSm2000.MC_GA_SE(3)]);
1066 xlim([2,6]);
1067 if i <= 2
1068     ylim([-0.025,0.02]);
1069 elseif i > 2
1070     ylim([-100,75]);
1071 end
1072 set(splot1_3,'LineStyle','none');
1073 set(splot1_3,'Marker','o');
1074 title('Schwartz and Smith (2000)','FontSize',11,'FontWeight','normal')
1075 xlabel('ln(n)','FontSize',11);
1076 ylabel('Bias with 90% CI','FontSize',11);
1077 hline3 = reffline([0 0]);
1078 set(hline3,'LineStyle',':');
1079 set(hline3,'LineWidth',0.5);
1080 set(hline3,'Color','k');
1081
1082 % Korn (2005)
1083 spl1_4 = subplot(2,2,4);
1084 splot1_4 = errorbar(log([n(1),n(2),n(3),n(4),n(5)]), ...
1085     [RESULTS.K1.N1.(j).Korn2005.BIAS(3), ...
1086     RESULTS.K1.N2.(j).Korn2005.BIAS(3),RESULTS.K1.N3.(j).Korn2005.BIAS(3), ...
1087     RESULTS.K1.N4.(j).Korn2005.BIAS(3),RESULTS.K1.N5.(j).Korn2005.BIAS(3)], ...
1088     SE_FACTOR .* [RESULTS.K1.N1.(j).Korn2005.MC_GA_SE(3), ...
1089     RESULTS.K1.N2.(j).Korn2005.MC_GA_SE(3), ...
1090     RESULTS.K1.N3.(j).Korn2005.MC_GA_SE(3), ...
1091     RESULTS.K1.N4.(j).Korn2005.MC_GA_SE(3), ...
1092     RESULTS.K1.N5.(j).Korn2005.MC_GA_SE(3)]);
1093 xlim([2,6]);
1094 if i <= 2
1095     ylim([-0.025,0.02]);
1096 elseif i > 2
1097     ylim([-100,75]);
1098 end
1099 set(splot1_4,'LineStyle','none');
1100 set(splot1_4,'Marker','o');
1101 title('Korn (2005)','FontSize',11,'FontWeight','normal');
1102 xlabel('ln(n)','FontSize',11);
1103 ylabel('Bias with 90% CI','FontSize',11);
```

```
1104     hline4 = reffline([0 0]);
1105     set(hline4, 'LineStyle', ':');
1106     set(hline4, 'LineWidth', 0.5);
1107     set(hline4, 'Color', 'k');
1108
1109     % clear helper variables
1110     clearvars spl1_1 splot1_1 splot1_11 spl1_2 splot1_2 splot1_12 spl1_3 splot1_3 ...
        splot1_13 spl1_4 splot1_4 splot1_14 hline1 hline2 hline3 hline4
1111 end
1112 clearvars i j
1113
1114 % plot 2: variance reduction
1115 for i = 1:4
1116
1117     % differentiate routes
1118     if i == 1
1119         j = 'C4';
1120     elseif i == 2
1121         j = 'C7';
1122     elseif i == 3
1123         j = 'P2A';
1124     elseif i == 4
1125         j = 'P3A';
1126     end
1127
1128     % create figure
1129     figure
1130
1131     % Black (1976)
1132     spl2_1 = subplot(2,2,1);
1133     plot2_1a = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1134         log([RESULTS.K1.N1.(j).Black1976.MC_AA_SE(3), ...
1135         RESULTS.K2.N1.(j).Black1976.MC_AA_SE(3), ...
1136         RESULTS.K3.N1.(j).Black1976.MC_AA_SE(3), ...
1137         RESULTS.K4.N1.(j).Black1976.MC_AA_SE(3), ...
1138         RESULTS.K5.N1.(j).Black1976.MC_AA_SE(3)]), 'o:');
1139     xlim([4,12]);
1140     if i <= 2
1141         ylim([-10,0]);
1142     elseif i > 2
1143         ylim([-2,8]);
1144     end
1145     title('Black (1976)', 'FontSize', 11, 'FontWeight', 'normal');
1146     xlabel('ln(k)', 'FontSize', 11);
1147     ylabel('ln(SE)', 'FontSize', 11);
1148     hold on
1149     splot2_1b = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1150         log([RESULTS.K1.N1.(j).Black1976.MC_AA_CV_SE(3), ...
1151         RESULTS.K2.N1.(j).Black1976.MC_AA_CV_SE(3), ...
1152         RESULTS.K3.N1.(j).Black1976.MC_AA_CV_SE(3), ...
1153         RESULTS.K4.N1.(j).Black1976.MC_AA_CV_SE(3), ...
1154         RESULTS.K5.N1.(j).Black1976.MC_AA_CV_SE(3)]), 'o:');
1155     legend('MC', 'MC CV', 'Location', 'northeast');
1156     hold off
```

