Electric-magnetic duality invariance implications for axion physics

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Abstract

Axions and axion-like particles are very well-motivated candidates for physics beyond the Standard model, which can be probed by multiple existing and projected experiments and astrophysical observations. Theoretical determination of the relevant low energy axion interactions is essential for guiding the corresponding experimental and observational efforts. In this thesis, we revisit the previous theoretical investigations in this direction. In particular, we show that, contrary to assertions in the literature, the main contribution to the axion-photon coupling need not be quantized in units proportional to e^2. We discuss a loophole in the argument for this quantization and then provide explicit counterexamples. Based on this, we construct a generic axion-photon effective Lagrangian and find that the axion-photon coupling may be dominated by previously unknown Wilson coefficients. We show that this result implies a significant modification of conventional axion electrodynamics and sets new targets for axion experiments. We find that the electromagnetic interactions of axions can violate the CP symmetry and that future experiments could be sensitive to the corresponding coupling. At the core of our theoretical analysis lies a critical reexamination of the interactions between axions and magnetic monopoles. We develop the effective field theory approach to the Zwanziger theory of quantum electromagnetodynamics and show that, contrary to claims in the literature, magnetic monopoles need not give mass to axions. Moreover, we find that a future detection of an axion or axion-like particle with certain parameters can serve as evidence for the existence of magnetically charged matter.

Besides studying the structure of the low energy axion interactions in the effective field theory approach, we explicitly construct new theoretical models for the axion which realize the newly found interactions. In these models, the PQ mechanism is realized through a coupling of the Peccei-Quinn complex scalar field to magnetically charged fermions at high energies. We consider both the cases of Abelian and non-Abelian magnetic charges. We show that these models indeed solve the strong CP problem and then integrate out heavy magnetic monopoles using the Schwinger proper time method. We find that the models discussed yield axion couplings to the Standard Model which are drastically different from the ones calculated within the KSVZ/DFSZ-type models. As a consequence, large part of the corresponding parameter space can be probed by various projected experiments. Moreover, the axion we introduce is consistent with the astrophysical hints for axions suggested both by the anomalous TeV-transparency of the Universe and by the excessive cooling of horizontal branch stars in globular clusters. We argue that the leading term for the cosmic axion abundance is not changed compared to the conventional pre-inflationary scenario for an axion decay constant $f_a > 10^{12}$ GeV.

Zusammenfassung

Axionen und axionähnliche Teilchen sind sehr motivierte Kandidaten für Physik jenseits des Standardmodells, die durch mehrere bestehende und geplante Experimente und astrophysikalische Beobachtungen untersucht werden können. Die theoretische Bestimmung der relevanten Niederenergie-Axion-Wechselwirkungen ist unentbehrlich, um einen Leitfaden für die entsprechenden experimentellen und beobachtenden Bemühungen zu erstellen. In dieser Arbeit überdenken wir die bisherigen theoretischen Untersuchungen in dieser Richtung. Insbesondere zeigen wir, dass entgegen den Behauptungen in der Literatur der Hauptbeitrag zur Axion-Photon-Kopplung nicht in zu e^2 proportionalen Einheiten quantisiert werden muss. Wir diskutieren eine Lücke im Argument für diese Quantisierung und liefern dann explizite Gegenbeispiele. Darauf beruhend konstruieren wir einen generischen Axion-Photon-effektiven Lagrange-Operator und stellen fest, dass die Axion-Photon-Kopplung möglicherweise von zuvor unbekannten Wilson-Koeffizienten dominiert wird. Wir zeigen, dass dieses Ergebnis eine signifikante Modifikation der konventionellen Axion-Elektrodynamik impliziert und neue Ziele für Axion-Experimente schafft. Wir finden, dass die elektromagnetischen Wechselwirkungen von Axionen die CP-Symmetrie verletzen können und dass zukünftige Experimente sensitiv zu der entsprechenden Kopplung sein könnten. Im Zentrum unserer theoretischen Analyse steht eine kritische Überprüfung der Wechselwirkungen zwischen Axionen und magnetischen Monopolen. Wir entwickeln den Ansatz der effektiven Feldtheorie zur Zwanziger-Theorie der Quantenelektromagnetodynamik und zeigen, dass magnetische Monopole im Gegensatz zu Behauptungen in der Literatur Axionen keine Masse verleihen müssen. Darüber hinaus stellen wir fest, dass ein zukünftiger Nachweis eines Axions oder eines axionähnlichen Teilchens mit bestimmten Parametern als Indiz für die Existenz magnetisch geladener Materie dienen kann.

Neben der Untersuchung der Struktur der niederenergetischen Axion-Wechselwirkungen im Ansatz der effektiven Feldtheorie konstruieren wir explizit neue theoretische Modelle für das Axion, die die neu gefundenen Wechselwirkungen realisieren. In diesen Modellen wird der PQ-Mechanismus durch eine Kopplung des komplexen Peccei-Quinn-Skalarfelds an magnetisch geladene Fermionen bei hohen Energien realisiert. Wir betrachten sowohl die Fälle Abelscher als auch nicht-Abelscher magnetischer Ladungen. Wir zeigen, dass diese Modelle tatsächlich das starke CP-Problem lösen und integrieren dann schwere magnetische Monopole unter Verwendung der Schwinger-Eigenzeitmethode aus. Wir stellen fest, dass die diskutierten Modelle Axionkopplungen zu den Teilchen des Standardmodells ergeben, die sich drastisch von denen unterscheiden, die innerhalb der KSVZ/DFSZ-Modelle berechnet wurden. Als Folge davon kann ein großer Teil des entsprechenden Parameterraums durch verschiedene geplante Experimente untersucht werden. Darüber hinaus ist das Axion, das wir einführen, konsistent mit den astrophysikalischen Hinweisen für Axionen, die sowohl von der anomalen TeV-Transparenz des Universums als auch von der übermäßigen Abkühlung Horizontalast-Sterne in Kugelsternhaufen nahegelegt werden. Wir argumentieren, dass der führende Term für die kosmische Axion-Häufigkeit im Vergleich zum konventionellen vorinflationären Szenario nicht geändert wird, falls die Axion-Zerfallskonstante größer als 10^{12} GeV ist.

This thesis is based on the publications:

- A. Sokolov, A. Ringwald, *Electromagnetic Couplings of Axions*, arXiv:2205.02605 [hep-ph] [1]

- A. Sokolov, A. Ringwald, *Photophilic hadronic axion from heavy magnetic monopoles*, *JHEP* 06 (2021) 123, arXiv:2104.02574 [hep-ph] [2]

- A. Sokolov, A. Ringwald, *Magnetic anomaly coefficients for QCD axion couplings*, *PoS EPS-HEP2021* (2022) 178, arXiv:2109.08503 [hep-ph] [3]

- J. Jaeckel, G. Rybka, L. Winslow, and the Wave-like Dark Matter Community, *Axion Dark Matter*, arXiv:2203.14923 [hep-ph] [4]

Other publications by the author:

- A. Sokolov, *Gravitational Wave Electrodynamics*, arXiv:2203.03278 [hep-ph]

- G. A. Pallathadka, F. Calore, P. Carenza, M. Giannotti, D. Horns, J. Kuhlmann, J. Majumdar, A. Mirizzi, A. Ringwald, A. Sokolov et al., *Reconciling hints on axion-like-particles from high-energy gamma rays with stellar bounds*, *JCAP* 11 (2021) 036, arXiv:2008.08100 [hep-ph]

- A. Sokolov, *Generic energy transport solutions to the solar abundance problem — a hint of new physics*, *JCAP* 03 (2020) 013, arXiv:1907.06928 [astro-ph.SR]

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Contents

Introduction

The Standard model of particle physics provides a very good description of the interactions of elementary particles. Its structure alone predicts many low energy symmetries which were never disproved by any experiment, such as for instance baryon number conservation or time reversal symmetry of quantum electrodynamics. Not all of the possible symmetries of the theory can be however inferred from the structure of the Standard model. In particular, this is the case of time reversal symmetry of Quantum Chromo-Dynamics (QCD). Namely, there is a special free parameter $\bar{\theta}$ in the Standard model which indicates whether this symmetry holds. Fortunately, there is also an experimentally accessible observable proportional to $\bar{\theta}$ – the neutron electric dipole moment. While any measured value of this observable would call for some explanation in terms of a more fundamental theory, it is especially challenging that the measurements of the neutron electric dipole moment reveal it to be consistent with zero with an unprecedented precision of $10^{-26}\ e \cdot \mathrm{cm}$ [5]. The question of why QCD is symmetric under time reversal constitutes the core of the so-called strong CP problem. As science aims to explain what we observe, one is tempted to hypothesize a new model where the neutron electric dipole moment is constrained to be practically zero. In particular, one of the ideas proposed is to drive this observable to zero dynamically by introducing a new pseudoscalar particle called axion, which is a pseudo Nambu-Goldstone boson associated to spontaneous breaking of the anomalous Peccei-Quinn (PQ) symmetry [6–9]. The great advantage of this mechanism is that the introduction of the axion can naturally solve not only the strong CP problem, but also a much more pressing problem of missing mass in the Universe, i.e. the axion is a perfect candidate for dark matter [10–12].

The problem of dark matter has its origins in the work by Fritz Zwicky back in the 1930s [13]. The latter astronomer investigated the motion of the galaxies of the Coma cluster and found that the real mass of the cluster is much larger than the mass associated to the visible matter only. Later, in the 1970s, it was found that the problem of the missing mass manifests itself also in the rotation curves of galaxies [14, 15]: stars on the galaxy outskirts move much faster than one would expect taking into account the gravitational pull of the baryonic matter only. Since then, a lot of different astronomical observations have been made which support the hypothesis of dark matter. Nowadays, there exists vast evidence that the baryonic matter makes up on average only around 20% of the total mass of galaxies. It is reassuring that the energy density of dark matter inferred from astronomical observations is consistent with the standard model of cosmology (ΛCDM), which successfully describes the features of the observed cosmic microwave background and the structure formation in the Universe[1], both these probes being highly dependent on the abundance of dark matter. Although given the fact that our knowledge about dark matter comes only from its gravitational interactions one could think that a suitable modification of gravity would resolve the problem, it is becoming increasingly difficult to describe all the spectrum of astronomical observations in this way. A much more simple explanation is that dark matter is composed of some non-relativistic objects of a new type, which could be either particles beyond the Standard model or primordial black holes. Primordial black holes are good candidates for dark matter, as they do not require any physics beyond the Standard model, however the models of their production in the early Universe normally require large fine-tuning [17]. While the latter fine-tuning problem can be resolved in some clever models [18], in this thesis we will discuss a different scenario where the cold dark matter is produced in a simple way via the so-called misalignment mechanism. The latter scenario is relevant for axion particles, the properties of which will be the main subject of this thesis.

Although many properties of axions have been extensively studied in the literature before, we found that some very important class of axion interactions has been missed. The reason is that while constructing the low energy effective axion theories and the corresponding high energy models, one has always paid attention only to the electric sector of the theory. For instance, in the case of electromag-

[1] Although ΛCDM has its own problems such as the Hubble tension [16], this does not affect the conclusion about the presence of dark matter.

netic interactions, using the terms we review in Chapter 1 sec. 1.1, the axion has always been coupled to the electric part of the helicity of the electromagnetic field. The magnetic part of the helicity is however by no means less important. What we find is that the coupling of the axion to this magnetic part is significantly different compared to the conventional axion-photon coupling and thus has to be thoroughly studied. In general, we find that the neglect of the magnetic sector of the theory is absolutely not justified. In fact, not only is it unjustified in the framework of low energy effective theory, but also in the construction of particular high energy models. For example, consider the well-known class of axion models called hadronic axion models [19, 20], where one introduces at least one new heavy quark beyond the Standard model. If one wants to keep the model as minimal as possible, one has to allow this heavy quark to carry magnetic charge, which means that one has to consider the magnetic sector of the theory. The reason for this is that due to the quantization of the electric charge observed in nature, one expects to have at least one particle with magnetic charge in the spectrum of the theory [21]. As no magnetic charge has yet been observed, one expects this magnetically charged particle to be quite heavy. This means that if we introduce a new heavy quark like we do in hadronic axion models, it is preferred that this quark carries magnetic charge. Note that the connection between the quantization of charge observed in nature and the existence of magnetically charged matter is actually very strong. Indeed, as it was found recently in a number of works [22, 23], in the quantum world like ours where the quantization of charge coexists with the force of gravity, the existence of magnetically charged particles is inevitable.

Thus, it is important to investigate axion interactions which arise from the magnetic sector of the theory. Although the magnetic sector has always been neglected in its full generality before, there have been a few works studying the interactions of axions with magnetic monopoles. In particular, it has been long believed that the interactions between axions and magnetic monopoles are necessarily induced by the Witten effect [24]. What we show is that there are more possibilities, so that the shift of the axion field need not induce electric charge on monopoles. The corresponding non-conventional electromagnetic couplings of axions enter the axion-photon effective field theory (EFT) whenever one admits the no global symmetries [22, 25–28] and completeness of the charge spectrum [22,23,27] conjectures of quantum gravity, since these conjectures imply the existence of magnetic monopoles with any charge allowed by the Dirac-Schwinger-Zwanziger (DSZ) quantization condition [21,29,30]. The same new interaction terms enter the low energy EFT of axion-like particles (ALPs) – pseudo Nambu Goldstone bosons of any spontaneously broken anomalous global $U(1)$ symmetry. We derive phenomenological consequences of the new electromagnetic couplings of axions and ALPs, showing that they would represent new, distinctive features, which are possible to detect in various axion experiments. Moreover, we argue that the detection of axions or ALPs with such features would provide circumstantial experimental evidence for the existence of magnetically charged matter.

Besides purely theoretical arguments for considering the new class of axion interactions mentioned above, there are also quite strong phenomenological reasons to investigate these interactions. Nowadays, there is a vast number of ideas how one can potentially discover axions, both directly in the laboratory and indirectly through astrophysical data. Many of them have been already put into practice and more are yet to be implemented in the future. Although the axion has not been discovered so far, its parameter space has been constrained and even some hints pointing to a particular range of parameters have been found [31–33]. Since most of the axion searches exploit especially its coupling to photons, it is very important to understand which values of this coupling are theoretically preferred. It turns out that large part of the parameter space of conventional axion models cannot be probed in the near-future experiments. Thus, it seems that these models cannot be falsified in the nearest future. In this thesis, we find that accounting for the magnetic sector of the theory allows one to significantly alleviate the latter problem. The new electromagnetic interactions of axions which we find are much stronger than the conventional one and are in an immediate reach of multiple projected experiments.

This thesis is structured as follows: in Chapter 1, we discuss the electric-magnetic duality symmetry and magnetic monopoles, both Abelian and non-Abelian; in Chapter 2, we briefly review the physics of axions; in Chapter 3, we elaborate our arguments about why the structure of the electromagnetic interactions of axions as presented in the literature has to be revised; in Chapter 4, we discuss an exhaustive quantum field-theoretical framework required to take into account the magnetic sector of the theory and develop an EFT approach to the theories with magnetic charges; in Chapter 5, we build a generic axion-photon EFT and classify the new electromagnetic interactions of axions, discussing some of the implications of the Witten-effect induced coupling; in Chapter 6, we build explicit axion models solving the strong CP problem which realize the newly found interactions and derive the low energy axion couplings in these models, studying both the models with heavy Abelian and non-Abelian magnetic charges; in Chapter 7, we discuss the phenomenology of the new electromagnetic couplings and their implications for axion search experiments; finally, in Conclusions, we sum up the most important results of this thesis.

This Introduction is partly written based on the publications [1, 2] of the author of this thesis.

Chapter 1

Electric-magnetic duality and magnetic monopoles

1.1 Electric-magnetic duality symmetry

The Maxwell equations for the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields are:

$$\boldsymbol{\nabla}\times\mathbf{H}-\dot{\mathbf{E}}=\mathbf{j}_e\,, \tag{1.1}$$

$$\boldsymbol{\nabla}\times\mathbf{E}+\dot{\mathbf{H}}=0, \tag{1.2}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H}=0\,, \tag{1.3}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}=\rho_e\,, \tag{1.4}$$

where ρ and $\mathbf{j}_e$ are electric charge and current densities, respectively. In the space-time regions where $\rho_e = 0$, $\mathbf{j}_e = 0$, these equations have a symmetry with respect to the rotations in the $(\mathbf{E}, \mathbf{H})$ plane:

$$\begin{aligned} \mathbf{E} &\rightarrow \mathbf{E}\cos\theta+\mathbf{H}\sin\theta\,, \\ \mathbf{H} &\rightarrow \mathbf{H}\cos\theta-\mathbf{E}\sin\theta\;. \end{aligned} \tag{1.5}$$

This symmetry is however not manifest in the action for the electromagnetic field:

$$S_{\rm EM} \;=\; \frac{1}{2}\int d^4x\,\left(\mathbf{E}^2-\mathbf{H}^2\right). \tag{1.6}$$

To see the electric-magnetic duality symmetry (1.5) in the Lagrangian description, one has to remember that the action for the electromagnetic field (1.6) is introduced as a functional of the four-potential $A_\mu = (A_0,\, \mathbf{A})$, which is connected to the physical fields as follows:

$$\mathbf{E} = -\dot{\mathbf{A}} - \boldsymbol{\nabla} A_0\,, \quad \mathbf{H} = \boldsymbol{\nabla}\times\mathbf{A}\,. \tag{1.7}$$

The description of a physical system in terms of the four-potential is redundant. To discuss transformations of A_μ describing the free electromagnetic field, we first fix the redundancy by requiring $A_0 = 0$ and $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$. Performing the Helmholtz decomposition of the vector-potential $\mathbf{A} = \mathbf{A}^{\mathrm{T}} + \mathbf{A}^{\mathrm{L}}$, where $\boldsymbol{\nabla}\cdot\mathbf{A}^{\mathrm{T}} = 0$ and $\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{L}} = 0$, we see that the dynamics is determined by the transverse component of the vector-potential $\mathbf{A}^{\mathrm{T}}$, in terms of which the action (1.6) can be rewritten as follows:

$$S_{\mathrm{EM}} \;=\; \frac{1}{2}\int d^4x\, \left\{ \left(\dot{\mathbf{A}}^{\mathrm{T}}\right)^2 - \left(\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{T}}\right)^2 \right\}. \tag{1.8}$$

Now, let us consider the following infinitesimal transformation of the physically meaningful component of $\mathbf{A}$ [34]:

$$\delta\mathbf{A}^{\mathrm{T}} = -\theta\, \boldsymbol{\nabla}^{-2}\, \boldsymbol{\nabla}\times\dot{\mathbf{A}}^{\mathrm{T}}\,, \tag{1.9}$$

where $\theta \ll 1$ and $(\boldsymbol{\nabla}^{-2}\boldsymbol{\nabla}\times) = -(\boldsymbol{\nabla}\times)^{-1}$ is the inverse curl operator, which is well-defined while acting on the transverse component of a given vector field. The transformation (1.9) leaves the action (1.8) invariant, not taking into account the boundary term which does not contribute to the equations of motion:

$$\delta S_{\mathrm{EM}} \;=\; -\frac{\theta}{2}\int d^3x\, \left\{ \dot{\mathbf{A}}^{\mathrm{T}}\cdot\nabla^{-2}\left(\boldsymbol{\nabla}\times\dot{\mathbf{A}}^{\mathrm{T}}\right) + \mathbf{A}^{\mathrm{T}}\left(\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{T}}\right) \right\}\Bigg|_{t_1}^{t_2}. \tag{1.10}$$

Thus, there exists an internal $U(1)$ symmetry of the electromagnetic field with respect to the transformations arising from the exponentiation of the infinitesimal transformation (1.9). Although such transformations are in general different from the transformations corresponding to the electric-magnetic duality symmetry (1.5), it turns out that these two kinds of variations of $\mathbf{A}^{\mathrm{T}}$ are equivalent as long as one assumes the equation of motion (1.1) to hold. Indeed, the variations of electric and magnetic fields corresponding to the transformation (1.9) in this case are:

$$\delta\mathbf{H} = \nabla\times\delta\mathbf{A}^{\mathrm{T}} = \theta\dot{\mathbf{A}}^{\mathrm{T}} = -\theta\mathbf{E}\,, \tag{1.11}$$

$$\delta\mathbf{E} = -\delta\dot{\mathbf{A}}^{\mathrm{T}} = \theta\,\boldsymbol{\nabla}^{-2}\boldsymbol{\nabla}\times\ddot{\mathbf{A}}^{\mathrm{T}} = \theta\mathbf{H}\,, \tag{1.12}$$

where in the last line we used the equation of motion (1.1) rewritten in terms of the vector-potential:

$$\ddot{\mathbf{A}}^{\mathrm{T}} = \boldsymbol{\nabla}^2\mathbf{A}^{\mathrm{T}}\,. \tag{1.13}$$

Exponentiation of the infinitesimal transformations (1.11) and (1.12) gives indeed the electric-magnetic duality rotations (1.5).

According to the Noether's theorem, the symmetry of the action (1.8) with respect to the transformations (1.9) gives rise to a conservation law. The conserved charge is known as the helicity of the electromagnetic field [30, 34–39] and is given by the following expression:

$$S_0 = \int d^3x\, s_0 = \frac{1}{2}\int d^3x\, \left\{\dot{\mathbf{A}}^{\mathrm{T}}\!\cdot\boldsymbol{\nabla}^{-2}\boldsymbol{\nabla}\times\dot{\mathbf{A}}^{\mathrm{T}} - \mathbf{A}^{\mathrm{T}}\!\cdot\boldsymbol{\nabla}\times\mathbf{A}^{\mathrm{T}}\right\} =$$
$$\frac{1}{2}\int d^3x\Big\{\mathbf{H}\cdot\boldsymbol{\nabla}^{-2}\boldsymbol{\nabla}\times\mathbf{H} + \mathbf{E}\cdot\boldsymbol{\nabla}^{-2}\boldsymbol{\nabla}\times\mathbf{E}\Big\}\,. \tag{1.14}$$

The two terms in Eq. (1.14) have similar bilinear structure in terms of $\mathbf{E}$ and $\mathbf{H}$ and their time derivatives cancel each other due to the similarity of the structure of the Maxwell equations (1.1) and (1.2) for $\dot{\mathbf{E}}$ and $-\dot{\mathbf{H}}$. Introducing the dual-symmetrized spin angular momentum density of the electromagnetic field $\mathbf{s}$, one can rewrite the conservation law in terms of the conservation of the four-current $\partial_\mu s^\mu = 0$.

The transformation (1.9) as well as the corresponding conserved charge density (1.14) are given by spatially non-local expressions. It is often convenient to deal with local expressions instead, which can be achieved by introducing a second (electric) four-potential $B_\mu = (B_0, \mathbf{B})$. The latter four-potential is related to the physical fields as follows:

$$\mathbf{E} = -\boldsymbol{\nabla}\times\mathbf{B}\,, \quad \mathbf{H} = -\dot{\mathbf{B}} - \boldsymbol{\nabla}B_0\,. \tag{1.15}$$

The conserved charge (1.14) can then be rewritten in the following simple form:

$$S_0 = \int d^3x \, s_0 = -\frac{1}{2}\int d^3x \, (\mathbf{H}\cdot\mathbf{A} - \mathbf{E}\cdot\mathbf{B}) \;, \tag{1.16}$$

while the corresponding three-current density, which equals to the dual-symmetrized spin angular momentum density of the electromagnetic field, is:

$$\mathbf{s} = -\frac{1}{2}\left(\mathbf{E}\times\mathbf{A} + \mathbf{H}\times\mathbf{B}\right) \,. \tag{1.17}$$

While introducing a second four-potential for the electromagnetic field, one has to make sure that the number of degrees of freedom of the theory is not changed. The Lagrangian description of the theory of electromagnetic field in terms of two four-potentials was found by Zwanziger [40]. We will give a description of this theory in Chapter 4. A nice feature of this theory is that it allows one to introduce the coupling of electromagnetic field to magnetic currents $j_m^\mu = (\rho_m, \mathbf{j}_m)$, which is completely analogous to the coupling between the electromagnetic field and the electric currents $j_e^\mu = (\rho_e, \mathbf{j}_e)$:

$$\mathcal{L}_{\text{int}} \;=\; -j_e^\mu A_\mu - j_m^\mu B_\mu \,. \tag{1.18}$$

Indeed, the duality symmetry tells us that the existence of magnetic charges, and more generally dyons, is perfectly consistent with the structure of the classical theory, the Maxwell equations being extended as follows:

$$\boldsymbol{\nabla}\times\mathbf{H} - \dot{\mathbf{E}} = \mathbf{j}_e \,, \tag{1.19}$$

$$\boldsymbol{\nabla}\times\mathbf{E} + \dot{\mathbf{H}} = -\mathbf{j}_m, \tag{1.20}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H} = \rho_m \,, \tag{1.21}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \rho_e \,. \tag{1.22}$$

It turned out that the existence of magnetic monopoles and dyons is also fully consistent in the quantum theory. Moreover, not only is it fully consistent theoretically, but it also implies a restriction on the allowed charges of particles corresponding to the one observed in nature. Let us proceed to the next section, where we discuss the perfect fit of magnetic monopoles into the quantum theory as well as why the magnetic monopoles are probably the best motivated hypothetical particles beyond the Standard model.

1.2 Magnetic charges in quantum theory

In 1894, Pierre Curie noted that the existence of magnetic monopoles would be perfectly consistent with the classical theory of electromagnetism [41]. His motivation for considering magnetic charges stemmed from the desire to give magnetism an analogous status to the one of electricity. Now we know that this desire was never to come true: all magnetic phenomena observed so far can be perfectly described by the motion and interactions of electrically charged particles. However, the fundamental quantum theory which provided for such a successful electric description of the magnetic phenomena has also revolutionized our views on magnetic monopoles. Whereas Pierre Curie and his contemporaries regarded magnetic charges as a purely phenomenological construct, Paul Dirac argued in 1931 that the quantum theoretical formalism itself suggests their existence [21]. In particular, he pointed out that, in a consistent quantum theory, the change in the phase of a wave function around any closed curve must be the same modulo $2\pi n$ for all the wave functions, where n is an integer. In case $n = 0$ one recovers the standard gauge principle, introduced earlier by Hermann Weyl [42]. This is the case where there are only electrically but no magnetically charged particles in the theory and which is known to be realized in the Standard model of particle physics. The quantum theory itself however does not tell us any reason for why only the case $n = 0$ should be realized in nature, so that for a generic quantum mechanical model of particle physics one would expect that any n is allowed. As Dirac showed, the $n \neq 0$ case corresponds to a model with both electric and magnetic monopoles involved. Moreover, he found that the corresponding electric (q) and magnetic (g) gauge charges are necessarily related via the quantization condition $qg = 2\pi n$, which conveniently explains quantization of the electric charge observed in nature. Note that this is still the simplest explanation for the latter phenomenon to date, since it follows directly from the formalism of quantum mechanics given one does not put any additional restrictions on the quantum states of the theory.

One can wonder if the results which Dirac obtained in the framework of quantum mechanics can be derived in the more fundamental formalism of quantum field theory. Does a generic quantum field theory of gauge interactions predict quantization of the electric gauge charge? The answer is positive and it was found by Daniel Zwanziger [43, 44]. In order to address this question, Zwanziger had to revisit the very basis of any Poincaré-invariant quantum field theory – irreducible unitary representations of the Poincaré group under which particles of the theory can transform. One-particle irreducible representations were studied in 1939 by Eugene Wigner [45], however what Zwanziger found is that there exist also two-particle irreducible representations. The latter are parameterized by an angular momentum variable, which is quantized. The two-particle irreducible representations correspond to pairs of particles, each pair containing both an electric and a magnetic monopole. The quantized angular momentum variable for a given pair is proportional to the product of the corresponding electric and magnetic charges, hence one automatically recovers charge quantization. Even more, a quantum field theory of gauge interactions which allows for the two-particle irreducible representations was explicitly constructed in the case of the Abelian gauge group, both in Hamiltonian and Lagrangian formulations, in the works by Julian Schwinger and Daniel Zwanziger [40, 46]. The latter authors also showed that in an Abelian gauge theory a particle can have both electric and magnetic charges, i.e. it can be a dyon, in which case the quantization condition is generalized: $q_i g_j - q_j g_i = 2\pi n$ for any pair of particles (i, j) [29,30]. This means the right statement is not that the known charged particles have no magnetic charge, as it is usually claimed, but rather that their magnetic charges happen to be proportional to their electric charges. One of the essential features of both Schwinger and Zwanziger formulations is the introduction of two four-potentials for the description of the photon field, one of which couples to the electric and another to the magnetic currents. The advantage of the Zwanziger formulation is that it is based on a Lagrangian which preserves locality and which treats electric and magnetic variables symmetrically. We will review Zwanziger's Lagrangian formulation and take advantage of it later in this thesis.

So far we have been discussing generic magnetic monopoles, i.e. essentially the possibility of having two different kinds of gauge charges deeply rooted in the formalism of both quantum mechanics and quantum field theory. Let us now consider 't Hooft-Polyakov magnetic monopoles – topological solitons arising in the spontaneously broken symmetry phase of purely electric non-Abelian gauge theories, discovered by Gerardus 't Hooft and Alexander Polyakov in 1974 [47, 48]. These topological solitons were called magnetic monopoles because they create a monopole-like magnetic field far from their cores. Note however that they are more complicated constructs compared to their fundamental counterparts which we discussed earlier. It is important that the difference can reveal itself even at energies much lower than the inverse monopole core size, for which one would expect 't Hooft-Polyakov monopoles to behave similarly to fundamental ones due to the identical monopole-like configuration of the long-range magnetic field. The reason for this is that the instanton effects of the full non-Abelian theory are not suppressed on monopoles [49], thus contributing an extra rotor degree of freedom to the EFT describing infrared (IR) physics [50]. The best known phenomenological implication of this extra degree of freedom is the Rubakov-Callan effect [49, 51]: as Valery Rubakov and Curtis Callan showed in the beginning of 1980s, the Grand Unified Theory (GUT) magnetic monopole can induce proton decay at a strong interaction rate. If one is interested only in the processes which do not involve the rotor degree of freedom, the 't Hooft-Polyakov monopoles behave similarly to Dirac monopoles in the IR, i.e. their EFT is given by the above-mentioned Zwanziger theory [52]. Since 't Hooft-Polyakov monopoles (more generally, Julia-Zee dyons [53]) are an inevitable prediction of GUTs, they represent a well motivated case for the existence of magnetic monopoles (more generally, dyons). Explicit constructions show that such dyons can be bosonic as well as fermionic [54–56]. In this work, we will not adhere to any particular GUT, keeping our discussion as generic as possible.

From what has been already discussed, we see that the existence of magnetically charged matter would fit very well both in the structure of quantum mechanics, completing the gauge principle, and in the structure of relativistic quantum theory, completing the irreducible unitary representations of the Poincaré group realized in nature. The observed quantization of the electric charge would be explained. Moreover, the existence of magnetic monopoles would be a natural consequence of the unification of fundamental interactions, if the latter unification takes place. This is however not an exhaustive list of arguments in support of the existence of magnetically charged particles. Another strong motivation comes from our understanding of gravity: the consistency of a quantum theory of the latter was shown to imply a number of restrictions on the structure of admissible field theories. In particular, it was argued that there can be no global symmetries in a consistent theory which includes quantum gravity [22, 25–28] and that in such a theory, the charge spectrum is complete [22, 23, 27]. These conjectures were shown to imply the existence of magnetic monopoles with any magnetic charge allowed by the DSZ quantization condition [22, 23].

