Higher-Derivative Supergravity and String Cosmology

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Abstract

In this thesis we study theories with $\mathcal{N} = 1, D = 4$ supersymmetry including higher-derivative operators for chiral multiplets. In particular, we initiate a systematic analysis of these higher-derivative operators and corrections to the scalar potential required by supersymmetry. For theories formulated in flat superspace the full scalar potential can be derived from a generalized Kähler potential where the additional corrections arise from higher powers of the chiral auxiliary field. Consequently, the equations of motion for the auxiliary field admit several solutions. However, we demonstrate that in the context of effective field theories there exists only a single solution which is compatible with the principles of effective field theory. For supergravity we develop new tools that allow us to compute the component actions, but also simplify the determination of the linearized on-shell theory. We then classify the leading order and next-to-leading order superspace derivative operators and determine the component forms of a subclass thereof. Equipped with the higher-derivative actions in flat and curved superspace we proceed to investigate the vacuum structure of the theory. In particular, we study the properties of supersymmetric Minkowksi and AdS_4 vacua and also comment on non-supersymmetric vacua. In the second part of this thesis we turn to Calabi-Yau orientifold compactifications of type IIB string theory with background fluxes. We derive four-derivative terms for the volume modulus from ten-dimensional $(\alpha')^3\mathcal{R}^4$-corrections and match these to a particular higher-derivative operator which we computed in the first part. Thereby, we can indirectly infer new F^4-type corrections to the scalar potential. Lastly, we study the relevance of this new correction for moduli stabilization and inflation. In particular, we demonstrate the possibility of stabilizing all Kähler moduli model-independently just by $(\alpha')^3$-corrections. Finally, we comment on realizations of plateau-type inflationary models via the new F^4-correction in the context of the Large Volume Scenario with K3-fibered compactification geometries.

Zusammenfassung

In dieser Dissertation werden Theorien mit $\mathcal{N} = 1$ Supersymmetrie in vier Raumzeitdimensionen untersucht unter Berücksichtigung höherer Ableitungsoperatoren für chirale Multiplets. Insbesondere initiieren wir eine systematische Analyse von diesen höheren Ableitungsoperatoren und Korrekturen zum skalaren Potential, welche durch Supersymmetrie verlangt werden. Für Theorien, welche im flachen Superraum formuliert sind, zeigen wir, dass das vollständige skalare Potential aus einem verallgemeinerten Kählerpotential abgeleitet werden kann und die zusätzlichen Korrekturen zum skalaren Potential entstehen durch höhere Potenzen des chiralen Hilfsfeldes. Daher erlauben die Bewegungsgleichungen dieses Hilfsfeldes in solchen Theorien eine Vielzahl an Lösungen. Wir zeigen, dass im Kontext von effektiven Feldtheorien nur eine eindeutige Lösung existiert, welche die Prinzipien bzw. Axiome von effektiven Feldtheorien erfüllt. Im Kontext von Supergravitation entwickeln wir verschiedene neue Methoden und Werkzeuge, um die entsprechenden Komponentenwirkungen zu berechnen, aber auch um die Berechnung der linearisierten, auf der Massenschale gelegenen Theorie zu erleichtern. Weiterhin klassifizieren wir die Superraum-Ableitungsoperatoren führender und subdominanter Ordnung und bestimmen die Komponentenwirkungen einer Unterklasse davon. Basierend auf diesen Resultaten untersuchen wir die Vakuumstruktur der Theorien mit höheren Ableitungsoperatoren. Insbesondere bestimmen wir Eigenschaften der supersymmetrischen Minkowski und Anti-de Sitter Vakua sowie der nicht-supersymmetrischen Vakua. Im zweiten Teil dieser Dissertation widmen wir uns dem Studium von Calabi-Yau Orientifold Kompaktifizierungen mit Flüssen von Typ IIB Stringtheorie. Wir leiten Vierableitungsoperatoren für den Volumenmodulus aus zehndimensionalen $(\alpha')^3\mathcal{R}^4$-Korrekturen her und gleichen diese ab mit einem der supersymmetrischen Ableitungsoperatoren aus dem ersten Teil dieser Dissertation. In dieser Weise können wir indirekt neue F^4-artige Korrekturen zum skalaren Potential bestimmen. Weiterhin untersuchen wir die Relevanz dieser neuen Korrekturen für Modulistabilisierung und kosmologische Inflation. Insbesondere beweisen wir die Existenz eines Vakuums, in welchem alle Kähler-Moduli modellunabhängig stabilisiert sind und zwar nur durch die $(\alpha')^3$-Korrekturen in führender Ordnung. Zuletzt untersuchen wir Realisierungen von Plateau-artigen Modellen für Inflation mittels der neuen F^4-Korrektur im Kontext des sogenannten Large Volume Scenario bezüglich Kompaktifizierungsgeometrien, welche durch Faserbündel über der K3-Mannigfaltigkeit gegeben sind.

This thesis is based on the following publications:

- D. Ciupke “Scalar Potential from Higher Derivative $\mathcal{N} = 1$ Superspace”, [arXiv: 1605.00651]
- B. Broy, D. Ciupke, F. Pedro and A. Westphal “Starobinsky-Type Inflation from α'-Corrections“, JCAP **1601** (2016) 001
- D. Ciupke and L. Zarate ”Classification of Shift-Symmetric No-Scale Supergravities“, JHEP **11** (2015) 179
- D. Ciupke, J. Louis and A. Westphal “Higher-Derivative Supergravity and Moduli Stabilization”, JHEP **10** (2015) 094

Contents

Chapter 1

Introduction

Higher-derivative operators have a long history in theoretical physics dating back to early works by Ostrogradski [1]. While in several cases they exhibit ghost-like behavior which, in turn, has given them the reputation of being ill-defined, in some situations they lead to interesting dynamics, in particular, in the context of gravity. More specifically, they have attracted the attention of cosmologists who have applied them to models of early-universe inflation [2, 3] and dynamical dark energy [4]. Generally we have to sharply distinguish between ultraviolet-complete (UV) theories with higher-derivatives and effective field theories (EFT) with higher-derivatives. In the former case, the higher-derivative operators must not lead to ghost-like instabilities and this requirement induces severe constraints on the possible operators which might be present.[1] In effective field theories all operators consistent with the symmetries must be included in the theory and, hence, higher-derivative operators are generically present, even if they (naively) lead to ghosts in the spectrum. In situations where we explicitly compute EFTs from UV physics higher-derivative operators arise e.g. from integrating out heavy fields. As long as the higher-derivatives are not treated as dynamical degrees of freedom but instead as small perturbations of the leading order Lagrangian, then the principles of effective field theory are respected and the higher-derivatives do not lead to ghost-like instabilities [6, 7]. Instead the apparent ghost-like nature of higher-derivative terms is attributed to the truncation of a non-local infinite-derivative UV theory to a local effective theory.

One may now proceed to study those effective theories where higher-derivative operators have a significant impact on the properties and features of the theory. The special situation that we are going to be interested in here are theories with supersymmetry. Supersymmetric theories are not only of interest for phenomenology, but also because supersymmetry places constraints on the form of the Lagrangian and the respective quantum corrections and thereby leads to restricted dynamics. In particular, supersymmetry enforces so-called non-renormalization theorems [8]. Furthermore, supersymmetric theories arise from compactifications of string theory and are, therefore, a starting point for string phenomenology and string cosmology, and we will return to the context of string theory in a moment. The interface between

[1] For instance, an example is presented in [5].

higher-derivative operators and supersymmetric theories, more precisely global and local $\mathcal{N} = 1$ supersymmetry in $D = 4$ spacetime dimensions, is the topic of this thesis. Among the possible representations of the supersymmetry algebra here we focus on chiral multiplets which contain a complex scalar, a Weyl spinor and an auxiliary complex scalar. While the higher-derivative sector for the gravity multiplet has been studied in several works [9–12], only a few investigations about higher-derivative operators for the chiral multiplets exist [13–24]. An important observation, which was already made in [13, 14], is that supersymmetric higher-derivative operators for the chiral multiplets induce corrections to the scalar potential given by higher monomials of the chiral auxiliary field. Therefore, the complete scalar potential and, hence, the vacuum structure can only be fully understood in a higher-derivative theory. However, due to the lack of an action for the general higher-derivative theory the scalar potential and its supersymmetric and non-supersymmetric vacua are unknown. The additional corrections to the scalar potential are not only crucial for determining the vacuum structure. Moreover, they generically are as important as non-renormalizable operators in the Kähler potential or the Planck-suppressed corrections to the scalar potential that are manifestly present in supergravity. In sum this renders them relevant in the context of supersymmetry breaking, moduli stabilization and inflation.

Ultimately, we are interested in the on-shell (higher-derivative) theories obtained after solving the equations of motion for the auxiliary fields and reinserting the solution in the Lagrangian. However, due to the appearance of the higher-derivative corrections the equations of motion are now higher polynomial equations which admit several solutions and, in turn, induce several theories describing mutually inequivalent dynamics [21]. While in several works the beneficial aspects of the different theories emerging from a particular higher-derivative operator were studied, the fact that the appearance of multiple theories is in conflict with basic principles of classical physics has not been addressed so far. In particular, in having a family of theories to choose from the classical dynamics of the fields are not unique once initial conditions are specified.

Following the above outline, the central problems that we want to address in this thesis regarding $\mathcal{N} = 1$ higher-derivative theories read:

I. *Gain a systematic understanding of higher-derivative operators and their corrections to the scalar potential. Classify these operators and compute their component forms.*

II. *Understand and resolve the problem of the emergence of multiple on-shell theories.*

III. *Characterize the structure of the supersymmetric and the non-supersymmetric vacua and their corresponding moduli spaces in the general higher-derivative theory.*

Before we turn to the results that this thesis offers, let us proceed by motivating and introducing the second topic of this thesis. As we already mentioned $\mathcal{N} = 1$

supergravity prominently arises as the low energy EFT of certain string compactifications. For instance, Calabi-Yau orientifold compactifications of type IIB string theory belong to this class [25]. A generic property of effective supergravities for string compactifications is the existence of a plethora of massless fields. The stabilization of these massless fields is crucial and a necessary prerequisite to study inflation or particle phenomenology. At the level of the supergravity the appearance of massless fields is enforced by the no-scale condition which, in a particular version, is equivalent to the absence of a scalar potential. In turn, interactions which may make the fields massive are generated only after inclusion of perturbative α'- and g_s-corrections.[2] Unfortunately, these perturbative corrections are, at least for the orientifold compactifications, only poorly understood. For the particular case of Calabi-Yau orientifold compactifications of type IIB string theory after inclusion of background fluxes [26] the remaining massless modes include the Kähler moduli. Ten-dimensional eight-derivative $(\alpha')^3$-corrections which appeared for instance in [27] lead to a modification of the effective 4D supergravity after compactification and induce a non-trivial scalar potential for the Kähler moduli [28]. While this correction breaks the no-scale property it does not suffice for a stabilization. The prominent stabilization proposals instead rely on non-perturbative corrections [29, 30] which have the disadvantage of being highly dependent on the particular compactification geometry considered. To test the properties of string vacua more universally we are motivated to find a model-independent stabilization mechanism. Since perturbative α'- and g_s-corrections to the 4D effective supergravity such as the aforementioned corrections computed in [28] are, in principle, model-independent a more complete understanding of these terms may possibly lead to a model-independent stabilization mechanism. Due to the no-scale condition present at leading order, the higher-derivative operators are generically of interest here and their role in the effective 4D supergravity obtained from string compactifications has not been investigated yet. This leads us to the fourth question that we would like to answer in the context of this thesis:

IV. *Can we compute 4D higher-derivative operators from perturbative α'- or g_s-corrections to 10D type IIB supergravity? What effect do they have on moduli stabilization and inflation? To what extent do we need to answer I. and II. to successfully answer this question?*

Let us now proceed to outline the results of this thesis in the attempt to answer and address the main problems which we formulated above. We first turn to the general discussion of higher-derivative theories with $\mathcal{N}$ supersymmetry, i.e. problems I. to III., following the references [31, 32].

For the situation of global supersymmetry we demonstrate that the complete scalar potential can be derived from a superspace-Lagrangian given in terms of a (pseudo-) Kähler potential $\mathcal{K}$ together with the ordinary superpotential of the two-derivative theory and additional constraints. $\mathcal{K}$ depends on additional chiral multiplets which are determined by the additional constraints. In particular, their

[2]Further interaction terms might also be generated at the non-perturbative level.

scalar components are given by the auxiliary fields. Note that the higher-derivative part of $\mathcal{K}$ already appeared in [15, 16] under the name of effective auxiliary field potential. In the aforementioned theory the auxiliary fields can either remain algebraic or obtain kinetic terms. We discuss the equations of motion for the auxiliary fields in detail. On the one hand, we argue that even if they obtain kinetic terms, the auxiliary fields must still be treated as algebraic degrees of freedom, since they generically obtain masses at the cut-off scale Λ of the EFT. On the other hand, we clarify the appearance of multiple solutions to the respective equations of motion. We demonstrate in generality that among the multiple on-shell theories there exists a unique physical theory compatible with the principles of effective field theory. The remaining solutions violate the decoupling principle which states that irrelevant operators must decouple from the infrared (IR) dynamics, for a review on EFT see [33]. Therefore, we regard them as unphysical artifacts of a truncation of an infinite-derivative UV-theory to a local EFT with finitely many operators. Thus, the nature of the additional solutions is analogous to the appearance of ghost-like modes in local EFTs with higher-derivatives [6, 7]. We also validate our interpretation by explicitly studying the truncation for the example of the one-loop Wess-Zumino model computed in [18].

Next we turn to the general study of higher-derivative operators for chiral multiplets in supergravity. Here we adapt the conventions of old minimal supergravity following [34]. Our analysis is split into two parts: Firstly, we study conceptual and computational aspects of the higher-derivative operators. We collect these statements in an algorithm that simplifies the computation of the linearized on-shell action of a particular operator.[3] Notably, this algorithm synthesizes the following observations. On the one hand, it is crucial that auxiliary fields are integrated out in the Einstein-frame and, on the other hand, the linearized on-shell action does not require solving the equations of motion of the auxiliary fields. In the second part of the analysis we classify all leading and next-to-leading order higher-derivative operators for the chiral multiplets and determine the component versions of the subclass of operators which induce four-derivative terms for the chiral scalars in the linearized on-shell action. This result is model-independent and widely applicable to any case study in which the leading order corrections from higher-derivative operators for a specific (ungauged) matter-coupled supergravity might be relevant.

Moreover, we address the vacuum structure of the general higher-derivative theory. Firstly, we turn to the supersymmetric Minkowski and AdS_4-vacua. In particular, we pay special attention to the curvature constraints enforced by the necessary existence of Killing spinors on the spacetime background [35] and their compatibility with the full scalar potential of higher-derivative supergravity. While the supersymmetric Minkowski vacua are unaffected by the presence of the higher-derivatives, as also noted in [13], the supersymmetric AdS_4-vacua and the non-supersymmetric vacua are altered. In particular, we argue that typically these vacua should not admit any moduli space. Therefore, the higher-derivative operators are generically

[3] In general the on-shell action is a non-local action containing infinitely many operators. In particular, the linearized action refers to the on-shell action truncated at linear order in the coupling of the operator.

important in the context of moduli stabilization. Finally, we discuss the form of the higher-derivative operators for the special case where the leading order (two-derivative) theory is given by a shift-symmetric no-scale model. These models appear in the context of the low-energy effective descriptions of certain string compactifications. In particular, we demonstrate that the no-scale condition leads to the vanishing of many leading-order contributions to the scalar potential. In this context we also briefly review the results of [36] where the shift-symmetric no-scale models were classified.

Finally, we turn to the discussion of the role of 4D higher-derivative operators and perturbative $(\alpha')^3$-corrections in the context of orientifold compactifications in IIB string theory following [31], thereby providing a possible answer to the fourth question. A subsector of the $(\alpha')^3$-corrections to IIB supergravity is given by a contraction of four Riemann tensors, schematically denoted as $\mathcal{R}^4$ [27]. This fully known term was used to infer new corrections to the Kähler potential of the Kähler moduli in [28]. By performing a Kaluza-Klein (KK) decomposition we determine four-derivative terms for a volume deformation from the $\mathcal{R}^4$-invariant. We then match these four-derivative terms to a particular (supersymmetric) higher-derivative operator. Thereby, we can infer new F^4-type corrections to the scalar potential. These descend from only partially known ten-dimensional $(\alpha')^3$-corrections involving the 3-form gauge potentials of IIB supergravity. We then proceed to study the minima of the scalar potential at order $(\alpha')^3$ that is including also the correction determined in [28]. We demonstrate the presence of a model-independent non-supersymmetric minimum where all Kähler moduli are fixed. However, the existence of the minimum requires that the Calabi-Yau threefold has a positive Euler number and that an overall undetermined numerical prefactor of the F^4-term is negative. Computation of this prefactor would require a full matching of the $(\alpha')^3$-corrections to a combination of all higher-derivative operators for supergravity which we classified in the first part. This step will be performed in future work. In sum, these results suggest the existence of a large class of new non-supersymmetric vacua in the landscape where stabilization occurs purely from perturbative α'-corrections. Computationally these vacua are more easily accessible since they only depend on topological numbers of the threefold. Should the sign of the F^4-term turn out to be negative, they would constitute a framework in which general aspects of string vacua could be studied.

Lastly, we investigate the possibility that the F^4-term yields a viable potential for inflation driven by a Kähler modulus following [37].[4] For simplicity we consider a setup where Kähler moduli stabilization occurs in the setup of the Large Volume Scenario (LVS) [30]. Here we also include the special KK- and winding-mode string-loop corrections to the potential discussed in [41–43]. Furthermore, we employ the particular $K3$-fibered geometry considered in [39], since in this case the fiber modulus is a flat direction and receives a potential only after including the F^4-term or the aforementioned string-loop corrections. In turn, this field can be made parametrically light and, thus, may play the role of the inflaton. Here we propose a stabilization of the fiber modulus via an interplay between the F^4-term and

[4] For earlier works on Kähler moduli inflation see [38–40].

the string-loop corrections. In this case the F^4-term leads to viable plateau-type potentials similar to the proposed inflationary scenario of [44].

This thesis is organized as follows. In chapter 2 we discuss higher-derivative theories in flat and curved superspace and determine the vacuum structure of these theories. In chapter 3 we study Calabi-Yau orientifold flux-compactifications of IIB string theory and the emergence of 4D higher-derivative operators from $(\alpha')^3$-corrections. In chapter 4 we investigate the implications of the novel F^4-type correction for moduli stabilization and inflation. Thereafter, in chapter 5 we conclude this thesis. In appendix A we present additional demonstrations regarding general theories with higher-derivatives as well as a catalog of component identities for supergravity and, lastly, the classification of higher-derivative operators for supergravity. Finally, in appendix B we provide additional technical proofs involving the F^4-term descending from $(\alpha')^3$-corrections.

Chapter 2

$\mathcal{N} = 1, D = 4$ Supersymmetry, Supergravity and Higher-Derivative Operators

In this chapter we follow mostly the reference [32]. More precisely, sec. 2.3, sec. 2.4, sec. 2.5 and sec. 2.6 are taken from this reference. The exceptions in these sections are sec. 2.3.4, sec. 2.5.7 and sec. 2.6.3 which review content of [31]. Lastly, sec. 2.7 reviews the reference [36], but also contains a discussion from [32].

2.1 A Primer on Higher-Derivative Operators

We begin this section by giving a brief overview of how higher-derivatives arise in the context of effective field theory, which problems theories with higher-derivatives have to face and how these problems can be resolved by appealing to EFT principles. Since these subjects are independent of whether the theory is supersymmetric or not, we explicitly prepone this discussion.

As an example let us consider an ordinary two-derivative theory of two real scalar fields l, h subject to the Lagrangian

$$\mathcal{L}_{h+l} = -\frac{1}{2}\partial_a h \partial^a h - \frac{1}{2}\partial_a l \partial^a l - \frac{1}{2}m_l^2 l^2 - \frac{1}{2}m_h^2 h^2 - m_{hl} l h \ , \tag{2.1}$$

where we use $a = 0, \dots, 3$ as a label for Minkowski-space. Suppose that $m_l \ll m_h$ that is l is much lighter than h. If we are interested in physics at scales much smaller than m_h, then it suffices to discuss an EFT obtained by integrating out h. At the classical level we integrate out h by solving its respective equation of motion, which yields

$$h = -\frac{m_{hl}}{m_h^2}\left(1 - \frac{\Box}{m_h^2}\right)^{-1} l = -\frac{m_{hl}}{m_h^2}\sum_{i=0}^{\infty}(-1)^i \left(\frac{\Box}{m_h^2}\right)^i l \tag{2.2}$$

and reinsert this result back into eq. (2.1).[5] This way we obtain a non-local theory which includes an infinite tower of higher-derivatives acting on l. Since we cannot include all possible quantum corrections, we would violate the principles of effective field theory by taking into account the complete infinite sum of higher-derivatives. Therefore, and for computational reasons we need to obtain a local theory and, hence, we must truncate the infinite tower to a finite number of terms. This procedure indirectly introduces new degrees of freedom into the theory. Naively, these emerge from the fact that now the field equations for l are not second order anymore but include higher-derivatives acting on l. In the respective Cauchy problem, these higher-derivatives require specification of additional initial data and, therefore, correspond to new degrees of freedom. For instance if we keep terms up to $\mathcal{O}(1/m_h^8)$ then a new field given by

$$\phi_\Box \equiv \Box l \tag{2.3}$$

enters the theory as a dynamical degree of freedom. In [1] it was noticed that such types of degrees of freedom can be reformulated in a two-derivative language, but the higher the order of derivatives acting on l can be found in the Lagrangian the more degrees of freedom are required for the reformulation. In addition, the new degrees of freedom generically have the wrong sign of the kinetic term and, therefore, constitute ghosts, that cause harmful instabilities in both the classical as well as quantum version of the theory. Thus, local theories with higher-derivatives are usually ruled out as candidates for UV-complete theories, see for instance [45, 46] for a discussion in the context of gravity. However, generic EFTs must include higher-derivatives, unless there is a symmetry which forbids them all, and they also have to be local theories. Moreover, in particular examples in which we can see how EFTs descend from ultraviolet physics as in our example above, we know that the UV theory is fully well-behaved and that there are no ghosts in its spectrum. Therefore, the higher-derivative degrees of freedom must be unphysical. One may ask which principle or computational tool allows us to make the above observations manifest? Fortunately, the answer was already given in [6, 7]. The authors of these references proposed to restore the principles of EFT by imposing that the solutions to the equations of motion are perturbative in control parameters of the EFT, such as the inverse cut-off scale Λ^{-1}. In [6] it was shown that thereby the harmful ghostlike degrees of freedom are effectively removed from the spectrum and that the theory is well-behaved. In particular, the higher-derivatives then correct the dynamics of the ordinary two-derivative theory only by Λ-suppressed corrections just as it is expected for an EFT.

To illustrate further why the interpretation of [6, 7] is necessary, we turn again to our example in eq. (2.1) to make another observation about the higher-derivative degrees of freedom. Let us canonically normalize the field $\phi_\Box$ as defined in eq. (2.3). Up to some numerical factor, the canonical normalization amounts to introducing

$$\phi_c \sim \frac{m_{lh}}{m_h^4} \Box l \ . \tag{2.4}$$

[5]The geometric series converges, since we are assuming that we are interested at energies much smaller than m_h^2 and, therefore, $(\Box/m_h^2)l \ll 1$.

From eq. (2.1) we see that the mass of ϕ_c is given by m_h up to some $\mathcal{O}(1)$ constant or a possible sign factor. Now, if we consider ϕ_c to be part of the physical spectrum then we have a direct contradiction to the fact that we started by integrating out h to obtain a low-energy EFT for the light degree of freedom l. In other words once we identify ϕ_c as a degree of freedom we should immediately eliminate it from the spectrum by integrating it out. However, integrating out this field is nothing else, but solving the equations of motion for l perturbatively in powers of $1/m_h^2$ and, hence, equivalent to the proposal of [6, 7].

We now turn to theories with supersymmetry. After establishing the usual notations and conventions we proceed to discuss supersymmetric theories with higher-derivatives. There we will encounter new problems which we analyze and discuss in the spirit of this primer.

2.2 Basic Notions of $\mathcal{N}=1, D=4$ Supersymmetry

In this section we review the basics of $\mathcal{N}=1, D=4$ supersymmetry, the superspace formalism and the construction of supersymmetric actions. The supersymmetry algebra is a generalization of the Poincaré algebra that contains, for the case of $\mathcal{N}=1, D=4$, two additional generators which are given by a pair of left- and right-handed Weyl spinors Q_α and $\bar{Q}_{\dot\alpha}$. The crucial part of the supersymmetry algebra is then given by

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2P_{\alpha\dot\alpha} \ , \qquad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\beta}, \bar{Q}_{\dot\alpha}\} = 0 \ , \tag{2.5}$$

where $P_{\alpha\dot\alpha}$ is the generator of spacetime translations. It is convenient to discuss supersymmetric theories in superspace, which is an eight-dimensional super-manifold that simultaneously describes spacetime as well as Grassmann-space. For theories with global $\mathcal{N}=1$ supersymmetry the respective superspace is a generalization of Minkowski space and has a flat geometry. We begin by recapitulating the basics on flat superspace and theories with global supersymmetry, adopting the notations and conventions of [34]. We choose flat superspace to be parametrized by the variables

$$z^A = (x^a, \theta^\alpha, \bar\theta_{\dot\alpha}) \ , \qquad a = 0, \ldots, 3 \ , \qquad \alpha, \dot\alpha = 1, 2 \ . \tag{2.6}$$

In the following, let us denote $a, b, c, \ldots$ as Minkowski space indices and $\alpha, \beta, \dot\alpha, \dot\beta, \ldots$ as flat Grassmann space indices. Moreover, we introduce integration measures over flat superspace as follows

$$\mathrm{d}^8 z = \mathrm{d}^4 x\, \mathrm{d}^4\theta = \mathrm{d}^4 x\, \mathrm{d}^2\theta\, \mathrm{d}^2\bar\theta \ , \qquad \mathrm{d}^6 z = \mathrm{d}^4 x\, \mathrm{d}^2\theta \ , \qquad \mathrm{d}^6 \bar{z} = \mathrm{d}^4 x\, \mathrm{d}^2\bar\theta \ . \tag{2.7}$$

Supersymmetric field theories are built by considering fields that map to superspace, the so-called superfields. Via the Grassmann-algebra these objects have a finite θ, $\bar\theta$ expansion which for a generic superfield f reads

$$\begin{aligned} f(x,\theta,\bar\theta) = {} & f_0(x) + \theta f_1(x) + \bar\theta f_2(x) + \theta\theta f_3(x) + \bar\theta\bar\theta f_4(x) + \theta\sigma_a\bar\theta f_5^a(x) \\ & + \theta\theta\bar\theta f_6(x) + \theta\bar\theta\bar\theta f_7(x) + \theta\theta\bar\theta\bar\theta f_8(x) \ , \end{aligned} \tag{2.8}$$

where σ_a denote the Pauli σ-matrices, see appendix A.1 for the conventions we use here. Superfields already transform as representations of the supersymmetry algebra and are therefore ideal candidates to construct supersymmetric theories. However, a general superfield forms a reducible representation. To find more interesting representations we have to consider constrained superfields. The constraints have to be implemented in such a way, that the constrained superfields still form representations of the supersymmetry algebra. This is achieved by considering the so-called superspace-derivatives, which are given by

$$D_A = (\partial_a, D_\alpha, \bar{D}^{\dot\alpha}) \ , \tag{2.9}$$

where ∂_a denotes the usual spacetime-derivative and the spinorial components of D_A read

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^a_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial x^a} \qquad \text{and} \qquad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^a_{\alpha\dot\alpha}\frac{\partial}{\partial x^a} \ . \tag{2.10}$$

These derivatives form the following algebra

$$\begin{aligned} \{D_\alpha, \bar{D}_{\dot\beta}\} &= -2i\sigma^a_{\alpha\dot\beta}\partial_a \ , \\ \{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = [D_\alpha, \partial_a] = [\bar{D}_{\dot\alpha}, \partial_a] = 0 \ . \end{aligned} \tag{2.11}$$

The spinorial components D_α and $\bar{D}_{\dot\alpha}$ have the property that they anticommute with Q_α and $\bar{Q}_{\dot\alpha}$. Therefore, when we impose constraints by acting with spinorial superspace derivatives on superfields it is guaranteed that the constrained superfields form representations of the supersymmetry algebra. For instance by imposing the constraints

$$\bar{D}_{\dot\alpha}\Phi = D_\alpha\bar\Phi = 0 \ , \tag{2.12}$$

we obtain superfields which are denoted as chiral and anti-chiral respectively. A superfield V which obeys $V = V^\dagger$ is denotes as real superfield. Of interest are also real superfields L which additionally satisfy the constraint

$$D^2 L = \bar{D}^2 L = 0 \ , \tag{2.13}$$

where we introduced the conventions

$$D^2 = D^\alpha D_\alpha \ , \qquad \bar{D}^2 = \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha} \ . \tag{2.14}$$

Superfields with the property in eq. (2.13) are denoted as real linear superfields. In the following we are mostly interested in chiral superfields. Eq. (2.12) introduces relations among the components of the generic θ-expansion for Φ as in eq. (2.8) and can be simplified to

$$\Phi = A + \sqrt{2}\theta\chi + \theta^2 F + i\theta\sigma^a\bar\theta\partial_a A - \tfrac{i}{\sqrt{2}}\theta\theta\partial_a\chi\sigma^a\bar\theta + \frac{1}{4}\theta^2\bar\theta^2\Box A \ , \tag{2.15}$$

where A is a complex scalar, χ a Weyl spinor and F a complex scalar, which is called auxiliary field. We abbreviate the component-expansion for chiral superfields as

$\Phi = (A, \chi, F)$. Similarly, eq. (2.13) leads to an expansion for L which is determined by a real scalar L together with a two-form B_2 and a Majorana-spinor η. In tab. 2.1 we collect some of the important supersymmetry multiplets and their component fields. We have also included the gravity and a higher-derivative multiplet in this list. The former will be explained in more detail when we discuss supergravity and the latter we will investigate now.

Multiplet	Superfield	Content
Gravity (old minimal)	$R, G_a, W_{\alpha\beta\gamma}$	(g_{mn}, ψ_m, M, b_m)
Real Linear	L	(L, η, B_2)
Chiral	Φ	(A, χ, F)
Chiral (higher-derivative)	Ψ	$(\bar{F}, -i\sigma^a\partial_a\bar{\chi}, \Box\bar{A})$

Table 2.1: *Various off-shell representations of the $\mathcal{N} = 1$ supersymmetry algebra in $D = 4$ spacetime dimensions. The respective superfields and components are also displayed.*

Let us introduce the special superfields

$$\bar{\Psi} \equiv -\tfrac{1}{4}D^2\Phi \ , \qquad \Psi \equiv -\tfrac{1}{4}\bar{D}^2\bar{\Phi} \ , \tag{2.16}$$

which will be of interest later on during the discussion of higher-derivatives. From the commutation relations in eq. (2.11) we find that the superspace derivatives obey the identities

$$D_\alpha D_\beta D_\gamma = \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta}\bar{D}_{\dot\gamma} = 0 \ , \tag{2.17}$$

$$D_\alpha D_\beta = \tfrac{1}{2}\epsilon_{\alpha\beta}D^2 \ , \quad \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta} = -\tfrac{1}{2}\epsilon_{\dot\alpha\dot\beta}\bar{D}^2 \ . \tag{2.18}$$

Eq. (2.17), in turn, implies that Ψ is a chiral and $\bar{\Psi}$ an anti-chiral superfield. More precisely, the component expansion of $\bar{\Psi}$ reads

$$\bar{\Psi} = F - \sqrt{2}i\partial_a\chi\sigma^a\bar{\theta} + \bar{\theta}^2\Box A - i\theta\sigma^a\bar{\theta}\partial_a F + \tfrac{1}{\sqrt{2}}\bar{\theta}^2\theta\Box\chi + \tfrac{1}{4}\theta^4\Box F \ . \tag{2.19}$$

Therefore, we can abbreviate the component form as $\bar{\Psi} = (F, i\sigma^a\partial_a\chi, \Box A)$ and, similarly, $\Psi = (\bar{F}, -i\sigma^a\partial_a\bar{\chi}, \Box\bar{A})$.

We are now in a position to start discussing supersymmetric Lagrangians. Generally, supersymmetric Lagrangians are constructed from $\theta^2\bar{\theta}^2$-components of arbitrary superfields. Equivalently, we can first project an arbitrary superfield f onto its chiral component by acting with $\bar{D}^2$ and then take the θ^2-component of this truncated chiral superfield to obtain a supersymmetric Lagrangian. However, if we want to keep only terms with at most two derivatives then it is convenient to distinguish these two terms and write them down independently. For instance, we may write an action for a chiral superfield Φ in the following way

$$\mathcal{L}_0[\Phi, \bar{\Phi}] = \int \mathrm{d}^4\theta K(\Phi, \bar{\Phi}) + \int \mathrm{d}^2\theta W(\Phi) + h.c. \ , \tag{2.20}$$

where K is a real function and can be understood as a Kähler potential of a respective Kähler manifold which the chiral scalars locally parametrize. The object W is a chiral superfield and denoted as superpotential. $\mathcal{L}_0$ describes the general theory of an ungauged chiral multiplet with at most two derivatives.

2.3 Higher-Derivative Actions in Flat $\mathcal{N} = 1, D = 4$ Superspace

So far we have considered the Lagrangian to include at most two derivatives. Under this assumption the general theory for ungauged chiral multiplets is of the form in eq. (2.20). However, following our overview in sec. 2.1 in a generic EFT also higher-derivative operators have to be present. In the superspace formalism higher-derivatives are realized by acting with superspace-derivatives on superfields. In fact, it can be demonstrated that D_A are the only objects that fully (anti-) commute with the supersymmetry generators [16] and, therefore, the only way to obtain higher-derivatives in superspace. In conclusion, the general superspace effective action for a theory describing an ungauged chiral multiplet is of the form

$$\begin{aligned} S_{\text{gen}} = &\int \mathrm{d}^8 z\, \mathcal{K}(\Phi, \bar{\Phi}, D_A\Phi, D_B\bar{\Phi}, D_A D_B \Phi, \dots) \\ &+ \int \mathrm{d}^6 z \mathcal{W}(\Phi, \partial_a\Phi, \bar{D}^2\bar{\Phi}, \partial_a\partial_b\Phi, \dots) + h.c. \,, \end{aligned} \tag{2.21}$$

where the dots indicate a dependence on higher superspace-derivatives acting on Φ or $\bar{\Phi}$. To guarantee that the above action is supersymmetric $\mathcal{W}$ must only depend on chiral superfields.[6] Furthermore, we have introduced new symbols $\mathcal{K}$ for the $\mathrm{d}^4\theta$-integrand and $\mathcal{W}$ for the $\mathrm{d}^2\theta$-integrand to distinguish these objects from the Kähler potential K and superpotential W in eq. (2.20).

The general action in eq. (2.21) is a rather complicated quantity which depends on infinitely many higher-derivative superfields. However, many higher-derivative operators in S_{gen} contribute only kinetic terms for the component fields. In this section we focus purely on the special subclass of higher-derivative operators which manifestly contribute to the scalar potential. We will show that, if we allow the action to depend only on this subclass of operators, then the structure of the action simplifies considerably.

Let us demonstrate, how we can obtain the general form of the off-shell superspace effective action under the condition that all terms in this action manifestly contribute to the scalar potential. From now on let us refer to this action as S_{eff}. This action first appeared in [15, 16] where it was determined in the context of Wilsonian effective actions and computed for a notable example, namely the one-loop Wess-Zumino model in [15, 17, 18]. To determine S_{eff} we evaluate S_{gen} at the condition

$$\partial_a\Phi = \partial_a\bar{\Phi} = 0 \,. \tag{2.22}$$

[6] Indeed, $\partial_a\Phi$ and $\partial_a\partial_b\Phi$ are chiral superfields due to eq. (2.11).

Since the spinorial super-derivatives commute with ∂_a, all operators which we set to zero via the above condition contribute only to the kinetic part of the Lagrangian and are, therefore, irrelevant in our analysis. Moreover, note that the constraint in eq. (2.22) respects supersymmetry. When evaluating the general action in eq. (2.21) at the condition in eq. (2.22) the resulting action S_{eff} greatly simplifies [15, 16]. Taking an EFT perspective we give an alternative derivation of S_{eff} in appendix A.2. There we also demand that S_{eff} does not exhibit any redundancy in the operators that it includes. Briefly summarized appendix A.2 consists of two parts. Firstly, we show that a dependence of the action on the superspace derivatives acting on Φ and $\bar{\Phi}$ is restricted by the (anti-)commutation relations in eq. (2.11). Secondly, using integration by parts identities we reduce the effective action even further, such that finally one is left with

$$S_{\text{eff}} = \int \mathrm{d}^8 z\, \mathcal{K}(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) + \int \mathrm{d}^6 z\, W(\Phi) + h.c. \; . \tag{2.23}$$

Up to some minor differences regarding possible redundancies of operators, this coincides with the action that was already obtained in [15, 16].

Let us pause a moment to clarify the physical meaning of the additional degree of freedom associated with Ψ. As displayed in eq. (2.19) the fermionic- and θ^2-components of Ψ are given by higher-derivatives of the chiral scalar and chiral fermion $\sigma^a \partial_a \bar{\chi}$ and $\Box \bar{A}$. In the sense of the Ostrogradski-procedure these higher-derivatives constitute additional degrees of freedom [1]. However, since we are discussing theories with off-shell supersymmetry the number of bosonic and fermionic degrees of freedom must match and, furthermore, allow a description in terms of an appropriate multiplet. This matching is achieved when taking into account the auxiliary field $\bar{F}$, such that the collection of component fields fits nicely into the chiral multiplet Ψ.

2.3.1 General Scalar Potential in Higher-Derivative Theory

It is now straightforward to generalize the result in eq. (2.23) to the case of a collection of n_c chiral superfields $\Phi^i, i = 1, \ldots, n_c$. We denote the respective components as $\Phi^i = (A^i, \chi^i, F^i)$ and

$$\begin{aligned} \Psi^i &= -\tfrac{1}{4} \bar{D}^2 \bar{\Phi}^i = (\bar{F}^i, -i\sigma^a \partial_a \bar{\chi}^i, \Box \bar{A}^i) \; , \\ \bar{\Psi}^{\bar{\jmath}} &= -\tfrac{1}{4} D^2 \Phi^{\bar{\jmath}} = (F^{\bar{\jmath}}, i\sigma^a \partial_a \chi^{\bar{\jmath}}, \Box A^{\bar{\jmath}}) \; . \end{aligned} \tag{2.24}$$

The appropriate multi-field generalization of eq. (2.23) is now readily obtained and reads

$$\mathcal{L}_{\text{eff}} = \int \mathrm{d}^4\theta\, \mathcal{K}(\Phi^i, \bar{\Phi}^{\bar{\jmath}}, \Psi^k, \bar{\Psi}^{\bar{l}}) + \int \mathrm{d}^2\theta\, W(\Phi^i) + h.c. \; , \tag{2.25}$$

which should be completed by adding a Lagrange multiplier whose equation of motion is equivalent to eq. (2.24).

Next, let us determine the component version of $\mathcal{L}_{\text{eff}}$. We understand the Lagrangian in eq. (2.25) as an ordinary theory with Kähler potential $\mathcal{K}$ and superpotential W, such that the respective Kähler manifold is $2n_c$-dimensional, together with the additional constraints in eq. (2.24). Therefore, the θ-integration in eq. (2.25) can be performed straightforwardly. We can simply use the component Lagrangian of the two-derivative theory displayed in [34] and insert the constraints in eq. (2.24) for the components of $\Psi, \bar{\Psi}$. This procedure yields

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\mathcal{K}_{,A^i\bar{A}^{\bar{\jmath}}}\,\partial_a A^i\partial^a\bar{A}^{\bar{\jmath}} - \mathcal{K}_{,A^iF^j}\partial_a A^i\partial^a F^j - \mathcal{K}_{,\bar{A}^{\bar{\imath}}\bar{F}^{\bar{\jmath}}}\,\partial_a\bar{A}^{\bar{\imath}}\partial^a\bar{F}^{\bar{\jmath}} \\ & -\mathcal{K}_{,F^i\bar{F}^{\bar{\jmath}}}\,\partial_a F^i\partial^a\bar{F}^{\bar{\jmath}} + \mathcal{K}_{,A^iF^j}F^i\Box A^j + \mathcal{K}_{,\bar{A}^{\bar{\imath}}\bar{F}^{\bar{\jmath}}}\,\bar{F}^{\bar{\imath}}\Box\bar{A}^{\bar{\jmath}} \\ & +\mathcal{K}_{,F^i\bar{F}^{\bar{\jmath}}}\,\Box A^i\Box\bar{A}^{\bar{\jmath}} + \mathcal{K}_{,A^i\bar{A}^{\bar{\jmath}}}\,F^i\bar{F}^{\bar{\jmath}} + F^iW_{,A^i} + \bar{F}^{\bar{\imath}}\bar{W}_{,\bar{A}^{\bar{\jmath}}}\ ,\end{aligned} \tag{2.26}$$

where the superfields $\mathcal{K}$ and derivatives thereof are understood as being evaluated at their respective scalar component. As before, we display only the bosonic terms here. Inspecting eq. (2.26) we observe that F^i are now propagating degrees of freedom. The second derivatives of $\mathcal{K}$ control the kinetic terms for the scalars and, hence, determine whether the auxiliary fields are propagating and, in particular, ghostlike or not. We return to the discussion of the propagating auxiliary fields in sec. 2.3.5. For now we do not make any further assumptions about the kinetic terms.

We are intrigued to find a geometric understanding of eq. (2.26) and eq. (2.25). A first guess would be that our theory is described by a $2n_c$-dimensional (pseudo-) Kähler geometry $\mathcal{M}_p$.[7] However, the constraints in eq. (2.24) break the respective target space reparametrization-invariance of the $2n_c$-dimensional geometry, and indeed it is easily seen that eq. (2.26) does not support a reparametrization-invariance with respect to a $2n_c$-dimensional (pseudo-) Kähler geometry. Nevertheless, the ordinary reparametrization-invariance with respect to the chirals scalars $A^i, \bar{A}^{\bar{\jmath}}$ parametrizing an n_c-dimensional complex manifold $\mathcal{M}_0$ must be maintained. Suppose that we are in a situation where we integrate out the auxiliary fields. We will justify this assumption later on in sec. 2.3.5. Via the interactions in eq. (2.26) the solutions to the equations of motion for the auxiliary fields formally read

$$F^i = F^i[A^i, \bar{A}^{\bar{\jmath}}, \partial_a A^i, \partial_a\bar{A}^{\bar{\jmath}}, \dots]\ , \tag{2.27}$$

where the dots indicate a dependence on higher spacetime-derivatives of the chiral scalars. Reparametrization-invariance with respect to $\mathcal{M}_0$ must, in particular, individually hold for the scalar potential in eq. (2.26). To determine the scalar potential we have to truncate F^i given in eq. (2.27) so that only a dependence on $A^i, \bar{A}^{\bar{\jmath}}$ remains.[8] Then target space reparametrization-invariance necessarily requires that

$$\omega_F = F^i(A,\bar{A})\mathrm{d}A^i + \bar{F}^{\bar{\imath}}(A,\bar{A})\mathrm{d}\bar{A}^{\bar{\imath}} \tag{2.28}$$

constitutes a one-form on $\mathcal{M}_0$. For clarity, note that F^i and $\bar{F}^{\bar{\imath}}$ in the above are the truncated versions of eq. (2.27) for which we continue to use the same symbol. From eq. (2.28) we learn that it is natural to discuss the scalar potential in eq. (2.26)

[7] A pseudo-Kähler manifold obeys the same conditions as a Kähler manifold, but instead of being equipped with a positive definite metric it is endowed with an indefinite bilinear form. This situation occurs when the auxiliary fields are ghostlike or remain algebraic degrees of freedom.

