Four Essays on Real Estate Markets

ggf. kumulative Dissertation

Universität Hamburg
Fakultät Wirtschafts- und Sozialwissenschaften

Dissertation

Zur Erlangung der Würde einer Doktorin/eines Doktors der
Wirtschafts- und Sozialwissenschaften
"Dr. rer. pol."
(gemäss der PromO vom 24. August 2010)

vorgelegt von
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Hamburg, 2016

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Datum der Disputation: 27.01.2016

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Acknowledgments

I owe enormous debt to the many people who have kindly offered comments, ideas or have otherwise helped during the process of creating this volume.

First and foremost I would like to thank my advisor (Doktorvater) Michael Funke for his generous supporting and outstanding supervision that made my PhD experience at university Hamburg productive and stimulating. I am also very grateful to the members of my final defence committee, Thomas Straubhaar and Ulrich Fritsche for their generosity and kindness. I will forever be thankful to Yu-Fu Chen for his scientific advice and knowledge and many insightful discussions and suggestions. A good support system and harmonious working environment is critical to surviving in graduate school. I would therefore like to give a big thanks to all my colleagues, Mariia Belaia, Boris Blagov, Sercan Eraslan, Nicole Glanemann, Kristin Göbel, Yu Hao, Christine Kniephoff, Kirsten Kuhk, Ivan Lozev, Petar Mihaylovski, Michael Paetz, Annette Pauls, Jana Peters and Hao Weng for many wonderful times. I will never forget tons of interesting topics and discussions during our lunch breaks and fun activities we have done together.

Special thanks should also be given to many extraordinary scientists and researchers I met in conferences and summer school, Yongheng Deng, Oded Galor, Eric Girardin, Stephanie Schmitt-Grohé and Harry X. Wu for their excellent presentations, interesting lectures and talks which gave me great encouragement and inspiration for my own research.

Finally, I would like to thank my parents, my big sister and my best friends for their trust and patience with me in the past 4 years. Without them my PhD time would have been twice as hard as it was.

1

Introduction

1.1 Roadmap

With good reason, the global financial crisis arising from the American housing market in 2007 has been widely referred to as the "Great Recession". In 2007–2009, nearly $20 trillion worth of financial assets owned by US households was destroyed. Industry production fell by 15 per cent, the largest fall since the Second World War. Meanwhile, the labour market deteriorated seriously and the unemployment rate rose from 4.7 per cent to 10 per cent. In a world of global financial markets, crises can no longer be viewed as isolated phenomena. Beyond US borders, many european countries also experienced a significant economic downturn. Access to loans was limited, the survival of many banks became uncertain and the equity markets tumbled. Consumer confidence in the euro area fell to record low levels and households held back on discretionary spending. On the other side of the world, although Asian countries were less exposed to the financial crisis as their financial markets and investors are less integrated with the US, the subsequent influence on the real economy was still severe because of the sizeable trade effects. As a result, global GDP declined by 2 per cent in 2009. It has been estimated that between 50 million and 100 million people around the world either fell into or were prevented from escaping extreme poverty due to the crisis (Goodwin et al., 2013). Indeed, financial crises are not a new phenomenon. From time to time the world has been hit by severe crises, such as the Nordic banking crisis in the early 1990s, Japan's "Lost Decade" of 1991–2000 and the East Asian Financial Crisis in 1997. The recent financial crisis will most probably not be the last.

One common feature of these crises is that the underlying causes were associated closely with boom–bust cycles in real estate markets (Bordo and Jeanne, 2002; Crowe et al., 2013; Hartmann, 2015; Reinhart and Rogoff, 2009). On the one hand, real estate has become an important factor in the real economy as it makes a major contribution to GDP and provides prosperity and jobs in most countries. In addition, it also serves as provision for old age and protection against inflation. A widespread downturn in property prices will have a significant impact on the real economy. On the other hand, construction projects and residential real estate purchases are usually credit-financed. Leveraged banks rely heavily on real estate collateral to reduce the risk of much of their household and commercial lending. As a result, real estate is also deemed an important factor in financial stability. It is clear that careful research is needed to analyse the pre-crisis issues and ways of preventing a similar crisis in the future.

The past eight years have witnessed an increasing interest in comprehensive study of the root causes, key events and responses related to the global financial crisis of 2007–2009. A considerable number of economic models and methods have been designed specifically to anatomize the financial crisis. As a part of the growing body of literature, the focus of this study centres on the old question of what we can learn from this crisis. However, this thesis attempts to provide different insights by employing a newly developed empirical method and economic concepts.

There are several lessons that we can learn from the financial crisis. First, countries should watch for early warning and now-casting hints signalling a future crisis. As the crisis in many countries was caused by excessive increases in property prices and/or rapid credit growth, one of the major tasks is to develop renewed empirical methods and econometric tests to detect excessive asset price developments. This is of particular importance because these empirical procedures also aim to flag levels of financial and housing risks that require a pre-emptive macroprudential policy response. Broadly speaking, macroprudential policy is seen as aiming at financial stability. In terms of the specific goals of macroprudential policy, the general view is that it is all about limiting the risk and costs of systemic crises. Since boom-bust cycles in real estate markets have been major factors in systemic financial crises and therefore need to be at the forefront of macroprudential policy (Hartmann, 2015). This requires that policymakers can identify emerging bubbles in the housing market in real-time with a certain confidence.

Second, policymakers should also be mindful of the channels through which the financial crisis erupted. Since the explosion of the crisis in 2007, most of the blame has been placed on the regulatory authorities and investment banks. The criticisms are primarily based on shortcomings in the implementation of policies, failures in the supervision of financial markets and opaque financial products, such as a collateralized debt obligation (CDO). Nevertheless, in the instance of the subprime mortgage catastrophe, we should not point the finger only at them. Rather, this crisis was the collective creation of the world's central banks, investment banks, homeowners, mortgage lenders and investors (Petroff, 2007). Against this background, the study also attempts to scrutinize the housing investment behaviour of households, especially those who are homeowners, within two theoretical frameworks: (i) the recently proposed risky steady state modelling approach and (ii) dynamic life cycle model incorporating housing affordability constraints, as explained in more detail below.

1.2 Contents and Structure

Broadly speaking, there are two classes of economic models – theoretical and empirical. Theoretical models help economists determine logically complicated causes and influences among the numerous interacting elements in an economy. However, no matter how good a theory may be, it is certain to be an incomplete representation of even the key features of "reality" and will be replaced by a better theory in future. Thus, imposing empirical models could assist us to verify the qualitative predictions of theoretical models and convert them to precise, numerical outcomes (Ouliaris, 2011). As outlined above, this thesis consists of two empirical and two theoretical chapters investigating the real estate markets from different perspectives and over various time horizons, with the purpose of building road maps of reality and enhancing our understanding of the real estate markets in the pre- and post- crisis world.

More specifically, chapter 2, "Real-time Warning Signs of Emerging and Collapsing Chinese House Price Bubbles", attempts to answer the question of whether there are residential real estate bubbles in China. This question is particularly important as China has experienced an extraordinary housing market boom since 2000. In the aftermath of the global financial crisis of 2007–2009, property was further boosted by China's huge financial crisis stimulus package.

This increase has led to concerns that China is vulnerable to a housing market shock. Thus, the study employs the newly developed recursive unit root tests to identify the beginning and the end of potential speculative bubbles in the Chinese housing price cycle. This persuasive strategy for identifying and dating multiple bubbles in real time has been pioneered by Phillips and Yu (2011) and Phillips et al. (2012). Their point of departure is the observation that the explosive real estate of bubbles is very different from random walk behaviour. Correspondingly, they have developed a new recursive econometric methodology, interpreting mildly explosive unit roots as a hint of bubbles. Applying this strategy to the Chinese housing market for the period 2005–2010, the results of this study signal a heightened probability of an emerging Chinese housing price bubble in 2009–2010. In other years, significant signs of unsustainable overvaluation are not displayed.

Record low interest rates since the global financial crisis have driven house prices up in many countries. The examples include Germany, where the real estate market started to surge and house prices in larger cities have experienced large mark-ups since 2010. On the one hand, the prevailing expectation that central banks in advanced economics will not tighten monetary policy in the near future has played a role in this development, as low interest rates can fuel excess borrowing and push asset prices ever higher. On the other hand, the euro area crisis matters. Not only are German households acquiring more real estate, but foreigners see Germany as a safe haven. In light of previous experience of how the bursting of a real estate bubble triggered the recession in the US, there are increasing worries that Germany might be destined to a similar fate. Thus, the aim of chapter 3 is to detect the beginning and the end of potential speculative bubbles in Germany. In particular, the study employs the same statistical test strategy introduced in chapter 2 and house price data over the sample period 1971–2013 to assess the renewed momentum in the German housing market. Overall, actual house prices are found not to be disconnected significantly from underlying economic fundamentals and there is no evidence of an emerging speculative housing bubble at the present time in Germany. Further to this, the study also investigates house price developments across other OECD countries. In contrast to Germany, the majority of OECD countries, such as Ireland, New Zealand, Spain, the Netherlands, the UK and the US, experienced strong house price growth in the early 2000s, which cumulated in 2007–2008 into an astounding burst of speculative house price bubbles. Against this background, the test statistics are also calculated for these countries, aiming to assess the genuine validity and reliability of the univariate screening toolkit. The test evidence delivers timely warnings of underlying misalignments, vulnerabilities and tail risks that redisposed the international housing market to the financial crisis in 2007–2009. This gives us confidence in the potential applicability of the proposed testing strategy to the German housing market.

As the crisis emerged from the US housing market, risky house prices are deemed to play a significant role in the global economic collapse. To study the implications of long-term risky house prices for the housing market, we employ the risky steady state concept proposed by Coeurdacier et al. (2011) in chapter 4. Traditionally, steady state refers to the deterministic steady state to which the economic system gravitates when future shocks are assumed to be zero. In contrast, the risky or stochastic steady state is one in which the system comes to rest, when agents know that future shocks will continue to occur based on certain known distributions of those shocks. The latter has a wider scope and greater application. If risk-averse agents are aware of the existence of future shocks hitting the economy, they will anticipate the convergence of economic variables to some stochastic steady state, which incorporates information about expected future risk and the corresponding optimal decisions. Against this background, the main purpose of this chapter is to understand the investors' attitude towards long-horizon house price risks. This study develops and calibrates a theoretical model analysing the impact of stochastic labour income, risky interest rates and house prices on housing investment choice. According to the model, the precautionary saving effect, the risk premium elicited by interest rate risk and house price risk and the crowding out effect are well reflected in the approximation equilibrium function. This implies that riskier countries tend to have larger investment in housing and accumulate more financial assets than safer ones. Further to this, the study also provides empirical evidence of positive relationships between housing investment level and risk level across OECD countries.

The final chapter addresses the issue of housing investment from the perspective of housing affordability in a dynamic life-cycle modelling framework. This study is basically related to two strands of literature. The first strand concerns the affordability problem. Owner-occupied housing is the single largest expenditure item in the budgets of most families. A small percentage change in housing prices will have a large impact on a household's consumption decisions and asset allocation. The collapse of the global financial markets in 2007–2009 sends a clear sign that careful research is necessary to assess the extent to which the mortgage market must be restructured to deal with households' "ability to borrow". The second strand includes theories typified by Cocco (2005) and Yao and Zhang (2005a). In their models, households' optimal housing, consumption and portfolio decisions are analysed in a realistic, dynamic life cycle model calibrated using US data. By explicitly incorporating the affordability constraints in the mortgage market, this study attempts to assess quantitatively the impact of the affordability constraints on households' optimal consumption, mortgage, portfolio choices and poverty status over the lifetime. Meanwhile, we also investigate the interaction between borrower-based macroprudential policies and social policies aimed at improving poverty and fostering home ownership and credit availability. Based on our simulation results, the mortgage affordability constraints are shown to have a significant impact in the prime and subprime mortgage markets. Moreover, the sensitivity analysis confirms the findings in the baseline case and emphasises that there is a non-linear (hump-shaped) relationship between affordability degrees and housing-caused poverty, particular in the age groups above 40. This implies that in light of the age profile of households and features of mortgage credit markets, the magnitudes of the borrower-based macroprudential policies are needed to be carefully assessed in order to minimise the potential conflicts with other social policies.

2

Real-time Warning Signs of Emerging and Collapsing Chinese House Price Bubbles[1]

2.1 Introduction

Issues related to Chinese house prices have become an international concern. China's extraordinary real estate boom began in the early 2000s and was further boosted in 2009 by China's huge financial crisis stimulus package. In the aftermath of the global financial crisis in 2007–2009, the Chinese government urged banks to increase lending. Buyers took advantage of looser real estate lending terms and lower mortgage rates. Increasing rates of urbanization, rising income, and rapid economic growth have also contributed to high real estate demand. Furthermore, the expansionary monetary policy stance has not only boosted house prices but has also generated a shift in house price expectations and spurred excessive risk-taking in the banking sector.[2] As a result, real estate in many cities has become unaffordable for broad sections of the population in China.

[1]This chapter is co-authored by Michael Funke and published as Chen and Funke (2013).

[2]For the impact of the monetary policy stance on the banking sector, see Altunbas et al. (2010). Chinese banks are now much more exposed to the property market than they were in the early 2000s, with real estate loans now accounting for about 20% of total loans.

Ultimately, house prices have also become an important and topical issue for Chinese policymakers.[3] The property sector now makes up about 12% of GDP. Furthermore, property is a sizable component of household and corporate balance sheets. Therefore, a sudden collapse in house prices may have negative spillover effects on the overall macroeconomic situation and may pose macroeconomic and financial stability risks.[4] Just as a quick reminder, the build-up of property price overvaluations triggered the Asian financial crisis of the late 1990s. In response to the sustained run-up in house prices, therefore, the Chinese government imposed in spring 2010 several market-cooling measures and restrictions intended to bring house prices down to a "reasonable level". In addition, the People's Bank of China benchmark mortgage lending rate was raised in summer 2011. As a result, multiple indicators suggested a slight market downturn in 2011. It must be pointed out that it remains an open question whether the latest market dip may be a short-term episode since high and rising real estate prices may be in line with market fundamentals.

Recent research has also focused on central banks' incentives. Kocherlakota and Shim (2007) demonstrate that the utility-maximizing central bank's response to house price increases is conditioned on the real time probability of a future house price collapse. If this is high ex-ante, pro-active corrective action is optimal. Otherwise the central bank shows forbearance towards instability.

The uncertainties in defining a sustainable house price level and identifying emerging housing bubbles in real time have not lessened substantially in past decades. Even worse, it may turn out not to be very useful to identify bubbles in real time. Even if statistically significant bubble characteristics are found and monetary policymakers are confident that a speculative housing bubble has emerged, the question of the timeliness of the policy response remains. The problem is the timing of the detection of the bubble relative to the timing of its collapse. The risk is that the subsequent interest and/or macroprudential policy response occurs not long before

[3]There has been a considerable debate among economists on the evolution of Chinese property prices and the empirical evidence remains at best ambiguous, varying with the selected empirical methodology. For example, Wu et al. (2010) have argued that a real estate bubble has emerged in recent years, spurred by the fiscal stimulus after the great recession. In contrast, Ren et al. (2012) have found no evidence to support the existence of speculative price bubbles in China. The fragility of the results likely stems from the inherent difficulty of identifying bubbles. A review of traditional econometric tests for asset price bubbles is available in Gürkaynak (2008) and Mikhed and Zemčík (2009).

[4]See Ciarlone (2011) and Chen et al. (2014).

the bubble collapses on its own. Given the lags associated with monetary policy, the resulting contractionary effects of the pro-active policy tightening would occur just when the bubble bursts, worsening rather than mitigating the effects of the bubble's collapse. Thus, those seeking to identify significant warning signs of future housing bubbles may turn out to be the Don Quixotes of housing research. Is it therefore time to call off the quest?

The plan of the chapter is as follows. Section 2.2 reviews some theoretical and econometric issues related to housing valuation and bubble identification. Section 2.3 proceeds by discussing the data and the results of the econometric diagnostics. Section 2.5 draws some conclusions.

2.2 Theoretical and Econometric Considerations in Relation to Detecting Property Price Bubbles

In the first stage, we need to define bubble periods. Based on this, we can then identify inflated house prices and bubble periods. Rational house price bubbles can arise because of the indeterminate aspect of solutions to rational expectations models. The house price that agents are prepared to pay today depends on the expected house price at some point in the future. But the latter depends on the expected house price even further in the future. The resulting process governing house prices does not pin down a unique house price level unless, somewhat arbitrarily, a transversality condition has to be imposed to obtain a unique solution. However, in general, the possibility that house prices may systematically deviate from their fundamental value cannot be ruled out. Even if risk-neutral agents are perfectly rational, the actual house price may contain a bubble element, and thus there can be a divergence between the house price and its fundamental value. The resulting real estate bubble is an upward house price movement over an extended range that then suddenly collapses.[5]

Our goal is to find how house prices evolve over time, given the behaviour of fundamentals. Time is discrete. In the modelling framework, fundamental house prices H_t can be represented

[5] Martin and Ventura (2011) have recently presented a rational bubble model with investor sentiment shocks and imperfect financial markets. In their framework, the size of the bubble depends upon investor sentiment. On the other hand, financial frictions allow efficient and inefficient investments to coexist. Introducing financial frictions can thus explain why bubbles can temporarily lead to expansions in the capital stock and in GDP although a bubble is nothing but a pyramid scheme. This happens when the bubble raises the net worth of efficient investors, allowing them to increase investment.

as follows:

$$H_t = \Big(\frac{1}{1+r}\Big)E_t(R_t + H_{t+1}), \tag{2.1}$$

where E_t is the expectations operator, R_t is the rent, and r is the discount rate. To solve the model, we need to eliminate the term involving the expectation of the future value of the endogenous variable. It is straightforward to show that the fundamental house price ${H_t}^F$ can be solved under rational expectations by repeated forward substitution. This implies

$${H_t}^F = \sum_{j=1}^{\infty}\Big(\frac{1}{1+r}\Big)^j E_t(R_{t+1}). \tag{2.2}$$

The logic of equation 2.2 is that house market prices contain expectations of future rents. No specific assumptions are made about the process followed by R_t.[6] The rational bubble components B_t follow

$$B_t = \Big(\frac{1}{1+r}\Big)E_t(B_{t+1}). \tag{2.3}$$

Solving for H_t finally yields

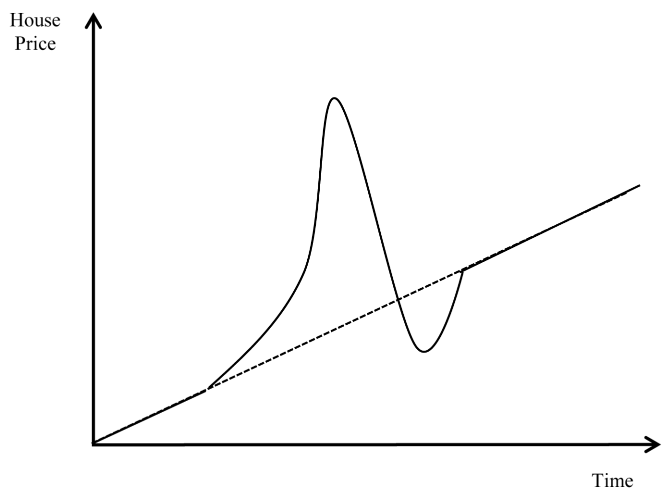
$$H_t = {H_t}^F + B_t. \tag{2.4}$$

Equation 2.4 breaks up house prices into a "fundamental" and a "bubble" component. Without a bubble, house prices equal the fundamental value ${H_t}^F$. Under bubble conditions house prices may show an explosive behaviour inherent in B_t.[7] What kind of house price bubble is B_t? Mathematically, the explosive bubble term is a *deus ex machina* arising as an alternative solution to the process governing house prices. The origin of the bubble cannot be explained, and only the dynamics of the bubble are given by the model. If a bubble is present in the house

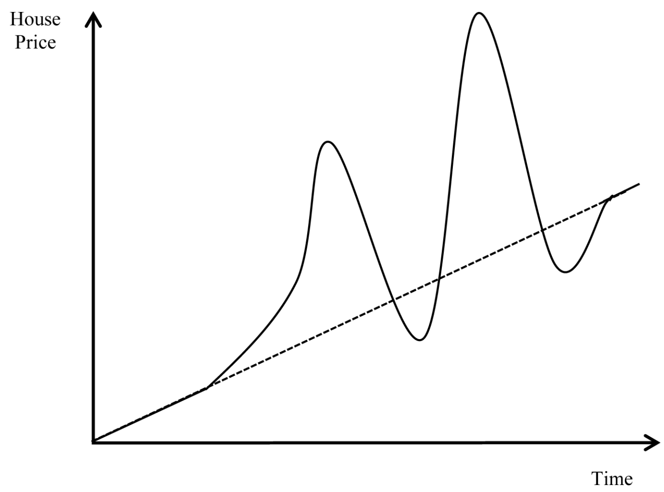
[6] Much of the modelling appeal is clarity, not realism. Because of the complexity of the fundamental $E_t(R_{t+j})$ and the lack of agreement about its key ingredients, the frameworks stop short of being a fully specified model.

[7] One implication of rational house price bubbles is that they cannot be negative, i.e. $B_t<0$. This is because the growing bubble term falls at a faster rate than house prices increase and thus a negative bubble ultimately ends in a zero house price. Rational agents realize that and know that the bubble must eventually burst. By backward induction, the bubble must then burst immediately, as no investor will pay the "bubble premium" in the earlier periods.

price, equation 2.4 requires that any rational investor must expect the bubble to grow. If this is the case, and if B_t is strictly positive, this builds the stage for speculative investor behaviour: a rational investor is willing to buy an "overpriced" house, since he/she believes that through price increases he/she will be sufficiently compensated for the extra payment he/she has to make as well as the risk of the bubble bursting. In that sense, the house price bubble is a self-fulfilling expectation. Eventually, the bubble implodes, house prices fall with a sharp correction and deleveraging occurs.



Panel A



Panel B

Figure 2.1: Stylised One-off Bubble vs periodically Collapsing Bubbles.

Next we discuss how the theoretical framework can be linked to an econometric testing strategy. In the econometric literature, identifying a bubble in real time has proved challenging. In addition, severe econometric problems result from finite samples. Standard unit root and cointegration tests may be able to detect one-off exploding speculative bubbles, as in panel A of Figure 2.1, but are unlikely to detect periodically collapsing bubbles, as in panel B of Figure 2.1. In other words, efforts to identify significant warning signs of future housing bubbles have been impeded by the necessity to spot multiple starting and ending points. The reason is that traditional unit root tests are not well equipped to handle changes from $I(0)$ to $I(1)$ and back to $I(0)$. This makes detection by cointegration techniques harder, due to bias and kurtosis (Evans, 1991).

A nuanced and persuasive approach to identification and dating multiple bubbles in real time has recently been pioneered by Phillips and Yu (2011) and Phillips et al. (2012).[8] The idea is to spot speculative bubbles as they emerge, not just after they have collapsed. Their point of departure is the observation that the explosive property of bubbles is very different from random walk behaviour. Correspondingly, they have developed a new recursive econometric methodology interpreting mildly explosive unit roots as a hint for bubbles. If we consider the typical difference of stationary vs trend stationary testing procedures for a unit root, we usually restrict our attention to regions of "no more than" a unit root process, i.e. an autoregressive process where $\rho \leq 1$. In contrast, Phillips and Yu (2011) model mildly explosive behaviour by an autoregressive process with a root ρ that exceeds unity but is still in the neighbourhood of unity. The basic idea of their approach is to recursively calculate right-sided unit root tests to assess evidence for mildly explosive behaviour in the data. The test is a right-sided test and therefore differs from the usual left-sided tests for stationarity. More specifically, consider the following autoregressive specification estimated by recursive least squares:

$$x_t = \mu + \rho x_{t-1} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t \sim \text{i.i.d}N(0, \sigma^2). \tag{2.5}$$

The usual $H_0 : \rho = 1$ applies, but unlike the left-sided tests which have relevance for a sta-

[8] The diagnostic for multiple speculative bubbles modifies a previous method for identifying one-off bubbles suggested in Phillips et al. (2011). A different class of tests for identifying periodically collapsing bubbles based on Markov-switching models has been explored in Funke et al. (1994) and van Norden and Schaller (2002), among others.

tionary alternative, Phillips and Yu (2011) have $H_1 : \rho > 1$, which, with $\rho = 1 + c/k_n$, where $c > 0$, $k_n \longrightarrow \infty$ and $k_n/n \longrightarrow 0$, allows for their mildly explosive cases. Phillips and Yu (2011) argue that their tests have discriminatory power, because they are sensitive to the changes that occur when a process undergoes a change from a unit root to a mildly explosive root or vice versa. This sensitivity is much greater than in left-sided unit root tests against stationary alternatives. But this is not all. It should be added that bubbles usually collapse periodically. Therefore, standard unit root tests have limited power in detecting periodically collapsing bubbles.[9] To overcome this drawback, Phillips and Yu (2011) have suggested using the supremum of recursively determined Dickey-Fuller (*DF*) t-statistics. The estimation is intended to identify the time period where the explosive property of the bubble component becomes dominant in the price process. The test is applied sequentially on different subsamples. The first subsample contains observations from the initial sample and is then extended forward until all observations of the complete sample are included. The beginning of the bubble is estimated as the first date when the *DF* t-statistic is greater than its corresponding critical value of the right-sided unit root test. The end of the speculative bubble will be determined as the first period when the *DF* t-statistic is below the aforementioned critical value.

Formally, Phillips et al. (2011, 2012) suggest calculating a sequence of *DF* tests. Let $\hat{\rho}_\tau$ denote the OLS estimator of ρ and $\hat{\sigma}_{\rho,\tau}$ the usual estimator for the standard deviation of $\hat{\rho}_\tau$ using the subsample $\{y_1, \cdots, y_{[\tau T]}\}$. The forward recursive *DF* test of H_0 against H_1 is given by

$$\sup DF(r_0) = \sup_{r_0 \leq \tau \leq 1} DF_\tau, \tag{2.6}$$

where $DF_\tau = \frac{\hat{\rho}_\tau - 1}{\hat{\sigma}_{\rho,\tau}}$. Note that the *DF* statistic is computed for the asymmetric interval $[r_0, 1]$. In applications, r_0 will be set to start with a sample fraction of reasonable size. The limiting distribution is

$$\sup_{r_0 \leq \tau \leq 1} DF_\tau \xrightarrow{D} \sup_{r_0 \leq \tau \leq 1} \frac{\int_0^\tau W(r)dW(r)}{\int_0^\tau W(r)^2 dr}, \tag{2.7}$$

where "$\xrightarrow{D}$" denotes convergence in distribution and W is a standard Wiener process. Analo-

[9] Busetti and Taylor (2004), Kim et al. (2002) and Leybourne et al. (2006) have shown that traditional unit root tests have low power in the case of gradually changing persistence and/or the existence of persistence breaks.

gously, the augmented supADF (SADF) test can be derived. In addition, Phillips et al. (2012) have suggested employing the "generalized" supADF (GSADF) test as a dating mechanism. The GSADF diagnostic is also based on the idea of sequential right-tailed ADF tests, but the diagnostic extends the sample sequence to a more flexible range. Instead of fixing the starting point of the sample, the GSADF test changes the starting point and ending point of the sample over a feasible range of windows. Phillips et al. (2012) demonstrate that the moving sample GSADF diagnostic outperforms the SADF test based on an expanding sample size in detecting explosive behaviour in multiple bubble episodes and seldom gives false alarms, even in relatively modest sample sizes. The reason is that the GSADF test covers more subsamples of the data. In the next section of the chapter we shall apply these two bubble dating algorithms to locate periodic explosive sub-periods.[10] They also show that the diagnostics perform accurately even with relatively small sample sizes. This gives us confidence in the potential applicability of the proposed testing strategy to Chinese house price data under real-time conditions, as shown below.

2.3 Data and Estimation Results

Prior to the econometric analysis, we briefly describe the data set. Our data set for mainland China covers nationwide nominal house prices (H_t) and the price-to-rent ratio ($\frac{H_t}{R_t}$) over the period 2003Q1–2011Q4. This period coincides with China's peak phase of urbanization and the private housing market boom.[11]

Figure 2.2 documents the magnitude of the nationwide surge in Chinese house prices. At

[10]Skipping, for the sake of brevity, further technical details, the interested reader is referred to the above-mentioned papers introducing the right-tailed unit root testing strategy. A technical supplement providing a complete set of mathematical derivations of the limit theory underlying the unit root tests is available at http://sites.google.com/site/shupingshi/TN_GSADFtest.pdf?attredirects=0&d=1.

[11]Reliable Chinese house price indices are hard to come by. The official 70 cities house price index published by the Chinese National Bureau of Statistics (NBS) is mistrusted and has been widely criticized for underestimating house price inflation. Given the suspicion and criticism, the NBS suspended publication of the housing data in February 2011. See http://online.wsj.com/article/SB10001424052748703373404576147792827651116.html for more details. Therefore, we employ the house price and price-to-rent data in Igan and Loungani (2012). They pay particular attention to data coverage and computation leading to discrepancies among different data sources. Longer time series of Chinese house price data may not improve the results since China has experienced a regime shift in the housing market in the late 1990s. Indeed, until the late-1990s, the allocation of apartment units to most urban households was determined by employers, primarily government institutions and state-owned enterprises.

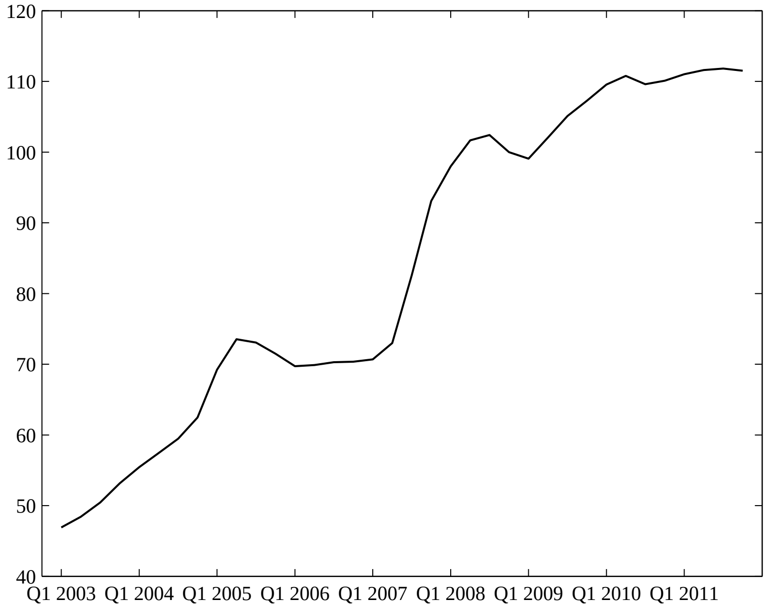


Figure 2.2: Nationwide Chinese House Price Cycles, 2003Q1-2011Q4.

Note: Seasonally adjusted Index 2008Q4=100. Source: Igan and Loungani (2012).

first glance, the plot of the time series appears to justify the expression "speculative housing bubble". Chinese house prices rose rapidly until 2005. They accelerated again sharply in 2008, fuelled by the fiscal stimulus package, low interest rates and massive credit expansion.[12] Chinese house prices soon regained a steep upward trend until mid-2010 when, against the risk of a speculative bubble in the housing market, the Chinese government announced a number of measures to cool the market. The campaign intensified in 2011. The measures included (i) increasing down payments for first-time buyers' mortgages from 20% to 30%, and for second homes from 50% to 60%; (ii) a total ban on mortgages for third home purchases; (iii) introduction of new restraints on house purchases by non-locals; (iv) introduction of new property taxes in Shanghai and Chongqing: between 0.4% and 0.6% in Shanghai, and between 0.5% and

[12]On the surface, the Chinese house price increases seem to share many of the features of the Japanese property price bubble in the 1980s. This does not in any way imply that a Chinese bubble, were it to exist, would collapse like the Japanese one. China is still years behind pre-bubble Japan and has abundant room for driving its maturing export-driven economy into one more geared towards consumption. Furthermore, Chinese banks are still majority-owned by the state and therefore policy restraints aimed at deflating bubble periods would be more effective in China than in Japan. Therefore, China is hardly a Japan in the making.

1.2% on luxury homes in Chongqing; (v) elimination of mortgage discounts for first-time home buyers; and (vi) raising of the benchmark interest rate to 6.56% in July 2011. Subsequently, the pace of house price increases began to slow.

In order to identify speculative house price bubbles, the fundamental part of house prices has to be separated from the speculative part. There are various ways to estimate the fundamental value of house prices. The asset pricing equation 2.2 suggests looking at the Chinese price-to-rent ratio as a yardstick, i.e. house price changes should be in line with rent changes, given constant interest rates. A corollary of this is that the price-to-rent ratio ($\frac{H_t}{R_t}$) should be constant over time in the absence of a speculative bubble. When house prices are low relative to rent, future increases in house prices are likely to be high. Thus, the price-to-rent ratio ($\frac{H_t}{R_t}$) can be viewed as "an indicator of valuation in the housing market" (Gallin, 2008).[13]

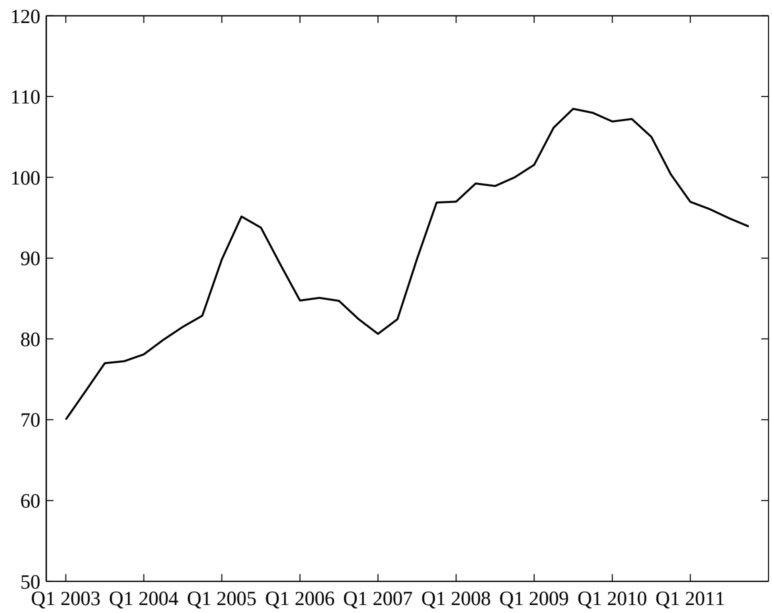


Figure 2.3: Nationwide Chinese Price-to-Rent Ratio, 2003Q1-2011Q4.