What are the ways to probe magnetic monopoles experimentally? Many direct search techniques have been proposed [57], such as searches for monopoles bound in matter, searches in cosmic rays, searches at colliders and, in the case of some GUT monopoles, searches via the catalysis of nucleon decay. None of the direct detection experiments have however yielded a conclusive signal so far. Moreover, it is quite difficult to derive accurate exclusion limits on the monopole mass due to the large theoretical uncertainty. In fact, the quantum field theory of magnetic monopoles is an essentially non-perturbative theory and there is still no reliable method to calculate cross-sections of the quantum field theory processes involving magnetic charges. The interpretation of the indirect searches for virtual monopoles at colliders [58] suffers from the same problem. In this thesis, we will point out a new possible signature for virtual magnetic monopoles, which has the advantage of being independent of any non-rigorous statements within the non-perturbative theory of magnetic charges. In particular, we will show that there is a certain modification of free electrodynamics which, if experimentally detected, would favor the existence of magnetic monopoles. A complication is that such a modification must involve a new hypothetical particle – the axion or, more generally, an ALP.

Till now, we considered only the magnetic monopoles charged under the $U(1)_{\mathrm{EM}}$ gauge group of electromagnetism. However, the electromagnetic field is not unique in this sense and any other $U(1)$ gauge field could be coupled to magnetic monopoles as well, which would be described in a complete analogy to the magnetic charges of electromagnetism. A particularly important example of such non-electromagnetic

magnetic charges can arise in the description of non-Abelian gauge theories, such as QCD. Indeed, in the Abelian 't Hooft gauges, the non-Abelian gauge symmetry of the Yang-Mills theory is reduced to the number of Abelian subgroups. The gauge fixing procedure introduces singularities which turn out to be magnetically charged under the surviving $U(1)$ gauge groups. Understanding the dynamics of these magnetic monopoles can shed some light on the dynamics of the Yang-Mills theory and QCD in particular. The dual superconductor picture of confinement suggests that these magnetic charges play a very important role in the IR. Studying magnetic charges can help one to understand better various phenomena in low energy QCD.

Considering magnetic monopoles which are charged under different gauge groups, not necessarily the $U(1)_{\mathrm{EM}}$ of electromagnetism, brings us to the next section, where we will discuss the general case for which the magnetic monopoles could be charged under non-Abelian gauge symmetries as well.

1.3 Non-Abelian magnetic monopoles

As it was discussed in the previous section, the consistency condition for a theory with both electric and magnetic currents is:

$$eg = 2\pi n\,, \;\; n \in \mathbb{Z}\,. \tag{1.23}$$

With the advent of the Standard model of particle physics, this condition was extended [59] to include all possible types of magnetic charges $\vec{Q}_{M_i}$ in the theory:

$$\exp\left(i\sum_{i=1}^{r}\vec{Q}_{M_i}\,\vec{\mathcal{H}}\right) = 1\,, \tag{1.24}$$

where $\mathcal{H}_k \equiv e_k \cdot h_k$ are Cartan generators of the Lie algebra $\mathcal{G}$ of rank r of the gauge group multiplied by the corresponding electric charges e_k. In case of a non-Abelian gauge theory, e_k are equal to the gauge couplings of the theory. For the Standard model, at low energies, we have $\mathcal{G} = \mathfrak{su}(3) \oplus \mathfrak{u}(1)$, which means that a magnetically charged particle has generally Abelian as well as non-Abelian magnetic charges. In this theory, the minimal magnetic charge corresponding to the electromagnetic subgroup, is still $g = 2\pi/e$, although there are now fractionally charged quarks. The reason is that quarks interact strongly with the monopole that has a color magnetic charge, compensating the would-be observable phase which results from the electromagnetic interaction. In particular, for a down-type quark the quantization condition (1.24) can be written as:

$$\xi g_s t_3 + \zeta\sqrt{3}\, g_s t_8 - \frac{e}{3}\, g = 2\pi \cdot \mathrm{diag}\,(n_1, n_2, n_3)\,, \tag{1.25}$$

where $\xi, \zeta \in \mathbb{R}$, $n_1, n_2, n_3 \in \mathbb{Z}$, $t_3 = \lambda_3/2$, $t_8 = \lambda_8/2$; λ_a are Gell-Mann matrices; g_s is the strong coupling. Coexistence of a monopole with charged leptons requires $eg = 2\pi m$, $m \in \mathbb{Z}$. Then Eq. (1.25) can be solved with respect to the coefficients ξ, ζ:

$$\xi = \frac{2\pi}{g_s} \cdot (2n_1 + n_3 + m)\,, \quad \zeta = -\frac{2\pi}{g_s} \cdot \left(n_3 + \frac{m}{3}\right)\,. \tag{1.26}$$

Note that the quantization condition for up-type quarks is satisfied automatically as long as Eq. (1.25) holds, for their electric charges differ by one elementary charge e from those of the down-type quarks. One can see that $m = 1$, which corresponds to the minimal Dirac magnetic charge, is still possible, although the magnetic monopole must carry non-Abelian magnetic charge as well. The latter is not necessary in the case $m = 3$ where viable solutions include $\xi = \zeta = 0$, which means vanishing non-Abelian magnetic charge.

Having discussed the Abelian magnetic monopoles and the generic quantization condition pertinent to both Abelian and non-Abelian magnetic charges, let us outline the status of the theory of the latter. First, we note that the condition (1.24) can be expressed in a simple way using the language of the Lie group theory. In particular, Goddard, Nuyts and Olive [60] showed that the condition (1.24) in a theory with gauge group G can be regarded as a one-to-one correspondence between the magnetic charges of monopoles in this theory and the weights of the Langlands dual gauge group G^V, which is now also known as the GNO group. For example, the gauge group of electromagnetism is self-dual in this sense: $(U(1))^V = U(1)$; and the GNO group corresponding to the gauge theory of QCD can be inferred from the following identity: $(SU(3)/\mathbb{Z}_3)^V = SU(3)$. Based on the derived relation between magnetic charges and the dual gauge group G^V, which is completely analogous to the relation between electric charges and the gauge group G, Goddard, Nuyts and Olive suggested that magnetic monopoles of a gauge theory with a group G generally transform in the representations of the group G^V. The above conjecture, known as the GNO conjecture, obviously holds in the case of the Abelian group $G = U(1)$, for which the Zwanziger theory mentioned earlier in this Chapter can be constructed. The GNO conjecture for the non-Abelian monopoles, in its stronger form known as the Montonen-Olive conjecture [61], has recently been proven by Kapustin and Witten [62] for a twisted $\mathcal{N} = 4$ supersymmetric Yang-Mills (YM) theory. In this thesis, we assume that the GNO conjecture holds for the gauge theory of QCD as well, inspired by the findings of Hong-Mo, Faridani and Tsun [63] that the classical (nonsupersymmetric) YM equations possess a generalized dual symmetry similar to the electric-magnetic $\mathbb{Z}_2$ symmetry of the Zwanziger theory mentioned above. Let us also note, that although non-Abelian magnetic charges are often introduced as emergent from spontaneous breaking of some larger gauge symmetry, the results by Goddard, Nuyts and Olive do not depend on such a construction and can be as well stated for generic magnetic monopoles defined in the fiber bundle framework of Wu and Yang [64].

This Chapter is partly written based on the publications [1, 2] of the author of this thesis.

Chapter 2

Axions and axion-like particles

2.1 Strong CP problem

In QCD, configurations of the pure gauge, i.e. vacuum, gluon fields A^a_μ, where $a = 1..8$, can be classified by the winding number $\mathcal{K} \in \mathbb{Z}$, which is given by the following expression:

$$\mathcal{K} = \frac{g^2}{32\pi^2} \int K_0(x)\, d^3x\,, \tag{2.1}$$

where $K_0(x)$ is the Chern-Simons charge density, which is the zeroth component of the Chern-Simons four-current:

$$K^\mu = 2\epsilon^{\mu\nu\rho\lambda} \left(A^a_\nu\, \partial_\rho A^a_\lambda + \frac{g_s}{3}\, f^{abc} A^a_\nu A^b_\rho A^c_\lambda \right)\,, \tag{2.2}$$

where g_s is the strong coupling constant, $\epsilon^{0123} = 1$, and f^{abc} are the structure constants of $\mathfrak{su}(3)$. Such classification of the vacuum configurations implies that the space of pure gauge gluon fields is topologically a circle, with $\mathcal{K}$ indicating the number of times a given configuration winds around this circle. Invariance of the theory with respect to "large" gauge transformations, i.e. the gauge transformations that can change $\mathcal{K}$, as well as independently the cluster decomposition principle are well-known to yield the following structure of the quantum theory vacuum state:

$$|\theta\rangle = \sum_{n\in\mathbb{Z}} e^{in\theta}\, |n\rangle\,, \tag{2.3}$$

where $|n\rangle$ are the formal vacuum states corresponding to the classical configurations with definite $\mathcal{K}$, and $\theta \in \big[0,\, 2\pi\big)$.

Note that θ is an additional parameter of the theory, the value of which can influence various physical processes. To be more precise, this parameter enters the QCD Lagrangian in the following way:

$$\mathcal{L}_\theta \;=\; (\theta - \mathrm{Arg}\det M)\, \frac{g_s^2}{32\pi^2}\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}\,, \tag{2.4}$$

where M is the quark mass matrix,

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \tag{2.5}$$

is the QCD field strength tensor, and for any tensor $B_{\mu\nu}$ its Hodge dual is defined as $B^d_{\mu\nu} = \epsilon_{\mu\nu\lambda\rho} B^{\lambda\rho}/2$. Let us introduce the notation $\bar{\theta} = \theta - \mathrm{Arg}\det M$ for simplicity. Although the term (2.4) is a total derivative proportional to $\partial_\mu K^\mu$, it yields in general a non-vanishing contribution to the action due to the instanton processes, i.e. non-triviality of the boundary integral. Moreover, the term (2.4) violates the CP symmetry which is otherwise conserved in QCD. This violation of CP yields generally non-vanishing electric dipole moment of neutron proportional to $\bar{\theta}$ [65]:

$$d_n = 2.4\,(1.0) \cdot 10^{-16}\,\bar{\theta}\, e{\cdot}\mathrm{cm}\,. \tag{2.6}$$

The latter theoretically calculated value of the dipole moment is to be compared with the experimental measurement, which yields [5]:

$$|d_n| < 1.8 \cdot 10^{-26} e{\cdot}\mathrm{cm}\,. \tag{2.7}$$

The comparison implies $|\bar{\theta}| \lesssim 10^{-10}$, which means that the CP symmetry is actually conserved with a very good accuracy. The question of why QCD is symmetric under time reversal constitutes the core of the so-called strong CP problem. To get rid of the extreme fine-tuning of the CP-violating parameter of the theory, one is tempted to hypothesize a new model where the latter parameter is constrained to be practically zero due to the properties of the model.

2.2 Axions

One of the ideas proposed to explain the absence of CP violation in QCD is to drive the $\bar{\theta}$-parameter to zero dynamically by introducing a new pseudoscalar particle called axion, which is a pseudo Nambu-Goldstone boson associated to spontaneous breaking of anomalous Peccei-Quinn (PQ) symmetry [6–9]. The great advantage of this mechanism is that the introduction of the axion can naturally solve not only the strong CP problem, but also a much more pressing problem of missing mass in the Universe, i.e. the axion is a perfect candidate for dark matter [10–12], as we will outline later in this Chapter.

The PQ mechanism yields the following axion coupling to gluons:

$$\mathcal{L} \ = \ \left(\frac{a}{f_a} - \bar{\theta}\right) \frac{g_s^2}{32\pi^2} \, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu} \,, \tag{2.8}$$

where a is the axion field and f_a is the axion decay constant – a parameter of the axion models directly related to the high energy scale of the PQ symmetry breaking v_a. Due to the instanton processes, the latter coupling does not have the symmetry with respect to arbitrary shifts of the axion field $a \to a + C$ for $C \in \mathbb{R}$, which means that such interaction with topologically non-trivial gluon field fluctuations generates a non-flat contribution to the axion potential. It is straightforward to show that the minimum of this potential is CP-conserving [66]. Thus the axion dynamically relaxes the value of $\bar{\theta}_{\text{eff}} \equiv \langle a \rangle / f_a - \bar{\theta}$ to zero solving the strong CP problem.

The axion potential can be calculated explicitly using the dilute instanton gas approximation at high temperatures ($T > 1$ GeV), chiral perturbation theory at low temperatures ($T \lesssim \Lambda_{\text{QCD}} \simeq 0.2$ GeV) or lattice QCD methods [67] in the intermediate temperature region. In particular, at low temperatures, below the QCD scale Λ_{QCD}, the chiral perturbation theory gives the following result for the axion mass:

$$m_a^2 \ \simeq \ \frac{f_\pi^2 m_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2} \,, \tag{2.9}$$

so that it is inversely proportional to the axion decay constant f_a. Note that m_π is the pion mass, f_π is the pion decay constant, and m_u and m_d are the up- and down-quark masses. The most precise computation to date [68] yields:

$$m_a \; = \; 5.691(51)\,\mu\text{eV}\,(10^{12}\text{GeV}/f_a)\,. \tag{2.10}$$

The expression for the axion mass in Eq. (2.9) is a robust prediction of the PQ mechanism. However, one should keep in mind that some extensions of the mechanism predict axions that are parametrically lighter [69–71], or heavier [72–87].

Details of particular axion models can vary. The first axion model proposed, which is the PQWW model [6–9], identified the axion field with a phase of the Higgs in a two-Higgs-doublet model (2HDM) and was ruled out experimentally soon after the proposal. Then the KSVZ [19, 20] and DFSZ [88, 89] axion models were constructed, which were called invisible, because interactions of the corresponding axion particles with the Standard model are very faint. Such faint they are that even after four decades of exploration the parameter space of these models is still largely terra incognita. Appeal of the invisible models is their simplicity: the DFSZ model exploits the 2HDM just as in the case of the PQWW axion but the axion is now identified with the phase of a new Standard-model-singlet complex scalar field which couples to the Higgses at high energies; while the KSVZ model exploits coupling of a new Standard-model-singlet complex scalar field, the phase of which is identified with the axion, to a new heavy quark. Over the years, there have been attempts of constructing axion models which would be more "visible" than the DFSZ and KSVZ models, however it always turned out that simplicity was to be sacrificed. For example, in the clockwork axion model [90], in order to get an enhancement of the axion-photon coupling by six orders of magnitude compared to the KSVZ model, one has to introduce at least 13 new scalar fields. A similar enhancement by six orders of magnitude in all couplings to the Stadard model particles is achievable in the $\mathbb{Z}_\mathcal{N}$ axion model [69, 70], but it requires $\mathcal{N} = 45$ copies of the SM. Although quite non-minimal from the theory side, such an enhancement would allow one to explain some uneven astrophysical observations concerning cooling of the horizontal branch stars in globular clusters [33] and anomalous TeV-transparency of the Universe [31, 32], not to mention that such photophilic axions can be well probed experimentally in the nearest future.

The main properties of the QCD axion, such as its lightness and feeble derivative or anomalous interactions, stem from its pseudo Nambu-Goldstone nature. This informs us about the low energy processes that would be affected by the existence of this hypothetical particle and thus which detection strategies we should pursue. Since our detectors are particularly sensitive to the electromagnetic (EM) interactions, most of the axion experiments take advantage of the axion-photon coupling,

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4}\, aF^{\mu\nu}F^{d}_{\mu\nu} = g_{a\gamma\gamma} a\, \mathbf{E}\cdot\mathbf{H}\,, \tag{2.11}$$

where $F^{\mu\nu}$ is the field strength tensor of Quantum Electrodynamics (QED) and $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields. With this interaction included the Maxwell equations get modified as follows [91],

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \rho - g_{a\gamma\gamma}\mathbf{H}\cdot\boldsymbol{\nabla} a\,, \tag{2.12}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H} = 0\,, \tag{2.13}$$

$$\boldsymbol{\nabla}\times\mathbf{E} = -\frac{\partial\mathbf{H}}{\partial t}\,, \tag{2.14}$$

$$\boldsymbol{\nabla}\times\mathbf{H} = \frac{\partial\mathbf{E}}{\partial t} + \mathbf{J} - g_{a\gamma\gamma}\left(\mathbf{E}\times\boldsymbol{\nabla} a - \frac{\partial a}{\partial t}\mathbf{H}\right)\,. \tag{2.15}$$

In general the axion-photon coupling has a contribution which depends on the fermionic content of the specific model and another contribution originating from the axion mixing with neutral mesons [92],

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)\,. \tag{2.16}$$

Here, E and N are the EM and QCD anomaly coefficients and $\alpha = e^2/(4\pi)$ is the fine structure constant. The original KSVZ model predicts $E/N = 0$ while the original DFSZ model predicts $E/N = 8/3$. Due to their simplicity, the latter models are normally taken as benchmarks.

Although the axion-photon coupling is the most popular one to search for in experiments, the Lagrangian (2.11) is actually not specific to axions. The same kind of coupling is shared by any ALP, which need not solve the strong CP problem. To discover the QCD axion, one has to probe its coupling to gluons (2.8). At low energies, the latter coupling induces interactions of axions with the electric dipole moments of nucleons. The corresponding Lagrangian is,

$$\mathcal{L}_{aN\gamma} = -\frac{i}{2} g_{aN\gamma} a \bar{\Psi}_N \sigma_{\mu\nu} \gamma_5 \Psi_N F^{\mu\nu} \,, \tag{2.17}$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$ and the nucleon N can be the neutron n or the proton p. The coupling constants $g_{aN\gamma}$ corresponding to each of the nucleons depend only on the PQ scale f_a and are given by the following expression [57]:

$$g_{ap\gamma} = -g_{an\gamma} = -(3.7 \pm 1.5) \times 10^{-3} \left(\frac{1}{f_a}\right) \frac{1}{\text{GeV}} \,. \tag{2.18}$$

Axions are also predicted to couple to leptons at tree level in a wide range of models, such as those based on Grand Unified Theories (GUTs). Even though it is challenging to probe these interactions with current experiments, a promising avenue is to study the influence they exhibit on various astrophysical processes, where the interactions of axions with electrons can play a crucial role. The interaction Lagrangian with leptons reads,

$$\mathcal{L}_{a\ell\ell} = \frac{C_\ell}{2 f_a} \, \partial_\mu a \, \bar{\Psi}_\ell \gamma^\mu \gamma_5 \Psi_\ell \,, \tag{2.19}$$

where the coefficient C_ℓ depends on the particular lepton flavor under consideration. Note that in hadronic axion models, such as KSVZ, the couplings to leptons are suppressed, since they are generated only at the loop level through the axion-photon coupling (2.16), whereas DFSZ constructions predict tree-level couplings to leptons.

2.3 Axion-like particles

ALPs are the particles which need not solve the strong CP problem, i.e. couple to gluons via the interaction term (2.8), but which have couplings with the structure similar to the structure of the couplings discussed in the previous section. From the EFT perspective, the similarity of the structure is ensured by these particles being pseudoscalars and having the shift symmetry $a \to a + 2\pi n v_a$, $n \in \mathbb{Z}$, where v_a is some high energy scale. In field-theoretic models, ALPs emerge as Nambu-Goldstone bosons of spontaneously broken global $U(1)$ symmetries. In string theory, ALPs emerge as Kaluza-Klein zero modes of antisymmetric tensor fields living in ten dimensions [93–95]. Apart from string theory ALPs, examples of ALPs are the majoron, which arises as Nambu-Goldstone boson of the broken $U(1)$ lepton number symmetry [96,97], or the familon, which is associated to the breaking of global family symmetries [98–100].

For ALPs, the relation between the mass and the decay constant (2.9) need not be satisfied. Moreover, explicit expressions for the couplings, such as (2.16) or (2.18), can in general be different from the axion case discussed in the previous section. Note that the structure of the axion Maxwell equations (2.12)-(2.15) is unchanged.

2.4 Axions as dark matter candidates

In this section, we will consider axions as dark matter candidates. First of all, a good dark matter candidate should be stable over the cosmological timescales. Axions have a decay channel into two photons due to the coupling (2.16). Assuming $E/N = 0$ for certainty and taking advantage of Eq. (2.9), one obtains the following expression for the axion decay time:

$$\tau_a = \frac{1}{\Gamma_{a\to\gamma\gamma}} = \frac{64\pi}{g_{a\gamma\gamma}^2 m_a^3} \simeq \frac{64\pi^3 m_\pi^2 f_\pi^2}{\alpha^2 m_a^5}\,, \tag{2.20}$$

where $\Gamma_{a\to\gamma\gamma}$ is the corresponding decay rate. The decay time being large on cosmological timescales then implies $m_a < O\,(10)$ eV. This upper bound on the mass means that we should consider a non-thermal mechanism of dark matter production, otherwise it would be hard to produce the required dark matter abundance and keep the dark matter cold.

It turns out that in the case of axions, non-thermal mechanisms for dark matter production do exist. Let us consider the evolution of the axion field in the expanding Universe [10–12]. There are two important epoques during this evolution: the PQ phase transition and the QCD crossover. At the PQ phase transition, the PQ symmetry $U(1)_{\rm PQ}$ becomes spontaneously broken and the axion field takes some random value in the vacuum manifold $a_i \in [0,\, 2\pi v_a)$, these values being uncorrelated between different Hubble patches. At the QCD crossover, the PQ symmetry $U(1)_{\rm PQ}$

becomes explicitly broken by instanton effects, which yield a non-flat potential for the axion field. In the dilute instanton gas approximation, this potential is given by the following expression:

$$V(a,T) = \chi(T)\cdot\left[1-\cos\left(\frac{a}{f_a}\right)\right]\,, \tag{2.21}$$

where $\chi(T) = f_a^2 m_a^2(T)$ is the topological susceptibility of QCD, and T is the temperature of the early Universe thermal bath.

Let us approximate the potential assuming that the axion field is close to the minimum of its potential: $V(a,T) = m_a^2 a^2/2$. In this case, the equation of motion for the axion field in the expanding Universe is:

$$\ddot{a} + 3H(T)\,\dot{a} + m_a^2(T)\,a = 0\,, \tag{2.22}$$

where $H(T)$ is the Hubble parameter. The equation (2.22) is analogous to the equation for a harmonic oscillator with frequency m_a and friction $3H$. Before the QCD crossover, the friction term obviously dominates, and the axion field is frozen at its initial value $a_i = \theta_i f_a$, where $\theta \in [0,\, 2\pi)$ is usually called the misalignment angle. During the crossover, the axion mass starts to increase and given the decrease of the Hubble parameter with time, at some point $m_a(T_{\text{roll}}) \sim 3H(T_{\text{roll}})$ and the oscillations start, where T_{roll} is the corresponding temperature. Knowing the dependence of the topological susceptibility on temperature, which can be found from lattice QCD simulations, one can determine the value of T_{roll} as a function of zero-temperature axion mass $m_a \equiv m_a(0)$. The mean energy density of the oscillating axion field is:

$$\rho_a(T_{\text{roll}}) = \left\langle \frac{\dot{a}^2}{2} + \frac{m_a^2(T_{\text{roll}})a^2}{2} \right\rangle = m_a^2(T_{\text{roll}})\, f_a^2\, \langle\theta_i^2\rangle\,. \tag{2.23}$$

Using $n_a(T)/s(T) = \text{const}$, where $s(T)$ is the entropy density of the early Universe thermal bath, and the relation between T_{roll} and m_a, one can obtain the expression for the present day energy density of the axion field normalized to the critical energy density $\rho_c = 3M_{pl}^2 H_0^2/8\pi$, where M_{pl} is the Planck mass and $H_0 \equiv 100h\,\text{km/s/Mpc}$

is the Hubble constant:

$$\Omega_a h^2 \simeq 0.12 \left(\frac{28\ \mu\text{eV}}{m_a}\right)^{7/6} \frac{\langle\theta_i^2\rangle}{4.62}\,, \tag{2.24}$$

while the observed cold dark matter abundance satisfies

$$\Omega_{\text{CDM}} h^2 = 0.12\,. \tag{2.25}$$

The mechanism of the generation of the axion dark matter which we discussed in the previous paragraph is called the misalignment mechanism. It is easy to see that the oscillating axion field has the equation of state corresponding to the cold dark matter, as the pressure of such dark matter is zero:

$$p_a(T) = \left\langle \frac{\dot{a}^2}{2} - \frac{m_a^2(T) a^2}{2} \right\rangle = 0\,. \tag{2.26}$$

To determine the masses of axions which could allow for $\Omega_a = \Omega_{\text{CDM}}$, one has to find the value of the average square of the initial misalignment angle $\langle\theta_i^2\rangle$. The latter value depends on the point in the cosmological history where the PQ phase transition occurs. If the PQ symmetry is broken during inflation and is never restored, the value of the initial misalignment angle is homogeneous and completely random. In this case, which is normally called the pre-inflationary scenario, $\langle\theta_i^2\rangle$ in Eq. (2.24) is an arbitrary number between 0 and $(2\pi)^2$, which means that axions could comprise dark matter for any mass $m_a \lesssim 10^{-4}$ eV. In the other case, where the PQ symmetry is broken spontaneously after inflation and which is normally called the post-inflationary scenario, the Hubble patches with the different values of θ_i reenter the horizon later in the history of the Universe, and one has to average over the random uniformly distributed values of θ_i. Taking into account the unharmonicities of the potential (2.21), one obtains $\langle\theta_i^2\rangle = 4.62$. Naively, this could mean that the mass of the dark matter axions in the post-inflationary scenario is fixed, however the situation is not so simple. Indeed, in the latter scenario, due to the difference in the initial misalignment angles between different regions, there arise topological defects, which emit axions in the process of their evolution and thus make an additional contribution to the axion dark matter abundance. The evolution of these topological defects is an essentially non-linear phenomenon which is quite challenging to study even numerically. We will not deal with the post-inflationary scenario in this thesis.

This Chapter is partly written based on the publication [4] of the author of this thesis.

Chapter 3

Structure of the axion-photon coupling – need for revision

3.1 Previous arguments supporting quantization

As it was discussed in the previous Chapter, the axion is the pseudo Nambu-Goldstone boson of the spontaneously broken $U(1)_{\rm PQ}$ Peccei-Quinn (PQ) symmetry [6–9]. Since the PQ symmetry is anomalous, the low energy axion Lagrangian generally contains non-derivative couplings of the axion to CP-odd combinations of the gauge fields of the low energy Standard model:

$$\mathcal{L}_a \supset -\frac{1}{4}\, g_{a\gamma\gamma}\, a\, F^{\mu\nu}F^d_{\mu\nu} + \frac{a g_s^2}{32\pi^2 f_a}\, G^{a\,\mu\nu}G^{d\,a}_{\mu\nu}\,, \tag{3.1}$$

where a is the axion field, $F_{\mu\nu}$ ($G_{\mu\nu}$) is the field strength tensor of the QED (QCD) gauge field, $g_{a\gamma\gamma}$ is the axion-photon coupling, g_s is the coupling constant of QCD, f_a is the axion decay constant; summation over the index $a = 1 \ldots 8$ for gluons is implied and for any tensor $B_{\mu\nu}$ its Hodge dual is defined as $B^d_{\mu\nu} = \epsilon_{\mu\nu\lambda\rho}B^{\lambda\rho}/2$, where $\epsilon^{0123} = 1$.

Both axion-photon and axion-gluon couplings are probed in various experiments. Interactions of the axion with photons are particularly well constrained. In fact, a large parameter region on the $(g_{a\gamma\gamma}, f_a)$ plane has been excluded already and there are a lot of new different experiments planned which are going to explore the axion-photon coupling further in the nearest future. A natural question then arises: where should we look in the first place? What are the best motivated values for $g_{a\gamma\gamma}$ and f_a from the theoretical viewpoint? In the last years, it has been claimed by many authors that this question can be answered by considering a consistency condition for the axion EFT [101–103]. The latter condition takes advantage of the fact that the axion is essentially an angular variable with a period $2\pi v_a$, where v_a is the PQ symmetry breaking scale, so that the effective low energy action must be symmetric under the following shifts:

$$a \to a + 2\pi v_a n, \; n \in \mathbb{Z} . \tag{3.2}$$

Let us review the argument of Refs. [101,102]. First, since the topological charge of QCD,

$$Q_t = \frac{g_s^2}{32\pi^2} \int d^4x \, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu} , \tag{3.3}$$

is an integer, symmetry of the axion-gluon interaction under the transformation (3.2) requires

$$f_a = v_a / N_{\rm DW}, \;\; N_{\rm DW} \in \mathbb{Z} , \tag{3.4}$$

in which case under the transformation (3.2), the action changes by $2\pi k$, $k \in \mathbb{Z}$, and the path integral is unchanged. Second, since one cannot generically exclude the presence of magnetic monopoles at high energies, it has been claimed that the Witten effect [104] makes the term

$$\theta_{\rm em} \cdot \frac{e^2}{16\pi^2} \int d^4x \, F^{\mu\nu} F^d_{\mu\nu} , \tag{3.5}$$

which enters the QED action, physically relevant. The parameter $\theta_{\rm em}$ is cyclic with a period that depends on the global structure of the Standard model gauge group, see Ref. [105]. For the following argument, it is important that the period of $\theta_{\rm em}$ is always an integer multiple of 2π. Identification of the two similar structures in Eqs. (3.1) and (3.5) restricts the values of the axion-photon coupling $g_{a\gamma\gamma}$ due to the periodicity of the axion field Eq. (3.2):

$$g_{a\gamma\gamma} = \frac{E}{N} \cdot \frac{e^2}{8\pi^2 f_a} , \tag{3.6}$$

where $N = N_{\rm DW}/2$, $E \in \mathbb{Z}$, and we used Eq. (3.4) in order to relate $g_{a\gamma\gamma}$ to f_a. The authors of Refs. [101, 102] then proceed to argue that any contribution to $g_{a\gamma\gamma}$ that is not quantized, i.e. which does not satisfy Eq. (3.6), must be proportional to the mass of the axion squared and can be significantly larger than the order of magnitude of the quantized contribution $e^2/(8\pi^2 f_a)$ only in non-minimal models which introduce new unnecessary energy scales and/or particles.

Let us highlight the step in the derivation where one identifies the two similar $F^{\mu\nu}F^d_{\mu\nu}$ structures in Eqs. (3.1) and (3.5) in the presence of magnetic monopoles. Physically, it is equivalent to stating that electromagnetic interactions between axions and magnetic monopoles are necessarily induced by the Witten effect. What we found is that the latter statement has actually never been consistently derived; moreover, it does not necessarily hold. Before we explain the loophole that has been overlooked, let us briefly review the Witten effect and its low energy description since they are central to the following discussion.

3.2 Witten effect and its low energy description

The Witten effect is an effect in a theory with spontaneously broken non-Abelian gauge symmetry derived in 1979 by Edward Witten [104]. The latter author showed that if the full non-Abelian $SO(3)$ theory with coupling constant $\bar{g}$ and field strength $G_{\mu\nu}$ has a CP-violating parameter θ in the Lagrangian:

$$\mathcal{L}_\theta = -\frac{\theta\bar{g}^2}{32\pi^2}\, G^{a\,\mu\nu}G^{d\,a}_{\mu\nu}\,, \;\; \theta \neq 2\pi n\,, \tag{3.7}$$

where $n \in \mathbb{Z}$, then 't Hooft-Polyakov monopoles of the broken phase of such a theory get an additional contribution δq to their electric charges $q = m\bar{g} + \delta q$, $m \in \mathbb{Z}$:

$$\delta q = -\frac{\theta\bar{g}}{2\pi}\, k, \;\; k \in \mathbb{Z}\,, \tag{3.8}$$

which is not quantized in units of $\bar{g}$, but which is proportional to the CP-violating parameter θ. One can wonder if there exists a low energy Lagrangian, which would account for the Witten effect, i.e. which would ensure that any monopole-like magnetic field comes together with the monopole-like electric field of strength corresponding to the non-quantized electric charge δq from Eq. (3.8). One would normally write such a Lagrangian in the following form[1]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta e^2}{32\pi^2}F^{\mu\nu}F^d_{\mu\nu}\,, \tag{3.9}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, A_ν is the electromagnetic four-potential and e is the low energy electric gauge coupling. The non-trivial topology of the A_ν field in the vicinity of the magnetic monopole ensures that the second term of Eq. (3.9) has physical importance. Indeed, if one imposes

$$\partial_\mu F^{d\,\mu\nu} = \frac{4\pi k}{e} j^\nu_m\,, \;\; k \in \mathbb{Z}\,, \tag{3.10}$$

where j^ν_m is the current of magnetic monopoles and $4\pi/e$ is the minimal allowed charge of the $SO(3)$ 't Hooft-Polyakov monopole, then one obtains the following equations of motion corresponding to the Lagrangian (3.9):

$$\partial_\mu F^{\mu\nu} = -\frac{\theta e}{2\pi}\, k\, j^\nu_m\,. \tag{3.11}$$

Comparing the latter equation with the expression (3.8) for the non-quantized contribution to the electric charge of the 't Hooft-Polyakov monopole, we see that the low energy Lagrangian (3.9) does allow us to account for the Witten effect.