[8] This truncation is achieved by evaluating eq. (2.27) at the conditions $\partial_a A^i = \partial_a\bar{A}^{\bar{\jmath}} = \cdots = 0$ where the dots indicate all possible higher spacetime-derivatives of the chiral scalars.

in the context of the cotangent-bundle $T^*\mathcal{M}_0$. Target space invariance requires, furthermore, that $\mathcal{K}$ transforms as a scalar on the cotangent-bundle or in other words that it constitutes a zero-form on $T^*\mathcal{M}_0$. Note that this requirement is by no means automatically guaranteed but restricts the possible choices of $\mathcal{K}$.

Checking the invariance of the kinetic terms in eq. (2.26) under reparametrizations of $A^i, \bar{A}^{\bar{\jmath}}$ is considerably more involved. In particular, the transformation behavior of the auxiliary fields must necessarily differ from eq. (2.28) when including the dependence of F^i on spacetime-derivatives of the chiral scalars which was indicated in eq. (2.27). The general discussion of these transformation properties is rather involved and we omit their discussion here.[9]

One may wonder whether the manifold $\mathcal{M}_0$ is still endowed with a Kähler structure. In eq. (2.26) the kinetic terms for the chiral scalars are multiplied by a complicated metric which in general is not even hermitian. In the usual sense of a non-linear sigma model we, therefore, do not have a Kähler structure here. However, since we are interested in the general scalar potential and not in the general two- or higher-derivative component action, it is interesting to identify a geometric meaning of the scalar potential alone. Indeed, the scalar potential induced by $\mathcal{K}$ is given as the pseudo-norm of the one-form ω_F with respect to the bilinear form $\mathcal{K}_{,A^i\bar{A}^{\bar{\jmath}}}$. This object indeed defines a pseudo-Kähler structure on $\mathcal{M}_0$ and in the limit where all higher-derivative operators vanish this structure reproduces the Kähler structure of the ordinary two-derivative theory.

Finally, let us make a comment regarding Kähler-invariance. Since eq. (2.25) is a theory of chiral multiplets only, we observe that $\mathcal{L}_{\text{eff}}$ enjoys a respective extended Kähler invariance. The corresponding Kähler-transformations are of the form

$$\mathcal{K}(\Phi^i, \bar{\Phi}^{\bar{\jmath}}, \Psi^k, \bar{\Psi}^{\bar{l}}) \longrightarrow \mathcal{K}(\Phi^i, \bar{\Phi}^{\bar{\jmath}}, \Psi^k, \bar{\Psi}^{\bar{l}}) + G(\Phi^i, \Psi^j) + \bar{G}(\bar{\Phi}^{\bar{\jmath}}, \bar{\Psi}^{\bar{l}}) \ , \tag{2.29}$$

where G is an arbitrary holomorphic function and $\bar{G}$ the respective anti-holomorphic function.

2.3.2 Alternative Higher-Derivative Lagrangian

Let us mention a second higher-derivative Lagrangian, which is of interest. This Lagrangian also describes the general effective scalar potential but it does not induce kinetic terms for the auxiliary fields. Moreover, the requirement of target space-invariance is, at least in parts, more easily realized here. We construct this Lagrangian from eq. (2.25) via the following instruction. Inside each operator in $\mathcal{K}$, which includes at least one factor of $\Psi^i\bar{\Psi}^{\bar{\jmath}}$, we replace this factor by a term $D^\alpha\Phi^i D_\alpha\Phi^j \bar{D}_{\dot{\alpha}}\bar{\Phi}^{\bar{k}}\bar{D}^{\dot{\alpha}}\bar{\Phi}^{\bar{l}}$ via the last identity in eq. (A.13). If this factor appears more than once we perform this procedure only for one of them. The resulting Lagrangian

[9] However, the proper transformation behavior will place constraints on the allowed operators. In particular, for a generic $\mathcal{K}$ it is not guaranteed that one can always choose the transformation behavior of the auxiliary fields appropriately. There might exist situations where it is necessary to add further superspace higher-derivative operators which we excluded by means of eq. (2.22).

can then be cast into the form[10]

$$\mathcal{L}'_{\text{eff}} = \int \mathrm{d}^4\theta \, [K(\Phi,\bar\Phi) + \mathcal{F}(\Phi,\bar\Phi,\Psi) + \bar{\mathcal{F}}(\Phi,\bar\Phi,\bar\Psi)] + \int \mathrm{d}^2\theta \, W(\Phi) + \int \mathrm{d}^2\bar\theta \, \bar W(\bar\Phi)$$
$$+ \frac{1}{16}\int \mathrm{d}^4\theta \, T_{ij\bar k\bar l}(\Phi,\bar\Phi,\Psi,\bar\Psi)\, D^\alpha\Phi^i D_\alpha\Phi^j \bar D_{\dot\alpha}\bar\Phi^{\bar k}\bar D^{\dot\alpha}\bar\Phi^{\bar l} \; . \tag{2.30}$$

The object $T_{ij\bar k\bar l}$ is a superfield and in order to support reparametrization-invariance it transforms as a tensor of $\mathcal{M}_0$ [21]. From eq. (2.30) we observe that

$$T_{ij\bar k\bar l} = T_{ji\bar k\bar l} = T_{ji\bar l\bar k} \; . \tag{2.31}$$

Furthermore this object has to be a hermitian tensor to ensure reality of the Lagrangian. Let us emphasize that $\mathcal{L}'_{\text{eff}}$ and $\mathcal{L}_{\text{eff}}$ are in general distinct and only the respective scalar potentials coincide. More precisely, $\mathcal{L}'_{\text{eff}}$ and $\mathcal{L}_{\text{eff}}$ differ by purely kinetic superspace-operators.

For completeness we also determine the component form of $\mathcal{L}'_{\text{eff}}$. To this end it is necessary to compute the last integral in eq. (2.30). Using eq. (2.10) and the θ-expansion for the (anti-) chiral superfield in eq. (2.15) one finds by direct computation

$$\begin{aligned} &D^\alpha\Phi^i D_\alpha\Phi^j \bar D_{\dot\alpha}\bar\Phi^{\bar k}\bar D^{\dot\alpha}\bar\Phi^{\bar l}|_{\text{bos}} = \\ &16\left[(\partial_a A^i\partial^a A^j)(\partial_b\bar A^{\bar k}\partial^b\bar A^{\bar l}) - 2F^i\bar F^{\bar k}(\partial_a A^j\partial^a\bar A^{\bar l}) + F^iF^j\bar F^{\bar k}\bar F^{\bar l}\right]\theta^2\bar\theta^2 \; , \end{aligned} \tag{2.32}$$

which, in turn, implies that the component version of $\mathcal{L}'_{\text{eff}}$ reads

$$\begin{aligned} \mathcal{L}'_{\text{eff}} = &-(K_{,A^i\bar A^{\bar\jmath}} + \mathcal{F}_{,A^i\bar A^{\bar\jmath}} + \bar{\mathcal{F}}_{,A^i\bar A^{\bar\jmath}})\partial_a A^i\partial^a\bar A^{\bar\jmath} - \bar{\mathcal{F}}_{,A^iF^j}\partial_a A^i\partial^a F^j \\ &- \mathcal{F}_{,\bar A^{\bar\imath}\bar F^{\bar\jmath}}\partial_a\bar A^{\bar\imath}\partial^a\bar F^{\bar\jmath} + \bar{\mathcal{F}}_{,A^iF^j}F^i\Box A^j + \mathcal{F}_{,\bar A^{\bar\imath}\bar F^{\bar\jmath}}\bar F^{\bar\imath}\Box\bar A^{\bar\jmath} \\ &+ (K_{,A^i\bar A^{\bar\jmath}} + \mathcal{F}_{,A^i\bar A^{\bar\jmath}} + \bar{\mathcal{F}}_{,A^i\bar A^{\bar\jmath}})F^i\bar F^{\bar\jmath} + F^iW_{,i} + \bar F^{\bar\imath}\bar W_{,\bar\imath} \\ &+ T_{ij\bar k\bar l}\left[(\partial_a A^i\partial^a A^j)(\partial_b\bar A^{\bar k}\partial^b\bar A^{\bar l}) - 2F^i\bar F^{\bar k}(\partial_a A^j\partial^a\bar A^{\bar l}) + F^iF^j\bar F^{\bar k}\bar F^{\bar l}\right] \; . \end{aligned} \tag{2.33}$$

Even though the above Lagrangian depends on $\partial_a F$, inside the equations of motion for the auxiliary fields this dependence cancels out and the resulting equations are purely algebraic. More precisely the equations of motion for the auxiliary fields read

$$\begin{aligned} &\mathcal{F}_{,\bar A^{\bar\imath}\bar A^{\bar\jmath}\bar F^{\bar n}}\left(\partial_a\bar A^{\bar\imath}\partial^a\bar A^{\bar\jmath} + \bar F^{\bar\imath}\Box\bar A^{\bar\jmath}\right) + (K_{,A^i\bar A^{\bar n}} + \mathcal{F}_{,A^i\bar A^{\bar n}} + \bar{\mathcal{F}}_{,A^i\bar A^{\bar n}})F^i \\ &+ 2\mathcal{F}_{,\bar A^{\bar\imath}\bar F^{\bar n}}\Box\bar A^{\bar\imath} + \bar W_{,\bar n} + \mathcal{F}_{,A^i\bar A^{\bar\jmath}\bar F^{\bar n}}F^i\bar F^{\bar\jmath} + 2T_{ij\bar k\bar l}F^i\left(F^j\bar F^{\bar l} - \partial_a A^j\partial^a\bar A^{\bar l}\right) \\ &+ T_{ij\bar k\bar l,\bar F^{\bar n}}\left[(\partial_a A^i\partial^a A^j)(\partial_b\bar A^{\bar k}\partial^b\bar A^{\bar l}) - 2F^i\bar F^{\bar k}(\partial_a A^j\partial^a\bar A^{\bar l}) + F^iF^j\bar F^{\bar k}\bar F^{\bar l}\right] = 0 \; . \end{aligned} \tag{2.34}$$

In the next section we will analyze the equations of motion for the F^i and the scalar potential more closely.

[10] The factor of 1/16 is introduced purely for convenience here.

2.3.3 On-Shell Action and Effective Field Theory

So far we have discussed superspace actions for the general scalar potential which are off-shell by construction. Ultimately, one is interested in the on-shell action and, hence, in the solution to the equations of motion for the auxiliary fields. In this section we study the equations of motion for the auxiliary fields in the context of effective field theory and, in particular, clarify the apparent presence of multiple on-shell theories.

In the following, for the sake of simplicity and brevity we set $n_c = 1$ and perform the discussion using the Lagrangian in eq. (2.26). The fact that this Lagrangian may include kinetic terms for F is of no relevance here. We conduct a brief separate discussion of these kinetic terms in sec. 2.3.5. To emphasize the difference between terms which are already present in the two-derivative theory and those that originate from the higher-derivatives we split $\mathcal{K}$ up as follows

$$\mathcal{K}(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) = K(\Phi, \bar{\Phi}) + \mathbb{F}(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) \ , \tag{2.35}$$

such that $\mathbb{F}$ is at least linear in Ψ and/or $\bar{\Psi}$.[11] The respective off-shell scalar potential in eq. (2.26) reads

$$V = -K_{,A\bar{A}}|F|^2 - FW_{,A} - \bar{F}\bar{W}_{,\bar{A}} - |F|^2\mathbb{F}_{,A\bar{A}}(A, \bar{A}, \bar{F}, F) \ . \tag{2.36}$$

We immediately observe that the corrections to the scalar potential of the two-derivative theory are at least cubic in the auxiliary field. The on-shell scalar potential is obtained by determining the solution to the equation of motion for F, evaluating the solution at $\partial_a A = \partial_a \bar{A} = 0$ and finally inserting the result into the expression above. As we already mentioned we still have to integrate out $F, \bar{F}$ even if a kinetic term for these fields is present. Thereby, terms proportional to $\partial_a F, \partial_a \bar{F}$ only induce additional kinetic operators for $A, \bar{A}$ after solving the equations of motion for $F, \bar{F}$ and, hence, do not have to be taken into account for the discussion of the scalar potential. Therefore, we can simply ignore all kinetic contributions inside the equations of motion for F and $\bar{F}$. In particular, the equations of motion for $\bar{F}$ then read

$$K_{,A\bar{A}}F + \bar{W}_{,\bar{A}} + F\mathbb{F}_{,A\bar{A}}(A, \bar{A}, \bar{F}, F) + |F|^2\mathbb{F}_{,A\bar{A}\bar{F}}(A, \bar{A}, \bar{F}, F) = 0 \ . \tag{2.37}$$

Contrary to the situation of the ordinary two-derivative theory where the equation of motion for F is just a linear algebraic equation, in the general higher-derivative theory we have to deal with a general algebraic equation, which does not even have to be polynomial. This is due to the fact that in general $\mathbb{F}$ can be given by an infinite power series in Ψ and $\bar{\Psi}$. However, in a local theory this sum has to be finite and therefore an upper bound on the number of derivatives must exist, let us denote it as $2N$, such that

$$\mathbb{F} = \sum_{1 \leq n+m \leq N} \mathcal{T}_{nm}(\Phi, \bar{\Phi})\Psi^n\bar{\Psi}^m \ , \tag{2.38}$$

[11] The object $\mathbb{F}$ was already introduced in [15, 16] and denoted as the effective auxiliary field potential (EAFP).

where we assume that also the coefficient functions $\mathcal{T}_{ij}$ are truncated appropriately. When inserting eq. (2.38) into eq. (2.37), we observe that the equations of motion for F allow for up to $N+1$ solutions and, hence, we obtain up to $N+1$ distinct on-shell Lagrangians. In general these Lagrangians describe inequivalent dynamics for the scalar field (and chiral fermion) and, thus, we loose predictability of the classical dynamics.

In the following, we analyze the behavior of the different on-shell Lagrangians taking a bottom-up perspective. Let us assume our theory is an effective field theory valid up to some cut-off scale Λ and that all operators consistent with the symmetries are present. The relevant mass-dimensions of the fields and quantities in the Lagrangian are given by

$$\begin{aligned} [A] = [\bar{A}] = \Lambda \ , \qquad [F] = [\bar{F}] = \Lambda^2 \ , \\ [K] = [\mathbb{F}] = \Lambda^2 \ , \qquad [W] = [\bar{W}] = \Lambda^3 \ . \end{aligned} \tag{2.39}$$

It is convenient to expand $\mathcal{K}$ and W in inverse powers of this cut-off scale. The lowest order terms of this expansion read

$$\begin{aligned} K(A,\bar{A}) &= |A|^2 + \mathcal{O}(\Lambda^{-1}) \ , \\ \mathbb{F}(A,\bar{A},\bar{F},F) &= \frac{|A|^2}{\Lambda^2}(TF + \bar{T}\bar{F}) + \cdots + \mathcal{O}(\Lambda^{-3}) \ , \\ W(A) &= \Lambda^3 \left[w_0 \frac{A}{\Lambda} + w_1 \frac{A^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-3}) \right] \ , \end{aligned} \tag{2.40}$$

where for brevity we displayed only those leading order terms which contribute to the scalar potential. For instance, we did not explicitly display all operators of order $\mathcal{O}(\Lambda^{-1})$ and $\mathcal{O}(\Lambda^{-2})$ in $\mathbb{F}$, since some operators do not contribute in eq. (2.37) but only modify the kinetic terms of the on-shell Lagrangian. We observe that when performing the limit $\Lambda \to \infty$ in the above off-shell Lagrangian, we recover the IR renormalizable two-derivative theory described by a canonical Kähler potential and a superpotential with at most cubic terms, just as demanded by the decoupling principle. Let us now analyze the behavior of the respective on-shell theories.

After inserting eq. (2.40) into eq. (2.37) we expand the equation of motion in powers of Λ. It is easy to see that this expansion starts at order Λ^2. Therefore, the formal solution of eq. (2.37) can be written as

$$F = \Lambda^2 \sum_{n=0} \frac{F_{(n)}}{\Lambda^n} \ , \tag{2.41}$$

where $F_{(n)}$ has mass dimension n. After inserting the above form of F into eq. (2.37) we find that at leading order in Λ we have to solve the following equation

$$F_{(0)} + w_0 + F_{(0)}\mathbb{F}_{,A\bar{A}}(0,0,\bar{F}_{(0)},F_{(0)}) + |F_{(0)}|^2\,\mathbb{F}_{,A\bar{A}\bar{F}}(0,0,\bar{F}_{(0)},F_{(0)}) = 0 \ . \tag{2.42}$$

This equation already suffices to conceptually understand the behavior of the plethora of solutions to eq. (2.37). Let us clarify that a solution for F is uniquely determined once the solution for $F_{(0)}$ is fixed via eq. (2.42). This is due to the fact that we solve the equations of motion perturbatively in Λ. In particular, when expanding eq. (2.37) the next-to-leading order in Λ yields an equation which is linear in $F_{(1)}$ and, hence, uniquely fixes $F_{(1)}$. This linear structure persists at higher orders such that the solution is uniquely fixed. For the discussion of the solutions to eq. (2.42) we distinguish theories with and without tadpole-like terms in the superpotential.

Superpotential with Tadpole Terms

Suppose that $w_0 \neq 0$ in eq. (2.40), such that the ordinary two-derivative theory includes constant and tadpole-like terms in the on-shell Lagrangian. Then all $(N+1)$ solutions to eq. (2.42) are non-zero and the higher-derivative operators induce $\mathcal{T}_{nm}(0,0)$-dependent corrections to the constant and tadpole term in the on-shell Lagrangian. In fact, every off-shell higher-derivative, regardless of the number of superspace-derivatives, contributes to these terms. Thus, we have a sensitivity to an infinite number of operators.

Absence of Tadpole Terms

Let us now look at the theories with $w_0 = 0$. In this case eq. (2.42) admits one solution $F_{(0)} = 0$ as well as up to N additional solutions with $F_{(0)} \neq 0$. For $F_{(0)} = 0$ we find a scalar potential which agrees with the scalar potential of the two-derivative theory up to operators of mass dimension three. More precisely, it reproduces the quadratic term of the two-derivative theory, but induces $\mathcal{T}_{nm}$-dependent corrections starting at cubic order in A and $\bar{A}$ and, thus, can be regarded as contributing to the on-shell Lagrangian at subleading order. The remaining N solutions with $F_{(0)} \neq 0$, however, contribute constant and linear terms to the scalar potential, which in the infrared-regime are dominant over the quadratic terms of the two-derivative theory. In this sense they cannot be regarded as correcting the Lagrangian of the ordinary two-derivative theory at subleading order and contradict the fact that the leading order infrared dynamics are captured by K and W and, therefore, violate the decoupling principle. To emphasize this further let us consider a superpotential $W(A) \sim \Lambda^3 (A/\Lambda)^n$ for some $n > 3$. Then the scalar potential of the two-derivative theory consists purely of irrelevant mass dimension $(2n-2)$-operators, but the solutions with $F_{(0)} \neq 0$ still induce all possible relevant operators. In particular, in this situation the IR-dynamics are entirely governed by the higher-derivative contribution and not by the ordinary two-derivative part of the Lagrangian.

In summary, the above analysis shows that there always exists a unique theory given by $F_{(0)} = 0$ which is in agreement with the principles of EFT, unless the theory has spurious tadpole-like terms. The remaining theories are unphysical and we regard them as artifacts of a truncation of an infinite sum of higher-derivatives. We will substantiate this interpretation in the next section. Note that the above discussion is reminiscent of our arguments in sec. 2.1. Similarly, we saw that ghost-like degrees of freedom arise by truncating an infinite series of higher-derivative terms to a finite sum and their appearance violates the principles of EFT as well.

2.3.4 One-Loop Wess-Zumino Model

Following up on the general discussion of the previous section we now present an explicit example which allows us to study the truncation of an infinite sum of higher superspace-derivative operators to a finite sum and the structure of the equations of motion for the F. The only example known so far for which an infinite-derivative action of the form in eq. (2.30) has been determined is the given by the one-loop Wess-Zumino model. Indeed, following up on the earlier works [15, 17] in [18] an action of the form as in eq. (2.30) was computed. Let us begin by briefly summarizing the result of this reference. The Wess-Zumino model describes a renormalizable theory of a single chiral superfield with a Kähler potential and superpotential of the form

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi} \ , \qquad W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\Phi^3 \ . \tag{2.43}$$

The authors of [18] derived one-loop corrections of the form in eq. (2.30) with

$$\mathcal{F} = 0 \ , \qquad T(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) = \frac{1}{(W_{\Phi\Phi}\bar{W}_{\bar{\Phi}\bar{\Phi}})^2}\, \mathcal{G}\left(\frac{\Psi\bar{\Psi}}{(W_{\Phi\Phi}\bar{W}_{\bar{\Phi}\bar{\Phi}})^2}\right) \ , \tag{2.44}$$

where $T \equiv T_{\Phi\Phi\bar{\Phi}\bar{\Phi}}$ and $\mathcal{G}$ is a real-valued analytic function such that the coefficients in its respective series expansion are non-vanishing at all orders, see [18] for the explicit form of $\mathcal{G}$. Since the Lagrangian includes an infinite sum of higher-derivative operators, the theory is non-local.

From eq. (2.44) we find that additional powers of $\Psi\bar{\Psi}$ go hand in hand with powers of

$$\epsilon \equiv (W_{\Phi\Phi}\bar{W}_{\bar{\Phi}\bar{\Phi}})^{-2} \ . \tag{2.45}$$

Therefore, ϵ controls the couplings of the expansion of T. In this sense, it plays a role analogous to the inverse cut-off scale Λ^{-1} of a generic EFT, as we discussed it in the previous section. Therefore, even though we are not discussing an effective field theory here, the important structure can be analyzed nevertheless. Turning now to the component Lagrangian, the infinite series in powers of $|F|^2$ that emerges from eq. (2.44) is correspondingly controlled by

$$\epsilon = |m + \lambda A|^{-4} \ . \tag{2.46}$$

Using the explicit form of the function G we performed a numerical analysis of the non-polynomial equations of motion for F. As a result we find that there exists a unique solution which is analytic in ϵ.[12]

We regard the non-local theory in eq. (2.44) as a UV-completion of a local theory which is defined by truncating the infinite tower in $\Psi\bar{\Psi}$ to a finite number of terms. In addition, since ϵ is non-polynomial, it is mandatory to truncate this quantity as well in order to obtain a fully local theory. Therefore, it would be necessary to expand ϵ in some small parameter and truncate this expansion at an appropriate order. While this step can be performed in a straightforward way we omit this detail here,

[12] The numerical analysis was performed with the help of Mathematica 10.

as it does not provide additional insight into the structure of the series in higher-derivatives. Therefore, we simply continue to use the symbol ϵ as controlling the couplings of the $|F|^2$-expansion.

Let us now proceed to study the equations of motion for F in the truncated theory. In the following, let G_n denote the truncation of the series expansion of G at order n. Ignoring the kinetic terms, eq. (2.34) reads

$$F + \bar{W}' + 2\epsilon F|F|^2 G_n(\epsilon|F|^2) + \epsilon^2 F|F|^4 G_n'(\epsilon|F|^2) = 0 \ . \tag{2.47}$$

Firstly, we observe that G_n induces monomial terms in $|F|^2$ up to degree n and, hence, eq. (2.47) allows for up to $(2n+3)$ independent solutions. To solve eq. (2.47) we employ the following redefinition of the auxiliary field

$$F = \bar{W}' f \ . \tag{2.48}$$

By inserting this redefinition into eq. (2.47) we find that f has to be real-valued and, hence, eq. (2.47) reduces to

$$f + 1 + 2\epsilon f^3 |W'|^2 G_n(\epsilon f^2 |W'|^2) + \epsilon^2 f^5 |W'|^4 G_n'(\epsilon f^2 |W'|^2) = 0 \ . \tag{2.49}$$

To solve this equation we make an ansatz of the form

$$f = \sum_{i=-1}^{\infty} \epsilon^{i/2} f_i \ , \tag{2.50}$$

such that eq. (2.49) at lowest order in ϵ reads

$$f_{-1} + f_{-1}^3 |W'|^2 G_n(f_{-1}^2 |W'|^2) + f_{-1}^5 |W'|^4 G_n'(f_{-1}^2 |W'|^2) = 0 \ . \tag{2.51}$$

Since G_n is a polynomial of degree n with non-vanishing coefficients we see that only the branch given by $f_{-1} = 0$ is analytic. All other solutions, which are defined at lowest order by the remaining $2n+2$ solutions of eq. (2.51) and necessarily fulfill $f_{-1} \neq 0$, are non-analytic in ϵ for any n. These arguments are in perfect agreement with the discussion of eq. (2.42) even though, as we already mentioned we are not dealing with an EFT here.

Now we generally expect to be able to compute observables with higher precision by including additional higher-order operators. Indeed, since the unique solution of the non-local theory was analytic, the analytic solution of the truncated theory is able to reproduce the Lagrangian of the non-local theory at order ϵ^{n+1} and, thus, mimics the non-local theory with better precision for larger n. However, regardless of the order of the truncation the non-analytic theories fail to reproduce the non-local theory to that specific order. We explicitly checked this for the first components in the expansion in eq. (2.50).

2.3.5 Propagating Auxiliary Fields

Finally, let us return to the possibility that the auxiliary fields obtain a kinetic term. More precisely, when inspecting the Lagrangians in eq. (2.25) and eq. (2.26) we observe that depending on the signature of $\mathcal{K}_{,F^i\bar{F}^{\bar{\jmath}}}$ the auxiliary fields may constitute propagating degrees of freedom. The degrees of freedom associated with F^i are related via supersymmetry to higher-derivative degrees of freedom given by $\sigma^a\partial_a\chi^i$ and $\Box A^i$. As discussed in sec. 2.1 these degrees of freedom are usually unphysical ghosts. We may consider this already as strong evidence that the propagating auxiliary fields are of a similar nature. Additionally, in the spirit of the last paragraph in sec. 2.1 we argue that these degrees of freedom are generically unphysical, independent of the signature of the kinetic terms. This argument was already presented in [20] for a particular operator of kinetic type, but applies equally in our discussion. For simplicity we consider theories with $n_c = 1$, a generalization to arbitrary n_c follows immediately. Keeping track of the mass dimensions, a generic expansion of $\mathcal{K}$ reads

$$\begin{aligned}\mathcal{K}(\Phi,\Phi^\dagger,\Psi,\Psi^\dagger) = K(\Phi,\Phi^\dagger) + \Big(\mathcal{T}_{10}(\Phi,\Phi^\dagger)\Psi + \frac{1}{\Lambda^2}\mathcal{T}_{20}(\Phi,\Phi^\dagger)\Psi^2 + \text{h.c.}\Big)\\ + \frac{1}{\Lambda^2}\mathcal{T}_{11}(\Phi,\Phi^\dagger)|\Psi|^2 + \dots\ ,\end{aligned} \tag{2.52}$$

where the dots indicate higher-order terms in $\Psi, \Psi^\dagger$ and the superfields $\mathcal{T}_{10}, \mathcal{T}_{11}$ and $\mathcal{T}_{20}$ are dimensionless. The last term in the above expression yields the first contribution to the kinetic term for F. In the component version given in eq. (2.26) it reads

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{\Lambda^2}\mathcal{T}_{11}(\Phi,\Phi^\dagger)\partial_a F\partial^a\bar{F} + \dots\ . \tag{2.53}$$

Independently of the details of $\mathcal{T}_{11}$ the canonically normalized scalar field, let us denote it by $\tilde{F}$, therefore picks up a factor of $1/\Lambda$. At leading order in the expansion of $\mathcal{K}$ in Ψ and $\Psi^\dagger$ this reads $\Lambda\tilde{F} \sim F$. Thus, when recasting the scalar potential as given in eq. (2.36) in terms of $\tilde{F}$ we pick up an additional factor of Λ^2 in the terms that are at least quadratic in F and $\bar{F}$ respectively. Generically, we therefore expect $\tilde{F}$ to have a mass of order Λ. This is in contradiction with the assumption that we are dealing with a low-energy effective field theory describing physics below the cut-off Λ and indicates that $\tilde{F}$ is not a physical degree of freedom and should be integrated out. Hence, we must continue to treat the auxiliary fields as algebraic degrees of freedom. In this case the kinetic terms in eq. (2.53) yield kinetic and higher-derivative terms for $A, \bar{A}$ after eliminating $F, \bar{F}$ via their respective equations of motion.

We now end the discussion of the effective scalar potential in global supersymmetry and turn to the case of local supersymmetry.

2.4 Superspace Formulation of $\mathcal{N} = 1, D = 4$ Supergravity

So far we have considered theories with global $\mathcal{N} = 1$ supersymmetry which are formulated in flat superspace. We now turn to $\mathcal{N} = 1$ supergravity which is the correct framework to study effective actions obtained from string compactifications. Before we can discuss higher-derivative theories of supergravity we first have to establish the basics conventions and notation, which will be the subject of this section. In the next section we then review the construction of the two-derivative action for supergravity coupled to chiral multiplets. Afterwards, we turn to the investigation of higher-derivatives.

Over the years several formalisms to construct supergravity actions have been engineered including superspace-techniques as well as superconformal methods. Even though more elegant versions of curved superspace, such as $U(1)$-superspace [47] or conformal superspace [48], exist, we continue to use the ordinary Wess and Bagger superspace-formalism and, hence, adopt the conventions and notations of [34]. This has the advantage that we can directly compare our results to the existing literature on higher-derivative supergravity, in particular to [20], but also to the results of the rigid theory in the preceding sections. The superspace-formulation of supergravity is highly reminiscent of the construction of ordinary gravity and involves studying the differential geometry of curved superspace. In this section we begin by reviewing the formalism and basic notions of the differential geometry of curved superspace along the lines of [34].

We choose curved superspace to be locally parametrized by the variables

$$z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}}) \ , \qquad m = 0, \ldots, 3 \ , \qquad \mu, \dot{\mu} = 1, 2 \ . \tag{2.54}$$

In the following $m, n, \ldots$ denote curved spacetime indices and $\mu, \nu, \ldots$ $(\dot{\mu}, \dot{\nu}, \ldots)$ curved Grassmannian indices. The convention of summing over superspace indices reads

$$\mathrm{d}z^M \omega_M = \mathrm{d}x^m \omega_m + \mathrm{d}\theta^\mu \omega_\mu + \mathrm{d}\bar{\theta}_{\dot{\mu}} \omega^{\dot{\mu}} \ . \tag{2.55}$$

The geometry of curved superspace is described by a super-vielbein E_M^A together with a connection $\Omega_{MA}{}^B$. The vielbein converts curved superspace indices to local flat superspace indices. Moreover, the connection allows us to introduce a super-covariant derivative, which for instance acts on a vector field V^A as

$$\mathcal{D}_M V^A = \partial_M V^A + (-1)^{mb} V^B \Omega_{MB}{}^A \ , \tag{2.56}$$

where m, b take values $0(1)$ if M, B are vector (spinor) indices. This covariant derivative is the curved superspace analogue of the flat superspace derivative in eqs. (2.9), (2.10). Naturally, we can define a torsion

$$T_{MN}{}^A = \mathcal{D}_N E_M^A - (-1)^{nm} \mathcal{D}_M E_N^A \ , \tag{2.57}$$

and, similarly, the super-curvature tensor

$$\begin{aligned} R_{NMA}{}^B = {} & \partial_N \Omega_{MA}{}^B - (-1)^{nm} \partial_M \Omega_{NA}{}^B + (-1)^{n(m+a+c)} \Omega_{MA}{}^C \Omega_{NC}{}^B \\ & - (-1)^{m(a+c)} \Omega_{NA}{}^C \Omega_{MC}{}^B \ . \end{aligned} \tag{2.58}$$

The torsion and curvature are the relevant objects required to express any geometric quantity of curved superspace. In particular they determine the curved superspace analogue of the (anti-) commutation relations in eq. (2.11). More precisely, we have

$$(\mathcal{D}_C \mathcal{D}_B - (-1)^{bc} \mathcal{D}_B \mathcal{D}_C) V^A = (-1)^{d(c+b)} V^D R_{CBD}{}^A - T_{CB}{}^D \mathcal{D}_D V^A \ . \tag{2.59}$$

The algebra of super-covariant derivatives plays an important role later in the discussion of higher-derivatives and we will make extensive use of it.

In analogy to ordinary gravity, the vielbein and the connection and, hence, the torsion and curvature describe the gravitational degrees of freedom. These objects contain a large number of component superfields, which can be reduced to a minimal set of superfields by imposing constraints on the torsion. These constraints have to be chosen, such that they reproduce the flat SUSY algebra in the rigid limit and such that they allow to consistently define covariantly chiral superfields. Here we follow the conventions of old minimal supergravity, see [34] for the constraints on the torsion. The next step involves solving the respective Bianchi identities for the curvature and torsion. Certain components of the torsion and curvature remain non-vanishing, some of which are displayed in appendix A.3, and they can be expressed entirely in terms of the vielbein as well as the following superfields

$$R\ , \quad \bar{R}\ , \quad G_a\ , \quad W_{\alpha\beta\gamma}\ , \quad \bar{W}_{\dot\alpha\dot\beta\dot\gamma}\ . \tag{2.60}$$

The superfields R and $W_{\alpha\beta\gamma}$ are covariantly chiral, that is they satisfy

$$\bar{\mathcal{D}}_{\dot\alpha} R = \bar{\mathcal{D}}_{\dot\alpha} W_{\alpha\beta\gamma} = 0\ , \tag{2.61}$$

while $\bar{R}$, $\bar{W}_{\dot\alpha\dot\beta\dot\gamma}$ denote their conjugate superfields, which are covariantly anti-chiral. Moreover, G_a is a real superfield $\bar{G}_a = G_a$. Note also that $W_{\alpha\beta\gamma}$ and $\bar{W}_{\dot\alpha\dot\beta\dot\gamma}$ are completely symmetric tensors.

As explained in [34] one may utilize the gauge symmetry to partially gauge-fix the superfields in eq. (2.60) as well as the super-vielbein. For instance, we may fix the higher components of E_M^A, but leave the $\theta = \bar\theta = 0$ component unfixed. This procedure yields

$$E_M^A|_{\theta=\bar\theta=0} = \begin{pmatrix} e_m^a & \frac{1}{2}\psi_m^\alpha & \frac{1}{2}\bar\psi_{m\dot\alpha} \\ 0 & \delta_\mu^\alpha & 0 \\ 0 & 0 & \delta_{\dot\alpha}^{\dot\mu} \end{pmatrix}\ , \tag{2.62}$$

where e_m^a denotes the graviton and ψ_m^α the gravitino. Similarly, gauge-fixing the superfields R and G_a leaves only their $\theta = \bar\theta = 0$ components as degrees of freedom, which are denoted as

$$R| = -\tfrac{1}{6}M\ , \qquad G_a| = -\tfrac{1}{3}b_a\ , \tag{2.63}$$

where from now on we use the convention $R| \equiv R|_{\theta=\bar\theta=0}$ for any superfield.[13] In eq. (2.63) we find a complex auxiliary scalar M as well as a real auxiliary vector b_a. These auxiliary fields are necessary in order to match the off-shell counting of bosonic and fermionic degrees of freedom. Altogether the gravitational multiplet encompasses the component fields $(e_m^a, \psi_m^\alpha, M, b_a)$.

In the following we are interested in the coupling of covariantly chiral multiplets to supergravity. In analogy to flat superspace, covariantly chiral multiplets are defined by the condition $\bar{\mathcal{D}}_{\dot\alpha}\Phi^i = 0$ and have the following components

$$A^i = \Phi^i|\ , \qquad \chi_\alpha^i = \tfrac{1}{\sqrt{2}}\mathcal{D}_\alpha\Phi^i|\ , \qquad F^i = -\tfrac{1}{4}\mathcal{D}^2\Phi^i|\ , \tag{2.64}$$

[13] Correspondingly, the lowest component of $W_{\alpha\beta\gamma}$ is given as a symmetrized combination of the gravitino [34].

where $\mathcal{D}^2 = \mathcal{D}^\alpha \mathcal{D}_\alpha$. Similarly, we use the notation $\bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}$. It is convenient to introduce a new Grassmann variable Θ_α, such that

$$\Phi^i = A^i + \sqrt{2}\Theta^\alpha \chi^i_\alpha + \Theta^\alpha\Theta_\alpha F^i \ . \tag{2.65}$$

Correspondingly we can introduce a differentiation and integration with respect to Θ_α. In particular we have that

$$\int \mathrm{d}^2\Theta \, f \equiv -\tfrac{1}{4}\mathcal{D}^2 f| \tag{2.66}$$

for any superfield f. Via the Bianchi identities one can determine the Θ-expansion of the superfield R. Displaying only the bosonic terms it reads

$$R = -\tfrac{1}{6}\left[M + \Theta^2\left(-\tfrac{1}{2}\mathcal{R} + \tfrac{2}{3}|M|^2 + \tfrac{1}{3}b_a b^a - i e_a^m \mathcal{D}_m b^a\right)\right] \ , \tag{2.67}$$

where $\mathcal{R}$ denotes the scalar (spacetime) curvature. The formalism of this section provides the tools to compute the component form for actions of matter-coupled supergravity. In particular it will be necessary to determine components of superfields with several covariant derivatives. The general rule here is to iteratively apply the (anti-) commutation relations in eq. (2.59) until the number of covariant derivatives has reduced enough such that the respective component can be related to already known objects such as the components in eqs. (2.64), (2.67). In appendix A.3 we apply this algorithm to several superfields, whose components are important in the following sections.

2.4.1 Ordinary Matter-Coupled Supergravity

An action for supergravity coupled to chiral matter can be constructed following the methods of flat superspace. We begin by reviewing the two-derivative Lagrangian in order to familiarize ourselves with the derivation of the respective component action. Furthermore, we revert back to some key formulas later on during the discussion of higher-derivative operators.

Given some scalar superfield U, we can construct a super-diffeomorphism invariant action via the integral

$$S_U = \frac{1}{\kappa^2}\int \mathrm{d}^8 z E U \ , \tag{2.68}$$

where E denotes the super-determinant of the super-vielbein, $\mathrm{d}^8 z$ the measure on curved superspace and $\kappa = M_p^{-1}$ the inverse Planck mass, which in the following we set to one.[14] To ensure that S_U is real, we have to consider a real scalar superfield U. Matter-coupled supergravity is constructed from the following superfield

$$U_{(0)} = -3\mathrm{e}^{-K(\Phi,\bar\Phi)/3} + \frac{W(\Phi)}{2R} + \frac{\bar W(\bar\Phi)}{2\bar R} \ , \tag{2.69}$$

[14] If one is interested in the precise mass-scales appearing, it is straightforward to reintroduce κ at any point.

where K is the Kähler potential and W the superpotential. For the purpose of computing the component form of the action it is convenient to rewrite the action by expressing it via a chiral integral. In flat superspace we can replace the $\mathrm{d}^2\bar\theta$ measure by the object $\bar D^2$, which projects an arbitrary superfield onto a chiral superfield. Due to the more complicated algebra of derivatives in eq. (2.59) the object $\bar{\mathcal{D}}^2$ does not project onto chiral superfields. Instead the proper curved space generalization is given by $(\bar{\mathcal{D}}^2 - 8R)$, which indeed has the property

$$\bar{\mathcal{D}}_{\dot\alpha}(\bar{\mathcal{D}}^2 - 8R)\mathcal{S} = 0 \tag{2.70}$$

for any scalar superfield S.[15] Owing to this property the object $(\bar{\mathcal{D}}^2 - 8R)$ is denoted as chiral projector. Hence, to construct an action for supergravity we have to replace the $\mathrm{d}^2\bar\theta$ integration by the chiral projector. Altogether, the action can be rewritten as

$$S_{(0)} = \int \mathrm{d}^4x \int \mathrm{d}^2\Theta\, \mathcal{E}\left[\tfrac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-\frac{1}{3}K(\Phi,\bar\Phi)} + W(\Phi)\right] + h.c.\ , \tag{2.71}$$

where the object $\mathcal{E}$, denoted as chiral density, is a chiral superfield and enjoys the expansion

$$\mathcal{E} = e(1 - \Theta^2\bar M)\ , \tag{2.72}$$

where e denotes the determinant of the vielbein e_m^a and we displayed only the bosonic components.

We now review the derivation of the component version of eq. (2.71). In the following, we ignore the fermionic terms to shorten notation. Using eq. (2.66), eq. (2.72) as well as the components in eq. (2.64) and eq. (2.67) one finds

$$\begin{aligned}\mathcal{L}_{(0)}/e = &-\tfrac{3}{32}\mathcal{D}^2\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}| - \tfrac{3}{32}\bar{\mathcal{D}}^2\mathcal{D}^2\mathrm{e}^{-K/3}| - \tfrac{1}{2}\bar M\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}| - \tfrac{1}{2}M\mathcal{D}^2\mathrm{e}^{-K/3}| \\ &+ W_{,i}F^i + \bar W_{,\bar\jmath}\bar F^{\bar\jmath} - W\bar M - \bar W M + \mathrm{e}^{-K/3}|\left(-\tfrac{1}{2}\mathcal{R} - \tfrac{1}{3}|M|^2 + \tfrac{1}{3}b_a b^a\right)\ .\end{aligned} \tag{2.73}$$

To proceed, we have to compute the respective $\theta = \bar\theta = 0$-components of the superfields which appear in the above Lagrangian. Furthermore, we observe that $\mathcal{L}_{(0)}$ is not expressed in the Einstein frame. To obtain an Einstein frame action we perform a Weyl-rescaling of the metric as follows

$$g_{mn} \longrightarrow \tilde g_{mn} = g_{mn}\,\mathrm{e}^{-K/3}|\ . \tag{2.74}$$

After replacing the superfields with their component versions the resulting Weyl-transformed Lagrangian reads

$$\begin{aligned}\mathcal{L}_{(0)}/e = &-\tfrac{1}{2}\mathcal{R} - \tfrac{3}{4}e^{2K/3}\partial_m(e^{-K/3})\partial^m(e^{-K/3}) + \tfrac{1}{3}b_a b^a + \text{total derivative} \\ &+ e^{2K/3}(-\bar M W - M\bar W + W_i F^i + \bar W_{\bar\jmath}\bar F^{\bar\jmath}) + K_{i\bar\jmath}\,e^{K/3}F^i\bar F^{\bar\jmath} \\ &- \tfrac{1}{3}e^{K/3}(M + K_{\bar\jmath}\bar F^{\bar\jmath})(\bar M + K_i F^i) - (K_{i\bar\jmath} - \tfrac{1}{3}K_i K_{\bar\jmath})\partial_m A^i\partial^m\bar A^{\bar\jmath} \\ &- \tfrac{i}{3}b^a e_a^m(K_i\partial_m A^i - K_{\bar\jmath}\partial_m\bar A^{\bar\jmath})\ .\end{aligned} \tag{2.75}$$

[15] Eq. (2.70) still holds for tensor superfields of the type $\mathcal{S}_{\alpha_1\ldots\alpha_n}$.

