Fermion families and soft supersymmetry breaking from flux in six dimensions.

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"Divide each difficulty into as many parts
as is feasilbe and necessary to resolve it."

René Descartes, Discours de la méthode

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Abstract

In this thesis, we study compactifications from six to four dimensions on the orbifold $T^2/\mathbb{Z}_2$. By choosing this simple famework, we are able to study effects generically present in compactifications of higher-dimensional theories, while still working in a well-defined and manageable setting, where a detailed examination of all contributions is possible. Progess in understanding effects pertaining to compactification has applications to String Theory and orbifold GUT constructions. We investigate three models.

The first is a model of global supersymmetry in six dimensions, where fixed point localised FI terms for a bulk U(1) arise from one-loop diagrams of charged bulk fields. These FI terms generically break supersymmetry and the gauge symmetry. In addition, charged bulk fields induce anomalies in the bulk and at the fixed points. We expand previous work by considering spontaneous gauge symmetry breaking at fixed points as a method to cancel some localised FI terms. We also construct the field content for which all bulk and fixed point anomalies can be cancelled by a Green-Schwarz mechanism.

The second model we examine is a 6d supergravity with a U(1) gauge symmetry in which we turn on background flux. While it is known that the flux can help in stabilising the moduli together with a KKLT-type superpotential, we investigate anomaly cancellation in this setup for the first time. From the Green-Schwarz term we can read off one-loop corrections to the gauge kinetic function. They play an important role in finding realistic vacua while complete moduli stabilisation is achieved through a combination of D- and F-terms. While the spectrum and localisation of charged bulk fields were constructed before, on the torus and on an orbifold without Wilson lines, we present the orbifold case with Wilson lines in a consistent manner. The Wilson lines give strong criteria for the (de)localisation of the bulk field. The multiplicity of even and odd parity states also depends on the Wilson lines, while the mass spectrum does not.

We construct a third model with an SO(10) $\times$ U(1) bulk gauge symmetry, with flux in the U(1). Such a setup opens up the possibility to embed the Standard Model matter in a bulk **16**-plet charged under the fluxed U(1), while the gauge and Higgs fields are not charged. As a consequence of the flux, the charged bulk field possesses a multiplicity of chiral zero modes that are not subject to the gauge symmetry breaking, which is effected by Wilson lines. At tree level, supersymmetry is broken at the compactification scale in the U(1) charged sector, but remains intact in the uncharged sector. This leads to a tree-level spectrum akin to "Split Supersymmetry" with heavy sleptons and squarks and light gauginos and higgsinos. All anomalies can be cancelled. We also study flavour mixing in this setup, which is determined by the localisation properties of the bulk field. It is not possible to obtain satisfactory flavour mixing from fixed point superpotentials involving the SM fields. We conjecture that mixing with vector-like exotic states, that are generically present in our model, can lead to a phenomenologically viable flavour sector.

Zusammenfassung

Diese Dissertation befasst sich mit Kompaktifizierungen auf der Orbifold $T^2/\mathbb{Z}_2$ von sechs nach vier Dimensionen. Die Wahl dieses relativ einfachen Ansatzes erlaubt es uns, Effekte in kompaktifizierten Theorien zu studieren, da das Umfeld stets wohldefiniert und übersichtlich bleibt, sodass eine detaillierte Untersuchung möglich ist. Ein besseres Verständnis von solchen Effekten hat Einfluss auf die String-Theorie und auf GUT Konstruktionen auf Orbifolds. Wir untersuchen drei Modelle.

Das erste Modell besitzt eine globale Supersymmetrie in sechs Dimensionen, wobei Ein-Loop-Diagramme geladener Bulkfelder an den Fixpunkten lokalisierte FI-Terme einer U(1) erzeugen. Im Allgemeinen brechen solche FI-Terme die Eich- und Supersymmetrie. Zudem erzeugen die Bulkfelder eine Anomalie, im Bulk und an den Fixpunkten. Wir erweitern vorherige Betrachtungen, indem wir an den Fixpunkten spontane Symmetriebrechung zulassen, um die FI-Terme aufzuheben. Außerdem finden wir den Feldinhalt, für den ein Green-Schwarz-Mechanismus alle Bulk- und Fixpunktanomalien aufheben kann.

Beim zweiten Modell handelt es sich um eine 6d Supergravitation mit einer U(1) Eichsymmetrie, in der wir einen Hintergrundfluss anschalten. Es ist bekannt, dass der Fluss, zusammen mit einem KKLT-artigen Superpotential, eine Rolle in der Modulistabilisierung spielen kann. Wir untersuchen die Anomaliekanzellierung in diesem Rahmen zum ersten Mal. Am Green-Schwarz-Term können die Ein-Loop-Korrekturen zur Eichkinetischen Funktion abgelesen werden. Diese sind für die Existenz realistischer Vakua relevant, nachdem alle Moduli durch eine Kombination aus D- und F-Termen stabilisiert wurden. Das Spektrum und die Verteilung der geladenen Bulkfelder sind für den Torus und die Orbifold ohne Wilsonlinien bereits bekannt, wir geben zudem jedoch die konsistenten Ausdrücke für die Orbifold mit Wilsonlinien an. Die Wilsonlinien haben eine starken Einfluss auf die (De-)Lokalisierung des Feldes. Auch die Multiplizität der Paritätseigenzustände hängt von den Wilsonlinien ab, das Massenspektrum jedoch nicht.

Unser drittes Modell verfügt über eine SO(10) $\times$ U(1) Eichsymmetrie, mit Fluss in der U(1). Dieser Ansatz erlaubt es, die Materie des Standardmodells aus einem Bulk-**16**-Plet zu erhalten, das, im Gegensatz zu den Eich- und Higgsfeldern, unter der U(1) geladen ist. Durch den Fluss erhält das geladene Bulkfeld eine Mehrzahl chiraler masseloser Moden, die von der Eichsymmetriebrechung durch Wilsonlinien nicht belangt werden. Im geladenen Sektor ist die Supersymmetrie an der Kompaktifizierungsskala gebrochen, während sie im ungeladenen Sektor intakt bleibt. Folglich ist das Massenspektrum in erster Näherung dem "Split Supersymmetry"-Ansatz ähnlich, in dem die sLeptonen und sQuarks schwer, die Gauginos und Higgsinos jedoch leicht, sind. Alle Anomalien können aufgehoben werden. Wir untersuchen auch die Flavourmischung in diesem Ansatz, die durch die Verteilung des Bulkfeldes bestimmt wird. Es ist nicht möglich, alleine durch Fixpunkt-Superpotentiale, die nur Standardmodellfelder enthalten, realistische Flavourmischung zu erhalten. Wir mutmaßen, dass Mischungseffekte mit vektorartigen exotischen Zuständen, die in unserem Modell generisch auftreten, die Flavourmischung zufriedenstellend reproduzieren können.

This thesis is based on the following publications:

- W. Buchmüller, M. Dierigl, F. Rühle, J. Schweizer, **“Chiral fermions and anomaly cancellation on orbifolds with Wilson lines and flux”**, Phys.Rev.D92 (2015) no. 10, 105031
- W. Buchmüller, M. Dierigl, F. Rühle, J. Schweizer, **“Split symmetries”**, Phys.Lett.B750 (2015) 615-619
- W. Buchmüller, M. Dierigl, F. Rühle, J. Schweizer, **“de Sitter vacua from an anomalous gauge symmetry”**, Phys.Rev.Lett 116 (2016) no.22, 221303
- W. Buchmüller, M. Dierigl, F. Rühle, J. Schweizer, **“de Sitter vacua and supersymmetry breaking in six-dimensional flux compactifications”**, Phys.Rev.D94 (2016) no.2, 025025

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1 Introduction

Ever since the Age of Enlightenment, understanding the fundamental laws of nature has been a human endeavour. Many physicists believe that nature can be described from a single unified framework. There are two avenues to formulating new ideas in this context: On the one hand, there is the "bottom-up" approach, where one takes consolidated knowledge and expands it incrementally to incorporate phenomena that were yet unexplained. Let us call out the prediction of the charm quark by Glashow, Iliopolus and Maiani [1] from the peculiar smallness of the Kaon mass splitting to bear testimony to the efficiency of this approach. On the other hand, there is the "top-down" approach, where a framework or theory is postulated based on a principle, rather than a phenomenon, and the theory's predictions are subsequently compared with the experiment. Usually not constructed to solve specific issues, advances by this approach are paradigm shifts, altering our understanding of nature on a profound level. For example, Einstein most certainly did not consider the perihelion precession of Mercury when he proposed General Relativity [2]. Yet, his theory, based on the revolutionary concept of general coordinate invariance, explained the phenomenon that had challenged Newtonian gravity for half a century at the time [3].

Both approaches to develop physics have proven invaluable in the past and from both approaches, we are lead to investigate the topic of this thesis, flux compactifications in six dimensions. Let us first explain why we choose six dimensional supergravity as a component of a top-down development of a new framework called String Theory in the first part of the introduction and then elucidate the link to the bottom-up approach looking to explain the phenomenon of Grand Unification in the second part of the introduction.

From the top-down, we have String Theory (ST), tackling the unification of the Standard Model of particle physics with the ΛCDM, the standard model of cosmology. The prior is successful on the smallest scales and is formulated as a local quantum field theory (QFT), whereas the latter's success on the largest scales is based on General Relativity (GR), notoriously defying a QFT description. ST attempts to reconcile these two branches of modern physics by doing away with the idea of point-like particles as the core elements of nature, replacing them with the notion of extended strings. This circumvents localised interactions, which lead to divergences on the quantum level that cannot be controlled in GR. Within ST, higher order quantum corrections to GR are calculable and finite [4], making it a theory of quantum gravity.

It has been shown that ST is well described by QFT at energies below the string scale M_s, which must in turn be lower than the energy scale of gravity, the Planck scale M_P. This QFT description is highly constrained – only five versions exist, Type I, Type IIA and IIB, Heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ and Heterotic SO(32). Their mutual independence is a matter of debate. Many dualities relating the different formulations have been constructed, leading to M-theory and F-theory as overarching frameworks for ST.

A shared feature in all of these theories is that they can be described as supergravities in ten dimensions, eleven dimensions in the case of M-theory and, technically, twelve dimensions in the case of F-theory. However, at energy scales up to those tested in modern high-energy colliders, space-time has only four dimensions. The discrepancy between living in only four dimensions whereas ST predicts more can be amended by choosing a space-time background with a number of compact dimensions, whose length scale is smaller than those tested in high-energy experiments. While the ten-dimensional theory is highly constrained, there are no first principles governing the choice of compactification background. The search for appropriate manifolds is therefore guided by phenomenological considerations in the four-dimensional effective theory after compactification. As an example, if the effective action is to retain some degree of supersymmetry, the compactification manifold has to allow for a covariantly constant spinor, constraining the choice to Ricci-flat Calabi-Yau manifolds. Even then, plethora of different compactifications can be considered.

The most eminent problem in these compactifications is moduli stabilisation. Moduli are metric degrees of freedom describing continuous deformations of the compact manifold. They are important, because their vacuum values govern the geometry of the compact space, which in turn influences many parts of the effective theory after compactification. In the four-dimensional theory they appear as scalar fields and, due to their intimate relation with symmetries in the compact space, their potentials are often flat [5]. Their values then are unconstrained and would generically be different in different locations, leading to spatially varying properties of the four-dimensional effective theory, which is unacceptable. Stabilising the moduli means lifting the flatness of their potential, rendering them massive and giving them a definitive constant vacuum expectation value. Obviously, complete stabilisation of the moduli is therefore a requirement for a realistic compactification of ST. Closely related to moduli stabilisation is the search for appropriate vacua in ST and supergravity. All mechanisms for moduli stabilisation will affect the scalar potential, whose vacuum value appears as the cosmological constant (CC). The CC is known to be positive but extremely close to zero from the direct measurement of the accelerated expansion of the universe [6] and from fits of ΛCDM parameters to CMB observations [7].

Two well-known methods to stabilise moduli are fluxes (see [8] for a review) and non-perturbative effects, e.g., from hidden sector gaugino condensation [9] or world-sheet instantons [10, 11, 12]. Non-perturbative effects can lead to a specifically shaped superpotential as described by Kachru, Kallosh, Linde and Trivedi (KKLT) [13]. The resulting potential features only anti de Sitter (AdS) vacua, i.e., vacua with a negative energy density. Several approaches are known to "uplift" the KKLT proposal to Minkowski or de Sitter space, such as anti-D3 branes [13, 14], D-terms induced by magnetic flux [15, 16, 17] or a gauged R-symmetry [18, 19], as well as F-term uplifts with matter fields [20, 21]. All of them, as well as generalisations of the original KKLT framework, like the Large Volume Scenario [22] or Kähler Uplift [23, 24], require additional degrees of freedom below the compactification scale to obtain a phenomenologically viable vacuum energy.

Often in these constructions, researchers consider the problematic part of the theory in isolation and neglect the other sectors of the theory, unless these sectors are required for the specific solution to the problem at hand. A reduced approach is necessary, since the full theory is complicated and dealing with all issues at once would be an overwhelming task. An opportunity might be missed by isolating the different aspects of the theory, though. It is conceivable that a challenge to one sector of a given construction can be solved by taking into account complications from another sector of the theory. To study such interactions between the different parts of the theory, we have to work with a simpler model than ten-dimensional supergravity.

We study six-dimensional supergravity, whose form with general gauge group and matter couplings has first been constructed in the 1980s [25]. Our focus is on models with two extra dimensions, because this case is quite special. It is the lowest dimension allowing for a classical bulk flux while keeping Poincaré invariance in 4d. Bulk anomalies exist in six dimensions, which is not the case in 5d. Moreover, the minimal supergravity in six dimensions features eight real supercharges, i.e., it has an $\mathcal{N} = 2$ extended supersymmetry. Going to even higher dimensions would imply a larger supersymmetry algebra, rendering the theory more complicated. Another reason to study the 6d case is the consideration of anisotropic string compactifications, where the effective theory is six-dimensional at intermediate energies. Such constructions have been created in ST [26, 27, 28] and F-theory [29]. A pioneering work in the study of six dimensional flux compactification is [30].

In our flux compactification of 6d supergravity we find exactly what we alluded to above. The issues of anomaly cancellation and uplifting the AdS vacuum of a KKLT brane superpotential are intertwined. The starting point is a model of fluxed 6d supergravity, similar to the one considered in [17]. The flux has a major impact on the solution of the internal space equations of motion of the charged matter, leading to an altered spectrum for the states in the effective 4d action and non-trivial wave functions across the bulk [30, 31]. Notably, the spectrum is M-fold degenerate, where M is the number of flux quanta, and the fermionic part of the spectrum is chiral. We project the wave functions to the orbifold and investigate the degeneracy, with special attention to the case when Wilson lines are present in addition to the flux. A projection of the wave functions to general $T^2/\mathbb{Z}_n$ was performed in [32]. However, the authors of that work did not recognise the impact of the orbifolding on the flux quantisation, casting some doubt on their results. The charged bulk field creates an anomaly, both in the bulk and at the fixed points. We implement the mechanism of anomaly cancellation by the Green-Schwarz mechanism [33], with special attention to the impact of the flux on the process. From the Green-Schwarz term we obtain some of the one-loop corrections to the action. We compute the effective action, with attention paid to the dimensionful constants in the process. A key feature resulting form anomaly cancellation is the modification of the gauge-kinetic function, which obtains a dependence on a second modulus; such corrections are also known to arise at the one-loop level in the string literature [34, 35]. In our case, the correction is such that the gauge-kinetic function can turn negative in part of the moduli space. This would render the theory unstable, which is why the physical moduli space is constrained to the region where the gauge-kinetic function is positive. Constraining the moduli fields in this way is dynamically stable, i.e., the dynamics of the system will never lead to a violation of the constraint if it was fulfilled before. In the physical region of the moduli space, the D-term potential, i.e., the scalar potential induced by the gauge sector, turns out to be positive definite. Upon the addition of a KKLT-type superpotential, all moduli are stabilised. Because the D-term potential is positive, it naturally raises the scalar potential and realistic vacua are possible, if one tunes the superpotential parameters. It would be imprecise to call our mechanism an "uplift", as that term implies the addition of something to achieve the effect. Unlike previous constructions, our model does not involve additional degrees of freedom below the compactification scale, while still being able to stabilise all moduli. With a fully specified effective action that has all moduli stabilised, we are able to derive the masses of all low-energy degrees of freedom and study their dependence on the compactification volume. Our results show that corrections to the gauge-kinetic function can bear major relevance for moduli stabilisation, a correlation that was not widely known. It will be interesting to see whether our mechanism can be realised in a full ST model.

From the bottom-up perspective, we are also lead to study flux compactifications of 6d supergravity. The consolidated knowledge providing our starting point is the Standard Model of particle physics (SM). It explains all experimental results from high-energy colliders, up to minor tensions. The phenomena guiding us in this approach are therefore more conceptual in nature than actual measurements at an experiment. They are two aspects of the SM that could be waved aside as mere coincidences, but that could as well be the beacons guiding us to a deeper understanding of particle physics. We are talking about flavour and grand unification.

With "flavour" we refer to the fact that there are three generations of SM fermions that differ only in their coupling to the Higgs boson. There is no fundamental reason why there should be more than one generation, nor is there an obvious reason why there should not be more than three. A consecutive fourth generation has been excluded through the observation of the Higgs boson with production and decay rates as expected in a three generation SM [36]. The multiplicity of the fermion generations remains one of the biggest mysteries in particle physics. Even the mixing between the generations is a puzzle keeping a whole sub-field of particle physics active. A straightforward construction of the

SM Yang-Mills theory with three generations of matter leaves their mixing angles and phase unconstrained. Without any organising principle, one would expect the angles to be random and therefore sizeable, but the mixing in the quark sector is small, as the small Wolfenstein parameter $\lambda \approx 0.22$ [37, 38] signalises. In contrast, the mixing among leptons was found to be larger than expected randomly [39], close to a "tri-bi-maximal" scenario [40]. This is sometimes related to the smallness of the neutrino masses, which can be explained by more elaborate schemes than a tiny Yukawa coupling. In the see-saw mechanism [41, 42] small neutrino masses are generated by combining sizeable Yukawa couplings with a Majorana mass for right-handed neutrinos, that is generated at a high scale. The mass eigenstates are then mostly left-handed and their masses are hierarchically suppressed. While such models have explicit predictions, such as neutrino-less double beta decay, they remain hard to test.

Grand unification refers to the fact that all three gauge couplings of the SM become comparably strong at a scale $M_{\text{GUT}} \approx 10^{16}$GeV. This unification is even more effective if there are fermions with the quantum number of gauge bosons at some mass scale that is not too large [43]. This fact provides motivation, beyond the Hierarchy Problem of the Higgs boson's mass, to study supersymmetric models. Because the couplings eventually become equally strong, one can conceive of a Grand Unified Theory (GUT), which has a simple gauge group that is broken to the SM at M_{GUT}. While SU(5) is the smallest group that can accommodate the SM gauge group, $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$, and therefore was the first to be studied [44], other groups are interesting as well [45]. The matter sector of the SM points towards SO(10) as the prime candidate for a GUT group, because all fermions in a SM generation can be incorporated in a single irreducible spinor representation of SO(10) [46]. This would explain the anomaly freedom of the SM fermion spectrum, since SO(10) is anomaly-free in four dimensions.

Such a large gauge group entails challenges to the model building, though. A very prominent issue is dubbed "Doublet-Triplet Splitting" (DTS) problem, from its materialisation in SU(5) GUT models. It describes the issue that, while the matter sector can conveniently be described by complete GUT multiplets, the Higgs and gauge sectors cannot. In the gauge sector this is a consequence of GUT symmetry breaking, but from the GUT perspective there are no reasons why the Higgs, described as a **5**-plet of SU(5) in the original implementation, should be split into a nearly massless doublet, the SM Higgs, while the mass for the colour triplet Higgs bosons has to be much larger to comply with experimental bounds, e.g., from proton decay [47]. While GUT gauge symmetry breaking can be implemented through a 4d Higgs mechanism with an appropriate scalar potential, just like electro-weak symmetry breaking, it is questionable why the same symmetry breaking mechanism should be implemented at two vastly different energy scales. The DTS problem can also be solved by introducing sufficiently complicated scalar potentials. However, GUT potentials that solve the DTS problem, break the GUT gauge symmetry at a high scale and the electro-weak symmetry at a low scale seem very contrived and baroque. For supersymmetric SO(10), it is unclear whether potentials with vacua that fully solve these issues even exist.

The picture changes, if we allow for higher dimensions at the GUT scale. Gauge symmetry and supersymmetry breaking can then be effected through compactification on an orbifold, i.e., a manifold with singular fixed points, by symmetry breaking boundary conditions [48, 49] or, equivalently, by Wilson lines in the compact space [50, 51, 52]. These ideas have previously been applied to study SU(5) in five dimensions [53, 54, 55] and SO(10) in six dimensions [56, 57]. To successfully break SO(10) to the SM, two extra dimensions are necessary [56], because a Wilson line will only break the symmetry to a maximally symmetric subgroup. For SO(10) the relevant subgroups are the original Georgi-Glashow SU(5) [44] with an additional U(1) factor, $\mathrm{SU}(5)_{\mathrm{GG}} \times \mathrm{U}(1)_{\mathrm{X}}$ and the Pati-Salam group [58], $\mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Combining both of these breaking schemes yields the "flipped SU(5)" [59, 60] $\mathrm{SU}(5)_{\mathrm{fl}} \times \mathrm{U}(1)_{\mathrm{X}'}$. The intersection of these groups is exactly the SM gauge group, enhanced by an additional U(1) factor. Doublet-Triplet splitting is easy in higher dimensional setups: Assigning the appropriate boundary condition to the Higgs field, the doublet remains massless while the triplet only exists as a Kaluza-Klein state with a mass of the order of the compactification scale [53].

Six-dimensional models with an SO(10) bulk gauge group and their possible bulk matter content have been discussed before [56, 61], also with respect to the anomaly in such models [62]. The introduction of SM matter at the fixed points opens up the possibility of mixing with bulk states [63] to achieve realistic flavour mixing. Introducing the SM matter as a bulk field, the symmetry breaking boundary conditions will project out most of the spectrum. However, if SM matter is located at the fixed points where the gauge symmetry is broken already and the integrity of the matter multiplet is granted, there is no clear-cut reason why it should furbish an irreducible representation of the entire bulk gauge symmetry vis-à-vis the remaining, broken gauge symmetry. It would be more attractive to have the matter multiplet live in the bulk. To achieve this, one needs to protect the matter fermions from the symmetry breaking Wilson lines.

Incorporating a classical bulk flux in the $\mathrm{U}(1)_A$ of an $\mathrm{SO}(10) \times \mathrm{U}(1)_A$ bulk gauge group provides interesting new opportunities for orbifold GUT model building. We first study the symmetry breaking procedures and their implications for bulk matter representations in absence of the flux. In a second step, we consider the mixed gauge bulk and brane anomalies induced by a charged **16**-plet in the bulk. Then we investigate the implications of the flux on the matter sector of the model, with special attention on a realistic fermion spectrum. There are two major effects of the flux: On the one hand, the flux will lead to a "protection" from Wilson lines, meaning that even in the presence of SO(10) breaking gauge backgrounds, representations charged w.r.t. the flux will be present in the low-energy spectrum in full. Is is therefore viable to include the SM matter as a bulk **16**-plet that is charged under $\mathrm{U}(1)_A$. On the other hand, the flux will induce a multiplicity of charged bulk states [64], providing multiple generations from just a single charged **16**-plet. The Yukawa couplings that are needed to provide masses to the chiral SM spectrum arise at the fixed points, but the localisation of the wave functions is such that there is no satisfactory flavour mixing. Fortunately, there are heavy vector-like states; mixing with those can create phenomenologically viable mass matrices. In addition, the flux provides a direct source of supersymmetry breaking in the $U(1)_A$ charged sector [30]. This leads to GUT scale masses for the supersymmetric partners of the quarks and leptons, while the gauge and Higgs bosons' partners remain massless at tree level – a picture akin to "Split Supersymmetry" [65, 66] or "Spread Supersymmetry" [67], but different in a key issue: These original proposals have only one light Higgs doublet, which is a problem when matching the effective action to the supersymmetric action at scales beyond $\sim 10^{10}$GeV [68], while our theory leaves an effective Two-Higgs-Doublet-Model at low energies, where the matching issue can be circumvented [69]. If the mechanism stabilising the moduli that appear in the gauge-kinetic function of the SO(10) and its subgroup is not supersymmetric, the F-terms of the moduli superfields will endow the gauginos with a mass. In any way, supersymmetry breaking will be transported to the gaugino and higgsino sector by some mechanism, such as Gravity Mediation [70] or Anomaly Mediation [71, 72]. To find the exact low-energy mass spectrum, one has to implement such a mediation mechanism explicitly, which is yet to be done in our model.

From both, the top-down String Theory perspective and the bottom-up orbifold GUT perspective the flux compactification of 6d supergravity is an interesting research topic. The flux provides a number of functionalities that we will elucidate in this thesis.

The first chapter is based on a work by Lee, Nilles and Zucker [73] and discusses a globally supersymmetric orbifold compactification without a bulk flux. Their focus is on the Fayet-Iliopolus terms induced by tadpole diagrams of a charged bulk field and their effect. The author of this thesis learned about extended supersymmetry in higher dimensions, compactification and anomaly cancellation by studying their work and expanding it. Therefore, a review of the paper and the expansions we developed is given in Chapter 2. While we do not follow their construction of the localisation behaviour of charged bulk fields due to the quantum correction induced FI terms, the inclusion of this effect would improve our own constructions. This provides further motivation to include their work as part of our thesis.

In the second chapter, we construct a model of six-dimensional supergravity with a background flux and a charged hypermultiplet in the bulk [30]. In such a setup, the interplay of the flux and a non-perturbative superpotential for moduli stabilisation has been considered in [17]. We expand these considerations by treating anomaly cancellation, which has a major impact on the result. The quantum corrections that can be read off the anomaly cancelling terms support the moduli stabilisation by providing a positive contribution to the vacuum energy. We achieve a tunably small cosmological constant only with elements that are present in the model anyways, exploiting a correlation rather than sculpting a solution from additional pieces. A detailed study of 6d flux compactification is presented in Chapter 3. Special attention was paid to charged matter on the orbifold, where we considered the complications from the quaternionic Kähler structure in a simpler context and argue they are not relevant for the model. Moreover, we studied in detail the wave functions of charged matter on the orbifold, including the influence Wilson lines have on their localisation and multiplicity. Another focus of our research was the effective action, minding the dimensionful constants, as well as the vacua that are possible in our setup, which include (meta-) stable models with vanishing or positive cosmological constant.

The fluxed GUT construction is presented in Chapter 4. It is based on orbifold GUT models investigated in [56, 61, 62, 63]. Novel in our work is the inclusion of a bulk flux in this context. It opens up the possibility to have SM matter from bulk fields and conveniently generates multiple SM generations. Moreover, supersymmetry is broken at a high scale, explaining its persistent absence in experimental searches. A central topic of our research was flavour mixing, which arises at the fixed points. As a consequence of the localisation behaviour of the wave functions in the internal space, it is realistic only after considering mixing with vector-like exotics that are a natural part of the construction.

Finally, in Chapter 5 we conclude, reviewing what we achieved and providing an overview of what remains to be done.

We are well aware that a thesis is rarely read from one end to the other, but that a well-written thesis is rather a resource to be re-read in parts. Therefore, we transfer the facts that a prospective reader probably knows, but a novice reader might not be aware of, to Appendix A. On top of that, we tried to keep the chapters and sections of this thesis somewhat self-contained. To achieve this, some information had to be included in more than one location; we kindly ask the reader to bear with these minor repetitions, should he read the entire thesis in a short amount of time. Beyond the appendix introducing the basics of our research, there are two appendices providing detailed computations that we deemed too extensive to include in the main text. Appendix B presents the careful compactification of 6d supergravity in the flux background along with a related proof showing that one can always define the constants of the theory such that the radion is stabilised at unity, $\langle r \rangle = 1$. Appendix C showcases the difficulties encountered in the simplest case when taking into account the quaternionic Kähler structure, which is imposed on the hypermultiplet sector by 6d supergravity. It also serves to support our claim that the corrections from this enormous complication can safely be neglected in the models we construct.

We fluctuate freely between component notation and form language, using whatever allows the clearest presentation and employ the differential geometry conventions of [74]. All products of forms are understood to be wedge products.

Generalising a model of global 6d supersymmetry 2

In a first project, whose results were not published, we generalised a model proposed by Lee, Nilles and Zucker [73]. The model features global supersymmetry in six dimensions, broken to $\mathcal{N} = 1$ supersymmetry in four dimensions through compactification on $T^2/\mathbb{Z}_2$. Tadpole diagrams involving bulk scalar fields charged under a U(1) generate Fayet-Iliopolus terms localised at the branes. These FI terms are cancelled by localised fluxes. The focus of the original work [73] was the impact of the localised flux on the wave function and mass spectrum of charged bulk scalars, which is of interest for couplings that arise at the fixed points. The advantage of using localised flux for tadpole cancellation is that the ground state leaves both the 4d supersymmetry and the gauge symmetry intact. However, the setup requires anomaly cancellation and therefore a Green-Schwarz (GS) term needs to be included. The model presented there is not consistent, though, as the given field content fails to allow for anomaly cancellation.

We expand this framework by allowing for a non-vanishing sum of FI terms as well as allowing for other options of U(1) breaking. Fixed point localised charged scalars can obtain a vacuum expectation value, thereby cancelling parts of the tadpole induced FI terms, but also spontaneously breaking the U(1) symmetry. Further, we show that there is a bulk field content for which all bulk and fixed point anomalies, including the gravitational and mixed anomalies, are cancelled.

2.1. Review of "Spontaneous localisation of bulk fields: the six-dimensional case"

The model proposed in [73] is a model of global supersymmetry in six dimensions compactified on $T^2/\mathbb{Z}_2$. It features a U(1) gauge symmetry with charged bulk hypermultiplets and brane-localised charged chiral multiplets. With compactification in mind, we formulate the theory in terms of 4d superfields [75].

In six dimensions the smallest spinor has eight real degrees of freedom. Therefore, minimal supersymmetry in six dimensions features eight supercharges, which we call $\mathcal{N} = 2$ because there are twice as many supercharges as in minimal 4d supersymmetry. Such a counting is in intuitive agreement with the $\mathrm{SU}(2)_R$ symmetry in this class of models.

The 6d gauge multiplet consists of a 6d vector A_M and a 6d right-handed Weyl spinor Ω, along with an $\mathrm{SU}(2)_R$ triplet of auxiliary fields $\vec{D}$. Upon compactification, the internal components of the vector become a complex 4d scalar, while the 6d gaugino decomposes into two 4d Weyl spinors, the left-handed Ω_L and the right-handed Ω_R. One can consistently group the 4d spinor Ω_R with the external components A_μ of the vector and one auxiliary field $D = D_3$ to form a 4d vector multiplet V, while the remaining degrees of freedom assemble into a 4d chiral multiplet Φ with the complex scalar $\phi = (A_6 + iA_5)/\sqrt{2}$ and the complex auxiliary field $F_\phi = (D_1 + iD_3)/\sqrt{2}$

$$V = \left(A_\mu, \Omega_R, D\right), \quad \Phi = \left(\phi, \Omega_L, F_\phi\right). \tag{2.1}$$

Note that this embedding of 6d degrees of freedom into 4d superfields differs from the original formulation of the theory. Most prominently, the 4d vector field constructed there has an auxiliary component $-D_3 + F_{56}$ (cf. Eq. (36) in [73]). Still, both formulations describe the same theory, because the resulting 6d and 4d Lagrangian are identical after all auxiliary fields have been eliminated.

6d bulk matter is described by hypermultiplets, each containing four real scalars and a left-handed 6d Weyl spinor. A single hypermultiplet of charge q can be written as two 4d chiral multiplets of opposite charge

$$H = (h, \psi, F), \quad H^c = (h^c, \psi^c, F^c). \tag{2.2}$$

Its coupling to the bulk vector multiplet is then given concisely by introducing the complex derivative $\partial = \partial_5 - i\partial_6$ [75][1,2]

$$\begin{aligned} \mathcal{L}_6 = &\left[\int \mathrm{d}^2\theta \left(\frac{1}{4} W_\alpha W^\alpha + H^c(\partial + \sqrt{2} g q \Phi) H\right) + \text{h.c.}\right] \\ &+ \int \mathrm{d}^4\theta \left(\bar{\partial} V \partial V + \bar{\Phi}\Phi + \sqrt{2} V \left(\partial\bar{\Phi} + \bar{\partial}\Phi\right) + \bar{H} e^{2gqV} H + \bar{H}^c e^{-2gqV} H^c + g\xi V\right). \end{aligned} \tag{2.3}$$

In this notation, the bosonic fields A_M, $h^{(c)}$ have mass dimension 2, the fermionic fields $\Omega_{R,L}, \psi^{(c)}$ have mass dimension 5/2 and the gauge coupling g has mass dimension -1. Upon compactification an appropriate re-scaling of the fields with the compactification scale is required to obtain canonical mass dimensions in 4d.