3.3 Loophole in the previous arguments

Suppose now that we know nothing about the high energy non-Abelian theory and want to justify the Witten effect merely by means of the low energy EFT. In this EFT, we introduce the topological term, which coincides with the second term of the low energy Lagrangian Eq. (3.9). The coefficient $\theta e^2/(32\pi^2)$ is fixed by topology. Derivation of the Witten effect then proceeds on the lines reviewed in the previous section. Let us however stress an essential assumption that enters such

[1] Note the difference in the coefficients of the θ-terms in Eqs. (3.5) and (3.9). The reason is that defining the Abelian theory of Eq. (3.9) we made a definite choice for the underlying non-Abelian gauge group $SO(3) = SU(2)/\mathbb{Z}_2$. The spectrum of line operators in the non-Abelian theory with this gauge group fixes the period of θ to be 4π [105], so that an extra factor of 2 accumulates in the denominator of the corresponding θ-term.

considerations. Namely, one assumes that the electromagnetic field is quantized in terms of the four-potential A_μ in the presence of magnetic monopoles. The latter quantization procedure is only valid if all the magnetic monopoles of the theory are treated as merely quasi-classical external sources. Only then is the second term in Eq. (3.9) topological. This description of nature, where the magnetic monopoles are non-dynamical, is incomplete. An exhaustive quantization of the system with magnetic charges has to follow the works by Schwinger and Zwanziger [40, 46]. Note that the particular case of GUT monopoles is not an exception [52].

The interactions between axions and magnetic monopoles have never been derived from a high energy theory, such as a GUT, but only from the low energy axion EFT, namely from the first term in the Lagrangian (3.1) – see Ref. [24], which is followed by all other works on the subject. Such derivation suffers from the same supposition as that discussed in the previous paragraph. Quantum dynamical effects of the monopoles are neglected. This means for example that the loop effects of magnetic charges cannot be reliably accounted for. Such truly quantum field-theoretical effects involving magnetic monopoles can however play a very important role in low energy physics: in particular, as we show in the following Chapters, they dominate the axion-photon coupling and can lead to novel experimental signatures for axions. Thus, the conventional derivation of the axion-monopole Witten effect induced interactions from the low energy EFT (3.1) has a limited range of applicability.

What are the ultraviolet (UV) models in which it is possible to prove that the interactions between axions and magnetic monopoles are the ones induced by the Witten effect? We can find the answer to this question by deliberately preserving the full analogy to the case of constant θ. Note that the second term on the right-hand side of the Eq. (3.9) is derived from the θ-term (3.7) of the corresponding high energy Lagrangian, either by requiring that the Witten effect is reproduced, as we did in sec. 3.2, or by simply neglecting the contribution from the heavy gauge fields in the symmetry broken phase. The latter consideration allows us to substitute θ with the axion field, but only *both* at low and high energy scales:

$$\mathcal{L}_a^{\mathrm{UV}} = -\frac{a\bar{g}^2}{32\pi^2 f_a}\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu} \quad \Rightarrow \quad \mathcal{L}_a^{\mathrm{IR}} = -\frac{ae^2}{32\pi^2 f_a} F^{\mu\nu} F^d_{\mu\nu}\,, \tag{3.12}$$

where the notations are adopted from sec. 3.2. The logic outlined in (3.12) is possible only in a theory where the PQ scale v_a is much larger than the GUT scale: $v_a \gg v_{\mathrm{GUT}}$. It is thus not applicable to the vast majority of axion GUT models (starting from the model of Ref. [106]), where one normally identifies some part of the PQ field with the scalar Higgs fields which govern the breaking of the unifying gauge group.

In closing, we see that there is really no robust theoretical argument showing that the electromagnetic interactions between axions and magnetic monopoles are necessarily induced by the Witten effect. Thus, the argument for the axion-photon coupling quantization from sec. 3.1 is inconclusive. But then, how can one infer the structure of the axion-photon coupling relevant for experimental searches? The answer to this question follows straightforwardly from the nature of the loophole discussed: we have to consider a proper quantum field theory with magnetic charges, e.g. Zwanziger theory. Let us proceed to the next Chapter, where we will outline the main features of the Zwanziger theory and develop an EFT approach to it.

This Chapter is written based on the publication [1] of the author of this thesis.

Chapter 4

Quantum electromagnetodynamics

4.1 Zwanziger theory

Quantum electromagnetodynamics (QEMD) is the quantum field theory describing interactions of electric charges, magnetic charges and photons. Local-Lagrangian QEMD was constructed by Zwanziger [30]. In the latter theory, the photon is described by two four-potentials A_μ and B_μ, which are regular everywhere. The gauge group $U(1)$ of electrodynamics is substituted with the new one $U(1)_{\mathrm{E}} \times U(1)_{\mathrm{M}}$, where the electric (E) and magnetic (M) factors act in the standard way on A_μ and B_μ, respectively. One fixes the gauge freedom and restricts the physical states by requiring that they be vacuum states with respect to the free scalar fields[1] $(n \cdot A)$ and $(n \cdot B)$, where $n^\mu = (0, \vec{n})$ is an arbitrary fixed spatial vector. The right number of degrees of freedom of the photon is preserved due to the special form of the equal-time commutators between the potentials:

$$[A^\mu(t,\vec{x}), B^\nu(t,\vec{y})] = i\epsilon^{\mu\nu}{}_{\rho 0}\, n^\rho\, (n\cdot\partial)^{-1}(\vec{x}-\vec{y}) \ , \tag{4.1}$$

$$[A^\mu(t,\vec{x}), A^\nu(t,\vec{y})] = [B^\mu(t,\vec{x})\, , B^\nu(t,\vec{y})] = -i\, ({g_0}^\mu n^\nu + {g_0}^\nu n^\mu)\, (n\cdot\partial)^{-1}(\vec{x}-\vec{y}) \ , \tag{4.2}$$

[1]We use the following simplified notations: $a \cdot b = a_\mu b^\mu$.

where $(n\cdot\partial)^{-1}(\vec{x}-\vec{y})$ is the kernel of the integral operator $(n\cdot\partial)^{-1}$ satisfying $n\cdot\partial\,(n\cdot\partial)^{-1}(\vec{x})=\delta(\vec{x})$:

$$(n\cdot\partial)^{-1}(\vec{x})=\frac{1}{2}\int_{-\infty}^{\infty}\delta^3\left(\vec{x}-\vec{n}s\right)\varepsilon(s)ds\,, \tag{4.3}$$

$\varepsilon(s)$ is the signum function. The commutation relations Eqs. (4.1), (4.2) thus make the theory essentially different from the simple case of the gauge theory with two electric $U(1)$ gauge groups, used e.g. in models with a hidden photon. The two four-potentials are not independent and their relation absorbs the non-locality which is inherent to any quantum field theory with both electric and magnetic charges. The Lagrangian of the Zwanziger theory is local and is given by the expression[2]:

$$\begin{aligned}\mathcal{L} \;=\; \frac{1}{2n^2}\left\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \;-\right.\\ \left.[n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2\right\} \;-\; j_e\cdot A \;-\; j_m\cdot B \;+\; \mathcal{L}_G\,,\end{aligned} \tag{4.4}$$

where j_e and j_m are electric and magnetic currents, respectively, and $\mathcal{L}_G$ is the gauge-fixing part:

$$\mathcal{L}_G \;=\; \frac{1}{2n^2}\left\{[\partial\,(n\cdot A)]^2+[\partial\,(n\cdot B)]^2\right\}\,. \tag{4.5}$$

The Lagrangian (4.4) is invariant under those $SO(2)$ transformations which rotate the two-vectors (A,B) and (j_e,j_m) simultaneously. This symmetry ensures that the absolute directions in the gauge charge space (q,g) are not observable. Another important symmetry is however not manifest in the Lagrangian (4.4) – the Lorentz-invariance seems to be lost. This appearance is in fact deceptive. The reason is intimately connected to the non-perturbativity of the theory and to the DSZ quantization condition. It was shown in Refs. [107,108] that, after all the quantum corrections are properly accounted for, the dependence on the vector n_μ in the action S factorizes into a linking number L_n, which is an integer, multiplied by the combination of charges entering the quantization condition $q_ig_j-q_jg_i$, which is 2π times an integer. Since S contributes to the generating functional as $\exp(iS)$, this Lorentz-violating part does not play any role in physical processes. The same result has been obtained directly at the level of amplitudes in the toy model where the

[2]The notations are further simplified: $(a\wedge b)^{\mu\nu}=a^\mu b^\nu-a^\nu b^\mu$, $(a\cdot G)^\nu=a_\mu G^{\mu\nu}$.

magnetic charge is made perturbative [109].

4.2 Classical limit and its peculiarities

Let us now show that the classical limit of the theory with the Lagrangian (4.4) indeed corresponds to classical electromagnetism with magnetic currents. The classical equations of motion for the potentials corresponding to the Lagrangian (4.4) are:

$$\frac{n\cdot\partial}{n^2}\left(n\cdot\partial A^\mu \;-\; \partial^\mu n\cdot A \;-\; n^\mu\partial\cdot A \;-\; \epsilon^\mu{}_{\nu\rho\sigma}n^\nu\partial^\rho B^\sigma\right) \;=\; j_e^{\,\mu}\;, \tag{4.6}$$

$$\frac{n\cdot\partial}{n^2}\left(n\cdot\partial B^\mu \;-\; \partial^\mu n\cdot B \;-\; n^\mu\partial\cdot B \;-\; \epsilon^\mu{}_{\nu\rho\sigma}n^\nu\partial^\rho A^\sigma\right) \;=\; j_m^{\,\mu}\;. \tag{4.7}$$

They are first-order equations in the time derivative, which allows the two different four-potentials to describe a sole particle – the photon. To transform these equations, it is convenient to use the identity

$$X = \frac{1}{n^2}\left\{[n\wedge(n\cdot X)] \;-\; [n\wedge(n\cdot X^d)]^d\right\}\;, \tag{4.8}$$

which holds for any antisymmetric tensor X. Namely, assume $X = F$, where F is the field strength tensor introduced such that $n\cdot F = n\cdot(\partial\wedge A)$ and $n\cdot F^d = n\cdot(\partial\wedge B)$. Then, recalling that the scalar expressions nA and nB are free fields by definition, one can transform Eqs. (4.6), (4.7) into the Maxwell equations with magnetic currents:

$$\partial_\mu F^{\mu\nu} = j_e^{\,\nu}\;, \tag{4.9}$$

$$\partial_\mu F^{d\,\mu\nu} = j_m^{\,\nu}\;. \tag{4.10}$$

Thus the Lagrangian (4.4) gives us the correct classical equations of motion for the electromagnetic field.

What remains to be seen is whether the classical equations of motion for the charged particles are recovered. Classical expressions for the electric and magnetic currents are:

$$j_e^\nu(x) = \sum_i q_i \int \delta^4(x - x_i(\tau_i))\,dx_i^\nu\;, \tag{4.11}$$

$$j_m^\nu(x) = \sum_i g_i \int \delta^4(x - x_i(\tau_i))\, dx_i^\nu \,, \tag{4.12}$$

where $x_i(\tau_i)$ is the trajectory of the i-th particle. Supplementing the Lagrangian (4.4) with the standard kinetic terms for the particles, one obtains the following classical equations of motion for the i-th particle:

$$\frac{d}{d\tau_i}\left(\frac{m_i u_i}{(u_i^2)^{1/2}}\right) = (q_i\,[\partial\wedge A(x_i)] + g_i\,[\partial\wedge B(x_i)]\,)\cdot u_i\,, \tag{4.13}$$

where $u_i^\mu = dx_i^\mu/d\tau_i$. The way the electromagnetic field strength tensor was introduced above ($n{\cdot}F = n{\cdot}(\partial\wedge A)$ and $n{\cdot}F^d = n{\cdot}(\partial\wedge B)$) and Eqs. (4.9), (4.10) suggest that

$$\partial\wedge A = F + (n{\cdot}\partial)^{-1}(n\wedge j_m)^d\,, \tag{4.14}$$

$$\partial\wedge B = F^d - (n{\cdot}\partial)^{-1}(n\wedge j_e)^d\,, \tag{4.15}$$

so that the final expression describing the classical force exerted on the i-th particle by the electromagnetic field is:

$$\begin{aligned}\frac{d}{d\tau_i}\left(\frac{m_i u_i}{(u_i^2)^{1/2}}\right) = \quad & \left(q_i F(x_i) + g_i F^d(x_i)\,\right)\cdot u_i \\ & - \sum_j (q_i g_j - g_i q_j)\, n{\cdot}\int (n{\cdot}\partial)^{-1}(x_i - x_j)\ (u_i\wedge u_j)^d\, d\tau_j\,. \end{aligned} \tag{4.16}$$

This expression correctly accounts for the Lorentz force law only if the non-local term in the second row does not contribute. It is easy to see that the latter term indeed cannot play any role in classical dynamics, since the support of the kernel $(n{\cdot}\partial)^{-1}(x_i - x_j)$ is restricted by the condition

$$\vec{x}_i(\tau) - \vec{x}_j(\tau) = \vec{n}s\,, \tag{4.17}$$

which contains three equations, but only two independent variables and is thus satisfied only for exceptional trajectories. At the points of these trajectories where Eq. (4.17) is satisfied, Eq. (4.13) should be solved by continuity, which makes it basically equivalent to the conventional equation for the Lorentz force given by the first row of Eq. (4.16). As it was mentioned before, the full quantum dynamics does not depend on the choice of $\vec{n}$, so that the appearance of the non-local $\vec{n}$-dependent term in Eq. (4.16) is a mere artifact of the classical approximation. For instance, in the path integral formulation, exceptional trajectories form a measure zero subset of all trajectories and thus do not contribute to physical amplitudes. The practical prescription which one can use for deriving the classical equations of motion is simple: in the resulting equations, one should omit any singular terms proportional to $(n\cdot\partial)^{-1}G$, where G is some function of the currents.

4.3 EFT approach to QEMD

Let us consider the QEMD Lagrangian (4.4) from the EFT perspective. In particular, we want to find all independent marginal operators respecting the gauge invariance of the theory and preserving the number of degrees of freedom of QEMD. Such operators can be constructed from the gauge invariant tensors and the vector n_μ. For now, we will not consider the operators containing the gauge currents j_e and j_m, which will be discussed in the next section. We find six classes of dimension four operators, each class containing operators of the form $\mathrm{tr}(X\cdot Y)$ and $(n\cdot X)(n\cdot Y)$, where X and Y can stand for any of the two tensors $\partial\wedge A$ and $\partial\wedge B$. From the identity (4.8), one can find the relation between the operators pertaining to the same class:

$$\mathrm{tr}(X\cdot Y) = \frac{2}{n^2}\left[(n\cdot X^d)(n\cdot Y^d) - (n\cdot X)(n\cdot Y)\right] . \tag{4.18}$$

Let us name the classes depending on the pair (X, Y):

$$\begin{aligned}
&x \quad \text{for} \quad (\partial\wedge A,\, \partial\wedge B)\,, \qquad &&y \quad \text{for} \quad (\partial\wedge A,\, [\partial\wedge B]^d\,)\,,\\
&\alpha \quad \text{for} \quad (\partial\wedge A,\, \partial\wedge A)\,, \qquad &&\beta \quad \text{for} \quad (\partial\wedge B,\, \partial\wedge B)\,,\\
&a \quad \text{for} \quad (\partial\wedge A,\, [\partial\wedge A]^d\,)\,, \qquad &&b \quad \text{for} \quad (\partial\wedge B,\, [\partial\wedge B]^d\,)\,.
\end{aligned}$$

The members of the same class are distinguished by indices:

$$\begin{aligned}
x_1 &\equiv \frac{2}{n^2}\,(n\cdot(\partial\wedge A))\,(n\cdot(\partial\wedge B))\ ,\\
x_2 &\equiv \frac{2}{n^2}\,(n\cdot(\partial\wedge A)^d\,)\,(n\cdot(\partial\wedge B)^d\,)\,,
\end{aligned}$$

$$x_+ \equiv x_1 + x_2 = \frac{2}{n^2} \left\{ (n\cdot(\partial \wedge A)) \, (n\cdot(\partial \wedge B)) + (n\cdot(\partial \wedge A)^d \,) \, (n\cdot(\partial \wedge B)^d \,) \right\} \, ,$$

$$x_- \equiv x_1 - x_2 = -\mathrm{tr} \left((\partial \wedge A) \, (\partial \wedge B) \right) \, ,$$

where we used Eq. (4.18); indices are assigned analogously for the operators in the other five classes. In each of the classes x, y, α or β, the basis is formed by any two members. The classes a and b each contain only one operator, since $a_1 = -a_2 = a_-/2$, $a_+ = 0$ and $b_1 = -b_2 = b_-/2$, $b_+ = 0$. Disregarding the source terms, there are thus 10 independent gauge-invariant dimension four operators in the Zwanziger theory, which we choose to be x_1, x_-, y_+, y_-, α_1, α_-, β_1, β_-, a_-, b_-. From these, only three enter the Lagrangian (4.4), the free part of which can be rewritten as follows:

$$\mathcal{L}_\gamma \;=\; -\frac{1}{4} \left(y_+ + \alpha_1 + \beta_1 \right) \, . \tag{4.19}$$

Let us see which operators can be added to this Lagrangian without conflicting with the structure of the theory. The inclusion of the terms

$$x_- = -\mathrm{tr} \left((\partial \wedge A) \, (\partial \wedge B) \right) \, , \tag{4.20}$$

$$\alpha_- = -\mathrm{tr} \left((\partial \wedge A) \, (\partial \wedge A) \right) \, , \tag{4.21}$$

$$\beta_- = -\mathrm{tr} \left((\partial \wedge B) \, (\partial \wedge B) \right) \tag{4.22}$$

is incompatible with the number of degrees of freedom in QEMD, since these operators give rise to second order time derivatives of the four-potentials A_μ or B_μ in the classical equations of motion. There is no such problem with the four remaining independent operators, three of which correspond to the total derivative terms in the Lagrangian:

$$a_- = -\mathrm{tr} \left\{ (\partial \wedge A) \, (\partial \wedge A)^d \right\} \, , \tag{4.23}$$

$$b_- = -\mathrm{tr} \left\{ (\partial \wedge B) \, (\partial \wedge B)^d \right\} \, , \tag{4.24}$$

$$y_- = -\mathrm{tr} \left\{ (\partial \wedge A) \, (\partial \wedge B)^d \right\} \, , \tag{4.25}$$

and thus do not contribute to the equations of motion. The last operator from our basis is:

$$x_1 = \frac{2}{n^2}\,(n\cdot(\partial\wedge A))\,(n\cdot(\partial\wedge B))\ , \tag{4.26}$$

which does modify the equations of motion and should be added to the Zwanziger Lagrangian (4.4) in the EFT approach. Note that since the two four-potentials A_μ and B_μ have different parities[3], the operators a_-, b_- and x_1 are CP-odd, while the operator y_- is CP-even. This means that one can expect the operator x_1 to be responsible for CP-violation in QEMD. Let us proceed to the next section to see that x_1 is directly related to the Witten effect.

4.4 CP-violation in QEMD

Contrary to QED, the theory of QEMD has an intrinsic source of CP-violation. The reason is that the magnetic charge changes its sign under any of the discrete transformations C, P or T [110], so that a dyon with charges (q, g) is mapped into a dyon with charges $(-q, g)$ under a CP-transformation. The spectrum of charges is not CP-invariant if there exists a state (q, g) while its CP-conjugate state $(-q, g)$ is missing. In this case, it is impossible to define a CP transformation in such a way that the theory is invariant under it [111]. Note that due to the DSZ quantization condition,

$$q_i g_j - q_j g_i = 2\pi n, \quad n\in\mathbb{Z}, \tag{4.27}$$

and our choice for the gauge charges carried by the electron $(e, 0)$, any magnetic charge must be quantized in the units of the minimal magnetic charge $g_0 = 2\pi/e$:

$$g_i = n_i^m g_0\,, \quad n_i^m\in\mathbb{Z}\,. \tag{4.28}$$

The case of electric charges is however different: what one can infer from the quantization condition (4.27) applied to dyons with charges (q_1, g_1) and (q_2, g_2) is that only the difference of some multiples of the electric charges of dyons is quantized: $n_2^m q_1 - n_1^m q_2 = ne$, $n\in\mathbb{Z}$. The latter condition leads to the quantization of the electric charges themselves only if $q_1 = -q_2$ and $g_1 = g_2$, i.e. only if the theory is CP-invariant. Thus, absolute values of the electric charges introduce a CP-violating

[3]Parities of A_μ and B_μ can be inferred for instance from Eqs. (4.14) and (4.15).

parameter θ into the theory:

$$q_i = \left(n_i^e + \frac{\theta}{2\pi} n_i^m\right) \cdot e \,, \quad n_i^e \in \mathbb{Z} \,. \tag{4.29}$$

Since only the total value of the charge, and not any separate contribution, is physical, the parameter θ introduced in this way is defined on the unit circle $\theta \in [0, 2\pi)$. The additional contribution to the electric charge which is proportional to θ is in perfect consistency with Eq. (3.8) derived from the Witten effect, which means that in the particular case of 't Hooft-Polyakov monopoles the parameter θ is the vacuum angle of the full non-Abelian theory.

Let us now find the connection between the CP-violation in QEMD discussed in the previous paragraph and the CP-violating operator x_1 introduced in the previous section. We will show that it is possible to remove θ from the definition of charges (4.29) at the cost of adding the operator x_1 with an appropriate coefficient to the kinetic part of the Lagrangian as well as modifying the coefficient in front of the $[n\cdot(\partial \wedge A)]^2$ term. First, we redefine the electric current $j_e \to \bar{j}_e$ so that it contains only the contribution proportional to $n_i^e \, e$. The QEMD Lagrangian becomes:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2n^2}\Big\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \;- \\ & [n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2\Big\} \;-\; \left(\bar{j}_e + \frac{e^2\theta}{4\pi^2}\, j_m\right)\cdot A \;-\; j_m \cdot B \,.\end{aligned} \tag{4.30}$$

Next, we make the following $SL(2,\mathbb{R})$ transformation in the space of four-potentials:

$$\begin{pmatrix} A \\ B \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ -\frac{e^2\theta}{4\pi^2} & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} . \tag{4.31}$$

The first row of the Lagrangian (4.30), which corresponds to the operator y_+ from the previous section, is not affected by this transformation. The second row is transformed yielding the conventional source terms and an extra x_1 term as promised:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2n^2}\Big\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \;- \\ & \left(1 + \frac{e^4\theta^2}{16\pi^4}\right)[n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2 + \frac{e^2\theta}{2\pi^2}\,(n\cdot(\partial\wedge A))\,(n\cdot(\partial\wedge B))\Big\} \;-\end{aligned}$$

$$\bar{j}_e\cdot A \;-\; j_m\cdot B\,, \tag{4.32}$$

which can be rewritten more compactly in our operator notation:

$$\mathcal{L} \;=\; -\frac{1}{4}\left(y_+ + \left(1+\frac{e^4\theta^2}{16\pi^4}\right)\alpha_1 + \beta_1 - \frac{e^2\theta}{2\pi^2}\,x_1\right) - \bar{j}_e\cdot A \;-\; j_m\cdot B\,. \tag{4.33}$$

Several important comments are in order. First, note that the periodicity of θ is no longer explicit in the Lagrangian (4.33). In fact, to see the symmetry under $\theta \to \theta + 2\pi$ transformation, we have to account for the implicit dependence of the four-potential B_μ on θ arising from the transformation (4.31). The term

$$\begin{aligned}\frac{1}{4}\cdot\frac{e^2\theta}{2\pi^2}\,x_1 \;=\; \frac{e^2\theta}{4\pi^2 n^2} & \quad (n\cdot(\partial\wedge A))\,(n\cdot(\partial\wedge B)) \\ & = \frac{e^2\theta}{4\pi^2 n^2}\,(n\cdot F)\,(n\cdot F^d) \;=\; -\frac{e^2\theta}{16\pi^2}\mathrm{tr}\left(FF^d\right)\,,\end{aligned} \tag{4.34}$$

is similar to the conventional QED θ-term (3.5), but is by no means symmetric under the transformation $\theta \to \theta + 2\pi$ by itself. We see that in the theory where magnetic currents are properly included in the Lagrangian of the theory, not only does the term (3.5) lose its total derivative structure, but it is also no longer topological.

The second comment which we would like to make is about Lorentz-invariance of QEMD with CP-violation. Although the Lagrangian (4.33) contains an extra term with n_μ-dependence, added to the Zwanziger Lagrangian (4.4), and a change in the coefficient in front of the α_1 term, it is clear that the theory is Lorentz-invariant, since one can get rid of the unusual n_μ-dependence by performing a $SL(2,\mathbb{R})$ transformation of the potentials. Since it is always possible to get rid of the x_1 term in this way, we see that the three operators y_+, α_1 and β_1 entering the Zwanziger Lagrangian (4.19) are indeed the only independent gauge-invariant four-dimensional operators which are relevant for the kinetic part of QEMD. The possible CP-violation is most elegantly accounted for in the expression (4.30) for the QEMD Lagrangian, since in this form the periodicity of the θ-parameter is made obvious. The latter form of the QEMD Lagrangian is also convenient for finding the extension of QEMD which incorporates axions – the endeavor we accomplish in Chapter 5.

4.5 QEMD of 't Hooft-Polyakov monopoles

Let us now consider the QEMD of 't Hooft-Polyakov monopoles. After introducing the CP-violating parameter θ into the Zwanziger theory, we identified it with the instanton angle of the UV non-Abelian theory through the Witten effect. Still, the modified Zwanziger Lagrangian (4.30) misses some of the effects associated with the 't Hooft-Polyakov monopoles, since, as we discussed in Chapter 1 sec. 1.2, the latter monopoles cannot be modeled by simple point-like magnetic field sources even in the IR.

Consider instanton effects of the UV non-Abelian theory. At low energies, in the symmetry-broken phase, they are known to be suppressed everywhere, but on 't Hooft-Polyakov monopoles [49]. As a result, some of the good symmetries of the low energy EFT can be violated by unsuppressed instanton-induced effects on the monopole. The most famous example is the Rubakov-Callan effect [49, 51]: the decay of a proton catalyzed by a monopole. A consistent QEMD of 't Hooft-Polyakov monopoles has to account for such instanton effects. To satisfy this requirement, we introduce an extra degree of freedom $\phi\left(x^{\mu}\right)$ into QEMD, which interacts with the electric current j_e^{μ} via the following Lagrangian: $\mathcal{L} = \left(j_e \cdot \partial\right)\phi$. The field ϕ does not contribute to the classical equations of motion, as it should be for a variable describing instanton effects. As the latter effects are localized on the monopole, we require that the interaction Hamiltonian $\mathcal{H} = -\left(\mathbf{j}_e \cdot \boldsymbol{\nabla}\right)\phi$ vanishes outside the monopole core, so that $\boldsymbol{\nabla}\phi$ is zero everywhere but on the monopole. The latter localization property also means that in our low energy EFT, only s-wave fermions can interact with ϕ, because wave-functions of scalars and higher partial wave fermions vanish on the monopole due to the centrifugal barrier [112, 113] [4].

Let us now show that the interaction Hamiltonian $\mathcal{H}$ introduced in the previous paragraph can provide a valid description for the Rubakov-Callan effect. For this, we take advantage of the work by Joseph Polchinski [50]. In the latter work, the author showed that the Rubakov-Callan effect can be described as an interaction between s-wave fermions and a rotor coordinate $\alpha(t)$. One can show that this description is equivalent to ours as long as one identifies $\alpha(t)$ with the temporal dependence of $e\phi$.

[4] In this section, we assume the magnetic monopole to be a scalar particle, since to our knowledge this is the only case which has been studied in the literature on the Rubakov-Callan effect.

For the sake of comparison, consider the case of a left-handed Weyl fermion χ interacting with an $SU(2)$ monopole. The only part of the electric current contributing to $\mathcal{H}$ is associated to s-wave fermions $j_e^i = e\,\bar{\chi}^k_{(s)}\sigma^i\chi^k_{(s)}/\,2$, where k is a flavor index, so that the theory can be reduced to (1+1) dimensions:

$$H = -\int d^3x\;(\mathbf{j}_e\cdot\boldsymbol{\nabla})\,\phi = -\frac{e}{2}\int\limits_0^{+\infty} dr\,\left(\xi_+^\dagger\xi_+ - \xi_-^\dagger\xi_-\right)\partial_r\phi = \int\limits_{-\infty}^{+\infty} dr\,\psi_k^\dagger\alpha q'(r)\psi_k\,, \quad (4.35)$$

where, following the notations of Ref. [50], we define $\xi_\pm$ spinors as charge eigenstates, $\psi_k(\pm r)\equiv\xi_\pm^{(k)}(r)$; $q(r\,|\,r<-r_0)=1/2$ and $q(r\,|\,r>r_0)=-1/2$; r_0 is the size of the monopole core; we omitted the terms which are suppressed by the high energy scale $1/r_0$. One sees that the interaction Hamiltonian (4.35) is equivalent to the one used in Ref. [50] [5]. Thus, to account for the Rubakov-Callan effect, the source term of the QEMD Lagrangian has to be modified as follows: $-j_e\cdot A \;\;\rightarrow\;\; -j_e\cdot(A-\partial\phi)$. In the case of non-zero θ, one obtains:

$$\mathcal{L}\;\supset\;-\left(\bar{j}_e + \frac{e^2\theta}{4\pi^2}\,j_m\right)\cdot(A-\partial\phi)\;. \quad (4.36)$$

The term $e^2\theta\,(j_m\cdot\partial)\,\phi/4\pi^2$ corresponds to the θ-term in the worldline action for the collective coordinate $e\phi$:

$$S_{[e\phi]}\supset\sum_i\int\limits_{\gamma_i}\frac{\theta}{2\pi}\,d(e\phi)\,, \quad (4.37)$$

where γ_i are the monopole worldlines.

There is yet another way to understand why in the case of 't Hooft-Polyakov monopoles, the θ-term of the QEMD Lagrangian (4.30) has to be modified. In particular, consider the case of a varying θ. It is clear that in the full non-Abelian theory the coupling of the new pseudoscalar field θ to GG^d is legitimate. However, varying θ in the Lagrangian (4.30) would be inconsistent with electric charge conservation: the gauge invariance of the theory would require $\partial_\mu\theta=0$. A way to resolve this paradox is to introduce a Stückelberg field ϕ localized on the monopole, so that $j_m\cdot(A-\partial\phi)$ is gauge invariant [114], which leads us again to the coupling (4.36). Note that the

[5] The other two terms in the Hamiltonian of the model of Ref. [50] containing only the collective coordinate α and its canonical momentum Π correspond to the potential and kinetic energy terms for ϕ, respectively.

dependence of the vacuum energy on θ, which was calculated in Ref. [50] using the low energy theory with the interaction Hamiltonian (4.35), agrees with the high energy non-Abelian theory result obtained in dilute instanton gas approximation $V(\theta) \propto -\cos\theta$. A similar dependence was found in Ref. [115] where the authors took advantage of the worldline action (4.37) and computed the self-energy of ϕ.