Next we successively integrate out the auxiliary fields. The equations of motion for the auxiliary vector yield

$$b^a_{(0)} = \tfrac{i}{2}\eta^{ab}e^m_b(K_i\partial_m A^i - K_{\bar{\jmath}}\partial_m \bar{A}^{\bar{\jmath}}) \ . \tag{2.76}$$

Inserting this into the component Lagrangian we find

$$\begin{aligned}\mathcal{L}_{(0)}/e = &-\tfrac{1}{2}\mathcal{R} - K_{i\bar{\jmath}}\,\partial_m A^i\partial^m\bar{A}^{\bar{\jmath}} - \tfrac{1}{3}e^{K/3}(M + K_{\bar{\jmath}}\bar{F}^{\bar{\jmath}})(\bar{M} + K_iF^i)\\ &+ K_{i\bar{\jmath}}\,e^{K/3}F^i\bar{F}^{\bar{\jmath}} + e^{2K/3}(-\bar{M}W - M\bar{W} + W_iF^i + \bar{W}_{\bar{\jmath}}\bar{F}^{\bar{\jmath}}) \ .\end{aligned} \tag{2.77}$$

The equations of motion for M are solved by

$$\bar{M}_{(0)} = -(K_iF^i + 3\bar{W}e^{K/3}) \ , \tag{2.78}$$

which, when inserted back into $\mathcal{L}_{(0)}$, yields

$$\begin{aligned}\mathcal{L}_{(0)}/e = &-\tfrac{1}{2}\mathcal{R} - K_{i\bar{\jmath}}\,\partial_m A^i\partial^m\bar{A}^{\bar{\jmath}} + K_{i\bar{\jmath}}\,e^{K/3}F^i\bar{F}^{\bar{\jmath}} + 3|W|^2e^K\\ &+ e^{2K/3}(D_iWF^i + D_{\bar{\jmath}}\bar{W}\bar{F}^{\bar{\jmath}}) \ .\end{aligned} \tag{2.79}$$

Lastly the equations of motion for the chiral auxiliary fields read

$$\bar{F}^{\bar{\jmath}}_{(0)} = -e^{K/3}K^{i\bar{\jmath}}D_iW \ , \tag{2.80}$$

which results in the familiar scalar potential

$$V_{(0)} = e^K(K^{i\bar{\jmath}}D_iWD_{\bar{\jmath}}\bar{W} - 3|W|^2) \ , \tag{2.81}$$

where $D_iW = W_i + K_iW$ denotes the Kähler-covariant derivative.

2.5 Higher-Derivatives in Curved $\mathcal{N} = 1, D = 4$ Superspace

2.5.1 Higher-Derivative Supergravity: Preliminaries

We now proceed to discuss higher-derivative operators in curved superspace. To shorten notation we consider only a single chiral field Φ from now on. The multi-field generalization is straightforward and can always be performed in the final component Lagrangian. Later on in sec. (2.5.7) we present an explicit multi-field example. A generic supergravity Lagrangian including higher-derivative operators can be constructed from a superspace-integral of the form in eq. (2.68). More precisely, the general higher-derivative theory reads

$$\begin{aligned}\mathcal{L}_{hd} = \int \mathrm{d}^4\theta E\Big[&-3U(\Phi,\bar{\Phi},R,\bar{R},G_a,W_{\alpha\beta\gamma},\bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}},\mathcal{D}_AR,\mathcal{D}_A\Phi,\mathcal{D}_A\bar{\Phi},\dots)\\ &+\frac{1}{2R}W(\Phi,R,\dots) + \frac{1}{2R^\dagger}\bar{W}(\bar{\Phi},\bar{R},\dots)\Big] \ ,\end{aligned} \tag{2.82}$$

where the superpotential W is allowed to depend on those higher-derivative superfields which are chiral and the dots indicate further super-covariant derivatives acting on the relevant superfields. Firstly, let us emphasize the importance of the dependence of U and W on the gravitational superfields $R, \bar{R}, G_a, W_{\alpha\beta\gamma}, \bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}}$ and derivatives thereof. Even if we only care about higher-derivatives for the chiral multiplets, these superfields must be included in the Lagrangian as they are related to higher-derivative operators for the multiplets $\Phi, \bar{\Phi}$ via the algebra of super-covariant derivatives. A simple illustrative example is given by the operator $\mathcal{D}^4\Phi$, which by means of eq. (2.70) and, hence, implicitly eq. (2.11), we can rewrite as $\mathcal{D}^4\Phi = 8\bar{R}\mathcal{D}^2\Phi$.

As before in the global case, we are interested in higher-derivative operators that contribute to the effective scalar potential for chiral multiplets. The result of flat superspace, that the general scalar potential can be derived from the superspace-Lagrangian in eq. (2.25), does not hold in curved superspace. The reason is the existence of the additional complex auxiliary field M, which can give rise to new corrections to the scalar potential.[16] The complex auxiliary field M is also (and together with the Weyl-rescaling) responsible for the difference between $V_{(0)}$ in eq. (2.81) and its rigid limit. An attempt to classify these new corrections to the scalar potential would eventually have to overcome the complications that are induced by the algebra of super-covariant derivatives. More precisely, for all operators which we discuss in the next sections, the algebra of super-covariant derivatives induces relations between these higher-derivative operators and corrections to the scalar potential.[17] As a particular example, consider the (flat superspace) operator $D_a\Phi D^a\bar{\Phi}$. From eq. (2.22) we know that this operator does not correct the scalar potential. However, the appropriate curved superspace generalization $\mathcal{D}_a\Phi\mathcal{D}^a\bar{\Phi}$ does contribute to the scalar potential as shown in [20].

In conclusion, we will not attempt to derive the general form of the scalar potential in curved superspace. Nevertheless, let us conjecture a possible Lagrangian that should yield the complete scalar potential. Naively, all we have to do is to allow for a generic dependence on F, $\bar{F}$ but also on M and $\bar{M}$. Therefore, we consider the Lagrangian in eq. (2.82) with

$$U = \mathrm{e}^{-\frac{1}{3}K(\Phi,\bar{\Phi},\Psi,\bar{\Psi},R,\bar{R})} \, , \qquad \Psi = (\bar{\mathcal{D}}^2 - 8R)\bar{\Phi} \, , \qquad \bar{\Psi} = (\mathcal{D}^2 - 8\bar{R})\Phi \, . \qquad (2.83)$$

Note that we could also choose K to be a function of $\mathcal{D}^2\Phi, \bar{\mathcal{D}}^2\bar{\Phi}$ instead of $\Psi, \bar{\Psi}$. This way the dependence of F and $\bar{F}$ is more clean, but the price to pay is that $\mathcal{D}^2\Phi, \bar{\mathcal{D}}^2\bar{\Phi}$ are not covariantly (anti-) chiral which in turn complicates the computation of the component action. We leave the verification/falsification of the above Lagrangian to future research. Instead we now turn our attention to particular higher-derivative operators. More precisely, the goal of the following sections is to classify the leading order and next-to-leading order higher-derivative operators and determine their component forms.

[16] Due to the index structure of b_a, the auxiliary vector purely couples to kinetic terms and, hence, does not affect the scalar potential.

[17] There exist operators which do not contribute to V and are obtained by taking linear combinations of the operators which we discuss later on.

Let us first demonstrate that we do not have to include higher-derivatives in the superpotential. To see this we rewrite eq. (2.82) as a chiral integral in the spirit of eq. (2.71), which yields

$$\mathcal{L}_{hd} = \int \mathrm{d}^2\Theta\, \mathcal{E}\left[\tfrac{3}{8}(\bar{\mathcal{D}}^2 - 8R)U + W\right] + h.c.\ . \tag{2.84}$$

Higher-derivative superfields appearing in the superpotential must necessarily be covariantly chiral. However, an arbitrary chiral superfield $\mathcal{C}$ can always be written as $\mathcal{C} = (\bar{\mathcal{D}}^2 - 8R)\mathcal{S}$ for an appropriate scalar superfield $\mathcal{S}$ and, hence, be absorbed into a term inside U. Thus, unless explicitly stated otherwise we discuss higher-derivative operators as contributing to U.[18]

From now on we consider a generic effective supergravity with a cut-off scale $\Lambda \leq M_p$. There are two different situations of interest

$$(1):\ \Lambda = M_p\ , \qquad (2):\ \Lambda < M_p\ . \tag{2.85}$$

In the first case the effective operators are generated by Planckian physics. For instance they may be induced directly from string theory. The second case could, for instance, correspond to an effective supergravity where heavy fields are integrated out whose mass is not much smaller than the Planck mass. In this situation the terms in the higher-derivative Lagrangian are suppressed by Λ and/or M_p. In the following we are more interested in scenario (1) and, hence, we do not distinguish between Λ and M_p any further, but just collectively assume that operators are Λ-suppressed. Should one be interested in case (2), then the proper mass scales can always be reintroduced at a later stage. In particular, operators involving the gravitational superfields should always be M_p-suppressed. In conclusion, we expand the superfield U in eq. (2.82) in inverse powers of Λ and truncate this series at an appropriate order. In effective supergravities descending from string compactifications the Kähler potential is typically given by a non-local function. This, in turn, suggests that the couplings of the higher-derivative operators may also be given by non-local functions. Therefore, in the following we distinguish higher-derivative operators only by their dependence on

$$(\mathcal{D}_A, \quad R, \quad \bar{R}, \quad G_a, \quad W_{\alpha\beta\gamma}, \quad \bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}})\ , \tag{2.86}$$

but we explicitly do not distinguish operators that differ by a dependence on Φ and $\bar{\Phi}$ alone. To make this statement more clear, let us define a higher-derivative operator of order N as an operator where the collective mass-dimension of the objects in eq. (2.86) appearing in this operators is given by $\Lambda^{N/2}$. The mass-dimensions of the individual objects read

$$\begin{gathered}[\mathcal{D}_\alpha] = [\bar{\mathcal{D}}_{\dot{\alpha}}] = \Lambda^{1/2}\ , \qquad [\mathcal{D}_a] = [R] = [\bar{R}] = [G_a] = \Lambda\ , \\ [W_{\alpha\beta\gamma}] = [\bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}}] = \Lambda^{3/2}\ .\end{gathered} \tag{2.87}$$

[18] However, it is sometimes convenient to analyze certain operators via corrections to W, we will later on turn to explicit examples of this.

Class	# Operators	Section	Form
$N = 2$	2	sec. 2.5.5	eq. (2.101)
$N = 4$, HC	3*	sec. 2.5.2	eq. (2.89)
$N = 4$, EH	9	sec. 2.5.6	tab. 2.3

Table 2.2: *Overview of different classes of operators. Here HC stands for higher curvature operators, EH for operators which induce purely Einstein-Hilbert ordinary gravity. Visible are also the sections in which these classes are discussed and the equations respectively the table in which the form of these operators is displayed. The number of independent higher curvature operators depends on whether matter-coupling is taken into account or not. If not then the super Gauss-Bonnet theorem shown in eq.* (2.90) *reduces the number of operators further.*

For the convenience of the reader we now give an outline of the results of the remaining sections. In the following we simplify our discussion by explicitly distinguishing between those operators which induce higher-curvature terms and, hence, higher-derivatives for the gravitational sector and those that do not. Let us from now on refer to the former class of operators as higher-curvature operators. The higher-curvature operators will be the subject of the next section. Afterwards, we turn to the analysis of higher-derivative operators which are not higher-curvature operators. We classify the $N = 2$ and $N = 4$ operators of this type and determine their component actions in sec. 2.5.5 and sec. 2.5.6. For the sake of clarity we provide a brief outline in tab. 2.2 which includes references to the respective sections, the number of independent operators which result from our analysis as well as key formulas.

2.5.2 Higher-Curvature Superspace Operators

We begin our analysis by discussing first those higher-derivative operators that include higher-curvature terms in their component forms. Note that these have been studied in the past [9–12, 49] and we briefly summarize the essential information on them here.[19] In the later sections, the only exception being appendix A.5, we do not include them in the analysis anymore. In ordinary gravity the leading order four-derivative corrections to the Einstein-Hilbert term are of the form [45, 46]

$$S_{\mathcal{R}^2} = \int \mathrm{d}^4x\, e \left(\lambda_1 \mathcal{R}^2 + \lambda_2 \mathcal{R}_{mn}\mathcal{R}^{mn} + \lambda_3 \mathcal{R}_{mnpq}\mathcal{R}^{mnpq} + \lambda_4 \Box \mathcal{R}\right) \ , \tag{2.88}$$

where $\mathcal{R}_{mn}$ and $\mathcal{R}_{mnpq}$ denote the (spacetime-) Ricci and Riemann tensors. The last term is a total derivative and, hence, can be ignored here. Using the Gauss-Bonnet theorem, which relates a particular linear combination of the above operators to a topological invariant, one can in fact simplify the four-derivative action to include

[19] Note that higher-curvature operators have also been studied in new minimal supergravity [50] and in $U(1)$-superspace [51, 52].

merely the $\mathcal{R}^2$ and $\mathcal{R}_{mn}\mathcal{R}^{mn}$ operators. A generalization of $S_{\mathcal{R}^2}$ to supergravity is given by the action [11]

$$S'_{\mathcal{R}^2} = \int \mathrm{d}^8 z E \left(c_1 R\bar{R} + c_2 G^a G_a + \frac{c_3}{R} W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} + h.c. \right) . \tag{2.89}$$

The component version of the first operator includes $\mathcal{R}^2$-terms, while that of the second includes $\mathcal{R}^2$- and $\mathcal{R}_{mn}\mathcal{R}^{mn}$-terms and that of the third is given by the square of the Weyl-tensor.[20] In particular, one can demonstrate the superspace-version of the Gauss-Bonnet theorem [9–11]

$$\int \mathrm{d}^8 z E \left(16 R\bar{R} + 8 G^a G_a + \frac{2}{R} W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} + h.c. \right) = 32\pi^2 \chi , \tag{2.90}$$

where χ denotes the Euler number.

Furthermore, one may also consider operators with derivatives of the gravitational superfields, such as $\mathcal{D}^2 R$ and $\mathcal{D}_a G^a$.[21] However, these operators do not include higher-curvature terms in their respective component expressions. We return to the discussion of these operators in the context of higher-derivative operators for the chiral multiplets, in particular in appendix A.4.

2.5.3 Non-Minimal Coupling and Integrating out Fields

Next we investigate an important conceptual question which naturally arises in the context of effective field theories with non-minimal couplings to gravity and higher-curvature terms. More precisely, we discuss the difference between those higher-curvature terms which are present in the off-shell theory and those that arise by integrating out fields (both propagating as well as auxiliary fields). So far the higher-curvature superspace operators, in particular those in eq. (2.89), induce higher-curvature terms purely off-shell. These have to be contrasted to those operators which induce higher-curvature terms after integrating out auxiliary fields. To illustrate what this means, consider as a first example a (not necessarily supersymmetric) theory of a collection of scalar fields $\phi_1, \ldots, \phi_n$ coupled to gravity subject to the Lagrangian

$$\mathcal{L}/e = -\tfrac{1}{2}\mathcal{R} + \tilde{\mathcal{L}}(\phi_1, \ldots, \phi_n) . \tag{2.91}$$

Now, suppose one of the scalars, say ϕ_1, has a mass M much larger than the masses of the remaining scalars, that is $M \gg m_2, \ldots, m_n$, and we want to integrate it out to obtain an effective theory for physics at scales much smaller than M. We can integrate ϕ_1 out in any frame of our choice, such as the Einstein frame or any other frame, in which ϕ_1 couples non-minimally to $\mathcal{R}$. However, while integrating out in the Einstein frame results in a Lagrangian of the form

$$\mathcal{L}_{EFT}/e = -\tfrac{1}{2}\mathcal{R} + \mathcal{L}_{\text{effective}}(\phi_2, \ldots, \phi_n) , \tag{2.92}$$

[20] A matter-coupled version of eq. (2.89) was investigated in [12].
[21] In fact these two operators are related via the identity $\mathcal{D}^2 R - \bar{\mathcal{D}}^2 \bar{R} = 4i\mathcal{D}_a G^a$.

in another frame with non-minimal coupling between ϕ_1 and $\mathcal{R}$ we find

$$\mathcal{L}'_{EFT}/e = f(\mathcal{R}) + \mathcal{L}'_{\text{effective}}(\mathcal{R}, \phi_2, \ldots, \phi_n) \ , \tag{2.93}$$

for some particular function $f(\mathcal{R})$. It is well-known that such a theory of gravity can be recast into the form of an Einstein-Hilbert term minimally coupled to a real scalar by performing a Weyl-transformation. One might wonder, how this additional degree of freedom emerged, since all we did was to choose a different frame prior to integrating out ϕ_1. In fact, this additional degree of freedom is nothing but ϕ_1, which is reintroduced into the spectrum and by performing the Weyl-transformation we retain the original theory in eq. (2.91). This example shows that degrees of freedom should always be integrated out in the Einstein frame, as otherwise one might obtain $f(\mathcal{R})$ theories, which are merely dual descriptions of the to-be-integrated-out degrees of freedom.

Let us now analyze a second example, where the previous issue arises in the context of integrating out auxiliary fields, and which is relevant in the discussion of higher-derivative supergravity. Consider the following action

$$S_g = \int \mathrm{d}^8 z E(g(R) + \bar{g}(\bar{R})) \ . \tag{2.94}$$

At the superspace level one can demonstrate that this action is equivalent to an ordinary Einstein-Hilbert superspace action coupled to a covariantly chiral superfield Σ [53, 54], thus, displaying a superspace generalization of the duality between $f(\mathcal{R})$ theories of gravity and Einstein-Hilbert gravity coupled to a real scalar field. This equivalence can also be understood at the component level. The off-shell component form of S_g does not contain any higher-curvature terms, but includes a coupling of the auxiliary field M to the scalar curvature.

On the one hand, we may rewrite the action in the Einstein frame by performing a Weyl-transformation. Thereby the auxiliary field M, since it enters in the Weyl-factor, picks up a kinetic term and, hence, constitutes the complex scalar of the new chiral superfield Σ. On the other hand, we could choose to integrate out M before performing a Weyl-transformation. This way we obtain a particular $f(\mathcal{R})$-theory. Both procedures agree with each other after using the duality between $f(\mathcal{R})$ theories and ordinary general relativity coupled to a real scalar. In this sense we have a situation similar to the previous example in eq. (2.91), i.e. higher-curvature terms emerge when auxiliary fields are integrated out in a frame in which they are non-minimally coupled to $\mathcal{R}$. Again we interpret these $f(\mathcal{R})$-theories as dual descriptions, which encode the dynamics of the auxiliary fields. At this point we could stop our analysis if these propagating auxiliary fields would be part of the physical spectrum. However, quite similar to the discussion in sec. 2.3.5, these propagating auxiliary fields are, despite having the correct sign of the kinetic terms, unphysical in the context of effective field theory as they generically have a mass at the cutoff-scale of the EFT.[22] This is explicitly demonstrated in [32].

[22] Note also that the fermionic component of Σ is given by a higher-derivative of the gravitino [54], which constitutes an additional degrees of freedom via the standard Ostrogradski procedure. This

Altogether, we conclude that we should avoid the description of (unphysical) propagating auxiliary degrees of freedom via higher-curvature terms. This can most conveniently be done by choosing to perform a Weyl-transformation to the Einstein frame before any auxiliary field is integrated out.

2.5.4 Component Form of Operators and On-Shell Results

After discussion of the former conceptual points we now turn to the analysis of those higher-derivative operators which do not induce higher-curvature terms. To begin with it is necessary to introduce some computational tools and formulas, which comprise the topic of this section. In particular, we present an algorithm that allows to determine the final on-shell component version of a given operator. In the upcoming sections we then apply these formulas to determine the component forms of the $N = 2$ and $N = 4$ higher-derivative operators.

In the following we consider a particular higher-derivative operator $\mathcal{O}$ coupled to the general two-derivative theory given in eq. (2.71). To this end we regard the following Lagrangian

$$\mathcal{L}_{\mathcal{O}} = \mathcal{L}_{(0)} + \hat{\mathcal{L}}_{\mathcal{O}} \,, \qquad \text{where} \qquad \hat{\mathcal{L}}_{\mathcal{O}} = \frac{3}{4}\int \mathrm{d}^2\Theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\,\mathcal{O} + h.c. \,. \tag{2.95}$$

To reduce the computational effort for determining the component Lagrangian of a specific operator, it is convenient to note that [55]

$$\int \mathrm{d}^2\Theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{O} + h.c. = \int \mathrm{d}^2\Theta\, \mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{O}^\dagger + h.c. + \text{total derivative} \,. \tag{2.96}$$

In turn, this implies that it is not necessary to consider $\mathcal{O}$ as a real operator in eq. (2.95), but that instead it is sufficient to consider a complex operator $\mathcal{O}$. Therefore, in the following we always simply use complex operators $\mathcal{O}$ without adding $\mathcal{O}^\dagger$. With this in mind, we rewrite the higher-derivative contribution in eq. (2.95) as

$$\begin{aligned}\hat{\mathcal{L}}_{\mathcal{O}}/e = &-\tfrac{3}{16}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{O}| - \tfrac{3}{4}\bar{M}\bar{\mathcal{D}}^2\mathcal{O}| - \tfrac{1}{4}M\mathcal{D}^2\mathcal{O}| \\ &+ \mathcal{O}|\left(-\tfrac{1}{2}\mathcal{R} - \tfrac{1}{3}|M|^2 + \tfrac{1}{3}b_a b^a - i\mathcal{D}_m b^m\right) + h.c. \,.\end{aligned} \tag{2.97}$$

Next let us provide an algorithm to determine the respective component Lagrangian, which respects the principles of EFT.

I. We begin by computing the respective components version of the objects appearing in eq. (2.97).

II. Perform the Weyl-rescaling of the spacetime-metric. Depending on the operator $\mathcal{O}$, the Weyl-rescaling can be affected by the terms in eq. (2.97) and, thus, differ from the two-derivative version in eq. (2.74).

should alert us, that care has to be taken with the proper interpretation of our theory. As we have mentioned, in the EFT-context degrees of freedom that are associated with (usually ghost-like) higher-derivatives are unphysical since they emerge from truncating a more fundamental non-local theory that contains an infinite series of higher-derivatives.

III. Then we integrate out the auxiliary fields following the methods explained in sec. 2.3.3, 2.3.5. More precisely one has to expand the solution in inverse powers of Λ and solve the equations of motion order by order in Λ^{-1}. If some auxiliary field receives a kinetic term, we continue to treat it as an algebraic degree of freedom, inspired by the results in sec. 2.3.5 and sec. 2.5.3, and again determine the solution to the equations of motion by applying perturbation theory in Λ^{-1}.

IV. It is convenient to truncate the solutions for the auxiliary fields at the highest order in Λ^{-1} which appears in eq. (2.97). After insertion of the truncated solution back into the Lagrangian, terms which exceeds the maximal mass dimension should be neglected.

Let us make some remarks regarding this algorithm. Firstly, we should emphasize once more the importance of performing the Weyl-rescaling before integrating out the auxiliary fields. This point was already stressed and exemplified in the previous section. Furthermore, in principle we would have to apply this algorithm also to the two-derivative part of the Lagrangian. In particular, this would include an appropriate truncation of the expressions for K and W. However, in the context of effective supergravities describing the low-energy 4D dynamics of string compactifications, it is useful to keep K and W arbitrary and, furthermore, to allow higher-derivative operators to be multiplied by arbitrary functions of the chiral fields.

In the next sections we discuss the $N = 2$ and $N = 4$ operators. The operators we consider are multiplied by an arbitrary coupling function T which carries a dependence on $\Phi, \bar{\Phi}$ and on the cut-off scale Λ. More precisely, the coupling carries mass dimension $T \sim \Lambda^{-N}$ with $N = 2, 4$. According to the above algorithm it suffices to consider the component Lagrangian for a particular operator only up to order T (i.e. Λ^{-N}), since terms of order $\mathcal{O}(T^2)$ would receive corrections from operators of higher order. Therefore, in the following we restrict ourselves to determine the on-shell form of these operators only at linear order in T. In this case the computation of the on-shell component Lagrangian simplifies considerably, the reason being a special property of the theory in eq. (2.95) which we explain now. Generically the presence of $\hat{\mathcal{L}}_{\mathcal{O}}$ in eq. (2.95) affects the solutions of the equations of motion of the auxiliary fields. We expand the auxiliary fields in powers of T such that

$$\begin{aligned} b^a &= b^a_{(0)} + b^a_{(1)} + \mathcal{O}(\Lambda^{-2N}) \ , \\ M &= M_{(0)} + M_{(1)} + \mathcal{O}(\Lambda^{-2N}) \ , \\ F &= F_{(0)} + F_{(1)} + \mathcal{O}(\Lambda^{-2N}) \ , \end{aligned} \tag{2.98}$$

where $b^a_{(0)}, M_{(0)}$ and $F_{(0)}$ are given in eqs. (2.76), (2.78), (2.80) and $b^a_{(1)}, M_{(1)}, F_{(1)}$ are linear in T and depend on the details of $\hat{\mathcal{L}}_{\mathcal{O}}$. The aforementioned special property of the theory can now by stated as follows: *The linearized on-shell Lagrangian is obtained by inserting $b^a_{(0)}, M_{(0)}$ and $F_{(0)}$ in eq.* (2.95). In other words the terms involving $b^a_{(1)}, M_{(1)}$ or $F_{(1)}$ precisely cancel out. Furthermore, the linearized terms arise only from $\hat{\mathcal{L}}_{\mathcal{O}}$ or via the Weyl-rescaling in $\mathcal{L}_{(0)}$.

This observation can be understood from the structure of the ordinary two-derivative Lagrangian. Let us now explicitly demonstrate this for the terms involving the auxiliary vector, the argument follows immediately also for the remaining auxiliary fields. Generically the presence of $\hat{\mathcal{L}}_{\mathcal{O}}$ corrects the Weyl-factor Ω as follows

$$\Omega = \mathrm{e}^{-K/3} + \Lambda^{-N}\,\delta\Omega(M, \bar{M}, b_a, F, \bar{F}, \Phi, \bar{\Phi}) \ , \tag{2.99}$$

where $\delta\Omega$ depends on the details of the higher-derivative operator $\mathcal{O}$. Now the terms in $\mathcal{L}_{(0)}$ displayed in eq. (2.75) that involve the auxiliary vector can, after performing the Weyl-transformation with respect to Ω given in eq. (2.99), be rewritten as

$$\mathcal{L}_{(0)} \supset \frac{1}{3\Omega}\mathrm{e}^{-K/3}(b_a b^a - 2 b_a b^a_{(0)}) + \mathcal{O}(\delta\Omega) \ , \tag{2.100}$$

where the $\mathcal{O}(\delta\Omega)$ terms indicate the contributions to the Lagrangian that carry a dependence on the Weyl-factor. These terms appear only when $\delta\Omega$ explicitly depends on b_a. Inserting the expansion in eq. (2.98) into eq. (2.100) we find that the terms involving $b^a_{(1)}$ precisely cancel and, hence, the only corrections linear in T are obtained via Ω. Again let us emphasize that this argument can be made for M and F in precisely the same way. Ultimately, the reason for this cancellation is the quadratic form of $\mathcal{L}_{(0)}$. The above observation greatly simplifies the computation of the on-shell action, since we do not have to determine the solution to the equations of motion for the auxiliary fields, but merely insert $b^a_{(0)}$, $M_{(0)}$ and $F_{(0)}$ in $\hat{\mathcal{L}}_{\mathcal{O}}$ and in the corrections induced by $\delta\Omega$ in $\mathcal{L}_{(0)}$. In fact, since it is straightforward to obtain the on-shell theories, in the following we display most of the component forms off-shell.

2.5.5 Component Forms of $N = 2$ Operators

Following the general discussion of the previous section, we now turn to a systematic study of higher-derivative operators. In the previous sections we introduced the concept of a higher-derivative operator of order N. The lowest possible order is $N = 2$. There exist three distinct operators we can construct at this order and they read

$$\mathcal{O}_{(1)} = T\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi \ , \qquad \mathcal{O}_{(2)} = T\mathcal{D}^2\Phi \ , \qquad \mathcal{O}_{(3)} = TR \ , \tag{2.101}$$

where the coupling function $T = T(\Phi/\Lambda, \bar{\Phi}/\Lambda)$ is an arbitrary function of the chiral and anti-chiral superfields and is chosen to have the appriopriate mass-dimension. It turns out that, similar to their rigid counterparts, the operators $\mathcal{O}_{(1)}$ and $\mathcal{O}_{(2)}$ are equivalent to each other. This equivalence can be demonstrated via integration by parts identities which we display in appendix A.4. We now illustrate the general algorithm of the previous section at the example of $\mathcal{O}_{(1)}$. Later on we also turn to the third operator $\mathcal{O}_{(3)}$.

Let us now follow the general prescription of the previous section to compute the component form of $\mathcal{O}_{(1)}$ together with the ordinary two-derivative Lagrangian $\mathcal{L}_{(0)}$ given in eq. (2.71). Here we explicitly include the kinetic terms into the analysis. Following the algorithm presented in the previous section, as a first step we need to compute the relevant quantities in eq. (2.97). The $\theta = \bar{\theta} = 0$-component of $\mathcal{O}_{(1)}$ is purely fermionic. Furthermore, we need the following quantities

$$\mathcal{D}^2\mathcal{O}_{(1)}| = 16TF^2 \ , \qquad \bar{\mathcal{D}}^2\mathcal{O}_{(1)}| = -16T(\partial A)^2 \ , \tag{2.102}$$

where we made use of eq. (A.24). Furthermore, it is necessary to determine the $\mathcal{D}^2\bar{\mathcal{D}}^2$-component of $\mathcal{O}_{(1)}$. Direct computation ignoring fermionic terms yields

$$\begin{aligned}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{O}_{(1)}| = \Big(& T_{\bar{\Phi}}\bar{\mathcal{D}}^2\bar{\Phi}(\mathcal{D}^2\Phi)^2 + 4T_{\bar{\Phi}}\mathcal{D}^2\Phi\mathcal{D}_\alpha\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}^\alpha\Phi - 2T\mathcal{D}^2\Phi\mathcal{D}_\alpha\bar{\mathcal{D}}^2\mathcal{D}^\alpha\Phi \\ & + 2T_\Phi\mathcal{D}^2\Phi\bar{\mathcal{D}}^{\dot{\alpha}}\mathcal{D}_\alpha\Phi\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}^\alpha\Phi + 4T\mathcal{D}^2\bar{\mathcal{D}}^{\dot{\alpha}}\mathcal{D}_\alpha\Phi\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}^\alpha\Phi\Big)\Big| \ .\end{aligned} \tag{2.103}$$

Inserting the component expressions in eqs. (A.24), (A.26) and (A.29) in the above formula we find

$$\begin{aligned}\tfrac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{O}_{(1)}| = & -4T_{\bar{A}}|F|^2F - \tfrac{8}{3}TMF^2 + 8T\partial_mF\partial^mA + \tfrac{4}{3}T\bar{M}\partial_mA\partial^mA \\ & + \tfrac{16}{3}iTFb_ae_a^m\partial_mA + 8T_{\bar{A}}F\partial_mA\partial^m\bar{A} + 4T_AF\partial_mA\partial^mA \ .\end{aligned} \tag{2.104}$$

We are now equipped with the necessary quantities and can proceed to determine the overall component form of $\mathcal{O}_{(1)}$. It is convenient to decompose the result as follows

$$\hat{\mathcal{L}}_{\mathcal{O}_{(1)}}/e = -V_{\mathcal{O}_{(1)}} + \mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(1)}} \ , \tag{2.105}$$

where the individual parts of the Lagrangian are given by

$$\begin{aligned}V_{\mathcal{O}_{(1)}} &= -4TMF^2 - 12|F|^2T_{\bar{A}}F + \text{h.c.} \ , \\ \mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(1)}} &= 8T\bar{M}(\partial A)^2 - 24T\partial F\partial A - 24T_{\bar{A}}F|\partial A|^2 - 12T_AF(\partial A)^2 \\ &\quad - 16ib_mTF\partial^mA + \text{h.c.} \ .\end{aligned} \tag{2.106}$$

Following the general prescription of sec. 2.4.1 let us now determine the linearized on-shell Lagrangian. To begin with we rewrite the Lagrangian in the Einstein frame. Since $\hat{\mathcal{L}}_{\mathcal{O}}$ does not contain couplings to the scalar curvature, the Weyl transformation continues to be given by eq. (2.74). The next step consists of integrating out the auxiliary fields. As demonstrated in the previous section, we obtain the linearized on-shell action by simply replacing the auxiliary fields in $\hat{\mathcal{L}}_{\mathcal{O}_{(1)}}$ by the solutions of the ordinary two-derivative theory given in eqs. (2.76), (2.78) and (2.80). Altogether, the final linearized on-shell Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathcal{O}_{(1)}} &= e(\mathcal{L}_{(0)} - V_{(1)} - \mathcal{L}_{\text{der}}) \ , \\ V_{(1)} &= 12\mathrm{e}^{5K/3}(K^{A\bar{A}})^2(D_AW)^2\Big[K^{A\bar{A}}D_{\bar{A}}\bar{W}(\bar{T}_A - \tfrac{1}{3}\bar{T}K_A) + \bar{T}\bar{W}\Big] + h.c. \\ \mathcal{L}_{\text{der}} &= 24\mathrm{e}^{2K/3}K^{A\bar{A}}(\partial A)^2\Big[D_{\bar{A}}\bar{W}(\tfrac{1}{3}TK_A + \tfrac{1}{2}T_A) + K_{A\bar{A}}T\bar{W}\Big] \\ &\quad + 24T\mathrm{e}^{2K/3}K^{A\bar{A}}D_{\bar{A}}\bar{W}\Box A + h.c. \ .\end{aligned} \tag{2.107}$$

Here we introduced the abbreviation $\Box = \mathcal{D}^m\mathcal{D}_m$ and $D_AW = W_A + K_AW$ and $D_{\bar{A}}\bar{W}$ denote the Kähler-covariant derivatives. The index $A, \bar{A}$ stands for derivatives with respect to the scalar fields $A, \bar{A}$ and are not to be confused with the superspace indices. In the above $\mathcal{L}_{(0)}$ is given by the kinetic terms inside eq. (2.79) together with the scalar potential in eq. (2.81). Let us make a few remarks regarding the above result. The linearized Lagrangian contains only two-derivative terms. However, when using the full solution to the equations of motion for F one finds a non-local theory, which in particular includes an infinite sum of higher-derivatives. Due to the mixing between M and F in eq. (2.105) the linear correction to the scalar potential looks rather involved. Besides the corrections of the type $|F_{(0)}|^2 F_{(0)}$, which survive the rigid limit and can also be inferred from the lowest order contribution to eq. (2.26), we, furthermore, find terms of the type $F_{(0)}^2 \bar{W}$. Roughly speaking, these can be read as describing a mixing between the gravitational piece and the F-term piece of the ordinary (two-derivative) scalar potential.

It remains to discuss the operator $\mathcal{O}_{(3)}$ in eq. (2.101). It is conceptually straightforward to perform the computation and, therefore, we display only the final result here. Altogether, we find the following off-shell component expression

$$\hat{\mathcal{L}}_{\mathcal{O}_{(3)}}/e = -\tfrac{1}{2}\mathcal{R}\Omega_{\mathcal{O}_{(3)}} - V_{\mathcal{O}_{(3)}} + \mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(3)}} \ , \tag{2.108}$$

where we introduced the abbreviations

$$\begin{aligned}
\Omega_{\mathcal{O}_{(3)}} &= -\tfrac{1}{3}TM + \tfrac{1}{2}T_{\bar{A}}\bar{F} + h.c. \ , \\
V_{\mathcal{O}_{(3)}} &= -\tfrac{1}{6}|M|^2(T_{\bar{A}}\bar{F} - \tfrac{1}{3}TM) - \tfrac{1}{2}T_{A\bar{A}}M|F|^2 + \tfrac{1}{6}T_A M^2 F + h.c. \ , \\
\mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(3)}} &= (\tfrac{1}{3}b_a b^a - ie^m_a \mathcal{D}_m b^a)(-\tfrac{1}{3}TM + \tfrac{1}{2}T_{\bar{A}}\bar{F}) + \tfrac{1}{2}MT_{\bar{A}\bar{A}}(\partial\bar{A})^2 \ , \\
&\quad + \tfrac{1}{2}MT_{\bar{A}}(\Box\bar{A} + \tfrac{2}{3}ib^a e^m_a \partial_m \bar{A})) + h.c. \ .
\end{aligned} \tag{2.109}$$

Note that setting T constant reproduces the result obtained in [54]. In particular, we observe that the Einstein-Hilbert term is modified by the presence of $\mathcal{O}_{(3)}$. This, in turn, implies that the Weyl-rescaling is affected. The on-shell Lagrangian can now be obtained readily, but note that now one has to take into account the modified Weyl-factor. The resulting on-shell form is rather lengthy and, hence, we do not display it here.

2.5.6 Component Forms of $N = 4$ Operators

At order $N = 4$ the number of allowed operators increases significantly. We perform the explicit classification of these operators in appendix A.4. Let us briefly summarize the content of this appendix here. We begin by writing down the list of all allowed operators in tab. A.1. However, many operators in this list are redundant and can be recast into combinations of other operators. The main tools to identify equivalences between operators are, on the one hand, the algebra of covariant derivatives in eq. (2.11) and, on the other hand, integration by parts identities in curved superspace. These are introduced and explained in detail in appendix A.4. We then use these identities to determine a minimal set of mutually inequivalent operators. More precisely, this set contains 9 operators. This minimal set of operators can be

Label	Operator	Real	∂^4-Terms
$\mathcal{O}_{(4\|2)}$	$\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}$	✓	✓
$\mathcal{O}_{(3\|1)}$	$\mathcal{D}^2\Phi\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$		
$\mathcal{O}_{(3\|3)}$	$\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}$		✓
$\mathcal{O}_{(2\|1)}$	$\mathcal{D}^2\Phi\bar{\mathcal{D}}^2\bar{\Phi}$	✓	✓
$\mathcal{O}_{(2\|2)}$	$(\mathcal{D}^2\Phi)^2$		
$\mathcal{O}_{(2\|3)}$	$\mathcal{D}_a\Phi\mathcal{D}^a\bar{\Phi}$	✓	✓
$\mathcal{O}_{(R\|1)}$	$R\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$		✓
$\mathcal{O}_{(R\|2)}$	$R\mathcal{D}^2\Phi$		✓
$\mathcal{O}_{(R\|3)}$	R^2		

Table 2.3: *Particular minimal choice of four-superspace-derivative operators which are mutually distinct and cannot be related to each other. The individual operators are understood as being multiplied by a superfield $T(\Phi, \bar{\Phi})$ and $\bar{T}(\Phi, \bar{\Phi})$ for their conjugate parts. In the last two rows we indicated whether the operator is real- or complex-valued and whether it contributes four-derivatives terms for the chiral scalar in the linearized on-shell Lagrangian.*

chosen in several different but equivalent ways, only the total number of operators is fixed. Here we make a particular choice of these operators which is displayed in tab. 2.3. In this list we also indicate whether the operators are real-valued, that is $\mathcal{O} = \bar{\mathcal{O}}$, and whether they induce four-derivative terms for the chiral scalar in the linearized on-shell Lagrangian.

Operators which induce ∂^4-terms off-shell or at the linearized on-shell level are of particular interest to us and, therefore, from now on we constrain our discussion to this subclass only. These operators will play a special role in chapter 3 where we apply them in the context of Kaluza-Klein reduction of IIB supergravity. In tab. 2.3 we find six operators which induce four-derivative terms of the aforementioned type. In the following we determine the component form of all operators of this class, the results are displayed below. Let us make a few comments regarding this procedure. Firstly, for the sake of brevity we omit the details of intermediate results, such as the $\mathcal{D}^2\bar{\mathcal{D}}^2$-components of the various operators. However, in appendix A.3 we collect all component identities which are required to compute these $\mathcal{D}^2\bar{\mathcal{D}}^2$-components. In our survey below we indicate which component-identities are necessary for determining the individual $\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{O}|$-components. Secondly, for those operators which are real-valued we take the coupling function T to be real-valued as well. For operators which are not real we assume that T is complex-valued. Finally, since the resulting component expressions are rather lengthy, we decompose the Lagrangians into different blocks. More precisely, we split them into a sum of the Einstein-Hilbert term, the scalar potential as well as four- and two-derivative terms. The only operator for which we do not apply this decomposition is the first in the list, since its component expression is particularly simple.

Operator $\mathcal{O}_{(4|2)}$:

This operator was already studied in several papers [13, 21, 22]. The respective component version is particularly simple and reads

$$\tfrac{1}{48}\hat{\mathcal{L}}_{\mathcal{O}_{(4|2)}}/e = T|F|^4 + T(\partial A)^2(\partial\bar{A})^2 - 2T|F|^2|\partial A|^2 \; . \tag{2.110}$$

We return to the discussion of this operator in sec. 2.5.7 where we generalize it to multi-field case and compute the respective on-shell version.