An orbifold permits a number of chiral brane superfields $H_I^i = (h_I^i, \psi_I^i, F_I^i)$ of charge q_{Ii} at each fixed point ζ_I. They have a regular $\mathcal{N} = 1$ kinetic term, which allows a standard coupling to the 4d vector superfield V. In addition, brane superpotentials $\mathcal{W}_I(H|_{\zeta_I}, H^c|_{\zeta_I}, H_I^i)$, that depend on bulk fields and the fields localised at the brane, are

[1] Note that the expression in [75] requires a partial integration to reproduce the canonical kinetic term for the 6d gauge field. In situations with flux, partial integration can be tricky. We therefore give a modified expression that circumvents the problem, as no partial integration is necessary. In the case without flux, our expression is equivalent to the one given in [75].

[2] We added a possible FI term, which was not considered in [75].

possible. Only bulk fields whose profile in the internal space does not vanish at a given fixed point can contribute to the superpotential there. All in all, the brane Lagrangian is

$$\mathcal{L}_{\text{brane}} = \sum_I \delta^2(y-\zeta_I) \int \mathrm{d}^4\theta \left(\sum_i \bar{H}_I^i e^{2q_{Ii}gV} H_I^i + \left[\bar\theta^2\, \mathcal{W}_I(H|_{\zeta_I}, H^c|_{\zeta_I}, H_I^i) + \text{h.c.}\right]\right). \quad (2.4)$$

The shorthand notation $\delta^2(y-\zeta_I)$ stands for the delta functions $\delta(x_5 - x_5^I)\delta(x_6 - x_6^I)$ peaking at the fixed point ζ_I. We do not discuss the option of introducing brane-localised gauge interactions or vector fields.

In the case without brane superpotentials the equations of motion for the auxiliary fields are given by

$$\begin{aligned} F^* &= \left(\bar\partial_5 - i\bar\partial_6 - \sqrt{2}gq\phi\right)h^c, \quad F^{c*} = \left(\partial_5 - i\partial_6 + \sqrt{2}gq\phi\right)h, \quad F_I^{i*} = 0 \\ F_\phi^* &= \sqrt{2}gqhh^c, \qquad D = F_{56} + g\xi + gq\left(|h| - |h_c|^2\right) + \sum_{I,i}\delta^2(y-\zeta_I)gq_{Ii}|h_I^i|^2. \end{aligned} \quad (2.5)$$

Writing $D_M = \partial_M + igqA_M$ and $\bar{D}_M = \partial_M - igqA_M$ and eliminating the auxiliary fields with (2.5) one finds the bosonic part of the Lagrangian

$$\begin{aligned} \mathcal{L}_6 = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu 5}F^{\mu 5} - \frac{1}{2}F_{\mu 6}F^{\mu 6} - \sum_{I,i}\delta^2(y-\zeta_I)|D_\mu h_I^i|^2 \\ &- |D_\mu h|^2 - |\bar{D}_\mu h^c|^2 - \left|\left(D_5 - iD_6\right)h\right|^2 - \left|\left(\bar{D}_5 - i\bar{D}_6\right)h^c\right|^2 - 2g^2q^2|h|^2|h^c|^2 \\ &- \frac{1}{2}\left(F_{56} + g\xi + gq(|h|^2 - |h_c|^2) + \sum_{I,i}\delta^2(y-\zeta_I)\, gq_{Ii}|h_I^i|^2\right)^2. \end{aligned} \quad (2.6)$$

From this expression the fermionic terms can be reproduced by supersymmetry. Note that the terms quartic in bulk scalars could be rewritten as $\mathcal{L}_6 \supset -g^2/2(|h|^2 + |h^c|^2)^2$. The fact that F- and D-term contributions to the scalar potential of h, h^c organise into a sum of squares is reminiscent of the $\mathrm{SU}(2)_R$ symmetry present in $\mathcal{N} = 2$ supersymmetry. Also, the apparent coupling of F_{56} to h, h^c cancels after rewriting

$$\begin{aligned} \left|\left(D_5 - iD_6\right)h\right|^2 &= |D_5 h|^2 + |D_6 h|^2 - gqF_{56}|h|^2, \\ \left|\left(\bar{D}_5 - i\bar{D}_6\right)h^c\right|^2 &= \left|\bar{D}_5 h^c\right|^2 + \left|\bar{D}_6 h^c\right|^2 + gqF_{56}|h^c|^2. \end{aligned} \quad (2.7)$$

Finally, the the D-term induces a coupling of F_{56} to the brane scalars, which might seem odd at first. Nevertheless, it is required as the supersymmetrisation of the gaugino-fermion-scalar coupling $\mathcal{L}_{\text{brane}} \supset gq_{Ii}\Omega_R\psi_I^i {h_I^i}^*$, because the supersymmetry variation of the gaugino involves $\delta_\epsilon\Omega_R \supset \epsilon\, F_{56}/2$.

This concludes our description of the Lagrangian for a bulk U(1) coupled to one bulk hypermultiplet and a number of brane-localised chiral multiplets. An extension to several bulk hypermultiplets with different charges is straightforward: Wherever the multiplets H, H^c or their scalar components appear, a sum has to be included; wherever a coupling of these fields includes the gauge coupling g, the field's charge q has to be appropriated.

In a supersymmetric theory, a Fayet-Iliopolus (FI) term for a U(1) symmetry is radiatively induced if charged chiral multiplets are present. For the model in question, with a UV cutoff Λ, they are given by [73, 76, 77]

$$\xi = \xi_{\text{bulk}} + \xi_{\text{branes}} \tag{2.8}$$

with

$$\xi_{\text{bulk}} = \frac{1}{4}\operatorname{tr}(q)\left(\frac{\Lambda^2}{16\pi^2} + \frac{1}{4}\frac{\ln\Lambda^2}{16\pi^2}(\partial_5^2 + \partial_6^2)\right)\sum_I \delta^2(y - \zeta_I) \tag{2.9}$$

$$\xi_{\text{brane}} = \frac{\Lambda^2}{16\pi^2}\sum_I \operatorname{tr}(q_I)\delta^2(y - \zeta_I). \tag{2.10}$$

$\operatorname{tr}(q)$ is the sum of bulk U(1) charges and $\operatorname{tr}(q_I)$ the sum of charges of the fields localised at the fixed point ζ_I. A convenient reshuffling of terms gives

$$\xi = \sum_I \left(\xi_I + \xi''(\partial_5^2 + \partial_6^2)\right)\delta^2(y - \zeta_I), \tag{2.11a}$$

where

$$\xi_I = \frac{\Lambda^2}{16\pi^2}\left(\frac{1}{4}\operatorname{tr}(q) + \operatorname{tr}(q_I)\right), \quad \xi'' = \frac{1}{16}\frac{\ln\Lambda^2}{16\pi^2}\operatorname{tr}(q). \tag{2.11b}$$

These terms vanish if the charges add up to zero both in the bulk and at each fixed point individually.

To keep supersymmetry unbroken all F- and D-terms need to vanish in the background. Inspecting Eq. (2.5), we see that the FI terms will in general break supersymmetry by introducing a localised D-term potential. To preserve supersymmetry and the U(1) gauge theory below the compactification scale, these localised FI terms can be cancelled by a localised background flux

$$\langle F_{56}\rangle = -g\xi. \tag{2.12}$$

Such a background flux is a consistent solution to the equations of motion and affects the charged bulk fields through their internal space equation of motion. The main result of [73] is the study of the (de)localisation effects introduced by such a background. We will not repeat the results, but refer the interested reader to the original publication.

The model studied in [73] has a sum of charges that vanishes globally, but not locally

$$\operatorname{tr}(q) + \sum_I \operatorname{tr}(q_I) = 0, \text{ which leads to } \sum_I \xi_I = 0, \tag{2.13}$$

and no charged field with a vacuum expectation value. The authors claim that this way, mixed 4d gauge-gravity anomalies vanish, since they are proportional to the sum of all $U(1)$ charges. We will later show that the requiring the relation (2.13) implies the vanishing of the brane-induced mixed anomaly indeed. The mixed gravity-gauge anomaly from the bulk is not proportional to the sum of charges, however; it does therefore not vanish even if Eq. (2.13) is fulfilled.

We now compactify the theory on the torus orbifold $T^2/\mathbb{Z}_2$. The construction of this manifold is described in Appendix A.2. The process of compactification is elucidated in Appendix A.3.

We are free to assign orbifold parities as long as the entire Lagrangian is $\mathbb{Z}_2$ invariant. A standard choice of parities is easily given in terms of the 4d superfield description

$$V \xrightarrow{\mathbb{Z}_2} V, \quad \phi \xrightarrow{\mathbb{Z}_2} -\phi, \quad H \xrightarrow{\mathbb{Z}_2} H, \quad H^c \xrightarrow{\mathbb{Z}_2} -H^c. \tag{2.14}$$

Keeping in mind that the internal space derivatives, ∂_5 and ∂_6, and therefore ∂ and $\bar{\partial}$ introduced above, are of odd parity the consistency of this setup can easily be checked by inspecting Eq. (2.3). Because entire 4d superfields are given a definite parity, the compactified action is guaranteed to obey $\mathcal{N} = 1$ supersymmetry.

The volume factor picked up by integrating out the internal space is $V_2 = L^2/2$. Ergo, the re-scaling factors required to have the fields canonically normalised in 4d are

$$A_\mu \to \frac{L}{\sqrt{2}} A_\mu, \quad h \to \frac{L}{\sqrt{2}} h, \quad F_{56} \to \frac{L}{\sqrt{2}} F_{56}, \quad g \to \frac{\sqrt{2}}{L} g. \tag{2.15}$$

If we truncate to massless states, i.e., writing only $\mathbb{Z}_2$ even terms and neglecting the Kaluza-Klein tower, the bosonic 4d Lagrangian is

$$\begin{aligned} \mathcal{L}_4 = &-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D_\mu h|^2 - \sum_{I,i} |D_\mu h_I^i|^2 + g F_{56} |h|^2 \\ &- \frac{1}{2} \left(F_{56} + g\xi + g|h|^2 + \sum_{I,i} g q_{Ii} |h_I^i|^2 \right)^2 + \sum_{I \neq J, i, j} g^2 q_{Ii} q_{Jj} |h_I^i|^2 |h_J^j|^2 . \end{aligned} \tag{2.16}$$

The last term in the first row is the $\mathbb{Z}_2$ even part of $|(D_5 - iD_6)h|^2$ and the last term in the second line eliminates cross terms between fields localised at different fixed points.

2.2. Generalisations: Spontaneous symmetry breaking and anomaly cancellation

In this section we present two generalisations to the considerations describe above. On the one hand, we allow for spontaneous symmetry breaking. Through the D-term potential, the localised fields will naturally develop a background value that cancels some contributions to the FI terms. On the other hand, we study anomaly cancellation, including the gravitational and mixed gauge-gravity anomalies. From the requirements, that the irreducible anomaly vanishes and the remaining anomalies can be cancelled through a Green-Schwarz mechanism, strict conditions on the bulk spectrum can be derived. Lee, Nilles and Zucker aim to keep both supersymmetry and the U(1) gauge theory unbroken below the compactification scale. The D-term is given by the FI terms along with charged fields and the internal space U(1) field strength,

$$D = F_{56} + g\xi + g|h|^2 + \sum_{I,i} g q_{Ii} |h_I^i|^2. \tag{2.17}$$

In order to prevent a non-zero D-term, which would imply supersymmetry breaking, a non-trivial background flux is their only option to cancel the FI term contributions. Maybe if we relax our goals and only plan to preserve supersymmetry, but not the U(1), suitable scalar field vacuum expectation values (vevs) could cancel the problematic terms?

The FI terms receive two contributions, both proportional to delta functions, or their derivatives, peaked at the fixed points. Bulk fields are ruled out as compensation for these FI terms, because their mass-less state is constant across the extra dimensions, as long as there is no background flux. Localisation of the bulk zero mode, which could potentially allow it to influence brane-localised effects, only arises as a secondary effect of the localised flux in the original work [73]. As we want to find alternatives to such a flux, we do not consider bulk fields with localised wave functions any further.

The fields h_I^i, that are brane-localised from the start, are more suitable. Eq. (2.5) is key to see that the non-derivative part of the FI terms can be cancelled by brane-localised vevs, if every fixed point comes with at least one localised field of appropriate charge. Setting $q_I^0 = -\operatorname{sgn}(\xi_I)$ we can cancel all ξ_I (in the notation of equation (2.11)) by requiring

$$\left\langle |h_I^0|^2 \right\rangle = \xi_I = \frac{\Lambda^2}{16\pi^2}\left(\frac{1}{4}\operatorname{tr}(q) + \operatorname{tr}(q_I)\right). \tag{2.18}$$

In the case without brane superpotential, h_I^0 naturally takes this background value as it minimises the potential. The vacuum expectation value necessary to cancel the FI terms could also be distributed among a number of fields at each fixed point, which would not change our conclusions.

The logarithmically divergent piece of the FI term, ξ'', can not be cancelled in this fashion, though. Its dependence on the *derivative* of delta functions prevents well-behaved fields within the 6d theory to provide similar contributions. We have not found an alternative to localised flux to cancel this part of the FI terms.

In a different vein, anomalies generically appear in quantum theories. A short review of the most important facts on anomalies in higher dimensions and on orbifolds can be found in Appendix A.5.

To consistently treat the anomalies in six dimensions we have to include the gravitational sector, which was neglected so far. We assume a trivial extension of the model described in this chapter, where we add nothing but the gravity-tensor multiplet of 6d supergravity to the field content and couple the theory minimally to gravity. More information about 6d supergravity and its field content can be found in Appendix A.1.

Actually, in order to gauge a symmetry of 6d supergravity it must be an isometry of the scalar manifold, which is of quaternionic Kähler type. The embedding of the gauge symmetry in this manifold determines the scalar sigma model metric as well as the scalar potential. Deriving these is complicated and they only lead to sub-leading corrections to the effective theory. We tested this explicitly for the simplest case of a single hypermultiplet and a U(1) gauge symmetry, see Appendix C. Extending these considerations to the case with multiple charged fields is very complicated, especially since the product of several quaternionic Kähler manifolds is *not* itself a quaternionic Kähler manifold. In conjunction with their sub-leading nature, we do not deem the corrections interesting enough to work them out.

The anomaly polynomial for the theory described in Section 2.1 has two contributions[3],

$$I_8 = \frac{\beta}{2} I_8^{(b)} + \alpha \sum_p I_6^{(p)} \Delta_2^{(p)}, \tag{2.19}$$

where the loop factors are $\beta = -i/(2\pi)^3$ and $\alpha = i/(2\pi)^2$. The first contribution is from bulk fields and describes an irreducible gravitational anomaly as well as gauge and mixed gauge-gravity anomalies. The second contribution is from the fixed points and receives contributions from both bulk fields and brane-localised fields. $\Delta_2^{(p)} = \delta(y - \zeta_p)\mathrm{d}x^5\mathrm{d}x^6$ is a two-form version of the delta function peaked at the fixed point ζ_p.

The bulk anomaly is given by [78, 79]

$$I_8^{(b)} = -\frac{H-245}{5760}\left[\operatorname{tr} R^4 + \frac{5}{4}\left(\operatorname{tr} R^2\right)^2\right] - \frac{1}{16}\left[\left(\operatorname{tr} R^2\right)^2 - \frac{1}{6} m \operatorname{tr} R^2 F^2 - \frac{2}{3} h F^4\right]. \tag{2.20}$$

The irreducible gravitational anomaly fixes the number of bulk hypermultiplets to be $H = 245$. It does not matter how many of these fields are charged under the U(1), though. The mixed part of the anomaly can be cancelled by a Green-Schwarz mechanism, if the coefficients $m = \sum_i q_i^2$ and $h = \sum_i q_i^4$ are such that the second term can be factorised into two 4-forms $I_8^{(b)} = X_4 Y_4$. The sums in m and h run over all bulk hypermultiplets and q_i are their charges.

The fixed point anomaly is given by [80]

$$I_6^{(p)} = -\frac{1}{6}\left[\frac{1}{8} m_p F \operatorname{tr} R^2 - h_p F^3\right]. \tag{2.21}$$

The coefficients are $m_p = 1/4 \sum_i q_i + \sum_j q_{pj}$ and $h_p = 1/4 \sum_i q_i^3 + \sum_j q_{pj}^3$, where i runs over all bulk hypermultiplets and j over all localised chiral multiplets at the fixed point p. In order to cancel the localised anomaly it too has to be factorisable, too, albeit as a four-form times a two-form $I_6^{(p)} = X_4 Y_2^{(p)}$. Note that the four-form X_4 needs to be the *same* four-form as for the bulk anomaly if one wishes to cancel both contributions to the

[3] To avoid confusing notation, we change the variable enumerating fixed points from I to p.

anomaly with a single Green-Schwarz term.

Using the Ansatz

$$X_4 = \operatorname{tr} R^2 - aF^2, \quad Y_4 = \operatorname{tr} R^2 - bF^2, \quad Y_2^{(p)} = c^{(p)} F, \tag{2.22}$$

and equating coefficients with the expressions (2.20) and (2.21), b can be eliminated. Subsequently, one finds two conditions relating a and the charge sums m, h, m_p and h_p,

$$a = \frac{m}{12} \pm \frac{1}{12}\sqrt{m^2 - 96h}, \quad a\, m_p = 8h_p. \tag{2.23}$$

These conditions are fulfilled for 96 bulk hypermultiplets of unit charge ($q_i = 1, i = 1, \ldots, 96$), if all of the bulk-localised fields have same unit charge $q_{pj} = 1$. This gives

$$m = h = 96, \Rightarrow\ a = \frac{96}{12} = 8, \Rightarrow\ m_p = h_p. \tag{2.24}$$

So the charged bulk spectrum is fixed to 96 hypermultiplets of charge $q_i = 1$ by requiring that the bulk anomaly factorises, while the total number of bulk hypermultiplets has to be 245 to subdue the irreducible gravitational anomaly, leaving us with $245 - 96 = 149$ uncharged bulk hypermultiplets. In contrast, there can be an arbitrary number of localised chiral multiplets, as long as their charges are of unit magnitude $q_{pj} = \pm 1$. We could not find further solutions to the anomaly cancellation conditions, even though they might exist.

We have shown that requiring a vanishing sum of charges as in Eq. (2.13) is not sufficient to guarantee the absence of mixed gauge-gravity anomalies. Instead, bulk anomaly cancellation fixes the number of charged hypermultiplets and constrains the charges of the brane fields. A Green-Schwarz mechanism, which will break the U(1), is inevitable for a consistent quantisation of this model.

We expanded the model proposed in [73] by dropping their original requirement of a U(1) symmetric ground state. This opens the possibility of cancelling parts of the tadpoles through vacuum expectation values of brane-localised fields instead of localised fluxes. The tadpoles proportional to derivatives of delta functions can not be cancelled in this way, though. They are more problematic anyways, since their effect on the wave function needs to be regularised, e.g., by introducing a finite brane width. This hints to additional effects once one resolves the orbifold fixed points.

Another issue in this setup are anomalies. The field content given in [73] is not sufficient to cancel the arising anomalies through a Green-Schwarz mechanism and is therefore inconsistent. We constructed both the bulk and brane anomalies and gave a field content for which the irreducible anomaly vanishes and the reducible anomalies can be cancelled by a single Green-Schwarz term. While anomaly cancellation fixes the bulk hypermultiplet content, the number of brane fields is unconstrained, as long as they all carry a charge of unit magnitude. From the large number of charged fields necessarily present one would expect some of them to receive a vev, once the present model is embedded into a more realistic scenario. Therefore, the $U(1)$ bulk gauge symmetry is generically broken spontaneously.

Flux compactification of six-dimensional supergravity 3

In this chapter we describe a compactification of 6d supergravity with a U(1) gauge group and bulk flux. The flux impacts virtually all aspects of the theory. It influences anomaly cancellation through the fermionic spectrum of the theory, where it creates a multiple massless states upon compactification, akin to Landau levels in solid state physics. Moreover, the flux induces a field-dependent FI term in the 4d effective action which plays a major role in stabilising the moduli of our model. All in all, our setup brings together many aspects that are usual ingredients in theories with extra dimensions. A detailed study of the model reveals that these aspects are highly interdependent.

This chapter begins with a study of the classical effective action without matter in Section 3.1. We will then study the matter sector separately in Section 3.2, focusing on the construction of the solutions to the internal space equations of motion, or "wave functions", in the flux background on the torus and on the orbifold. Then we look at the gauge anomaly cancellation in the flux background and find the effective action including the Green-Schwarz term in Section 3.3. Section 3.4 is where we add a non-perturbative fixed point superpotential to our theory and stabilise all moduli in a Minkowski or de Sitter vacuum. We then go on to study the mass spectrum after compactification and moduli stabilisation in Section 3.5, before giving a brief chapter summary in Section 3.6.

3.1. Six-dimensional supergravity with a background flux

In this section we describe the classical part of a 6d supergravity with a constant background flux in the extra dimensions. We compactify the model on a background space $\mathcal{M}_4 \times T^2/\mathbb{Z}_2$, where $\mathcal{M}_4$ is 4d Minkowski space and $T^2/\mathbb{Z}_2$ is a torus orbifold. Details on the construction of the latter can be found in Appendix A.2.

The background geometry is described by the metric

$$g_{MN} = \begin{pmatrix} r^{-2}(g_4)_{\mu\nu} & 0 \\ 0 & r^2(g_2)_{mn} \end{pmatrix}, \quad g_2 = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}, \tag{3.1}$$

where we measure the internal space coordinates w.r.t. a typical length scale L, i.e., $(x_5, x_6) = (y_1, y_2)L$. The moduli $\tau_{1,2}$ are real fields describing the shape of the torus. We combine them to a complex $\tau = \tau_1 + i\tau_2$. The factor of r^{-2} in front of the 4d metric is convenient, as it gives standard expressions for the moduli kinetic terms and renders the Einstein-Hilbert term field independent. Should r not be stabilised at $\langle r\rangle = 1$, a constant Weyl transformation by $\langle r\rangle^2$ becomes necessary to obtain the proper measure of 4d distances.

The volume of the internal space is given by

$$V_2 = \frac{1}{2}\langle r\rangle^2 L^2. \tag{3.2}$$

We can always choose the scale L such that $\langle r\rangle = 1$ and will assume this in the following. In Appendix B.2 we show that a compactification with a different choice of L, leading to $\langle r\rangle \neq 1$, can be rewritten in terms of the proper length scale, $L' = \langle r\rangle L$, through constant re-scaling of the metric, some fields and constants, such that the volume modulus in the primed expression fulfils $\langle r'\rangle = 1$.

The action we start from is the standard 6d supergravity with a $U(1)$ bulk gauge field, as given in [25]

$$\begin{aligned} S = \int \Bigg[& \frac{M_6^4}{2}\left(*R_6 - \mathrm{d}\phi \wedge *\mathrm{d}\phi\right) - \frac{e^{2\phi}}{4M_6^4 g_6^4} H \wedge *H - \frac{e^{\phi}}{2g_6^2} F \wedge *F \\ & - \frac{M_6^4}{2} g_{\alpha\beta}\, \mathrm{D}\psi^\alpha \wedge *\mathrm{D}\psi^\beta - M_6^8 \frac{g_6^2}{2} e^{-\phi} V(\psi) \Bigg]. \end{aligned} \tag{3.3}$$

We only give the bosonic terms, as all other terms are then fixed by supersymmetry. The gravitational sector consists of one gravity-tensor multiplet, where R_6 is the Ricci scalar, ϕ the dilaton and H the field strength of the two-form field B. The two-form will be part of a Green-Schwarz mechanism. Therefore, it will be required to shift under local Lorentz and gauge transformations. H has to remain gauge invariant, so we include Chern-Simons forms for gravity $\omega_{3L} = \mathrm{tr}\left(\omega \mathrm{d}\omega + 2/3\,\omega^3\right)$ and gauge interactions ω_{3G} in its definition

$$H = \mathrm{d}B - X_3^{(0)}, \quad X_3^{(0)} = \omega_{3L} - \omega_{3G}, \tag{3.4}$$

where $\omega_{3G} = AF$ for this chapter, where we only study a single $U(1)$.

$F = \mathrm{d}A$ is the field strength of the $U(1)$ vector field. With compact dimensions present, a classical background flux is at our disposal, as explained in Appendix A.4. It arises from a gauge field background

$$\langle A\rangle = -f y_2 \mathrm{d}y_1, \quad \Rightarrow \quad \langle F\rangle = f dy_1 dy_2, \tag{3.5}$$

on top of which we can add Wilson lines W_I at will. The magnitude of the flux is subject to quantisation; on the orbifold $T^2/\mathbb{Z}_2$ we have

$$f = -\frac{4\pi}{q} N, \quad N \in \mathbb{Z}. \tag{3.6}$$

We find the negative sign relating f to the flux quantum number N convenient, as it will give left-handed fermionic zero modes for $N > 0$ (see Section 3.2).

The scalar fields ψ^α furnish a quaternionic Kähler manifold whose geometry fixes the scalar metric $g_{\alpha\beta}$ and the potential $V(\psi)$. As the full treatment of such a geometry is highly involved and the study of the simplest case shows effects that go beyond the results from global supersymmetry starting only from $\mathcal{O}(M_{\mathrm{Pl}}^{-2})$ after compactification (see Appendix C) we will not engage with the details of the scalar sector. Instead, we trust the higher-order corrections to be negligible, which should be granted as long as no 6d bulk scalar obtains a large vacuum expectation value. The leading order terms for bulk fields are the subject of Section 3.2, which is why we will not consider the hypermultiplet sector for the remainder of this section.

Given $\langle r\rangle = 1$, the relevant constants of the 4d theory can be read off directly from the expression one obtains after compactifying the action (3.3) on $\mathcal{M} \times T^2/\mathbb{Z}_2$. We find the Planck mass M_P, the 4d bare gauge coupling[1] g_4 and a useful dimensionless parameter ℓ to be

$$M_P^2 = \frac{L^2}{2} M_6^4 = V_2 M_6^4, \quad g_4^{-2} = \frac{L^2}{2} g_6^{-2} = V_2 g_6^{-2}, \quad \ell = g_4 M_4 L. \tag{3.7}$$

The computation (sans 6d scalar fields) is presented in detail in Appendix B.1, where we introduce the combinations

$$s = r^2 e^{\phi}, \quad t = r^2 e^{-\phi}, \tag{3.8}$$

as well as the decomposition

$$\mathrm{d}B = \mathrm{d}\hat{B} + \mathrm{d}b\, v_2, \tag{3.9}$$

where $\hat{B}$ is a two-form in 4d, while b is a 4d pseudoscalar and v_2 is the volume form of the internal space. Note that in the compactification, $H_{\mu n r}$ also includes contributions of the form $\partial_n B_{r\mu}$, which contribute to the shift behaviour of b. This can be linked to the fact that the two-form B is not globally defined in the flux background [17].

[1] Both, the 4d coupling and the 4d metric are called "g_4". We will write out the indices of the metric as in $(g_4)_{\mu\nu}$ wherever ambiguities could arise.

We obtain Eq. (B.14) for the effective 4d Lagrangian before including the effects of anomaly cancellation. The two-form $B_{\mu\nu}$ in 4d can be dualised to a pseudo-scalar c by adding a Lagrange multiplier term

$$S_c = \int \frac{1}{2g_4^2}\, c\, \mathrm{d}(H - AF), \tag{3.10}$$

where A and F are now the gauge field in 4d and its field strength, respectively. After eliminating H through its equations of motion we find the classical action

$$\begin{aligned} S^4 = \int & \frac{M_P^2}{2} R_4 - \frac{M_P^2}{4\tau_2^2} \mathrm{d}\tau \wedge * \mathrm{d}\bar{\tau} - \frac{s}{2g_4^2} F \wedge *F - \frac{c}{2g_4^2} F \wedge *F \\ & - \frac{M_P^2}{4s^2} \mathrm{d}s \wedge * \mathrm{d}s - \frac{M_P^2}{4s^2} \mathrm{d}c \wedge * \mathrm{d}c \\ & - \frac{M_P^2}{4t^2} \mathrm{d}t \wedge * \mathrm{d}t - \frac{M_P^2}{4t^2} \left(\mathrm{d}b + A\frac{2f}{\ell^2} \right) \wedge * \left(\mathrm{d}b + A\frac{2f}{\ell^2} \right) - \frac{M_P^4}{2} \frac{g_4^2}{s} \frac{f^2}{t^2 \ell^4}. \end{aligned} \tag{3.11}$$

The action neatly organises into the template of a $\mathcal{N} = 1$ 4d supergravity given in terms of the complex fields

$$T = \frac{1}{2}(t + ib), \quad S = \frac{1}{2}(s + ic), \quad U = \frac{1}{2}(\tau_2 + i\tau_1). \tag{3.12}$$

The Kähler potential and gauge kinetic function are

$$K = -M_P^2 \ln\left(T + \bar{T} + iX^T V\right) - M_P^2 \ln\left(S + \bar{S}\right) - M_P^2 \ln\left(U + \bar{U}\right), \tag{3.13}$$

$$H = 2S. \tag{3.14}$$

The gauge shift of the T field and the D-term it induces are

$$X^T = -i\frac{f}{\ell^2}, \quad D = iK_T X^T = -\frac{f}{t\ell^2}. \tag{3.15}$$

The flux induces a shift behaviour of the pseudo-scalar degree of freedom b in the compactified theory, whereby it effects a field-dependent FI term. Since $\langle t \rangle > 0$ by definition this corresponds to spontaneous D-term breaking of the $\mathcal{N} = 1$ supersymmetry that remains after compactification. Moreover, the pseudoscalar b can be absorbed in the vector boson by a 4d gauge transformation

$$A \mapsto A - \frac{\ell^2}{2f} \mathrm{d}b, \tag{3.16}$$

upon which the kinetic term of b turns into a mass term for the vector boson, whose mass is given by

$$m_A^2 = \frac{g_4^2}{s} \frac{M_P^2}{4t^2} \frac{4f^2}{\ell^4} = \frac{1}{M_P^2 V_2^2} \frac{f^2}{4g_4^2 t} \tag{3.17}$$

after canonically normalising its kinetic term. This classical vector boson mass is a consequence of the Chern-Simons term contained in $H = \mathrm{d}B + A \wedge F$ in combination with the background flux, $\langle F \rangle$.

We neglected two major ingredients of the full theory so far: The hypermultiplet sector and anomaly cancellation. The supergravity structure of the action allows us to formulate an expectation on the hypermultiplet sector. The compactification on the orbifold should give a chiral multiplet of spontaneously broken $\mathcal{N} = 1$ supergravity as a low-energy remnant. In Section 3.2 we investigate the fate of a charged 6d hypermultiplet and see whether our expectations are answered. Anomaly cancellation is the topic of Section 3.3; more precisely we will cancel the reducible gauge anomalies in the flux background. The Green-Schwarz term that cancels the anomaly will affect the dualisation of H and, consequently, the complex field S as well as the gauge-kinetic function. These modifications will be of paramount importance to stabilise all moduli of the model in a Minkowski or de Sitter vacuum, as discussed in Section 3.4.

3.2. Wave functions in the flux background

In this section, we turn our attention to charged hypermultiplets in the flux background. A hypermultiplet in 6d contains a left-handed 6d Weyl spinor and four scalar degrees of freedom that form a quaternionic coordinate of a quaternionic-Kähler manifold. Quaternionic-Kähler (qK) manifolds are not Kähler in the sense that their holonomy, $\mathrm{Sp}(n) \times \mathrm{Sp}(1)_R$, is not contained in $\mathrm{U}(2n)$. Instead, the locally defined $\mathrm{Sp}(1)_R$ connection spoils key properties physicists cherish in a Kähler manifold: A product of qK manifolds is not qK itself and no Kähler potential exists for them from which one could obtain the Kähler metric by differentiation. Instead, the target space metric (and the scalar potential) has to be constructed explicitly on a case by case basis. Ergo, one can not simply study a single hypermultiplet and generalise the result to a number of hypermultiplets in a straightforward manner as it is possible for other kinds of Kähler manifolds. This is particularly problematic as the cancellation of the pure gravity and the mixed gauge-gravity anomaly in 6d requires a large number of charged and uncharged multiplets, as we found in Section 2.2, and therefore a large qK manifold.

Appendix C demonstrates the derivation of the quaternionic Kähler geometry for a single charged hypermultiplet. Let us repeat the result after compactification on a torus without flux here:

$$S \supset \int \mathrm{d}^4 x \, e_4 \left[D_\mu \bar{z} D^\mu z + D_\mu \bar{c} D^\mu c - \frac{g_4^2}{s} \left(|z|^2 + |c|^2 \right)^2 + \mathcal{O}(M_4^{-2}) \right] . \tag{3.18}$$

We see that the leading order terms have the well-known structure of global $\mathcal{N} = 2$ supersymmetry. This is not a surprise, as the limit $M_P \to \infty$ makes the $\mathrm{Sp}(1)_R$ connection globally defined and reduces a qK manifold to a hyperkähler manifold, which has all the nice features of Kähler geometry. As long as none of the scalar components of the hypermultiplet condense with an expectation value of the order of the Planck scale, it is safe to work in this limit. Hence, we will neglect the higher order corrections, saving us the trouble of dealing with a non-linear sigma model in the matter sector.