4.6 Scattering in QEMD

4.6.1 Basic features of scattering

Dirac quantization condition [21] tells us that for any particle with magnetic charge g, the following identity holds:

$$eg = 2\pi n\,, \quad n \in \mathbb{Z}\,. \tag{4.38}$$

Since the unit of electric charge is small $e \simeq 0.3$, the interaction between a given electric and a given magnetic charge as well as the interaction between any two given magnetic charges are strong. This means that the perturbation theory is of no help while calculating the scattering amplitudes involving magnetically charged particles. The scattering theory has to be built differently. For an illustration, let us consider again the work by D. Zwanziger [30], where he finds a local Lagrangian for QEMD. As it was mentioned in sec. 4.1, due to the presence of the fixed vector n in its formulation, this Lagrangian is not Lorentz-invariant. It was shown both in path integral formalism [107, 108] and in a toy model with perturbative magnetic charge [109], that one has to take into account all quantum corrections to recover Lorentz-invariance of the theory. Although formal Feynman rules can be formulated, the tree-level (or any finite order) amplitudes do not make sense in this case, since they are not even Lorentz-invariant. The Lagrangian approach is not useful for studying the scattering of electric and magnetic charges.

An option which is left and which has been successfully pursued in the literature is to assume that the magnetic charge is static, i.e. that electric particles scatter on the classical electromagnetic field created by the heavy magnetic monopole. In this case, it is convenient to make use of the axial symmetry of the problem and expand the scattering amplitude as a series of partial waves, each of which corresponds to the fixed value of total angular momentum j.

Beyond non-perturbativity, another important property of the existing field-theoretical constructions of QEMD is non-locality. While Zwanziger theory Lagrangian (4.4) is local, the connection between the two four-potentials A and B describing the electromagnetic field in this formulation is non-local, see Eqs. (4.1) and (4.2). Another formulation of QEMD due to J. Schwinger [46] reveals its non-local properties already at the level of the theory Hamiltonian.

So why is QEMD essentially non-local? To answer this question, let us investigate a system of particles with charges (e_i, g_i) which move in the asymptotically distant past or future [44]. Such particles can be approximated as moving along the straight lines $\lim_{t\to\pm\infty} x_i^\mu(t) = u_i^\mu t$ with the constant four-velocity u_i^μ. The electromagnetic field which is spatially far from particle trajectories can be calculated classically. Solving the Maxwell equations

$$\partial_\mu F^{\mu\nu} = j_e^{\,\nu} \,, \tag{4.39}$$

$$\partial_\mu F^{d\,\mu\nu} = j_m^{\,\nu} \,, \tag{4.40}$$

one obtains

$$\lim_{t\to\pm\infty} F(x) = F^{\text{free}}(x) + \int d^4y \; G(x-y) \left(\partial\wedge j_e - (\partial\wedge j_m)^d\right) \,, \tag{4.41}$$

where $G(x-y)$ is the Green function (retarded or advanced) for the wave equation, $F^{\text{free}}(x)$ is a solution of the free Maxwell equations. Due to the simplicity of the motion of the particles in the asymptotic region, the integral in the right-hand side of Eq. (4.41) is nothing more than the well-known Liénard-Wiechert solution for uniformly moving electric charge plus the dual solution for the magnetic charge:

$$\lim_{t\to\pm\infty} F(x) = F^{\text{free}}(x) + \frac{1}{4\pi}\sum_i \frac{e_i(x\wedge u_i) - g_i(x\wedge u_i)^d}{\left[(x\cdot u_i)^2 - x^2\right]^{3/2}} \,. \tag{4.42}$$

The source contribution to F vanishes as t^{-2}, so that the corresponding energy-momentum tensor $T_{\mu\nu}$ scales as t^{-4}. Since the volume scales as t^3, total energy and momentum associated to the charges vanish for $t\to\pm\infty$, however there is a finite contribution to the angular momentum tensor:

$$M^{\mu\nu} = \lim_{t\to\pm\infty} \int d^3x \, \left(x^\mu T^{\nu 0} - x^\nu T^{\mu 0} \right) \,, \tag{4.43}$$

where $T = (F{\cdot}F + F^d{\cdot}F^d)/2$. Let us keep only the source contribution to F since we are interested in the self-fields of the charges. After a straightforward calculation, one arrives at the following expression for the asymptotic angular momentum tensor:

$$M^{\mu\nu} = \sum_{i>j} \pm \frac{e_i g_j - e_j g_i}{4\pi} \, \frac{\epsilon^{\mu\nu}{}_{\kappa\lambda} u_i^\kappa u_j^\lambda}{\left[(u_i{\cdot}u_j)^2 - 1\right]^{3/2}} \,, \tag{4.44}$$

where the sum is over all possible pairs of particles. The term for each pair comes with the plus sign if the pair is incoming (in-state) and with the minus sign if the pair is outgoing (out-state). In the non-relativistic limit, summing over all pairs of particles (i,j) with the relative velocities $\vec{v}_{ij}$ one obtains:

$$M^{0k} = 0 \,, \quad \vec{J} = \sum_{i>j} \pm \frac{e_i g_j - e_j g_i}{4\pi} \, \frac{\vec{v}_{ij}}{|\vec{v}_{ij}|} \,, \tag{4.45}$$

where $J_k = \epsilon_{klm} M^{lm}/2$ is angular momentum. The angular momentum does not depend on the values of the relative speeds and agrees with the expression for the angular momentum of the static system of electric and magnetic charges.

The presence of extra angular momentum in the in- and out-states associated to the pairs of particles implies that these pairs are quantum mechanically entangled: the quantum state of the pair cannot be reduced to the product of one-particle states. This non-locality is connected to the failure of the quantum-field-theoretical Lagrangian methods in providing the scattering matrix: note that the usage of quantum fields for calculating scattering processes is motivated by the cluster decomposition principle [116], which is basically the principle of locality. This means that for the calculation of the electric-magnetic S-matrix we have to resort to on-shell methods.

Another important property of the scattering amplitudes in QEMD is the absence of the crossing symmetry. This property follows immediately from the non-locality we have just discussed. Since each given pair of particles (i,j) in the in- or out-state is entangled as long as $e_i g_j - e_j g_i \neq 0$, one cannot CPT-transform one particle from the pair without affecting the other. Moreover, all the incoming pairs have different sign in front of their additional angular momenta compared to all the outgoing pairs. This means that no single particle from the in-state can be transferred to the out-state without modifying the amplitude unless all particles in the in- and out-states are either purely electric or purely magnetic.

4.6.2 Pairwise helicity

The asymptotic properties of in- and out-states can be inferred from the analysis of the irreducible projective unitary representations of the Poincaré group [45]. Normally, in- and out-states of the S-matrix approach the product of states transforming under one-particle irreducible representations of the Poincaré group. In fact, this is the basis of the normal "electric" quantum field theory, where the particles are defined by these representations. For example, any state of a single massive particle in the reference frame where it is at rest generically has extra degrees of freedom associated to the rotations in the $SU(2)$ double covering of the $SO(3)$ rotation group. All the massive particles are then parameterized not only by their mass, but also by their spin s which fixes the $(2s+1)$-dimensional representation of the Poincaré group. Similarly, it can be derived that the massless particles are parameterized by an extra degree of freedom called helicity which is an integral multiple of $1/2$.

Let us apply Wigner's method to the particle states in QEMD. Allowed extra degrees of freedom will follow from the little group transformations, i.e. the Poincaré group actions which do not affect particle momenta. Wigner classified all the little groups for one-particle states. Let us now follow Zwanziger [44] and consider multi-particle states, too. An arbitrary two-particle state can be transformed into COM frame (or simply the frame where momenta are collinear) via a Lorentz transformation:

$$|p_1, p_2\rangle \quad \longrightarrow \quad |k_1, k_2\rangle \ , \tag{4.46}$$

so that $\vec{k}_1 \upuparrows \vec{k}_2$. It is now easy to see that the little group which preserves both momenta is the $U(1)$ subgroup of the Poincaré group: both momenta are invariant under rotations around the axis directed along $\vec{k}_1$. This means that one can associate an extra helicity with each electric-magnetic pair which is called pairwise helicity q_{ij}.

Pairwise helicity parameterizes a finite-dimensional representation of the little group and thus is an integral multiple of $1/2$. Under a generic Lorentz transformation Λ, any two-particle state must transform as follows:

$$U(\Lambda)|p_1, p_2; \sigma_1, \sigma_2; q_{12}\rangle \;=\; e^{iq_{12}\phi_{12}}\, D_{\sigma_1'\sigma_1}(W_1)\, D_{\sigma_2'\sigma_2}(W_2)\, |\Lambda p_1, \Lambda p_2; \sigma_1', \sigma_2'; q_{12}\rangle\,, \tag{4.47}$$

where σ_1, σ_2 are little group parameters (spins, their projections or helicities) of the one-particle states $|p_1\rangle$ and $|p_2\rangle$, respectively; W_1 and W_2 are the corresponding little groups and $D(W_i)$ are their representations. Extension of the transformation law (4.47) to generic multi-particle states is straightforward. Indeed, there is no subgroup of the Poincaré group that would leave more than two momenta invariant, which means that a generic n-particle state transforms as a product of n one-particle states and $n(n-1)/2$ two-particle states.

Let us now connect the pairwise helicity q_{12} to the observables in QEMD. Consider a pair of massive spinless particles with charges (e_1, g_1) and (e_2, g_2) in the in- or out-state. In the frame where the first particle is at rest $p_1 = (m, 0, 0, 0)$ and the second is moving along the z-axis $p_2 = (\sqrt{m_2^2 + \vec{p}^2}, 0, 0, p)$, the expression for non-vanishing component of the angular momentum tensor derived in subsec. 4.6.1 is

$$M_{12} = \pm\frac{e_1 g_2 - e_2 g_1}{4\pi}\,, \tag{4.48}$$

where plus sign is for in-state and minus sign for the out-state. Thus, under the rotation around the z-axis the two-particle state transforms as follows:

$$|p_1, p_2\rangle \quad\longrightarrow\quad e^{iM_{12}\phi_{12}}\, |p_1, p_2\rangle\ . \tag{4.49}$$

Comparing this transformation with the transformation given by eq. (4.47), we obtain the value for the pairwise helicity in terms of electric and magnetic charges:

$$q_{12} = \pm\frac{e_1 g_2 - e_2 g_1}{4\pi}\,. \tag{4.50}$$

Note that we have derived the Dirac quantization condition once again, since the representation theory restricts pairwise helicity q_{12} to the integral multiples of 1/2:

$$q_{12} = \frac{n}{2} \quad \Rightarrow \quad e_1 g_2 - e_2 g_1 = 2\pi n \,, \quad n \in \mathbb{Z} \,. \tag{4.51}$$

4.6.3 Electric-magnetic S-matrix

Now that we determined the Lorentz transformations of in- and out-states of electric-magnetic scattering, we can infer the corresponding transformations of the S-matrix:

$$\begin{aligned} S(p'_1, \ldots, p'_m | p_1, \ldots, p_n) \equiv \langle p'_1, \ldots, p'_m; - | p_1, \ldots, p_n; + \rangle = \\ \langle p'_1, \ldots, p'_m; - | U(\Lambda)^\dagger \, U(\Lambda) | p_1, \ldots, p_n; + \rangle = \\ e^{i(\Sigma_+ + \Sigma_-)} \prod_{i=1}^{m} \mathcal{D}(W_i)^\dagger \prod_{i'=1}^{n} \mathcal{D}(W_{i'}) \; S(\Lambda p'_1, \ldots, \Lambda p'_m | \Lambda p_1, \ldots, \Lambda p_n) \,, \end{aligned} \tag{4.52}$$

where

$$\Sigma_+ \equiv \sum_{i>j}^{n} \mu_{ij} \phi_{ij} \,, \qquad \Sigma_- \equiv \sum_{i'>j'}^{m} \mu_{i'j'} \phi_{i'j'} \,, \tag{4.53}$$

(i, j) and (i', j') are indices for incoming and outgoing particles, respectively, and $\mu_{ab} = (e_a g_b - e_b g_a)/4\pi$. Thus, S-matrix transforms as follows:

$$\begin{aligned} S(\Lambda p'_1, \ldots, \Lambda p'_m | \Lambda p_1, \ldots, \Lambda p_n) \;=\; e^{-i(\Sigma_+ + \Sigma_-)} \prod_{i=1}^{m} \mathcal{D}(W_i) \cdot \\ \prod_{i'=1}^{n} \mathcal{D}(W_{i'})^\dagger \, S(p'_1, \ldots, p'_m | p_1, \ldots, p_n) \,. \end{aligned} \tag{4.54}$$

The transformation law (4.54) restricts the explicit form of the S-matrix in each particular process. To see what these restrictions are, it is convenient to work with spinor-helicity variables, since they transform under the little group in a simple way. S-matrix depends on momenta p_i which transform in the $(1/2, 1/2)$ representation of the Lorentz group (note the isomorphism $so(1,3) \simeq su(2) \oplus su(2)$). This means that every momentum can be represented as a bispinor:

$$p^{\alpha\dot{\alpha}} = \sigma_\mu^{\alpha\dot{\alpha}} p^\mu = \begin{pmatrix} p^0 - p^3 & -p_1 + i p_2 \\ -p_1 - i p_2 & p_0 + p_3 \end{pmatrix} \,, \tag{4.55}$$

where $\sigma_\mu = (1, \vec{\sigma})$. Note that $\det p^{\alpha\dot{\alpha}} = m^2$. For massless particles, $\det p^{\alpha\dot{\alpha}} = 0$, which ensures that one can decompose the momentum matrix into the product of two spinors:

$$p^{\alpha\dot{\alpha}} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}} \equiv p\rangle[p\,. \tag{4.56}$$

For massive particles, the momentum can be represented as rank two matrix, which means that it can be decomposed into the sum of two rank one matrices as follows:

$$p^{\alpha\dot{\alpha}} = \lambda^{\alpha}_{I}\tilde{\lambda}^{\dot{\alpha}\,I} \equiv p_I\rangle[p^I\,, \quad I = 1,2\,. \tag{4.57}$$

In the spinor-helicity variables, we will denote the momentum of the ith particle p_i simply by i. Then our building blocks for the S-matrix are $i\rangle$, $j_I\rangle$, $[i$ and $[j^I$, where i runs over all the massless particles and j runs over all the massive ones. Under the action of the corresponding little groups, these spinors transform in a simple way:

$$i\rangle \;\to\; e^{i\phi/2}|i\rangle\,, \quad [i \;\to\; e^{-i\phi/2}[i\,, \quad j_I\rangle \;\to\; W^I_K|j_I\rangle\,, \quad [j^I \;\to\; (W^\dagger)^K_I[j^I\,, \tag{4.58}$$

where $W \in SU(2)$. For simplification, let us work in the out-out framework, in which all of the particles in the process are formally considered outgoing. This is possible if we always keep in mind which particles are really incoming and do not pair particles one of which is in the in-state and another is in the out-state. Helicities and spins of the particles in the scattering process put a constraint on how many helicity spinors of each type we are supposed to have in the S-matrix. For example, if the particle 1 is massless and has helicity h_1, we will have n spinors $1\rangle$ and m spinors $[1$, such that $m-n = 2h_1$. The massive particle of spin s enters the S-matrix as a symmetric rank $2s$ tensor which forms an irreducible spin-s representation of the $SU(2)$ group. For this reason, the S-matrix is always symmetrized over I indices. Keeping this in mind, we will omit them and write massive spinors in bold, like $\mathbf{i}\rangle$.

Let us first consider an example which involves only electrically charged particles. In this case, we already have all the helicity spinors necessary to construct the S-matrix. Our example will be a massive particle decaying into the two massless particles. Suppose that the massive particle has spin 1/2. The S-matrix can then be given by the following expressions:

$$S \propto \langle\mathbf{1}2\rangle[23]^n\langle 23\rangle^m \quad \text{or} \quad [\mathbf{1}2][23]^n\langle 23\rangle^m \quad \text{or} \quad 2 \leftrightarrow 3\,, \tag{4.59}$$

from which we infer that precisely one of the massless particles must be a fermion. In general, it is clear that using this method one can find no-go theorems (or selection rules) for various processes. In particular, using the same setup of one massive and two massless particles, one can prove that a massive spin one particle cannot decay to a pair of photons and that a massive spin three particle cannot decay to a pair of gravitons [117].

Let us now adapt this method to the electric-magnetic scattering [118]. In this case, we have to account for the extra little group transformations associated to the pairwise helicities. Since these transformations are $U(1)$ rotations, similar to the massless particle case, we have to introduce new spinor helicity variables associated with null momenta. The required null momenta must be linear combinations of the particle momenta in the pair, so that they have the same Lorentz transformation properties. It is especially easy to build the reference null momenta in the COM reference frame:

$$\left(k_{ij}^{\pm}\right)^{\mu} = p_c\,(1,0,0,\pm 1)\,, \tag{4.60}$$

where p_c is the COM momentum of the (i,j) pair. Boosted to any other frame, the null momenta are given by the following covariant expressions in terms of the particle momenta:

$$p_{ij}^{+} = \frac{1}{E_i^c + E_j^c}\left[\left(E_j^c + p_c\right)p_i - \left(E_i^c - p_c\right)p_j\right]\,, \tag{4.61}$$

$$p_{ij}^{-} = \frac{1}{E_i^c + E_j^c}\left[\left(E_i^c + p_c\right)p_j - \left(E_j^c - p_c\right)p_i\right]\,. \tag{4.62}$$

Now that we defined the pairwise null momenta, it is straightforward to define the corresponding spinor helicity variables, in full analogy to the case of the massless particles discussed above:

$$\left(p_{ij}^{\pm}\right)^{\alpha\dot{\alpha}} = \sigma_{\mu}^{\alpha\dot{\alpha}}\left(p_{ij}^{\pm}\right)^{\mu} = p_{ij}^{\pm}\rangle[p_{ij}^{\pm}\,. \tag{4.63}$$

Their little group transformations are given by the following $U(1)$ rotations:

$$p_{ij}^{\pm}\rangle \;\to\; e^{\pm i\phi/2}|p_{ij}^{\pm}\rangle\,, \quad [p_{ij}^{\pm} \;\to\; e^{\mp i\phi/2}[p_{ij}^{\pm}\,. \tag{4.64}$$

Note that $p_{ij}^{+}\rangle$ and $p_{ij}^{-}\rangle$ have opposite pairwise helicities $\pm 1/2$. Explicit contractions with the other spinors in the massless limit are:

$$[p_{ij}^{+} i] = \langle i p_{ij}^{+}\rangle = [\eta_i p_{ij}^{-}] = \langle p_{ij}^{-} \eta_i\rangle = 0\,, \tag{4.65}$$

$$[p_{ij}^{-} i] = \langle i p_{ij}^{-}\rangle = \sqrt{2p_c}[\eta_i p_{ij}^{+}] = \sqrt{2p_c}\langle p_{ij}^{+} \eta_i\rangle = 2p_c\,, \tag{4.66}$$

where η_i are Parity-conjugate massless spinors which appear in the massless limit of the massive spinors $\mathbf{i}$ [117]. Note that in constructing the S-matrix we require that the helicity weights under each individual particle as well as the pairwise helicity weights are matched for both the initial and the final states, since only the diagonal Lorentz transformation for which each particle and each pair of particles are transformed simultaneously is physical.

Let us construct the S-matrix of a massive vector decaying to two different massless fermions with the pairwise helicity $\mu_{23} = -1$. For the massive vector, we need two spinors $\mathbf{1}\rangle$. For the massless fermions, it is enough to take spinors $2\rangle$ and $3\rangle$. Now, there are four spinor indices from the normal spinors which need to be contracted with the pairwise spinors. In total, we should have 4 pairwise spinors, three of which should have negative helicity and one positive, since we require that $\mu_{23} = -1$. The scattering matrix for positive helicity fermions is then:

$$S\left(\mathbf{1}^{s=1}|2^{-1/2}, 3^{-1/2}\right)_{\mu_{23}=-1} \sim \langle 2p_{23}^{-}\rangle\langle p_{23}^{+}3\rangle\langle \mathbf{1}p_{23}^{-}\rangle^2\,, \tag{4.67}$$

up to a little group invariant. We also see that the decay to the different helicity fermions $h_2 = -h_3 = 1/2$ is forbidden in this case, since $[p_{23}^{-}3] = 0$.

Now let us consider the same example where the pairwise helicity of the massless fermions is $\mu_{23} = -2$. In this case, the situation is the opposite: the S-matrix for the same helicity fermions vanishes, while for the different helicity fermions it does not vanish:

$$S\left(\mathbf{1}^{s=1}|2^{-1/2}, 3^{+1/2}\right)_{\mu_{23}=-2} \sim \langle 2p_{23}^{-}\rangle[p_{23}^{+}3]\langle \mathbf{1}p_{23}^{-}\rangle^2\,, \tag{4.68}$$

up to a little group invariant. Same helicity fermions are forbidden in this case since $\langle p_{23}^{-}3\rangle = [p_{23}^{+}2] = 0$.

Finally, let us deal with a more general case of a massive particle decaying into two other massive particles. The S-matrix is a contraction of the massive part:

$$\left(\langle \mathbf{1}|^{2s_1}\right)^{\{\alpha_1\ldots\alpha_{2s_1}\}}\ \left(\langle \mathbf{2}|^{2s_2}\right)^{\{\beta_1\ldots\beta_{2s_2}\}}\ \left(\langle \mathbf{3}|^{2s_3}\right)^{\{\gamma_1\ldots\gamma_{2s_3}\}} \tag{4.69}$$

with a massless part involving the pairwise spinors:

$$S^q_{\{\alpha_1\ldots\alpha_{2s_1}\}\{\beta_1\ldots\beta_{2s_2}\}\{\gamma_1\ldots\gamma_{2s_3}\}} = \sum_{i=1}^{C} a_i \left(|p_{23}^-\rangle^{\hat{s}-\mu_{23}}\ |p_{23}^+\rangle^{\hat{s}+\mu_{23}}\right)_{\{\alpha_1\ldots\alpha_{2s_1}\}\{\beta_1\ldots\beta_{2s_2}\}\{\gamma_1\ldots\gamma_{2s_3}\}}\ , \tag{4.70}$$

where $\hat{s} = s_1 + s_2 + s_3$ and C counts all the ways to group the spinors into α, β, γ indices. Since we cannot have negative powers of pairwise spinors, a selection rule follows:

$$|q| \leq \hat{s}\,, \tag{4.71}$$

which restricts the charges and spins of individual particles.

This Chapter is partly written based on the publication [1] of the author of this thesis.

Chapter 5

Generic low energy axion-photon EFT

5.1 Anomalous axion-photon interactions

Let us now find the extension of QEMD which incorporates axions. We first limit ourselves to the CP-conserving axion interactions, which means that the dimension five operators containing the axion field are obtained from the CP-odd dimension four operators of QEMD: a_-, b_- and x_1. Axion EFT must be symmetric under the transformation $a \to a + 2\pi v_a n$, $n \in \mathbb{Z}$, which suggests that we use the operator $j_m A$ instead of x_1, since $a x_1$ would not have the discrete shift symmetry required, as outlined in Chapter 4 sec. 4.4. The operator $j_m A$ corresponds to the Witten-effect induced axion interaction and we postpone its discussion until the next section, limiting ourselves to the pure axion-photon couplings first. Thus, the Lagrangian for a generic CP-conserving axion-photon EFT is:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2n^2}\Big\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \;-\\ & [n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2\Big\} \;-\\ & \frac{1}{4}\,g_{a\mathrm{AA}}\,a\,\mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge A)^d\right\} \;-\; \frac{1}{4}\,g_{a\mathrm{BB}}\,a\,\mathrm{tr}\left\{(\partial\wedge B)\,(\partial\wedge B)^d\right\}\,,\end{aligned} \tag{5.1}$$

or written in a more compact operator notation:

$$\mathcal{L} \;=\; -\frac{1}{4}\,(y_+ + \alpha_1 + \beta_1) + \frac{1}{4}\,g_{a\mathrm{AA}}\,a\,a_- + \frac{1}{4}\,g_{a\mathrm{BB}}\,a\,b_-\,. \tag{5.2}$$

The coefficients $g_{a\mathrm{AA}}$ and $g_{a\mathrm{BB}}$ cannot be determined by symmetry arguments, since both a_- and b_- terms are total derivatives, which ensures shift symmetry regardless of their coefficients. However, to compute the coefficients, we can take advantage of the fact that these terms arise from the anomalous divergence of the Peccei-Quinn current, so that $g_{a\mathrm{AA}}$ and $g_{a\mathrm{BB}}$ are determined by the $U(1)_{\mathrm{PQ}}\,(U(1)_{\mathrm{E}})^2$ and $U(1)_{\mathrm{PQ}}\,(U(1)_{\mathrm{M}})^2$ anomalies, respectively[1]:

$$g_{a\mathrm{AA}} = \frac{Ee^2}{4\pi^2 v_a}\,, \quad E = \sum_{\psi} q_\psi^2 \cdot d(C_\psi)\ , \tag{5.3}$$

$$g_{a\mathrm{BB}} = \frac{Mg_0^2}{4\pi^2 v_a}\,, \quad M = \sum_{\psi} g_\psi^2 \cdot d(C_\psi)\ , \tag{5.4}$$

where E and M are electric and magnetic anomaly coefficients, respectively; q_ψ and g_ψ are electric and magnetic charges of heavy PQ-charged fermions ψ in units of e and g_0, respectively; $d(C_\psi)$ is the dimension of the color representation of ψ. Due to the DSZ quantization condition, $g_0 \gg e$ so that the Wilson coefficient $g_{a\mathrm{BB}}$ is expected to dominate the axion-photon coupling.

Let us now consider the CP-violating axion interactions. We have not yet taken advantage of the CP-even four-dimensional operator y_-, which can be coupled to the axion since the resulting CP-odd five-dimensional operator ay_- respects the axion shift symmetry. The corresponding term in the Lagrangian is:

$$\mathcal{L}_{\not{CP}} \supset -\frac{1}{2}\,g_{a\mathrm{AB}}\,a\,\mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge B)^d\right\}\,, \tag{5.5}$$

where the coefficient $g_{a\mathrm{AB}}$ is determined by the $U(1)_{\mathrm{PQ}}\,U(1)_{\mathrm{E}}\,U(1)_{\mathrm{M}}$ anomaly. Note that the latter anomaly is non-zero only in the case where the spectrum of dyons violates CP. In this case, the intrinsic CP-violation of high energy QEMD is transferred to the low energy axion-photon EFT after integrating out heavy dyons. As we show later in Chapter 6 sec. 6.3, the coefficient $g_{a\mathrm{AB}}$ is given by the following

[1] For the detailed derivation, see Chapter 6 sec. 6.3.

expression:

$$g_{a\mathrm{AB}} = \frac{D e g_0}{4\pi^2 v_a}\,, \quad D = \sum_\psi q_\psi g_\psi \cdot d(C_\psi)\,, \tag{5.6}$$

where D is the mixed electric-magnetic CP-violating anomaly coefficient, which depends on the spectrum of heavy PQ-charged dyons. The DSZ quantization condition ensures $g_0 \gg e$, so that the CP-violating axion-photon coupling $g_{a\mathrm{AB}}$ is naturally suppressed compared to the CP-conserving $g_{a\mathrm{BB}}$ coupling, but dominates over the CP-conserving $g_{a\mathrm{AA}}$ coupling: $g_{a\mathrm{BB}} \gg |g_{a\mathrm{AB}}| \gg g_{a\mathrm{AA}}$.

Not only do the values of the anomaly coefficients E, M and D depend on the details of the UV model, but also the value of the minimal magnetic charge g_0 does. While we used $g_0 = 2\pi/e$ for pure QEMD in Chapter 4 sec. 4.4, the real low energy theory describing nature involves also the $SU(3)_c$ color gauge group, and the quarks charged under this group have minimal electric charge $|e_0| = e/3$. Naively, this implies that the minimal magnetic charge is $g_0 = 2\pi/|e_0| = 6\pi/e$. However, this is true only if the magnetic monopoles are Abelian, i.e. if they do not carry color magnetic charge. As we discussed in Chapter 1 sec. 1.3, if the monopoles are to the contrary non-Abelian, i.e. if they carry also color magnetic charge[2], the DSZ quantization condition generalizes to include such extra magnetic charges [59, 122, 123] and allows for a minimal $U(1)_\mathrm{M}$ magnetic charge similar to the one of pure QEMD: $g_0 = 2\pi/e$. In Chapter 6, we build an axion model with heavy PQ-charged fermions ψ_i carrying $SU(3)_\mathrm{M}$ color magnetic charges and show that it indeed solves the strong CP problem. In the explicit calculations, it will be convenient to parameterize the minimal magnetic charge g_0 by an integer ζ:

$$g_0 = \frac{2\pi\zeta}{e}\,, \quad \zeta = \begin{cases} 3\,, & \psi_i \in U(1)_\mathrm{E}\times U(1)_\mathrm{M}\times SU(3)_\mathrm{E} \\ 1\,, & \psi_i \in U(1)_\mathrm{E}\times U(1)_\mathrm{M}\times SU(3)_\mathrm{M} \end{cases}\,. \tag{5.7}$$

As we derived the axion-photon couplings (5.1) and (5.5) from general symmetry arguments, the field a entering our EFT need not be the QCD axion, but could as well correspond to a generic ALP. In this case, Eqs. (5.3), (5.4) and (5.6) need not hold. Nevertheless, the scaling of the corresponding ALP-photon couplings with electric and magnetic elementary charges e and g_0 given in Eqs. (5.3), (5.4) and (5.6)

[2]Note that contrary to the Abelian case, there are no non-Abelian dyons, i.e. particles which carry both color electric and color magnetic charges [119–121].

persists for any ALP, because with our normalisation of A_μ and B_μ four-potentials, the former four-potential always enters the interaction Lagrangian with a factor of e while the latter one always enters the interaction Lagrangian with a factor of g_0. This means that for a generic ALP, one still expects the above-mentioned hierarchy of couplings: $g_{a\mathrm{BB}} \gg |g_{a\mathrm{AB}}| \gg g_{a\mathrm{AA}}$.

5.2 Witten-effect induced axion interaction

Let us return to the discussion of the CP-conserving $\mathcal{O} = a\,(j_m \cdot A)$ operator of a generic axion EFT. The coefficient in front of this operator is determined by the discrete shift symmetry requirement. The corresponding term in the Lagrangian is obtained by the substitution $\theta \to a/v_a$ in Eq. (4.30):

$$\mathcal{L} \supset -\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\, j_m\right) \cdot A\,. \tag{5.8}$$

Indeed, the results on the periodicity of θ obtained in Chapter 4 sec. 4.4 show that the axion field has the required discrete shift symmetry $a \to a + 2\pi v_a n$, $n \in \mathbb{Z}$. Note also that, as we discussed in Chapter 4 sec. 4.4 for the analogous case of the θ-parameter, the latter symmetry would no longer be explicit if we were to redefine the fields and move the axion dependence into the kinetic part of the Lagrangian.