Operator $\mathcal{O}_{(3|3)}$:

Computation of this operator requires using the component identities in eq. (A.26) and eq. (A.29). Altogether, we find

$$\begin{aligned}
\hat{\mathcal{L}}_{\mathcal{O}_{(3|3)}}/e &= -V_{\mathcal{O}_{(3|3)}} + \mathcal{L}^{(4\text{-der})}_{\mathcal{O}_{(3|3)}} + \mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(3|3)}} \; , \\
V_{\mathcal{O}_{(3|3)}} &= -16|F|^2(TMF + \bar{T}\bar{M}\bar{F}) \; , \\
\mathcal{L}^{(4\text{-der})}_{\mathcal{O}_{(3|3)}} &= -24T_{\bar{A}}(\partial A)^2(\partial\bar{A})^2 - 24T(\partial A)^2(\Box\bar{A} + \tfrac{2}{3}ib^m\partial_m\bar{A}) + \text{h.c.} \\
\mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(3|3)}} &= -16TMF|\partial A|^2 - 48T\bar{F}(\partial_m A\partial^m F + \tfrac{2i}{3}Fb^m\partial_m A) \\
&\quad - 24T_A|F|^2(\partial A)^2 + \text{h.c.} \; .
\end{aligned} \tag{2.111}$$

Operator $\mathcal{O}_{(2|1)}$:

This operator additionally requires new components displayed in eq. (A.27). The final result reads

$$\begin{aligned}
\hat{\mathcal{L}}_{\mathcal{O}_{(2|1)}}/e &= -\tfrac{1}{2}\mathcal{R}\Omega_{\mathcal{O}_{(2|1)}} - V_{\mathcal{O}_{(2|1)}} + \mathcal{L}^{(4\text{-der})}_{\mathcal{O}_{(2|1)}} + \mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(2|1)}} \; , \\
\tfrac{1}{16}\Omega_{\mathcal{O}_{(2|1)}} &= -2T|F|^2 \; , \\
\tfrac{1}{16}V_{\mathcal{O}_{(2|1)}} &= 6T_{A\bar{A}}|F|^4 + 4|F|^2(T_A MF + T_{\bar{A}}\bar{M}\bar{F}) + 8T|M|^2|F|^2 \; , \\
\tfrac{1}{16}\mathcal{L}^{(4\text{-der})}_{\mathcal{O}_{(2|1)}} &= -6T(\Box A - \tfrac{2}{3}ib^m\partial_m A)(\Box\bar{A} + \tfrac{2}{3}ib^m\partial_m\bar{A}) \; , \\
\tfrac{1}{16}\mathcal{L}^{(2\text{-der})}_{\mathcal{O}_{(2|1)}} &= -3T_{AA}|F|^2(\partial A)^2 - 9T_{\bar{A}}|F|^2(\Box\bar{A} + \tfrac{2i}{3}b^m\partial_m\bar{A}) \\
&\quad - 4T_{\bar{A}}(MF(\partial\bar{A})^2 + \tfrac{3}{2}\bar{F}\partial_m F\partial^m\bar{A}) - \tfrac{1}{3}T|F|^2 b_a b^a \\
&\quad + \tfrac{2}{3}T|M|^2|\partial A|^2 - 4TMF(\Box\bar{A} + \tfrac{2i}{3}b^m\partial_m\bar{A}) \\
&\quad - 4TM\partial_m F\partial^m\bar{A} - 2iT\bar{F}b^m\partial_m F - 3T\bar{F}\Box F + h.c. \; .
\end{aligned} \tag{2.112}$$

Note that here we applied partial integration to terms of the form $\partial_m\bar{M}\partial^m A$.

Operator $\mathcal{O}_{(2|3)}$:

For this operator we find the following component form

$$
\begin{aligned}
\hat{\mathcal{L}}_{\mathcal{O}_{(2|3)}}/e &= -\tfrac{1}{2}\mathcal{R}\Omega_{\mathcal{O}_{(2|3)}} - V_{\mathcal{O}_{(2|3)}} + \mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(2|3)}} + \mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(2|3)}} \ , \\
\Omega_{\mathcal{O}_{(2|3)}} &= 4T|\partial A|^2 \\
V_{\mathcal{O}_{(2|3)}} &= -\tfrac{4}{3}T|F|^2|M|^2 \\
\mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(2|3)}} &= -3T_{\bar{A}}\left[|\partial A|^2(\Box\bar{A} + \tfrac{2}{3}ib^m\partial_m\bar{A}) + 2\partial^m\bar{A}\,\partial^n A\,\mathcal{D}_m\mathcal{D}_n\bar{A}\right] \\
&\quad - 3T_{\bar{A}\bar{A}}|\partial A|^2(\partial\bar{A})^2 + 3T\partial^m A\partial^n\bar{A}\left[\mathcal{R}_{mn} + \tfrac{2}{9}b_m b_n + \tfrac{2}{3}i\mathcal{D}_n b_m\right] \\
&\quad - 3T\partial_m A\,\mathcal{D}^m(\Box\bar{A} + \tfrac{2}{3}ib^n\partial_n\bar{A}) + \text{h.c.} \\
\mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(2|3)}} &= T_{\bar{A}}\left[MF(\partial\bar{A})^2 + \bar{M}\bar{F}|\partial A|^2 - 6\bar{F}\partial_m F\partial^m\bar{A} - 4i|F|^2 b^m\partial_m\bar{A}\right] \\
&\quad - 3T_{A\bar{A}}|F|^2|\partial A|^2 - T\left(\tfrac{1}{3}|\partial A|^2|M|^2 + \tfrac{4}{3}|F|^2 b_a b^a + 3|\partial F|^2\right) \\
&\quad + T(FM\Box\bar{A} + M\partial_m F\partial^m\bar{A} - F\partial_m M\partial^m\bar{A}) \\
&\quad + \tfrac{4}{3}Tib^m(FM\partial_m\bar{A} + 3\bar{F}\partial_m F) + \text{h.c.} \ .
\end{aligned}
\tag{2.113}
$$

Note that $\mathcal{O}_{(2|3)}$ was already studied in [20] for the special case with T being a constant. Here we displayed the component form for the generalized operator where T is an arbitrary function of Φ and $\bar{\Phi}$. The computation of the above result requires knowledge of several additional superfield component identities. These are displayed in eq. (A.29) and eq. (A.30). While these identities were already computed in [20] we recalculated them as a cross-check.[23]

Let us compare the above result with [20]. Overall we find a remarkable agreement, the only difference with the latter reference being a minus sign in the scalar potential that can be traced back to a minus-sign difference in eq. (A.29).

Operator $\mathcal{O}_{(R|1)}$:

$$
\begin{aligned}
\hat{\mathcal{L}}_{\mathcal{O}_{(R|1)}}/e &= -\tfrac{1}{2}\mathcal{R}\Omega_{\mathcal{O}_{(R|1)}} - V_{\mathcal{O}_{(R|1)}} + \mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(R|1)}} + \mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(R|1)}} \ , \\
\Omega_{\mathcal{O}_{(R|1)}} &= 2T(\partial A)^2 + 2\bar{T}(\partial\bar{A})^2 \\
V_{\mathcal{O}_{(R|1)}} &= \tfrac{2}{3}TM^2F^2 + 2T_{\bar{A}}M|F|^2F + \text{h.c.} \\
\mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(R|1)}} &= 2T(\partial A)^2(\tfrac{1}{3}b_a b^a - i\mathcal{D}_m b^m) + \text{h.c.} \\
\mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(R|1)}} &= 4T_{\bar{A}}MF|\partial A|^2 + 2T_A MF(\partial A)^2 + 4TM\partial_m F\partial^m A \\
&\qquad + \tfrac{8i}{3}TMFb^m\partial_m A + \text{h.c.} \ .
\end{aligned}
\tag{2.114}
$$

[23]In particular, we found a minus-sign difference and a typo in that reference.

Operator $\mathcal{O}_{(R|2)}$:

$$
\begin{aligned}
\hat{\mathcal{L}}_{\mathcal{O}_{(R|2)}}/e &= -\tfrac{1}{2}\mathcal{R}\Omega_{\mathcal{O}_{(R|2)}} - V_{\mathcal{O}_{(R|2)}} + \mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(R|2)}} + \mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(R|2)}} \,, \\
\Omega_{\mathcal{O}_{(R|2)}} &= -2T_{\bar{A}}|F|^2 - 2T(\Box A - \tfrac{2i}{3}b^m\partial_m A + \tfrac{1}{3}MF) + \text{h.c.} \\
V_{\mathcal{O}_{(R|2)}} &= 2MF\left(T_{A\bar{A}}|F|^2 + \tfrac{1}{3}T_A MF + \tfrac{2}{3}T_{\bar{A}}\bar{M}\bar{F} + \tfrac{2}{9}T|M|^2\right) + \text{h.c.} \\
\mathcal{L}^{(\text{4-der})}_{\mathcal{O}_{(R|2)}} &= -2T(\Box A - \tfrac{2i}{3}b^m\partial_m A)(\tfrac{1}{3}b_a b^a - i\mathcal{D}_m b^m) + \text{h.c.} \\
\mathcal{L}^{(\text{2-der})}_{\mathcal{O}_{(R|2)}} &= -2MF\left[T_A(\Box A - \tfrac{2i}{3}b^m\partial_m A) + T_{\bar{A}}(\bar{A} + \tfrac{2i}{3}b^m\partial_m\bar{A})\right] \\
&\quad - 2T_{\bar{A}}\left[|F|^2(\tfrac{1}{3}b_a b^a - i\mathcal{D}_m b^m) - \tfrac{2}{3}|M|^2|\partial A|^2 + 2M\partial_m F\partial^m\bar{A}\right] \\
&\quad - 2MF\left[T_{\bar{A}\bar{A}}(\partial\bar{A})^2 + \tfrac{1}{3}T(\tfrac{1}{3}b_a b^a - i\mathcal{D}_m b^m)\right] - 2TM\Box F \\
&\quad + \tfrac{4}{3}T\left[M\partial_m\bar{M}\partial^m A - iMb^m\partial_m F + \tfrac{2}{3}i|M|^2 b^m\partial_m A\right] + \text{h.c.} \,.
\end{aligned}
\tag{2.115}
$$

Comments and Remarks

Let us make a few remarks regarding the above list of component forms for the operators as well as the result of the classification in tab. 2.3. Firstly, out of the six operators (that induce four-derivative component terms in the linearized on-shell action) there are two which do not have a rigid counterpart. More precisely, these are given by $\mathcal{O}_{(R|1)}$ and $\mathcal{O}_{(R|2)}$ which indeed identically vanish in the rigid limit $M_p \to 0$.[24] The rigid limit of the remaining four operators is given by

$$
\begin{aligned}
\mathcal{O}_{(4|2)} &\longrightarrow D_\alpha\Phi D^\alpha\Phi\bar{D}_{\dot{\alpha}}\bar{\Phi}\bar{D}^{\dot{\alpha}}\bar{\Phi} \\
\mathcal{O}_{(3|3)} &\longrightarrow \bar{D}_{\dot{\alpha}}D_\alpha\Phi D^\alpha\Phi\bar{D}^{\dot{\alpha}}\bar{\Phi} \\
\mathcal{O}_{(2|1)} &\longrightarrow D^2\Phi\bar{D}^2\bar{\Phi} \\
\mathcal{O}_{(2|3)} &\longrightarrow D_a\Phi D^a\bar{\Phi} \,.
\end{aligned}
\tag{2.116}
$$

The component version of each of these operators contains four-derivative terms, for $\mathcal{O}_{(4|2)}$ and $\mathcal{O}_{(2|1)}$ this is also clear from eq. (2.33) and eq. (2.26). As a consistency check we can compare this result with [19]. The latter reference includes a classification of those dimension-eight operators for global $\mathcal{N} = 1$ supersymmetry that induce four-derivative component terms, the result being a minimal list of four operators of this class. Since we also find four operators of this type from our classification above, our results are fully consistent with [19].

Secondly, one may perform a consistency check of the component operators with the expectations from linearized supergravity. In that case the Lagrangian is described via a coupling to a Ferrara-Zumino multiplet, see e.g. [56–59]. The authors of [20] already showed that the linearized version (in an expansion in M and b_a) of $\mathcal{O}_{(2|3)}$ (for T constant) agrees with the expectations from linearized gravity. From

[24] For this purpose we consider the situation where $\Lambda \ll M_p$. While $\mathcal{O}_{(R|1)}$ and $\mathcal{O}_{(R|2)}$ are suppressed by at least a factor of M_p^2, the remaining four operators are purely Λ-suppressed.

the results in [20] we can also perform this check for the linearized version of $\mathcal{O}_{(2|1)}$ after setting T to be constant. Indeed we find that eq. (2.112) matches with a certain Ferrara-Zumino multiplet. Note that for the remaining operators we cannot apply the formulas in [20] and, therefore, it would be necessary to recalculate the general form of the Ferrara-Zumino multiplet.

Furthermore, let us make some comments regarding reparametrization- and Kähler-invariance of the operators we discussed so far. Firstly, analogous to the discussion in sec. 2.3.1 ensuring reparametrization-invariance for the operators in eq. (2.101) and tab. 2.3 is non-trivial. While reparametrization-invariance for the contributions to the scalar potential can be made manifest more easily, the kinetic terms are harder to understand. We omit the details of this discussion here for the following reason. Ultimately, we are interested in situations where we compute higher-derivative operators directly from UV-physics and, hence, the operators must be reparametrization-invariant by construction. For instance, we may integrate out heavy fields or compute quantum corrections. More specifically, we will be interested in KK-compactifications of ten-dimensional IIB supergravity where 10D higher-derivative corrections source 4D higher-derivative operators. In that case, the 10D action is not manifestly supersymmetric and the 4D supersymmetric completion is far from obvious. Here our results are of particular importance, since now a full matching to parts of the component Lagrangian can be performed and thereby, at least in principle, a full set of manifestly supersymmetric operators inferred. Naturally, reparametrization-invariance is always guaranteed in that case. However, the discussion of target space reparametrization-invariance is necessary in situations where we take a purely bottom-up EFT approach and attempt to write down all possible operators consistent with the symmetries.

Secondly, we now briefly discuss Kähler-invariance for the higher-derivative operators. Recall that an important feature of the two-derivative theory given in eq. (2.68) and eq. (2.69) is an invariance under a combination of a super-Weyl and a Kähler transformation [9, 47]. For instance, the component form in eq. (2.107) does not exhibit this ordinary Kähler-invariance. Generically theories of supergravity with higher-derivative operators allow for a larger set of transformations which leave the action invariant, one might refer to these as generalized Kähler transformations. In some form this was already visible in the context of flat superspace in eq. (2.29). A concrete example in supergravity was discussed in [20], where an explicit improvement term given by $\mathcal{O}_{(G|2)}$ in tab. A.1 was added to the particular higher-derivative Lagrangian given by $\mathcal{O}_{(2|3)}$. In general we do not expect that a particular higher-derivative operator happens to be invariant under the restricted, two-derivative Kähler-transformations by itself. In fact this applies only to those operators, which are either super-Weyl invariant or have an explicit dependence on K and W in the coupling function T, that allows for a cancellation against their respective super-Weyl weight. For instance, the operator $\mathcal{O}_{(4|2)}$ is super-Weyl-invariant [22] and, hence, supports the ordinary Kähler-invariance. However, note that since the two-derivative Kähler invariance allows to cast the component Lagrangian in a rather simple form, it is interesting to find higher-derivative operators with this property. We leave this to future research.

2.5.7 $N = 4$ On-Shell Example

We now focus on the operator $\mathcal{O}_{(4|2)}$ to present an on-shell example, which will also be of importance in chapter 3. In this section we follow the reference [31]. We have already seen that it is particularly easy to compute the respective component form, the result being considerably more tractable compared to the remaining operators listed in tab. 2.3. In particular, the form of this operators implies that the bosonic terms depend only on T and no derivatives of T appear. Furthermore, let us highlight that this operator is easily made invariant under reparametrizations of the target space of the chiral scalars. In addition, it is even super-Weyl invariant [22] such that it is supports an invariance with respect to ordinary Kähler-transformations.

We now derive the on-shell form of $\mathcal{O}_{(4|2)}$ and perform this computation for the multi-field case. The appropriate multi-field version of $\mathcal{O}_{(4|2)}$ reads

$$\mathcal{O}_{(4|2)} = \frac{1}{48} T_{ij\bar{k}\bar{l}} \mathcal{D}_\alpha \Phi^i \mathcal{D}^\alpha \Phi^j \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{k}} \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}^{\bar{l}} \ . \tag{2.117}$$

To support target space invariance with respect to the n_c-dimensional complex manifold parametrized by A^i and $\bar{A}^{\bar{\jmath}}$ the superfield $T_{ij\bar{k}\bar{l}}$ must transform as a tensor of this complex manifold and must obey the identities displayed in eq. (2.31) [21]. The respective component version of this operators reads

$$\begin{aligned} \hat{\mathcal{L}}_{\mathcal{O}_{(4|2)}}/e = {} & T_{ij\bar{k}\bar{l}} (\partial_m A^i \partial^m A^j)(\partial_n \bar{A}^{\bar{k}} \partial^n \bar{A}^{\bar{l}}) - 2 T_{ij\bar{k}\bar{l}} F^i \bar{F}^{\bar{k}} (\partial_m A^j \partial^m \bar{A}^{\bar{l}}) \\ & + T_{ij\bar{k}\bar{l}} F^i F^j \bar{F}^{\bar{k}} \bar{F}^{\bar{l}} \ . \end{aligned} \tag{2.118}$$

Let us now investigate the situation in which only $\mathcal{O}_{(4|2)}$ corrects the two-derivative theory in eq. (2.75) and, hence, we consider

$$\mathcal{L}_{\mathcal{O}_{(4|2)}} = \mathcal{L}_{(0)} + \hat{\mathcal{L}}_{\mathcal{O}_{(4|2)}} \ . \tag{2.119}$$

From our results in sec. 2.5.4 we know that in order to determine the linearized on-shell Lagrangian all we have to do is perform the Weyl rescaling and replace F^i by $F^i_{(0)}$ displayed in eq. (2.80). In this case the presence of $\mathcal{O}_{(4|2)}$ does not affect the Weyl factor and, hence, the Weyl transformation continues to be given by eq. (2.74). Therefore, the linearized on-shell Lagrangian reads

$$\begin{aligned} \mathcal{L}_s/e = & -\tfrac{1}{2}\mathcal{R} - \left(K_{i\bar{k}} + 2 \mathrm{e}^K T^{\bar{l}}{}_i{}^j{}_{\bar{k}} \, D_j W \, D_{\bar{l}} \bar{W} \right) \partial_m A^i \partial^m \bar{A}^{\bar{k}} \\ & + T_{ij\bar{k}\bar{l}} (\partial_m A^i \partial^m A^j)(\partial_n \bar{A}^{\bar{k}} \partial^n \bar{A}^{\bar{l}}) - V(A, \bar{A}) \ , \end{aligned} \tag{2.120}$$

where the scalar potential is of the form

$$V = V_{(0)} + V_{(1)} \ . \tag{2.121}$$

Here $V_{(0)}$ is given by eq. (2.81), while

$$V_{(1)} = -\mathrm{e}^{2K} T^{\bar{\imath}\bar{\jmath}kl} D_{\bar{\imath}} \bar{W} D_{\bar{\jmath}} \bar{W} D_k W D_l W \ . \tag{2.122}$$

We observe that the metric multiplying the two-derivative term receives a correction here. From eq. (2.120) we read off its form

$$\delta G_{i\bar{k}} = 2\mathrm{e}^{K} T^{\bar{l}}{}_{i}{}^{j}{}_{\bar{k}}\, D_j W\, D_{\bar{l}} \bar{W} \ . \tag{2.123}$$

In general, this correction renders the metric non-Kähler. Indeed, in [13] the following special case was investigated

$$T_{ij\bar{k}\bar{l}} = \frac{T}{2}\left(K_{i\bar{k}}K_{j\bar{l}} + K_{i\bar{l}}K_{j\bar{k}}\right) \ , \tag{2.124}$$

where T was chosen as a constant. In this case the hermitian connection has non-vanishing torsion and, thus, the metric multiplying the two-derivative term cannot be Kähler.

To summarize, in this section we classified the leading and next-to leading order higher-derivative operators for chiral multiplets and determined the component form of those operators which induce four-derivative terms for the component fields at the linear level. We now proceed to discuss possible applications of these results.

2.6 Structure of Vacua of Higher-Derivative Theories

The form of the scalar potential determines the structure of the vacua of the theory. Equipped with S_{eff} in eq. (2.25) for the case of rigid $\mathcal{N} = 1$ and the results of sec. 2.5 for the case of supergravity, it is interesting to study the effect that the higher-derivatives have on the vacua of the theory. We initiate this discussion by turning first to supersymmetric vacua. Supersymmetry preservation requires additional conditions on the structure of the backgrounds that can be used to derive properties thereof.

2.6.1 Supersymmetric Vacua in Rigid $\mathcal{N} = 1$

Let us start by discussing supersymmetric vacua of rigid $\mathcal{N} = 1$. The off-shell supersymmetry transformation of the chiral fermions read [34]

$$\delta_\zeta \chi^i = i\sqrt{2}\sigma^a \bar{\zeta} \partial_a A^i + \sqrt{2}\zeta F^i \ , \tag{2.125}$$

where ζ is the parameter of the supersymmetry transformation.[25] Supersymmetric vacua, therefore, are defined by

$$\langle \partial_a A^i \rangle = 0 \ , \qquad \langle F^i \rangle = 0 \ . \tag{2.126}$$

[25]Naturally, the off-shell supersymmetry transformations of the chiral multiplet are unaffected by the presence of higher-derivatives. These transformations are defined entirely by the supersymmetry transformation of a general superfield together with the chirality constraint in eq. (2.12).

The equations of motion for the chiral auxiliary fields derived from eq. (2.26) after evaluating at $\langle\partial_a A^i\rangle = \langle\partial_a F^i\rangle = 0$ read

$$-\mathcal{K}_{A^i\bar{A}^{\bar{\jmath}}}\bar{F}^{\bar{\jmath}} - \mathcal{K}_{A^k\bar{A}^{\bar{\jmath}}F^i}F^k\bar{F}^{\bar{\jmath}} - W_{A^i} = 0\ . \tag{2.127}$$

Evaluating the above equation at the supersymmetric condition $\langle F^i\rangle = 0$ and assuming that the derivatives of $\mathcal{K}$ in the above equation are regular at the supersymmetric point, we find that

$$\langle W_{A^i}\rangle = 0\ , \qquad \langle V\rangle = 0\ . \tag{2.128}$$

In other words the supersymmetric vacua of the general higher-derivative theory are identical to the vacua of the two-derivative theory. Note that this was also shown based on a conjectured form of the higher-derivative scalar potential in [13]. Here, we demonstrated this by means of the explicit form of the scalar potential. Note furthermore, that the associated moduli spaces of the supersymmetric backgrounds for the general higher-derivative and for the ordinary two-derivative theories precisely coincide.

2.6.2 Supersymmetric Vacua in $\mathcal{N} = 1$ Supergravity

We now turn to supersymmetric vacua of $\mathcal{N} = 1$ supergravity. The supersymmetry variation of the chiral fermion now reads [34]

$$\delta_\zeta\chi^i = i\sigma^m\bar{\zeta}(\sqrt{2}\partial_m A^i - \psi_m\chi) + \sqrt{2}\zeta F^i\ . \tag{2.129}$$

Therefore, for supersymmetric backgrounds we find the condition $\langle F^i\rangle = 0$. In addition, the supersymmetry variation of the gravitino [34]

$$\delta_\zeta\psi_m^\alpha = -2\mathcal{D}_m\zeta^\alpha + ie_m^c\left[\frac{1}{3}M(\epsilon\sigma_c\bar{\zeta})^\alpha + b_c\zeta^\alpha + \frac{1}{3}b^d(\zeta\sigma_d\bar{\sigma}_c)^\alpha\right] \tag{2.130}$$

has to vanish in a supersymmetric background. This requires that the spacetime background admits four independent Killing spinors. In particular, maximally symmetric spacetimes allow for the existence of four Killing spinors [35]. More precisely, these supersymmetric backgrounds are either M_4, in which case $M = 0$, or AdS_4, in which case $M \neq 0$ [35]. dS_4 on the other hand does not allow for Killing spinors and, hence, supersymmetry is always broken. Since we are discussing off-shell theories, this result holds regardless of the structure of the Lagrangian and, hence, applies both to the ordinary two-derivative as well as to the generic higher-derivative theory. The Killing spinors also determine the curvature of the background. This can be seen easily by analyzing the supersymmetry variations of the superfield R. In particular, since R is covariantly chiral, preservation of supersymmetry requires that the component $R|_{\Theta^2}$ in eq. (2.67) vanishes. This is precisely the case if

$$\langle\mathcal{R}\rangle = \tfrac{4}{3}\langle|M|^2\rangle\ . \tag{2.131}$$

The above equation can also be derived from the integrability condition for the Killing spinors.[26] Moreover, eq. (2.131) establishes a relation between the cosmological constant and the auxiliary scalar M in the vacuum. For the ordinary two-derivative theory, the on-shell scalar potential, which in turn sets the value of the cosmological constant, precisely agrees with eq. (2.131) at the level of the Einstein equations.

Minkowski Vacua

Let turn first to the analysis of the Minkowski vacua. These are characterized by the conditions

$$\langle F^i \rangle = 0 \ , \qquad \langle M \rangle = 0 \ . \tag{2.132}$$

A generic higher-derivative theory schematically obtains a scalar potential of the form

$$\begin{aligned} V &= V_{\text{off}}^{(0)} + V_{hd}(M, \bar{M}, F^i, \bar{F}^{\bar{\jmath}}, A^i, \bar{A}^{\bar{\jmath}}) \ , \\ V_{\text{off}}^{(0)} &= e^{2K/3}(\bar{M}W + M\bar{W} - W_i F^i - \bar{W}_{\bar{\jmath}} \bar{F}^{\bar{\jmath}}) - K_{i\bar{\jmath}} e^{K/3} F^i \bar{F}^{\bar{\jmath}} \\ &\quad + \tfrac{1}{3} e^{K/3}(M + K_{\bar{\jmath}} \bar{F}^{\bar{\jmath}})(\bar{M} + K_i F^i) \ , \end{aligned} \tag{2.133}$$

which, in turn, yields the following equations of motion for $\bar{F}^{\bar{\jmath}}$

$$K_{i\bar{\jmath}} e^{K/3} F^i + e^{2K/3} \bar{W}_{\bar{\jmath}} - \frac{1}{3} e^{K/3} K_{\bar{\jmath}} (\bar{M} + K_i F^i) + \frac{\partial}{\partial \bar{F}^{\bar{\jmath}}} V_{hd} = 0 \ . \tag{2.134}$$

Now, from our results in sec. 2.5 we know that V_{hd} is at least cubic in a combined expansion in powers of $M, \bar{M}, F^i$ and $\bar{F}^{\bar{\jmath}}$. Therefore, eq. (2.132) implies that $\langle \frac{\partial}{\partial \bar{F}^{\bar{\jmath}}} V_{hd} \rangle = 0$ and, hence, eq. (2.134) at the supersymmetric vacuum reads

$$\langle \bar{W}_{\bar{\jmath}} \rangle = 0 \ . \tag{2.135}$$

Similarly, the equations of motion for M lead us to the condition that $\langle W \rangle = 0$. Note that, automatically we also find that $\langle \frac{\partial}{\partial A^i} V \rangle = \langle \frac{\partial}{\partial \bar{A}^{\bar{\jmath}}} V \rangle = 0$. In total, the supersymmetric Minkowski vacua in eq. (2.132) are equivalently defined by the conditions

$$\langle \bar{W}_{\bar{\jmath}} \rangle = \langle W \rangle = 0 \ . \tag{2.136}$$

Therefore, the supersymmetric M_4 vacua as well as their corresponding moduli spaces of the general higher-derivative and of the ordinary two-derivative theories are identical. Again these results are in agreement with [13]. Note that from eq. (2.136) it follows that the moduli space is defined by a set of holomorphic equations. Hence, the moduli space is given by a complex submanifold of the Kähler manifold and, in turn, Kähler itself.

[26] Note that the existence of a single Killing spinor already demands that the background is an Einstein-manifold.

Anti-de Sitter Vacua

Let us now turn to the AdS_4 vacua. Compared to the Minkowski case the AdS_4 vacua require more effort to understand. We characterize these vacua by the conditions

$$\langle F^i \rangle = 0 \ , \qquad \langle M \rangle \neq 0 \ , \qquad \langle \mathcal{R} \rangle = \tfrac{4}{3} \langle |M|^2 \rangle \ , \qquad \langle V_i \rangle = \langle V_{\bar{\jmath}} \rangle = 0 \ . \tag{2.137}$$

In the ordinary two-derivative theory these properties are equivalent to $\langle D_i W \rangle = 0$ and $\langle W \rangle \neq 0$.

In the higher-derivative theory it is not a priori clear whether the conditions in eq. (2.137) can still simultaneously be satisfied. Let us begin by investigating the curvature constraint in eq. (2.131). In appendix A.5 we demonstrate that eq. (2.131) is a possible consequence of the equations of motion of M in a general higher-derivative supergravity after solving the higher-curvature Einstein equations. The explicit analysis is performed for a Lagrangian supporting the complete scalar potential and an $\mathcal{R} + \mathcal{R}^2$ gravity. More precisely, the presence of the $\mathcal{R}^2$-term implies that at least two solutions to the equations of motion for M exist, one which satisfies eq. (2.131) and another which violates this equation and, hence, corresponds to a non-supersymmetric vacuum.[27] Here we are not interested any further in this possible second solution and whether it is physically viable and in agreement with the principles of EFT. We simply conclude that eq. (2.131) can automatically be satisfied and does not have to be included in the list of conditions in eq. (2.137).

Let us now analyze how the remaining conditions in eq. (2.137) are affected by the equations of motion of the auxiliary fields. Firstly, the equation of motion for $\bar{M}$ after evaluating at eq. (2.137) reads

$$\left\langle e^{2K/3} W + \frac{1}{3} e^{K/3} M + \frac{\partial}{\partial \bar{M}} V_{hd} \right\rangle = 0 \ . \tag{2.138}$$

Furthermore, the equations of motion for the chiral auxiliary fields as given in eq. (2.134) can be simplified by using eq. (2.138) and are of the form

$$\left\langle D_{\bar{\jmath}} \bar{W} - \mathrm{e}^{-2K/3} \left(K_{\bar{\jmath}} \frac{\partial}{\partial M} - \frac{\partial}{\partial \bar{F}^{\bar{\jmath}}} \right) V_{hd} \right\rangle = 0 \ . \tag{2.139}$$

The equations of motion in eq. (2.138) and in eq. (2.139) have several consequences. Firstly, the value of the cosmological constant is determined by $\langle M \rangle$ via eq. (2.131) and, hence, might differ from the two-derivative result. Secondly, eq. (2.139) is in general satisfied for $\langle D_i W \rangle \neq 0$. Therefore, the position of the supersymmetric vacuum is shifted. Moreover, in general we expect $\langle V_i \rangle = \langle V_{\bar{\jmath}} \rangle = 0$ to be independent conditions and no longer satisfied just by means of eqs. (2.138),(2.139). However, certainly one should test this expectation for explicit examples which we leave to future investigations.

In turn, generically the AdS_4-vacua of the higher-derivative theory should not admit any flat directions. In particular, in situations where V_{hd} constitutes a small

[27] Note that further higher-curvature terms may lead to additional non-supersymmetric solutions, but the supersymmetric solution where eq. (2.131) is satisfied should still be allowed.

correction to the ordinary scalar potential and, hence, the supersymmetric vacuum is shifted to a nearby position, we expect that the higher-derivative corrections lift any flat direction which may have existed in the two-derivative theory. This observation is also in agreement with the existing literature on $(\mathcal{N}=1, D=3)$ superconformal field theories (SCFT) which are expected to be dual to AdS_4-supergravities via the AdS/CFT correspondence [60]. The moduli space of the AdS_4 vacua corresponds to the space of deformations of exactly marginal operators in the respective SCFT. In particular, as stated in [61] generically one expects that there are no such deformations in the $(\mathcal{N}=1, D=3)$ SCFTs and, therefore, no moduli spaces in the dual AdS_4 vacua.

Of course the above arguments merely describe the general expectation and do not represent strict bounds on the moduli space. It would be interesting to check whether rigorous statements about the moduli spaces can be made. In particular, one may try to derive a bound on the dimension, similar to the discussion in the ordinary two-derivative theory where the moduli space has dimension $\leq n_c$ [62]. To perform such a discussion it would be necessary to make an explicit analysis for the general higher-derivative theory (including also the dependence on the chiral auxiliary fields) which is outside the scope of this thesis. We leave these issues to future research.

In summary, the supersymmetric AdS_4 vacua are defined by eq. (2.139) and eq. (2.138) together with the conditions $\langle V_i \rangle = \langle V_{\bar{j}} \rangle = 0$ and $\langle M \rangle \neq 0$. As in the ordinary theory the curvature constraint in eq. (2.131) is automatically satisfied on-shell.

2.6.3 Structure of Non-Supersymmetric Vacua

In general not much about the structure of the non-supersymmetric vacua can be said. However, let us offer at least a few observations here. As before we conduct this discussion separately for higher-derivative theories with global and local $\mathcal{N}=1$ supersymmetry.

Non-Supersymmetric Vacua in Rigid Theory

Let us first turn to the situation in globally supersymmetric theories. If supersymmetry is broken, then necessarily

$$\langle F^i \rangle \neq 0 \,, \tag{2.140}$$

at least for one value of i. As in the respective two-derivative theory, also in the higher-derivative theory the on-shell scalar potential is necessarily positive as required by the supersymmetry algebra.

Let us consider the situation of an EFT in which all possible higher-derivative corrections allowed by the symmetries are present. If the higher order corrections to V in powers of the auxiliary fields are small perturbations of the ordinary two-derivative scalar potential, then we expect that the vacuum will be shifted to a nearby field value. Suppose that the non-supersymmetric vacuum of the two-derivative theory has a flat direction, then the contributions from higher powers in F^i generically constitute the leading order term in this direction and may lift its flatness. However, the flatness will be preserved if it is enforced by a symmetry, such as a perturbatively unbroken shift-symmetry. A second class of models where the flat directions may not be lifted are those in which supersymmetry breaking occurs via a spontaneously broken R-symmetry [63]. These models always have a flat direction, the R-axion, associated with the Goldstone boson of the broken R-symmetry.

Given that the higher-derivative corrections to the scalar potential lift certain flat directions, these may either be stabilized or left as tachyonic directions. The structure of the general scalar potential does not allow to make model-independent statements and, therefore, one has to perform a case-by-case study.

Non-Supersymmetric Vacua in Supergravity

Let us now turn to the study of non-supersymmetric vacua in higher-derivative supergravity. Firstly, eq. (2.140) is no longer a necessary condition for supersymmetry breaking, since higher-curvature terms alone can induce supersymmetry breaking. For instance for theories of $f(\mathcal{R})$-gravity the vacuum structure can be understood more easily via the dual description in terms of a real scalar which may feature a non-supersymmetric minimum. In appendix A.5 we present an explicit example where after setting $\langle F^i \rangle = 0$ for all i there exists a non-supersymmetric vacuum generated by the presence of an $\mathcal{R}^2$-term. In that case the $\mathcal{R}^2$-term modifies the equations of motion for M which now admits a solution where the Killing spinor equation is not satisfied. Another example of this effect was discussed in [54]. However, let us emphasize again that supersymmetry breaking of this type is a feature of higher-curvature gravity, and is not related to the higher-derivative terms for the chiral scalar.

As in the global case the presence of new corrections to the scalar potential may lift additional flat directions, but in general it is hard to make model-independent statements.

2.7 Example: Shift-Symmetric No-Scale Models

So far we presented a catalog of higher-derivative operators and investigated their effects on the vacuum structure. It is interesting to study the form of the higher-derivative operators for an explicit example. In this section we discuss the special case of so-called no-scale models which originally appeared in [64]. For simplicity, here we define no-scale models as supergravities with the property $V_{(0)} \equiv 0$.[28] As

[28]Note that the notion of no-scale model can be generalized to models with a semi-positive or negative scalar potential. Furthermore, there is also a weaker definition of the no-scale property where one only requires that the Kähler potential obeys eq. (2.142) while the superpotential is

long as the no-scale property is not induced by a symmetry, and we will comment on this possibility in a moment, then higher-derivative operators generally violate it. In this case the scalar potential is completely composed of higher-derivative contributions.

In string compactifications one often deals with no-scale supergravities which additionally enjoy a Peccei-Quinn shift-symmetry and we turn more closely to their discussion in the next chapter. The shift-symmetry implies that the Lagrangian is invariant under the transformation

$$\Phi^i - \bar{\Phi}^{\bar{\imath}} \longrightarrow \Phi^i - \bar{\Phi}^{\bar{\imath}} + iC^i \ , \qquad i = 1, \ldots, n_c \ , \tag{2.141}$$

where C^i are real constants. By performing a Kähler transformation it is always possible to locally choose a constant superpotential. In this case an invariance under the shifts in eq. (2.141) must satisfied by K individually and, in turn, K must be a function of the real parts of A^i only. In turn, the defining condition for a no-scale model reads[29]

$$K^{i\bar{\jmath}} K_i K_{\bar{\jmath}} = 3 \ . \tag{2.142}$$

No-scale supergravities with an underlying shift-symmetry have been classified in [36] both for chiral multiplets as well as for real linear multiplets. Let us briefly summarize the main results of this paper here. Recall that we defined real linear multiplets in the context of global supersymmetry and the content of these multiplets is displayed in tab. 2.1. In particular, shift-symmetric models of chiral multiplets are dual to general theories of real linear multiplets.

The task of classifying all shift-symmetric no-scale models formulated in terms of chiral multiplets amounts to finding all allowed Kähler potentials which satisfy eq. (2.142). It is convenient to rephrase this problem by reducing eq. (2.142) to a simpler equation. To this end we rewrite the Kähler potential as follows

$$K = -3 \ln(Y) \ . \tag{2.143}$$

As shown in [36, 65] eq. (2.142) is satisfied, if and only if Y is a solution to the homogeneous Monge-Ampere equation which reads

$$\det(Y_{i\bar{\jmath}}) = 0 \ . \tag{2.144}$$

The general solution to this equation can at most be given in a semi-explicit form, and this solution is displayed in [36]. Let us contrast this result to the classification of no-scale models for real linear multiplets for which the general solution takes a fully explicit and rather simple form. Among the chiral no-scale models characterized by eq. (2.144) there exists a special subclass of solutions where the isometry group of the Kähler manifold includes an additional Killing vector associated with dilatations. In this case Y is given by a homogeneous function of degree one. Remarkably, these are the only type of no-scale models which arise from geometric string compactifications

arbitrary.

[29] A generalized notion of no-scale model is obtained by choosing a different constant on the r.h.s. of the below equation.

and we display explicit examples in the next chapter. Let us emphasize again that among all solutions to eq. (2.144) the homogeneous functions of degree one are extremely special. This implies that the vast majority of no-scale models do not arise from string compactifications, at least not from geometric compactifications in a perturbative regime in α' and g_s. This concludes the brief overview of the results of [36].

Before turning to the higher-derivative operators for no-scale models, let us make a remark which follows readily from the results in [36], but has not been explicitly stated in that reference. One may ask whether the no-scale property is equivalent to a symmetry of the theory which, therefore, must imply the existence of a hypothetical no-scale Killing vector. We now argue that the results of [36] suggest otherwise. Firstly, in [66] it was proposed that the no-scale property may be related to an underlying scaling (i.e. dilatation) symmetry. However, this cannot be a candidate for a no-scale symmetry as it only applies to the special subclass of no-scale models where Y is a homogeneous function. Furthermore, even in this subclass there are counter-examples in the context of type II orientifold flux-compactifications as discussed in [36]. One may argue for the existence of another Killing vector associated with the no-scale property that we did not take into account yet. To explore this possibility it is instructive to regard the single field case where there is a unique no-scale theory determined by $K = -3\ln(A + \bar{A})$ [64]. The respective Kähler manifold is given by the coset $SU(1,1)/U(1)$ whose isometry group coincides with the modular group $SL(2,\mathbb{R})$. Therefore, besides the Killing vectors associated to rescalings and shifts we find a third Killing vector corresponding to inversions. Since the other two Killing vectors are not related to the no-scale property, this third Killing vector is the only remaining option for a generator of a hypothetical no-scale symmetry.[30] However, the inversion is not necessarily a symmetry for models with more than a single field as can be checked for explicit examples. In total, this suggests that the no-scale property in eq. (2.142) is not equivalent to a symmetry of the theory.

We now turn our attention to higher-derivative supergravity and the form of the operators in tab. 2.3 for shift-symmetric no-scale models. To understand the structure of the higher-derivative corrections it is important to note that the no-scale condition in eq. (2.142) leads to the following simplification of eq. (2.78)

$$M_{(0)} = 0 \ . \tag{2.145}$$

Eq. (2.145), in turn, implies that the linearized on-shell Lagrangians for the operators in tab. 2.3 simplify considerably and, in particular, that at leading order several operators do not contribute to the scalar potential. More specifically, among the operators which contribute four-derivative terms at the linearized level we find that only $\mathcal{O}_{(4|2)}$ and $\mathcal{O}_{(2|1)}$ induce new terms in the scalar potential. In fact, both operators lead to the same correction which is of the form $|F|^4$. More generally, the only corrections at the linearized level which can arise are given by higher powers of the F-terms. Altogether, these F-term corrections generally induce a scalar potential and, thereby, lift the no-scale property $V = 0$.

[30] The shift-symmetry is not a necessary feature of no-scale models, since a large number of counter-examples exist such as the coset $SU(n,1)/U(n)$.

Chapter 3

Type IIB Orientifold Flux-Compactifications and α'-Corrections

In the last chapter we investigated higher-derivative theories of chiral multiplets in $\mathcal{N} = 1, D = 4$ flat and curves superspace. Furthermore, we analyzed the vacuum structure of these higher-derivative theories. In particular, the presence of additional corrections to the scalar potential induced the higher-derivative operators is capable of lifting flat directions. Therefore, these corrections are of particular relevance in the discussion of vacua of string compactifications which typically exhibit a large number of flat directions. In this chapter we perform an explicit reduction of higher-derivative operators from ten-dimensional $(\alpha')^3$-corrections in the context of Calabi-Yau orientifold compactifications of IIB string theory reviewing the work [31], but also extending parts of this reference by including additional information. We begin by reviewing the essential formalism of IIB supergravity, orientifold compactifications and the resulting low-energy effective $\mathcal{N} = 1, D = 4$ supergravity.

The low-energy effective action of IIB superstring-theory is captured by $D = 10$ IIB supergravity. IIB supergravity has the maximal amount of supercharges possible in $D = 10$ spacetime-dimensions corresponding to $\mathcal{N} = 2$ Majorana-Weyl spinors. Therefore, the supersymmetry algebra allows only for a gravity multiplet, which encompasses the following component fields [67]

$$\text{NS-NS}: \quad (g_{(10)}\ , B_2\ , \phi)\ , \qquad \text{R-R}: \quad (C_0\ , C_2\ , C_4)\ , \tag{3.1}$$

where $g_{(10)}$ denotes the 10D spacetime metric, ϕ is a real scalar denoted as the dilaton, B_2 is a two-form and C_p are p-form gauge potentials. Here we only displayed the bosonic components. We associate the following field strengths to the gauge potentials

$$H_3 = dB_2\ , \qquad F_1 = dC_0\ , \qquad F_3 = dC_2\ , \qquad F_5 = dC_4\ . \tag{3.2}$$

It is convenient to introduce the following quantities

$$S = C_0 + ie^{-\phi} \ , \qquad G_3 = F_3 - SH_3 \ ,$$
$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \ . \tag{3.3}$$

The leading-order two-derivative action for IIB supergravity in the string frame reads [67]

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}e^{-2\phi}\Big(\mathcal{R}_{(10)} + 4(\partial\phi)^2\Big) - \frac{i}{8\kappa_{10}^2}\int e^{\phi}C_4 \wedge G_3 \wedge \bar{G}_3$$
$$- \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\left[\frac{1}{2}|F_1|^2 + \frac{1}{12}|G_3|^2 + \frac{1}{480}|\tilde{F}_5|^2\right] \ , \tag{3.4}$$

where $\mathcal{R}_{(10)}$ denotes the ten-dimensional scalar curvature and κ_{10} sets the 10D Planck scale. This action is incomplete in the sense that it does not automatically lead to the self-duality condition $\tilde{F}_5 = *\tilde{F}_5$, which has to be additionally enforced at the level of the equations of motion.