Note: Seasonally adjusted Index 2008Q4=100. Source: Igan and Loungani (2012).

Figure 2.3 shows the Chinese nationwide house price-to-rent ratio from 2003Q1 to 2011Q4. A mere look at the plot of this time series indicates that the price-to-rent ratio increased until 2010

[13]Also see Case and Shiller (2003) and Himmelberg et al. (2005).

and has decreased since. It should be noted that a rising price-to-rent ratio is only a necessary but not a sufficient condition for speculative misalignment from fundaments. Below we therefore test for significant overvaluation using the recursive testing procedure suggested by Phillips et al. (2012).

Identifying speculative bubbles is no easy task even in mature markets with long time series. In China, time series for house prices and in particular for the price-to-rent ratio are short. Phillips et al. (2012) have demonstrated that higher-frequency data significantly improves the finite sample power of recursive tests. Taking this into account, we have first generated monthly price-to-rent ratios using the proportional Denton (1971) method.[14]

Next we employ the recursive right-tailed ADF statistics to scrutinize for speculative bubbles in Chinese housing markets. For the SADF and GSADF tests, r_0 has to be chosen. If the number of observations is small, r_0 needs to be large enough to ensure there are enough observations for initial estimation. In our application, we choose $r_0 = 0.3$ and $r_0 = 0.4$, respectively.[15] The finite sample critical values are obtained via Monte Carlo simulations with 2,000 iterations. Observations above the respective critical values signal a warning to policymakers as when to start to "lean against the wind" in order to restrain undesirable and unsustainable trends. All computations were generated using a program in MATLAB.

Figures 2.4–2.7 provide an overall picture of Chinese house price valuation over the sample period under consideration. The dotted red lines in Figures 2.4–2.7 show the recursively calculated univariate backward ADF and SADF statistic sequences, respectively. The blue and green lines show the associated critical values. The graphs lend themselves to several conclusions. Firstly, the GSADF tests flag a statistically significant periodic misalignment in 2009–2010. The periodic bubble period is short but exceeds the minimum time span $\log(n)$ suggested by Phillips et al. (2012), where n is the sample size. It is noticeable that this confirms the preliminary results from glancing at Figure 2.2. Secondly, as expected, the SADF diagnostic turns

[14] The Denton procedure is a standard tool for compiling higher-frequency data. The technique generates monthly series which are both consistent with the quarterly data (i.e. the average of the monthly indices is equal to the quarterly indices) and as close as possible to the movements of a monthly reference series. The monthly house price index of the Chinese National Bureau of Statistics is used as the indicator series. The interpolation problem is nonlinear and can be solved using standard optimization procedures, as discussed by Bloem et al. (2001) and Denton (1971).

[15] In robustness checks, we used several r_0s and find that the results are not particularly sensitive to the precise choice. The qualitative results also remain unchanged when the logged price-to-rent ratio is used for the diagnostic tests.

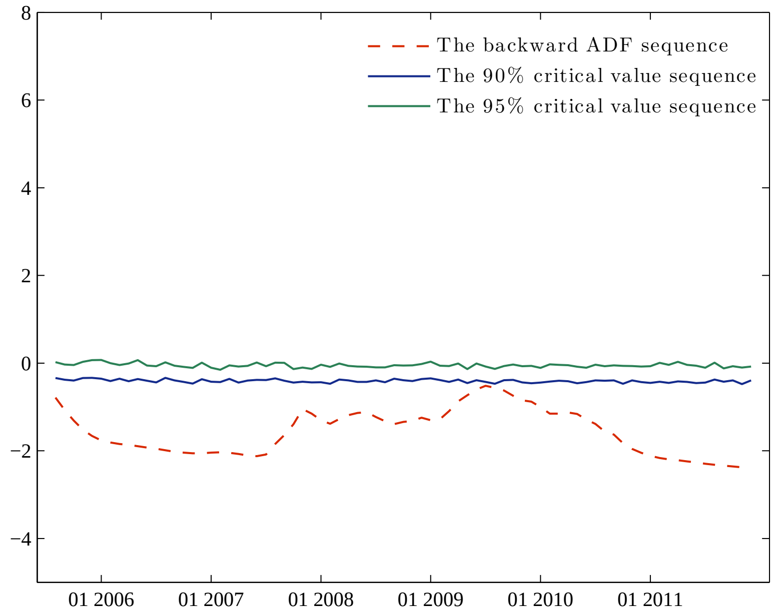


Figure 2.4: Recursive Calculation of the SADF Test for r_0=0.3.

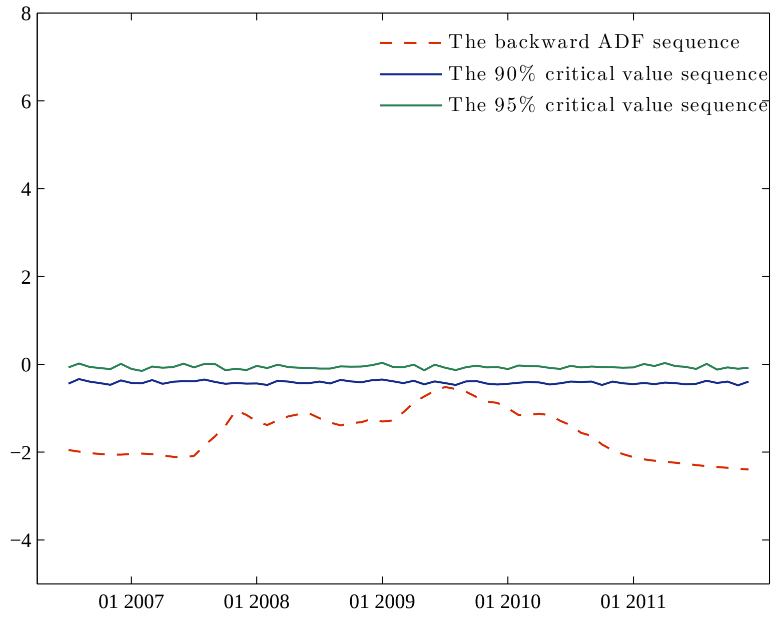


Figure 2.5: Recursive Calculation of the SADF Test for r_0=0.4.

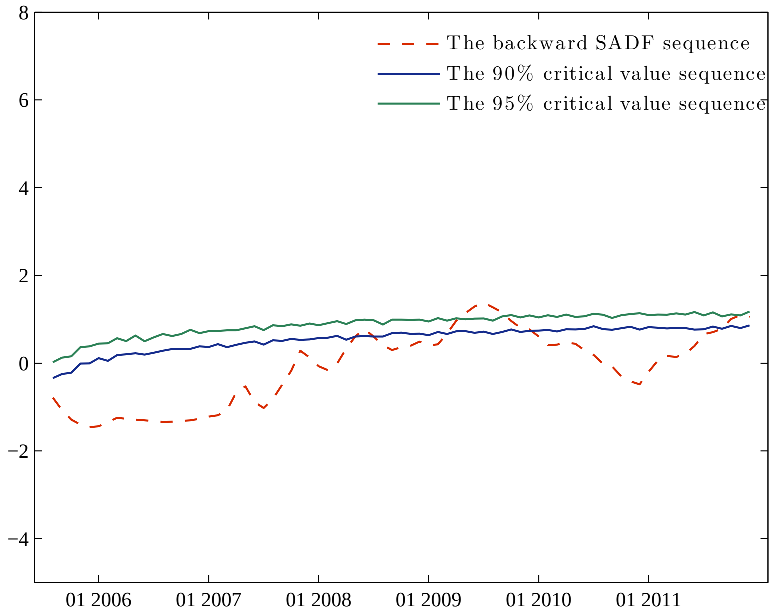


Figure 2.6: Recursive Calculation of the GSADF Test for r_0=0.3.

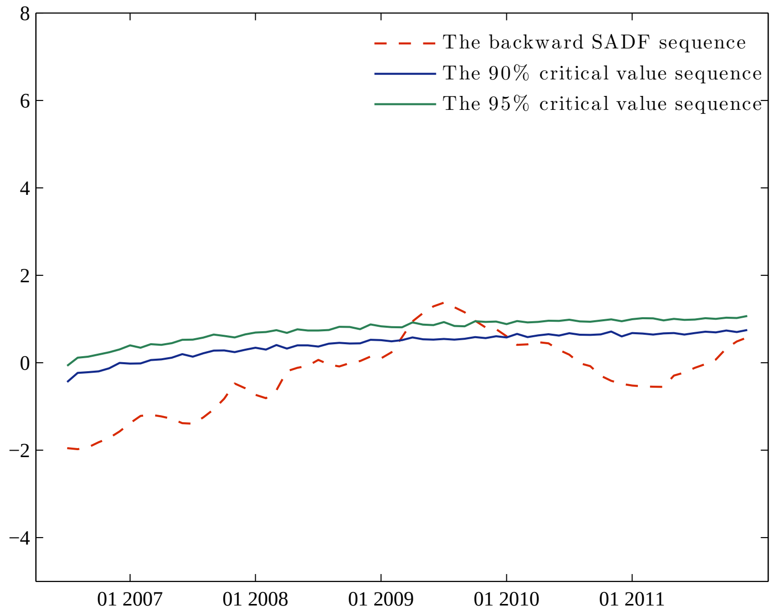


Figure 2.7: Recursive Calculation of the GSADF Test for r_0=0.4.

out to be more conservative in detecting exploding sub-periods. Thirdly, except for that, sub-period house prices were not overly and significantly disconnected from fundamentals. Thus, the administrative measures to dampen house price inflation appear to be having the desired effect. Finally, it is an encouraging sign that the testing procedure is able to give warnings even when the speculative bubble period is short-lived.

It is worth emphasizing that price-to-rent indices have obvious disadvantages and shortcomings. Certainly, it is true that the indices provide information about the dynamics of the price-to-rent ratio over time. However, they do not provide any information about the actual level of the price-to-rent ratio. Therefore, we additionally provide information about gross rental yields ($\frac{R_t}{H_t}$) across major Chinese cities and various market segments from 2005 to 2011. The gross rental yield is the rent over the course of one year, expressed as a percentage of the purchase price of the property. While this supplementary shorthand measure may not resolve all our interpretation difficulties, it may give us a better sense of where we are currently going in China. The disaggregated data also provide an important comparison with the nationwide trend and therefore round up the image.

Gross rental yields across cities have been quite heterogeneous, as is clear from the cross-city, cross-time data in Figure 2.8.[16] Although yields are correlated across most cities, aggregate Chinese house price changes clearly mask sharp regional differences. In 2005, rental yields in all categories of Beijing property were above 9%. In Shanghai, returns were lower than in Beijing, with gross rental yields ranging from 5.4% to 7%. In 2011, rental yields in Beijing were below 3%, and in Shanghai below 3.5%. The data send a clear message – during the period of study, property prices have been climbing steeply, while rents have not moved much.[17] The degree of price misalignment is particularly pronounced in the mass markets of a number of coastal cities like Beijing and Shanghai. The substantial heterogeneity in house prices and the house price-to-income ratio dynamics highlight the complexity of an appropriate policy response in situations where asset prices are not rising uniformly. The heterogeneity and idiosyncratic pattern may reflect the fact that city-level house prices include significant local variables. This is particularly true for so-called "superstar cities", where local circumstances can result in a prolonged period

[16] At the city-level, rental and price information for different market segments is even more limited and only selected annual data is available. Therefore, formal bubble tests cannot be employed.

[17] Chengdu is an exception. For reasons unknown so far, yields appear healthy there. On the other hand, this may also represent just a statistical artifact.

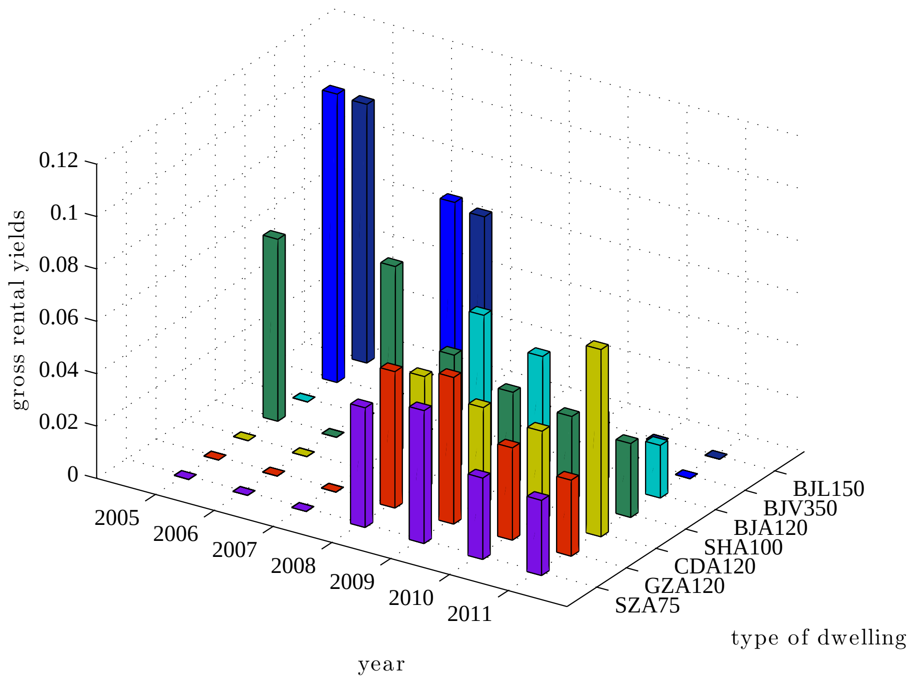


Figure 2.8: Gross rental Yields across Chinese Cities and Market Segments, 2005-2011.

Note: The yield is defined as the gross annual rental income, expressed as a percentage of property purchase price. The yields are constructed by compiling and processing transaction-level data from a variety of market sources. Only resale apartments and houses are researched. Yields for newly built properties are not included. No data is available for the year 2006. BJL150: Beijing luxury apartments 150 m2; BJV350: Beijing villas 350 m^2; BJA120: Beijing apartments 120 m^2; SHA100: Shanghai apartments 100 m^2; CDA120: Chengdu apartments 120 m^2; GZA120: Guangzhou apartments 120 m^2; SZA75: Shenzhen apartments 75 m^2. Source: Global Property Guide Research (`http://www.globalpropertyguide.com`).

of higher than average growth in house prices (see Himmelberg et al., 2005).

While there is no sign of significant nationwide overvaluation in Figures 2.4–2.7 after introduction of the cooling measures in 2010–2011, there are still signs that house prices in some coastal cities and market segments are disconnected from fundamentals. Overall, these results are consistent with the extant, rather scant empirical literature on the dynamics of Chinese city-level house prices. For example, Ahuja et al. (2010) have also concluded that, over the period 2000Q1–2009Q4, Chinese house prices were not significantly higher than would be justified by underlying fundamentals, while signs of overvaluation were present in some cities' mass-market and luxury segments. The balance of nationwide econometric and cross-city descriptive evidence points towards the conclusion that the period of market overheating cooled off at the end of 2011 but remains at a high level. But at least prices have risen so high that it is inconceivable that they will continue to rise further.

Another natural temptation is to compare the gross rental yields in China to those of other countries. This can provide a more condensed picture of the Chinese housing market. Last, but not least, we therefore provide the cross-country gross rental yields for 2011 (Figure 2.9). This may allow for a comprehensive picture and balanced assessment of the Chinese housing market.

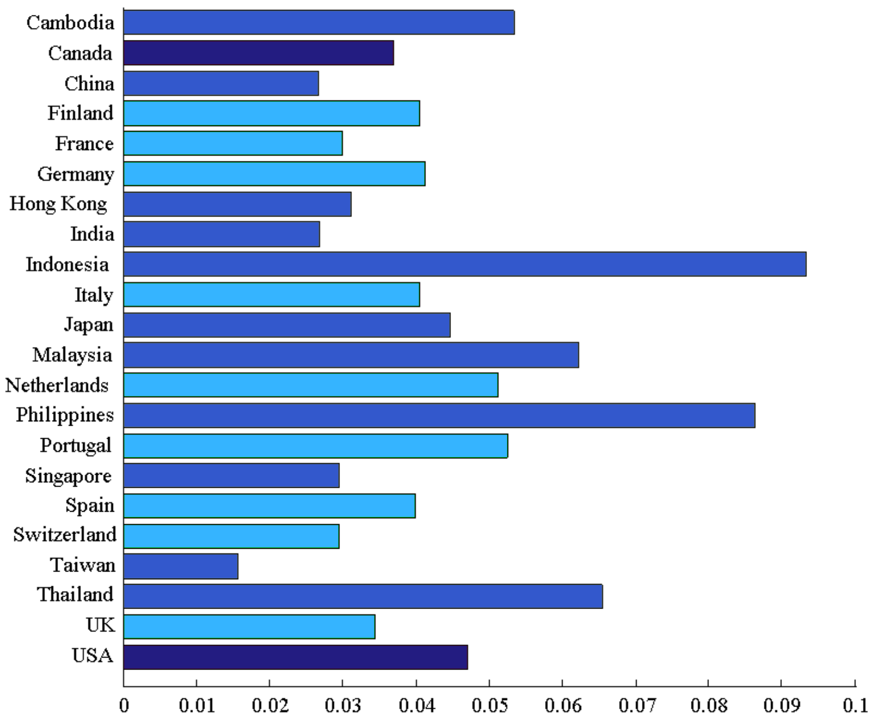


Figure 2.9: Gross rental Yields in 2011: An international Comparison.

Note: The gross annual rental income for a 100–150 m^2 apartment in a premier city location, expressed as a percentage of the purchase price. Only high-quality resale apartments and houses are included. The yields are constructed by compiling and processing transaction-level data from a variety of market sources. Source: Global Property Guide Research (`http://www.globalpropertyguide.com`).

Several descriptive results are obtained. The first thing to note is the considerable variation across countries. Yields below 3% are usually considered to be a sign of an overvalued market, leading early warning signals to flash red. By international comparison, China had rather low rental yields in 2011. The same is true for Taiwan, where yields have reached unsustainably low levels. After three years of unbroken house price rises, gross rental yields are unusually low, at an average of 2.8%. One trigger for rising Taiwanese property prices is speculation about future investment by mainland Chinese. At the other end of the scale are Indonesia and the Philippines. Despite high growth rates in recent years, the housing market in Indonesia has faltered. Some of the major factors that have made a decisive contribution to this development include high mortgage rates, high tax rates and restrictions on foreign ownership. Similarly, housing markets in the Philippines were held back by several obstacles, including high taxation, fake land titles and high transaction costs. Superficially, yields on property therefore look attractive. Property in the United States is now relatively inexpensive from an international perspective.[18] All in all, the evidence in Figure 2.9 provides a more nuanced understanding of Chinese house price developments. The evidence also indicates that in several countries the ongoing housing downturn still has further to go.

2.4 Wrapping Up: Signalling Chinese House Price Bubbles with Time Series Methods

Few areas have received the same amount of focus and scrutiny over the last couple of years as property prices. The collapse of the financial markets and the need for additional regulatory and macroprudential policies has overturned previously accepted wisdom about risk and self-regulation in a market economy. Monetary policymakers have two different strategies to deal with a possible asset price bubble: the "conventional" strategy and an "activist" strategy. A central bank following the conventional strategy does not attempt to use monetary policy to influence the speculative component of asset prices, on the assumption that it has little ability to do so and that any attempt will only result in suboptimal economic performance in the medium term. Instead, the central bank responds to asset price movements, whether driven by fundamentals or not, only to the degree that those movements have implications for future output and inflation. In contrast, an activist strategy takes extra action by tightening policy

[18] However, house prices in the US were pushed up by consumers who borrowed heavily, while China´s house prices were pushed up by high savings and a lack of alternative investment. On the other hand, this may not resolve the problem in the long-run since this is at least partially the result of distorted financial markets in China. So any liberalisation of financial markets may render high house prices unsustainable.

beyond what the conventional strategy would suggest. This requires that policymakers can identify emerging bubbles in real time with reasonable confidence.

In this chapter we have employed the newly developed testing strategy pioneered by Phillips and Yu (2011) and Phillips et al. (2012) aimed at identifying explosive bubbles in real time. We believe that this new approach to identifying growing bubbles and their collapse will make a significant impact in constructing early warning systems, and we have therefore used the method as a signpost for periodically collapsing Chinese housing bubbles.[19] The results flash a heightened probability of an emerging Chinese house price bubble in 2009–2010. During other years, the Chinese housing market does not display significant signs of unsustainable overvaluation. Another contribution of this chapter lies in its comprehensive approach. To measure and benchmark Chinese house prices, the study presents and analyses several datasets and measures of house price overvaluation. In focusing on various measures, this chapter provides empirical shape and substance to the multifaceted concept of house price bubbles. One conclusion is that the considerable house price variation across Chinese cities requires differentiated local policy responses to trigger price corrections.

[19]It is paramount to remember that we rely on limited observations. While the test results yield reasonable results, more work is needed to confirm our findings. Data availability limits the number of observations available for a more definitive evaluation. Further research with longer time series is therefore desirable to corroborate our assessment.

3

Renewed Momentum in the German Housing Market: Real-Time Monitoring of Boom vs. Bubble?[1]

3.1 Introduction

After more than a decade of stagnating or even falling house prices, the German real estate market started to surge in 2010 and house prices in larger cities have experienced large mark-ups. On the one hand, the prevailing expectations that central banks in advanced economies will not tighten monetary policy in the near future play a role in this development, as low interest rates can fuel excess borrowing and push asset prices ever higher. On the other hand, the euro crisis matters. Not only are German households acquiring more real estate, but foreigners see Germany as a safe haven. In light of increasing house prices and the previous experience of how the bursting of real-estate bubbles triggered 2007-2009 recessions in several countries, there are increasing concerns that Germany might be destined for a similar fate. Unbounded enthusiasm could be a real danger in this context. History is replete with examples of plenty of prolonged

[1] This chapter is co-authored by Michael Funke.

periods of low interest rates that encouraged speculative housing bubbles.

Prior to the global recession of 2007-2009 and the associated disruptions in financial markets, asset price bubbles were often considered as a sideshow to macroeconomic fluctuations. The global recession demonstrated painfully that this dominant pre-crisis presumption was dangerously wrong. A rapidly growing literature is now seeking to remedy this shortcoming and has begun to address this knowledge gaphead-on. In particular, Agnello and Schuknecht (2011), Claessens et al. (2009), Hirata et al. (2013) and Igan and Loungani (2012) have taken a global perspective and have provided an assessment of the linkages between house prices and real economic activity. Drehmann et al. (2012) have recently characterised empirically the financial cycle and its relationship with the business cycle. The analysis shows that the medium-term financial cycle is a different phenomenon from the business cycle. Furthermore, the length and amplitude of financial cycles have increased markedly since the mid-1980s. The IMF (2003) has documented the information content of house prices for both business cycles and systemic banking crises with serious macroeconomic dislocations. These studies also discuss the surprisingly high synchronization of house price downturns as observed during the global financial crises, which is likely to have exacerbated the deep recession.[2]

At the same time, a new empirical literature on early warning indicators has emerged.[3] This literature reflects a desire to better identify speculative bubbles in real time. Since boom-and-bust cycles possibly lead to serious serious financial and macroeconomic strains, central banks have reconsidered their monetary policy strategies with regard to asset bubbles. Prior to the global recession 2007-2009, the European Central Bank (2002) had expressed doubts about the ability to detect bubbles with a sufficient degree of certainty. A first change of course occurred in 2005, when the European Central Bank (2005) argued that, firstly, there are a number of tools to detect asset bubbles and, secondly, emerging asset bubbles should be taken into consideration when making interest rate decisions. In the light of the global recession the European Central Bank (2010) has finally acknowledged that the case for pre-emptive monetary policy responses to emerging asset bubbles has been strengthened. In light of the recent momentum in

[2]Another relevant strand of literature concerns the role of housing within dynamic stochastic general equilibrium (DSGE) models. See, for example, Funke and Paetz (2013), Iacoviello and Minetti (2008), Iacoviello and Neri (2010). This literature is beyond the scope of the brief review presented in this section.

[3]See, for example, Alessi and Detken (2011), Crespo Cuaresma (2010), Gerdesmeier et al. (2012) and the literature cited therein.

German house prices, the question of house price bubbles is also a matter of concern for the Deutsche Bundesbank. In the same spirit as the European Central Bank, the Deutsche Bundesbank (2012) has emphasized that a combination of low interest rates and high liquidity may pose a considerable danger to financial stability. Furthermore, easy monetary policy and especially unconventional monetary policy that lowers interest rates all along the yield curve facilitate low risk premiums. Therefore, monetary policymakers should deploy micro- and macro-prudential policy tools to cool down housing markets in case of emerging price escalations.

Against this backdrop, our study complements and extends the existing literature in several ways. In particular, we employ a new statistical test pioneered by Phillips and Yu (2011) and Phillips et al. (2012) and up-to-date house prices data to assess the renewed momentum in the German housing market. We find no evidence of an emerging speculative housing bubble in Germany at the present moment. It goes without saying that this is just a snapshot of the current situation and no clean bill of health can be given for the future.

The structure of this chapter is as follows. Section 3.2 reviews some theoretical and econometric issues related to housing valuation and bubble identification. In section 3.3, we introduce the house price database. In section 3.4, we proceed by discussing the results of the real-time econometric diagnostics. The final section concludes with a summary and suggestions for further research.

3.2 Modelling and Testing Strategy

In the first stage, we need to define bubble periods. Based on this, we can then identify inflated house prices and bubble periods. The classical literature on rational bubbles derives conditions under which bubbles can occur when all agents are perfectly rational. Classical rational house price bubbles can arise because of the indeterminate aspect of solutions to rational expectations models. The house price that agents are prepared to pay today depends on the expected house price at some point in the future. But the latter depends on the expected house price even further in the future. The resulting process governing house prices does not pin down a unique house price level unless, somewhat arbitrarily, a transversality condition has to be imposed to obtain a unique solution. However, in general, the possibility that house prices may systematically deviate from their fundamental value cannot be ruled out. Even if risk-neutral agents are perfectly rational, the actual house price may contain a bubble element, and thus there can be a divergence between the house price and its fundamental value. The resulting real estate bubble is an upward house price movement over an extended range that then suddenly collapses.

Our goal is to ascertain how house prices evolve over time, given the behaviour of fundamentals. Time is discrete. In the modelling framework, fundamental house prices H_t can be represented as follows:

$$H_t = \left(\frac{1}{1+r}\right)E_t(R_t + H_{t+1}), \tag{3.1}$$

where E_t is the expectations operator, R_t is the rental value at time t, and r is the discount rate. To solve the model, we need to eliminate the term involving the expectation of the future value of the endogenous variable. It is straightforward to show that the fundamental house price H_t^F can be solved under rational expectations by repeated forward substitution. This implies

$$H_t^F = \sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j E_t(R_{t+1}). \tag{3.2}$$

The logic of equation 3.2 is that house market prices contain expectations of future rents. No specific assumptions are made about the process followed by R_t. The rational bubble components B_t follow

$$B_t = \left(\frac{1}{1+r}\right)E_t(B_{t+1}). \tag{3.3}$$

Solving for H_t finally yields

$$H_t = H_t^F + B_t. \tag{3.4}$$

Equation 3.4 breaks up house prices into a "fundamental" and a "bubble" component. Without a bubble, house prices equal the fundamental value H_t^F. Under bubble conditions house prices may show an explosive behaviour inherent in B_t. If B_t is strictly positive, this builds the stage for speculative investor behaviour: a rational investor is willing to buy an "overpriced" house, since he/she believes that future price increases will sufficiently compensate him/her for both the extra payment he/she has to make and the risk of the bubble bursting. In that sense, the house price bubble is a self-fulfilling prophesy. Eventually the bubble implodes, house prices fall with a sharp correction, and deleveraging occurs. In recent years, a new generation of behavioural models capable of generating bubbles has emerged. This literature is quite broad, so we will touch on only a few important papers here. The unifying feature behind this class of model is bounded rationality for at least one group of agents. In the behavioural models, a bubble may arise when asset prices overreact to a potentially informative signal about fundamentals. Behavioural models can be classified into three categories. Firstly, differences of opinion and short sale constraints may generate asset bubbles. Scheinkman and Xiong (2003) provide a dynamic model, in which optimistic investors exhibit bounded rationality and fail to take into account that other agents in the economy may have more pessimistic views about an asset but cannot sell that asset due to short sale constraints. Secondly, feedback trading mechanisms may allow bubbles to grow for a period of time before they eventually collapse. An example of a model that contains feedback traders is Hong and Stein (1999). The model includes two groups of traders - news watchers and feedback traders. Neither group is completely rational. News watchers do not condition on past prices. On the other hand, feedback traders do not observe the signals about the fundamentals and condition their trading decisions entirely on past asset price changes. Thirdly, biased self-attribution may lead to asset price bubbles. The term self-attribution was coined by research emanating from the field of psychology. Biased self-attribution leads agents to take into account signals that confirm their beliefs and dismiss as noise signals that contradict their beliefs. Daniel et al. (1998) have formulated a comprehensive model with noisy signals and agents suffering from biased self-attribution. As a result they grow overconfident, which leads to the formation of a bubble.[4]

Next we discuss how the theoretical frameworks can be linked to an econometric testing strategy. In the econometric literature, identifying a emerging bubble in real time has proved challenging and remains an elusive task. In addition, subtle econometric problems result from finite samples. Standard unit root and cointegration tests may be able to detect one-off exploding speculative bubbles, but are unlikely to detect periodically collapsing bubbles.[5] The reason

[4] A frequent argument against behavioural models is that the presence of rational investors in the market should stabilise prices. Remarkably, the models of DeLong et al. (1990) and Abreu and Brunnermeier (2003) show that under certain conditions rational arbitrageurs may even amplify rather than eliminate the asset mispricing.

[5] Figure 2.1 in chapter 2 illustrates, at the risk of oversimplification, the taxonomy and conceptual differences

is that traditional unit root tests are not well equipped to handle changes from $I(0)$ to $I(1)$ and back to $I(0)$. This makes detection by cointegration techniques all the more difficult, due to bias and kurtosis (Evans, 1991).[6]

A nuanced and persuasive approach to identification and dating multiple bubbles in real time has recently been pioneered by Phillips and Yu (2011) and Phillips et al. (2012).[7] The idea is to spot speculative bubbles as they emerge, not just after they have collapsed. Their point of departure is the observation that the explosive property of bubbles is very different from random walk behaviour. Correspondingly, they have developed a new recursive econometric methodology interpreting mildly explosive unit roots as a hint for bubbles. If we consider the typical difference of stationary vs trend stationary testing procedures for a unit root, we usually restrict our attention to regions of "no more than" a unit root process, i.e. an autoregressive process where $\rho \leq 1$. In contrast, Phillips and Yu (2011) model mildly explosive behaviour by an autoregressive process with a root ρ that exceeds unity but is still in the neighbourhood of unity. The basic idea of their approach is to recursively calculate right-sided unit root tests to assess evidence for mildly explosive behaviour in the data. The test is a right-sided test and therefore differs from the usual left-sided tests for stationarity. More specifically, consider the following autoregressive specification estimated by recursive least squares:

$$x_t = \mu + \rho x_{t-1} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t \sim \text{i.i.d}N(0, \sigma^2). \tag{3.5}$$

The usual $H_0 : \rho = 1$ applies, but unlike the left-sided tests which have relevance for a stationary alternative, Phillips and Yu (2011) have $H_1 : \rho > 1$, which, with $\rho = 1 + c/k_n$, where $c > 0$, $k_n \longrightarrow \infty$ and $k_n/n \longrightarrow 0$, allows for their mildly explosive cases.[8] Phillips and Yu (2011) argue that their tests have discriminatory power, because they are sensitive to the changes that occur when a process undergoes a change from a unit root to a mildly explosive root or vice versa. This sensitivity is much greater than in left-sided unit root tests against stationary alter-

between a one-off bubble versus periodically collapsing bubbles.

[6]For a survey of traditional econometric bubble tests, see Gürkaynak (2008).

[7]The diagnostic for multiple speculative bubbles modifies a previous method for identifying one-off bubbles suggested in Phillips et al. (2011). A different class of tests for identifying periodically collapsing bubbles based on Markov-switching models has been explored in Funke et al. (1994) and van Norden and Schaller (2002), among others.

[8]The H_1 hypothesis is motivated by the theory of rational asset bubbles, which claims that asset prices should be explosive in the presence of an asset bubble. See Diba and Grossman (1987, 1988).

natives. But this is not all. It should be added that bubbles usually collapse periodically. Therefore, standard unit root tests have limited power in detecting periodically collapsing bubbles.[9] To overcome this drawback, Phillips et al. (2012) have suggested using the generalized supremum of recursively determined Dickey-Fuller (*DF*) t-statistics. The estimation is intended to identify the time period where the explosive property of the bubble component becomes dominant in the price process. The test is applied sequentially on different subsamples. The first subsample contains observations from the initial sample and is then extended forward until all observations of the complete sample are included. The beginning of the bubble is estimated as the first date when the *DF* t-statistic is greater than its corresponding critical value of the right-sided unit root test. The end of the speculative bubble will be determined as the first period when the *DF* t-statistic is below the aforementioned critical value. In other words, as long as the statistic has crossed the critical values, a bubble is deemed to be imminent.