Before we study the behaviour of charged hypermultiplets in a flux background, let us look at some related cases. On the torus without flux, a Wilson line gauge background is still possible. Implementing the Wilson lines, $W_i = \exp(iq\alpha_i)$, as twists for our charged field, the general decomposition reads

$$\phi(y) = e^{iq(\alpha_1 y_1 + \alpha_2 y_2)} \sum_{k,l \in \mathbb{Z}} a_{rs} e^{2\pi i(ky_1 + ly_2)} = \sum_{k,l \in \mathbb{Z}} \phi_{kl}(y), \tag{3.19}$$

which also serves as a definition of the mode function $\phi_{kl}(y)$. By our choice of a gauge where the Wilson lines appear as modified boundary conditions, the background gauge field vanishes and the Kaluza-Klein (KK) equation for the mode functions is

$$(g_2^{mn} \partial_m \partial_n)\, \phi_{kl}(y) = \Delta_2 \phi_{kl} = -m_{kl}^2 \phi_{kl}. \tag{3.20}$$

From this we easily find the eigenvalues of the internal space Laplacian [81],

$$m_{kl}^2 = \frac{1}{\tau_2} |(2\pi l + q\alpha_2) - \tau(2\pi k + q\alpha_1)|^2. \tag{3.21}$$

The mass dimension of this expression is zero as a consequence of our choice of dimensionless coordinates. To obtain the KK mass spectrum in physical coordinates one has to restore the physical dimensions, where the compactification volume enters the mass formula as

$$m_{\mathrm{phys}}^2 = V_2^{-1} m^2. \tag{3.22}$$

The expression (3.21) is quadratic in both mode numbers k and l. This is the standard behaviour of KK masses.

To find the fermion masses, we have to find the spectrum of the squared Dirac operator in the compact space[2]. It is convenient to decompose the 6d Weyl fermion into a tensor product of 4d and 2d Weyl fermions like

$$\psi_L(x^\mu, y_m) = \psi_{4L}(x^\mu) \otimes \psi_{2R}(y_m) + \psi_{4R}(x^\mu) \otimes \psi_{2L}(y_m), \tag{3.23}$$

as we can now concentrate on the 2d components. The pairing of 4d and 2d chiralities is fixed by the 6d chirality through the relation

$$\Gamma^7 = \gamma_5 \otimes \sigma_3, \tag{3.24}$$

[2]The Dirac operator itself does not permit eigenfunctions, which is obvious from its off-diagonal shape in the chiral base of the Clifford algebra.

where Γ^7, γ_5 and σ_3 are the chirality operators in six, four and two dimensions, respectively. The Dirac operator squared is then

$$(\Gamma^m D_m)^2 = g_2^{mn} D_m D_n - \frac{1}{2}\Gamma^m\Gamma^n \left[D_n, D_m\right] = \Delta_2 - \frac{1}{2}\Gamma^m\Gamma^n \left[D_n, D_m\right]. \tag{3.25}$$

As long as the covariant derivatives in the internal space commute, the mass spectra for fermions and bosons coincide and supersymmetry is not broken by the gauge background. Without flux this is the case and the fermionic mass spectrum is given by Eq. (3.21).

The case with flux is quite different [30]. As the background gauge field is not constant, the covariant derivatives no longer commute. Instead, for the Hermitian operators $iD_{1,2}$ we have

$$\langle A\rangle = -f y_2 \mathrm{d}y_1 \;\Rightarrow\; [iD_1, iD_2] = -iqf. \tag{3.26}$$

Just like the position and momentum operators of elementary quantum mechanics, the covariant derivatives on the fluxed torus have an imaginary, constant commutator. Moreover, the Laplacian can be rearranged in a way analogous to the harmonic oscillator [17, 30]

$$\Delta_2 = g_2^{mn} D_m D_n = -\frac{1}{\tau_2}\left((\tau_2 iD_1)^2 + (iD_2 - \tau_1 iD_1)^2\right) = -\frac{1}{\tau_2}\hat{H}, \tag{3.27}$$

which reveals that the operators on the torus corresponding to quantum mechanic's position and momentum are not exactly iD_1 and iD_2, but rather

$$\hat{x} \sim \tau_2 iD_1, \quad \hat{p} \sim iD_2 - \tau_1 iD_1. \tag{3.28}$$

Furthermore, one can read off the constants describing the harmonic oscillator: m and ω follow directly from the expression for $\hat{H}$ in Eq. (3.27), while $\hbar$ is given through the commutator of $\hat{x}$ and $\hat{p}$,

$$m = \frac{1}{2}, \quad \omega = 2, \quad \hbar = -i[\hat{x}, \hat{p}] = -i\tau_2[iD_1, iD_2] = -\tau_2 qf > 0. \tag{3.29}$$

Should one pick a negative number of flux quanta, making the quantity corresponding to the Planck constant negative, the interpretation as position and momentum operators have to be switched, i.e., $\hat{p} \sim \tau_2 iD_1$ and $\hat{x} \sim iD_2 - \tau_1 iD_1$.

Knowing the expressions that map to ω and $\hbar$, we can already give the scalar mass spectrum, which depends on the number of flux quanta on the torus M,

$$m_n^2 = \frac{\hbar\omega}{\tau_2}\left(n + \frac{1}{2}\right) = 4\pi|M|\left(n + \frac{1}{2}\right). \tag{3.30}$$

It is peculiar that the spectrum of squared masses is linear in the mode number n. Moreover, the spectrum does not have a scalar zero mode; the lowest-lying scalar is at

$m^2 = 2\pi|M|V_2^{-1}$, which is above the compactification scale. The fermionic spectrum differs from Eq. (3.30), because the covariant derivatives do not commute in the presence of flux. From Eq. (3.25) we see that the fermion masses receive a chirality-dependent shift [17]

$$(\Gamma^m D_m) = \Delta_2 - 2\pi M\sigma_3 \quad \Rightarrow m_n^2 = 4\pi|M|\left(n + \frac{1}{2} \mp \frac{1}{2}\right). \tag{3.31}$$

For one of the chiralities, zero modes emerge. Which one is determined by the sign of the flux; this is why we chose $f < 0$, because we now have left-handed zero modes in 4d.

Following the algebraic treatment of the quantum harmonic oscillator, we define creation and annihilation operators

$$a \sim \hat{x} + i\hat{p} = \tau D_1 - D_2, \quad a^\dagger \sim \hat{x} - i\hat{p} = -\bar{\tau} D_1 + D_2. \tag{3.32}$$

As expected, the Laplacian (3.27) can be expressed through a and $a^\dagger$,

$$\Delta_2 = -\frac{\hbar\omega}{\tau_2}\left(a^\dagger a + \frac{1}{2}\right) = 4\pi|M|\left(a^\dagger a + \frac{1}{2}\right). \tag{3.33}$$

The ground state is defined by

$$a\,|\phi_0\rangle = 0. \tag{3.34}$$

Before we solve this equation to obtain an explicit expression for $\phi_0(y)$, we show even more clearly that the fermionic zero modes are left-handed in 4d. The fermionic ground state solves the massless Dirac equation in the compact space,

$$i\Gamma^m D_m \begin{pmatrix} \psi_{2L} \\ \psi_{2R} \end{pmatrix} = \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix} \begin{pmatrix} \psi_{2L} \\ \psi_{2R} \end{pmatrix} = 0. \tag{3.35}$$

Taking into account Eq. (3.34), a straightforward solution is given by

$$\psi_0 = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}. \tag{3.36}$$

It is right-handed in 2d which implies, through the relation (3.23), that its associated 4d spinor must be left-handed.

To find the ground state wave function we solve Eq. (3.34) in position space, following the computation presented in [31], but in different conventions. The boundary conditions for a field on the torus with Wilson lines $W_i = \exp(iq\alpha_i)$ and a flux given by Eq. (3.26) are

$$\phi(y + \lambda_1) = e^{iq\alpha_1}\phi(y), \quad \phi(y + \lambda_2) = e^{iq(\alpha_2 + fy_1)}\phi(y). \tag{3.37}$$

The boundary conditions (3.37) give a general expression for the wave function that is

$$\phi(y) = e^{iq(\alpha_1 y_1 + \alpha_2 y_2)} \sum_n f_n(y_2) e^{2\pi i n y_1}, \tag{3.38}$$

where $f_n(y_2)$, which does not depend on y_1, fulfils a recurrence relation

$$f_n(y_2 + 1) = f_{n-M}(y_2). \tag{3.39}$$

The ground state equation

$$\langle y|a|\phi_0\rangle = (\tau D_1 - D_2)\, \phi_0(y) = 0 \tag{3.40}$$

leads to a differential equation for $f_n(y_2)$, which reads[3]

$$f_n(y_2)' = f_n(y_2) \cdot 2\pi i \left[-\tau\, (n - M(y_2 + \eta_1)) + M\eta_2\right], \tag{3.41}$$

where we rephrased the Wilson lines as $\eta_m = q\alpha_m/M$. From this we find

$$f_n(y_2) = k_n \cdot \exp\left\{2\pi i \left[\frac{\tau}{2M}\, (n - M(y_2 + \eta_1))^2 + M\eta_2 y_2\right]\right\}. \tag{3.42}$$

The constant k_n is fixed through the recurrence relation (3.39). We find

$$k_n = e^{-2\pi i n \eta_2}, \tag{3.43}$$

which gives the final expression

$$f_n(y_2) = \exp\left\{2\pi i \left[\frac{\tau}{2M}\, (n - M(y_2 + \eta_1))^2 - \eta_2(n - My_2)\right]\right\}. \tag{3.44}$$

Not all n are independent as n and $n+M$ are related to each other via a translation in y_2. We can therefore split the index $n = M\nu + j$ into an index $\nu \in \mathbb{Z}$ that gathers all contributions linked through the recurrence relation (3.39) and a sum over $j \in \{0, \dots, |M|-1\}$, labelling independent pieces that can have independent amplitudes.

All in all, the zero mode wave function is a superposition of the independent pieces, each normalised by a normalisation factor $\mathcal{N} = \sqrt[4]{2M\tau_2/(\tau\bar{\tau})}$,

$$\phi_0^{(j)}(y) = \mathcal{N} e^{-iM(\eta_1 y_1 + \eta_2 y_2)} \sum_\nu e^{2\pi i M \left[\frac{\tau}{2}\left((\nu + \frac{j}{M}) - (y_2 + \eta_1)\right)^2 - \eta_2\left((\nu + \frac{j}{M}) - y_2\right) + (\nu + \frac{j}{M}) y_1\right]}. \tag{3.45}$$

The terms can be rearranged to give rise to one of Jacobi's theta functions

$$\phi_0^{(j)}(y) = \mathcal{N} e^{iq\alpha_1(y_1 + \tau y_2) - i\pi M \tau y_2^2}\, \theta\begin{bmatrix} j/M \\ -\frac{q}{2\pi}(\alpha_1\tau - \alpha_2) \end{bmatrix}\big(M(y_1 + \tau y_2), -M\tau\big), \tag{3.46}$$

[3] Here, the prime denotes a derivative w.r.t. y_2.

Table 3.1.: *The ranges of j that give linearly independent wave functions on the orbifold $T^2/\mathbb{Z}_2$ for different Wilson line configurations.*

k_1	k_2	Even	#	Odd	#	Total
0	0	$j \in \{0, \ldots, N\}$	$N+1$	$j \in \{1, \ldots, N-1\}$	$N-1$	$2N$
0	1	$j \in \{0, \ldots, N-1\}$	N	$j \in \{1, \ldots, N\}$	N	$2N$
1	0	$j \in \{0, \ldots, N-1\}$	N	$j \in \{0, \ldots, N-1\}$	N	$2N$
1	1	$j \in \{0, \ldots, N-1\}$	N	$j \in \{0, \ldots, N-1\}$	N	$2N$

where we reverted to the original Wilson line parameters α_m and use the definition [82]

$$\theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(x, y) = \sum_n e^{\pi i y (n+\alpha)^2} e^{2\pi i (n+\alpha)(x+\beta)}. \tag{3.47}$$

This expression was previously constructed for the square torus in [30] and for the general case in [31].

The previous expression describes the distribution of the scalar and fermionic ground state on the fluxed torus. To project these wave functions to the orbifold $T^2/\mathbb{Z}_2$, we have to construct linear combinations with definite behaviour under the parity operation. Straightforwardly taking the linear combinations $\phi_\pm^{(j)}(y) \propto \phi_0^{(j)}(y) \pm \phi_0^{(j)}(-y)$ yields

$$\phi_+^{(j)}(y; k_1, k_2) = \mathcal{N}' e^{2\pi i N \tau y_2^2} \sum_{\nu \in \mathbb{Z}} e^{2\pi i N \left(\nu - \frac{j}{2N}\right)^2} e^{-i\pi\left(\nu - \frac{j}{2N}\right)(k_1\tau - k_2)} \times \\ \cos\left[2\pi\left(2N\left(\nu - \frac{j}{2N}\right) - \frac{k_1}{2}\right)(y_1 + \tau y_2)\right], \tag{3.48}$$

where $\mathcal{N}'$ is an adjusted normalisation coefficient. The parity odd linear combination has the same structure with a sine instead of a cosine. Here, we re-introduced the number of flux quanta on the orbifold $N = -M/2 > 0$ and used the quantisation of Wilson lines on the orbifold $\alpha_m = k_m \pi / q$ with $k_m \in \{0, 1\}$.

How many linearly independent wave functions are there for a given N? On the torus, one can count the number of values j can take and arrives at an $|M|$-fold degeneracy in a straightforward manner. On the orbifold, matters are more subtle. The ranges of j that give linearly independent wave functions for the different Wilson line configurations are summarised in Table 3.1.

In the case without Wilson lines, $k_m = 0$, one can easily see that the wave functions with j and $j' = 2N - j$ are not linearly independent by first sending $j \mapsto 2N - j$ and then shifting the summation index by $\nu \mapsto -\nu + 1$. These transformations combined give $\nu - j/(2N) \mapsto -(\nu - j/(2N))$, which changes nothing but the sign of the argument of the trigonometric function. Using the (anti)symmetry of the cosine (sine), we see that

$$\phi_\pm^{(2N-j)}(y; 0, 0) = \pm\phi_\pm^{(j)}(y; 0, 0). \tag{3.49}$$

Therefore, independent even wave functions have $j \in \{0, \ldots, N\}$ while independent odd wave functions have $j \in \{1, \ldots, N-1\}$, giving $N+1$ zero modes with even and $N-1$ zero modes with odd parity. The counting of independent orbifold wave functions with even and odd parity in absence of Wilson lines has previously appeared in [17]. This adds up to a grand total of $(N+1)+(N-1) = 2N = |M|$ degrees of freedom, which, reassuringly, is the number of degrees of freedom we started with. The odd parity modes can be projected out by choosing chiral boundary conditions upon compactification, leaving a total of $N+1$ fermionic left-handed zero modes in the low-energy effective theory.

In the case when $k_1 = 0, k_2 = 1$, one can perform the same shifts with the summation index, however now there is an additional phase $\exp[i\pi k_2(\nu - j/(2N))]$ involved. It still allows a mapping of $\phi_{\pm}^{(2N-j)}$ onto $\phi_{\pm}^{(j)}$ because a global phase is not observable, but in the borderline case $j = N$ this phase amounts to a sign flip, which critically affects the behaviour of the wave function

$$\phi_{\pm}^{(2N-N)}(y;0,1) = \mp\phi_{\pm}^{(N)}(y;0,1) = \phi_{\pm}^{(N)}(y;0,1). \tag{3.50}$$

For $j = N$ the even wave function vanishes identically while the odd wave function does not. Hence, $j \in \{0, \ldots, N-1\}$ for even and $j \in \{1, \ldots, N\}$ for odd wave functions. This tallies up to N independent wave functions each and gives the expected total of $2N = |M|$ degrees of freedom .

In the case when $k_1 = 1, k_2 = 0$ considerations similar to the case without Wilson lines show that, due to k_1 appearing along with the summation index inside the trigonometric function, it is now $j' = 2N - j - 1$ which is equivalent to j. Therefore, we have

$$\phi_{\pm}^{(2N-j-1)}(y;1,0) = \pm\phi_{\pm}^{(j)}(y;1,0), \tag{3.51}$$

which gives independent even or odd wave functions for $j \in \{0, \ldots, N-1\}$. If the second Wilson line is present, such that $k_1 = 1, k_2 = 1$, these considerations still hold, even though the relation (3.51) now involves a complex phase instead of a sign. Again the count of degrees of freedom gives N even and N odd independent modes; the overall degeneracy of the system is conserved.

It is remarkable that the mass of the states does not depend on their orbifold parity; there are a number of odd-parity zero modes to the internal space Dirac equation. We can remove them through chiral boundary conditions

$$\begin{aligned} \psi_{4L}(x^\mu) \otimes \psi_{2R}(y_m) &= +\psi_{4L}(x^\mu) \otimes \psi_{2R}(-y_m), \\ \psi_{4R}(x^\mu) \otimes \psi_{2L}(y_m) &= -\psi_{4R}(x^\mu) \otimes \psi_{2L}(-y_m), \end{aligned} \tag{3.52}$$

which correspond to an additional $\mathrm{SU}(2)_R$ transformation associated with the orbifold parity. Moreover, unlike for uncharged fields, Wilson lines do not affect the mass spectrum at all. Using the orbifold base of one-cycles introduced in Appendix A.2, we can relate combinations of the orbifold parity and the Wilson lines to parities at fixed points. The

Table 3.2.: *Parities for charged fields at the individual fixed points ζ_i induced by Wilson lines $W_i = e^{i\pi k_i}$ and the orbifold parity P.*

(P, W_1, W_2)	ζ_1	ζ_2	ζ_3	ζ_4
$(+,+,+)$	$+$	$+$	$+$	$+$
$(+,+,-)$	$+$	$+$	$-$	$-$
$(+,-,+)$	$+$	$-$	$+$	$-$
$(+,-,-)$	$+$	$-$	$-$	$+$

(P, W_1, W_2)	ζ_1	ζ_2	ζ_3	ζ_4
$(-,+,+)$	$-$	$-$	$-$	$-$
$(-,+,-)$	$-$	$-$	$+$	$+$
$(-,-,+)$	$-$	$+$	$-$	$+$
$(-,-,-)$	$-$	$+$	$+$	$-$

correspondences are summarised in Table 3.2.

To visualise the behaviour of the wave function in the flux background, Figure 3.1 shows the absolute squared for the positive parity wave functions for $N = 3$. Without Wilson lines (Fig. 3.1a) we have $N + 1 = 4$ independent wave functions that do not vanish at any fixed point. In the other cases (Figs. 3.1b - d) we have $N = 3$ independent modes each. The empty circles in the plots represent the fixed points where the respective wave function vanishes. Their locations are in perfect agreement with Table 3.2, an additional reason to consider the Wilson lines in the orbifold one-cycle base, where this behaviour is clear from the start. It is astonishing that on the torus, j only shifts the location of the wave function profile without altering it [31], while on the orbifold wave functions with different j can have tremendously different shapes.

From the $\mathcal{N} = 1$ structure of the effective 4d Lagrangian we found in the previous section, one can formulate a clear expectation on the hypermultiplets: The remnants after compactification and truncation to the lowest KK state should give massless charged chiral multiplets with canonical kinetic terms and Killing vectors

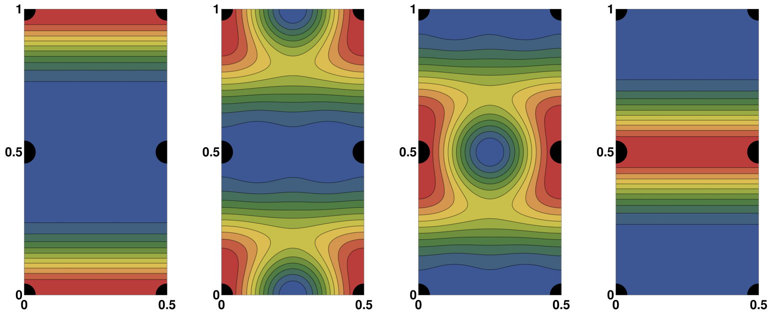
$$\Delta K = \sum_i \bar{\phi}_i e^{2g_4 qV} \phi_i, \quad X^{\phi_i} = -iq\bar{\phi}_i. \tag{3.53}$$

Through their Killing vectors, the scalar components of these multiplets appear in the D-term potential. The field-dependent FI term then gives a supersymmetry breaking mass to the scalars according to

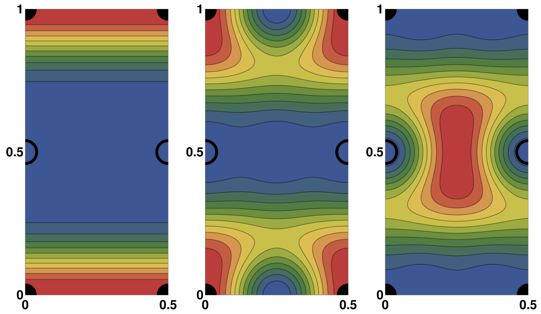
$$V_D = \frac{M_P^4 g_4^2}{2s} \left(-\frac{f}{t\ell^2} + \sum_i q \frac{|\phi_i|^2}{M_P^2} \right)^2 \Rightarrow m_{\phi_i}^2 = -\frac{qf}{tsL^2} = \frac{4\pi N}{L^2}, \tag{3.54}$$

where we used $\langle st \rangle = 1$ in the last step. The mass we expect from the $\mathcal{N} = 1$ supergravity is in perfect agreement with the ground state mass we computed directly in Eq. (3.30), if we keep in mind that on the orbifold $|M| = 2N$. It is reassuring that the hypermultiplet sector also abides to the $\mathcal{N} = 1$ supergravity structure, even if the supersymmetry is broken by the flux.

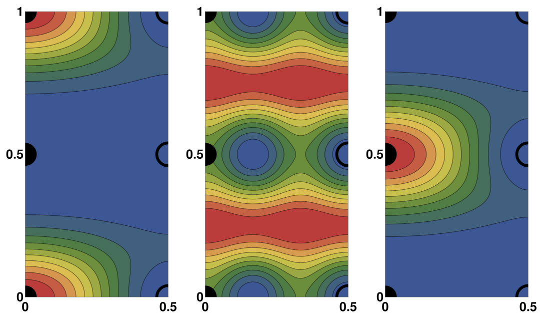
Strictly speaking, such a truncation to the lowest KK level is hard to justify. Even though they are mass degenerate, we assign the lowest mass even parity scalars to a



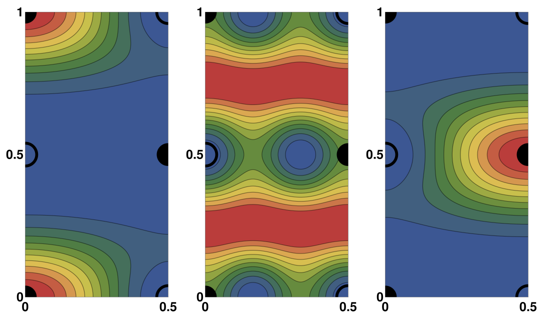
(a) *P-even wave functions* $|\phi_+(y;0,0)|^2$*. The parity is even at all fixed points.*



(b) *P-even wave functions* $|\phi_+(y;0,1)|^2$ *that vanish at* ζ_3 *and* ζ_4*.*



(c) *P-even wave functions* $|\phi_+(y;1,0)|^2$ *that vanish at* ζ_2 *and* ζ_4*.*



(d) *P-even wave functions* $|\phi_+(y;1,1)|^2$ *that vanish at* ζ_2 *and* ζ_3*.*

Figure 3.1.: *P-even wave functions* $|\phi_+(y;k_1,k_2)|^2$ *for* $N = 3$ *flux quanta on the orbifold. We show the absolute squared of the wave function on an arbitrary scale. The wave functions vanish at fixed points represented by hollow circles, while solid circles are fixed points where the wave function is non-vanishing. The configuration of wave function zeroes is in agreement with the fixed point parities summarised in Table 3.2.*

massless 4d chiral multiplet, which we call lowest KK level, while the odd parity scalars are assigned to a massive 4d chiral multiplet that we call first excited KK level. Capturing the full KK tower in the effective $\mathcal{N} = 1$ supergravity is possible by means of a superpotential

$$W_{\mathrm{KK}} = \sum_n \frac{m}{2}\sqrt{n+1}\,\Phi_+^{(n+1)}\Phi_-^{(n)}, \tag{3.55}$$

where the 4d chiral multiplets $\Phi_\pm^{(n)}$ are given through

$$\Phi_+^{(n)} = \left(\phi_+^{(n)}, \psi_L^{(n)}, F_+^{(n)}\right), \quad \Phi_-^{(n)} = \left(\phi_-^{(n)}, \psi_R^{(n)c}, F_-^{(n)}\right). \tag{3.56}$$

The investigation of the effects from resummation of the KK tower is a work in progress [83] and will not be part of this thesis.

3.3. Anomaly cancellation and effective action

Quantum field theories with chiral fermions need to ensure that no anomalies arise in gauged symmetries, or cancel the anomaly through a Green-Schwarz mechanism. In our model of 6d supergravity, the 6d gaugino and hyperino are both chiral. As long as we restrict ourselves to a pure U(1) gauge theory, the gauginos only contribute to gravitational anomalies, while the hyperinos are involved in pure gravity, mixed, and pure gauge anomalies. Appendix A.5 summarises the most important facts about anomalies and their cancellation in higher dimensions and in the presence of fixed points.

The bulk anomaly can be deduced either by direct computation [62] or through the anomaly polynomial [78, 79]. Let us focus on the pure gauge anomaly, which is that of a single charged 6d Weyl fermion

$$\delta\Gamma_f = -\int \mathcal{A} = -\int \Lambda F \wedge \left(\frac{\beta}{2} F \wedge F + \alpha F \wedge \delta_0 v_2\right), \tag{3.57}$$

with the loop factors $\beta = -q^4/(2\pi)^3$ and $\alpha = q^3/(2\pi)^2$. v_2 is the internal space volume two-form and δ_0 localises the fixed point anomaly equally at all four fixed points

$$\delta_0 = \frac{1}{4}\sum_I \delta^2(y - \zeta_I). \tag{3.58}$$

With foresight we defined the field strength of B such that its gauge variation is $\delta \mathrm{d}B = -\mathrm{d}\Lambda \wedge F$. Now, the Green-Schwarz term that cancels the anomaly is readily guessed, due to the conveniently factorised form we gave in (3.57). It is

$$S_{\mathrm{GS}} = -\int \left(\frac{\beta}{2} A \wedge F + \alpha A \wedge \delta_0 v_2\right) \wedge \mathrm{d}B. \tag{3.59}$$

Discarding total derivatives after performing a gauge variation we see that

$$\delta S_{\mathrm{GS}} = \int \left(\frac{\beta}{2} A \wedge F + \alpha A \wedge \delta_0 v_2 \right) \wedge \mathrm{d}\Lambda F = -\delta \Gamma_f . \tag{3.60}$$

What happens to the anomaly upon compactification? If we compactify without a background flux, we can set the field strength equal to its 4d components $F = \hat{F}$ and the bulk contributions to the anomaly (3.57) vanish. The remaining

$$\mathcal{A}_4|_{f=0} = \alpha \Lambda \hat{F} \wedge \hat{F} \tag{3.61}$$

is the anomaly of a single charged 4d Weyl fermion. It is the consequence of chiral boundary conditions, which create a single chiral 4d fermion zero mode from a 6d Weyl fermion. As we have seen above, the situation changes in the presence of flux: A charged Weyl fermion in 6d leads to a number of chiral zero-mode fermions after compactification. Inserting the decomposition of the field strength two-form in the flux background, $F = \hat{F} + f v_2$, into the expression for the 6d anomaly, we find

$$\mathcal{A}_4 = \left(\alpha + \frac{3}{2} \beta f \right) \Lambda \hat{F} \wedge \hat{F} . \tag{3.62}$$

The contribution proportional to f captures the additional flux-induced chiral degrees of freedom.

To see that the Green-Schwarz term (3.59) still cancels the anomalous transformation of the fermionic effective action, we have to include it in the bosonic action and compactify. Most of the bosonic compactification was discussed in Section 3.1, but the Green-Schwarz term gives a crucial correction. Details on its compactification can be found in Appendix B.1, we only repeat the most important facts here.

Using the decomposition of $\mathrm{d}B$, given in Eq. (3.9), the Green-Schwarz term is compactified to

$$S^4_{\mathrm{GS}} = -\int \left(\ell^2 \frac{\beta}{2} \hat{A} \wedge \hat{F} \wedge \mathrm{d}b + \left(\alpha + \frac{\beta}{2} f \right) \hat{A} \wedge \mathrm{d}\hat{B} \right) . \tag{3.63}$$

We can use the fact that $\hat{A} \wedge \hat{A} = 0$ due to the Abelian nature of the gauge group to replace $\mathrm{d}\hat{B}$ with $\hat{H} = \mathrm{d}\hat{B} + \hat{A} \wedge \hat{F}$ in the second term of Eq. (3.63). We can then dualise $\hat{H}$ to a pseudoscalar degree of freedom through a Lagrange multiplier term

$$S_c = \int \frac{1}{2 g_4^2} c \, \mathrm{d} \left(\hat{H} - \hat{A} \hat{F} \right) , \tag{3.64}$$

which ensures the closedness of $\mathrm{d}\hat{B}$. The altered equations of motion for $\hat{H}$ are given in

Eq. (B.18). It leads to a bosonic action (we drop the hats)

$$S_b^4 = \int \Bigg[\frac{M_P^2}{2} R_4 - \frac{M_P^2}{4\tau_2^2} \mathrm{d}\tau \wedge *\mathrm{d}\bar{\tau} - \frac{s}{2g_4^2} F \wedge *F - \frac{c + g_4^2 \beta \ell^2 b}{2g_4^2} F \wedge F \\ - \frac{M_P^2}{4t^2} \mathrm{d}t \wedge *\mathrm{d}t - \frac{M_P^2}{4t^2} \left(\mathrm{d}b + \frac{2f}{\ell^2} A \right) \wedge * \left(\mathrm{d}b + \frac{2f}{\ell^2} A \right) - \frac{M_P^4}{2} \frac{g_4^2}{s} \left(\frac{f}{\ell^2 t} \right)^2 \\ - \frac{M_P^2}{4s^2} \mathrm{d}s \wedge *\mathrm{d}s - \frac{M_P^2}{4s^2} \left(\mathrm{d}c + 2g_4^2 \left(\alpha + \frac{\beta}{2} f \right) A \right) \wedge * \left(\mathrm{d}c + 2g_4^2 \left(\alpha + \frac{\beta}{2} f \right) A \right) \Bigg]. \tag{3.65}$$

From this expression, we can read off the gauge variations

$$\delta b = -\frac{2f}{\ell^2} \Lambda, \quad \delta c = -2g_4^2 \left(\alpha + \frac{\beta}{2} f \right) \Lambda, \tag{3.66}$$

which imply an overall gauge variation of the bosonic action, that originates solely in the topological term $\propto F \wedge F$ and reads

$$\delta S_b^4 = -\frac{\delta c + g_4^2 \beta \ell^2 \delta b}{2g_4^2} F \wedge F = \left(\left(\alpha + \frac{\beta}{2} f \right) + \beta f \right) \Lambda F \wedge F = \mathcal{A}_4 = -\delta \Gamma_f^4. \tag{3.67}$$

We see that it precisely cancels the shift of the fermionic effective action, which is given by the 4d anomaly (3.62)

As was the case with just the classical terms, this action invites the definition of complex fields

$$T = \frac{1}{2}(t + ib), \quad S = \frac{1}{2}(s + ic), \quad U = \frac{1}{2}(\tau_2 + i\tau_1), \tag{3.68}$$

and a formulation in terms of $\mathcal{N} = 1$ supergravity. We have taken into account part of the one-loop corrections by including the Green-Schwarz term. It induces a shift of the pseudoscalar c, which is a Killing vector for S in supergravity terms, and it affects the gauge-kinetic function, whose imaginary part we read off the $F \wedge F$ term. A full one-loop calculation should preserve supersymmetry and therefore include all other terms induced by these changes. If we take supersymmetry at face value, we can construct the effective action as $\mathcal{N} = 1$ supergravity with a Kähler potential

$$K = -M_P^2 \log \left(T + \bar{T} + iX^T V \right) - M_P^2 \log \left(S + \bar{S} + iX^S V \right) - M_P^2 \log \left(U + \bar{U} \right) \tag{3.69a}$$

where the Killing vectors X^T, X^S are

$$X^T = -i\frac{f}{\ell^2}, \quad X^S = -ig_4^2 \left(\alpha + \frac{\beta}{2} f \right) = -ig_4^2 \alpha (1 + N) \tag{3.69b}$$

and a gauge-kinetic function

$$H = 2(S + g_4^2 \ell^2 \beta T). \tag{3.69c}$$

Compared to Eq. (3.65), the supergravity defined by Eqs. (3.69) has a modified gauge-kinetic function and D-term potential. The bosonic effective action is

$$\begin{aligned} S_b^4 = \int \Bigg[& \frac{M_P^2}{2} - \frac{M_P^2}{(U+\bar{U})^2} \mathrm{d}U \wedge *\mathrm{d}\bar{U} - \frac{s + g_4^2 \beta \ell^2 t}{2 g_4^2} F \wedge *F - \frac{c + g_4^2 \beta \ell^2 b}{2 g_4^2} F \wedge F \\ & - \frac{M_P^2}{(T+\bar{T})^2} \mathrm{D}T \wedge *\mathrm{D}\bar{T} - \frac{M_P^2}{(S+\bar{S})^2} \mathrm{D}S \wedge *\mathrm{D}\bar{S} - \sum_j \mathrm{D}\phi_+^j \wedge *\mathrm{D}\bar{\phi}_+^j \\ & - \frac{M_P^4}{2} \frac{g_4^2}{s + g_4^2 \beta \ell^2 t} \Bigg(- \frac{f}{t \ell^2} - g_4^2 \frac{(\alpha + \beta f/2)}{s} + \sum_j \frac{q |\phi_+^j|^2}{M_P^2} \Bigg) \Bigg], \end{aligned} \tag{3.70}$$

where we added the scalar components of a single chiral multiplet according to our findings in the previous section, as given in Eq. (3.53).