```

1157
1158 % Schwartz (1997)
1159 spl2_2 = subplot(2,2,2);
1160 splot2_2a = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1161     log([RESULTS.K1.N1.(j).Schw1997.MC_AA_SE(3), ...
1162     RESULTS.K2.N1.(j).Schw1997.MC_AA_SE(3), ...
1163     RESULTS.K3.N1.(j).Schw1997.MC_AA_SE(3), ...
1164     RESULTS.K4.N1.(j).Schw1997.MC_AA_SE(3), ...
1165     RESULTS.K5.N1.(j).Schw1997.MC_AA_SE(3)]), 'o:');
1166 xlim([4,12]);
1167 if i <= 2
1168     ylim([-10,0]);
1169 elseif i > 2
1170     ylim([-2,8]);
1171 end
1172 title('Schwartz (1997)', 'FontSize', 11, 'FontWeight', 'normal');
1173 xlabel('ln(k)', 'FontSize', 11);
1174 ylabel('ln(SE)', 'FontSize', 11);
1175 hold on
1176 splot2_2b = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1177     log([RESULTS.K1.N1.(j).Schw1997.MC_AA_CV_SE(3), ...
1178     RESULTS.K2.N1.(j).Schw1997.MC_AA_CV_SE(3), ...
1179     RESULTS.K3.N1.(j).Schw1997.MC_AA_CV_SE(3), ...
1180     RESULTS.K4.N1.(j).Schw1997.MC_AA_CV_SE(3), ...
1181     RESULTS.K5.N1.(j).Schw1997.MC_AA_CV_SE(3)]), 'o:');
1182 legend('MC', 'MC CV', 'Location', 'northeast');
1183 hold off
1184
1185 % Schwartz and Smith (2000)
1186 spl2_3 = subplot(2,2,3);
1187 splot2_3a = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1188     log([RESULTS.K1.N1.(j).SchwSm2000.MC_AA_SE(3), ...
1189     RESULTS.K2.N1.(j).SchwSm2000.MC_AA_SE(3), ...
1190     RESULTS.K3.N1.(j).SchwSm2000.MC_AA_SE(3), ...
1191     RESULTS.K4.N1.(j).SchwSm2000.MC_AA_SE(3), ...
1192     RESULTS.K5.N1.(j).SchwSm2000.MC_AA_SE(3)]), 'o:');
1193 xlim([4,12]);
1194 if i <= 2
1195     ylim([-10,0]);
1196 elseif i > 2
1197     ylim([-2,8]);
1198 end
1199 set(gca, 'XTick', (2:2:12));
1200 title('Schwartz and Smith (2000)', 'FontSize', 11, 'FontWeight', 'normal');
1201 xlabel('ln(k)', 'FontSize', 11);
1202 ylabel('ln(SE)', 'FontSize', 11);
1203 hold on
1204 splot2_3b = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1205     log([RESULTS.K1.N1.(j).SchwSm2000.MC_AA_CV_SE(3), ...
1206     RESULTS.K2.N1.(j).SchwSm2000.MC_AA_CV_SE(3), ...
1207     RESULTS.K3.N1.(j).SchwSm2000.MC_AA_CV_SE(3), ...
1208     RESULTS.K4.N1.(j).SchwSm2000.MC_AA_CV_SE(3), ...
1209     RESULTS.K5.N1.(j).SchwSm2000.MC_AA_CV_SE(3)]), 'o:');
1210 legend('MC', 'MC CV', 'Location', 'northeast');

```

```

1211     hold off
1212
1213     % Korn (2005)
1214     spl2_4 = subplot(2,2,4);
1215     splot2_4a = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1216         log([RESULTS.K1.N1.(j).Korn2005.MC_AA_SE(3), ...
1217             RESULTS.K2.N1.(j).Korn2005.MC_AA_SE(3), ...
1218             RESULTS.K3.N1.(j).Korn2005.MC_AA_SE(3), ...
1219             RESULTS.K4.N1.(j).Korn2005.MC_AA_SE(3), ...
1220             RESULTS.K5.N1.(j).Korn2005.MC_AA_SE(3)]), 'o:');
1221     xlim([4,12]);
1222     if i <= 2
1223         ylim([-10,0]);
1224     elseif i > 2
1225         ylim([-2,8]);
1226     end
1227     set(gca, 'XTick', (2:2:12));
1228     title('Korn (2005)', 'FontSize', 11, 'FontWeight', 'normal');
1229     xlabel('ln(k)', 'FontSize', 11);
1230     ylabel('ln(SE)', 'FontSize', 11);
1231
1232     hold on
1233     splot2_4b = plot(log([k(1),k(2),k(3),k(4),k(5)]), ...
1234         log([RESULTS.K1.N1.(j).Korn2005.MC_AA_CV_SE(3), ...
1235             RESULTS.K2.N1.(j).Korn2005.MC_AA_CV_SE(3), ...
1236             RESULTS.K3.N1.(j).Korn2005.MC_AA_CV_SE(3), ...
1237             RESULTS.K4.N1.(j).Korn2005.MC_AA_CV_SE(3), ...
1238             RESULTS.K5.N1.(j).Korn2005.MC_AA_CV_SE(3)]), 'o:');
1239     legend('MC', 'MC CV', 'Location', 'northeast');
1240     hold off
1241
1242     % clear helper variables
1243     clearvars spl2_1 splot2_1a splot2_1b splot2_11a splot2_11b spl2_2 splot2_2a ...
1244         splot2_2b splot2_12a splot2_12b spl2_3 splot2_3a splot2_3b splot2_13a ...
1245         splot2_13b spl2_4 splot2_4a splot2_4b splot2_14a splot2_14b
1246
1247     end
1248     clearvars i j
1249
1250     % clear path variables
1251     clearvars path exportfile_path
1252
1253     %% End stopwatch
1254     tend = toc(tstart);
1255     tendrem = tend - floor(tend/3600)*3600;
1256     fprintf('Elapsed time is %d hours, %d minutes, and %f ...
1257         seconds.\n', floor(tend/3600), floor(tendrem/60), rem(tendrem, 60));
1258     clearvars tstart tend tendrem

```

## C.2 MC functions

### C.2.1 Black (1976) one-factor model

```
1 function [MC_GA,MC_GA_SE,CFS,MC_AA,MC_AA_SE,MC_AA_CV,MC_AA_CV_SE,VR] = ...
2     Black1976_MC(xi_0,a_star,sigma_xi,dt,Z,K,r,t)
3
4 % Description of function Black1976_MC
5 %   The function Black1976_MC runs a MC simulation for the Black (1976)
6 %   one-factor model and determines MC prices for a geometric Asian option,
7 %   an arithmetic Asian option, the closed-form solution for a geometric
8 %   Asian option as well as the MC control variate price of an arithmetic
9 %   Asian option. Furthermore, MC standard deviations are determined.
10
11 fprintf('\t\tBlack (1976)...')
12
13 % determine size of input matrix Z
14 [n,k] = size(Z); %#ok<ASGLU>
15
16 % determine size of input vector K
17 l = length(K);
18
19 % simulate price paths
20 [time,S] = Black1976_simulate(xi_0,a_star,sigma_xi,dt,Z); %#ok<ASGLU>
21
22 % initialize variables
23 MC_geo_payoff = zeros(l,k);
24 MC_GA = zeros(l,1);
25 MC_GA_SE = zeros(l,1);
26 CFS = zeros(l,1);
27 MC_arm_payoff = zeros (l,k);
28 MC_AA = zeros(l,1);
29 MC_AA_SE = zeros(l,1);
30 X = zeros(l,k);
31 Y = zeros(l,k);
32 MC_AA_CV = zeros(l,1);
33 MC_AA_CV_SE = zeros(l,1);
34 VR = zeros(l,1);
35
36 %% Geometric Asian option
37 % geometric Asian option: determine geometric mean at time t
38 G_T = geomean(S,1);
39
40 % loop over strike prices
41 for i = 1:l
42
43     % geometric Asian option: determine payoff at time t
44     MC_geo_payoff(i,:) = max(G_T - K(i),0);
45
46     % geometric Asian option: determine MC price
47     MC_GA(i) = exp(-r * t) * mean(MC_geo_payoff(i,:));
48
```