Formally, Phillips et al. (2011) and Phillips et al. (2012) suggest calculating a sequence of *DF* tests. Let $\hat{\rho}_\tau$ denote the OLS estimator of ρ and $\hat{\sigma}_{\rho,\tau}$ the usual estimator for the standard deviation of $\hat{\rho}_\tau$ using the subsample $\{\mathrm{y}_1, \cdots, \mathrm{y}_{[\tau \mathrm{T}]}\}$. The forward recursive *DF* test of H_0 against H_1 is given by

$$\sup DF(\mathrm{r}_0) = \sup_{\mathrm{r}_0 \leq \tau \leq 1} DF_\tau, \tag{3.6}$$

where $DF_\tau = \frac{\hat{\rho}_\tau - 1}{\hat{\sigma}_{\rho,\tau}}$. Note that the *DF* statistic is computed for the asymmetric interval $[\mathrm{r}_0, 1]$. In applications, r_0 will be set to start with a sample fraction of reasonable size. The limiting distribution is

$$\sup_{\mathrm{r}_0 \leq \tau \leq 1} DF_\tau \xrightarrow{\mathrm{D}} \sup_{\mathrm{r}_0 \leq \tau \leq 1} \frac{\int_0^\tau \mathrm{W}(\mathrm{r})\mathrm{d}\mathrm{W}(\mathrm{r})}{\int_0^\tau \mathrm{W}(\mathrm{r})^2 \mathrm{d}\mathrm{r}}, \tag{3.7}$$

where "$\xrightarrow{\mathrm{D}}$" denotes convergence in distribution and W is a standard Wiener process. Analogously, the augmented supADF (SADF) test can be derived. Thereby, the optimal lag length of the AR(k)-process is chosen using the Akaike information criterion. In addition, Phillips et al. (2012) have suggested employing the "generalized" supADF (GSADF) test as a dating

[9]Busetti and Taylor (2004), Kim et al. (2002) and Leybourne et al. (2006) have shown that traditional unit root tests have low power in the case of gradually changing persistence and/or the existence of persistence breaks.

mechanism. The GSADF diagnostic is also based on the idea of sequential right-tailed ADF tests, but the diagnostic extends the sample sequence to a more flexible range. Instead of fixing the starting point of the sample, the GSADF test changes the starting point and ending point of the sample over a feasible range of windows. In other words, it calculates the right-tailed DF statistic in a more flexible recursive manner. In particular, it varies not only the number of observations but also varies the initial observation of each regression. The supDF statistic is then used to pinpoint the presence of periodic bubbles. The supDF statistic is obtained by taking the supremum twice with respect to the fractional window size of the regression and the ending fraction of the sample. In order to identify the beginning and end dates of a housing bubble, the supDF statistic can then be compared with the corresponding critical value. Phillips et al. (2012) demonstrate that the moving sample GSADF diagnostic outperforms the SADF test based on an expanding sample size in detecting explosive behaviour in multiple bubble episodes and seldom gives false alarms, even in relatively modest sample sizes. The reason is that the GSADF test covers more subsamples of the data.[10] For these reasons the continuous scale GSADF test becomes the method of choice in our application and we shall apply the GSADF test to monitor periodic explosive sub-periods under real-time conditions, as shown below.

3.3 The Dynamics of German House Prices vis-à-vis other OECD Countries

The section begins by presenting the most recent house price surge in Germany in the context of the experiences of other OECD countries.[11] A graphical tool that is very helpful in highlighting recent house price developments is a 3-dimensional scatter plot of house price developments across OECD countries for 2011, 2012 and 2013.

Figure 3.1 mirrors the experiences of various economies in these years. The following stylised facts are noteworthy. First, within the OECD countries there are large divergences. Cases of

[10] In the interests of brevity, further technical details are not presented here. The interested reader is referred to the above-mentioned papers introducing the right-tailed unit root testing strategy. A technical supplement providing a complete set of mathematical derivations of the limit theory underlying the unit root tests is available at `http://sites.google.com/site/shupingshi/TN_GSADFtest.pdf?attredirects=0&d=1`.

[11] The seasonally-adjusted quarterly house price dataset employed in this chapter stems from the Organization for Economic Cooperation and Development (OECD) which is a widely watched multi-country house price database.

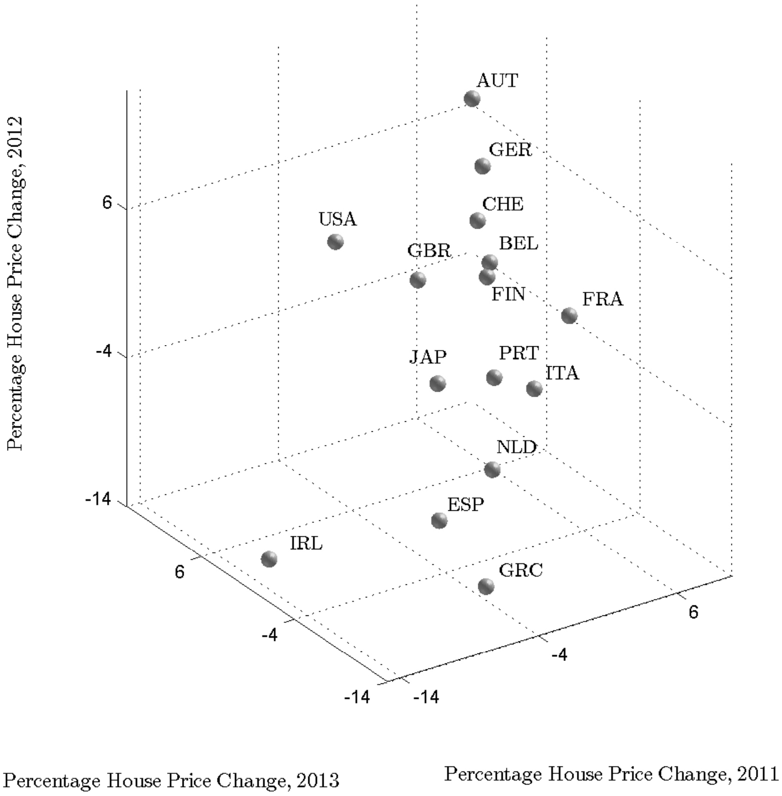


Figure 3.1: Recent House Price Changes across OECD Countries, 2011-2013.

rapidly rising house prices co-exist along with cases of constant or even falling house prices. Housing markets are depressed in southern Euroland, notably in Greece, Portugal and Spain. House prices are also falling fast in Ireland and the Netherlands. This contrasting performance of housing markets reflects the broader trend towards a two-speed Euro area. Second, in several countries including Germany a strong positive house price dynamic has prevailed over the period 2011–2013. Several mechanisms are at work. The renewed momentum in the German housing market was triggered by positive prospects for German GDP growth and employment as well as historically low mortgage financing rates.[12] Furthermore, the Euro crisis triggered an international flight to attractive safe assets. It is for these reasons that lingering worries about a

[12] We use the term momentum in a purely time series context. In finance momentum also has a cross-sectional notion, denoting the fact that if some assets exhibit higher returns than others at time t, they will continue to exhibit higher returns than the other assets in the future,see for example, Jegadeesh and Titman (1993).

German house price bubble have emerged.[13] Third, as in Germany house prices have climbed towards new heights in Austria and Switzerland.[14]

All in all, one can conclude that Germany is one of a few countries constituting special cases. Of course, strong house price increases in a few years are not necessarily evidence of an overvaluation. To address this issue, one has to put the current period of house price increases into historical perspective. Furthermore, it is necessary to relate house prices to their putative underlying determinants. To this end, Figure 3.2 and 3.3 present seasonally adjusted quarterly time series for German nominal and real house prices and the associated price-to-rent ratio for 1971Q1–2013Q4, respectively. Over the last 30 years, nominal house prices in Germany have been growing rather moderately, whereas real house prices have been stagnating or even declining. German house prices-both in nominal and real terms-have only started to rise since 2010.[15]

Consequently, German house prices have been moving in opposite direction to those in other countries: while in the majority of OECD countries the early 2000s had been characterized by a strong house price increase (especially, in Ireland, Spain, the Netherlands and the UK), which culminated 2007-2008 in a spectacular burst of speculative house price bubbles, starting from 1995 German house prices have been going down and have only recently recovered from their protracted decline. Another summary measure used to get an indication of over

[13]The Deutsche Bundesbank (2013) has pointed out that house prices in German cities have risen so strongly since 2010 that a possible overvaluation cannot be ruled out any more. The IMF has also warned that "loose liquidity conditions in the banking sector may lead to excessive asset price increases" in Germany (IMF, 2012, p. 39).

[14]It is therefore not surprising that the Austrian and Swiss housing markets are also under close surveillance. The overall assessment of the market development is that the Swiss house price level clearly lies in the risk zone. For example, The *UBS Swiss Real Estate Bubble Index 2014Q1* indicates a clear correction potential. The overall assessment is that the risks of high prices triggering a substantial subsequent price correction are high. See http://www.ubs.com/global/en/wealth_management/wealth_management_research/bubble_index.html. The Austrian National Bank has recently diagnosed an increasing degree of overvaluation in Austrian property prices (by 20% in the second quarter of 2013). See http://www.oenb.at/dms/oenb/Publikationen/Volkswirtschaft/MOP-GEWI/2013/Monetary-Policy-and-the-Economy-Q4-13/chapters/mop_2013_q4_analyses2.pdf.

[15]It is well known that house price developments are uneven. At present, Germany experiences a wide range of appreciation in house prices, with house prices in the largest cities increasing at a faster pace. Therefore, one might argue that closer inspection should be placed on city-level house price developments. Yet, this argument is not very conclusive. This is because macro-prudential policy measures would have nationwide effects in all geographic areas of the country, not just in those areas where house prices are rising rapidly. Therefore, a widely held view is that macro-prudential and monetary policies should focus only on aggregate economic conditions because they cannot control or target the conditions of particular geographic regions.

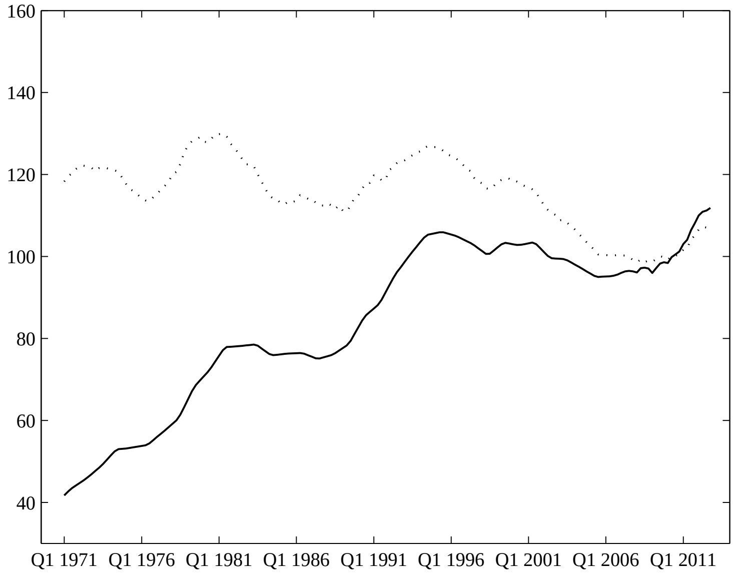


Figure 3.2: German Nominal and Real House Prices, 1971Q1-2013Q4, Indices 2010=100.

Note: The solid (dashed) line represents the seasonally-adjusted quarterly nominal (real) house price index. Real house prices are deflated by the CPI.

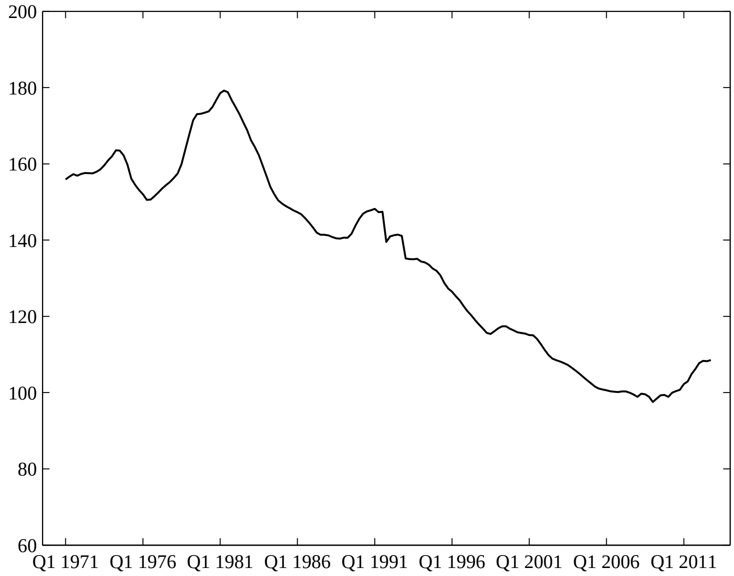


Figure 3.3: German Price-To-Rent-Ratio, 1971Q1-2013Q4, Index 2010=100.

or undervaluation is the price-to-rent ratio (the nominal house price index divided by the rent component of the consumer price index). This measure, which is akin to a price-to-dividend ratio in the stock market, could be interpreted as the cost of owning versus renting a house. When house prices are too high relative to rents, potential buyers find it more advantageous to rent, which should in turn exert downward pressure on house prices. Unlike in many other countries, the price-to-rent ratio in Germany steadily declined until 2010 when the ratio began to rebound.

What does this mean for macro-prudential market surveillance? Systemic risk in the housing market has to be addressed preemptively at an early stage of the bubble. However, preemption is difficult in the context of tail events that are experienced after long time intervals of moderate house price changes during which public memory of past asset price bubbles has faded. In the next section of the chapter we shall implement the recursive GSADF bubble dating algorithm outlined above to monitor periodic explosive periods in real time.

3.4 Real-time Monitoring of periodically Collapsing Bubbles

Could Germany be heading for a housing bubble? In order to identify speculative house price bubbles, the fundamental part of house prices has to be separated from the speculative part. There are various ways to estimate the fundamental value of house prices. The asset pricing equation 3.2 suggests looking at the German price-to-rent ratio as a yardstick, i.e. house price changes should be in line with rental changes, given constant interest rates. A corollary of this is that the price-to-rent ratio should be constant over time in the absence of a speculative bubble. When house prices are low relative to rent, future increases in house prices are likely to be high. Thus, the price-to-rent ratio can be viewed as "an indicator of valuation in the housing market" (Gallin, 2008, p. 635). In the following, we will therefore apply the real-time dating method to the price-to-rent ratio behaviour to detect emerging bubbles using quarterly data from 1971Q1 to 2013Q4. A delicate point of the procedure is the choice of the fractional window size of the regression. Suppose the minimum number of observation used in any regression is r_0T, for some fraction $r_0 \in (0,1)$. So far, no automatic algorithm for the selection of r_0 is available. In our application, we choose r_0 =0.4. Robustness testing indicates that the pictures painted

Price-to-Rent Ratio	ESP	GBR	GER	IRL	NLD	NZL	USA
				Summary Statistics			
Sample Size	172	172	172	172	172	172	172
Min	24.66	48.05	97.53	23.18	45.33	39.15	88.55
Date(min)	1971Q1	1971Q1	2009Q1	1973Q2	1985Q3	1971Q1	1997Q1
Max	122.22	112.62	179.22	151.66	110.32	111.74	127.28
Date(max)	2006Q4	2007Q4	1981Q2	2004Q3	2008Q3	2007Q2	2006Q1
				Test Statistics			
GSADF	4.13	3.31	1.63	8.41	10.08	9.13	12.05
				Finite Sample Critical Values			
	90%		95%		99%		
	1.15		1.42		1.99		

Table 3.1: Summary Statistics and the GSADF Statistic, 1971Q1-2013Q4.

Note: Critical values of GSADF test are obtained from 2,000 Monte Carlo simulations with a sample size of 172.

by Figure 3.4-3.10 below do not change for changes in r_0.[16] The beginning of the bubble is estimated as the first date when the backward SADF statistic is greater than its corresponding critical value. The end of the speculative bubble will be determined as the first period when the backward SADF statistic is below the aforementioned critical value. The finite sample critical values are obtained via Monte Carlo simulations with 2,000 iterations. These simulations incorporate the sampling uncertainty of the data generating process. We rely on the critical values to determine the optimal thresholds. All calculations have been executed in the MATLAB programming environment.

Table 3.1 reports some summary descriptive statistics for the price-to-rent ratio, including sample size, sample minimum, date of the minimum, sample maximum, date of the maximum, as well as the GSADF statistic based on the entire sample for the seven OECD countries considered. Figure 3.4 provides the real-time house price bubble barometer for Germany. The

[16] 172 observations and r_0 =0.4 yield a minimum window size of n =68. Then employing the algorithms, we obtain the backward SADF sequence from 1987Q3 onwards. The choice of r_0 may also be thought of as a trade-off between efficiency and robustness.

dashed red line shows the recursively calculated GSADF statistic sequence, along with the associated 95 percent (green line) and 99 percent (blue line) critical values, respectively. The black solid line gives the real house price index.

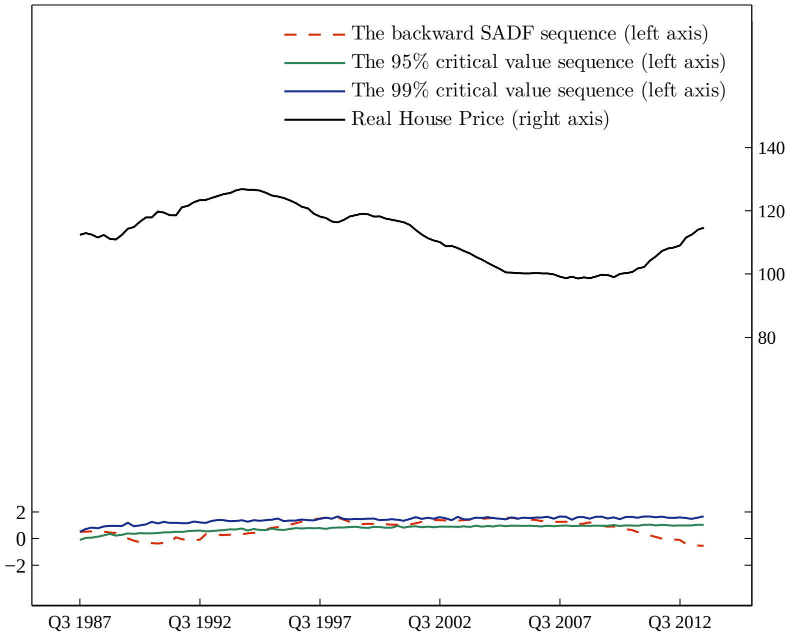


Figure 3.4: Recursive Calculation of the GSADF Test for Germany.

At first glance it turns out that the continuously evolving GSADF statistics signal no statistically significant periodic misalignment at the 1 percent significance level. In other words, German house prices were out of the significant danger zone. It is noticeable that this confirms the preliminary results illustrated in Figure 3.3.[17]

But things are not that simple. Unfortunately, early warning indicators don´t "make" definite diagnoses; they supplement a careful housing market monitoring and reduce the level of monitoring uncertainty. While after the global recession 2007-2009 real-time warning systems of housing bubbles were a much sought-after diagnostic tool, there is also a lot of scepticism on the ability to monitor housing crises or, more generally, any type of financial crises in real time. This scepticism stems from the alleged poor out-of-sample performance of many early warn-

[17] At the very most, the procedure flashed some borderline red "flags" in the mid 1990s which coincides with the concurrent increase in house prices.

ing models. Diagnostics are rarely 100 percent accurate, so false positives and false negatives can occur. Notwithstanding the sophistication of the statistical toolbox described above, any proposed real-time warning indicator is certain to face challenges in generating "misses" rather than "hits". It is therefore an open question whether the line of enquiry presented above proves empirically fruitful. A reliable real-time warning indicator would correctly call all bubbles and would not issue bubble announcements unnecessarily. Erroneous misses represent a failure to call a bubble (false negative type I error), while erroneous hits generate a false alarm (false positive type II error). It should be borne in mind that there is an inherent trade-off between type I and type II errors which are both functions of the chosen significance level. Changing the significance level to allow more housing bubbles to be picked up necessarily raises the likelihood of false bubble alarms. Traditionally, monetary policymakers tended to have a stronger preference for missing crises than to act on noisy signals. The global financial crisis 2007-2009 may have changed that. In other words, policymaker concerned with avoiding housing bubbles may now choose to minimise type I errors even if this entails unnecessary macro-prudential policy intervention. One rationale behind this could be that monetary policymakers are willing to take a "bubble insurance" and to accept a possible false alarm rather than be taken by surprise by a financial crisis. In other words, since the global financial crisis a gradual policy change from a "benign neglect" towards a "leaning against the wind" strategy has occurred. This shift of policy implies that now, more than ever, monetary policymakers are willing to dampen asset bubbles at the early stage of their formation.[18]

One simple way of assessing the genuine validity and reliability of the univariate screening toolkit is to calculate the statistics across a range of countries known to have experienced boom/bust episodes in the global recession 2007-2009.[19] In defence of our real-time warning

[18]Given the difficulties of detecting emerging housing bubbles in real time, the situation policymakers are facing is one of Knightian uncertainty. The associated question on the optimal dynamic path of monetary policy is of great interest, but lies beyond the scope of this chapter. Agur and Demertzis (2013) have recently shown that financial stability objectives make optimal monetary policy more aggressive, i.e. monetary policy tightens as soon as bank risk profiles increase. In other words, the optimal approach to dealing with unknown unknowns is to move away from the danger zone. For an axiomatic foundation of Knightian uncertainty, see Gilboa and Schmeidler (1989).

[19]In most countries only one (most recent) house price boom-bust-cycle can be analysed. Thus although the sample is long enough for sound econometric analysis, the informational content along the time dimension is selective. However, it is reassuring that the indicator matches the two turning points for housing busts in Spain in 1991Q4 and the UK in 1989Q3 documented in IMF (2003), p. 91.

signal we have therefore also calculated the test statistics for Ireland, Spain, the Netherland, the UK, the U.S., and New Zealand. This allows one to determine the accuracy of the indicator, i.e. the cross-country comparison provides a rough indication of the type I and type II error rates of our real-time monitoring toolkit. It may also help to dispel misconceptions that people have about early warning indicators. Again we have applied the real-time dating method to the price-to-rent ratio behaviour to detect emerging bubbles using quarterly data from 1971Q1 to 2013Q4 (left axis). In addition, the real house price indices are also plotted (black solid lines; right axis). The results of our screening indicator´s ability to correctly identify bubble periods are available in Figure 3.5–3.10 below.

Casting the net more widely for illustrative purposes, and looking across several "housing bubble countries", the following results warrant attention. The visual inspection of Figure 3.5–3.10 shows the fundamental suitability of the GSADF house price bubble early warning indicator. Note that despite the simple methodology employed the real-time predictive content is remarkably good and delivers a cohesive picture. In all countries the statistic signalled the build-up of risk and forthcoming trouble in real time with fairly good accuracy. This early warning in all countries leads one to reject the existence of type I error. On the other hand, the indicator is apparently fraught with type II errors. Examples are Ireland, the Netherland and New Zealand, where the signals flashed at the end of the 1990s and/or the beginning of the year 2002 but these warning signs did not culminate into bursting bubble until 2007-2008. Therefore a country may be vulnerable in the sense that the GSADF statistic is signaling trouble, yet a bursting bubble may be averted through good luck and/or good policies. On the other hand, synchronized house price shocks across countries may reinforce each other and may lead to a significant increase in the probability of a bursting housing bubble in one country, conditional on a bursting housing bubble occurring in another country and exposure to the foreign cycle. Finally, it should be noted that the probability of a crisis typically increases the larger the house price increase and the longer the duration of the boom is. This mechanism linking asset booms to crisis is clearly visible for the U.S. indicator in Figure 3.9.

To summarize, the flag-raising GSADF statistic in Figure 3.9 indicates that the synchronized global crisis 2007-2009 originated in the U.S. with the unravelling of the subprime U.S. mortgage market and has quickly spread to the European countries, due to asset price linkages and in particular the process of securitization and reinsurance in the derivatives market across banks

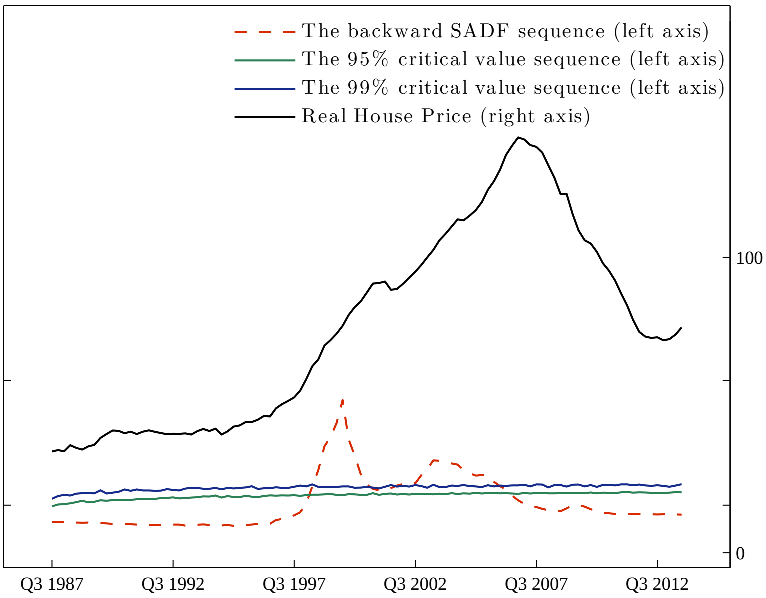


Figure 3.5: Recursive Calculation of the GSADF Test for Ireland.

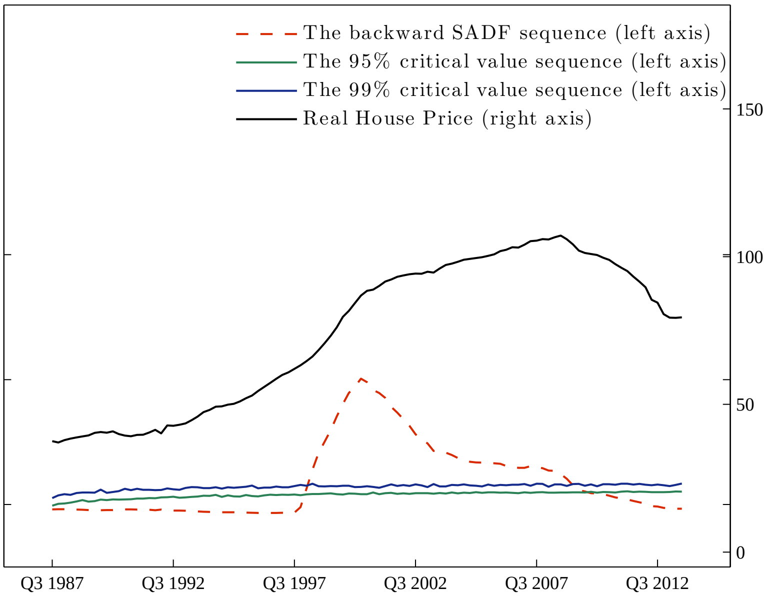


Figure 3.6: Recursive Calculation of the GSADF Test for the Netherlands.

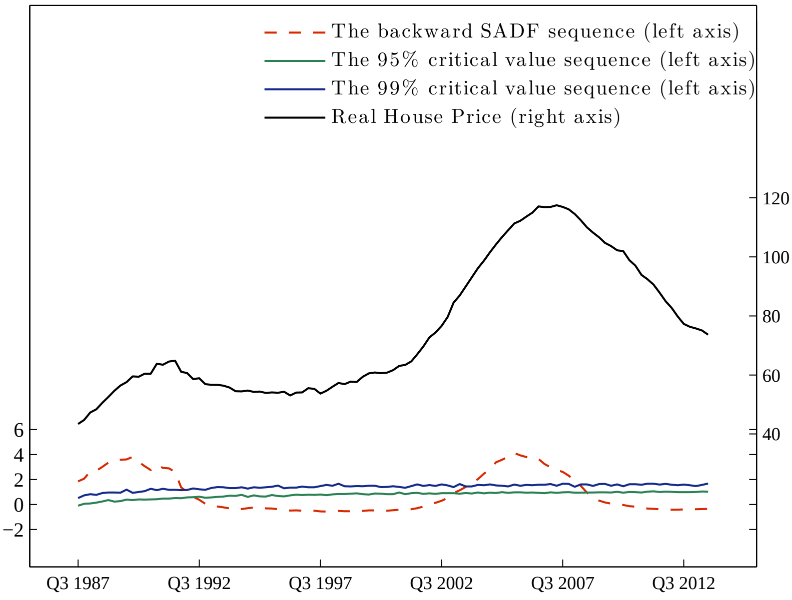


Figure 3.7: Recursive Calculation of the GSADF Test for Spain.

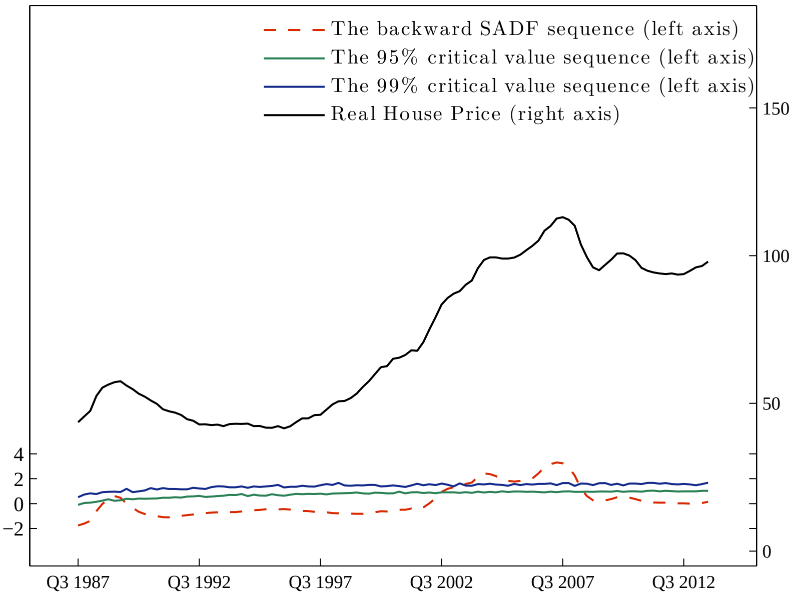


Figure 3.8: Recursive Calculation of the GSADF Test for the UK.

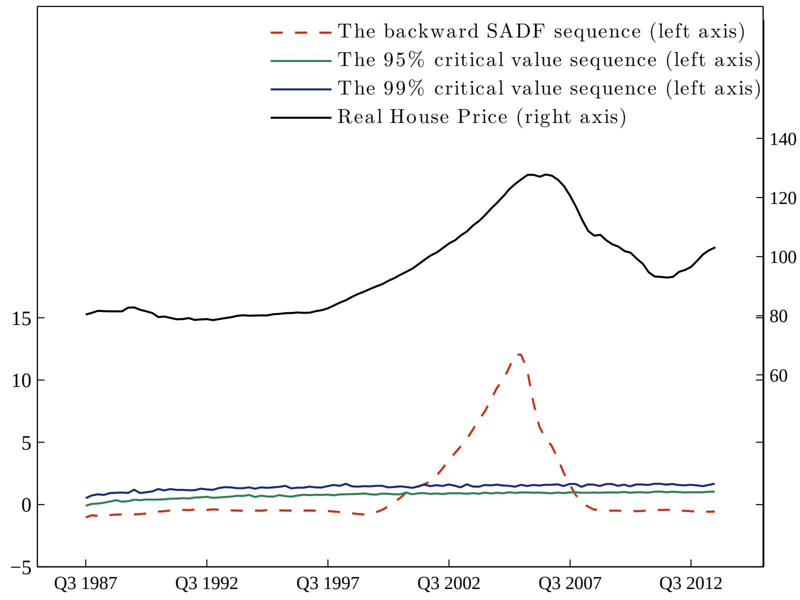


Figure 3.9: Recursive Calculation of the GSADF Test for the US.

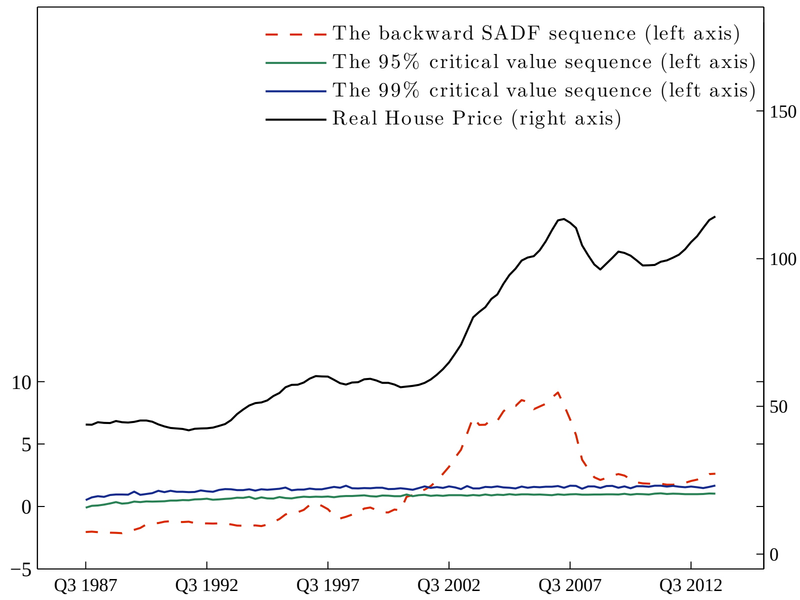


Figure 3.10: Recursive Calculation of the GSADF Test for New Zealand.

worldwide. This has triggered credit crunches and consequent economic crises in various advanced countries. In addition, informational cascades and herding by agents, unregulated off-balance sheet vehicles and/or correlated risk premiums across countries have also transmitted the U.S. shock to other countries. This shift-contagion has led to the global recession 2007-2009.