The quantum corrections as captured by the Green Schwarz term give a correction to the scalar mass[4]

$$m_+^2 \propto -\frac{f}{t\ell^2} - g_4^2 \alpha \frac{1+N}{s} = m^2|_t + m^2|_s. \tag{3.71}$$

The classical contribution to the mass is positive, $m^2|_t > 0$ as $f < 0$, while the quantum correction gives a negative contribution, $m^2|_s < 0$. We see that the background flux stabilises the charged scalar, which would otherwise develop a vacuum expectation value from its D-term potential.

The vector boson mass is also affected through the quantum corrections. Now, both axions shift under gauge transformations. To find the linear combination that is absorbed, we introduce the field χ through the transformation $A \mapsto A + \mathrm{d}\chi$ and demand that all mixing terms between $\mathrm{d}b$, $\mathrm{d}c$, $\mathrm{d}\chi$ and A vanish. This fixes χ in terms of b and c as

$$\chi = \mathcal{N} \left(\frac{iX^T}{t_0^2} b + \frac{iX^S}{s_0^2} c \right), \quad \text{with} \quad \mathcal{N}^{-1} = \left| X^T / t_0 \right|^2 + \left| X^S / s_0^2 \right|^2, \tag{3.72}$$

and leaves a linear combination,

$$a = iX^T\, c - iX^S\, b, \tag{3.73}$$

which does not shift under gauge transformations as pseudoscalar degree of freedom in the Lagrangian. The vector boson mass is

$$m_A^2 = \frac{g_4^2 M_P^2}{s + g_4^2 \beta \ell^2 t} \left((iX^T/t)^2 + (iX^S/s)^2 \right) = \frac{g_4^2 M_P^2}{s + g_4^2 \beta \ell^2 t} \left(\frac{f^2}{\ell^4 t^2} + \frac{g_4^4 (\alpha + \beta f/2)^2}{s^2} \right); \tag{3.74}$$

it, too, has classical and quantum contributions. Unlike for the scalar, both contributions are positive, though. The classical limit (cf. Eq. (3.17)) can be recovered if we set the loop

[4] When discussing masses, we assume the moduli s and t to be stabilised and do not differentiate between the field and its vacuum expectation value.

factors α and β to zero. We also can set the flux to zero, $f = 0$, and see that the resulting vector boson mass originates purely from the 4d anomaly.

3.4. Moduli stabilisation

A general issue in compactifications is the stabilisation of the moduli, e.g., scalar fields that arise from internal components of the metric. Their potential is generically either flat or of runaway type, which means that the potential asymptotes to zero from above for large field values. This is the case we found in our effective action, where the D-term potential,

$$V_D = \frac{M_P^4}{2} \frac{g_4^2}{s + g_4^2 \beta \ell^2 t} \left(-\frac{f}{t\ell^2} - g_4^2 \alpha \frac{N+1}{s} \right)^2 , \tag{3.75}$$

only features inverse powers of the moduli t and s and neither involves the pseudoscalars b or c, nor the complex modulus U.

There is a constraint on the moduli field values from the stability of the theory: The gauge kinetic function can become zero for positive values of the moduli due to the negative contribution linear in t – remember that β is negative. As a gauge theory with vanishing or negative gauge coupling is not well-defined, we require

$$\mathrm{Re}(H) = h = s + g_4^2 \ell^2 \beta t > 0 \quad \Rightarrow \quad s > \frac{g_4^2 \ell^2 q^4}{(2\pi)^3} t. \tag{3.76}$$

When the field values approach the critical line along which $h = 0$, the D-term potential diverges[5] ,

$$V_D \propto \frac{1}{h} \xrightarrow{h \to 0} \infty. \tag{3.77}$$

Therefore, the constraint (3.76) is dynamically stable in the sense that once the moduli are in the field space region where it is satisfied, they will remain there indefinitely. After stabilisation, the effective gauge coupling is given by $g_{\text{eff}}^2 = g_4^2 / \langle h \rangle$.

On the physical moduli space, the D-term potential is always positive. For the D-term to vanish, the positive and negative contributions would have to cancel, which implies

$$D = 0 \quad \Rightarrow \quad -\frac{f}{t\ell^2} = g_4^2 \alpha \frac{1+N}{s} \quad \Rightarrow \quad s = \frac{g_4^2 \ell^2 q^4}{(2\pi)^3} \frac{N+1}{2N} t. \tag{3.78}$$

Comparing this expression with the constraint (3.76), we see that for $N \geq 1$, the D-term is indeed never zero.

We did not involve the hypermultiplet degrees of freedom in Eq. (3.75), because they

[5] Small values of h imply a strong coupling, but the argument that the field values possible for the moduli are constrained by the relation (3.76) should hold nevertheless.

are stabilised at zero as long as their mass is positive[6]. The scalar mass is proportional to the sum of the FI terms, whose positivity we just showed. We can therefore concentrate on the complex fields S, T and U, having ensured that no hypermultiplet scalar vacuum expectation values develop.

In a first step, we stabilise the two complex moduli S and T and assume that the complex modulus $U = 1$. The real parts of S and T obtain a runaway potential from the compactification, while their imaginary parts have a continuous shift symmetry. We are free to introduce a 4d superpotential that involves these fields; in 6d this superpotential lives at the fixed points and interferes neither with any of the bulk dynamics nor with the compactification. Past research in moduli stabilisation has shown superpotentials to arise from non-perturbative effects such as hidden sector gaugino condensation [9] or world-sheet instantons [10, 11, 12]. We will not specify how our superpotential arises but instead focus on the vacua we can find. The superpotential has to be a holomorphic, gauge invariant function of the scalar fields, since we do not gauge an R symmetry. Since both, S and T shift under gauge transformations, the superpotential can only depend on the neutral linear combination

$$Z = \frac{1}{2}(z + i\tilde{c}) = -iX^T S + iX^S T. \tag{3.79}$$

We chose the signs such that $z \geq 0$. Note that, up to a sign, the axion $\tilde{c}$ is exactly the linear combination of pseudoscalars that remains dynamical after the Stückelberg mechanism, see Eq. (3.73).

Taking the standard KKLT form of the superpotential [13], our effective action is a $\mathcal{N} = 1$ supergravity with

$$\begin{aligned} K &= -M_P^2 \log\left(T + \bar{T} + iX^T V\right) - M_P^2 \log\left(S + \bar{S} + iX^S V\right) - M_P^2 \log\left(U + \bar{U}\right), \\ H &= 2S + 2g_4^2 \ell^2 \beta T = h_S S + h_T T, \quad X^T = -i\frac{f}{\ell^2},\ X^S = -ig_4^2 \alpha(N+1) \\ W &= W_0 + W_1 e^{aZ}. \end{aligned} \tag{3.80}$$

W.l.o.g., W_0 and W_1 are real constants and with foresight we demand that $W_0 \cdot W_1 < 0$. Setting $M_P = 1$, the general form of the scalar potential is

$$V = V_F + V_D = e^K \left(D_i W\, K^{i\bar{j}} \bar{D}_{\bar{j}} \bar{W} - 3|W|^2\right) + \frac{1}{2}\frac{g_4^2}{h} D^2. \tag{3.81}$$

We use the Kähler covariant derivative $D_i W = W_i + K_i W$, denote the real part of the gauge-kinetic function as h and write

$$D = iK_i X^i = -\frac{i}{t}X^T - \frac{i}{s}X^S, \quad E = -\frac{i}{t}X^T + \frac{i}{s}X^S > 0. \tag{3.82}$$

[6]Strictly speaking the field is stabilised also if its mass vanishes due to the $|\phi_+|^4$-contribution to the potential.

The Kähler potential is of no-scale type, i.e., $K_i K^{i\bar{j}} K_{\bar{j}} = 3$, which removes the contribution $\propto |W|^2$ from the scalar potential. Using the notation introduced in Eq. (3.82) we can write the scalar potential as

$$V = \frac{st}{2}(D^2 + E^2)A - EB + \frac{g_4^2}{2h}D^2, \tag{3.83}$$

where the superpotential parameters are featured indirectly through

$$\begin{aligned} A &= |\partial_Z W|^2 = a^2 W_1^2 e^{-az} \\ B &= (\partial_Z W)\bar{W} + W(\partial_{\bar{Z}}\bar{W}) = -2aW_1\left(W_1 e^{-az} + W_0 e^{-az}\cos\left(\frac{a}{2}\tilde{c}\right)\right). \end{aligned} \tag{3.84}$$

Instead of fixing the superpotential parameters W_0, W_1 and a and then looking for a minimum in the potential, we require a vacuum that stabilises all moduli with a small or vanishing cosmological constant, i.e., we demand

$$\partial_S V = 0, \quad \partial_T V = 0, \quad V = \epsilon \gtrsim 0, \tag{3.85}$$

and reverse-engineer, which values for W_0, W_1 and a give a corresponding potential. The term in the potential, that features the axion $\tilde{c}$ is

$$V_{\tilde{c}} = 2aE\, W_0 W_1 e^{-az}\cos\left(\frac{a}{2}\tilde{c}\right) \tag{3.86}$$

with $E > 0$. $\tilde{c}$ is stabilised at the origin as long as the constant and exponential contributions in the superpotential have opposite signs. We set $\tilde{c} = 0$ in the following and focus on the scalars s and t.

Computing the derivatives of the potential is tedious, but straightforward. Since both $s > 0$ and $t > 0$ we can introduce convenient new derivative operators $\partial_\pm = s\partial_S \pm t\partial_T$, which act on Z as $\partial_+ Z = stE$ and $\partial_- Z = stD$. The conditions (3.85) then are equivalent to

$$\epsilon = V = \frac{st}{2}(D^2 + E^2)A - EB + \frac{g^2}{2h}D^2, \tag{3.87a}$$

$$0 = \partial_+ V = \left(\frac{a}{2}s^2t^2 E(D^2 + E^2) - stE^2\right)A + \left(\frac{a}{2}stE - 1\right)EB - 3\frac{g^2}{2h}D^2, \tag{3.87b}$$

$$0 = \partial_- V = \left(\frac{a}{2}s^2t^2(D^2 + E^2) + stE\right)A + \left(\frac{a}{2}stE + 1\right)B + E\frac{g^2}{h} - \frac{g^2}{2h^2}D\tilde{h}, \tag{3.87c}$$

with $\tilde{h} = \frac{1}{2}(h_S s - h_T t)$. These equations can be solved for A, B and a if one takes $\langle s\rangle$ and $\langle t\rangle$ as given parameters. Introducing $\rho = D/E \in [0, 1)$, the solutions read

$$A = -\frac{1}{2h^2 st(1-\rho^2)}\left(h_T t\rho + h(2 - \rho + \rho^2) + h^2\frac{2\epsilon}{E^2}\right), \tag{3.88a}$$

$$B = -\frac{E}{4h^2(1-\rho)}\left(h_T t\rho(1+\rho^2) + h(2 - \rho + \rho^2 - \rho^3 + 3\rho^4) + h^2\frac{8\epsilon}{E^2}\right), \tag{3.88b}$$

$$a = \frac{2E(1-\rho^2)\left(h_T t\rho + h(2-\rho-3\rho^2)\right)}{st\left(E^2\left[h_T t\rho(1+\rho^2) + h(2-\rho+5\rho^2-\rho^3-\rho^4)\right] + 8h^2\rho\epsilon\right)}. \tag{3.88c}$$

We are now in a position to pick the location in moduli space, where we want our minimum to be located and subsequently reconstruct the superpotential parameters that will stabilise the moduli there. Fixing the model parameters N, g_4 and L and inserting values for $\langle s\rangle$ and $\langle t\rangle$ in Eqs. (3.88), we can compute the values of A, B and a. Those are, in turn, related to the superpotential parameters W_0, W_1 via Eqs. (3.84). Note that not all combinations of $\langle s\rangle$, $\langle t\rangle$, N, g_4 and L give meaningful values for A, B or a. We can always choose $\langle s\rangle = \kappa$, $\langle t\rangle = \kappa^{-1}$, such that $\langle st\rangle = 1$, leaving $\kappa^2 = \langle s\rangle/\langle t\rangle = e^{2\phi}$ as a free parameter. Appendix B.2 shows how any vacuum with $\langle st\rangle \neq 1$ can be transformed to comply with this case. Empirically, the stabilisation works well if[7]

$$g_4^2 L \approx \frac{40}{3}\kappa. \tag{3.89}$$

Let us give an example for a stabilisation with a Minkowski vacuum, i.e., cosmological constant $\epsilon = 0$. For

$$\kappa = 3/5, \quad N = 3, \quad L = 200, \quad \text{and} \quad g_4 = 0.2 \tag{3.90}$$

we obtain

$$A = 2.2\times 10^{-3}, \quad B = -3.3\times 10^{-4}, \tag{3.91}$$

which corresponds to superpotential parameters

$$W_0 = -3.65\times 10^{-3}, \quad W_1 = 2.07\times 10^{-3}, \quad az = 4.51. \tag{3.92}$$

We do not give a numerical value for a itself, because it appears in the exponential together with the scalar z, which involves the Killing vectors X^S and X^T. The Killing vectors are numerically small, in turn rendering the value of a unintuitively large. The scale $\mathcal{O}(10^{-3})M_P$ for the W_i suits our interpretation of the superpotential as an effect from even higher dimensions. These further extra dimensions would have to be compactified at a scale L_s^{-1}, lying somewhere between L^{-1} and M_P. Generic superpotential parameters are expected to be of the order of the compactification scale cubed, L_s^{-3}; from the W_i one would thus expect $L_s^{-1} \approx 10^{-1}M_P$.

It is extraordinary to find Minkowski minima, as the non-perturbative superpotential generically produces an anti-de Sitter (AdS) vacuum. An additional "uplift" sector then increases the negative cosmological constant to give Minkowski or de Sitter spacetime. A crucial difference to the standard supergravity constructions can be found in our gauge kinetic function. By taking into account anomaly cancellation during our compactification

[7] This relation was noticed by M. Dierigl.

we use a one-loop corrected expression that has a negative contribution linear in the modulus T. Such corrections are also known to arise in String Theory [34, 35]. The importance of this correction in our construction can be witnessed in the expression for A, given in Eq. (3.88a).

From its definition, A is manifestly positive. Since $h > 0$ and $\rho < 1$, the only two contributions that can assure the positivity of A are the first and the last term in the parentheses. The first is proportional to h_T, while the last term is proportional to the cosmological constant (CC) ϵ. One of the two has to be negative to obtain a meaningful value for A. Hence, if h_T was absent or positive, $A > 0$ would imply a negative CC, $\epsilon < 0$. This shows the direct connection between the quantum correction of the gauge kinetic function and the vanishing or positive CC in our model.

So far, we just assumed a fixed value for the modulus U. It does not receive any potential from the compactification. As it is uncharged, it can be included in the superpotential in a very straightforward manner. We write

$$W = W_0 + W_1 E^{-aZ} + W_2 e^{-\tilde{a}U}. \tag{3.93}$$

The scalar potential then is

$$V = \frac{st}{2\tau_2}(D^2 + E^2)A + \frac{\tau_2}{st}\tilde{A} - \frac{1}{\tau_2}EB - \frac{1}{st}\tilde{B} + \frac{g^2}{2h}D^2, \tag{3.94}$$

with the superpotential parameters included in

$$\begin{aligned}
A &= |\partial_Z W|^2 = a^2 W_1^2 e^{-az}, \\
\tilde{A} &= |\partial_U W|^2 = \tilde{a}^2 W_2^2 e^{-\tilde{a}\tau_2} \\
B &= (\partial_Z W)\bar{W} + W(\partial_{\bar{Z}}\bar{W}) \\
&= -2aW_0 W_1 e^{-\frac{a}{2}z} \cos\left(\frac{a}{2}\tilde{c}\right) - 2aW_1 W_2 e^{-\frac{a}{2}z - \frac{\tilde{a}}{2}\tau_2} \cos\left(\frac{a}{2}\tilde{c} - \frac{\tilde{a}}{2}\tau_1\right), \\
\tilde{B} &= (\partial_U W)\bar{W} + W(\partial_{\bar{U}}\bar{W}) \\
&= -2\tilde{a}W_0 W_2 e^{-\frac{\tilde{a}}{2}\tau_2} \cos\left(\frac{\tilde{a}}{2}\tau_1\right) - 2\tilde{a}W_1 W_2 e^{-\frac{a}{2}z - \frac{\tilde{a}}{2}\tau_2} \cos\left(\frac{a}{2}\tilde{c} - \frac{\tilde{a}}{2}\tau_1\right).
\end{aligned} \tag{3.95}$$

The conditions for a Minkowski or de Sitter vacuum with all scalars stabilised are given by

$$\partial_S V = 0, \quad \partial_T V = 0, \quad \partial_U V = 0, \quad V = \epsilon. \tag{3.96}$$

Once again, one can introduce the linear combinations $\partial_\pm$ as in the two moduli case, but now also $\partial_0 = \tau_2 \partial_U$ is useful. The conditions for a vacuum with a CC ϵ and all moduli stabilised then are

$$\begin{aligned}
\epsilon = V &= \frac{st}{2\tau_2}(D^2+E^2)A + \frac{\tau_2}{st}\tilde{A} - \frac{1}{\tau_2}EB - \frac{1}{st}\tilde{B} + \frac{g^2}{2h}D^2, && \text{(3.97a)}\\
0 = \partial_+ V &= \frac{st}{\tau_2}\left(-\frac{a}{2}stE(D^2+E^2) - 2E^2\right)A + \frac{1}{\tau_2}\left(\frac{a}{2}stE^2 + E\right)B \\
&\quad - E\big(A\tilde{A}\big)^{1/2} - \frac{2\tau_2}{st}\tilde{A} + \frac{2}{st}\tilde{B} - 3\frac{g^2}{2h}D^2 && \text{(3.97b)}\\
0 = \partial_- V &= \frac{st}{\tau_2}\left(-\frac{a}{2}stD(D^2+E^2) + DE\right)A + \frac{1}{\tau_2}\left(\frac{a}{2}stDE - D\right)B \\
&\quad - D\big(A\tilde{A}\big)^{1/2} + \frac{g^2}{h}DE - \frac{g^2}{2h^2}D^2\tilde{h}, && \text{(3.97c)}\\
0 = \partial_0 V &= -\frac{st}{2u^2}(D^2+E^2)A + \frac{1}{u^2}EB - \frac{\tau_2}{st}\tilde{a}\tilde{A} - \frac{1}{\tau_2}E\big(A\tilde{A}\big)^{1/2} + \frac{\tilde{a}}{2st}\tilde{B}. && \text{(3.97d)}
\end{aligned}$$

Solving these equations gives an expression for A, which is just the same as for the two moduli case, Eq. (3.88a), multiplied by τ_2. The other parameters, $B, \tilde{A}, \tilde{B}, a$ no longer allow neat representations in terms of s, t, τ_2, D and E. Our reasoning for the connection between $h_T < 0$ and Minkowski vacua rests exclusively on the expression for A. It can therefore be transferred directly from the two moduli case.

Concerning numerical solutions, we are also in a slightly less elegant situation than in the two moduli case: There, we had three conditions and three real parameters to fix, whereas now we have one more condition, $\partial_U V = 0$, but in turn our superpotential involves two new degrees of freedom, $\tilde{a}$ and W_2. In order to find well-defined solutions we have to fix one of the parameters by hand, which we choose to be $\tilde{a}$. While the argument for the stabilisation of the axions mostly carries over, caution is required. There is a cosine term that involves a combination of $\tilde{c}$ and τ_1,

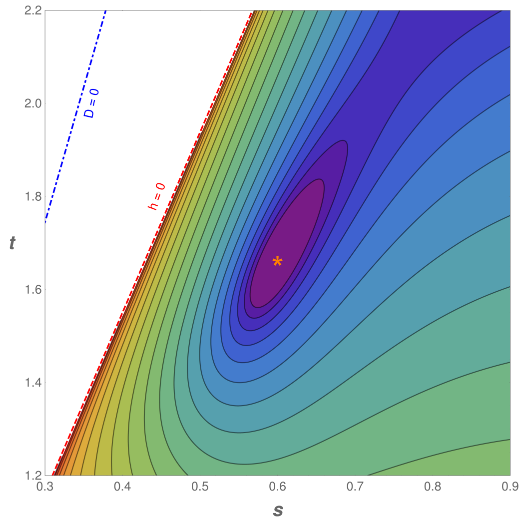
$$V \supset \frac{az + \tilde{a}\tau_2}{st\tau_2} 2W_1W_2 e^{-\frac{a}{2}z - \frac{\tilde{a}}{2}\tau_2} \cos\left(\frac{a}{2}\tilde{c} - \frac{\tilde{a}}{2}\tau_1\right), \tag{3.98}$$

which might destabilise the axions. We assume both $\tilde{c}$ and τ_1 to be stabilised at the origin as long as W_0 carries a sign opposite to that of W_1 and W_2, but we have to check this statement after determining the actual values for the superpotential parameters.

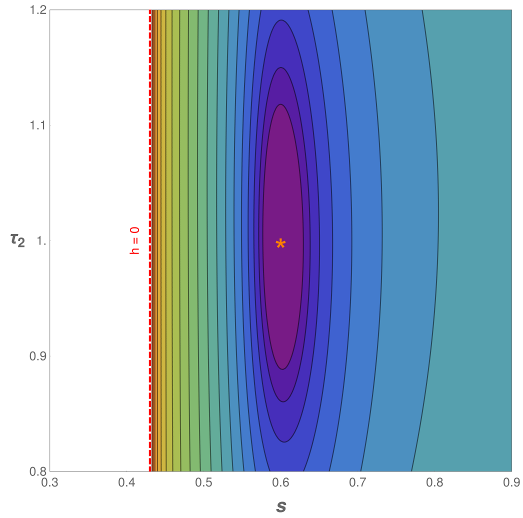
Picking up our example from the two moduli case we choose $s = \kappa = 3/5$, $t = \kappa^{-1} = 5/3$ and $\tau_2 = 1$, while $N = 3$, $g_4 = 0.2$ and $L = 200$ along with $\tilde{a} = 2\pi/3$ to find

$$W_0 = -2.94 \times 10^{-3}, \quad W_1 = 1.16 \times 10^{-2}, \quad W_2 = 2.83 \times 10^{-3}, \quad az = 9.44. \tag{3.99}$$

We show the potential close to its minimum in Figure 3.2. The red dashed line is where the gauge-kinetic function vanishes; it borders the physical moduli space. In addition, the blue dot-dashed line shows where the D-term would vanish, if the moduli could take arbitrary values. The explicit check shows that the axions are stabilised indeed.



(a) *The potential in the s-t-plane with $\tau_2 = 1$.*



(b) *The potential in the s-τ_2-plane with $t = \frac{5}{3}$.*

Figure 3.2.: *Contour plots of the scalar potential close to its minimum in two different planes of the moduli space. The contours are distributed logarithmically. The orange star marks the location of the minimum.*

3.5. Mass spectrum of the low-energy effective action

As numerical values are available for all supergravity parameters, we can study the mass spectrum of the effective action. In the following, we explain how to determine the masses and give numerical values for our example minimum with parameters given in Eqs. (3.90) and (3.99). For the bosons, this is a somewhat straightforward exercise: Their kinetic terms need to be canonically normalised, which can be achieved by the transformations

$$\begin{aligned} &s \mapsto \frac{M_P}{\sqrt{2}} \log(s), \quad c \mapsto \frac{1}{\sqrt{2\langle s\rangle}} c, \quad t \mapsto \frac{M_P}{\sqrt{2}} \log(t), \quad b \mapsto \frac{1}{\sqrt{2\langle t\rangle}} b, \\ &\tau_2 \mapsto \frac{M_P}{\sqrt{2}} \log(\tau_2), \quad \tau_1 \mapsto \frac{1}{\sqrt{2\langle \tau_2\rangle}} \tau_1, \quad A \mapsto \frac{g_4}{\sqrt{h}} A. \end{aligned} \tag{3.100}$$

The axion masses can be computed from the Hessian matrix of the scalar potential, which has one zero eigenvalue that corresponds to the axion "eaten" by the gauge field in a Stückelberg mechanism, and whose other eigenvalues are

$$m^2_{\text{axion},1} = \left(9.45 \times 10^{-3} M_P\right)^2, \quad m^2_{\text{axion},2} = \left(4.12 \times 10^{-3} M_P\right)^2. \tag{3.101}$$

In a similar fashion, computing the eigenvalues of the Hessian matrix for the scalars s, t and τ_2 we find the moduli masses

$$\begin{aligned} &m^2_{\text{mod},1} = \left(1.2 \times 10^{-2} M_P\right)^2, \quad m^2_{\text{mod},2} = \left(4.14 \times 10^{-3} M_P\right)^2, \\ &m^2_{\text{mod},3} = \left(3.40 \times 10^{-3} M_P\right)^2. \end{aligned} \tag{3.102}$$

The positively charged hypermultiplet scalar obtains a mass from the D-term potential,

$$m_+^2 = \frac{g_4^2 M_P^2}{s + g_4^2 \beta \ell^2 t} \left(-\frac{qf}{t\ell^2} - g_4^2 q\alpha \frac{N+1}{s} \right) = \left(8.33 \times 10^{-3} M_P \right)^2. \tag{3.103}$$

Finally, we can absorb a linear combination of the axions via the Stückelberg mechanism and read off the vector boson mass, which we gave before on Page 35,

$$m_A^2 = \frac{g_4^2 M_P^2}{s + g_4^2 \beta \ell^2 t} \left(\frac{f^2}{\ell^4 t^2} + \frac{g_4^4 (\alpha + \beta f/2)^2}{s^2} \right) = \left(2.15 \times 10^{-3} M_P \right)^2. \tag{3.104}$$

The charged scalar and vector boson masses each have a classical contribution that is volume-suppressed, as can be seen by the inverse powers of ℓ appearing, and a quantum contribution from anomaly cancellation, which, at first sight, seems to be independent of the compactification scale. To find the true dependence on the compactification scale, one has to keep in mind the empirical relation between the compactification scale and the bare coupling, given in Eq. (3.89). It implies that the coupling scales like $g_4 \sim L^{-1/2}$, giving a uniform scaling behaviour[8] of the terms in Eqs. (3.103) and (3.104),

$$m_+^2 \sim L^{-2}, \quad m_A^2 \sim L^{-3}. \tag{3.105}$$

Moreover, one can deduce the scaling behaviour of the D-term potential, which also is the scaling behaviour of the F-term potential, because we tune the two to give a vanishing CC, and from there the scaling behaviour of the superpotential parameters,

$$V_D \sim L^{-3} \Rightarrow V_F \sim L^{-3} \Rightarrow W_0, W_1, W_2 \sim L^{-3/2}, a \sim L. \tag{3.106}$$

Since the moduli and axion masses are given through derivatives of the potentials, they scale like

$$m_{\text{mod}}^2 \sim m_{\text{axion}}^2 \sim V \sim L^{-3}. \tag{3.107}$$

To find the fermion masses, more work is required. In broken supergravity, the gravitino mixes with a linear combination of the other fermions in the theory, the goldstino. In general we have

$$\mathcal{L}_f \supset \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \bar{\sigma}_\nu \partial_\rho \psi_\sigma - m_{3/2} \psi_\mu \sigma^{\mu\nu} \psi_\nu - \bar{m}_{3/2} \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu + \psi_\mu \sigma^\mu \bar{\chi} + \chi \sigma^\mu \bar{\psi}_\mu, \tag{3.108}$$

where the gravitino mass is given by

$$m_{3/2} = e^{\langle K \rangle /2} \langle W \rangle \tag{3.109}$$

and the goldstino direction in field space is given by the vacuum expectation values of the

[8] M. Dierigl was the first in our group to study the relation between the masses and the length scale L.

D- and F-terms,

$$\chi = -\frac{g}{2}\langle D\rangle\lambda - \frac{i}{\sqrt{2}}e^{\langle K\rangle/2}\langle D_i W\rangle\psi^i. \tag{3.110}$$

The goldstino can be eliminated from the action by a supergravity transformation [84],

$$\psi_\mu \mapsto \psi_\mu - \frac{\sqrt{2}}{\sqrt{3}m_{3/2}}\partial_\mu\eta + \frac{i}{\sqrt{6}}\bar{\eta}\bar{\sigma}_\mu, \quad \eta = \frac{i\sqrt{2}}{\sqrt{3}\bar{m}_{3/2}}\chi, \tag{3.111}$$

after which the Lagrangian (3.108) is

$$\mathcal{L}_f \supset \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - m_{3/2}\psi_\mu\sigma^{\mu\nu}\psi_\nu - \bar{m}_{3/2}\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + i\bar{\eta}\bar{\sigma}^\mu\partial_\mu\eta + \bar{m}_{3/2}\eta\eta + m_{3/2}\bar{\eta}\bar{\eta}. \tag{3.112}$$

The terms that involve η are such that, when they are added to the regular fermion action for λ and ψ^i, the goldstino is eliminated from the action entirely. The contributions to the fermion mass matrix from the terms $\bar{m}_{3/2}\eta\eta$ + h.c. prove crucial to find the physical mass spectrum for the fermions. This is evident, as we know that the fermion sector has a massless degree of freedom, the goldstino; however, before diagonalising the gravitino sector, no zero eigenvalue can be found in the mass matrix. We give the expressions for the fermion sector with the goldstino eliminated in[9] [85].

The fermions remaining after the super-Higgs mechanism are a massive gravitino and three mixed states of the gaugino and the modulini. Their masses can be evaluated numerically from the expressions in [85] and are

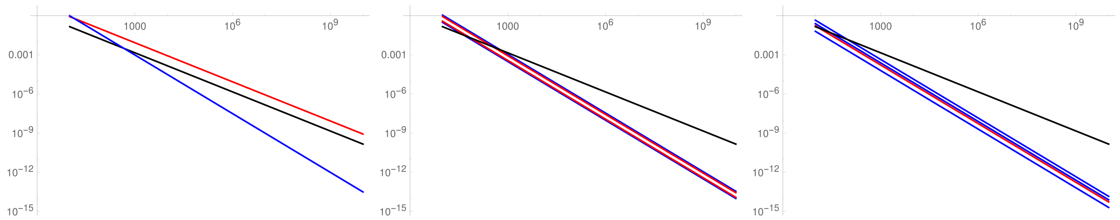
$$m_{3/2} = 1.84 \times 10^{-3} M_P, \tag{3.113}$$
$$m_{1/2} = \left\{4.92 \times 10^{-3},\ 2.69 \times 10^{-3},\ 6.96 \times 10^{-4}\right\} M_P. \tag{3.114}$$

Again, we can deduce their behaviour under scaling of the extra dimensions from the behaviour of the superpotential parameters, giving

$$m_{3/2} \sim L^{-3/2}, \quad m_{1/2} \sim L^{-3/2}. \tag{3.115}$$

Figure 3.3 shows the scaling behaviour for compactification scales from close to the Planck scale to some intermediate scale. Beyond $L \approx 10^{10} M_P^{-1}$ the numerical precision of our code is insufficient to reliably solve the system of equations (3.97). Most masses scale like the gravitino mass, which sets the scale of supersymmetry breaking. With larger extra dimensions, all masses except for the charged scalar's mass fall below the compactification scale. Strictly speaking, the charged scalar is therefore not a part of the low-energy effective theory.

[9] This part of our work was done mostly by M. Dierigl and W. Buchmüller, we therefore do not provide further details here.



(a) *Scaling of vector (blue) and charged scalar (red) masses.* **(b)** *Scaling of moduli (blue) and axion (red) masses.* **(c)** *Scaling of modulini (blue) and gravitino (red) masses.*

Figure 3.3.: *Scaling behaviour of different parts of the low-energy action in doubly logarithmic plots. The masses are plotted in Planck units against the length scale L. For easier comparison, the compactification scale is shown in each plot as the black line. To the left, we show the charged scalar mass in red and the vector boson mass in blue. Their different scaling in L is evident. In the centre, the moduli masses are shown as blue lines, the axion masses as red lines. On the right, the modulini masses are shown as blue line, whereas the gravitino mass is the red line.*

3.6. Chapter summary

We construct a six-dimensional supergravity with a background flux. Its consistent reduction to four dimensions is non-trivial. Special attention had to be paid to the dimensionful parameters of the theory, as well as to finding the proper coordinate frame for the four-dimensional theory.