3.1 IIB Compactifications, O-Planes, D-Branes and Background Fluxes

To obtain theories in $D = 4$ spacetime dimensions we have to study compactifications of IIB supergravity. In the simplest case, this means that we look for solutions of the $D = 10$ supergravity where the ten-dimensional metric describes a product

$$\mathcal{M}_{(10)} = \mathcal{M}_{(4)} \times X_6 \ , \tag{3.5}$$

such that $\mathcal{M}_{(4)}$ is an arbitrary four-dimensional spacetime and X_6 some compact manifold. Naturally, this has to happen in such a way, that the theory lifts to a solution of IIB string theory. A generic X_6 will almost always break all of the supersymmetry and, therefore, the respective $D = 4$ theory has no supersymmetric structure. To preserve at least some of the supercharges X_6 has to allow for the existence of globally defined non-vanishing spinors. A particular example are manifolds with $SU(3)$ holonomy group, which admit one covariantly constant spinor. These manifolds are referred to as Calabi-Yau manifolds and can equivalently be characterized by the property that the Ricci tensor vanishes identically. However, since IIB supergravity has a supersymmetry algebra with $\mathcal{N} = 2$ $D = 10$ supercharges, Calabi-Yau-compactifications preserve two $D = 4$ spinors which, in turn, leads a 4D theory with $\mathcal{N} = 2$ supersymmetry. Note that preservation of supersymmetry in four dimensions does not require that X_6 is Calabi-Yau. In fact, there is a much broader class of compactifications that yield $(\mathcal{N} = 2, D = 4)$ where X_6 is only required to have an $SU(3)$-structure group [68–72]. In this case the $(\mathcal{N} = 2, D = 4)$ theory, contrary to the Calabi-Yau compactifications, is in general gauged. However, the Calabi-Yau compactifications have the advantage of being computationally more tractable and, therefore, we constrain ourselves to this class of compactifications here.

To obtain theories with $\mathcal{N} = 1$ in $D = 4$ it is necessary to modify our ansatz in eq. (3.5). The required additional ingredients of our theory are D-branes and orientifold O-planes. D-branes are dynamical extended objects in $\mathcal{M}_{(10)}$ and constitute BPS-states of IIB supergravity. They naturally arise in string theory as hypersurfaces in $\mathcal{M}_{(10)}$ on which open strings end. Furthermore, they carry charge under p-form gauge potentials in the R-R sector. To consistently define the R-R charges of D-branes on a compact space it is necessary to introduce additional localized objects, the orientifold O-planes. The O-planes are extended objects in $\mathcal{M}_{(10)}$ as well, but arise after performing an orientifold projection of type II string theory. The orientifold projection is a discrete transformation which includes world-sheet parity as well as a target space symmetry σ, which has to be an involution on the compact space.[31] Moreover, it reduces the amount of supersymmetry that is preserved in the four-dimensional theory such that we obtain $\mathcal{N} = 1$ supergravity in $D = 4$. For IIB with X_6 being a Calabi-Yau threefold the orientifold projection can be chosen such that the theory either includes $O3/O7$- or $O5/O9$-planes [25]. Henceforth, we discuss only the $O3/O7$-plane case. Given these orientifold planes, we are in a position to also include $D3/D7$-branes in our theory. Positive tension induced by R-R charge from the D-brane sector can now be canceled with negative tension of the O-planes leading to well-defined solutions of IIB supergravity [26]. In general, in the presence of the aforementioned localized sources the ten-dimensional solution is described by a warped metric, which reproduces eq. (3.5) in the limit of weak warping. Thus, the compact dimensions are described by a manifold that is conformally Calabi-Yau.

One is now in a position to work out the effective $\mathcal{N} = 1$ supergravity which describes such orientifold $O3/O7$-compactifications. However, as it turns out, one generally obtains large numbers of massless chiral (and/or real linear) multiplets whose constituents are geometric moduli of the compact manifold and axions descending from the various p-form gauge potentials in ten dimensions. To find phenomenologically more desirable theories, it is necessary to include additional building blocks, that allow to generate interactions and, in turn, make the chiral multiplets massive. To this end we consider background fluxes for the three-form field strengths F_3 and H_3, which constitute the final ingredient of the compactification data. In [26] a warped solution of type IIB supergravity was found which includes $O3$-planes as well as $D3$- and $D7$-branes and which admits quantized fluxes for an imaginary self-dual three-form

$$* G_3 = iG_3 \ . \tag{3.6}$$

The flux conditions are of the form

$$\frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi N_1 \ , \qquad \frac{1}{2\pi\alpha'}\int_B H_3 = -2\pi N_2 \ , \tag{3.7}$$

where A, B denote special 3-cycles and N_1, N_2 are integers displaying the units of flux. The presence of the fluxes also modifies the cancellation condition between the

[31] Target space symmetry here means that σ describes a map between $\mathcal{M}_{(10)}$ and $\mathcal{M}_{(10)}$.

tension of O-planes and D-branes. Some of the geometric moduli of the Calabi-Yau background, and we turn more closely to their discussion in a moment, are described by 3-forms, which acquire a scalar potential via eq. (3.7) and, hence, become massive.

3.1.1 $\mathcal{N} = 1$ Spectrum of $O3$-Compactifications

More generally, the spectrum obtained from Calabi-Yau-compactifications of IIB with $O3/O7$ orientifold-planes was worked out in [25]. To understand the $\mathcal{N} = 1$ spectrum one has to study the Dolbeault-cohomology of X_6. On one hand, the cohomology tells us how the ten-dimensional p-form gauge potentials decompose into 4D and 6D constituents, but it also determines the harmonic forms on X_6 [73]. The harmonic forms, or equivalently the zero modes of the internal Laplacian, are in a one-to-one correspondence with the geometric moduli of X_6. Thus, it is evident that geometric moduli and the axionic components of the p-form gauge potentials combine into $\mathcal{N} = 2$-multiplets in the case of pure Calabi-Yau compactifications and into $\mathcal{N} = 1$-multiplets for the orientifold compactifications. Additionally, under the orientifold projection the cohomology decomposes into even and odd parts. The Dolbeault-cohomology for Calabi-Yau threefolds is relatively simple and can be described by the Hodge-diamond, which reads

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \quad . \tag{3.8}$$

The unit entries are related to the unique $(3,0)$-form Ω on the threefold, while the only non-trivial entries $h^{1,1}, h^{2,1}$ describe the dimension of $H^{1,1}(X_6,\mathbb{Z})$ and $H^{2,1}(X_6,\mathbb{Z})$. Under the involution σ the cohomology decomposes as

$$\begin{aligned} H^{1,1}(X_6,\mathbb{Z}) &= H^{1,1}_{+}(X_6,\mathbb{Z}) \oplus H^{1,1}_{-}(X_6,\mathbb{Z}) \ , \qquad h^{1,1} = h^{1,1}_{+} + h^{1,1}_{-} \\ H^{2,1}(X_6,\mathbb{Z}) &= H^{2,1}_{+}(X_6,\mathbb{Z}) \oplus H^{2,1}_{-}(X_6,\mathbb{Z}) \ , \qquad h^{2,1} = h^{2,1}_{+} + h^{2,1}_{-} \ . \end{aligned} \tag{3.9}$$

In the remainder of this thesis, we assume that the orientifold projection acts such that

$$h^{2,1}_{+} = 0 \ , \qquad h^{1,1}_{-} = 0 \ , \tag{3.10}$$

in which case the spectrum simplifies further. This choice corresponds to the situation studied in [26, 28]. The $\mathcal{N} = 1$ spectrum then contains the following chiral multiplets [25]

$$(T_i \ , U_I \ , S) \ , \qquad \text{where} \quad i = 1, \dots, h^{1,1}_{+} \ , \quad I = 1, \dots, h^{2,1}_{-} \ . \tag{3.11}$$

We refer to T_i as Kähler moduli and U_I as complex-structure moduli while S is denoted as axio-dilaton. The scalar components of these chiral multiplets read

$$T_i = \tau_i + i\rho_i \ , \qquad U_I = z_I \ , \qquad S = C_0 + ie^{-\phi} \ , \tag{3.12}$$

where ρ_i are axions descending from C_4 in 10D. To define the components of the complex-structure moduli [73] we have to introduce a canonical basis $(A^{\mathcal{A}}, B_{\mathcal{B}})$, where $\mathcal{A}, \mathcal{B} = 0, \ldots, h^{2,1}$ for the homology spaces $H_3(X_6, \mathbb{Z})$ with a dual basis $(\alpha_{\mathcal{A}}, \beta^{\mathcal{B}})$ of the respective cohomological space, such that

$$\int_{A^{\mathcal{B}}} \alpha_{\mathcal{A}} = \delta_{\mathcal{A}}^{\mathcal{B}} \ , \qquad \int_{B_{\mathcal{A}}} \beta^{\mathcal{B}} = -\delta_{\mathcal{A}}^{\mathcal{B}} \ . \tag{3.13}$$

The periods of the unique (3,0)-form Ω with respect to the above basis read

$$Z^{\mathcal{A}} = \int_{A^{\mathcal{A}}} \Omega \ , \qquad \mathcal{G}_{\mathcal{A}} = \int_{B_{\mathcal{A}}} \Omega \ . \tag{3.14}$$

Now $Z^{\mathcal{A}}$ describe projective coordinates for the space of deformations of the complex-structure of X_6. The scalars z_I can then be described by affine coordinates, for instance, via

$$z_I = \frac{Z^I}{Z^0} \ , \qquad I = 1, \ldots, h^{2,1} \ . \tag{3.15}$$

It remains to define the real parts of the Kähler moduli. These are given by the Einstein frame volumes of four-cycles in $H_{1,1}(X_6, \mathbb{Z})$. In particular, let us choose a basis of the respective dual cohomological space $H^{1,1}(X_6, \mathbb{Z})$ as $\hat{D}_i$ such that the Kähler 2-form enjoys the expansion

$$J = \sum_{i=1}^{h^{1,1}} \hat{t}^i \hat{D}_i \ , \tag{3.16}$$

where $\hat{t}^i$ are the string frame two-cycle volumes. Then we can define the triple intersection numbers as

$$k_{ijk} = \frac{1}{6} \int_{X_6} \hat{D}_i \wedge \hat{D}_j \wedge \hat{D}_k \ . \tag{3.17}$$

The total string frame volume of the Calabi-Yau is, hence, given by

$$\hat{\mathcal{V}} = \int_{X_6} J \wedge J \wedge J = \frac{1}{6} k_{ijk} \hat{t}^i \hat{t}^j \hat{t}^k \ . \tag{3.18}$$

The string frame volumes of the four-cycles are of the form

$$\hat{\tau}_i = \frac{\partial \mathcal{V}}{\partial \hat{t}^i} = \frac{1}{2} k_{ijk} \hat{t}^j \hat{t}^k \ . \tag{3.19}$$

The Einstein frame volumes can now be obtained via a Weyl-rescaling of the 4D metric. The relation between Einstein- and string frame metric in the $D = 10$ theory is given by

$$g^{(E)}_{\mathcal{M}\mathcal{N}} = e^{-\phi/2} g^{(S)}_{\mathcal{M}\mathcal{N}} \ , \tag{3.20}$$

where we use $\mathcal{M}, \mathcal{N} = 0, \ldots, 9$ as ten-dimensional spacetime-indices. In particular, the Einstein frame volumes, which we simply denote by unhatted objects, read

$$\mathcal{V} = \hat{\mathcal{V}} e^{-3\phi/2} \ , \qquad t^i = e^{-\phi/2} \hat{t}^i \ , \qquad \tau_i = e^{-\phi} \hat{\tau}_i \ . \tag{3.21}$$

The fact that we explicitly distinguish between Einstein- and string frame variables may seem awkward at the moment, but will be of importance later on during the discussion of α'-corrections to the effective action of IIB supergravity. Before proceeding let us also display the following identity

$$\mathcal{V} = \frac{1}{3}\tau_i t^i \ , \tag{3.22}$$

of which we will occasionally make use later on.

3.1.2 Effective Action of $O3$-Compactifications with Fluxes

Being equipped with the structure of the chiral multiplets in the resulting $\mathcal{N}=1$, $D=4$ theory, let us now display the form of the effective action. In the following, we assume that background fluxes for an imaginary self-dual G_3 of the form as in [26] are present. The two-derivative effective action for this case was determined in [25, 26]. The Kähler potential and superpotential are of the form

$$\begin{aligned} K(T_i, U_I, S) &= -\ln(-i(S-\bar{S})) - 2\ln(\mathcal{V}) - \ln\left(-i\int_{X_6}\Omega\wedge\bar{\Omega}\right) \ , \\ W(U_I, S) &= \int_{X_6}\Omega\wedge G_3 \ . \end{aligned} \tag{3.23}$$

Notably, the above Kähler potential obeys the no-scale properties

$$K^{T_i\bar{T}_{\bar{\jmath}}}K_{T_i}K_{\bar{T}_{\bar{\jmath}}} = 3 \ , \qquad K^{X_{\hat{\imath}}\bar{X}_{\bar{\jmath}}}K_{X_{\hat{\imath}}}K_{\bar{X}_{\bar{\jmath}}} = 4 \ , \tag{3.24}$$

where we abbreviate $X_{\hat{\imath}} = (T_i, S)$. The first no-scale property, in turn, implies that the scalar potential is non-negative. In particular, the scalar potential exhibits a (non-trivial) dependence on the complex-structure moduli and the axio-dilaton, which is induced by the three-form flux, and, moreover, features a generic supersymmetric vacuum, in which U_I and S become massive. However, in this vacuum the Kähler moduli remain flat directions.

3.2 Perturbative Corrections to Type IIB Supergravity

Several different proposals for the stabilization of the Kähler moduli have been made over the years [29, 30, 74, 75]. In essence, all of these approaches involve non-perturbative corrections to W coming either from Euclidean $D3$-brane instantons or from gaugino condensation, see [76] for an exception. However, the Euclidean $D3$-brane instantons only occur when wrapping so-called rigid divisors and, hence, generically allow to stabilize only a small subset of Kähler moduli. An explicit example where not all Kähler moduli can be stabilized in this way was presented in

[77]. Additionally, the computation of these instanton corrections is a tedious task and, hence, models with several Kähler moduli are sparse.

A generic class of corrections are perturbative α'- and g_s-corrections as well as non-perturbative world-sheet instanton-corrections. It is only after inclusion of these corrections that the respective low-energy effective $D = 10$ supergravity encodes artifacts of stringy behaviour and is distinct from the ordinary IIB supergravity displayed in eq. (3.4). Moreover, regardless of the structure of X_6 we expect perturbative corrections to descend from the $D = 10$ theory and, furthermore, at least in principle to be fully calculable and model-independent. Still, it remains an open problem to prove or disprove that perturbative corrections alone can generate supersymmetric or non-supersymmetric minima for the $\mathcal{N} = 1$ Kähler moduli. At least some of the perturbative corrections have been computed, for instance in [28, 78, 79]. In this section we display the ten-dimensional form of perturbative α'- and g_s-corrections focusing on the closed string sector. Perturbative corrections for the localized sources are also of relevance and, for instance, need to be taken into account in the analysis of [26], but we omit their details in the context of this thesis. Note that recently additional apparent α^2-corrections to the Kähler potential for the Kähler moduli were inferred from F-theory [80]. These corrections are related to a redundancy in the underlying M-theory description [81] and can be absorbed via field-redefinitions [80].

The leading order perturbative corrections to S_{IIB} given in eq. (3.4) appear at the eight-derivative level and, in particular, encompass tree-level corrections of order $(\alpha')^3$. Modularity of these eight-derivative terms enforces that these corrections are multiplied by appropriate modular forms which, therefore, capture the behavior of g_s- and world-sheet instanton-corrections as well. However, the precise form of the eight-derivative corrections is only partially known. In an expansion of the component fields of the NS-NS sector of IIB supergravity, the tree level-corrections are known at the level of quartic terms [27], while the one-loop terms are known up to quintic order [82]. In particular, the tensor structure of the quartic terms is identical for the tree-level and one-loop corrections in IIB string theory.[32]

More precisely, the quartic tree-level eight-derivative corrections to the effective action in eq. (3.4) read [27, 82–86]

$$\begin{aligned} S^{(\alpha')^3}_{\text{tree}} &= \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}e^{-2\phi}\frac{\zeta(3)}{3\cdot 2^{11}}(\alpha')^3 J_0(\bar{\mathcal{R}}) \ , \\ J_0(\bar{\mathcal{R}}) &= t_8 t_8 \bar{\mathcal{R}}^4 + \frac{1}{8}\epsilon_{10}\epsilon_{10}\bar{\mathcal{R}}^4 \ . \end{aligned} \tag{3.25}$$

Here $\bar{\mathcal{R}}$ collectively denotes a tensor with four 10D spacetime indices given by

$$\bar{\mathcal{R}}_{\mathcal{MNPQ}} = \mathcal{R}_{\mathcal{MNPQ}} + \mathcal{D}_{[\mathcal{M}}H_{\mathcal{N}]\mathcal{PQ}} \ , \tag{3.26}$$

where $H_{\mathcal{NPQ}}$ denote components of H_3 and $\mathcal{D}_{\mathcal{M}}$ the 10D spacetime-covariant derivative. The tensors $\bar{\mathcal{R}}$ in eq. (3.25) are contracted with certain tensors t_8 and ϵ_{10}, see

[32] This is to be contrasted with the situation in type IIA, where the one-loop term has a different tensor structure.

[27] for their explicit form. Let us also mention the one-loop results of [82], since they shed light onto the expected form of corrections involving H_3. In particular, at one-loop these authors demonstrated that the replacement

$$\mathcal{R}_{\mathcal{MN}}{}^{\mathcal{PQ}} \to \mathcal{R}(\Omega_+)_{\mathcal{MN}}{}^{\mathcal{PQ}} = \mathcal{R}_{\mathcal{MN}}{}^{\mathcal{PQ}} + \mathcal{D}_{[\mathcal{M}} H_{\mathcal{N}]}{}^{\mathcal{PQ}} + \frac{1}{2} H_{[\mathcal{M}}{}^{\mathcal{PS}} H_{\mathcal{N}]\mathcal{S}}{}^{\mathcal{Q}} \ , \tag{3.27}$$

in the regular one-loop terms almost completely captures the dependence on H_3 up to quintic order, with additional terms arising in the odd-odd spin structure sector. In particular, due to supersymmetry we expect that the same replacement should also be valid for the tree-level terms, which would lead us to consider $J_0(\mathcal{R}(\Omega_+))$. Regardless of the specific tensor structure, these arguments strongly support the presence of the following schematic terms

$$H_3^2 \mathcal{R}^3 \ , \qquad H_3^4 \mathcal{R}^2 \ , \qquad H_3^6 \mathcal{R} \ , \qquad H_3^8 \ . \tag{3.28}$$

However, in compactifications with H-flux as in eq. (3.7) it is evident that these terms contribute to the scalar potential of the effective $\mathcal{N}=1, D=4$ supergravity. Since we neither know the exact form of these terms nor their R-R complements, we cannot compute these corrections to the 4D theory directly. Nevertheless, in [28] an indirect argument to infer at least some of the 4D terms was presented. We now turn our attention to this argument and see whether it can be extended to infer additional α'-corrections to the scalar potential.

3.2.1 α'-Corrections to 4D Supergravity

The pure $\mathcal{R}^4$-term in eq. (3.25) which we obtain when performing the limit $H_3 \to 0$ will in the following be denoted by $J_0(\mathcal{R})$. Historically, these corrections first appeared from graviton four-point scattering amplitudes [87] and the computation of the β-function of the sigma-model [88–92]. We are interested in the possible implications of these eight-derivative corrections for the four-dimensional compactified theory. In particular, in pure Calabi-Yau threefold compactifications $J_0(\mathcal{R})$ yields a correction to the prepotential describing the special Kähler geometry of the complexified Kähler deformations which constitute a subsector of the hypermultiplets in the respective $\mathcal{N}=2, D=4$ theory [93]. Since it is already a manifest correction to the $\mathcal{N}=2$ theory, it persists after introducing $O3/O7$-planes and, therefore, corrects the Kähler potential for the Kähler moduli in the $\mathcal{N}=1$ theory. However, since the orientifold projection drastically changes the spectrum, this correction has to be adapted to the proper $\mathcal{N}=1$ variables. This step was performed in [28], where also the resulting scalar potential after inclusion of 3-form fluxes was displayed. In particular, the directions T_i now receive a non-trivial scalar potential, which as argued in [28] is partially induced by conjectured 10D terms of the form

$$H_3^2 \mathcal{R}^3 \ , \qquad \mathcal{D} H_3 \mathcal{D} H_3 \mathcal{R}^2 \ . \tag{3.29}$$

In sum, the contributions to V were indirectly determined by corrections to the Kähler potential which correspond to corrections to the two-derivative kinetic terms induced by the 10D $\mathcal{R}^4$-terms.[33] The corrections to V descending from the terms in eq. (3.29) correspond schematically to regular $|F|^2$-terms in the off-shell Lagrangian of the respective $\mathcal{N}=1, D=4$ supergravity, where the symbol F collectively denotes the chiral auxiliary fields and is not to be confused with the p-form field strengths of IIB supergravity. For brevity we suppress the details of the index structure of the $|F|^2$-term here. Furthermore, also terms of the form $H_3^4\mathcal{R}^2$ are included in the eight-derivative corrections to IIB supergravity and we expect that these terms correspond to $|F|^4$-terms in the off-shell 4D theory.[34] Therefore, a possible off-shell completion of these corrections is provided by 4D higher-derivative operators.[35] These corrections will be the topic of the remainder of this chapter. Let us begin by laying out our strategy on how to compute them.

Firstly, we know that eq. (3.23) describes a theory, such that the Kähler moduli are subject to a shift-symmetric no-scale model. Hence, our results in sec. 2.7 imply that the allowed $(\alpha')^3$-corrections to V are of the form[36]

$$|F|^4\ , \qquad F^2|F|^2\ , \qquad \text{at} \quad \mathcal{O}(\mathcal{D}^4)\ . \tag{3.30}$$

From the list of operators in tab. 2.3 and the respective component forms which are displayed in sec. 2.5.6 the operators $\mathcal{O}_{(4|2)}$ and $\mathcal{O}_{(2|1)}$ generate $|F|^4$-terms. These operators also induce four-derivative terms for the chiral scalars at the level of the linearized on-shell action. Since these four-derivative terms do not depend on F^i or $\bar{F}^{\bar{\jmath}}$, they must descend from 10D terms which purely depend on the Riemann tensor, and, hence, from $J_0(\mathcal{R})$ in eq. (3.25). In turn, it is possible to compute these four-derivatives terms exactly. Therefore, our strategy is to compute the four-derivative terms for T_i originating from $J_0(\mathcal{R})$ and, afterwards, perform a matching to the supersymmetric higher-derivative operators in tab. 2.3. For simplicity, we perform the matching only for a single operator. Due to its particularly simple form the operator $\mathcal{O}_{(4|2)}$ which we already studied in greater detail in sec. 2.5.7 is predestined for this task. By supersymmetry we can then extract the $|F|^4$-term via eq. (2.122). Since we only match $\mathcal{O}_{(4|2)}$ and not $\mathcal{O}_{(2|1)}$ or any other operator in tab. 2.3 we can do this identification only up to some numerical constant.

Before we turn to the explicit computation in the next section let us make further remarks. Firstly, let us clarify that additional higher-derivative operators besides $\mathcal{O}_{(4|2)}$ are expected to be present. We discuss these further in sec. 3.2.4. Secondly,

[33] One should mention that the analysis in [28] neglects the effects of the $O7$-planes as well as the warping induced by the presence of the background flux. To properly include these effects it is convenient to start from F-theory, where recently these $(\alpha')^3$-corrections were computed and are indeed corrected by the presence of the $O7$-planes [79].

[34] Note that in the situation with localized sources and background fluxes turned on we expect these contributions to be present. On the other hand, in the context of $\mathcal{N}=2$ compactifications these corrections will be absent as no scalar potential for the moduli is generated. Indeed the corrections to the potential that will be computed in this section vanish when turning off fluxes.

[35] Note that also higher-derivative couplings in compactifications to $(\mathcal{N}=2, D=4)$-supergravity have been investigated, for instance in [94].

[36] Let us emphasize that this would equally hold if other leading order corrections to the scalar potential coming from higher-derivative corrections of different origin, such as g_s-corrections, were determined.

one may wonder whether the correction to V in the 4D theory induced by the $H_3^4\mathcal{R}^2$-term can also be captured via a new term in the Kähler potential. In appendix B.2 we investigate and comment on this possibility further. However, absent knowledge of the precise form of the H_3-dependent terms in eq. (3.25) this question cannot be fully answered.

3.2.2 KK-Reduction of Type IIB $(\alpha')^3\mathcal{R}^4$-Corrections

We now perform the explicit reduction of the four-derivative terms for Kähler-class deformations from the $(\alpha')^3\mathcal{R}^4$ corrections in eq. (3.25). The following derivation is in many ways analogous to the computation in [28]. Before turning to the explicit analysis let us stress again that we focus on deriving the overall functional form of the tensor $T_{ij\bar{k}\bar{l}}$ in eq. (2.120) and omit the details of numerical factors. While it would, in principle, be possible to perform a full discussion of the four-derivative terms and conduct an exact operator-matching using the result of sec. 2.5.6, our approach here is more modest.

Our starting point is the eight-derivative correction $J_0(\mathcal{R})$ given in eq. (3.25) which we displayed in the string frame.[37] There exists a basis of 26 independent contractions of four Riemann tensors [95] and we use this basis to expand $J_0(\mathcal{R})$. Since we are not interested in keeping track of numerical factors, the coefficients of the expansion of $J_0(\mathcal{R})$ are irrelevant here. From now on we simply argue within this basis of 26 terms to obtain the functional form of the possible four-derivative terms.

Next let us discuss the form of the ten-dimensional metric, which constitutes the starting point of the KK-reduction. We make the following simplifications. Firstly, we will not compute the coupling of gravity to the higher-derivatives of the Kähler moduli and, therefore, set the four-dimensional part of the metric to a Minkowski-form. Secondly, we will set the warping-factor to unity. Note that, in general this warping-factor is a non-trivial function after inclusion of background fluxes, but is expected to correct our results only at a subleading order. Lastly, for simplicity we conduct the analysis with a single Kähler-type deformation present, that is

$$h_+^{1,1} = h^{1,1} = 1 \ . \tag{3.31}$$

Using these assumptions the ten-dimensional metric reads

$$\mathrm{d}s^2_{(10)} = g_{\mathcal{M}\mathcal{N}}\mathrm{d}x^{\mathcal{M}}\mathrm{d}x^{\mathcal{N}} = \eta_{mn}\mathrm{d}x^m\mathrm{d}x^n + g_{\hat{m}\hat{n}}\mathrm{d}y^{\hat{m}}\mathrm{d}y^{\hat{n}} \ , \tag{3.32}$$

where as before $\mathcal{M},\mathcal{N} = 0,\dots,9$ and $y^{\hat{m}}, \hat{m} = 1,\dots,6$ denote real coordinates on the compact manifold X_6. It is convenient to decompose the 6D metric as follows

$$g_{\hat{m}\hat{n}} = \mathrm{e}^{2u(x)}h_{\hat{m}\hat{n}}(y) \ . \tag{3.33}$$

[37] Note that in a proper discussion of the $(\alpha')^3$-corrections to the low-energy effective 4D supergravity of the scalars we would also have to incorporate higher-derivative corrections for the dilaton as given in [84]. In particular, these higher-derivative corrections were included in the analysis of [28]. However, for our purposes it suffices to consider the dilaton at the two-derivative level and simply use the results of [28] for the correction to the two-derivative terms for the volume modulus.

Here the volume measured by the background metric $h_{\hat{m}\hat{n}}$ is normalized to unity, which is achieved by setting $(2\pi\alpha') = 1$. This way the Planck constants in ten and four dimensions coincide $\kappa_{10}^{-2} = \kappa_4^{-2}$. The single volume modulus in the string frame is normalized as $\mathrm{e}^{6u} = \hat{\mathcal{V}}$.

The $\mathcal{R}^4$-term in eq. (3.25) modifies the ten-dimensional Einstein equations. In particular, the Einstein equations along the six-dimensional directions are of the form [96]

$$\mathcal{R}_{\alpha\bar{\beta}} \sim (\alpha')^3 \partial_\alpha \partial_{\bar{\beta}} Q \ , \tag{3.34}$$

where we introduced local complex coordinates $(z^\alpha, \bar{z}^{\bar{\beta}})$ with $\alpha, \bar{\beta} = 1, 2, 3$ on the internal manifold. The indices here are not to be confused with the 4D Weyl-spinor indices. Furthermore, Q denotes the six-dimensional Euler integrand, that is $\int d^6y \sqrt{g} Q = \chi(X_6)$, where $\chi(X_6) = 2(h^{1,1} - h^{2,1})$ denotes the Euler characteristic of X_6. In general, a Ricci-flat metric does not solve eq. (3.34). Instead we can formally solve eq. (3.34) by

$$h_{\hat{m}\hat{n}} = h^{(0)}_{\hat{m}\hat{n}} + (\alpha')^3 h^{(1)}_{\hat{m}\hat{n}} \ , \tag{3.35}$$

where $h^{(0)}$ is a Ricci-flat metric solving the zeroth-order Einstein equations and $h^{(1)}$ solves eq. (3.34) at order $(\alpha')^3$. In turn, we expect that also the deformations receive $(\alpha')^3$-corrections. Now, $h^{(1)}$ only needs to be considered for the reduction of the leading order $(\alpha')^0$ terms in eq. (3.4). However, none of these terms induce 4D four-derivative corrections. Therefore, it is not necessary to take into account the correction $h^{(1)}$, since it corrects the four-derivative terms only at order $(\alpha')^6$. Thus, without loss of generality we can safely ignore the correction $h^{(1)}$ and treat h as Ricci-flat.

To determine the curvature invariants in $J_0(\mathcal{R})$ we first need to compute the components of the Riemann tensor. In the following we use the conventions

$$\begin{aligned} \mathcal{R}^{\mathcal{M}}{}_{\mathcal{N}\mathcal{P}\mathcal{Q}} &= \partial_{\mathcal{P}} \Gamma^{\mathcal{M}}_{\mathcal{Q}\mathcal{N}} - \partial_{\mathcal{Q}} \Gamma^{\mathcal{M}}_{\mathcal{P}\mathcal{N}} + \Gamma^{\mathcal{R}}_{\mathcal{Q}\mathcal{N}} \Gamma^{\mathcal{M}}_{\mathcal{P}\mathcal{R}} - \Gamma^{\mathcal{R}}_{\mathcal{P}\mathcal{N}} \Gamma^{\mathcal{M}}_{\mathcal{Q}\mathcal{R}} \ , \\ \Gamma^{\mathcal{M}}_{\mathcal{P}\mathcal{N}} &= \frac{1}{2} g^{\mathcal{M}\mathcal{Q}} \left(\partial_{\mathcal{P}} g_{\mathcal{N}\mathcal{Q}} + \partial_{\mathcal{N}} g_{\mathcal{P}\mathcal{Q}} - \partial_{\mathcal{Q}} g_{\mathcal{P}\mathcal{N}} \right) \ . \end{aligned} \tag{3.36}$$

Up to symmetries there are only two non-vanishing independent pieces of the Riemann tensor computed with respect to the metric in eq. (3.32). They are given by

$$\begin{aligned} \mathcal{R}_{\hat{m}m\hat{n}n} &= -g_{\hat{m}\hat{n}} (\partial_m u \partial_n u + \partial_m \partial_n u) \ , \\ \mathcal{R}_{\hat{k}\hat{m}\hat{n}\hat{p}} &= \mathrm{e}^{2u} \mathcal{R}^{(h)}_{\hat{k}\hat{m}\hat{n}\hat{p}} + (\partial u)^2 (g_{\hat{k}\hat{p}} g_{\hat{m}\hat{n}} - g_{\hat{k}\hat{n}} g_{\hat{p}\hat{m}}) \ . \end{aligned} \tag{3.37}$$

Here $\mathcal{R}^{(h)}_{\hat{k}\hat{m}\hat{n}\hat{p}}$ denotes the Riemann tensor components of the background metric h. Furthermore, the Riemann tensor allows to compute the Ricci-tensor as well as the scalar curvature

$$\begin{gathered} \mathcal{R}_{mn} = -6(\partial_m u \partial_n u + \partial_m \partial_n u) \ , \qquad \mathcal{R}_{\hat{m}\hat{n}} = -g_{\hat{m}\hat{n}} (6(\partial u)^2 + \Box u) \ , \\ \mathcal{R} = -42(\partial u)^2 - 12\Box u \ . \end{gathered} \tag{3.38}$$

It is evident that in the KK-reduction of $J_0(\mathcal{R})$ we obtain terms with up to eight derivatives acting on u. For our purposes it suffices to determine the terms with four-derivatives. After computation of all 26 basis elements in [95] we find the following four-derivative terms

$$\begin{aligned} J_0(\mathcal{R}) \supset \mathrm{e}^{-4u}\Big[&\alpha_1(\partial u)^4 + \alpha_2\Box u(\partial u)^2 + \alpha_3(\Box u)^2 + \alpha_4(\partial_m\partial_n u)(\partial^m\partial^n u) \\ &+ \alpha_5(\partial_m\partial_n u)(\partial^m u)(\partial^n u)\Big]\mathcal{R}^{(h)}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}_{(h)}^{\hat{k}\hat{m}\hat{n}\hat{p}} \ , \end{aligned} \tag{3.39}$$

for some constants α_i. Since for a Calabi-Yau $\mathcal{R}^{(h)}_{\hat{m}\hat{n}} = 0$, the only non-vanishing contraction of two Riemann tensors is given by $\mathcal{R}^{(h)}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}_{(h)}^{\hat{k}\hat{m}\hat{n}\hat{p}}$. We see that five different four-derivative terms appear here. However, in the four-dimensional action these terms are not mutually independent but certain terms are related by partial integration.[38] In a full reduction it would be necessary to jointly discuss all five operators in eq. (3.39). However, here we are interested only in the first term in eq. (3.39), which is the only ∂^4-term that needs to be matched to the Lagrangian in eq. (2.120). It is convenient to express the Riemann-tensor square with respect to $g_{\hat{m}\hat{n}}$ again. Up to derivatives we have

$$\mathcal{R}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}^{\hat{k}\hat{m}\hat{n}\hat{p}} = \mathrm{e}^{-4u}\mathcal{R}^{(h)}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}_{(h)}^{\hat{k}\hat{m}\hat{n}\hat{p}} + \dots \tag{3.40}$$

In the action the integration now splits into four- and six-dimensional parts, such that

$$S_{(\partial u)^4} = -\frac{1}{2\kappa_4^2}\int \mathrm{d}^4x\sqrt{-g}\,\mathrm{e}^{-2\phi_0}\,\alpha_1\,(\partial u)^4\int_{X_6}\mathrm{d}^6y\sqrt{g}\,\mathcal{R}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}^{\hat{k}\hat{m}\hat{n}\hat{p}} \ . \tag{3.41}$$

It is convenient to rewrite the integral over the compact dimensions as follows

$$\int_{X_6}\mathrm{d}^6y\sqrt{g}\,\mathcal{R}_{\hat{k}\hat{m}\hat{n}\hat{p}}\mathcal{R}^{\hat{k}\hat{m}\hat{n}\hat{p}} \sim \int_{X_6} c_2\wedge J \ , \tag{3.42}$$

where c_2 is the second Chern class of the Calabi-Yau threefold and J its Kähler form. This can be checked directly using local complex coordinates.[39] With respect to these coordinates we have

$$c_2 = \frac{1}{2}\left(\mathrm{Tr}\mathcal{R}^2 - (\mathrm{Tr}\mathcal{R})^2\right) \ , \qquad J \ = ig_{\alpha\bar{\beta}}\mathrm{d}z^\alpha\wedge\mathrm{d}\bar{z}^{\bar{\beta}} \ , \tag{3.43}$$

where $\mathcal{R}$ is the curvature two-form. The traces of the curvature two-form are given by

$$\begin{aligned} \mathrm{Tr}\mathcal{R} &= \mathcal{R}^\alpha{}_{\alpha\beta\bar{\gamma}}\,\mathrm{d}z^\beta\wedge\mathrm{d}\bar{z}^{\bar{\gamma}} \ , \\ \mathrm{Tr}\mathcal{R}^2 &= \mathcal{R}^\alpha{}_{\beta\gamma\bar{\delta}}\mathcal{R}^\beta{}_{\alpha\epsilon\bar{\zeta}}\,\mathrm{d}z^\gamma\wedge\mathrm{d}\bar{z}^{\bar{\delta}}\wedge\mathrm{d}z^\epsilon\wedge\mathrm{d}\bar{z}^{\bar{\zeta}} \ . \end{aligned} \tag{3.44}$$

[38] For example via partial integration $(\partial_m\partial_n u)(\partial^m u)(\partial^n u)$ can be recast as a combination of $(\partial u)^4$ and $\Box u(\partial u)^2$.

[39] To prove eq. (3.42) it is also helpful to note the relation $\sqrt{\det(h_{\hat{m}\hat{n}})} = \det(g_{\alpha\bar{\beta}})$, which links the volume forms of the two different coordinate systems to each other.

On a Calabi-Yau we have $\mathrm{Tr}\mathcal{R} = 0$ and, hence, we find that eq. (3.42) holds. Furthermore, from eq. (3.42) it is evident that

$$\int_{X_6} c_2 \wedge J \geq 0 \ . \tag{3.45}$$

Here equality holds, if and only if X_6 has constant holomorphic sectional curvature [97]. For Kähler manifolds with constant holomorphic sectional curvature c the Riemann tensor must necessarily take the form [98]

$$\mathcal{R}_{\alpha\bar{\beta}\gamma\bar{\delta}} = -\frac{c}{2}\left(g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} + g_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}}\right) \tag{3.46}$$

and, thus, for Calabi-Yau manifolds $c = 0 = \mathcal{R}_{\alpha\bar{\beta}\gamma\bar{\delta}}$. This is only possible if X_6 is a torus T^6.

The term in eq. (3.41) is expressed in the string frame. In order to transform to the Einstein frame we need to determine the proper Weyl factor, which can be read off from the results of [28]. The two-derivative part of the bosonic action is given by[40]

$$S = -\frac{1}{2\kappa_4^2}\int \mathrm{d}^4x\sqrt{-g}\,\mathrm{e}^{-2\phi_0}\left(\mathrm{e}^{6u} + \frac{\xi}{2}\right)\mathcal{R}^{(4)} + \ldots \tag{3.47}$$

where $\mathcal{R}^{(4)}$ denotes the scalar curvature in four dimensions and ξ parametrizes the leading α'-corrections and reads [28]

$$\xi = -\frac{(\alpha')^3\zeta(3)\chi(X_6)}{2(2\pi)^3} \ . \tag{3.48}$$

In turn, the 4D Weyl-transformation is of the form

$$g^{(E)}_{mn} = \mathrm{e}^{-\phi_0/2}\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) g_{mn} \ , \tag{3.49}$$

where the combinatorial number ξ was rescaled by a dilaton-dependent factor as follows

$$\hat{\xi} = \xi g_s^{-3/2} \ . \tag{3.50}$$

The Weyl rescaling implies that also couplings of the 4D Riemann tensor to the Kähler deformation contribute to the four-derivative term for u. Again these terms are functionally indistinguishable from eq. (3.41) since they necessarily need to be multiplied by the same curvature-invariant on X_6 given in eq. (3.42).[41] After performing the Weyl transformation we also have to rediagonalize the kinetic terms for the scalar fields. This is achieved by recasting all the string frame volumes in terms of Einstein frame volumes as in eq. (3.21), such that we are left with the proper $\mathcal{N} = 1$ coordinates displayed in eq. (3.12).

[40] We promote η_{mn} to an arbitrary Lorentzian metric g_{mn} here.

[41] A coupling of the $\mathcal{N} = 2$ vector multiplets to four-dimensional curvature invariants is forbidden by supersymmetry [27] and, hence, we expect these couplings also to be absent in the $\mathcal{N} = 1$ sector. However, a coupling of the four dimensional Riemann-tensor to derivatives of the Kähler moduli might be present.

Furthermore, we can expand $J = \hat{t}\hat{D}$, where $\hat{D}$ is the single $(1,1)$-form in $H^{1,1}(X_6, \mathbb{Z})$. Hence, the integral on the r.h.s. of eq. (3.42) reads

$$\int_{X_6} c_2 \wedge J = \hat{t} \int_{X_6} c_2 \wedge \hat{D} \equiv \Pi \, \hat{t} \ , \tag{3.51}$$

where Π is an integer number encoding the topological information of the second Chern class. We are now in a position to read off the final form of the four-derivative term. Up to terms involving derivatives of the dilaton we can recast eq. (3.41) in terms of the proper $\mathcal{N} = 1$ coordinates in eq. (3.12)

$$S_{(\partial u)^4} \sim -\frac{1}{2\kappa_4^2} \int \mathrm{d}^4x \sqrt{-g^{(E)}} g_s^{-3/2} \Pi \, t \, (\partial \tau)^4 \left(\frac{\partial}{\partial \tau} K_{(0)} \right)^4 \ , \tag{3.52}$$

where $K_{(0)} = -2 \ln(\hat{\mathcal{V}})$ denotes the leading order no-scale Kähler potential. Finally, we can match this result to the Lagrangian in eq. (2.120) and read off the coupling

$$T = \lambda(\alpha')^3 (\Pi \, t) \left[\frac{1}{2i}(S - \bar{S}) \right]^{3/2} \left(\frac{\partial}{\partial \tau} K_{(0)} \right)^4 \ , \tag{3.53}$$

where λ denotes the overall unknown numerical factor. Matching to eq. (2.122) find the following correction to the scalar potential

$$V_{(1)} = -e^{2K_{(0)}} \lambda (\Pi \, t) g_s^{-3/2} \left(\frac{\partial}{\partial \tau} K_{(0)} \right)^4 |D_T W|^4 \ . \tag{3.54}$$

Note that this result holds for any superpotential, even if it includes non-perturbative corrections. Moreover, we truncated $V_{(1)}$ at the linearized level and, therefore, did not include the $(\alpha')^3$-corrections to the Kähler potential descending from eq. (3.47) into the above formula, since they enter only at order $(\alpha')^6$. We conclude this section by remarking that the result in eq. (3.54) is in agreement with [99], where a naive estimate for the volume dependence of the potential induced by the $\mathcal{R}^2 G_3^4$ terms was found to be $\mathcal{V}^{-11/3}$.

