Abstract

This thesis deals with the asymptotic properties of weighted voting games (WVGs) when there are many 'small' voters. It comprises three main parts.

Part I is concerned with *Penrose's Limit Theorem* (PLT). This research goes back to a presumption of L.S. Penrose concerning an asymptotic property of some sequences of WVGs with an increasing number of voters: under certain conditions the ratio between the voting power of any two voters (according to various measures of voting powers) approaches the ratio of their weights. So far there has been no rigorous proof of PLT for any non-trivial class of cases and counterexamples to Penrose's claim can be constructed. Part I introduces the concept of *q-chains* of weighted voting games and considers the question whether for a given q-chain and a given power index the PLT holds true. It provides sufficient conditions for the two most prominent power indices - the Shapley-Shubik and the Banzhaf index. The main result with respect to the Shapley-Shubik index (Theorem 3.4) states that given a *non-atomic* q-chain PLT holds for those voters for which the chain is *replicative.* Also, the PLT is proved with respect to the Banzhaf index for an important class of WVG-sequences with quota 1/2 (Theorem 3.13). Finally, the thesis contains an analogue of the last mentioned result for weighted decision rules that admit abstention as a *tertium quid* (Theorem 4.13).

Part II is concerned with the asymptotic behaviour of some global quantities of WVGs. Here, the setup is such that there are two kinds of voters: a fixed (possibly empty) set of *major voters* with fixed weights (the atomic part), and a growing population of *minor voters* with weights converging uniformly to zero (the non-atomic part). The question under consideration is what happens when the number of minor voters tends to infinity. First, the analysed quantity is *complaisance* introduced by J.S. Coleman in 1971 as the 'power of a collectivity to act'. Here, the the decision making body in binary WVGs (Theorem 7.2) and ternary WVGs (Theorem 7.5) is considered as a 'preference-aggregating machine'. Second, decision-making is

assumed as 'truth-tracking' such that there is a 'right answer' but the voters only have partial information and imperfect competence for detecting the truth. The *Condorcet jury theorem* considers a quantity called the *collective competence*, i.e. the probability of the decision-making body to arrive at the correct decision. This part of the thesis extends the celebrated theorem to general q-chains (Theorem 8.5).

Part III develops numerical methods for computing the quantities considered in Part I and II. The standard approach of evaluating WVGs exactly is the method of *generating functions* known from combinatorics, however, it shows insurmountable demand in storage in large WVGs. Part III shows how to overcome this difficulty by using methods designed for sparse matrices (Section 11.5 and source code in Chapter 15). Chapter 12 establishes a foundation of the widespread but merely heuristically stated approximation methods for WVGs by proving the validity of the normal distribution as an approximation tool.