We then studied wave functions for charged fields in the bulk. Some material, including the outline of our computation for the torus wave functions can be found in the literature. The projection of the wave functions to the orbifold, along with understanding their multiplicities is somewhat novel; the authors of [32] attempted a similar approach for general $T^2/\mathbb{Z}_n$ orbifolds. Yet, we found their notation confusing and they did not properly account for the altered flux quantisation conditions on the various orbifolds.

To further make our model consistent, we include a Green-Schwarz term cancelling the gauge anomaly. It represents part of the one-loop corrections to the theory and modifies some key aspects of the effective action after compactification. The altered gauge-kinetic function has a zero for finite values of the moduli, which helps in moduli stabilisation. Moreover, the GS term induces a second field-dependent FI term. We trust that there is a field content cancelling also the gravitational and mixed gauge/gravity anomalies, like in the flux-less case, cf. Section 2.2.

To stabilise the moduli, we add a non-perturbative superpotential. de Sitter vacua are possible without further model building ingredients. The one-loop corrected gauge-kinetic function plays a major role in their emergence. Parallel to the analytic investigation, MATHEMATICA code was written to numerically evaluate the expressions and find interesting examples. In the end, a KKLT-type superpotential can cooperate with the D-term potential from the compactification to stabilise all moduli in the model.

Finally, the possibility to evaluate the entire action allows us to find the mass spectrum of the low-energy effective action. We find all boson and fermion masses, as well as their behaviour under a controlled rescaling of the size of the extra dimensions.

There are several things that could be done to give an even more complete picture of 6d flux compactification. First, we did not include the localised FI terms that charged bulk fields induce and that were described by Nilles et. al. in [73]. If they are cancelled with localised flux, they will modify the wave function in the internal space, possibly leading to a different localisation behaviour. Second, the separation of the charged sector into KK levels is questionable. A resummation of the effects from the charged bulk fields gives a clearer picture. It is a work in progress and will not be part of this thesis.

Split Symmetries: An $SO(10)$ model 4

In this chapter we describe the flux compactification of an $SO(10) \times U(1)_A$ GUT model. The flux provides the exciting option of having the Standard Model matter emerge from a charged bulk field. In previous orbifold GUT constructions this was not possible because bulk fields are affected by the Wilson lines that break the gauge symmetry. As a consequence, some of the bulk field components obtain masses of the order of the compactification scale, while others remain light. This is a useful mechanism to solve the Doublet-Triplet Splitting problem of GUT constructions. We studied charged fields in the flux background in Section 3.2, finding that a charged field in the flux background can be subject to Wilson lines without obtaining a mass. The flux will also create a multiplicity of zero modes, which can be used to obtain three generations of Standard Model fermions from a single bulk field. Furthermore, the flux background breaks supersymmetry at a large scale in the charged sector, while keeping it intact in the uncharged sectors.

The compactification in a flux background explains why the fermions in nature come in multiple generations that each fill an entire **16**-plet of SO(10), whereas the bosons are in single "split" multiplets of the GUT symmetry. Also, spectra with large masses for the superpartners of the Standard Model fermions but with light gauginos were dubbed "split" supersymmetry. Hence, for each bulk field either the GUT gauge symmetry or supersymmetry is "split" by the gauge field background.

We begin this chapter by motivating our choice for the bulk field content of the model from the 6d SO(10) anomaly in Section 4.1. There, we also study the effects of the gauge symmetry breaking Wilson lines on the bulk fields. In Section 4.2, we give the Green-Schwarz term that cancels all bulk and fixed point anomalies in absence of gauge symmetry breaking and derive an effective 4d action without the matter fields. Section 4.3 is devoted to the phenomenology of the matter sector. It is there that we construct the zero mode spectrum of the matter fields, which includes vector-like exotic states besides the three generations of Standard Model matter fermions promised above. We also study the mixing between the generations, arising from fixed-point superpotentials. The section is closed by a brief comment on supersymmetry breaking in our model. Finally, Section 4.4 provides a summary of this chapter and gives some ideas on how to refine the model we present.

4.1. Field content and gauge symmetry breaking

In this section, we present the setup of an SO(10) orbifold GUT, compactified on $T^2/\mathbb{Z}_2$. As a bulk gauge symmetry, we choose the group to be simply SO(10), which we expand in the next section. We let considerations in the context of the pure SO(10) anomaly guide our choice of bulk field content. Besides the vector multiplet in the adjoint of SO(10), we allow for bulk fields in the representations **16**, $\mathbf{16}^*$ and **10**.

Owing to the absence of a third order Casimir invariant and the tracelessness of its generators, SO(10) is anomaly free in four dimensions, where the anomaly polynomial is a six-form. This is not the case in six dimensions. Here, the anomaly polynomial is an eight-form and a fourth order Casimir operator does exist in SO(10). The pure SO(10) anomaly polynomial is [78, 79]

$$\begin{aligned} I_8 &= \frac{\beta}{24}\Big(\operatorname{Tr}\tilde{F}^4 - \sum_i s_i \operatorname{tr}_i \tilde{F}^4\Big) \\ &= \frac{\beta}{24}\Big((2 - s_{\mathbf{10}} + s_{\mathbf{16}} + s_{\mathbf{16}^*})\operatorname{tr}\tilde{F}^4 + \frac{3}{16}(6 - s_{\mathbf{10}})\big(\operatorname{tr}\tilde{F}^2\big)^2\Big), \end{aligned} \tag{4.1}$$

where $\beta = -1/(2\pi)^3$, Tr is the trace in the adjoint, i enumerates the other representations, for which s_i is the multiplicity and tr_i is the respective trace. Going to the second line, we expressed all traces in the **16** through group-theoretical relations, i.e., $\operatorname{tr} \equiv \operatorname{tr}_{\mathbf{16}}$ [79]. While the first factor is irreducible and therefore has to vanish in a consistent theory, the second term could in principle be cancelled through a Green-Schwarz mechanism. Still, it is convenient to choose a bulk field content that makes the entire expression (4.1) vanish. This fixes the number of **10**-plets to be $s_{\mathbf{10}} = 6$, we denote them by $H_1, \ldots, H_6$. In order to keep the remnants at the fixed points vector-like after compactification, we choose equal numbers of **16**-plets and $\mathbf{16}^*$-plets, giving $s_{\mathbf{16}} = s_{\mathbf{16}^*} = 2$. We denote the **16**-plets by ψ, Ψ and the $\mathbf{16}^*$-plets by ψ^c, Ψ^c. Such a bulk spectrum has previously been used in an orbifold GUT construction in [63].

In orbifold compactifications it is possible to break the bulk gauge symmetry by embedding non-trivial twists, related to the torus lattice translations, into the gauge group. It has been shown that these symmetry breaking boundary conditions can be described equivalently by Wilson lines [50]. This mechanism will always break to a maximally symmetric subgroup, as long as the Wilson lines in the independent directions on the orbifold commute [86]. In order to obtain an extended Standard Model gauge group $\mathcal{G}'_{\mathrm{SM}} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_X$, we can break the SO(10) to the Pati-Salam group $\mathcal{G}_{\mathrm{PS}} = \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ [58] in one direction and to the Georgi-Glashow $\mathcal{G}_{\mathrm{GG}} = \mathrm{SU}(5) \times \mathrm{U}(1)_x$ [44] in the other direction. This is effected by twists, given in the vector representation [56]

$$T_1 = P_{\mathrm{PS}} = \operatorname{diag}(-\sigma_0, -\sigma_0, -\sigma_0, \sigma_0, \sigma_0)\,, \quad T_2 = P_{\mathrm{GG}} = \operatorname{diag}(\sigma_2, \sigma_2, \sigma_2, \sigma_2, \sigma_2)\,, \tag{4.2}$$

where σ_0 is the 2×2 unit matrix. These twists leave the entire SO(10) intact at the fixed point at the origin, $\zeta_1 \equiv \zeta_I$, but reduce the gauge symmetry to $\mathcal{G}_{\rm PS}$ at $\zeta_2 \equiv \zeta_{\rm PS}$ and to $\mathcal{G}_{\rm GG}$ at $\zeta_3 \equiv \zeta_{\rm GG}$. Consequently, both twists are applied at the fourth fixed point, $\zeta_4 \equiv \zeta_{\rm fl}$, leading to a "flipped SU(5)" gauge group $\mathcal{G}_{\rm fl} = {\rm SU}(5) \times {\rm U}(1)_{X'}$ with a twist

$$T_1 T_2 = T_2 T_1 = P_{\rm fl} = {\rm diag}\left(-\sigma_2, -\sigma_2, -\sigma_2, \sigma_2, \sigma_2\right). \tag{4.3}$$

In order to break the extended supersymmetry down to $\mathcal{N}=1$ in 4d, we have to assign opposite parities to different parts of the 6d supermultiplets. They can conveniently be grouped in pairs of 4d $\mathcal{N}=1$ superfields [75], where a 6d vector multiplet gives a 4d vector and a 4d chiral supermultiplet and a 6d hypermultiplet gives two 4d chiral multiplets (cf. Section 2.1). By applying the parities at the 4d superfield level, we keep the $\mathcal{N}=1$ supersymmetry manifest. If we use P_i to denote the projection operators at the four fixed points, with P_I the 10×10 identity matrix and $P_{\rm PS,GG,fl}$ corresponding to the gauge group at the respective fixed points, we can ensure to project all coset vectors and scalars out of the 6d vector multiplet by setting

$$P_i A(x,\zeta_i - y) P_i^{-1} = A(x,\zeta_i + y), \quad P_i \Sigma(x,\zeta_i - y) P_i^{-1} = -\Sigma(x,\zeta_i + y). \tag{4.4}$$

Here, $A = (A_\mu, \lambda_R)$ is the 4d vector multiplet and $\Sigma = ((A_6 + iA_5)/\sqrt{2}, \lambda_L)$ the 4d chiral multiplet (we only give the on-shell degrees of freedom here). Regarding the matter fields, we can choose the signs in the boundary conditions freely and thus write, with $\eta_i^\alpha = \pm 1$,

$$P_i \Phi_\alpha(x,\zeta_i - y) = \eta_i^\alpha \Phi_\alpha(x,\zeta_i + y), \quad P_i \Phi_\alpha^c(x,\zeta_i - y) = -\eta_i^\alpha \Phi_\alpha^c(x,\zeta_i + y). \tag{4.5}$$

No sum is understood on the r.h.s. of Eq. (4.5) and the Φ_α enumerate the bulk spectrum of 6 **10**-plets, 2 **16**-plets and 2 **16***-plets.

The action of the boundary conditions is related to the picture of orbifold Wilson lines, where some fields in a **10** or **16** "feel" the Wilson line and others don't. To construct the effective Wilson line background for each component, we first have to study the decomposition of the SO(10) representations under the broken gauge groups, which are readily available in the literature [87]. For the Pati-Salam group we have

$$\begin{aligned}
\mathbf{45} &\rightarrow (\mathbf{15},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3}) \oplus (\mathbf{6},\mathbf{2},\mathbf{2})\\
\mathbf{10} &\rightarrow (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\\
\mathbf{16} &\rightarrow (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\mathbf{4}^*,\mathbf{1},\mathbf{2})
\end{aligned} \tag{4.6}$$

and for the SU(5) $\times$ U(1) groups the decomposition is

$$\begin{aligned}
\mathbf{45} &\rightarrow \mathbf{24}_0 \oplus \mathbf{10}_4 \oplus \mathbf{10}^*_{-4} \oplus \mathbf{1}_0\\
\mathbf{10} &\rightarrow \mathbf{5}_2 \oplus \mathbf{5}^*_{-2}\\
\mathbf{16} &\rightarrow \mathbf{5}^*_3 \oplus \mathbf{10}_{-1} \oplus \mathbf{1}_{-5}
\end{aligned} \tag{4.7}$$

We can deduce the action of the SO(10) Wilson lines on the matter representations easily by considering the kinetic terms, which have symmetry structures $\mathbf{16}^\dagger\,\mathbf{45}\,\mathbf{16}$ and $\mathbf{10}^\dagger\,\mathbf{45}\,\mathbf{10}$. It is known how the Wilson lines act on the $\mathbf{45}$: We constructed them in such a way that the representations containing the SM gauge bosons have even parities, while those in the cosets have odd parities. However, the entire kinetic term has to have an even parity, allowing us to use the decomposition of the kinetic terms to learn about the parities of the matter representations without much ado.

For the Pati-Salam group we look at the couplings involving the odd $(\mathbf{6},\mathbf{2},\mathbf{2})$, and for the SU(5) cases we study the terms with the odd $\mathbf{10}_4$. Decomposing the kinetic terms according to Eqs. (4.6), we find the relevant contributions for the Pati-Salam case

$$\mathbf{16}^\dagger\,\mathbf{45}\,\mathbf{16} \supset (\mathbf{4}^*,\mathbf{2},\mathbf{1})^\dagger\,(\mathbf{6},\mathbf{2},\mathbf{2})\,(\mathbf{4},\mathbf{1},\mathbf{2}), \tag{4.8a}$$

$$\mathbf{10}^\dagger\,\mathbf{45}\,\mathbf{10} \supset (\mathbf{1},\mathbf{2},\mathbf{2})^\dagger\;\;(\mathbf{6},\mathbf{2},\mathbf{2})\,(\mathbf{6},\mathbf{1},\mathbf{1}), \tag{4.8b}$$

and using Eqs. (4.7) the relevant coset couplings for the SU(5) cases are

$$\mathbf{16}^\dagger\,\mathbf{45}\,\mathbf{16} \supset \left(\mathbf{5}^*_3\right)^\dagger\,\mathbf{10}_4\,\mathbf{10}_{-1} + \left(\mathbf{10}_{-1}\right)^\dagger\mathbf{10}_4\mathbf{1}_{-5}, \tag{4.9a}$$

$$\mathbf{10}^\dagger\,\mathbf{45}\,\mathbf{10} \supset \left(\mathbf{5}_2\right)^\dagger\,\mathbf{10}_4\,\mathbf{5}^*_{-2}, \tag{4.9b}$$

where we displayed only as many terms as are required to make our point.

From Eq. (4.8a), we see that the Wilson line has to act in opposite ways on $(\mathbf{4},\mathbf{2},\mathbf{1})$ and $(\mathbf{4}^*,\mathbf{1},\mathbf{2})$, because $(\mathbf{6},\mathbf{2},\mathbf{2})$ has odd parity at ζ_{PS} while the entire term must be even. With the same reasoning, Eq. (4.8b) implies reversed Wilson line parities for the $(\mathbf{6},\mathbf{1},\mathbf{1})$ and $(\mathbf{1},\mathbf{2},\mathbf{2})$ within the $\mathbf{10}$ of SO(10). In the SU(5) case, we see from Eq. (4.9a) that once we define the parity of the $\mathbf{10}_{-1}$ within the $\mathbf{16}$, the parities of the $\mathbf{5}^*_3$ and $\mathbf{1}_{-5}$ are fixed to be opposite. Along the same lines, Eq. (4.9b) implies contrary parities for the $\mathbf{5}_2$ and $\mathbf{5}^*_{-2}$ contained in the $\mathbf{10}$ of SO(10). We are still free to pick the phases η_i^α and thereby choose which representations are not projected out, but it is not possible to retain, for example, an entire $\mathbf{16}$. This statement has a caveat that is central to our model: A background flux can protect fields from the Wilson lines. We will explore the implications of this fact in Section 4.3

For each bulk field, we now have two signs to choose freely. Once we decide how the Wilson lines act on the $(\mathbf{4},\mathbf{1},\mathbf{2})$ and $\mathbf{10}_{-1}$, the parities for the remaining fields inside the $\mathbf{16}$ are all fixed. The same is true for the $\mathbf{10}$, where we can choose the parities of the $(\mathbf{1},\mathbf{2},\mathbf{2})$ and $\mathbf{5}_2$ to determine the behaviour of all other components at the fixed points. Table 4.1 shows which effective Wilson line configuration each component of the $\mathbf{16}$ is subjected to for our four possible choices. Naturally, we have the same freedom to choose the parity for the fields contained in the $\mathbf{16}^*$, whose zero modes will be the conjugate representations of those in the last row of Table 4.1.

We can arrange for a vector-like spectrum remaining after compactification, e.g., by assigning parities such that we retain fields with the quantum numbers of a lepton doublet

Table 4.1.: *The effective Wilson line background* (k_1, k_2) *for the Standard Model representations contained in the* $\mathbf{16}$ *of* $\mathrm{SO}(10)$*, as determined by the choice of parity* p *for the* $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ *and* $\mathbf{10}_{-1}$*. We refer to representations of the Pati-Salam group by their* $\mathrm{SU}(4)$ *representation and suppress the* $\mathrm{U}(1)$ *charge of the* $\mathrm{SU}(5)$ *representations. The (torus) Wilson lines relate to the* k_i *by* $W_i = \exp(iq\pi k_i)$.

Choice of parity		$\mathbf{4}^-, \mathbf{10}^-$	$\mathbf{4}^+, \mathbf{10}^-$	$\mathbf{4}^-, \mathbf{10}^+$	$\mathbf{4}^+, \mathbf{10}^+$
$Q_L \in$	$\mathbf{4},\ \mathbf{10}$	$(1,1)$	$(0,1)$	$(1,0)$	$(0,0)$
$U_R \in$	$\mathbf{4}^*, \mathbf{10}$	$(0,1)$	$(1,1)$	$(0,0)$	$(1,0)$
$D_R \in$	$\mathbf{4}^*, \mathbf{5}^*$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$L_L \in$	$\mathbf{4},\ \mathbf{5}^*$	$(1,0)$	$(0,0)$	$(1,1)$	$(0,1)$
$E_R \in$	$\mathbf{4}^*, \mathbf{10}$	$(0,1)$	$(1,1)$	$(0,0)$	$(1,0)$
$\nu_R \in$	$\mathbf{4}^*, \mathbf{1}$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
zero mode for		D_R, ν_R	L_L	U_R, E_R	Q_L

L_L from ψ and its conjugate L_L^c from ψ^c, as well as colour triplets and SM singlets D_R, N_R from Ψ and their conjugates D_R^c, N_R^c from Ψ^c. This has the advantage that no fixed point anomalies are induced by the zero mode spectrum. Moreover, the right-handed scalars $N_R \in \Psi, N_R^c \in \Psi^c$ can break the $B-L$ symmetry at a high or intermediate scale. For the $\mathbf{10}$-plets, we can choose the parities such that we retain two $\mathrm{SU}(2)_L$ doublets from H_1 and H_2 which we identify with the MSSM Higgs doublets, $H_u \in H_1$, $H_d \in H_2$. A useful choice for the remaining $\mathbf{10}$-plets is to leave two colour triplets $D_{1,2} \in H_{3,4}$ and two colour anti-triplets $D_{1,2}^c \in H_{5,6}$, again forming vector-like states that do not contribute to the 4d anomaly and that can be given a large mass. In this way, the Doublet-Triplet-Splitting problem is avoided and our bulk matter does not induce a fixed point anomaly.

4.2. Anomaly cancellation

In order to use the advantages of a flux compactification we have to expand our bulk gauge group to $\mathrm{SO}(10) \times \mathrm{U}(1)_A$, because a flux embedded in a non-Abelian gauge group leads to tachyonic states [30]. To be definitive in our notation, we denote the $\mathrm{SO}(10)$ gauge fields and field strength with a tilde, $\tilde{F} = \mathrm{d}\tilde{A} + i\tilde{A} \wedge \tilde{A}$, and the $\mathrm{U}(1)_A$ gauge fields with a prime, $F' = \mathrm{d}A'$. For the $\mathrm{U}(1)_A$ vector multiplet we apply the same boundary conditions as for the $\mathrm{SO}(10)$ vector multiplet, see Eq. (4.4), while the projection operators are trivial in that case, $P_i = 1$ within the $\mathrm{U}(1)_A$. We assign charge q to the $\mathbf{16}$-plet ψ and no charge to all other bulk matter fields. This will induce a number of SM fermion generations upon compactification in the flux background. It also leads to an anomaly. The corresponding bulk and fixed point anomaly polynomials are given by [78, 79]

$$I_8^b = \frac{\beta}{24} \operatorname{tr} \left(6\tilde{F}^2 + q^2 F'^2\right) q^2 F'^2, \quad I_8^f = \frac{\alpha}{24} \operatorname{tr} \left(3\tilde{F}^2 + q^2 F'^2\right) q F' \, \delta_0 v_2, \tag{4.10}$$

where $\beta = -1/(2\pi)^3$, $\alpha = 1/(2\pi)^2$, v_2 is the volume form on the internal space and $\delta_0 = 1/4 \sum \delta(y - \zeta_i)$ is a sum of δ functions distributed evenly among the fixed points ζ_i. The bulk part of the anomaly polynomial is already neatly factorised, showing that the corresponding anomaly can be cancelled by a standard Green-Schwarz mechanism [33]. The brane part can be cancelled by allowing localised contributions to the two-form's shift, as discussed in [88]. We have not taken into account the gauge symmetry breaking at the fixed points when we wrote the fixed point anomaly in Eq. (4.10). In a work in preparation we show that the fixed point anomalies as well as mixed gauge-gravity anomalies, which are not included in our discussion, can be cancelled even when the symmetry breaking is considered [89]. Some fixed point fields in non-SO(10) representations are necessary to do so. This is to be expected from a comparison with anisotropic string compactifications, like the one in [26], where an intermediate 6d SU(6) GUT was obtained from the heterotic string. There, the cancellation of the fixed point anomalies also involved localised fields.

Disregarding the normalisation, the anomaly derived from the anomaly polynomials (4.10) amounts to

$$\mathcal{A}_6 = -\delta\Gamma_6^f = \bar{\beta}\left(\tilde{\omega}_3 + \gamma A'F'\right) F'\,\mathrm{d}\Lambda' + \bar{\alpha}\left(\tilde{\omega}_3 + 2\gamma A'F'\right)\delta_0\, v_2 \mathrm{d}\Lambda', \tag{4.11}$$

where we introduced $\bar{\beta} = 6q^2\beta$, $\bar{\alpha} = 3q\alpha$ and $\gamma = q^2 d_r/6$, with d_r the dimension of the charged bulk representation, i.e., $d_r = 16$ for our construction. We also introduced the Chern-Simons 3-form for the SO(10), $\tilde{\omega}_3 = \mathrm{tr}\left(\tilde{A}\mathrm{d}\tilde{A} + 2i/3\,\tilde{A}^3\right)$. By virtue of the freedom to add local counter terms to the anomaly, we achieve that $\mathcal{A}_6$ is independent of the SO(10) gauge parameter $\tilde{\Lambda}$. The anomaly can be cancelled by a Green-Schwarz term

$$S_{\mathrm{GS}} = -\int \bar{\beta}\left(\tilde{\omega}_3 + \gamma A'F' + \rho\gamma A'\delta_0\, v_2\right)\mathrm{d}B, \tag{4.12}$$

where $\rho = \bar{\alpha}/\bar{\beta}$. The shift of the two-form field B is given by

$$\delta\mathrm{d}B = -\delta X_3 = -\mathrm{d}\left(\Lambda'F' + \rho\Lambda'\delta_0\, v_2\right), \tag{4.13}$$

and its field strength by $H = \mathrm{d}B + X_3$, with $X_3 = A'F' + \rho A'\delta_0\, v_2$.

The 4d anomaly can be obtained by compactifying the 6d anomaly (4.11) on $T^2/\mathbb{Z}_2$, which gives

$$\mathcal{A}_4 = -\delta\Gamma_4^f = \frac{\bar{\beta}}{2}\,(f + 2\rho)\left(\tilde{\omega}_3 + 2\gamma\hat{A}\hat{F}\right)\mathrm{d}\hat{\Lambda}, \tag{4.14}$$

where $\hat{A}$ is the 4d part of A' etc. The process to obtain the 4d effective action from the 6d supergravity action

$$S = \int\left[\frac{M_6^4}{2}(R - \mathrm{d}\phi\wedge *\mathrm{d}\phi) - \frac{e^{2\phi}}{4M_6^4 g_6'^4}H\wedge *H - \frac{e^{\phi}}{2g_6'^2}F'\wedge *F' - \frac{e^{\phi}}{2\tilde{g}_6}\,\mathrm{tr}\left(\tilde{F}\wedge *\tilde{F}\right)\right] \tag{4.15}$$

is similar to the case with only a U(1) gauge group, which was described in detail in Chapter 3 and Appendix B. We only give the bosonic terms as the fermionic part can be reconstructed by supersymmetry. Moreover, we do not explicitly include the matter sector, because it would involve a tremendously complicated quaternionic Kähler manifold. By the arguments given in Appendix C, we assume the corrections a detailed treatment might reveal will not impact any of our conclusions. The spectra for the matter sectors, both charged and uncharged under $\mathrm{U}(1)_A$, were constructed in Section 3.2. Since the two-form B only shifts under the $\mathrm{U}(1)_A$, we include the $\mathrm{U}(1)_A$ gauge coupling g_6' in its kinetic term.

Two subtleties in the compactification concern the two-form $\mathrm{d}B = \mathrm{d}b\, v_2 + \mathrm{d}\hat{B}$. In the purely U(1) case, we included internal derivatives of off-diagonal components in the field strength and recognised their impact on the transformation behaviour of the pseudoscalar b, which was first discussed in [17]. With the larger gauge group $\mathrm{SO}(10)\times\mathrm{U}(1)_A$, the inclusion of these components would also require an understanding of mixed index components in the SO(10), which could contribute to the Chern-Simons term. As such an understanding eludes us so far, we neglected these mixed index contributions in compactifying the anomaly to obtain Eq. (4.14). To remain consistent and still cancel the 4d anomaly with the compactified Green-Schwarz term, we have to truncate the mixed index contributions also for the $U(1)_A$ contributions and for the two-form. As a second issue, the internal components of the field strength

$$H = \mathrm{d}B + X_3 \supset (\mathrm{d}b + f\hat{A} + \rho\delta_0\hat{A})\, v_2 \tag{4.16}$$

include localised terms and therefore the corresponding kinetic term in the action (4.15) contains terms proportional to delta functions squared. On the level of the effective action, these contributions to the action have to be regularised and we assume a regularisation to be possible without spoiling anomaly cancellation. In a full UV complete theory, this issue has to be treated by resolving the orbifold singularities.

After dualising the 4d components of the three-form $\hat{H}$ to a 4d pseudoscalar with a Lagrange multiplier term $S_c = (2g_4')^{-1}\int c\,\mathrm{d}\big(\hat{H} - X_3\big)$, the effective action is given by

$$\begin{aligned} S_4 = \int \Bigg[&\frac{M_P^2}{2}R_4 - \frac{M_P^2}{4s^2}\mathrm{d}s\wedge *\mathrm{d}s - \frac{M_P^2}{4t^2}\mathrm{d}t\wedge *\mathrm{d}t - \frac{M_P^2}{4\tau_2}\mathrm{d}\tau\wedge *\mathrm{d}\bar{\tau} \\ &- \frac{s}{2\tilde{g}_4^2}\,\mathrm{tr}\left(\tilde{F}\wedge *\tilde{F}\right) - \frac{s}{2g_4'^2}\hat{F}'\wedge *\hat{F}' - \frac{M_P^2}{2}\frac{g_4'^2}{s}\frac{f^2}{t^2\ell^4} \\ &- \frac{M_P^2}{4t^2}\left(\mathrm{d}b + \frac{f+2\rho}{\ell^2}\hat{A}'\right)\wedge *\left(\mathrm{d}b + \frac{f+2\rho}{\ell^2}\hat{A}'\right) \\ &- \frac{M_P^2}{4s^2}\left(\mathrm{d}c + 2g_4'^2\frac{\bar{\beta}}{2}(f+2\rho)\gamma\hat{A}'\right)\wedge *\left(\mathrm{d}c + 2g_4'^2\frac{\bar{\beta}}{2}(f+2\rho)\gamma\hat{A}'\right) \\ &- \frac{\bar{\beta}}{2}\ell^2\, b\left(\tilde{F}^2 + \gamma\hat{F}'^2\right) - \frac{1}{2g_4'^2}\,c\,\hat{F}'^2. \end{aligned} \tag{4.17}$$

The appearance of $\ell = g_4' M_P L$ is due to the introduction of the proper scaling for b, see Eq. (B.13) and the discussion leading to it. The axionic sector, making up the last three lines in the action (4.17), is quite similar to the purely U(1) case: A linear combination of the axions b and c is absorbed by the $\mathrm{U}(1)_A$ gauge bosons in a Stückelberg mechanism. The orthogonal combination will then provide a coupling between the massive $\mathrm{U}(1)_A$ and the Standard Model, whose field strength is what remains of the SO(10) field strength $\tilde{F}^2$ after considering symmetry breaking. Again, the gauge-kinetic function of the U(1) receives contributions from both axions, as witnessed by both axions coupling to $\hat{F}'^2$, which is why we assume moduli stabilisation as described in (3.4) to be possible without affecting the SO(10) sector of the theory.

The precise supergravity embedding of the SO(10) is unclear at this point, because the SO(10) kinetic term, involving the real part of the corresponding gauge-kinetic function, seems to be at odds with the $\tilde{F}^2$ term, from which we would like to read off the gauge-kinetic function's imaginary part. The prior is proportional to s, which gave the real part of the complex scalar S in Chapter 3, while the latter is linear in b, which is associated to the complex scalar T. For the U(1) gauge interaction, we find that the gauge-kinetic function is a linear combination of these two fields. It is therefore to be expected that also the gauge-kinetic function for the SO(10) depends on both. This issue can possibly be resolved by taking into account the mixed-index contributions to the Chern-Simons forms that we truncated in our derivation of Eq. (4.17).

Reading the axionic shifts off their kinetic terms in Eq. (4.17), we find

$$\delta b = -\frac{f+2\rho}{\ell^2}\hat{\Lambda}, \quad \delta c = -2g_4'^2\frac{\bar{\beta}}{2}(f+2\rho)\gamma\hat{\Lambda}. \tag{4.18}$$

Using these expressions it is easy to show that the anomaly (4.14) is cancelled by the gauge transformation of the bosonic action (4.17).

4.3. Matter from a charged bulk field

In Section 4.1 we constructed an SO(10) anomaly-free bulk field content and studied the effects of symmetry-breaking Wilson lines on the various representations that are present in the bulk. Of special interest in this context are the **16**-plets of SO(10), since they contain fields with the quantum numbers of Standard Model matter. Most degrees of freedom are projected out of the low-energy spectrum by the Wilson lines as shown in Table 4.1, where we construct the effective Wilson line backgrounds for the individual SM fields, which depend on our choice of two phases.

"Projecting out" implies that a state with an odd parity at some of the fixed points no longer corresponds to a massless field in 4d. As we have shown in Section 3.2, this is not true for charged fields in presence of a background flux. In that case, the Wilson line configuration determines the localisation of the field in the internal space, but its mass is not affected. This means that in a fluxed background, a charged hypermultiplet, transforming as a **16**-plet under SO(10), will spawn several chiral fermionic fields in 4d, each of which transforms with the quantum numbers of a full SM generation. The number of flux quanta then determines the number of SM generations.

Table 4.2.: *Projections at the Pati-Salam and Georgi-Glashow fixed points $\zeta_{\mathrm{PS}}, \zeta_{\mathrm{GG}}$ and the resulting 4d zero modes. We display the representations with even parity. The subscript in the first column is the $\mathrm{U}(1)_A$ charge. All fields are even at the $\mathrm{SO}(10)$ fixed point $\zeta_{\mathrm{SO}(10)}$. Lower case letters represent SM fields and upper case letters are exotics.*

$\mathrm{SO}(10)_{q_A}$ field	even at ζ_{PS}	even at ζ_{GG}	4d zero modes
$\psi \;= \mathbf{16}_1$	$(\mathbf{4},\; \mathbf{2}, \mathbf{1})$	$\mathbf{5}^*_3,\;\; \mathbf{1}_{-5}$	$3 \times (q_l, u_r, d_r, l_l, e_r, \nu_r) + L_L$
$\psi^c = \mathbf{16}^*_0$	$(\mathbf{4}^*, \mathbf{2}, \mathbf{1})$	$\mathbf{5}_{-3}, \mathbf{1}_5$	L^c_L
$\Psi \;= \mathbf{16}_0$	$(\mathbf{4},\; \mathbf{1}, \mathbf{2})$	$\mathbf{5}^*_3,\;\; \mathbf{1}_{-5}$	D_R, N_R
$\Psi^c = \mathbf{16}^*_0$	$(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$	$\mathbf{5}_{-3},\; \mathbf{1}_5$	D^c_R, N^c_R
$H_1 = \mathbf{10}_0$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$\mathbf{5}_2$	H_u
$H_2 = \mathbf{10}_0$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$\mathbf{5}^*_{-2}$	H_d

We want to use this effect to study a novel embedding of matter for orbifold GUT constructions. If we charge one single $\mathbf{16}$, say ψ, under the anomalous $\mathrm{U}(1)_A$, its wave function in the internal space will be given by Eq. (3.48). The multiplicity of SM generations can then originate from having $N = 3$ flux quanta present in the internal space. While this does not explain why there are three generations per se, it gives a reasoning why there is a finite number of generations. Because different SM fields within the $\mathbf{16}$ will feel different Wilson line backgrounds, their wave functions will differ as well.