Overall, the evidence in Figure 3.5–3.10 delivers timely warnings of underlying misalignments, vulnerabilities, and tail risks that predisposed the international housing markets to the crisis 2007-2009. This gives us confidence in the potential applicability of the proposed testing strategy to German house price data in Figure 3.4. Lacking a gold standard procedure for monitoring periodically collapsing house price bubbles in real time, an early warning bubble test with high sensitivity can be considered as a reliable indicator when its result is negative, since it rarely misses true positives among those who are actually positive. Put differently, highly sensitive diagnostics have few false-negative results and are therefore most useful to rule out a beginning decoupling of house prices from their underlying fundamentals. Such highly sensitive diagnostics should particularly be used when we need to detect house price exaggerations and flag vulnerabilities in real time. Finally, the estimation results can also be interpreted as an indirect validation of the main argument put forth in Reinhart and Rogoff (2009) celebrated book *This Time is Different*. Therein they have provocatively argued that there are strong regularities attached to financial crises, which are therefore predictable based on economic fundamentals.

3.5 Wrapping Up: Real-time Monitoring of Risk with univariate Time Series Methods

It is sometimes alleged that monetary policy is closer to art than to science because it is frequently confronted to new, poorly anticipated and poorly understood, developments and shocks. It is claimed that in such situations common sense and experience are more powerful tools than a slavish adherence to theoretical and econometric models.

Since the global recession 2007-2009, the emphasis on systemic risk assessment and macro stress tests has gained importance. When rapid increases in house prices occur concerns are frequently voiced that prices may have lost touch with the underlying fundamentals. In such a circumstance, there is the fear a bubble may be developing that may eventually burst. This can potentially impart ripple effects throughout the rest of the economy. The main objective of this chapter is not to pretend that a simple model can predict emerging bubbles perfectly, but rather to show that even a parsimonious univariate statistical toolbox can do a good job at indicating housing market vulnerabilities in real time. To this end we have employed the state-of-the-art GSADF unit root tests suggested by Phillips and Yu (2011) and Phillips et al. (2012) as a barometer. The methodology offers a simple and straightforward real-time monitoring of housing cycles. Based on the GSADF statistic, so far there is no reason to believe that a German housing bubble is emerging.

It is important to stress that, just as any other methodology for monitoring house price bubbles in real time, this one is not a panacea. Nevertheless it is hoped that it will help to move the debate forward on this vital topic. However, whether this line of enquiry will ultimately prove fruitful and paves the way for early enough macro-prudential policies will probably continue to be a subject of debate. In any case, results suggested here should be interpreted carefully and should only be considered as part of a suite of indicators used in a complementary manner.

4

What Difference Does Long-Horizon Uncertainty Make? Housing Investment Choices in the Presence of Risky Steady State House Prices[1]

4.1 Introduction

The global financial crisis in 2007–2009 led to increasing interest among academic researchers and policymakers alike in the dynamics of house prices. For many consumers, housing is the most important asset in their portfolio and a better understanding is required when making intertemporal investment decisions. To this purpose, we first provide a narrative description of the impact of the global financial crisis upon long-run house price growth. In the forecasting literature, substantially less attention has been dedicated to long-horizon forecasts. One reason for this might be the methodological challenges confronted in forecasting by taking future

[1]I record my gratitude to Mahadeva (Oxford Institute for Energy Studies, University of Oxford) for helpful comments and suggestions on an earlier draft of this chapter.

uncertainty into account.[2] Mueller and Watson (2013) generate a set of state-of-the-art econometric tools for evaluating long-run forecast uncertainty by focusing upon the low-frequency shape of the spectrum of a time series. Instead of predicting the exact point, they construct predictive sets that move towards probability distribution prediction. Unlike housing market models that relate house prices to a set of other variables in a structural modelling framework, they suggest a nonstructural approach. This univariate frequency-domain approach is a simple and effective way to make forecasts when long-run causal relationships are less clear.[3]

In light of the challenges of obtaining reliable long-horizon predictions, We employ this technique to understand the long-run risk of real house prices across OECD countries. The probability distributions associated with the 25- and 50-year-horizon predictions are available in Figures 4.1 and 4.2, respectively. The probability distributions can be interpreted as a measure of how uncertain long-horizon house price forecasts are. Apparently, long-run risks imply a high degree of uncertainty around the predicted average growth rate of real house prices. The difference between the dashed red and solid blue lines is that the former refers to the sample period 1970Q1–2007Q4, while the latter uses the full sample up to 2013Q4.[4] It is shown that adding the after-crisis data tends to reduce the standard deviations of the long-run growth rate for most countries.[5] At the same time, predicted long-run house price growth rates are uneven. In some countries, such as the US, the UK, Japan, France and Ireland, expected growth decreases, while in Germany, Finland and Switzerland, increments are clearly presented.

So far, we have had an impression of the impact of the global financial crisis on long-run real house price growth. Nevertheless, how to think about such risky long-run house prices from a

[2]Predictions are usually classified according to the timescale involved in the prediction. Short-term (high frequency) and long-term (low frequency) are the usual categories but the actual meaning of each will vary according to the economic question analysed. In the application below, we choose a forecasting horizon of 25 years and 50 years, respectively.

[3]It goes without saying that the main shortcoming of univariate time series methods is that they are purely statistical, mechanical filters. On the other hand, they only require time series data on real house prices, which makes them very easy to implement for a wide range of countries. For a recent structural modelling approach forecasting the US housing market, see Kouwenberg and Zwinkels (2014).

[4]The seasonally adjusted quarterly house price dataset employed in this chapter stems from the Organization for Economic Cooperation and Development (OECD), which is a widely watched multi-country house price database.

[5]Following Mueller and Watson (2013), the prediction sets are constructed by using the $I(d)$ model with $d = 0$. Precise estimates for d are not readily available. Therefore, we also investigate the robustness and sensitivity of the results obtained by using the flat Bayes prior $d \in [-0.4; 1.4]$ suggested by Mueller and Watson (2013). The results match those in Figures 4.1 and 4.2. Supplementary graphs are available upon request.

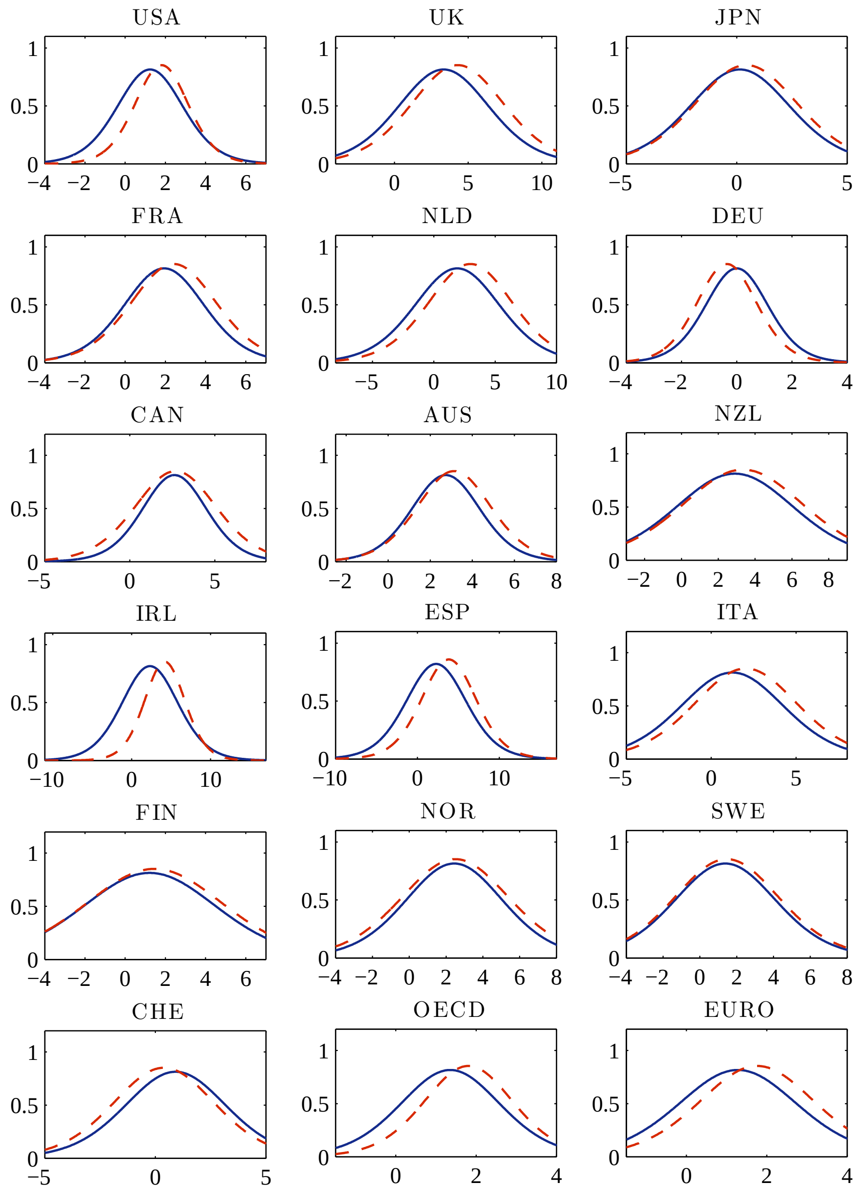


Figure 4.1: 25-Year-Ahead Predictive Density of Real House Price Growth across Countries.

Note: Solid blue lines are predicted based on the after-crisis HPI 1970Q1–2013Q4 and the dashed red lines are based on pre-crisis HPI 1970Q1–2007Q4.

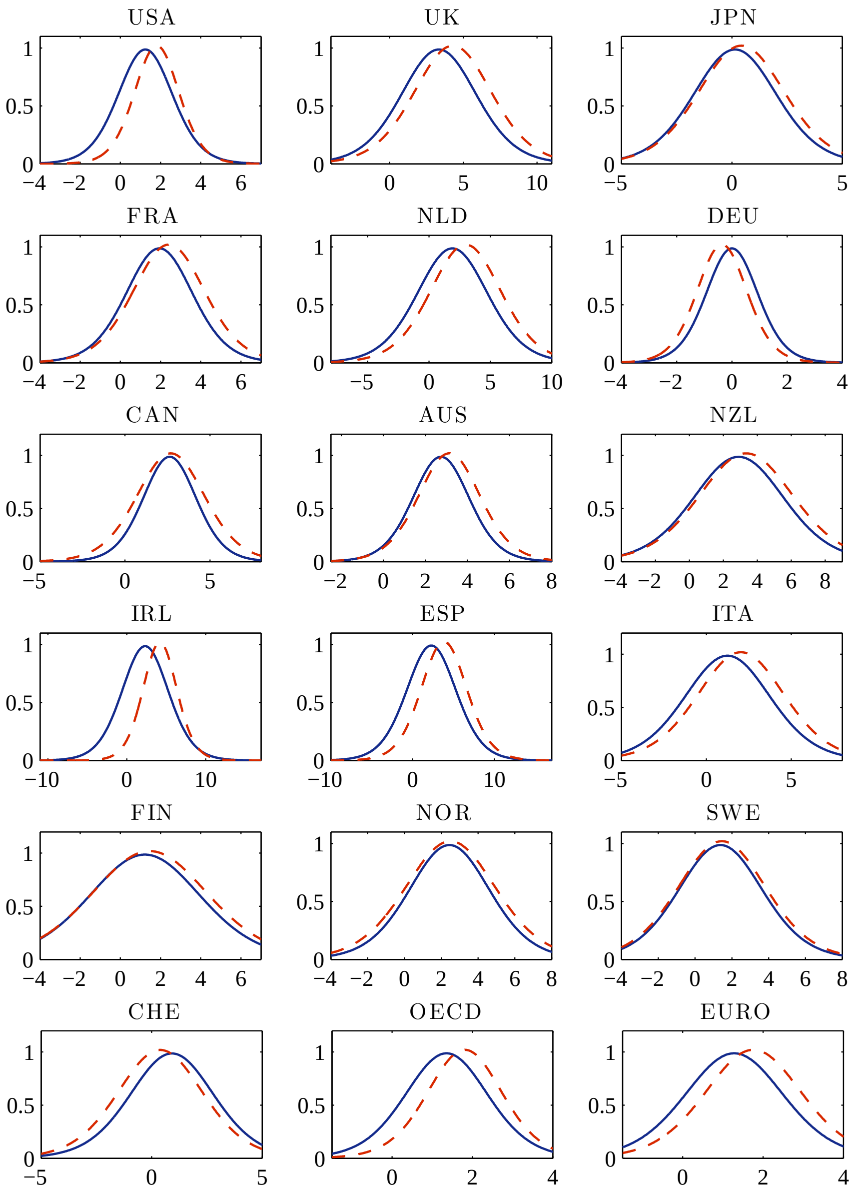


Figure 4.2: 50-Year-Ahead Predictive Density of Real House Price Growth across Countries.

Note: Solid blue lines are predicted based on the after-crisis HPI 1970Q1–2013Q4 and the dashed red lines are based on pre-crisis HPI 1970Q1–2007Q4.

conceptual standpoint and to understand the implications for the housing market? To do this, we employ the concept of a risky steady state proposed by Coeurdacier et al. (2011). It is well known that many nonlinear dynamic macroeconomics problems do not have analytical solutions and can only be approximated numerically. One of the most commonly used methodologies is the standard perturbation. Perturbation methods used to solve dynamic stochastic general equilibrium (DSGE) models are similar in spirit to Taylor series approximations.[6] Based on function derivatives, approximations are taken around a specific point or value of the parameter domain in order to approximate the function's corresponding value when these values are perturbed away by small degrees from that point around which the approximation is taken. This solution method lends itself very well to the approximation of the exact solution of the nonlinear systems of DSGE models for two reasons. First and foremost, the assumption that shocks (perturbations) are not too large in a real world economic system is a reasonable one most of the time. Second, while the general discussion on perturbation methods does not prescribe a preferred point around which the approximation should be taken, dynamic economic systems exhibiting a steady state provide a reasonable value around which the approximation should be taken.

In the most recent literature, this point has been given additional attention in that researchers typically consider two possible candidate steady state equilibrium points around which to form the approximation. One of them is the traditional deterministic steady state to which the economic system gravitates when future shocks are assumed to be zero, while the other steady state, suitably called the risky or stochastic steady state, is the one where the system comes to rest, when agents know that future shocks will continue to occur based on the certain known distributions of those shocks.[7] In this chapter, we want to draw attention to the second-order approximation of the equilibrium conditions to solve the stochastic steady state in a portfolio problem.

In fact, the risky steady state was first introduced by Juillard and Kamenik (2005). They argue that when perturbation methods are applied to stochastic general equilibrium models,

[6]Standard perturbation relies on implicit function theorems, Taylor series expansions and techniques from bifurcation and singularity theory. Judd (1998) explains explicitly how to use these techniques to approximate the policy functions of dynamically stable stochastic control models near the steady state of their deterministic counterparts.

[7]De Groot (2013) extends this to general settings as a matrix quadratic problem.

because of nonlinearities, the centre of the ergodic distribution of the endogenous variables can be away from the deterministic steady state, making it not the best point around which to take the approximation.[8] Coeurdacier et al. (2011) believe that risk-averse agents are aware of the existence of future shocks hitting the economy. Therefore, they anticipate the convergence of economic variables to some stochastic steady state, which incorporates information about expected future risk and the corresponding optimal decisions.

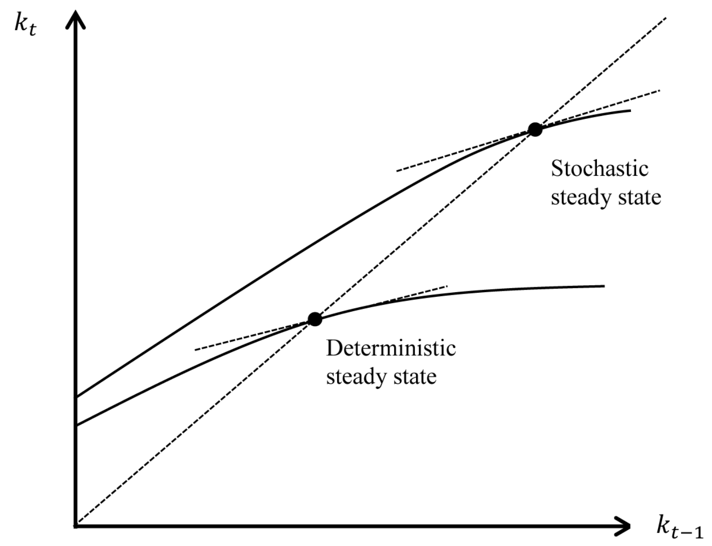


Figure 4.3: Deterministic vs. Stochastic Steady State: Decision Rules for Capital Accumulation.

Borrowing the example of a standard stochastic growth model presented by Coeurdacier et al. (2011), this concept is well illustrated in Figure 4.3. Anticipated uncertainty leads to precautionary capital accumulation, which raises the level of the stock of capital in the stochastic steady state than that in the deterministic steady state. Specifically, the stochastic steady state is defined as the point where agents decide to stay in the absence of shocks, but taking into account the likelihood of future shocks. By contrast, the deterministic steady state is the point where agents decide to stay in the absence of shocks and ignoring future shocks. Unfortunately, the risky steady state cannot be found analytically for most DSGE models. In addition, there are

[8]Juillard (2011) uses the simple asset pricing model for which there exists a closed form solution to compare the accuracy of the approximation around the deterministic steady state with the one around the risky steady state. He finds that the approximation of the solution appears as quite different depending whether the approximation taken around the deterministic steady state or risky steady state.

only a few numerical methods available in previous studies. Juillard (2011) proposes a numerical algorithm to find the risky steady state, which is truly mathematically challenging because of the nonlinearity. Alternatively, Coeurdacier et al. (2011) suggest another feasible strategy, which consists of postulating a linear decision rule for control variables around unknown risky steady states and their identifications along with the coefficients simultaneously.

The main purpose of this chapter is to provide a risky steady state framework to illustrate how long-horizon house price uncertainty effects housing investment choice. The model is designed to highlight the role of housing investment choice in the presence of stochastic labour income, a risky interest rate and risky house prices. Setting aside general equilibrium considerations, these risk sources will be treated as exogenous random processes, since in a risky steady state framework the mechanism of the calculation strategy can be more clearly demonstrated throughout this chapter. According to the model, the precautionary saving effect, the risk premium elicited by interest rate risk and house price risk and the crowding out effect are well reflected in the approximation equilibrium function. This implies that riskier countries will tend to have larger investment in housing and accumulate more financial assets than safer ones. In addition, we provide a numerical analysis of the theoretical model to show the impacts of a country's risk level on an agent's consumption and investment decision rules. Finally, we create proxy variables to denote the housing investment level and risk level of a country from three different perspectives, with which we provide evidence of positive relationships between them across 13 OECD countries.

The remainder of this chapter is divided into five sections. Section 4.2 describes the details of the extended model and derives our solution system, with which we compute the risky steady states and postulated coefficients of the decision rule endogenously. Section 4.3 discusses the results of the numerical experiments. The empirical evidence is presented in section 4.4. Finally, section 4.5 concludes.

4.2 Modelling Framework

To illustrate the idea, we present an extended model along the lines of the work by Coeurdacier et al. (2011) and use their notation for convenience. A representative household lives forever and has preferences over current and future consumption and housing. In each period t, the individual needs to choose the amounts of non-housing consumption good c_t and housing service h_t. The price per unit of housing at time t is denoted by p_t, while the price of c_t is fixed and normalised to one. The representative individual's intertemporal utility is

$$E_t[\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)], \tag{4.1}$$

where $E_t[\cdot]$ is the expectation operator based on the information available in period t, and β is the time discount factor for the future utility stream $u(c_t, h_t)$. The individual starts in period $t \geq 1$ with net worth NW_t given by

$$NW_t = HW_t + NHW_t, \tag{4.2}$$

where HW_t and NHW_t represent housing wealth and non-housing wealth, respectively. Given the housing price in period t and housing investment from the previous period, the corresponding housing wealth in period t is then given as $p_t h_{t-1}$ assuming only one financial asset ω_{t-1} is involved in the non-housing investment from the last period at the risky interest rate r_t.[9] Therefore, the individual's net worth at the beginning of period t can be rewritten as

$$NW_t = p_t h_{t-1} + r_t \omega_{t-1}. \tag{4.3}$$

The resultant resources available for consumption and investment in period t, Q_t, are defined as the sum of labour income plus net worth

$$Q_t = y_t + p_t h_{t-1} + r_t \omega_{t-1}. \tag{4.4}$$

Finally, in any period the individual faces the budget constraint

$$c_t + p_t h_t + \omega_t = y_t + p_t h_{t-1} + r_t \omega_{t-1}. \tag{4.5}$$

[9]Here, we ignore the maintenance cost of housing for simplicity. Another obvious omission from the analysis is that non-housing assets are not taken into account. Endogenous labour supply, the non-separability of c_t and h_t and bequest considerations are also ignored here.

The meaning of the intertemporal budget constraint is straightforward. The right-hand side, again, represents the total resources of the household in period t. Ignoring the portfolio adjustment costs, these resources can be used for consumption, housing wealth and non-housing wealth accumulation, which is given by the left-hand side of the equation.[10] Putting general equilibrium considerations aside, we assume y_t, r_t and p_t are exogenous variables and are log-normally distributed stochastic AR(1) processes

$$\ln(y_t) = (1-\rho_y)\bar{l}_y + \rho_y \ln(y_{t-1}) + u_{y,t+1}, \tag{4.6}$$

$$\ln(r_t) = (1-\rho_r)\bar{l}_r + \rho_r \ln(r_{t-1}) + u_{r,t+1}, \tag{4.7}$$

$$\ln(p_t) = (1-\rho_p)\bar{l}_p + \rho_p \ln(p_{t-1}) + u_{p,t+1}, \tag{4.8}$$

where $\bar{l}_y$, $\bar{l}_r$ and $\bar{l}_p$ are the means of $\ln(y_t)$, $\ln(r_t)$ and $\ln(p_t)$, $|\rho_y| < 1$, $|\rho_r| < 1$, and $|\rho_p| < 1$ are the AR(1) coefficients and the mean-zero random terms are defined as $u_{y,t} \sim N(0, {\sigma_{u,y}}^2)$, $u_{r,t} \sim N(0, {\sigma_{u,r}}^2)$ and $u_{p,t} \sim N(0, {\sigma_{u,p}}^2)$ for $t \geq 1$. The correlations between all three variables are set to zero for simplicity. Given equations 4.1–4.5, the agent's problem is to maximise his or her discounted expected utility subject to the intertemporal budget constraint, given his or her initial values of asset holdings. Rearranging equation 4.1 gives us the value function of the individual's intertemporal consumption and investment problem

$$V_t(X_t) = \max_{Y_t} \Big\{ u(c_t, h_t) + \beta E_t[V_{t+1}(X_{t+1})] \Big\},$$

where

$$X_t = (t, y_t, r_t, p_t, \omega_{t-1}, h_{t-1}),$$
$$Y_t = (c_t, h_t) \quad \text{for} \quad t \geq 0. \tag{4.9}$$

State vector X_t consists of the investor's labour income, the return on risky assets, the price per unit of housing services, the amount of existing risky non-housing assets and the size of existing housing. The first-order conditions of the value function with respect to c_t and h_t are given by

[10] The difference $p_t(h_t - h_{t-1})$ in the budget constraint implies that we allow households to move up or down the housing ladder. For an empirical study of housing mobility and downsizing in older age in the US and the UK, see Banks et al. (2012).

the following three equations

$$\frac{1}{p_t} = \frac{\partial u/\partial c_t}{\partial u/\partial h_t}, \tag{4.10}$$

$$1 = \beta E_t[\frac{\partial u/\partial h_{t+1}}{\partial u/\partial h_t} r_{t+1} \frac{p_t}{p_{t+1}}], \tag{4.11}$$

$$1 = \beta E_t[\frac{\partial u/\partial c_{t+1}}{\partial u/\partial c_t} r_{t+1}], \tag{4.12}$$

The interpretation of equations 4.10–4.12 is as follows. Optimal behaviour requires equating the marginal utilities of housing services in period t and the expected discounted marginal utilities of housing services in period $t + 1$ with the relative housing prices in t and $t + 1$. Notice that house price in $t + 1$ is also discounted by the risky interest rate r_{t+1}. For ease of exposition and without loss of generality, the individual's preferences over the non-housing consumption good and housing services are parameterised as

$$u(c_t, h_t) = \frac{({c_t}^{1-\alpha} {h_t}^{\alpha})^{1-\gamma}}{1-\gamma}, \tag{4.13}$$

where α measures the relative importance of housing services versus non-housing and γ is the coefficient of relative risk aversion. By inserting equation 4.13 into 4.10–4.12, we have

$$h_t = M(\frac{{c_t}^{\alpha}}{p_t})^{\frac{1}{1-\alpha}}, \tag{4.14}$$

$$1 = \beta E_t[r_{t+1}(\frac{c_{t+1}}{c_t})^{-\gamma(1-\alpha)}(\frac{h_{t+1}}{h_t})^{-\gamma\alpha+\alpha-1}\frac{p_t}{p_{t+1}}], \tag{4.15}$$

$$1 = \beta E_t[r_{t+1}(\frac{c_{t+1}}{c_t})^{-\gamma(1-\alpha)-\alpha}(\frac{h_{t+1}}{h_t})^{-\gamma\alpha}], \tag{4.16}$$

where

$$M = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{4.17}$$

Essentially, equations 4.15 and 4.16 are equivalent if we insert 4.14 into both of them. Therefore, one could focus on one of these two equations, say, equation 4.16 for further analysis. We define

$$g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t) \equiv \beta r_{t+1}\Big(\frac{c_{t+1}}{c_t}\Big)^{\Sigma}\Big(\frac{h_{t+1}}{h_t}\Big)^{\Pi} - 1,$$

where

$$\begin{aligned} \Sigma &= -\gamma(1-\alpha) - \alpha, \\ \Pi &= -\alpha\gamma. \end{aligned} \tag{4.18}$$

Then, equation 4.16 could be rewritten as

$$E_t[g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t)] = 0. \tag{4.19}$$

In order to take risk into account, equation 4.19 is replaced by its second-order Taylor expansion Ψ around the expected future variables

$$\begin{aligned} 0 &= E_t[g(c_{t+1}, c_t, r_{t+1}, h_{t+1}, h_t)] \\ &\approx \Psi[E_t(c_{t+1}), E_t(r_{t+1}), E_t(h_{t+1}), c_t, h_t], \end{aligned} \tag{4.20}$$

where

$$\begin{aligned} \Psi = {} & \beta E_t(r_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi} - 1 \\ & + \frac{\beta(\Sigma-1)\Sigma}{2}\mathrm{Var}_t(c_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi}\frac{E_t(r_{t+1})}{E_t(c_{t+1})^2} \\ & + \frac{\beta(\Pi+1)\Pi}{2}\mathrm{Var}_t(h_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi}\frac{E_t(r_{t+1})}{E_t(h_{t+1})^2} \\ & + \beta\Sigma\Pi\mathrm{Cov}_t(c_{t+1}, h_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi}\frac{E_t(r_{t+1})}{E_t(c_{t+1})E_t(h_{t+1})} \\ & + \beta\Sigma\mathrm{Cov}_t(c_{t+1}, r_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi}\frac{1}{E_t(c_{t+1})} \\ & + \beta\Pi\mathrm{Cov}_t(h_{t+1}, r_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Sigma}\Big(\frac{E_t(h_{t+1})}{h_t}\Big)^{\Pi}\frac{1}{E_t(h_{t+1})}. \end{aligned} \tag{4.21}$$

Multiplying equation 4.20 by the non-zero term $\beta^{-1}\mathrm{E_t}(\mathrm{r_{t+1}})^{-1}\left(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\right)^{-\Sigma}\left(\frac{\mathrm{E_t}(\mathrm{h_{t+1}})}{\mathrm{h_t}}\right)^{-\Pi}$ and simplifying further allows us to obtain

$$\begin{aligned}&\frac{1}{\beta}\Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{-\Sigma}\Big(\frac{\mathrm{E_t}(\mathrm{h_{t+1}})}{\mathrm{h_t}}\Big)^{-\Pi}\\ &=\mathrm{E_t}(\mathrm{r_{t+1}})\Big[1+\frac{(\Sigma-1)\Sigma}{2}\frac{\mathrm{Var_t}(\mathrm{c_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})^2}+\frac{(\Pi+1)\Pi}{2}\frac{\mathrm{Var_t}(\mathrm{h_{t+1}})}{\mathrm{E_t}(\mathrm{h_{t+1}})^2}\\ &+\Sigma\Pi\frac{\mathrm{Cov_t}(\mathrm{c_{t+1}},\mathrm{h_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{h_{t+1}})}\Big]+\Sigma\frac{\mathrm{Cov_t}(\mathrm{r_{t+1}},\mathrm{c_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})}+\Pi\frac{\mathrm{Cov_t}(\mathrm{r_{t+1}},\mathrm{h_{t+1}})}{\mathrm{E_t}(\mathrm{h_{t+1}})}.\end{aligned}\tag{4.22}$$

We assume that agents are aware of future shocks hitting the economy and anticipating the economic variables converging to some stochastic steady states, i.e. the ergodic distribution of these variables. Instead of the deterministic steady state $(\mathrm{X}^*, \mathrm{Y}^*)$, from now on we consider the local behaviour of an economy around the risky steady state $(\bar{\mathrm{X}}, \bar{\mathrm{Y}})$.[11] Therefore, at the risky steady state, this approximation becomes

$$\begin{aligned}\frac{1}{\beta}&=\bar{\mathrm{r}}\Big[1+\frac{(\Sigma-1)\Sigma}{2}\frac{\overline{\mathrm{Var}_t}(\mathrm{c_{t+1}})}{\bar{\mathrm{c}}^2}+\frac{(\Pi+1)\Pi}{2}\frac{\overline{\mathrm{Var}_t}(\mathrm{h_{t+1}})}{\bar{\mathrm{h}}^2}+\Sigma\Pi\frac{\overline{\mathrm{Cov}_t}(\mathrm{c_{t+1}},\mathrm{h_{t+1}})}{\bar{\mathrm{c}}\bar{\mathrm{h}}}\Big]\\ &+\Sigma\frac{\overline{\mathrm{Cov}_t}(\mathrm{r_{t+1}},\mathrm{c_{t+1}})}{\bar{\mathrm{c}}}+\Pi\frac{\overline{\mathrm{Cov}_t}(\mathrm{r_{t+1}},\mathrm{h_{t+1}})}{\bar{\mathrm{h}}},\end{aligned}\tag{4.23}$$

where $\overline{\mathrm{Var}_t}(\cdot)$ and $\overline{\mathrm{Cov}_t}(\cdot)$ denote the second-order moments evaluated at the risky steady state. More specific, variance and covariance in period t are evaluated at the risky steady state according to $\overline{\mathrm{Var}_t}(\mathrm{c_{t+1}}) = \overline{\mathrm{Cov}_t}(\mathrm{c_{t+1}}, \mathrm{c_{t+1}} \mid \mathrm{c_t} = \bar{\mathrm{c}})$ and $\overline{\mathrm{Cov}_t}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}}) = \overline{\mathrm{Cov}_t}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}} \mid \mathrm{c_t} = \bar{\mathrm{c}}, \mathrm{r_t} = \bar{\mathrm{r}})$. The same occurs for $\overline{\mathrm{Var}_t}(\mathrm{h_{t+1}})$, $\overline{\mathrm{Cov}_t}(\mathrm{c_{t+1}}, \mathrm{h_{t+1}})$ and $\overline{\mathrm{Cov}_t}(\mathrm{h_{t+1}}, \mathrm{r_{t+1}})$. The pattern can now be inferred. Compared with Coeurdacier et al.'s (2011) results, we find common features and differences. In the absence of risk, the return on financial investment must be equal to the inverse of time preference, which is given by $\bar{\mathrm{r}} = \frac{1}{\beta}$. The second term $\frac{1}{2}(\Sigma-1)\Sigma\overline{\mathrm{Var}_t}(\mathrm{c_{t+1}})/\bar{\mathrm{c}}^2$ in square brackets is the so-called precautionary saving term. Coeurdacier et al. (2011) point out that if uncertainty over future consumption increases, risk-averse

[11] For the rest of the chapter, Z^* and $\bar{\mathrm{Z}}$ denote the deterministic and risky steady states for any variable Z, respectively.

agents will sacrifice consumption in the current period to ensure future consumption at the desired higher level. They also state that when financial assets are risky, an additional stabilising force on the consumption path is at work. This is reflected in the first term out of square brackets $\Sigma\overline{\text{Cov}_t}(r_{t+1}, c_{t+1})/\bar{c}$, i.e., the risk premium term elicited from the financial market.

Our extended model generates extra three terms at the equilibrium: $\frac{(\Pi+1)\Pi}{2}\overline{\text{Var}_t}(h_{t+1})/\bar{h}^2$, $\Sigma\Pi\overline{\text{Cov}_t}(c_{t+1}, h_{t+1})/\bar{c}\bar{h}$ and $\Pi\overline{\text{Cov}_t}(r_{t+1}, h_{t+1})/\bar{h}$. The first one is similar to the precautionary effect. It implies that when future uncertainty in the housing market grows, clients anticipate the convergence of housing investment to a higher level in the long run. The second term comes from the risk premium associated with risky house prices. This stresses the fact that in the housing market, house price risk serves as the third stabilising force other than precautionary savings and the risky interest rate, since the covariance between consumption and housing reduces the persistence of shocks in the housing market. When the economy reaches the risky steady state, countries with higher house price risk tend to invest more in housing than safer ones. The last term in our equilibrium is similar to the crowding out effect highlighted by Cocco (2005). Unlike other risky assets, house price risk can hardly be avoided for most households, since everyone wants to purchase a home eventually. Nevertheless, with limited resources, participation in the financial market is crowded out by risky house prices. Investment in housing assets increases alone with the covariance between housing investment and financial asset return at the risky steady state, as shown in equation 4.23. Note that the extended model renders an explicit analytical solution impossible. Therefore, we employ numerical techniques in the next section.