3.2.3 Multiple Kähler Deformations and Form of Full Potential

In the previous section we conducted the KK-reduction with a single Kähler class deformation turned on. However, we expect the inferred form of the correction to the scalar potential $V_{(1)}$ in eq. (3.54) to hold also in the case of arbitrarily many Kähler moduli, and in this section we explain in detail what this statement precisely means. This holds only as long as the superpotential is well-approximated by the leading constant flux-superpotential along the T_i directions. Our argument heavily relies on some technical observations, which are proven in appendix B.1. When arbitrarily many Kähler-class deformations are present, we no longer know the precise structure of the coupling tensor in eq. (3.53). One may conjecture that it is proportional to

$$T_{ijkl} \sim K_{(0),i} K_{(0),j} K_{(0),k} K_{(0),l} \ , \tag{3.55}$$

but it might also be that the Kähler metric $K_{(0),i\bar{\jmath}}$ or an even more complicated tensor appears. However, even though the coupling tensor computed for arbitrary $h^{1,1}$ might be different from eq. (3.55), there is evidence that the induced correction to the scalar potential can be inferred from the computation with $h^{1,1} = 1$ without loss of generality. To see this we use of the results of appendix B.1, which we now briefly summarize. In the large volume limit, and under the assumption that the superpotential is constant, the correction to the scalar potential in eq. (2.122) behaves as

$$V_{(1)} = -\frac{|W|^4}{\mathcal{V}^4} T_{(0)}{}^{\bar{\imath}\bar{\jmath}kl} K_{(0),\bar{\imath}} K_{(0),\bar{\jmath}} K_{(0),k} K_{(0),l} + \dots \ , \tag{3.56}$$

where $T_{(0)}$ is the coupling tensor truncated to the leading order term in the large volume limit. From the above index structure it is clear that $T_{(0)}$ must be a tensor of the Kähler manifold defined by the Kähler potential $K_{(0)}$. We assume that its tensor structure is derived from $K_{(0)}$, which means that any indexed quantity appearing in $T_{(0)}$ is related to derivatives of $K_{(0)}$ and possibly contractions with the inverse Kähler metric, see appendix B.1 for more details. This already exhausts all plausible tensors which one may write down for this Kähler geometry. In appendix B.1 we study eq. (3.56) in detail and provide evidence for the following statement: If $T_{(0)}$ does not involve any scalar function and, hence, only consists of objects with at least one index, then $V_{(1)} \sim \mathcal{V}^{-4}$ up to some constant. Thus, an additional dependence of $V_{(1)}$ on $\mathcal{V}$ or τ_i can only be generated by scalar functions appearing in $T_{(0)}$.

When reducing J_0 with an arbitrary number of Kähler-type deformations turned on, the four-derivative terms are again obtained from those contractions where two out of the four Riemann tensors have indices along the internal directions and, thus, contribute a factor $\int c_2 \wedge J$. The remaining indices yield contracted metrics or derivatives. We infer that the general coupling tensor should be of the form

$$T_{kl\bar{\imath}\bar{\jmath}} \sim \left(\int_{X_6} c_2 \wedge J \right) \mathcal{T}_{kl\bar{\imath}\bar{\jmath}} \tag{3.57}$$

where $\mathcal{T}$ is a tensor, that consists purely of indexed quantities. As we consider terms at order $(\alpha')^3$ this tensor is a tensor in the geometry defined by $K_{(0)}$. Thus, we can apply the results of the appendix B.1 and conclude that the functional behavior of eq. (3.56) is captured by $\int c_2 \wedge J$, which was already present in the computation with $h^{1,1} = 1$.

With these results at hand, we can now proceed to display the final form of the correction to the scalar potential. Again we expand $J = \sum_{i=1}^{h^{1,1}} \hat{t}^i \hat{D}_i$, where $\hat{D}_i$ form a basis of the Dolbeault-cohomology $H^{1,1}(X_6, \mathbb{Z})$ such that

$$\int_{X_6} c_2 \wedge J = \sum_{i=1}^{h^{1,1}} \hat{t}^i \int_{X_6} c_2 \wedge \hat{D}_i \equiv \sum_{i=1}^{h^{1,1}} \Pi_i \hat{t}^i \ , \tag{3.58}$$

where as before Π_i are some integer topological numbers. Finally, putting all the information together we conclude

$$V_{(1)} = -\hat{\lambda} g_s^2 \frac{|W|^4}{\mathcal{V}^4} \sum_{i=1}^{h^{1,1}} \Pi_i t^i \ , \tag{3.59}$$

where $\hat{\lambda} = g_s^{-3/2}\hat{\lambda}_0$ and $\hat{\lambda}_0$ denotes the undetermined overall numerical factor and is simply a real number.

To conclude this section let us also mention the additional two-derivative terms in the Lagrangian in eq. (2.120) which accompany the four-derivative terms and eq. (3.59). Absent knowledge of the precise form of the coupling tensor of the higher-derivative operator under investigation, we cannot display the explicit form of this two-derivative correction. We can, however, comment on its ten-dimensional origin. Necessarily this term will involve $\int c_2 \wedge J$ and feature two-derivative terms for T_i. Therefore, it must be induced by corrections of the NS-NS sector of the type

$$\mathcal{R}^3 H_3^2 \ , \tag{3.60}$$

which as we already mentioned are also in part responsible for the correction to the scalar potential of [28].

3.2.4 Survey of Corrections and Higher-Derivative Operators

So far we have matched the operator $\mathcal{O}_{(4|2)}$ to the four-derivative terms descending from the ten-dimensional $(\alpha')^3$-correction $J_0(\mathcal{R})$. We may ask ourselves which other corrections arise from $J_0(\mathcal{R}(\Omega_+))$ with $\mathcal{R}(\Omega_+)$ given in eq. (3.27) or from possible additional corrections in the NS-NS or R-R sector.[42]

Let us begin by giving a survey of all terms which arise just from $J_0(\mathcal{R})$ and which additional corrections to the scalar potential are required for their supersymmetrization. In principle, we obtain from $J_0(\mathcal{R})$ terms without derivatives as well as two-derivative, four-derivative, six-derivative and eight-derivative terms for the volume moduli. The coefficient functions of these terms are built from topological integrals over the compact dimensions. For instance, the two-derivative terms require that three out of the four Riemann tensors in $J_0(\mathcal{R})$ form a topological integrand over X_6, the result being the Euler integrand Q or equivalently the third Chern class $c_3(X_6)$. As we have seen in the previous sections, the four-derivative terms are multiplied by an integral over $c_2(X_6) \wedge J$. Similarly, the coefficient of the six-derivative terms is given by an integral over a single Riemann tensor with indices in the compact directions. Therefore, the corresponding topological integrand must involve the first Chern class $c_1(X_6)$. Since $c_1(X_6)$ vanishes at this order of the calculation, we expect these terms to be absent. Finally, the eight-derivative terms require exhausting all four Riemann tensors in $J_0(\mathcal{R})$ and, hence, their coefficient is given by the volume of the compact dimensions.

In tab. 3.1 we collect the possible terms that may arise from $J_0(\mathcal{R})$, the respective coefficient function induced by the remaining Riemann-tensors with indices along the directions of X_6, the possible off-shell corrections to the scalar potential which may thereby be induced and the corresponding ten-dimensional terms which generate

[42]Note that also S_{IIB} in eq. (3.4) yields α'-corrections after transforming to the Einstein frame in the four-dimensional action. In particular, the H_3^2 term in eq. (3.4) then contributes to the $(\alpha')^3|F|^2$-corrections of [28].

Type	Coefficient	δV via SUSY	δV induced by
1	0	0	$\mathcal{R}^4$
∂^2	c_3	$\|F\|^2$	$H_3^2\mathcal{R}^3$, ...
∂^4	$c_2 \wedge J$	$\|F\|^4$	$H_3^4\mathcal{R}^2$, ...
∂^6	0	0	–
∂^8	$J \wedge J \wedge J$	0	–

Table 3.1: *Survey of different terms descending from $J_0(\mathcal{R})$. The type of correction refers to how many derivatives of the Kähler moduli the individual contractions yield, and the coefficient which invariant on X_6 the remaining Riemann-tensors form. We also displayed which type of corrections to the 4D scalar potential δV are related to these derivative terms by supersymmetry and which class of ten-dimensional $(\alpha')^3$ terms may induce δV.*

these corrections to V. Let us give a few clarifications regarding tab. 3.1. Firstly, in the absence of warping effects $J_0(\mathcal{R})$ does not induce terms in the scalar potential. Suppose such a correction to the scalar potential would exist, then this correction would already be present in the pure Calabi-Yau compactifications. However, in the respective $\mathcal{N} = 2, D = 4$ theory, no potential for the moduli can be generated and all α'-corrections merely renormalize the definition of the tree-level moduli. Secondly, note that the reason why the additional six- and eight-derivative terms cannot induce corrections to the scalar potential can be understood from eq. (2.26).

In the NS-NS sector it remains to discuss additional terms in the ten-dimensional action with varying powers of H_3. Instead of giving a survey of possible terms descending from these various 10D terms, we investigate the possible 4D higher-derivative operators, since, ultimately, additional corrections have to be matched to 4D higher-derivative operators. Let us make a small survey of possible operators now. To begin with, operators at the level of two-superspace derivatives include $\mathcal{O}_{(1)}$ and $\mathcal{O}_{(3)}$. Both operators are forbidden by world-sheet parity, as they would imply flux-induced corrections to the scalar potential descending from 10D terms with an odd number of H_3. Therefore, the leading order higher-derivative operators are induced at the order $\mathcal{O}(\mathcal{D}^4)$. We have already discussed $\mathcal{O}_{(4|2)}$ and $\mathcal{O}_{(2|1)}$ which are the only operators at that order that receive four-derivative terms from $(\alpha')^3$-terms and induce corrections to the scalar potential. In both cases the induced correction is of the form $|F|^4$ and, thus, coincides. Further operators at this order may induce terms of the type $F^2|F|^2$, but we expect these to be functionally indistinguishable from the $|F|^4$ terms. Operators involving the equivalent of more than four spinorial super-covariant derivatives are also allowed, but by means of eq. (2.145) they will induce pure F-term corrections. Again via world-sheet parity the only possible corrections of this type are of the form

$$|F|^6 \; , \qquad |F|^8 \; , \tag{3.61}$$

where also rearrangements thereof which arise by replacing F by $\bar{F}$ and vice versa are allowed. The total number of possible F-terms is, of course, bounded by the fact that these terms must descend from eight-derivative corrections in ten dimensions. Supersymmetry relates the quantities in eq. (3.61) to four-derivative terms for the Kähler moduli. For dimensional reasons these four-derivative terms must necessarily depend on F^i and, therefore, cannot be induced by $J_0(\mathcal{R})$. Instead they descend from contractions of the form $\mathcal{R}^3 H_3^2$ and $\mathcal{R}^2 H_3^4$ respectively. For instance, we find that

$$\begin{array}{ccc}
\mathcal{R}^3 H_3^2 & \xrightarrow{\ \mathcal{M}_4 \times X_6\ } & |F|^2 c_1(X_6)(\partial u)^4 \\
{\scriptstyle \text{10D SUSY}}\ \updownarrow & & \updownarrow\ {\scriptstyle \text{4D SUSY}} \\
\mathcal{R} H_3^6 & \xrightarrow{\ \mathcal{M}_4 \times X_6\ } & |F|^6 c_1(X_6)
\end{array}$$

and, since at leading order $c_1(X_6) = 0$ we expect the $|F|^6$-term to be absent. In turn, the single remaining type of correction (induced by pure H_3^8-terms) reads

$$\left(\int_{X_6} J \wedge J \wedge J \right) |F|^8 \sim \frac{|W|^8}{\mathcal{V}^7} \,, \tag{3.62}$$

and only depends on the overall volume.

Let us now make a few remarks regarding further terms which we did not capture so far. To begin with there exist corrections with additional derivatives of the dilaton. These terms do not contribute to the scalar potential, but are important for the consistency of the equations of motion. More precisely the presence of the R^4 terms demands the addition of terms of the type $R^3(\nabla\tau)^2$ [28]. Additional terms are expected to arise in the R-R sector. In particular, this encompasses terms involving the self-dual five-form $\tilde{F}_5$. In compactifications with imaginary self-dual 3-form flux also $\tilde{F}_5$ receives a background flux but only indirectly via warping effects [26]. Therefore, in a weak-warping approximation these terms are expected to be subleading.[43]

[43] However, in principle warping-induced corrections to the scalar potential are relevant, since naive dimensional arguments suggest that these contribute at $\mathcal{O}(\mathcal{V}^{-11/3})$ [99]. A proper accounting of such effects is outside the scope of this thesis and will be left to future investigations.

Chapter 4

Application to String Cosmology

Having determined the novel $(\alpha')^3$ corrections to the four-dimensional scalar potential in eq. (3.59) we would like to study their relevance for moduli stabilization and inflation. Here we begin by investigating the possibility of stabilizing Kähler moduli via the new correction following ref. [31]. In sec. 4.2 we instead review content from [37].

To begin with it is necessary to display the full scalar potential. We assume that the complex structure moduli U_I and the dilaton S are stabilized and sit in the supersymmetric minimum generated by the G_3-flux. The theory describing the dynamics of the Kähler moduli is obtained by replacing U_I and S by their vacuum expectation values in eq. (3.23). We also now include the $(\alpha')^3$-corrections to the Kähler potential of [28]. Furthermore, the Kähler potential can receive additional string-loop corrections induced by the exchange of strings between localized sources, in particular of strings winding cycles in the intersection locus of stacks of $D7$-branes or carrying KK-momentum. These have been explicitly computed for toroidal orientifolds, such as $T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ in [41] and for arbitrary Calabi-Yau threefolds their functional form has been inferred in [42]. Altogether the Kähler and superpotential then read

$$\begin{aligned} K &= \ln(g_s) - 2\ln(\mathcal{V} + \tfrac{1}{2}\hat{\xi}) + \delta K^{KK}_{(g_s)} + \delta K^{W}_{(g_s)} + \dots \ , \\ W &= W_0 = \left\langle \int_{X_6} G_3 \wedge \Omega \right\rangle \ , \end{aligned} \tag{4.1}$$

where $\hat{\xi}$ was given in eq. (3.50) and eq. (3.48), and the dots denote the Kähler potential of the U_I. Furthermore, $\mathcal{V}$ was given in eqs. (3.18), (3.21). The corrections $\delta K^{KK}_{(g_s)}$ and $\delta K^{W}_{(g_s)}$ in eq. (4.1) denote the leading order string-loop corrections. Their general form for arbitrary Calabi-Yau threefolds has been argued to be [42]

$$\delta K^{KK}_{(g_s)} \sim g_s \sum_{i=1}^{h^{1,1}} \frac{C_i(a_{ij}t^j)}{\mathcal{V}} \ , \qquad \delta K^{W}_{(g_s)} \sim \sum_{i=1}^{h^{1,1}} \frac{D_i(a_{ij}t^j)^{-1}}{\mathcal{V}} \ . \tag{4.2}$$

The first term is interpreted as coming from exchange KK-modes between $D3/D7$-branes and $O3/O7$-planes, while the latter is induced by the exchange of strings winding one-cycles in the intersection locus of stacks of $D7$-branes. The coefficients

C_i and D_i are expected to be functions of the complex structure moduli and the dilaton. However, since we assume the latter have already been stabilized, we treat C_i, D_i as constants. The matrix a_{ij} consists of combinatorial constants.

The scalar potential derived from eq. (4.1) including the $|F|^4$-term in (3.59) can be split up as follows

$$V = V_{(\alpha')} + V_{(g_s)} + V_{(1)} \ . \tag{4.3}$$

The first term describes the scalar potential obtained from the Kähler potential in eq. (4.1) without string-loop corrections. The pure $(\alpha')^3$-piece reads

$$V_{(\alpha')} = g_s \frac{3\hat{\xi}|W|^2}{4\mathcal{V}^3} \ . \tag{4.4}$$

When expanding the string-loop contribution to the potential, one obtains the following terms at leading order [43]

$$V_{(g_s)} = \sum_{i=1}^{h^{1,1}} \frac{|W|^2}{\mathcal{V}^2} \left[g_s^3 C_i^2 K_{(0),ii} - 2 g_s \delta K^W_{(g_s),\tau_i} \right] \ . \tag{4.5}$$

We observe that the dominant term in inverse volume is given by $V_{(\alpha')}$, followed by $V_{(g_s)}$ and finally $V_{(1)}$ which, roughly speaking, is suppressed by a factor of $\mathcal{V}^{1/3}$ with respect to $V_{(g_s)}$. Furthermore, the g_s-corrections have a relative factor of $g_s^{1/2}$ and $g_s^{5/2}$ with respect to $V_{(1)}$.

Let us make a final remark regarding the Kähler potential in eq. (4.1) noted in [36]. The Kähler potential including the tree-level $(\alpha')^3$-corrections parametrized by $\hat{\xi}$ is still of a no-scale form, since it is given by the logarithm of a homogeneous function when including the dilaton. The respective no-scale property reads

$$K^{X_{\hat{\imath}}\bar{X}_{\bar{\jmath}}} K_{X_{\hat{\imath}}} K_{\bar{X}_{\bar{\jmath}}} = 4 \ , \tag{4.6}$$

where as before $X_{\hat{\imath}} = (T_i, S)$. However, this no-scale property no longer holds when including the above g_s-corrections, or the 1-loop corrections or world-sheet instanton corrections to $J_0(\mathcal{R})$.

4.1 Perturbative Moduli Stabilization

As we have already mentioned it is interesting to study whether a fully perturbative stabilization of the Kähler moduli is possible. In the following we will entertain the possibility that all Kähler moduli are stabilized purely by $(\alpha')^3$-corrections instead of the non-perturbative corrections to the superpotential. Here we take only the $(\alpha')^3$-corrections in eq. (4.3) into account and will neglect eq. (3.62) as well as the string-loop corrections, which can be sufficiently suppressed by choosing moderately small values of g_s. Altogether, the scalar potential then reads

$$V = \frac{3\hat{\xi}|W|^2}{4\mathcal{V}^3} - \hat{\lambda}|W|^4 \frac{\Pi_i t^i}{\mathcal{V}^4} \ . \tag{4.7}$$

To shorten notation we absorbed the additional factor of g_s coming from the e^K prefactor into $|W|^2$ here.

For $\hat{\lambda} < 0$ we will now show that V has a non-supersymmetric AdS minimum for any orientifolded Calabi-Yau threefold with $\xi < 0$ where *all* four-cycles are fixed as

$$\langle \tau_i \rangle = \mathcal{C}\,\Pi_i \;, \qquad \text{with} \qquad \mathcal{C} = \frac{44\hat{\lambda}|W|^2}{9\hat{\xi}} \;. \tag{4.8}$$

The volume in this minimum is given by

$$\langle \mathcal{V} \rangle = \tfrac{1}{3}\mathcal{C}\,\Pi_k\langle t^k \rangle = \frac{44}{27}\left\langle \int c_2 \wedge J \right\rangle \frac{\hat{\lambda}|W|^2}{\hat{\xi}} \;\sim\; \Pi_k\langle t_0^k \rangle \left(\frac{\hat{\lambda}|W|^2}{\hat{\xi}}\right)^{3/2} \;, \tag{4.9}$$

where $\langle t_0^i \rangle$ do not depend on $\mathcal{C}$, but are implicit functions of the Π_i. Moreover, positivity of the four-cycles requires that $\Pi_i > 0$ for all $i = 1, \ldots, h^{1,1}$. When we choose our variables in the Kähler cone, then all $t^i \geq 0$ independently from each other. Therefore, eq. (3.45) implies that $\Pi_i \geq 0$, so we only have to require that $\Pi_i \neq 0$.

In order to prove the existence of this minimum it is sufficient to show that the potential in eq. (4.7) is minimal as a function of the two-cycle volumes t^i as it is then also minimal in terms of the four-cycle volumes τ_i. The first derivatives of eq. (4.7) read

$$\frac{\partial V}{\partial t^i} = \frac{|W|^2}{\mathcal{V}^5}\left[-\frac{3}{4}\hat{\xi}\tau_i\left(t^i\tau_i\right) - \frac{1}{3}\hat{\lambda}|W|^2\Pi_i\left(t^j\tau_j\right) + 4\hat{\lambda}|W|^2\tau_i\left(\Pi_j t^j\right)\right] \;, \tag{4.10}$$

where we used eq. (3.22). Inserting the values of the four-cycle volumes given in eq. (4.8) one finds that indeed $\langle \partial V/\partial t^i \rangle = 0$. From eq. (3.22) we also obtain the first equality in eq. (4.9). To determine the overall dependence of $\langle \mathcal{V} \rangle$ on $\mathcal{C}$, note that the two-cycles are implicitly defined via eq. (3.19), which at the extremal point is given by

$$k_{ijk}\langle t^j \rangle \langle t^k \rangle = 2\mathcal{C}\Pi_i \;. \tag{4.11}$$

This implies $\langle t^i \rangle = \sqrt{\mathcal{C}}\langle t_0^i \rangle$, where t_0^i do not depend on $\mathcal{C}$. With this we obtain the scaling of the volume with respect to $|W|, \hat{\xi}$ and $\hat{\lambda}$ in eq. (4.9).

It remains to analyze the matrix of second derivatives. In general it reads

$$\begin{aligned} \frac{\partial^2 V}{\partial t^i \partial t^j} = \frac{|W|^2}{\mathcal{V}^6}\Big[& 9\hat{\xi}\mathcal{V}\tau_i\tau_j + 4\hat{\lambda}|W|^2\mathcal{V}\left(\tau_i\Pi_j + \Pi_i\tau_j\right) - 20\hat{\lambda}|W|^2(\Pi_k t^k)\tau_i\tau_j \\ & + \frac{\partial \tau_j}{\partial t^i}\left(4\hat{\lambda}|W|^2\mathcal{V}(\Pi_k t^k) - \frac{9}{4}\hat{\xi}\mathcal{V}^2\right)\Big] \;. \end{aligned} \tag{4.12}$$

Making use of eq. (3.22) we find that at the extremal point this simplifies to

$$\left\langle \frac{\partial^2 V}{\partial t^i \partial t^j} \right\rangle = a\Pi_i\Pi_j + b k_{ijk}\langle t^k \rangle \;, \qquad a = -\frac{8\hat{\lambda}|W|^4\mathcal{C}}{\langle \mathcal{V} \rangle^5} \;, \qquad b = \frac{9}{44}\frac{\hat{\xi}|W|^2}{\langle \mathcal{V} \rangle^4} \;. \tag{4.13}$$

For $\lambda < 0$ and $\chi(M_3) > 0$ we see that $a > 0$ and $b < 0$. For any vector with components x_i we have $(x_i\Pi_i)(x_j\Pi_j) \geq 0$ and so $a\Pi_i\Pi_j$ is a positive-semidefinite matrix. The matrix $k_{ijk}t^k$ was studied in [73] and shown to have signature $(1, h^{1,1} - 1)$. In other words there exists an orthogonal decomposition of the $h^{1,1}$-dimensional vector space into a one-dimensional subspace, on which $k_{ijk}t^k$ is positive definite and an $(h^{1,1} - 1)$-dimensional complement on which it is negative definite. Here orthogonality is defined with respect to the inner product determined by $k_{ijk}t^k$. The one-dimensional subspace is spanned by the vector with components t^i, as the volume has to be positive. Since we have $b < 0$ the signature of $bk_{ijk}\langle t^k\rangle$ reads $(h^{1,1} - 1, 1)$. On the $(h^{1,1} - 1)$-dimensional subspace the sum $a\Pi_i\Pi_j + bk_{ijk}\langle t^k\rangle$ must hence be positive-definite. On the one-dimensional subspace we find

$$\langle t^i\rangle \left\langle \frac{\partial^2 V}{\partial t^i \partial t^j} \right\rangle \langle t^j\rangle = -\frac{22\hat{\lambda}|W|^4\mathcal{C}}{3\langle\mathcal{V}\rangle^5} \left(\Pi_k\langle t^k\rangle\right)^2 > 0 \,, \tag{4.14}$$

which shows that the matrix of second derivatives is also positive definite there.

It remains to be shown, that the matrix (4.13) is positive definite on the whole space. A generic non-zero vector with components x^i can be decomposed as $x^i = \mu\langle t^i\rangle + x^i_\perp$, where $\mu \in \mathbb{R}$ and $x^i_\perp$ is the component of x^i in the subspace orthogonal to the one-dimensional space spanned by $\langle t^i\rangle$. Since

$$\Pi_i\Pi_j\langle t^j\rangle \sim \Pi_i \sim k_{ijk}\langle t^j\rangle\langle t^k\rangle \tag{4.15}$$

we have the following orthogonality relations

$$x^i_\perp k_{ijk}\langle t^j\rangle\langle t^k\rangle = x^i_\perp \Pi_i\Pi_j\langle t^j\rangle = 0 \,. \tag{4.16}$$

With this we find

$$\begin{aligned} &x^i \left\langle \frac{\partial^2 V}{\partial t^i \partial t^j} \right\rangle x^j \\ &= x^i_\perp \left(a\Pi_i\Pi_j + bk_{ijk}\langle t^k\rangle\right) x^j_\perp + \mu^2\langle t^i\rangle \left(a\Pi_i\Pi_j + bk_{ijk}\langle t^k\rangle\right) \langle t^j\rangle > 0 \,, \end{aligned} \tag{4.17}$$

since the matrix is positive on the respective subspaces. We conclude that the matrix in eq. (4.13) is positive definite.

In addition we have to establish that the locus specified in eq. (4.8) is a minimum of the potential, which includes also the dilaton as well as the complex structure moduli. The answer can be easily obtained in the spirit of [74]. Indeed the potential including the dilaton and complex-structure moduli reads [28]

$$\begin{aligned} V = {} & \mathrm{e}^K (G^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} + G^{S\bar{S}} D_S W D_{\bar{S}} \bar{W}) \\ & + \mathrm{e}^K \frac{\xi}{2\mathcal{V}} (W D_{\bar{S}} \bar{W} + \bar{W} D_S W) + V_{(\alpha')} + V_{(1)} \,, \end{aligned} \tag{4.18}$$

with W given in eq. (3.23). The first term in the above potential is positive definite and has a $\mathcal{V}^{-2}$ behavior at large volume. At the extremal condition $D_I W = D_S W = 0$, it vanishes identically and is positive around this value. Since it dominates over the subleading $\mathcal{O}(\mathcal{V}^{-3})$ and $\mathcal{O}(\mathcal{V}^{-11/3})$ terms coming from $V_{(\alpha')} + V_{(1)}$, eq. (4.8) represents a minimum of the full potential. Of course also the dilaton and complex structure moduli will receive higher-derivative corrections. However, these terms have a subleading volume-dependence compared to the first terms in eq. (4.18) and, thus, do not spoil the argument.

Furthermore, supersymmetry is broken in the minimum given in eq. (4.9) which can be seen as follows. Since $\mathcal{O}_{(4|2)}$ corrects the scalar potential only via F-term contributions, the conditions for supersymmetry breaking of the higher-derivative theory defined in eq. (2.119) are identical to the supersymmetry breaking conditions in the ordinary two-derivative theory. Suppose supersymmetry was unbroken, then one would be able to determine the position of the minimum from eq. (2.137). However, necessarily any solution to eq. (2.137) would be $\hat{\lambda}$-independent, which is not satisfied for our minimum. Thus, supersymmetry is indeed broken in the vacuum in eq. (4.9). Up to numerical factors the value of the potential in the minimum reads

$$\langle V \rangle \sim \frac{\hat{\xi}}{|W|^7} \left(\frac{\hat{\xi}}{\hat{\lambda}} \right)^{9/2} . \tag{4.19}$$

We can estimate the gravitino mass from the ordinary two-derivative theory. At leading order it reads

$$m_{3/2} \sim \mathrm{e}^{K/2} |W| \sim \frac{|W|}{\mathcal{V}} \sim \frac{\hat{\xi}^{3/2}}{\hat{\lambda}^{3/2} |W|^2 \Pi_i \langle t_0^i \rangle} . \tag{4.20}$$

Let us compare the gravitino mass with the string scale and Kaluza-Klein scale [99]

$$m_s \sim \frac{1}{\sqrt{\mathcal{V}}} , \qquad m_{KK} \sim \frac{1}{\mathcal{V}^{2/3}} . \tag{4.21}$$

Direct computation reveals that

$$\frac{m_{3/2}}{m_s} \sim \frac{\hat{\xi}^{3/4}}{\hat{\lambda}^{3/4} \sqrt{|W| \Pi_i \langle t_0^i \rangle}} . \tag{4.22}$$

Furthermore, from eq. (4.11) we find that roughly $\langle t_0^i \rangle \sim \sqrt{\Pi_i}$. Let $\tilde{\Pi}$ denote a typical value for the topological numbers Π_i, then we can estimate

$$\Pi_i \langle t_0^i \rangle \sim h^{1,1} \tilde{\Pi}^{3/2} . \tag{4.23}$$

The size of the topological numbers Π_i is roughly of the same order as $\chi(X_6)$. Toric examples from the Kreuzer-Skarke list [100] typically yield values for $\tilde{\Pi}$ ranging between $\mathcal{O}(1)$ numbers up to numbers of a few hundred [101]. Furthermore, we can estimate the size of $\hat{\lambda}$ by the combinatorial part of $\hat{\xi}$. In other words we roughly expect that $|\hat{\lambda}| \sim |\hat{\xi}/\chi(X_6)|$. Altogether, the scale-quotients read

$$\begin{aligned} \frac{m_{3/2}}{m_s} &\sim \mathrm{e}^{-\langle K_{cs} \rangle/4} g_s^{-1/4} \frac{\chi(X_6)^{3/4}}{\sqrt{|W_0| \, h^{1,1} \, \tilde{\Pi}}} \lesssim \mathcal{O}(10^{-1}) , \\ \frac{m_{3/2}}{m_{KK}} &\sim \frac{\chi(X_6)^{1/2}}{(h^{1,1})^{1/3} \sqrt{\tilde{\Pi}}} < 1 . \end{aligned} \tag{4.24}$$

To obtain more accurate expressions for $m_{3/2}/m_s$ and $m_{3/2}/m_{KK}$, it will be necessary to compute $\hat{\lambda}$ and study the minimum for explicit examples. Note furthermore, that we need $m_{3/2}/m_{KK} \ll 1$ in order to ensure that higher superspace-derivative corrections and, hence, higher-corrections to the scalar potential of the type $|F|^n$ with $n > 4$ are under control [102]. This can be achieved best by choosing a geometry with $\chi(X_6) \sim \mathcal{O}(1)$ and $h^{1,1} \gg 1$. The fact that a smallness of $m_{3/2}/m_{KK}$ is not universally satisfied for an arbitrary compactification geometry is induced by our stabilization mechanism. More precisely, we balance two terms at order $(\alpha')^3$ that roughly speaking are different terms in an expansion in $|F|^n$. However, we know from sec. 3.2.4 that there is only a single additional term in this expansion given by $(\alpha')^3|F|^8$ which is displayed in eq. (3.62). While this correction only depends on the overall volume and, therefore, looks rather harmless, its effect on the stabilization should be checked. Nevertheless, this is evidence that even if $m_{3/2}/m_{KK} \sim \mathcal{O}(1)$ the series $(\alpha')^3|F|^n$ may be under control.

Let us finish this section with some remarks. Firstly let us stress again that the stabilization of the four-cycle volumes proposed here does not require any non-perturbative effects, but occurs purely from considering the leading order $(\alpha')^3$-corrections in the potential. Note, furthermore, that for very special cases it might happen that some $\Pi_i = 0$ for particular values of i. In that case the overall volume is still stabilized at a positive value and string-loop or other α'-corrections may shift the minimum to a point at which all four-cycles are positive and the overall volume is roughly the same. Lastly, the requirement that $\xi < 0$ amounts, in the absence of $O7$-planes, to the condition $\chi(X_6) > 0$. In [79] it was proposed that when taking into account the $O7$-planes ξ is shifted via the following replacement

$$\chi(X_6) \longrightarrow \chi(X_6) + 2\int_{X_6} D_{O7}^3 \,, \tag{4.25}$$

where D_{O7} is the Poincare-dual to the divisor which the $O7$-plane wraps. The above shift affects our proposed stabilization scenario in a beneficial manner. If we demand that $\chi(X_6)$ is positive then, since the contribution from the $O7$-planes enters as a positive number, the sign of ξ remains negative. Note that this is opposite to the situation in LVS where the $O7$-plane contribution can, in principle, flip the sign of ξ and is, therefore, potentially harmful. Furthermore, if the $O7$-plane contribution is large enough, it might even be possible that $\xi < 0$ even though $\chi(X_6) < 0$. In turn, this may provide access to models with a small number of Kähler moduli but large number of complex-structure moduli with respect to our proposed stabilization scenario.

Moreover, from the analysis of [79] we expect that the $|F|^4$-term is also shifted in the presence of $O7$-planes.[44] We conjecture that from an F-theory derivation including the $O7$-planes the structure of the prefactor in eq. (3.59) will be shifted

[44] It might also be that additional $D7$-brane contributions exist. In particular, it is possible that an approach in the spirit of [79], where the α'-corrections are inferred by compactifying an auxiliary twelve-dimensional supergravity and not from the M-theory description of F-theory, is not capable of capturing such terms.

as follows

$$\int_{X_6} c_2 \wedge J \longrightarrow \int_{X_6} c_2 \wedge J + O7\text{-plane contributions} \ . \tag{4.26}$$

We leave the derivation of the precise form of these corrections to future research.

4.2 Inflation from $(\alpha')^3$-Corrections

While the perturbative stabilization scenario of sec. 4.1 is capable of lifting the flatness of all Kähler moduli model-independently, a necessary requirement of this setup is that $\chi(X_6) > 0$ (given that we ignore the additional contributions from the $O7$-planes of [79]). For the sake of simplicity, it would be desirable to discuss explicit examples with a small number of Kähler moduli. However, for $\chi(X_6) > 0$ we have $h^{1,1} > h^{2,1}$. Only a few explicit threefolds with small $h^{1,1}$ and $h^{2,1}$ are known [103–105] and these Calabi-Yau manifolds are not included in the list of [100] but instead arise after taking a quiotent with respect to a discrete automorphism group. In turn, these examples are computationally less tractable. Moreover, having a small number of complex-structure moduli restricts our freedom to choose the flux-superpotential and, hence, our ability to tune the value of the cosmological constant after uplifting. Thus, to study the potential relevance of the $|F|^4$-term for inflationary model building, we find it instructive to look at other stabilization scenarios, where the number of Kähler moduli can be small without either giving up the tools of toric geometry or our ability to tune the value of the cosmological constant. One option is certainly given by the Large Volume Scenario [39, 74, 75]. Here we follow and summarize the results of [37]. See also [106] for a similar application of the $|F|^4$-term to inflationary model building in the context of LVS. To make sure that we find a parametrically light field in our spectrum it is convenient to use the geometry proposed in [39] which corresponds to a threefold which is a $K3$-fibration together with a blow-up divisor. Explicit realizations for such a geometry with two blow-up divisors were constructed in [107]. Note that we do not revisit the basics of inflation here, see [108–110] for some complete reviews on the subject (with particular emphasis on inflation in string theory).

Large Volume Scenario for $K3$-fibered Threefold

We begin by reviewing the essentials of the stabilization in the spirit of [39, 74, 75] required for our discussion. In addition to the scalar potential in eq. (4.3) we assume that the superpotential receives non-perturbative corrections from gaugino condensation or Euclidean D-brane instantons, which read

$$W = W_0 + \sum_{i=1}^{h^{1,1}} A_i \mathrm{e}^{-a_i T_i} \ . \tag{4.27}$$

The scalar potential now reads

$$
\begin{aligned}
V &= V_{(\alpha')} + V_{(g_s)} + V_{(1)} + V_{np} \ , \\
V_{np} &= \mathrm{e}^K K_0^{i\bar{j}} \Big(a_i a_j A_i \bar{A}_{\bar{j}} \, \mathrm{e}^{-(a_i T_i + a_j \bar{T}_{\bar{j}})} - a_i A_i \, \mathrm{e}^{-a_i T_i} \bar{W}_0 K_{0,\bar{j}} \\
&\qquad - a_j \bar{A}_{\bar{j}} \, \mathrm{e}^{-a_j \bar{T}_{\bar{j}}} W_0 K_{0,i} \Big) \ .
\end{aligned}
\tag{4.28}
$$

Here we use $K_0 = -2 \ln \mathcal{V}$. As we have mentioned not every Kähler modulus may receive a non-perturbative correction of this type, but moduli corresponding to rigid divisors certainly can receive non-vanishing non-perturbative correction of the above type.

We now assume that the volume in terms of the four-cycles volumes is of the form [39, 75]

$$
\mathcal{V} = \alpha \left(\sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right) \ , \tag{4.29}
$$

where τ_1 is associated with the volume of the $K3$-fiber, τ_2 controls the overall volume and τ_3 denotes the blow-up and corresponds to a rigid divisor. We can write down undetermined intersection numbers, which formally yield a volume of the above type by choosing

$$
\mathcal{V} = \lambda_1 t^1 (t^2)^2 + \lambda_3 (t^3)^3 \ . \tag{4.30}
$$

The relations between four- and two-cycles, therefore, read

$$
\tau_1 = \lambda_1 (t^2)^2 \, , \quad \tau_2 = 2\lambda_1 t^1 t^2 \, , \quad \tau_3 = 3\lambda_3 (t^3)^2 \ . \tag{4.31}
$$

Let us assume that we have chosen a basis such that $t^3 \leq 0$ and, therefore, that we are not in the Kähler cone. Consequently, we must have that $t^1 \geq 0$ and $t^2 \geq 0$. Inverting eq. (4.31) yields

$$
t^1 = \frac{\tau_2}{2\sqrt{\lambda_1 \tau_1}} \, , \quad t^2 = \sqrt{\frac{\tau_1}{\lambda_1}} \, , \quad t^3 = -\sqrt{\frac{\tau_3}{3\lambda_3}} \ . \tag{4.32}
$$

From these identities we indeed obtain eq. (4.29) where $\alpha = \frac{1}{2} \lambda_1^{-1/2}$ and $\gamma = \sqrt{\frac{4}{27}} \sqrt{\lambda_1} \lambda_3^{-3/2}$. Next, let us briefly explain how the overall volume modulus $\mathcal{V}$ is stabilized now. Firstly, the limit of large volume requires that

$$
\tau_1, \tau_2 \gg \tau_3 \ , \tag{4.33}
$$

and, therefore, the relevant part of the superpotential reads

$$
W \simeq W_0 + A_3 \mathrm{e}^{-a_3 T_3} \ . \tag{4.34}
$$

Now, to obtain V_{np} it is necessary to compute the Kähler metric and minimize the axionic component of T_3. Afterwards, one is left with the following potential [39]

$$
V_{LVS}(\mathcal{V}, \tau_3) = g_s \left[\frac{8 a_3^2 A_3^2}{3\alpha\gamma} \frac{\sqrt{\tau_3}}{\mathcal{V}} \mathrm{e}^{-2a_3\tau_3} - 4 W_0 a_3 A_3 \frac{\tau_3}{\mathcal{V}^2} \mathrm{e}^{-a_3\tau_3} + \frac{3\hat{\xi} |W_0|^2}{4\mathcal{V}^3} \right] \ , \tag{4.35}
$$

where we did not include $V_{(g_s)}$ and $V_{(1)}$ just yet. It is evident that τ_1 is left as a flat direction in V_{LVS}. The LVS-minimum sits at exponentially large volume where

$$\langle\tau_3\rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma}\right)^{2/3} \quad , \quad \langle\mathcal{V}\rangle = \frac{3\alpha\gamma}{4a_3A_3} W_0\sqrt{\langle\tau_3\rangle}\mathrm{e}^{a_3\langle\tau_3\rangle} \; . \tag{4.36}$$

This minimum is expected to be stable under perturbative corrections as long as all higher-order corrections enter with at most $\mathcal{O}(1)$ coefficients [99]. From now on, we assume that τ_3 and $\mathcal{V}$ are sitting at their minimum in eq. (4.36) and focus on the flat direction τ_1, for which additional perturbative corrections, both $\delta V_{(g_s)}$ and $V_{(1)}$, are important. Moreover, as the size of these corrections is subleading compared to V_{LVS} we naively expect τ_1 to be the lightest modulus and, hence, τ_1 possibly constitutes a candidate to successfully drive inflation.

Analysis of F^4 and g_s-Corrections

Since we are interested in using τ_1 as an inflaton candidate, it is convenient to first perform the canonical normalization, which was already displayed in [39]. After replacing $\tau_2 = \tau_2(\tau_1, \mathcal{V}, \tau_3)$ via (4.29) in the Lagrangian, the relevant kinetic terms are given by

$$\mathcal{L} \supset -\frac{3}{8\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1 + \frac{1}{2\tau_1\mathcal{V}}\partial_\mu\tau_1\partial^\mu\mathcal{V} - \frac{1}{2\mathcal{V}^2}\partial_\mu\mathcal{V}\partial^\mu\mathcal{V} + \dots \; . \tag{4.37}$$

Therefore, the canonically normalized inflaton φ (at leading order in α' and g_s) is related to τ_1 as follows

$$\tau_1 = e^{2\varphi/\sqrt{3}} \; . \tag{4.38}$$

The scalar potential for the geometry defined in eq. (4.29) and eq. (4.32) then reads

$$\begin{aligned} V(\varphi) = V_{LVS} &+ \frac{g_s^3|W_0|^2}{\mathcal{V}^2}\left((C_1^{KK})^2 e^{-4\varphi/\sqrt{3}} + 2(\alpha\, C_2^{KK})^2\frac{e^{2\varphi/\sqrt{3}}}{\mathcal{V}^2}\right) \\ &- g_s^2\hat{\lambda}\frac{|W_0|^4}{\mathcal{V}^4}\left(\Pi_1 e^{-2\varphi/\sqrt{3}}\mathcal{V} + \Pi_2\lambda_1^{-1/2}e^{\varphi/\sqrt{3}}\right) \; , \end{aligned} \tag{4.39}$$

where the string-loop terms were already given in [39]. Here we assume configurations where the stacks of $D7$-branes only wrap those four-cycles associated with τ_2 or τ_3. In this case the winding-mode contributions to $V_{(g_s)}$ identically vanish. Recall that this contribution is generated via the exchange of strings winding one-cycles in the intersection of four-cycles which the stacks of $D7$-branes wrap. However, since the four-cycle associated with τ_3 only intersects with itself, we only need to take into account a possible intersection of the four-cycles associated to τ_1 and τ_2. Since we assumed that the $D7$-branes do not wrap τ_1 there are no such one-cycles present. Furthermore, we infer from the positivity of the two-cycle volumes that

$$\Pi_1 \geq 0 \; , \qquad \Pi_2 \geq 0 \; . \tag{4.40}$$

In the situation with $\hat{\lambda} < 0$ the F^4-term stabilizes τ_1. More generally, for $\hat{\lambda} < 0$ the F^4-term is capable of stabilizing all flat directions in LVS, since after minimizing with respect to the blow-up cycle the LVS scalar potential is given by an effective $1/\mathcal{V}^3$-potential with a negative prefactor and, hence, the stabilization mechanism from sec. 4.1 can be directly applied. However, while the $K3$-fiber volume modulus is stabilized, it does not constitute a viable candidate for the inflaton, since the F^4-potential is given by a growing exponential. A scalar potential of this type is too steep in order to fulfill the current observational bounds on inflation and, hence, this scenario is observationally ruled out [111, 112]. Therefore, in the following we assume that $\hat{\lambda} > 0$ in which case we can generate minima by means of an interplay between the F^4-term and g_s-corrections and the F^4-potential is given by a decaying exponential term.