```

49     % geometric Asian option: determine MC standard error
50     MC_GA_SE(i) = exp(-r * t) * std(MC_geo_payoff(i,:)) / sqrt(k);
51
52     % geometric Asian option: determine closed-form solution
53     CFS(i) = Black1976_GAoption(K(i),xi_0,a_star,sigma_xi,r,t);
54 end
55 clearvars i
56
57 %% Arithmetic Asian option
58 % arithmetic Asian option: determine arithmetic mean at time t
59 A_T = mean(S,1);
60
61 % loop over strike prices
62 for i = 1:l
63
64     % arithmetic Asian option: determine payoff at time t
65     MC_arm_payoff(i,:) = max(A_T - K(i),0);
66
67     % arithmetic Asian option: determine MC price
68     MC_AA(i) = exp(-r * t) * mean(MC_arm_payoff(i,:));
69
70     % arithmetic Asian option: determine MC standard error
71     MC_AA_SE(i) = exp(-r * t) * std(MC_arm_payoff(i,:)) / sqrt(k);
72 end
73 clearvars i
74
75 %% Arithmetic Asian option: control variate
76 % loop over strike prices
77 for i = 1:l
78
79     % arithmetic Asian option (CV): determine individual prices for each path
80     X(i,:) = exp(-r * t) .* max(G_T(1,:) - K(i),0);
81     Y(i,:) = exp(-r * t) .* max(A_T(1,:) - K(i),0);
82
83     % arithmetic Asian option (CV): determine covariance between X and Y
84     covCV = cov(Y,X);
85     betaCV = covCV(1,2) ./ covCV(2,2);
86
87     % arithmetic Asian option (CV): determine MC price
88     MC_AA_CV(i) = mean(Y(i,:)) - betaCV .* (mean(X(i,:)) - CFS(i));
89
90     % arithmetic Asian option (CV): determine MC standard error
91     MC_AA_CV_SE(i) = std(Y(i,:) - betaCV * X(i,:)) ./ sqrt(k);
92
93     % arithmetic Asian option (CV): determine variance reduction
94     %   VR(i) = (covCV(1,2) ./ (sqrt(covCV(1,1)) .* sqrt(covCV(2,2)))).^2 * 100;
95     VR(i) = (MC_AA_SE(i)^2 - MC_AA_CV_SE(i)^2) / MC_AA_SE(i)^2 * 100;
96 end
97
98 fprintf('done.\n')
99
100 end

```

### C.2.2 Schwartz (1997) one-factor model

```
1 function [MC_GA,MC_GA_SE,CFS,MC_AA,MC_AA_SE,MC_AA_CV,MC_AA_CV_SE,VR] = ...
2     Schw1997_MC(xi_0,kappa_xi,a_star,sigma_xi,dt,Z,K,r,t)
3
4 % Description of function Schw1997_MC
5 %   The function Schw1997_MC runs a MC simulation for the Schwartz (1997)
6 %   one-factor model and determines MC prices for a geometric Asian option,
7 %   an arithmetic Asian option, the closed-form solution for a geometric
8 %   Asian option as well as the MC control variate price of an arithmetic
9 %   Asian option. Furthermore, MC standard deviations are determined.
10
11 fprintf('\t\tSchwartz (1997)...')
12
13 % determine size of input matrix Z
14 [n,k] = size(Z); %#ok<ASGLU>
15
16 % determine size of input vector K
17 l = length(K);
18
19 % simulate price paths
20 [time,S] = Schw1997_simulate(xi_0,kappa_xi,a_star,sigma_xi,dt,Z); %#ok<ASGLU>
21
22 % initialize variables
23 MC_geo_payoff = zeros(l,k);
24 MC_GA = zeros(l,1);
25 MC_GA_SE = zeros(l,1);
26 CFS = zeros(l,1);
27 MC_arm_payoff = zeros (l,k);
28 MC_AA = zeros(l,1);
29 MC_AA_SE = zeros(l,1);
30 X = zeros(l,k);
31 Y = zeros(l,k);
32 MC_AA_CV = zeros(l,1);
33 MC_AA_CV_SE = zeros(l,1);
34 VR = zeros(l,1);
35
36 %% Geometric Asian option
37 % geometric Asian option: determine geometric mean at time t
38 G_T = geomean(S,1);
39
40 % loop over strike prices
41 for i = 1:l
42
43     % geometric Asian option: determine payoff at time t
44     MC_geo_payoff(i,:) = max(G_T - K(i),0);
45
46     % geometric Asian option: determine MC price
47     MC_GA(i) = exp(-r * t) * mean(MC_geo_payoff(i,:));
48
49     % geometric Asian option: determine MC standard error
50     MC_GA_SE(i) = exp(-r * t) * std(MC_geo_payoff(i,:)) / sqrt(k);
51
```

```

52     % geometric Asian option: determine closed-form solution
53     CFS(i) = Schw1997_GAoption(K(i),xi_0,kappa_xi,a_star,sigma_xi,r,t);
54 end
55 clearvars i
56
57 %% Arithmetic Asian option
58 % arithmetic Asian option: determine arithmetic mean at time t
59 A_T = mean(S,1);
60
61 % loop over strike prices
62 for i = 1:l
63
64     % arithmetic Asian option: determine payoff at time t
65     MC_arm_payoff(i,:) = max(A_T - K(i),0);
66
67     % arithmetic Asian option: determine MC price
68     MC_AA(i) = exp(-r * t) * mean(MC_arm_payoff(i,:));
69
70     % arithmetic Asian option: determine MC standard error
71     MC_AA_SE(i) = exp(-r * t) * std(MC_arm_payoff(i,:)) / sqrt(k);
72 end
73 clearvars i
74
75 %% Arithmetic Asian option: control variate
76 % loop over strike prices
77 for i = 1:l
78
79     % arithmetic Asian option (CV): determine individual prices for each path
80     X(i,:) = exp(-r * t) .* max(G_T(1,:) - K(i),0);
81     Y(i,:) = exp(-r * t) .* max(A_T(1,:) - K(i),0);
82
83     % arithmetic Asian option (CV): determine covariance between X and Y
84     covCV = cov(Y,X);
85     betaCV = covCV(1,2) ./ covCV(2,2);
86
87     % arithmetic Asian option (CV): determine MC price
88     MC_AA_CV(i) = mean(Y(i,:)) - betaCV .* (mean(X(i,:)) - CFS(i));
89
90     % arithmetic Asian option (CV): determine MC standard error
91     MC_AA_CV_SE(i) = std(Y(i,:)- betaCV * X(i,:)) ./ sqrt(k);
92
93     % arithmetic Asian option (CV): determine variance reduction
94     % VR(i) = (covCV(1,2) ./ (sqrt(covCV(1,1)) .* sqrt(covCV(2,2))))).^2 * 100;
95     VR(i) = (MC_AA_SE(i)^2 - MC_AA_CV_SE(i)^2) / MC_AA_SE(i)^2 * 100;
96 end
97
98 fprintf('done.\n')
99
100 end

```

### C.2.3 Schwartz and Smith (2000) two-factor model