The term (5.8) is not gauge invariant unless $\partial_\mu a = 0$, which tells us that our axion EFT has to be modified. A way to restore the gauge invariance is to introduce a Stückelberg field ϕ into the Lagrangian:

$$\mathcal{L} \supset -\left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\, j_m\right) \cdot (A - \partial\phi)\ . \tag{5.9}$$

As we discussed in Chapter 4 sec. 4.5, such an extra degree of freedom ϕ arises naturally while considering the case of 't Hooft-Polyakov monopoles, where it plays the role of the dyon collective coordinate. Let us then consider the case where the interaction Lagrangian (5.9) corresponds to the low energy phase of a non-Abelian gauge theory. Comparing Eq. (5.9) with the θ-term (4.36) of the low energy phase, we see that the CP-violating θ-parameter of a non-Abelian theory is simply substituted with the axion field $\theta \to a/v_a$, so that the $a\,(j_m \cdot A)$ operator corresponds to the aGG^d operator at high energies. This means that the coupling (5.9) describes Witten-effect induced axion interactions. Indeed, as we discussed in Chapter 3 sec. 3.2, one expects the Witten-effect kind of coupling between axions and monopoles whenever one considers the spontaneously broken symmetry phase of a high energy non-Abelian theory with aGG^d term.

Contrary to the three anomalous axion couplings described in the previous section, the coupling (5.9) does not respect the continuous shift symmetry $a \to a + C$, where C is an arbitrary constant. This means that the latter coupling generates a non-flat contribution to the potential for the axion field. Since the axion coupling (5.9) arises in the low energy EFT of a non-Abelian theory with aGG^d interaction, such a contribution to the axion potential is not unexpected: in fact, it has to correspond to the potential created by instantons of the high energy non-Abelian theory. As it was discussed in the end of Chapter 4 sec. 4.5, explicit calculations [50, 115] support the latter correspondence. Note, however, that contrary to the claim made in Ref. [115], additional contribution to the axion potential need not arise in *every* theory of an axion coupled to an Abelian gauge field whenever there are monopoles magnetically charged under the latter field. The axion mass is generated *not* by magnetic monopoles, but always by instantons, even if these instantons happen to live on the monopole worldvolume in the low energy EFT. Indeed, consider the simplest example of a QEMD theory (4.4) which has no extra degrees of freedom. In such a theory, there cannot exist a consistent Witten-effect induced axion coupling, although there can exist the anomalous axion-photon couplings discussed in the previous section. Thus, such a theory has both axions and magnetic monopoles interacting with the Abelian gauge field, but no axion mass is generated through these interactions, which contradicts the statement of Ref. [115]. In general, we see that the anomalous axion-photon couplings and the Witten-effect induced axion coupling are independent. The Witten-effect induced coupling arises only in theories which have instanton degrees of freedom, e.g. in the spontaneously broken symmetry phase of a high energy non-Abelian gauge theory.

5.3 Axion Maxwell equations

Having analyzed different axion-photon interactions in the previous two sections, we are now ready to collect them all together in a generic axion-photon EFT Lagrangian:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2n^2}\Big\{[n\cdot(\partial\wedge B)]\cdot[n\cdot(\partial\wedge A)^d] \;-\; [n\cdot(\partial\wedge A)]\cdot[n\cdot(\partial\wedge B)^d] \;- \\ & [n\cdot(\partial\wedge A)]^2 \;-\; [n\cdot(\partial\wedge B)]^2\Big\} \;-\; \frac{1}{4}\,g_{a\mathrm{AA}}\,a\,\mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge A)^d\right\} \;- \\ & \frac{1}{4}\,g_{a\mathrm{BB}}\,a\,\mathrm{tr}\left\{(\partial\wedge B)\,(\partial\wedge B)^d\right\} \;-\frac{1}{2}\,g_{a\mathrm{AB}}\,a\,\mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge B)^d\right\} \;- \\ & \left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\,j^{\phi}_m\right)\cdot(A-\partial\phi) \;-\; j_m\cdot B \;+\; \mathcal{L}_G\;, \end{aligned} \tag{5.10}$$

or written in a more compact operator notation:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}\Big(y_+ + \alpha_1 + \beta_1 - g_{a\mathrm{AA}}\,a\,a_- - g_{a\mathrm{BB}}\,a\,b_- - 2\,g_{a\mathrm{AB}}\,a\,y_-\Big) \;- \\ & \left(\bar{j}_e + \frac{e^2 a}{4\pi^2 v_a}\,j^{\phi}_m\right)\cdot(A-\partial\phi) - j_m\cdot B + \mathcal{L}_G\;, \end{aligned} \tag{5.11}$$

where we denoted the part of the magnetic current j_m carrying an instanton degree of freedom ϕ by j^{ϕ}_m. For instance, j^{ϕ}_m can correspond to a current of 't Hooft-Polyakov monopoles. Let us remind the reader that $\mathcal{L}_G$ is the gauge-fixing Lagrangian given by Eq. (4.5), $\bar{j}_e$ is the part of the electric current which is quantized in units of elementary electric charge and y_+, α_1, β_1, a_-, b_-, y_- are the QEMD operators defined in sec. 4.3. Note that since we derived the Lagrangian (5.11) from general symmetry arguments, the field a entering our EFT need not be the QCD axion, but could as well correspond to a generic ALP.

Let us derive the classical equations of motion corresponding to the Lagrangian (5.11). For this, we follow the standard procedure outlined in Chapter 4 sec. 4.2. Varying over the two four-potentials, we obtain:

$$\begin{aligned} & \frac{n\cdot\partial}{n^2}\left(n\cdot\partial A^{\mu} \;-\; \partial^{\mu} n\cdot A \;-\; n^{\mu}\partial\cdot A \;-\; \epsilon^{\mu}{}_{\nu\rho\sigma}n^{\nu}\partial^{\rho}B^{\sigma}\right) - \\ & \qquad g_{a\mathrm{AA}}\,\partial_{\nu}a\,\left\{(\partial\wedge A)^d\right\}^{\nu\mu} - g_{a\mathrm{AB}}\,\partial_{\nu}a\,\left\{(\partial\wedge B)^d\right\}^{\nu\mu} - \frac{e^2 a}{4\pi^2 v_a}\,j^{\phi\,\mu}_m \;=\; \bar{j}^{\,\mu}_e\;, \end{aligned} \tag{5.12}$$

$$\frac{n\cdot\partial}{n^2}\left(n\cdot\partial B^\mu \;-\; \partial^\mu n\cdot B \;-\; n^\mu\partial\cdot B \;-\; \epsilon^\mu{}_{\nu\rho\sigma}n^\nu\partial^\rho A^\sigma\right) -$$
$$g_{a\mathrm{BB}}\,\partial_\nu a\left\{(\partial\wedge B)^d\right\}^{\nu\mu} - g_{a\mathrm{AB}}\,\partial_\nu a\left\{(\partial\wedge A)^d\right\}^{\nu\mu} \;=\; j_m^\mu\,. \tag{5.13}$$

Then, transitioning to the description in terms of the field strength tensor F, we find the following axion Maxwell equations:

$$\partial_\mu F^{\mu\nu} - g_{a\mathrm{AA}}\,\partial_\mu a\,F^{d\,\mu\nu} + g_{a\mathrm{AB}}\,\partial_\mu a\,F^{\mu\nu} - \frac{e^2 a}{4\pi^2 v_a}\,j_m^{\phi\,\nu} \;=\; \bar{j}_e^{\,\nu}\,, \tag{5.14}$$
$$\partial_\mu F^{d\,\mu\nu} + g_{a\mathrm{BB}}\,\partial_\mu a\,F^{\mu\nu} - g_{a\mathrm{AB}}\,\partial_\mu a\,F^{d\,\mu\nu} \;=\; j_m^{\,\nu}\,. \tag{5.15}$$

Note that the terms proportional to $(n\cdot\partial)^{-1}\,(n\wedge j_m)^{\mu\nu}$ and $(n\cdot\partial)^{-1}\,(n\wedge j_e)^{\mu\nu}$ do not contribute to the classical equations of motion, as it was discussed in Chapter 4 sec. 4.2 [3]. Eqs. (5.14) and (5.15) are to be supplemented by the following equation of motion for the axion field:

$$\left(\partial^2 - m_a^2\right) a \;=\; -\frac{1}{4}\,\left(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}\right) F_{\mu\nu}F^{d\,\mu\nu} - \frac{1}{2}\,g_{a\mathrm{AB}}F_{\mu\nu}F^{\mu\nu}, \tag{5.16}$$

where the right-hand side is obtained by varying the Lagrangian (5.11) with respect to the axion field and transitioning to the description in terms of the field strength tensor F. According to the discussion of the previous section, the axion mass m_a receives an additional contribution from the Witten-effect induced interaction in the case $j_m^\phi \neq 0$.

Let us now bring Eqs. (5.14), (5.15) and (5.16) into the form convenient for their experimental study. First, we set $j_m = j_m^\phi = 0$, since there are no magnetic monopoles in the laboratory. Second, we expand the electromagnetic field in powers of the anomalous axion-photon couplings $g_{a\mathrm{AA}}$, $g_{a\mathrm{BB}}$ and $g_{a\mathrm{AB}}$, keeping only the zeroth and the first orders $F = F_0 + F_a$. At zeroth order, Eqs. (5.14), (5.15) and (5.16) decouple and give the ordinary Maxwell equations as well as the homogeneous Klein-Gordon equation for the axion field. At first order, Eqs. (5.14), (5.15) and (5.16)

[3] That such terms cannot contribute to the classical equations of motion is also clear from the fact that the interaction of axions with electromagnetic field cannot depend on the kind of currents sourcing the latter field: for instance, in a given setting the axion field could be causally disconnected from these currents. In fact, one can obtain the axion Maxwell equations (5.17), (5.18) and (5.19) by using an even simpler two-potential framework of Ref. [39] which does not involve currents at all.

yield:

$$\partial_\mu F_a^{\mu\nu} - g_{a\mathrm{AA}}\, \partial_\mu a\, F_0^{d\,\mu\nu} + g_{a\mathrm{AB}}\, \partial_\mu a\, F_0^{\mu\nu} \;=\; 0\,, \tag{5.17}$$

$$\partial_\mu F_a^{d\,\mu\nu} + g_{a\mathrm{BB}}\, \partial_\mu a\, F_0^{\mu\nu} - g_{a\mathrm{AB}}\, \partial_\mu a\, F_0^{d\,\mu\nu} \;=\; 0\,, \tag{5.18}$$

$$\left(\partial^2 - m_a^2\right) a \;=\; -\frac{1}{4}\left(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}\right) F_{0\,\mu\nu} F_0^{d\,\mu\nu} - \frac{1}{2}\, g_{a\mathrm{AB}} F_{0\,\mu\nu} F_0^{\mu\nu}\,, \tag{5.19}$$

so that F_a is an axion-induced part of the electromagnetic field sourced by the following effective electric and magnetic currents:

$$j^\nu_{e,\,\mathrm{eff}} = g_{a\mathrm{AA}}\, \partial_\mu a\, F_0^{d\,\mu\nu} - g_{a\mathrm{AB}}\, \partial_\mu a\, F_0^{\mu\nu}\,, \tag{5.20}$$

$$j^\nu_{m,\,\mathrm{eff}} = -g_{a\mathrm{BB}}\, \partial_\mu a\, F_0^{\mu\nu} + g_{a\mathrm{AB}}\, \partial_\mu a\, F_0^{d\,\mu\nu}\,, \tag{5.21}$$

which depend on the external field F_0 created in the laboratory. Eqs. (5.20) and (5.21) extend the results of the axion EFT of Ref. [91], which yields

$$j^\nu_{e,\,\mathrm{eff}} = g_{a\mathrm{AA}}\, \partial_\mu a\, F_0^{d\,\mu\nu} \quad \text{and} \quad j^\nu_{m,\,\mathrm{eff}} = 0\,. \tag{5.22}$$

As we discussed in sec. 5.1, in an axion model with a generic spectrum of heavy PQ-charged dyons the couplings satisfy $g_{a\mathrm{BB}} \gg |g_{a\mathrm{AB}}| \gg g_{a\mathrm{AA}}$, so that the additional terms we obtain dominate the conventional contribution to the effective currents.

Finally, in terms of electric and magnetic fields, Eqs. (5.17), (5.18) and (5.19) are given by the following expressions:

$$\boldsymbol{\nabla}\times\mathbf{H}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{AA}}\left(\mathbf{E}_0\times\boldsymbol{\nabla}a - \dot{a}\mathbf{H}_0\right) + g_{a\mathrm{AB}}\left(\mathbf{H}_0\times\boldsymbol{\nabla}a + \dot{a}\mathbf{E}_0\right)\,, \tag{5.23}$$

$$\boldsymbol{\nabla}\times\mathbf{E}_a + \dot{\mathbf{H}}_a = -g_{a\mathrm{BB}}\left(\mathbf{H}_0\times\boldsymbol{\nabla}a + \dot{a}\mathbf{E}_0\right) - g_{a\mathrm{AB}}\left(\mathbf{E}_0\times\boldsymbol{\nabla}a - \dot{a}\mathbf{H}_0\right)\,, \tag{5.24}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H}_a = -g_{a\mathrm{BB}}\, \mathbf{E}_0\cdot\boldsymbol{\nabla}a + g_{a\mathrm{AB}}\, \mathbf{H}_0\cdot\boldsymbol{\nabla}a\,, \tag{5.25}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}_a = g_{a\mathrm{AA}}\, \mathbf{H}_0\cdot\boldsymbol{\nabla}a - g_{a\mathrm{AB}}\, \mathbf{E}_0\cdot\boldsymbol{\nabla}a\,, \tag{5.26}$$

$$\left(\partial^2 - m_a^2\right) a \;=\; \left(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}\right) \mathbf{E}_0\cdot\mathbf{H}_0 + g_{a\mathrm{AB}}\left(\mathbf{E}_0^2 - \mathbf{H}_0^2\right)\,, \tag{5.27}$$

where $\mathbf{E}_a$ and $\mathbf{H}_a$ are axion-induced electric and magnetic fields, while $\mathbf{E}_0$ and $\mathbf{H}_0$ are background electric and magnetic fields created in the detector. Note that to study the propagation of light with Eqs. (5.23)–(5.27), it is convenient not to perform the expansion of the electromagnetic fields $\mathbf{E}_\gamma$ and $\mathbf{H}_\gamma$, in which case the equations are the same, but with the substitutions $\mathbf{E}_a$, $\mathbf{E}_0 \rightarrow \mathbf{E}_\gamma$ and $\mathbf{H}_a$, $\mathbf{H}_0 \rightarrow \mathbf{H}_\gamma$.

This Chapter is written based on the publication [1] of the author of this thesis.

Chapter 6

General hadronic axion

6.1 KSVZ-like models are intrinsically biased

Normally, construction of the KSVZ-like axion models proceeds as follows. One introduces a complex scalar field Φ, which breaks the $U(1)_{\mathrm{PQ}}$ symmetry spontaneously after relaxing to its non-zero vacuum expectation value $v_a/\sqrt{2}$. For consistency with observations, v_a must correspond to some high energy scale. Moreover, in order to solve the strong CP problem, the $U(1)_{\mathrm{PQ}}$ symmetry has to be color anomalous. This is achieved via introducing a new vector-like colored fermion $\psi = \psi_L + \psi_R$, so that $U(1)_{\mathrm{PQ}}$ acts differently on the two chirality components of ψ. These requirements lead naturally to the following Lagrangian:

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + y\left(\Phi\,\bar{\psi}_L \psi_R + \mathrm{h.c.}\right) - \lambda_\Phi \left(|\Phi|^2 - \frac{v_a^2}{2}\right)^2, \tag{6.1}$$

where y and λ_Φ are some dimensionless constants and $\mathcal{D}_\mu$ is a covariant derivative encoding the interaction of ψ with the gauge fields of the Standard model. Note that as a result of the spontaneous symmetry breaking ψ gets a mass $m = yv_a/\sqrt{2}$, which is very large for reasonable values of y. What remains as a low energy degree of freedom is a pseudo Nambu-Goldstone boson of the spontaneous symmetry breaking a – the axion – which can be thought of as an angular mode of Φ. The axion interacts with the gauge fields of the Standard model through loops of ψ. In particular, as it was discussed in Chapter 2, at low energies where the relevant gauge bosons are photons and gluons, these interactions are given by the axion field times the expressions for axial electromagnetic and color anomalies, respectively, with some coefficients:

$$\mathcal{L}_a \supset -\frac{1}{4}\, g^0_{a\gamma\gamma}\, a\, F^{\mu\nu} F^d_{\mu\nu} - \frac{a g_s^2}{32\pi^2 f_a}\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}\,, \tag{6.2}$$

where

$$f_a = \frac{v_a}{2N}\,, \quad g^0_{a\gamma\gamma} = \frac{E}{N}\cdot\frac{e^2}{8\pi^2 f_a}\,, \tag{6.3}$$

and the parameters E and N are called electromagnetic and color anomaly coefficients, respectively. They depend on the representation of ψ under the gauge symmetries of the Standard model. Since the latter representation is unknown, E and N can in principle vary considerably, which gives rise to an uncertainty in the parameter $g^0_{a\gamma\gamma}$ [124]. This uncertainty translates into a band on the plot of possible axion-photon couplings $g_{a\gamma\gamma}$ versus axion mass m_a, see fig. 6.1, due to the following relations[1]:

$$g_{a\gamma\gamma} = g^0_{a\gamma\gamma} - \frac{e^2}{12\pi^2 f_a}\cdot\frac{4m_d + m_u}{m_u + m_d}\,, \quad m_a = \frac{m_\pi f_\pi \sqrt{m_u m_d}}{(m_u + m_d)\, f_a}\,. \tag{6.4}$$

Let us now elaborate why the construction presented in the previous paragraph contains an implicit far-reaching assumption. While allowing the new heavy fermion ψ to be charged under any possible gauge symmetry at hand is indeed the most generic option, it is not consistently implemented in the KSVZ-like models. The reason is that these models consider only electric representations of the gauge groups. Meanwhile, as it was shown back in 1931 by Dirac [21], gauge interactions in the quantum theory need not be limited to the electric ones: gauge charges can be electric as well as magnetic. Although we do not see magnetic charges at low energies, their existence is actually indirectly evidenced by the quantization of the electric charge observed in nature. Indeed, as Dirac found, quantum theory requires that the electric gauge charge e is related to the magnetic one g as follows: $eg = 2\pi n\,, n \in \mathbb{Z}$, so that the charges are quantized. Moreover, as it was advocated in ref. [23] and mentioned in Chapter 1, there is a mounting evidence supporting the conjecture that charge quantization is not only a necessary but also a sufficient condition for the magnetic monopoles to exist. Quantum field theory coupled to gravity suggests that any possible electric or magnetic charge should have a physical realization. The

[1] m_u, m_d, m_π are masses of u-quark, d-quark and pion, respectively; f_π is pion decay constant.

construction of KSVZ-like models is thus too restrictive, so that the gauge charges assigned to the heavy fermion ψ are not generic and the resulting predictions for axion couplings are biased.

6.2 Axion-photon coupling from the axial anomaly of magnetic currents

Let us now relax the above-mentioned assumption on the representations of the heavy fermion ψ under the gauge groups of the Standard model and consider a truly generic setting. In this setting, ψ is a dyon, i.e. a particle carrying both electric and magnetic charges. We limit our investigation to axion phenomenology at low energies, so that the relevant gauge interactions are electromagnetic and color ones. For simplicity, in this section we do not consider the case where ψ has a color magnetic charge, which is studied in detail later in this Chapter. Since this means we do not modify the strong sector of the model, phenomenology of the strong interactions of axions is fully analogous to the one in the KSVZ model: in particular, the strong CP problem is solved and the relation between axion mass m_a and decay constant f_a is standard. What is modified are axion-photon interactions. Let us proceed to derive the latter interactions from the UV theory with the Lagrangian given by eq. (6.1). As it was shown by Zwanziger [40], a local quantum field theory with both electric and magnetic charges necessarily involves two vector-potentials, A_μ and B_μ, each having the standard coupling to the corresponding current, electric or magnetic, respectively. The covariant derivative entering eq. (6.1) is thus $\mathcal{D}_\mu = \partial_\mu - e_0\, q_e A_\mu - g_0\, q_m B_\mu$, where $e_0 = e/3$ and g_0 are elementary electric and magnetic charges, respectively. Due to the Dirac quantization condition, $g_0 = 2\pi/e_0$.

Below the PQ symmetry breaking scale, one can expand

$$\Phi = (v_a + \rho)\exp{(ia/v_a)}/\sqrt{2}\,, \tag{6.5}$$

where ρ is a heavy radial mode and a is a pseudo Nambu-Goldstone boson (axion). The terms in the resulting Lagrangian which are relevant for the low energy phenomenology are:

$$\mathcal{L} \;\supset\; i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + \frac{y v_a}{\sqrt{2}} \left\{ \exp\left(\frac{ia}{v_a}\right) \bar{\psi}_L \psi_R + \text{h.c.} \right\} . \tag{6.6}$$

We perform then an axial rotation of the fermion $\psi \to \exp\left(ia\gamma_5/2v_a\right) \cdot \psi$, after which there arise an anomalous term $\mathcal{L}_\text{F}$ from the transformation of the measure of the path integral and a derivative coupling of a to the axial current of ψ:

$$\mathcal{L} \;\supset\; i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + \frac{y v_a}{\sqrt{2}}\, \bar{\psi}\psi - \frac{\partial^\mu a}{2 v_a}\, \bar{\psi}\gamma_\mu \gamma_5 \psi - \mathcal{L}_\text{F} . \tag{6.7}$$

The axial anomaly in a theory with dyons was found in ref. [111], from where one infers the expression for the anomalous term:

$$\mathcal{L}_\text{F} \;=\; \frac{a \cdot d(C_\psi)}{16\pi^2 v_a} \cdot \left\{ \left(e_0^2 q_e^2 - g_0^2\, q_m^2 \right) F^{\mu\nu} F^d_{\mu\nu} - 4\pi q_m q_e F_{\mu\nu} F^{\mu\nu} \right\} . \tag{6.8}$$

The dimension $d(C_\psi)$ of the color representation of ψ in the numerator on the right-hand side comes from summing over the color indices. We corrected a mistake which crawled into the coefficient of the last term on the right-hand side in the original expression for the anomalous current in ref. [111]. This term is CP-violating and arises only if there is CP-violation in the UV theory. As it was shown in Chapter 4, a generic theory with dyons does violate CP, unless for any dyon with charges (q_e, q_m) there is another dyon with charges (q_e, $-q_m$). Anyway, be there CP-violation or not, it is easy to see that the second term on the right-hand side of eq. (6.8) dominates over all other terms: if we assume $q_e, q_m \sim 1$, its coefficient is by a factor of $4\pi^2/e_0^4$ larger than the coefficient in front of the first term and by a factor of π/e_0^2 – than the coefficient in front of the third term. At the leading order, as long as $m = y v_a/\sqrt{2}$ is very large, the derivative axion coupling from eq. (6.7) does not contribute to the low energy Lagrangian of axion-photon interactions, for the same reason as in the KSVZ-like models[2]. The leading effect in the interactions between axion and photons is thus given by the following Lagrangian:

$$\mathcal{L}_{a\gamma} = \frac{1}{4}\, \tilde{g}_{a\gamma\gamma}\, a F^{\mu\nu} F^d_{\mu\nu}, \quad \tilde{g}_{a\gamma\gamma} = \frac{q_m^2\, d(C_\psi)\, g_0^2}{4\pi^2 v_a} , \tag{6.9}$$

[2] The reason is the behaviour of the Ward identities for the correlation functions which involve the axial current of ψ in the limit $m \to \infty$.

where we took into account that the contribution to the axion-photon coupling $\tilde{g}_{a\gamma\gamma}$ from the strong sector, see eq. (6.4), is now absolutely negligible. The axion-photon coupling can be rewritten in terms of the magnetic anomaly coefficients M_ψ:

$$\tilde{g}_{a\gamma\gamma} = \frac{M}{N} \cdot \frac{g_0^2}{8\pi^2 f_a}\,, \qquad M = \sum_\psi M_\psi = \sum_\psi q_m^2(\psi) \cdot d(C_\psi)\ , \tag{6.10}$$

where $q_m(\psi) \in \mathbb{Z}$ are magnetic charges of new heavy fermions ψ.

The effective axion-photon Lagrangian (6.8) is however not fully satisfactory, as it seems to lack the axion shift symmetry $a \to a + 2\pi n v_a$, $n \in \mathbb{Z}$. The reason for this is simple. As it was argued in Chapter 3, to obtain reliable EFT from the UV model containing magnetic monopoles and dyons, one has to stick to the QEMD formalism discussed in Chapter 4 until the very end of the calculation. In fact, the fully consistent EFT derived from the Zwanziger theory has to contain two four-potentials, not one as in Eq. (6.8). Let us proceed to the next section where we derive the effective Lagrangian using the Zwanziger theory from start till finish and show that the relevant anomaly coefficients are given by the expressions derived in this section.

6.3 Calculation of the anomaly coefficients

Let us calculate the anomaly coefficients E, M and D, which enter the expressions (5.3), (5.4) and (5.6) for the axion-photon couplings. We start with a high-energy QEMD theory and integrate out heavy fermions ψ carrying PQ charges. In the PQ-symmetric phase, the Lagrangian includes the following terms for each of the fermions ψ:

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + y\left(\Phi\, \bar{\psi}_L \psi_R + \text{h.c.}\right)\,, \tag{6.11}$$

where y is a dimensionless Yukawa constant, $\mathcal{D}_\mu = \partial_\mu - e_0\, q_\psi A_\mu - g_0\, g_\psi B_\mu$ is a covariant derivative including both magnetic and electric four-potentials multiplied by the corresponding electric and magnetic charges, and Φ is the PQ complex scalar field.

As in the previous section, for energies below the PQ scale, we expand

$$\Phi = (v_a + \rho) \exp\left(ia/v_a\right)/\sqrt{2}\,, \tag{6.12}$$

perform an axial rotation of the fermion ψ, and obtain

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + \frac{y v_a}{\sqrt{2}}\,\bar{\psi}\psi - \frac{\partial^\mu a}{2 v_a}\,\bar{\psi}\gamma_\mu\gamma_5\psi - \mathcal{L}_{\mathrm{F}}\,. \tag{6.13}$$

The fermion ψ gets its mass $m = y v_a/\sqrt{2}$, which we assume to be large compared to the energy scales probed in experiments. In the large m limit, the derivative axion coupling from eq. (6.13) does not contribute to the low energy Lagrangian of axion-photon interactions due to the Sutherland-Veltman theorem [125, 126]. The axion-photon couplings are thus given by the anomalous terms $\mathcal{L}_{\mathrm{F}}$ which can be calculated using the Fujikawa method [127]. In particular, following Fujikawa, we apply the gauge invariant heat kernel regularization to the path integral measure which yields:

$$\mathcal{L}_{\mathrm{F}} \;=\; -\frac{a}{v_a}\cdot \lim_{\substack{\Lambda\to\infty \\ x\to y}} \mathrm{tr}\left\{\gamma_5 \exp\left(\not{\mathcal{D}}^2/\Lambda^2\right)\delta^4(x-y)\right\}. \tag{6.14}$$

We then notice that the commutators eqs. (4.1) and (4.2) vanish if the four-potentials are taken at the same space-time point. This means that these commutators do not contribute to the expression for $\not{\mathcal{D}}^2$, which becomes:

$$\not{\mathcal{D}}^2 = \mathcal{D}^2 - i\gamma_\mu\gamma_\nu\left(e_0 q_\psi\,\partial^\mu A^\nu + g_0 g_\psi\,\partial^\mu B^\nu\right). \tag{6.15}$$

It is then convenient to express the delta-function as a superposition of plane waves: $\delta^4(x-y) = \int d^4k\, e^{ik(x-y)}/\left(2\pi\right)^4$, each of which shifts the derivative operator $\mathcal{D}_\mu \to \mathcal{D}_\mu + ik_\mu$. Taking into account eq. (6.15), we obtain:

$$\begin{aligned}\mathcal{L}_{\mathrm{F}} \;=\; -\frac{a}{v_a}\cdot \lim_{\Lambda\to\infty}\int \frac{d^4k}{\left(2\pi\right)^4}\,\mathrm{tr}\Big\{\gamma_5 \exp\Big(&-i\gamma_\mu\gamma_\nu\left(e_0 q_\psi\,\partial^\mu A^\nu + g_0 g_\psi\,\partial^\mu B^\nu\right)/\Lambda^2 + \\ &\left(\mathcal{D}+ik\right)^2/\Lambda^2\Big)\Big\}\,. \end{aligned}\tag{6.16}$$

Any terms in the integrand which are $o(1/\Lambda^4)$ vanish after performing the integration and sending $\Lambda \to \infty$. Taylor expanding the exponent, we are then left with a finite number of terms, which after taking the trace, the integral and the limit simplify into:

$$\mathcal{L}_\text{F} \;=\; \frac{a\, d(C_\psi)}{8\pi^2 v_a} \cdot \epsilon_{\mu\nu\lambda\rho}\Big(e_0 q_\psi\, \partial^\mu A^\nu + g_0 g_\psi\, \partial^\mu B^\nu\Big)\left(e_0 q_\psi\, \partial^\lambda A^\rho + g_0 g_\psi\, \partial^\lambda B^\rho\right), \tag{6.17}$$

where $d(C_\psi)$ is the dimension of the color representation of ψ. Using the notations of sec. 4, eq. (6.17) can be rewritten as follows:

$$\mathcal{L}_\text{F} \;=\; -\frac{a\, d(C_\psi)}{16\pi^2 v_a} \cdot \Big(q_\psi^2 e_0^2\, \mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge A)^d\right\} + g_\psi^2 g_0^2\, \mathrm{tr}\left\{(\partial\wedge B)\,(\partial\wedge B)^d\right\} + 2\, q_\psi g_\psi\, e_0 g_0\, \mathrm{tr}\left\{(\partial\wedge A)\,(\partial\wedge B)^d\right\}\Big). \tag{6.18}$$

If there are multiple dyons ψ, each of them gives a similar contribution to the resulting axion-photon Lagrangian. Thus, the expressions for the axion-photon couplings $g_{a\mathrm{AA}}, g_{a\mathrm{BB}}, g_{a\mathrm{AB}}$ and the corresponding anomaly coefficients E, M, D are:

$$g_{a\mathrm{AA}} = \frac{E e_0^2}{4\pi^2 v_a}, \quad E = \sum_\psi q_\psi^2 \cdot d(C_\psi)\,, \tag{6.19}$$

$$g_{a\mathrm{BB}} = \frac{M g_0^2}{4\pi^2 v_a}, \quad M = \sum_\psi g_\psi^2 \cdot d(C_\psi)\,, \tag{6.20}$$

$$g_{a\mathrm{AB}} = \frac{D e_0 g_0}{4\pi^2 v_a}, \quad D = \sum_\psi q_\psi g_\psi \cdot d(C_\psi)\,. \tag{6.21}$$

Note that these expressions for the anomaly coefficients agree with the results of the previous section.

6.4 Solution to the strong CP problem

Now that we derived the effective Lagrangian and explicit expressions for the axion-photon couplings in the case of Abelian magnetic charges, it is time to consider the case where the heavy magnetic monopoles carry also non-Abelian magnetic charges, in particular if they are magnetically charged under the $SU(3)_c$ of QCD. We remind the reader that we had a general discussion of the non-Abelian magnetic monopoles in Chapter 1 sec. 1.3.