To be concrete, let us choose the orbifold parities such that without flux, a lepton doublet would remain, i.e., the $\mathbf{4}^+, \mathbf{10}^-$ column of table 4.1. We can then infer the exact localisation for each SM field in the internal space by cross-referencing its effective Wilson line background with Fig. 3.1. For the lepton doublet, four instead of three zero modes arise. This is useful because the fourth zero mode can form a vector-like representation with the zero mode from the $\mathbf{16}^*$-plet ψ^c, for which we assign the parities such that a conjugate lepton doublet remains. Concerning the other pair of bulk $\mathbf{16}$ and $\mathbf{16}^*$, Ψ and Ψ^c, we choose their parities such that overall, a vector-like colour triplet and two SM singlets remain. The colour triplet will be useful in obtaining realistic flavour mixing, while the SM singlet can be used to break $B - L$ symmetry and initiate a see-saw mechanism.

The six $\mathbf{10}$ plets are all uncharged under the $\mathrm{U}(1)_A$, which is why most of their components are projected out. Two of them, H_1 and H_2, will provide the two Higgs doublets of the MSSM. To achieve this, we project both fields to their $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ components at the Pati-Salam fixed point, and then project H_1 to its $\mathbf{5}_2$ but H_2 to its $\mathbf{5}^*_{-2}$ at the Georgi-Glashow fixed point. The intersection of these representations is exactly two $\mathrm{SU}(2)_L$ doublets of opposite hypercharge, as required for the MSSM. It is not important to which representations the remaining $\mathbf{10}$-plets $H_3, \ldots, H_6$ are projected, as long as we keep their 4d massless degrees of freedom vector-like, so that they can get a large mass. Table 4.2 summarises all parity assignments and the zero modes resulting from our choice. The key feature are the three generations of SM fermions from ψ thanks to the flux in conjunction with vector-like exotics that will not induce a 4d anomaly.

An interesting feature in connection with the SM fermion generations is flavour mixing or, equivalently, the structure of the Yukawa coupling matrices. While the extended supersymmetry in 6d forbids a bulk superpotential, the effective 4d action is not restricted in such a way. Each orbifold fixed point can therefore be endowed with an independent localised superpotential. Their sum then forms the 4d superpotential. There are two contributions to the Yukawa couplings that are relevant in our case: On the one hand, we can directly write down Yukawa couplings at each fixed point individually. On the other hand, the SM fields can also mix with vector-like exotic states in appropriate representations through further terms in the localised superpotentials.

The superpotentials at the four fixed points can be general couplings, invariant under the gauge group that remains locally unbroken. Each independent combination of fields carries a coupling strength as a free parameter. The SO(10) invariant couplings $\psi\,\psi\,H_{1,2}$ and $\psi\,\psi\,\Psi^c\Psi^c$ generate all terms, which involve only the SM matter[1] fields along with fields that obtain a vacuum expectation value. The general expression for the fixed point superpotential is, with $\delta_i = \delta^2(y-\zeta_i)$,

$$\begin{aligned} W_{\text{fp}} = \;& \delta_I\big(h_u^I\,\psi\psi\,H_1 + h_d^I\,\psi\psi\,H_2 + h_n^I\,\psi\psi\Psi^c\Psi^c\big) \\ &+\delta_{\text{PS}}\big(h_u^{\text{PS}}\,\mathbf{4}\,\mathbf{4}^*\,\Delta_1 + h_d^{\text{PS}}\mathbf{4}\,\mathbf{4}^*\Delta_2 + h_n^{\text{PS}}\mathbf{4}^*\mathbf{4}^*FF\big) \\ &+\delta_{\text{GG}}\big(H_u^{\text{GG}}\,\mathbf{10}\,\mathbf{10}\,H_{\mathbf{5}} + h_d^{\text{GG}}\,\mathbf{5}^*\mathbf{10}\,H_{\mathbf{5}^*} + h_\nu^{\text{GG}}\mathbf{5}^*\nu_r\,H_{\mathbf{5}} + h_n^{\text{GG}}n^c n^c\,NN\big) \\ &+\delta_{\text{fl}}\big(h_u^{\text{fl}}\,\tilde{\mathbf{5}}^*\tilde{\mathbf{10}}\,H_{\tilde{\mathbf{5}}^*} + h_d^{\text{fl}}\,\tilde{\mathbf{10}}\,\tilde{\mathbf{10}}\,H_{\tilde{\mathbf{5}}} + h_e^{\text{fl}}\tilde{\mathbf{5}}^*e_r\,H_{\tilde{\mathbf{5}}} + h_n^{\text{fl}}\,\tilde{\mathbf{10}}\,\tilde{\mathbf{10}}\,\tilde{T}^*\tilde{T}^*\big). \end{aligned} \tag{4.19}$$

Here, we denote the components of the matter field by their SU(4) representation at the Pati-Salam fixed point ζ_{PS} and by their SU(5) representations at the Georgi-Glashow fixed point ζ_{GG} (without a tilde) and at the flipped SU(5) fixed point ζ_{fl} (with a tilde). The SU(5) singlets in the Georgi-Glashow and flipped SU(5) embeddings are ν_r, which is a right-handed neutrino, and e_r, which is the right-handed electron. Furthermore, the relevant Higgs field representations at $(\zeta_{\text{PS}}, \zeta_{\text{GG}}, \zeta_{\text{fl}})$ are, in that order: $H_1 \supset (\Delta_1, H_5, H_{\tilde{5}^*})$, $H_2 \supset (\Delta_2, H_{5^*}, H_{\tilde{5}})$ and $\Psi^c \supset (F, N, \tilde{T}^*)$. $\Delta_{1,2}$ are bidoublets $(\mathbf{1},\mathbf{2},\mathbf{2})$, F is a four-plet $(\mathbf{4},\mathbf{2},\mathbf{1})$, N is a SU(5) singlet and T^* is a conjugate ten-plet $\mathbf{10}^*$.

If a field's parity is odd at a given fixed point, it can not appear in the superpotential couplings localised there. We can cross-reference[2] Table 4.1 with Table 3.2 to find the fixed point interactions that are possible. Most of the couplings given in Eq. (4.19) vanish by this consideration and we are left with a much reduced superpotential

$$\begin{aligned} W_{\text{fp}} = \;& \delta_I\big(h_u^I\,\psi\psi\,H_1 + h_d^I\,\psi\psi\,H_2 + h_n^I\,\psi\psi\Psi^c\Psi^c\big) \\ &+\delta_{\text{GG}}\big(h_\nu^{\text{GG}}\mathbf{5}^*n^c\,H_{\mathbf{5}} + h_n^{\text{GG}}n^c n^c\,NN\big) + \delta_{\text{fl}}\big(h_e^{\text{fl}}\tilde{\mathbf{5}}^*e^c H_{\tilde{\mathbf{5}}}\big). \end{aligned} \tag{4.20}$$

The fixed point Yukawa couplings depend only on the four parameters $h_u^I, h_d^I, h_\nu^{\text{GG}}$ and h_e^{fl}, while the right-handed neutrinos' Majorana masses depend on the two parameters h_n^I, h_N^{GG}. It is remarkable that the delocalisation effect of the Wilson lines reduces the possibilities for the flavour sector by such an amount. Note that the precise form of the

[1] We include three right-handed neutrinos in our definition of SM matter.

[2] Keep in mind that $k_i = 1$ means $W_i = -$.

superpotential depends on our choice for the matter field's parities η^i.

The coupling parameters h define the overall strength of the Yukawa couplings, but they are universal for all generations. The relative strength of the Yukawa couplings follows from the amplitudes of the different zero modes at the fixed points. Writing the effective 4d superpotential as

$$W_{4d} = H_u\, q_l^i Y_{ij}^u u_r^j + H_d\, q_l^i Y_{ij}^d d_r^j + H_d\, l_l^i Y_{ij}^e e_r^j + H_u\, l_l^i Y_{ij}^\nu \nu_r^j + \nu_r^i \nu_r^j M_{ij}^\nu \frac{\langle N\rangle^2}{M_c}, \tag{4.21}$$

where i, j are family indices, the Yukawa matrices Y^u, Y^d, Y^e Y^ν, as well as the right-handed Majorana mass matrix M^ν can be computed from wave function overlaps. As an example, the down-type quark Yukawa couplings, only arise from the SO(10) fixed point. This is because at each of the other fixed points either one or both of the wave functions for q_l ($\psi_+(y;0,1)$) or d_r ($\psi_+(y;1,0)$) are odd. The explicit 3×3 matrix of Yukawa couplings can be obtained from the expressions for the wave functions (3.48)

$$Y_{jk}^d = h_d^I\, \psi_+^{(j)}(\zeta_I;0,1)\psi_+^{(k)}(\zeta_I;1,0). \tag{4.22}$$

Computing all required overlaps in this way, the matrices in the superpotential (4.21) are given by

$$Y^u = \quad h_u^I \begin{pmatrix} 4.3 & 1.31+0.75i & 0.09+0.15i \\ 3.12+1.8i & 0.63+1.09i & 0.15i \\ 0.37+0.65i & 0.26i & -0.02+0.03i \end{pmatrix}, \tag{4.23a}$$

$$Y^d = \quad h_d^I \begin{pmatrix} 4.3 & 1.51 & 0.19 \\ 3.12+1.8i & 1.09+0.63i & 0.14+0.08i \\ 0.37+0.65i & 0.13+0.23i & 0.02+0.03i \end{pmatrix}, \tag{4.23b}$$

$$\begin{aligned} Y^e = \quad & h_d^I \begin{pmatrix} 4.3 & 1.31+0.75i & 0.09+0.15i \\ 3.6+0.i & 1.09+0.63i & 0.07+0.13i \\ 0.75+0.i & 0.23+0.13i & 0.02+0.03i \\ 0.08+0.i & 0.02+0.01i & 0 \end{pmatrix} \\ & + h_e^{\text{fl}} \begin{pmatrix} 0 & 0.01\ -0.02i & -0.07+0.04i \\ 0.\ -0.03i & -0.13+0.23i & 0.65\ -0.37i \\ 0.\ +0.15i & 0.63\ -1.09i & -3.12+1.8i \\ 0.\ -0.18i & -0.75+1.31i & 3.72\ -2.15i \end{pmatrix}, \end{aligned} \tag{4.23c}$$

$$Y^\nu = \quad h_u^I \begin{pmatrix} 4.3 & 1.51 & 0.19 \\ 3.6 & 1.26 & 0.16 \\ 0.75 & 0.26 & 0.03 \end{pmatrix} + h_\nu^{\text{GG}} \begin{pmatrix} 0. & 0.03 & 0.08 \\ 0.03 & 0.26 & 0.75 \\ 0.16 & 1.26 & 3.6 \end{pmatrix}, \tag{4.23d}$$

$$M^\nu = \quad h_n^I \begin{pmatrix} 5.33 & 1.87 & 0.24 \\ 1.87 & 0.66 & 0.08 \\ 0.24 & 0.08 & 0.01 \end{pmatrix} + h_n^{\text{GG}} \begin{pmatrix} 0.01 & 0.08 & 0.24 \\ 0.08 & 0.66 & 1.87 \\ 0.24 & 1.87 & 5.33 \end{pmatrix}. \tag{4.23e}$$

The lepton Yukawa matrix has a 4×3 structure, because our choice of parities leads to four zero modes with the quantum numbers of a lepton doublet in ψ. One linear combination of these doublets will pair up with the conjugate lepton doublet that is the zero mode of ψ^c and obtain a mass of the order of the compactification scale.

Even though the fixed point superpotential (4.20) is highly constrained, it has some remarkable features. The down-quark Yukawa couplings arise from a single interaction at the SO(10) fixed point. In contrast, the charged lepton Yukawa couplings have a second contribution from the coupling in the flipped SU(5). As a result, typical mass relations in GUTs, such as bottom-tau unification, do not apply to our construction.

The superpotential (4.21) also immediately suggests an implementation of the see-saw mechanism, which we want to outline. From the two contributions to the neutrino Dirac masses one is the same as for the up-type quarks, up to phases. As the other contribution arises at a separate fixed point, large cancellations are not expected. Therefore, we estimate the Dirac neutrino masses to be of similar magnitude as the up-type quark masses. This implies that the heaviest neutrino has a Dirac mass comparable to the top quark, which is a standard assumption in see-saw mechanisms. A Majorana mass for the right-handed neutrinos is generated by non-renormalisable interactions at the SO(10) and Georgi-Glashow fixed points. They come suppressed with one power of the compactification scale, M_c. If the $B-L$ breaking vacuum expectation value, $\langle N\rangle$, is not much smaller than the compactification scale, the right-handed neutrino mass will be of similar magnitude as well. Diagonalising the neutrino mass matrix then leads to a very small mass for the mostly left-handed neutrinos.

A unique prediction for the mass hierarchy of quarks and the CKM matrix can be derived from Eqs. (4.23). This is because the quark Yukawa matrices are both proportional to only one parameter each, which influences the overall magnitude of the coupling but not the mixing between different fields. Diagonalising Y^u and Y^d by a bi-unitary transformation each, the CKM matrix is the product of the left-handed diagonalisation matrices,

$$Y^u = U_l^u Y^u_{\text{diag}} U_r^u, \quad Y^d = U_l^d Y^d_{\text{diag}} U_r^d \;\Rightarrow\; V_{\text{CKM}} = U_l^{u\dagger} U_l^d. \tag{4.24}$$

The masses of the quarks are given by the eigenvalues inside $Y^{u,d}_{\text{diag}}$ times the appropriate Higgs field's vacuum expectation value. While the parameters h_u^I, h_d^I, which govern the overall scale of the quark masses, are not predicted in our model, approximate mass relations can be determined as the ratios of eigenvalues. We find a near degeneracy between the first two generations, but a much heavier third generation. In detail, the predictions for the tree-level, bare mass parameters are

$$\frac{m_c}{m_u} \approx 1.73, \quad \frac{m_t}{m_c} \approx 660, \quad \frac{m_s}{m_d} \approx 1.24, \quad \frac{m_b}{m_s} \approx 1097. \tag{4.25}$$

Obviously, there need to be further contributions to the first two generation's masses in order to give realistic hierarchies, as the measured masses [39] give quite different ratios, e.g., $m_t/m_c \approx 140, m_c/m_u \approx 570$. Still the mere fact that our tree-level considerations reproduce a mass hierarchy is noteworthy.

The mixing between the quark generations is also not realistic right away, as can be seen from the magnitude of the CKM matrix' entries,

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.4947 & 0.8691 & 0.0002 \\ 0.8691 & 0.4947 & 0.0002 \\ 0.0003 & 0.0001 & 1 \end{pmatrix}, \tag{4.26}$$

where we sorted the entries such that the first row/column corresponds to the field with the smallest mass. While further contributions to the quark mass sector are definitely needed to render it realistic, one qualitative aspect of the measured flavour sector is reproduced: Our model predicts some mixing between the first two generations and very little mixing with the third generation. To what extend this feature still exists after further contributions are considered has to be seen in an explicit construction.

Note that these results on the quark sector are a consequence of our choice of parities for the charged field, which forbid quark Yukawa couplings at most of the fixed points. A different choice, such as a positive parity for the **10**-plet at the Georgi-Glashow fixed point, would allow contributions proportional to the parameters h_u^{GG} and h_d^{fl} in Eq. (4.19). Whether the additional freedom in the quark masses and mixing granted by this, or any other, choice of parities can lead to a satisfactory phenomenology is the subject of future studies [89]. For the remainder of this thesis, we assume the parities to be chosen such that a lepton doublet arises as additional zero mode beyond the three SM generations.

Studying the phenomenology that arises from just the fixed point superpotential terms involving only SM fields, we have concluded that further contributions to the flavour sector are required in order to render it realistic. From Table 4.2, we see that there are vector-like exotics with the quantum numbers of a lepton doublet and a down quark. The SM fields can couple to these exotic states in various combinations. Such a mixing was studied previously in the context of an SO(10) orbifold GUT [90]. There, the three Standard Model generations are introduced as fixed-point localised fields, which gives flavour-diagonal Yukawa matrices, but also allows different mixing with the vector-like states for different generations. The mixing parameters can then be constrained by the requirement of a phenomenologically viable flavour sector. In our construction, all generations originate from a single GUT representation. Therefore, the mixing with the exotic states again depends on the amplitude of the wave function at the fixed point. Example superpotential operators that lead to mixing between the down quarks and vector-like exotics are

$$\begin{aligned} W_{\mathrm{fp}}^i &\supset {h'}_d^i\, \psi \Psi H_2 + m_d^i\, \psi \Psi^c \\ &\supset \sum_{j=1}^{3} \Big({h'}_d^i\, v_2 \psi_+^{(j)}(\zeta_i; 0,1)\; q_l^j D_R + m_d^i\, \psi_+^{(j)}(\zeta_i; 1,0)\; d_r^j D_R^c \Big), \end{aligned} \tag{4.27}$$

where q_l^j, d_r^j are the three generations of left-handed quark doublets and right-handed down quarks, respectively, and D_R, D_R^c are exotic states with the quantum numbers of a down quark, or its conjugate. Because of the interactions in Eq. (4.27), the quark mass matrix is padded, i.e., it is now a 4×4 matrix. Mass eigenstates therefore are a mixture between the SM fermions and the exotic state. The entries in the fourth row are of the order of the compactification scale, while the other entries arise only after electro-weak symmetry breaking and are therefore much smaller. Such a structure leads to one very massive state and three states that correspond to the down quarks of the standard model. Even though the massive state is not observable, the mixing among the light fields is greatly influenced compared to the case where the vector-like state is not taken into account. In addition to the interaction shown in Eq. (4.27), which has the same structure as a SM Yukawa interaction, there can also be couplings to vector-like states from the additional **10**-plets, $H_3, \ldots, H_6$, from operators of the form $\psi H_n \Psi^c$. These couplings involve the $B-L$ breaking vacuum expectation values $\langle \Psi^c \rangle$ instead of the electro-weak $v_{1,2}$. The lepton sector will therefore also feature a mixing with vector-like exotics.

We see that heavy vector-like states that are present in our construction can affect the mass spectrum and mixing of the SM-like effective theory. However, the mixing operators are not entirely free. Just like for the Yukawa couplings of the fixed point superpotentials, there is one free parameter per coupling and fixed point. The relative importance of the operator for the various generations is given by the amplitude of the wave function at the respective fixed point. Whether these effects can lead to a realistic flavour sector is the subject of further study.

As the fixed point contributions to the Yukawa couplings is supersymmetric, the wave functions are the same for the bosonic and fermionic components of the bulk field and the mixing with vector-like exotics can also be implemented at the superfield level, our flavour sector provides "minimal flavour violation", in the sense that flavour mixing is the same for the fermions and their superpartners. A full study of the flavour sector, including the mixing with vector-like exotics and also studying other choices for the matter parities would be very interesting.

As a final topic, let us talk about supersymmetry breaking. While the quarks and leptons are massless at tree level, up to electro-weak symmetry breaking, the flux generates masses of the order of the compactification scale for the associated scalars. For three flux quanta and a compactification close to the GUT scale, $L = 200 M_P^{-1}$, we have computed the mass spectrum in Section 3.5. The resulting mass for the charged scalars was slightly larger than the compactification scale,

$$m_0 = 8.33 \times 10^{-3} M_P. \tag{4.28}$$

In our model, this supersymmetry breaking mass is universal for all squarks and sleptons, i.e., for all sfermions. This is a scenario widely used in phenomenological studies of supersymmetry breaking. However, without a concrete model of supersymmetry breaking it would be difficult to argue for a general scalar mass m_0, valid for all scalars in the theory except Higgs bosons. Interestingly enough, this is precisely what we find.

While the truncation of off-diagonal terms in the derivation of the effective action (4.17) obfuscates the precise form of the gauge-kinetic function for the SO(10) or SM gauge interactions, we expect it to depend on the moduli fields S and T in a linear fashion. If these moduli are not stabilised in a supersymmetric manner, the corresponding F-terms will break supersymmetry. As a consequence, the gauginos will then receive masses of the order of the gravitino mass $m_{3/2}$. With superpartners this heavy, the running of the gauge couplings will be very similar to the SM case. The fact that gauge coupling unification is only approximate in the SM is sometimes given as an argument for supersymmetry at low-ish energies. However, during compactification and GUT symmetry breaking, non-universal threshold effects to the gauge couplings arise which can mend the situation [91].

Besides the SM fields, the higgsinos remained light in the discussion so far. This is a consequence of them being protected by a Peccei-Quinn symmetry. Higher-order effects, like anomaly mediation [71, 72], will induce a suppressed supersymmetry breaking mass for the higgsinos. The fact that the higgsinos are light is a fortunate circumstance. Scenarios with just the SM spectrum at low energies are known to have trouble to match the effective action to the supersymmetric parent theory at scales beyond $M_S \sim 10^{12}\text{GeV}$ [68]. However, unlike in the SM, there are two Higgs doublets at low energies in our model. In such a Two-Higgs-Doublet-Model, the matching conditions to an MSSM spectrum can be fulfilled up to very high scales, if the higgsinos are light [69].

4.4. Chapter summary

In this chapter, we constructed an orbifold GUT model with an $\mathrm{SO}(10)\times\mathrm{U}(1)_A$ bulk gauge group and a background flux. The SO(10) bulk anomaly guided our choice of a bulk spectrum. We derived the action of the gauge symmetry breaking Wilson lines on the matter representations from considerations of the coset pieces of the kinetic terms. Charging just one **16**-plet in the flux background induces mixed $\mathrm{SO}(10)\times\mathrm{U}(1)_A$ and pure $\mathrm{U}(1)_A$ anomalies, both in the bulk and on the fixed points. We showed explicitly that the arising anomaly can be cancelled, if gauge symmetry breaking is neglected; for the case including gauge symmetry breaking we refer to future work [89]. The flux protects the charged representation from having its fermionic components projected out of the zero-mode spectrum. This effect is used to obtain three generations of Standard Model fermionic matter in addition to vector-like exotic representations from the bulk fields. Finally, we looked at the various contributions to the Yukawa couplings of the effective action. Our model is not constrained by the regular GUT mass relations, such as bottom-tau unification. However, a realistic mixing pattern does not emerge from simply implementing brane-localised superpotentials. The mixing with vector-like heavy states is another effect to consider in this context, but it is constrained by the common origin of all three generations, too. Whether this contribution is enough to render the flavour sector realistic remains to be seen.

While some details of the construction are still to be worked out, the general picture is quite clear. From our setup, we obtain an effective Standard Model with two light Higgs doublets, universal scalar and gaugino masses of the order of the compactification scale and suppressed higgsino masses. $B - L$ breaking can be achieved naturally and induces a see-saw mechanism that explains the small neutrino masses. The flavour sector of the theory is highly predictive.

We have left several aspects in the dark that should be investigated in the future. While we have derived an effective 4d action, including anomaly cancellation, using a consistent truncation, that truncation seems to be at odds with supergravity: The gauge-kinetic function of the SO(10) can not be read off Eq. (4.17) straightforwardly. We suspect this to be a consequence of the neglected mixed-index contributions to the SO(10) Chern-Simons term during compactification. Their inclusion would presumably give a full supergravity embedding in which, e.g., the precise values for the gaugino masses are known after moduli stabilisation.

A second issue is that flavour mixing is not yet satisfactory in our model. All the required pieces appear in a natural manner, but no phenomenologically viable fermion sector emerges. While we argue that mixing with vector-like representations could save the day, we remain short on an actual implementation.

Supersymmetry breaking is automatic for the matter fields thanks to the flux and, unless the moduli are stabilised supersymmetrically, carried to the gauginos at tree level through the gauge-kinetic function. The mediation mechanism by which the higgsinos obtain their mass is not included, however. This piece can affect the assessment, whether the matching conditions between the supersymmetric and supersymmetry-broken actions can be fulfilled.

Finally, six-dimensional supergravity is a non-renormalisable quantum field theory and requires a UV completion. It would be a great success, if our model allowed for a String Theory embedding. In this context, it is worthwhile to reconsider our bulk spectrum, which we defined from purely 6d considerations. Spectra obtained from coset spaces of E_8 provide only three **10**-plets, whereas the multiplicities for the **16** and $\mathbf{16}^*$ that appear are either 2 and 2 or 3 and 1 [61, 92]. If all fields live in the bulk, these spectra imply an irreducible bulk anomaly and are therefore inconsistent. Hence, some of the matter has to be located at the fixed points. Thinking towards a UV embedding of our model, it would be interesting to construct a variant thereof, where there are three **10**-plets and one charged **16** in the bulk and all other fields are brane-localised. Such an distribution of fields would lead to a different phenomenology and therefore give a similar, but different, model.

5 Conclusion

In this thesis, we studied constructions of six-dimensional supergravity with a U(1) background flux, compactified on the orbifold $T^2/\mathbb{Z}_2$. On the one hand, such models are useful toy theories to gain insight into the intricate interactions between different elements usually encountered in String Theory while working in an overseeable context. On the other hand, background flux had not yet been considered as an ingredient for orbifold GUT constructions, a situation we amend in our work.

In our analysis of a model with a U(1) gauge group, presented in Chapter 3, we found that a careful treatment of anomaly cancellation affects the seemingly unrelated topic of moduli stabilisation. Technically, we deduce part of the one-loop corrections to the effective action from the Green-Schwarz term that cancels the anomaly. The key result is a correction to the gauge-kinetic function of the fluxed gauge group. The sign of the one-loop contribution is fixed and the requirement of a positive gauge-kinetic function leads to a restriction on the field ranges for some of the moduli fields. Because of that, the D-term potential is positive definite on the physical moduli space. In conjunction with a fine-tuned superpotential of KKLT type, this allows for realistic values for the cosmological constant without the need to add a special "uplift" sector. The fact that corrections to gauge-kinetic functions akin to the one we find are known to arise also in one-loop String Theory computations [35] is a hopeful sign that our result might be important in a more general context.

The effects the background flux has on a charged bulk field make it an interesting ingredient in orbifold GUT building: Firstly, the flux provides a number of chiral fermionic zero modes from a single charged hypermultiplet in the bulk. Secondly, no components of the bulk field are projected out, even in the presence of symmetry breaking Wilson lines. Finally, supersymmetry is broken spontaneously in the charged sector of the theory. By introducing a single charged **16** of SO(10) into the bulk with three flux quanta we obtain three entire generations of Standard Model matter. While the fermions are massless, their superpartners obtain masses of the order of the compactification scale. Scenarios with light fermions and heavy scalars were studied independently from UV considerations. All bulk and fixed point anomalies can be cancelled by choosing an appropriate bulk spectrum and by adding a Green-Schwarz term. The additional bulk fields lead to vector-like exotics after compactification, which can help to provide viable flavour mixing.

Our work can be refined in a number of ways:

Firstly, we did not take into account the fixed point localised Fayet-Iliopolus terms that are generated by charged matter in the bulk [73]. This effect and the way in which it is treated, e.g., by cancelling the localised FI terms with localised flux, has an impact on the localisation and the spectrum of charged bulk fields and thus on a number of results presented in this thesis.

Secondly, the charged field's Kaluza-Klein spectrum in the flux background can not cleanly be separated into discrete levels. Our truncation to the chiral superfield with the lowest mass is therefore questionable. A resummation of the entire tower of states would provide a more reliable low-energy effective theory. These considerations are a work in progress [83].

Thirdly, while we use ingredients that are readily available in String Theory, we have not provided an actual embedding of our models into a UV complete setup. While our toy models work well, it is improbable that they arise in a full String Theory computation without modifications. These modifications, be they constraints on the charged bulk spectrum, additional effects from fixed point localised states, or a moduli superpotential with a different form, can easily spoil the achievements of our constructions.

Finally, we left several key pieces undefined in the GUT model of Chapter 4. Its phenomenological predictions will crucially depend on the implementations of the see-saw mechanism, the mixing between the Standard Model fermions and the vector-like exotics and the way supersymmetry breaking is mediated to the higgsinos. Without including these components, we can not claim to have presented a complete model. Our plan is to provide a more detailed construction in the future [89].

Despite all the things that remain to be done, we feel that we have made progress towards a deeper understanding of the effects of flux in orbifold compactifications. From the top-down perspective, one-loop contributions to the gauge-kinetic function might be considered more thoroughly in future studies of String Theory phenomena. Moreover, we have shown that a careful analysis of all ingredients of a model can reveal correlations that would be missed by reverting to general considerations in lieu of a definite construction. From the bottom-up approach, we constructed a model where a background flux can explain the appearance of multiple Standard Model fermion generations that appear in complete GUT group representations, while the bosonic fields are single and do not form representations of a larger group. All in all, six-dimensional flux compactifications prove to be a useful laboratory to experiment on theories and we are confident that our results will help to better understand how to link the framework of String Theory with realistic constructions of Grand Unified Theories.

Basics A

A.1. Supergravity in six dimensions

There are great introductory texts on supergravity. Therefore, we do not offer our own pedagogical approach, but rather direct the novice reader to a textbook on the subject, such as the book "Supergravity" by Freedman and van Proeyen [93]. This section serves to remind the reader of the most important facts.

The theory we want to study is the six-dimensional supergravity with eight supercharges, in 4d counting dubbed $\mathcal{N} = 2$. In six dimensions, this is the minimal amount of supersymmetry, since the smallest spinor of $SO(1, 5)$ has eight real degrees of freedom. In addition to a single gravity-tensor multiplet, the theory features an arbitrary number of vector- and hypermultiplets. Their field content is summarised in table A.1.

The general expression for six-dimensional supergravity with gauged symmetries and a matter sector has first been worked out in [25]. We only give the bosonic part of the action, the fermionic terms are then uniquely defined through supersymmetry. With the determinant of the vielbein e_6, the Ricci scalar R_6, the dilaton ϕ, the three-form field strength H, the gauge field strength two-form F and scalars ψ the action reads

$$S = \int e_6 \left[\frac{M_6^4}{2} \left(*R_6 - \mathrm{d}\phi \wedge *\mathrm{d}\phi \right) - \frac{e^{2\phi}}{4 M_6^4 g_6^4} H \wedge *H - \frac{e^{\phi}}{2 g_6^2} F \wedge *F - \frac{M_6^4}{2} g_{\alpha\beta}\, \mathrm{D}\psi^\alpha \wedge *\mathrm{D}\psi^\beta \right] . \tag{A.1}$$

This action contains the six dimensional Planck mass M_6 and the inverse six dimensional

Table A.1.: *Field content of 6d supergravities.*

Multiplet	Bosonic fields		Fermionic fields	
Gravity-Tensor multiplet	6d metric	$g_{(MN)}$	Gravitino	Ψ_M
	2-Form	$B_{[MN]}$	Dilatino	χ
	Dialton	ϕ		
Vector multiplet	Vector	A_M	Gaugino	Λ
Hypermultiplet	Scalars	ψ^α	Hyperino	ζ

gauge coupling g_6^{-1} as dimensionful constants. D are covariant derivatives given by

$$\mathrm{D} = d - iX_a^\alpha A^a \,, \tag{A.2}$$

with the gauge fields A^a and the killing vectors X_a^α. The fact that the scalars need to span a quaternionic Kähler manifold with metric $g_{\alpha\beta}$, containing the gauge group within its isometries, is a crucial constraint on the matter sector of such a theory. We are, however, interested in effects on the low-energy effective theory after compactification to four dimensions. The non-trivial geometry of the scalar manifold only adds sub-leading terms to the effective action, suppressed by powers of the 4d Planck mass. Therefore, it is sufficient to neglect this major complication and assume that a quaternionic Kähler manifold with a large enough isometry exists[1]. The action (A.1) is invariant under a set of transformations given in [25]. The precise form of these transformations bears no relevance to this work, so we do not repeat them here.

Realistic models need to be compactified to four dimensions. The minimal spinor of $SO(1,5)$ decomposes into two, four component spinors of $SO(1,3)$. Compactifying a six-dimensional supergravity on a Ricci-flat two dimensional manifold, leaving the full 6d spinor covariantly constant, would result in a four-dimensional supergravity with eight supercharges, i.e., $\mathcal{N}=2$ supergravity. This only permits vector-like fermion pairs and is in conflict with the chiral fermion spectrum we observe in nature. In order to construct a theory more suitable for particle physics, we need to break at least one of the 4d supersymmetries at a high scale. Compactification on a non-flat manifold is one option to do so. Particularly simple in this respect are torus orbifolds. Within this class compactification on the orbifold $T^2/\mathbb{Z}_2$, described in detail in Section A.2, is the simplest choice. The behaviour of the fields under the orbifold parity operation can be chosen such, that the zero modes furnish an effective $\mathcal{N}=1$ supersymmetric theory. This is in concordance with the interpretation of the orbifold fixed points carrying localised curvature, which no longer allows a full 6d Weyl spinor to be covariantly constant.

The effective theory after compactification of 6d supergravity on $T^2/\mathbb{Z}_2$ is $\mathcal{N}=1$ supergravity in four dimensions. This framework has been studied in great detail, condensed in a famous book by Wess and Bagger [84]. To fully define the theory, three functions need to be specified: First, the real *Kähler potential* fixes the kinetic terms of chiral multiplets. Here, we also include the *Killing vectors* that define which isometries of the Kähler manifold are gauged. Second, the holomorphic *gauge-kinetic function* determines the kinetic terms for vector multiplets. Finally, the holomorphic *superpotential* gives couplings beyond those governed by gauge symmetry and the Kähler geometry. The high degree of symmetry present in supergravity allows for this amazing distillation of information.