```
1 function [MC_GA,MC_GA_SE,CFS,MC_AA,MC_AA_SE,MC_AA_CV,MC_AA_CV_SE,VR] = ...
2     SchwSm2000_MC(xi_0,a_star,sigma_xi,chi_0,kappa_chi,lambda_chi,sigma_chi, ...
3     rho,dt,Z1,Z2,K,r,t)
4
5 % Description of function SchwSm2000_MC
6 % The function SchwSm2000_MC runs a MC simulation for the Schwartz and
7 % Smith (2000) two-factor model and determines MC prices for a geometric
8 % Asian option, an arithmetic Asian option, the closed-form solution for
9 % a geometric Asian option as well as the MC control variate price of an
10 % arithmetic Asian option. Furthermore, MC standard deviations are
11 % determined.
12
13 fprintf('\t\tSchwartz and Smith (2000)...')
14
15 % determine size of input matrix Z1
16 [n,k] = size(Z1); %#ok<ASGLU>
17
18 % determine size of input vector K
19 l = length(K);
20
21 % simulate price paths
22 [time,S] = SchwSm2000_simulate(xi_0,a_star,sigma_xi,chi_0,kappa_chi, ...
23     lambda_chi,sigma_chi,dt,Z1,Z2); %#ok<ASGLU>
24
25 % initialize variables
26 MC_geo_payoff = zeros(l,k);
27 MC_GA = zeros(l,1);
28 MC_GA_SE = zeros(l,1);
29 CFS = zeros(l,1);
30 MC_arm_payoff = zeros(l,k);
31 MC_AA = zeros(l,1);
32 MC_AA_SE = zeros(l,1);
33 X = zeros(l,k);
34 Y = zeros(l,k);
35 MC_AA_CV = zeros(l,1);
36 MC_AA_CV_SE = zeros(l,1);
37 VR = zeros(l,1);
38
39 %% Geometric Asian option
40 % geometric Asian option: determine geometric mean at time t
41 G_T = geomean(S,1);
42
43 % loop over strike prices
44 for i = 1:l
45
46     % geometric Asian option: determine payoff at time t
47     MC_geo_payoff(i,:) = max(G_T - K(i),0);
48
49     % geometric Asian option: determine MC price
50     MC_GA(i) = exp(-r * t) * mean(MC_geo_payoff(i,:));
51
```

```

52     % geometric Asian option: determine MC standard error
53     MC_GA_SE(i) = exp(-r * t) * std(MC_geo_payoff(i,:)) / sqrt(k);
54
55     % geometric Asian option: determine closed-form solution
56     CFS(i) = SchwSm2000_GAoption(K(i),xi_0,a_star,sigma_xi,chi_0,kappa_chi, ...
57         lambda_chi,sigma_chi,rho,r,t);
58 end
59 clearvars i
60
61 %% Arithmetic Asian option
62 % arithmetic Asian option: determine arithmetic mean at time t
63 A_T = mean(S,1);
64
65 % loop over strike prices
66 for i = 1:l
67
68     % arithmetic Asian option: determine payoff at time t
69     MC_arm_payoff(i,:) = max(A_T - K(i),0);
70
71     % arithmetic Asian option: determine MC price
72     MC_AA(i) = exp(-r * t) * mean(MC_arm_payoff(i,:));
73
74     % arithmetic Asian option: determine MC standard error
75     MC_AA_SE(i) = exp(-r * t) * std(MC_arm_payoff(i,:)) / sqrt(k);
76 end
77 clearvars i
78
79 %% Arithmetic Asian option: control variate
80 % loop over strike prices
81 for i = 1:l
82
83     % arithmetic Asian option (CV): determine individual prices for each path
84     X(i,:) = exp(-r * t) .* max(G_T(1,:) - K(i),0);
85     Y(i,:) = exp(-r * t) .* max(A_T(1,:) - K(i),0);
86
87     % arithmetic Asian option (CV): determine covariance between X and Y
88     covCV = cov(Y,X);
89     betaCV = covCV(1,2) ./ covCV(2,2);
90
91     % arithmetic Asian option (CV): determine MC price
92     MC_AA_CV(i) = mean(Y(i,:)) - betaCV .* (mean(X(i,:)) - CFS(i));
93
94     % arithmetic Asian option (CV): determine MC standard error
95     MC_AA_CV_SE(i) = std(Y(i,:) - betaCV * X(i,:)) ./ sqrt(k);
96
97     % arithmetic Asian option (CV): determine variance reduction
98     %   VR(i) = (covCV(1,2) ./ (sqrt(covCV(1,1)) .* sqrt(covCV(2,2)))).^2 * 100;
99     VR(i) = (MC_AA_SE(i)^2 - MC_AA_CV_SE(i)^2) / MC_AA_SE(i)^2 * 100;
100 end
101
102 fprintf('done.\n')
103
104 end

```

### C.2.4 Korn (2005) two-factor model