Before turning to the numerical analysis, we need to rewrite the second-order expansion for computational convenience. We define

$$f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t) \equiv \beta r_{t+1}\Big(\frac{c_{t+1}}{c_t}\Big)^{\Delta}\Big(\frac{p_t}{p_t+1}\Big)^{\Lambda} - 1,$$

where

$$\Delta = -\Big\{\frac{\gamma}{1-\alpha}[\alpha^2 + (1-\alpha)^2] + \alpha\Big\},$$
$$\Lambda = -\frac{\alpha\gamma}{1-\alpha}. \tag{4.24}$$

Collecting the terms, another form of the approximation of our equilibrium around the risky steady state could be given as follows[12]

$$\begin{aligned}
&\frac{1}{\beta}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda} \\
&= E_t(r_{t+1})\Big[1 + \frac{(\Delta-1)\Delta}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^2} \\
&- \Delta\Lambda\frac{Cov_t(c_{t+1}, p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})}\Big] + \Delta\frac{Cov_t(c_{t+1}, r_{t+1})}{E_t(c_{t+1})}. \qquad (4.25)
\end{aligned}$$

In fact, this is equivalent to equation 4.22, which provides information between future consumption and housing investment. However, equation 4.25 contains only one nonpredetermined variable, consumption c_t, which makes our numerical exercise easier, since both interest rate r_t and housing price p_t are assumed to be exogenous stochastic processes. Following Coeurdacier et al.'s (2011) strategy, we also postulate a linear decision rule for ω_t

$$\omega_t = \bar{\omega} + G_{\omega\omega}(\omega_{t-1} - \bar{\omega}) + G_{\omega r}(r_t - \bar{r}) + G_{\omega y}(y_t - \bar{y}) + G_{\omega p}(p_t - \bar{p}), \qquad (4.26)$$

where $\bar{\omega}$ is the unknown risky steady state value for financial assets and $G_{\omega\omega}$, $G_{\omega r}$, $G_{\omega y}$ and $G_{\omega p}$ are the four coefficients needed to be calculated endogenously. By inserting equations 4.14 and 4.26 into budget constraint 4.5 and linearising it, one obtains the approximations of conditional expectation and variance of consumption

$$\begin{aligned}
E_t(c_{t+1}) &= K_1\bar{\omega}(G_{\omega\omega} - 1) + K_1G_{\omega r}\bar{r} + K_1G_{\omega y}\bar{y} + K_1G_{\omega p}\bar{p} \\
&+ K_1(1 - G_{\omega y})E_t(y_{t+1}) + K_1(\omega_t - G_{\omega r})E_t(r_{t+1}) \\
&- K_1G_{\omega\omega}\omega_t + [K_1(h_t - G_{\omega p}) + K_2]E_t(p_{t+1}) + K_3, \qquad (4.27)
\end{aligned}$$

and

$$\begin{aligned}
Var_t(c_{t+1}) &= {K_1}^2(1 - G_{\omega y})^2 Var_t(y_{t+1}) + {K_1}^2(\omega_t - G_{\omega r})^2 Var_t(r_{t+1}) \\
&+ [K_1(h_t - G_{\omega p}) + K_2]^2 Var_t(p_{t+1}), \qquad (4.28)
\end{aligned}$$

[12] The calculation details are given in Appendixes A.

where

$$\Gamma = \frac{\alpha}{1-\alpha}, \mathrm{K}_1 = \frac{1}{1+\Gamma}, \mathrm{K}_2 = \frac{\Gamma}{1+\Gamma}, \mathrm{K}_3 = \frac{1-\ln \mathrm{M}}{1+\Gamma}.$$

At the same time, we also know the conditional covariance of consumption and the interest rate and housing price

$$\mathrm{Cov}_t(c_{t+1}, r_{t+1}) = \mathrm{K}_1(\omega_t - \mathrm{G}_{\omega r})\mathrm{Var}_t(r_{t+1}), \tag{4.29}$$

$$\mathrm{Cov}_t(c_{t+1}, p_{t+1}) = [\mathrm{K}_1(h_t - \mathrm{G}_{\omega p}) + \mathrm{K}_2]\mathrm{Var}_t(p_{t+1}). \tag{4.30}$$

y_t, r_t and p_t are assumed to be three exogenous variables and autocorrelated lognormally distributed stochastic processes. Their risky steady states are calculated as follows[13]

$$\bar{y} = e^{\bar{l}_y + \frac{1}{2}\frac{\sigma_{u,y}{}^2}{1-\rho_y{}^2}}, \tag{4.31}$$

$$\bar{r} = e^{\bar{l}_r + \frac{1}{2}\frac{\sigma_{u,r}{}^2}{1-\rho_r{}^2}}, \tag{4.32}$$

$$\bar{p} = e^{\bar{l}_p + \frac{1}{2}\frac{\sigma_{u,p}{}^2}{1-\rho_p{}^2}}, \tag{4.33}$$

and their corresponding conditional expectations are

$$\mathrm{E}_t(y_{t+1}) = e^{(1-\rho_y)\bar{l}_y + \rho_y \ln y_t + \frac{\sigma_{u,y}{}^2}{2}}, \tag{4.34}$$

$$\mathrm{E}_t(r_{t+1}) = e^{(1-\rho_r)\bar{l}_r + \rho_r \ln r_t + \frac{\sigma_{u,r}{}^2}{2}}, \tag{4.35}$$

$$\mathrm{E}_t(p_{t+1}) = e^{(1-\rho_p)\bar{l}_p + \rho_p \ln p_t + \frac{\sigma_{u,p}{}^2}{2}}. \tag{4.36}$$

Compared with conditional expectations, it is apparent that in the case of the risky steady state, the response to positive and negative shocks is stronger. In addition, their conditional variances

[13]The risky steady states of y_t, p_t and r_t are the unconditional expectations of the associated ergodic distribution. Following the assumption, they are lognormal-distributed processes, i.e., $y_t \sim \ln \mathrm{N}(\bar{l}_y, \frac{\sigma_{u,y}{}^2}{1-\rho_y{}^2})$, $r_t \sim \ln \mathrm{N}(\bar{l}_r, \frac{\sigma_{u,r}{}^2}{1-\rho_r{}^2})$ and $p_t \sim \ln \mathrm{N}(\bar{l}_p, \frac{\sigma_{u,p}{}^2}{1-\rho_p{}^2})$. Therefore, $\mathrm{E}(y_t) = e^{\bar{l}_y + \frac{1}{2}\frac{\sigma_{u,y}{}^2}{1-\rho_y{}^2}}$, $\mathrm{E}(r_t) = e^{\bar{l}_r + \frac{1}{2}\frac{\sigma_{u,r}{}^2}{1-\rho_r{}^2}}$, $\mathrm{E}(p_t) = e^{\bar{l}_p + \frac{1}{2}\frac{\sigma_{u,p}{}^2}{1-\rho_p{}^2}}$.

are given as

$$\mathrm{Var_t(y_{t+1})} = \mathrm{e}^{2(1-\rho_y)\bar{l}_y + 2\rho_y \ln y_t + \sigma_{u,y}{}^2}(\mathrm{e}^{\sigma_{u,y}{}^2} - 1), \tag{4.37}$$

$$\mathrm{Var_t(r_{t+1})} = \mathrm{e}^{2(1-\rho_r)\bar{l}_r + 2\rho_r \ln r_t + \sigma_{u,r}{}^2}(\mathrm{e}^{\sigma_{u,r}{}^2} - 1), \tag{4.38}$$

$$\mathrm{Var_t(p_{t+1})} = \mathrm{e}^{2(1-\rho_p)\bar{l}_p + 2\rho_p \ln p_t + \sigma_{u,p}{}^2}(\mathrm{e}^{\sigma_{u,p}{}^2} - 1). \tag{4.39}$$

Finally, we have the following local conditions to identify the risky steady states and coefficients assumed in equation 4.26

$$\widehat{\Phi}(\bar{S}) = 0, \tag{4.40}$$

$$\frac{\partial \widehat{\Phi}}{\partial S_t}|_{S_t=\bar{S}} = 0, \tag{4.41}$$

where

$$S_t = (X_t, Y_t), \quad \bar{S} = (\bar{X}, \bar{Y}).$$

In our model, the system can be written as a five-dimension equation system.[14] The solutions of this system are the values of $\bar{\omega}$, $G_{\omega\omega}$, $G_{\omega y}$, $G_{\omega r}$ and $G_{\omega p}$, with which we are able to calculate $\bar{c}$ and $\bar{h}$ as follows

$$\bar{c} = \bar{y} + \bar{\omega}(\bar{r} - 1), \tag{4.42}$$

$$\bar{h} = M\Big(\frac{\bar{c}^{\alpha}}{\bar{p}}\Big)^{\frac{1}{1-\alpha}} \quad \text{with} \quad M = \Big(\frac{\alpha}{1-\alpha}\Big)^{\frac{1}{1-\alpha}}. \tag{4.43}$$

4.3 Numerical Analysis

In this section, we provide a numerical analysis of the theoretical model using MATLAB. By changing the standard deviations of labour income, the risky interest rate and house prices, the discussion focuses on the variations of consumption and investment at the risky steady states. Specifically, we design three experiments to analyse the decision rules given the different levels of aggregate income risk, financial market risk and housing market risk. The purpose of first ex-

[14] See Appendixes A for the calculation details.

periment is to replicate part of Coeurdacier et al.'s (2011) earlier work and extend the numerical analysis to test whether their conclusions are still robust in the presence of a risky housing market. Moreover, by changing the standard deviation of the risky interest rate and house prices, the last two experiments aim to demonstrate the risky housing-related effects, as presented in equation 4.23.

Parameter		Value	
Time discount parameter β		0.96	
Curvature parameter γ		3.0	
Preference for housing α		0.2	
Risky steady state $\bar{y}$		0.8	
Risky steady state $\bar{r}$		1.027	
Risky steady state $\bar{p}$		0.8	
AR(1) coefficient ρ_y		0.9	
AR(1) coefficient ρ_r		0.9	
AR(1) coefficient ρ_p		0.9	
	Experiment 1: Aggregate Income	Experiment 2: Financial Market	Experiment 3: Housing Market
Sd.of labour income σ_y	(0, 0.1)	0.02	0.02
Sd.of risky interest rate σ_r	0.02	(0.01, 0.1)	0.02
Sd.of labour house price σ_p	0.07	0.07	(0, 0.1)

Table 4.1: Baseline Parameters in the Model.

To do this, we begin by discussing a baseline setup of the parameters in the model. Instead of estimating key parameters from the original data ourselves, we rely upon the US data-based estimations already widely used by most researchers. The time discount parameter β is standard; we set it at 0.96, which follows Cocco et al. (2005). The curvature parameter γ is equal to 3, below the upper bound of 10 considered to be plausible by Mehra and Prescott (1985). Parameter α measures the relative importance of housing services versus non-housing, and it is fixed at 0.2, which is approximatively equal to the average proportion of household housing expenditure in the Consumer Expenditure Survey 2001 suggested by Yao and Zhang (2005a) (see also U.S. Department of Labor, Bureau of Labor Statistics, 2003). Similar to Cocco (2005), we set the standard deviation of labour income σ_y at 0.02 in the second and third experiments. As such, the test scale of σ_y in the first experiment will include this value as well and this is fixed between 0 and 0.1. In order to make our results comparable with those of Coeurdacier et al. (2011), we follow their suggestion and fix the mean of labour income $\bar{y}$ at 0.8 and $\mathrm{AR}(1)$ coefficient ρ_y at 0.9. Recall that in the absence of risk, the usual Euler equation implies $\bar{r} = \frac{1}{\beta}$, but under the consideration of the risky steady state, the value of $\bar{r}$ is no longer equal to the inverse of the time discount parameter and it varies away from $\frac{1}{\beta}$. Thus, we set $\bar{r}$ at 1.027 similar to Coeurdacier et al. (2011). In the specification of the house price process, we follow Nagaraja et al. (2011), who provide a sophisticated autoregressive approach to predict the parameter of risky house prices. Their estimations for the standard deviation of house prices and the $\mathrm{AR}(1)$ coefficient are 0.07 and 0.9, respectively.[15] The values of the baseline parameters and test ranges in each experiment are given in Table 4.1.

In Figure 4.4, the results of experiments 1–3 are presented in the three panels. By allowing the standard deviation of stochastic income σ_y to change between 0 and 0.1, the first panel shows the impact of labour income risk on consumption and investment decisions at the risky steady state. The overall qualitative feature of the variations is consistent with Coeurdacier et al. (2011) results: consumption, financial assets and housing investment all increase monotonously alone with the risk level of labour income at the risky steady state. A rise in consumption is not a surprising outcome, since the precautionary effect is at work. People living in riskier countries usually anticipate bigger shocks hitting the economy. Typically, risk-averse agents will save more resources in exchange for a stable life in the long run. Therefore, higher future uncertainty caused either by income risk or by financial market/housing market risk induces higher consumption at the risky steady state. These patterns can be observed in the first graph of each panel.

Meanwhile, some quantitative differences are also worth noticing. In our experiments, the variations of aggregate income risk, financial risk and housing risk have a relatively small impact on the risky steady states of consumption. For instance, in the first experiment, it goes up from

[15] Cocco's (2005) estimation of standard deviation of house price is 0.062, which is also close to Nagaraja et al.'s (2011) value.

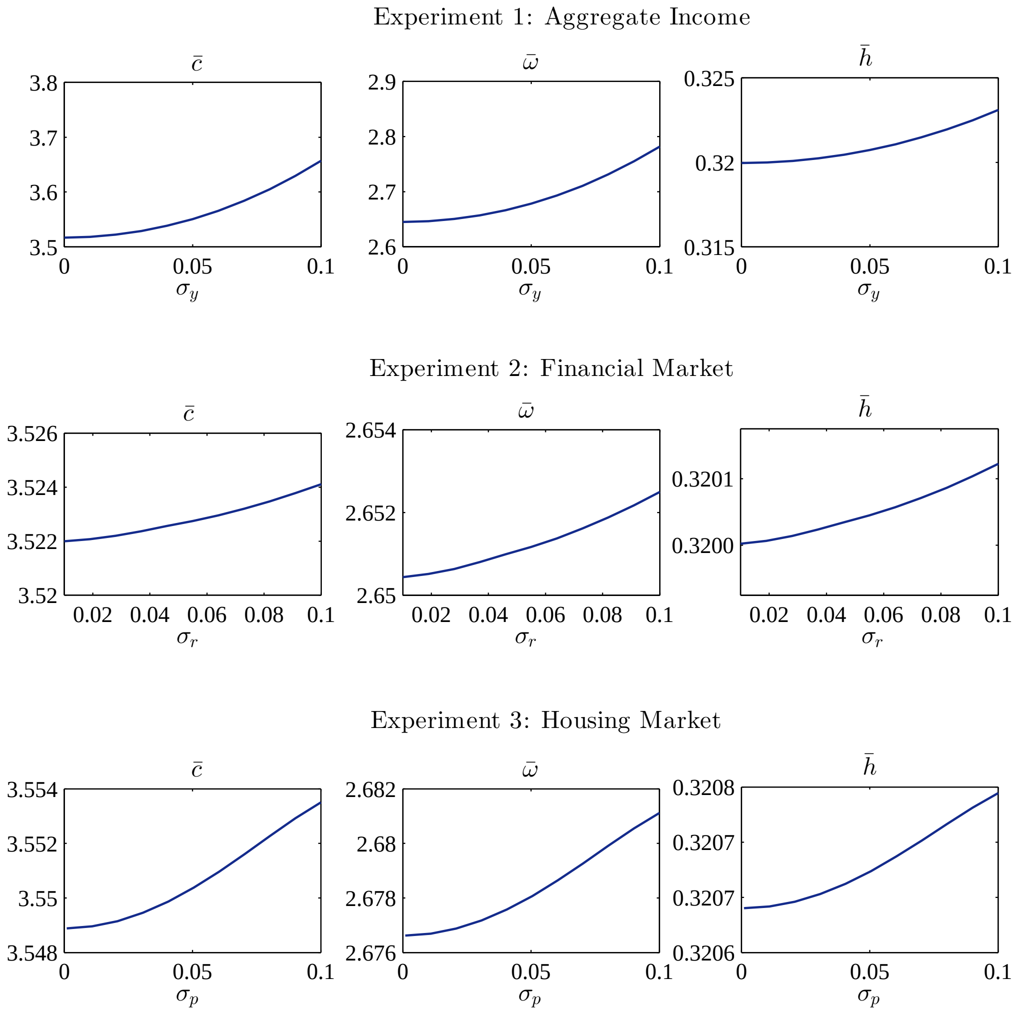


Figure 4.4: Results of the Numerical Analysis.

3.5166 to 3.5506, only a 0.97% increase, when aggregate risk changes from 0.01 to 0.05. These small dispersions lie on the risk premium effects induced by the risky interest rate and house prices. Coeurdacier et al. (2011) extend their numerical analysis by differentiating the precautionary effect and risk premium effect separately. Under the assumption that financial returns are risk-free, small changes to labour income risk have a strong impact on the risky steady state for net foreign assets; however, this feature will disappear if the assets are risky, since a risky financial return reduces the persistence of shocks on risky assets compared with non-stochastic scenarios and acts as an additional stabilising force on the consumption path.[16] In our model, we also include the housing market under the assumption that house prices change randomly. In this scenario, the risk premium effect associated with risky house prices is working. Similar to risky financial returns, uncertainty in house prices lowers the persistence of shocks on housing assets and also has a stabilising effect on consumption. This smoothing caused by the housing market has already been pointed out by Hurst and Stafford (2004). They use an empirical study to document the extent to which homeowners use housing equity to smooth their consumption over time.

As for financial assets level, we find it positively associated with the country's risk levels evaluated in three aspects. This is also highlighted roughly by Coeurdacier et al. (2011). Influenced by the level of aggregate income risk in the economy, the precautionary motivation induces a well-defined risky steady state for financial assets. Therefore, a riskier country tends to accumulate more wealth than safer one in the long-term. However, the extents of accumulated wealth are different from case to case. If the aggregate income risk level changes from 0 to 0.1, this causes a 4.97% increase in financial assets (second graph, panel 1). With the same amount of risk changing in the housing market, our model predicts lower growth from 2.6766 to 2.6799, a 0.12% increase (second graph, panel 3). This difference can be explained by the so-called crowding out effect induced by risky housing. House price risk can be substantial, but unlike other risky assets that people can avoid, most households keep investing in housing in order to own their home eventually (Banks et al., 2010).

Participation in the financial market is squeezed out by risky housing.[17] Although long-horizon uncertainty triggers more financial wealth accumulation, the increment will be smaller if this uncertainty comes from the housing market. Following a similar logic, we are also able to explain the changing of housing investment when the standard deviation of the risky interest rate increases, as shown in the third graph, panel 2. As the financial market becomes more volatile, households will adjust their investment strategy by investing less in risky assets and more in housing. High uncertainty in the financial market then serves as a motivation to invest

[16] In Coeurdacier et al.'s (2011) numerical example, the risky steady state for consumption $\bar{c}$ jumps from 0.4 to 2.0 when σ_y varies from 0.01 to 0.05, a 400% increase under the assumption of a risk-free financial return. However, when financial return is risky, $\bar{c}$ changes only from 0.995 to 1.017, a 2.2% increase.

[17] This is similar to Cocco's (2005) crowding out effect. He provides empirical evidence to show that house price risk can crowd out stockholdings, and this effect is significant for investors that have limited financial wealth.

in the housing market. Moreover, another investment incentive can be identified in our experiments as well. With increasing house price risk, the model predicts that housing investment also grows. This impact is at work through the channel of the hedging effect associated with future house price risk (Han, 2010). If households expect higher uncertainty in the housing market, they may have an incentive to invest more in housing assets, since these serve as an insurance against price fluctuations for future movements up the housing ladder (Banks et al., 2010). By comparing the second experiment with the third one, the general impression is that uncertainty rooted in the housing market has a stronger impact on household consumption and investment decisions than that rooted in the financial market.

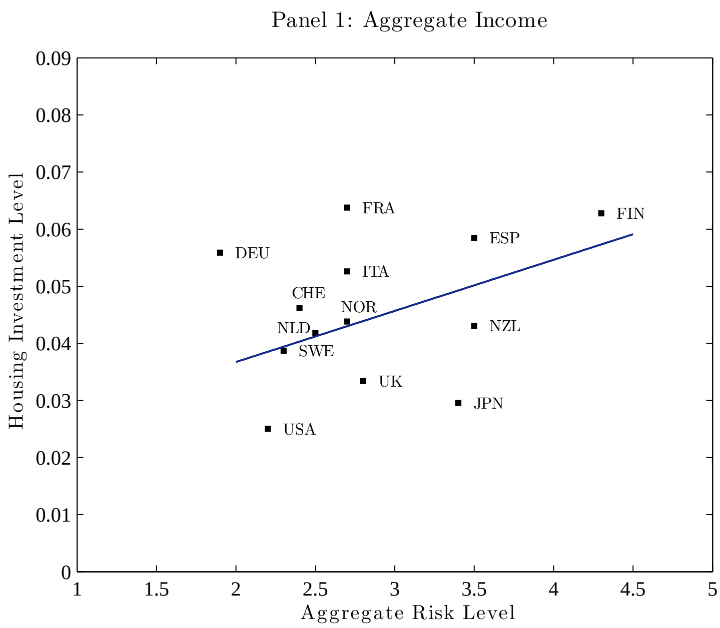
4.4 Empirical Analysis

In the previous section, we used three numerical experiments to discuss the effects of certain parameters qualitatively and quantitatively. Next, we present some tentative empirical evidence for each idea. The model implies that, at equilibrium with the expectation of future risk, a country with a higher risk level tends to invest more in housing assets than a safer one. In this sense, we wish to visualise this relationship between risk level and housing investment level by creating a scatter plot of two proxy variables from three different perspectives across OECD countries. More specifically, we use the 25-year-ahead predicted standard deviations of the real GDP growth rate to represent a country's aggregate risk level. These predictions are constructed by the $I(0)$ model following the earlier work of Mueller and Watson (2013) using the real GDP growth rate 1970Q1–2010Q4 for 13 OECD countries.[18] Their method aims to quantify the uncertainty in the long-run forecasting of economic variables. against this background, past history-based standard deviations could be considered to be proxies of countries' aggregate risk levels. In the same spirit, we construct a country's risk level in the financial market and housing market by predicting the 25-year-ahead standard deviations of the real interest rate and real house price index growth rates, respectively.

To keep the forecast results as consistent as possible, our samples of the real interest rate and real house price index are taken between 1970Q1 and 2010Q4 as well. Given the potential reverse causality, the proxy variable for housing investment level needs to be measured carefully.

[18] Owing to the availability of data in OECD's iLibrary, we use the data sample between 1970Q1–2011Q4 for 13 OECD countries.

To do this, we use OECD data to compute the housing asset-to-GDP ratio in 2011 to capture the level of housing investment in that year. According to the modelling setup, this study takes the risk resources as given and the standard reversal causality issue is less of a concern in our discussion. The measures of risk used in the scatter plots are forecasted from historical data between 1970Q1 and 2010Q4, which are most unlikely to be affected by the housing asset level evaluated in 2011. Finally, the relationships between housing investment and the three different risk measures are demonstrated in Figure 4.5, panels 1–3. The general impression is that there are positive associations across OECD countries.[19] Agents living in riskier countries are aware of the existence of future uncertainty and therefore anticipate bigger shocks hitting the economy. For instance, in 2010, based on historical information between 1970 and 2010, agents from Spain presuppose their housing market risk as high as 28. In such a case, they tend to invest more in housing compared with a safer county such as Sweden (22.4) or Switzerland (18.6). In Spain, the housing asset-to-GDP ratio was almost 6% in 2011.



Meanwhile, housing investment in the same year accounted for 3.6% and 4.8% of GDP in

[19]We could also use 50-year-ahead predicted standard deviation and this will not change the distribution of the points in the scatter plot too much. The relationship will remain positive.

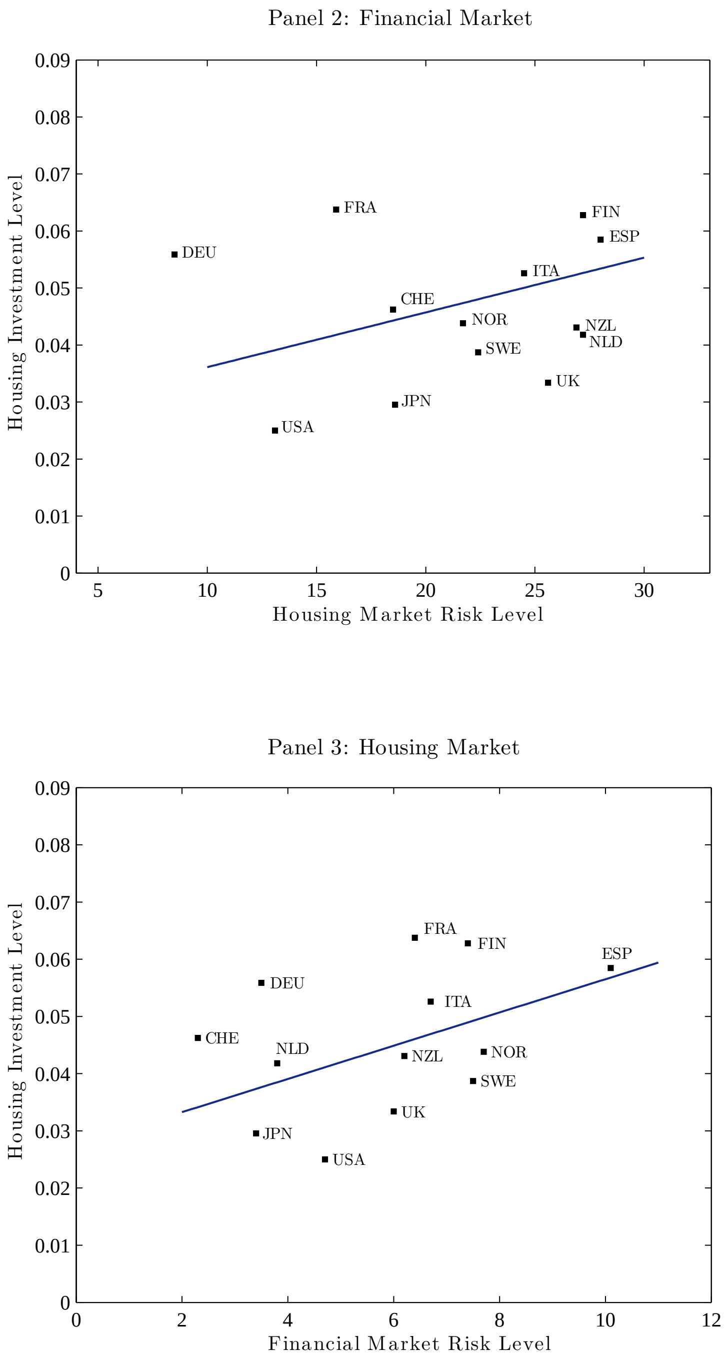


Figure 4.5: Relationship between Housing Investment Level and Risk Level Evaluated from Three Aspects across OECD Countries.

Sweden and Switzerland, respectively (panel 3 in Figure 4.5). A similar positive relationship can also be observed in the financial market in panel 2 of Figure 4.5, which provides empirical evidence of the crowding out effect triggered by risky house prices. Unlike other risky assets, purchasing a home is the single most important financial decision that cannot be bypassed for typical households (Han, 2010). Therefore, they prefer housing assets to financial assets, since most people want to own their home eventually and thereby create insurance demand for housing ownership (Banks et al., 2010).

4.5 Conclusion

This chapter analyses the impact of long-horizon house price uncertainty on housing investment choices in the risky steady state framework. Typically, risk-averse agents will spend less on risky assets as volatility increases. However, in this study we show that housing investment in the presence of risky steady state house prices is an exception to this rule. Taking future risk into account, the precautionary saving effect, crowding out effect and risk premium elicited by interest rate risk and house price risk are well reflected in our approximated risky steady state equilibrium function. Thus, the model stresses analytically and conceptually why long-horizon house price uncertainty acts as an incentive to invest more in housing assets in a riskier country. Moreover, the numerical results also show the extents to which housing investment will go up when uncertainty changes at different scales, measured by different aspects: aggregate income risk, the risky interest rate and risky house prices. This also stresses that in a volatile economy housing serves not only as a durable consumption good but also as a self-insurance instrument, through which a household has leverage against future house price fluctuations. Finally, the empirical evidence across OECD countries sheds light on the role of the risky housing market in an agent's intertemporal consumption and investment decisions and broadly confirms our discussions of the theoretical results within the risky steady state framework.

What is missing in the modelling setup above and what warrants further investigation? Our model above makes the extreme assumption that house prices are an exogenous $\mathrm{AR}(1)$ process. However, the true stochastic process is likely to be much more complex than the one we have assumed, involving higher-order autoregressive or moving average terms (Cocco, 2005). Another important limitation is the assumption of the correlation between aggregate income and house prices. We focus mainly on the covariance of consumption with housing investment, house prices and the interest rate. The fact that we ignored this correlation may have an important impact on the covariance terms in the model. Further ignored features are financial frictions and the resulting borrowing constraints. At a more fundamental level, general equilibrium effects are missing in the simple representative agent model. Conventional wisdom is that house price fluctuations are driven by technology, tastes and various macroeconomic shocks in general equilibrium models. We leave such richer modelling frameworks for future research.

5

Optimal Life-Cycle Mortgage and Portfolio Choices in the Presence of the Affordability Constraint[1]

5.1 Introduction

In the past decades, the housing affordability issue has received an increasing amount of attention worldwide. In some emerging markets and developed countries, it has become the major topic on the housing policy agenda. The discussions mainly focus on new measurements of affordability, the structure of housing finance, mortgage instrument design and mortgage regulation. The public concern over housing affordability is attributed to two main reasons. First of all, owner-occupied housing is the single largest expenditure item in the budgets of most families. The average household devotes roughly one-quarter of its income to housing expenditure, while poor and young households commonly devote half of their income to housing. These high proportions imply that small percentage changes in housing prices will have large impacts on households' non-housing consumption and asset allocation. Second, the early years of this

[1] I record my gratitude to Cocco for kindly offering me the Matlab codes for his earlier paper (Cocco, 2005).

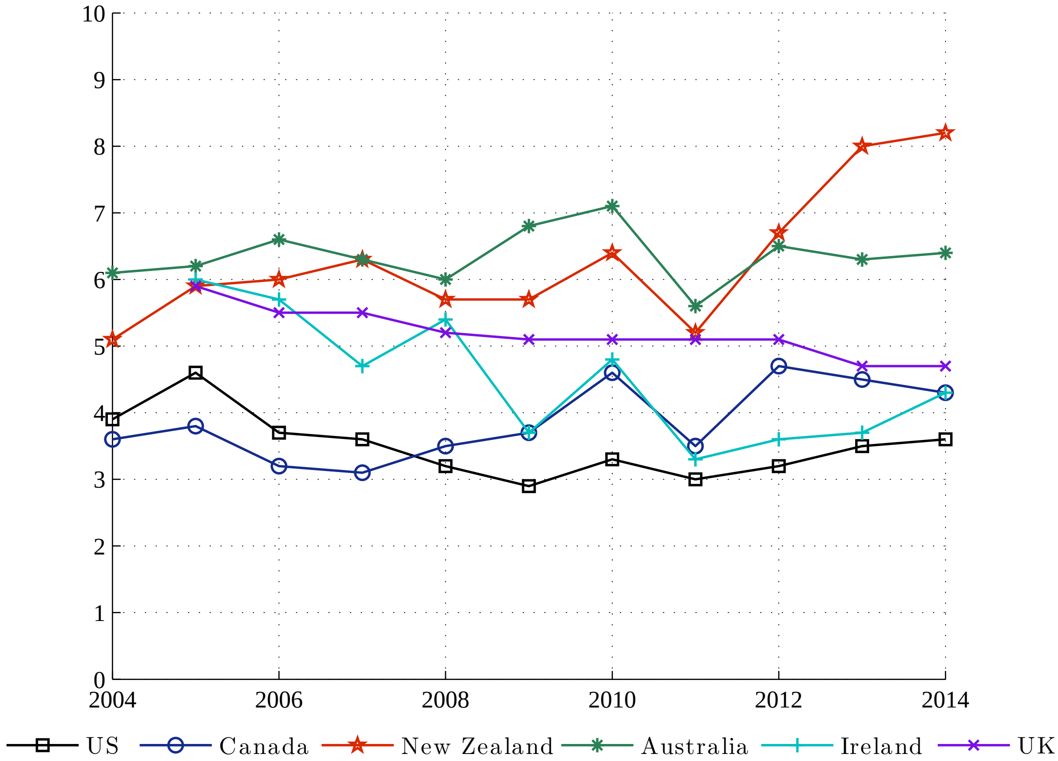


Figure 5.1: Housing Affordability across Countries: 2004–2014.

Note: The annual housing affordability in major markets is calculated by the *Demographia International Housing Affordability Survey 2015*. It uses the "Median Multiple" – the median house price divided by the gross annual median household income - to assess housing affordability. This indicator is widely used for evaluating urban markets and is recommended by the World Bank and the United Nations. Housing affordability ratings are assigned as follows using the Median Multiple: above 5.1: severely unaffordable; 4.1–5.0: seriously unaffordable; 3.1–4.0: moderately unaffordable; below 3.1: affordable. The 11^{th} *Demographia International Housing Affordability Survey 2015* provides housing affordability ratings on 86 major markets (with a population of over 1,000,000) and an overall total of 378 markets. This figure shows the average annual Median Multiples in 6 nations in 2004–2014.

century were characterized by unprecedented instability in house prices globally (Quigley and Raphael, 2004). There is substantial evidence of a growing housing affordability problem as well as a widening of differences across regions (see Figure 5.1). This crisis could further induce a severe problem in economic activity and raise concerns about the sustainability of the boom. The recent global financial crisis in 2007-09 triggered by subprime lending has added to these concerns (Gan and Hill, 2009).