Inflationary Dynamics

Depending on whether the fiber-volume is shrinking or growing, both terms in the F^4-piece give rise to viable plateau-like potentials. We can distinguish two different regimes

$$\text{(I)}: \quad \tau_1 \lesssim 1 \, , \qquad \text{(II)}: \quad \tau_1 \gg 1 \, . \tag{4.41}$$

In both cases either the decaying or growing exponentials in eq. (4.39) are negligible. After inclusion of a Minkowski-uplift and a rescaling of τ_1 which corresponds to shifting the minimum to $\varphi = 0$, the scalar potential is of the form

$$V_{\text{inf}} \simeq V_0 (1 - e^{\kappa\varphi})^2 \, , \tag{4.42}$$

and, therefore, formally of the plateau-type as in [44]. In tab. 4.1 we collect the values of κ as well as the approximate values for the spectral index n_s and the tensor-to-scalar ratio r for regime (I) and (II). In scenario (I) the plateau lies to the right of the

	Scenario (I)	Scenario (II)
Exponent	$\kappa = -\frac{2}{\sqrt{3}}$	$\kappa = \frac{1}{\sqrt{3}}$
n_s	~ 0.97	~ 0.97
r	$\mathcal{O}(10^{-3})$	$\mathcal{O}(10^{-3})$ to $\mathcal{O}(10^{-2})$

Table 4.1: *Exponential slope of scalar potential in eq. (4.42) for inflationary scenarios in eq. (4.41) and their predictions for $N = 50$ to $N = 60$ e-folds.*

minimum, while in scenario (II) it lies to the left. In [37] explicit numerical examples were computed which fulfill all necessary constraints. One finds that a moderate tuning of the string-loop coefficients and a small hierarchy between Π_1 and Π_2 of $\mathcal{O}(100)$ is necessary to satisfy all required conditions. With explicit realizations of the $K3$-fibered geometry as in [107] one may now check whether hierarchies in the values of Π_1, Π_2 can be achieved. We leave this to future investigations.

Chapter 5

Conclusions

Briefly summarized, this thesis consists of three parts. In the first part of this thesis we initiated a systematic study of supersymmetric theories in four-dimensional flat and curved $\mathcal{N}=1$-superspace including higher-derivative operators for chiral multiplets with particular emphasis on possible corrections to the scalar potential. In the second part we employed the results for the component form of the higher-derivative operator in eq. (2.120) to indirectly infer new purely $\mathcal{N}=1$ $(\alpha')^3$-corrections to the low-energy effective action of IIB Calabi-Yau orientifold compactifications with imaginary self-dual background fluxes. These corrections were determined by performing a Kaluza-Klein reduction of the leading order $(\alpha')^3\mathcal{R}^4$-corrections to ten-dimensional IIB supergravity descending from IIB string theory. Finally, in the third part we studied the implications of the new $(\alpha')^3$-corrections for Kähler moduli stabilization and inflation.

Following this short outline let us now describe the results of this thesis in detail. The first part of the thesis was dedicated to the systematic analysis of new terms in the scalar potential induced by higher-derivative operators. For the situation of rigid supersymmetry we derived a superspace action for the general scalar potential given in eq. (2.25). This action can be understood as an ordinary theory given by a $2n_c$-dimensional (pseudo-) Kähler potential and superpotential together with the supplementary constraints in eq. (2.24). The additional chiral multiplets are higher-derivative multiplets whose scalar components are given by the chiral auxiliary fields. However, note that the constraints in eq. (2.24) are incompatible with a $2n_c$-dimensional target space reparametrization-invariance. Instead, at least for the discussion of the scalar potential, the action is geometrically understood in the context of the cotangent-bundle over the complex manifold parametrized by the chiral scalars. In general the metric on this manifold is not even hermitian. In eq. (2.30) and eq. (2.33) we also displayed an alternative higher-derivative Lagrangian which, contrary to the aforementioned theory, does not induce kinetic terms for the auxiliary fields.

While the aforementioned off-shell theories can be written down fully explicitly, obtaining the on-shell theory is a daunting task, since the equations of motion for the auxiliary fields are now arbitrary algebraic equations. The algebraic nature of the auxiliary fields holds even if they obtain kinetic terms, since as we demonstrated in sec. 2.3.5 their masses sit at the cut-off scale of the EFT. For local theories the equations of motion for F^i are polynomial and, hence, induce a multiplet of on-shell theories. In the context of effective field theory we demonstrated that there exists only a single solution that yields a physically viable Lagrangian. We interpret the remaining solutions as artifacts of a truncation of an infinite sum of higher-derivative operators. This interpretation was also supported by the explicit example of the one-loop Wess-Zumino model. The fact that the discussion of the additional solutions is in analogy to the situation of ghostlike higher-derivative degrees of freedom in EFT can also be understood from supersymmetry, since the auxiliary fields sit in the same multiplet as the (ghost-like) higher-derivative degrees of freedom.

For $\mathcal{N} = 1$ old-minimal supergravity we conjectured a possible extension of eq. (2.25) given in eq. (2.83). However, due to the complicated form of the algebra of super-covariant derivatives we did not attempt to prove this conjecture. Instead, we classified the leading ($N = 2$) and next-to-leading order ($N = 4$) higher-derivative operators for the chiral multiplets including a short survey on higher-curvature operators as well. The classification of the $N = 4$ operators was substantially more involved, the results being displayed in tab. 2.3. To compute the component actions of higher-derivative operators we developed several tools. Firstly, we provided a catalog of component identities for higher-derivative superfield in appendix A.3 extending the results of [20]. Secondly, we developed an algorithm to compute the on-shell component action. In particular, we showed that for the computation of the linearized (in the coupling of the higher-derivative operator) on-shell action it suffices to simply insert the leading order solutions for the auxiliary fields in the Lagrangian and, therefore, it is not necessary to solve the corrected equations of motion for the auxiliary fields. Thus, from there on we simply displayed the component results off-shell, as the respective linearized on-shell theories are obtained readily, the only exception being an illustrative example in sec. 2.5.5. The component forms of the $N = 2$ operators and a subclass of $N = 4$ are displayed in sec. 2.5.5 and sec. 2.5.6. These results are model-independent and universally applicable for computing leading-order higher-derivative corrections to a generic supergravity with chiral multiplets.

Furthermore, we discussed two particular applications of the aforementioned results. Firstly, we investigated the vacuum structure of the (general) higher-derivative theories. On the one hand, we found that for the supersymmetric Minkowski vacua nothing changes compared to the two-derivative case. For the supersymmetric AdS_4-vacua, on the other hand, the presence of higher-derivative operators has an effect. While we found that the Killing spinor equation is still automatically satisfied, we generically expect that the conditions $\langle F^i \rangle = 0$ do not ensure anymore that the scalar potential is extremal. Therefore, generically the supersymmetric AdS_4-vacua should not admit any moduli space. This is also in agreement with corresponding observations for the dual three-dimensional SCFTs [61]. For non-supersymmetric minima there are two consequences worth mentioning. On the one hand, the presence of the higher-derivative operators may lift flat directions and, on the other hand, supersymmetry-breaking can be induced by higher-curvature operators and no longer requires the non-vanishing of the F-terms. This was demonstrated explicitly for an $\mathcal{R} + \mathcal{R}^2$-gravity in appendix A.5. More precisely, here supersymmetry was broken since the scalar curvature was no longer compatible with the Killing spinors. Secondly, we investigated the form of the higher-derivative corrections for shift-symmetric no-scale models. We found that the no-scale condition leads to the vanishing of several contributions and, in particular, the scalar potential is only corrected by monomials in the chiral auxiliary fields.

In the second part of this thesis we discussed the low-energy effective supergravity obtained from compactifications of IIB string theory on Calabi-Yau threefolds with $O3$-planes and background fluxes. String-theory induces higher-derivative α'-corrections to ten-dimensional IIB supergravity. The leading-order terms coming from four-graviton scattering arise at the eight-derivative level, i.e. $(\alpha')^3$ order. These terms modify the 4D supergravity after compactification in several ways, in particular, via corrections to the Kähler potential of the Kähler moduli [28]. Here we demonstrated the existence of a new class of corrections descending from the $(\alpha')^3$-corrections given by 4D higher-derivative operators for the Kähler moduli. In particular, by performing the KK-reduction of the explicitly known 10D pure $\mathcal{R}^4$-subsector of the total $(\alpha')^3$-corrections we computed four-derivative terms for the Kähler moduli. Via matching to $\mathcal{O}_{(4|2)}$ we inferred new corrections to the scalar potential, which partially descend from so far unknown quartic terms in the field strength H_3 of the NS-NS two-form. While the KK-reduction was performed for a single volume modulus, we argued that the form of the scalar potential can be inferred in general, making use of the no-scale structure present at leading order. The result is displayed in eq. (3.59) and depends on the second Chern class of the threefold. The topological information of the second Chern class did so far not enter in the $\mathcal{N} = 1$ action and contains information which is independent on the intersection numbers, the Hodge numbers or the Euler number.[45] Since the new correction exhibits an explicit dependence on all Kähler moduli, it is of interest for Kähler moduli stabilization and inflation. In this regard it is similar to the string-loop corrections of [42]. However, for phenomenological purposes it has several advantages. Firstly, contrary to the g_s-corrections, which have been conjectured in [42] but so far were not computed for any explicit threefold, the F^4-term is fully calculable and the topological numbers Π_i can be readily determined via the tools of toric geometry. Moreover, eq. (3.59) has a much simpler structure and dependence on the Kähler moduli compared to the string-loop corrections. Here we have already made use of these benefits. In particular, we proved that the F^4-term taken together with the known $(\alpha')^3$-corrections leading corrections to the scalar potential of [28] lead to a model-independent non-supersymmetric AdS_4 minimum, where all Kähler moduli are fixed and proportional to the topological numbers Π_i. The existence of this minimum requires that $\chi(X_6) > 0$ and $\hat{\lambda} < 0$, where $\hat{\lambda}$ is proportional to an undetermined numerical constant multiplying the F^4-term.

To assess the importance for building models of Kähler-modulus inflation we investigated a particular example in sec. 4.2 corresponding to a special $K3$-fibered

[45] However, note that the second Chern class and the topological numbers Π_i have appeared in the context of mirror symmetry [113–115].

geometry proposed in [39, 75] where the volume stabilization is of the LVS-type [74, 75]. While the F^4-term is capable of stabilizing the inflaton-candidate, it does not lead to the correct pattern of inflationary observables. Therefore, we focused on the situation of $\hat{\lambda} > 0$, in which case the F^4-term can give rise to viable plateau-type potentials and minima can be generated by an interplay between the F^4-term and KK-mode contributions of the g_s-corrections conjectured in [42]. Explicit numerical examples require a hierarchy between the topological numbers Π_i. In future studies one may look for explicit $K3$-fibered threefolds which realize such a hierarchy.

Let us outline possible future directions and open questions which result from this thesis. In particular, the results in chapter 2 may be generalized to gauged theories with vector multiplets. Furthermore, it would be interesting to understand how the target space invariance, in particular for the theory in eq. (2.25), is restored at the level of the full Lagrangian. Regarding higher-derivative theories with $\mathcal{N} = 1$ supergravity one may investigate the conjecture that the Lagrangian in eq. (2.83) already captures the general scalar potential. Furthermore, one may test the expectations of the effect of higher-derivatives on supersymmetric AdS_4-vacua and their moduli spaces for explicit examples.

Regarding chapter 3 in the future one may perform a full KK-reduction of the 10D $\mathcal{R}^4$-terms which would involve determining the precise numerical coefficients in eq. (3.39) and the (kinetic) couplings between the 4D curvature tensor and the volume deformation. Afterwards, one must match the result to the general linear combination of the operators displayed in tab. 2.3 using the component forms computed in sec. 2.5.6 to make the result manifestly supersymmetric. Thereby, one would be able to explicitly determine the numerical value of $\hat{\lambda}$ which, in turn, would allow to test the validity of the model-independent $(\alpha')^3$-induced Kähler moduli stabilization. Moreover, this way one would compute additional two-derivative terms for the Kähler moduli as well as couplings between the Kähler moduli and the 4D curvature tensor. The latter corrections are also interesting in the context of Kähler moduli inflation and explicit inflationary realizations could be tested against these terms. In addition, being equipped with the result of an explicit reduction one may conduct a comparison with the respective results for pure Calabi-Yau compactifications and the respective $\mathcal{N} = 2$ higher-derivative couplings [94]. Finally, recall that we neglected the warping of the background in our KK-reduction. Warping-induced contributions to the low-energy effective action have recently received attention for instance in [116]. Thus, it would be interesting to test our approximation in the future. Lastly, it might also be of interest to compute higher-derivative operators for other multiplets or in other corners of string theory.

Appendix A

Superspace Identities and Classification of Operators

The following appendices are taken from or review content of the reference [32]. In particular, they include many of the computational tools but also supplementary proofs and demonstrations for chapter 2.

A.1 Spinor Conventions and Identities

In this thesis we use the spinor notations and conventions of [34], which we briefly review in this appendix. In addition we provide some important formulas which prove useful for several computations. We use the following convention for raising and lowering the Weyl-spinor indices

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta \ , \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta \ , \qquad \epsilon^{\alpha\beta}\epsilon_{\beta\alpha} = 2 \ . \tag{A.1}$$

Furthermore, as in [34] we use a different symbol for Pauli-matrices with raised indices

$$\bar{\sigma}_a^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma_{a\beta\dot{\beta}} \ . \tag{A.2}$$

The Pauli-matrices convert tensor superfields $V_{\alpha\dot{\alpha}}$ and vector superfields V_a into each other as follows

$$V_a = -\tfrac{1}{2}\bar{\sigma}_a^{\dot{\alpha}\alpha}V_{\alpha\dot{\alpha}} \ , \qquad V_{\alpha\dot{\alpha}} = \sigma^a_{\alpha\dot{\alpha}}V_a \ . \tag{A.3}$$

The following identities are necessary to perform some of our computations

$$Tr(\sigma_a\bar{\sigma}_b) = -2\eta_{ab} \tag{A.4}$$

$$\sigma^a_{\alpha\dot{\alpha}}\bar{\sigma}_a^{\dot{\beta}\beta} = -2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}} \tag{A.5}$$

$$\sigma_a\bar{\sigma}_b\sigma_c - \sigma_c\bar{\sigma}_b\sigma_a = 2i\epsilon_{abcd}\sigma^d \tag{A.6}$$

$$Tr(\sigma_a\bar{\sigma}_b\sigma_c\bar{\sigma}_d) = 2(\eta_{ab}\eta_{cd} - \eta_{ac}\eta_{bd} + \eta_{ad}\eta_{bc} - i\epsilon_{abcd}) \ . \tag{A.7}$$

A.2 Derivation of Superspace Action for General Scalar Potential

We begin in this appendix by deriving the superspace effective scalar potential which we presented in sec. 2.3. More precisely our goal is to simplify eq. (2.21) assuming that we only consider operators in the action that manifestly contribute to the scalar potential. To this end we evaluate the general action at the supersymmetric condition in eq. (2.22). Therefore, we can ignore all operators in S_{gen} which involve the spacetime-derivatives ∂_a. Moreover, the condition in eq. (2.22) also restricts the dependence on the spinorial component of the superspace-derivatives. To begin with, all mixed-type combinations of spinorial superspace-derivatives acting on Φ or $\bar{\Phi}$ vanish when evaluated at eq. (2.22). With mixed-type we mean that the combination involves at least one power of D_α as well as $\bar{D}_{\dot{\beta}}$. This can be seen iteratively. For two superspace-derivatives the possible combinations are given by $D_\alpha \bar{D}_{\dot{\beta}} \bar{\Phi}$ and $\bar{D}_{\dot{\beta}} D_\alpha \Phi$. Making use of eq. (2.11) one indeed finds that these terms vanish at the condition in eq. (2.22). The possible terms with three superspace-derivatives are given by

$$\begin{gathered} D_\alpha \bar{D}_{\dot{\beta}} \bar{D}_{\dot{\gamma}} \bar{\Phi} \ , \quad D_\alpha D_\beta \bar{D}_{\dot{\gamma}} \bar{\Phi} \ , \quad \bar{D}_{\dot{\alpha}} D_\beta \bar{D}_{\dot{\gamma}} \bar{\Phi} \ , \quad \bar{D}_{\dot{\alpha}} \bar{D}_{\dot{\beta}} D_\gamma \Phi \ , \\ D_\alpha \bar{D}_{\dot{\beta}} D_\gamma \Phi \ , \quad \bar{D}_{\dot{\alpha}} D_\beta D_\gamma \Phi \ . \end{gathered} \tag{A.8}$$

Again via eq. (2.11) one finds that these terms vanish after inserting eq. (2.22). From eq. (2.17) we learn that terms with more than three superspace-derivatives are simply further superspace derivatives acting on the terms in eq. (A.8) and, thus, the claim holds in general. Furthermore, eq. (2.17) implies that we have to consider only terms with at most two superspace-derivatives acting on Φ or $\bar{\Phi}$. Finally, eq. (2.18) shows that it suffices to consider a dependence of S_{eff} on $D_\alpha \Phi$, $\bar{D}_{\dot{\alpha}} \bar{\Phi}$ as well as $D^2 \Phi$ and $\bar{D}^2 \bar{\Phi}$.

After the above considerations we can express the effective superspace action evaluated at eq. (2.22) as follows

$$\begin{aligned} S_{\text{eff}} = & \int \mathrm{d}^8 z \, \mathcal{K}(\Phi, \bar{\Phi}, D^2\Phi, D_\alpha \Phi D^\alpha \Phi, \bar{D}^2 \bar{\Phi}, \bar{D}_{\dot{\alpha}} \bar{\Phi} \bar{D}^{\dot{\alpha}} \bar{\Phi}) \\ & + \int \mathrm{d}^6 z \, W(\Phi, \bar{D}^2 \bar{\Phi}) + \int \mathrm{d}^6 \bar{z} \, \bar{W}(\bar{\Phi}, D^2 \Phi) \ . \end{aligned} \tag{A.9}$$

Let us turn more closely to W. We consider the superpotential to be a power series in Φ and $\bar{D}^2 \bar{\Phi}$. An arbitrary term in this series is of the form

$$\Phi^k (\bar{D}^2 \bar{\Phi})^l = \bar{D}^2 (\Phi^k (\bar{D}^2 \bar{\Phi})^{l-1} \bar{\Phi}) \ , \qquad k, l \in \mathbb{N}_0 \ , \tag{A.10}$$

where equality holds due to eq. (2.12) and eq. (2.17). Using the following identity

$$\int \mathrm{d}^6 z \bar{D}^2 f(x, \theta, \bar{\theta}) = -4 \int \mathrm{d}^8 z f(x, \theta, \bar{\theta}) \ , \tag{A.11}$$

where f is an arbitrary superfield, we observe that the dependence of W on $\bar{D}^2\bar{\Phi}$ can be entirely absorbed into $\mathcal{K}$. Therefore, we confine the discussion of S_{eff} to $\mathcal{K}$. Moreover, we focus on the bosonic part of S_{eff} from now on. In this case we can simplify $\mathcal{K}$ even further. To see this, as a first step we compute

$$(D_\alpha \Phi D^\alpha \Phi)^2\Big|_{\text{bos}} = 0 \ . \tag{A.12}$$

The above identity can either be computed directly or alternatively be derived from integration by parts identities in superspace such as eq. (A.13). Consequently in $\mathcal{K}$ we only have to consider terms with up to a single factor of $D_\alpha\Phi D^\alpha\Phi$ and $\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi}$. Up to boundary terms as well as mixed type superspace-derivative terms, which yield purely kinetic contributions, the following integration by parts identities hold for arbitrary functions T [46]

$$\begin{aligned}
&\int \mathrm{d}^8 z D^2\Phi\, T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \\
&= \int \mathrm{d}^8 z\, (D_\alpha\Phi D^\alpha\Phi)\, \partial_\Phi T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \ , \\
&\int \mathrm{d}^8 z\, \bar{D}^2\bar{\Phi}\, T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \\
&= -\int \mathrm{d}^8 z\, (\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi})\, \partial_{\bar{\Phi}} T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \ , \\
&\int \mathrm{d}^8 z\, D^2\Phi \bar{D}^2\bar{\Phi}\, T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \\
&= -\int \mathrm{d}^8 z\, (D_\alpha\Phi D^\alpha\Phi)(\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi})\, \partial_\Phi \partial_{\bar{\Phi}} T(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) \ .
\end{aligned} \tag{A.13}$$

Thus, we infer that the factors $D_\alpha\Phi D^\alpha\Phi$ and $\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi}$ in $\mathcal{K}$ can always be recast into additional factors of $D^2\Phi$ and $\bar{D}^2\bar{\Phi}$ respectively. The equivalence between the superfields $D_\alpha\Phi D^\alpha\Phi$ and $\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi}$ as well as $D^2\Phi$ and $\bar{D}^2\bar{\Phi}$ can also be understood from the fact that while $D^2\Phi$ ($\bar{D}^2\bar{\Phi}$) are anti-chiral (chiral) superfields, $D_\alpha\Phi D^\alpha\Phi$ and $\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi}$ are complex linear superfields, that is they satisfy

$$D^2(D_\alpha\Phi D^\alpha\Phi) = \bar{D}^2(\bar{D}_{\dot\alpha}\bar{\Phi}\bar{D}^{\dot\alpha}\bar{\Phi}) = 0 \ . \tag{A.14}$$

In total we find that, without loss of generality, the general superspace action for the effective scalar potential is of the form

$$S_{\text{eff}} = \int \mathrm{d}^8 z\, \mathcal{K}(\Phi, \bar{\Phi}, D^2\Phi, \bar{D}^2\bar{\Phi}) + \int \mathrm{d}^6 z\, W(\Phi) + \int \mathrm{d}^6 \bar{z}\, \bar{W}(\bar{\Phi}) \ . \tag{A.15}$$

In sec. 2.3.1 we proceed to generalize this action to the multi-field case and to discuss its respective component version.

[46] These integration by parts identities can also be understood as arising in the rigid limit of the curved superspace identities in appendix A.4.

A.3 Component Identities for Superfields in Curved Superspace

In this appendix we provide the necessary formulas to compute the component actions for matter coupled supergravity in sec. 2.4.1, in particular for the four covariant derivative operators in sec. 2.5.6. We start by giving a catalog of component identities for higher super-covariant derivatives acting on the superfields $(\Phi, \bar{\Phi}, R, \bar{R}, G_{\alpha\dot{\alpha}})$. A useful list of component identities that goes beyond the formulas presented in [34] was already given in [20]. Our results below partially overlap with that reference, but we also compute new identities which are required for the discussion in sec. (2.5.6). Here we derive all components starting from the solution to the Bianchi identities and the algebra of super-covariant derivatives. As a cross-check we also rederive component identities which appeared in [20]. We find some minor disagreements with the results in that reference on some component identities which we indicate explicitly later on.

The tool for the computation of component identities are the (anti-) commutation relations in eq. (2.59), which relate the covariant derivatives to the torsion defined in eq. (2.57) and the super-curvature tensor in eq. (2.58). After imposing the constraints of old minimal supergravity on the torsion and solving the Bianchi identities, the only non-zero components of the torsion are given by [34]

$$\begin{aligned}
&T_{\alpha\dot{\alpha}}{}^{a} = T_{\dot{\alpha}\alpha}{}^{a} = 2i\sigma^{a}_{\alpha\dot{\alpha}} \\
&T_{\dot{\alpha}a}{}^{\alpha} = -T_{a\dot{\alpha}}{}^{\alpha} = -iR\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_{a}^{\dot{\beta}\alpha} \\
&T_{\alpha a}{}^{\dot{\alpha}} = -T_{a\alpha}{}^{\dot{\alpha}} = -i\bar{R}\epsilon_{\alpha\beta}\bar{\sigma}_{a}^{\dot{\alpha}\beta} \\
&T_{\beta a}{}^{\alpha} = -T_{a\beta}{}^{\alpha} = \tfrac{i}{8}\bar{\sigma}_{a}^{\dot{\gamma}\gamma}(\delta^{\alpha}_{\gamma}G_{\beta\dot{\gamma}} - 3\delta^{\alpha}_{\beta}G_{\gamma\dot{\gamma}} + 3\epsilon_{\beta\gamma}G^{\alpha}_{\dot{\gamma}}) \\
&T_{\dot{\beta}a}{}^{\dot{\alpha}} = -T_{a\dot{\beta}}{}^{\dot{\alpha}} = \tfrac{i}{8}\bar{\sigma}_{a}^{\dot{\gamma}\gamma}(\delta^{\dot{\alpha}}_{\dot{\gamma}}G_{\gamma\dot{\beta}} - 3\delta^{\dot{\alpha}}_{\dot{\beta}}G_{\gamma\dot{\gamma}} + 3\epsilon_{\dot{\beta}\dot{\gamma}}G^{\dot{\alpha}}_{\gamma})\ ,
\end{aligned} \tag{A.16}$$

as well as the components T^{α}_{ab} and $T^{\dot{\alpha}}_{ab}$ which we do not display here as they are not required. Additionally, we need certain components of the curvature tensor. The following list contains some of the frequently used components

$$\begin{aligned}
&R_{\delta\gamma\beta\alpha} = 4(\epsilon_{\delta\beta}\epsilon_{\gamma\alpha} + \epsilon_{\gamma\beta}\epsilon_{\delta\alpha})\bar{R} \\
&R_{\dot{\delta}\dot{\gamma}\dot{\beta}\dot{\alpha}} = 4(\epsilon_{\dot{\delta}\dot{\beta}}\epsilon_{\dot{\gamma}\dot{\alpha}} + \epsilon_{\dot{\gamma}\dot{\beta}}\epsilon_{\dot{\delta}\dot{\alpha}})R \\
&R_{\delta\dot{\gamma}\beta\alpha} = R_{\dot{\gamma}\delta\beta\alpha} = -\epsilon_{\delta\beta}G_{\alpha\dot{\gamma}} - \epsilon_{\delta\alpha}G_{\beta\dot{\gamma}} \\
&R_{\delta\dot{\gamma}\dot{\beta}\dot{\alpha}} = R_{\dot{\gamma}\delta\dot{\beta}\dot{\alpha}} = -\epsilon_{\dot{\gamma}\dot{\beta}}G_{\delta\dot{\alpha}} - \epsilon_{\dot{\gamma}\dot{\alpha}}G_{\delta\dot{\beta}} \\
&R_{\dot{\delta}c\dot{\beta}\dot{\alpha}} = -R_{c\dot{\delta}\dot{\beta}\dot{\alpha}} = -\tfrac{1}{2}\bar{\sigma}_{c}^{\dot{\gamma}\gamma}\Big[i(\epsilon_{\dot{\delta}\dot{\beta}}\epsilon_{\dot{\gamma}\dot{\alpha}} + \epsilon_{\dot{\delta}\dot{\alpha}}\epsilon_{\dot{\gamma}\dot{\beta}})\bar{\mathcal{D}}_{\dot{\epsilon}}G^{\dot{\epsilon}}_{\gamma} + \tfrac{i}{2}(\epsilon_{\dot{\delta}\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\beta}} + \epsilon_{\dot{\delta}\dot{\beta}}\bar{\mathcal{D}}_{\dot{\gamma}})G_{\gamma\dot{\alpha}} \\
&\qquad\qquad + \tfrac{i}{2}(\epsilon_{\dot{\delta}\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\alpha}} + \epsilon_{\dot{\delta}\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\gamma}})G_{\gamma\dot{\beta}}\Big]\ .
\end{aligned} \tag{A.17}$$

Additionally, the following component will be required

$$R_{\alpha\dot{\alpha}ab} = 2i\epsilon_{abcd}\,\sigma^{d}_{\alpha\dot{\alpha}}G^{c}\ , \tag{A.18}$$

which is not it displayed explicitly in [34] but can be computed directly from the Bianchi identities given in that reference. It is also useful to note the following relations

$$\mathcal{D}^{\alpha}G_{\alpha\dot{\alpha}} = \bar{\mathcal{D}}_{\dot{\alpha}}\bar{R} \ , \qquad \bar{\mathcal{D}}^{\dot{\alpha}}G_{\alpha\dot{\alpha}} = \mathcal{D}_{\alpha}R \ . \tag{A.19}$$

From the components of the torsion and curvature we also derive the useful identities

$$T_{\dot{\alpha}a}{}^{\dot{\alpha}} = T_{\alpha a}{}^{\alpha} = 2iG_a \ , \qquad R_{\dot{\alpha}\beta\alpha}{}^{\beta} = 3G_{\alpha\dot{\alpha}} \ . \tag{A.20}$$

Furthermore, by using the Bianchi identities in [34] we deduce the following equations

$$\mathcal{D}_{\alpha}G_{\beta\dot{\beta}} - \mathcal{D}_{\beta}G_{\alpha\dot{\beta}} = \epsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot{\beta}}\bar{R} \ , \tag{A.21}$$

$$\bar{\mathcal{D}}_{\dot{\alpha}}G_{\beta\dot{\beta}} - \bar{\mathcal{D}}_{\dot{\beta}}G_{\beta\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\mathcal{D}_{\beta}R \ . \tag{A.22}$$

We now proceed to present a catalog of those component identities for the chiral superfields which are required for the computation of the component forms of the higher-derivative operators in eq. (2.101) and tab. 2.3. The lowest-order terms $\mathcal{D}_{\alpha}\Phi|$ and $\mathcal{D}^2\Phi|$ were already given in eq. (2.64). Since we are interested only in the bosonic components, let us identify the fermionic terms. In general, components with an odd number of spinorial covariant derivatives acting on Φ or $\bar{\Phi}$ are fermionic. A simple example is given by

$$\tfrac{1}{\sqrt{2}}\mathcal{D}_{\alpha}\mathcal{D}^2\Phi| = -\tfrac{4}{3}\chi_{\alpha}\bar{M} \ , \tag{A.23}$$

which can be checked by means of eq. (2.70).

The bosonic terms are those with an even number of spinorial covariant derivatives. In the remainder of this appendix we display only the bosonic terms of the components. In particular, at the level of two-superspace derivatives we find

$$\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}_{\alpha}\Phi| = \{\bar{\mathcal{D}}_{\dot{\alpha}},\mathcal{D}_{\alpha}\}\Phi| = -T_{\alpha\dot{\alpha}}{}^{A}\mathcal{D}_{A}\Phi| = -2i\sigma^a_{\alpha\dot{\alpha}}\mathcal{D}_a\Phi| = -2i\sigma^a_{\alpha\dot{\alpha}}e_a^m\partial_m A \ , \tag{A.24}$$

where we used eq. (A.16) and we displayed only the bosonic terms. It is also useful to note the following identities

$$\mathcal{D}_{\alpha}\mathcal{D}_{\beta}\Phi = \tfrac{1}{2}\epsilon_{\alpha\beta}\mathcal{D}^2\Phi \ , \qquad \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\beta}}\bar{\Phi} = -\tfrac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\mathcal{D}}^2\bar{\Phi} \ . \tag{A.25}$$

Components of $\mathcal{O}(\mathcal{D}^4)$ Acting on $\Phi, \bar{\Phi}$

Furthermore, there are several components involving four spinorial super-covariant derivatives which are of relevance for the computation of the component action of the operators in sec. 2.5.6. Using eq. (2.70) and the components of torsion and curvature in eq. (A.16) we find the following identities

$$
\begin{aligned}
\mathcal{D}^2\mathcal{D}^2\Phi| &= \tfrac{16}{3}F\bar{M} \\
\bar{\mathcal{D}}^2\bar{\mathcal{D}}^2\bar{\Phi}| &= \tfrac{16}{3}\bar{F}M \\
\mathcal{D}_\alpha\bar{\mathcal{D}}^2\mathcal{D}^\alpha\Phi| &= -\tfrac{16}{3}MF \\
\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}| &= 16\Box\bar{A} + \tfrac{32}{3}ib^m\partial_m\bar{A} + \tfrac{32}{3}\bar{M}\bar{F} \\
\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^2\bar{\Phi}| &= \tfrac{8}{3}iM\sigma^a_{\alpha\dot\alpha}e_a^m\partial_m\bar{A} \\
\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2\Phi| &= 8i\sigma^a_{\alpha\dot\alpha}e_a^m(\partial_mF - \tfrac{1}{3}\bar{M}\partial_mA) \ .
\end{aligned}
\tag{A.26}
$$

Components of $\mathcal{O}(\mathcal{D}^6)$ Acting on $\Phi, \bar{\Phi}$

We also need certain components with six spinorial super-covariant derivatives, which read

$$
\begin{aligned}
\tfrac{1}{64}\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\mathcal{D}}^2\bar{\Phi}| &= \tfrac{1}{3}\bar{F}\left(\tfrac{1}{2}\mathcal{R} - \tfrac{4}{3}|M|^2 - \tfrac{1}{3}b_ab^a + i\mathcal{D}_mb^m\right) - \tfrac{1}{3}M\Box\bar{A} \\
&\quad - \tfrac{2i}{9}Mb^m\partial_m\bar{A} \\
\tfrac{1}{64}\bar{\mathcal{D}}^2\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}| &= \tfrac{1}{3}\bar{F}\left(\tfrac{1}{2}\mathcal{R} - 2|M|^2 - \tfrac{1}{3}b_ab^a + i\mathcal{D}_mb^m\right) - \Box\bar{F} - \tfrac{1}{3}M\Box\bar{A} \\
&\quad + \tfrac{2}{3}\partial_mM\partial^m\bar{A} + \tfrac{2i}{3}b^m(\partial_m\bar{F} - M\partial_m\bar{A}) \ .
\end{aligned}
\tag{A.27}
$$

Components of R

For the convenience of the reader we display some of the relevant components of the superfield R here.

$$
\begin{aligned}
R| &= -\tfrac{1}{6}M \\
\mathcal{D}_aR| &= -\tfrac{1}{6}e_a^m\partial_mM \\
\mathcal{D}^2R| &= \tfrac{2}{3}\left(-\tfrac{1}{2}\mathcal{R} + \tfrac{2}{3}|M|^2 + \tfrac{1}{3}b_ab^a - i\mathcal{D}_mb^m\right) \\
\bar{\mathcal{D}}^2\mathcal{D}^2R| &= -\tfrac{8}{3}\Box M + \tfrac{16}{9}ib^m\partial_mM + \tfrac{16}{9}M\left(\tfrac{1}{2}\mathcal{R} - \tfrac{2}{3}|M|^2 - \tfrac{1}{3}b_ab^a + i\mathcal{D}_mb^m\right) \ .
\end{aligned}
\tag{A.28}
$$

Components with $\mathcal{D}_a$ Acting on $\Phi, \bar{\Phi}$

In addition, several components involving super-covariant derivatives of $\mathcal{D}_a\bar{\Phi}$ and $\mathcal{D}_a\Phi$ are relevant, in particular for the computation of $\mathcal{O}_{(2|3)}$ in tab. 2.3. The important identities involving two spinorial covariant derivatives are given by

$$
\begin{aligned}
\mathcal{D}^2\mathcal{D}_a\bar{\Phi}| &= \tfrac{2}{3}\bar{M}e_a^m\partial_m\bar{A} \\
\bar{\mathcal{D}}^2\mathcal{D}_a\Phi| &= \tfrac{2}{3}Me_a^m\partial_m A \\
\mathcal{D}^\alpha\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^a\Phi| &= \tfrac{i}{3}MF\bar{\sigma}^{a\dot\alpha\alpha} \\
\mathcal{D}^2\mathcal{D}_a\Phi| &= -\tfrac{8i}{3}Fb_a - \tfrac{2}{3}\bar{M}e_a^m\partial_m A - 4e_a^m\partial_m F \\
\mathcal{D}^\alpha\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^a\bar{\Phi}| &= -2i\bar{\sigma}_b^{\dot\alpha\alpha}(\tfrac{1}{3}\epsilon_{cd}{}^{ab}b^c e_m^d\partial^m\bar{A} + e_m^b e_n^a\mathcal{D}^m\partial^n\bar{A}) - \tfrac{i}{3}\bar{\sigma}^{a\dot\alpha\alpha}\bar{M}\bar{F}\ .
\end{aligned}
\tag{A.29}
$$

The above components were also determined in [20]. Note that we find agreement with the results in that reference, apart from the last component identity which differs by a minus sign in the first and last term.

Finally, there are also components with four spinorial covariant derivatives acting on $\mathcal{D}_a\bar{\Phi}$ and $\mathcal{D}_a\Phi$ and they read

$$
\begin{aligned}
\tfrac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{D}_a\Phi| &= -\tfrac{1}{6}(-\tfrac{1}{2}\mathcal{R} + \tfrac{5}{6}|M|^2 + \tfrac{1}{3}b_c b^c - i\mathcal{D}_m b^m)\partial_a A \\
&\quad - \tfrac{i}{9}MFb_a - \tfrac{1}{6}M\partial_a F \\
\tfrac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{D}_a\bar{\Phi}| &= e_a^m\Big[\mathcal{D}_m(\Box\bar{A} + \tfrac{2}{3}ib^n\partial_n\bar{A}) - \tfrac{i}{3}\bar{F}\bar{M}b_m + \tfrac{1}{3}\bar{F}\partial_m\bar{M} \\
&\quad - (\mathcal{R}_{mn} + \tfrac{2}{9}b_m b_n + \tfrac{2}{3}i\mathcal{D}_n b_m - \tfrac{1}{3}\epsilon_{pmqn}\mathcal{D}^q b^p)\partial^n\bar{A} \\
&\quad + \tfrac{5}{6}\bar{M}\partial_m\bar{F} + (\tfrac{1}{12}\mathcal{R} + \tfrac{1}{12}|M|^2 + \tfrac{1}{6}b_c b^c - \tfrac{i}{6}\mathcal{D}_n b^n)\partial_m\bar{A}\Big]\ .
\end{aligned}
\tag{A.30}
$$

Note that the first identity differs from the result in [20] by a factor multiplying the first term.

Components of G_a

For the computation of higher-components of the superfield $\mathcal{D}_a\Phi$ and its conjugate we need certain components of G_a. The relevant identities read

$$
\mathcal{D}^2 G_a| = \tfrac{2}{3}(\bar{M}b_a - ie_a^m\partial_m\bar{M})\ , \tag{A.31}
$$

$$
\begin{aligned}
\bar{\sigma}_b^{\dot\beta\beta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_d| &= \mathcal{R}_{bd} - \eta_{bd}(\tfrac{1}{6}\mathcal{R} + \tfrac{1}{9}|M|^2 + \tfrac{1}{9}b_a b^a) + \tfrac{2}{9}b_b b_d \\
&\quad - \tfrac{2i}{3}e_b^m\mathcal{D}_m b_d + \tfrac{1}{3}\epsilon_{abcd}\,e_m^c\mathcal{D}^m b^a\ .
\end{aligned}
\tag{A.32}
$$

These results perfectly agree with [20].

A.3.1 Derivation of Component Identities

Let us now give some of the derivations of the component identities which listed in the previous section. Here we constrain ourselves to the discussion of the more involved computations. In particular, the identities in eq. (A.26) are easily obtained by means of eq. (2.70) together with the (anti-)commutation relations and the components of the torsion and curvature and, hence, do not discuss them any further here. Instead let us demonstrate the identities in eq. (A.27). We rewrite the first component expression using eq. (2.70) as follows

$$\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\mathcal{D}}^2\bar{\Phi}| = 8\mathcal{D}^2R|\,\bar{\mathcal{D}}^2\bar{\Phi}| \;+\; 8R|\,\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}| \;. \qquad \text{(A.33)}$$

Inserting the required components we directly arrive at the result in eq. (A.27). The second identity in eq. (A.27) reads

$$\bar{\mathcal{D}}^2\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}| = \bar{\mathcal{D}}^2\mathcal{D}^2(\bar{\mathcal{D}}^2 - 8R)\bar{\Phi}| + 8\bar{\Phi}|\,\bar{\mathcal{D}}^2\mathcal{D}^2R| + 8\bar{\mathcal{D}}^2\bar{\Phi}|\,\mathcal{D}^2R| \;. \qquad \text{(A.34)}$$

Here, only the first term on the r.h.s. needs further attention, the remaining terms can be read off from eq. (A.28). Since $(\bar{\mathcal{D}}^2 - 8R)\bar{\Phi}$ is a covariantly chiral superfield, we need to determine the Θ-expansion of this chiral superfield and then use eq. (A.26). Fortunately, this expansion was already given in [34] and we simply insert the respective components here. Altogether, we then find the result in eq. (A.27).

Next, we turn to eq. (A.30) together with the necessary auxiliary results in eqs. (A.31), (A.32). Let us mention again that these have already been derived in [20], but as a cross-check we rederive them here. In particular, since we find disagreement regarding the first identity in eq. (A.30), it important to explain why we obtain a different result. This identity can be computed rather straightforwardly by using the algebra of super-covariant derivatives together with the torsion components. More precisely, we find

$$\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{D}_a\Phi| = -4(\mathcal{D}^2R|\,\mathcal{D}_a\Phi| + R|\,\mathcal{D}^2\mathcal{D}_a\Phi|) \;. \qquad \text{(A.35)}$$

After insertion of the required component identities we arrive at the displayed result in eq. (A.30).

Next, we turn to the auxiliary results in eqs. (A.31), (A.32). Firstly, by acting with a super-covariant derivative $\mathcal{D}^\alpha$ on eq. (A.21) we find eq. (A.31). Alternatively, we can act with $\bar{\mathcal{D}}_{\dot\alpha}$ on eq. (A.21) which yields

$$\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha G_{\beta\dot\beta} - \bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\beta G_{\alpha\dot\beta} = -\tfrac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\bar{\mathcal{D}}^2\bar{R} \;. \qquad \text{(A.36)}$$

Analogously, from eq. (A.22) we derive the following identity

$$\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\alpha}G_{\beta\dot\beta} - \mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\beta}G_{\beta\dot\alpha} = \tfrac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\mathcal{D}^2R \;. \qquad \text{(A.37)}$$

We can decompose $\bar\sigma_b^{\dot\beta\beta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_d$ into symmetric and anti-symmetric components as

$$\bar\sigma_b^{\dot\beta\beta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_d = \tfrac{1}{2}(\bar\sigma_b^{\dot\beta\beta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_d + \bar\sigma_d^{\dot\delta\delta}\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\delta G_b) + \tfrac{1}{2}(\bar\sigma_b^{\dot\beta\beta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_d - \bar\sigma_d^{\dot\delta\delta}\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\delta G_b) \;. \qquad \text{(A.38)}$$

The anti-symmetric part can be easily computed by using eqs. (A.36), (A.37) and the commutation relations.[47] We find

$$\bar\sigma_b^{\dot\beta\beta}\bar\sigma_d^{\dot\delta\delta}(\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta G_{\delta\dot\delta} - \bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\delta G_{\beta\dot\beta}) = 4i(\mathcal{D}_d G_b - \mathcal{D}_b G_d) - 4\epsilon_{adcb}\mathcal{D}^c G^a \;. \qquad \text{(A.39)}$$

[47]It is also necessary to use the σ-matrix trace identity $Tr(\sigma_a\bar\sigma_b\sigma_c\bar\sigma_d) = 2(\eta_{ab}\eta_{cd} - \eta_{ac}\eta_{bd} + \eta_{ad}\eta_{bc} - i\epsilon_{abcd})$.