As aesthetic as supergravity is, the supersymmetry must be broken in a realistic theory, simply because we have not seen any hint for superpartners up to energies of a few TeV.

[1] This assumption is backed by the existence of classes of quaternionic Kähler manifolds whose isometries contain Sp(n), SO(n) or SU(n) subgroups for arbitrary n [25].

We will not go into much detail on supersymmetry breaking and once again refer the novice reader to textbooks on the subject. Still, we feel it is useful to mention that there are two, not mutually exclusive, options to break supersymmetry [84]. The first is dubbed "D-term breaking", which proceeds through the gauge sector of the theory. The auxiliary field of a vector multiplet, D, develops a vacuum expectation value. The gaugino, whose supersymmetry transformation can be written as

$$\delta_\epsilon \lambda = iD\epsilon + \sigma^{\mu\nu} F_{\mu\nu}\epsilon, \tag{A.3}$$

then transforms non-linearly under supersymmetry. Thus D can be interpreted as an order parameter of supersymmetry breaking. In a similar fashion, "F-term breaking" proceeds via the auxiliary field of a chiral multiplet. If F obtains a vacuum expectation value, the chiral fermion no longer transforms linearly

$$\delta_\epsilon \chi = i\sqrt{2}\sigma^\mu \bar{\epsilon} \partial_\mu A + \sqrt{2} F\epsilon, \tag{A.4}$$

and supersymmetry is broken. As a consequence of the non-linear transformations of the fermion in the supersymmetry breaking sector, it can be removed from the theory by choosing an appropriate supergravity transformation parameter. In such a "super-Higgs" mechanism, the degrees of freedom that are "gauged away" turn into the longitudinal components of a now massive gravitino. If supersymmetry breaking is effected by both, D and F-terms, the degrees of freedom that are "eaten" by the gravitino is a linear combination of the corresponding fermions.

A.2. The orbifold $T^2/\mathbb{Z}_2$

This section describes the construction of the torus orbifold $T^2/\mathbb{Z}_2$. In the process, we fix some notation and introduce concepts required to understand the importance of gauge field backgrounds in the compact space.

A torus is a compact two-dimensional space that can be obtained from its universal covering space, the real plane $\mathbb{R}^2$ with coordinates y_1, y_2. To do so, we require a lattice of discrete translations, which can be constructed from two basis vectors $\lambda_{1,2}$

$$\lambda = n_1\lambda_1 + n_2\lambda_2. \tag{A.5}$$

$n_{1,2}$ are arbitrary integers. The base lattice vectors correspond to elementary torus translations $t_{1,2}$. The crucial step in going from extended $\mathbb{R}^2$ to the torus is to identify points in the plane that can be related to each other by a lattice vector

$$y \sim y + \lambda \tag{A.6}$$

Objects defined on the universal covering space can be projected to the torus as well, by requiring

$$\phi(y+\lambda) = T_1^{n_1} T_2^{n_2} \phi(y). \tag{A.7}$$

Here, the translation operators $T_{1,2}$ form a representation of the translations and describe how translation by a basis vector of the lattice changes the physical object ϕ. Therefore, it suffices to describe the *fundamental domain* of the torus, to which all other points in the plane can be mapped by lattice translations, along with the action of the translation operators on the objects in our theory. The fundamental domain is always well-defined, but there are infinitely many equivalent versions of it[2]. For convenience, we orient our fundamental domain such that one of its corners is at the origin and its edges align with the lattice vectors, i.e., we choose

$$t_1 : (y_1, y_2) \mapsto (y_1 + L_1, y_2), \quad t_2 : (y_1, y_2) \mapsto (y_1, y_2 + L_2). \tag{A.8}$$

This fundamental domain is depicted in Figure[3] A.1. This space can be described by a metric

$$g_{mn} = \frac{r^2}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_2^2 + \tau_1^2 \end{pmatrix}. \tag{A.9}$$

The shape of the torus is captured in $\tau = \tau_1 + i\tau_2$, where $|\tau|$ encodes the ratio of lengths and τ_1 encodes the angle between the basis vectors $\lambda_{1,2}$. The coordinates $y_{1,2}$ now run over the same interval $[0, L)$. The area of the torus is governed by r^2 and L, as

$$V_2 = \int \mathrm{d}^2 y \sqrt{g} = \int \mathrm{d}^2 y \, r^2 = r^2 L^2 \tag{A.10}$$

To go from the torus to the orbifold, we introduce another action on the covering space: a parity

$$p : y \mapsto -y. \tag{A.11}$$

This corresponds to half a rotation around the origin. The orbifold is what remains of the torus after identifying $y \sim -y$ and has half the area of the torus. In contrast to the translations, which affected all points on the real plane, there are four locations on the torus, which are identified with themselves, possibly up to a lattice translation. These points are the *fixed points* ζ_I of the orbifold. They lie at

$$\zeta_1 = (0,0), \quad \zeta_2 = (L/2, 0), \quad \zeta_3 = (0, L/2), \quad \zeta_4 = (L/2, L/2). \tag{A.12}$$

[2]For example, from a given fundamental domain a new representation can be constructed via translation by an infinitesimal vector.

[3]The discussion of “orbifold one-cycles” below as well as Figures A.1 and A.2 are due to M. Dierigl.

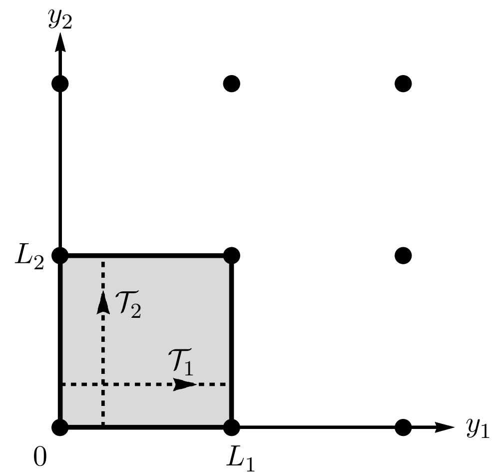


Figure A.1.: *The fundamental domain of the torus, the lattice of translation vectors and the standard representation of the torus one-cycles $\mathcal{T}_{1,2}$.*

Again, as with translations objects defined on the torus can be projected to the orbifold with

$$\phi(-y) = P\phi(y). \tag{A.13}$$

The group of transformations acting on the universal covering space is the semi-direct product $\mathbb{Z}^2 \rtimes \mathbb{Z}_2$ with an algebra

$$pt_m p = (t_m)^{-1}, \quad p^2 = 1. \tag{A.14}$$

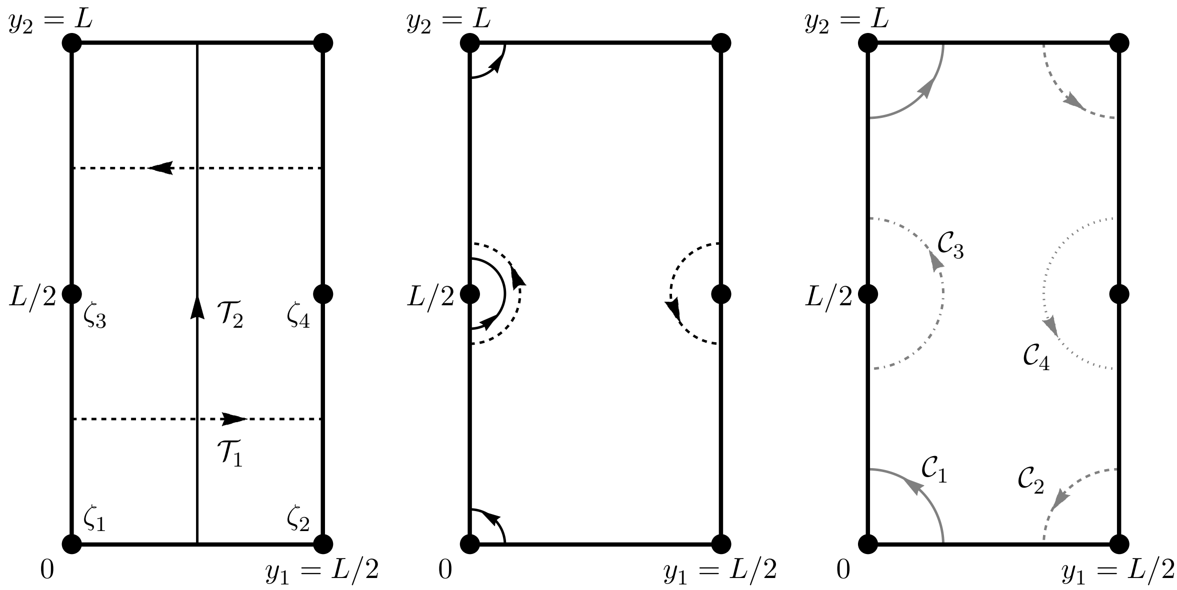
Of course, the representation of the space group on the objects in our theory has to fulfil the very same properties.

There is a neat correspondence between the generators of the space group and the one-cycles on the torus/the orbifold. Each translation operator of the Torus $T_{1,2}$ generates a one-cycle $\mathcal{T}_{1,2}$ on the torus. Their projection onto the orbifold is shown in Figure A.2a.

On the orbifold, we can define four one-cycles $\mathcal{C}_i$ each wrapping around one fixed point. Of these four, three are linearly independent. One can express the torus one-cycles $\mathcal{T}_{1,2}$ in terms of the $\mathcal{C}_i$ as

$$\mathcal{T}_1 = \mathcal{C}_3 + \mathcal{C}_4\,, \quad \mathcal{T}_2 = \mathcal{C}_1 + \mathcal{C}_3. \tag{A.15}$$

This becomes clear after continually deforming the $\mathcal{T}_i$ in such a way that they wrap around two of the orbifold fixed points each, as illustrated in Figure A.2b. Additionally, the orbifold parity operation corresponds to following the one-cycle $\mathcal{C}_1$. The generators of the space group T_1, T_2, P can thus be matched to orbifold one-cycles.



(a) *Torus one-cycles projected onto the orbifold.* **(b)** *Deformation of torus one-cycles towards the fixed points.* **(c)** *The orbifold cycles around the fixed points.*

Figure A.2.: *The fundamental domain of $T^2/\mathbb{Z}_2$ with several bases for one-cycles. Comparing b and c illustrates that $\mathcal{T}_1 = \mathcal{C}_3 + \mathcal{C}_4$ and $\mathcal{T}_2 = \mathcal{C}_1 + \mathcal{C}_3$. Also, one can see that $\mathcal{C}_4 = -(\mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3)$.*

A.3. A recipe for compactification

In this Section, we try to give an overview of the elementary concepts involved in describing a theory with extended and compact dimensions and the method required to obtain the low-energy effective action below the compactification scale. Many of the ideas presented can also be found in the excellent introduction by Quiros [94].

First of all, we need to define our background spacetime. In order to display all features we need in this thesis, we will take $\mathcal{M}_4 \times T^2/\mathbb{Z}_2$, i.e., 4d Minkowski space times the orbifold discussed in the previous section, as our background. This spacetime is six-dimensional, ergo our action is of the form

$$S = \int \mathrm{d}^6 x \mathcal{L}_6 \tag{A.16}$$

and our Lagrangian has mass dimension six. All fields that appear in our theory also have to be defined in the full six dimensions, with the exception of localised fields, whose Lagrangian involves a delta function peaking at one of the fixed points. Due to the mass dimension of the Lagrangian, the kinetic terms of the fields also have to have mass dimension six. If one wants to keep the 4d canonical dimension of the fields, a dimensionful parameter has to be introduced to accompany the two (one) derivative operator for bosons (fermions). Alternatively, one can work with fields of higher mass dimension than is usual in 4d field theory, e.g., with scalars of mass dimension 2 and 6d chiral fermions of mass dimension 5/2, and rescale the fields after compactification. Note that also in the first case, the fields have to be rescaled, but with a dimensionless constant.

Once one has constructed the expression for the six-dimensional action, all space-time indices need to be divided into the "external" indices $\mu = 0, 1, 2, 3$ and the "internal" indices $m = 5, 6$. Combinatorical factors may appear for objects with multiple indices, such as field strengths. Special attention must be paid to the kinetic terms, as derivatives also carry a space-time index. We will follow the procedure for a scalar field, where the kinetic term is

$$S \supset \int \mathrm{d}^6 x \, \partial_M \phi \, \partial^M \phi = \int \mathrm{d}^6 x \, \left(\partial_\mu \phi \, \partial^\mu \phi + \partial_m \phi \, \partial^m \phi \right). \tag{A.17}$$

A sum of differential operators invites a product Ansatz and indeed, writing

$$\phi = \hat{\phi}(x^\mu) \cdot \tilde{\phi}(x_m), \tag{A.18}$$

we obtain

$$S \supset \int \mathrm{d}^6 x \, \left(|\tilde{\phi}(x_m)|^2 \partial_\mu \hat{\phi} \, \partial^\mu \hat{\phi} + |\hat{\phi}(x^\mu)|^2 \partial_m \tilde{\phi} \, \partial^m \tilde{\phi} \right). \tag{A.19}$$

The "wave function" of the field in the internal space, $\tilde{\phi}$, has to fulfil periodicity conditions

$$\tilde{\phi}(t_g(x_m)) = T_g \tilde{\phi}(x_m), \tag{A.20}$$

where t_g is a member of the space group of the internal space and T_g is a representation of the space group on the field space. The operators T_g are often called "twists". The space group for the orbifold $T^2/\mathbb{Z}_2$ has three generators, as discussed in Appendix A.2. Therefore, up to three non-trivial twists may be present for a given field. As a representation of the space group, the twists have to fulfil the space-group algebra. Moreover, they have to leave the action invariant. As an interesting example, twists that do not commute with the gauge symmetry can break the latter to a maximally symmetric subgroup.

Once the twists are specified, one can solve the internal space equation of motion. Since the extra dimensions are spatial, the metric has a positive determinant and the relevant equation is an elliptic partial differential equation of second order. Such an equation has smooth solutions[4], whose eigenvalues all carry the same sign. Moreover, the wave functions need to be normalisable. Labelling different solutions with an index i we have

$$g^{mn} \partial_m \partial_n \tilde{\phi}_i(x_m) = m_i^2 \tilde{\phi}_i(x_m) \quad \text{with} \quad \int \mathrm{d}^2 y \sqrt{g_2} \, |\tilde{\phi}_i(x_m)|^2 = 1. \tag{A.21}$$

To each of these solutions, we have to associate an independent 4d mode $\hat{\phi}_i(x_\mu)$. Using these properties, we can eliminate the derivatives w.r.t. internal space coordinates in Eq. (A.19) in trade for a sum over all solutions. Once we don't have any internal space

[4]Singular solutions might exist if covariant derivatives in the equation of motion involve singular gauge field backgrounds.

derivatives any more we can execute the integration over the internal space and obtain an effective action for a whole tower of 4d fields

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{g_4}\, V_2 \sum_i \left(\partial_\mu \hat{\phi}_i \, \partial^\mu \hat{\phi}_i + \frac{m_i^2}{V_2} |\hat{\phi}_i|^2 \right). \tag{A.22}$$

This result can be interpreted as follows: Modes with different momenta in the internal space appear as independent modes in the 4d action. The larger the (quantised) momentum in the internal space, the larger the effective 4d mass. The mass spectrum of these "Kaluza-Klein" modes is determined by the spectrum of the internal space equations of motion. It is quantised due to the compact nature of the internal space, which imposes (quasi)periodic boundary conditions.

The procedure is similar for fermions: Again the full 6d Spinor, which has eight components, can be decomposed into a a four-dimensional and a two-dimensional spinor, joined by a tensor product. For a left-handed 6d Weyl spinor, that fulfills $\Gamma_7 \psi = -\psi$, the chiralities of the 4d and 2d spinors needs to be opposite, because of the relation $\Gamma_7 = \gamma_5 \otimes \sigma_3$, where σ_3 is the 2d chirality operator. This gives a decomposition

$$\psi(x_M) = \psi_{4L} \otimes \psi_{2R} + \psi_{4R} \otimes \psi_{2L}. \tag{A.23}$$

The 6d Dirac matrices can also be written as tensor products of the regular 4d Dirac and 2×2 matrices

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma_5 \\ i\gamma_5 & 0 \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} 0 & \gamma_5 \\ -\gamma_5 & 0 \end{pmatrix}. \tag{A.24}$$

One can then separate the equations of motion for ψ_4 and ψ_2, solve the latter and retrieve a Kaluza-Klein mass just as for the scalar case discussed above.

Once the dependence on the internal space coordinates has been solved and the extra dimensions are integrated out, we are left with an effective 4d action and compactification is complete. Often, one truncates the tower of massive states, neglecting states with masses of the order of the compactification scale, or above. The lowest-lying truncated state or the compactification scale, whichever is lower, defines the upper limit for the validity of the effective theory.

Note that the boundary conditions have major impact on the spectrum of the internal space equations of motion. For example, setting a negative value for the parity operation will typically not allow an eigenvalue $m_i^2 = 0$. Fields of negative parity are therefore generically truncated away and not a part of the low-energy effective action. Exceptions are possible though, e.g., charged fermions in a flux background do have a zero mode, independent of their parity.

A.4. $U(1)$ gauge backgrounds on $T^2/\mathbb{Z}_2$

This section introduces the reader to gauge field backgrounds on the torus and its $\mathbb{Z}_2$ orbifold. We restrict our attention to the Abelian case, as this is what we apply in our model. Even though we also use gauge field backgrounds in the $SO(10)$, they are Wilson lines, embedded in $U(1)$ subgroups of the $SO(10)$, and can therefore be treated with the formulae presented here. In this section we set the length scale of the extra dimensions $L = 1$.

One fundamental property of gauge fields is the freedom to perform gauge transformations, that act as

$$A \to A - \mathrm{d}\Lambda. \tag{A.25}$$

Where A is the gauge field one-form and Λ an arbitrary function. This kind of transformation does not alter the field-strength associated to the gauge field. Therefore, all gauge fields associated to each other by a transformation (A.25) belong to the same so-called orbit, describing the same physical situation. Gauge fields that can be brought to the form $A \equiv 0$ are called "pure gauge" configurations.

The twists of charged fields can be embedded into the gauge group. For a charged field, a $U(1)$ group representative can easily be given through a real parameter α_m as in

$$T_m = e^{iq\alpha_m}. \tag{A.26}$$

A shift $y_1 \to y_1 + 1$ would therefore endow a charged field with a phase $\exp(iq\alpha_1)$. Nonequivalent parameters fulfil $\alpha_m \in [0, 2\pi/q)$. To identify the phases created by general lattice shifts as gauge transformations, we assert

$$e^{iq\Lambda(y+\lambda)}\phi(y+\lambda) = e^{iq\Lambda(y)}\phi(y), \tag{A.27}$$

where $\Lambda(y)$ is the local gauge parameter. Combining this with

$$\phi(y+\lambda) = T_\lambda\phi(y) = e^{iq(n_1\alpha_1+n_2\alpha_2)}\phi(y) \tag{A.28}$$

we find that in order to identify T_λ as a gauge transformation, Λ has to fulfil

$$\Lambda(y+\lambda) = -(n_1\alpha_1 + n_2\alpha_2)\Lambda(y), \tag{A.29}$$

which can be solved by

$$\Lambda(y) = -(\alpha_1 y_1 + \alpha_2 y_2). \tag{A.30}$$

In turn, this gauge parameter represents a constant gauge field background because

$$A = -d\Lambda = \alpha_m \mathrm{d}y_m. \tag{A.31}$$

So, non-trivial twists T_m embedded in a gauged $U(1)$ can be transported to a constant gauge field background through a gauge transformation (A.30).

The periodicity conditions on the torus also affect the gauge parameter, which has to fulfil

$$e^{iq\Lambda(y+\lambda)} = e^{iq\Lambda(y)}, \quad \Rightarrow \quad \Lambda(y+\lambda) = \Lambda(y) + \frac{2\pi k}{q}, \quad k \in \mathbb{Z}. \tag{A.32}$$

This also implies that constant gauge fields on the torus are *not* pure gauge configurations, since they cannot be gauged away, unless $A = 2\pi k_m/q \, \mathrm{d}y_m$ with $k_m \in \mathbb{Z}$. In the overall picture this is consistent because non-trivial twists of charged fields have physical consequences, therefore they should not correspond to a pure gauge background, which has no impact on the fields. A central quantity related to gauge configurations in the compact space is the Wilson line. It is defined as a path-ordered exponential

$$W = \mathcal{P} \exp \left[iq \int_C A \right], \tag{A.33}$$

where C is the integration path. Of major interest are the Wilson lines associated to one-cycles, i.e., paths whose endpoints are identified under the action of the space group. Since $W \in U(1)$, we find the following behaviour under gauge transformations for Wilson lines along one-cycles

$$W \to e^{iq\Lambda(y)} W e^{-iq\Lambda(y+\lambda)} = W e^{iq(n_1\alpha_1 + n_2\alpha_2)}. \tag{A.34}$$

Wilson lines along one-cycles can be used to determine the phase a charged field picks up under lattice translations. This will become useful once we include the orbifold parity operation, where the gauge field background is more involved. Note that the Wilson line along a one-cycle is invariant under continuous deformation of the path[5].

Having looked at the situation on the torus, we now include the parity operation that creates the orbifold. Using the space group algebra (A.14) and the fact that $U(1)$ is Abelian, we find that

$$P^2 = 1 \Rightarrow P = \pm 1 = e^{iq\alpha_p} \tag{A.35}$$

and

$$P\,T_m\,P = (T_m)^{-1} \Rightarrow P P\,T_m = (T_m)^{-1} \Rightarrow T_m = (T_m)^{-1} \Rightarrow T_m = \pm 1 = e^{iq\alpha_m} \tag{A.36}$$

The phases picked up by charged fields on the orbifold are quantised for parity and trans-

[5] This is no longer true in the presence of a bulk flux. This provides a major motivation to study the orbifold one-cycles introduced by M. Dierigl. See [95] for a detailled presentation.

lations, $\alpha_m, \alpha_p \in \pi\mathbb{Z}/q$. These phases can be related neatly to the one-cycles $\mathcal{C}_i$ introduced in Appendix A.2.

We now construct a map from the α, describing twists in the torus one-cycle language, to a new set of variables $c_i \in \mathbb{Z}/q$, that capture the Wilson lines in the orbifold one-cycle notation,

$$W_i = \exp\left(iq \oint_{\mathcal{C}_i} A\right) = e^{iq\pi c_i}, i = 1, \ldots, 4. \tag{A.37}$$

The parity operation directly corresponds to following the one-cycle $\mathcal{C}_1$, so we can assign the twist T_P to it. We therefore have

$$T_p = e^{iq\alpha_p} = W_1 = e^{iq\pi c_1} \quad \Rightarrow \quad c_1 = \frac{\alpha_p}{\pi} \in \mathbb{Z}/q. \tag{A.38}$$

The translation along $\mathcal{T}_2$ is equivalent to following $\mathcal{C}_1 + \mathcal{C}_3$. From this and the knowledge of W_1 we can infer

$$T_2 = e^{iq\alpha_2} = W_1 \cdot W_3 = e^{iq\alpha_p} e^{iq\pi c_3} \quad \Rightarrow \quad c_3 = \frac{\alpha_2 - \alpha_p}{\pi} \in \mathbb{Z}/q. \tag{A.39}$$

The same procedure for $\mathcal{T}_1 = \mathcal{C}_3 + \mathcal{C}_4 = -\mathcal{C}_1 - \mathcal{C}_2$ yields

$$T_1 = e^{iq\alpha_1} = W_1 \cdot W_2 = e^{iq\alpha_p} e^{iq\pi c_2} \quad \Rightarrow \quad c_2 = \frac{\alpha_1 - \alpha_p}{\pi} \in \mathbb{Z}/q. \tag{A.40}$$

Finally, because the fourth one-cycle in not independent, we see that

$$\prod_i W_i = 1 \quad \Rightarrow \quad \sum_i c_i = 0 \quad \Rightarrow \quad c_4 = \frac{\alpha_p - \alpha_1 - \alpha_2}{\pi} \in \mathbb{Z}/q. \tag{A.41}$$

We have related the boundary conditions of charged fields to the Wilson lines obtained by integrating along the orbifold one-cycles. Since the Wilson lines are $W_i = \pm 1$ and represent boundary conditions to charged fields we will sometimes speak of "parities" at individual fixed points.

It is notable that the gauge field background leading to these Wilson lines can *not* be the constant gauge field discussed in the first part of this Section. This becomes obvious after considering the internal space covariant derivative $D_m = \partial_m + iqA_m$, which has to be odd under the orbifold parity; a property that a constant field can not support. One can however construct vortex configurations that give any combination of W_i possible on the orbifold. We will not discuss these configurations any further and refer the reader to [95] for more detail.

All backgrounds discussed so far do not lead to a background flux, since in all cases $\langle F \rangle = \mathrm{d}\langle A \rangle = 0$. Backgrounds with $\langle F \rangle = \mathrm{d}\langle A \rangle = f v_2 = \text{const.} \neq 0$ are possible, but $f \in \mathbb{R}$ is subject to quantisation. Let us write

$$\langle A \rangle = -f y_2 \mathrm{d}y_1. \tag{A.42}$$

To consistently project this background to the torus, we require that a lattice translation is equivalent to a gauge transformation, which in turn is restricted by Equation (A.29) if charged fields are present. Therefore, we have

$$\langle A(y+\lambda)\rangle = \langle A(y)\rangle - \mathrm{d}\Lambda(y+\lambda) \quad \Rightarrow \quad \mathrm{d}\Lambda = n_2 f \mathrm{d}y_1. \tag{A.43}$$

Also, the gauge parameter has to fulfil a periodicity condition (A.32), which gives with $\lambda = (1,1)$

$$\Lambda(y+\lambda) - \Lambda(y) = \int_y^{y+\lambda} \mathrm{d}\Lambda = \int_0^1 f \mathrm{d}y_1 = f \in \frac{2\pi\mathbb{Z}}{q}. \tag{A.44}$$

This gives integral numbers of flux quanta on the torus

$$\frac{q}{2\pi}\int_{T^2}\langle F\rangle = \frac{qf}{2\pi} = M \in \mathbb{Z} \tag{A.45}$$

and, due to the reduced area of the orbifold, twice the flux density on the orbifold

$$\frac{q}{2\pi}\int_{T^2/\mathbb{Z}_2}\langle F\rangle = \frac{qf}{4\pi} = -N \in \mathbb{Z}. \tag{A.46}$$

We define $f \propto -N$ in order to obtain left-handed fermionic zero modes for positive flux number N on the orbifold. The Wilson lines W_i along the orbifold one-cycles can easily be superimposed on such a flux background, which was the original intention in constructing them.

A.5. Anomalies on orbifolds

In this appendix, we briefly review anomaly cancellation by the Green-Schwarz mechanism. It is by no means a complete introduction to the topic, but should be understood as a brief reminder for the knowledgeable reader.

Upon quantising a field theory, amplitudes with fermions in the loop can create effective operators that do not adhere to the classical symmetries of the theory. Such an amplitude is called an anomaly. They arise in even dimensions at the one-loop level and turn the theory inconsistent if the broken symmetry is a gauge symmetry. If a theory has an anomalous gauge symmetry, it can possibly be made consistent by introduction of the Green-Schwarz mechanism: A two-form is introduced into the theory, with a gauge transformation and couplings to a Chern-Simons form, such that the variation of the total action remains gauge invariant, despite the non-trivial transformation of the fermionic part.

A concise way to determine the consistent form of the anomaly for a theory is to construct the gauge invariant anomaly polynomial [4]. While the anomaly $\mathcal{A}_d$ in d dimensions is a d-form, the anomaly polynomial I_{d+2} is a d+2 form. The two are related by the descent equations through a series of gauge variations and exterior derivatives. The anomaly polynomial is closed, which locally implies the existence of a $d+1$ form fulfilling $I_{d+2} = \mathrm{d}I^{(0)}_{d+1}$. $I^{(0)}_{d+1}$ is not gauge invariant, $\delta I^{(0)}_{d+1} \neq 0$, but its variation is again closed $\mathrm{d}\delta I^{(0)}_{d+1} = 0$. In turn, this means that locally there is a d form $I^{(1)}_d$, such that $\delta I^{(0)}_{d+1} = \mathrm{d}I^{(1)}_d$. This d-form is the anomaly we are looking for.

The expression for the anomaly derived from the anomaly polynomial is guaranteed to fulfil the Wess-Zumino consistency conditions. It is not unique, though, since local counter-terms can be added to change the form of the anomaly terms. One can use these counter terms to concisely demonstrate the Green-Schwarz (GS) mechanism. For the sake of clarity, let us specify $d = 6$, i.e., we demonstrate the GS mechanism for the bulk anomaly of a 6d theory. A prerequisite for anomaly cancellation is that the anomaly polynomial factorises,

$$I_8 = X_4 Y_4, \tag{A.47}$$

where all products of differential forms are understood to be wedge products. The anomaly derived from such an anomaly polynomial can be written as

$$I^{(1)}_6 = \left(\frac{1}{2} + \xi\right) X^{(1)}_2 Y_4 + \left(\frac{1}{2} - \xi\right) X_4 Y^{(1)}_2. \tag{A.48}$$

A choice of local counter terms fixes the ambiguity ξ and allows us to set one of the contributions to zero by setting, e.g., $\xi = 1/2$. Since the anomaly is the gauge transformation of the one-loop fermionic action, we add a piece to the bosonic action, such that the total action is again gauge invariant. The relevant GS counter-term is

$$S_{\mathrm{GS}} = -\int Y_4 B_2 = \int Y_3 \mathrm{d}B_2 \tag{A.49}$$

where B_2 is a two form that shifts under gauge transformations as $\delta B_2 = X^{(1)}_2$. In order for the field strength of B_2 to be gauge invariant, it has to include a Chern-Simons term $H_3 = dB_2 - X^{(0)}_3 \Rightarrow \delta H_3 = 0$. This ensures that the bosonic action cancels the anomaly through the variation of the GS term, while all other terms remain gauge invariant. Note that the $X^{(0)}_3$ can be a sum of Chern-Simons forms for different gauge connections or even for the spin connection, if gravitational anomalies are present.

While the Green-Schwarz mechanism is a neat tool to restore consistency, there is a price to pay. The gauge symmetry in question will be broken by the coupling to B_2 in a variant of the Stückelberg mechanism. If one insists on keeping the gauge symmetry intact, the spectrum of the theory has to be such that no anomalies arise, which heavily constrains the fermionic spectrum of the theory.

On a 6d torus orbifold, in addition to the regular bulk anomaly, there are brane-localised anomalies. They, too, can be derived from an anomaly polynomial, which is a six-form.

The full anomaly polynomial can be written as

$$I_8 = I_8^{(b)} + \sum_p I_6^{(p)} \Delta_2^{(p)}, \tag{A.50}$$

where p enumerates the fixed points and $\Delta_2^{(p)} = \delta^2(y - \zeta_p) \mathrm{d}^2 y$ is a two-form delta function peaking at the fixed point ζ_p. The brane anomalies can be cancelled by the GS mechanism as well, if they factorise $I_6^{(p)} = X_4^{(p)} Y_2^{(p)}$. However, if one wishes to cancel both brane and bulk anomalies with a single two-form B_2, the four-form factor of all brane anomalies and of the bulk anomaly have to be identical, $\forall p : X_4^{(p)} = X_4$. The GS term cancelling all anomalies then reads

$$S_{\mathrm{GS}} = -\int \Big(Y_4 + \sum_p Y_2^{(p)} \Delta_2^{(p)} \Big) B_2 = \int \Big(Y_3^{(0)} + \sum_p \Big(Y_1^{(0)} \Big)^{(p)} \Delta_2^{(p)} \Big) \mathrm{d}B_2. \tag{A.51}$$

$Y_1^{(0)}$ are forms that appear in the descent of the brane anomaly polynomial, analogous to $Y_3^{(0)}$ for the bulk anomaly. The behaviour of B_2 under gauge transformations does not change, by virtue of the common four-form factor X_4 in the anomaly polynomial.

Details on the compactification of 6d supergravity B

B.1. Compactifcation of the Bososnic 6d Lagrangean on $T^2/\mathbb{Z}_2$

The expression we start with is

$$e_6^{-1}\mathcal{L}_B^6 = \frac{M_6^4}{2}\left(R_6 - \mathrm{d}\phi\wedge *\mathrm{d}\phi\right) - \frac{1}{4M_6^4 g_6^4}e^{2\phi}H\wedge *H - \frac{1}{2g_6^2}e^{\phi}F\wedge *F. \tag{B.1}$$

Later, we will add a Green-Schwarz term. We do not include the hypermultiplets here, since their compactification is mostly independent and, aside from the flux-induced mass, very standard.