Housing affordability issues have been increasingly documented in the research literature.

The previous works mainly focus on the definitions and measures of affordability and attempt to examine it in terms of different aspects, such as the deposit gap, borrowing capacity, house-price-to-income ratio and rent-to-income ratio. Bourassa (1996) proposes a borrowing-constraint-based measuring method and applies it to household survey data from 1989 - 1990 for Australia. He finds that only small percentages of households would have been able to afford homeownership without putting themselves into poverty. Gan and Hill (2009) develop a new approach to the construction of affordability indexes that links to the concept of value-at-risk from the finance literature. As such, the whole distribution of households is taken into account, rather than focusing on either low-income households or the median. Taltavull and Tang (2012) attempts to use a combination of two conventional affordability measures (the rent-toincome ratio and the residual income standards) to examine the affordability problem of social tenants in the English housing association sector. His analysis confirms the affordability problem in London, where nearly half of the existing housing association tenants fall well below the poverty line.

Despite the fact that a wide range of affordability measures have been developed and applied in different contexts throughout the international arena, "affordability" is still an ambiguous concept as there is no precise and unified definition within the popular press and academic circles. The main reason is that the affordability problem is influenced by a large number of issues in real economic activities. From a more preliminary conceptual point of view, Quigley and Raphael (2004) try to sort out these disparate issues, such as the distribution of housing prices, the distribution of housing quality, the distribution of income, the ability of households to borrow, the public affecting housing markets, the conditions affecting the supply of new or refurbished housing and the choices that people make about how much housing to consume relative to other goods. By doing so, they attempt to sketch policies that might improve the affordability for both homeowners and renters.

However, in response to the recent global financial crisis, it is particularly important to note that the problem of the affordability of mortgages for homeowners accompanied by spiking levels of repossessions, negative equity and bank losses may lead to severe consequences in both mortgage markets and housing markets (Bramley, 2011). Thus, the current affordability measurements centre on the "ability to borrow". In other words, the ability to afford property ownership depends on the household income and the mortgage loan: the higher the household income and/or the lower the mortgage loan, the more affordable the property (Taltavull et al., 2011). Against this background, in the US, major affordability indexes monitored by a variety of organizations are defined as a household's ability to qualify for conventional mortgage financing. For instance, the National Association of Realtors (NAR) index measures the ratio of 25% of the median monthly income to the monthly repayments on a fixed-rate mortgage on the median house at the current interest rates. The US Department of Housing and Urban Development (HUD) index computes the ratio of the median family income to the income required to qualify for a conventional mortgage on the median- valued house sold.[2] In this context, some economists analyse particularly the interrelationships between housing affordability and the mortgage market. For instance, Taltavull et al. (2011) analyse empirically the relationship between mortgage liquidity and housing affordability in Northern Ireland during the boom - bust cycle in the residential property market. They find that the affordability has been driven by the deregulation of the mortgage market, contributing to the rise in house prices and affordability pressures during the market up cycle in Northern Ireland.

Our study is basically related to two strands of literature. The first strand, concerning affordability, has been introduced in the preceding paragraph. The second relevant strand of literature addresses dynamic life-cycle portfolio choices in the presence of owner-occupied housing. The prior research on owner-occupied housing and portfolio choices began with the work of Grossman and Laroque (1990), who focus on the impact of the transaction cost of illiquid durable goods. Brueckner (1997) and Flavin and Yamashita (2002) use a static, one-period, mean-variance framework to study the relation between housing and investors' optimal holding of financial assets. The studies by Cocco (2005) and Yao and Zhang (2005a) are the first to investigate optimal housing, consumption and portfolio decisions in a dynamic life-cycle context. Other researchers extend their work in various directions. Van Hemert (2006) integrates the bond market and allows for an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM) and a combination of the two (a hybrid mortgage). Hu (2005) and Yao and Zhang (2005b) consider more realistic scenarios in the housing market, including costly mortgage refinancing and default penalties. Unlike these previous papers, we extend the earlier work of

[2]The third major index in the US is the National Association of Home Builders (NAHB) index. It measures the fraction of dwellings sold that could be purchased by the median household with 28% of the household income.

Cocco (2005) by adding the housing affordability constraint explicitly in an attempt to study the relationship between households' affordability and mortgage loan and to explore how the housing affordability constraint affects households' optimal consumption, mortgage, portfolio choices and poverty status at different ages.

Our findings are twofold. On the one hand, we confirm the robustness of the main results of Cocco (2005) and Hu (2005). They argue that integrating housing into standard life-cycle models can partially resolve the portfolio composition puzzle.[3] For many younger homeowners, investment in housing keeps their cash-on-hand low. They are relatively poor and need to convert all their spendable resources into housing assets, leaving less/no savings for financial investment. Housing investments crowd out stock investments. As homeowners age, housing equity (the house value net of debt) and human capital are more important relative to stocks and bonds for future consumption, since households on average have less mortgage liability in this period of life. Consequently, older homeowners are more willing to take risks in their liquid financial portfolio (Cocco, 2005). In the presence of the affordability constraint, we show that their findings still hold qualitatively. Quantitatively, our simulation results are more successful in matching the observations in the data.[4]

On the other hand, our study also makes a contribution to the affordability literature. According to the simulation results, the housing affordability constraint has a significant impact on optimal consumption, portfolio composition, mortgage choices and poverty status. Irresponsible borrowing easily puts households into poverty, especially those under the age of 40. Under the assumption that the affordability constraint is strictly taken into account, households manage to maintain a fairly smooth consumption pattern and keep the mortgage debt burden spread more evenly throughout their lifetime. Another purpose of this chapter is to assess quantitatively the influence of important factors concerning housing affordability in the model. More specifically, we focus primarily on the mortgage market and distinguish two types: "contracted" and "liberalized" (Taltavull et al., 2011). The contracted mortgage market is close

[3]Basic (standard) theoretical portfolio choice models without housing predict that the risky asset share is expected to decline with age. Nevertheless, much empirical evidence is at odds with the predictions: risky asset shares among young people are generally found to be low and hump-shaped, increasing or constant with age. Furthermore, young and poor households are usually borrowing-constrained and tend to hold less stock. (See Ameriks and Zeldes, 2004; Campbell, 2006; Guiso et al., 2003; Haliassos and Michaelides, 2002; Heaton and Lucas, 2000). These contradictions are regarded as "the composition puzzles".

[4]For detailed explanations, please see section 5.3.1.

to the traditional prime (standard) mortgage market, in which the borrower cost is primarily driven by the down payment alone, given that the minimum credit history requirements are satisfied. By contrast, a liberalized mortgage is similar to a subprime mortgage and it is normally made out to higher-risk borrowers who buy pricey houses relative to their income level and make little or no down payment yet have a high-interest rate. The main benefits of this type of mortgage are the increased numbers of homeowners and the opportunity for these homeowners to create wealth (Chomsisengphet and Pennington-Cross, 2006). Therefore, in our simulation exercises, we use the down-payment ratio and mortgage rate to characterize these mortgages' lending patterns. Instead of deriving the optimal mortgage contract, as in Piskorski and Tchistyi (2010, 2011), we attempt to illustrate quantitatively the impact of the affordability constraint in the subprime mortgage market as well and compare it with that in the prime market.

In the numerical experiments, we focus specifically on loan-to-value (LTV) and loan-to-income (LTI) ratio. The main reason is that these measures are of particular importance in the sense of macroprudential policies aimed at reducing and mitigating systemic risks. They are usually used according to whether they are aimed at borrowers (LTV and LTI ratios), banks' assets or liabilities (limits on credit growth, foreign currency credit growth and reserve requirements) and policies that encourage counter-cyclical buffers (counter-cyclical capital, dynamic provisioning and profits distribution restrictions)(Claessens et al., 2013). Recently, a new empirical literature on the effectiveness macroprudential regulatory policies seems to increasingly suggest that borrower-based instruments may be more effective in containing real estate bubbles than bank-based instruments(Hartmann, 2015). While macroprudential policies aim at financial stability, there are clear implications for housing affordability. As house prices and debt levels trend increasingly upwards, so too residential housing becomes less affordable, particular for first-time buyers. In the short-run, macroprudential policy tools tend to make credit less accessible, however, they should make house prices more affordable in the long-run and reduce the risks of a sharp housing downturn (Spencer, Deputy Governor and Head of Financial Stability of the Reserve Bank of New Zealand, speech to the Business NZ Council, Wellington, 2013). Against this background, we also attempt to scrutinise the interaction between borrower-based macroprudential policies and social policies aimed at improving poverty status and fostering home ownership and credit availability. On average, the optimal LTI and

LTV ratios in the subprime market decline strongly in the presence of affordability restrictions, keeping consumption, loans and poverty status fairly stable over the lifetime. Moreover, the mechanism of the affordability constraint works more effectively in the subprime market than in the prime market. Finally, our sensitivity analysis confirms the robustness of the main findings in the baseline model. It also emphasises that the magnitudes of the constraint influence households' behaviour in different ways. According to the results, we find a non-linear (hump-shaped) relationship between affordability degrees and poverty status. The reason is that a less strict affordability restriction allows people to accumulate more resources on hand, especially after the age of 40. As such, they could use them to smooth the consumption and rebalance their portfolios, which reduce the risk of putting themselves into poverty. The chapter proceeds by first presenting the life-cycle modelling set-up in section 5.2. In section 5.3, we provide the parameterization of the model and conduct a simulation and sensitivity analysis. Finally, section 5.4 concludes.

5.2 Model

In this section, we extend a realistic dynamic life-cycle model to study how the housing affordability constraint affects households' non-housing and housing consumption, portfolio choices in financial markets and poverty status throughout their lifetime. We consider a partial equilibrium model in which the households take all the prices as given.

5.2.1 Preferences

Putting overlapping generation aspects aside, we only need one index t to denote adult age. We assume that a representative household enters the model at the age of 30 and works for the first K periods and leaves our model after T periods. We allow for uncertainty during the household's life by taking the age-specific conditional survival probabilities π_t into account, where π_t denotes the probability that the household will be alive in period $t+1$, conditional on being alive in the previous period t (Hubbard et al., 1995). In each period, the household needs to choose nondurable goods consumption C_t and housing services H_t. While the former needs to be purchased anew in every period, housing services depend on the size of the house stock that the household chooses to own. More specifically, a household receives one unit of housing service for every unit of owned housing stock. The household is assumed to be concerned about its expected discounted lifetime utility from both housing services and non-durable goods. The preferences are given by:

$$E_1[\sum_{t=1}^{T} \beta^{t-1}(\prod_{j=0}^{t-1} \pi_j)u(C_t, H_t)], \quad \text{where} \quad u(C_t, H_t) = \frac{({C_t}^{1-\theta}{H_t}^{\theta})^{1-\gamma}}{1-\gamma}. \tag{5.1}$$

β is the time discount factor, γ is the coefficient of relative risk aversion and θ determines the housing service shares in a static model.

5.2.2 Labour Income

In each working period, the household receives an exogenous stochastic stream of labour income Y_t. Let $y_t \equiv \ln Y_t$, and y_t is commonly modelled as the sum of a permanent component and a transitory shock ε_t. The permanent component consists of a deterministic function $f(t, Z_t)$ of age t and individual characteristics Z_t and of a persistent income component υ_t following an AR(1) process:[5]

$$\upsilon_t = \phi \upsilon_{t-1} + u_t. \tag{5.2}$$

Therefore:

$$y_t \equiv \ln Y_t = f(t, Z_t) + \upsilon_t + \varepsilon_t, \quad \forall t \leq K, \tag{5.3}$$

where $\varepsilon_t \sim$ i.i.d$N(0, {\sigma_\varepsilon}^2)$, $u_t \sim$ i.i.d$N(0, {\sigma_u}^2)$ and u_t is uncorrelated with ε_t. Since most of the uncertainty related to future labour income in retirement has been resolved, the income at this stage of life is then modelled simply as a constant fraction of the permanent labour income in the last working period:

$$y_t = \ln(\zeta) + f(K, Z_K) + \upsilon_K, \quad \forall K < t \leq T, \tag{5.4}$$

[5] This assumption follows Hubbard et al. (1995). They estimate a general first-order autoregressive process and find the auto-correlation coefficient to be very close to one, indicating that $\phi \approx 1$. For convenience, we will use this value in the numerical analysis.

where ζ is the replacement ratio. We will explain the detailed numerical implementation of stochastic labour income in section 5.3.1.

5.2.3 Housing

Owner-occupied housing differs from other financial assets in that housing is both a durable consumption good and an investment instrument. Unlike traditional liquid financial assets, the housing asset enters an investor's wealth accumulation in a complicated way because it involves a down payment, mortgage debt and adjustment and maintenance costs. The choice about how much housing and which house to buy is a joint consumption-Cinvestment decision. To purchase a house, a fraction of the market house value is required as a down payment and the rest of the cost can be financed with a mortgage loan. Notice that mortgage payments over a long horizon are needed out of an uncertain stream of labour income, making the post-mortgage-payment income lower and more volatile. Once a house has been bought, there is less flexibility in housing expenditure since the maintenance costs and transaction costs associated with frequent trades can be very large. Therefore, we assume that the maintenance costs and transaction costs are equal to a proportion ψ and λ of the current house value, respectively. The price per unit of housing in period t is denoted by P_t, such that the market value of a house of size H_t is then equal to $P_t H_t$.[6] For computational simplicity, we model the house price as an exogenous stochastic process with a deterministic exponential trend. Let $p_t = \ln(P_t)$, then the detrended log price of housing per unit is defined as:

$$p_t{}' \equiv p_t - gt. \tag{5.5}$$

Although the house price is assumed to be an exogenous source, we should not neglect its role in the model. First of all, the house price is an important determinant of the affordability of homeownership. Some quantitative measures of housing affordability can be viewed as a relationship between home prices and household incomes. Rising home prices impede prospective households' accumulation of a down payment and raise other required housing expenses. Second, the price of housing in a given region is frequently affected by labour income shocks in

[6]Since we focus only on homeowners' consumption-investment behaviour in the model, we assume that there is a minimum house size H^{min} for each household. By doing so, the rental market is eliminated: $H_t \geq H^{min}, \quad \forall 1 \leq t \leq T$, see Cocco (2005).

the same region (Linneman and Megbolugbe, 1992). Cocco (2005) assumes that cyclical fluctuations in house prices are perfectly correlated with aggregate (permanent) labour income shocks:

$$\upsilon_t = \kappa_\upsilon p_t{}', \tag{5.6}$$

where κ_υ is the regression coefficient. However, in our baseline case, we abstract from this complication to make the model tractable.

5.2.4 Financial Assets

We assume that there are three financial assets in this model. A riskless asset, called bonds, has a constant gross real return $R_f = (1 + r_f)$. The risky asset, called stock, has a stochastic gross real return $R_t{}^s$, and its excess return is given by:

$$R_t{}^s - R_f = \mu + \eta_t, \tag{5.7}$$

where μ is the mean equity premium and η_t is the innovation to excess returns in period t and is assumed to be $\eta_t \sim$ i.i.d$N(0, \sigma_\eta{}^2)$. We denote the amount of bonds and stocks that the investor has in period t by B_t and S_t, respectively, and assume that the investor faces the following borrowing and short-sale constraints:[7]

$$B_t \geq 0, \tag{5.8}$$
$$S_t \geq 0. \tag{5.9}$$

Let α_t represent the proportion of liquid assets invested in stocks over the sum of bonds and stocks in period t. Then, these two constraints imply that $\alpha_t \in [0, 1]$ for $\forall$t. The third financial asset is the mortgage loan M_t. To become a homeowner, an investor needs to make a down payment, which is assumed to be a proportion δ of the housing value at loan initiation. To finance the rest, the investor can borrow a mortgage loan against his house with a constant

[7]The borrowing constraint 5.8 ensures that the investor's allocation to bills is non-negative on all the dates. It prevents the investor from capitalizing or borrowing against future labour income or retirement wealth. The short-sale constraint 5.9 ensures that the investor's allocation to equities is non-negative on all the dates.

gross real rate $R_M = (1 + r_M)$. The mortgage is assumed to mature at T, such that the entire balance is paid off in the terminal period. Therefore, a newly initiated mortgage satisfies:

$$0 \leq M_t \leq (1 - \delta)P_t H_t, \quad \forall 1 \leq t \leq T. \tag{5.10}$$

For convenient numerical implementation, we allow for costless renegotiation of the desired level of debt in each period. By doing this, the mortgage will be treated as a control variable but not a state variable in the computation.[8]

5.2.5 Housing Affordability Constraint

Investors – in other words, home buyers – must have sufficient wealth and income to gain access to mortgage loans. Earlier studies of housing tenure choice in the US (Linneman and Wachter, 1989) and Australia (Bourassa, 1995) show that households' borrowing constraints are significant determinants of their access to homeownership. A useful starting point for understanding housing affordability is the concept of an affordability constraint, which is similar to the "borrowing constraint" and "affordable limit" introduced by Bourassa (1995) and Gan and Hill (2009), respectively. The affordability constraint is defined as the ratio of the allowable loan to the maximum income allocated to the mortgage. More formally, the mortgage loan M_t is deemed affordable for a household with gross income Y_t in each period if it satisfies:

$$\frac{M_t}{\omega Y_t} \leq \text{Affordability Constraint (AC)}, \quad \forall 1 \leq t \leq T, \tag{5.11}$$

where ω denotes the maximum proportion of gross income required to qualify for a mortgage. Otherwise, the house is deemed unaffordable. Naturally, the consideration of ω is that its value might vary with income. However, ω does not seem to change much empirically with the income level. Piazzesi et al. (2007) find that in the US the lowest income quintile spends roughly the same percentage as the highest income quintile based on the data from the consumer expenditure survey for the years 1984–2002. We thus assume that the household buys a house and

[8] In fact, households could consider the mortgage debt as an additional resource, so that they can use it either for non-housing consumption and participation in the equity market or to invest in the housing market. However, under the assumption that mortgage debt is costlessly renegotiated in each period, an investor will never hold bonds and debt simultaneously.

the present values of the maximum affordable labour income stream and mortgage loan are:

$$\sum_{n=1}^{T} \frac{\omega E_1[Y_n]}{{R_f}^{n-1}}, \tag{5.12}$$

$$\sum_{n=1}^{T} \frac{E_1[M_n]}{{R_M}^{n-1}}. \tag{5.13}$$

Accordingly, the affordability constraint in period t is obtained as follows:

$$\frac{M_t}{\omega Y_t} \leq \frac{R_f - (\frac{1}{R_f})^{T-1}}{R_M - (\frac{1}{R_M})^{T-1}} \frac{R_M - 1}{R_f - 1}(AC), \quad \forall 1 \leq t \leq T. \tag{5.14}$$

In other words, the mortgage–income ratio should not be larger than ωAC in each period in the model.

5.2.6 Budget Constraints and Dynamic Optimization

In each period t, the household uses its total spendable resource or *cash-on-hand* Q_t, which consists of stochastic labour income Y_t and liquid wealth LW_t, to purchase non-housing goods C_t, to adjust its housing stock H_t or to invest in liquid assets S_t and B_t. It is expected that the household invests in liquid assets after paying for consumption and housing. The period t budget constraint is given by:

$$Q_t = \begin{cases} C_t + B_t + S_t + \psi P_t H_{t-1} - M_t, & \text{No Adjust} \\ C_t + B_t + S_t + P_t H_t - P_t H_{t-1} + (\psi + \delta + \lambda) P_t H_t - M_t, & \text{Adjust} \end{cases} \tag{5.15}$$

where

$$Q_t = LW_t + Y_t, \tag{5.16}$$

$$LW_t = B_{t-1} R_f + S_{t-1} {R_t}^s - M_{t-1} R_M. \tag{5.17}$$

The household maximizes its expected discounted lifetime utility 5.1 subject to budget constraints 5.15–5.17, the borrowing constraint for a mortgage 5.10, the affordability constraint 5.14

and no short sales for bonds 5.8 and stocks 5.9 on all the dates. A household's state depends on its beginning-of-period liquid wealth, realized labour income, current house price and size of the existing housing, that is:

$$X_t \equiv \{LW_t, Y_t, P_t, H_{t-1}\}. \tag{5.18}$$

Meanwhile, the control variables of this problem are given as:

$$A_t \equiv \{C_t, H_t, M_t, B_t, S_t\}. \tag{5.19}$$

Accordingly, the Bellman equation of the household's intertemporal consumption and investment problem can be written as:

$$V_t(X_t) = \max_{A_t} \left\{ \frac{\left({C_t}^{1-\theta}{H_t}^{\theta}\right)^{1-\gamma}}{1-\gamma} + \beta E_t[V_{t+1}(X_{t+1})] \right\}, \quad \forall 1 \leq t \leq T. \tag{5.20}$$

For simplicity of implementation, it is possible to standardize this maximization problem such that some state variables vanish. The standardization of the life-cycle portfolio choice model is most commonly performed by normalizing the problem by the permanent component of labour income ${Y_t}'$ (Cocco et al., 2005; Gomes and Michaelides, 2003, 2005; Polkovnichenko, 2007).[9] As a result of this standardization, the vectors of the state and control variables become:

$$x_t = \{q_t, {h_t}'\}, \tag{5.21}$$

$$a_t = \{c_t, h_t, m_t, a_t\}, \tag{5.22}$$

where $q_t = Q_t/{Y_t}'$ is the wealth–income ratio, ${h_t}' = P_t H_{t-1}/{Y_t}'$ is the household's beginning-of period house-value-to-income ratio, $c_t = C_t/{Y_t}'$ is the consumption-to-income ratio, $h_t = P_t H_t/{Y_t}'$ is the house-value-to-income ratio, $m_t = M_t/{Y_t}'$ is the mortgage-to-income ratio and $\alpha_t = S_t/(S_t + B_t)$ is the proportion of liquid assets invested in stocks over the sum of bonds and stocks. However, there is no general analytical solution to this optimization problem. We can only solve it numerically using backward induction (Judd, 1998).[10]

[9] According to the setting up of labour income, ${Y_t}' = \exp(\upsilon_t)$, where υ_t is the permanent income shock.

[10] The detailed standardizing of the optimal problem and the numerical techniques are given in Appendix B.

5.3 Numerical Analysis

Before we start the numerical analysis, a detailed parameterization needs to be specified. Fortunately, previous researchers have already estimated the relevant parameters based on data in the US, which are widely used to solve the life-cycle portfolio selection problem in the presence of housing.[11] Therefore, we rely fully on their results instead of estimating our own. In the baseline case, we set the annual discount factor β as 0.96 and the coefficient of relative risk aversion γ as 5. The weight that housing carries in the instantaneous utility function is set as 0.1, and the conditional survival probabilities π_t are parameterized based on the data from the National Centre for Health Statistics (Cocco, 2005). We assume that the household makes the decision annually starting at the age of 30 and leaves the model at the age of 80.[12]

The household's 50 years of life are divided into two stages: working and retirement. During the first stage (age 30–65), the household's income is determined by a permanent component and a transitory shock. The deterministic function $f(t, Z_t)$ is assumed to be additively separable in t and Z_t. The vector Z_t represents personal characteristics other than age, and the fixed household effect includes marital status and household size. To obtain the profiles for the numerical solution, we follow Cocco et al. (2005), who fit a third-order polynomial to the age dummies for each education group: no high school degree, high school degree and college degree. The coefficients are estimated using the Panel Study of Income Dynamics (PSID) data. In the baseline model, we use the wage profile of a household with a college degree. After retirement (age 65–80), the household receives a constant annual income determined by the replacement ratio ζ. It is calibrated as the ratio of the average of the labour income variable defined for retirees to the average of the labour income in the last working year prior to retirement. The error structure of the labour income process is obtained by following Carroll and Samwick's (1997) method, and the variance of permanent and transitory labour income shocks are taken from Cocco (2005) and set as 0.019^2 and 0.133^2, respectively. Based on these values, Figure 5.2 illustrates the average household's income by age for 50,000 simulation trials.

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[11]Campbell and Cocco (2015), Cocco (2005), Cocco et al. (2005), Hu (2005) and Yao and Zhang (2005a) estimate the relevant parameters based on US data.

[12]Since we eliminate the rent market and assume that the household holds at least H^{min} housing stock before entering the model, it is reasonable to set the starting age of the household as 30. This assumption is also consistent with Yao and Zhang's (2005a) findings. Before 30, the fraction of households owning a house is almost zero.

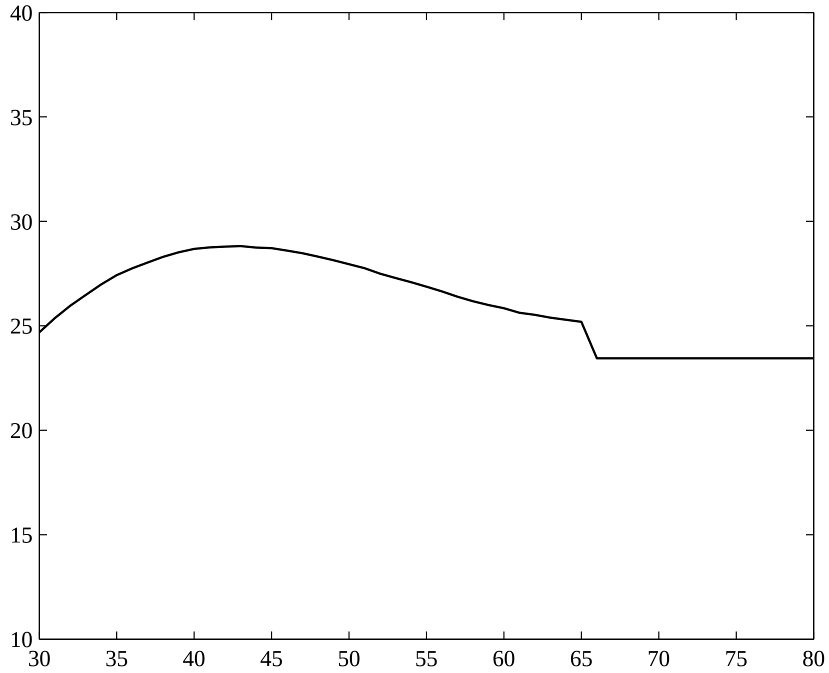

Figure 5.2: Simulated Household Income by Age.

In the calibration of the house price, we again follow Cocco's (2005) work. He estimates the annual growth rate of the log real house price as 1.59% and the standard deviation of the detrended log house price as 0.062. Nevertheless, he also points out that part of the increase is probably due to an improvement in the quality of housing, which cannot be accounted for using PSID data. Thus, a lower value of 1% is taken into account in his simulation. As such, we use this value in the baseline case as well. Despite the fact that the aggregate labour income is correlated with cyclical fluctuations in house prices, we set it as zero in the benchmark case for simplicity.

The transaction cost caused by trading a house is set at 10%, which is approximatively consistent with Smith et al.'s (1988) result of 8%–10%. We let the maintenance cost and down payment equal 1% and 30% of the value of the house, respectively. In general, mortgage loans require a higher rate than bonds, since they bear long-term interest rate risk, default risk and prepayment risk (Hu, 2005). In this model, we set the risk-free rate r_f as 2% per year and the annual mortgage

Parameter and Description	Value
Time discount parameter β	0.96
Curvature parameter γ	5.0
Preference for housing θ	0.1
Retirement age K	65
Maximum age T	80
Replace rate ζ	0.9388
Variance of permanent shocks ${\sigma_u}^2$	0.019^2
Variance of transitory shocks ${\sigma_\varepsilon}^2$	0.133^2
Real log house price growth g	0.01
Variance of detrended log house price ${\sigma_{p'}}^2$	0.062^2
Transaction cost λ	0.10
Maintenance cost ψ	0.01
Down payment δ	0.30
Mortgage rate r_M	0.04
Risk-free rate r_f	0.02
Mean equity premium μ	0.04
Variance of stock return ${\sigma_\eta}^2$	0.157^2
Max. prop. of gross income req. to qual. for a mortgage ω	0.30
Autoregression parameter ϕ	1.0

Table 5.1: Baseline Parameters.

rate r_m as 4%, which gives a 2% mortgage premium.[13] The excess return on stocks is the sum of two components: the mean equity premium μ, which we set as 4% following Fama and French (2002), and a stochastic element η_t, the annual standard deviation of which is parameterized to 0.157 based on Standard & Poor's 500 Index (Campbell and Viceira, 2002). As for the maximum proportion of gross income required to qualify for a conventional mortgage, we follow

[13]Most papers estimate that the average spread between the conventional mortgage rate and the Treasury bill rate is between 2.29% and 3.01% (Hu, 2005). Since we eliminate a number of potential risks in the model, we use a much lower annual mortgage rate premium of 2%.

the construction of the affordability index in the US and fix it at 30%. This value is also consistent with the conventional identification of people with the housing affordability problem, which is based on whether they pay more than a certain fixed percentage (25% or 30%) of their income for housing (Stone, 1990). Table 5.1 summarizes the parameters used in the baseline case.

5.3.1 Baseline Simulation Results

Our simulations begin with a baseline case. First of all, we simulate the exogenous stochastic labour income, housing prices and stock return based on the setting up performed in the previous section. Using the Bellman equation, we compute the household's optimal consumption, housing stock, mortgage and portfolio choices. The special features of life-cycle models, such as the household's finite horizon and age-dependent labour incomes, indicate that the policy function does not converge to a steady state. Thus, this problem can only be solved by backwards induction. To calculate the optimal decisions for the next period, we need to update the (standardized) cash-on-hand and housing stock at the beginning of each period. Once the optimal matrix of decision rules is derived, we may use it to generate 50,000 simulated optimal paths from age 30 to age 80. Finally, the statistical averages are obtained by simulations.

Our prime purpose is to examine the impact of the housing affordability constraint upon an investor's optimal consumption, mortgage, portfolio choices and poverty status over a lifetime. In accordance with the baseline parameters, simulation results are calculated for two scenarios in the prime (standard) mortgage market: with (scenario 1) and without (scenario 2) the affordability constraint. Table 5.2 shows the summary of the evolutions of the mean shares for various assets relative to the total assets, average consumption, income, mortgage-to-total-assets ratio, LTI, LTV and fraction of investors lying below the average poverty line in all the age groups. The overall qualitative feature of the evolutions of the portfolio composition is roughly consistent with Cocco's (2005) results. First of all, in both scenarios, stock- and bond-holdings are much less important than real estate and human capital when they are measured relative to the total assets. After retirement, real estate and human capital together account for 98% and 92% of the total assets in both scenarios. Secondly, our results also show that human capital is the most important component of wealth at all ages. Early in life, the shares of human capital in the total assets in scenarios 1 and 2 are as high as 69.9% and 74%. As the household ages, the fractions become smaller, and in the last age group they are both around 42%. Another similarity is that we find increasing portfolio shares invested in stock over the lifetime as well. Stockholding rises from 0.35%(3.9%) to 0.72%(7.3%) with (without) consideration of the affordability constraint as the household ages.

Asset	Scenario 1: With Affordability Constraint				Scenario 2: Without Affordability Constraint			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.0035	0.0045	0.0063	0.0072	0.0390	0.0056	0.0229	0.0730
Bonds	0.0015	0.0016	0.0021	0.0028	0.0004	0.0001	0.0012	0.0020
Real Estate	0.2959	0.3268	0.3848	0.5565	0.2200	0.2007	0.2317	0.5100
Human Capital	0.6991	0.6670	0.6068	0.4335	0.7400	0.7936	0.7442	0.4150
Variables								
Consumption	25.27	25.90	24.11	21.00	58.68	62.52	49.23	22.01
Income	26.95	28.64	26.50	23.55	26.95	28.61	26.46	23.54
Mortgage-to-Total-Assets-Ratio	0.0129	0.0142	0.0153	0.0220	0.1126	0.1059	0.1188	0.0920
LTI	0.3489	0.3438	0.3361	0.3229	3.0100	2.1400	2.1301	1.6900
LTV	4.39%	4.35%	3.99%	4.3%	51%	53%	51%	24%
% of Inv. Below Poverty Line	19.5%	19.75%	20.16%	19.3%	63.8%	18.8%	21.4%	33.2%

Table 5.2: Shares of Assets and Variables by Age in the Prime Mortgage Market.

Note: In this table, financial asset is the sum of stocks, bonds and real estate. Total asset is the sum of the financial asset and human capital. Human capital is the present discounted value of future income with the annual discounted rate of 5% following Heaton and Lucas (2000). For each age group, we calculate the average stock, bonds, real estate and human capital proportions in the total assets. The poverty line is calculated as 60% of the contemporary median household income, which includes earnings, retirement income, dividends, interest and income from real estate (OECD and US Department of Commerce).