Suppose there exist a vector-like fermionic magnetic monopole $\psi = \psi_L + \psi_R$ which transforms under an anomalous PQ symmetry $U(1)_{PQ}$ [6, 7] and a complex scalar field Φ which breaks the PQ symmetry spontaneously at some high energy scale v_a. For a moment, assume that ψ transforms in a fundamental representation of the QCD gauge group, i.e. it is a new quark. As far as we do not consider the electromagnetic interaction, such model with a new quark is an exact analog of the KSVZ axion model and thus it provides a solution to the strong CP problem in the same way the KSVZ model does. The aim of this section is then to show that the model with the non-Abelian color-magnetic monopole solves the strong CP problem as well. The high-energy Lagrangian in this case includes the following terms:

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu\partial_\mu\psi + \bar{\psi}\gamma^\mu C_\mu\psi + y\left(\Phi\,\bar{\psi}_L\psi_R + \text{h.c.}\right) - \lambda_\Phi\left(|\Phi|^2 - \frac{v_a^2}{2}\right)^2, \tag{6.22}$$

where C_μ is a connection on a GNO group $SU(3)$ multiplied by the corresponding magnetic coupling: $C_\mu = g_{\text{m}}\, t_a\, C_\mu^a$. In the broken phase, there exists a pseudo Goldstone boson a (axion), which can be introduced via the polar decomposition of the PQ scalar field $\Phi = \frac{1}{\sqrt{2}}\,(v_a + \sigma)\cdot\exp\left(-ia/v_a\right)$ near the vacuum. Let us dispose of the axion dependence in the Yukawa term by performing a chiral rotation of the fermions $\psi \to \exp\left(ia\gamma_5/2v_a\right)\cdot\psi$. Omitting the terms containing a heavy radial field σ, one then obtains:

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu\partial_\mu\psi + \bar{\psi}\gamma^\mu C_\mu\psi + \frac{y v_a}{\sqrt{2}}\,\bar{\psi}\psi - \frac{\partial^\mu a}{2v_a}\,\bar{\psi}\gamma_\mu\gamma_5\psi + \mathcal{L}_{\text{F}}, \tag{6.23}$$

where $\mathcal{L}_{\text{F}}$ is a Fujikawa contribution coming from the transformation of the fermion measure in the path integral, i.e. the density of the index of the Dirac operator $\gamma^\mu D_\mu = \gamma^\mu(\partial_\mu - C_\mu)$. By the Atiyah-Singer index theorem, the latter is equal to the characteristic class of the GNO group bundle, so that:

$$\mathcal{L}_{\text{F}} = \frac{a}{16\pi^2 v_a}\,\text{tr}\, C^{\mu\nu}C^d_{\mu\nu}, \tag{6.24}$$

where $C_{\mu\nu}$ is the curvature of the GNO group connection and $C^d_{\mu\nu} = \epsilon_{\mu\nu\lambda\rho}\, C^{\lambda\rho}/2$, $\epsilon^{0123} \equiv 1$.

In order to see that such a model provides a solution to the strong CP problem, we invoke Abelian gauge fixing introduced by 't Hooft [128]. In the Abelian gauges there arise singularities corresponding to effective color magnetic currents which result in the violation of the non-Abelian Bianchi identities (VNABI) [129]. The time reversal violating term of the QCD action can then be expanded as follows:

$$\mathcal{S}_{\rm QCD} \supset -\frac{\bar\theta g_s^2}{32\pi^2}\int d^4x \sum_{a=1}^{8} G^{a\,\mu\nu}G^{d\,a}_{\mu\nu} = -\frac{\bar\theta g_s^2}{32\pi^2}\times$$

$$\left\{\int d^4x\ \epsilon_{\mu\nu\lambda\rho}\,\partial^\mu \sum_{a,b,c=1}^{8}\left(A_a^\nu\,G_a^{\lambda\rho} - \frac{1}{3}\,g_s f_{abc}\,A_a^\nu A_b^\lambda A_c^\rho\right) - 2\int d^4x \sum_{a=1}^{8} A_\nu^a\left(D_\mu G^{d\,\mu\nu}\right)_a\right\}, \qquad (6.25)$$

where $G^a_{\mu\nu}$ (A^a_μ) are components of the non-Abelian field strength tensor $G_{\mu\nu}$ (four-potential A_μ) of QCD, f_{abc} are $\mathfrak{su}(3)$ structure constants, $\bar\theta$ is QCD vacuum angle, $G^{d\,a}_{\mu\nu} = \epsilon_{\mu\nu\lambda\rho}\,G^{a\,\lambda\rho}/2$.

Let us consider the first term on the right-hand side of Eq. (6.25). Since all the singularities characteristic of the Abelian 't Hooft gauges arise in the diagonal part of the gluon field, i.e. in the components A^3_μ and A^8_μ, the terms of the integrand which contain solely off-diagonal fields can be safely integrated with the use of the Stokes theorem:

$$\int d^4x\ \epsilon_{\mu\nu\lambda\rho}\,\partial^\mu \sum_{a,b,c=1}^{8}\left(A_a^\nu\,G_a^{\lambda\rho} - \frac{1}{3}\,g_s f_{abc}\,A_a^\nu A_b^\lambda A_c^\rho\right) =$$

$$\int d^4x\ \epsilon_{\mu\nu\lambda\rho}\,\partial^\mu \sum_{\alpha=3,8}\ \sum_{b,c=1}^{8}\left(A_\alpha^\nu \mathscr{G}_\alpha^{\lambda\rho} + 2\,g_s f_{\alpha bc}\,A_\alpha^\nu A_b^\lambda A_c^\rho\right) + \int\limits_{\Omega_\infty} dS^\mu\,\mathfrak{K}_\mu\left[A_{\text{off-diag}}\right], \qquad (6.26)$$

where $\mathscr{G}_\alpha^{\mu\nu} = \partial^\mu A_\alpha^\nu - \partial^\nu A_\alpha^\mu$ ($\alpha = 3, 8$) are Abelian field strength tensors. As it is derived both from theoretical considerations [130] and lattice calculations [131], in the Abelian gauges off-diagonal gluons obtain finite mass, which means that the functional $\mathfrak{K}_\mu\left[A_{\text{off-diag}}\right]$ vanishes at the surface at infinity, Ω_∞. For the same reason the integrand in Eq. (6.26) proportional to $\partial^\mu(A_\alpha^\nu A_b^\lambda A_c^\rho)$ is restricted to arbitrarily small surfaces around the singularities after application of the Stokes theorem and finally integrates to zero due to regularity of the off-diagonal fields.

Equation (6.25) can now be rewritten in the following way:

$$\int d^4x \sum_{a=1}^{8} G^{a\,\mu\nu}G^{d\,a}_{\mu\nu} = \int d^4x \sum_{\alpha=3,8} \mathscr{G}^{\alpha\,\mu\nu}\mathscr{G}^{d\,\alpha}_{\mu\nu} +$$

$$2\int d^4x\left(\sum_{\alpha=3,8} A_{\alpha\,\nu}\partial_\mu \mathscr{G}^{d\,\mu\nu}_{\alpha} - \sum_{a=1}^{8} A_{a\,\nu}\left(D_\mu G^{d\,\mu\nu}\right)_a\right). \tag{6.27}$$

Let us show that the VNABI, $D_\mu G^{d\,\mu\nu}$, is diagonal in color space, so that the second row in Eq. (6.27) equals to zero. First, note that the only contribution to VNABI comes from singularities, where topological defects associated with the monopoles hamper commutation of partial derivatives, so that in the expression for a commutator of covariant derivatives,

$$[D_\rho, D_\lambda] = -iG_{\rho\lambda} + [\partial_\rho, \partial_\lambda]\,, \tag{6.28}$$

the second term on the right does not vanish. After taking advantage of Eq. (6.28) and Jacobi identities for partial as well as covariant derivatives, the expression for VNABI can be simplified [132]:

$$D_\mu G^{d\,\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\rho\lambda}\left[D^\mu, G^{\rho\lambda}\right] = \frac{1}{2}\,\epsilon_{\mu\nu\rho\lambda}\left[\partial^\rho, \partial^\lambda\right] A^\mu = \partial_\rho \mathscr{G}^{d\,\rho\nu}, \tag{6.29}$$

where in the last step only diagonal gluons survive. One can see that the diagonal form of VNABI is ensured by its linearity in the A_μ field. We note that the second term on the right-hand side of Eq. (6.25) is then nothing but a manifestation of the Witten effect [104]: QCD monopoles are dyons with color electric charges proportional to the vacuum angle $\bar\theta$.

Due to the identities Eqs. (6.27) and (6.29) the CP violating term of the QCD Lagrangian reduces in the Abelian gauges to

$$-\frac{\bar\theta g_s^2}{32\pi^2}\sum_{\alpha=3,8} \mathscr{G}^{\alpha\,\mu\nu}\mathscr{G}^{d\,\alpha}_{\mu\nu}\,, \tag{6.30}$$

which involves now only Abelian four-potentials. By the analogous transformation of the Fujikawa contribution (6.24) to the axion Lagrangian (6.23), i.e. choosing the same Abelian gauge in the GNO gauge group, one obtains the term for the interaction of the axion with the Abelian dual four-potentials:

$$\mathcal{L}_{\mathrm{F}} = \frac{a g_{\mathrm{m}}^2}{32\pi^2 v_a}\sum_{\alpha=3,8} C^{\alpha\,\mu\nu} C^{d\,\alpha}_{\mu\nu}, \tag{6.31}$$

where the axion field is assumed to be constant and homogeneous, since this is a vacuum expectation value of it which is a key to the PQ mechanism. Now that we have abelianized the relevant terms, we are in the realm of the Zwanziger theory, so that the electric and magnetic four-potentials can be related due to the dual symmetry[3], $C^{\alpha}_{\mu\nu} = \mathscr{G}^{d\,\alpha}_{\mu\nu}$, which yields:

$$\mathcal{L}_{\rm QCD} \supset -\frac{v_a g_s^2\,\bar\theta + a g_{\rm m}^2}{32\pi^2 v_a} \sum_{\alpha=3,8} \mathscr{G}^{\alpha\,\mu\nu}\mathscr{G}^{d\,\alpha}_{\mu\nu}\,. \tag{6.32}$$

Physically, this is just an instantiation of the fact that the $U(1)$ electric and magnetic fields enter the expressions (6.30) and (6.31) symmetrically, as products $\vec{E}\cdot\vec{B}$. The standard PQ mechanism is now in order: redefinition of the pseudo Goldstone axion field $a \to a - v_a\bar\theta\, g_s^2/g_{\rm m}^2$ absorbs the $\bar\theta$-term into the axion-gluon term and subsequent application of the Vafa-Witten theorem [133] ensures $\langle a\rangle = 0$. The strong CP problem is thus solved.

6.5 Calculation of the effective Lagrangian

Let us return to the original Lagrangian (6.22) and derive the corresponding low energy physical phenomena. For that, we use a linear decomposition of the PQ field, $\Phi = (v_a + \sigma + ia)/\sqrt{2}$, where a is a pseudo Goldstone axion field.[4] Below the PQ scale, the field σ decouples and we are left with the Lagrangian involving axion and heavy monopoles:

$$\mathcal{L} \supset i\bar\psi\gamma^\mu\partial_\mu\psi + \bar\psi\gamma^\mu C_\mu\psi + \frac{yv_a}{\sqrt{2}}\,\bar\psi\psi + \frac{iy}{\sqrt{2}}\,a\bar\psi\gamma_5\psi\,, \tag{6.33}$$

where C_μ now also includes the electromagnetic four-potential and corresponds in general to the connection on either of the two GNO gauge groups, Abelian or non-Abelian, discussed in Chapter 1 sec. 1.3. The aim of this section is to integrate out the heavy field ψ. The beauty of the pseudoscalar interaction is that in this case

[3]The existence of the Standard model quarks – given the absence of their magnetic partners – obviously violates the electric-magnetic symmetry of this $U(1)^2$ Zwanziger-like theory. However, these quarks are known to be massive. This means they have no relevance for the instanton vacuum effects which are responsible for the generation of the $\bar\theta$-term

[4]σ and a introduced here are different from the fields denoted by the same letters in Sec. 6.4, but there should be no confusion, since different notations are restricted to different sections.

the calculations can be done exactly, without the need of perturbative expansion in the coupling constant. In order to get an effective Lagrangian at low energy we use the proper time method [134] developed by Schwinger. The effective pseudoscalar current is

$$J_a = i \left\langle C \left| \bar{\psi}(x)\gamma_5\psi(x) \right| C \right\rangle = -\, i\, \frac{y v_a}{\sqrt{2}} \int_0^\infty ds\, e^{-isy^2 v_a^2/2}\ \mathrm{tr}\left[\langle x|\gamma_5 e^{-i\hat{H}s}|x\rangle \right], \quad (6.34)$$

with the proper time Hamiltonian

$$\hat{H} = -\left(\not{\hat{p}} - \not{C}(\hat{x})\right)^2 = -\left(\hat{p}_\mu - C_\mu(\hat{x})\right)^2 + \frac{1}{2}\,\sigma^{\mu\nu} C_{\mu\nu}(\hat{x})\,, \quad (6.35)$$

where $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]$, $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + \left[C_\mu, C_\nu\right]$ and $\not{a} \equiv a_\mu \gamma^\mu$.

First, our goal is to evaluate the matrix element entering Eq. (6.34), which modulo γ_5 denotes the probability amplitude of returning to the same point x^μ in Minkowski space after proper time s. Note that since we are interested in the phenomenology at energies much less than the PQ scale v_a and the fluctuations of heavy fields ψ are possible only at the spatial and temporal extent $\sim v_a^{-1}$, external gauge fields in the following calculation can be considered constant. Our calculation of the pseudoscalar current then closely follows that performed by Schwinger [134], although we are considering a generic non-Abelian GNO group connection instead of the electromagnetic four-potential. We solve the Heisenberg equations of motion in a constant field $C_{\mu\nu}$,

$$\frac{d\hat{\pi}_\mu}{ds} = i\left[\hat{H}, \hat{\pi}_\mu\right] = 2\, C_{\mu\nu} \hat{\pi}^\nu\,, \quad (6.36)$$

$$\frac{d\hat{x}_\mu}{ds} = i\left[\hat{H}, \hat{x}_\mu\right] = 2\,\hat{\pi}_\mu\,, \quad (6.37)$$

and find the generalized momentum $\hat{\pi}_\mu = \hat{p}_\mu - C_\mu$ and position $\hat{x}_\mu$ as a function of proper time s:

$$\hat{\pi}_\mu(s) = e^{2s\, C_{\mu\nu}}\, \hat{\pi}^\nu(0)\,, \quad (6.38)$$

$$\hat{x}_\mu(s) = \hat{x}_\mu(0) + 2\left(C^{\mu\lambda}\right)^{-1} \cdot e^{sC^{\lambda\nu}} \sinh sC_{\nu\rho} \cdot \hat{\pi}^\rho(0)\,. \quad (6.39)$$

Next, with the use of Eqs. (6.38) and (6.39) we rewrite the Hamiltonian (6.35) in terms of position operators $\hat{x}_\mu(s)$ and $\hat{x}_\mu(0)$:

$$\hat{H} \supset -\frac{1}{4}\left(\sinh sC_{\kappa\lambda}\right)^{-1} C_{\lambda\nu}C^{\nu\rho}\left(\sinh sC^{\rho\sigma}\right)^{-1} \times \\ [\hat{x}_\kappa(s), \hat{x}^\sigma(0)] + \frac{1}{2}\sigma_{\mu\nu}C^{\mu\nu}, \tag{6.40}$$

leaving only the terms that do not vanish after taking the matrix element

$$\langle x(0)|\hat{H}|x(s)\rangle \propto \langle x(0)|x(s)\rangle \,. \tag{6.41}$$

Note that the exponents coming from Eqs. (6.38), (6.39) contract into the identity matrix due to antisymmetricity of the field strength tensor $C_{\mu\nu}$. The commutator in Eq. (6.40) is easily calculated with the help of Eq. (6.39) and canonical commutation relations. Since the Hamiltonian (6.40) is a generator of proper time translations, one can write now a differential equation for the sought-after matrix element:

$$i\partial_s\langle x(0)|x(s)\rangle = \langle x(0)|\hat{H}|x(s)\rangle = \langle x(0)|x(s)\rangle \times \\ \left\{\frac{i}{2}C_{\mu\nu}\coth sC^{\mu\nu} + \frac{1}{2}\sigma_{\mu\nu}C^{\mu\nu}\right\}. \tag{6.42}$$

The solution is:

$$\langle x(0)|x(s)\rangle = A\,\frac{\mathrm{pf}\,C_{\alpha\beta}}{\mathrm{pf}\sinh sC_{\alpha\beta}}\cdot\exp\left(-\frac{is}{2}\sigma_{\mu\nu}C^{\mu\nu}\right), \tag{6.43}$$

where $A = -i/(4\pi)^2$ is an integration constant which is calculated by matching with the elementary case of vanishing field strength $\mathscr{G}^\alpha = 0$. A skew-symmetric four-by-four matrix has two pairs of opposite sign eigenvalues, which we denote as $\pm\Lambda_1$, $\pm\Lambda_2$ in the particular case of $C_{\alpha\beta}$. The trace entering Eq. (6.34) can be now rewritten in the following form:

$$\mathrm{tr}\left[\langle x|\gamma_5 e^{-i\hat{H}s}|x\rangle\right] = -\frac{i}{16\pi^2}\,\mathrm{tr}_c\left[\frac{\Lambda_1\Lambda_2}{\sinh s\Lambda_1\sinh s\Lambda_2}\times \\ \mathrm{tr}_\gamma\left\{\gamma_5\exp\left(-\frac{is}{2}\sigma_{\mu\nu}C^{\mu\nu}\right)\right\}\right], \tag{6.44}$$

where we have explicitly separated traces over colour (tr_c) and spinor (tr_γ) indices.

Sums over the spinor indices can be performed using simple algebraic relations, namely $(\sigma_{\mu\nu}C^{\mu\nu})^2 = 8I_1 + 8i\gamma_5 I_2$, $\gamma_5^2 = 1$, $\mathrm{tr}\,\gamma_5 = \mathrm{tr}\,\sigma_{\mu\nu} = \mathrm{tr}\,\gamma_5\sigma_{\mu\nu} = 0$, where $I_1 \equiv C_{\mu\nu}C^{\mu\nu}/4$, $I_2 \equiv \epsilon_{\mu\nu\lambda\rho}\,C^{\mu\nu}C^{\lambda\rho}/8$:

$$\mathrm{tr}_\gamma \left\{\gamma_5 \exp\left(-\frac{is}{2}\sigma_{\mu\nu}C^{\mu\nu}\right)\right\} = 4i\,\mathrm{Im}\cosh sX = \\ 4\sinh\left(s\,\frac{X+X^*}{2}\right)\sinh\left(s\,\frac{X-X^*}{2}\right), \qquad (6.45)$$

where $X \equiv i\sqrt{2}\cdot\sqrt{I_1 + iI_2}$. Quite conveniently, by solving the characteristic equation for the matrix $C_{\alpha\beta}$, which has the form $\Lambda^4 + 2I_1\Lambda^2 - I_2^2 = 0$, one can infer that

$$\Lambda_1 = \frac{X+X^*}{2}\,, \quad \Lambda_2 = \frac{X-X^*}{2}\,, \qquad (6.46)$$

and the overall expression for the current simplifies into

$$J_a = \frac{iyv_a}{4\sqrt{2}\pi^2}\,\mathrm{tr}_c(I_2)\int_0^\infty ds\, e^{-isy^2v_a^2/2} = \frac{1}{16\sqrt{2}\pi^2 yv_a}\,\epsilon_{\mu\nu\lambda\rho}\,\mathrm{tr}_c\big(C^{\mu\nu}C^{\lambda\rho}\big)\,. \qquad (6.47)$$

Finally, we calculate the trace over color indices and expand in terms of the electromagnetic and color gauge fields:

$$J_a = \frac{1}{8\sqrt{2}\pi^2 yv_a} \times \begin{cases} 3g_1^2 F^{V\,\mu\nu}F^{V\,d}_{\mu\nu} + \dfrac{g_m^2}{2}\left(G^V\right)^{a\,\mu\nu}\left(G^V\right)^{d\,a}_{\mu\nu}\,, \\ 3g_2^2 F^{V\,\mu\nu}F^{V\,d}_{\mu\nu} + \dfrac{g_s^2}{2}\,G^{a\,\mu\nu}G^{d\,a}_{\mu\nu}\,, \end{cases} \qquad (6.48)$$

where summation over $a = 1\ldots8$ is implied. We also introduced notation for the dual gluon fields $\left(G^V\right)^{a\,\mu\nu}$, which are components of the connection on the color GNO group, and the dual electromagnetic field strength tensor $F^{V\,\mu\nu}$.

For concreteness, in phenomenological applications, we limit ourselves to the two minimal magnetic charge assignments: a pure Abelian magnetic monopole with a charge $6\pi/e$ and a non-Abelian color-magnetic monopole with an Abelian magnetic charge $2\pi/e$, which correspond respectively to the cases $m = 3$ and $m = 1$ discussed after Eq. (1.26) in Chapter 1 sec. 1.3. For the non-Abelian case, we will consider only magnetic charges transforming in the fundamental representation of $SU(3)$ with the coupling constant $2\pi/g_s$, bearing in mind that the higher representation GNO monopoles are unstable due to the Brandt-Neri-Coleman analysis [135, 136].

Thus, $g_1 = 2\pi/e$ and $g_2 = 6\pi/e$ are the two cases, corresponding to the stable non-Abelian monopole and the minimal Abelian one, respectively. The effective axion Lagrangian is then given by the following expression:

$$\mathcal{L}_{\rm eff} \supset \frac{y}{\sqrt{2}}\, a J_a = \frac{a}{16\pi^2 v_a} \times \begin{cases} \dfrac{3}{4\alpha^2}\, e^2 F^{V\,\mu\nu} F^{V\,d}_{\mu\nu} + \dfrac{1}{8\alpha_s^2}\, g_s^2 \left(G^V\right)^{a\,\mu\nu} \left(G^V\right)^{d\,a}_{\mu\nu}\,, \\ \dfrac{27}{4\alpha^2}\, e^2 F^{V\,\mu\nu} F^{V\,d}_{\mu\nu} + \dfrac{1}{2}\, g_s^2\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}\,, \end{cases} \tag{6.49}$$

where we introduced the fine-structure constant $\alpha = e^2/4\pi$ and its QCD analogue $\alpha_s = g_s^2/4\pi$.

6.6 Axion couplings

6.6.1 Axion couplings to gauge bosons

Let us introduce the axion decay constant $f_a = 4\alpha_s^2 v_a$ ($f_a = v_a$), for the case of the non-Abelian (Abelian) monopole. Assuming the dual symmetry of a Zwanziger-like theory describing diagonal gluons and photons, we obtain the relation between the magnetic and electric $U(1)$ field strength tensors $\mathscr{G}^{V\,\alpha} = \mathscr{G}^{d\,\alpha}$ ($\alpha = 3, 8$) and $F^V = F^d$. The effective Lagrangian Eq. (6.49) can then be rewritten in the following form:

$$\mathcal{L}_{\rm eff} \supset \begin{cases} -\dfrac{1}{4}\left(g^0_{a\gamma}\right)_1 a\, F^{\mu\nu} F^d_{\mu\nu} - \dfrac{a g_s^2}{32\pi^2 f_a}\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu} + \mathcal{L}_{\rm off}\,, \\ -\dfrac{1}{4}\left(g^0_{a\gamma}\right)_2 a\, F^{\mu\nu} F^d_{\mu\nu} + \dfrac{a g_s^2}{32\pi^2 f_a}\, G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}\,, \end{cases} \tag{6.50}$$

where

$$g^0_{a\gamma} = \begin{cases} 3\alpha_s^2/\left(\pi\alpha f_a\right)\,, \\ 27/\left(4\pi\alpha f_a\right)\,, \end{cases} \tag{6.51}$$

is a coupling of axion to photons. For convenience, we separated some axion-gluon interactions into $\mathcal{L}_{\rm off}$, which is given by the following expression:

$$\mathcal{L}_{\rm off} = \frac{a g_s^2}{32\pi^2 f_a} \times \left(G^{a\,\mu\nu} G^{d\,a}_{\mu\nu} - \sum_{\alpha=3,8} \mathscr{G}^{\alpha\,\mu\nu} \mathscr{G}^{d\,\alpha}_{\mu\nu} + \left(A \to A^V\right) \right) =$$

$$-\frac{g_s^2\, \partial^\mu a}{16\pi^2 f_a}\, \epsilon_{\mu\nu\rho\lambda} \times \left\{ \sum_{i,j,k\,\in\,\mathcal{I}_{\rm off}} \left(A_i^\nu\, \partial^\rho A_i^\lambda + \frac{1}{3}\, g_s\, f_{ijk} A_i^\nu A_j^\lambda A_k^\rho \right) + \right.$$

$$\sum_{\alpha=3,8} \sum_{j,k\,\in\mathcal{I}_{\rm off}} g_s f_{\alpha jk} A^{\nu}_{\alpha} A^{\lambda}_{j} A^{\rho}_{k} \;+\; \left(A \to A^V\right) \Bigg\}\,, \qquad (6.52)$$

where $\mathcal{I}_{\rm off} = [1;7]/\{3\}$[5]. Note that each of the interactions presented in Eq. (6.52) contains two or three off-diagonal (dual) gluon fields. Restricting our study in what follows to the field of low energy QCD, we neglect contribution from these terms. The reason are strong indications [130,131,137,138] of Abelian dominance in QCD below the energies of 1 GeV, which means that the processes involving off-diagonal gluons are suppressed in the IR. Moreover, as we will show in sec. 6.6.2, the term (6.52) is exactly zero in the classical approximation. Let us note, however, that in the future it would be very interesting to study if the quantum effects can generate non-zero $\mathcal{L}_{\rm off}$, because, although such effects are expected to be small in IR, they would be a very distinctive feature of the model we discuss.

The effective Lagrangian Eq. (6.50) without the term $\mathcal{L}_{\rm off}$ has the form of the conventional axion effective Lagrangian. As we will show, however, the corresponding axion particle has couplings with the Standard model particles which differ a lot from the ones calculated in DFSZ and KSVZ models. In particular, the coupling to photons $g_{a\gamma}$ is enhanced by many orders of magnitude compared to the conventional models. Namely, after the standard chiral rotation of quarks

$$q \to \exp\left(i\gamma_5\,\frac{aM_q^{-1}}{2f_a{\rm tr}M_q^{-1}}\right)\cdot q, \quad M_q = {\rm diag}\left(m_u, m_d\right), \qquad (6.53)$$

which eliminates the GG^d term, is performed, one finds that the coupling to photons is

$$g_{a\gamma} = g^0_{a\gamma} - \frac{\alpha}{3\pi f_a}\left(\frac{4m_d+m_u}{m_d+m_u}\right) \simeq g^0_{a\gamma}, \qquad (6.54)$$

so that it is practically not affected by the quark masses. In the conventional notation used to parameterize the strength of the axion-photon coupling,

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}\cdot\frac{E}{N}\,, \qquad (6.55)$$

[5] By this notation we mean all integers from 1 to 7 excluding 3.

our model predicts

$$\frac{E}{N} = \begin{cases} 6\alpha_s^2/\alpha^2\,, \\ \\ 27/(2\alpha^2)\,, \end{cases} \tag{6.56}$$

so that the coupling gets increased by 5-6 orders of magnitude. Bearing in mind that the standard expression for the axion mass,

$$m_a = \frac{m_\pi f_\pi \sqrt{m_u m_d}}{(m_u + m_d)\, f_a}\,, \tag{6.57}$$

is derived from the conventional axion-gluon coupling and thus holds in our case automatically as long as $\mathcal{L}_{\text{off}}$ is small, we plot the axion-photon coupling as a function of axion mass and decay constant in Fig. 6.1 together with the hints and existing as well as projected constraints from various experiments and astrophysical observations.[6] For reference, we show axion-photon couplings in KSVZ models with heavy fermions in one representation of the Standard model gauge group [124] and in DFSZ model.

In Fig. 6.1, possible values for the axion-photon coupling in the model with the non-Abelian monopole are organized in a vertically hatched band, while the model with the minimal Abelian monopole yields a single line inside this band. The band denotes the uncertainty we estimate for the model with the non-Abelian monopole, which is associated to the dependence of the first line of Eq. (6.51) on the strong coupling α_s in the IR. The state of the art in studies of the behavior of the latter was discussed in detail in a recent review [149], where it was shown that there exists a definition of α_s in the IR, which is analytic, independent of the choice of renormalization scheme or gauge, universal, based on first principles and IR-finite (see Table 5.4 in Ref. [149]). This choice of definition for IR α_s corresponds to the so-called effective charges α_{g_1}, α_{F_3} and α_τ, which are directly related to the observables of low energy QCD. The measurements show that the IR strong coupling

[6] Hints and most constraints are discussed in detail in Ref. [139]. We present updated astrophysical constraints from Ref. [140] together with the constraints derived from Chandra data on NGC 1275 [141] (see however Ref. [142]), as well as constraints derived from the data on SN1987A from the GRS instrument of the SMM satellite [143], constraints from NuSTAR data on super star clusters [144], constraints from GBT data on neutron stars in the Galactic Center [145], and projected constraints from advanced LIGO [146]. Constraints from ADMX SLIC [147] search for dark matter axions include three very narrow close exclusion regions which are impossible to resolve in our plot. SHAFT constraint is discussed in Ref. [148].

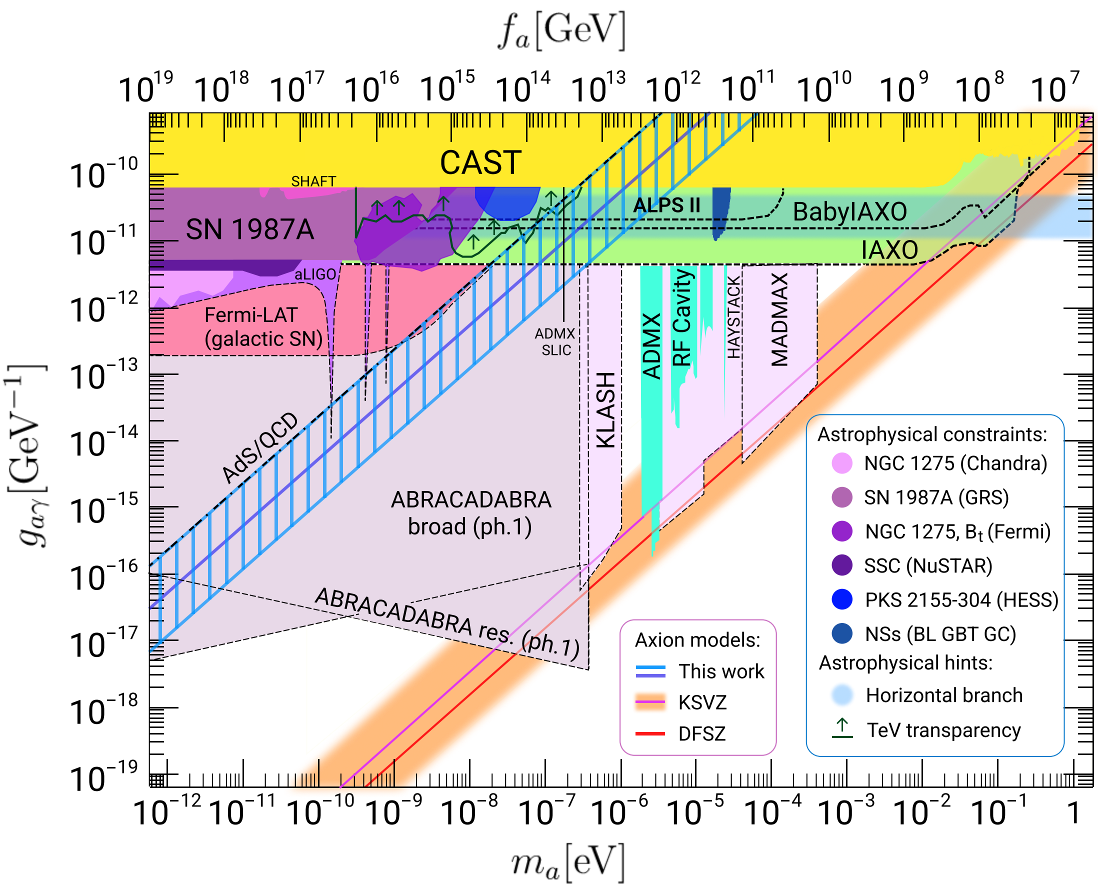

Figure 6.1: Comparison between the axion-photon couplings predicted in the conventional DFSZ- and KSVZ-like models (orange band) and the ones predicted in the general hadronic models (blue band). Axion-photon coupling is plotted as a function of axion mass and decay constant together with the existing and projected (dashed lines) constraints on the corresponding parameter space from experiments as well as from astrophysical data. Note that the haloscope experiments searching for light axions ($m_a \ll \mu$eV) are blind to the new axion-photon couplings of the general hadronic models, as it will be discussed in Chapter 7 sec. 7.3. Astrophysical hints are also shown. The dash-dotted line corresponds to the model with the non-Abelian monopole where the IR strong coupling α_s value calculated in the AdS/QCD framework is adopted. The line in the center of the vertically hatched band corresponds to the model with the minimal Abelian monopole. For further discussion, see main text.