```
1 function [MC_GA,MC_GA_SE,CFS,MC_AA,MC_AA_SE,MC_AA_CV,MC_AA_CV_SE,VR] = ...
2     Korn2005_MC(xi_0,kappa_xi,a_star,sigma_xi,chi_0,kappa_chi,lambda_chi, ...
3     sigma_chi,rho,dt,Z1,Z2,K,r,t)
4
5 % Description of function Korn2005_MC
6 %   The function Korn2005_MC runs a MC simulation for the Korn (2005)
7 %   two-factor model and determines MC prices for a geometric Asian option,
8 %   an arithmetic Asian option, the closed-form solution for a geometric
9 %   Asian option as well as the MC control variate price of an arithmetic
10 %   Asian option. Furthermore, MC standard deviations are determined.
11
12 fprintf('\t\tKorn (2005)...')
13
14 % determine size of input matrix Z1
15 [n,k] = size(Z1); %#ok<ASGLU>
16
17 % determine size of input vector K
18 l = length(K);
19
20 % simulate price paths
21 [time,S] = Korn2005_simulate(xi_0,kappa_xi,a_star,sigma_xi,chi_0,kappa_chi, ...
22     lambda_chi,sigma_chi,dt,Z1,Z2); %#ok<ASGLU>
23
24 % initialize variables
25 MC_geo_payoff = zeros(l,k);
26 MC_GA = zeros(l,1);
27 MC_GA_SE = zeros(l,1);
28 CFS = zeros(l,1);
29 MC_arm_payoff = zeros(l,k);
30 MC_AA = zeros(l,1);
31 MC_AA_SE = zeros(l,1);
32 X = zeros(l,k);
33 Y = zeros(l,k);
34 MC_AA_CV = zeros(l,1);
35 MC_AA_CV_SE = zeros(l,1);
36 VR = zeros(l,1);
37
38 %% Geometric Asian option
39 % geometric Asian option: determine geometric mean at time t
40 G_T = geomean(S,1);
41
42 % loop over strike prices
43 for i = 1:l
44
45     % geometric Asian option: determine payoff at time t
46     MC_geo_payoff(i,:) = max(G_T - K(i),0);
47
48     % geometric Asian option: determine MC price
49     MC_GA(i) = exp(-r * t) * mean(MC_geo_payoff(i,:));
50
51     % geometric Asian option: determine MC standard error
```

```

52     MC_GA_SE(i) = exp(-r * t) * std(MC_geo_payoff(i,:)) / sqrt(k);
53
54     % geometric Asian option: determine closed-form solution
55     CFS(i) = Korn2005_GAoption(K(i),xi_0,kappa_xi,a_star,sigma_xi,chi_0, ...
56         kappa_chi,lambda_chi,sigma_chi,rho,r,t);
57 end
58 clearvars i
59
60 %% Arithmetic Asian option
61 % arithmetic Asian option: determine arithmetic mean at time t
62 A_T = mean(S,1);
63
64 % loop over strike prices
65 for i = 1:l
66
67     % arithmetic Asian option: determine payoff at time t
68     MC_arm_payoff(i,:) = max(A_T - K(i),0);
69
70     % arithmetic Asian option: determine MC price
71     MC_AA(i) = exp(-r * t) * mean(MC_arm_payoff(i,:));
72
73     % arithmetic Asian option: determine MC standard error
74     MC_AA_SE(i) = exp(-r * t) * std(MC_arm_payoff(i,:)) / sqrt(k);
75 end
76 clearvars i
77
78 %% Arithmetic Asian option: control variate
79 % loop over strike prices
80 for i = 1:l
81
82     % arithmetic Asian option (CV): determine individual prices for each path
83     X(i,:) = exp(-r * t) .* max(G_T(1,:) - K(i),0);
84     Y(i,:) = exp(-r * t) .* max(A_T(1,:) - K(i),0);
85
86     % arithmetic Asian option (CV): determine covariance between X and Y
87     covCV = cov(Y,X);
88     betaCV = covCV(1,2) ./ covCV(2,2);
89
90     % arithmetic Asian option (CV): determine MC price
91     MC_AA_CV(i) = mean(Y(i,:)) - betaCV .* (mean(X(i,:)) - CFS(i));
92
93     % arithmetic Asian option (CV): determine MC standard error
94     MC_AA_CV_SE(i) = std(Y(i,:) - betaCV * X(i,:)) ./ sqrt(k);
95
96     % arithmetic Asian option (CV): determine variance reduction
97     %   VR(i) = (covCV(1,2) ./ (sqrt(covCV(1,1)) .* sqrt(covCV(2,2)))).^2 * 100;
98     VR(i) = (MC_AA_SE(i)^2 - MC_AA_CV_SE(i)^2) / MC_AA_SE(i)^2 * 100;
99 end
100
101 fprintf('done.\n')
102
103
104 end

```

## C.3 Simulation functions

### C.3.1 Black (1976) one-factor model

```
1 function [time,S] = Black1976_simulate(xi_0,a_star,sigma_xi,dt,Z)
2
3 % Description of the function Black1976_simulate
4 %   The function Black1976_simulate simulates the path of the Black (1976)
5 %   model. The model assumes that the log spot price follows an arithmetic
6 %   Brownian motion (ABM). This function provides an exact simulation of
7 %   the ABM path in vectorized version.
8
9 % Black (1976) one-factor model:  $\xi = \ln(S)$ 
10
11 % variable definitions
12 % n = number of time steps
13 % Z = matrix of given normally distributed random numbers
14
15 % determine size of input matrix Z
16 [n,k] = size(Z);
17
18 % initialize vectors
19 time = (0:dt:n*dt)';
20 S = zeros(n+1,k);
21 S_int = zeros(n,k);
22
23 % define first observation of S
24 S(1,:) = exp(xi_0);
25
26 % adjust Z-values by specified mean and variance
27 Z_xi = Z .* sigma_xi * sqrt(dt) + a_star * dt;
28
29 % simulate path
30 parfor i = 1:k
31     S_int(:,i) = S(1,i) .* exp(cumsum(Z_xi(:,i)));
32 end
33 clearvars i
34
35 % merge S_int and S
36 S(2:end,:) = S_int;
37
38 end
```

### C.3.2 Schwartz (1997) one-factor model

```

1 function [time,S] = Schw1997_simulate(xi_0,kappa_xi,a_star,sigma_xi,dt,Z)
2
3 % Description of the function Schw1997_simulate
4 % The function Schw1997_simulate simulates the path according to the
5 % Schwartz (1997) one-factor model. This is an exact simulation of the
6 % path in vectorized version.
7
8 % Schwartz (1997) one-factor model:  $\xi = \ln(S)$ 
9
10 % variable definitions
11 % n = number of time steps
12 % Z = matrix of given normally distributed random numbers
13
14 % determine size of input matrix Z
15 [n,k] = size(Z);
16
17 % initialize vectors
18 time = (0:dt:n*dt)';
19 xi = zeros(n+1,k);
20 Z1 = zeros(n,k);
21 xi_int = zeros(n,k);
22
23 % get part of time vector
24 time_int = time(2:end);
25
26 % define first observation of X
27 xi(1,:) = xi_0;
28
29 % adjust random standard normal variable with zero mean and time changing
30 % volatility (scaled, time-changed Wiener process)
31 parfor i = 1:k
32     Z1(:,i) = sqrt(diff(exp(2 .* kappa_xi .* time) - 1) ./ (2 .* kappa_xi)) .* Z(:,i);
33 end
34 clearvars i
35
36 % simulate path of the OU process
37 parfor i = 1:k
38     xi_int(:,i) = xi(1,i) .* exp(-kappa_xi .* time_int) + a_star .* (1 - exp(-kappa_xi ...
39         .* time_int)) + sigma_xi .* exp(-kappa_xi .* time_int) .* cumsum(Z1(:,i));
40 end
41 clearvars i
42
43 % merge xi_int and xi
44 xi(2:end,:) = xi_int;
45
46 % transform to S
47 S = exp(xi);
48 end

```

### C.3.3 Schwartz and Smith (2000) two-factor model