The compactification manifold is the orbifold $T^2/\mathbb{Z}_2$. We factor r^2, the volume modulus, out of the background metric, which reads

$$G = \begin{pmatrix} r^{-2}G_4 & \\ & r^2 G_2\end{pmatrix}, \quad G_2 = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_2^2+\tau_1^2\end{pmatrix}, \quad e_6 = e_4 r^{-2}. \tag{B.2}$$

We combine the shape moduli τ_1, τ_2 to a complex $\tau = \tau_2 + i\tau_1$. Next, we use the following decomposition of the 6d Ricci scalar, 3-form and 2-form field strengths, which follows from setting to zero all internal derivatives as well as tensor components with an uneven number of internal space indices:

$$\begin{aligned}
R_6 &\to r^2\left(R_4 - r^{-4}\,\partial_\mu r^2\partial^\mu r^2 - 1/(2\tau_2^2)\,\partial_\mu\tau\partial^\mu\bar\tau\right)\\
\mathrm{d}\phi\wedge *\mathrm{d}\phi &\to r^2\partial_\mu\phi\,\partial^\mu\phi\\
F\wedge *F &\to r^4 F_{\mu\nu}F^{\mu\nu} + r^{-4}F_{mn}F^{mn}\\
H\wedge *H &\to r^6 H_{\mu\nu\rho}H^{\mu\nu\rho} + 3r^{-2}H_{\mu nr}H^{\mu nr}
\end{aligned} \tag{B.3}$$

This gives, with a factor $(1/2)L^2$ from the integration over the compact space[1] and ap-

[1] We keep track of the factor of 2 from the orbifolding by putting in in parentheses.

propriate combinatorical factors,

$$\begin{aligned} e_4^{-1}\mathcal{L}_B^4 = &\left(\frac{1}{2}\right)L^2 r^{-2}\frac{M_6^4}{2}r^2\left(R_4-\partial_\mu\phi\,\partial^\mu\phi-\frac{1}{r^4}\partial_\mu r^2\partial^\mu r^2-\frac{1}{2\tau_2^2}\partial_\mu\tau\,\partial^\mu\bar{\tau}\right)\\ &-\left(\frac{1}{2}\right)L^2 r^{-2}\frac{1}{4g_6^2}e^\phi\left(r^4F_{\mu\nu}F^{\mu\nu}+r^{-4}F_{mn}F^{mn}\right)\\ &-\left(\frac{1}{2}\right)L^2r^{-2}\frac{1}{24M_6^4g_6^4}e^{2\phi}\left(r^6H_{\mu\nu\rho}H^{\mu\nu\rho}+3r^{-2}H_{\mu nr}H^{\mu nr}\right). \end{aligned} \tag{B.4}$$

We now introduce the fields

$$t=r^2e^{-\phi}\text{ and }s=r^2e^\phi. \tag{B.5}$$

The Lagrangean then reads

$$\begin{aligned} e_4^{-1}\mathcal{L}_B^4 = &\frac{M_6^4\left(\frac{1}{2}\right)L^2}{2}\left(R_4-\frac{1}{2t^2}\partial_\mu t\,\partial^\mu t-\frac{1}{2s^2}\partial_\mu s\,\partial^\mu s-\frac{1}{2\tau_2^2}\partial_\mu\tau\,\partial^\mu\bar{\tau}\right)\\ &-\frac{\left(\frac{1}{2}\right)L^2}{4g_6^2}s\,F_{\mu\nu}F^{\mu\nu}-\frac{\left(\frac{1}{2}\right)L^2}{4g_6^2}\frac{1}{st^2}F_{mn}F^{mn}\\ &-\frac{1}{24M_6^4\left(\frac{1}{2}\right)L^2}\left(\frac{\left(\frac{1}{2}\right)L^2}{g_6^2}\right)^2s^2\,H_{\mu\nu\rho}H^{\mu\nu\rho}-\frac{1}{8M_6^4\left(\frac{1}{2}\right)L^2}\left(\frac{\left(\frac{1}{2}\right)L^2}{g_6^2}\right)^2\frac{1}{t^2}H_{\mu nr}H^{\mu nr}. \end{aligned} \tag{B.6}$$

Identifying the Planck mass from the constant in front of R_4 and the 4d gauge coupling from the constant factor in front of the field strength we define

$$M_P^2=M_6^4\left(\frac{1}{2}\right)L^2,\quad g_4^2=\frac{g_6^2}{\left(\frac{1}{2}\right)L^2}. \tag{B.7}$$

With these definitions, the Langrangean takes the more compact form

$$\begin{aligned} e_4^{-1}\mathcal{L}_B^4 = &\frac{M_P^2}{2}R_4-\frac{M_P^2}{4t^2}\partial_\mu t\,\partial^\mu t-\frac{M_P^2}{4s^2}\partial_\mu s\,\partial^\mu s-\frac{M_P^2}{4\tau_2^2}\partial_\mu\tau\,\partial^\mu\bar{\tau}\\ &-\frac{s}{4g_4^2}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4g_4^2}\frac{1}{st^2}F_{mn}F^{mn}\\ &-\frac{1}{24M_P^2g_4^4}s^2H_{\mu\nu\rho}H^{\mu\nu\rho}-\frac{1}{8M_P^2g_4^4}\frac{1}{t^2}H_{\mu nr}H^{\mu nr}. \end{aligned} \tag{B.8}$$

Turning now to the internal components of tensors, we first look at the quantization condition for the background flux:

$$\frac{2\pi(-N)}{q}=\int\langle F\rangle=\left(\frac{1}{2}\right)L^2\langle F\rangle\ \Rightarrow\ \langle F\rangle=\frac{(2)2\pi(-N)}{qL^2}=\frac{f}{L^2}\,(=F_{mn}) \tag{B.9}$$

For the internal components of the three-form $H=\mathrm{d}B+A\wedge F$ we factor out an unknown mass scale $\tilde{M}$ and write ("[2]" represents the factor of two in the shift behaviour of b from contributions like $\partial_n B_{r\mu}$)

$$H_{\mu nr} = \tilde{M}^2 \partial_\mu b \epsilon_{nr} + A_\mu \frac{[2]f}{L^2} \epsilon_{nr} = \tilde{M}^2 \left(\partial_\mu b + A_\mu \frac{[2]f}{\tilde{M}^2 L^2} \right) \epsilon_{nr}. \tag{B.10}$$

Plugging this into the Lagrangean we have

$$\begin{aligned} e_4^{-1} \mathcal{L}_B^4 = & \frac{M_P^2}{2} R_4 - \frac{M_P^2}{4t^2} \partial_\mu t \, \partial^\mu t - \frac{M_P^2}{4s^2} \partial_\mu s \, \partial^\mu s - \frac{M_P^2}{4\tau_2^2} \partial_\mu \tau \, \partial^\mu \bar{\tau} \\ & - \frac{s}{4g_4^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4g_4^2} \frac{1}{st^2} \frac{f^2}{L^4} - \frac{1}{24 M_P^2 g_4^4} s^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \\ & - \frac{1}{4t^2} \frac{\tilde{M}^4}{M_P^2 g_4^4} \left(\partial_\mu b + A_\mu \frac{[2]f}{\tilde{M}^2 L^2} \right) \left(\partial^\mu b + A^\mu \frac{[2]f}{\tilde{M}^2 L^2} \right). \end{aligned} \tag{B.11}$$

In order to introduce the complex field $T = \frac{1}{2}(t + i\, b)$ the kinetic terms of t and b need to match. This fixes the scale $\tilde{M}$ to be

$$M_P^2 = \frac{\tilde{M}^4}{M_P^2 g_4^4} \Rightarrow \tilde{M}^2 = M_P^2 g_4^2. \tag{B.12}$$

Additionally, we use the shorthand

$$\ell = g_4 M_P L \tag{B.13}$$

which is a dimensionless combination of constants. Now, the Langragean is

$$\begin{aligned} e_4^{-1} \mathcal{L}_B^4 = & \frac{M_P^2}{2} R_4 - \frac{M_P^2}{4s^2} \partial_\mu s \, \partial^\mu s - \frac{M_P^2}{4\tau_2^2} \partial_\mu \tau \, \partial^\mu \bar{\tau} \\ & - \frac{s}{4g_4^2} F_{\mu\nu} F^{\mu\nu} - \frac{g_4^2 M_P^4}{2st^2} \frac{f^2}{\ell^4} - \frac{1}{24 M_P^2 g_4^4} s^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \\ & - \frac{M_P^2}{4t^2} \partial_\mu t \, \partial^\mu t - \frac{M_P^2}{4t^2} \left(\partial_\mu b + A_\mu \frac{[2]f}{\ell^2} \right) \left(\partial^\mu b + A^\mu \frac{[2]f}{\ell^2} \right). \end{aligned} \tag{B.14}$$

The last line invites the definition of the field $T = \frac{1}{2}(t + i\, b)$, which has a Killing vector

$$X^T = -i \frac{[2]f}{2\ell^2}. \tag{B.15}$$

Having dealt with the purely classical contributions to the Lagrangean, we now turn our attention to the Green-Schwarz term that cancels all 6d and 4d anomalies. It is given by

$$S_{\text{GS}} = -\int \left(L^2 \frac{\beta}{2} \left(\hat{A} \wedge \hat{F} \wedge (g_4 M_P)^2 \mathrm{d}b + \frac{f}{L^2} \hat{A} \wedge \mathrm{d}\hat{B} \right) + \alpha \hat{A} \wedge \hat{\mathrm{d}} \hat{B} \right). \tag{B.16}$$

Here, α and β are constants, $\hat{A}$ and $\hat{F}$ are the 4d components of the $U(1)$ vector field and field strength respectively and the purely 4d component of the three-form is given by $\hat{H} = \mathrm{d}\hat{B} + \hat{A} \wedge \hat{F}$. So far, we marked the factor $1/2$ from the orbifolding by explicit brackets. In the Green-Schwarz term the orbifolding effects the factor $1/2$ that comes with β. From here on we will not mark the orbifold factor anymore, since it does not appear explicitly anymore. Replacing $\mathrm{d}\hat{B}$ through the equation $\mathrm{d}\hat{B} = \hat{H} - \hat{A} \wedge \hat{F}$, all terms involving $\hat{H}$ can be expressed in 4d differential forms as (we drop the hats)

$$S^H = \int -\frac{1}{4} \frac{s^2}{M_P^2 g_4^4} H \wedge *H - \left(\frac{f}{2}\beta + \alpha\right) A \wedge H - \sigma c \, \mathrm{d}(H - A \wedge F). \tag{B.17}$$

The last term introduces a Lagrange multiplier field c that ensures $\mathrm{d}(\mathrm{d}B) = 0$ and σ is a constant to be fixed later on. Next, we employ the equation of motion for H to replace it as a dynamical degree of freedom in favor of c,

$$\begin{aligned} \frac{1}{2} \frac{s^2}{M_P^2 g_4^4} * H &= -\sigma \mathrm{d}c + \left(\frac{f}{2}\beta + \alpha\right) A, \\ *H &= -\frac{2 M_P^2 g_4^4}{s^2} \sigma \left(\mathrm{d}c - \frac{1}{\sigma}\left(\frac{f}{2}\beta + \alpha\right) A\right). \end{aligned} \tag{B.18}$$

Plugging this into S^H we have

$$S^H = \int -\sigma^2 g_4^4 \frac{M_P^2}{s^2} \left(\mathrm{d}c - \frac{1}{\sigma}\left(\frac{f}{2}\beta + \alpha\right) A\right) \wedge * \left(\mathrm{d}c - \frac{1}{\sigma}\left(\frac{f}{2}\beta + \alpha\right) A\right) - \sigma \mathrm{d}c \wedge A \wedge F. \tag{B.19}$$

Once again, we are eager to define a complex field $S = \frac{1}{2}(s \pm i\, c)$ and use this relation to fix the constant $\sigma = \pm 1/(2 g_4^2)$. In principle, both signs are possible for σ, even when considering anomaly cancellation. σ drops out of the gauge variation of the bosonic action, since it appears in both, the $c\, F \wedge F$ term and the gauge shift in c.

Gathering the terms with c, the term with b from the GS term and the classical contributions to the Lagrangean we reach the result

$$\begin{aligned} e_4^{-1} \mathcal{L}_B^4 = \; & \frac{M_P^2}{2} R_4 - \frac{M_P^2}{(\tau + \bar{\tau})^2} \partial_\mu \tau \partial^\mu \bar{\tau} - \frac{s}{4 g_4^2} F_{\mu\nu} F^{\mu\nu} - \frac{g_4^2 M_P^4}{2 s t^2} \frac{f^2}{\ell^4} \\ & - \frac{M_P^2}{(T + \bar{T})^2} \mathrm{D}_\mu T \, \mathrm{D}^\mu \bar{T} - \frac{M_P^2}{(S + \bar{S})^2} \mathrm{D}_\mu S \, \mathrm{D}^\mu \bar{S} - \frac{2 g_4^2 \left(-\sigma c + \frac{\beta}{2} \ell^2 b\right)}{4 g_4^2} F_{\mu\nu} \tilde{F}^{\mu\nu}, \end{aligned} \tag{B.20}$$

where $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. This expression is readily reproduced as part of a 4d $\mathcal{N} = 1$ SuGra (which involves other terms) with $T = \frac{1}{2}(t + i\, b)$, $S = \frac{1}{2}(s - i\, 2 g_4^2 \sigma c)$ and

$$\begin{aligned} K &= -M_P^2 \log(T + \bar{T} + i X^T V) - M_P^2 \log(S + \bar{S} + i X^S V) - M_P^2 \log(\tau + \bar{\tau}), \\ X^T &= -i[2] f / (2 \ell^2), \quad X^S = -i g_4^2 (\alpha + \beta f / 2), \\ H &= 2 \cdot \left(S + g_4^2 \beta \ell^2 T\right). \end{aligned} \tag{B.21}$$

The Kähler potential was read of the scalar kinetic terms, the gauge-kinetic function was read of the $F\tilde{F}$ term and the Killing vectors follow from the axion shifts. The bosonic Lagrangean of this SuGra is, according to Wess & Bagger [84][2] (who, compared to our expression, absorb a factor of g_4 into A_μ)

$$\begin{aligned} e_4^{-1}\mathcal{L}_B^4 = \frac{M_P^2}{2}R_4 &- \frac{s+g_4^2\beta\ell^2 t}{4g_4^2}F_{\mu\nu}F^{\mu\nu} - \frac{-2g_4^2\sigma c + g_4^2\beta\ell^2 b}{4g_4^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ &- \frac{M_P^2}{(T+\bar{T})^2}\mathrm{D}_\mu T\,\mathrm{D}^\mu\bar{T} - \frac{M_P^2}{(S+\bar{S})^2}\mathrm{D}_\mu S\,\mathrm{D}^\mu\bar{S} - \frac{M_P^2}{(\tau+\bar{\tau})^2}\partial_\mu\tau\partial^\mu\bar{\tau} \\ &- \frac{M_P^4 g_4^2}{2}\frac{1}{s+g_4^2\beta\ell^2 t}\left(-\frac{[2]f}{2t\ell^2} - \frac{g_4^2(\alpha+f\beta/2)}{s}\right)^2. \end{aligned} \tag{B.22}$$

Defining $S = \frac{1}{2}(s + i\,c)$ fixes $\sigma = -(2g_4^2)^{-1}$. The other sign for σ is possible as well, however the meaningful linear combination of modulus and axion would then be $s - i\,c$.

Taking $\sigma = -(2g_4^2)^{-1}$, we have

$$\begin{aligned} e_4^{-1}\mathcal{L}_B^4 = \frac{M_P^2}{2}R_4 &- \frac{s+g_4^2\beta\ell^2 t}{4g_4^2}F_{\mu\nu}F^{\mu\nu} - \frac{c + g_4^2\beta\ell^2 b}{4g_4^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ &- \frac{M_P^2}{(T+\bar{T})^2}\mathrm{D}_\mu T\,\mathrm{D}^\mu\bar{T} - \frac{M_P^2}{(S+\bar{S})^2}\mathrm{D}_\mu S\,\mathrm{D}^\mu\bar{S} - \frac{M_P^2}{(\tau+\bar{\tau})^2}\partial_\mu\tau\partial^\mu\bar{\tau} \\ &- \frac{M_P^4 g_4^2}{2}\frac{1}{s+g_4^2\beta\ell^2 t}\left(-\frac{[2]f}{2t\ell^2} - \frac{g_4^2(\alpha+f\beta/2)}{s}\right)^2. \end{aligned} \tag{B.23}$$

B.2. Proof that $\langle r\rangle = 1$ w.l.o.g.

It seems obvious that the theory is not affected by our choice of a length scale L to measure the extra dimensions; we set this scale *ad hoc*, after all. Intuitively, one would expect that, after stabilizing the moduli and finding the *dynamical* scale of the extra dimensions, we can rescale the parameters in such a way that the scale L represents the *physical* scale of the extra dimensions and the radion field, merely describing fluctuations of this scale, is stabilized at unity. It will be very reassuring to find our intuition verified.

To do so, we start from the compactified action (B.23), include the hypermultiplet terms and assume $\langle r\rangle = r_0 \neq 1$. First, we perform a Weyl rescaling of the 4d metric in order to measure lengths appropriately, i.e., with the pullback of the 6d metric to 4d. Such a Weyl rescaling implies

$$(g_4)_{\mu\nu} \mapsto r_0^2(g_4)_{\mu\nu}, \quad (g_4)^{\mu\nu} \mapsto r_0^{-2}(g_4)^{\mu\nu}, \quad \sqrt{-g_4} \mapsto r_0^4\sqrt{-g_4}, \quad R_4 \mapsto r_0^{-2}R_4. \tag{B.24}$$

[2] Beware of a sign error in front of the ϵFF term in Appendix G!

With these replacements, the action reads

$$\begin{aligned}\mathcal{S}_B^4 = \int \mathrm{d}^4x\sqrt{-g_4}\Bigg[& r_0^2\frac{M_P^2}{2}R_4 - \frac{s+g_4^2\beta\ell^2 t}{4g_4^2}F_{\mu\nu}F^{\mu\nu} - \frac{c+g_4^2\beta\ell^2 b}{4g_4^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ & - r_0^2\frac{M_P^2}{(T+\bar{T})^2}\mathrm{D}_\mu T\,\mathrm{D}^\mu\bar{T} - r_0^2\frac{M_P^2}{(S+\bar{S})^2}\mathrm{D}_\mu S\,\mathrm{D}^\mu\bar{S} - r_0^2\frac{M_P^2}{(\tau+\bar{\tau})^2}\partial_\mu\tau\partial^\mu\bar{\tau} \\ & - r_0^4\frac{M_P^4}{2}\frac{g_4^2}{s+g_4^2\beta\ell^2 t}\left(-\frac{[2]f}{2t\ell^2} - \frac{g_4^2(\alpha+f\beta/2)}{s}\right)^2 \\ & - r_0^2 D_\mu\phi_+\left(D^\mu\phi_+\right)^\dagger - r_0^4 m_+^2|\phi_+|^2 - r_0^4\frac{g_4^2 q^2}{2s}|\phi_+|^4\Bigg].\end{aligned} \tag{B.25}$$

Now, we introduce a new set of parameters to describe the compactification scale. In order to have $\langle r'\rangle = 1$ while keeping physical quantities like the volume of the compact dimensions constant, we need to rescale the length parameter $L \mapsto L' = \langle r\rangle L = r_0 L$. Other parameters that are linked with L through Eqs. (B.7) and (B.13) transform accordingly, which is

$$L \mapsto L' = r_0 L, \quad M_P \mapsto M_P' = r_0 M_P, \quad g_4^2 \mapsto r_0^{-2} g_4'^2, \quad \ell \mapsto \ell' = g_4' M_P' L' = r_0\ell. \tag{B.26}$$

Also fields that contain r and their associated axions as well as fields that were rescaled with L after compactification need to be transformed, i.e.,

$$s \mapsto s' = r_0^{-2}s, \quad c \mapsto c' = r_0^{-2}c, \quad t \mapsto t' = r_0^{-2}t, \quad b \mapsto r_0^{-2}b, \quad \phi_+ \mapsto r_0^{-1}\phi_+. \tag{B.27}$$

Neither the physical volume $V_2 = \langle r\rangle^2 L^2/2$, nor the effective gauge coupling $g_{\text{eff}}^2 = g_4^2/h$ are affected by this rescaling. The latter involves the real part of the gauge-kinetic function h, linear in the moduli s and t, whose rescaling cancels that of the bare coupling.

In primed parameters, the radion modulus is necessarily stabilized at $\langle r'\rangle = 1$, as

$$\langle r'\rangle^4 = \langle s't'\rangle = r_0^{-4}\langle st\rangle = r_0^{-4}\langle r\rangle^4 = 1. \tag{B.28}$$

Furthermore, it is easy to check that the Weyl rescaled action (B.25) takes the form (B.23) if expressed in primed parameters.

One might be worried that a change befalls the scalar mass, which transforms as $m_+^2 \mapsto m_+'^2 = r_0^2 m_+^2$, but there is a subtelty involved: In the unprimed system masses are given in terms of the unprimed (and unphysical) Planck mass, while in the primed system the masses are instead given in terms of the physical Planck mass. What is to stay constant when going from the unprimed to the primed system is the ratio

$$\frac{m_+^2}{M_P^2} \mapsto \frac{m_+'^2}{M_P'^2} = \frac{m_+^2 r_0^2}{M_P^2 r_0^2} = \frac{m_+^2}{M_P^2} \tag{B.29}$$

which is unaffected by the transformation indeed.

With these transformations we have shown explicitly that for any value of $\langle r \rangle$ one can rescale the parameters of the theory such that $\langle r' \rangle = 1$. We use this to argue that it is possible, without any loss of generality, to choose an L such that $\langle r \rangle = 1$. Morevoer, this implies that for any vacuum with $\langle st \rangle \neq 1$ there exists an equivalent parameterization with $\langle s't' \rangle = 1$ and it is the latter in which L' gives the size of the extra dimensions. While we use $\langle st \rangle = 1$ in the search for superpotential parameters, we checked numerically that, coming from a vacuum with $\langle st \rangle \neq 1$, rescaling the theory according to Eqs. (B.26) and (B.27) the moduli are stabilized at $\langle s't' \rangle = 1$ with the same superpotential parameters and scalar masses, now given in terms of the physical Planck mass.

The quaternionic structure of $\mathrm{Sp}(1,1)/\mathrm{Sp}(1)\times\mathrm{Sp}(1)$ C

Scalars in supergravity can be interpreted as coordinates of Kähler manifolds. Gauge symmetries have to be isometries of the corresponding metric. The more supersymmetry generators are present, the more constrained the Kähler geometry is. In $\mathcal{N}=1$ supergravity the scalar metric can be derived from the Kähler potential, a real function that can be chosen freely. Kähler manifolds of complex dimension n have a holonomy that is contained in $U(2n)$. For $\mathcal{N}=2$ supergravity, the appropriate geometry is of quaternionic Kähler type. A quaternionic Kähler manifold has a holonomy in $\mathrm{Sp}(n)\times\mathrm{Sp}(1)_R \nsubseteq U(2n)$.

Strictly speaking, a quaternionic Kähler (qK) manifold is therefore not Kähler. This is why the metric of a qK manifold can not be derived from a real function, but needs to be constructed in another way. In addition, while a product of Kähler manifolds is again Kähler, this property does not extend to qK manifolds. Hence, one cannot deduce the structure of complicated qK manifolds from studying simple specimen. However, we anticipate that the corrections due to the scalar geometry generically arise at the same level for all qK manifolds. Moreover, the study of a simple example can highlight the difficulties one would encounter treating a realistic model. We want to study the simplest interesting case of a qK manifold, which describes a single hypermultiplet charged under a local $U(1)$ symmetry, i.e.,

$$\mathcal{M}=\frac{\mathrm{Sp}(1,1)}{\mathrm{Sp}(1)\times\mathrm{Sp}(1)_R}. \tag{C.1}$$

We follow the construction in [25], which is specified to this particular manifold in [96].

The coordinate of $\mathcal{M}$ is the quaternion

$$t=\phi^1\mathbf{1}+\phi^2\mathbf{i}+\phi^3\mathbf{j}+\phi^4\mathbf{k}, \tag{C.2}$$

where the ϕ_i are real fields, and $\mathbf{1},\mathbf{i},\mathbf{j},\mathbf{k}$ fulfill the quaternion algebra. The quaternion base can be expressed through the well-known Pauli matrices as follows (i is the regular imaginary unit)

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{i} = -i\sigma_3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \ \mathbf{j} = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ \mathbf{k} = -i\sigma_1 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}. \tag{C.3}$$

With this, we write a coset representative as

$$L = \gamma^{-1} \begin{pmatrix} \mathbf{1} & t^\dagger \\ t & \Lambda(t) \end{pmatrix}, \quad L^{-1} = \gamma^{-1} \begin{pmatrix} \mathbf{1} & -t^\dagger \\ -t & \Lambda(t)^{-1} \end{pmatrix}, \tag{C.4}$$

where we defined

$$\gamma = \left(\mathbf{1} - t^\dagger t\right)^{1/2}, \quad \Lambda(t) = \gamma \left(\mathbf{I} - tt^\dagger\right)^{-1/2} \overset{n=1}{=} \mathbf{1}. \tag{C.5}$$

We decided to include the first expression given for $\Lambda(t)$, which is for a more general manifold $\mathrm{Sp}(n,1)/\mathrm{Sp}(n) \times \mathrm{Sp}(1)_R$, because of the great simplification in the case $n = 1$.

From the coset representative, one obtains the Maurer-Cartan form $L^{-1}\partial_\alpha L$, where ∂_α are derivatives w.r.t. ϕ^α. The explicit form of the Maurer-Cartan form allows us to read of the pullbacks of the Vielbein and spin connections for $\mathcal{M}$ as

$$L^{-1}\partial_\alpha L = \begin{pmatrix} W_\alpha^{AB} & V_\alpha^{aA\dagger} \\ V_\alpha^{aA} & W_\alpha^{ab} \end{pmatrix} = \gamma^{-2} \begin{pmatrix} \frac{1}{2}\left(\partial_\alpha t^\dagger t - t^\dagger \partial_\alpha t\right) & \partial_\alpha t^\dagger \\ \partial_\alpha t & \frac{1}{2}\left(\partial_\alpha (t^\dagger t) - 2t\partial_\alpha t^\dagger\right) \end{pmatrix} \tag{C.6}$$

where capital letters belong to the $\mathrm{Sp}(1)_R$ and lowercase letters to the other $\mathrm{Sp}(1)$. Having obtained the vielbein of our manifold, we simply have to concentrate the incides with the symplectic metric $\epsilon_{AB}\epsilon_{ab}$ of $\mathrm{Sp}(1) \times \mathrm{Sp}(1)_R$ to find the metric

$$g_{\alpha\beta} = V_\alpha^{aA} V_\beta^{bB} \epsilon_{ab}\epsilon_{AB} = 2\gamma^{-4}\delta_{\alpha\beta} = \frac{2}{\left(1 - |\phi|^2\right)^2}\delta_{\alpha\beta}, \tag{C.7}$$

where we introduced $|\phi|^2 = \left(\phi^1\right)^2 + \left(\phi^2\right)^2 + \left(\phi^3\right)^2 + \left(\phi^4\right)^2$. As a next step, we compute the potentials arising from gauging a $U(1)$ subgroup of either of the Sp(1) symmetries. The difference between gauging a subgroup of the R symmetry is the choice of Killing vectors and the form of the prepotential [25].

If we gauge a non-R $U(1)$ symmetry, an appropriate choice of Killing vector is

$$\xi^1 = -\phi^3, \ \xi^2 = \phi^4, \ \xi^3 = \phi^1, \ \xi^4 = -\phi^2. \tag{C.8}$$

Note that the two complex scalars obtained by defining

$$z = \phi^1 + i\phi^3, \quad c = \phi^2 + i\phi^4, \tag{C.9}$$

have opposite $U(1)$ charges. With the Killing vectors ξ^α and the connection W_α^{AB} we obtain the prepotential components as

$$C^x = gT^x_{AB}W_\alpha^{AB}\xi^\alpha, \tag{C.10}$$

where the T^x_{AB} are Sp(1) generators as given in Eq. (C.3). From the C^x the potential finally follows as

$$V(\phi) = C^xC^x = g^2\gamma^{-4}|\phi|^4 = g^2\frac{|\phi|^4}{\left(1-|\phi|^2\right)^2} = g^2\frac{\left(|z|^2+|c|^2\right)^2}{\left(1-|z|^2-|c|^2\right)^2}. \tag{C.11}$$

If we gauge a $U(1)$ subgroup of the R symmetry, the complex scalars z and c introduced in Eq. (C.9) have to have the same charge. This means the Killing vectors are

$$\xi_R^2 = -\phi^3,\ \xi_R^2 = -\phi^4,\ \xi_R^3 = \phi^1,\ \xi_R^4 = \phi^1. \tag{C.12}$$

Furthermore, the prepotential components are now found as

$$C_R^x = g'\left(T^x_{AB}W_\alpha^{AB}\xi_R^\alpha - 1\right), \tag{C.13}$$

giving a potential of

$$V_R(\phi) = C_R^xC_R^x = g'^2\gamma^{-4} = g'^2\frac{1}{\left(1-|\phi|^2\right)^2} = g'^2\frac{1}{\left(1-|z|^2-|c|^2\right)^2}. \tag{C.14}$$

The purely scalar part of the 6d supergravity action is [97]

$$S \supset \int \mathrm{d}^6x e_6\left[\frac{M_6^4}{2}g_{\alpha\beta}D_M\phi^\alpha D^M\phi^\beta - \frac{M_6^8}{2}g_6^2e^{-\varphi}V(\phi)\right], \tag{C.15}$$

where e_6 is the determinant of the Vielbein, φ the dilaton, $D_M\phi^\alpha = \partial_M\phi^\alpha - A_M\xi^\alpha$ with the Killing vectors given by Eq. (C.8) (Eq. (C.12)), and the potential given by Eq. (C.11) (Eq. (C.14)) depending on whether the gauge group is a subgroup of the R symmetry (in parantheses) or not. The fields ϕ^α are dimensionless and we pulled the 6d gauge coupling g_6, which has mass dimension -1, out of the expression for the potential. Compactifying this expression on a torus is straightforward, if one only takes the zero modes into account, which we do. We obtain

$$S \supset \int \mathrm{d}^4x e_4\left[\frac{M_4^2}{2}g_{\alpha\beta}D_\mu\phi^\alpha D^\mu\phi^\beta + \frac{M_4^2}{2}g_{\alpha\beta}g^{mn}\xi^\alpha\xi^\beta A_mA_n - M_4^4\frac{g_4^2}{s}V(\phi)\right] \tag{C.16}$$

where g^{mn} is the inverse of the internal space metric, $s = r^2e^\varphi$ a real scalar and the relations between the 6d Planck mass and gauge coupling are given in Eq (B.7). We will drop the term with the internal space components of the vector field, since it is not important to the point we are making. Introducing the linear combinations z and c as before, see Eq. (C.9), and writing out the scalar metric, Killing vectors and the potential for the case where we do not gauge a part of the R symmetry, the scalar action reads

$$S \supset \int \mathrm{d}^4 x e_4 \left[\frac{2M_4^2}{2} \frac{D_\mu \bar{z} D^\mu z + D_\mu \bar{c} D^\mu c}{\left(1 - |z|^2 - |c|^2\right)^2} - M_4^4 \frac{g_4^2}{s} \frac{\left(|z|^2 + |c|^2\right)^2}{\left(1 - |z|^2 - |c|^2\right)^2} \right]. \tag{C.17}$$

Rescaling the fields z and c such that the leading part of their kinetic term becomes canoncial, i.e., $z \to M_4 z$ and $c \to M_4 c$, we find

$$S \supset \int \mathrm{d}^4 x e_4 \left[\frac{D_\mu \bar{z} D^\mu z + D_\mu \bar{c} D^\mu c - \frac{g_4^2}{s} \left(|z|^2 + |c|^2\right)^2}{\left(1 - |z|^2/M_4^2 - |c|^2/M_4^2\right)^2} \right]. \tag{C.18}$$

Expanding the denominator according to $1/(1-x) = 1 + x + \ldots$ we find the expansion in $1/M_4$ to be

$$S \supset \int \mathrm{d}^4 x e_4 \left[\left(D_\mu \bar{z} D^\mu z + D_\mu \bar{c} D^\mu c - \frac{g_4^2}{s} \left(|z|^2 + |c|^2\right)^2 \right) \left(1 + \frac{|z|^2 + |c|^2}{M_4^2} + \mathcal{O}\left(M_4^{-4}\right) \right) \right]. \tag{C.19}$$

The quaternionic Kähler structure of the scalar manifold yields corrections to the globally supersymmetric case (cf. Eq. (2.6)) at the level of dimension six operators. As long as one studies only effective actions up to dimension five operators, the intricacies of the scalar sector in $\mathcal{N} = 2$ supergravity will not be important. The moduli sector, albeit describing scalar fields in 4d, is not affected by these corrections, as the moduli only adopt their scalar nature upon compactification. Thus one can argue that, as long as there are no large vacuum expectation values for any scalar component of a hypermultiplet, one can even consider operators of higher dimension in other sectors of the theory without working out the entire scalar structure. This is a great relief when building models, since the quaternionic Kähler structure necessitates a case-by-case analysis which is very cumbersome. Especially so, since the irreducible gravitational anomaly requires a large number of scalar fields to be present. One could not even divide the problem by simply writing down the interesting part of the scalar sector and postulating a number of uncharged, gravitationally coupled "spectator" fields to fulfill the requirements of anomaly cancellation, because the product of two quaterionic Kähler manifolds is not generally qK itself.

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Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Hamburg, den 21. September 2016 Unterschrift