α_s defined in such a way freezes at low energies. The freezing behavior of IR α_s is also supported by the success of the AdS/QCD technique in the description of hadron properties [150]. Moreover, the value of the IR strong coupling calculated in AdS/QCD, $\alpha_{\text{AdS}}(0) = \pi$, is consistent with the values $\alpha_{g_1}(0)$ and $\alpha_{F_3}(0)$ [7]. All this convinces us to assume that the AdS/QCD value of IR strong coupling is a

[7] Although the effective charge $\alpha_\tau(0)$ is different, it is known that it contains an unsubtracted pion pole.

relevant one, that is why we highlight the corresponding values of $g_{a\gamma}$ in Fig. 6.1 with a dash-dotted line. However, bearing in mind that low energy QCD is still largely terra incognita, we allow for uncertainty in α_s which results in a band in Fig 6.1 where the lower edge $\alpha_s(0) = 0.7$ is chosen. Such choice is suggested by the observation in Ref. [149] that most of the values of $\alpha_s(0)$ in the literature are clustered around $\alpha_s(0) \sim 3$ (close to the AdS/QCD value) and $\alpha_s(0) \sim 0.7$, not taking into account the decoupling solution $\alpha_s(0) = 0$ disfavored for a number of reasons [151, 152]. Let us note as well that too large values of α_s are disfavored by calculations in Ref. [153], where it was shown that the magnetic coupling (i.e. the coupling inverse to α_s) never gets too small in pure $SU(2)$ gluodynamics, these results being extended to the pure $SU(3)$ case in Ref. [154].

Finally, let us mention that there is yet another source of uncertainty in our predictions, both for the models with Abelian and non-Abelian monopoles, which is associated with the $U(1)$ magnetic charges of the monopoles. Whereas we consider them to be minimal in each of the model, they are in principle not constrained by the stability arguments. This means that $g_{a\gamma}$ can be further increased in Fig. 6.1 for the general hadronic axion models discussed in this chapter.

6.6.2 Axion-gluon coupling in the classical approximation

In this section, we show that the axion-gluon coupling in the model with a heavy non-Abelian monopole preserves its universality in the classical approximation, i.e. it is given by the expression:

$$- \frac{a g_s^2}{32\pi^2 f_a} G^{a\,\mu\nu} G^{d\,a}_{\mu\nu}, \tag{6.58}$$

so that $\mathcal{L}_{\text{off}} = 0$ in Eq. (6.50), at least classicaly. We use the formalism of loop space variables pioneered by Polyakov [155] and developed with the focus on the electric-magnetic dual symmetry of the YM theory by Hong-Mo, Faridani and Tsun [63]. Central object of the formalism is the parallel phase transport along the loop $\xi(s)$, $s \in [0, 2\pi]$ from one point s_1 to another s_2:

$$\Phi_\xi(s_2, s_1) = P_s \exp\left(i g_s \int_{s_1}^{s_2} ds\, A_\mu\left(\xi(s)\right) \dot{\xi}^\mu(s) \right), \tag{6.59}$$

where P_s is the Dyson ordering. Loop derivative of the holonomy defines the Polyakov variables:

$$F_\mu[\xi|s] = \frac{i}{g_s}\Phi_\xi^{-1}(2\pi,0)\cdot\frac{\delta\Phi_\xi(2\pi,0)}{\delta\xi^\mu(s)}\,, \tag{6.60}$$

which are known to constitute a valid set for a full description of the YM field [156, 157]. It was shown in Ref. [63] that another complete set of variables is better suited for dealing with the electric-magnetic dual symmetry of the classical YM theory, namely:

$$E_\mu[\xi|s] = \Phi_\xi(s,0)\,F_\mu[\xi|s]\,\Phi_\xi^{-1}(s,0)\,, \tag{6.61}$$

which can be connected to the local quantities by the expression:

$$\omega^{-1}(x)\;G^d_{\mu\nu}(x)\,\omega(x) = \frac{2}{N}\,\epsilon_{\mu\nu\rho\sigma}\int\delta\xi ds\,E^\rho[\xi|s]\,\dot\xi^\sigma(s)\,\dot\xi^{-2}(s)\,\delta(x-\xi(s))\,, \tag{6.62}$$

where $\omega(x)$ is an arbitrary local $SU(3)$ matrix and N is a normalization factor. The dual (magnetic) variables $E^{(d)}_\mu$ were shown to be related to the electric ones E_μ in the pure YM theory in the following way:

$$\begin{aligned}\omega^{-1}(\eta(t))\,E^{(d)}_\mu[\eta|t]\;\omega(\eta(t)) = \frac{2}{N}\,\epsilon_{\mu\nu\rho\sigma}\dot\eta^\nu(t)\int\delta\xi ds\,E^\rho[\xi|s]\,\cdot\\ \dot\xi^\sigma(s)\,\dot\xi^{-2}(s)\,\delta(\xi(s)-\eta(t))\,,\end{aligned} \tag{6.63}$$

while the inverse transformation is:

$$\begin{aligned}&\omega(\eta(t))\,E_\mu[\eta|t]\;\omega^{-1}(\eta(t)) =\\ &\quad-\frac{2}{N}\,\epsilon_{\mu\nu\rho\sigma}\dot\eta^\nu(t)\int\delta\xi ds\,E^{(d)\,\rho}[\xi|s]\,\dot\xi^\sigma(s)\,\dot\xi^{-2}(s)\,\delta(\xi(s)-\eta(t))\,.\end{aligned} \tag{6.64}$$

Since in the derivation of the axion effective Lagrangian external fields can be considered constant and homogeneous, as discussed in sec. 6.5, we can apply Eqs. (6.63) and (6.64) in order to find the relation between the expression (6.58) and its dual analogue, constructed from the GNO group connection, in the classical theory. The calculation proceeds as follows:

$$
\begin{aligned}
&\int d^4x\, a(x)\, G^{a\,\mu\nu}(x)\, G^{d\,a}_{\mu\nu}(x) = \\
&2\int d^4x\, a(x)\,\mathrm{tr}\left\{\omega^{-1}(x)\, G_{\mu\nu}(x)\,\omega\,(x)\,\omega^{-1}(x)\; G^{d}_{\mu\nu}\,(x)\;\omega\,(x)\right\} = \\
&\frac{8}{N}\int d^4x\,\delta\xi ds\, a(x)\,\mathrm{tr}\left\{\omega^{-1}(x)\; G^{d}_{\mu\nu}\,(x)\;\omega\,(x)\, E^{\mu}\,[\xi|s]\right\}\dot{\xi}^{\nu}(s)\,\dot{\xi}^{-2}(s)\,\delta(x-\xi(s)) = \\
&\frac{16}{N^2}\,\epsilon_{\mu\nu\rho\sigma}\int \delta\eta dt\,\delta\xi ds\, a(\eta(t))\,\mathrm{tr}\left\{E^{\rho}\,[\eta|t]\, E^{\mu}\,[\xi|s]\right\}\cdot \\
&\qquad\qquad\qquad \dot{\eta}^{\sigma}(t)\,\dot{\eta}^{-2}(t)\,\dot{\xi}^{\nu}(s)\,\dot{\xi}^{-2}(s)\,\delta(\eta(t)-\xi(s)) = \\
&\frac{8}{N}\int \delta\eta dt\, a(\eta(t))\,\mathrm{tr}\left\{E^{\mu}\,[\eta|t]\;\omega^{-1}(\eta(t))\, E^{(d)}_{\mu}\,[\eta|t]\;\omega\,(\eta(t))\right\}\dot{\eta}^{-2}(t) = \\
&\frac{8}{N}\int \delta\eta dt\, a(\eta(t))\,\mathrm{tr}\left\{\,\omega\,(\eta(t))\, E^{\mu}\,[\eta|t]\;\omega^{-1}(\eta(t))\, E^{(d)}_{\mu}\,[\eta|t]\,\right\}\dot{\eta}^{-2}(t) = \\
&-\frac{16}{N^2}\,\epsilon^{\mu\nu\rho\sigma}\int \delta\eta dt\,\delta\xi ds\, a(\eta(t))\,\mathrm{tr}\left\{E^{(d)}_{\rho}\,[\eta|t]\, E^{(d)}_{\mu}\,[\xi|s]\right\}\cdot \\
&\qquad\qquad\qquad \dot{\eta}^{\sigma}(t)\,\dot{\eta}^{-2}(t)\,\dot{\xi}^{\nu}(s)\,\dot{\xi}^{-2}(s)\,\delta(\eta(t)-\xi(s)) = \\
&-\int d^4x\, a(x)\,\left(G^{V}\right)^{a\,\mu\nu}(x)\,\left(G^{V}\right)^{d\,a}_{\mu\nu}(x)\;,
\end{aligned}
\tag{6.65}
$$

where we took advantage of Eqs. (6.62), (6.63) and (6.64), as well as of the cyclic property of the trace. The last identity follows automatically as far as one notices that the third and the sixth lines of the Eq. (6.65) are identical but for the overall sign and electric-magnetic variables interchange. Now, one can clearly see that classically we recover the universal axion-gluon coupling even in the model with the non-Abelian magnetic monopole:

$$
\mathcal{S}_{\text{eff, classical}} \supset \int d^4x\,\frac{ag_s^2}{32\pi^2 f_a}\,\left(G^{V}\right)^{a\,\mu\nu}\left(G^{V}\right)^{d\,a}_{\mu\nu} = -\int d^4x\,\frac{ag_s^2}{32\pi^2 f_a}\,G^{a\,\mu\nu}G^{d\,a}_{\mu\nu}\,. \tag{6.66}
$$

6.6.3 Axion couplings to fermions

Let us consider the following couplings of axions with matter:

$$
g_{ai} \equiv C_{ai} m_i / f_a\,, \tag{6.67}
$$

where m_i is the mass of fermion i, which correspond to the following terms in the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} \supset C_{ai} \frac{\partial_\mu a}{2f_a} \bar{\psi}_i \gamma^\mu \gamma_5 \psi_i \,. \tag{6.68}$$

As electrons do not carry PQ charge in the general hadronic models we consider, the axion-electron coupling g_{ae} is generated radiatively [158, 159]:

$$g_{ae} = g^0_{a\gamma} \cdot \frac{3\alpha}{2\pi} m_e \ln \frac{f_a}{m_e} \,, \tag{6.69}$$

where $g^0_{a\gamma}$ is given by the expressions (6.51), for the models with the minimal Abelian or non-Abelian monopole, and we took into account that the term associated to the axion-pion mixing is negligibly small compared to the leading contribution[8]. We find that the experiments and astrophysical observations probing axion-electron interactions do not yield new constraints on the model. Indeed, the CAST bound [160] on the axion-photon coupling, $g_{a\gamma} < 0.66 \cdot 10^{-10}$ GeV^{-1}, constrains the phenomenologically viable region for axion-electron coupling: $g_{ae} < 1.2 \cdot 10^{-16} \ln f_a/m_e$. This constraint is stronger than any existing or projected bound from interaction with electrons. As to the interactions of the axion with nucleons, it turns out that contributions from radiatively generated axion-quark couplings are non-negligible and actually enhance axion-nucleon couplings with respect to the conventional DFSZ case in much of the parameter space. One can find that the coefficients C_{ap} and C_{an} are

$$C_{ap} = -0.47 - 0.39\, \delta c_d + 0.88\, \delta c_u \,, \tag{6.70}$$

$$C_{an} = -0.02 + 0.88\, \delta c_d - 0.39\, \delta c_u \,, \tag{6.71}$$

where the numerical coefficients were calculated in [161] and the radiatively generated quark couplings read as follows:

$$\delta c_u = g^0_{a\gamma} f_a \cdot \frac{8\alpha}{27\pi} \ln \frac{f_a}{m_N} \,, \tag{6.72}$$

[8] Note that in principle the axion-fermion couplings discussed in this section have to be calculated within the full non-perturbative QEMD formalism of Chapter 5; however, as it was discussed in Chapter 4 sec. 4.6, there exist no reliable methods for such an exact calculation, so we assume that the result can be approximated by the one obtained in the QED framework.

$$\delta c_d = g^0_{a\gamma} f_a \cdot \frac{\alpha}{54\pi} \ln \frac{f_a}{m_N} \,, \tag{6.73}$$

where m_N is the nucleon mass. Constraints on axion-neutron interactions are more stringent than constraints on interactions with protons. We plot g_{an} as a function of axion mass and decay constant in Fig. 6.2 together with the constraint from neutron star cooling [162] and the projected reach of the CASPEr Wind experiment [163]. For reference, we show the neutron-axion coupling in DFSZ models, the range of which is constrained by the requirement of perturbative unitarity of the Yukawa couplings of the Standard model fermions [164]. Note that the slope of the DFSZ band in Fig. 6.2 is different from the slope of the band corresponding to the axion model of this paper. The difference arises because, in the DFSZ case, one obtains a linear dependence of the coupling on the axion mass, $g_{an} \propto m_a$, characteristic of the tree-level couplings to quarks, while in the case of our model the linear dependence is superseded by a nonlinear one, $g_{an} \propto m_a \ln(\text{const}/m_a)$, due to the radiative origin of the coupling.

In Fig. 6.2, we show also the CAST bound [160] which is translated to a constraint on the axion-neutron coupling with the use of Eqs. (6.71)-(6.73). Uncertainty in the prediction of the axion-neutron coupling in the axion models of this Chapter comes from the uncertainty in the prediction of the axion-photon coupling, the latter being discussed at length in sec. 6.6.1.

6.7 Axion dark matter

Let us discuss if the axions we propose can comprise dark matter. In order to avoid the cosmological magnetic monopole problem [165, 166], i.e. overproduction of monopoles during the hot Big Bang epoch, we will set their masses and therefore the axion decay constant f_a to be larger than the reheating temperature. This means that we have to deal only with the pre-inflationary scenario of axion dark matter production, which hinges upon the misalignment mechanism [10–12]. Our model with a heavy Abelian magnetic monopole charged electrically under $SU(3)$ will then give exactly the same dark matter abundance as in the case of the KSVZ model. This follows from the fact that the axion-gluon couplings are identical in the

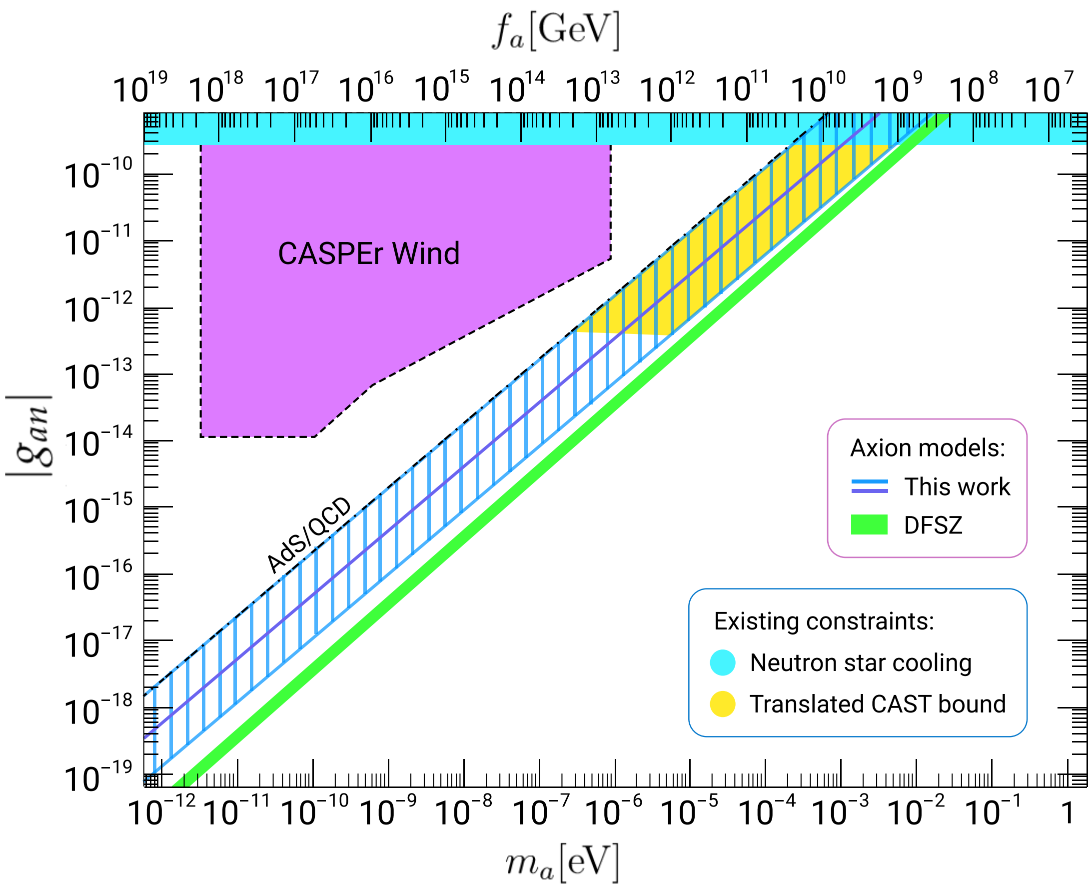

Figure 6.2: Axion-neutron coupling as a function of axion mass and decay constant for various axion models together with the existing and projected (dashed lines) constraints on the corresponding parameter space from experiments as well as from astrophysical data. The dash-dotted line corresponds to the model of this Chapter with the non-Abelian monopole where the IR strong coupling α_s value calculated in the AdS/QCD framework is adopted. The line in the center of the vertically hatched band corresponds to the model with the minimal Abelian monopole. For further discussion, see main text.

latter two models. Meanwhile, calculation of the dark matter abundance is generally not so simple in the case of our model with a non-Abelian magnetic monopole. Note that while Abelian dominance suggests that the low temperature axion mass $m_a(f_a)$ in this case is given approximately by the familiar expression for the standard QCD axion, at higher temperatures, $T \gtrsim 1$ GeV, the axion mass can differ significantly from the standard case. The cosmic axion abundance resulting from the misalignment production mechanism ρ_a^{mis} is inversely proportional to the square root of the axion mass at the moment where oscillations of the axion field start:

$$\rho_a^{\text{mis}} \propto \frac{f_a}{\sqrt{m_a(T_{\text{roll}})}} \cdot \mathcal{F}\left(T_{\text{roll}}\right), \tag{6.74}$$

where $T_{\rm roll}$ is the temperature at which $m_a(T_{\rm roll}) = 3H(T_{\rm roll})$, H being the Hubble expansion rate, and $\mathcal{F}$ a fixed function of temperature. Due to Abelian dominance, we expect that $\rho_a^{\rm mis}$ does not change too much with respect to the conventional QCD axion models if $T_{\rm roll} < 1$ GeV. The latter condition can be recast into the form $m_a(\text{GeV}) < 3H(\text{GeV})$, which yields $f_a > 10^{12}$ GeV assuming the axion mass at 1 GeV is not much off the values given in Ref. [67]. Combining it with the CAST bound, we see that in much of the allowed parameter space axions produced via the misalignment mechanism have approximately the same abundance as axions with the same mass in KSVZ and DFSZ-like models. The case $f_a \lesssim 10^{12}$ GeV is more difficult: in order to infer the abundance of cosmic axions in the model with a non-Abelian monopole in this case, one has to calculate the axion mass as a function of temperature in the energy range where there is no Abelian dominance.

This Chapter is written based on the publications [2, 3] of the author of this thesis.

Chapter 7

New experimental targets for axions and magnetic monopoles

7.1 General lessons

Let us discuss the phenomenology of the new electromagnetic couplings of axions and ALPs found in the previous Chapters. In our discussion, we will always consider first the general case of ALPs, i.e. Nambu-Goldstone bosons of an arbitrary spontaneously broken global $U(1)$ symmetry, which by definition include QCD axions as a special case, and only then make quantitative predictions in particular axion models.

Due to the scaling of the new $g_{a\mathrm{BB}}$ and $g_{a\mathrm{AB}}$ couplings with the elementary electric and magnetic charge units found in Chapter 5 sec. 5.1, in any model where $g_{a\mathrm{AB}} \neq 0$, one expects the ratio $g_{a\mathrm{BB}}/|g_{a\mathrm{AB}}|$ to be proportional to a large number $g_0/e \gg 1$. This means that the possible effects associated to the $g_{a\mathrm{AB}}$ coupling play the dominant role only for those observables, which do not get any sizable contribution from the $g_{a\mathrm{BB}}$ coupling. As we will discuss in the next sections, such observables do exist and can be probed in various experiments by studying the interactions of ALPs with polarized light, searching for electric dipole moments of charged particles and a fifth force or by looking for light ALP dark matter in an external magnetic field with haloscopes.

Still, for most of the processes involving ALP-photon interactions, the dominant effect is associated to the $g_{a\mathrm{BB}}$ coupling. Symmetry of Eq. (5.27) with respect to the interchange of the $g_{a\mathrm{AA}}$ and $g_{a\mathrm{BB}}$ couplings suggests that in any process of creation or disappearance of a certain number of ALPs, the effect of the $g_{a\mathrm{BB}}$ coupling is analogous to the effect of the conventional $g_{a\gamma\gamma}$ coupling[1]. This means that the rates of such processes as ALP-photon conversion in external electromagnetic fields [167], ALP decay, ALP emission through Primakoff effect [168] or photon coalescence, are all given by the conventional expressions, but with the $g_{a\gamma\gamma}$ coupling substituted by the $g_{a\mathrm{BB}}$ one.

The same simple rule of replacing the $g_{a\gamma\gamma}$ coupling with the $g_{a\mathrm{BB}}$ coupling in conventional expressions applies to the dispersion relation of light in an ALP background. Indeed, after we omit the subdominant $|g_{a\mathrm{AB}}| \ll g_{a\mathrm{BB}}$ coupling and put $\mathbf{E}_a$, $\mathbf{E}_0 \to \mathbf{E}_\gamma$ and $\mathbf{H}_a$, $\mathbf{H}_0 \to \mathbf{H}_\gamma$, the axion Maxwell equations (5.23)–(5.26) become invariant under the interchange of the couplings $g_{a\mathrm{AA}}$ and $g_{a\mathrm{BB}}$ supplemented by the electric-magnetic duality transformation $\mathbf{E}_\gamma \to \mathbf{H}_\gamma$, $\mathbf{H}_\gamma \to -\mathbf{E}_\gamma$. Since the propagation of light is electric-magnetic duality invariant, the $g_{a\mathrm{BB}}$ coupling enters the dispersion relation in the same way as the conventional $g_{a\mathrm{AA}}$ coupling. It can also be explicitly checked that the form of the second-order differential equations for $\mathbf{E}_\gamma$ and $\mathbf{H}_\gamma$ does not change.

Let us consider the existing constraints on the ALP-photon $g_{a\gamma\gamma}$ coupling which take advantage of the effects discussed in the previous two paragraphs. It is now clear that the same constraints hold also for the new $g_{a\mathrm{BB}}$ coupling and the corresponding search strategies need not be updated. In particular, this is the case of astrophysical and cosmological constraints [139], helioscope searches [169–171], light-shining-through-wall (LSW) [172–178] and axion interferometry [146, 179–182] experiments as well as the ones of those haloscope searches, where the ALP Compton wavelength fits into the experimental apparatus and thus where the interaction between an ALP and a magnetic field can be described as an ALP-photon conversion in an external field. We present the corresponding constraints on the $g_{a\mathrm{BB}}$ coupling in Fig. 7.1. Note that as $|g_{a\mathrm{AB}}| \ll g_{a\mathrm{BB}}$, the same constraints obviously hold for $|g_{a\mathrm{AB}}|$ and $\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$.

The qualitative distinction between the new $g_{a\mathrm{BB}}$ coupling and the conventional $g_{a\gamma\gamma}$ coupling arises whenever a given process cannot be described by Eq. (5.27)

[1]This statement need not hold for loop effects, as Eq. (5.27) is classical.

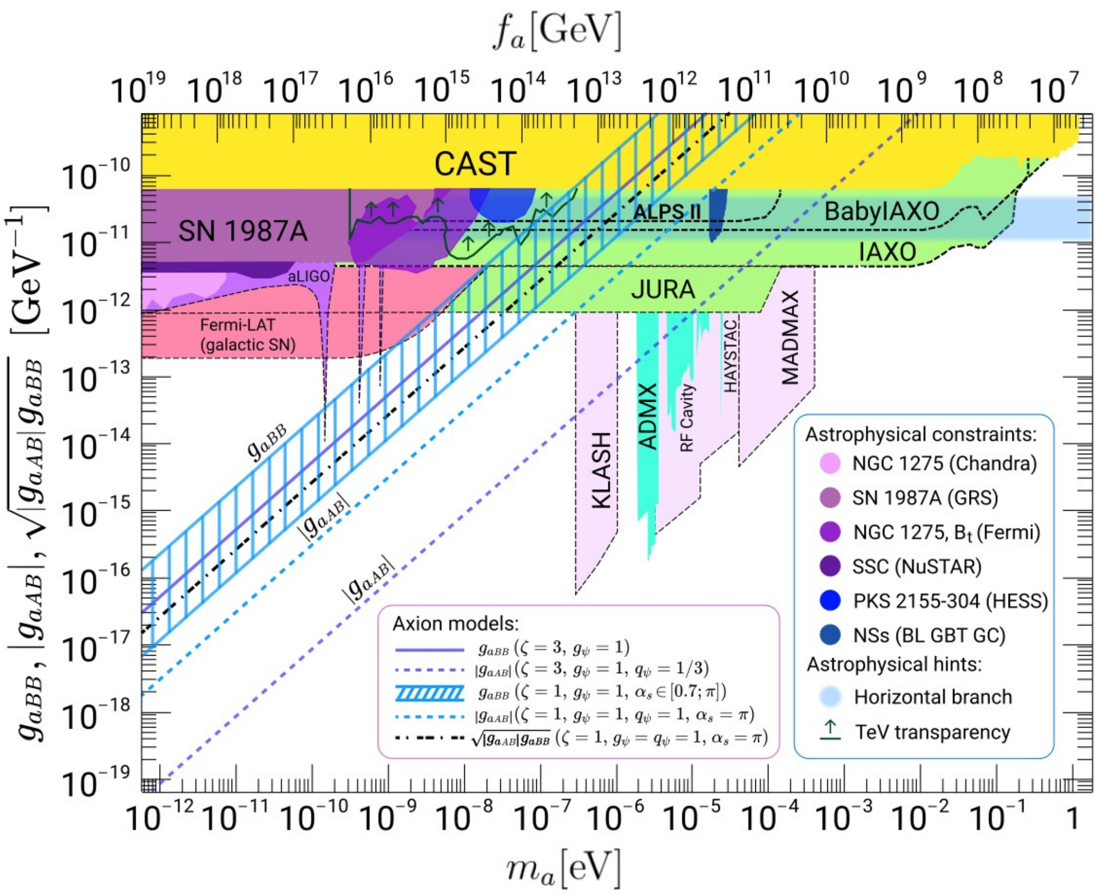

Figure 7.1: Existing and projected (dashed lines) constraints on the parameter space of ALP-photon $g_{a\mathrm{BB}}$ and $g_{a\mathrm{AB}}$ couplings versus ALP mass and decay constant together with the lines corresponding to $g_{a\mathrm{BB}}$ (solid), $|g_{a\mathrm{AB}}|$ (dashed) and $\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$ (dash-dotted) in different hadronic axion models with one heavy PQ-charged fermion ψ with the parameters given in a box and $N_{\mathrm{DW}} = 1$. Astrophysical hints are also shown. For further discussion, see main text.

and involves observables which are not invariant under the electric-magnetic duality symmetry. In this case, the values of these observables derived from the axion Maxwell equations (5.23)–(5.26) are not symmetric with respect to the interchange $g_{a\mathrm{AA}} \leftrightarrow g_{a\mathrm{BB}}$ of the two couplings, so that there is a qualitative difference in the effects of these couplings. We give a particular example where the latter difference plays a crucial role in sec. 7.3. In particular, in the latter section, we discuss haloscope experiments searching for light ALP dark matter ($m_a \ll \mu$eV). We find that in this case, the constraints obtained for the $g_{a\gamma\gamma}$ coupling need not hold for the $g_{a\mathrm{BB}}$ coupling. This means that to probe the latter coupling, these experiments should exploit a search strategy which is different from the one normally used.

To be more specific, when we make quantitative predictions in the next sections, we will consider a particular kind of ALP: the axion particle arising in general hadronic models discussed in the previous Chapter. In this case, we will take advantage of Eqs. (5.3), (5.4) and (5.6) for the axion-photon couplings. As we discussed in Chapter 5 sec. 5.1, there are two families of axion models where the new electromagnetic couplings $g_{a\mathrm{AB}}$ and $g_{a\mathrm{BB}}$ can arise: those with Abelian ($\zeta = 3$) and those with non-Abelian ($\zeta = 1$) magnetic monopoles, cf. Eq. (5.7). In the former case, the axion decay constant f_a is obviously related to the QCD anomaly coefficient N and PQ scale v_a in the standard way: $f_a = v_a/2N$, while in the latter case, as it was shown in Chapter 6, the relation is non-standard: $f_a = 2\alpha_s^2 v_a/N$, where $\alpha_s = g_s^2/4\pi$. Using these relations, we plot the lines corresponding to $g_{a\mathrm{BB}}$, $|g_{a\mathrm{AB}}|$ and $\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$ [2] in different hadronic axion models. For simplicity, we choose the models having only one heavy vector-like PQ-charged fermion ψ, which transforms trivially under the $SU(2)_L$ gauge group of the weak interactions and in the fundamental representation under the color $SU(3)_c$ gauge group (electric $SU(3)_\mathrm{E}$ in the Abelian monopole case or magnetic $SU(3)_\mathrm{M}$ in the non-Abelian monopole case), and has charges q_ψ and g_ψ, see Fig. 7.1 and the legend therein. In these models, the QCD anomaly coefficient is $N = 1/2$, so that $N_\mathrm{DW} = 1$. In the non-Abelian monopole case $\zeta = 1$, there is an uncertainty associated to our ignorance of the exact value of α_s at low energies [149], see the discussion in Chapter 6 sec. 6.6.1. Note that the axion models populate a region of the parameter space, which in the analogous plot for the $g_{a\mathrm{AA}}$ coupling would be extensively probed by existing and projected haloscope experiments searching for light ALP dark matter ($m_a < \mu$eV), see Fig. 6.1. However, as we discussed briefly in the previous paragraph and will elaborate later in sec. 7.3, the constraints from such haloscopes cannot be translated to Fig. 7.1. Also, note that the conventional KSVZ and DFSZ axion lines are obviously missing from the plot in Fig. 7.1, since this plot depicts the $g_{a\mathrm{BB}}$ and $|g_{a\mathrm{AB}}|$ couplings, but not the $g_{a\mathrm{AA}}$ coupling.