```
1 function [time,S] = SchwSm2000_simulate(xi_0,a_star,sigma_xi,...
2     chi_0,kappa_chi,lambda_chi,sigma_chi,dt,Z1,Z2)
3
4 % Description of the function SchwSm2000_simulate
5 % The function SchwSm2000_simulate simulates a path according to the
6 % Schwartz and Smith (2000) two-factor model. This is an exact simulation
7 % of the path in vectorized version.
8
9 % Schwartz and Smith (2000) two-factor model:  $X = \ln(S) = \xi + \chi$ 
10
11 % variable definitions
12 % n = number of time steps
13 % Z1 = matrix of given normally distributed random numbers
14 % Z2 = matrix of given normally distributed random numbers with correlation
15 % of rho_dt with Z1
16
17 % determine size of input matrix Z1
18 [n,k] = size(Z1);
19
20 % initialize vectors
21 time = (0:dt:n*dt)';
22 xi = zeros(n+1,k);
23 xi_int = zeros(n,k);
24 chi = zeros(n+1,k);
25 chi_int = zeros(n,k);
26 Z_chi = zeros(n,k);
27
28 % get part of time vector
29 time_int = time(2:end);
30
31 % simulate long-term equilibrium xi (follows an ABM)
32 % define first observation of xi
33 xi(1,:) = xi_0;
34
35 % adjust random standard normal numbers with specified mean and standard
36 % deviation
37 Z_xi = Z1 .* sigma_xi .* sqrt(dt) + a_star .* dt;
38
39 % simulate path of xi
40 parfor i = 1:k
41 %     xi(2:n+1,i) = xi(1,i) + cumsum(Z_xi(:,i));
42     xi_int(:,i) = xi(1,i) + cumsum(Z_xi(:,i));
43 end
44 clearvars i
45
46 % merge xi_int and xi
47 xi(2:end,:) = xi_int;
48
49 % simulate short-term variations chi (follows an OU process)
50 % define first observation of chi
51 chi(1,:) = chi_0;
```

```
52
53 % adjust random standard normal variable with zero mean and time changing
54 % volatility (scaled, time-changed Wiener process)
55 parfor i = 1:k
56     Z_chi(:,i) = sqrt(diff(exp(2 .* kappa_chi .* time) - 1) ./ (2 .* kappa_chi)) .* ...
        Z2(:,i);
57 end
58 clearvars i
59
60 % simulate path of the OU process
61 parfor i = 1:k
62 %     chi(2:n+1,i) = chi(1,i) .* exp(-kappa_chi .* time(2:n+1)) - lambda_chi / ...
        kappa_chi .* (1 - exp(-kappa_chi .* time(2:n+1))) + sigma_chi .* exp(-kappa_chi .* ...
        time(2:n+1)) .* cumsum(Z_chi(:,i));
63     chi_int(:,i) = chi(1,i) .* exp(-kappa_chi .* time_int) - lambda_chi / kappa_chi .* ...
        (1 - exp(-kappa_chi .* time_int)) + sigma_chi .* exp(-kappa_chi .* time_int) ...
        .* cumsum(Z_chi(:,i));
64 end
65 clearvars i
66
67 % merge chi_int and chi
68 chi(2:end,:) = chi_int;
69
70 % get simulated path for X
71 X = xi + chi;
72
73 % transform to S
74 S = exp(X);
75
76 end
```

### C.3.4 Korn (2005) two-factor model

```
1 function [time,S] = Korn2005_simulate(xi_0,kappa_xi,a_star,sigma_xi,...
2     chi_0,kappa_chi,lambda_chi,sigma_chi,dt,Z1,Z2)
3
4 % Description of the function Korn2005_simulate
5 % The function Korn2005_simulate simulates a path according to the
6 % Korn (2005) two-factor model. This is an exact simulation of the
7 % path in vectorized version.
8
9 % Korn (2005) two-factor model:  $X = \ln(S) = \xi + \chi$ 
10
11 % variable definitions
12 % n = number of time steps
13 % Z1 = matrix of given normally distributed random numbers
14 % Z2 = matrix of given normally distributed random numbers with correlation
15 % of rho_dt with Z1
16
17 % determine size of input matrix Z1
18 [n,k] = size(Z1);
19
20 % initialize vectors
21 time = (0:dt:n*dt)';
22 xi = zeros(n+1,k);
23 xi_int = zeros(n,k);
24 Z_xi = zeros(n,k);
25 chi = zeros(n+1,k);
26 chi_int = zeros(n,k);
27 Z_chi = zeros(n,k);
28
29 % get part of time vector
30 time_int = time(2:end);
31
32 % simulate long-term equilibrium xi (follows an OU process)
33 % define first observation of xi
34 xi(1,:) = xi_0;
35
36 % adjust random standard normal variable with zero mean and time changing
37 % volatility (scaled, time-changed Wiener process)
38 parfor i = 1:k
39     Z_xi(:,i) = sqrt(diff(exp(2 .* kappa_xi .* time) - 1) ./ (2 .* kappa_xi)) .* Z1(:,i);
40 end
41 clearvars i
42
43 % simulate path of xi
44 parfor i = 1:k
45 %     xi(2:n+1,i) = xi(1,i) .* exp(-kappa_xi .* time(2:n+1)) + a_star .* (1 - ...
46 %     exp(-kappa_xi .* time(2:n+1))) + sigma_xi .* exp(-kappa_xi .* time(2:n+1)) .* ...
47 %     cumsum(Z_xi(:,i));
48     xi_int(:,i) = xi(1,i) .* exp(-kappa_xi .* time_int) + a_star .* (1 - exp(-kappa_xi .*
49 %     time_int)) + sigma_xi .* exp(-kappa_xi .* time_int) .* cumsum(Z_xi(:,i));
50 end
51 clearvars i
```