Nevertheless, there are some magnitude differences between our results and Cocco's (2005). Although Cocco (2005) predicts an increasing life-cycle share of stock investments, the predictions of stockholdings are much less successful in matching the observations and the predicted values are on average higher than those in the data. In the presence of an affordable mortgage, our model predicts much lower shares of stocks across different age groups. Close to retirement (age 50–65), the household's stockholdings on average are 0.63%, whereas Cocco predicts 9.9% in the same age group, which is much higher than the observation in the data of 1.9%.[14] Despite our outcome at the baseline also showing mismatching with the empirical evidence, the further investigation into the sensitivity of the affordability constraint degrees in section 5.3.3 shows more promising results.[15]

Now we focus on the quantitative differences between scenario 1 and scenario 2. In the presence of the affordability constraint, mortgage debt is restrained not only by the household's labour income but also by the house value owned in each period. To avoid excess borrowing from banks, the household needs to assess its "ability" during the whole dynamic optimization process. As a result, the household manages to maintain a fairly smooth consumption pattern and keep the mortgage debt burden spread more evenly across different age groups. More specifically, according to the simulation results in scenario 1, the optimal consumption tracks closely the change in labour income and exhibits a humpshaped distribution with peaks in the household's 50s. After retirement, consumption decreases by 20% because of the reduction in labour income. When ignoring affordability, the consumption also has a hump shape, yet it is accompanied by much higher volatility throughout the lifetime: it falls from the peak value of 62.52 between the ages of 40–50 to 22.01 after retirement. Housing equity is well known as a mechanism to smooth consumption over time (Hurst and Stafford, 2004). According to the simulation, this mechanism only functions well under the consideration of mortgage affordability. Likewise, the mortgage-to-total-assets ratio, LTI and LTV on average are much smaller in scenario 1 than in scenario 2. Taking the LTV for example, if banks allow a household to increase the level of mortgage debt without any limitation, the LTV in early life is as high as 53% and declines slowly to 24% after retirement. Notice that the maximal LTV in the simulation is approximatively equal to 60% given a 30% down payment. In addition, we assume that all the households enter the model with a certain level of housing assets. It is reasonable that the simulated LTV ratios without the affordability constraint are below 60%.[16]

[14]The mean portfolio shares by age in the PSID data are reported by Cocco (2005, Table 9). The average stockholding shares related to the total assets in the age groups <35, 35–50, 50–65 and >65 are 0.2%, 0.7%, 1.9% and 2.9%.

[15]If the affordability constraint is relaxed (ω =0.5), the average stockholding in the age group 50–65 increases to 1.2%, see section 5.3.3, Table 5.4.

[16]On average, the LTV is roughly 45.8% before retirement based on the statistics for housing from the Survey

Consistent with the behaviour of the LTV, the poverty status of households is also strongly influenced by the affordability constraint. To illustrate this, we first compute the average poverty line for each age group. Following the OECD and the US Department of Commerce, the poverty line is calculated as 60% of the contemporary median household income including earnings, retirement income, dividends, interest and income from real estate. Then, we calculate the average percentage of households living below the poverty line in different age groups. Without consideration of the affordability constraint, irresponsible borrowing can easily puts younger people into poverty: 63.8% of households in the 30–40 age group lie below the poverty line. Although the living situation improves as households age, some of them might suffer from poverty again after retirement. On the other hand, if people take their "ability to borrow" into consideration, this percentage drops strongly across the different age groups and fluctuates between 19.3% and 20.1%. As a matter of fact, housing-caused poverty is also considered to be a measure of housing affordability. Instead of using a certain fixed percentage for the mortgage, housing-caused poverty is a sliding scale of affordability, which varies with income and household type.[17] In this context, our results also reflect the fact that even when households consider their capacity to borrow strictly throughout their lifetime, poverty cannot be eliminated. This implies that policy makers need to take into account more than one affordability measure for new housing reforms.[18]

5.3.2 Effect of the Affordability Constraint in the Subprime Mortgage Market

Another purpose of this study is to assess quantitatively the influence of the housing affordability constraint when households confront different mortgage markets. Besides the prime mortgage market, there is a liberalized mortgage market known as the subprime mortgage market. This type of mortgage lending aims to increase the numbers of homeowners and the opportunities for those homeowners to create wealth. However, it comes at a price and is normally described as high-cost lending (Chomsisengphet and Pennington-Cross, 2006). As a result, the interest rates for subprime loans are substantially higher than those for prime loans. According

of Consumer Finances (SCF) in the US, see Yao and Zhang (2005b).

[17] This measure is first introduced by Stone (1990, 2010) as "shelter poverty". Another similar concept, "housing-induced poverty" is discussed by Kutty (2005).

[18] It is also discussed by Taltavull et al. (2011).

to the report of the Freddie Mac Primary Mortgage Market Survey, the difference between the prime and the subprime market is two to three percentage points. Thus, we set the mortgage rate in the subprime market as 7% and the down payment ratio as 10% to match the features of the subprime market. On the other hand, the deregulation of the mortgage market has acted to increase the average debt burden while reducing the average down payment and it also seems to have contributed to the rise in house prices (Gan and Hill, 2009). As such, we also differentiate the prime and subprime mortgage markets by considering different magnitudes of volatility of house prices and set the standard deviations of the detrended log house price in the prime and subprime market as 0.062 and 0.1145, respectively.[19]

Asset	Scenario 3: Prime Mortgage Market δ=30%, r_M=4%, $\sigma_{p'}$=0.062				Scenario 4: Subprime Mortgage Market δ=10%, r_M=7%, $\sigma_{p'}$=0.1145			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Consumption	25.27	25.90	24.11	21.00	24.68	26.43	24.55	23.34
House-Value-to-Income-Ratio	8.05	7.89	8.43	8.02	3.96	3.71	4.14	7.29
Liquid-Wealth-to-Income-Ratio	0.13	0.14	0.18	0.15	0.064	0.053	0.056	0.14
LTI	0.35	0.34	0.33	0.32	0.45	0.47	0.44	0.35
LTV	4.39%	4.35%	3.99%	4.3%	11.63%	10.58%	10.55%	7.08%

Table 5.3: Evolution of Variables by Age in the Prime and Subprime Mortgage Markets with the Affordability Constraint.

Note: This table reports the households' average optimal non-housing consumption, house-value-to-income ratio, liquid-wealth-to-income ratio, LTI and LTV based on 50,000 simulated optimal paths in the prime and subprime mortgage markets.

Table 5.3 reports the primary results of the numerical experiment in the subprime mortgage market (scenario 4). For comparison purposes, we also publish part of the results in the prime market in this table and name it scenario 3. Firstly, we notice that the evolutions of the average optimal consumption have similar patterns in the two markets: they evolve smoothly and exhibit hump shapes with peaks in the households' 50s. We also notice that the housing

[19] $\sigma_{p'}$ is estimated as 0.1145 by Campbell and Cocco (2015).

affordability restriction influences the housevalue- to-income ratio and the liquid-wealth-to-income ratio in the prime and subprime markets in different ways. In the former, the two ratios are quasi-hump-shaped or hump-shaped and peak just before retirement. The house-value-to-income ratio starts at round 8 and reaches the peak value of 8.43 around retirement. Young households accumulate wealth slowly until retiring. After the age of 65, their wealth declines as they draw down the wealth accumulated during their working years to supplement their retirement income to pay for housing and non-housing goods expenses (Yao and Zhang, 2005b).

In the subprime mortgage market, the affordability constraint crowds out the advantage of the lower down-payment rate for young households. Compared with scenario 3, there is a significant drop in the house-value-to-income ratio in scenario 4. It begins at 3.96 and increases with fluctuations to 4.14. After retirement, it reaches the peak value of 7.29. In each period, households need to evaluate their capability to borrow before upgrading their housing investment. Since this investment becomes more expensive in the subprime market, younger households cannot keep their house value as high as that in the prime market. Not surprisingly, the deregulation in the mortgage market (subprime mortgage) has relaxed the borrowing constraint (a larger LTI in scenario 4 than in scenario 3), but the growing burden imposed by the high mortgage interest rate confines households' capability to accumulate house value; therefore, it causes a low house-value-to-income ratio but with an accompanying high LTI and LTV in the subprime market throughout the households' lifetime.

Finally, to examine the effectiveness of the affordability constraint in the subprime mortgage market, we also calculate the households' average optimal consumption, house-value-to-income ratio, LTI, LTV and housing-caused poverty in the same manner as in Table 5.2 and a brief summary is reported in Table B.1 (see Appendix B). Unsurprisingly, the affordability constraint also has a strong impact on households' investment behaviour, and as a stabilizer it is more effective in the subprime market than in the prime market. Taking the LTV for example, in scenario 6 Table B.1 (without the affordability constraint), it exhibits a monotonous falling pattern with the peak value of 63.61% in households' 30s. Meanwhile, in scenario 5 (with the affordability constraint), a similar trend is found but with significantly lower values. It starts with the peak value of 11.63% and declines to 7.08% as the household ages. Likewise, the optimal consumption decreases on average by 70 percentage points before retirement, when compared with the prime market, in which the corresponding value is only 55 percentage points.

5.3.3 Sensitivity Analysis

To check the robustness of our findings and the effects of certain parameters of the model, we also conduct simulation experiments for alternative parameterizations. Specifically, we consider different degrees of the mortgage affordability constraint by changing the value of ω. In fact, the thresholds of the housing-cost-to-income ratio have increased in the US over time. The Housing and Community Development Act of 1974 set this ratio as 25%; the Omnibus Budget Reconciliation Act of System of 1981 increased it to 30%; in 1979 and 1983 Congress enacted laws establishing a system of preferences for housing assistance and one of the criteria was a housing cost burden in excess of 50% of income (Kutty, 2005).[20] As such, we consider the cases of lower (ω =0.25) and higher (ω =0.5) values of the maximum proportion of gross income required to qualify for a mortgage.

Table 5.4 compares the mean portfolio shares, average optimal mortgage-to-total-assets ratio, LTI, LTV, liquid-wealth-to-income ratio and percentage of households living below the poverty line for these cases. The overall impression is that the portfolio shares in both cases evolve with a similar pattern to that in the baseline case. When the constraint becomes less strict (ω =0.50), a household has access to a higher loan such that it becomes more active in the equity market and accordingly more liquid assets are accumulated, resulting in a higher liquid-wealth-to-income ratio. Meanwhile, a less strict affordability constraint implies a heavier debt burden. Therefore, the mortgage-to-total-assets ratio, LTI and LTV are on average higher than those in the baseline case and in the case of ω =0.25.

One surprising result is that the average percentage of households living below the poverty line for all the age groups in the relaxed constraint (ω =0.50) case are approximately equal to or even smaller than those in the other cases, including the baseline case and the case without the affordability constraint. This finding indicates that there is a non-linear (hump-shaped) relation between affordability degrees and poverty status, particular in the age groups above 40 (see Figure 5.3). Relaxing constraint (higher ω) on credit allow people to hoard more resource to

[20]Nevertheless, the 30% of income standard is still widely used in the US. According to the 2006 American Community Survey (ACS), 37% of owners with mortgages and 16% of owners without mortgages spend 30% or more of their income on housing costs (Schwartz and Wilson, 2008).

Asset	ω=0.25				ω=0.50			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.0026	0.0038	0.0052	0.0050	0.0082	0.0088	0.0111	0.0120
Bonds	0.0014	0.0017	0.0019	0.0018	0.0012	0.0014	0.0019	0.0021
Real Estate	0.3071	0.3477	0.3998	0.5698	0.2598	0.2712	0.3282	0.5291
Human Capital	0.6887	0.6466	0.5929	0.4233	0.7308	0.7186	0.6588	0.4569
Variables								
Mortgage-to-Total-Assets-Ratio	0.0102	0.0105	0.0120	0.0150	0.0243	0.0279	0.0300	0.0336
LTI	0.2800	0.2634	0.2637	0.2328	0.63	0.62	0.61	0.48
LTV	3.3%	3.0%	2.9%	2.8%	10.0%	10.3%	9.2%	8%
Liquid-Wealth-to-Income-Ratio	0.11	0.14	0.16	0.1	0.25	0.23	0.26	0.19
% of Inv. Below Poverty Line	19.5%	19.6%	19.9%	19.2%	19.65%	19.5%	19.3%	17.2%

Table 5.4: Different Degrees of Affordability Constraint in the Prime Mortgage Market.

Note: This table reports the average portfolio shares of various assets relative to total assets and the average optimal mortgage-to-total-assets ratio, LTI, LTV, liquid-wealth-to-income ratio and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the prime mortgage market. The other parameters are held the same as in the baseline case.

smooth their consumption and rebalance the portfolios. Recall that if the constraint is ignored entirely (scenario 2 in Table 5.2), the heavy debt burden squeezes out the benefits obtained in the equity market, and younger homeowners will find themselves trapped in poverty. This implies that in light of the age profile of households and feature of credit markets, the magnitudes of the affordability constraint are needed to be taken into account.

Another important component of our model is the housing asset, and as such we examine the impact of the affordability constraint for different parameters related to housing as well, including the preference for housing θ, real log house price growth rate g and standard deviation of the detrended log house price $\sigma_p{}'$. When the importance of housing increases, due to the limited access to mortgage loans, households have to reduce their participation in the equity market (smaller stockholdings) in exchange for more investment in real estate. Households' av-

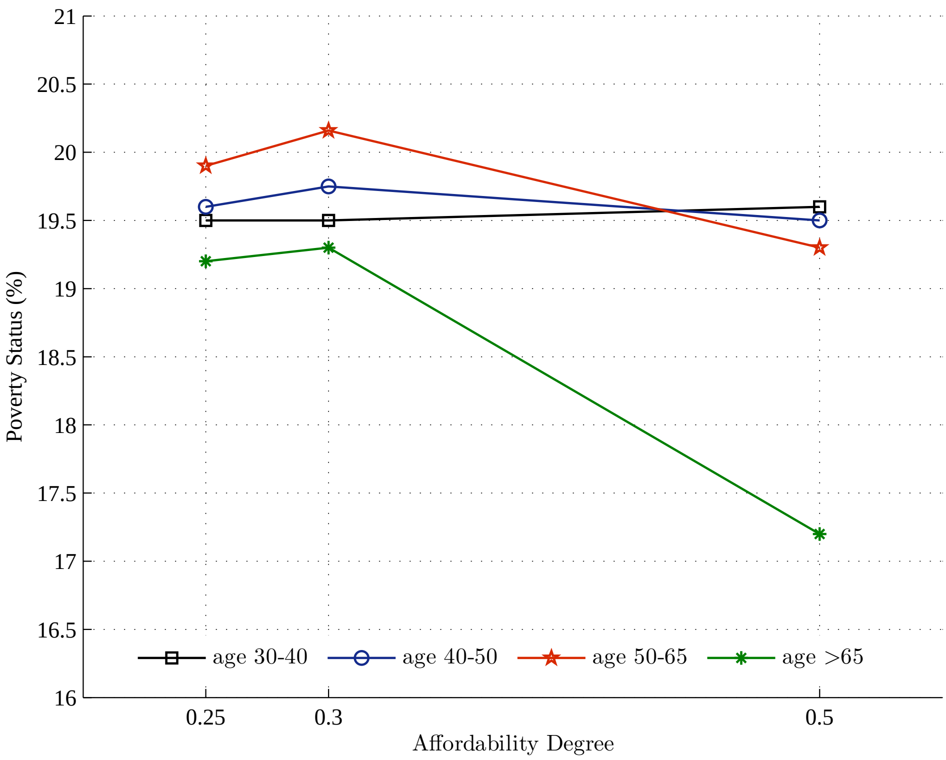

Figure 5.3: Hump-shaped Relation between Affordability Degrees and Poverty Status.

Note: Poverty status is calculated as the average percentage of households living below the corresponding poverty lines in different age groups.

erage stockholding and bondholding decrease while their housing asset and their house-value-to-income ratio increase throughout their lifetime. On the other hand, without the limitation on borrowing, it is no longer necessary for households to quit the stock market. However, massive investment in both equity and housing markets easily brings them into severe poverty not only early in life but also after retirement (see Appendix B, Table B.2).

Similar to this, a higher real log house price growth rate motivates people to invest more in real estate, but the affordability constraint as a stabilizer again weakens the boost in housing and stockholding. The housing asset is well known as a self-insurance instrument against future risk. If households expect higher house price risk, they have a much stronger incentive to invest in real estate, resulting in a lower level of stockholding at the same time. This mechanism of the housing asset is well documented by many researchers, such as Banks et al. (2012), Cocco

(2005) and Han (2010). However, in the presence of the affordability constraint, households' investment behaviour is strictly restricted, making the hedging effect and crowding-out effect of house price risk less straightforward.

The fixed return to riskless bonds r_f also has an impact on our affordability constraint. For a given mortgage rate, the lower r_f is, the less strict the restriction is and vice versa. Accordingly, we set r_f at a lower (0.01) and a higher value (0.03), respectively, and the overall impression is that in the presence of affordability the patterns of the average portfolio composition, LTI, LTV, house-value-to-income ratio, liquid-value-to-income ratio, poverty status and non-housing goods consumed resemble those in the baseline case. However, there are also some quantitative differences that are worth noticing. For instance, the share of bondholding increases across all the age groups, since a higher r_f indicates a higher return on bonds. On the other hand, a higher r_f also implies a stricter affordability constraint. As such, in the case with a higher r_f, the average LTI and LTV drop and consequently the living quality improves (fewer households living below the poverty line) in all the age groups. In addition, the magnitude of these variations becomes much larger in the case without considering the affordability limitation.

Last but not least, we are also interested in the effect of the mortgage rate r_M in the model. In the baseline case, we assume that mortgage lenders require a positive spread between the mortgage rate and the risk-free rate for compensation, because of the potential risks of default and refinancing. We now examine the effects of a lower mortgage premium, given the risk-free rate. In particular, we set r_M as 2% and 6%, which give a 0% and 4% mortgage premium, in the prime and subprime mortgage markets, respectively. As we expected, with limited access to credit, the patterns of the optimal average portfolio composition, credit borrowing behaviour, poverty status and consumption do not change much compared with the baseline case in both mortgage markets (see Appendix B, Table B.3). The reason is that the lower mortgage rate has a dual effect on households' investment behaviour in the presence of the affordability constraint. On the one hand, the lower mortgage rate means "cheaper" loans, and as such households are willing to enlarge their housing stock. On the other hand, a small r_M implies a more restricted borrowing constraint (according to equation 5.14), which leads to less mortgage borrowing. Our simulation results show that the mortgage, LTI and mortgage-to-total-assets ratio across different age groups decrease slightly in both markets when r_M decreases, indicating that the affordability effect overtakes the benefit of "cheaper" loans. However, if mortgage lenders allow households to borrow loans beyond their ability, we reach different conclusions. Apparently, households have more incentive to invest in real estate at a lower mortgage rate/smaller mortgage premium. As a consequence, the average mortgage, LTI and mortgage-to-total-assets ratio rise throughout the lifetime. In early life, a mortgage borrowed from the prime market increases from 80 to 113.8, which gives an increment of the LTI from 3.01 to 4.25. After retirement, the LTI declines slightly but is still as high as 3.13. Meanwhile, in the subprime market, the LTV ratio increases from 63.61% to 67.72% early in life, when the mortgage rate r_M decreases from 7% to 6%. For all the alternative scenarios discussed in this section, the overall impression is that our sensitivity analysis confirms the robustness of the findings in the baseline case. Further, it emphasizes that the affordability constraint, along with its degree, is an important determinant of households' investment behaviour and poverty status caused by housing.

5.4 Conclusion

Housing affordability issues have been documented increasingly in the recent research literature; nevertheless, most studies focus mainly on the empirical measurement and examination of the affordability problem of homeowners or renters. The collapse of the global financial markets in 2007 sends us a clear sign that careful research is necessary to assess the extent to which the mortgage market must be restructured to deal with these problems. As such, we attempt to investigate the housing affordability problem in a well-developed, calibrated, rational and dynamic life-cycle modelling framework. By incorporating explicitly the affordability constraint in the mortgage market, we focus primarily on households' "ability to borrow". Our simulation results indicate that the affordability constraint has a significant impact on households' optimal non-housing consumption, LTI, LTV, portfolio shares and poverty situation with age. Especially for young households that take out mortgage debt irresponsibly, the heavy burden imposed by housing investment significantly reduces the total spendable resources on hand, easily putting them into poverty, whereas when households assess their borrowing ability more carefully, they can maintain a fairly smooth consumption pattern and evenly spread mortgage burden. Furthermore, this mechanism works more effectively in the subprime market than in the prime market. Additionally, we conduct simulation experiments with alternative parametrizations and our sensitivity analysis confirms the robustness of the findings in the baseline case. One surprising result is that we find a non-linear relationship between affordability degrees and poverty status. After the age of 40, relaxing constraints (higher ω) on credit allow people to hoard more resource to smooth their consumption and rebalance the portfolios. As such, they could obtain benefits from both equity and housing markets. Although the average LTV increases slightly across all the age groups, it is not a threat to the living quality after their retirement. Meanwhile, this study also serves as scenarios tests intended to investigate the impact of borrower-based macroprudential policies on social policies aimed at improving poverty status and fostering home ownership and credit availability.

The previous works show that the dynamic life-cycle model of portfolio choice and housing offers a useful framework to study consumption behaviour and investment choices. The advantage of this type of modelling is that many realistic scenarios can be framed well and studied quantitatively, such as a rental market for housing services (Yao and Zhang, 2005a), a fixed stock market participation fee (Cocco, 2005), a refinancing charge, a default penalty (Yao and Zhang, 2005b) and mortgage default decisions under an adjustable/fixed rate, inflation and taxation (Campbell and Cocco, 2015). The present study can be added to the list of these scenario studies. However, some limitations of the dynamic life-cycle model need to be noted as well. Although many of them have been acknowledged and discussed before, in terms of the affordability constraint we have several concerns. First, labour income is the foundation of the construction of the affordability constraint in our model. Since we consider a partial equilibrium model, income is naturally assumed to be an exogenous and independent stochastic process. However, the true mechanism is far more complex than the one that we have used. For instance, if the income is correlated positively with the house price, housing becomes riskier, because it is not as good a hedge against labour income risk. Households prefer liquid financial assets to housing assets (Cocco, 2005). However, if this correlation is neglected, we expect a positive relationship between income and housing stock. A further limitation is that we assume that households evaluate their borrowing/affording capacity using the same affordability degrees in each period (annually), which is highly unlikely in reality. Since mortgage payment is usually determined by a standard annuity formula, renegotiation is associated with high costs that could affect households' intertemporal decisions.

The model is able to illustrate the impact of the affordability constraint throughout the lifetime; noting the limitations mentioned above, we do not suggest that it should be rigorously applied to housing investment activities. However, considering our numerical experiments as suggestive evidence, it is apparent that sound standards for affordability assessment are needed to reduce the mortgage payment problems and avoid the incidence of mortgage default. In a deeper sense, this is also propitious to contain systemic risks and the macroeconomic costs of financial instability. A possible extension of this study is to consider more than one affordability assessment while updating the calibration of the model by using multi-country OECD data for a more comprehensive and comparative analysis.

A

Supplements Chapter 4

Derivation of Equation 4.25:

The Euler equation is given by

$$1 = \beta E_t[r_{t+1}(\frac{c_{t+1}}{c_t})^{\Delta}(\frac{p_t}{p_{t+1}})^{\Lambda}],$$

with

$$\Delta = -\Big\{\frac{\gamma}{1-\alpha}[\alpha^2 + (1-\alpha)^2] + \alpha\Big\},$$
$$\Lambda = -\frac{\alpha\gamma}{1-\alpha}. \tag{A.1}$$

We define

$$f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t) \equiv \beta r_{t+1}\Big(\frac{c_{t+1}}{c_t}\Big)^{\Delta}\Big(\frac{p_t}{p_{t+1}}\Big)^{\Lambda} - 1, \tag{A.2}$$

The Euler equation A.1 can be rewritten as

$$E_t[f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t)] = 0. \tag{A.3}$$

Following Coeurdacier et al.'s (2011) risky steady state strategy, we replace equation A.3 with its second-order Taylor expansion Φ around the expected future variable

$$\begin{aligned} 0 &= \mathrm{E_t}[\mathrm{f}(\mathrm{c_{t+1}}, \mathrm{c_t}, \mathrm{r_{t+1}}, \mathrm{p_{t+1}}, \mathrm{p_t})] \\ &\approx \Phi[\mathrm{E_t}(\mathrm{c_{t+1}}), \mathrm{E_t}(\mathrm{r_{t+1}}), \mathrm{E_t}(\mathrm{p_{t+1}}), \mathrm{c_t}, \mathrm{p_t}], \end{aligned} \tag{A.4}$$

where

$$\begin{aligned} \Phi &= \beta \mathrm{E_t}(\mathrm{r_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} - 1 \\ &+ \frac{\beta(\Delta-1)\Delta}{2} \mathrm{Var_t}(\mathrm{c_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})^2} \\ &+ \frac{\beta(\Lambda+1)\Lambda}{2} \mathrm{Var_t}(\mathrm{p_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{p_{t+1}})^2} \\ &- \beta\Delta\Lambda \mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{p_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{p_{t+1}})} \\ &+ \beta\Delta \mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{1}{\mathrm{E_t}(\mathrm{c_{t+1}})}. \end{aligned} \tag{A.5}$$

Multiplying equation A.5 by the non-zero $\beta^{-1}\mathrm{E_t}(\mathrm{r_{t+1}})^{-1}\Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{-\Delta}\Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{-\Lambda}$ term gives us

$$\begin{aligned} 0 &\approx \widehat{\Phi}[\mathrm{E_t}(\mathrm{c_{t+1}}), \mathrm{E_t}(\mathrm{r_{t+1}}), \mathrm{E_t}(\mathrm{p_{t+1}}), \mathrm{c_t}, \mathrm{p_t}] \\ &= 1 - \frac{1}{\beta \mathrm{E_t}(\mathrm{r_{t+1}})} \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{-\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{-\Lambda} \\ &+ \frac{(\Delta-1)\Delta}{2} \frac{\mathrm{Var_t}(\mathrm{c_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})^2} + \frac{(\Lambda+1)\Lambda}{2} \frac{\mathrm{Var_t}(\mathrm{p_{t+1}})}{\mathrm{E_t}(\mathrm{p_{t+1}})^2} \\ &- \Delta\Lambda \frac{\mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{p_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{p_{t+1}})} + \Delta \frac{\mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{r_{t+1}})}. \end{aligned} \tag{A.6}$$

By moving the second term to the left and multiplying both sides by $\mathrm{E_t}(\mathrm{r_{t+1}})$, equation A.6 will be transferred into

$$\frac{1}{\beta}\Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{-\Delta}\Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{-\Lambda}$$

$$= E_t(r_{t+1})\Big[1 + \frac{(\Delta-1)\Delta}{2}\frac{Var_t(c_{t+1})}{E_t(c_{t+1})^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{Var_t(p_{t+1})}{E_t(p_{t+1})^2}$$
$$- \Delta\Lambda\frac{Cov_t(c_{t+1}, p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})}\Big] + \Delta\frac{Cov_t(c_{t+1}, r_{t+1})}{E_t(c_{t+1})}. \tag{A.7}$$

At the risky steady state, this approximation becomes

$$\frac{1}{\beta} = \bar{r}\Big[1 + \frac{(\Delta-1)\Delta}{2}\frac{\overline{Var_t}(c_{t+1})}{\bar{c}^2} + \frac{(\Lambda+1)\Lambda}{2}\frac{\overline{Var_t}(p_{t+1})}{\bar{p}^2}$$
$$- \Delta\Lambda\frac{\overline{Cov_t}(c_{t+1}, p_{t+1})}{\bar{c}\bar{p}}\Big] + \Delta\frac{\overline{Cov_t}(c_{t+1}, r_{t+1})}{\bar{c}}. \tag{A.8}$$

Second-Order Approximation of the Euler Equation:

The second-order expansion of Euler equation around the expected future variable is given by

$$\begin{aligned}
0 &= E_t[f(c_{t+1}, c_t, r_{t+1}, p_{t+1}, p_t)] \\
&\approx \Phi[E_t(c_{t+1}), E_t(r_{t+1}), E_t(p_{t+1}), c_t, p_t] \\
&= f(E_t(c_{t+1}), E_t(r_{t+1}), E_t(p_{t+1}), c_t, p_t) \\
&+ \frac{1}{2}f_{c_{t+1}c_{t+1}}E_t(c_{t+1} - E_t(c_{t+1}))^2 \\
&+ \frac{1}{2}f_{p_{t+1}p_{t+1}}E_t(p_{t+1} - E_t(p_{t+1}))^2 \\
&+ \frac{1}{2}f_{r_{t+1}r_{t+1}}E_t(r_{t+1} - E_t(r_{t+1}))^2 \\
&+ f_{c_{t+1}p_{t+1}}E_t[(c_{t+1} - E_t(c_{t+1}))(p_{t+1} - E_t(p_{t+1}))] \\
&+ f_{c_{t+1}r_{t+1}}E_t[(c_{t+1} - E_t(c_{t+1}))(r_{t+1} - E_t(r_{t+1}))] \\
&+ f_{r_{t+1}p_{t+1}}E_t[(r_{t+1} - E_t(r_{t+1}))(p_{t+1} - E_t(p_{t+1}))] \\
&= \beta E_t(r_{t+1})\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{\Lambda} - 1 \\
&+ \frac{1}{2}f_{c_{t+1}c_{t+1}}Var_t(c_{t+1}) + \frac{1}{2}f_{p_{t+1}p_{t+1}}Var_t(p_{t+1}) \\
&+ \frac{1}{2}f_{r_{t+1}r_{t+1}}Var_t(r_{t+1}) + f_{c_{t+1}p_{t+1}}Cov_t(c_{t+1}, p_{t+1}) \\
&+ f_{c_{t+1}r_{t+1}}Cov_t(c_{t+1}, r_{t+1}) + f_{r_{t+1}p_{t+1}}Cov_t(r_{t+1}, p_{t+1})
\end{aligned}$$

$$
\begin{aligned}
&= \beta \mathrm{E_t}(\mathrm{r_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} - 1 \\
&+ \frac{\beta(\Delta-1)\Delta}{2} \mathrm{Var_t}(\mathrm{c_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})^2} \\
&+ \frac{\beta(\Lambda+1)\Lambda}{2} \mathrm{Var_t}(\mathrm{p_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{p_{t+1}})^2} \\
&- \beta\Delta\Lambda \mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{p_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{\mathrm{E_t}(\mathrm{r_{t+1}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{p_{t+1}})} \\
&+ \beta\Delta \mathrm{Cov_t}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}}) \Big(\frac{\mathrm{E_t}(\mathrm{c_{t+1}})}{\mathrm{c_t}}\Big)^{\Delta} \Big(\frac{\mathrm{p_t}}{\mathrm{E_t}(\mathrm{p_{t+1}})}\Big)^{\Lambda} \frac{1}{\mathrm{E_t}(\mathrm{c_{t+1}})}. \qquad \text{(A.9)}
\end{aligned}
$$

Derivation of the Solution System and the Risky Steady State:

The optimality conditions are given by a five-dimension system. The first equation is equation A.8

$$
\begin{aligned}
0 &= 1 - \frac{1}{\beta \bar{\mathrm{r}}} + \frac{(\Delta-1)\Delta}{2} \frac{\overline{\mathrm{Var_t}}(\mathrm{c_{t+1}})}{\bar{\mathrm{c}}^2} + \frac{(\Lambda+1)\Lambda}{2} \frac{\overline{\mathrm{Var_t}}(\mathrm{p_{t+1}})}{\bar{\mathrm{p}}^2} \\
&- \Delta\Lambda \frac{\overline{\mathrm{Cov_t}}(\mathrm{c_{t+1}}, \mathrm{p_{t+1}})}{\bar{\mathrm{c}}\bar{\mathrm{p}}} + \Delta \frac{\overline{\mathrm{Cov_t}}(\mathrm{c_{t+1}}, \mathrm{r_{t+1}})}{\bar{\mathrm{c}}\bar{\mathrm{r}}}. \qquad \text{(A.10)}
\end{aligned}
$$

The remaining equations are given as follows

$$
\begin{aligned}
\frac{\mathrm{d}\widehat{\Phi}}{\mathrm{d}\omega_{\mathrm{t-1}}}\Big|_{\mathrm{S_t}=\bar{\mathrm{s}}} &= \frac{\partial\widehat{\Phi}}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\omega_{\mathrm{t-1}}} + \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{c_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{c_{t+1}})}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\omega_{\mathrm{t-1}}} \\
&+ \frac{\partial\widehat{\Phi}}{\partial\mathrm{c_t}} \frac{\partial\mathrm{c_t}}{\partial\omega_{\mathrm{t-1}}} = 0, \qquad \text{(A.11)} \\
\frac{\mathrm{d}\widehat{\Phi}}{\mathrm{dy_t}}\Big|_{\mathrm{S_t}=\bar{\mathrm{s}}} &= \frac{\partial\widehat{\Phi}}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\mathrm{y_t}} + \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{c_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{c_{t+1}})}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\mathrm{y_t}} \\
&+ \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{c_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{c_{t+1}})}{\partial\mathrm{E_t}(\mathrm{y_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{y_{t+1}})}{\partial\mathrm{y_t}} + \frac{\partial\widehat{\Phi}}{\partial\mathrm{c_t}} \frac{\partial\mathrm{c_t}}{\partial\mathrm{y_t}} \\
&+ \frac{\partial\widehat{\Phi}}{\partial\mathrm{Var_t}(\mathrm{y_{t+1}})} \frac{\partial\mathrm{Var_t}(\mathrm{y_{t+1}})}{\partial\mathrm{y_t}} = 0, \qquad \text{(A.12)} \\
\frac{\mathrm{d}\widehat{\Phi}}{\mathrm{dr_t}}\Big|_{\mathrm{S_t}=\bar{\mathrm{s}}} &= \frac{\partial\widehat{\Phi}}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\mathrm{r_t}} + \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{c_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{c_{t+1}})}{\partial\omega_{\mathrm{t}}} \frac{\partial\omega_{\mathrm{t}}}{\partial\mathrm{r_t}} \\
&+ \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{c_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{c_{t+1}})}{\partial\mathrm{E_t}(\mathrm{r_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{r_{t+1}})}{\partial\mathrm{r_t}} + \frac{\partial\widehat{\Phi}}{\partial\mathrm{E_t}(\mathrm{r_{t+1}})} \frac{\partial\mathrm{E_t}(\mathrm{r_{t+1}})}{\partial\mathrm{r_t}}
\end{aligned}
$$

$$+\frac{\partial\widehat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial r_t}+\frac{\partial\widehat{\Phi}}{\partial \mathrm{Var}_t(r_{t+1})}\frac{\partial \mathrm{Var}_t(r_{t+1})}{\partial r_t}=0, \tag{A.13}$$

$$\frac{d\widehat{\Phi}}{dp_t}|_{S_t=\bar{s}}=\frac{\partial\widehat{\Phi}}{\partial\omega_t}\frac{\partial\omega_t}{\partial p_t}+\frac{\partial\widehat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial\omega_t}\frac{\partial\omega_t}{\partial p_t}$$
$$+\frac{\partial\widehat{\Phi}}{\partial E_t(c_{t+1})}\frac{\partial E_t(c_{t+1})}{\partial E_t(p_{t+1})}\frac{\partial E_t(p_{t+1})}{\partial p_t}+\frac{\partial\widehat{\Phi}}{\partial E_t(p_{t+1})}\frac{\partial E_t(p_{t+1})}{\partial p_t}$$
$$+\frac{\partial\widehat{\Phi}}{\partial c_t}\frac{\partial c_t}{\partial p_t}+\frac{\partial\widehat{\Phi}}{\partial \mathrm{Var}_t(p_{t+1})}\frac{\partial \mathrm{Var}_t(p_{t+1})}{\partial p_t}+\frac{\partial\widehat{\Phi}}{\partial p_t}=0. \tag{A.14}$$