The symmetric component of $\bar{\sigma}_b^{\dot{\beta}\beta}\bar{\mathcal{D}}_{\dot{\beta}}\mathcal{D}_\beta G_d$ can be derived by computing

$$\eta^{ac}R_{abcd} = \tfrac{1}{16}\eta^{ac}\bar{\sigma}_a^{\dot{\alpha}\alpha}\bar{\sigma}_b^{\dot{\beta}\beta}\bar{\sigma}_c^{\dot{\gamma}\gamma}\bar{\sigma}_d^{\dot{\delta}\delta}R_{\alpha\dot{\alpha}\beta\dot{\beta}\gamma\dot{\gamma}\delta\dot{\delta}} \tag{A.40}$$

from the formula displayed in [34] which is determined by the solution to the Bianchi identities. To simplify the resulting expression we use trace identities for the σ-matrices and eq. (A.36) such that

$$\begin{aligned}&\frac{1}{2}\left(\bar{\sigma}_b^{\dot{\beta}\beta}\bar{\mathcal{D}}_{\dot{\beta}}\mathcal{D}_\beta G_d + \bar{\sigma}_d^{\dot{\delta}\delta}\bar{\mathcal{D}}_{\dot{\delta}}\mathcal{D}_\delta G_b\right)\\&= R_{bd} - \eta_{bd}\left[12R\bar{R} + 2G_aG^a - \tfrac{1}{4}\left(\mathcal{D}^2R + \bar{\mathcal{D}}^2\bar{R}\right)\right] + 2G_bG_d + i(\mathcal{D}_bG_d + \mathcal{D}_dG_b)\ .\end{aligned} \tag{A.41}$$

Altogether, this demonstrates eq. (A.32). We are now in a position to derive the missing identity in eq. (A.30). Making iterative use of the (anti-)commutation relations we find

$$\begin{aligned}\mathcal{D}^2\bar{\mathcal{D}}^2\mathcal{D}_a\bar{\Phi} = &-\mathcal{D}^2\bar{\mathcal{D}}^{\dot{\alpha}}(T_{\dot{\alpha}a}{}^{\dot{\beta}}\bar{\mathcal{D}}_{\dot{\beta}}\bar{\Phi}) - \mathcal{D}^2(R_{\dot{\alpha}a\dot{\delta}}{}^{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\delta}}\bar{\Phi}) - \mathcal{D}^2(T_{\dot{\alpha}a}{}^{\gamma}\mathcal{D}_\gamma\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi})\\&+ \mathcal{D}^2(T_{\dot{\alpha}a}{}^{\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\gamma}}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}) + \mathcal{D}^\alpha(-T_{\alpha a}{}^{\gamma}\mathcal{D}_\gamma\bar{\mathcal{D}}^2\bar{\Phi} + T_{\alpha a}{}^{\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\gamma}}\bar{\mathcal{D}}^2\bar{\Phi})\\&+ R_{\alpha a\delta}{}^{\alpha}\mathcal{D}^\delta\bar{\mathcal{D}}^2\bar{\Phi} + T_{\alpha a}{}^{\gamma}\mathcal{D}_\gamma\mathcal{D}^\alpha\bar{\mathcal{D}}^2\bar{\Phi} - T_{\alpha a}{}^{\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\gamma}}\mathcal{D}^\alpha\bar{\mathcal{D}}^2\bar{\Phi} + \mathcal{D}_a\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}\ .\end{aligned} \tag{A.42}$$

This precisely coincides with the respective result in [20]. To determine the final component form one has to compute the individual terms. The first and second term in eq. (A.42) require most effort. The remaining terms in eq. (A.42) are easier to compute and we omit their details here. After some work, using the algebra of super-covariant derivatives and eqs. (A.16), (A.17), (A.22) we find

$$\begin{aligned}&-\mathcal{D}^2\bar{\mathcal{D}}^{\dot{\alpha}}(T_{\dot{\alpha}a}{}^{\dot{\beta}}\bar{\mathcal{D}}_{\dot{\beta}}\bar{\Phi})| - \mathcal{D}^2(R_{\dot{\alpha}a\dot{\delta}}{}^{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\delta}}\bar{\Phi})|\\&= i\mathcal{D}^2G_a|\,\bar{\mathcal{D}}^2\bar{\Phi}| + iG_a|\,\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\Phi}| + 64i\mathcal{D}_bG_a|\,\mathcal{D}^b\bar{\Phi}| - 16\mathcal{D}^b\bar{\Phi}|\,\bar{\sigma}_b^{\dot{\beta}\beta}\bar{\mathcal{D}}_{\dot{\beta}}\mathcal{D}_\beta G_a|\ .\end{aligned} \tag{A.43}$$

To obtain the final component expression, it remains to insert the required component formulas which, in particular, encompass eqs. (A.31), (A.32). Finally, we arrive at the displayed component form in eq. (A.30) which agrees with the result in [20].

A.4 Classification of $N = 4$ Operators

In this appendix we classify the possible four superspace-derivative operators and reduce them to a minimal set of relevant operators by using the commutation relations in eq. (2.11) as well as integration by parts identities. As mentioned in sec. 2.5.1 we have to include also the gravitational superfields R, G_a and $W_{\alpha\beta\gamma}$ into the analysis. R and G_a count as the equivalent of two and $W_{\alpha\beta\gamma}$ as the equivalent of three spinorial covariant derivatives. Operators which are directly related via the identity $\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\}\Phi \sim \mathcal{D}_a\Phi$, via eq. (A.25) or via the chirality condition in eq. (2.70) are easy to identify and, therefore, for the sake of brevity we do not need to distinguish them explicitly here.

We conduct the classification stepwise by listing those operators first which depend only on super-covariant derivatives acting on $\Phi, \bar{\Phi}$ (and, hence, do not involve the gravitational superfield). Any operator of this type can be labeled by the number of chiral or anti-chiral superfields on which covariant derivatives act. This number ranges between four and one. All remaining operators that we did not capture yet carry an explicit dependence on the superfields $R, \bar{R}$ and G_a. The results are displayed in table A.1. Note that we did not include higher-curvature operators in table A.1. These were already displayed in eq. (2.89) and briefly discussed in sec. 2.5.2.[48]

The list of operators in table A.1 is highly degenerate and displays several redundant operators. One of the tools that allow us to identify redundant operators is the algebra of super-covariant derivatives given in eq. (2.11). Making repetitive use of eq. (2.11) and occasionally also of eq. (A.19) we deduce the following list of relations among the operators in table A.1

$$\begin{aligned}
\mathcal{O}_{(1|1)} &= -\tfrac{5}{2}\mathcal{O}_{(R|\mathcal{D})} - 10i\mathcal{O}_{(G|1)} - \tfrac{1}{2}\mathcal{O}_{(1|5)} \\
\mathcal{O}_{(1|5)} &= 8\mathcal{O}_{(1|2)} + 8\mathcal{O}_{(R|2)} - 8\mathcal{O}_{(R|\mathcal{D})} \\
\mathcal{O}_{(1|4)} &= -8\mathcal{O}_{(R|2)} + 8\mathcal{O}_{(R|\mathcal{D})} \\
\mathcal{O}_{(1|3)} &= 16\mathcal{O}_{(1|2)} + 16\mathcal{O}_{(R|2)} + 32i\mathcal{O}_{(G|1)} - 8\mathcal{O}_{(R|\mathcal{D})} \\
\mathcal{O}_{(2|5)} &= -8\mathcal{O}_{(G|2)} - 2\mathcal{O}_{(2|7)} \\
\mathcal{O}_{(2|6)} &= -8\mathcal{O}_{(R|1)} \\
\mathcal{O}_{(R|\mathcal{D}^2)} - \text{h.c.} &= 4i\mathcal{O}_{(G|\mathcal{D})} \ .
\end{aligned} \tag{A.44}$$

In total this reduces the list in table A.1 by seven operators. In the following we make the particular choice to delete the operators $\{\mathcal{O}_{(1|5)}, \mathcal{O}_{(1|4)}, \mathcal{O}_{(1|3)}, \mathcal{O}_{(1|1)}, \mathcal{O}_{(2|7)}, \mathcal{O}_{(2|6)}, \mathcal{O}_{(G|\mathcal{D})}\}$ from the list of relevant operators. To indicate this we marked these operators in blue color in table A.1.

To further reduce the number of independent operators, we now apply a second tool, namely integration by parts identities in curved superspace. To this end, it is necessary to use the fact that the following integrals are entirely given by superspace-surface terms [47]

$$\int d^8z E\mathcal{D}_\alpha V^\alpha \ , \qquad \int d^8z E\bar{\mathcal{D}}^{\dot\alpha} V_{\dot\alpha} \ , \qquad \int d^8z E\mathcal{D}_a V^a \ , \tag{A.45}$$

where V^A is an arbitrary covariant superfield. As a first example we now consider

$$V^\alpha = S(\Phi, \bar{\Phi})\mathcal{D}^\alpha\Phi \ , \tag{A.46}$$

where S is an arbitrary scalar superfield, which depends on the superfields $\Phi, \bar{\Phi}$. For the above V^α we find that up to boundary terms

$$0 = \int d^8z E\mathcal{D}_\alpha(S(\Phi,\bar{\Phi})\mathcal{D}^\alpha\Phi) = \int d^8z E(-S(\Phi,\bar{\Phi})\mathcal{D}^2\Phi + \frac{\partial S}{\partial\Phi}\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi) \ , \tag{A.47}$$

[48]One may wonder why the superfield $W_{\alpha\beta\gamma}$ does not appear in table A.1. The reason for this is that, since $W_{\alpha\beta\gamma}$ has mass dimension three, we have to contract two of its indices to build a scalar operator. However, as $W_{\alpha\beta\gamma}$ is completely symmetric, such operators vanish identically.

Label	Form of Operator	Real
$\mathcal{O}_{(4\|1)}$	$(\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi)^2$	
$\mathcal{O}_{(4\|2)}$	$\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	✓
$\mathcal{O}_{(3\|1)}$	$\mathcal{D}^2\Phi\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(3\|2)}$	$\bar{\mathcal{D}}^2\bar\Phi\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(3\|3)}$	$\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	
$\mathcal{O}_{(2\|1)}$	$\mathcal{D}^2\Phi\bar{\mathcal{D}}^2\bar\Phi$	✓
$\mathcal{O}_{(2\|2)}$	$(\mathcal{D}^2\Phi)^2$	
$\mathcal{O}_{(2\|3)}$	$\mathcal{D}_a\Phi\mathcal{D}^a\bar\Phi$	✓
$\mathcal{O}_{(2\|4)}$	$\mathcal{D}_a\Phi\mathcal{D}^a\Phi$	
$\mathcal{O}_{(2\|5)}$	$\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	
$\mathcal{O}_{(2\|6)}$	$\mathcal{D}^2\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	
$\mathcal{O}_{(2\|7)}$	$\mathcal{D}^\alpha\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	
$\mathcal{O}_{(1\|1)}$	$i\bar\sigma^{a\dot\alpha\alpha}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_a\mathcal{D}_\alpha\Phi$	
$\mathcal{O}_{(1\|2)}$	$\mathcal{D}_a\mathcal{D}^a\Phi$	
$\mathcal{O}_{(1\|3)}$	$\bar{\mathcal{D}}^2\mathcal{D}^2\Phi$	
$\mathcal{O}_{(1\|4)}$	$\mathcal{D}_\alpha\bar{\mathcal{D}}^2\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(1\|5)}$	$\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(R\|1)}$	$R\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(R\|2)}$	$R\mathcal{D}^2\Phi$	
$\mathcal{O}_{(R\|3)}$	R^2	
$\mathcal{O}_{(\bar R\|1)}$	$\bar R\mathcal{D}_\alpha\Phi\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(\bar R\|2)}$	$\bar R\mathcal{D}^2\Phi$	
$\mathcal{O}_{(G\|1)}$	$G_a\mathcal{D}^a\Phi$	
$\mathcal{O}_{(G\|2)}$	$G_{\alpha\dot\alpha}\mathcal{D}^\alpha\Phi\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	✓
$\mathcal{O}_{(R\|\mathcal{D}^2)}$	$\mathcal{D}^2R$	
$\mathcal{O}_{(R\|\mathcal{D})}$	$\mathcal{D}_\alpha R\mathcal{D}^\alpha\Phi$	
$\mathcal{O}_{(G\|\mathcal{D})}$	$\mathcal{D}^aG_a$	✓

Table A.1: *List of operators at four superspace-derivative level. Operators whose $\theta = \bar\theta = 0$-component is not real-valued have to be completed by their conjugate expressions at the level of the action as in eq. (2.82). The individual operators are understood as being multiplied by a superfield $T(\Phi, \bar\Phi)$ (and $\bar T(\Phi, \bar\Phi)$ for their conjugate parts, in case that the operator is not real). Operators, which are displayed in blue color can be recast in terms of other (black) operators by means of the algebra of covariant derivatives eq. (2.11). Note that we omitted several operators here, for which it can be seen quite easily that they are redundant and, therefore, be recast in terms of other operators.*

which together with the analogous result for $V_{\dot\alpha} = \bar{S}(\Phi,\bar\Phi)\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi$ shows that the operators $\mathcal{O}_{(1)}$ and $\mathcal{O}_{(2)}$ in eq. (2.101) are equivalent.

Next we turn to integration by parts identities at the four-superspace derivative level. Different identities arise from choosing distinct superfields $V^a, V^\alpha, V_{\dot\alpha}$. However, we only need to classify superfields with an undotted spinor index V^α, since for any superfield $V_{\dot\alpha}$ there exists a conjugate superfield V^α, whose integration by parts identity expresses the conjugate of the integration by parts identity for $V_{\dot\alpha}$. Superfields with spacetime index V^a fulfill the relation

$$\mathcal{D}_a V^a = -\tfrac{i}{4}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\}V^{\alpha\dot\alpha} = -\tfrac{i}{4}(\mathcal{D}_\alpha \tilde{V}^\alpha - \bar{\mathcal{D}}^{\dot\alpha}\tilde{V}_{\dot\alpha}) \ , \tag{A.48}$$

where $\tilde{V}^\alpha = \bar{\mathcal{D}}_{\dot\alpha}V^{\alpha\dot\alpha}$ and $\tilde{V}_{\dot\alpha} = -\mathcal{D}^\alpha V_{\alpha\dot\alpha}$ and, hence, yield integration by parts identities that can be rewritten in terms of the spinorial superfields $\tilde{V}^\alpha, \tilde{V}_{\dot\alpha}$. Our task is, therefore, to classify all possible higher-derivative spinorial superfields V^α, such that the collective mass-dimension of the objects $(\mathcal{D}_A, R, \bar{R}, G_a, W_{\alpha\beta\gamma}, \bar{W}_{\dot\alpha\dot\beta\dot\gamma})$ appearing in V^α is given by $\Lambda^{3/2}$. Again care must be taken, since some of these superfields are related via the algebra of covariant derivatives in eq. (2.11). To give some examples, consider $V^\alpha = \bar{\mathcal{D}}^{\dot\alpha}G^\alpha_{\dot\alpha}$ which is equivalent to $\mathcal{D}^\alpha R$ via eq. (A.19). Further examples are given by

$$V^\alpha = \bar{\mathcal{D}}^2\mathcal{D}^\alpha\Phi = 8R\mathcal{D}^\alpha\Phi \ , \qquad V^\alpha = \mathcal{D}^\alpha\bar{\mathcal{D}}^2\bar\Phi = -8G^\alpha_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi + 2\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi \ . \tag{A.49}$$

We collect the remaining, independent integration by parts identities in table A.2.

V^α	Equivalence of operators
$T(\Phi,\bar\Phi)\mathcal{D}_\beta\Phi\mathcal{D}^\beta\Phi\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(4\|1)} \simeq 0$
$T(\Phi,\bar\Phi)\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(3\|2)} \simeq \mathcal{O}_{(4\|2)} + \mathcal{O}_{(3\|3)}$
$T(\Phi,\bar\Phi)\bar{\mathcal{D}}^2\bar\Phi\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(2\|5)} \simeq \mathcal{O}_{(2\|1)} + \mathcal{O}_{(3\|2)}$
$T(\Phi,\bar\Phi)\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(2\|7)} \simeq \mathcal{O}_{(3\|3)} + \mathcal{O}_{(2\|3)}$
$T(\Phi,\bar\Phi)\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi\mathcal{D}^\alpha\bar{\mathcal{D}}_{\dot\alpha}\bar\Phi$	$\mathcal{O}_{(3\|3)} \simeq \mathcal{O}_{(2\|4)} + \mathcal{O}_{(2\|6)}$
$T(\Phi,\bar\Phi)\mathcal{D}^2\Phi\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(\bar{R}\|1)} \simeq \mathcal{O}_{(3\|1)} + \mathcal{O}_{(2\|2)}$
$T(\Phi,\bar\Phi)\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}^\alpha\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	$\mathcal{O}_{(1\|5)} \simeq \mathcal{O}_{(2\|7)}$
$T(\Phi,\bar\Phi)R\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(R\|\mathcal{D})} \simeq \mathcal{O}_{(R\|2)} + \mathcal{O}_{(R\|1)}$
$T(\Phi,\bar\Phi)\bar{R}\mathcal{D}^\alpha\Phi$	$\mathcal{O}_{(\bar{R}\|1)} \simeq \mathcal{O}_{(\bar{R}\|2)}$
$T(\Phi,\bar\Phi)\mathcal{D}^\alpha R$	$\mathcal{O}_{(R\|\mathcal{D}^2)} \simeq \mathcal{O}_{(R\|\mathcal{D})}$
$T(\Phi,\bar\Phi)G^\alpha_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\bar\Phi$	$\mathcal{O}_{(G\|1)} \simeq \mathcal{O}_{(G\|2)} + \mathcal{O}_{(R\|\mathcal{D})}$

Table A.2: *Table of independent integration by parts identities. Here the symbol $\simeq$ is understood as indicating that the particular operator can be recast as a combination of other operators, up to some numerical coefficients. Here we display only those identities which give rise to a set of linearly independent constraints.*

It can be explicitly checked that the resulting system of constraints is non-degenerate by computing the rank of the corresponding matrix, and, hence, that each identity is independent from the others. In total we find 11 constraints, which together with the redundancy coming from the use of the commutation relations reduces the number of relevant operators down to a set of 9. According to the identities in table A.2, there still is some freedom left in choosing this set of operators.[49] Here we make the following choice of a basis of independent operators

$$\{\mathcal{O}_{(4|2)}, \mathcal{O}_{(3|1)}, \mathcal{O}_{(3|3)}, \mathcal{O}_{(2|1)}, \mathcal{O}_{(2|2)}, \mathcal{O}_{(2|3)}, \mathcal{O}_{(R|1)}, \mathcal{O}_{(R|2)}, \mathcal{O}_{(R|3)}\} \ . \tag{A.50}$$

Note that six of these operators induce four-derivative terms for the chiral scalar off-shell. These are precisely the operators, which involve an equal number of spinorial and anti-spinorial covariant derivatives and they read

$$\{\mathcal{O}_{(4|2)}, \mathcal{O}_{(3|3)}, \mathcal{O}_{(2|1)}, \mathcal{O}_{(2|3)}, \mathcal{O}_{(R|1)}, \mathcal{O}_{(R|2)}\} \ . \tag{A.51}$$

The minimal list of operators in eq. (A.50) is the result of the classification of the $N = 4$ operators. One may now proceed to compute the component versions of these operators. In sec. 2.5.6 we display the component forms of the operators in eq. (A.51).

A.5 Curvature Constraint from Killing Spinor in Higher-Derivative Supergravity

In this appendix we investigate the properties of supersymmetric vacua in higher-derivative $\mathcal{N} = 1$ supergravity more closely and pay special attention to the constraint on the scalar curvature of the background required by the existence of Killing spinors. More precisely, as we already stated in sec. 2.6.2 the preservation of supersymmetry in vacua of $\mathcal{N} = 1$ supergravity demands

$$\langle \mathcal{R} \rangle = \tfrac{4}{3} \langle |M|^2 \rangle \ , \tag{A.52}$$

which also follows from the integrability condition for the Killing spinors and is associated with the vanishing of the gravitino variation in the vacuum. For the ordinary two-derivative supergravity eq. (A.52) is indeed satisfied on-shell, that is after solving the Einstein equations, integrating out the auxiliary field M and evaluating the action at the supersymmetric condition $\langle F^i \rangle = 0$. In this appendix we investigate eq. (A.52) for the general higher-derivative supergravity. To this end, it would, in principle, be necessary to determine the component form of the complete higher-derivative action displayed in eq. (2.82). However, we will now demonstrate that it suffices to analyze a simpler Lagrangian. Since we are interested in the Lagrangian at the supersymmetric points, we want to evaluate the action at $\langle F^i \rangle = 0$. Higher-derivative operators for the chiral superfields, such as operators involving $\mathcal{D}_A \Phi^i$ and $\mathcal{D}_A \bar{\Phi}^{\bar{\jmath}}$, neither contribute to the scalar potential nor to the gravitational

[49] For instance, we find that the following operators are equivalent to each other $\mathcal{O}_{(3|2)} \longleftrightarrow \mathcal{O}_{(3|3)}$ and $\mathcal{O}_{(2|3)} \longleftrightarrow \mathcal{O}_{(G|1)} \longleftrightarrow \mathcal{O}_{(G|2)}$. In fact the last equivalence was identified as a generalized Kähler transformation in [20] and used to simplify the component Lagrangian.

part of the action after evaluating at $\langle F^i \rangle = 0$. Therefore, without loss of generality it is sufficient to ignore these operators here. What remains are higher-order operators involving only the gravitational superfields.[50] We are interested on examining whether the general form of the scalar potential, which is now corrected by higher-powers of M and $\bar{M}$, still allows the condition in eq. (A.52) to be satisfied. To this end it is convenient to discuss the following Lagrangian

$$\mathcal{L} = \int \mathrm{d}^4\theta E \left(-3U(\Phi, \bar{\Phi}, R, \bar{R}) + \frac{W(\Phi)}{2R} + \frac{\bar{W}(\bar{\Phi})}{2\bar{R}} \right) . \tag{A.53}$$

Note that there is an infinite tower of purely gravitational higher-order operators which we excluded here. The excluded operators involve super-covariant derivatives acting on $R, \bar{R}$ and the superfields G_a or $W_{\alpha\beta\gamma}$. In particular, these operators can also contribute higher monomials in M and $\bar{M}$ to the scalar potential. However, any contribution of that type can equivalently be generated via operators of the form $R^n \bar{R}^m$ and, therefore, be rewritten via operators involving only $R, \bar{R}$. These considerations are also in agreement with the conjectured action in eq. (2.83). Therefore, by choosing the Lagrangian in eq. (A.53) we only constrain the allowed higher-curvature terms, but not the form of the scalar potential. More specifically, eq. (A.53) implies off-shell only a $\mathcal{R}+\mathcal{R}^2$ gravity. Naturally, the form of the higher-curvature terms has an effect on the vacuum-structure of the theory [117]. For instance, this can be seen when we rewrite the $f(\mathcal{R})$-degree of freedom in terms of a scalar field, which may induce supersymmetry breaking [53, 54]. We will show now that while additional non-supersymmetric vacua may exist due to the presence of the $\mathcal{R}^2$-term, there still exists a supersymmetric vacua where eq. (A.52) holds. We return to the discussion of the higher-curvature terms at the end of this appendix.

The component version of eq. (A.53) can easily be computed following the steps in sec. 2.4.1. Evaluating the result at the vacuum conditions

$$\langle F^i \rangle = \langle \partial_a A^i \rangle = \langle \partial_a \bar{A}^{\bar{\jmath}} \rangle = \langle \partial_a M \rangle = \langle b_a \rangle = \cdots = 0 \ , \tag{A.54}$$

we arrive at the component Lagrangian

$$\begin{aligned} \mathcal{L}/e &= -\tfrac{1}{2}\Omega\mathcal{R} - \tfrac{3}{4}U_{M\bar{M}}\mathcal{R}^2 - V_J \ , \\ \Omega &= U + MU_M + \bar{M}U_{\bar{M}} - 4|M|^2 U_{M\bar{M}} \ , \\ V_J &= W\bar{M} + \bar{W}M + \tfrac{1}{3}|M|^2\Xi \ , \\ \Xi &= U - 2MU_M - 2\bar{M}U_{\bar{M}} + 4|M|^2 U_{M\bar{M}} \ . \end{aligned} \tag{A.55}$$

For clarity we set the kinetic terms for M to zero, since in an EFT we expect that we must integrate M out and, hence, contributions to $\mathcal{L}$ involving $\partial_a M$ generate kinetic terms for the chiral scalars.[51] Furthermore, we do not explicitly indicate anymore that all quantities are understood as being evaluated in the supersymmetric vacuum.

[50] The couplings of these operators are allowed to depend on the chiral multiplets.

[51] It is also possible that higher-derivatives terms for the spacetime-metric are introduced this way. However, for the maximally symmetric solutions to the Einstein equations these terms are irrelevant as well.

After performing the Weyl transformation to the Einstein frame we arrive at the following action

$$\mathcal{L}/e = -\tfrac{1}{2}\mathcal{R}_E - \tfrac{3}{4}U_{M\bar{M}}\mathcal{R}_E^2 - V_E\ , \qquad V_E = \frac{V_J}{\Omega^2}\ , \tag{A.56}$$

where $\mathcal{R}_E$ denotes the Einstein frame scalar curvature. Let us first have a look at the respective Einstein equations for eq. (A.56). In the vacuum $\langle V_E \rangle$ sets the value of the cosmological constant via

$$\Lambda = -V_E\ . \tag{A.57}$$

Since we have evaluated the Lagrangian at eq. (A.54) the coupling $U_{M\bar{M}}$ does not carry any dependence on the space-time metric and, hence, the Einstein equations read

$$9\,U_{M\bar{M}}\Box\mathcal{R}_E + \mathcal{R}_E = -4\Lambda\ , \tag{A.58}$$

where we used the general Einstein equations for $\mathcal{R}^2$-gravity as displayed in [45, 46]. Since we are looking for maximally symmetric backgrounds we set $\Box\mathcal{R}_E = 0$ and, hence, we obtain

$$\mathcal{R}_E = -4\Lambda = 4V_E\ , \tag{A.59}$$

which coincides with the solution of the ordinary Einstein-equations. At the level of eq. (A.59) the curvature constraint in eq. (A.52) can thereby be recast as

$$V_J = -\frac{1}{3}\Omega|M|^2\ . \tag{A.60}$$

It remains to check whether this condition is indeed fulfilled after replacing M in V_J via the solution to its respective equations of motion. The equations of motion for $\bar{M}$ read

$$W + \frac{1}{3}M\Xi + \frac{1}{3}|M|^2\frac{\partial\Xi}{\partial\bar{M}} - 2\frac{V_J}{\Omega}\frac{\partial\Omega}{\partial\bar{M}} + \frac{3}{4}\Omega^2 U_{M\bar{M}\bar{M}}\mathcal{R}_E^2 = 0\ . \tag{A.61}$$

Let us instead analyze a real-valued version of this equation. For instance, we may investigate

$$\bar{M}\frac{\partial\mathcal{L}}{\partial\bar{M}} + M\frac{\partial\mathcal{L}}{\partial M} = 0\ . \tag{A.62}$$

Eliminating the terms involving the superpotential via eq. (A.55) and replacing $\mathcal{R}_E$ via eq. (A.59) we find that the above expression reduces to

$$\begin{aligned}&\frac{1}{3}|M|^2\left(\Xi + \bar{M}\frac{\partial\Xi}{\partial\bar{M}} + M\frac{\partial\Xi}{\partial M}\right) + \frac{V_J}{\Omega}\left(\Omega - 2\bar{M}\frac{\partial\Omega}{\partial\bar{M}} - 2M\frac{\partial\Omega}{\partial M}\right)\\ &+ 12\left(\frac{V_J}{\Omega}\right)^2\left(MU_{MM\bar{M}} + \bar{M}U_{M\bar{M}\bar{M}}\right) = 0\ .\end{aligned} \tag{A.63}$$

We read this equation as a quadratic equation determining V_J/Ω. Inserting the expressions for Ξ, Ω displayed in eq. (A.55) into eq. (A.63) yields

$$\mathcal{E}\left(\frac{1}{3}|M|^2 + \frac{V_J}{\Omega}\right) + \left(MU_{MM\bar{M}} + \text{h.c.}\right)\left(\frac{4}{3}|M|^4 + 8|M|^2\frac{V_J}{\Omega} + 12\frac{V_J^2}{\Omega^2}\right) = 0\ , \tag{A.64}$$

where $\mathcal{E}$ is an expression in $M, \bar{M}, U$ and derivatives thereof. It is now easily seen that one of the two solutions of eq. (A.64) is precisely given by eq. (A.60). This concludes the demonstration that the curvature constraint as displayed in eq. (A.52) can still be satisfied for eq. (A.53).

There exists a second solution of eq. (A.64) which leads to a violation of eq. (A.60) and, hence, corresponds to a non-supersymmetric vacuum. As we mentioned in the beginning of this appendix the presence of higher-curvature terms changes the vacuum structure of the theory. Here we see that the presence of the $\mathcal{R}^2$-term leads to the existence of this second vacuum. The reformulation of $\mathcal{R}+\mathcal{R}^2$ gravity in terms of an additional scalar degree of freedom features a scalar potential with a plateau region. In the asymptotic regime of the plateau the value of the scalar potential is positive and, hence, in the respective vacuum supersymmetry is broken. If we had allowed for additional higher-curvature terms in $\mathcal{L}$ then further vacua might have appeared. However, we expect that the supersymmetric vacuum where eq. (A.60) is satisfied is still allowed.

Appendix B

Derivation of Structure of F^4-Term

B.1 Kähler Moduli Space and Coupling Tensor

In this as well as the next appendix we follow the reference [31]. Here we study the correction to the scalar potential induced by the higher-derivative operator in eq. (3.56) for the geometry of the Kähler moduli at leading order in the large volume limit.[52] This geometry is defined by

$$K_{(0)} = -2\ln(\mathcal{V}) \ , \qquad W = W_0 \ , \tag{B.1}$$

where $\mathcal{V}$ is displayed in eq. (3.18) and eq. (3.21). In this appendix we set $K_{(0)} = K$ and $T_{(0)} = T$ for the sake of brevity. Up to factors the relevant object that we want to investigate reads

$$\mathcal{Z} \equiv T_{ij\bar{k}\bar{l}} K^i K^j K^{\bar{k}} K^{\bar{l}} \ , \tag{B.2}$$

where $K^i = K^{i\bar{\jmath}} K_{\bar{\jmath}}$ and $K^{i\bar{\jmath}}$ denotes the inverse Kähler metric. Due to the shift-symmetry of K we may replace anti-holomorphic by holomorphic indices. $\mathcal{Z}$ is the quantity describing the $|F|^4$-correction in eq. (3.56) and we want to investigate the possible functional forms that it may take.

We will now provide evidence, but not a rigorous proof, for the following claim. If T_{ijkl} consists purely of quantities carrying at least one index, in other words that no scalar functions appear in T_{ijkl}, then $\mathcal{Z}$ is a constant. However, if non-constant scalar functions, such as K or the scalar curvature of the Kähler manifold $R_{ij}K^{ij}$, appear, then this statement is no longer true. Here R_{ij} denotes the Ricci tensor on the Kähler manifold.

Let us begin by investigating the possible structure of T_{ijkl}. Since the superpotential is a constant, we can assume that the coupling tensor is built entirely out of derivatives of K. The following list contains the simplest conceivable objects that

[52] Some of the below results can also be found in [118].

can be constructed this way:

$$T_{ijkl} = K_{ik}K_{jl} + K_{il}K_{jk} \tag{B.3}$$

$$T_{ijkl} = K_i K_j K_k K_l \tag{B.4}$$

$$T_{ijkl} = K_i K_k K_{jl} + \text{symmetrized} \tag{B.5}$$

$$T_{ijkl} = R_{ijkl} = K_{ijkl} - K_{ijm}K^{mn}K_{nkl} \tag{B.6}$$

$$T_{ijkl} = R_{ik}R_{jl} + R_{il}R_{jk} \tag{B.7}$$

$$T_{ijkl} = R_{ik}K_{jl} + \text{symmetrized} \tag{B.8}$$

$$T_{ijkl} = R_{ik}K_j K_l + \text{symmetrized} \tag{B.9}$$

$$T_{ijkl} = K_j \nabla_l R_{ik} + \text{symmetrized} \tag{B.10}$$

$$T_{ijkl} = \nabla_j \nabla_l R_{ik} + \text{symmetrized} \; . \tag{B.11}$$

The symmetrization is chosen such that T_{ijkl} obeys the identities in eq. (2.31). Moreover, in the above R_{ijkl} denotes the Riemann tensor and ∇_k the covariant derivative. We will show that for any four-tensor in the upper list of choices $\mathcal{Z}$ is a constant. For the tensors in eq. (B.3) to eq. (B.5) this simply follows from the no-scale condition $K^i K_i = 3$.[53]

The following identity is essential in order to prove our claim

$$K_{i_1 \ldots i_n j_1 \ldots j_m} K^{i_1} \ldots K^{i_n} \propto K_{j_1 \ldots j_m} \; . \tag{B.12}$$

This relation can be shown stepwise. To begin with note that $\mathcal{V}$ is a homogeneous function of degree (3/2) in the four-cycle volumes τ_i. According to Euler's theorem for homogeneous functions it, thus, has to satisfy

$$\frac{3}{2}\mathcal{V} = \sum_i \tau_i \mathcal{V}_i \; . \tag{B.13}$$

Taking iterative derivatives of this equation we obtain

$$\sum_i \tau_i \mathcal{V}_{i j_1 \ldots j_n} = \frac{3-2n}{2} \mathcal{V}_{j_1 \ldots j_n} \; . \tag{B.14}$$

With this we can prove the following auxiliary result[54]

$$K_{i_1 \ldots i_n} K^{i_1} \ldots K^{i_n} = \text{const} \; . \tag{B.15}$$

First note that we have

$$K^i = K^{ij} K_j = -\tau_i \; . \tag{B.16}$$

[53]Note that, if we choose T_{ijkl} according to eq. (B.6), then $\mathcal{Z}$ describes the holomorphic sectional curvature along K^i.

[54]For $n = 2$ this simply corresponds to the no-scale condition.

In general the derivative is of the form

$$K_{i_1\ldots i_n} = -\frac{2}{\mathcal{V}}\mathcal{V}_{i_1\ldots i_n} + \frac{2}{\mathcal{V}^2}(\mathcal{V}_{i_1\ldots i_{n-1}}\mathcal{V}_{i_n} + \text{symmetrized}) + \ldots \\ + 2\frac{(-1)^n(n-1)!}{\mathcal{V}^n}\mathcal{V}_{i_1}\ldots\mathcal{V}_{i_n} \ . \tag{B.17}$$

For each term a successive insertion of eq. (B.14) yields precisely the correct power of $\mathcal{V}$, since there are always as many products of derivatives of $\mathcal{V}$ in the numerator as there are powers of $\mathcal{V}$ in the denominator. Thus, one is left with a combinatorial constant for each term. We conclude that eq. (B.15) is satisfied.

Now we are in a position to show the following

$$K_{i_1\ldots i_n j}K^{i_1}\ldots K^{i_n} \propto K_j \ . \tag{B.18}$$

This can be seen via induction in n. For $n=1$ the above can simply be checked using eq. (B.14). Suppose the statement is true for $(n-1)$. Then, taking the derivative of eq. (B.15) with respect to τ_j, we obtain

$$K_{i_1\ldots i_n j}K^{i_1}\ldots K^{i_n} = -K_{j i_2\ldots i_n}K^{i_2}\ldots K^{i_n} - K_{i_1 j i_3\ldots i_n}K^{i_1}K^{i_3}\ldots K^{i_n} - \ldots \ . \tag{B.19}$$

Thus, since the statement is true for $(n-1)$, one infers that eq. (B.18) holds. Now we are in a position to generalize this statement for eq. (B.12). Again the proof uses induction: For $n=1$ this can be directly deduced by taking derivatives of eq. (B.18). For arbitrary n successive differentiation of eq. (B.18) yields eq. (B.12), if eq. (B.12) holds for $(n-1)$.

Now let us consider for example $\mathcal{Z}$ with T_{ijkl} given by eq. (B.6), then iterative use of eq. (B.12) yields

$$\mathcal{Z} \propto K_i K^{ij} K_j + \text{const.} \ , \tag{B.20}$$

which again gives a constant due to the no-scale property. Similarly one can show that $\mathcal{Z}$ is a constant for the choices in eq. (B.7), (B.8), (B.9). The cases of eq. (B.10) and eq. (B.11) require a little more effort, but can be derived making use of properties, such as $(\partial_k K^{ij})K_{ij} = -K^{ij}K_{ijk}$.

B.2 F^4-Term from Kähler-Correction?

In this appendix we investigate whether $V_{(1)}$ as given by eq. (3.54) could also be captured or induced by a correction to the two-derivative theory that is to K or W. While this question is of no further relevance to the results of this thesis, it would be important for an exact matching of the result of the KK-reduction of the various 10D $(\alpha)^3$-corrections to a manifestly supersymmetric 4D theory.

If $V_{(1)}$ could be described as a correction to the two-derivative theory, then possibly only via a new contribution to the Kähler potential since W has to be holomorphic. In addition we have to guarantee Kähler-invariance. Hence, the correction to the Kähler potential has to be a function of $G \equiv K + \ln W + \ln \bar{W}$. The corrected Kähler potential K_c is then of the form

$$K_c = K_0 + (\alpha')^3 \delta K(Q, T + \bar{T}) \ , \tag{B.21}$$

where $K_0 = -2\ln(\mathcal{V})$ is the tree-level Kähler potential. For convenience we choose $Q \equiv e^G$, and T collectively denotes the Kähler moduli. Note that in eq. (B.21) we do not need to include the $\hat{\xi}$-correction since it already is of order $(\alpha')^3$. For simplicity in the following we set $\alpha' = 1$. Let us assume that δK is an analytic function. In order to reproduce the scalar potential for the theory at $\mathcal{O}(|W|^2)$ the lowest order coefficient of the series expansion of δK in Q has to vanish, such that

$$\delta K(Q, T + \bar{T}) = Q\, K_{(1)}(T + \bar{T}) + \mathcal{O}(Q^2) \ . \tag{B.22}$$

Including only the Kähler moduli and ignoring again the $\hat{\xi}$-correction the scalar potential has the form[55]

$$V = e^{K_c} |W|^2 (K_c^{ij} K_{c,i} K_{c,j} - 3) \ . \tag{B.23}$$

We now want to compute the terms in V which are quartic in $|W|$. In other words these are all terms of order $\mathcal{O}(Q^2)$. To this end we compute the following expansion

$$e^{K_c}|W|^2 = Q + Q^2 K_{(1)} + \mathcal{O}(Q^3) \ . \tag{B.24}$$

Furthermore, the Kähler metric reads

$$\begin{aligned} K_{c,ij} = K_{0,ij} &+ Q(K_{0,ij}K_{(1)} + K_{0,i}K_{0,j}K_{(1)} + K_{0,i}K_{(1),j}) \\ &+ Q(K_{0,j}K_{(1),i} + K_{(1),ij}) + \mathcal{O}(Q^2) \ . \end{aligned} \tag{B.25}$$

We find that the inverse Kähler metric is given by

$$\begin{aligned} K_c^{ij} = K_0^{ij} &- Q(K_0^{ij}K_{(1)} + K_0^i K_0^j K_{(1)} + K_0^i K_0^{jk} K_{(1),k}) \\ &- Q(K_0^j K_0^{ik} K_{(1),k} + K_0^{ik} K_0^{jl} K_{(1),ij}) + \mathcal{O}(Q^2) \ . \end{aligned} \tag{B.26}$$

Now we are in a position to determine the scalar potential at order Q^2. We find that

$$V_{Q^2} = -Q^2 (6K_{(1)} + 4K_0^i K_{(1),i} + K_0^i K_0^j K_{(1),ij}) \ , \tag{B.27}$$

where we made extensive use of the no-scale property of K_0. Now, V_{Q^2} has to match $V_{(1)}$ as given by eq. (3.59). This yields

$$6K_{(1)} + 4K_0^i K_{(1),i} + K_0^i K_0^j K_{(1),ij} = \hat{\lambda} \int c_2 \wedge J \ . \tag{B.28}$$

We read this equation as an inhomogeneous linear partial differential equation for $K_{(1)}$. The solution can always be decomposed into an arbitrary solution to the respective homogeneous differential equation as well as a particular solution to the

[55] As in the previous appendix we do not need to distinguish between holomorphic and antiholomorphic indices.

inhomogeneous one. A particular solution to the inhomogeneous differential equation is given by

$$K_{(1)} = \tfrac{4}{31}\hat{\lambda}\Pi_k t^k \ . \tag{B.29}$$

To check that eq. (B.29) indeed solves eq. (B.28) we have to make use of the identity

$$2\tau^i \frac{\partial t^j}{\partial \tau^i} = t^j \ , \tag{B.30}$$

which can be checked by using

$$K_{0,ij} = \frac{1}{2}\frac{t^i t^j}{\mathcal{V}^2} - \frac{1}{\mathcal{V}}\frac{\partial t^j}{\partial \tau^i} \ , \tag{B.31}$$

as well as

$$K_0^i = -\tau^i \ , \tag{B.32}$$

and finally it is also necessary to note that

$$K_{0,i} = -\frac{t^i}{\mathcal{V}} \ . \tag{B.33}$$

So far we have found that the correction in eq. (B.29) indeed reproduces $V_{(1)}$ in eq. (3.54). However, it demands also a new correction to the two-derivative kinetic term via the formula of the Kähler metric in eq. (B.25). In particular, this includes a term in the Lagrangian

$$e^{-1}\mathcal{L} \supset \tfrac{4}{31}\hat{\lambda} Q \left(\partial_\mu T^i \partial^\mu \bar{T}^{\bar{\jmath}}\right) \partial_{T^i}\partial_{\bar{T}^{\bar{\jmath}}} \left[\int c_2 \wedge J\right] \ . \tag{B.34}$$

To fully confirm that the correction to the Kähler potential is allowed, it would be necessary to determine whether the above two-derivative terms can indeed emerge from the KK-reduction of the 10D $(\alpha')^3$-corrections. Since they descend from only partially known terms involving the field-strength H_3, we cannot provide a full answer to this question here. Nevertheless, let us make a small remark. We expect that the above correction can only emerge from a ten-dimensional term that involves covariant-derivatives acting on Riemann-tensors. If the replacement for $\mathcal{R}(\Omega_+)$ for the Riemann-tensor displayed in eq. (3.27) would already fully capture the dependence on H_3, then we naively expect the required terms with covariant derivatives acting on the Riemann-tensor to be absent. However, let us emphasize again that a full analysis is mandatory to completely understand the required terms.

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Eidesstattliche Erklärung:

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Hamburg, den

(David Ciupke)