```
49
50 % merge xi_int and xi
51 xi(2:end,:) = xi_int;
52
53 % simulate short-term variations chi (follows an OU process)
54 % define first observation of chi
55 chi(1,:) = chi_0;
56
57 % adjust random standard normal variable with zero mean and time changing
58 % volatility (scaled, time-changed Wiener process)
59 parfor i = 1:k
60     Z_chi(:,i) = sqrt(diff(exp(2 .* kappa_chi .* time) - 1) ./ (2 .* kappa_chi)) .* ...
        Z2(:,i);
61 end
62 clearvars i
63
64 % simulate path of the OU process
65 for i = 1:k
66     % chi(2:n+1,i) = chi(1,i) .* exp(-kappa_chi .* time(2:n+1)) - lambda_chi / ...
        kappa_chi .* (1 - exp(-kappa_chi .* time(2:n+1))) + sigma_chi .* exp(-kappa_chi .* ...
        time(2:n+1)) .* cumsum(Z_chi(:,i));
67     chi_int(:,i) = chi(1,i) .* exp(-kappa_chi .* time_int) - lambda_chi / kappa_chi .* ...
        (1 - exp(-kappa_chi .* time_int)) + sigma_chi .* exp(-kappa_chi .* time_int) ...
        .* cumsum(Z_chi(:,i));
68 end
69 clearvars i
70
71 % merge chi_int and chi
72 chi(2:end,:) = chi_int;
73
74 % get simulated path for X
75 X = xi + chi;
76
77 % transform to S
78 S = exp(X);
79
80 end
```

## C.4 Closed-form solution functions

### C.4.1 Black (1976) one-factor model

```
1 function [call_price] = Black1976_GAoption(K,xi_0,a_star,sigma_xi,r,t)
2
3 % Description of the function Black1976_GAoption
4 %   The function Black1976_GAoption determines the price of a geometric
5 %   average call option for the Black (1976) one-factor model.
6
7 % determine the mean of G(T;0,T)
8 mu_geo = xi_0 + 1/2 * a_star * t;
9
10 % determine standard deviation of G(T;0,T)
11 sigma_geo = sqrt(1/3 * sigma_xi^2 * t);
12
13 % determine d1
14 d1 = (mu_geo - log(K) + sigma_geo^2) / sigma_geo;
15
16 % determine d2
17 d2 = d1 - sigma_geo;
18
19 % determine call price
20 call_price = exp(mu_geo + 1/2 * sigma_geo^2 - r * t) * normcdf(d1) - exp(-r * t) * K * ...
    normcdf(d2);
21
22 end
```

### C.4.2 Schwartz (1997) one-factor model

```
1 function [call_price] = Schw1997_GAoption(K,xi_0,kappa_xi,a_star,sigma_xi,r,T)
2
3 % Description of the function Schw1997_GAoption
4 % The function Schw1997_GAoption determines the price of a
5 % geometric average call option for the Schwartz (1997) one-factor model.
6
7 % determine the mean of G(T;0,T)
8 mu_geo = xi_0 / (kappa_xi * T) * (1 - exp(-kappa_xi * T)) + a_star - a_star / ...
          (kappa_xi * T) * (1 - exp(-kappa_xi * T));
9
10 % determine standard deviation of G(T;0,T)
11 sigma_geo = sqrt(sigma_xi^2 / (2 * kappa_xi^3 * T^2) * (2 * kappa_xi * T + 4 * ...
                  exp(-kappa_xi * T) - exp(-2 * kappa_xi * T) - 3));
12
13 % determine d1
14 d1 = (mu_geo - log(K) + sigma_geo^2) / sigma_geo;
15
16 % determine d2
17 d2 = d1 - sigma_geo;
18
19 % determine call price
20 call_price = exp(mu_geo + 1/2 * sigma_geo^2 - r * T) * normcdf(d1) - exp(-r * T) * K * ...
              normcdf(d2);
21
22 end
```

### C.4.3 Schwartz and Smith (2000) two-factor model

```
1 function [call_price] = SchwSm2000_GAoption(K,xi_0,a_star,sigma_xi, ...
2     chi_0,kappa_chi,lambda_chi,sigma_chi,rho,r,T)
3
4 % Description of the function SchwSm2000_GAoption
5 % The function SchwSm2000_GAoption determines the price of a
6 % geometric average call option for the Schwartz and Smith (2000) two-
7 % factor model.
8
9 % determine the mean of G(T;0,T)
10 mu_geo = xi_0 + chi_0 / (kappa_chi * T) * (1 - exp(-kappa_chi * T)) + 1/2 * a_star * T ...
    + lambda_chi / (kappa_chi^2 * T) * (1 - kappa_chi * T - exp(-kappa_chi * T));
11
12 % determine standard deviation of G(T;0,T)
13 sigma_geo = sqrt(1/3 * sigma_xi^2 * T + (rho * sigma_xi * sigma_chi) / (kappa_chi^3 * ...
    T^2) * (kappa_chi^2 * T^2 + 2 * kappa_chi * T * exp(-kappa_chi * T) + 2 * ...
    exp(-kappa_chi * T) - 2) ...
14 + sigma_chi^2 / (2 * kappa_chi^3 * T^2) * (2 * kappa_chi * T + 4 * exp(-kappa_chi ...
    * T) - exp(-2 * kappa_chi * T) - 3));
15
16 % determine d1
17 d1 = (mu_geo - log(K) + sigma_geo^2) / sigma_geo;
18
19 % determine d2
20 d2 = d1 - sigma_geo;
21
22 % determine call price
23 call_price = exp(mu_geo + 1/2 * sigma_geo^2 - r * T) * normcdf(d1) - exp(-r * T) * K * ...
    normcdf(d2);
24
25 end
```

## C.4.4 Korn (2005) two-factor model