[2] The $\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$ line is relevant for LSW searches, see Eq. (7.2) and discussion in sec. 7.2.

7.2 Purely laboratory-based experiments

A particularly clean way to measure the $g_{a\mathrm{AB}}$ coupling is provided by LSW experiments [183]. As one can see from Eq. (5.27), contrary to the CP-conserving couplings, the CP-violating $g_{a\mathrm{AB}}$ coupling allows interaction between ALPs and light polarized in a plane perpendicular to the external magnetic field. As one can control the polarization of the incoming light in a LSW experiment, it is straightforward to artificially turn off the CP-conserving ALP-photon interaction on the photon to ALP conversion side before the wall. Using $g_{a\mathrm{BB}} \gg |g_{a\mathrm{AB}}| \gg g_{a\mathrm{AA}}$, we find the following LSW probabilities corresponding to different linear polarizations of incoming light:

$$P(\gamma_{\parallel} \to a \to \gamma) \simeq 16\,\frac{(g_{a\mathrm{BB}}\omega H_0)^4}{m_a^8}\,\sin^4\left(\frac{m_a^2\,L_{H_0}}{4\omega}\right), \tag{7.1}$$

$$P(\gamma_{\perp} \to a \to \gamma) \simeq 16\,\frac{(g_{a\mathrm{AB}}\omega H_0)^2(g_{a\mathrm{BB}}\omega H_0)^2}{m_a^8}\,\sin^4\left(\frac{m_a^2\,L_{H_0}}{4\omega}\right), \tag{7.2}$$

where $\gamma_{\parallel}$ ($\gamma_{\perp}$) denotes the incoming light with frequency $f = \omega/(2\pi)$ and with polarisation parallel (perpendicular) to the magnetic field H_0, which is supposed to be transverse to the direction of the light beam and which is sustained in a cavity of length L_{H_0}, both before and behind the wall. Clearly, from a detection of LSW with some probability $P(\gamma_{\parallel} \to a \to \gamma)$, one can determine the $g_{a\mathrm{BB}}$ coupling in a first measurement. In a second measurement, one can also search for LSW via $\gamma_{\perp} \to a \to \gamma$. Then, for the case of axions, using Eqs. (5.4), (5.6) and (5.7), we see that the coupling $g_{a\mathrm{AB}}$ can be determined from the following ratio:

$$\frac{P(\gamma_{\perp} \to a \to \gamma)}{P(\gamma_{\parallel} \to a \to \gamma)} \simeq \frac{g_{a\mathrm{AB}}^2}{g_{a\mathrm{BB}}^2} = \left(\frac{D}{M}\frac{e}{g_0}\right)^2 = 4\left(\frac{D}{\zeta M}\right)^2\alpha^2 \simeq 2.13\times 10^{-4}\left(\frac{D}{\zeta M}\right)^2. \tag{7.3}$$

For example, the experiment ALPS II ($H_0 = 5.3\,\mathrm{T}$, $L_{H_0} = 105.6\,\mathrm{m}$, $\omega = 1.17\,\mathrm{eV}$) has the capability to search for LSW using incoming light with both polarisations, $\gamma_{\parallel}$ and $\gamma_{\perp}$ [184]. For both of them, ALPS II has the projected sensitivity $P_{\mathrm{sens}} \approx 10^{-29}$ to the corresponding LSW probabilities. This would allow for the detection of a light, $m_a \lesssim 10^{-4}\,\mathrm{eV}$, axion featuring a CP-conserving coupling $(g_{a\mathrm{AA}} + g_{a\mathrm{BB}}) \simeq g_{a\mathrm{BB}} \gtrsim 2\times 10^{-11}\,\mathrm{GeV}^{-1}$ via $\gamma_{\parallel} \to a \to \gamma$, as can be inferred from Eq. (7.1). If this newly discovered axion features also a CP-violating coupling, then the latter has to be in the range

$$|g_{a\mathrm{AB}}| = 2\alpha(|D|/\zeta M)g_{a\mathrm{BB}} \gtrsim 3\times 10^{-13}\,\mathrm{GeV}^{-1}(|D|/\zeta M)\,. \tag{7.4}$$

If $|D| \simeq M$, to detect such a coupling via $\gamma_\perp \to a \to \gamma$ requires a sensitivity improvement by four order of magnitudes, to $P_{\text{sens}} \sim 10^{-33}$, as can be seen from Eqs. (7.1) (7.2), and (7.3). Intriguingly, such a sensitivity has been argued to be achievable by the next generation LSW experiment JURA (also known as ALPS III [185,186]). Indeed, see Fig. 7.1, where we showed the $g_{a\mathrm{BB}}$ ($\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$) parameter space probed by JURA according to Eq. (7.1) (Eq. (7.2)) together with the lines corresponding to $g_{a\mathrm{BB}}$ and $\sqrt{|g_{a\mathrm{AB}}|\, g_{a\mathrm{BB}}}$ in the axion model with $|D| = M = \zeta = 1$. Thus, our considerations show that an eventual detection of an ALP by ALPS II would strongly motivate the construction of JURA. After all, an experimental verification of Eq. (7.3) would allow a deep view into the UV, provide strong evidence for the existence of heavy dyons and even an insight into their spectrum via the ratio $|D|/\zeta M$.

Although LSW experiments can probe the $g_{a\mathrm{AB}}$ coupling, we saw that the effects of the $g_{a\mathrm{BB}}$ coupling are dominant and are expected to be discovered first. To the contrary, there exist purely-laboratory experiments which are primarily sensitive to the $g_{a\mathrm{AB}}$ coupling. These are the experiments which probe CP-violating observables, since only the $g_{a\mathrm{AB}}$ coupling violates CP. One can probe the corresponding CP-violating effects by searching for electric dipole moments of charged particles, such as electrons, protons and muons [187]. Moreover, the CP-violating axion-photon coupling can be probed in various experiments searching for fifth force or monopole-dipole axion-induced interactions [188], since one expects the $g_{a\mathrm{AB}}$ coupling to radiatively induce CP-violating interactions between axions and charged fermions f of the form $g_f a\bar{f}f$. Naively, one could argue that there exist strong constraints on the $g_{a\mathrm{AB}}$ coupling already, analogously to the constraints obtained in Ref. [189]. Note however, that although all the couplings of the ALP EFT (5.10) are small, the theory is still essentially non-perturbative, so predicting the exact values for the discussed CP-violating observables is not straightforward and requires further investigation.

7.3 Haloscope experiments

As we mentioned in sec. 7.1, the effect of the dominant $g_{a\mathrm{BB}}$ coupling on the ALP-photon conversion in an external electromagnetic field is the same as the effect of the conventional $g_{a\gamma\gamma}$ coupling. This means that in the case of the haloscopes that search for cosmic ALPs with masses $m_a \gtrsim (0.1-1)\,\mu$eV, such as ADMX [190], CAPP [191], HAYSTAC [192], KLASH [193], MADMAX [194], ORGAN [195] and others, all the constraints obtained for the $g_{a\gamma\gamma}$ coupling are valid also for the $g_{a\mathrm{BB}}$ coupling. Indeed, in this case, the Compton wavelength of an ALP $\lambda_a = 2\pi/m_a$ is comparable to the physical size of the utilized detectors, or even smaller, and thus one can use a particle-like description of the process.

The same cannot be stated in the case where ALPs have large Compton wavelengths λ_a compared to the length scale L of the experiment. In particular, this is the case of light cosmic ALPs with masses $m_a \ll \mu$eV, which one aims to detect with such haloscope experiments as ABRACADABRA [196,197], ADMX SLIC [147], DM Radio [198], SHAFT [199], and others. In these experiments, one maintains a constant magnetic field $\mathbf{H}_0$ in a laboratory and searches for an ALP-induced oscillating magnetic field $\mathbf{H}_a$. Note that due to the condition $\lambda_a \gg L$, interactions of ALPs with the field $\mathbf{H}_0$ cannot be described as a conventional ALP-photon conversion phenomenon. To determine the expected magnitude of the induced fields in this case, one has to use the axion Maxwell equations (5.23)–(5.26). The latter equations can be significantly simplified since most of the terms on the right-hand side are normally suppressed. Indeed, considering the case of axions and assuming $E \simeq M \simeq |D|$, the axion-photon couplings satisfy $g_{a\mathrm{AA}}/g_{a\mathrm{BB}} \simeq (e/g_0)^2 \lesssim 2\cdot 10^{-4}$ and $|g_{a\mathrm{AB}}|/g_{a\mathrm{BB}} \simeq e/g_0 \lesssim 10^{-2}$. Moreover, the cosmic axions that form dark matter have typical velocities $v_a \sim 10^{-3}$, so that the gradient of the oscillating axion field is suppressed with respect to its time derivative: $|\boldsymbol{\nabla} a| \sim 10^{-3}\,\dot{a}$.

Leaving only the first three dominant terms, we obtain the following simplified axion Maxwell equations:

$$\boldsymbol{\nabla}\times\mathbf{H}_a - \dot{\mathbf{E}}_a = 0\,, \tag{7.5}$$

$$\boldsymbol{\nabla}\times\mathbf{E}_a + \dot{\mathbf{H}}_a = -g_{a\mathrm{BB}}\,(\mathbf{H}_0\times\boldsymbol{\nabla}a + \dot{a}\mathbf{E}_0) + g_{a\mathrm{AB}}\dot{a}\mathbf{H}_0\,, \tag{7.6}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H}_a = 0\,, \tag{7.7}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}_a = 0\,, \tag{7.8}$$

where we included the dominant effect arising from an external electric field $\mathbf{E}_0$. Note that all the existing haloscopes use an external magnetic field $\mathbf{H}_0$ instead, partly because the dominant effect for the usually considered $g_{a\mathrm{AA}}$ coupling is due to the term with an external magnetic, but not electric, field and partly because it is technologically challenging to sustain a large enough electric field in a big enough volume. It is clear from Eq. (7.6) that if the latter technological problem is solved, so that $E_0 \gtrsim 10^{-2}\,(|D|/\zeta M)\,H_0$ in the CP-violating case or $E_0 \gtrsim 10^{-3}\,H_0$ in the CP-conserving case, the $g_{a\mathrm{BB}}\dot{a}\mathbf{E}_0$ term will allow one to search for dark matter axions in an external electric field with the sensitivity which is not worse than the one of the conventional searches conducted in an external magnetic field.

Returning to the case of the existing haloscopes where $\mathbf{E}_0 = 0$, $\mathbf{H}_0 \neq 0$, we see that the axion Maxwell equations (7.5)–(7.8) have significantly different structure compared to the conventional axion Maxwell equations which take into account solely the $g_{a\mathrm{AA}}$ coupling. While in the latter case axions generate an effective electric current $\mathbf{j}^e_{\mathrm{eff}} = -g_{a\mathrm{AA}}\dot{a}\mathbf{H}_0$, in the former case an effective magnetic current is generated:

$$\mathbf{j}^m_{\mathrm{eff}} = g_{a\mathrm{BB}}\mathbf{H}_0\times\boldsymbol{\nabla}a - g_{a\mathrm{AB}}\dot{a}\mathbf{H}_0\,. \tag{7.9}$$

Note that in the case $M \simeq |D|$, the term proportional to the CP-violating $g_{a\mathrm{AB}}$ coupling is dominant. To the contrary, if the underlying UV theory is CP-conserving, then $D = 0$ and $g_{a\mathrm{AB}} = 0$, so that only the term proportional to the $g_{a\mathrm{BB}}$ coupling contributes.

Let us now find experimental implications of the magnetic current (7.9). Applying the curl differential operator to the Eqs. (7.5) and (7.6), and using the other equations from the system (7.5)–(7.8), we obtain:

$$\Delta\mathbf{E}_a - \ddot{\mathbf{E}}_a = \boldsymbol{\nabla}\times\mathbf{j}^m_{\mathrm{eff}}\,, \tag{7.10}$$

$$\Delta\mathbf{H}_a - \ddot{\mathbf{H}}_a = \partial\mathbf{j}^m_{\mathrm{eff}}/\partial t\,. \tag{7.11}$$

The leading terms contributing to the right-hand side are:

$$\boldsymbol{\nabla}\times\mathbf{j}_{\text{eff}}^{m} = g_{a\text{BB}}\,(\boldsymbol{\nabla}a\cdot\boldsymbol{\nabla})\,\mathbf{H}_0 - g_{a\text{AB}}\dot{a}\,\boldsymbol{\nabla}\times\mathbf{H}_0\,, \tag{7.12}$$

$$\partial\mathbf{j}_{\text{eff}}^{m}/\partial t = g_{a\text{BB}}\mathbf{H}_0\times\boldsymbol{\nabla}\dot{a} - g_{a\text{AB}}\ddot{a}\mathbf{H}_0\,. \tag{7.13}$$

The axion dark matter field is given by the following expression:

$$a(t,\mathbf{r}) = a_0\cos(\omega_a t - \mathbf{k}_a\cdot\mathbf{r})\ , \tag{7.14}$$

where $\omega_a = m_a$ and $\mathbf{k}_a = m_a\mathbf{v}_a$. The leading CP-conserving effect then depends on the direction of the axion wind and thus experiences modulations with the periods of one sidereal day T_d and one sidereal year T_y due to the rotation of the Earth around its axis and around the Sun, respectively. To find the axion-induced $\mathbf{E}_a$ and $\mathbf{H}_a$ fields, Eqs. (7.10) and (7.11) have to be solved for a particular geometry of a given haloscope experiment.

Let us illustrate the general features of the solution by considering the example of a very long solenoid of radius R with magnetic field $\mathbf{H}_0$ directed along the z-axis. In this case, Eqs. (7.10) and (7.11) become:

$$\Delta\mathbf{E}_a - \ddot{\mathbf{E}}_a = -\left(g_{a\text{BB}}\,\partial_\rho a\,\mathbf{e}_z + g_{a\text{AB}}\dot{a}\,\mathbf{e}_\phi\right)H_0\,\delta(\rho - R)\ , \tag{7.15}$$

$$\Delta\mathbf{H}_a - \ddot{\mathbf{H}}_a = g_{a\text{BB}}\mathbf{H}_0\times\boldsymbol{\nabla}\dot{a} - g_{a\text{AB}}\ddot{a}\mathbf{H}_0\,, \tag{7.16}$$

where we work in cylindrical coordinates (ρ,ϕ,z) with unit vectors $(\mathbf{e}_\rho,\mathbf{e}_\phi,\mathbf{e}_z)$. Let us parameterize the direction of the axion wind

$$\hat{\boldsymbol{v}}_a = (\sin\theta\cos(\phi-\xi),\,-\sin\theta\sin(\phi-\xi),\,\cos\theta) \tag{7.17}$$

in cylindrical coordinates by two angles θ and ξ. Assuming $T_d \gg 2\pi/\omega_a$, which corresponds to $m_a \gg 5\cdot 10^{-20}$ eV, we neglect the terms proportional to $\dot{\xi}$ and $\dot{\theta}$ in the Eqs. (7.15) and (7.16). It is then straightforward to obtain the solutions of these equations in terms of Bessel functions. All we need however are these solutions in the limit $\omega_a R \ll 1$, as we are interested in axions with large Compton wavelengths. In the latter limit, solutions to Eqs. (7.15), (7.16) with physical boundary conditions are:

$$\mathbf{E}_a = \begin{cases} \frac{1}{2}\, a_0\, \omega_a \rho\, H_0 \Big(g_{a\mathrm{AB}}\, \mathbf{e}_\phi - g_{a\mathrm{BB}}\, v_a \sin\theta\, \cos(\phi-\xi)\, \mathbf{e}_z \Big) \sin\omega_a t\,, \quad \rho < R \\[2ex] \frac{1}{2}\, a_0\, \omega_a \frac{R^2}{\rho} H_0 \Big(g_{a\mathrm{AB}}\, \mathbf{e}_\phi - g_{a\mathrm{BB}}\, v_a \sin\theta\, \cos(\phi-\xi)\, \mathbf{e}_z \Big) \sin\omega_a t\,, \quad \rho > R \end{cases} , \tag{7.18}$$

$$\mathbf{H}_a = \begin{cases} \frac{1}{2}\, a_0\, (\omega_a R)^2\, H_0 \Big(g_{a\mathrm{AB}}\, \mathbf{e}_z + g_{a\mathrm{BB}}\, v_a \sin\theta \left\{ \cos(\phi-\xi)\, \mathbf{e}_\phi + \right. \\[1ex] \qquad\qquad \left. \sin(\phi-\xi)\, \mathbf{e}_\rho \right\} \Big) \Big(\ln\omega_a R + \frac{\rho^2}{2R^2} \Big) \cos\omega_a t\,, \quad \rho < R \\[2ex] \frac{1}{2}\, a_0\, (\omega_a R)^2\, H_0 \Big(g_{a\mathrm{AB}}\, \mathbf{e}_z + g_{a\mathrm{BB}}\, v_a \sin\theta \left\{ \cos(\phi-\xi)\, \mathbf{e}_\phi + \right. \\[1ex] \qquad\qquad\qquad \left. \sin(\phi-\xi)\, \mathbf{e}_\rho \right\} \Big) \ln\omega_a\rho\, \cos\omega_a t\,, \quad \rho > R \end{cases} . \tag{7.19}$$

It is immediately clear that $E_a \gg H_a$, which means that in an experiment with the geometry of our example one has to search for axion-induced electric, but not magnetic, fields. Moreover, it turns out that this feature persists for any other possible geometry as well. Indeed, our Eqs. (7.10) and (7.11) can be rendered equivalent to the equations studied in Ref. [200] by substituting $\mathbf{E}_a \to \mathbf{H}_a$, $\mathbf{H}_a \to -\mathbf{E}_a$ and $\mathbf{j}^m_{\text{eff}} \to \mathbf{j}^e_{\text{eff}}$. In the latter work, it was found that for any haloscope geometry with characteristic length scale L, equation involving the time derivative of the effective current yields solutions which are suppressed by powers of $\omega_a L \ll 1$ with respect to the solutions of the equation involving the curl of this current. In our case, this means that in any haloscope probing $m_a \ll \mu$eV, the axion-induced magnetic field $\mathbf{H}_a$ is suppressed with respect to the axion-induced electric field $\mathbf{E}_a$.

Note however that all the existing as well as many projected haloscopes searching for such light axions – such as ABRACADABRA, ADMX SLIC, DM Radio, SHAFT, ... – aim to measure only the axion-induced magnetic, but not electric, fields. Thus, the constraints on the conventional $g_{a\mathrm{AA}}$ coupling obtained by these experiments do not hold for the dominant axion-photon couplings $g_{a\mathrm{AB}}$ and $g_{a\mathrm{BB}}$. The latter couplings can be probed by future haloscopes equipped with electric field sensors. One haloscope of such kind has already been proposed, see Refs. [201, 202]. We hope that our work will encourage more experimental effort in this direction, as we provided a sound theoretical motivation for such an endeavor. Since the electromagnetic fields generated in a haloscope by the $g_{a\mathrm{BB}}$ and $g_{a\mathrm{AB}}$ couplings are qualitatively different from the fields generated by the conventional $g_{a\mathrm{AA}}$ coupling, for which $H_a \gg E_a$ [200], the first detection of cosmic axions with electric, but not magnetic, sensor haloscope would not only constitute the discovery of axions and dark matter, but also provide a circumstantial experimental evidence for the existence of heavy magnetically charged particles. Furthermore, as one can see from Eq. (7.18), the direction of the detected electric field could allow one to infer the ratio $g_{a\mathrm{AB}}/g_{a\mathrm{BB}} = 2\alpha(D/\zeta M)$ and thus get information about the spectrum of dyons in the UV.

Finally, to compare different extensions of electrodynamics which could be probed by haloscope experiments, it is convenient to reexpress the axion Maxwell equations in terms of the axion-induced polarization and magnetization vectors [200, 203], which are defined as follows:

$$\boldsymbol{\nabla}\times\mathbf{H}_a - \dot{\mathbf{E}}_a = \frac{\partial \mathbf{P}^e_{\text{eff}}}{\partial t} + \boldsymbol{\nabla}\times\mathbf{M}^e_{\text{eff}}\,, \tag{7.20}$$

$$\boldsymbol{\nabla}\times\mathbf{E}_a + \dot{\mathbf{H}}_a = -\frac{\partial \mathbf{P}^m_{\text{eff}}}{\partial t} + \boldsymbol{\nabla}\times\mathbf{M}^m_{\text{eff}}\,, \tag{7.21}$$

$$\boldsymbol{\nabla}\cdot\mathbf{H}_a = -\boldsymbol{\nabla}\cdot\mathbf{P}^m_{\text{eff}}\,, \tag{7.22}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}_a = -\boldsymbol{\nabla}\cdot\mathbf{P}^e_{\text{eff}}\,. \tag{7.23}$$

Note that along with the ordinary effective electric polarization and magnetization vectors $\mathbf{P}^e_{\text{eff}}$ and $\mathbf{M}^e_{\text{eff}}$, we introduced effective magnetic polarization and magnetization vectors $\mathbf{P}^m_{\text{eff}}$ and $\mathbf{M}^m_{\text{eff}}$, which describe electromagnetic properties of an effective medium consisting of magnetically charged particles. The explicit expressions for the effective polarization and magnetization vectors corresponding to a generic axion in an external electromagnetic field are:

$$\mathbf{P}^e_{\text{eff}} = -g_{a\mathrm{AA}}\, a\mathbf{H}_0 + g_{a\mathrm{AB}}\, a\mathbf{E}_0\,, \tag{7.24}$$

$$\mathbf{M}^m_{\text{eff}} = g_{a\mathrm{BB}}\, a\mathbf{H}_0 + g_{a\mathrm{AB}}\, a\mathbf{E}_0\,, \tag{7.25}$$

$$\mathbf{M}^e_{\text{eff}} = -g_{a\mathrm{AA}}\, a\mathbf{E}_0 - g_{a\mathrm{AB}}\, a\mathbf{H}_0\,, \tag{7.26}$$

$$\mathbf{P}^m_{\text{eff}} = g_{a\mathrm{BB}}\, a\mathbf{E}_0 - g_{a\mathrm{AB}}\, a\mathbf{H}_0\,. \tag{7.27}$$

The fact that it is possible to bring the axion Maxwell equations (5.23)–(5.26) into the form (7.20)–(7.23) suggests that the effects of axions in external electromagnetic fields are analogous to the effects of a certain medium consisting of both electrically and magnetically charged particles. From what has been discussed before, it is clear that for a generic axion, effects of the magnetic polarization (magnetization) vectors $\mathbf{P}^m_{\text{eff}}$ ($\mathbf{M}^m_{\text{eff}}$) in an external magnetic field dominate the effects of the electric polarization (magnetization) vectors $\mathbf{P}^e_{\text{eff}}$ ($\mathbf{M}^e_{\text{eff}}$). Such asymmetry between the effects exhibited by electric and magnetic constituents of the effective medium is to be contrasted with the case of gravitational wave electrodynamics [204], where there exists an electric-magnetic $U(1)$ duality symmetry rendering electric and magnetic variables equivalent. This difference in the symmetry properties of the two theories can be easily understood from the fact that the axion-photon interactions are fundamentally mediated by heavy charged particles which break the duality symmetry, while in General Relativity, the interaction between gravity and electromagnetic field is direct and independent of any charges. For an experimentalist, this distinction signifies a substantial difference in the distribution of the induced electromagnetic fields inside the haloscope. Indeed, as it was discussed before, for a generic light axion ($m_a \ll \mu$eV, $g_{a\mathrm{BB}} \gg |g_{a\mathrm{AB}}| \gg g_{a\mathrm{AA}}$) in an external magnetic field one expects $\mathbf{j}^m_{\text{eff}} \gg \mathbf{j}^e_{\text{eff}}$ and thus $E_a \gg H_a$, while for a gravitational wave in an external magnetic field, due to the electric-magnetic duality symmetry, one obtains $\mathbf{j}^m_{\text{eff}} \sim \mathbf{j}^e_{\text{eff}}$ and thus $E_a \sim H_a$.

This Chapter is written based on the publication [1] of the author of this thesis.

Conclusions

As it was asserted by J. Polchinski [23], "the existence of magnetic monopoles seems like one of the safest bets that one can make about physics not yet seen". Indeed, in Chapter 1, we reviewed a number of theoretical arguments which together provide an overwhelming theoretical evidence for magnetic monopoles, like there probably exists for no other kind of hypothetical particles. There are nevertheless multiple practical challenges behind the experimental study of monopoles. While some of the discussed arguments for the existence of monopoles do not restrict their masses, another part of these arguments suggests that monopoles are super heavy, with masses well beyond the energy reach of the present-day collider experiments. Thus, although monopoles almost certainly exist from the theoretical point of view, it might be difficult to directly probe them with existing and near-future experiments.

In this thesis, we developed a framework which allows one to study the indirect effects magnetic monopoles could exhibit on the interactions of other particles. In particular, in Chapter 4 we applied the EFT approach to QEMD by classifying all the possible marginal operators consistent with the symmetries and degrees of freedom of the theory. We found the CP-violating operator responsible for the Witten effect and showed that it does not have a total-derivative structure, contrary to the case of the conventional QED approach. Moreover, we incorporated the Rubakov-Callan effect for the 't Hooft-Polyakov monopoles into the QEMD framework by means of introducing a Stückelberg scalar into the Lagrangian and showing that the latter scalar corresponds to the instanton degree of freedom known as the dyon collective coordinate.

We then showed that our research in QEMD allows one to introduce novel electromagnetic couplings for axions and ALPs in the EFT approach. As we discussed in Chapter 2, axions and ALPs form a class of very well-motivated candidates for physics beyond the Standard model, since different particles of this class are indispensable ingredients to various new physics models and theories which solve the long-standing physics puzzles, such as the quantization of gravity (string theory), the strong CP problem (models implementing the PQ mechanism), the smallness of neutrino masses (models with spontaneously broken global lepton number symmetry) etc. Furthermore, axions and ALPs are perfect candidates for cold dark matter. All this suggests that the experimental searches for these particles are of paramount importance. As the main search strategies exploit the electromagnetic interactions of axions and ALPs, we decided to analyze all such interactions, and in Chapter 5, we found that instead of the only one kind of coupling considered in the literature, there are actually four different types of axion-photon couplings. The three new couplings are associated to the effects of magnetic monopoles and dyons – two of the couplings are generated by virtual monopoles and dyons, while the remaining one is the Witten-effect induced coupling which describes the interaction vertex including an axion, a photon and a 't Hooft-Polyakov monopole. Although the Witten-effect induced coupling has been described in the literature before, it has never been distinguished from the conventional axion-photon coupling. We clarified the physics of the Witten-effect induced coupling showing that it is not generic to all possible models containing monopoles, but only to the models which contain an instanton degree of freedom localized in the monopole worldvolume, such as the models with 't Hooft-Polyakov monopoles. We also found the scaling of the different axion-photon couplings with the elementary electric and magnetic charges, and showed that the two new electromagnetic couplings of axions associated to the virtual monopoles and dyons dominate the conventional axion-photon coupling.

Based on our theoretical analysis of the axion extension to QEMD, in Chapter 7 we proposed experiments which could probe magnetic monopoles indirectly, in particular through the influence virtual monopoles would exhibit on the interactions of axions and ALPs with an electromagnetic field. We found that the new electromagnetic couplings of axions give rise to unique experimental signatures in LSW experiments and in some kinds of axion searches, such as haloscopes searching for low-mass axions ($m_a \ll \mu$eV). In the latter case, we showed that the best sensitivity to the new electromagnetic couplings of axions could be achieved by measuring an induced oscillating electric field, instead of an induced oscillating magnetic field, contrary to the setup of existing experiments. The case of low mass axions is particularly interesting, since, as it can be seen from Fig. 7.1, the simplest axion models predict rather large $g_{a\mathrm{BB}}$ and $g_{a\mathrm{AB}}$ couplings, which can be probed by many projected experiments. Thus, we encourage the development of electric sensor haloscopes which would search for axion dark matter in the corresponding parameter region.

In the case of the LSW experiments and the ALPS II experiment in particular, we found that due to the new electromagnetic couplings of axions, ALPS II is sensitive to the QCD axion. Moreover, it probes the very interesting region in the parameter space of the $g_{a\mathrm{BB}}$ coupling between the QCD axion and photons where there have been claimed several astrophysical hints, see Fig. 7.1. The axions with the $g_{a\mathrm{BB}}$ couplings in the latter parameter region could also form the cold dark matter, which hypothesis can be probed by future haloscope experiments with electric field sensors. Moreover, in the case of the detection of the axion signal in ALPS II due to the $g_{a\mathrm{BB}}$ coupling, we found that the projected LSW experiments of the next generation, such as JURA, would be sensitive to the corresponding CP-violating $g_{a\mathrm{AB}}$ axion-photon coupling in the channel where the incoming light is polarized perpendicular to the magnetic field. The ratio of the two couplings would give information about the spectrum of heavy dyons.

Apart from unveiling these intriguing experimental applications, our work reconsidered several questions in axion theory. First, in Chapter 3 we showed that contrary to the existing statements, the main contribution to the axion-photon coupling need not be quantized in units proportional to e^2. Second, contrary to what has been advocated recently in the literature, we found that magnetic monopoles of an Abelian gauge field need not give mass to axions coupled to this gauge field. We showed that the axions do get mass in theories with magnetic monopoles only if these monopoles carry an additional instanton degree of freedom interacting with the axion a, e.g. in the case of the spontaneously broken symmetry phase of a non-Abelian gauge theory with aGG^d term in the Lagrangian, where G (G^d) is the (dual) field strength tensor of the non-Abelian gauge field. This axion mass is generated via the conventional mechanism through instantons of the non-Abelian theory, which however live on the 't Hooft-Polyakov monopoles in the low energy phase. Finally, we found that the interaction between axions and an electromagnetic field need not preserve CP. In particular, as long as one has heavy dyons in the high energy theory, there exists a natural source of CP-violation which can be transferred to low energy physics through axion-photon interactions. We discussed experiments which can probe this new CP-violating axion coupling.

In Chapter 6, we built novel models of axion which provide UV-completions to the axion-photon EFTs discussed in this thesis. In particular, we introduced two new families of hadronic axion models which involve a very heavy vector-like fermion magnetically charged either under the full non-Abelian symmetry of the low energy Standard model or only under its electromagnetic subgroup. In the case of the models with non-Abelian magnetic monopoles, we assumed that these monopoles transform in the representations of the Langlands dual (GNO) group of the $SU(3)_c$ QCD gauge group, in accord with the GNO conjecture. We showed that both models with the Abelian and non-Abelian magnetic monopoles realize the PQ mechanism and thus solve the strong CP problem. We calculated the low energy axion couplings in these models. We argued that in the models involving non-Abelian magnetic monopoles there could arise a deviation from the property of universality of the axion-gluon coupling, however we showed that the latter coupling recovers its universal form in the classical approximation. If the quantum corrections are not negligible, difference in the electric dipole moment coupling with respect to the conventional QCD axion models can offer an exciting opportunity of distinguishing the model involving non-Abelian magnetic monopoles from other QCD axion models in the experiments such as CASPEr Electric [163].

These Conclusions are partly written based on the publications [1,2] of the author of this thesis.

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Eidesstattliche Erklärung / Declaration on oath

Hiermit versichere ich an Eides statt, die vorliegende Dissertationsschrift selbst verfasst und keine anderen als die angegebenen Hilfsmittel und Quellen benutzt zu haben.

Hamburg, den 04.07.2022

Unterschrift des Doktoranden