In order to obtain the exact expression of this equation system, we need to deduce the partial derivatives of $\widehat{\Phi}$ given by equation A.6 with respect to ω_t, c_t, p_t, $E_t(c_{t+1})$, $E_t(r_{t+1})$, $E_t(p_{t+1})$, $\mathrm{Var}_t(y_{t+1})$, $E_t(r_{t+1})$ and $E_t(p_{t+1})$

$$\frac{\partial\widehat{\Phi}}{\partial\omega_t}=(\Delta-1)\Delta\frac{{K_1}^2(\omega_t-G_{\omega r})\mathrm{Var}_t(r_{t+1})}{E_t(c_{t+1})^2}+\Delta\frac{K_1\mathrm{Var}_t(r_{t+1})}{E_t(c_{t+1})E_t(r_{t+1})}, \tag{A.15}$$

$$\frac{\partial\widehat{\Phi}}{\partial c_t}=-\frac{\Delta}{\beta E_t(r_{t+1})c_t}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda}, \tag{A.16}$$

$$\frac{\partial\widehat{\Phi}}{\partial p_t}=\frac{\Lambda}{\beta E_t(r_{t+1})p_t}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda}, \tag{A.17}$$

$$\frac{\partial\widehat{\Phi}}{\partial E_t(c_{t+1})}=\frac{\Delta}{\beta E_t(r_{t+1})E_t(c_{t+1})}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda}$$
$$+(\Delta-1)\Delta\frac{K_1(c_{t+1})}{E_t(c_{t+1})^3}+\Delta\Lambda\frac{\mathrm{Cov}_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})^2E_t(p_{t+1})}$$
$$-\Delta\frac{\mathrm{Cov}_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})^2E_t(r_{t+1})}, \tag{A.18}$$

$$\frac{\partial\widehat{\Phi}}{\partial E_t(r_{t+1})}=\frac{\Lambda}{\beta E_t(r_{t+1})^2}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda}$$
$$-\Delta\frac{\mathrm{Cov}_t(c_{t+1},r_{t+1})}{E_t(c_{t+1})E_t(r_{t+1})^2}, \tag{A.19}$$

$$\frac{\partial\widehat{\Phi}}{\partial E_t(p_{t+1})}=-\frac{\Lambda}{\beta E_t(r_{t+1})E_t(p_{t+1})}\Big(\frac{E_t(c_{t+1})}{c_t}\Big)^{-\Delta}\Big(\frac{p_t}{E_t(p_{t+1})}\Big)^{-\Lambda}$$
$$-\Lambda(\Lambda+1)\frac{\mathrm{Var}_t(p_{t+1})}{E_t(p_{t+1})^3}+\Delta\Lambda\frac{\mathrm{Cov}_t(c_{t+1},p_{t+1})}{E_t(c_{t+1})E_t(p_{t+1})^2}, \tag{A.20}$$

$$\frac{\partial\widehat{\Phi}}{\partial \mathrm{Var_t}(\mathrm{y_{t+1}})} = \frac{(\Delta-1)\Delta}{2}\frac{\mathrm{K_1}^2(1-\mathrm{G_{\omega y}})^2}{\mathrm{E_t}(\mathrm{c_{t+1}})^2}, \tag{A.21}$$

$$\frac{\partial\widehat{\Phi}}{\partial \mathrm{Var_t}(\mathrm{r_{t+1}})} = \frac{(\Delta-1)\Delta}{2}\frac{\mathrm{K_1}^2(\omega_t-\mathrm{G_{\omega r}})^2}{\mathrm{E_t}(\mathrm{c_{t+1}})^2} + \frac{\mathrm{K_1}(\omega_t-\mathrm{G_{\omega r}})}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{r_{t+1}})}, \tag{A.22}$$

$$\begin{aligned}\frac{\partial\widehat{\Phi}}{\partial \mathrm{Var_t}(\mathrm{p_{t+1}})} &= \frac{(\Delta-1)\Delta}{2}\frac{[\mathrm{K_1}(\mathrm{h_t}-\mathrm{G_{\omega p}})+\mathrm{K_2}]^2}{\mathrm{E_t}(\mathrm{c_{t+1}})^2} + \frac{1}{2}\frac{\Lambda(\Lambda+1)}{\mathrm{E_t}(\mathrm{p_{t+1}})^2} \\ &\quad - \Delta\Lambda\frac{\mathrm{K_1}(\mathrm{h_t}-\mathrm{G_{\omega p}})+\mathrm{K_2}}{\mathrm{E_t}(\mathrm{c_{t+1}})\mathrm{E_t}(\mathrm{p_{t+1}})}. \end{aligned} \tag{A.23}$$

Next, we differentiate the conditional expectation of consumption 4.27 in terms of the expected values of $\mathrm{y_{t+1}}, \mathrm{r_{t+1}}, \mathrm{p_{t+1}}$

$$\frac{\partial \mathrm{E_t}(\mathrm{c_{t+1}})}{\partial \mathrm{E_t}(\mathrm{y_{t+1}})} = \mathrm{K_1}(1-\mathrm{G_{\omega y}}), \tag{A.24}$$

$$\frac{\partial \mathrm{E_t}(\mathrm{c_{t+1}})}{\partial \mathrm{E_t}(\mathrm{r_{t+1}})} = \mathrm{K_1}(\omega_t-\mathrm{G_{\omega r}}), \tag{A.25}$$

$$\frac{\partial \mathrm{E_t}(\mathrm{c_{t+1}})}{\partial \mathrm{E_t}(\mathrm{p_{t+1}})} = \mathrm{K_1}(\mathrm{h_t}-\mathrm{G_{\omega p}})+\mathrm{K_2}, \tag{A.26}$$

$$\frac{\partial \mathrm{E_t}(\mathrm{c_{t+1}})}{\partial \omega_t} = \mathrm{K_1}[(\mathrm{E_t}(\mathrm{r_{t+1}})-\mathrm{G_{\omega\omega}})]. \tag{A.27}$$

We also need the differentiations of expectation and variances in income, the interest rate and housing prices with respect to $\mathrm{y_t}$, $\mathrm{r_t}$ and $\mathrm{p_t}$

$$\frac{\partial \mathrm{E_t}(\mathrm{y_{t+1}})}{\partial \mathrm{y_t}} = \mathrm{E_t}(\mathrm{y_{t+1}})\frac{\rho_y}{\mathrm{y_t}}, \tag{A.28}$$

$$\frac{\partial \mathrm{E_t}(\mathrm{r_{t+1}})}{\partial \mathrm{r_t}} = \mathrm{E_t}(\mathrm{r_{t+1}})\frac{\rho_r}{\mathrm{r_t}}, \tag{A.29}$$

$$\frac{\partial \mathrm{E_t}(\mathrm{p_{t+1}})}{\partial \mathrm{p_t}} = \mathrm{E_t}(\mathrm{p_{t+1}})\frac{\rho_p}{\mathrm{p_t}}, \tag{A.30}$$

$$\frac{\partial \mathrm{Var_t}(\mathrm{y_{t+1}})}{\partial \mathrm{y_t}} = 2\mathrm{Var_t}(\mathrm{y_{t+1}})\frac{\rho_y}{\mathrm{y_t}}, \tag{A.31}$$

$$\frac{\partial \mathrm{Var_t}(\mathrm{r_{t+1}})}{\partial \mathrm{r_t}} = 2\mathrm{Var_t}(\mathrm{r_{t+1}})\frac{\rho_r}{\mathrm{r_t}}, \tag{A.32}$$

$$\frac{\partial \mathrm{Var_t}(\mathrm{p_{t+1}})}{\partial \mathrm{p_t}} = 2\mathrm{Var_t}(\mathrm{p_{t+1}})\frac{\rho_p}{\mathrm{p_t}}. \tag{A.33}$$

After substituting equation 4.26 into the budget constraint, we obtain the derivatives of c_t to y_t, r_t, p_t and ω_{t-1}

$$\frac{\partial c_t}{\partial y_t} = K_1(1 - G_{\omega y}), \tag{A.34}$$
$$\frac{\partial c_t}{\partial r_t} = K_1(\omega_{t-1} - G_{\omega r}), \tag{A.35}$$
$$\frac{\partial c_t}{\partial p_t} = K_1(h_t - G_{\omega p}) + K_2, \tag{A.36}$$
$$\frac{\partial c_t}{\partial \omega_{t-1}} = K_1(r_t - G_{\omega\omega}). \tag{A.37}$$

Finally, we differentiate ω_t again with respect to y_t, r_t, p_t and ω_{t-1}

$$\frac{\partial \omega_t}{\partial y_t} = G_{\omega y}, \tag{A.38}$$
$$\frac{\partial \omega_t}{\partial r_t} = G_{\omega r}, \tag{A.39}$$
$$\frac{\partial \omega_t}{\partial p_t} = G_{\omega p}, \tag{A.40}$$
$$\frac{\partial \omega_t}{\partial \omega_{t-1}} = G_{\omega\omega}. \tag{A.41}$$

Inserting equations A.15–A.41 into equations A.10–A.14 and evaluating at the risky steady state, we obtain an equation system containing five unknown variables: $\overline{\omega}$, $G_{\omega\omega}$, $G_{\omega y}$, $G_{\omega r}$, $G_{\omega p}$, with which we are able to calculate $\overline{c}$ and $\overline{h}$ as follows

$$\overline{c} = \overline{y} + \overline{\omega}(\overline{r} - 1), \tag{A.42}$$
$$\overline{h} = M\Big(\frac{\overline{c}^{\alpha}}{\overline{p}}\Big)^{\frac{1}{1-\alpha}}. \tag{A.43}$$

Optimal Housing Investment:

An agent's maximisation problem gives us the optimal housing investment

$$h_t = M\Big(\frac{{c_t}^{\alpha}}{p_t}\Big)^{\frac{1}{1-\alpha}} \quad \text{with} \quad M = \Big(\frac{\alpha}{1-\alpha}\Big)^{\frac{1}{1-\alpha}}. \tag{A.44}$$

Taking $\ln$ on both sides of equation A.44 leads to

$$\ln h_t = \ln M + \frac{\alpha}{1-\alpha} \ln c_t + \frac{1}{\alpha - 1} \ln p_t, \tag{A.45}$$

which is approximately a lognormally distributed process under our assumptions. Expectation and variance of $\ln h_t$ can be computed immediately as follows

$$E[\ln h_t] = \ln M + \frac{\alpha}{1-\alpha} E[\ln c_t] + \frac{1}{\alpha - 1} E[\ln p_t], \tag{A.46}$$

$$\operatorname{Var}[\ln h_t] = \left(\frac{\alpha}{1-\alpha}\right)^2 \operatorname{Var}[\ln c_t] + \frac{1}{(1-\alpha)^2} \operatorname{Var}[\ln p_t] - \frac{2\alpha}{(1-\alpha)^2} \operatorname{Cov}[\ln c_t, \ln p_t]. \tag{A.47}$$

Variance of h_t is

$$\operatorname{Var}[h_t] = e^{2E[\ln h_t] + \operatorname{Var}[\ln h_t]} \left(e^{\operatorname{Var}[\ln h_t]} - 1 \right). \tag{A.48}$$

This implies that an increase in house price risk will increase housing investment volatility in our model.

B

Supplements Chapter 5

Standardizing and Backward Induction:

We simplify the household's optimization problem by standardizing its continuous choice variables with the permanent component of labour income $Y_t{}'$. Let $c_t = C_t/Y_t{}'$ be the consumption-to-income ratio, $b_t = B_t/Y_t{}'$ the bonds-to-income ratio, $s_t = S_t/Y_t{}'$ the stocks-to-income ratio, $\alpha_t = S_t/(S_t + B_t)$ the proportion of liquid assets invested in stocks over the sum of bonds and stocks, $h_t = P_t H_t/Y_t{}'$ the house-value-to-income ratio and $m_t = M_t/Y_t{}'$ the mortgage-to-income ratio. By assuming the Cobb-Douglas utility function and proportional housing maintenance and transition costs, we ensure that the numeracy good consumption, housing service, mortgage loan and portfolio rules c_t, h_t, m_t, α_t are independent of the household's income level. Consequently, the relevant state variables for the household's problem can be written as $x_t = \{q_t, h_t{}'\}$ where $q_t = Q_t/Y_t{}'$ is the wealth-to-income ratio and $h_t{}' = P_t H_{t-1}/Y_t{}'$ is the household's beginning-of-period house-value-to-income ratio. Similarly, the standardized budget constraint, mortgage borrowing and affordability constraint are given as:

$$q_t = \begin{cases} c_t + b_t + s_t + \psi {h_t}' - m_t, & \text{No Adjust} \\ c_t + b_t + s_t + h_t - {h_t}' + (\psi + \delta + \lambda)h_t - m_t, & \text{Adjust} \end{cases} \tag{B.1}$$

where

$$0 \leq m_t \leq (1-\delta)h_t, \tag{B.2}$$

$$\frac{M_t}{Y_t} = \frac{m_t}{\exp(f(t, Z_t))\exp(\varepsilon_t)} \leq \omega \frac{R_f - (\frac{1}{R_f})^{T-1}}{R_M - (\frac{1}{R_M})^{T-1}} \frac{R_M - 1}{R_f - 1}. \tag{B.3}$$

Defining $v_t(a_t) = \frac{V_t(X_t)}{(\frac{{Y_t}'}{{P_t}^{\theta}})^{1-\gamma}}$ as the value function after standardization, the Bellman equation can be written as follows:

$$v_t(x_t) = \max_{a_t} \left\{ \frac{({c_t}^{1-\theta}{h_t}^{\theta})^{1-\gamma}}{1-\gamma} + \beta E_t[v_{t+1}(x_{t+1})] \right\}, \quad \forall 1 \leq t \leq T, \tag{B.4}$$

s.t.

$$c_t > 0, h_t > 0, m_t \geq 0, b_t \geq 0, s_t \geq 0,$$

and equations B.1-B.3.

The above problem can only be solved numerically using backward induction (Judd, 1998). To compare our results with Cocco's (2005), we follow his way, using Gaussian quadrature methods to approximate the density functions of the exogenous random variables. For instance, the aggregate labour income process is approximated by a three-state transition probability matrix. Then, we discretize the state-space and the variables over which the choices are made with equally spaced grids. In the terminal period $(T+1)$, for each possible combination of the state variables, we can calculate the corresponding utility. According to the terminal condition, the utility function coincides with the value function, that is, $v_{T+1}(x_{T+1}) = u_{T+1}(x_{T+1})$.

In every period t prior to $T + 1$, we obtain firstly the utility associated with the different choice of control variables. Then, we calculate the value function at t, which is equal to the current utility plus the expected discounted continuation value associated with the choices made, and the given values of the state variables. Notice that the value function in each period t is only computed at discrete grid points, whereas it probably also needs to be evaluated between these points in the next period $t - 1$ (backward), indicating that we have to compute them using interpolation and possibly extrapolation methods. This dynamic optimization process repeated recursively goes backwards until $t = 1$.

Table B1: Evolution of Variables by Age in the Subprime Mortgage Market.

Asset	Scenario 5: With Affordability Constraint				Scenario 6: Without Affordability Constraint			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Consumption	24.68	26.43	24.55	23.34	145.11	141.45	71.05	27.04
House-Value-to-Income-Ratio	3.96	3.71	4.14	7.29	24.61	20.26	10.45	7.63
LTI	0.4546	0.4745	0.4358	0.3533	16.16	11.40	4.61	1.46
LTV	11.63%	10.58%	10.55%	7.08%	63.61%	56.20%	40.0%	26.07%
% of Inv. Below Poverty Line	19.3%	19.25%	19.17%	19.4%	49.06%	19.47%	21.75%	20.27%

Table B.1: Evolution of Variables by Age in the Subprime Mortgage Market.

Note: This table reports the households' average optimal non-housing consumption, house-value-to-income ratio, LTI, LTV and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the subprime mortgage market.

Table B2: Robustness Analysis in the Prime Mortgage Market: Preference for Housing θ.

Asset	With Affordability Constraint							
	Baseline θ=0.1				θ=0.3			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.0035	0.0045	0.0062	0.0071	0.0028	0.0035	0.0044	0.0049
Bonds	0.0015	0.0016	0.0022	0.0028	0.0013	0.0014	0.0014	0.0010
Real Estate	0.2961	0.3272	0.3848	0.5565	0.5813	0.6267	0.6800	0.8000
Human Capital	0.6989	0.6667	0.6068	0.4226	0.4144	0.3682	0.3100	0.1932
Variables								
Consumption	25.28	25.91	24.11	21.00	20.01	20.52	18.81	16.87
House-Value-to-Income-Ratio	8.05	7.89	8.43	8.02	26.6	27.4	28.81	27.17
% of Inv. Below Poverty Line	19.48%	19.75%	20.16%	19.3%	19.78%	20.01%	20.41%	19.43%
Asset	Without Affordability Constraint							
	Baseline θ=0.1				θ=0.3			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
Stocks	0.0390	0.0056	0.0229	0.0730	0.0450	0.0250	0.047	0.0370
Bonds	0.0004	0.0001	0.0012	0.0020	0.0012	0.0014	0.0013	0.0017
Real Estate	0.2214	0.2007	0.2317	0.5100	0.5971	0.6267	0.6782	0.8130
Human Capital	0.7400	0.7936	0.7442	0.4150	0.3561	0.3467	0.2732	0.1483
Variables								
Consumption	58.68	62.52	49.23	22.01	21.88	21.62	20.56	17.99
House-Value-to-Income-Ratio	5.86	4.06	4.14	8.08	31.77	29.08	33.34	34.33
% of Inv. Below Poverty Line	63.8%	18.8%	21.4%	33.2%	54.72%	31.16%	52.82%	53%

Table B.2: Robustness Analysis in the Prime Mortgage Market: Preference for Housing θ.

Note: This table reports the average portfolio shares of various assets relative to the total assets and the average optimal non-housing consumption, house-value-to-income ratio and percentage of households living below the poverty line based on 50,000 simulated optimal paths in the prime mortgage market. The other parameters are set as the same values as in the baseline case.

Table B3: Robustness Analysis: Mortgage Rate r_M.

Asset	Panel A: Prime Mortgage Market							
	Baseline r_M=0.04				r_M=0.02			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
	With Affordability Constraint							
Stocks	0.0035	0.0045	0.0063	0.0072	0.0025	0.0041	0.0052	0.0074
Bonds	0.0015	0.0016	0.0021	0.0028	0.0015	0.0017	0.0019	0.0031
Real Estate	0.2959	0.3268	0.3848	0.5565	0.3193	0.3588	0.4135	0.5842
Human Capital	0.6991	0.6670	0.6068	0.4335	0.6765	0.6352	0.5793	0.4076
Variables								
LTI	0.3489	0.3438	0.3361	0.3229	0.2350	0.2346	0.2203	0.2496
LTV	4.39%	4.35%	3.99%	4.3%	3%	2.6%	2.3%	2.8%
% of Inv. Below Poverty Line	19.48%	19.75%	20.16%	19.3%	19.5%	19.77%	20.05%	19.4%
Variables	Without Affordability Constraint							
Real Estate	0.22	0.20	0.23	0.51	0.29	0.28	0.32	0.54
Mortgage	80.42	61.39	56.38	40.1	113.82	92.84	93.32	73.82
LTI	3.01	2.14	2.13	1.69	4.25	3.25	3.53	3.13
Asset	Panel B: Subprime Mortgage Market							
	Baseline r_M=0.07				r_M=0.06			
	30-40	40-50	50-65	>65	30-40	40-50	50-65	>65
	With Affordability Constraint							
Stocks	0.0009	0.0011	0.0012	0.0059	0.0018	0.0014	0.0018	0.0054
Bonds	0.0017	0.0016	0.0019	0.0077	0.0017	0.0017	0.0017	0.0073
Real Estate	0.1722	0.1874	0.2372	0.4941	0.1723	0.1877	0.2376	0.4945
Human Capital	0.8249	0.8098	0.7595	0.4921	0.8240	0.8091	0.7588	0.4925
Variables								
LTI	0.4546	0.4745	0.4358	0.3533	0.4393	0.4325	0.4148	0.3258
LTV	11.63%	10.58%	10.55%	7.08%	11.2%	11.6%	10.03%	6.6%
% of Inv. Below Poverty Line	19.3%	19.25%	19.17%	19.4%	19.37%	19.27%	19.23%	19.38%
Variables	Without Affordability Constraint							
LTV	63.61%	56.20%	40.0%	26.07%	67.72%	54.2%	40.0%	27.2%

Table B.3: Robustness Analysis: Mortgage Rate r_M.

Note: This table reports the average portfolio shares of various assets relative to the total assets and the average optimal LTI, LTV, real estate, mortgage and percentage of households living below the poverty line based on 50,000 simulated optimal paths in both mortgage markets. The other parameters are set as the same values as in the baseline case.

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Hiermit erkläre ich, Xi Chen, dass ich keine kommerzielle Promotionsberatung in Anspruch genommen habe. Die Arbeit wurde nicht schon einmal in einem früheren Promotionsverfahren angenommen oder also ungenügend beurteilt.

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Abstract

In the past decades, the issues in real estate have received an increasing amount of attention worldwide. One of the main reasons is that the boom bust cycles in real estate markets were attributed to the underlying causes of numerous financial crises, such as the Nordic banking crisis in the early 1990s, Japan's "Lost Decade" of 1991–2000, the East Asian Financial Crisis in 1997 and the recent global financial crisis erupted in the American 2007 (Bordo and Jeanne, 2002; Crowe et al., 2013; Hartmann, 2015; Reinhart and Rogoff, 2009). Real estate is deemed to be an important factor in the real economy as it makes a major contribution to GDP and provides prosperity and jobs in most countries. In addition, real estate also plays a crucial role in the financial system, since construction projects and residential property purchases are usually credit-financed. Leveraged banks rely heavily on real estate collateral to reduce the risk of much of their household and commercial lending. A widespread downturn in property prices will have a severe impact on the financial stability.

Since the explosion of the global financial crisis in 2007, a considerable number of economic models and methods have been designed specifically to anatomize the root causes and key events of it. As a part of the growing body of literature, the focus of this study centres on the old question of what we can learn from this crisis. However, this thesis attempts to provide different insights by employing a newly developed empirical method and economic concepts. There are several lessons that we can learn from the financial crisis. First, countries should watch for early warning and now-casting hints signalling a future crisis. As the crisis in many countries was caused by excessive increases in property prices and/or rapid credit growth, one of the major tasks is to develop renewed empirical methods and econometric tests to detect excessive asset price developments. Meanwhile, these methods are of particular importance in the sense of macroprudential policy. Broadly speaking, macroprudential policy is seen as aiming at financial stability. In terms of the specific goals of macroprudential policy, the general view is that it is all about limiting the risk and costs of systemic crises. Since boom-bust cycles in real estate markets have been major factors in systemic financial crises and therefore need to be at the forefront of macroprudential policy (Hartmann, 2015). Against this background, we employ the newly developed recursive unit root tests to spot the beginning and the end of potential speculative bubbles in the Chinese and German housing price cycle. Because both countries have experienced large mark-up in the aftermath of the global financial crisis of 2007-2009, there

are increasing worries that they might be destined to a similar fate. Nevertheless, we find that actual house prices are not to be disconnected significantly from underlying economic fundamentals and there is no evidence of an emerging speculative housing bubble at the present time in China and Germany. Further to this, we also investigate house price developments across other OECD countries. In contrast to Germany, the majority of OECD countries, such as Ireland, New Zealand, Spain, the Netherlands, the UK and the US, have experienced strong house price growth in the early 2000s, which cumulated in 2007–2008 into an astounding burst of speculative house price bubbles. Aiming to assess the genuine validity and reliability of the univariate screening toolkit, we also provide the test statistics for these countries. The test results deliver timely warnings of underlying misalignments, vulnerabilities and tail risks that redisposed the international housing market to the financial crisis in 2007–2009.

The second lesson we could learn is that policymakers should also be mindful of the channels through which the financial crisis erupted. Since the explosion of the crisis in 2007, most of the blame has been placed on the regulatory authorities and investment banks. Nevertheless, in the instance of the subprime mortgage catastrophe, we should not point the finger only at them. Rather, this crisis was the collective creation of the world's central banks, homeowners, mortgage lenders and investors (Petroff, 2007). Against this background, this thesis also attempts to scrutinize the housing investment behaviour of households. Based on Coeurdacier et al.'s(2011) earlier work, we develop a stylised stochastic model to show that risky steady state house prices have a significant impact on housing investment choice. With increasing risk from aggregate income, the financial market and the housing market, the model predicts that agents tend to invest more in housing and financial assets. We also address the issue of housing investment from the perspective of housing affordability in a dynamic life-cycle modelling framework. The main purpose is to assess quantitatively the impact of the affordability constraints on households' optimal consumption, mortgage, portfolio choices and poverty status over the lifetime. Meanwhile, we also investigate the interaction between borrower-based macroprudential policies and social policies aimed at improving poverty and fostering home ownership and credit availability. Through studies of different scenarios, we find that the affordability constraints are crucial determinants of households' investment behaviour and their poverty status in both prime and subprime mortgage markets. Moreover, the sensitivity analysis implies that in light of the age profile of households and features of mortgage credit markets, the magnitudes of the borrower-based macroprudential policies are needed to be carefully assessed in order to minimise the potential conflicts with other social policies.

Zusammenfassung

In den vergangenen Jahrzehnten haben die Immobilienprobleme zunehmende Aufmerksamkeit weltweit erhalten. Einer der Hauptgründe dafür ist, dass der Boom-Bust-Zyklus auf den Immobilienmärkten den eigentlichen Ursachen der zahlreichen Finanzkrisen zugeordnet wurde, wie zum Beispiel der nordischen Bankenkrise in den frühen 1990er Jahren, Japans "verlorenem Jahrzehnt" von 1991 bis 2000, der ostasiatischen Finanzkrise im Jahr 1997 und der jüngsten globalen Finanzkrise, die in Amerika 2007 ausbrach (Bordo and Jeanne, 2002; Crowe et al., 2013; Hartmann, 2015; Reinhart and Rogoff, 2009). Immobilien gelten als ein wichtiger Faktor in der Realwirtschaft, da sie einen wichtigen Beitrag zum BIP leisten und zu Wohlstand und Arbeitschancen in den meisten Ländern beitragen. Darüber hinaus spielen Immobilien auch eine entscheidende Rolle im Finanzsystem, da Bauvorhaben und der Kauf von Wohneigentum in der Regel mit Hilfe von Krediten finanziert werden. Mit Fremdkapital finanzierte Banken verlassen sich stark auf Immobiliensicherheiten, um das Risiko der Privathaushalts- und Gewerbekredite zu reduzieren. Ein umfassender Rückgang der Immobilienpreise hat einen starken Einfluss auf die Stabilität des Finanzsystems.

Seit dem Ausbruch der globalen Finanzkrise im Jahr 2007, wurden eine beträchtliche Anzahl von Wirtschaftsmodellen und Methoden speziell entwickelt, um die Ursachen und deren Schlüsselereignisse analysieren zu können. Als Teil der wachsenden Ansammlung von Literatur fokussiert sich diese Studie auf die alte Frage, was wir aus dieser Krise lernen können. Diese Doktorarbeit versucht allerdings unterschiedliche Einblicke durch Einsatz einer neu entwickelten empirischen Methode und wirtschaftlicher Konzepte anzubieten. Es gibt einige Lektionen, die wir aus der Finanzkrise lernen können. Zunächst sollten die Länder die Frühwarnungen und kurzfristigen Hinweise, die eine künftige Krise signalisieren, dringend beachten. Da die Krise in vielen Ländern von übermäßigem Immobilien-Preisanstieg und/oder rasantem Kreditwachstum verursacht wurde, ist es eine der wichtigsten Aufgaben, erneuerte empirische Methoden und ökonometrische Tests zu entwickeln, um einen übermäßigen Anstieg der Vermögenspreise zu erkennen. Zugleich sind diese ökonometrischen Tests im Sinne der makroprudenziellen Politik von besonderer Bedeutung, weil Boom-Bust-Zyklen an den Immobilienmärkten die Hauptfaktoren in systemischen Finanzkrisen darstellen (Hartmann, 2015). Allgemein gesprochen kann makroprudenzielle Politik als Ausrichtung auf Finanzstabilität gesehen werden. In Bezug auf die spezifischen Ziele der makroprudenziellen Politik ist die allgemeine

Sicht, dass es sich um die Begrenzung der Risiken und Kosten der systemischen Krisen handelt. Vor diesem Hintergrund verwenden wir die neu entwickelten rekursiven Einheitswurzeltests, um den Anfang und das Ende der möglichen Spekulationsblasen im chinesischen und deutschen Immobilienpreiszyklus herauszufinden. Da beide Länder starke Preiserhöhungen nach der globalen Finanzkrise von 2007-2009 erlebt haben, gibt es zunehmende Sorge, dass ihnen ein ähnliches Schicksal bestimmt sein könnte. Allerdings finden wir, dass die aktuellen Immobilienpreise nicht von den wirtschaftlichen Fundamentalfaktoren deutlich zu trennen sind, und es gibt zurzeit keine Hinweise auf eine aufkommende spekulative Immobilienblase in China und Deutschland. Überdies haben wir Immobilienpreisentwicklung in den anderen OECD-Ländern geprüft. Im Gegensatz zu Deutschland haben die meisten OECD-Länder, wie Irland, Neuseeland, Spanien, die Niederlande, Großbritannien und die USA, ein starkes Immobilienpreiswachstum in den frühen 2000er Jahren erlebt, das in den Jahren 2007-2008 kumulierte und zu dem erstaunlichen Platzen von spekulativen Immobilienpreisblasen geführt hat. Um die echte Gültigkeit und Zuverlässigkeit des eindimensionalen Screening-Toolkit zu belegen, legen wir auch die Untersuchungsstatistiken für diese Länder vor. Der Untersuchungsnachweis liefert rechtzeitige Warnungen vor zugrunde liegender Falschausrichtung, Schwachstellen und Restrisiken, die den internationalen Immobilienmarkt in der Finanzkrise der Jahre 2007-2009 fehlgelenkt haben.

In der zweiten Lektion erfahren wir, dass die politischen Entscheidungsträger sich der Kanäle bewusst sein sollten, durch die die Finanzkrise ausgebrochen ist. Seit dem Ausbruch der Krise im Jahr 2007 wurden die Vorwürfe überwiegend gegen die Regulierungsbehörden und Investmentbanken erhoben. Dennoch sollten wir im Falle der Subprime-Hypothekenkrise nicht nur mit dem Finger auf diese zeigen. Diese Krise war eigentlich die kollektive Erschaffung der weltweiten Zentralbanken, Hausbesitzer, Kreditgeber und Investoren (Petroff, 2007). Vor diesem Hintergrund versucht diese Arbeit auch das Investitionsverhalten der Haushalte in Immobilien eingehend zu untersuchen. Basierend auf Coeurdacier et al.'s (2011) früherer Arbeit entwickeln wir einem stilisierten stochastischen Modell um zu zeigen, dass riskante stationäre Immobilienpreise einen wesentlichen Einfluss auf das Wahlverhalten bei Immobilieninvestition haben. Mit zunehmendem Risiko aus dem Gesamteinkommen, dem Finanzmarkt und dem Immobilienmarkt prognostiziert das Modell, dass Agenten tendenziell mehr in Immobilien und Finanzanlagen investieren. Wir befassen uns auch mit der Frage der Immobilieninvestitionen aus der Sicht der Immobilienerschwinglichkeit in einem dynamischen Lebenszyklus des Modellierungsrahmens. Der Hauptzweck besteht darin, die Auswirkungen der Erschwinglichkeitsbeschränkungen auf den optimalen Konsum, die Portfolioauswahl, die Hypothek und den Armutsstatus der privaten Haushalte über die gesamte Lebensdauer quantitativ zu beurteilen. Außerdem untersuchen wir auch die Interaktion zwischen Kreditnehmerbasierte makroprudenzielle Maßnahmen und sozialpolitischen Maßnahmen zur Verbesserung der Armut und zur Förderung von Wohneigentum sowie Kreditverfügbarkeit. Durch Untersuchungen von verschiedenen Szenarien verdeutlichen die Ergebnisse der Modellkalibrierung, dass die Erschwinglichkeitseinschränkungen entscheidende Faktoren für Investitionsverhalten der Haushalte und ihren Armutstatus im Prime und im Subprime-Hypothekenmarkt sind. Darüber hinaus impliziert die Sensitivitätsanalyse, dass angesichts des Altersprofiles von Haushalten und die Grundzüge von Hypothekenmärkten die Einflußgrößen der Kreditnehmerbasierten makroprudenziellen Maßnahmen sorgfältig beurteilt werden müssen, um mögliche Konflikte mit anderen Sozialpolitiken zu minimieren.