```

1 function [call_price] = Korn2005_GAoption(K,xi_0,kappa_xi,a_star,sigma_xi, ...
2     chi_0,kappa_chi,lambda_chi,sigma_chi,rho,r,t)
3
4 % Description of the function Korn2005_GAoption
5 % The function Korn2005_GAoption determines the price of a
6 % geometric average call option for the Korn (2005) two-factor model.
7
8 % determine the mean of G(T;0,T)
9 mu_geo = xi_0 / (kappa_xi * t) * (1 - exp(-kappa_xi * t)) + chi_0 / (kappa_chi * t) * ...
    (1 - exp(-kappa_chi * t)) + a_star / (kappa_xi * t) * (exp(-kappa_xi * t) + ...
    kappa_xi * t - 1) ...
10 + lambda_chi / (kappa_chi^2 * t) * (1 - kappa_chi * t - exp(-kappa_chi * t));
11
12 % determine standard deviation of G(T;0,T)
13 sigma_geo = sqrt(sigma_xi^2 / (2 * kappa_xi^3 * t^2) * (2 * kappa_xi * t + 4 * ...
    exp(-kappa_xi * t) - exp(-2 * kappa_xi * t) - 3) ...
14 + (2 * rho * sigma_xi * sigma_chi) / (kappa_xi^2 * kappa_chi^2 * (kappa_xi + ...
    kappa_chi) * t^2) * (kappa_xi * (kappa_xi + kappa_chi) * exp(-kappa_chi * t) ...
15 + kappa_xi * kappa_chi * t * (kappa_xi + kappa_chi) + kappa_chi * (kappa_xi + ...
    kappa_chi) * exp(-kappa_xi * t) ...
16 - kappa_xi * kappa_chi * exp(-(kappa_xi + kappa_chi) * t) - (kappa_xi + ...
    kappa_chi)^2 + kappa_xi * kappa_chi) ...
17 + sigma_chi^2 / (2 * kappa_chi^3 * t^2) * (2 * kappa_chi * t + 4 * exp(-kappa_chi ...
    * t) - exp(-2 * kappa_chi * t) - 3));
18
19 % determine d1
20 d1 = (mu_geo - log(K) + sigma_geo^2) / sigma_geo;
21
22 % determine d2
23 d2 = d1 - sigma_geo;
24
25 % determine call price
26 call_price = exp(mu_geo + 1/2 * sigma_geo^2 - r * t) * normcdf(d1) - exp(-r * t) * K * ...
    normcdf(d2);
27
28 end

```

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## APPENDIX A

# SUMMARIES

*according to article 6(5) of the PromO*

### A.1 Abstract

This cumulative dissertation consists of three individual essays that are generally concerned with the following two, broader topics: financial risk management and derivative pricing.

Concerning financial risk management, the first two essays study the cross-hedging of dry bulk Capesize ship price risks using freight derivatives. The first essay focuses on Forward Freight Agreements (FFAs) as hedge instruments and empirically compares the hedge effectiveness of a structural pricing model (SPM)-based hedging approach and a classical minimum-variance cross-hedging approach to derive the desired hedge exposure. The results show that the SPM-based hedging approach consistently outperforms the classical minimum-variance cross-hedging approach in terms of variance reduction. The second essay focuses on freight options as hedge instruments and empirically assesses the hedge effectiveness of different freight option-based cross-hedging strategies using several risk-, downside-risk-, as well as return-based measures. The results suggest that freight options generally qualify quite well as cross-hedge instrument for dry bulk Capesize ship price risks and that one-sided, option-based hedging strategies prove to be beneficial compared to the classical two-sided hedging strategies in case the market development does not require any downside-risk protection.

Concerning derivative pricing, the third essay focuses on pricing of Asian options for affine Gaussian diffusions. A general pricing framework to derive closed-form solutions for continuously monitored geometric Asian options for affine  $n$ -factor Gaussian

diffusions is developed and specifically applied to three mean-reversion commodity pricing models. In a numerical example, the derived closed-form solutions turn out to be accurate. Additionally, the geometric Asian option is used as control variate in a Monte Carlo (MC) simulation in order to price an arithmetic Asian option. This yields considerable variance reduction which can be translated into substantial computation-time savings. Finally, an extension to forward-start Asian options is outlined as this type of option are quite common in commodity markets.

## A.2 Zusammenfassung

Diese kumulative Dissertation besteht aus drei Aufsätzen, die sich generell mit den folgenden zwei Themen befassen: finanzielles Risikomanagement sowie Derivatebewertung.

Die ersten beiden Aufsätze befassen sich im Bereich des finanziellen Risikomanagements mit der Absicherung von bilanziellen Preisrisiken von Capesize Schüttgutfrachtern mit Hilfe von Frachtderivaten. Der erste Aufsatz konzentriert sich dabei auf Forward Freight Agreements (FFAs) als Hedgeinstrumente und vergleicht empirisch die Hedgeeffizienz zweier Ansätze zur Bestimmung des gewünschten Absicherungsgrades. Der erste Ansatz basiert auf einem strukturellen Preismodell, das aus historischen Schiffstransaktionen geschätzt wird. Der zweite Ansatz basiert auf der klassischen Minimum-Varianz-Optimierung. Die Ergebnisse der Studie zeigen, dass der Ansatz basierend auf dem strukturellen Preismodell kontinuierlich besser hinsichtlich der erzielten Varianzreduktion abschneidet als der klassische Minimum-Varianz-Ansatz. Der zweite Aufsatz konzentriert sich auf Frachtoptionen als Hedgeinstrumente und untersucht empirisch die Hedgeeffektivität verschiedener Hedgestrategien, die auf Frachtoptionen basieren. Die Hedgeeffektivität wird dabei nicht nur in Form von Varianzreduktion gemessen sondern auch mit Hilfe von verschiedenen Downside-Risikomaßen sowie renditebasierten Maßen. Die Ergebnisse der Studie zeigen, dass sich Frachtoptionen generell als Hedgeinstrument zur Absicherung von bilanziellen Schiffspreisrisiken für Capesize Schüttgutfrachter eignen. Insbesondere sind optionsbasierte, einseitige Hedgestrategien gegenüber klassischen, zweiseitigen Hedgestrategien im Vorteil, wenn die Marktentwicklung eigentlich keine Downside-Absicherung erfordert.

Hinsichtlich der Derivatebewertung befasst sich der dritte Aufsatz mit der Bewertung

asiatischer Optionen für affine Gauß-Diffusionen. Es wird ein generellerer Bewertungsansatz entwickelt, der es erlaubt geschlossene Lösungsformeln für stetig geometrisch-gemittelte asiatische Optionen für affine  $n$ -Faktor Gauß-Diffusionen herzuleiten. Der generelle Bewertungsansatz wird dann konkret auf drei ‘mean-reversion’-Modelle angewendet. In einem numerischen Beispiel werden die hergeleiteten, geschlossenen Lösungsformeln via Monte Carlo (MC)-Simulation für akkurat befunden. Darüber hinaus wird die geometrisch asiatische Option als ‘control variate’ in einer MC-Simulation angewendet, um eine arithmetisch asiatische Option zu bewerten. Dies führt zu einer erheblichen Varianzreduktion, die in substanzielle Rechenzeiterparnisse übersetzt werden kann. Zum Abschluss wird noch eine Erweiterung auf asiatische ‘forward-start’ Optionen skizziert, da diese in Rohstoff- und Verbrauchsgütermärkten recht verbreitet sind.



## APPENDIX B

# LIST OF PUBLICATIONS

*according to article 6(5) of the PromO*

As of the submission date, no publications have emerged from this dissertation yet. However, it is planned to submit shortened versions of selected essays to academic journals.